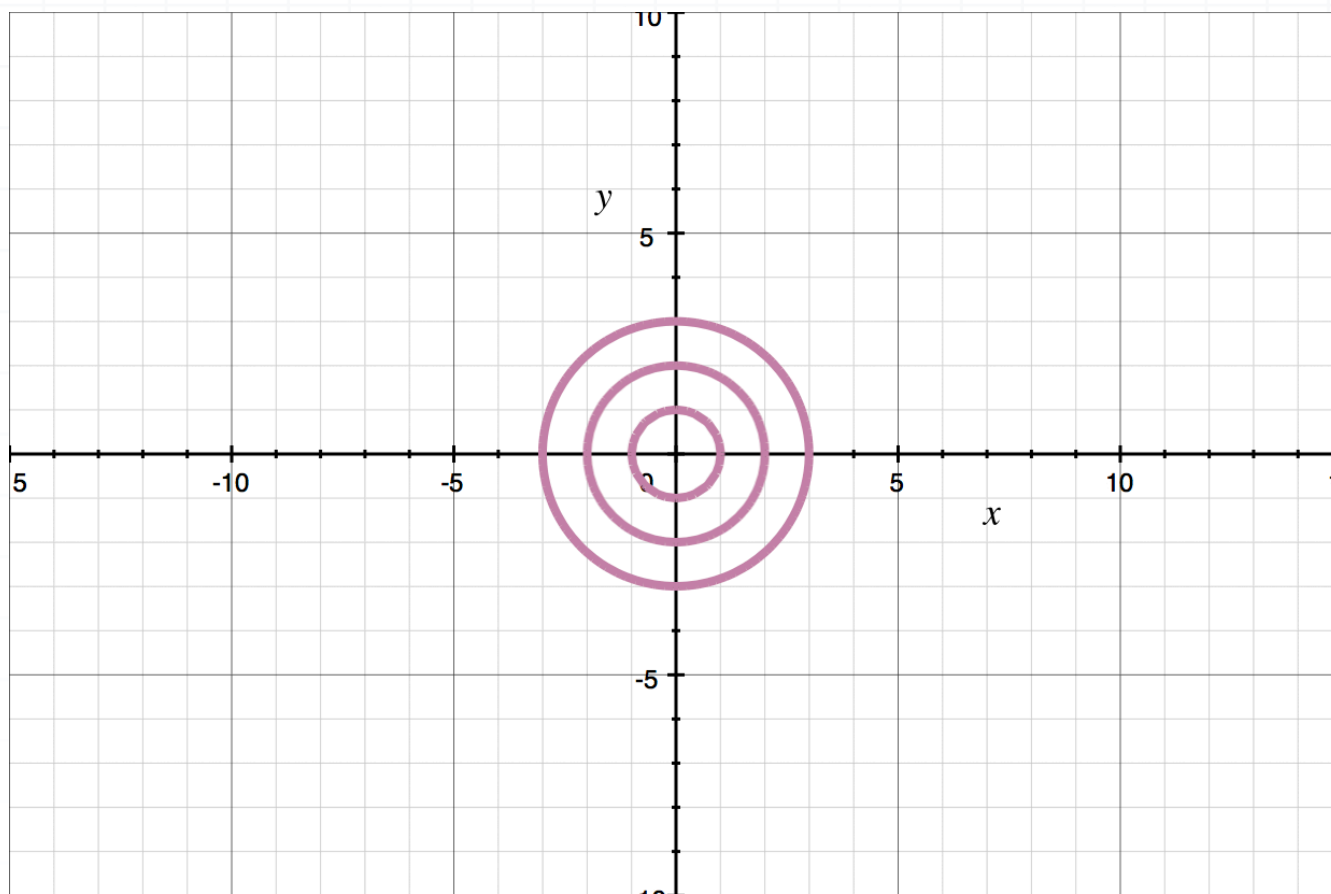


Topic: Sketching level curves of multivariable functions**Question:** Which function gives the equation for these level curves?**Answer choices:**

A $f(x, y) = \sqrt{x^2 - y^2}$

B $f(x, y) = \sqrt{x + y^2}$

C $f(x, y) = \sqrt{x^2 + y^2}$

D $f(x, y) = \sqrt{x^2 + y}$



Solution: C

The level curves pictured are circles, which means we need to look in the answer choices for the equation of a circle. We know that the equation of a circle is

$$x^2 + y^2 = r^2$$

where r is the radius of the circle. These level curves each have a different radius, so in effect, we can replace the radius with some arbitrary constant c .

