How Small Are Our Big Data:

Turning the 2016 Surprise into a 2020 Vision

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Department of Statistics, Harvard University

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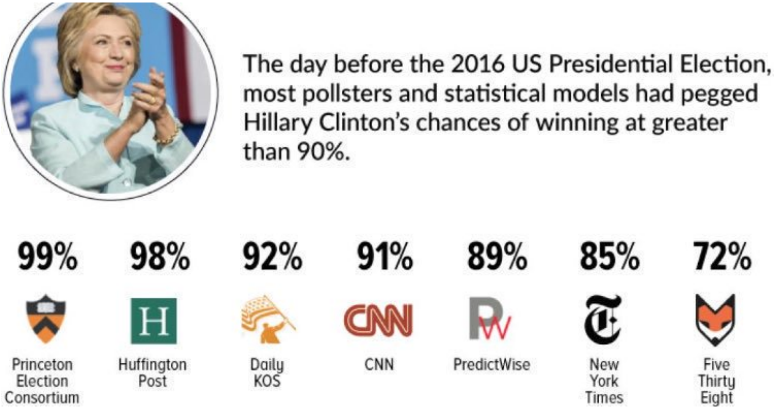
- Many thanks to **Stephen Ansolabehere and Shiro Kuriwaki** for the CCES (**Cooperative Congressional Election Study**) data and analysis on 2016 US election.

A Painful Reminder ... (from internet)

Xiao-Li Meng
Department of Statistics, Harvard University



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$\Longleftrightarrow$

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Estimatinng Trump's share: $\mu_N = \text{Ave}(X_j)$ by sample average:

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Actual estimation error

$$\begin{aligned}\hat{\mu}_n - \mu_N &= \frac{\text{Ave}(R_j X_j)}{\text{Ave}(R_j)} - \text{Ave}(X_j) \\ &= \left[\frac{\text{Ave}(R_j X_j) - \text{Ave}(R_j)\text{Ave}(X_j)}{\sigma_R \sigma_X}\right] \times \frac{\sigma_R}{\text{Ave}(R_j)} \times \sigma_X\end{aligned}$$

Data quality, quantity, and uncertainty

Because $\sigma_R^2 = f(1-f)$, $f = \text{Ave}\{R_j\} = \frac{n}{N}$, we have

$$\text{Error} = \underbrace{\hat{\rho}_{R,X}}_{\textbf{Data Quality}} \times$$

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Data Defect Index (d.d.i.)

Mean Squared Error (MSE)

$$\mathrm{MSE}(\hat{\mu}_n) = \mathsf{E}_R(\hat{\rho}^2) \times \frac{N-n}{n} \times \sigma_X^2$$

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- For Simple Random Sample (SRS): $D_I = (N-1)^{-1}$
- For probabilistic samples in general: $D_I \propto N^{-1}$
- Deep trouble when D_I does not vanish with N^{-1};
- or equivalently when $\hat{\rho}$ does not vanish with $N^{-1/2}$ ...

A Law of Large Populations (LLP)

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If $\rho = \mathrm{E}_R(\hat{\rho}) \neq 0$, then on average, the relative error $\uparrow \sqrt{N}$:

$$\frac{\text{Actual Error}}{\text{Benchmark SRS Standard Error}} = \sqrt{N-1}\hat{\rho}$$

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Paradigm shift for "Big Data":

$$\mathit{From} \quad \underbrace{\frac{\sigma}{\sqrt{n}}}_{\mathit{random\ \ error}} \quad \mathit{to} \quad \underbrace{\hat{\rho}\sqrt{N}}_{\mathit{relative\ \ systemtic\ \ bias}}$$

Effective Sample Size

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The *Effective Sample Size* n_{eff} of a "Big Data" set

Equate its MSE to that from a SRS with size n_{eff}:

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What matters is the relative size $f = n/N$

$$n_{\rm eff} = \frac{n}{1+(1-f)[(N-1)D_I-1]} \approx \frac{f}{1-f}\frac{1}{\hat{\rho}^2}.$$

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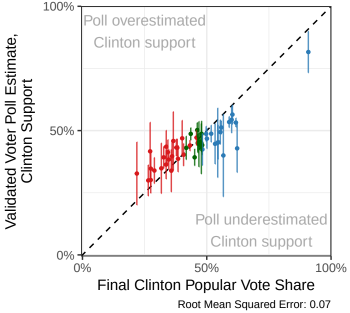

Reasonable predictions for Clinton's Vote Share

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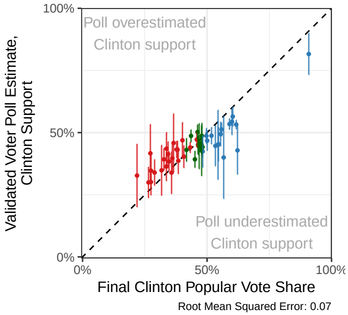

