Topic: Integral of a vector function

Question: Find the integral of the vector function.

$$\int_0^{\pi} 7t^2\mathbf{i} - e^{2t}\mathbf{j} + \sin(3t)\mathbf{k}\ dt$$

Answer choices:

A $\quad \dfrac{7\pi^3}{3}\mathbf{i} - \left(\dfrac{1}{2}e^{2\pi} - \dfrac{1}{2}\right)\mathbf{j} + \dfrac{2}{3}\mathbf{k}$

B $\quad \dfrac{7\pi^3}{3}\mathbf{i} - \left(e^{2\pi} - 1\right)\mathbf{j} + \dfrac{2}{3}\mathbf{k}$

C $\quad \dfrac{7\pi^3}{3}\mathbf{i} - \left(\dfrac{1}{2}e^{2\pi} - \dfrac{1}{2}\right)\mathbf{j}$

D $\quad \dfrac{7\pi^3}{3}\mathbf{i} - \left(e^{2\pi} - 1\right)\mathbf{j}$

Solution: A

First we'll rewrite the integral by splitting apart the terms.

$$\int_0^{\pi} 7t^2\mathbf{i} - e^{2t}\mathbf{j} + \sin(3t)\mathbf{k}\ dt$$

$$\int_0^{\pi} 7t^2\ dt\mathbf{i} - \int_0^{\pi} e^{2t}\ dt\mathbf{j} + \int_0^{\pi} \sin(3t)\ dt\mathbf{k}$$

Integrate and then evaluate over the interval.

$$\frac{7}{3}t^3\Big|_0^{\pi}\mathbf{i} - \frac{1}{2}e^{2t}\Big|_0^{\pi}\mathbf{j} - \frac{1}{3}\cos(3t)\Big|_0^{\pi}\mathbf{k}$$

$$\left[\frac{7}{3}(\pi)^3 - \frac{7}{3}(0)^3\right]\mathbf{i} - \left[\frac{1}{2}e^{2(\pi)} - \frac{1}{2}e^{2(0)}\right]\mathbf{j} - \left[\frac{1}{3}\cos(3(\pi)) - \frac{1}{3}\cos(3(0))\right]\mathbf{k}$$

$$\frac{7\pi^3}{3}\mathbf{i} - \left[\frac{1}{2}e^{2\pi} - \frac{1}{2}(1)\right]\mathbf{j} - \left[\frac{1}{3}(-1) - \frac{1}{3}(1)\right]\mathbf{k}$$

$$\frac{7\pi^3}{3}\mathbf{i} - \left(\frac{1}{2}e^{2\pi} - \frac{1}{2}\right)\mathbf{j} + \frac{2}{3}\mathbf{k}$$

This is the integral of the vector function.

Topic: Integral of a vector function

Question: Find the integral of the vector function.

$$\int_0^{\frac{\pi}{2}} \frac{1}{t+3}\mathbf{i} + \sin t \cos t\mathbf{j} + t^3\mathbf{k}\ dt$$

Answer choices:

A $\ln\left(\frac{6+\pi}{2}\right)\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{\pi^4}{16}\mathbf{k}$

B $\ln\left(\frac{6+\pi}{6}\right)\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{\pi^4}{16}\mathbf{k}$

C $\ln\left(\frac{6+\pi}{2}\right)\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{\pi^4}{64}\mathbf{k}$

D $\ln\left(\frac{6+\pi}{6}\right)\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{\pi^4}{64}\mathbf{k}$

Solution: D

First we'll rewrite the integral by splitting apart the terms.

$$\int_0^{\frac{\pi}{2}} \frac{1}{t+3}\mathbf{i} + \sin t \cos t\mathbf{j} + t^3\mathbf{k}\ dt$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{t+3}\ dt\mathbf{i} + \int_0^{\frac{\pi}{2}} \sin t \cos t\ dt\mathbf{j} + \int_0^{\frac{\pi}{2}} t^3\ dt\mathbf{k}$$

