Relinking of Social Ties in a Quarantined Network*

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Abstract. The Covid-19 Pandemic has essentially put a stop to the whole world. In a time where human interaction can be potentially lethal, social ties have been disrupted like never before. With countries under lockdown, most people have been quarantined at their homes and their only connection with the outside world is through technology and social media.
Under such trying times, university students throughout the world have undergone suffering in not only their studies but also their support networks. In this report, we will analyze the friendship network of a Liberal Arts College and try to create a pseudo network based on their social ties in quarantine. After getting survey results of 300 students from Habib University and linking the individuals based on whether they have turned to the same coping mechanism in such crisis or not, we try to find out whether small world characteristics can be visualized in our pseudo network or not.
We start off by summarising the four famous small world networks i.e. Erdos-Renyi, Watt Strogatz, Jon Kleinberg and Barbarasi-Albert models. We then move on to show our findings from the survey and build pesuodo networks to study the small world effect in them. We will explore how the social ties have formed in the support network of the students due to the mandatory quarantine caused by the Covid-19 Pandemic. We will then go on to study the mediums through which the changes, if any, have occurred.

Keywords: Covid 19 · Support Network · Friendship Network · Quarantine · Social Media During Quarantine

1 Introduction

Human beings are believed to be joined in a social network that exhibits "small world" phenomenon. This property of networks represents the nodes being joined in such a way that the distance between any two nodes is 'small' or, according to another definition, less than six degrees. The small world effect signifies the abundance of short paths in a graph. Researchers believe that networks built on social interactions between humans exhibit small world properties. While the physical interaction between humans decreased excessively due to the outbreak

* Supported by organization x.

of the novel Corona Virus in 2020, individuals re-linked their emotional support network by interacting with new people in virtual spaces

1.1 Erdos-Renyi Model

Erdos-Renyi Network is named after two famous Hungarian mathematicians who published a series of eight papers between 1959 and 1968 in which they established a new branch of mathematics known as random graph theory. Erdos-Renyi graph is a random graph in which there are 'n' number of nodes and we place an edge between all the pairs of nodes based on the probability 'p'. If p=1, we obtain an N-regular lattice in which every node is connected to all other nodes. If p=0, we obtain N isolates not connected to any other nodes. The value of this probability 'p', which usually lies between 0 and 1, is the main deciding factor of the shape of the graph. One of the key characteristics of ER Networks is that neither outliers nor a hub is expected. The chances of clusters forming, high density of links in a particular node's immediate neighborhood, is fairly low in ER Networks. Though if the graph is large, a small value of 'p' suffices for the graph to have a giant component i.e. there exists a path that connects most of the pair of nodes to one another. The path length of the graph grows logarithmically to the increase in the number of nodes hence maintaining a small value of the average path length of the graph

1.2 Watts-Strogatz Model

The Watts-Strogatz Model was proposed in 1998 by Duncan J. Watts and Steven Strogatz. This model is based on the random graph model and exhibits small-world properties. It differentiates itself from the ER model by accounting for clustering and keeping a short avg path length.

This model starts off with a Regular Ring Lattice. P is the probability of an edge randomly re-linking. When P=1, all the edges are re-linked and the graph will resemble a random graph with low clustering and a short avg path length. The W-S model re-links only a few edges and this effectively shortens the average path compared the longer path length in a regular lattice. Subsequently, a high clustering is maintained as the number of re-linked edges is kept low. This also leads to a high variance in the degree distribution [6].

1.3 Jon Kleinberg

Kleinberg built his model on the work of Watts and Strogatz. According to him, short paths not only exist but with limited knowledge of the global network, we can find them. Kleinberg's basic model contains a 2-d grid as its base. It has long ranged random links add between any of the nodes u and v with a probability of $d^{-2}(u,v)$, which is the inverse square of the lattice between u and v. In this model, for each node, there exists an undirected local link to four of its grid neighbours and a single random long ranged link. In this setting, according to Kleinberg, a simple greedy algorithm with only local information can find routes between any source and destination using only $O(log^2 n)$ [2].

