Topic: Maximum rate of change and its direction

Question: Find the maximum rate of change and its direction.

$$f(x, y) = 2x^2y - 3y^3$$

at $P(1,1)$

Answer choices:

A $\quad ||\nabla f(1,1)|| = \sqrt{65}$ and $\nabla f(1,1) = \langle 4,7 \rangle$

B $\quad ||\nabla f(1,1)|| = \sqrt{65}$ and $\nabla f(1,1) = \langle -4,7 \rangle$

C $\quad ||\nabla f(1,1)|| = \sqrt{65}$ and $\nabla f(1,1) = \langle 4, -7 \rangle$

D $\quad ||\nabla f(1,1)|| = \sqrt{65}$ and $\nabla f(1,1) = \langle -4, -7 \rangle$

Solution: C

The maximum rate of change of a function at the point $P(x, y)$ is given by

$$||\nabla f(x, y)|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

so we'll need to start by finding first order partial derivatives.

$$\frac{\partial f}{\partial x} = 4xy$$

and

$$\frac{\partial f}{\partial y} = 2x^2 - 9y^2$$

Evaluating at $P(1,1)$ gives

$$\frac{\partial f}{\partial x}(1,1) = 4(1)(1)$$

$$\frac{\partial f}{\partial x}(1,1) = 4$$

and

$$\frac{\partial f}{\partial y}(1,1) = 2(1)^2 - 9(1)^2$$

$$\frac{\partial f}{\partial y}(1,1) = -7$$

Plugging these into the formula for maximum rate of change, we get

$$||\nabla f(1,1)|| = \sqrt{(4)^2 + (-7)^2}$$

$$||\nabla f(1,1)|| = \sqrt{16 + 49}$$

$$||\nabla f(1,1)|| = \sqrt{65}$$

The direction of the maximum rate of change is given by the gradient vector.

$$\nabla f(x, y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

Since we already know the value of the partial derivatives once they're evaluated at $P(1,1)$, we can say that the direction of the maximum rate of change is

$$\nabla f(1,1) = \langle 4, -7 \rangle$$

Topic: Maximum rate of change and its direction

Question: Find the maximum rate of change and its direction.

$$f(x, y) = 5x^2 \cos y - 5x^4$$

at $P(-2,0)$

Answer choices:

A $\quad ||\nabla f(-2,0)|| = 140$ and $\nabla f(-2,0) = \langle 140,0 \rangle$

B $\quad ||\nabla f(-2,0)|| = \sqrt{140}$ and $\nabla f(-2,0) = \langle 140,0 \rangle$

C $\quad ||\nabla f(-2,0)|| = 140$ and $\nabla f(-2,0) = \langle 0,140 \rangle$

D $\quad ||\nabla f(-2,0)|| = \sqrt{140}$ and $\nabla f(-2,0) = \langle 0,140 \rangle$

Solution: A

The maximum rate of change of a function at the point $P(x, y)$ is given by

$$||\nabla f(x, y)|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

so we'll need to start by finding first order partial derivatives.

$$\frac{\partial f}{\partial x} = 10x\cos y - 20x^3$$

and

$$\frac{\partial f}{\partial y} = -5x^2\sin y$$

Evaluating at $P(-2,0)$ gives

$$\frac{\partial f}{\partial x}(-2,0) = 10(-2)\cos 0 - 20(-2)^3$$

$$\frac{\partial f}{\partial x}(-2,0) = 140$$

and

$$\frac{\partial f}{\partial y}(-2,0) = -5(-2)^2\sin 0$$

$$\frac{\partial f}{\partial y}(-2,0) = 0$$

Plugging these into the formula for maximum rate of change, we get

$$||\nabla f(-2,0)|| = \sqrt{(140)^2 + (0)^2}$$

$$||\nabla f(-2,0)|| = \sqrt{(140)^2}$$

$$||\nabla f(-2,0)|| = 140$$

The direction of the maximum rate of change is given by the gradient vector.

$$\nabla f(x, y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

Since we already know the value of the partial derivatives once they're evaluated at $P(-2,0)$, we can say that the direction of the maximum rate of change is

$$\nabla f(-2,0) = \langle 140,0 \rangle$$

Topic: Maximum rate of change and its direction

Question: Find the maximum rate of change and its direction.

$$f(x, y) = 3y^2 - \sin(xy) + 4e^x$$

at $P(0,3)$

Answer choices:

A $||\nabla f(0,3)|| = 10\sqrt{13}$ and $\nabla f(0,3) = \langle 18,1 \rangle$

B $||\nabla f(0,3)|| = 5\sqrt{13}$ and $\nabla f(0,3) = \langle 18,1 \rangle$

C $||\nabla f(0,3)|| = 10\sqrt{13}$ and $\nabla f(0,3) = \langle 1,18 \rangle$

D $||\nabla f(0,3)|| = 5\sqrt{13}$ and $\nabla f(0,3) = \langle 1,18 \rangle$

Solution: D

The maximum rate of change of a function at the point $P(x, y)$ is given by

$$||\nabla f(x, y)|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

so we'll need to start by finding first order partial derivatives.

$$\frac{\partial f}{\partial x} = -y\cos(xy) + 4e^x$$

and

$$\frac{\partial f}{\partial y} = 6y - x\cos(xy)$$

Evaluating at $P(0,3)$ gives

$$\frac{\partial f}{\partial x}(0,3) = -3\cos(0 \cdot 3) + 4e^0$$

$$\frac{\partial f}{\partial x}(0,3) = 1$$

and

$$\frac{\partial f}{\partial y}(0,3) = 6(3) - 0\cos(0 \cdot 3)$$

$$\frac{\partial f}{\partial y}(0,3) = 18$$

Plugging these into the formula for maximum rate of change, we get

$$||\nabla f(0,3)|| = \sqrt{(1)^2 + (18)^2}$$

$$||\nabla f(0,3)|| = \sqrt{1 + 324}$$

$$||\nabla f(0,3)|| = 5\sqrt{13}$$

The direction of the maximum rate of change is given by the gradient vector.

$$\nabla f(x, y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

Since we already know the value of the partial derivatives once they're evaluated at $P(0,3)$, we can say that the direction of the maximum rate of change is

$$\nabla f(0,3) = \langle 1,18 \rangle$$