Data Structures and Algorithms

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Reference Books

- A. V. Aho, J. E. Hopcroft, J. D. Ullman, *“Data Structures and Algorithm”*, Pearson.
- D. Samanta, *“Classic Data Structure”*, PHI.
- S. Lipscutz, *“Data Structures with C”*, TMH.
- R. Kruse, C.L. Tondo, B. Leung, S. Mogalla, *“Data Structures and Program Design in C”*, Pearson.
- D. E. Knuth, *“The Art of Computer Programming”*, Addison-Wesley
- S. Chattopadhyay, D.G. Dastidar, M. Chattopadhyay, *“Data Structures through C Language”*, BPB Publications.

Introduction

Algorithms

- A *finite* set of instructions executed in *sequence* in *finite time*
- Algorithm for finding GCD

step 1: read two positive integers **x** and **y**

step 2: divide **x** by **y** to get remainder **r** and quotient **q**

step 3: if **r** is zero go to step **7**

step 4: assign **y** to **x**

step 5: assign **r** to **y**

step 6: go to step 2

step 7: **y is the required GCD**, print **y**

step 8: stop

Algorithms

- Properties of an algorithm
  - *Input*
  - *Output*
  - *Finiteness*
    - For all input data, the algorithm must terminate after a finite number of steps
  - *Definiteness*
    - Clear and unambiguous steps
  - *Effectiveness*
    - Steps must be very basic

Algorithms

- Expressing an algorithm
  - Natural language, flowchart, programming languages
- Designing an algorithm
  - Innovative exercise, no methodology to automatically generate algorithms
  - Use of new techniques and strategies for good algorithms
    - Depends heavily on the organization of data
- Analyzing an algorithm
  - Validation
  - Complexity evaluation
    - Both time and space

Abstract Data Type

- Data type defines a set of values and permitted operations
- Built-in data types are not enough for most applications
  - Create new data types in terms of structure
    - Operations applicable to structure variables can not be specified
  - Concept of abstract data type (ADT) introduced
    - A mathematical model with collection of operations defined on that model

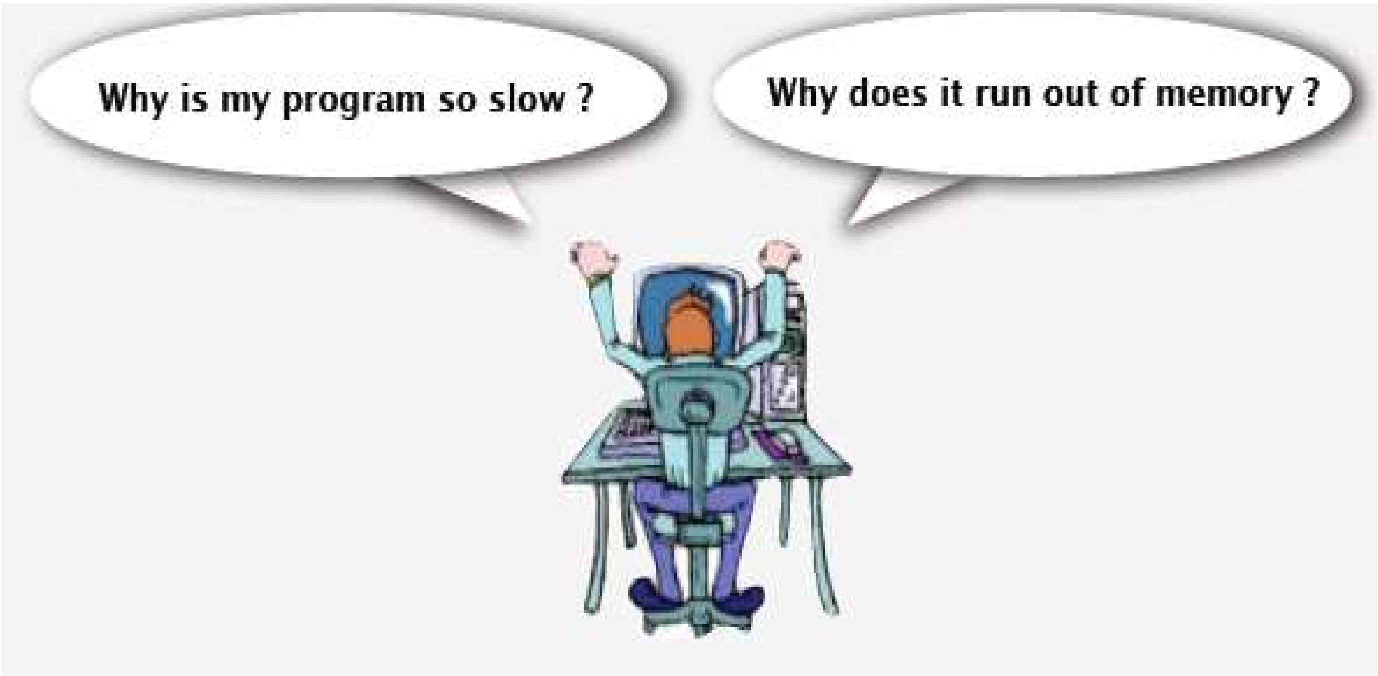
Abstract Data Type

- Define a new data type "SET"
- Operations on ADT "SET"
  - assign (SET A, SET B)
  - SET union (SET A, SET B)
  - SET Intersection (SET A, SET B)
  - int cardinality (SET A)
- ***No limit*** on the number of operations
- ***Implementation aspect*** is not considered

Data Structure

- A scheme to organize data
  - Stack, queue, tree, graph etc.
- Affects the performance of a program for different tasks
- Choice of data structure depends on:
  - Nature of data
  - Processes to be performed on the data

Analysis of Algorithms



Analysis of Algorithms

- Used to compare number of algorithms and choose the best one
- Typically, two quantitative metrics:
  – Space complexity
    - Run time storage requirement
  – Time complexity
    - Time required to complete execution
      – Usually depends on ***input size***, i.e., time complexity is a ***function of input size***

Analysis of Algorithms

- Types of analysis
  - Best Case
    - *Lower bound* on cost
    - Determined by *"easiest"* input
    - Provides a goal for all inputs
  - Worst Case
    - *Upper bound* on cost
    - Determined by most *"difficult"* input
    - Provides a guarantee for all inputs
  - Average Case
    - *Expected cost* for *random input*
    - Provides a way to predict performance

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Analysis of Algorithms

- Time complexity
  - Very difficult to compute exact time
    - Several factors influence execution time which are outside the domain of programmers
      - Programs are translated to machine code
  - Define a function f(n) that gives an estimate of volume of work done by the algorithm on input size n

Analysis of Algorithms

- Big-Oh notation
  - $T(n)=O(\ f(n)\ )$ if $T(n) \le c\, f(n)$

    for some constant, $c > 0$, and $n \ge n_0$

    f is an upper bound for T

    ie for sufficiently large ***n***

  - If $T(0)=0$, $T(1)=4$, and in general $T(n)=(n+1)^2$, then $T(n)=O(n^2)$
    - Let $n_0=1$, $c=4$, i.e., $\forall n \ge 1$, $(n+1)^2 \le 4n^2$

Analysis of Algorithms

- **Constant factors may be ignored**
  - $\forall\ k > 0,\ kf$ is $O(f)$
- **Higher powers grow faster**
  - – n^r is $O(n^s)$ if $0 \le r \le s$

Fastest growing term dominates a sum

e.g., $3n^3 + 2n^2$ is $O(n^3)$

c + cn + cnlogn is O(nlogn)

Polynomial's growth rate is determined by leading term

– If T is a polynomial of degree d,
then T is $O(n^d)$

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Analysis of Algorithms

- *T* is *O(g)* is transitive
  - If T is O(g) and g is O(h) then T is O(h)
- Product of upper bounds is upper bound for the product
  - If f is O(g) and h is O(r) then fh is O(gr)
- Two additional notations
- $\Omega(g(n))$
  → $T(n) \geq c\, g(n)$

  for some constant, c, and $n > n_0$

  $g(n)$ **is a lower bound for** T
- $\theta(g(n))$, **for some constants, c_1, c_2 and $n > n_0$**
  → $c_2 g(n) \geq T(n) \geq c_1\, g(n)$

  Best and worst case complexities are same

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Analysis of Algorithms

- Simple statement

```
S=p+q
```

  - Time Complexity is O(1)

- Simple loops

```
for(i=0;i<n;i++) { s=p+q; }
```

  - Time complexity is n O(1) or O(n)

- Nested loops

```
for(i=0;i<n;i++)
    for(j=0;j<n;j++) { s=p+q; }
```

This part is *O(n)*

  - Time Complexity is n O(n) or $O(n^2)$

Analysis of Algorithms

- Loop index doesn't vary linearly

```
h = 1;
while ( h <= n ) {
        s;
        h = 2 * h;
        }
```

  - `h` takes values 1, 2, 4, … until it exceeds n
  - There are $1 + \log_2 n$ iterations
  - Complexity O(log n)

Analysis of Algorithms

- Loop index depends on outer loop index

```
for(j=0;j<n;j++)
    for(k=0;k<j;k++){
      s;
      }
```

  - Inner loop executed
    - 1, 2, 3, …., n times

$$\sum_{i=1} i = \frac{n(n+1)}{2}$$

$\therefore$ Complexity $O(n^2)$

Analysis of Algorithms

- Common computing times are

  $O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n)$

- *log n*
  - Logarithmic algorithm, cuts down the problem to smaller one
- *n*
  - Linear algorithm
- *n log n*
  - Breaks the large problem into sub-problems, solve sub-problems independently, combine the results
- *2^n*
  - Exponential running time, not suitable for practical use

The LIST ADT

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

LIST ADT

- Ordered sequence of data items called elements
  - $A_1, A_2, A_3, \ldots, A_N$ is a list of size N
- Size of an empty list is 0
- First element is A_1 called "head"
- Last element is A_N called "tail"

Operations ?

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

LIST ADT

- **Operations**
  - PrintList
  - Search
  - FindKth
  - Insert
  - Delete
  - Reverse
  - Sorting
  - MakeEmpty

LIST ADT

- **Example:**

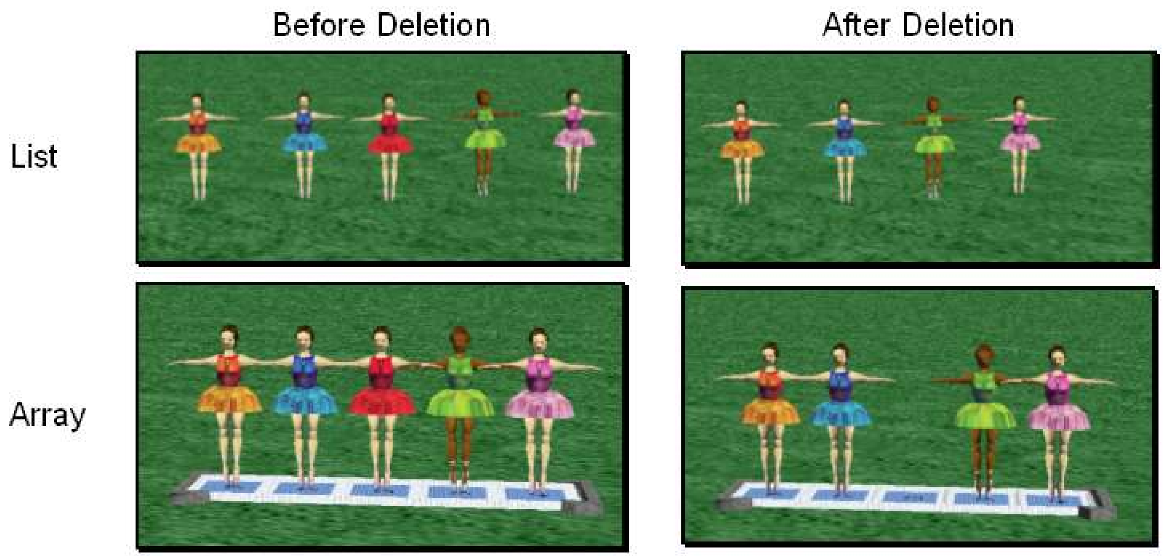
  the elements of a list are

  34, 12, 52, 16, 12

  - Search (52) → 3
  - Insert (20, 3) → 34, 12, 20, 52, 16, 12
  - Delete (52) → 34, 12, 20, 16, 12
  - FindKth (3) → 20

LIST ADT

- You can see the difference between arrays and lists when you delete items.



Array Implementation of LIST

- Need to define a size for array
  - High overestimate (waste of space)

- Operations Running Times

PrintList Search	O(N)
Insert Delete	O(N) **(on avarage half needs to be moved)**
FindKth Next Previous	O(1)

Array Implementation of LIST

```
int search (int a[], int n, int val)
{
int i=0;
for (i=0; i<n;i++)
        if (a[i] = = val)
                break;
        if (i = = n)
                return -1;
        else
                return i;
}
```

If a match is found come out of the for loop

Val not found in a[ ]

Return the index of val in a[ ]

Best Case: **O(1)**

Average Case:

number of comparisons could be 1,2,..,n depending on the position of val.

=(1+2+…+n)/n

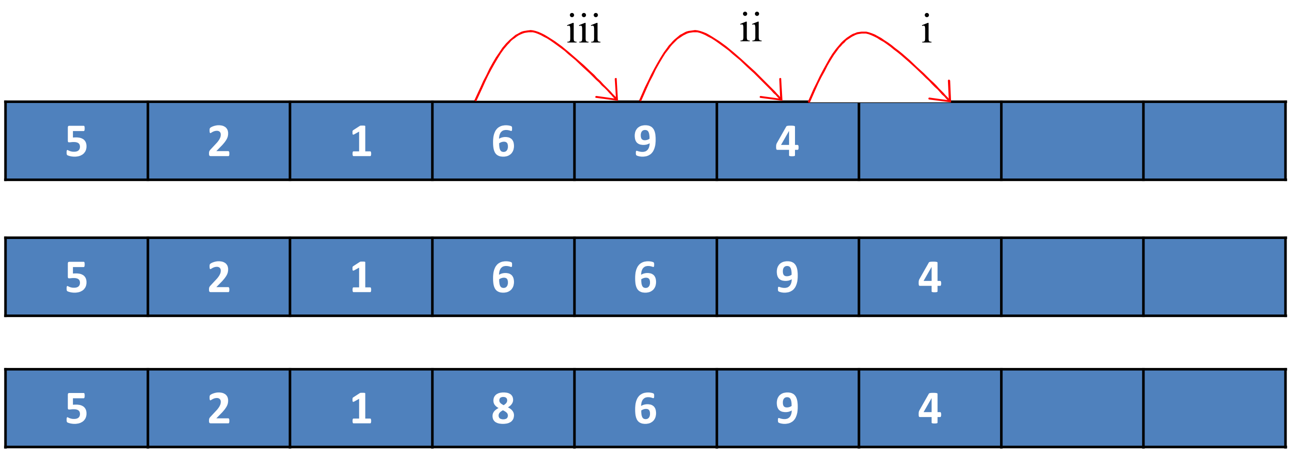
=(n+1)/2

=**O(n)**

Worst Case: **O(n)**

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Array Implementation of LIST



Initial array; arrow shows shifting of elements

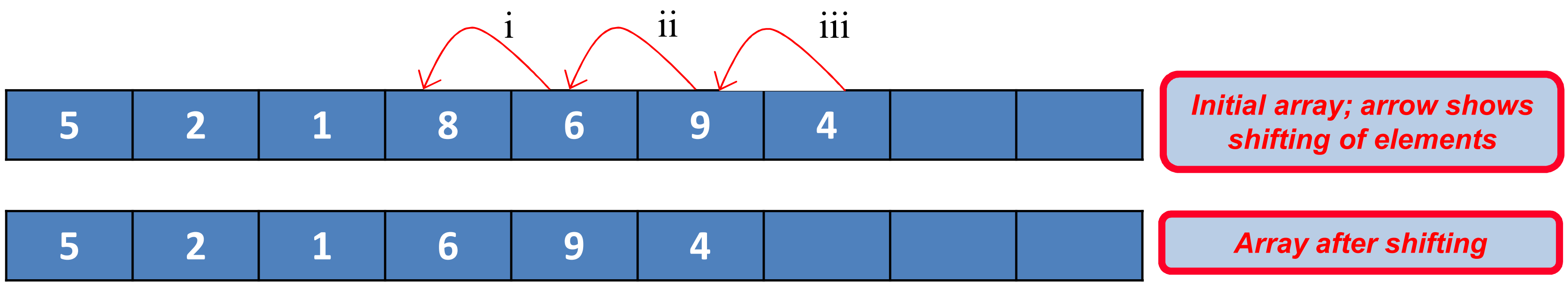
Array after shifting

Array after inserting 8 at position 3

```
void insert (int a [], int n, int j, int val)
{
    int i;
    for ( i = n-1; i >= j; i-- )
        a[i+1] = a[i];
    a[j] = val;
}
```

Shifting of elements to the right

Array Implementation of LIST

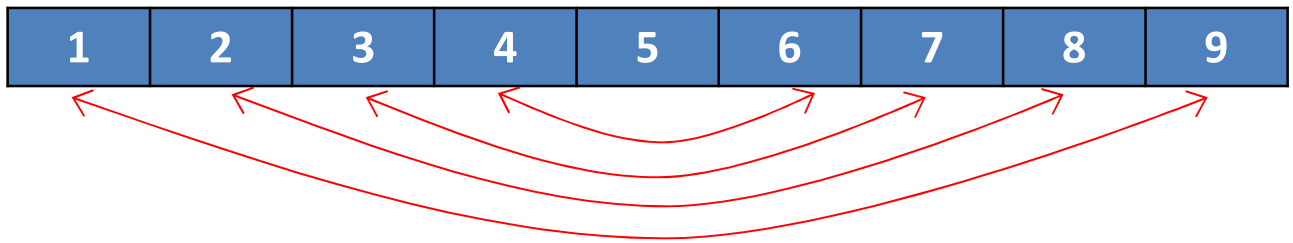



```
Void delete (int a [], int n, int j)
{
    int i;
    for ( i = j+1; i < n; i++ )
        a[i-1] = a[i];
}
```

Shifting of elements to the left

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Array Implementation of LIST



Arrows indicate required exchange

```
void reverse (int a [], int n)
{
    int i, temp;
    for ( i = 0; i < n/2 ; i++ )
        {
                temp = a [i];
                a [i] = a [n-1-i];
                a [n-1-i] = a [i];
        }
}
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Polynomials using Arrays

- Polynomial, P(x) of degree n is:

  $P(x) = a_0 + a_1x + a_2x^2 + \dots\dots + a_nx^n$
- Treat Polynomials as ADT
  - *Operations*
    - **Initialization**
    - **Copy**
    - **Add**
    - **Multiply**
    - **Evaluate**

Polynomials using Arrays

- Representation:

```
typedef struct poly
{
        float coeff [1000];
        int degree;
} poly
```

- Polynomial 1 + 4x^2 + 2x^8 is stored as:

1	0	4	0	0	0	0	0	2

Wasteful representation, most of the elements are Zero

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Polynomials using Arrays

- Store only **non-zero** terms $a_i x^i$
- Define the terms:

```
typedef  struct term
{
        float  coeff;
        int  expo;
} term;
```

- Now, define the polynomial:

```
typedef  struct poly
{
        term  a[ 1000];
        int  no_of_terms;
}poly;
```

Polynomials using Arrays

- Polynomial $1 + 4x^2 + 2x^8$ is stored as:

1 , 0	4 , 2	2 , 8	……

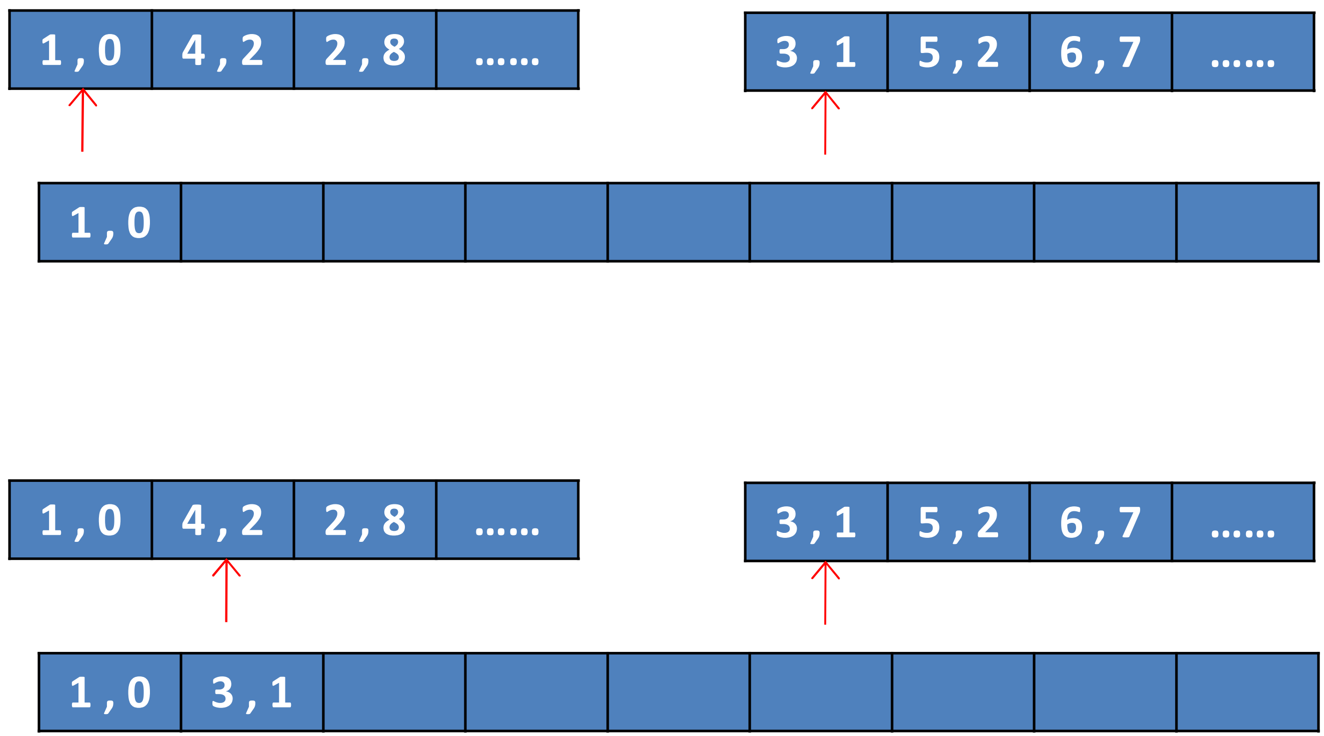
- Addition of polynomials $1 + 4x^2 + 2x^8$ and $3x + 5x^2 + 6x^7$

1 , 0	4 , 2	2 , 8	……

3 , 1	5 , 2	6 , 7	……

Polynomials using Arrays

- Addition of polynomials:



Polynomials using Arrays

- Addition of polynomials:

1 , 0	4 , 2	2 , 8	……

3 , 1	5 , 2	6 , 7	……

1 , 0	3 , 1	9 , 2						

1 , 0	4 , 2	2 , 8	……

3 , 1	5 , 2	6 , 7	……

1 , 0	3 , 1	9 , 2	6 , 7					

Polynomials using Arrays

- Addition of polynomials:

1 , 0	4 , 2	2 , 8	……

3 , 1	5 , 2	6 , 7	……

1 , 0	3 , 1	9 , 2	6 , 7	2 , 8				

Polynomials using Arrays

```
for ( i = 0, j = 0, k=0; ( i < P1→no_of_terms) && ( j < P2→no_of_terms); k++)
{
      if (P1→ a [i].expo == P2→ a[j].expo)
      {
            P3→a[k].coeff = P1→a[i].coeff + P2→a[j].coeff;
            P3→a[k].expo = P1→a[i].expo;
            i++; j++;
      }
      else if (P1→ a [i].expo < P2→ a[j].expo)
      {
            copy the term in P1 to P3;          i++;
      }
      else
      {
            copy the term in P2 to P3;          j++;
      }
}
```

For remaining terms in P1 and P2

```
if (i < P1→no_of_terms)
      for (l = i; l < P1→no_of_terms; l++, k++)
      {
            copy the terms in P1 to P3;
      }
else
      for (l = j; l < P2→no_of_terms; l++, k++)
      {
            copy the terms in P2 to P3;
      }
P3→no_of_terms = k;
```

Complexity: O(m+n), m and n are degree of the polynomials

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Polynomials using Arrays

- Very large numbers can not be stored in variables of type "int" or "long"!!!!
  - 80 digit number is greater than the maximum value in "long"

- A number can be represented as a polynomial
  - e.g., **$12345 = 1 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$**
  - Equivalent to ***$P(x) = x^4 + 2x^3 + 3x^2 + 4x + 5$, for $x=10$***

Polynomials using Arrays

- An integer of *n* digits is a polynomial of degree *n-1*:

  P(x)=$\sum_{i=0,n-1} a_i x_i$ for $x=10$, $0 \le a_i \le 9$

- 200002000000008000090000000001 can be represented as:

1 , 0	9 , 9	9 , 2	8 , 14	2 , 22	2 , 27

Sparse Matrix

- Most of the elements are **zero** in a matrix

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 2 & 0 & 5 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

- Two dimensional array representation is **inefficient**
- Solution:
  – Store only ***non-zero elements***

Sparse Matrix

- Treat a sparse matrix as a **ordered LIST** of non-zero elements
- Information regarding each element:
  - row, col, val
- Define the elements:

```
typedef struct element
{
        int row, col, val;
} element;
```

Sparse Matrix

- Define the sparse matrix:

```
typedef struct sparsemat
{
        int no_of_nonzero_elements;
        int no_of_rows, no_of_cols;
        element a [100];
}sparsemat;
```

- Example matrix can be represented as follows:

0, 2, 1	1, 0, 2	1, 2, 5	2, 3, 3	3, 1, 4

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Sparse Matrix

- Saving in Space
  - Space required to store m x n matrix of integers is

    m x n x (size of an integer)
  - Space required in ordered LIST

    3 x p x (size of an integer), p is size of array
  - LIST is *advantageous* if

    3 x p < m x n i.e., **p < m x n/3**

Sparse Matrix

- Finding number of non-zero elements in each column:

```
for ( i = 0; i < s→no_of_cols; i ++)
{
    count = 0;
    for (j = 0; j < s→no_of_nonzero_elements; j ++)
        if (s→a[j].col  == i)
            count ++;
    printf (“number of no-zero elements in column %d is %d”, i, count);
}
```

Number of comparisons = p x n
In worst case p = O(mn)
So, complexity is O (mn^2)

However, a function using two dimensional representation takes O(mn) time !!!!!!!

Sparse Matrix

- Use of programming trick helps to reduce the complexity
- Use an array column so that column [i] stores number of elements in *i^{th}* column

```
for ( i = 0; i < s→no_of_nonzero_elements; i ++)
{
            j = s→a[i].col;
            col [j] = col [j] + 1;
}
```

Complexity is now O(mn)

Multi-dimensional Array

- Computer memory is one dimensional
  - Multi-dimensional arrays are represented as one dimensional array

  e.g., int a[5][10][4][10][5] may be stored as b[10000]

A difficult question:

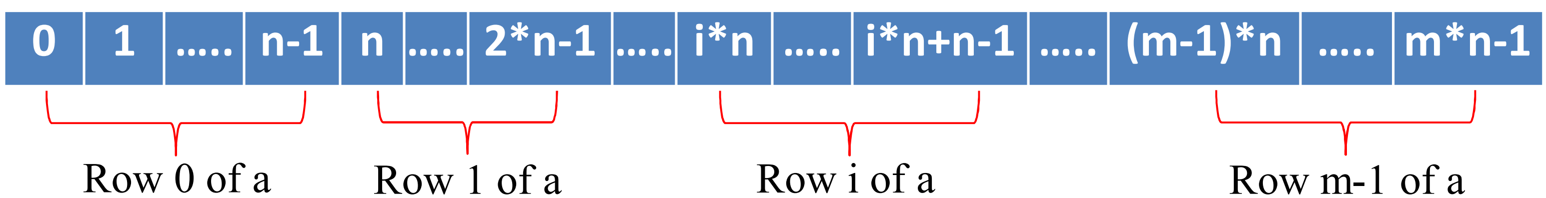
Which element in "b" contains a particular element of "a"??

- Consider the following example:

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Multi-dimensional Array

- Two dimensional array a[m][n] converted to one dimensional array b[mxn] as follows.



- This is **row-major** ordering
- Element a[i][j] is mapped to $(i \times n + j)^{th}$ element in array b

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Multi-dimensional Array

- Three dimensional array a[2][3][2] looks as follows:

(0,0,0) (0,0,1) (0,1,0) (0,1,1) (0,2,0) (0,2,1)

(1,0,0) (1,0,1) (1,1,0) (1,1,1) (1,2,0) (1,2,1)

- To reach a[i][j][k], go to a[i][0][0]
  - Number of elements between a[0][0][0] and a[i][0][0] is **i x n x p**
- From a[i][0][0] go to a[i][j][0]
  - Number of elements **j x p**
- From a[i][j][0] go to a[i][j][k]
  - Number of elements **k**
- So, a[i][j][k] mapped to i x n x p + j x p + k

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Multi-dimensional Array

- Consider the following m-dimensional array

$$a[u_0][u_1]....[u_{m-1}]$$

- The position of $a[i_0][i_1]....[i_{m-1}]$ is

$$\sum_{j=0}^{m-2}\left(i_j * \prod_{i=j+1}^{m-1} u_i\right) + i_{m-1}$$

Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Linked Lists

Limitations of Arrays

- Simple, Fast

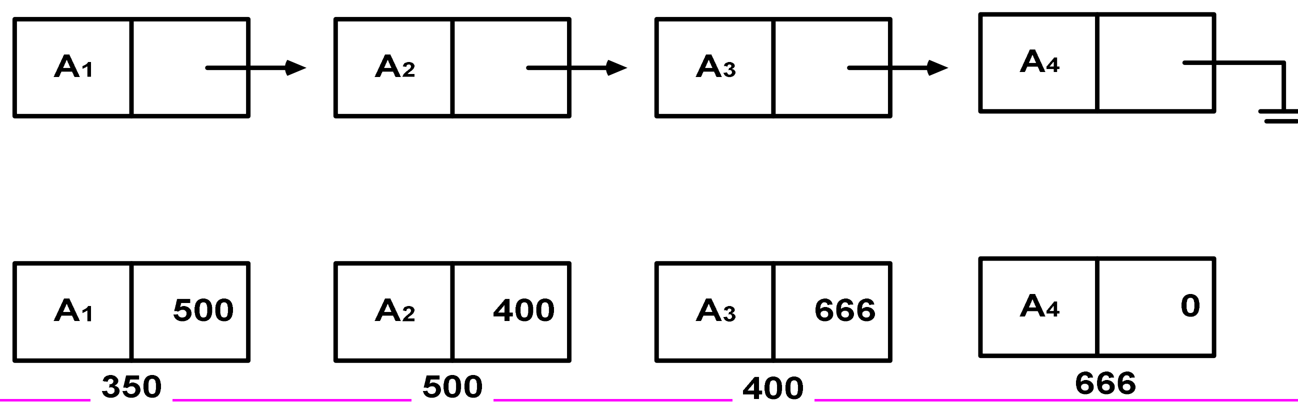
  but

- Must specify size at construction time
  - Construct an array with space for n
    - n = twice your estimate of largest collection
  - Actual size is much less than n, wastage of space
  - Tomorrow you'll need $n+1$, overflow
- Shifting of elements during insertion and deletion

  – **More flexible system?**

Linked Lists

- Flexible space use
  - Dynamically allocate space for each element as needed
- Series of nodes
  - Each node of the list contains
    - the data item
    - a pointer to the next node
- Avoids the linear cost of insertion and deletion !



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked Lists

- Define a node

```
typedef Struct node
{
    int val;
    struct node *next;
} node;
```

Recursive type definition

- Create the Header

```
node* create_header( int item )
{
    Node* header = (node*) malloc( sizeof(node) );
    header->val = item;
    header->next = NULL;
    return header;
}
```

Header keeps track of the entire list; Carefully handle the header

Linked Lists

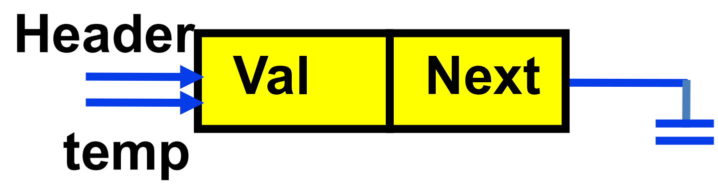
- Creation of list with n nodes:

void create_list (node *header, int n) {

node *temp = header, *last;

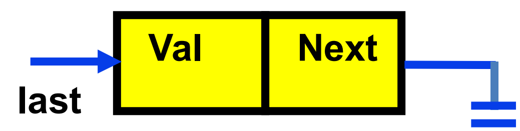
For (i = 0; i < n; i++)

{



last = (node*) malloc (sizeof (node));

New node is created



scanf (“%d”, &(last→val));

last→next = NULL;

temp→next = last;

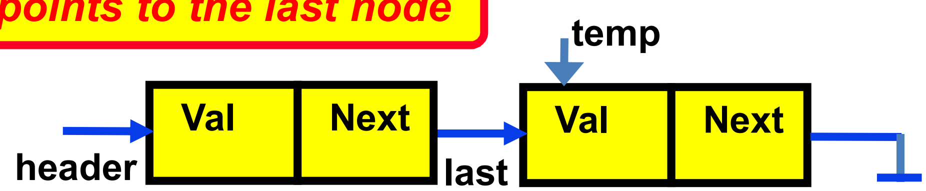
Attached to the end of the list

temp = last;

temp and last points to the last node

}

}



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked Lists

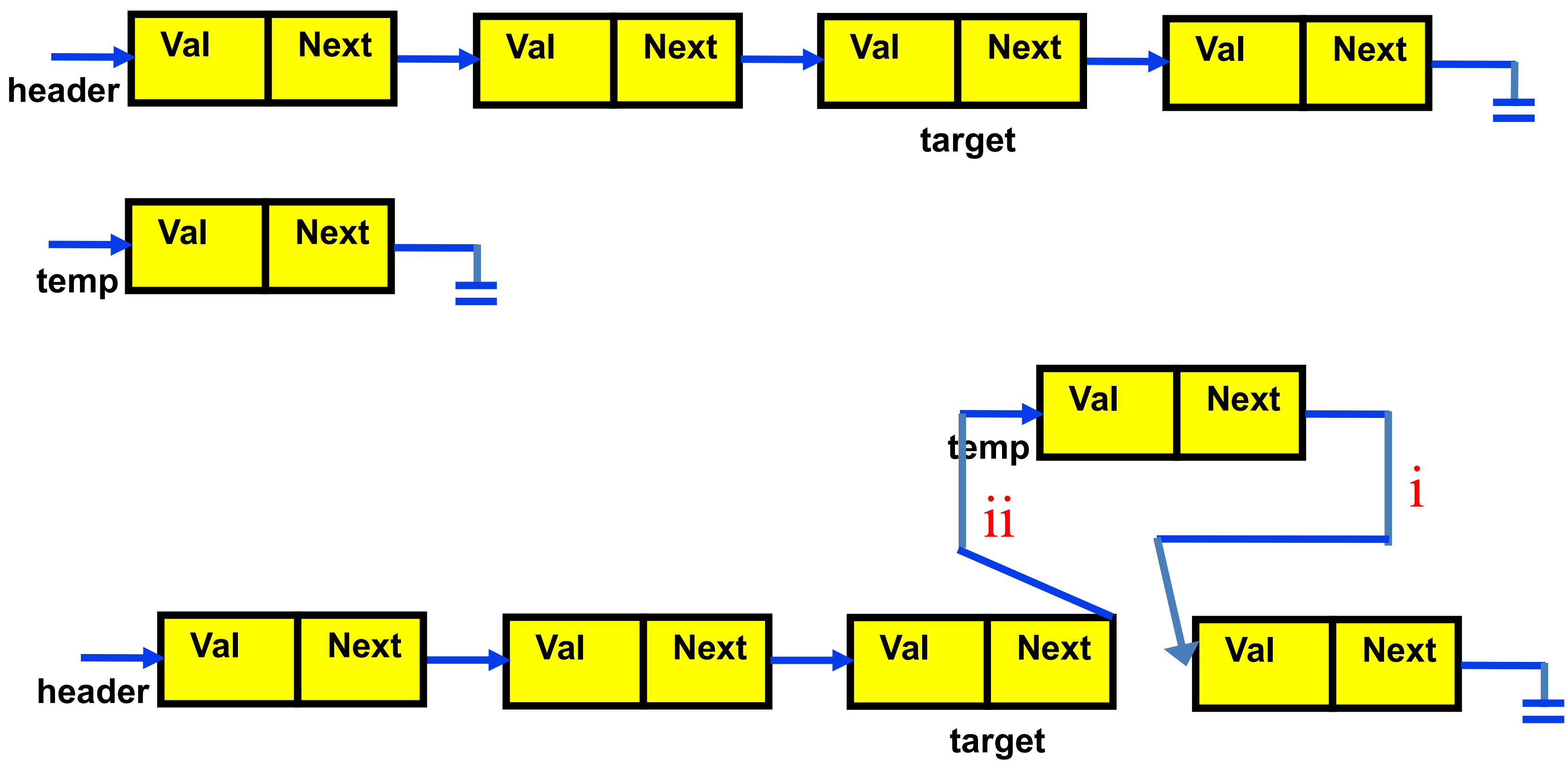
- Printing content of a list is simple…

```
Void printlist (node *header)
{
        node *temp=header;
        while (temp != NULL)
        {
                printf (“%d”, temp→val);
                temp = temp → next;
        }
}
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked Lists

- Insert a node in a linked list



Linked Lists

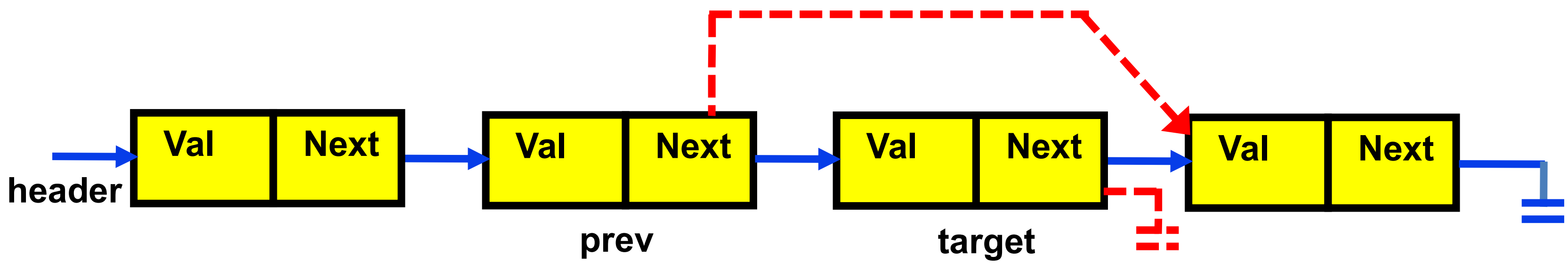
```
void insert (node * header, int i, node *t)  {
     int k;    node *temp = header;
     if ( i == 0)
     {
          t→next = temp;
          header = t;
          return;
     }
for ( k = 1; ( k < i )  &&  ( temp != NULL );  k++)
          temp = temp→next;
If ( ( temp == NULL )  &&  i > 0)    return;
t→next  =  temp→next;
temp→next  =  t;
return;   }
```

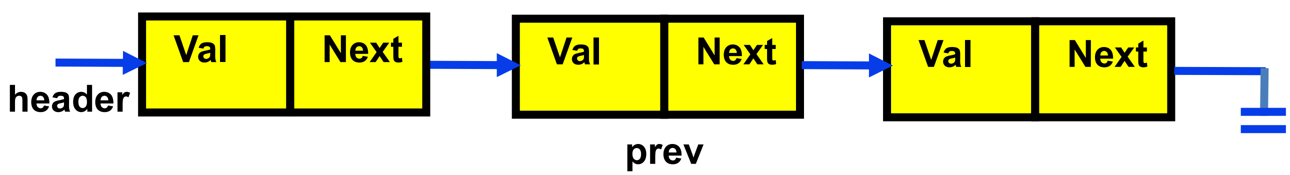
Insert at the first position

Move to the target node in question

Linked Lists

- Delete a node from linked list





Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked Lists

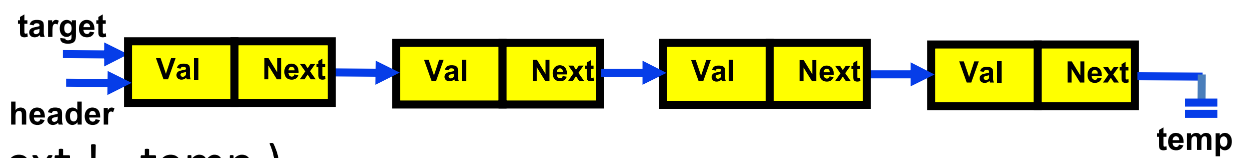
```
void delete ( node * header, int k)   {
     int  i; node *temp = header, *target;
     If ( k == 0 )
     {
          header = header→next;    free ( temp );       return;
     }
     target = temp→next;
     for ( i =  1; i < k && target != NULL; i ++)
     {
          temp = target;      target = target→next;
     }
     if ( target == NULL && i > 0)             return;
     temp→next = target→next;       target→next = NULL;
     free (target);         return;              }
```

Linked Lists

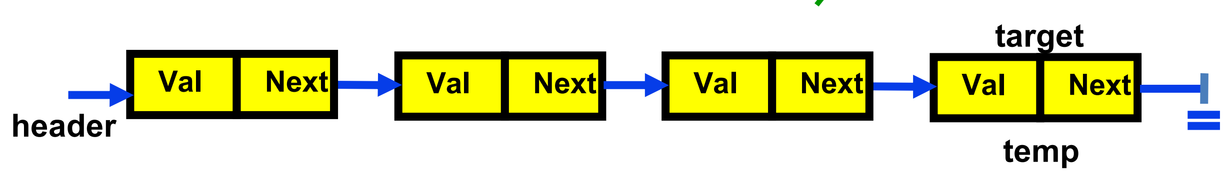
- Printing a linked list in reverse order

```
Void printreverse ( node *header ) {
    node * target, *temp = NULL;
    if ( header == NULL)     return;
    while ( temp != header)
    {
        target = header;
        while ( target→next != temp )
            target = target→next;
        printf ( “%d”, target→val );
        temp = target;
    }
}
```

Node n is printed after visiting n nodes
Node n-1 is printed after visiting n-1 nodes
So, complexity=n(n+1)/2 = O(n^2)



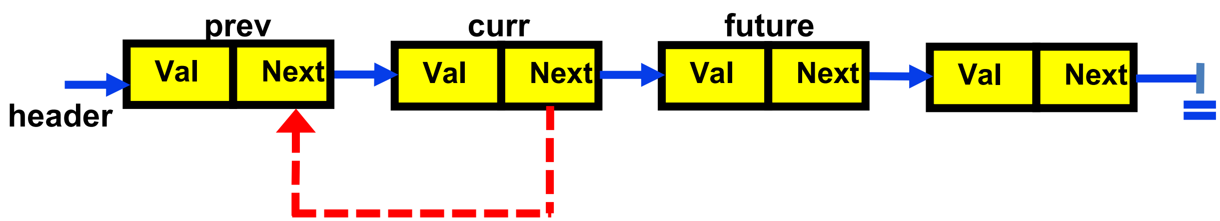
After one iteration of outer while loop

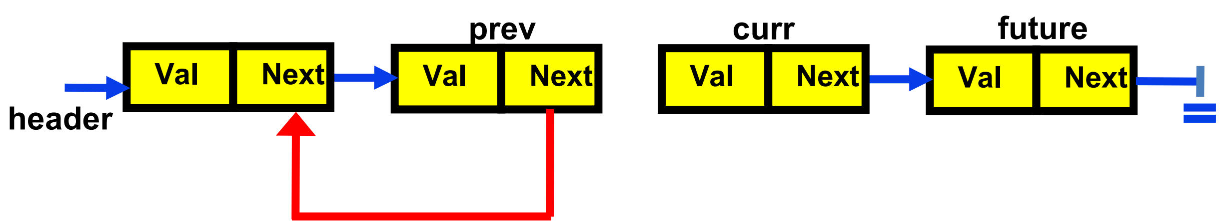


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked Lists

- Reverse a linked list



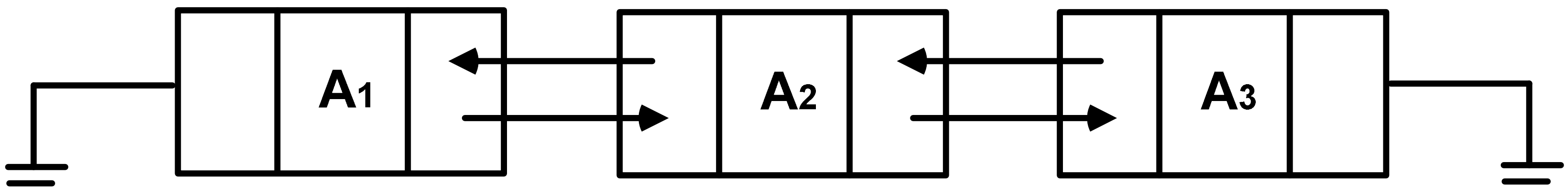


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked Lists

```
node * reverse ( node * header)     {
          node *prev = NULL, *future, *curr = header;
          future = curr→next;
          while ( curr→next != NULL)
          {
                    curr→next = prev;
                    prev = curr;
                    curr = future;
                    future = future→next;
          }
          curr→next = prev;
          return ( curr );                    }
```

Doubly Linked Lists



- Traversing list backwards
  - not easy with **regular lists**
- Insertion and deletion more pointer fixing
- Deletion is easier
  - **Previous node** is easy to find

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Doubly Linked Lists

- Define a node:

```
typedef struct node
{
        int  val;
        struct node *prev;
        struct node *next;
}  node;
```

Doubly Linked Lists

- Insert a node after target node:

```
Void insert ( node *header, node *new, node *target)
{
        new→next = target→next;
        new→prev = target;

                if ( target→next != NULL)
                        target→next→prev = new;
        target→next = new;
}
```

Set the pointers of node "new"

Inserting at the end

Doubly Linked Lists

- Delete a node pointed to by target:

```
void  delete( node *header, node *target)            {
        if ( target != header )
                target→prev→next = target→next;
        else
        {
                header = target→next;
                header→prev = NULL;
        }
        if ( target→next != NULL )
                target→next→prev = target→prev;
        free ( target );     }
```

Deleting the first node

Deleting the last node

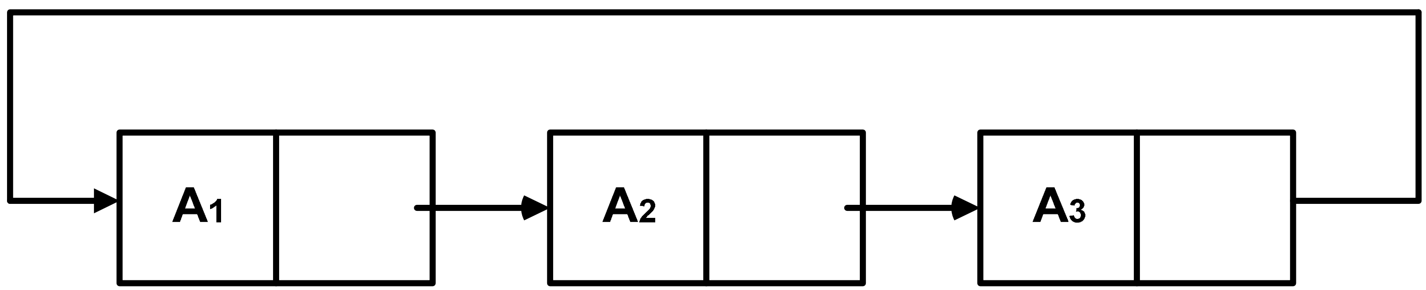
Doubly Linked Lists

- Reverse a doubly linked list:

```
void reverse ( node *header)        {
    node *last = header, *start = header;
    while ( last→next != NULL)      last = last→next;    Move to the last node
    while ( start != last )
        {
            swap ( start→val , last→val);
            start = start→next;
            if ( start == last)   break;     Check for even number of nodes
            last = last→prev;
        }
}
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Circular Linked Lists



- Last node points to the first node
- Traversing a circular linked list
  - Different than singly linked list
    - NULL pointer is missing
    - Save the starting pointer and traverse until the next field of a node becomes equal to the start node

Circular Linked Lists

- Identify a circular list by the pointer to the last node
  - Insertion at the start or end of a list takes O(1) time
  - Concatenating two lists also takes O(1) time

Josephus Problem

- *"n"* children arranged a in a circle
  - Children are numbered in clockwise fashion
- Choose a lucky number "m"
- Start counting from child 1 in clockwise fashion
  - The ***m*** m^{th} child is eliminated
- Start the next round from the *child next to the eliminated child*
  - Continue until you are left with one child who is the **winner**

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Josephus Problem

- Define a child:

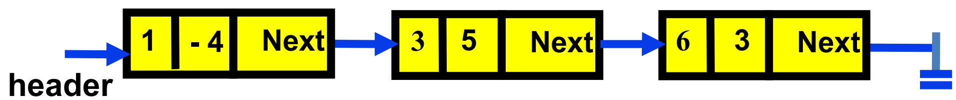
```
typedef struct child
{
        int position;
        struct child *nextchild;
} child;
```

Josephus Problem

```
int findwinner ( int n, int m)
{
    create a circular linked list with n children;
    while ( the list contain more than one child)
    {
        set a counter to zero;
        go to the next child and increment the counter as long as it is less
        than m;
        delete the current child;
    }
    get the position of the only child in the list;
    return the position;     }
```

Polynomials

- The polynomial $P(x) = 3x^6 + 5x^3 - 4x$ can be represented as:

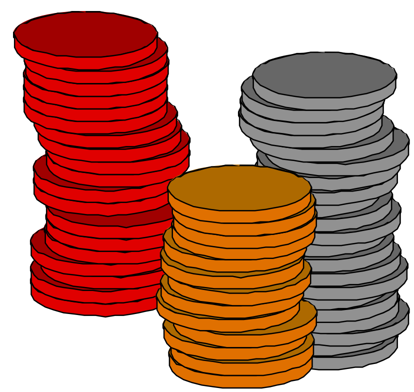



```
typedef struct poly
{
        int expo, coeff;
        struct poly * next;
}
```

Stacks

What is a Stack

- Special form of linear list
- Principle: Last In First Out
  – the last element inserted is the first one to be removed
- Like a plate stacker

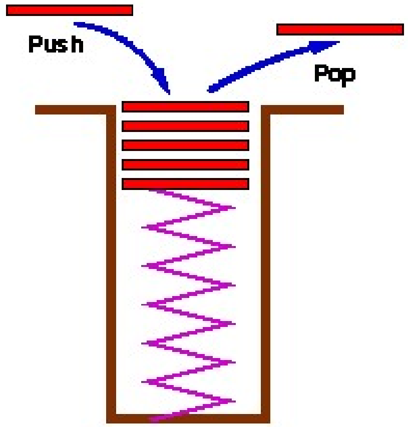


Stack Applications

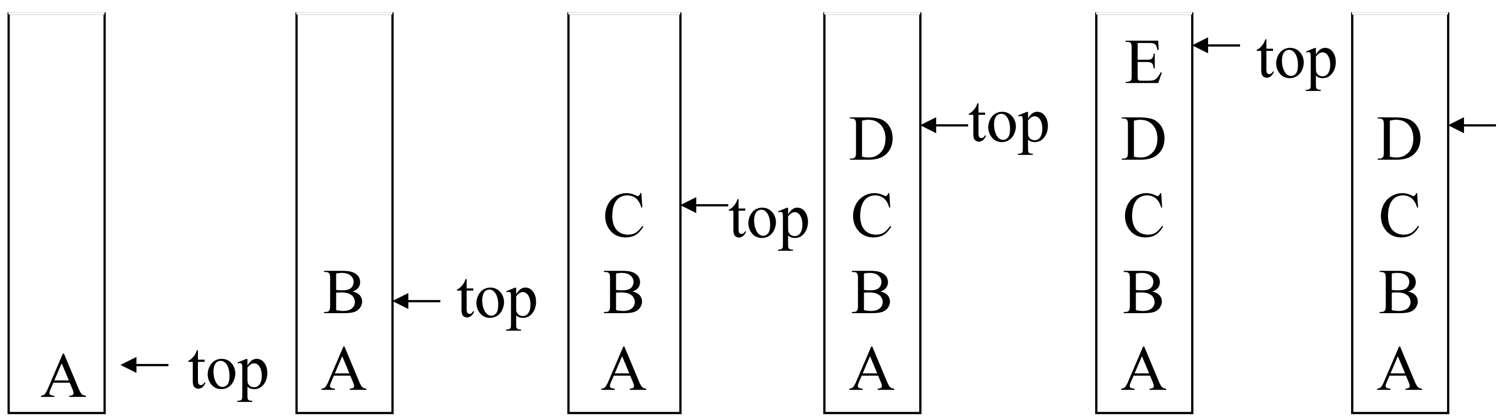
- Real life
  - Pile of books
  - Plate trays
- More applications related to computer science
  - Recursive function calls
  - Evaluating expressions

Stack Operations

- construct a stack (usually empty)
- check if it is empty
- Push: add an element to the top
- Top: retrieve the top element
- Pop: remove the top element



Last In First Out



Array Implementation of Stack

- Allocate an array of some size (pre-defined)
  - MAX_STACK_SIZE elements in stack
  - Bottom stack element stored at position 0
  - Last element in the stack is the *top*
- *Define the stack:*

```
typedef struct stack
{
        int a [MAX_STACK_SIZE];
        int top;
} stack;
```

Array Implementation of Stack

- Push Operation

```
void push ( stack *s, int item)
{
        if ( s→top == MAX_STACK_SIZE - 1 )   return;    Stack is full
        else
                {
                        s→top = s→top + 1;        Increment the top
                        s→a [s→top] = item;       Insert the element
                }
}
```

Array Implementation of Stack

- Pop Operation

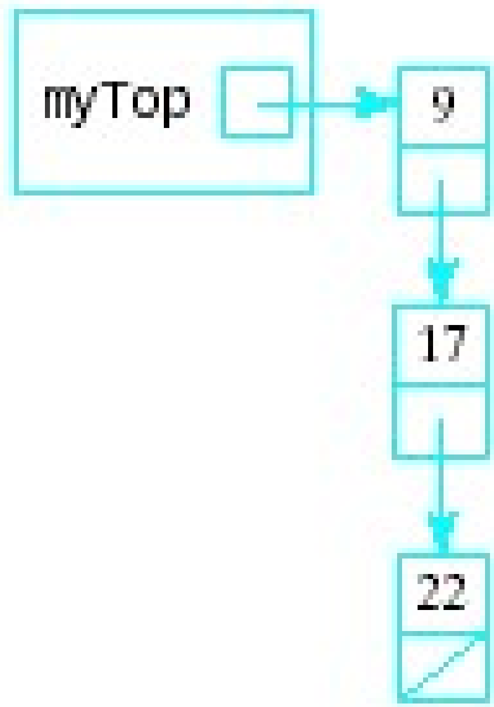
```
int pop ( stack *s, int *x )
{
        if ( s→top == -1)  return 0;        Stack is empty
        else
                {
                        *x = s→a[s→top];    Remove topmost element
                        s→top = s→top - 1;  Decrement the top
                        return 1;
                }
}
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked List Implementation

- Define the nodes:

```
typedef struct node
{
        int item;
        struct node *next;
} node;
```



- Define the top:

```
typedef struct stack
{
        node *mytop;
} stack;
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked List Implementation

- Push Operation

```
void push ( stack *s, int data )
{
        node *newnode = ( node *) malloc ( sizeof ( node ) );
        newnode→item = data;
        newnode→next = s→mytop;        New node inserted
        s→mytop = newnode;        New node becomes the top
}
```

Linked List Implementation

- Pop Operation

```
int pop ( stack *s, int *x )
{
        node *temp;
        if ( s→mytop == NULL )
        {       x = NULL;       return 0;       }

        *x = s→mytop→item;
        temp = s→mytop;
        s→mytop = s→mytop→next;
        free ( temp );    return 1;
}
```

Empty stack

Remove item from top

Advance top to the next node

Application of Stacks

Function Calls

Consider events when a function begins execution

- *Stack frame* is created
- Copy of stack frame pushed onto run-time stack
- Arguments copied into parameter spaces
- Control transferred to starting address of body of function

Bhaskar Sardar, Information Technology Department, Jadavpur University, India

Function Calls

When function terminates

- Run-time stack popped
  - Removes stack frame of terminated function
  - exposes stack frame of previously executing function

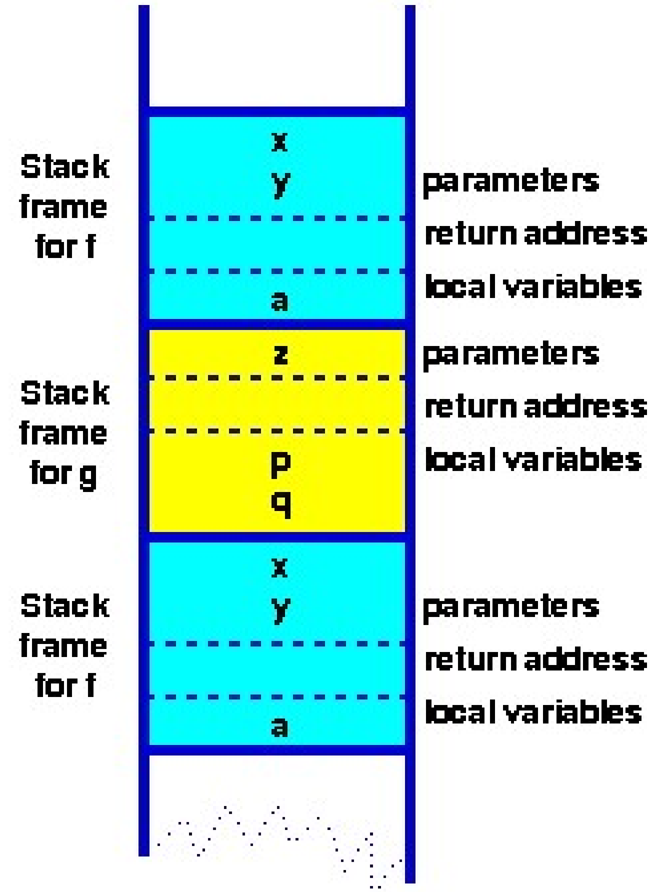
- Stack frame used to restore environment of interrupted function
- Interrupted function resumes execution

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Function Calls

```
function f( int x, int y) {
   int a;
   if ( term_cond ) return …;
   a = ….;
   return g( a );
   }

function g( int z ) {
   int p, q;
   p = …. ; q = …. ;
   return f(p,q);
   }
```



Context for execution of f

Evaluation of Arithmetic Expression

INFIX	**POSTFIX**	**PREFIX**
A + B	A B +	+ A B
A * B + C	A B * C +	+ * A B C
A * (B + C)	A B C + *	* A + B C
A - (B - (C - D))	A B C D---	-A-B-C D
A - B - C - D	A B-C-D-	---A B C D

- Most compilers convert an expression in ***infix*** notation to ***postfix*** notation
- Advantage:
  - ✓ expressions can be written without parentheses

Evaluation of Arithmetic Expression

- Evaluating Postfix Expression
  - *"By hand" (Underlining technique)*:
    - Scan the expression from left to right to find an operator.
    - Locate ("underline") the last two preceding operands and combine them using this operator.
    - Repeat until the end of the expression is reached.

$\rightarrow$2 3 4 + 5 6 - - *

$\rightarrow$2 $\underline{3\ 4\ +}$ 5 6 - - *

$\rightarrow$2 7 5 6 - - *

$\rightarrow$2 7 $\underline{5\ 6\ -}$ - *

$\rightarrow$2 7 -1 - *

$\rightarrow$2 $\underline{7\ -1\ -}$ *

$\rightarrow$2 8 *

$\rightarrow$$\underline{2\ 8\ *}$

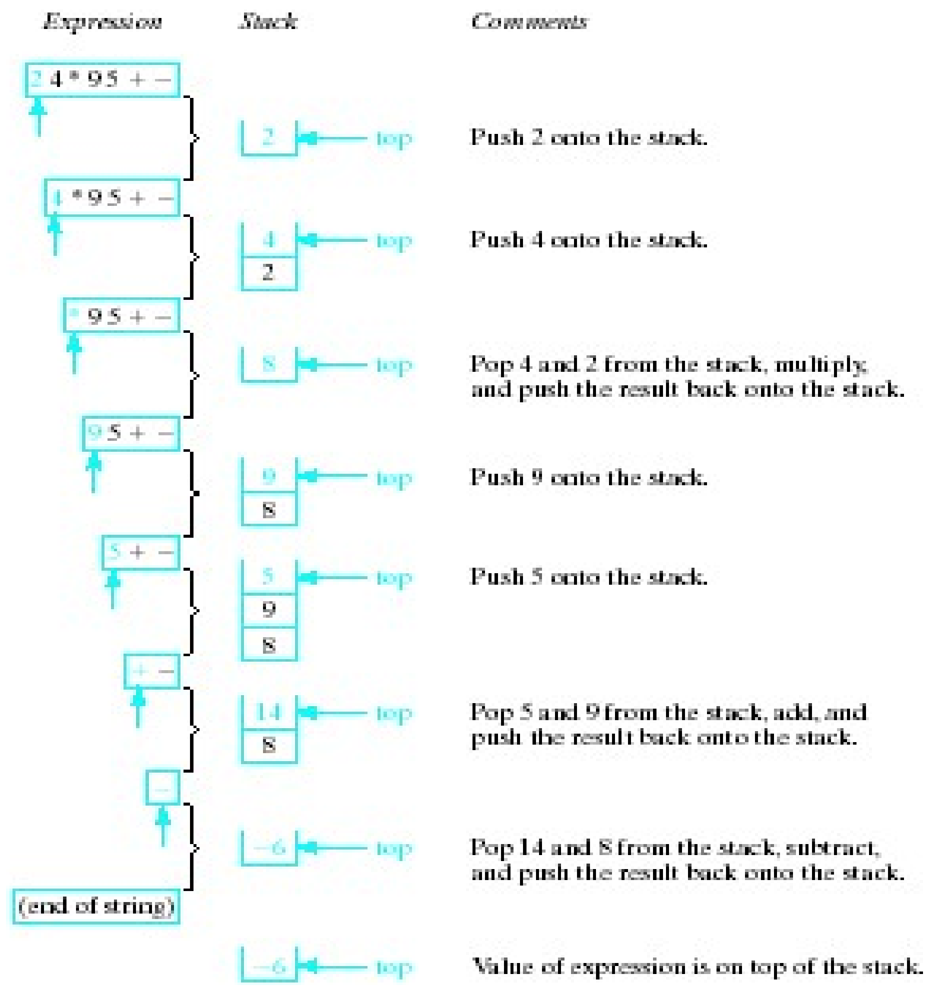
$\rightarrow$16

Evaluation of Arithmetic Expression

- Evaluating Postfix Expression
  1. Initialize an empty stack
  2. Repeat the following until the end of the expression is encountered
     1. Get the next element from the expression
     2. Operand – push onto stack
        Operator – do the following
        1. **Pop 2 values from stack**
        2. **Apply operator to the two values**
        3. **Push resulting value back onto stack**
  3. When end of expression encountered, value of expression is the (only) number left in stack

Evaluation of Arithmetic Expression

- Evaluating Postfix Expression



Evaluation of Arithmetic Expression

- Infix to Postfix Conversion
  - *By hand:* "Fully parenthesize-move-erase" method:
    - Fully parenthesize the expression.
    - Replace each right parenthesis by the corresponding operator.
    - Erase all left parentheses.

```
A * B + C  →  ((A * B) + C)        A * (B + C)  →  (A * (B + C) )
           →  ((A B * C +                       →  (A (B C + *
           →  A B * C +                         →  A B C + *
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Evaluation of Arithmetic Expression

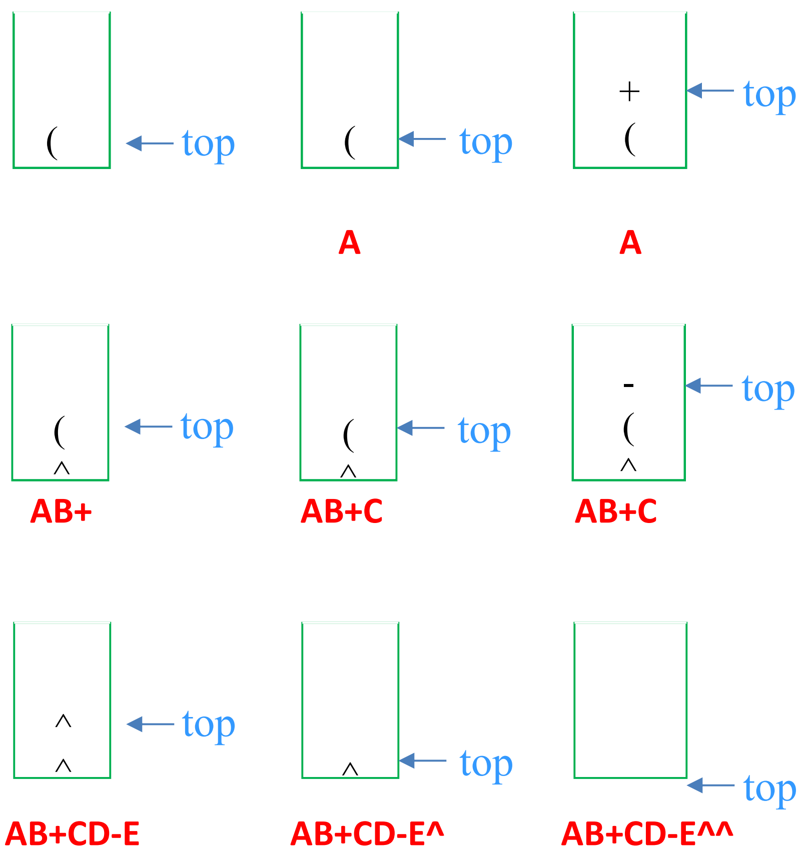
1. Initialize an empty stack of operators
2. While !end of expression
   a) Get next input "token" from infix expression
   b) If token is …
      i. operand display it
      ii. operator
         if operator has higher priority than top of stack
            push token onto stack
         else
            pop and display top of stack
            repeat comparison of token with top of stack
      iii. "(" : push onto stack
      iv. ")" : pop and display stack elements until
            "(" occurs, do not display it
3. When end of infix reached, pop and display stack items until empty

Operator	In-Stack Priority	Input Priority
+, -	1	1
*, /	2	2
^	3	4
(	0	4

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Evaluation of Arithmetic Expression

- (A+B)^(C-D)^E



Queues

What is a queue

- Stores a set of elements in a particular order
- Stack principle: FIRST IN FIRST OUT
- = FIFO
- It means: the first element inserted is the first one to be removed
- Example

- The first one in line is the first one to be served

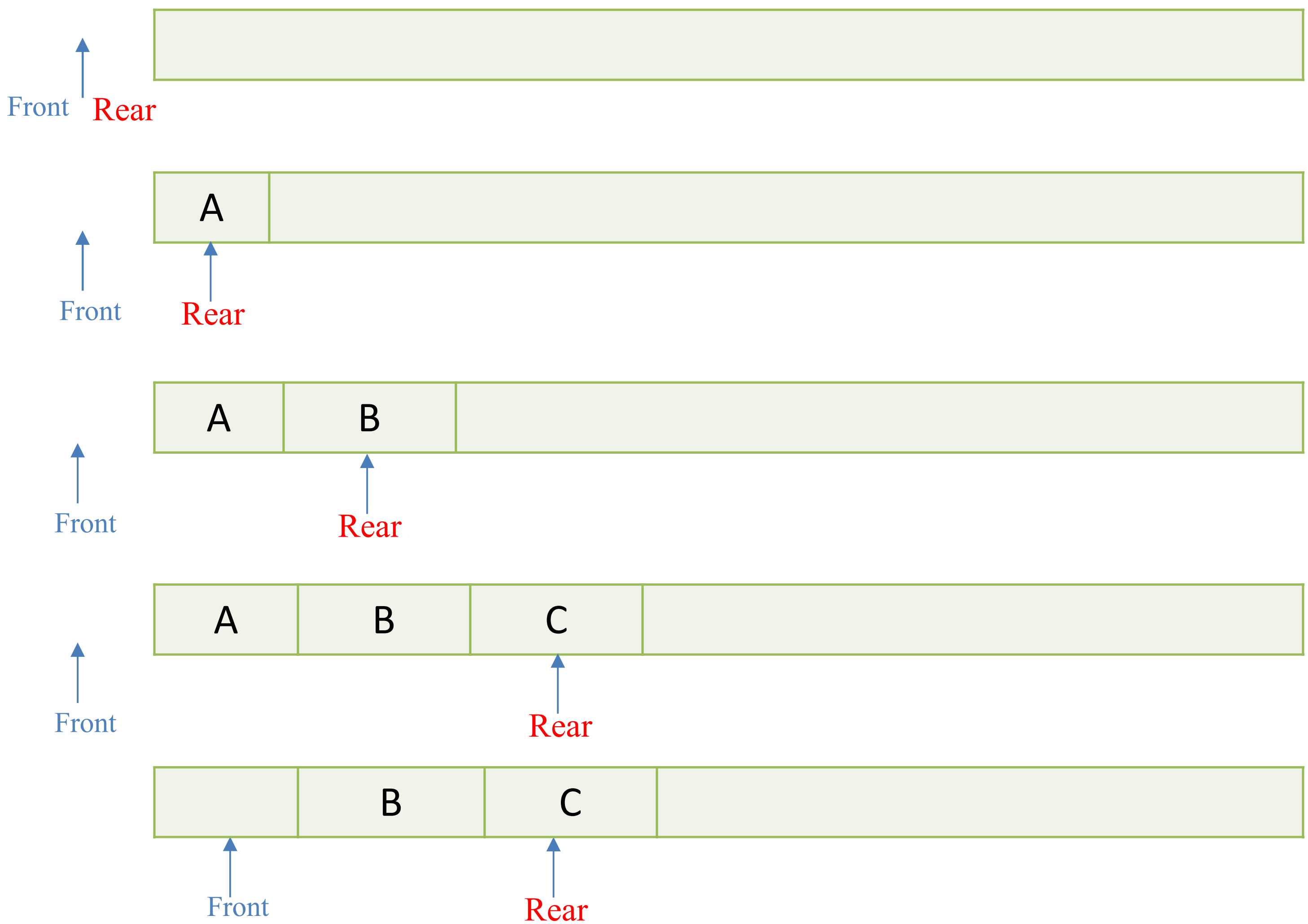
Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Queue Applications

- Real life examples
  - Waiting in line
  - Waiting on hold for tech support

- Applications related to Computer Science
  - Threads
  - Job scheduling (e.g. Round-Robin algorithm for CPU allocation)

First In First Out



Job Scheduling

front	rear	Q[0]	Q[1]	Q[2]	Q[3]	Comments
-1	-1					queue is empty
-1	0	J1				Job 1 is added
-1	1	J1	J2			Job 2 is added
-1	2	J1	J2	J3		Job 3 is added
0	2		J2	J3		Job 1 is deleted
1	2			J3		Job 2 is deleted

Queue Operations

- Create a queue
- Check if a queue is full or not
- Check if a queue is empty or not
- Add an element to a queue (enqueue)
- Remove an element from queue (dequeue)

Array Implementation of Queue

- As with the array-based stack implementation, the array is of fixed size
  - A queue of maximum N elements
- Slightly more complicated
  - Need to maintain track of both front and rear
- Define the queue:

```
typedef struct queue
{
        int a[MAX_SIZE];
        int rear, front;
} queue;
```

Array Implementation of Queue

- Enqueue Operation

```
int enqueue (queue *q, int data)
{
        if (q→rear == MAX_SIZE – 1)
                return 0;
        else
        {
                q→rear == q→rear + 1;
                q→a[q→rear] = data;
                return 1;
        }
}
```

Queue is full

Increment the rear

Insert the element

Array Implementation of Queue

- Dequeue Operation

```
int dequeue (queue *q, int *data)
{
        if (q→rear == q→front)
        {
                q→front = -1;
                q→rear = -1;
                *data  = NULL;
                return 0;
        }
        else
        {
                q→front = q→front + 1;
                *data = q→a[q→front];
                return 1;
        }
}
```

Queue is empty

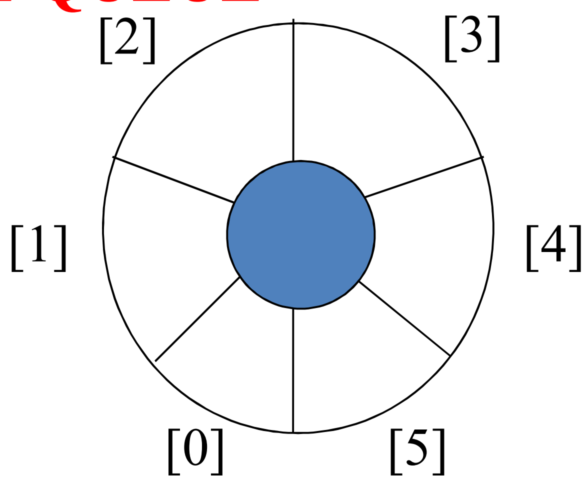
Increment the front

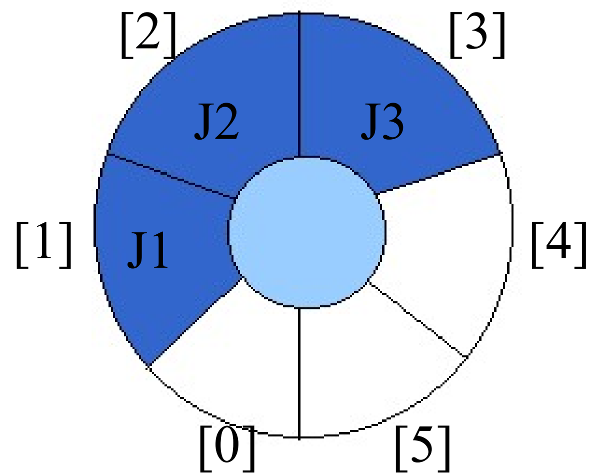
Remove the element

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Circular Queue

EMPTY QUEUE





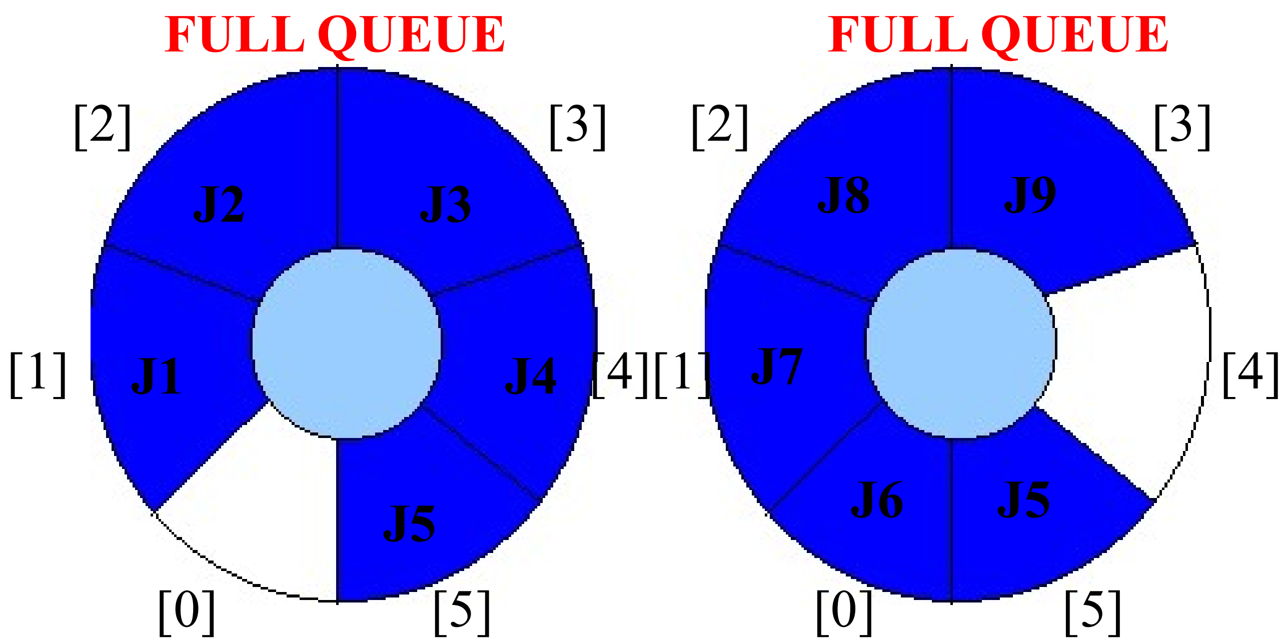
front = 0
rear = 0

front = 0
rear = 3

Can be seen as a circular queue

Circular Queue

Leave one empty space when queue is full. Why?



front =0
rear = 5

front =4
rear =3

How to test when queue is empty?
How to test when queue is full?

