

CS528

Cilk

Slides are adopted from

<http://supertech.csail.mit.edu/cilk/>

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Cilk

- Developed by **Leiserson at CSAIL, MIT**
 - **Chapter 27, Multithreaded Algorithm, Introduction to Algorithm, Coreman, Leiserson and Rivest**
- Initiated a startup: Cilk Plus
 - Added Cilk_for Keyword, Cilk Reduction features
 - Acquired by Intel, Intel uses Cilk Scheduler
- Addition of 6 keywords to standard C
 - Easy to install in linux system
 - With gcc and pthread

Cilk

- In 2008, ACM SIGPLAN awarded **Best influential paper of Decade**
 - **The Implementation of the Cilk-5 Multithreaded Language**, PLDI 1998
- PLDI 2008 Best paper Award
 - Reducers and Other Cilk++ Hyperobjects , PLDI 2008

Cilk : Biggest principle

- Programmer should be responsible for
 - Exposing the parallelism,
 - Identifying elements that can safely be executed in parallel
- Work of run-time environment (scheduler) to
 - Decide during execution how to actually divide the work between processors
- Work Stealing Scheduler
 - Proved to be good scheduler
 - Now also in GCC, Intel CC, **Intel acquire Cilk++**

Fibonacci

```
int fib (int n) {  
  if (n<2) return (n);  
  else {  
    int x,y;  
    x = fib(n-1);  
    y = fib(n-2);  
    return (x+y);  
  }  
}
```

C elision

Cilk code

```
cilk int fib (int n) {  
  if (n<2) return (n);  
  else {  
    int x,y;  
    x = spawn fib(n-1);  
    y = spawn fib(n-2);  
    sync;  
    return (x+y);  
  }  
}
```

Cilk is a *faithful* extension of C. A Cilk program's *serial elision* is always a legal implementation of Cilk semantics. Cilk provides *no* new data types.

Basic Cilk Keywords

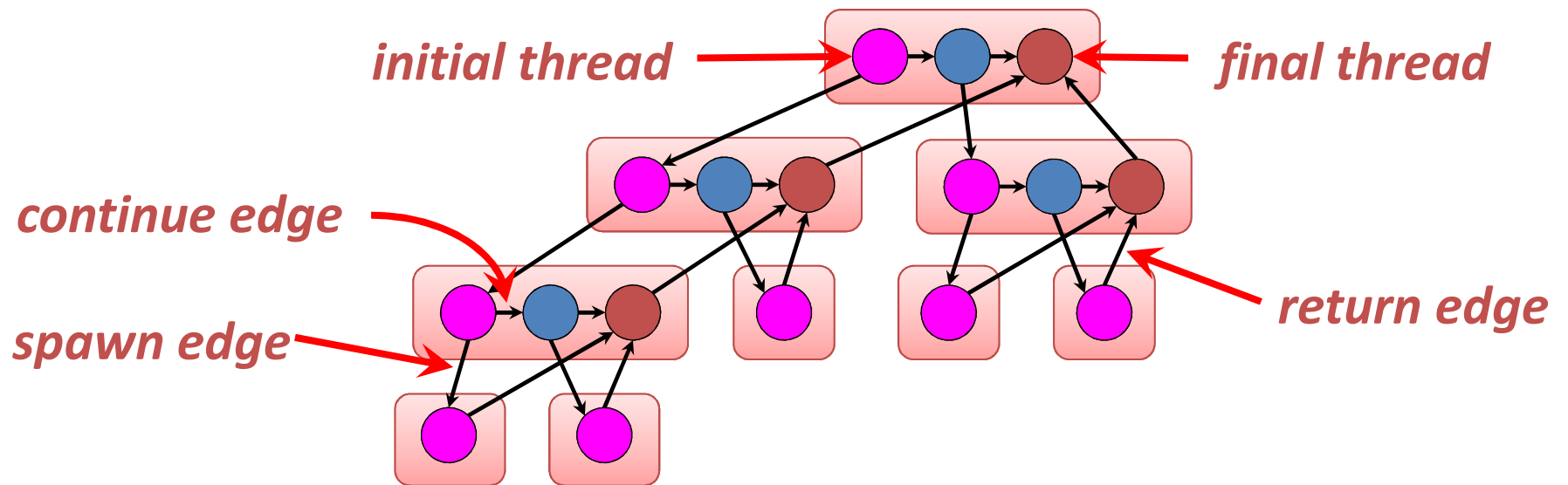
```
cilk int fib (int n) {  
    if (n<2) return (n);  
    else {  
        int x,y;  
        x = spawn fib(n-1);  
        y = spawn fib(n-2);  
        sync;  
        return (x+y);  
    }  
}
```

Identifies a function as a *Cilk procedure*, capable of being spawned in parallel.

The named *child* Cilk procedure can execute in parallel with the *parent* caller.

Control cannot pass this point until all spawned children have returned.

Multithreaded Computation

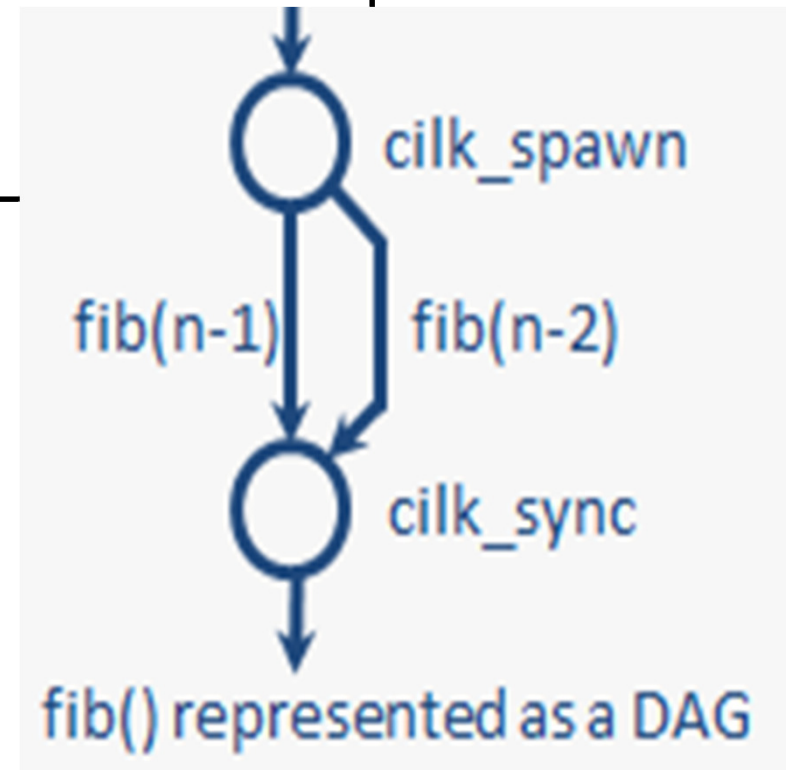


- The dag $G = (V, E)$ represents a parallel instruction stream.
- Each vertex $v \in V$ represents a *(Cilk) thread*: a maximal sequence of instructions not containing parallel control (**spawn**, **sync**, **return**).
- Every edge $e \in E$ is either a *spawn* edge, a *return* edge, or a *continue* edge.

