BINARY HEAPS

OBJECTIVES

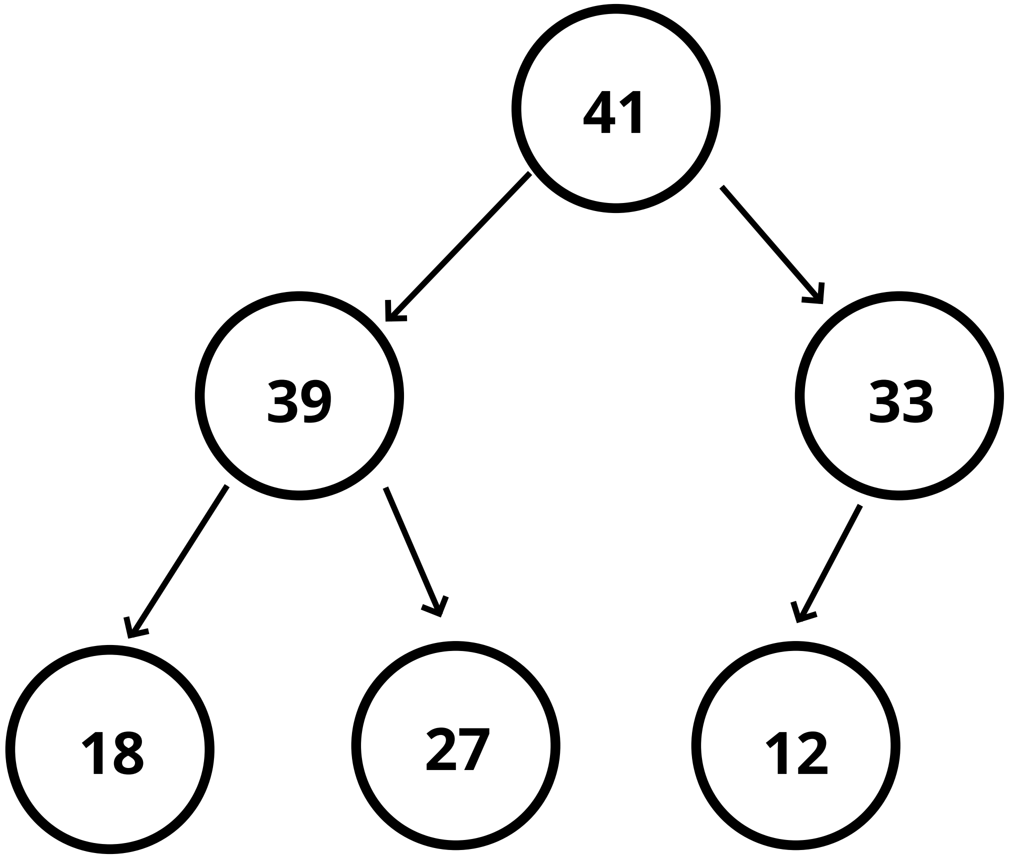
- Define what a binary heap is
- Compare and contrast min and max heaps
- Implement basic methods on heaps
- Understand where heaps are used in the real world and what other data structures can be constructed from heaps

WHAT IS A BINARY HEAP?

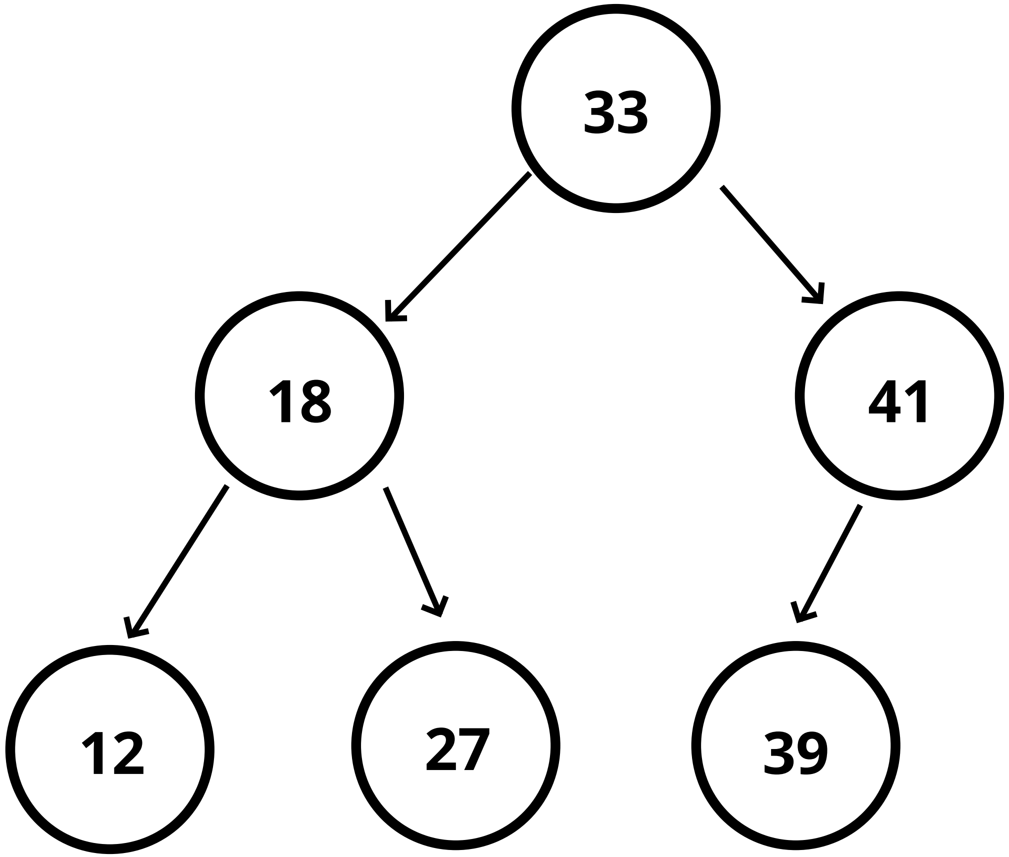
Very similar to a binary search tree, but with some different rules!

In a **MaxBinaryHeap**, parent nodes are always larger than child nodes. In a **MinBinaryHeap**, parent nodes are always smaller than child nodes

WHAT DOES IT LOOK LIKE?



