Experiment: 03

Date: 06.04.2022

Program Name:

a) Write a menu-driven MATLAB program for obtaining Degenerate and Non-degenerate Basic Solutions, Basic Feasible Solutions.
b) Using your code get the results of the following problems:
Find all basic solutions for the system of simultaneous equations. Determine the degenerate and non-degenerate basic solutions and basic feasible solutions separately.

$$2x_1 + 3x_2 + 4x_3 = 5, 3x_1 + 4x_2 + 5x_3 = 6$$
$$2x_1 + x_2 + 4x_3 = 11, 3x_1 + x_2 + 5x_3 = 14$$
$$3x_1 + x_2 + 5x_3 + x_4 = 12, 2x_1 + 4x_2 + x_3 + 2x_5 = 8$$
$$2x_1 + 6x_2 + 2x_3 + x_4 = 3, 6x_1 + 4x_2 + 4x_3 + 6x_4 = 22$$

c) Obtain all extreme points and the corresponding optimal solution of the following LPP:

$$\text{Maximize } 5x_1 + 3x_2 \quad subject\ to,\ 3x_1 + 5x_2 \le 15, 5x_1 + 2x_2 \le 10$$
$$\text{Maximize } 2x_1 + x_2 \quad subject\ to,\ x_1 + 2x_2 \le 10, x_1 + x_2 \le 6, x_1 - x_2 \le 2, x_1 - 2x_2 \le 1$$

Theory:

1. **Basic Solution:** Given a system of m simultaneous linear equations n variables (unknowns) $(m < n)$.

$$AX = b\ , X^T \epsilon\ R^n$$

Where, A is an $(m \times n)$ matrix of rank m .
Let, B be any $(m \times m)$ sub-matrix formed by m linearly independent columns of A .
Then a solution obtained by setting $(n - m)$ variables not associated with the columns of B , equal to zero and solving the resulting system is called a Basic Solution to the given system of equations.

Here,

i. The m variables which may be all different from Zero (0) , are called Basic Variables.
The $(m \times m)$ non-singular sub-matrix B is called basic matrix whose columns are called basis vactors.

ii. **If B is basic submatrix then the basic solution to the system is ,**

$$X_B = B^{-1}b$$

2. **Degenerate Basic Solution :** **A Basic Solution to the system is called Degenerate if one or more of the Basic Variables vanish (i.e., value of the variables is zero (0)).**

3. **Non-Degenerate Basic Solution :** **A Basic Solution to the system is called Non-Degenerate if none of the Basic Variables vanishes .**

4. **Feasible Solution:** **A Feasible solution to a L.P.P. is a set of values of the variables, which satisfy all the constraints and all the non-negative restrictions of the variables, is known as the feasible solution (F.S.) to the LPP.**

5. **Basic Feasible Solution (B.F.S.) :** **A Feasible Solution (F.S.) to a LPP which is also a Basic Solution to the problem is called a Basic Feasible Solution (B.F.S.) to the L.P.P. .**

Program Code:

❖ **Program Code : For Build a function gauss_elimination (gauss_elimination.m)**

```
%ANINDYA NAG
%UG/02/BTCSE/2018/005

function x = gauss_elimination(A,b)
  [m,n] = size(A);
  if ( m == n)
      Aug =[A b];
      for k = 1 : (n-1)
          [big ip] = max(abs(Aug(k:m,k)));
          ipr = ip +k-1 ;
            if (ipr ~= k)
               Aug([k ipr],:) =Aug([ipr k],:);
            end
            for i = (k +1) : n
                factor = Aug(i,k)/Aug(k,k);
                for j=1 : (n+1)
                   Aug(i,j) =(Aug(i,j)-(factor*Aug(k,j)));
                end

            end
      end
      x = [];
      x(n) = Aug(n,n+1)/ Aug(n,n);

      for i=(n-1):-1: 1
          Sum = 0;
          for j = n:-1:(i+1)
              Sum = Sum + Aug(i,j)*x(j) ;
          end
         x(i) =( Aug(i,n+1) -Sum)/Aug(i,i);
      end
  end

end
```

