Topic: Limit of a vector function

Question: Find the limit of the vector function.

$$\lim_{t \to 0}\left((t+4)\mathbf{i}+3\mathbf{j}+\frac{2t}{\sin t}\mathbf{k}\right)$$

Answer choices:

A $4\mathbf{i}+3\mathbf{j}+2\mathbf{k}$

B $4\mathbf{i}+3\mathbf{j}$

C $-4\mathbf{i}-3\mathbf{j}$

D $-4\mathbf{i}-3\mathbf{j}-2\mathbf{k}$

Solution: A

We'll rewrite the limit as three separate limits.

$$\lim_{t\to 0}\left((t+4)\mathbf{i}+3\mathbf{j}+\frac{2t}{\sin t}\mathbf{k}\right)$$

$$\lim_{t\to 0}(t+4)\mathbf{i}+\lim_{t\to 0}3\mathbf{j}+\lim_{t\to 0}\frac{2t}{\sin t}\mathbf{k}$$

Evaluate the first limit.

$$(0+4)\mathbf{i}+\lim_{t\to 0}3\mathbf{j}+\lim_{t\to 0}\frac{2t}{\sin t}\mathbf{k}$$

$$4\mathbf{i}+\lim_{t\to 0}3\mathbf{j}+\lim_{t\to 0}\frac{2t}{\sin t}\mathbf{k}$$

Evaluate the second limit.

$$4\mathbf{i}+3\mathbf{j}+\lim_{t\to 0}\frac{2t}{\sin t}\mathbf{k}$$

If we evaluate the last limit at $t=0$, we'll get a $0/0$ value, which is indeterminate. So we'll use L'Hospital's rule to simplify the function, and then we'll evaluate the limit.

$$4\mathbf{i}+3\mathbf{j}+\lim_{t\to 0}\frac{2}{\cos t}\mathbf{k}$$

$$4\mathbf{i}+3\mathbf{j}+\frac{2}{\cos(0)}\mathbf{k}$$

$$4\mathbf{i}+3\mathbf{j}+\frac{2}{1}\mathbf{k}$$

$$4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$

This is the limit of the vector function.

Topic: Limit of a vector function

Question: Find the limit of the vector function.

$$\lim_{t \to 0} \left(\ln(2t+e)\mathbf{i} + \frac{6}{\cos t}\mathbf{j} + (t^3 - 1)\mathbf{k} \right)$$

Answer choices:

A $\mathbf{i} - \mathbf{k}$

B $\mathbf{i} + 6\mathbf{j} - \mathbf{k}$

C $-\mathbf{i} - 6\mathbf{j} + \mathbf{k}$

D $-\mathbf{i} + \mathbf{k}$

Solution: B

We'll rewrite the limit as three separate limits.

$$\lim_{t\to 0}\left(\ln(2t+e)\mathbf{i}+\frac{6}{\cos t}\mathbf{j}+(t^3-1)\mathbf{k}\right)$$

$$\lim_{t\to 0}\ln(2t+e)\mathbf{i}+\lim_{t\to 0}\frac{6}{\cos t}\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

Evaluate the first limit.

$$\ln(2(0)+e)\mathbf{i}+\lim_{t\to 0}\frac{6}{\cos t}\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

$$\ln(e)\mathbf{i}+\lim_{t\to 0}\frac{6}{\cos t}\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

$$1\mathbf{i}+\lim_{t\to 0}\frac{6}{\cos t}\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

$$\mathbf{i}+\lim_{t\to 0}\frac{6}{\cos t}\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

Evaluate the second limit.

$$\mathbf{i}+\frac{6}{\cos(0)}\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

$$\mathbf{i}+\frac{6}{1}\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

$$\mathbf{i}+6\mathbf{j}+\lim_{t\to 0}(t^3-1)\mathbf{k}$$

Evaluate the third limit.

$$\mathbf{i} + 6\mathbf{j} + (0^3 - 1)\mathbf{k}$$

$$\mathbf{i} + 6\mathbf{j} - 1\mathbf{k}$$

This is the limit of the vector function.

Topic: Limit of a vector function

Question: Find the limit of the vector function.

$$\lim_{t\to 0}\left(\ln(t^2+e)\mathbf{i}+(4t^2+2)\mathbf{j}+\frac{6t}{\sin(2t)}\mathbf{k}\right)$$

Answer choices:

A $-\mathbf{i}-2\mathbf{j}-6\mathbf{k}$

B $\mathbf{i}+2\mathbf{j}+6\mathbf{k}$

C $-\mathbf{i}-2\mathbf{j}-3\mathbf{k}$

D $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$

Solution: D

We'll rewrite the limit as three separate limits.

$$\lim_{t \to 0}\left(\ln(t^2+e)\mathbf{i}+(4t^2+2)\mathbf{j}+\frac{6t}{\sin(2t)}\mathbf{k}\right)$$

$$\lim_{t \to 0}\ln(t^2+e)\mathbf{i}+\lim_{t \to 0}(4t^2+2)\mathbf{j}+\lim_{t \to 0}\frac{6t}{\sin(2t)}\mathbf{k}$$

Evaluate the first limit.

$$\ln(0^2+e)\mathbf{i}+\lim_{t \to 0}(4t^2+2)\mathbf{j}+\lim_{t \to 0}\frac{6t}{\sin(2t)}\mathbf{k}$$

$$\ln(e)\mathbf{i}+\lim_{t \to 0}(4t^2+2)\mathbf{j}+\lim_{t \to 0}\frac{6t}{\sin(2t)}\mathbf{k}$$

$$1\mathbf{i}+\lim_{t \to 0}(4t^2+2)\mathbf{j}+\lim_{t \to 0}\frac{6t}{\sin(2t)}\mathbf{k}$$

$$\mathbf{i}+\lim_{t \to 0}(4t^2+2)\mathbf{j}+\lim_{t \to 0}\frac{6t}{\sin(2t)}\mathbf{k}$$

Evaluate the second limit.

$$\mathbf{i}+(4(0)^2+2)\mathbf{j}+\lim_{t \to 0}\frac{6t}{\sin(2t)}\mathbf{k}$$

$$\mathbf{i}+2\mathbf{j}+\lim_{t \to 0}\frac{6t}{\sin(2t)}\mathbf{k}$$

If we evaluate the last limit at $t = 0$, we'll get a $0/0$ value, which is indeterminate. So we'll use L'Hospital's rule to simplify the function, and then we'll evaluate the limit.

$$\mathbf{i} + 2\mathbf{j} + \lim_{t \to 0} \frac{6}{2\cos(2t)}\mathbf{k}$$

$$\mathbf{i} + 2\mathbf{j} + \frac{6}{2\cos(2(0))}\mathbf{k}$$

$$\mathbf{i} + 2\mathbf{j} + \frac{6}{2(1)}\mathbf{k}$$

$$\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

This is the limit of the vector function.