APPLICATIONS OF GENERATIVE ADVERSARIAL NETWORKS TO NATURAL HANDWRITING AND MUSIC CREATION

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Abstract. We apply the recent concept of a generative adversarial network (GAN) to two tasks: natural handwriting emulation and music creation. Because it is of great interest to mathematically generalize neural networks (NNs), we will also present some abstract definitions that can serve as templates for NNs and GANs.

1. Introduction

In this paper, we exhibit two applications of generative adversarial networks (*1*), and we attempt to generalize various aspects of them, including the definition of a neural network. Before describing GANs, we will introduce some preliminary notation and definitions in the next sections. Later, we will proceed to applications of GANs in natural handwriting emulation and music creation. Because the generalization of neural networks is of great interest, along the way we will take the opportunity to propose general mathematical notions. The reader will be provided with both code to the GANs and instructions for playing with our GANs. We will also give some examples of outputs of our GANs in both applications. For natural handwriting images are provided; for natural music creation we provide links where MIDI files are stored so that the reader can perhaps be impressed.

2. Definitions and Notation

Much of the mathematical probability and statistics preliminaries we include here are borrowed from "Probability & Mathematical Statistics"(*2*). We assume that the reader is familiar with the definition of a σ-field on a set S. We also assume that the reader knows that a random variable is a function X with domain in the sample space S of a repeatable experiment and with co-domain in a previously chosen set T. Suppose that $\mathcal{F}$ is a σ-field on a set S. Let P be a probability function on $\mathcal{F}$. Recall that P and X together induce a random variable P_X on the collection of all inverse images under X of subsets of T given by

$$P_X(B) := P[X^{-1}(B)].$$

Note that this definition does not restrict us to the case when X is a real variable. For example, X could more generally, be vector-valued, matrix valued, or operator valued.

Next, suppose that X is a random variable that represents data that we have generated as a result of a collection and analysis algorithm. Then we may use D in place of X and denote p_X instead by p_{data}.

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If instead X is a randomly generated noise variable Z, then we may use Z as notation for X and denote p_Z instead by p_z throughout the paper.

Notation 2.1 (Prediction ($\sim$))**.** Let $m, n \in \mathbb{N}$, and suppose that $\alpha \in [0, 1]$. Let S be a non-empty set for outcomes of some experiment or action. Suppose that $\mathbf{X}$ is an $m \times n$-real matrix valued random variable on S; i.e., assume that $\mathbf{X} : S \to M_{m\times n}$, where $M_{m\times n}$ denotes the collection of all real matrices of size $m \times n$. Assume (for simplicity) that $\mathbf{X}$ is a continuous random variable. Let P be a probability function on a σ-field $\mathcal{F}$ on S, and let $B \subset S$. Define the random variable $C_\alpha \circ \mathbf{X}$ by $(C_\alpha \circ \mathbf{X})(s) = 1$ if the probability that $s \in B$ is $\geq \alpha$ and $(C_\alpha \circ \mathbf{X})(s) = 0$ otherwise.

Let $p_{\alpha,\mathbf{X}}$ more briefly denote the probability function induced by p and $C_\alpha \circ \mathbf{X}$. Let $\mathbf{X}_0 \in \mathbf{X}(S)$. Then the notation $\mathbf{X}_0 \sim_\alpha p(\mathbf{X}_0)$, or simply, $\mathbf{X}_0 \sim p(\mathbf{X}_0)$ in the case that $\alpha = 1/2$, denotes the phrase "The value $p_{\alpha,X}(\mathbf{X}_0)(s)$ allows us to conclude that $s \in B$, with probability (i.e., the value of $p_{\alpha,\mathbf{X}}$) of truth of this statement being $\geq \alpha$." Another way we may say this statement is, "$p_{\alpha,\mathbf{X}}(\mathbf{X}_0)(s)$ predicts s is in B with probability (i.e, value of $p_{\alpha,\mathbf{X}}[X_0(s)]$) being $\geq 1/2$."

Remark 2.2. Note, the entries of $\mathbf{X}(s)$ for some outcome $s \in S$ are called *predictors*.

To improve our predictions in the specific applications of GANs discussed in this paper, we must first find how to quantify inaccuracy in our predictions. To do so, we attempt to define an error function in the context of the above definition of prediction. Later in this paper, we will specialize this idea to the more specific applications of Generative Adversarial Networks we are interested in, natural handwriting and music creation.

