Topic: Scalar triple product to prove vectors are coplanar

Question: Which of the following statements is true?

Answer choices:

A In order for three vectors to be coplanar, their scalar triple product must be greater than 0.

B In order for three vectors to be coplanar, their scalar triple product must be less than 0.

C In order for three vectors to be coplanar, their scalar triple product must be 0.

D In order for three vectors to be coplanar, their scalar triple product cannot be 0.

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Solution: C

The scalar triple product $\left|a \cdot (b \times c)\right|$ of three vectors $a\langle a_1, a_2, a_3\rangle$, $b\langle b_1, b_2, b_3\rangle$ and $c\langle c_1, c_2, c_3\rangle$ will be 0 when the vectors are coplanar.

This means that answer choices A, B, and D cannot be true, since in each case the scalar triple product $\left|a \cdot (b \times c)\right|$ is not equal to 0.

The answer is C because the scalar triple product $\left|a \cdot (b \times c)\right|$ of three vectors must be 0 in order for the vectors to be coplanar.

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Topic: Scalar triple product to prove vectors are coplanar

Question: Say whether or not the vectors are coplanar.

$$a\langle 1,2,-1\rangle$$

$$b\langle 0,4,-1\rangle$$

$$c\langle 1,1,1\rangle$$

Answer choices:

A Yes, the vectors are coplanar because the scalar triple product isn't 0.

B Yes, the vectors are coplanar because the scalar triple product is 0.

C No, the vectors aren't coplanar because the scalar triple product is 0.

D No, the vectors aren't coplanar because the scalar triple product isn't 0.

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Solution: D

If the scalar triple product of the vectors is 0, it means they're coplanar, which means that they lie in the same plane. To find the value of the scalar triple product, we'll start by taking the cross product $b \times c$ using $b\langle 0,4,-1\rangle$ and $c\langle 1,1,1\rangle$.

$$b \times c = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$b \times c = \mathbf{i} \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$b \times c = \mathbf{i}\left(b_2c_3 - b_3c_2\right) - \mathbf{j}\left(b_1c_3 - b_3c_1\right) + \mathbf{k}\left(b_1c_2 - b_2c_1\right)$$

$$b \times c = \mathbf{i}\left[(4)(1) - (-1)(1)\right] - \mathbf{j}\left[(0)(1) - (-1)(1)\right] + \mathbf{k}\left[(0)(1) - (4)(1)\right]$$

$$b \times c = \mathbf{i}(4+1) - \mathbf{j}(0+1) + \mathbf{k}(0-4)$$

$$b \times c = 5\mathbf{i} - \mathbf{j} - 4\mathbf{k}$$

$$b \times c = \langle 5,-1,-4\rangle$$

Then we'll take the dot product of $a\langle 1,2,-1\rangle$ and $b \times c = \langle 5,-1,-4\rangle$.

$$\left|a \cdot (b \times c)\right| = (1)(5) + (2)(-1) + (-1)(-4)$$

$$\left|a \cdot (b \times c)\right| = 5 - 2 + 4$$

$$\left|a \cdot (b \times c)\right| = 7$$

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Since the scalar triple product isn't 0, it means the vectors aren't coplanar.

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Topic: Scalar triple product to prove vectors are coplanar

Question: Say whether or not the vectors are coplanar.

$$a\langle 3,-3,4\rangle$$

$$b\langle 2,-2,-2\rangle$$

$$c\langle 1,-1,1\rangle$$

Answer choices:

A Yes, the vectors are coplanar because the scalar triple product isn't 0.

B Yes, the vectors are coplanar because the scalar triple product is 0.

C No, the vectors aren't coplanar because the scalar triple product is 0.

D No, the vectors aren't coplanar because the scalar triple product isn't 0.

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Solution: B

If the scalar triple product of the vectors is 0, it means they're coplanar, which means that they lie in the same plane. To find the value of the scalar triple product, we'll start by taking the cross product $b \times c$ using $b\langle 2,-2,-2\rangle$ and $c\langle 1,-1,1\rangle$.

$$b \times c = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$b \times c = \mathbf{i}\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$b \times c = \mathbf{i}\left(b_2c_3 - b_3c_2\right) - \mathbf{j}\left(b_1c_3 - b_3c_1\right) + \mathbf{k}\left(b_1c_2 - b_2c_1\right)$$

$$b \times c = \mathbf{i}\left[(-2)(1) - (-2)(-1)\right] - \mathbf{j}\left[(2)(1) - (-2)(1)\right] + \mathbf{k}\left[(2)(-1) - (-2)(1)\right]$$

$$b \times c = \mathbf{i}(-2-2) - \mathbf{j}(2+2) + \mathbf{k}(-2+2)$$

$$b \times c = -4\mathbf{i} - 4\mathbf{j}$$

$$b \times c = \langle -4,-4,0\rangle$$

Then we'll take the dot product of $a\langle 3,-3,4\rangle$ and $b \times c = \langle -4,-4,0\rangle$.

$$\left|a \cdot (b \times c)\right| = (3)(-4) + (-3)(-4) + (4)(0)$$

$$\left|a \cdot (b \times c)\right| = -12 + 12 + 0$$

$$\left|a \cdot (b \times c)\right| = 0$$

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Since the scalar triple product is 0, it means the vectors are coplanar.

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