

Discrete Optimization

The Knapsack Problem:
Greedy Algorithms

Goals of the Lecture

- ▶ Compare different greedy algorithms
 - knapsack example

What is Greedy?

- ▶ Assume the it's “easy” to build a feasible solution
- ▶ Key Idea:
 - Build a solution by picking items one at a time
- ▶ Called: greedy algorithms or heuristics

The Temple is Collapsing



\$1 Million
2kg



\$1 Million
2kg



\$1 Million
2kg



\$10 Million
5kg



\$10 Million
5kg

Which items to take?



\$13 Million
8kg



\$7 Million
3kg

Maximum
Capacity
10kg

Knapsack Greedy Algorithms

- ▶ Idea 1:

- More items is best, start with small ones and take as many as you can

Idea: The More Items the Better



\$1 Million
2kg

\$1 Million
2kg

\$1 Million
2kg

\$10 Million
5kg

\$10 Million
5kg

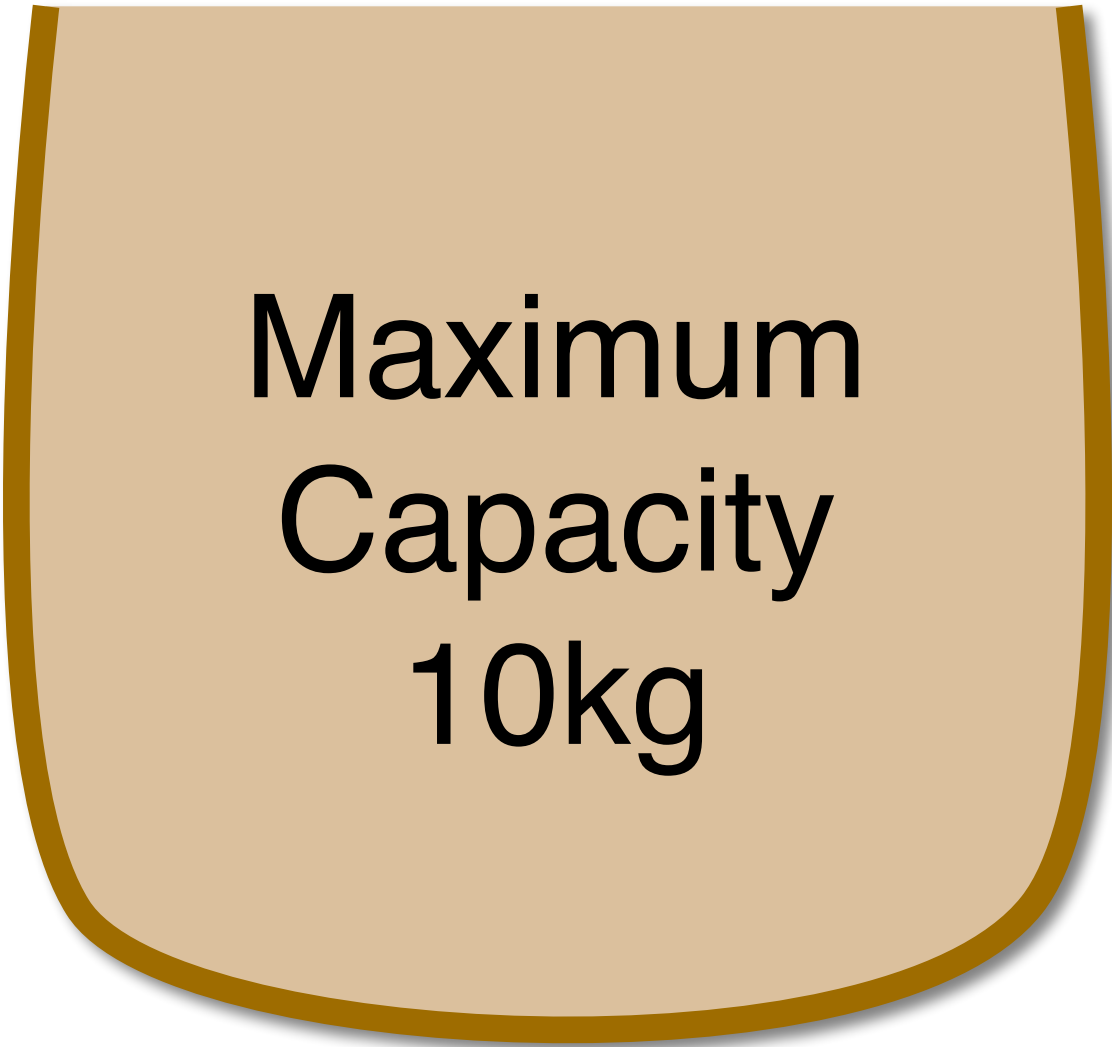
Total: \$10 Million



\$13 Million
8kg



\$7 Million
3kg



Knapsack Greedy Algorithms

- ▶ Idea 1: (\$10 Million)
 - More items is best, start with small ones and take as many as you can
- ▶ Idea 2:
 - Valuable items are best, start with the most valuable items

