

SUBJECT: MATHEMATICS

CLASS : IX

MAX. MARKS : 80

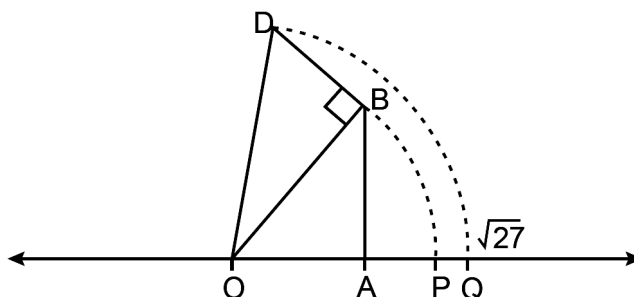
DURATION : 3 HRS

General Instruction:

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION – A**Questions 1 to 20 carry 1 mark each.**

1. Kevin's work to represent $\sqrt{27}$ on a number line is shown. In the number line, arc DQ is drawn using OD as the radius.



Looking at Kevin's work, Sonia and Rakesh made following statements.

Sonia: $OA = 5$ units, $AB = BD = 1$ unit

Rakesh: $OB = 26$ units and $AB = 1$ unit

Who is correct?

- (a) Only Sonia (b) Only Rakesh (c) Both of them (d) Neither of them

Ans. (a) Only Sonia

2. The number $1.101001000100001\dots$ is
- (a) a natural number (b) a whole number
- (c) a rational number (d) an irrational number

Ans. (d) an irrational number

3. Which of the following statements is/are correct?
- (i) Every integer is a rational number.
- (ii) Every rational number is an integer.
- (iii) A real number is either a rational or irrational number.
- (iv) Every whole number is a natural number.
- (a) (ii) (b) (iii) (c) (i) and (iii) (d) all of these
- Ans. (c) (i) and (iii)

4. The value of $\left[9(64^{1/3} + 125^{1/3})\right]^{1/4}$ is equal to
- (a) 9 (b) 3 (c) 81 (d) $9\sqrt[4]{9}$
- Ans. (b) 3

5. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to
(a) 3 (b) $2x$ (c) 0 (d) 6

Ans. (d) 6

$$p(x) + p(-x) = x + 3 + (-x) + 3 = 6$$

6. Factors of $3x^2 - x - 4$ are
(a) $(x - 1)$ and $(3x - 4)$ (b) $(x + 1)$ and $(3x - 4)$
(c) $(x + 1)$ and $(3x + 4)$ (d) $(x - 1)$ and $(3x + 4)$

Ans. (b) $(x + 1)$ and $(3x - 4)$

$$3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$$

$$= x(3x - 4) + 1(3x - 4) = (x + 1)(3x - 4)$$

7. $x = 5, y = -2$ is a solution of the linear equation
(a) $2x + y = 9$ (b) $2x - y = 12$ (c) $x + 3y = 1$ (d) $x + 3y = 0$

Ans. (b) $2x - y = 12$

Substituting $x = 5$ and $y = -2$ in LHS of $2x - y = 12$,
we have

$$\text{LHS} = 2 \times 5 - (-2) = 10 + 2 = 12 = \text{RHS}$$

8. If point $(3, 0)$ lies on the graph of the equation $2x + 3y = k$, then the value of k is
(a) 6 (b) 3 (c) 2 (d) 5

Ans. (a) 6

On putting $x = 3$ and $y = 0$ in the equation $2x + 3y = k$, we have

$$2 \times 3 + 3 \times 0 = k$$

$$\Rightarrow 6 + 0 = k \Rightarrow k = 6$$

9. If we multiply or divide both sides of a linear equation with a non-zero number, then the solution of the linear equation:

- (a) changes (b) remains the same
(c) changes in case of multiplication only (d) changes in case of division only.

Ans. (b) remains the same

If we multiply or divide both sides of a linear equation with a non-zero number, then graph will be same in both cases. Thus, the solution of the linear equation remains the same.

10. On plotting the points $O(0, 0)$, $A(4, 0)$, $B(4, 5)$, $C(0, 5)$ and joining OA , AB , BC and CO which of the following figure is obtained ?

- (a) Square (b) Rectangle (c) Trapezium (d) Rhombus.

Ans. (a) Square

11. The points in which abscissa and ordinate have different signs will lie in

- (a) I and II quadrants. (b) II and III quadrants.
(c) I and III quadrants. (d) II and IV quadrants.

Ans. (d) II and IV quadrants.

12. If the difference between two complementary angles is 10° , then the angles are

- (a) $50^\circ, 60^\circ$ (b) $50^\circ, 40^\circ$ (c) $80^\circ, 10^\circ$ (d) $35^\circ, 45^\circ$

Ans. (b) $50^\circ, 40^\circ$

Let an angle be x . Then other angle $= x - 10^\circ$. Since the two angles are complementary, so

$$x + x - 10^\circ = 90^\circ$$

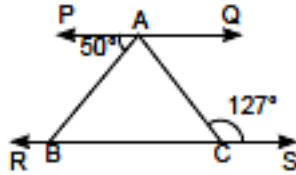
$$\Rightarrow 2x = 90^\circ + 10^\circ = 100^\circ$$

$$\Rightarrow x = 100/2$$

So, one angle $= 50^\circ$. Then, other angle $= x - 10^\circ$

$$= 50^\circ - 10^\circ = 40^\circ$$

13. In the given figure, $PQ \parallel RS$ and $\angle ACS = 127^\circ$, $\angle BAC$ is



- (a) 53° (b) 77° (c) 50° (d) 107°

Ans. (b) 77°

Since $PQ \parallel RS$, so

$$\Rightarrow \angle PAC = \angle ACS$$

(\because Alternate interior angles)

$$\Rightarrow \angle PAB + \angle BAC = 127^\circ$$

$$\Rightarrow 50^\circ + \angle BAC = 127^\circ$$

$$\Rightarrow \angle BAC = 77^\circ$$

14. If $\triangle ABC \cong \triangle PQR$ and $\triangle ABC$ is not congruent to $\triangle RPQ$, then which of the following is not true?

