

US Rates

US Vol Primer: A Guide for the Perplexed

Primer

US Rates Volatility – A Guide for the Perplexed

The objective of this primer is to be a reference note on rates volatility for a wide range of investment professionals with differing degrees of experience, from those new to the topic and for more experienced professionals.

We recognize we are being a bit too ambitious. Some parts of this report may be too simple for some, and some too complicated for others. Please feel free to reach out if you have difficulties with some of the material. Feedback is also always highly appreciated. We like to think of this primer as a work in progress, with a greater level of detail and complexity likely to be added over time.

11 March 2024

Rates Strategy
United States

Table of Contents

Contents	2
Rates Volatility Basics	4
The swaption grid and its dynamic	12
Using vol to leverage a macro view	20
Trading volatility as an asset class	30
Appendices	39

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Timestamp: 11 March 2024 06:39AM EDT

Contents

1. [Rates Volatility Basics](#)

- a. Rate options definitions
- b. Equity vs rates volatility
- c. Overview of the rates volatility market
 - Exchange traded / OTC
 - Vanilla / Exotics
- d. Buyers & Sellers
- e. Long vol exposures & Greeks

2. [The swaption grid and its dynamic – 4 rotations](#)

- a. How to identify the main grid rotations (PCA & HPCA)
- b. Broader directionality and the grid first principal component (HPCA1)
- c. 2nd Rotation: Gamma / Vega (HPCA2)
 - Implied vs delivered volatility
- d. 3rd Rotation: Left / Right (HPCA3)
- e. 4th Rotation: Skew and the volatility cube
 - Implied probability distributions

3. [Using volatility to leverage a macro view](#)

- a. Directional duration exposures
 - Limited (spreads & flies) vs. unlimited downside (1x2 & ladders)
 - Another look at skew
- b. Curve exposure (3rd rotation)
 - Bear/Bull-Flatteners/Steeptenars vs. Caps/Floors
- c. Short vs intermediate expiries (2nd rotation)
- d. Conditional spread trades (listed vs OTC vol)
 - Puts/payers and calls/receivers

4. [Trading volatility as an asset class](#)

- a. Event pricing
- b. Delivered volatility and walk vol
- c. Short gamma as yield enhancement ...
 - ... and how it reinforces the range
- d. Long vega as positive carry macro hedge & Formosa issuance
- e. Left vs right side vol (3rd rotation)
- f. Shorter vs longer expiries (2nd rotation)
 - Calendar spreads
 - Forward vol

5. Appendices

- a. Resources
 - Mercury Resources
 - Bloomberg vol cube
 - Bloomberg pricing
 - Flows
 - Swaption flows and SDR
 - Formosa supply
- b. Swaption valuation
 - Normal model & Greeks
 - Black Model
 - Black, Normal & normalized vol
 - SABR model
- c. PDFs and CDFs
- d. Forward Volatility



1. Rates Volatility Basics

The objective for this primer is to be a reference note on rates volatility for a wide range of investment professionals with differing degrees of experience, for those new to the topic to more experienced professionals.

We recognize we are being a bit too ambitious. Some parts may be too simple for some, and some too complicated for others. Please feel free to reach out if you have difficulties with some of the material. Feedback is always highly appreciated, and we will try to improve on it in subsequent versions.

a. Rate options definitions

Let's start with defining what an option is and understanding the relationship between options and volatility. An investor that buys an option is purchasing a financial instrument that gives him the right to buy or sell an underlying asset at a pre-determined price sometime in the future. To define more precisely what this investor is buying we need to know:

Underlying: The underlying asset. This can really be pretty much anything: gold, oil, an index of equities. In this primer we will concentrate on options where the underlying is tied to interest rates, e.g., a Treasury bond, a Treasury futures contract, or an interest rate swap. Indeed, we will focus mostly on the latter.

Expiry: When the option expires and when the investor decides whether to exercise the right to buy or sell the asset.

Strike: The level at which the investor agrees to buy or sell the underlying asset. If this level is the forward price of the underlying asset at the time the option expires, the option is said to be at-the-money (ATM), otherwise the option is said to be out-of-the-money (OTM).

What is an interest rate swap?

Let's break a bit here to define precisely the type of underlying asset we will focus on in this primer. An interest rate (IR) swap is an instrument that exchanges floating rate payments linked to an interest reference rate (SOFR, OIS, LIBOR, etc.) with fixed rate payments. The fixed rate on an IR swap is set to make the present value (PV) of the discounted fixed and floating cash flows (CF) add up to zero at inception.

An investor may decide to pay the fixed rate and receive the floating leg (if he thinks that rates are going to increase, he will want to pay the current rate going forward and receive a floating rate that will increase over time as rates increase) or receive the fixed rate and pay the floating (if he thinks rates will decrease).

Two types of swaptions...

Investors have two choices when buying a rates option:

- They can purchase an option that gives them the right to pay the fixed rate (RTP) on a given IR swap at pre-determined fixed rate (the strike) at some point in the future (expiry) and receive a floating rate, aka, a payer option.
- Or they can choose to buy an option that gives them the right to receive the fixed rate (RTR) with a pre-determined strike and expiry, and pay a floating rate, aka a receiver option.

Options on IR swaps are commonly called swaptions and they exist in two flavors: swaption payers and swaption receivers.

An example of an investor's

Here we focus on a slightly more concrete example. Let's assume our investor decides he wants to:

Buy \$100m notional of a 1y10y swaption payer with an ATM strike

The expiry of the option is in 1y, and the strike on the option is the ATM forward rate, i.e., the expected level for the 10y rate in 1y. Our investor thinks that rates are going higher over the next year and wants an asset that benefits from that scenario.

The cost and the payoff profile of this option ... "there is no free lunch"!

Our investor now needs to understand how much this option will cost. To figure out the price the investor basically needs to use the golden rule of finance: "There is no free lunch" (i.e., the price of the option is going to be the PV of the expected payoff).

The terminal payoff (the expected payoff profile at the expiry of the option) for the 1y10y ATM swaption payer above is represented (abstractly) in Exhibit 1. If the underlying (the 10y swap rate – x-axis) at the expiry of the option is above the option strike (K), our investor should exercise the option (i.e., pay a fixed rate on the swap = option strike and receive a floating that is higher than the option strike).

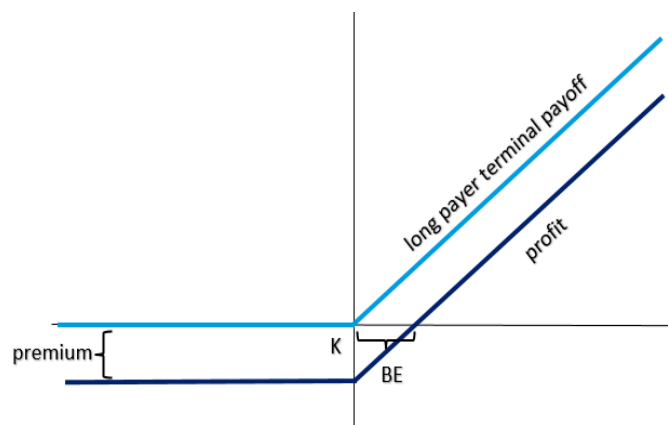
Up to a certain level – defined as the breakeven rate (BE) on the trade – our investor is only clawing back the premium that was paid upfront. Beyond the BE, the investor starts to have positive P&L on the position. Now, common wisdom has it that for each formula in a research note we risk losing half of the audience ... we have delayed the first formula as much as we could, but it is starting to be inevitable. So, here it goes ... the present value of the payoff in Exhibit 1 can be represented by:

$$PV01(T_x; x, y) \cdot N \cdot \{ w \cdot (S_{xy}(T_x) - K) \}_+$$

Where T_x is the option expiry (1y), S_{xy} the spot swap rate (10y rate) observed at time T_x with fixed payment dates $T_{x+1} \dots T_y$, N is the notional invested on the trade (\$100m); K is the option strike; and w is equal to +1 for swaption payers and -1 for receivers. The curly brackets $\{ \}_+$ represent the maximum between zero and its argument (i.e., the option is exercised only when the P&L is positive). Finally, the $PV01$ represents the present value (the value today) of a series of \$1 CFs received at the fixed payment dates $T_{x+1} \dots T_y$.

Exhibit 1: Terminal payoff of a long ATM payer position

Payer swaptions position for a rising rate (x-axis) environment

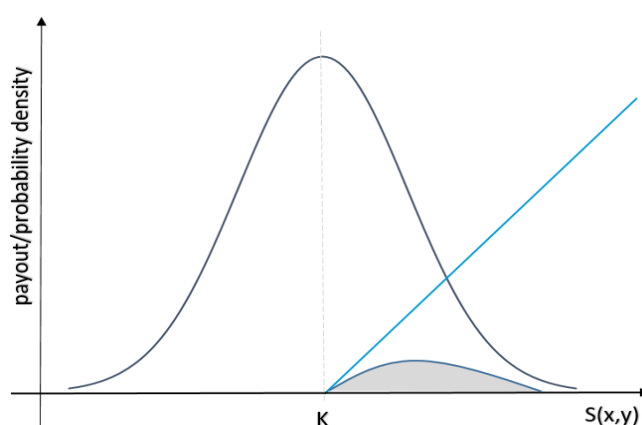


Source: BofA Global Research

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Exhibit 2: Terminal payoff weighted by probability distribution

Value of the option \approx PV of terminal payoff \times probability distribution



Source: BofA Global Research

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Now, Exhibit 1 represents the expected payoff (y-axis) contingent on the level of the underlying (the 10y rate – x-axis) at the expiry of the option (in 1y). However, our investor is trying to value the option today.

To calculate the expected payoff of the trade today, our investor needs to weigh the expected terminal payoff in Exhibit 1, with today's expectation for the distribution of the underlying at the expiry of the option. Exhibit 2 represents this weighing. The value of the option today is the present value of the integral of the grayed-out area.

The price of the option vs. volatility

The intuition that is important to get from Exhibit 2 is that there is a relationship between the value of the option and the probability distribution for the underlying.

The terminal payoff expressed in basis points (bp) is unchanged over time. If the underlying (10y rate) at the expiry of the option is 10bp above the strike (10bp in-the-money), the terminal payoff is 10bp. In dollars, the terminal payoff may change slightly as the PV01 of the underlying changes with the dynamic of rates ($P\&L \text{ in } \$ = \text{bp P\&L} * PV01 * \text{Notional}$), but expressed in bp, the terminal payoff is unchanged.

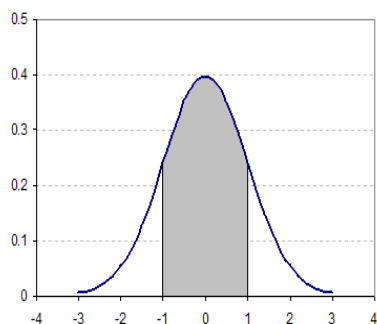
Therefore, the main driver of the value of the option is the shape of the probability distribution of the underlying. The wider the distribution, the higher the value of the option (the higher the premium our investor needs to pay). This is significant, because it means is that we can talk about the value of the option either: (1) in dollar terms; or (2) in terms of the width of the distribution of the underlying. The two express the same thing, and there is a continuous bijective (and therefore monotonic) function that converts one expression of the option value into the other (\$ into bp or bp into \$).

Let's break here again to understand exactly what the distribution of the underlying means using a Gaussian (a.k.a., Normal) distribution as an example. A normal distribution can be represented by two parameters: the mean of the distribution and its width represented by the standard deviation (μ and σ , and respectively). We are interested in understanding the σ , since μ , for the purpose of this discussion, is the forward level of the underlying (the ATM strike). In Exhibit 3–Exhibit 5 we illustrate what different levels of σ represent in terms of the area of the distribution around the mean. In practice we refer to the width of any given distribution by its 1σ value, i.e., the range of values for the underlying that encompasses 68.2% of the expected range of outcomes.

By definition, σ is the volatility of the underlying and we accomplish here what we set out to do in this section: "Define options and understand the relationship between options and volatility." The volatility of the underlying asset is the option value!

Exhibit 3: Normal distribution - 1σ

68.2% of the distribution with 1σ

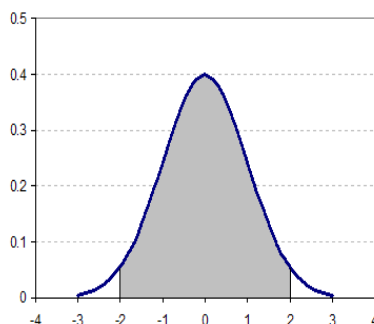


Source: BofA Global Research

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Exhibit 4: Normal distribution - 2σ

95.5% of the distribution with 2σ

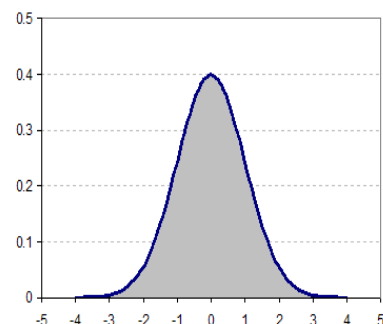


Source: BofA Global Research

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Exhibit 5: Normal distribution - 3σ

99.7% of the distribution with 3σ



Source: BofA Global Research

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In practice, traders, sale professionals and strategists talk about the price of options interchangeably in both basis points vol and cents (value of the option in dollars divided by the notional on the position). Strategists tend to use bp vol more often, while traders tend to use cents (on the other side of the trader's conversation there are clients that want to know exactly how much they need to pay for the position in dollars).

Finally.... An even more concrete example!

Fortunately, our investor can use the Bloomberg's SWPM screen to price his option. If the investor is long 100m notional of a 1y expiry on a 10y underlying payer swaption, with strike $K = 3.727\%$ (see Exhibit 6), the premium that he/she needs to pay for the option can be expressed both in dollars and bp vol terms:

- \$3.432m or 3.432% of notional (\$3.432m premium / \$100m notional)
- 110bp annualized vol, which given a 1y10y forward of c.3.727% corresponds to 68.2% of the expected range of outcomes for the 10y rate in 1y between 2.627% and 4.827% (372.7bp strike \pm 110bp vol)

Exhibit 6: Long 100m 1y10y payer ATM

Exposure: short delta, long vega, long gamma, short theta

Tenor	1y10y	NPV (\$)	3,432,356	DV01	-36,451
Notional	100m	Premium (%)	3.432	Gamma (1bp)	230
Strike	3.727%	Yield Value (bp)	44.1	Vega (1bp)	31,203
ATM Fwd	3.727%	Implied vol (bp)	110.0	Theta (1day)	-4,107

Source: BofA Global Research; Bloomberg

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It is sometimes useful to convert annualized vol into daily vol. Volatility scales with the square root of time (c.252 business days in a 1y expiry). To convert from annualized to daily bp vol we divide the c.110bp annualized vol by the $\sqrt{252} \approx 16$, or 6.93bp/day. The market "expects" the underlying rate will move by 6.93bp per day.

The screen also shows a 44.1bp breakeven (BE) on the position, i.e., with a PV01 of c.\$77.8k on the underlying (10y rate), it takes a c.44.1bp underperformance beyond the ATM strike for the investor to recover the premium paid.

b. Equities vs rates volatility

Since late '21, we have seen a pickup of interest in rates volatility from clients that generally have a background in equity vol. There is a significant overlap between the two fields, but also some differences that are important to understand.

A key difference between equity and rates volatility is that that equity vol is assumed to have a lognormal distribution and expressed in percentage terms (i.e., % vol), whereas rates vol is assumed to be normally distributed (or mixed, as we will see much later in [Appendix B](#)) and expressed in basis point terms (i.e., bp vol).

Let's run through some numbers to understand what this means. Say an equity investor buys a 100m exposure in the S&P 1y forward, and a rates investor buys 100m exposure on the 10y rate 1y forward. These investors want to compare the distribution for the expected performance of the two positions over the next year (let's assume for simplicity that expected returns are zero).

These are not options, just forwards, but from the equity and rates vol markets our investors can gauge the level of implied volatility priced for these underlyings. Let's assume 15.2% implied vol for 1y forwards on the S&P, and 110bp for 1y10y vol. What does this mean for the expected performance of these two positions (we are going to neglect dividends and coupons here also)?

- The 15.2% equity vol means that at the end of 1y, our equity investor should expect his position to be worth between 84.8m and 115.2m with 68.2% probability.
- The 110bp rates vol means that at the end of 1y, our rates investor should expect the 10y rate to be between 2.627% and 4.827% with 68.2% probability. This is useful information, but relatively meaningless to compare to equity performance.



To get to something sensible, our rates investor needs to scale the 110bp vol by the PV01 of the position, or c.77.8k. When he does that, he will realize that the volatility of his portfolio is c.8.56m, i.e., our rates investor should expect his position to be worth between 91.44m and 108.56m with 68.2% probability. In lognormal terms, our rates investor has 8.56% vol on expected returns.

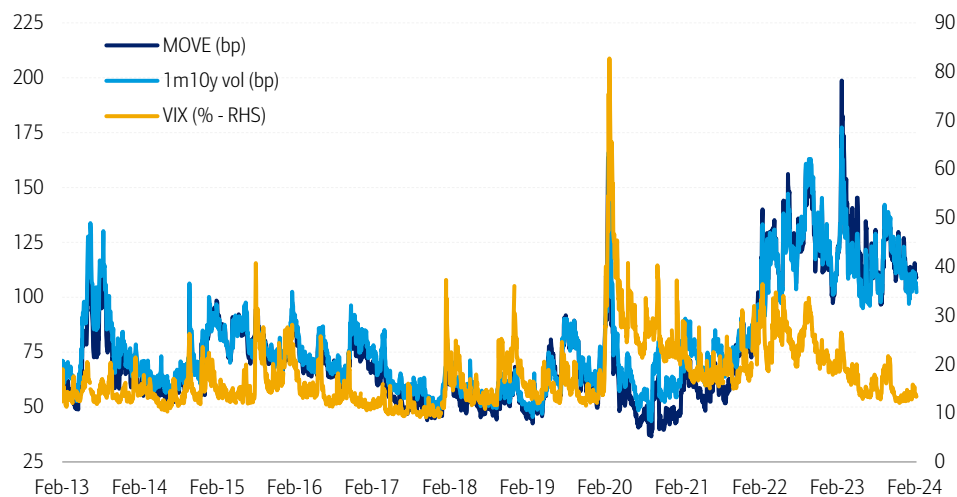
As an aside, our rates investor could have felt tempted to calculate the lognormal vol for rates, which is 110bp/372.7bp, or c.29.5%, which looks a lot higher than the equity vol, but unfortunately is also meaningless to compare performance.

