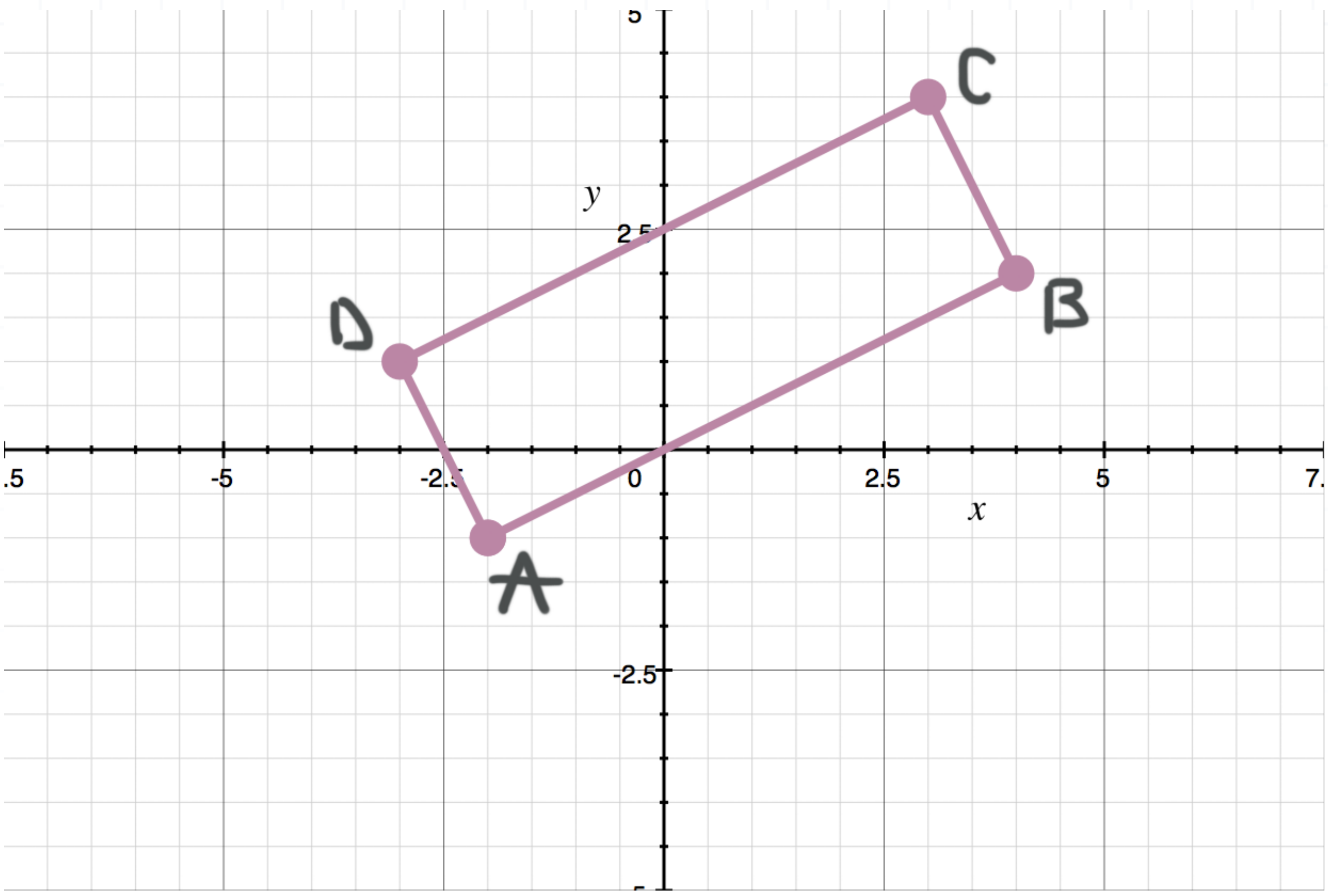
Calculus 3 Workbook

Change of variables

krista king MATH

JACOBIAN FOR TWO VARIABLES

1. Find the Jacobian of the transformation that rotates the rectangle $ABCD$, given by $A(-2,-1)$, $B(4,2)$, $C(3,4)$, and $D(-3,1)$, clockwise about the origin in such a way that AB will lie on the x-axis.



