FUTURE INSTITUTE OF TECHNOLOGY

TECHNICAL REPORT WRITING

Student Name: Anirban Manna .

DEPT. : C.S.E .

Year: 2nd ,

Semester: 4th ,

University Roll No.: 34200121045 .

University Regn. No: 213420100110015 (2021-22) .

Paper Name: Mathematics .

Paper Code: PCC-CS401 .

TEACHER SIGNATURE :. ________________________________ **.**

COUNTING TECHNIQUES

ABSTRACT

Counting techniques are an essential part of mathematics used in various fields, including probability theory, combinatorics, and computer science. There are times when the sample space or event space is very large, that it isn't feasible to write it out. In that case, it helps to have mathematical tools for counting the size of the sample space and event space. These tools are known as counting techniques. They are used to determine the number of possible outcomes in a given situation. Counting techniques can be used to solve problems related to permutations, combinations, and probability including the multiplication rule, and the addition rule. These techniques are widely used in mathematics, statistics, and computer science, and are essential tools for solving problems in various fields, In this report, we will discuss the different types of counting techniques - inclusion and exclusion, pigeon-hole principle, permutation and Combination, and their applications.

INTRODUCTION

It is essential to understand the number of all possible outcomes for a series of events. In daily life, many times one needs to find out the number of all possible outcomes for a series of events. The different ways in which 10-lettered PAN numbers can be generated in such a way that the first five letters are capital alphabets and the next four are digits and the last is again a capital letter. This can be calculated by using the mathematical theory of Counting. The fundamental counting rule, permutation rule, and combination rule is used by Counting. The fundamental counting rule, permutation rule, and combination rule are used by Counting. To know this technique, this report will help you. In optimization, counting techniques are used to find the optimal solutions to problems, such as the optimal allocation of resources.

PROBLEM DEFINITION

In daily life, many times one needs to find out the number of all possible outcomes for a series of events. For instance, in how many ways can a panel of judges comprising of 6 men and 4 women be chosen from among 50 men and 38 women? How many different 10-lettered PAN numbers can be generated such that the first five letters are capital alphabets, the next four are digits and the last is again a capital letter? For solving these problems, the mathematical theory of counting is used. Counting mainly encompasses the fundamental counting rule, the permutation rule, and the combination rule. So as we can see counting techniques give us the freedom to calculate or assume anything, any probability. The multiplication rule, addition rule, permutations, and combinations are the basic counting techniques that one can use to determine the number of possible outcomes in a given scenario. By understanding these techniques, one can solve complex problems and make informed decisions.

DISCUSSION

There are many types of Counting Techniques. We will discuss many types briefly and also in a short way. The multiplication rule, addition rule, permutations, and combinations are the basic counting techniques that one can use to determine the number of possible outcomes in a given scenario.

- **Types of Counting Techniques :**

1. The Rules of Sum and Product

The Rule of Sum and Rule of Product are used to decompose difficult counting problems into simple problems.

The Rule of Sum − If a sequence of tasks $T_1, T_2, \ldots, T_m$ can be done in $w_1, w_2, \ldots w_m$. ways respectively (the condition is that no tasks can be performed simultaneously), then the number of ways to do one of these tasks is $w_1 + w_2 + \cdots + w_m$. If we consider two tasks A and B which are disjoint (i.e., $A \cap B = \emptyset$), then mathematically $|A \cup B| = |A| + |B|$

The Rule of Product − If a sequence of tasks $T_1, T_2, \ldots, T_m$ can be done in $w_1, w_2, \ldots w_m$ ways respectively and every task arrives after the occurrence of the previous task, then there are $w_1 \times w_2 \times \cdots \times w_m$ ways to perform the tasks. Mathematically, if a task B arrives after task A, then $|A \times B| = |A| \times |B|$

2. Multiplication Principle:

This principle is used to count the number of ways in which a sequence of events can occur. If an event can occur in m ways and another event can occur in n ways, then the number of ways in which both events can occur is m X n.