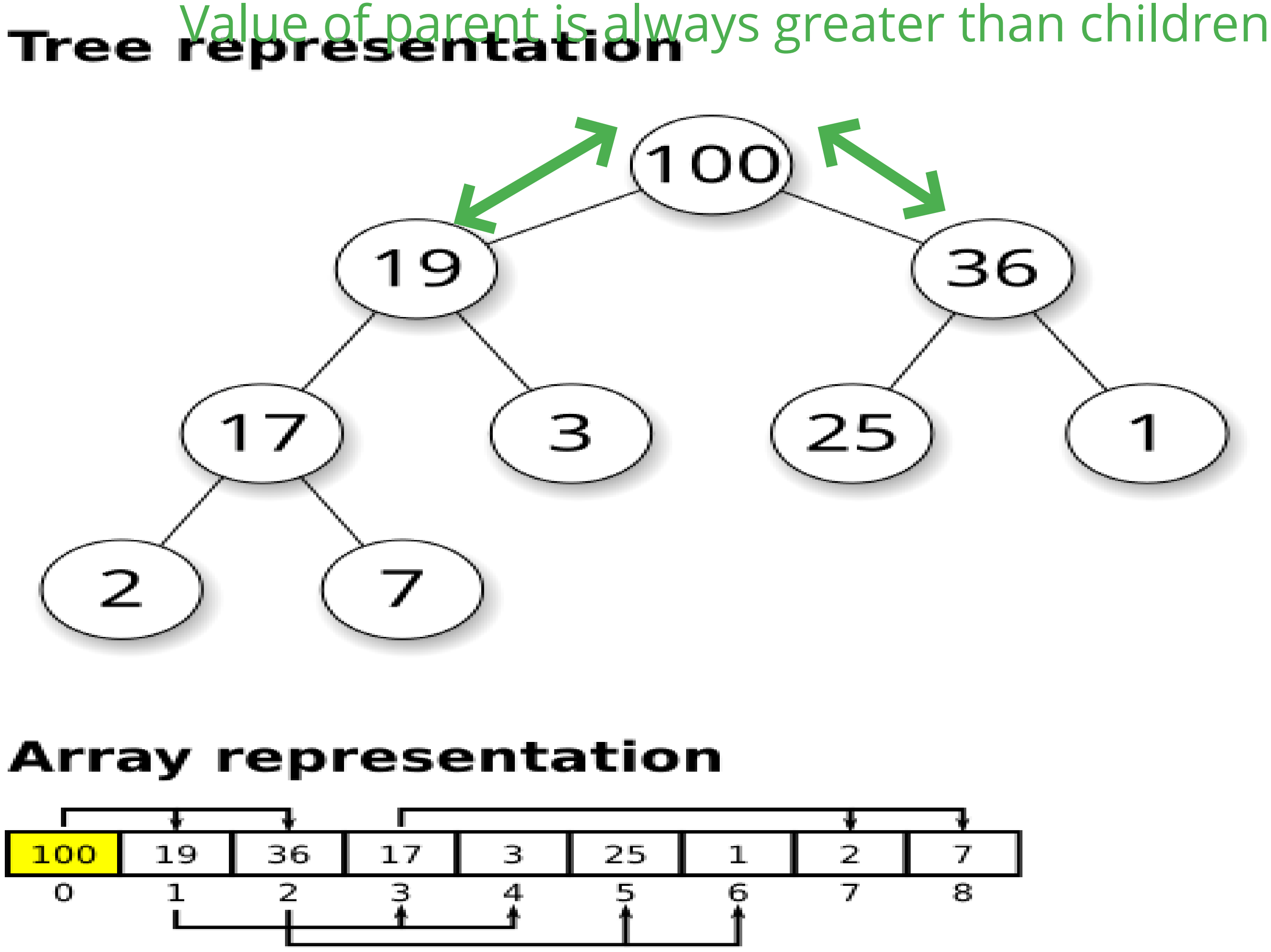
NOT A BINARY HEAP

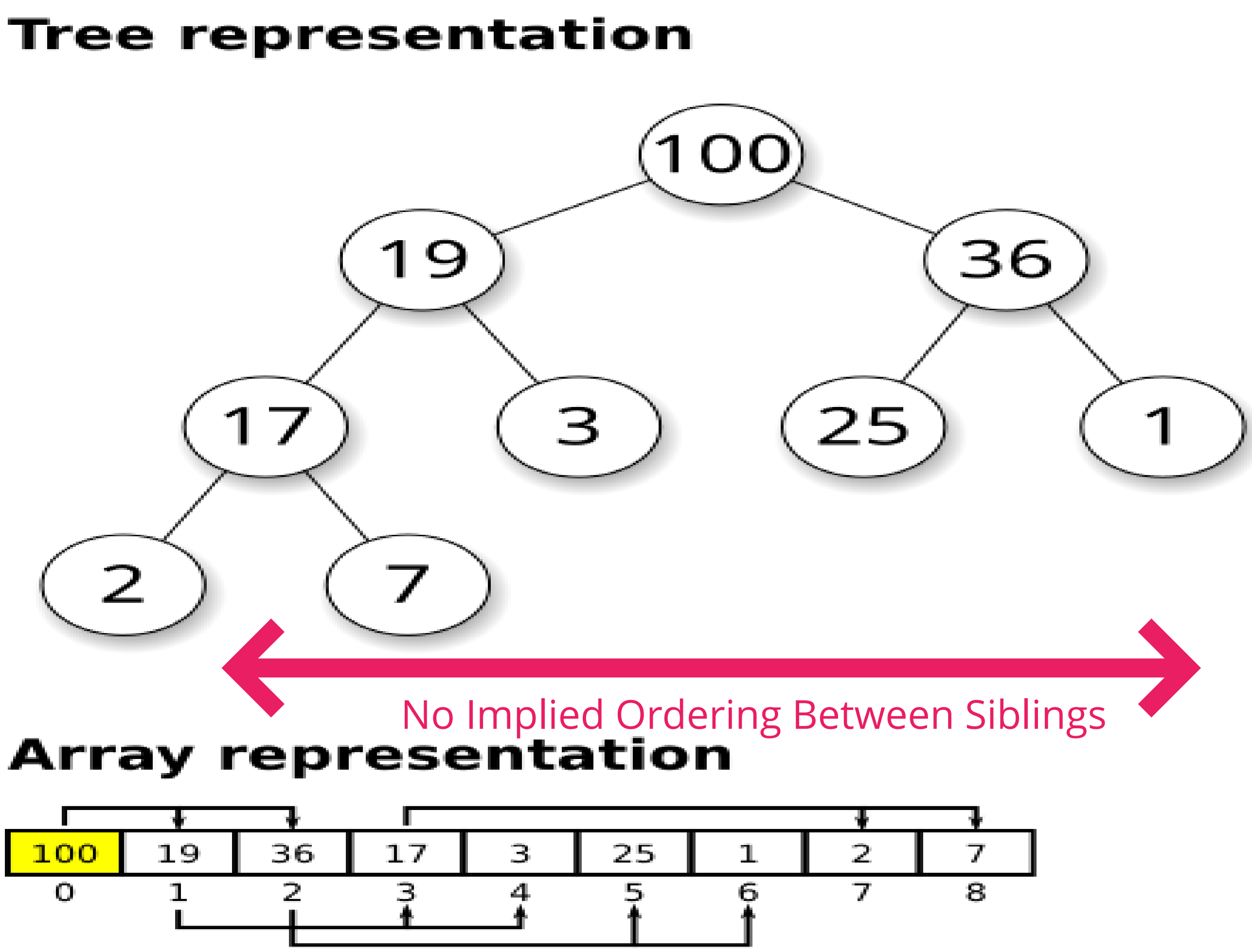


MAX BINARY HEAP

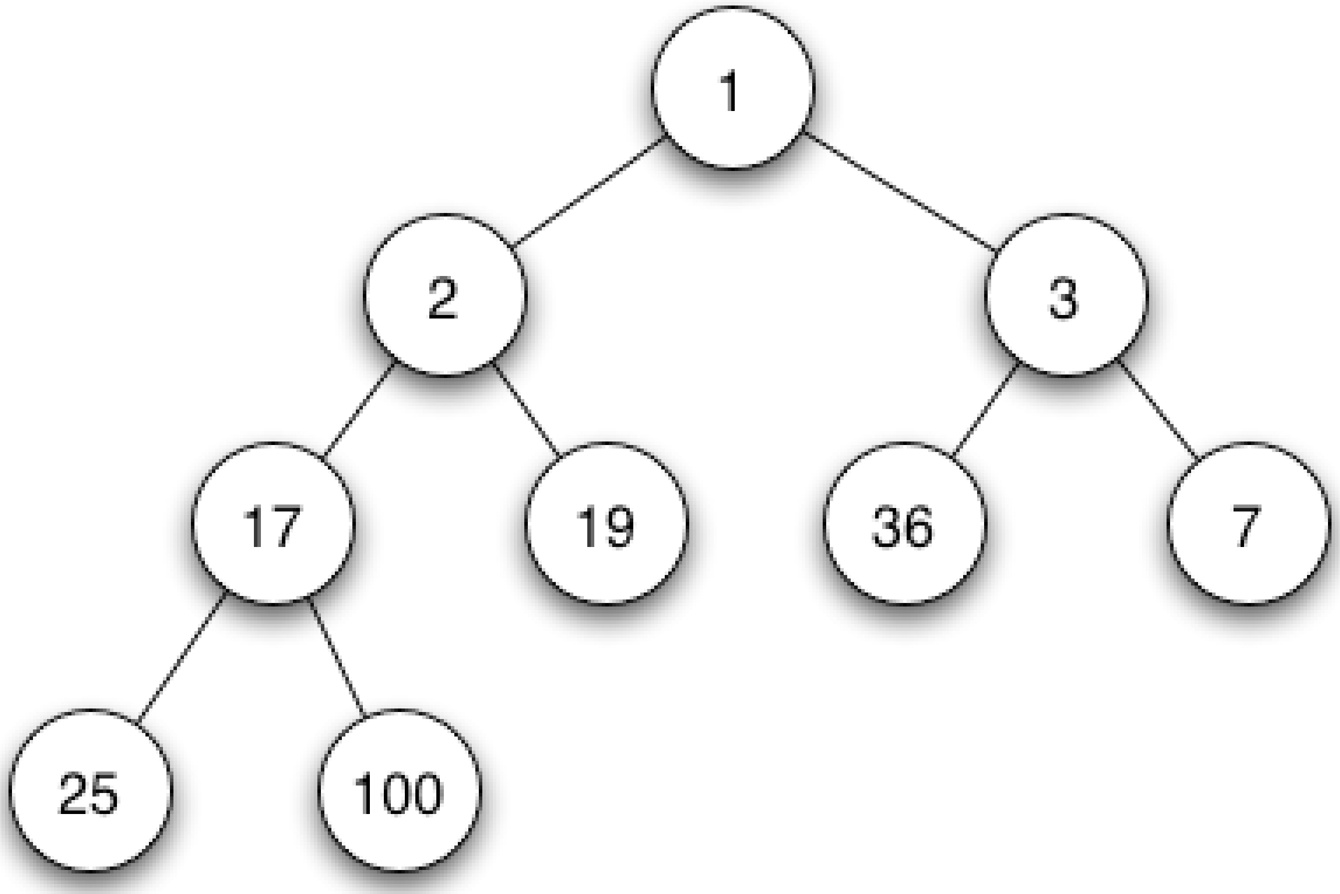
- Each parent has at most two child nodes
- The value of each parent node is **always** greater than its child nodes
- In a max Binary Heap the parent is greater than the children, but there are no guarantees between sibling nodes.
- A binary heap is as compact as possible. All the children of each node are as full as they can be and left children are filled out first

Value of parent is always greater than children




Tree representation
100
19
36
17
3
25
1
2
7
No Implied Ordering Between Siblings
Array representation
100 19 36 17 3 25 1 2 7
0 1 2 3 4 5 6 7 8

A MIN BINARY HEAP



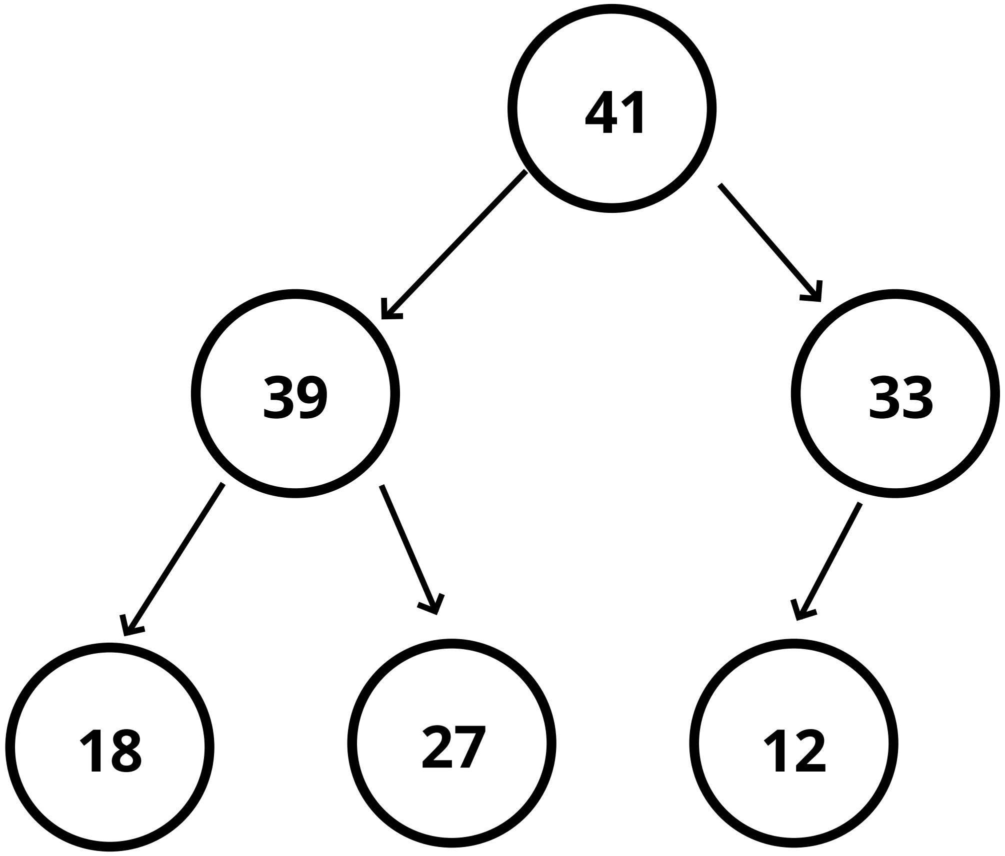
Why do we need to know this?

Binary Heaps are used to implement Priority Queues, which are **very** commonly used data structures

They are also used quite a bit, with **graph traversal** algorithms

We'll come back to this!

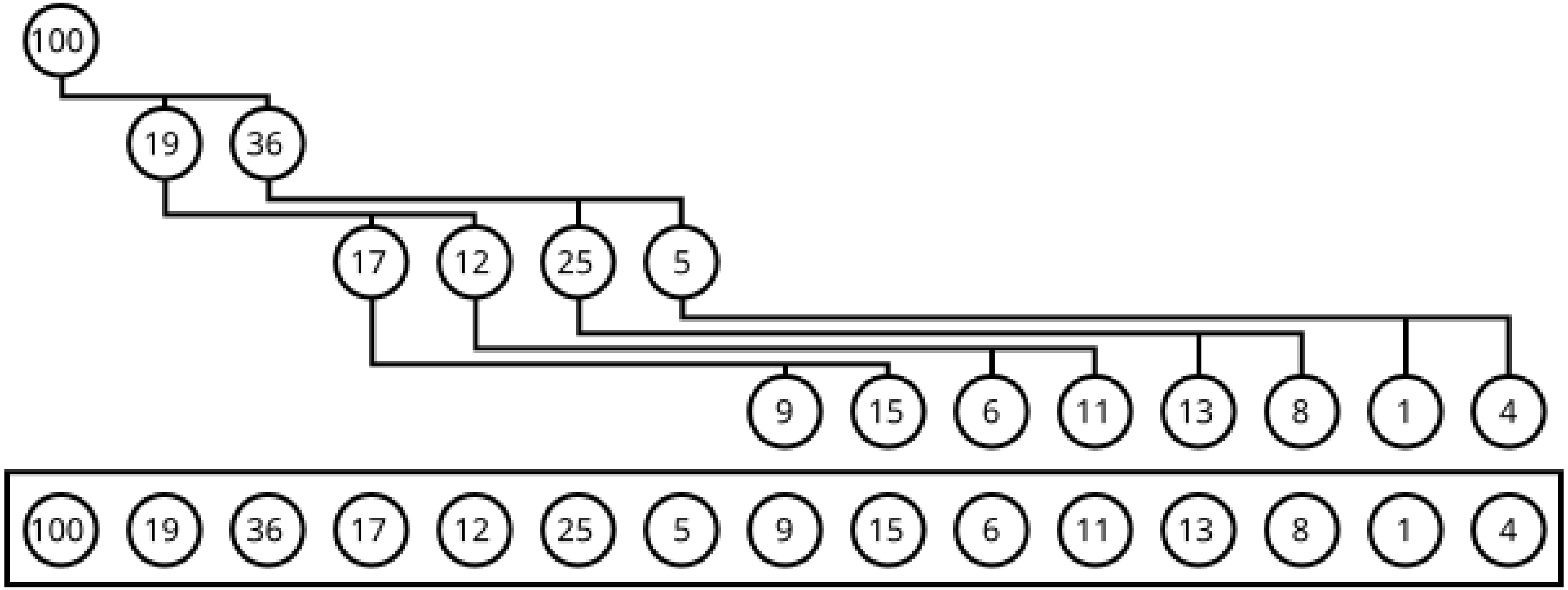
REPRESENTING HEAPS



THERE'S AN EASY WAY OF STORING A BINARY HEAP...

A LIST/ARRAY

REPRESENTING A HEAP



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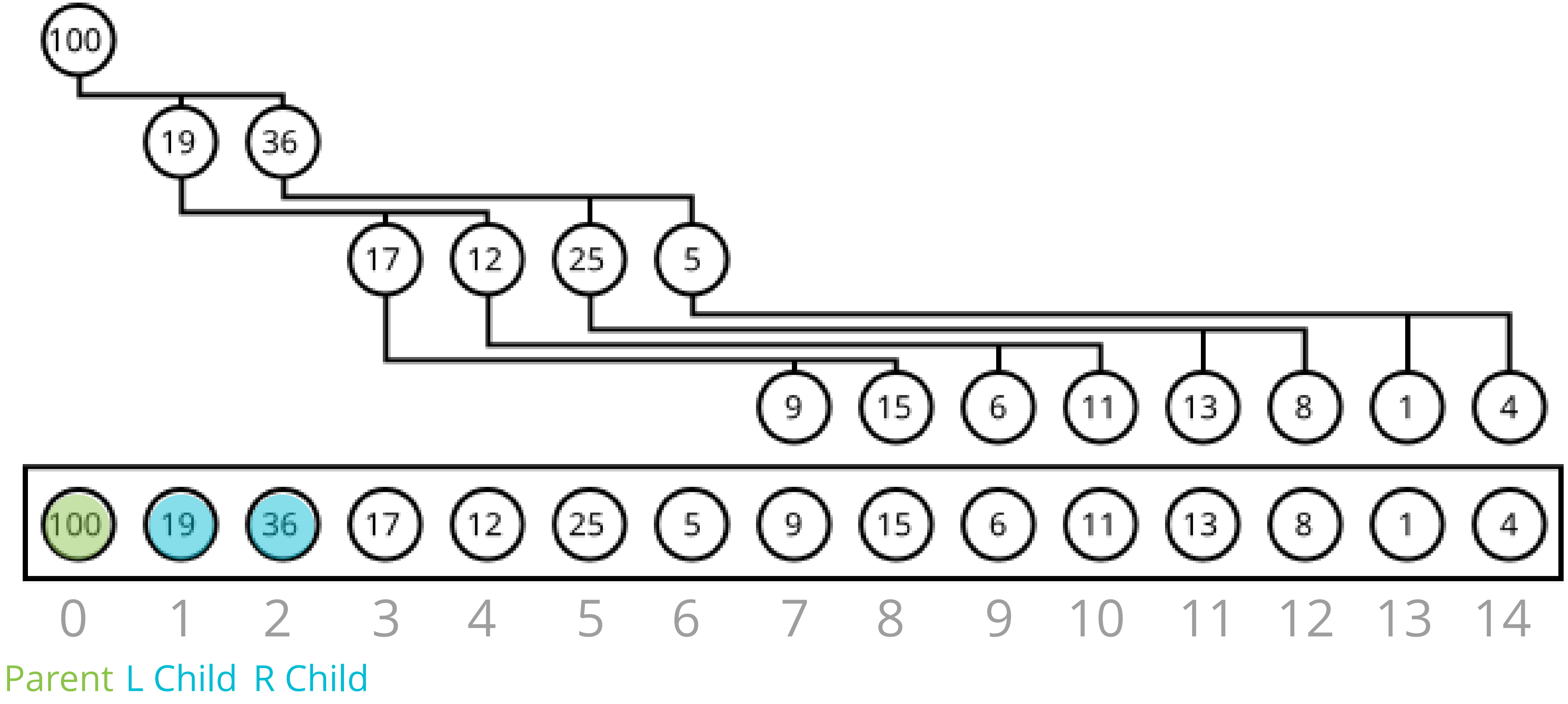