1.4 Barabasi-Albert

Barabasi Albert is a minimal model that can generate scale free networks by recognizing that growth and preferential attachments that coexist in real networks. It is also called as BA or scale free model. Unlike the random networks, Barabasi-Albert model is a real world network. The growth element refers to the network having continuously increasing number of nodes rather than fixed nodes. The preferential attachment property refers to a new node joining links with already popular nodes. The more the number of nodes attached to a node, the greater the probability of a new node forming link with that node. This implies that if a new node had to choose between a 2 degree or a 4 degree node, it is twice as likely to choose the 4 degree one. This will result in the formation of hubs, as the more connected nodes are more likely to get edges from the newly added nodes. We can also say that the formation of hubs are the result of rich gets richer phenomenon. The average path length of the graph grows exponentially to the number of nodes hence maintaining the small world property of short average path length. [3].

2 Hypothesis

For this research, we are going to have a set of hypothesis. Our main hypothesis is that any two individuals sharing an edge between them, irrespective of the weight, have a chance of interacting with each other in the virtual space. The more the weight of the edge between them, the higher their chance of meeting in the virtual space. Our second hypothesis, which is a set of hypothesis, looks upon the three tiers of graphs and expect them to follow small world properties. We also hypothesize that as we move from each tier to the next, we would end up finding lesser neighbors connected to each individual.

3 Methodology

To assess the re-linking of social ties in quarantine and to study the support network that individuals have created, we developed a questionnaire aimed at students of Habib University.

According to the answers of the questionnaire, each node (i.e student) is then linked to another node with a weighted edge. The weight of the edges are determined according to the number of similar answers that they have given. From this data, we will build a pseudo network and then several sub-networks.

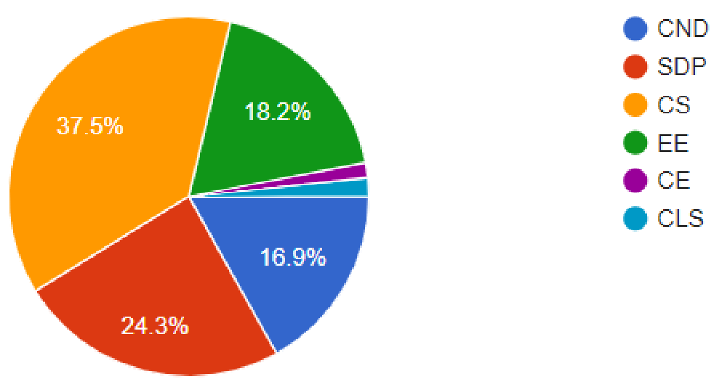
The tiers will be divided with respect to the weights of the edges. In our first tier, we study all the edges with weight equal to or above 3. In the second tier, we only create a network containing edges of weights equal to or above 6. And in the final tier, we study the network that is formed by edges having weight equal to 9, i.e. all the replies of the two individuals forming the edge are same.

After having built multiple tiers of pseudo network, we visualize each layer and study their properties trying to evaluate whether their properties fit the criteria of small world or not. So for each network, we calculate its average path length and global clustering coefficient.

