Topic: Type I and II regions

Question: Say whether the region is type I or II, then find the volume given by the double integral, if D is the triangle bounded by $x=1$, $y=1$, and $y=-x+4$.

$$\iint_D x^2 \, dA$$

Answer choices:

A 8

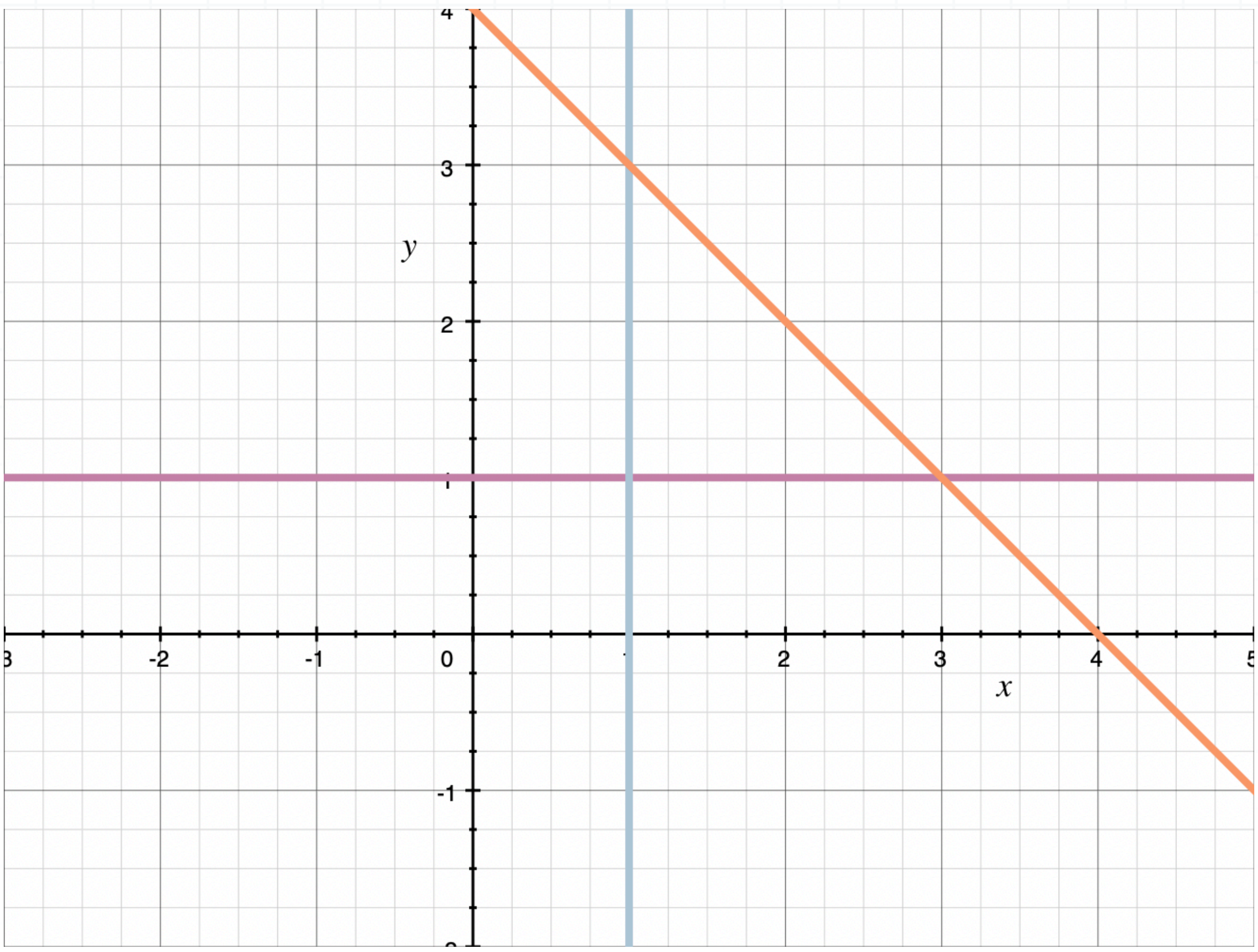
B 6

C 2

D 12

Solution: B

A sketch of the region D is



We can cut the region into uniform slices both horizontally and vertically, which means we can evaluate it as type I or type II. Let's do this as a type II region.

