Experiment: 04

Date: 13.04.2022

Program Name:

1. Solve the following problem manually by simplex method:

$$Max: 3x_1 + 5x_2 + 4x_3$$
$$subject\ to, 2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0$$

$Ans: (x_1 = 89/41;\ x_2 = 50/41;\ x_3 = 62/41);\ max = 765/41$

2. Write MATLAB program for simplex method to solve a general linear programming problem:

$$Maximize\ c_1x_1 + c_2x_2 + \cdots + c_nx_n$$
$$subject\ to, a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

$where\ b_i \geq 0;\ i = 1,2,\dots,m;\ x_j \geq 0;\ j = 1,2,\dots,n;\ m \leq n$

Out put:

a) Optimal Solution, Optimal value
b) Solution is Unbounded (If $z_j - c_j \leq 0$ corresponding to some non-basic column and corresponding column element $x_{ij} \leq 0$ for all $i = 1,2,\dots,m$.)
c) Alternative optimal Solution (Alternate solution exists: (i) If there is an optimal basic feasible solution to a LPP and for some non-basic column $z_j - c_j = 0$ and corresponding $x_{ij} < 0$, for all i then **non-basic alternate optimum solution** will exist. (ii) If there is an optimal basic feasible solution to a LPP and for some non-basic column $z_j - c_j = 0$ and corresponding $x_{ij} > 0$ for at least one $i = 1,2,\dots,m$ then **basic alternate optimum solution** will exist.)

3. Using the program developed, find the solution of the following problems:

a) $max: 2x_1 + 4x_2 + x_3 + x_4$

$Subject\ to, x_1 + 3x_2 + x_4 \leq 4$

$2x_1 + x_2 \leq 3$

$x_2 + 4x_3 + x_4 \leq 3$

$x_j \geq 0, j = 1,2,3,4$

b) $min: -3x_1 - 4x_2$

$Subject\ to, x_1 - x_2 \leq 1$

$-x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

Simplex Algorithm:

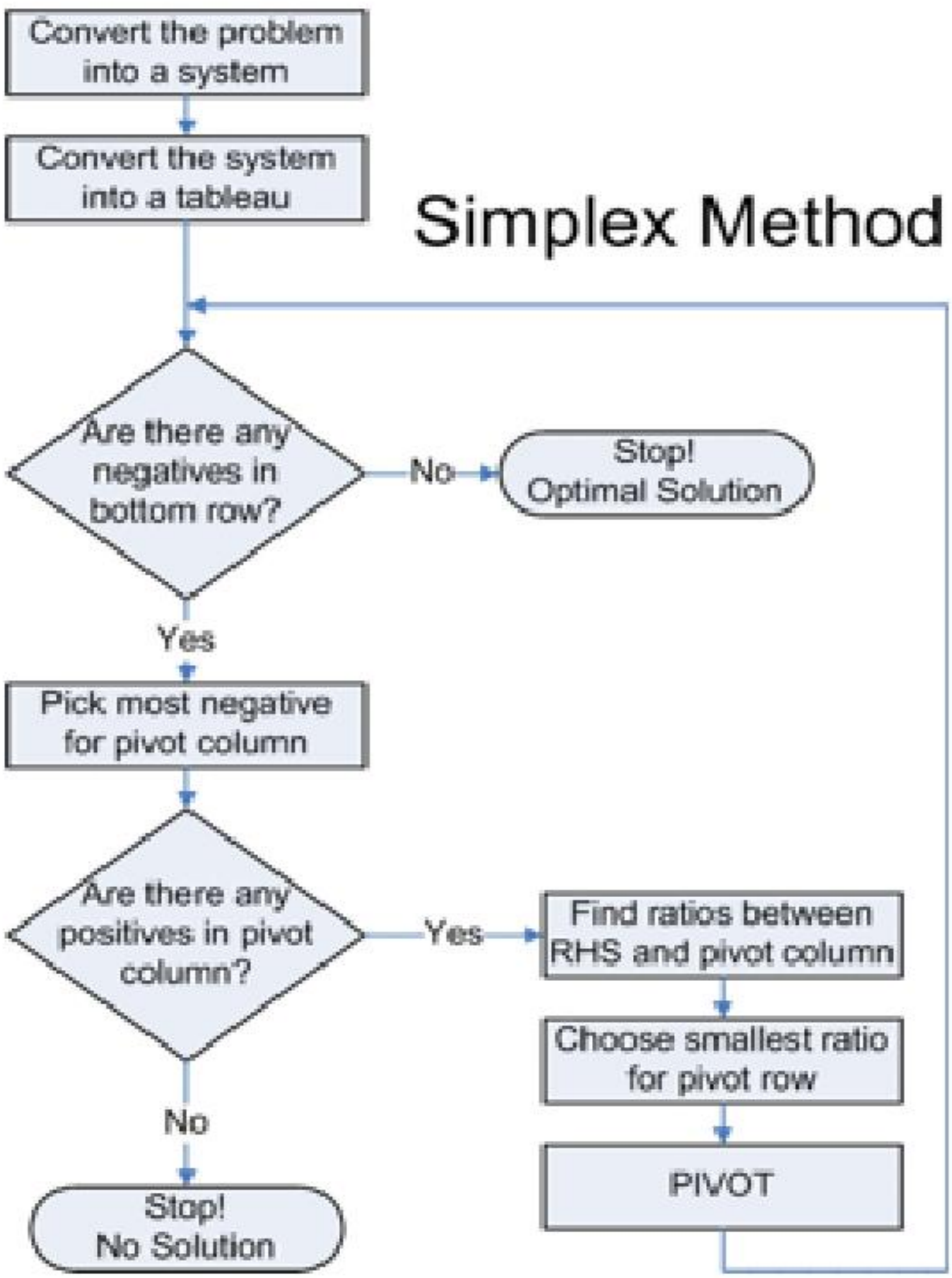
(Assuming the problem is in Standard form with maximization type objective function)

