Calculus 3
Final Exam Solutions

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MATH

Calculus 3 Final Exam Answer Key

1. (5 pts)	A	■	C	D	E
2. (5 pts)	A	B	■	D	E
3. (5 pts)	A	B	C	■	E
4. (5 pts)	A	B	C	D	■
5. (5 pts)	A	B	C	■	E
6. (5 pts)	■	B	C	D	E
7. (5 pts)	■	B	C	D	E
8. (5 pts)	A	B	■	D	E

9. (15 pts) $d=\sqrt{\frac{629}{30}}$

10. (15 pts) $(-2,-1)$, $(0,0)$, and $(2,-1)$

11. (15 pts) $y=\pm\frac{\sqrt{25-2x^2}}{2}$

12. (15 pts) 4

Calculus 3 Final Exam Solutions

1. B. To find the third-order partial derivative f_{zxy}, first find the first-order partial derivative f_z.

$$f(x, y, z) = x^4y^2z + xy^2$$

$$f_z = x^4y^2$$

Find the second-order partial derivative f_{zx} by treating y and z as constants while differentiating f_z with respect to x.

$$f_{zx} = 4x^3y^2$$

Find the third-order partial derivative f_{zxy} by treating x and z as constants while differentiating f_{zx} with respect to y.

$$f_{zxy} = 8x^3y$$

2. C. Convert the vector $\vec{v} = \langle c, d \rangle = \langle 1, -1 \rangle$ into a unit vector.

$$\vec{u} = \left\langle \frac{c}{\sqrt{c^2 + d^2}}, \frac{d}{\sqrt{c^2 + d^2}} \right\rangle$$

$$\vec{u} = \left\langle \frac{1}{\sqrt{1^2 + (-1)^2}}, \frac{-1}{\sqrt{1^2 + (-1)^2}} \right\rangle$$

$$\vec{u} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$$

$$\vec{u} = \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$$

Find the first-order partial derivatives of $f(x, y) = -2ye^{3x} + x^2y$.

$$\frac{\partial f}{\partial x} = -6ye^{3x} + 2xy$$

$$\frac{\partial f}{\partial y} = -2e^{3x} + x^2$$

Evaluate the partial derivatives at $(0, -2)$, since we're looking for the directional derivative $D_u f(0, -2)$.

$$\frac{\partial f}{\partial x}(0, -2) = -6(-2)e^{3(0)} + 2(0)(-2)$$

$$\frac{\partial f}{\partial x}(0, -2) = 12$$

and

$$\frac{\partial f}{\partial y}(0, -2) = -2e^{3(0)} + 0^2$$

$$\frac{\partial f}{\partial y}(0, -2) = -2$$

Plugging everything into the formula for the directional derivative, we get

$$D_u f(x, y) = a\left(\frac{\partial f}{\partial x}\right) + b\left(\frac{\partial f}{\partial y}\right)$$

$$D_u f(0, -2) = \frac{\sqrt{2}}{2}(12) - \frac{\sqrt{2}}{2}(-2)$$

$$D_u f(0, -2) = \frac{12\sqrt{2}}{2} + \frac{2\sqrt{2}}{2}$$

$$D_u f(0, -2) = 6\sqrt{2} + \sqrt{2}$$

$$D_u f(0, -2) = 7\sqrt{2}$$

3. D. Find the gradient vector of each of the functions $f(x, y) = 2x^2y - y^2$ and $g(x, y) = 2xy^2 + y^2$.

$$\nabla f(x, y) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

$$\nabla f(x, y) = 4xy\mathbf{i} + (2x^2 - 2y)\mathbf{j}$$

and

$$\nabla g(x, y) = \frac{\partial g}{\partial x}\mathbf{i} + \frac{\partial g}{\partial y}\mathbf{j}$$

$$\nabla g(x, y) = 2y^2\mathbf{i} + (4xy + 2y)\mathbf{j}$$

Then the gradient vector $\nabla(fg)$ is

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(fg)=(2x^2y-y^2)(2y^2\mathbf{i}+(4xy+2y)\mathbf{j})$$

