Assignment-3

1. **For a group of 7 people, find the probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays, assuming that all seasons are equally likely.**

Answer:

Given that seasons are equally likely and all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays

Total outcomes: Each person is allotted a season out of 4. Hence 48 possibilities

Number of Outcomes that one or more season has no student having their birthday:

Using Inclusion and Exclusion, $C_1^4 \times 38 - C_2^4 \times 28 - C_3^4 \times 18$

[i.e., Exclude 1 season - Exclude 2 season + exclude 3 seasons, also note that we can't exclude all the 4 seasons]

Probability that one or more season has no student having their birthday:

$(C_1^4 \times 38 - C_2^4 \times 28 - C_3^4 \times 18)/48 = 0.377$

Required probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays: 1-0.377=0.623

2. **Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday?**

Answer:

Using Inclusion – Exclusion:

$A_i = \{Alice\ has\ not\ any\ of\ classes\ at\ i - th\ day\ in\ week\}$

$$\mathrm{P}\left(\bigcap_{i=1}^{5} A_i^{C}\right) = \mathrm{P}\left(\bigcup_{i=1}^{5} A_i^{C}\right) = 1\text{-}\mathrm{P}\left(\bigcup_{i=1}^{5} A_i\right)$$

$$= 1\text{-}\big(\mathrm{P}(A_1)+\ldots\ldots+\mathrm{P}(A_5)\text{-}\mathrm{P}(A_1 \cap A_2)\text{-}\ldots.\text{-}\mathrm{P}(A_1 \cap A_5) + \mathrm{P}(A_2 \cap \ldots.\cap A_5)\big)$$

$$= 1 - \sum_{k=1}^{5} (\text{-}1)^{k+1} \binom{5}{k} \mathrm{P}(A_1 \cap \ldots.\cap A_k)$$

$$= 1\text{-} \sum_{k=1}^{5} (\text{-}1)^{k+1} \binom{5}{k} \frac{C_7^{30\text{-}6k}}{C_7^{30}}$$