2. Find the Jacobian of the transformation which converts the ellipse $5\sqrt{2}x^2 + 6\sqrt{2}xy + 8x + 5\sqrt{2}y^2 - 8y = 0$ into the ellipse with center at the origin, and x- and $y$semi-axes 2 and 1 respectively. Use a rotation counterclockwise by $\pi/4$, and then move it by 2 to the positive direction of the x-axis.

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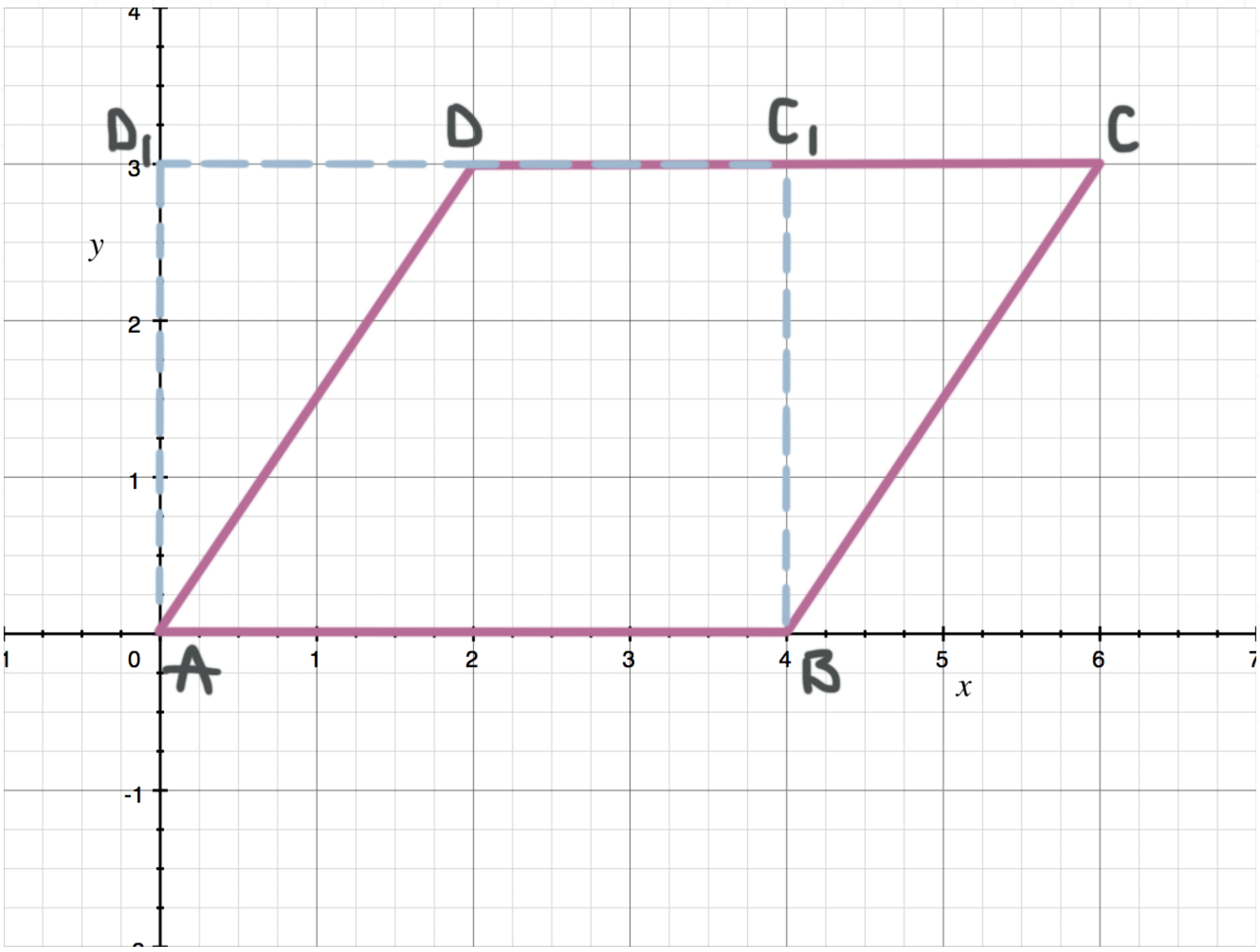
3. Find the Jacobian of the linear transformation that converts the parallelogram $ABCD$, given by $A(0,0)$, $B(4,0)$, $C(6,3)$, $D(2,3)$, into the rectangle ABC_1D_1 with the same base and height.