$$+(2xy^2+y^2)(4xy\mathbf{i}+(2x^2-2y)\mathbf{j})$$

At the point $(-1,3)$, $\nabla(fg)$ becomes

$$\nabla(fg)=(2(-1)^2(3)-3^2)(2(3)^2\mathbf{i}+(4(-1)(3)+2(3))\mathbf{j})$$

$$+(2(-1)(3)^2+3^2)(4(-1)(3)\mathbf{i}+(2(-1)^2-2(3))\mathbf{j})$$

$$\nabla(fg)=(6-9)(2(9)\mathbf{i}+(-4(3)+6)\mathbf{j})+(-2(9)+9)(-4(3)\mathbf{i}+(2-6)\mathbf{j})$$

$$\nabla(fg)=-3(18\mathbf{i}+(-12+6)\mathbf{j})+(-18+9)(-12\mathbf{i}+(-4)\mathbf{j})$$

$$\nabla(fg)=-3(18\mathbf{i}-6\mathbf{j})-9(-12\mathbf{i}-4\mathbf{j})$$

$$\nabla(fg)=-54\mathbf{i}+18\mathbf{j}+108\mathbf{i}+36\mathbf{j}$$

$$\nabla(fg)=54\mathbf{i}+54\mathbf{j}$$

4. E. We always integrate from the inside out, which means we'll integrate first with respect to r.

$$\int_0^\pi\int_0^2 r^3\ dr\ d\theta$$

$$\int_0^\pi \frac{r^4}{4}\Big|_{r=0}^{r=2}\ d\theta$$

$$\int_0^\pi \frac{2^4}{4}-\frac{0^4}{4}\ d\theta$$

$$\int_0^\pi 4\ d\theta$$

Integrate with respect to θ.

$$4\theta\Big|_0^\pi$$

$$4\pi - 4(0)$$

$$4\pi$$

5. D. To find the Jacobian of the transformation, we'll find the partial derivatives of $x = r^2\cos\theta$ and $y = r^2\sin\theta$.

$$\frac{\partial x}{\partial r} = 2r\cos\theta$$

$$\frac{\partial x}{\partial \theta} = -r^2\sin\theta$$

and

$$\frac{\partial y}{\partial r} = 2r\sin\theta$$

$$\frac{\partial y}{\partial \theta} = r^2\cos\theta$$

Plug the partial derivatives into the formula for the Jacobian.

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} 2r\cos\theta & -r^2\sin\theta \\ 2r\sin\theta & r^2\cos\theta \end{vmatrix}$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = (2r\cos\theta)(r^2\cos\theta) - (2r\sin\theta)(-r^2\sin\theta)$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = 2r^3\cos^2\theta + 2r^3\sin^2\theta$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = 2r^3(\cos^2\theta + \sin^2\theta)$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = 2r^3(1)$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = 2r^3$$

6. A. First we'll find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$.

$$\overrightarrow{AB} = \langle 3-(-1), 6-2, 7-5 \rangle$$

$$\overrightarrow{AB} = \langle 4,4,2 \rangle$$

and

$$\overrightarrow{AC} = \langle 2-(-1), 9-2, 6-5 \rangle$$

$$\overrightarrow{AC} = \langle 3,7,1 \rangle$$

and

$$\overrightarrow{AD} = \langle 1-(-1), 4-2, -5-5 \rangle$$

$$\overrightarrow{AD} = \langle 2,2,-10 \rangle$$

Find the cross product $\overrightarrow{AB} \times \overrightarrow{AC}$,

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ AB_1 & AB_2 & AB_3 \\ AC_1 & AC_2 & AC_3 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i} \begin{vmatrix} AB_2 & AB_3 \\ AC_2 & AC_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} AB_1 & AB_3 \\ AC_1 & AC_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} AB_1 & AB_2 \\ AC_1 & AC_2 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = (AB_2AC_3 - AB_3AC_2)\mathbf{i} - (AB_1AC_3 - AB_3AC_1)\mathbf{j}$$

