**Topic**: Curvature

**Question**: Find the curvature of the vector function.

$$r(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + 2t\mathbf{k}$$

**Answer choices**:

A $\kappa(t) = \dfrac{1}{\sqrt{5}}$

B $\kappa(t) = \sqrt{5}$

C $\kappa(t) = \dfrac{1}{5}$

D $\kappa(t) = 5$

**Solution**: C

First we'll find the derivative of the vector function.

$$r(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + 2t\mathbf{k}$$

$$r'(t) = -\sin t\mathbf{i} + \cos t\mathbf{j} + 2\mathbf{k}$$

Then we'll find the magnitude of this derivative.

$$\left|r'(t)\right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left|r'(t)\right| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (2)^2}$$

$$\left|r'(t)\right| = \sqrt{\left(\sin^2 t + \cos^2 t\right) + 4}$$

$$\left|r'(t)\right| = \sqrt{(1) + 4}$$

$$\left|r'(t)\right| = \sqrt{5}$$

And now we can use everything we just found to find the unit tangent vector.

$$T(t) = \frac{r'(t)}{\left|r'(t)\right|}$$

$$T(t) = \frac{-\sin t\mathbf{i} + \cos t\mathbf{j} + 2\mathbf{k}}{\sqrt{5}}$$

$$T(t) = -\frac{\sin t}{\sqrt{5}}\mathbf{i} + \frac{\cos t}{\sqrt{5}}\mathbf{j} + \frac{2}{\sqrt{5}}\mathbf{k}$$

$$T(t) = -\frac{1}{\sqrt{5}}\sin t\mathbf{i} + \frac{1}{\sqrt{5}}\cos t\mathbf{j} + \frac{2}{\sqrt{5}}\mathbf{k}$$

To find the curvature of the vector function, we'll first need to find the derivative of this unit tangent vector.

$$T(t) = -\frac{1}{\sqrt{5}}\sin t\mathbf{i} + \frac{1}{\sqrt{5}}\cos t\mathbf{j} + \frac{2}{\sqrt{5}}\mathbf{k}$$

$$T'(t) = -\frac{1}{\sqrt{5}}\cos t\mathbf{i} - \frac{1}{\sqrt{5}}\sin t\mathbf{j} + (0)\mathbf{k}$$

Then we'll find the magnitude of this derivative.

$$\left|T'(t)\right| = \sqrt{\left[T'(t)_1\right]^2 + \left[T'(t)_2\right]^2 + \left[T'(t)_3\right]^2}$$

$$\left|T'(t)\right| = \sqrt{\left(-\frac{1}{\sqrt{5}}\cos t\right)^2 + \left(-\frac{1}{\sqrt{5}}\sin t\right)^2 + (0)^2}$$

$$\left|T'(t)\right| = \sqrt{\frac{1}{5}\cos^2 t + \frac{1}{5}\sin^2 t}$$

$$\left|T'(t)\right| = \sqrt{\frac{1}{5}\left(\cos^2 t + \sin^2 t\right)}$$

$$\left|T'(t)\right| = \sqrt{\frac{1}{5}(1)}$$

$$\left|T'(t)\right| = \frac{1}{\sqrt{5}}$$

And now we can use everything we just found to solve for the curvature of the vector function.

$$\kappa(t) = \frac{\left| T'(t) \right|}{\left| r'(t) \right|}$$

$$\kappa(t) = \frac{\frac{1}{\sqrt{5}}}{\sqrt{5}}$$

$$\kappa(t) = \frac{1}{\sqrt{5}\sqrt{5}}$$

$$\kappa(t) = \frac{1}{5}$$

This is the curvature of the vector function.

**Topic**: Curvature

**Question**: Find the curvature of the vector function.

$$r(t) = 4t\mathbf{i} - 3\cos t\mathbf{j} - 3\sin t\mathbf{k}$$

**Answer choices**:

A $\kappa(t) = \frac{1}{25}$

B $\kappa(t) = \frac{3}{5}$

C $\kappa(t) = \frac{1}{5}$

D $\kappa(t) = \frac{3}{25}$

**Solution**: D

First we'll find the derivative of the vector function.

$$r(t) = 4t\mathbf{i} - 3\cos t\mathbf{j} - 3\sin t\mathbf{k}$$

$$r'(t) = 4\mathbf{i} + 3\sin t\mathbf{j} - 3\cos t\mathbf{k}$$

Then we'll find the magnitude of this derivative.

$$\left|r'(t)\right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left|r'(t)\right| = \sqrt{(4)^2 + (3\sin t)^2 + (-3\cos t)^2}$$

$$\left|r'(t)\right| = \sqrt{16 + 9\sin^2 t + 9\cos^2 t}$$

$$\left|r'(t)\right| = \sqrt{16 + 9\left(\sin^2 t + \cos^2 t\right)}$$

$$\left|r'(t)\right| = \sqrt{16 + 9(1)}$$

$$\left|r'(t)\right| = \sqrt{25}$$

$$\left|r'(t)\right| = 5$$

And now we can use everything we just found to find the unit tangent vector.

