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PRACTICE PAPER 13 (2024–25)
CHAPTER-09 CIRCLES (ANSWERS)

SUBJECT: MATHEMATICS	**MAX. MARKS : 40**
CLASS : IX	**DURATION : 1½ hrs**

General Instructions:
(i).   **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
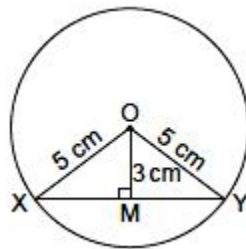(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. Given a circle of radius 5 cm and centre O. OM is drawn perpendicular to the chord XY. If OM = 3 cm, then length of chord XY is
   (a) 4 cm   (b) 6 cm   (c) 8 cm   (d) 10 cm
   Ans: Since the perpendicular drawn from the centre of a circle to a chord bisects the chord, so XM = MY
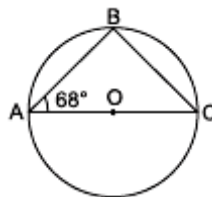

   In right-angled $\triangle$OMX, we have
   $(OX)^2 = (OM)^2 + (XM)^2$ (By Pythagoras theorem)
   $\Rightarrow (5)^2 = (3)^2 + (XM)^2 \Rightarrow (XM)^2 = 25 - 9 = 16 \Rightarrow XM = 4$ cm
   So, the length of chord $XY = 2XM = 2 \times 4 = 8$ cm
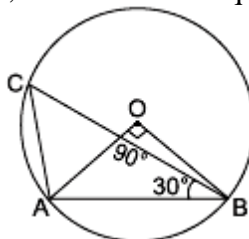 $\therefore$ Correct option is (c).

2. In the given figure, O is centre of the circle, $\angle BAO = 68°$, AC is diameter of the circle, then measure of $\angle BCO$ is

   (a) 22°   (b) 33°   (c) 44°   (d) 68°
   Ans: (a) 22°

3. In figure, $\angle AOB = 90°$ and $\angle ABC = 30°$, then $\angle CAO$ is equal to

(a) 30°　　　　　　　　(b) 45°　　　　　　　(c) 90°　　　　　　　(d) 60°

Ans: We have $\angle ACB = \dfrac{1}{2} \angle AOB = \dfrac{1}{2} \times 90° = 45°$

Using angle sum property of triangle in ΔCAB, we get

∠CAB = 105°

Since OA = OB (∵ Radii of the circle)

⇒ ∠OBA = ∠OAB

Using angle sum property of triangle in ∆AOB, we get ∠OAB = 45°

Now, ∠CAO = ∠CAB – ∠OAB

= 105° – 45° = 60°

∴ Correct option is (d).

4. In the given figure, ∠DBC = 55°, ∠BAC = 45° then ∠BCD is
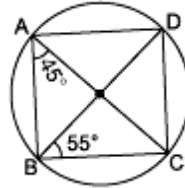
(a) 45°　　　　　(b) 55°　　　　　(c) 100°　　　　　　　(d) 80°

Ans: We have ∠BAC = ∠BDC
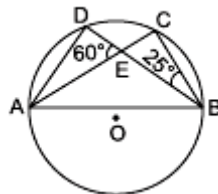
(∵ Angles in the same segment of a circle are equal)

⇒ ∠BDC = 45°

Using angle sum property of triangle in DBDC, we get

∠DBC + ∠BDC + ∠BCD = 180° ⇒ 55° + 45° + ∠BCD = 180° ⇒ ∠BCD = 80°

∴ Correct option is (d).

5. In the given figure, O is the centre of the circle, ∠CBE = 25° and ∠DEA = 60°. The measure of ∠ADB is

(a) 90°　　　　　　　(b) 85°　　　　　(c) 95°　　　　　　　(d) 120°

Ans:

We have ∠DEA = ∠CEB = 60° (Vertically opposite angles)

Using angle sum property of triangle in DCEB, we have

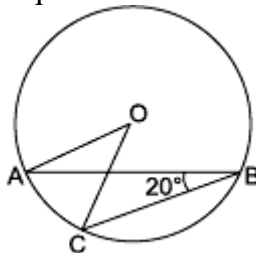∠CEB + ∠CBE + ∠ECB = 180° ⇒ 60° + 25° + ∠ECB = 180° ⇒ ∠ECB = 95°

Now, ∠ADB = ∠ACB (∵ Angles in the same segment of a circle are equal)
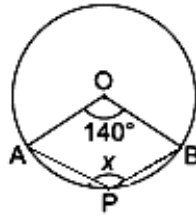
⇒ ∠ADB = 95°

∴ Correct option is (c).

