Q1)

$$y = \frac{1}{x^2}$$

$x_1 = 2, \quad x_2 = 100$

$\therefore \quad \Delta x = (x_2 - x_1) = (100 - 2) = 98$

$\therefore \quad \dfrac{\Delta y}{\Delta x} = \dfrac{dy}{dx} \quad$ or, $\quad \Delta y = \dfrac{dy}{dx}(\Delta x)$

where $\quad y = \dfrac{1}{x^2}$

$\therefore \quad \Delta y = \dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)\Delta x \quad$ or $\quad \left(-2 \times \dfrac{1}{x^3}\right)\Delta x$

where $\Delta x = 98, \quad x = 2.$

or, $\quad \Delta y = \left(-2 \times \dfrac{1}{8} \times 98\right) = -24.5$

$\therefore \quad \boxed{\Delta y = -24.5}$

Q2) let the currency notes = Rupee (Rs)

$\therefore$ Total money = 590 Rs. — (i).

Ratio of 50 Rs, 20 Rs = 3 : 5

$\therefore$ Number of 50 Rs notes = $3x$

Number of 20 Rs notes = $5x$

Total number of notes = 25

$\therefore$ Number of 10 Rs notes = $25 - (3x + 5x) = (25 - 8x)$

$\therefore$ Total amount of 50 Rs = $(50 \times 3x) = 150x$ — (ii)

Total amount of 20 Rs = $(20 \times 5x) = 100x$ — (iii)

Total amount of 10 Rs = $10\ (25 - 8x) = 250 - 80x$ — (iv)

$\therefore$ From (i), (ii), (iii), (iv), we get

$\therefore$ $150x + 100x + 250 - 80x = 590$ or $x = 2$

$\therefore$ No: of 50 Rs notes = $3(2) = 6$, 20 Rs notes = 10, 10 Rs notes = 9

Q3) let the number of winners $= x$

Total participants $= 63$

$\therefore$ Total no. of participant did not won $= 63 - x$

Total prize money $=$ 3000 Rs.

winners gets Rs 100 and participants who did not win gets Rs 25.

$$\therefore \quad 100x + 25(63 - x) = 3000.$$

$$\text{or,} \quad 75x + 1575 = 3000$$

$$\text{or,} \quad x = 19$$

$\therefore$ The number of winners $= 19$.

Q 4. let the multiple be x

$\therefore$ The three consicutive multiples of 11 are

$11x$, $11(x+1)$, $11(x+2)$

Total sum $= 363$

$\therefore$ $11x + 11(x+1) + 11(x+2) = 363$

or, $33x + 33 = 363$

or, $x = \frac{\cancel{330}\ 10}{\cancel{33}} = 10$

$\therefore$ Theree Consicutive multiples are:

$\boxed{110,\ 121,\ 132}$

Q5) let the greater digit $= y$ and smaller $= x$

$\therefore$ Number formed $= (x + 10y)$ — (i)

difference between the number $= 3$

$\therefore$ $y - x = 3$ — (ii)

also (original) + (interchanged digit num) $= 143$

$\therefore$ $(x + 10y) + (y + 10x) = 143$

or, $x + y = 13$ — (iii)

From (ii), (iii) we get $2y = 16$ or $y = 8$, $x = 5$

$\therefore$ Number $= 85$ $\therefore$ Similarly taking the vice versa of the logic, the potential number could also be 58

6)

$$5x - 2(2x - 7) = 2(3x - 1) + \frac{7}{2}$$

or, $$(x + 14) = 6x - 2 + \frac{7}{2}$$

or, $$5x = 16 - \frac{7}{2}$$

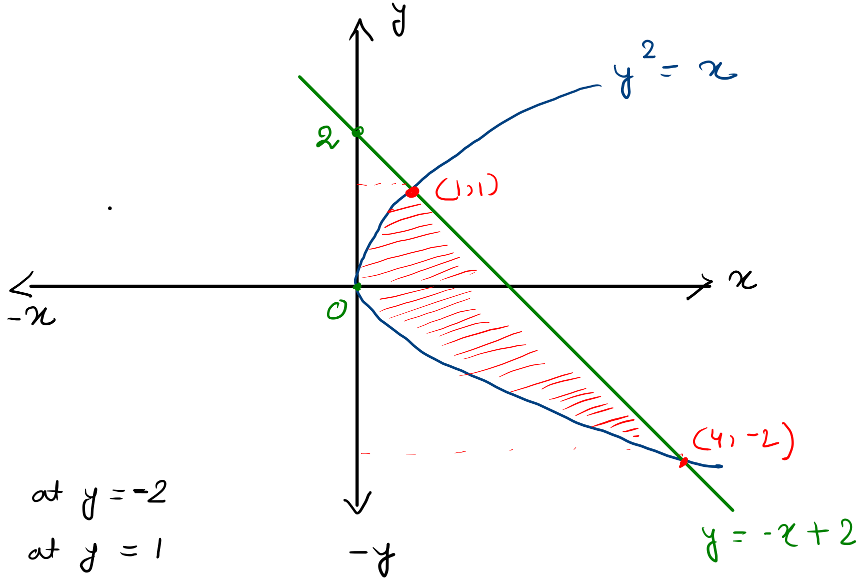
or, $$5x = \frac{32 - 7}{2} = \frac{25}{2}$$

or, $$x = \frac{5}{2}$$

7) Given: $y^2 = x$ —(i)

$y = -x + 2$ —(ii)

$\therefore$ (i) + (ii), we get



$$y^2 + y - 2 = 0$$

or, $y = \dfrac{-1 \pm \sqrt{1+8}}{2}$

$\therefore$ $\boxed{y = -2, 1}$ $\therefore$ $x = 4$ at $y = -2$

$x = 1$ at $y = 1$

$\therefore$ Required area

$$A = \int_{-2}^{1} \left((2-y) - y^2\right) dy = \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$$

or, $A = \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - \frac{4}{2} + \frac{8}{3}\right) = \frac{9}{2}$ sq. unit.

8) From statement 1, we get

$$10x + 5y + 4z = 212 \quad \text{--- (i)}$$

From statemet 2, we get

$$20x + 30y + 40z = 920 \quad \text{--- (ii)}$$

From statement 3, we get

$$40x + 10y + 70z = 810 \quad \text{--- (iii)}$$

$\therefore$ We have 3 equations and 3 variable, so we can solve this system of linear equation as:

$$AX = B$$

where $A = \begin{pmatrix} 10 & 5 & 4 \\ 20 & 30 & 40 \\ 40 & 10 & 70 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $B = \begin{pmatrix} 212 \\ 920 \\ 810 \end{pmatrix}$

$\therefore$ Solving this through row-echlon way, we get:

$$\boxed{x = 10, \quad y = 20, \quad z = 3}$$

To proof: $\tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right) = \left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$

L.H.S

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{1-2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$\because \cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}\right] = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\cancel{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^{\cancel{2}}}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right] = \tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$$

(dividing the numerator and denominator by $\cos\frac{x}{2}$)

$$\therefore \tan^{-1}\left[\frac{\tan\pi/4+\tan\pi/4\tan\frac{x}{2}}{1-\tan\pi/4\tan x/2}\right] = \tan^{-1}\left[\tan\left(\pi/4+\frac{x}{2}\right)\right] = \left(\pi/4+\frac{x}{2}\right) = R.H.S$$

$\because \tan(x+y) = \dfrac{\tan x+\tan y}{1-\tan x\tan y}$