Projections of the curve

Sometimes the easiest way to sketch a three-dimensional curve is to sketch its projections on the xy-, xz-, and yz-coordinate planes.

Think about the projections of a curve as the shadows they cast against the coordinate planes. You can also think about them as the view of the curve from the coordinate planes. In other words, if you're standing squarely parallel to the xy-coordinate plane, what you see of the curve is the projection of the curve on the xy-coordinate plane.

Once we have the projections of the curve on each of the coordinate planes, we can use them to draw the three-dimensional graph.

Example

Sketch the projections of the curve and use them to sketch the three-dimensional curve.

$$r(t) = \left\langle t, t^2, t^2 + 1 \right\rangle$$

We'll convert the vector function to three parametric equations.

$$x = t$$

$$y = t^2$$

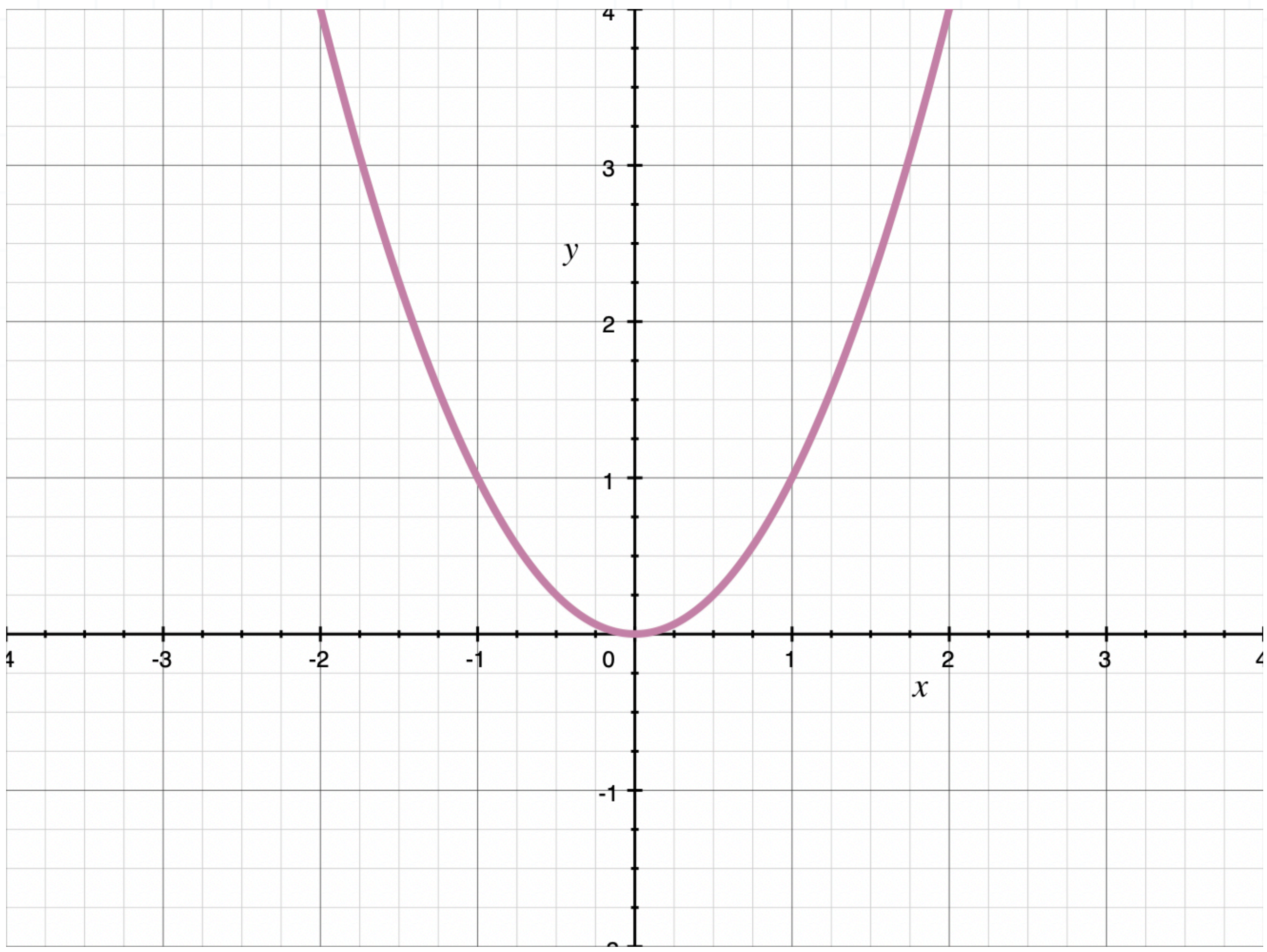
$$z = t^2 + 1$$

To find the projection on the xy-coordinate plane, we need to find an equation in terms of only x and y, which we'll do by plugging $x = t$ into $y = t^2$.

$$y = t^2$$

$$y = x^2$$

We'll sketch this curve in the xy-coordinate plane.