JACOBIAN FOR THREE VARIABLES

■ 1. Find the Jacobian of the transformation that rotates the space clockwise about the y-axis by $\pi/6$.

■ 2. Find the Jacobian of the transformation to spherical coordinates that converts the ellipsoid to the unit sphere (a sphere with center at the origin and radius 1).

$$\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{9} = 1$$

■ 3. Find the following transformations:

1. the transformation that converts the given cylinder to a circular cylinder with radius 1 and an axis parallel to the z-axis

$$\frac{x^2}{5} + \frac{z^2}{4} = 1$$

2. the transformation that converts to cylindrical coordinates

Then write down the composition of these two transformations and find its Jacobian.

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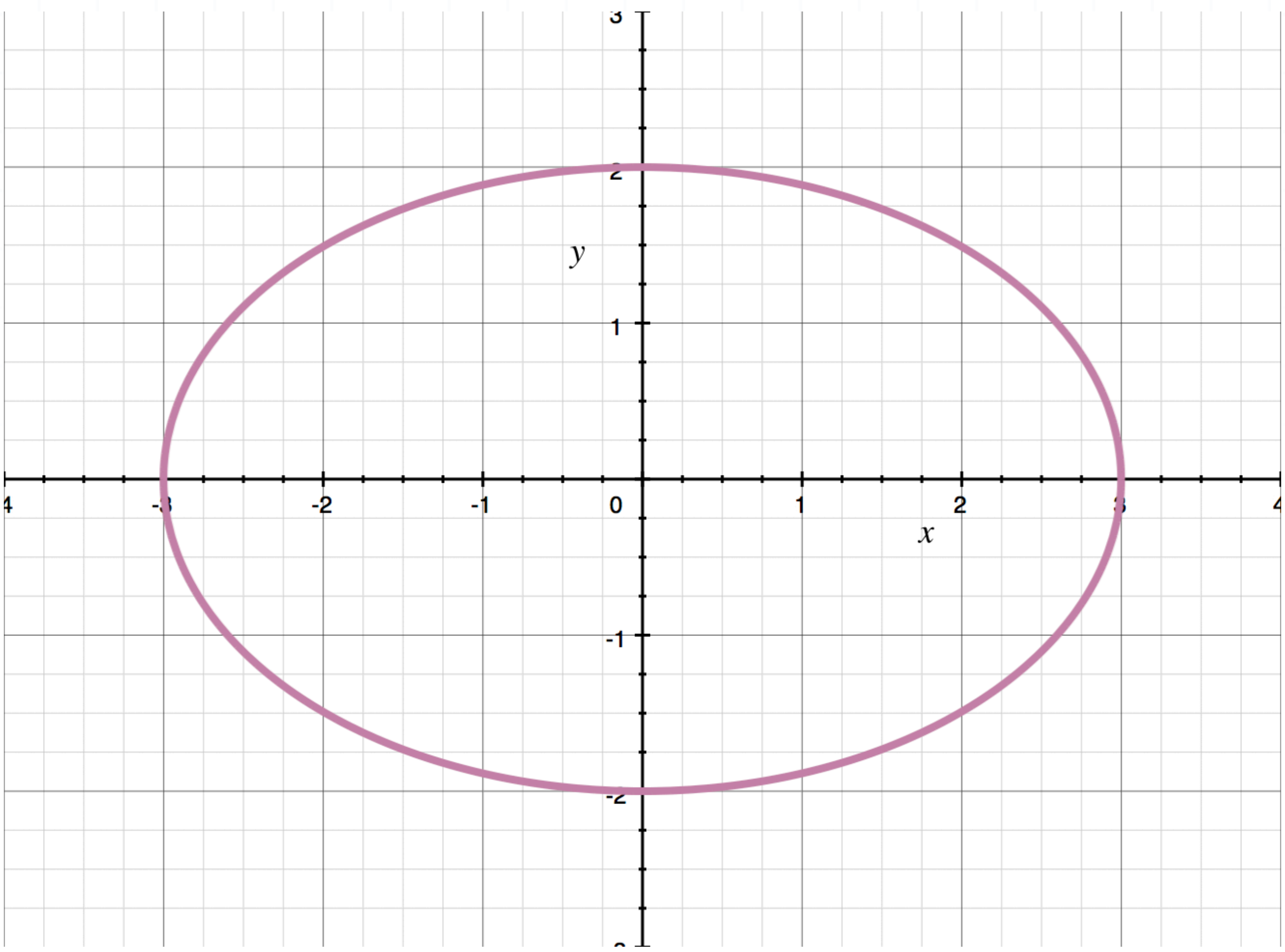
EVALUATING DOUBLE INTEGRALS

