Velocity, acceleration and speed given position

Given a position function

$$r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$$

the velocity function is the derivative of position, and the acceleration function is the derivative of velocity (which means acceleration is also the second derivative of position).

Position vector	$r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$
Velocity vector	$v(t) = r'(t) = r'(t)_1\mathbf{i} + r'(t)_2\mathbf{j} + r'(t)_3\mathbf{k}$
Acceleration vector	$a(t) = v'(t) = r''(t) = r''(t)_1\mathbf{i} + r''(t)_2\mathbf{j} + r''(t)_3\mathbf{k}$

We can also find speed by taking the magnitude of the velocity function.

$$s = \left|v(t)\right| = \sqrt{\left[v(t)_1\right]^2 + \left[v(t)_2\right]^2 + \left[v(t)_3\right]^2}$$

Example

Find the velocity and acceleration functions and speed if the position function is given by $r(t) = 4t^2\mathbf{i} + t^3\mathbf{j} + \sin(2t)\mathbf{k}$.

We'll take the derivative of position to find velocity.

$$v(t) = r'(t) = 8t\mathbf{i} + 3t^2\mathbf{j} + 2\cos(2t)\mathbf{k}$$

Now we'll take the derivative of velocity to find acceleration.

$$a(t) = v'(t) = 8\mathbf{i} + 6t\mathbf{j} - 4\sin(2t)\mathbf{k}$$

Finally, we'll go back to the velocity function we found earlier and find its magnitude in order to get speed.

$$s = \left|v(t)\right| = \sqrt{\left[v(t)_1\right]^2 + \left[v(t)_2\right]^2 + \left[v(t)_3\right]^2}$$

$$s = \left|v(t)\right| = \sqrt{\left[8t\right]^2 + \left[3t^2\right]^2 + \left[2\cos(2t)\right]^2}$$

$$s = \sqrt{64t^2 + 9t^4 + 4\cos^2(2t)}$$

We can summarize our findings as

Velocity function	$v(t) = 8t\mathbf{i} + 3t^2\mathbf{j} + 2\cos(2t)\mathbf{k}$
Acceleration function	$a(t) = 8\mathbf{i} + 6t\mathbf{j} - 4\sin(2t)\mathbf{k}$
Speed	$s = \sqrt{64t^2 + 9t^4 + 4\cos^2(2t)}$