SINCE 2007
TathaGat
LEADERS IN TEST-PREP

IMPORTANT CONCEPTS FOR CAT

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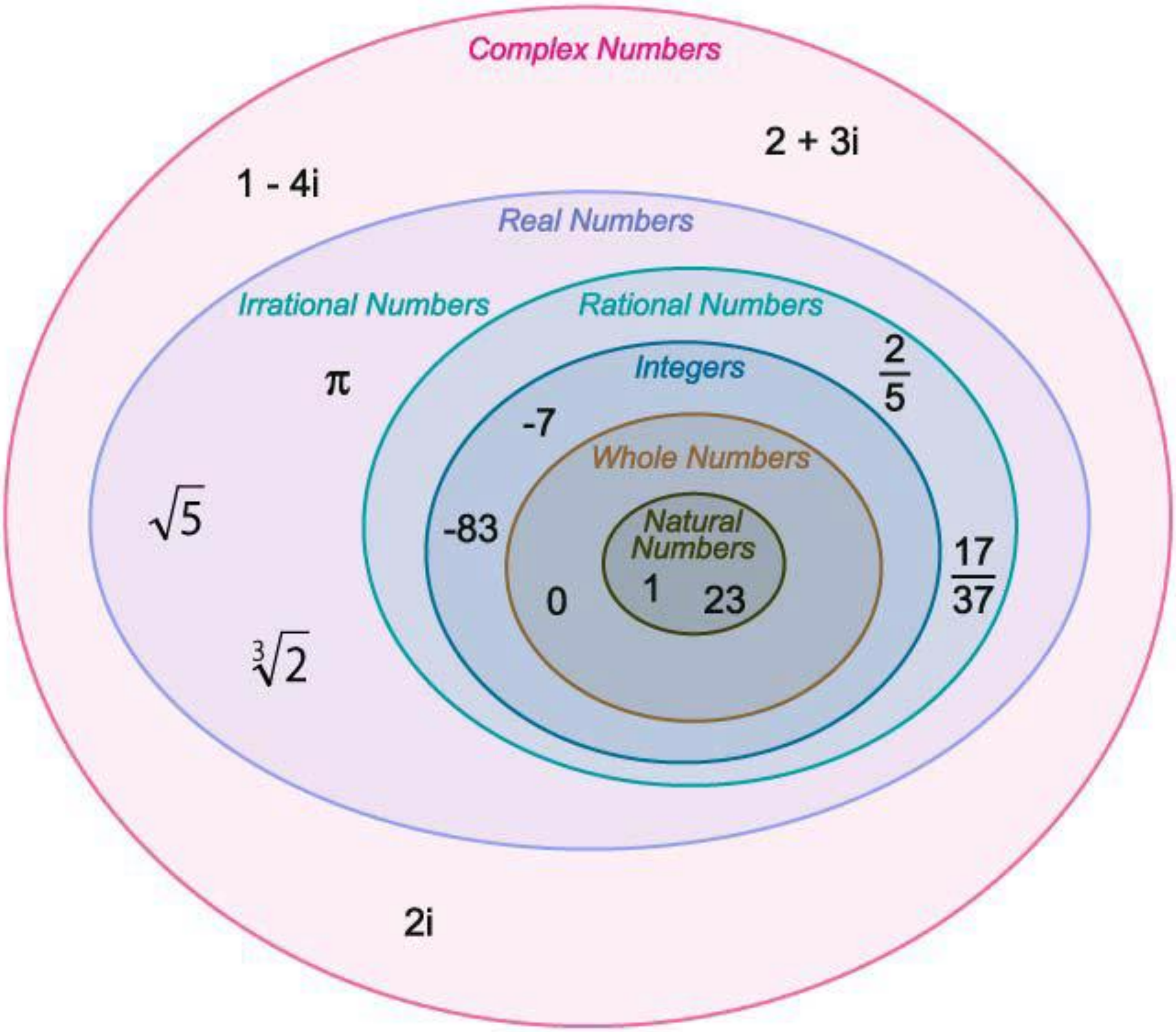
Number Theory



•Integers
Numbers, such as -1, 0, 1, 2, and 3, that have no fractional part. Integers include the counting numbers (1, 2, 3, …), their negative counterparts (-1, -2, -3, …), and 0.

•Whole & Natural Numbers
The terms from 0,1,2,3,….. are known as Whole numbers. Natural numbers do not include 0.

•Factors
Positive integers that divide evenly into an integer. Factors are equal to or smaller than the integer in question. 12 is a factor of 12, as are 1, 2, 3, 4, and 6.

•Factor Foundation Rule
If *a* is a factor of *b*, and *b* is a factor of *c*, then *a* is also a factor of *c*. For example, 5 is a factor of 25 and 25 is a factor of 625. Therefore, 5 is also a factor of 625.

•Multiples
Multiples are integers formed by multiplying some integer by any other integer. For example, 6 is a multiple of 3 (2×3), as are 12 (4×3), 18 (6×3), etc. In addition 3 is also a multiple of itself i.e. 3 (1×3). Think of multiples as equal to or larger than the integer in question

•Prime Numbers
A positive integer with exactly two factors: 1 and itself. The number 1 does not qualify as prime because it has only one factor, not two. The number 2 is the smallest prime number; it is also the only even prime number. The numbers 2, 3, 5, 7, 11, 13 etc. are prime.

•Prime Factorization
Prime factorization is a way to express any number as a product of prime numbers. For example, the prime factorization of 30 is $2 \times 3 \times 5$. Prime factorization is useful in answering questions about divisibility.

•Greatest Common Factor
Greatest Common FACTOR refers to the largest factor of two (or more) integers. Factors will be equal to or smaller than the starting integers. The GCF of 12 and 30 is 6 because 6 is the largest number that goes into both 12 and 30.

•Least Common Multiple (LCM)
Least Common Multiple refers to the smallest multiple of two (or more) integers. Multiples will be equal to or larger than the starting integers. The LCM of 6 and 15 is 30 because 30 is the smallest number that both 6 and 15 go into.

•Odd & Even Numbers
Any number divisible by 2 is even and not divisible by 2 is odd.

•Product of n consecutive integers and divisibility
The product of n consecutive integers is always divisible by n! Given $5\times6\times7\times8$, we have $n = 4$ consecutive integers. The product of $5\times6\times7\times8$ (=1680), therefore, is divisible by $4! = 5\times6\times7\times8 = 24$.

