Calculus 3 Workbook

Triple integrals in cylindrical coordinates

krista king MATH

CYLINDRICAL COORDINATES

1. Evaluate the triple integral given in cylindrical coordinates, where $f(r,\theta,z)=(3r-12z^2)\cos\theta$.

$$\int_{-1}^{1}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_{2}^{3} f(r,\theta,z)\ r\ dr\ d\theta\ dz$$

2. Identify the solid given by the following iterated integral in cylindrical coordinates.

$$\int_{-4}^{6}\int_{0}^{\pi}\int_{0}^{2} f(r,\theta,x)\ r\ dr\ d\theta\ dx$$

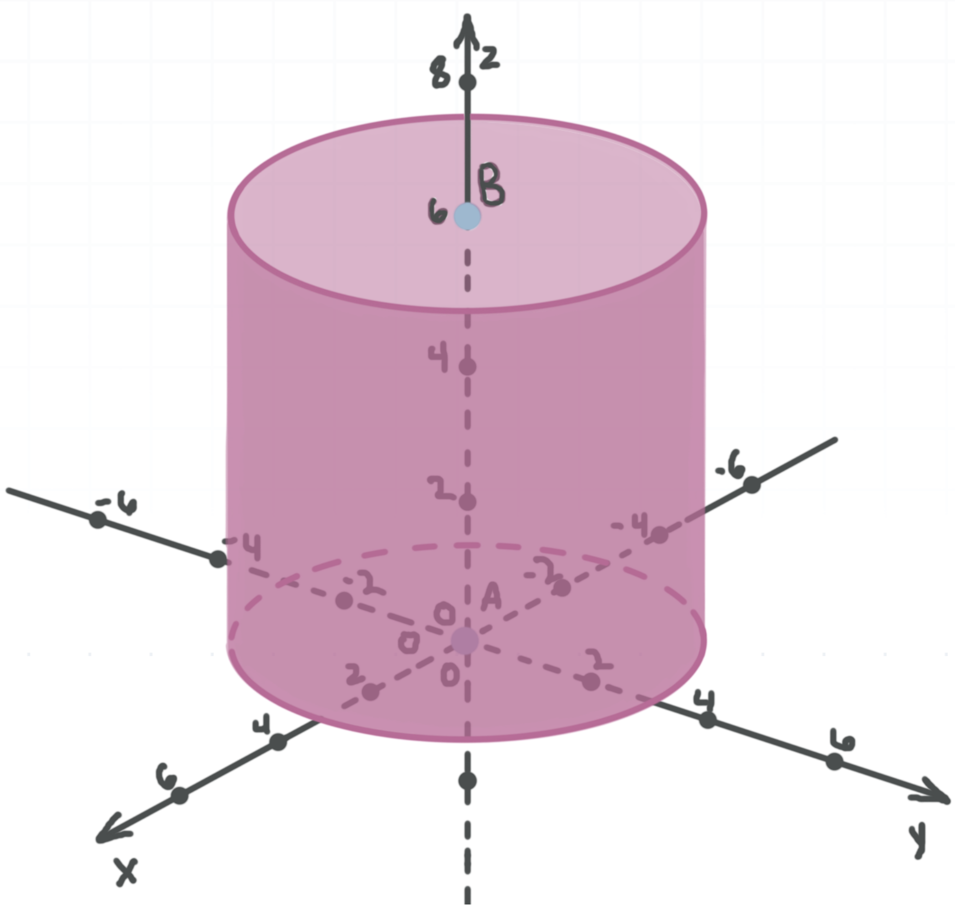
3. Identify the solid given by the iterated improper integral in cylindrical coordinates.

$$\int_{2}^{\infty}\int_{0}^{2\pi}\int_{0}^{\sqrt{2y-4}} f(r,\theta,y)\ r\ dr\ d\theta\ dy$$

