

Team notebook

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1 DisjointSets

```
#include <template.hpp>
```

```
1 struct DisjointSetTree {
2     ll comp_count;
3     vector<ll> parent, comp_size;
4     set<ll> roots;
5
6     DisjointSetTree(int n) {
7         comp_count = n;
8         parent.resize(n);
9         comp_size.resize(n, 1);
10        iota(parent.begin(), parent.end(), 0);
11        for (int i = 0; i < n; i++) {
12            roots.insert(i);
13        }
14
15        int find(int u) {
16            if (parent[u] == u)
17                return parent[u];
18            return parent[u] = find(parent[u]);
19        }
20
21        bool merge(int u, int v) {
22            u = find(u), v = find(v);
23            if (u == v)
24                return false;
25            parent[u] = v;
26            comp_size[v] += comp_size[u];
27            comp_size[u] = 0;
28            roots.erase(u);
29            comp_count--;
30            return true;
31        }
32    }
```

};

```

class DynamicConnectivity {
    void __dfs(int v, int l, int r, vector<long long>& res) {
        long long last_ans = answer;
        int state = save_ptr;
        for (auto query : tree[v])
            merge(query);
        if (l == r - 1)
            res[l] = answer;
        else {
            int m = (l + r) / 2;
            __dfs(v * 2 + 1, l, m, res);
            __dfs(v * 2 + 2, m, r, res);
        }
        while (save_ptr != state)
            rollback();
        answer = last_ans;
    };

```

public:

```

int size_nodes;
int size_query;

```

```

struct Node {
    long long parent, comp_size = 1;
};
long long answer = 0;
vector<Node> data;
vector<long long*> saved_object;
vector<long long> saved_value;
int save_ptr = 0;

```

```

struct Query {
    int u, v;
    Query(pair<int, int> p = {0, 0}) {
        u = p.first, v = p.second;
    }
};

```

```

vector<vector<Query>> tree;

```

```

DynamicConnectivity(int n = 600000, int q = 300000) {
    size_nodes = n;
    size_query = q;
    int tree_size = 1;

```

```

while (tree_size < q)
    tree_size <= 1;
data = vector<Node>(n);
tree = vector<vector<Query>>(2 * tree_size);
saved_object = vector<long long*>(4 * q);
saved_value = vector<long long>(4 * q);
for (int i = 0; i < n; i++) {
    data[i].parent = i;
}

```

```

// Storing the initial answer
answer = n;

```

}

```

void change(long long& x, long long y) {
    saved_object[save_ptr] = &x;
    saved_value[save_ptr] = x;
    x = y;
    save_ptr++;
}

```

```

void rollback() {
    save_ptr--;
    (*saved_object[save_ptr]) = saved_value[save_ptr];
}

```

```

int find(int x) {
    if (data[x].parent == x)
        return x;
    return find(data[x].parent);
}

```

```

void merge(const Query& q) {
    int x = find(q.u);
    int y = find(q.v);
    if (x == y)
        return;
    if (data[x].comp_size < data[y].comp_size)
        swap(x, y);
    change(data[y].parent, x);
    change(data[x].comp_size, data[x].comp_size + data[y].comp_size);
    // Changing the Answer on query
    change(answer, answer - 1);
}

```

```

void add(int l, int r, Query edge, int node = 0, int x = 0, int y =
-1) {
    if (y == -1)
        y = size_query;
    if (l >= r)
        return;
    if (l == x && r == y)
        tree[node].push_back(edge);
    else {
        int m = (x + y) / 2;
        add(l, min(r, m), edge, node * 2 + 1, x, m);
        add(max(m, l), r, edge, node * 2 + 2, m, y);
    }
}

vector<long long> solve(int v = 0, int l = 0, int r = -1) {
    if (r == -1)
        r = size_query;
    vector<long long> vec(size_query);
    if (size_query > 0)
        __dfs(v, l, r, vec);
    return vec;
}

DynamicConnectivity(int n, vector<Query> queries)
: DynamicConnectivity(n, queries.size()) {
    map<pair<int, int>, int> last;
    for (int i = 0; i < size_query; i++) {
        pair<int, int> p(queries[i].u, queries[i].v);
        if (last.count(p)) {
            add(last[p], i, queries[i]);
            last.erase(p);
        } else {
            last[p] = i;
        }
    }
    for (auto x : last)
        add(x.second, size_query, x.first);
};

```

2 DynamicProgramming

```

#include <template.hpp>

class LineContainer {
private:
    struct Line {
        mutable long long slope, constt, p;
        bool operator<(const Line &o) const {
            return slope < o.slope;
        }
        bool operator<(long long x) const {
            return p < x;
        }
    };
    multiset<Line, less<>> lines;
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    bool __is_max_query = false;
    const long long inf = LLONG_MAX;
    long long __div(long long a, long long b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool __intersect(multiset<Line>::iterator x, multiset<Line>::iterator
y) {
        if (y == lines.end()) {
            x->p = inf;
            return false;
        }
        if (x->slope == y->slope)
            x->p = x->constt > y->constt ? inf : -inf;
        else
            x->p = __div(y->constt - x->constt, x->slope - y->slope);
        return x->p >= y->p;
    }

public:
    LineContainer(bool is_max = false) {
        this->__is_max_query = is_max;
    }
    void add(long long slope, long long constt) {
        if (!__is_max_query) {
            slope = -slope;
            constt = -constt;
        }
        auto z = lines.insert({slope, constt, 0}), y = z++, x = y;
        while (__intersect(y, z))