Fib: Cilk++ Version

```
int fib(int n) {  
    if (n < 2) return n;  
    int x=cilk_spawn fib(n-1);  
    int y = fib(n-2);  
    cilk_sync;  
    return x + y;  
}
```

Not available in
Cilk



For loop in Cilk

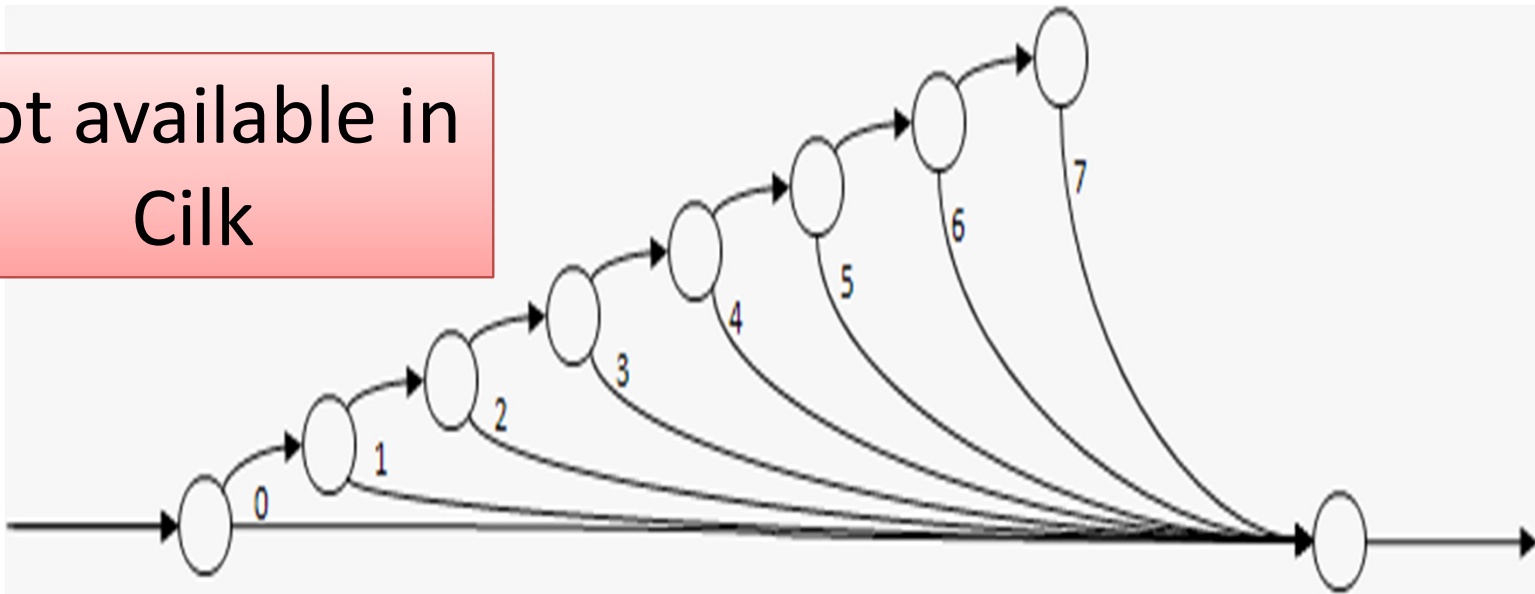
```
for (int i = 0; i < 8; ++i)  
    do_work(i);
```

Serial

```
for (int i = 0; i < 8; ++i)  
    cilk_spawn do_work(i);  
cilk_sync;
```

Parallel

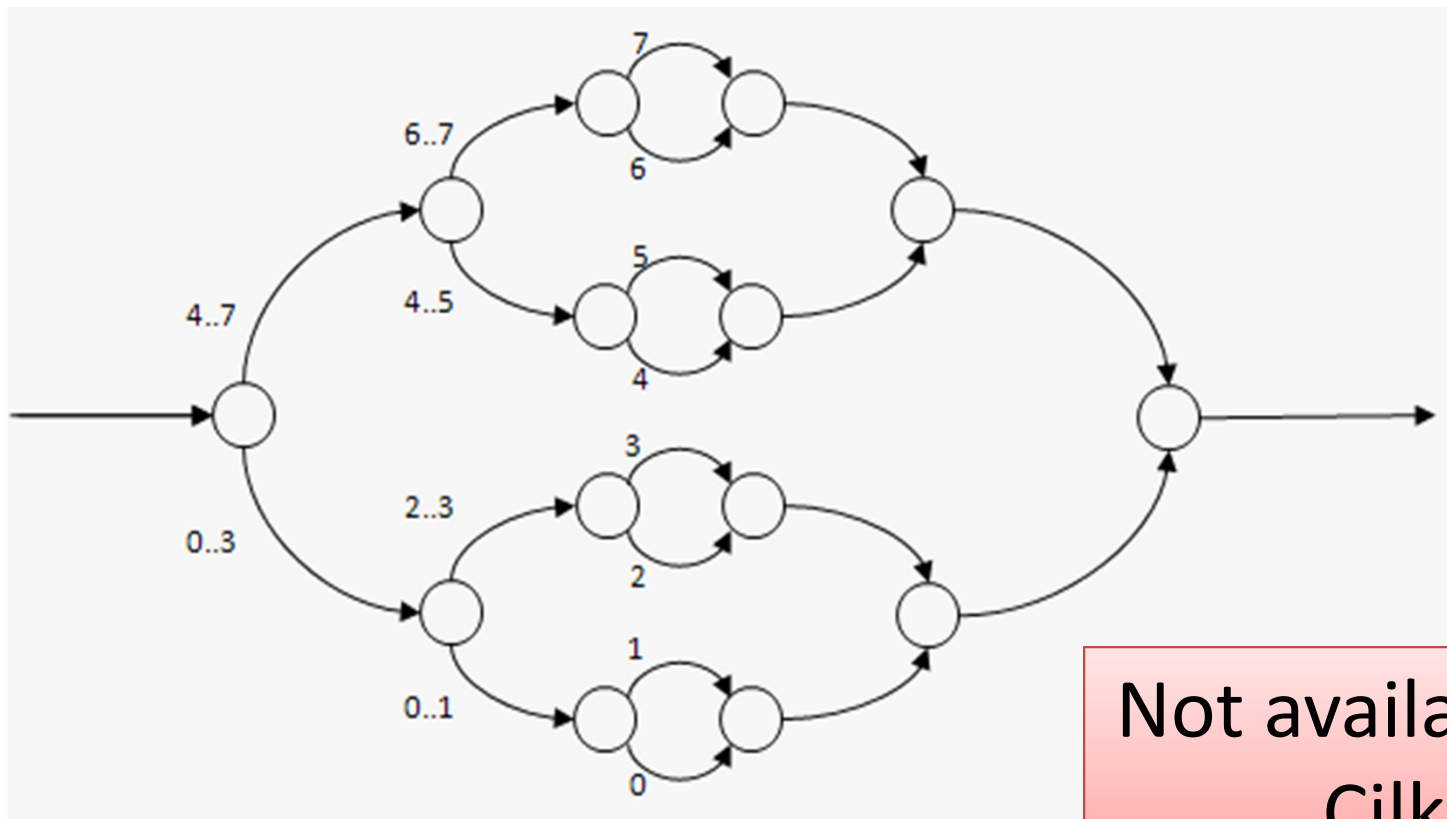
Not available in
Cilk



Loop_for in Cilk++

```
cilk_for (int i=0; i<8; ++i) {  
    do_work(i);  
} // No sync required; auto sync
```

Parallel



Not available in
Cilk

Cilk Run Time Scheduler

- Distributed load balancing
 - Receiver initiated
- Work stealing : Free processor steal a task of busy processor
- When ever a process spawns a new process,
 - This processor starts executing the spawned one
 - Parent goes to waiting/suspend mode
 - Parent can be transferred to other processor