The way to properly compare equity and rates vol, is therefore to reduce rates vol to the volatility of the portfolio returns (scaling by the PV01). Our equity investor can expect 15.2% volatility in dollar return terms, and our rates investor only c.8.56%.

Charts like Exhibit 7 (which compares the MOVE index – a weighted average of the vol for 1m expiry option on 2y, 5y, 10y and 30y SOFR options – with 1m10y swaption vol and the VIX index for equity volatility) are frequently used to illustrate the relative dynamic of equity and rates vol but are to some extent meaningless in our view.

Exhibit 7: Move index (bp), 1m10y swaption vol (bp), and the VIX index (%)

Rates vol at the highest levels in a decade, equity vol are relatively anchored levels



Source: BofA Global Research; Bloomberg

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c. Overview of the rates volatility market

It should now be clear that trading volatility is akin to putting a price on uncertainty and on the distribution of risks around baseline expectations (the forwards).

The rates market offers different ways of trading volatility. There are two distinct platforms to trade rates volatility: listed and over the counter (OTC). In the listed market clients face an exchange, while in the OTC market clients use a broker-dealer. The key difference is that the OTC market offers investors more flexibility, while listed products are more standardized. In general, the OTC market also allows investors to access longer expiries, while in the listed market investors find liquidity in expiries only up to c.6m.

The listed market offers investors a variety of products. For example, options on Fed funds futures which allow investors to trade the uncertainty around the monetary policy trajectory, or options on treasury futures which allow investors to trade the distribution of outcomes across different sectors of the curve.

OTC markets also offer a broad range of products, from the simplest caps & floors to vanilla-style European options (exercised at expiry only) and American options (exercised at any time prior to the expiry, and therefore containing more optionality trading at a premium to European options). There is also the opportunity to trade exotic products like Bermudan options (exercised at discrete periods over the lifetime of the option), Asian options (which generally pay an average of the underlying over a given period and are therefore path dependent), binary options (pay a fixed amount or nothing depending on a set of conditions at expiry), etc.

In this primer we will focus for the most part on OTC European style swaptions, with limited excursions into Bermudan options, caps and floors, and options on UST futures.

d. Buyers and sellers

Understanding the dynamic of supply and demand (sellers and buyers) in the rates vol market, and how different types of investors generally use this market, is critical to understanding the dynamic of the volatility grid – the grid that reflects the price of ATM options across different expiries and underlyings (tails).

Exhibit 8: Buyers and sellers (demand and supply) of volatility across the grid

Different market participants drive the dynamic of different sectors of the volatility grid

	Sub 4y Tails		5-9y tails		10y+ tails	
	Demand	Supply	Demand	Supply	Demand	Supply
Gamma (sub 1y expiries)	Fast money: CB related trades	Fast money: CB related trades	Fast money, esp. curve trades	Fast money, esp. curve trades	Fast money: curve trades	Fast money: e.g. curve, calendar trades
	MBS hedging	Callable Issuance (esp. GSEs)	MBS hedging	Systematic short vol	MBS hedging	Systematic short vol
	Corporates: to cap funding costs		Corporates: rate lock pre issuance	Callable issuance	Corporates: rate lock pre issuance	Convexity selling on purchases of long-dated bonds
Vega (2y+ expiries)	Corporates: to cap or reduce funding costs	Callable issuance	LDI hedging	Callable issuance	LDI hedging	Callable issuance
			Systematic Long vol		Systematic Long vol	

Source: BofA Global Research

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In Exhibit 8 we illustrate schematically how and where different types of investors use the rates volatility market. Before discussing some of these in more detail, however, it is important to note two key distinctions between the rates vol market and a more orthodox supply/demand market like the Treasury market:

- **Vol supply** – Contrary to the Treasury market, where supply is managed by the US Treasury in a relatively regular and predictable manner, there are no regular and predictable sellers in the rates volatility market. Sellers of volatility tend to be opportunistic and reflect the view for some level of mispricing.
- **Vol demand** – Likewise on the demand side, whereas in the Treasury market there is a structural incentive to buy duration (long duration is generally long carry in an upward sloping curve), in the rates vol market long positions are generally negative carry (investors generally buy vol because they think something may happen that is not accurately priced, not because they think forwards will be realized).

Having noted these key differences (which tend to enhance the significance of the role of broker-dealers in the dynamic of the market), let's move on to some of the key buyers and sellers and what drives them (only a few of the ones mentioned in Exhibit 8):

- **ULC** – The top left of the vol grid dynamic is generally driven by fast money accounts expressing views on the Fed policy trajectory or hedging different scenarios for the Fed. In periods where the Fed is on-hold (both at the peak and trough of the cycle, but more significantly on the latter) volatility on the left side of the grid tends to collapse to the cycle lows.
- **URC** – There are a few significant types of flows on the top right corner of the grid, including: (1) programmatic gamma selling where investors (generally asset managers) sell straddles or strangles on short expiries (generally 1-3m) in longer dated tails (mostly 10y) to harvest vol premium and enhance the yield of portfolios; (2) MBS hedging where mortgage portfolios rebalance their vol and duration exposures (mortgage portfolios are short the prepayment option and are therefore negatively convex, i.e., they need to pay fix – short the market – on selloffs and receive fix – long the market – on rallies); and (3) curves and calendar trades where investors position for the potential for the left side of the grid to out/under-perform the right side, or for gamma (expiries up to c.1y) to out/under-perform intermediate expiries (> c.1y and < c.5y) or longer dated expiries (c.5y or beyond).
- **LLC** – At the bottom left corner of the grid, the flows that tend to be more significant are: (1) callable issuance, e.g., callable GSE debt – the issuance of callable bonds allows Agencies to better match the characteristics of their portfolios of mortgage loan – which effectively supply vol to the market; and (2) corporate demand for caps to reduce their funding costs.
- **LRC** – There are a couple of key flows that drive the dynamic of the lower right corner: (1) Formosa issuance, i.e., corporates that issue callable bonds with relatively long lockouts (5y or longer) to meet the demand from Taiwanese life insurance companies; and (2) The hedging of Bermudan portfolios. We will focus on both flows later in this primer when we discuss long vega positions as a positive carry macro hedge. For now, let's just register that Formosa issuance is a net supply of vega to the grid, and that if you are long Bermudan vol and hedge your vega exposure by shorting European swaption vega, you are going to need to buy back some of that European vega on rallies (Bermudan vega collapses on rallies faster than European vega as callable bonds are likely to get called earlier).

e. Long vol exposures & Greeks

In this section we examine the four main greeks of a long option position: delta, vega, gamma and theta.

Delta

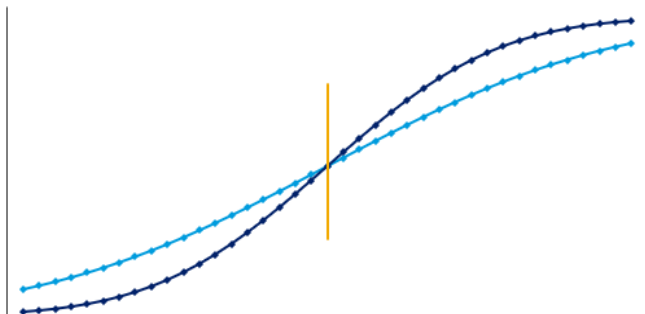
Delta measures the sensitivity of the price of the option (the option premium) to the underlying forward rate. The delta of an ATM option is 50%. For deep in the money payers/receivers the delta approaches +/-1, i.e., for a 1bp move in underlying the value of the option increases by 1bp * the PV01 of the underlying swap rate. For deep OTM payers/receivers, the delta approaches 0. In Exhibit 9 we show schematically the profile for the delta of payers (between 0 and 1 for the y axis) of shorter/longer expiries between deep OTM (left on x-axis) and deep ITM (right on x-axis) forward levels.

Vega

Vega measures the sensitivity of the price of the option to changes in the implied volatility. Vega for payers/receivers decreases as the option gets closer to expiry and increases as the underlying forward gets closer to the ATM strike (see Exhibit 10). Vega also increases with the maturity of the underlying.

Exhibit 9: Delta profile for payers with shorter (dark blue) vs longer (light blue) expiries

Delta 50% for ATM payers, approaches +1 for deep ITM payers (right on x-axis), and 0 for deep OTM payers (left on x-axis)

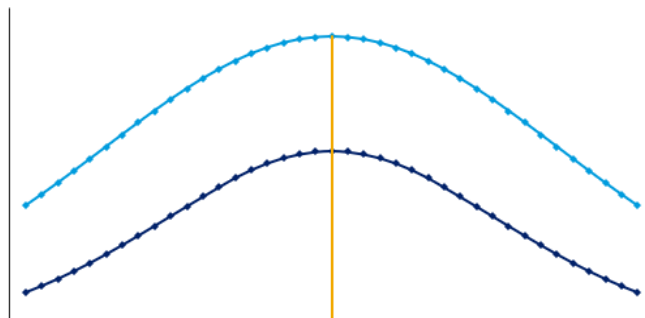


Source: BofA Global Research

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Exhibit 10: Vega profile for options with shorter vs longer expiries

Vega for payers/receivers decreases as the option gets closer to expiry (dark vs light blue) and increases as the underlying fwd gets closer to the ATM strike



Source: BofA Global Research

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Gamma

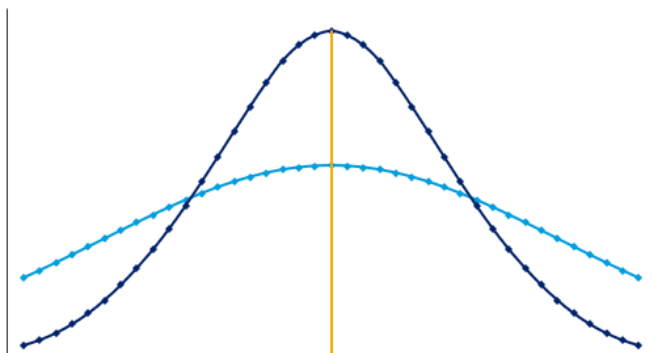
Delta is not constant and changes as the forwards changes. Gamma represents the rate of change of delta as a function of the changes in the forward rate. At the money options have the highest gamma because the delta of ATM options shows the highest sensitivity to changes in the underlying forward (see Exhibit 11). As delta converges to +/-1 for deep ITM payers/receivers (see Exhibit 9), gamma will decrease.

Gamma of shorter expiry options is higher than that of longer expiry options. As expiry approaches the option loses extrinsic value and retains intrinsic value (amount by which the option is ITM). Close to expiry, therefore, delta can have extreme swings (largest gamma) between 0 over OTM levels for the underlying and +/-1 for ITM levels.

Because longer dated expiry options have higher vega exposures, and gamma increases for shorter date expiries, market convention defines expiries up to 1y as the gamma sector, and expiries beyond 1y as the vega sector of the grid (as we did in Exhibit 8).

Exhibit 11: Gamma profile for options w/ shorter vs longer expiries

Gamma highest for ATM & for shorter (dark blue) vs longer (light blue) expiries

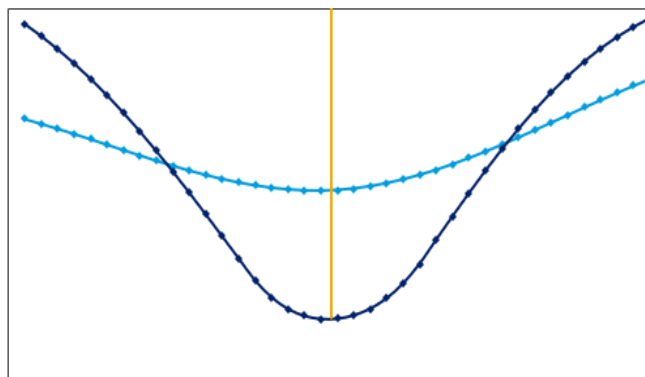


Source: BofA Global Research

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Exhibit 12: Theta profile for options w/ shorter vs longer expiries

Theta highest for ATM & for shorter (dark blue) vs longer (light blue) expiries



Source: BofA Global Research

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Theta

Theta measures the decay in the time value of the option, i.e., the change of the value of the option over time as all other factors remain constant. The loss of time value accelerates as the expiry approaches (see Exhibit 12). Options closer to ATM with shorter expiries have higher theta, as on both accounts there is more urgency for the underlying to move into ITM levels before expiration.

2. The swaption grid and its dynamic

With the preliminaries out of the way, let's focus now on the dynamic of the ATM European swaption grid, i.e., the dynamic of the 2-dimensional grid that contains the prices (in normal vol terms, i.e., in basis points) of options with different expiries on different underlyings (rows = expiries & columns = underlyings or tails).

We show an example of this grid in Exhibit 13. Our old friend 1y10y tenor is sort of the benchmark point on this grid, and you can see 1y10y vol is trading c.110bp.

Exhibit 13: ATM European swaption grid (normal vol in bps)

1y10y normal vol c.110bp

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	108	129	126	121	113	105	86
3M	116	126	124	120	113	106	91
6M	126	129	124	118	113	108	94
1Y	136	132	127	118	114	110	97
2Y	128	124	121	115	111	107	94
3Y	122	119	116	111	107	103	91
4Y	117	114	111	106	103	99	88
5Y	112	109	107	103	99	96	85
10Y	93	92	90	86	84	81	73
15Y	82	80	79	75	73	70	64
30Y	68	67	66	63	62	57	54

Source: BofA Global Research

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Exhibit 14: Segmentation of the rates volatility grid

Static correlation clusters on the ATM volatility surface

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	1	1	1	2	2	2	2
3M	1	1	1	2	2	2	2
6M	1	1	1	2	2	2	2
1Y	1	1	1	2	2	2	2
2Y	3	3	3	4	4	4	4
3Y	3	3	3	4	4	4	4
4Y	3	3	3	4	4	4	4
5Y	3	3	3	5	5	5	5
10Y	3	3	3	5	5	5	5
15Y	3	3	3	5	5	5	5
30Y	3	3	3	5	5	5	5

Source: BofA Global Research

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a. How to identify the main grid rotations

Here we show how the grid generally moves. Now, there's about $7 \times 11 = 77$ tenors on our grid, that is 77 degrees of freedom if these tenors were all uncorrelated, and there could be more had we added more granularity.

Clearly, we not going to try to understand how every single tenor of the grid moves ... fortunately there is a lot of correlation between the 77 tenors and effectively the number of degrees of freedom can be expected to be much (much) lower. Significantly also, not all 77 tenors trade on a regular basis. Effectively, practitioners extrapolate the tenors that trade to construct the broader grid with algos of varying complexity. If 10 tenors trade, and the extrapolation algo has 3 degrees of freedom, effectively there are at best 13 degrees of freedom on our grid (if those 10 tenors were perfectly uncorrelated in their dynamic, which is far from true).

These problems call for the use of dimensional reduction techniques in the analysis of the volatility grid, and that is indeed the standard approach. The starting point for analysts is usually a standard PCA (see [A hitchhikers guide to RV on the UST curve](#)). From there, we can add a higher degree of complexity, for example by considering clusters of correlations in the volatility grid (static – see Exhibit 14 – or dynamic). Drivers for these correlations include the activity of market participants in different sectors of the grid, and the directionality of the vol dynamic on the curve dynamic (see details of an Hierarchical PCA approach in [Understanding Principal Components](#)).

When applied to the vol grid, both these approaches produce a set of orthogonal principal components which correspond to main grid rotations. In the analysis below we use the latter (HPCA) to gain intuition on the main rotations of the ATM grid.

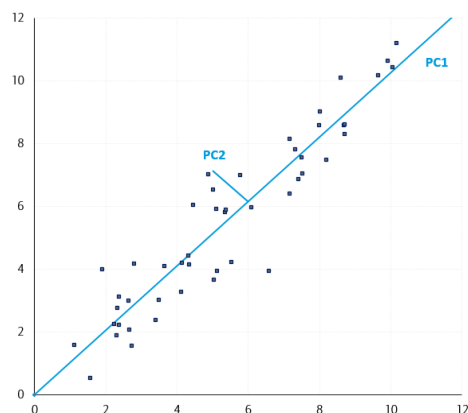
Small parenthesis on Principal components

We won't be exhaustive here in introducing Principal Components Analysis.

Mathematically the principal components are the eigenvectors of the covariance matrix, sorted by decreasing magnitude of the corresponding eigenvalues. Intuitively the principal components are a set of orthogonal vectors that describe the main directions of movement in a dataset (see Exhibit 15 for as simple 2-dimensional data set).

Exhibit 15: Principal components in a simple 2-dimensional data set

The first principal component describes the direction of largest variance in the data set ($y = x$). The second principal component is orthogonal to the first ($y = -x + 12$... the scale is slightly distorted in the chart) and describes a lower degree of variance of the data set.



Source: BofA Global Research

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The intuition behind principal components for a 2-dimensional data set is therefore rather straightforward. Fortunately, the same seems to be the case for a 3-dimensional data set, particularly in the rates space (see Exhibit 16). Unfortunately, as the dimensionality of the data set increases, it becomes progressively more difficult to interpret the meaning of the principal components. In the vol grid, as we noted above, we can easily have 70-80 tenors (dimensions).

Interpretability of the principal components is not the only issue that affects the use of principal components analysis on the volatility grid. Other problems include: (1) the directionality between the volatility grid and the yield curve; and (2) how to make the model responsive to changing market conditions while minimizing the estimation errors.

HPCA helps with some of these issues, particularly in data sets that exhibit clusters of correlations. We use it below, with a static segmentation of the vol grid with 5 clusters (see Exhibit 14): upper left corner (ULC – cluster 1), upper right corner (URC – cluster 2), lower left corner (LLC – cluster 3), intermediate vega (IV – cluster 4) and lower right corner (LRC – cluster 5).

b. Broader directionality and the first principal component

The first hierarchical principal component (HPC1) of the vol grid still describes a parallel rotation of the grid (up or down), as the loadings all have the same sign and broadly similar magnitudes (see Exhibit 17).