```

```

        z = lines.erase(z);
        if (x != lines.begin() && __intersect(--x, y))
            __intersect(x, y = lines.erase(y));
        while ((y = x) != lines.begin() && (--x)->p >= y->p)
            __intersect(x, lines.erase(y));
    }
    long long query(long long x) {
        assert(!lines.empty());
        auto l = *lines.lower_bound(x);
        return (l.slope * x + l.constt) * (__is_max_query ? 1 : -1);
    }
};

void dp_sos(vll &arr) {
    const int bitsize = 20;
    for (int i = 0; i < bitsize; ++i)
        for (int mask = 0; mask < (1 << bitsize); ++mask)
            if (mask & (1 << i))
                arr[mask] += arr[mask ^ (1 << i)];
}

```

3 FastFourier

```
#include <template.hpp>
```

```

class Polynomial {
    static const int root = 973800541;
    static const int root_1 = 595374802;
    static const int root_pw = 1 << 20;
    static const ll MOD = 998244353;

    static ll __mod_pow(ll a, ll n) {
        int res = 1;
        for (a %= MOD; n > 0; n >>= 1) {
            if (n & 1)
                res = (res * 1ll * a) % MOD;
            a = (a * 1ll * a) % MOD;
        }
        return res;
    }

public:

```

```

    int order;
    vll coeff;

    explicit Polynomial(vll coefficients) {
        order = coefficients.size() - 1;
        coeff = coefficients;
        this->resize(order);
    }

    void resize(int order) {
        int size;
        for (size = 1; size < order + 1; size *= 2)
            ;
        coeff.resize(size);
    }

    void ntt(bool invert = false) {
        int n = coeff.size();
        for (int i = 1, j = 0; i < n; i++) {
            int bit = n >> 1;
            for (; j & bit; bit >>= 1)
                j ^= bit;
            j ^= bit;
            if (i < j)
                swap(coeff[i], coeff[j]);
        }
        for (int len = 2; len <= n; len <= 1) {
            int wlen = invert ? root_1 : root;
            for (int i = len; i < root_pw; i <= 1)
                wlen = (int)(1LL * wlen * wlen % MOD);
            for (int i = 0; i < n; i += len) {
                int w = 1;
                for (int j = 0; j < len / 2; j++) {
                    int u = coeff[i + j],
                        v = (ll)((coeff[i + j + len / 2] * 1ll * w) % MOD);
                    coeff[i + j] = u + v < MOD ? u + v : u + v - MOD;
                    coeff[i + j + len / 2] = u - v >= 0 ? u - v : u - v +
                        MOD;
                    w = (int)((w * 1ll * wlen) % MOD);
                }
            }
        }
        if (invert) {
            int n_1 = __mod_pow(n, MOD - 2);
            for (ll &x : coeff)

```

```

        x = (1ll)((x * 1ll * n_1) % MOD);
    }
}

friend Polynomial operator*(const Polynomial &a, const Polynomial &b)
{
    Polynomial x(a), y(b);
    int order = a.order + b.order;
    x.resize(order), y.resize(order);
    x.ntt(), y.ntt();
    int size = x.coeff.size();
    vll poly(size);
    for (int i = 0; i < size; i++)
        poly[i] = (x.coeff[i] * y.coeff[i]) % MOD;
    Polynomial res(poly);
    res.ntt(true), res.order = order;
    return res;
}

friend Polynomial operator^(const Polynomial &a, ll pow) {
    Polynomial x(a);
    int order = a.order * pow;
    x.resize(order);
    x.ntt();
    int size = x.coeff.size();
    vll poly(size);
    for (int i = 0; i < size; i++)
        poly[i] = __mod_pow(x.coeff[i], pow);
    Polynomial res(poly);
    res.ntt(true), res.order = order;
    return res;
}
};

int main() {
    Polynomial a({1, 2, 3});
    Polynomial b({1, 1});
    Polynomial c = a * b;
    for (int i = 0; i <= c.order; i++)
        cout << c.coeff[i] << endl;
}

```

4 FlowAlgorithms

```

#include "template.hpp"

class Dinics {
public:
    typedef int FT;           // can use float/double static
    const FT INF = 1e9;       // maximum capacity
    static const FT EPS = 0;  // minimum capacity/flow change
    int nodes, src, dest;
    vector<int> dist, q, work;
    struct Edge {
        int to, rev;
        FT f, cap;
    };
    vector<vector<Edge>> g;
    bool dinic_bfs() {
        fill(dist.begin(), dist.end(), -1);
        dist[src] = 0;
        int qt = 0;
        q[qt++] = src;
        for (int qh = 0; qh < qt; qh++) {
            int u = q[qh];
            for (int j = 0; j < (int)g[u].size(); j++) {
                Edge &e = g[u][j];
                int v = e.to;
                if (dist[v] < 0 && e.f < e.cap)
                    dist[v] = dist[u] + 1;
                q[qt++] = v;
            }
        }
        return dist[dest] >= 0;
    }
    int dinic_dfs(int u, int f) {
        if (u == dest)
            return f;
        for (int &i = work[u]; i < (int)g[u].size(); i++) {
            Edge &e = g[u][i];
            if (e.cap <= e.f)
                continue;
            int v = e.to;
            if (dist[v] == dist[u] + 1) {
                FT df = dinic_dfs(v, min(f, e.cap - e.f));
                if (df > 0) {

