Topic: Acute angle between the lines

Question: Find the acute angle between the lines.

$$x - 2y = 1$$

$$2x - y = 3$$

Answer choices:

A 12°

B 37°

C 45°

D 53°

Solution: B

To find the acute angle between two lines, we'll first convert them to standard vector format, so that we can use the formula for the angle between two vectors. In this case, the line $x - 2y = 1$ will become $a = \langle 1, -2 \rangle$ and the line $2x - y = 3$ will become $b = \langle 2, -1 \rangle$. Now we can use

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

to find the angle between the vectors. $a \cdot b$ is the dot product of the vectors, and $|a|$ and $|b|$ are their lengths. The lengths can also be denoted by D_a and D_b, and are found using the distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We'll start by finding the dot product.

$$a \cdot b = (1)(2) + (-2)(-1)$$

$$a \cdot b = 2 + 2$$

$$a \cdot b = 4$$

Now we'll find the length of each vector, using the origin $(0,0)$ as (x_1, y_1). The length of a is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D_a = \sqrt{(1 - 0)^2 + (-2 - 0)^2}$$

$$D_a = \sqrt{1+4}$$

$$D_a = |a| = \sqrt{5}$$

And the length of b is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D_b = \sqrt{(2-0)^2 + (-1-0)^2}$$

$$D_b = \sqrt{4+1}$$

$$D_b = |b| = \sqrt{5}$$

We'll plug everything we've found into the formula for the angle between the vectors.

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\cos\theta = \frac{4}{\sqrt{5}\sqrt{5}}$$

$$\cos\theta = \frac{4}{5}$$

$$\theta = \arccos\frac{4}{5}$$

$$\theta \approx 37^\circ$$

Remember, if the angle is greater than 90°, it is not acute. This just means we accidentally found the obtuse angle between the two vectors. If that's the case, we can find the acute by subtracting the obtuse angle from 180°.

Topic: Acute angle between the lines

Question: Find the acute angle between the lines.

$$-3x - 1y = 2$$

$$4x + 5y = 5$$

Answer choices:

A 147°

B 123°

C 57°

D 33°

Solution: D

To find the acute angle between two lines, we'll first convert them to standard vector format, so that we can use the formula for the angle between two vectors. In this case, the line $-3x - 1y = 2$ will become $a = \langle -3, -1 \rangle$ and the line $4x + 5y = 5$ will become $b = \langle 4,5 \rangle$. Now we can use

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

to find the angle between the vectors. $a \cdot b$ is the dot product of the vectors, and $|a|$ and $|b|$ are their lengths. The lengths can also be denoted by D_a and D_b, and are found using the distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We'll start by finding the dot product.

$$a \cdot b = (-3)(4) + (-1)(5)$$

$$a \cdot b = -12 - 5$$

$$a \cdot b = -17$$

Now we'll find the length of each vector, using the origin $(0,0)$ as (x_1, y_1). The length of a is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D_a = \sqrt{(-3 - 0)^2 + (-1 - 0)^2}$$

$$D_a = \sqrt{9+1}$$

$$D_a = |a| = \sqrt{10}$$

And the length of b is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D_b = \sqrt{(4-0)^2 + (5-0)^2}$$

$$D_b = \sqrt{16+25}$$

$$D_b = |b| = \sqrt{41}$$

We'll plug everything we've found into the formula for the angle between the vectors.

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\cos\theta = \frac{-17}{\sqrt{10}\sqrt{41}}$$

$$\cos\theta = \frac{-17}{\sqrt{410}}$$

$$\theta = \arccos\frac{-17}{\sqrt{410}}$$

$$\theta \approx 147^\circ$$

Remember, if the angle is greater than 90°, it is not acute. This just means we accidentally found the obtuse angle between the two vectors. If that's the case, we can find the acute by subtracting the obtuse angle from 180°.

$$\theta = 180^\circ - 147^\circ$$

$$\theta \approx 33^\circ$$

Topic: Acute angle between the lines

Question: Find the acute angle between the lines.

$$-8x - 4y = 7$$

$$11x + 7y = 2$$

Answer choices:

A 15°

B 84°

C 6°

D 174°

Solution: C

To find the acute angle between two lines, we'll first convert them to standard vector format, so that we can use the formula for the angle between two vectors. In this case, the line $-8x-4y=7$ will become $a=\langle -8,-4\rangle$ and the line $11x+7y=2$ will become $b=\langle 11,7\rangle$. Now we can use

$$\cos\theta=\frac{a\cdot b}{|a||b|}$$

to find the angle between the vectors. $a\cdot b$ is the dot product of the vectors, and $|a|$ and $|b|$ are their lengths. The lengths can also be denoted by D_a and D_b, and are found using the distance formula

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

We'll start by finding the dot product.

$$a\cdot b=(-8)(11)+(-4)(7)$$

$$a\cdot b=-88-28$$

$$a\cdot b=-116$$

Now we'll find the length of each vector, using the origin $(0,0)$ as (x_1,y_1). The length of a is

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$D_a=\sqrt{(-8-0)^2+(-4-0)^2}$$

$$D_a = \sqrt{64 + 16}$$

$$D_a = |a| = \sqrt{80}$$

And the length of b is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D_b = \sqrt{(11 - 0)^2 + (7 - 0)^2}$$

$$D_b = \sqrt{121 + 49}$$

$$D_b = |b| = \sqrt{170}$$

We'll plug everything we've found into the formula for the angle between the vectors.

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\cos\theta = \frac{-116}{\sqrt{80}\sqrt{170}}$$

$$\cos\theta = \frac{-116}{\sqrt{13{,}600}}$$

$$\theta = \arccos\frac{-116}{\sqrt{13{,}600}}$$

$$\theta \approx 174^\circ$$

Remember, if the angle is greater than 90°, it is not acute. This just means we accidentally found the obtuse angle between the two vectors. If that's the case, we can find the acute by subtracting the obtuse angle from 180°.

$$\theta = 180^\circ - 174^\circ$$

$$\theta \approx 6^\circ$$