The ULC lags slightly in the dynamic represented by the first principal component dynamic recently, as it is generally the case in regimes where the Fed is on-hold (the HPCA was calibrated on one year of data over '23).

The dynamic of HPC1 may be represented by the 1y10y vol dynamic, which is generally seen as a benchmark point on the grid. This helps us illustrate a key feature of the HPC1 dynamic, which is its directionality with the level of rates. Indeed, the main drivers of HPC1 are expected to be the broader macro and risk backdrop, which fundamentally implies a strong directionality between the level of vol and forwards (see Exhibit 18). However, a material portion of this directionality comes from the fact that the normal volatility (σ_N , in basis points) is in fact explicitly directional with the forwards (F):

$$\sigma_N \approx F \cdot \sigma_B$$

where σ_B the lognormal volatility (% vol) as we discuss in [Appendix B](#).

Exhibit 16: The three principal components of a 3-dimensional data set of 2y, 5y and 10y rates also have an intuitive interpretation

The first principal component describes a parallel move on the curve (level). The second principal component describes a steepening/flattening move on the curve (slope). The third principal component describes a change in the concavity of the curve (underperformance/outperformance of the belly versus the wings). The first principal component generally typically explains 80-90% of the variance of the data set

	PCA1	PCA2	PCA3
2yT	51%	77%	-38%
5yT	60%	-1%	80%
10yT	61%	-64%	-47%
Interpretation	Level	Slope	Curvature
Eigenvalues	0.22	0.03	0.00
% Var explained	87%	100%	100%

Source: BofA Global Research

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Exhibit 17: First principal component (HPCA1)

First rotation = parallel (up/down) move on the grid, although the ULC is underrepresented in the dynamic (usually the case with a Fed on-hold))

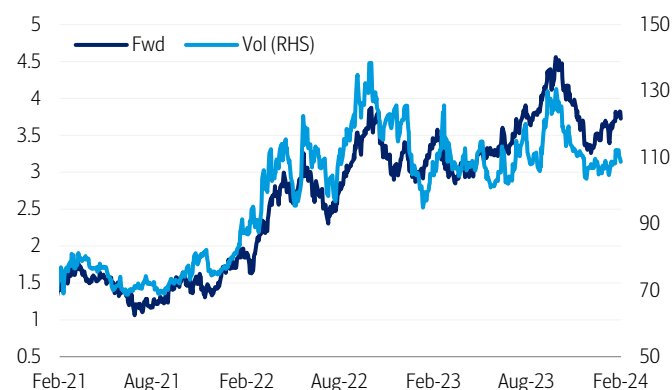
	1y	2y	3y	5y	7y	10y	30y
1m	1%	1%	1%	8%	10%	11%	10%
3m	1%	1%	1%	8%	10%	11%	10%
6m	1%	1%	1%	9%	11%	11%	9%
1y	1%	1%	1%	11%	11%	10%	9%
2y	9%	12%	12%	13%	13%	14%	13%
3y	12%	13%	13%	13%	14%	14%	13%
4y	13%	13%	14%	14%	14%	14%	13%
5y	13%	14%	14%	13%	13%	13%	13%
10y	13%	13%	13%	13%	13%	13%	13%
15y	13%	13%	13%	13%	13%	13%	13%
30y	13%	12%	12%	13%	13%	13%	13%

Source: BofA Global Research

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Exhibit 18: 1y10y vol vs the 1y10 forward rate

1y10y vol highly directional with the forwards



Source: BofA Global Research; Bloomberg

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Why is the directionality of HPCA1 with rates a problem?

The whole idea of applying principal components on the volatility grid is so that we can reduce 70-80 vol tenors to 2-3 principal components that explain 80-90% of the dynamic of the data entire data set (i.e., 80-90% of the information content of the grid).

Generally, one then applies the transpose of the PCA transformation to these 2-3 principal components to obtain the models for the dynamic of the 70-80 tenors in the original data set. The residual of the actual dynamic vs the model corresponds to the roughly 20-30% of the dynamic left behind (not explained by the first 2-3 components), and it is usually seen as a mean reverting residual.

The problem with the directionality of the first principal component of vol on the rates dynamic is that when our investor decides to trade on the mean reverting signals implied by these residuals (e.g., sell vol when the residual between actual – model > 1 StdDev deviation, or buy vol when residual < -1 StdDev the view here is that vols will mean revert to the model implied levels), he/she is trading fundamentally a rate dynamic and not a vol dynamic -- and to trade a rate dynamic, the investor might as well trade rates outright.

A way out of this predicament is to regress the first 2-3 principal components of the vol grid on the principal components of the forwards. These model principal components therefore represent the portion of the vol principal components that is driven by the rates dynamic. We can then apply the transpose of the PCA transformation to these model principal components, to obtain the models for the for the dynamic of the 70-80 tenors in the original data set. The residual of the actual dynamic vs these models is now a purer volatility signal (the residual of the vol dynamic to the models for vol that are implied by the rates dynamic). When our investor trades the mean reverting signals implied by these residuals, he/she is trading more of a vol signal and less a rates signal.

c. Gamma / Vega (HPCA2)

The second hierarchical principal component (HPCA2, i.e., the second main rotation of the volatility grid – see Exhibit 19) corresponds currently to a rotation between gamma (clusters 1 & 2) vs. intermediate expiries and vega (clusters 3, 4 and 5).

Trading the first principal component generally implies expressing a directional view (long or short vol, although to some extent one can also trade a cheap tenor vs a rich tenor based on the first principal component alone).

The trades the best fit with the second principal component, however, are calendar spreads of forward vol positions (the former is to some extent a proxy for the latter). Let's focus on calendar spreads for a moment and leave forward vol positions for later.

Calendar spreads

In a calendar spread our investor is short (or long) a shorter expiry (e.g., 1m10y) vs long (or short) a longer dated expiry (e.g., 1y10y vol). Generally, the notionals on each leg are chosen to make the structure insensitive to parallel moves on the grid (i.e., vega neutral, such that if implied vol in 1m10y goes up/down by the same amount as in 1y10y, the trades position is broadly flat P&L). The idea is that if our investor wanted to trade the grid directionally (i.e., with net vega on the position) he/she should be trading the first principal component... not the second!

Trading calendars spreads implies trading a view on the term structure of volatility (the shape of the vol grid across a given underlying – see Exhibit 20). Our investor:

- may think that the grid is too flat or even inverted, in which case he may want to short gamma (e.g., 1m10y) vs. intermediates (e.g., 1y10y). This is a highly positive carry position since theta is in favor of our investor (he is short the leg with the highest negative theta) ...
- ... or vice versa, the investor may think that the grid is too steep, in which case he may want to buy gamma and sell intermediates, which is a highly negative carry position.

Inversions of the term structure of volatility generally reflect periods of high uncertainty and stress around the macro backdrop ... and high delivered vol (more details on this just below). Because fading the inversion is positive carry, these inversions tend to not be long lasting. The post Covid period was exceptional in this sense as the inversion of the term structure of volatility has been quite persistent.

Exhibit 19: Second principal component (HPCA2)

Second rotation = under/over-performance of gamma vs intermediate expiries and vega

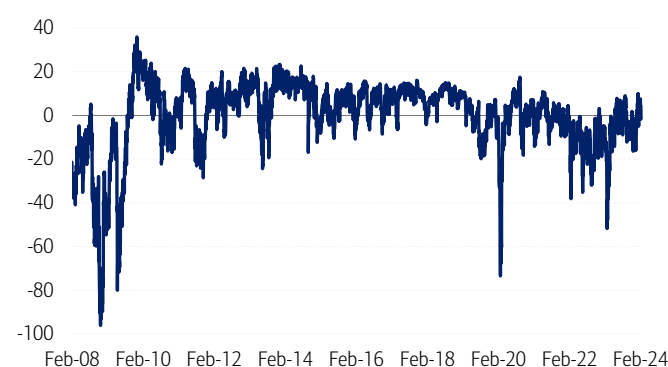
	1y	2y	3y	5y	7y	10y	30y
1m	25%	25%	25%	8%	10%	11%	11%
3m	25%	26%	26%	8%	11%	12%	10%
6m	26%	26%	26%	10%	12%	12%	10%
1y	25%	23%	21%	11%	11%	11%	9%
2y	-1%	-1%	-1%	-3%	-3%	-3%	-3%
3y	-1%	-1%	-1%	-3%	-3%	-3%	-3%
4y	-1%	-1%	-1%	-3%	-3%	-3%	-3%
5y	-1%	-1%	-1%	-6%	-6%	-6%	-6%
10y	-1%	-1%	-1%	-6%	-6%	-6%	-6%
15y	-1%	-1%	-1%	-6%	-6%	-6%	-6%
30y	-1%	-1%	-1%	-6%	-6%	-6%	-6%

Source: BofA Global Research

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Exhibit 20: 1m10y vs 1y10y volatility spread

Historically inversions if the term structure of volatility tend to fade quickly, but the post-covid period as seen a persistence of these inversions



Source: BofA Global Research; Bloomberg

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Implied vs delivered volatility

A large part of the dynamic of gamma vs intermediates is driven by the delivered (aka realized) volatility of the underlying. The delivered vol (e.g., 1m10y delivered vol) is calculated as the StdDev of daily returns of the underlying (e.g., 1m10y forward) over a given historical window (e.g., 21 business days for 1m delivered vol or 63 business days for 3m) ... * sqrt(252) to annualize the volatility.

Ratios or spreads of implied to delivered vol measure the premium (or discount) of implied volatility levels (forward looking vol) relative to the recent delivered volatility of the underlying (backward looking vol – see [Vol in delayed landing scenarios](#) & Exhibit 21 and Exhibit 22). Our investor can look at implied to delivered vol ratios to help make a call on the expectations for delivered going forward. Ratios >1, and a view for relatively anchored delivered volatility around recent levels, create scope to harvest the vol premium in gamma space by shorting gamma vol and hedging the delta of the position. Indeed, some investors sell gamma systematically to enhance the yield of portfolios.

There's quite a bit more on this topic later in this primer. For now, the concepts to retain are: (1) the understanding of how delivered vol drives much of the dynamic of the gamma sector vs intermediates; and (2) the concept of vol premium and a broad understanding of how it can be harvested as a yield enhancement strategy for portfolios.

Exhibit 21: Implied vs. Delivered (21d) Ratio

Ratios of Implied to 21d delivered still < 100% across the grid

	1y	2y	3y	5y	7y	10y	30y
1m	100%	96%	91%	88%	87%	86%	86%
3m	89%	87%	86%	85%	85%	86%	91%
6m	83%	84%	83%	83%	84%	87%	93%
1y	83%	85%	85%	84%	86%	90%	96%
2y	85%	85%	85%	87%	88%	91%	95%

Source: BofA Global Research

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Exhibit 22: 3m Z-Scores for Implied vs. Delivered (21d) Ratio

Ratios of implied/21d delivered fair/cheap across the grid

	1y	2y	3y	5y	7y	10y	30y
1m	-0.2	-0.5	-0.8	-1.1	-1.2	-1.3	-1.3
3m	-0.2	-0.6	-0.8	-1.0	-1.0	-1.1	-0.9
6m	-0.4	-0.6	-0.8	-0.9	-0.9	-0.9	-0.7
1y	-0.4	-0.6	-0.7	-0.8	-0.8	-0.7	-0.5
2y	-0.8	-0.9	-0.9	-0.8	-0.8	-0.7	-0.4

Source: BofA Global Research

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d. Left / Right (HPCA3)

Generally (see [Understanding Principal Components](#)), the third hierarchical principal component (HPC3, i.e., the third main rotation of the volatility grid corresponds to a rotation between the left side of the grid (clusters 1 & 3) vs. the right side (clusters 2, 4 and 5). With the Fed on hold as is the case currently, the third component corresponds to a left vs right dynamic but within the URC of the grid (see Exhibit 23).

Trading the third principal component generally implies expressing a view on the curve dynamic. The curve can only move in one of 4 ways: (1) bull steepen & bear flatten in periods where monetary policy drives most of the dynamic of the curve (i.e., Fed tightening and easing); and (2) bull flatten & bear steepen when the dynamic is led by the belly or backend of the curve and by macro fundamentals rather than Fed policy.

The dynamic of the left side vs right side of the grid (see Exhibit 24) is generally directional to the curve dynamic and can therefore be mapped to different stages of the cycle (i.e., directionality of the third principal component of the vol grid on the second principal component of the forwards).

Exhibit 23: Second principal component (HPCA3)

Third rotation = under/over-performance of the left side vs the right side of the grid

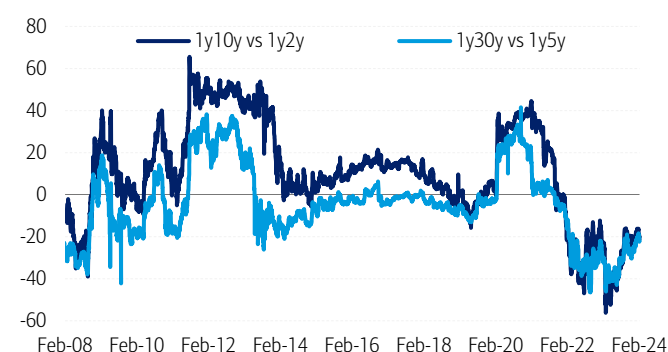
	1y	2y	3y	5y	7y	10y	30y
1m	0%	0%	0%	41%	27%	8%	-23%
3m	0%	0%	0%	40%	23%	-1%	-28%
6m	0%	0%	0%	30%	10%	-11%	-31%
1y	0%	0%	0%	0%	-14%	-23%	-35%
2y	0%	0%	0%	0%	0%	0%	0%
3y	0%	0%	0%	0%	0%	0%	0%
4y	0%	0%	0%	0%	0%	0%	0%
5y	0%	0%	0%	0%	0%	0%	0%
10y	0%	0%	0%	0%	0%	0%	0%
15y	0%	0%	0%	0%	0%	0%	0%
30y	0%	0%	0%	0%	0%	0%	0%

Source: BofA Global Research

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Exhibit 24: Left vs right side dynamic

Spread of 1y10y vs 1y2y vol, and 1y30y vs 1y5y vol



Source: BofA Global Research

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Roadmap for curve and vol over the cycle

The roadmap for the dynamic of the yield curve and volatility over the economic cycle usually goes like this:

- **Early cycle = Bear steepening = Right side vol outperforms:** Fundamentals improve out of the recession, and backend yields push higher even as monetary policy stays on hold. The curve bear steepens, and the right side of the vol grid gets bid even as left side vol stays relatively anchored by on-hold policy.

- **Mid cycle = Bear flattening = Left side vol outperforms:** The economy improves to such an extent that on-hold guidance loses credibility at a 3-5y horizon. The belly un-anchors, and the curve starts to bear flatten first in 5s30s (short the belly vs the wings – see Exhibit 25) and a couple of quarters later 2s10s (investors shift from short the belly to 2s10s bear flatteners at this point – see Exhibit 26).

In vol space, market participants may use the richness of the right-side vs the left-side to pre-empt the shift of curve dynamic to bear flattening. Bear flatteners sell right side vol (payers) vs belly or left side and see significant pickups vs the fwds (more on this later...). Generally, the vol grid starts to see a bid for belly vol before the belly un-anchors.

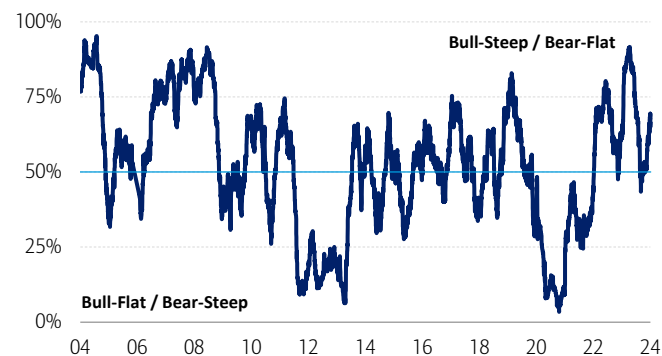
- **Late cycle = flattening with higher bull flattening frequency = Higher vol:** Policy rates overshoot neutral rate expectations and financial conditions tighten. Macroeconomic data dispersion starts to increase, which supports vol broadly, but also starts to support backend yields (more mixed data) even as the monetary policy may continue to tighten. This dynamic increases the frequency of bull flattening moves in the broader flattening dynamic of the late cycle.

Early in the late cycle left side vol is generally rich vs the right. Investors can use this to pre-empt the change in curve dynamic and buy cheap receivers on the right vs rich receivers on the left in conditional bull flatteners (which pickup to the fwds - they sell rich vol on the left to buy cheap vol on the right... again, more on this later).

- **Slowdown / recession = Bull steepening = Lower left side vol:** As the slowdown starts to materialize and recession odds increase, monetary policy shifts first to on-hold and subsequently to outright policy easing (rate cuts). The curve dynamic shifts to bull steepening. In vol, we see underperformance of the left side of the grid vs the right side.

