

SUBJECT: MATHEMATICS

MAX. MARKS : 40

CLASS : IX

DURATION : 1½ hrs

General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

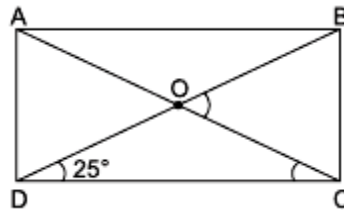
SECTION – A

Questions 1 to 10 carry 1 mark each.

1. A diagonal of a rectangle is inclined to one side of the rectangle at 25° . The acute angle between the diagonals is

(a) 55° (b) 50° (c) 40° (d) 25°

Ans: Given, $\angle ODC = 25^\circ$



Since ABCD is a rectangle, so diagonals are equal.

$$\Rightarrow AC = BD \Rightarrow \frac{1}{2} AC = \frac{1}{2} BD \Rightarrow OC = OD$$

$\Rightarrow \angle ODC = \angle OCD$ (\because Angles opposite to equal sides are equal)

But $\angle BOC = \angle ODC + \angle OCD$ (Exterior angle property)

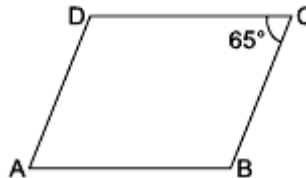
$$\Rightarrow \angle BOC = \angle ODC + \angle ODC \Rightarrow \angle BOC = 2\angle ODC$$

$$\Rightarrow \angle BOC = 2 \times 25^\circ = 50^\circ$$

So, the acute angle between the diagonals is 50° .

\therefore Correct options is (b).

2. In the given figure, ABCD is a parallelogram. If $\angle C = 65^\circ$, then $(\angle B + \angle D)$ is equal to



(a) 180° (b) 115° (c) 155° (d) 230°

Ans. (d) 230°

Since ABCD is a parallelogram, so opposite angles are equal. Thus, $\angle B = \angle D$ and $\angle A = \angle C = 65^\circ$.

Using angle sum property of a quadrilateral, we have

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 65^\circ + \angle B + 65^\circ + \angle D = 360^\circ$$

$$\Rightarrow \angle B + \angle D = 360^\circ - 130^\circ = 230^\circ$$

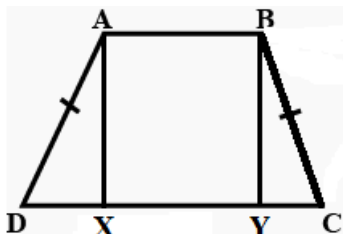
3. Given a quadrilateral ABCD, and diagonals AC and BD bisect each other at P such that $AP = CP$ and $BP = DP$. Also $\angle APD = 90^\circ$, then quadrilateral is a

- (a) rhombus (b) trapezium (c) parallelogram (d) rectangle
 Ans: (a) rhombus

4. Diagonals of a rectangle ABCD intersect at O. If $\angle AOB = 70^\circ$, then $\angle DCO$ is
 (a) 70° (b) 110° (c) 35° (d) 55°
 Ans: (d) 55°

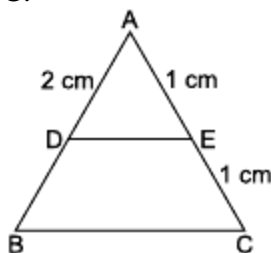
5. Given a trapezium ABCD, in which $AB \parallel CD$ and $AD = BC$. If $\angle D = 70^\circ$, then $\angle C$ will be
 (a) 70° (b) 110° (c) 20° (d) none of these
 Ans. (a) 70°

Draw $AX \perp DC$ and $BY \perp DC$.