$$T(t) = \frac{r'(t)}{\left|r'(t)\right|}$$

$$T(t) = \frac{4\mathbf{i} + 3\sin t\mathbf{j} - 3\cos t\mathbf{k}}{5}$$

$$T(t) = \frac{4}{5}\mathbf{i} + \frac{3}{5}\sin t\mathbf{j} - \frac{3}{5}\cos t\mathbf{k}$$

To find the curvature of the vector function, we'll first need to find the derivative of this unit tangent vector.

$$T(t) = \frac{4}{5}\mathbf{i} + \frac{3}{5}\sin t\mathbf{j} - \frac{3}{5}\cos t\mathbf{k}$$

$$T'(t) = (0)\mathbf{i} + \frac{3}{5}\cos t\mathbf{j} + \frac{3}{5}\sin t\mathbf{k}$$

Then we'll find the magnitude of this derivative.

$$\left|T'(t)\right| = \sqrt{\left[T'(t)_1\right]^2 + \left[T'(t)_2\right]^2 + \left[T'(t)_3\right]^2}$$

$$\left|T'(t)\right| = \sqrt{(0)^2 + \left(\frac{3}{5}\cos t\right)^2 + \left(\frac{3}{5}\sin t\right)^2}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}\cos^2 t + \frac{9}{25}\sin^2 t}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}\left(\cos^2 t + \sin^2 t\right)}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}(1)}$$

$$\left|T'(t)\right| = \frac{3}{5}$$

And now we can use everything we just found to solve for the curvature of the vector function.

$$\kappa(t) = \frac{|T'(t)|}{|r'(t)|}$$

$$\kappa(t) = \frac{\frac{3}{5}}{5}$$

$$\kappa(t) = \frac{3}{5 \cdot 5}$$

$$\kappa(t) = \frac{3}{25}$$

This is the curvature of the vector function.

**Topic**: Curvature

**Question**: Find the curvature of the vector function.

$$r(t) = 6\sin t\mathbf{i} - 8t\mathbf{j} + 6\cos t\mathbf{k}$$

**Answer choices**:

A $\kappa(t) = \dfrac{3}{5}$

B $\kappa(t) = \dfrac{3}{50}$

C $\kappa(t) = \dfrac{3}{25}$

D $\kappa(t) = \dfrac{3}{10}$

**Solution**: B

First we'll find the derivative of the vector function.

$$r(t) = 6\sin t\mathbf{i} - 8t\mathbf{j} + 6\cos t\mathbf{k}$$

$$r(t) = 6\cos t\mathbf{i} - 8\mathbf{j} - 6\sin t\mathbf{k}$$

Then we'll find the magnitude of this derivative.

$$\left| r'(t) \right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left| r'(t) \right| = \sqrt{(6\cos t)^2 + (-8)^2 + (-6\sin t)^2}$$

$$\left| r'(t) \right| = \sqrt{36\cos^2 t + 64 + 36\sin^2 t}$$

$$\left| r'(t) \right| = \sqrt{64 + 36\left(\cos^2 t + \sin^2 t\right)}$$

$$\left| r'(t) \right| = \sqrt{64 + 36(1)}$$

$$\left| r'(t) \right| = \sqrt{100}$$

$$\left| r'(t) \right| = 10$$

And now we can use everything we just found to find the unit tangent vector.

$$T(t) = \frac{r'(t)}{\left| r'(t) \right|}$$

$$T(t) = \frac{6\cos t\mathbf{i} - 8\mathbf{j} - 6\sin t\mathbf{k}}{10}$$

$$T(t) = \frac{6}{10}\cos t\mathbf{i} - \frac{8}{10}\mathbf{j} - \frac{6}{10}\sin t\mathbf{k}$$

$$T(t) = \frac{3}{5}\cos t\mathbf{i} - \frac{4}{5}\mathbf{j} - \frac{3}{5}\sin t\mathbf{k}$$

To find the curvature of the vector function, we'll first need to find the derivative of this unit tangent vector.

$$T(t) = \frac{3}{5}\cos t\mathbf{i} - \frac{4}{5}\mathbf{j} - \frac{3}{5}\sin t\mathbf{k}$$

$$T'(t) = -\frac{3}{5}\sin t\mathbf{i} - (0)\mathbf{j} - \frac{3}{5}\cos t\mathbf{k}$$

Then we'll find the magnitude of this derivative.

$$\left|T'(t)\right| = \sqrt{\left[T'(t)_1\right]^2 + \left[T'(t)_2\right]^2 + \left[T'(t)_3\right]^2}$$

$$\left|T'(t)\right| = \sqrt{\left(-\frac{3}{5}\sin t\right)^2 + (0)^2 + \left(-\frac{3}{5}\cos t\right)^2}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}\sin^2 t + \frac{9}{25}\cos^2 t}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}\left(\sin^2 t + \cos^2 t\right)}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}(1)}$$

$$\left|T'(t)\right| = \frac{3}{5}$$

And now we can use everything we just found to solve for the curvature of the vector function.

$$\kappa(t) = \frac{\left|T'(t)\right|}{\left|r'(t)\right|}$$

$$\kappa(t) = \frac{\frac{3}{5}}{10}$$

$$\kappa(t) = \frac{3}{5 \cdot 10}$$

$$\kappa(t) = \frac{3}{50}$$

This is the curvature of the vector function.