

Simplex Algorithm:

(Assuming the problem is in Standard form with maximization type objective function)

1. Start
2. Read the technological coefficients matrix $\mathbf{a} = [\mathbf{a}_{ij}]$, $i = 1, 2, \dots, m; j = 1, 2, \dots, n$
3. Read the right hand side vector $\mathbf{b} = [\mathbf{b}_i]$ for $i = 1, 2, \dots, m$
4. Read the cost vector $\mathbf{c} = [\mathbf{c}_j]$ for $j = 1, 2, \dots, n$
5. Initialize solution $x_1 = 0, x_2 = 0, \dots, x_n = 0$
6. Identify the basic variables in the initial basic feasible solution (\mathbf{x}_B) and corresponding cost coefficients (\mathbf{c}_B); Initialize $\mathbf{x}_B = \mathbf{b}$
7. Construct indicator vector whose elements are $(z_j - c_j)$ values, where $\mathbf{z}_j = \mathbf{c}_B \mathbf{a}_j - \mathbf{c}_j$, \mathbf{a}_j is the j -th column of \mathbf{a}
8. Construct the initial simplex tableau \mathbf{tab} ;
(Hint: you can write $\mathbf{a} = [\mathbf{a} \ \mathbf{b}; \text{indicator vector } \mathbf{c}_B \mathbf{x}_B]$)
9. If all $(z_j - c_j) \geq 0$ i.e., $\mathbf{tab}_{end,j} \geq \mathbf{0}$ and no artificial variable is in basis at a positive value then the solution optimal; Terminate the program
elseif, all $(z_j - c_j) \geq 0$ i.e., $\mathbf{tab}_{end,j} \geq \mathbf{0}$ and at least one artificial variable is in basis at a positive value then the problem is infeasible; Terminate the program
elseif, any $(z_j - c_j) < 0$ i.e., $\mathbf{tab}_{end,j} < \mathbf{0}$ then go to **Step 10**
10. Find the most negative value and its corresponding column index j' in indicator vector;
If all elements of \mathbf{tab}_j are ≤ 0 , then the problem is unbounded;
Otherwise go to **Step 11**
11. For each row find $\frac{\mathbf{x}_{Bi}}{\mathbf{tab}_{ij'}}$, ($\mathbf{tab}_{ij'} > \mathbf{0}$) and find minimum of them, indicate that row as i' ;
New basic variable is j' -th variable and departing variable is i' -th variable;
Key element is $\mathbf{tab}(i', j')$
Construct the new simplex tableau using the following steps:
 - a) Create $\mathbf{tab_temp} = \mathbf{tab}$
 - b) for new i' -th row: $\mathbf{tab}_{i'j} = \mathbf{tab_temp}_{i'j} / \mathbf{tab}_{i'j'}$
 - c) for new j' -th column: $\mathbf{tab}_{ij'} = \mathbf{0}$, for $i \neq i'$
 - d) for other positions of the table:

$$\mathbf{tab}_{ij} = \mathbf{tab_temp}_{ij} - \frac{(\mathbf{tab_temp}_{i'j})(\mathbf{tab_temp}_{ij'})}{\mathbf{tab}_{i'j'}}, i \neq i', j \neq j'$$

Return to Step 9