Linear approximation in two variables

We can use the linear approximation formula

$$L(x, y) = f(a, b) + \frac{\partial f}{\partial x}(a, b)(x - a) + \frac{\partial f}{\partial y}(a, b)(y - b)$$

(a, b) is the given point

$f(a, b)$ is the value of the function at (a, b)

$\frac{\partial f}{\partial x}(a, b)$ is the partial derivative of f with respect to x at (a, b)

$\frac{\partial f}{\partial y}(a, b)$ is the partial derivative of f with respect to y at (a, b)

to find an approximation of the function at the given point (a, b).

Example

Find the linear approximation of the multivariable function at the given point.

$$f(x, y) = 6x^3 - 2xy^2$$

at $(1,2)$

The problem tells us that $(a, b) = (1,2)$, so we need to find $f(a, b) = f(1,2)$.

$$f(1,2) = 6(1)^3 - 2(1)(2)^2$$

$$f(1,2) = 6 - 2(4)$$

$$f(1,2) = -2$$

Then we need to find the partial derivatives of the function with respect to x and y.

$$\frac{\partial f}{\partial x} = 18x^2 - 2y^2$$

$$\frac{\partial f}{\partial x}(1,2) = 18(1)^2 - 2(2)^2$$

$$\frac{\partial f}{\partial x}(1,2) = 10$$

and

$$\frac{\partial f}{\partial y} = -4xy$$

$$\frac{\partial f}{\partial y}(1,2) = -4(1)(2)$$

$$\frac{\partial f}{\partial y}(1,2) = -8$$

Plugging the slope in each direction, (a, b), and $f(a, b)$ into the linear approximation formula, we get

$$L(x, y) = -2 + (10)(x - 1) + (-8)(y - 2)$$

$$L(x, y) = -2 + 10x - 10 - 8y + 16$$

$$L(x, y) = 10x - 8y + 4$$

This is the linear approximation of the given function at the given point.