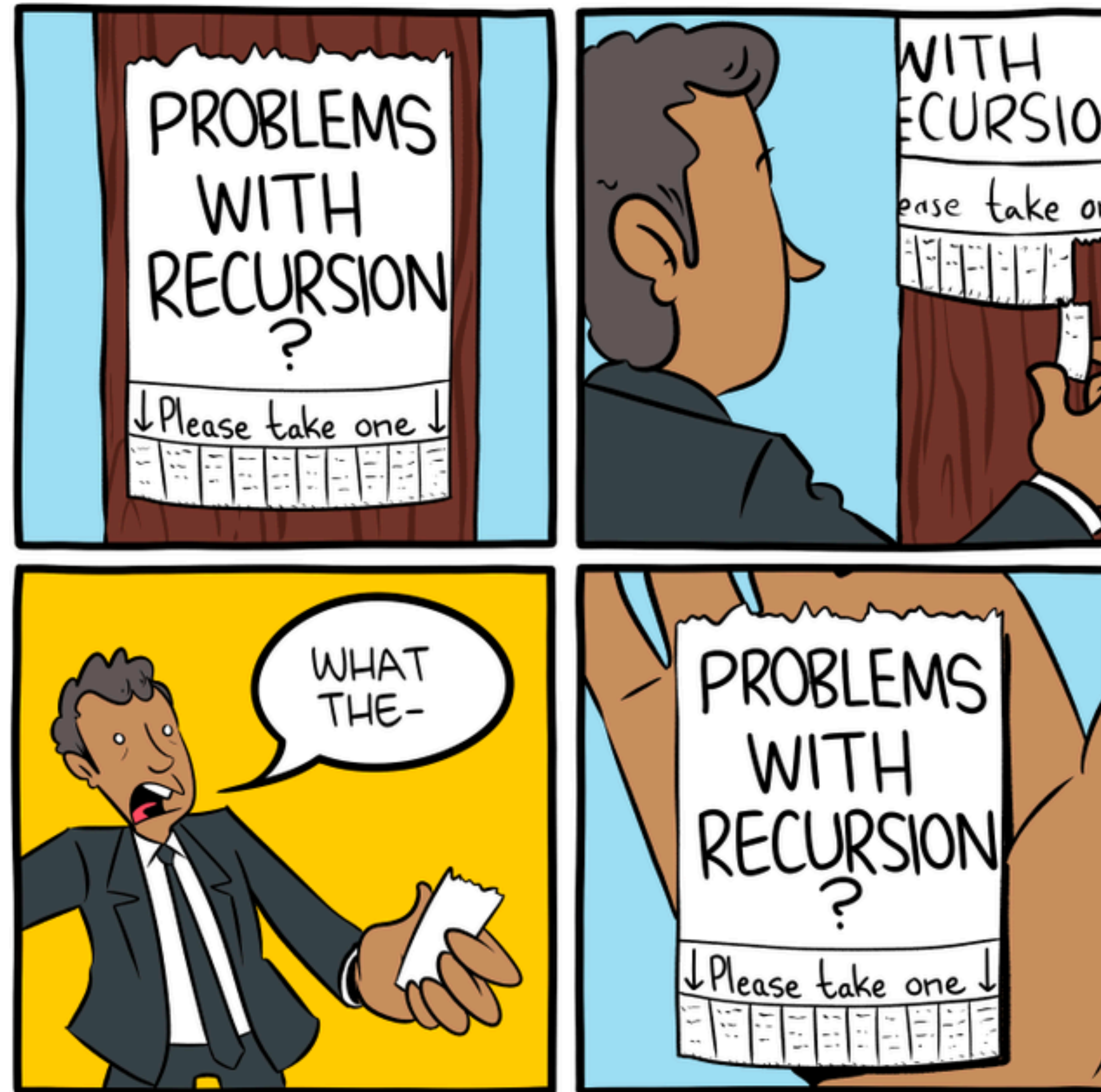


# Discussion 3



[smbc-comics.com](http://smbc-comics.com)

[www.yellkey.com/whatever](http://www.yellkey.com/whatever)

# Midterm 1

- Congrats! First exam done
- Everyone learns at their own pace
  - don't compare yourself to others
- Lower exam averages are normal in CS classes

# Midterm 1

- Midterm 1 is a small part of 61a
  - 13% Midterm 1
  - 16% Midterm 2
  - 25% Final
  - 45% Assignments

# Midterm 1: Resources

- Advising appointments
  - <https://cs61a.org/articles/advising/>
- CSM
  - Type @879 into piazza search bar
    - Title: "[CSM] Section Signups Reminder"
- Course Staff

# Feedback!

- Go a little faster
  - I agree
- Responses to student questions are a little long



