Assignment-8

1. A crime is committed by one of two suspects, A and B. Initially, there is equal evidence against both of them. In further investigation at the crime scene, it is found that the guilty party had a blood type found in 10% of the population. Suspect A does match this blood type, whereas the blood type of Suspect B is unknown. (a) Given this new information, what is the probability that A is the guilty party? (b) Given this new information, what is the probability that B's blood type matches that found at the crime scene?

Answer:

Define events

A: {A is guilty}

B: {B is Guilty}

MA = {A's Blood matches the guilty party}

MB = {B's Blood matches the guilty party}

(a) we want to calculate P(A/MA).

Use Byes rule to Calculate

$$P(A|MA) = \frac{P(MA|A)P(A)}{P(MA|A)P(A) + P(MA|B)P(B)} = \frac{\left(\frac{1.1}{2}\right)}{\left(\frac{1.1}{2}\right) + \left(\frac{1}{10} \times \frac{1}{2}\right)} = \frac{\left(\frac{1.1}{2}\right)}{\left(\frac{1.1}{2}\right) + \left(\frac{1}{20}\right)}$$

$$P(A|MA) = \frac{\left(\frac{1.1}{2}\right)}{\left(\frac{1.1}{2}\right) + \left(\frac{1}{20}\right)} = \frac{\left(\frac{1.1}{2}\right)}{\left(\frac{11+1}{20}\right)} = \frac{1.1}{2} \times \frac{20}{12} = \frac{11}{12}$$

(b) We want to calculate P(MB|MA). Use Lots to obtain

P(MB|MA) = P(MB|MA, A). P(A|MA) + P(MB|MB, A). P(B|MA)

$$= \left(\frac{1}{10}\right) \times \left(\frac{10}{11}\right) + \frac{1.1}{11} = \frac{2}{11}$$