CHANGING TRIPLE INTEGRALS TO CYLINDRICAL COORDINATES

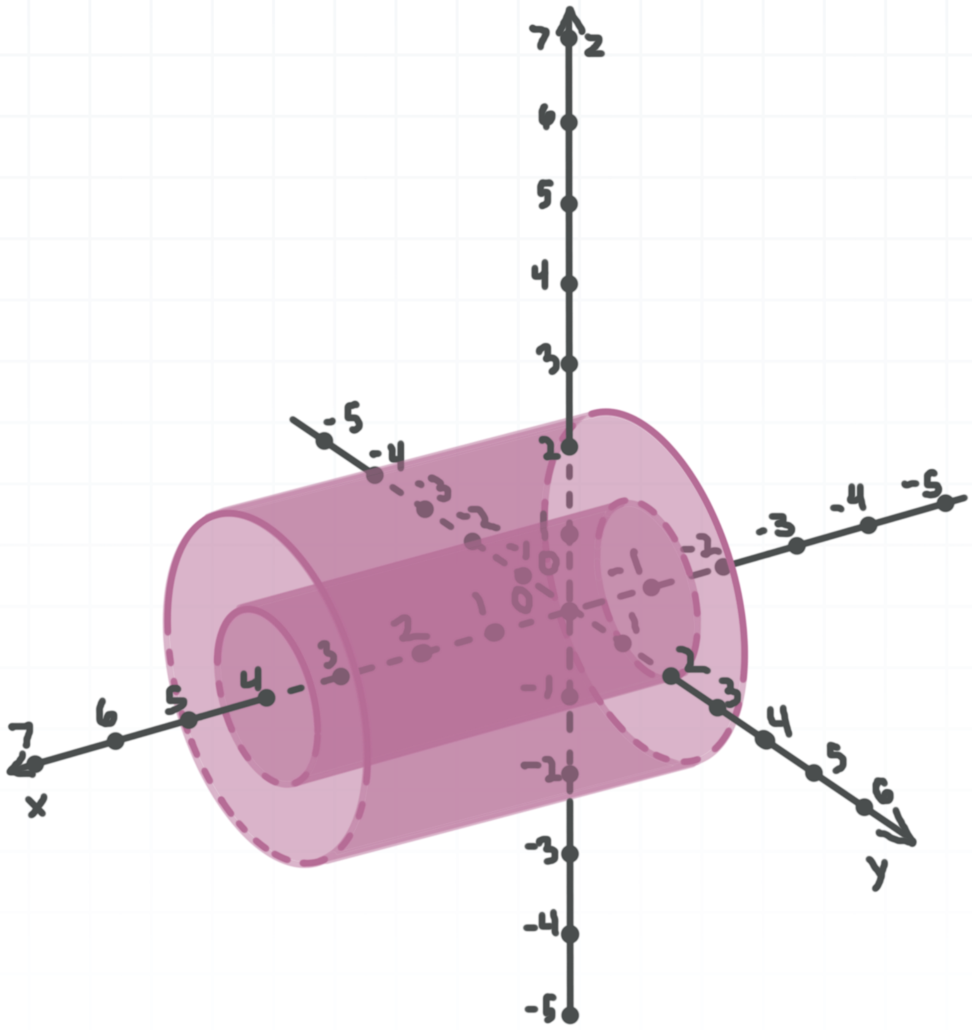
■ 1. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the right circular cylinder with radius 3, height 6, and a base that lies in the xy-plane with center at the origin.