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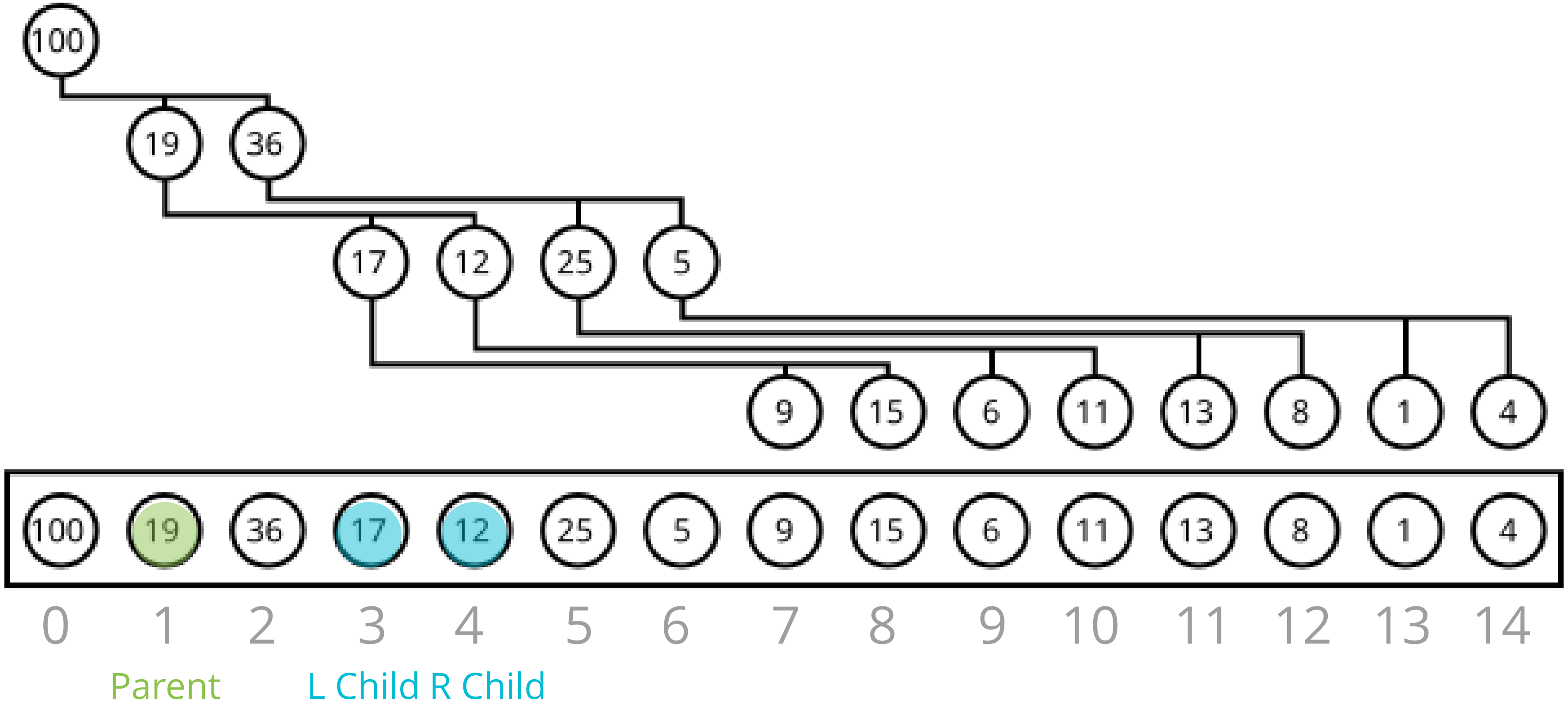

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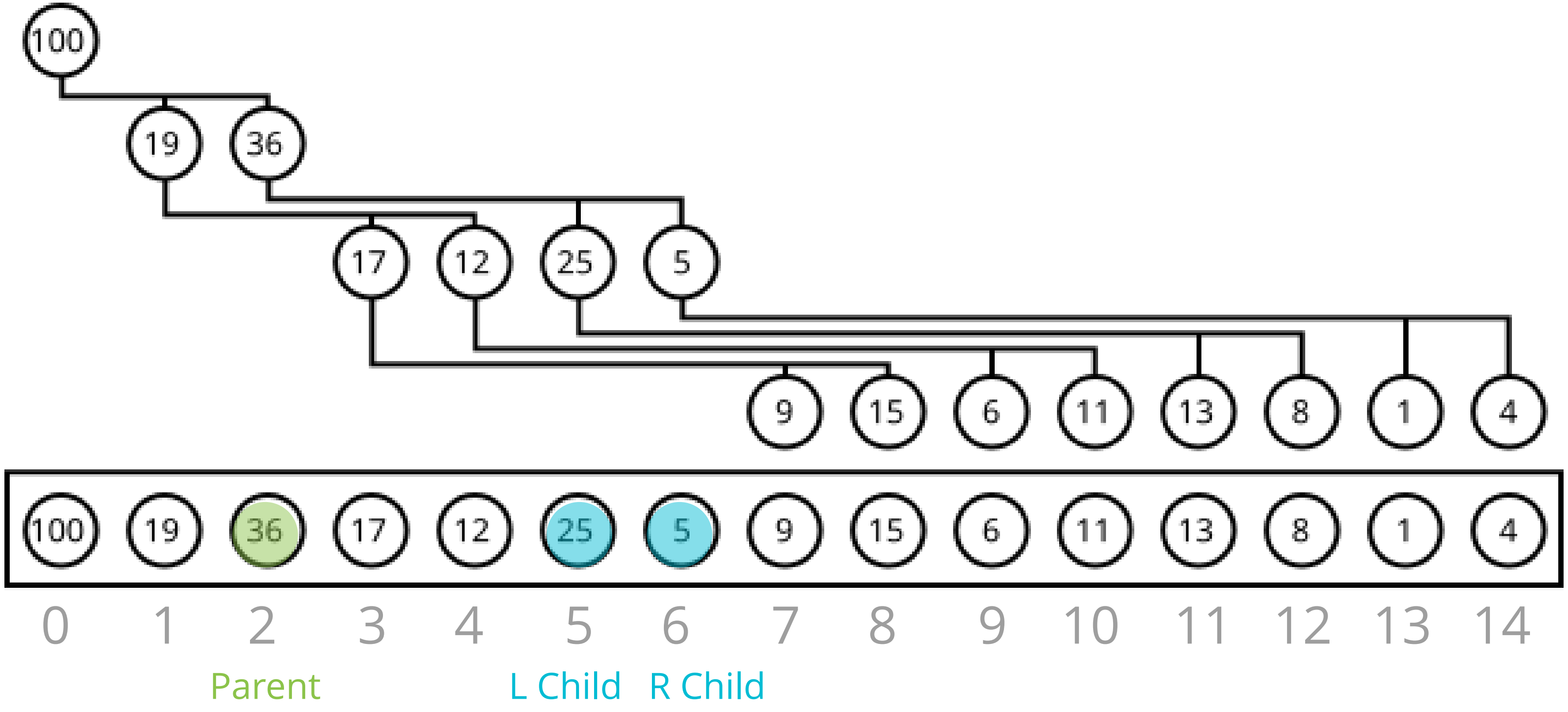

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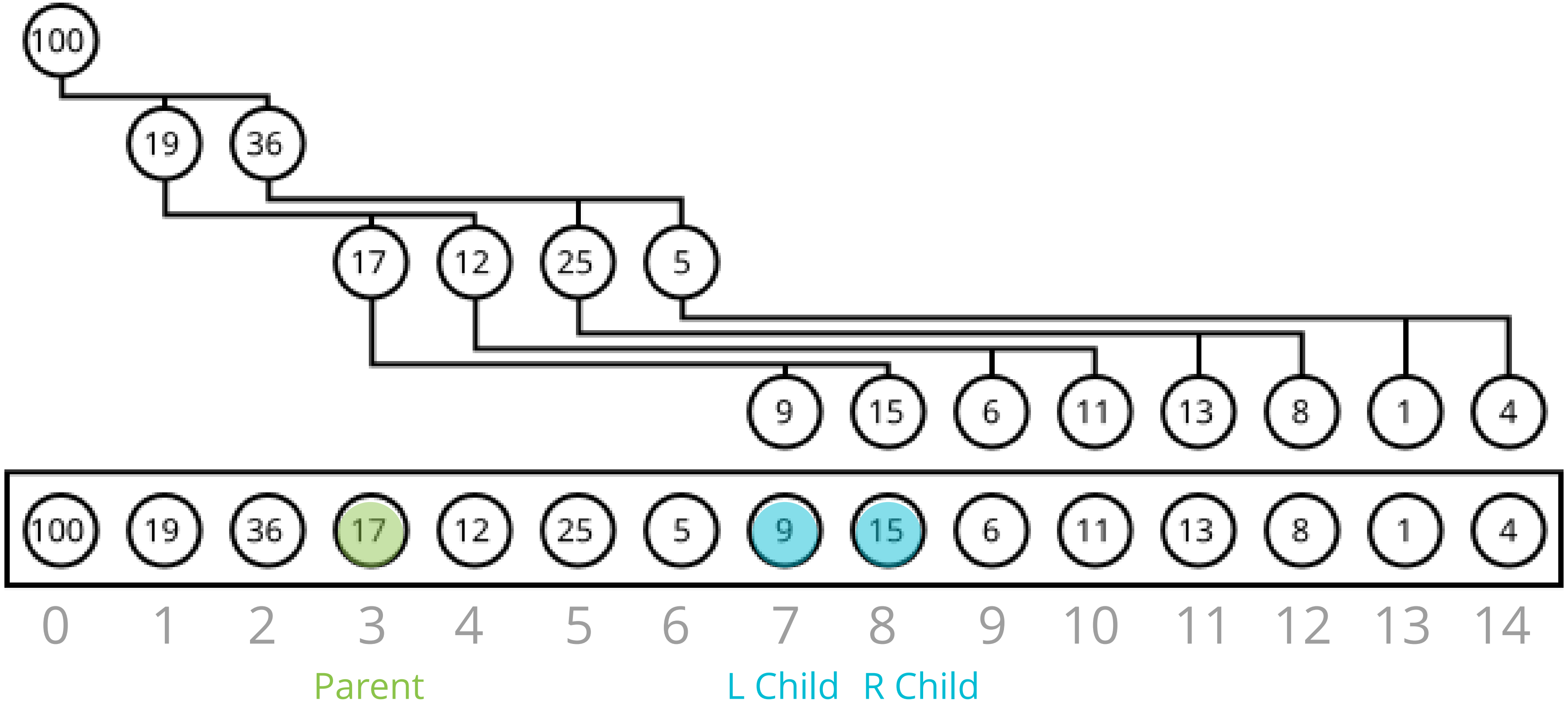
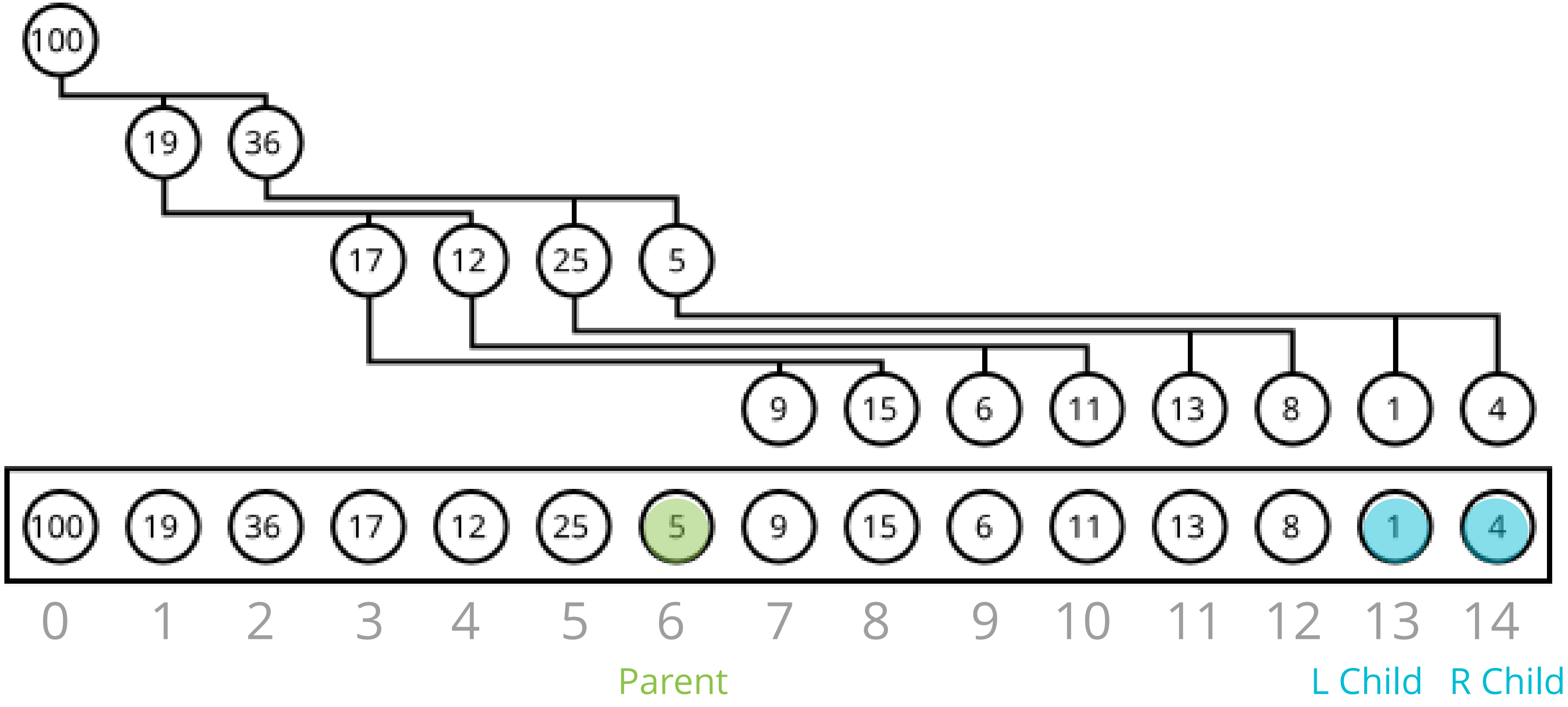