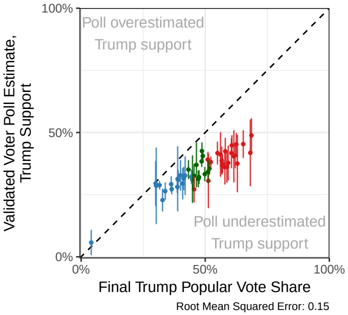

Reasonable predictions for Clinton's Vote Share

Serious underestimation of Trump's Vote Share

Assessing $\hat{\rho}$ using state-level data

Xiao-Li Meng
Department of Statistics, Harvard University

Let μ_N be the true share, and $\hat{\mu}_n$ the estimated share. Then

$$\hat{\rho} = \frac{\hat{\mu}_n - \mu_N}{\sqrt{\frac{N-n}{n}\sigma^2}}, \quad \& \quad \sigma^2 = \mu_N(1-\mu_N)$$

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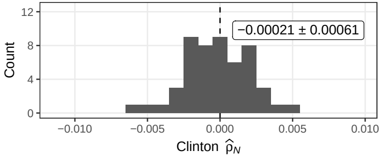

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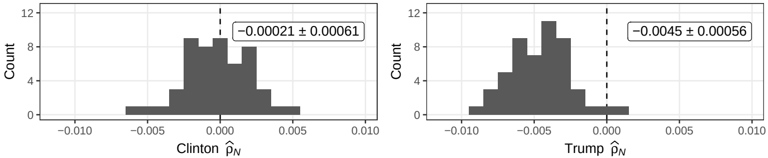

Clinton: $\hat{\rho} \approx -0.0002 \pm 0.0006$ Trump: $\hat{\rho} \approx -0.0045 \pm 0.0006$

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When $\hat{\rho} = -0.005 = -1/200$, $D_I = 1/40000$, and hence

$$n_{\text{eff}} = \frac{f}{1-f}\frac{1}{D_I} = \frac{1}{99} \times 40000 \approx 404!$$

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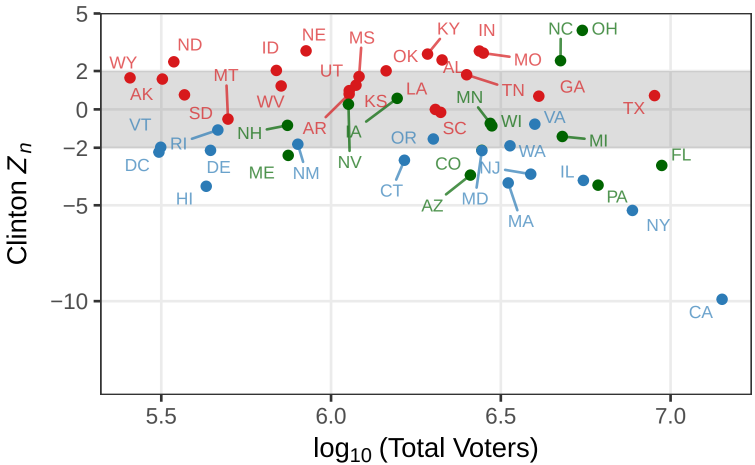
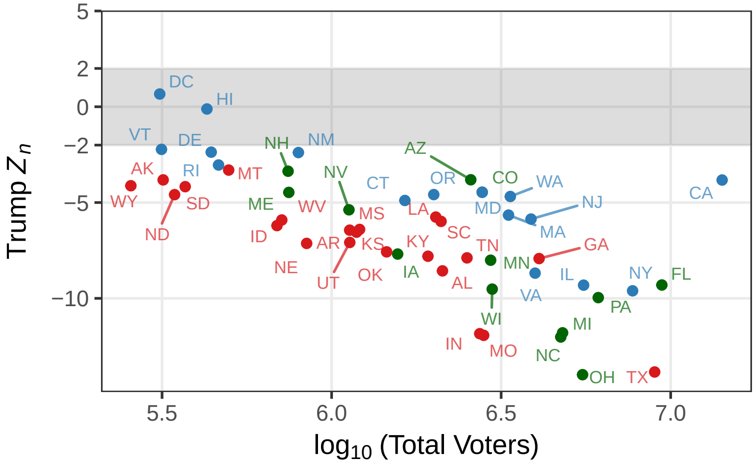
- For $N = 230,000,000$

$$\textbf{Relative Error} = -75.8$$

Visualizing LLP: Actual Coverage for Clinton



Visualizing LLP: Actual Coverage for Trump



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The Big Data Paradox:

If we do not pay attention to data quality, then

The bigger the data,

the surer we fool ourselves.

Lessons Learned ...

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- Lesson 5: **We may all have had too much "confidence" in big size ...**

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More Lessons From ...

HARVARD DATA SCIENCE REVIEW

A Telescopic, Microscopic, and Kaleidoscopic View of Data Science