Exhibit 25: 5s30s curve directionality

5y unanchored in '13 and '21 as the Fed signaled policy tightening, supporting vol in the belly of the grid

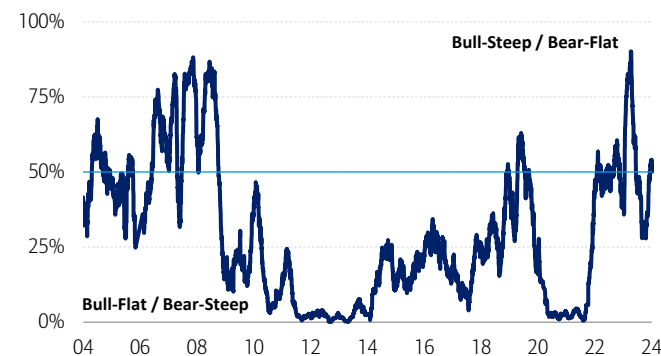


Source: BofA Global Research

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Exhibit 26: 2s10s curve directionality

Frontend of the curve un-anchored after the 5y sector and closer to liftoff in '14 and '21



Source: BofA Global Research

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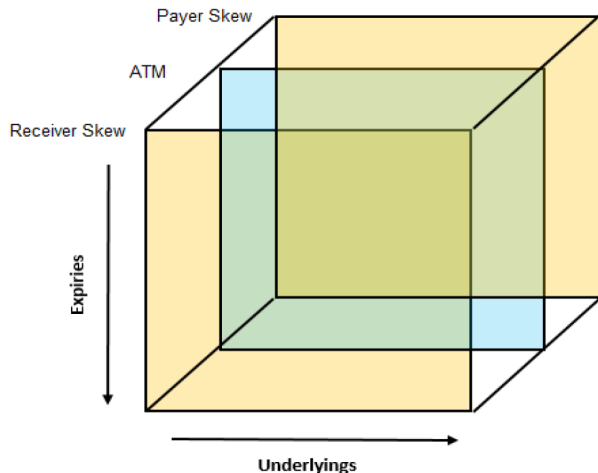
e. Skew and the volatility cube

Let's now abandon the PCA framework... as we noted before, the first 3 principal components explain 80-90% of the broader dynamic of the ATM volatility surface. Around the ATM vol surface, there are a series of layers that contains the vol levels for each tenor of the ATM grid corresponding to option strikes above (payers) and below (receivers) the ATM forwards. This is what is commonly referred to as the volatility cube (see Exhibit 27).

The vol skew for a given tenor on the grid is defined as the spread between that tenor vol for an OTM strikes, and its ATM strike vol. In Exhibit 28 we show the transversal sections of the vol cube for three tenors on the volatility surface, i.e., the shape of the volatility smile for 1m10y, 1y1y and 1y10y tenors.

Exhibit 27: Vol cube

The volatility skew creates a cube around the ATM volatility surface



Source: BofA Global Research

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The rotations in the dynamic of the volatility skew are directional with the dynamic of the underlying (as rates sell off payer skew is bid vs receiver skew, and vice versa) and the level of ATM vol. In fact, as we will see later, the shape of the smile is contingent on the volatility of volatility (see [Appendix B](#)).

There are therefore quite a few issues with RV type analysis on the dynamic of the skew, and we address some of these later in this primer. For now, we will focus on the problem of defining a broad metric for skew across the grid. Here's the issue: we want to compare skew across the grid to find out where it may be rich or cheap. However, it does not make sense to compare +100bp payer skew for 1y10y vol with +100bp payer skew for 1m1y vol as the magnitude a 100bp move relative to the dynamic of these two tenors is very different. One solution to this problem (we will discuss others later) is to look at the OTM strike for each tenor on the grid that corresponds to a 25% delta for the option (for both receivers and payers).

In Exhibit 29 and Exhibit 30 we show those grids for receivers (25% of delta) and payers (usually referred as 75% of delta). Receivers seems to be rich vs payers on the left side of the grid (e.g., 1y1y 25% delta > 75% delta), while payers seem rich vs receivers on the belly and right side of the grid (e.g., 1y10y 75% delta > 25% delta).

Exhibit 29: Skew for 25% of delta

Receivers rich for short expiries on the left/belly on 3m Z-Scores

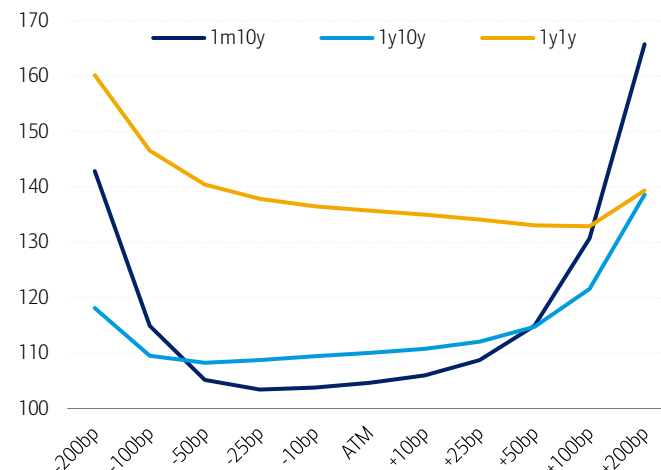
	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	9.8	8.4	6.5	3.0	0.8	-1.2	-1.1
3M	14.8	11.6	9.5	5.0	2.2	-1.0	-1.0
6M	15.5	11.0	9.5	5.6	2.9	-0.4	-1.0
1Y	9.4	6.8	5.6	2.8	1.0	-1.7	-1.9
2Y	-2.0	-2.8	-2.6	-3.2	-3.5	-4.0	-3.7
3Y	-6.2	-6.6	-5.8	-5.6	-5.5	-5.4	-4.7
4Y	-6.6	-6.6	-5.8	-5.8	-5.8	-5.8	-4.9
5Y	-6.8	-6.4	-5.8	-6.0	-6.0	-6.2	-5.1
10Y	-5.7	-5.3	-4.9	-5.5	-5.9	-6.4	-4.9
15Y	-5.1	-4.7	-4.3	-5.0	-5.4	-5.9	-4.7
30Y	-7.4	-6.8	-5.9	-6.2	-6.1	-6.5	-7.7

Source: BofA Global Research

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Exhibit 28: Volatility smile for 1m10y, 1y1y and 1y10y tenors

Vol smile is a transversal section of the vol cube for each tenor of the grid



Source: BofA Global Research

BofA GLOBAL RESEARCH

Exhibit 30: Skew for 75% of delta

Payers Cheap on short expiries on left/belly on 3m Z-Scores

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	-9.1	-7.0	-4.7	-0.8	1.3	3.2	2.7
3M	-13.1	-8.7	-5.9	-0.6	2.1	4.9	4.4
6M	-9.7	-5.3	-3.4	0.8	3.4	6.3	6.1
1Y	-2.8	0.3	1.5	4.2	5.7	7.9	7.3
2Y	7.7	8.4	8.5	8.7	9.1	9.6	8.7
3Y	11.4	11.9	11.6	11.6	11.3	11.4	9.8
4Y	12.8	13.0	12.7	12.7	12.5	12.2	10.3
5Y	13.9	13.7	13.5	13.4	13.2	12.6	10.4
10Y	15.4	15.2	15.1	15.2	14.9	14.4	11.6
15Y	15.4	15.2	15.0	15.1	14.5	14.4	11.8
30Y	17.5	17.2	17.0	16.9	16.8	15.0	12.7

Source: BofA Global Research

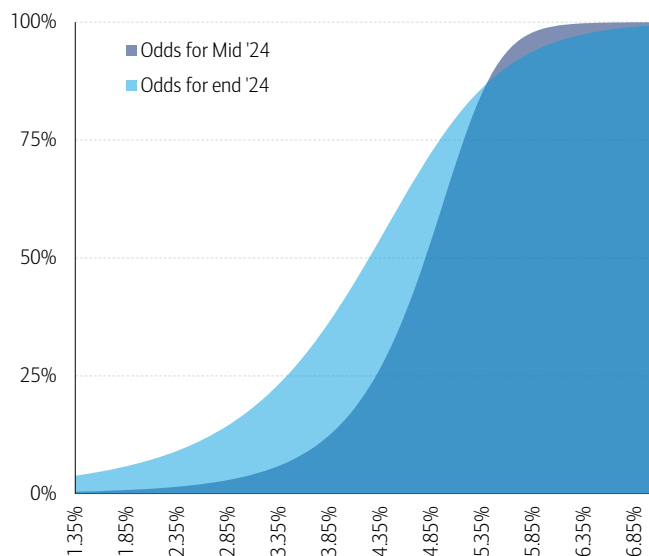
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Implied probability distributions

From every tenor on the volatility grid, we can use the shape of the smile to derive the implied distribution function for the underlying, i.e., the probability of seeing different yield levels for the underlying at the expiry of the option (see Exhibit 31 & Exhibit 32 for the 1y and 10y rate CDFs at mid and end '24, and [Appendix C](#) for a broad description of how these CDFs are derived).

Exhibit 31: CDF for 1y SOFR rates at mid-24 and end-24

1y rate cumulative distribution functions

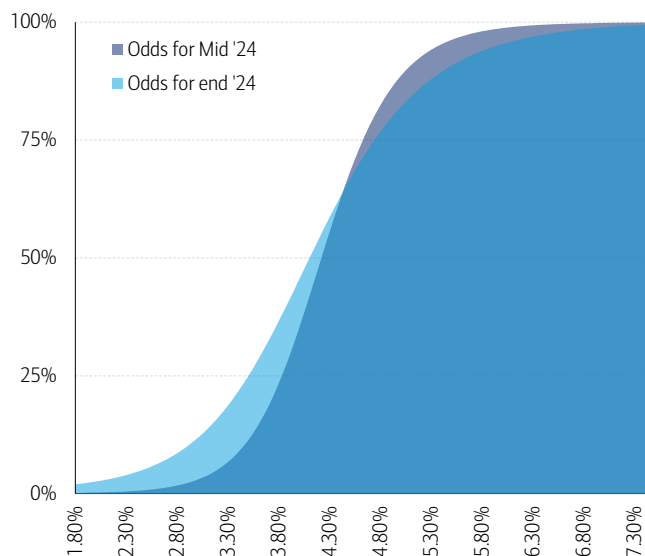


Source: BofA Global Research

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Exhibit 32: CDF for 10y SOFR rates at mid-24 and end-24

10y rate cumulative distribution functions



Source: BofA Global Research

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3. Using vol to leverage a macro view

In this section we are going to start to be a bit more practical (finally!) and discuss some of the basic ways in which options can be used to leverage a given macro view. We map this section around the main grid rotations in the previous section and will start by discussing how to leverage a directional view (first rotation), before moving on to a view on the dynamic of rates short-term vs medium-term (short vs intermediate expiries = second rotation), and the yield curve (left vs the right side of the grid = third rotation).

We will discuss these structures from a payer perspective (and assume our investor has a short bias) but all the discussion can be easily transposed into receiver structures.

a. Directional duration exposures

The most basic directional exposure can be achieved by buying a payer/receiver as an expression of a short/long duration bias. These positions reflect: (1) The potential for the market to underperform/outperform the forwards, i.e., they are short/long delta; and (2) long vol; and (3) have downside limited to the upfront premium.

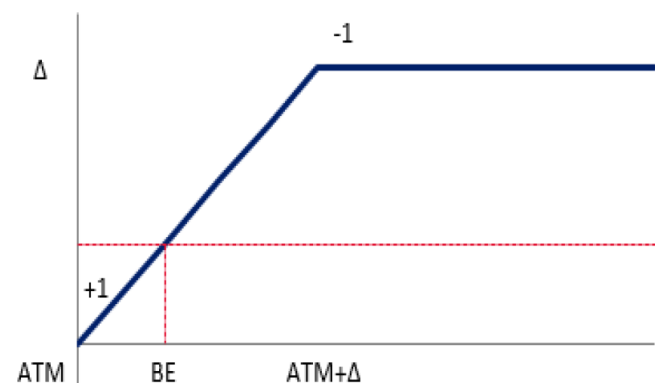
The latter point is important: as we move on in order of complexity it is useful to separate different structures between those that have limited downside, and those that have unlimited downside.

Limited downside

Let's assume that our investor wants to limit the risk on the structures to the premium paid and that: (1) either the investor does not see potential for unlimited upside for duration shorts, or (2) wants to cheapen the cost of the structure. Payer spreads and flies are some of the basic structures that fit within these constraints. We show the terminal payoffs for these two types of structures in Exhibit 33 and Exhibit 34.

Exhibit 33: Terminal payoff for payer spreads

Structure: (1,-1)

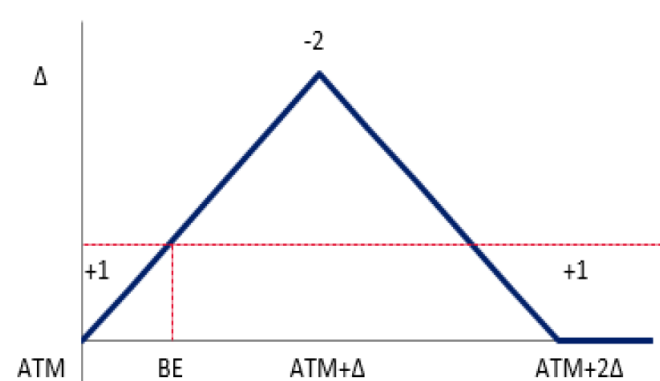


Source: BofA Global Research

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Exhibit 34: Terminal payoff for payer flies

Structure: (1,-2,1)



Source: BofA Global Research

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In a payer spread, our investor buys an ATM payer (may also be an OTM payer) and sells another payer with the same notional – hence a (1,-1) structure – but with a strike higher than the first one (say $ATM+\Delta$). The maximum upside on the position is therefore Δ in basis points (or Δ * the terminal PV01 of the position in dollars), and the investor sees that max upside for yields $> ATM+\Delta$.

In a payer fly, our investor buys an ATM payer (may again be an OTM payer), sells a higher strike payer (say $ATM+\Delta$) with 2x the notional of the first leg, and buys a further OTM payer ($ATM+2\Delta$) with the same notional as the first leg – hence a (1,-2,1) structure. The maximum upside on the position is therefore Δ in basis points (or Δ * the terminal PV01 of the position in dollars), but our investor sees that max upside only for a terminal yield of $ATM+2\Delta$.

Let's go through an example to understand what kind of exposures these structures have at inception: a 100m 1y10y payer spread with strikes ATM/ATM+30bp (i.e., $\Delta=30\text{bp}$). We model each leg independently in Bloomberg and show these models in Exhibit 6 (ATM leg) and Exhibit 35 (ATM+30bp leg). The only difference between the two are the strikes, and the fact that the investor is long payer in the former, and short the payer in the latter (although we model all the legs from the long side), with the same notional on each leg.

Exhibit 35: Long 100m 1y10y payer ATM+30bp

Combine a long exposure in 100m 1y10y payer ATM with a short exposure in 100m 1y10y payer ATM+30bp to structure a payer spread

Tenor	1y10y	NPV (\$)	2,448,066	DV01	-28,962
Notional	100m	Premium (%)	2.448	Gamma (1bp)	225
Strike	4.027%	Yield Value (bp)	31.5	Vega (1bp)	30,115
ATM Fwd	3.727%	Implied vol (bp)	111.9	Theta (1day)	-4,174

Source: BofA Global Research; Bloomberg

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The maximum upside on this position is $30\text{bp} \times \text{the terminal PV01 of the position}$ (c.77.8k currently), or c.2.33m. In Exhibit 36 we show the net exposure for the two legs of the payer spread position. We note that our investor is:

- Short delta in the long ATM leg (short the market), and long delta in the ATM+30bp leg, with a short net delta on the position at inception of c.-7.5k.
- Long vega in the long ATM leg (long an option), and short vega in the ATM+30bp leg, with a long net vega on the position at inception of c.1.1k.

On the net, therefore, our investor is still short delta and long vega in a payer spread, but these exposures are obviously much smaller vs. the ones corresponding to an outright long the ATM payer swaption.

Exhibit 36: Exposures for a long 100m 1y10y payer spread ATM/ATM+30bp

Investor still short delta and long vega (albeit smaller exposure vs outright payer)

NPV (\$)	984,290	DV01	-7,489
Premium (%)	0.9843	Gamma (1bp)	4
		Vega (1bp)	1,089
		Theta (1day)	68

Source: BofA Global Research; Bloomberg

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For context, for a payer fly with 30bp strikes reveals a position that (see Exhibit 38) is net only slightly short delta (by c.-0.8k) and, significantly, now net short vega (by c.-1.7k).

Exhibit 37: Long 100m 1y10y payer ATM+60bp

Combine long ATM + 2x short ATM+30bp + long ATM+60 for a 100m 1y10y payer fly

Tenor	1y10y	NPV (\$)	1,738,418	DV01	-22,286
Notional	100m	Premium (%)	1.738	Gamma (1bp)	202
Strike	4.327%	Yield Value (bp)	22.4	Vega (1bp)	27,303
ATM Fwd	3.727%	Implied vol (bp)	115.4	Theta (1day)	-3,988

Source: BofA Global Research; Bloomberg

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Exhibit 38: Exposures for a long 100m 1y10y payer fly ATM/ATM+30bp/ATM+60bp

The fly is only slightly short delta at inception, and significantly it is short vega

NPV (\$)	274,641	DV01	-814
Premium (%)	0.2746	Gamma (1bp)	-19
		Vega (1bp)	-1,723
		Theta (1day)	254

Source: BofA Global Research; Bloomberg

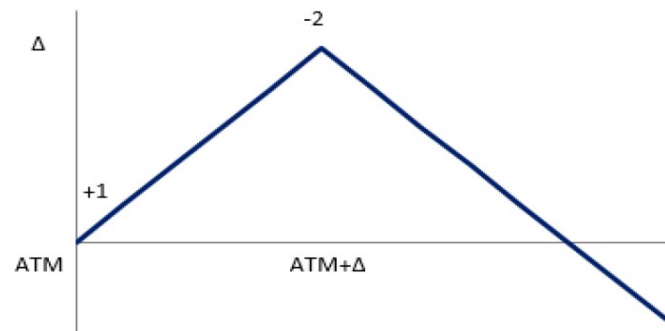
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Unlimited downside

Now let's assume that our investor thinks that the underperformance of the market relative to the forwards is capped beyond a certain level, i.e., he/she is comfortable selling scenarios where yields selloff beyond that level. The investor may be able to tolerate unlimited downside on his structure beyond a certain level. Payer 1x2 or ladders are some of the structures he may be willing to consider in this case. We show the terminal payoffs for these two types of structures in Exhibit 39 and Exhibit 40.