■ 1. Find the Jacobian of the transformation and use it to find the area of the ellipse with center at the origin, semi-major axis along the x-axis with length 3, and semi-minor axis along the y-axis with length 2.

$$x = 3r\cos\phi$$

$$y = 2r\sin\phi$$



■ 2. Find the Jacobian of the transformation and use it to find the double integral of the function $f(x, y) = x^2 + y^2$ over the circle with center at $(-2,3)$ and radius 2.

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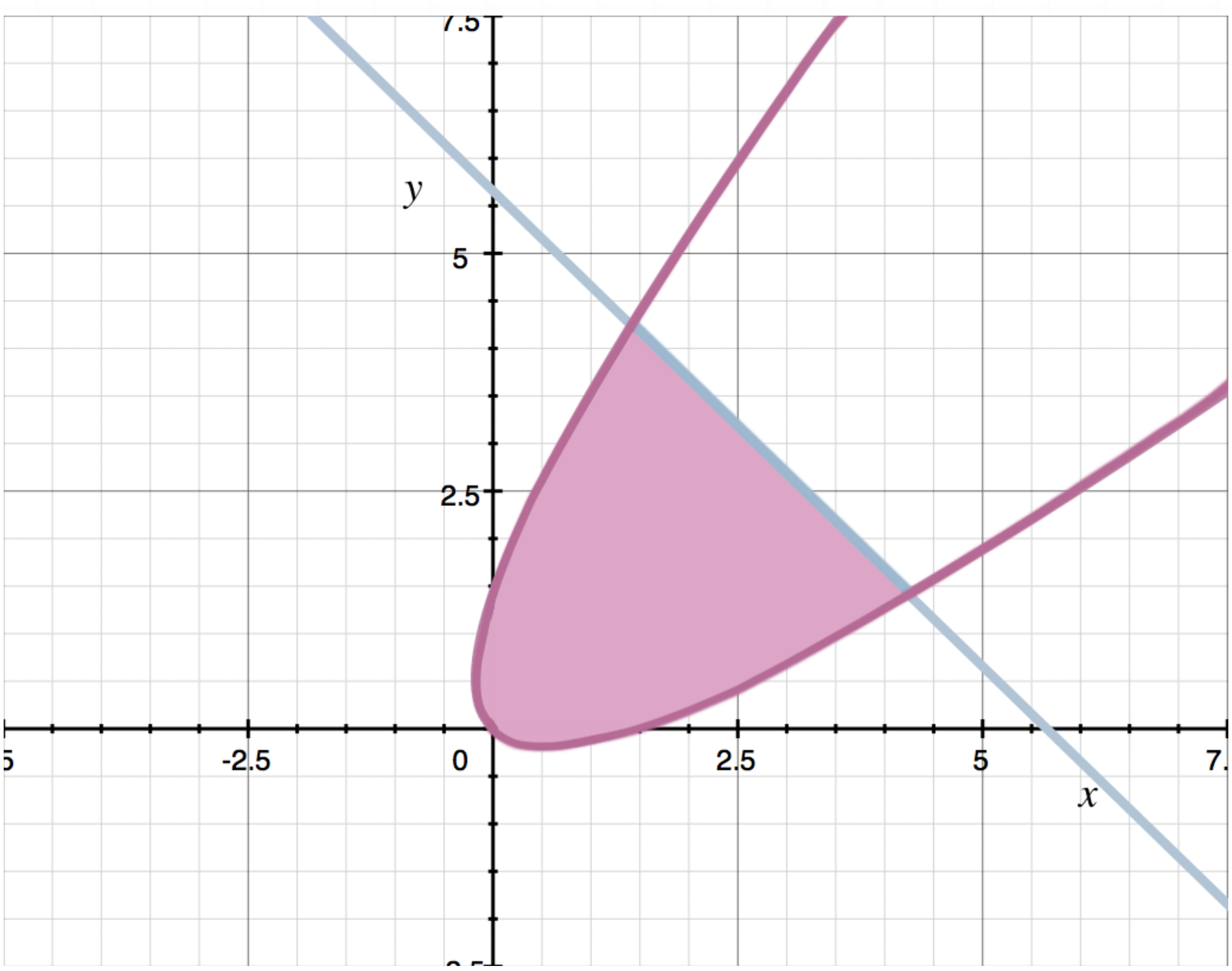
$$x = -2 + r\cos\phi$$

