Topic: Arc length of a vector function

Question: Find the arc length of the vector function.

$$r(t) = \frac{1}{3}t^3\mathbf{i} + \frac{4}{5}t^{\frac{5}{2}}\mathbf{j} + t^2\mathbf{k}$$

when $0 \leq t \leq 1$

Answer choices:

A $\frac{1}{4}$

B $\frac{4}{3}$

C $\frac{1}{3}$

D $\frac{3}{4}$

Solution: B

First we'll turn the vector equation into parametric equations.

$r(t) = \frac{1}{3}t^3\mathbf{i} + \frac{4}{5}t^{\frac{5}{2}}\mathbf{j} + t^2\mathbf{k}$ becomes

$$x = \frac{1}{3}t^3$$

$$y = \frac{4}{5}t^{\frac{5}{2}}$$

$$z = t^2$$

Then we'll take the derivative of these.

$$\frac{dx}{dt} = t^2$$

$$\frac{dy}{dt} = 2t^{\frac{3}{2}}$$

$$\frac{dz}{dt} = 2t$$

Our limits of integration are given by $0 \le t \le 1$, so we can plug all of this into the arc length formula and integrate.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

$$L = \int_0^1 \sqrt{(t^2)^2 + \left(2t^{\frac{3}{2}}\right)^2 + (2t)^2}\ dt$$

$$L=\int_0^1 \sqrt{t^4+4t^3+4t^2}\ dt$$

$$L=\int_0^1 \sqrt{t^2\left(t^2+4t+4\right)}\ dt$$

$$L=\int_0^1 t\sqrt{(t+2)^2}\ dt$$

$$L=\int_0^1 t(t+2)\ dt$$

$$L=\int_0^1 t^2+2t\ dt$$

$$L=\frac{1}{3}t^3+t^2\Big|_0^1$$

Evaluate over the interval.

$$L=\left[\frac{1}{3}(1)^3+(1)^2\right]-\left[\frac{1}{3}(0)^3+(0)^2\right]$$

$$L=\frac{1}{3}+1$$

$$L=\frac{4}{3}$$

This is the arc length of the vector function.

Topic: Arc length of a vector function

Question: Find the arc length of the vector function.

$$r(t) = \sin t\mathbf{i} + 5t\mathbf{j} + \cos t\mathbf{k}$$

when $0 \leq t \leq 1$

Answer choices:

A $\sqrt{26}$

B $5\sqrt{13}$

C 5

D $2\sqrt{13}$

Solution: A

First we'll turn the vector equation into parametric equations.

$r(t) = \sin t\mathbf{i} + 5t\mathbf{j} + \cos t\mathbf{k}$ becomes

$$x = \sin t$$

$$y = 5t$$

$$z = \cos t$$

Then we'll take the derivative of these.

$$\frac{dx}{dt} = \cos t$$

$$\frac{dy}{dt} = 5$$

$$\frac{dz}{dt} = -\sin t$$

Our limits of integration are given by $0 \le t \le 1$, so we can plug all of this into the arc length formula and integrate.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

$$L = \int_0^1 \sqrt{(\cos t)^2 + (5)^2 + (-\sin t)^2}\ dt$$

$$L = \int_0^1 \sqrt{\cos^2 t + 25 + \sin^2 t}\ dt$$

$$L = \int_0^1 \sqrt{25 + \left(\sin^2 t + \cos^2 t\right)} \ dt$$

$$L = \int_0^1 \sqrt{25 + 1} \ dt$$

$$L = \int_0^1 \sqrt{26} \ dt$$

$$L = \sqrt{26}t \Big|_0^1$$

Evaluate over the interval.

$$L = \sqrt{26}(1) - \sqrt{26}(0)$$

$$L = \sqrt{26}$$

This is the arc length of the vector function.

Topic: Arc length of a vector function

Question: Find the arc length of the vector function.

$$r(t) = 3\cos t\,\mathbf{i} + 4t\,\mathbf{j} + 3\sin t\,\mathbf{k}$$

when $0 \leq t \leq 1$

Answer choices:

A $4\sqrt{3}$

B $\sqrt{15}$

C $\sqrt{17}$

D 5

Solution: D

First we'll turn the vector equation into parametric equations.

$r(t) = 3\cos t\mathbf{i} + 4t\mathbf{j} + 3\sin t\mathbf{k}$ becomes

$$x = 3\cos t$$

$$y = 4t$$

$$z = 3\sin t$$

Then we'll take the derivative of these.

$$\frac{dx}{dt} = -3\sin t$$

$$\frac{dy}{dt} = 4$$

$$\frac{dz}{dt} = 3\cos t$$

Our limits of integration are given by $0 \le t \le 1$, so we can plug all of this into the arc length formula and integrate.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

$$L = \int_0^1 \sqrt{(-3\sin t)^2 + (4)^2 + (3\cos t)^2}\ dt$$

$$L = \int_0^1 \sqrt{9\sin^2 t + 16 + 9\cos^2 t}\ dt$$

$$L = \int_0^1 \sqrt{16 + 9\left(\sin^2 t + \cos^2 t\right)}\ dt$$

$$L = \int_0^1 \sqrt{16 + 9(1)}\ dt$$

$$L = \int_0^1 \sqrt{25}\ dt$$

$$L = \int_0^1 5\ dt$$

$$L = 5t\Big|_0^1$$

Evaluate over the interval.

$$L = 5(1) - 5(0)$$

$$L = 5$$

This is the arc length of the vector function.