6. In figure, if ∠ABC = 20°, then ∠AOC is equal to:

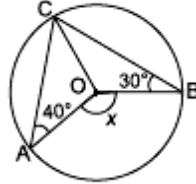(a) 20°　　　　　(b) 40°　　　　　(c) 60°　　　　　　　(d) 10°

Ans: (b) 40°

7. In the given figure, value of x is

   (a) 140°   (b) 70°   (c) 110°   (d) 280°

Ans: (c) 110°

8. In the given figure, O is the centre of the circle. The value of x is


   (a) 140°   (b) 70°   (c) 290°   (d) 210°

Ans: (a) 140°

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. Assertion (A): If O is the centre of the circle as shown in figure, then $\angle CBD = 55°$.
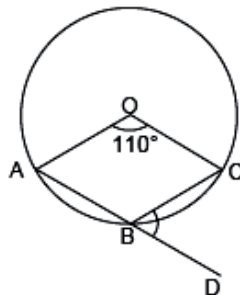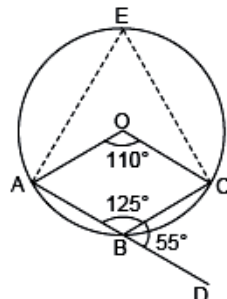

   Reason (R): Exterior angle of cyclic quadrilateral is equal to interior opposite angle

Ans: (a) Both A and R are true and R is the correct explanation of A.

O is the centre of the circle and $\angle AOC = 110°$.
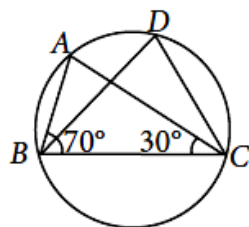
Take point E on major arc, join AE and EC.

$\angle AEC = \dfrac{1}{2} \angle AOC = 55°$ [∵ Arc ABC subtends $\angle AEC$ in the alternate segment and $\angle AOC$ at the centre]

But $\angle AEC = \angle CBD$ [Exterior angle of cyclic quadrilateral is equal to interior opposite angle]

∴ $\angle CBD = 55°$

10. Assertion (A): In the given figure, $\angle ABC = 70°$ and $\angle ACB = 30°$. Then, $\angle BDC = 80°$.

Reason (R): Angles in the same segment of a circle are equal.
Ans: (a) Both A and R are true and R is the correct explanation of A.
Consider the ∆ABC, the sum of all angles will be 180°.
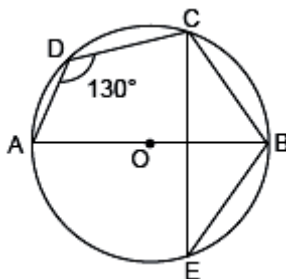∠ABC + ∠BAC + ∠ACB = 180°
⇒ 70° + ∠BAC + 30° = 180°
⇒ ∠BAC = 180° - 100°= 80°
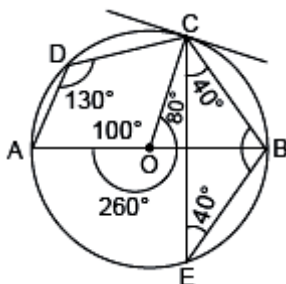We know that angles in the same segment of a circle are equal.
So, ∠BDC = ∠BAC = 80°

SECTION – B
Questions 11 to 14 carry 2 marks each.