4 Analyses and Findings

4.1 Network Data

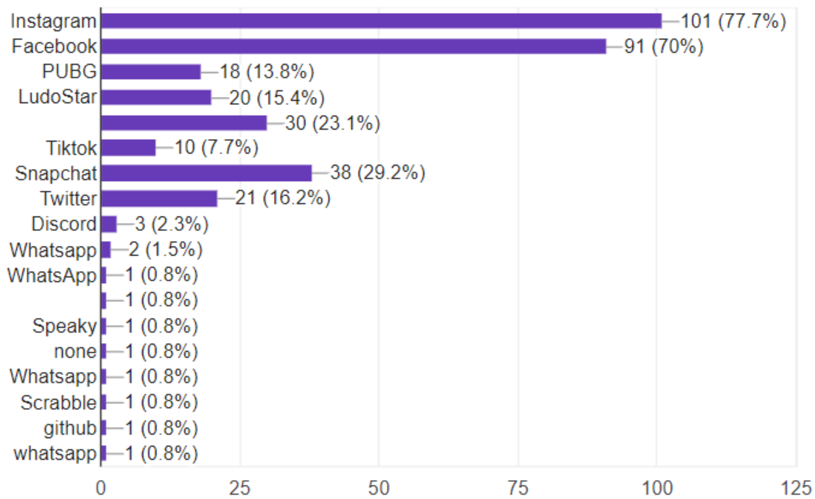
Of the 300 responses we collected, 42.4% were men and 56.9% were women. The rest preferred not to tell. We received a very diverse response in terms of the students' majors.



As seen in the chart above, the majority of our respondents were from the Dhanani School of Science and Engineering with the majors CS, EE and CE. The rest were from the School of Arts, Humanities and Social Sciences with 24.3% majoring in SDP. So it is safe to say that all of the majors offered at Habib were represented.

In order to link the students according to their quarantine habits, we had to see how they're mental health and social life had been affected. 81.2% respondents admitted that their mental health suffered during the quarantine, however only 11.7% took online therapy to cope with it.

32.3% of our respondents admitted to have reconsidered their current friends and 24.1% of our respondents had lost friends since the lockdown started. For the friends they were still in touch with, 63.7% of our respondents relied heavily on social media to stay in touch with them during this difficult time. 30.7% of our participants actively tried to interact with new people and make new friends. Following are the applications used most by our respondents in order to meet new people;



Furthermore, 28% of the respondents did end up making new friends during quarantine from their own institution.

4.2 Mapping the Data

In order to map this data on a network, we used a combination of python libraries i.e Pandas, Matplotlib, Seaborn and Networkx.

We create an edge list as a list of 3-tuples (x,y,z) where x and y represents the two individuals under observation and z represents the weight. The tuples are going to be symmetric since our graph is an undirected one.

We start off by computing the weight for all the 44,850(300C2) combination of nodes. For each node x, we iterate the loop over all the other 299 nodes. In each of these iterations, we loop over each of the 9 columns of the survey and if their answers match, we increment the z counter.

(x, y, z) of each tuple represents the two nodes and their weights computed. We create a new empty Graph, G, Using the Networkx library below and add the nodes and edges as shown on the next page.

The Gender of the respondents was not considered in the analyses. While all the other columns were compared if the exact same answers were given, the column where we asked the participants to fill in the application they used to make new friends, we regarded the answer as same even if one of their used application matched

The following piece of code shows the graph forming process

```
G=nx.Graph()
for i in range(0,300):
  for j in range(i+1,301):
    z = 0
    for k in range(0,6):
      if df.loc[i][k]==df.loc[j][k]:
        z=z+1
    for k in range(7,9):
      if df.loc[i][k]==df.loc[j][k]:
        z=z+1
    same=False
    x=''
    y=''
    x = df.loc[i][k]
    y = df.loc[j][k]
    x=str(x)
    y=str(y)
    x=x.split(';')
    y=y.split(';')

    for a in x:
        for b in y:
            if a==b:
                same=True;

    if (same==True):
      z=z+1
    G.add_weighted_edges_from([(i,j,z)])
```

