Some extensions of the CAPM for individual assets

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A R T I C L E   I N F O

A B S T R A C T

There is ample evidence that stock returns exhibit non-normal distributions with high skewness and excess kurtosis. Experimental evidence has shown that investors like positive skewness, dislike extreme losses and show high levels of prudence. This has motivated the introduction of the four-moment capital asset pricing model (CAPM). This extension, however, has not been able to successfully explain average returns. Our paper argues that a number of pitfalls may have contributed to the weak and conflicting empirical results found in the literature. We investigate whether conditional models, whether models that use individual stocks rather than portfolios and whether models that extend both the moment and factor dimension can improve on more traditional static, portfolio-based, mean–variance models. More importantly, we find that the use of a scaled coskewness measure in cross-section regression is likely to be spurious because of the possibility for the market skewness to be close to zero, at least for some periods. We provide a simple solution to this problem.

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1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has been a major player in empirical asset pricing for more than half a century. Its simplicity and theoretical appeal appear to have strongly outweighed the paucity of empirical evidence in its favour. The extant body of existing empirical work has largely rejected the CAPM (Black, Jensen, & Scholes, 1972; Douglas, 1969; Fama & French, 1992; Fama & MacBeth, 1973; Lintner, 1965). Of course, there is no shortage of explanations. One of these is that beta is not the only relevant systematic risk. Basu (1977) finds that the earnings–price ratio has additional explanatory power for average returns. Similar conclusions were reached by Banz (1981) for market capitalization, Bhandari (1988) for leverage, and Rosenberg, Reid, and Lanstein (1985) for the book-to-market ratio.

The quest for explaining average stock returns has led to extensions of the traditional CAPM in two main directions. The first was based on the fact that the CAPM is actually a conditional model. As Jagannathan and Wang (1996) emphasise, the CAPM may not hold unconditionally even if the CAPM held conditionally. Conditional versions of the CAPM have had limited incremental success (Lettau & Ludvigson, 2001).

A second direction focused on the specification of the empirical model by adding more proxies for systematic risks. The most notable study in this direction is Fama and French (1993), who introduce a

three-factor model with the market portfolio and two other factors: SMB (the return of a portfolio of small minus big capitalisation stocks) and HML (the return of a portfolio of high minus low book-to-market ratio stocks). Carhart (1997) extends the Fama and French three-factor model by adding a momentum factor based on previous stock performance, thereby providing the most popular model in empirical asset pricing. Carhart's model is criticised by Lewellen, Nagel and Shanken (2005) who observe that the abundance of models capable of explaining the CAPM's empirical failures suggests that it is relatively easy to discover variables correlated with size and book-to-market ratio, and also that the R-squared is an inappropriate model test measure.

While the Fama–French–Carhart approach extension focuses on using portfolios other than the market, some studies argued that the problem lies in the limits of covariance (beta) risk to fully represent systematic risk and not in the choice of the market or non-market portfolios. Kraus and Litzenberger (1976) pioneered extensions based on increasing the moments of the investor optimisation problem, and introduced skewness into the CAPM. Harvey and Siddique (2000a, 2000b) used a conditional skewness formulation. The fourth moment, kurtosis, was introduced by Dittmar (2002).

Single factor (market portfolio) multi-moment models are theoretically more appealing than multi-factor single moment models. Multi-moments models are grounded in theory, and can be derived either from a utility optimisation or a statistical optimisation perspective. A stylized fact in finance is that stock returns are far from normally distributed. There is evidence that returns are both skewed and leptokurtic (Taylor, 2005). However, the standard CAPM implies elliptically distributed returns and/or investors that have a quadratic utility function. Consequently, the inclusion of skewness and kurtosis in asset pricing

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models should help investigators mitigate the limitations of the mean–variance-only approach. Adopting a multi-moment approach, therefore, enriches the internal validity of asset pricing models by allowing for a richer set of systematic risks. In other words, the traditional CAPM might be misspecified, and thus might not be able to explain satisfactorily the cross-section of average stock returns.

From a utility theory perspective, the standard CAPM imposes a quadratic utility function on investors. Theoretical arguments suggest the preference of economic agents for positive skewness and aversion to large losses (Fang & Lai, 1997; Kahneman & Tversky, 1979). In Kahneman and Tversky (1979), prospect theory investors attribute more weight to losses than to gains. More flexible utility functions have been shown to be consistent with skewness preference and kurtosis aversion (Arditti, 1967; Dittmar, 2002; Kraus & Litzenberger, 1976).

In practice, the distribution of returns and the shape of the tails of such a distribution have become a matter of concern for investors and regulators in recent years. Indeed, since the October 1987 stock market crash, extreme gains and losses have become common features of financial markets. The world has seen major crises in 1997 (the Asian financial crisis), 2000 (the high-tech bubble crisis), 2008 (the credit crunch), and 2010 (the sovereign debt crisis). Higher-moments based models are therefore more likely to capture the crises-related systematic risks exhibited by modern financial markets.

