

#### **Assignment- 4**

##### **1. Is it possible that an event is independent of itself? If so, when?**

###### **Answer:**

The only events that are independent of themselves are those with probability either 0 or 1. That follows from the fact that a number is its own square if and only if it's either 0 or 1. The only way a random variable  $X$  can be independent of itself is if for every measurable set  $A$ , either

$$P(X \in A) = 1 \text{ or } P(X \in A) = 0$$

##### **2. Is it always true that if $A$ and $B$ are independent events, then $A^c$ and $B^c$ are independent events? Show that it is, or give a counterexample.**

###### **Answer:**

Let  $A$  and  $B$  be independent events, and let  $A$  and  $C$  be independent events.

Since  $A$  and  $B$  and  $A$  and  $C$  are independent, i.e.

$$P(A \cap B) = P(A)P(B) \quad \text{and} \quad P(A \cap C) = P(A)P(C)$$

According to the definition of independent events,  $A$  and  $B \cup C$  are independent if and only if

$$P(A \cap (B \cup C)) = P(A)P(B \cup C)$$