DAA - Quiz 2 - Online - 24-March - 12.00 - 13.30

1. Consider the complete graph on 4 vertices whose all edge weights are 2. How many different MSTs are possible.

   (a) 12
   (b) 14
   (c) 16 the number of spanning trees in a complete graph is n^{n-2}.
   (d) 20

2. Consider the cycle graph on n vertices with all edges weights 4. How many different MSTs are possible.

   (a) n If we remove any one edge in C_n, we get T_n and there are n such possibilities.
   (b) $n-1$
   (c) $\sqrt{n}$
   (d) $\frac{n}{2}$

3. Which of the following is(are) true

   (a) Both BFS and DFS can answer shortest path problem `DFS cannot compute spath`
   (b) Both BFS and DFS can be used for acyclicity testing `missing edges can help`
   (c) Both BFS and DFS can test whether a graph is bipartite `odd cycle testing can be done with both`
   (d) Test for even cyclicity is possible with BFS but not with DFS. `with both bfs/dfs, there is no straightforward algorithm`

   (a) only b
   (b) b and c
   (c) all are true
   (d) only c

4. Pick all that are false

   (a) SPath on weighted graphs can be computed using BFS in polynomial-time `NO. if weights are` 2^n`, then the size of the graph after transformation using edge sub-division technique is exponential and thus any algo is expo`
   (b) DFS can answer longest path in trees in polynomial-time `True. if we run two DFS; the first DFS to get the first deepest node and if we run second DFS from the first deepest node, we get the second deepest node; thus lpath`
   (c) BFS can answer longest path in trees in polynomial-time `True. similar to the above, two bfs can answer lpath`
   (d) SPath on weighted graphs can be computed in using BFS in exponential-time `True. Reason given in option 1`

   (a) a and c
   (b) all are false
   (c) only c
   (d) only a

5. A graph G is a tree if and only if

   (a) G is connected and G is acyclic
   (b) G is acyclic and has $(n-1)$ edges
   (c) G is connected and has $(n-1)$ edges
   (d) G has $(n-1)$ edges and there exists a unique path between any pair of vertices

   (a) only d
   (b) a and b
   (c) a,b,c
   (d) all are true `using MI, the above observations can be proved`

6. Pick all that are true

   (a) Test for 2-colorability can be answered using DFS `odd cycle testing`
   (b) Test for 2-colorability can be answered using BFS `odd cycle testing`
   (c) Test for 3-colorability can be answered using DFS `pick an independent set and remove it from the graph and check whether the remaining graph is bipartite using dfs/bfs.`
   (d) Test for 3-colorability can be answered using BFS

   (a) only b
   (b) b and d
   (c) a and b
   (d) all are true

7. Which of the following statements are true

   (a) For a given graph, BFS tree is unique (from a fixed vertex, when we run BFS, we get exactly one BFS tree) `since the order of exploration of neighbors can be different, we get more than on BFS tree, false)`
   (b) For a given graph, DFS tree is unique (false, the reason same as above)
   (c) For a given graph, more than one BFS tree is possible (true)
   (d) For a given graph, more than one DFS tree is possible (true)

   (a) c and d
   (b) a and b
   (c) only c
   (d) all are true

8. The asymptotic tight complexity of checking whether a given graph has C_4 or not, is

   (a) $\theta(n+m)$
   (b) $\theta(n^2)$
   (c) $\theta(m\Delta^2)$, where Δ is the maximum degree of the graph for each edge $e = \{u, v\}$, remove $N_G(u) \cap N_G(v)$. Now, check whether there is an edge $\{w, z\}$ such that $w \in N_G(u)$ and $z \in N_G(v)$.
   (d) $\theta(n^2 \log n)$

9. BFS can answer

(a) even cyclicity but not odd cycles (there is no straightforward algorithm using just BFS)
(b) cycles of specific length $l \geq 3$ (there is no straightforward algorithm using just BFS, some additional computation is needed.)
(c) odd-cycle free testing (true)
(d) even-cycle free testing (there is no straightforward algorithm using just BFS)

