Derivative of a vector function

To find the derivative of a vector function, we just need to find the derivatives of the coefficients when the vector function is in the form

$$r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$$

With the vector function in this form, the derivative is

$$r'(t) = r'(t)_1\mathbf{i} + r'(t)_2\mathbf{j} + r'(t)_3\mathbf{k}$$

If the vector function is in the form

$$r(t) = \langle r(t)_1, r(t)_2, r(t)_3 \rangle$$

we can just attach each of the direction numbers to $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ to transform it into the form

$$r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$$

and then take the derivatives of the coefficients. Alternately, we can just take the derivatives of each direction number, leaving the function in its original form. Make sure to give an answer that matches the form of the original vector function. In other words,

Given,	**the answer should be:**
$r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$	$r'(t) = r'(t)_1\mathbf{i} + r'(t)_2\mathbf{j} + r'(t)_3\mathbf{k}$
$r(t) = \langle r(t)_1, r(t)_2, r(t)_3 \rangle$	$r'(t) = \langle r'(t)_1, r'(t)_2, r'(t)_3 \rangle$

Let's do an example so that we can practice finding the derivative of a vector function.

Example

Find the derivative of the vector function.

$$r(t) = \langle t^2, e^{3t}, t^3 \sin(4t) \rangle$$

The given function is the same as

$$r(t) = t^2\mathbf{i} + e^{3t}\mathbf{j} + t^3 \sin(4t)\mathbf{k}$$

No matter which way we write it, we just need to replace each coefficient with its derivative. In this particular example, we'll need to use product rule to find the derivative of the coefficient on $\mathbf{k}$.

$$r'(t) = 2t\mathbf{i} + e^{3t}(3)\mathbf{j} + [3t^2(\sin(4t)) + t^3(\cos(4t)(4))]\mathbf{k}$$

$$r'(t) = 2t\mathbf{i} + 3e^{3t}\mathbf{j} + (3t^2 \sin(4t) + 4t^3 \cos(4t))\mathbf{k}$$

Since the question gave the original vector function in terms of its direction numbers, we'll give the derivative in that form in our answer.

$$r'(t) = \langle 2t,\ 3e^{3t},\ 3t^2 \sin(4t) + 4t^3 \cos(4t) \rangle$$