Definition 2.3 (Error Function)**.** Let $\mathbf{X}$ denote an $m \times n$-real matrix-valued (datum) variable that we would like to use to classify whether or not a given outcome s of an experiment is or is not in a given set B in the sample space S. Let P be a probability function on a σ-field $\mathcal{F}$ on S. We say that a given function is an *error function induced by the variable* $\mathbf{X}$ *and* P and denote such an error function by $\mathbb{E}_{\mathbf{X}\sim\mathbf{P}(\mathbf{X})}$if and only if the following conditions are satisfied:

(1)
$$\mathbb{E}_{\mathbf{X}\sim\mathbf{P}(\mathbf{X})} : \mathbb{R} \to \mathbb{R},$$

(2) A given input $d \in \mathbb{R}$ for this function is the output of $f \circ P$, where f is a one-to-one function on $[0, 1]$,
(3) $\mathbb{E}_{\mathbf{X}\sim\mathbf{P}(\mathbf{X})}(d) = 0$ when the input d corresponds to the statement $\mathbf{X} \sim_1 P(\mathbf{X})$;
(4) A large value of $\mathbb{E}_{\mathbf{X}\sim\mathbf{P}(\mathbf{X})}(d)$ for some input d corresponds to the statement that $\mathbf{X} \sim_\alpha P(\mathbf{X})$ only for small values of $\alpha \geq 0$.

In order to make predictions, we must use some mathematical model. For the purpose of GANs, the model we will use is based on the human brain's layered neuron activity. We propose the following way of mathematically defining a neural network:

Definition 2.4 ((Generalized) Neural Network)**.**

(1) We call (V, E, W) a *neural network*, or more briefly an NN, if and only if the following statements hold:

(a) (V, E) is a digraph.
(b) W is a function whose domain is E.
(c) $\forall (u, v) \in E$ such that u is the terminal vertex of an edge in E, we have that u is a real number.
(d) $\forall (u, v) \in E,\ W(u, v) : \text{Range}(u) \to \text{Dom}(v)$.

(2) An NN (V, E, W) is called *real-valued* if and only if $\forall (u, v) \in E$ such that v is the terminal vertex of an edge in E, we have that v is a real number.
(3) Furthermore, the vertices of an NN that are both the terminal edge of a some vertex in V and the initial edge of another vertex in V are also called *neurons*.

Recall that graphs can be connected or disconnected, or they can be simple or non-simple. Since an NN (V, E, W) has an underlying graph (V, E), then NNs can be called connected, disconnected, simple, or non-simple, in exactly the same manner depending on the nature of the underlying graph.

However practically speaking, we note that in reasonable applications the underlying graph of an NN should be connected. Since an NN is a mathematical model inspired by how a brain works, a disconnected NN would be like two or more separate brains that don't interact with each other (not of interest in this paper). We remark that some neural networks, called recurrent ones, can have subgraphs of infinite length. However, in the applications examined in this paper, recurrency is not a feature of our GAN's.

For the applications that we are interested in, the vertices of a given neural network that are functions will have domains and ranges in Euclidean space.

Definition 2.5 (Euclidean NN and notation within it)**.**

(1) We call an NN (V, E, W) *Euclidean* if and only if each $v \in V$ is either a real constant or $v : \mathbb{R}^m \to \mathbb{R}^n$ for some positive integers m, n and, $\forall (u, v) \in E$, we have that $W(u, v)$ is either a real number or a column vector of real numbers whose number of components is the same as the dimension of the domain of v.
(2) Let (V, E, W) be an Euclidean NN, and let $v \in \mathbb{N}$ be the cardinality of V. Denote the vertices by $v_1, v_2, \ldots, v_k$.
   (a) We denote by $I := \{0\,..\,k\}$, the set of all integers between and including 0 and k.
   (b) With respect to $m_0, m_1, m_2, \ldots, m_k, n_0, n_1, n_2, \ldots, n_k \in \mathbb{W}$, the set of all positive integers, we impose that $m_i\ \forall\, i$ is the dimension of the domain of v_i vertex and also impose that n_i is the dimension of the range of v_i.
   (c) That is, we suppose that each vertex v_i of V here satisfies

$$v_i : \mathbb{R}^{m_i} \to \mathbb{R}^{n_i}$$

   (d) If $i, j \in \{1..k\}$ and $(v_i, v_j) \in E$, then we denote $W(v_i, v_j)$ more briefly by $w_{i,j}$.

