Topic: Cross product of two vectors

Question: Find the cross product.

$$a\langle 1,-1,1\rangle$$

$$b\langle -2,1,2\rangle$$

Answer choices:

A $c\langle 3,-4,1\rangle$

B $c\langle -3,-4,-1\rangle$

C $c\langle -3,4,-1\rangle$

D $c\langle 3,4,1\rangle$

Solution: B

The cross product $a \times b$ of two vectors $a\langle a_1, a_2, a_3\rangle$ and $b\langle b_1, b_2, b_3\rangle$ is given by

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\left(a_2b_3 - a_3b_2\right) - \mathbf{j}\left(a_1b_3 - a_3b_1\right) + \mathbf{k}\left(a_1b_2 - a_2b_1\right)$$

If we plug our vectors $a\langle 1, -1, 1\rangle$ and $b\langle -2, 1, 2\rangle$ into this last formula, we get

$$a \times b = \mathbf{i}\left[(-1)(2) - (1)(1)\right] - \mathbf{j}\left[(1)(2) - (1)(-2)\right] + \mathbf{k}\left[(1)(1) - (-1)(-2)\right]$$

$$a \times b = \mathbf{i}(-2 - 1) - \mathbf{j}(2 + 2) + \mathbf{k}(1 - 2)$$

$$a \times b = -3\mathbf{i} - 4\mathbf{j} - \mathbf{k}$$

We'll convert this into standard vector form $c\langle c_1, c_2, c_3\rangle$ to get the cross product of the vectors.

$$c\langle -3, -4, -1\rangle$$

Topic: Cross product of two vectors

Question: Find the cross product.

$a\langle 4,2,0\rangle$

$b\langle -1,-3,1\rangle$

Answer choices:

A $c\langle -2,4,10\rangle$

B $c\langle -2,-4,10\rangle$

C $c\langle 2,4,-10\rangle$

D $c\langle 2,-4,-10\rangle$

Solution: D

The cross product $a \times b$ of two vectors $a\langle a_1, a_2, a_3\rangle$ and $b\langle b_1, b_2, b_3\rangle$ is given by

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\left(a_2b_3 - a_3b_2\right) - \mathbf{j}\left(a_1b_3 - a_3b_1\right) + \mathbf{k}\left(a_1b_2 - a_2b_1\right)$$

If we plug our vectors $a\langle 4,2,0\rangle$ and $b\langle -1, -3, 1\rangle$ into this last formula, we get

$$a \times b = \mathbf{i}\left[(2)(1) - (0)(-3)\right] - \mathbf{j}\left[(4)(1) - (0)(-1)\right] + \mathbf{k}\left[(4)(-3) - (2)(-1)\right]$$

$$a \times b = \mathbf{i}(2 - 0) - \mathbf{j}(4 - 0) + \mathbf{k}(-12 + 2)$$

$$a \times b = 2\mathbf{i} - 4\mathbf{j} - 10\mathbf{k}$$

We'll convert this into standard vector form $c\langle c_1, c_2, c_3\rangle$ to get the cross product of the vectors.

$$c\langle 2, -4, -10\rangle$$

Topic: Cross product of two vectors

Question: Find the cross product.

$$a\langle 6,7,-5\rangle$$

$$b\langle 8,7,-11\rangle$$

Answer choices:

A $c\langle -42,-22,-14\rangle$

B $c\langle -112,106,98\rangle$

C $c\langle -21,13,-7\rangle$

D $c\langle -112,-106,98\rangle$

Solution: C

The cross product $a \times b$ of two vectors $a\langle a_1, a_2, a_3\rangle$ and $b\langle b_1, b_2, b_3\rangle$ is given by

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\left(a_2b_3 - a_3b_2\right) - \mathbf{j}\left(a_1b_3 - a_3b_1\right) + \mathbf{k}\left(a_1b_2 - a_2b_1\right)$$

If we plug our vectors $a\langle 6,7,-5\rangle$ and $b\langle 8,7,-11\rangle$ into this last formula, we get

$$a \times b = \mathbf{i}\left[(7)(-11) - (-5)(7)\right] - \mathbf{j}\left[(6)(-11) - (-5)(8)\right] + \mathbf{k}\left[(6)(7) - (7)(8)\right]$$

$$a \times b = \mathbf{i}(-77 + 35) - \mathbf{j}(-66 + 40) + \mathbf{k}(42 - 56)$$

$$a \times b = -42\mathbf{i} + 26\mathbf{j} - 14\mathbf{k}$$

$$a \times b = 2(-21\mathbf{i} + 13\mathbf{j} - 7\mathbf{k})$$

$$a \times b = -21\mathbf{i} + 13\mathbf{j} - 7\mathbf{k}$$

We'll convert this into standard vector form $c\langle c_1, c_2, c_3\rangle$ to get the cross product of the vectors.

$c\langle -21,13,-7\rangle$