- (a) $BC = PQ$ (b) $AC = PR$ (c) $QR = BC$ (d) $AB = PQ$

Ans. (a) $BC = PQ$

Given, $\triangle ABC \cong \triangle PQR$

Thus, the corresponding sides are equal

Hence, $AB = PQ$, $BC = QR$ and $AC = PR$

Therefore, $BC = PQ$ is not true for the triangles.

15. A diagonal of a rectangle is inclined to one side of the rectangle at 25° . The acute angle between the diagonals is

- (a) 55° (b) 50° (c) 40° (d) 25°

Ans: Given, $\angle ODC = 25^\circ$

Since ABCD is a rectangle, so diagonals are equal.

$$\Rightarrow AC = BD \Rightarrow \frac{1}{2} AC = \frac{1}{2} BD \Rightarrow OC = OD$$

$$\Rightarrow \angle ODC = \angle OCD (\because \text{Angles opposite to equal sides are equal})$$

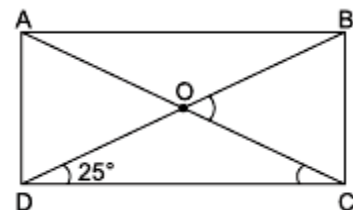
But $\angle BOC = \angle ODC + \angle OCD$ (u2235 Exterior angle property)

$$\Rightarrow \angle BOC = \angle ODC + \angle ODC \Rightarrow \angle BOC = 2\angle ODC$$

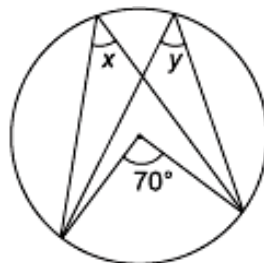
$$\Rightarrow \angle BOC = 2 \times 25^\circ = 50^\circ$$

So, the acute angle between the diagonals is 50° .

\therefore Correct options is (b).



16. In the given figure, value of y is



- (a) 35° (b) 140° (c) $70^\circ + x$ (d) 70°

Ans: (a) 35°

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. So,

$$y = \frac{1}{2} \times 70^\circ = 35^\circ$$

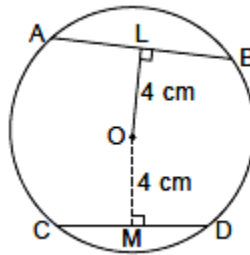
17. Given a chord AB of length 5 cm, of a circle with centre O. OL is perpendicular to chord AB and OL = 4 cm. OM is perpendicular to chord CD such that OM = 4 cm. Then CM is equal to
 (a) 4 cm (b) 5 cm (c) 2.5 cm (d) 3 cm

Ans: (c) 2.5 cm

Since OL = OM = 4 cm, so

AB = CD (\because Chords equidistant from the centre of a circle are equal in length)

\Rightarrow CD = 5 cm



Since the perpendicular drawn from the centre of a circle to a chord bisects the chord, so

$$CM = MD = \frac{1}{2} CD \Rightarrow CM = 2.5 \text{ cm}$$

18. The area of an equilateral triangle is $16\sqrt{3} \text{ cm}^2$, then half of the perimeter of the triangle is
 (a) 12 cm (b) 6 cm (c) 10 cm (d) 8 cm

Ans. (a) 12 cm

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
 (c) Assertion (A) is true but Reason (R) is false.
 (d) Assertion (A) is false but Reason (R) is true.
19. **Assertion (A):** The decimal expansion of a rational number is either terminating or non-terminating recurring.
Reason (R): Every number with a non-terminating recurring decimal expansion can be expressed in the form p/q and $q \neq 0$ where p and q are integers.
 Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
20. **Assertion (A):** The quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral whose diagonals are perpendicular is a rectangle.
Reason (R): The line segment in a triangle joining the midpoint of any two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side and the quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral is a parallelogram.
 Ans: (a) Both A and R are true and R is the correct explanation of A.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Express $0.\overline{57}$ in the form p/q where p and q are integers where $q \neq 0$.
 Ans. Let $x = 0.\overline{57}$
 Then, $x = 0.575757 \dots \dots$ (i)

Since the repeating block 57 has two digits, we multiply x by 100 to get

$$100x = 57.5757.... \dots \text{ (ii)}$$

Subtracting (i) from (ii), we get

$$99x = 57 \Rightarrow x = \frac{57}{99} = \frac{19}{33}$$

OR

Simplify: $\left(\frac{3125}{243}\right)^{\frac{-4}{5}}$

Ans.

$$\left(\frac{3125}{243}\right)^{\frac{-4}{5}} = \left(\frac{243}{3125}\right)^{\frac{4}{5}} = \left(\frac{3^5}{5^5}\right)^{\frac{4}{5}} = \left[\left(\frac{3}{5}\right)^5\right]^{\frac{4}{5}} = \left(\frac{3}{5}\right)^4 = \frac{81}{625}$$

22. Find the coordinates of a point:

(i) whose ordinate is 6 and lies on the y -axis

(ii) whose abscissa is -3 and lies on the x -axis.