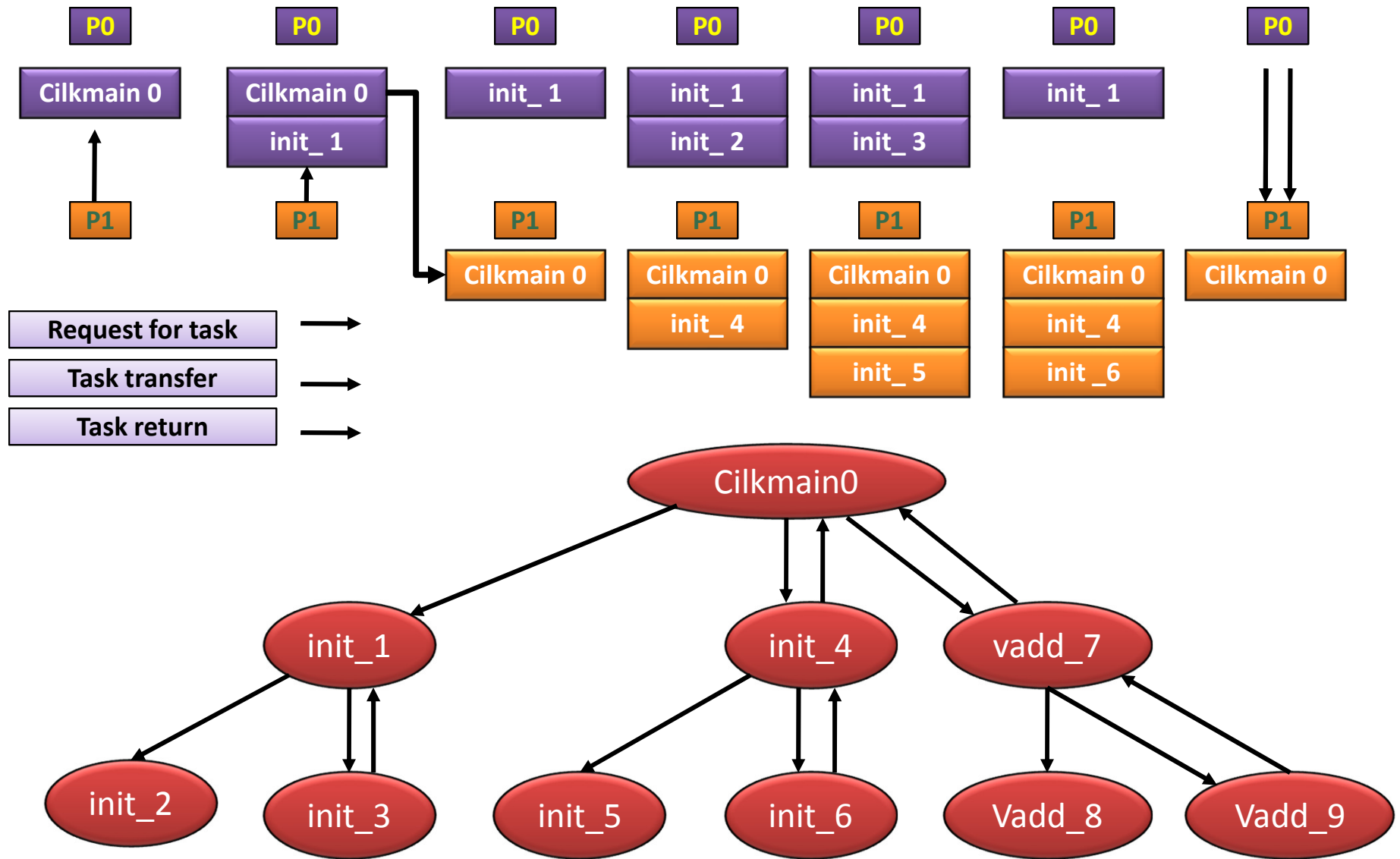
Work stealing

- Work stealing algorithm is receiver initiated algorithm
- Technique commonly used for load balancing
- Thief processor (Idle processor)
 - Steal work from other processor
 - Victim is selected randomly
- Victim processor (From a set of busy processor)
 - Work is stolen from these processor

Work stealing

- Optimal algorithm for load balancing
 - If select victim randomly algorithm is Optimal
- **Proved**
- Basic assumption in work stealing
 - All the memory access are take same time
 - **UMA (Uniform Memory Access): shared memory**
 - Can be feasible iff
 - Task transfer time is same for all pair of processors
 - Communication bandwidth is same for all pair of processors

