Indian Institute of Information Technology
Design and Manufacturing, Kancheepuram
Chennai 600 127, India
An Autonomous Institute under MHRD, Govt of India
An Institute of National Importance
COM 209T Design and Analysis of Algorithms

End Semester
09-May-2016
Duration: 1hr
Marks: 50

Roll No: Name:

0. Name the conference in which Prof.Cook and Prof.Levin met and shared their NP experience

1 Light Dose

(1 mark each)

1. Solve $T(n) = 3T(\frac{n}{4}) + n^2$.

2. Solve $T(n) = 2T(\sqrt{n}) + 1$, $T(1) = 1$

3. Solve $T(n) = 4T(\frac{n}{4}) + n^{1.001} \log n$.

4. Is Heap sort a stable sort. If not, present a counter example.

5. Trace quick sort algorithm for the input $\{4, -1, 7, 6, 3, 2, 4\}$. Present only the first level trace.

6. Show that TSP runs in time $O(n^2 \cdot 2^n)$. Present only the analysis.

7. Do OBST and Assembly line scheduling belong to class NP. Justify.

8. Present a linear-time algorithm to find a longest path in a tree.

9. Recall the $L[v]$ computation performed as part of DFS tree. **Claim:** u is an AP if there exists a child v of u such that $L[u] < L[v]$. Prove or disprove.

10. Show that the number of ways of multiplying a chain of n matrices is $\Omega(2^n)$.

11. Draw a graph with 2 articulation points, 2 bridges, and 4 biconnected components.

12. Let G be a connected graph on n nodes. What is the lower bound and the upper bound for the number of biconnected components.

13. Draw a graph on 6 nodes with maximum number of strongly connected components.

14. Write the decision version of Matrix Chain Multiplication problem. Clearly mention, the input and the question to be asked.

2 Medium Dose

(2 marks each)

1. Mention two approaches using which one can show P=NP.

2. Researcher-1: Claims 3-SAT incurs $\Omega(1.5^n)$.
   Researcher-2: Claims Vertex Cover has $O(n^{10})$ deterministic algorithm.
   What can you infer/what would be the consequence of these two statements.

3. Consider a path on 5 vertices. Compute $L[v]$ and identify all articulation points and bridges using $L[]$.

4. An integer array of size n is filled in such a way that the first half of the array contains odd numbers in the range $[1..n]$ in **increasing order** and the second half contains even numbers in the range $[1..n]$ in **increasing order**. Is this a worst case/best case/average case input of Quick Sort. What would be the run-time of Quick Sort if it is given as an input. Justify.

5. Present a greedy algorithm to find a maximum independent set in a connected graph. Your algorithm must run in deterministic polynomial time and need not give optimum always.

6. Solve $T(n) = nT(n-1) + n(n-1)T(n-2)$.

7. Recall the regularity condition check done as part of Case-3 of Master's theorem. What is the significance of the check $c < 1$. What would happen if $c = 3$.

8. Solve: $T(n) = 2T(\frac{n}{2} + 3) + 1$, $T(1) = 1$.

3 Strong Dose

(2.5 marks each)

1. Consider the Fibonacci Series; $A1$: Iterative algorithm that prints Fibonacci Series. $A2$: Recursive algorithm that prints Fibonacci Series.

   - What is the asymptotic tight complexity (theta notation) of $A1$ and $A2$. You need not present the algorithm.

   - Is this problem (printing Fibonacci series) a candidate for Dynamic Programming. Justify.

- If DP exists, what would be the run-time of DP based algorithm. What is the size of the table maintained by DP.

2. Show that Vertex Cover is NP-complete by presenting a reduction from clique problem. Assume clique problem is NP-Hard.

3. `Coin change problem:` Given a set $\{1, 2, 5, 10\}$ denominations, the objective is to give a change for Rupees x using the minimum number of denominations. Present a greedy algorithm and a **proof of correctness** to show that greedy indeed gives optimum always.

   - Algorithm and its analysis

   - Proof of Correctness

   - Will the above greedy work if denominations are different. For example: $\{2, 3, 5\}$.

4. Consider the coin change problem again with the same set of denominations ($\{1, 2, 5, 10\}$);

   - Is coin change problem a candidate for dynamic programming. Justify.

   - Mention recursive sub problem with base cases if DP exists.

   - Trace your DP approach for $x = 7, 13, 15$.

5. What would be the number of comparisons incurred by the merge sort for the following inputs

   - To sort 6 elements in best case. Mention one input sequence meeting your number.

   - To sort 6 elements in worst case. Mention one input sequence meeting your number.

   - To sort 6 elements in average case.

6. Show that any algorithm to search an element x in a sorted array incurs $\Omega(\log n)$ in worst case. Mention two search algorithms for which the worst case input takes $\theta(\log n)$.