(a) only b
(b) c and d
(c) only c
(d) only a

10. Given a graph G and an edge $e = \{u, v\}$, what is the complexity of checking whether there is a short cycle containing e

    (a) $\theta(m^2)$
    (b) $\theta(m+n)$ run BFS at u and obtain the spath between u and v, spath together with e yields scycle
    (c) $\theta(2^m)$
    (d) $\theta(n \log n)$

11. Which one of the following yields MST

    (a) For each non-cut edge e, remove the max weight edge from the cycle containing e
    (b) Choose an arbitrary edge e, if e belongs to a cycle C, then remove max weight edge from C. Repeat.
    (c) Perform BFS and return BFS tree `false, not always`
    (d) Choose a Max weighted cycle, remove max weight edge from the cycle. Repeat.

    (a) All are true
    (b) a and b
    (c) a and c
    (d) a,b,d

12. Consider a graph G on 6 vertices with edges $\{\{1,2\},\{1,5\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{3,5\},\{3,6\},\{4,6\}\}$. It is proposed to perform BFS and the output is given in level order. Pick all that correspond to some BFS tree. BFS is run at vertex 1.

    (a) $1, 2, 3, 5, 4, 6$
    (b) $1, 5, 3, 2, 4, 6$
    (c) $1, 5, 4, 2, 3, 6$
    (d) $3, 5, 4, 1, 2, 3$

    (a) a and b
    (b) only a
    (c) only b
    (d) a,b and c

13. In the graph given above, we wish to perform DFS and the output is given in level order. DFS is run at vertex 1.

    (a) $1, 5, 3, 6, 4, 2$
    (b) $1, 2, 4, 6, 3, 5$
    (c) $1, 3, 6, 5, 4, 2$
    (d) $1, 2, 3, 5, 6, 4$

    (a) a and b
    (b) only c
    (c) all are true
    (d) c and d

14. The maximum number of edges in a graph in which every induced cycle is C_4 is

    (a) $\binom{n}{2}$
    (b) $\binom{n}{4}$
    (c) n^2
    (d) $\frac{n^2}{4}$. `complete bipartite graphs with` $n/2$ `vertices on each side`

15. Pick all that are true.

    (a) Krukal's algorithm maintains acyclicity as its invariant and on stopping the algorithm, we get connectedness. `True`
    (b) Prim's algorithm maintains connectedness as its invariant and on stopping the algorithm we get acyclicity. `True`
    (c) Prim's algorithm ensures connectedness and as a consequence acyclicity at each iteration. `True`
    (d) Kruskal's algorithm ensures acyclicity and as a consequence connectedness at each iteration. `False, only at the termination we get connectedness, it is a forest at other iterations`

    (a) a and b
    (b) a,b,c
    (c) all are true
    (d) only d

16. The linear-time algorithm to compute longest paths in trees invokes

    (a) DFS twice `True`
    (b) DFS exactly once
    (c) BFS once and DFS once `bfs to get the first deepest node followed by dfs to get the second deepest node, True`
    (d) BFS twice `True`

17. Given an edge weighted graph with all weights distinct, pick all that are FALSE

    (a) Prim's algorithm output and BFS tree output are same. `False, BFS does not weights into account`
    (b) Second best MST is unique `False;` $w(a, b) = 2, w(a, c) = 3, w(a, d) = 9, w(b, c) = 7, w(c, d) = 5$ `this graphs has two MSTs of weight 14.`

(c) MST is unique `True`

(d) MST is unique if G is acyclic. `False, if` G `is disconnected, then we will only get a forest, not a tree`

(a) a and b

(b) a,b,d

(c) all are false

(d) only b

18. In the context of Articulation points (cut-vertices), pick all that are true.

    (a) v is a cut-vertex if there exists a branch at v such that there is no back edge from any descendant(v) to any ancestor(v) `True`

    (b) the above property must be true for each branch at v `Not required for each branch, False`

    (c) the root of the DFS tree is an AP if the degree of root is at least 2. `True`

    (d) v is a cut-vertex if v does not belong to a cycle. `False, two cycles intersecting at a vertex, the intersecting vertex is an AP`