$$\iiint_E (x^2 + y^2)2^z \ dV$$



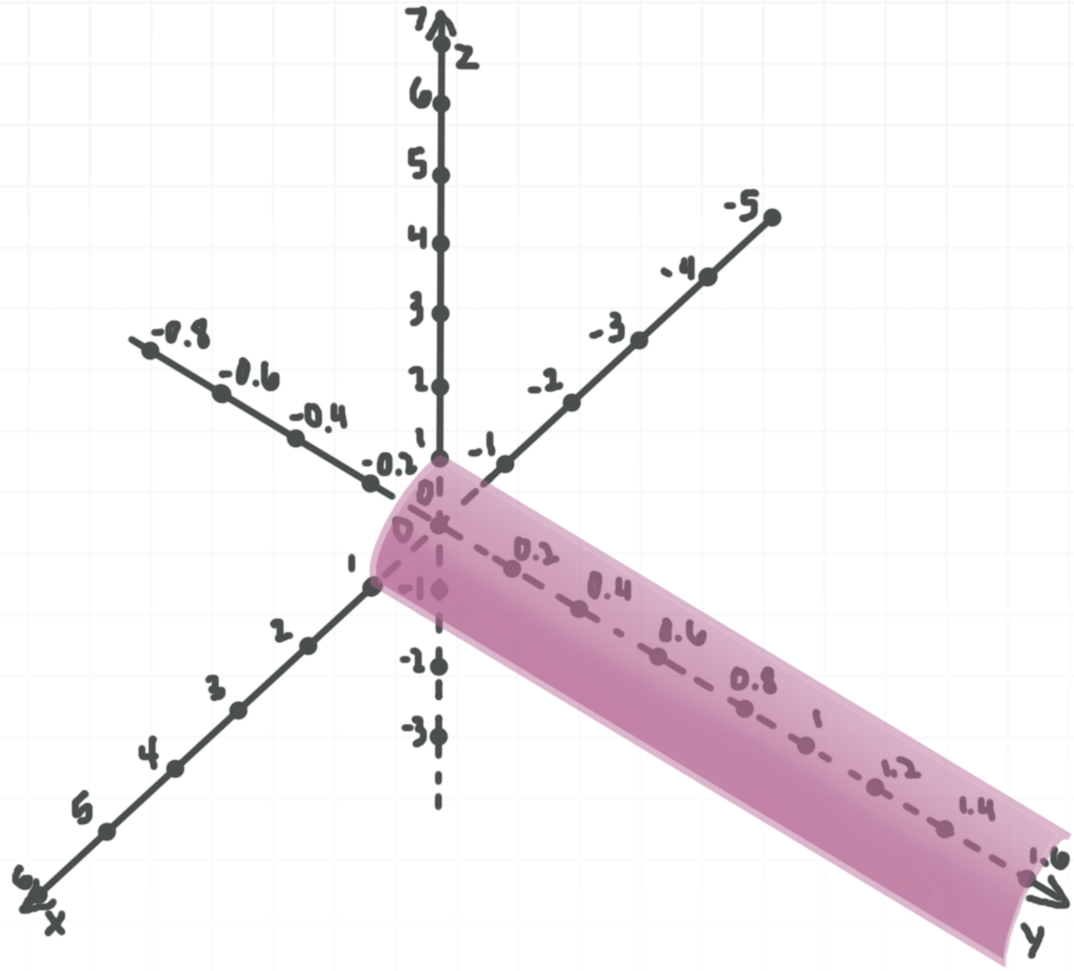
■ 2. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the set of points between right circular cylinders with radius 1 and 2, height 5, cylinder axes are x -axis, and bases that lie in the planes $x = -1$ and $x = 4$.

$$\iiint_E \frac{x+y+z}{y^2+z^2}\ dV$$



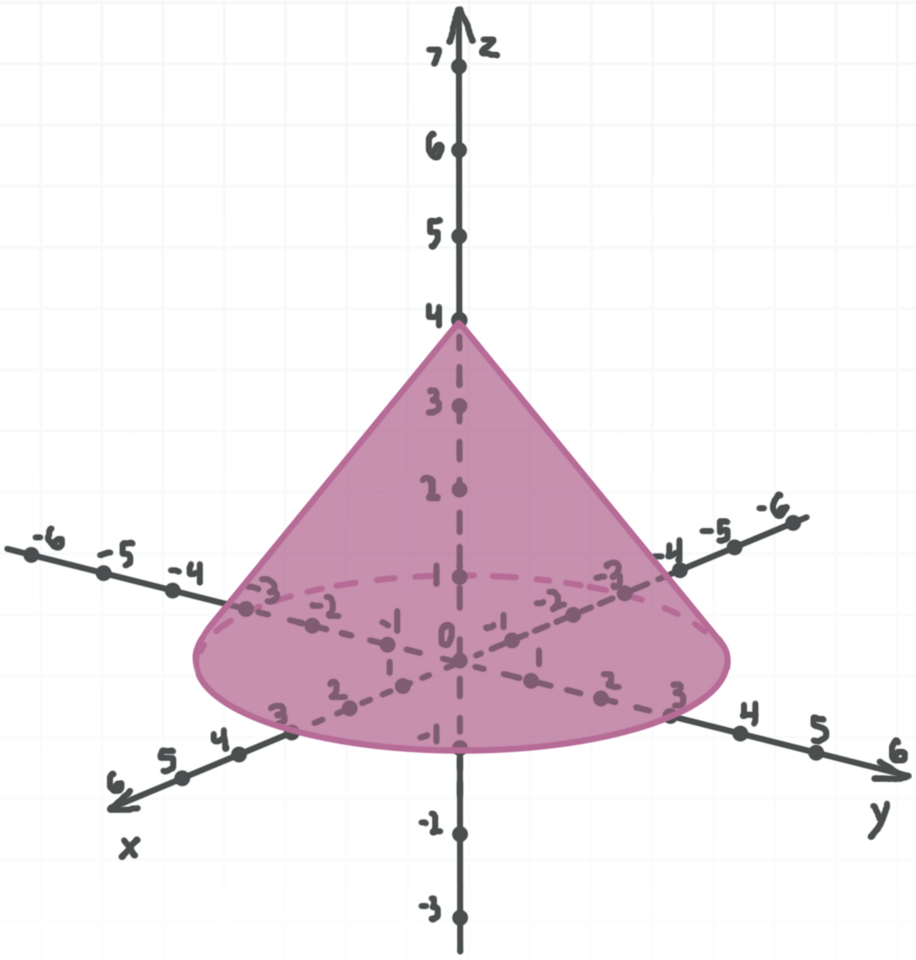
3. Evaluate the improper triple integral by changing it to cylindrical coordinates, where E is part of the cylinder $x^2+z^2=1$ that lies in the first octant.