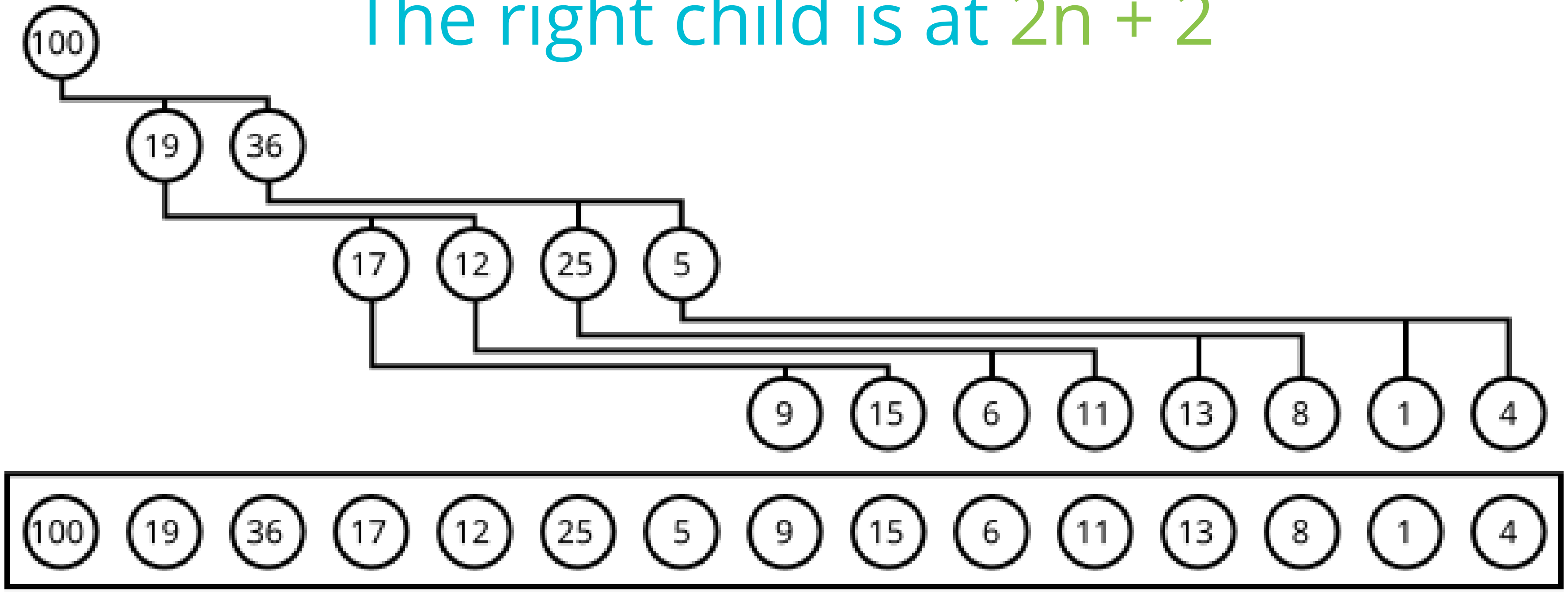
REPRESENTING A HEAP



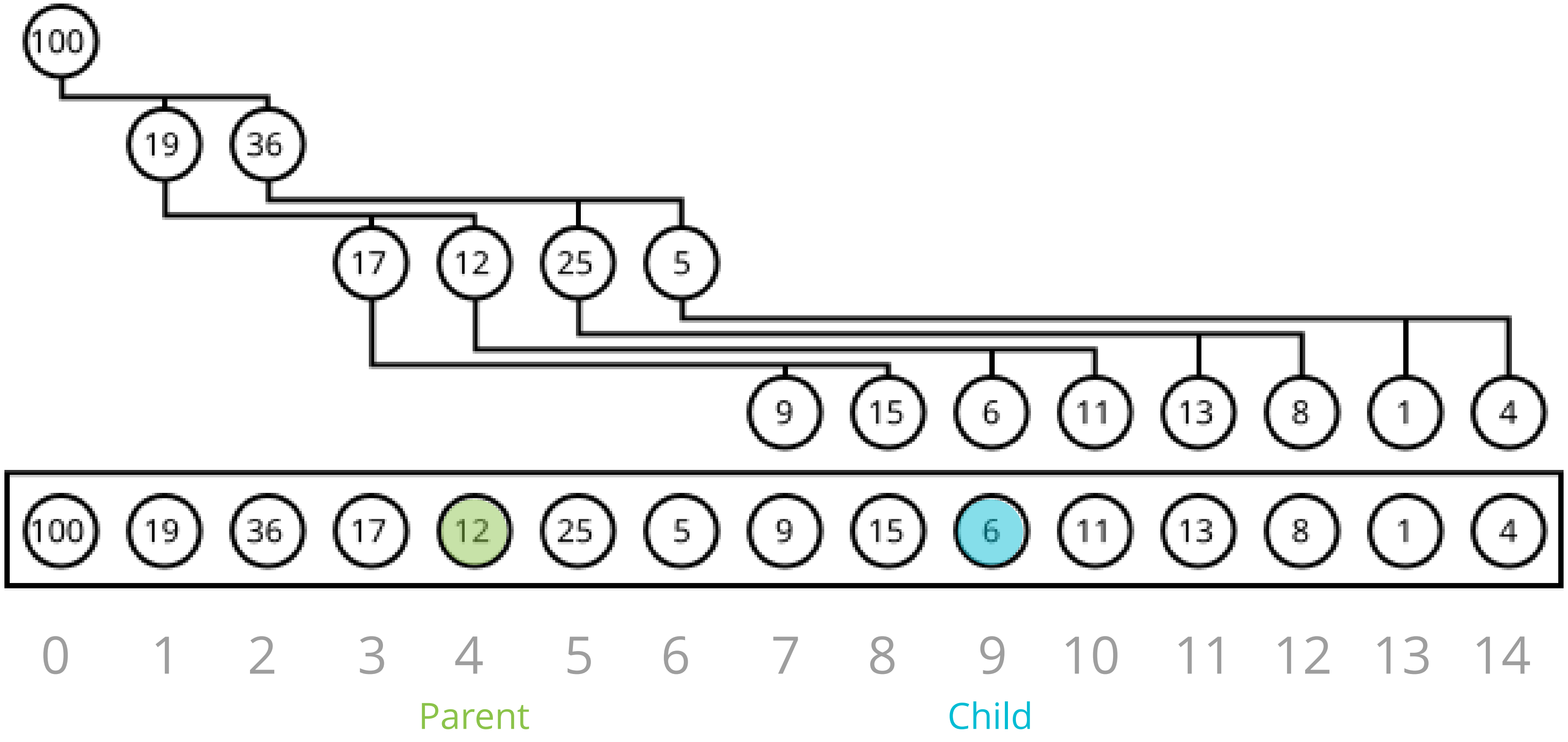
For any index of an array *n*...

The left child is stored at 2n + 1

The right child is at 2n + 2



WHAT IF WE HAVE A CHILD NODE AND WANT TO FIND ITS PARENT?



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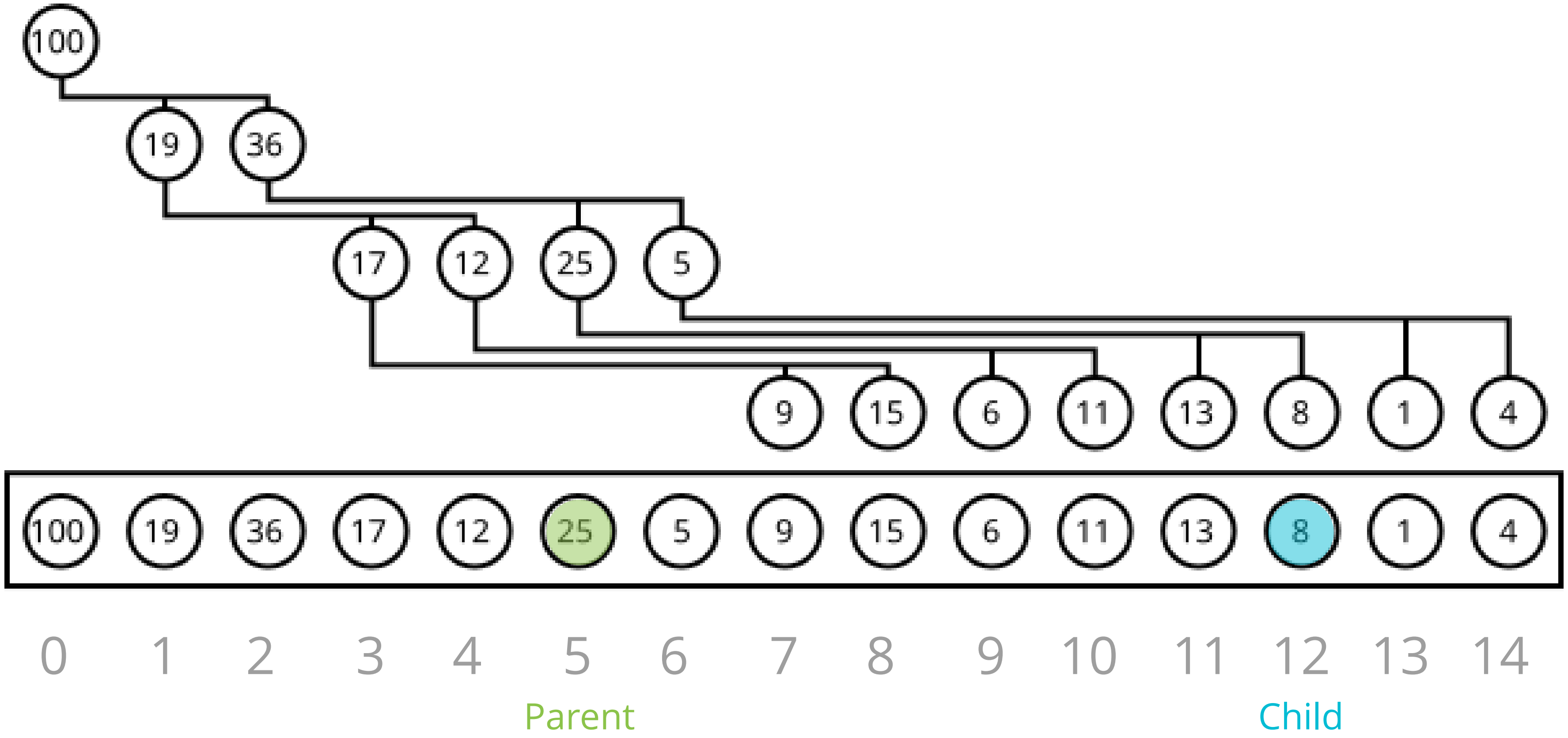
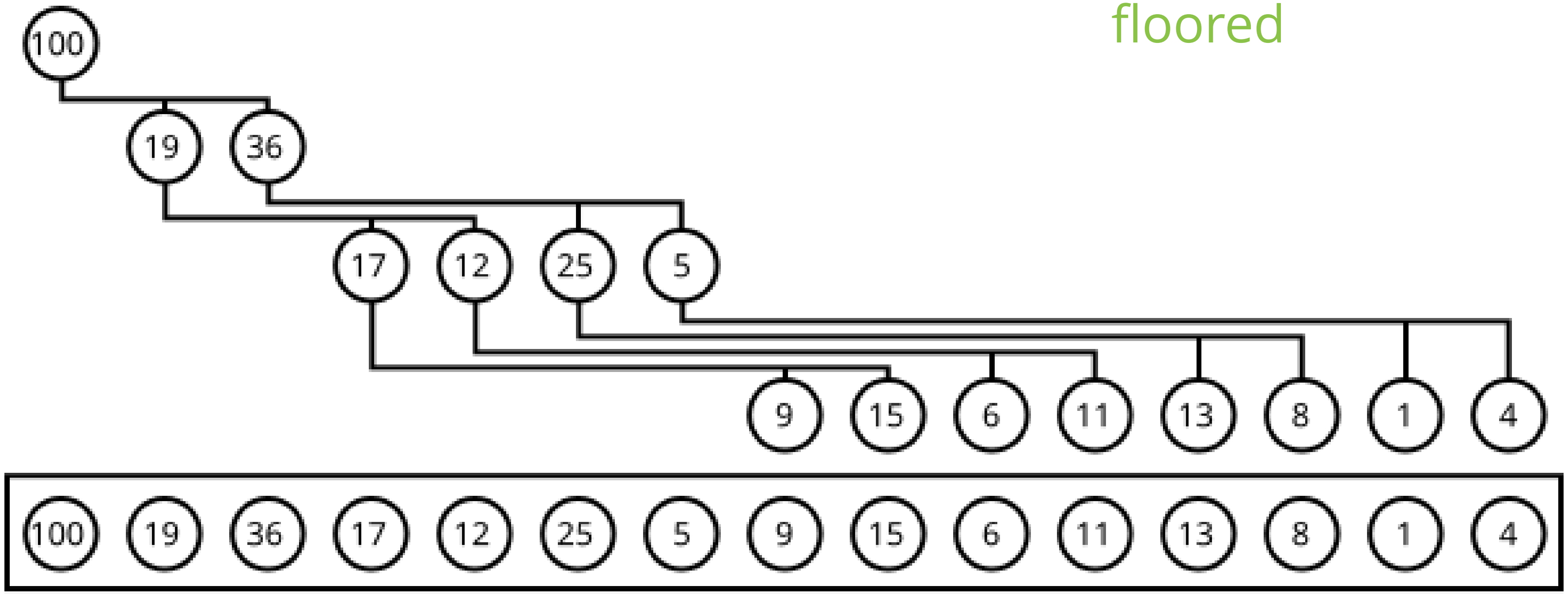


For any child node at index *n*...

