Assignment-01

1. If $z(x+y) = x^2+y^2$, then Show that

$$\left[\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]^2 = 4\left[1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]$$

2. If $z = (1-2xy+y^2)^{-\frac{1}{2}}$, Prove that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = y^2 z^3$

3. If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ Prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

4. If $V = f(r)$, where $r = \sqrt{x^2+y^2+z^2}$, Prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$$

5. If $u = \log(x^3+y^3+z^3-3xyz-3xyz)$, Show that

(a) $$\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)^2 = \frac{9}{(x+y+z)^2}$$

(b) $$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

6. If $x^x y^y z^z = c$, show that $\frac{\partial^2 z}{\partial x \partial y} = -[x \log_e x)^{-1}$,

When $x = y = z$.

7. If $V = r^m$, where $r = \sqrt{x^2+y^2+z^2}$, Show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1) r^{m-2}$$

8. If $u = \sin^{-1}\left[\dfrac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right]$, then using

Euler's theorem, Prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = -3\tan u$

9. If $u = \sin^{-1}\left[\dfrac{x^2+y^2}{x+y}\right]$, Prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$

& Show that

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = \tan^3 u$$

10 Verify the Euler's theorem for the function
$z = (x^{\frac{1}{2}}+y^{\frac{1}{2}})(x^n+y^n)$

11. If $u = \log \dfrac{x^5+y^5+z^5}{x^2+y^2+z^2}$, Prove that

$$xu_x + yu_y + zu_z = 3.$$

12. If $u = \dfrac{x^3y^3z^3}{x^3+y^3+z^3} + \log\left[\dfrac{xy+yz+zx}{x^2+y^2+z^2}\right]$ find

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$$

13. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, Prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

& $x^2\dfrac{\partial^2 u}{\partial x^2} + y^2\dfrac{\partial^2 u}{\partial y^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} = \sin 4u - \sin 2u$

$= 2\cos 3u \sin u$

14. If $w = x^2+y^2$, $x = r-s$, $y = r+s$, find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s.

15. If $H = f(y-z, z-x, x-y)$, Prove that

$$\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}=0$$

16. If $u = x \log yx$, where $x^3+y^3+3xy=1$, find $\frac{du}{dx}$

17. find $\frac{dy}{dx}$ if $x^4+y^4=4a^2xy$

18. If $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$, $w = \frac{z}{x-y}$

Prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$

19. If $u^3+v^3+w^3 = x+y+z$, $u^2+v^2+w^2 = x^3+y^3+z^3$, $u+v+w = x^2+y^2+z^2$, then

Prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$

20. Evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$, where $u = x+y-z$
$v = x-y+z$
$w = x^2+y^2+z^2-2yz$