$$\iiint_E 2e^{-x^2-y^2-z^2}\ dV$$



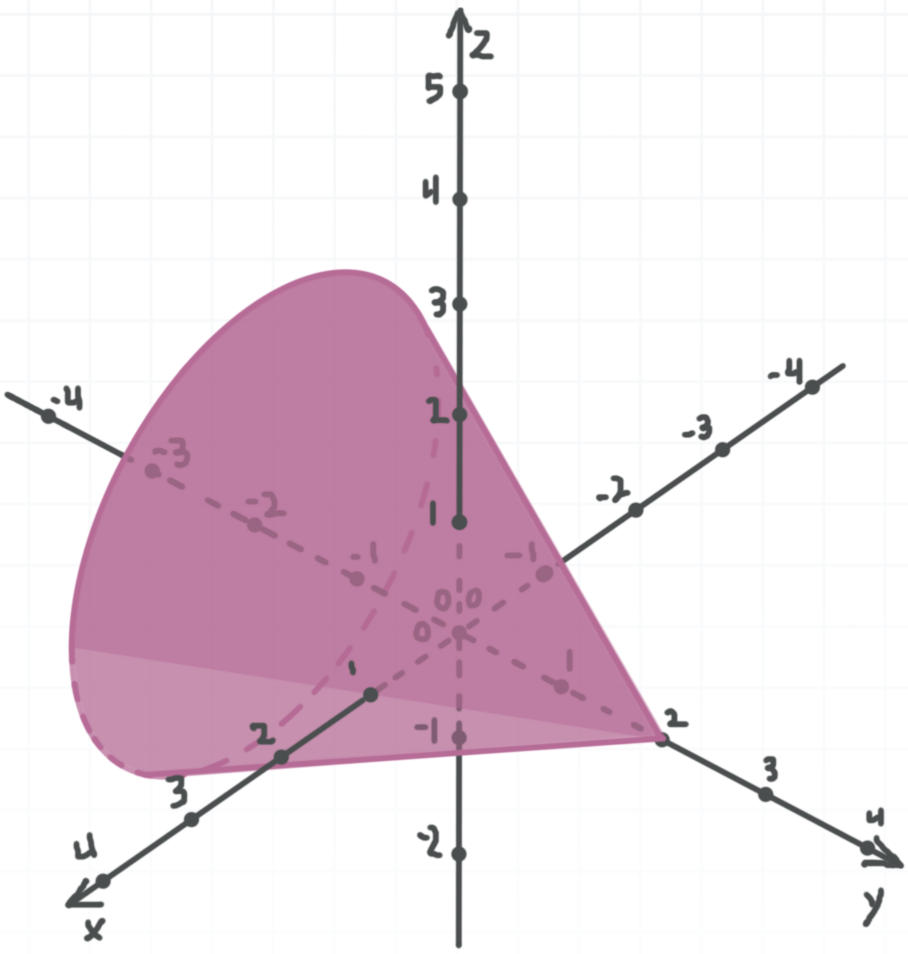
■ 4. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the right circular cone with radius 3, vertex at the point $(0,0,4)$, and a base that lies in the xy-plane with its center at the origin.