```

```

        e.f += df, g[v][e.rev].f -= df;
        return df;
    }
}
return 0;
}
Dinics(int n) : dist(n, 0), q(n, 0), work(n, 0), g(n), nodes(n) {
} // *** s->t (cap); t->s (rcap)
void addEdge(int s, int t, FT cap, FT rcap = 0) {
    g[s].push_back({t, (int)g[t].size(), 0, cap});
    g[t].push_back({s, (int)g[s].size() - 1, 0, rcap});
} // ***
FT maxFlow(int _src, int _dest) {
    src = _src, dest = _dest;
    FT result = 0, delta;
    while (dinic_bfs()) {
        fill(work.begin(), work.end(), 0);
        while ((delta = dinic_dfs(src, INF)) > EPS)
            result += delta;
    }
    return result;
}
};

class HopcroftKarp {
public:
    static const int INF = 1e9;
    int U, V, nil;
    vector<int> pairU, pairV, dist;
    vector<vector<int>> > adj;
    bool bfs() {
        queue<int> q;
        for (int u = 0; u < U; u++)
            if (pairU[u] == nil)
                dist[u] = 0, q.push(u);
            else
                dist[u] = INF;
        dist[nil] = INF;
        while (not q.empty()) {
            int u = q.front();
            q.pop();
            if (dist[u] >= dist[nil])
                continue;
            for (int v : adj[u])

```

```

                if (dist[pairV[v]] == INF)
                    dist[pairV[v]] = dist[u] + 1, q.push(pairV[v]);
            }
            return dist[nil] != INF;
        }
    }
    bool dfs(int u) {
        if (u == nil)
            return true;
        for (int v : adj[u])
            if (dist[pairV[v]] == dist[u] + 1)
                if (dfs(pairV[v])) {
                    pairV[v] = u, pairU[u] = v;
                    return true;
                }
        dist[u] = INF;
        return false;
    }
}

public:
    HopcroftKarp(int U_, int V_) {
        nil = U = V = max(U_, V_);
        adj.resize(U + 1);
        dist.resize(U + 1);
        pairU.resize(U + 1);
        pairV.resize(V);
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
    }
    int maxMatch() {
        fill(pairU.begin(), pairU.end(), nil);
        fill(pairV.begin(), pairV.end(), nil);
        int res = 0;
        while (bfs())
            for (int u = 0; u < U; u++)
                if (pairU[u] == nil && dfs(u))
                    res++;
        return res;
    }
};

```

5 Geometry

```
#include "template.hpp"

class Point {
public:
    typedef long long coord_t;
    coord_t x, y;

    Point(coord_t coord_x = 0, coord_t coord_y = 0) {
        this->x = coord_x;
        this->y = coord_y;
    }
    Point(pair<coord_t, coord_t> coord) {
        this->x = coord.first;
        this->y = coord.second;
    }
    static coord_t area(const Point &a, const Point &b, const Point &c) {
        // Area function: area < 0 = clockwise, area > 0 counterclockwise
        return a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
    };
    static coord_t area(const vector<Point> &polygon) {
        int n = polygon.size();
        coord_t ans = 0;
        for (int i = 0; i < n; i++) {
            ans += polygon[i].x * polygon[(i + 1) % n].y -
                polygon[i].y * polygon[(i + 1) % n].x;
        }
    }
    friend bool operator<(const Point &a, const Point &b) {
        return (a.x != b.x) ? a.x < b.x : a.y < b.y;
    }
    friend bool operator==(const Point &a, const Point &b) {
        return (a.x == b.x) && (a.y == b.y);
    }
    friend istream &operator>>(istream &in, Point &p) {
        in >> p.x >> p.y;
        return in;
    }
    friend ostream &operator<<(ostream &out, Point &p) {
        out << p.x << " " << p.y;
        return out;
    }
    static coord_t sq_dist(const Point &a, const Point &b) {
        return (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
    }
};
```

```
static coord_t cross(const Point &O, const Point &A, const Point &B) {
    return (A.x - O.x) * (B.y - O.y) - (A.y - O.y) * (B.x - O.x);
}
static coord_t dot(const Point &O, const Point &A, const Point &B) {
    return (A.x - O.x) * (B.x - O.x) + (A.y - O.y) * (B.y - O.y);
}

static vector<Point> convex_hull(vector<Point> &a) {
    if (a.size() <= 3)
        return a;
    int n = a.size(), k = 0;
    sort(a.begin(), a.end());
    vector<Point> result(2 * n);
    for (int i = 0; i < n; ++i) {
        while (k >= 2 && cross(result[k - 2], result[k - 1], a[i]) <=
            0)
            k--;
        result[k++] = a[i];
    }
    for (int i = n - 1, t = k + 1; i > 0; --i) {
        while (k >= t && cross(result[k - 2], result[k - 1], a[i - 1])
            <= 0)
            k--;
        result[k++] = a[i - 1];
    }
    result.resize(k - 1);
    return result;
}
};
```

