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PRACTICE PAPER 12 (2024-25)

CHAPTER-10 HERON'S FORMULA (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : IX **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The length of each side of an equilateral triangle having an area of $9\sqrt{3}$ cm^2 is

(a) 8 cm (b) 36 cm (c) 4 cm (d) 6 cm

Ans: (d) 6 cm

$\frac{\sqrt{3}}{4}a^2 = 9\sqrt{3} \Rightarrow a = 6cm$

2. The perimeter of an equilateral triangle is 60 m. The area is

(a) $10\sqrt{3}$ m^2 (b) $15\sqrt{3}$ m^2 (c) $20\sqrt{3}$ m^2 (d) $100\sqrt{3}$ m^2

Ans: (d) $100\sqrt{3}$ m^2

As side of an equilateral triangle = $\frac{60}{3} = 20m$

$\therefore$ Area $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 20^2 = 100\sqrt{3}m^2$

3. The height of an equilateral triangle is 6 cm. Its area is

(a) $12\sqrt{3}$ cm^2 (b) $6\sqrt{3}$ cm^2 (c) $12\sqrt{3}$ cm^2 (d) 18 cm^2

Ans: (a) $12\sqrt{3}$ cm^2

Height of equilateral triangle = $\frac{\sqrt{3}}{2} \times Side \Rightarrow 6 = \frac{\sqrt{3}}{2} \times Side$

$\Rightarrow Side = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12}{3} \times \sqrt{3} = 4\sqrt{3}\,cm$

Now, Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (Side)^2 = \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2 = \frac{\sqrt{3}}{4} \times 48 = 12\sqrt{3}\,cm^2$

4. The lengths of three sides of a triangle are 20 cm, 16 cm and 12 cm. The area of the triangle is

(a) 96 cm^2 (b) 120 cm^2 (c) 144 cm^2 (d) 160 cm^2

Ans: (a) 96 cm^2

Here, $a = 20\,cm, b = 16\,cm \text{ and } c = 12\,cm$

$s = \frac{a+b+c}{2} = \frac{20+16+12}{2} = 24\,cm$

By Heron's formula, we have Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-20)(24-16)(24-12)} = \sqrt{24 \times 4 \times 8 \times 12} = \sqrt{6 \times 4 \times 4 \times 4 \times 4 \times 6} = 6 \times 4 \times 4 = 96\,cm^2$

5. If the side of rhombus is 10 cm and one diagonal is 12 cm, then area of rhombus is
(a) 96 cm^2 (b) 48 cm^2 (c) 72 cm^2 (d) 80 cm^2
Ans: (a) 96 cm^2

6. The area of an equilateral triangle with side $4\sqrt{3}$ cm is
(a) 20 cm^2 (b) $20\sqrt{3}$ cm^2 (c) 18.784 cm^2 (d) 20.784 cm^2
Ans: (d) 20.784 cm^2

7. The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be
(a) 24 cm^2 (b) 40 cm^2 (c) 48 cm^2 (d) 80 cm^2
Ans: (a) 24 cm^2
Altitude of right triangle = $\sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6$ cm
Area of right triangle = $\frac{1}{2} \times 8 \times 6 = 24$ cm^2

8. Sides of a triangle are 8 cm, 11 cm and 13 cm. Then value of 's' is
(a) 19 cm (b) 20 cm (c) 21.5 cm (d) 16 cm
Ans: (d) 16 cm
Perimeter (2s) = 8 + 11 + 13 = 32 cm $\Rightarrow$ s = 16 cm

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** Area of an equilateral triangle having each side 4 cm is $4\sqrt{3}$ cm^2
Reason (R): Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (Side)^2$
Ans: (a) Both A and R are true and R is the correct explanation of A.

10. **Assertion (A):** Area of a triangle is 6 cm^2 whose sides are 3 cm, 4 cm and 5 cm respectively.
Reason (R): Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
Ans: (a) Both A and R are true and R is the correct explanation of A.