$$\iiint_E 48x^2y^2(z+2)\ dV$$



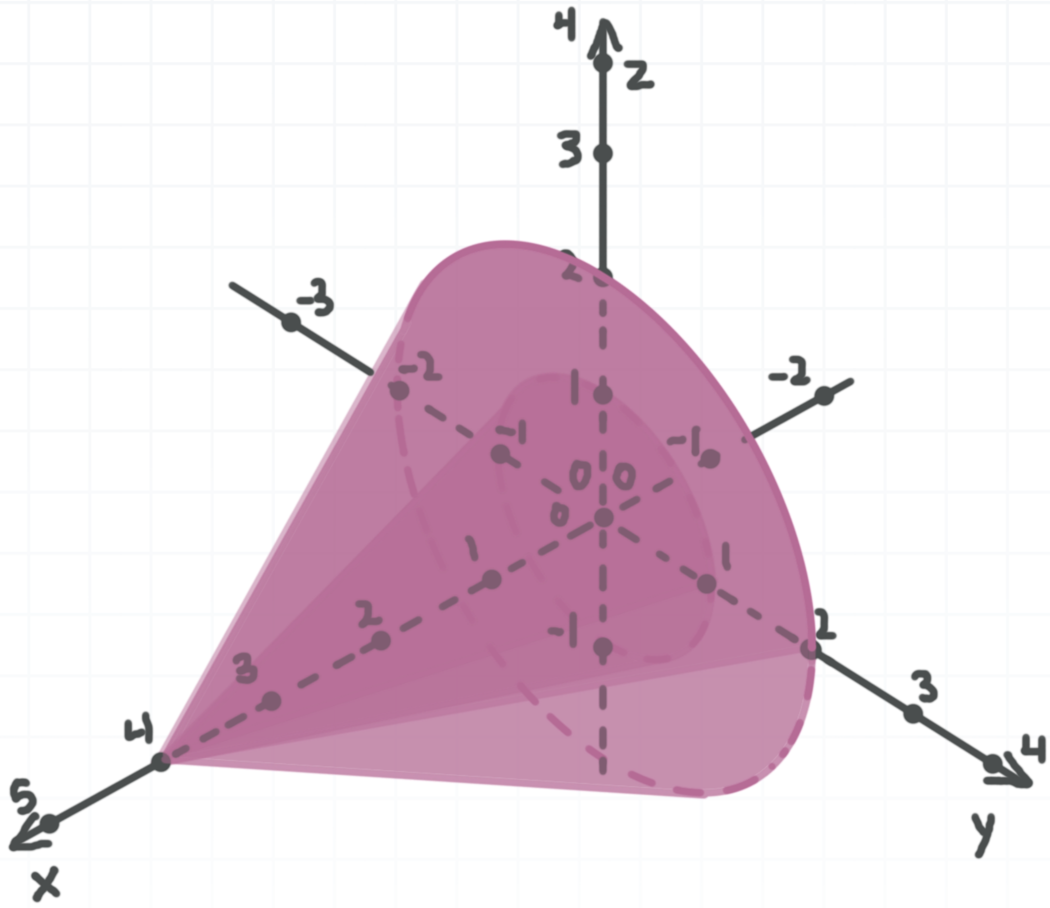
■ 5. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the right circular cone with radius 2, vertex at the point $(0,2,0)$, and a base that lies in the plane $y=-2$ with its center at the point $(0,-2,0)$.

$$\iiint_E \frac{(x+z)^2}{(y+4)}\ dV$$



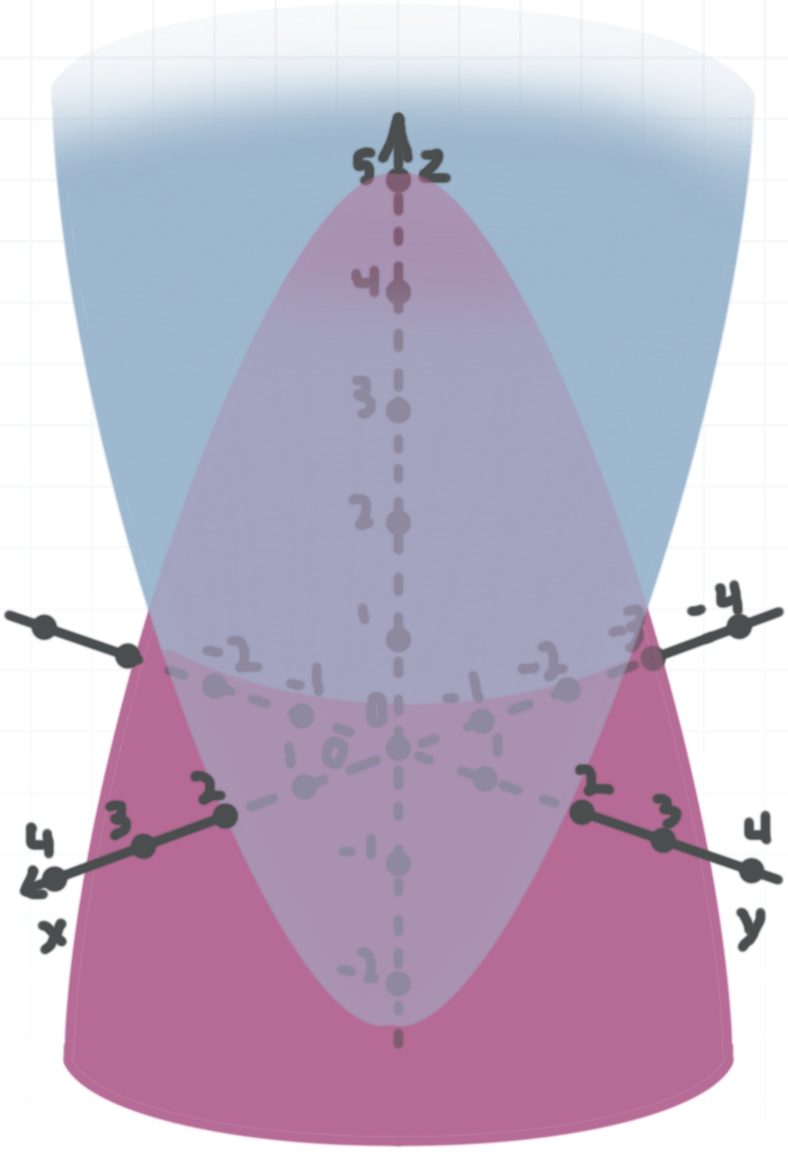
■ 6. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the set of points between two right circular cones with radii 1 and 2, vertexes at the point $(4,0,0)$, and bases that lie in the yz-plane with center at the origin.

