Calculus 3 Workbook

Line integrals

krista king MATH

LINE INTEGRAL OF A CURVE

■ 1. Calculate the line integral over c, where c is the circle that lies in the plane $z=3$, with center on the z-axis and radius 4.

$$\int_c x^2+y^2+z^2\ ds$$

■ 2. Calculate the line integral P over c, where c is the part of the graph of the vector function $\vec{r}(t)$ between the points $(-2,6,-2)$ and $(4,9,1)$.

$$\vec{r}(t)=\langle 2t,t^2+5,t-1\rangle$$

$$P=\int_c (y-z^2)\sqrt{5+x^2}\ ds$$

■ 3. Calculate the improper line integral over c, where c is the line of intersection of the surfaces $z-x^2-y^2+2y+1=0$ and $x-y-1=0$.

$$\int_c \frac{1}{(1+8(x-1)y)^2}\ ds$$

LINE INTEGRAL OF A VECTOR FUNCTION

■ 1. Calculate the line integral of the vector function $\vec{F}(x,y)=\langle x+y,x-y\rangle$ over the curve $\vec{r}(t)=\langle t^2-1,t^2+1\rangle$ for $-2\le t\le 3$.

■ 2. Calculate the line integral of the vector function $\vec{F}(x,y,z)=\langle xyz,-z,y\rangle$ over c, where c is the ellipse that lies in the plane $x=-4$ with the center on the x-axis, a semi-axis of 2 in the y-direction, and a semi-axis of 5 in the z -direction.

■ 3. Calculate the improper line integral of the vector function $\vec{F}(x,y,z)$ over the curve $\vec{r}(t)=\langle e^t,-e^{-t},2t\rangle$ for $t\ge 0$.

$$\vec{F}(x,y,z)=\left\langle y^2,\frac{3}{x^2},2xy^2z\right\rangle$$

POTENTIAL FUNCTION OF A CONSERVATIVE VECTOR FIELD

■ 1. Determine whether or not the vector field is conservative.

$$\vec{F}(x,y,z)=\left\langle \ln(2y+z),\frac{2x}{2y+z},\frac{x}{2y+z}\right\rangle$$

■ 2. Find the potential function of the vector field.

$$\vec{F}(x,y)=\langle \cos(x-3y)+5,-3\cos(x-3y)-8\rangle$$

■ 3. Find the potential function of the vector field.

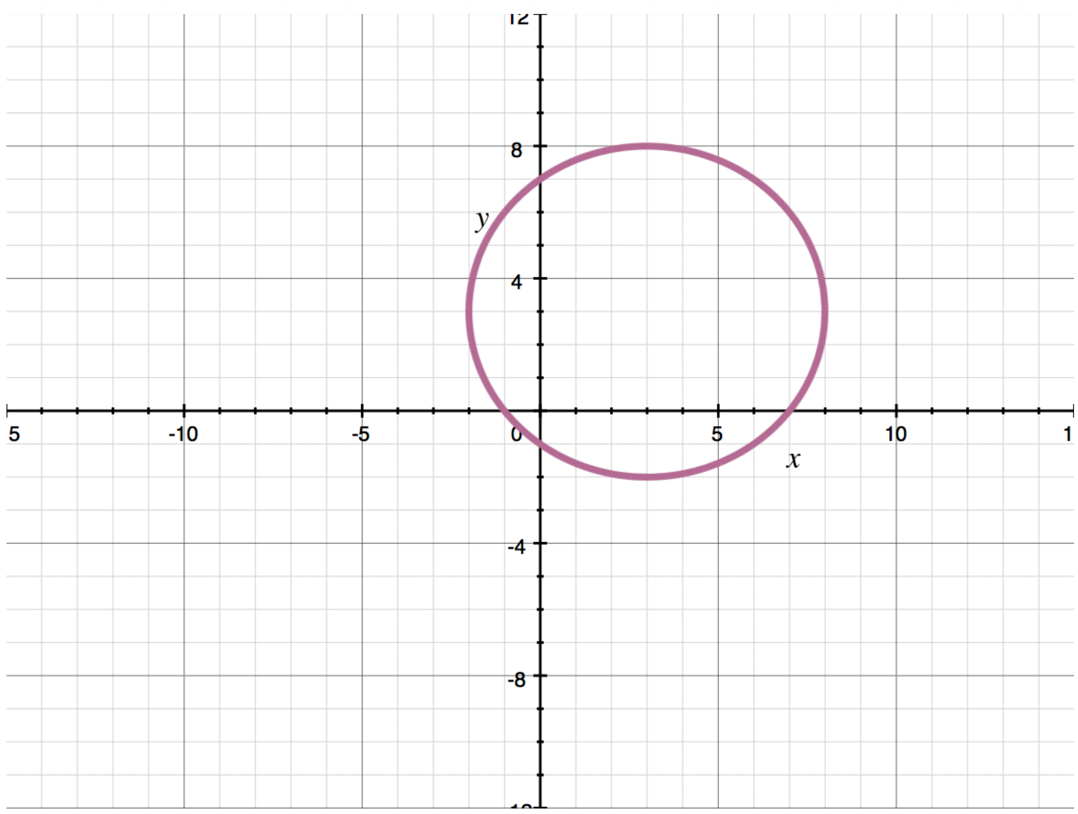
$$\vec{F}(x,y,z)=\langle z^2\,2^{x+4y}\ln 2,\ z^2\,2^{x+4y+2}\ln 2,\ z\,2^{x+4y+1}-6z^2\rangle$$