Circular Queue

- Enqueue Operation

```
void enqueue (queue *q, int data)
{
        q→rear = (q→rear +1) % MAX_SIZE;
        if (q→front == q→rear)
                return;
        q→a[q→rear] = item;
}
```

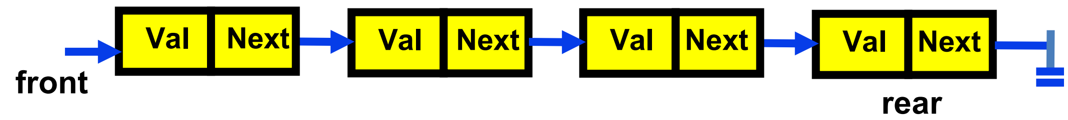
Circular Queue

- Dequeue Operation

```
void dequeue (queue *q, int *data)
{
        if (q→front  ==  q→rear)
                return;
        q→front  =  (q→front+1) % MAX_SIZE;
        *data  =  q→a[q→front];
}
```

Linked List Implementation

- Define the nodes




```
typedef struct node
{
        int item;
        struct node *next;
} node;
```

- Define the rear and front

```
typedef struct queue
{
        node *rear;
        node *front;
}
```

Linked List Implementation

- Enqueue Operation

```
void enqueue (queue *q, int data)
{
        node *temp = (node *) malloc (size of (queue));
        temp→item = data;
        temp→next = NULL;
        if (q→front == NULL)
        {
                q→front = temp;
                q→rear = temp;
                return;
        }
        q→rear→next = temp;
        q→rear = temp;
}
```

New node created

Queue is empty

Insert at the rear

New node becomes rear

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Linked List Implementation

- Dequeue Operation

```
int dequeue (queue *q, int *data)
{
        node *temp;
        if (q→front == NULL)
        {
                data = NULL;
                return 0;
        }
        temp = q→front;
        *data = temp→item;
        q→front = q→front→next;
        if (q→rear == temp)
                q→rear = NULL;
        free (temp);
        return 1;
}
```

Queue is empty

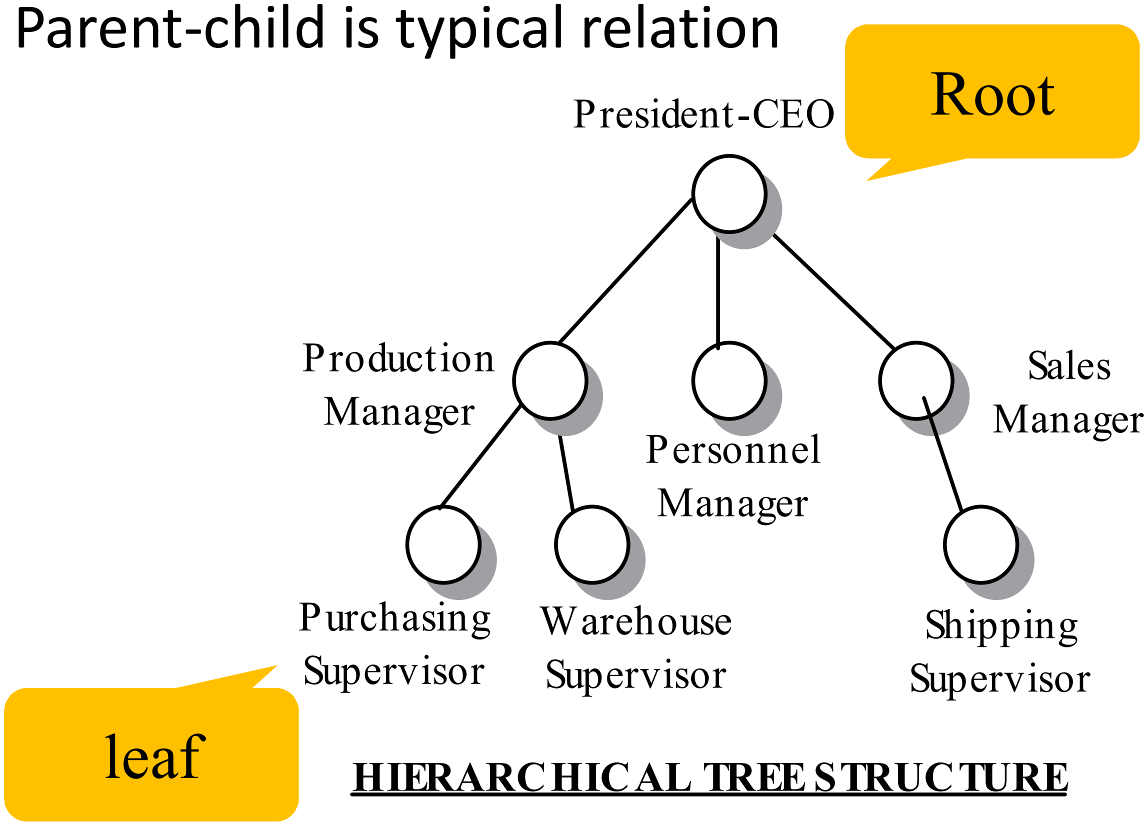
Only node in the Queue

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Trees

Introduction

- Non-linear data structure
- Depicts **hierarchical** relationship between the nodes
  - Parent-child is typical relation

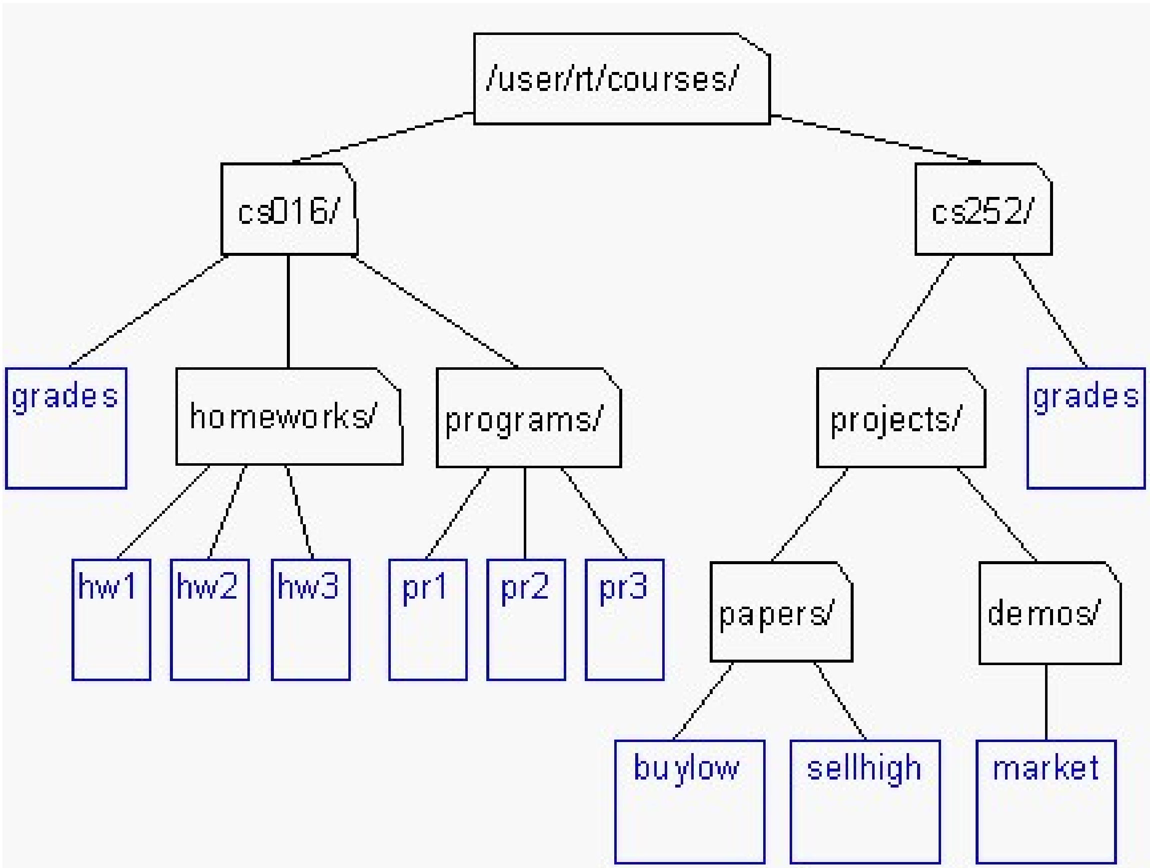


HIERARCHICAL TREE STRUCTURE

Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Another Example

- Unix / Windows file structure



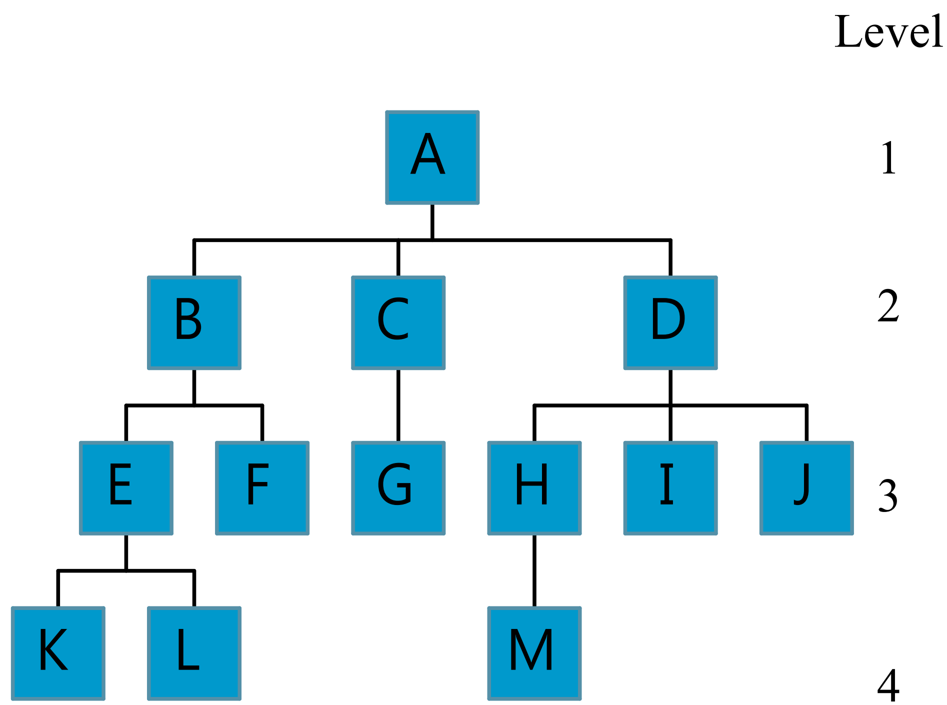
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Definition

- A tree T is a finite set of one or more nodes such that:
  - There is a specially designated node called the **root**.
  - The remaining nodes are partitioned into $n>=0$ disjoint sets $T_1, ..., T_n$, where each of these sets is a tree.
  - We call $T_1, ..., T_n$ the **subtrees** of the root.

Terminology

- **Degree** of a node: number of nodes connected of that node
  - The node with degree **1** is a **leaf or terminal node**.
- Children of the same parent are ***siblings***.
- **Ancestors** of a node: all the nodes along the path from the root to the node.
- **Level**: level of a node is one more than its parent. **Root node** has a level **1**.
- **Height** of a tree: **maximum level** of any node

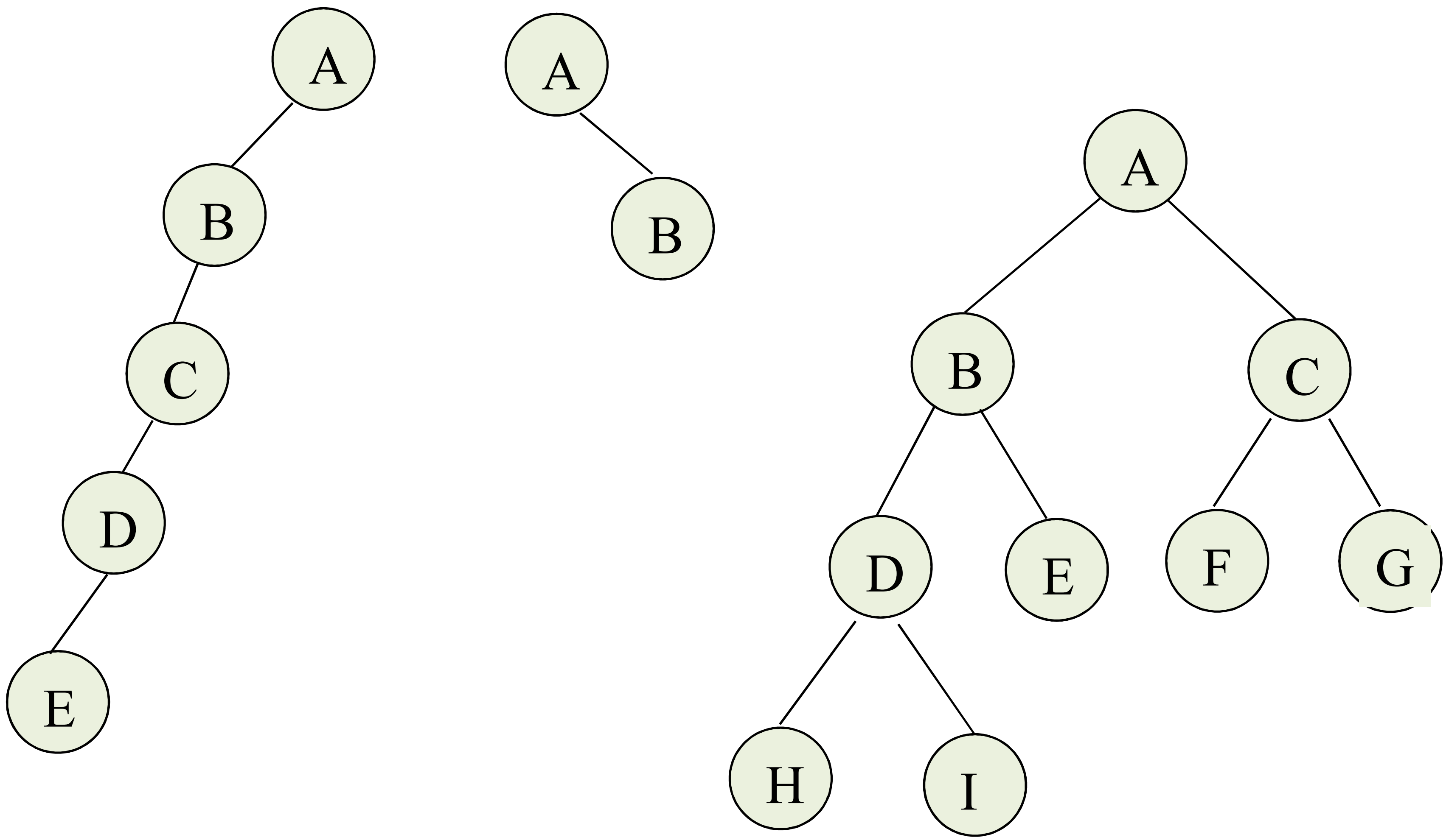


Binary Tree

- A special class of trees: max number of child for each node is **2**.
- Recursive definition: A binary tree is a finite set of nodes that is either empty or consists of a root and two **disjoint binary trees** called *the left subtree* and *the right subtree*.
- Any tree can be transformed into binary tree.
  - by left child-right sibling representation
- Total number of binary trees possible with n nodes is $^{2n}C_n - {}^{2n}C_{n-1}$

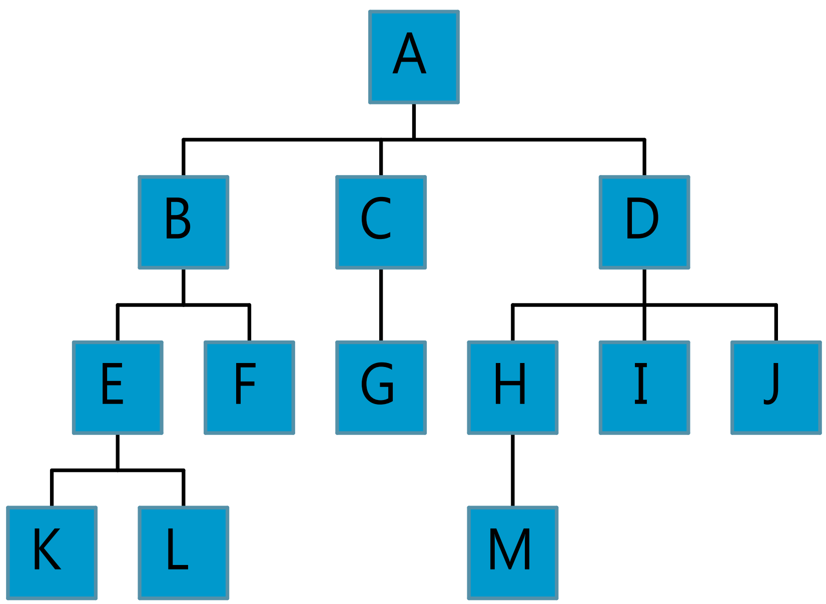
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Examples

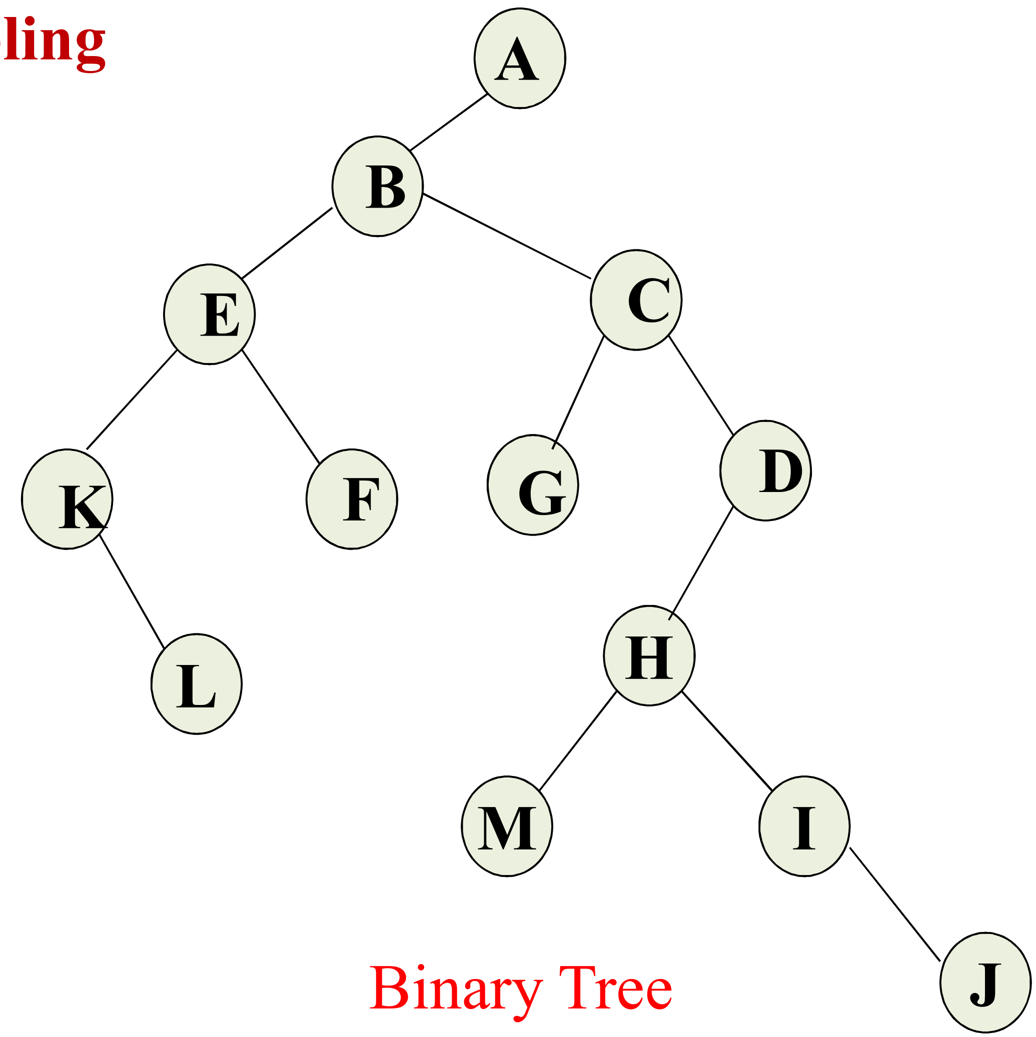


Examples

Left Child - Right Sibling



General Tree



Binary Tree

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Properties

- The maximum number of nodes on level **i** of a binary tree is 2^{i-1}, i>=1.
- The maximum number of nodes in a binary tree of depth **h** is 2^h-1, h>=1.

Prove by induction

$$\sum_{i=1}^{h} 2^{i-1} = 2^h - 1$$

In other words
h=log(n+1)

Properties

For any nonempty binary tree, T, if n_0 is the number of leaf nodes and n_2 the number of nodes with 2 children, then $n_0=n_2+1$

proof:

Let n and B are the total number of nodes & branches in T.

Let n_0, n_1, n_2 represent the nodes with no children, single child, and two children respectively.

$$n= n_0+n_1+n_2,\ B+1=n,\ B=n_1+2n_2 ==>$$

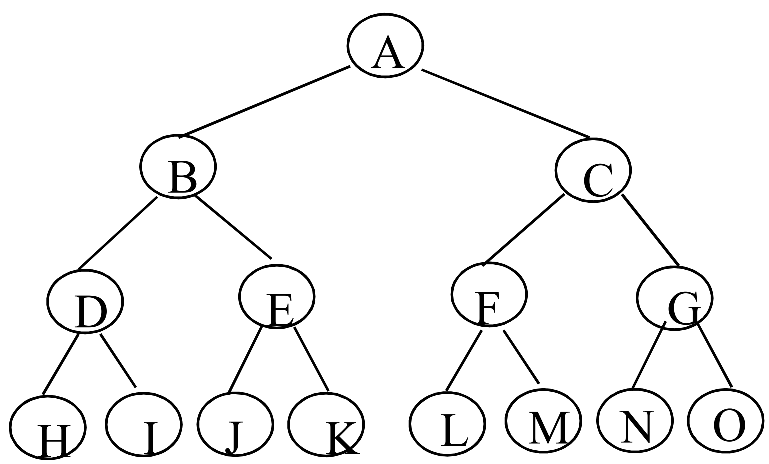
$$n_1+2n_2+1= n,$$

$$n_1+2n_2+1= n_0+n_1+n_2 ==> n_0=n_2+1$$

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

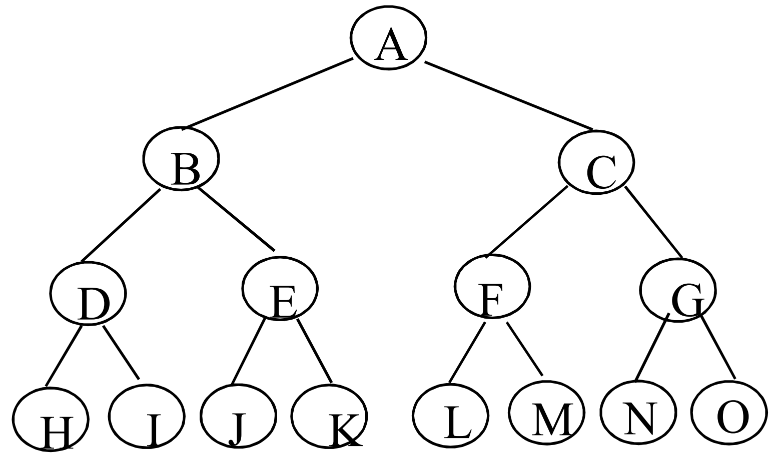
Full Binary Tree

- If all non-leaf nodes of a binary tress have **exactly two non-empty children** and **all leaf nodes are at the same level**.
- A full binary tree of depth *h* is a binary tree of depth *h* having $\mathbf{2^h - 1}$ nodes, $h>=1$.

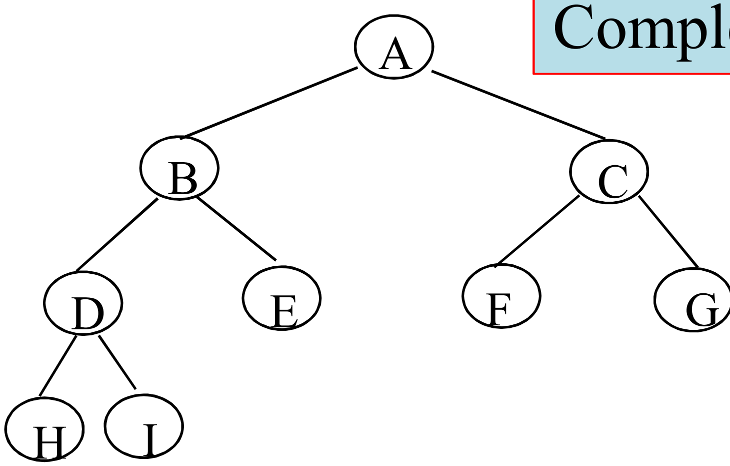


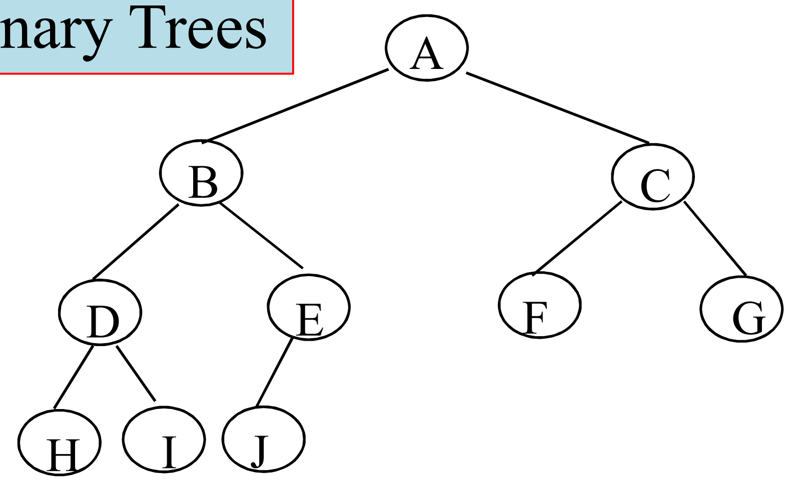
Complete Binary Tree

- A full binary tree or Full up to level h-1 and if any node at **level h-1 has one child**, that must be a **left child**.



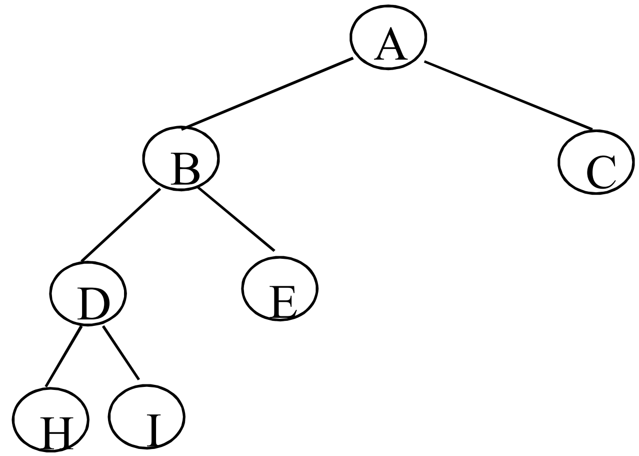
Complete Binary Trees

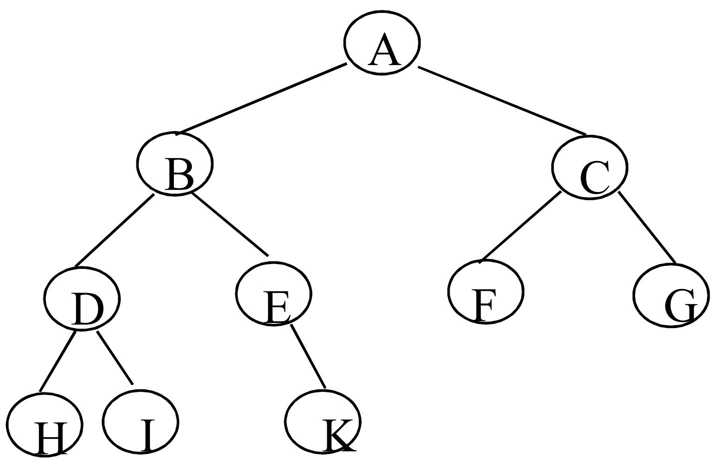




Complete Binary Tree

- A full binary tree or Full up to level h-1 and if any node at **level h-1 has one child**, that must be a **left child**.

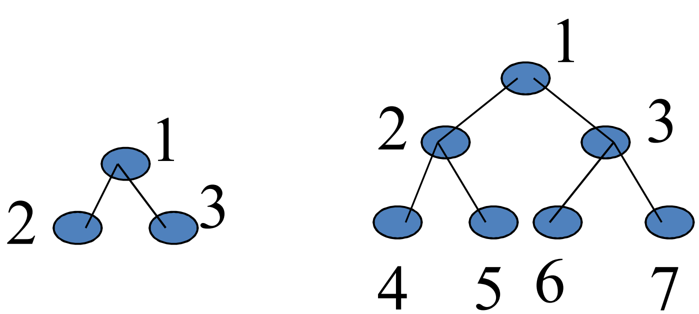




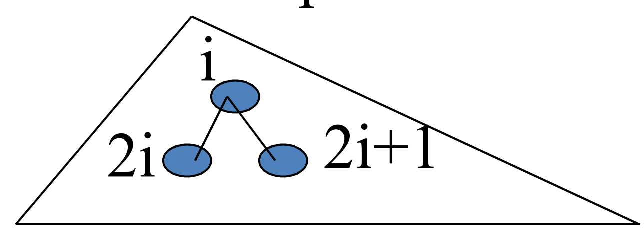
Non-Complete Binary Trees

Binary Tree Representation

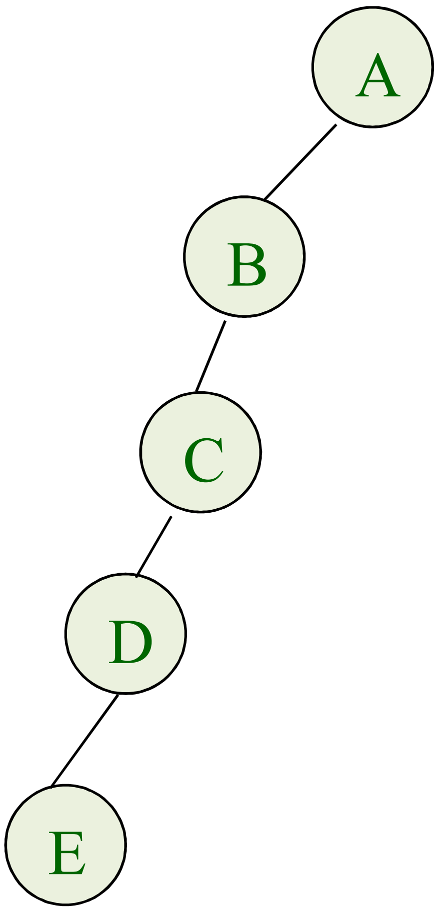
- If a complete binary tree with n nodes is represented sequentially, then for any node with index i, $1<=i<=n$, we have:
    - *parent*(i) is at $i/2$ if $i!=1$. If $i=1$, i is at the root and has no parent.
    - *leftChild*(i) is at $2i$ if $2i<=n$. If $2i>n$, then i has no left child.
    - *rightChild*(i) is at $2i+1$ if $2i+1<=n$. If $2i+1>n$, then i has no right child.



Relationships between labels of children and parent:

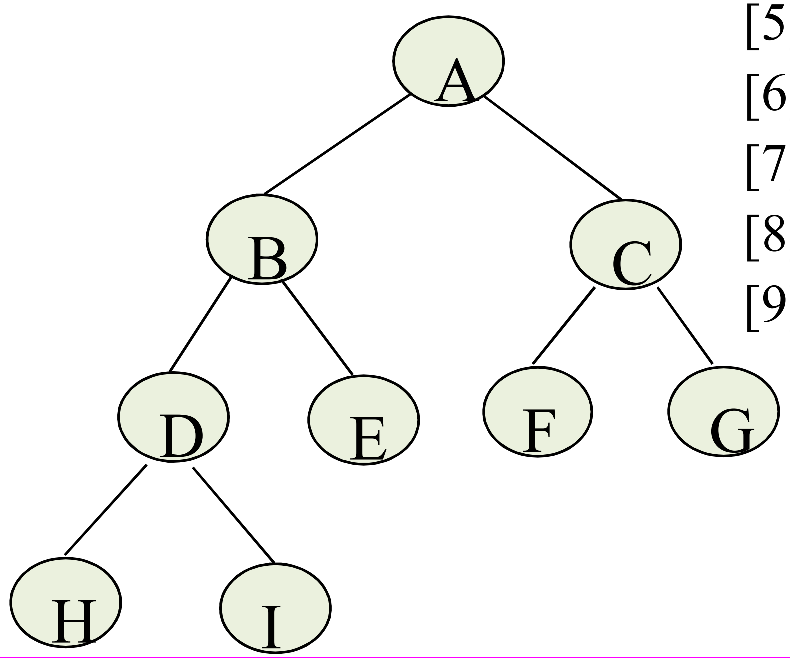


Binary Tree Representation



[1]	A
[2]	B
[3]	--
[4]	C
[5]	--
[6]	--
[7]	--
[8]	D
[9]	--
.	.
[16]	E

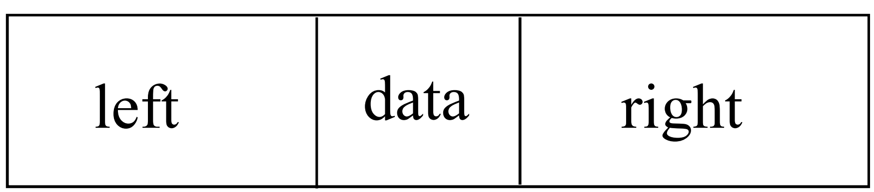
Wastage of space

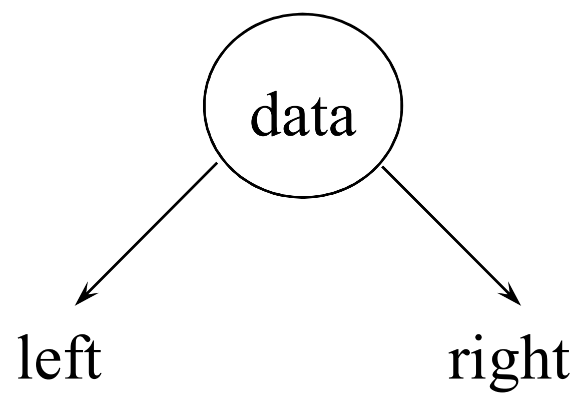


[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

Linked Representation

```
typedef struct btnode
    {
        int data;
        btnode *lchild, *rchild;
    }btnode;
```





Creation of Binary Tree

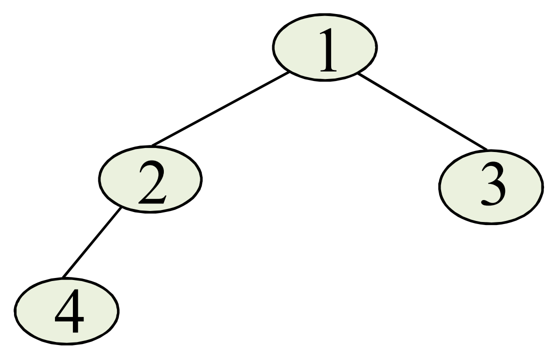
```
main ()
{
    btnode *Root=NULL;
    btnode *p, *q;

    p=(btnode*)malloc(sizeof(btnode));
    p→data=1;
    p→lchild=p→rchild=NULL;
    Root=p;          q=p;

    p=(btnode*)malloc(sizeof(btnode));
    p→data=2;
    p→lchild=p→rchild=NULL;
    q→lchild=p;          q=p;
```

```
p=(btnode*)malloc(sizeof(btnode));
p→data=3;
p→lchild=p→rchild=NULL;
q→lchild=p;
q=p;

q=Root;
p=(btnode*)malloc(sizeof(btnode));
p→data=4;
p→lchild=p→rchild=NULL;
q→rchild=p;
}
```



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Binary Tree Traversal

- Traversal is the process of visiting every node once
- Let l, R, and r denotes moving left, visiting the node, and moving right.
- Six possible combinations of traversal
  - lRr, lrR, Rlr, Rrl, rRl, rlR
- Adopt convention that we traverse left before right, only 3 traversals remain
  - lRr, lrR, Rlr
  - inorder, postorder, preorder

Binary Tree Traversal

Inorder Traversal

1. Traverse left subtree
2. Visit the root
3. Traverse right subtree

Postorder Traversal

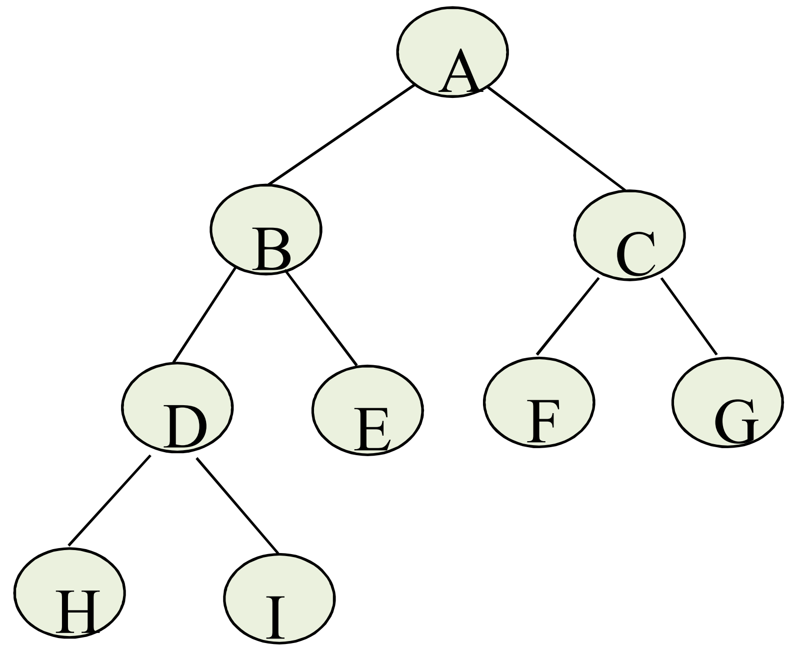
1. Traverse left subtree
2. Traverse right subtree
3. Visit the root

Preorder Traversal

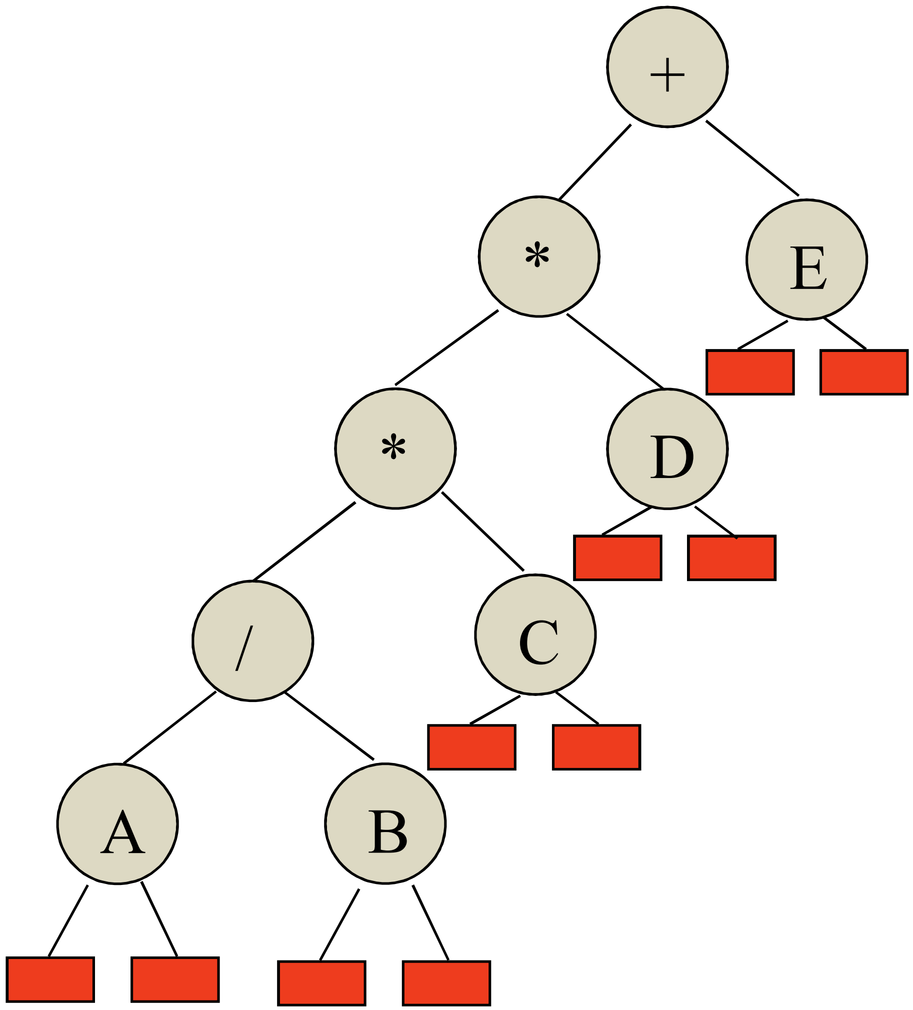
1. Visit the root
2. Traverse left subtree
3. Traverse right subtree

Binary Tree Traversal

- Inorder:
  H D I B E A F C G
- Postorder:
  H I D E B F G C A
- Preorder:
  A B D H I E C F G



Binary Tree Traversal



Inorder
A / B * C * D + E
infix expression

postorder
A B / C * D * E +
postfix expression

preorder
+ * * / A B C D E
prefix expression

Binary Tree Traversal

```
void inorder(btnode *Root)
{
    if (Root) {
        inorder(Root->lchild);
        printf("%d",Root->data);
        inorder(Root->rchild);
    }
}
```


Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Binary Tree Traversal

```
void postorder(btnode *Root)
{
    if (Root) {
        postorder(Root->lchild);
        postorder(Root->rchild);
        printf("%d",Root->data);
    }
}
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Binary Tree Traversal

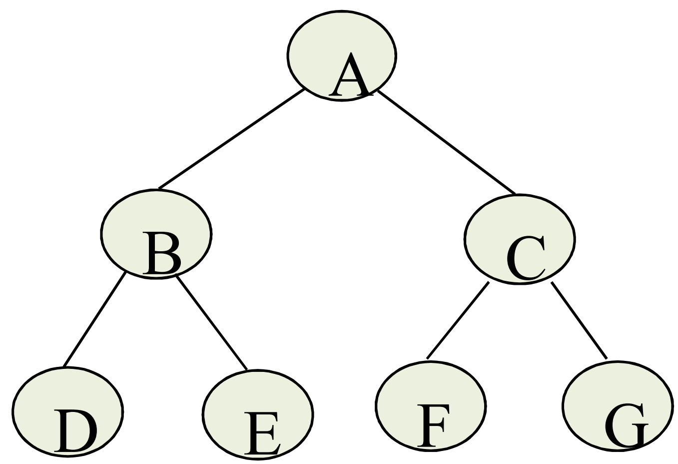
```
void preorder(btnode *Root)
{
    if (Root) {
        printf("%d",Root->data);
        preorder(Root->lchild);
        preorder(Root->rchild);
    }
}
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Reconstruction of Binary Tree

- It is impossible to reconstruct binary tree from inorder or preorder or postorder traversals alone.
- However, if inorder and preorder traversals are given, a unique binary tree can be reconstructed

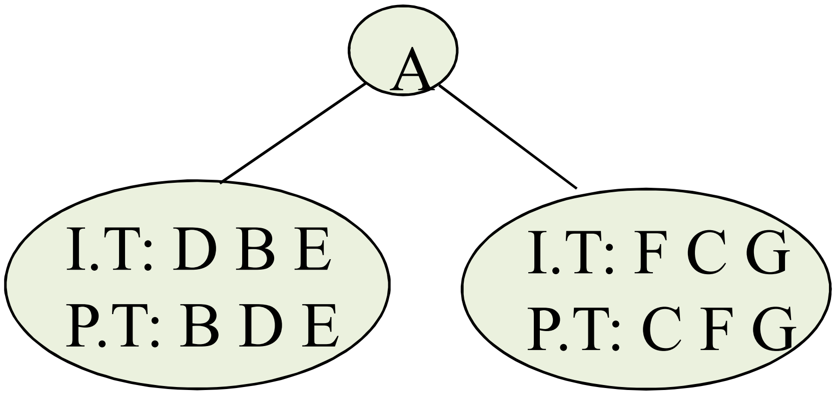
Reconstruction of Binary Tree

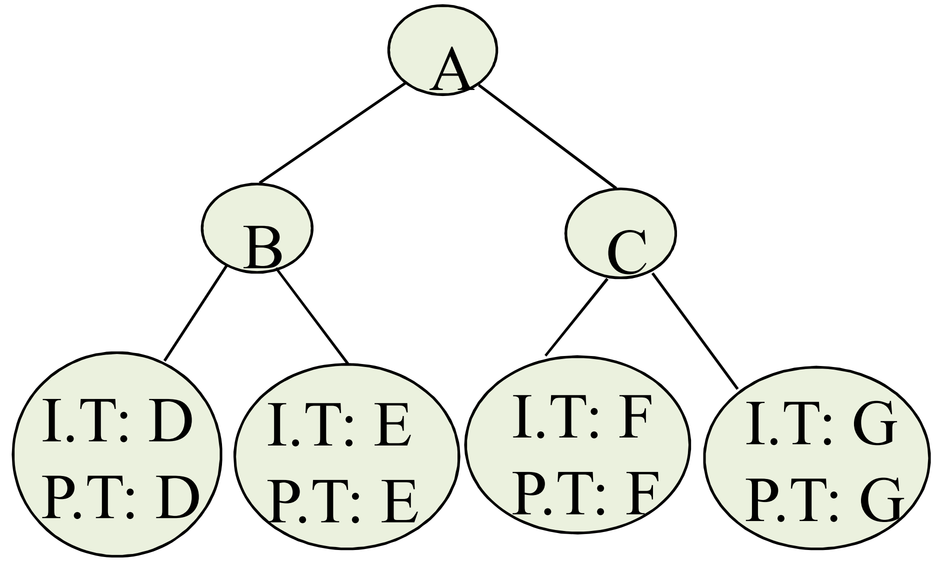


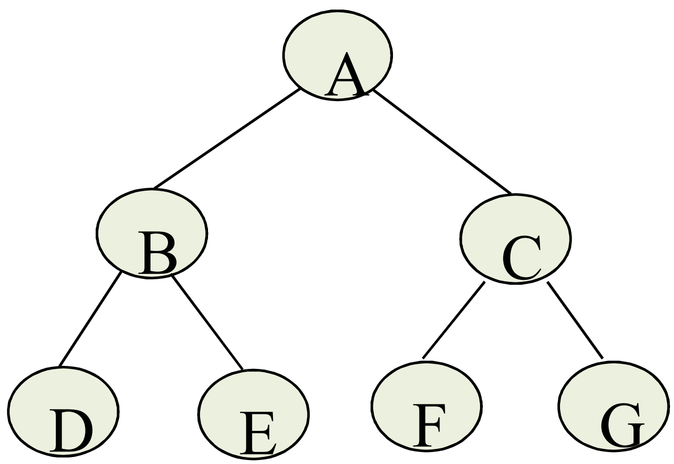
Inorder: D B E A F C G

Preorder: A B D E C F G

Reconstruction of Binary Tree



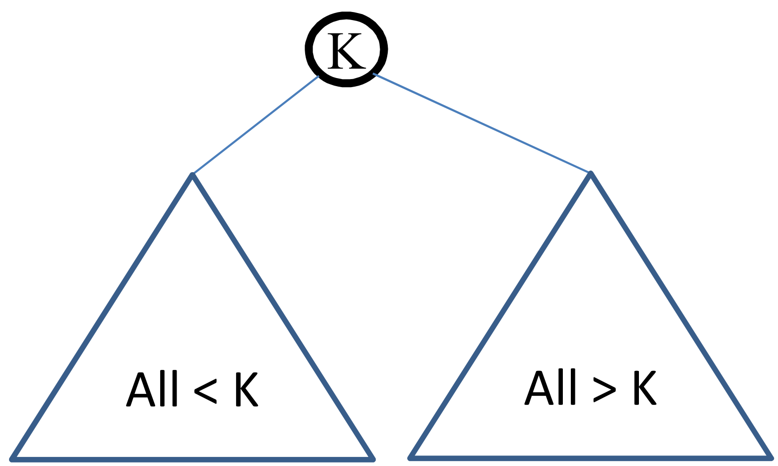




Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Binary Search Trees

- Definition
  - The keys in a nonempty left subtree (right subtree) are smaller (larger) than the key in the root of subtree.
  - The left and right subtrees are also binary search trees.

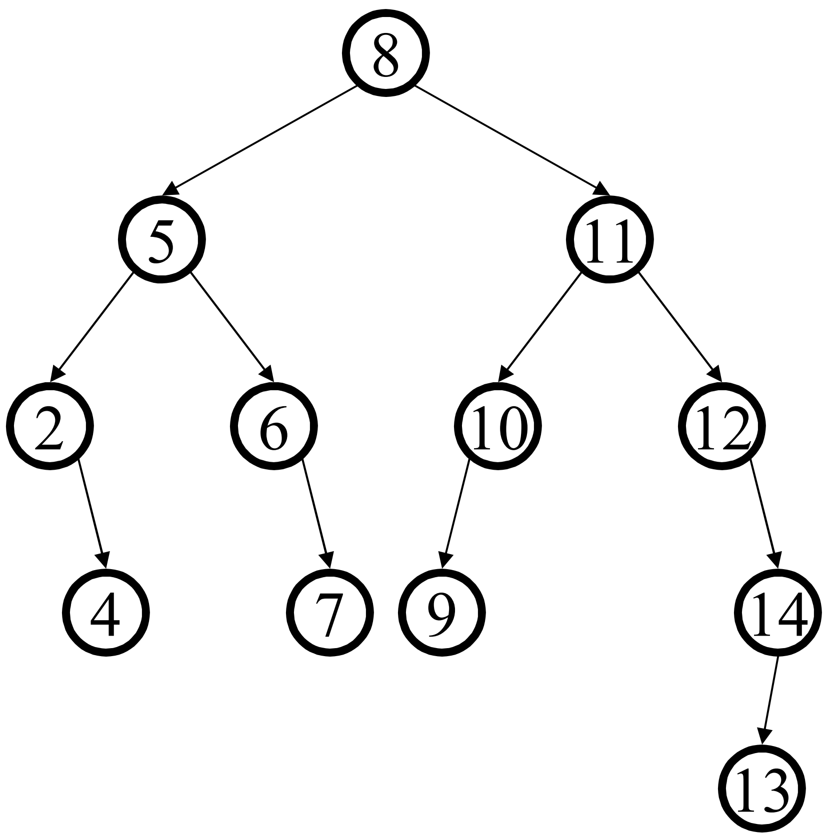


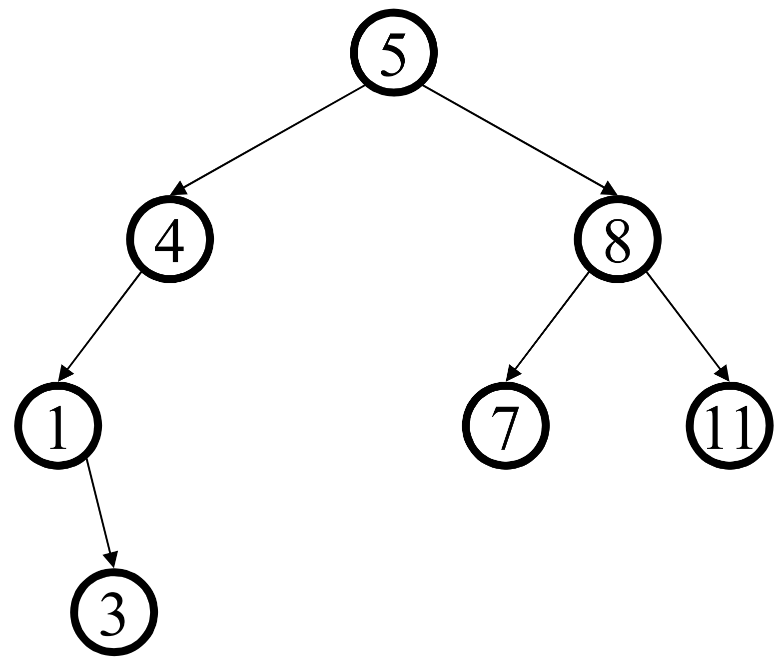
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Binary Search Trees

- Binary Search Trees (BST) are a type of Binary Trees with a special organization of data.
- Leads to O(log n) complexity for searches, insertions and deletions in certain types of BST (balanced trees).
  - O(h) in general

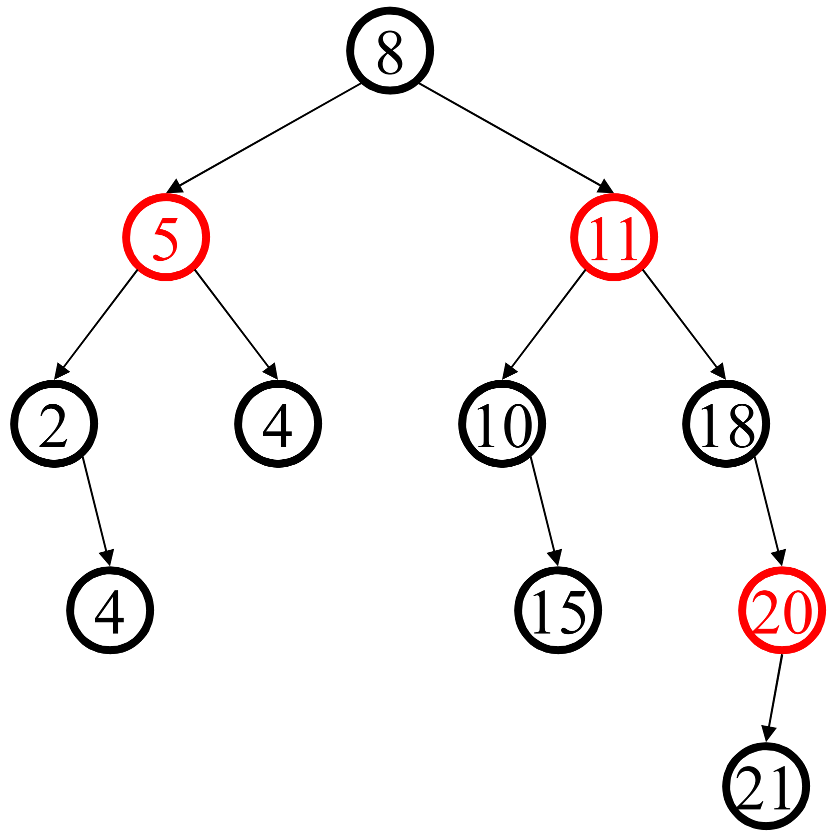
Examples and Counter Examples



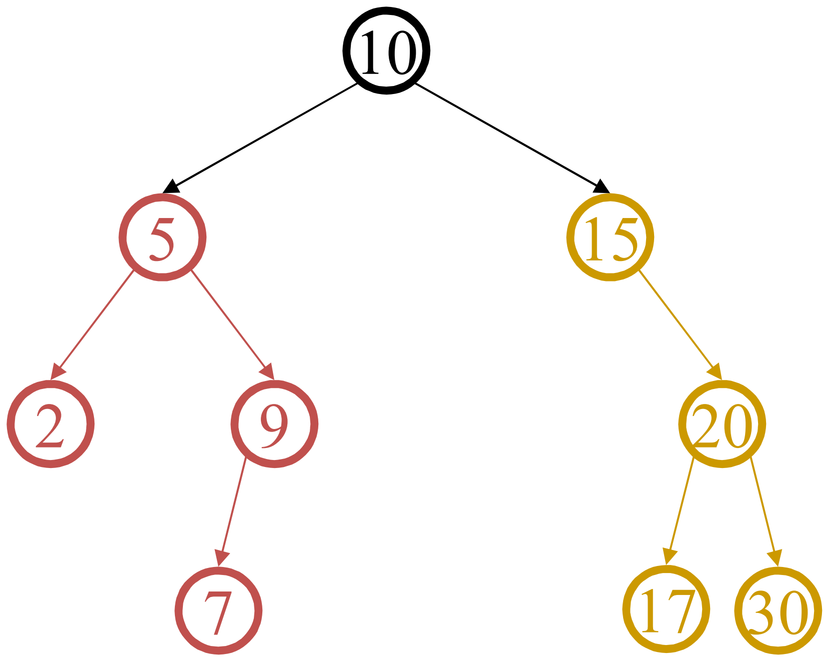


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Examples and Counter Examples



Inorder Traversal

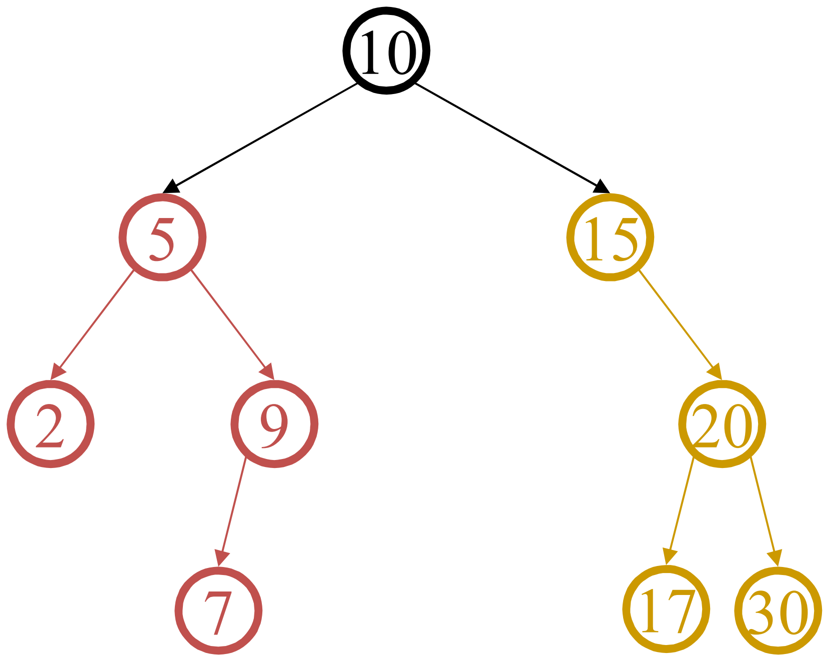


2→5→7→9→10→15→17→20→30

What does this guarantee with a BST?

Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

rRl Traversal



30 →20 →17 →15 →10→9→7→5→2

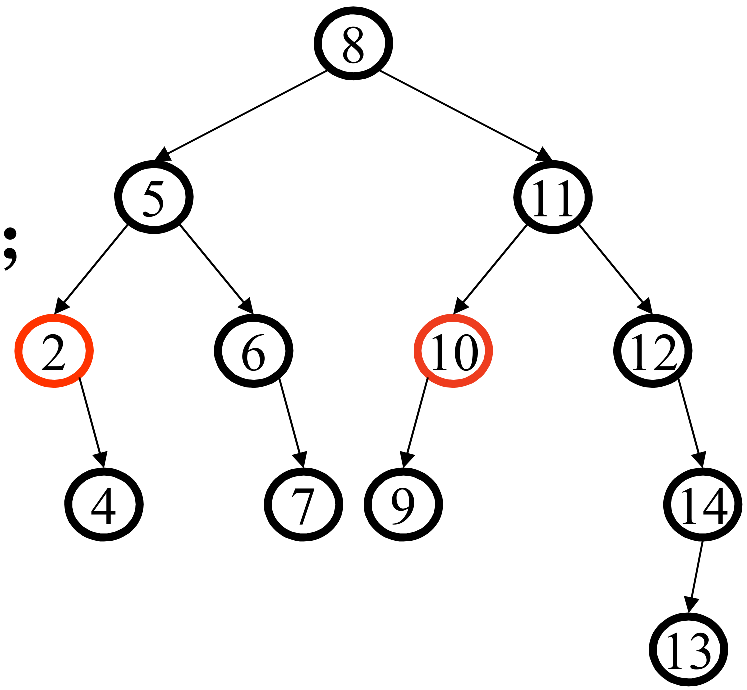
What does this guarantee with a BST?

BST Representation

```
typedef struct bstnode
{
    int data;
    bstnode *lchild, *rchild;
}bstnode;
```

Searching in a BST

```
bstnode *search(int key, bstnode * root)
{
 if (root == NULL) return root;
 else if (key < root→data)
  return search(key, root→lchild);
 else if (key > root→data)
  return find(key, root→rchild);
 else
  return root;
}
```

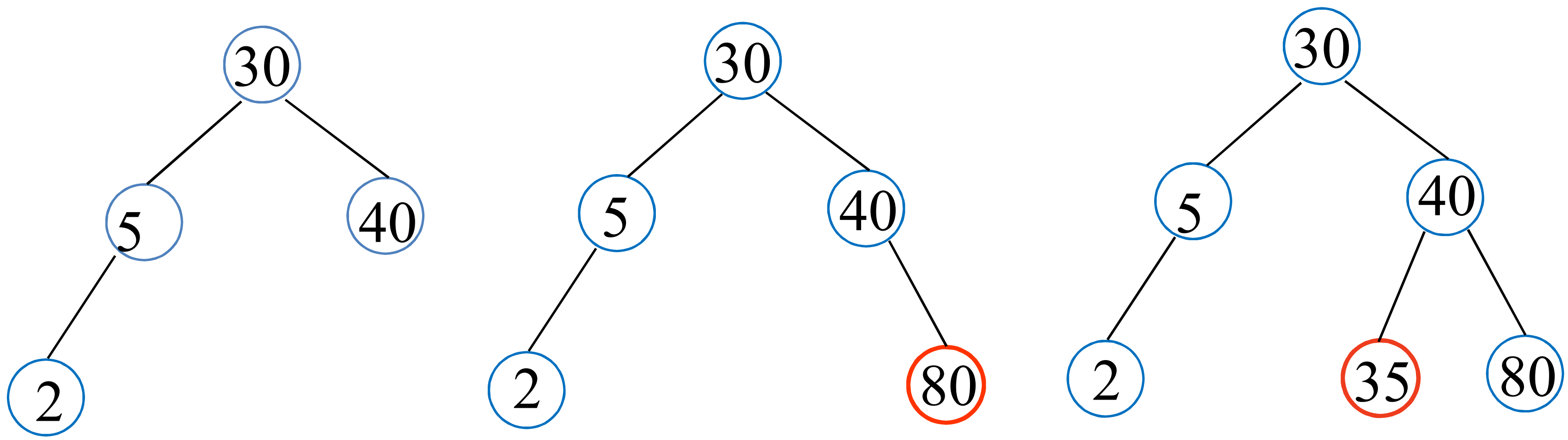


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into a BST

- based on comparisons of the new item and values of nodes in the BST
- **starting at the root** probe **down** the tree till you find a node whose left or right pointer is empty and is a logical place for the new value
- In other words, all inserts take place at a leaf or at a leaflike node – a node that has only one null subtree.

Insertion into a BST



Insert 80

Insert 35

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into a BST

```
void insert(bstnode * newnode, bstnode * root)
{
        if (root→data > newnode→data)
        {
                if (root→lchild == NULL)
                        root→lchild=newnode;
                else
                        insert( newnode, root→lchild );
        }
        else
        {
                if (root→rchild == NULL)
                        root→rchild=newnode;
                else
                        insert( newnode, root→rchild );
        }
}
```

Insertion into a BST

- The order of supplying the data determines where it is placed in the BST , which determines the shape of the BST
- Create BSTs from the same set of data presented each time in a different order:

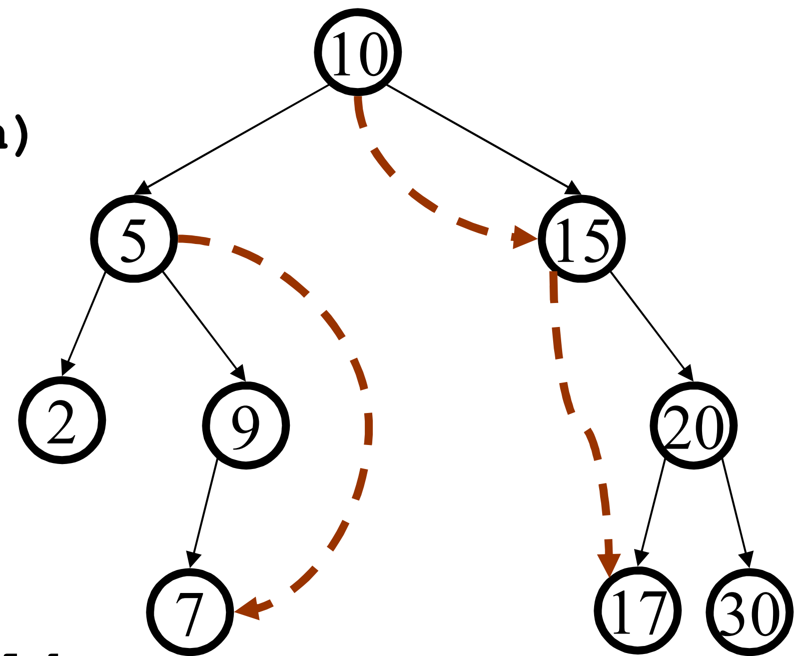
a) 17 4 14 19 15 7 9 3 16 10

b) 9 10 17 4 3 7 14 16 15 19

c) 19 17 16 15 14 10 9 7 4 3 **can you guess this shape?**

Inorder Successor

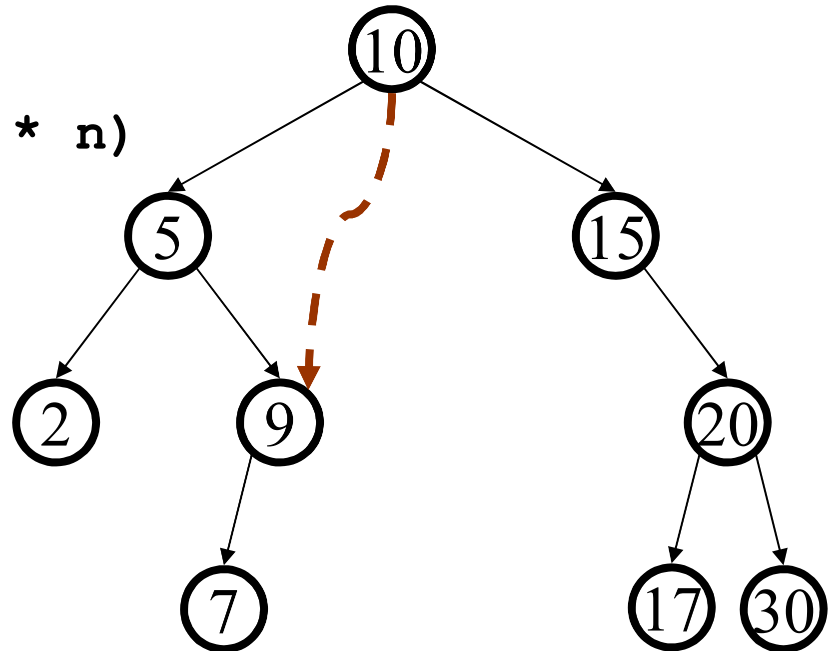
```
bstnode * successor(bstnode * n)
{
   bstnode *iosuccessor;
        if (n→rchild == NULL)
            iosuccessor= ???;
        else
        {
              iosuccessor=n→rchild;
              while (iosuccessor→lchild != NULL)
                    iosuccessor=iosuccessor→lchild;
        }
return iosuccessor;
}
```



How many children can the inorder successor of a node have?

Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Inorder Predecessor




```
bstnode * predecessor(bstnode * n)
{
   bstnode *iopredecessor;
        if (n→lchild == NULL)
            iopredecessor= ???;
        else
        {
              iopredecessor=n→lchild;
              while (iopredecessor→rchild != NULL)
              iopredecessor=iopredecessor→rchild;
        }
return iopredecessor;
}
```

How many children can the inorder predecessor of a node have?

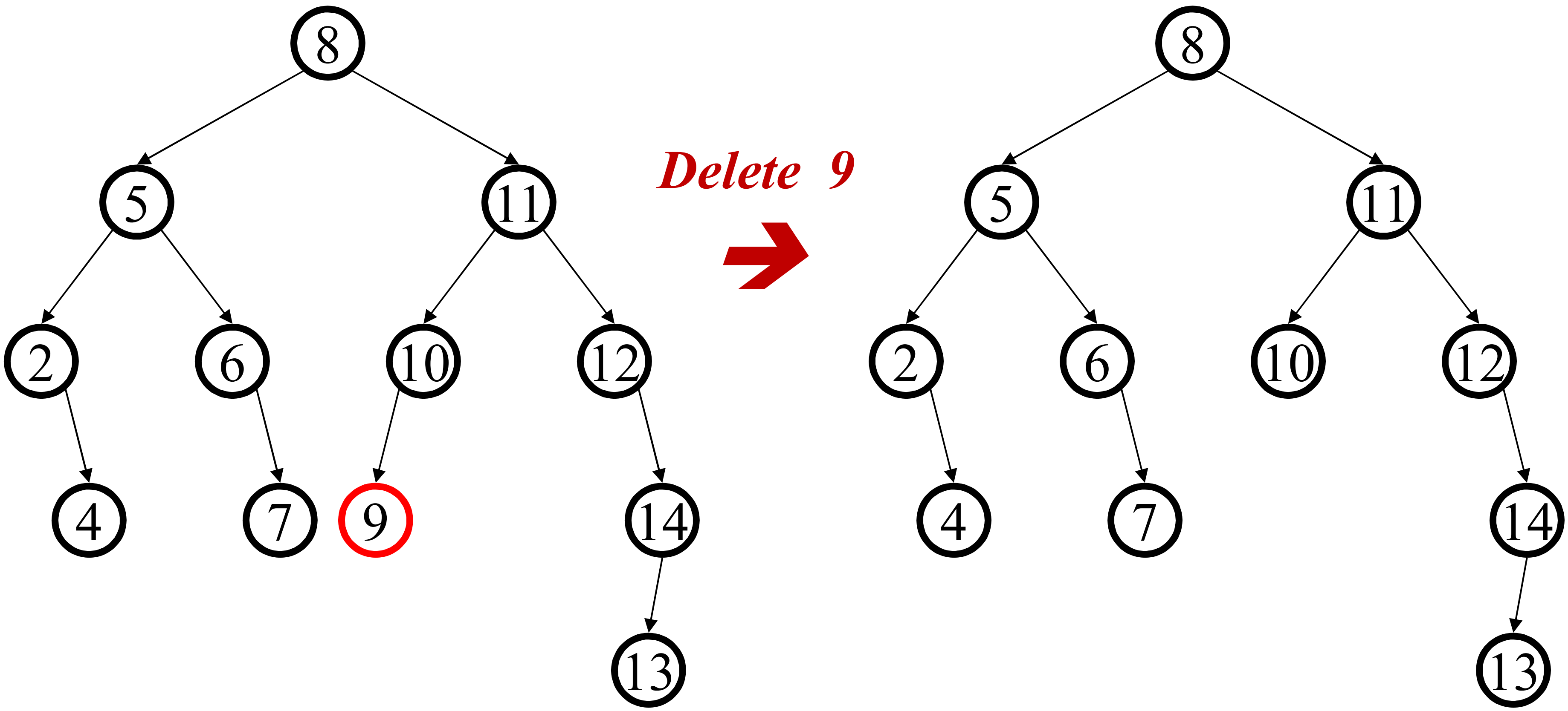
Delete a Node from a BST

- Following are the possible cases when we delete a node:
  - The node to be deleted has no children.
    - Set the respective pointer of its parent to NULL.
  - The node to be deleted has only a right subtree.
    - Attach **respective pointer** of node's parent to **right subtree**.
  - The node to be deleted has only a left subtree.
    - Attach **respective pointer** of node's parent to **left subtree**.
  - The node to be deleted has two subtrees.
    - Replace node's data with data in **inorder successor** (predecessor) and delete the **inorder successor** (predecessor).

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from a BST

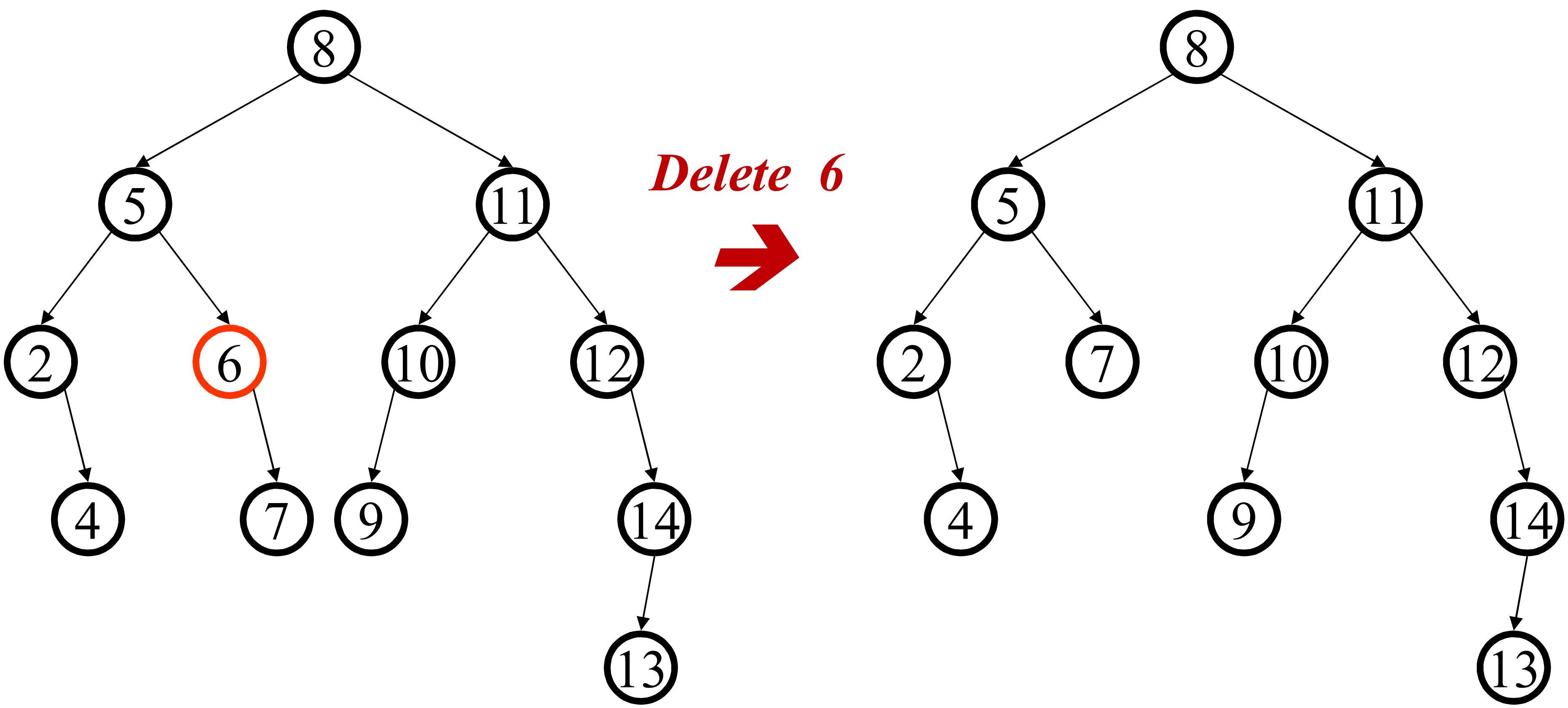
Case 1: deleting a node with 2 EMPTY SUBTREES



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from a BST

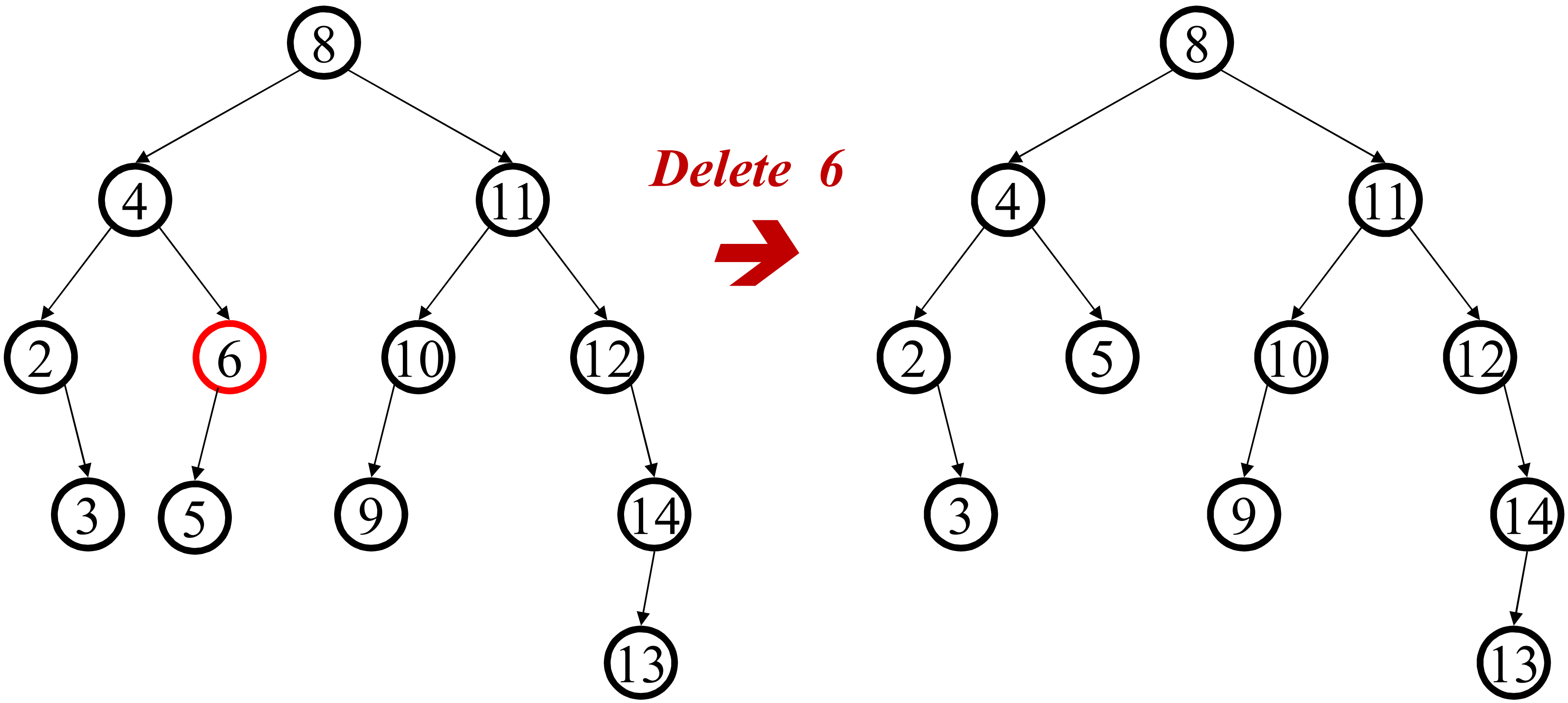
Case 2: deleting a node with only RIGHT SUBTREES



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from a BST

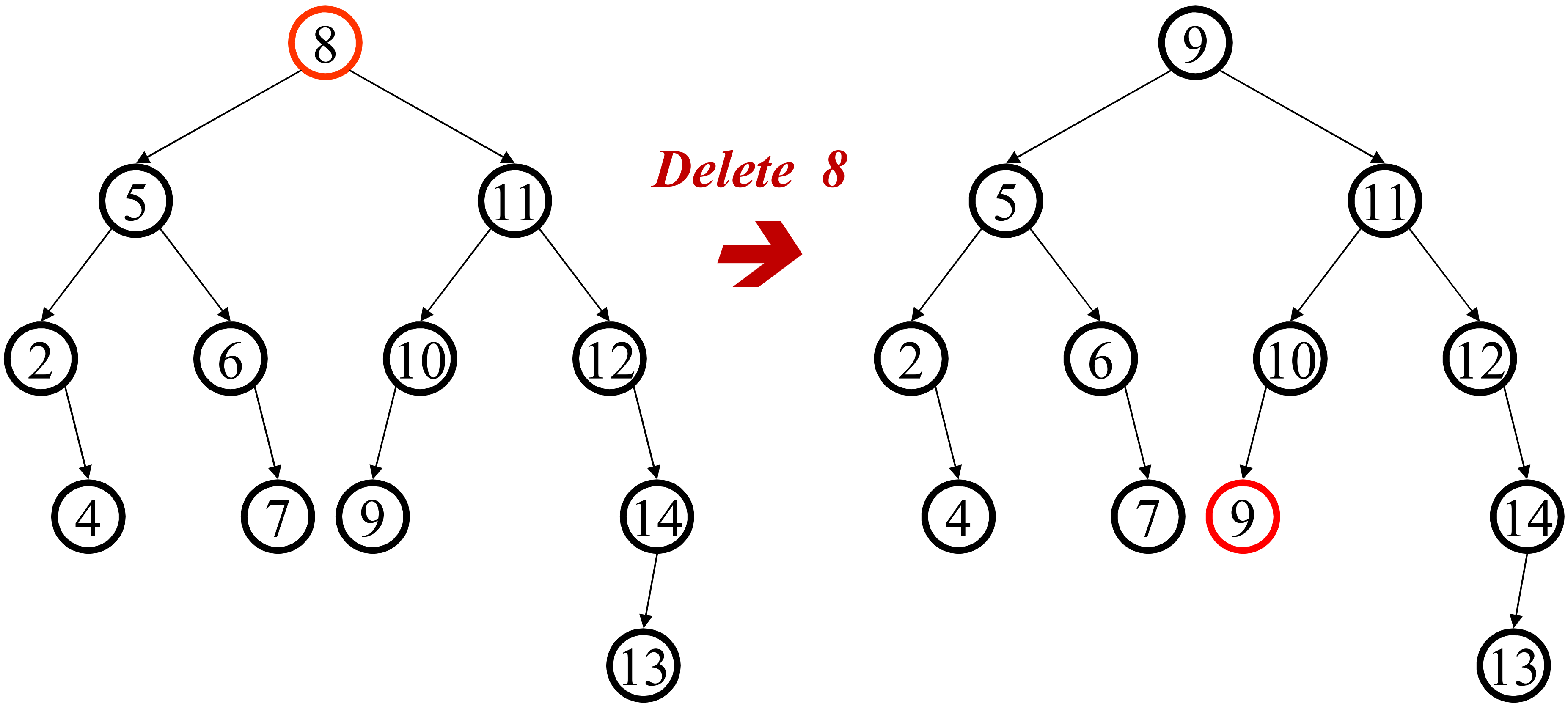
Case 3: deleting a node with only LEFT SUBTREES



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from a BST

Case 4: deleting a node with 2 NON-EMPTY SUBTREES



Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Delete a Node from a BST

```
bstnode * delete (bstnode * root, bstnode * n, bstnode * parent)
{
    bstnode *iosuccessor, *retval;
    if (n→lchild != NULL && n→rchild != NULL)
    {
        successor (n, &parent, &iosuccessor);
        n→data = iosuccessor→data;
        n=iosuccessor;
    }
    if (n→lchild == NULL)
    {
        if (parent != NULL)
                {
```

This function takes node to be deleted, its parent as argument. When the function terminates iosuccessor contains the inorder successor of n, parent becomes the parent of iosuccessor

Now delete the inorder successor

n does not have left subtree

Check for deleting root

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from a BST

```
            if (parent→lchild == n)
                    parent→lchild=n→rchild;
            else
                    parent→rchild=n→rchild;
            retval=root;
        }
        else
            retval=n→rchild;
}
if (n→rchild == NULL)
        {
            if (parent != NULL)
```

Check to see whether n belongs to the left subtree of parent

Deleting root

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from a BST

```
        {
                if (parent→lchild == n)
                        parent→lchild=n→lchild;
                else
                        parent→rchild=n→lchild;
                retval=root;
        }
        else
                retval=n→lchild;    Deleting root
    }
return retval;
}
```

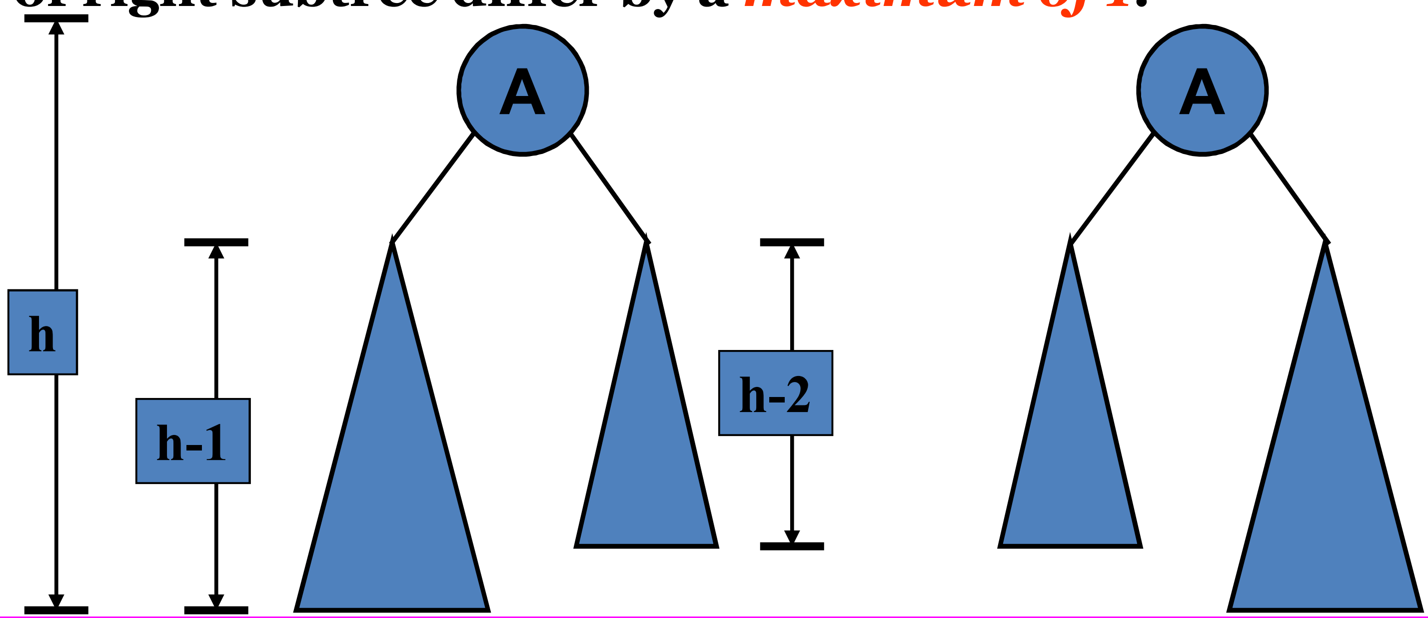
Analysis of BST Operations

- The complexity of operations **`search`**, **`insert`** and **`delete`** in BST is **O(h)** , where **h= O(log n)**, the height of BST.