Our contribution is twofold. First, we integrate both directions, extending the factors as well as the moments, and use these in a conditional setting. We recognise that the truth can be complex. First, the true market portfolio is unobservable and replacing it with a proxy can lead to missing factors that may well be correlated with portfolios such as Fama and French (1993) small-minus-big (SMB) and high-minus-low (HML) portfolios. Second, using a static (unconditional) CAPM when the true model is conditional can also give rise to a second factor (Jagannathan & Wang, 1996). Thus, even if investors were optimising in a mean–variance world, beta alone may not be sufficient to explain average returns. Third, investors may well be optimising in a mean–variance–skewness–kurtosis world, which would give rise to missing systematic risks from the empirical CAPM. Using a four-moment CAPM would therefore mitigate this limitation.

Our second contribution is to demonstrate the advantage of using individual stocks, rather than portfolios of stocks, in empirical tests of asset pricing models. The common practice when testing asset pricing models is to build portfolios of stocks and then investigate the return-beta relationship in cross-sectional regressions. More recently, Ang, Liu, and Schwarz (2008) suggest that focusing instead on individual stocks leads to more efficient tests of factor pricing. The common practice in empirical asset pricing of forming portfolios has been motivated by an attempt to reduce beta estimation errors, as doing so reduces idiosyncratic risk. However, Ang et al. (2008) argue that the reduction in standard errors of the estimated betas does not lead to more precise estimates of the risk premia. Rather, forming portfolios causes a lower dispersion of the estimated betas and a loss of information that, together, result in higher standard errors in the premia estimates. These authors find that the annualized beta premium is significant and positive when testing for individual stocks, whereas the construction of portfolios results in a negative and insignificant beta premium. Furthermore, with individual assets there is greater beta dispersion and therefore more information available for the cross-sectional estimation of the risk premium, leading to better precision (Kim, 1995). Finally, the focus on individual assets is more consistent with the single period investment assumed in the CAPM.

Our paper is similar in its purpose to Chung, Johnson, and Schill (2006) and Lambert and Hubner (2013). Chung et al. (2006) use up to 10 higher co-moments using portfolios as test assets. We take a different route by limiting ourselves to coskewness and cokurtosis. These moments are intuitive and easily interpretable from an investor's perspective. Our choice is also driven by the need to achieve parsimony—which helps alleviate error-in-variable and multicollinearity problems. Like

our paper, Lambert and Hubner (2013) avoid using moments higher than the fourth, but use portfolios rather than individual assets. Their approach is based on building mimicking portfolios that represent investment strategies in a similar fashion to Fama and French (1993). Although this approach has its merits, we believe our individual asset testing approach avoids the limitations associated with building portfolios, such as transaction cost and liquidity considerations. Moreover, our contribution is in the spirit that the existence of competing approaches can enrich the asset pricing literature by providing alternative ways of testing asset pricing models.

The remainder of the paper is structured as follows. In Section 2, the general four-moment CAPM is briefly outlined. Section 3 discusses the literature underpinning the higher-moment CAPM. In Sections 4 and 5, respectively, the data and the methodology are introduced. Section 6 presents the empirical results. Finally, Section 7 summarises and concludes.

2. The four-moment CAPM

The CAPM states that the expected excess return on any stock i is proportional to systematic risk (beta).

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_{i,m} \tag{1}$$

where E is the expectation operator, R_i is the return to stock i, R_m is the return to the market portfolio, R_f is the risk-free rate, and $\beta_{i,m}$ is the scaled covariance between the returns of stock i and the market.

The four-moment CAPM has been derived in a variety of ways (see for example Fang & Lai, 1997; Jurczenko & Maillet, 2001). Both expected utility optimisation (Jurczenko & Maillet, 2001) and mean–variance–skewness–kurtosis frontier optimisation (Athayde & Flores, 1997) have been employed. Let R_{it} be the simple return of asset or portfolio i at time t and R_{ft} be the return to the risk free asset at time t. Denote the mean return of portfolio p as $E(R_{pt})$. Then, the variance, skewness and kurtosis of a portfolio p are given, respectively, by

$$\sigma^2(R_p) = E[R_p - E(R_p)]^2 \tag{2}$$

$$s^3(R_p) = E[R_p - E(R_p)]^3 \tag{3}$$

$$k^4(R_p) = E[R_p - E(R_p)]^4 \tag{4}$$

We may also be interested in the contribution of a given asset to total (portfolio) skewness and kurtosis. Coskewness and cokurtosis are the counterparts of covariance. The coskewness and cokurtosis between asset i and portfolio p are defined as follows:

$$Cos(R_i, R_p) = E\left\{[R_i - E(R_i)][R_p - E(R_p)]^2\right\} \tag{5}$$

$$Cok(R_i, R_p) = E\left\{[R_i - E(R_i)][R_p - E(R_p)]^3\right\} \tag{6}$$

An asset that exhibits positive coskewness tends to perform well during volatile periods and is therefore considered less risky, whereas an asset with positive cokurtosis tends to suffer larger losses when the market is volatile and is therefore considered more risky. A mean–variance–skewness–kurtosis optimisation (see for example Jurczenko & Maillet, 2001) implies the following four-moment CAPM:

$$\begin{aligned} E(R_i) - R_f &= \lambda_\beta \frac{E(r_i \cdot r_m)}{E(r_m^2)} + \lambda_s \frac{E(r_i \cdot r_m^2)}{E(r_m^3)} + \lambda_k \frac{E(r_i \cdot r_m^3)}{E(r_m^4)} \\ &= \lambda_\beta \beta_i + \lambda_s \gamma_i + \lambda_k \delta_i \end{aligned} \tag{7}$$

where $r \equiv R - E(R)$.