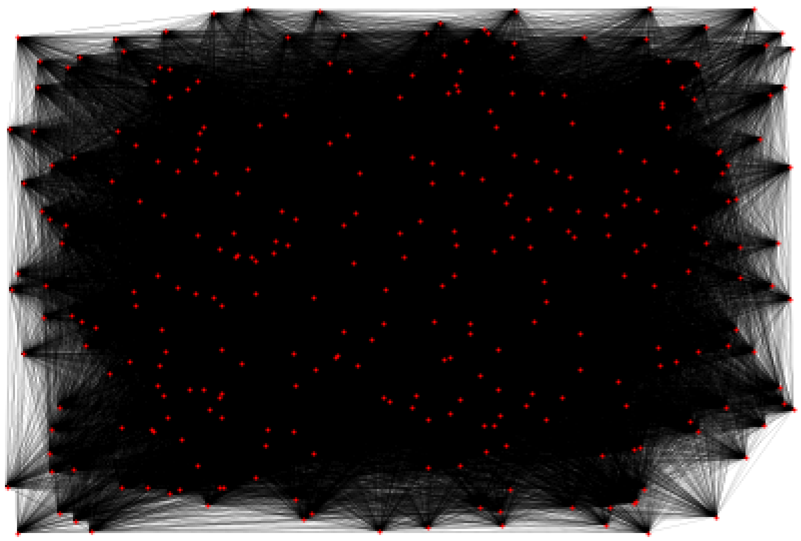
Next we construct another Graph, G_1, consisting of only edges with weight 3 or above. We create a sub-graph from our original graph with a condition on the weight of the edges

```
G_1 = nx.Graph()

for (u,v,d) in G.edges(data=True):

  if d['weight'] >= 3:
    G_1.add_weighted_edges_from([(u,v,d['weight'])])
```

Our graph G_1 is visualized below



The nodes are represented by red circles and the edges are represented by the black lines. As we can apparently see, the graph that is produced is highly dense, clustered and a strongly connected one. All these properties of this graph were checked with the built-in methods of 'networkx'.The total number of edges in this graph are 36442 and the graph is connected in a single giant component. Moreover, the total number of nodes in this graph are 297 compared with the 300 nodes we began with. Hence, we see that 3 nodes were such that they did not have even 3 similarities with any of the other participants. The average node degree, the average number of individuals linked to each, is 245. The number of edges required to be removed from the graph to make it a disconnected component are 86. This shows the high strength of the connectivity of G_1. While the two most important properties with respect to the small world properties have the following values:

Average Path length is: 1.170943670943671
Average clustering is: 0.8741738659738874
The python library 'networkx' has a couple of functions to classify whether the graph is a 'small world' or not by comparing the graph with an equivalent random graph. We calculate the small world coefficient (sigma) of the above graph as $sigma = C/Cr/L/Lr$.
where L and C are the average shortest path and average clustering coefficient respectively. Lr and Cr are the average shortest path and average clustering coefficient of an equivalent random graph.

The Small World Coefficient (sigma) came out to be **1.01**.

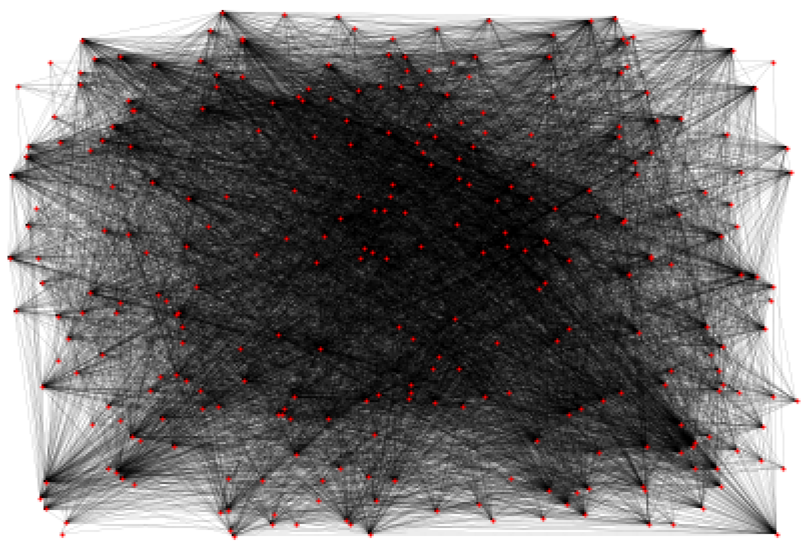
A value of sigma greater than 1 is classified as a small world network [2]. Hence, this particular graph can be classified as a small world network. Similarly, we constructed a graph, G_2, with weight 6 or above.

```
G_2 = nx.Graph()

for (u,v,d) in G.edges(data=True):

  if d['weight'] >= 6:
    G_2.add_weighted_edges_from([(u,v,d['weight'])])
```