•Sum of n consecutive integers and divisibility
There are two cases, depending upon whether n is odd or even:
If n is odd, the sum of the integers is always divisible by n. Given 5+6+7, we have $n = 3$ consecutive integers. The sum of 5+6+7 (=18), therefore, is divisible by 3.

If n is even, the sum of the integers is never divisible by n. Given 5+6+7+8, we have $n = 4$ consecutive integers. The sum of 5+6+7+8 (=26), therefore, is not divisible by 4.

•**MANTRA**: Square of any natural number can be written in the form of 3n or 3n+1. Also, square of any natural number can be written in the form of 4n or 4n+1.

•**MANTRA**: Square of a natural number can only end in 0, 1, 4, 5, 6 or 9. Second last digit of a square of a natural number is always even except when last digit is 6. If the last digit is 5, second last digit has to be 2.

•**MANTRA**: Any prime number greater than 3 can be written as $6k\pm1$.

•**MANTRA:** The fifth power of any number has the same units place digit as the number itself.

•**MANTRA:** Cube on any natural number can be written in the form of 7n , 7n +1 or 7n -1 Also, cubw of any natural number can be written in the form 9n , 9n -1 or 9n + 1 .

•**MANTRA:** If two numbers a and b are given, and their LCM and HCF are L and H respectively,
then $L \times H = a \times b$.

•**MANTRA:** LCM of fractions = $\frac{\textbf{\textit{LCM of Numerators}}}{\textbf{\textit{HCF of Denominators}}}$

•**MANTRA:** HCF of fractions = $\frac{\textbf{\textit{HCF of Numerators}}}{\textbf{\textit{LCM of numerators}}}$

Note: Fractions should be in the lowest form.

•**MANTRA** The least number leaving remainder 'r' in each case when divided by 'x', 'y' and 'z' = **(LCM of x, y, z) + r**

•**MANTRA** The series of such numbers will be **(LCM of x, y, z) × n + r**

Divisibilty Rules

•**Divisibility Rules for the numbers of the form $10^n – 1$.**

For checking divisibility by 'p', which is of the format of $10^n – 1$, sum of blocks of size 'n' needs to be checked (Blocks should be considered from the least significant digit i.e the right side). If the sum is divisible by p, then the number is divisible by p.

•**Check if a number (N = abcdefgh) is divisible by 9**

9 is $10^1 – 1$

Sum of digits is done 1 at a time = a + b + c + d + e + f + g + h = X
If X is divisible by 9, N is divisible by 9
Also, N is divisible by all factors of 9. Hence the same test works for 3.

•Check if a number (N = abcdefgh) is divisible by 99

99 is $10^2 - 1$
Sum of digits is done 2 at a time = ab + cd + ef + gh = X
If X is divisible by 99, N is divisible by 99
Also, N is divisible by all factors of 99. Hence the same test works for 9, 11 and others.

•Check if a number (N = abcdefgh) is divisible by 999

999 is $10^3 - 1$
Sum of digits is done 3 at a time = ab + cde + fgh = X
If X is divisible by 999, N is divisible by 999
Also, N is divisible by all factors of 999. Hence the same test works for 27, 37 and others.

•Divisibility Rules for the numbers of the form $10^n + 1$.

For checking divisibility by 'p', which is of the format of 10n + 1, *alternating sum* of blocks of size 'n' needs to be checked (Blocks should be considered from the least significant digit ie the right side). If the alternating sum is divisible by p, then the number is divisible by p.

•Check if a number (N = abcdefgh) is divisible by 11

11 is $10^1 + 1$
Alternating sum of digits is done 1 at a time = a - b + c - d + e - f + g - h = X
If X is divisible by 11, N is divisible by 11

•Check if a number (N = abcdefgh) is divisible by 101

101 is $10^2 + 1$
Alternating sum of digits is done 2 at a time = ab - cd + ef - gh = X
If X is divisible by 101, N is divisible by 101

•Check if a number (N = abcdefgh) is divisible by 1001
1001 is $10^3 + 1$
Sum of digits is done 3 at a time = ab - cde + fgh = X
If X is divisible by 1001, N is divisible by 1001
Also, N is divisible by all factors of 1001. Hence the same test works for 7, 11, 13 and others.

FACTOR THEORY

•In general, for any composite number C, which can be expressed as $\mathbf{C = a^m \times b^n \times c^p \times \ldots}$, where a, b, c, … are all prime factors and m, n, p are positive integers, then:

•Number of factors is equal to
$\mathbf{(m + 1)(n + 1)(p + 1) \ldots}$

•If N is not a perfect square, No. of ways of writing N as a product of two natural numbers :

$\mathbf{\frac{1}{2} \{(p + 1)(q + 1)(r + 1) \ldots\}}$

•If N is a perfect square, No. of ways in which N can be expressed as a product of two different factors:
$\mathbf{\frac{1}{2} \{(p + 1)(q + 1)(r + 1) \ldots - 1\}}$ ways

and as a product of two factors
$\mathbf{\frac{1}{2} \{(p + 1)(q + 1)(r + 1) \ldots + 1\}}$ ways

•Sum of the factors of N

$\mathbf{=\{(1 + a^1 + a^2 + a^3 + \ldots + a^m)(1+b+b^2+\ldots+b^n)(1+c+c^2+\ldots+c^p)\ldots}$

Remainders

•When a number 'n' is the divisor, the remainder 'r' can take any integral value from 0 to n-1. The remainder can also be considered to be (r-n).

For example, remainder when 2^3 is divided by 9 is 8 but it can also be considered as (8-9)= -1.

•Theorem1: $a^n + b^n$ is divisible by a + b when n is **ODD**.

•Theorem 2: $a^n - b^n$ is divisible by a + b when n is **EVEN**.

•Theorem 3: $a^n - b^n$ is **ALWAYS** divisible by a − b.

•Remainders Are Multiplicative

•Remainder when a product of numbers is divided by a number is the same as the product of remainders when the numbers are individually divided by the number.

Remainder $(a\times b)/c$ = $Rem(a/c)\times Rem(b/c)$

For example :
What will be the remainder when 1225 × 4121 × 566 is divided by 9.

Rem (1225 × 4121 × 566)/9

Rem(1225/9)×Rem(4121/9)×Rem(566/9)

Rem (1×8×8)/9

Final Remainder 1.

•Remainders Are Additive

Remainder of a sum of numbers when divided by another number is the same as the sum of remainders when the numbers are individually divided by the number.

Remainder $(a+b)/c$ = $Rem(a/c)+(Rem(b/c)$

For example:
What will be the remainder when 1225+4121+566 is divided by 9.