Its parent is at index (n-1)/2

floored



DEFINING OUR CLASS

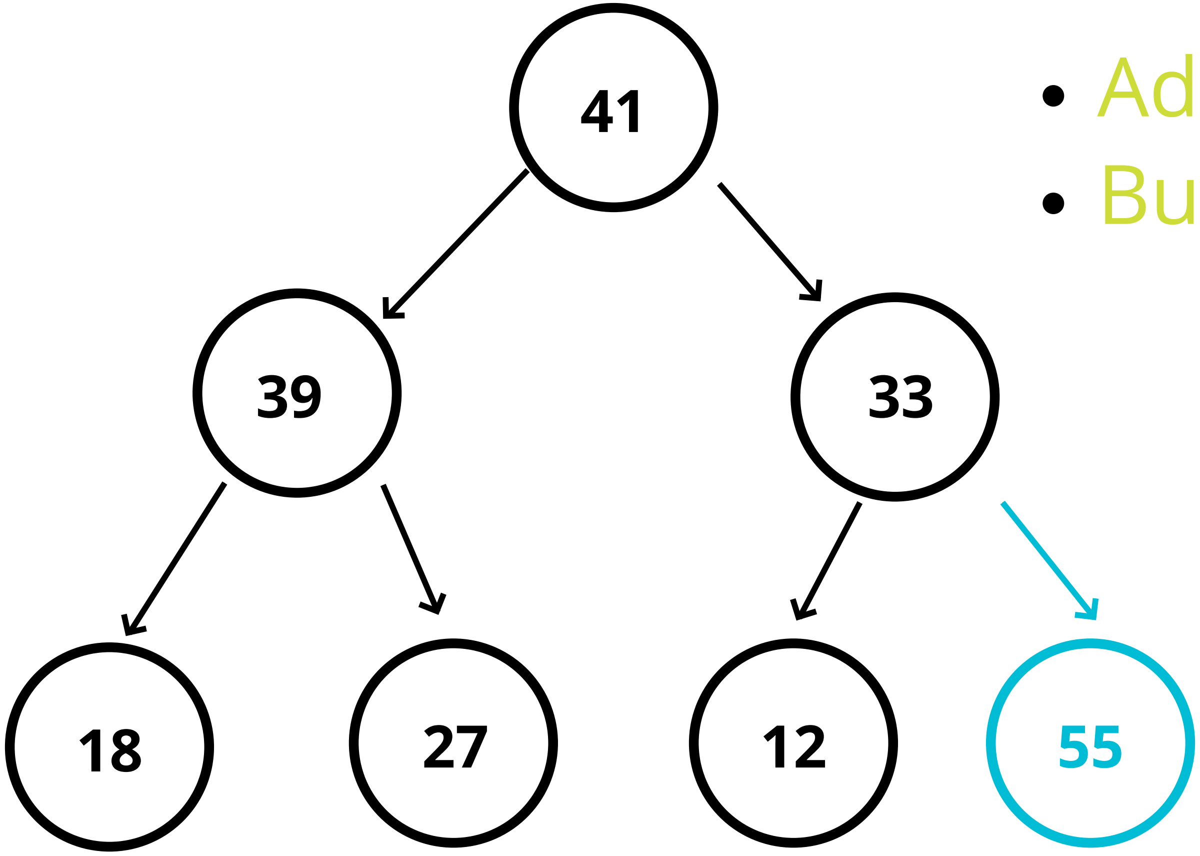
Class Name:

MaxBinaryHeap

Properties:

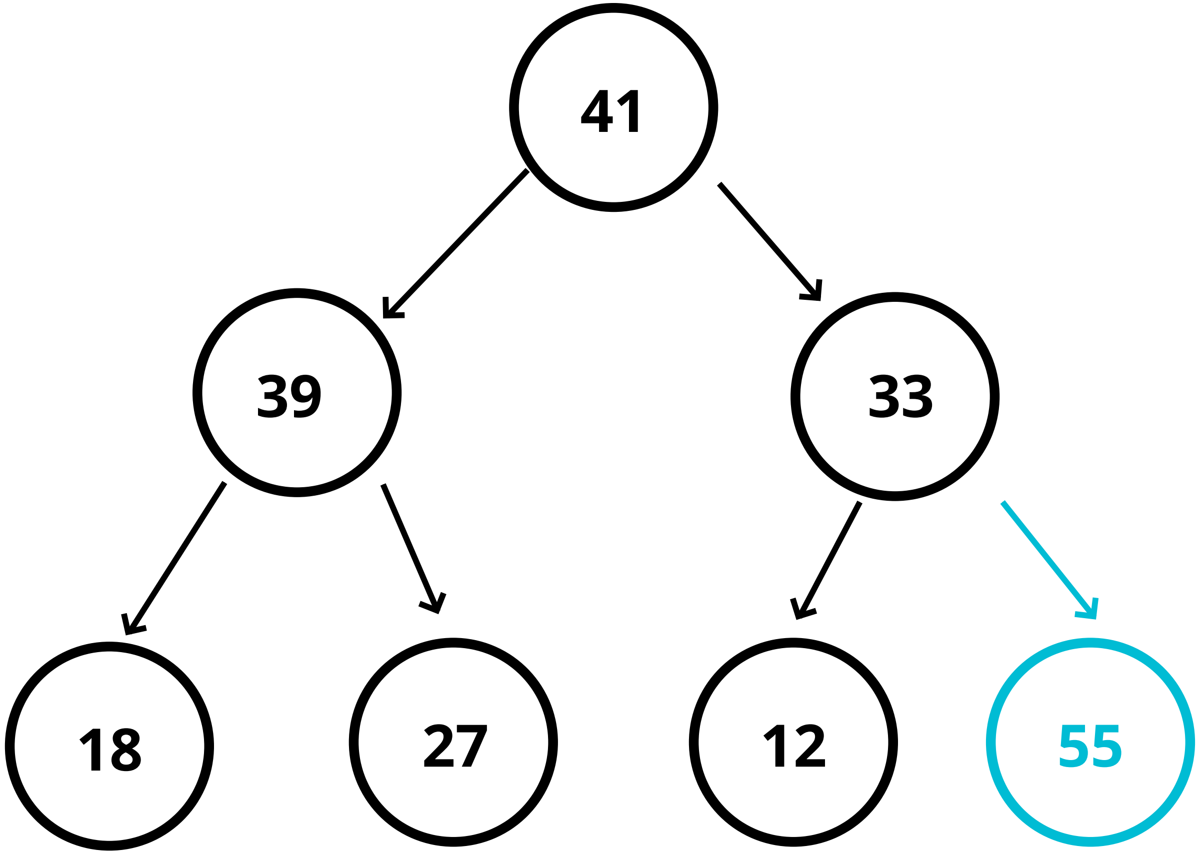
values = []

Adding to a MaxBinaryHeap



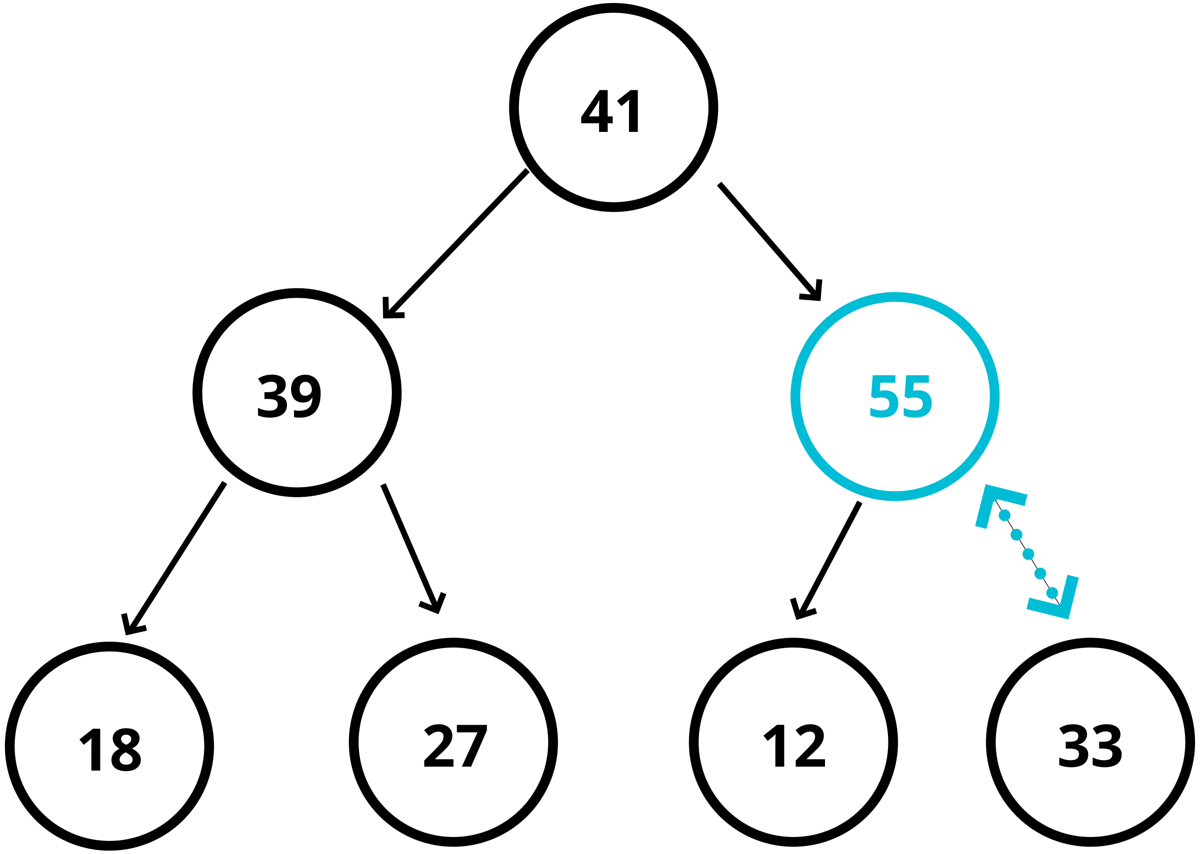
- Add to the end
- Bubble up

ADD TO THE END



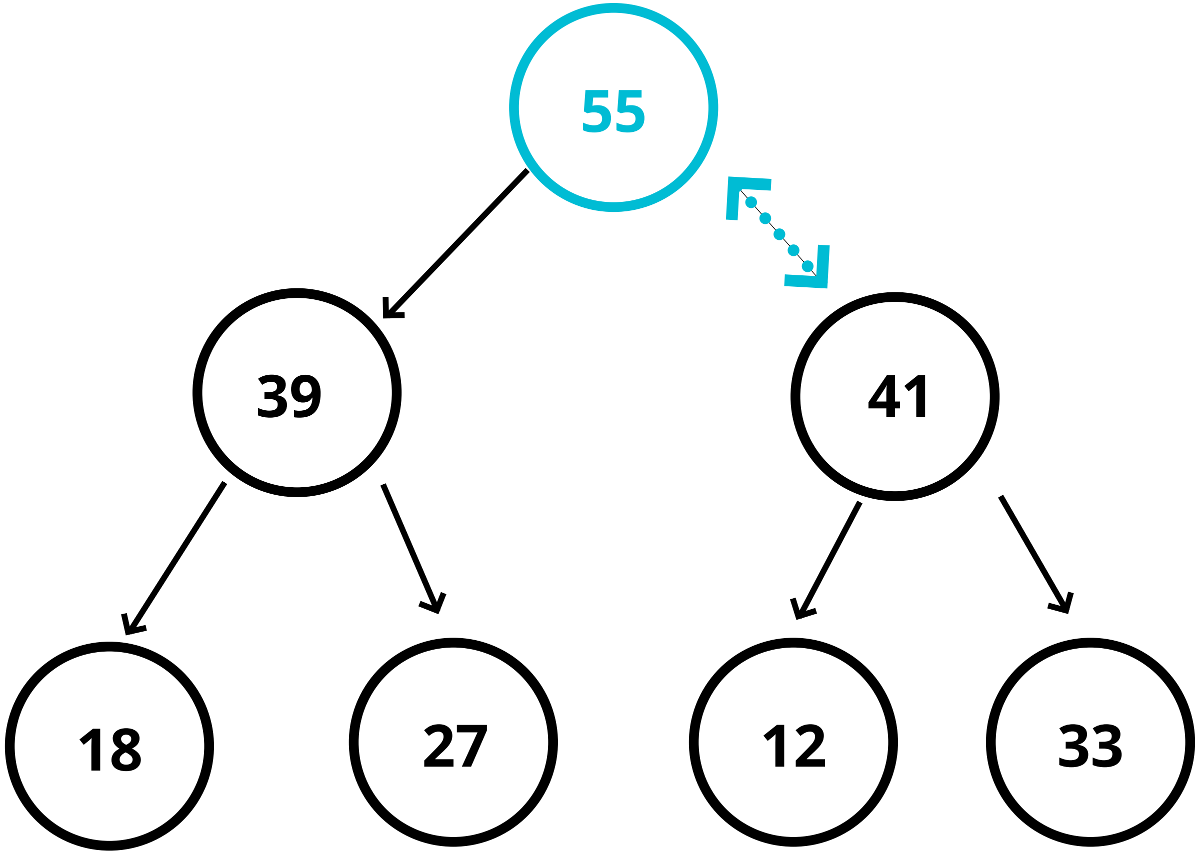
[41,39,33,18,27,12,55]