In $\triangle AXD$ and $\triangle BYC$, we have
 $AD = BC$ (Given)
 $\angle AXD = \angle BYC$ (Each 90°)
 $AX = BY$ (Distance between parallel sides)
 So, $\triangle AXD \cong \triangle BYC$ (RHS congruence rule)
 Thus, $\angle D = \angle C$ (CPCT)
 Hence, $\angle C = 70^\circ$

6. In the given figure, find BD, if $DE \parallel BC$.



- (a) 2 cm (b) 1 cm (c) 3 cm (d) none of these
 Ans. (a) 2 cm
 $AD = DB$ by the converse of the Mid-point Theorem

7. Four points A,B,C,D are joined together in order and we noticed $AB = CD = 5$ cm and also, AB is parallel to CD then the quadrilateral obtained is a
 (a) rhombus (b) trapezium (c) parallelogram (d) rectangle
 Ans. (c) parallelogram
8. Two angles of a quadrilateral are 60° and 70° and other two angles are in the ratio 8 : 15, then the remaining two angles are
 (a) $140^\circ, 90^\circ$ (b) $100^\circ, 130^\circ$ (c) $80^\circ, 150^\circ$ (d) $70^\circ, 160^\circ$
 Ans: (c) $80^\circ, 150^\circ$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

9. **Assertion (A):** If the diagonal of a parallelogram are equal, then it is a rectangle.

Reason (R): The diagonals of parallelogram bisect each other at right angles.

Ans. (c) Assertion (A) is true but reason (R) is false.

A rectangle is a parallelogram whose diagonals are equal and bisect each other. Here, only Assertion is true.

10. **Assertion (A):** The quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral whose diagonals are perpendicular is a rectangle.

Reason (R): The line segment in a triangle joining the midpoint of any two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side and the quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral is a parallelogram.

Ans: (a) Both A and R are true and R is the correct explanation of A.

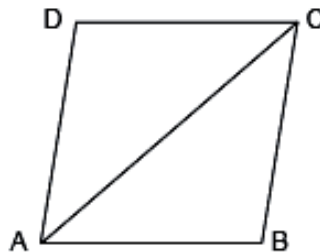
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Prove that a diagonal of a parallelogram divides it into two congruent triangles.

Ans: Given: ABCD is a parallelogram.

To prove: $\triangle ABC \cong \triangle ADC$



Proof: In $\triangle ABC$ and $\triangle ADC$,

$AB = DC$ (Opposite sides of parallelogram)

$BC = AD$ (Opposite sides of parallelogram)

$AC = AC$ (Common)

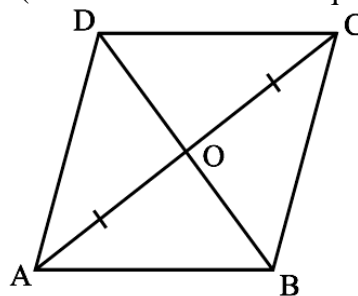
$\therefore \triangle ABC \cong \triangle ADC$ (SSS congruence rule)

\therefore Diagonal AC divides parallelogram ABCD into two congruent triangles ABC and CDA.

12. Show that the diagonals of a rhombus are perpendicular to each other.

Ans. Consider the rhombus ABCD (see the below figure).

We know that $AB = BC = CD = DA$ (Sides of rhombus are equal)



Now, in $\triangle AOD$ and $\triangle COD$,

$OA = OC$ (Diagonals of a parallelogram bisect each other)

$OD = OD$ (Common)

$AD = CD$

Therefore, $\triangle AOD \cong \triangle COD$ (by SSS congruence rule)

This gives, $\angle AOD = \angle COD$ (CPCT)

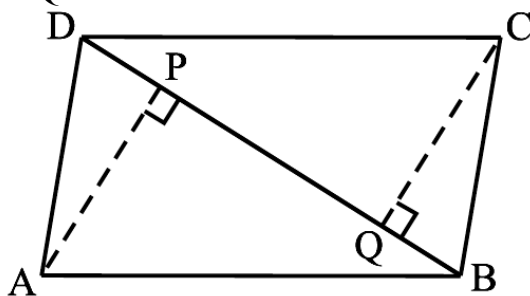
But, $\angle AOD + \angle COD = 180^\circ$ (Linear pair)

$\Rightarrow 2\angle AOD = 180^\circ \Rightarrow \angle AOD = 90^\circ$

So, the diagonals of a rhombus are perpendicular to each other.

13. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see the below). Show that

(i) $\triangle APB \cong \triangle CQD$ (ii) $AP = CQ$



Ans. Given : ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.

(a) In $\triangle APB$ and $\triangle CQD$, we have

$\angle ABP = \angle CDQ$ [Alternate angles]

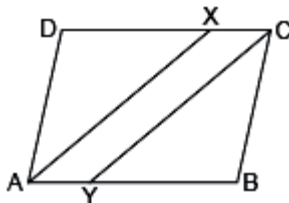
$AB = CD$ [Opposite sides of a parallelogram]

$\angle APB = \angle CQD$ [Each = 90°]

$\therefore \triangle APB \cong \triangle CQD$ [ASA congruence]

(b) So, $AP = CQ$ [CPCT]

14. In the given figure, ABCD is a parallelogram and line segments AX and CY bisect the angles A and C respectively. Show that $AX \parallel CY$.



Ans: AX bisects $\angle A$

$$\therefore \angle XAB = \frac{1}{2} \angle DAB \quad \dots(i)$$

CY bisects $\angle C$.

$$\therefore \angle XCY = \angle DCB \quad \dots(ii)$$

Also, $\angle DAB = \angle DCB$ (Opposite angles of parallelogram)

$$\Rightarrow \frac{1}{2} \angle DAB = \frac{1}{2} \angle DCB$$

$$\Rightarrow \angle XAB = \angle XCY$$

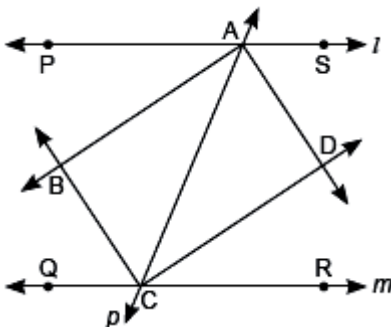
$$\Rightarrow XC \parallel AY \quad \text{(Parts of parallel lines are parallel)}$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Two parallel lines l and m are intersected by a transversal p . Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Ans:



We have $\angle PAC = \angle ACR$ (Alternate interior angles as $l \parallel m$ and p is transversal)

$$\frac{1}{2} \angle PAC = \frac{1}{2} \angle ACR$$

$\Rightarrow \angle BAC = \angle ACD$ (As BA and DC are bisectors of $\angle PAC$ and $\angle ACR$ respectively)

But these are alternate angles. This shows that $AB \parallel CD$

Similarly, $BC \parallel AD$

\Rightarrow Quadrilateral ABCD is a parallelogram. ... (i)

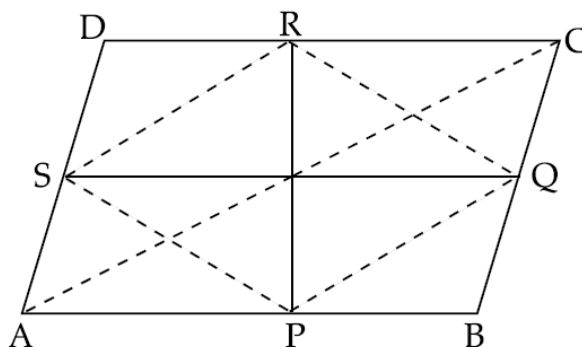
Now, $\angle PAC + \angle CAS = 180^\circ$ (Linear pair axiom)

$$\Rightarrow \frac{1}{2} \angle PAC + \frac{1}{2} \angle CAS = 90^\circ \Rightarrow \angle BAC + \angle CAD = 90^\circ \Rightarrow \angle BAD = 90^\circ \quad \dots (ii)$$

From (i) and (ii), we can say that ABCD is a rectangle.

