INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY, HYDERABAD

FIRST YEAR

REAL ANALYSIS, DIGITAL SYSTEMS, DICRETE MATHEMATICS, COMPUTER PROGRAMMING

College Notes

Author: Animesh Sinha

September 4, 2019

Chapter 1

Particle Physics for Data Scientists

1.1 The Preliminaries

1.1.1 Problems with the standard model

What makes us unhappy?

- Matter and Antimatter inequivalence.
- 19 Arbitrary constants.
- Why is Gravity so weak.

1.1.2 The Particles we want to detect

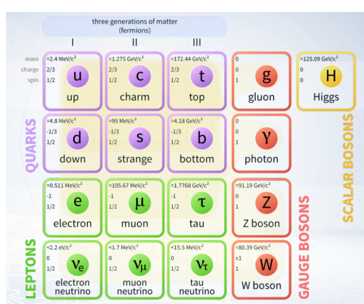


Figure 1.1: List of Particles of different types

The following are types of particles we want to defect.

- muon
- pion
- electron
- kon
- proton

1.1.3 The Experiments in LHC

There are 4 major detectors

- ALICE (A Large Ion Collider Experiment) is a heavy-ion detector on the Large Hadron Collider (LHC) ring. It is designed to study the physics of strongly interacting matter at extreme energy densities, where a phase of matter called quark-gluon plasma forms.

- ATLAS (A Toroidal LHC ApparatuS) is one of two general-purpose detectors at the Large Hadron Collider (LHC). It investigates a wide range of physics, from the search for the Higgs boson to extra dimensions and particles that could make up dark matter. Although it has the same scientific goals as the CMS experiment, it uses different technical solutions and a different magnet-system design. It has a cylindrical structure and measures particles in all directions.

- CMS (Compact Muon Solenoid) is a general-purpose detector at the Large Hadron Collider (LHC). It has a broad physics programme ranging from studying the Standard Model (including the Higgs boson) to searching for extra dimensions and particles that could make up dark matter. Although it has the same scientific goals as the ATLAS experiment, it uses different technical solutions and a different magnet-system design.

- LHCb (Large Hadron Collider Beauty) experiment specializes in investigating the slight differences between matter and antimatter by studying a type of particle called the "beauty quark", or "b quark". It is a single arm forward spectrometer.

We smash bunches of protons ('events'), record the pixels ('hits'), reconstruct trajectories ('jets', 'showers', 'tracks'), and we perform Statistical analysis on them.

A Trigger System is a system that uses criteria to rapidly decide which events in a particle detector to keep when only a small fraction of the total can be recorded.

1.1.4 Simulation Package

- http://www.genie-mc.org/

- http://home.thep.lu.se/Pythia/: Nutrino Simulations

- GEANT4: http://geant4.web.cern.ch/: Particles interacting with matter.

- FLUKA: http://www.fluka.org/fluka.php

1.1.5 Feynman Diagrams

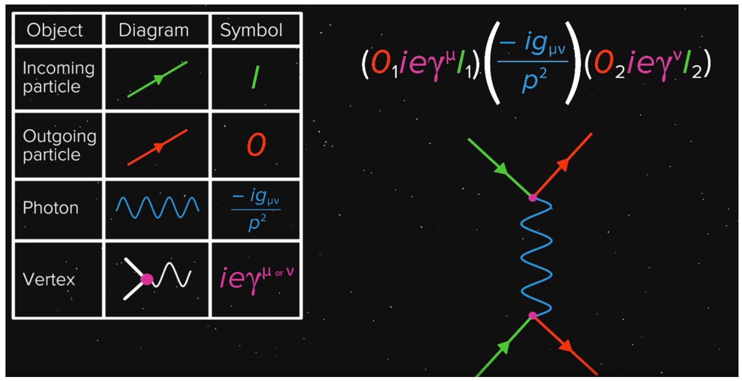
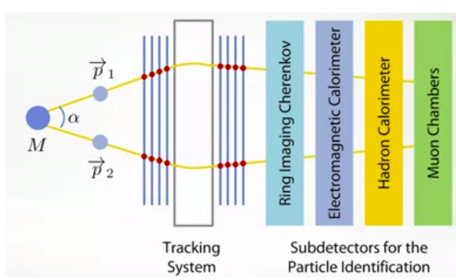
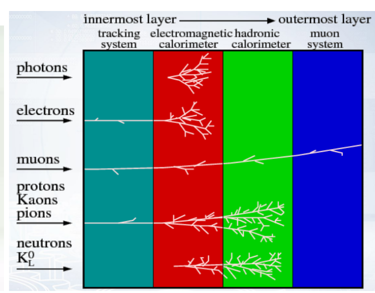


Figure 1.2: A Sample Feynman Diagram for scattering of Two electron by the transfer of one Photon

1.2 The Large Hadron Collider Setup



(a) Detector Setup of the LHC



(b) The tracks made in the Chambers

1.2.1 Tracking System

The first system of detectors, stands before particle collision area. Important for parameter estimation.