  - **But**, the BST can take a linear shape and the operations will become **O (n)**

Height Balanced or AVL Trees

- **Unbalanced Binary Search Trees are bad. Worst case: operations take O(n).**
- **Height Balanced or AVL (Adelson-Velskii & Landi) trees maintain balance.**
  - **For each node in BST, height of left subtree and height of right subtree differ by a *maximum of 1*.**



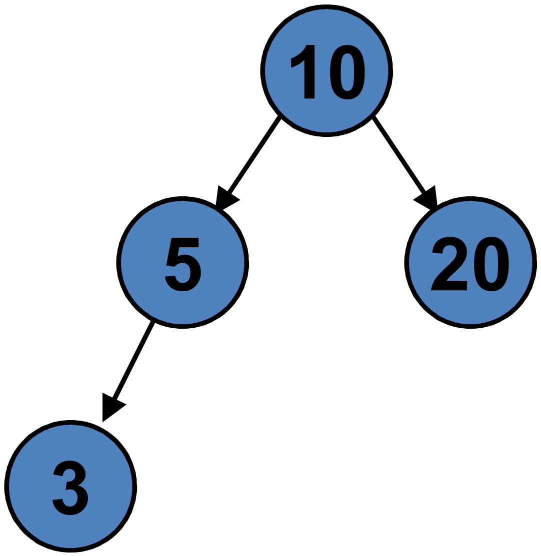
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

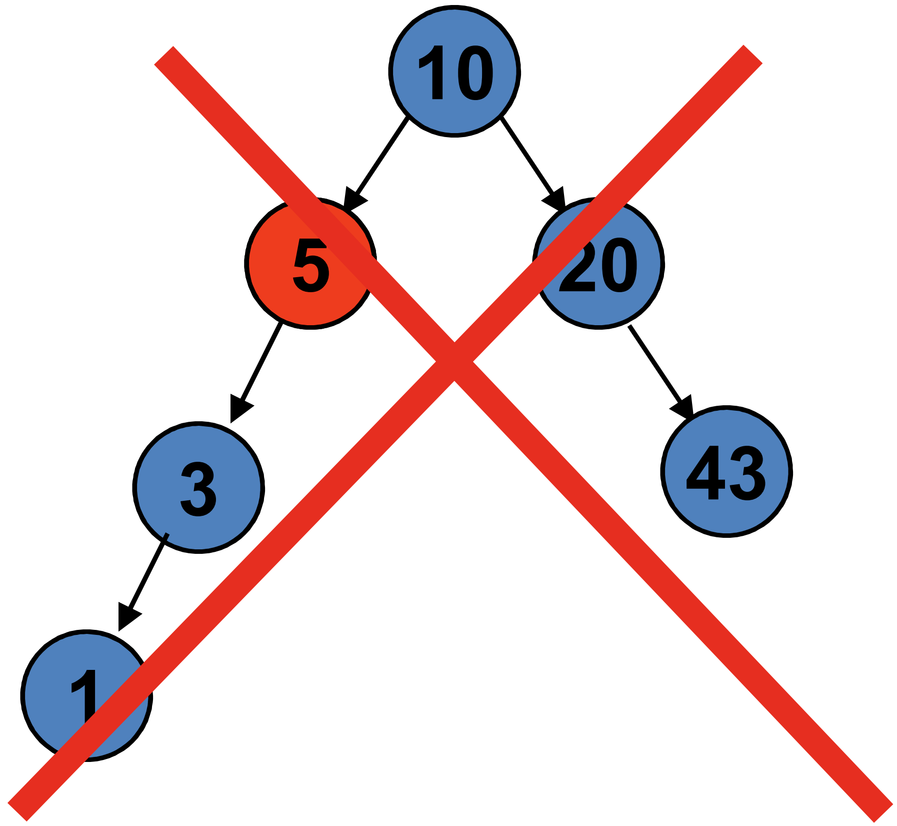
Height Balanced or AVL Trees

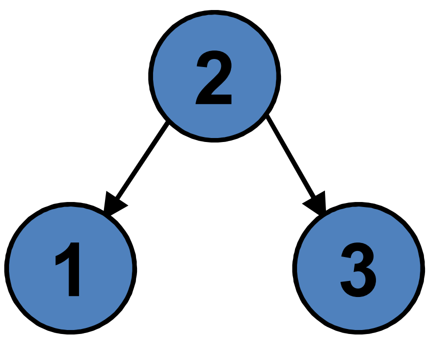
```
typedef struct avlnode
{
    int data;
    int balancefactor;
    bstnode *lchild, *rchild;
}avlnode;
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Examples of AVL trees

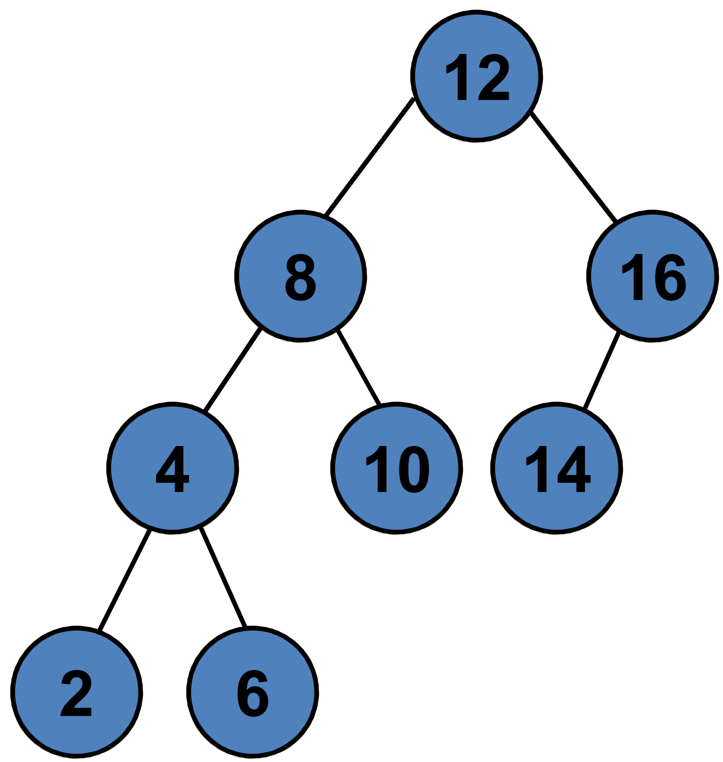






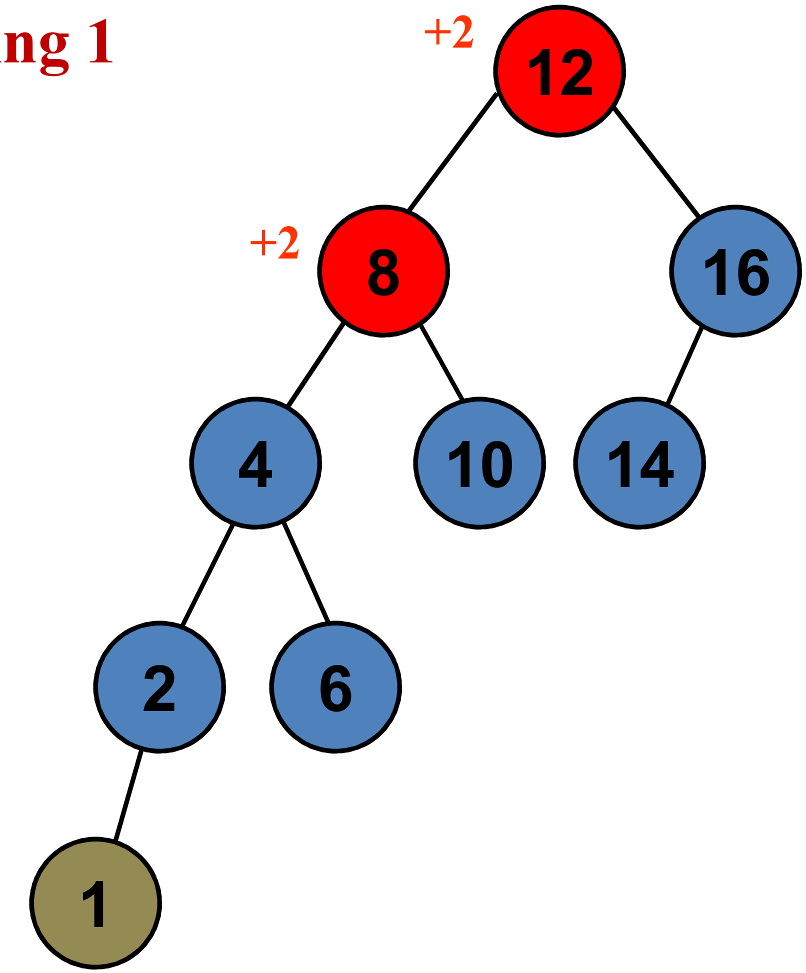
Insertion into an AVL Tree

Insert 1 in the following AVL tree



Insertion into an AVL Tree

After inserting 1



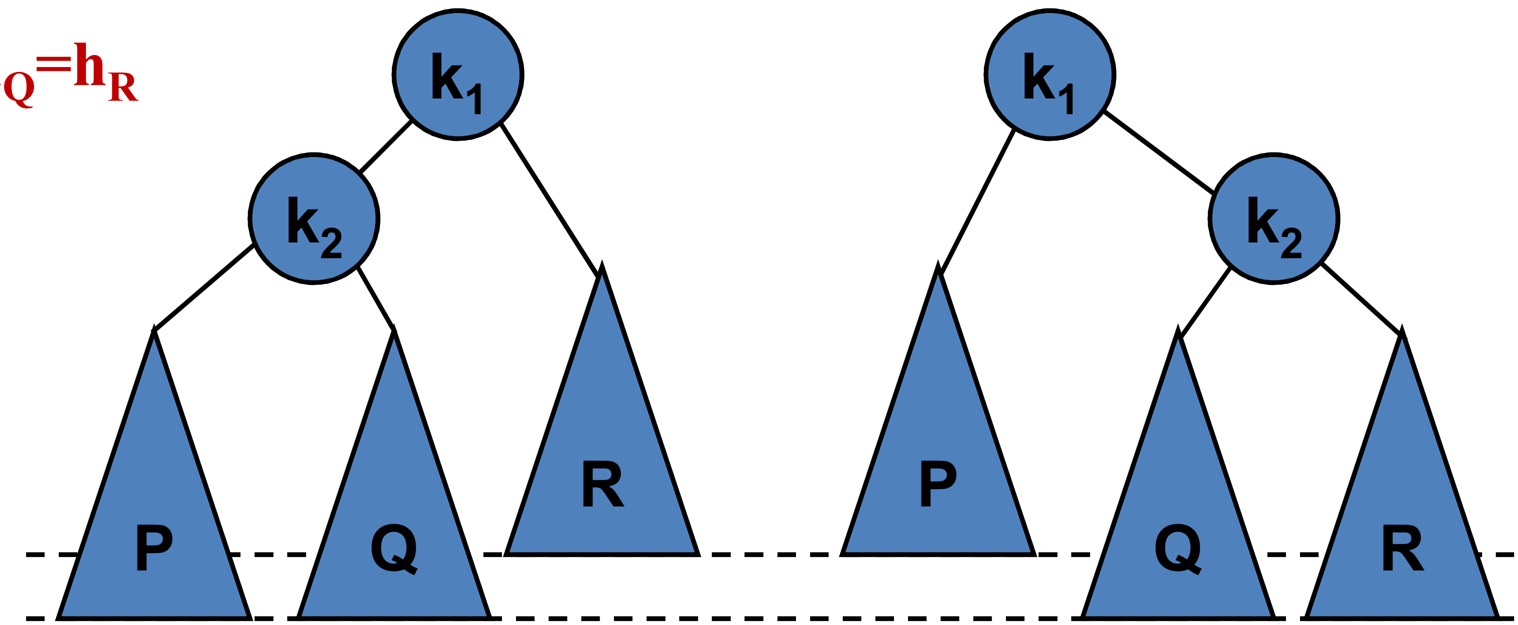
Insertion into an AVL Tree

- Insertion of a node into AVL tree may result in ***imbalance***.
- To ensure balance condition, after insertion of a new node, back up the path from the inserted node to root and calculate **balance factor** for each node.
  - **If the balance condition does not hold in a certain node, we do one of the following rotations:**
    - **Single rotation**
    - **Double rotation**

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into an AVL Tree

$h_P = h_Q = h_R$



Possible cases of insertions that may result imbalance

- An insertion into the subtree:
  - **Case 1:** insert into P **(outside)**
  - **Case 2:** insert into Q **(inside)**
- An insertion into the subtree:
  - **Case 3:** insert into Q **(inside)**
  - **Case 4:** insert into R **(outside)**

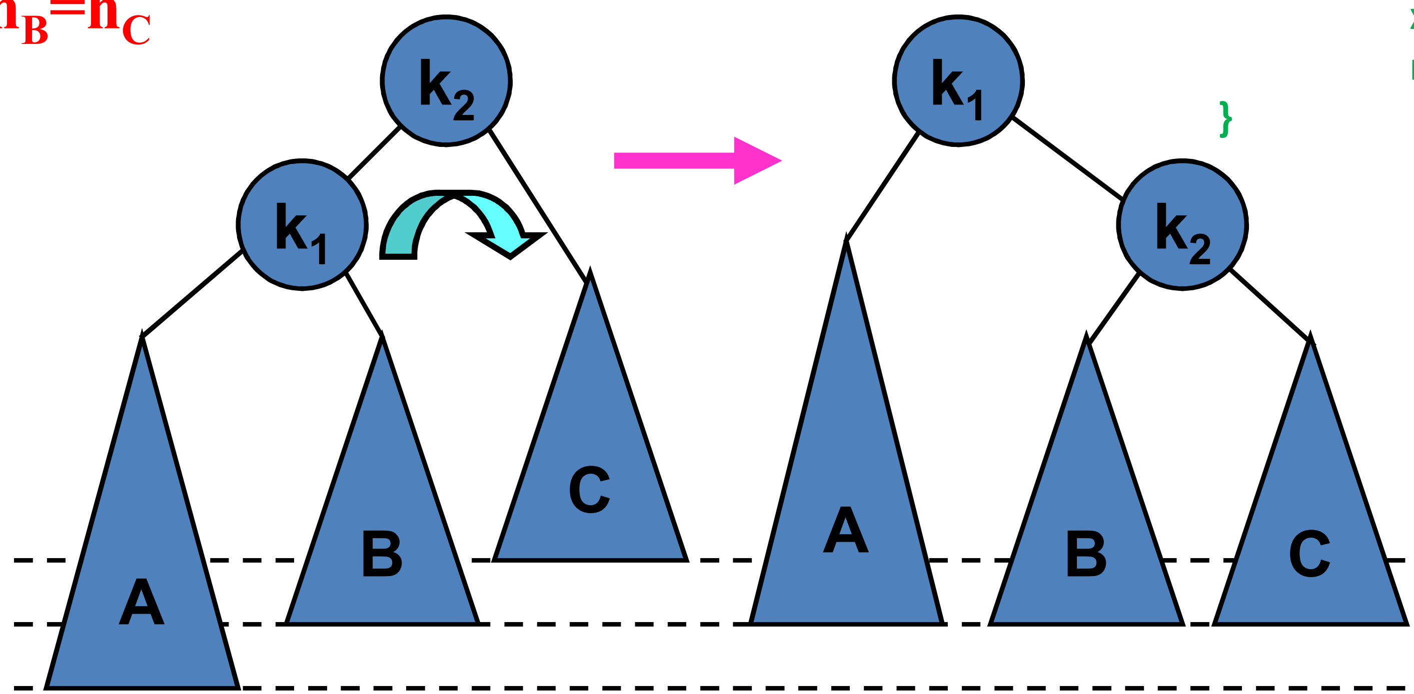
Insertion into an AVL Tree

Case 1: Single Rotation

```
avlnode* rotateright (avlnode *y)
{
        avlnode *x;
        x=y→lchild;
        y→lchild=x→rchild;
        x→rchild=y;
        return x;
}
```

$h_A = h_B + 1$

$h_B = h_C$



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

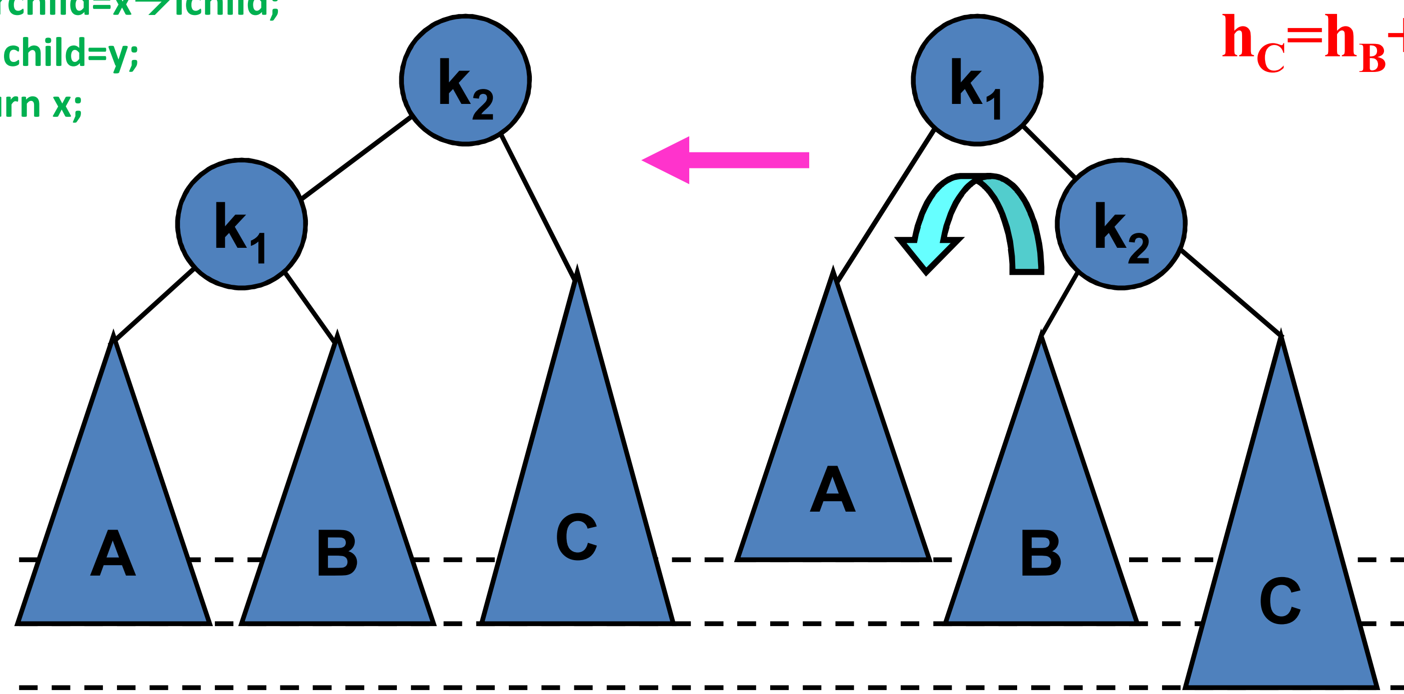
Insertion into an AVL Tree

Case 4: Single Rotation

```
avlnode* rotateleft (avlnode *y)
{
        avlnode *x;
        x=y→rchild;
        y→rchild=x→lchild;
        x→lchild=y;
        return x;
}
```

$h_A = h_B$

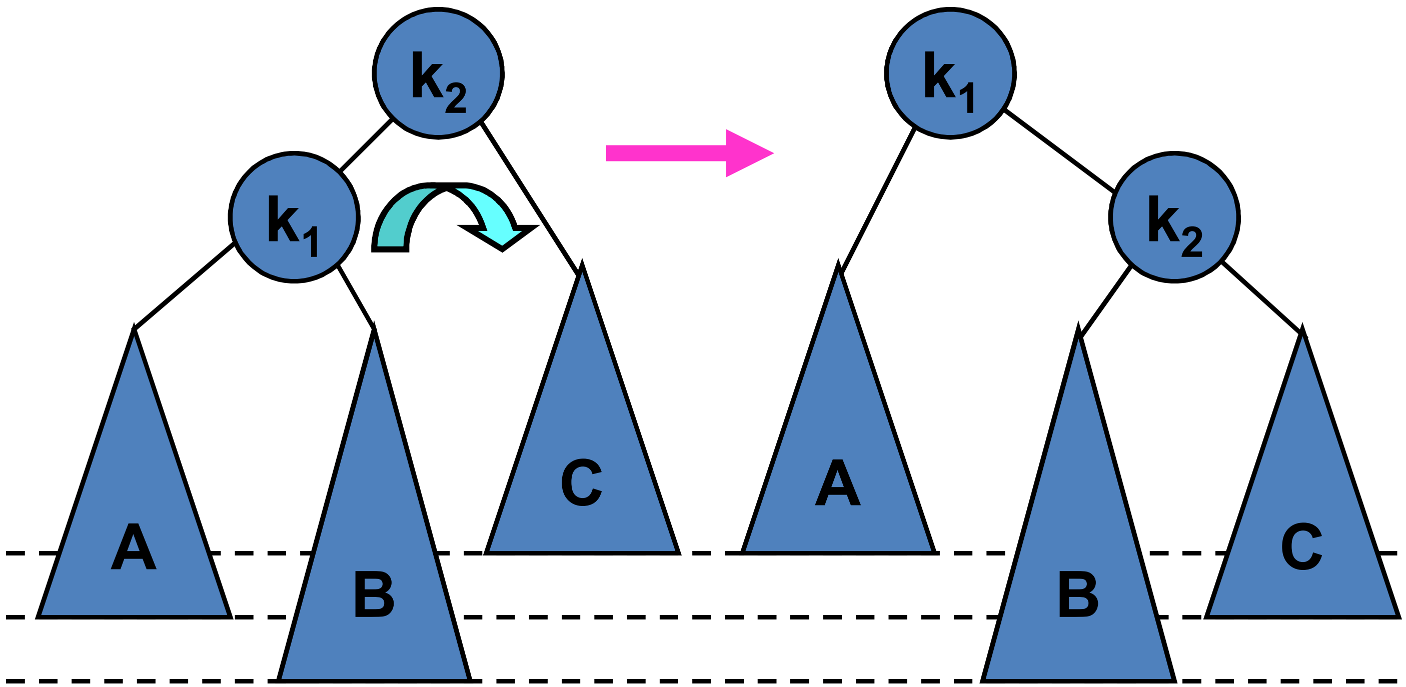
$h_C = h_B + 1$



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into an AVL Tree

Case 2 & 3 (inside case): Single Rotation does not work



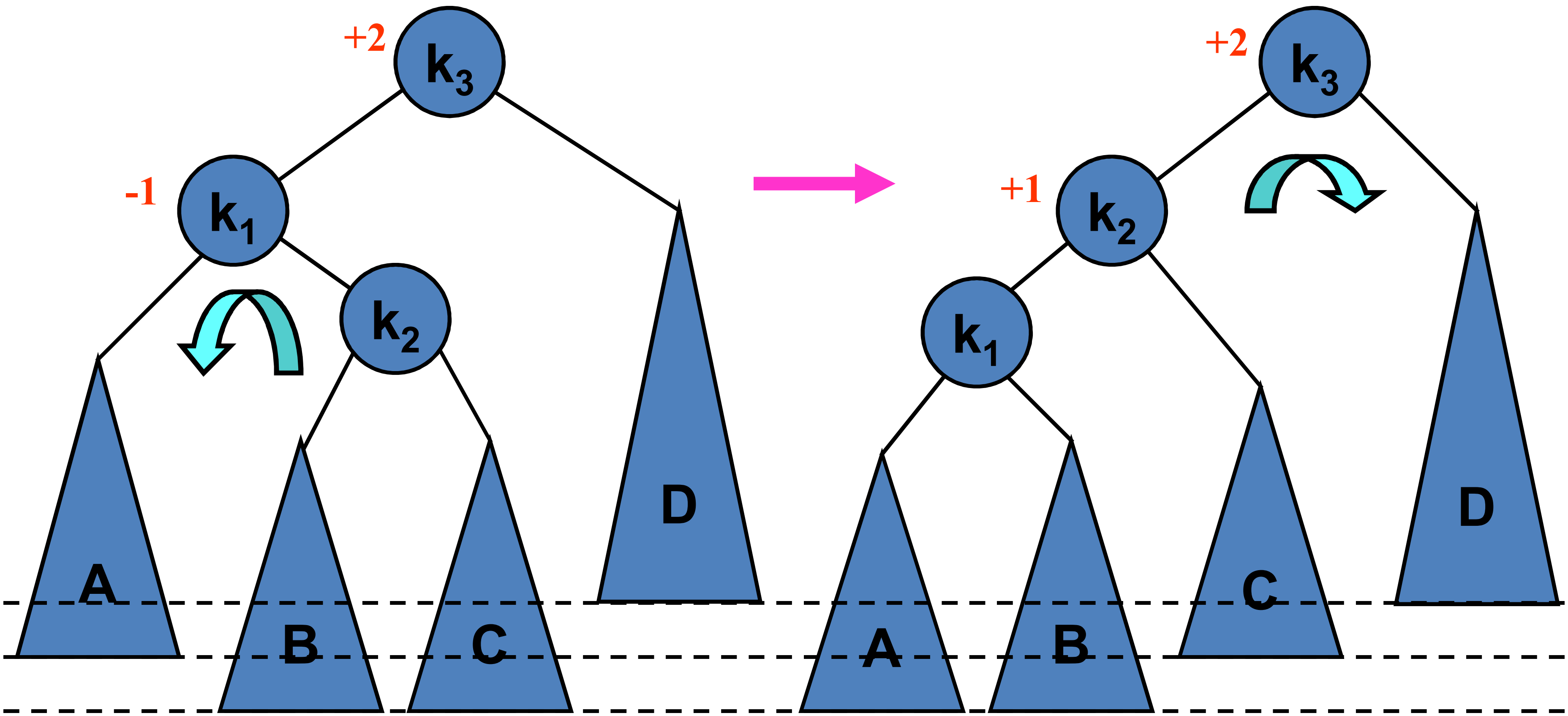
$h_B = h_A + 1$

$h_A = h_C$

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into an AVL Tree

Case 2 & 3 (inside case): Double Rotation

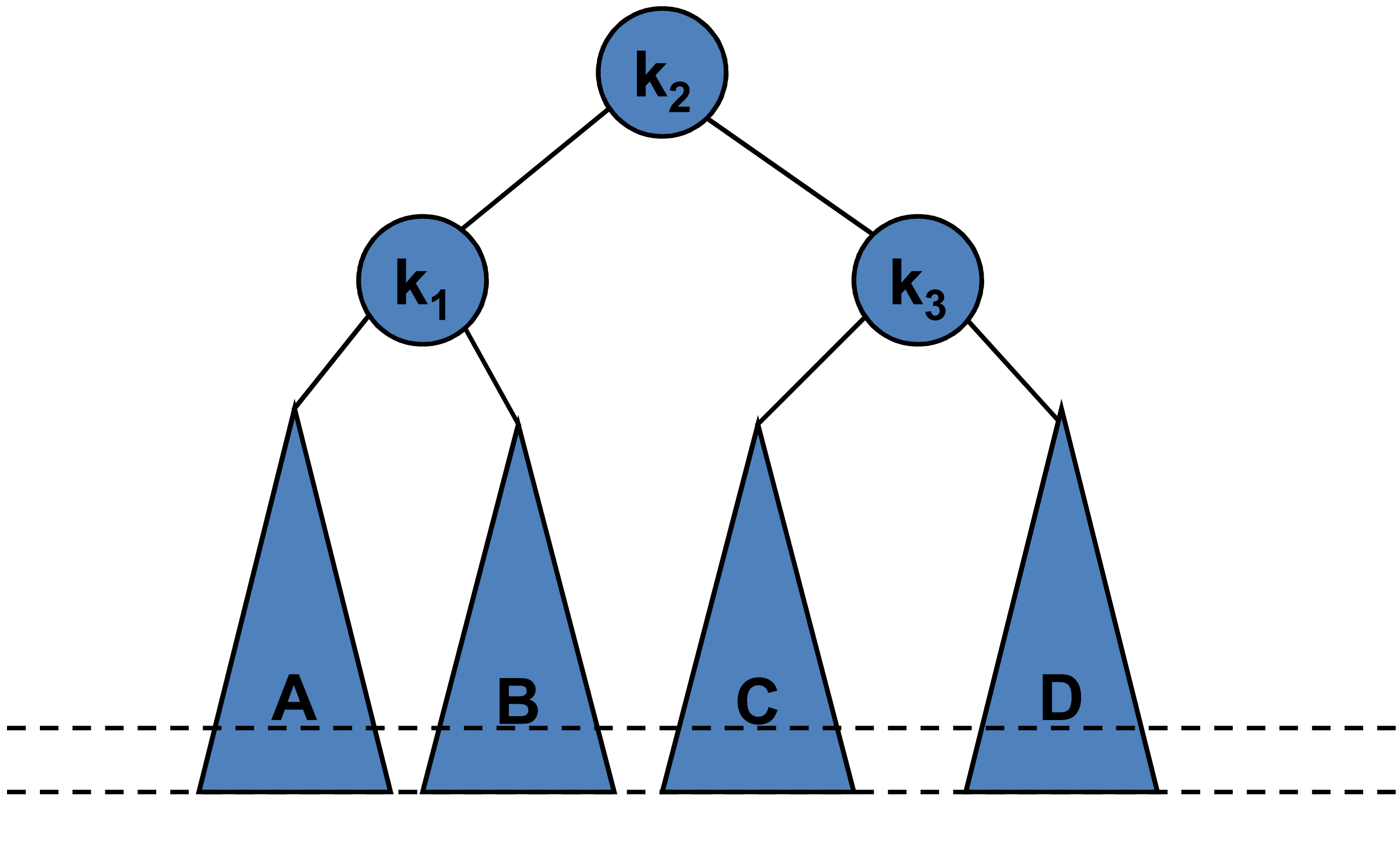


$h_A = h_B = h_C = h_D$

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

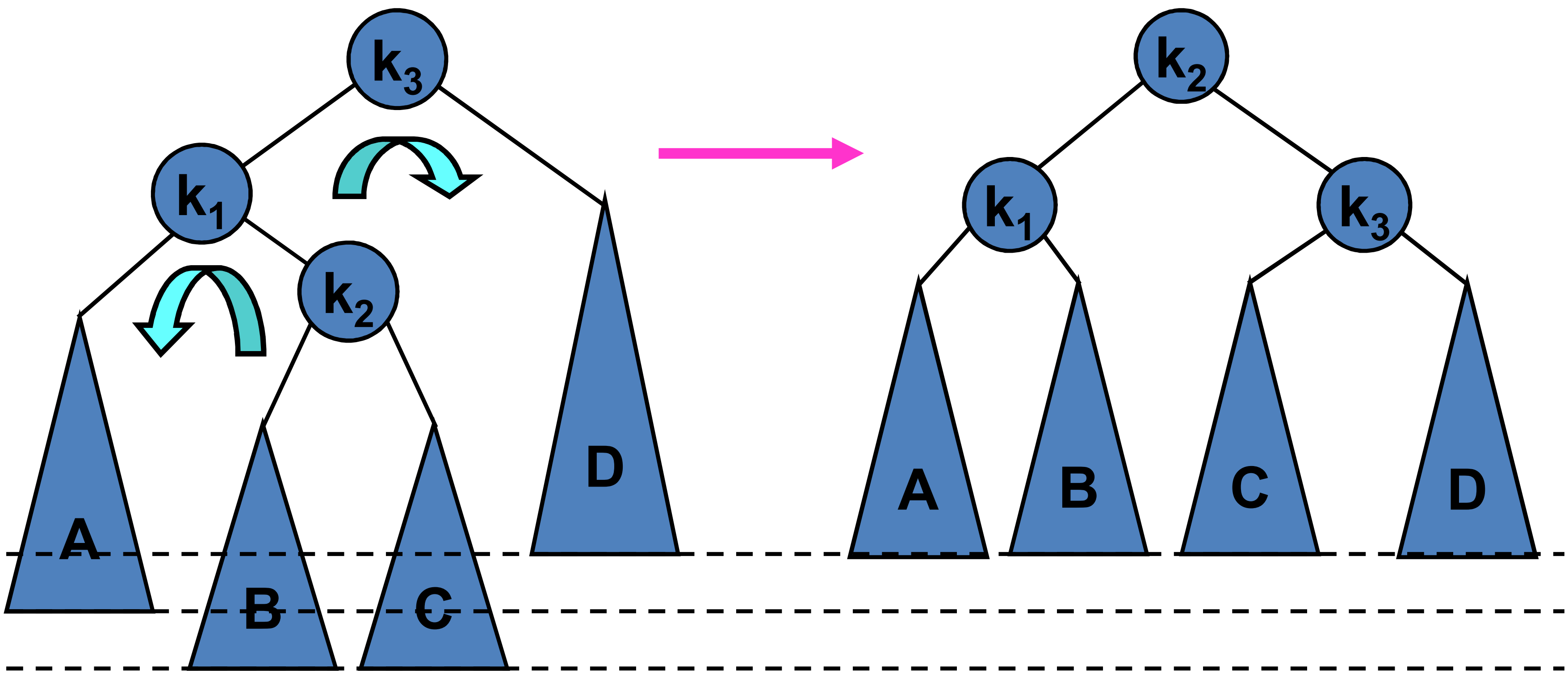
Insertion into an AVL Tree

Case 2 & 3 (inside case): Double Rotation



Insertion into an AVL Tree

Double Rotation

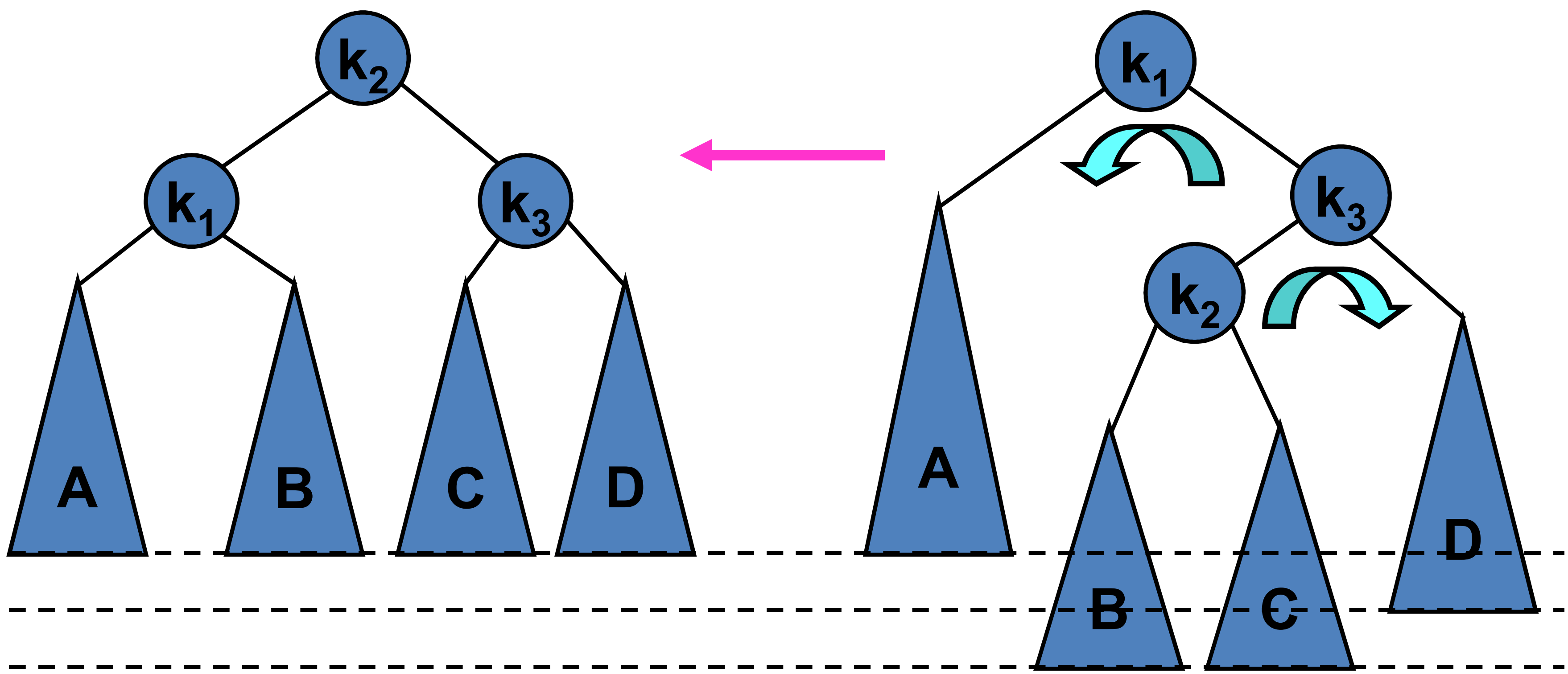


$h_A = h_B = h_C = h_D$

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into an AVL Tree

Double Rotation

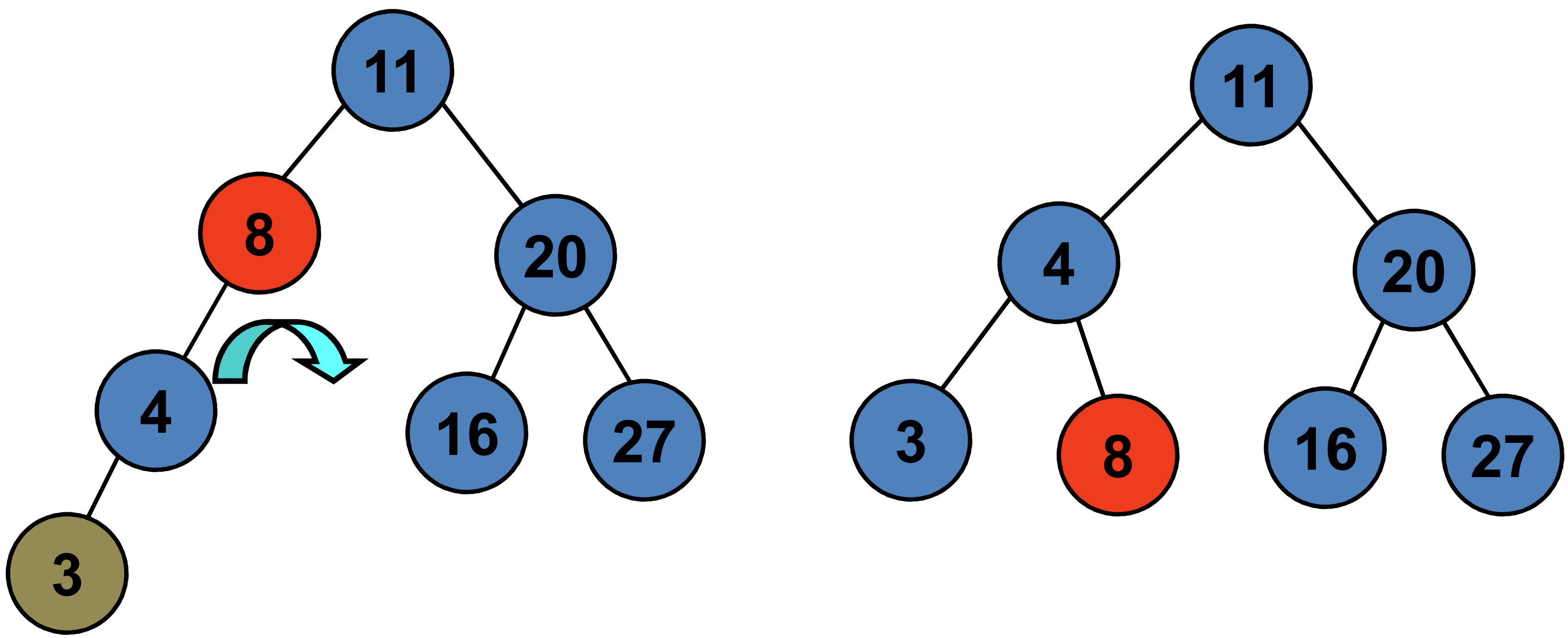


$h_A = h_B = h_C = h_D$

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into an AVL Tree

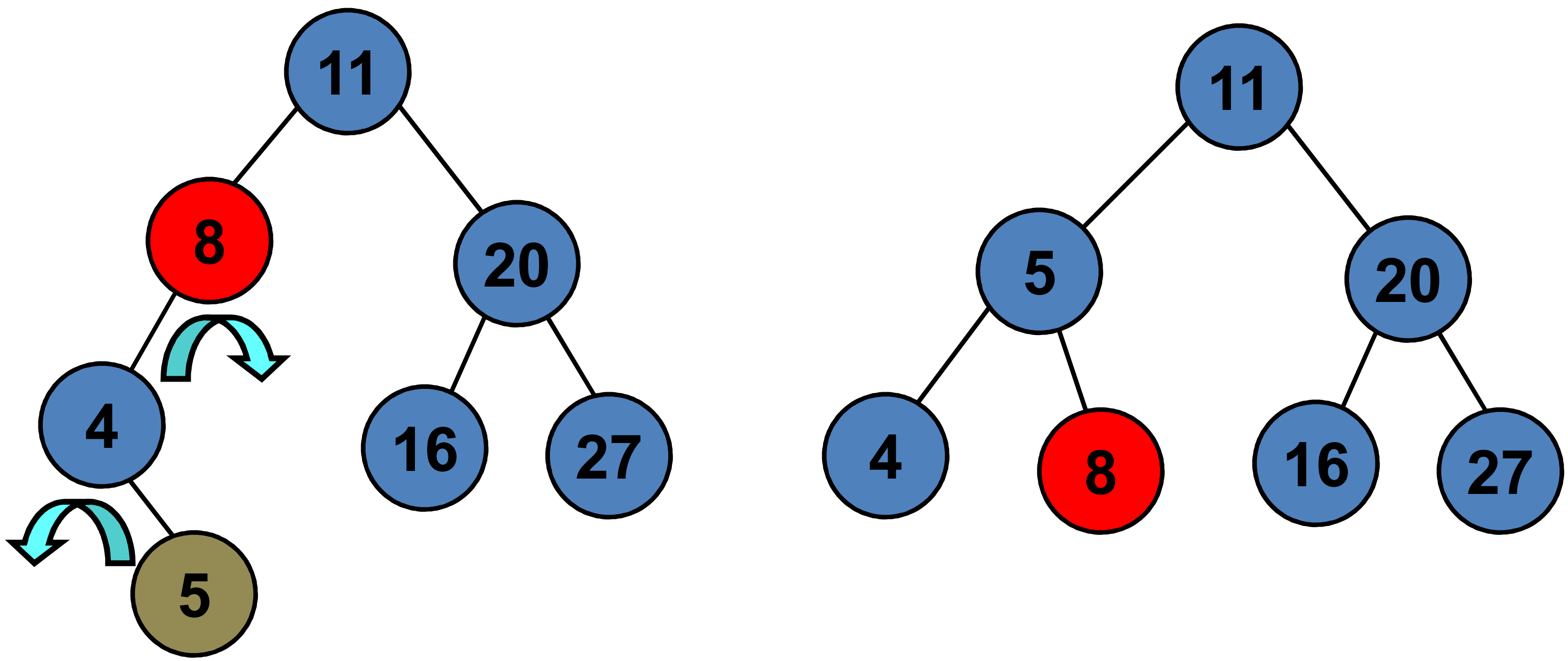
Insert 3 into the AVL tree



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Insertion into an AVL Tree

Insert 5 into the AVL tree



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from AVL Tree

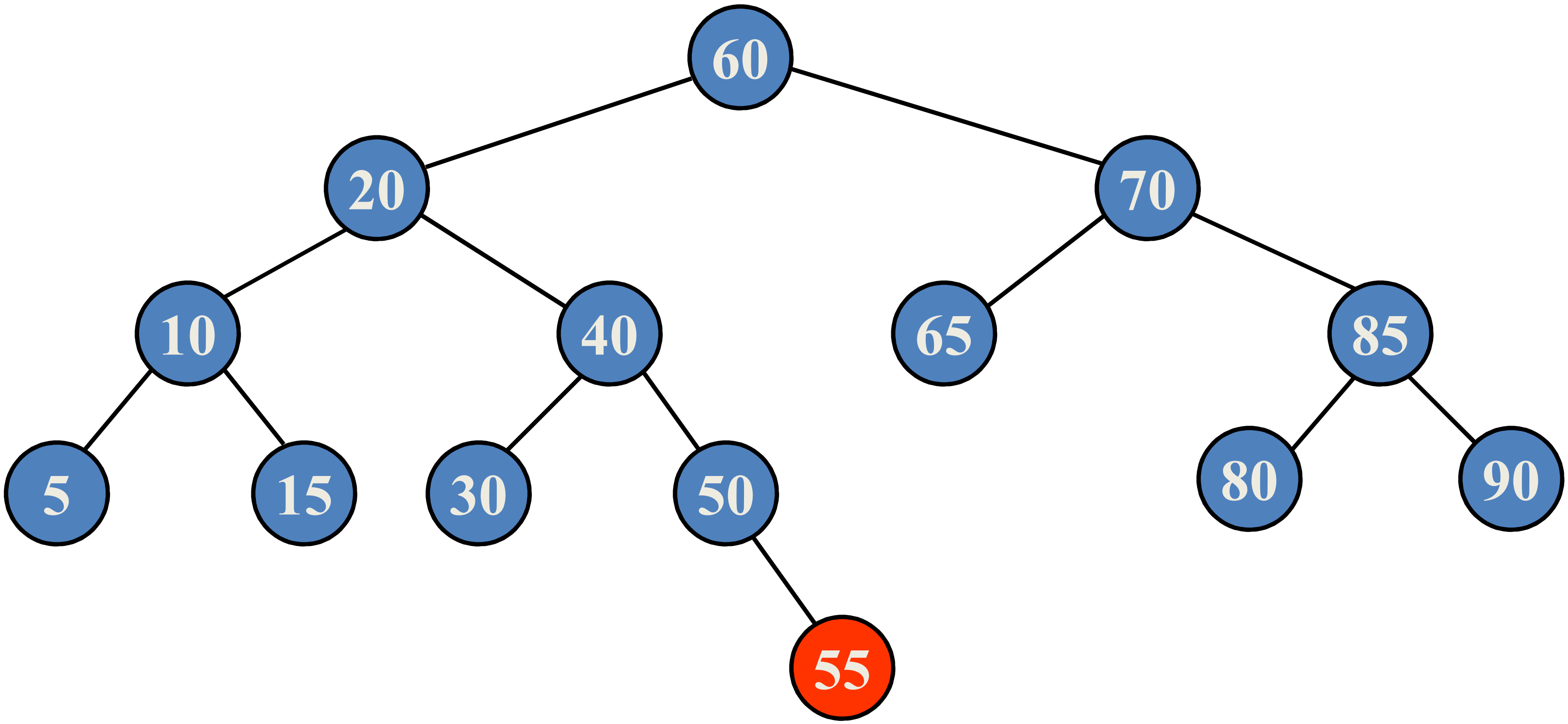
- Deleting a node from an AVL Tree is similar to that of deleting a node from a binary search tree. However, it may **unbalance** the tree.
- Starting from the deleted node, check all the nodes in the path up to the root for the first unbalance node.
  - Use appropriate **single or double rotation**.
  - May need to continue searching for unbalanced nodes all the way to the root.

Delete a Node from AVL Tree

- Deletion:
  - Case 1: if X is a **leaf**, delete X
  - Case 2: if X has **1 child**, use it to replace X
  - Case 3: if X has **2 children**, replace X with its **inorder predecessor** (and recursively delete it)
- Rebalancing

Delete a Node from AVL Tree

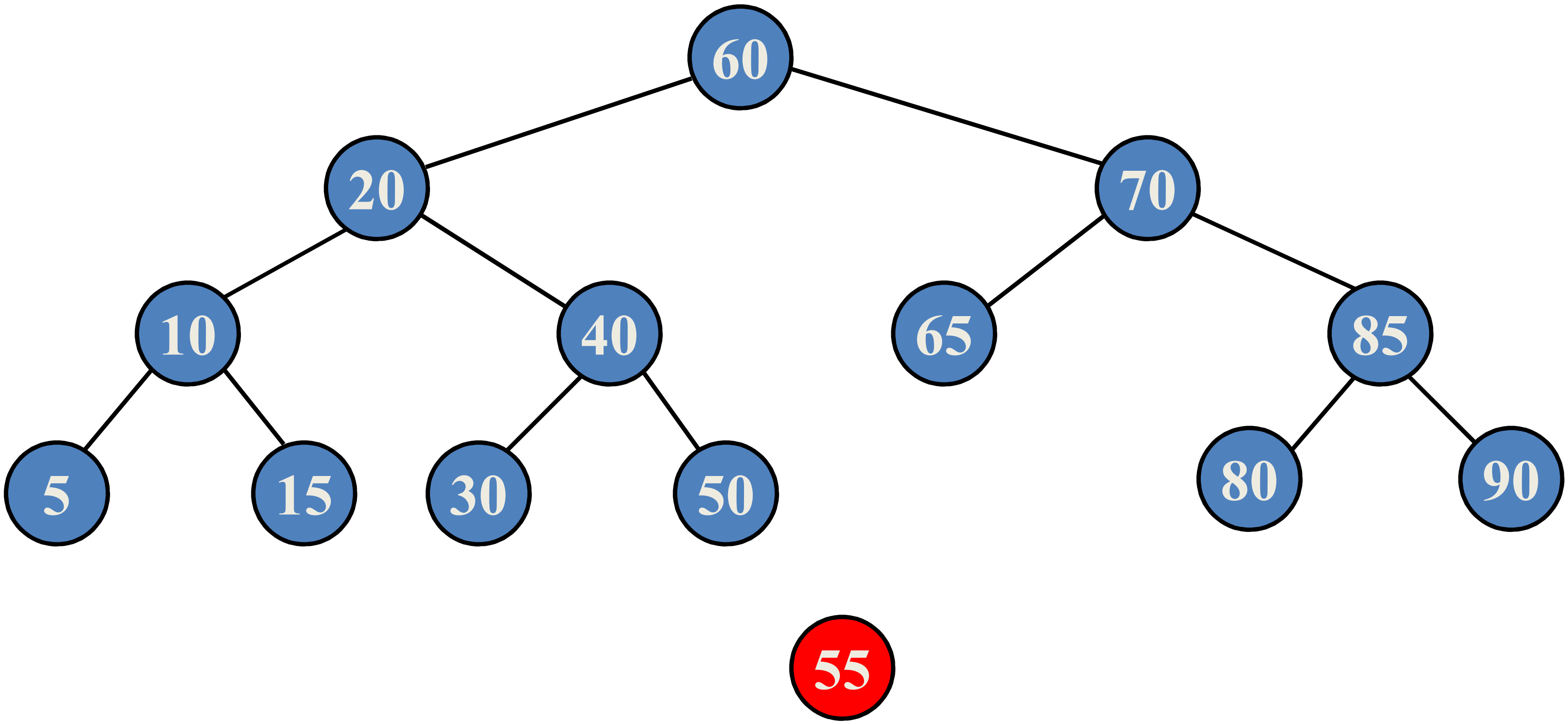
Delete 55



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

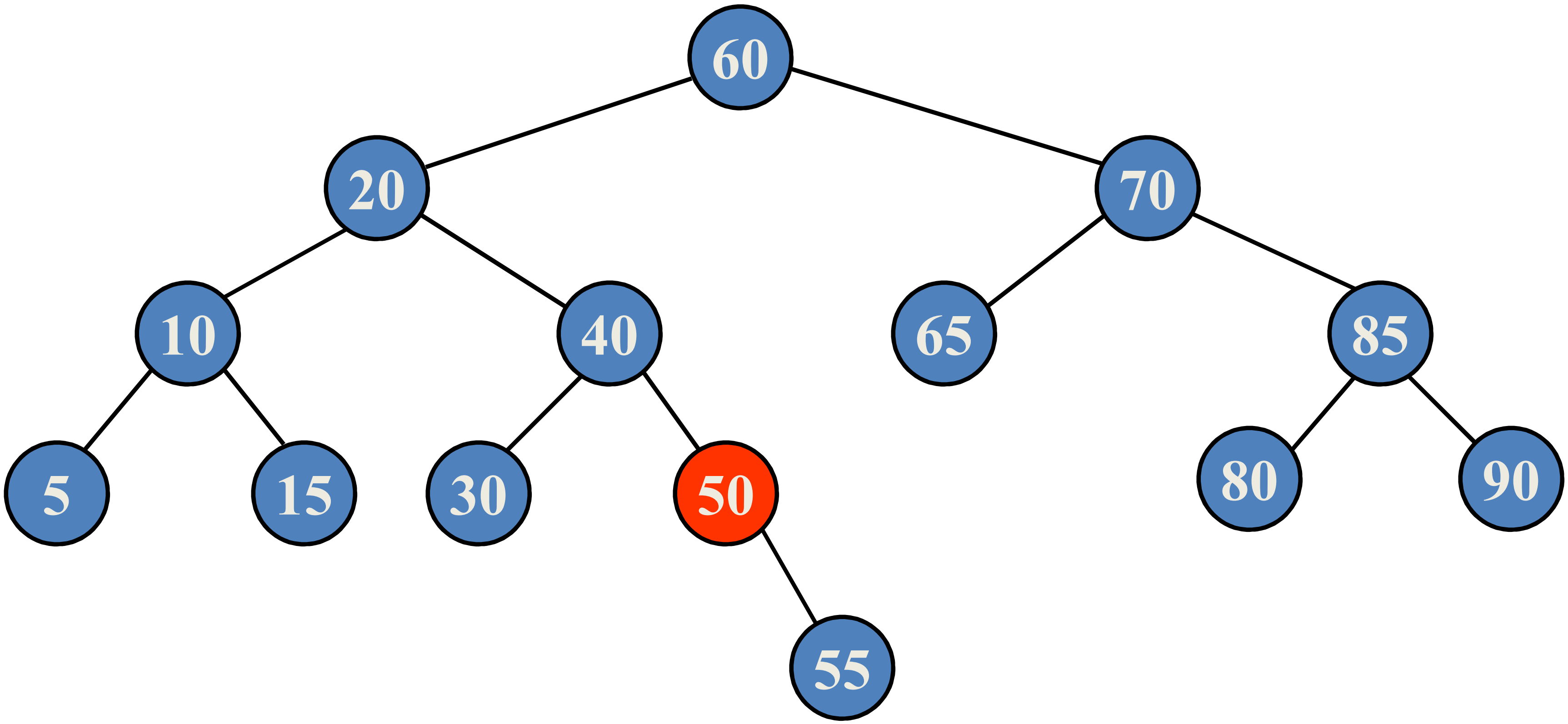
Delete a Node from AVL Tree

Delete 55



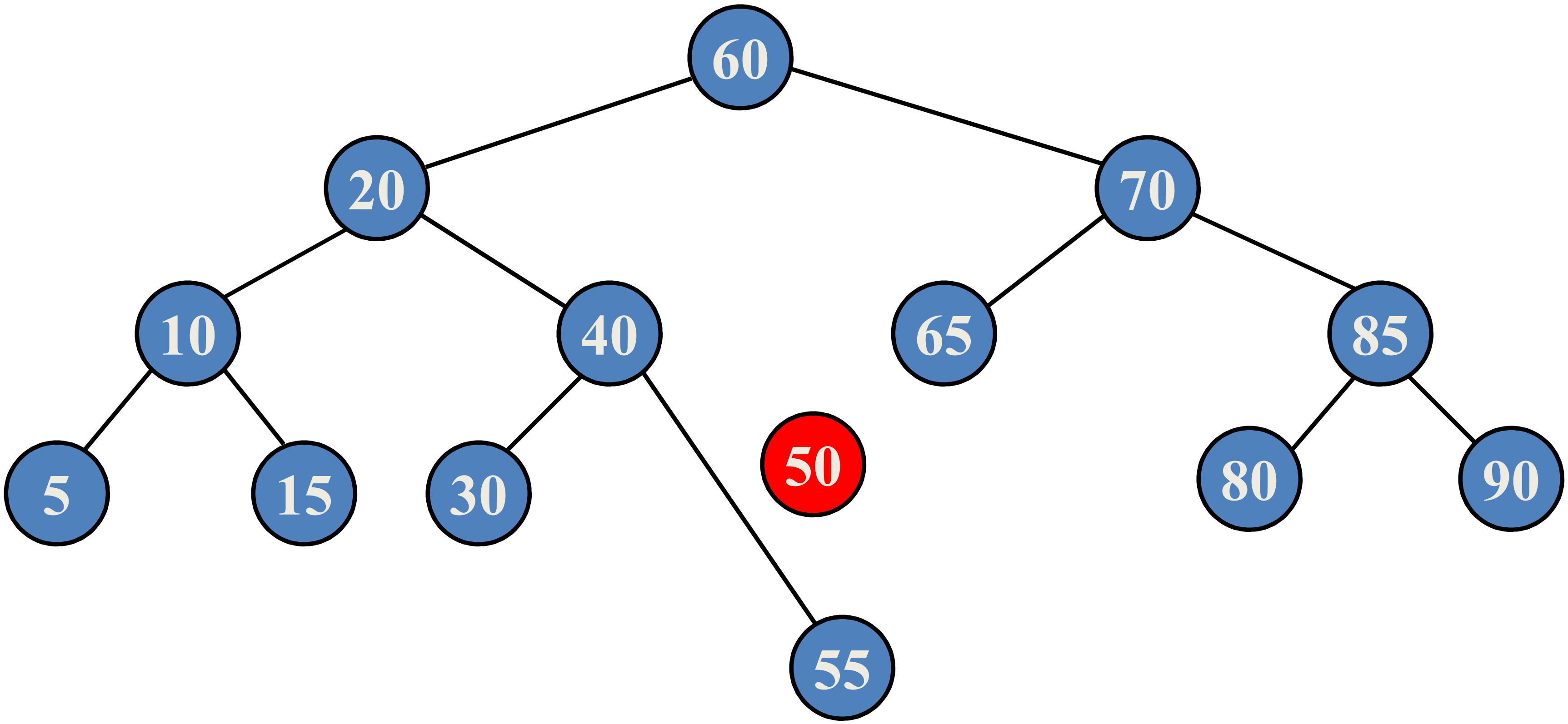
Delete a Node from AVL Tree

Delete 50



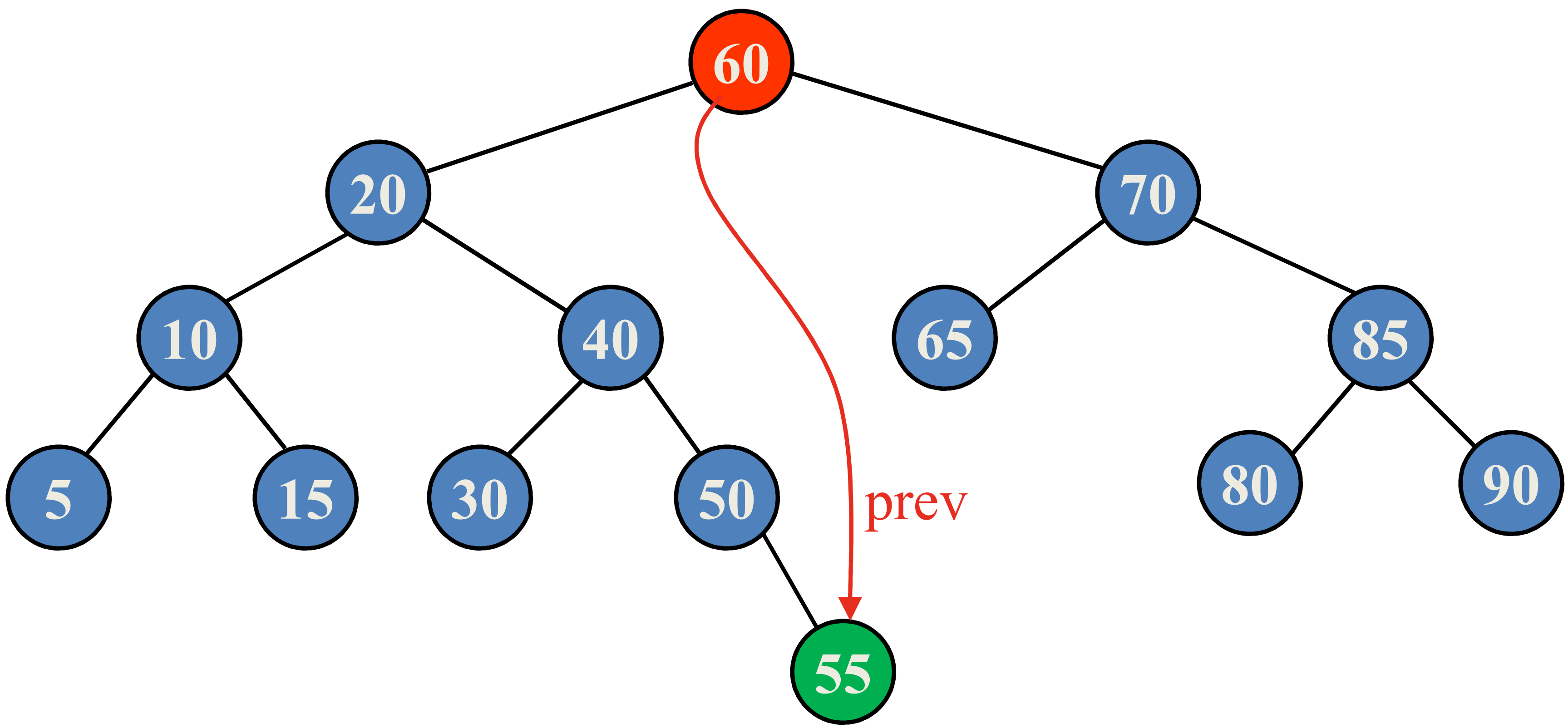
Delete a Node from AVL Tree

Delete 50



Delete a Node from AVL Tree

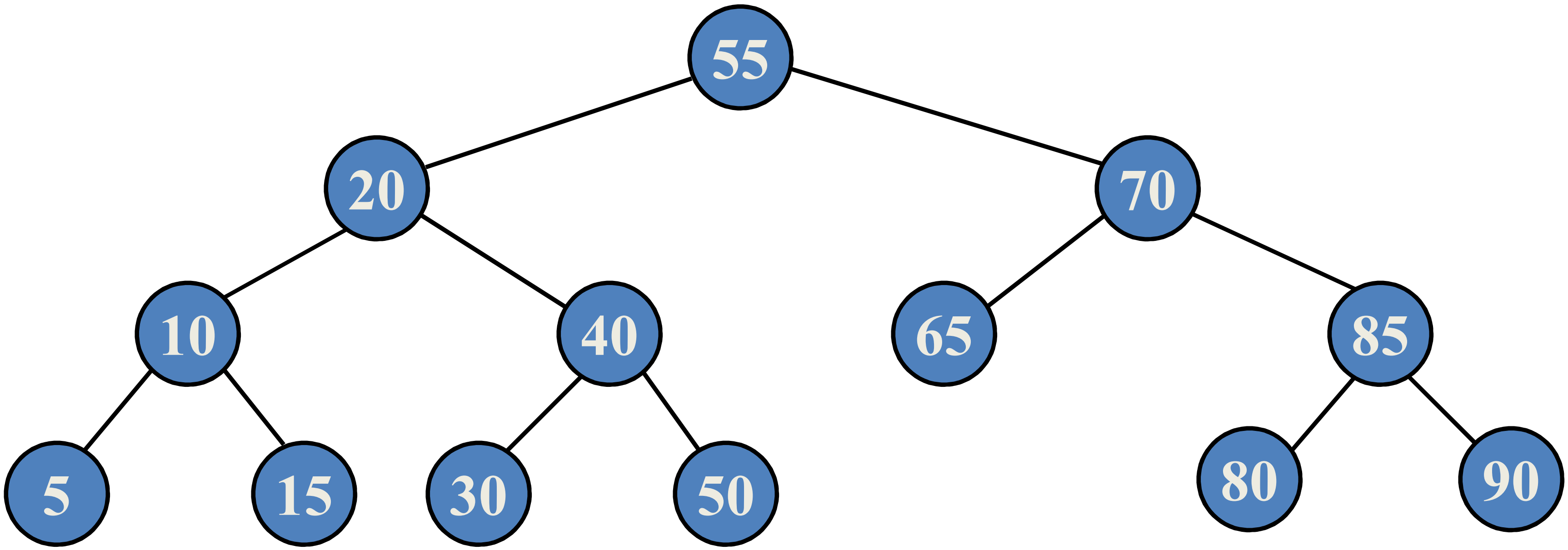
Delete 60



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

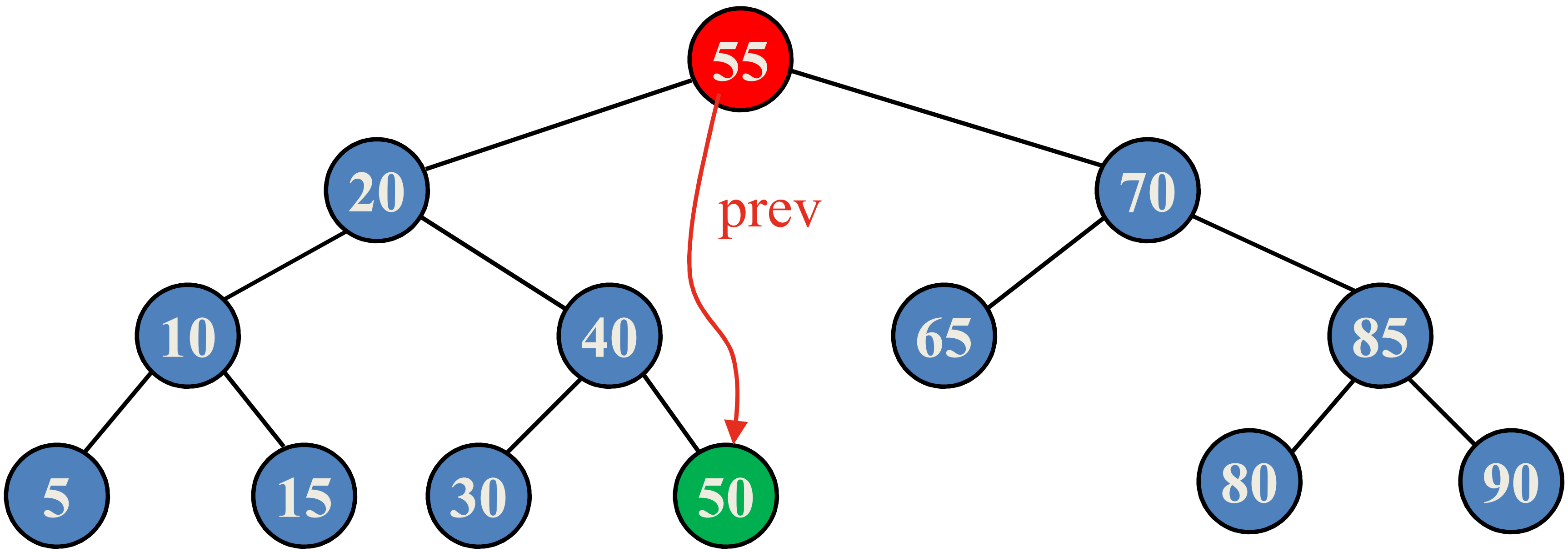
Delete a Node from AVL Tree

Delete 60



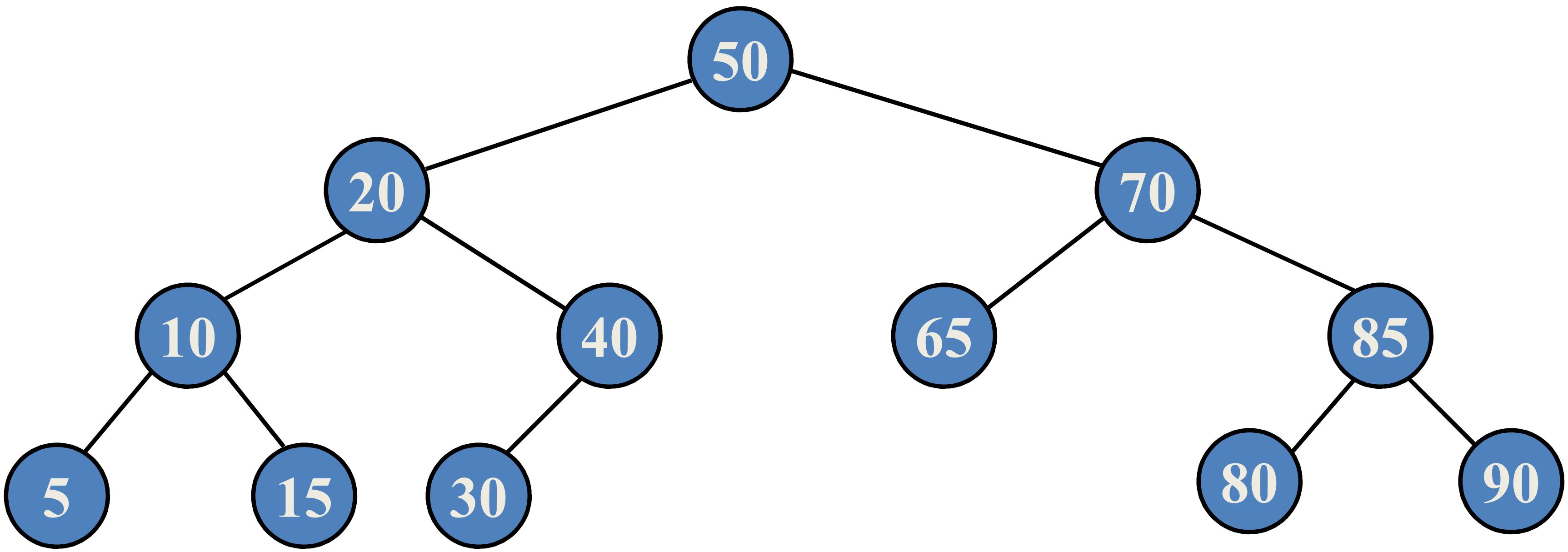
Delete a Node from AVL Tree

Delete 55



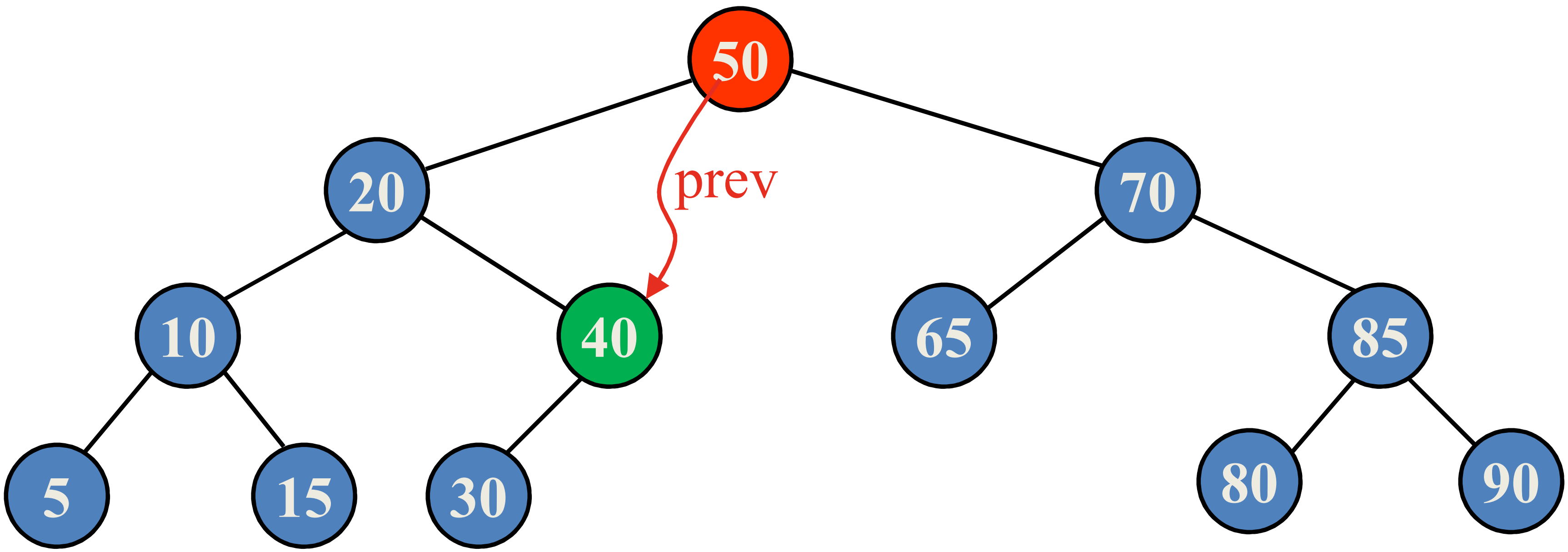
Delete a Node from AVL Tree

Delete 55



Delete a Node from AVL Tree

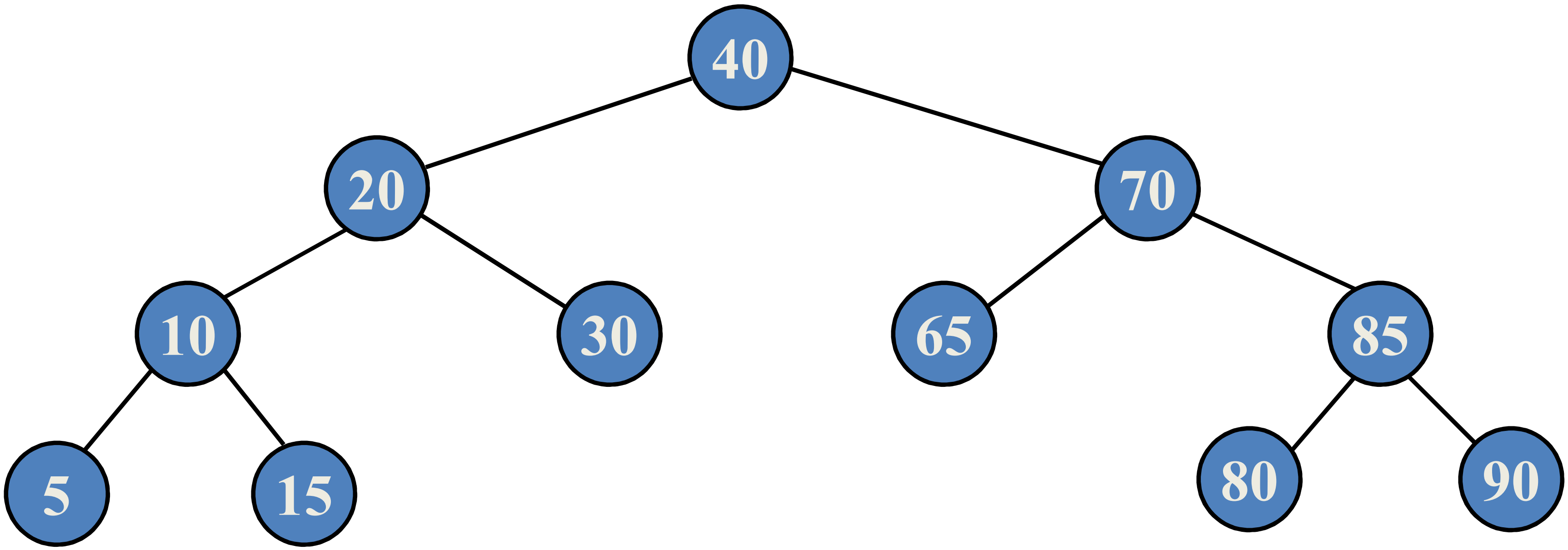
Delete 50



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from AVL Tree

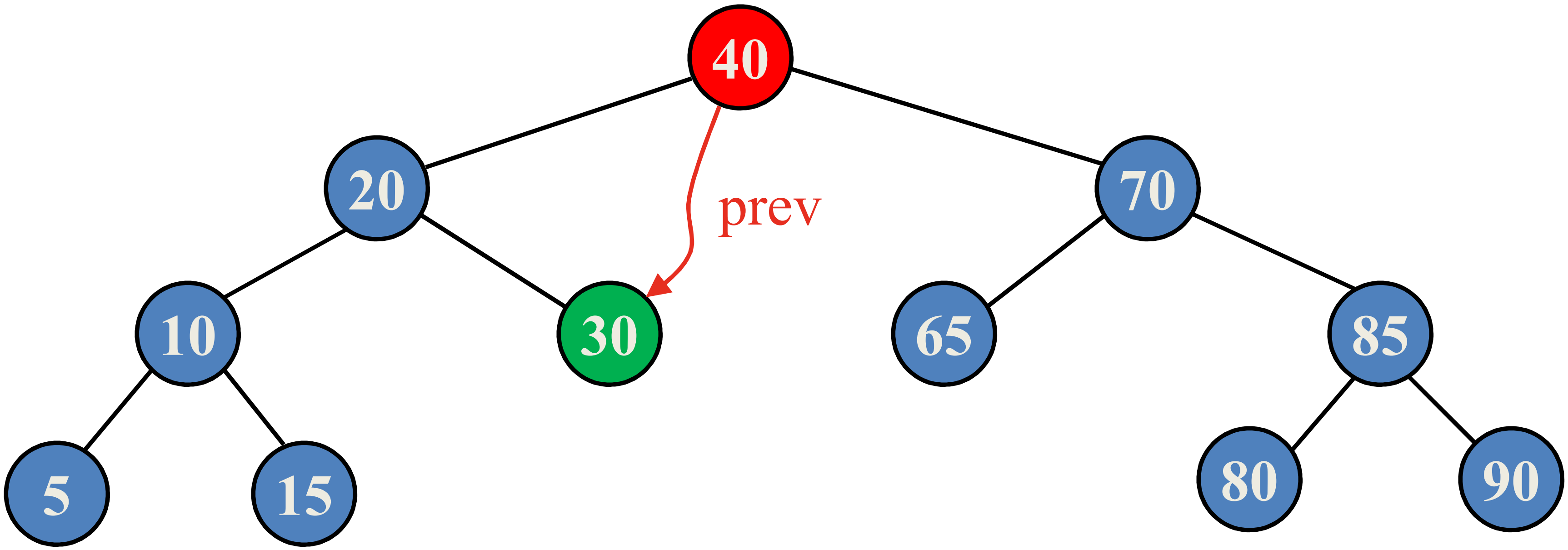
Delete 50



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from AVL Tree

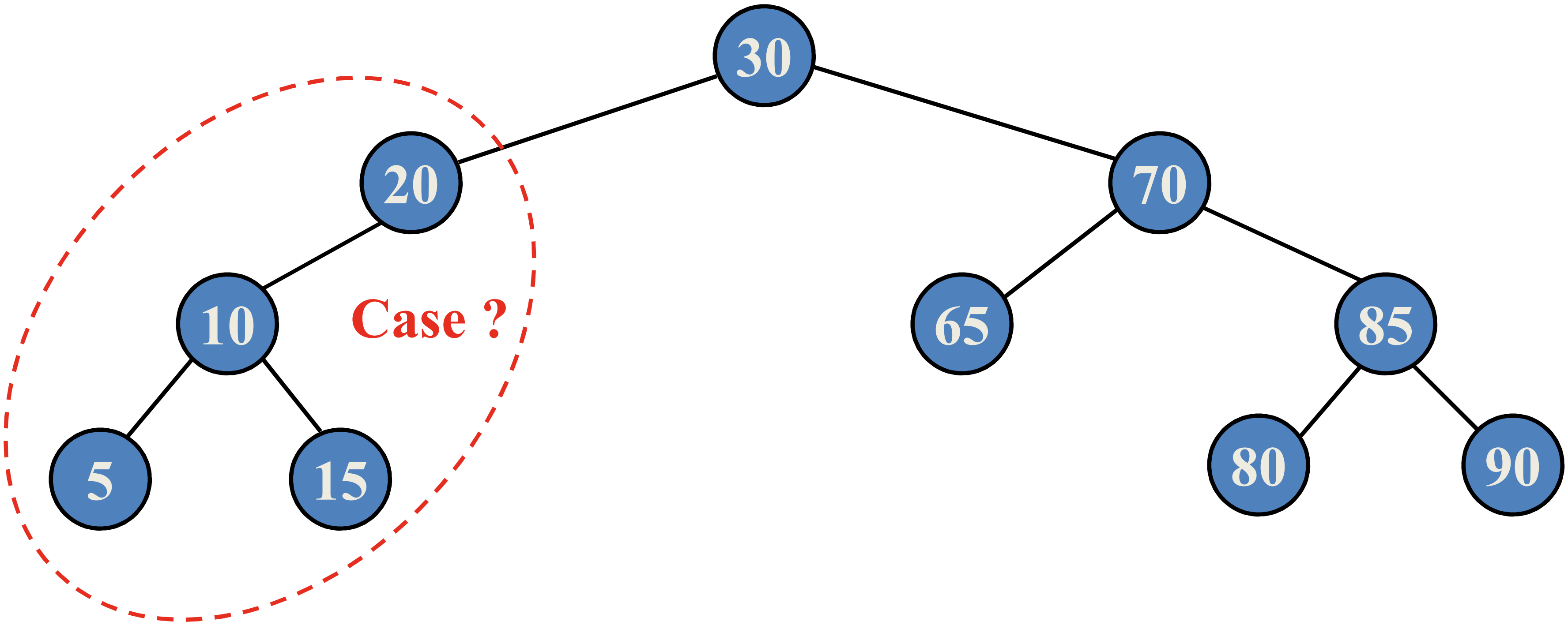
Delete 40



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from AVL Tree

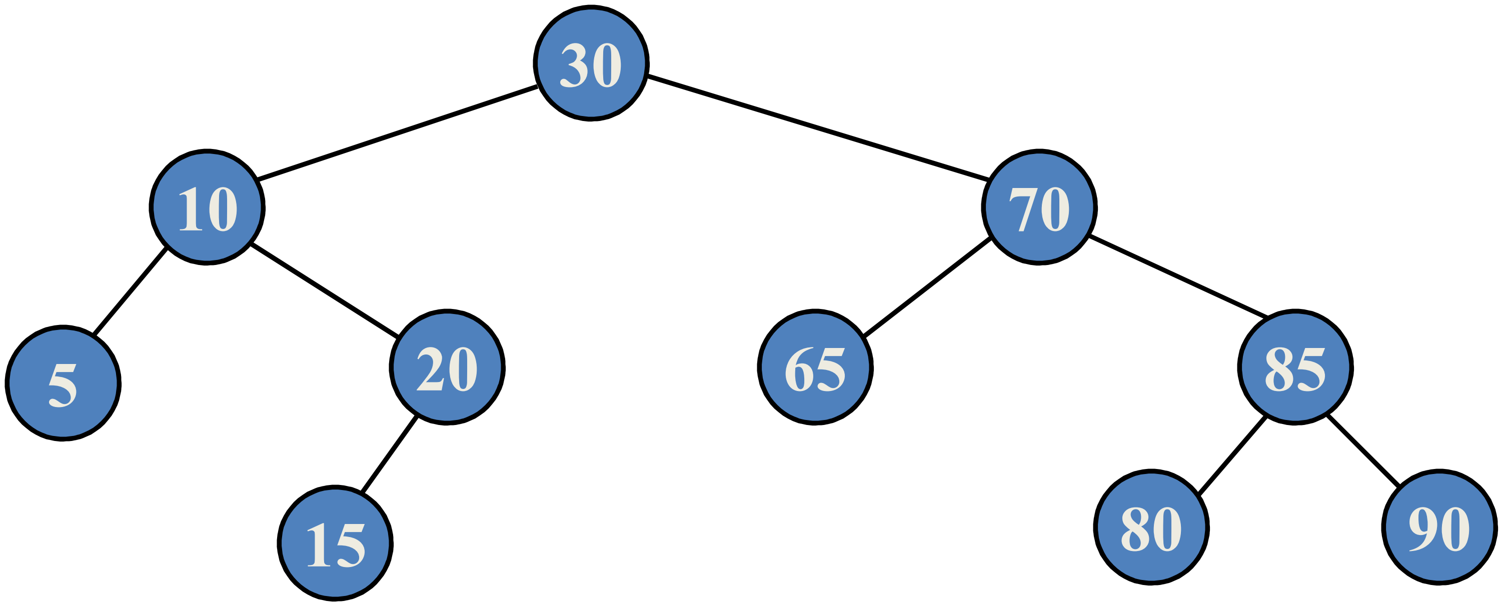
Delete 40



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Delete a Node from AVL Tree

After Rebalancing



Graph Algorithms

Definitions

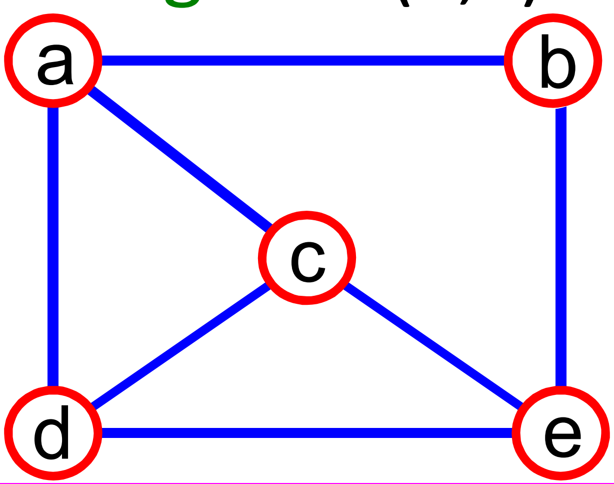
- A graph G = (V,E) is composed of:

    V: Finite, non-empty set of vertices

    E: set of edges connecting the vertices in V

    = Subset of VxV

- An edge e = (u,v) is a pair of vertices.



V= {a, b, c, d, e}

E= {(a,b), (a,c), (a,d), (b,e), (c,d), (c,e), (d,e)}

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Definitions

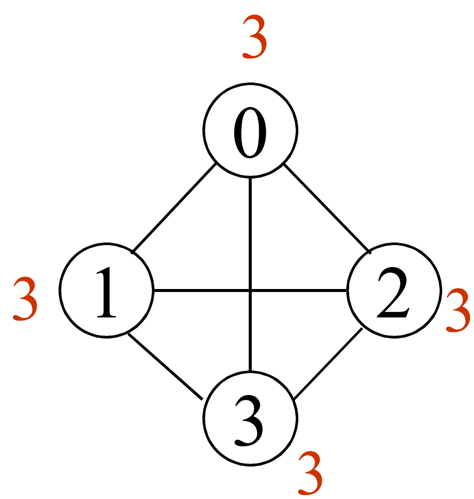
- An **undirected graph** is one in which the pair of vertices in a edge is **unordered**, $(v_0, v_1) = (v_1,v_0)$
- A **directed graph** is one in which each edge is a **directed** pair of vertices, $<v_0, v_1>$!= $<v_1,v_0>$
- If (v_0, v_1) is an edge in an undirected graph,
  - v_0 and v_1 are *adjacent*
  - The edge (v_0, v_1) is incident on vertices v_0 and v_1
- If $<v_0, v_1>$ is an edge in a directed graph
  - v_0 is *adjacent to* v_1, and v_1 is *adjacent from* v_0

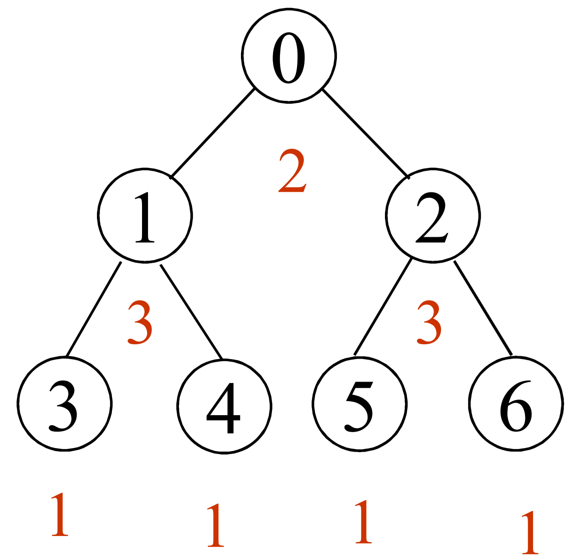
Definitions

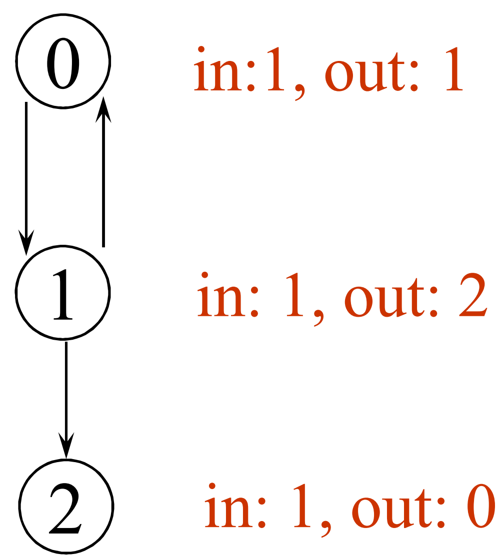
- The degree of a vertex is the number of edges incident to that vertex
- For directed graph,
  - the in-degree of a vertex v is the number of edges that have v as the end vertex
  - the out-degree of a vertex v is the number of edges that have v as the start vertex
  - if d_i is the degree of a vertex i in a graph G with n vertices and e edges, the number of edges is

$$e = \left(\sum_{0}^{n-1} d_i \right) / 2$$

Definitions



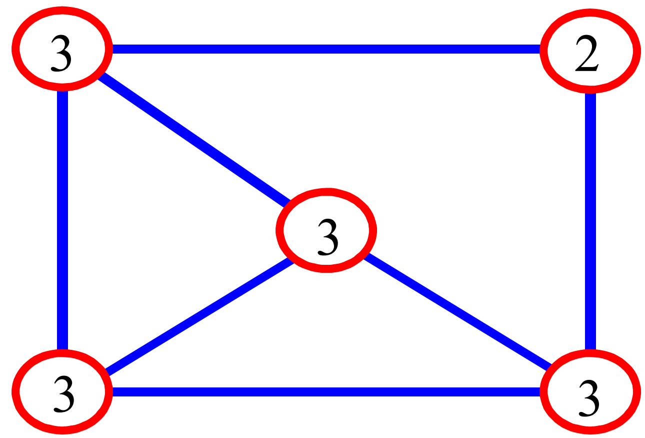


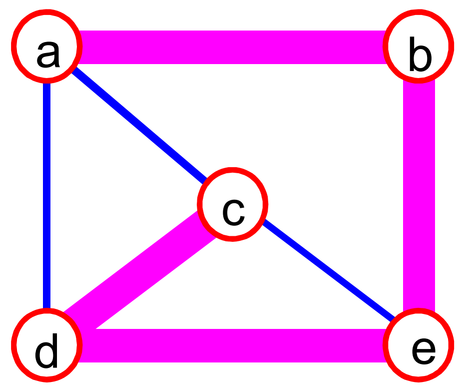


Bhaskar Sardar,* Information Technology Department, *Jadavpur University, India

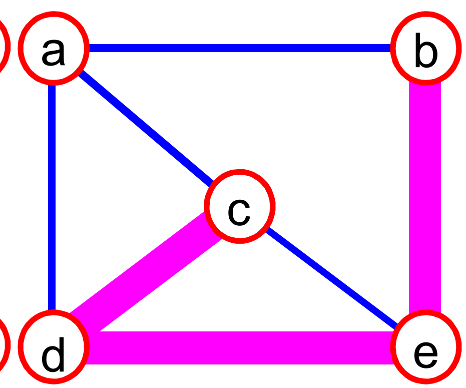
Definitions

- path: sequence of vertices $v_1, v_2, . . . v_k$ such that consecutive vertices v_i and v_{i+1} are adjacent.





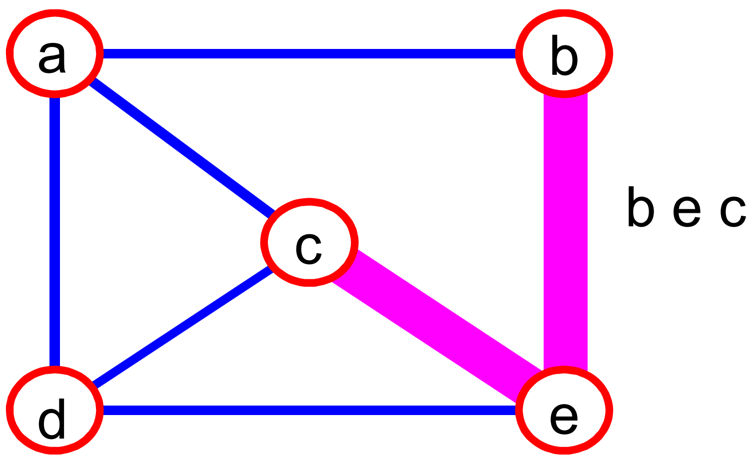
a b e d c

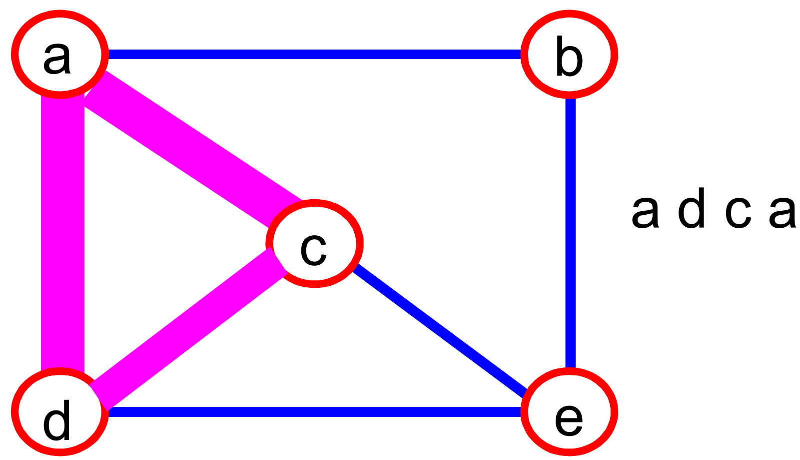


b e d c

Definitions

- simple path: no repeated vertices
- cycle: simple path, except that the last vertex is the same as the first vertex

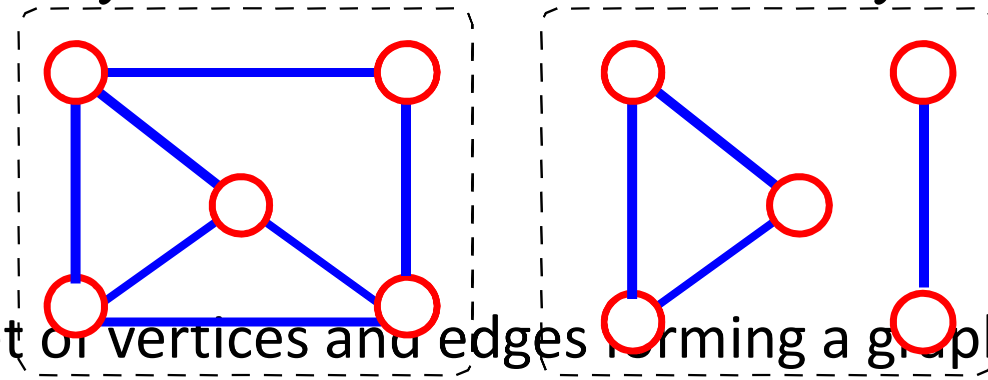




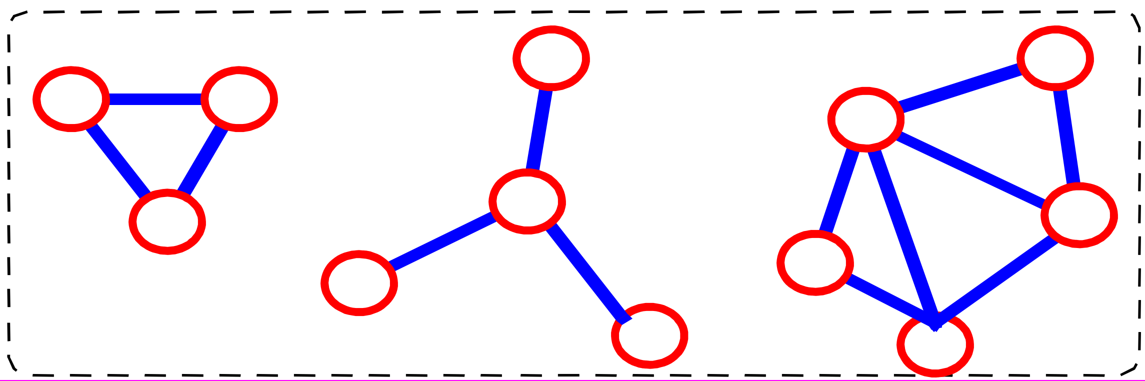
Bhaskar Sardar, Information Technology Department, Jadavpur University, India

Definitions

- connected graph: any two vertices are connected by some path



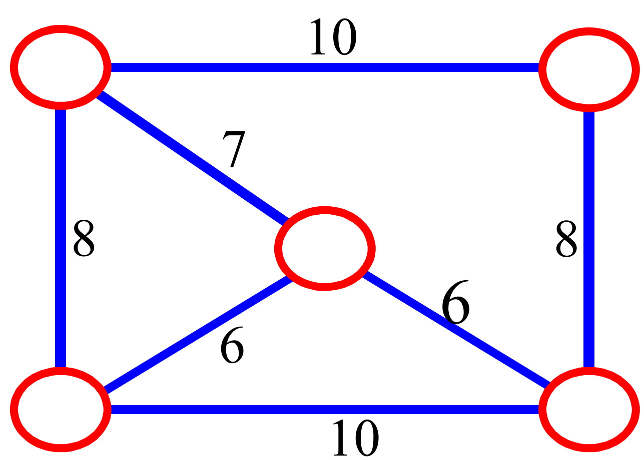
- subgraph: subset of vertices and edges forming a graph
- connected component: collection of subgraphs which are not connected. e.g., the graph below has 3 connected components.



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Definitions

- A *weighted graph* associates weights with the edges
  - e.g., a road map: edges might be weighted with distance



Definitions

- We will typically express running times in terms of $|E|$ and $|V|$
  - If $|E| \approx |V|^2$ the graph is *dense*
  - If $|E| \approx |V|$ the graph is *sparse*

- If you know you are dealing with dense or sparse graphs, different data structures may make sense

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Graph Representation

- Adjacency Matrix
- Adjacency Lists
- Incidence Matrix

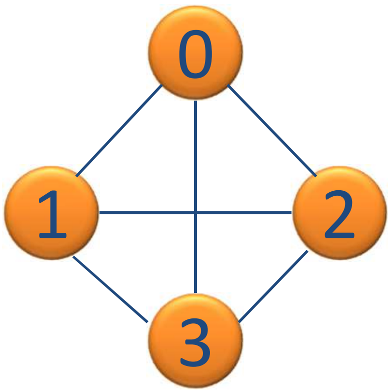
Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Adjacency Matrix

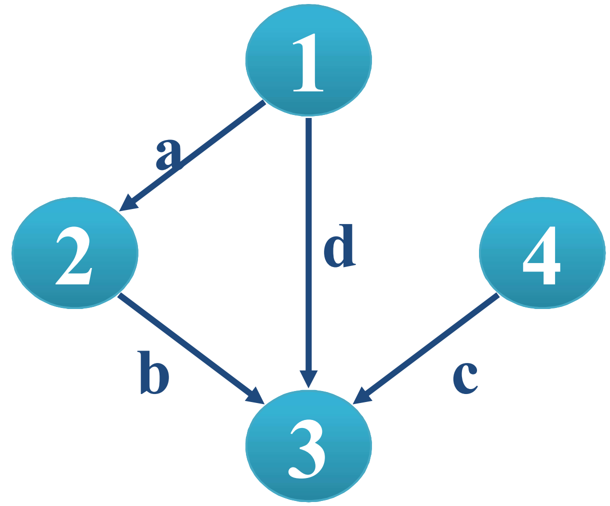
- Let G=(V,E) be a graph with n vertices.
- The adjacency matrix of G is a two-dimensional nxn array, say adj_mat
  - ✓ If the edge (v_i, v_j) is in E(G), adj_mat[i][j]=1
  - ✓ If there is no such edge in E(G), adj_mat[i][j]=0

- The adjacency matrix for an undirected graph is symmetric; the adjacency matrix for a directed graph need not be symmetric

Adjacency Matrix



A	0	1	2	3
0	0	1	1	1
1	1	0	1	1
2	1	1	0	1
3	1	1	1	0



A	1	2	3	4
1	0	1	1	0
2	0	0	1	0
3	0	0	0	0
4	0	0	1	0

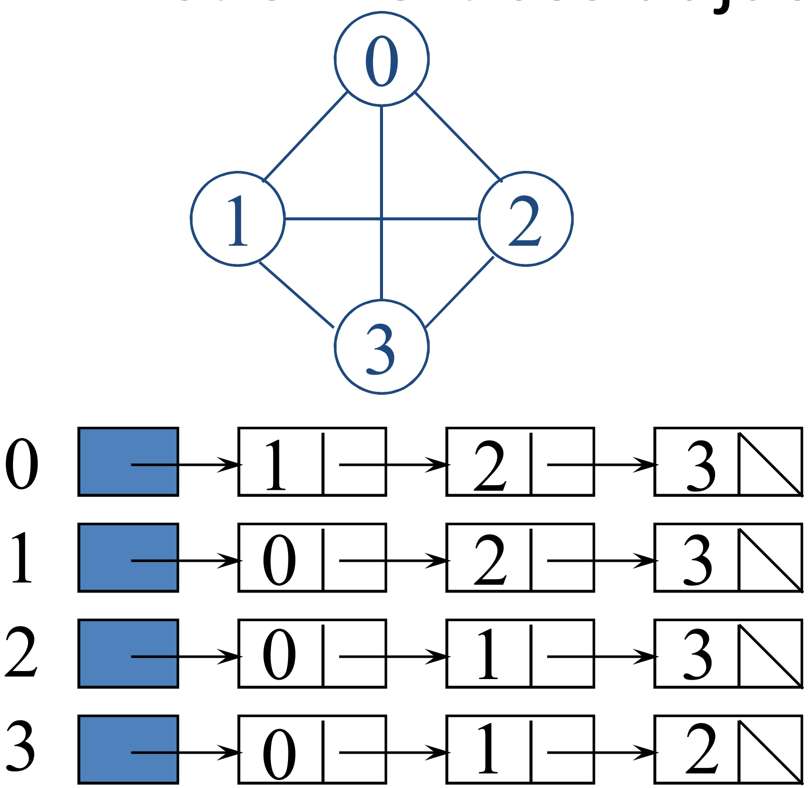
Adjacency Matrix

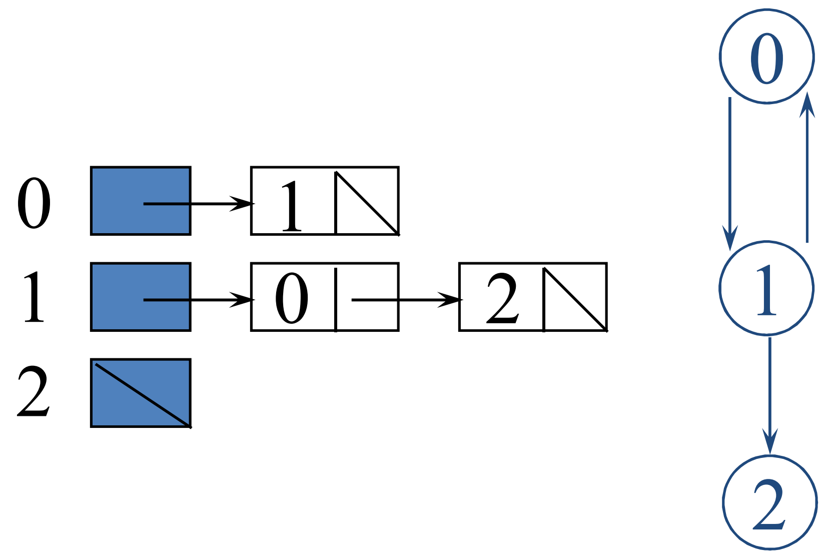
- From the adjacency matrix, to determine the connection of vertices is easy
  - The degree of a vertex i is the number of 1's in i^{th} row
  - For a directed graph, the number of 1's in i^{th} row is the out_degree, while the number of 1's in i^{th} column in_degree.
- **Time:** to list all vertices adjacent to u: O(V).
- **Time:** to determine if $(u, v) \in E$: O(1).
- **Space:** O(V^2).
  - Not memory efficient for large graphs.
- ***Parallel edges*** cannot be represented
- Can store weights instead of bits for weighted graph.

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Adjacency Lists

- Adjacency list: for each vertex $v \in V$, store a list of vertices adjacent to v





Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Adjacency Lists