It follows that for every security i, the expected excess return can be written as a linear function of the three co-moments of the asset returns

with the market portfolio. The coefficients λ_β, λ_s and λ_k are interpreted as risk premia. A positive risk premium for beta is expected as investors require higher returns for higher systematic beta risk. As for gamma, if the market portfolio returns have negative skewness, investors should prefer assets with lower coskewness and dislike assets with higher coskewness. If market portfolio returns have positive skewness, investors prefer assets with high coskewness which, as a result, become more valuable and therefore a negative coefficient for λ_s is expected as investors are willing to forego some returns for positive skewness. Finally, a positive risk premium is expected for systematic kurtosis as investors require higher compensation for assets with a greater probability of extreme outcomes (Fang & Lai, 1997).

3. CAPM extensions in the literature

Extensions of the mean variance framework have a long history in the academic literature. Arditti and Levy (1972) show that non-increasing absolute risk aversion implies investor preference for positive skewness, while Rubinstein (1973) provides a model in which expected returns are equal to the weighted sum of higher co-moments. Scott and Horvath (1980) shows that risk-averse investors with decreasing marginal utility have a positive preference for mean and skewness and a negative preference for variance and kurtosis, and thus risk-averse investors prefer higher returns and skewness, and lower variance and kurtosis.

Kraus and Litzenberger (1976) derive a three-moment CAPM by adding coskewness risk to the standard CAPM and apply it for portfolios of stocks double-sorted on beta and systematic coskewness over the period 1936–1970 using the Fama and MacBeth (1973) methodology. They find an insignificant negative intercept, a significant positive beta premium (larger than that obtained in a model when beta is the only explanatory), and a market premium for gamma which is significant and negative, consistent with expectations. However, Friend and Westerfield (1980) find results that partly contradict the findings of Kraus and Litzenberger, with a significant non-zero intercept and a time-varying coefficient for coskewness.

Lim (1989) tests the three-moment CAPM using a generalised method of moments (GMM) approach and shows that investors prefer coskewness when market returns are positively skewed, and dislike coskewness when market returns are negatively skewed. Harvey and Siddique (2000a, 2000b), test whether the inclusion of various measures of conditional coskewness improves the pricing errors in the three-factor model of Fama and French and the standard CAPM. They show that the adjusted R-squared statistics of both models improve.

Fang and Lai (1997) test the four-moment CAPM on portfolios triple-sorted on beta, coskewness and cokurtosis over three distinct five-year periods where the factor loadings are estimated using time series regressions of the cubic market model.[1] The results show a substantial improvement in R-squared for the four-moment CAPM compared to the two- and three-moment CAPM. Most importantly, the risk premia for beta and cokurtosis are significant and positive for the three sub-periods. Athayde and Flores (1997) test a four-moment CAPM for Brazilian stocks using GMM over the period 1996–1997. They conclude that skewness, rather than kurtosis, plays the most important role for Brazilian stocks. Hwang and Satchell (1999) estimate an unconditional four-moment CAPM for emerging market stocks over the period 1985–1997 using GMM and conclude that higher moments can add explanatory power to model returns for emerging markets, though with variations across countries.

Dittmar (2002) estimates a conditional four-moment CAPM using a stochastic discount factor approach. Two models are implemented: one with the equity market index as a market proxy of wealth and another

including human labour wealth. The model terms are found significant and the pricing errors are significantly reduced when human capital is included in the four-moment CAPM. Tan (1991) applies a three-moment CAPM on a sample of mutual funds and finds results that do not support the three-moment CAPM (an intercept significantly greater than zero, an insignificant beta risk premium, and an incorrect positive sign for coskewness). Hasan and Kamil (2014) test a higher-moment CAPM with coskewness, cokurtosis, market capitalization and book-to-market to model stock returns in Bangladesh, and find that coskewness and cokurtosis are weakly negatively and positively related to returns, respectively.

Lambert and Hubner (2013) test an extension of the four-moment CAPM for US returns over the period 1989–2008. When testing an augmented three-factor model of Fama and French with coskewness and cokurtosis on the 25 portfolios sorted on size and book-to-market, they find that low book-to-market portfolios are positively related to high cokurtosis risk and small sized portfolios are positively related to low coskewness, that is, they are more exposed to coskewness risk. In the cross-section they find that the augmented model reduces the pricing errors and has a higher R-squared than the simple CAPM. Kostakis, Muhammad, and Siganos (2012) test the higher-moment CAPM for UK stocks over the period 1986–2008 and find that coskewness demands a negative risk premium whereas stocks with higher cokurtosis yield higher returns on average. In particular, coskewness and cokurtosis have additional explanatory power to covariance risk, size, book-to-market and momentum factors. The alpha or unexplained return of portfolios with negative coskewness and positive cokurtosis is not eliminated in time series after controlling for size, value and the momentum factors of Carhart (1997).