Rem (1225/9) + Rem(4121/9) +Rem (566/9)

Rem (1+8+8)/9 = 8

•Remainder Theorem - Factor Theorem:

To identify whether a given expression is a factor of another expression, we can take help of Remainder Theorem.
According to the remainder theorem, when any expression f(x) is divided by (x − a), the remainder is f(a).

An expression is said to be a factor of another expression only when the remainder is 0 when the latter is divided by the former. (x − a) is a factor of f(x) if and only if f(a) = 0.

•EULER'S THEOREM

Number of numbers which are less than $N = a^p\times b^q\times c^r$....and co-prime to it are,

$\varphi(N)$ (Euler's totient)= $N(1-1/a)(1-1/b)(1-1/c)$....

Euler's theorem states that if M and N are co-prime positive integers, then ,
$Rem[(M^{\varphi(N)}/N] = 1$

•MANTRA: Sum of numbers less than and co-prime to a number N : $\mathbf{N\times\varphi(N)/2}$

For Example :

•Find the remainder when 5^9 is divided by 24.

Here 5 and 24 are co-prime so
$\varphi(24) = 24(1-1/2)(1-1/3) = 8$
$[24 = 2^3 \times 3]$
Rem $\{5^{\varphi(24)}\}/24 = 1$
Rem $\{5^8\}/24 = 1$
Rem $(5^8 \times 5)/24 = 5$

•Find the remainder when 5^{38} is divided by 63.

5 and 63 are co primes,
$63 = 3^2 \times 7$
So $\varphi(63) = 63(1-1/3)(1-1/7)$
$= 36$
Therefore , remainder($5^{38}/63$) = remainder ($5^{36} \times 5^2/63$)
$= 1 \times 5 \times 5 = 25$

•FERMAT'S THEOREM
If P is a prime number and N is prime to P, then $(N^P - N)$ is divisible by P

What is the remainder when $(N^7 - N)$ is divided by 7
Answer : 0

•WILSON'S THEOREM
If P is a prime no. then remainder (P-1)!+1/P=0, which means that remainder when (P-1)! Is divided by P is P-1

For example: remainder when 40! Is divided by 41
= 41-1
= 40

•MANTRA: Remainder when (p-2)! Is divided by p is 1 and remainder when (p-3)! Is divided by p is (p-1)/2 .

•MANTRA: Any single digit number written (P-1) times is divisible by P, where P is a prime number >5.
Examples:
222222 is divisible by 7
444444….. 18 times is divisible by 19

•MANTRA: Number of ways of writing a number N as a product of two co-prime numbers $= \mathbf{2^{(n-1)}}$
where, n is the number of prime factors of a number

•MANTRA: Product of all the factors of = $\mathbf{N^{(Number\ of\ factors\ of\ N/2)}}$

Number of ending zeroes in a factorial (n!)

•Number of zeroes is given by the sum of the quotients obtained by successive division of 'n' by 5.
What is the number of ending zeroes in 134!
[134/5] = 26
[26/5] = 5
[5/5] = 1
Number of ending zeroes = 26 + 5 + 1 = 32

Time Speed and Distance

•Some Standard Definitions and Concepts:

The speed of a body is defined as the distance covered by it unit time.

Speed = Distance / Time
Time – Distance / Speed
Distance = Speed x Time

Note:

• 1 km/hr = 5/18 m/s Conversely 1 m/s = 18/5 km/hr
• If the distance covered is constant then the speed is inversely proportional to time
• If the time is constant the distance covered is directly proportional to the speed
• If the speed is constant the distance covered is directly proportional to the time
• If a train crosses a lamp post the distance travelled by the train during the time taken to cross the lamp post is equal to the length of the train
• If a train crosses a bridge the distance travelled by the train during the time taken to cross the bridge is equal to the sum of the length of the train and the length of the bridge.

• Average Speed

$$Average.speed = \frac{Total.dis\tan ce.travelled}{Total.time.taken}$$

$$= \frac{d_1 + d_2 + d_3 + ...d_n}{t_1 + t_2 + t_3 + ...t_n} = \frac{s_1t_1 + s_2t_2 + s_3t_3 + ... + s_nt_n}{t_1 + t_2 + t_3 + ... + t_n} = \frac{d_1 + d_2 + d_3 + ... + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + ... + \frac{d_n}{s_n}}$$

• In a journey travelled with different speeds, if the distance covered in each stage is constant, the average speed is the harmonic mean of the different speeds.

Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr Then the average speed during the whole journey is 2xy/(x+y) km/hr

• In a journey travelled with different speeds, if the time travelled in each stage is constant, the average speed is the arithmetic mean of the different speeds.

If a man travelled for certain time at the speed of x km/hr and travelled for equal amount of time at the speed of y km/hr then The average speed during the whole journey is (x+y)/2 km/hr

•Relative Speed:

•**Case 1:** When one object is moving and the other is stationary the relative speed between them is the speed of the moving object. For ex. when a train crosses a stationary person on a platform the relative speed of the train and the person will be the speed of the train

•**Case 2:** When two objects are moving in opposite directions the relative speed between them is the sum of their speeds. For ex. the relative speed between two trains moving towards each other is the sum of their individual speeds.

•**Case3:** When two bodies are moving in the same direction the realtive speed between them is the difference of their speeds. For ex. the relative speed between two trains moving in the same direction on parallel tracks is the difference between their individual speeds.

Circular tracks:

• When two persons are running around a circular track in the same direction, the difference in the distances covered by the faster and slower person between two meeting points is equal to the perimeter of the circular track and the relative speed is equal to difference in speeds of two persons.

• When two persons are running around a circular track in opposite directions, the relative speed is equal to the sum of their speeds and from one meeting point to another, the sum of the distances travelled by them is equal to the perimeter of circular track.

• Two persons starting a race on a circular track at the same time and from the same point, will meet for the first time when the faster person gains one complete round over the slower person. The time taken for this = length of track /relative speed.

• Two persons starting a race on a circular track at the same time and from the same point, will meet for the first time at the starting point after a time which is the LCM of time taken by each one of them to complete one lap of the track.

• Three persons starting a race on a circular track and from the same starting point, will meet for the first time after the start at a time which is equal to the LCM of the time taken by the fastest to gain a complete round over each of the other two.