```
typedef struct adjvertex
{
        int vertex;
        struct node *next;
}adjvertex;
```

```
typedef struct graph
{
        int no_of_Vertices;
        adjvertex *adjlist [100];
}graph;
```


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Adjacency Lists

- How much storage is required?
  - The *degree* of a vertex *v* = # incident edges
    - Directed graphs have in-degree, out-degree
  - For directed graphs, # of items in adjacency lists is

    Σ out-degree(v) = |E|

    takes O(V + E) storage
  - For undirected graphs, # items in adjacency lists is

    Σ degree(v) = 2 |E|

    also O(V + E) storage
- So: Adjacency lists take O(V+E) storage

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Incidence Matrix

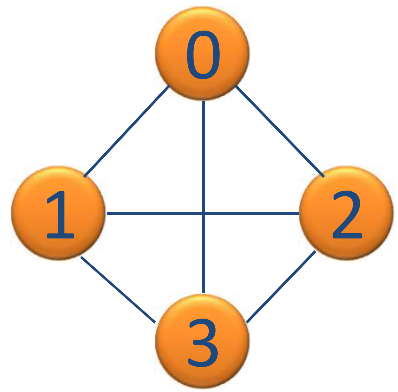
- Consider a matrix $A = (a_{ij})$, rows corresponds to vertices, column corresponds to edges.
- For undirected graph:
  - a_{ij}= 1 if e_j is incedent to v_i

    = 0 otherwise
- For directed graph:
  - a_{ij}= 1 if e_j is incedent out of v_i

    = -1 if e_j is incedent into v_i

    = 0 otherwise

Incidence Matrix



A	(0,1)	(0, 2)	(0, 3)	(1, 2)	(1, 3)	(2, 3)
0	1	1	1	0	0	0
1	1	0	0	1	1	0
2	0	1	0	1	0	1
3	0	0	1	0	1	1

Graph Traversal

- Given: a graph $G = (V, E)$, directed or undirected
- Goal: systematically explore every vertex and every edge
- Ultimately: build a tree on the graph
  - Pick a vertex as the root
  - Choose certain edges to produce a tree

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Graph Traversal

- Depth First Search
  - Once a possible path is found, continue the search until the end of the path
  - Think of a Stack
- Breadth First Search
  - Start several paths at a time, and advance in each one step at a time
  - Think of a Queue

Depth First Search

- We start at vertex ***s***, and mark ***s*** "visited". Next we label ***s*** as our current vertex called ***u***.
- Now we travel along an arbitrary edge (***u***, ***v***).
- If edge (***u***, ***v***) leads us to an already visited vertex ***v*** we return to ***u***.
- If vertex ***v*** is unvisited, we move to ***v***, mark ***v*** "visited", set ***v*** as our current vertex, and repeat the previous steps.

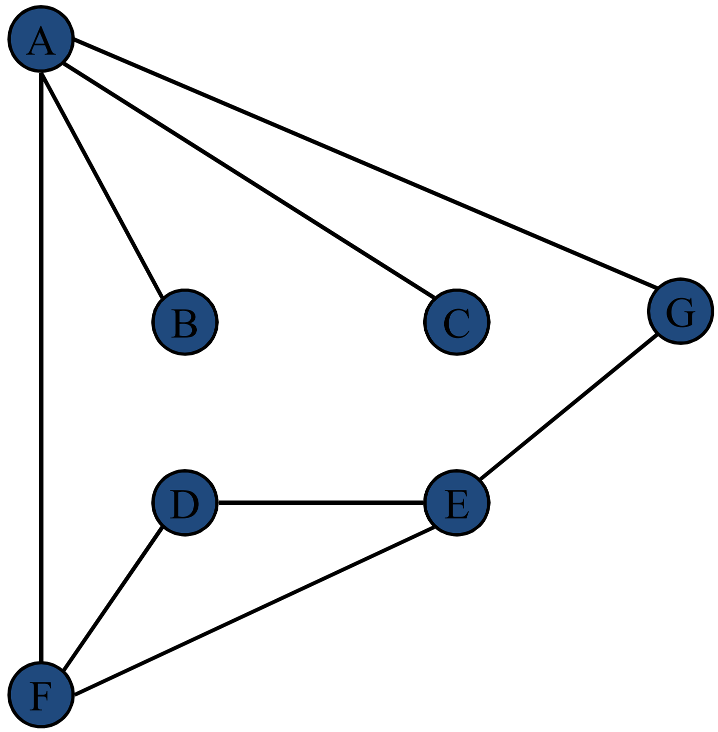
Depth First Search

```
void DFS (int start)
{
        int v;
        adjvertex *adj;
        visited [start] = 1;
        printf (“%d”, start);
        adj = g→adjlist [start];
        while (adj ! = NULL)
        {
                v = adj→vertex;
                if (! visited [v])
                        DFS (v);
                adj = adj →next;
        }
}
```

Total running time: O(V+E)

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search

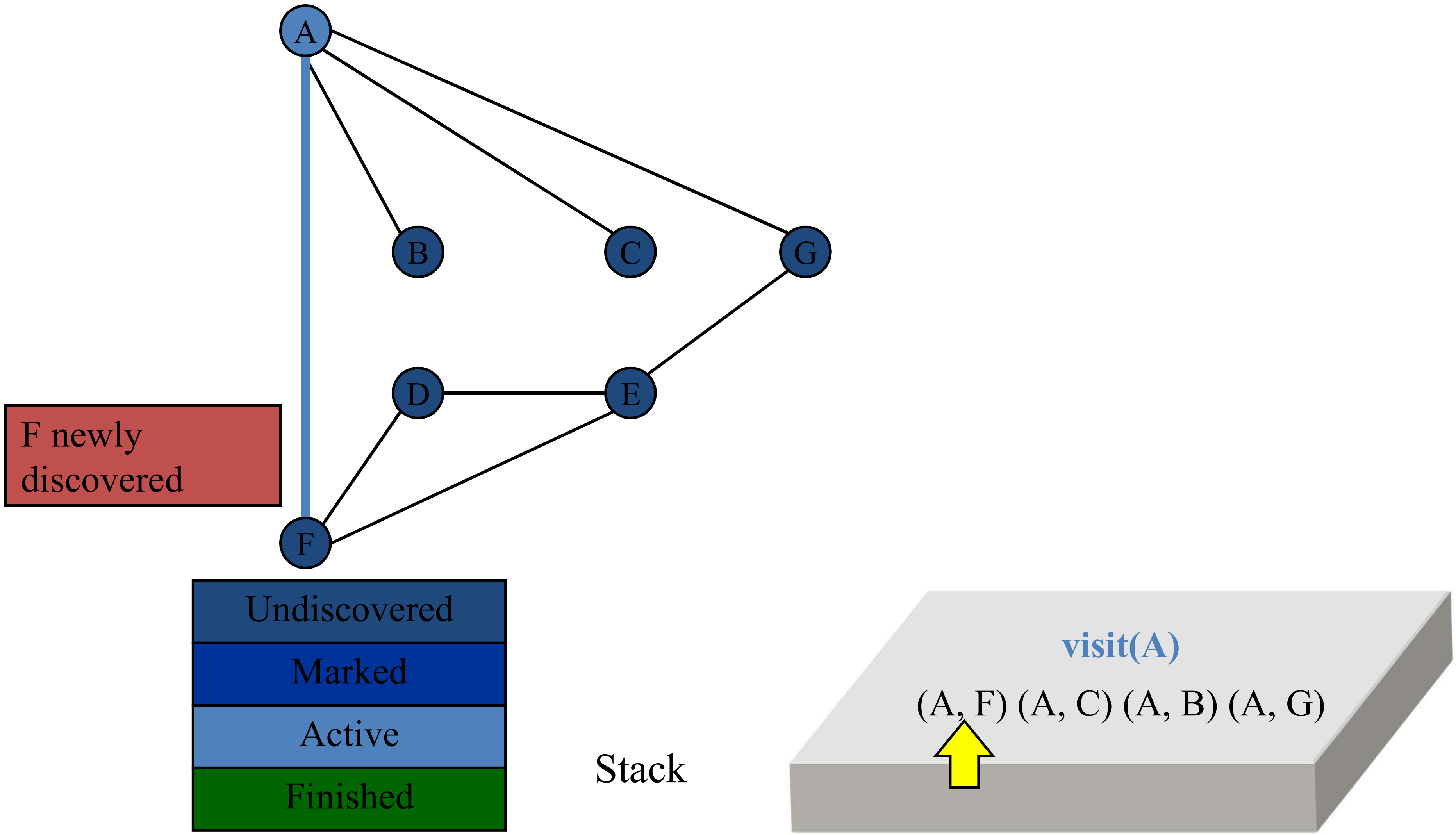


Adjacency Lists

```
A:   F C B G
B:   A
C:   A
D:   F E
E:   G F D
F:   A E D
G:   E A
```

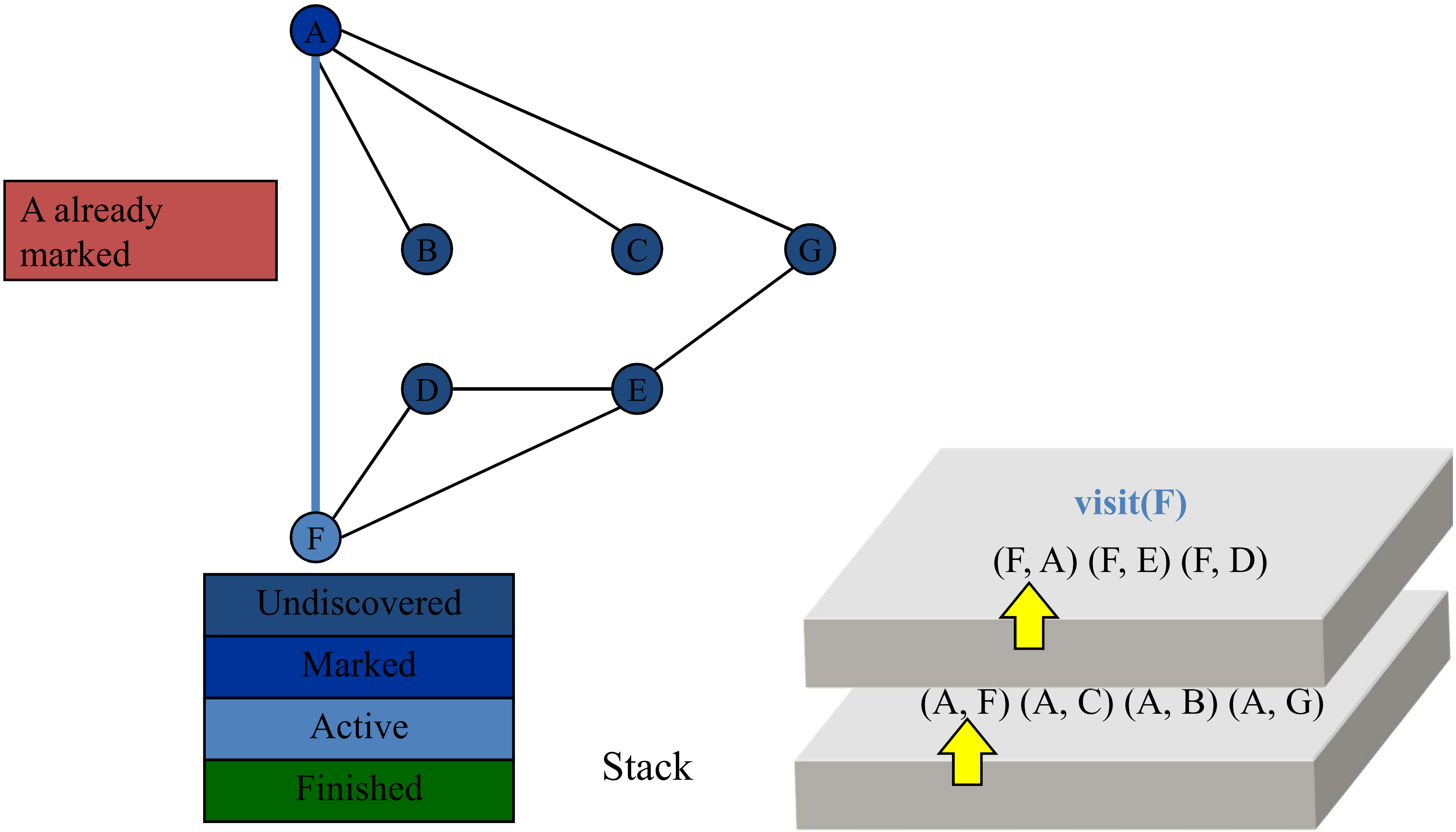

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



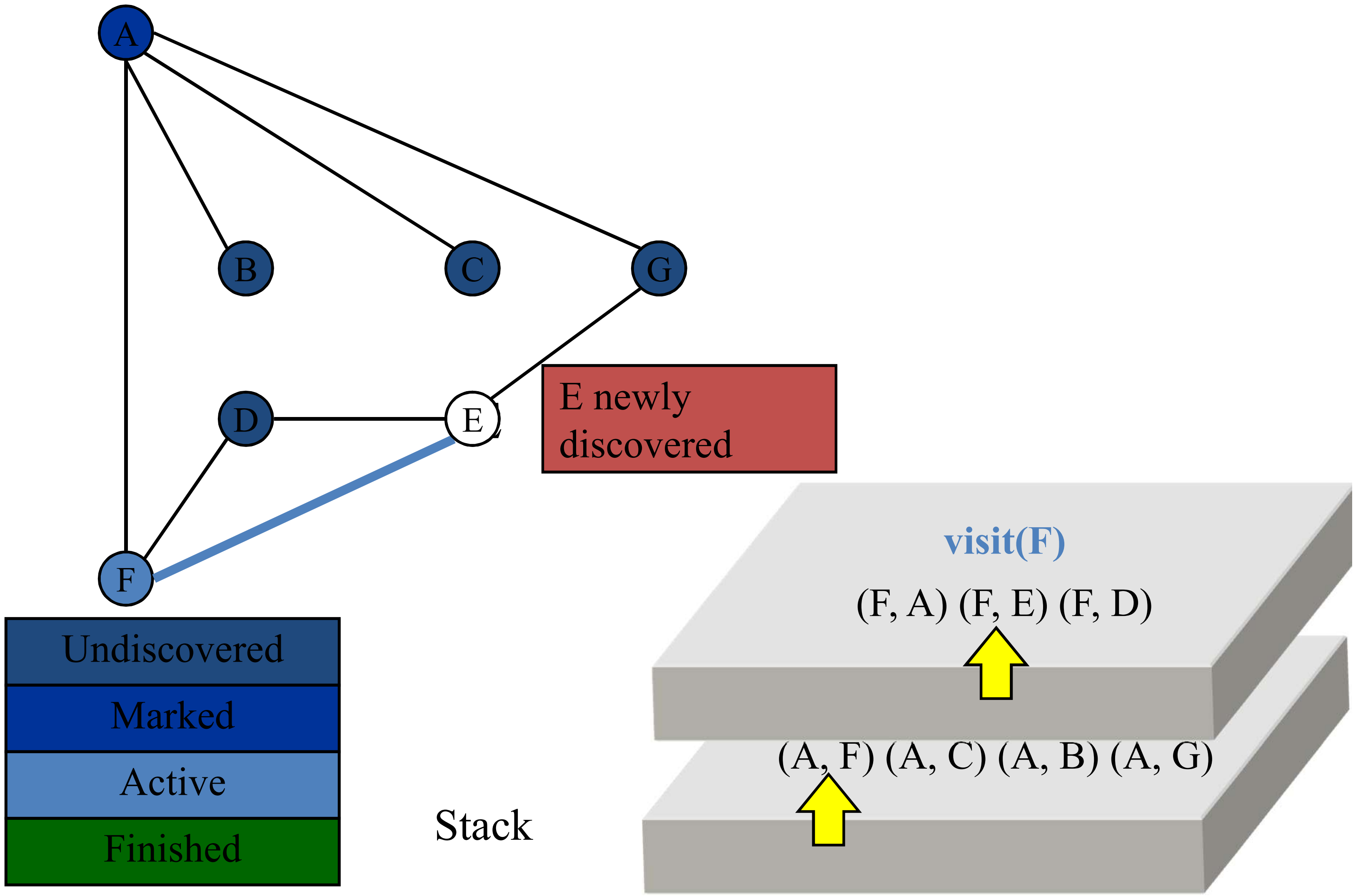
Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Depth First Search



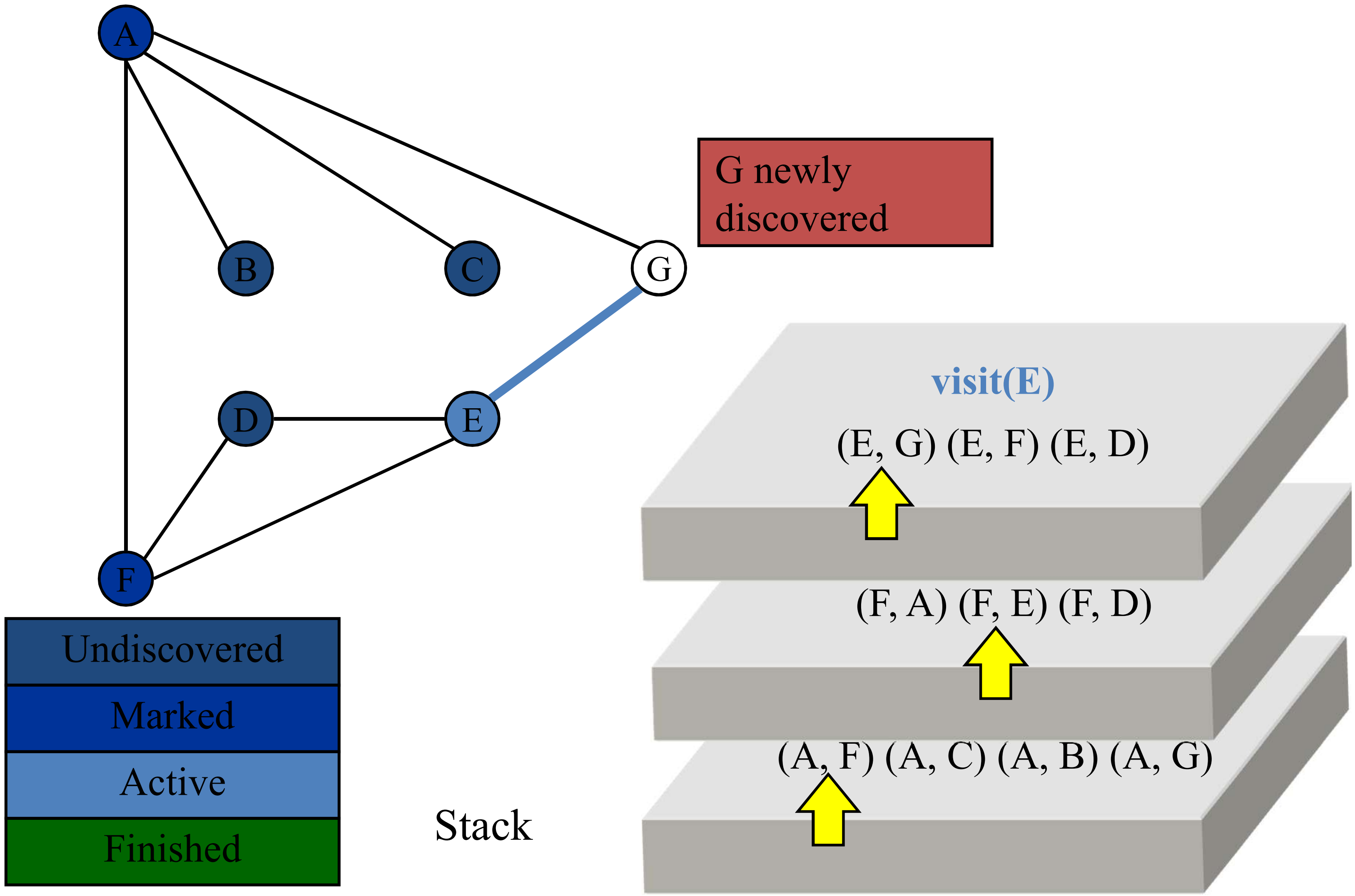
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



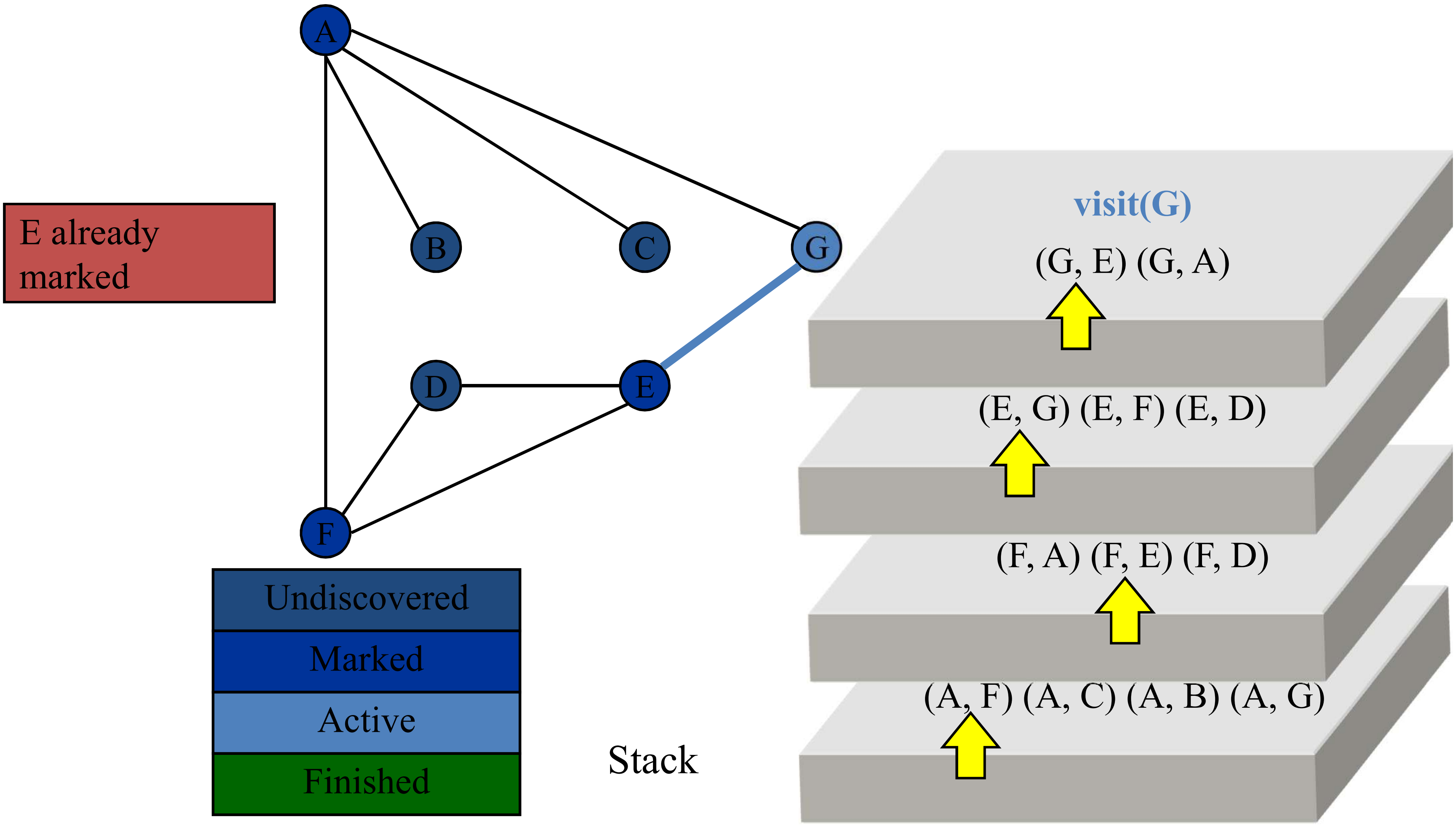
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



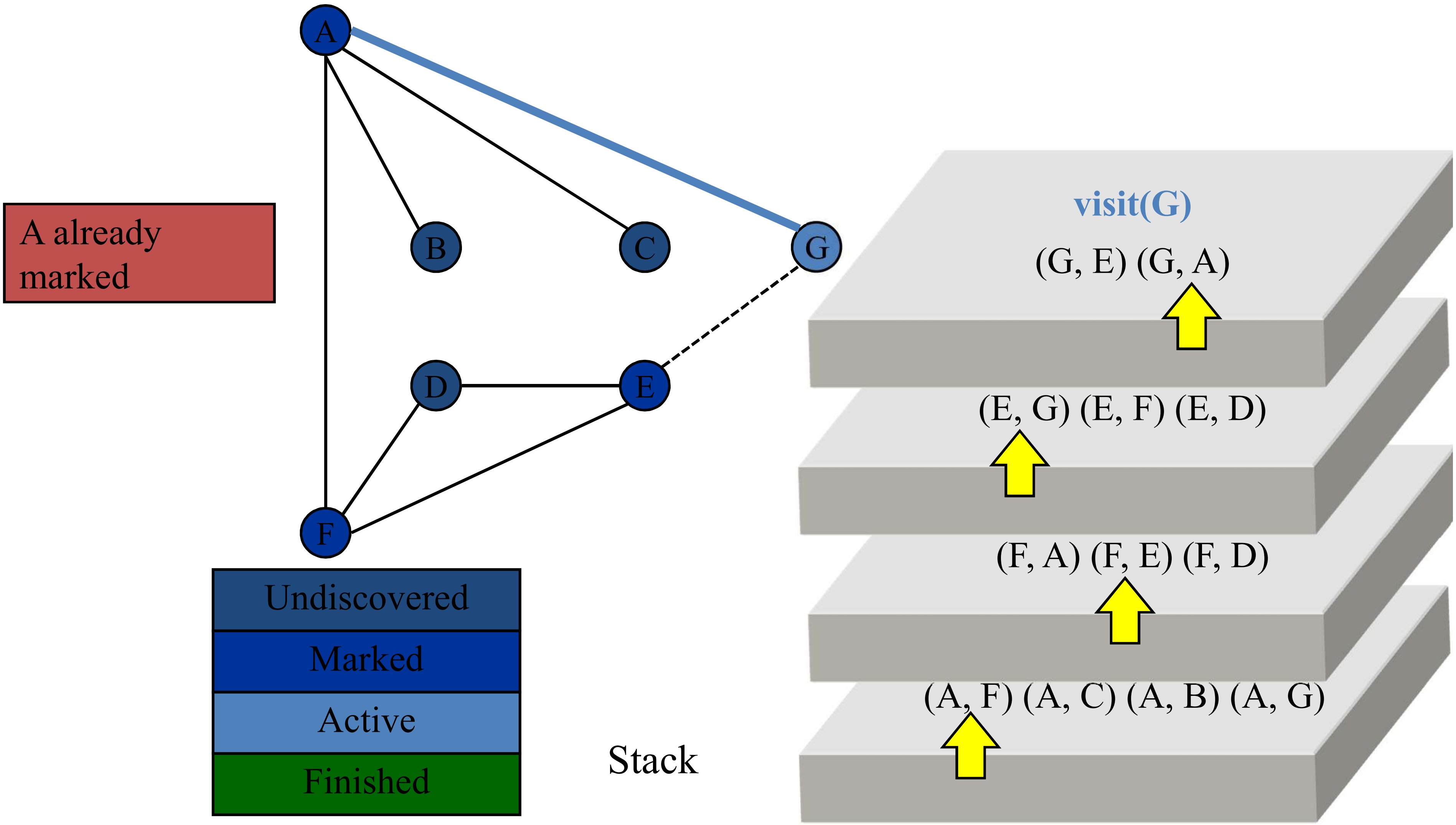
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



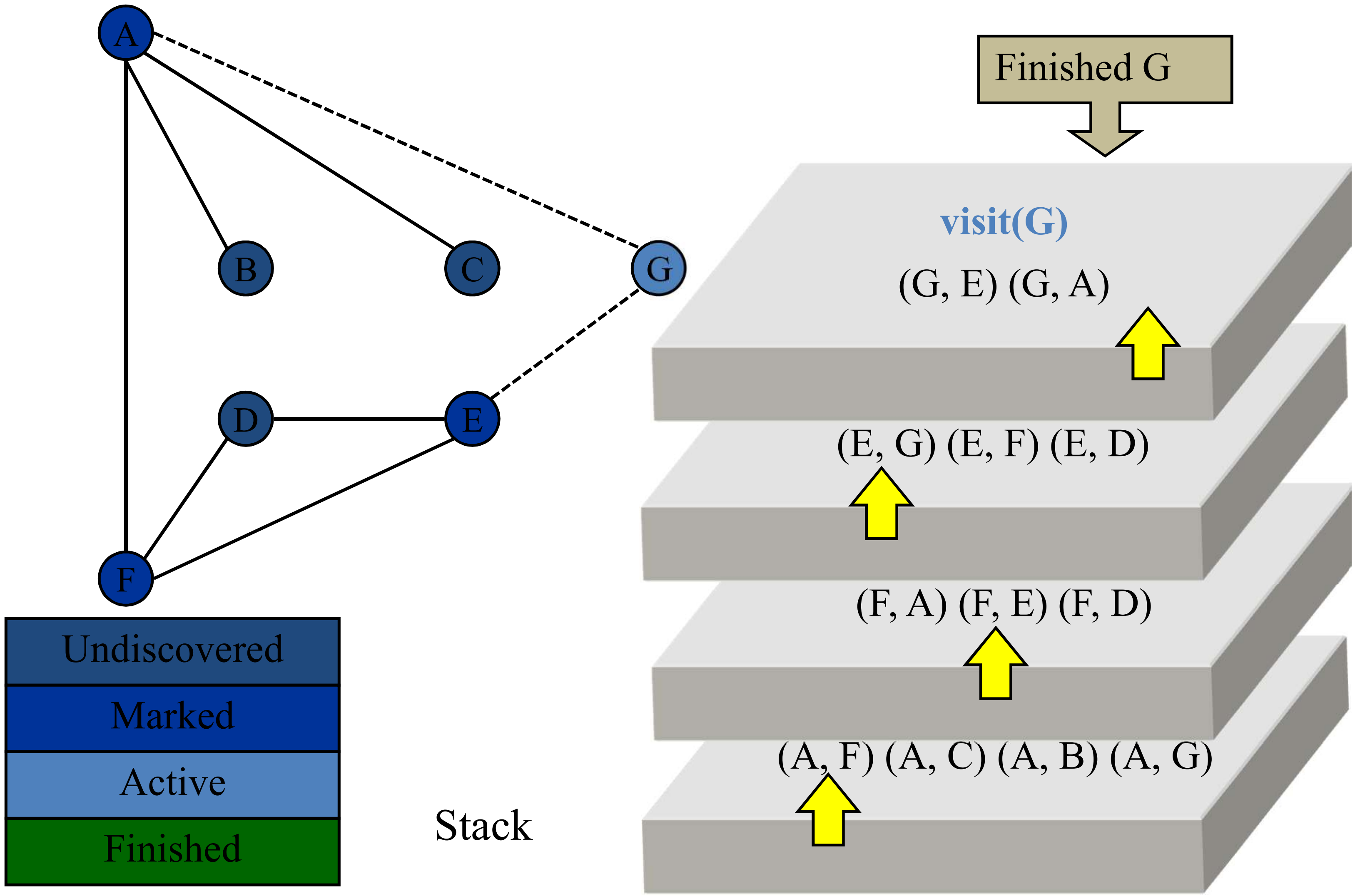
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search

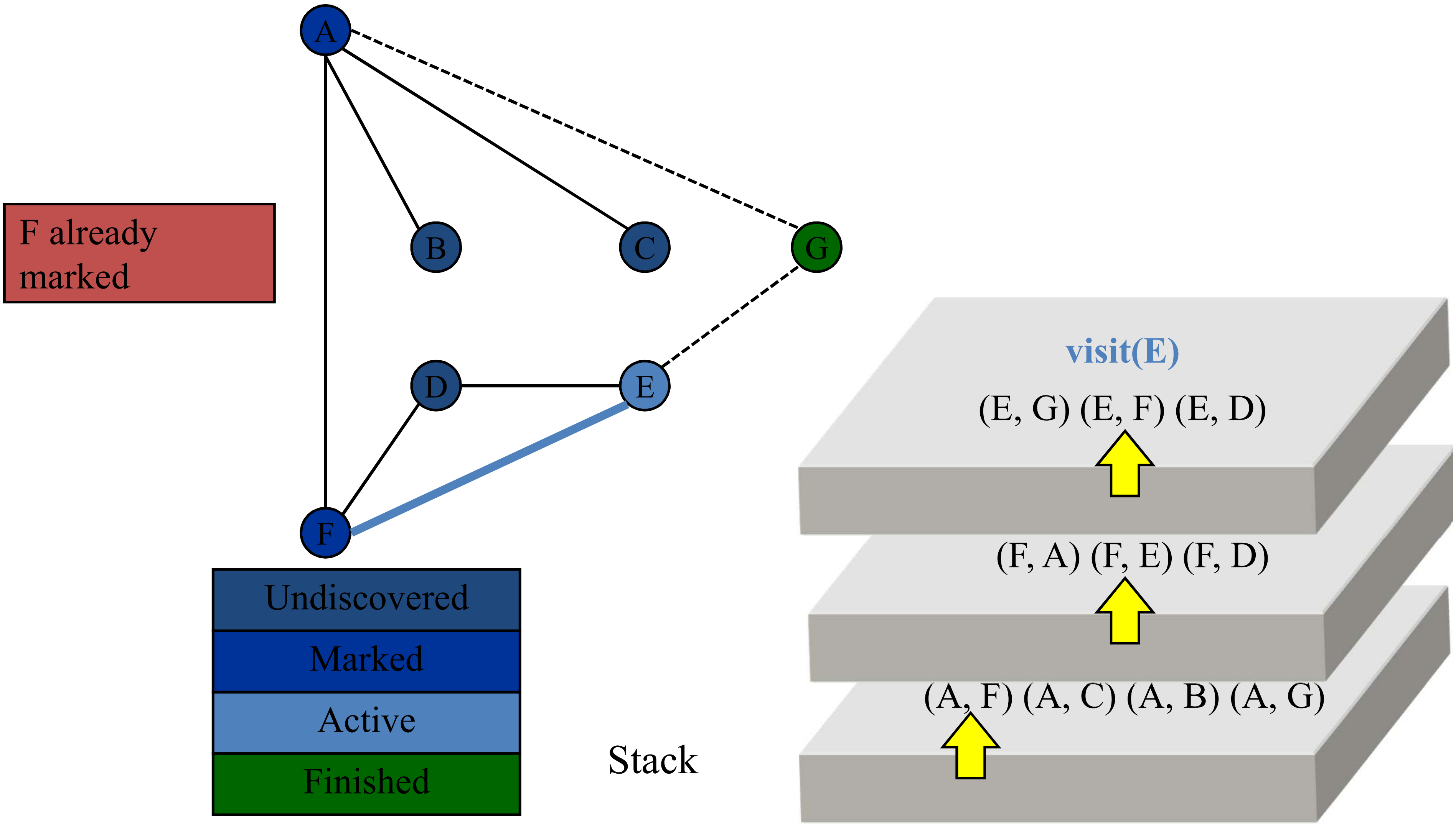


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search

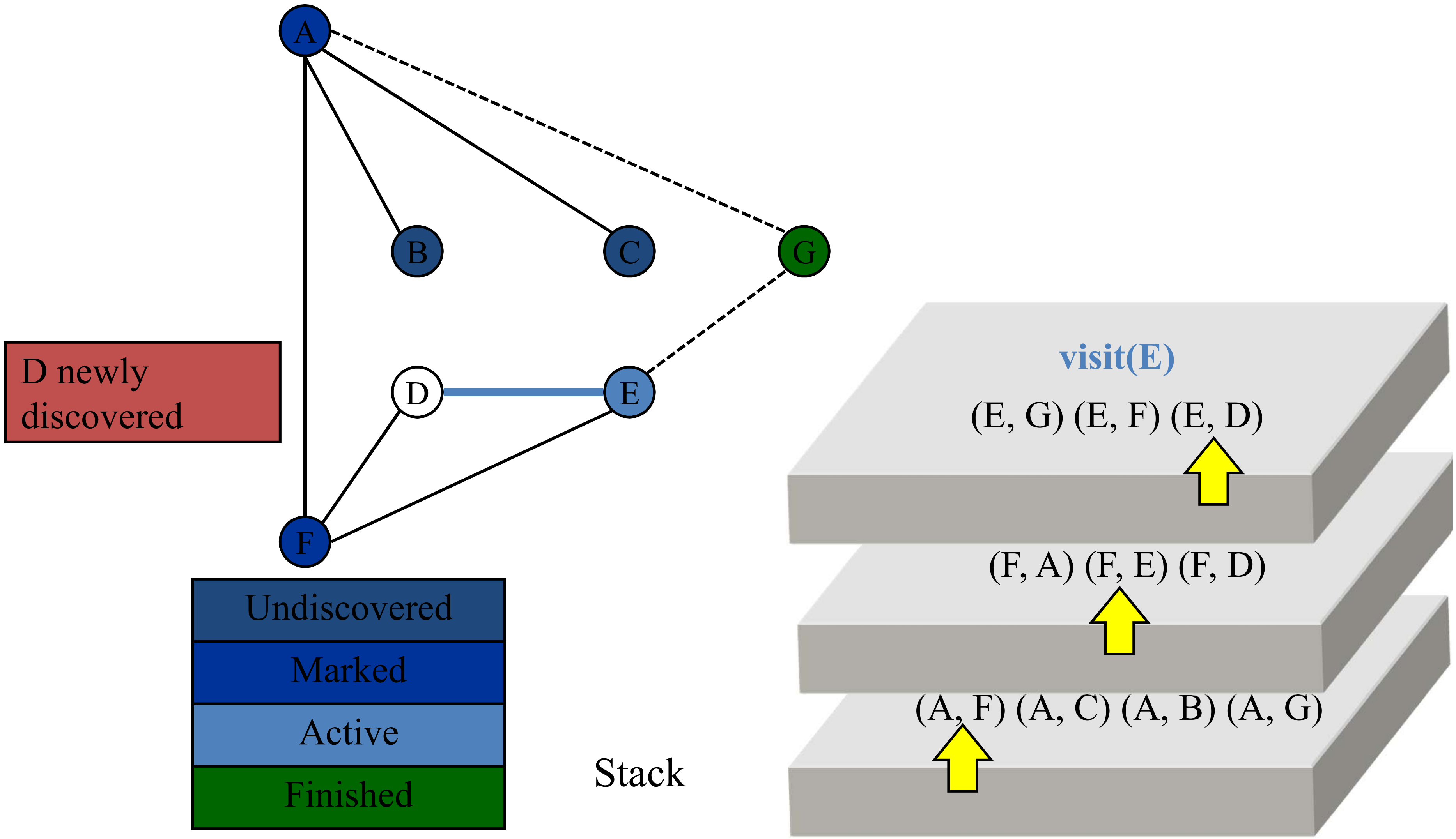


Depth First Search



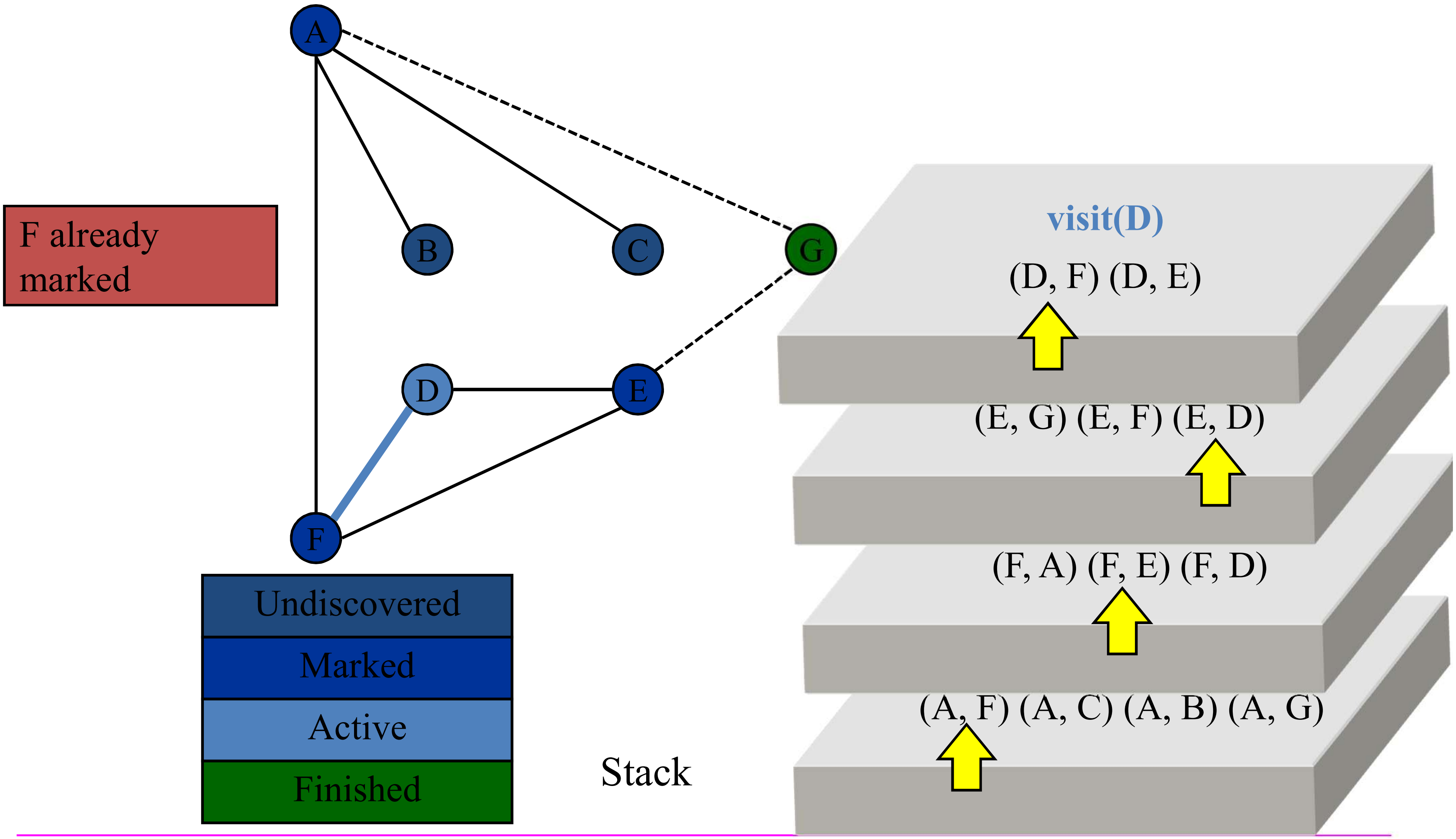
Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Depth First Search



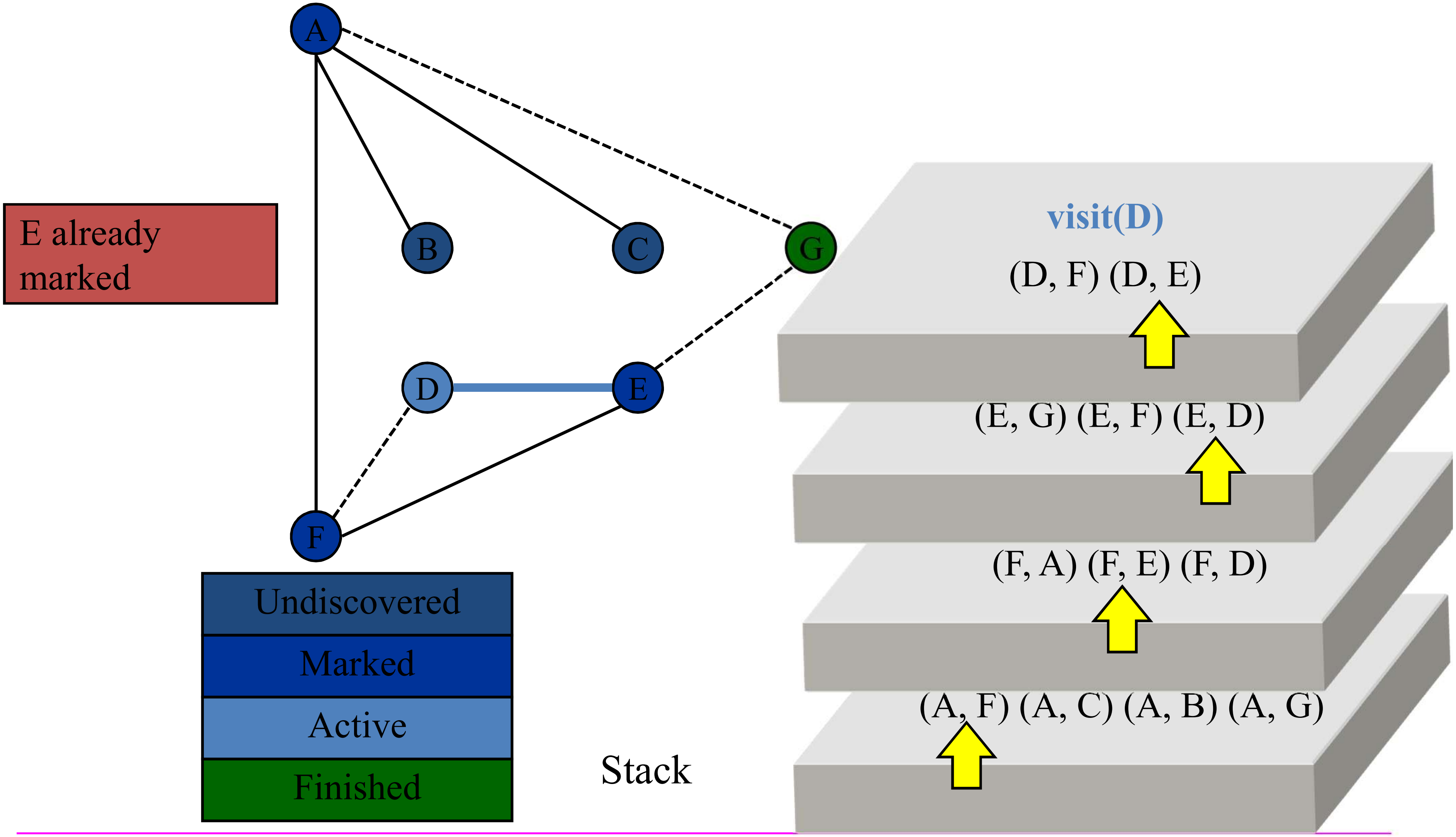
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search

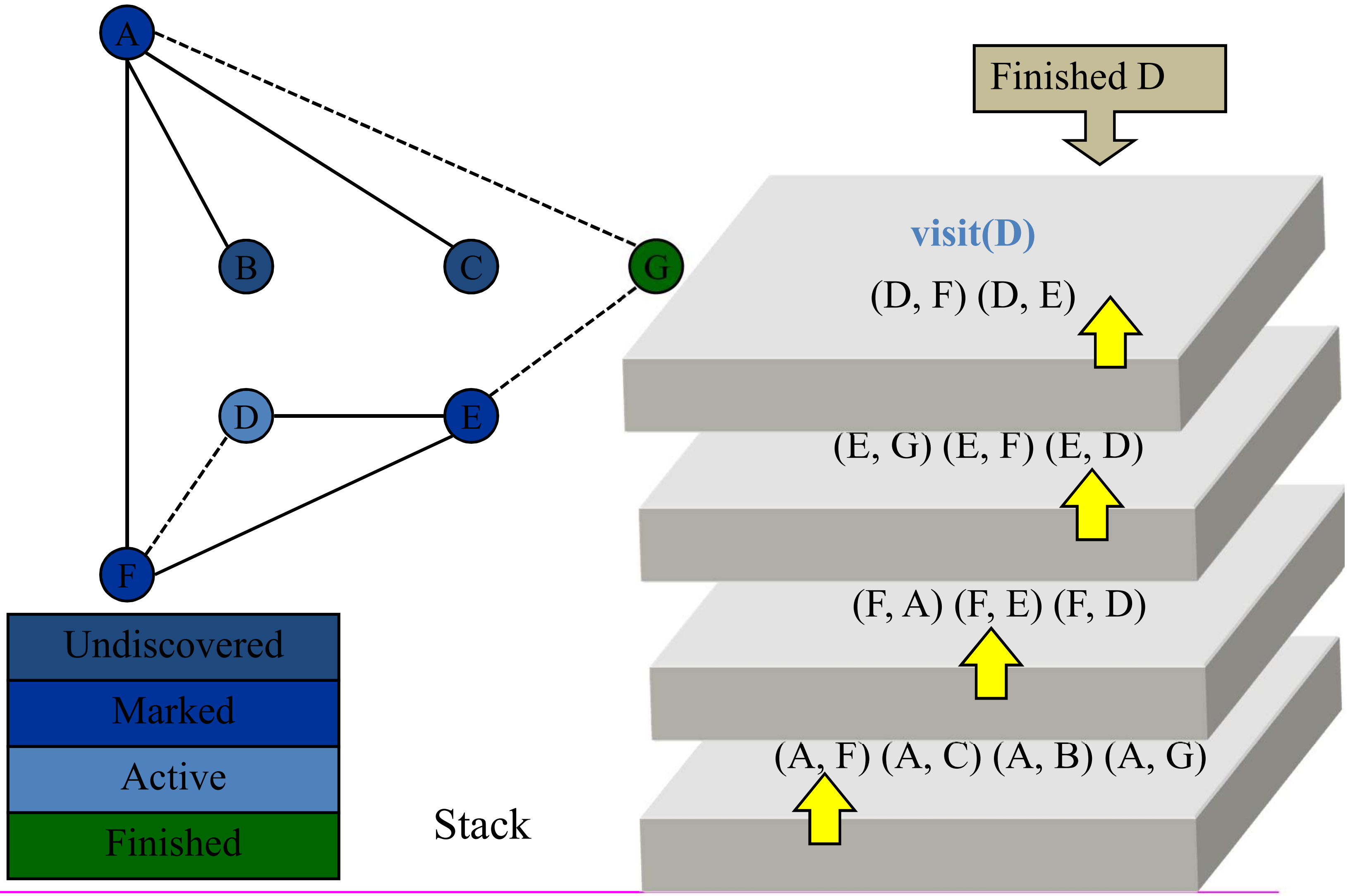


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search

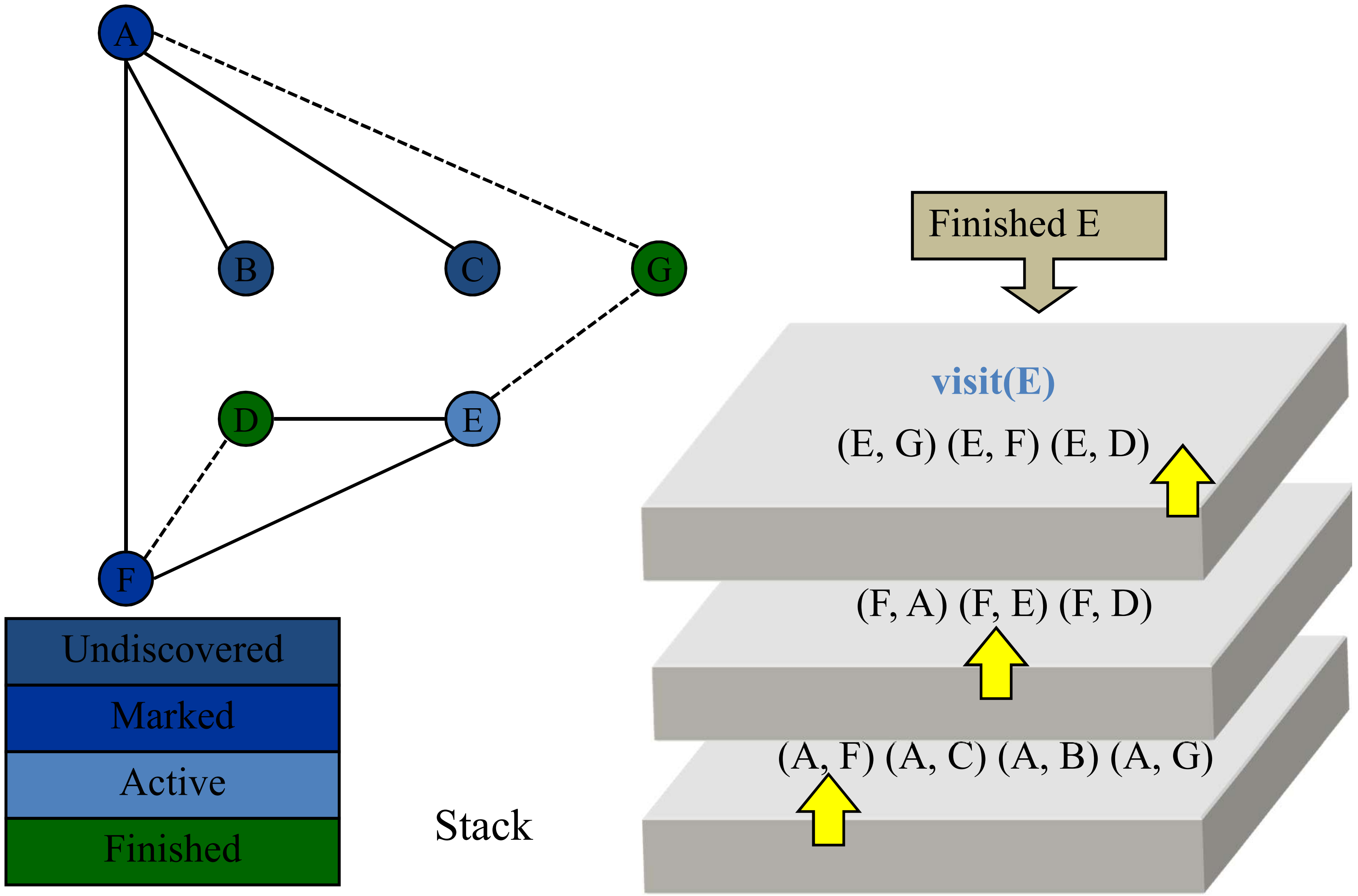


Depth First Search



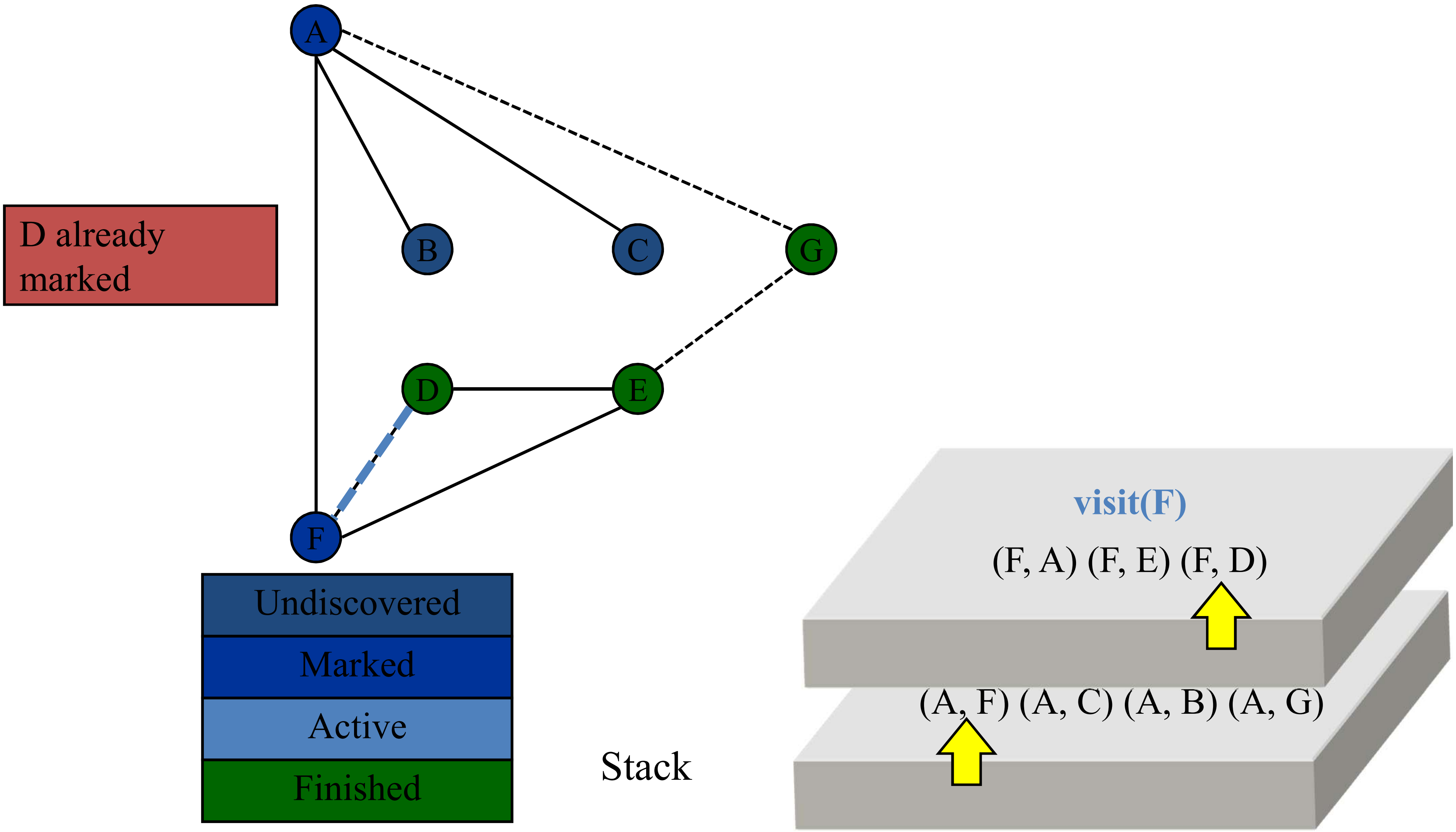
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



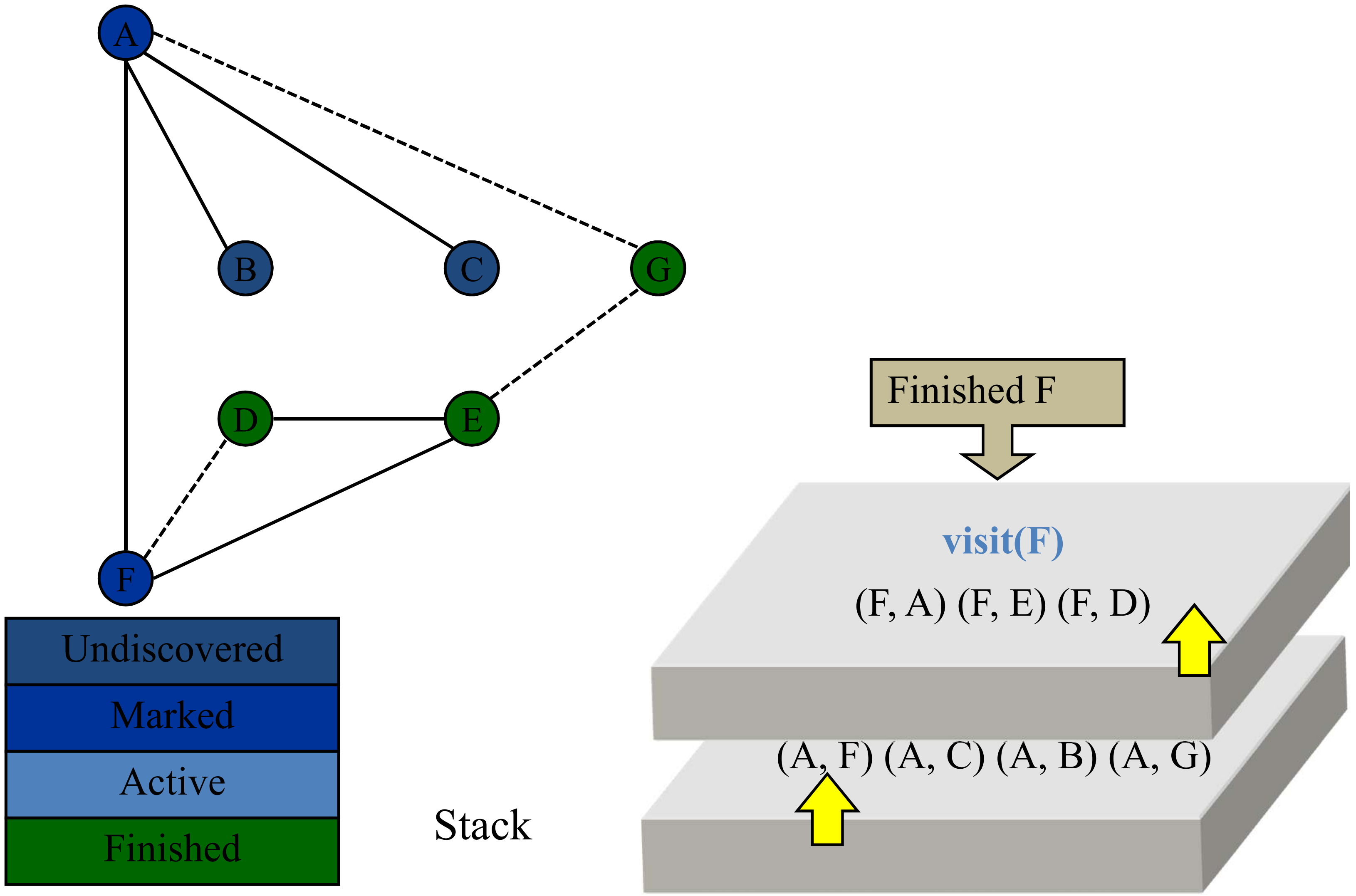
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search

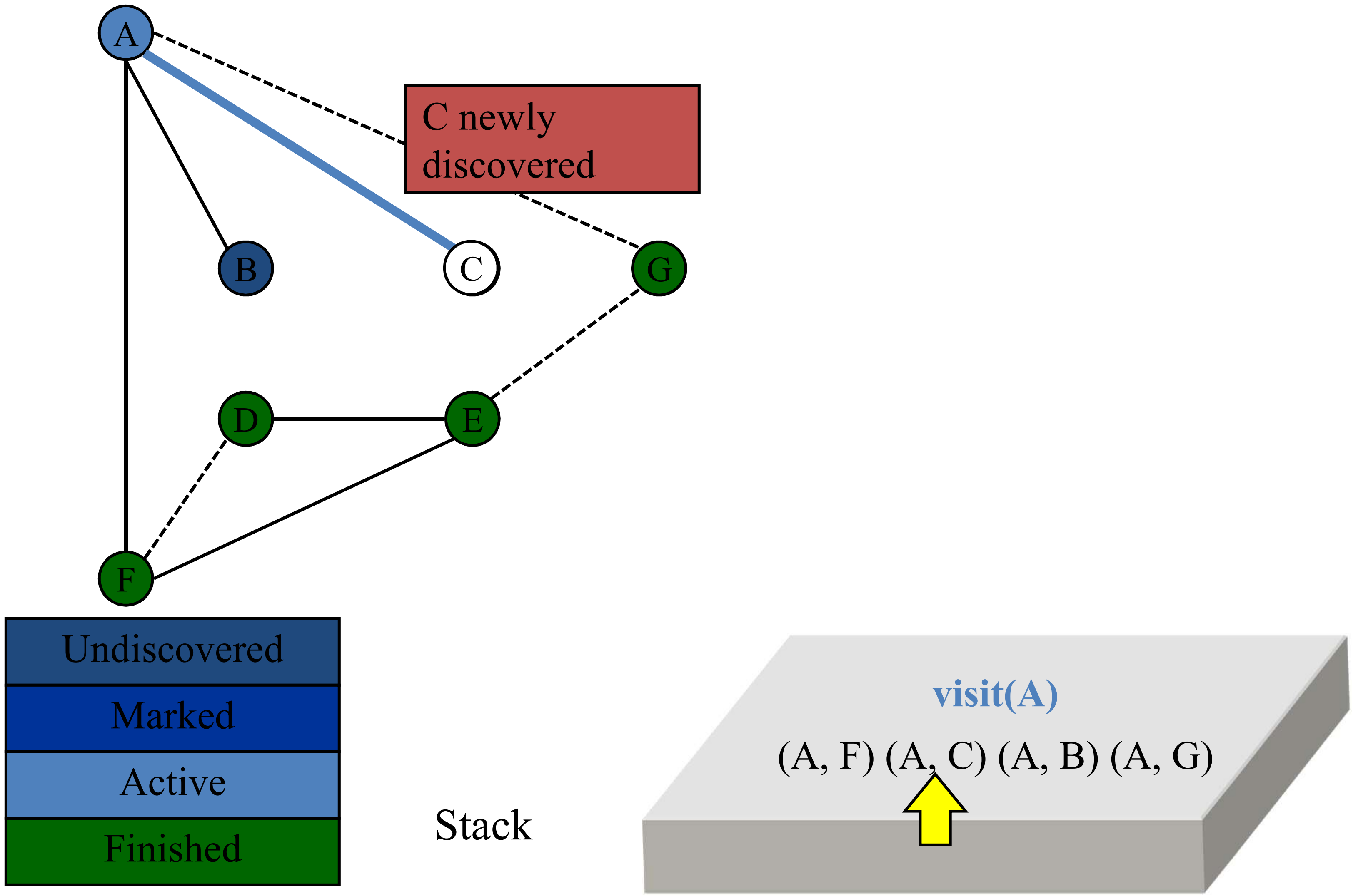


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search

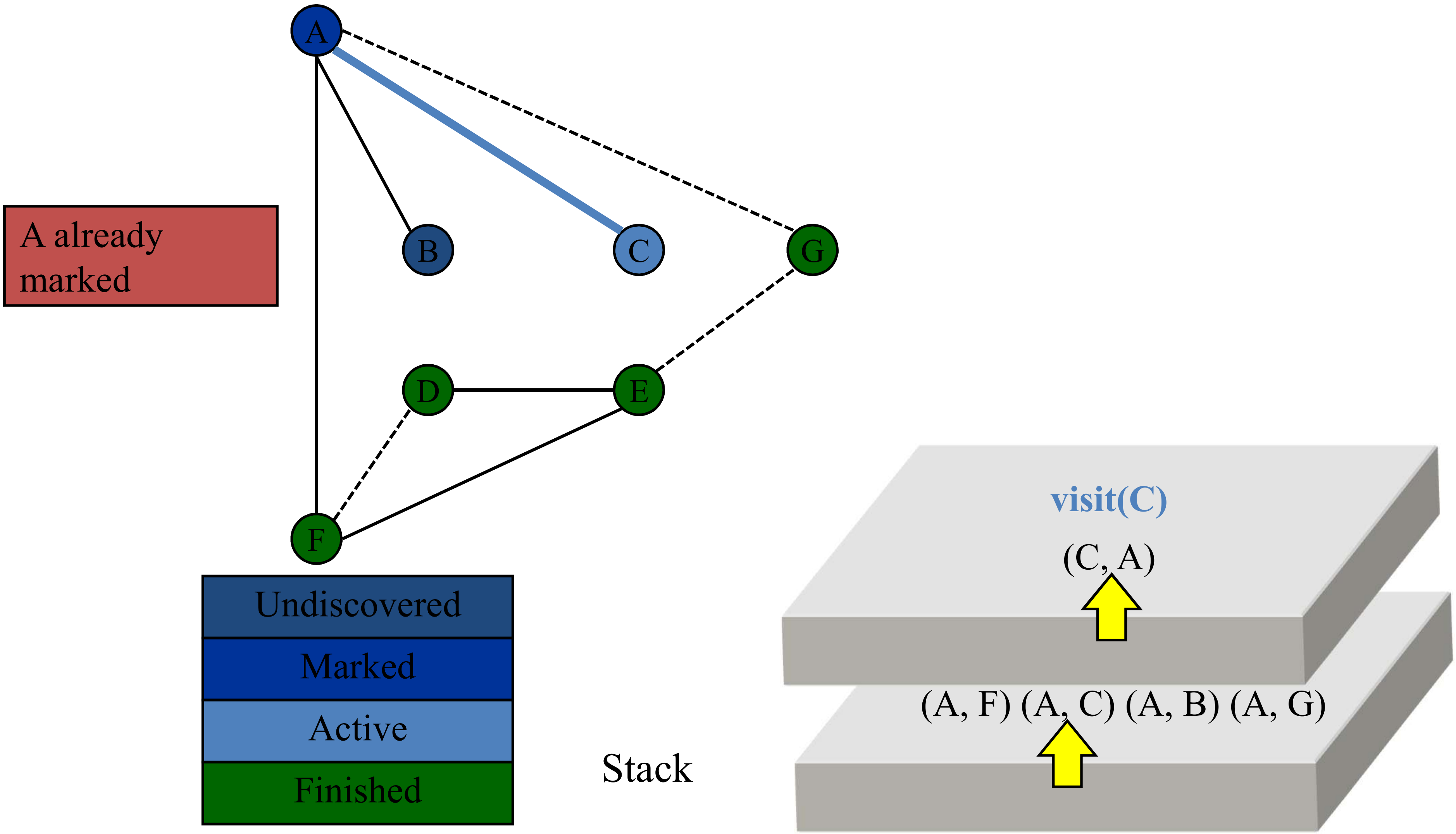


Depth First Search



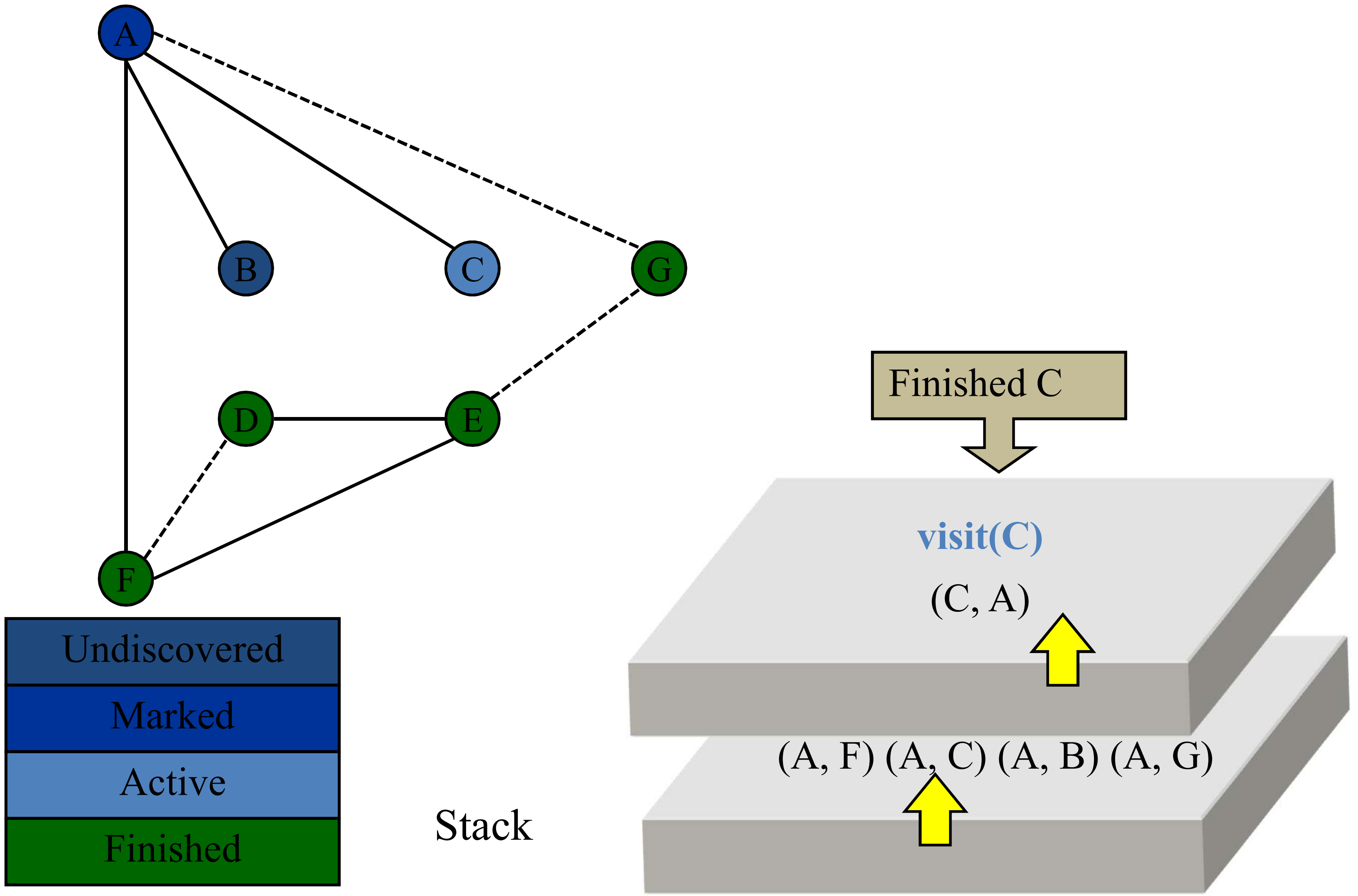
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



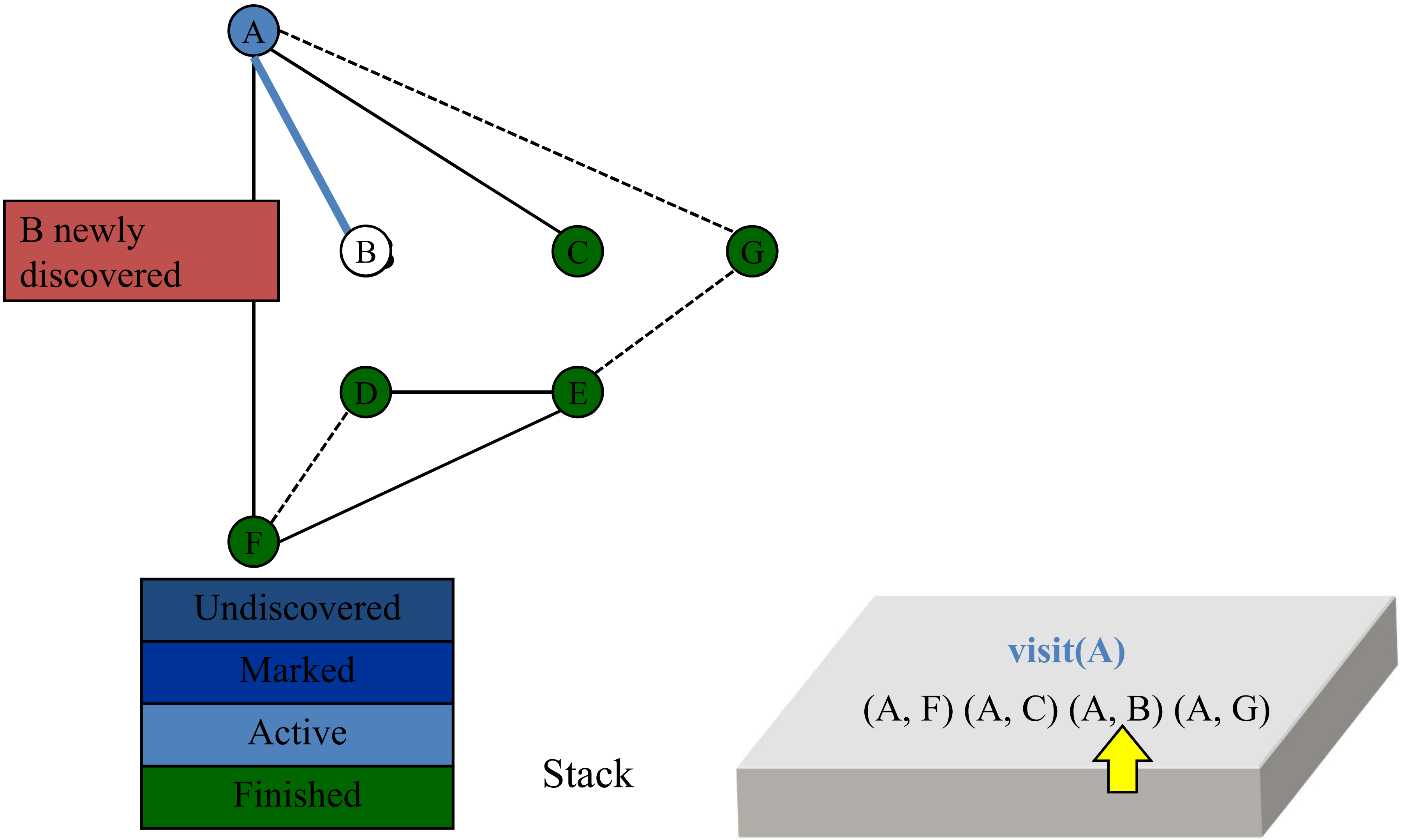
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



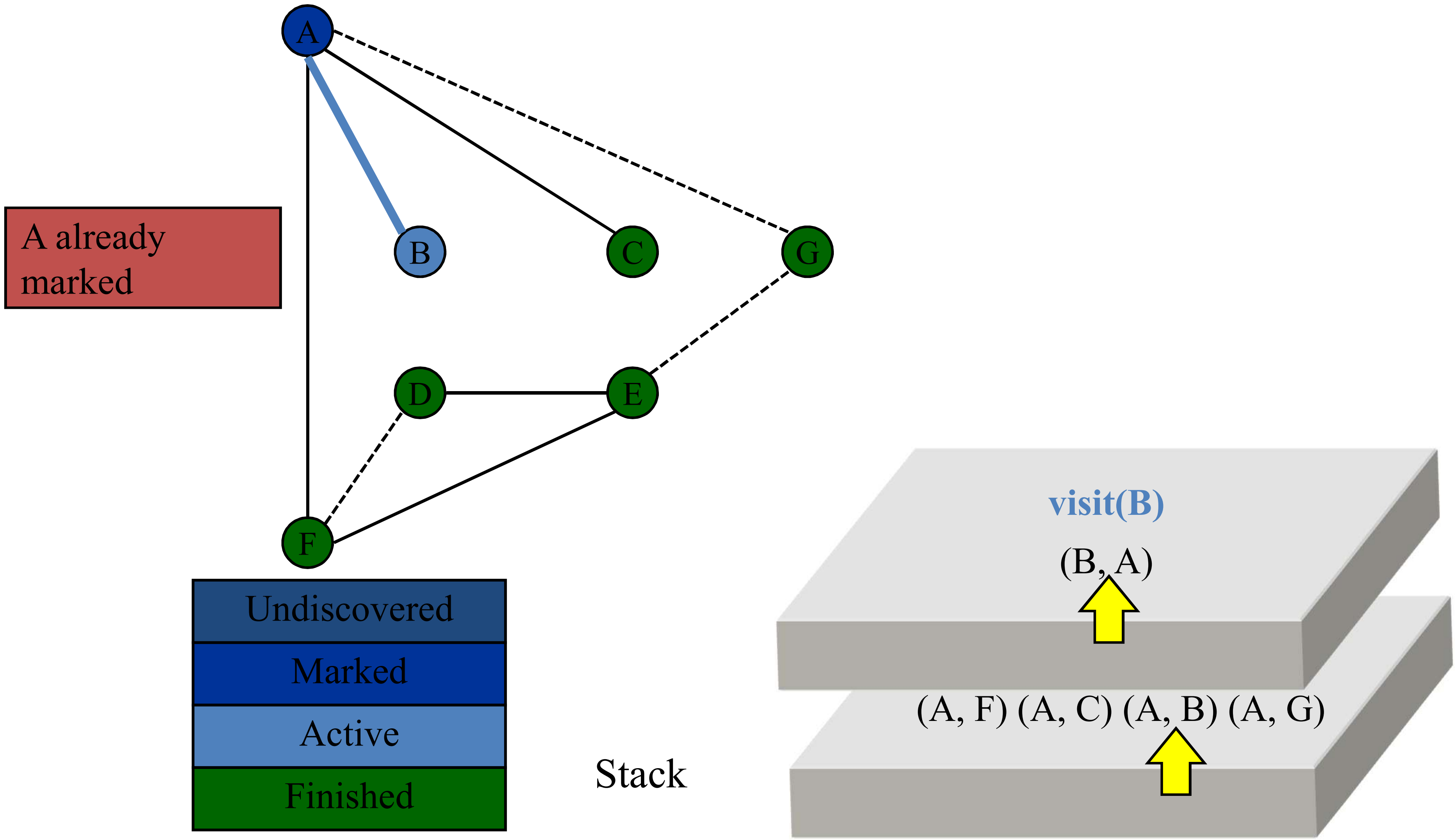
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



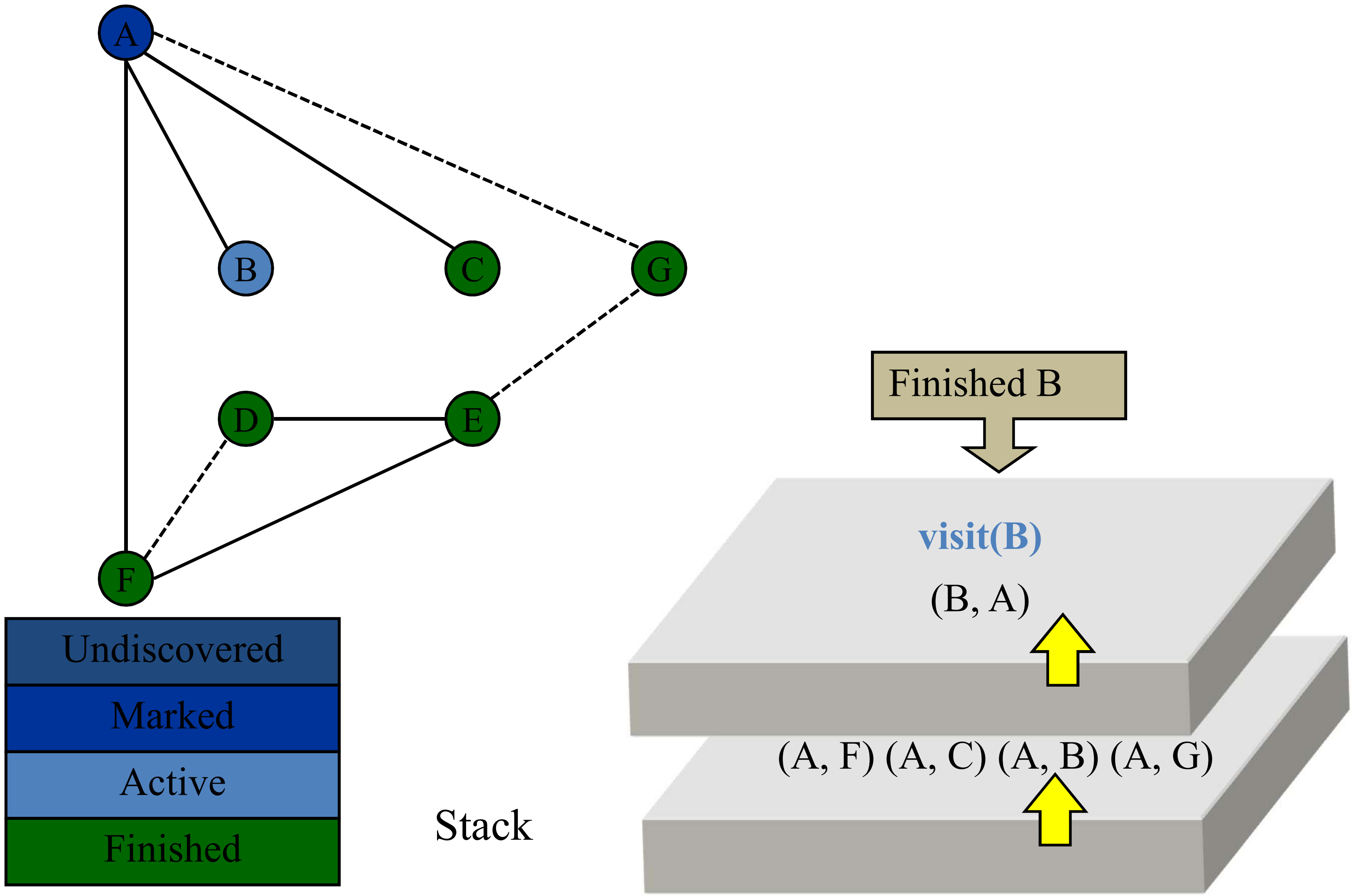
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



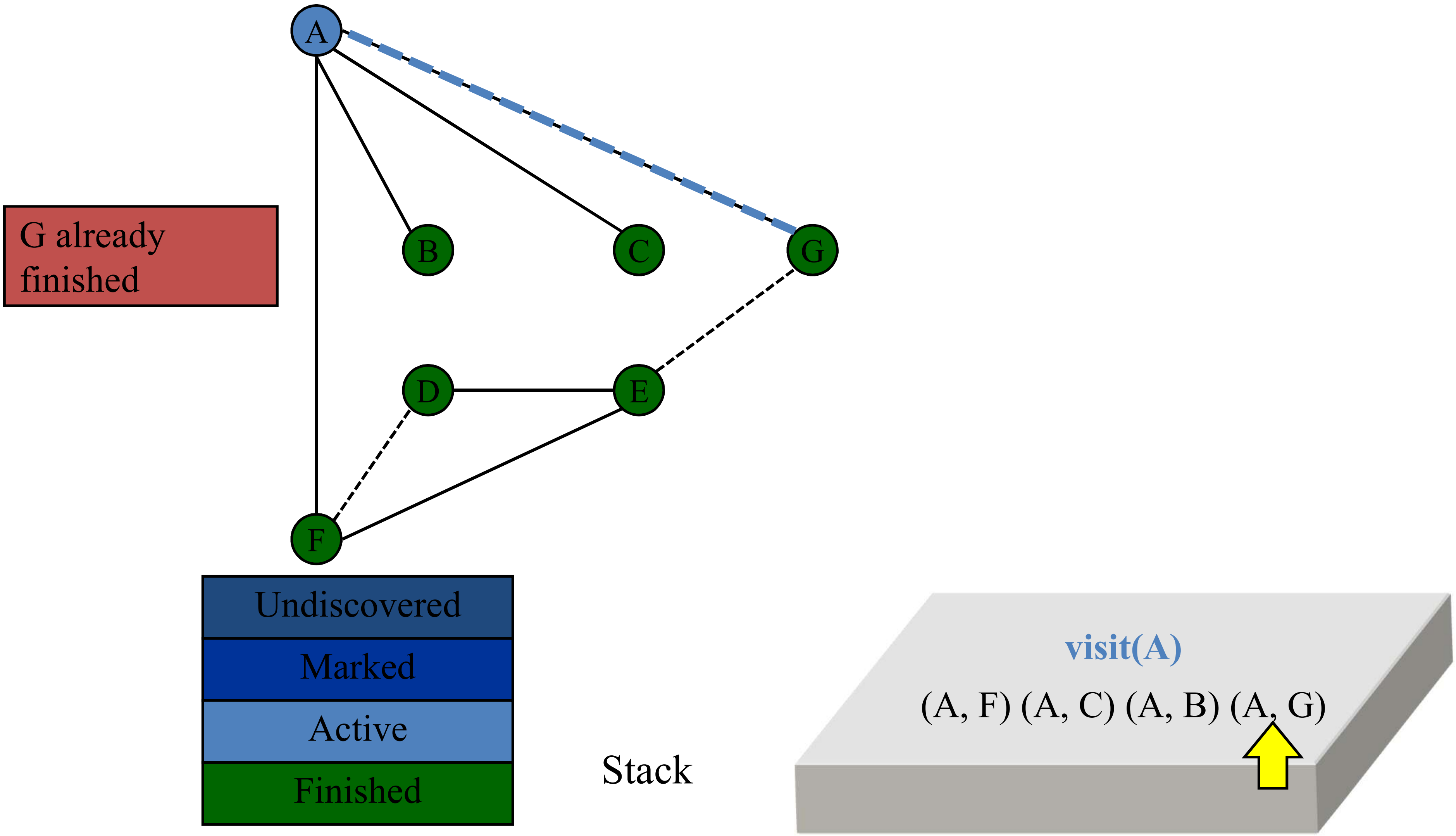
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



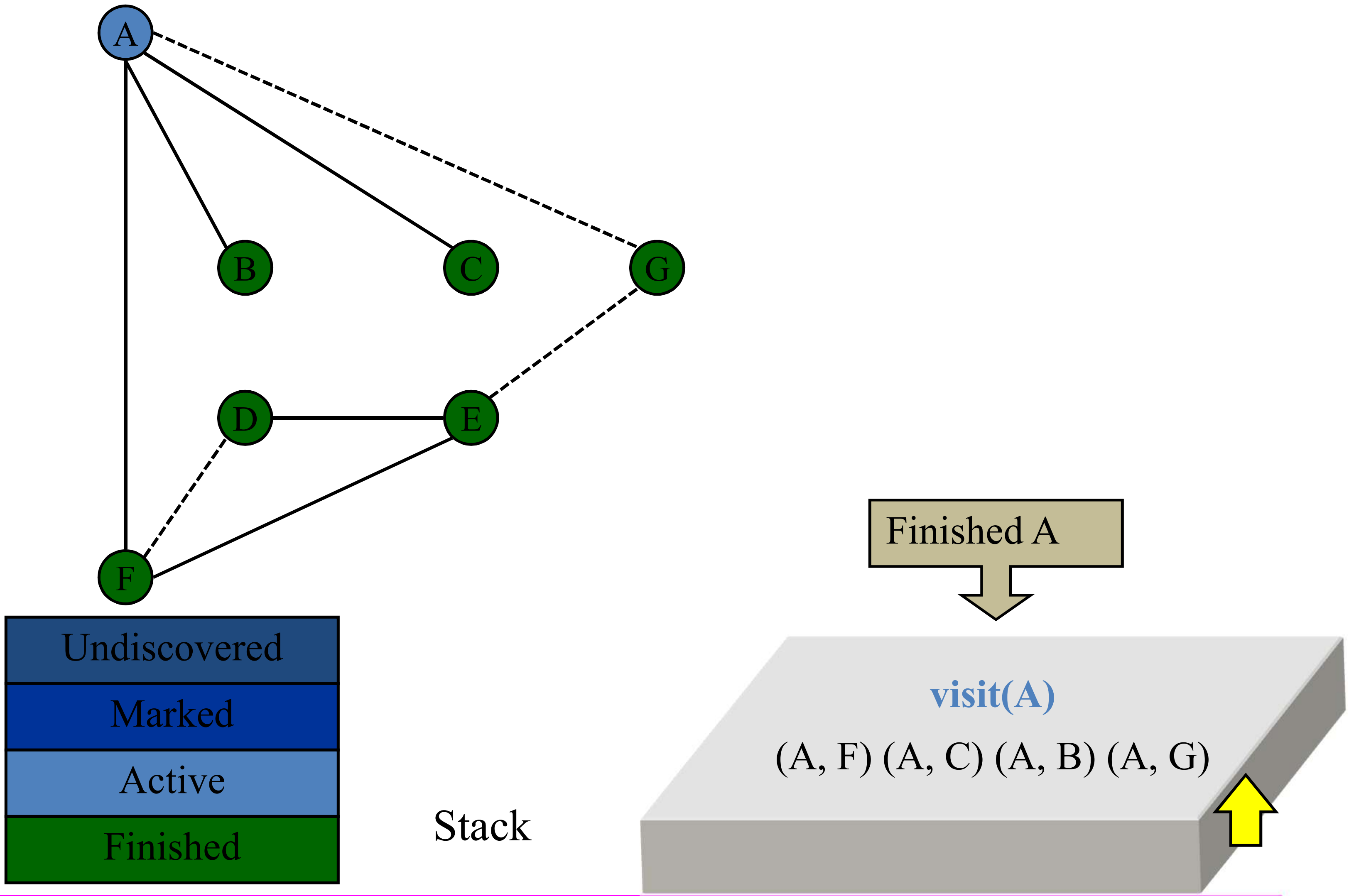
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



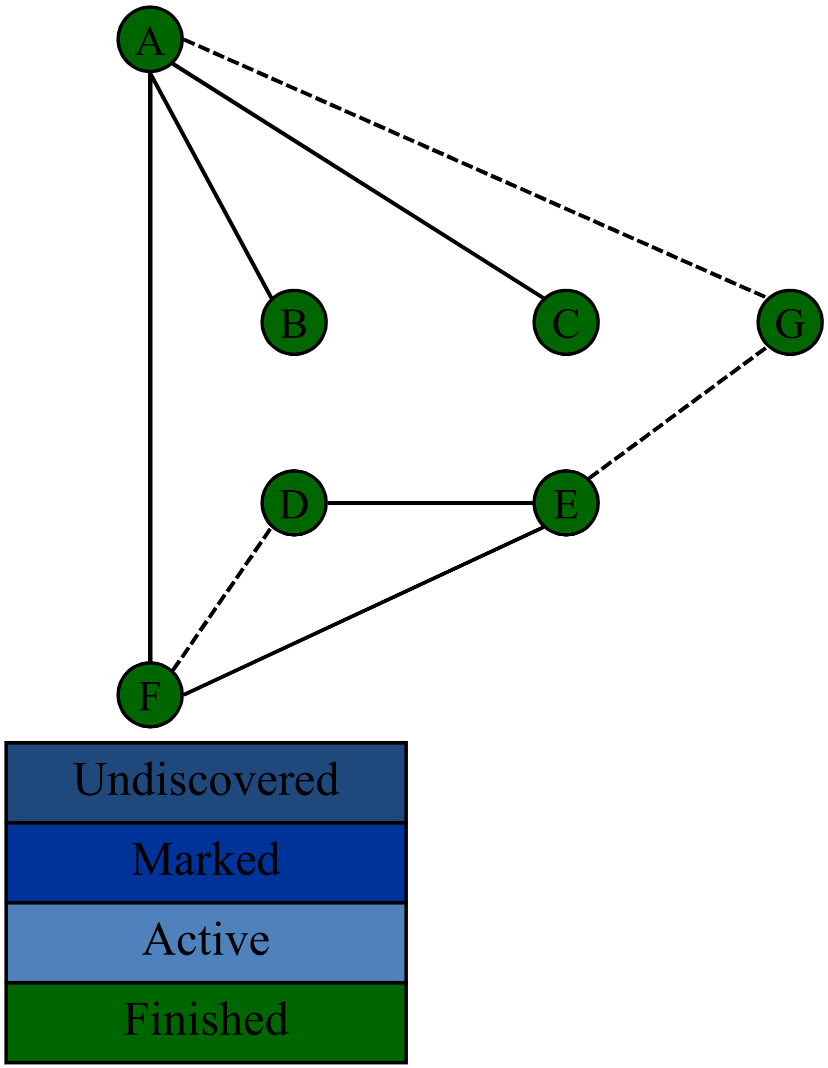
Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Depth First Search



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Depth First Search



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search

- Breadth-First Search (BFS) traverses a graph, and in doing so defines a tree with several useful properties.
- The starting vertex *s* has level 0, and, as in DFS, defines that point as an “anchor.”
- In the first round, all of the edges that are only one edge away from the anchor are visited.
- These vertices are placed into level 1;
- In the second round, all the new edges from level 1 that can be reached are visited and placed in level 2.
- This continues until every vertex has been assigned a level.
- The label of any vertex ***v*** corresponds to the length of the shortest path from ***s*** to ***v***.

Breadth First Search