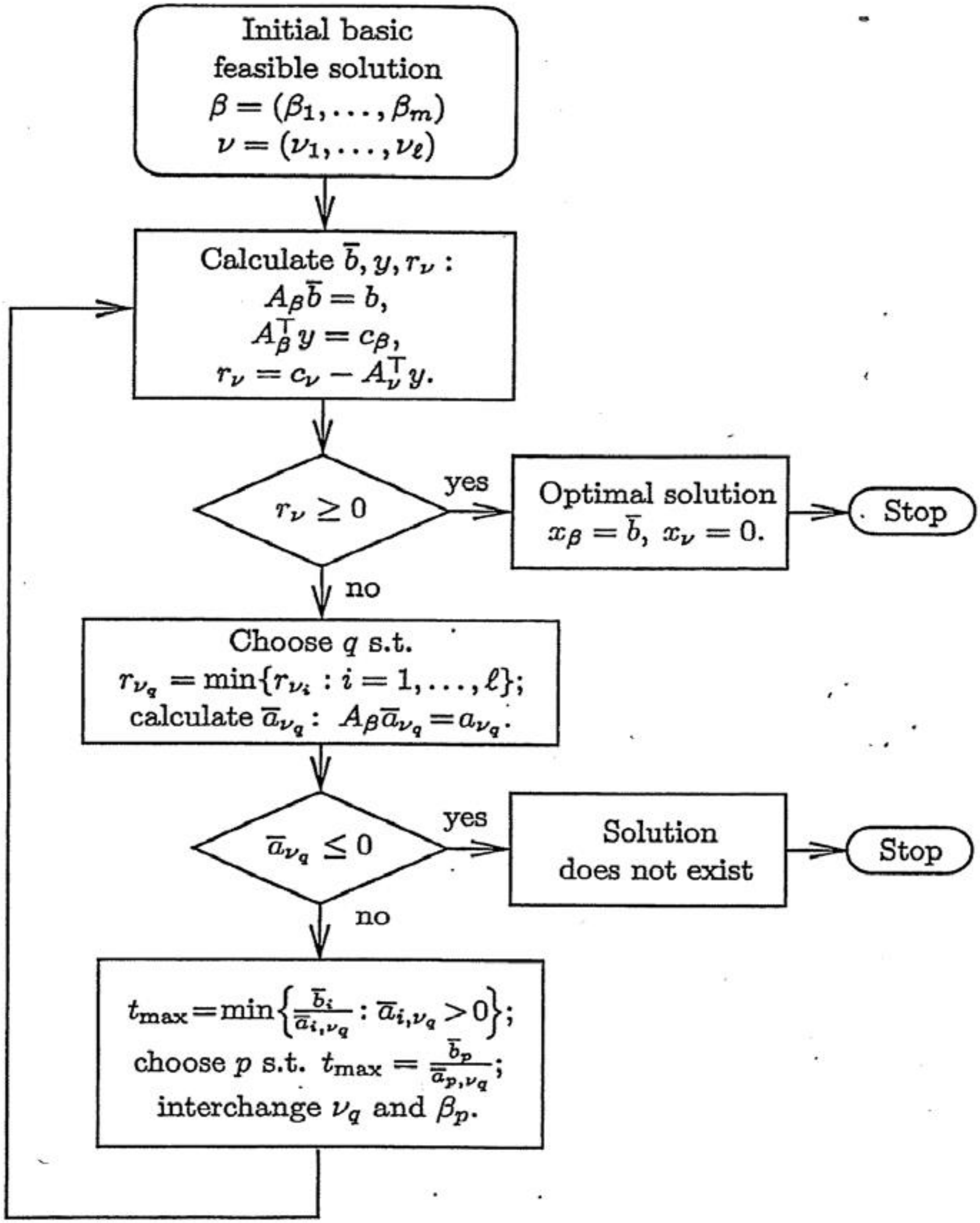
1. Start
2. Read the technological coefficients matrix $\boldsymbol{a} = [\boldsymbol{a_{ij}}], i = 1,2, \dots, m; j = 1,2, \dots, n$
3. Read the right hand side vector $\boldsymbol{b} = [\boldsymbol{b_i}]$ for $i = 1,2, \dots, m$
4. Read the cost vector $\boldsymbol{c} = [\boldsymbol{c_j}]$ for $j = 1,2, \dots, n$
5. Initialize solution $x_1 = 0, x_2 = 0, \dots, x_n = 0$
6. Identify the basic variables in the initial basic feasible solution $(\boldsymbol{x_B})$ and corresponding cost coefficients $(\boldsymbol{c_B})$; Initialize $\boldsymbol{x_B} = \boldsymbol{b}$
7. Construct indicator vector whose elements are $(z_j - c_j)$ values, where $\boldsymbol{z_j} = \boldsymbol{c_B a_j} - \boldsymbol{c_j}$, $\boldsymbol{a_j}$ is the j-th column of a
8. Construct the initial simplex tableau $\boldsymbol{tab}$;
   (Hint: you can write $a = [a\ b; indicator\ vector\ c_B x_B]$)
9. If all $(z_j - c_j) \geq 0$ i.e., $\boldsymbol{tab_{end,j}} \geq \boldsymbol{0}$ and no artificial variable is in basis at a positive value then the solution optimal; Terminate the program