The weights we assign each neuron change the network's influence on these neurons. Sometimes, we do not want a neuron to be included in the calculation for training so the network does not gain dependency on one neuron. Analogously to the human brain we have certain neurons removed from the graph.

Remark 2.6 (Drop-out). A weight of zero represents a vertex or neuron has *dropped-out* for some training cycle. Since the weight is a scalar multiplied by the input vector, when that weight is zero, that input will be ignored.

Definition 2.7 (Perceptron)**.** A *perceptron* is a neural network consisting of a single layer of weights and with identity functions as the vertex set.

Remark 2.8. As an effect of this definition, perceptrons may be modeled as linear predictors, or as functions $\mathbf{q}(\mathbf{x}) \sim \mathbf{p}(\mathbf{x})$. Later in the paper we will use this notation to refer to perceptrons.

We define neural networks as above to create a general mathematical structure for the theory of a largely implementation-based algorithm. Allowing each neuron to be any function and generalizing weights allows for a general theoretical neural network definition. A GAN uses two neural networks which are consequently trained in tandem. We will define the two separate neural networks before we define the full GAN.

Definition 2.9 (Discriminator)**.** A *discriminator* is a type of neural network that determines authenticity of input data. It can be modeled by a function

$$D : \mathbb{R}^n \to \mathbb{R},$$

together with the other properties of a more general NN.

The discriminator is a neural network that takes a vector to check if it is part of the same set as the data it was trained on. It outputs a probability that the vector is a part of its training set.

Definition 2.10 (Generator)**.** A *generator* is a type of neural network that produces new data based on training, and can be described as a function

$$G : \mathbf{z} \to \mathbb{R}^n$$

where $\mathbf{z}$ is a noise vector.

The generator is some neural network that takes in some kind of random noise and outputs things that look like the training data. Thus, when we train our generator in tandem with our discriminator we can evoke the idea of counterfeiting, in the sense that the outputs of the generator try to fool the input to the discriminator.

3. Definition of a Generative Adversarial Network

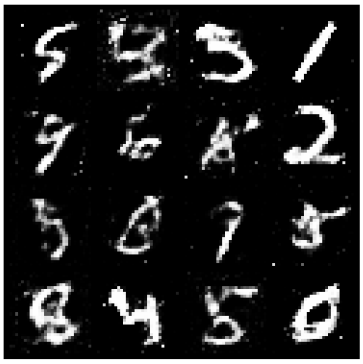
Definition 3.1 (Generative Adversarial Network)**.** We define a *generative adversarial network* (GAN) in terms of a two player min-max game between a discriminator D and a generator G with value function

$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x}\sim p_{data}(\mathbf{x})}[\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z}\sim p_{\mathbf{z}}(\mathbf{z})}[\log(1 - D(G(\mathbf{z})))] \quad (1)$$

where we train D to maximize the probability of correctly assigning labels to both training data and data that comes from G. In turn we train G to minimize $\log(1 - D(G(\mathbf{z})))$.

4. Results

4.1. **Natural Handwriting.** In the first application of GANs we attempted to produce handwritten letters and numbers in a manner that looks human written. We achieved partial results, based upon and modified from open source work by Diego Gomez[1]. Many changes were made in order to preserve outputs long term, as well as laying foundations for cohesive data inputs. For example, work was first done using MNIST as the input data, which produced very legible numbers.



The code for the handwriting portion and instructions on how to run it can be found at https://github.com/Onionchi/GANdwriting

Listed here is an example in Python of how to train an actual neural network.

```
for epoch in range(num_epochs):
  for n_batch, (real_batch,_) in enumerate(data_loader):

    # 1. Train Discriminator
    real_data = Variable(images_to_vectors(real_batch))
    if torch.cuda.is_available(): real_data = real_data.cuda()
    # Generate fake data
    fake_data = generator(noise(real_data.size(0))).detach()
    # Train D
    d_error, d_pred_real, d_pred_fake =
           train_discriminator(d_optimizer, real_data, fake_data)

    # 2. Train Generator
    # Generate fake data
    fake_data = generator(noise(real_batch.size(0)))
    # Train G
    g_error = train_generator(g_optimizer, fake_data)
    # Log error
    logger.log(d_error, g_error, epoch, n_batch, num_batches)
```

