Chennai Mathematical Institute

Time Series Analysis (Endsem Exam)

December 18, 2021

Marks: 30 **Time: 30 Minutes (10:00 AM to 10:30 AM)**

Important Instructions

This is a multiple choice question paper with possibly more than one correct answer to a question. The marks will be given only for the choice of **all correct** answers to a question. No partial marks for incomplete choice of correct answers.

Please ensure that your answers reach the email ID **mas1221@gmail.com** by not later than **10:35 am**. Submissions beyond 10:35 am will be given **zero** marks as the endsem score.

Send your answer sheet as a PDF with the file name **your name endsem.pdf**; for example, **yash jain endsem.pdf**.

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1. Which of the following can be inferred from the $AR\,(1)$ model with intercept $\alpha = 0.8$ and coefficient $\beta = 0.2$?

   (a) The autocorrelation function $\rho(k)$ decays exponentially to zero.

   (b) The variance of the process is equal to 0.04 σ^2, where σ^2 is the variance of the white noise.

   (c) The mean of the process is equal to 1.

   (d) The time series is not stationary. **[2]**

2. Which of the following processes is stationary? Here, $\{\epsilon_t\} \sim WN(0, \sigma^2)$.

   (a) $Y_t = 0.3 + Y_{t-1} + \epsilon_t - \epsilon_{t-1}$, with $Y_0 = 0 = \epsilon_0$

   (b) $Y_t = Y_{t-1} + \epsilon_t - \epsilon_{t-1}$, with $Y_0 = 0 = \epsilon_0$

   (c) An $MA(2)$ process with $\theta_1 = e^{-3}$ and $\theta_2 = 1.01$

   (d) $Y_t = \alpha + \epsilon_t$ **[4]**

3. High frequency data is required to study

   (a) daily volatility
   (b) weekly volatility
   (c) intraday volatility
   (d) monthly volatility **[1]**

4. We say that the $ARCH$ effect is present in a stationary $ARMA$ model if

   (a) the residuals are uncorrelated.
   (b) the squared residuals are uncorrelated.
   (c) the residuals are correlated.
   (d) the squared residuals are correlated. **[2]**

5. Consider the stationary AR (2) model $Y_t = 0.01 + 0.02Y_{t-2} + \epsilon_t$, where $\epsilon_t \sim N(0, 1)$. Let $\{Y_1, Y_2, ..., Y_{100}\}$ be the given data with $Y_{99} = 0.02$ and $Y_{100} = -0.01$. Let FEV denote the forecast error variance. Then

   (a) $\hat{Y}_{101} = 0.0104$ with $FEV = 1$.
   (b) $\hat{Y}_{101} = 0.0098$ with $FEV = 1.0004$.
   (c) $\hat{Y}_{102} = 0.0098$ with $FEV = 1$.
   (d) $\hat{Y}_{102} = 0.0104$ with $FEV = 1.0004$. **[6]**

6. Let $\{Y_t\} \sim ARIMA$ $(1, 3, 1)$; that is, $Y_t^{(3)} = \alpha + \beta Y_{t-1}^{(3)} + \theta\epsilon_{t-1} + \epsilon_t$. Then

   (a) $Y_t = \alpha + (1 + \beta)Y_{t-1} - 3(1 - \beta)Y_{t-2} + 3(1 + \beta)Y_{t-3} - \beta Y_{t-4} + \theta\epsilon_{t-1} + \epsilon_t$.
   (b) $Y_t = \alpha + (3 + \beta)Y_{t-1} - 3(1 + \beta)Y_{t-2} + (3\beta + 1)Y_{t-3} - \beta Y_{t-4} + \theta\epsilon_{t-1} + \epsilon_t$.
   (c) $Y_t = \alpha + (3 - \beta)Y_{t-1} + 3(1 + \beta)Y_{t-2} + (\beta + 1)Y_{t-3} - \beta Y_{t-4} + \theta\epsilon_{t-1} + \epsilon_t$.
   (d) $Y_t = \alpha - (3 + \beta)Y_{t-1} + 3(1 + \beta)Y_{t-2} + (\beta + 1)Y_{t-3} + \beta Y_{t-4} + \theta\epsilon_{t-1} + \epsilon_t$.

   [5]

7. Consider the model, due to Roll, given by $P_t = P_t^* + I_t(\frac{S}{2})$, where $\{I_t\}$ is a sequence of i.i.d. random variables with $P[I_t = +1] = 0.5 = 1 - P[I_t = -1]$, S is the BAS. Suppose that the true price P_t^* is an $MA(1)$ process given by,

$$P_t^* = \epsilon_{t-1} + \epsilon_t,$$

   where $\{\epsilon_t\} \sim WN(0, \sigma^2)$. Also, assume that the two sequences $\{I_t\}$ and $\{\epsilon_t\}$ are independent. Then

(a) $Correlation\ (\Delta P_t, \Delta P_{t-1}) = -\frac{(S^2/4)}{\sigma^2+(S^2/2)}$.

(b) $Correlation\ (\Delta P_t, \Delta P_{t-1}) = -\frac{(S^2/2)}{2\sigma^2+(S^2/4)}$.

(c) $Correlation\ (\Delta P_t, \Delta P_{t-1}) = -\frac{(S^2/4)}{\sigma^2+(S^2/4)}$.

(d) $Correlation\ (\Delta P_t, \Delta P_{t-1}) = -\frac{(S^2/4)}{2\sigma^2+(S^2/2)}$. **[5]**

8. Let $\{Y_t\}$ be a given stationary time series with stationary mean μ, variance δ^2 and the auto-covariance function $\lambda(k)$, $k \geq 0$. Let $\{Y_t^{(1)}\}$ be the first difference series of $\{Y_t\}$. Then $\{Y_t^{(1)}\}$ is stationary with

(a) $V[Y_t^{(1)}] = 2\{\delta^2 + \lambda(1)\}$ and $Cov(Y_t^{(1)}, Y_{t+k}^{(1)}) = \lambda(k) - \lambda(k-1) + \lambda(k+1)$.

(b) $V[Y_t^{(1)}] = \{\delta^2 - \lambda(1)\}$ and $Cov(Y_t^{(1)}, Y_{t+k}^{(1)}) = 2\lambda(k) + \lambda(k-1) - \lambda(k+1)$.

(c) $V[Y_t^{(1)}] = \{\delta^2 + \lambda(1)\}$ and $Cov(Y_t^{(1)}, Y_{t+k}^{(1)}) = \lambda(k) - 2\lambda(k-1) + \lambda(k+1)$.

(d) $V[Y_t^{(1)}] = 2\{\delta^2 - \lambda(1)\}$ and $Cov(Y_t^{(1)}, Y_{t+k}^{(1)}) = 2\lambda(k) - \lambda(k-1) - \lambda(k+1)$.

[5]

THE END