$$+(AB_1AC_2 - AB_2AC_1)\mathbf{k}$$

Since $\overrightarrow{AB} = \langle 4,4,2 \rangle$ and $\overrightarrow{AC} = \langle 3,7,1 \rangle$, we get

$$\overrightarrow{AB} \times \overrightarrow{AC} = [(4)(1)-(2)(7)]\mathbf{i} - [(4)(1)-(2)(3)]\mathbf{j} + [(4)(7)-(4)(3)]\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = (4-14)\mathbf{i} - (4-6)\mathbf{j} + (28-12)\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = -10\mathbf{i} + 2\mathbf{j} + 16\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \langle -10,2,16 \rangle$$

Finally, we'll take the dot product of $\overrightarrow{AD} = \langle 2,2,-10 \rangle$ and the cross product we just found, which will give us the volume of the parallelepiped.

$$|\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})| = |(2)(-10) + (2)(2) + (-10)(16)|$$

$$|\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})| = |-20 + 4 - 160|$$

$$|\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})| = |-176|$$

$$|\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})| = 176$$

7. A. First we'll turn the vector equation

$$r(t) = \left(2 - \frac{5}{2}t\right)\mathbf{i} - (1 - 2t)\mathbf{j} + \sqrt{2}t\mathbf{k}$$

into parametric equations.

$$x = 2 - \frac{5}{2}t$$

$$y = -1 + 2t$$

$$z = \sqrt{2}t$$

Then we'll take the derivative of these.

$$\frac{dx}{dt} = -\frac{5}{2}$$

$$\frac{dy}{dt} = 2$$

$$\frac{dz}{dt}=\sqrt{2}$$

Since we're told that we'll start at $t=0$ and move in the direction of decreasing t, the limits of integration are given by $[-t,0]$. Now we can plug everything we have into the arc length formula and integrate.

$$L=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\ dt$$

$$L=\int_{-t}^0 \sqrt{\left(-\frac{5}{2}\right)^2+(2)^2+(\sqrt{2})^2}\ dt$$

$$L=\int_{-t}^0 \sqrt{\frac{25}{4}+4+2}\ dt$$

$$L=\int_{-t}^0 \sqrt{\frac{25}{4}+\frac{24}{4}}\ dt$$

$$L=\int_{-t}^0 \sqrt{\frac{49}{4}}\ dt$$

$$L=\frac{7}{2}t\Big|_{-t}^0$$

Evaluate over the interval.

$$L=\frac{7}{2}(0)-\frac{7}{2}(-t)$$

$$L = \frac{7}{2}t$$

Now we can set $L = s$ and then solve for t.

$$s = \frac{7}{2}t$$

$$t = \frac{2}{7}s$$

Now reparametrize the curve by substituting this value for t into the vector function.

$$r(t) = \left(2 - \frac{5}{2}t\right)\mathbf{i} - (1 - 2t)\mathbf{j} + \sqrt{2}t\mathbf{k}$$

$$r(t(s)) = \left(2 - \frac{5}{2}\left(\frac{2}{7}s\right)\right)\mathbf{i} - \left(1 - 2\left(\frac{2}{7}s\right)\right)\mathbf{j} + \sqrt{2}\left(\frac{2}{7}s\right)\mathbf{k}$$

$$r(t(s)) = \left(2 - \frac{5}{7}s\right)\mathbf{i} - \left(1 - \frac{4}{7}s\right)\mathbf{j} + \frac{2\sqrt{2}}{7}s\mathbf{k}$$

8. C. Given $F(x, y, z) = 2xy^2z^3\mathbf{i} + 2x^2yz^3\mathbf{j} + 3x^2y^2z^2\mathbf{k}$, and remembering that

$$F(x, y, z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$

we can say

$$f_x = 2xy^2z^3$$

$$f_y = 2x^2yz^3$$

$$f_z = 3x^2y^2z^2$$

We can integrate $f_x = 2xy^2z^3$ to find f.

