DS 100: Principles and Techniques of Data Science **Date: May 3, 2019**

Exam Review

Name:

Sampling

1. A political scientist is interested in answering a question about a country composed of three states A, B and C. These states have exactly 100, 200, and 300 voting adults respectively. Within each state, assume that there are 10 towns and the population of voting adults in each state is split among its 10 towns uniformly. So for example, state A will have 10 towns, each with $\frac{100}{10} = 10$ voting adults. In addition, assume that each town has an equal number of male and female voting adults. So for example, state A will have 5 male and 5 female voting adults in each town.

   To answer this question, a political survey is administered by randomly sampling 3, 8, and 12 voting adults from each town without replacement in each state, respectively.

   (a) Which sampling plan was used in the survey?

   (a) cluster sampling (b) stratified sampling (c) quota sampling (d) census

   Solution: B. The strata are the towns in each state and from each town, we randomly sample adults.

   (b) How many strata are there in total?

   Solution: 30 strata in total, 10 per state, where each strata is a town.

   (c) What is the probability that the sample generated from the sampling strategy from part (a) will comprise of all females?

   Solution: State A - 10 towns, each containing 10 voting adults, 5 female, 5 male
   State B - 10 towns, each containing 20 voting adults, 10 female, 10 male
   State C - 10 towns, each containing 30 voting adults, 15 female, 15 male
   For each town in state A, we sample 3 voting adults, so P(all_female_A) =

   $$(\frac{\binom{5}{3}}{\binom{10}{3}})^{10}$$

   . For state B, P(all_female_B) =

   $$(\frac{\binom{10}{8}}{\binom{20}{8}})^{10}$$

. For state C, P(all_female_C) =

$$(\frac{\binom{15}{12}}{\binom{30}{12}})^{10}$$

. Therefore, the probability that the sample comprises of all females is P(all_female_A) * P(all_female_B) * P(all_female_C) =

$$(\frac{\binom{5}{3}}{\binom{10}{3}})^{10} * (\frac{\binom{10}{8}}{\binom{20}{8}})^{10} * (\frac{\binom{15}{12}}{\binom{30}{12}})^{10}$$

Alternate Solution: An alternate way to look at the P(all_female_A) is that for each town in state A, we have 10 voting adults and we are sampling 3 voting adults. We want each one of these adults to comprise of all females. Therefore, P(all_female_A) =

$$(\frac{5}{10} * \frac{4}{9} * \frac{3}{8})^{10}$$

as we are sampling without replacement. We repeat this idea for states B and C and then multiply the 3 probabilities together to obtain the solution above.

EDA & Visualization

2. For each of the following scenarios, determine which plot type is *most* appropriate to reveal the distribution of and/or the relationships between the following variable(s). For each scenario, select only one plot type. Some plot types may be used multiple times.

A. histogram B. pie chart C. bar plot D. line plot

E. side-by-side boxplots F. scatter plot G. stacked bar plot H. overlaid line plots

(a) Sale price and number of bedrooms for houses sold in Berkeley in 2010.

Solution: E. Side-by-side Boxplots. We might imagine using a scatter plot since we are plotting the relationship between two numeric quantities. However because the number of bedrooms is an integer and most houses will only have a small number, we are likely to encounter *over-plotting* in the scatter plot. Therefore side-by-side boxplots are likely to be most informative.

(b) Sale price and date of sale for houses sold in Berkeley between 1995 and 2015.

Solution: F. Scatter Plot. Here we are plotting two numeric quantities with sufficient spread on each axis.

(c) Infant birth weight (grams) for babies born at Alta Bates hospital in 2016.

Solution: A. Histogram. Here we are plotting the distribution of a likely large number of observations and therefore a histogram would be most appropriate.

(d) Mother's education-level (highest degree held) for students admitted to UC Berkeley in 2016.

Solution: C. Bar Plot. Here we want to visualize counts of a categorical variable.

(e) SAT score and HS GPA of students admitted to UC Berkeley in 2016.

Solution: F. Scatter Plot. Here we are visualizing the relationship between two continuous quantities.

(f) The percentage of female student admitted to UC Berkeley each year from 1950 to 2000.

Solution: D. Line plot. This allows us to see the trends over time.

