Limit of a vector function

To find the limit of a vector function,

$$r(t) = a(t)\mathbf{i} + b(t)\mathbf{j} + c(t)\mathbf{k}$$

we'll need to take the limit of each term separately.

$$\lim_{t \to x} \left[a(t)\mathbf{i} + b(t)\mathbf{j} + c(t)\mathbf{k} \right]$$

$$\lim_{t \to x} a(t)\mathbf{i} + \lim_{t \to x} b(t)\mathbf{j} + \lim_{t \to x} c(t)\mathbf{k}$$

$$a(x)\mathbf{i} + b(x)\mathbf{j} + c(x)\mathbf{k}$$

Example

Find the limit of the vector function.

$$\lim_{t \to 0} \left[(t^2 - 2)\mathbf{i} + \ln(t + e)\mathbf{j} + \frac{4t}{\sin t}\mathbf{k} \right]$$

We'll take the limit of each term separately.

$$\lim_{t \to 0} \left(t^2 - 2\right)\mathbf{i} + \lim_{t \to 0} \ln(t + e)\mathbf{j} + \lim_{t \to 0} \frac{4t}{\sin t}\mathbf{k}$$

Evaluating the first two terms as $t \to 0$, we get

$$\left(0^2 - 2\right)\mathbf{i} + \ln(0 + e)\mathbf{j} + \lim_{t \to 0} \frac{4t}{\sin t}\mathbf{k}$$

$$-2\mathbf{i} + \ln(e)\mathbf{j} + \lim_{t\to 0} \frac{4t}{\sin t}\mathbf{k}$$

$$-2\mathbf{i} + 1\mathbf{j} + \lim_{t\to 0} \frac{4t}{\sin t}\mathbf{k}$$

$$-2\mathbf{i} + \mathbf{j} + \lim_{t\to 0} \frac{4t}{\sin t}\mathbf{k}$$

Because the third term gives $0/0$ when $t \to 0$, we have to use L'Hospital's rule, replacing the numerator and denominator with their derivatives.

$$-2\mathbf{i} + \mathbf{j} + \lim_{t\to 0} \frac{4}{\cos t}\mathbf{k}$$

Evaluating as $t \to 0$, we get

$$-2\mathbf{i} + \mathbf{j} + \frac{4}{\cos 0}\mathbf{k}$$

$$-2\mathbf{i} + \mathbf{j} + \frac{4}{1}\mathbf{k}$$

$$-2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$$

This is the limit of the vector function.