There are several layers of sensors that measure hits, and allow us to recognise particle trajectories. There is also a Magnetic Field that allows us to measure momentum (using radius of curvature in the field).

We have the following conservation equationsm wgeb a particle D^0 with mass m breaks into a K^- with mass m_1 and a π^+ with mass m_2, and they go away from each other at angle α:

$$
\begin{aligned}
E_m &= E_1 + E_2 && (1.1)\\
\hat{p_m} &= \hat{p_1} + \hat{p_2} && (1.2)\\
E^2 &= p^2c^2 + m^2c^4 && (1.3)\\
M^2 &= m_1^2 + m_2^2 + \frac{2}{c^4}(E_1E_2 - p_1p_2c^2cos\alpha) && (1.4)
\end{aligned}
$$

Problem: Track Pattern Recognition

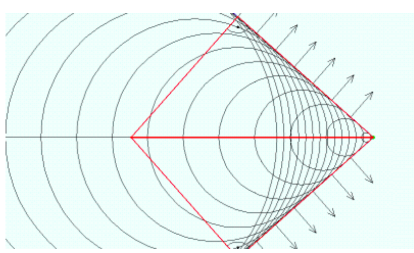
Recognizing hits that belong to the same track. Currently we have the following methods.

- Half Transform and Kalman Filtering. (Statistical, computationally cheaper.)
- Hopfield Neural Netorks. (Denby Peterson and Cellular Automaton.)
- Convolutional Nerual Networks (classify result as correct or wrong). Recurrent Neural Netorks (predict the next hit location).

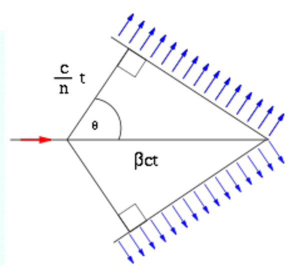
We can then combine these particle tracks into decays.

1.2.2 Ring Imaging Chernekov Detector (RICH)

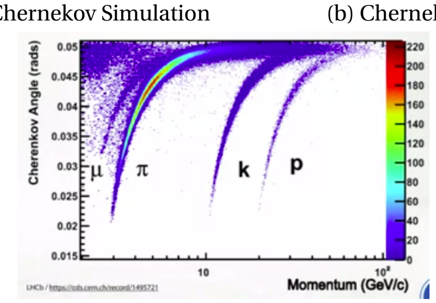
This is the first detector because it does not affect the flight of the particle.



(a) Chernekov Simulation



(b) Chernekov Angles



(c) Chernekov Radiation Graphs

Figure 1.4: Registers in Processor Design

Here is the angle of the Chernokov radiation, derivable using simple geometric means and some special relativity in terms of the momentum.

$$p = \frac{mc\beta}{\sqrt{1 - v^2/c^2}} \tag{1.5}$$

$$cos\theta = \frac{1}{n\beta} = \frac{\sqrt{p^2 + m^2c^2}}{np} \tag{1.6}$$

From the figure above, and the equation for it's analytical feel, that we can figure out the mass of the particle and thereby which particle it is.

1.2.3 Ring Imaging Chernekov Detector (RICH)

Electromagnetic calorimeter - stops everything other than muons and quarks. Hadron calorimeter - stops the quarks.

Chapter 2

Quantum Field Theory

2.1 Classical Field Theory

2.1.1 What is a field?

A field ϕ is a quantity (eg. Density, Spin, Charge) defined at every point in a manifold M (spacetime, Minkowski space usually).

$$\phi : M \leftarrow S$$

So the field is any function from the space to a Target Space.

- Here S can be Scalar Field, where $S = \Re$. It good for modeling Higs Boson, Charge Density, Magnetisation density, etc.
- Or it can be a vector field $S = \Re^n$. It's good for modelling Pions, Elecromagnetic fields, etc.
- We can also have $S = S^2$, which is the surface of a sphere, this is used for the σ-model and modelling Quantum Magnets.

We will restrict our attention to fields whose Classical Dynamics are obtained by applying Variational Principal applied to an Action Functional (Lagrangian Fields). These encode symetries well.

2.1.2 Our Lagrangian Field

We break our vector field down into several scalar fields, $\Phi_a(x)$; $a = 1, 2, 3 \ldots, N$.

Our action functional involves the Lagrange density $\mathscr{L}$ which is a function of

$$S(\Omega) = \int_\Omega \mathscr{L}(\partial_\mu \Phi_a) d^4x \ ; \ d^4x = dx_0 dx_1 dx_2 dx_3.$$

Here $\Omega \in \mu_{1,3}$ is a measurable subset of spacetime, the region where $\mathscr{L}$ is defined.