6 GraphAlgorithms

```
#include "template.hpp"

class Graph {
public:
    enum NodeColor { VISITED, VISITING, UNVISITED };
    struct Node {
        int index;
        vpl adjacent;
        NodeColor color = UNVISITED;
    };
};
```

```

vector<Node> list;
int n;
Graph(int n) {
    list.resize(n);
    for (int i = 0; i < n; i++)
        list[i].index = i;
    this->n = n;
}
void add_edge(int u, int v, long long w = 1) {
    list[u].adjacent.emplace_back(v, w);
    list[v].adjacent.emplace_back(u, w);
}

pair<vll, vll> dijkstra(vll from) {
    vll dist(n, INT64_MAX), parent(n, INT32_MAX);
    priority_queue<pll, vpl, greater<>> q;
    for (auto index : from)
        dist[index] = 0, q.emplace(0, index);
    while (!q.empty()) {
        pll top = q.top();
        q.pop();
        if (top.first > dist[top.second])
            continue;
        for (auto edge : list[top.second].adjacent)
            if (top.first + edge.second < dist[edge.first])
                dist[edge.first] = top.first + edge.second,
                parent[edge.first] = top.second,
                q.emplace(top.first + edge.second, edge.first);
    }
    return {dist, parent};
}

// Returns sorted vector of indices
vector<int> topological_sort() {
    vector<int> in_degree(list.size(), 0), result;
    result.reserve(list.size());
    for (auto node : list)
        for (auto route : node.adjacent)
            in_degree[route.first - 1]++;
    queue<int> process;
    for (int i = 0; i < list.size(); i++) {
        if (in_degree[i] == 0) {
            process.push(i);
            result.push_back(i);
        }
    }
}

```

```

    }
    while (!process.empty()) {
        int processing = process.front();
        process.pop();
        for (auto route : list[processing].adjacent) {
            in_degree[route.first - 1]--;
            if (in_degree[route.first - 1] == 0) {
                process.push(route.first - 1);
                result.push_back(route.first - 1);
            }
        }
    }
    return result;
}

mll components() {
    vbl visited(n);
    mll result(0);
    for (int i = 0; i < n; i++) {
        if (visited[i])
            continue;
        vll component;
        stack<ll> process;
        process.push(list[i].index);
        component.push_back(i);
        visited[i] = true;
        while (!process.empty()) {
            ll processing = process.top();
            process.pop();
            for (pll neighbor : list[processing].adjacent)
                if (!visited[neighbor.first])
                    process.push(neighbor.first),
                    component.push_back(neighbor.first),
                    visited[neighbor.first] = true;
        }
        result.push_back(component);
    }
    return result;
}

pair<vll, vll> bellman_ford(vll from) {
    vll distances(n, INT64_MAX);
    vll parent(n, INT32_MAX);
    // Bellman Ford Algorithm
    for (ll &i : from)

```



```

    distances[i] = 0;
    for (int i = 0; i < n - 1; i++) {
        for (int source = 0; source < n - 1; source++) {
            if (distances[source] == INT64_MAX)
                continue;
            for (const auto &edge : list[source].adjacent) {
                ll sink = edge.first;
                if (distances[source] + edge.second < distances[sink]) {
                    distances[sink] = distances[source] + edge.second;
                    parent[sink] = source;
                }
            }
        }
    }
    // Checking for negative cycles and putting -1 if it exists.
    for (ll source = 0; source < n - 1; source++) {
        for (const auto &edge : list[source].adjacent) {
            ll sink = edge.first;
            if (distances[source] + edge.second < distances[sink]) {
                for (ll i : from)
                    distances[i] = -1;
                return {distances, parent};
            }
        }
    }
    return {distances, parent};
}

vector<vector<long long>> floyd_warshall() {
    vector<vector<long long>> distances(n, vector<long long>(n,
        INT64_MAX));
    for (int i = 0; i < n; i++)
        distances[i][i] = 0;
    for (int i = 0; i < n; i++)
        for (auto route : list[i].adjacent)
            distances[i][route.first] = route.second;
    for (int k = 0; k < n; k++) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                if (distances[i][k] == INT64_MAX ||
                    distances[k][j] == INT64_MAX)
                    continue;
                distances[i][j] =
                    min(distances[i][j], distances[i][k] +
                        distances[k][j]);
            }
        }
    }
}

```

```

    }
    }
    return distances;
}

pair<ll, vll> prims_mst() {
    priority_queue<pll, vpl, greater<>> routes;
    vll costs(n);
    vbl visited(n, false);
    for (int i = 0; i < n; i++) {
        if (!visited[i])
            routes.emplace(INT32_MAX, i);
        while (!routes.empty()) {
            pll best = routes.top();
            routes.pop();
            if (!visited[best.second])
                costs[best.second] = best.first;
            visited[best.second] = true;
            for (const auto &path : list[best.second].adjacent)
                if (!visited[path.second])
                    routes.push(path);
        }
    }
    ll sum = accumulate(costs.begin(), costs.end(), 0);
    return {sum, costs};
}
};

```

7 MergeSortTree

```
#include "template.hpp"
```

```

template <typename Type>
class MergeSortTree {
protected:
    int size;
    vector<Type> data;
    vector<vector<int>> tree_idx;
    vector<vector<Type>> tree_val;
    long long inversions;

```

```

vector<pair<int, Type>> merge(const vector<int> &arr1,
                             const vector<int> &arr2) {
    int n = arr1.size(), m = arr2.size();
    vector<pair<int, Type>> result(n + m);
    for (int x = 0, y = 0; x < n || y < m;) {
        if (x < n && (y >= m || data[arr1[x]] <= data[arr2[y]]))
            result.push_back(arr1[x++]);
        else
            result.push_back(arr2[y++]), inversions += n - x;
    }
    return move(result);
}

int order_fn(const Type &value, const vector<Type> &arr) {
    return lower_bound(arr.begin(), arr.end(), value) - arr.begin();
}

public:
explicit MergeSortTree(const vector<Type> &list) {
    for (size = 1; size < list.size(); size *= 2)
        ;
    tree_idx.resize(2 * size);
    tree_val.resize(2 * size);
    for (int i = 0; i < list.size(); i++)
        tree_idx[i + size].emplace_back(i, list[i]);
    for (int i = size - 1; i > 0; --i)
        tree_idx[i] = merge(tree[i << 1], tree[i << 1 | 1]);
    for (int i = 0; i < 2 * size; i++)
        for (int el : tree_idx[i])
            tree_val[i].push_back(data[el]);
}

int order_of_key(int l, int r, Type value) {
    int result = 0;
    for (l = l + size, r = r + size; l < r; l >>= 1, r >>= 1) {
        if (l & 1)
            result += order_fn(value, tree_val[l++]);
        if (r & 1)
            result += order_fn(value, tree_val[--r]);
    }
    return result;
}

int key_of_order(int l, int r, int order, int node = 0, int x = 0,
                 int y = -1) {