3. Pigeonhole Principle :

In 1834, German mathematician, Peter Gustav Lejeune Dirichlet, stated a principle that he called the drawer principle. Now, it is known as the pigeonhole principle.The pigeonhole principle is a simple, yet beautiful and useful idea. Given a set A of pigeons and a set B of pigeonholes, if all the pigeons fly into a pigeonhole and there are more pigeons than holes, then one of the pigeonholes has to contain more than one pigeon.

The pigeonhole principle states that if n items are put into m containers, with

$n > m$, then at least one container must contain more than one item.

4. The Inclusion-Exclusion principle

The Inclusion-exclusion principle computes the cardinal number of the union of multiple non-disjoint sets. For two sets A and B, the principle states −

$$|A \cup B| = |A| + |B| - |A \cap B|$$

For three sets A, B, and C, the principle states −

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

The generalized formula -

$$|\bigcup_{i=1}^{n} A_i| = \sum_{1\le i<j<k\le n} |A_i \cap A_j| + \sum_{1\le i<j<k\le n} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{\backslash n-1} |A_1$$

∩⋯∩A2|

5. Pascal's Identity

Pascal's identity, first derived by Blaise Pascal in 17th century, states that the number of ways to choose k elements from n elements is equal to the summation of number of ways to choose (k-1) elements from (n-1) elements and the number of ways to choose elements from n-1 elements.

Mathematically, for any positive integers k and n:

$$^{n}C_{k} = {}^{n-1}C_{k} - {}^{1+n-1}C_{k}$$

6. **Permutations:**

Permutations are arrangements of a set of objects in a particular order. The number of permutations of n distinct objects taken r at a time is given by

$$n_{P_r} = \frac{n!}{(n-r)!}$$

Permutations are used to solve problems where order matters, such as arranging people in a line or selecting a committee with a particular order of members.

Some important formulas of permutation

- If there are n elements of which a_1 are alike of some kind, a2 are alike of another kind; a3 are alike of the third kind and so on and a_r are of r^{th} kind, where (a1+a2+...ar)=n
  Then, number of permutations of these n objects is = n!/[(a1!(a2!)…(ar!)]

- Number of permutations of n distinct elements taking n elements at a time = $nPn=n!$.
- The number of permutations of n dissimilar elements when r specified things always come together is − $r!(n-r+1)!$
- The number of circular permutations of n different elements taken x elements at time = npx/x
- The number of circular permutations of n different things = npn/n

7. Combination

Combinations are selections of objects from a set without regard to their order. The number of combinations of n distinct objects taken r at a time is given by

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Combinations are used to solve problems where order does not matter, such as selecting a team of players from a group.

Example :

There are 6 men and 5 women in a room. In how many ways we can choose 3 men and 2 women from the room?

Solution :

The number of ways to choose 3 men from 6 men is 6C3

and the number of ways to choose 2 women from 5 women is 5C2

Hence, the total number of ways is −

$$^6C_3 \times {}^5C_2 = 20 \times 10 = 200$$

Here are a few types of Techniques. So now we now to know where and when we need to use them. In easy words, Applications of Counting Techniques.

- **Applications:**

Probability Theory: Counting techniques are used in probability theory to determine the number of possible outcomes of an event. For example, if a fair coin is tossed three times, the number of possible outcomes is 2^3=8.

Combinatorics: Combinatorics is a branch of mathematics that deals with the study of counting techniques. It is used in computer science, cryptography, and other fields.

Optimization: Counting techniques are used in optimization problems to determine the optimal solution. For example, in scheduling problems, counting techniques can be used to determine the number of possible schedules.

CONCLUSION

In conclusion, counting techniques refer to mathematical methods used to count the number of possible outcomes in a given situation. These techniques are important in various fields, including probability theory, statistics, computer science, and combinatorics. Counting techniques are essential in solving various problems that involve counting, such as gambling, lottery, and card games. They are also used in scientific research to estimate the probability of certain events and in computer science to analyse the complexity of algorithms.

Overall, mastering counting techniques is crucial for understanding various mathematical concepts and solving complex problems in different fields.

BIBLIOGRAPHY

- CLASS NOTE
- https://www.tutorialspoint.com/
- https://www.javatpoint.com/
- https://chat.openai.com/