Integrate, using u-substitution to find the integral of $\sin t \cos t$.

$$\ln|t+3|\Big|_0^{\frac{\pi}{2}}\mathbf{i} + \int_0^{\frac{\pi}{2}} \sin t \cos t\ dt\mathbf{j} + \frac{1}{4}t^4\Big|_0^{\frac{\pi}{2}}\mathbf{k}$$

$u = \sin t$ and $\frac{du}{dt} = \cos t$, so $du = \cos t\ dt$, or $dt = \frac{du}{\cos t}$

$$\ln|t+3|\Big|_0^{\frac{\pi}{2}}\mathbf{i} + \int_{t=0}^{t=\frac{\pi}{2}} u \cos t\left(\frac{du}{\cos t}\right)\mathbf{j} + \frac{1}{4}t^4\Big|_0^{\frac{\pi}{2}}\mathbf{k}$$

$$\ln|t+3|\Big|_0^{\frac{\pi}{2}}\mathbf{i} + \int_{t=0}^{t=\frac{\pi}{2}} u\ du\mathbf{j} + \frac{1}{4}t^4\Big|_0^{\frac{\pi}{2}}\mathbf{k}$$

$$\ln|t+3|\Big|_0^{\frac{\pi}{2}}\mathbf{i} + \frac{1}{2}u^2\Big|_{t=0}^{t=\frac{\pi}{2}}\mathbf{j} + \frac{1}{4}t^4\Big|_0^{\frac{\pi}{2}}\mathbf{k}$$

Back-substitute and evaluate over the interval.

$$\ln|t+3|\Big|_0^{\frac{\pi}{2}}\mathbf{i} + \frac{1}{2}\sin^2 t\Big|_0^{\frac{\pi}{2}}\mathbf{j} + \frac{1}{4}t^4\Big|_0^{\frac{\pi}{2}}\mathbf{k}$$

$$\left[\ln\left|\frac{\pi}{2}+3\right|-\ln|0+3|\right]\mathbf{i}+\left[\frac{1}{2}\sin^2\left(\frac{\pi}{2}\right)-\frac{1}{2}\sin^2(0)\right]\mathbf{j}+\left[\frac{1}{4}\left(\frac{\pi}{2}\right)^4-\frac{1}{4}(0)^4\right]\mathbf{k}$$

$$\left[\ln\left(\frac{\pi}{2}+\frac{6}{2}\right)-\ln 3\right]\mathbf{i}+\left[\frac{1}{2}(1)-\frac{1}{2}(0)\right]\mathbf{j}+\frac{\pi^4}{64}\mathbf{k}$$

$$\left[\ln\left(\frac{6+\pi}{2}\cdot\frac{1}{3}\right)\right]\mathbf{i}+\frac{1}{2}\mathbf{j}+\frac{\pi^4}{64}\mathbf{k}$$

$$\ln\left(\frac{6+\pi}{6}\right)\mathbf{i}+\frac{1}{2}\mathbf{j}+\frac{\pi^4}{64}\mathbf{k}$$

This is the integral of the vector function.

Topic: Integral of a vector function

Question: Find the integral of the vector function.

$$\int_0^{\frac{\pi}{4}} \sec^2 t\mathbf{i} + \frac{1}{t+5}\mathbf{j} + \sin t\cos^2 t\mathbf{k}\ dt$$

Answer choices:

A $\quad \mathbf{i} + \ln\left(\frac{20+\pi}{4}\right)\mathbf{j} + \frac{\sqrt{2}-4}{12}\mathbf{k}$

B $\quad \mathbf{i} + \ln\left(\frac{20+\pi}{20}\right)\mathbf{j} + \frac{\sqrt{2}}{12}\mathbf{k}$

C $\quad \mathbf{i} + \ln\left(\frac{20+\pi}{20}\right)\mathbf{j} - \frac{\sqrt{2}-4}{12}\mathbf{k}$

