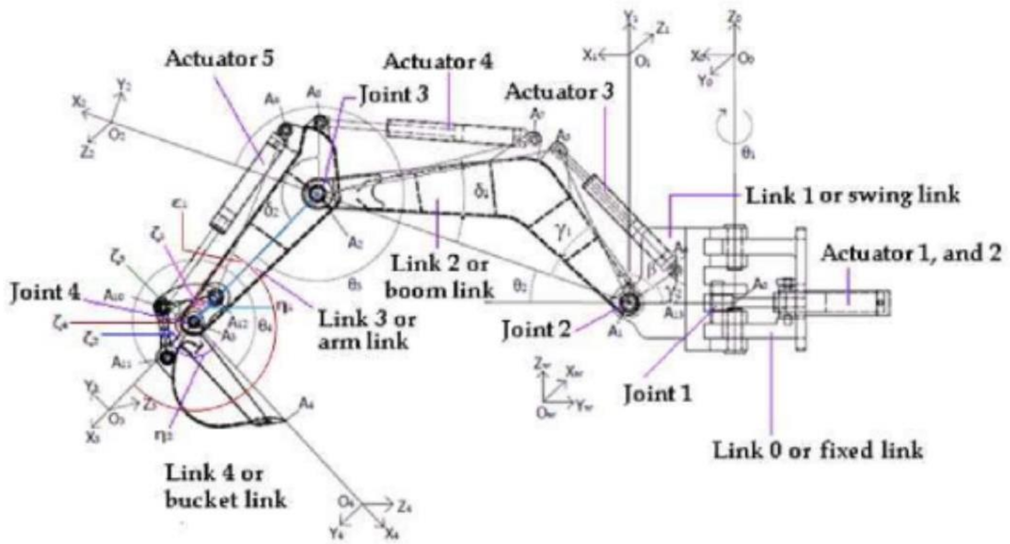
The Diagram of the Robotic Excavator Arm is given below:-

Necessary Data



For the required study, Upper and Lower carriage is supposed to be ignored.

Problem Statements

Problem 1:- Identifying the links, joints and End Effectors.

Solution: Referring to the diagram of the arm given above

Links →
- Link 0 or fixed link (Dead link)
- Link 1 or swing link
- Link 2 or boom link
- Link 3 or arm link

Joints → Revolute joints
- Joint 1
- Joint 2
- Joint 3
- Joint 4

End Effector → Link 4 or bucket link

Problem 2:- Describing the purpose of the given link.

Solution:-

Link 0 or fixed link = Purpose is to act as a rigid support base to the entire arrangement.

Link 1 or swing link
Link 2 or boom link
Link 3 or arm link

= Provides support to initiate ~~constraints~~ constrained ~~moving~~ mechanical arrangement of the arm, which is relatively moving to each other

Link 4 or bucket link = Acts as the end effector and helps the robot interact with the environment.

Problem 3:- Describing the type of end effector.

Solution:- From the given diagram, it is evident that the end effector used in this case is Gripper type

Grippers are end effectors which are used to grasp or hold any object. Specifically in this case, it is a mechanical hand or Scoop to hold the mud excavated.

Problem 4: Brief description of the type of actuator used.

Solution:- The actuator used in the robotic arm excavator is ~~Hydrauli~~ Hydraulic Actuator.

Hydraulic actuators used fluid filled cylinder piston mechanism The to and fro oscillation of the piston assisted by the hydraulic fluid initiates relative motion in straight line between the connecting members.

Problem 5: Explaining the configuration, with respect to types of joints.

Solution: As per as the arrangement, all three ~~arms~~ joints of the arm has 1-dof (degree of freedom) revolute joints which represents, RRR configuration or Articulated or Anthromorphic Configuration.

Problem 6 :- Determining the Degrees of Freedom using the ~~Kutzwatch~~ Kutzbach Formula.

Solution :- According to the Kutzbach Equation, we know,

No. of degrees of freedom be D

$$D = 3(L-1) - 2J - H$$

where

L = Number of links of the arms
J = Number of joints of the arm
H = Numbers of Higher Pair

From the given diagram, we get

L = 4 [Link 0, Link 1, Link 2, Link 3]
J = 4 [Joint 1, 2, 3, 4]
H = 0 [Given, all members of surface to surface contact]

$\therefore$ $D = 3(4-1) - 2\times(4) - 0$

$\Rightarrow D = 3(3) - 2(4) - 0$

$\Rightarrow D = 9 - 8 = 1$

$\therefore$ The degree of freedom (D.O.F) of the excavator arm is 1.