Ans. For the point (x, y) , x represents abscissa and y represents ordinate. Hence,

(i) The coordinates of a point whose abscissa is zero lies on the y -axis.

Therefore, required coordinates = $(0, 6)$.

(ii) The coordinates of a point whose ordinate is zero and lies on the x -axis. Therefore, required coordinates = $(-3, 0)$

23. Solve the equation, $x - 10 = 25$ and state which axiom do you use here.

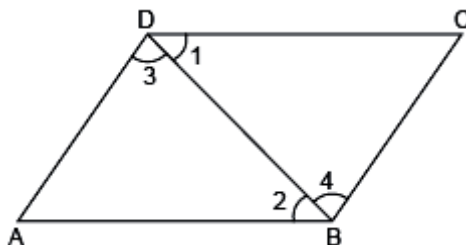
Ans. $x - 10 = 25$

On adding 10 both sides, we have

$$x - 10 + 10 = 25 + 10 \Rightarrow x = 35$$

Here, we use Euclid axiom, "If equal be added to the equal, the whole are equal."

24. In the given figure, it is given that $\angle 1 = \angle 4$ and $\angle 3 = \angle 2$



By which Euclid's axiom, it can be shown that if $\angle 2 = \angle 4$, then $\angle 1 = \angle 3$.

Ans. Given $\angle 1 = \angle 4$, and $\angle 3 = \angle 2$ and $\angle 2 = \angle 4$

Thus from Euclid's axiom, if things which are equal to same thing are equal to one another.

Here, $\angle 1 = \angle 4$

But $\angle 2 = \angle 4 \Rightarrow \angle 1 = \angle 2$... (i)

Again, given $\angle 3 = \angle 2$... (ii)

From (i) and (ii), we get

$$\angle 1 = \angle 3$$

25. The radius and height of a cone are in the ratio 3 : 4. If its volume is 301.44 cm^3 , what is its radius? (Use $\pi = 3.14$)

Ans. Here, radius of cone be $3x$ cm and height of the cone be $4x$ cm

$$\text{Volume of cone} = 301.44 \text{ cm}^3$$

$$\Rightarrow \frac{1}{3} \pi r^2 h = 301.44$$

$$\Rightarrow \frac{1}{3} \times 3.14 \times (3x)^2 \times 4x = 301.44$$

$$\Rightarrow \frac{1}{3} \times 3.14 \times 9x^2 \times 4x = 301.44$$

$$\Rightarrow x^3 = \frac{301.44 \times 3}{3.14 \times 9 \times 4} = 8$$

$$\Rightarrow x = 2 \text{ cm}$$

$$\therefore \text{Radius of cone} = 3x = 3 \times 2 = 6 \text{ cm}$$

OR

Find the volume of metal used to construct a hollow metallic sphere of internal and external diameters as 10 cm and 13 cm respectively (Use $\pi = 3.14$)

Ans. External diameter of hollow metallic sphere = 13 cm

External radius = $R = 13/2$ cm

Internal diameter of hollow metallic sphere = 10 cm

Internal radius = $r = 10/2 = 5$ cm

$$\begin{aligned} \text{Volume of metal used} &= \frac{4}{3}\pi(R^3 - r^3) \\ &= \frac{4}{3} \times 3.14 \left[\left(\frac{13}{2}\right)^3 - (5)^3 \right] \\ &= \frac{4}{3} \times 3.14 \times \left[\frac{2197}{8} - \frac{1000}{8} \right] \\ &= \frac{4}{3} \times 3.14 \times \frac{1197}{8} \\ &= 626.43 \text{ cm}^3 \end{aligned}$$

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. Simplify by rationalising the denominator: $\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$

Ans.

$$\begin{aligned} \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} &= \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{16 \times 3} + \sqrt{9 \times 2}} = \frac{4\sqrt{3} + 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}} \times \frac{4\sqrt{3} - 3\sqrt{2}}{4\sqrt{3} - 3\sqrt{2}} \\ &= \frac{16(\sqrt{3})^2 + 20\sqrt{6} - 12\sqrt{6} - 15(\sqrt{2})^2}{(4\sqrt{3})^2 - (3\sqrt{2})^2} = \frac{16 \times 3 + (20 - 12)\sqrt{6} - 15 \times 2}{16 \times 3 - 9 \times 2} \\ &= \frac{48 + 8\sqrt{6} - 30}{48 - 18} = \frac{18 + 8\sqrt{6}}{30} = \frac{2(9 + 4\sqrt{6})}{30} = \frac{9 + 4\sqrt{6}}{15} \end{aligned}$$

27. Find the value of k , if $x + k$ is factor of the polynomials:

(i) $x^3 - (k^2 - 1)x + 3$ (ii) $-4x^3 + 4x^2 + 4kx - k$

Ans. (i) Let $p(x) = x^3 - (k^2 - 1)x + 3$

$$p(-k) = (-k)^3 - (k^2 - 1)(-k) + 3 = 0$$

$$\Rightarrow -k^3 + k^3 - k + 3 = 0$$

$$\Rightarrow k = 3$$

(ii) Let $p(x) = -4x^3 + 4x^2 + 4kx - k$

$$p(-k) = -4(-k)^3 + 4(-k)^2 + 4k(-k) - k = 0$$

$$\Rightarrow 4k^3 + 4k^2 - 4k^2 - k = 0$$

$$\Rightarrow 4k^3 - k = 0 \Rightarrow k(4k^2 - 1) = 0$$

$$\Rightarrow \text{Either } k = 0 \text{ or } 4k^2 - 1 = 0 \Rightarrow (2k)^2 - 1^2 = 0$$

$$\Rightarrow (2k - 1)(2k + 1) = 0 \Rightarrow k = \frac{1}{2} \text{ or } -\frac{1}{2}$$

Hence, for the given $p(x)$, $k = 0, \frac{1}{2} \text{ or } -\frac{1}{2}$.