(g) SAT score for males and females of students admitted to UCB from 1950 to 2000

Solution: E. side-by-side boxplots. This allows us to see the distributions of SAT scores per gender and year.

Estimation

3. Suppose that we try to predict a donkey's weight, y_i from its sex alone. There are 3 different sexes of donkeys, so the sex variable has values: gelding, stallion, and female). Consider the following model consisting of dummy variables:

$$y_i = \beta_F D_{F,i} + \beta_G D_{G,i} + \beta_S D_{S,i}$$

where the dummy variable $D_{F,i} = 1$ if the i^{th} donkey is female and $D_{F,i} = 0$ otherwise. The dummy variables D_G and D_S are dummies for geldings and stallions, respectively.

Prove that if we using the following loss function:

$$L(\beta_F, \beta_G, \beta_S) = \sum_{i=1}^{n} \left(y_i - (\beta_F D_{F,i} + \beta_G D_{G,i} + \beta_S D_{S,i})\right)^2$$

then the loss minimizing value $\hat{\beta}_F = \bar{y}_F$ where $\bar{y}_F$ is the average weight of the female donkeys.

Solution: The summation that we are minimizing can be split into three separate sums because only one of the dummy variables is 1 for any observation. That is, when $D_{F,i} = 1$ then $D_{G,i} = 0$ and $D_{S,i} = 0$.

$$\begin{aligned} &\min_{\beta_F, \beta_G, \beta_S} \sum_{i=1}^{n} (y_i - (\beta_F D_{F,i} + \beta_G D_{G,i} + \beta_S D_{S,i}))^2 \\ &= \sum_F (y_i - \beta_F)^2 + \sum_G (y_i - \beta_G)^2 + \sum_S (y_i - \beta_S)^2 \end{aligned}$$

This implies that we can minimize over β_F separately, i.e.,

$$\min_{\beta_F} \sum_F (y_i - \beta_F)^2$$

We can differentiate with respect to β_F to get

$$\sum_F -2(y_i - \beta_F)$$

Set this to 0 and solve for β_F

$$\frac{1}{\#F} \sum_F y_i = \hat{\beta}_F$$

Optimization

4. Fix the following buggy Python implementation of gradient descent:

```
def grad_descent(X, Y, theta0, grad_func, max_iter = 1000, alpha):
    """X: A 2D array, the feature matrix.
    Y: A 1D array, the response vector.
    theta0: A 1D array, the initial parameter vector.
    grad_func: Maps a parameter vector, a feature matrix, and a
        response vector to the gradient of some loss function at the
        given parameter value.
    alpha: Learning rate at each step. The return value is a 1D
        array."""
    theta = theta0
    for t in range(1, max_iter+1):
        grad = grad_func(theta, X, Y)
        theta = theta0 + 1/alpha * grad
    return grad
```

Solution: The last two lines need to change:

```
def grad_descent(X, Y, theta0, grad_func, max_iter = 1000, alpha):
    """X: A 2D array, the feature matrix.
    Y: A 1D array, the response vector.
    theta0: A 1D array, the initial parameter vector.
    grad_func: Maps a parameter vector, a feature matrix, and a
        response vector to the gradient of some loss function at
        the given parameter value.
    alpha: Learning rate at each step. The return value is a 1D
        array."""
    theta = theta0
    for t in range(1, max_iter+1):
        grad = grad_func(theta, X, Y)
        theta = theta - alpha * grad
    return theta
```

5. Suppose you are given a dataset $\{(x_i, y_i)\}_{i=1}^n$ where $x_i \in \mathbb{R}$ is a one dimensional feature and $y_i \in \mathbb{R}$ is a real-valued response. You use $E[Y|X] = \theta(x_i)$ to model the data with β as the model parameter. You choose to use the following regularized empirical risk:

$$L(\beta) = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \theta(x_i)\right)^2 + \lambda\beta^2$$

(a) This regularized empirical risk is best described as:

(a) mean absolute error with L^2 regularization.
(b) mean squared error with L^2 regularization.
(c) mean squared error with L^1 regularization.
(d) empirical Huber risk with λ regularization.

Solution: B. We are minimizing the mean squared error with L^2 regularization, which is shown above.