```
void BFS (int start)  {
    int v, result;      queue q;          adjvertex *adj;
    visited [start] = 1;                  enqueue (start, &q);
    while ((result = dequeue (&v, &q)) != -1)              {
            printf ("%d", v);     adj = g→adjlist [v];
            while (adj != NULL) {
                    if (! visited [adj→vertex])
                    {
                            visited [adj→vertex] = 1;
                            enqueue (adj→vertex, &q);
                    }
                    adj = adj→next;
            }           }           }
```

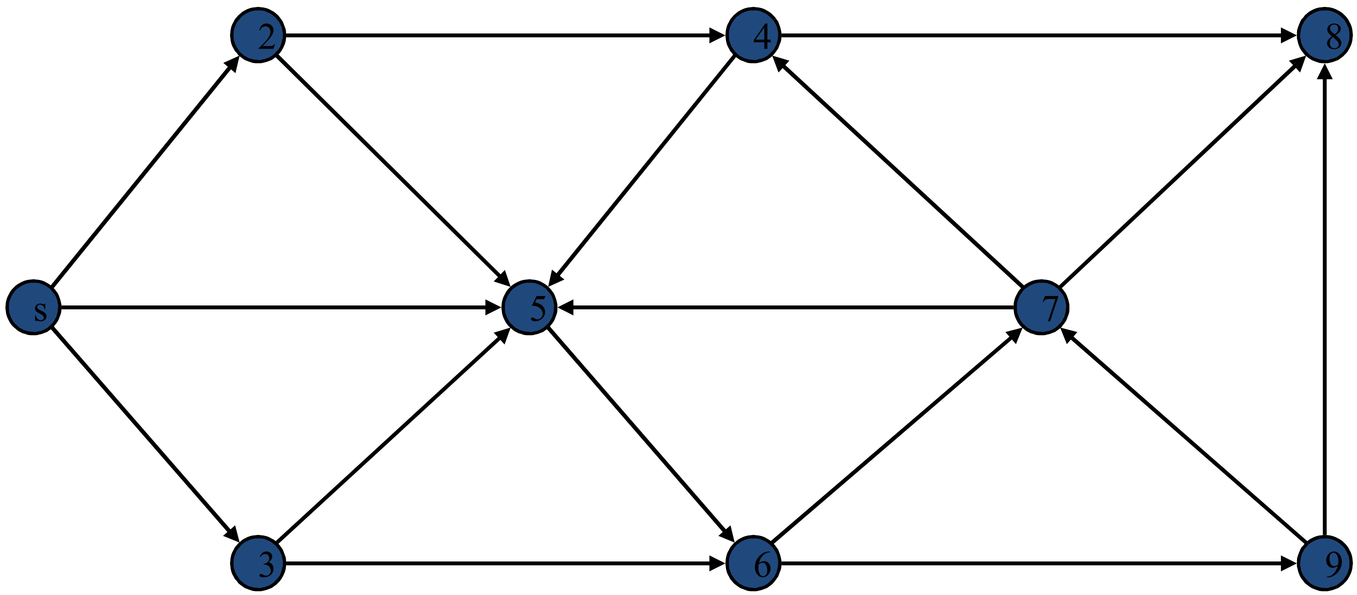
Nodes dequeued

Nodes enqueued exactly once

Total running time: O(V+E)

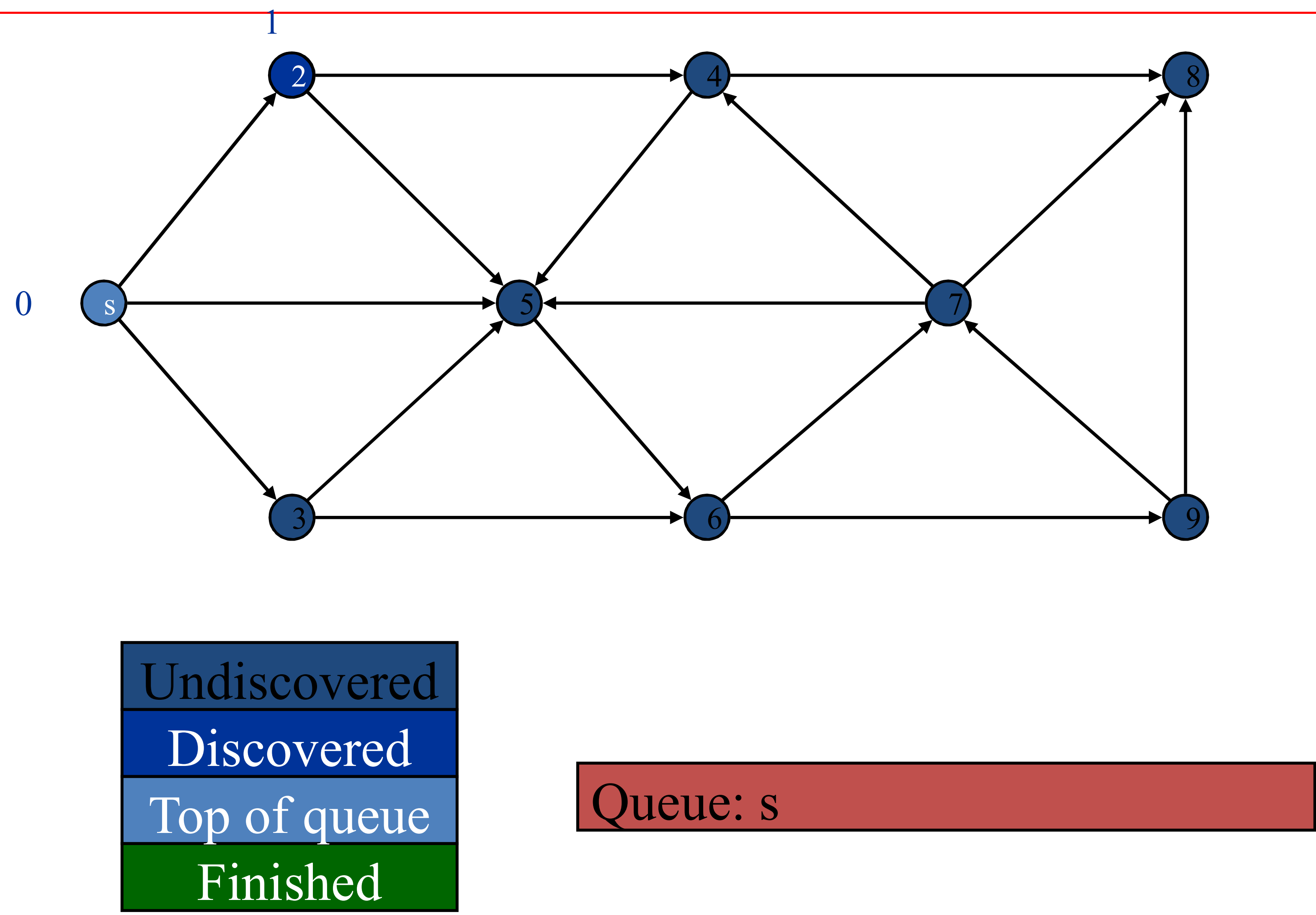
Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Breadth First Search



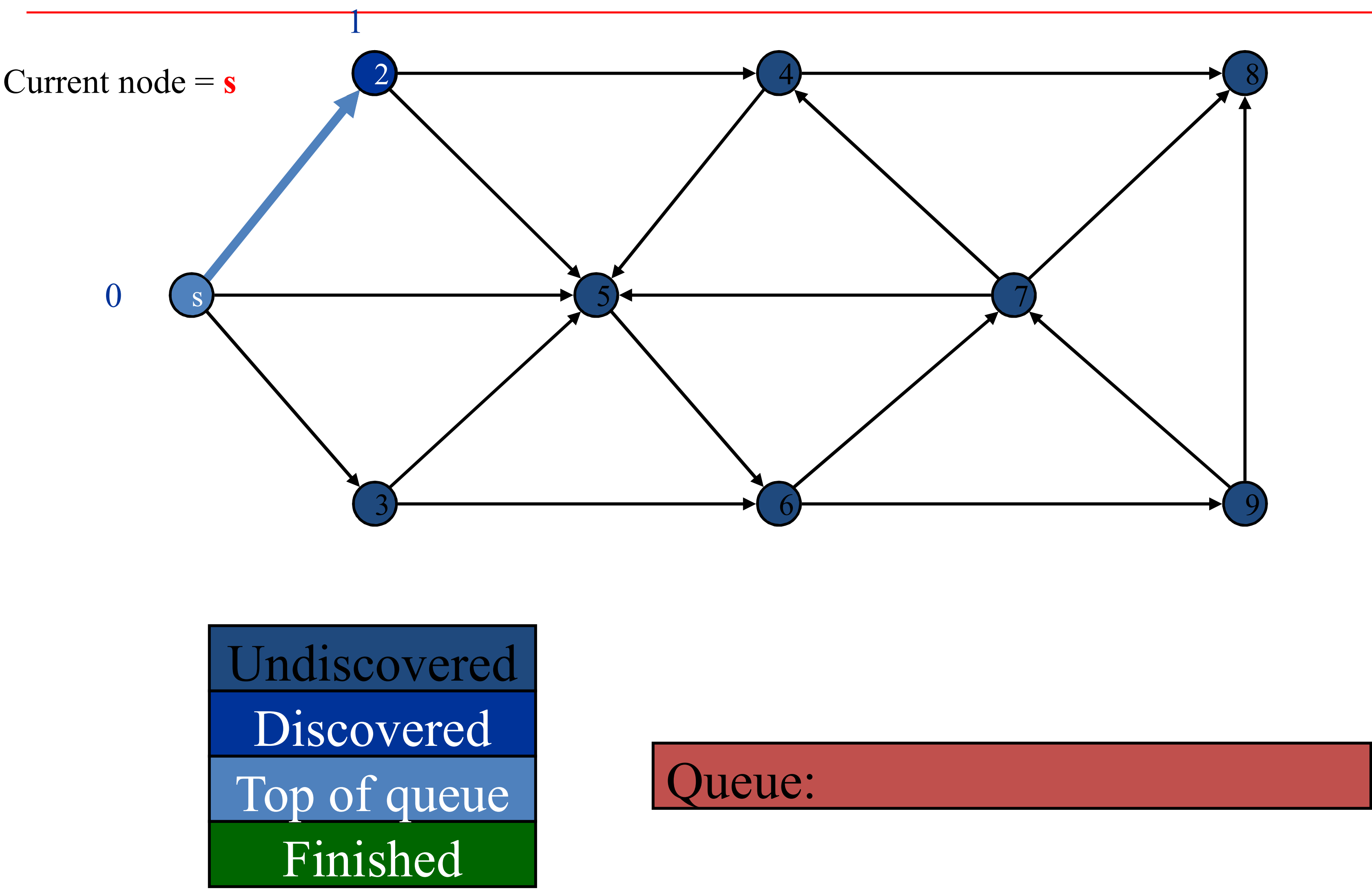
Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Breadth First Search



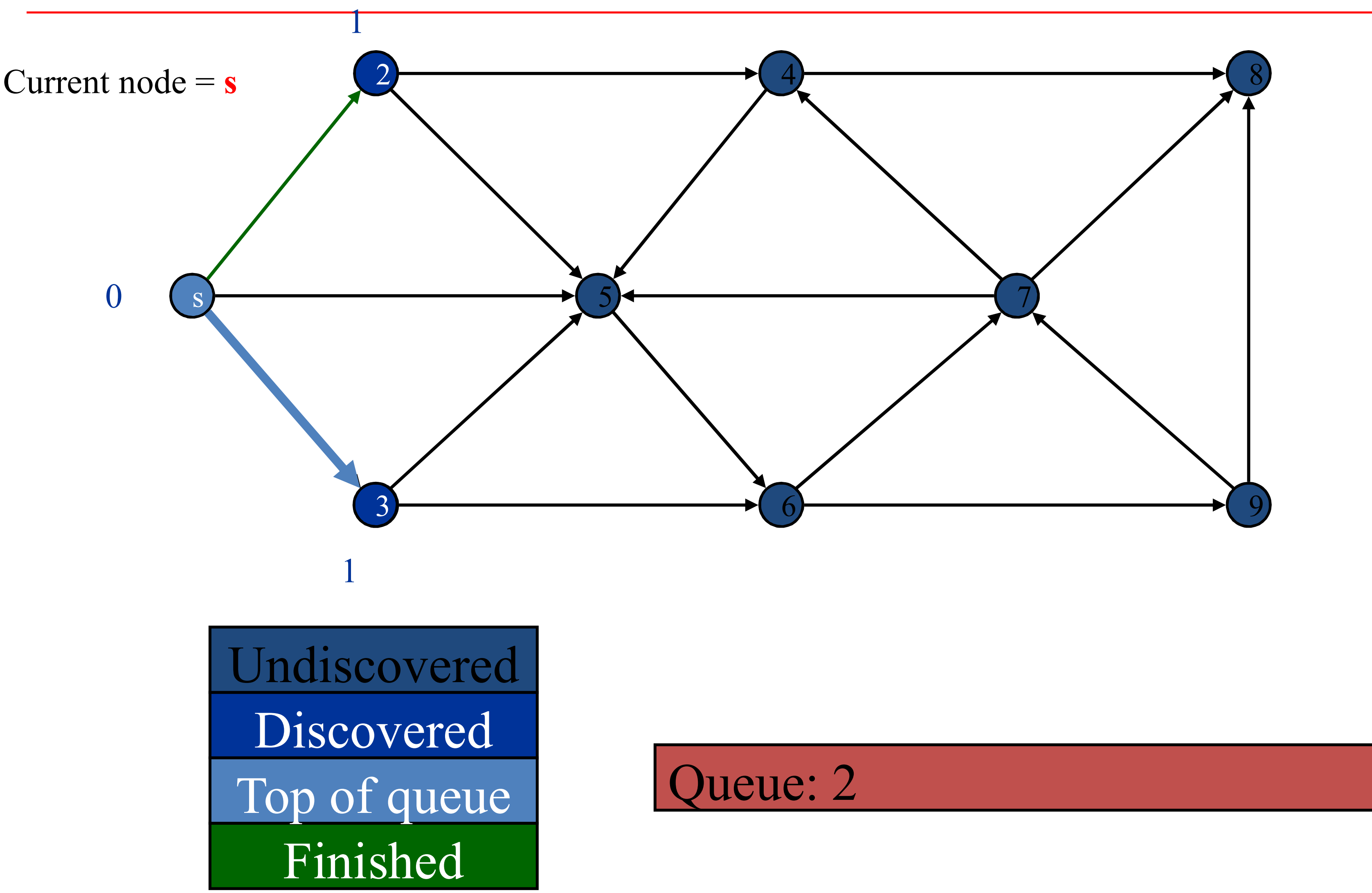
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search

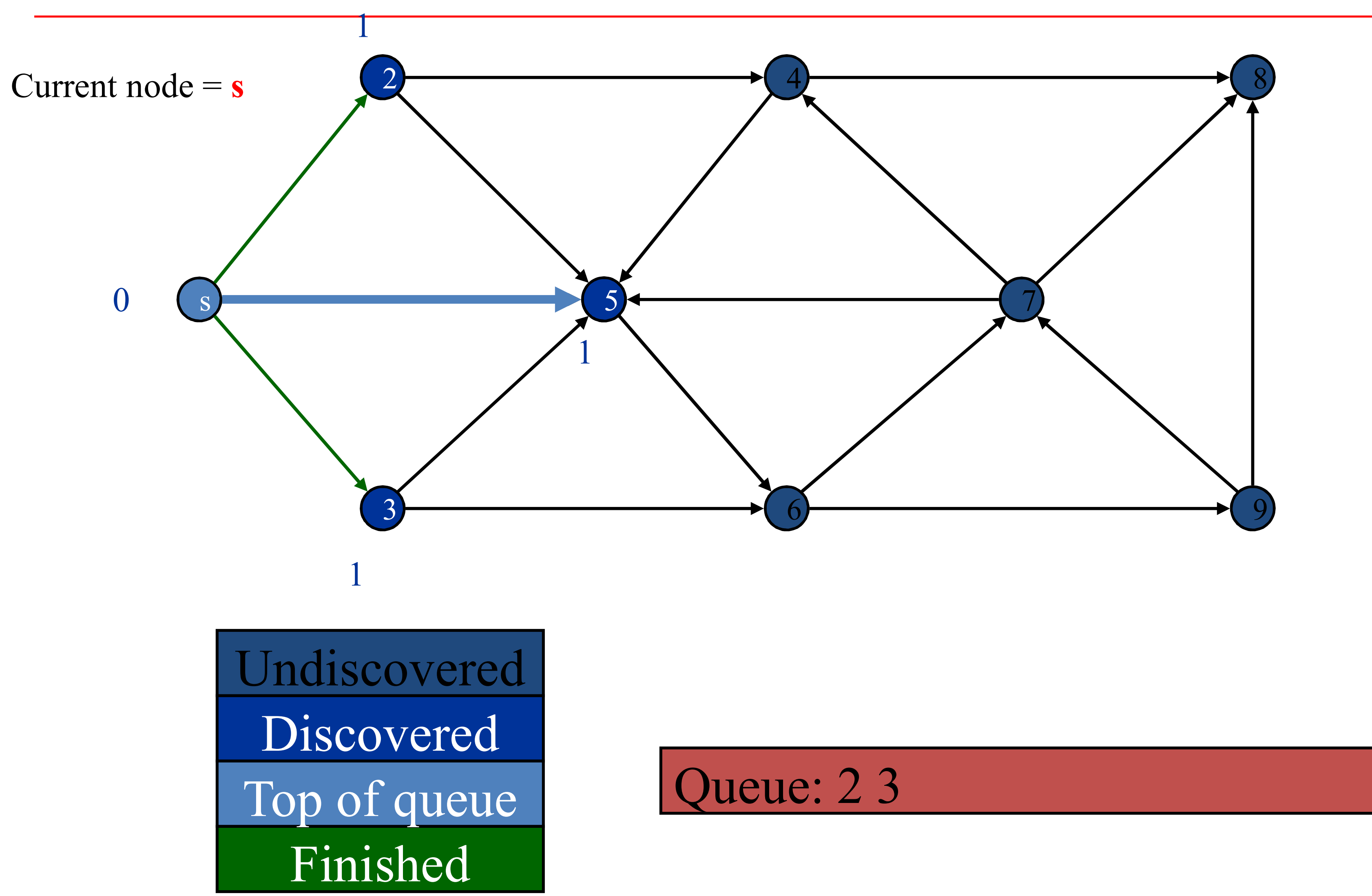


Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Breadth First Search



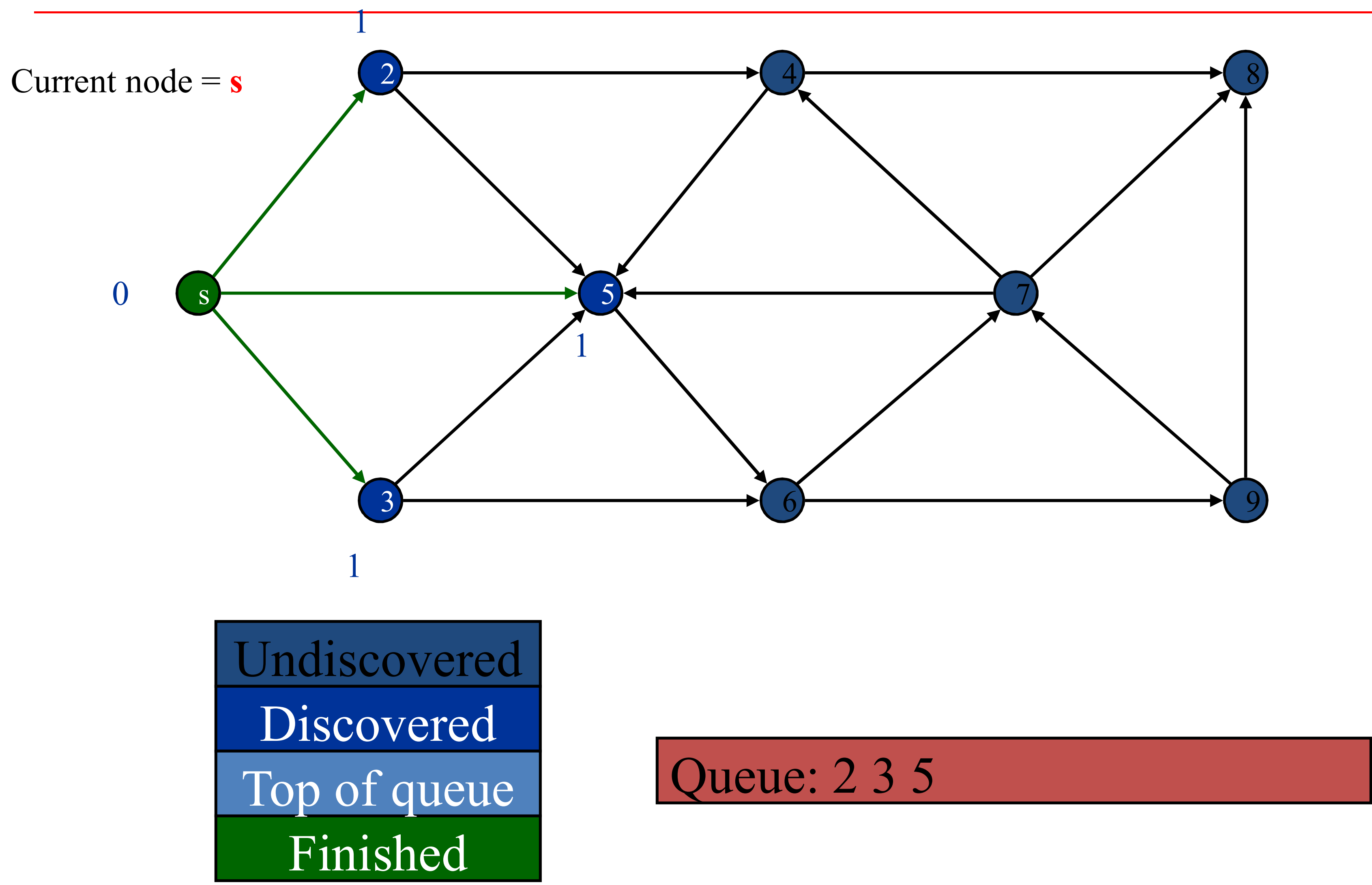
Breadth First Search



Queue: 2 3

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

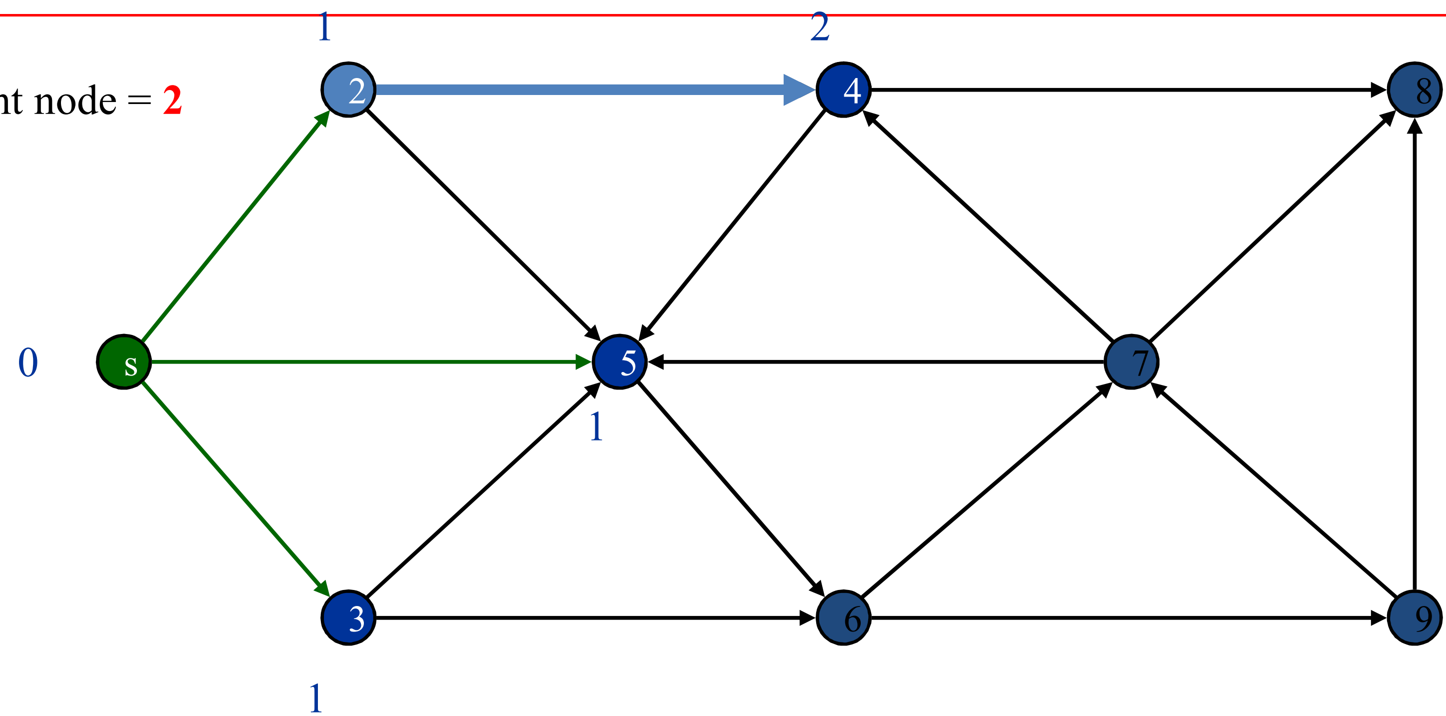
Breadth First Search



Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Breadth First Search

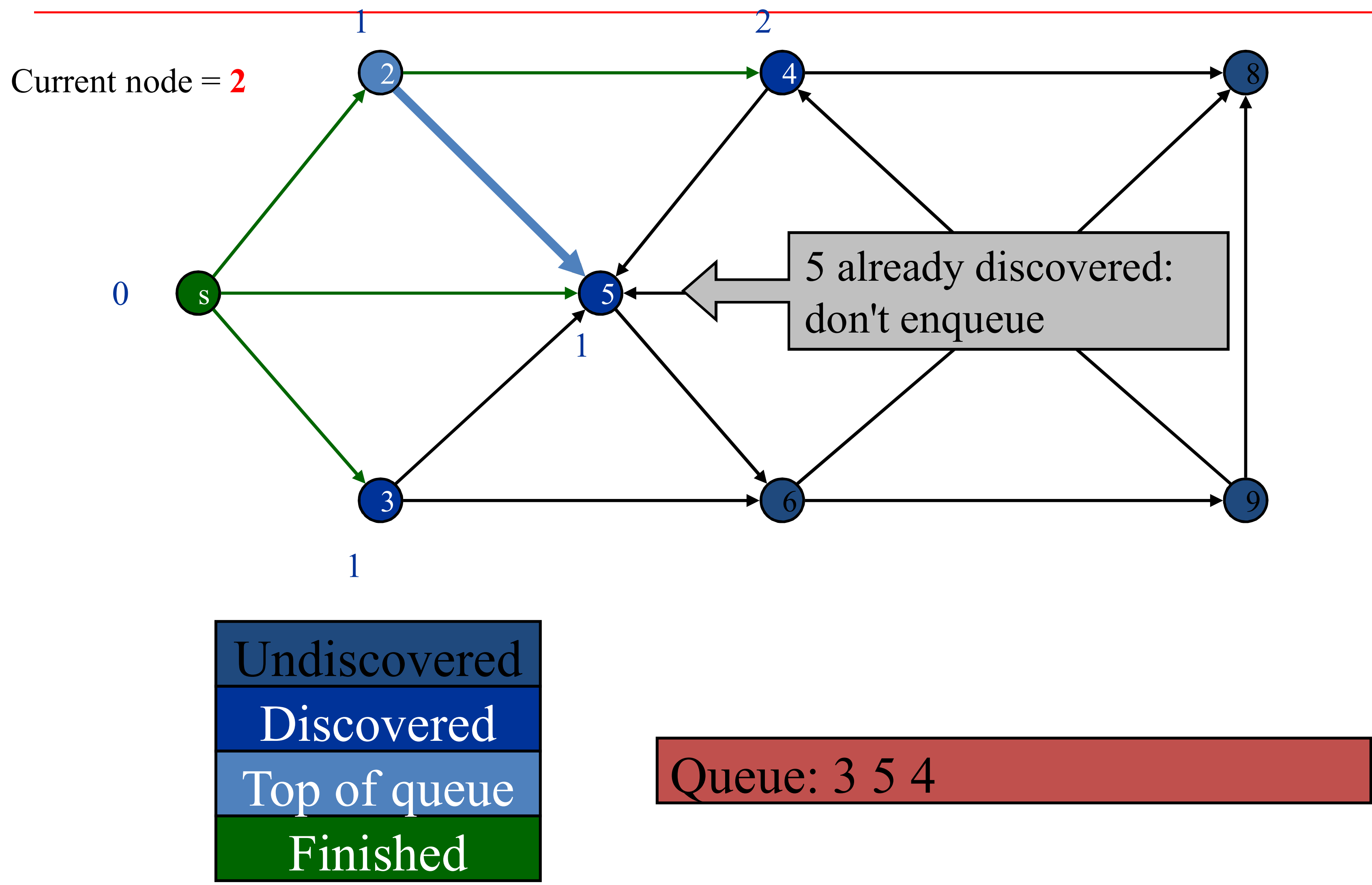
Current node = **2**



Undiscovered
Discovered
Top of queue
Finished

Queue: 3 5

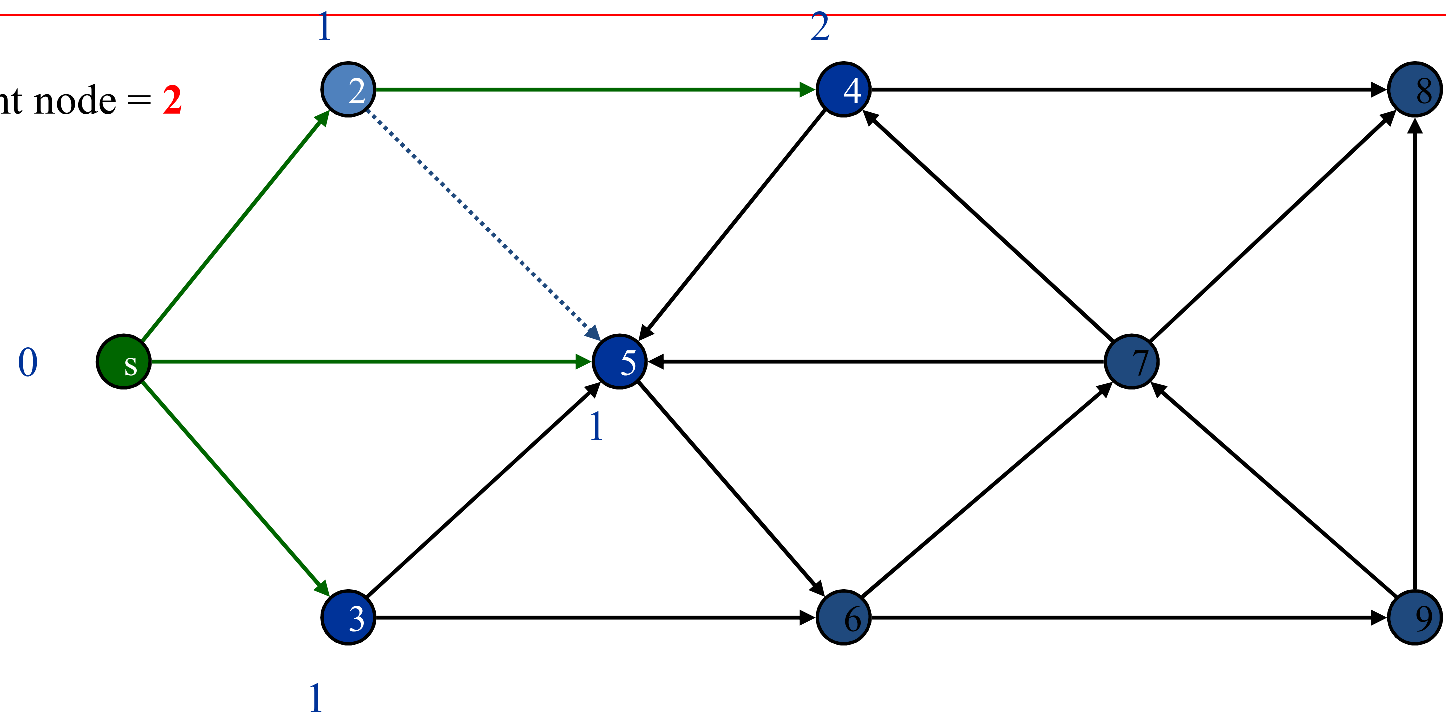
Breadth First Search



Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Breadth First Search

Current node = **2**

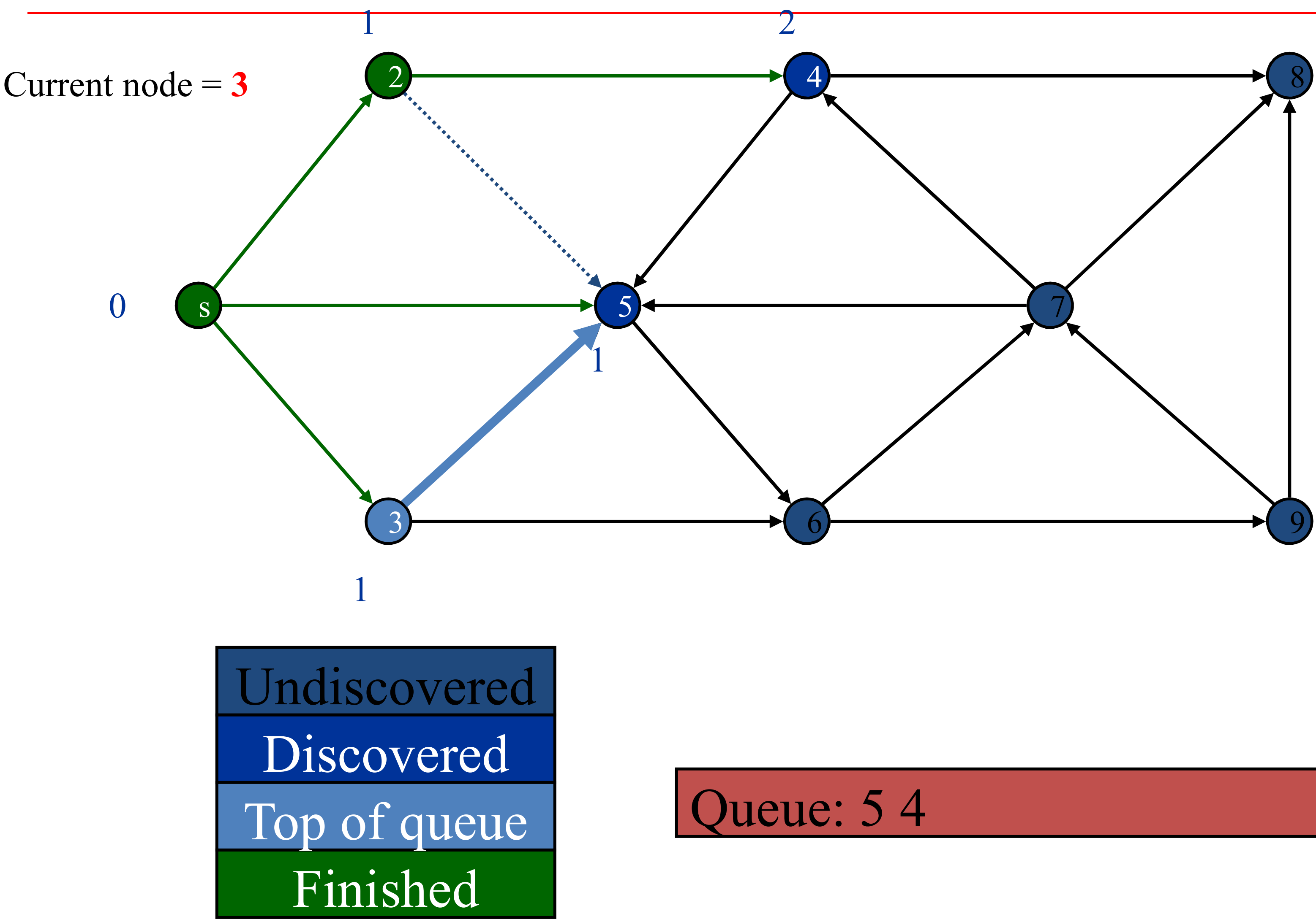


Undiscovered
Discovered
Top of queue
Finished

Queue: 2 3 5 4

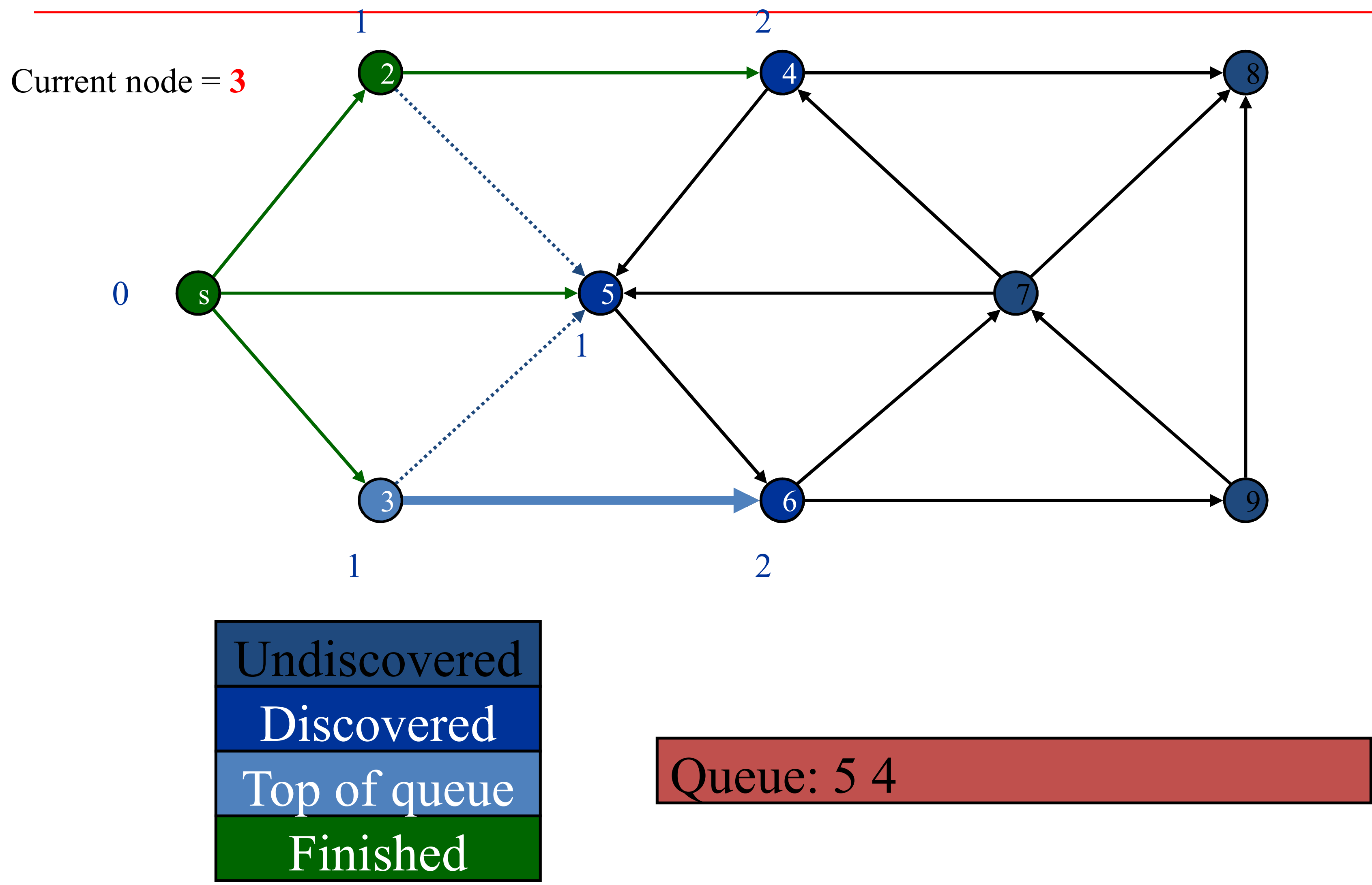
Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Breadth First Search



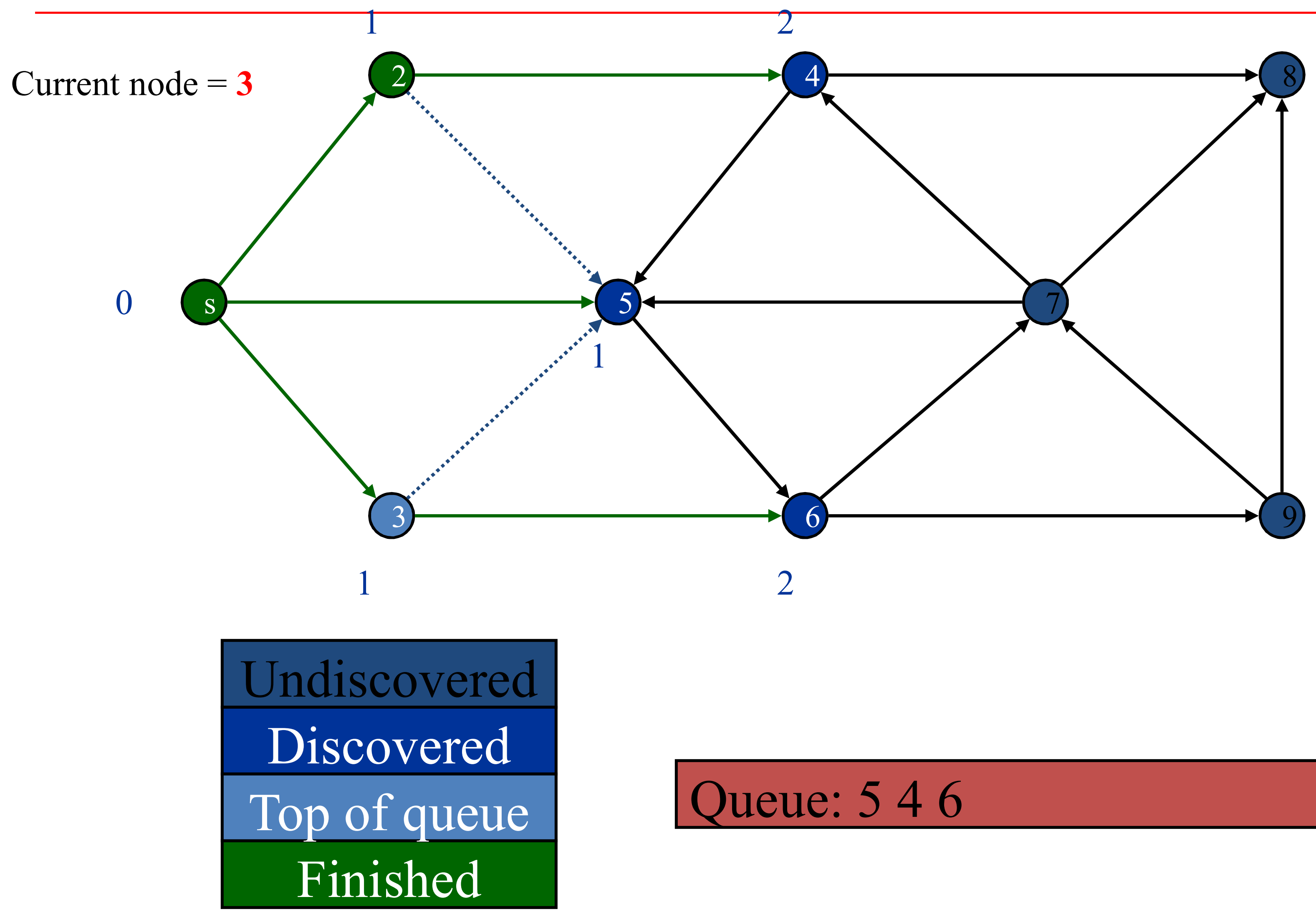
Queue: 5 4

Breadth First Search



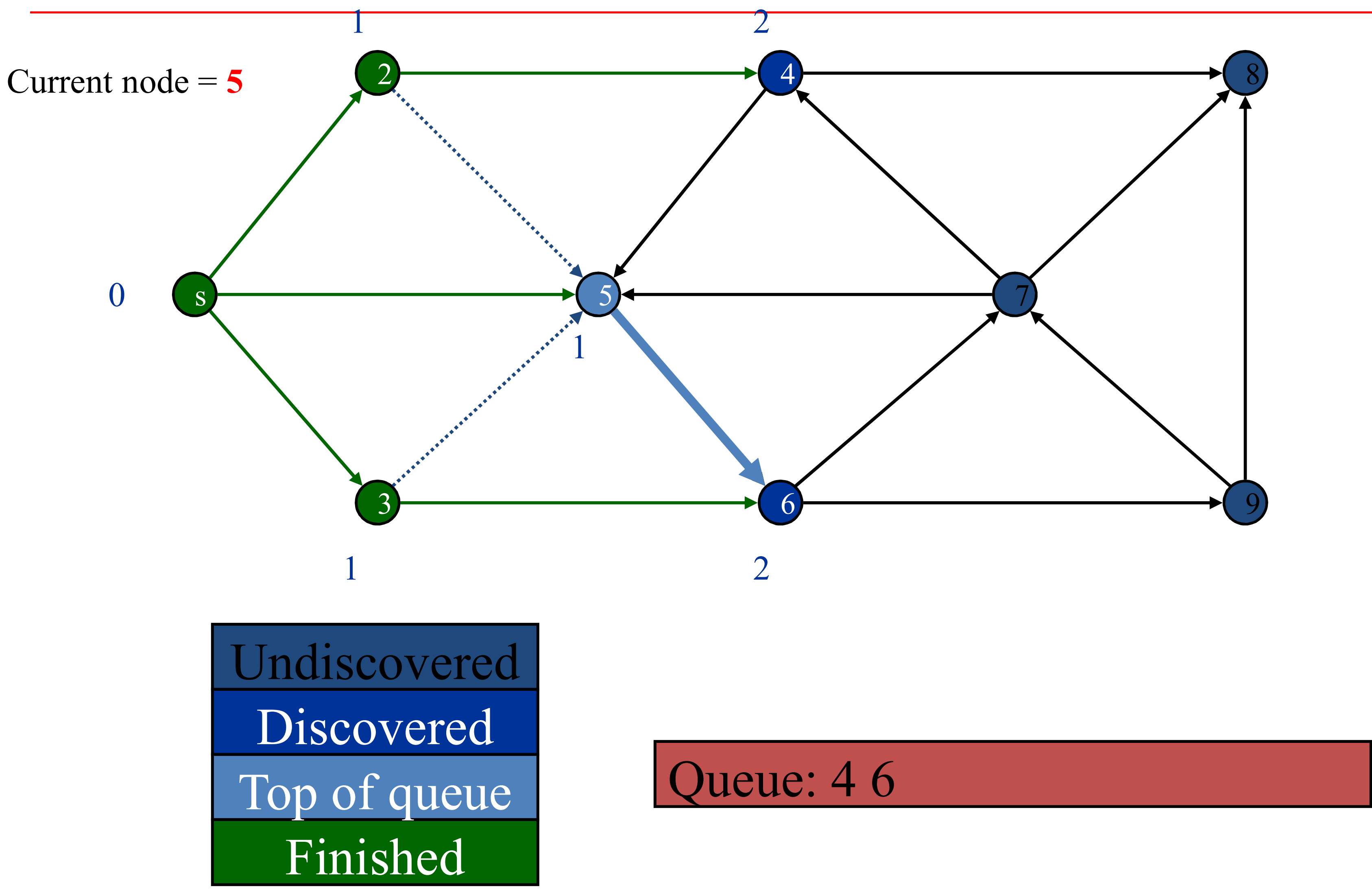
Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Breadth First Search



Queue: 5 4 6

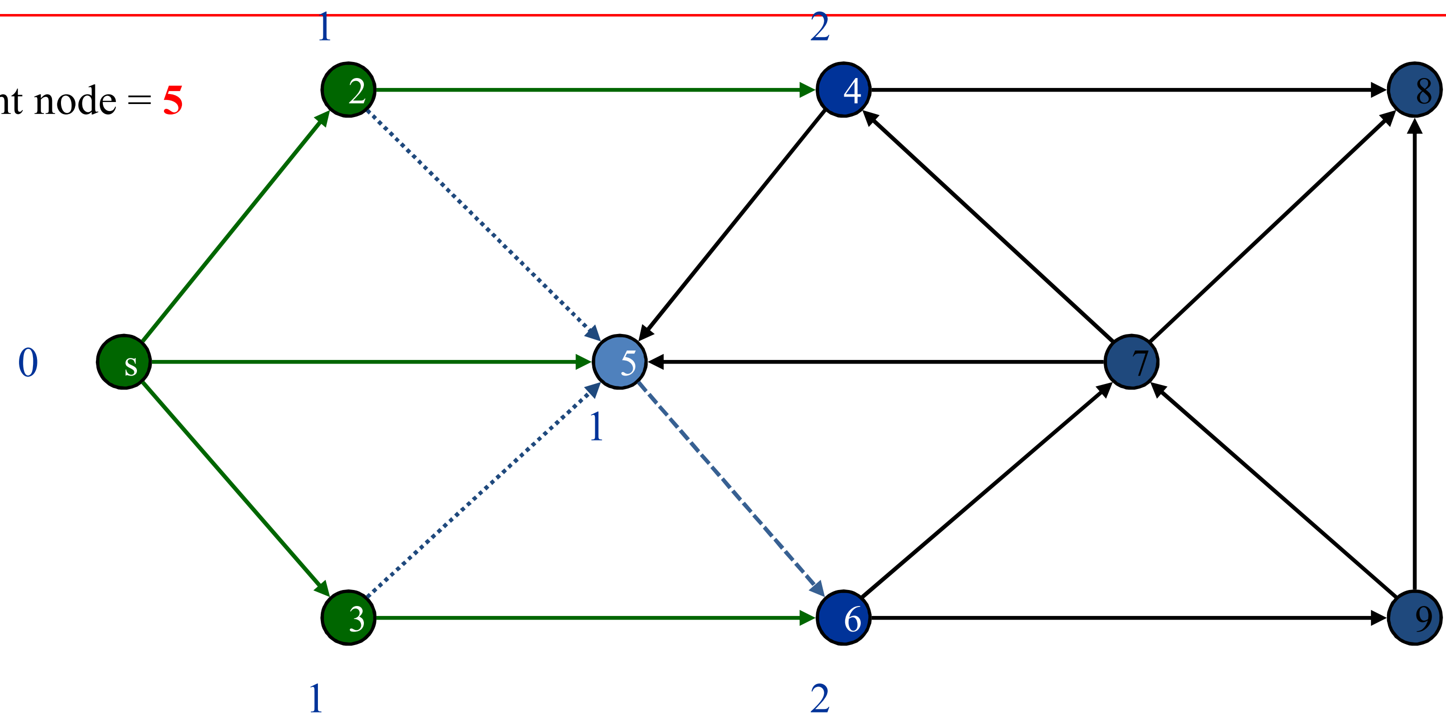
Breadth First Search



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search

Current node = **5**

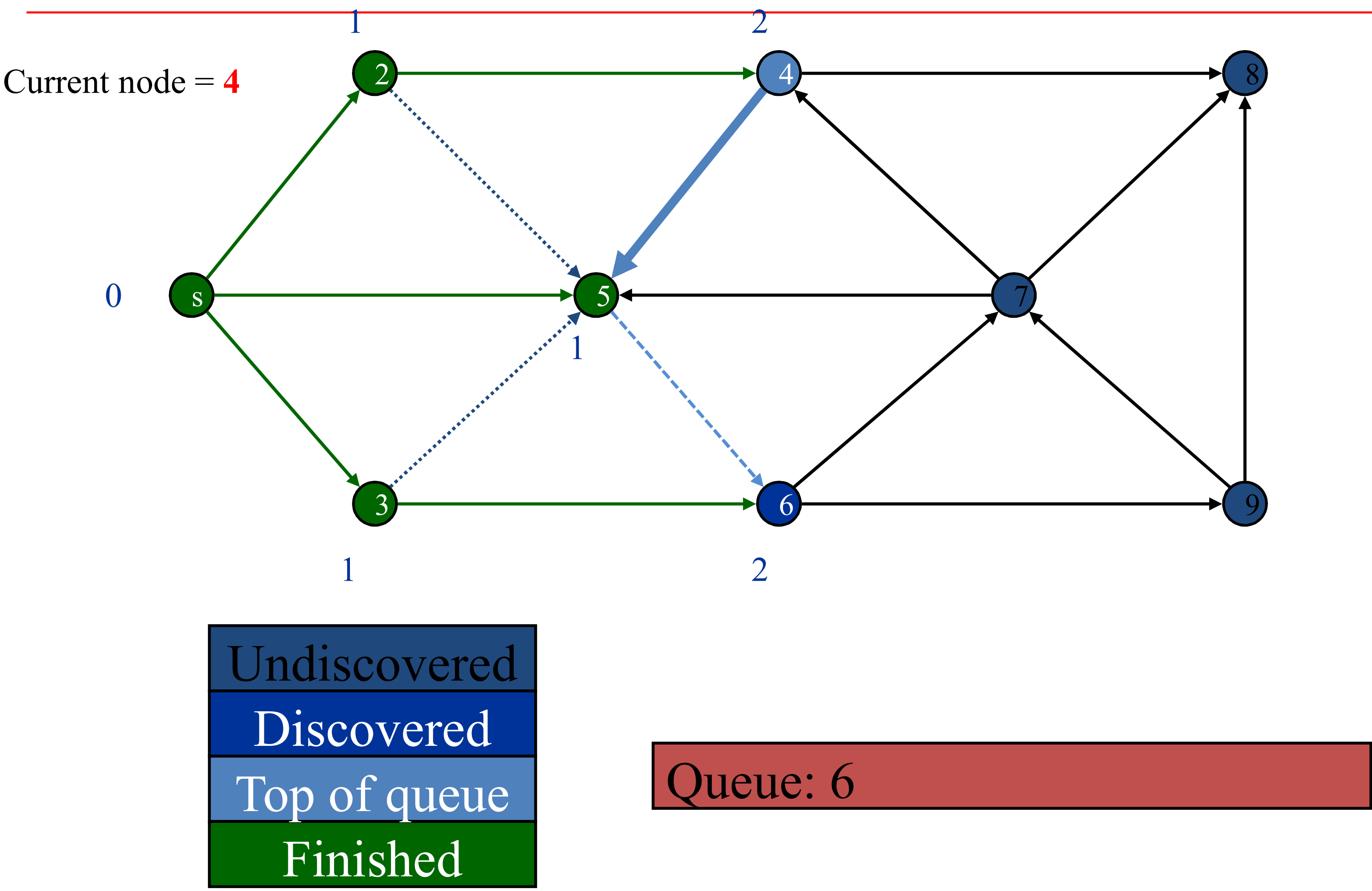


Undiscovered
Discovered
Top of queue
Finished

Queue: 4 6

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

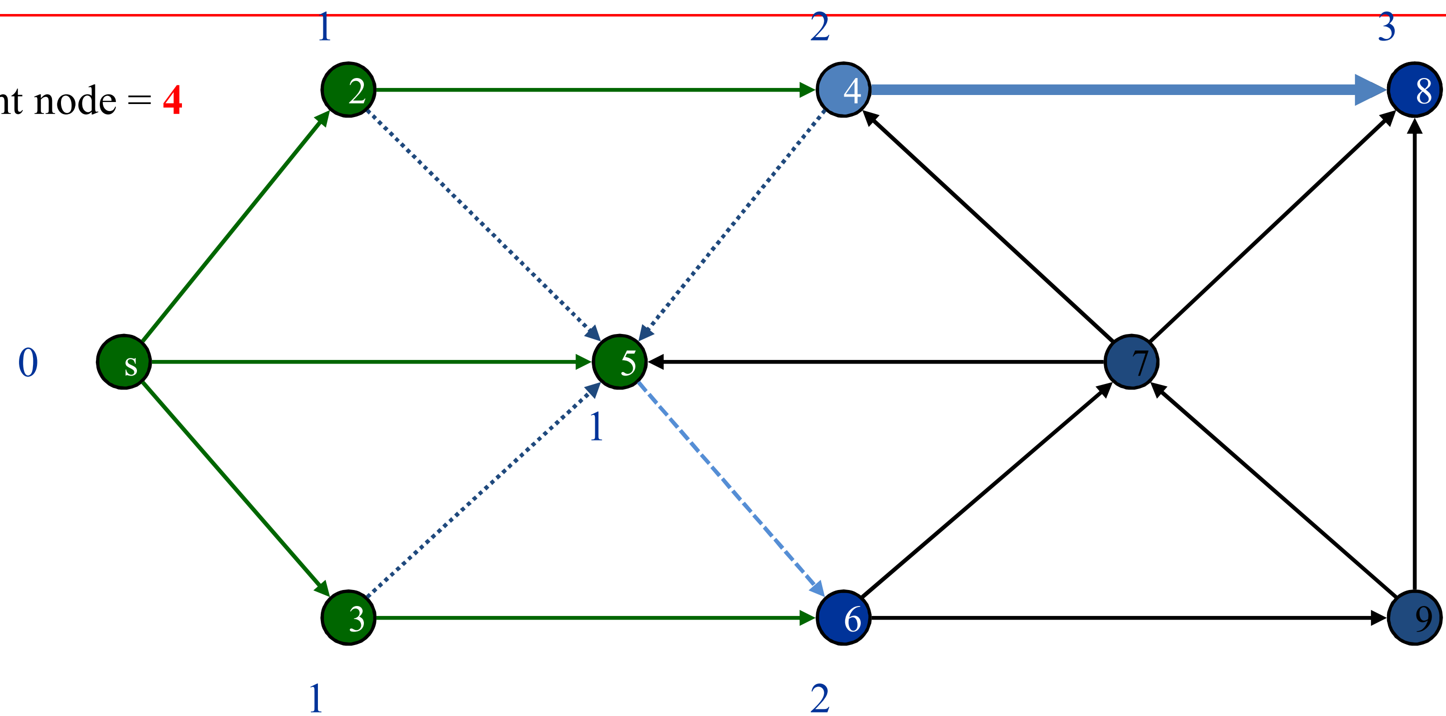
Breadth First Search



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search

Current node = **4**

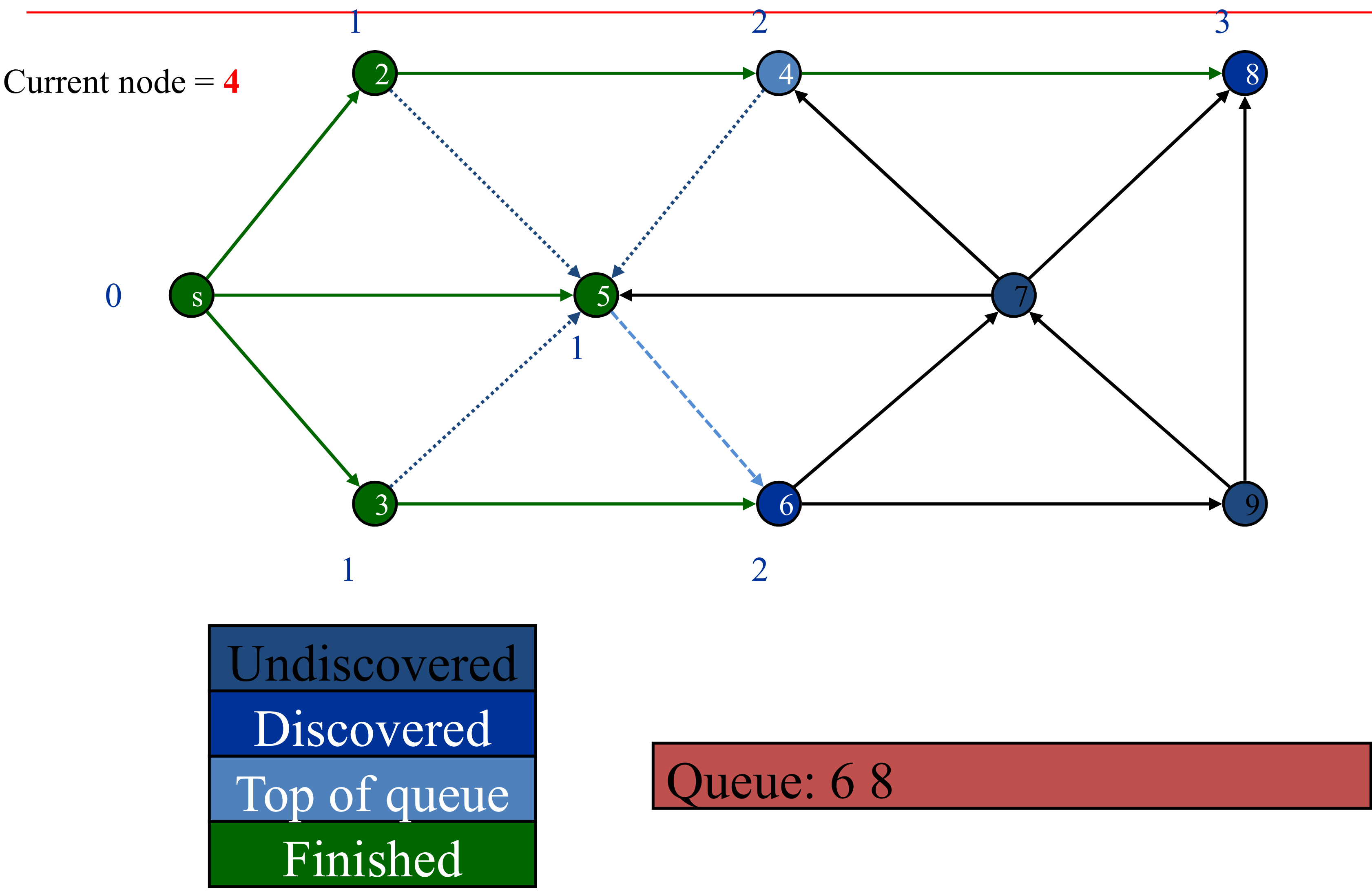


Undiscovered
Discovered
Top of queue
Finished

Queue: 6

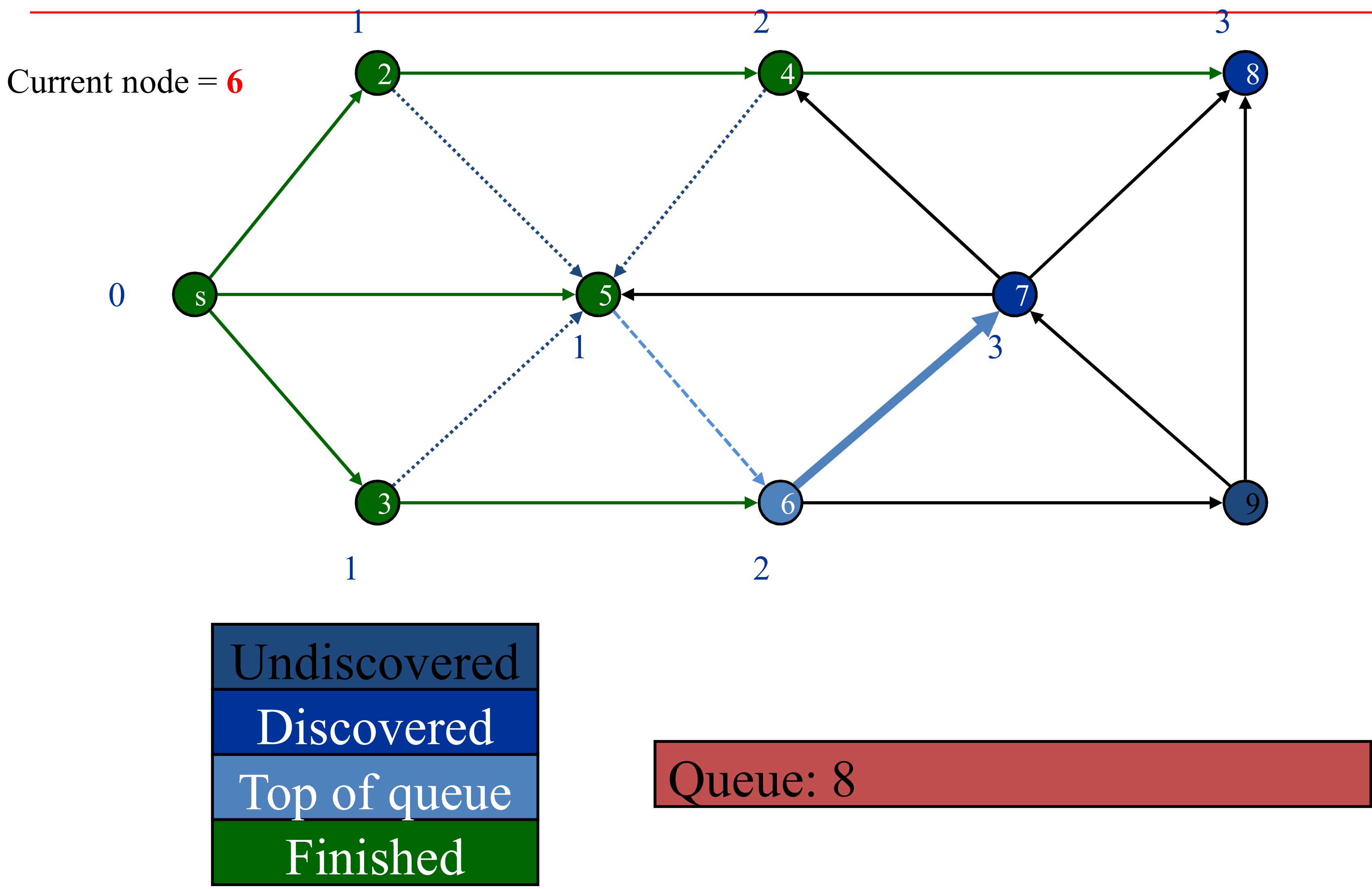
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search



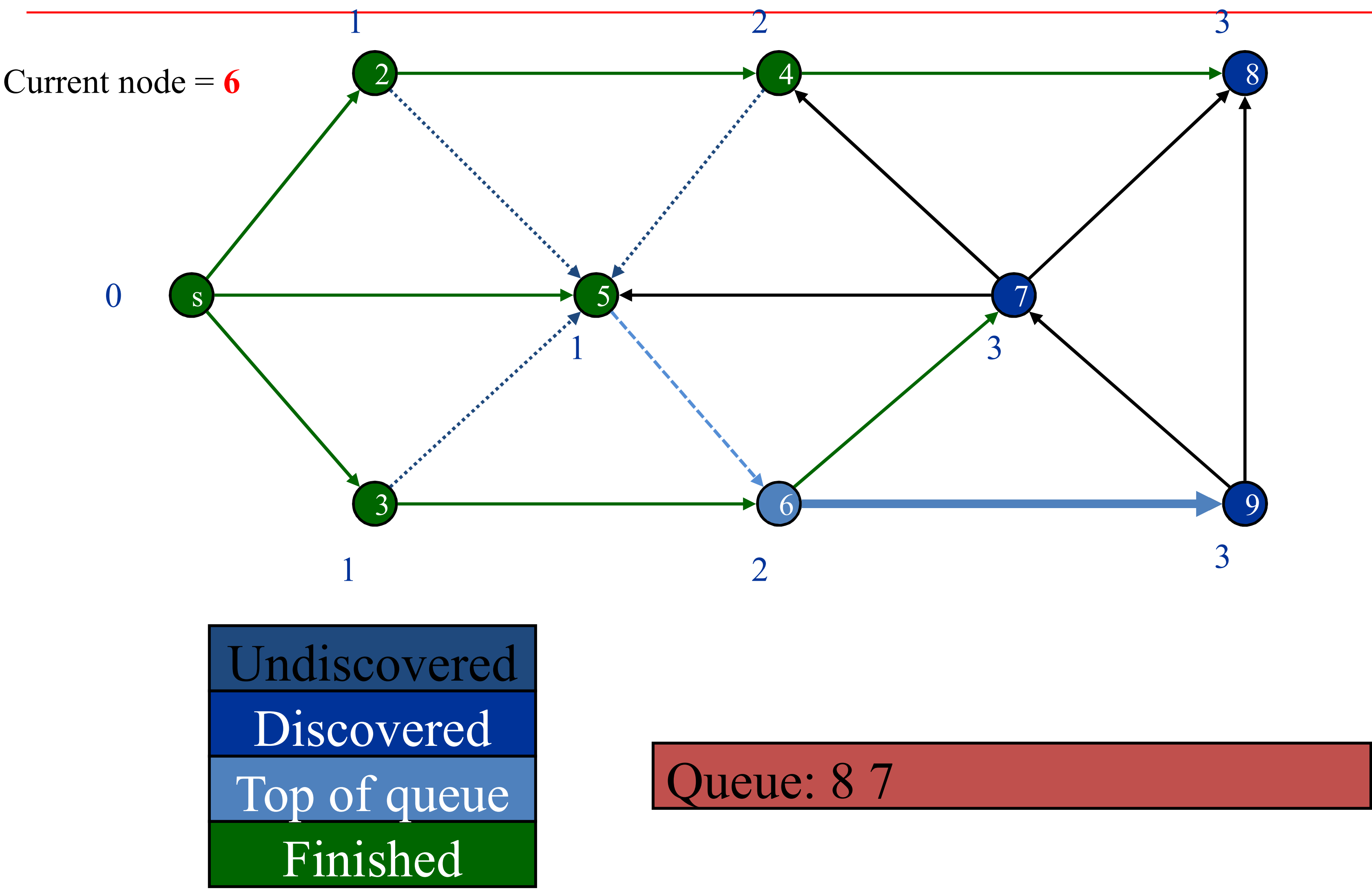
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search



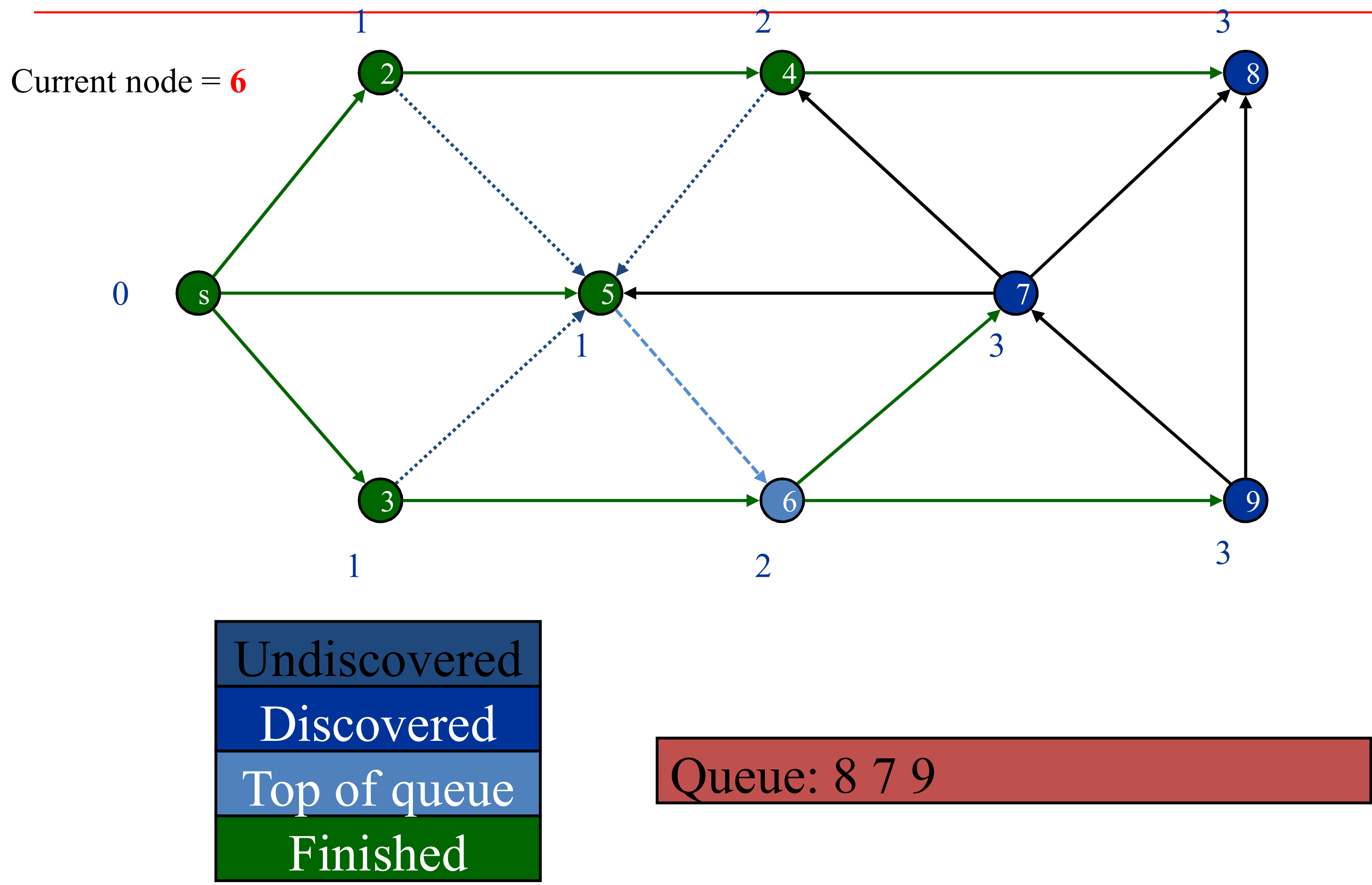
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

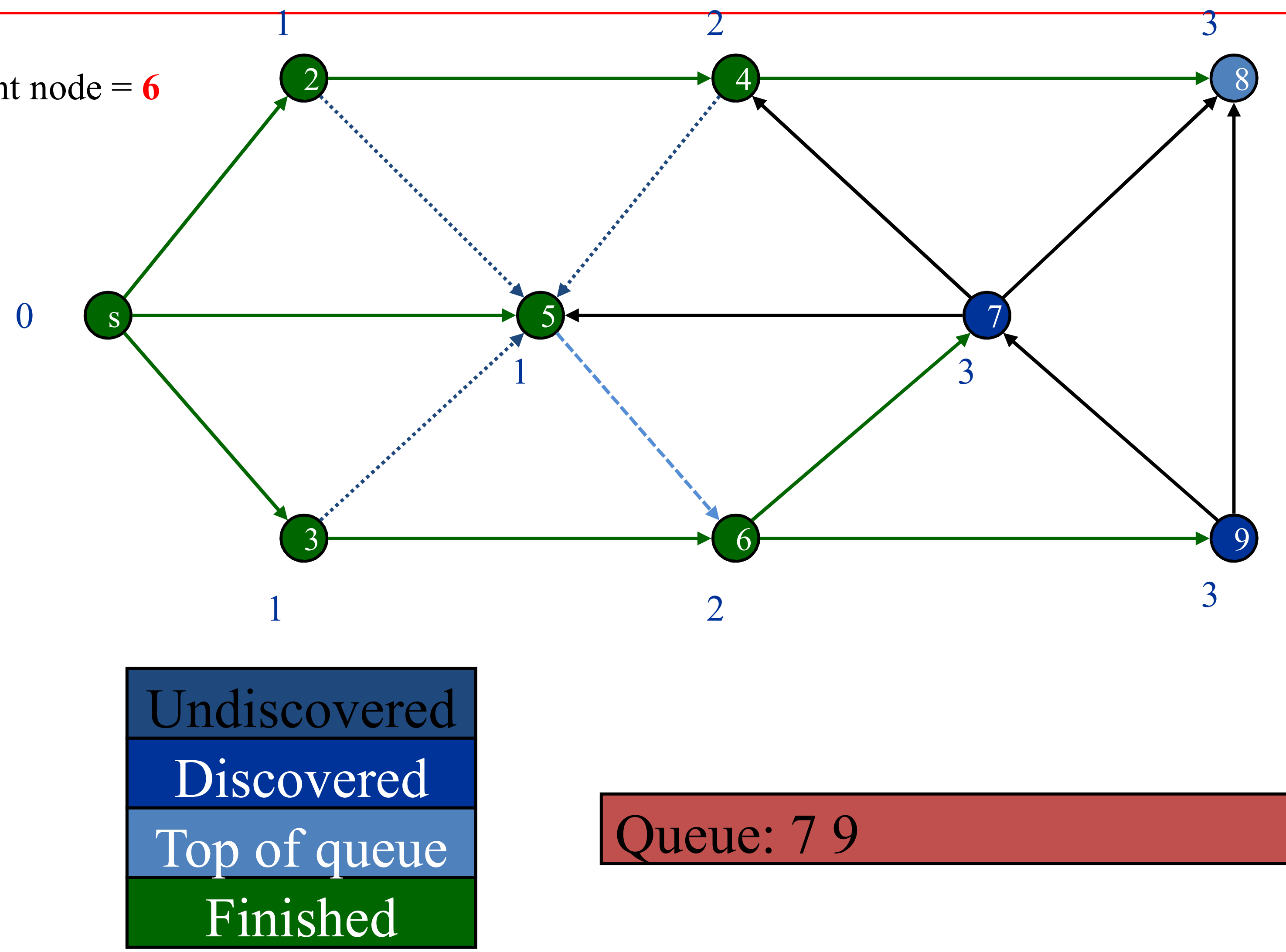
Breadth First Search



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search

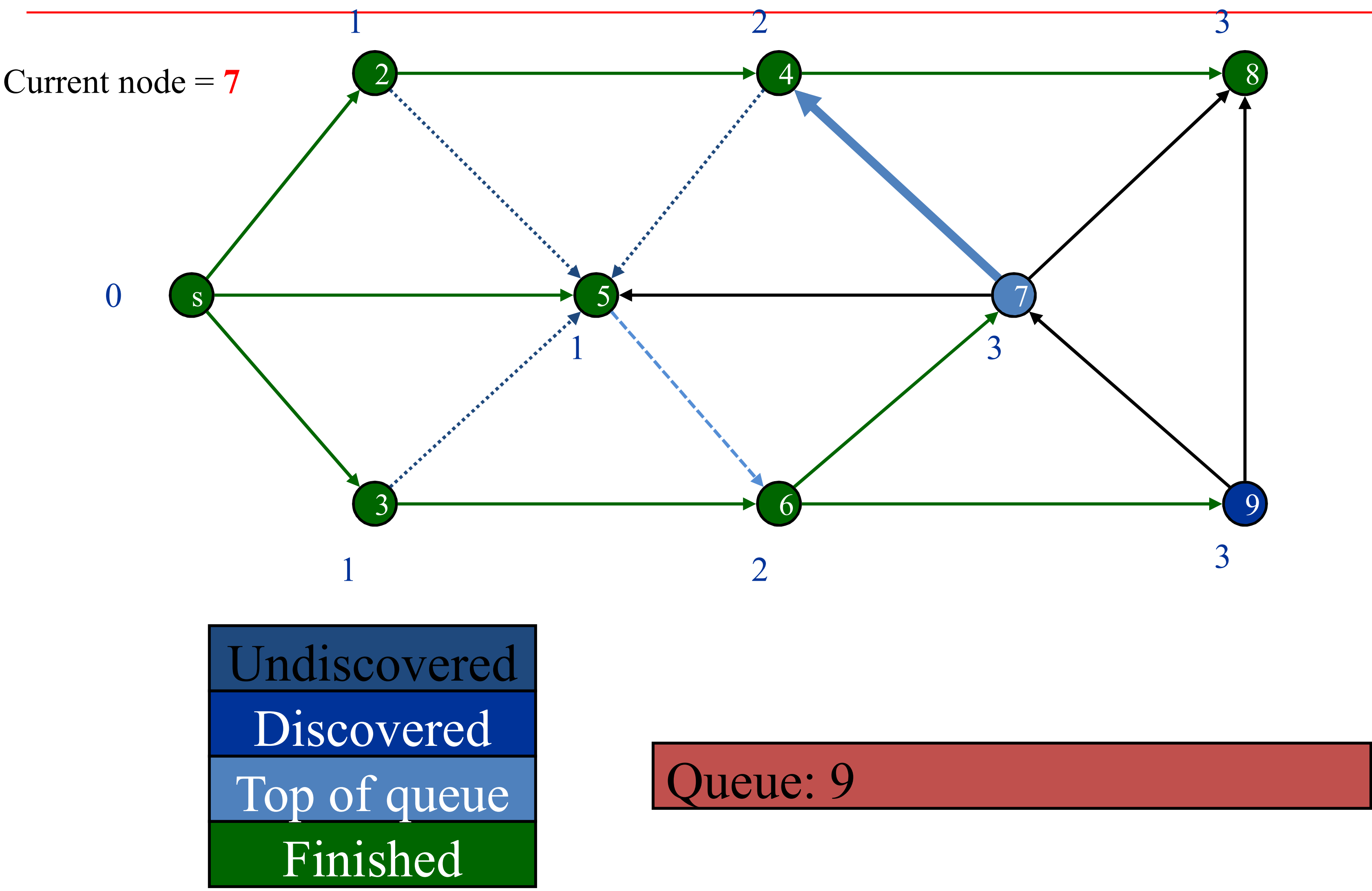
Current node = **6**



Queue: 7 9

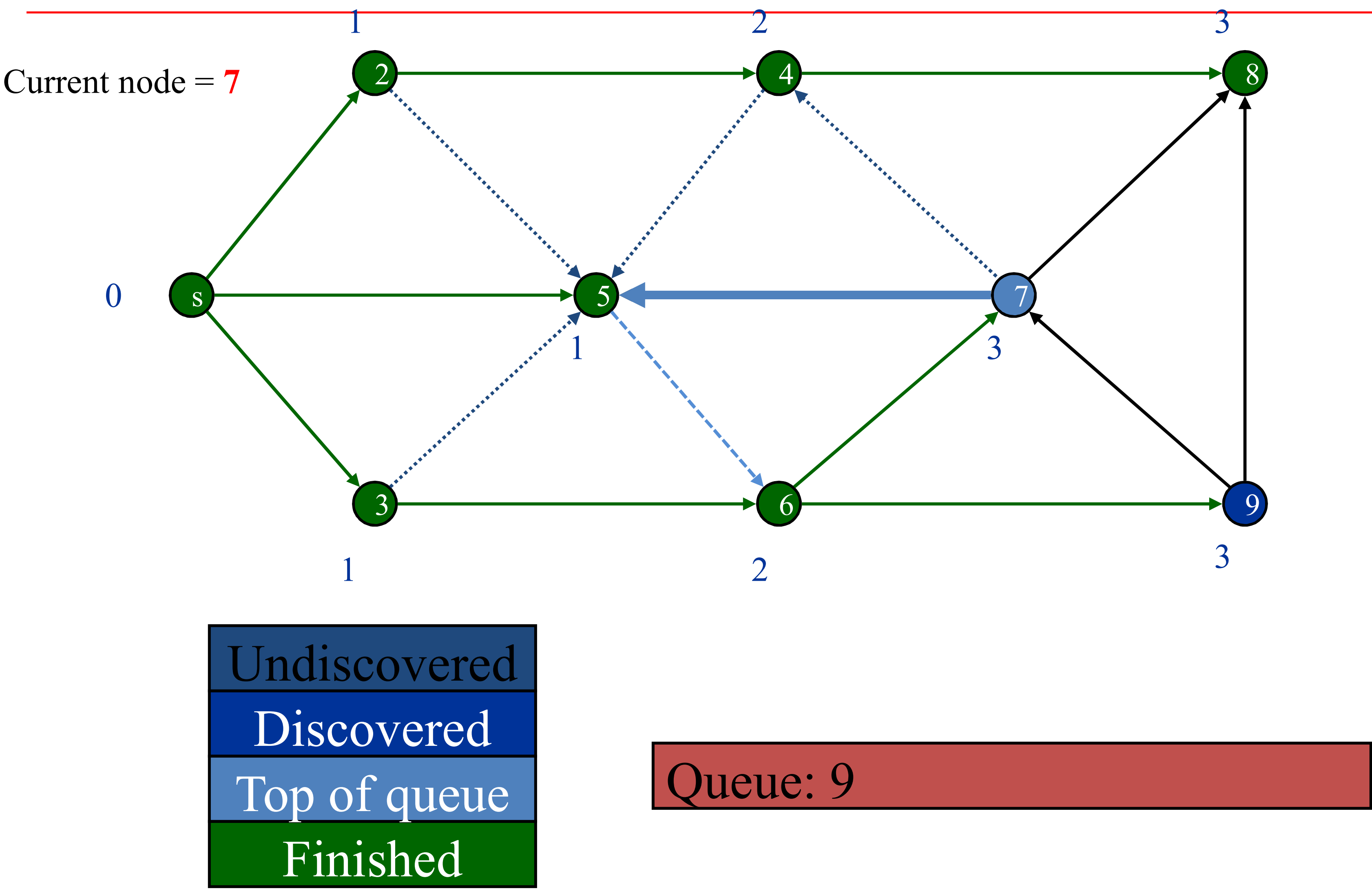
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search



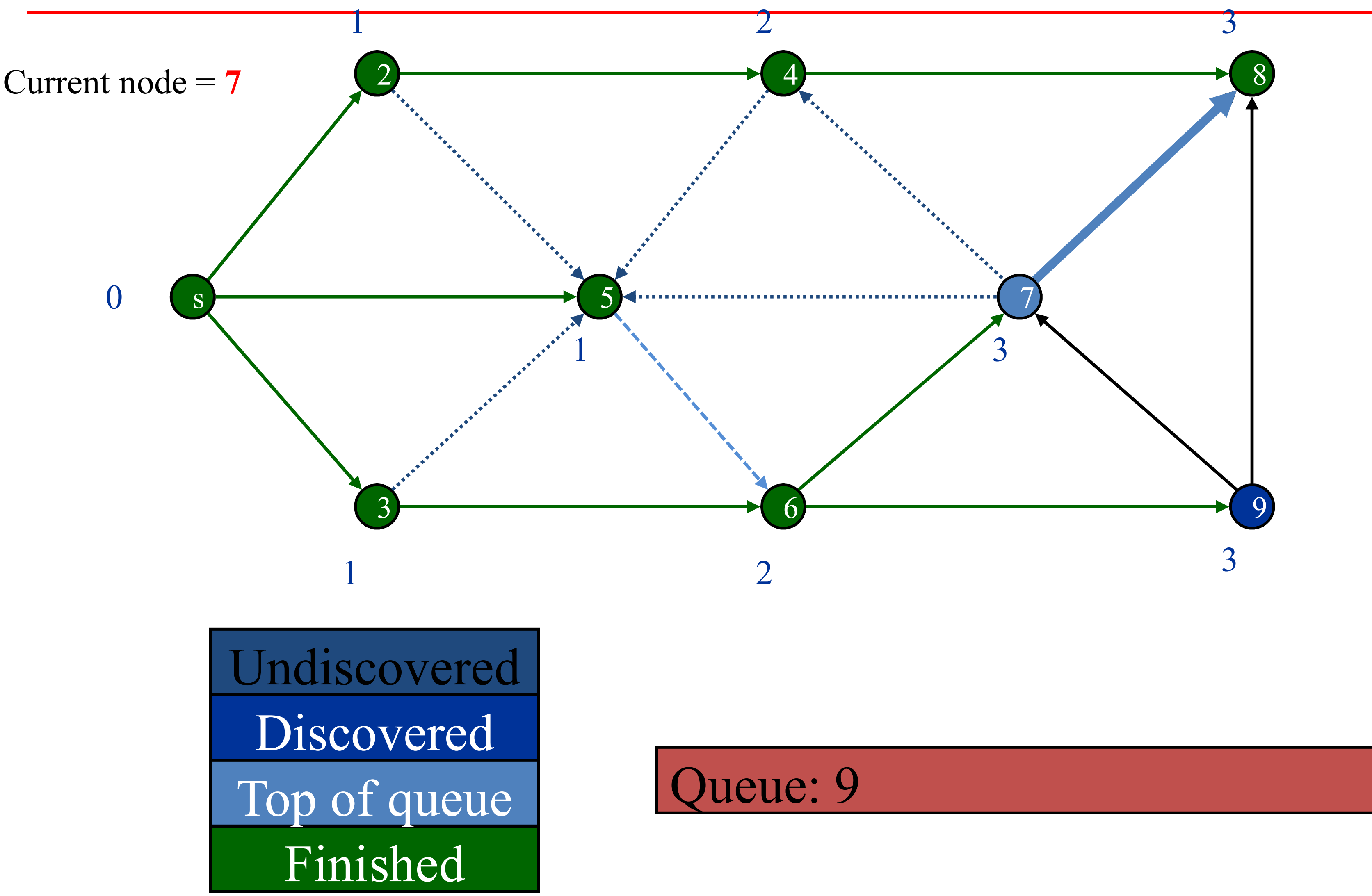
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search



Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

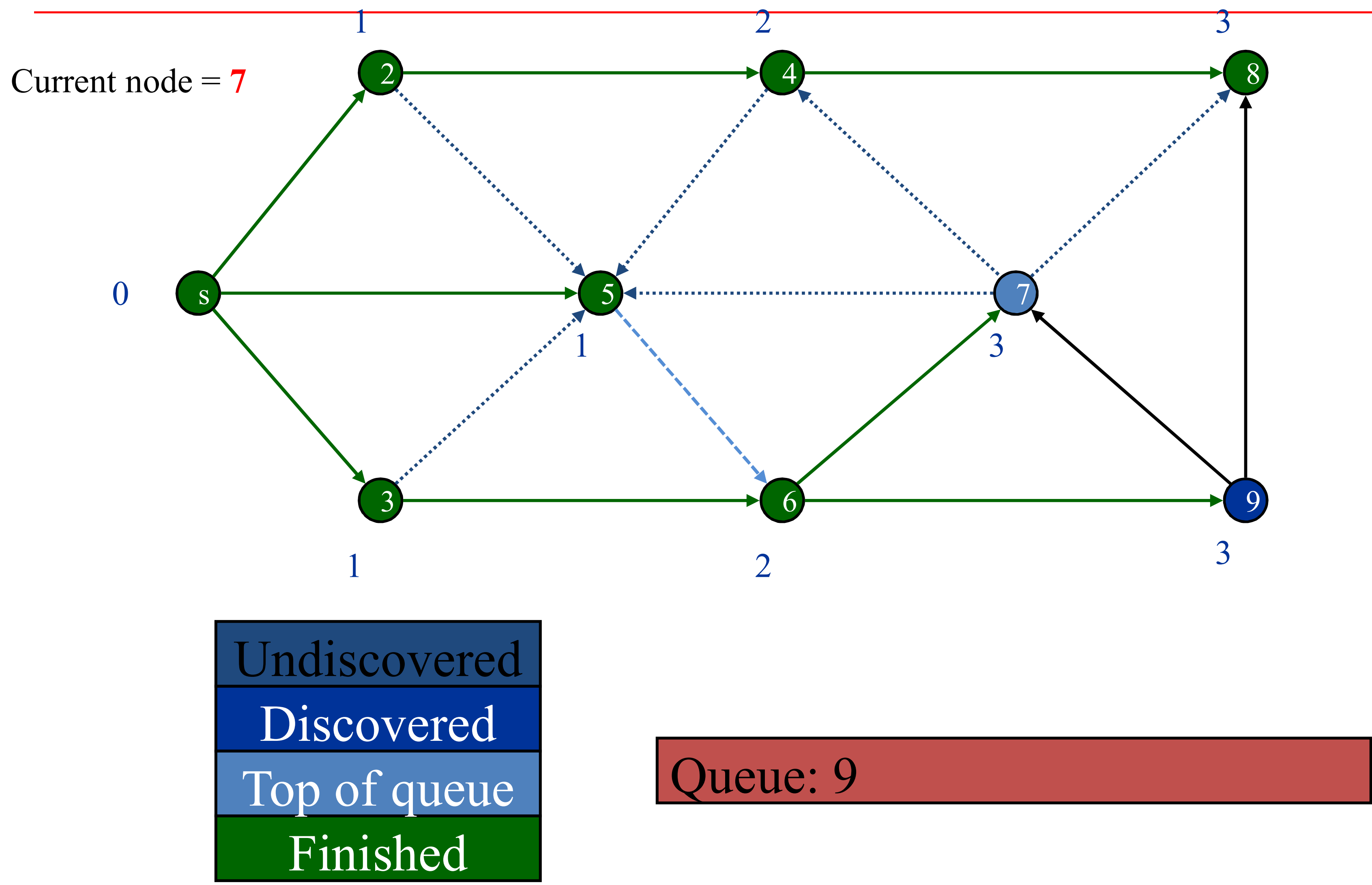
Breadth First Search



Queue: 9

