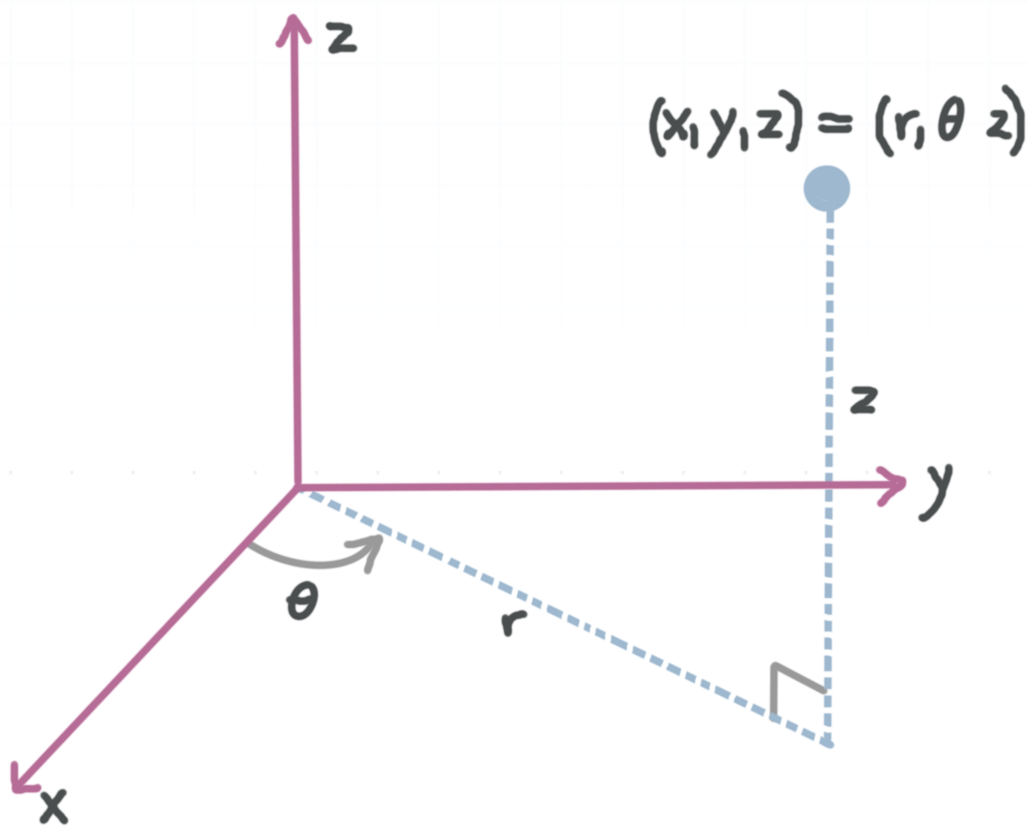
Cylindrical coordinates

Like cartesian (or rectangular) coordinates and polar coordinates, cylindrical coordinates are just another way to describe points in three-dimensional space.

Cylindrical coordinates are like polar coordinates, just in three-dimensional space instead of two-dimensional space. Since polar coordinates in two dimensions are given as (r, θ), cylindrical coordinates have us add a value for z to account for the third dimension, so cylindrical coordinates are given as (r, θ, z).



So if **rectangular coordinates** are given as (x, y, z), where x is the distance of (x, y, z) from the origin along the x-axis, y is the distance of (x, y, z) from the origin along the y-axis, and z is the distance of (x, y, z) from the origin along the z-axis, then **cylindrical coordinates** are given as (r, θ, z), where r is the distance of $(r, \theta, 0)$ from the origin, θ is the angle between r (the line connecting (r, θ, z) to the origin) and the positive direction of the x-axis, and z is the the distance of (r, θ, z) from the origin along the z-axis.

To convert between cylindrical coordinates and rectangular coordinates, we'll use

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

Let's try an example where we convert rectangular coordinates to cylindrical coordinates.

Example

Convert the rectangular point $(1,1,1)$ to cylindrical coordinates.

We'll plug $(1,1,1)$ into the conversion formulas to get

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

$$1 = r\cos\theta \qquad 1 = r\sin\theta \qquad 1 = z$$

$$r = \frac{1}{\cos\theta} \qquad r = \frac{1}{\sin\theta}$$

Because we have two equations both defined for r, we can set them equal to each other.

$$\frac{1}{\cos\theta} = \frac{1}{\sin\theta}$$

$$\cos\theta = \sin\theta$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

Since we found two values for θ, we'll have two cylindrical points that can represent the given rectangular point.

For $\theta = \pi/4$,

$$r = \frac{1}{\cos\frac{\pi}{4}}$$

$$r = \frac{1}{\frac{\sqrt{2}}{2}}$$

$$r = \frac{2}{\sqrt{2}}$$

$$r = \sqrt{2}$$

For $\theta = 5\pi/4$,

$$r = \frac{1}{\cos\frac{5\pi}{4}}$$

$$r = \frac{1}{-\frac{\sqrt{2}}{2}}$$

$$r = -\frac{2}{\sqrt{2}}$$

$$r = -\sqrt{2}$$

So the rectangular point $(1,1,1)$ is equivalent to the cylindrical points

$$\left(\sqrt{2},\frac{\pi}{4},1\right)$$

$$\left(-\sqrt{2},\frac{5\pi}{4},1\right)$$

Let’s try an example where we convert cylindrical coordinates to rectangular coordinates.

Example

Convert the cylindrical point $(2,\pi,3)$ to a rectangular point.

From the cylindrical coordinate $(r,\theta,z)=(2,\pi,3)$, we know $r=2$, $\theta=\pi$, and $z=3$, so from our conversion formulas we find

$$x=r\cos\theta \qquad y=r\sin\theta \qquad z=z$$

$$x=2\cos\pi \qquad y=2\sin\pi \qquad z=3$$

$$x=2(-1) \qquad y=2(0)$$

$$x=-2 \qquad y=0$$

Putting these values together, we can say that the cylindrical point $(2,\pi,3)$ is the same as the rectangular point $(-2,0,3)$.