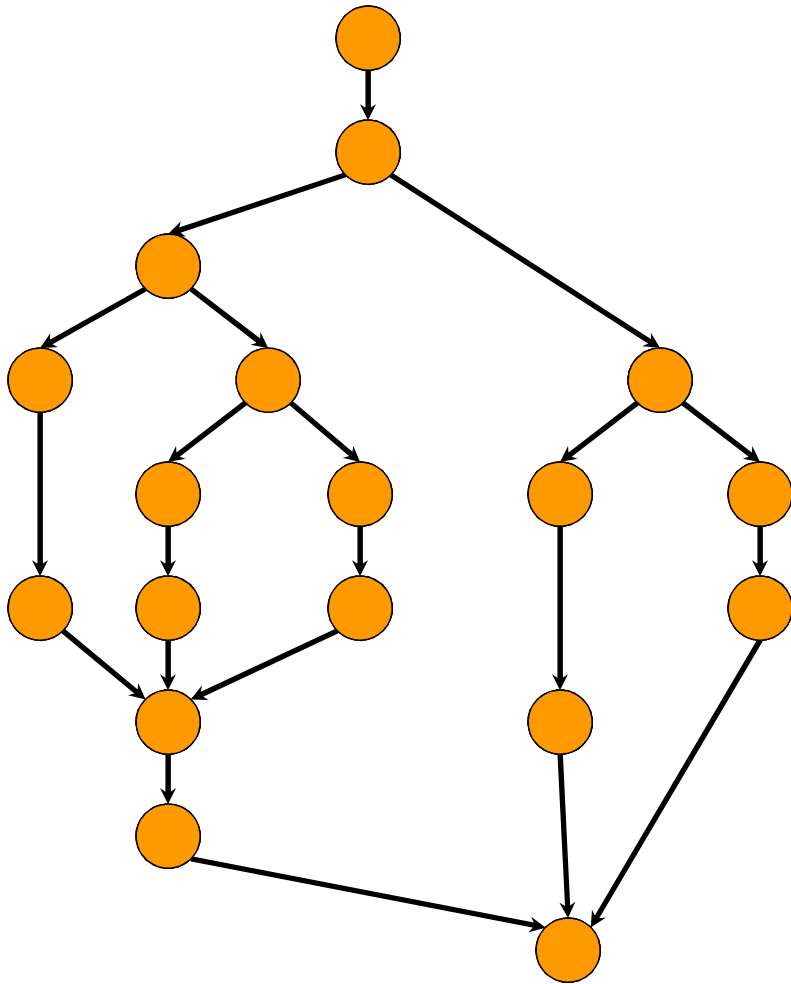
Cilk Run Time Scheduler



Algorithmic Complexity Measures

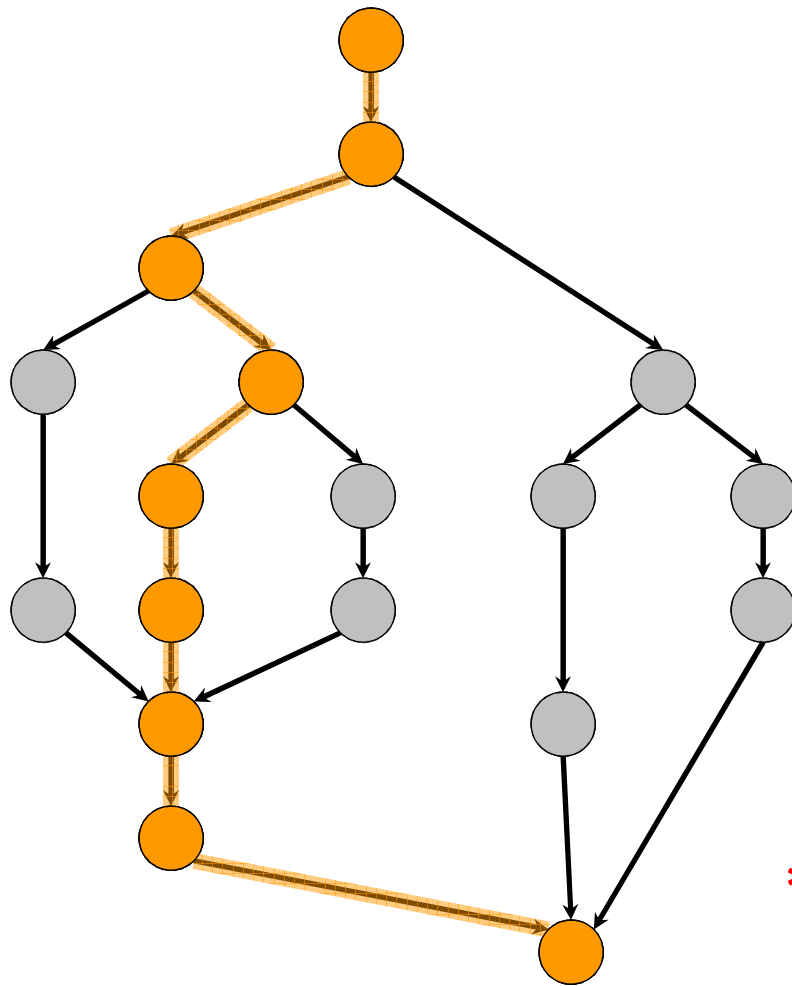
T_P = execution time on P processors

T_1 = *work*



Algorithmic Complexity Measures

T_P = execution time on P processors



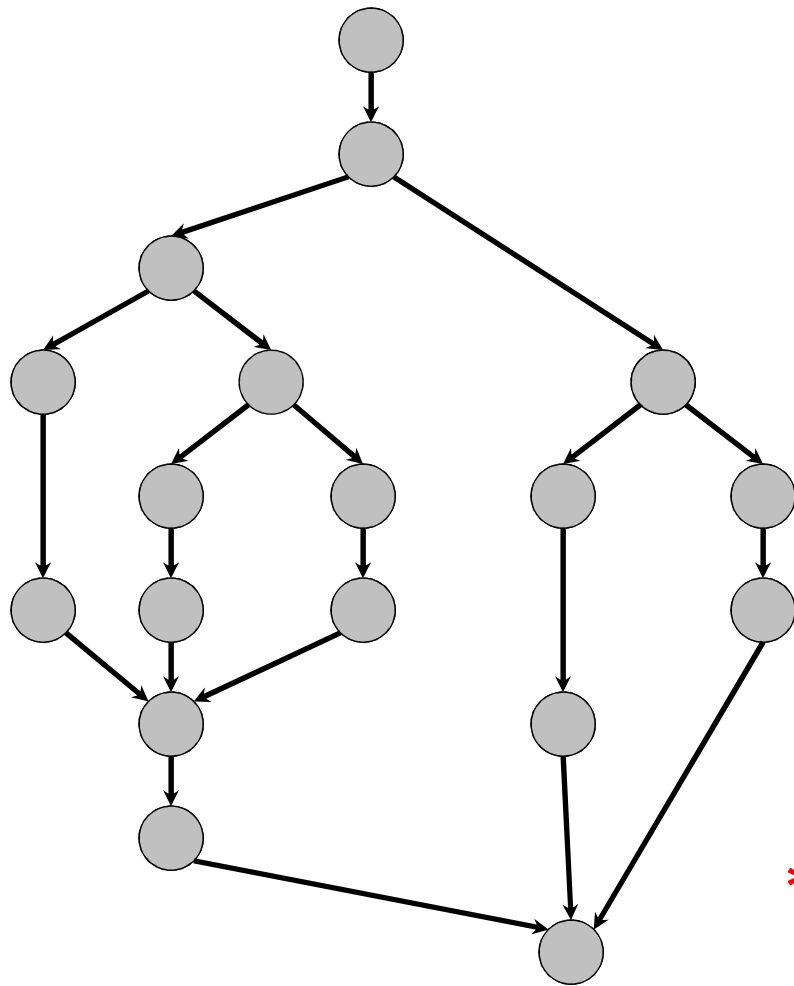
$$T_1 = \textit{work}$$

$$T_\infty = \textit{span}^*$$

* Also called *critical-path length* or *computational depth*.

Algorithmic Complexity Measures

T_p = execution time on P processors



$$T_1 = \textit{work}$$

$$T_\infty = \textit{span}^*$$

LOWER BOUNDS

- $T_p \geq T_1/P$
- $T_p \geq T_\infty$

* Also called *critical-path length* or *computational depth*.

Speedup

Definition: $T_1/T_P = \text{speedup}$ on P processors.

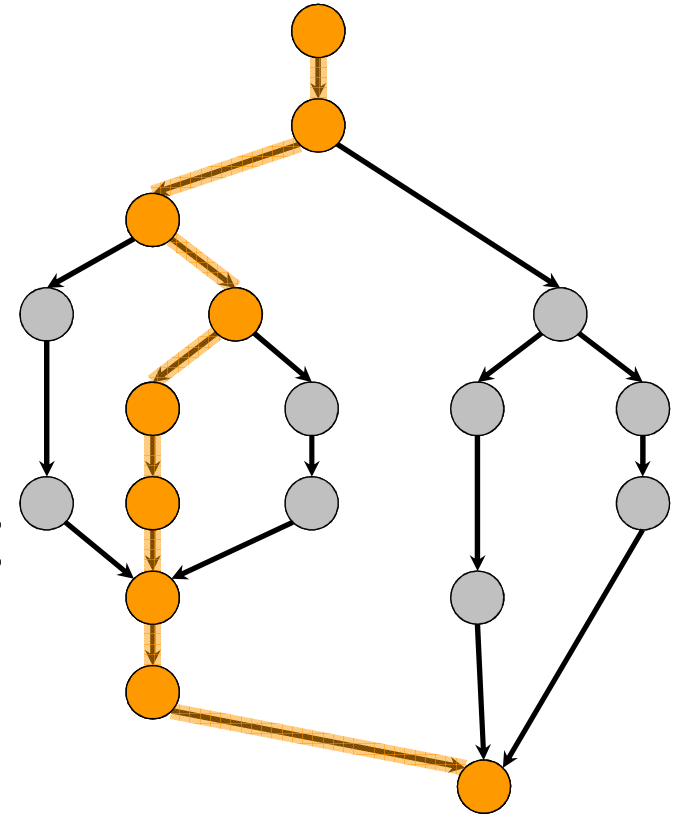
If $T_1/T_P = \Theta(P) \cdot P$, we have *linear speedup*;
 $= P$, we have *perfect linear speedup*;
 $> P$, we have *superlinear speedup*,
which is not possible in our model, because
of the lower bound $T_P \geq T_1/P$.