28. Show that the points A (1, 2), B (-1, -16) and C (0, -7) lie on the graph of the linear equation $y = 9x - 7$.

Ans. $y = 9x - 7$

or $9x - y = 7$... (i)

On putting $x = 1, y = 2$ in (i), we have

$$9 \times 1 - 2 = 7 \Rightarrow 9 - 2 = 7$$

$$\Rightarrow 7 = 7, \text{ true.}$$

Therefore, (1, 2) is a solution of linear equation $y = 9x - 7$.

On putting $x = -1, y = -16$ in (i), we have

$$9 \times (-1) - (-16) = 7 \Rightarrow -9 + 16 = 7$$

$$\Rightarrow 7 = 7, \text{ true.}$$

Therefore, (-1, -16) is a solution of linear equation $y = 9x - 7$.

On putting $x = 0, y = -7$ in (i), we have

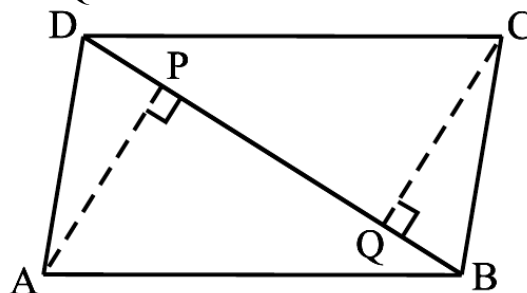
$$9 \times 0 - (-7) = 7 \Rightarrow 0 + 7 = 7$$

$$\Rightarrow 7 = 7, \text{ true.}$$

Therefore, (0, -7) is a solution of linear equation $y = 9x - 7$.

29. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see the below). Show that

(i) $\triangle APB \cong \triangle CQD$ (ii) $AP = CQ$



Ans. Given : ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.

(a) In $\triangle APB$ and $\triangle CQD$, we have

$\angle ABP = \angle CDQ$ [Alternate angles]

$AB = CD$ [Opposite sides of a parallelogram]

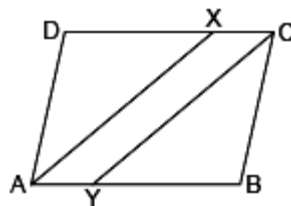
$\angle APB = \angle CQD$ [Each = 90°]

$\therefore \triangle APB \cong \triangle CQD$ [ASA congruence]

(b) So, $AP = CQ$ [CPCT]

OR

In the given figure, ABCD is a parallelogram and line segments AX and CY bisect the angles A and C respectively. Show that $AX \parallel CY$.



Ans: AX bisects $\angle A$

$$\therefore \angle XAB = \frac{1}{2} \angle DAB \quad \dots (i)$$

CY bisects $\angle C$.

$$\therefore \angle XCY = \angle DCB \quad \dots (ii)$$

Also, $\angle DAB = \angle DCB$ (Opposite angles of parallelogram)

$$\Rightarrow \frac{1}{2} \angle DAB = \frac{1}{2} \angle DCB$$

$$\Rightarrow \angle XAB = \angle XCY$$

$$\Rightarrow XC \parallel AY \quad (\text{Parts of parallel lines are parallel})$$

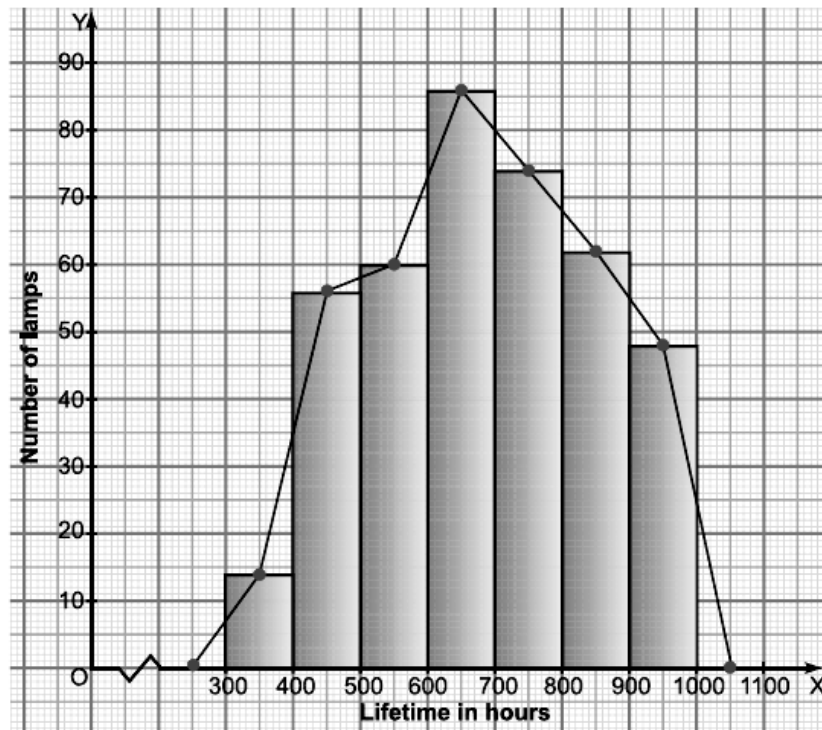
30. The following table gives the lifetimes of 400 neon lamps:

Lifetime (in hours)	Number of lamps
300 – 400	14
400 – 500	56
500 – 600	60
600 – 700	86
700 – 800	74
800 – 900	62
900 – 1000	48

(i) Represent the given information with the help of a histogram and a frequency polygon.