If two people are running on a circular track with speeds in ratio a:b where a and b are co-prime, then

• They will meet at a+b distinct points if they are running in opposite direction.
• They will meet at |a-b| distinct points if they are running in same direction

If two people are running on a circular track having perimeter l, with speeds m and n,

• The time for their first meeting = l / (m +n) (when they are running in opposite directions)
• The time for their first meeting = l / (|m-n|) (when they are running in the same direction

Boats & Streams:

• Speed of a boat upstream = speed of boat in still water - speed of the stream

• Speed of a boat downstream = speed of boat in still water + speed of stream

• Speed of boat in still water = (downstream speed + upstream speed) / 2

• Speed of stream = (downstream speed − upstream speed) / 2

Clocks:
• Angle covered by a minute hand in 1minute = 6 degrees

• Angle covered by an hours hand in 1 minute = 0.5 degrees

• Angle covered by an hours hand in 1 hour = 30 degrees

• In one minute, a minute hand gains 5.5 degrees over the hours hand.

• In one hour, every angle is made twice by the two hands of the clock.

• The hands make an angle of 0 or coincide with each other once in every 655/11 minutes.

Some other results which might be useful:
• Hands of a clock meet at a gap of 65 5/11 minutes.
• The meetings take place at 12:00:00, 1:05:5/11, 2:10:10/11 … and so on.
• Hands of a clock meet 11 times in 12 hours and 22 times in a day.
• Hands of a clock are perfectly opposite to each other (i.e. 180 degrees) 11 times in 12 hours and 22 times a day. {Same as above}
• Any other angle is made 22 times in 12 hours and 44 times in a day

•**Concept of Head start:**
• In a race of x metres, if A gives B a head start of y metres then B will start the race y metre ahead of A i.e. if A runs for entire x metres then B will only run for x-y metres. In a race of x metres, if A gives B a head start of t seconds then A will start the race t seconds after B.

Gyan: Given that the distance between two points is constant, then

• If the speeds are in *Arithmetic Progression*, then the times taken are in *Harmonic Progression*
• If the speeds are in *Harmonic Progression*, then the times taken are in *Arithmetic Progression*

•If a person P starts from A and heads towards B and another person Q starts from B and heads towards A and they meet after a time 't' then, $t = \sqrt{xy}$

Where x = time taken (after meeting) by P to reach B and y = time taken (after meeting) by Q to reach A.

• A and B started at a time towards each other. After crossing each other, they took T1 hrs, T2 hrs respectively to reach their destinations. If they travel at constant speeds S1 and S2 respectively all over the journey, Then

$$\frac{S1}{S2} = \sqrt{\frac{T2}{T1}}$$

Geometry

•Basic properties of lines and planes.

1. One and only one line passes through two distinct points .
2. Infinite number of lines pass through a given point. These lines are called concurrent lines.
3. The intersection of two distinct lines is a point.
4. Three or more points are said to be collinear if they lie on a line, otherwise they are said to be non collinear.
5. There are infinite number of planes passing through any given lines.
6. There is exactly one plane passing through three non -collinear points.
7. The intersection of two planes is a line.
8. If two lines are perpendicular to the same line they are parallel to each other.

•Euclid's Axioms :

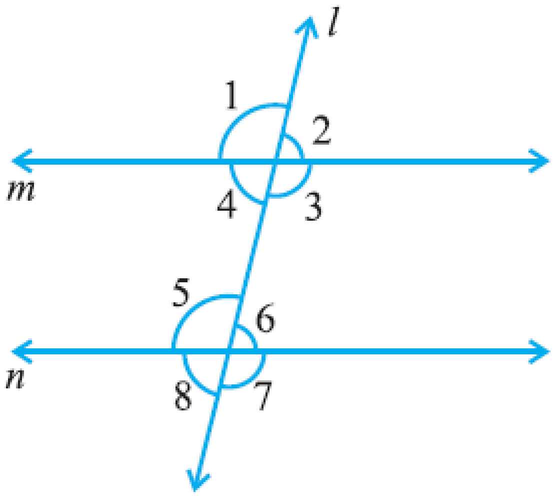
1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

•Euclid's Five Postulates :

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight lines segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All Right Angles are congruent.
5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two Right Angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the Parallel Postulate.

•Supplementary angles: - angles x and angle y are supplementary if x + y = 180 degree

•Complimentary angles:- angles x and angle y are complimentary if x + y = 90



1. Vertically Opposite angles
   $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 6$ and $\angle 8$, $\angle 5$ and $\angle 7$
   Each of the angles in all the pairs is always equal in measure, i.e,
   $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 6 = \angle 8$, $\angle 5 = \angle 7$
2. Corresponding angles
   $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
   When m || n,
   $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$
3. Alternate interior angles
   $\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$
   When m || n,
   $\angle 3 = \angle 5$, $\angle 4 = \angle 6$
4. Alternate exterior angles
   $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$
   When m || n, $\angle 1 = \angle 7$, $\angle 2 = \angle 8$

So, in the diagram if m || n, then $\angle 1 = \angle 3 = \angle 5 = \angle 7$ and $\angle 2 = \angle 4 = \angle 6 = \angle 8$. Also, sum of interior angles on the same side of the transversal (line l) will always be 180, given that m|| n. Converse of all the above is also true, i.e, when $\angle 1 = \angle 3 = \angle 5 = \angle 7$ or $\angle 2 = \angle 4 = \angle 6 = \angle 8$ or $\angle 3 + \angle 6 = 180°$ or $\angle 4 + \angle 5 = 180°$, then m || n.

- Sum of angles in any triangle is 180°
- Angle Bisector is a line that divides an angle in two equal parts.

Triangles

• Three types of triangles: Acute (all angles less than 90°), Right Angle (one angle is 90°), Obtuse (one angle is more than 90°). Angle opposite to the larger side is always greater than angle opposite to smaller side.

•Sum of two sides is greater than third. So, in a ΔABC

AB + BC > AC,

AB + AC > BC

And AC + BC > AB

• **Gyan:** For acute angle triangle: - $AB^2 + BC^2 > AC^2$, $AB^2 + AC^2 > AB^2$ and $BC^2 + AC^2 > AB^2$

• **Gyan:** For right angle triangle: - $AB^2 + BC^2 = AC^2$, where AC is hypotenuse

• **Gyan:** For obtuse angle triangle: - $AB^2 + BC^2 < AC^2$, where AC is the largest side

• There is one more way of classifying the triangles as Scalene (none of the sides are equal), Isosceles (two sides are equal) and equilateral triangle (all of the sides are equal)

• **Congruency of triangles:** - Two or more triangles are congruent when they are equal in all aspects (shape, size and everything)

•Conditions for congruency

1. **SSS congruency:** - When all the sides of the given two or more triangles are equal, then the triangles are congruent.
2. **SAS congruency:** - When two sides of the given two or more triangles are equal and angle formed by these two sides are also equal, then also the two triangles are congruent.
3. **ASA congruency:** - When two angles of the given two or more triangles are equal and side between these two angles are also equal, then also the two triangles are congruent.
4. **RHS congruency:** - If in two or more triangles we have right angle, equal hypotenuse and one of the other sides also equal, then the triangles will be congruent.