We need to solve $y = -x + 4$ for x, and we find $x = 4 - y$. This equation, along with $x = 1$, define the limits of integration with respect to x. If we look at the sketch of the region, we can see that y is defined on $[1,3]$. Putting all of this into a double integral, we get

$$\iint_D x^2 \, dA$$

$$\int_1^3 \int_1^{4-y} x^2 \, dx \, dy$$

Integrate with respect to x and evaluate over the interval.

$$\int_1^3 \frac{1}{3}x^3 \Big|_{x=1}^{x=4-y} \, dy$$

$$\int_1^3 \frac{1}{3}(4-y)^3 - \frac{1}{3}(1)^3 \, dy$$

$$\int_1^3 \frac{1}{3}(4-y)^3 - \frac{1}{3} \, dy$$

Integrate with respect to y and evaluate over the interval.

$$-\frac{1}{12}(4-y)^4 - \frac{1}{3}y \Big|_1^3$$

$$\left(-\frac{1}{12}(4-3)^4 - \frac{1}{3}(3)\right) - \left(-\frac{1}{12}(4-1)^4 - \frac{1}{3}(1)\right)$$

$$\left(-\frac{1}{12} - 1\right) - \left(-\frac{81}{12} - \frac{1}{3}\right)$$

$$-\frac{13}{12} + \frac{85}{12}$$

$$\frac{72}{12}$$

$$6$$

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$$\iint_D x^2 \, dA$$

Answer choices:

A $\frac{189}{16}$

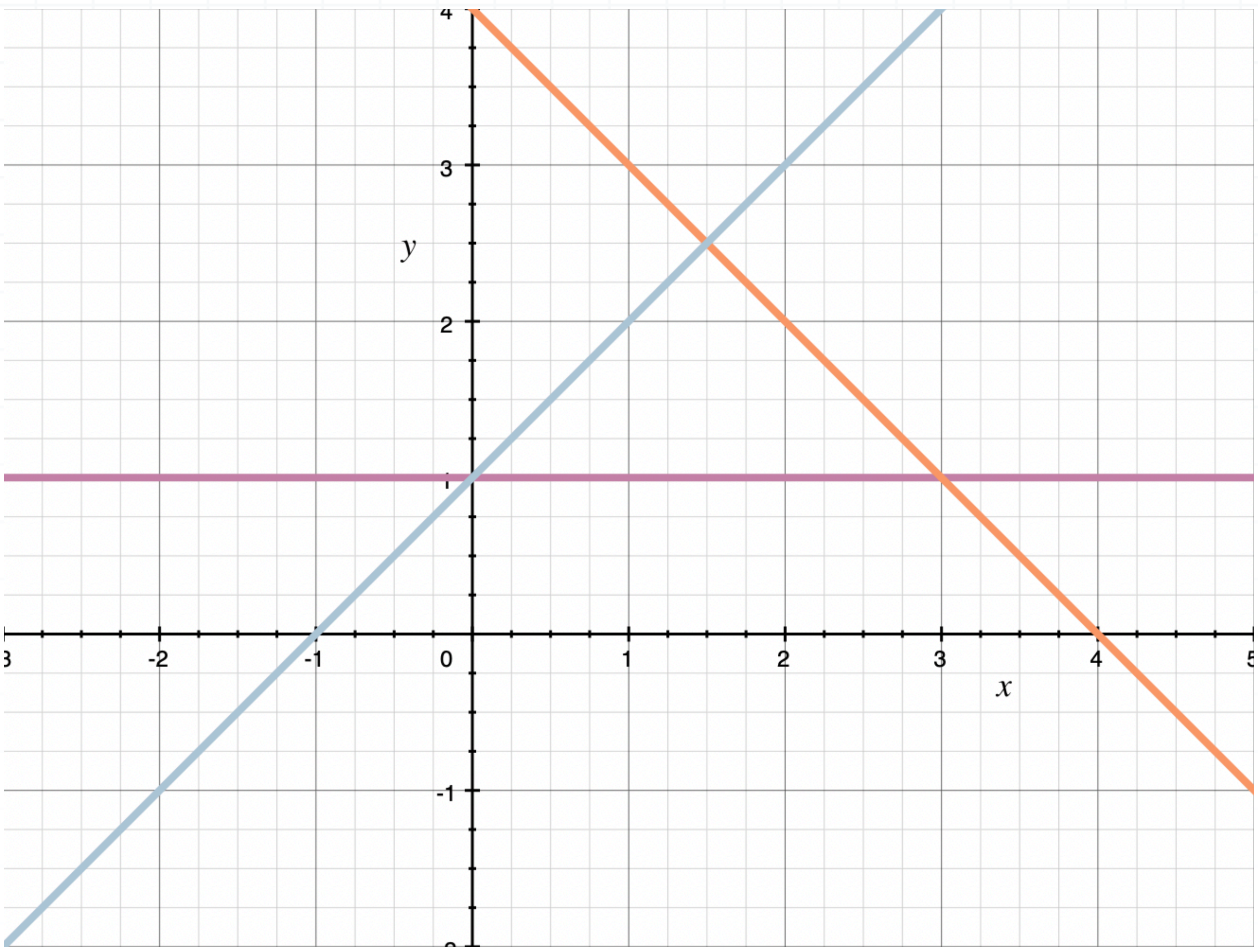
B $\frac{378}{32}$

C $\frac{189}{64}$

D $\frac{189}{32}$

Solution: D

A sketch of the region D is



We can cut the region into uniform slices both horizontally and vertically, which means we can evaluate it as type I or type II. Let's do this as a type II region.

