Social Networks: Assignment #2

Due on Saturday, December 26, 2025

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Question 1

Structural Index

(a) Explain what each of these functions represents (what information they provide about the network). In doing so, describe what a high value versus a low value of each function indicates.

1. **Global Communication Efficiency (f_1)**

$$f_1(G) = \frac{1}{n(n-1)} \sum_{u \neq v} \frac{1}{d(u,v)} \tag{1}$$

   This function measures the average speed and ease with which information spreads across the organization. A **high value** indicates a "flat" structure where employees can reach anyone quickly with few intermediaries, facilitating rapid decision-making. A **low value** suggests a siloed or highly hierarchical organization where information travels slowly through many bottlenecks.

2. **Collaborative Redundancy and Local Density (f_2)**

$$f_2(G) = \frac{1}{n} \sum_{i \in V} \sum_{j \in N(i)} \left(p_{ij} + \sum_{\substack{q \in N(i) \\ q \neq j}} p_{iq} p_{qj} \right)^2 \tag{2}$$

   This metric represents the intensity of local team clusters and how much an employee's attention is reinforced by a tight-knit circle. A **high value** indicates strong, redundant team structures where social capital is high and the network is resilient to individual absences. A **low value** indicates sparse local connections, suggesting employees may be working in isolation without a strong peer support system.

3. **Structural Heterogeneity and Degree Entropy (f_3)**

$$f_3(G) = \frac{1}{n} \sum_{v \in V} \frac{d(v)}{\sum_{u \in V} d(u)} \log \left(\frac{d(v)}{\sum_{u \in V} d(u)} \right) \tag{3}$$

   This formula measures the diversity of influence and connectivity within the workforce. A **high value** (in magnitude) indicates a diverse range of roles, featuring both specialized individual contributors and highly connected "hubs" or bridge-builders. A **low value** suggests a very uniform network where every employee has a similar number of contacts, which can result in a lack of clear organizational "connectors."

4. **Average Digital Presence (f_4)**

$$f_4(G) = \frac{1}{n} \sum_{v \in V} H(v) \tag{4}$$

   This represents the average amount of time employees spend working online. A **high value** indicates a highly digitally-active or remote-first workforce, though if extreme, it

may point toward meeting fatigue or burnout. A **low value** suggests an organization that operates primarily offline (such as manual labor or face-to-face services) or is currently in a low-activity phase.

5. **Total Organizational Connectivity (f_5)**

$$f_5(G) = \frac{1}{n} \sum_{v \in V} \sum_{u \in N(v)} w_{vu} \tag{5}$$

This sums and averages the raw weights of all interactions to measure the total volume of work flowing through the network. A **high value** indicates high-intensity collaboration and "heavy" workloads across the organization's links. A **low value** indicates "thin" relationships where, despite being connected, the actual strength and frequency of interaction between colleagues is minimal.

(b) Determine which of these functions is a Structural Index (SI) and which ones are not. Provide a brief justification for your answer

1. **Global Communication Efficiency (f_1)**: Yes, SI. Depends only on shortest path distances between nodes; measures how efficiently information can flow through the network.

2. **Collaborative Redundancy / Local Density (f_2)**: Yes, SI. If the proportions p_{ij} are derived from intrinsic edge weights w_{ij} of the weighted graph, f_2 depends purely on the weighted network topology and reflects relative connectivity patterns dictated by the network structure.

3. **Structural Heterogeneity / Degree Entropy (f_3)**: Yes, SI. Uses node degrees only; measures diversity in connectivity and roles within the network.

4. **Average Digital Presence (f_4)**: No, not SI. Depends on dynamic activity of nodes ($H(v)$), which is an external attribute not determined by network topology.

5. **Total Organizational Connectivity (f_5)**: Yes, SI. If the edge weights w_{ij} are intrinsic to the weighted graph structure, f_5 depends purely on the weighted topology and reflects the total connectivity encoded by the network structure.