$$x^2 + y^2 = c^2$$

The level curves displayed in this question are likely given by $c = 1, 2, 3$, such that

$$x^2 + y^2 = 1^2$$

$$x^2 + y^2 = 2^2$$

$$x^2 + y^2 = 3^2$$

We want to solve the equation for c , so we get

$$c^2 = x^2 + y^2$$

$$c = \sqrt{x^2 + y^2}$$

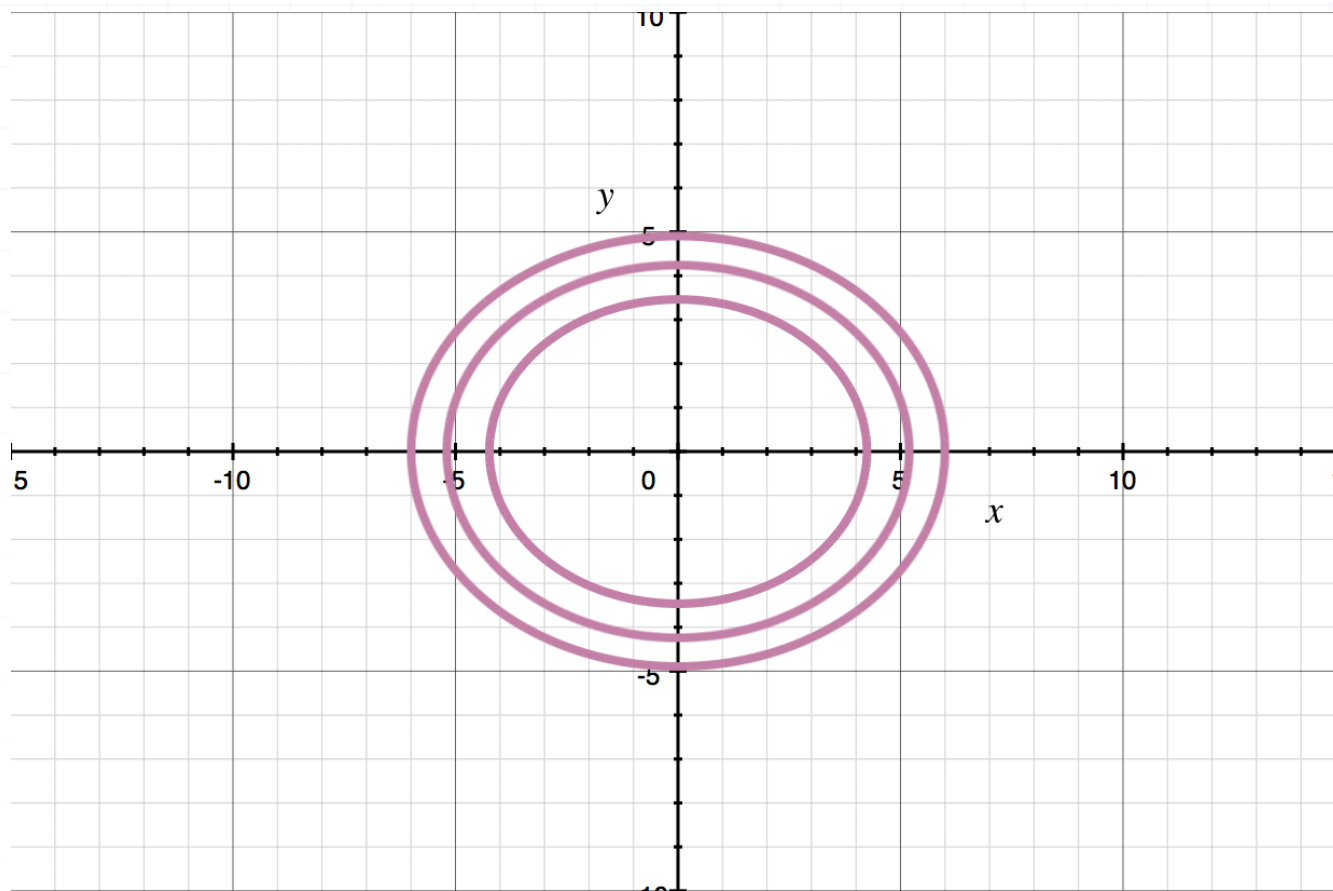
If we want to give an equation that represents the level curves in general, regardless of the value of c , then we just replace c with $f(x, y)$, and we get



$$f(x, y) = \sqrt{x^2 + y^2}$$

This is the equation that represents the level curves.



Topic: Sketching level curves of multivariable functions**Question:** Which function gives the equation for these level curves?**Answer choices:**

- A $f(x, y) = \frac{1}{3}x + \frac{1}{2}y^2$
- B $f(x, y) = \frac{1}{3}x^2 - \frac{1}{2}y^2$
- C $f(x, y) = \frac{1}{3}x^2 + \frac{1}{2}y$
- D $f(x, y) = \frac{1}{3}x^2 + \frac{1}{2}y^2$



Solution: D

The level curves pictured are ellipses, which means we need to look in the answer choices for the equation of an ellipse. We know that the equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

These level curves each have a different radius, and if we change the constant on the right side of this equation, it changes the radius. So in effect, we can replace 1 with some arbitrary constant c .