POTENTIAL FUNCTION OF A CONSERVATIVE VECTOR FIELD TO EVALUATE A LINE INTEGRAL

■ 1. Calculate the line integral of the conservative vector field $\vec{F}(x,y)$ over the curve $\vec{r}(t) = \langle 9\arctan^2 t, t^4 - 2t^2 + 2 \rangle$ between $(0,2)$ and $(\pi^2,5)$.

$$\vec{F}(x,y) = \left\langle \frac{y}{\sqrt{x}}, 2(y + \sqrt{x}) \right\rangle$$

■ 2. Calculate the line integral of the conservative vector field $\vec{F}(x,y) = \langle x^2 + y^2, 2xy + 1 \rangle$ over the part of the circle with center at $(3,3)$ and radius 5, that lies in the first quadrant, with clockwise rotation.



■ 3. Calculate the line integral of the conservative vector field $\vec{F}(x, y, z) = \langle y^2, 2xy, (1+z)^{-1} \rangle$ over the curve $\vec{r}(t) = \langle \sin(\pi t^2), t^3e^{t-1}, (t-2)^2 \rangle$ for $1 \le t \le 2$.

INDEPENDENCE OF PATH

1. Check if the line integral of the vector field $\vec{F}(x, y)$ is independent of path for any curve connecting the points $(2,0)$ and $(0,2)$. If it *is* independent of path, then prove it. If not, give a counterexample.

$$\vec{F}(x, y) = \left\langle \frac{4y}{x^2 + y^2}, \frac{-4x}{x^2 + y^2} \right\rangle$$

2. Check if the line integral of the vector field $\vec{F}(x, y)$ is independent of path for any curve that lies within the rectangle given by $1 < x < 5$ and $1 < y < 5$, and that connects the points $(2,4)$ and $(4,2)$.

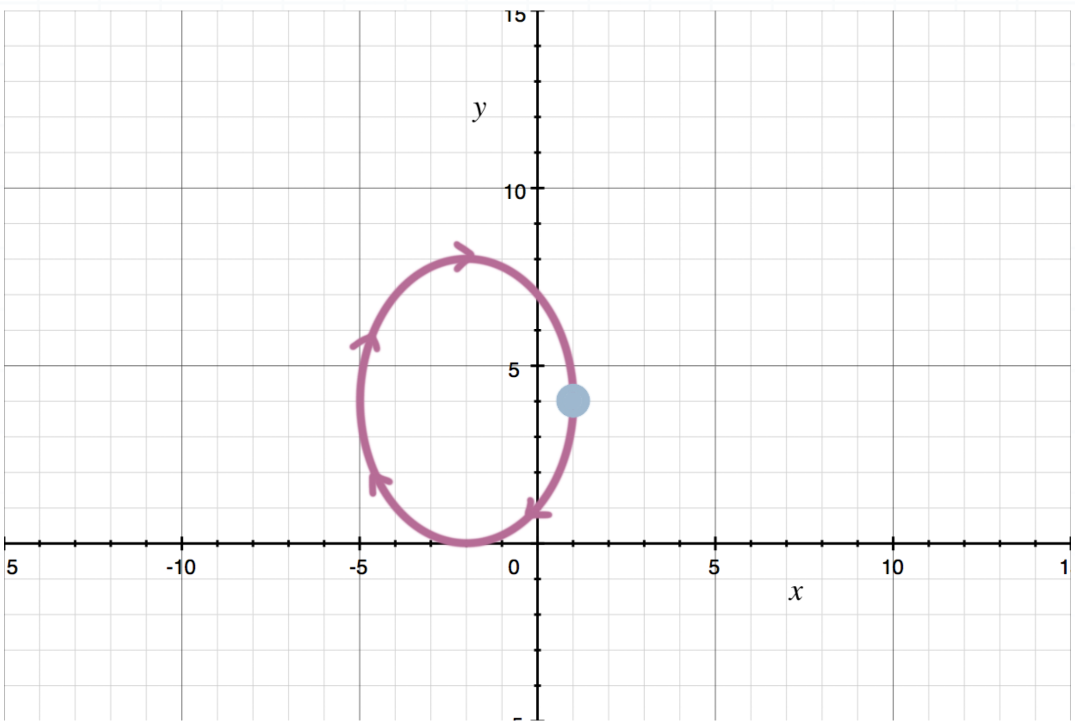
$$\vec{F}(x, y) = \left\langle \frac{2(x-1)}{(x^2 - 2x + y^2 + 1)^2}, \frac{2y}{(x^2 - 2x + y^2 + 1)^2} \right\rangle$$