Our graph G_2 is visualized below



The first look shows the graph clearly less denser than the previous one. Although the clustering seems to be lesser than the previous graph but no isolates can be found. We verify the properties of the graph with the built-in methods of 'networkx'. Total number of edges are 9226 and the graph is connected in a single giant component. The number of nodes in the graph is the same value, 297, that we obtained in G_1. The average node degree of G_2 is 62.The number of edges required to be removed from the graph to make it a disconnected component are 4. As we can see this value dropped from 84 to 4 as we moved from our first tier to the second tier. Although the graph is currently a connected one, as we further increase our cutoff criteria, we expect the graph to break into a few segments. While the two most important properties with respect to the small world properties have the following values:

Average Path length is: 2.06997906997907
Average clustering is: 0.5632137984175112

Here, we use the other small world coefficient, omega, provided by the 'networkx' library. Omega compares the graph with an equivalent lattice graph. We calculate the small world coefficient (omega) of the above graph as $omega = Lr/L - C/Cl$
where L and C are the average shortest path and average clustering coefficient respectively. Lr and Cl are the average shortest path of an equivalent random graph and average clustering coefficient of an equivalent lattice graph.
The Small World Coefficient (omega) came out to be **-0.627**.

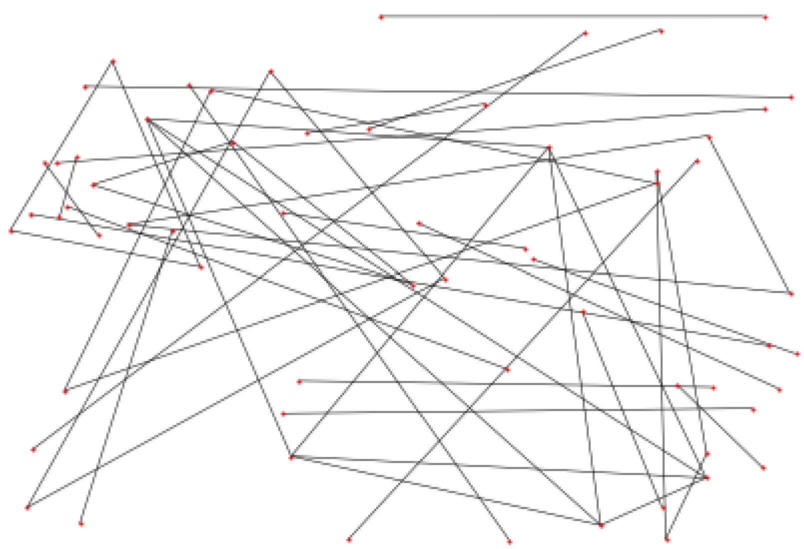
Values close to 0 means that the graph has small world properties. Values close to 1 means that the graph is closer to a random graph and values closer to -1 means that the graph has a lattice shape [5].
As our value of omega, **-0.627**, is closest to the lattice shape, we say that G_2 has more lattice-shaped properties. Since only one out of two properties of small world is being satisfied, the path length is no longer 'short', we say that the graph is no longer a small world graph. We can second this by reviewing our average path value which has risen to 2.

Finally, we made a graph, G_3, consisting of edges with weight 9 i.e all their answers were similar.

```
G_3 = nx.Graph()

for (u,v,d) in G.edges(data=True):

  if d['weight'] == 9:
    G_3.add_weighted_edges_from([(u,v,d['weight'])])
```



As we can observe, the number of edges in the graph dropped drastically. Neither high clustering nor a strongly connected component can be observed. We verify the properties of the graph with the built-in methods of Networkx. The total number of nodes remaining in G_3 are 63 and the average node degree is 1.5. Total number of edges are 48 and the graph is a disconnected one, broken into 27 components. Since the graph is no longer a connected one, the value of shortest path can not be computed and thus it is safe to say that this graph does not exhibit small world characteristics.

