Topic: Angle between two vectors

Question: Find the angle between the vectors.

$$a = \langle 2,0,-1 \rangle$$

$$b = \langle -1,4,2 \rangle$$

Answer choices:

A 113°

B 247°

C 293°

D 67°

Solution: A

The angle between two vectors a and b can be given by

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$

where $a \cdot b$ is the dot product

where $|a|$ is the length of the vector a (can also be called D_a)

where $|b|$ is the length of the vector b (can also be called D_b)

We'll start by finding the dot product. To find the dot product of two vectors, we just multiply like coordinates together and then add them to each other.

$$a \cdot b = (2)(-1) + (0)(4) + (-1)(2)$$

$$a \cdot b = -2 + 0 - 2$$

$$a \cdot b = -4$$

Next we'll find the length of each vector using the distance formula. We'll use the origin $(0,0,0)$ as (x_1, y_1, z_1), and we'll take (x_2, y_2, z_2) from the direction numbers of the vector.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The length of $a = \langle 2,0,-1 \rangle$ is

$$D_a = \sqrt{(2-0)^2 + (0-0)^2 + (-1-0)^2}$$

$$D_a = \sqrt{4 + 0 + 1}$$

$$D_a = |a| = \sqrt{5}$$

The length of $b = \langle -1,4,2 \rangle$ is

$$D_b = \sqrt{(-1 - 0)^2 + (4 - 0)^2 + (2 - 0)^2}$$

$$D_b = \sqrt{1 + 16 + 4}$$

$$D_b = |b| = \sqrt{21}$$

Plugging everything we've found into the formula gives the angle between the vectors.

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$

$$\cos(\theta) = \frac{-4}{\sqrt{5}\sqrt{21}}$$

$$\cos(\theta) = \frac{-4}{\sqrt{105}}$$

$$\theta = \arccos \frac{-4}{\sqrt{105}}$$

$$\theta = 113^\circ$$

Topic: Angle between two vectors

Question: Find the angle between the vectors.

$$a = \langle 3, -2, 1 \rangle$$

$$b = \langle 5, 3, 1 \rangle$$

Answer choices:

A 297°

B 243°

C 63°

D 117°

Solution: C

The angle between two vectors a and b can be given by

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$

where $a \cdot b$ is the dot product

where $|a|$ is the length of the vector a (can also be called D_a)

where $|b|$ is the length of the vector b (can also be called D_b)

We'll start by finding the dot product. To find the dot product of two vectors, we just multiply like coordinates together and then add them to each other.

$$a \cdot b = (3)(5) + (-2)(3) + (1)(1)$$

$$a \cdot b = 15 - 6 + 1$$

$$a \cdot b = 10$$

Next we'll find the length of each vector using the distance formula. We'll use the origin $(0,0,0)$ as (x_1, y_1, z_1), and we'll take (x_2, y_2, z_2) from the direction numbers of the vector.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The length of $a = \langle 3, -2, 1 \rangle$ is

$$D_a = \sqrt{(3 - 0)^2 + (-2 - 0)^2 + (1 - 0)^2}$$

$$D_a = \sqrt{9+4+1}$$

$$D_a = |a| = \sqrt{14}$$

The length of $b = \langle 5,3,1 \rangle$ is

$$D_b = \sqrt{(5-0)^2 + (3-0)^2 + (1-0)^2}$$

$$D_b = \sqrt{25+9+1}$$

$$D_b = |b| = \sqrt{35}$$

Plugging everything we've found into the formula gives the angle between the vectors.

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$

$$\cos(\theta) = \frac{10}{\sqrt{14}\sqrt{35}}$$

$$\cos(\theta) = \frac{10}{\sqrt{490}}$$

$$\theta = \arccos\frac{10}{\sqrt{490}}$$

$$\theta = 63^\circ$$

Topic: Angle between two vectors

Question: Find the angle between the vectors.

$$a = \langle 4, -4, -5 \rangle$$

$$b = \langle -2, 4, -3 \rangle$$

Answer choices:

A 283°

B 103°

C 257°

D 77°

Solution: B

The angle between two vectors a and b can be given by

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$

where $a \cdot b$ is the dot product

where $|a|$ is the length of the vector a (can also be called D_a)

where $|b|$ is the length of the vector b (can also be called D_b)

We'll start by finding the dot product. To find the dot product of two vectors, we just multiply like coordinates together and then add them to each other.

$$a \cdot b = (4)(-2) + (-4)(4) + (-5)(-3)$$

$$a \cdot b = -8 - 16 + 15$$

$$a \cdot b = -9$$

Next we'll find the length of each vector using the distance formula. We'll use the origin $(0,0,0)$ as (x_1, y_1, z_1), and we'll take (x_2, y_2, z_2) from the direction numbers of the vector.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The length of $a = \langle 4, -4, -5 \rangle$ is

$$D_a = \sqrt{(4-0)^2 + (-4-0)^2 + (-5-0)^2}$$

$$D_a = \sqrt{16 + 16 + 25}$$

$$D_a = |a| = \sqrt{57}$$

The length of $b = \langle -2,4,-3 \rangle$ is

$$D_b = \sqrt{(-2-0)^2 + (4-0)^2 + (-3-0)^2}$$

$$D_b = \sqrt{4 + 16 + 9}$$

$$D_b = |b| = \sqrt{29}$$

Plugging everything we've found into the formula gives the angle between the vectors.

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$

$$\cos(\theta) = \frac{-9}{\sqrt{57}\sqrt{29}}$$

$$\cos(\theta) = \frac{-9}{\sqrt{1{,}653}}$$

$$\theta = \arccos\frac{-9}{\sqrt{1{,}653}}$$

$$\theta = 103^\circ$$