$$y = 3 + r\sin\phi$$

■ 3. Find the Jacobian of the transformation, and use it to find the area bounded by the curves $x^2 - 2xy + y^2 - \sqrt{2}x - \sqrt{2}y = 0$ and $x + y = 4\sqrt{2}$.

$$x = \frac{\sqrt{2}}{2}(u + v)$$

$$y = \frac{\sqrt{2}}{2}(v - u)$$



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EQUATIONS OF THE TRANSFORMATION

■ 1. Identify the equation obtained from $f(x, y, z) = x^2 + y^2 + z^2 - 2$ by applying the transformation.

$$x = \sin u + \cos u$$

$$y = \sin u - \cos u$$

$$z = \sqrt{u + v + w}$$

■ 2. Find the inverse transformation and determine its Jacobian.

$$u = x - 2y + 1$$

$$v = -3x + y + 2$$

■ 3. Find the inverse transformation $x(r, \phi)$ and $y(r, \phi)$ and determine its Jacobian.

$$r = \sqrt{\frac{x^2 + y^2}{4}}$$

$\phi = \arctan \dfrac{y}{x}$ for $x \neq 0$

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IMAGE OF THE SET UNDER THE TRANSFORMATION

■ 1. Identify the surface obtained from the unit sphere (a sphere with center at the origin and radius 1 by applying the transformation.

$$x = u - 2v + 2$$

$$y = u + v + w$$

$$z = u + v - w + 4$$

■ 2. Identify the shape obtained from the parallelogram $ABCD$, where $A(-2,2)$, $B(-2,5)$, $C(-3,6)$, and $D(-3,3)$, by applying the transformation.

$$u = -x - 2$$

$$v = \frac{x + y}{3}$$

■ 3. Identify the solid obtained from the set of interior points of the circular cylinder $x^2 + y^2 = 9$, with $1 \le z \le 5$, when the following transformation is applied.

$$u = \sqrt{x^2 + y^2}$$

$$v = \arctan \frac{y}{x}$$

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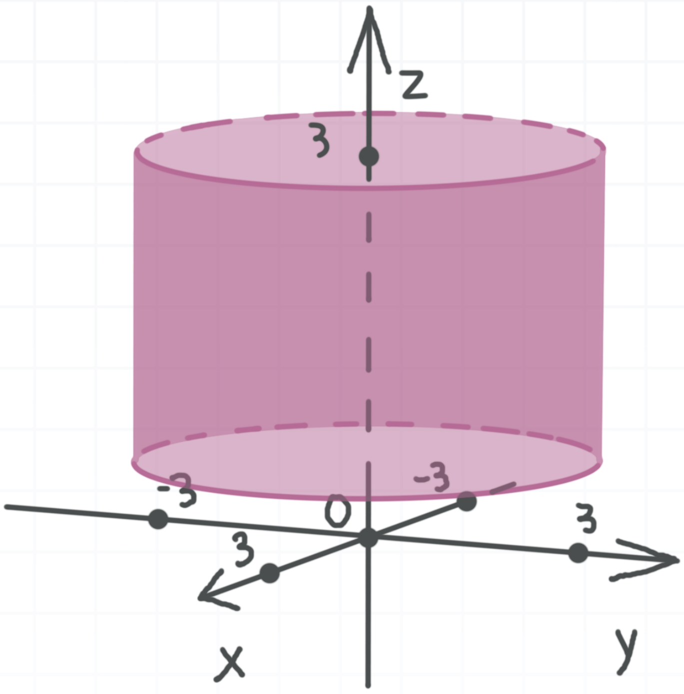
$w = z - 1$



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