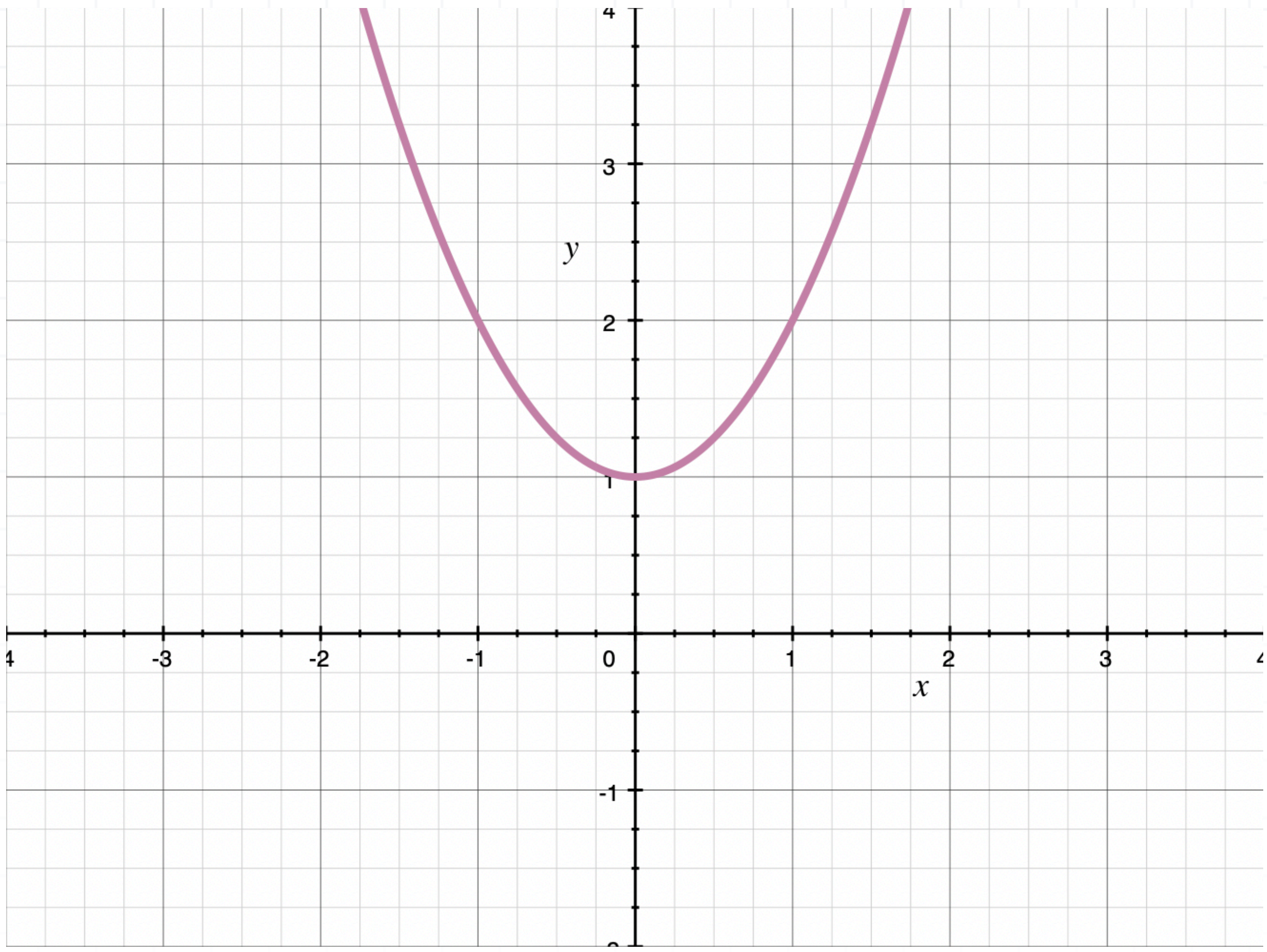


To find the projection on the xz-coordinate plane, we need to find an equation in terms of only x and z, which we'll do by plugging $x = t$ into $z = t^2 + 1$.

$$z = t^2 + 1$$

$$z = x^2 + 1$$

We'll sketch this curve in the xz-coordinate plane.