7. Present $O(2^n \cdot m)$ algorithm to check whether a given graph is 3-colorable or not.

8. Present $O(n^3)$ algorithm to find the existence of C_4 in a connected graph.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 209T Design and Analysis of Algorithms	Quiz 2 Make up 13-Mar-2016 Duration: 1hr Marks: 15

Roll No: Name:

1. (4 marks) Present two different ways of constructing a max-heap and analyze in detail the construction cost.

2. (1.5 marks) How do you use max-heap as a black box to sort a sequence of numbers in decreasing order.

3. (1.5 marks) What is the size of the recursive subproblem of max-heapify() routine used as part of bottom-up approach.

4. (4 marks) Present a dynamic programming based approach for constructing Optimal Binary Search tree. Clearly mention, why is this problem a candidate for dynamic programming. Also, analyse the run-time.

5. (4 marks) Present a lower bound for best case and average case of any sorting algorithm.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 209T Design and Analysis of Algorithms	Quiz 1 10-Feb-2016 Duration: 1hr Marks: 15

Roll No: Name:

0. Expand CLRS

1. (1 mark) For the input $(4, -5, 4, 7, 9, 2, 1, 3)$, trace insertion sort algorithm with clearly mentioning the output of each iteration.

2. (1 mark) For the input $(4, -5, 4, 7, 9, 2, 1, 3)$, perform **first level** trace of quick sort algorithm with pivot element '3'. Mention the output of each iteration. Note: No Need to perform trace on recursive subproblems.

3. (1 mark) What is the solution to the recurrence $T(n) = 2T(n/2)+1$, $T(2) = 1$. Mention one algorithmic problem that has this $T(n)$ as its time complexity.

4. (3 marks) Solve using master theorem.

   - $T(n) = 2T(\frac{n}{0.25}) + n^2 \log n$

   - $T(n) = 5T(n/4) + n \log \log n$

   - $T(n) = 4T(n/2) + n^{2.000001} \log n$

5. (2 marks) $T(n) = T(n/2) + T(n/3) + T(n/5) + n$. Find $g(n)$ and $h(n)$ such that $T(n) = \Omega(g(n))$ and $T(n) = O(h(n))$. Justify your bounds.

6. (2 marks) Consider a variant of merge sort in which the input array is split into 3 equal parts, further 3 equal parts are sorted recursively and their recursive sub-solution is combined to get a sorting sequence for the input array. Write the recurrence with clearly mentioning the base value. Solve the recurrence using recurrence tree method.

7. (2 marks) Present an efficient algorithm that, given an integer array outputs a minimum and a maximum element. Estimate the number of comparisons required using step count analysis. Do not present asymptotic analysis. Example: for the input array of Q1, the output is -5 and 9.

8. (1 mark) Order them based on asymptotic growth. $n^{O(n)}, n^{O(1)}, n!$

9. (2 marks) Solve: $T(n) = \sqrt{T(n-1) \times T(n-2)}$

Space for Rough Work

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 209T Design and Analysis of Algorithms	Quiz 2 18-Mar-2016 Duration: 1hr Marks: 15

Roll No: Name:

0. Who discovered the paradigm 'Divide and Conquer'

1. (1 mark) Create a min-heap for the sequence 100, -1, 45, -5, 10, -50.

2. (1 mark) Where can you find a maximum element in a min-heap and what is the asymptotic tight complexity.

3. (1.5 marks) Create a max-heap on 5 nodes such that each insert violates max heap property. Is it possible to generalize and construct a heap on n-nodes such that each insert violates max-heap property. Justify.

4. (2 marks) Consider the decision tree T (used in lower bound argument) of sorting problem.
   - What is the length of the shortest path in T. What does it signify.
   - What is the length of the longest path in T. What does it signify.

5. (1.5 marks) Given l-max heaps with n_i denotes the size of i^{th} max heap. How efficiently can you merge l-max heaps into one max-heap. Justify that your algorithm is indeed efficient.

6. (2 marks) For the keyword list {int, float, while, do}. How many Binary Search Trees are there. If frequencies (probabilities) are same for all key words and non-key words (words that are not in the list), which BST yields the least search cost. Draw one such BST.

7. (4 marks) **Knapsack-variant**: Given n objects $x_1, \ldots, x_n$ with weights $w_1, \ldots, w_n$, and capacity of knapsack is W. The objective is to pick $S \subseteq \{x_1, \ldots, x_n\}$ such that S is maximum subject to the constraint that $\sum_{x_i \in S} w_i \leq W$. NOTE: there is NO profit component in this problem.

   - What is the optimal substructure for this problem. Is this a candidate problem for Dynamic Programming.

   - Mention the recursive subproblem.

- What is the size of the table maintained by Dynamic programming. What is the run-time of the algorithm in asymptotic sense.

