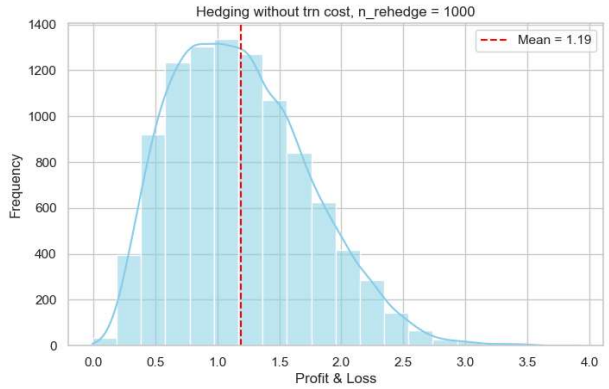
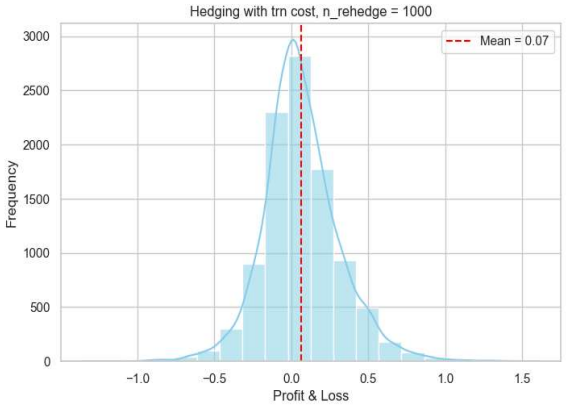
Efficient Delta Hedging with Transaction Costs

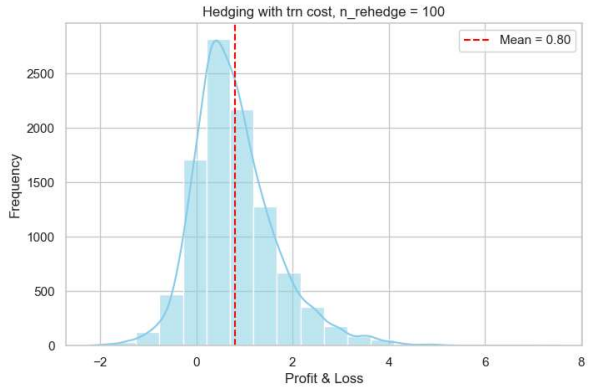
1. Introduction

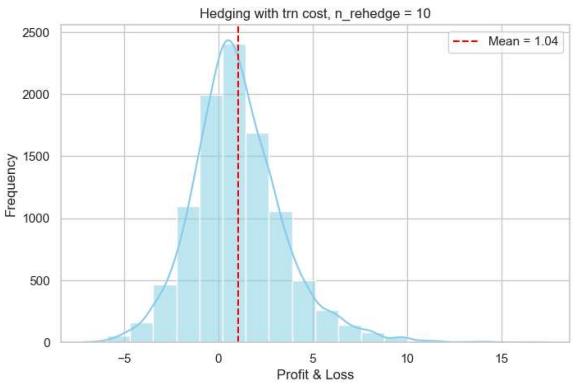
This report examines **delta-hedging strategies** for a 1-year at-the-money (ATM) call option on a stock. The **objective** is to determine which hedging approach yields the **highest expected P/L** with the **lowest standard deviation** when re-hedging the option under different conditions.