3. Determine whether the line integral of the vector field $\vec{F}(x, y, z)$ is independent of path for any curve that connects any two points within the vector field's domain.

$$\vec{F}(x, y, z) = \langle x \ln(x^2 + y^2 + z^2 - 9), y \ln(x^2 + y^2 + z^2 - 9), z \ln(x^2 + y^2 + z^2 - 9) \rangle$$

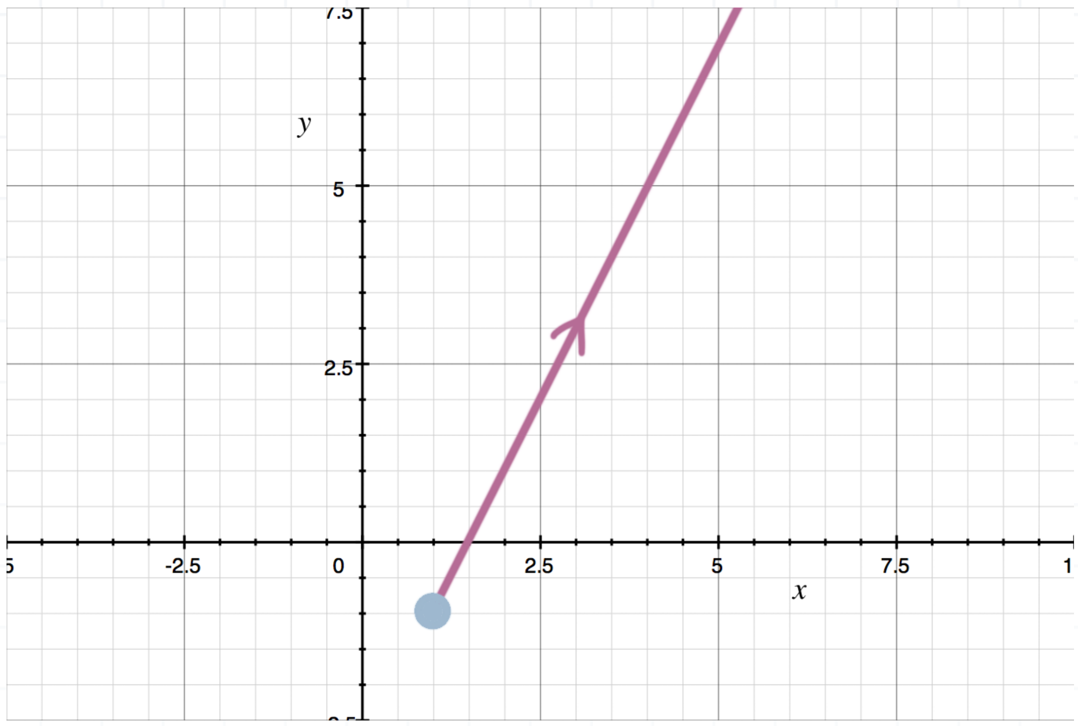
WORK DONE BY A FORCE FIELD

■ 1. Calculate the work done by the force field $\vec{F}(x, y) = \langle 25x^2 + 9y^2 + 1, x - y - 3 \rangle$ to move an object clockwise along the ellipse centered at $(-2,4)$ with semi-axis of 3 in the x-direction and semi-axis of 5 in the y-direction.