```

```

    if (y == -1)
        y = size;
    if (x + 1 == y)
        return tree_idx[node][0];
    int last_in_query_range = upper_bound(tree_idx[2 * node].begin(),
                                           tree_idx[2 * node].end(), r -
                                           1) -
                             tree_idx[2 * node].begin();
    int first_in_query_range = lower_bound(tree_idx[2 * node].begin(),
                                           tree_idx[2 * node].end(), l) -
                              tree_idx[2 * node].begin();
    int m = last_in_query_range - first_in_query_range;
    if (m >= k)
        return key_of_order(l, r, order, node << 1, x, (x + y) / 2);
    else
        return key_of_order(l, r, order - m, node << 1 | 1, (x + y) /
                              2, y);
}
};

```

8 Miscellaneous

```
#include "template.hpp"
```

```

ll binary_search(ll TOP, ll BOT, function<bool(ll)> check) {
    ll result = 0;
    for (ll top = 1e5, bot = 0, mid = bot + (top - bot) / 2; bot <= top;
         mid = bot + (top - bot) / 2) {
        if (check(mid) && !check(mid - 1)) {
            result = mid;
            break;
        }
        (check(mid)) ? (top = mid - 1) : (bot = mid + 1);
    }
}

ll gcd(ll a, ll b, ll &x, ll &y) {
    int g = a;
    x = 1, y = 0;
    if (b)
        g = gcd(b, a % b, y, x), y -= a / b * x;
    return g;
}

```

```

}
ll mod_inverse(ll a, ll mod) {
    ll x, y;
    gcd(a, mod, x, y);
    return (x + mod) % mod;
}

long long _inv = 0;
void _merge(int A[], int start, int mid, int end) {
    int result[end - start];
    for (int x = start, y = mid; x < mid || y < end;) {
        if (x < mid && (y >= end || A[x] <= A[y])) {
            result[x + y - start - mid] = A[x];
            x++;
        } else {
            result[x + y - start - mid] = A[y];
            y++;
            _inv += mid - x;
        }
    }
    for (int i = start; i < end; i++)
        A[i] = result[i - start];
}

void sort(int A[], int start, int end) {
    if (start >= end - 1)
        return;
    sort(A, start, (start + end) / 2);
    sort(A, (start + end) / 2, end);
    _merge(A, start, (start + end) / 2, end);
}

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
            tree_order_statistics_node_update>
ordered_set;

```

9 MobiusSieve

```
#include "template.hpp"
```

```

class Multiplicative {
// This is the definition for PHI
#define fn_prime_values(prime) (prime - 1)
#define fn_non_coprime(num, prime) (fn[num] * prime)
public:
    ll size;
    vector<ll> fn;
    vector<ll> primes;
    vector<ll> lowest_prime_factor;
    Multiplicative(ll size) {
        size = size;
        lowest_prime_factor = vector<ll>(size, 0);
        fn = vector<ll>(size, 0);
        // https://stackoverflow.com/questions/34260399
        // linear sieve
        for (ll i = 2; i < size; i++)
            lowest_prime_factor[i] = i;
        // put any specific initialization code here like
        // multiplicativeFn[1] = 1;
        for (ll i = 2; i < size; i++) {
            if (lowest_prime_factor[i] == i) {
                fn[i] = fn_prime_values(i);
                primes.push_back(i);
            }
            for (auto p : primes) {
                ll ith_multiple = i * p;
                if (ith_multiple >= size)
                    break;
                lowest_prime_factor[ith_multiple] =
                    min(lowest_prime_factor[i], p);

                if (i % p) {
                    fn[ith_multiple] = fn[i] * fn[p];
                } else {
                    fn[ith_multiple] = fn_non_coprime(i, p);
                    break;
                }
            }
        }
    }
};

```

10 PalindromicTree

```

#include "template.hpp"
#define MAXN 1000
class PalindromicTree {
public:
    struct Node {
        int start, end;
        int length;
        int insert_edge[26];
        int suffix_edge;
    };

    Node root1, root2;
    Node tree[MAXN];
    int curr_node, ptr, size;
    string s;

    void insert(int idx) {
        int tmp = curr_node;
        while (true) {
            int curLength = tree[tmp].length;
            if (idx - curLength >= 1 and s[idx] == s[idx - curLength - 1])
                break;
            tmp = tree[tmp].suffix_edge;
        }
        if (tree[tmp].insert_edge[s[idx] - 'a'] != 0) {
            curr_node = tree[tmp].insert_edge[s[idx] - 'a'];
            return;
        }
        ptr++;
        tree[tmp].insert_edge[s[idx] - 'a'] = ptr;
        tree[ptr].length = tree[tmp].length + 2;
        tree[ptr].end = idx;
        tree[ptr].start = idx - tree[ptr].length + 1;
        tmp = tree[tmp].suffix_edge;
        curr_node = ptr;
        if (tree[curr_node].length == 1) {
            tree[curr_node].suffix_edge = 2;
            return;
        }
        while (true) {
            int cur_length = tree[tmp].length;