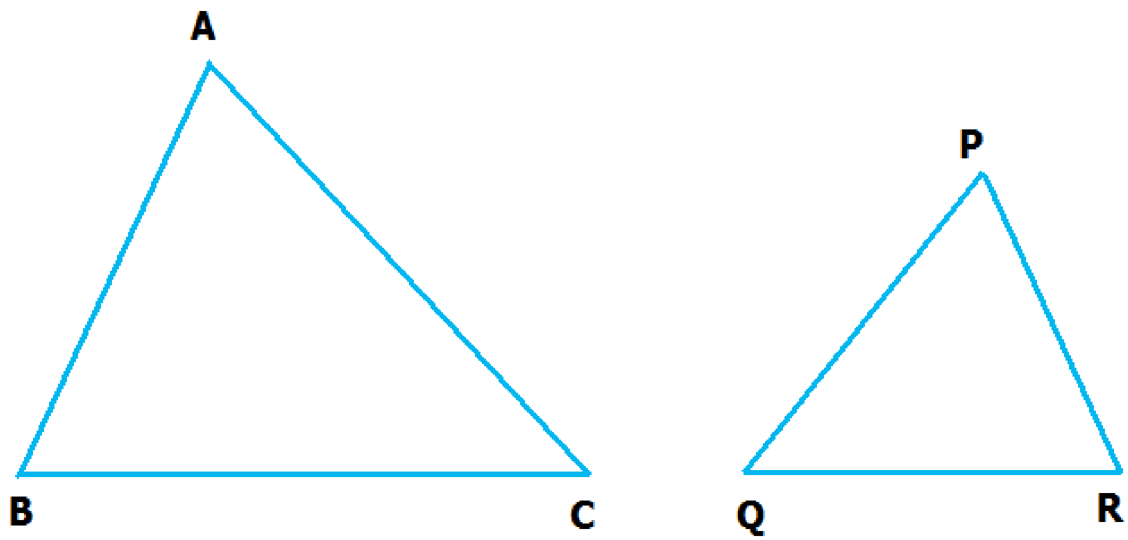
•Similarity of triangles

Two triangles will be similar if: -

1. Their corresponding angles are equal,
2. Their corresponding sides are in same proportion,
3. Also, if one angle of a triangle is equal to one angle of another triangle and the sides including the angle are in proportion, then also triangles are similar.

Notice that when ΔPQR ~ ΔABC, then sides of ΔPQR fall on corresponding equal sides of ΔABC and so is the case for the angles. That is, PQ covers AB, QR covers BC and RP covers CA; P covers A, Q covers B and R covers C. Also, there is a one-one correspondence between the vertices. That is, P corresponds to A, Q to B, R to C and so on which is written as

P →□A, Q →□B, R →□C



Here, $\frac{AB}{PR} = \frac{BC}{QR} = \frac{AC}{PQ}$, so we will write ΔABC ~ ΔPRQ and not ΔABC ~ ΔPQR. This was a simple case but in more complex cases it is always helpful.

Note: - For two similar triangles not only the corresponding sides are in the same proportion, but all corresponding medians, altitudes, angle bisectors are also in same proportion.

Also, ratio of area of similar triangles is equal to the ratio f square of corresponding sides, i.e., if ΔABC ~ Δ DEF, then

$$\frac{ar(ABC)}{ar(DEF)} = \frac{(AB)^2}{(DE)^2} = \frac{(BC)^2}{(EF)^2} = \frac{(AC)^2}{(DF)^2}$$

•**Few more important terms**:

•**Centroid:** - It is intersection points of the three medians of the triangle. Centroid divided the median in the ratio of 2:1. If AD is median of the triangle ABC and G is the centroid, then AG: GD = 2 : 1

•**Circum-centre:** - Intersection point of all the perpendicular bisectors of the sides of the triangle. It is also the centre of the circle which circumscribes the triangle. Circum-centre is denoted by 'R'.

•**In-centre:** - Intersection point of the three angle bisectors. It is also centre of the circle which is inscribed inside the triangle, i.e., it touches all the sides of the triangle. In-radius is denoted by 'r'.

•**Orthocentre:** - Intersection point of the three altitudes of the triangle.

•Apollonius Theorem: -

If in a triangle ABC, AD is median, the as per Apollonius theorem

$AB^2 + AC^2 = 2(AD^2 + CD^2)$

•Angle Bisector Theorem: If AD is angle bisector of angle BAC in triangle ABC, then AB/AC = BD/CD

•Area of two triangles having their base on the same line and also the third vertex on the same line have ratio of their areas as ratio of the length of their bases.

Quadrilaterals

•Trapezium:

1. One pair of opposite sides of quadrilateral is parallel.
2. In case of isosceles trapezium, sides which are not parallel are equal

•Kite:

1. If ABCD is a quadrilateral then AB = BC and CD = DA.
2. Diagonals are perpendicular to each other.
3. One of the diagonals will be bisected by the other. In the above example, AC is bisected by BD

•Parallelogram:

1. Both pairs of opposite sides of quadrilaterals are parallel.
2. A diagonal of a parallelogram divides it into two congruent triangles and diagonals bisect each other
3. In a parallelogram, opposite sides and angles are equal.
4. Sum of adjacent angles is 180°

•Rhombus:

1. Both pairs of opposite sides of quadrilaterals are parallel.
2. A diagonal of a parallelogram divides it into two congruent triangles and diagonals are perpendicular bisector of each other
3. In a parallelogram, all the sides are equal.
4. Sum of adjacent angles is 180°
5. Opposite angles are equal

•Rectangle:

1. It's a parallelogram having all angles equal to 90°
2. All properties of parallelogram are also applicable

•Square:

1. Rhombus having all angles 90°
2. All properties of rhombus are applicable

•Note that a square, rectangle and rhombus are all parallelograms.

- A square is a rectangle and also a rhombus.
- A parallelogram is a trapezium.
- A kite is not a parallelogram.
- A trapezium is not a parallelogram (as only one pair of opposite sides is parallel
- in a trapezium and we require both pairs to be parallel in a parallelogram).
- A rectangle or a rhombus is not a square.

