Topic: Gradient vectors

Question: Which function produces the gradient vector at the given point?

$$\nabla f(x, y, z) = 22\mathbf{i} - 20\mathbf{j} + 26\mathbf{k}$$

at $(-1,2,-3)$

Answer choices:

A $f(x, y, z) = x^2 + 2y^2 - 3z^2 + 4xyz$

B $f(x, y, z) = x^2 - 2y^2 - 3z^2 - 4xyz$

C $f(x, y, z) = x^2 - 2y^2 + 3z^2 + 4xyz$

D $f(x, y, z) = x^2 + 2y^2 + 3z^2 - 4xyz$

Solution: B

Apply the definition of gradient vector to each answer choice. Answer choice B gives

$$\nabla f(x,y,z)=\frac{\partial f}{\partial x}\mathbf{i}+\frac{\partial f}{\partial y}\mathbf{j}+\frac{\partial f}{\partial z}\mathbf{k}$$

$$\nabla f(x,y,z)=\nabla\left(x^2-2y^2-3z^2-4xyz\right)$$

$$\nabla f(x,y,z)=\frac{\partial\left(x^2-2y^2-3z^2-4xyz\right)}{\partial x}\mathbf{i}+\frac{\partial\left(x^2-2y^2-3z^2-4xyz\right)}{\partial y}\mathbf{j}$$

$$+\frac{\partial\left(x^2-2y^2-3z^2-4xyz\right)}{\partial z}\mathbf{k}$$

$$\nabla f(x,y,z)=(2x-4yz)\mathbf{i}+(-4y-4xz)\mathbf{j}+(-6z-4xy)\mathbf{k}$$

Evaluate $\nabla f(-1,2,-3)$.

$$\nabla f(-1,2,-3)=\left(2(-1)-4(2)(-3)\right)\mathbf{i}-\left(-4(2)-4(-1)(-3)\right)\mathbf{j}+\left(-6(-3)-4(-1)(2)\right)\mathbf{k}$$

$$\nabla f(-1,2,-3)=(-2+24)\mathbf{i}+(-8-12)\mathbf{j}+(18+8)\mathbf{k}$$

$$\nabla f(-1,2,-3)=22\mathbf{i}-20\mathbf{j}+26\mathbf{k}$$

Topic: Gradient vectors

Question: Find $\nabla(f/g)$.

$$f(x,y)=x^2y$$

$$g(x,y)=xy^2$$

Answer choices:

A $\nabla\left(\frac{f}{g}\right)=\frac{1}{y}\mathbf{i}+\frac{x}{y^2}\mathbf{j}$

B $\nabla\left(\frac{f}{g}\right)=\frac{1}{y^2}\mathbf{i}+\frac{x}{y}\mathbf{j}$

C $\nabla\left(\frac{f}{g}\right)=\frac{1}{x}\mathbf{i}-\frac{x}{y^2}\mathbf{j}$

D $\nabla\left(\frac{f}{g}\right)=\frac{1}{y}\mathbf{i}-\frac{x}{y^2}\mathbf{j}$

Solution: D

First we'll find $\nabla f(x,y)$ and $\nabla g(x,y)$.

$$\nabla f(x,y)=\frac{\partial\left(x^2y\right)}{\partial x}\mathbf{i}+\frac{\partial\left(x^2y\right)}{\partial y}\mathbf{j}$$

$$\nabla f(x,y)=2xy\mathbf{i}+x^2\mathbf{j}$$

and

$$\nabla g(x,y)=\frac{\partial\left(xy^2\right)}{\partial x}\mathbf{i}+\frac{\partial\left(xy^2\right)}{\partial y}\mathbf{j}$$

$$\nabla g(x,y)=y^2\mathbf{i}+2xy\mathbf{j}$$

Plug into the formula.

$$\nabla\left(\frac{f}{g}\right)=\frac{g\nabla f-f\nabla g}{g^2}$$

$$\nabla\left(\frac{f}{g}\right)=\frac{xy^2\left(2xy\mathbf{i}+x^2\mathbf{j}\right)-x^2y\left(y^2\mathbf{i}+2xy\mathbf{j}\right)}{\left(xy^2\right)^2}$$

$$\nabla\left(\frac{f}{g}\right)=\frac{2x^2y^3\mathbf{i}+x^3y^2\mathbf{j}-x^2y^3\mathbf{i}-2x^3y^2\mathbf{j}}{x^2y^4}$$

$$\nabla\left(\frac{f}{g}\right)=\frac{1}{y}\mathbf{i}-\frac{x}{y^2}\mathbf{j}$$

Topic: Gradient vectors

Question: Find $\nabla(fg)$ at $(2,-2)$.

$$f(x,y)=x^2-y$$

$$g(x,y)=x+y^2$$

Answer choices:

A $\quad \nabla(fg)=30\mathbf{i}-30\mathbf{j}$

B $\quad \nabla(fg)=36\mathbf{i}-2\mathbf{j}$

C $\quad \nabla(fg)=12\mathbf{i}+36\mathbf{j}$

D $\quad \nabla(fg)=12\mathbf{i}-36\mathbf{j}$

Solution: A

First find the gradient vector of each function.

$$\nabla f(x, y) = \frac{\partial \left(x^2 - y\right)}{\partial x}\mathbf{i} + \frac{\partial \left(x^2 - y\right)}{\partial y}\mathbf{j}$$

$$\nabla f(x, y) = 2x\mathbf{i} - \mathbf{j}$$

and

$$\nabla g(x, y) = \frac{\partial \left(x + y^2\right)}{\partial x}\mathbf{i} + \frac{\partial \left(x + y^2\right)}{\partial y}\mathbf{j}$$

$$\nabla g(x, y) = \mathbf{i} + 2y\mathbf{j}$$

Plug into the formula.

$$\nabla (fg) = f\,\nabla g + g\,\nabla f$$

$$\nabla (fg) = \left(x^2 - y\right)\left(\mathbf{i} + 2y\mathbf{j}\right) + \left(x + y^2\right)\left(2x\mathbf{i} - \mathbf{j}\right)$$

$$\nabla (fg) = \left(2^2 - (-2)\right)\left(\mathbf{i} + 2(-2)\mathbf{j}\right) + \left(2 + (-2)^2\right)\left(2(2)\mathbf{i} - \mathbf{j}\right)$$

$$\nabla (fg) = (6)\left(\mathbf{i} - 4\mathbf{j}\right) + (6)\left(4\mathbf{i} - \mathbf{j}\right)$$

$$\nabla (fg) = 30\mathbf{i} - 30\mathbf{j}$$