Exhibit 39: Terminal payoff for payer 1x2

Structure: (1,-2)

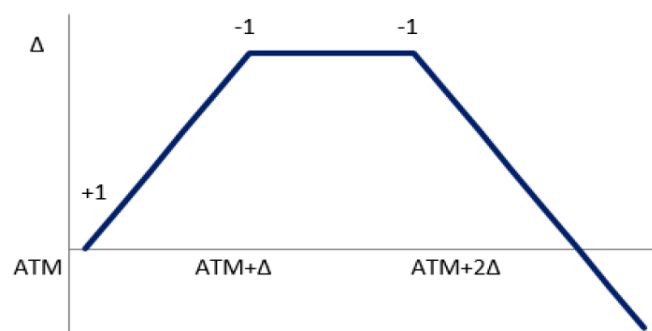


Source: BofA Global Research

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Exhibit 40: Terminal payoff for payer ladders

Structure: (1,-1,-1)



Source: BofA Global Research

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Critically, because these structures have unlimited downside, there is always a gap between strikes (i.e., a Δ) that can make these structures costless.

In a payer 1x2, our investor buys an ATM payer (may also be an OTM payer) and sells another payer with 2x the notional of the first leg – a (1,-2) structure – but with a strike higher than the first one (say $ATM+\Delta$)... such that the premium he pays for the long leg equals the premium he receives for the short leg. The maximum upside on the position is therefore Δ in basis points (or $\Delta \times$ the terminal PV01 of the position in dollars), and the investor sees that max upside only at $ATM+\Delta$. The structure is costless.

In a payer ladder, our investor buys an ATM payer (may again be an OTM payer), sells a higher strike payer (say $ATM+\Delta$) with the same notional of the first leg, and sells a further OTM payer ($ATM+2\Delta$) with again the same notional as the first leg – a (1,-1,-1) structure. The maximum upside on the position is therefore Δ in basis points (or $\Delta \times$ the terminal PV01 of the position in dollars), and our investor sees that max upside for a terminal yield between $ATM+\Delta$ and $ATM+2\Delta$. The structure is costless.

In terms of exposures, for costless strikes on both 1x2 and ladders (c.60bp and 42bp strike gaps, respectively) we see:

- Payer 1x2: net c.-23.5k vega & c.8k delta (the net of the exposures in Exhibit 6 and 2 * the exposures in Exhibit 37 – see Exhibit 41). The buyer of the long 1x2 payer is receiving c.44k on the position which is in the context of the bid/offer for these positions.
- Payer ladder: net c. -22k vega & 7.5k delta (let's leave this one for homework ... calculate exposures for 100m 1y10y payer ATM - $ATM+42bp$ - $ATM+84bp$ for 23 Feb '24 and check that on the net the structure has the exposures in Exhibit 42).

As you can immediately see in both these structures our investor is on the net long delta and short vol at inception... rather the opposite of the exposure he desired when he considered buying a swaption payer (i.e., long vol and short delta). Indeed, these structures incur significant mark-to-market losses in sharp moves close to trade inception in the direction of our macro view (higher vol and higher rates). These structures generally leverage expectations for more gradual moves where the market underperforms the forwards only to a limited degree.

Exhibit 41: Exposures for a long costless 1y10y payer 1x2

Net of 100m 1y10y payer ATM - 200m 1y10y payer ATM+60bp

NPV (\$)	-44,479	DV01	8,122
		Gamma (1bp)	-175
		Vega (1bp)	-23,402
		Theta (1day)	3,869

Source: BofA Global Research; Bloomberg

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Exhibit 42: Exposures for a long costless 100m 1y10y payer ladder

Net of 100m 1y10y payer ATM - 100m 1y10y payer ATM+42bp - 100m 1y10y payer ATM+84bp

NPV (\$)	-21,423	DV01	7,494
Premium (%)	-0.0214	Gamma (1bp)	-165
		Vega (1bp)	-22,312
		Theta (1day)	3,723

Source: BofA Global Research; Bloomberg

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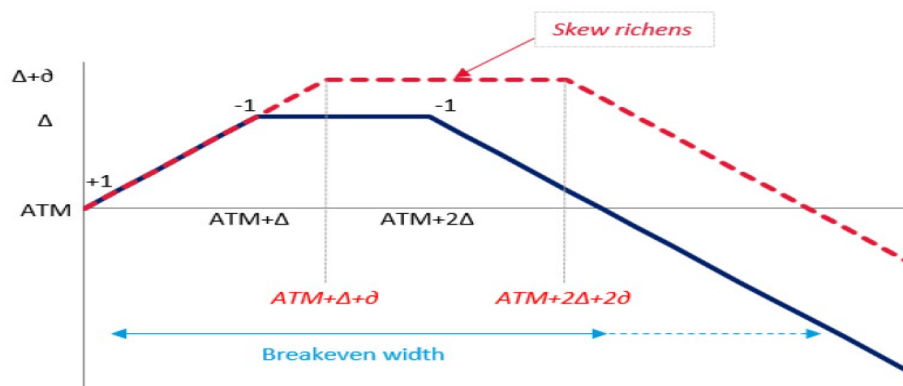
Another look at skew

We come back to the skew to discuss another way to measure skew RV across the grid in a consistent way such we can compare signals for different tenors.

Let's consider the payer ladder structure. As we noted, because these structures have unlimited downside, we can always find a strike gap (i.e., a Δ) such that the structure is costless. Now consider what would happen to that strike gap if the ATM vol stayed constant but OTM vols richened (see answer in Exhibit 43).

Exhibit 43: Breakeven width for payer ladders as skew richens

Breakeven width widens as skew richens and tightens as skew cheapens



Source: BofA Global Research

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If OTM strikes richen relative to the price for the ATM leg, our investor receives more money for the two strikes he is selling than what he pays for the ATM. And, on the net, the investor will receive a premium on the position. To make the structure costless again, he needs to widen the strike gaps from Δ to $\Delta+\delta$, i.e., the breakeven width increases from 3Δ to $3(\Delta+\delta)$. Conversely, had the skew become cheaper the breakeven width would have tightened.

It should be clear now that we can use the dynamic of these breakeven widths over time for each tenor on the vol grid as a proxy for the dynamic of the skew. In Exhibit 44 we show the breakeven widths obtained from costless payer ladders across the grid, and in Exhibit 45 we show the 3m Z-scores for these payer ladders after removing the directionality of the breakeven widths on the level of ATM vol and the forwards. In this way we obtain a consistent metric for payer skew RV across the grid and are able to remove some of the directionality of the skew on the forwards and the level of ATM vol to obtain a purer skew RV signal (the two problems we had identified earlier).

There are several added benefits from this approach:

- This skew RV metric is tied to specific positions. It is therefore unambiguous what position should be used to explore the rich/cheap signals.
- The ratio of the BE width to the implied vol for each tenor measures the probability of falling on the negative P&L side of the structure, e.g., 127bp breakeven for 1y10y (c.3x the 42bp strike gap above – see Exhibit 42 and Exhibit 44) cautioned by 110bp for 1y10y vol (see Exhibit 13) suggest a 1.155 Z-Score or 12.4% implied probability of a selloff beyond the downside breakeven.
- The same approach can then be applied to receiver ladders, to calculate the ratios of payer to receiver BE widths and Z-Scores of these ratios, and in this way gauge RV for payers vs receivers skew (see [Vol in delayed landing scenarios](#)).

Exhibit 44: Breakeven Widths for Costless Payer Ladders

127bp breakeven width for 1y10y payer ladders

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	28	35	35	35	34	32	27
3M	49	57	58	60	59	59	51
6M	81	87	86	86	86	87	77
1Y	131	133	131	127	126	127	114
2Y	203	201	197	190	185	180	161
5Y	326	320	313	303	296	284	245
10Y	427	420	415	407	398	384	327

Source: BofA Global Research

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Exhibit 45: 3m Z-Score Payer Skew (w/o direction w/ fwds & ATM)

Payer skew fair/rich broadly across the grid

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	0.1	0.5	0.7	0.4	0.5	0.5	0.4
3M	0.3	0.0	0.2	0.3	0.4	0.7	1.0
6M	0.4	0.6	0.7	0.5	0.6	0.8	0.9
1Y	0.4	0.6	0.8	0.6	0.3	0.3	0.5
2Y	1.6	1.1	1.3	1.1	0.5	0.2	-0.4
5Y	1.6	0.1	0.1	0.1	0.0	-0.8	0.5
10Y	0.6	-0.5	-0.7	-1.1	-1.3	-1.4	-1.2

Source: BofA Global Research

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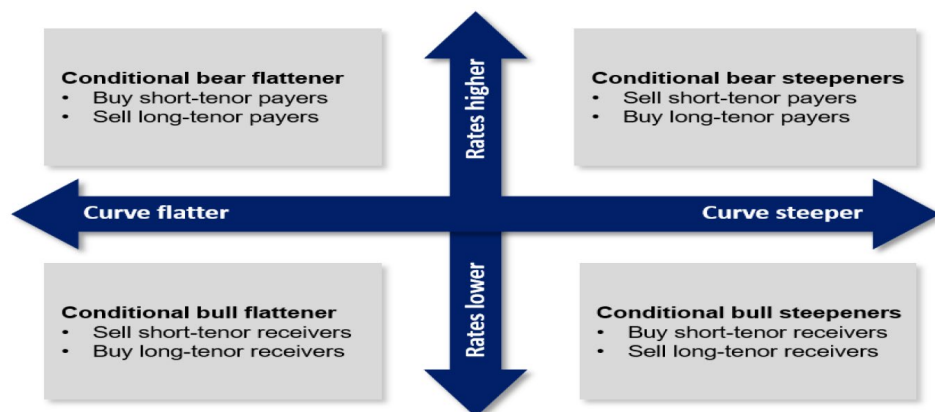
More broadly also, this type of approach is valid for any structure that has unlimited downside (e.g., we can calculate the dynamic of the breakeven widths for payer 1x2 also). It can also be applied to structures with limited downside, but in those we need to define the cost of the structure as a function of the price of the ATM leg (e.g., by calculating daily the strike gaps that recover 70% of the price of the ATM for payer spreads or flies).

b. Curve exposure

Conditional curve trades reflect expectations for the dynamic of the left side vs the right side of the grid (the third principal component discussed in the previous section). The view on the curve and volatility can be leveraged in four different ways (see Exhibit 46)

Exhibit 46: Conditional curve positions

Conditional curve trades isolate portions of the range of outcomes, e.g., a 1y fwd 2s10s steepener is not exposed (terminally) to bullish rate scenarios (both bull steepening and flattening moves) and has the main risk isolated to a bear flattening dynamic (generally driven by a hawkish repricing of policy expectations)



Source: BofA Global research

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- Bear steepener: buying right side payers finance with left side payers (e.g., long 1y10y payers financed by selling 1y2y payers = 1y fwd 2s10s bear steepener)
- Bear flattener: buying left side payers finance with right side payers (e.g., long 1y2y payers financed by selling 1y10y payers = 1y fwd 2s10s bear flattener)
- Bull steepener: buying left side receivers finance with right side receivers (e.g., long 1y2y receivers financed by selling 1y10y receivers = 1y fwd 2s10s bull steepener)
- Bull flattener: buying right side receivers financed with left side receivers (e.g., long 1y10y receivers financed by selling 1y2y receivers = 1y fwd 2s10s bull flattener)

Bear flatteners & bull steepeners are generally frontend / monetary policy driven, while bear steepeners & bull flatteners tend to be more macro driven.

How to structure one of these trades

The key to the positions is whether they benefit from selling rich vol on the short leg while buying cheap vol on the long leg. When that happens, these positions allow an entry level on the curve at a pickup to the forwards.

Let's take an example. Say our investor has a bear steepening bias on the 2s10s curve and is checking whether a 1y fwd 2s10s bear steepener makes sense. To structure the position, the investor must:

- Buy a 1y10y swaption payer with 78k '01 terminal exposure (c.100m notional)
- Sell a 1y2y swaption payer with 78k '01 terminal exposure (c.430m notional)

The key is that if the two options are exercised in a selloff, our investor is left in a curve steepener with zero terminal net delta between the 10y and the 2y leg. We modeled the 100m 1y10y payer leg in Exhibit 6, and the 1y2y leg in Exhibit 47. The position is net receiving c.680k on (receiving 4.11m NPV on the short 1y2y payer and paying 3.43m NVP for the 1y10y payer – see Exhibit 48).

Exhibit 47: 430m 1y2y payer ATM

Short this exposure in a 1y fwd 2s10s bear steepener

Tenor	1y2y	NPV (\$)	4,111,782	DV01	-37,464
Notional	430m	Premium (%)	0.956	Gamma (1bp)	210
Strike	3.837%	Yield Value (bp)	53.0	Vega (1bp)	31,150
ATM Fwd	3.837%	Implied vol (bp)	132.0	Theta (1day)	-4,920

Source: BofA Global Research; Bloomberg

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Exhibit 48: Net exposure of long 100m 1y10y payer ATM & short 430m 1y2y payer ATM

Long position receives c.680k net

NPV (\$)	-679,426	DV01	1,013
		Gamma (1bp)	20
		Vega (1bp)	53
		Theta (1day)	813

Source: BofA Global Research; Bloomberg

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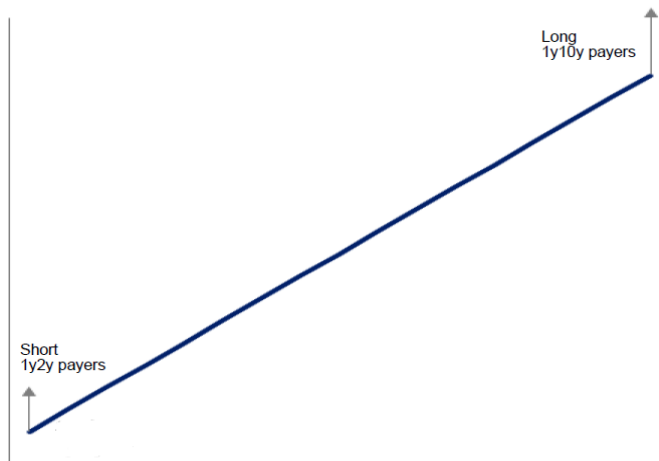
Generally, these structures trade costless so our investor has c.680k to spend on adjusting the strikes on either the long payer (down) or the short payer (up) to make the structure costless. There are two options here. Exhibit 49 represents the shape of the 1y fwd 2s10s curve, our investor has the option to:

- Spend more on the payer being purchased and therefore pushing the strike on the long leg in the money, i.e., lower (see Exhibit 50)
- Receiving less on the payer being sold and therefore pushing the strike on the short leg out of the money, i.e., higher (see Exhibit 51).



Exhibit 49: Representation of the 1y fwd 2s10s curve ATM

When our investor receives on a bear steepener, he has the option to push the strike on the long payer down, or the short payer up

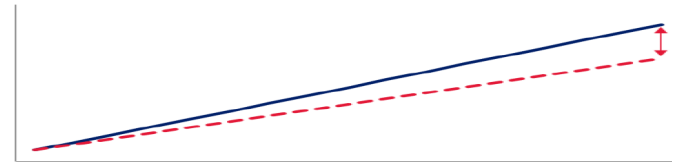


Source: BofA Global research

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Exhibit 50: Adjust strike on 1y10y payer down

Long 10y leg in the money earlier in a selloff

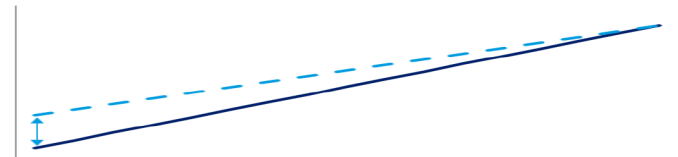


Source: BofA Global research

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Exhibit 51: Adjust strike on 1y2y leg up

Short 2y leg in of the money later in a selloff



Source: BofA Global research

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In the former, our investor sees the long 10y leg in the money earlier in a selloff, while in the latter he sees the short 2y leg in-the-money later in a selloff. Either way, the position picks up to the fwds, i.e., the position locks an entry level for the 2s10s curve flatter than the forwards. However, generally investors are more conservative and risk averse, and the bias is for building some cushion on the position for adverse moves, i.e., adjust the strike on the short leg (1y2y leg in the 1y fwd 2s10s bear steepener) higher.

We model again the 1y2y leg to match the NPV of the 1y10y leg in Exhibit 52. A +17bp in the strike of the short 1y2y payer leg matches the NPV of the long 1y10y ATM payer leg and makes the conditional 1y fwd 2s10s bear steepener costless. We show the exposures for the bear steepener in Exhibit 53. Our investor enters the position at a 17bp pickup to the forwards (enters the bear steepener at a 17bp flatter level relative to the ATM curve – see Exhibit 54).

Exhibit 52: Long 430m 1y2y payer ATM+17bp

A +17bp shift in the short 1y2y payer leg matches the NPV of the long 1y10y ATM payer leg, and makes the conditional 1y fwd 2s10s bear steepener costless

Tenor	1y2y	NPV (\$)	3,474,995	DV01	-33,699
Notional	430m	Premium (%)	0.808	Gamma (1bp)	211
Strike	4.007%	Yield Value (bp)	44.8	Vega (1bp)	30,894
ATM Fwd	3.837%	Implied vol (bp)	131.7	Theta (1day)	-4,959

Source: BofA Global Research; Bloomberg

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Exhibit 53: Exposures for a costless 1y fwd 2s10s conditional bear steepener

Net exposure of a long 100m 1y10y payer ATM and a short 1y2y ATM payer ATM+17bp

NPV (\$)	-42,639	DV01	-2,753
		Gamma (1bp)	19
		Vega (1bp)	309
		Theta (1day)	852

Source: BofA Global Research; Bloomberg

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In Exhibit 54 & Exhibit 55 we show the picks to the forwards (and gives) for costless conditional bear steepeners and bull flatteners (generally macro driven moves), and in Exhibit 56 & Exhibit 57 for bear flatteners and bull steepeners (generally policy driven, with Fed tightening driving the former and easing driving the latter).