D $\quad \mathbf{i} + \ln\left(\frac{20+\pi}{4}\right)\mathbf{j} + \frac{\sqrt{2}}{12}\mathbf{k}$

Solution: C

First we'll rewrite the integral by splitting apart the terms.

$$\int_0^{\frac{\pi}{4}} \sec^2 t\mathbf{i} + \frac{1}{t+5}\mathbf{j} + \sin t\cos^2 t\mathbf{k}\ dt$$

$$\int_0^{\frac{\pi}{4}} \sec^2 t\ dt\mathbf{i} + \int_0^{\frac{\pi}{4}} \frac{1}{t+5}\ dt\mathbf{j} + \int_0^{\frac{\pi}{4}} \sin t\cos^2 t\ dt\mathbf{k}$$

Integrate, using u-substitution to find the integral of $\sin t\cos^2 t$.

$$\tan t\Big|_0^{\frac{\pi}{4}}\mathbf{i} + \ln|t+5|\Big|_0^{\frac{\pi}{4}}\mathbf{j} + \int_0^{\frac{\pi}{4}} \sin t\cos^2 t\ dt\mathbf{k}$$

$u = \cos t$ and $\dfrac{du}{dt} = -\sin t$, so $du = -\sin t\ dt$, or $dt = -\dfrac{du}{\sin t}$

$$\tan t\Big|_0^{\frac{\pi}{4}}\mathbf{i} + \ln|t+5|\Big|_0^{\frac{\pi}{4}}\mathbf{j} + \int_{t=0}^{t=\frac{\pi}{4}} \sin t u^2\left(-\frac{du}{\sin t}\right)\mathbf{k}$$

$$\tan t\Big|_0^{\frac{\pi}{4}}\mathbf{i} + \ln|t+5|\Big|_0^{\frac{\pi}{4}}\mathbf{j} - \int_{t=0}^{t=\frac{\pi}{4}} u^2\ du\mathbf{k}$$

$$\tan t\Big|_0^{\frac{\pi}{4}}\mathbf{i} + \ln|t+5|\Big|_0^{\frac{\pi}{4}}\mathbf{j} - \frac{1}{3}u^3\Big|_{t=0}^{t=\frac{\pi}{4}}\mathbf{k}$$

Back-substitute and evaluate over the interval.

$$\tan t\Big|_0^{\frac{\pi}{4}}\mathbf{i} + \ln|t+5|\Big|_0^{\frac{\pi}{4}}\mathbf{j} - \frac{1}{3}\cos^3 t\Big|_0^{\frac{\pi}{4}}\mathbf{k}$$

$$\left(\tan\frac{\pi}{4}-\tan 0\right)\mathbf{i}+\left(\ln\left|\frac{\pi}{4}+5\right|-\ln|0+5|\right)\mathbf{j}-\left(\frac{1}{3}\cos^3\frac{\pi}{4}-\frac{1}{3}\cos^3 0\right)\mathbf{k}$$

$$(1-0)\mathbf{i}+\left[\ln\left(\frac{\pi}{4}+\frac{20}{4}\right)-\ln 5\right]\mathbf{j}-\left[\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3-\frac{1}{3}(1)\right]\mathbf{k}$$

$$\mathbf{i}+\left[\ln\left(\frac{20+\pi}{4}\right)-\ln 5\right]\mathbf{j}-\left[\frac{1}{3}\left(\frac{2\sqrt{2}}{8}\right)-\frac{1}{3}\right]\mathbf{k}$$

$$\mathbf{i}+\left[\ln\left(\frac{20+\pi}{4}\cdot\frac{1}{5}\right)\right]\mathbf{j}-\left(\frac{\sqrt{2}}{12}-\frac{4}{12}\right)\mathbf{k}$$

$$\mathbf{i}+\ln\left(\frac{20+\pi}{20}\right)\mathbf{j}-\frac{\sqrt{2}-4}{12}\mathbf{k}$$

This is the integral of the vector function.