Topic: Domain of a multivariable function

Question: Find the domain of the multivariable function.

$$f(x, y) = \frac{\sqrt{x^2 + y}}{x - 4}$$

Answer choices:

A $\quad y > -x^2$ and $x \neq 4$

B $\quad y < -x^2$ and $x \neq 4$

C $\quad y \geq -x^2$ and $x \neq 4$

D $\quad y \leq -x^2$ and $x \neq 4$

Solution: C

The domain of a function $f(x, y)$ is the set of all values of x and y that can be plugged into the function and yield valid results.

In the given function,

$$f(x, y) = \frac{\sqrt{x^2 + y}}{x - 4}$$

there are two situations in which we'd be unable to evaluate the function:

a negative result under the square root, or

a 0 result in the denominator.

Therefore, the domain of the function is the set of all values of x and y that avoid either of these scenarios.

First, to avoid taking the square root of a negative number, the quantity under the square root sign must be non-negative (but it can be 0), that is:

$$x^2 + y \geq 0$$

which can be rearranged to $y \geq -x^2$.

Next, to avoid dividing by 0, the denominator of the fraction must not be equal to 0:

$$x - 4 \neq 0$$

Therefore, $x \neq 4$.

Topic: Domain of a multivariable function

Question: Find the domain of the multivariable function.

$$f(x, y) = x\ln(x + 3y)$$

Answer choices:

A $\quad y > -\frac{x}{3}$

B $\quad y > \frac{x}{3}$

C $\quad y < -\frac{x}{3}$

D $\quad y < \frac{x}{3}$

Solution: A

The domain of a function $f(x, y)$ is the set of all values of x and y that can be plugged into the function and yield valid results.

In the given function,

$$f(x, y) = x \ln(x + 3y)$$

there is only one situation in which we'd be unable to evaluate the function:

a 0 or negative result inside the natural logarithm.

We would run into problems if the value of $x + 3y$ were 0 or negative, since the natural logarithm ($\ln$) can only accept positive values. As long as we can avoid that situation, we should be able to successfully evaluate $f(x, y)$ for any other values.

Therefore, the domain of our function is the set of values of x and y such that $x + 3y > 0$, which we can rearrange like this:

$$x + 3y > 0$$

$$3y > -x$$

$$y > -\frac{x}{3}$$

Topic: Domain of a multivariable function

Question: Find the domain of the multivariable function.

$$f(x, y) = \frac{\sqrt{-2x}}{3y^2 - 3}$$

Answer choices:

A $x \geq 0$ and $y \neq 1$

B $x \leq 0$ and $y \neq 1$

C $x \geq 0$ and $y \neq 1$ and $y \neq -1$

D $x \leq 0$ and $y \neq 1$ and $y \neq -1$

Solution: D

The domain of a function $f(x, y)$ is the set of all values of x and y that can be plugged into the function and yield valid results.

In the given function,

$$f(x, y) = \frac{\sqrt{-2x}}{3y^2 - 3}$$

there are two situations in which we'd be unable to evaluate the function:

a negative result under the square root, or

a 0 result in the denominator.

To avoid taking the square root of a negative number, the expression under the square root sign, $-2x$, must be non-negative:

$$-2x \geq 0$$

$$x \leq 0$$

To avoid dividing by 0, we must make sure the denominator of the fraction $3y^2 - 3$ is non-zero.

$$3y^2 - 3 \neq 0$$

$$3y^2 \neq 3$$

$$y^2 \neq 1$$

$$y \neq \pm 1$$

The function $f(x, y)$ can be evaluated as long as all of these conditions are true. Therefore, the domain of $f(x, y)$ is

$x \leq 0$ and $y \neq 1$ and $y \neq -1$