Intersection of a line and a plane

If a line and a plane intersect one another, the intersection will be a single point.

To find the point of intersection, we'll

substitute the values of x, y and z from the equation of the line into the equation of the plane and solve for the parameter t

take the value of t and plug it back into the equation of the line

This will give us the coordinates of the point of intersection.

Example

Find the point where the line intersects the plane.

The line is given by $x = -1 + 2t$, $y = 4 - 5t$, and $z = 1 + t$

The plane is given by $2x - 3y + z = 3$

Our first step is to plug the values for x, y and z given by the equation of the line into the equation of the plane.

$$2(-1 + 2t) - 3(4 - 5t) + (1 + t) = 3$$

$$-2 + 4t - 12 + 15t + 1 + t = 3$$

$$20t = 16$$

$$t = \frac{16}{20}$$

$$t = \frac{4}{5}$$

Now we'll plug the value we found for t back into the equation of the line.

$$x = -1 + 2\left(\frac{4}{5}\right)$$

$$x = \frac{3}{5}$$

and

$$y = 4 - 5\left(\frac{4}{5}\right)$$

$$y = 0$$

and

$$z = 1 + \left(\frac{4}{5}\right)$$

$$z = \frac{9}{5}$$

Putting these values together, we can say the point of intersection of the line and the plane is the coordinate point

$$\left(\frac{3}{5}, 0, \frac{9}{5}\right)$$

If we want to double-check ourselves, we can plug this coordinate point back into the equation of the plane.

$$2\left(\frac{3}{5}\right) - 3(0) + \left(\frac{9}{5}\right) = 3$$

$$\frac{6}{5} + \frac{9}{5} = 3$$

$$\frac{15}{5} = 3$$

$$3 = 3$$

Since $3 = 3$ is true, we know that the point we found is a true intersection point with the plane.