Calculus 3 Workbook

Vector functions and space curves

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MATH

DOMAIN OF A VECTOR FUNCTION

1. Find the domain of the vector function.

$$\overrightarrow{F}(t,s) = \left\langle \sqrt{ts}, \frac{t}{s}, e^{t^2+s^2} \right\rangle$$

2. Find the domain of the vector function.

$$\overrightarrow{F}(x,y) = \ln(x+y-3)\cdot\mathbf{i} + \sqrt{2x-2}\cdot\mathbf{j} + \sqrt{6-y}\cdot\mathbf{k}$$

3. Find the domain of the vector function.

$$\overrightarrow{F}(x,y,z) = \sqrt{4-x^2-y^2-z^2}\cdot\mathbf{i} + \frac{2x-y}{x+y+z-4}\cdot\mathbf{j}$$

LIMIT OF A VECTOR FUNCTION

■ 1. Find the limit of the vector function.

$$\lim_{t\to 0,\, s\to 1} \vec{F}(t,s)$$

$$\vec{F}(t,s) = \left\langle \sqrt{s^2 - t^2}, \frac{\sin 3t}{t} + 3s^2, \frac{(t^2 - 2t - 3)(s^2 - 1)}{s - 1} \right\rangle$$

■ 2. Find the limit of the vector function.

$$\lim_{x\to \infty,\, y\to \infty} \vec{F}(t,s)$$

$$\vec{F}(x,y) = xy\,e^{-(x^2+y^2)} \cdot \mathbf{i} + \frac{\sin(x+y)}{x+y} \cdot \mathbf{j} + \frac{x}{y^4} \cdot \mathbf{k}$$

■ 3. Find the limit of the vector function.

$$\lim_{x\to 3,\, y\to \infty,\, z\to 1} \vec{F}(x,y,z)$$

$$\vec{F}(x,y,z) = (x^2y - 3xyz + z^2 - x + 3y - 3z + 5)\mathbf{i} + \ln\frac{x+y}{z+y}\mathbf{j}$$

SKETCHING THE VECTOR EQUATION

■ 1. Identify and sketch the curve that represents $\vec{r}(t) = \langle 3 - 5t, 2t + 1, -3t \rangle$.

■ 2. Identify and sketch the curve representing the graph of the vector function $\vec{r}(t) = \langle 5 \sin t, 3 \cos t, -2 \rangle$.

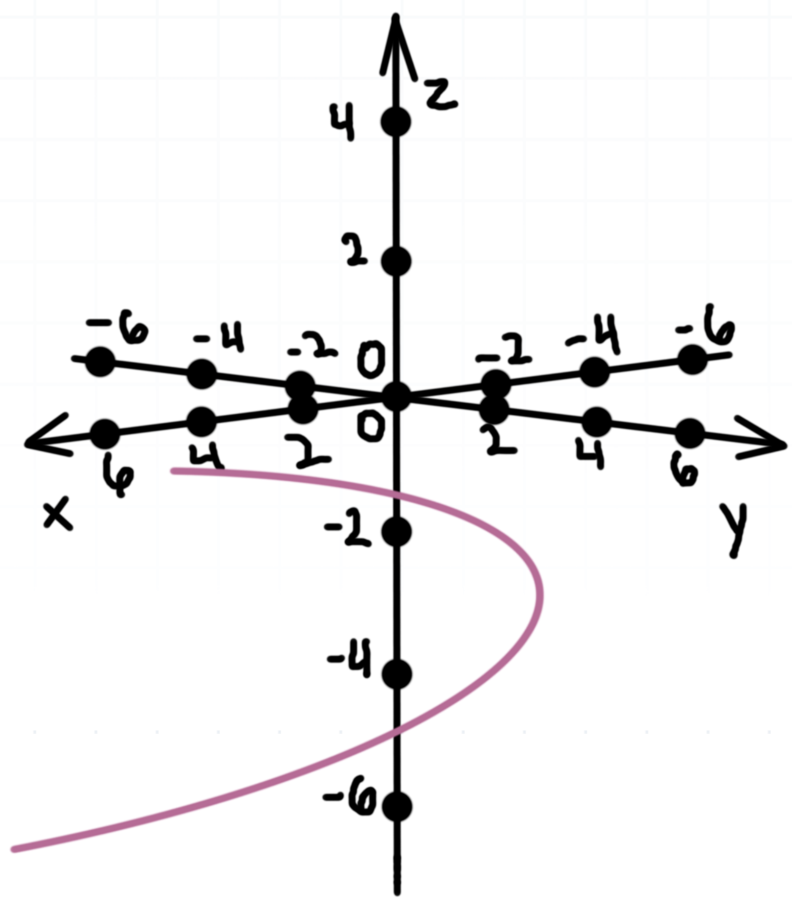
■ 3. Identify and sketch the surface representing the graph of the vector function.