- Does there exist a polynomial-time algorithm. Justify intuitively if it exists.

8. (2 marks) Present an algorithm to find a maximum and second maximum in less than $3\lceil \frac{n}{2} \rceil - 2$ comparisons. Trace your algorithm with a suitable example. Also, anaylse its time complexity using step count method.

Indian Institute of Information Technology
Design and Manufacturing, Kancheepuram
Chennai 600 127, India
An Autonomous Institute under MHRD, Govt of India
An Institute of National Importance
COM 209T Design and Analysis of Algorithms

End Semester
19-April-2017
Duration: 3 hrs
Marks: 40

Roll No: Name:

0. Samgatha T-Shirt is 200 GSM, what does GSM stand for

1 Medium Dose

1.5 marks each

1. Calculate the step count. Also, write the tight asymptotic analysis of the following code.

```
for i = 1 to n-1
   for j = i+1 to n
       int k=1, step=1
          while (k <= j)
             k = k * 2
             step++
        print step
```

2. Solve: $T(n) = 2T(\sqrt{n}) + n \log n$. Assume suitable base values.

3. Use Master's Theorem

- $T(n) = 7T(\frac{n}{2}) + n^2$

- $T(n) = 16T(\frac{n}{4}) + n^2$

- $T(n) = 4T(\frac{n}{4}) + n^2 \log n$

4. Solve: $T(n) = T(\frac{n}{5}) + T(\frac{4n}{5}) + n$. Assume suitable base values.

5. Arrange the following running times in increasing order of asymptotic growth;
$4^{\log_4 n}, 4^{O(1)}, 3^n, 2^{n^2}, 3^{\log_2 n}, n^{O(1)}$

6. Trace insertion sort and quick sort for the input 8 2 7 3 1 5 6 4

7. Where do you find min, second min and third min in a Max Heap, what is the asymptotic tight complexity of finding the same in worst case.

8. Are Heap sort and Quick sort stable ? Justify.

9. Is it true that, if $f(n) = O(g(n))$ and $f(n) \neq \Omega(g(n))$, then $f(n) = o(g(n))$.

10. Construct two examples such that for the first one, the MERGE() of merge sort incurs the least number of comparisons and for the second example, it incurs the maximum number of comparisons. Note: the example must incur least/greatest in every level of recursion.

11. Draw a graph with 4 Articulation points, 4 bridges and 5 biconnected components. Use the minimum number of vertices to accomplish this task.

12. Consider the cycle graph on 5 vertices (C_5). Trace $L[\ ]$ algorithm and compute Articulation points, bridges and biconnected components.

13. Perform topological sorting on the graph constructed from the vertex set `Courses =` $\{DM, DAA, CO, CN, CA, VLSI, HPC, IDC\}$ and the edge set (represents the relationship 'pre-requisite')
$(CO, CA), (CA, VLSI), (CA, HPC), (DAA, ADSA), (DM, ADSA), (CO, HPC), (DM, CN), (DM, DAA)$

14. For the above graph, list all strongly connected components. Do you see any interesting observation.

15. Show that 3-coloring is in NP. What about 2-coloring ? Note: Define the problem precisely before attempting a solution.

16. Show that TSP runs in $\Omega(2^n \cdot n^2)$.

17. Professor NPC discovers the following two reductions:
`Reduction 1:` Longest Path $\leq^p$ Shortest Path;
`Reduction 2:` Shortest Path $\leq^p$ Longest Path. What do you infer from these reductions.

18. For OBST, write the decision version and the verification version. Clearly mention, the input and the question to be asked. Between the two, which version belongs to NP.

19. Show that Clique is NP-Hard by considering vertex cover as the candidate NP-Hard problem. You must clearly mention problem definitions before attempting a reduction.

20. For each of the following scenarios, mention one computational problem

    - Respects overlapping subproblem property but not optimal substructure. Justify your answer.
    - Respects optimal substructure but not overlapping subproblem property. Justify your answer.
    - Respects neither overlapping subproblem property nor optimal substructure. Justify your answer.

2 Strong Dose

2 marks each

1. `Problem 1:` List all cycles in a graph, `Problem 2:` List all C_4's in a graph. `Problem 3:` Finding the largest cycle in a graph. What are the status of these problems (P / NP-complete / some other). Justify your answer. Note: Appropriate problem definition is must to comment about the complexity status.

2. Show that number of ways of multiplying a sequence of n-matrices is $\Omega(2^n)$. What about the number of BSTs for a set of n-key words. Prove your answers.

3. `Coin Change:` For the denominations $(1, 2, 3, 4)$ with unlimited supply of each, present a greedy algorithm to give a change for the integer x. Present a proof of correctness to show that your algorithm is indeed optimum (think of a natural optimization parameter). Does this problem belong to NP.