$$\iiint_E x^2 - y^2 - z^2 \ dV$$



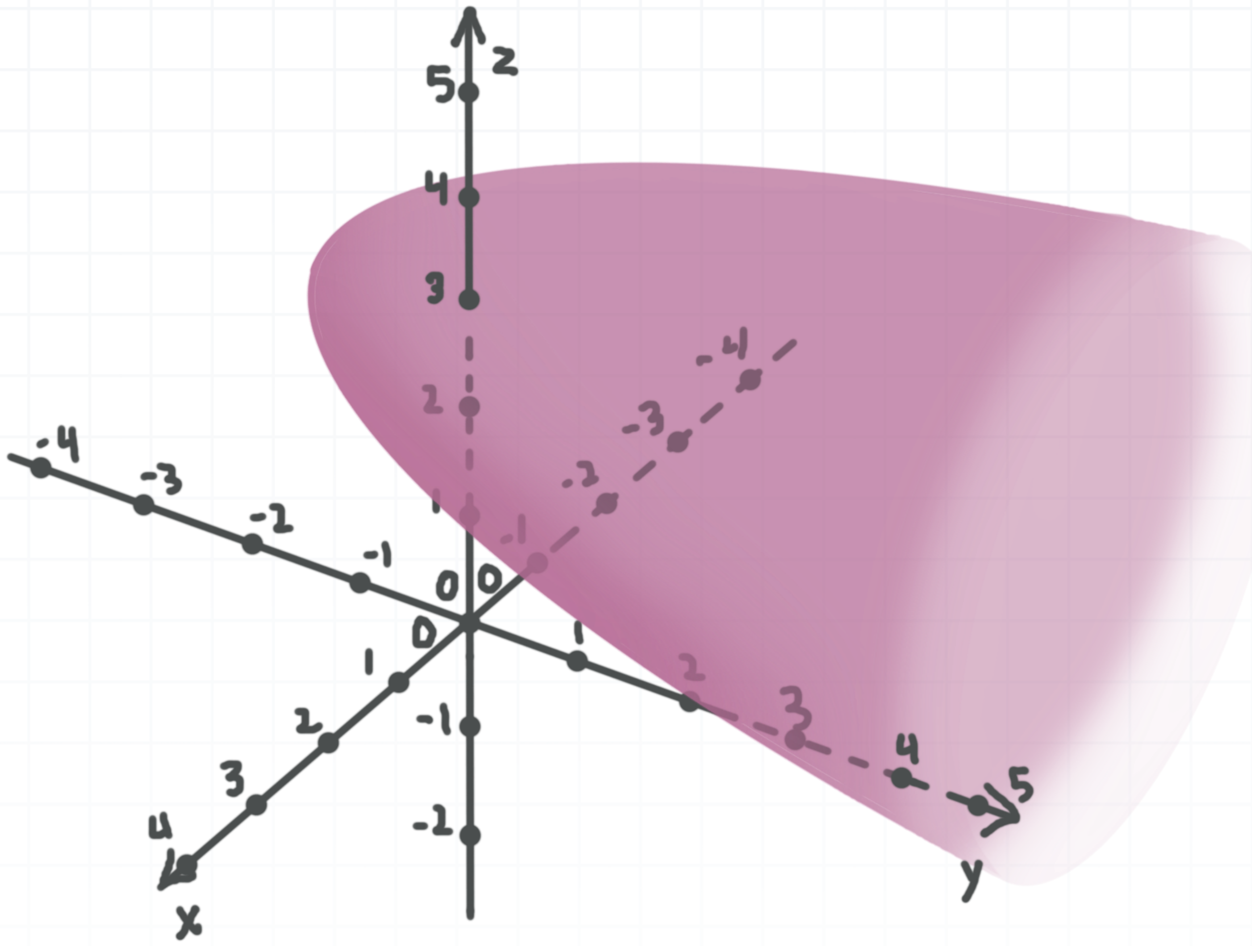
■ 7. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the solid bounded by the surfaces $x^2 + y^2 + z - 5 = 0$ and $x^2 + y^2 - z - 3 = 0$.