Exhibit 54: Bear Steepener (pickup to forwards for ATM)

Buy longer maturity payer, sell shorter maturity payer

ATM	2/5s	2/10s	2/30s	5/10s	5/30s	10/30s
3M	2.4	7.7	12.8	5.2	10.6	5.8
6M	5.8	11.6	18.5	6.0	13.2	7.9
1Y	10.6	17.3	26.5	6.7	16.9	10.8
1.5Y	11.2	19.4	30.6	8.3	20.4	12.8
2Y	10.6	20.0	32.3	9.7	23.1	14.2

Source: BofA Global Research

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Exhibit 55: Bull Flatteners (pickup to forwards for ATM)

Buy longer maturity receiver, sell shorter maturity receiver

ATM	2/5s	2/10s	2/30s	5/10s	5/30s	10/30s
3M	2.6	7.1	12.0	5.0	9.9	5.4
6M	6.0	10.8	17.1	5.6	12.2	7.1
1Y	10.4	15.7	24.1	6.2	15.3	9.7
1.5Y	10.6	17.1	27.1	7.3	18.1	11.2
2Y	9.5	17.3	28.4	8.3	20.0	12.2

Source: BofA Global Research

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Exhibit 56: Bear Flatteners (pickup to forwards for ATM)

Sell longer maturity payer, buy shorter maturity payer

ATM	2/5s	2/10s	2/30s	5/10s	5/30s	10/30s
3M	-2.4	-7.7	-12.8	-5.2	-10.6	-5.8
6M	-5.8	-11.6	-18.5	-6.0	-13.2	-7.9
1Y	-10.6	-17.3	-26.5	-6.7	-16.9	-10.8
1.5Y	-11.2	-19.4	-30.6	-8.3	-20.4	-12.8
2Y	-10.6	-20.0	-32.3	-9.7	-23.1	-14.2

Source: BofA Global Research

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Exhibit 57: Bull Steepener (pickup to forwards for ATM)

Sell longer maturity receiver, buy shorter maturity receiver

ATM	2/5s	2/10s	2/30s	5/10s	5/30s	10/30s
3M	-2.6	-7.1	-12.0	-5.0	-9.9	-5.4
6M	-6.0	-10.8	-17.1	-5.6	-12.2	-7.1
1Y	-10.4	-15.7	-24.1	-6.2	-15.3	-9.7
1.5Y	-10.6	-17.1	-27.1	-7.3	-18.1	-11.2
2Y	-9.5	-17.3	-28.4	-8.3	-20.0	-12.2

Source: BofA Global Research

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Because left side vol trades rich to the right, bear steepeners and bull flatteners pickup to the forwards currently (they sell rich left side vol). Normally investors stay away from conditional curve trades that giveup to the forwards. Our investor currently sees c.17bp pickup to the forwards for costless 1y fwd 2s10s bear steepeners and may judge that to be attractive.

Alternatively...

This is just food for thought, and really falls outside of the scope of this primer, but alternatively our investor may consider 1y fwd 2s10s curve caps. A curve cap positions for a steepening of the curve (a curve floor positions for a flattening) and it is an option on the curve level itself. Because in a curve cap an investor is long an option on the curve, the maximum downside on the position is the upfront premium (whereas the conditional bear steepener is exposed to potentially significant downside in bear-flattening scenarios).

An important component of the pricing of the cap relative to the conditional curve trades above, is that to price the cap we need to take into consideration the correlation between the dynamic of the 2y and the 10y leg. The vol for the cap is the sum of vols for the 2y and 10y legs plus the correlation term:

$$\sigma_{2s10s} = \sqrt{(\sigma_{2y})^2 + (\sigma_{10y})^2 + 2 \cdot \rho \cdot \sigma_{2y} \cdot \sigma_{10y}}$$

So implicitly, in the curve cap trade, our investor would also be expressing a view on the level of the implied correlation.

c. Short vs. intermediate expiries

A directional view may also be expressed across expiries reflecting a view on the second principal component of the grid discussed in [Section 2.c](#). One way to express this view is through payers or receiver calendars.

Let's assume that our investor, while optimistic about the economy near-term (say over a 3m horizon), believes that medium-term rates may be lower. In other words, he sees a relatively downside or symmetric balance of risks medium term (say 1y horizon) but tilted to the upside in yields near term (c.3m). In this scenario the bias should be long payers in 10y tails with expiries c.3m, but the investor may be willing to finance (partially or fully) that position by selling medium term payers with expiries c.1y. Let's then move into a concrete example: buy 3m10y payers financed by selling 1y10y payers.

How is this position weighted? If our investor is expressing a directional view, the choice would generally be to delta weight the legs terminally (i.e., similar notionals on the 3m10y and 1y10y legs). This position is short delta (investor is short the market at inception), long gamma (our investor is long the shorter expiry leg) and short vega (our investor is short the longer expiry leg) Thus, in a way our investor is short fwd vol (which we will discuss this later).

An upward sloping term structure for the volatility grid generally implies that an investor needs to pay to enter a short ATM gamma leg vs a long ATM intermediate expiry spread, although carry on the position is highly favorable ... and vice-versa for a long ATM gamma leg vs short ATM intermediate expiry, i.e., investor likely receives but structure has unfavorable carry.

If our investor needs to pay for a long intermediate vs short gamma position (generally the case unless the term structure of volatility is inverted, as we noted already), he/she may still make the structure costless by:

- Capping the upside on the position (buying 1y10y receiver spreads) and therefore reducing the cost on the long leg
- Or adjusting the strike on the long leg OTM (and hence paying less for the long leg)

In the structure that fits with our investor's macro view (long 3m10y payers vs short 1y10y payers), the expectation is that he/she will receive on the structure. The 1y10y ATM payer exposure has been modeled in Exhibit 6. In Exhibit 58 we model a long 3m10y payer ATM leg, and show the net exposure between a 100m long 3m10y ATM payer vs 100m short 1y10y ATM payer in Exhibit 59.

Exhibit 58: Long 100m 3m10y ATM payer

On the calendar, investor pays NPV 1.68m on long 3m10y ATM payer and receives NPV 3.43m on short 1y10y ATM payer, i.e., he receives net 1.75m on the calendar position

Tenor	3m10y	NPV (\$)	1,680,807	DV01	-39,009
Notional	100m	Premium (%)	1.681	Gamma (1bp)	559
Strike	3.830%	Yield Value (bp)	21.0	Vega (1bp)	15,857
ATM Fwd	3.830%	Implied vol (bp)	106.0	Theta (1day)	-9,104

Source: BofA Global Research; Bloomberg

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Exhibit 59: Net exposure for long 100m 3m10y ATM payer vs short 100m 1y10y ATM payer

Investor receives net 1.75m on the calendar position

NPV (%)	-1,751,548	DV01	-2,558
		Gamma (1bp)	330
		Vega (1bp)	-15,347
		Theta (1day)	-4,997

Source: BofA Global Research; Bloomberg

BofA GLOBAL RESEARCH

Our investor can choose to receive 1.75m on the position or adjust the strike on the short leg to cushion the position against adverse moves (higher rates medium term). As we saw in Exhibit 37, if our investor sells a 100m 1y10y payer ATM+60bp (instead of an ATM payer) he will receive NPV of c.1.74m, and that roughly offsets the premium paid for the 3m10y leg (see Exhibit 60).

Exhibit 60: Net exposure for long 100m 3m10y ATM payer vs short 100m 1y10y ATM+60bp payer

Investor is short delta, long gamma, short vega and short theta in the calendar spread at inception

NPV (\$)	-57,610	DV01	-16,723
		Gamma (1bp)	357
		Vega (1bp)	-11,446
		Theta (1day)	-5,116

Source: BofA Global Research; Bloomberg

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d. Conditional spread trades

Spread trades leverage expectations for the relative under- or over-performance - of two yield curves, e.g., nominal vs real yields in inflation breakeven, Libor vs USTs in swap spreads back in the day, or currently SOFR swaps vs USTs. Focusing on the latter, we see that our investor can express expectations for widening of the swap spread by:

- Paying 100k '01 on 10y SOFR swaps (short the market)
- Buying 100k '01 of 10y notes (long the market)

To express a tightening view, our investor would have to receive on the swap vs shorting the 10y note. Our investor can also overlay a view on the relative cheapness of futures vs cash by shifting the cash leg into futures (buying 100k of '01 in 10y note futures), thus entering what is called an invoice spread position (the "invoice" comes from the invoice yield for the cash leg implied by the futures price, i.e., the CTD yield implied by the futures price).

Historically, spreads (swap vs cash) have shown some directionality to rates, wider on selloffs and tighter on rallies. The broader dynamic of spreads, and their directionality with rates, is driven by a series of factors, including corporates swapping fixed for floating, and the bias in hedging activity flows towards the swap leg.

This directionality can be explored through options through a conditional invoice spread position, i.e., puts/payers or calls/receivers. Let's say our investor believes spreads may widen near-term and therefore focus on the former (the rationale for the latter – conditional spread tightener – is constructed along similar lines).

In puts/payers (conditional spread widener) our client is long the payer leg (short the market) and finances the position by selling puts (if a long position in a put contract is short the market – reflects a view for lower UST futures prices, or higher yields – a short position in a put contract is long the market and offsets the exposure on the payer leg):

- Buy a 7m10y swaption payer (Sep expiry)
- Sell a TYU4 put (Sep expiry)

The notionals on each leg should be such that there is an offset between: (1) the terminal '01 of the swap leg (the underlying on the payer); and (2) the '01 of CTD at the invoice yield implied by the strike on the treasury futures option leg (the underlying is the futures contracts, which delivers on the CTD).

If - the operative word is if, as this is not always the case - the premium our investor receives on the futures option > than the premium he spends on the payer, he can adjust the strike on the futures lower (corresponding CTD invoice yield higher) such that he enters a conditional invoice spread widener at a pickup to the forwards (he could have also adjusted the strike on the payer lower, but as we noted earlier, the preference is generally to lose money later in a selloff rather than make money sooner).

Any potential pickup to the forwards comes in large part from selling rich vol in futures options vs buying cheap vol in swaptions. Futures options trade on the exchange (CBOT) while swaptions trade OTC, so to a large extent puts/payers and calls/receivers arbitrage the vol differentials between board vol and OTC vol.



4. Trading volatility as an asset class

In the previous section we discussed how our investor can use options payers and receivers to express a macro view. For the most part these implied a directional exposure to express a view on the dynamic of rates, curve, or the path of monetary policy. Here we focus on taking a view on vol itself, but we will continue to explore the same rotations that we saw as key in the grid dynamic in previous sections.

Before we get into the core of this discussion, however, let's do a shallow dive into the concepts of event pricing and delivered volatility.

a. Event pricing

Broadly, in long vol positions investors are waiting for something to happen (generally negative carry), while in short vol positions investors are hoping for nothing to happen outside of what is already priced and for fwds to be realized (generally positive carry).

When we cast vol in this context, it follows that vol levels should be contingent on the number of event days (where the market may deliver surprises and absorb these surprises) that are contained in any given expiry, rather than calendar days. Weekdays should have a higher event weight than weekends in this context (given that there are few economic releases on weekends). However, Fridays should be worth slightly more (as the market is less capable of absorbing any potential surprises on weekends). Broadly, days with significant events (e.g., non-farm payroll releases or elections) should have a higher event weight).

Now, imagine that the chances of a material shock to the market are all concentrated on a particular data release at a particular date and time (this could be an important economic release or another major event with material implications for the outlook like an election). While this is a relatively drastic assumption, let's run with it and assume also for simplicity that: (1) this release happens in 12 days, such that 1-week expiry options reflect none of the risk to this release, but that 2-week expiry options fully reflect this risk; and (2) that there are liquid prices for 1- and 2-week options.

In Exhibit 61 we use these assumptions to extract: (1) the daily breakeven vol per unit of event weight; and (2) the event weight that the volatility market is assigning to the release. Every weekday is worth 1 event weight except for the release day which is worth an unknown (x) event weight. Therefore:

- from 1-week options we calculate the daily BE vol per unit of event weight over the first week (5.47bp in the example) and extrapolate the same daily BE for standard event days over the second week (all even days except the release day) ...
- ... and calculate the event weight associated to the release that recovers the price of the 2-week option (4.4 event weight)

Exhibit 61: Inferring the vol the market is assigning to a particular event

Pricing the event weight associated to a data release or other material event

	Th	F	Sa	Su	M	T	W	Th	F	Sa	Su	M	T	W	Th		
Calendar days	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	---->	14
Event days	1				1	1	1	1	1			1	X	1	1	---->	9+X
	1 Week Option: $\sigma(1w) = 10bp$																
Daily B/E	n															---->	$\sigma(1w) = n * \sqrt{5}$
	Week 2: $\sigma(1w) = 20bp$																$n = 10 / \sqrt{5} \sim 5.47bp$
Daily	5.47				5.47	5.47	5.47	5.47	5.47			5.47	?	5.47	5.47	---->	$\sigma(2w) = 5.47 * \sqrt{9 + X}$
																	$X = (20 / 5.47)^2 - 9$
																	$X \sim 4.4 \text{ event days}$

Source: BofA Global Research

BofA GLOBAL RESEARCH

The vol associated with the release is the product of the daily breakeven vol per unity of event weight (5.47bp) and the square root of the event weight (4.4), i.e., more than twice the daily breakeven associated to a regular event day. Our investor can make a call on whether this is too cheap or expensive (buy or sell 2-week options vs 1-week options, respectively, vega weighted) given his view on the likelihood of a surprise materializing on this release.

b. Delivered vol and walk vol

Realized volatility is a backward-looking measure of the volatility of the underlying asset. As noted above (see [Section 2.c](#)), it is calculated as the standard deviation of daily changes of the underlying over a given historical window (the daily normalized vol) and annualized by taking the product of this number to the square root of business days in one year (a factor of c.16).

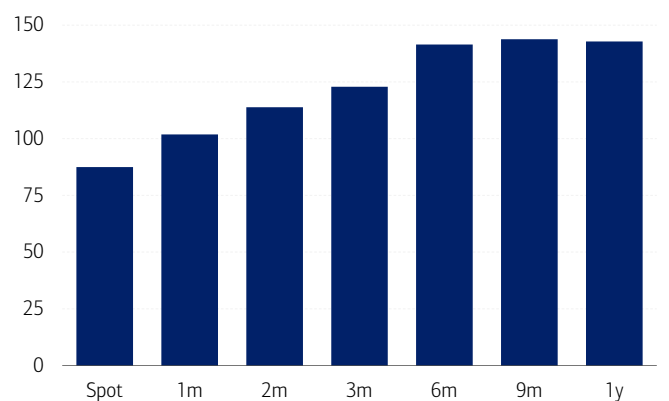
As an aside, realized log vol (%vol – see [Section 1.b](#)) is calculated similarly, but instead of daily differences – $r(t)-r(t-1)$ – it is calculated from the standard deviation of log returns – $\ln(r(t)/r(t-1))$ – and annualized by taking the product of this number to the square root of the number of business days in one year (c.16).

Walk volatility

For shorter expiries the realized normal vol noted above may be appropriate, but for longer expiries it starts to make sense to reflect in the delivered vol calculation the term structure of volatility. Say our investor buys a 1y10y payer today... in 3m he holds a 9m10y payer, in 6m he holds 6m10y payer, etc. Therefore, our investor is not exposed to the vol of the 1y10y swap only, but to a range of vols that reflect the term structure of the delivered vol for the same underlying.

Exhibit 62: 3m realized normal vol (annualized) for 1y rates fwd

1y1y options experience different delivered options as expiry approaches



Source: BofA Global research

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Exhibit 63: 1y1y delivered (3m) walk vol example calculation

1y1y delivered walk vol = $\sqrt{17,501}$ = 132.3bp vs 142.8 1y1y delivered vol

Months	Delivered vol tenor		Period	Time Weights		Variance
1m	Spot	87.5	0.5m	0.5/12	0.0417	319
	1m1y	101.8	1m	1/12	0.0833	864
2m	2m1y	113.8	1m	1/12	0.0833	1078
3m	3m1y	122.8	2m	2/12	0.1667	2515
4m						
5m	6m1y	141.5	3m	3/12	0.25	5004
6m						
7m						
8m	9m1y	143.8	3m	3/12	0.25	5170
9m						
10m						
11m	1y1y	142.8	1.5m	1.5/12	0.125	2551
12m						
Sum				12/12	1	17501

Source: BofA Global research

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The concept of walk volatility reflects this term structure, by time-weighting each of the intermediate expiries between spot and 1y (see Exhibit 63). Ratios of implied to realized walk vol are generally considered to be a more realistic gauge for value in implied vol (and the vol premium that may be harvested in shorter expiries – more on this next) than ratios to standard delivered vol.



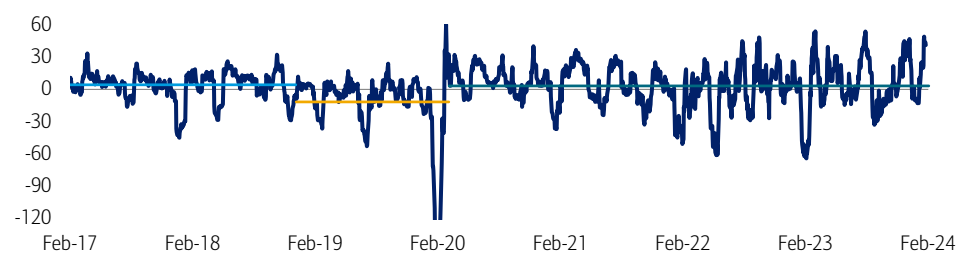
c. Short gamma strategies as yield enhancement...

Short gamma strategies try to harvest the vol premium in implied over (forward looking) delivered vols (see Exhibit 64). In this strategy, investors sell strangles, generally in 1-3m expiries in 10y tails, and use the premiums received as a yield enhancement. Generally, portfolios run this system somewhat systematically, but obviously de-allocate to the strategy around calendar-driven even risks or periods of high uncertainty/diffused risk.

Significantly, periods of high uncertainty/diffused risk generally drive an inversion of the term structure of volatility (gamma trading rich vs intermediates and vega - the second rotation discussed above). Despite the richness in gamma on the grid, these high-vol periods are less than ideal to harvest gamma. In a recent note we looked specifically at CB policy as one of the key drivers of rates vol and short gamma strategies, and we noted that systematic short gamma strategies are more dependent on CB predictability than on the point in the cycle (see [Systematic vol selling](#)).

