Calculus 3 Workbook

Arc length and curvature

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MATH

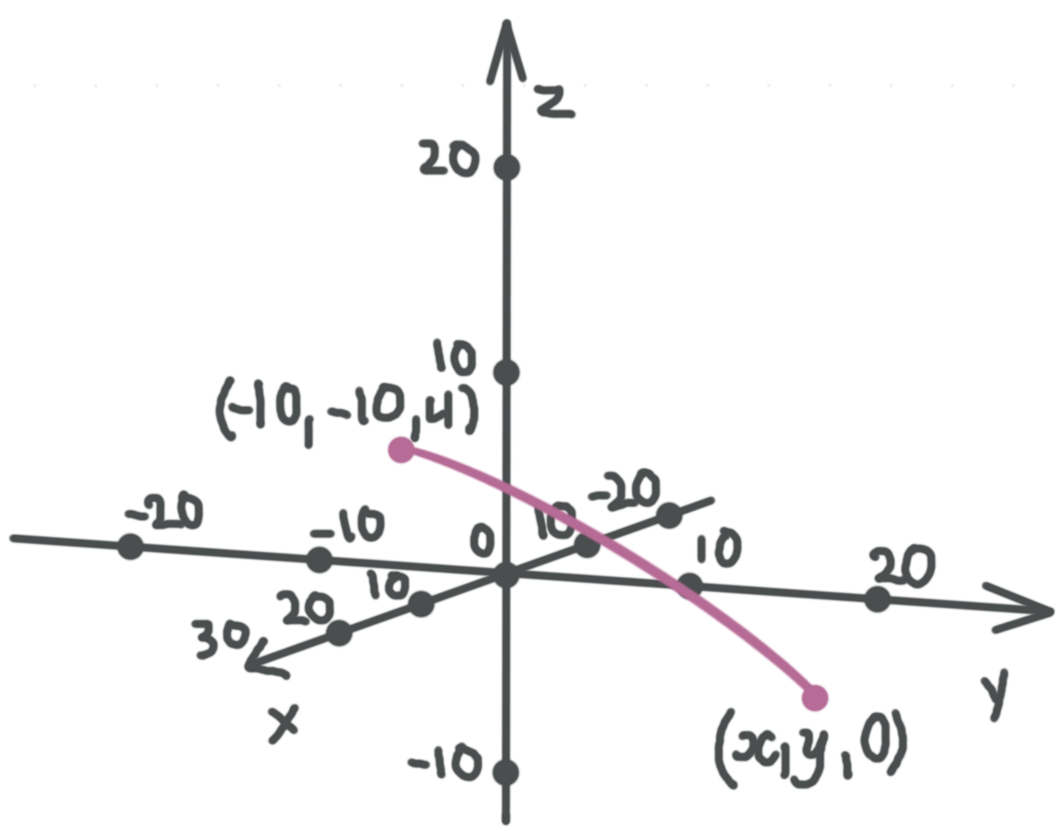
ARC LENGTH OF A VECTOR FUNCTION

■ 1. Confirm the formula for the arc length $L = 2\pi R$ around the circle by considering the circle's equation as the vector function in polar coordinates, where R is the radius of the circle.

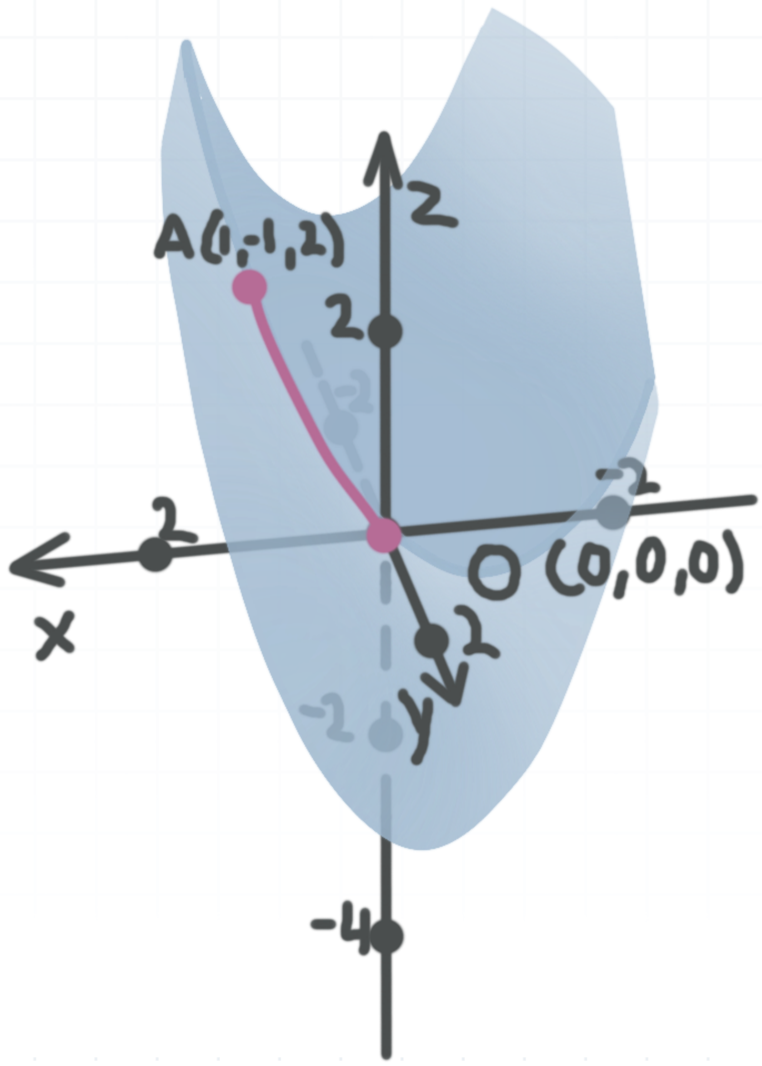
$\vec{r}(\phi) = \langle R\cos\phi, R\sin\phi\rangle$ with $0 \le \phi \le 2\pi$