$$\int f_x \, dx = f$$

$$\int 2xy^2z^3 \, dx = f$$

$$f = x^2y^2z^3 + g(y, z)$$

To find $g(y, z)$, we'll take partial derivatives with respect to y and z.

$$f_y = 2x^2yz^3 + g_y(y, z)$$

$$f_z = 3x^2y^2z^2 + g_z(y, z)$$

We found $f_y = 2x^2yz^3$ and $f_z = 3x^2y^2z^2$ earlier, so we can set those equal to these new partial derivative equations.

$$2x^2yz^3 + g_y(y, z) = 2x^2yz^3$$

$$g_y(y, z) = 0$$

and

$$3x^2y^2z^2 + g_z(y, z) = 3x^2y^2z^2$$

$$g_z(y, z) = 0$$

Then integrate $g_y(y,z)=0$ and $g_z(y,z)=0$ to get $g(y,z)$.

$$g(y,z)=h(z)$$

$$g(y,z)=k(y)$$

Now we equate $g(y,z)=h(z)$ and $g(y,z)=k(y)$.

$$h(z)=k(y)$$

Since there are no z terms, $h(z)=0$, and since there are no y terms, $k(y)=0$. Therefore we can say that $g(y,z)=0$. This means that $f=x^2y^2z^3+g(y,z)$ is actually just $f=x^2y^2z^3$.

Using this information and the endpoints of the line segment $(1,3,2)$ and $(4,2,5)$, we can find the line integral.

$$\int_c \nabla f\cdot dr=f_2(x_2,y_2,z_2)-f_1(x_1,y_1,z_1)$$

$$\int_c \nabla f\cdot dr=(4)^2(2)^2(5)^3-(1)^2(3)^2(2)^3$$

$$\int_c \nabla f\cdot dr=8{,}000-72$$

$$\int_c \nabla f\cdot dr=7{,}928$$

9. We have to start by converting the parametric equations to a vector equation. Since we have $x=-2+t$, $y=1-2t$, and $z=-5t+3$, we get

$$r = (-2 + t)\mathbf{i} + (1 - 2t)\mathbf{j} + (-5t + 3)\mathbf{k}$$

Now we'll rearrange the vector equation until it matches the format $r = r_0 + tv$.

$$r = -2\mathbf{i} + t\mathbf{i} + \mathbf{j} - 2t\mathbf{j} + 3\mathbf{k} - 5t\mathbf{k}$$

$$r = (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + (t\mathbf{i} - 2t\mathbf{j} - 5t\mathbf{k})$$

$$r = (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + t(\mathbf{i} - 2\mathbf{j} - 5\mathbf{k})$$

Matching this to $r = r_0 + tv$ gives us $r_0(-2,1,3)$ and $v\langle 1, -2, -5\rangle$. We'll rename the vector $v\langle 1, -2, -5\rangle$ to $a\langle 1, -2, -5\rangle$. We'll set a aside for a moment and work on the vector b, which connects the given point $(-1, -3,5)$ to the point on the line, $r_0(-2,1,3)$.

$$b\langle -1 - (-2), -3 - 1, 5 - 3\rangle$$

$$b\langle 1, -4, 2\rangle$$

Now we'll find the cross product of a and b.

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$a \times b = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$a \times b = (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$

$$a \times b = [(-2)(2) - (-5)(-4)]\mathbf{i} - [(1)(2) - (-5)(1)]\mathbf{j} + [(1)(-4) - (-2)(1)]\mathbf{k}$$

$$a \times b = (-4 - 20)\mathbf{i} - (2 + 5)\mathbf{j} + (-4 + 2)\mathbf{k}$$

$$a \times b = -24\mathbf{i} - 7\mathbf{j} - 2\mathbf{k}$$

$$a \times b = \langle -24, -7, -2 \rangle$$

Then we need the magnitude of the cross product of a and b.

$$|a \times b| = \sqrt{(-24)^2 + (-7)^2 + (-2)^2}$$

$$|a \times b| = \sqrt{576 + 49 + 4}$$

$$|a \times b| = \sqrt{629}$$

We also need the magnitude of $a\langle 1, -2, -5 \rangle$.