```

```

            if (idx - cur_length >= 1 and s[idx] == s[idx - cur_length - 1])
                break;
            tmp = tree[tmp].suffix_edge;
        }
        tree[curr_node].suffix_edge = tree[tmp].insert_edge[s[idx] - 'a'];
    }

    PalindromicTree(string st) {
        root1.length = -1, root1.suffix_edge = 1, root2.length = 0,
        root2.suffix_edge = 1, tree[1] = root1, tree[2] = root2, ptr = 2;
        curr_node = 1, s = st, size = st.size();
        for (int i = 0; i < size; i++)
            insert(i);
    }

    vpl get_palindromes() {
        vpl res(ptr - 2);
        for (int i = 3; i <= ptr; i++)
            res[i - 2] = {tree[i].start, tree[i].end};
        return res;
    }
};

```

11 SegmentTree

```

#include <template.hpp>

template <typename Type>
class LazySegtree {
    int size;
    vector<Type> tree, lazy;
    Type _default;
    function<Type(Type, Type)> _operation;
    function<Type(Type, Type)> _setter;

    void split(int node) {
        lazy[2 * node] = _setter(lazy[2 * node], lazy[node]);
        tree[2 * node] = _setter(tree[2 * node], lazy[node]);
        lazy[2 * node + 1] = _setter(lazy[2 * node + 1], lazy[node]);
        tree[2 * node + 1] = _setter(tree[2 * node + 1], lazy[node]);
        lazy[node] = _default;
    }

```

```

}
void merge(int node) {
    tree[node] = _operation(tree[2 * node], tree[2 * node + 1]);
}

public:
LazySegtree(int n, const function<Type(Type, Type)> &op,
             const function<Type(Type, Type)> &set, const Type
             identity) {
    for (size = 1; size < n; size <= 1)
        ;
    _setter = set, _operation = op, _default = identity;
    tree.assign(2 * size, _default);
    lazy.assign(2 * size, _default);
}

void modify(int l, int r, Type delta, int node = 1, int x = 0, int y
            = -1) {
    if (y == -1)
        y = size;
    if (r <= x || l >= y)
        return;
    if (l <= x && y <= r) {
        lazy[node] = _setter(lazy[node], delta);
        tree[node] = _setter(tree[node], delta);
        return;
    }
    split(node);
    modify(l, r, delta, 2 * node, x, (x + y) / 2);
    modify(l, r, delta, 2 * node + 1, (x + y) / 2, y);
    merge(node);
}

Type query(int l, int r, int node = 1, int x = 0, int y = -1) {
    if (y == -1)
        y = size;
    if (r <= x || l >= y)
        return _default;
    if (l <= x && y <= r) {
        return tree[node];
    }
    split(node);
    Type lres = query(l, r, 2 * node, x, (x + y) / 2);
    Type rres = query(l, r, 2 * node + 1, (x + y) / 2, y);
    merge(node);
    return _operation(lres, rres);
}

```

```

}
};

template <typename Type>
class ImplicitSegupdate {
    struct Node {
        Type data = 0;
        Node *l_ptr = nullptr, *r_ptr = nullptr;
        Node *l_child() {
            if (l_ptr == nullptr)
                l_ptr = new Node, r_ptr = new Node;
            return l_ptr;
        }
        Node *r_child() {
            if (r_ptr == nullptr)
                l_ptr = new Node, r_ptr = new Node;
            return r_ptr;
        }
    };
    int size;
    Node *root;
    function<Type(Type, Type)> _setter;

public:
    ImplicitSegupdate(int n, const function<Type(Type, Type)> &set) {
        for (size = 1; size < n; size <= 1)
            ;
        _setter = set;
        root = new Node;
    }

    void modify(int l, int r, Type delta, Node *node = nullptr, int x = 0,
                int y = -1) {
        if (node == nullptr)
            node = root, y = size;
        if (r <= x || l >= y)
            return;
        if (l <= x && y <= r) {
            node->data = _setter(node->data, delta);
            return;
        }
        modify(l, r, delta, node->l_child(), x, (x + y) / 2);
        modify(l, r, delta, node->r_child(), (x + y) / 2, y);
    }

    Type query(int p, Node *node = nullptr, int x = 0, int y = -1) {
        if (node == nullptr)

```

```

        node = root, y = size;
    if (x == p && y == p + 1) {
        return node->data;
    }
    if (x <= p && p < (x + y) / 2)
        return _setter(node->data,
            query(p, node->l_child(), x, (x + y) / 2));
    else
        return _setter(node->data,
            query(p, node->r_child(), (x + y) / 2, y));
}

};

class PersistentSegtree {
    struct Node {
        int l, r, val;
        Node() {
            l = r = val = 0;
        }
    };
    int node_size, query_size;
    int curr;
    vector<int> root;
    vector<Node> seg;

    PersistentSegtree(int n, int q) {
        node_size = n, query_size = q;
        seg.resize(2 * (n + q * (log2(n) + 1)));
        root = vector<int>(query_size + 10);
        curr = 1, seg[curr].l = seg[curr].r = seg[curr].val = 0;
    }

    int _new_node(int val, int l, int r) {
        seg[curr].val = val, seg[curr].l = l, seg[curr].r = r;
        return curr++;
    }

    int insert(int cur, int idx, int val, int lo, int hi) {
        if (idx < lo || idx > hi)
            return cur;
        else if (lo == hi)
            return _new_node(val, 0, 0);
        int mid = (lo + hi) >> 1;
        int pos = _new_node(-1, insert(seg[cur].l, idx, val, lo, mid),
            insert(seg[cur].r, idx, val, mid + 1, hi));
        seg[pos].val = max(seg[seg[pos].l].val, seg[seg[pos].r].val);
        return pos;
    }
};