Young, Christoffersen, and Jacobs (2010) examine a higher-moment CAPM in which the moments are estimated using daily data for S&P 500 Index options. They find that stocks with high exposure to change in implied market volatility and market skewness yield lower returns whereas stocks with higher sensitivity to kurtosis yield higher returns. Heaney, Lan, and Treepongkaruna (2012) test whether coskewness and cokurtosis are priced for US equity returns over the period 1963–2010, using individual assets as opposed to portfolios. They find little evidence that the higher moments are priced and show that these are encompassed by size and book-to-market factors. In particular, size tends to eliminate the significance of cokurtosis, which is found unexpectedly to be negatively rewarded, and coskewness varies over time. Furthermore, Moreno and Rodriguez (2009) analyse the returns of US mutual funds over the period 1962–2006 and find that coskewness is a priced risk factor for mutual funds. The findings show that funds that invest in stocks with more negative coskewness tend to yield higher average returns. Finally, Doan, Lin, and Zurbruegg (2010) analyse a higher-moment CAPM for US and Australian stocks and find that returns are sensitive to higher moments, and that higher moments explain a portion of returns not explained by the Fama and French factors. Thus, there is now a significant body of literature on the higher-moment CAPM which underpins the importance of considering skewness and kurtosis when modelling asset returns, though the results of empirical studies do not as yet offer conclusive support for the model.

4. Data and method

We use the monthly returns CRSP data for all common stocks listed on the NYSE, AMEX and NASDAQ exchanges for the period January 1926 to December 2010 (NASDAQ from January 1972). The empirical tests are conducted for the sample 1930–2010, and for the subsample 1980–2010. In the cross-sectional regressions used in the empirical tests, only stocks with 24 months of returns are included, as this is required for our short window beta computation.

Table 1 reports summary statistics for the cross-section of stock returns at 10-year intervals over the sample period. The number of stocks increased over the years from less than 1000 stocks before

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[1] The cubic model assumes that excess returns are generated by $R_{it} = \alpha_0 + \alpha_{1i}R_{mt} + \alpha_{2i}R_{mt}^2 + \alpha_{3i}R_{mt}^3$, where R_{it} and R_{mt} are excess returns, $R_{mt}^2 = (R_{mt} - E(R_{mt}))^2$ and $R_{mt}^3 = (R_{mt} - E(R_{mt}))^3$.

Table 1
Summary statistics for the cross-section of stock returns over the period 1930–2010.

December (Year)	1940	1950	1960	1970	1980	1990	2000	2010
Number of stocks	852	986	1159	2242	4733	7507	9681	7820
Mean excess return	0.54%	2.00%	0.39%	−1.55%	1.63%	−0.50%	0.62%	1.84%
t-statistic	7.17***	50.02***	7.03***	−36.0***	40.72***	−15.34***	16.11***	42.85***
Standard deviation	0.022	0.013	0.019	0.020	0.028	0.028	0.038	0.038
Skewness	1.14	0.28	−7.10	−0.70	1.18	0.47	1.34	0.71
Kurtosis (excess)	9.46	0.65	146.79	1.06	3.04	20.09	9.65	18.77

Notes: This table reports summary statistics for the cross-sectional regression of stocks in December of the years 1940, 1950, 1960, 1970, 1980, 1990, 2000 and 2010, showing the number of stocks, the mean excess return, *t*-statistics, standard deviations, skewness and excess kurtosis.

1950 to a peak of 9681 in December 2000. The number of stocks appears to be driven by the trend of industrialization and the economic boom period from the 1980 to 2000. We also observe that the standard deviation of the returns across stocks has increased since the 1980s, thus suggesting two distinct volatility regimes: before and after 1980. The normality of the distribution of returns, that is the joint hypothesis that skewness and excess kurtosis are equal to zero, is rejected in the cross-section for each period in the table.

The testing of the CAPM is straightforward. Re-writing Eq. 1 as

$$E(R_i) - R_f = \alpha + \lambda_\beta \beta_{i,m} \tag{8}$$

where $\lambda_\beta = E[(R_m) - R_f]$ is the risk premium, the null hypotheses to test are that the intercept is zero and that the risk premium is positive and equal to the average historical market excess return.

$$\begin{cases} \alpha = 0 \\ \lambda_\beta > 0 \\ \lambda_\beta = [(R_m) - R_f] \end{cases} \tag{9}$$

Testing the four-moment CAPM requires a small modification of Eq. 7. As $E(r_m^3)$ may be zero (the distribution of the market portfolio can be symmetric), to avoid dividing by zero the model is represented as

$$E[(R_i) - R_f] = \lambda_* \frac{E(r_i.r_m)}{E(r_m^2)} + \lambda_s E(r_i.r_m^2) + \lambda_k \frac{E(r_i.r_m^3)}{E(r_m^4)} \tag{10}$$

$$= \lambda_* \beta_i + \lambda_s s_i + \lambda_k k_i$$

where λ_s is the premium for coskewness, rather than standardized coskewness.