a) Program Code :(Write a menu-driven MATLAB program for obtaining Degenerate and Non-degenerate Basic Solutions, Basic Feasible Solutions.)

```
%ANINDYA NAG
%UG/02/BTCSE/2018/005
clc;
clearvars;
A = input('Enter the coefficients matrix :  ');
b = input('Enter the right hand side vector :  ');
[m,n] = size(A);
combo = nchoosek(1:n,m);
[m1 n1] = size(combo);
nbs = nchoosek(n,m);
Deg= [];
Non_Deg=[];
B_F_S=[];
for i = 1 :nbs
     x = zeros(n,1);
     bas_mat = [ ];
     for j = 1 :n1
        bas_mat = [bas_mat A(:,combo(i,j))];
     end
     bas_mat;
     y = gauss_elimination(bas_mat,b);
     if( max(y) ~= Inf )
         for j = 1 : n1
             x(combo(i,j),1) = y(j);
         end
         disp('Basic solution is:  ');
         x
         if( min(x) >= 0 )
             B_F_S = [B_F_S x];
             disp(' This is a Basic Feasible solution.');
         else
            disp (' This is a Basic Infeasible solution.');
         end

         if(y == 0)
              Deg = [Deg x];
              disp('This is Degenerate Basic solution.');
         else
              Non_Deg =[Non_Deg x];
```

```
            disp ('This is Non Degenerate Basic solution.'
);
         end
     else
         disp( 'In this case no Basic solution exist.  ' );
     end
end

disp('D: Degenerate Solution, ND: Non Degenerate Solution,
BFS: Basic Feasible Solution');
choice = menu( ' Choose','D',' ND','BFS');

switch choice
     case 1
          [D1,D2]=size(Deg);
          if (D1 < 1)
              disp('No Degenerate Solution..')
          else
             disp(Deg) ;
          end
     case  2
         [ND1,ND2]=size(Non_Deg);
         if (ND1 < 1)
             disp('No Non Degenerate Solution..')
         else
            disp(Non_Deg) ;
         end
     case 3
          [BFS1,BFS2]=size(B_F_S);
          if (BFS1 < 1)
             disp('No Basic Feasible Solution..')
          else
             disp(B_F_S) ;
          end

 end
```

b) **Using your code get the results of the following problems:**
Find all basic solutions for the system of simultaneous equations. Determine the degenerate and non-degenerate basic solutions and basic feasible solutions separately.

$$2x_1 + 3x_2 + 4x_3 = 5, 3x_1 + 4x_2 + 5x_3 = 6$$
$$2x_1 + x_2 + 4x_3 = 11, 3x_1 + x_2 + 5x_3 = 14$$
$$3x_1 + x_2 + 5x_3 + x_4 = 12, 2x_1 + 4x_2 + x_3 + 2x_5 = 8$$
$$2x_1 + 6x_2 + 2x_3 + x_4 = 3, 6x_1 + 4x_2 + 4x_3 + 6x_4 = 22$$

Output: For $2x_1 + 3x_2 + 4x_3 = 5, 3x_1 + 4x_2 + 5x_3 = 6$

```
Enter the coefficients matrix :  [2 3 4;3 4 5]
Enter the right hand side vector :  [5;6]
Basic solution is:

x =

  -2.0000
   3.0000
        0

 This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

  -0.5000
        0
   1.5000

 This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

     0
    -1
```

2

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
D: Degenerate Solution, ND: Non Degenerate Solution, BFS: Basic Feasible Solution

1.For Choosing D :
No Degenerate Solution..

2.For Choosing ND :

-2.0000	-0.5000	0
3.0000	0	-1.0000
0	1.5000	2.0000

3.For Choosing BFS :
No Basic Feasible Solution..

Output: For $2x_1 + x_2 + 4x_3 = 11, 3x_1 + x_2 + 5x_3 = 14$

Enter the coefficients matrix : [2 1 4;3 1 5]
Enter the right hand side vector : [11;14]
Basic solution is:

x =

3.0000
5.0000
0

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0.5000
0

2.5000

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0
-1
3

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
D: Degenerate Solution, ND: Non Degenerate Solution, BFS: Basic Feasible Solution

1.For Choosing D :

No Degenerate Solution..