$\mathscr{L}$ is a function if Φ_a, $\partial_\mu \Phi_a$, $\partial_\mu \partial_\nu \Phi_a$, and so on, but we will only take the first derivative, so:

$$\mathscr{L} = \mathscr{L}(\Phi_a, \partial_\mu \Phi_a) \tag{2.1}$$

We assume that this functional is stationary under small perturbations, i.e. only the second derivative varies, not till the first derivatives ⚠. This puts some conditions on Φ_as.

2.1.3 Extracting Equations of Motion

$$\frac{\partial \mathscr{L}}{\partial \Phi_a} - \frac{\partial}{\partial x^\mu} \frac{\partial \mathscr{L}}{\partial(\partial_\mu \Phi_a)} = 0 \tag{2.2}$$

2.1.4 Example: Klein Gordon Field

$$\mathscr{L} = \frac{1}{2}\dot{\Phi}^2 - \frac{1}{2}(\nabla\Phi)^2 - \frac{1}{2}\mu^2\Phi^2 \tag{2.3}$$

Chapter 3

General Relativity

3.1 Preliminaries

3.1.1 What is flat space

Space is flat if there exists a way to chose the metric tensor, such that it's the knonecker delta. $\eta^{x,y} = \delta_{x,y}$.

A piece of flat paper that is folded is still flat, it has no curvature, it can be made flat again and the notion of distance is still the same as when it was flat. This is just extrinsic curvature, which comes from the way it's embedded in a higher dimentional space.

3.1.2 Tensor Analysis

When we take the components as scalar that sum to it, the components are contravarient components.

$$V = V^1 \hat{e}_1 + V^2 \hat{e}_2 + V^3 \hat{e}_3 \tag{3.1}$$

And if we find the components using dot products, we get the covarient components.

$$V_n = V \cdot \hat{e}_n = \Sigma_n V^n (e_n \cdot e_m) \tag{3.2}$$

These are exactly the same thing when we are in the Cartesian coordinates.

Transformation Rules of Tensors

For contravarient index:

$$dy^m = \frac{\partial y^m}{\partial x^n} dx^n \tag{3.3}$$

$$\frac{\partial S}{\partial y^m} = \frac{\partial x^n}{\partial y^m} \frac{\partial S}{\partial x^n} \tag{3.4}$$

For covarient index:

$$dy^m = \frac{\partial y^m}{\partial x^n} dx^n \tag{3.5}$$

$$\frac{\partial S}{\partial y^m} = \frac{\partial x^n}{\partial y^m} \frac{\partial S}{\partial x^n} \tag{3.6}$$

For Mixed tensors (Rank 2):

$$(W')^m_n = \frac{\partial y^m}{\partial x^p} \frac{\partial x^q}{\partial y^n} W^p_q \tag{3.7}$$

For Covarient tensors (Rank 2):

$$(W')_{mn} = \frac{\partial x^p}{\partial y^m} \frac{\partial x^q}{\partial y^n} W_{pq} \tag{3.8}$$

For contravarient tensors (Rank 2):

$$(W')^{mn} = \frac{\partial y^m}{\partial x^p} \frac{\partial y^n}{\partial x^q} W^{pq} \tag{3.9}$$

Addition, Multiplication, Contraction

We only add tensor if they have indices of the same kind, that is:

$$T^{m_1,m_2...m_k}_{n_1,n_2...n_l} + S^{m_1,m_2...m_k}_{n_1,n_2...n_l} = (T+S)^{m_1,m_2...m_k}_{n_1,n_2...n_l} \tag{3.10}$$

We can multiply any two tensors (We do get tensors of higher rank):

$$V^m \otimes W_n = X^m_n \tag{3.11}$$
$$V^m \otimes W^n = X^{mn} \tag{3.12}$$
$$V_m \otimes W_n = X_{mn} \tag{3.13}$$

The generalization of inner product of two vectors to tensors is called it's contraction. Proof/Intuition of contraction:

$$(V^m W_m)' = \frac{\partial}{\partial}(V^a W_b) \tag{3.14}$$

When we contract an upper with a lower index, we reduce the number of indices (rank) by 2, as both of them become dummy indices, and we sum over them.

The Metric Tensor

$$ds^2 = g(x)_{mn} dx^m dx^n \tag{3.15}$$

The metric tensor is always symetric. Everyone agrees on the length of every vector, though not on the individual components, when viewed from different frames.

Transformation of the metric tensor:

$$g_{mn}(x)dx^m dx^n = g'_{pq}dy^p dy^q = g_{mn}(x) = \mathbf{g_{pq}}\frac{\partial \mathbf{x^m}}{\partial \mathbf{x^n}}\frac{\partial \mathbf{y^p}}{\partial \mathbf{y^q}}dy^p dy^q \tag{3.16}$$

We have two metric tensors, one with covarient and one with contravarient indices, defined as:

$$g_{mn}g^{np} = \delta^p_m \tag{3.17}$$

So, one is the inverse of the other.