11. In the given figure, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.

Ans: Join OC

Reflex ∠AOC = 2 × ∠ADC = 2 × 130° = 260°
∴ ∠AOC = 100°
Now, ∠COB = 180° − 100° = 80°
$\angle CEB = \frac{1}{2} \angle COB$
⇒ $\angle CEB = \frac{1}{2} \times 80° = 40°$
Also, ∠CEB = ∠ECB = 40°   (∵ BC = BE given)
∴ ∠CBE = 180º – (40º + 40º) = 100°

12. In the given figure, ∆ABC is equilateral. Find ∠BDC and ∠BEC.

Ans: ∠BAC = 60°   [∵ ΔABC is an equilateral triangle]

∴ ∠BAC = ∠BDC   [∵ Angles in the same segment of a circle are equal]
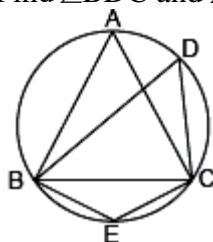
⇒ ∠BDC = 60°

Now, ▢DBEC is a cyclic quadrilateral

∴ ∠BDC + ∠BEC = 180°   [∵ Opposite angles of a cyclic quadrilateral are supplementary]

60° + ∠BEC = 180° ⇒ ∠BEC = 180° − 60° = 120°

13. In the given figure, ABCE is a cyclic quadrilateral and O is the centre of circle. If ∠AEC = 110°, then find (a) ∠ABC (b) ∠ADC
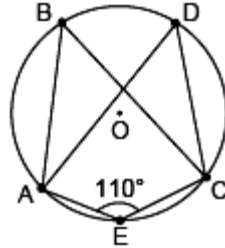
Ans: In cyclic quadrilateral ABCE,

∠ABC + ∠AEC = 180°   (Opp. ∠s of a cyclic quadrilateral)

⇒ ∠ABC + 110° = 180°

∴ ∠ABC = 70°

∠ABC = ∠ADC   (Angles in the same segment)

∴ ∠ADC = 70°

14. If two chords of a circle are equally inclined to the diameter passing through their point of intersection, prove that the chords are equal.

Ans: Two chords AB and AC of a circle are equally inclined to diameter AOD, i.e. ∠DAB = ∠DAC
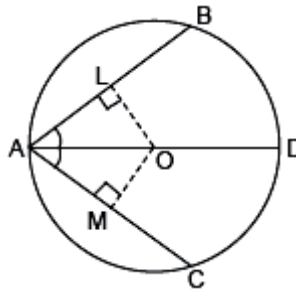
Draw OL ⊥ AB and OM ⊥ AC

In ΔOLA and ΔOMA

∠OLA = ∠OMA   (each 90°)

AO = AO   (common)

∠OAL = ∠OAM   (given)

ΔOLA ≅ ΔOMA   (AAS rule)
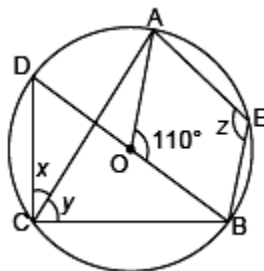
   OL = OM   (CPCT)

   AB = AC   (chords equidistant from the centre are equal)

SECTION – C
Questions 15 to 17 carry 3 marks each.

15. In the given figure, O is the centre of the circle and ∠AOB = 110°, find the value of *x*, *y* and *z*.

Ans: Given: A circle with centre O in which $\angle AOB = 110°$
Now, $\angle AOB = 2\angle ACB = 2y$
(Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$\Rightarrow y = \dfrac{1}{2}\angle AOB = \dfrac{1}{2} \times 110° = 55°$

From the given figure, ACBE is a cyclic quadrilateral
$\therefore \angle ACB + \angle AEB = 180°$ (Sum of opposite angles of a cyclic quadrilateral is 180°)
$\Rightarrow y + z = 180°$
$\Rightarrow z = 180° - 55° = 125°$
Now, $\angle DOA + \angle AOB = 180°$ (Linear pair)
$\Rightarrow \angle DOA = 180° - \angle AOB = 180° - 110° = 70°$
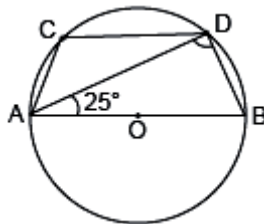Also, $\angle DOA = 2\angle DCA$
(Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$\Rightarrow \angle DCA = \dfrac{1}{2} \angle DOA$

$\Rightarrow x = \dfrac{1}{2} \times 70° = 35°$

$\therefore x = 35°, y = 55°$ and $z = 125°$

16. In the given figure, AB is diameter of the circle with centre O and CD || AB. If $\angle DAB = 25°$, then find the measure of $\angle CAD$.