```

```

    }
};

```

12 String Algorithms

```

class KMPstring {
    string pattern;
    vll lps;

public:
    explicit KMPstring(const string &pattern) {
        this->pattern = pattern;
        ll m = pattern.size();
        lps = vll(m + 1, 0);
        ll i = 0, j = -1;
        lps[0] = -1;
        while (i < m) {
            while (j >= 0 && pattern[i] != pattern[j])
                j = lps[j];
            i++, j++;
            lps[i] = j;
        }
    }

    vll match(const string &text) {
        ll n = text.size(), m = pattern.size();
        vll matches, m_length(n);
        ll i = 0, j = 0;
        while (i < n) {
            while (j >= 0 && text[i] != pattern[j])
                j = lps[j];
            i++, j++;
            m_length[i - 1] = j;
            if (j == m) {
                matches.push_back(i - m);
                j = lps[j];
            }
        }
        return move(matches); // or m_length
    }
};

class SuffixArray {

```

```

public:
    string s;
    int n, __log_n;
    vector<int> sa;           // Suffix Array
    vector<vector<int>> ra;    // Rank Array
    vector<vector<int>> _lcp;  // Longest Common Prefix
    vector<int> __msb, __dollar;

    SuffixArray(string st) {
        n = st.size();
        __log_n = log2(n) + 1;
        ra = vector<vector<int>>(__log_n, vector<int>(n));
        sa = vector<int>(n);

        __msb = vector<int>(n);
        int mx = -1;
        for (int i = 0; i < n; i++) {
            if (i >= (1 << (mx + 1)))
                mx++;
            __msb[i] = mx;
        }
        this->s = st;
        build_SA();
    }

    void __counting_sort(int l, int k) {
        int maxi = max(300, n);
        vector<int> count(maxi, 0), temp_sa(n, 0);
        for (int i = 0; i < n; i++) {
            int idx = (i + k < n ? ra[l][i + k] : 0);
            count[idx]++;
        }
        for (int i = 0, sum = 0; i < maxi; i++) {
            int t = count[i];
            count[i] = sum;
            sum += t;
        }
        for (int i = 0; i < n; i++) {
            int idx = sa[i] + k < n ? ra[l][sa[i] + k] : 0;
            temp_sa[count[idx]++] = sa[i];
        }
        sa = temp_sa;
    }

    void build_SA() {

```

```

        for (int i = 0; i < n; i++)
            ra[0][i] = s[i];
        for (int i = 0; i < n; i++)
            sa[i] = i;
        for (int i = 0; i < __log_n - 1; i++) {
            int k = (1 << i);
            if (k >= n)
                break;
            __counting_sort(i, k);
            __counting_sort(i, 0);
            int rank = 0;
            ra[i + 1][sa[0]] = rank;
            for (int j = 1; j < n; j++)
                if (ra[i][sa[j]] == ra[i][sa[j - 1]] &&
                    ra[i][sa[j] + k] == ra[i][sa[j - 1] + k])
                    ra[i + 1][sa[j]] = rank;
                else
                    ra[i + 1][sa[j]] = ++rank;
        }
    }

    void build_LCP() {
        _lcp = vector<vector<int>>(__log_n, vector<int>(n));
        for (int i = 0; i < n - 1; i++) { // Build the LCP array in
            O(NlogN)
            int x = sa[i], y = sa[i + 1], k, ret = 0;
            for (k = __log_n - 1; k >= 0 && x < n && y < n; k--) {
                if ((1 << k) >= n)
                    continue;
                if (ra[k][x] == ra[k][y])
                    x += 1 << k, y += 1 << k, ret += 1 << k;
            }
            if (ret >= __dollar[sa[i]] - sa[i])
                ret = __dollar[sa[i]] - sa[i];
            _lcp[0][i] = ret; // LCP[i] shouldnt exceed __dollar[sa[i]]
        } // __dollar[i] : index of __dollar to the right of i.
        _lcp[0][n - 1] = 10 * n;
        for (int i = 1; i < __log_n; i++) { // O(1) RMQ structure in
            O(NlogN)
            int add = (1 << (i - 1));
            if (add >= n)
                break; // small optimization
            for (int j = 0; j < n; j++)
                if (j + add < n)
                    _lcp[i][j] = min(_lcp[i - 1][j], _lcp[i - 1][j + add]);
            else

```

```

        _lcp[i][j] = _lcp[i - 1][j];
    }
}

int lcp(int x, int y) {
    // 0(1) LCP. x & y are indexes of the suffix in sa!
    if (x == y)
        return __dollar[sa[x]] - sa[x];
    if (x > y)
        swap(x, y);
    y--;
    int idx = __msb[y - x + 1], sub = (1 << idx);
    return min(_lcp[idx][x], _lcp[idx][y - sub + 1]);
}

bool equal(int i, int j, int p, int q) {
    if (j - i != q - p)
        return false;
    int idx = __msb[j - i + 1], sub = (1 << idx);
    return ra[idx][i] == ra[idx][p] &&
        ra[idx][j - sub + 1] == ra[idx][q - sub + 1];
} // Note : Do not forget to add a terminating $
};