   elseif, all $(z_j - c_j) \geq 0$ i.e., $\boldsymbol{tab_{end,j}} \geq \boldsymbol{0}$ and at least one artificial variable is in basis at a positive value then the problem is infeasible; Terminate the program
   elseif, any $(z_j - c_j) < 0$ i.e., $\boldsymbol{tab_{end,j}} < \boldsymbol{0}$ then go to **Step 10**
10. Find the most negative value and it's corresponding column index $\boldsymbol{j'}$in indicator vector;
    If all elements of tab_j are ≤ 0, then the problem is unbounded;
    Otherwise go to **Step 11**
11. For each row find $\frac{\boldsymbol{x_{Bi}}}{\boldsymbol{tab_{ij'}}}$, $(\boldsymbol{tab_{ij'}} > \boldsymbol{0})$ and find minimum of them, indicate that row as $\boldsymbol{i'}$;
    New basic variable is $\boldsymbol{j'}$-th variable and departing variable is $\boldsymbol{i'}$-th variable;
    Key element is $\boldsymbol{tab(i', j')}$
    Construct the new simplex tableau using the following steps:
    a) Create $\boldsymbol{tab\_temp} = \boldsymbol{tab}$
    b) for new i'-th row: $\boldsymbol{tab_{i'j}} = \boldsymbol{tab\_temp_{i'j}} / \boldsymbol{tab_{i'j'}}$
    c) for new j'-th column: $\boldsymbol{tab_{ij'}} = \boldsymbol{0}, \boldsymbol{for\ i \neq i'}$
    d) for other positions of the table:

$$\boldsymbol{tab_{ij}} = \boldsymbol{tab\_temp_{ij}} - \frac{(\boldsymbol{tab\_temp_{i'j}})(\boldsymbol{tab\_temp_{ij'}})}{\boldsymbol{tab_{i'j'}}}, \boldsymbol{i \neq i', j \neq j'}$$

Return to Step 9

Simplex Flowchart:




Initial basic feasible solution
$\beta = (\beta_1, \ldots, \beta_m)$
$\nu = (\nu_1, \ldots, \nu_\ell)$
Calculate $\overline{b}, y, r_\nu$:
$A_\beta \overline{b} = b,$
$A_\beta^\top y = c_\beta,$
$r_\nu = c_\nu - A_\nu^\top y.$
$r_\nu \geq 0$
yes
Optimal solution
$x_\beta = \overline{b},\ x_\nu = 0.$
Stop
no
Choose q s.t.
$r_{\nu_q} = \min\{r_{\nu_i} : i = 1, \ldots, \ell\};$
calculate $\overline{a}_{\nu_q}$: $A_\beta \overline{a}_{\nu_q} = a_{\nu_q}$.
$\overline{a}_{\nu_q} \leq 0$
yes
Solution does not exist
Stop
no
$t_{\max} = \min\left\{\frac{\overline{b}_i}{\overline{a}_{i,\nu_q}} : \overline{a}_{i,\nu_q} > 0\right\};$
choose p s.t. $t_{\max} = \frac{\overline{b}_p}{\overline{a}_{p,\nu_q}};$
interchange ν_q and β_p.

2. Program Code :(Write MATLAB program for simplex method to solve a general linear programming problem:)

$$\begin{aligned} &\text{Maximize } c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ &\text{subject to, } a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1 \\ &\qquad a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2 \\ &\qquad \dots\dots\dots\dots\dots\dots \\ &\qquad a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m \end{aligned}$$

$$\text{where } b_i \ge 0;\ i = 1,2,\dots,m;\ x_j \ge 0;\ j = 1,2,\dots,n;\ m \le n$$

Program Code:

```
%ANINDYA NAG
%UG/02/BTCSE/2018/005
clc;
clear all;
%This program is for solving problems on the simplex method
%where the problem is maximization type and all the constraints are less than equal (<= )
type.
m=input('How many constraints?\n');
n=input('How many variables?\n');
c_nb=input('Enter the cost vector: ');
for j=1:n
   %c_nb(j,1)=input('');
   xnb_ind(1,j)=j;
   x(j,1)=0;
end
b=input('Enter the right hand side vector: ');
for i=1:m
   %b(i,1)=input('');
   c_b(i,1)=0;
   xb_ind(i,1)=n+i;
   x(n+i,1)=0;
end
a=input('Enter the coefficient matrix: ');
% for i=1:m
%    for j=1:n
%        a(i,j)=input('');
%    end
% end
x=zeros(n+m,1);
for j=1:n
   indicator(1,j)=c_b'*a(:,j)-c_nb(j,1);
```

```
end
[ind,KC]=min(indicator);
tab_old=[a b; indicator c_b'*b];
while(ind < 0)
for i=1:m
   if (tab_old(i,KC)>0)
      ratio(i,1)=b(i,1)/tab_old(i,KC);
   else
      ratio(i,1)=10^6;
   end
end
[min_ratio,KR]=min(ratio);
temp=xb_ind(KR,1);
xb_ind(KR,1)=xnb_ind(1,KC);
xnb_ind(1,KC)=temp;
if (min_ratio==10^6)
   disp('Problem is unbounded');
else
   KE=tab_old(KR,KC);
   for i=1:m+1
      for j=1:n+1
         if (i==KR && j==KC)
            tab_new(i,j)=1/KE;
         elseif(i==KR)
            tab_new(i,j)=tab_old(i,j)/KE;
         elseif(j==KC)
            tab_new(i,j)=-tab_old(i,j)/KE;
         else
           tab_new(i,j)=(KE*tab_old(i,j)-tab_old(KR,j)*tab_old(i,KC))/KE;
         end
      end
   end
end
for i=1:m
   x(xb_ind(i,1),1)=tab_new(i,n+1);
end
for j=1:n
   x(xnb_ind(1,j),1)=tab_new(m+1,j);
end
tab_new;
indicator=tab_new(m+1,:);
[ind,KC]=min(indicator);
tab_old = tab_new;
end
x
```

