Decimal Number System

The number system that we use in our day-to-day life is the decimal number system. Decimal number system has base 10 as it uses 10 digits from 0 to 9. In decimal number system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands, and so on.

Each position represents a specific power of the base (10). For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position. Its value can be written as

$$(1 \times 1000) + (2 \times 100) + (3 \times 10) + (4 \times 1)$$
$$(1 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (4 \times 10^0)$$
$$1000 + 200 + 30 + 4$$
$$1234$$

As a computer programmer or an IT professional, you should understand the following number systems which are frequently used in computers.

S.No.	Number System and Description
1	**Binary Number System** Base 2. Digits used : 0, 1
2	**Octal Number System** Base 8. Digits used : 0 to 7
3	**Hexa Decimal Number System** Base 16. Digits used: 0 to 9, Letters used : A- F

Binary Number System

Characteristics of the binary number system are as follows −

- Uses two digits, 0 and 1
- Also called as base 2 number system
- Each position in a binary number represents a **0** power of the base (2). Example 2^0
- Last position in a binary number represents a **x** power of the base (2). Example 2^x where **x** represents the last position - 1.

Example

Binary Number: 10101_2

Calculating Decimal Equivalent −

Step	Binary Number	Decimal Number

Step 1	10101_2	$((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	10101_2	$(16 + 0 + 4 + 0 + 1)_{10}$
Step 3	10101_2	21_{10}

Note − 10101_2 is normally written as 10101.

Octal Number System

Characteristics of the octal number system are as follows −

- Uses eight digits, 0,1,2,3,4,5,6,7
- Also called as base 8 number system
- Each position in an octal number represents a **0** power of the base (8). Example 8^0
- Last position in an octal number represents a **x** power of the base (8). Example 8^x where **x** represents the last position - 1

Example

Octal Number: 12570_8

Calculating Decimal Equivalent −

Step	Octal Number	Decimal Number
Step 1	12570_8	$((1 \times 8^4) + (2 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (0 \times 8^0))_{10}$
Step 2	12570_8	$(4096 + 1024 + 320 + 56 + 0)_{10}$
Step 3	12570_8	5496_{10}

Note − 12570_8 is normally written as 12570.

Hexadecimal Number System

Characteristics of hexadecimal number system are as follows −

- Uses 10 digits and 6 letters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Letters represent the numbers starting from 10. A = 10. B = 11, C = 12, D = 13, E = 14, F = 15
- Also called as base 16 number system
- Each position in a hexadecimal number represents a **0** power of the base (16). Example, 16^0
- Last position in a hexadecimal number represents a **x** power of the base (16). Example 16^x where **x** represents the last position - 1

Example

Hexadecimal Number: $19FDE_{16}$

Calculating Decimal Equivalent −

Step	Binary Number	Decimal Number
Step 1	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$
Step 2	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$
Step 3	$19FDE_{16}$	$(65536+ 36864 + 3840 + 208 + 14)_{10}$
Step 4	$19FDE_{16}$	106462_{10}

Note − $19FDE_{16}$ is normally written as 19FDE.

There are many methods or techniques which can be used to convert numbers from one base to another. In this chapter, we'll demonstrate the following −

- Decimal to Other Base System
- Other Base System to Decimal
- Other Base System to Non-Decimal
- Shortcut method - Binary to Octal
- Shortcut method - Octal to Binary
- Shortcut method - Binary to Hexadecimal
- Shortcut method - Hexadecimal to Binary

Decimal to Other Base System

Step 1 − Divide the decimal number to be converted by the value of the new base.

Step 2 − Get the remainder from Step 1 as the rightmost digit (least significant digit) of the new base number.

Step 3 − Divide the quotient of the previous divide by the new base.

Step 4 − Record the remainder from Step 3 as the next digit (to the left) of the new base number.

Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.

The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.

