Maximum curvature

Before we can find maximum curvature of a vector function $r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$, we first have to find curvature $\kappa(t)$. To find the curvature $\kappa(t)$ of a vector function $r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$, we'll use the equation

$$\kappa(t) = \frac{\left|T'(t)\right|}{\left|r'(t)\right|}$$

where $\left|T'(t)\right|$ is the magnitude of the derivative of the unit tangent vector $T(t)$, which we can find using

$$\left|T'(t)\right| = \sqrt{\left[T'(t)_1\right]^2 + \left[T'(t)_2\right]^2 + \left[T'(t)_3\right]^2}$$

where $T(t)$ is the unit tangent vector, which we can find using

$$T(t) = \frac{r'(t)}{\left|r'(t)\right|}$$

where $r'(t)$ is the derivative of the vector function and where $\left|r'(t)\right|$ is the magnitude of the derivative of the vector function, which we can find using

$$\left|r'(t)\right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

In other words, in order to find $\kappa(t)$, we'll

1. Find $r'(t)$, and use it to

2. Find $|r'(t)|$, and then use $r'(t)$ and $|r'(t)|$ to

3. Find $T(t)$, and then use it to

4. Find $T'(t)$, and then use it to

5. Find $|T'(t)|$, and then use $|r'(t)|$ and $|T'(t)|$ to

6. Find $\kappa(t)$

Once we have curvature, we'll take its derivative $\kappa'(t)$. We'll set the derivative equal to 0 and solve for t. If there's only one value for t, that value is the one associated with maximum curvature. If there's more than one value for t, we'll use the second derivative test to determine which one represents maximum curvature.

Example

Find maximum curvature of the vector function with the given curvature.

$$\kappa(t) = 8t^2 - 4t$$

First, we'll find the derivative of $\kappa(t)$.

$$\kappa(t) = 8t^2 - 4t$$

$$\kappa'(t) = 16t - 4$$

Next we'll set $\kappa'(t) = 0$ and solve for t.

$$0 = 16t - 4$$

$$-16t = -4$$

$$t = \frac{-4}{-16}$$

$$t = \frac{1}{4}$$

Since we found just one value for t, we know that maximum curvature occurs when $t = 1/4$.