Ans: AB is the diameter of the circle with centre O and CD || AB. Also, $\angle DAB = 25°$
Now, $\angle ADB = 90°$ [Angle in a semicircle]
$\angle BAD = \angle ADC = 25°$ [Alternate interior angles]
$\therefore \angle BDC = 90° + 25° = 115°$
Now, $\angle BDC + \angle BAC = 180°$ [opp. $\angle s$ of cyclic quadrilateral]
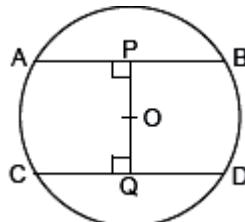$\Rightarrow 115° + \angle BAC = 180°$
$\Rightarrow \angle BAC = 180° - 115° = 65°$
Now, $\angle BAC = \angle BAD + \angle CAD \Rightarrow 65° = 25° + \angle CAD$
$\therefore \angle CAD = 65° - 25° = 40°$

17. In the given figure, O is the centre of the circle with radius 5 cm. OP $\perp$ AB, OQ $\perp$ CD, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ.

Ans: Join OA and OC.
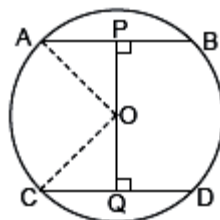
Since OP ⊥ AB, therefore, P will be the midpoint of AB. Similarly, Q will be the mid-point of CD.
In right-angled ∆OAP, $OP^2 + AP^2 = OA^2$

$$\Rightarrow OP^2 = OA^2 - AP^2 = 25 - 9 = 16 \begin{bmatrix} OA = radius = 5cm \\ AP = \dfrac{1}{2}(AB) = \dfrac{1}{2} \times 6 = 3cm \end{bmatrix}$$

∴ OP = 4 cm
Similarly, in ∆OQC,
OC = radius = 5 cm
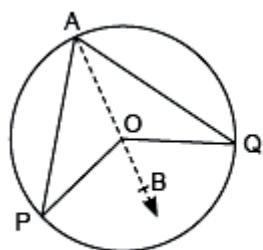CQ = $\dfrac{1}{2}$ (CD) = 4 cm
$OQ^2 = OC^2 - CQ^2 = 25 - 16 = 9$
∴ OQ = 3 cm
∴ PQ = OP + OQ = 4 + 3 = 7 cm
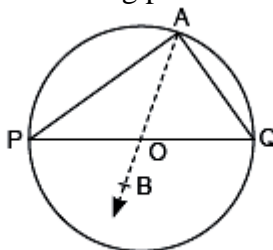
SECTION – D
Questions 18 carry 5 marks.

18. Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
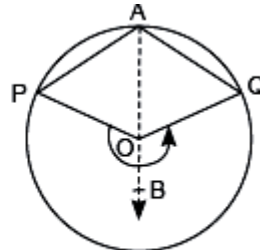
Given : Given an arc PQ of a circle subtending angles POQ at the centre O and ∠PAQ at a point A on the remaining part of the circle.

case (i)   case (ii)   case (iii)