- 16.** Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Ans. Join SP, PQ, QR, RS and AC



In $\triangle DAC$, $RS \parallel AC$

and $RS = \frac{1}{2} AC$ (Mid-point theorem) ... (i)

In $\triangle ABC$, $PQ \parallel AC$

and $PQ = \frac{1}{2} AC$ (Mid-point theorem) ... (ii)

From (i) and (ii), we get

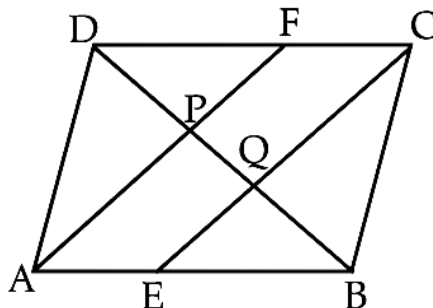
$RS \parallel PQ$ and $RS = PQ$

or, PQRS is a parallelogram.

Since, diagonals of a parallelogram bisect each other.

\therefore PR and QS bisect each other.

- 17.** In the figure, ABCD is a parallelogram. E and F are the mid-points of sides AB and CD respectively. Show that the line segments AF and EC trisect the diagonal BD.



Ans. According to the question, E and F are the midpoints of sides AB and CD.

$\therefore AE = EB$ and $CF = FD$

In the parallelogram opposite sides are equal, so $AB = CD$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} CD$$

$\therefore AE = CF$

Again, $AB \parallel CD \Rightarrow AE \parallel FC$

Hence, AECF is a parallelogram.

In $\triangle ABP$, E is the mid-point of AB and $EQ \parallel AP$.

\therefore Q is the mid-point of BP. (By converse of mid-point theorem)

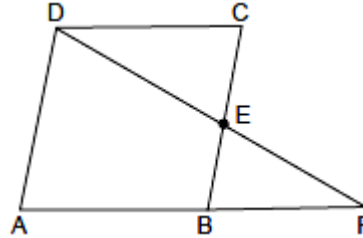
Similarly, P is the mid-point of DQ.

$\therefore DP = PQ = QB$

\therefore Line segments AF and EC trisect the diagonal BD.

OR

ABCD is a parallelogram and E is the mid-point of side BC. DE and AB on producing meet at F. Prove that $AF = 2AB$.



Ans: In $\triangle DCE$ and $\triangle BFE$,

$CE = EB$ (E is mid-point of BC)

$\angle DCE = \angle FBE$ (Alternate interior angles as $CD \parallel AF$)

$\angle DEC = \angle BEF$ (Vertically opposite angles)

$\therefore \triangle DCE \cong \triangle BFE$ (ASA congruence rule)

$\therefore DE = EF$ (CPCT)

\Rightarrow E is mid-point of DF.

In $\triangle ADF$,

E is mid-point of DF. (Proved above)

and $AD \parallel BE$ (As $AD \parallel BC$)

\Rightarrow B is mid-point of AF. (By converse of mid-point theorem)

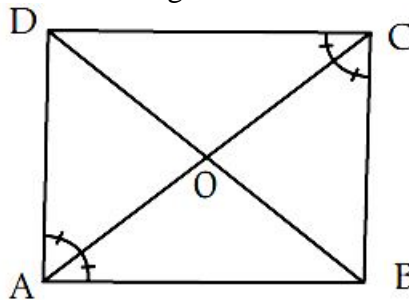
$\therefore AB = BF \Rightarrow AF = 2AB$

SECTION – D

Questions 18 carry 5 marks.

- 18.** ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that: (i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Ans, Given : ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.



(a) In $\triangle ABC$ and $\triangle ADC$, we have

$\angle BAC = \angle DAC$ [Given]

$\angle BCA = \angle DCA$ [Given]

$AC = AC$ [common]

$\therefore \triangle ABC \cong \triangle ADC$ [by ASA congruence rule]

$\therefore AB = AD$ and $CB = CD$ [CPCT]

We know that in a rectangle opposite sides are equal,

i.e., $AB = DC$ and $BC = AD$

$\therefore AB = BC = CD = DA$

Hence, ABCD is a square.