Exhibit 64: Harvesting volatility premium

Spread of 1m10y vol to forward looking 1m10y delivered vol



Source: BofA Global Research; Bloomberg

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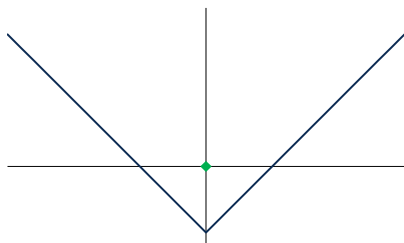
... and how they may reinforce the range

A key feature of periods of significant short gamma flow is the pull to the ATM that dealers hedging of these positions may create.

As portfolios engage in short gamma strategies, dealers are left long vol. Let's analyze what this implies based on exposures to straddles (exposures to strangles would be more realistic, but the dynamic is similar). At inception, a long straddle position is flat delta, but if rates selloff the deal is progressively short the market (payers ITM, receivers OTM). Dealers then need to receive on the underlying to neutralize the delta and capture the P&L (dynamic delta hedging). In a subsequent rally, the loss in the payer leg is more than offset by the gain in the delta hedge + the increase in value in the receiver swaption. As rates rally the straddle position may now be net long the market, and the delta needs to be rebalanced again but in this case by paying the underlying.

Exhibit 65: ATM long straddle position ...

No delta ...

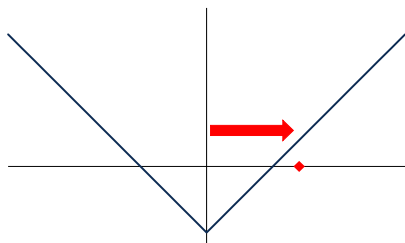


Source: BofA Global Research

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Exhibit 66: ... After selloff payer leg ITM ...

... short delta, receive in swaps

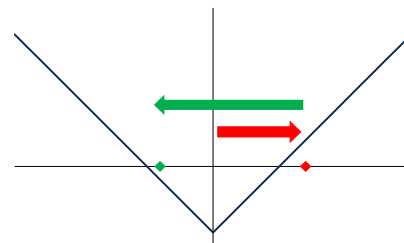


Source: BofA Global Research

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Exhibit 67: ... After rally receiver leg ITM

... realize swap P&L ... pay on swap to hedge delta



Source: BofA Global Research

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It is easy to understand, therefore, how the concerted and relatively mechanical action of dealers in the face of a significant accumulation of short gamma positions by the street may drive a pull to the ATM for the underlying.

d. Long vega as positive carry macro hedge & Formosa issuance

From the discussion above: (1) ideally investors would prefer to be long vol and positive carry, i.e., long a positive carry macro hedge (insofar as long vol positions generally perform well in periods of enhanced macro uncertainty); and (2) that these two exposures (long vol and positive carry) are almost contradictory.

There is, however, a slight anomaly on the US grid where vols for longer dated expiries in 10y and 30y tails tend to trade under intermediate expiries, which allows for some level of approximation to this type of exposure (long vol with positive vol carry, albeit still with negative net carry).

A material part of this anomaly is created by what is known as Formosa-linked supply, i.e., the demand for longer dated callable bonds (typically 30y ZC with 5y lockouts) by Taiwanese lifers. US corporates issue these bonds, and because they buy the call option in these structures (long vol) they can offer a yield pickup vs. bullets to Taiwanese lifers.

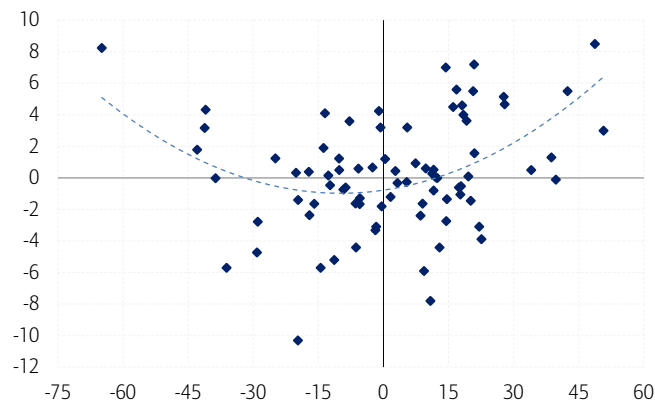
The issuing entities are therefore left with a long Bermudan vol exposure as they issue Formosa bonds, and to hedge this long vol position they generally sell longer dated European swaption vol. The implications of this hedging are significant for the grid:

- The downward pressure on longer dated expiries created by this hedging allows for a rollup of long vega positions on the grid (note in Exhibit 13 the vol rollup between 5y30y vol of c.85bp and 1y30 vol c.97bp, a rollup of c.3bp/year)
- Because of the mismatch between the dvega/dr profile of Bermuda and European vols, this hedge needs to be rebalanced for significant moves in the underlying. This generally means Bermudan portfolios need to bid back the vol they sold on rallies (Bermudan vol collapses more aggressively on rallies than European vol, as it becomes more likely that bonds get called), and sellers of vega on selloffs.

On the net, therefore, Formosa issuance allows: (1) for a positive rollup in longer dated vega; and (2) drives a bid for vega on rallies (allowing it to act as a macro hedge as generally rates rally with deteriorating fundamentals); while (3) on selloffs the selling of vega by Bermudan portfolios is at least partially offset by the directionally between normal vol and the forwards. The last two allow for at times a positive convex profile for the dynamic of vol with the underlying (see Exhibit 68). Formosa volumes have been lower over the last couple of years, but the rollup has persisted (see Exhibit 69).

Exhibit 68: Changes of 5y30y vol (y-axis) vs 5y30y fwds (x-axis)

Slight positive convex profile

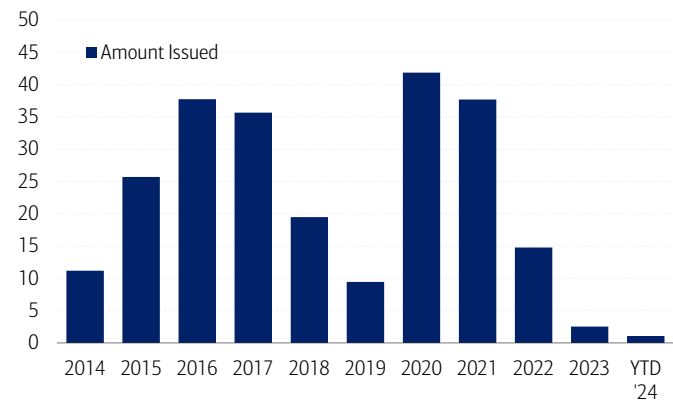


Source: BofA Global research

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Exhibit 69: Total issuance per year since 2014 (\$bn)

Formosa issuance YTD c.\$1.1bn



Source: BofA Global Research, Bloomberg

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A long vega position, with positive vol carry, and at times a positive dvega/dr profile is generally seen as a relatively attractive macro hedge by portfolios.



e. Left vs right side vol

With short gamma and long vega discussions out of the way (together a proxy for the second rotation for the grid described above ... and we will come back to the two again in the last section of this primer when we discuss forward), it is time to go back to the third rotation and discuss the left vs right side dynamic.

As noted above (see [Section 2.d](#)), a large part of the left vs right side dynamic over the cycle is driven by Fed policy, particularly the left side. To illustrate this, we show in Exhibit 70 and Exhibit 71 the average levels for the US grid from Apr '20 - Sep '21 (with the Fed easing) and from Oct '21 - July '22 (with the Fed tightening), respectively. In easing periods, vols push lower directionally broadly across the grid with the left side leading the way, while in tightening periods, vols push broadly higher with the left side leading the outperformance.

Exhibit 70: Average levels for the grid from Apr '20 to Sep '21 (under Fed easing)

Left side cheap vs the right side

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	15.5	21.1	30.3	47.1	56.9	65.6	73.5
3M	17.0	23.6	33.3	49.5	59.1	67.4	74.7
6M	19.3	27.5	36.9	51.6	59.9	67.2	72.9
1Y	26.4	35.3	43.6	55.0	61.3	67.0	70.7
2Y	43.5	50.1	54.1	60.5	64.0	67.2	67.9
3Y	55.8	58.8	60.7	63.6	65.4	67.2	66.0
4Y	61.3	62.8	63.5	65.1	66.0	66.7	64.2
5Y	64.4	65.0	65.5	66.3	66.3	66.2	62.6
10Y	65.1	64.5	64.3	63.7	63.1	62.1	56.4
15Y	60.4	59.8	59.4	59.3	58.1	57.4	51.7
30Y	52.1	52.0	51.3	50.1	49.8	48.9	47.0

Source: BofA Global Research

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Exhibit 71: Average levels for the grid from Oct '21 to July '22 (under Fed tightening)

Left side rich vs the right side

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	90.3	106.7	105.8	103.9	100.6	98.1	90.6
3M	97.5	105.5	107.8	101.6	97.7	95.7	88.5
6M	99.3	105.6	104.9	100.3	96.1	93.0	86.0
1Y	107.3	106.7	103.6	99.2	94.8	91.2	83.0
2Y	108.0	104.7	101.9	96.5	92.2	88.3	79.1
3Y	103.2	99.5	96.7	92.9	89.1	85.2	75.3
4Y	98.7	95.8	93.3	89.3	86.2	82.5	72.3
5Y	93.9	91.0	88.7	85.8	83.5	80.0	69.6
10Y	76.9	75.7	74.0	72.9	70.5	68.4	59.2
15Y	66.7	66.4	65.4	64.6	62.1	60.3	53.8
30Y	55.7	54.8	53.6	52.6	51.5	50.2	47.2

Source: BofA Global Research

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Let's assume that we are at the end of the tightening cycle, vols on the left side are still trading rich to the right side, and our investor believes that the Fed may shift policy communication and signal rate cuts over the next 6-12m.

On the grid, this view implied a normalization of the left side vs the right side of the grid. Our investor may then sell straddles/strangles on left side to finance buying straddles/strangles on the left, e.g., sell 1y2y vol vs 1y10y vol. The notionals on each leg are defined to make the structure insensitive to a parallel vol move across the grid (vega weighted). We model the two legs in Exhibit 72 and Exhibit 73.

Exhibit 72: Exposures for 100m 1y10y ATM straddle

Long vega, long gamma, short theta ... small residual delta from lognormality of underlying

Tenor	1y10y	NPV (\$)	6,864,711	DV01	4,326
Notional	100m	Premium (%)	6.865	Gamma (1bp)	557
Strike	3.727%	Implied vol (bp)	110.0	Vega (1bp)	62,283
				Theta (1day)	-8,235

Source: BofA Global research; Bloomberg

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Exhibit 73: Exposures for 430m 1y2y ATM straddle

430m notional for 1y2y straddles offsets the 62k vega on the 100m 1y10y straddle

Tenor	1y2y	NPV (\$)	8,223,563	DV01	2,235
Notional	430m	Premium (%)	1.912	Gamma (1bp)	462
Strike	3.837%	Implied vol (bp)	132.0	Vega (1bp)	62,183
				Theta (1day)	-9,866

Source: BofA Global research; Bloomberg

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Selling rich left side vs cheap right side vol generally implies that our investor will receive on this position (see Exhibit 74). At current levels the position has marginally positive vol carry, i.e., vol rolls down on the short vol leg (c.3bp over 6m from 132bp for 1y2y to 129bp for 6m2y – see Exhibit 13) by slightly more than on the long leg (c.2bp over 6m from 110bp for 1y10y to 108 for 6m10y).

Exhibit 74: Long 100m 1y10y straddles vs 430m 1y2y straddles vega weighted

Position leverages the outperformance of 1y10y vs 1y2y vol. Residual vega at inception ... investor receives 1.36m on the position

NPV (\$)	-1,358,852	DV01	2,091
		Gamma (1bp)	95
		Vega (1bp)	99
		Theta (1day)	1,631

Source: BofA Global research; Bloomberg

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f. Shorter vs. longer expiries

In this last section of this primer we come back to the second rotation on the grid (short vs intermediates rotation) after having discussed these independently when we examined short gamma as a yield enhancement strategy (see [Section 4.c](#)) and long vega as a macro hedge (see [Section 4.d](#)). For this discussion we consider two types of proxies for forward vol: (1) calendar spreads; and (2) fwd vol exposures expressed through mid-curve vs swaptions or swaption triangles.

Calendar spreads

In a calendar spread an investor sells/buys a shorter expiry straddles/strangles and buys/sells a longer expiry straddles/strangles, both with the same underlying rate. The position is vega weighted such that it is insensitive to parallel moves on the vol grid.

As noted in [Section 3.c](#), in an upward sloping term structure for the volatility grid, generally our investor needs to pay to enter a short ATM gamma leg vs a long ATM intermediate expiry calendar spread, although carry on the position is highly favorable (vice-versa for a long ATM gamma leg vs short ATM intermediate expiry, i.e., investor likely receives but structure has unfavorable carry).

Consider the similar view argued in [Section 3.c](#), i.e., our investor sees risks of higher yield near term (over the next 3m) but a relatively symmetric / downside tilt of the balance of risks medium term (1y horizon). In this case he may want to be short intermediate vol (1y10y) vs long gamma (3m10y), vega weighted at inception (i.e., same vega on each leg such that the structure is insensitive to parallel moved on the grid). As we noted, in an upward sloping vol term structure our investor generally receives a premium for this structure (by being short rich vol vs long cheap vol). An alternative to make this structure costless is to sell strangles in 1y10y leg vs straddles in 3m10y, optimizing the width of the strangles to make the structure costless. We will leave the modeling of these as homework ... pricing as of 23 Feb '23 (use Exhibit 58 and Exhibit 6 if you want to save yourself some time) reflects 195m notional on the 3m10y leg vs 100m on the 1y10y leg, and net receiving c.125k.

Forward vol

We finally reached the last section of this primer ... forward vol! It should be relatively easy to see how a calendar spreads, where our investor is long intermediate expiries vs short gamma, are to some extent a proxy for a forward vol position (either long – if buying the longer expiry vs. the shorter one – or short – if buying the short expiry vs. selling the longer one). However, the position has also a series of other exposures (e.g., long theta or short gamma at inception), that make its P&L profile more complicated than what would be desired in an outright forward vol exposure (where ideally our investor would only be sensitive to the forward implied vol).



There are contracts that mitigate some of these exposures and give investors a cleaner forward vol exposure (these are known as Forward Vol Agreements = FVA). Here we discuss how this FVA exposure may to some extent be replicated using vanilla swaptions (this is not a perfect replication as the FVA can never be replicated exactly with a collection of swaptions).

Let's assume that our investor wants exposure to 4y forward 1y10y volatility. To replicate this exposure with swaptions our investor needs to be:

- Long the variance from spot to 5y of a series of 10y rate CFs starting in 5y and ending in 15y
- Short the variance from spot to 4y of the same CFs (10y rate CFs that start in 5y and end in 15y)

Now, the long leg is clearly a 5y10y vanilla swaption leg. The short leg, however, is what is called a mid-curve swaption and it is really an option on a forward (in this case, a 4y option on a series of 10y rate cash flows that starting in 5y and ending in 15y, i.e., a 4y option on the cash flows of a 1y forward 10y rate). Because the CFs on both legs need to be the same, the strikes on the swaption and mid-curve legs need to be the same.

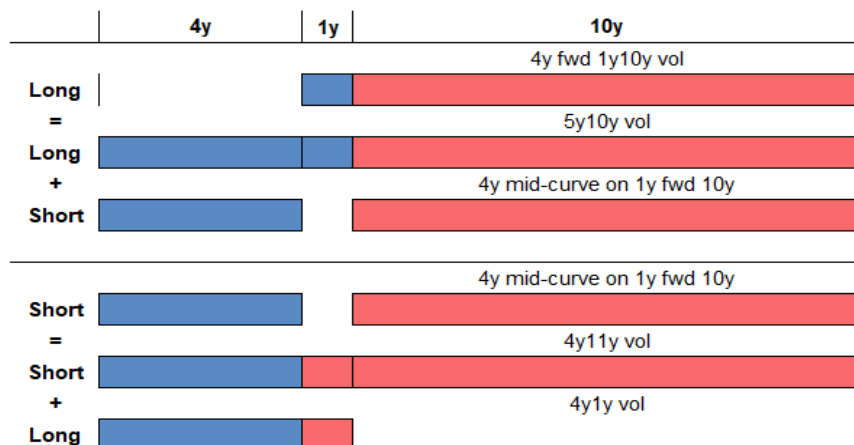
Now, mid-curves trade OTC and are relatively liquid, but the short mid-curve exposure (a 4y option on the 1y10y forward) can be further replicated with two swaptions by:

- selling a 4y11y swaption
- buying a 4y1y swaption

On the net, therefore, we replicated (imperfectly) the FVA exposure with 3 swaptions (see Exhibit 75 ... we leave the derivation of the forward vol formula for [Appendix D](#)), with the strikes on each of these 3 legs necessarily the same. Now, it should be easy to see how even in a relatively steep curve the ATM strikes for 4y11y and 5y10y should be roughly similar, but these will be very different from the ATM strike for the 4y1y leg.

Exhibit 75: Replication of a 4y fwd 1y10y vol exposure

y fwd 1y10y vol replicated as a long swaption vs midcurve or a portfolio of 3 vanilla swaptions



Source: BofA Global Research

BofA GLOBAL RESEARCH

We can measure RV forward vol position, by looking at the dynamic of the ratio of forward vol to spot vol across the grid (e.g., 1y forward 1y10y vol vs. 1y10y vol). We show 1y forward vol levels in Exhibit 76 and these ratios in Exhibit 77 (105bp for 1y fwd 1y1y fwd vol in Exhibit 76 / 110bp for 1y10y vol in Exhibit 13 = 95% ratio in Exhibit 77). We calculate the 3m Z-Score of these ratios in Exhibit 78. There's a clear relationship between RV in forward vol and the second rotation of the vol grid we discussed above ... generally, as gamma richens forward vol cheapens and vice-versa.

