Topic: Direction cosines and direction angles

Question: Find the direction cosines of $a = \langle 5,6 \rangle$.

Answer choices:

A $\cos\alpha = \frac{5}{\sqrt{61}}$ and $\cos\beta = \frac{6}{\sqrt{61}}$

B $\cos\alpha = \frac{5}{6}$ and $\cos\beta = \frac{6}{5}$

C $\cos\alpha = \frac{5}{\sqrt{11}}$ and $\cos\beta = \frac{6}{\sqrt{11}}$

D $\cos\alpha = \frac{5}{\sqrt{121}}$ and $\cos\beta = \frac{6}{\sqrt{121}}$

Solution: A

We'll start by finding the magnitude of a.

$$|a| = \sqrt{a_1^2 + a_2^2}$$

$$|a| = \sqrt{5^2 + 6^2}$$

$$|a| = \sqrt{25 + 36}$$

$$|a| = \sqrt{61}$$

Then the direction cosines of a are

$$\cos\alpha = \frac{a_1}{|a|} = \frac{5}{\sqrt{61}}$$

$$\cos\beta = \frac{a_2}{|a|} = \frac{6}{\sqrt{61}}$$

Topic: Direction cosines and direction angles

Question: Find the direction angles, in degrees, of $a = \langle -1, -2,3 \rangle$.

Answer choices:

A $\alpha \approx 105.5^\circ$, $\beta \approx 122.3^\circ$, and $\gamma \approx 143.3^\circ$

B $\alpha \approx 74.5^\circ$, $\beta \approx 57.7^\circ$, and $\gamma \approx 36.7^\circ$

C $\alpha \approx 105.5^\circ$, $\beta \approx 122.3^\circ$, and $\gamma \approx 36.7^\circ$

D $\alpha \approx 74.5^\circ$, $\beta \approx 57.7^\circ$, and $\gamma \approx 143.3^\circ$

Solution: C

We find the magnitude of the vector a using the distance formula.

$$|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$|a| = \sqrt{(-1)^2 + (-2)^2 + 3^2}$$

$$|a| = \sqrt{1 + 4 + 9}$$

$$|a| = \sqrt{14}$$

Plugging the vector's components and magnitude into the direction cosine formulas, we get

$$\cos\alpha = \frac{-1}{\sqrt{14}} \qquad \cos\beta = \frac{-2}{\sqrt{14}} \qquad \cos\gamma = \frac{3}{\sqrt{14}}$$

Now that we have the direction cosines, we can apply the inverse cosine to both sides of each equation to find the direction angles.

$$\alpha = \arccos\frac{-1}{\sqrt{14}} \qquad \beta = \arccos\frac{-2}{\sqrt{14}} \qquad \gamma = \arccos\frac{3}{\sqrt{14}}$$

$$\alpha \approx 105.5^\circ \qquad \beta \approx 122.3^\circ \qquad \gamma \approx 36.7^\circ$$

Topic: Direction cosines and direction angles

Question: Find the direction angles, in degrees, of the vector $m = 7\mathbf{i} - \mathbf{j} - 9\mathbf{k}$.

Answer choices:

A $\alpha \approx 127.7^\circ$, $\beta \approx 85.0^\circ$, and $\gamma \approx = 38.2^\circ$

B $\alpha \approx 52.3^\circ$, $\beta \approx 95.0^\circ$, and $\gamma \approx 141.8^\circ$

C $\alpha \approx 127.7^\circ$, $\beta \approx 95.0^\circ$, and $\gamma \approx 141.8^\circ$

D $\alpha \approx 52.3^\circ$, $\beta \approx 85.0^\circ$, and $\gamma \approx 38.2^\circ$

Solution: B

We find the magnitude of the vector m using the distance formula.

$$|m| = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

$$|m| = \sqrt{7^2 + (-1)^2 + (-9)^2}$$

$$|m| = \sqrt{49 + 1 + 81}$$

$$|m| = \sqrt{131}$$

Plugging the vector's components and magnitude into the direction cosine formulas, we get

$$\cos\alpha = \frac{7}{\sqrt{131}} \qquad \cos\beta = \frac{-1}{\sqrt{131}} \qquad \cos\gamma = \frac{-9}{\sqrt{131}}$$

Now that we have the direction cosines, we can apply the inverse cosine to both sides of each equation to find the direction angles.

$$\alpha = \arccos\frac{7}{\sqrt{131}} \qquad \beta = \arccos\frac{-1}{\sqrt{131}} \qquad \gamma = \arccos\frac{-9}{\sqrt{131}}$$

$$\alpha \approx 52.3^\circ \qquad \beta \approx 95.0^\circ \qquad \gamma \approx 141.8^\circ$$