Circles

•Minor arc and Major arc: - If P and Q are points on the circumference (periphery) of circle, and then smaller arc is termed as 'minor arc' and larger one as 'major arc'.

•Chord: - Line joining any two points on the circumference of the circle is chord.

•Diameter: - Chord passing through the centre of the circle. It is the largest chord of the circle. Diameter is twice the radius of a circle. When P and Q are ends of diameter, then both arcs are semi-circle.

•Segment: - The region between a chord and either of its arcs is called a segment of the circular region or simply a segment of the circle.

•Sector: - The region between an arc and the two radii, joining the centre to the end points of the arc is called a sector.

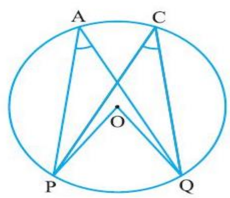
•Properties:-

1. Equal chords of a circle subtend equal angles at the centre. Reverse of it also holds true, i.e, if the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.
2. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres). Converse is also true, i.e., Chords equidistant from the centre of a circle are equal in length.
3. The perpendicular from the centre of a circle to a chord bisects the chord.
4. There is one and only one circle passing through three given non-collinear points.
5. The length of the perpendicular from a point to a line is the distance of the line from the point.
6. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
7. Angles in the same segment of a circle are equal and converse is also true, i.e., if a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle
8. Cyclic quadrilateral is a quadrilateral having all of its vertices on the circle.
9. The sum of either pair of opposite angles of a cyclic quadrilateral is 180º. Converse is also true, i.e., if the sum of a pair of opposite angles of a quadrilateral is 180º, the quadrilateral is cyclic.

10. **Alternate segment theorem:** Angle between any chord passing through the tangent point and tangent is equal to the angle subtended by the chord to any point on the other side of circumference (alternate segment)
11. **Ptolmey's theorem:** For a cyclic quadrilateral, the sum of products of two pairs of opposite sides equals the product of the diagonals.

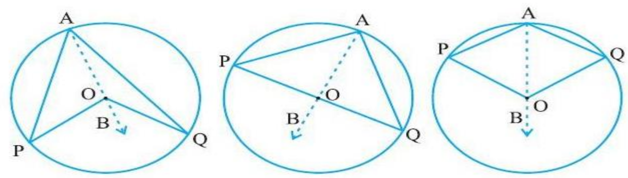
Point (vii)

$\angle PCQ = \angle PAQ$



Point (vi)

$\angle POQ = 2 \angle PAQ.$



•Tangents

•For any straight line we have only three possibilities: - it doesn't intersect the circle, it intersects the circle at two different points and it touches the circle at just one point.

•When a straight line intersects the circle as two points it is known as secant.

•When the straight line touches a circle at exactly one point, it is known as tangent to the circle at that point. A circle can have infinite number of tangents.

•There can be at most two tangents parallel to a secant.

•The common point of the circle and the tangent is known as point of contact.

•The tangent at any point of a circle is perpendicular to the radius through the point of contact.

•Number of tangents to a circle: -

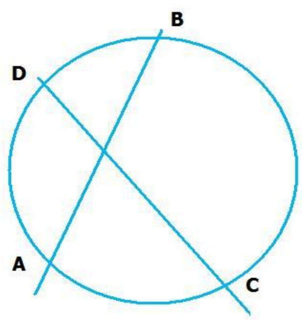
1. There is no tangent to a circle passing through a point lying inside the circle.
2. There is one and only one tangent to a circle passing through a point lying on the circle.
3. There are exactly two tangents to a circle through a point lying outside the circle

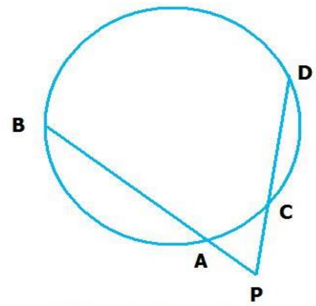
•Length of the segment of the tangent from an external point to the point of contact is length of the tangent.

•The lengths of tangents drawn from an external point to a circle are equal.

If AB and CD are chords of the circle intersecting at P, then

AP x BP = CP x DP



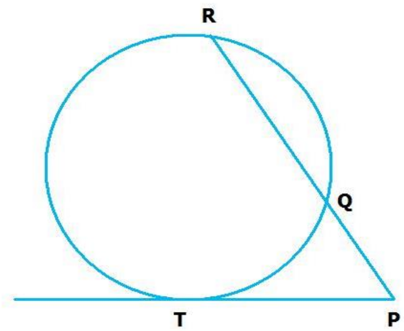


If AB and CD are chords such that they intersect outside circle at P, then also

PA x PB = PC x PD

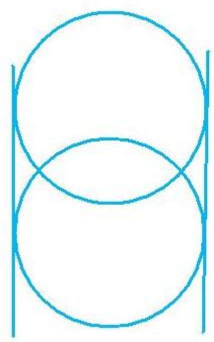
If PT is tangent to the circle with T as the point of contact and PQR is a secant, then

PT^2 = PQ x PR

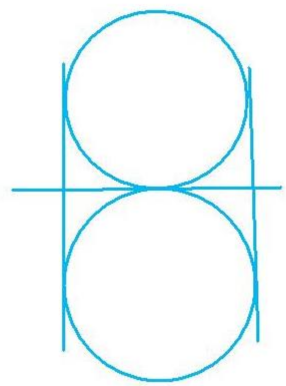


Common tangents to two circles

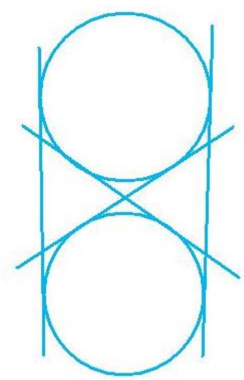
At most two when circles are intersecting at two different points