For the market portfolio:

$$E[(R_m) - R_f] = \lambda_* + \lambda_s s_m + \lambda_k \tag{11}$$

which implies $\lambda_* = E[(R_m) - R_f] - \lambda_s s_m - \lambda_k$. Substituting λ_* in Eq. 10, the final model obtains:

$$E[(R_i) - R_f] = \lambda_\beta \beta_i + \lambda_s(s_i - s_m \beta_i) + \lambda_k(k_i - \beta_i) \tag{12}$$

where $\lambda_\beta = E[(R_m) - R_f]$. This model has the advantage that it nests the CAPM. The main hypotheses to be tested for this model are that the price of beta is positive and equal to the market risk premium, the premium for (excess) coskewness is negative and the premium for (excess) cokurtosis is positive, that is:

$$\begin{cases} \lambda_\beta > 0 \\ \lambda_s < 0 \\ \lambda_k > 0 \end{cases}$$

The advantage of this formulation is that it can be compared with the standard CAPM, since the CAPM is a special case of the four-moment CAPM.

The CAPM and the four-moment CAPM are typically tested on portfolios. However, many authors (see, for instance, Ang et al., 2008; Kim, 1995) have criticized the use of portfolios to estimate the market premium, arguing that the spread in betas is effectively too small when portfolios are formed, leading to very large standard errors in the estimation of the risk premium. Further, Kim (1995) argues that when portfolios are formed, the behaviour of individual stocks is smoothed out, losing important information for the estimation of the risk premium in the process. We therefore use individual assets in our tests.

We address the limitations of static models by adopting the approach of Lewellen and Nagel (2006). We thus employ a two-step method. First, we conduct short-window (24 month) time series regressions of monthly individual asset excess returns over the market excess return to estimate conditional betas, coskewness, and cokurtosis as in Kraus and Litzenberger (1976). The average excess return for the individual stocks over the short windows is assumed to be their (conditional) expected excess return. In the second step, the average excess returns of individual stocks are regressed in the cross-section over the conditional co-moments (calculated over the same window as in the first step) to estimate the risk premia. The monthly conditional risk premia are then treated as time series observations, and hence tested using the Fama and MacBeth (1973) approach, i.e. the monthly conditional risk premia for each factor and the overall monthly conditional risk premia are averaged. However, because of potential autocorrelation and heteroscedasticity problems in the monthly risk premia, we also show results based on intercept only GMM estimation with Newey–West heteroscedasticity and autocorrelation consistent (HAC) standard errors. The models are conditional in so far as the factor loadings are obtained using rolling short windows.

4.1. The Fama and French factor augmented four-moment CAPM

The four-moment CAPM can be extended to include the effect of the Fama and French (1993) SMB and HML factors. The SMB factor represents the return of a portfolio of small-capitalization stocks minus the return on a portfolio of large-capitalization stocks. The HML factor represents the returns of a portfolio of high book-to-market stocks minus the return on a portfolio of low book-to-market stocks. The three-factor model of Fama and French is widely used in empirical studies of asset pricing. Therefore it would be interesting to augment the conditional four-moment CAPM with the SMB and HML factors in order to determine whether the Fama–French factors remain relevant in an asset pricing model once we correct for additional moment sensitivities.

The model adopted here is similar to that of Smith (2006) and Engle and Bali (2010), and is given by

$$R_i - R_f = \lambda_\beta \beta_i + \lambda_s(s_i - s_m \beta_i) + \lambda_k(k_i - \beta_i) + \lambda_{smb} \frac{E(r_i.smb)}{E(smb^2)}$$

$$+ \lambda_{hml} \frac{E(r_i.hml)}{E(hml^2)} + \varepsilon_i \tag{13}$$

The standardized covariances between the returns of stocks with the SMB and HML factors are obtained from univariate regressions.

Table 2
Test of the CAPM over the period 1930–2010.

	CAPM	4-Moment CAPM (adjusted)	4-Moment CAPM (unadjusted)	3-Moment CAPM (adjusted)	3-Moment CAPM (unadjusted)
α	0.0007	0.0006	0.0006	0.0006	0.0006
	(3.80)***/[1.74]*	(3.33)***/[1.53]	(3.33)***/[1.53]	(2.91)***/[1.33]	(2.91)***/[1.33]
λ_β	0.0067	0.0067	0.0062	0.0068	0.0061
	(22.14)***/[10.32]***	(23.88)***/[11.15]***	(11.78)***/[5.60]***	(23.14)***/[10.79]***	(17.10)***/[8.07]***
λ_s		−73.16	0.0010	−70.4126	0.0008
		(−14.27)***/[−7.00]***	(3.41)***/[1.73]*	(−17.08)***/[−8.01]***	(5.63)***/[2.81]***
λ_k		−0.0004	−0.0004		
		(−0.90)/[−0.45]	(−0.90)/[−0.45]		
Market risk premium		0.0073		0.0076	
$(\lambda_\beta + \lambda_s s_m + \lambda_k)$		(19.43)***/[9.58]***		(25.84)***/[12.20]***	
Market risk premium			0.0067		0.0068
$(\lambda_\beta + \lambda_s + \lambda_k)$			(23.88)***/[11.15]***		(23.14)***/[10.79]***