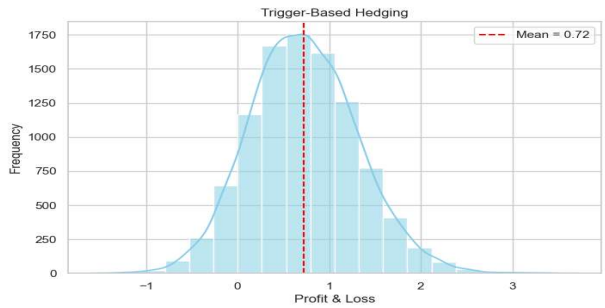
2. Results







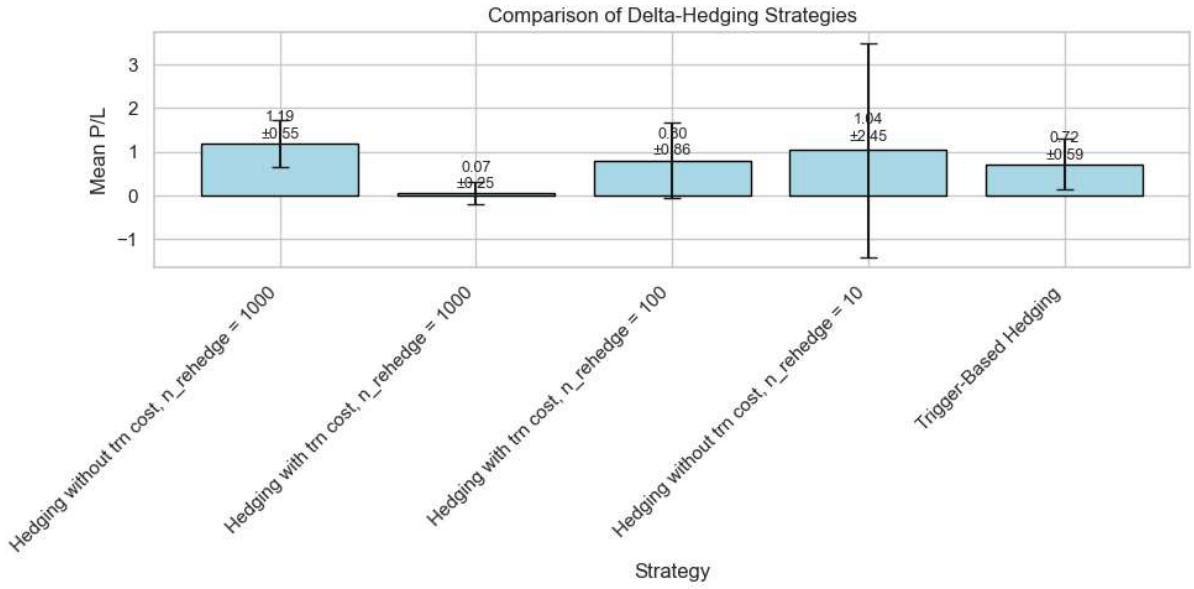




Below are the summarized results for **mean P&L** and **standard deviation**:

Strategy	Mean P/L	Std Dev
Hedging without trn cost, n_rehedge = 1000	1.1909	0.5482
Hedging with trn cost, n_rehedge = 1000	0.0654	0.2499
Hedging with trn cost, n_rehedge = 100	0.7961	0.8643
Hedging without trn cost, n_rehedge = 10	1.0386	2.4464
Trigger-Based Hedging	0.7220	0.5863

Key Obsevations and Plot:



- **Frequent Hedging (No Cost)**: Highest average P/L among all, but not realistic once costs are included. Here the mean value matches with theoretical value of C_r - C_i which in our case is `1.1899203401562488`.
- **Frequent Hedging (With Cost)**: Profitable but significantly lower due to cumulative transaction fees.
- **Reduced Hedging (100 rebalances)**: Decent compromise, higher mean P/L but larger variance.
- **Reduced Hedging (10 rebalances)**: Strong mean P/L but large standard deviation, indicating higher risk.
- **Trigger-Based**: Moderate mean P&L and moderate std, can be a good trade-off.

---

3. Analysis

1. **Transaction Costs**: Significantly erode P&L for frequent hedging, pushing the mean near zero or below.
2. **Reduced Hedging**: Saves on costs but can expose the portfolio to larger price movements between rebalances, raising the standard deviation.
3. **Trigger-Based**: Balances hedge accuracy and transaction costs, re-hedging only when Δ changes by more than 5% in absolute value, leading to fewer trades and a middle-range standard deviation.

---

4. Conclusion and Recommendation

1. **Highest Mean P&L**: Typically the **no-cost, frequent hedging** scenario, but that's idealized. Once cost is considered, **reduced** or **trigger-based** strategies can deliver **better** net returns.
2. **Lowest Risk (Std Dev)**: More frequent hedging naturally reduces risk but can push down net returns due to cost.
3. **Best Trade-Off**: If your priority is a **combination** of higher mean P&L **and** moderate risk, **Trigger-Based** or **Reduced Hedging** with ~100 rebalances often stands out.

Hence, the **superior** outcome under 0.1% transaction cost might be a **trigger-based** approach, depending on the tolerance for standard deviation. It yields higher average P&L than the 1,000-step strategy with costs, but at slightly more risk than frequent rebalancing