Topic: Volume of the parallelepiped from vectors

Question: Use the vectors to find the volume of the parallelepiped.

$$a\langle 1,-1,-1\rangle$$

$$b\langle -2,2,-3\rangle$$

$$c\langle 0,2,-1\rangle$$

Answer choices:

A 10

B 4

C 44

D 1

Solution: A

The volume of a parallelepiped is given by the scalar trip product of three vectors that define its edges. To find the scalar triple product, we'll take the cross product of the vectors b and c, and then take the dot product of the result and the vector a. The cross product of $b\langle -2,2,-3\rangle$ and $c\langle 0,2,-1\rangle$ is

$$b \times c = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$b \times c = \mathbf{i} \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$b \times c = \mathbf{i}\left(b_2c_3 - b_3c_2\right) - \mathbf{j}\left(b_1c_3 - b_3c_1\right) + \mathbf{k}\left(b_1c_2 - b_2c_1\right)$$

$$b \times c = \mathbf{i}\left[(2)(-1) - (-3)(2)\right] - \mathbf{j}\left[(-2)(-1) - (-3)(0)\right] + \mathbf{k}\left[(-2)(2) - (2)(0)\right]$$

$$b \times c = \mathbf{i}(-2+6) - \mathbf{j}(2+0) + \mathbf{k}(-4-0)$$

$$b \times c = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$$

$$b \times c = \langle 4, -2, -4\rangle$$

Now we'll take the dot product of $a\langle 1,-1,-1\rangle$ and $b \times c = \langle 4,-2,-4\rangle$ to find the volume of the parallelepiped.

$$\left|a \cdot (b \times c)\right| = (1)(4) + (-1)(-2) + (-1)(-4)$$

$$\left|a \cdot (b \times c)\right| = 4 + 2 + 4$$

$$\left| a \cdot (b \times c) \right| = 10$$

Topic: Volume of the parallelepiped from vectors

Question: Use the vectors to find the volume of the parallelepiped.

$$a\langle 4,5,-6\rangle$$

$$b\langle 6,-4,7\rangle$$

$$c\langle -4,-5,4\rangle$$

Answer choices:

A 60

B 76

C 92

D 108

Solution: C

The volume of a parallelepiped is given by the scalar trip product of three vectors that define its edges. To find the scalar triple product, we'll take the cross product of the vectors b and c, and then take the dot product of the result and the vector a. The cross product of $b\langle 6,-4,7\rangle$ and $c\langle -4,-5,4\rangle$ is

$$b \times c = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$b \times c = \mathbf{i} \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$b \times c = \mathbf{i}\left(b_2c_3 - b_3c_2\right) - \mathbf{j}\left(b_1c_3 - b_3c_1\right) + \mathbf{k}\left(b_1c_2 - b_2c_1\right)$$

$$b \times c = \mathbf{i}\left[(-4)(4) - (7)(-5)\right] - \mathbf{j}\left[(6)(4) - (7)(-4)\right] + \mathbf{k}\left[(6)(-5) - (-4)(-4)\right]$$

$$b \times c = \mathbf{i}(-16 + 35) - \mathbf{j}(24 + 28) + \mathbf{k}(-30 - 16)$$

$$b \times c = 19\mathbf{i} - 52\mathbf{j} - 46\mathbf{k}$$

$$b \times c = \langle 19, -52, -46\rangle$$

Now we'll take the dot product of $a\langle 4,5,-6\rangle$ and $b \times c = \langle 19,-52,-46\rangle$ to find the volume of the parallelepiped.

$$\left| a \cdot (b \times c) \right| = (4)(19) + (5)(-52) + (-6)(-46)$$

$$\left| a \cdot (b \times c) \right| = 76 - 260 + 276$$

$$\left| a \cdot (b \times c) \right| = 92$$

Topic: Volume of the parallelepiped from vectors

Question: Use the vectors to find the volume of the parallelepiped.

$$a\langle 6,2,3\rangle$$

$$b\langle -4,-3,-4\rangle$$

$$c\langle 6,3,-5\rangle$$

Answer choices:

A $\quad 176$

B $\quad 92$

C $\quad 38$

D $\quad 248$

Solution: B

The volume of a parallelepiped is given by the scalar trip product of three vectors that define its edges. To find the scalar triple product, we'll take the cross product of the vectors b and c, and then take the dot product of the result and the vector a. The cross product of $b\langle -4,-3,-4\rangle$ and $c\langle 6,3,-5\rangle$ is

$$b \times c = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$b \times c = \mathbf{i}\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$b \times c = \mathbf{i}\left(b_2c_3 - b_3c_2\right) - \mathbf{j}\left(b_1c_3 - b_3c_1\right) + \mathbf{k}\left(b_1c_2 - b_2c_1\right)$$

$$b \times c = \mathbf{i}\left[(-3)(-5) - (-4)(3)\right] - \mathbf{j}\left[(-4)(-5) - (-4)(6)\right] + \mathbf{k}\left[(-4)(3) - (-3)(6)\right]$$

$$b \times c = \mathbf{i}(15 + 12) - \mathbf{j}(20 + 24) + \mathbf{k}(-12 + 18)$$

$$b \times c = 27\mathbf{i} - 44\mathbf{j} + 6\mathbf{k}$$

$$b \times c = \langle 27,-44,6\rangle$$

Now we'll take the dot product of $a\langle 6,2,3\rangle$ and $b \times c = \langle 27,-44,6\rangle$ to find the volume of the parallelepiped.

$$\left| a \cdot (b \times c) \right| = (6)(27) + (2)(-44) + (3)(6)$$

$$\left| a \cdot (b \times c) \right| = 162 - 88 + 18$$

$$\left| a \cdot (b \times c) \right| = 92$$