Scanned with OKEN Scanner

Problem 7: Determining the Translational Matrix with Respect to the given data.

Frame P' is initially coincident with frame P. Frame P' is rotated about YB by 30 degrees, then about XB by 45 degree, then ZB by 60 degrees. ~~Find the origin~~ Finally the origin of {B} translated [XA, YA, ZA] T= [35,−10, 10]T. ~~Det~~ The entire Cartesian space is scaled by a factor of 2. Find in the order given PTP' (position of end effector w.r.t initial frame).

Solution: Given, angle of rotation about X axis = $\theta_x = 45°$

angle of rotation about Y axis = $\theta_y = 30°$

angle of rotation about z-axis = $\theta_z = 60°$

[The system is scaled by a factor of 2]

∴ Rotational matrix about x-axis,

$$^{P}_{P'}R_x = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

$$\Rightarrow {}^{P}_{P'}R_x = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° \\ 0 & \sin 45° & \cos 45° \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.707 & -0.707 \\ 0 & 0.707 & 0.707 \end{bmatrix}$$

Rotational matrix about Y axis,

$$^{P}_{P'}R_y = \begin{bmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 2 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{bmatrix}$$

$$\Rightarrow {}^{P}_{P'}R_y = \begin{bmatrix} \cos 30° & 0 & -\sin 30° \\ 0 & 2 & 0 \\ \sin 30° & 0 & \cos 30° \end{bmatrix} = \begin{bmatrix} 0.866 & 0 & -0.50 \\ 0 & 2 & 0 \\ 0.50 & 0 & 0.866 \end{bmatrix}$$

Similarly, Rotational matrix about z-axis,

$$^{P}_{P'}R_z = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow {}^{P}_{P'}R_z = \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.866 & 0 \\ 0.866 & 0.50 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

∴ The rotation matrix from frame P to P' is given as (taken as in given order)

$$ {}^{P}_{P'}R = {}^{P}_{P'}R_y \times {}^{P}_{P'}R_x \times {}^{P}_{P'}R_z $$

$$ \Rightarrow {}^{P}_{P'}R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.707 & -0.707 \\ 0 & 0.707 & 0.707 \end{bmatrix} \times \begin{bmatrix} 0.866 & 0 & -0.50 \\ 0 & 2 & 0 \\ 0.50 & 0 & 0.866 \end{bmatrix} \times \begin{bmatrix} 0.50 & -0.866 & 0 \\ 0.866 & 0.50 & 0 \\ 0 & 0 & 2 \end{bmatrix} $$

$$ \Rightarrow {}^{P}_{P'}R = \begin{bmatrix} 2\times 0.866+0 & 0 & -2\times 0.50 \\ -0.707\times 0.5 & 2\times 0.707 & -0.707\times 0.866 \\ 0.707\times 0.5 & 2\times 0.707 & 0.707\times 0.866 \end{bmatrix} \times \begin{bmatrix} 0.50 & -0.866 & 0 \\ 0.866 & 0.50 & 0 \\ 0 & 0 & 2 \end{bmatrix} $$

$$ \Rightarrow {}^{P}_{P'}R = \begin{bmatrix} 1.732 & 0 & -1 \\ -0.3535 & 1.414 & -0.612 \\ 0.3535 & 1.414 & 0.612 \end{bmatrix} \times \begin{bmatrix} 0.50 & -0.866 & 0 \\ 0.866 & 0.50 & 0 \\ 0 & 0 & 2 \end{bmatrix} $$

$$ \Rightarrow {}^{P}_{P'}R = \begin{bmatrix} 1.732\times 0.5 & -1.732\times 0.866 & -2 \\ -0.3535\times 0.5 + 0.866\times 1.414 & -0.3535\times -0.866 + 1.414\times 0.5 & -0.612\times 2 \\ 0.3535\times 0.5 + 1.414\times 0.866 & -0.866\times 0.3535 + 1.414\times 0.5 & 0.612\times 2 \end{bmatrix} $$

$$ \Rightarrow {}^{P}_{P'}R = \begin{bmatrix} 0.866 & -1.49991 & -2 \\ -0.17675+1.224 & 0.306+0.707 & -1.224 \\ 0.17675+1.224 & -0.306+0.707 & 1.224 \end{bmatrix} $$

$$ \Rightarrow {}^{P}_{P'}R = \begin{bmatrix} 0.866 & -1.5 & -2 \\ 1.04725 & 1.013 & -1.224 \\ 1.40075 & 0.401 & 1.224 \end{bmatrix} $$

