Topic: Scalar and vector projections

Question: Find the scalar projection.

b onto a

$$a = \langle 1, -2, 3 \rangle$$

$$b = \langle 4, 0, -1 \rangle$$

Answer choices:

A $\text{comp}_a b = \dfrac{1}{\sqrt{17}}$

B $\text{comp}_a b = \dfrac{7}{\sqrt{14}}$

C $\text{comp}_a b = \dfrac{1}{\sqrt{14}}$

D $\text{comp}_a b = \dfrac{7}{\sqrt{17}}$

Solution: C

In order to find the scalar projection of b onto a, we'll first find the dot product of the vectors we've been given. Since $a=\langle 1,-2,3\rangle$ and $b=\langle 4,0,-1\rangle$, we get

$$a\cdot b=(1)(4)+(-2)(0)+(3)(-1)$$

$$a\cdot b=4+0-3$$

$$a\cdot b=1$$

Since we're looking for the projection of b onto a, we'll find the magnitude of a, using the distance formula and the origin $(0,0,0)$ as (x_1,y_1,z_1).

$$|a|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$|a|=\sqrt{(1-0)^2+(-2-0)^2+(3-0)^2}$$

$$|a|=\sqrt{1+4+9}$$

$$|a|=\sqrt{14}$$

Now we'll plug these pieces into the formula for the scalar projection of b onto a.

$$\text{comp}_a b=\frac{a\cdot b}{|a|}$$

$$\text{comp}_a b=\frac{1}{\sqrt{14}}$$

Topic: Scalar and vector projections

Question: Find the scalar and vector projections.

b onto a

$$a = \langle 4,3,5 \rangle$$

$$b = \langle -5,5,6 \rangle$$

Answer choices:

A $\quad \text{comp}_a b = \dfrac{5}{\sqrt{2}} \qquad \text{proj}_a b = 2\mathbf{i} + \dfrac{3}{2}\mathbf{j} + \dfrac{5}{2}\mathbf{k}$

B $\quad \text{comp}_a b = \dfrac{5}{\sqrt{2}} \qquad \text{proj}_a b = \dfrac{2}{25}\mathbf{i} + \dfrac{3}{50}\mathbf{j} + \dfrac{1}{10}\mathbf{k}$

C $\quad \text{comp}_a b = \dfrac{25}{\sqrt{86}} \qquad \text{proj}_a b = 2\mathbf{i} + \dfrac{3}{2}\mathbf{j} + \dfrac{5}{2}\mathbf{k}$

D $\quad \text{comp}_a b = \dfrac{25}{\sqrt{86}} \qquad \text{proj}_a b = \dfrac{2}{25}\mathbf{i} + \dfrac{3}{50}\mathbf{j} + \dfrac{1}{10}\mathbf{k}$

Solution: A

In order to find the scalar projection of b onto a, we'll first find the dot product of the vectors we've been given. Since $a = \langle 4,3,5 \rangle$ and $b = \langle -5,5,6 \rangle$, we get

$$a \cdot b = (4)(-5) + (3)(5) + (5)(6)$$

$$a \cdot b = -20 + 15 + 30$$

$$a \cdot b = 25$$

Since we're looking for the projection of b onto a, we'll find the magnitude of a, using the distance formula and the origin $(0,0,0)$ as (x_1, y_1, z_1).

$$|a| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$|a| = \sqrt{(4 - 0)^2 + (3 - 0)^2 + (5 - 0)^2}$$

$$|a| = \sqrt{16 + 9 + 25}$$

$$|a| = \sqrt{50}$$

$$|a| = \sqrt{25 \cdot 2}$$

$$|a| = 5\sqrt{2}$$

Now we'll plug these pieces into the formula for the scalar projection of b onto a.

$$\text{comp}_a b = \frac{a \cdot b}{|a|}$$

$$\text{comp}_a b = \frac{25}{5\sqrt{2}}$$

$$\text{comp}_a b = \frac{5}{\sqrt{2}}$$

Then to find the vector projection, we'll plug everything we already have into the formula for the vector projection of b onto a. Since $a = \langle 4,3,5 \rangle$, we get

$$\text{proj}_a b = \left(\frac{a \cdot b}{|a|} \right) \frac{a}{|a|}$$

$$\text{proj}_a b = \left(\frac{25}{5\sqrt{2}} \right) \frac{4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}}{5\sqrt{2}}$$

$$\text{proj}_a b = \frac{25(4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})}{25(2)}$$

$$\text{proj}_a b = \frac{4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}}{2}$$

$$\text{proj}_a b = \frac{4}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$$

$$\text{proj}_a b = 2\mathbf{i} + \frac{3}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$$

Topic: Scalar and vector projections

Question: Find the scalar and vector projections.

b onto a

$$a = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

$$b = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$$

Answer choices:

A $\quad \text{comp}_a b = -\frac{1}{\sqrt{29}} \qquad \text{proj}_a b = \frac{4}{29}\mathbf{i} + \frac{3}{29}\mathbf{j} - \frac{1}{29}\mathbf{k}$

B $\quad \text{comp}_a b = \frac{1}{\sqrt{8}} \qquad \text{proj}_a b = \frac{1}{2}\mathbf{i} + \frac{3}{8}\mathbf{j} - \frac{1}{8}\mathbf{k}$

C $\quad \text{comp}_a b = \frac{1}{3} \qquad \text{proj}_a b = \frac{4}{9}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{1}{9}\mathbf{k}$

D $\quad \text{comp}_a b = -\frac{1}{\sqrt{26}} \qquad \text{proj}_a b = -\frac{2}{13}\mathbf{i} - \frac{3}{26}\mathbf{j} + \frac{1}{26}\mathbf{k}$

Solution: D

In order to find the scalar projection of b onto a, we'll first find the dot product of the vectors we've been given. Since $a = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $b = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$, we get

$$a \cdot b = (4)(2) + (3)(-4) + (-1)(-3)$$

$$a \cdot b = 8 - 12 + 3$$

$$a \cdot b = -1$$

Since we're looking for the projection of b onto a, we'll find the magnitude of a, using the distance formula and the origin $(0,0,0)$ as (x_1, y_1, z_1).

$$|a| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$|a| = \sqrt{(4 - 0)^2 + (3 - 0)^2 + (-1 - 0)^2}$$

$$|a| = \sqrt{16 + 9 + 1}$$

$$|a| = \sqrt{26}$$

Now we'll plug these pieces into the formula for the scalar projection of b onto a.

$$\text{comp}_a b = \frac{a \cdot b}{|a|}$$

$$\text{comp}_a b = -\frac{1}{\sqrt{26}}$$

Then to find the vector projection, we'll plug everything we already have into the formula for the vector projection of b onto a. Since $a = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, we get

$$\text{proj}_a b = \left(\frac{a \cdot b}{|a|}\right)\frac{a}{|a|}$$

$$\text{proj}_a b = \left(-\frac{1}{\sqrt{26}}\right)\frac{4\mathbf{i} + 3\mathbf{j} - \mathbf{k}}{\sqrt{26}}$$

$$\text{proj}_a b = \frac{-4\mathbf{i} - 3\mathbf{j} + \mathbf{k}}{26}$$

$$\text{proj}_a b = -\frac{4}{26}\mathbf{i} - \frac{3}{26}\mathbf{j} + \frac{1}{26}\mathbf{k}$$

$$\text{proj}_a b = -\frac{2}{13}\mathbf{i} - \frac{3}{26}\mathbf{j} + \frac{1}{26}\mathbf{k}$$