Experiment: 07

Date: 27.04.2022

Title: Assignment Problem Solution with Excel Solver.

Question:

1. A toy set is complete by packing five small toys. Each toy is packed by a worker on the packing line. So, five workers W1, W2, W3, W4 and W5 are required to complete the set. Each worker can pack any of the small toys (T1, T2, T3, T4 and T5). Cost of packing and revenue earned from an entire lot of toy sets with regard to each worker is given. Make an optimal assignment so as to maximize profit.

	Revenue					Cost				
	T1	T2	T3	T4	T5	T1	T2	T3	T4	T5
W1	50	70	80	60	75	30	50	30	35	45
W2	80	50	60	40	65	40	25	30	20	35
W3	85	55	70	65	75	40	30	35	30	40
W4	50	65	80	80	70	25	40	40	35	35
W5	55	67	82	70	81	25	30	41	38	42

Note that, Profit = Revenue - Cost

2. Construct an Excel Solver model and determine the optimum solution.
   [Mention each step involved in the model building]

Answer:

1. Formulate the associated Assignment Problem:

By definition, **Profit = Revenue – Cost.** Hence the profit matrix corresponding to the problem is given by:

To / From	Profit				
	T1	**T2**	**T3**	**T4**	**T5**
W1	**20**	**20**	**50**	**25**	**30**
W2	**40**	**25**	**30**	**20**	**30**
W3	**45**	**25**	**35**	**35**	**35**
W4	**25**	**25**	**40**	**45**	**35**
W5	**30**	**37**	**41**	**32**	**39**

Mathematical Model of an Assignment Problem:

Let $x_{ij} = \begin{cases} \mathbf{0}\ , \text{ if worker } i \text{ is not assigned to pack toy } j \\ \mathbf{1}, \quad \text{ if worker } i \text{ is assigned to pack toy } j \end{cases}$

C_{ij} = **the profit obtained if worker i pack toy j**

Then the mathematical model for an Assignment Problem is given by:

Find x_{ij} **so as to,**

$$Maximize\ \ Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$$

Subject To,

$$\sum_{j=1}^{n} x_{ij} = 1 \quad , \qquad i = 1, 2, 3, \dots\dots\dots\dots\dots, n \qquad (Supply\ Constraint)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad , \qquad j = 1, 2, 3, \dots\dots\dots\dots\dots, n \qquad (Demand\ Constraint)$$

$$x_{ij} \in \{0, 1\} \ , \ \forall \ \ i, j$$

2. Solution of Assignment Problem using Excel Solver:

In Excel Solver, the spreadsheet is the input and output medium for the Assignment problem. We use the following steps to formulate the following model in Excel:

Assignment model:

To / From	Profit				
	T1	T2	T3	T4	T5
W1	20	20	50	25	30
W2	40	25	30	20	30
W3	45	25	35	35	35
W4	25	25	40	45	35
W5	30	37	41	32	39

Step 1:

- **In the first step, create the corresponding cost matrix in an Excel spreadsheet. The table should contain workers, toys, and profit per unit of packaging toy j by worker i, the total worker and total toys. This is shown in the Excel matrix indicated by cells I3:P10.**

Step 2:

- **Check whether the problem is balanced or not by comparing the total worker and the total toys. If total worker is more than the total toys then add a dummy worker (i.e., a row in the cost matrix) with the profit values as zero. Similarly, if the total toys is more than the total worker then add a dummy toy (i.e., a column in the cost matrix) with the cost values as zero.**

Step 3:

- **At the bottom of this Assignment matrix, a linear model is constructed with decision variables (Cell: J15:N19), objective function (Cell: J23) and both supply and demand constraints (Cell: P15:P19 and J21:N21, resp.). This is shown in cells I14:P23.**

Step 4:

- **Objective function is to maximization the total Profit. A formula for computing the objective function value is created in a cell. Cell J23 would give the final value of maximize profit.**
  For its calculation, a formula has to be created one can use SUMPRODUCT function available in the excel spreadsheet as the following:
  Cell J23: =SUMPRODUCT(J15:N19,J4:N8)

Step 5:

- **Formula for computing the total toy packaging job for different workers are formulated in Cell P15:P19. The formulas are as follows:**
  Cell P15:=SUM(J15:N15),
  Cell P16:=SUM(J16:N16),
  Cell P17:=SUM(J17:N17),
  Cell P18:=SUM(J18:N18),
  Cell P19:=SUM(J19:N19)

Step 6:

- Formula for computing the total number of worker assigned for packaging different toys are formulated in **Cell J21:N21**. The formulas are as follows:
  Cell J21:=SUM(J15:J19),
  Cell K21:=SUM(K15:K19),
  Cell L21:=SUM(L15:L19),
  Cell M21:=SUM(M15:M19),
  Cell N21:=SUM(N15:N19)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1				Assingment Problem								Problem Data					
2				Problem Data								Profit					
3					Reveneue					T1	T2	T3	T4	T5		Supply	
4			T1	T2	T3	T4	T5		W1	20	20	50	25	30		1	
5		W1	50	70	80	60	75		W2	40	25	30	20	30		1	
6		W2	80	50	60	40	65		W3	45	25	35	35	35		1	
7		W3	85	55	70	65	75		W4	25	25	40	45	35		1	
8		W4	50	65	80	80	70		W5	30	37	41	32	39		1	
9		W5	55	67	82	70	81										
10									Demand	1	1	1	1	1			
11				Problem Data													
12					Cost							Solution					
13			T1	T2	T3	T4	T5					Profit					
14		W1	30	50	30	35	45			T1	T2	T3	T4	T5		Supply	
15		W2	40	25	30	20	35		W1							0	
16		W3	40	30	35	30	40		W2							0	
17		W4	25	40	40	35	35		W3							0	
18		W5	25	30	41	38	42		W4							0	
19									W5							0	
20				Profit = Revenue -Cost													
21									Demand	0	0	0	0	0			
22																	
23									Objective	0							
24																	

In Microsoft Excel, after entering the entire Assignment Problem data in the worksheet, the following steps would lead to a solution:

Step 1:

➢ Select **Data** menu in the toolbar.

Step 2:

➢ In Data menu, select **Solver** application.

Step 3:

➢ Open Solver application. In Solver parameters dialog box Enter **$J$23** in set target cell. Select purpose of **max** (depending on the type of the objective). Enter **$J$15:$N$19** in by changing cell box. To enter constraint equations, click on **Add** button.

Step 4:

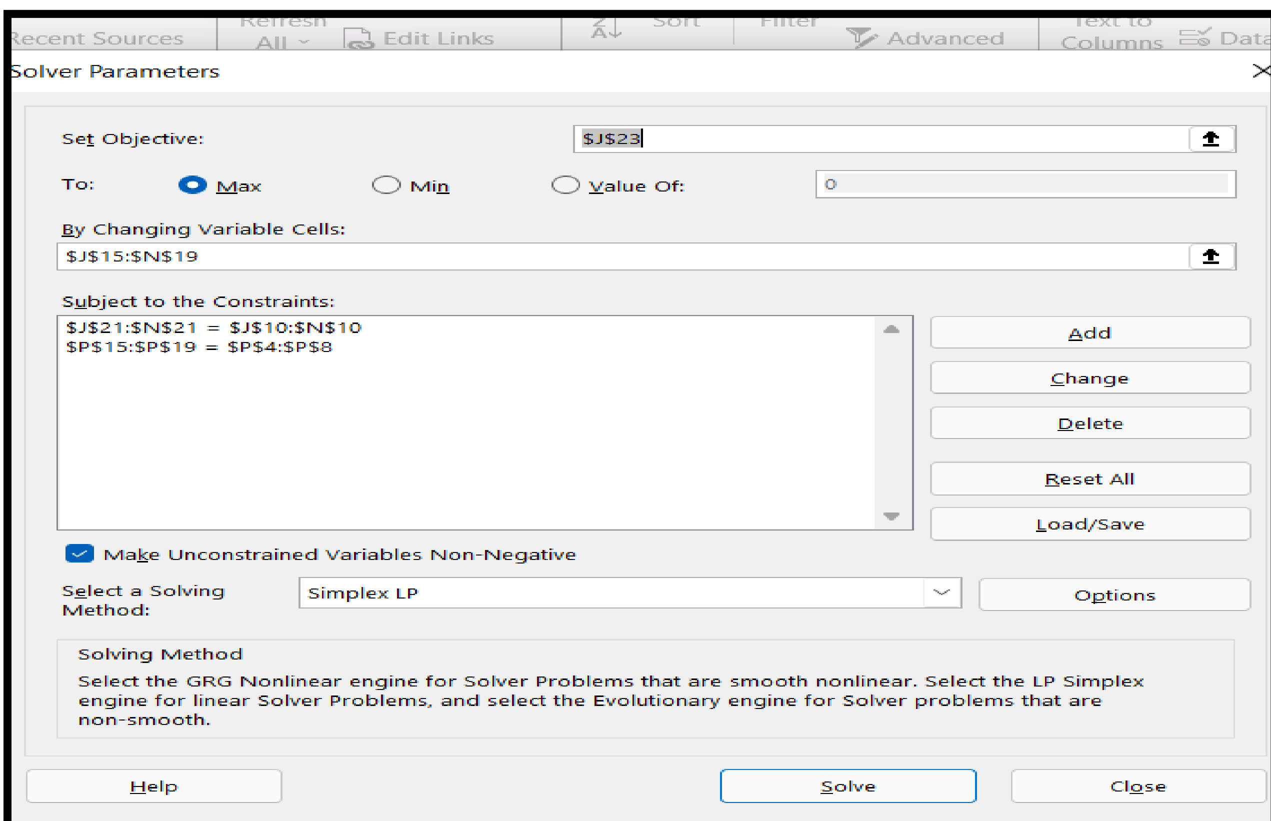
➢ When the **Add** constraint dialog box opens, it would have three boxes: first, cell reference; second, inequalities of $\leq$ and lastly, constraint box. For supply constraints, enter **$P$15:$P$19** in cell reference box; enter inequality of $=$ and **$P$4:$P$8** in the constraint box. Then click on Add to add demand constraints in the similar process. After entering all constraints, click **OK**.

Step 5:

➢ Now choose Options. Select Assume Non-Negative and Assume Linear Model (in **MS Excel 2007**). In **Excel 2010** onwards, select Make Unconstraint Variables Non-negative and select Simplex LP from the dropdown menu of Select a Solving Method. Click **OK**.

Step 6:

➢ When Solver parameters dialog box appear, click on **Solve**.



Step 7.:

➢ Finally, when final solution appears on the worksheet, select Keep Solver Solution and click **OK**.

Output:

Assignment Problem

Problem Data

Reveneue

	T1	T2	T3	T4	T5
W1	50	70	80	60	75
W2	80	50	60	40	65
W3	85	55	70	65	75
W4	50	65	80	80	70
W5	55	67	82	70	81

Problem Data

Cost

	T1	T2	T3	T4	T5
W1	30	50	30	35	45
W2	40	25	30	20	35
W3	40	30	35	30	40
W4	25	40	40	35	35
W5	25	30	41	38	42

Profit = Revenue -Cost

Problem Data

Profit

	T1	T2	T3	T4	T5	Supply
W1	20	20	50	25	30	1
W2	40	25	30	20	30	1
W3	45	25	35	35	35	1
W4	25	25	40	45	35	1
W5	30	37	41	32	39	1
Demand	1	1	1	1	1	

Solution

Profit

	T1	T2	T3	T4	T5	Supply
W1	0	0	1	0	0	1
W2	0	0	0	0	1	1
W3	1	0	0	0	0	1
W4	0	0	0	1	0	1
W5	0	1	0	0	0	1
Demand	1	1	1	1	1	

Objective 207

Conclusion:

The obtained optimal assignment of the assignment problem with the maximum profit is given by:

$$\boldsymbol{W_1 \rightarrow T_3}$$
$$\boldsymbol{W_2 \rightarrow T_5}$$
$$\boldsymbol{W_3 \rightarrow T_1}$$
$$\boldsymbol{W_4 \rightarrow T_4}$$
$$\boldsymbol{W_5 \rightarrow T_2}$$

$$\textbf{Maximum Profit } = \mathbf{50 + 30 + 45 + 45 + 37 = 207}$$