Notes: This table reports the results of monthly cross-sectional regressions of average stock returns (over 24 months) on the three factors of the four-moment CAPM, and on the single factor of the CAPM over the period 1930–2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. Conventional *t*-statistics are reported in parentheses, while HAC *t*-statistics are given in square brackets. Significant coefficients at the 1%, 5% and 10% levels are indicated with ***, ** and *, respectively. Beta, coskewness and cokurtosis are obtained as in Kraus and Litzenberger (1976).
Results are obtained from the following models:

$R_{i,t} - R_{f,t} = \alpha_t + \lambda_{\beta t}\,\beta_{i,t-1}$	CAPM
$R_{i,t} - R_{f,t} = \alpha_t + \lambda_{\beta t}\,\beta_{i,t-1} + \lambda_{st}(s_{i,t-1} - s_{m,t-1}\beta_{i,t-1}) + \lambda_{kt}(k_{i,t-1} - \beta_{i,t-1})$	Adjusted 4-moment CAPM
$R_{i,t} - R_{f,t} = \alpha_t + \lambda_{\beta t}\,\beta_{i,t-1} + \lambda_{st}(s_{i,t-1}/s_{m,t-1}) + \lambda_{kt}k_{i,t-1}$	Unadjusted 4-moment CAPM

5. Results

Our main results are summarised in Tables 2 to 4. The main highlight of these results is the sharp contrast between the conventional *t*-statistics and the HAC *t*-statistics. All corrected statistics are around half or less the value of standard *t*-statistics. As can be seen from the tables, this has major implications for the significance of the intercept term in the CAPM and extended CAPM models. All but one of the intercepts become insignificant once we correct the *t*-statistics.[2] More importantly, some risk premia also become insignificant after adjustment. Therefore, in the following discussion, we will rely solely on the HAC *t*-statistics to decide upon the significance of estimated risk premia.

5.1. Results of the test of the conditional CAPM on individual assets

The results for the conditional CAPM based on the full sample are given in Table 2. Two interesting points emerge. First, the conventional *t*-statistic suggests that the intercept is highly significant, thus leading to a rejection of the CAPM. But taking into account potential heteroscedasticity and autocorrelation shows that the intercept is insignificant. Given that the estimated market premium is positive and significant (0.67% per month, which is equivalent to 8.34% per year), the CAPM cannot be rejected. For the subsample from 1980–2010 shown in Table 3, the intercept remains insignificant and the beta premium has a positive and significant coefficient (0.59% per month), equal to a compounded return of 7.31% per year. The risk premium thus appears to have declined over the second period when compared to the first. Thus, the CAPM appears to hold quite well for individual assets as the estimated risk premium is consistent with theory.

5.2. Results of the test of the conditional four-moment CAPM on individual assets

All higher-moment CAPM models have insignificant intercepts (based on HAC standard errors). However, although beta is priced, the price skewness is also significant and has the expected sign in most cases. This rejects the standard CAPM in favour of a higher-moment CAPM. For the full sample shown in Table 2, the kurtosis premium of the four-moment CAPM is not significantly different from zero for

both the adjusted and unadjusted models. This suggests that a three-moment CAPM is more appropriate. This finding differs from the results of Heaney et al. (2012), who find significant coskewness and cokurtosis prices but an insignificant beta price. The result for the adjusted three-moment CAPM is shown in the fourth column of Table 2. The results do not differ substantially from those of the four-moment model, with an insignificant intercept, a beta price of 0.68%, and a negative coskewness price of − 70.41. The overall market premium is estimated at 0.76% per month, or 9.51% per year, which is larger than the 8.34% estimated for the simple CAPM.

As shown in Table 3, the risk premia declined over the period 1980–2010. Both beta and coskewness prices fell, but remain significant and with the expected sign. The market risk premium is also lower at 0.55% per month, or 6.80% per year. The market premium suggested by the higher-moment CAPM is now lower than that of the simple CAPM (0.59%).

It is worth noting that the importance of employing unscaled coskewness (that is, using the adjusted model). Although both adjusted and unadjusted models are equivalent mathematically, the model results are rather different. For example, in the last two columns of Table 2, the coskewness premium for the adjusted model is negative as expected, while it is positive and surprisingly significant in the unadjusted model (in the four-moment model it is positive but insignificant). We believe that such a result is due to the possible low market skewness at least for some periods, which could inflate the scale coskewness and hence produce erratic coskewness coefficients in the cross-sectional regression. For the subsample of 1980–2010 shown in Table 3, all skewness coefficients are negative, but are significant only for the adjusted models. In sum, the results show that both beta and coskewness are priced risks, but cokurtosis is not.

5.3. Results for the four-moment CAPM augmented with SMB and HML factors

The results for the augmented four-moment CAPM as well as the three-factor model of Fama and French (FF) are reported in Table 4. The models are tested for the full sample period of 1930–2010 and for the subsample period of 1980–2010. For the full sample period, the intercept is insignificant in the augmented model but significant in the FF model. This could be symptomatic of the missing coskewness in the FF model, which perhaps led to a greater and significant intercept. The beta price is significant in both models, but larger under the augmented

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[2] The constant reported in the tables is the average of the monthly conditional intercepts.

