Calculus 3 Final Exam

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MATH

Calculus 3 Final Exam

This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** Find the partial derivative f_z for $f(x, y, z) = 3x^2e^y \ln 2z$.

A $f_z = \dfrac{6x^2e^y}{z}$

B $f_z = \dfrac{3x^2e^y}{z}$

C $f_z = \dfrac{6xe^y}{z}$

D $f_z = \dfrac{12xe^y}{z}$

E $f_z = 12xe^y \ln z$

2. **(5 pts)** If $x = 2t - 5t^3$ and $y = t^2$, find dz/dt for $z = x - y^2$.

A $\dfrac{dz}{dt} = -19t^2 + 2$

B $\dfrac{dz}{dt} = -4t^3 - 15t^2 + 2$

C $\dfrac{dz}{dt} = -15t^2 - 4t + 2$

D $\dfrac{dz}{dt} = -15t^2 + 2t + 2$

E $\dfrac{dz}{dt} = -2t^2$

3. **(5 pts)** Find the gradient vector of $f(x,y) = 3x^2 + 3xy$ at $P(-1,2)$.

A $\langle -6,3 \rangle$

B $\langle 3,-3 \rangle$

C $\langle 0,-3 \rangle$

D $\langle -3,0 \rangle$

E $\langle 3,-6 \rangle$

4. **(5 pts)** Find the average value of the function $f(x,y,z) = 3x^2yz$ over a cube with side length 3, lying in the first octant with one vertex at $(0,0,0)$ and three sides on the coordinate axes.

A $\frac{81}{4}$

B $\frac{9}{2}$

C $\frac{4}{27}$

D 9

E $\frac{1}{27}$

5. **(5 pts)** Find the vector orthogonal to the plane that includes the vectors $\overrightarrow{a} = \langle 1,2,3 \rangle$ and $\overrightarrow{b} = \langle -1,2,1 \rangle$.

A $\langle -1, -1, -1 \rangle$

B $\langle 1,1,1 \rangle$

C $\langle -1, -1,1 \rangle$

D $\langle 1, -1, -1 \rangle$

E $\langle -2, -1,1 \rangle$

6. **(5 pts)** Find the unit tangent vector $T(t)$ to the curve $r(t) = (2 + t^2)\mathbf{i} + e^{-t}\mathbf{j} + \cos 2t\mathbf{k}$ where $t = 0$.

A $T(t) = -\mathbf{i}$

B $T(t) = \mathbf{i}$

C $T(t) = \mathbf{j}$

D $T(t) = -\mathbf{k}$

E $T(t) = -\mathbf{j}$

7. **(5 pts)** Find the arc length of the vector function $r(t) = 2\cos t\mathbf{i} - 2\sin t\mathbf{j} + t\mathbf{k}$ where $0 \leq t \leq 2\pi$.

A	$\sqrt{5}$	C	5	E	$\sqrt{5}\pi$
B	$2\sqrt{5}$	D	$2\sqrt{5}\pi$		

8. **(5 pts)** A ball is thrown into the air with initial velocity $v(t_0) = 6\mathbf{i} + 16\mathbf{k}$. The acceleration is given by $a(t) = 16\mathbf{j} - 32\mathbf{k}$. How far away is the ball from its initial position at $t = 1$?

A	8	C	6	E	14
B	10	D	$\sqrt{14}$		

9. **(15 pts)** Find the parametric equations for the line of intersection of the planes $x + y - z = 6$ and $2x - 3y + z = 7$.

10. **(15 pts)** Find the extrema of the function $f(x, y) = x^2 + y^2 - 1$ subject to the constraint $2x + y = 10$.

11. **(15 pts)** Use cylindrical coordinates to find the volume of the triple integral where E is the solid that lies within the cylinder $x^2 + y^2 = 9$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

$$\iiint_E 2 \, dV$$

12. **(15 pts)** Determine the curvature of $r(t) = 3t\mathbf{i} + 4\sin t\mathbf{j} - 4\cos t\mathbf{k}$.