```

13 TreesCentroids

```

class Tree {
public:
    struct Node {
        vector<Node*> adjacent;
        Node *parent = nullptr;
        long long start_time = 0, end_time = 0, subtree_size = 1;
        unsigned long depth = 0, height = 0;
        unsigned long index = INT32_MAX;
    };

    vector<Node> list;
    Node *root = nullptr;
    vector<vector<Node*>> __anc;

    Tree(int n = 1e5) {
        list.resize(n);
    }

```

```

        this->root = &list[0];
        for (int i = 0; i < n; i++)
            list[i].index = i;
    }

    void add_edge(int x, int y) {
        list[x].adjacent.push_back(&list[y]);
        list[y].adjacent.push_back(&list[x]);
    }

    Node *lca(Node *a, Node *b) {
        if (b->depth > a->depth)
            swap(a, b);
        for (int ptr = __anc[0].size() - 1; a->depth > b->depth && ptr >= 0; ptr--) {
            if (__anc[a->index][ptr] != nullptr &&
                __anc[a->index][ptr]->depth >= b->depth)
                a = __anc[a->index][ptr];
        }
        if (a == b)
            return a;
        for (long step = __anc[0].size() - 1; step >= 0; step--) {
            if (__anc[a->index][step] != __anc[b->index][step])
                a = __anc[a->index][step], b = __anc[b->index][step];
        }
        return a->parent;
    }

    Node *ancestor(Node *a, int degree) {
        ll target_depth = a->depth - degree;
        for (int ptr = __anc[0].size() - 1; a->depth > target_depth && ptr >= 0; ptr--) {
            if (__anc[a->index][ptr] != nullptr &&
                __anc[a->index][ptr]->depth >= target_depth)
                a = __anc[a->index][ptr];
        }
        return a;
    }

    int __build(Node *root = nullptr, int time = 0) {
        if (root == nullptr)
            root = this->root;
        root->start_time = time;
        for (auto child : root->adjacent) {
            if (child == root->parent)
                continue;

```



```

        child->parent = root;
        child->depth = root->depth + 1;
        time = __build(child, time + 1);
        root->height = max(root->height, child->height + 1);
        root->subtree_size += child->subtree_size;
    }
    root->end_time = time;
    return time;
}

void __build_lca_matrix() {
    int n = list.size();
    __anc = *new vector<vector<Node *>>(
        n, vector<Node *>(log2(n) + 1, nullptr));
    for (int i = 0; i < list.size(); i++)
        __anc[i][0] = list[i].parent;
    for (int level = 1; level < __anc[0].size(); level++)
        for (int i = 0; i < list.size(); i++) {
            if (__anc[i][level - 1] == nullptr)
                continue;
            __anc[i][level] = __anc[__anc[i][level - 1]->index][level - 1];
        }
}

};

class CentroidTree : public Tree {
private:
    vector<bool> __visited;
    vector<int> __dir_parents, __subtree_size;
    Tree base;

    void __dfs_centroid(int node) {
        __subtree_size[node] = 1;
        for (Node *next : base.list[node].adjacent)
            if (!__visited[next->index] && next->index !=
                __dir_parents[node]) {
                __dir_parents[next->index] = node;
                __dfs_centroid(next->index);
                __subtree_size[node] += __subtree_size[next->index];
            }
    }

    int __get_centroid(int x) {
        __dir_parents[x] = 0;
        __dfs_centroid(x);

```

```

        int sz = __subtree_size[x];
        while (true) {
            pair<int, int> mx = {0, 0};
            for (Node *next : base.list[x].adjacent)
                if (!__visited[next->index] && next->index !=
                    __dir_parents[x])
                    mx = max(mx, {__subtree_size[next->index],
                        next->index});
            if (mx.first * 2 > sz)
                x = mx.second;
            else
                return x;
        }
    }

    void __build_centroid(int node, Node *parent) {
        node = __get_centroid(node);
        list[node].parent = parent;
        __visited[node] = true;
        for (Node *next : base.list[node].adjacent)
            if (!__visited[next->index])
                __build_centroid(next->index, &list[node]);
    }

public:
    CentroidTree(Tree &tree) : Tree((int)tree.list.size()) {
        __visited = vector<bool>(tree.list.size());
        __subtree_size = vector<int>(tree.list.size());
        __dir_parents = vector<int>(tree.list.size());
        base = tree;
        __build_centroid(0, nullptr);
        for (auto el : list) {
            if (el.parent == nullptr)
                root = &list[el.index];
            else
                add_edge(el.index, el.parent->index);
        }
        __build(root);
    }

    ll diameter(Tree tree) {
        ll n = tree.list.size() + 1;
        vbl visited(n + 1, false);
        vll distances(n + 1, -1);

```

```

queue<pll> q;
q.push({tree.root->index, 0});
ll node_max = tree.root->index, distance_max = 0;
while (!q.empty()) {
    auto node = q.front();
    q.pop();
    if (node.second < distance_max) {
        distance_max = node.second;
        node_max = node.first;
    }

    for (auto neighbor : tree.list[node.first].adjacent) {
        if (!visited[neighbor->index]) {
            auto d = node.second + 1;
            q.push({neighbor->index, d});
            visited[neighbor->index] = 1;
        }
    }
}

```

```

}
visited = vbl(n + 1, false);
q.push({node_max, 0});
distance_max = 0;
while (!q.empty()) {
    auto node = q.front();
    q.pop();
    maximize(distance_max, node.second);
    for (auto neighbor : tree.list[node.first].adjacent) {
        if (!visited[neighbor->index]) {
            auto d = node.second + 1;
            q.push({neighbor->index, d});
            visited[neighbor->index] = 1;
        }
    }
}
}

```