4. For the above coin change problem, does Dynamic Programming exist ? Justify. If so, present a recursive subproblem with clearly mentioning base values. Trace your approach for $x = 10$. What would be the size of the recursion tree if a simple recursive algorithm is designed for this problem ? Let your analysis be with respect to arbitrary x.

5. How do you find a maximum independent in a graph using the underlying decision algorithm as a black box. Pay attention to details while presenting your algorithm.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 209T Design and Analysis of Algorithms	Quiz 1 7-Feb-2017 Duration: 1hr Marks: 15

Roll No: Name:

0. Who authored the famous text 'Sorting and Searching'

1. (1 mark) Trace insertion sort for the input 5 4 -1 6 7 1. Mention all intermediate steps.

2. (1 mark) Perform the first level (just the first iteration) trace of quick sort for the input 5 4 -1 6 7 1.

3. (1.5+1=2.5 marks) Apply Master theorem for

- $T(n) = 3T(\frac{n}{2}) + n^2 \log n$

- $T(n) = 7T(\frac{n}{2}) + O(1)$

4. (1.5 marks) Solve. $T(n) = 2T(\sqrt{n}) + 2^n$

5. (1.5 marks) Calculate the step count. Also, mention the tight asymptotic complexity.

```
begin

for i = 1 to n
    j=n
      while(j >=1)
        j=j/3

end
```

6. (2 marks) Between Iterative-Fibonacci-series() and Recursive-Fibonacci-series(), which one is asymptotically faster. Justify.

7. (1 mark) Are $n^{O(1)}$ and $O(n^1)$ same. Justify.

8. (1.5 marks) (i) $n^2 sinn = O(n)$, (ii) $n^2 sinn = \Omega(n)$. Of the two, which one is true. Justify.

9. (0.75+0.75+1.5 = 3 marks) Consider the following pseudocodes.

```
Dosomething1(int n, int A[])
  for i= 1 to n-1
    for j=i+1 to n
       if(A[j] < A[i])
         swap(A[j],A[i])


Dosomething2(int n, int A[])
for i= 1 to n
    for j= 1 to n-i
       if(A[j] > A[j+1])
             swap(A[j],A[j+1])
```

(i) What do Dosomething1 and Dosomething2 compute ?

(ii) Which is efficient between the two. Justify.

(iii) What are the loop invariants maintained by the two algorithms ?

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 209T Design and Analysis of Algorithms	Quiz 2 16-March-2017 Duration: 1hr Marks: 15

Roll No: Name:

0. Expand CLRS

1. (1 mark) Use $\theta(n)$ algorithm to construct a min-heap for the input sequence $(6, 5, 4, 3, 2, 1)$.

2. (2 marks) Show that for any n-size input sequence, min-heap construction in bottom-up fashion incurs $\theta(n)$ time. Any observation used as part of the proof must be proved separately.

3. (2 marks) Show that **any** algorithm to search an element in a sorted list incurs $\theta(\log n)$ in worst case.

4. (3 marks) For the keyword list $\{int, float, double, char\}$, how many different Binary Search Trees are there ? Suppose the probability of finding a keyword is $\frac{1}{2}$ and a non-keyword is also $\frac{1}{2}$, i.e., $P_i = Q_i = \frac{1}{2}$, how many optimum solutions are there ? Draw any two optimum solutions (draw the associated BST).

5. (2 marks) Recall the assembly-line scheduling: find a shortest path (fastest time) from the entry to exit. Can we use Dijkstra's shortest path algorithm to solve assembly-line scheduling. Justify with a good reasoning.

6. (2 marks) Recall 0/1-knapsack problem; present a counter example to each of the three greedy strategies discussed in class.

7. (3 marks) Consider the following case study:
Food items: `(Gulab jamun, Rasgulla, Vada, Juice, Cutlet, Carrot Halwa)`
Weights in grams (integers): $w_1, w_2, w_3, w_4, w_5, w_6$
Calories in grams (integers): $c_1, c_2, c_3, c_4, c_5, c_6$
Fat in grams (integers): $f_1, f_2, f_3, f_4, f_5, f_6$
Capacity of the stomach: 200 grams

   - An item when taken must be consumed fully. No partial consumption. The objective is to find a set of items that maximizes the calories of the food items consumed. If DP exists, then present the optimal substructure and recursive subproblem.

   - An item when taken must be consumed fully. No partial consumption. The objective is to find a set of items that minimizes the fat associated with the food items consumed. If DP exists, then present the optimal substructure and recursive subproblem.

   - What is the time complexity of the above two approaches if there are n food items. Does it run in polynomial time.

Extra Credit: (3 marks) Given n keywords, how many different BSTs are there. Prove your answer. Does it have any similarity with Matrix-Chain multiplication case study.