Topic: Unit vector in the direction of the given vector

Question: Find the unit vector in the direction of the given vector.

$$\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

Answer choices:

A $u = \frac{1}{9}\mathbf{i} + \frac{2}{9}\mathbf{j} - \frac{2}{9}\mathbf{k}$

B $u = -\frac{1}{9}\mathbf{i} - \frac{2}{9}\mathbf{j} + \frac{2}{9}\mathbf{k}$

C $u = \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

D $u = -\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$

Solution: C

A unit vector is a vector that has a magnitude of 1. To find the unit vector in the same direction as $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, we can start by finding the magnitude of $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ using the distance formula.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

We'll let (x_1, y_1, z_1) be the origin $(0,0,0)$, and we'll take (x_2, y_2, z_2) from the direction numbers of $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and say that $(x_2, y_2, z_2) = (1,2, -2)$. We'll plug both of these into the distance formula to find the magnitude.

$$D = \sqrt{(1 - 0)^2 + (2 - 0)^2 + (-2 - 0)^2}$$

$$D = \sqrt{1 + 4 + 4}$$

$$D = \sqrt{9}$$

$$D = 3$$

The formula for the unit vector is

$$u = \frac{x}{D}\mathbf{i} + \frac{y}{D}\mathbf{j} + \frac{z}{D}\mathbf{k}$$

where x, y and z are the direction numbers from the given vector

where D is the magnitude we found earlier using the distance formula

Plugging $(1,2, -2)$ and $D = 3$ into this formula gives

$$u = \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Topic: Unit vector in the direction of the given vector

Question: Find the unit vector in the direction of the given vector.

$$-4\mathbf{i} + \mathbf{j} - 9\mathbf{k}$$

Answer choices:

A $u = -\frac{4}{7\sqrt{2}}\mathbf{i} + \frac{1}{7\sqrt{2}}\mathbf{j} - \frac{9}{7\sqrt{2}}\mathbf{k}$

B $u = \frac{4}{7\sqrt{2}}\mathbf{i} - \frac{1}{7\sqrt{2}}\mathbf{j} + \frac{9}{7\sqrt{2}}\mathbf{k}$

C $u = -\frac{4}{98}\mathbf{i} + \frac{1}{98}\mathbf{j} - \frac{9}{98}\mathbf{k}$

D $u = \frac{4}{98}\mathbf{i} - \frac{1}{98}\mathbf{j} + \frac{9}{98}\mathbf{k}$

Solution: A

A unit vector is a vector that has a magnitude of 1. To find the unit vector in the same direction as $-4\mathbf{i}+\mathbf{j}-9\mathbf{k}$, we can start by finding the magnitude of $-4\mathbf{i}+\mathbf{j}-9\mathbf{k}$ using the distance formula.

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

We'll let (x_1, y_1, z_1) be the origin $(0,0,0)$, and we'll take (x_2, y_2, z_2) from the direction numbers of $-4\mathbf{i}+\mathbf{j}-9\mathbf{k}$ and say that $(x_2, y_2, z_2)=(-4,1,-9)$. We'll plug both of these into the distance formula to find the magnitude.

$$D=\sqrt{(-4-0)^2+(1-0)^2+(-9-0)^2}$$

$$D=\sqrt{16+1+81}$$

$$D=\sqrt{98}$$

$$D=\sqrt{49\cdot 2}$$

$$D=7\sqrt{2}$$

The formula for the unit vector is

$$u=\frac{x}{D}\mathbf{i}+\frac{y}{D}\mathbf{j}+\frac{z}{D}\mathbf{k}$$

where x, y and z are the direction numbers from the given vector

where D is the magnitude we found earlier using the distance formula

Plugging $(-4,1,-9)$ and $D = 7\sqrt{2}$ into this formula gives

$$u = -\frac{4}{7\sqrt{2}}\mathbf{i} + \frac{1}{7\sqrt{2}}\mathbf{j} - \frac{9}{7\sqrt{2}}\mathbf{k}$$

Topic: Unit vector in the direction of the given vector

Question: Find the unit vector in the direction of the given vector.

$$2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$$

Answer choices:

A $u = \frac{2}{38}\mathbf{i} - \frac{5}{38}\mathbf{j} + \frac{3}{38}\mathbf{k}$

B $u = -\frac{2}{\sqrt{38}}\mathbf{i} + \frac{5}{\sqrt{38}}\mathbf{j} - \frac{3}{\sqrt{38}}\mathbf{k}$

C $u = -\frac{2}{38}\mathbf{i} + \frac{5}{38}\mathbf{j} - \frac{3}{38}\mathbf{k}$

D $u = \frac{2}{\sqrt{38}}\mathbf{i} - \frac{5}{\sqrt{38}}\mathbf{j} + \frac{3}{\sqrt{38}}\mathbf{k}$

Solution: D

A unit vector is a vector that has a magnitude of 1. To find the unit vector in the same direction as $2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$, we can start by finding the magnitude of $2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ using the distance formula.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

We'll let (x_1, y_1, z_1) be the origin $(0,0,0)$, and we'll take (x_2, y_2, z_2) from the direction numbers of $2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ and say that $(x_2, y_2, z_2) = (2, -5, 3)$. We'll plug both of these into the distance formula to find the magnitude.

$$D = \sqrt{(2 - 0)^2 + (-5 - 0)^2 + (3 - 0)^2}$$

$$D = \sqrt{4 + 25 + 9}$$

$$D = \sqrt{38}$$

The formula for the unit vector is

$$u = \frac{x}{D}\mathbf{i} + \frac{y}{D}\mathbf{j} + \frac{z}{D}\mathbf{k}$$

where x, y and z are the direction numbers from the given vector

where D is the magnitude we found earlier using the distance formula

Plugging $(2, -5, 3)$ and $D = \sqrt{38}$ into this formula gives

$$u = \frac{2}{\sqrt{38}}\mathbf{i} - \frac{5}{\sqrt{38}}\mathbf{j} + \frac{3}{\sqrt{38}}\mathbf{k}$$