$$|a| = \sqrt{1^2 + (-2)^2 + (-5)^2}$$

$$|a| = \sqrt{1 + 4 + 25}$$

$$|a| = \sqrt{30}$$

Finally, we'll use the distance formula to find the distance from the point to the line.

$$d = \frac{|a \times b|}{|a|}$$

$$d = \frac{\sqrt{629}}{\sqrt{30}}$$

$$d = \sqrt{\frac{629}{30}}$$

10. To find the critical points of $f(x, y) = x^2y + x^2 + 2y^2 - 4$, we'll set the first-order partial derivatives equal to 0.

$$\frac{\partial f}{\partial x} = 2xy + 2x$$

$$2xy + 2x = 0$$

$$xy + x = 0$$

and

$$\frac{\partial f}{\partial y} = x^2 + 4y$$

$$x^2 + 4y = 0$$

$$4y = -x^2$$

$$y = -\frac{x^2}{4}$$

Plugging this value for y into $xy + x = 0$ gives

$$x\left(-\frac{x^2}{4}\right) + x = 0$$

$$x\left(-\frac{x^2}{4}+1\right)=0$$

$$x=0$$

and

$$-\frac{x^2}{4}+1=0$$

$$-\frac{1}{4}x^2=-1$$

$$x^2=4$$

$$x=\pm 2$$

So we know that $x=0$, $x=-2$, and $x=2$ are all critical points. Now we just need to solve for their associated y-values.

At $x=0$, the first critical point is $(0,0)$.

$$y=-\frac{0^2}{4}=0$$

At $x=-2$, the second critical point is $(-2,-1)$.

$$y=-\frac{(-2)^2}{4}=-1$$

At $x=2$, the third critical point is $(2,-1)$.

$$y=-\frac{2^2}{4}=-1$$

So the critical points of the function are $(-2,-1)$, $(0,0)$, and $(2,-1)$.

11. Because the solid is bounded by the xy-plane ($z=0$) and the paraboloid $z=-2x^2-4y^2+25$, we can say that those surfaces meet each other at

$$0=-2x^2-4y^2+25$$

$$4y^2=25-2x^2$$

$$y^2=\frac{25-2x^2}{4}$$

$$y=\pm\frac{\sqrt{25-2x^2}}{2}$$

Therefore,

$$-\frac{\sqrt{25-2x^2}}{2}\le y\le\frac{\sqrt{25-2x^2}}{2}$$

Let's check ourselves. We already know from the given integral that the bounds for x are given as $x=[-1,1]$. Plugging these bounds and the paraboloid which bounds the volume into a double integral would give

$$V=\int_{-1}^{1}\int_{-\frac{\sqrt{25-2x^2}}{2}}^{\frac{\sqrt{25-2x^2}}{2}}-2x^2-4y^2+25\ dy\ dx$$

If we integrated with respect to y, we'd get

$$V=\int_{-1}^{1}-2x^2y-\frac{4}{3}y^3+25y\Big|_{y=-\frac{\sqrt{25-2x^2}}{2}}^{y=\frac{\sqrt{25-2x^2}}{2}}\ dx$$

$$V=\int_{-1}^{1}-2x^2\frac{\sqrt{25-2x^2}}{2}-\frac{4}{3}\left(\frac{\sqrt{25-2x^2}}{2}\right)^3+25\frac{\sqrt{25-2x^2}}{2}$$

$$-\left[-2x^2\left(-\frac{\sqrt{25-2x^2}}{2}\right)-\frac{4}{3}\left(-\frac{\sqrt{25-2x^2}}{2}\right)^3+25\left(-\frac{\sqrt{25-2x^2}}{2}\right)\right]\ dx$$

$$V=\int_{-1}^{1}-x^2\sqrt{25-2x^2}-\frac{1}{6}(\sqrt{25-2x^2})^3+\frac{25}{2}\sqrt{25-2x^2}$$