(ii) How many lamps have a lifetime of 700 or more hours?

Ans. (i)



(ii) Number of lamps having life time 700 or more hours = $74 + 62 + 48 = 184$.

31. The marks obtained (out of 100) by a class of 80 students are given below:

Marks	10 – 20	20 – 30	30 – 50	50 – 70	70 – 100
Number of students	6	17	15	16	26

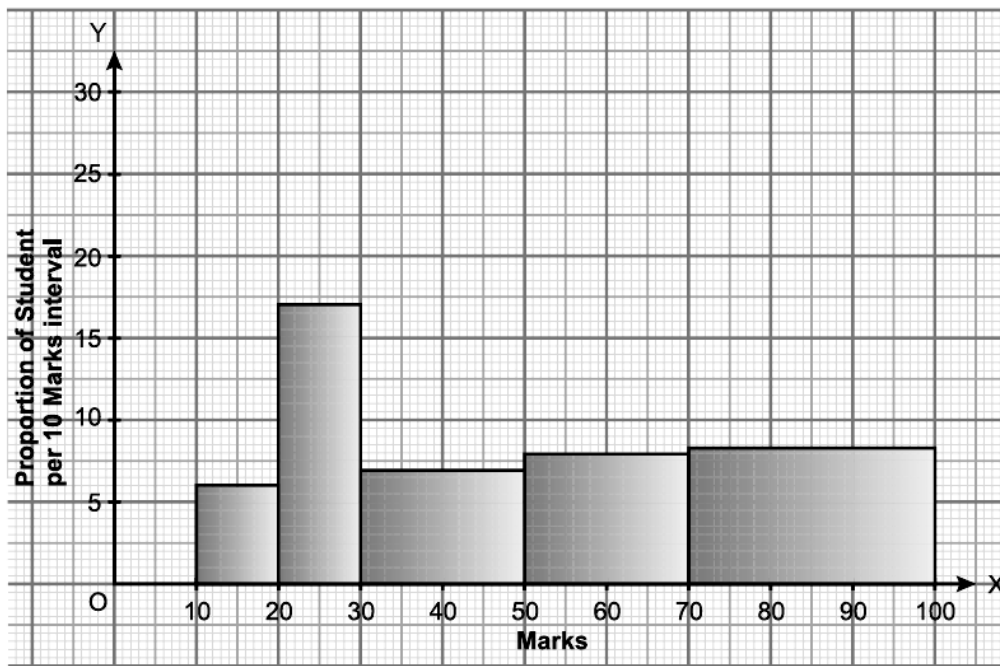
Construct a histogram to represent the data above.

Ans. In the given frequency distribution, the class intervals are not of equal width. Therefore, we would make modification in the lengths of the rectangle in the histogram so that the areas of rectangle are proportional to the frequencies. Thus we have:

Marks	Frequency	Class width	Adjusted Frequency
10 – 20	6	10	$\frac{10}{10} \times 6 = 6$
20 – 30	17	10	$\frac{10}{10} \times 17 = 17$
30 – 50	15	20	$\frac{10}{20} \times 15 = 7.5$

50 – 70	16	20	$\frac{10}{20} \times 16 = 8$
70 – 100	26	30	$\frac{10}{30} \times 26 = 8.67$

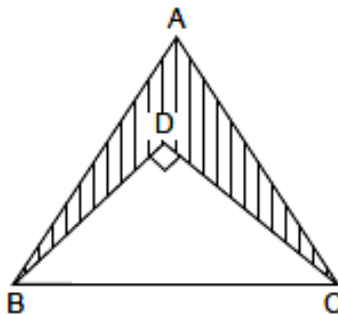
Now, we draw rectangles with lengths as given in the last column. The histogram of data is given below:



SECTION – D

Questions 32 to 35 carry 5 marks each.

32. In the given figure, $\triangle ABC$ is an equilateral triangle with side 10 cm and $\triangle BDC$ is right angled triangle with $\angle D = 90^\circ$. If $BD = 6$ cm, then find the area of the shaded portion. (Use $\sqrt{3} = 1.732$)



Ans. Given $\triangle ABC$ is an equilateral triangle with side 10 cm.

