Topic: Vector and parametric equations of a line

Question: Find the vector equation of the line.

Passing through $(2, -2, -1)$

Perpendicular to $5\mathbf{i} + 6\mathbf{j} - \mathbf{k} = 0$

Answer choices:

A $r = (5 + 2t)\mathbf{i} + (6 - 2t)\mathbf{j} + (-1 - t)\mathbf{k}$

B $r = (2 - 5t)\mathbf{i} + (-2 - 6t)\mathbf{j} + (-1 + t)\mathbf{k}$

C $r = (2 + 5t)\mathbf{i} + (-2 + 6t)\mathbf{j} + (-1 - t)\mathbf{k}$

D $r = (5 - 2t)\mathbf{i} + (6 + 2t)\mathbf{j} + (-1 + t)\mathbf{k}$

Solution: C

We'll start by converting the given point to its vector equivalent.

$$(2, -2, -1)$$

$$2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

We know we're looking for the line perpendicular to $5\mathbf{i} + 6\mathbf{j} - \mathbf{k} = 0$, which means we need the normal line to $5\mathbf{i} + 6\mathbf{j} - \mathbf{k} = 0$, which is $5\mathbf{i} + 6\mathbf{j} - \mathbf{k}$. The line we're looking for will be parallel to $5\mathbf{i} + 6\mathbf{j} - \mathbf{k}$.

Now we're ready to plug into the equation of a line, $r = r_0 + tv$, where r_0 is a point on the line, and where v is a vector parallel to the vector we want.

$$r = r_0 + tv$$

$$r = (2\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + t(5\mathbf{i} + 6\mathbf{j} - \mathbf{k})$$

$$r = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + 5t\mathbf{i} + 6t\mathbf{j} - t\mathbf{k}$$

$$r = (2\mathbf{i} + 5t\mathbf{i}) + (-2\mathbf{j} + 6t\mathbf{j}) + (-\mathbf{k} - t\mathbf{k})$$

$$r = (2 + 5t)\mathbf{i} + (-2 + 6t)\mathbf{j} + (-1 - t)\mathbf{k}$$

Topic: Vector and parametric equations of a line

Question: Find the parametric equations of the line that corresponds to the vector equation.

$$r = (-3 + t)\mathbf{i} + (8t)\mathbf{j} + (1 - 3t)\mathbf{k}$$

Answer choices:

A	$x = 1 + 3t$	$y = -8$	$z = 3 - t$
B	$x = -3 + t$	$y = 8t$	$z = 1 - 3t$
C	$x = 1 - 3t$	$y = 8$	$z = -3 + t$
D	$x = 3 - t$	$y = -8t$	$z = -1 + 3t$

Solution: B

Given a vector equation

$$r = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

the parametric equations are $x = a$, $y = b$ and $z = c$. So from the vector equation $r = (-3 + t)\mathbf{i} + (8t)\mathbf{j} + (1 - 3t)\mathbf{k}$, we get

$$x = -3 + t$$

$$y = 8t$$

$$z = 1 - 3t$$

Topic: Vector and parametric equations of a line

Question: Find the parametric equations of the line.

Passing through $(6,0,3)$

Perpendicular to $-\mathbf{i}+3\mathbf{j}+2\mathbf{k}=2$

Answer choices:

A $x=-6+t$ $y=-3t$ $z=-3-2t$

B $x=1-6t$ $y=-3$ $z=-2-3t$

C $x=-1+6t$ $y=3$ $z=2+3t$

D $x=6-t$ $y=3t$ $z=3+2t$

Solution: D

We'll start by converting the given point to its vector equivalent.

$$(6,0,3)$$

$$6\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}$$

$$6\mathbf{i} + 3\mathbf{k}$$

We know we're looking for the line perpendicular to $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} = 2$, which means we need the normal line to $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} = 2$, which is $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. The line we're looking for will be parallel to $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.

Now we're ready to plug into the equation of a line, $r = r_0 + tv$, where r_0 is a point on the line, and where v is a vector parallel to the vector we want.

$$r = r_0 + tv$$

$$r = (6\mathbf{i} + 3\mathbf{k}) + t(-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$

$$r = 6\mathbf{i} + 3\mathbf{k} - t\mathbf{i} + 3t\mathbf{j} + 2t\mathbf{k}$$

$$r = (6\mathbf{i} - t\mathbf{i}) + (3t\mathbf{j}) + (3\mathbf{k} + 2t\mathbf{k})$$

$$r = (6 - t)\mathbf{i} + 3t\mathbf{j} + (3 + 2t)\mathbf{k}$$

Now we can turn this into parametric equations. Given a vector equation

$$r = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

the parametric equations are $x = a$, $y = b$ and $z = c$. So from the vector equation $r = (6 - t)\mathbf{i} + 3t\mathbf{j} + (3 + 2t)\mathbf{k}$, we get

$$x = 6 - t$$

$$y = 3t$$

$$z = 3 + 2t$$