Calculus 3 Workbook

Parametric surfaces and areas

krista king MATH

POINTS ON THE SURFACE

■ 1. Find the points of the surface $\vec{r}(u, v)$ that lie on the z-axis.

$$\vec{r}(u, v) = \langle u^2 - 3v^2 - 1, 4u^2 - 9v^2 - 7, e^{u+v} \rangle$$

■ 2. Find the intersection point(s) of the surface $\vec{r}(u, v)$ and the line $x = y + 2 = z - 1$.

$$\vec{r}(u, v) = \langle \sin u + v, \cos u + v - 3, 2v + 7 + \sin u \rangle$$

■ 3. Identify the set of points of the surface $\vec{r}(u, v) = \langle u^2 + 2v^2, u, v + 2 \rangle$ that lie in the xy-plane.

SURFACE OF THE VECTOR EQUATION

■ 1. Identify the quadratic surface given as a vector function, where $u \in [0,2\pi]$ and $v \in (-\infty, \infty)$.

$$\vec{r}(u, v) = \langle 3 \sin u, 2v - 3, 5 \cos u \rangle$$

■ 2. Identify the quadratic surface given as a vector function, where $u \in [0,\pi]$ and $v \in [0,2\pi]$.

$$\vec{r}(u, v) = \langle -3 + 2 \cos u, 2 + 2 \sin u \cos v, 2 \sin u \sin v \rangle$$

■ 3. Identify the quadratic surface given as a vector function, where $u^2 + v^2 \leq 9$.

$$\vec{r}(u, v) = \langle v + 1, 5 + \sqrt{9 - u^2 - v^2}, u - 2 \rangle$$

PARAMETRIC REPRESENTATION OF THE SURFACE

■ 1. Consider the right circular cylinder with radius 5 and a cylindrical axis that's parallel to the z-axis and passes through $(2,-4,5)$. Find the parametrization of the part of the cylinder that lies above the xy-plane.

■ 2. Consider the plane $2x-3y+z-1=0$. Find the parametrization of the part of the plane that lies between the planes $y=-3$ and $y=3$.

■ 3. Consider the elliptic paraboloid $2(y+3)^2+4(z-2)^2-x-1=0$. Find the parametrization of the paraboloid for $x\leq 3$.

TANGENT PLANE TO THE PARAMETRIC SURFACE

■ 1. Find the equation of the tangent plane to the surface $\vec{r}(u, v) = \langle u + 2\cos v, u - 2\cos v, uv \rangle$ at the point $(4,0,\pi)$.

■ 2. Find the equation of the tangent plane(s) to the parametric surface $\vec{r}(u, v) = \langle u^2 + 2v, u - 2v, uv + 1 \rangle$ such that its normal vector $\vec{n}$ is parallel to the y-axis.

■ 3. Find the equation of the tangent plane(s) to the parametric surface $\vec{r}(u, v) = \langle v^2, u - v + 2, u^2 - 2 \rangle$ such that it's parallel to $3x - 24y + 2z - 1 = 0$.

AREA OF A SURFACE

■ 1. Find the area of the part of the surface $z = 2x + 2y - 1$ that lies within the rectangle given by $0 \le x \le \pi$ and $-1 \le y \le 1$.

■ 2. Find the area of the part of the surface $\vec{r}(u, v)$ that lies within the values of the parameters $-1 \le u \le 1$ and $0 \le v \le \sqrt{5}$.

$$\vec{r}(u, v) = \langle 2u - 3v + 1, 5u - v + 4, -u + 4v - 11 \rangle$$

■ 3. Find the area of the part of the surface $\vec{r}(u, v) = \langle 2\cos u, 5v + 3, 2\sin u \rangle$ that lies within the values of the parameters $\pi/6 \le u \le \pi/3$ and $0 \le v \le 3$.

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