Exhibit 76: 1y Forward Volatility

105.0bp for 1y fwd 1y10y

	1y	2y	5y	10y	30y
1m	136.6	129.6	116.3	107.3	95.3
3m	134.3	128.2	117.1	107.0	94.8
6m	130.7	126.2	116.7	106.5	94.6
1y	125.9	123.7	115.5	105.0	93.7
2y	122.8	120.5	110.7	100.9	90.4
5y	107.2	104.6	97.1	90.4	82.1
10y	88.4	86.7	81.4	77.0	71.8

Source: BofA Global Research

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Exhibit 77: 1y Fwd vol as % ATM

95% of ATM for 1y fwd 1y10y

	1y	2y	5y	10y	30y
1m	127%	100%	96%	103%	110%
3m	116%	102%	98%	101%	104%
6m	103%	98%	99%	99%	101%
1y	93%	94%	98%	95%	97%
2y	96%	97%	96%	95%	96%
5y	96%	96%	95%	94%	96%
10y	95%	95%	94%	95%	99%

Source: BofA Global Research

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Exhibit 78: 3m Z-Score 1y Fwd vol % ATM

1.4 Z-Score for 1y fwd 1y10y

	1y	2y	5y	10y	30y
1m	-0.1	0.4	0.9	1.5	2.2
3m	0.1	1.1	1.3	1.6	1.8
6m	0.6	1.1	1.7	1.6	1.8
1y	0.7	0.9	2.0	1.4	1.6
2y	1.8	2.0	2.0	2.0	2.0
5y	1.8	1.9	2.2	2.2	2.5
10y	1.9	2.0	1.8	1.9	2.0

Source: BofA Global Research

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Abbreviations:

ATM – At the Money
 BE – Breakeven
 bp – basis points
 c. – circa (approximately)
 CB – Central Banks
 CDF – Cumulative Distribution Function
 CF – Cash Flow
 FHLBs – Federal Home Loan Banks
 FHLM – Federal Home Loan Mortgage Corporation
 FNMA – Federal National Mortgage Association
 GSEs – Government sponsored enterprises (generally FNMA, FHLM, or FHLBs)
 HPCA – Hierarchical PCA
 IR – Interest Rate
 LIBOR – London Inter Bank Offer Rate
 LLC – Lower left corner
 LRC – Lower right corner
 OIS – Overnight Index Swap
 OTC – Over the Counter
 OTM – Out of the Money
 PCA – Principal Components Analysis
 PDF – Probability Distribution Function
 P&L – Profit and Loss
 PV – Present Value
 RTP – Right to Pay
 RTR – Right to Receive
 SOFR – Secured Overnight Financing Rate
 Sqrt – Square root
 StdDev – Standard Deviation
 ULC – Upper left corner
 URC – Upper right corner
 ZC – Zero Coupon

Appendices

In the appendices we provide details: (A) on some resources available for data, pricing of simple structures, tracking of flows; (B) a very cursory overview of option pricing; (C) the calculation of PDFs and CDFs from the option skew; (D) practical aspects of caps/floors pricing; and (E) forward volatility.

Appendix A: Resources

There are a variety of resources available to: (1) examine the dynamic of rates volatility over time (the ATM vol surface, the vol cube, etc.); (2) price some of the instruments; and (3) monitor some of the flows. We discuss some of these below.

A.1: Mercury Resources

Our research portal contains historical data for ATM and OTM implied and delivered vols, RV, closing skew, etc... some of the links include:

- Chart tool: markets.ml.com/chart-tool , for ATM & OTM implied vols
- Swaption heatmap: markets.ml.com/swaption-heat-map
- Swaption closing skew: markets.ml.com/swaption-closing-skew

Research also published regular updates on market dynamic, flows, and trade recommendations. Please see our reports: [US Vol – vol in delayed landing scenarios](#) from 26 Feb '24, and [Global Rates Vol in '24](#) from 21 Nov '23.

A.2: Bloomberg vol cube and pricing

Bloomberg provides resources for the visualization of the ATM grid (NSV screen) and the volatility cube (VCUB screen). From these users can graph the historical dynamic for selected tenors.

Bloomberg also provides pricing screens for basis structures under the SWPM screen, e.g., swaptions payer and receivers (SWPM -> Product -> Options -> Swaptions). In Exhibit 79 we show a pricing example for 100m 1y5y ATM payers.

Exhibit 79: Pricing example for 1y5y ATM payers using Bloomberg SWPM screen

Long 100m 1y5y ATM payers pricing parameters and exposures

91) Actions ▾		92) Products ▾		93) Views ▾		94) Info ▾		95) Settings ▾		Swap Manager ▾							
Solver (Premium) ▾				Load		Save		Trade ▾		CCP ▾							
3) Main		4) Details		5) Curves		6) Cashflow		7) Resets		8) Scenario		10) Risk		12) Matrix			
Deal		Option		Swaption		Counterparty		IRS CTRPARTY ▾		+ Ticker / IRS		20) Properties		Valuation Settings			
Style		European ▾		Notional		100M		USD		Curve Date		02/23/2024 ▾		Valuation		02/23/2024 ▾	
Position		Long Payer		Currency		USD		Strike		3.706685		%		Model		Normal ▾	
Type		1Y X 5Y		Expiration		03/03/2025 ▾		Delivery		Price (Cash)		0.00		Volatility Type		Normal ▾	
Swap Start		03/05/2025 ▾		Fee(Pay)		0.00		Fee Date		03/03/2025 ▾				CSA Coll Ccy		N/A ▾	
Swap End		03/05/2030 ▾		Fee Date		03/03/2025 ▾								OIS DC Stripping			
Notification Days		2 BD ▾						Premium Paid At Expiry									
Underlying																	
Leg 1: Fixed		Pay ▾		Leg 2: Float		Receive ▾											
Coupon		3.706685 %		Index		1D SOFRRATE											
Pay Freq		Annual ▾		Spread		0.000 bp											
Day Count		ACT/360 ▾		Leverage		1.00000											
Calc Basis		Money Mkt ▾		Reset Freq		Daily ▾											
				Pay Freq		Annual ▾											
				Day Count		ACT/360 ▾											
Market																	
Valuation Results																22) Calculators ▾	
ATM Strike		3.706685		Implied Vol (bp)		117.58		DV01		-20,345.71							
Yield Value (bp)		47.427		Underlying Prem		0.00000		Gamma (1bp)		124.06							
NPV Without Fee		2,023,435.65		Forward Prem		2.13619		Vega (1bp)		17,209.44							
NPV		2,023,435.65		Premium		2.02344		Theta (1-day)		-2,391.64							

Source: BofA Global research; Bloomberg

BofA GLOBAL RESEARCH

A.3: Flows

Volatility practitioners can monitor flows in options space through Bloomberg also.

Swaption flows and SDR

Swaption flows can be monitored using the SDR screen (SDR -> Rates -> Options). In our regular publications (e.g., [US Vol – vol in delayed landing scenarios](#) from 26 Feb '24) we monitor the these flows for payers (see Exhibit 80) and receivers (see Exhibit 81).

Exhibit 80: Payer volumes from 19-23 Feb '24

Demand for payers (\$m notionals)

Payers	1y	2y	3Y	4Y	5Y	7Y	10Y	20Y	30Y	
<1m	5080				440		520			6175
1m	4960	1600	240		1020	340	1761		240	11291
3m	7530	3130	671		2730	30	3454	10	1494	24217
6m	7646	690		84	440		906	795	437	12469
1y	6284	240	400	400	530	175	1414		536	9979
2y	2200					13	408	25	221	2867
3Y	250			480		241	451	68	50	1540
4Y						110	350	54		514
5Y						140	801	100	75	1116
7Y						170	100	104	175	549
10Y					50		369	313	195	927
20Y							240			240
30Y										
	33950	5660	1311	964	5210	1219	10774	1469	3423	71884

Source: BofA Global research; Bloomberg

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Exhibit 81: Receiver volumes from 19-23 Feb '24

Demand for receivers (\$m notionals)

Receivers	1y	2y	3Y	4Y	5Y	7Y	10Y	20Y	30Y	
<1m	1100			5	200		420			1745
1m	3870	100	244		770		2139		180	9103
3m	5890	1950	145		1240	200	2592	10	600	16694
6m	5026	1360	680	480	200		835	75	187	10812
1y	8713	700			2210	255	1800		1005	14683
2y	1153		960				408	25	146	2692
3Y	250			480		241	451	68	50	1540
4Y						110	180	54		344
5Y						140	751	100	75	1066
7Y						170	100	104	25	399
10Y					50		363	313	195	921
20Y							240			240
30Y										
	26002	4110	2030	965	4670	1116	10279	749	2463	60240

Source: BofA Global research; Bloomberg

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Formosa supply

Formosa supply can be monitored with the SRCH screen in Bloomberg, scanning for securities that match the relevant criteria (see Exhibit 82 for a SRCH query example).

Exhibit 82: SRCH query for callable bonds listed in Taipei

USD denominated callable bonds listed in Taipei

Actions		Settings		Fixed Income Search	
Build/Edit Search		My Searches		Example Searches	
Build with Criterion		Build by Merging Saved Searches			
1. Select Universe				As of 03/02/2024	
1) Asset Classes	Corporates, Governments			formosa	
2) Sources	All Securities			3,369,206 securities	
	Field	Boundaries	Selected Criteria	Matches	
3) And	Security Status	Include	Bonds: All	3,359,857	✕
3) And	Maturity Type	Include	(Callable)	530,780	✕
3) And	Currency	Include	(United States Dollar)	371,832	✕
3) And	Exchange / Listed Mark...	Include	(Taipei)	997	✕
3) And	Announce Date	>=	01/01/2011	997	✕
3) And					Fields

Source: BofA Global research; Bloomberg

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Appendix B: Swaption valuation

In this appendix we try to provide a very cursory overview of option pricing.

B.1: Swaption valuation general framework

From arbitrage free pricing, we can choose any freely traded instrument as our numeraire $n(T)$... the usual theorems then guarantee that there exists a probability measure such that the value $V(t)$ of any freely tradable deal divided by the numeraire is a martingale

$$V(t) = n(T) \cdot E \left\{ \frac{V(T)}{n(T)} | F_t \right\} \quad \forall T > t$$

For swaption pricing, we choose the underlying forward swap PV01 as our numeraire... it is a tradable asset (a collection of zero coupons). A time t , the value of a swaption with expiry T_x is:

$$V(t) = PV01(t, x, y) \cdot E \left\{ \frac{V(T_x)}{PV01(T_x, x, y)} | F_t \right\}$$

At the exercise date, the payoff is the value of the swap provided it is positive:

$$V(T_x) = PV01(T_x, x, y) \cdot [w \cdot (S_{x,y}(T_x) - K)]^+$$

So that we have:

$$V(t) = PV01(t, x, y) \cdot E \left\{ [w \cdot (S_{x,y}(T_x) - K)]^+ | F_t \right\}$$

The par swap $S_{x,y}(t)$ is the value of the freely tradable instrument (the difference of two zero coupons) divided by our numeraire (i.e., the swap PV01). So, the swap rate is a martingale under the swap measure:

$$S_{x,y}(t) = E \{ S_{x,y}(T) | F_t \} \quad \forall T > t$$

To complete the pricing, we need to assume a specific distribution for swap rate under the swap measure (we need to characterize the second and higher conditional moments):

- Normal model
- Black model
- SABR model
- etc...

B.2: Normal model & Greeks

In the Normal model the underlying diffuses from its forward according to a normal distribution. The dynamic of the forward rate is governed by:

$$dS_{x,y}(t) = \sigma_N \cdot dW(t)$$

$$S_{x,y}(0) = F$$

The distribution of the forward rates is independent of the absolute level of rates, and there is a non-zero probability that forward rates may become negative over time. The price of the swaption is:

$$Swaption(0, 1, K, \sigma_N, w) = PV01(0; x, y) \cdot \left\{ w \cdot (F - K) \cdot \Phi \left[\frac{w \cdot (F - K)}{\sigma_N \cdot \sqrt{T_x}} \right] + \sigma_N \cdot \sqrt{T_x} \cdot \phi \left[\frac{w \cdot (F - K)}{\sigma_N \cdot \sqrt{T_x}} \right] \right\}$$

Where $\Phi(x)$ is the normal CDF

$$\phi(x) \text{ is the normal density: } \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

For an ATM swaption we have:

$$\text{Swaption}(0,1,F,\sigma_N,w) = \text{PV01}(0;x,y) \cdot \sigma_N \cdot \sqrt{\frac{T_x}{2\pi}}$$

B.3: Black Model

Here, the underlying diffuses from its forward according to a lognormal distribution. The dynamic of the forward rate is governed by:

$$\begin{aligned} dS_{x,y}(t) &= S_{x,y}(t) \cdot \sigma_B \cdot dW(t) \\ S_{x,y}(0) &= F \end{aligned}$$

The distribution of the forward rates is dependent of the absolute level of rates. The price of the swaption is:

$$\begin{aligned} \text{Swaption}(0,1,K,\sigma_B,w) &= \text{PV01}(0;x,y) \cdot \text{Bl}(K,F,\sigma_B\sqrt{T_x},w) \\ \text{Where } \text{Bl}(K,F,v,w) &= F \cdot w \cdot \Phi(w \cdot d_1(K,F,v)) - K \cdot w \cdot \Phi(w \cdot d_2(K,F,v)) \\ d_{1,2} &= \frac{\ln \frac{F}{K} \pm \frac{v^2}{2}}{v} \end{aligned}$$

For an ATM swaption we have

$$\text{Swaption}(0,1,F,\sigma_B) = \text{PV01}(0;x,y) \cdot F \cdot \left[\Phi\left(\frac{\sigma_B\sqrt{T}}{2}\right) - \Phi\left(-\frac{\sigma_B\sqrt{T}}{2}\right) \right]$$

B.4: Black, Normal & normalized vol

ATM swaption price in the normal model:

$$P_N = \text{PV01} \cdot \sigma_N \cdot \sqrt{\frac{T_x}{2\pi}}$$

AMT swaption price in the Black model:

$$P_B = \text{PV01} \cdot F \cdot \left[\Phi\left(\frac{\sigma_B\sqrt{T}}{2}\right) - \Phi\left(-\frac{\sigma_B\sqrt{T}}{2}\right) \right]$$

$$\frac{\sigma_B\sqrt{T}}{2} \approx 0$$

Using a Taylor expansion around zero (ok for short expiry options):

$$P_B \approx \text{PV01} \cdot F \cdot \left[\Phi(0) - \Phi(0) + \phi(0) \frac{\sigma_B\sqrt{T}}{2} + \phi(0) \frac{\sigma_B\sqrt{T}}{2} \right] = \text{PV01} \cdot F \cdot \frac{\sigma_B\sqrt{T}}{\sqrt{2\pi}}$$

$$\text{since } \phi(0) = \frac{1}{\sqrt{2\pi}}$$

Hence

$$P_N = P_B \rightarrow \sigma_N \approx F \cdot \sigma_B$$

... is an acceptable approximation for short-expiry ATM options.

For longer expiries its preferable to use:

$$\sigma_N \approx \frac{F \cdot \sigma_B}{1 + \frac{T \cdot \sigma_B}{24}}$$

Similar formulas exist for non-ATM options.

Normal volatility advantages and disadvantages...

- Analysis based on normal volatility reveals directly the implied volatility and fwds.
- Normal vol can change just because the forwards have moved without change in underlying vol.
- More realistically, the distribution of forward rates exhibits both normal and lognormal characteristics, like a blended model. SABR blends the two models (β) while capturing the volatility of volatility (α) to fit the observed market smile.

B.5: SABR model

In the SABR model the dynamic of the underlying is weighted between a normal and a lognormal distribution using a stochastic volatility:

$$dS_{x,y}(t) = S_{x,y}(t)^\beta \cdot \sigma(t) \cdot dW_1(t)$$

$$S_{x,y}(0) = F$$

... where

$$d\sigma(t) = v \cdot \sigma(t) \cdot dW_2(t)$$

$$\sigma(0) = \alpha$$

... and

$$dW_2(t) \cdot dW_2(t) = \rho \cdot dt$$

The parameters of the SABR model define:

- **σ (volatility)** = controls the likelihood of the forward rate to move over time
- **v (vol of vol)** = controls the lognormal volatility of the volatility parameter and the convexity of the smile
- **β (mix of the distribution)** = controls the mix of the distribution of the returns of the underlying between lognormal (1) and normal (0)
- **ρ (correlation)** = controls the correlation between the moves in volatility and the underlying

The previous models assume constant volatility, but that is not how the market prices vol. The shape of the smile is controlled by the mixture ($\beta=1$ vol is proportional to the forwards, $\beta=0$ vol is independent of the forwards), the vol of vol (the higher the vol of vol, the more convex the smile) and the correlation parameters.

Appendix C: PDFs and CDFs

We can write the third equation of Appendix B.1 for the value of an option as an integral of the expected terminal payoff:

$$[w \cdot (S_{x,y}(T_x) - K)]^+$$

over the risk neutral probability density function (PDF) of the forward price of the underlying at the expiry of the option. From this it follows that the CDF for the underlying is related to the first derivative of the option price with respect to the strike, and the PDF for the underlying is related to the second derivative of the call price with respect to the strike (normalized to 1).

Appendix D: Forward Volatility

As we noted in [Section 4.f](#), we can replicate imperfectly a 4y fwd 1y10y positions through a long 5y10 swaption and a short 4y option on a 1y10y fwd (a midcurve):

$$(5 - 4) \cdot \sigma_{4y \text{ fwd } 1y10y}^2 = 5 \cdot \text{Var}(F_{5y}^{5y,10y}) - 4 \cdot \text{Var}(F_{4y}^{5y,10y})$$

... where:

$$\text{Var}(F_{5y}^{5y,10y}) = \sigma_{5y10y}^2$$

Now, the midcurve forward can be written as:

$$F_{4y}^{5y,10y} = (1 + \alpha) \cdot F_{4y}^{4y,11y} - \alpha \cdot F_{4y}^{4y,1y}$$

... where:

$$\alpha = DV01(4y1y)/DV01(5y10y)$$

Such that:

$$\text{Var}(F_{4y}^{5y,10y}) = (1 + \alpha)^2 \cdot \sigma_{4y11y}^2 + \alpha^2 \cdot \sigma_{4y1y}^2 - 2 \cdot \alpha \cdot (1 + \alpha) \cdot \rho \cdot \sigma_{4y11y} \cdot \sigma_{4y1y}$$

Options Risk Statement

Options and other related derivatives instruments are considered unsuitable for many investors. **Options strategy is by definition governed by a finite duration. The most severe risks associated with general options trading are total loss of capital invested and delivery/assignment risk, all which can occur in a short period.**

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