2.For Choosing ND:

3.0000	0.5000	0
5.0000	0	-1.0000
0	2.5000	3.0000

3.For Choosing BFS :

3.0000	0.5000
5.0000	0
0	2.5000

<u>Output</u>: For $3x_1 + x_2 + 5x_3 + x_4 = 12, 2x_1 + 4x_2 + x_3 + 2x_5 = 8$

Enter the coefficients matrix : [3 1 5 1 0;2 4 1 0 2]
Enter the right hand side vector : [12;8]
Basic solution is:

x =

```
4
0
0
0
0
```

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

```
4
0
0
0
0
```

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

```
4
0
0
0
0
```

This is a Basic Feasible solution.

This is Non Degenerate Basic solution.
Basic solution is:

x =

```
     4
     0
     0
     0
     0
```

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

```
         0
    1.4737
    2.1053
         0
         0
```

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

```
     0
     2
     0
    10
     0
```

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0
12
0
0
-20

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0
0
8.0000
-28.0000
0

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0
0
2.4000
0
2.8000

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0

0
0
12
4

This is a Basic Feasible solution.
This is Non Degenerate Basic solution.
D: Degenerate Solution, ND: Non Degenerate Solution, BFS: Basic Feasible Solution

1.For Choosing D :

No Degenerate Solution..

2.For Choosing ND :

Columns 1 through 8

4.0000	4.0000	4.0000	4.0000	0	0	0	0
0	0	0	0	1.4737	2.0000	12.0000	0
0	0	0	0	2.1053	0	0	8.0000
0	0	0	0	0	10.0000	0	-28.0000
0	0	0	0	0	0	-20.0000	0

Columns 9 through 10

0	0
0	0
2.4000	0
0	12.0000
2.8000	4.0000

3.For Choosing BFS:

4.0000	4.0000	4.0000	4.0000	0	0	0	0
0	0	0	0	1.4737	2.0000	0	0
0	0	0	0	2.1053	0	2.4000	0
0	0	0	0	0	10.0000	0	12.0000
0	0	0	0	0	0	2.8000	4.0000

<u>Output</u>: For $2x_1 + 6x_2 + 2x_3 + x_4 = 3, 6x_1 + 4x_2 + 4x_3 + 6x_4 = 22$

Enter the coefficients matrix : [2 6 2 4;6 4 4 6]
Enter the right hand side vector : [3;22]
Basic solution is:

x =

```
   4.2857
  -0.9286
        0
        0
```

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

```
   8.0000
        0
  -6.5000
        0
```

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

```
   5.8333
        0
        0
  -2.1667
```

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0
-2.0000
7.5000
0

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0
-3.5000
0
6.0000

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
Basic solution is:

x =

0
0
17.5000
-8.0000

This is a Basic Infeasible solution.
This is Non Degenerate Basic solution.
D: Degenerate Solution, ND: Non Degenerate Solution, BFS: Basic Feasible Solution

1.For Choosing D :

No Degenerate Solution..

2.For Choosing ND :

4.2857 8.0000 5.8333 0 0 0

-0.9286	**0**	**0**	**-2.0000**	**-3.5000**	**0**
0	**-6.5000**	**0**	**7.5000**	**0**	**17.5000**
0	**0**	**-2.1667**	**0**	**6.0000**	**-8.0000**

3.For Choosing BFS:

No Basic Feasible Solution..

c) Program Code: **Obtain all extreme points and the corresponding optimal solution of the following LPP:**

$\mathit{Maximize}\ \ 5x_1 + 3x_2\ \ \textit{subject to},\ \ 3x_1 + 5x_2 \le 15, 5x_1 + 2x_2 \le 10$

$\mathit{Maximize}\ \ 2x_1 + x_2\ \ \textit{subject to},\ \ x_1 + 2x_2 \le 10, x_1 + x_2 \le 6, x_1 - x_2 \le 2, x_1 - 2x_2 \le 1$

Program Code:

For $\mathit{Maximize}\ \ 5x_1 + 3x_2\ \ \textit{subject to},\ \ 3x_1 + 5x_2 \le 15, 5x_1 + 2x_2 \le 10$

```
%ANINDYA NAG
%UG/02/BTCSE/2018/005
clc;
clearvars;
A = input('Enter the coefficients matrix :  ');
b = input('Enter the right hand side vector :  ');
opt_value=0;
[m,n] = size(A);
combo = nchoosek(1:n,m);
[m1 n1] = size(combo);
nbs = nchoosek(n,m);
for i = 1 :nbs
     x = zeros(n,1);
     bas_mat = [ ];
     for j = 1 :n1
        bas_mat = [bas_mat A(:,combo(i,j))];
     end
     bas_mat;
     y = gauss_elimination(bas_mat,b);
     if( max(y) ~= Inf )
          for j = 1 : n1
              x(combo(i,j),1) = y(j);
          end
```

```
          %disp('Basic solution is:  ');
          x;
         if( min(x) >= 0 )
              %disp(' This is a Basic Feasible solution.');
              z= 5*x(1,1) + 3*x(2,1);
              if ( opt_value <= z)
                   opt_value = z;
                   opt_soln =x ;

              end
         end

     end
end
fprintf('Optimal Value is: %0.2f\n',opt_value)
fprintf('Optimal solution is: \n x1=%0.2f \n x2=%0.2f \n
x3=%0.2f \n x4=%0.2f
\n',opt_soln(1,1),opt_soln(2,1),opt_soln(3,1),opt_soln(4,1)
)
```

Output:

For ***Max*imize** $5x_1 + 3x_2$ ***subject to,*** $3x_1 + 5x_2 \leq 15, 5x_1 + 2x_2 \leq 10$

Enter the coefficients matrix : [3 5 1 0;5 2 0 1]
Enter the right hand side vector : [15;10]
Optimal Value is: 12.37
Optimal solution is:
x1=1.05
x2=2.37
x3=0.00
x4=0.00
>>

Program Code:

For *Maximize* $2x_1 + x_2$ *subject to,* $x_1 + 2x_2 \leq 10, x_1 + x_2 \leq 6, x_1 - x_2 \leq 2,$
$x_1 - 2x_2 \leq 1$

```
%ANINDYA NAG
%UG/02/BTCSE/2018/005
clc;
clearvars;
A = input('Enter the coefficients matrix :  ');
b = input('Enter the right hand side vector :  ');
opt_value=0;
[m,n] = size(A);
combo = nchoosek(1:n,m);
[m1 n1] = size(combo);
nbs = nchoosek(n,m);
for i = 1 :nbs
     x = zeros(n,1);
     bas_mat = [ ];
     for j = 1 :n1
        bas_mat = [bas_mat A(:,combo(i,j))];
     end
     bas_mat;
     y = gauss_elimination(bas_mat,b);
     if( max(y) ~= Inf )
          for j = 1 : n1
              x(combo(i,j),1) = y(j);
          end
          %disp('Basic solution is:  ');
          x;
         if( min(x) >= 0 )
              %disp(' This is a Basic Feasible solution.');
              z= 2*x(1,1) + x(2,1);
              if ( opt_value <= z)
                   opt_value = z;
                   opt_soln =x ;

              end
         end

     end
end
fprintf('Optimal Value is: %0.2f\n',opt_value)
```

```
fprintf('Optimal solution is: \n x1=%0.2f \n x2=%0.2f \n
x3=%0.2f \n x4=%0.2f
\n',opt_soln(1,1),opt_soln(2,1),opt_soln(3,1),opt_soln(4,1)
)
```

Output:

For $\boldsymbol{Maximize}\ \ 2x_1 + x_2 \quad \boldsymbol{subject\ to},\ \ x_1 + 2x_2 \le 10, x_1 + x_2 \le 6,$
$x_1 - \ x_2 \le 2,\ \ x_1 - 2x_2 \le 1$

Enter the coefficients matrix : [1 2 1 0 0;1 1 0 1 0;1 -2 0 0 1]
Enter the right hand side vector : [10;6;1]
Optimal Value is: 10.33
Optimal solution is:
x1=4.33
x2=1.67
x3=2.33
x4=0.00
>>