[1]https://github.com/diegoalejogm/gans

```
    # Display Progress
    if (n_batch) % 100 == 0:
      display.clear_output(True)
      # Display Images
      test_images = vectors_to_images(generator(test_noise)).data.cpu()
      logger.log_images(test_images, num_test_samples,
                        epoch, n_batch, num_batches);
      # Display status Logs
      logger.display_status(
        epoch, num_epochs, n_batch, num_batches,
        d_error, g_error, d_pred_real, d_pred_fake
      )
    # Model Checkpoints
    logger.save_models(generator, discriminator, epoch)
```

4.2. **Music Creation.** With the second application of GANs we attempted to produce new music based upon MIDI files from various classical composers. This involved two neural network models over the course of our research. Both of these models were useful in finding out more about GANs. Outputs to both models were generally not similar to music that would be considered "normal," but there was significant improvement between the two models. The model is based off starting code, which was originally an MNIST GAN that we heavily modified. [2]

The first model was a naive implementation of a GAN. This model simply took the input MIDI files and tried to produce new MIDI files that were similar. This led to results that generally fit in one of two categories.

(1) The MIDI file output would be a jumble of notes equivalent to someone randomly pushing keys on a keyboard.
(2) The MIDI file output would degrade into two notes one very high pitched and the other very low pitched. These notes would repeat at seemingly random intervals throughout.

We think that these two outcomes are due to a phenomenon known as modal collapse. More specifically over the process of training GANs, they will sometimes learn only one way of making output as indicated above.

The second model has many improvements over the first. The parameter that used to be a raw note value was changed to be a difference between notes over time. This way the neural network could try to learn some form of structure over time in music. This change was very successful and brought about new MIDI output that had segments resembling musical scales and arpeggios that might be features of human composed music. The following code includes the pre–processing performed on the data to add a note difference algorithm.

```
#Input: MidiFile Object, Output: Ordered Matrix with each part representing a
    note
def make_midi_matrix(mid_in, data_out):
    global min_time, max_time, min_channel, max_channel, min_note, max_note,
        min_attack, max_attack
```

[2]https://skymind.ai/wiki/generative-adversarial-network-gan

```
    first_loop = True #If we are in the first loop we will say the note is zero
        and store the note to get the difference
    for track in mid_in.tracks:
        for msg in track:
            if not first_loop:
                if not msg.is_meta and msg.type == "note_on":
                    command = (msg.channel, msg.note-prev_note, msg.velocity,
                        msg.time) #Get the note difference
                    min_time = min_time if min_time <= msg.time else msg.time
                    max_time = max_time if max_time >= msg.time else msg.time
                    min_channel = min_channel if min_channel <= msg.channel else
                        msg.channel
                    max_channel = max_channel if max_channel >= msg.channel else
                        msg.channel
                    data_out.append(command)
                    prev_note = msg.note
            else:
                if not msg.is_meta and msg.type == "note_on":
                    command = (msg.channel, 0, msg.velocity, msg.time) #Init line
                        one with a note base of 0
                    first_loop = False
                    min_time = min_time if min_time <= msg.time else msg.time
                    max_time = max_time if max_time >= msg.time else msg.time
                    min_channel = min_channel if min_channel <= msg.channel else
                        msg.channel
                    max_channel = max_channel if max_channel >= msg.channel else
                        msg.channel
                    prev_note = msg.note #Save the current cycles note for
                        differences
    return data_out
```

The above code and more is available at https://github.com/Aniranth/GAN_Project. Simply get Python 3 on your system and follow the instructions in the README file of the GitHub. You can also find sample MIDI files generated by several epochs of testing of the GAN at this link. The data used was acquired courtesy of http://www.jsbach.net/midi/ and http://www.shiningsilence.com/hpr/misc/

5. Further Research

There are many different directions that can be taken for further research. Currently for the application of handwriting we are trying to emulate the handwriting of Thomas Jefferson, as he has a large volume of annotated written work, available at the Library of Congress.

We would like to look further into the modal collapse that was experienced in the Music application. The authors would specifically like to consider a paper found late in the research as this paper developed.(*3*)

Another project the authors have significant personal interest in will be to implement the more generalized neural network that outputs functions rather than real numbers.

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References

1. I. J. Goodfellow *et al.*, *Generative Adversarial Networks*, arXiv: 1406.2661
2. D. Clahane, "Probability & Mathematical Statistics", Unpublished notes written for a class given at UC Riverside. Written by Dana Clahane, Fullerton College. Contact at dclahane@fullcoll.edu, 2006.
3. K. Li, J. Malik, *Implicit Maximum Likelihood Estimation*, 2018, eprint: arXiv:1809.09087.

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