To Prove: ∠POQ = 2∠PAQ
Construction: Join AO and extends it to B.
Proof: Consider three cases
case (i): When arc PQ is a minor arc.
case (ii): When arc PQ is a semicircle.
case (iii): When arc PQ is a major arc.
In all the three cases
Taking ∆AOQ
∠BOQ = ∠OAQ + ∠OQA   (Exterior angle of ∆ is equal to the sum of interior opposite angles)
Also OA = OQ   (radii of circle)
⇒ ∠OAQ = ∠OQA   (Angles opposite to equal sides)
⇒ ∠BOQ = ∠OAQ + ∠OAQ
⇒ ∠BOQ = 2∠OAQ   ...(i)
Similarly ∠BOP = 2∠OAP   ...(ii)
Adding (i) and (ii) we have
∠BOQ + ∠BOP = 2∠OAQ + 2∠OAP = 2(∠OAQ + ∠OAP)
⇒ ∠POQ = 2∠PAQ
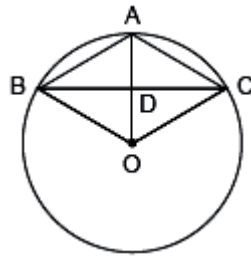Specially for case (iii) we can write reflex ∠POQ = 2∠PAQ

OR

In a circle of radius 18 cm, AB and AC are two chords such that AB = AC = 12 cm. Find the length of chord BC.
Ans: Given: A circle with centre O and two chords AB = AC = 12 cm
Radii = OA = OB = OC = 18 cm

Join OA, OB and OC
OA and BC intersect at D

In $\triangle OAB$ and $\triangle OAC$
OB = OC   (Radii of a circle)
OA = OA   (Common)
AB = AC   (Given)
$\therefore \triangle OAB \cong \triangle OAC$ (SSS congruence rule)
$\Rightarrow \angle OAB = \angle OAC$ ...(i) (CPCT)
In $\triangle DAB$ and $\triangle DAC$
AB = AC   (given)
DA = DA   (common)
$\angle DAB = \angle DAC$ [from (i)]
$\therefore \triangle DAB \cong \triangle DAC$ (SAS congruence rule)
$\Rightarrow \angle ADB = \angle ADC$ ...(ii) (CPCT)
Now $\angle ADC + \angle ADB = 180°$ (Linear pair)
$\Rightarrow \angle ADB = \angle ADC = 90°$
In right $\triangle ADC$ and $\triangle ODC$
$AC^2 = AD^2 + DC^2$ and $OC^2 = OD^2 + DC^2$
$\Rightarrow DC^2 = AC^2 - AD^2$ and $DC^2 = OC^2 - OD^2$
$\Rightarrow AC^2 - AD^2 = OC^2 - OD^2$
$\Rightarrow 12^2 - (18 - OD)^2 = 18^2 - OD^2$
$\Rightarrow 144 - 324 - OD^2 + 36OD = 324 - OD^2$
$\Rightarrow 36OD = 504 \Rightarrow OD = 14$ cm
Now, $DC^2 = OC^2 - OD^2 = 18^2 - (14)^2 = 324 - 196 = 128$
$\Rightarrow DC = 11.31$
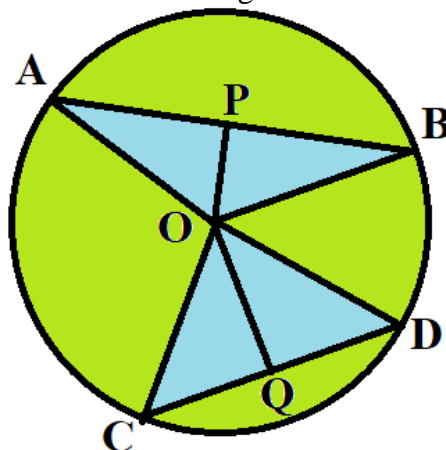Also DC = DB   (Perpendicular from the centre of the circle to the chord bisects the chord)
$\therefore$ BC = 2DC = 2 × 11.31 = 22.62 cm

SECTION – E (Case Study Based Questions)
Questions 19 to 20 carry 4 marks each.

19. Aditya seen one circular park in which two triangular ponds are there whose common vertex is the centre of the park. After coming back to home, he tried to draw the circular park on the paper. He draws a circle of radius 10 cm with the help of a compass and scale. He also draws two chords, AB and CD in such a way that the perpendicular distance from the center to AB and CD are 6 cm and 8 cm respectively. Now, he has some doubts that are given below.