# Recursion

1. **Base Case:** simplest function input
2. **Recursive Call** (smaller problem)
  - recursive leap of faith (trust yourself!)
3. **Solve the larger problem**

```
def factorial(n):  
    """Return the factorial of N, a positive integer."""  
    if n == 1:  
        return 1  
    else:  
        return n * factorial(n - 1)
```

# Q1: Recursive Multiplication

Write a function that takes two numbers m and n and returns their product. Assume m and n are positive integers. Use recursion, not mul or \*.

```
def multiply(m, n):  
    """ Takes two positive integers and returns their product using  
    recursion.  
    >>> multiply(5, 3)  
    15  
    """  
    "*** YOUR CODE HERE ***"
```

Hint:  $5 * 3 = 5 + (5 * 2) = 5 + 5 + (5 * 1)$ .



# Q1: Recursive Multiplication

```
def multiply(m, n):
```

$m=1?$

```
    if n==1:
```

```
        return m
```

```
    return multiply(m, n-1) + m
```

$$3 \times 1 = 3$$

$$1 \times 3 = 1 + 1 + 1 = 3$$

$$5 \times 3 = 5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3$$

Hint:  $5 * 3 = 5 + (5 * 2) = 5 + 5 + (5 * 1).$



# Q1: Recursive Multiplication

For the recursive case, what does calling `multiply(m - 1, n)` do?  
What does calling `multiply(m, n - 1)` do? Do we prefer one over the other?

# Q3: Find the Bug

Find the bug with this recursive function.

```
def skip_mul(n):  
    """Return the product of n * (n - 2) * (n - 4) * ...
```

```
>>> skip_mul(5) # 5 * 3 * 1
```

```
15
```

```
>>> skip_mul(8) # 8 * 6 * 4 * 2
```

```
384
```

```
"""  
if n == 1: return 1  
if n == 2:
```

```
    return 2
```

```
else:
```

```
    return n * skip_mul(n - 2)
```

1. Base Case
2. Recursive Call
3. Solve the larger problem

# Q5: Merge Numbers

Write a procedure `merge(n1, n2)` which takes numbers with digits in decreasing order and returns a single number with all of the digits of the two, in decreasing order. Any number merged with 0 will be that number (treat 0 as having no digits).

Use recursion

>>> `merge(31, 42)`

4321

>>> `merge(21, 0)`

21

>>> `merge(21, 31)`

3211

>>> `merge(54, 32)`  
5432

$\div 10$

// 10

3-4m



# Q5: Merge Numbers

```
def merge(n1, n2): # n1 // 10
    """ Merges two numbers by digit in decreasing order
```

```
>>> merge(31, 42)
```

```
4321
```

```
if n1 == 0:
```

```
    return n2
```

```
if n2 == 0:
```

```
    return n1
```

```
if n1 % 10 < n2 % 10:
```

```
    merge(n1 // 10, n2) * 10 + (n1 % 10)    # n1 % 10 at end
```

```
else:
```

```
    merge(n1, n2 // 10) * 10 + (n2 % 10)    # n2 % 10 at end
```

(31, 42)

if L → (3, 42)

else L → (3, 4)

if L → (0, 4)

(1, 2) 1 < 2

4321

(3, 2) 3 < 2

(3, 4) 3 < 4

R: 4

1  
2  
3



# Q4: Recursive Hailstone

Recall the hailstone function from Homework 2. First, pick a positive integer  $n$  as the start. If  $n$  is even, divide it by 2. If  $n$  is odd, multiply it by 3 and add 1. Repeat this process until  $n$  is 1. Write a recursive version of hailstone that prints out the values of the sequence and returns the number of steps.

```
>>> a = hailstone(10)
```

```
10
```

```
5
```

```
16
```

```
8
```

```
4
```

```
2
```

```
1
```

```
>>> a
```

```
7
```

# Q4: Recursive Hailstone

pick a positive integer  $n$  as the start. If  $n$  is even, divide it by 2. If  $n$  is odd, multiply it by 3 and add 1. Repeat this process until  $n$  is 1. Write a recursive version of hailstone that prints out the values of the sequence and returns the number of steps.

```
def hailstone(n):  
    """Print out the hailstone sequence starting at n, and return  
    the number of elements in the sequence.
```



# Q6: Merge Numbers

Write a function `is_prime` that takes a single argument `n` and returns `True` if `n` is a prime number and `False` otherwise.

Assume  $n > 1$ .

```
>>> is_prime(2)
```

```
True
```

```
>>> is_prime(16)
```

```
False
```

```
>>> is_prime(521)
```

```
True
```

# Q6: Merge Numbers

```
def is_prime(n):  
    """Returns True if n is a prime number and False otherwise.
```



# Feedback + Attendance

[www.yellkey.com/another](http://www.yellkey.com/another)

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