The given translational matrix $ {}^{P}_{P'}B = \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} = \begin{bmatrix} 35 \\ -10 \\ 10 \end{bmatrix} $

∴ The required homogenous matrix for position of end effector w.r.t to initial frame is,

$$ T = \begin{bmatrix} {}^{P}_{P'}R & B \end{bmatrix}^{P}_{P'} = \left[\begin{array}{ccc|c} 0.866 & -1.5 & -2 & 35 \\ 1.04725 & 1.013 & -1.224 & -10 \\ 1.40075 & 0.401 & 1.224 & 10 \\ \hline 0 & 0 & 0 & 2 \end{array}\right] $$

Problem 8: Executing The DH matrix for the same.

Solution:- The normal form of the DH matrix is given by

$$DH = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

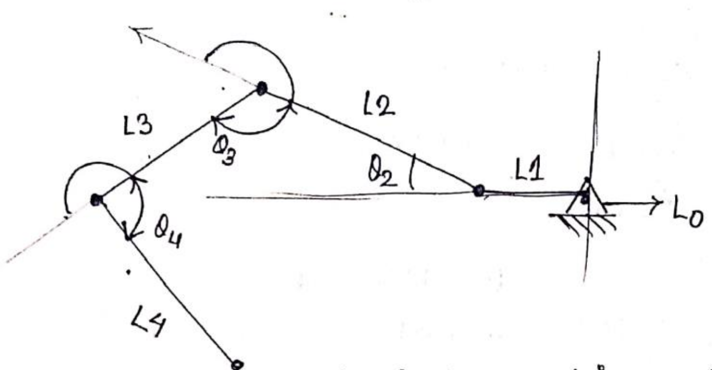
where a = link length
α = link twist
d_i = link offset
θ = Joint angle

Given

$a_{l_1} = 100$ mm ; $a_{l_2} = 240$ mm ; $a_{l_3} = 133.6$ mm ; $a_{l_4} = 52.8$ mm

$\cancel{\alpha_1 = 30^\circ;\ \alpha_2 = 30^\circ;\ \alpha_3 = 60^\circ;\ \alpha_4 = 30^\circ}$

$\theta_1 = 30^\circ;\ \theta_2 = 30^\circ;\ \theta_3 = 60^\circ;\ \theta_4 = 30^\circ$



Now, we know, α_i refers to link twist in DH matrices, which is nothing but the angle between axes of rotation of the revolute joint in the given arrangement. From the figure, as all the joints are revolving around parallel z-axes, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$

Finally d_i = link offset, [∵ All the links are ~~connected~~ coincident to each other]

$\therefore d_1 = d_2 = d_3 = d_4 = 0$

D-H parameters, =

	a	α	d	θ
1	100	0	0	30°
2	240	0	0	30°
3	133.6	0	0	60°
4	52.8	0	0	30°

$$\therefore \; {}^{1}_{0}[DH] = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \cos 0 & \sin 30^\circ \sin 0 & 100\cos 30^\circ \\ \sin 30^\circ & \cos 30^\circ \cos 0 & -\cos 30^\circ \sin 0 & 100\sin 30^\circ \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

[As all the factors are scaled by 2]

$$\Rightarrow {}_{1}[DH] = \begin{bmatrix} 0.866 & -0.5 & 0 & 86.60 \\ 0.50 & 0.866 & 0 & 50 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

And,

$${}^{2}_{1}[DH] = {}_{2}[DH] = \begin{bmatrix} C30^\circ & -S30^\circ C0^\circ & S30^\circ S0^\circ & 240C30^\circ \\ S30^\circ & C30^\circ C0^\circ & -C30^\circ S0^\circ & 240S30^\circ \\ 0 & S0^\circ & C0^\circ & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \begin{bmatrix} \cos\varphi = c\varphi \\ s\varphi = s\varphi \text{ (Let)} \end{bmatrix}$$

$$\Rightarrow {}_{2}[DH] = \begin{bmatrix} 0.866 & -0.5 & 0 & 207.84 \\ 0.50 & 0.866 & 0 & 120 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Further,