(b) Suppose you choose the model $E[Y|X] = \beta x_i^3$. Using the above objective function, derive the risk minimizing estimate for β.

Solution:

Step 1: Take the derivative of the loss function.

$$\frac{\partial}{\partial\beta}L(\beta) = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial}{\partial\beta}\left(y_i - \beta x_i^3\right)^2 + \frac{\partial}{\partial\beta}\lambda\beta^2 \tag{1}$$

$$= -\frac{2}{n}\sum_{i=1}^{n}\left(y_i - \beta x_i^3\right)x_i^3 + 2\lambda\beta \tag{2}$$

Step 2: Set derivative equal to zero and solve for β.

$$0 = -\frac{2}{n}\sum_{i=1}^{n}\left(y_i - \beta x_i^3\right)x_i^3 + 2\lambda\beta \tag{3}$$

$$\beta = \frac{1}{n\lambda}\sum_{i=1}^{n}\left(y_i - \beta x_i^3\right)x_i^3 \tag{4}$$

$$\beta = \frac{1}{n\lambda}\sum_{i=1}^{n}y_i x_i^3 - \beta\frac{1}{n\lambda}\sum_{i=1}^{n}x_i^6 \tag{5}$$

$$\beta\left(1 + \frac{1}{n\lambda}\sum_{i=1}^{n}x_i^6\right) = \frac{1}{n\lambda}\sum_{i=1}^{n}y_i x_i^3 \tag{6}$$

$$\tag{7}$$

Thus we obtain the final answer:

$$\boxed{\hat{\beta} = \frac{\frac{1}{n}\sum_{i=1}^{n}y_i x_i^3}{\left(\lambda + \frac{1}{n}\sum_{i=1}^{n}x_i^6\right)}} \tag{8}$$

Inference

6. **True or False.** Determine whether the following statements are true or false.

   (a) Suppose we have 100 samples drawn independently from a population. If we construct a 95% confidence interval for each sample, we expect 95 of them to include the **sample** mean.

   > **Solution: False.** All of them should include the sample mean.

   (b) We often prefer a pseudo-random number generator because our simulations results can be exactly reproduced by controlling the seed.

   > **Solution: True.** This is an essential aspect of reproducible data analyses and simulation studies.

7. Suppose we have a pandas Series called **`thePop`** which contains a census of **25000 subjects**. We also have a simple random sample of **400 individuals** saved in the Series **`theSample`**. We are interested in studying the behavior of the bootstrap procedure on the simple random sample. Fill in the blanks in the code below to construct **10000 bootstrapped estimates** for the **median**.

```
boot_stats = [

        ______________________

        .sample(n = _____________, replace = _____________)

        .______________________()


        for j in range(________________)
    ]
```

Solution:

```
boot_stats = [

        theSample
```

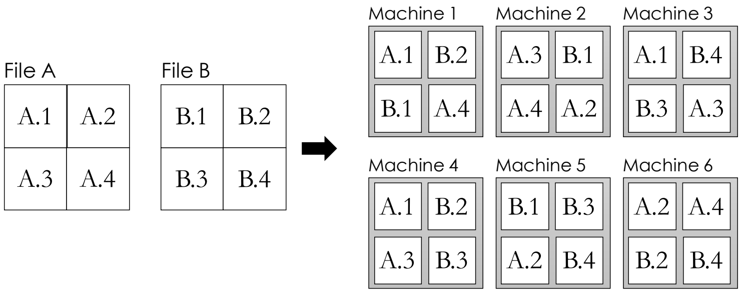
```
            .sample(n = 400, replace = True)

            .median()

            for j in range(10000)

            ]
```

8. Consider the following layout of the files `A` and `B` onto a distributed file-system of 6 machines.



Assume that all blocks have the same file size and computation takes the same amount of time.

(a) (1 point) If we were to lose machines $M1$, $M2$, and $M3$ which of the following file or files would we lose (select all that apply).

A. File `A` B. File `B` **C. We would still be able to load both files.**

(b) (1 point) If each of the six machines fail with probability p, what is the probability that we will lose block $B.1$ of file `B`.?

A. $3p$ **B.** p^3 C. $(1-p)^3$ D. $1-p^3$