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

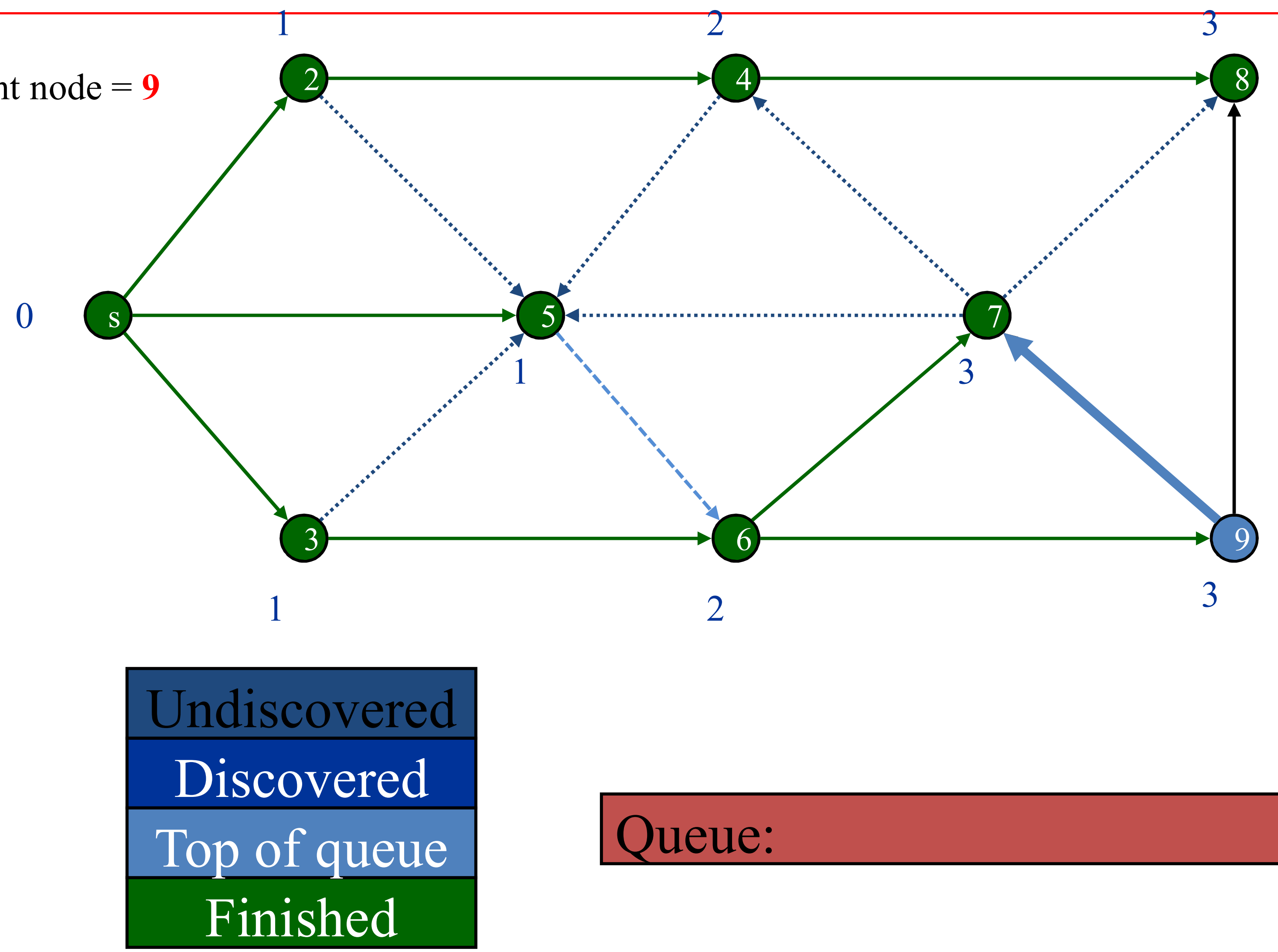
Breadth First Search



Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Breadth First Search

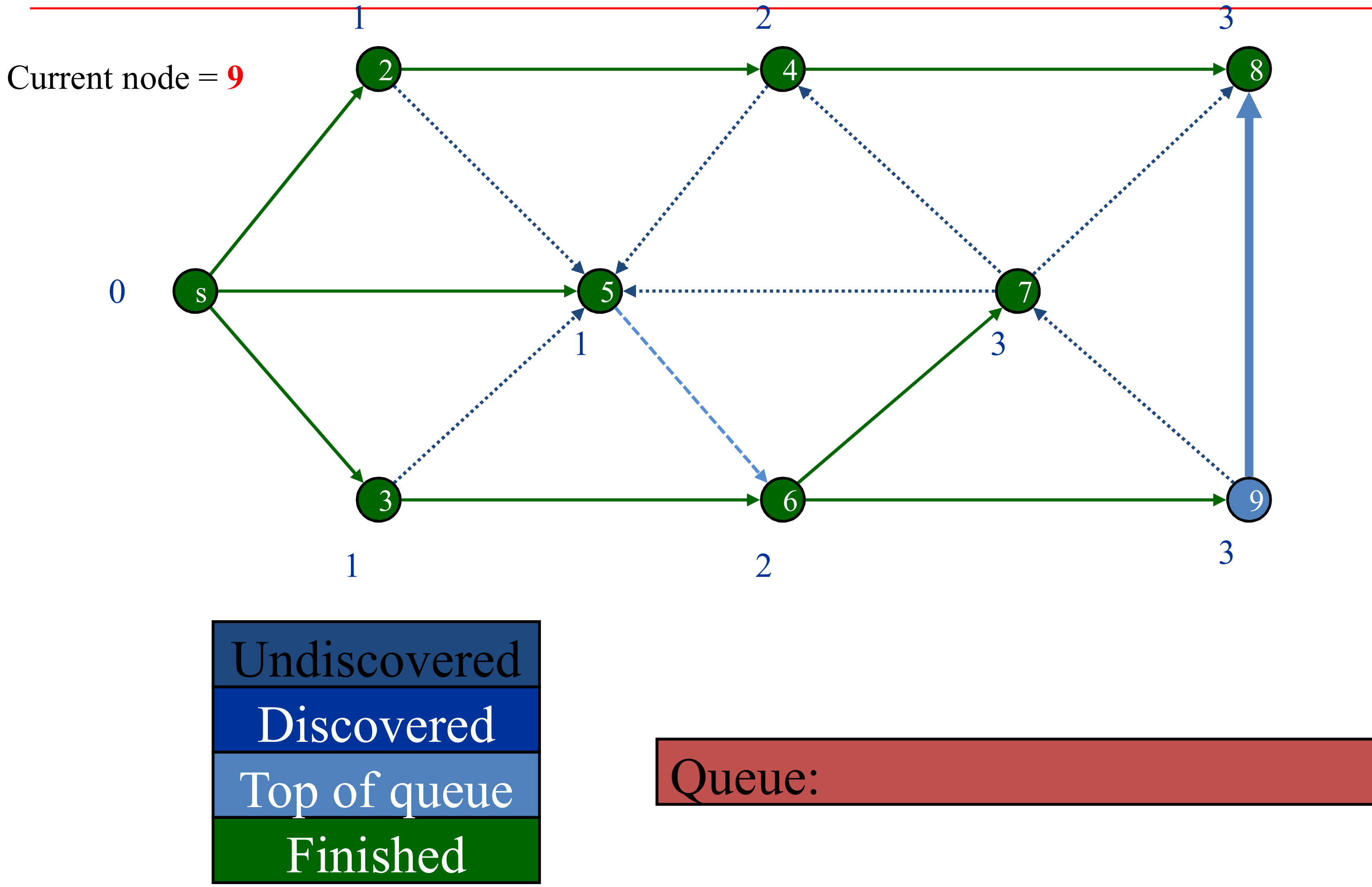
Current node = **9**



Queue:

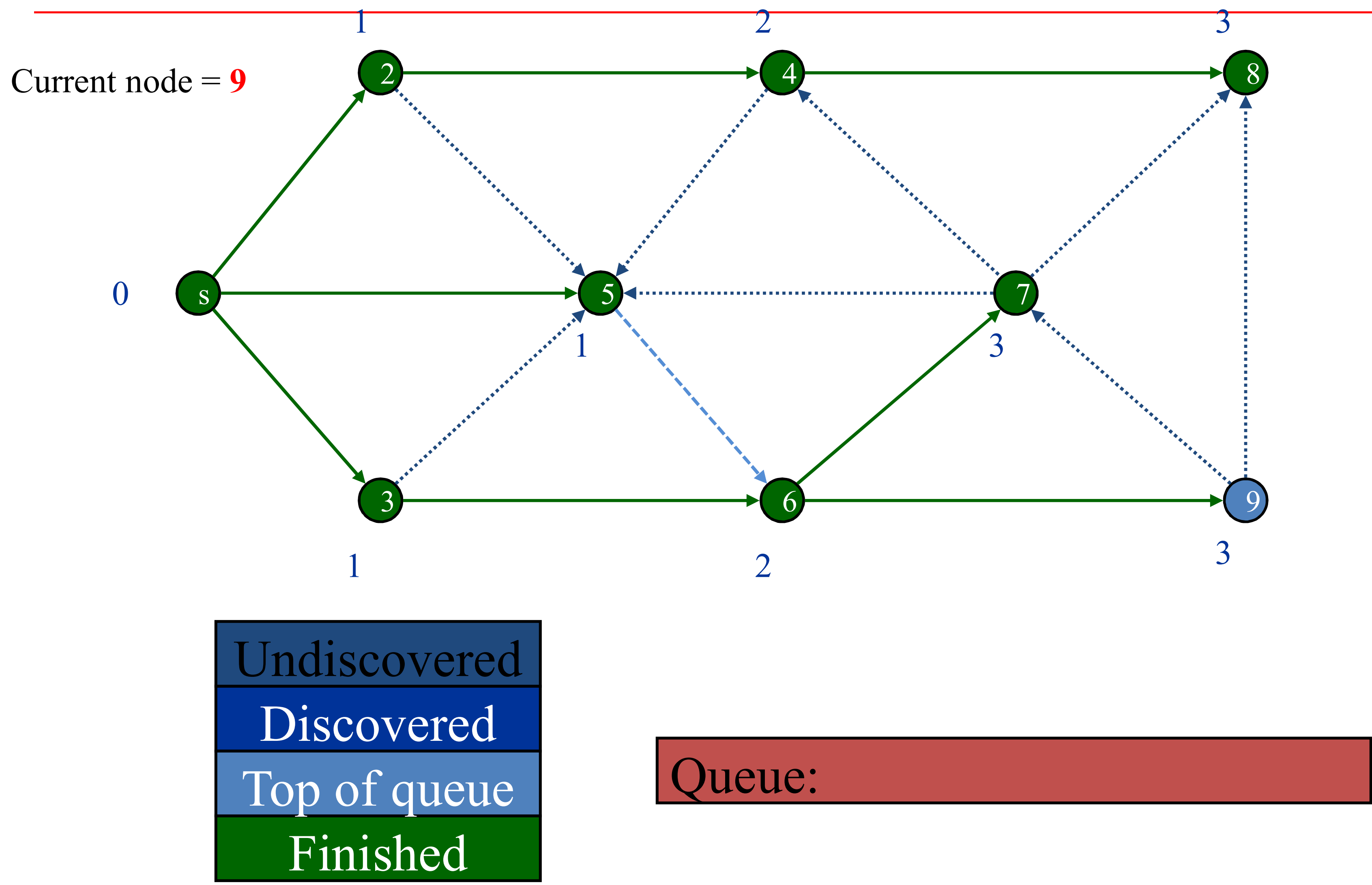
Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Breadth First Search



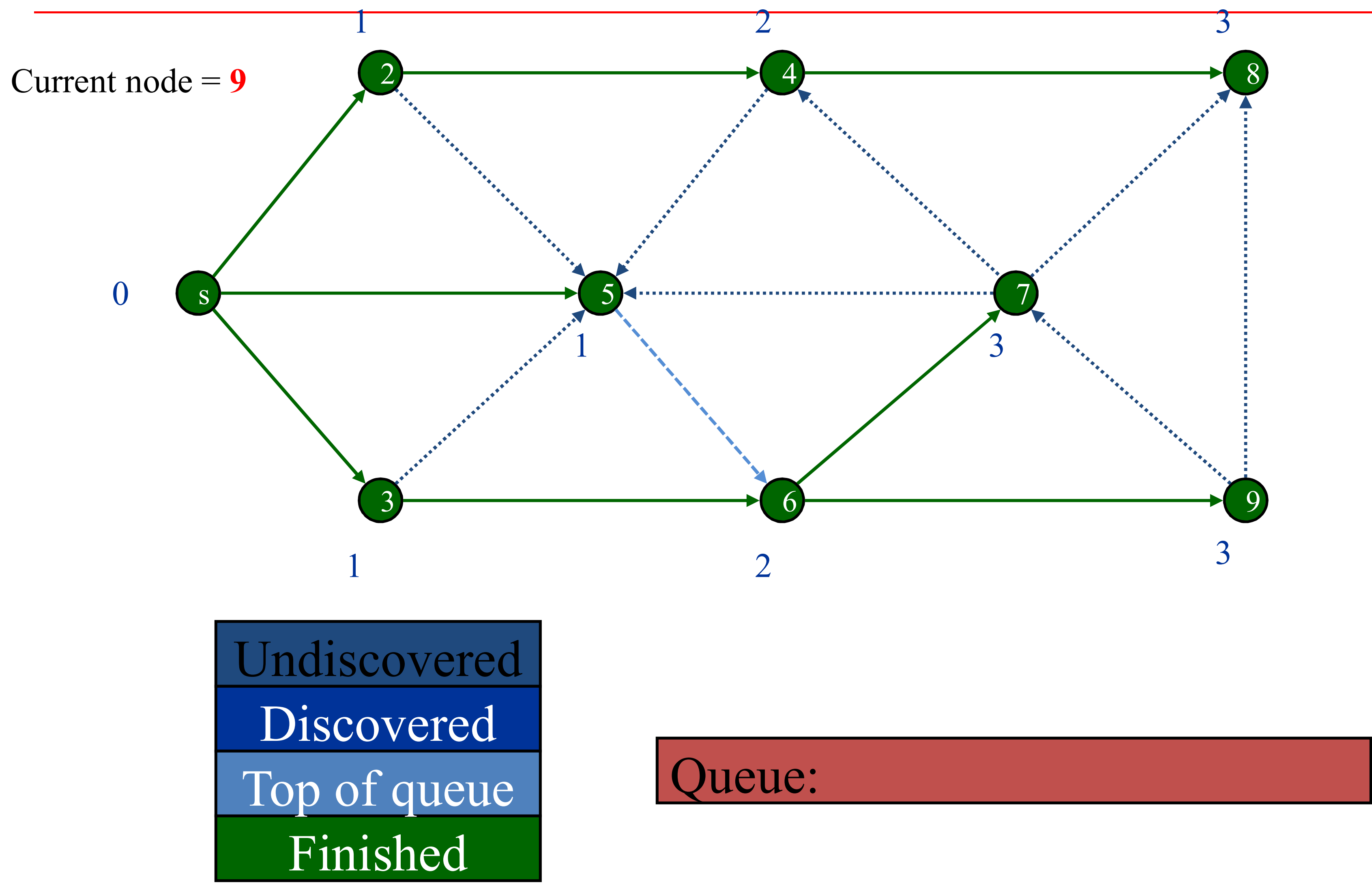
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Breadth First Search

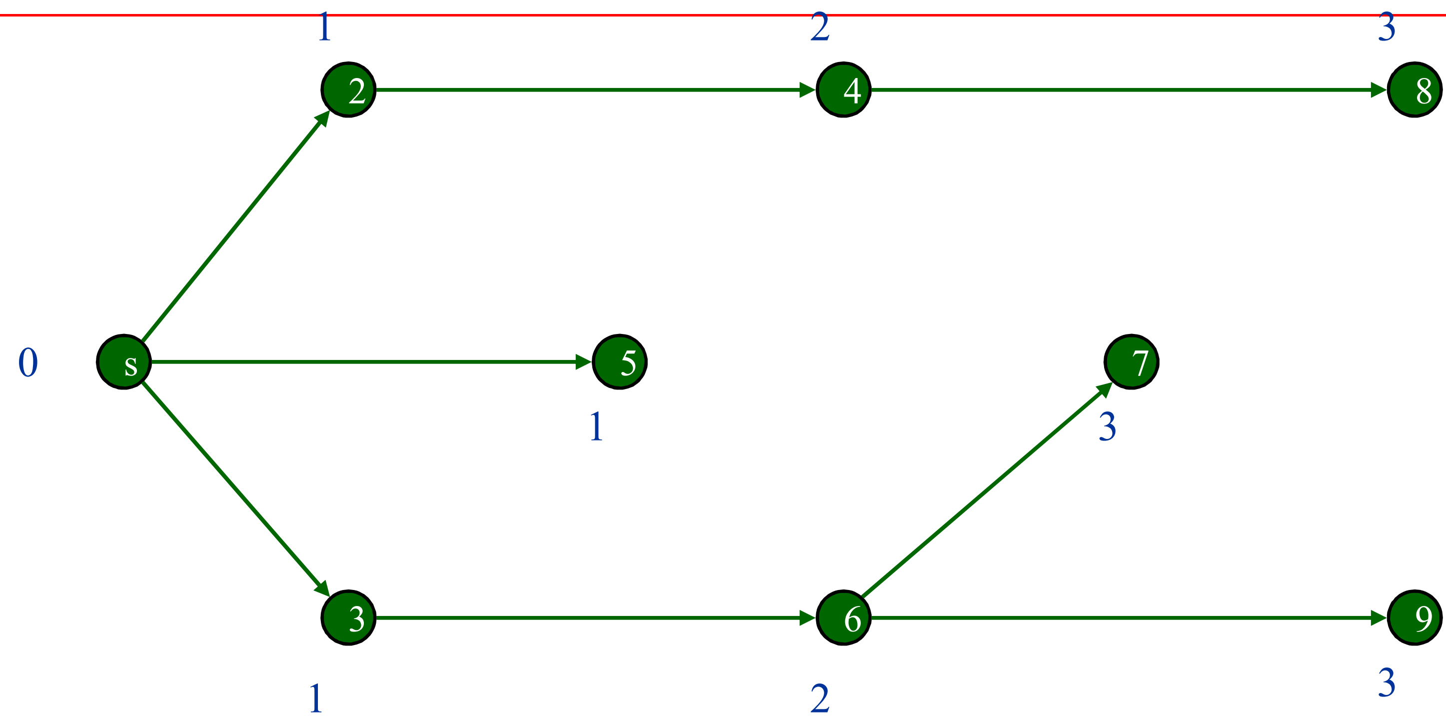


Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Breadth First Search



Breadth First Search

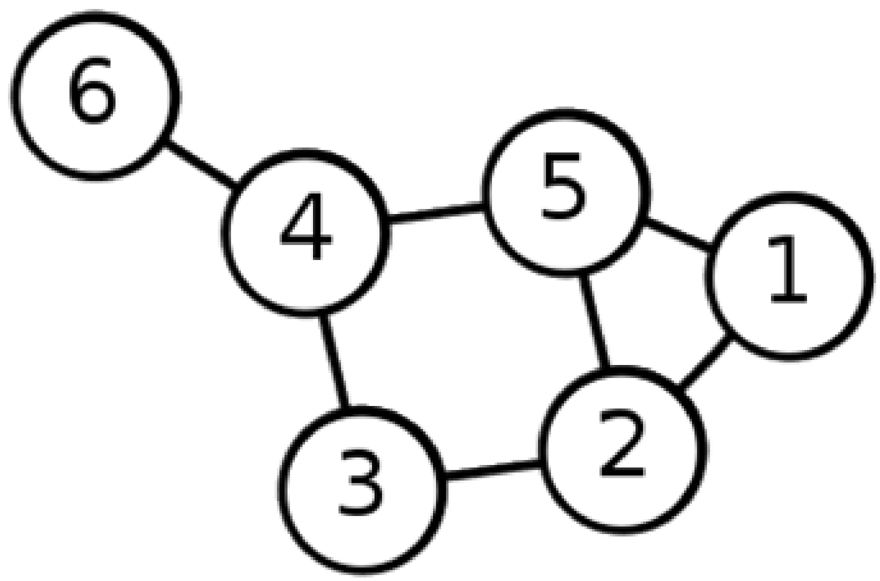


BFS Tree

Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Single Source Shortest Path Problem

- The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.



Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Dijkstra's Algorithm

solution to the single-source shortest path problem in graph theory.

Works on both directed and undirected graphs. However, all edges must have nonnegative weights.

Approach: Greedy : makes local optimum choice in each step hoping to reach global optimum.

Input: Weighted graph G={E,V} and source vertex $v \in V$, such that all edge weights are nonnegative

Output: Lengths of shortest paths (or the shortest paths themselves) from a given source vertex $v \in V$ to all other vertices

Dijkstra's Algorithm

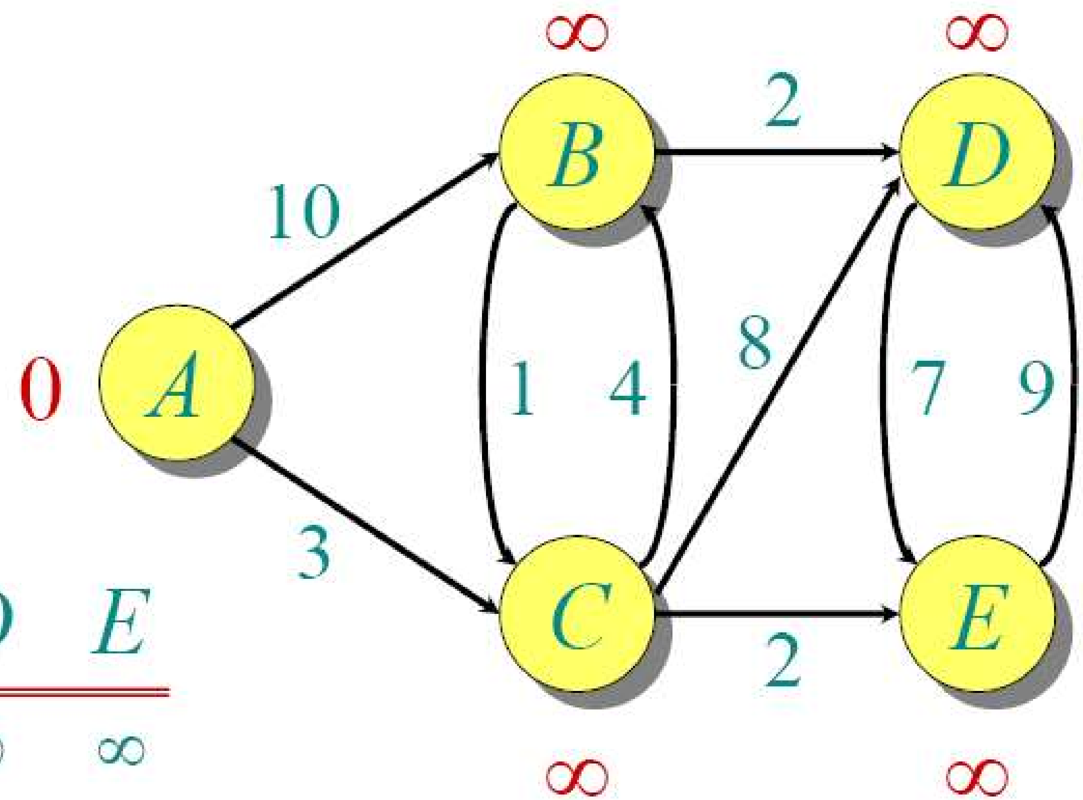
dist[s] ←0	(distance to source vertex is zero)
for all v ∈ V–{s}	
do dist[v] ←∞	(set all other distances to infinity)
S←∅	(S, the set of visited vertices is initially empty)
Q←V	(Q, the queue initially contains all vertices)
while Q ≠∅	(while the queue is not empty)
do u ← mindistance(Q,dist)	(select the element of Q with the min. distance)
S←S∪{u}	(add u to list of visited vertices)
for all v ∈ neighbors[u]	
do if dist[v] > dist[u] + w(u, v)	(if new shortest path found)
then d[v] ←d[u] + w(u, v)	(set new value of shortest path)
return dist	

Total running time: O(n^2)

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Dijkstra's Algorithm

Initialize:

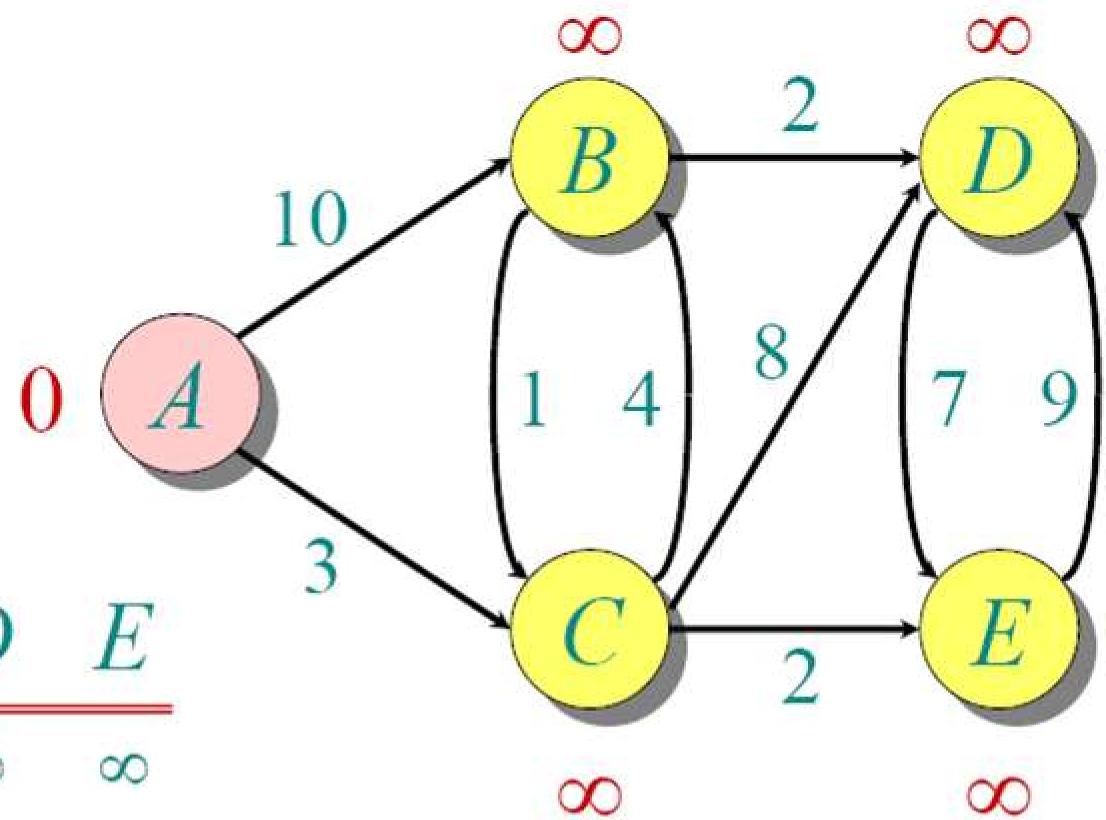


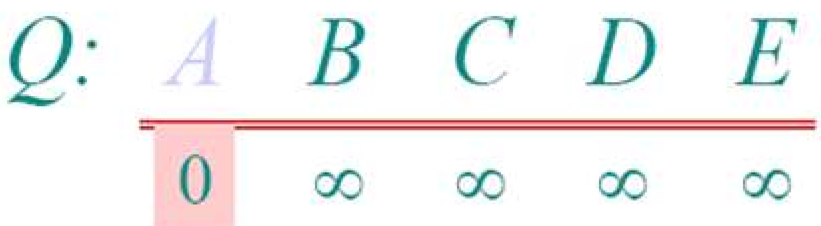
Q:	A	B	C	D	E
	0	∞	∞	∞	∞

S: {}

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

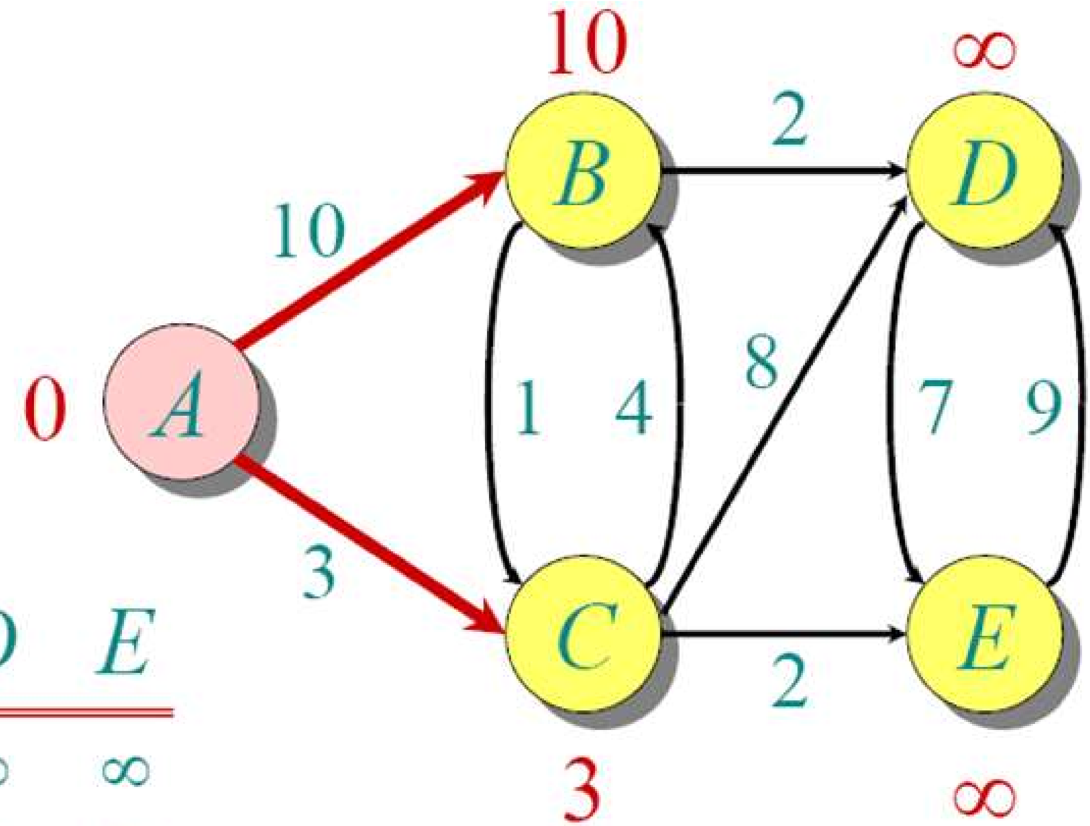
Dijkstra's Algorithm





Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Dijkstra’s Algorithm

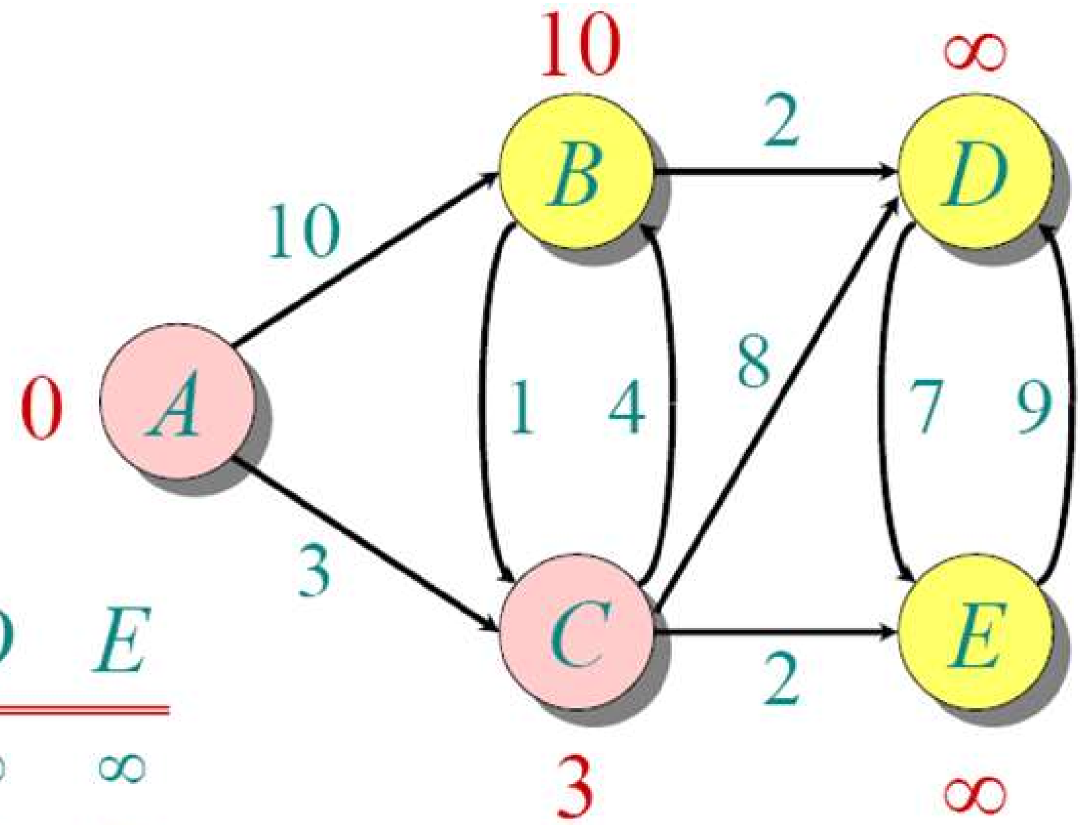


Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: $\{ A \}$

Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Dijkstra's Algorithm

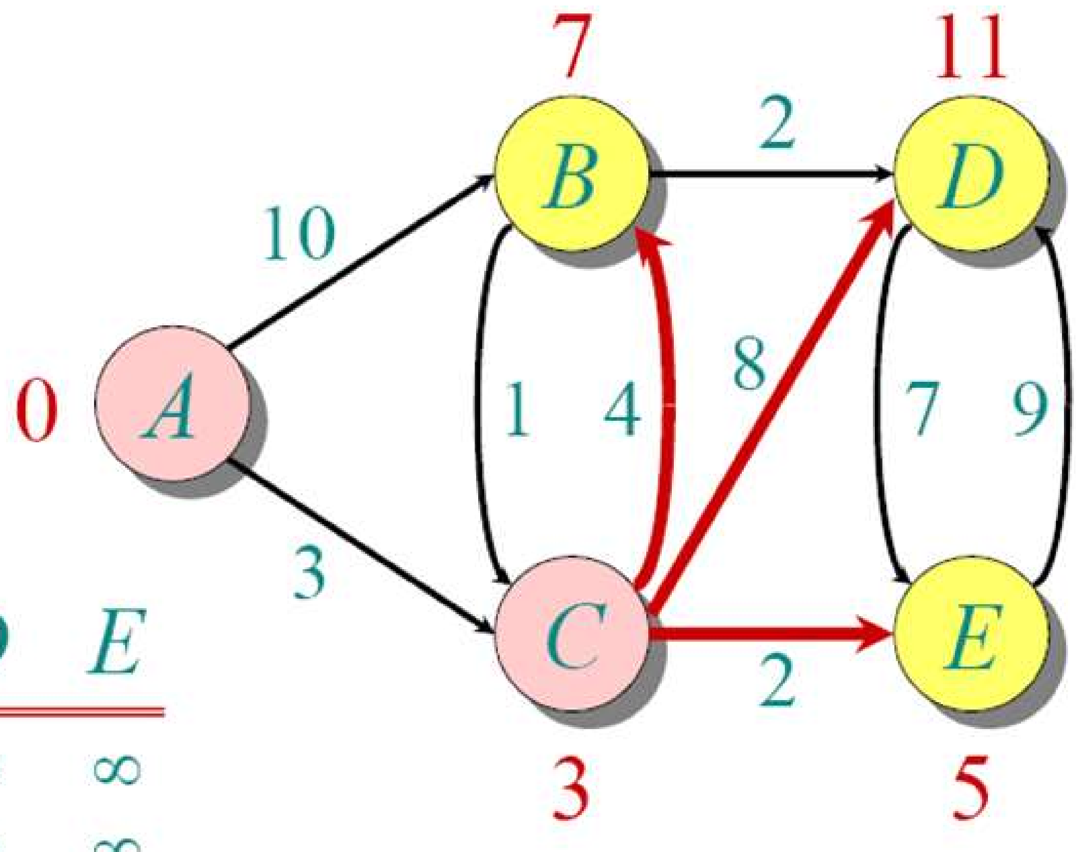


Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A, C }

Bhaskar Sardar, Information Technology Department, Jadavpur University, India

Dijkstra’s Algorithm

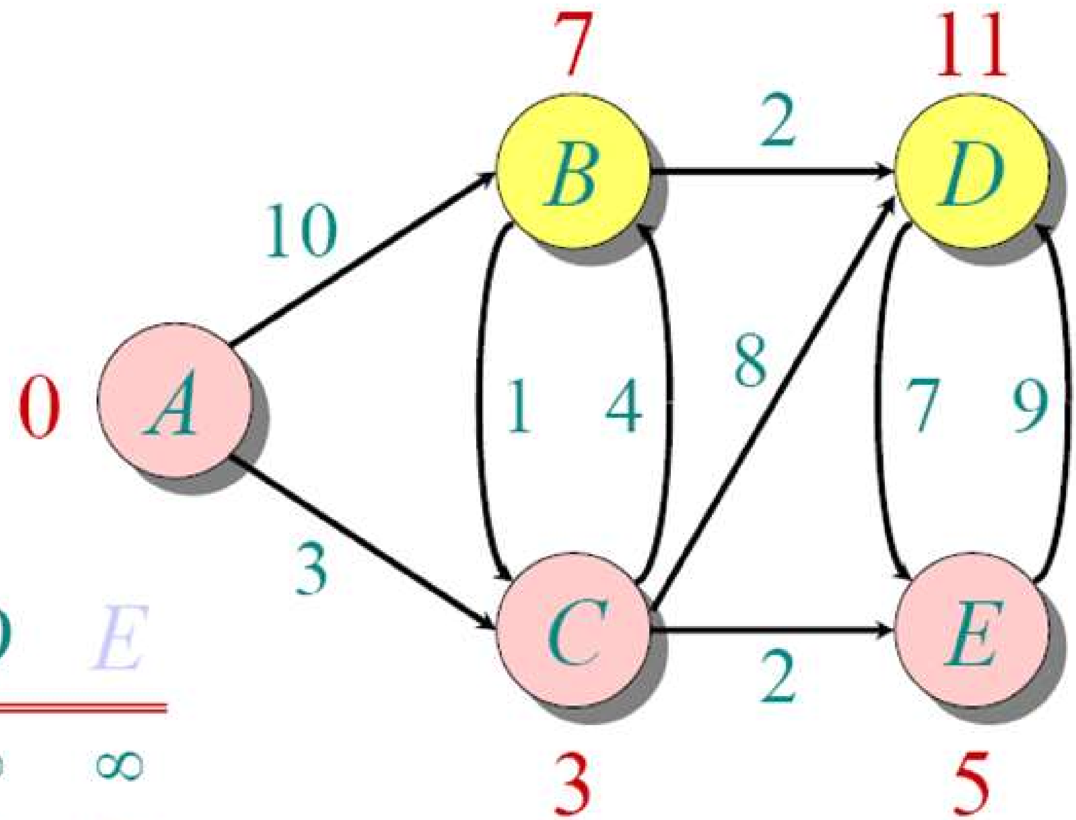


Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C }

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Dijkstra's Algorithm

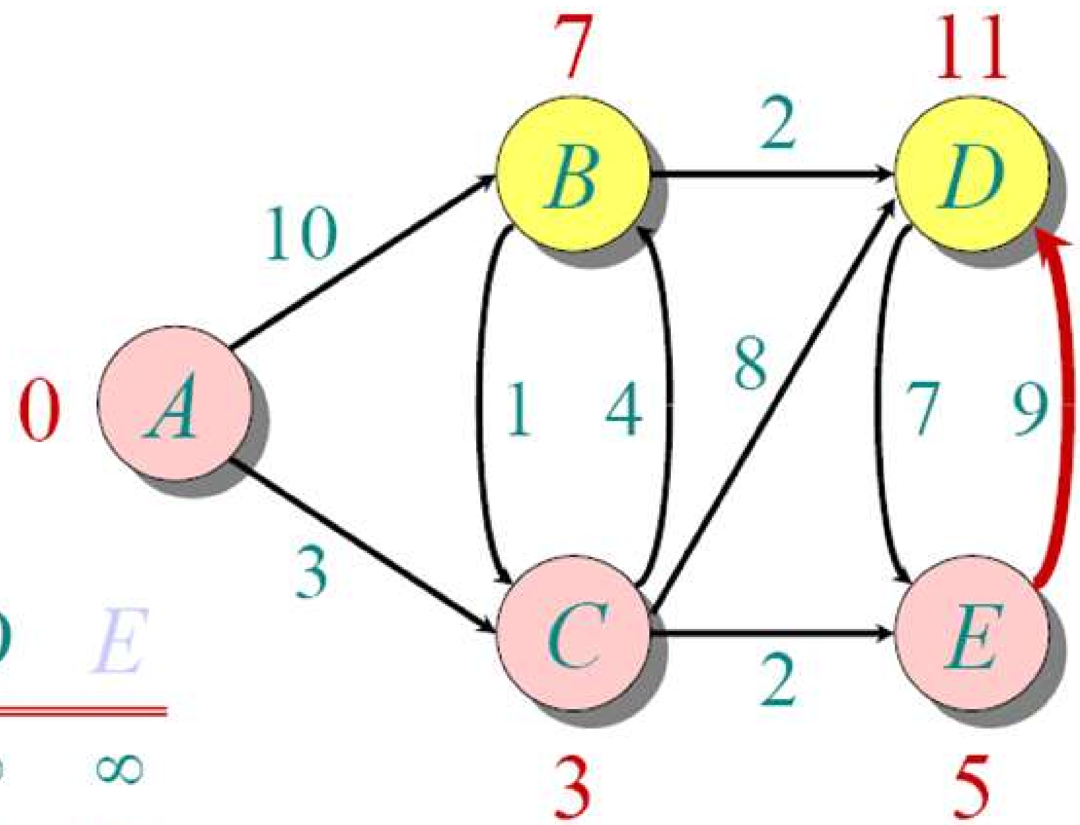


Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

$S: \{A, C, E\}$

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

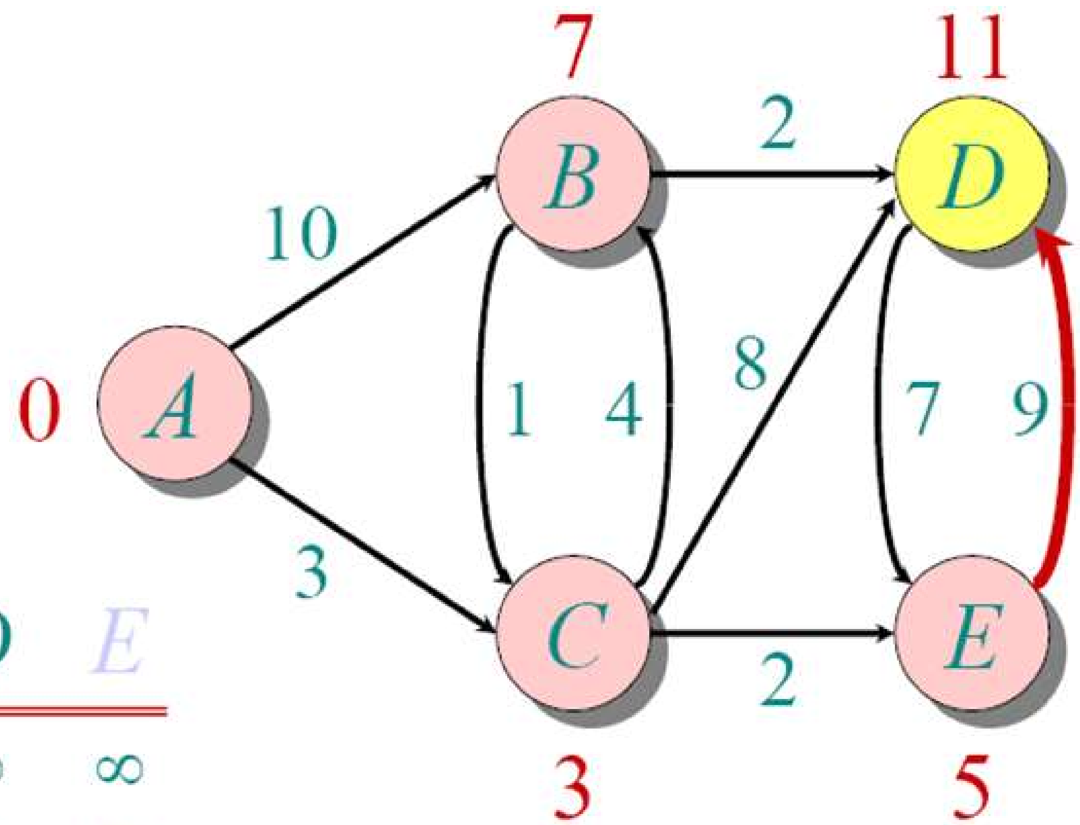
Dijkstra’s Algorithm



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E }

Dijkstra’s Algorithm

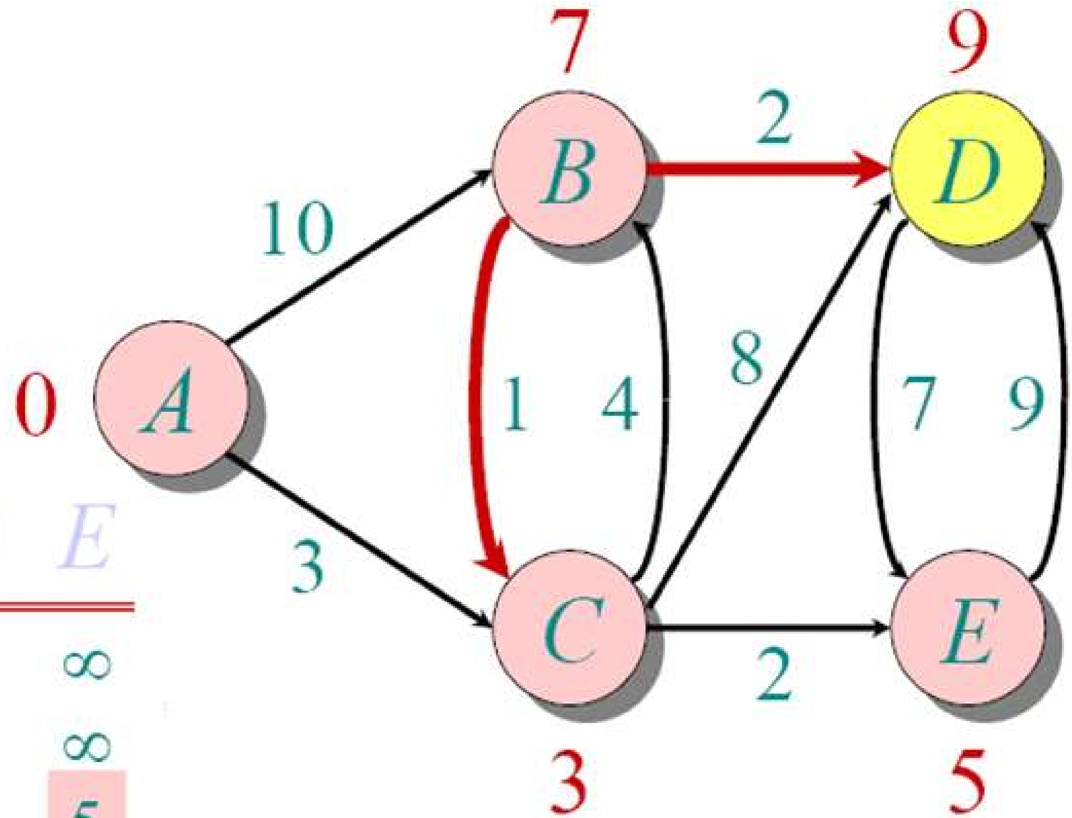


Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E, B }

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Dijkstra’s Algorithm



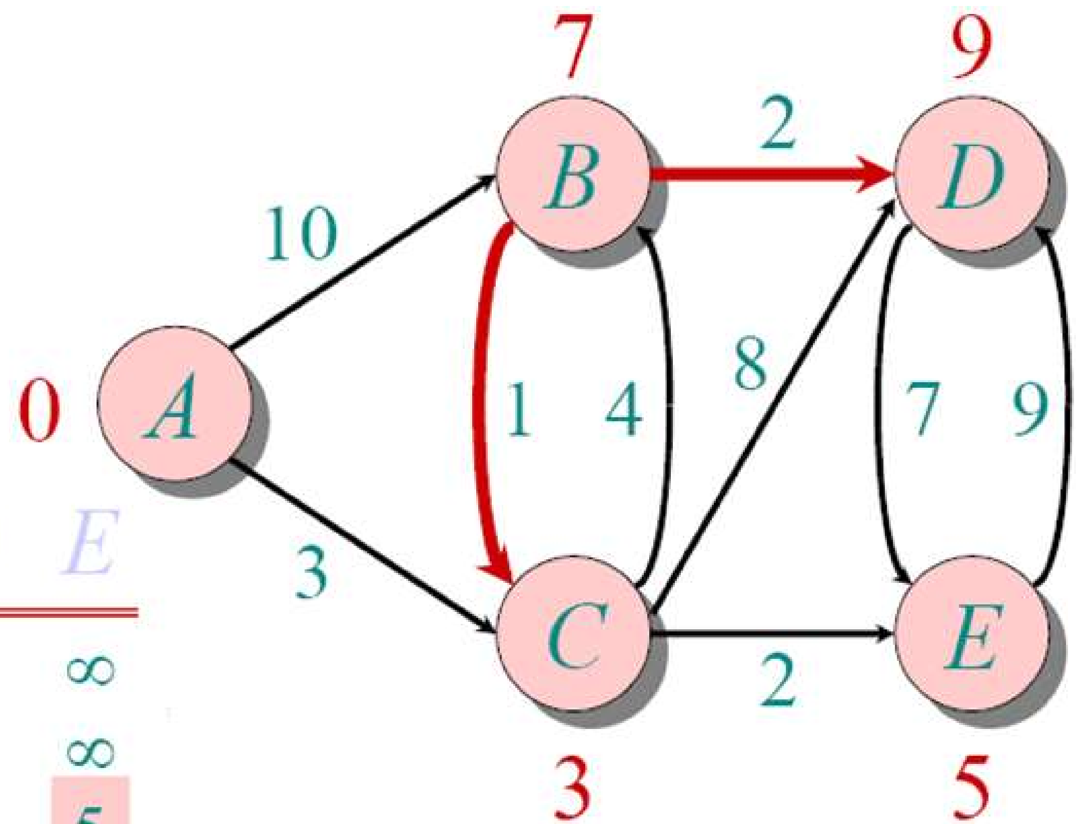
Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	