$$\iiint_E 3\sqrt{x^2 + y^2}\ dV$$



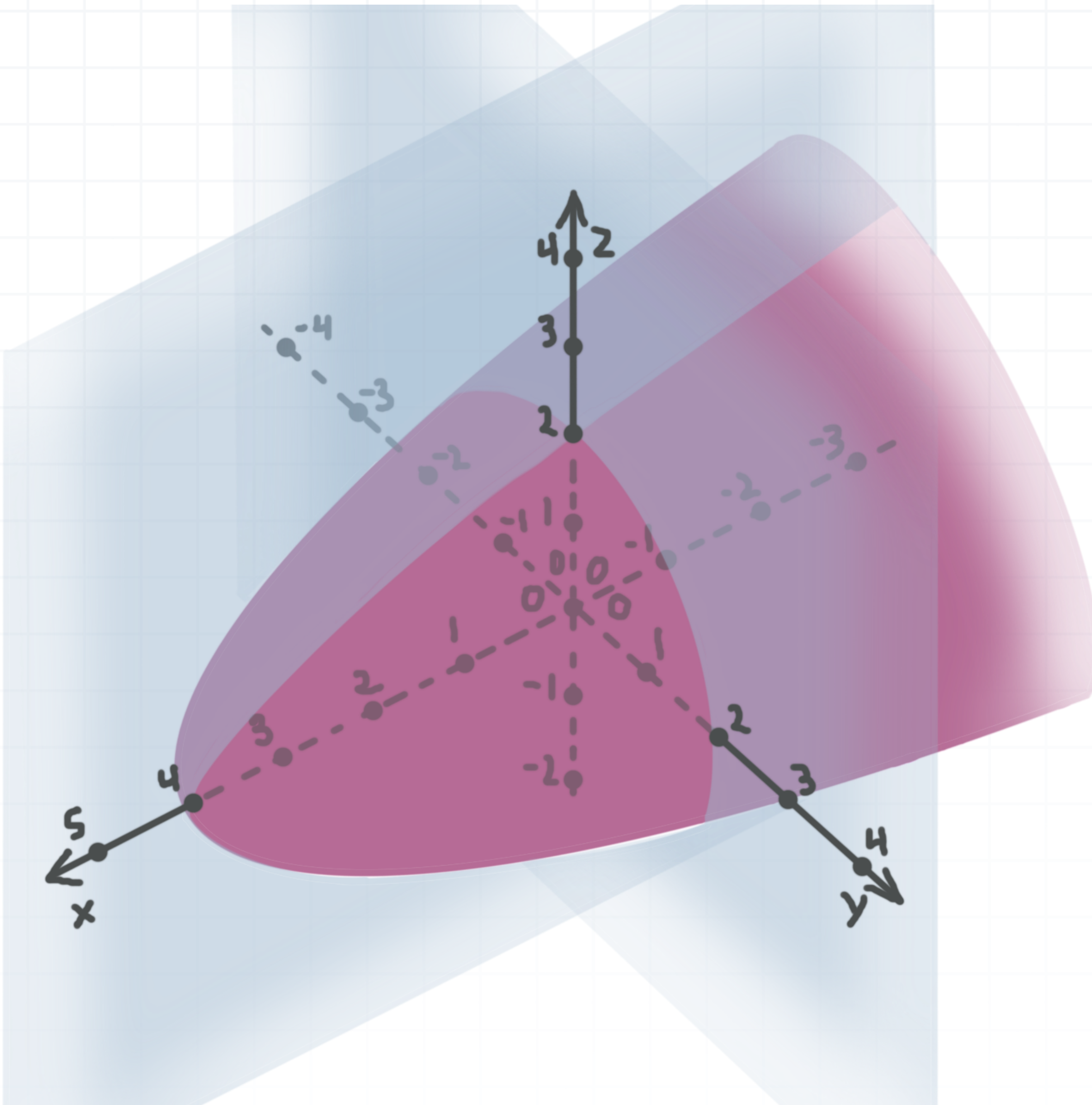
■ 8. Evaluate the triple improper integral by changing it to cylindrical coordinates, where E is the interior of the surface $(x+1)^2+(z-2)^2=y+2$.