Parallelism

Because we have the lower bound $T_p \geq T_\infty$, the maximum possible speedup given T_1 and T_∞ is

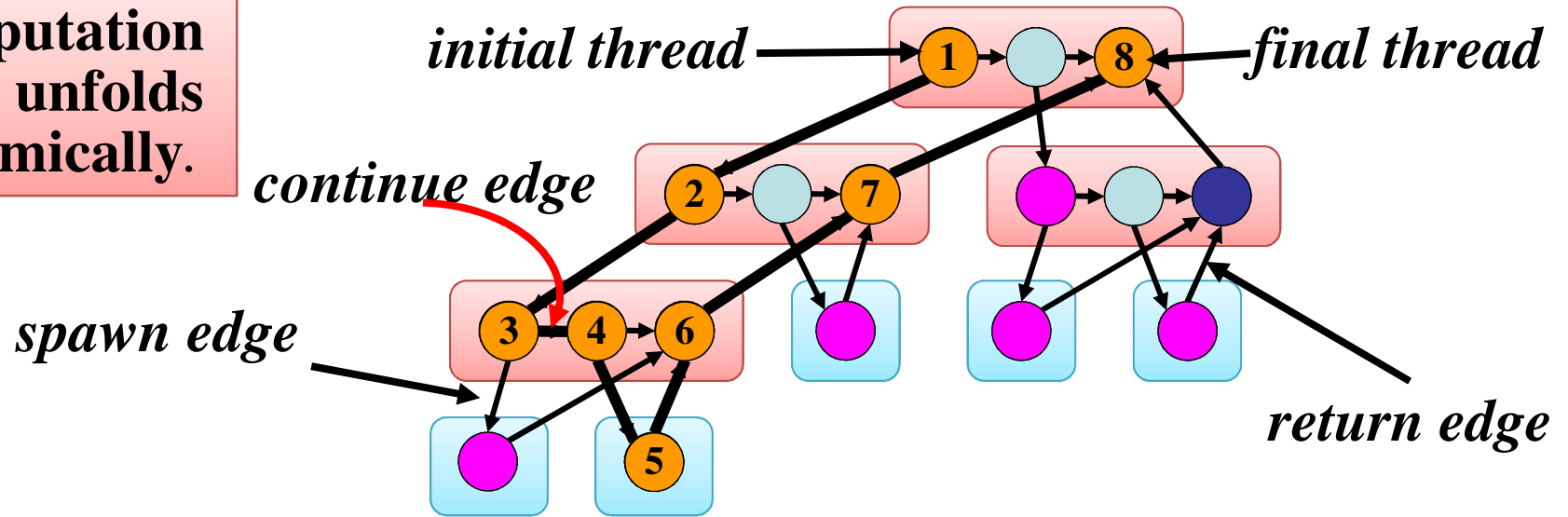
$$T_1/T_\infty = \textit{parallelism}$$

= the average amount of work per step along the span.



CILK Example: Fib(4)

Computation
DAG unfolds
dynamically.



Assume for simplicity that each Cilk thread in `fib()` takes unit time to execute.

Work: $T_1 = 17$

Span: $T_\infty = 8$

Parallelism: $T_1/T_\infty = 2.125$

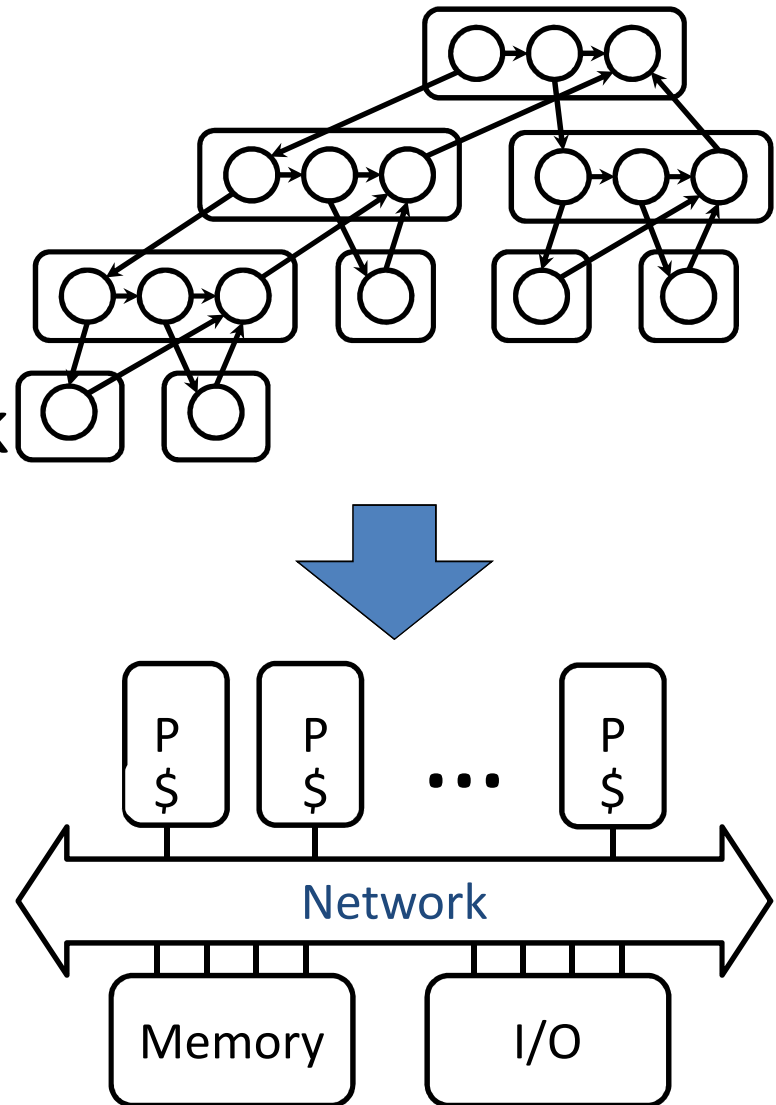
Using many more
than 2 processors
makes little sense.

Ref1: The Cilk System for Parallel Multithreaded Computing, MIT Phd Thesis

Ref2: The Implementation of the Cilk-5 Multithreaded Language, 1998 ACM SIGPLAN

Scheduling

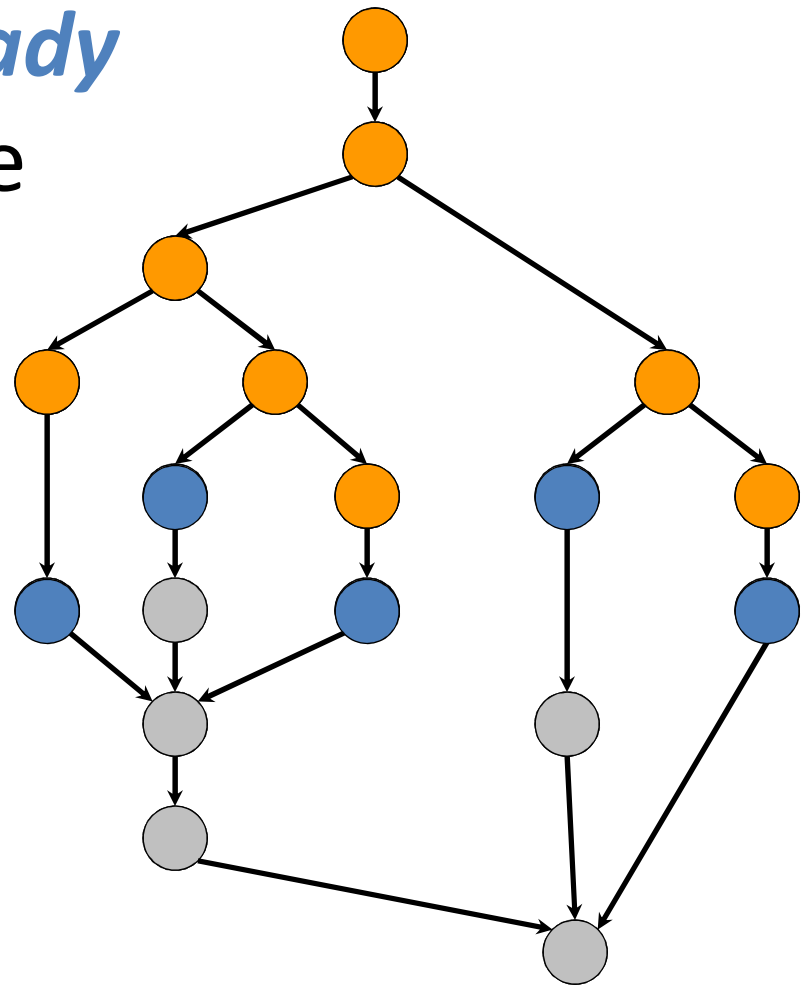
- Cilk allows the programmer to express *potential* parallelism in an application.
- The Cilk *scheduler* maps Cilk threads onto processors dynamically at runtime.
- Since *on-line* schedulers are complicated, we'll illustrate the ideas with an *off-line* scheduler.