At most three when circles are tangent to each other



Four common tangents when the circle don't intersect each other



Polygons

•In a polygon of ‘n’ no. of sides,

•Total number of diagonals = $\frac{n(n-3)}{2}$

•Exterior angle of a regular polygon = $\frac{360°}{n}$

•Interior angle of a regular convex polygon = 180 - $\frac{360°}{n}$

•Sum of all the exterior angles of a regular convex polygon = 360°

•Sum of interior angles of a n sided polygon = (n – 2) × 180°

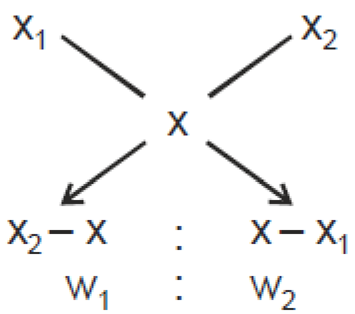
- **Few more important points:**

- Of all plane figures with a given perimeter, the circle has the greatest area.
- Of all plane figures with a given area, the circle has the least perimeter.
- Of all solids with a given surface area, the sphere has the greatest volume.
- Of all solids with a given volume, the sphere has the least surface area.
- Of all triangles with a common base and perimeter, the isosceles triangle has the greatest area.
- Of all triangles with a common base and area, the isosceles triangle has the smallest perimeter.
- If two triangles have the same base and the same perimeter, the one with the smaller difference in the lengths of its legs has the larger area.
- Of all triangles with a given perimeter, the equilateral triangle has the greatest area.
- Of all triangles with a given area, the equilateral triangle has the least perimeter.
- Of all n-gons inscribed in a given circle, the regular n-gon has the greatest area.
- Of all quadrilaterals with a given area, the square has the least perimeter.
- A quadrilateral with given sides has the greatest area when it can be inscribed in a circle.
- Of all quadrilateral prisms with a given volume, the cube has the least surface area.
- Given any n-gon which does not have all its sides of equal length, one can construct another n-gon of a larger area, with the same perimeter and with all sides of equal length.
- Given an acute-angled triangle, the vertices of the inscribed tri- angle with the smallest perimeter are the feet of the altitudes of the given triangle.

ARITHMETIC

Average/Mixture/Alligation

•Alligation is a method of calculating weighted averages. The ratio of the weights of the two items mixed will be inversely proportional to the difference of each of these two items from the average attribute of the resultant mixture.



$$\frac{w_1}{w_2} = \frac{(x_2 - x)}{(x - x_1)}$$

$$\text{Arithmetic Mean} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}$$

$$\text{Geometric Mean} = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots\ldots \times x_n}$$

Harmonic Mean =

$$\frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots\ldots + \frac{1}{x_n}\right)}$$

• Let K_o be the initial concentration of a solution and K is the final concentration after n dilutions is the original volume and x is the volume of the solution replaced each time, then

$$K = \frac{\mathbf{Ko(V-x)^n}}{\mathbf{V}}$$

PERCENTAGE/PROFIT AND LOSS/ INTERESTS

•Two successive percentage changes of **a%** and **b%** is an effective change of $\left(a + b + \frac{ab}{100}\right)$%

•If A is r% more/less than B, B is $(\frac{100r}{100 \pm r})$ less/more than A.

•Profit % = $\frac{Profit}{CP} \times 100$

•Loss% = $\frac{Loss}{CP} \times 100$

•SP = CP + P% of CP = **CP**$\left(1 + \frac{P}{100}\right)$

•Discount = **Marked Price – Selling Price**

•Discount % = $\frac{Discount}{Marked\ Price} \times 100$

•The selling price of two articles is same. If one is sold at X% profit and the other at loss of X%, then there is always a loss of $\frac{X^2}{100}$.

•P = Principal, A = Amount, I = Interest, n = no. of years, r% = rate of interest

•The Simple Interest (S.I.) = $\frac{P \times r \times n}{100}$

•If P is the principal kept at Compound Interest (C.I.) @ r% p.a., amount after n years

= **P**$\left(1 + \frac{r}{100}\right)^n$

•Amount = Principal + Interest

•Let **P** = Original Population, **P'** = Population after **n** years, **r%** = rate of anual growth Let P = Original Population, P′ = Population after n years, r% = rate of anual growth

P′ = $\mathbf{P}\left(1+\frac{r}{n}\right)^{n}$

•Difference between CI and SI for 2 and 3 years respectively:

$(CI)_2 - (SI)_2$ = $\mathbf{Pa^2}$ for two years
$(CI)_3 - (SI)_3$ = $\mathbf{Pa^2}$ (a + 3) for three years

a = r/100

•A principal amounts to X times in T years at S.I. It will become Y times in:

Years = $\frac{Y-1}{x-1} \times \boldsymbol{T}$

• A principal amounts to X times in T years at C.I. It will become Y times in:
Years = T × n, where n is given by $\mathbf{X^n = Y}$.

Permutation and Combination

•Number of ways of distributing 'n' identical things among 'r' persons such that each person may get any no. of things: $^{(n+r-1)}C_{(r-1)}$

•If out of n things, p are exactly alike of one kind, q exactly alike of second kind and r exactly alike of third kind and the rest are different, then the number of permutations of n things taken all at a time : $\frac{\boldsymbol{n}!}{\boldsymbol{p}!\boldsymbol{q}!\boldsymbol{r}!}$

•$\mathbf{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 \ldots\ldots {}^nC_n = 2^n}$

•Total number of ways in which a selection can be made by taking some or all out of (p + q + r + ...) items where p are of one type, q are of second type and r are of another type and so on = **{(p + 1) (q +1) (r + 1) …} – 1**

•The number of different relative arrangement for n different things arranged on a circle = **(n – 1)!**

•The number of ways in which **(m + n)** things can be divided into two groups containing m and n things respectively =$(\boldsymbol{m}+\boldsymbol{n})!/\boldsymbol{m}!\,\boldsymbol{n}!$

•If the numbers of things are equal, say **m = n**, total ways of grouping = $\frac{2\boldsymbol{m}!}{2!(\boldsymbol{m}!)^2}$

•Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$

•Odds in favour = $\dfrac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$

•Odds against = $\dfrac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$

•If two events are said to be mutually exclusive then if one happens, the other cannot happen and vice versa. In other words, the events have no simultaneous occurrence.

• In general P(A or B) = P(A) + P(B) – P(A∩B)

If A, B are mutually exclusive then
P(A ∩ B) = 0

If A, B are independent then
P(A ∩ B) = P(A).P(B)

• Additional law of probability:

If E and F are two mutually exclusive events, then the probability that either event E or event F will occur in a single trial is given by:
P(E or F) = P(E) + P(F)

If the events are not mutually exclusive, then P(E or F) = P(E) + P(F) – P(E and F together).

•Multiplication law of probability: If the events E and F are independent, then P(E and F) = P (E) × P(F)

Algebra

•An equation that can be written in the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation.

•In the equation, a, b, and c are real number coefficients and x is a variable. A quadratic equation written in the form $ax^2 + bx + c = 0$ is said to be in standard form. Sometimes, a quadratic equation is also called a second degree equation.