$$\text{Area of an equilateral triangle } \triangle ABC = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (10)^2 = 25\sqrt{3} \text{ cm}^2$$

$$= 25 \times 1.732 = 43.3 \text{ cm}^2$$

In right-angled $\triangle BDC$, $\angle D = 90^\circ$,

$BD = 6$ cm, $BC = 10$ cm

Using Pythagoras theorem,

$$BC^2 = BD^2 + DC^2$$

$$\Rightarrow 10^2 = 6^2 + DC^2$$

$$\Rightarrow DC = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64}$$

$$\Rightarrow DC = 8 \text{ cm}$$

The semi-perimeter of right-angled $\triangle BDC$,

$$s = (6 + 10 + 8)/2 = 24/2 = 12 \text{ cm}$$

Using Heron's formula,

$$\therefore \text{Area of } \triangle BDC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-6)(12-10)(12-8)}$$

$$= \sqrt{12 \times 6 \times 2 \times 4}$$

$$= \sqrt{6 \times 2 \times 6 \times 2 \times 2 \times 2} = \sqrt{6 \times 6 \times 2 \times 2 \times 2 \times 2}$$

$$= 6 \times 2 \times 2 = 24 \text{ cm}^2$$

$$\therefore \text{Area of shaded portion} = \text{ar}(\triangle ABC) - \text{ar}(\triangle BDC) = 43.3 - 24 = 19.3 \text{ cm}^2$$

OR

A gardener has to put double fence all around a triangular field with sides 120 m, 80 m and 60 m. In the middle of each of the sides, there is a gate of width 10 m.

(i) Find the length of wire needed for fencing.

(ii) Find the cost of fencing at the rate of ₹ 6 per metre.

(iii) Find the area of triangular field.

Ans. Perimeter of triangular field = 120 + 80 + 60 = 260 m

(i) Length of wire needed for single fencing

= 260 – 30 (to be left for gate on each side)

= 230 m

\therefore Total length of wire needed for double fencing = 2 × 230 = 460 m

(ii) Cost of fencing = ₹ 6 per metre

\therefore Total cost of fencing = 460 × 6 = ₹ 2760

(iii) Given $a = 120$ m, $b = 80$ m and $c = 60$ m

The semi-perimeter, $s = 260/2 = 130$ m

Using Heron's formula,

Area of triangular field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{130(130-120)(130-80)(130-60)}$$

$$= \sqrt{130 \times 10 \times 50 \times 70}$$

$$= 100\sqrt{13 \times 5 \times 7}$$

$$= 100\sqrt{455} = 100 \times 21.33 = 2133 \text{ m}^2$$

33. Find the value of a and b so that polynomial $p(x) = x^3 - 3x^2 - ax + b$ has $(x + 1)$ and $(x - 5)$ as factors.

Ans. The given polynomial is $p(x) = x^3 - 3x^2 - ax + b$.

If $(x + 1)$ and $(x - 5)$ are the factors of $p(x)$, then by factor theorem, $p(-1) = 0$ and $p(5) = 0$

i.e. $p(-1) = (-1)^3 - 3(-1)^2 - a(-1) + b = 0$

$$\Rightarrow -1 - 3 - a(-1) + b = 0$$

$$\Rightarrow a + b = 4$$

...(i)

$$\text{and } p(5) = 5^3 - 3(5)^2 - a(5) + b = 0$$

$$\Rightarrow 125 - 75 - a(5) + b = 0$$

$$\Rightarrow -5a + b = -50$$

$$\Rightarrow 5a - b = 50$$

...(ii)

Adding (i) and (ii), we get

$$6a = 54$$

$$\Rightarrow a = \frac{54}{6} = 9$$

From (i), we get $9 + b = 4$

$$\Rightarrow b = 4 - 9 = -5$$

Hence, $a = 9$ and $b = -5$.

OR

(a) Prove that: $(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$

(b) Factorise: $x^3 + 3x^2y + 3xy^2 + y^3 - 125$

Ans. (a) $(x - y) + (y - z) + (z - x) = x - y + y - z + z - x = 0$

Hence, from the identity,

if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$, we get

$$(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$$

$$(b) \quad x^3 + 3x^2y + 3xy^2 + y^3 - 125 = (x + y)^3 - (5)^3 \quad [\text{Using } a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3]$$

$$= (x + y - 5) [(x + y)^2 + (x + y)5 + 5^2] \quad [(a^3 - b^3) = (a - b)(a^2 + ab + b^2)]$$

$$= (x + y - 5)(x^2 + y^2 + 2xy + 5x + 5y + 25)$$

34. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m

(i) How many students can sit in the tent if a student, on an average, occupies $\frac{5}{7} \text{ m}^2$ on the ground?

(ii) Find the volume of the cone.

Ans. According to the question, Area of cloth = 165 m^2

Radius of conical tent = 5m

Area covered by 1 student = $\frac{5}{7} \text{ m}^2$

Curved surface area of cone = πrl

Thus, curved surface area of a conical; tent = πrl

$$\Rightarrow 165 = \frac{22}{7} \times 5 \times l$$

$$\Rightarrow l = \frac{165 \times 7}{22 \times 5} = \frac{21}{2} = 10.5 \text{ m}$$

(i)

$$\text{No. of students} = \frac{\text{Area of circular base of a cone}}{\text{Area covered by 1 student}}$$

$$\Rightarrow \text{No. of student} = \frac{\pi r^2}{\frac{5}{7}} = \frac{\left(\frac{22}{7} \times 5^2\right)}{\frac{5}{7}} = 22 \times 5 = 110$$

$$\text{No. of student occupies } \frac{5}{7} \text{ m}^2 \text{ of area} = 110$$