Greedy Scheduling

IDEA: Do as much as possible on every step.

Definition: A thread is *ready* if all its predecessors have *executed*.



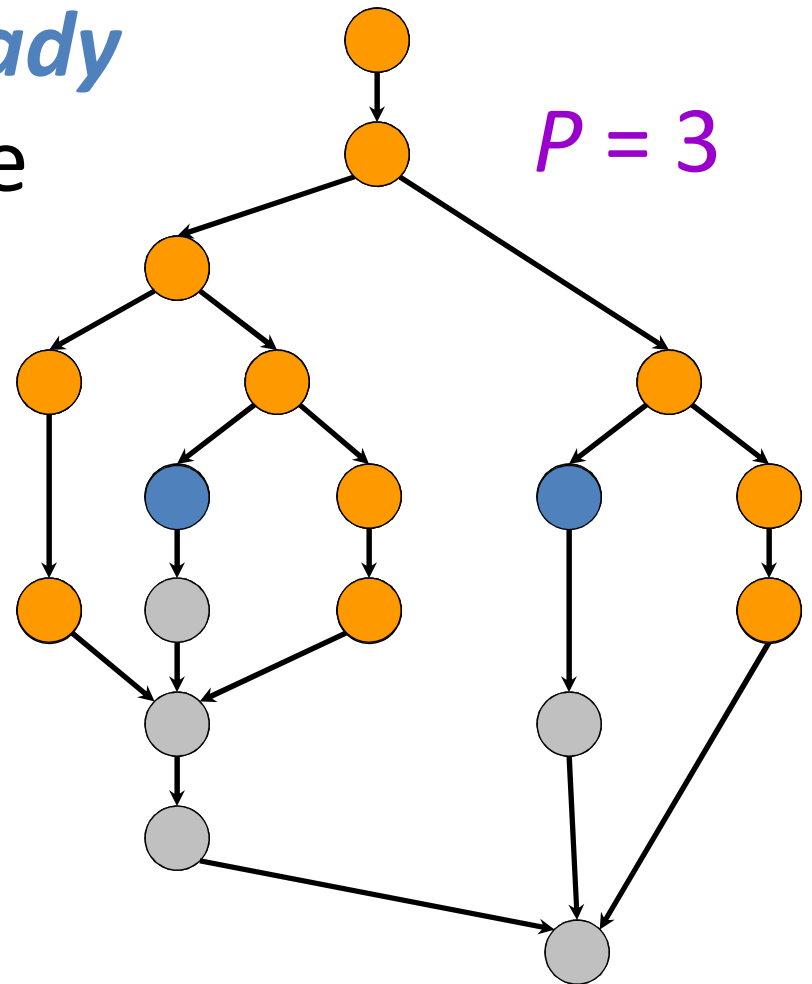
Greedy Scheduling

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Complete step

- $\geq P$ threads ready.
- Run any P .



Greedy Scheduling

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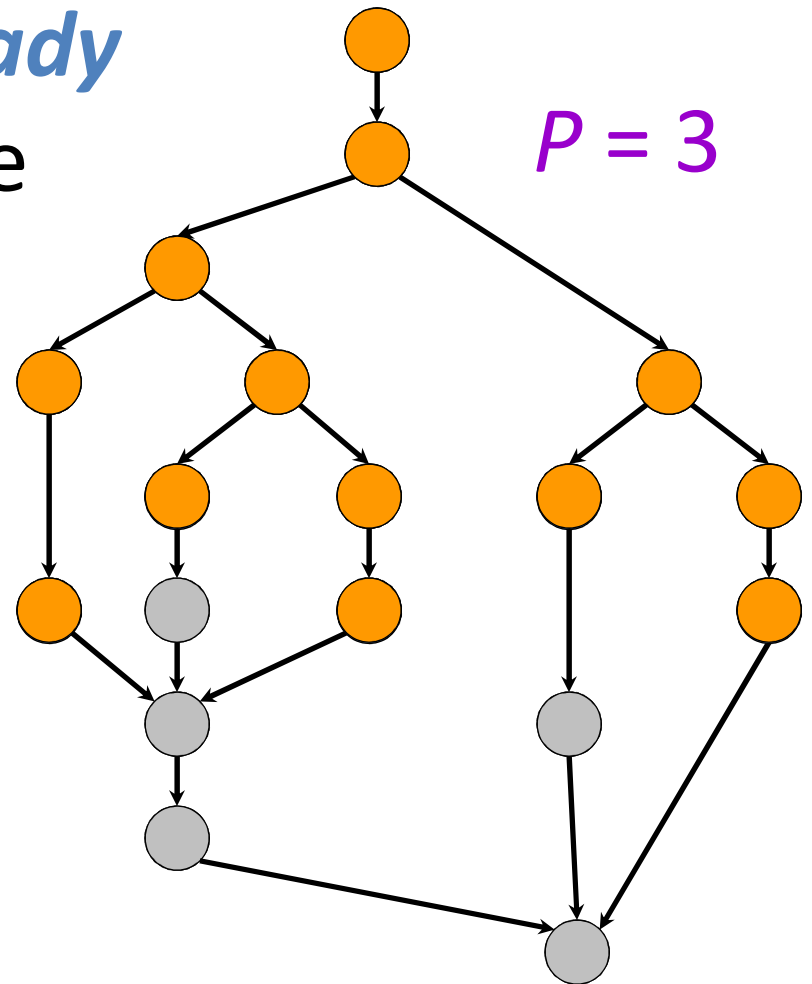
Definition: A thread is *ready* if all its predecessors have *executed*.

Complete step

- $\geq P$ threads ready.
- Run any P .

Incomplete step

- $< P$ threads ready.
- Run all of them.



Greedy-Scheduling Theorem

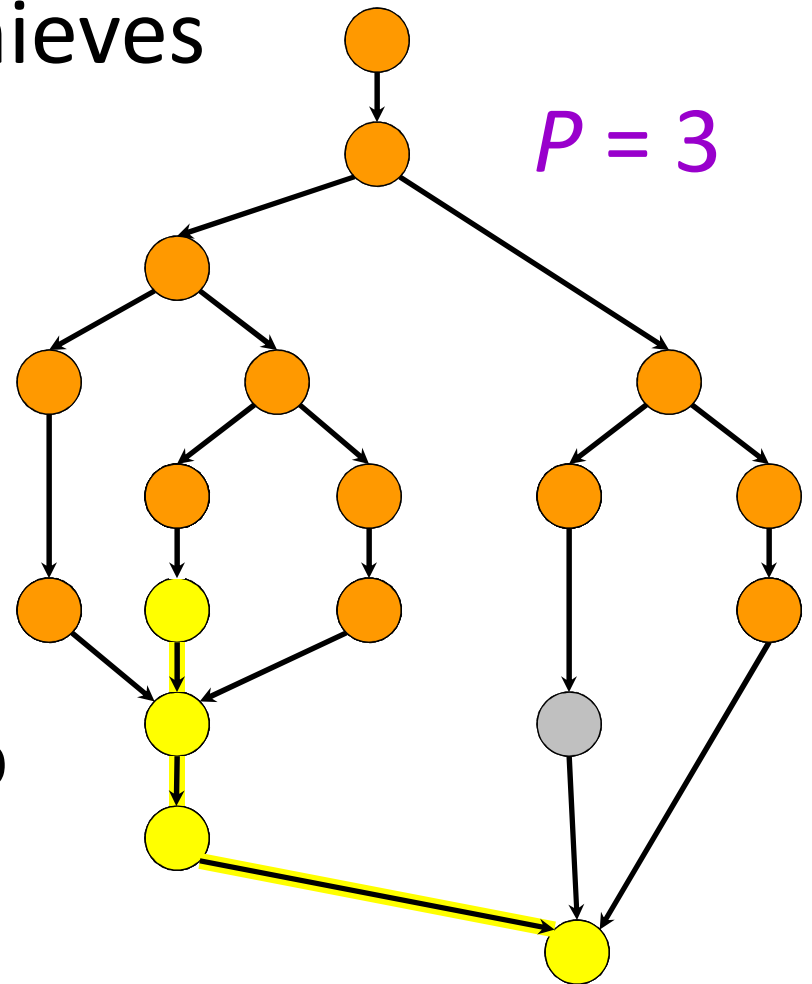
Theorem [Graham '68 & Brent '75].