			9	

S: { A, C, E, B }

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Dijkstra’s Algorithm



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

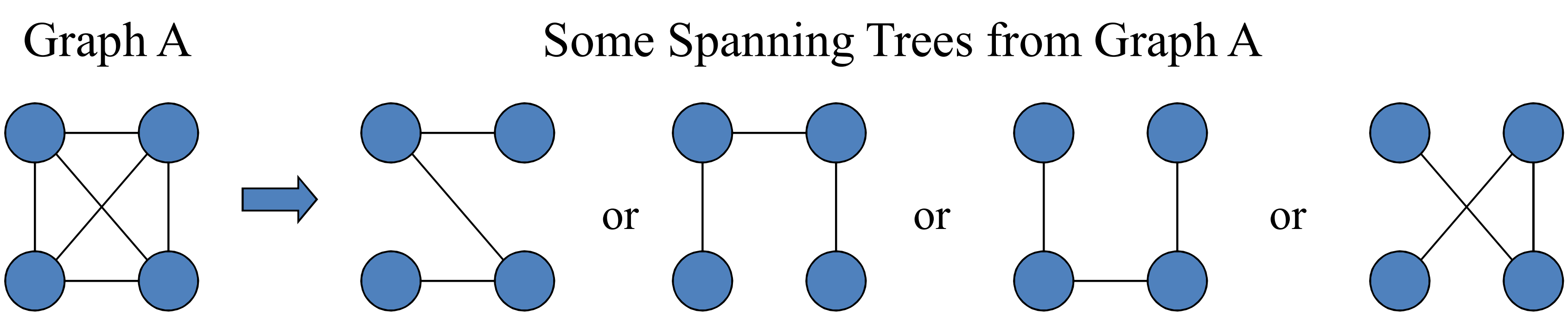
S: { A, C, E, B, D }

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Spanning Trees

A spanning tree of a graph is a subgraph that contains all the vertices and is a tree.

A graph may have many spanning trees.

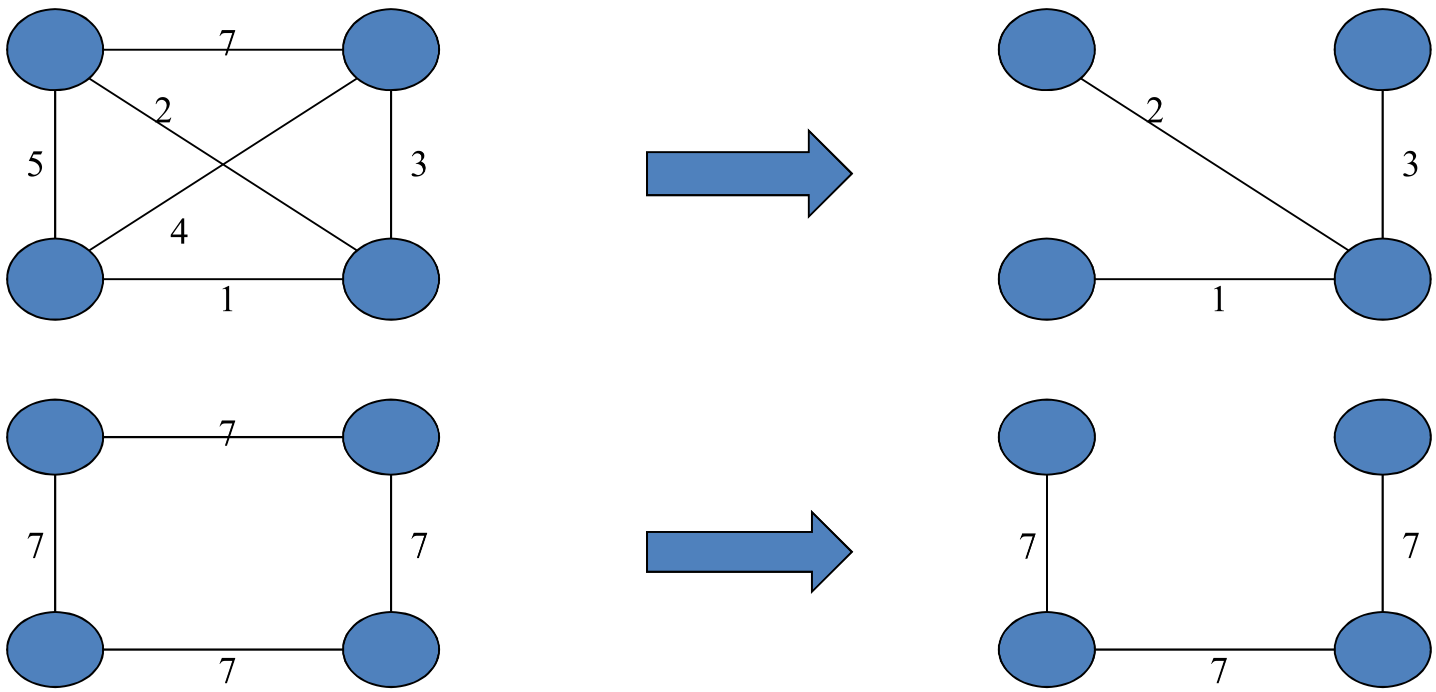


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Minimum Spanning Trees

The Minimum Spanning Tree for a given graph is the Spanning Tree of minimum cost for that graph.

Graph Minimum Spanning Tree



Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Algorithms for Obtaining the Minimum Spanning Tree

- Kruskal's Algorithm
- Prim's Algorithm

Kruskal's Algorithm

- This algorithm creates a forest of trees.
- Initially the forest consists of n single node trees (and no edges).
- At each step, we add one edge (the cheapest one) so that it joins two trees together.
- If it were to form a cycle, it would simply link two nodes that were already part of a single connected tree, so that this edge would not be needed.

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

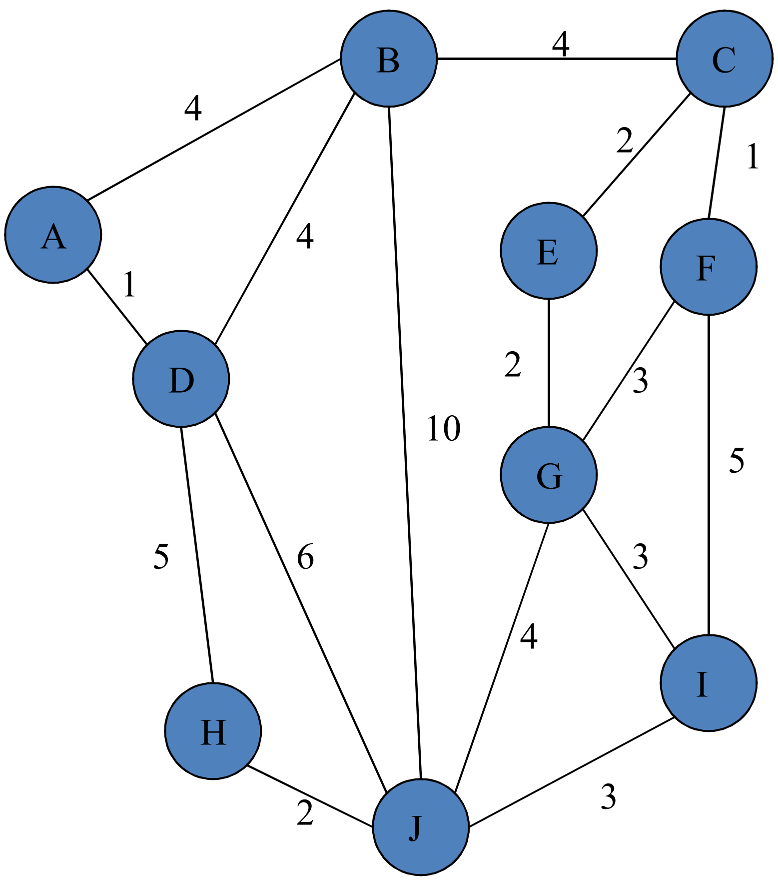
Kruskal's Algorithm

The steps are:

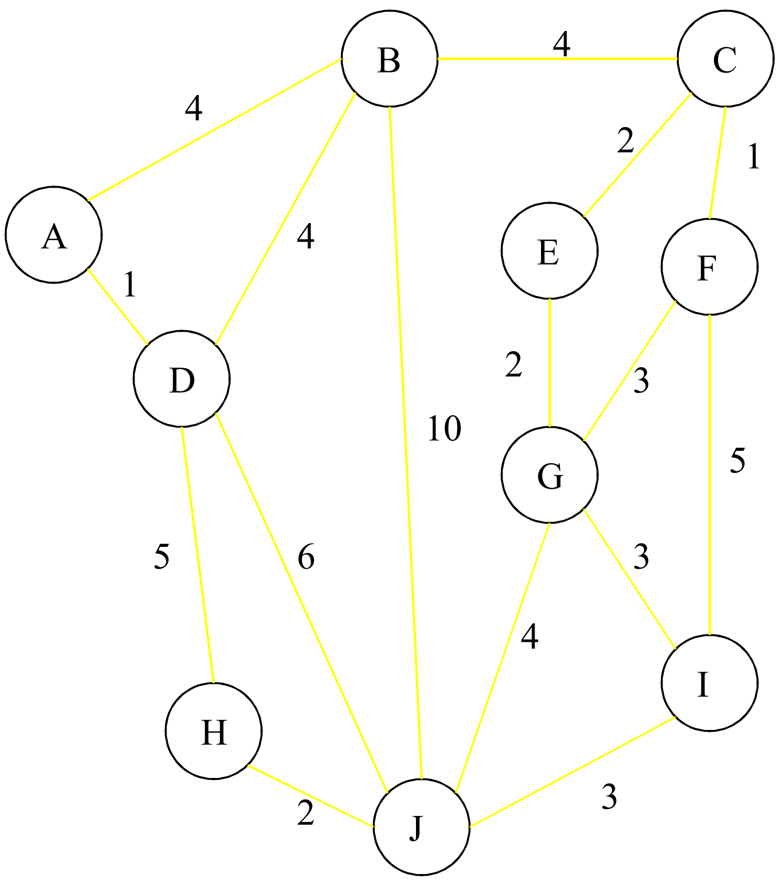
1. The forest is constructed - with each node in a separate tree.
2. Sort the edges.
3. Until we've added n-1 edges,
    3.1. Extract the next cheapest edge.
    3.2. If it forms a cycle, reject it.
    3.3. Else add it to the forest. Adding it to the forest will join two trees together.

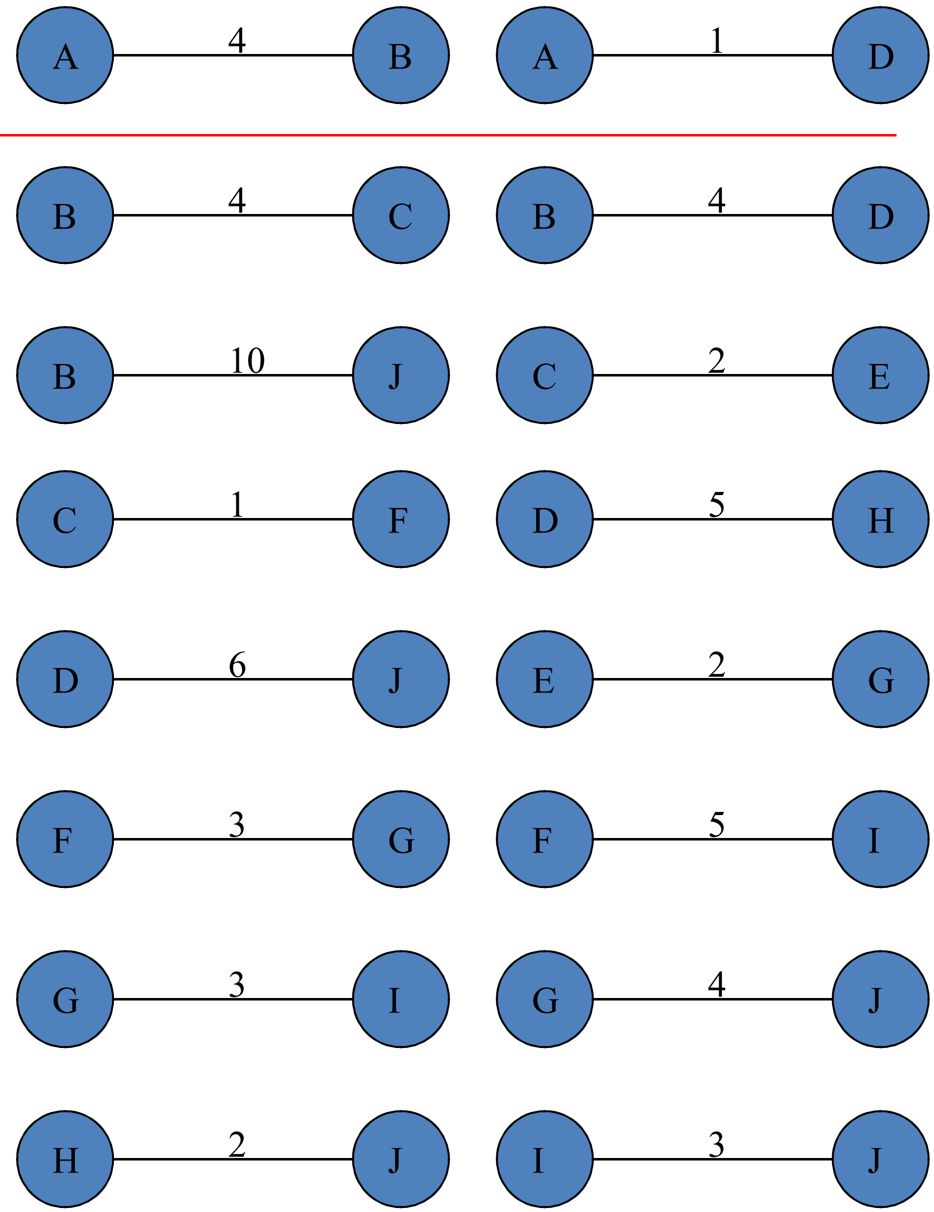
Every step will join two trees in the forest together, so that at the end, there will only be one tree in T.

Example Graph

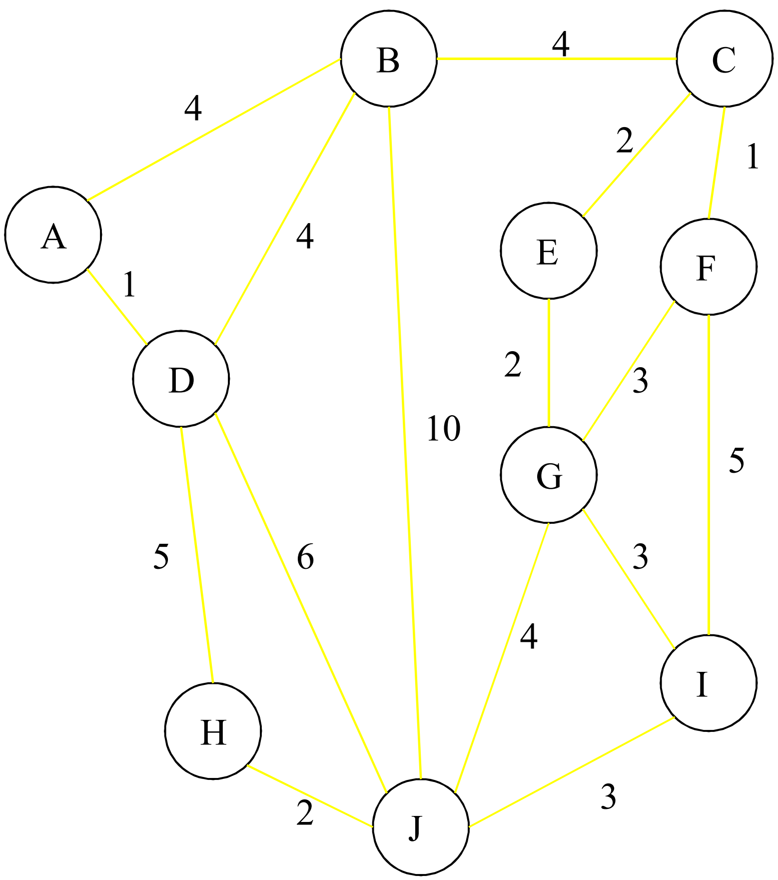


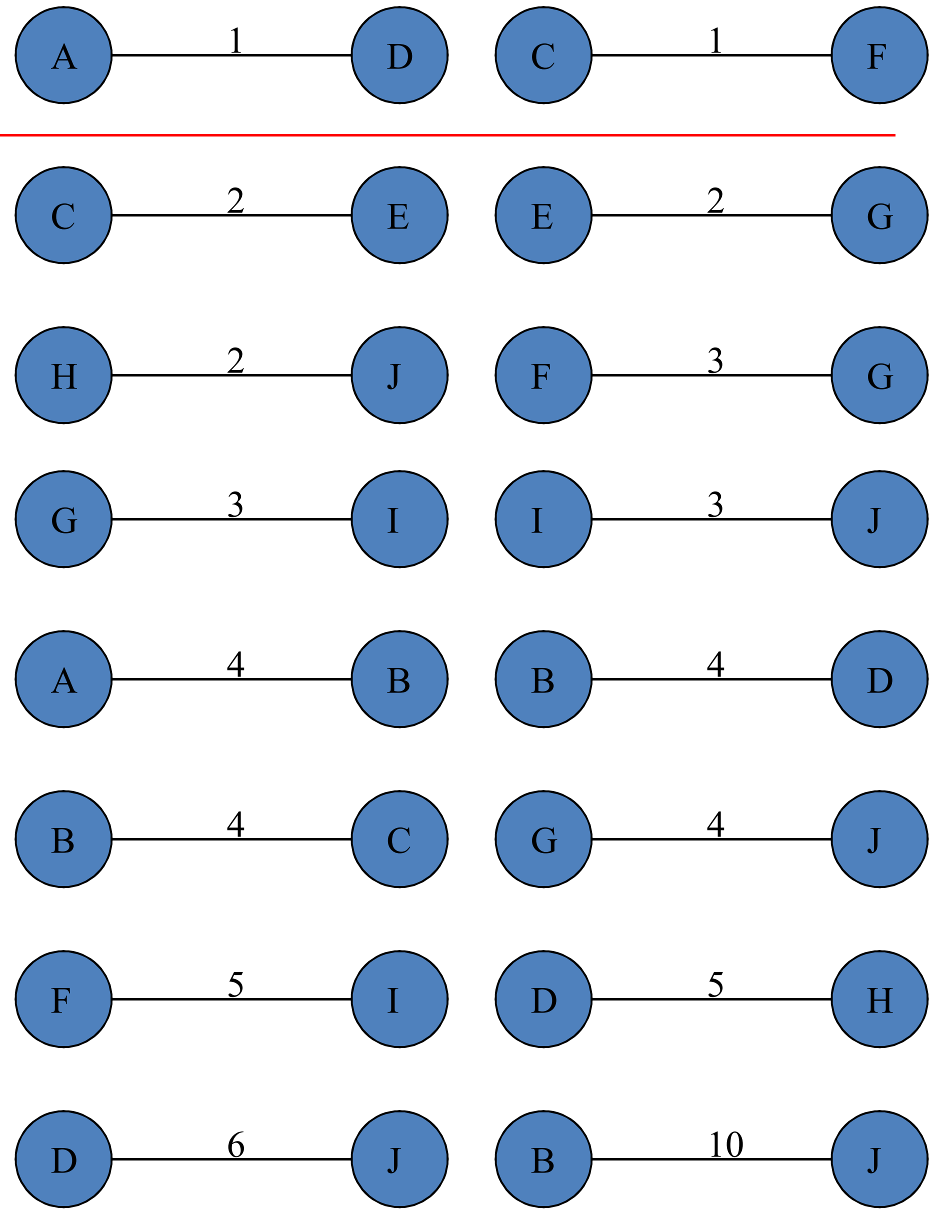
Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*


A
B
C
D
E
F
G
H
I
J
4
4
1
4
2
1
2
3
10
5
5
6
3
4
2
3


A 4 B
A 1 D
B 4 C
B 4 D
B 10 J
C 2 E
C 1 F
D 5 H
D 6 J
E 2 G
F 3 G
F 5 I
G 3 I
G 4 J
H 2 J
I 3 J

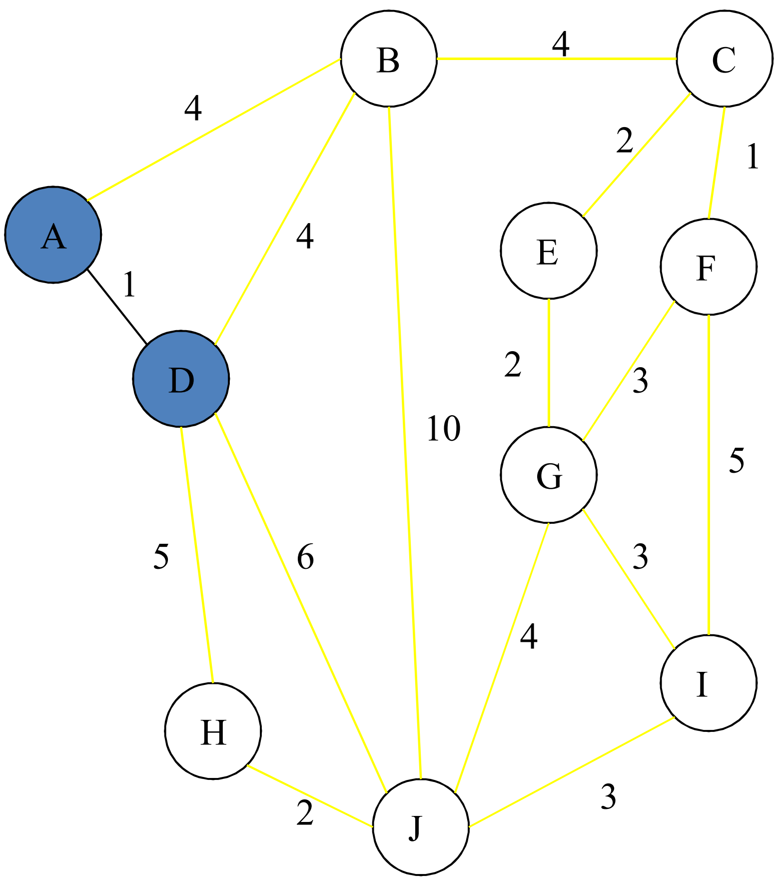
Sort Edges

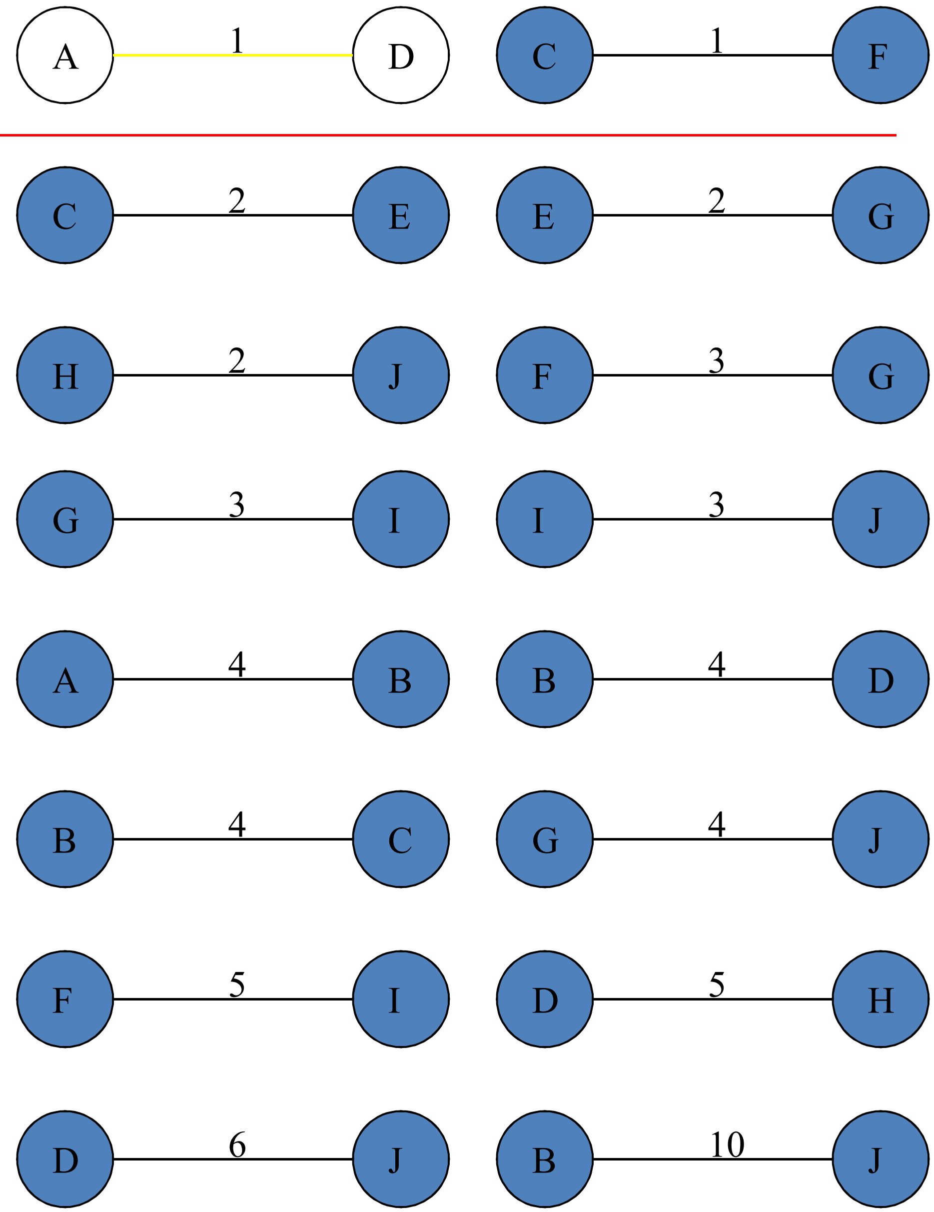




Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

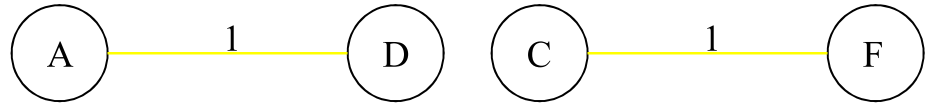
Add Edge

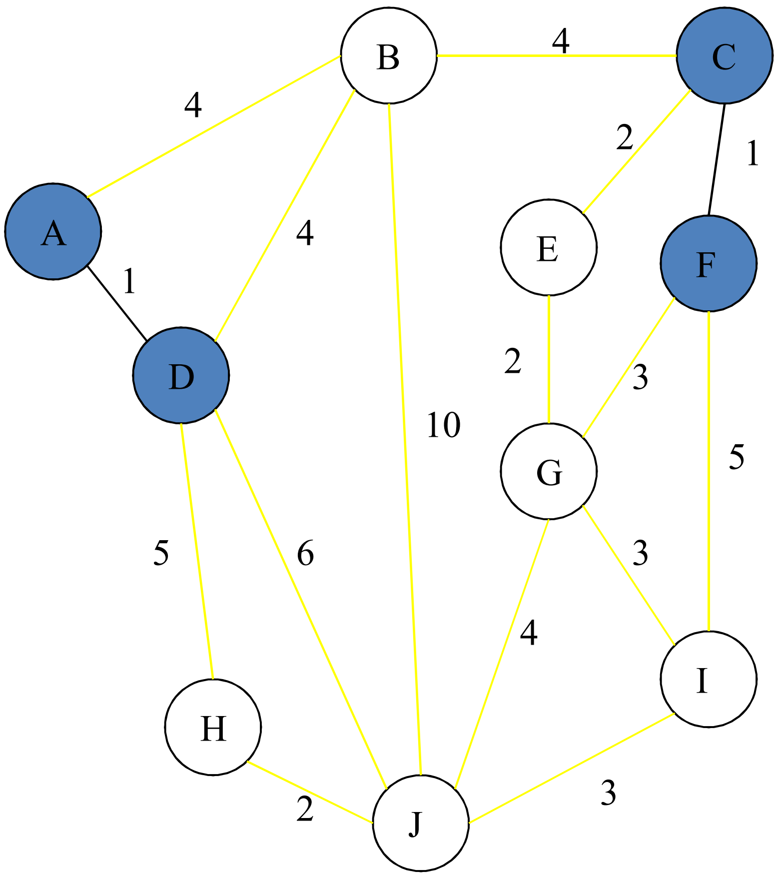


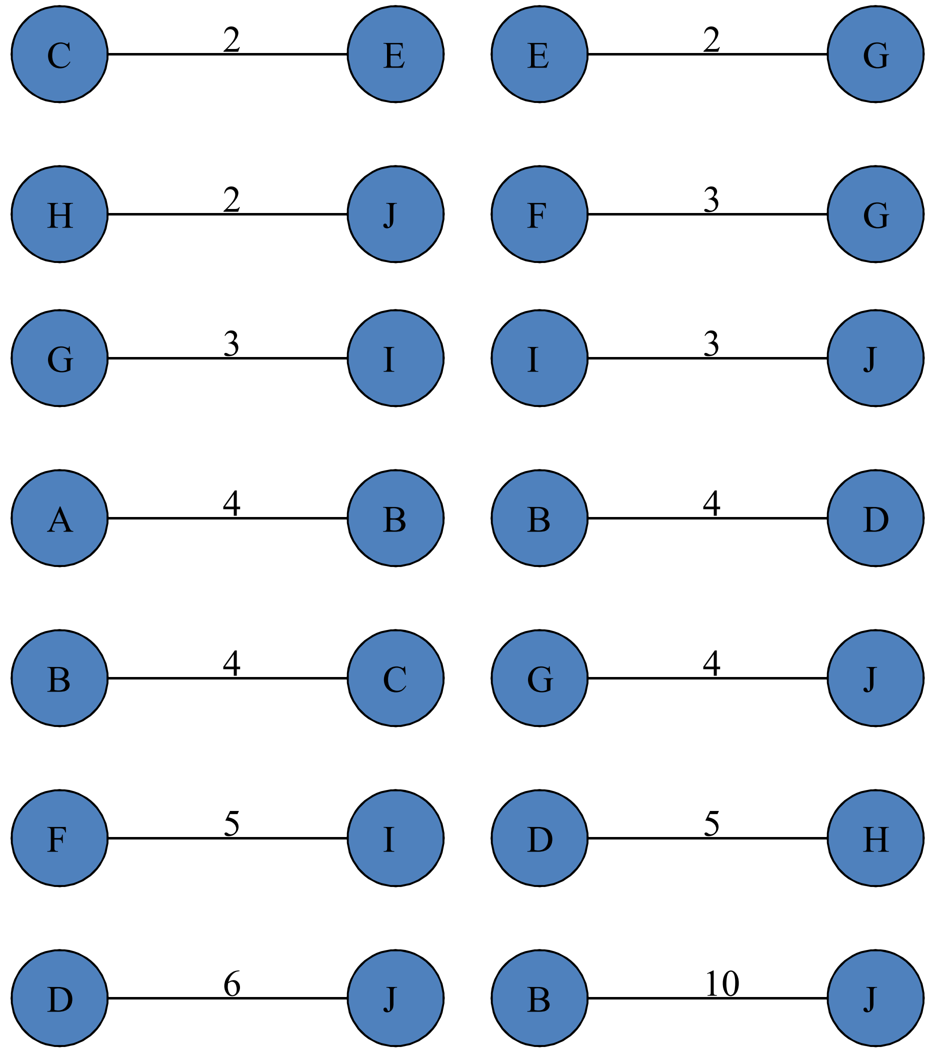


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Add Edge

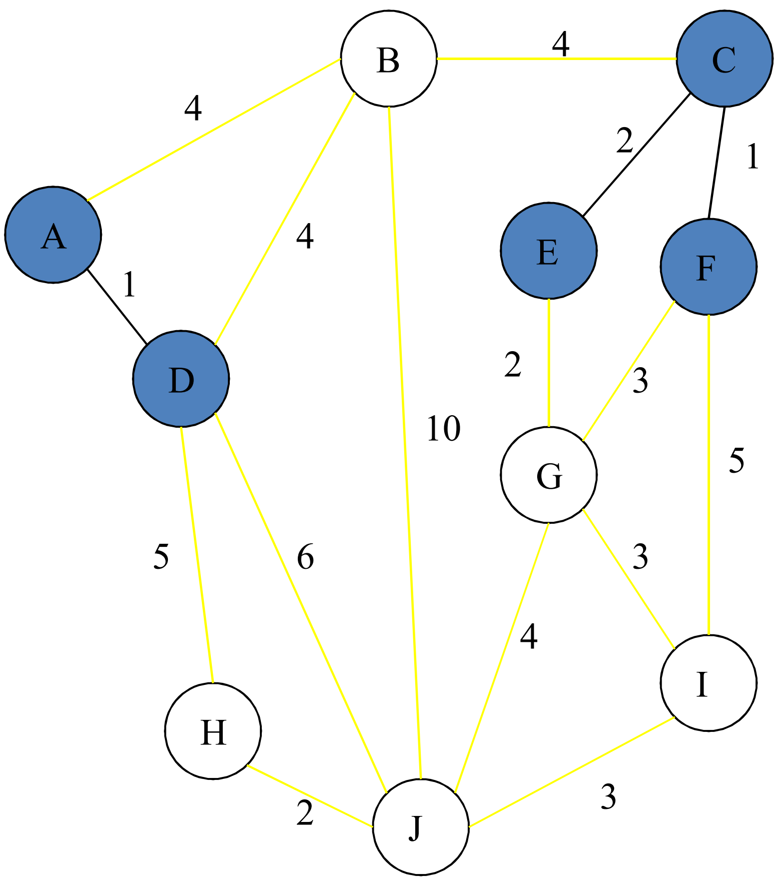


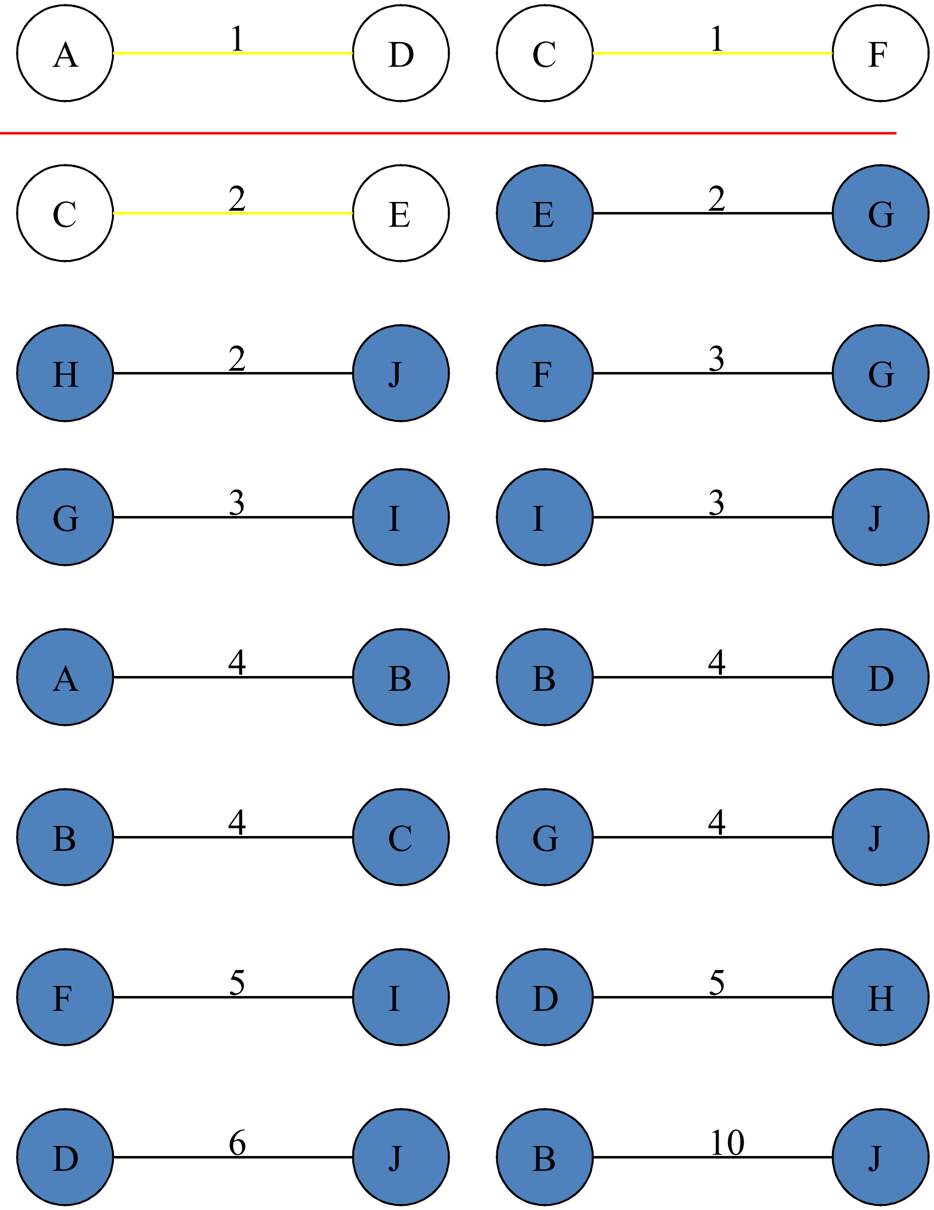




Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

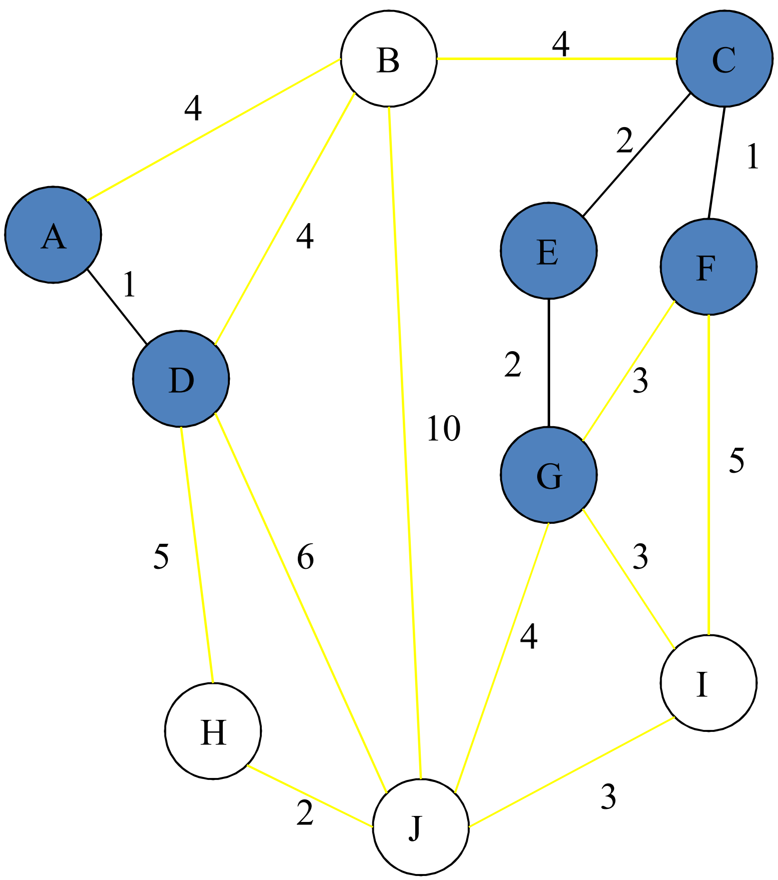
Add Edge



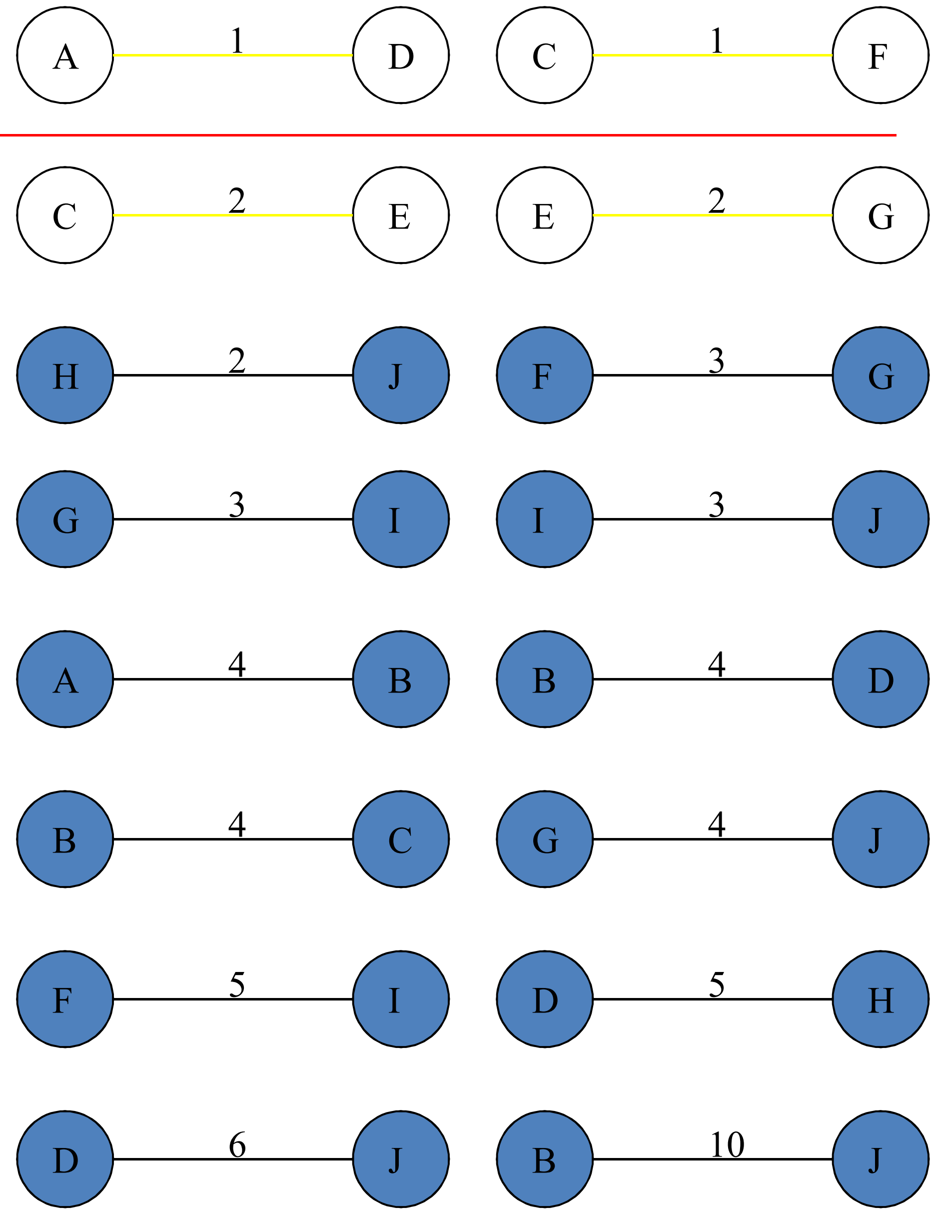


Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

Add Edge

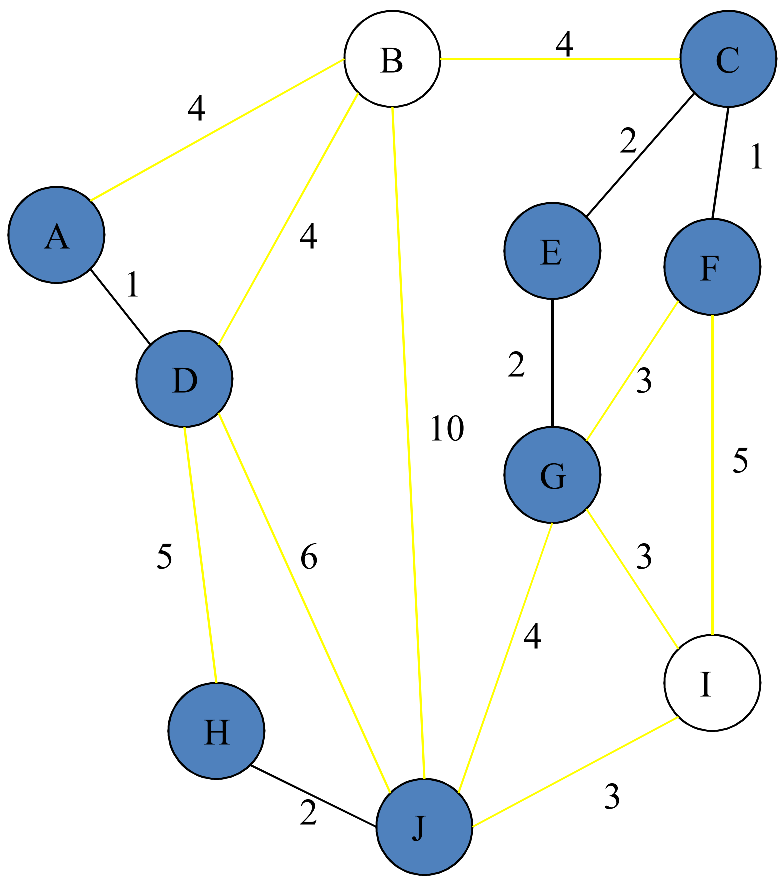


A
B
C
D
E
F
G
H
I
J
4
4
1
4
2
1
2
3
5
10
5
6
3
4
2
3

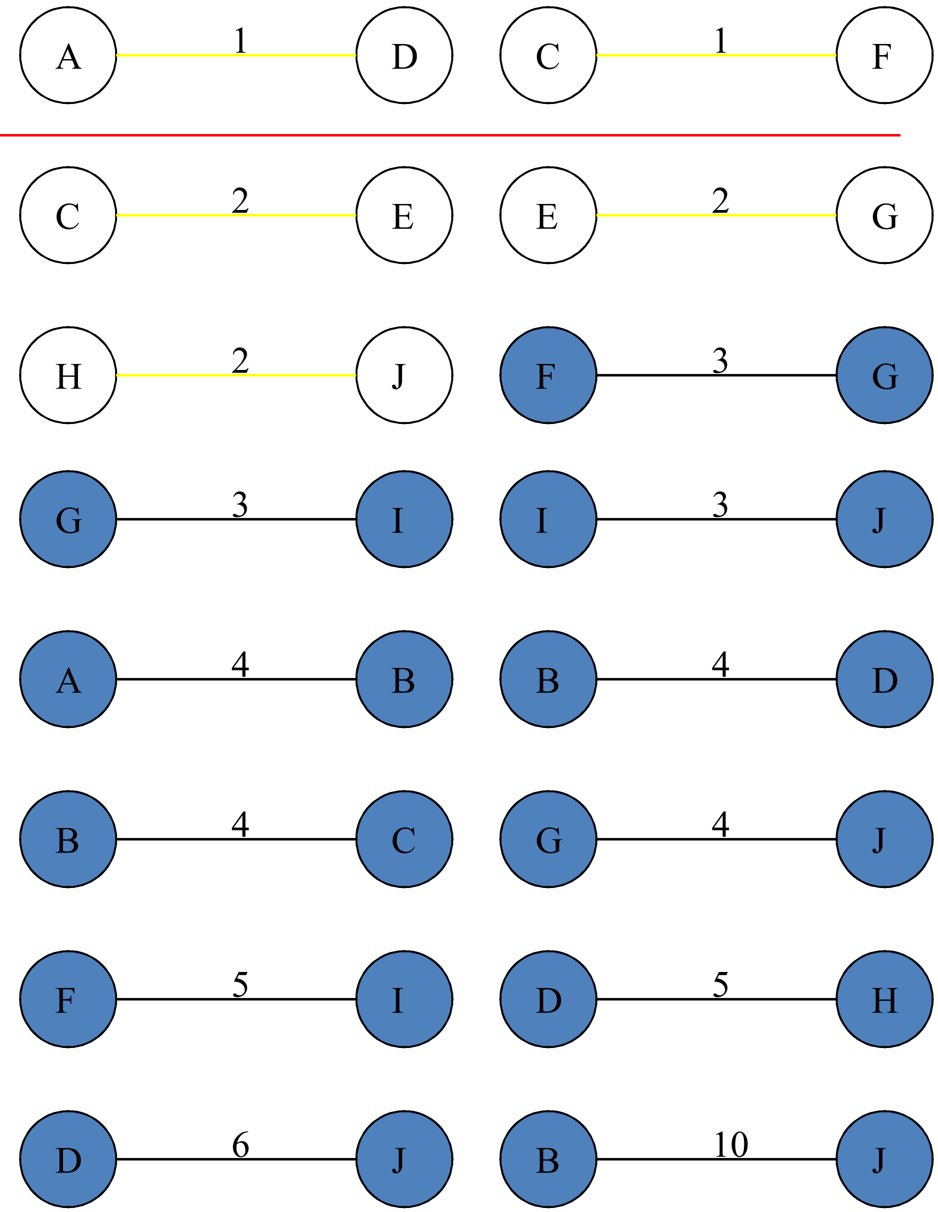


A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Add Edge



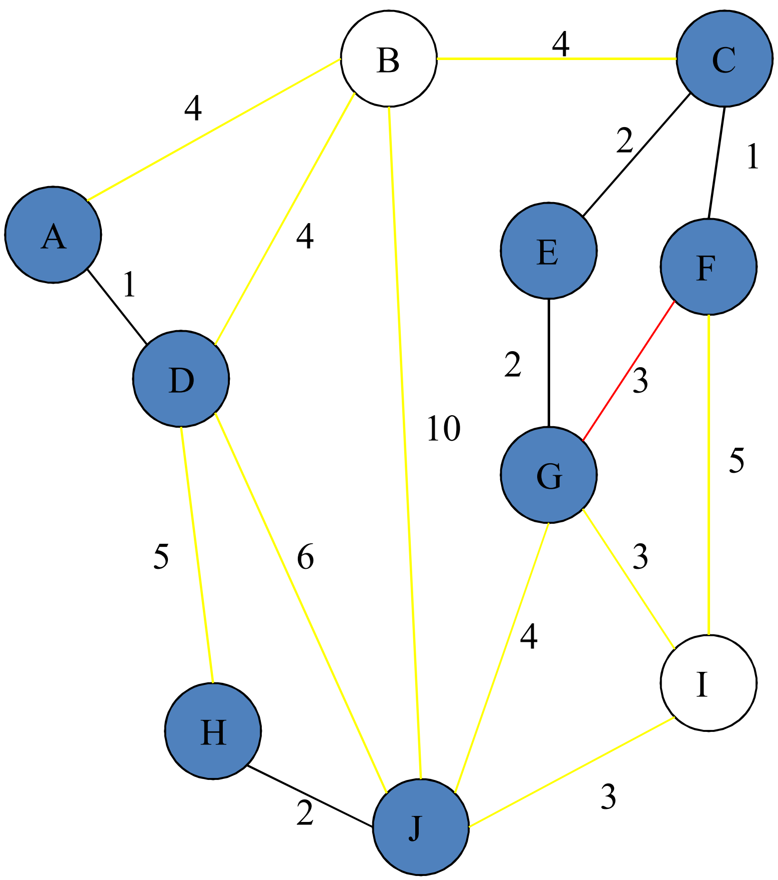
A
B
C
D
E
F
G
H
I
J
4
4
4
1
2
1
2
3
5
10
5
6
3
4
2
3

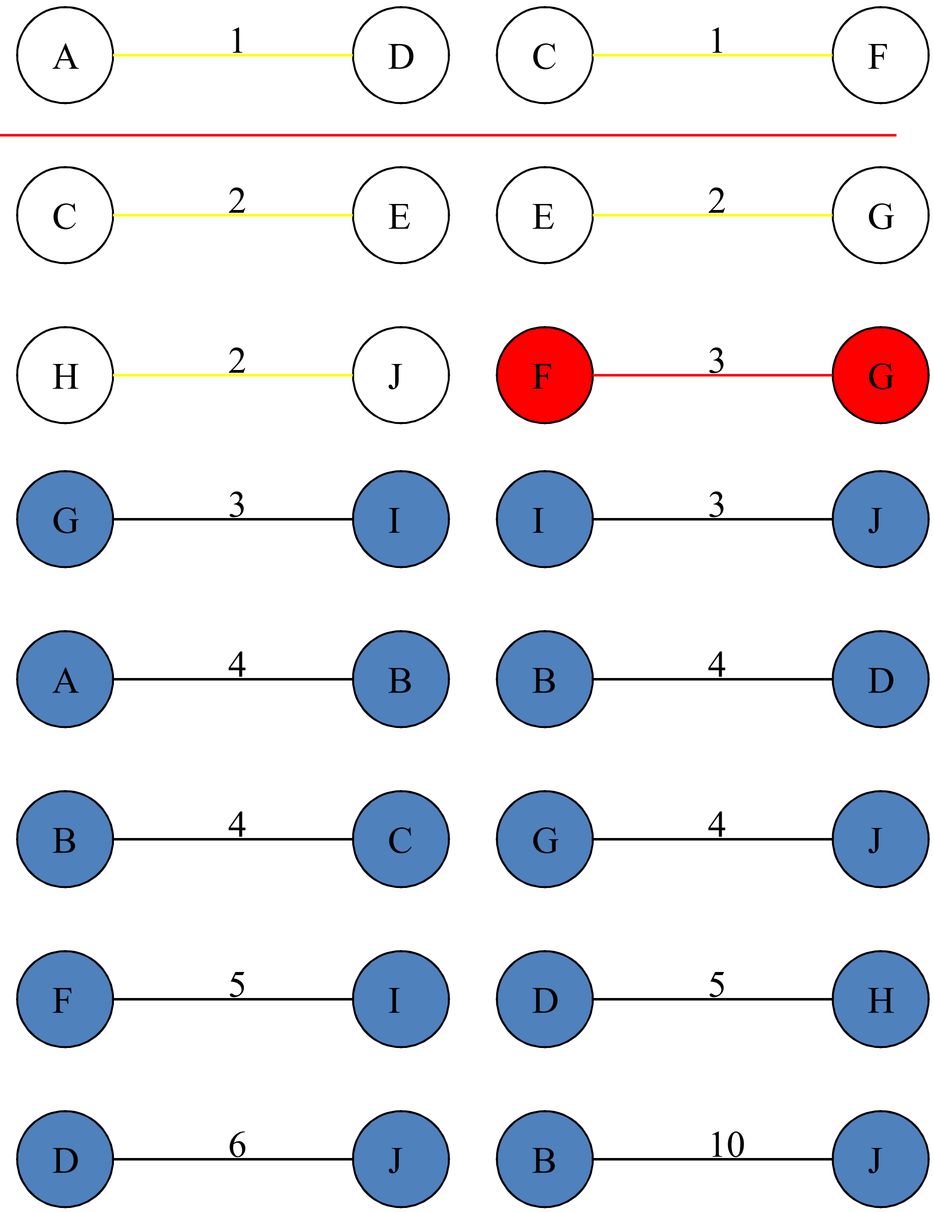


A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Cycle

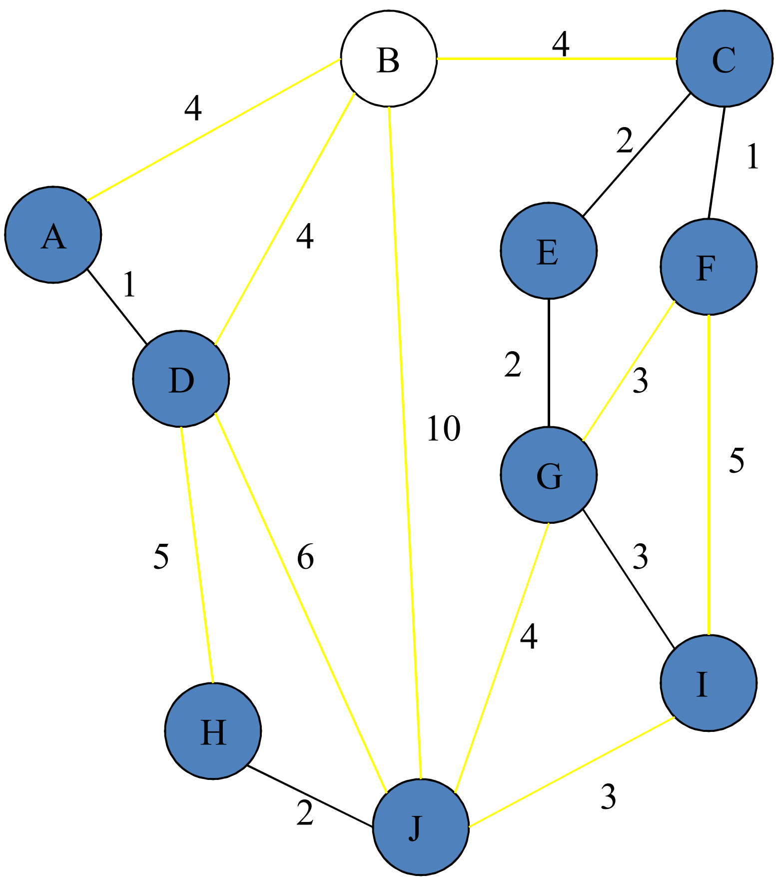
Don’t Add Edge

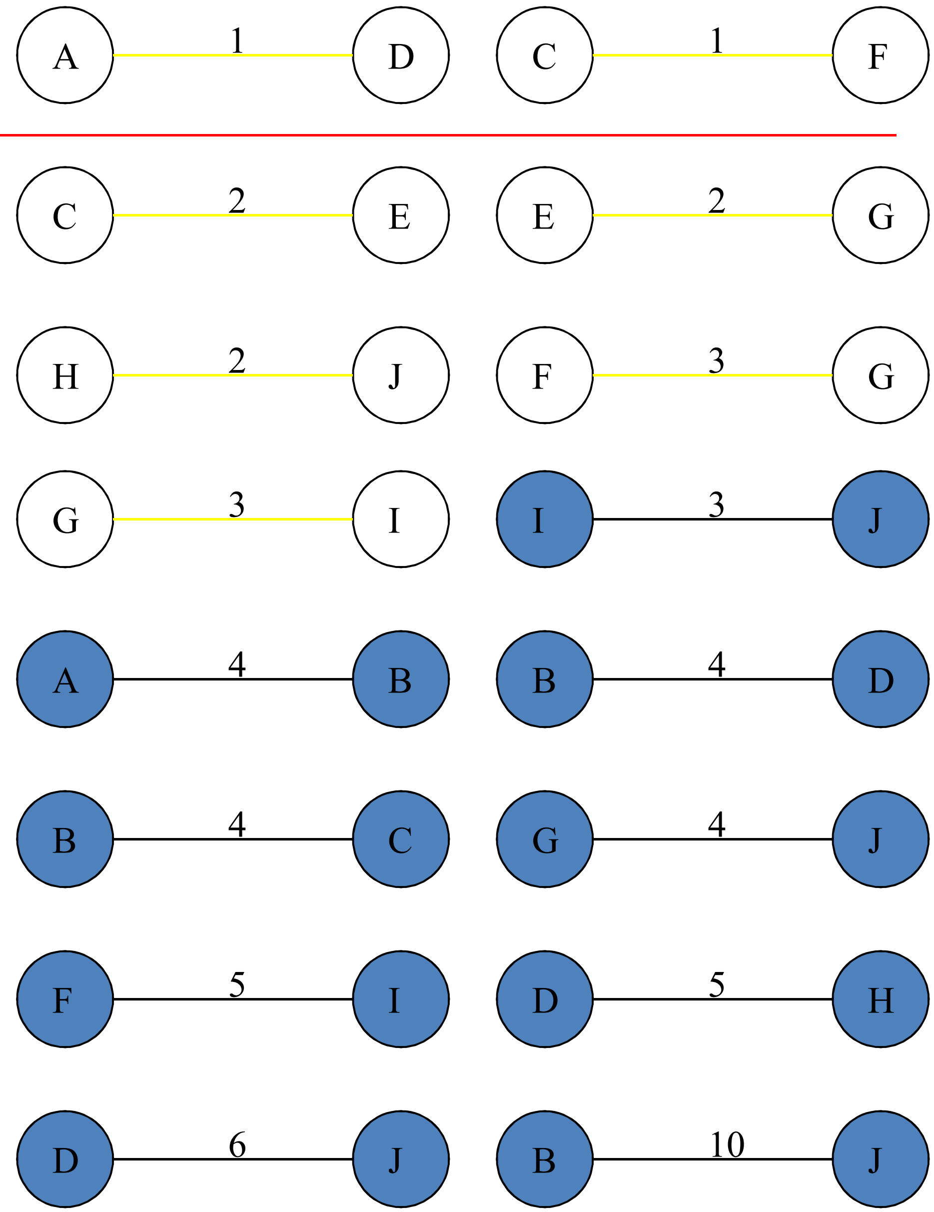




Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

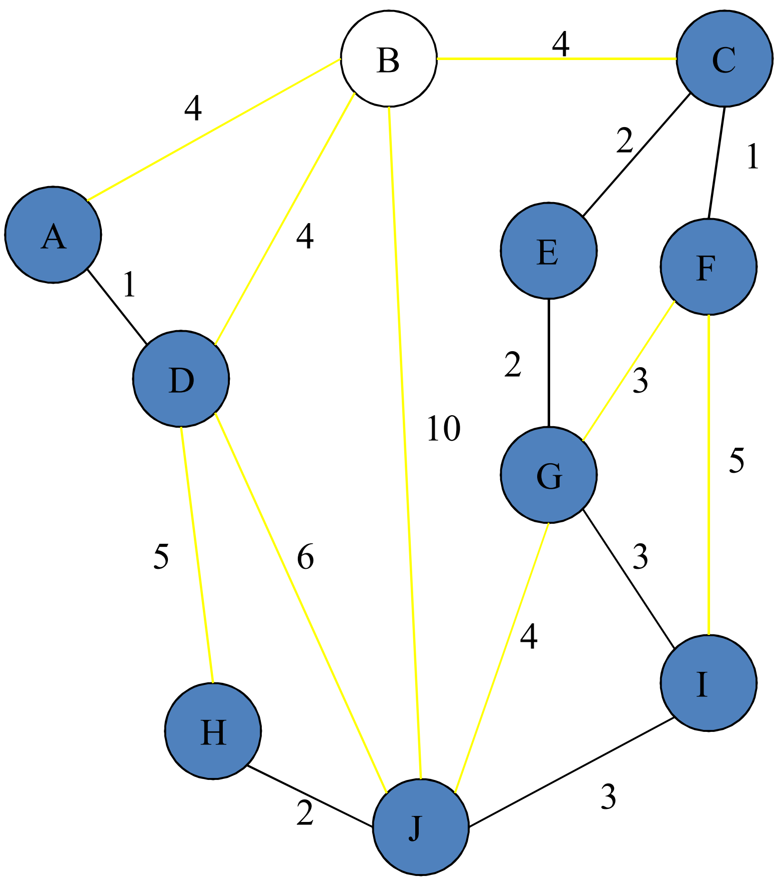
Add Edge

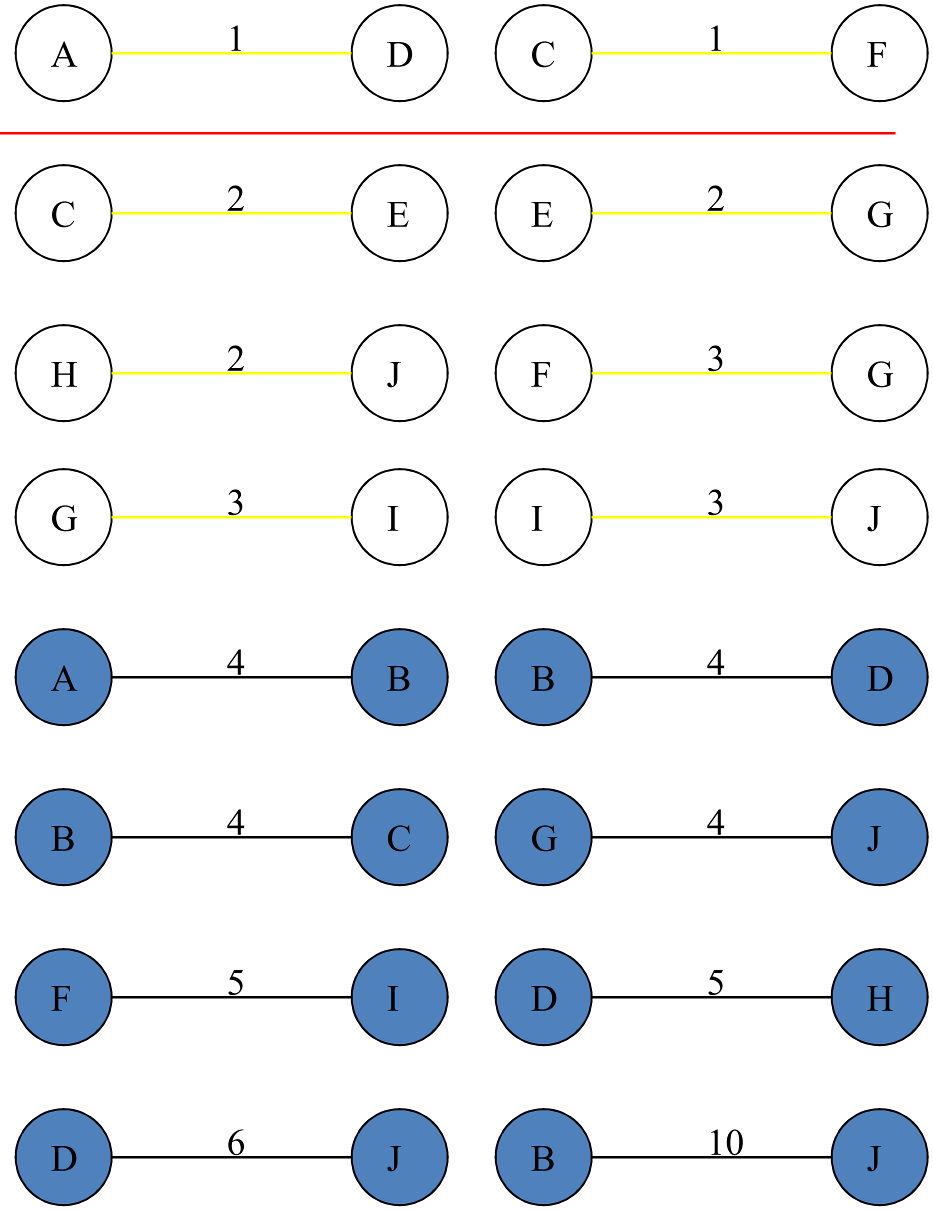




Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

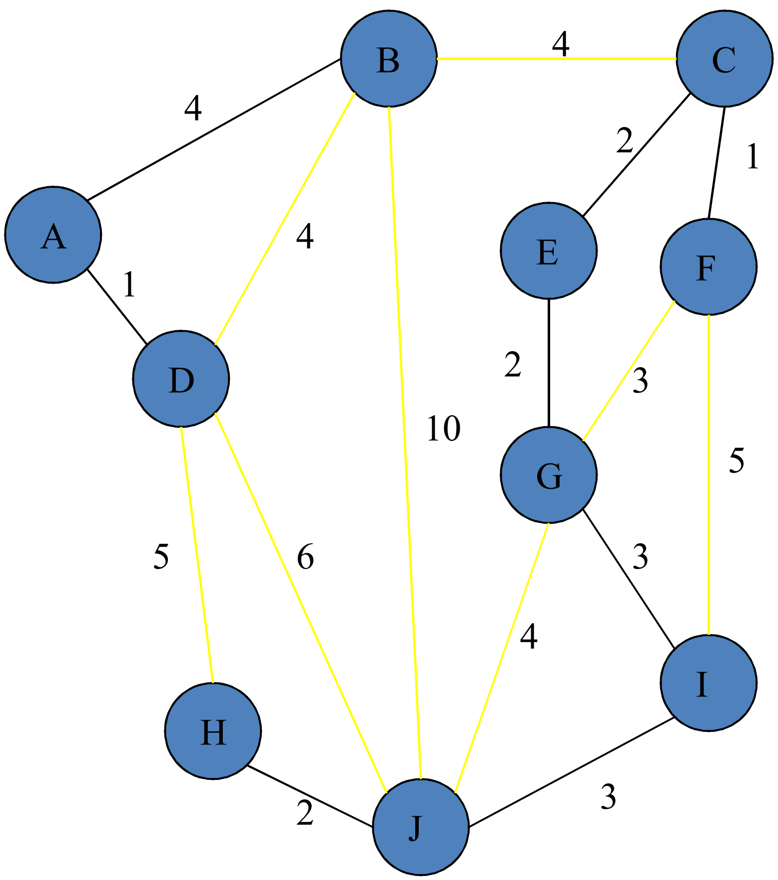
Add Edge