(ii) Height of a cone,

$$r^2 + h^2 = l^2$$

Where, r=radius of a cone

h=height of a cone

l=slant height of a cone

$$\Rightarrow (5)^2 + h^2 = (10.5)^2$$

$$\Rightarrow 25 + h^2 = 110.25$$

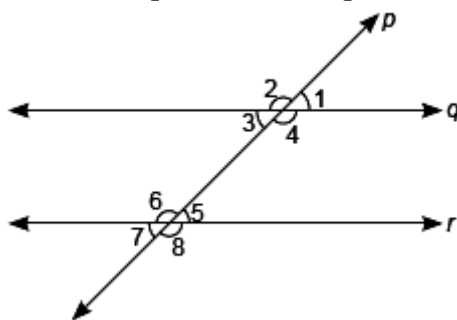
$$\Rightarrow h^2 = 110.25 - 25 = 85.25$$

$$\Rightarrow h = \sqrt{85.25} = 9.23 \text{ m}$$

Volume of a come = $(\frac{1}{3}) \pi r^2 h$

$$\text{Volume of a cone} = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9.23 = \frac{5076.5}{3} = 241.73 \text{ m}^3$$

35. (a) In the given figure, p is transversal to q and r . Given $q \parallel r$ and $\angle 1 = 75^\circ$. Find $\angle 6$ and $\angle 7$.



Ans. We have $\angle 1 = \angle 3$

$$\therefore \angle 3 = 75^\circ$$

$$\text{Now, } \angle 3 + \angle 6 = 180^\circ$$

(Vertically opposite angles)

(Given that $\angle 1 = 75^\circ$)

(Interior angles on the same side of transversal is

supplementary.)

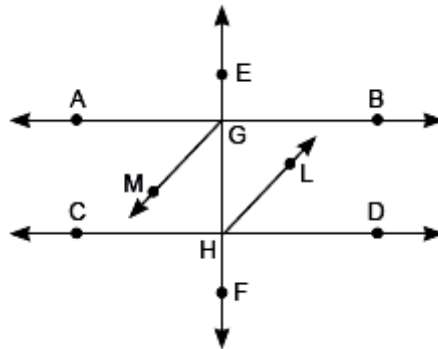
$$\Rightarrow \angle 6 = 180^\circ - \angle 3 = 180^\circ - 75^\circ = 105^\circ$$

Also, $\angle 3 = \angle 7$ (Corresponding angles)

$$\therefore \angle 7 = 75^\circ$$

Thus, $\angle 6 = 105^\circ$ and $\angle 7 = 75^\circ$.

(b) In the given figure, bisector GM and HL of alternate angles AGH and DHG respectively are parallel to each other. Prove that $AB \parallel CD$.



Ans. Since $GM \parallel HL$ and EF is transversal

$$\Rightarrow \angle MGH = \angle GHL \quad (\text{Alternate interior angles}) \dots (i)$$

Now, given GM is angle bisector of $\angle AGH$

$$\Rightarrow \angle MGH = \angle AGM = \frac{1}{2} \angle AGH \text{ and } HL \text{ is also angle bisector of } \angle DHG$$

$$\Rightarrow \angle GHL = \angle DHL = \frac{1}{2} \angle DHG$$

So from (i), we get

$$\frac{1}{2} \angle AGH = \frac{1}{2} \angle DHG$$

$$\Rightarrow \angle AGH = \angle DHG$$

But these are the alternate interior angles formed by the transversal EF with AB and CD .

Hence, $AB \parallel CD$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case Study – 1:

Christmas is celebrated on 25 December every year to remember the birth of Jesus Christ, who Christians believe is the son of God. Santa Claus, also known as the Father of Christmas, is a legendary character originating in western Christian culture and he brings gifts for everyone on Christmas. Let Santa Claus brings 3 chocolates for each child and 2 chocolates for each adult present at the Christmas party at Michael's home along with a Christmas cake. He distributes total 90 chocolates among all.



(a) How to represent the above situation in a linear equation in two variables by taking the number of children as x and the number of adults as y ? If the number of children is 10, then find the number of adults at the Christmas party.

(b) Find the value of k , if $x = 5$, $y = 1$ is a solution of the equation $5x + 7y = k$.

(c) Write the standard form of the linear equation $y - x = 7$.

Ans: (a) Here, the number of children is x and the number of adults is y at the Christmas party. Then, the linear equation in two variables for the given statement is,

$$3x + 2y = 90$$

Given, the number of children is 10.

Therefore, $x = 10$

Put $x = 10$ in the above equation, we get,

$$3(10) + 2y = 90 \Rightarrow 30 + 2y = 90 \Rightarrow 2y = 60 \Rightarrow y = 30$$

Thus, the number of adults at the christmas party is 30.

(b) Given: $5x + 7y = k$ and $x = 5$, $y = 1$

Substituting these values in the given equation, we get

$$5x + 7y = k$$

$$\Rightarrow 5 \times (5) + 7 \times (1) = k$$

$$\Rightarrow k = 25 + 7$$

$$\Rightarrow k = 32$$

(c) The standard form of the linear equation in two variables is $ax + by - c = 0$ where a , b and c are real numbers, and $a \neq 0$ and $b \neq 0$.