$${}^{3}_{2}[DH] = {}_{3}[DH] = \begin{bmatrix} C60^\circ & -S60^\circ C0^\circ & S60^\circ S0^\circ & 133.6C60^\circ \\ S60^\circ & C60^\circ C0^\circ & -C60^\circ S0^\circ & 133.6S60^\circ \\ 0 & S0^\circ & C0^\circ & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow {}_{3}[DH] = \begin{bmatrix} 0.5 & -0.866 & 0 & 66.8 \\ 0.866 & 0.5 & 0 & 115.6976 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Similarly,

$${}^{4}_{3}[DH] = {}_{4}[DH] = \begin{bmatrix} C30^\circ & -S30^\circ C0^\circ & S30^\circ S0^\circ & 52.8C30^\circ \\ S30^\circ & C30^\circ C0^\circ & -C30^\circ S0^\circ & 52.8S30^\circ \\ 0 & S0^\circ & C0^\circ & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow {}_{4}[DH] = \begin{bmatrix} 0.866 & -0.5 & 0 & 45.7248 \\ 0.5 & 0.866 & 0 & 26.4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\therefore [DH] = {}_{1}[DH] \times {}_{2}[DH] \times {}_{3}[DH] \times {}_{4}[DH]$$

By calculating, we get the final value of the DH matrix as

$$[DH] = 10^{3}\begin{bmatrix} -0.000866 & -\overset{0.0005}{\cancel{0.000499}} & 0 & 0.9934 \\ \overset{0.0005}{\cancel{0.000499}} & -0.000866 & 0 & 1.4891 \\ 0 & 0 & 0.0010 & 0 \\ 0 & 0 & 0 & 0.0160 \end{bmatrix}$$

$$\Rightarrow [DH] = \begin{bmatrix} -0.8660 & -0.50 & 0 & 993.4409 \\ 0.5 & -0.8660 & 0 & 1.4891\times 10^{3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix}$$

The MATLAB Code for the calculation of the required DH matrix is shown below:-

```
>> a=[ 0.866 -0.5 0 86.60;0.5 0.866 0 50; 0 0 1 0;0 0 0 2]

a =

    0.8660   -0.5000         0   86.6000
    0.5000    0.8660         0   50.0000
         0         0    1.0000         0
         0         0         0    2.0000

>> b=[ 0.866 -0.5 0 207.84;0.5 0.866 0 120; 0 0 1 0;0 0 0 2]

b =

    0.8660   -0.5000         0  207.8400
    0.5000    0.8660         0  120.0000
         0         0    1.0000         0
         0         0         0    2.0000

>> c=[0.5 -0.866 0 66.8;0.866 0.5 0 115.6976;0 0 1 0; 0 0 0 2]

c =

    0.5000   -0.8660         0   66.8000
    0.8660    0.5000         0  115.6976
         0         0    1.0000         0
         0         0         0    2.0000

>> d=[ 0.866 -0.5 0 45.7248;0.5 0.866 0 26.4; 0 0 1 0;0 0 0 2]
d =
```

