Topic: Sketching area

Question: The area between two polar curves is given by the double integral. Which of the following describes the area given by the double integral?

$$A = \int_{\pi}^{2\pi} \int_{1}^{1-\sin\theta} r \ dr \ d\theta$$

Answer choices:

A The intersection of the unit circle and the cardioid $r = 1 - \sin\theta$

B The union of the unit circle and the cardioid $r = 1 - \sin\theta$

C The region inside the unit circle and outside the cardioid $r = 1 - \sin\theta$

D The region outside the unit circle and inside the cardioid $r = 1 - \sin\theta$

Solution: D

Because we can see from the double integral that θ is defined on $\pi \leq \theta \leq 2\pi$, then, and the angles between π and 2π all lie below the x-axis where $y = 0$, and since $\sin\theta$ always represents the y-value, we know that

$$\sin\theta < 0$$

$$-\sin\theta > 0$$

$$1 - \sin\theta > 1$$

The graph of $r = 1 - \sin\theta$ is a cardioid and the graph of $r = 1$ is the unit circle (the circle centered at the origin with radius 1). With $1 - \sin\theta > 1$, we know that the cardioid lies outside of the unit circle.

Therefore, since in the given double integral r is given on the interval $r = 1$ to $r = 1 - \sin\theta$, we can say that the area represented by the double integral is the region outside the unit circle but inside the cardioid.

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Question: What are the boundaries of the region given in the double integral?

$$S = \int_{0}^{\frac{\pi}{3}} \int_{\frac{3}{2\cos\theta}}^{3} 2r \ dr \ d\theta$$

Answer choices:

A The intersection of the area of a circle with radius 3 and the region to the right of $x = 3/2$.

B The intersection of the area of a circle with radius 4 and the region to the left of $x = 2/3$.

C The intersection of the area of a circle with radius 3 and the area above $y = 3/2$.

D The intersection of the area of a circle with radius 4 and the area below $y = 2/3$.

Solution: A

The boundaries of the integral

$$S = \int_{\frac{3}{2\cos\theta}}^{3} 2r \ dr$$

indicate that the area is part of a circle with radius 3. Therefore, the equation of the circle is $x^2 + y^2 = 9$.

On the other hand, the lower boundary of the given integral indicates that the area is to the right of the vertical line $x = 3/2$, knowing that $r\cos\theta = 3/2$. Thus, the boundaries of the region are given by the intersection of the area of a circle with radius 3 and the region to the right of $x = 3/2$.

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Question: The area of the region A, which is part of an annulus, is defined by the double integral. What lines are the boundaries of the given region?

$$A = \int_0^{\frac{\pi}{3}} \tan^2\theta \ d\theta \int_a^{3a} r \ dr$$

Answer choices:

A The area between two circles for which one radius is twice the other, and the area is below the line $\theta = 2\pi/3$.

B The area between two circles for which one radius is twice the other, and the area is below the line $\theta = \pi$.

C The area between two circles for which one radius is three times the other, and the area is below the line $\theta = \pi/3$.

D The area between two circles for which one radius is three times the other, and the area is above the line $\theta = \pi/3$.

Solution: C

The boundaries of the second integral from

$$A = \int_0^{\frac{\pi}{3}} \tan^2 \theta \ d\theta \int_a^{3a} r \ dr$$

indicate that the region is part of an annulus formed by two circles for which one radius is three times the other. The boundaries of the first integral verify that the given segment of the annulus is limited to the line passing the origin and forming an angle of $\theta = \pi/3$.