Idea: More Valuable is Better



\$1 Million
2kg

\$1 Million
2kg

\$1 Million
2kg

\$10 Million
5kg

\$10 Million
5kg

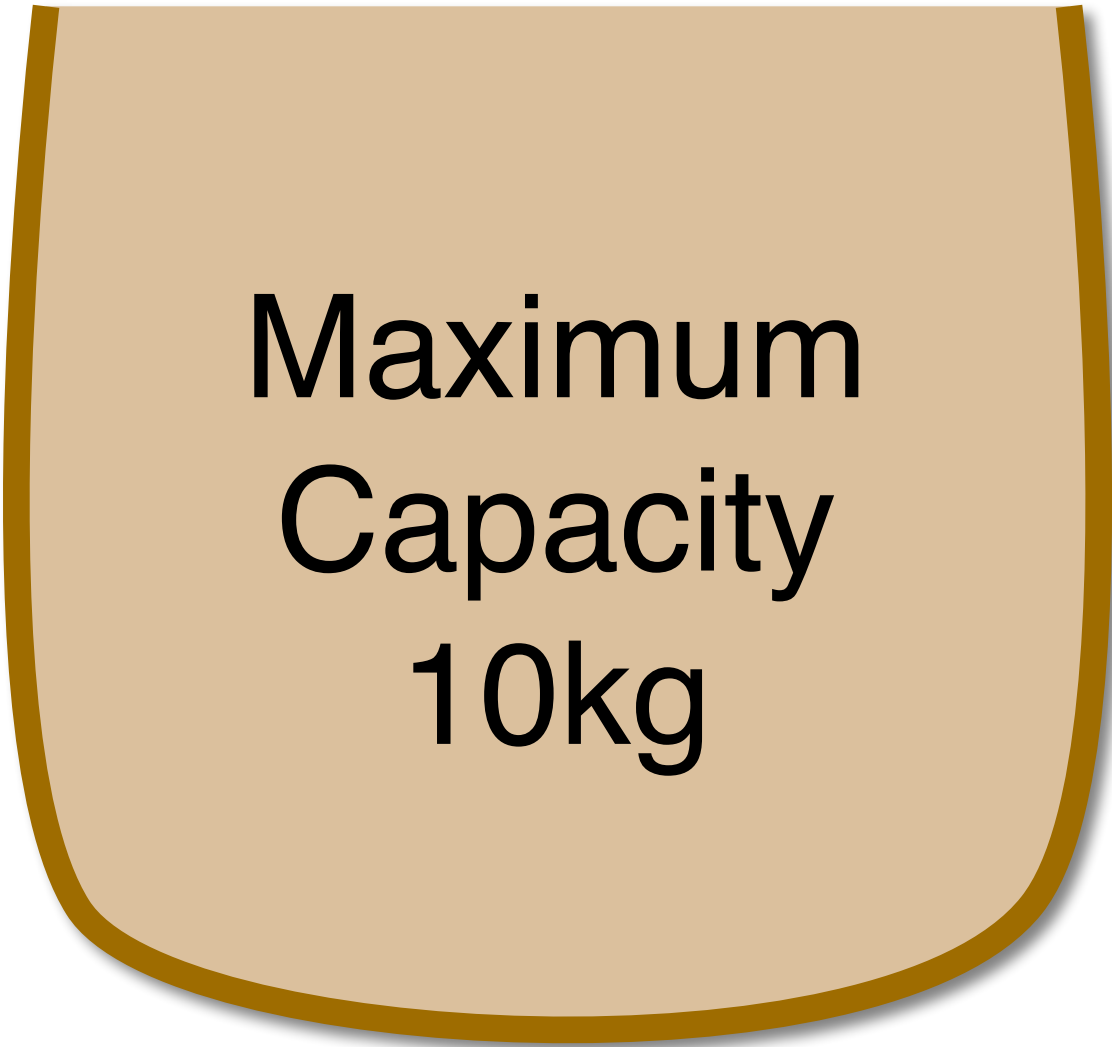
Total: \$14 Million



\$13 Million
8kg



\$7 Million
3kg




Maximum
Capacity
10kg

Knapsack Greedy Algorithms


- ▶ Idea 1: (\$10 Million)
 - More items is best, start with small ones and take as many as you can
- ▶ Idea 2: (\$14 Million)
 - Valuable items are best, start with the most valuable items
- ▶ Idea 3:
 - Value density! dollars per kilogram

Idea: The More Items the Better




0.5
M/kg


\$1 Million
2kg




\$1 Million
2kg



\$1 Million
2kg




\$10 Million
5kg




2.0
M/kg

\$10 Million
5kg

Total: \$1.8 Million



\$13 Million
8kg



\$7 Million
3kg

2.3
M/kg

Maximum
Capacity
10kg

Knapsack Greedy Algorithms

- Is \$18 million dollars the best we can do?
 - optimal?

Total: \$20 Million!



\$10 Million
5kg



\$10 Million
5kg

Maximum
Capacity
10kg

Greedy Algorithms Overview

- ▶ For one problem, there are **many** possible greedy algorithms.
 - some will do better than others
 - depends on the input!
- ▶ Advantages
 - quick to design and implement
 - can be very fast
- ▶ Problems
 - no quality guarantees (in general)
 - quality can vary widely on the input
 - problem feasibility needs to be “easy”

The Essence of this Class

- ▶ We can always start with greedy
- ▶ Going beyond greedy
 - Constraint Programming
 - Local Search
 - Mixed Integer Programming
- ▶ Ways to
 - reliably find feasible solutions
 - reliably build high-quality solutions
 - robust to different inputs
 - ideally, proving those solutions are the best

Until Next Time

Citations

Stone Foundation Tablet with Inscription of Gudea - 41221 ([http://commons.wikimedia.org/wiki/File:Sumerian _-
_Stone_Foundation_Tablet_with_Inscription_of_Gudea_-_Walters_41221_-_View_A.jpg](http://commons.wikimedia.org/wiki/File:Sumerian_-_Stone_Foundation_Tablet_with_Inscription_of_Gudea_-_Walters_41221_-_View_A.jpg)). Artist Unknown. Walters Art Museum [Public domain, CC-BY-SA-3.0 (<http://creativecommons.org/licenses/by-sa/3.0>)], via Wikimedia Commons

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