Example

Decimal Number: 29_{10}

Calculating Binary Equivalent −

Step	Operation	Result	Remainder
Step 1	29 / 2	14	1
Step 2	14 / 2	7	0
Step 3	7 / 2	3	1
Step 4	3 / 2	1	1
Step 5	1 / 2	0	1

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the Least Significant Digit (LSD) and the last remainder becomes the Most Significant Digit (MSD).

Decimal Number : 29_{10} = Binary Number : $11101_{2.}$

Other Base System to Decimal System

Step 1 − Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).

Step 2 − Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.

Step 3 − Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example

Binary Number: 11101_2

Calculating Decimal Equivalent −

Step	Binary Number	Decimal Number
Step 1	11101_2	$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	11101_2	$(16 + 8 + 4 + 0 + 1)_{10}$
Step 3	11101_2	29_{10}

Binary Number : 11101_2 = Decimal Number : 29_{10}

Other Base System to Non-Decimal System

Step 1 − Convert the original number to a decimal number (base 10).

Step 2 − Convert the decimal number so obtained to the new base number.

Example

Octal Number : 25_8

Calculating Binary Equivalent −

Step 1 - Convert to Decimal

Step	Octal Number	Decimal Number
Step 1	25_8	$((2 \times 8^1) + (5 \times 8^0))_{10}$
Step 2	25_8	$(16 + 5)_{10}$
Step 3	25_8	21_{10}

Octal Number : 25_8 = Decimal Number : 21_{10}

Step 2 - Convert Decimal to Binary

Step	Operation	Result	Remainder
Step 1	21 / 2	10	1
Step 2	10 / 2	5	0
Step 3	5 / 2	2	1
Step 4	2 / 2	1	0
Step 5	1 / 2	0	1

Decimal Number : 21_{10} = Binary Number : 10101_2

Octal Number : 25_8 = Binary Number : 10101_2

Shortcut Method − Binary to Octal

Step 1 − Divide the binary digits into groups of three (starting from the right).

Step 2 − Convert each group of three binary digits to one octal digit.

Example

Binary Number : 10101_2

Calculating Octal Equivalent −

Step	Binary Number	Octal Number
Step 1	10101_2	010 101
Step 2	10101_2	2_8 5_8
Step 3	10101_2	25_8

Binary Number : 10101_2 = Octal Number : 25_8

Shortcut Method − Octal to Binary

Step 1 − Convert each octal digit to a 3-digit binary number (the octal digits may be treated as decimal for this conversion).

Step 2 − Combine all the resulting binary groups (of 3 digits each) into a single binary number.

Example

Octal Number : 25_8

Calculating Binary Equivalent −

Step	Octal Number	Binary Number
Step 1	25_8	2_{10} 5_{10}
Step 2	25_8	010_2 101_2
Step 3	25_8	010101_2

Octal Number : 25_8 = Binary Number : 10101_2

Shortcut Method − Binary to Hexadecimal

Step 1 − Divide the binary digits into groups of four (starting from the right).

Step 2 − Convert each group of four binary digits to one hexadecimal symbol.

Example

Binary Number : 10101_2

Calculating hexadecimal Equivalent −

Step	Binary Number	Hexadecimal Number
Step 1	10101_2	0001 0101
Step 2	10101_2	1_{10} 5_{10}
Step 3	10101_2	15_{16}

Binary Number : 10101_2 = Hexadecimal Number : 15_{16}

Shortcut Method - Hexadecimal to Binary

Step 1 − Convert each hexadecimal digit to a 4-digit binary number (the hexadecimal digits may be treated as decimal for this conversion).

Step 2 − Combine all the resulting binary groups (of 4 digits each) into a single binary number.

Example

Hexadecimal Number : 15_{16}

Calculating Binary Equivalent −

Step	Hexadecimal Number	Binary Number
Step 1	15_{16}	1_{10} 5_{10}
Step 2	15_{16}	0001_2 0101_2
Step 3	15_{16}	00010101_2

Hexadecimal Number : 15_{16} = Binary Number : 10101_2