Topic: Compositions of multivariable functions

Question: Find $f(g(t))$.

$$f(x, y) = -3x^2y^2\cos(x+y)$$

$$g(t) = \left\langle t^2, t^3 \right\rangle$$

Answer choices:

A $\quad f(g(t)) = 3t^{10}\cos\left(t + t^3\right)$

B $\quad f(g(t)) = 3t^{10}\cos\left(t^2 + t\right)$

C $\quad f(g(t)) = -3t^{10}\cos\left(t^2 + t^3\right)$

D $\quad f(g(t)) = -3t^{10}\sin\left(t^2 + t^3\right)$

Solution: C

We're looking for the composition $f(g(t))$, which means we need to plug $g(t)$ into $f(x, y)$.

$$f(g(t)) = f\left(t^2, t^3\right)$$

Because

$$f(x, y) = -3x^2y^2\cos(x + y)$$

we'll be plugging $x = t^2$ and $y = t^3$ into $f(x, y)$.

$$f(t) = -3\left(t^2\right)^2\left(t^3\right)^2\cos\left(t^2 + t^3\right)$$

$$f(t) = -3\left(t^4\right)\left(t^6\right)\cos\left(t^2 + t^3\right)$$

$$f(t) = -3t^{10}\cos\left(t^2 + t^3\right)$$

Topic: Compositions of multivariable functions

Question: Find $h(f(x,y),g(x,y))$.

$$f(x,y)=x^2-y^2$$

$$g(x,y)=x^2+y^2$$

$$h(x,y)=\frac{x-y}{x+y}$$

Answer choices:

A $\quad h(x,y)=-\frac{y^2}{x^2}$

B $\quad h(x,y)=-\frac{x^2}{y^2}$

C $\quad h(x,y)=\frac{y^2}{x^2}$

D $\quad h(x,y)=\frac{x^2}{y^2}$

Solution: A

We're looking for the composition $h(f(x,y),g(x,y))$, which means we need to plug $f(x,y)$ and $g(x,y)$ into $h(x,y)$.

Because

$$h(x,y)=\frac{x-y}{x+y}$$

we'll be plugging $x=x^2-y^2$ and $y=x^2+y^2$ into $h(x,y)$.

$$h(f(x,y),g(x,y))=\frac{\left(x^2-y^2\right)-\left(x^2+y^2\right)}{\left(x^2-y^2\right)+\left(x^2+y^2\right)}$$

$$h(f(x,y),g(x,y))=\frac{x^2-y^2-x^2-y^2}{x^2-y^2+x^2+y^2}$$

$$h(f(x,y),g(x,y))=\frac{-2y^2}{2x^2}$$

$$h(f(x,y),g(x,y))=-\frac{y^2}{x^2}$$

Topic: Compositions of multivariable functions

Question: Given the following functions, which compositions are defined?

$$f(x, y) = x - 3y^2$$

$$g(x) = 1 - 3x^2$$

$$h(x, y) = 3x^2 - y$$

$$p(x, y) = x^3 - y^3$$

Answer choices:

A $f(g(x, y))$ and $g(f(x, y))$

B $g(f(x, y))$ and $g(p(x, y))$

C $h(p(x, y))$ and $h(f(x, y))$

D $p(g(x, y))$ and $g(f(x, y))$

Solution: B

Answer choice B is the only set of compositions where both are defined.

First composition:

$$g(f(x,y)) = g\left(x - 3y^2\right)$$

$$g(f(x,y)) = 1 - 3\left(x - 3y^2\right)^2$$

$$g(f(x,y)) = 1 - 3\left(x^2 - 6xy^2 + 9y^4\right)$$

$$g(f(x,y)) = -3x^2 + 18xy^2 - 27y^4 + 1$$

Second composition:

$$g(p(x,y)) = g\left(x^3 - y^3\right)$$

$$g(p(x,y)) = 1 - 3\left(x^3 - y^3\right)^2$$

$$g(p(x,y)) = 1 - 3\left(x^6 - 2x^3y^3 + y^6\right)$$

$$g(p(x,y)) = -3x^6 + 6x^3y^3 - 3y^6 + 1$$