BUBBLE UP



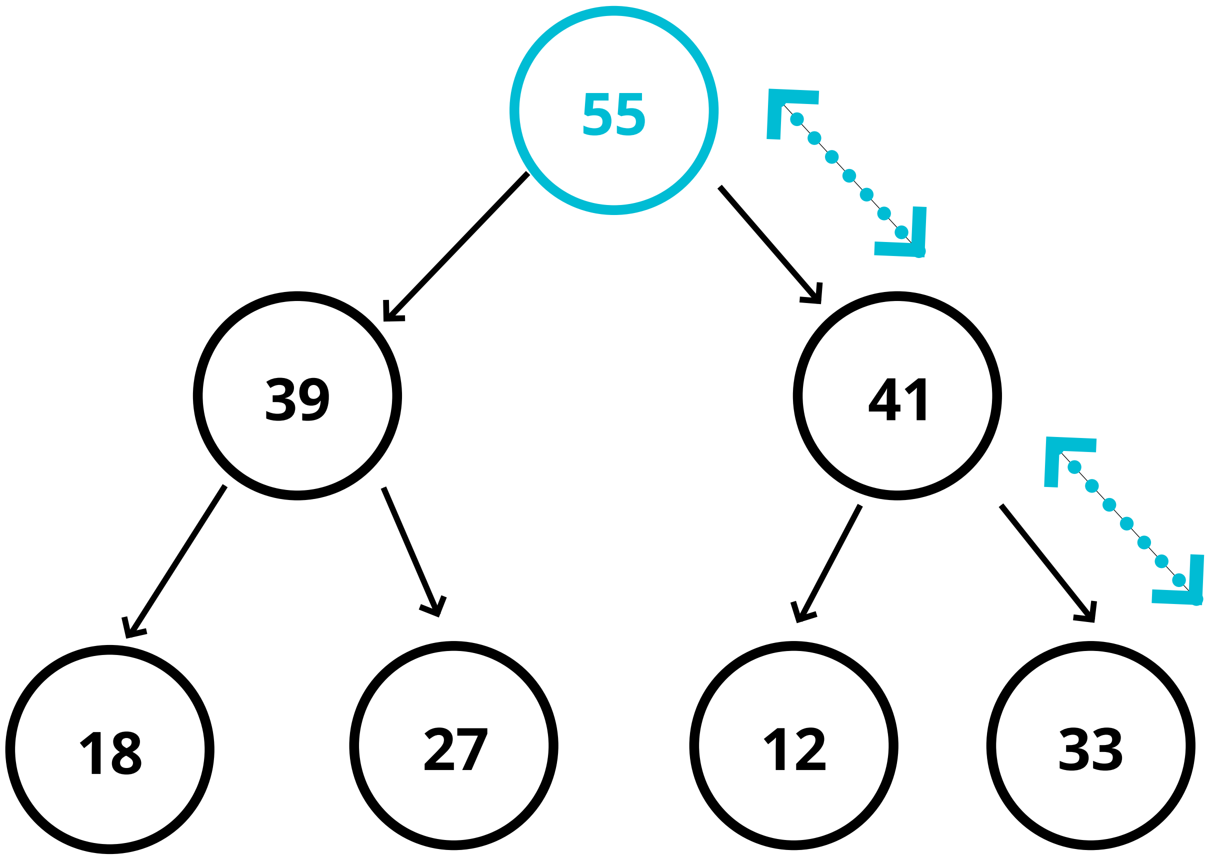
[41,39,55,18,27,12,33]

BUBBLE UP



[55,39,41,18,27,12,33]

Bubbling Up



INSERT PSEUDOCODE

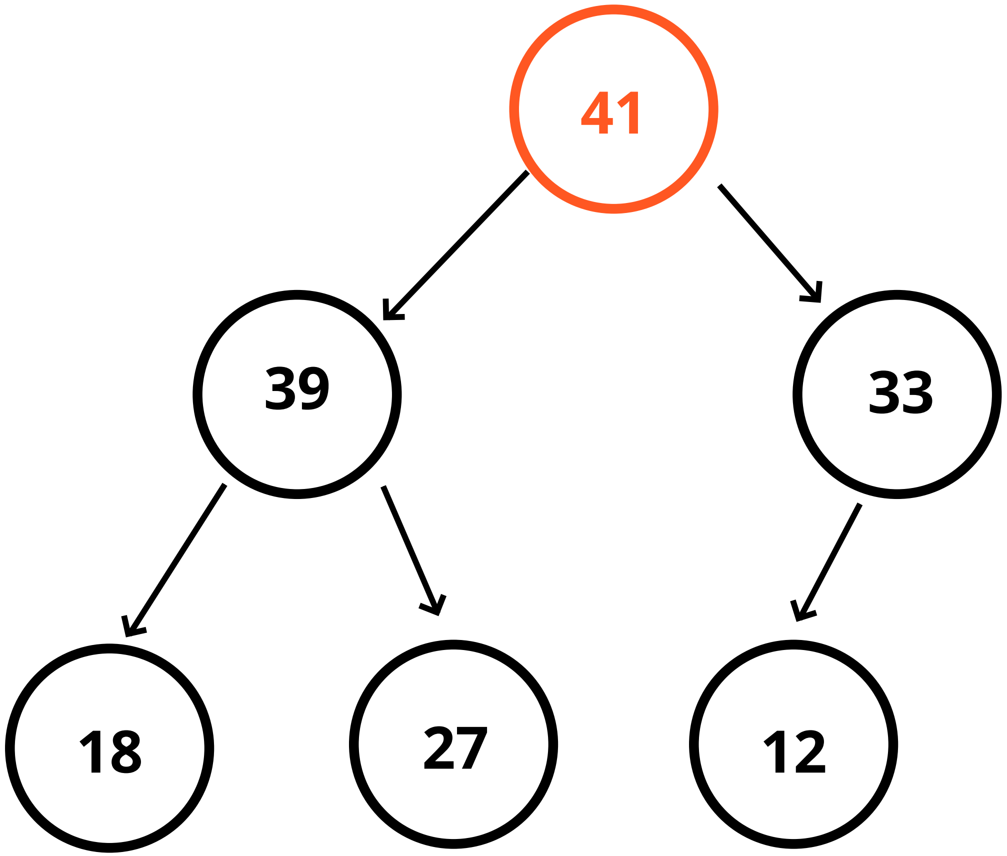
- Push the value into the values property on the heap
- Bubble the value up to its correct spot!

INSERT PSEUDOCODE

- Push the value into the values property on the heap
- Bubble Up:
    - Create a variable called index which is the length of the values property - 1
    - Create a variable called parentIndex which is the floor of (index-1)/2
    - Keep looping as long as the values element at the parentIndex is less than the values element at the child index
        - Swap the value of the values element at the parentIndex with the value of the element property at the child index
        - Set the index to be the parentIndex, and start over!

YOUR
TURN

REMOVING FROM A HEAP



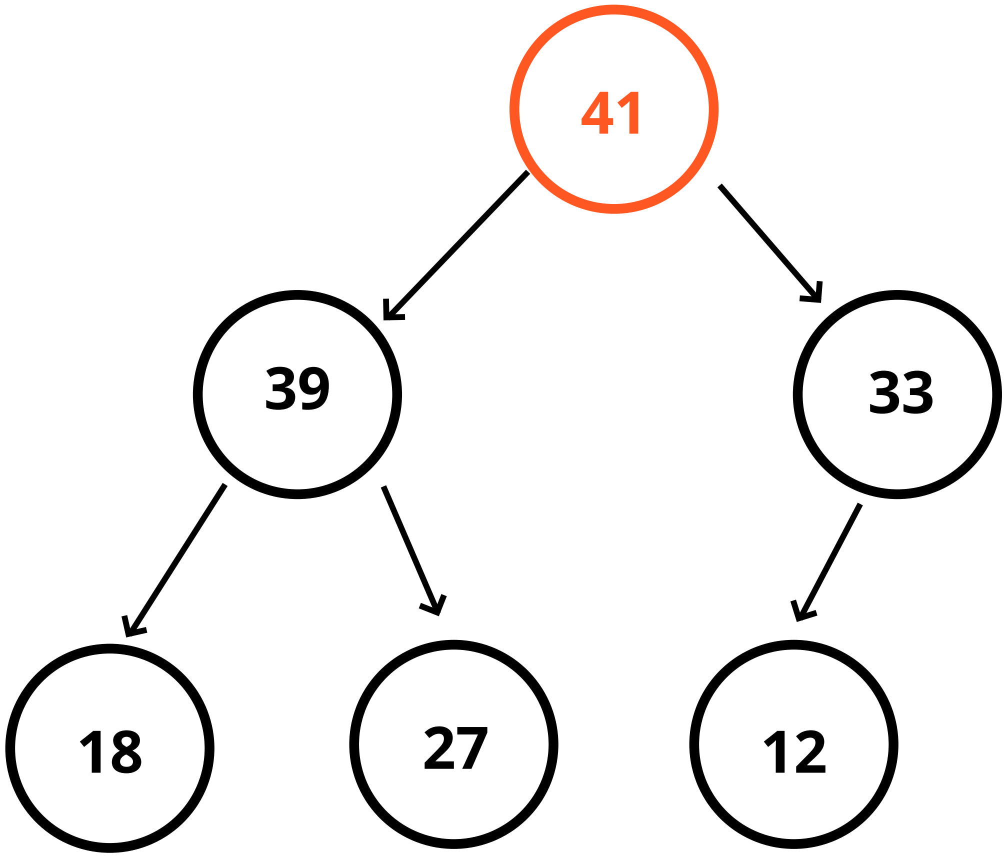
- Remove the root
- Replace with the most recently added
- Adjust (sink down)

Let's visualize this!

SINK DOWN?

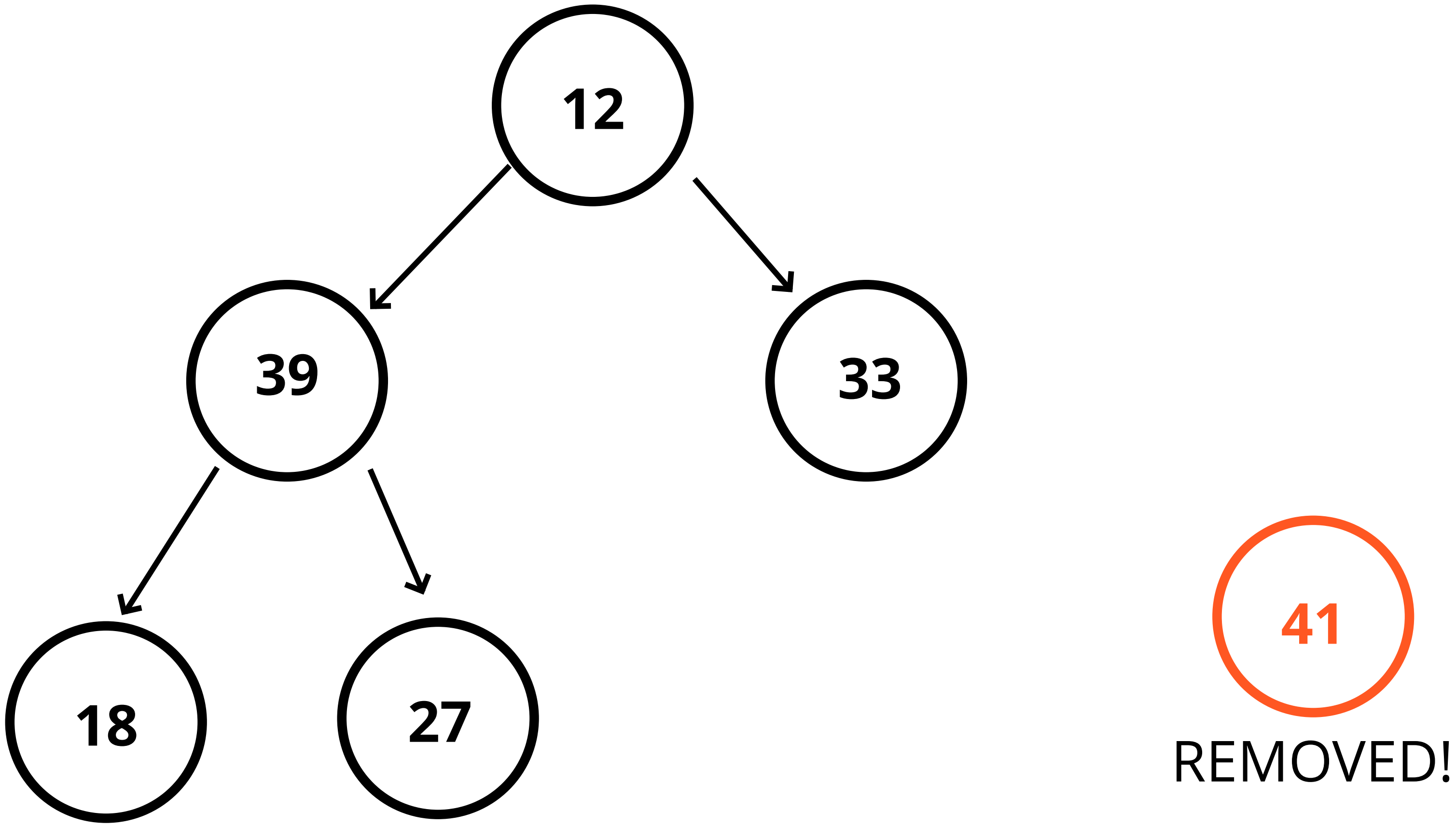
The procedure for deleting the root from the heap (effectively extracting the maximum element in a max-heap or the minimum element in a min-heap) and restoring the properties is called *down-heap* (also known as *bubble-down*, *percolate-down*, *sift-down*, *trickle down*, *heapify-down*, *cascade-down*, and *extract-min/max*).