$$-x^2\sqrt{25-2x^2}-\frac{1}{6}(\sqrt{25-2x^2})^3+\frac{25}{2}\sqrt{25-2x^2}\ dx$$

$$V=\int_{-1}^{1}-2x^2\sqrt{25-2x^2}-\frac{1}{3}(\sqrt{25-2x^2})^3+25\sqrt{25-2x^2}\ dx$$

$$V=\int_{-1}^{1}(25-2x^2)\sqrt{25-2x^2}-\frac{1}{3}(\sqrt{25-2x^2})^3\ dx$$

$$V=\int_{-1}^{1}(\sqrt{25-2x^2})^3-\frac{1}{3}(\sqrt{25-2x^2})^3\ dx$$

$$V=\int_{-1}^{1}\frac{2}{3}(\sqrt{25-2x^2})^3\ dx$$

Because we got back to the integral we were given, we know the bounds we found for y were correct.

$$y = \pm \frac{\sqrt{25 - 2x^2}}{2}$$

12. The given line integral is in the form

$$\oint_c P\ dx + Q\ dy$$

We'll change it to the form

$$\int\int_{R1} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\ dA + \int\int_{R2} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\ dA$$

From the line integral,

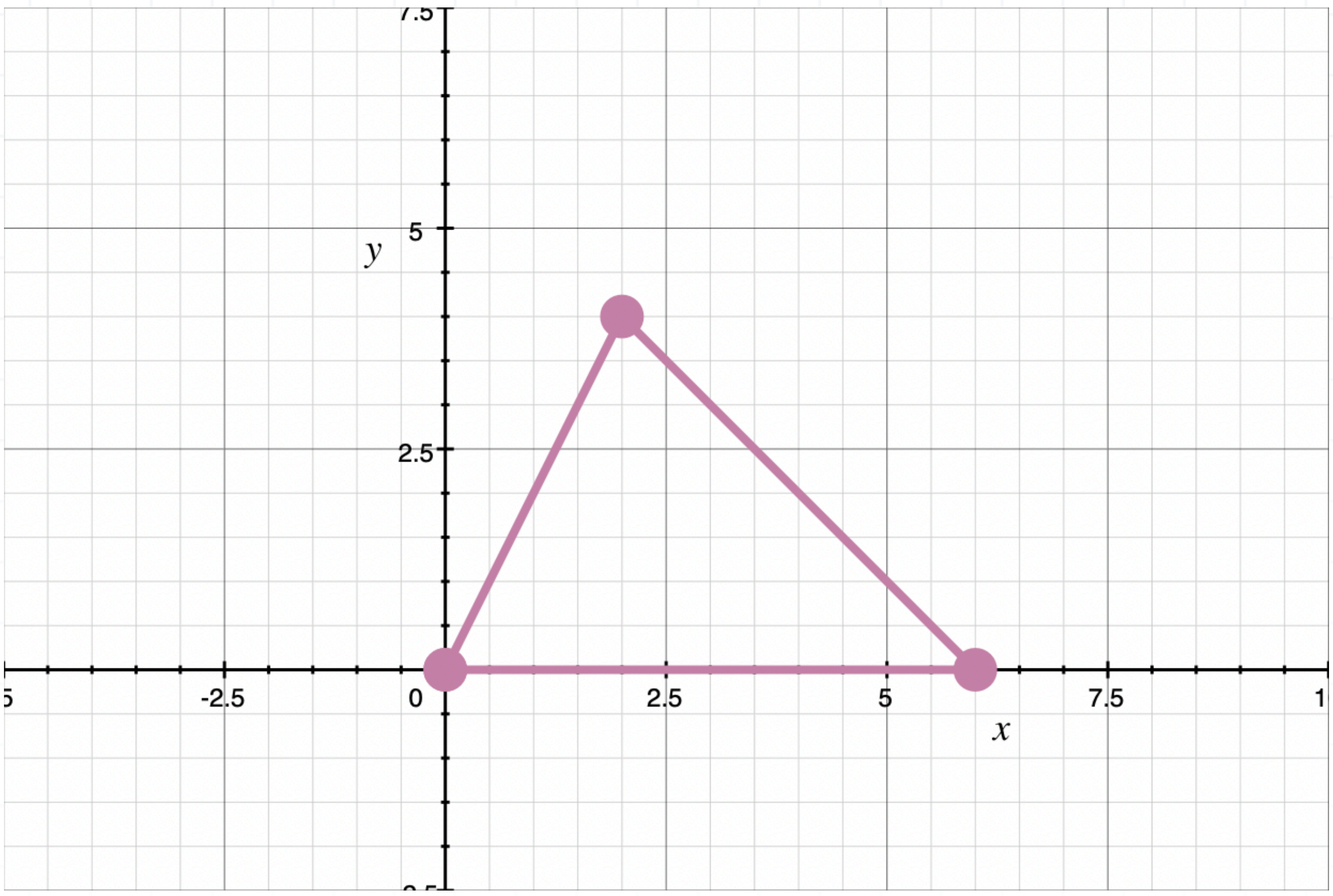
$$\oint_c 2 \sin x + y^2\ dx + 3x + e^{2y}\ dy$$