(i) Show that the perpendicular drawn from the Centre of a circle to a chord bisects the chord using any one triangle. (2)
(ii) What is the length of CD? (2)

OR

(ii) What is the length of AB? (2)
Ans: (i) In ΔAOP and ΔBOP
∠APO = ∠BPO (OP ⊥ AB)
OP = OP (Common)
AO = OB (radius of circle)
ΔAOP ≅ ΔBOP
∴ AP = BP (CPCT)
(ii) In right ΔCOQ
$CO^2 = OQ^2 + CQ^2$
$\Rightarrow 10^2 = 8^2 + CQ^2$
$\Rightarrow CQ^2 = 100 - 64 = 36$
$\Rightarrow CQ = 6$
$\Rightarrow CD = 2CQ = 12$ cm

OR

(ii)In right ΔAOB
$AO^2 = OP^2 + AP^2$
$\Rightarrow 10^2 = 6^2 + AP^2$
$\Rightarrow AP^2 = 100 - 36 = 64$
$\Rightarrow AP = 8$
$\Rightarrow AB = 2AP$
$\Rightarrow AB = 16$cm

20. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. The distance between Reshma and Salma and between Salma and Mandip is 6m each. In the given below figure Reshma's position is denoted by R, Salma's position is denoted by S and Mandip's position is denoted by M.
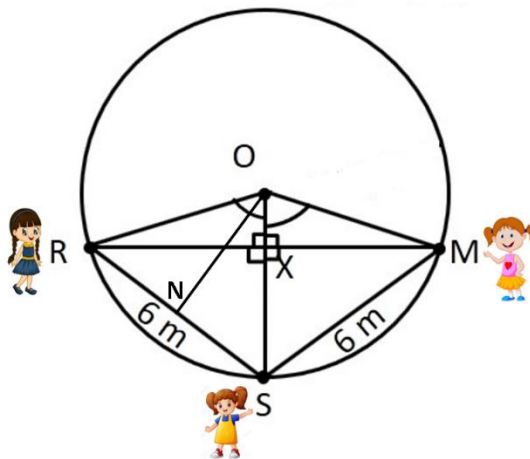
(i) Find the area of triangle ORS. [2]
(ii) What is the distance between Reshma and Mandip? [2]

OR

(ii) If BC is a diameter of a circle of centre O and OD is perpendicular to the chord AB of a circle, show that CA = 2OD. [2]

Ans: (i) $NR = NS = \dfrac{1}{6} = 3$ m

OR = OS = OM = 5m. (Radii of the circle)
In ΔORN, by Pythagoras theorem,
$ON^2 + NR^2 = OR^2$

$\Rightarrow ON^2 + (3^2) = (5)^2$

$\Rightarrow ON^2 = (25 - 9) = 16$

$\Rightarrow ON = 4m$

ORSM will be a kite (OR = OM and RS = SM). We know that diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangle is bisected by another diagonal

$\therefore \angle RXS$ will be of 90° and RX = XM

Area of $\Delta ORS = \dfrac{1}{2} \times ON \times RS = \dfrac{1}{2} \times 4 \times 6 = 12 \text{ m}^2$

(ii) Area of $\Delta ORS = \dfrac{1}{2} \times ON \times RS$

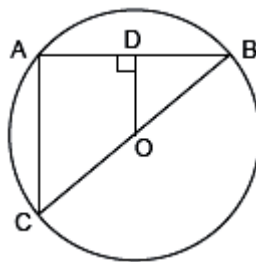$\Rightarrow \dfrac{1}{2} \times RX \times OS = \dfrac{1}{2} \times 4 \times 6$

$\Rightarrow RX \times 5 = 24$

$\Rightarrow RX = 4.8$

$\Rightarrow RM = 2RX = 2(4.8) = 9.6$ Therefore, the distance between Reshma and Mandip is 9.6 m.

OR

(ii)

Since OD $\perp$ AB

$\therefore$ D is the mid-point of AB   (perpendicular drawn from the centre to a chord bisects the chord)

O is centre $\Rightarrow$ O is the mid-point of BC.

In ΔABC, O and D are the mid-points of BC and AB, respectively.

$\therefore$ OD $\parallel$ AC and OD $= \dfrac{1}{2}$ AC   (mid-point theorem)

$\therefore$ CA = 2OD