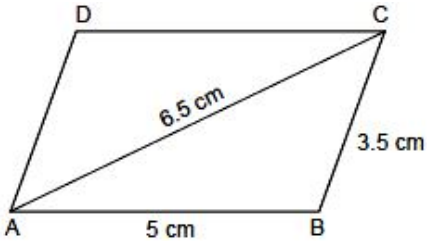
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The perimeter of an isosceles triangle is 32 cm. The ratio of equal side to the base is 3 : 2 Using Heron's formula, find the area of triangle.
Ans: Given perimeter of an isosceles triangle = 32 cm
$\Rightarrow 3x + 3x + 2x = 32$ [$\because$ Ratio of equal side to the base is 3 : 2]
$\Rightarrow 8x = 32 \Rightarrow x = 4$
$\therefore$ Sides of triangle are 12 cm, 12 cm and 8 cm
Semi-perimeter, $s = \frac{12+12+8}{2} = \frac{32}{2} = 16$ cm
$\therefore$ By Heron's formula, $Area\ of\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{16(16-12)(16-12)(16-8)} = \sqrt{16(4)(4)(8)} = 32\sqrt{2}\,cm^2$

12. Two adjacent sides of a parallelogram measures 5 cm and 3.5 cm. One of its diagonal measures 6.5 cm. Find the area of the parallelogram.

Ans: Let ABCD be the parallelogram with AB = 5 cm, BC = 3.5 cm and AC = 6.5 cm as shown in figure.



$\therefore$ Semi-perimeter of ΔABC, $s = \dfrac{5+3.5+6.5}{2} = \dfrac{15}{2} = 7.5$ cm

$\therefore$ By Heron's formula, $Area\ of\ \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7.5(7.5-5)(7.5-3.5)(7.5-6.5)} = \sqrt{7.5(2.5)(4)(1)} = 5\sqrt{3}\,cm^2$

We know that the diagonal of a parallelogram divides it into two congruent triangles of equal area.

$\therefore$ Area of parallelogram ABCD = 2 × ar(ΔABC) = $2 \times 5\sqrt{3} = 10\sqrt{3}$ cm^2

13. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Ans: Let the third side of the triangle be x.

Perimeter of the given triangle = 42 cm

$\Rightarrow$ 18 cm + 10 cm + x = 42 $\Rightarrow$ x = 14 cm

$\therefore$ Semi-perimeter of the triangle is $s = \dfrac{42}{2} = 21cm$

$\therefore$ By Heron's formula, $Area\ of\ \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)} = \sqrt{21(3)(11)(7)}$

$= 21\sqrt{11}\,cm^2$

14. The sides of a triangle are in the ratio 13 : 14 : 15 and its perimeter is 84 cm. Find the area of the triangle.

Ans: Given ratio of the sides of a triangle = 13 : 14 : 15

Let $a = 13k$, $b = 14k$ and $c = 15k$

Perimeter of triangle = 84 cm

$\Rightarrow 13k + 14k + 15k = 84$

$\Rightarrow 42k = 84 \Rightarrow k = \dfrac{84}{42} = 2$

So, the sides of a triangle are 13 × 2 = 26 cm, 14 × 2 = 28 cm and 15 × 2 = 30 cm

Its semi-perimeter, $s = \dfrac{84}{2} = 42$ cm

$\therefore$ By Heron's formula, $Area\ of\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-26)(42-28)(42-30)} = \sqrt{42(16)(14)(12)} = 336\,cm^2$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m, 15 m. The advertisements yield an earning of Rs. 2000 per m2 a year. A company hired one of its walls for 6 months. How much rent did it pay?

Ans: The sides of the triangle are of length 13 m, 14 m and 15 m.

∴ Semi-perimeter of the triangle is $s = \frac{13+14+15}{2} = \frac{42}{2} = 21\text{m}$

∴ By Heron's formula, *Area of* $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)} = \sqrt{21(8)(7)(6)} = 84\,\text{m}^2$

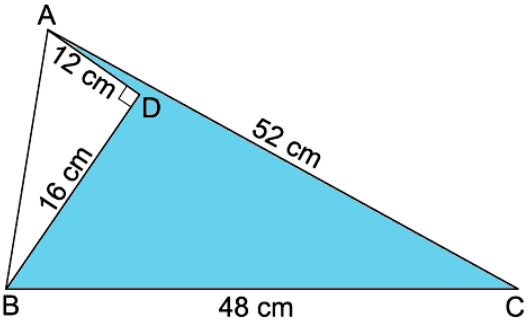
Now, The rent of advertisements per m^2 per year = Rs 2000

The rent of the wall with area 84 m^2 per year = Rs 2000 × 84 = Rs 168000

The rent of the wall with area 84 m^2 for 6 months = Rs $\frac{168000}{2}$ = Rs 84000

Hence, the rent paid by the company is Rs 84000.