Table 3
Test of the CAPM over the subsample period of 1980–2010.

	CAPM	4-Moment CAPM (adjusted)	4-Moment CAPM (unadjusted)	3-Moment CAPM (adjusted)	3-Moment CAPM (unadjusted)
α	0.0008	0.0004	0.0004	0.0004	0.0004
	(4.03)***/[1.92]*	(2.77)***/[1.32]	(2.77)***/[1.32]	(2.48)**/[1.18]	(2.48)**/[1.18]
λ_β	0.0059	0.0059	0.0063	0.0059	0.0064
	(18.98)***/[8.94]***	(19.10)***/[9.00]***	(15.48)***/[7.50]***	(19.19)***/[9.03]***	(22.01)***/[10.47]***
λ_s		−32.79	−0.0011	−35.57	−0.0004
		(−6.61)***/[−3.15]***	(−3.15)***/[−1.60]	(−8.99)***/[−4.38]***	(−3.36)***/[−1.63]
λ_k		0.0007	0.0007		
		(1.44)/[0.72]	(1.44)/[0.72]		
Market risk premium		0.0056		0.0055	
$(\lambda_\beta + \lambda_s s_m + \lambda_k)$		(10.61)***/[5.02]***		(14.45)***/[6.82]***	
Market risk premium			0.0059		0.0059
$(\lambda_\beta + \lambda_s + \lambda_k)$			(19.11)***/[9.00]***		(19.18)***/[9.03]***

Notes: This table reports the results of monthly cross-sectional regressions of average stock returns (over 24 months) on the three factors of the four-moment CAPM, and on the single factor of the CAPM over the period 1980–2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis and the conditional overall risk premium. Conventional t-statistics are reported in parentheses, while HAC t-statistics are given in square brackets. Significant coefficients at the 1%, 5% and 10% levels are indicated with ***, ** and *, respectively. Beta, coskewness and cokurtosis are obtained as in Kraus and Litzenberger (1976).
Results are obtained from the following models:

$R_{i,t} - R_{f,t} = \alpha_t + \lambda_{\beta t}\,\beta_{i,t-1}$	CAPM
$R_{i,t} - R_{f,t} = \alpha_t + \lambda_{\beta t}\,\beta_{i,t-1} + \lambda_{st}(s_{i,t-1} - s_{m,t-1}\beta_{i,t-1}) + \lambda_{kt}(k_{i,t-1} - \beta_{i,t-1})$	Adjusted 4-moment CAPM
$R_{i,t} - R_{f,t} = \alpha_t + \lambda_{\beta t}\beta_{i,t-1} + \lambda_{st}(s_{i,t-1}/s_{m,t-1}) + \lambda_{kt}k_{i,t-1}$	Unadjusted 4-moment CAPM

four-moment CAPM specification. Again, the omitted skewness factor may have biased the beta coefficient. The SMB price is identical and significant in both models, whereas the HML price is insignificant in both cases. Finally, the skewness price is negative and significant as expected, whereas kurtosis in not priced. Overall, the results for the full sample show that the size factor remains an important addition to the four-moment CAPM. In other words, coskewness and cokurtosis do not seem to be able to capture the size effect.

The subsample period leads to similar conclusions in terms of the significance of beta, coskewness and size. However, the scale of the risk prices appears to have changed in the subsample period. The price of beta declined from 0.58% to 0.43% (in the augmented four-moment CAPM), and the reward for skewness has declined in scale value from −49.47 to −10.32. The size premium has however increased from 0.16% to 0.28%. Overall, our results are not consistent with the findings of Heaney et al. (2012) who find that the higher moments are encompassed by SMB and HML.

6. The impact of using portfolios

In order to confirm the problems discussed above regarding empirical tests that use portfolios instead of individual assets, we repeated

the above analysis using 25 ME/MB sorted portfolios obtained from French website. The results are summarised in Table 5, which shows the Fama–MacBeth estimated average risk premia calculated using the same procedure as before. However, we only provide standard t-statistics since the statistics are based on a sample of 25 observations. Sul, Phillips, and Choi (2005) pointed out that the HAC estimators may be biased for small samples.

We estimate the simple CAPM, the adjusted four-moment CAPM, the FF augmented model and the simple FF model. All models are clearly rejected, having high and significant values of the intercept. These are abnormally large, reaching 1.43% in the case of the augmented four-moment CAPM for the period 1980–2010. Almost all of the premia are insignificant. The exceptions are the HML premium for the full sample under the FF model (although the premium is very low, it is highly significant with a t-statistic of 3.82), and the beta risk premium for the sub-period under the augmented four-moment CAPM, which is significant but (spuriously) negative. The results for the other specifications are similar and are available upon request. The portfolio results are thus interesting in their own right as they appear to explain the difficulties found in the literature to confirm different versions of the CAPM or multifactor model.

Table 4
Tests of the adjusted four-moment CAPM augmented with SMB and HML, and the three-factor model of Fama and French using short-window regressions, on individual assets over the period 1930–2010 and 1980–2010.