$$\vec{r}(t, s) = \langle 4 \sin t \cos s, 4 \sin t \sin s, 4 \cos t \rangle$$

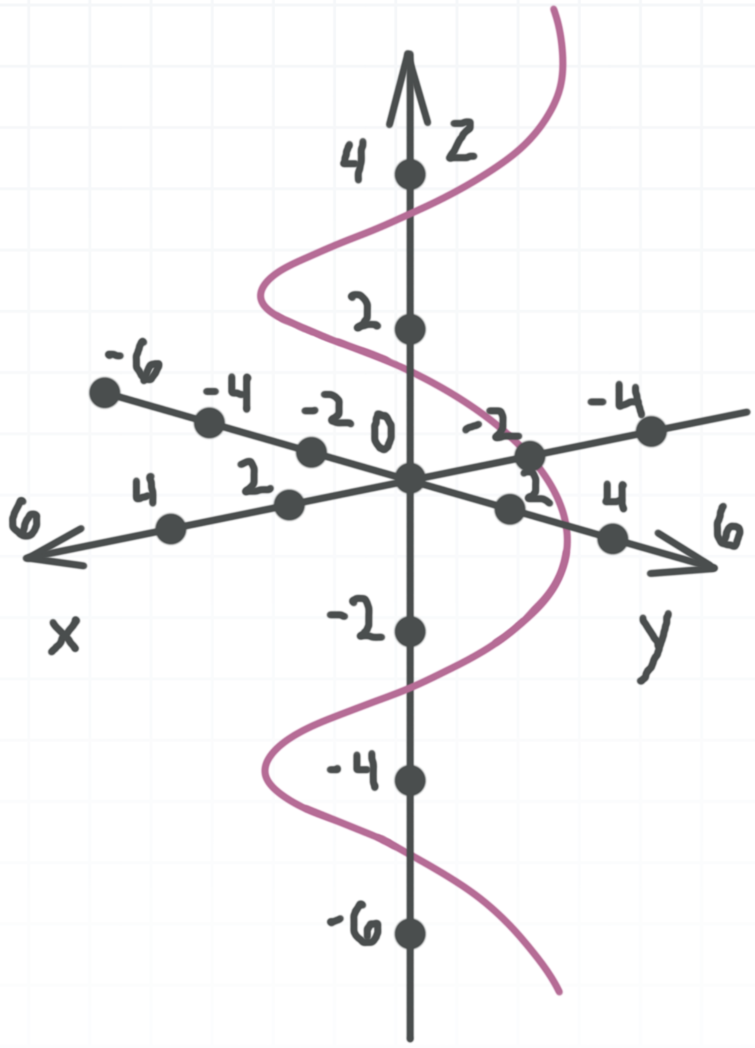
PROJECTIONS OF THE CURVE

■ 1. Identify and sketch the projections of the curve onto each of the major coordinate planes.

$$\vec{r}(t) = \left\langle t^2 - 1, \frac{t+4}{2}, t - 3 \right\rangle$$



■ 2. Identify and sketch the projections of the curve $\vec{r}(t) = \langle 2\cos t, 2\sin t, t + \pi \rangle$ onto each of the coordinate planes.



■ 3. Identify and sketch the projections of the surface onto each of the coordinate planes. Using the projections, identify the surface.

$$\vec{r}(u,v) = \left\langle 3\cos u, 3\sin u, \frac{v}{2} \right\rangle$$

VECTOR AND PARAMETRIC EQUATIONS OF A LINE SEGMENT

■ 1. Find the vector and parametric equation of the line segment AB, given $A(-4,2)$ and $B(1,5)$.

■ 2. Find the vector equation of the line segment AB, if $A(2,-1,3)$, $\overrightarrow{AB}$ is parallel to $\langle -2,2,1\rangle$, and B is the intersection point of the line AB with the xz-plane.

■ 3. Find the endpoints, midpoint, and the length of the line segment for $\vec{r}(t)=\langle 2-3t, 4+t, 2-5t\rangle$ with $0 \le t \le 1$.

VECTOR FUNCTION FOR THE CURVE OF INTERSECTION OF TWO SURFACES

■ 1. Find the vector function for the line of intersection of the two planes.

$$2x - y + 3z - 5 = 0$$

$$x + y - 2z + 1 = 0$$

■ 2. Find the vector function for the curve of intersection of two spheres.

$$x^2 + y^2 + z^2 = 5^2$$

$$(x - 3)^2 + y^2 + z^2 = 4^2$$

■ 3. Find the vector function for the curve of intersection of the elliptic cylinder and the plane.

$$\frac{(x - 2)^2}{3^2} + \frac{(y + 1)^2}{4^2} = 1$$

$$2x - 3y - z - 4 = 0$$

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