

Assignment-10

1. A chicken lays n eggs. Each egg independently does or doesn't hatch, with probability p of hatching. For each egg that hatches, the chick does or doesn't survive (independently of the other eggs), with probability s of survival. Let $N \leftarrow \text{Bin}(n, p)$ be the number of eggs which hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch but don't survive (so $X + Y = N$). Find the marginal PMF of X , and the joint PMF of X and Y . Are they independent?

Answer:

The probability that an egg hatches and chick survive is ps . We can consider each egg as a Bernoulli trial each with a success (hatching and surviving) probability ps . There are n independent trials, so $X \sim \text{Bin}(n, ps)$.

But I am trying to prove this more rigorously.

For any $1 \leq i \leq n$ we have

$$P(X = i) = \sum_{j=0}^n P(X = i | N = j) \cdot P(N = j) = \sum_{j=i}^n P(X = i | N = j) \cdot P(N = j) = \sum_{j=i}^n p^j (1-p)^{n-j}$$