	(1930–2010)		(1980–2010)	
	Adjusted 4M-CAPM with FF factors	Fama–French 3-factor model	Adjusted 4M-CAPM with FF factors	Fama–French 3-factor model
α	0.0007	0.0009	0.0002	0.0003
	(3.77)***/[1.72]*	(4.65)***/[2.13]**	(1.17)/[0.56]	(1.69)*/[0.81]
λ_β	0.0058	0.0053	0.0043	0.0041
	(22.83)***/[11.00]***	(20.00)***/[9.50]***	(14.78)***/[7.10]***	(15.28)***/[7.35]***
λ_s	−49.4709		−10.32	
	(−11.70)***/[−5.64]***		(−2.08)**/[−1.00]	
λ_k	0.0014		0.0018	
	(3.60)***/[1.86]*		(3.38)***/[1.65]*	
λ_{smb}	0.0016	0.0016	0.0028	0.0026
	(8.29)***/[3.86]***	(8.19)***/[3.78]***	(13.60)***/[6.44]***	(14.73)***/[7.00]***
λ_{hml}	−0.0004	−0.0005	−0.0003	0.0003
	(−2.97)***/[−1.46]	(−3.87)***/[−1.87]*	(−1.55)/[−0.74]	(−1.57)/[−0.75]

Notes: This table reports the results of monthly cross-sectional regressions of average stock returns (over 24 months) on the three factors of the adjusted four-moment CAPM, and on the three factors of the Fama and French model. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, SMB and HML. Conventional t-statistics are reported in parentheses, while HAC t-statistics are given in square brackets. Significant coefficients at the 1%, 5% and 10% levels are indicated with ***, ** and *, respectively.

Table 5
Test of the CAPM and four-moment CAPM based on 25 ME/BM portfolios.

Models	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$	SMB	HML
Panel A: 1930–2010							
CAPM	0.0075	−0.0001					
	(3.55)***	(0.02)					
Adjusted 4-moment CAPM	0.0068	−0.0000	50.31	−0.0033	−0.0009		
	(3.08)***	(−0.01)	(0.89)	(−0.46)	(−0.14)		
4-moment CAPM + FF	0.0091	−0.0026	16.37	0.0001		0.0013	0.0021
	(4.69)***	(−1.22)	(0.30)	(0.02)		(1.23)	(1.73)*
FF	0.0094	−0.0030				0.0012	0.0038
	(5.35)***	(−1.58)				(1.21)	(3.82)****
Panel B: 1980–2010							
CAPM	0.0107	−0.0042					
	(2.73)***	(−0.91)					
Adjusted 4-moment CAPM	0.0107	−0.0049	−4.44	−0.0055	−0.0063		
	(2.75)***	(−1.16)	(−0.04)	(−0.40)	(−0.61)		
4-moment CAPM + FF	0.0143	−0.0084	34.34	0.0091		−0.0007	0.0003
	(4.36)***	(−2.37)**	(0.34)			(−0.36)	(0.11)
FF	0.0124	−0.0067				0.0000	0.0032
	(3.92)***	(−1.91)*				(0.03)	(1.71)*

7. Conclusion

The results of the empirical tests of the conditional CAPM and conditional four-moment CAPM, along with tests of some alternative models, show that when modelling the returns of individual stocks, the risk premium is positive and significant, as expected from the CAPM, and that coskewness is significant and has the expected negative sign. The best model results are obtained when the four-moment CAPM is augmented to include the Fama–French small-minus-big (SMB) factor, such that all of the factors are significant and have the signs expected from theory, with sensitivity to the SMB factor exerting an important positive effect on the cross-section of returns. This suggests that the four-moment CAPM can indeed improve the performance of the standard CAPM. In particular, SMB sensitivity significantly improves the explanation of the cross-section of expected returns.

The use of individual assets in empirical asset pricing tests allows for a larger spread in systematic measures of risk such as beta. Researchers are therefore able to obtain more precise estimates of risk premia than is the case for portfolios of stocks. Furthermore, the use of a moving average to proxy for expected returns appears to improve the performance of asset pricing models. While far from perfect, this proxy appears to be a distinct improvement on realized returns. Indeed, the main reason why conditional models fail appears to be the use of realized returns as a proxy for expected returns. The use of conditional models highlights the need for a better proxy of expected returns in asset pricing.

Interestingly, the results support the simple CAPM when tested on individual stocks over the last 30 years. Furthermore, the four-moment CAPM appears to work well when the SMB factor is added. We find that all of the factors in such a model have the expected sign: beta demands a positive premium, coskewness has a negative premium, and cokurtosis has a positive premium. Interestingly, SMB retains its significance and has a positive risk premium in this model specification, and thus small stocks tend to earn higher returns even after accounting for the co-moments. Therefore, it appears that small stocks are characterised by some incremental risk such as a liquidity risk (though this is not tested in our paper). However, the HML factor has no relevance when testing individual stocks.

The results of our paper confirm the argument of Ang et al. (2008) and Avramov, Chordia, and Goyal (2005) that a conditional version of asset pricing models conducted for individual assets confirms a rational explanation of the cross-section of returns. Intriguingly, when considered together with higher moments, SMB is priced and perhaps related to a liquidity premium, whereas HML is not priced.

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