Topic: Symmetric equations for the line of intersection of two planes

Question: Find the symmetric equations for the line of intersection of the planes.

$$x + y + z = 1$$

$$x - y + z = 3$$

Answer choices:

A $\quad \dfrac{x-2}{2} = \dfrac{z}{2}, \quad y = -1$

B $\quad \dfrac{x-2}{2} = -\dfrac{z}{2}, \quad y = -1$

C $\quad \dfrac{x-2}{2} = \dfrac{z}{2}, \quad y = 1$

D $\quad \dfrac{x-2}{2} = -\dfrac{z}{2}, \quad y = 1$

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Solution: B

The symmetric equations for the line of intersection are given by

$$\frac{x - c_1}{v_1} = \frac{y - c_2}{v_2} = \frac{z - c_3}{v_3}$$

where $c(c_1, c_2, c_3)$ comes from a point on the line of intersection, and where $v(v_1, v_2, v_3)$ is the cross product of the normal vectors of the planes.

The normal vectors of the planes are given by their components, which means that the normal vectors of $x + y + z = 1$ and $x - y + z = 3$ are $a\langle 1,1,1\rangle$ and $b\langle 1, -1,1\rangle$, respectively. The cross product is given by

$$v = a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$v = a \times b = \mathbf{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$v = a \times b = \left(a_2b_3 - a_3b_2\right)\mathbf{i} - \left(a_1b_3 - a_3b_1\right)\mathbf{j} + \left(a_1b_2 - a_2b_1\right)\mathbf{k}$$

which means plugging in the normal vectors gives

$$v = \left[(1)(1) - (1)(-1)\right]\mathbf{i} - \left[(1)(1) - (1)(1)\right]\mathbf{j} + \left[(1)(-1) - (1)(1)\right]\mathbf{k}$$

$$v = (1 + 1)\mathbf{i} - (1 - 1)\mathbf{j} + (-1 - 1)\mathbf{k}$$

$$v = 2\mathbf{i} - 0\mathbf{j} - 2\mathbf{k}$$

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Now we'll need to find a point on the line of intersection, which we can do by setting $z=0$ in both equations, and then solving what remains as a system of equations. If the planes are

$$x+y+z=1$$

$$x-y+z=3$$

then setting $z=0$ gives

$$x+y=1$$

$$x-y=3$$

We can add the equations together to get

$$(x+y)+(x-y)=1+3$$

$$x+x+y-y=1+3$$

$$2x=4$$

$$x=2$$

Plugging $x=2$ back into $x+y=1$ gives

$$x+y=1$$

$$2+y=1$$

$$y=-1$$

If we put all these values together, we can say that $c(2,-1,0)$ is a point on the line of intersection.

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Now we'll put $v = 2\mathbf{i} - 0\mathbf{j} - 2\mathbf{k}$ and $c(2, -1{,}0)$ into the formula for the symmetric equations for the line of intersection.

$$\frac{x - c_1}{v_1} = \frac{y - c_2}{v_2} = \frac{z - c_3}{v_3}$$

$$\frac{x - 2}{2} = \frac{y - (-1)}{0} = \frac{z - 0}{-2}$$

Since we cannot divide by 0, we pull out the y equation as its own parametric equation, leaving the other two equations as symmetric equations.

$$\frac{x - 2}{2} = \frac{z - 0}{-2}, \quad y - (-1) = 0$$

$$\frac{x - 2}{2} = -\frac{z}{2}, \quad y + 1 = 0$$

$$\frac{x - 2}{2} = -\frac{z}{2}, \quad y = -1$$

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Topic: Symmetric equations for the line of intersection of two planes

Question: Find the symmetric equations for the line of intersection of the planes.

$$2x + 2y + 2z = 3$$

$$-2x - y - z = 3$$

Answer choices:

A $\quad x = -\frac{9}{2}, \quad -\frac{y-6}{2} = \frac{z}{2}$

B $\quad x = -\frac{9}{2}, \quad \frac{y-6}{2} = \frac{z}{2}$

C $\quad x = \frac{9}{2}, \quad -\frac{y-6}{2} = \frac{z}{2}$

D $\quad x = \frac{9}{2}, \quad \frac{y-6}{2} = \frac{z}{2}$

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Solution: A

The symmetric equations for the line of intersection are given by

$$\frac{x-c_1}{v_1}=\frac{y-c_2}{v_2}=\frac{z-c_3}{v_3}$$

where $c(c_1, c_2, c_3)$ comes from a point on the line of intersection, and where $v(v_1, v_2, v_3)$ is the cross product of the normal vectors of the planes.

The normal vectors of the planes are given by their components, which means that the normal vectors of $2x+2y+2z=3$ and $-2x-y-z=3$ are $a\langle 2,2,2\rangle$ and $b\langle -2,-1,-1\rangle$, respectively. The cross product is given by

$$v=a\times b=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$$v=a\times b=\mathbf{i}\begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix}-\mathbf{j}\begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix}+\mathbf{k}\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}$$

$$v=a\times b=\left(a_2b_3-a_3b_2\right)\mathbf{i}-\left(a_1b_3-a_3b_1\right)\mathbf{j}+\left(a_1b_2-a_2b_1\right)\mathbf{k}$$

which means plugging in the normal vectors gives

$$v=\left[(2)(-1)-(2)(-1)\right]\mathbf{i}-\left[(2)(-1)-(2)(-2)\right]\mathbf{j}+\left[(2)(-1)-(2)(-2)\right]\mathbf{k}$$

$$v=(-2+2)\mathbf{i}-(-2+4)\mathbf{j}+(-2+4)\mathbf{k}$$

$$v=0\mathbf{i}-2\mathbf{j}+2\mathbf{k}$$

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Now we'll need to find a point on the line of intersection, which we can do by setting $z = 0$ in both equations, and then solving what remains as a system of equations. If the planes are

$$2x + 2y + 2z = 3$$

$$-2x - y - z = 3$$

then setting $z = 0$ gives

$$2x + 2y = 3$$

$$-2x - y = 3$$

We can add the equations together to get

$$(2x + 2y) + (-2x - y) = 3 + 3$$

$$2x - 2x + 2y - y = 3 + 3$$

$$y = 6$$

Plugging $y = 6$ back into $2x + 2y = 3$ gives

$$2x + 2y = 3$$

$$2x + 2(6) = 3$$

$$2x + 12 = 3$$

$$2x = -9$$

$$x = -\frac{9}{2}$$

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If we put all these values together, we can say that

$$c\left(-\frac{9}{2},6,0\right)$$

is a point on the line of intersection.

Now we'll put $v=0\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ and $c(-9/2,6,0)$ into the formula for the symmetric equations for the line of intersection.

$$\frac{x-c_1}{v_1}=\frac{y-c_2}{v_2}=\frac{z-c_3}{v_3}$$

$$\frac{x-\left(-\frac{9}{2}\right)}{0}=\frac{y-6}{-2}=\frac{z-0}{2}$$

Since we cannot divide by 0, we pull out the x equation as its own parametric equation, leaving the other two equations as symmetric equations.

$$x-\left(-\frac{9}{2}\right)=0, \quad \frac{y-6}{-2}=\frac{z-0}{2}$$

$$x+\frac{9}{2}=0, \quad -\frac{y-6}{2}=\frac{z}{2}$$

$$x=-\frac{9}{2}, \quad -\frac{y-6}{2}=\frac{z}{2}$$

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Topic: Symmetric equations for the line of intersection of two planes

Question: Find the symmetric equations for the line of intersection of the planes.

$$2x + 4y + 5z = 7$$

$$3x - 5y + z = 8$$

Answer choices:

A $\quad \dfrac{x + \frac{67}{22}}{29} = \dfrac{y + \frac{5}{22}}{13} = \dfrac{z}{22}$