Any greedy scheduler achieves

$$T_p \leq T_1/P + T_\infty.$$

Proof.

- # complete steps $\leq T_1/P$, since each complete step performs P work.
- # incomplete steps $\leq T_\infty$, since each incomplete step reduces the span of the unexecuted dag by 1. ■



Optimality of Greedy

Corollary. Any greedy scheduler achieves within a factor of 2 of optimal.

Proof. Let T_p^* be the execution time produced by the optimal scheduler. Since $T_p^* \geq \max\{T_1/P, T_\infty\}$ (lower bounds), we have

$$\begin{aligned} T_p &\leq T_1/P + T_\infty \\ &\leq 2 \max\{T_1/P, T_\infty\} \\ &\leq 2T_p^* . \quad \blacksquare \end{aligned}$$

Linear Speedup

Corollary. Any greedy scheduler achieves near-perfect linear speedup whenever $T_1/T_\infty \gg P$

Proof. Since $T_1/T_\infty \gg P \Rightarrow T_\infty \ll T_1/P$, the Greedy Scheduling Theorem gives us

$$\begin{aligned} T_P &\leq T_1/P + T_\infty \\ &\approx T_1/P. \end{aligned}$$

Thus, the speedup is $T_1/T_P \approx P$. ■

Definition. The quantity $(T_1/T_\infty)/P$ is called the *parallel slackness*.

Cilk Performance

- Cilk's “work-stealing” scheduler achieves
 - $T_p = T_1/P + O(T_\infty)$ expected time (provably);
 - $T_p \approx T_1/P + T_\infty$ time (empirically).
- Near-perfect linear speedup if $P \ll T_1/T_\infty$.
- Instrumentation in Cilk allows the user to determine accurate measures of T_1 and T_∞ .
- The average cost of a **spawn** in Cilk-5 is only 2–6 times the cost of an ordinary C function call, depending on the platform.

Parallelizing Vector Addition

C

```
void vadd(float *A, float *B, int N)
    int i; for (i=0; i<n; i++) A[i]+=B[i];
}
```

Parallelization strategy:

1. Convert loops to recursion.

Parallelizing Vector Addition

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void vadd(float *A, float *B, int N)
    int i; for (i=0; i<n; i++) A[i]+=B[i];
}
```

C

```
void vadd(float *A, float *B, int N) {
    if (n<=BASE) {
        int i; for (i=0; i<n; i++) A[i]+=B[i];
    } else {
        vadd (A, B, n/2);
        vadd (A+n/2, B+n/2, n/2);
    }
}
```

Parallelization strategy:

1. Convert loops to recursion.
2. Insert Cilk keywords.

Parallelizing Vector Addition

C

```
void vadd(float *A, float *B, int N)
    int i; for (i=0; i<n; i++) A[i]+=B[i];
}
```

Cilk

```
cilk void vadd(float *A, float *B, int N) {
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    } else {
        spawn vadd (A, B, n/2);
        spawn vadd (A+n/2, B+n/2, n/2);
        sync;
    }
}
```

Parallelization strategy:

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        spawn vadd (A+n/2, B+n/2, n/2);
        sync;
    }
}
```

Parallelization strategy:

1. Convert loops to recursion.
2. Insert Cilk keywords.

Side benefit:

D&C is generally good for caches!

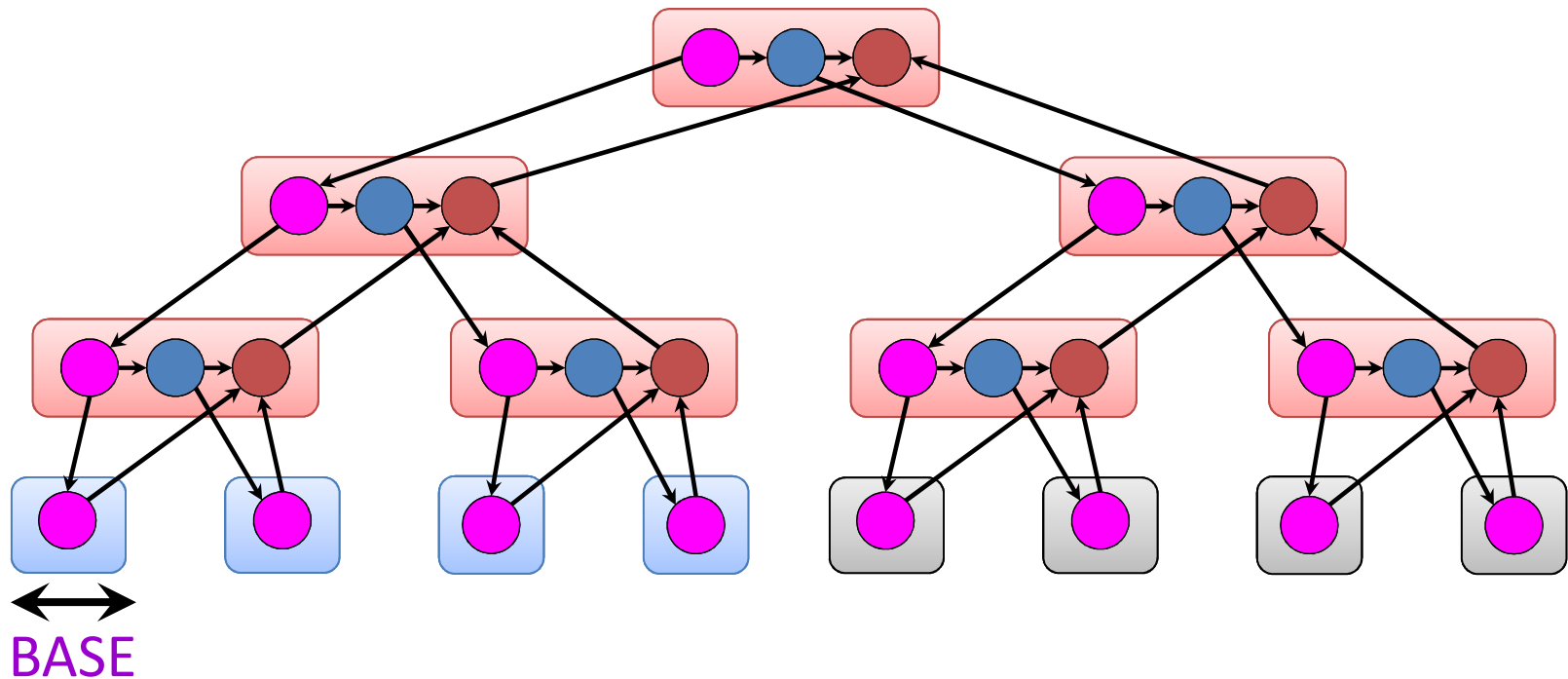
Vector Addition Analysis

To add two vectors of length n , where $\text{BASE} = \Theta(1)$:

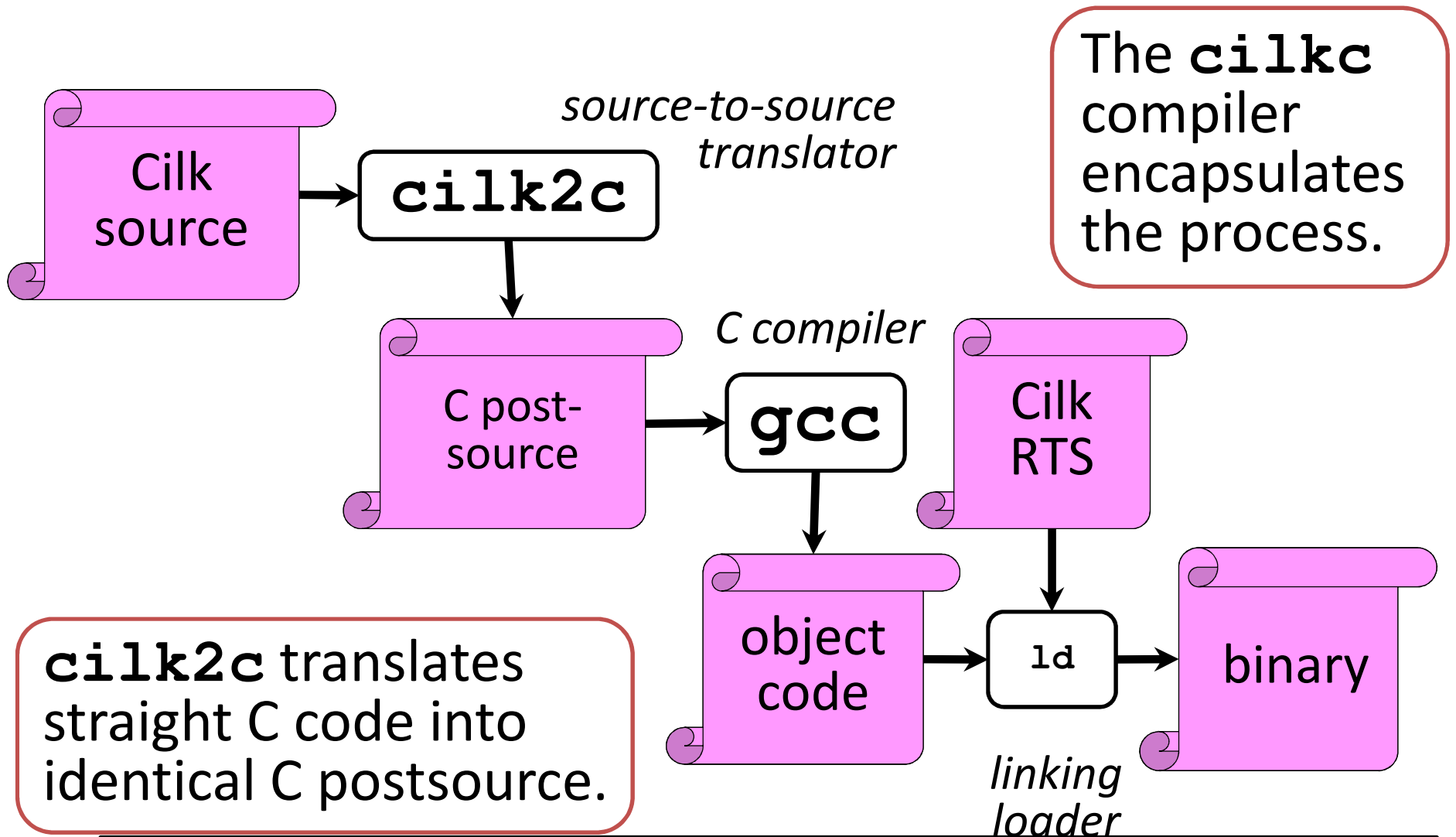
Work: $T_1 = ? \quad \Theta(n)$

Span: $T_\infty = ? \quad \Theta(\lg n)$

Parallelism: $T_1/T_\infty = ? \quad \Theta(n/\lg n)$



Compiling Cilk Program



```
$ cilkc fib.cilk -o fib
```

```
$ ./fib -proc 4 5 // run using 4 threads
```

Square-Matrix Multiplication

$$\begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix}$$

$C \qquad \qquad \qquad A \qquad \qquad \qquad B$

$$c_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$

Assume for simplicity that $n = 2^k$.

Recursive Matrix Multiplication

Divide and conquer —

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \times \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$
$$= \begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} \\ A_{21}B_{11} & A_{21}B_{12} \end{pmatrix} + \begin{pmatrix} A_{12}B_{21} & A_{12}B_{22} \\ A_{22}B_{21} & A_{22}B_{22} \end{pmatrix}$$

8 multiplications of $(n/2) \times (n/2)$ matrices.

1 addition of $n \times n$ matrices.

Matrix Multiply in Pseudo-Cilk

```
cilk void Mult(*C, *A, *B, n) {  
    float *T = Cilk_alloc(n*n*sizeof(float));  
    base case & partition matrices  
    spawn Mult(C11, A11, B11, n/2);  
    spawn Mult(C12, A11, B12, n/2);  
    spawn Mult(C22, A21, B12, n/2);  
    spawn Mult(C21, A21, B11, n/2);  
    spawn Mult(T11, A12, B21, n/2);  
    spawn Mult(T12, A12, B22, n/2);  
    spawn Mult(T22, A22, B22, n/2);  
    spawn Mult(T21, A22, B21, n/2);  
    sync;  
    spawn Add(C, T, n);  
    sync;  
    return;  
}
```

$$C = A \times B$$

*Absence of type
declarations.*

Matrix Multiply in Pseudo-Cilk

```
cilk void Mult(*C, *A, *B, n) {  
    float *T = Cilk_alloca(n*n*sizeof(float));  
    base case & partition matrices i  
    spawn Mult(C11, A11, B11, n/2);  
    spawn Mult(C12, A11, B12, n/2);  
    spawn Mult(C21, A21, B11, n/2);  
    spawn Mult(C22, A21, B12, n/2);  
    spawn Mult(T11, A12, B21, n/2);  
    spawn Mult(T12, A12, B22, n/2);  
    spawn Mult(T21, A22, B21, n/2);  
    spawn Mult(T22, A22, B22, n/2);  
    sync;  
    spawn Add(C, T, n);  
    sync;  
    return;  
}
```

$$C = A \times B$$

*Coarsen base cases
for efficiency.*

Matrix Multiply in Pseudo-Cilk

```
cilk void Mult(*C, *A, *B, n) {  
    float *T = Cilk_alloc(n*n*sizeof(float));  
    base case & partition matrices  
    spawn Mult(C11, A11, B11, n/2);  
    spawn Mult(C12, A11, B12, n/2);  
    spawn Mult(C22, A21, B12, n/2);  
    spawn Mult(C21, A21, B11, n/2);  
    spawn Mult(T11, A12, B21, n/2);  
    spawn Mult(T12, A12, B22, n/2);  
    spawn Mult(T22, A22, B22, n/2);  
    spawn Mult(T21, A22, B21, n/2);  
    sync;  
    spawn Add(C, T, n);  
    sync;  
    return;  
}
```

Also need a row-size argument for array indexing.

Submatrices are produced by pointer calculation, not copying of elements.

$$C = A \times B$$