(b) In $\triangle ABD$ and $\triangle CBD$, we have

$AD = CD$ [Side of a square]

$AB = BC$ [Side of a square]

$BD = BD$ [Common]

$\therefore \triangle ABD \cong \triangle CBD$ [by SSS congruence rule]

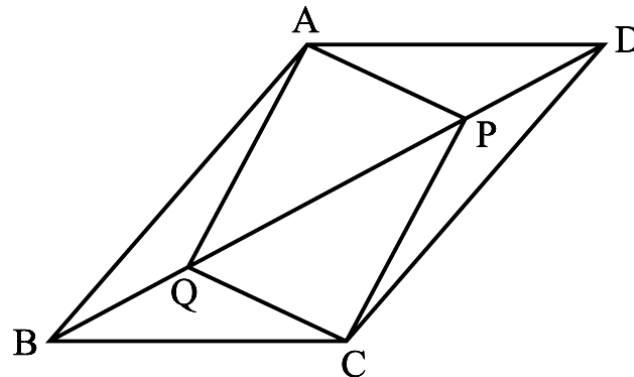
So, $\angle ABD = \angle CBD$ and $\angle ADB = \angle CDB$ [CPCT]

Since, the angles are equal with the diagonal.

Therefore, diagonal BD bisects $\angle B$ as well as $\angle D$.

OR

In parallelogram $ABCD$, two points P and Q are taken on diagonal BD such that $DP = BQ$ (see Figure). Show that :



(a) $\triangle APD \cong \triangle CQB$

(b) $AP = CQ$ (c) $\triangle AQB \cong \triangle CPD$

(d) $AQ = CP$

(e) $APCQ$ is a parallelogram

Ans. (a) In $\triangle APD$ and $\triangle CQB$, we have

$AD = BC$ [Opposite sides of a ||gm]

$DP = BQ$ [Given]

$\angle ADP = \angle CBQ$ [Alternate angles]

$\therefore \triangle APD \cong \triangle CQB$ [by SAS congruence rule]

(b) $\therefore AP = CQ$ [CPCT]

(c) In $\triangle AQB$ and $\triangle CPD$, we have

$AB = CD$ [Opposite sides of a ||gm]

$DP = BQ$ [Given]

$\angle ABQ = \angle CDP$ [Alternate angles]

$\therefore \triangle AQB \cong \triangle CPD$ [by SAS congruence rule]

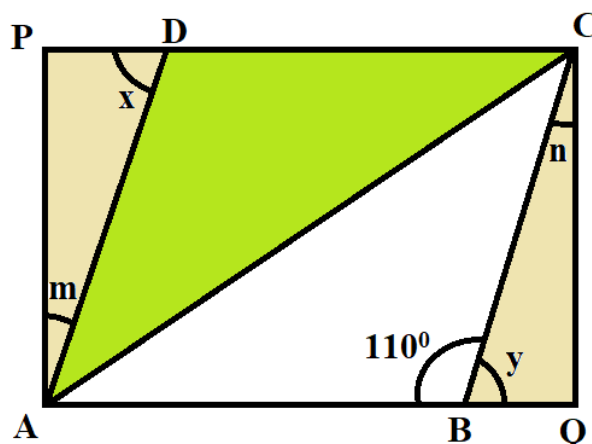
(d) $\therefore AQ = CP$ [CPCT]

(e) We know that if both pair of opposite sides are equal in quadrilateral, therefore, $APCQ$ is a parallelogram.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. In the middle of the city, there was a park $ABCD$ in the form of a parallelogram form so that $AB = CD$, $AB \parallel CD$ and $AD = BC$, $AD \parallel BC$. Municipality converted this park into a rectangular form by adding land in the form of $\triangle APD$ and $\triangle BCQ$. Both the triangular shape of land were covered by planting flower plants.



