Assignment 3, 23-March, 5 pm.

1. What is the complexity of checking whether a given graph is triangle-free. Present an algorithm and its analysis.
2. How do you use BFS as a black box to find single-source shortest paths (shortest paths from a fixed vertex to other vertices) in weighted graphs. Analyze the complexity of your algorithm interms of the input size.
3. Principle of optimality: any subpaths of a shortest path is shortest. Show that principle of optimality is not always true for longest paths. That is, subpaths of a longest path need not longest paths. Present a counter example.
4. How do you use BFS as a black box to test whether a given graph is 3-colorable or not (minimum number of colors to color the graph is 3). Present an algorithm and its analysis.
5. Which are the data structures used to implement (i) BFS (ii) DFS.
6. Show that slanting and cross edges do not exist in DFS, and back edges do not exist in BFS.
7. How do you find the shortest cycle in an unweighted graph. Present an efficient algorithm along with its analysis.
8. Present a greedy algorithm to find a longest cycle in an unweighted graph.
9. Show that 'max-degree' greedy and 'edge-coloring' greedy do not give optimum for policemen problem.
10. SECOND-BEST-MST: a spanning tree T' whose weight $W(T') = \min\{W(T'') \mid T''$ is a ST and T'' is not a MST$\}$. Present an algorithm with a tight analysis to compute second best MST.
11. Consider a weighted graph with all edge weights distinct. Is MST unique. How about second best MST.
12. Suppose there are negative edges incident on the source vertex and no negative edges in other part of the graph. Will Dijkstra work fine.
13. Consider a vertex weighted graph (no weights on edges, vertex weighted instead) with a fixed source. How do you use Dijkstra as a black box to find shortest paths from a fixed source.
14. Show that if a graph is acyclic and has $n - 1$ edges, then the graph is connected.
15. Present a linear time algorithm to compute a longest path in a tree.
16. Suppose, we run Prim's algorith from a vertex s. Does the tree output by Prim's algorithm give shortest path from s to all other vertices.
17. Which are the data structures used to implement (i) Prim's MST (ii) Kruskal's MST (iii) Dijkstra's SPATH
18. Given an unweighted graph, present two different greedy algorithms to find (i) a spanning tree with maximum number of leaves (ii) a spanning tree with minimum number of leaves.