    (a) a,c,d

    (b) all are true

    (c) a and c

    (d) b and d

19. The edge $\{u, v\}$ is a bridge if

    (a) no back edge from descendant(v) to v `False`

    (b) no back edge from v to ancestor(u) `False`

    (c) no back edge from descendant(v) to ancestor(v). Must be true for each branch at v. `False we may have back edge from` v `to the top, in this case` uv `is not a bridge`

    (d) no back edge from descendant(v) to ancestor(v). Must be true for at least one branch at v. `False`

20. In the context of topological ordering, pick all that are true

    (a) It is applied on directed acyclic graphs. `True`

    (b) To get ordering among vertices and ensuring pre-requisite relationship between vertices `True`

    (c) Can answer SPATH and LPATH in DAGs `True`

    (d) Runs in linear time in the input size `True`

    (a) All are true

    (b) only a

    (c) a and b

    (d) a,b,c

21. Consider a directed graph on 7 vertices with edge set $\{(1,2),(1,3),(2,4),(3,4),(3,5),(6,5),(7,5)\}$. Which of the following represents the topological ordering sequences of G

    (a) $1,2,3,4,6,7,5$

    (b) $6,7,1,2,3,4,5$

(c) $7, 1, 6, 3, 2, 5, 4$
(d) $3, 2, 1, 4, 5, 6, 7$

(a) b,c
(b) a,b
(c) only a
(d) a,b,c

22. In the context of MST, which one of the following is true.

(a) The running time of Prim's algorithm is $O(m \log n)$ and Krukal's algorithm is $O(m \log m)$ `True`
(b) In asymptotic sense, both algorithms are of same complexity `True,` $O(m \log m) = O(m \log n^2) = O(m \log n)$
(c) Krukal's algorithm is less efficient as it involves sorting as a subroutine. `False`
(d) Prim's algorithm is more efficient as it invokes find min as a subroutine. `False`

(a) only a
(b) a and b
(c) all are true
(d) a,c,d

23. Consider a graph on 4 vertices whose adjacency matrix is given below. $A[i, j]$ denotes the weights of the edge $\{i, j\}$. Assuming the Dijkstra is run at vertex a. Which of the following adjacency matrix is the result of Dijkstra's Algorithm.

	a	b	c	d
a	0	2	7	15
b	2	0	3	10
c	7	3	0	4
d	15	10	4	0

(a) The output matrix is

	a	b	c	d
a	0	2	7	9
b	2	0	3	7
c	7	3	0	4
d	9	7	4	0

(b) The output matrix is

	a	b	c	d
a	0	2	5	9
b	2	0	3	7
c	5	3	0	4
d	9	7	4	0

(c) The output matrix is

	a	b	c	d
a	0	2	5	15

b	2	0	3	7
c	5	3	0	4
d	15	7	4	0

(d) The output matrix is

	a	b	c	d
a	0	2	5	9
b	2	0	3	7
c	5	3	0	4
d	9	7	4	0

24. Pick the correct statements (i) Prims' algorithm can be used as Black box to get Max weight spanning tree, whereas Dijkstra's algorithm cannot be used to obtain LPATH `True` (ii) Dijkstra can detect negative weighted cycles in the graph `False` (iii) Dijkstra can be fine tuned to handle negative edge weights by linear scaling all edge weights by a threshold so that the resultant graph has all positive edge weights`False` (iv) Dijkstra works fine if negative edge weights are incident on the source vertex. `True`

   (a) (i),(ii)
   (b) (i),(iii),(iv)
   (c) (i),(iii) Linear scaling does not work.
   (d) all are false

25. Principle of optimality for SPATH states that subpaths of any shortest path is shortest. And for, LPATH it states that subpaths of any longest path is longest. Pick all that are true. (i) Principle of optimality for SPATH is true whereas for LPATH it is false `TRUE` (ii) There are graphs where principle of optimality for LPATH is true `YES, for example, Trees` (iii) Proof of correctness of Dijkstra is based on principle of optimality for SPATH. `True` (iv) There is no algorithm to compute LPATH. `False, trivial algorithm runs in` $n!$

   (a) (i), (ii), (iii)
   (b) only (iii)
   (c) (i) and (iii)
   (d) all are true

26. Let A be the adjacency matrix of an undirected unweighted graph G. What do A^2 entries represent. Pick the most appropriate choice.