(a) Show that $\triangle APD$ and $\triangle BQC$ are congruent. (2)

OR

What is the value of $\angle m$? (2)

(b) Which side is equal to PD? (1)

(c) Show that $\triangle ABC$ and $\triangle CDA$ are congruent. (1)

Ans: (a) In $\triangle APD$ and $\triangle BQC$

$AD = BC$ (given)

$AP = CQ$ (opposite sides of rectangle)

$\angle APD = \angle BQC = 90^\circ$

By RHS criteria $\triangle APD \cong \triangle CQB$

OR

In $\triangle APD$

$\angle APD + \angle PAD + \angle ADP = 180^\circ$

$\Rightarrow 90^\circ + (180^\circ - 110^\circ) + \angle ADP = 180^\circ$ (angle sum property of \triangle)

$\Rightarrow \angle ADP = m = 180^\circ - 90^\circ - 70^\circ = 20^\circ$

$\Rightarrow \angle ADP = m = 20^\circ$

(b) $\triangle APD \cong \triangle CQB$

Corresponding part of congruent triangle

side PD = side BQ

(c) In $\triangle ABC$ and $\triangle CDA$

$AB = CD$ (given)

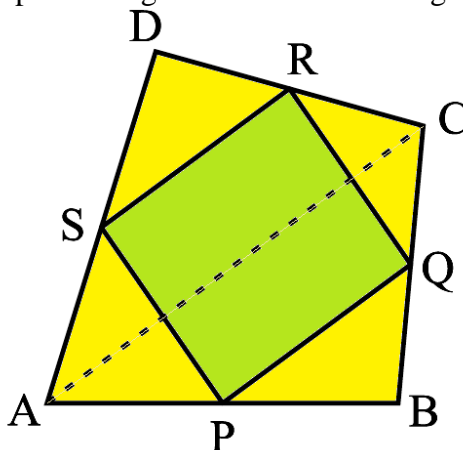
$BC = AD$ (given)

$AC = AC$ (common)

By SSS criteria $\triangle ABC \cong \triangle CDA$

20. Activity-based learning- ensures active engagement of learner with concepts and instructional materials. Learning is hands-on and experiential, providing learners the opportunity of learning through manipulation of materials and objects.

Teachers model the process, and students work independently to copy it. Kumar sir Maths teacher of class 9th wants to explain the properties of parallelograms in a creative way, so he gave students yellow colored paper in the shape of a quadrilateral and then ask the students to make a parallelogram from it by using paper folding and coloured it with green colour.



(a) How can a parallelogram be formed by using paper folding? (2)

(b) (i) If $\angle RSP = 30^\circ$, then find $\angle RQP$. (1)

(ii) If $SP = 3$ cm, Find the RQ. (1)

OR

(b) Find the value of $\angle R$ and $\angle S$ if $\angle P : \angle Q = 1 : 4$. (2)

Ans:

(a) By joining mid points of sides of a quadrilateral one can make parallelogram.

Now, S and R are mid points of sides AD and CD of $\triangle ADC$, P and Q are mid points of sides AB and BC of $\triangle ABC$, then by mid-point theorem $SR \parallel AC$ and $SR = \frac{1}{2}AC$

Similarly $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$.

Therefore $SR \parallel PQ$ and $SR = PQ$

A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Hence PQRS is parallelogram.

(b) (i) $\angle RQP = 30^\circ$ (Opposite angles of a parallelogram are equal.)

(ii) $RQ = 3\text{cm}$ (Opposite side of a parallelogram are equal.)

OR

(b) Since PQRS is a parallelogram, opposite angles are equal.

$\Rightarrow \angle P = \angle R$ and $\angle Q = \angle S$

Also, $\angle P : \angle Q = 1 : 4$

$\Rightarrow \angle P = \angle R = k$ and $\angle Q = \angle S = 4k$

Now, $\angle P + \angle Q + \angle R + \angle S = 360^\circ$ (Angle sum property of quadrilateral)

$\Rightarrow k + 4k + k + 4k = 360^\circ$

$\Rightarrow 10k = 360^\circ$

$\Rightarrow k = 36^\circ$

Hence, $\angle R = k = 36^\circ$ and $\angle S = 4k = 144^\circ$.

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