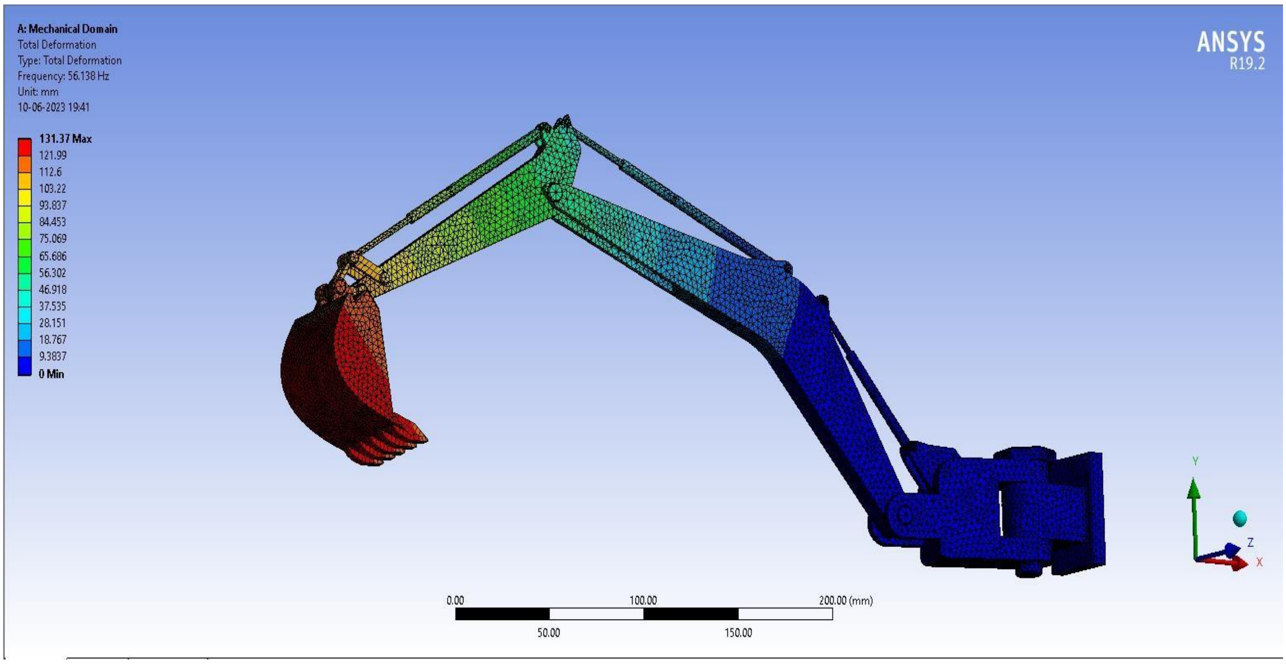
```
   0.8660   -0.5000        0   45.7248

   0.5000    0.8660        0   26.4000

        0         0   1.0000        0

        0         0        0   2.0000

>> m=a*b;

>> v=m*c;

>> DH=v*d


DH =

-0.866   -0.5     0        993.4409

0.5      -0.866   0        1489.1*10^3

0        0        1        0

0        0        0        16
```

SOLIDWORKS MODEL FILES LINK

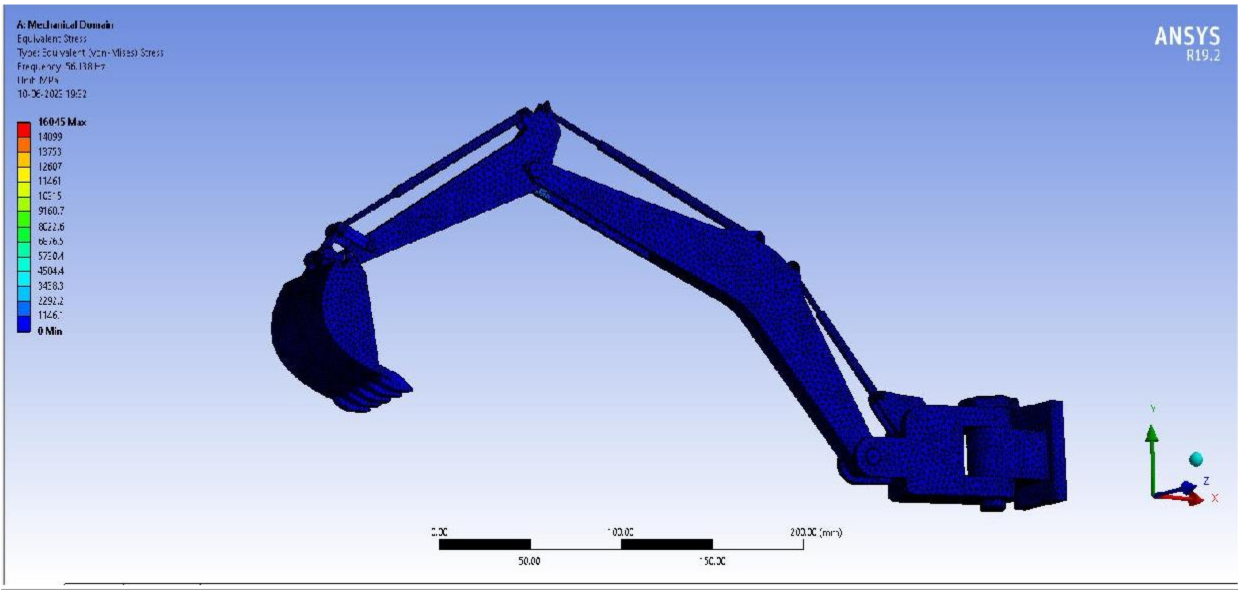
https://drive.google.com/drive/folders/1vYyu482vG2202i0bBe1KidiqcFcaXjE2?usp=sharing

ANSYS ANALYSIS SOLUTIONS

Total Deformation:



Equivalent Stress(Von-Mises Stress):



Equivalent Strain(Von-Mises Strain):