B $\quad \dfrac{x + \frac{67}{22}}{29} = \dfrac{y + \frac{5}{22}}{13} = -\dfrac{z}{22}$

C $\quad \dfrac{x - \frac{67}{22}}{29} = \dfrac{y - \frac{5}{22}}{13} = -\dfrac{z}{22}$

D $\quad \dfrac{x - \frac{67}{22}}{29} = \dfrac{y - \frac{5}{22}}{13} = \dfrac{z}{22}$

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Solution: C

The symmetric equations for the line of intersection are given by

$$\frac{x-c_1}{v_1}=\frac{y-c_2}{v_2}=\frac{z-c_3}{v_3}$$

where $c(c_1,c_2,c_3)$ comes from a point on the line of intersection, and where $v(v_1,v_2,v_3)$ is the cross product of the normal vectors of the planes.

The normal vectors of the planes are given by their components, which means that the normal vectors of $2x+4y+5z=7$ and $3x-5y+z=8$ are $a\langle 2,4,5\rangle$ and $b\langle 3,-5,1\rangle$, respectively. The cross product is given by

$$v=a\times b=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$$v=a\times b=\mathbf{i}\begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix}-\mathbf{j}\begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix}+\mathbf{k}\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}$$

$$v=a\times b=\left(a_2b_3-a_3b_2\right)\mathbf{i}-\left(a_1b_3-a_3b_1\right)\mathbf{j}+\left(a_1b_2-a_2b_1\right)\mathbf{k}$$

which means plugging in the normal vectors gives

$$v=\left[(4)(1)-(5)(-5)\right]\mathbf{i}-\left[(2)(1)-(5)(3)\right]\mathbf{j}+\left[(2)(-5)-(4)(3)\right]\mathbf{k}$$

$$v=(4+25)\mathbf{i}-(2-15)\mathbf{j}+(-10-12)\mathbf{k}$$

$$v=29\mathbf{i}+13\mathbf{j}-22\mathbf{k}$$

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Now we'll need to find a point on the line of intersection, which we can do by setting $z = 0$ in both equations, and then solving what remains as a system of equations. If the planes are

$$2x + 4y + 5z = 7$$

$$3x - 5y + z = 8$$

then setting $z = 0$ gives

[1] $2x + 4y = 7$

[2] $3x - 5y = 8$

We can multiply [1] by 3 and [2] by 2 to get

[3] $6x + 12y = 21$

[4] $6x - 10y = 16$

Now we can subtract [4] from [3] to get

$$(6x + 12y) - (6x - 10y) = 21 - 16$$

$$6x - 6x + 12y + 10y = 21 - 16$$

$$22y = 5$$

$$y = \frac{5}{22}$$

Plugging $y = 5/22$ back into $2x + 4y = 7$ gives

$$2x + 4y = 7$$

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$$2x+4\left(\frac{5}{22}\right)=7$$

$$2x+\frac{20}{22}=7$$

$$2x=\frac{154}{22}-\frac{20}{22}$$

$$2x=\frac{134}{22}$$

$$x=\frac{134}{44}$$

$$x=\frac{67}{22}$$

If we put all these values together, we can say that

$$c\left(\frac{67}{22},\frac{5}{22},0\right)$$

is a point on the line of intersection.

Now we'll put $v=29\mathbf{i}+13\mathbf{j}-22\mathbf{k}$ and $c\left(67/22,5/22,0\right)$ into the formula for the symmetric equations for the line of intersection.

$$\frac{x-c_1}{v_1}=\frac{y-c_2}{v_2}=\frac{z-c_3}{v_3}$$

$$\frac{x-\frac{67}{22}}{29}=\frac{y-\frac{5}{22}}{13}=\frac{z-0}{-22}$$

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$$\frac{x-\frac{67}{22}}{29}=\frac{y-\frac{5}{22}}{13}=-\frac{z}{22}$$

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