QUADRATIC FORMULA

If $ax^2 + bx + c = 0,\ a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

•The quantity $b^2 - 4ac$ is called the discriminant of a quadratic equation. The discriminant tells us whether the equation has real solutions, and also tells us how many roots of an equation exist. The discriminant is denoted by Δ(delta).

•For a quadratic equation $ax^2 + bx + c = 0$, the value of Δ determines the number of real roots.

1. $b^2 - 4ac > 0$: The roots are real and unequal. It means that curve intersects the x-axis in two distinct points.
2. $b^2 - 4ac = 0$: The roots are real and equal. It means that curve touches the x-axis at one point.
3. $b^2 - 4ac < 0$: The roots are imaginary. It means that the curve does not intersect the x-axis at all.
4. $b^2 - 4ac$ is a perfect square: The roots are rational and unequal.

•Let x_1 and x_2 be the roots of the quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$.
Then $\boldsymbol{x1 + x2 = -\frac{b}{a}}$ ***and*** $\boldsymbol{x1 \times x2 = \frac{c}{a}}$

- In the equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$, where $a_0, a_1, a_2, \ldots a_n \neq 0$,

  Sum of the roots $= -\frac{a_1}{a_0}$

  Sum of the products of the roots taken two at a time $= \frac{a_2}{a_0}$

  Sum of the products of the roots taken three at a time $= -\frac{a_3}{a_0}$

  •

  •

  Pr oduct of the roots $= (-1)^n \frac{a_n}{a_0}$

•In $ax^2 + bx + c$, if $a > 0$, The minimum value of $ax^2 + bx + c$ will be $y = \frac{4ac-b^2}{4a}$ at $x = -\frac{b}{2a}$

•In $ax^2 + bx + c$, if $a < 0$, The maximum value of $ax^2 + bx + c$ will be $y = \frac{4ac-b^2}{4a}$ at $x = -\frac{b}{2a}$

•If the roots of a quadratic equation are α and β the equation can be re-constructed as x^2 – (sum of roots) × x + (product of roots) = 0

•For the equation $ax^2 + bx + c = 0$

•If b = 0, the roots are equal in magnitude but opposite in sign.
•If a = c, the roots are reciprocals of each other.
•If a and c have the same sign but b has the opposite sign, the roots are positive.
•If a and b have the same sign but c has the opposite sign, the roots have opposite signs.

•In an equation with real coefficients imaginary roots occur in pairs i.e. if a + ib is a root of the equation f(x) = 0, then a - ib will also be a root of the same equation. For example, if 2 + 3i is a root of equation f(x) = 0, 2 - 3i is also a root.

•If the coefficients of an equation are all positive then the equation has no positive root.

•If the coefficients of even powers of x are all of one sign, and the coefficients of the odd powers are all of opposite sign, then the equation has no negative root.

•If the equation contains **only even** powers of x and the coefficients are all of the same sign, the equation has no real root.

•If the equation contains **only odd** powers of x, and the coefficients are all of the same sign, the equation has no real root except x = 0.

•**Descartes' Rule of Signs** : An equation f(x) = 0 cannot have more positive roots than there are changes of sign in f(x), and cannot have more negative roots than there changes of sign in f(- x).

•Sequence and Series

If a is the first term and d is the common difference,

- n^{th} term of the series $T_n = a + (n - 1)d$
- Sum of first n terms $S_n = n[2a + (n - 1)d]/ 2$

SOME SPECIAL RESULTS FOR ARITHMETIC PROGRESSIONS

- In an A.P. if p^{th} term is q and q^{th} term is p i.e. $T_p = q$ and $T_q = p$, then r^{th} term $T_r = p + q - r$.
- In an A.P. if $T_n = 1/m$, then $T_r = r/mn$ and $T_{mn} = 1$
- In an A.P. if $mT_m = nT_n$ then $(m + n)^{th}$ term i.e. $T_{m+n} = 0$
- In an A.P. if $T_p = q$ and $T_q = p$ then $T_{p+q} = 0$
- In an A.P. if the sum of p terms $S_p = q$ and the sum of q terms $S_q = p$ then sum of $(p + q)^{th}$ terms $S_{p+q} = -(p + q)$
- In an A.P. if the sum of p terms S_p = the sum of q terms S_q, then sum of $(p + q)^{th}$ terms $S_{p+q} = 0$.
- If the ratio $S_p/S_q = p^2/q^2$, then $T_p/T_q = (2p - 1)/(2q - 1)$.
- If the pth, qth, rth terms of an A.P. are a, b, c respectively, then $(q - r)a + (r - p)b + (p - q)c = 0$.

REMEMBER!

- In an AP, the sum of first term + last term = sum of second term + second last term = the sum of third term + third last term =.. and so on if the number of terms in the AP is even.
  If the number of terms in an AP are odd, the sum of first term + last term = sum of second term + second last term = the sum of third term + third last term = ..= 2 × the middle term.
- Numbers of terms in an AP = $\frac{\text{Last term} - \text{first term}}{\text{Common difference}} + 1$
- Arithmetic Mean between two numbers a and b = (a + b)/2
- If the same quantity is added to, or subtracted from, all the terms of an A.P., the resulting will again with be in A.P. with the same common difference as before.
- If all the terms of an A.P. are multiplied or divided by the same number, the resulting terms will again be in A.P.
- The sum of first n natural numbers = $\frac{n(n+1)}{2}$

If a is the first term and r is the common ratio of a geometric progression

- The n^{th} term of the series = ar^{n-1}.
- The Sum of n terms of a G.P. $S_n = \begin{cases} \dfrac{a(r^n - 1)}{r - 1} & \text{when } r > 1 \\ \dfrac{a(1 - r^n)}{1 - r} & \text{when } r < 1 \end{cases}$
- **Note:** The method to calculate the sum of a geometric progression is specially important as the same method is used to calculate the sum of arithmetico-geometric (AGP) series.
  Let $S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$. Multiplying by common ratio r on both sides, we get
  $rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$. Subtracting the first equation from the second, we get
  $(r - 1)S_n = ar^n - a \Rightarrow S_n = \dfrac{ar^n - a}{r - 1}$
- The sum of an infinite number of terms of a G.P. whose common ratio is less than 1 is equal to $\dfrac{a}{1-r}$.

- Geometric Mean between two numbers a and b = $\sqrt{ab}$
- If all the terms of a G.P. are multiplied or divided by the same quantity, the resulting will again be in G.P. with the same common ratio as before.
- If a, b, c, d,.... are in G.P., they are also in continued proportion, i.e. $\dfrac{a}{b} = \dfrac{b}{c} = \dfrac{c}{d} = \ldots = \dfrac{1}{r}$