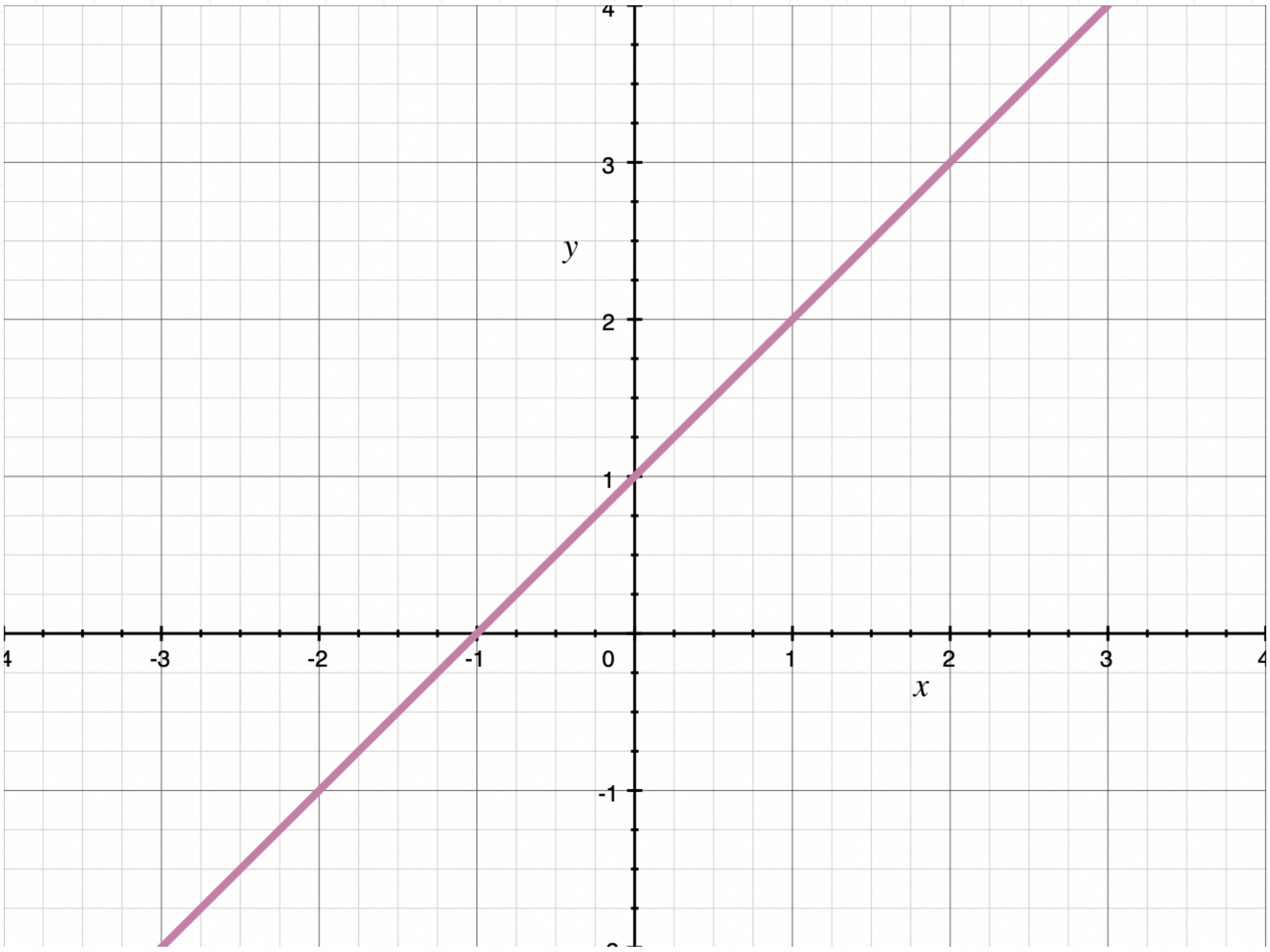


To find the projection on the yz-coordinate plane, we need to find an equation in terms of only y and z, which we'll do by plugging $y = t^2$ into $z = t^2 + 1$.

$$z = t^2 + 1$$

$$z = y + 1$$

We'll sketch this curve in the yz-coordinate plane.



Our final step is to use the projections to sketch the three-dimensional curve. We need a starting point. To find it, we'll set $t = 0$ in our parametric equations, and get

$$x = t$$

$$x = 0$$

and

$$y = t^2$$

$$y = 0^2$$

$$y = 0$$

and

$$z = t^2 + 1$$

$$z = 0^2 + 1$$

$$z = 1$$

Putting these values together, we get the point $(0,0,1)$. This means that our graph starts at $(0,0,1)$ and travels upwards in a parabolic shape from the xy- and xz-planar perspective. The three-dimensional graph is