The Average Clustering is: 0.36507936507936506

Having studied the three-tier networks above, we can say that as the similarities between the two individuals increased from 6 to 9, the graph of the individuals changed from a strongly connected one to a disconnected graph of 27 components. By brute force, we check the least value of edge at which the graph became disconnected. When we plotted G_2, we realized that only 4 edges needed to be removed in order for it to break. As expected, if we plotted the graph of edge weight greater than or equal to 7, the graph became disconnected into 4 pieces. This is the point where the graph is no longer a single strongly connected giant component

As we went from G_1 to G_2, we realized that the graph lost its small world properties. So by brute force method, we try to calculate the edge weight which is the last one to exhibit small world properties. We create another graph G_4 that contains the edges who have a weight above or equal to 5 and calculate its small world coefficient. The small world coefficient (sigma) of G_4 came out to be **1.09**. Since sigma greater than 1 refers classifies a graph as a small world [2], we can say that the graph is a small world graph indeed. Thus, we can say that the G_2 which mapped the edges who had weights greater than 6 is the first in the series that lost its small world properties.

5 Literature Review

While there have been several studies done about the psychological impact of Covid 19 and social distancing, we were not able to find any research done on the friendship network during quarantine and its properties.

One study was done in Kurdistan titled "The Impact of Social Media on Panic During the COVID-19 Pandemic in Iraqi Kurdistan: Online Questionnaire Study"

This paper discusses the mental health and the panic caused by COVID 19 in Kurdistan, Iraq. For this study, a questionnaire was prepared to be filled by the participants. 516 people from the social media participated in this online survey. After that, content analysis method was applied to analyze the data using SPSS software. According to the study, COVID-19 had a significant negative impact on the mental and psychological state of the participants. It was also evident that Facebook was social media that played a vital role in spreading panic amongst the participants. Also a positive statistical correlation was found between social media usage and panic spread because of COVID 19. Using all the data the authors collected, they were able to conclude the social media played an important role in spreading the panic and anxiety related to COVID 19. The results also showed that major chunk of youth aged 18-35 are facing psychological anxiety [1].

The research paper although does not directly validate or invalidate our study, but the fact that individuals moved to social media in order to find information fortifies our study that the social media usage increased which led to the rewiring of emotional support network of individuals

6 Conclusion

To conclude, since we are demonstrating a human behavior network, we obtain graphs that exhibited high clustering and short path length until the second tier. G_4 was the last graph to follow the small world characteristics. Hence, we can say that all the participants who gave 5 out of 9 similar answers are connected in a clustered,short path graph. Just like in physical spaces, human beings are held together in virtual spaces too. The common choices of their activities also increase the chances of the two individuals interacting with each other in the virtual space and ending up befriending each other. The results that we observe are an extension of homophily network which says that individuals with similar interests or activities are more likely to link with one another in social setting.

This study also emphasizes on a very important aspect of human nature. We conducted our research on 300 participants and until G_2, we had 297 participants who had 6 or more similarities with each other. Even, in G_3, we have 63 nodes who are connected with someone or the other This means that 63 out of 300 individuals took the exact same 9 choices with someone else in the study. Humans, having similar social setting, when faced with similar circumstances are expected to react to the situation in a similar way. As the data showed when the pandemic hit the world, the coping mechanisms and the building of the emotional support network of the students of Habib University followed a quite similar pattern to one other. Hence, we can expect the students of other universities, or even people in general, to behave/react to a situation in a similar way and adapt to the new situation to form their network.

The decline in the average node degree as we moved from G_1 to G_2 to G_3 proved another of our hypothesis that the more strict we make the comparison criteria, the lesser neighbors of each node would remain.

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