■ 2. A cannon ball is shot from the point $A(-10,-10,4)$. Its trajectory can be modeled by the vector function, where $t \ge 0$ is the time. Find the arc length of the ball's trajectory before it hits the ground $z = 0$.

$$\vec{r}(t) = \left\langle t-10, t-10, \frac{-t^2+20t+800}{200} \right\rangle$$



■ 3. Find the arc length of the curve that's the intersection of the cylinder $x^2 - y - z = 0$ and the plane $x + y = 0$, between $O(0,0,0)$ and $A(1,-1,2)$.



REPARAMETRIZING THE CURVE

■ 1. Reparametrize $\vec{r}(t)=\langle -3+t, 2+2t, 6-2t\rangle$ in terms of the arc length measured from $(-3,2,6)$ in the direction of increasing t.

■ 2. Reparametrize $\vec{r}(t)=\langle 4\cos 3t, -2t, 4\sin 3t\rangle$ in terms of arc length, measured from $(-4,2\pi,0)$.

■ 3. Reparametrize the curve $\vec{r}(t)=\langle 2e^{2t}, e^{2t}\rangle$ in terms of arc length measured from $t=0$. Use the parametrization to find the position after traveling 5 units.

CURVATURE

■ 1. Find the curvature of $f(x) = 2x^2 - 4$ at $x = 1$.

■ 2. Find the curvature of the vector function at $t = 0$.

$$\vec{r}(t) = \left\langle 2(2+t)^{3/2}, 6t, 2(2-t)^{3/2} \right\rangle$$

■ 3. Find the value(s) of t_0 such that the curvature of $\vec{r}(t) = \langle e^t + 5, 2e^t, -2e^t \rangle$ is 0 at $t = t_0$.

MAXIMUM CURVATURE

■ 1. Find the absolute maximum curvature $k(t)$ of $\vec{r}(t) = \langle 2 + \sin t, \ \cos(t + \pi) \rangle$ on the interval $[0,2\pi]$.

■ 2. Find the absolute minimum and maximum curvature $k(x)$ of the function $f(x) = \ln(6x)$ on the interval $(0,1]$.

■ 3. Find the absolute maximum curvature $k(t)$ of $\vec{r}(t) = \langle 3t + 1, 2.5t^2 - 3, 4 - 4t \rangle$ on the interval $(-\infty, \infty)$.

NORMAL AND OSCULATING PLANES

■ 1. Find the point(s) at which the normal plane to the curve $\vec{r}(t)$ is parallel to the y-axis, then find the equation(s) of the normal plane at each point.

$$\vec{r}(t) = \langle 3t^3 - 10t, t^3 - 6t^2 - 15t, 4t + 1 \rangle$$

■ 2. Find the equation of the osculating plane to $\vec{r}(t) = \langle 12 - 6t, 5t^2 - 10, 7 - 8t \rangle$ at the point $(0,10,-9)$.

■ 3. Use the binormal vector to prove that the graph of the vector function $\vec{r}(t)$ is a planar curve (a curve that lies in a single plane), then find the equation of the plane.

$$\vec{r}(t) = \langle 2\sin t - 2, \ \cos t + 1, 2\cos t + 5 \rangle$$

EQUATION OF THE OSCULATING CIRCLE

■ 1. Find the equation of the osculating circle to the curve $\vec{r}(t) = \langle 2 + 5\sin t, 5\cos t - 1 \rangle$ at an arbitrary point.

■ 2. Find the center and radius of the osculating circle to the curve $\vec{r}(t)$ at the point $(7,6)$.

$$\vec{r}(t) = \langle 4(5 - t)^{5/2} + 3, 24t - 90 \rangle$$

■ 3. Find the point(s) on the curve $\vec{r}(t) = \langle t^2 + 3, 2t - 5 \rangle$ where the osculating circle has a radius of 2.

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