Output :
For Question 1 :

1. Solve the following problem manually by simplex method:

$$Max: 3x_1 + 5x_2 + 4x_3$$
$$subject\ to, 2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0$$

$Ans: (x_1 = 89/41;\ x_2 = 50/41;\ x_3 = 62/41);\ max = 765/41$

Output:

```
How many constraints?
3
How many variables?
3
Enter the cost vector: [3;5;4]
Enter the right hand side vector: [8;10;15]
Enter the coefficient matrix: [2 3 0;0 2 5;3 2 4]

x =

    2.1707
    1.2195
    1.5122
    1.0976
    0.5854
    0.2683

>>
```

Conclusion :
We saw that, the solution we obtained by calculating manually by Simplex method and the solution we obtained by this program is the same.
And
The optimal solution:

$$x_1 = {}^{89}/_{41} = 2.1707$$
$$x_2 = {}^{50}/_{41} = 1.2195$$
$$x_3 = {}^{62}/_{41} = 1.5122$$

$$Z_{max} = 3x_1 + 5x_2 + 4x_3 = 3\left({}^{89}/_{41}\right) + 5\left({}^{50}/_{41}\right) + 4\left({}^{62}/_{41}\right) = {}^{765}/_{41}$$
$$= (3 \times 2.1707) + (5 \times 1.2195) + (4 \times 1.5122) = 18.6586$$

<u>3.(Using the program developed ,find the solution os the following problems:)</u>

<u>Output:</u>

a) For,

$$\begin{aligned} &max: 2x_1 + 4x_2 + x_3 + x_4 \\ &Subject\ to, x_1 + 3x_2 + x_4 \leq 4 \\ &\qquad\qquad 2x_1 + x_2 \leq 3 \\ &\qquad\qquad x_2 + 4x_3 + x_4 \leq 3 \\ &\qquad\qquad x_j \geq 0, j = 1,2,3,4 \end{aligned}$$

<u>Output:</u>

How many constraints?

3

How many variables?

4

Enter the cost vector: [2;4;1;1]

Enter the right hand side vector: [4;3;3]

Enter the coefficient matrix: [1 3 0 1;2 1 0 0;0 1 4 1]

x =

1.0000
1.0000
0.5000
0.3500
1.1000
0.4500
0.2500

>>

<u>The optimal solution:</u>

$$\begin{aligned} x_1 &= 1.0000 \\ x_2 &= 1.0000 \\ x_3 &= 0.5000 \\ x_4 &= 0.3500 \\ Z_{max} &= 2x_1 + 4x_2 + x_3 + x_4 \\ &= (2 \times 1.0000) + (4 \times 1.0000) + 0.5000 + 0.3500 \\ &= 6.85 \end{aligned}$$

Output:
b) For,

$$\begin{aligned} &min: -3x_1 - 4x_2 \\ &Subject\ to,\ x_1 - x_2 \leq 1 \\ &\qquad -x_1 + x_2 \leq 2 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Output:

How many constraints?

2

How many variables?

2

Enter the cost vector: [3;4]

Enter the right hand side vector: [1;2]

Enter the coefficient matrix: [1 -1;-1 1]

Problem is unbounded.