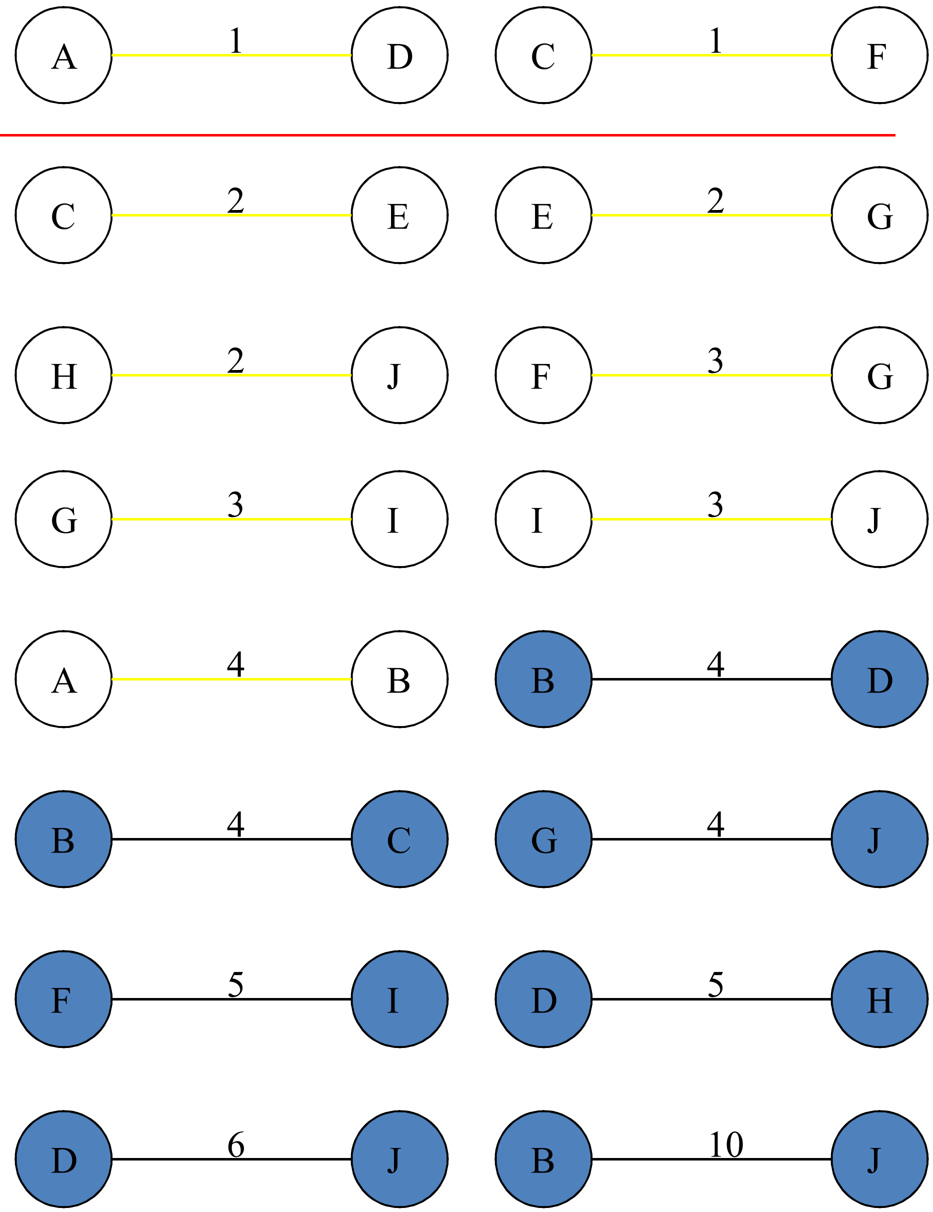




Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Add Edge

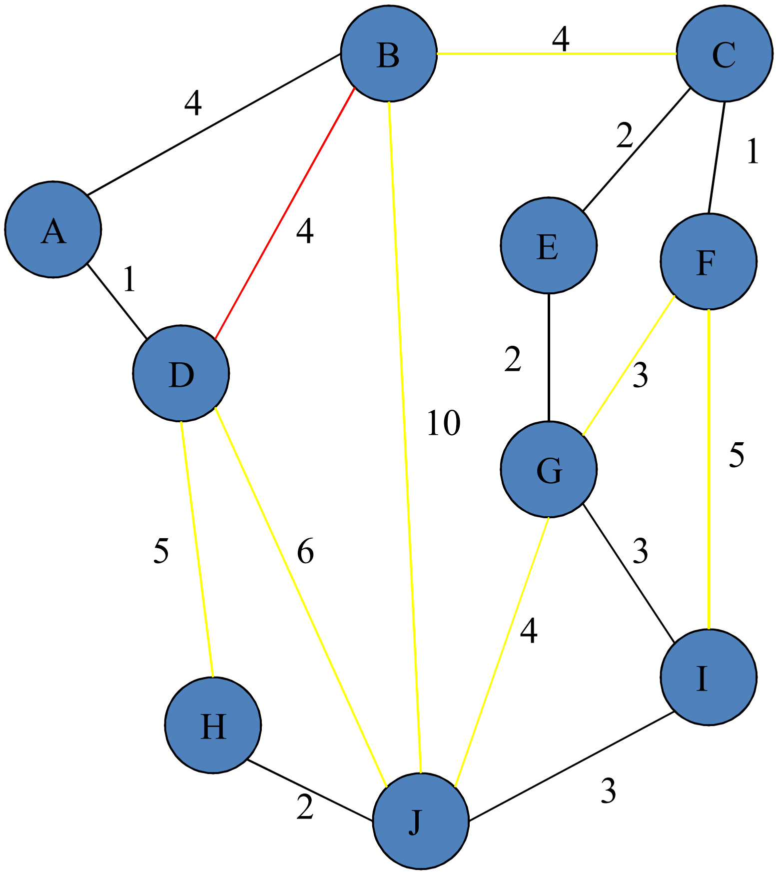


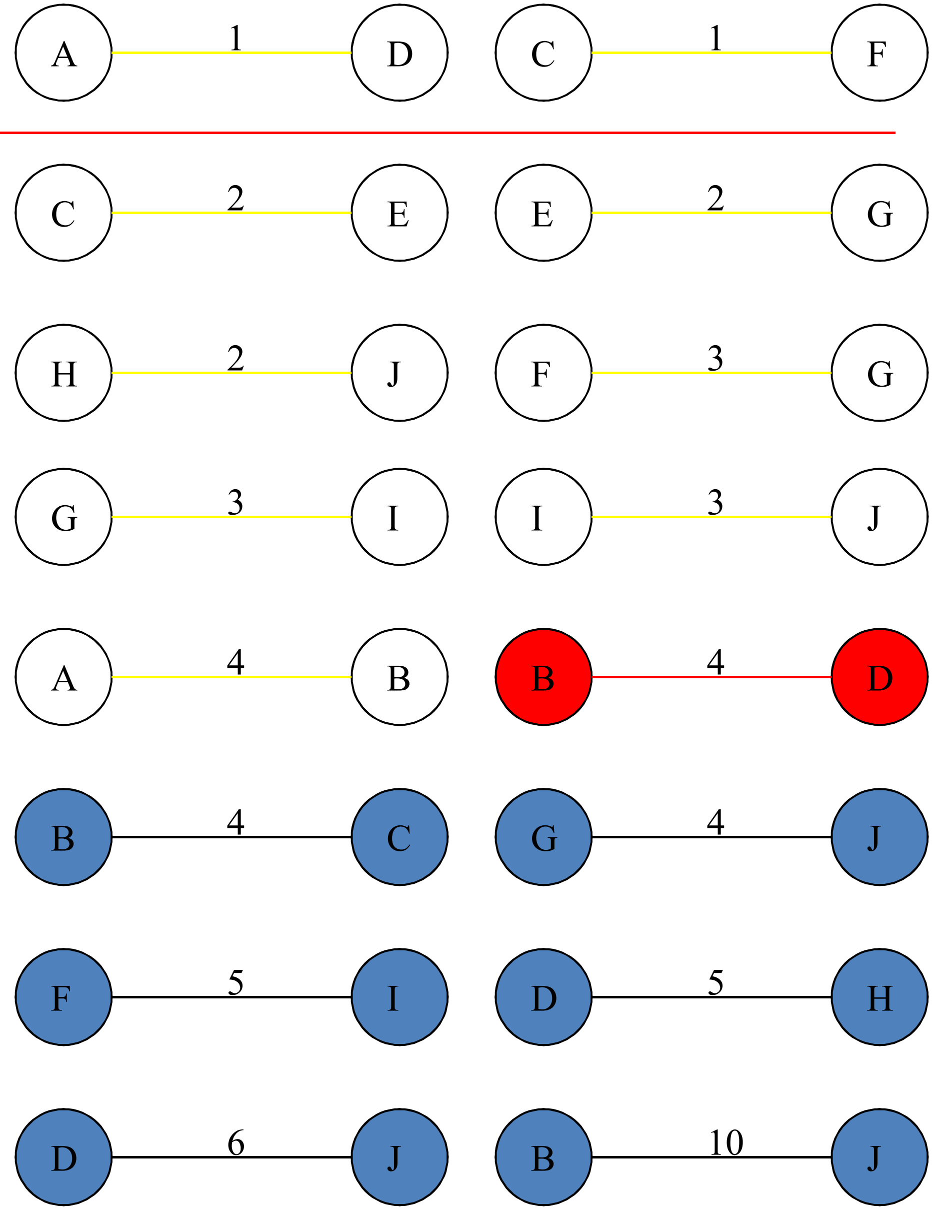


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Cycle

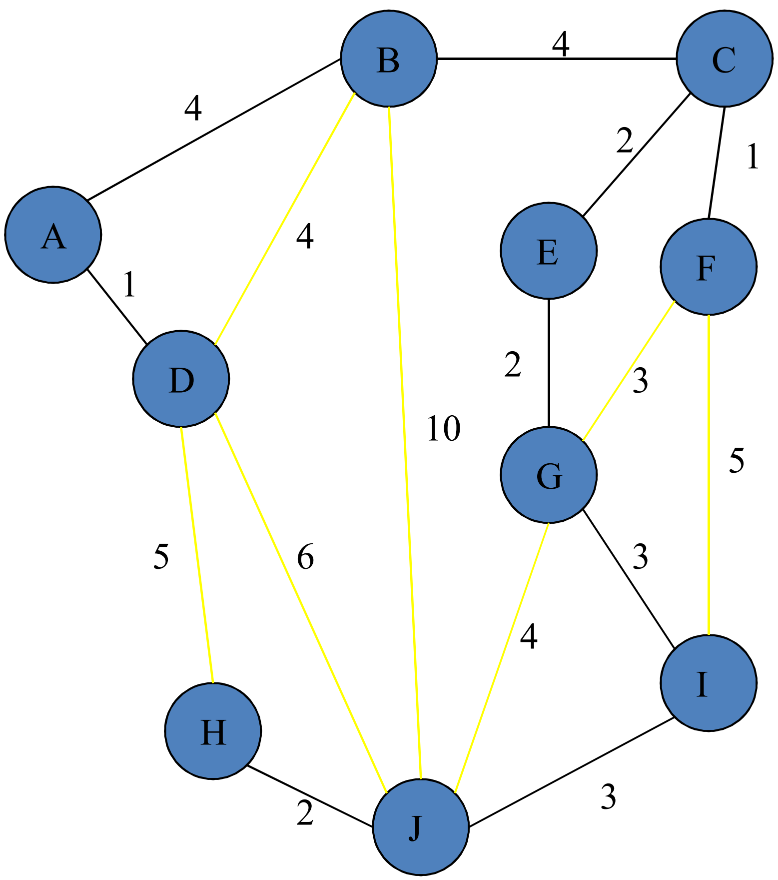
Don't Add Edge

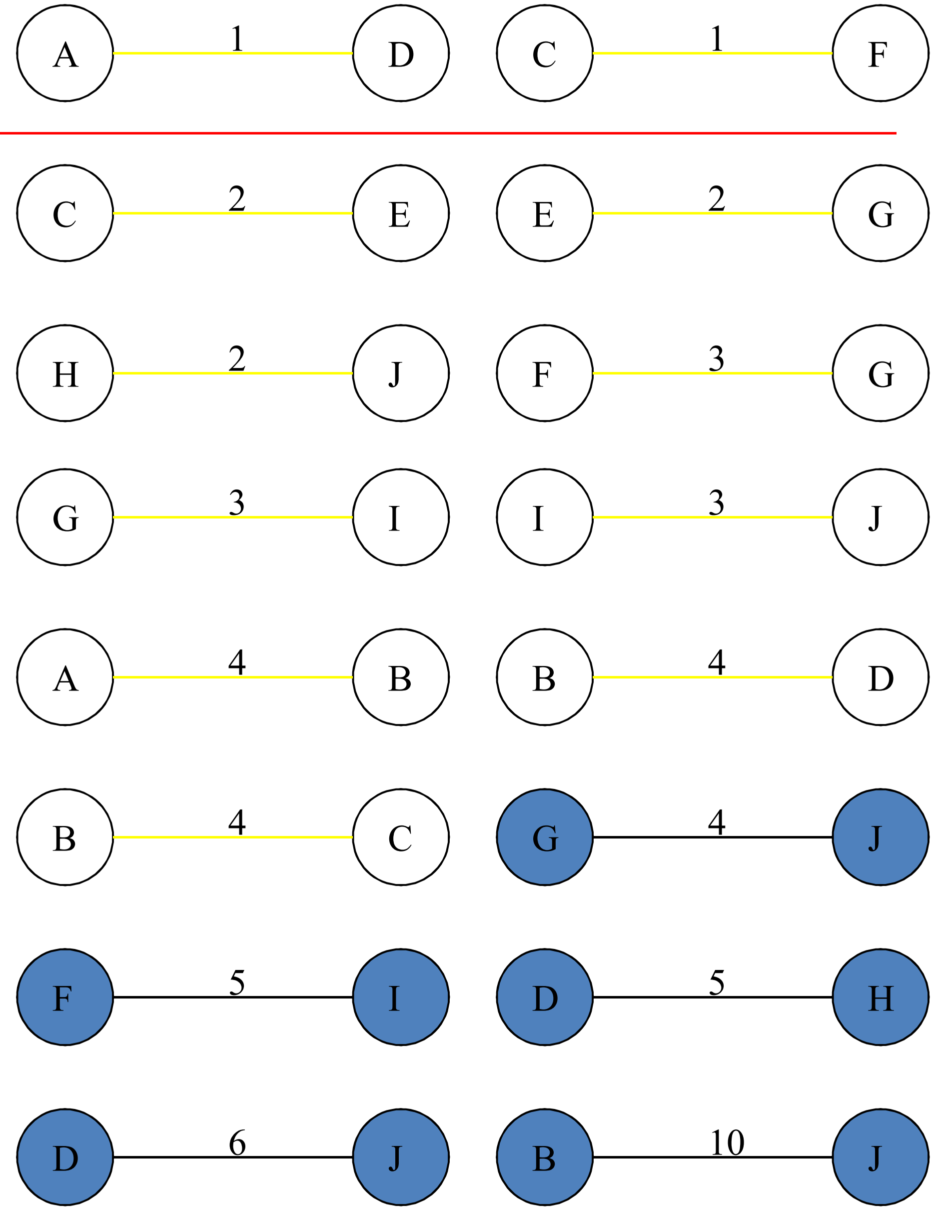




Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

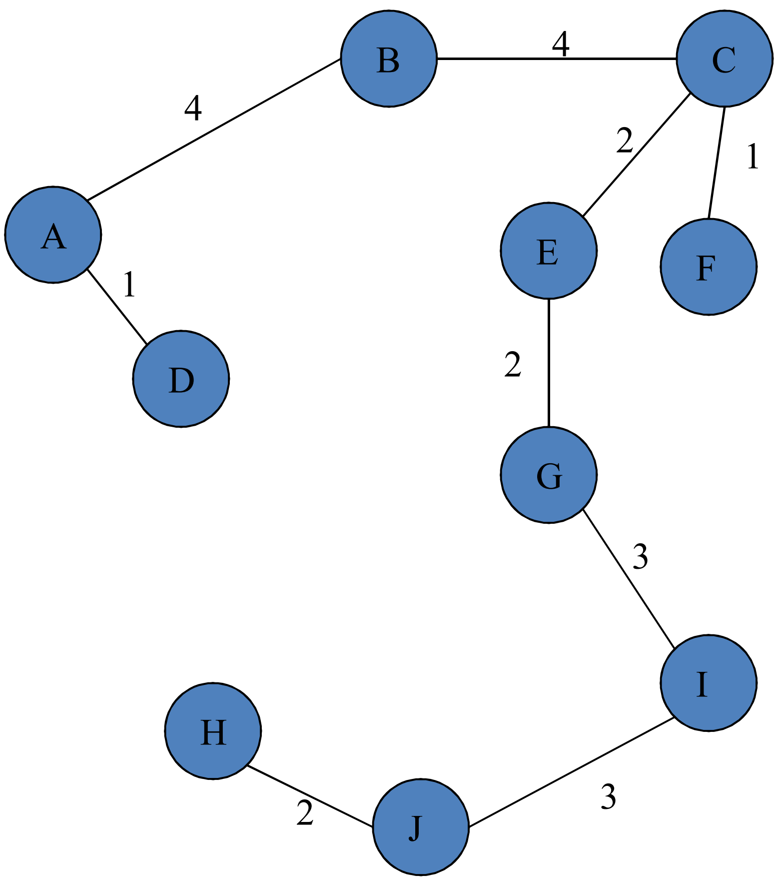
Add Edge



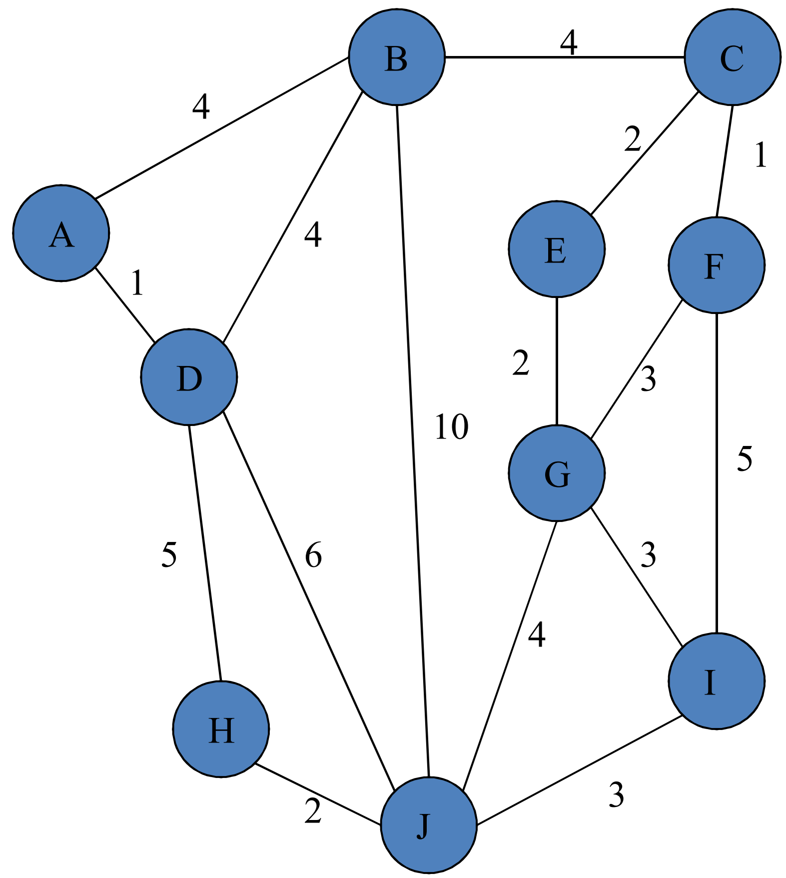


Bhaskar Sardar, **I**nformation **T**echnology **D**epartment, ***Jadavpur University, India***

Minimum Spanning Tree



Example Graph



Analysis of Kruskal's Algorithm

The algorithm starts with sorting edges (O (E log E)).

We consider all E edges.

For each edge we check for possibility of cycle (O(logn))

Total running time is O(ElogE) + O (Elogn)

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Prim's Algorithm

- This algorithm starts with one node.
- It then, one by one, adds a node that is unconnected to the new graph to the new graph.
- Each time selects the node whose connecting edge has the smallest weight out of the available nodes' connecting edges.

Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Prim's Algorithm

The steps are:

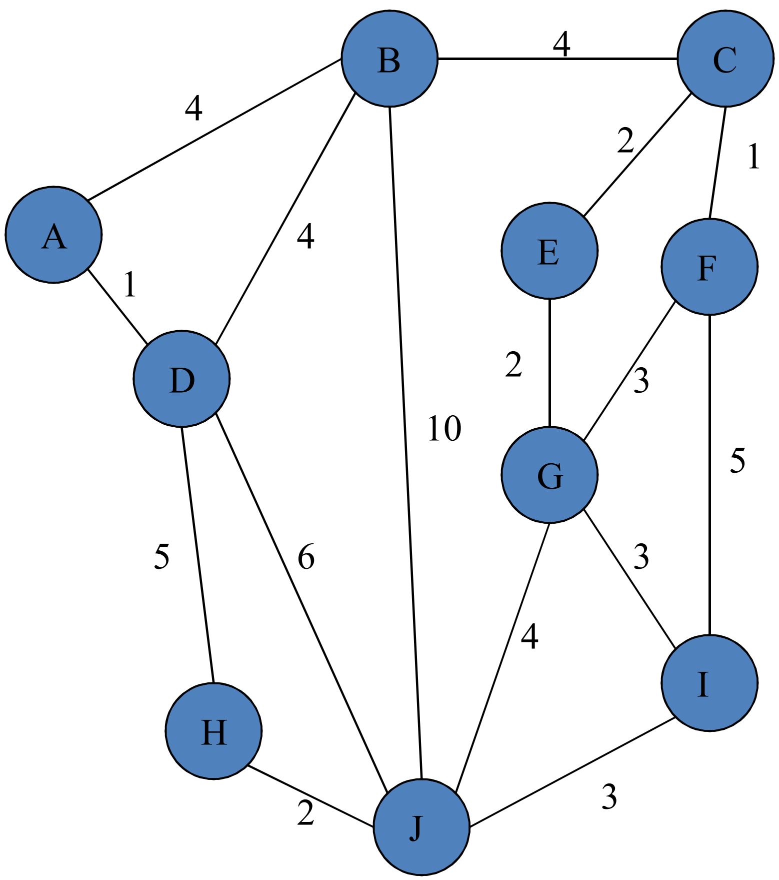
1. The new graph is constructed - with one node from the old graph.
2. While new graph has fewer than n nodes,
   2.1. Find the node from the old graph with the smallest connecting edge to the new graph,
   2.2. Add it to the new graph

Complexity: O(n^2)

In every step one node is added, so that at the end we will have one graph with all the nodes and it will be a minimum spanning tree of the original graph.

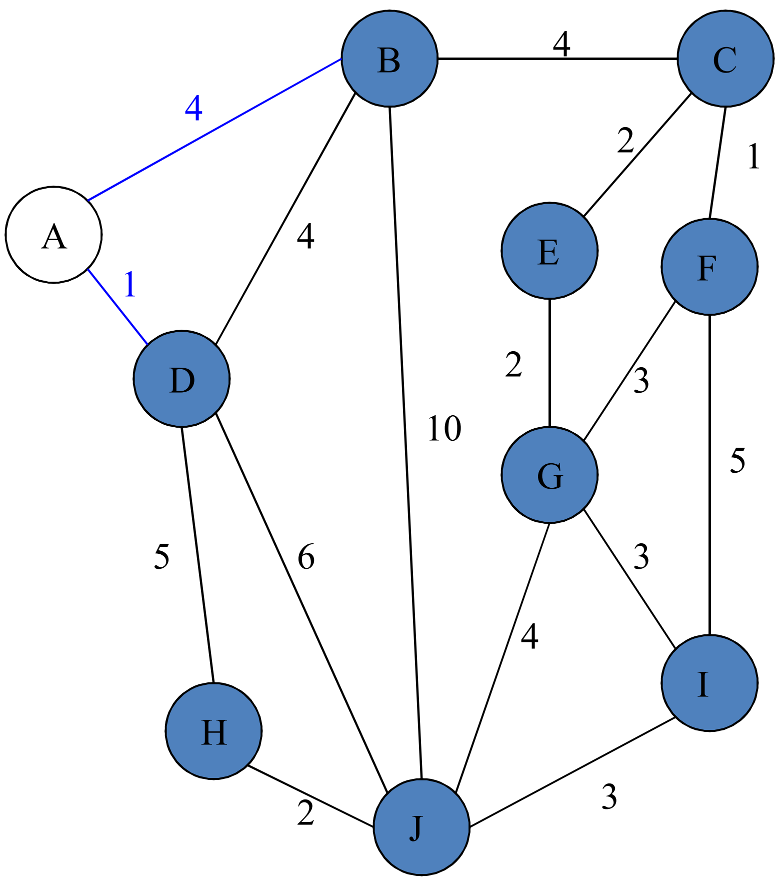
Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Example Graph

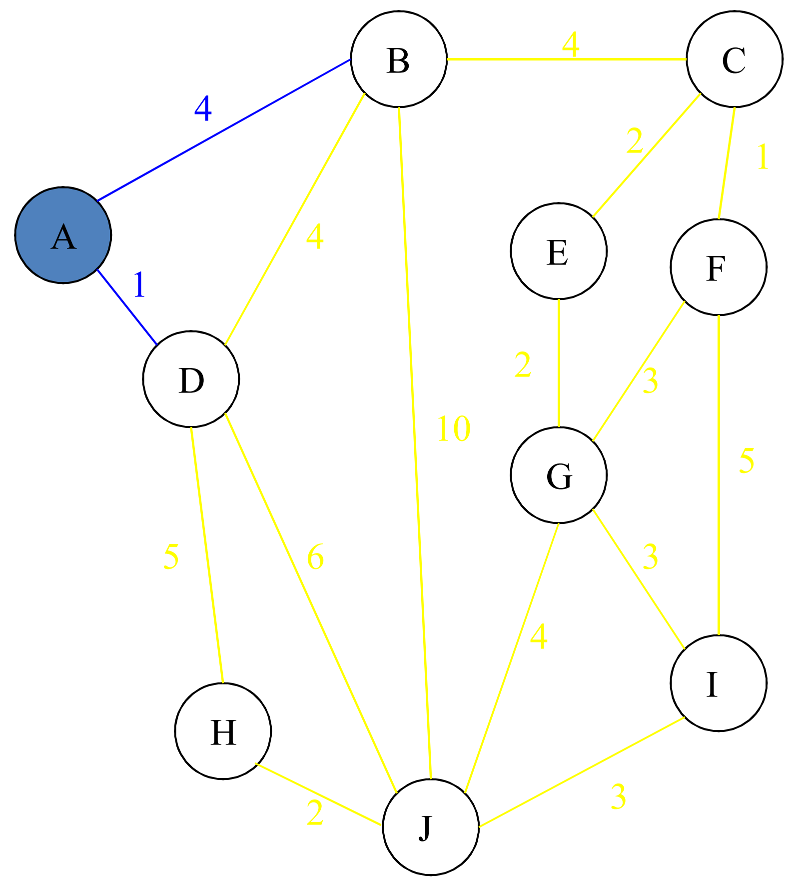


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

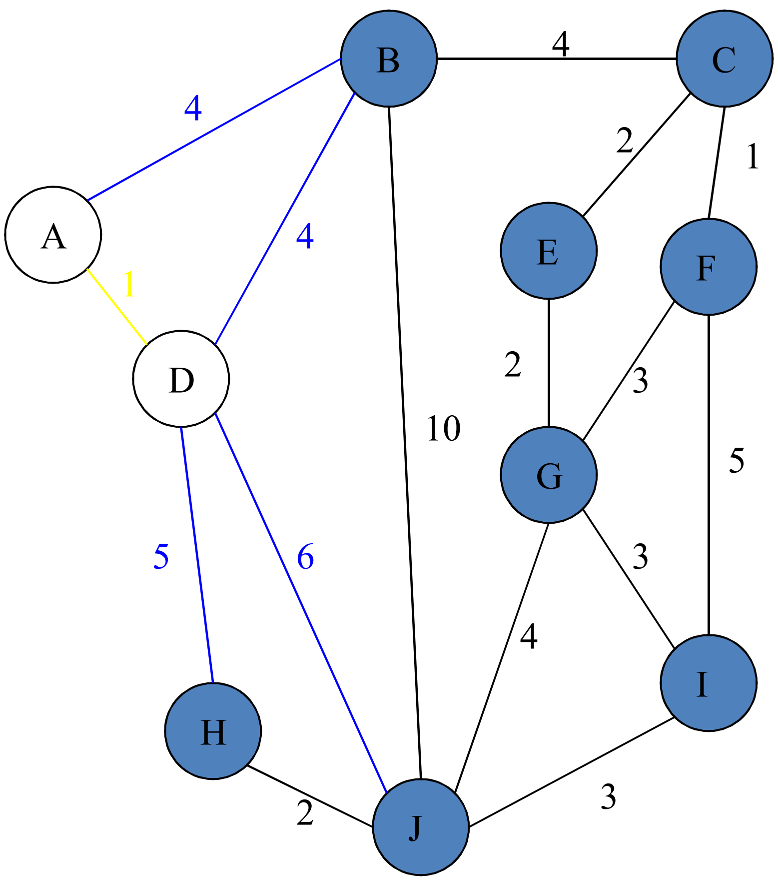
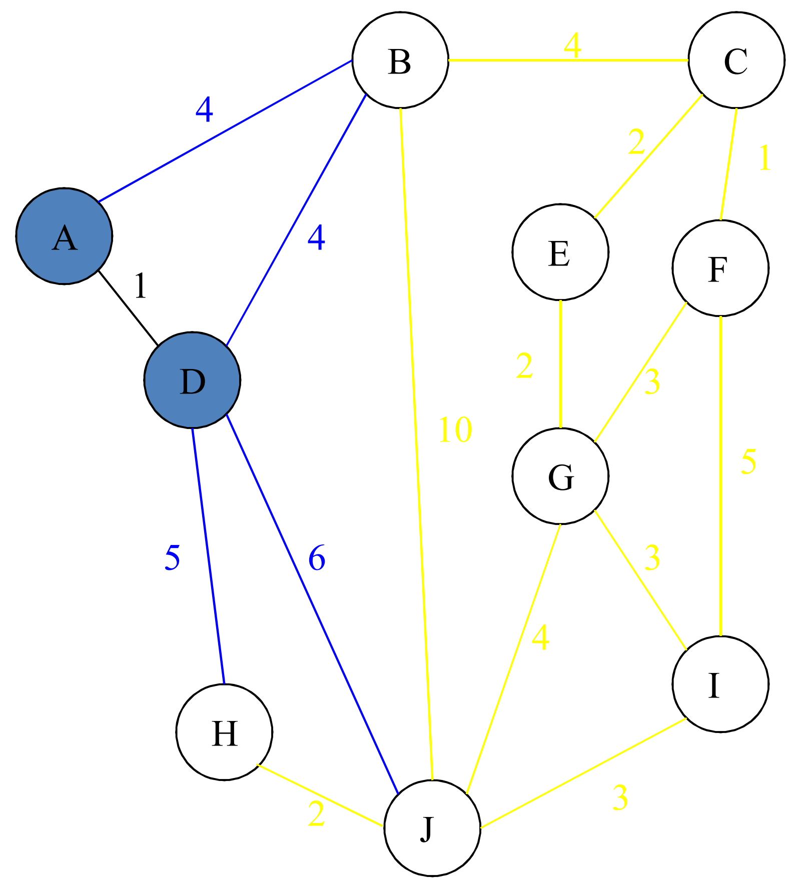
Example Graph



New Graph

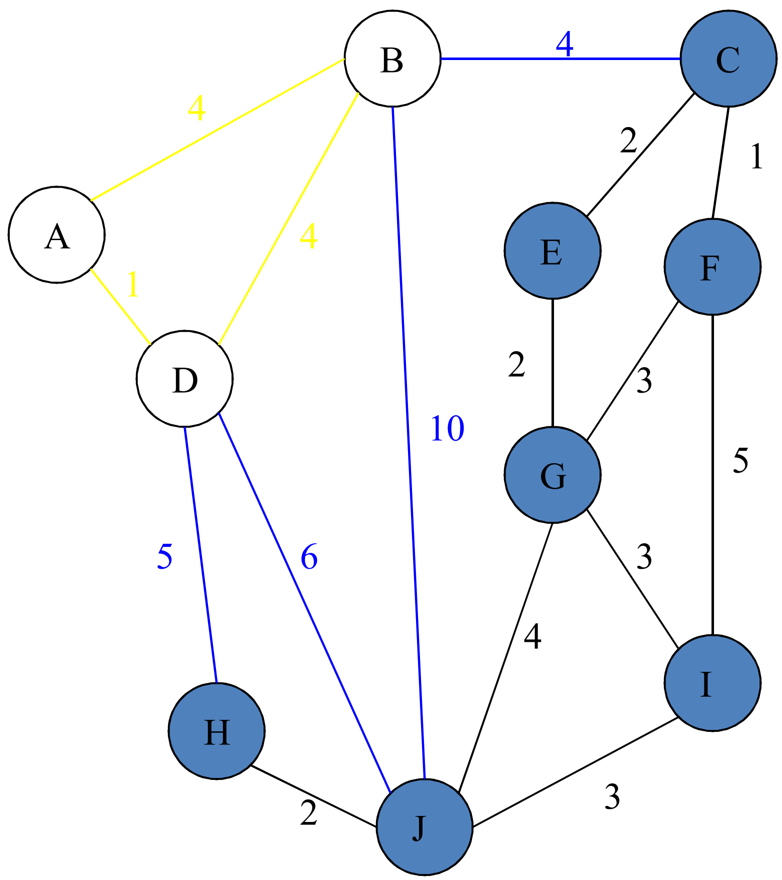


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

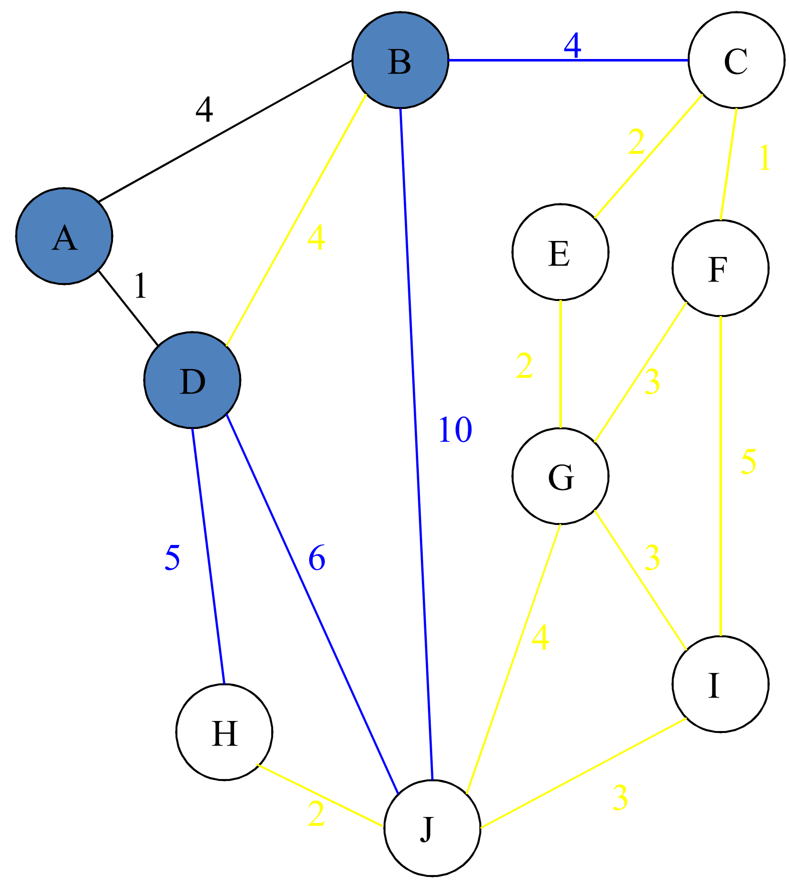
Example Graph	New Graph
	

Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

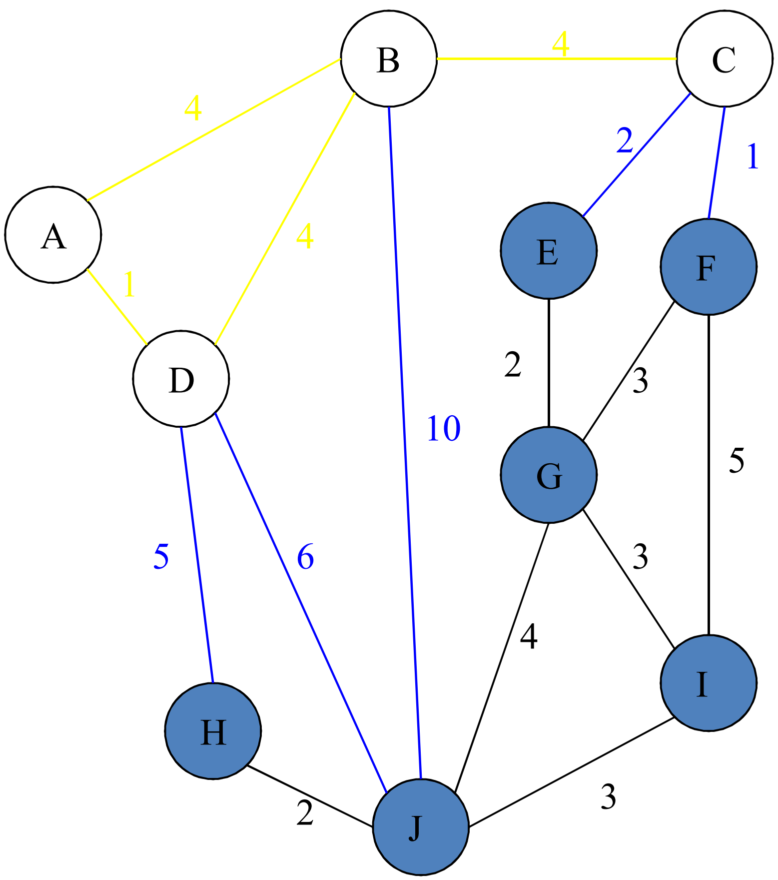
Example Graph



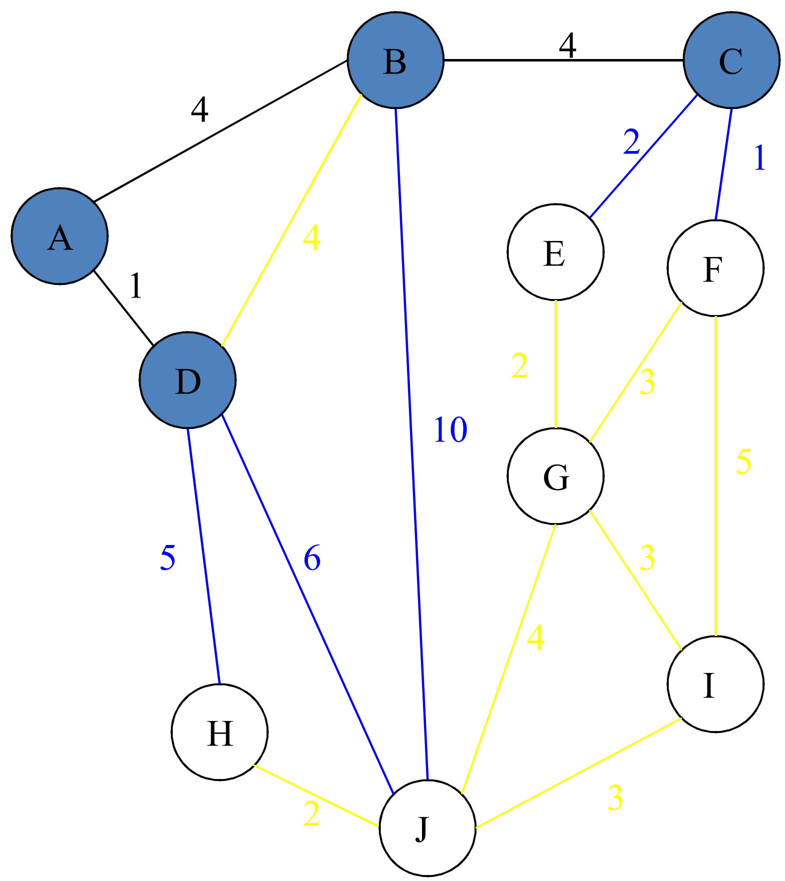
New Graph



Example Graph

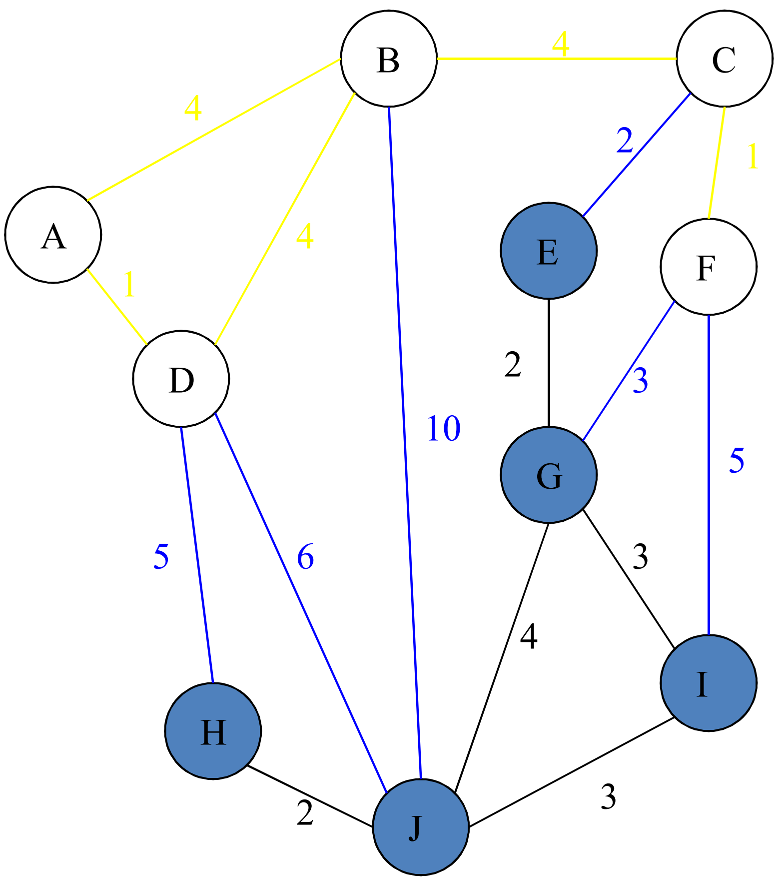


New Graph

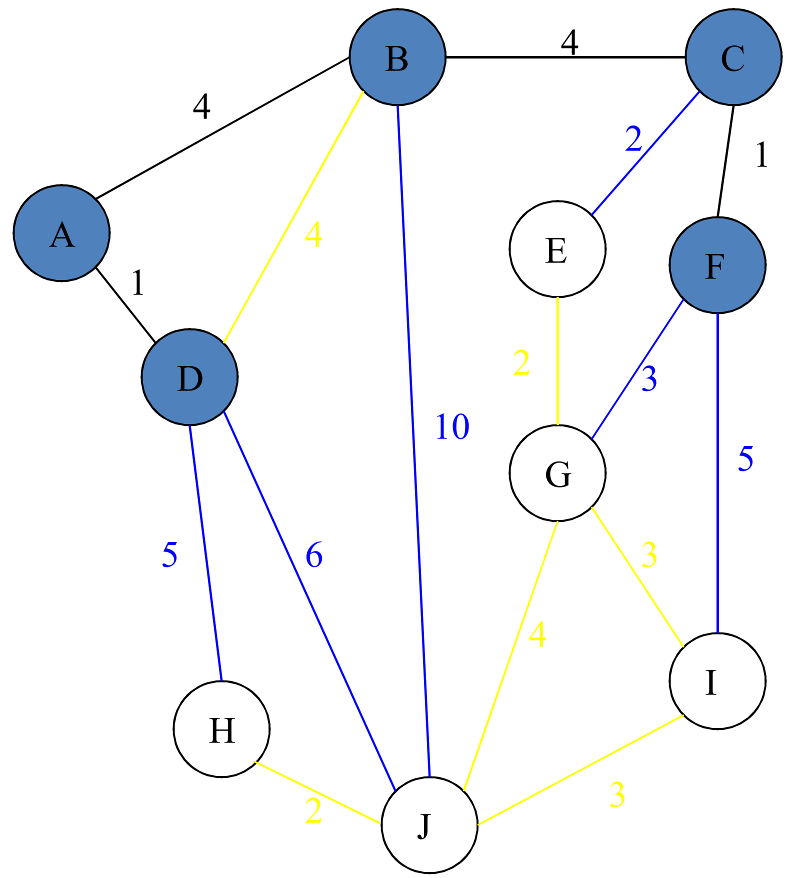


Bhaskar Sardar, Information Technology Department, *Jadavpur University, India*

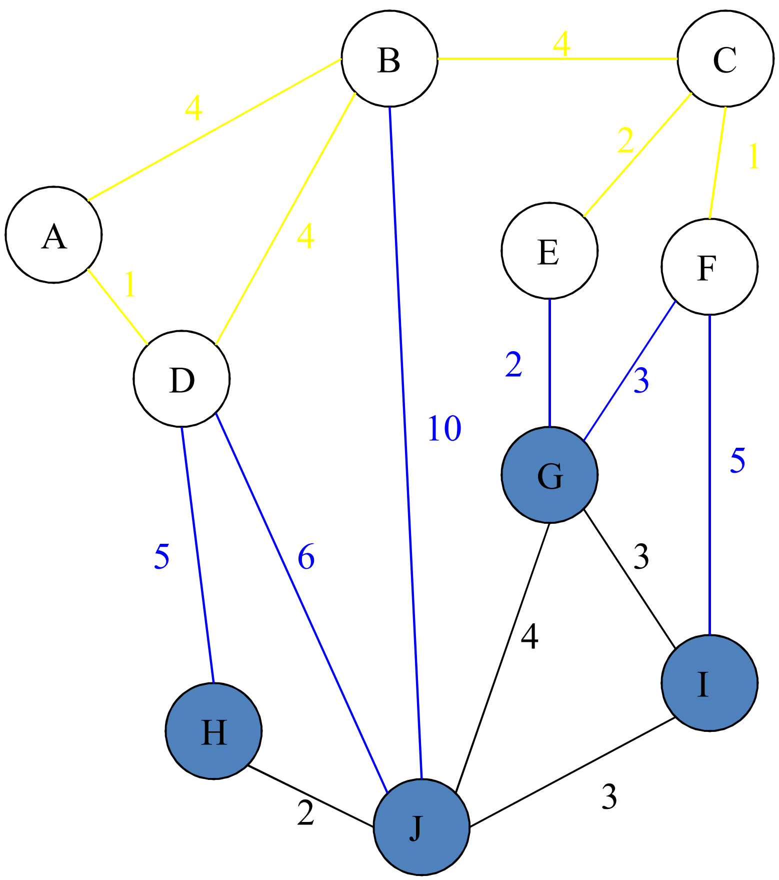
Example Graph



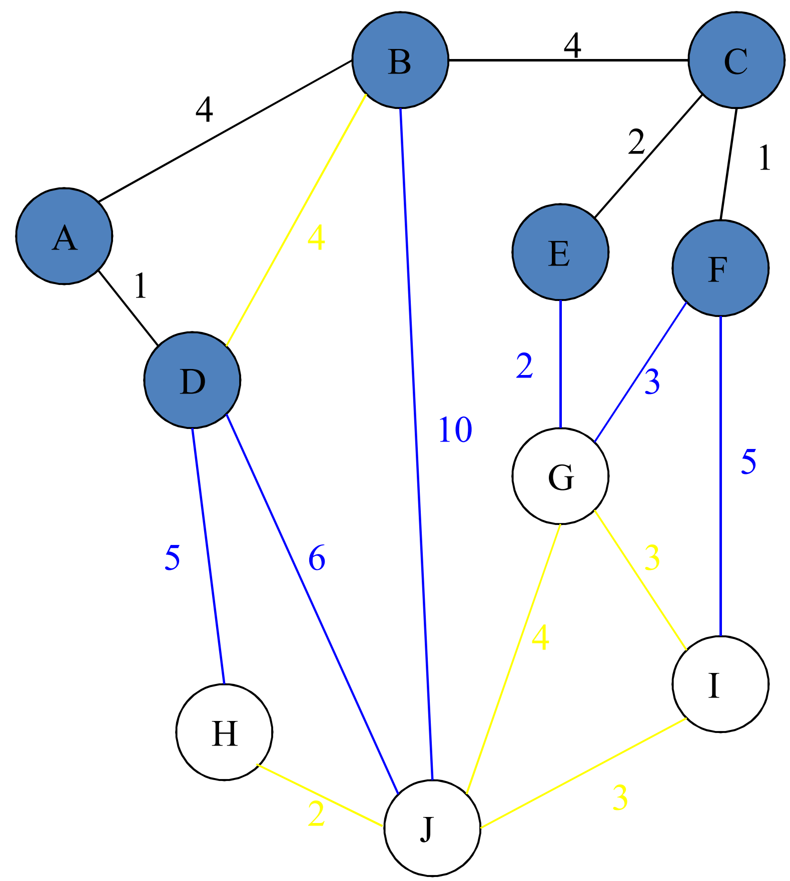
New Graph



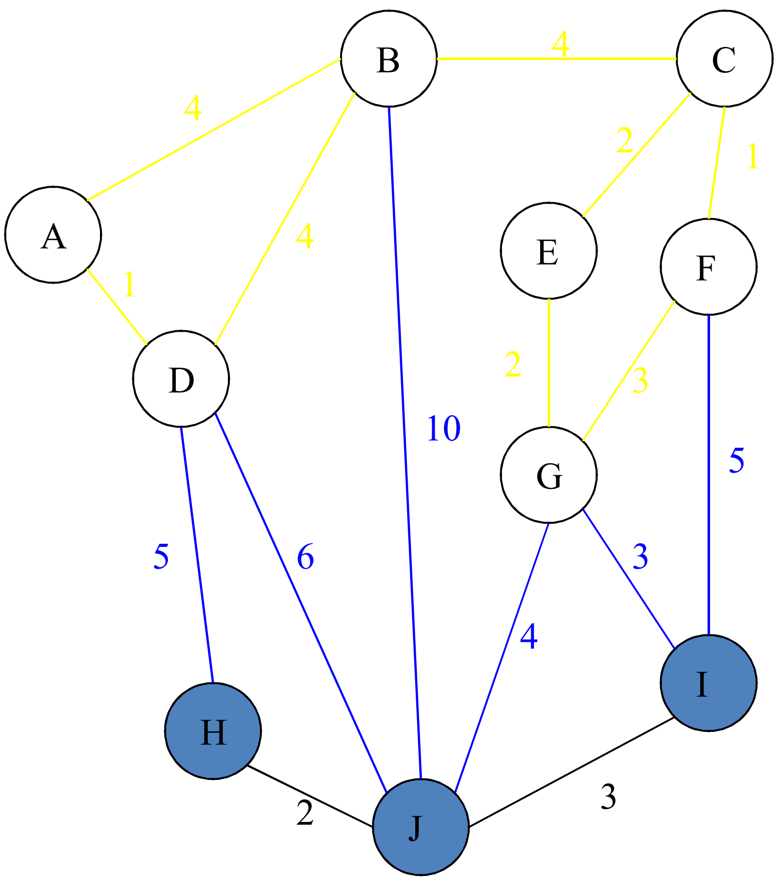
Example Graph



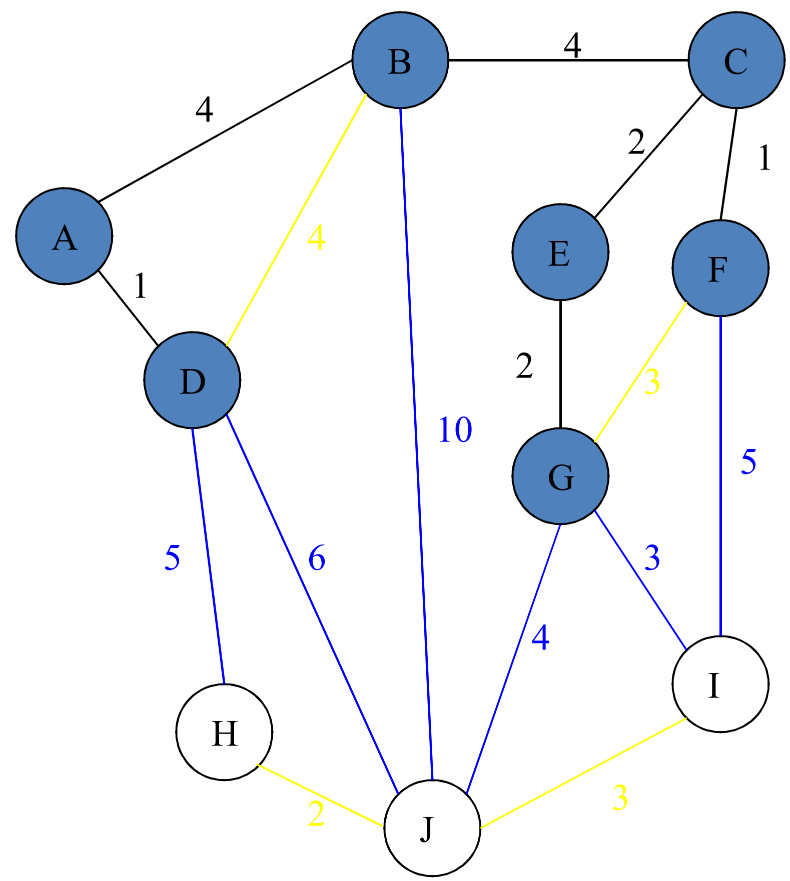
New Graph



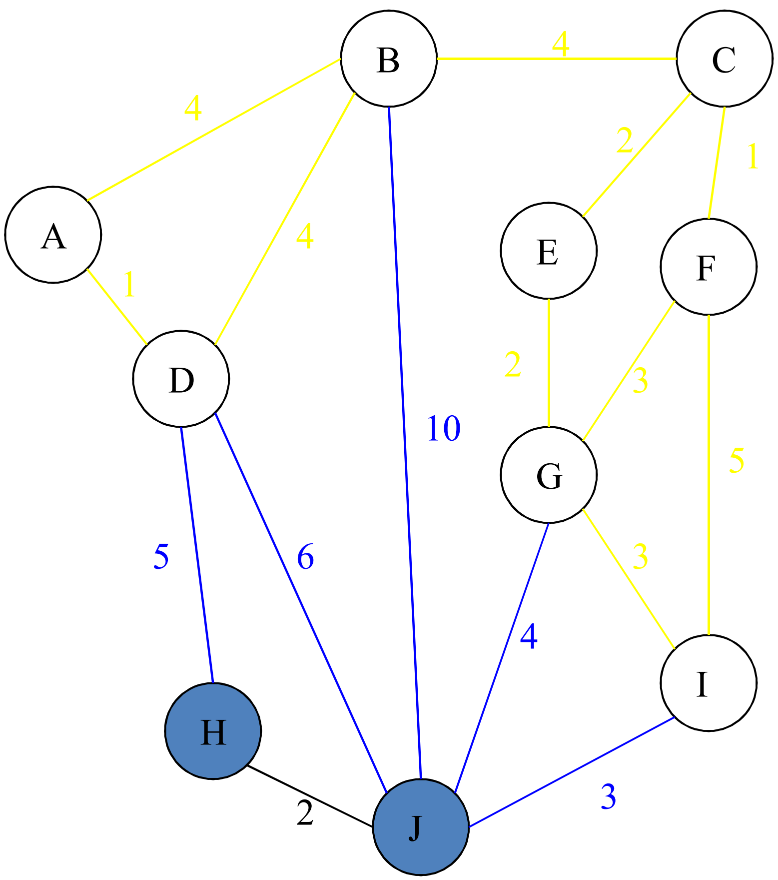
Example Graph

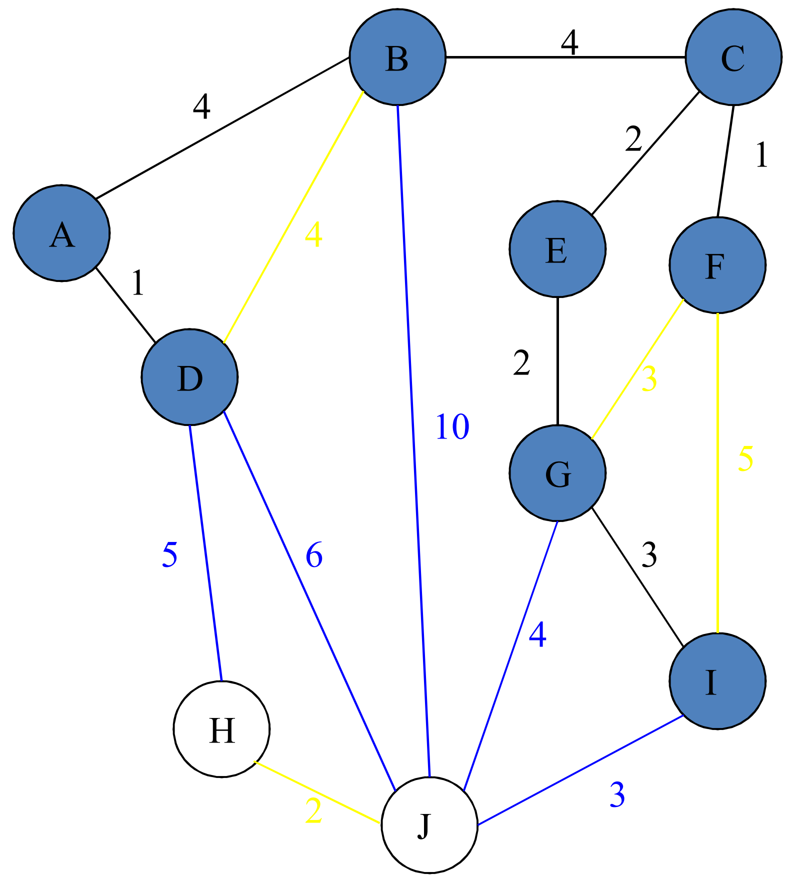


New Graph



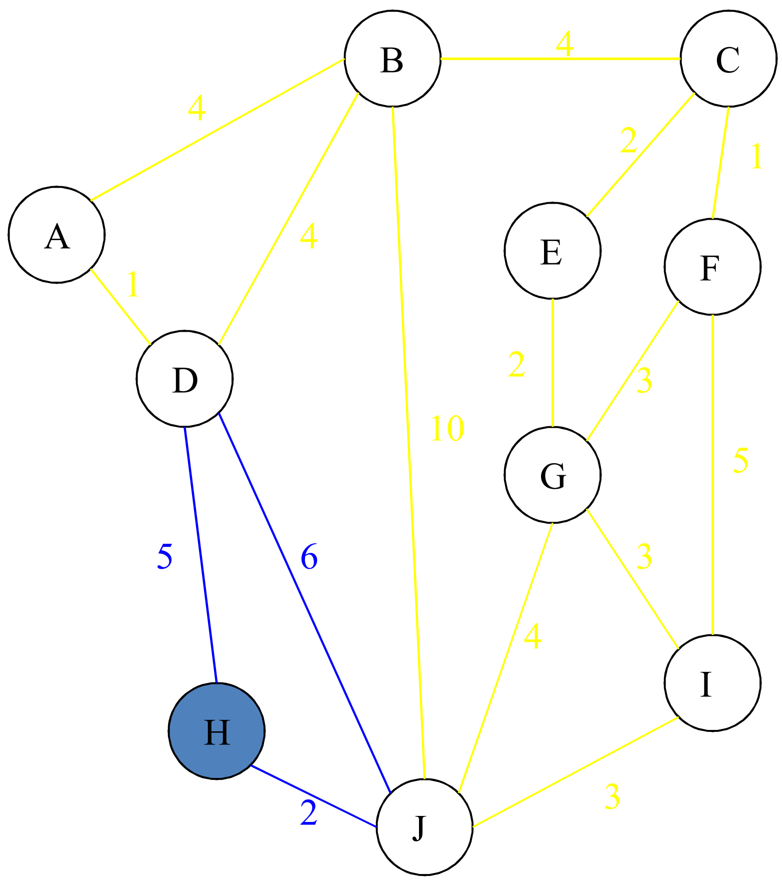
Example Graph	New Graph



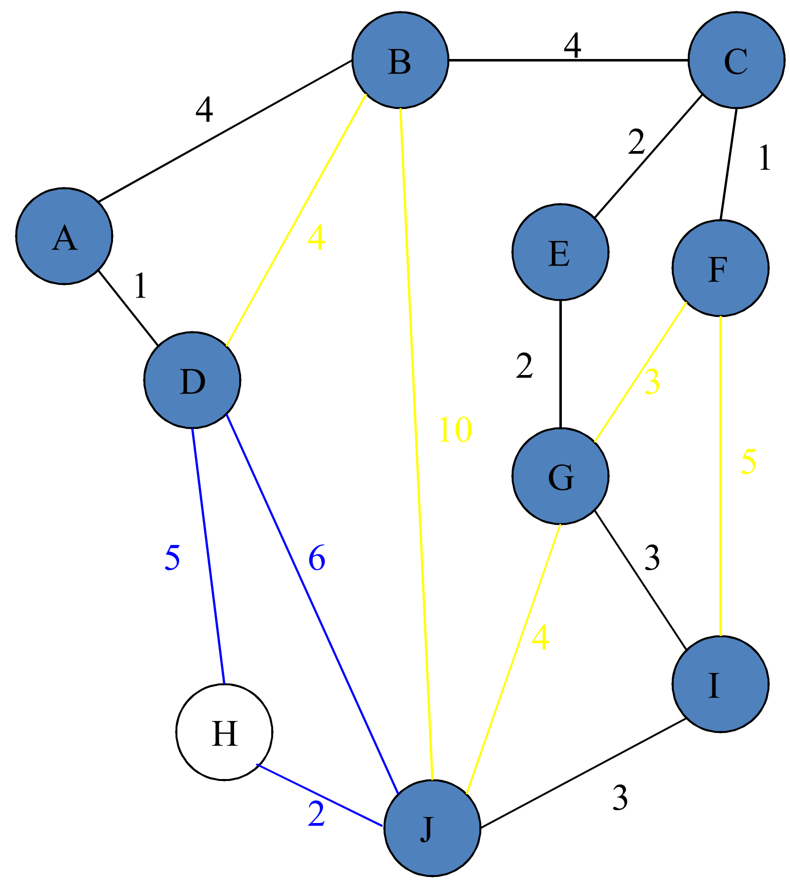


Bhaskar Sardar,* Information Technology Department, *Jadavpur University, India

Example Graph

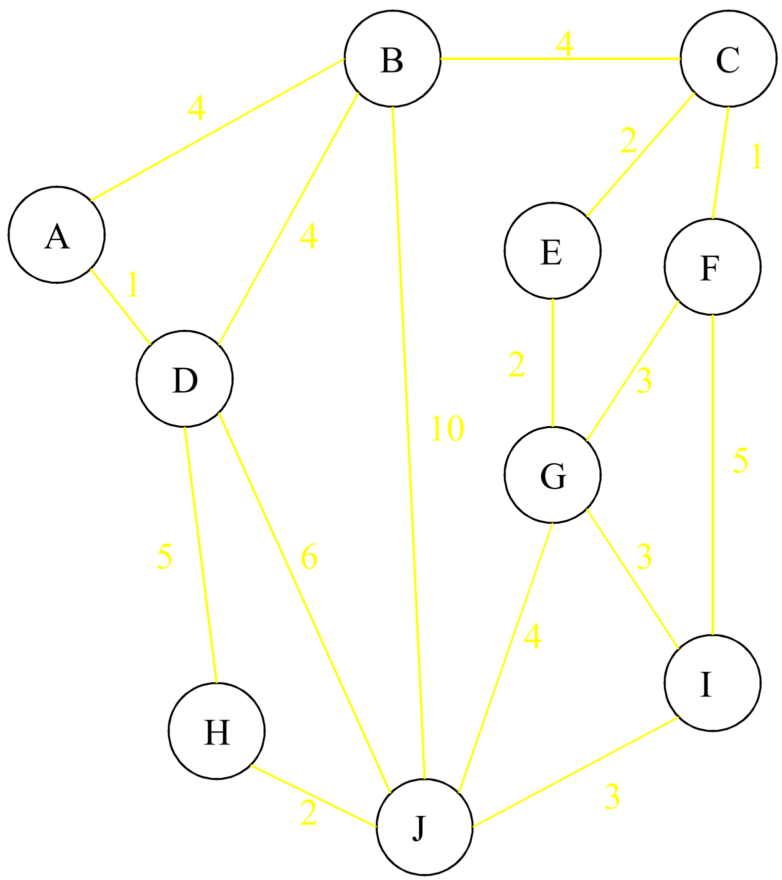


New Graph

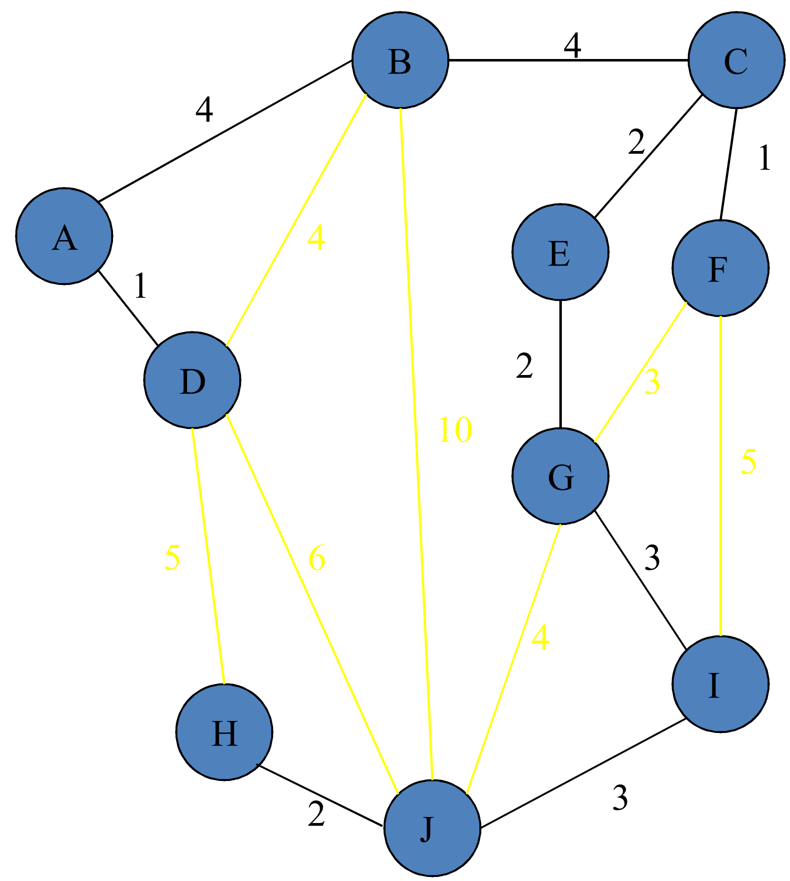


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Example Graph

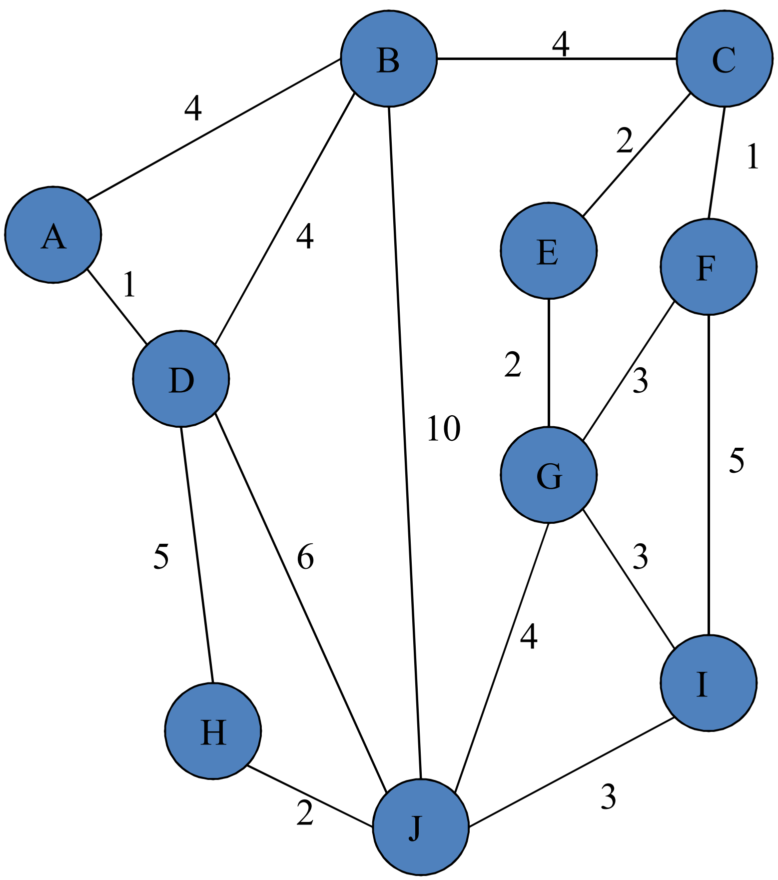


New Graph

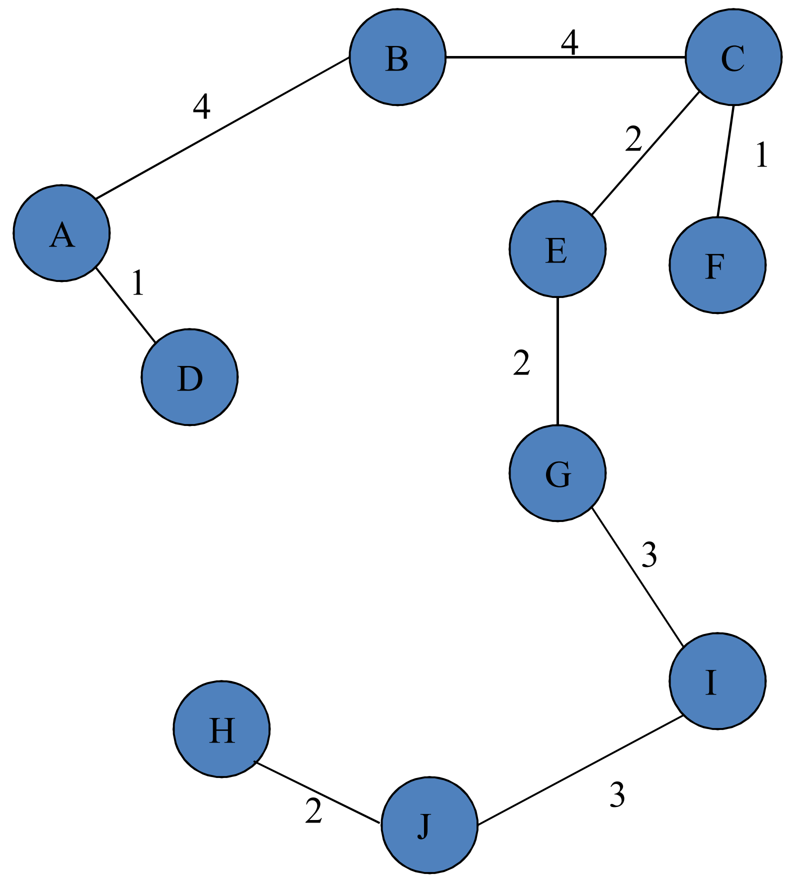


Bhaskar Sardar, Information Technology Department, ***Jadavpur University, India***

Example Graph



Minimum Spanning Tree



Any Doubt ?

- Please feel free to write to me:

bhaskargit@yahoo.co.in