we know that $Q(x, y) = 3x + e^{2y}$ and $P(x, y) = 2 \sin x + y^2$, so

$$\frac{\partial Q}{\partial x} = 3$$

$$\frac{\partial P}{\partial y} = 2y$$

A sketch of the region bounded by $(0,0)$, $(2,4)$, and $(6,0)$ is



The region needs to be divided in two by the vertical line $x = 2$ that runs through the vertex at $(2,4)$.

To calculate the limits for each of the two regions, we'll need to know the equations for the lines that form the three sides of the triangle. Just by looking at the graph, we can see that two of the lines are $y = 0$ and $y = 2x$. We can use the points $(2,4)$, and $(6,0)$ to solve for the third line using the formula for the equation of a line.

$$y = -x + 6$$

This means that the left side of the triangle is defined for y on $[0,2x]$ and for x on $[0,2]$. The right side of the triangle is defined for y on $[0, -x + 6]$ and for x on $[2,6]$.

Using Green's theorem to convert the line integral into double integrals gives

$$\oint_c P\ dx + Q\ dy = \iint_{R1} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\ dA + \iint_{R2} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\ dA$$

$$\oint_c 2 \sin x + y^2\ dx + 3x + e^{2y}\ dy$$

$$= \int_0^2 \int_0^{2x} 3 - 2y\ dy\ dx + \int_2^6 \int_0^{-x+6} 3 - 2y\ dy\ dx$$

Integrate with respect to y.

$$\int_0^2 3y - y^2 \Big|_{y=0}^{y=2x}\ dx + \int_2^6 3y - y^2 \Big|_{y=0}^{y=-x+6}\ dx$$

$$\int_0^2 3(2x) - (2x)^2 - (3(0) - 0^2)\ dx$$

$$+ \int_2^6 3(-x+6) - (-x+6)^2 - (3(0) - 0^2)\ dx$$

$$\int_0^2 6x - 4x^2\ dx + \int_2^6 -3x + 18 - (x^2 - 12x + 36)\ dx$$

$$\int_0^2 6x - 4x^2\ dx + \int_2^6 -x^2 + 9x - 18\ dx$$

Integrate with respect to x.

$$3x^2 - \frac{4}{3}x^3 \Big|_0^2 - \frac{1}{3}x^3 + \frac{9}{2}x^2 - 18x \Big|_2^6$$

$$3(2)^2-\frac{4}{3}(2)^3-\left(3(0)^2-\frac{4}{3}(0)^3\right)$$

$$-\frac{1}{3}(6)^3+\frac{9}{2}(6)^2-18(6)-\left(-\frac{1}{3}(2)^3+\frac{9}{2}(2)^2-18(2)\right)$$

$$12-\frac{32}{3}-72+162-108-\left(-\frac{8}{3}+18-36\right)$$

$$-\frac{32}{3}-6+\frac{8}{3}+18$$

$$-\frac{24}{3}+12$$

$$-8+12$$

$$4$$