REMOVE AND SWAP



[41,39,33,18,27,12]

REMOVE AND SWAP

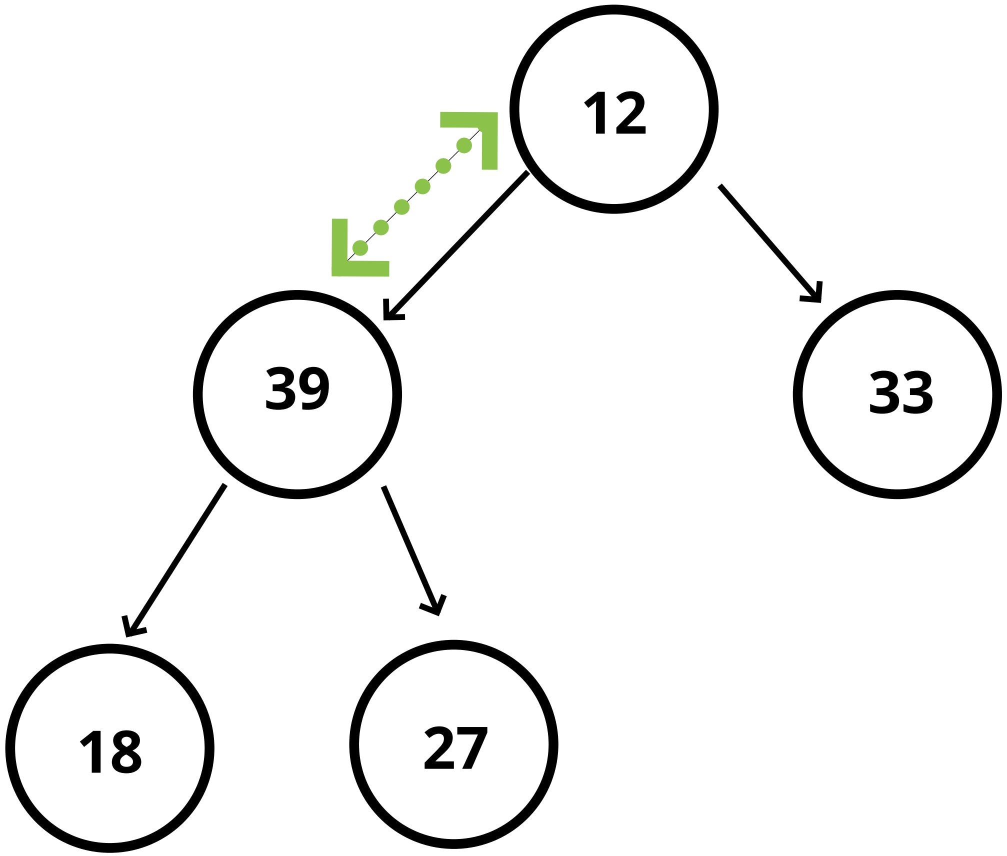


[41,39,33,18,27,12]

↓

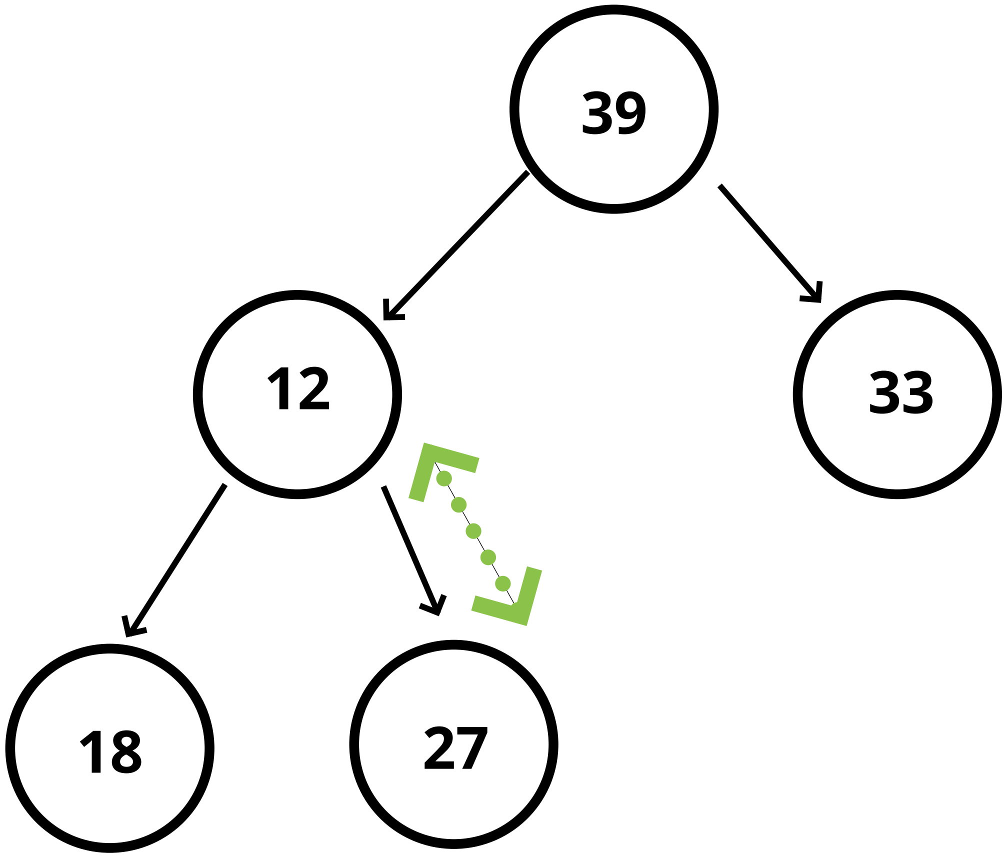
[12,39,33,18,27]

SINKING DOWN



[39,12,33,18,27]

SINKING DOWN



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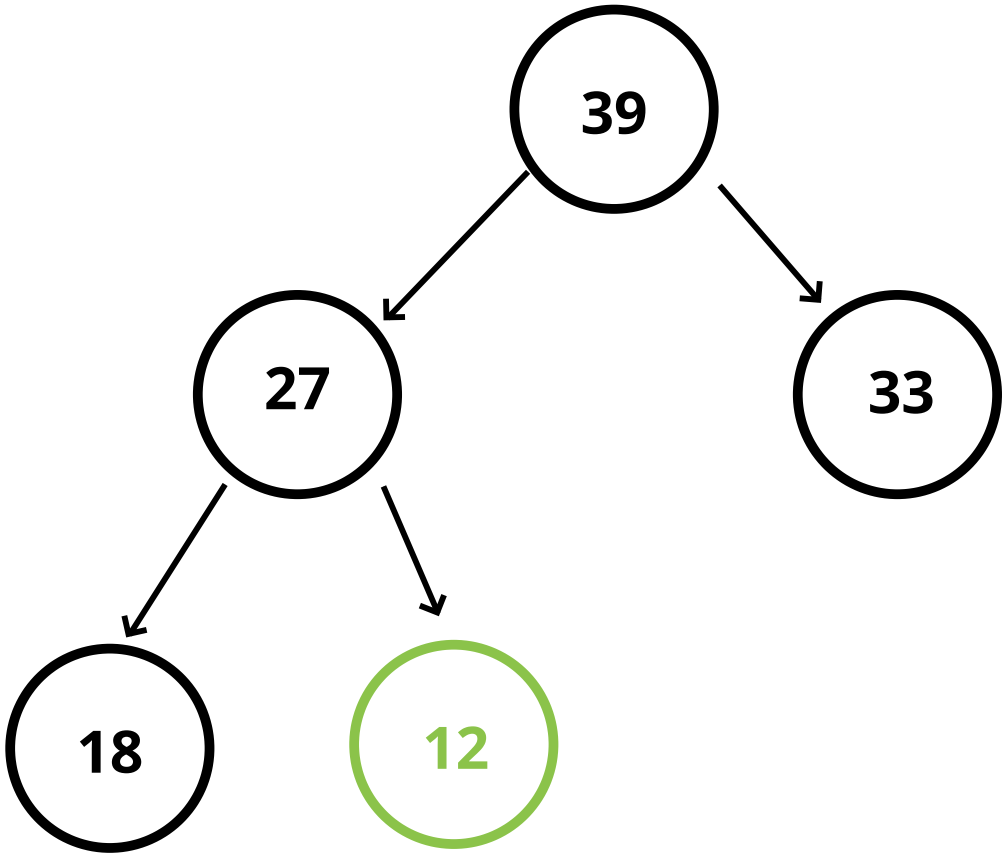
SINKING DOWN



[39,27,33,18,12]

REMOVING

(also called extractMax)

- Swap the first value in the values property with the last one
- Pop from the values property, so you can return the value at the end.
- Have the new root "sink down" to the correct spot...
    - Your parent index starts at 0 (the root)
    - Find the index of the left child: 2 * index + 1 (make sure its not out of bounds)
    - Find the index of the right child: 2*index + 2 (make sure its not out of bounds)
    - If the left or right child is greater than the element...swap. If both left and right children are larger, swap with the largest child.
    - The child index you swapped to now becomes the new parent index.
    - Keep looping and swapping until neither child is larger than the element.
    - Return the old root!

YOUR
TURN

BUILDING A PRIORITY QUEUE

WHAT IS A PRIORITY QUEUE?

A data structure where each element has a priority. Elements with higher priorities are served before elements with lower priorities.

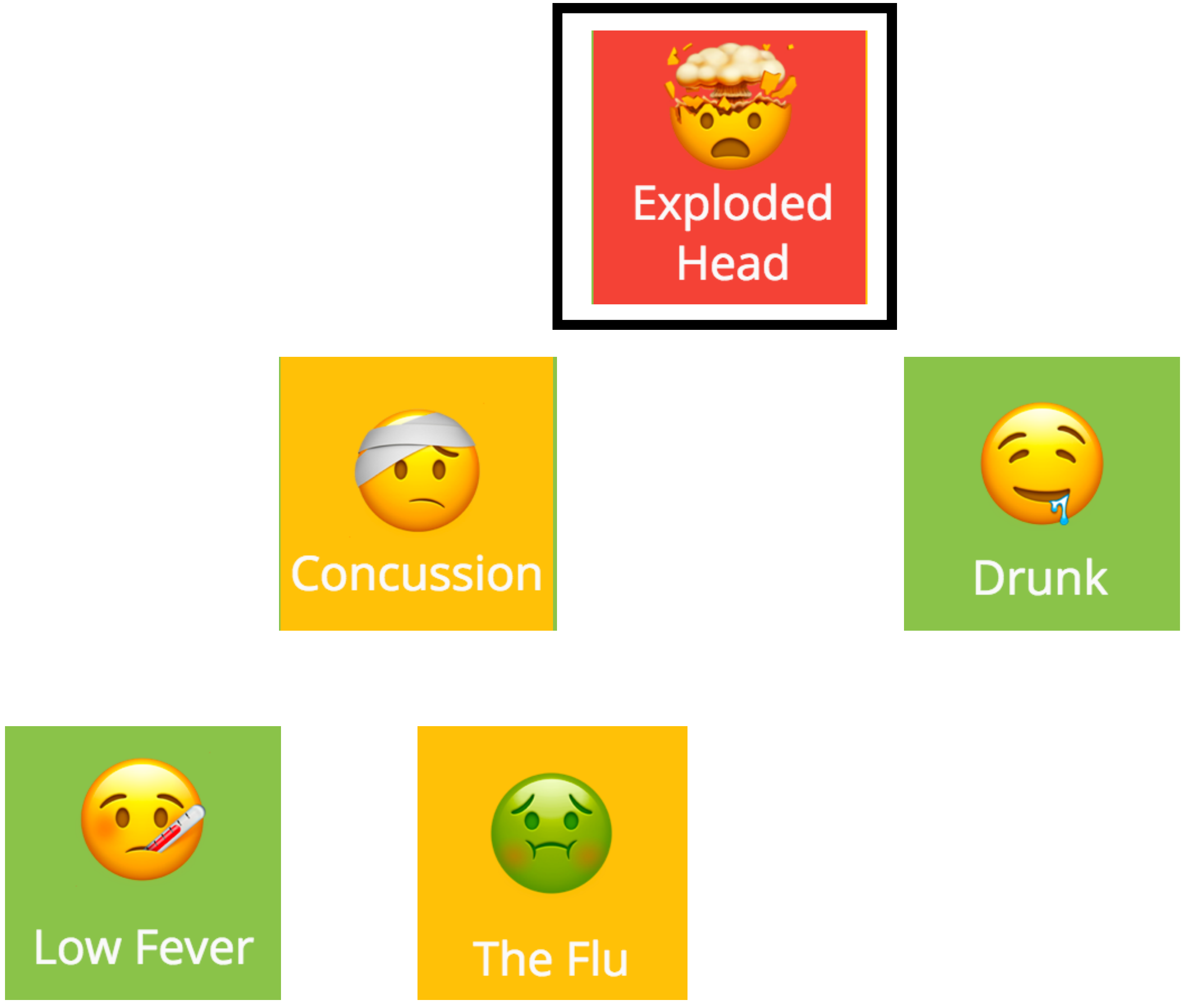
A NAIVE VERSION

Use a list to store all elements

priority: 3	priority: 1	priority: 2	priority: 5	priority: 4

Iterate over the entire thing to find the highest priority element.

NEXT TO
GET HELP



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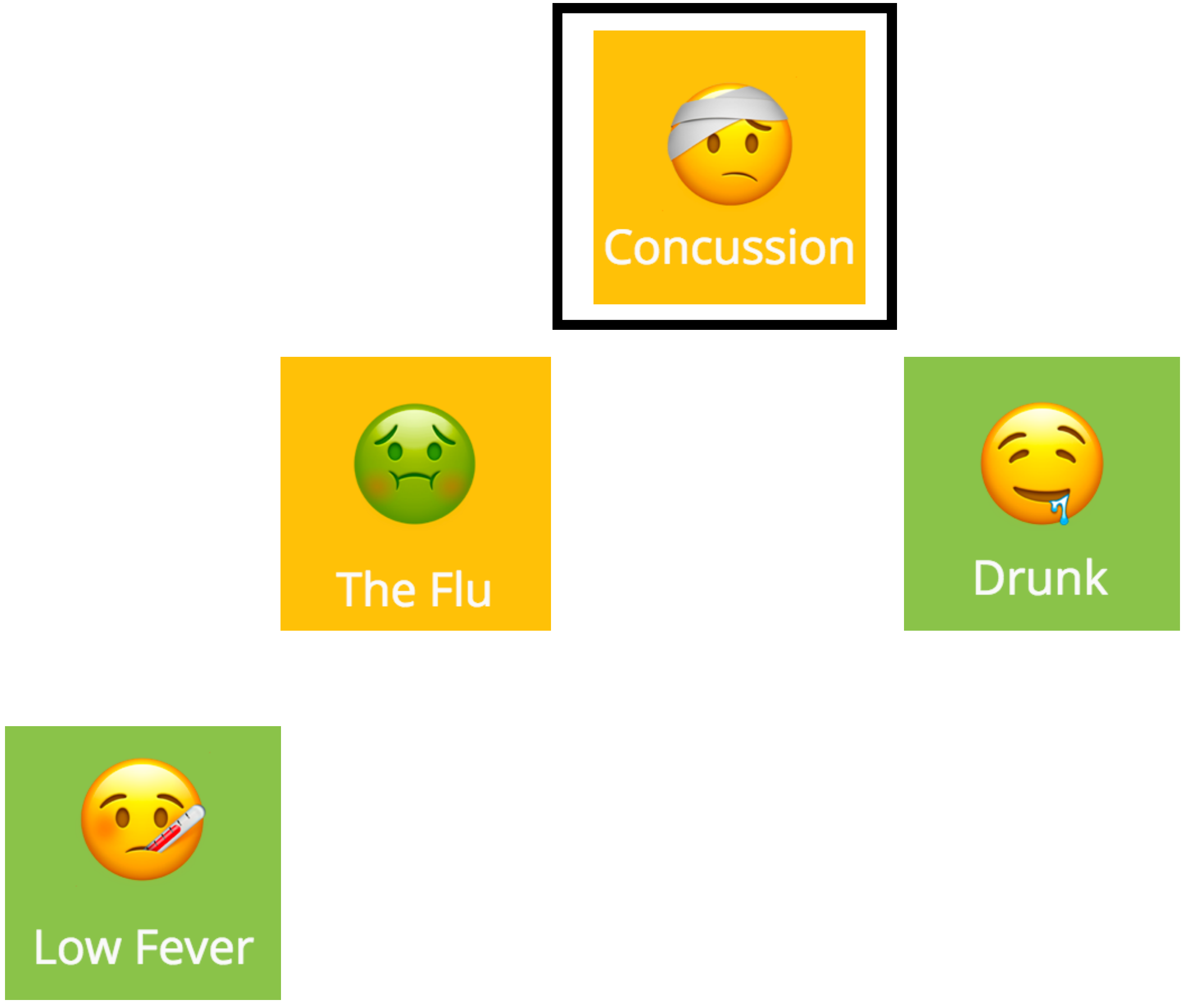


THE SAME AS BEFORE

Class Name:

PriorityQueue

Properties:

values = []

BUT ALSO...