16. Find the area of the shaded region in the figure given below.



Ans: In right angled Δ*ABD*,

$AB^2 = AD^2 + DB^2$ (Pythagoras Theorem)

$\Rightarrow AB^2 = 12^2 + 16^2$

$\Rightarrow AB^2 = 144 + 256$

$\Rightarrow AB^2 = 400 \Rightarrow AB = 20$ cm

Area of $\Delta ADB = \frac{1}{2} \times DB \times AD = \frac{1}{2} \times 16 \times 12 = 96\ \text{cm}^2$ ....(1)

In Δ*ACB*,

The sides of the triangle are of length 20 cm, 52 cm and 48 cm.

∴ Semi-perimeter of the triangle is $s = \frac{20+52+48}{2} = \frac{120}{2} = 60\,cm$

∴ By Heron's formula, *Area of* $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-52)(60-48)} = \sqrt{60(40)(8)(12)} = 480\,cm^2$ …(2)

Now, Area of the shaded region = Area of Δ*ACB* − Area of Δ*ADB* = 480 – 96 = 384 cm^2

Hence, the area of the shaded region in the given figure is 384 cm^2.

17. The perimeter of a triangle is 50 cm. One side of the triangle is 4 cm longer than the smallest side and the third side is 6 cm less than twice the smallest side. Find the area of the triangle.

Ans: Let *ABC* be any triangle with perimeter 50 cm.

Let the smallest side of the triangle be *x*.

Then the other sides be *x* + 4 and 2*x* − 6.

Now, *x* + *x* + 4 + 2*x* − 6 = 50 (â^µ perimeter is 50 cm)

$\Rightarrow 4x - 2 = 50 \Rightarrow 4x = 50 + 2$

$\Rightarrow 4x = 52 \Rightarrow x = 13$

∴ The sides of the triangle are of length 13 cm, 17 cm and 20 cm.

∴ Semi-perimeter of the triangle is $s = \frac{13+17+20}{2} = \frac{50}{2} = 25\,cm$

∴ By Heron's formula, *Area of* $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

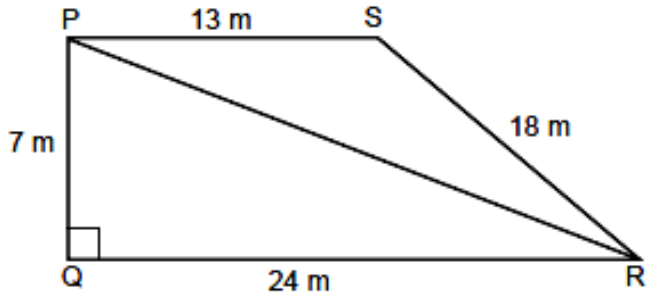
$= \sqrt{25(25-13)(25-17)(25-20)} = \sqrt{25(12)(8)(5)} = 20\sqrt{30}\,cm^2$

Hence, the area of the triangle is $20\sqrt{30}\,cm^2$

SECTION – D

Questions 18 carry 5 marks.

18. The students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes PQ, QR and RP; while the other group walked through PR, RS and SP as shown in figure:



These two groups cleaned the area enclosed within their lanes. If PQ = 7 m, QR = 24 m, RS = 18 m, SP = 13 m and $\angle Q = 90°$;

(i) Which group cleaned more area and by how much?

(ii) Find the total area cleaned by the students (neglecting the width of the lane).

Ans: (i) Given PQ = 7 m and QR = 24 m, $\angle Q = 90°$

Using Pythagoras theorem in right-angled ΔPQR,

we have $PR^2 = PQ^2 + QR^2$

PR = $\sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625}$ = 25 m

Therefore, first group has to clean the area of ΔPQR which is right-angled triangle.