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = c$$

If we want to give an equation that represents the level curves in general, regardless of the value of c , then we just replace c with $f(x, y)$, and we get

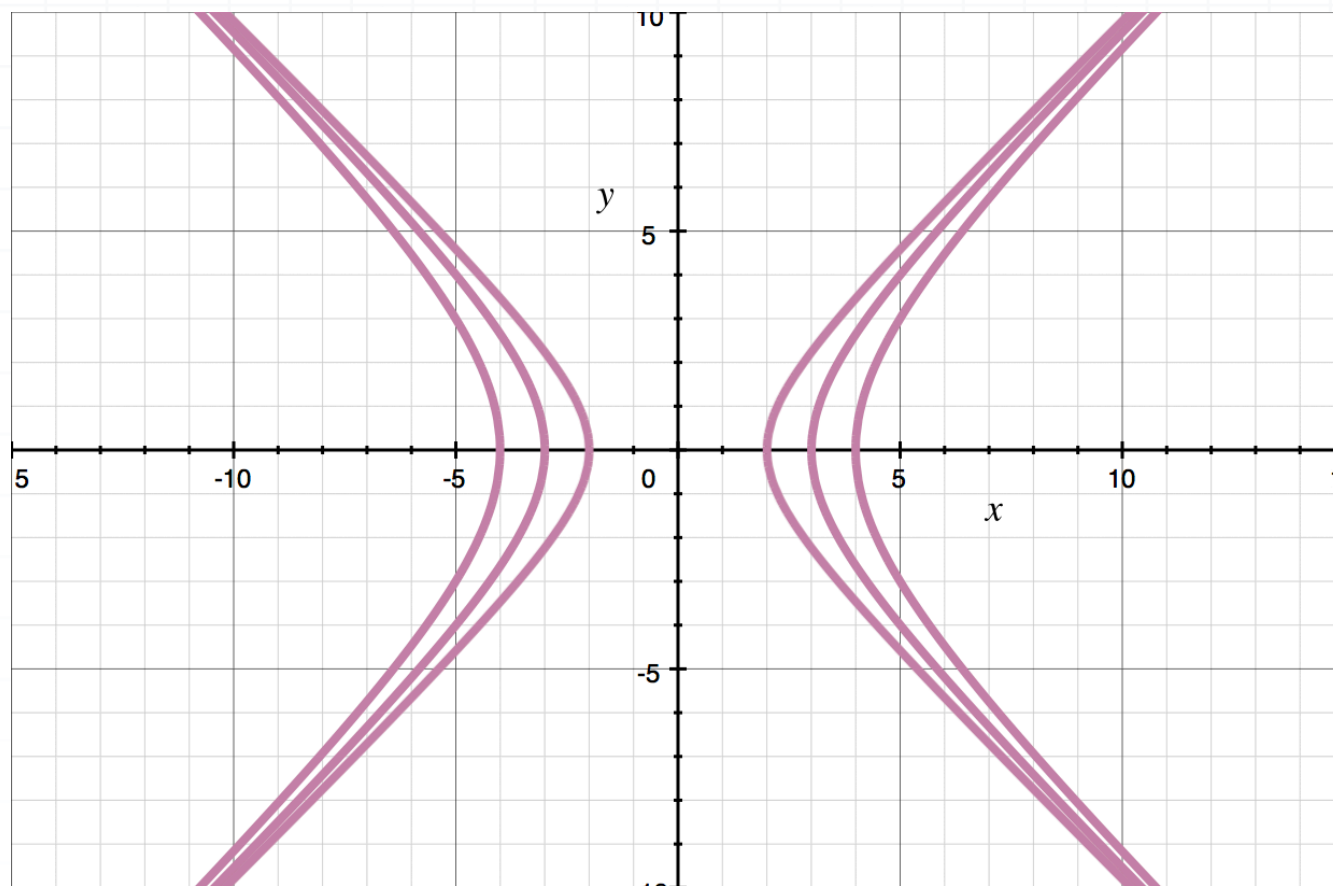
$$f(x, y) = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

The only equation that matches this format, where x and y are both squared and where we've got an addition sign between the terms instead of a subtraction sign, is

$$f(x, y) = \frac{1}{3}x^2 + \frac{1}{2}y^2$$

Therefore, this is the equation that represents the level curves.



Topic: Sketching level curves of multivariable functions**Question:** Which function gives the equation for these level curves?**Answer choices:**

- A $f(x, y) = x^2 - y^2$
- B $f(x, y) = x^2 + y^2$
- C $f(x, y) = x - y^2$
- D $f(x, y) = x^2 - y$



Solution: A

The level curves pictured are hyperbolas, which means we need to look in the answer choices for the equation of a hyperbola. We know that the equation of a hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{d}$$

These level curves each have a different directrix, so in effect, we can replace z/d with some arbitrary constant c .

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = c$$

The level curves displayed in this question are likely given by $c = 1, 2, 3$, such that

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 3$$

If we want to give an equation that represents the level curves in general, regardless of the value of c , then we just replace c with $f(x, y)$, and we get

$$f(x, y) = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$



The only equation that matches this format, where x and y are both squared and where we've got a subtraction sign between the terms instead of an addition sign, is

$$f(x, y) = x^2 - y^2$$

Therefore, this is the equation that represents the level curves.