Class Name:

Node

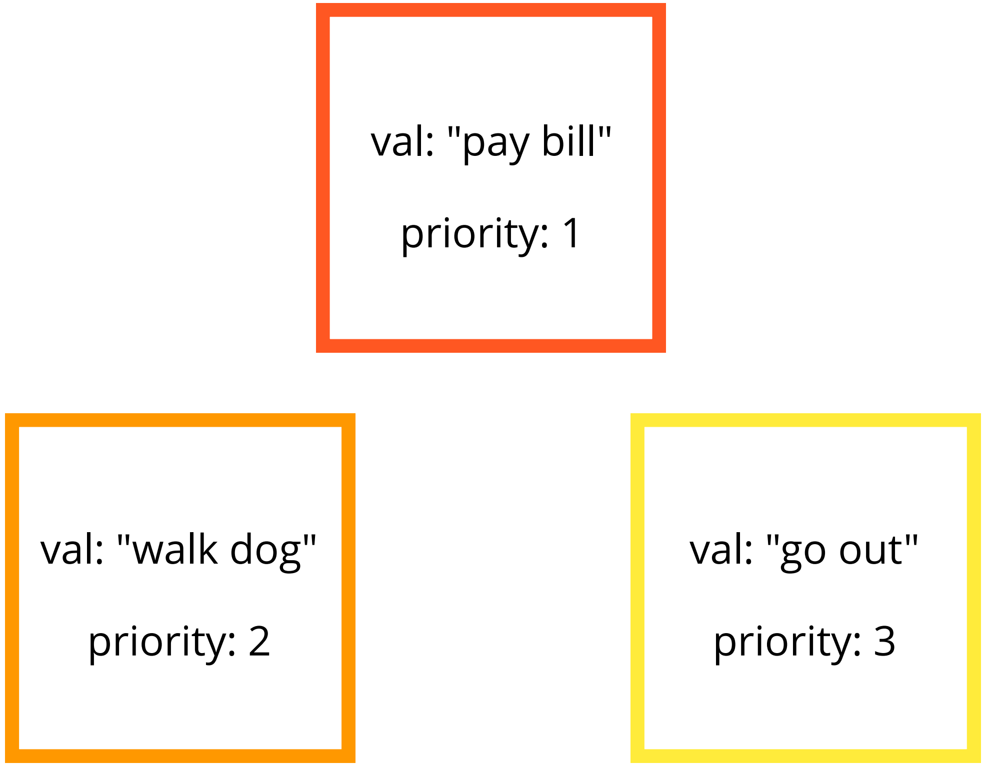
Properties:

val

priority

Val doesn't matter.

Heap is constructed using Priority



OUR PRIORITY QUEUE

- Write a Min Binary Heap - lower number means higher priority.
- Each Node has a val and a priority. Use the priority to build the heap.
- **Enqueue** method accepts a value and priority, makes a new node, and puts it in the right spot based off of its priority.
- **Dequeue** method removes root element, returns it, and rearranges heap using priority.

MaxHeapify

Converting an array into a MaxBinaryHeap

- Create a new heap
- Iterate over the array and invoke your **insert** function
- return the values property on the heap

YOUR
TURN

Heapsort

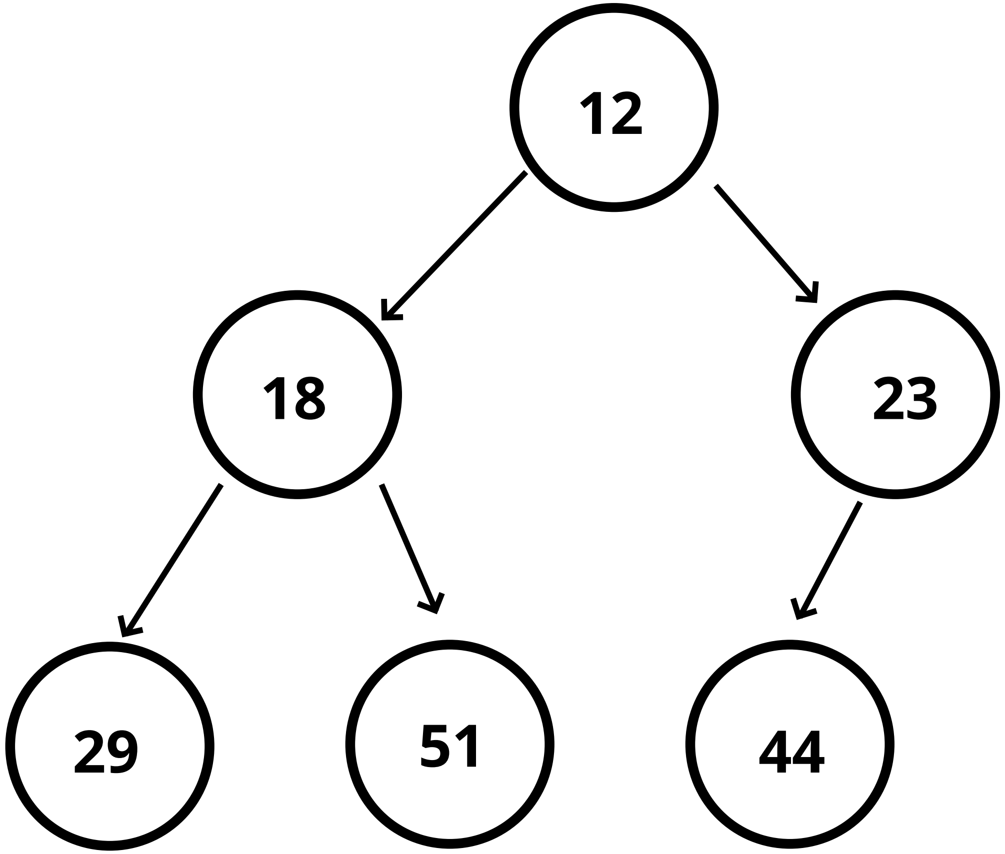
We can sort an array in **O(n log n)** time and **O(1)** space by making it a heap!

- Make the array a max heap (use **maxHeapify)**
- Loop over the array, swap the root node with last item in the array
- After swapping each item, run **maxHeapify** again to find the next root node
- Next loop you'll swap the root node with the second-to-last item in the array and run **maxHeapify** again.
- Once you've run out of items to swap, you have a sorted array!

Let's visualize this!

YOUR
TURN

MinBinaryHeap



Same idea, min values go upwards

Big O of Binary Heaps

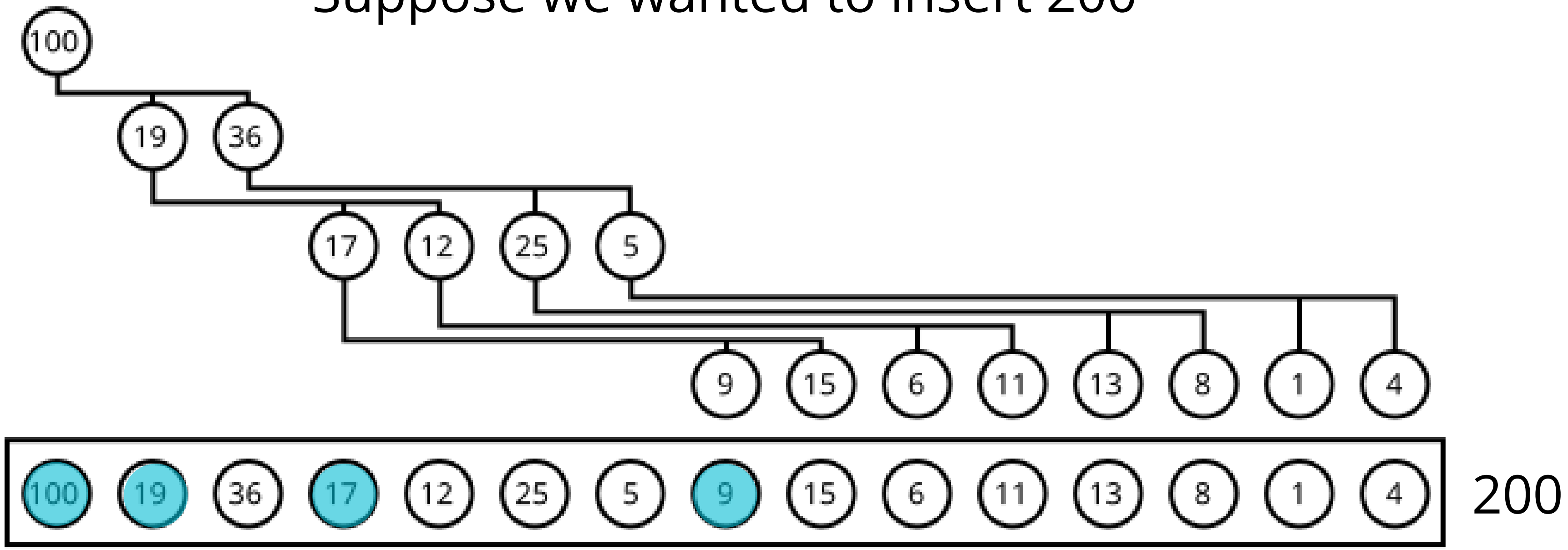
Insertion - **O(log N)**

Removal - **O(log N)**

Search - **O(N)**

WHY LOG(N)?

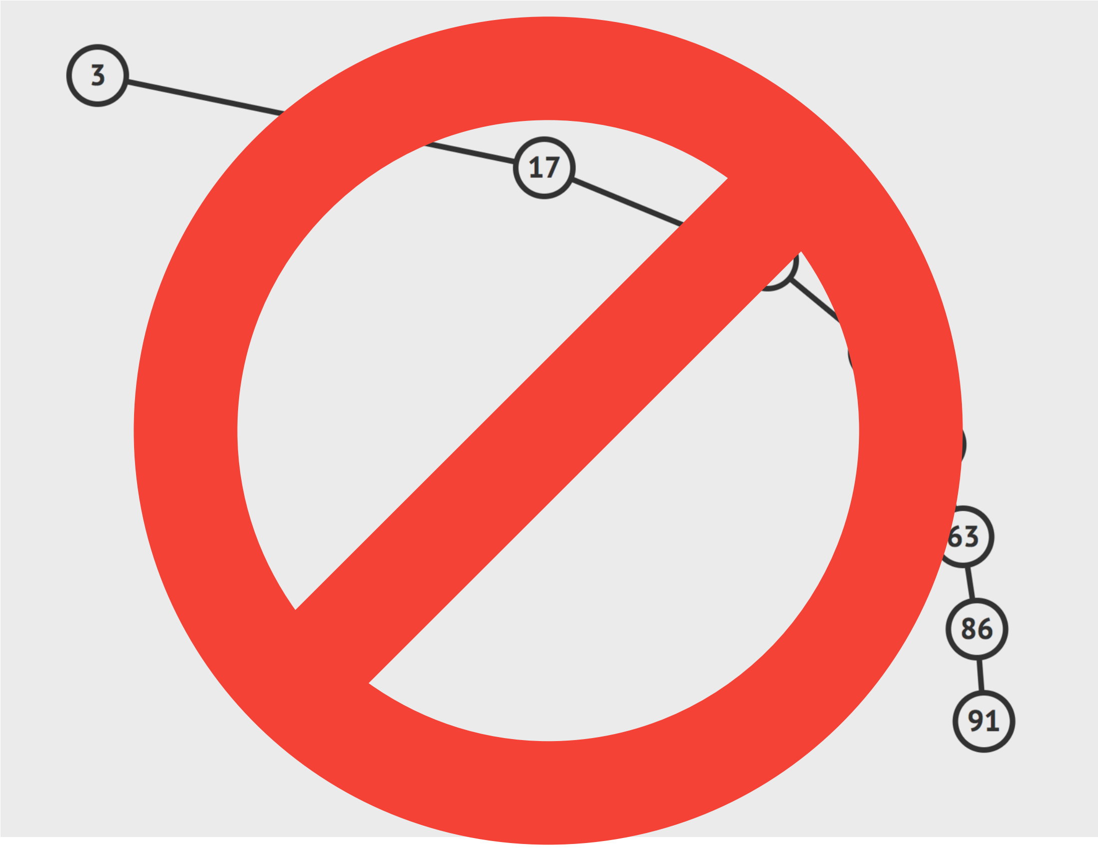
Suppose we wanted to insert 200



For 16 Elements....4 comparisons

WHAT ABOUT WORST CASE?

REMEMBER THIS DEPRESSING TREE?



NOT POSSIBLE WITH HEAPS!

RECAP

- Binary Heaps are very useful data structures for sorting, and implementing other data structures like priority queues
- Binary Heaps are either MaxBinaryHeaps or MinBinaryHeaps with parents either being smaller or larger than their children
- With just a little bit of math, we can represent heaps using arrays!