We need to solve $y = x + 1$ and $y = -x + 4$ for x, and we find $x = y - 1$ and $x = 4 - y$. These equations define the limits of integration with respect to x. If we look at the sketch of the region, we can see that y is defined on $[1,2.5]$. Putting all of this into a double integral, we get

$$\iint_D x^2 \, dA$$

$$\int_{1}^{\frac{5}{2}}\int_{y-1}^{4-y} x^2\ dx\ dy$$

Integrate with respect to x and evaluate over the interval.

$$\int_{1}^{\frac{5}{2}} \frac{1}{3}x^3\Big|_{x=y-1}^{x=4-y}\ dy$$

$$\int_{1}^{\frac{5}{2}} \frac{1}{3}(4-y)^3-\frac{1}{3}(y-1)^3\ dy$$

Integrate with respect to y and evaluate over the interval.

$$-\frac{1}{12}(4-y)^4-\frac{1}{12}(y-1)^4\Big|_{1}^{\frac{5}{2}}$$

$$-\frac{1}{12}\left(4-\frac{5}{2}\right)^4-\frac{1}{12}\left(\frac{5}{2}-1\right)^4-\left(-\frac{1}{12}(4-1)^4-\frac{1}{12}(1-1)^4\right)$$

$$-\frac{1}{12}\left(\frac{3}{2}\right)^4-\frac{1}{12}\left(\frac{3}{2}\right)^4+\frac{81}{12}$$

$$-\frac{27}{64}-\frac{27}{64}+\frac{432}{64}$$

$$\frac{378}{64}$$

$$\frac{189}{32}$$

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Question: Say whether the region is type I or II, then find the volume given by the double integral, if D is the triangle bounded by $y = 1$, $y = x + 1$, and $y = -x + 4$.

$$\iint_D x^2 + 1 \ dA$$

Answer choices:

A $\quad \dfrac{261}{32}$

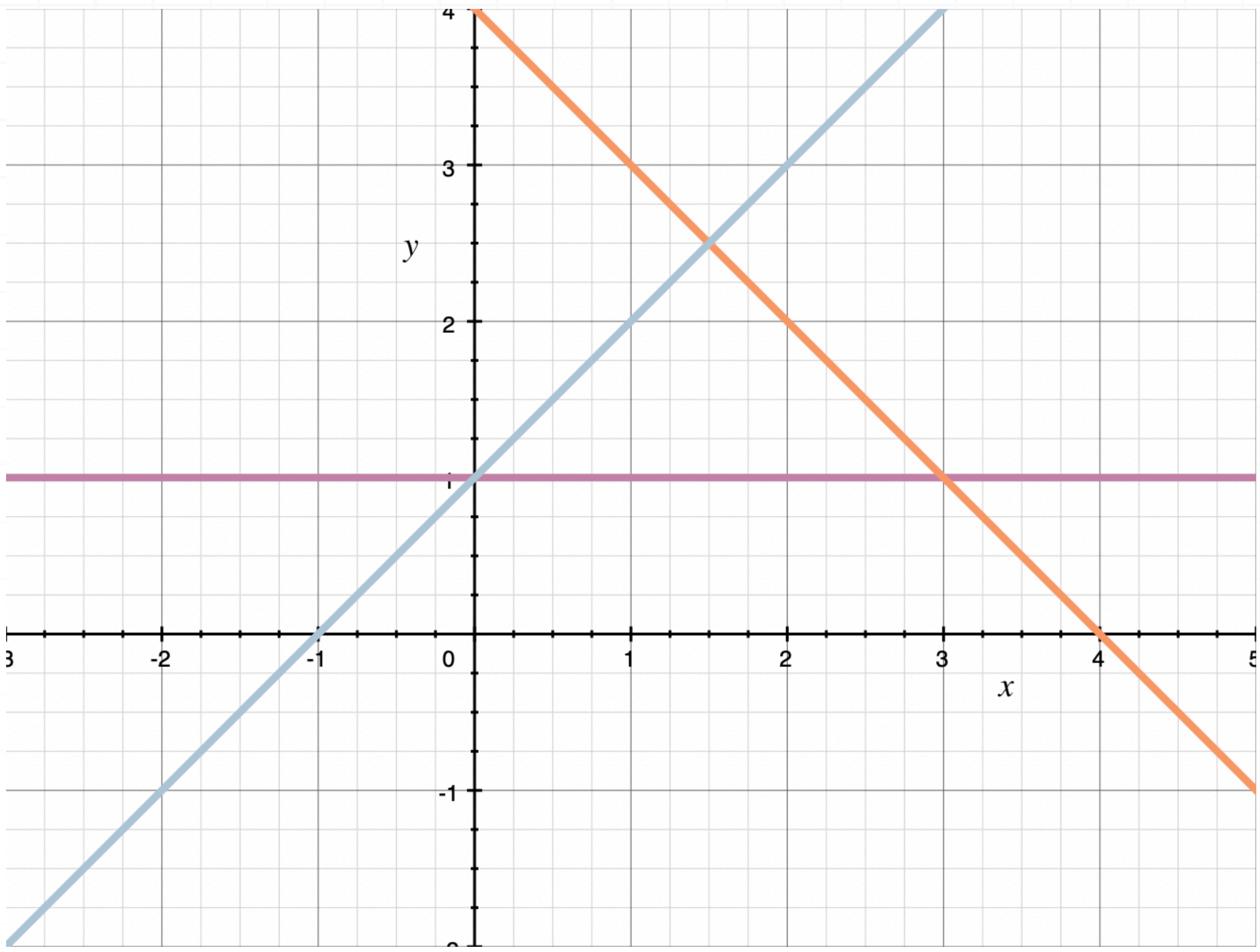
B $\quad \dfrac{522}{16}$

C $\quad \dfrac{261}{64}$

D $\quad \dfrac{522}{32}$

Solution: A

A sketch of the region D is



We can cut the region into uniform slices both horizontally and vertically, which means we can evaluate it as type I or type II. Let's do this as a type II region.

We need to solve $y = x + 1$ and $y = -x + 4$ for x, and we find $x = y - 1$ and $x = 4 - y$. These equations define the limits of integration with respect to x. If we look at the sketch of the region, we can see that y is defined on $[1,2.5]$. Putting all of this into a double integral, we get

$$\iint_D x^2 + 1 \ dA$$

$$\int_{1}^{\frac{5}{2}} \int_{y-1}^{4-y} x^2 + 1 \ dx \ dy$$

Integrate with respect to x and evaluate over the interval.

$$\int_{1}^{\frac{5}{2}} \frac{1}{3}x^3 + x \Big|_{x=y-1}^{x=4-y} \ dy$$

$$\int_{1}^{\frac{5}{2}} \frac{1}{3}(4-y)^3 + (4-y) - \left(\frac{1}{3}(y-1)^3 + (y-1)\right) \ dy$$

$$\int_{1}^{\frac{5}{2}} \frac{1}{3}(4-y)^3 + (4-y) - \frac{1}{3}(y-1)^3 - (y-1) \ dy$$

$$\int_{1}^{\frac{5}{2}} \frac{1}{3}(4-y)^3 - \frac{1}{3}(y-1)^3 - 2y + 5 \ dy$$

Integrate with respect to y and evaluate over the interval.

$$-\frac{1}{12}(4-y)^4 - \frac{1}{12}(y-1)^4 - y^2 + 5y \Big|_{1}^{\frac{5}{2}}$$

$$-\frac{1}{12}\left(4-\frac{5}{2}\right)^4 - \frac{1}{12}\left(\frac{5}{2}-1\right)^4 - \left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right)$$

$$-\left(-\frac{1}{12}(4-1)^4 - \frac{1}{12}(1-1)^4 - (1)^2 + 5(1)\right)$$

$$-\frac{1}{12}\left(\frac{8}{2}-\frac{5}{2}\right)^4 - \frac{1}{12}\left(\frac{5}{2}-\frac{2}{2}\right)^4 - \frac{25}{4} + \frac{25}{2} - \left(-\frac{1}{12}(81) + 4\right)$$

$$-\frac{1}{12}\left(\frac{3}{2}\right)^4-\frac{1}{12}\left(\frac{3}{2}\right)^4-\frac{25}{4}+\frac{25}{2}+\frac{81}{12}-4$$

$$-\frac{27}{64}-\frac{27}{64}-\frac{25}{4}+\frac{25}{2}+\frac{27}{4}-4$$

$$\frac{261}{32}$$