   (a) Diagonal entries represent degrees and other $A(i, j)$ entries represent paths (need not be simple; vertices can repeat) of length of at most 2 `True`
   (b) Diagonal entries are zero and it represents nothing `False`
   (c) Diagonal entries denote the number of cycles at each vertex `False`
   (d) Computation of A^2 incurs exponential-time in the input size `False`.

27. Diameter of a graph: Largest shortest path; of all SPaths choose the one with maximum weight. Pick the most appropriate choice.

   (a) The computation of Diameter and longest path are one and the same. `False`
   (b) The computation of Diameter does not obey the principle of optimality for SPATH `False`
   (c) Dijkstra as well as BFS can be used to compute Diameter of a graph. `True`

(d) Diameter is defined only for undirected graphs. `False, defined for DirectedGhs as well`

28. Suppose the given graph has the property that no two cycles overlap each other, that is there is no common edge between cycles. Pick the most appropriate choice.

    (a) Longest path is polynomial time whereas longest cycle is exponential time `False`

    (b) both are exponential time. `False`

    (c) both do not obey principle of optimality and hence there is no algorithm. `False`

    (d) Longest path and longest cycle can be computed in polynomial time. `True, since there is no overlap between cycles`

29. Pick the correct statement

    (a) Exactly one binary tree can be constructed from the given preorder and postorder `False`

    (b) More than one binary tree can be constructed from the given preorder and postorder `True`

    (c) To reconstruct the associated binary tree, one must need inorder, preorder, and postorder `False`

    (d) A binary tree cannot be reconstructed given its traversal. `False`

30. How many different binary search trees are there with 3 nodes. (Assume three nodes satisfy $A < B < C$)

    (a) 3

    (b) 4

    (c) 5

    (d) none

31. Let (x, y) represents the number of cut-vertices (articulation points) and the number of bridges (cut-edges) in a graph. The upper bounds for x and y are;

    (a) (n, n), where n is the number of vertices in the given graph

    (b) $(n-1, n-2)$

    (c) $(n-2, n-1)$

    (d) $(n, n-2)$

32. Let D_1 and D_2 denote the degree sequences of two different trees T_1 and T_2, respectively. Given that degree sum of D_1 equals the degree sum of D_2, which of the following is true.

    (a) $|V_1| = |V_2|$ $D_1 = D_2; 2(n_1 - 1) = 2(n_2 - 1); n_1 = n_2$

    (b) $|V_1| = |V_2| + 2$

    (c) $|V_1| = |V_2| + 1$

    (d) None

33. Given a tree T, the objective is to find the height of T and count the number of leaves in T,

    (a) Only pre-order traversal can be used.

    (b) Either pre-order or in-order.

    (c) All three traversals can be used.

    (d) Both height and the count on the number of leaves can not be computing using tree traversals.

34. Let (a, b) represents the number of labelled and unlabelled binary trees on '3' nodes. Then,

    (a) $(15, 5)$

(b) $(30, 5)$

(c) $(12, 4)$

(d) $(6, 3)$

35. Let $A = \{1, \ldots, n\}$ and $B = \{x \mid x \subseteq A\}$. A graph G is constructed on the set B with $V(G)$ being the elements of B. Two vertices x, y in $V(G)$ are adjacent if $x \subset y$. What is the minimum and maximum (in/out)degree of G. Minimum means, minimum over all in-degrees and out-degrees.

(a) MIN$= 2^{n/2} - 1$ MAX$= 2^n - 1$ `emptyset is a subset of every other set, thus max degree is` $2^n - 1$. `Consider a set containing` $n/2$ `elements, its outdegree (all sets containing this set) is` $2^{n/2} - 1$

(b) MIN$= 2^{n/2} - 1$ MAX$= 2^{n-1}$

(c) MIN$= 2^{n-1}$ MAX$= 2^n - 1$

(d) none of the above