$\therefore$ Area of ΔPQR = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 24 × 7

= 84 m^2

The second group has to clean the area of ΔPRS, which is scalene having sides 25 m, 18 m and 13 m.

Its semi-perimeter, $s = \frac{25+18+13}{2} = \frac{56}{2} = 28m$

$\therefore$ By Heron's formula, $Area\ of\ \Delta PRS = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{28(28-25)(28-18)(28-13)} = \sqrt{28(3)(10)(15)}$

$= 30\sqrt{14}\,cm^2 = 30 \times 3.74 = 112.2cm^2$

Clearly, the second group cleaned more area, i.e. 112.2 m^2 which is (112.2 – 84) = 28.2 m^2 more than the area cleaned by the first group.

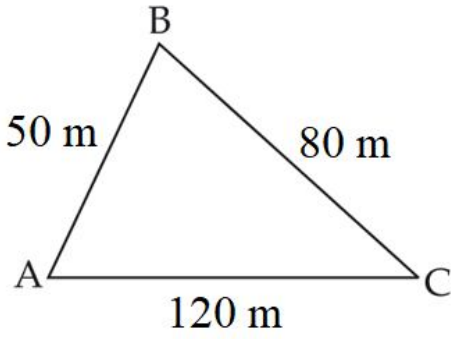
(ii) Total area cleaned by all the students

= 84 + 11.2 = 196.2 m^2

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. In my colony a park is situated in front of my house. This park has built in shape of triangle (ABC) with the following sides 120m, 80m and 50m. Now-a-days, some animals entered park and destroy and eat plants. So, our ward member of area has decided to put railing around the park for protecting plants and grass. Ward member ordered to a gardener to place a railing all around this park and maintain grass inside park. He also sanctioned an amount to improve park in a proper way for public of that colony. Costing is decided Rs. 10 per meter for railing around the park.



(i) What is the perimeter of the park? [1]
(ii) Calculate the semi-perimeter of triangle park, in which planting is needed? [2]
(iii) Calculate the area, in which planting is needed? [2]

OR

(iii) Find the cost of fencing it with barbed wire at the rate of Rs 20 per metre leaving a space 3m wide for a gate on one side. [2]

Ans: (i) Perimeter = 50 m + 80 m + 120 m = 250 m.
(ii) Semi-perimeter = Perimeter/2 = 250/2 = 125 m
(iii) By Heron's formula, *Area of* $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125(125-120)(125-80)(125-50)} = \sqrt{125(5)(45)(75)} = 375\sqrt{15}\, m^2$

OR

(iii) Perimeter of the park = AB + BC + CA = 250 m
Therefore, length of the wire needed for fencing = 250 m – 3 m (to be left for gate) = 247 m
And so the cost of fencing = Rs 20 × 247 = Rs 4940

20. Triangles are used in bridges because they evenly distribute weight without changing their proportions. When force is applied on a shape like a rectangle it would flatten out. Before triangles were used in bridges, they were weak and could not be very big. To solve that problem engineers would put a post in the middle of a square and make it more sturdy. Isosceles triangles were used to construct a bridge in which the base (unequal side) of an isosceles triangle is 4 m and its perimeter is 20 m.

(i) What is the length of equal sides? [1]
(ii) In a ΔABC it is given that base = 12 m and height = 5 m. Find its area. [1]
(iii) What is the area of the given isosceles triangle? [2]

OR

(iii) Find the cost of covering the border of one isosceles triangle at the rate of Rs 200 per metre. [2]

Ans:
(i) Perimeter = 20 m and one unequal side = 4 m
Each equal side = (20 – 4)/2 = 16/2 = 8 m
(ii) Area of ΔABC = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 12 \times 5 = 30\ m^2$

(iii) Semi-perimeter, $s = \frac{8+8+4}{2} = \frac{20}{2} = 10m$

$\therefore$ By Heron's formula, *Area of* $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(10-8)(10-8)(10-4)} = \sqrt{10(2)(2)(6)} = 4\sqrt{15}\, m^2$

OR

(iii) Perimeter = 20 m

Cost of 1 m = Rs. 200

Total cost of covering = Rs. 200 x 20 = Rs. 4000