Here, $y - x = 7$

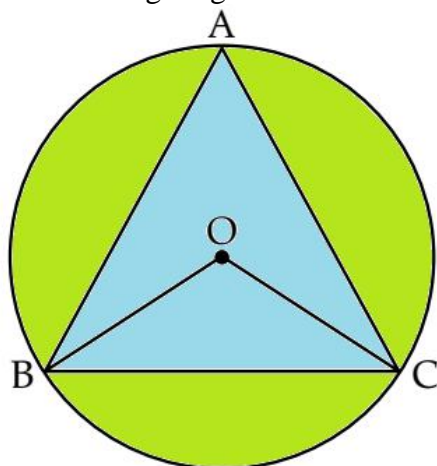
The standard form will be, $y - x = 7$

$$\Rightarrow -x + y - 7 = 0$$

$$\Rightarrow (-1)x + (1)y - 7 = 0$$

37. Case Study – 2:

One triangular shaped pond is there in a park marked by ABC. Three friends are sitting positions at A, B and C. They are studying in Class IX in an International. A, B and C are equidistant from each other as shown in figure given below.



(i) What is the value of $\angle BAC$? (1)

(ii) What will be the value of $\angle BOC$? (2)

OR

(ii) What will be the value of $\angle OBC$? (2)

(iii) Which angle will be equal to $\angle OBC$?

Ans: (i) $AB = BC = AC$ as per the given statement

$\therefore \triangle ABC$ is an equilateral triangle

$\therefore \angle BAC = 60^\circ$ (Angles of a equilateral triangle)

(ii) $\angle BOC = 2\angle BAC$ (\because Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.)

$$\therefore \angle BOC = 2 \times 60^\circ = 120^\circ$$

OR

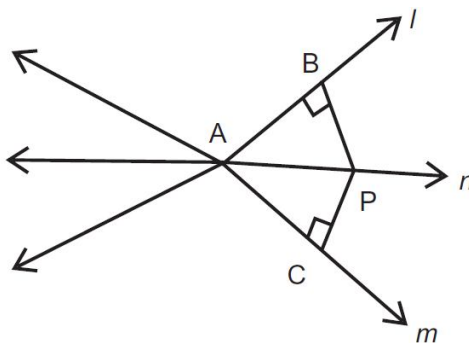
(ii) In $\triangle BOC$, $OB = OC$ (radii)
 $\Rightarrow \angle OBC = \angle OCB$ (\angle s opposite to equal sides are equal)
 $\Rightarrow \angle OBC + \angle OCB + \angle BOC = 180^\circ$ (Angle sum property of triangle)
 $\Rightarrow \angle OBC + \angle OBC + 120^\circ = 180^\circ$
 $\Rightarrow 2\angle OBC = 60^\circ \Rightarrow \angle OBC = 30^\circ$

38. Case Study – 3:

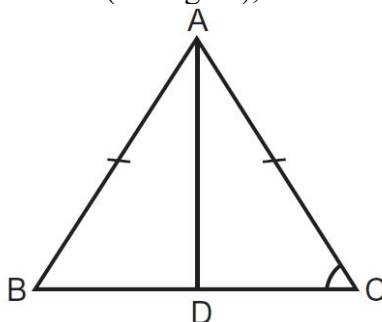
To check the understanding of the students of the class about IX the triangles, the Mathematics teacher write some questions on the blackboard and ask the students to read them carefully and answer the following question.



(a) In figure, P is a point equidistant from the lines l and m intersecting at point A, then find $\angle BAP$.

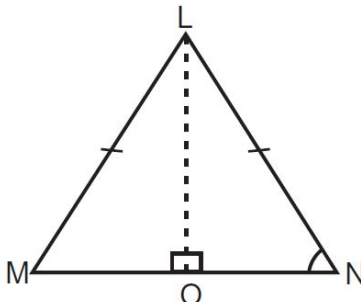


(b) In $\triangle ABC$, if $AB = AC$ and $BD = DC$ (see figure), then find $\angle ADC$.



OR

(b) $\triangle LMN$ is an isosceles triangle, where $LM = LN$ and LO , is an angle bisector of $\angle MLN$, Prove that point 'O' is the mid-point of side MN.



Ans. Ans. (a) Let us consider $\triangle PAB$ and $\triangle PAC$ (as shown in figure).
 Here, we have $PB = PC$ [Perpendicular distance]

$$\angle PBA = \angle PCA \text{ [Each } 90^\circ]$$

$$PA = PA \text{ [Common]}$$

$$\triangle PAB \cong \triangle PAC \text{ [By RHS congruence rule]}$$

$$\text{So, } \angle BAP = \angle CAP \text{ [By CPCT]}$$

$$\text{(b) We have, } AB = AC, BD = CD \text{ and } AD = AD$$

$$\therefore \triangle ABD = \triangle ACD \text{ [By SSS congruence rule]}$$

$$\angle ADB = \angle ADC \text{ [By CPCT]}$$

Since, BDC is a straight line.

$$\therefore \angle ADB + \angle ADC = 180^\circ \text{ [By SSS congruence rule]}$$

$$\Rightarrow 2\angle ADC = 180^\circ$$

$$\Rightarrow \angle ADC = 90^\circ$$

OR

$$\text{(b) Given: } LM = LN \text{ and } \angle MLO = \angle NLO$$

Since $\triangle LMN$ is an isosceles triangle and $LM = LN$

$$\therefore \angle M = \angle N \dots \text{(i)}$$

LO is an angle bisector of $\angle MLN$

$$\angle MLO = \angle NLO \dots \text{(ii)}$$

In $\triangle MLO$ and $\triangle NLO$, $\angle M = \angle N$

$$\text{i.e., } \angle OML = \angle ONL$$

$$LM = LN$$

$$\angle MLO = \angle NLO$$

$$\therefore \triangle MLO \cong \triangle NLO \text{ [By ASA congruence rule]}$$

$$\therefore OM = ON \text{ [By CPCT]}$$

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