■ 2. Find the work done by the force field $\vec{F}(x, y)$ to move an object infinitely along the line $y = 2x - 3$, starting from $(1, -1)$, in the positive direction of x.

$$\vec{F}(x, y) = \left\langle xe^{-y}, \frac{y+2}{x^3} \right\rangle$$

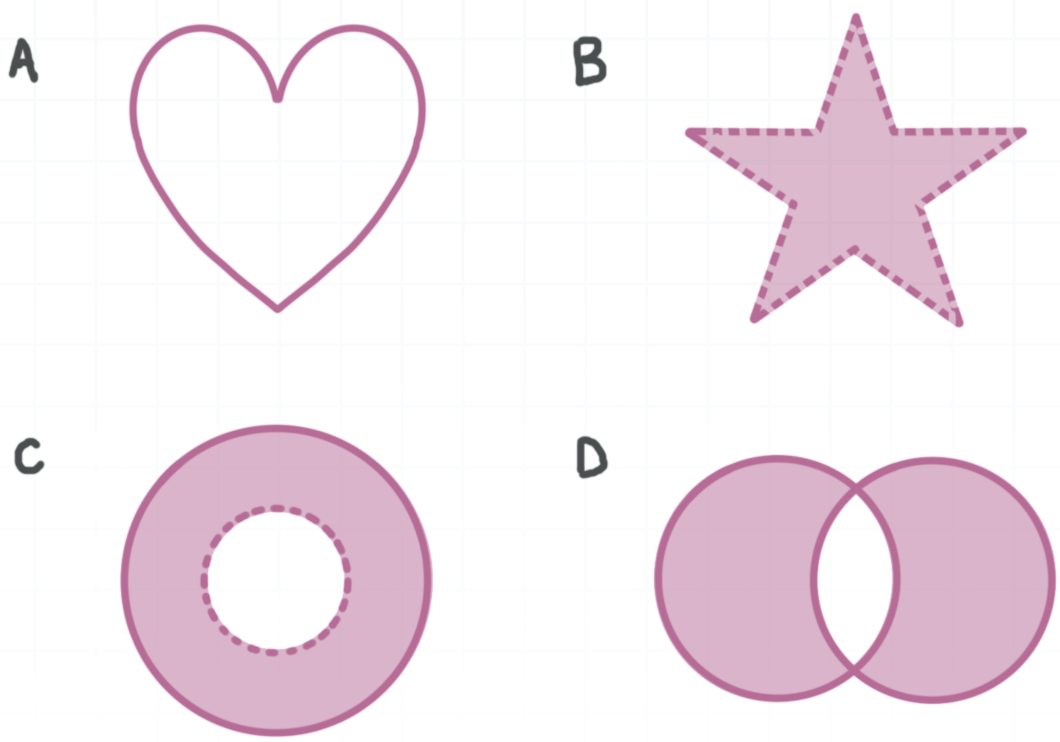


■ 3. Find the work done by the conservative force field $\overrightarrow{F}(x, y, z)$ to move an object between the four points $A(0,-1,2)$, $B(1,1,3)$, $C(2,3,0)$, and $D(0,2,1)$ (starting from A to B, then to C, and finally to D).

$$\overrightarrow{F}(x, y, z) = \langle 1 + 4x + yz + 3z^2, xz - 1, x(y + 6z) \rangle$$

OPEN, CONNECTED, AND SIMPLY CONNECTED

■ 1. Determine whether each set is open, closed, connected, or simply-connected.



■ 2. Find the domain D of the vector field $\vec{F}$, then determine whether it's open, closed, connected, or simply-connected.

$$\vec{F}(u, v) = \left\langle \sqrt{36 - 9u^2 - 4v^2}, \log_2(uv - v) \right\rangle$$

■ 3. Find the domain D of the vector field $\vec{F}$, then determine whether it's open, closed, connected, or simply-connected.

$$\vec{F}(x, y, z) = \left\langle \ln(4x - x^2 - y^2 - z^2), \frac{3x}{y^2 + z^2}, \frac{y}{x + 8} \right\rangle$$

W W W . K R I S T A K I N G M A T H . C O M