Finding area

You can use a double integral to find the area inside a polar curve. Assuming the function itself and the limits of integration are already in polar form, you'll be able to evaluate the iterated integral directly. Otherwise, if either the function and/or the limits of integration are in rectangular form, you'll need to convert to polar before you evaluate.

If you don't have limits of integration, often the best way to find them is to sketch the function so that you can identify the intervals for r and θ over which the function is defined.

Example

Find the area given by the double polar integral.

$$\int_0^{\frac{\pi}{2}} \int_1^3 r \ dr \ d\theta$$

Since the function and the limits of integration are already in terms of polar coordinates, we just need to evaluate the iterated integral. First we'll integrate with respect to r.

$$\int_0^{\frac{\pi}{2}} \int_1^3 r \ dr \ d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{2}r^2 \Big|_1^3 \ d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{2}(3)^2 - \frac{1}{2}(1)^2 \ d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{9}{2} - \frac{1}{2} \ d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{8}{2} \ d\theta$$

$$\int_0^{\frac{\pi}{2}} 4 \ d\theta$$

Now we'll integrate with respect to θ.

$$4\theta \Big|_0^{\frac{\pi}{2}}$$

$$4\left(\frac{\pi}{2}\right) - 4(0)$$

$$2\pi$$

This is the area defined by the double polar integral.