Topic: Derivative of a vector function

Question: Find the derivative of the vector function.

$$r(t) = 4t\mathbf{a} + 2t^3\mathbf{b} - \mathbf{c}$$

Answer choices:

A $r'(t) = 4\mathbf{a} + 6t^3\mathbf{b}$

B $r'(t) = 4\mathbf{a} + 3t^2\mathbf{b}$

C $r'(t) = 4\mathbf{a} + 2t^2\mathbf{b}$

D $r'(t) = 4\mathbf{a} + 6t^2\mathbf{b}$

Solution: D

To find the derivative, we'll differentiation with respect to t. We can differentiate each term individually.

$$r'(t) = 4\mathbf{a} + 2(3)t^2\mathbf{b} + (0)\mathbf{c}$$

$$r'(t) = 4\mathbf{a} + 6t^2\mathbf{b}$$

This is the derivative of the vector function, given in the same form as the original function.

Topic: Derivative of a vector function

Question: Find the derivative of the vector function.

$$r(t) = \ln\left(t^3\right)\mathbf{i} + 4\sin t\mathbf{j} + t\mathbf{k}$$

Answer choices:

A $r'(t) = \dfrac{3}{t^3}\mathbf{i} + 4\cos t\mathbf{j} + \mathbf{k}$

B $r'(t) = \dfrac{3}{t}\mathbf{i} - 4\cos t\mathbf{j} + \mathbf{k}$

C $r'(t) = \dfrac{3}{t}\mathbf{i} + 4\cos t\mathbf{j} + \mathbf{k}$

D $r'(t) = \dfrac{3}{t^3}\mathbf{i} - 4\cos t\mathbf{j} + \mathbf{k}$

Solution: C

To find the derivative, we'll differentiation with respect to t. We can differentiate each term individually.

$$r'(t) = \frac{1}{t^3}\left(3t^2\right)\mathbf{i} + 4\cos t\mathbf{j} + (1)\mathbf{k}$$

$$r'(t) = \frac{3t^2}{t^3}\mathbf{i} + 4\cos t\mathbf{j} + \mathbf{k}$$

$$r'(t) = \frac{3}{t}\mathbf{i} + 4\cos t\mathbf{j} + \mathbf{k}$$

This is the derivative of the vector function, given in the same form as the original function.

Topic: Derivative of a vector function

Question: Find the derivative of the vector function.

$$r(t) = \langle te^{4t}, \cos(3t), 5t^4 \rangle$$

Answer choices:

A $r'(t) = \langle e^{4t} + 4e^{4t}, 3\sin(3t), 20t^3 \rangle$

B $r'(t) = \langle e^{4t} + 4te^{4t}, -3\sin(3t), 20t^3 \rangle$

C $r'(t) = \langle e^{4t} + 4e^{4t}, -3\sin(3t), 20t^3 \rangle$

D $r'(t) = \langle e^{4t} + 4te^{4t}, 3\sin(3t), 20t^3 \rangle$

Solution: B

To find the derivative, we'll differentiation with respect to t. We can differentiate each term individually. We'll need to use product rule to take the derivative of te^{4t}.

$$r'(t) = \left\langle (1)\left(e^{4t}\right) + (t)\left(4e^{4t}\right), -(3)\sin(3t), 5(4)t^3 \right\rangle$$

$$r'(t) = \left\langle e^{4t} + 4te^{4t}, -3\sin(3t), 20t^3 \right\rangle$$

This is the derivative of the vector function, given in the same form as the original function.