Machine Data & Learning
Assignment 5 Part 1: POMDP

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Before the First Action

All that is known about the initial state is that it's red in color, so we start with the belief state of being in any red state with equal probability.

0.33333	0.33333	0.0	0.0	0.33333

For my roll number, the probability of moving in the right direction is 98% and opposite is 2%. Also, I use table 1 (0-indexed) to get observation probabilities, which makes the probability to correctly measure Red 0.8 and Green 0.95.

After Action 01

We go right and see red.

0.33333 × 0.02 + 0.33333 × 0.02	0.33333 × 0.98	0.33333 × 0.98	0.33333 × 0.02	0.33333 × 0.98

P(red) = 0.8(0.33 × 0.04 + 0.33 × 0.98 + 0.33 × 0.98) + 0.05(0.33 × 0.02 + 0.33 × 0.98) = 0.55

$$P\left(\frac{S}{red}\right) = P\left(\frac{red}{S}\right)\frac{P(S)}{P(red)}$$

So now the new probabilities will be:

$P(S) = P_{prev}(S) \times \frac{0.8}{0.55} = P_{prev}(S) \times \frac{16}{11}$ if S is a red state

$P(S) = P_{prev}(S) \times \frac{0.05}{0.55} = P_{prev}(S) \times \frac{1}{11}$ if S is a green state

0.01939393	0.47515151	0.02969696	0.00060606	0.47515151

After Action 02

We go left and see green.

0.0193939 × 0.98 + 0.4751515 × 0.98 = 0.48465449	0.0193939 × 0.02 + 0.0296969 × 0.98 = 0.02949408	0.0006060 × 0.98 + 0.4751515 × 0.02 = 0.01009691	0.0296969 × 0.02 + 0.4751515 × 0.98 = 0.46624240	0.4751515 × 0.02 + 0.0006060 × 0.02 = 0.00951515

P(green) = 0.2(0.48465449 + 0.02949408 + 0.00951515) + 0.95(0.01009691 + 0.46624240) = 0.5572550885

$$P\left(\frac{S}{green}\right) = P\left(\frac{green}{S}\right)\frac{P(S)}{P(green)}$$

So now the new probabilities will be:

$P(S) = P_{prev}(s) \times \frac{0.95}{0.5572550885} = P_{prev}(S) \times 1.70478479$ if S is a green state

$P(S) = P_{prev}(S) \times \frac{0.2}{0.5572550885} = P_{prev}(S) \times 0.35890206$ if S is a red state

0.17394349	0.01058548	0.01721306	0.79484295	0.00341500

After Action 03

We go left and see green.

0.17394349 × 0.98 + 0.01058548	0.01721306 × 0.98 + 0.17394349	0.79484295 × 0.98 + 0.01058548	0.00341500 × 0.98 + 0.01721306	0.00341500 × 0.02 + 0.79484295

× 0.98 = 0.18083839	× 0.02 = 0.02034766	× 0.02 = 0.77915780	× 0.02 = 0.00369096	× 0.02 = 0.01596515

$$P\left(\frac{S}{green}\right) = P\left(\frac{green}{S}\right)\frac{P(S)}{P(green)}$$

So now the new probabilities will be:

$P(S) = P_{prev}(S) \times \frac{0.95}{0.78713656735} = P_{prev}(S) \times 1.2069061957$ if S is a green state

$P(S) = P_{prev}(S) \times \frac{0.2}{0.78713656735} = P_{prev}(S) \times 0.35890206$ if S is a red state

0.18083839* 0.35890206	0.02034766* 0.35890206	0.77915780* 1. 20690619	0.00369096* 1. 20690619	0.01596515* 0.35890206

0.06490327	0.00730281	0.94037037	0.00445464	0.00572992

Conclusions

This is the belief state after each step taken:

0.33333	0.33333	0.0	0.0	0.33333

0.01939393	0.47515151	0.02969696	0.00060606	0.47515151

0.17394349	0.01058548	0.01721306	0.79484295	0.00341500

0.06490327	0.00730281	0.94037037	0.00445464	0.00572992

So, the most probable explanation is that it started in State **S5**, tried to move right and stayed in place at S5, then moved left to get to **S4** and left again to get to **S3**.