LPP formulation of a Two Person Zero-Sum Game:

Let us consider a two-person zero-sum game with payers A and B. Let A has m strategies $A_1, A_2, \dots, A_m$ and B has n strategies $B_1, B_2, \dots, B_n$. We assume that player A is the gainer and the player B is the looser. Then the corresponding pay-off matrix is given by $P = [a_{ij}]_{m\times n}$, where a_{ij} represent the amount of gain by player A if A use A_i strategy and player B use B_j strategy.

Now, let us assume that the probability of using A_i strategy by player A is p_i and the probability of using B_j strategy by player B is q_j. Then the linear programming problem corresponding to the game is given by:

For player A:

$$\text{min:} \quad \frac{1}{v} = X_1 + X_2 + \cdots + X_m$$

$$subject\ to\ \ a_{1j}X_1 + a_{2j}X_2 + \cdots + a_{mj}X_m \geq 1, \ \ j = 1,2,\dots,n$$

$$X_i = \frac{p_i}{v} \geq 0, i = 1,2,\dots,m$$

where v is the value of the game.

For player B:

$$\text{max:} \quad \frac{1}{v} = Y_1 + Y_2 + \cdots + Y_n$$

$$subject\ to\ \ a_{i1}Y_1 + a_{i2}Y_2 + \cdots + a_{in}Y_n \leq 1, \ \ i = 1,2,\dots,m$$

$$Y_j = \frac{q_j}{v} \geq 0, j = 1,2,\dots,n$$

Note: To formulate the LPP corresponding to a game, we convert the given pay-off matrix as a positive pay-off matrix where all the pay-off values are positive. To construct this matrix, we add the numeric value of the most negative element of the given matrix plus one to all the elements of the given matrix.

LP model of first problem from Exp-8:

Given pay-off matrix of the game is

		DU			
		DU1	DU2	DU3	DU4
UA	UA1	3	-2	1	4
	UA2	2	3	-5	0
	UA3	-1	2	-2	2
	UA4	-3	-5	4	1

We construct the positive pay-off matrix as:

		DU			
		DU1	DU2	DU3	DU4
UA	UA1	9	4	7	10
	UA2	8	9	1	6
	UA3	5	8	4	8
	UA4	3	1	10	7

LP model is given by:

For player B:

$$\max \frac{1}{v} = Y_1 + Y_2 + Y_3 + Y_4$$

$$\begin{aligned} \textit{Subject to}, \; & 9Y_1 + 4Y_2 + 7Y_3 + 10Y_4 \leq 1 \\ & 8Y_1 + 9Y_2 + Y_3 + 6Y_4 \leq 1 \\ & 5Y_1 + 8Y_2 + 4Y_3 + 8Y_4 \leq 1 \\ & 3Y_1 + Y_2 + 10Y_3 + 7Y_4 \leq 1 \\ & Y_j \geq 0, j = 1,2,3,4 \end{aligned}$$

Where value of the game is v and the optimal strategy for player B is (vY_1, vY_2, vY_3, vY_4).