Topic: Iterated integrals

Question: Evaluate the iterated integral.

$$\int_0^1 \int_0^1 x^3y^2 + e^y \ dy \ dx$$

Answer choices:

A $\quad e - \frac{11}{12}$

B $\quad e + \frac{11}{12}$

C $\quad e + 1$

D $\quad e - 1$

Solution: A

When we evaluate an iterated integral, we always start on the inside and work our way out. Since dy is on the inside and dx is on the outside, we'll start by integrating with respect to y. When we integrate with respect to y, we have to treat x like a constant.

$$\int_0^1 \int_0^1 x^3y^2 + e^y \ dy \ dx$$

$$\int_0^1 x^3 \left(\frac{y^3}{3}\right) + e^y \Bigg|_{y=0}^{y=1} \ dx$$

$$\int_0^1 \frac{x^3y^3}{3} + e^y \Bigg|_{y=0}^{y=1} \ dx$$

Now we can evaluate over the interval $[0,1]$.

$$\int_0^1 \left[\frac{x^3(1)^3}{3} + e^{(1)}\right] - \left[\frac{x^3(0)^3}{3} + e^{(0)}\right] \ dx$$

$$\int_0^1 \left(\frac{x^3}{3} + e\right) - (1) \ dx$$

$$\int_0^1 \frac{x^3}{3} + e - 1 \ dx$$

Now we'll integrate with respect to x and evaluate over the interval $[0,1]$.

$$\frac{x^4}{12} + xe - x \Big|_0^1$$

$$\left[\frac{(1)^4}{12} + (1)e - (1)\right] - \left[\frac{(0)^4}{12} + (0)e - (0)\right]$$

$$\frac{1}{12} + e - 1$$

$$-\frac{11}{12} + e$$

$$e - \frac{11}{12}$$

This is the volume given by the iterated integral.

Topic: Iterated integrals

Question: Evaluate the iterated integral.

$$\int_0^3 \int_1^2 2x^3e^{2y} - 3x^2y \; dy \; dx$$

Answer choices:

A -915.484

B $1,911.683$

C $-1,911.683$

D 915.484

Solution: D

When we evaluate an iterated integral, we always start on the inside and work our way out. Since dy is on the inside and dx is on the outside, we'll start by integrating with respect to y. When we integrate with respect to y, we have to treat x like a constant.

$$\int_0^3 \int_1^2 2x^3e^{2y} - 3x^2y\ dy\ dx$$

$$\int_0^3 2x^3\left(\frac{e^{2y}}{2}\right) - 3x^2\left(\frac{y^2}{2}\right)\Bigg|_{y=1}^{y=2} dx$$

Now we can evaluate over the interval $[1,2]$.

$$\int_0^3 2x^3\left(\frac{e^{2(2)}}{2}\right) - 3x^2\left(\frac{(2)^2}{2}\right) - \left[2x^3\left(\frac{e^{2(1)}}{2}\right) - 3x^2\left(\frac{(1)^2}{2}\right)\right]\ dx$$

$$\int_0^3 x^3e^4 - 6x^2 - \left(x^3e^2 - \frac{3x^2}{2}\right)\ dx$$

$$\int_0^3 x^3e^4 - 6x^2 - x^3e^2 + \frac{3x^2}{2}\ dx$$

$$\int_0^3 x^3e^4 - \frac{12x^2}{2} - x^3e^2 + \frac{3x^2}{2}\ dx$$

$$\int_0^3 \left(e^4 - e^2\right)x^3 - \frac{9x^2}{2}\ dx$$

Now we'll integrate with respect to x and evaluate over the interval $[0,3]$.

$$\left(e^4 - e^2\right)\frac{x^4}{4} - \frac{3x^3}{2}\Big|_0^3$$

$$\left(e^4 - e^2\right)\frac{(3)^4}{4} - \frac{3(3)^3}{2} - \left[\left(e^4 - e^2\right)\frac{(0)^4}{4} - \frac{3(0)^3}{2}\right]$$

$$\left(e^4 - e^2\right)\frac{81}{4} - \frac{81}{2}$$

$$\left(e^4 - e^2\right)\frac{81}{4} - (2)\frac{81}{4}$$

$$\left(e^4 - e^2 - 2\right)\frac{81}{4}$$

$$915.48$$

This is the volume given by the iterated integral.

Topic: Iterated integrals

Question: Find the area bounded by the given curves.

$$3 + 2\sin x$$

$$3 + 2\cos x$$

$$x = \frac{\pi}{3}$$

$$x = \frac{5\pi}{6}$$

Answer choices:

A $2\left(2 + 2\sqrt{3}\right)$

B $2\left(2 + 2\sqrt{2}\right)$

C $2\sqrt{3}$

D $\sqrt{3}$

Solution: C

Let $dy\ dx$ be the order of the integration. Then the area of the region is given by the iterated integral

$$A = \int_{\frac{\pi}{3}}^{\frac{5\pi}{6}} \int_{3+2\cos x}^{3+2\sin x} dy\ dx$$

Integrate with respect to y, then evaluate over the interval.

$$A = \int_{\frac{\pi}{3}}^{\frac{5\pi}{6}} y \Big|_{y=3+2\cos x}^{y=3+2\sin x} dx$$

$$A = \int_{\frac{\pi}{3}}^{\frac{5\pi}{6}} 3 + 2\sin x - (3 + 2\cos x)\ dx$$

$$A = \int_{\frac{\pi}{3}}^{\frac{5\pi}{6}} 3 + 2\sin x - 3 - 2\cos x\ dx$$

$$A = \int_{\frac{\pi}{3}}^{\frac{5\pi}{6}} 2\sin x - 2\cos x\ dx$$

Now integrate with respect to x, then evaluate over the interval.

$$A = -2\cos x - 2\sin x \Big|_{\frac{\pi}{3}}^{\frac{5\pi}{6}}$$

$$A = -2\cos\frac{5\pi}{6} - 2\sin\frac{5\pi}{6} - \left(-2\cos\frac{\pi}{3} - 2\sin\frac{\pi}{3}\right)$$

$$A = -2\left(-\frac{\sqrt{3}}{2}\right) - 2\left(\frac{1}{2}\right) - \left[-2\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$A = \sqrt{3} - 1 - \left(-1 - \sqrt{3}\right)$$

$$A = \sqrt{3} - 1 + 1 + \sqrt{3}$$

$$A = 2\sqrt{3}$$