$$\iiint_E xz10^{-y}\ dV$$



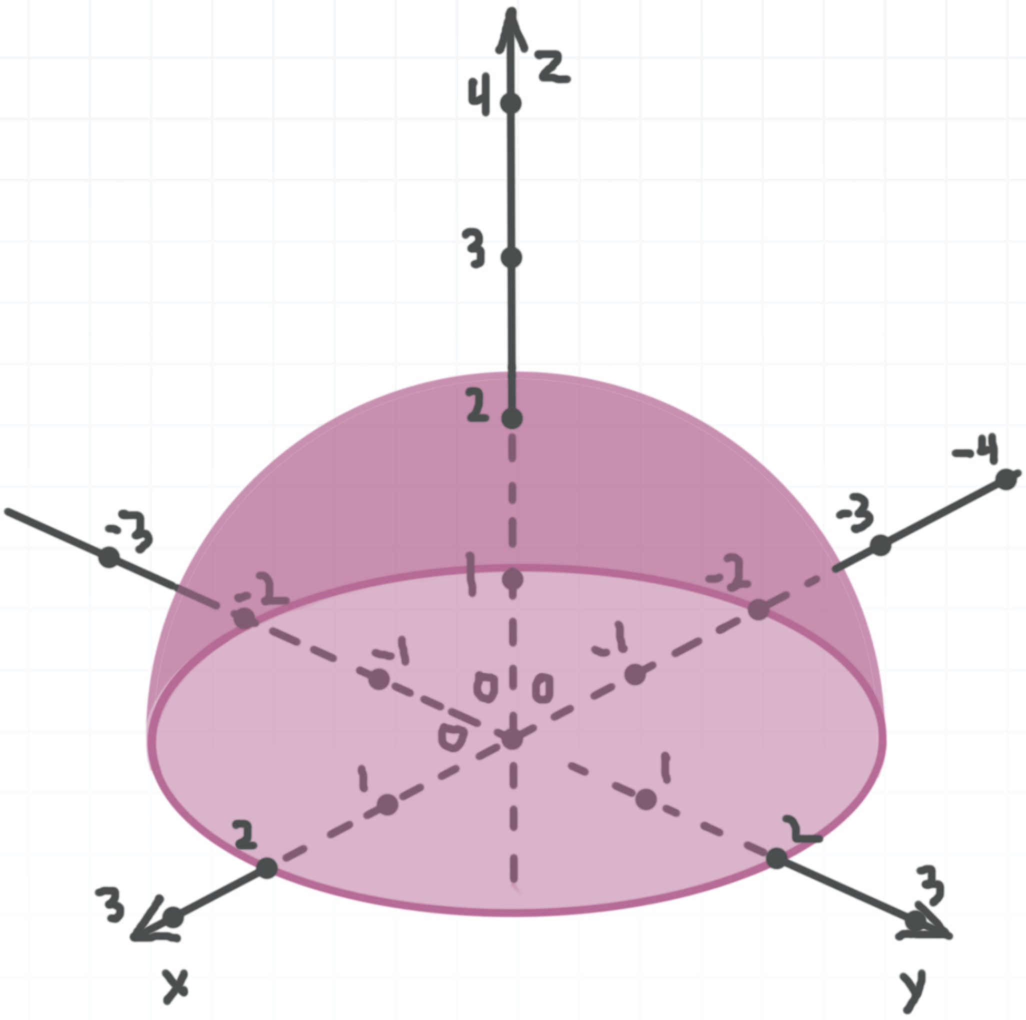
■ 9. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the interior of the surface $y^2 + z^2 + x - 4 = 0$ that lies within the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

$$\iiint_E x + 8yz \ dV$$



■ 10. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the hemisphere with center at the origin, radius 2, and $z \geq 0$.

$$\iiint_E 4x^2 + 4y^2 - 12z^2 \ dV$$

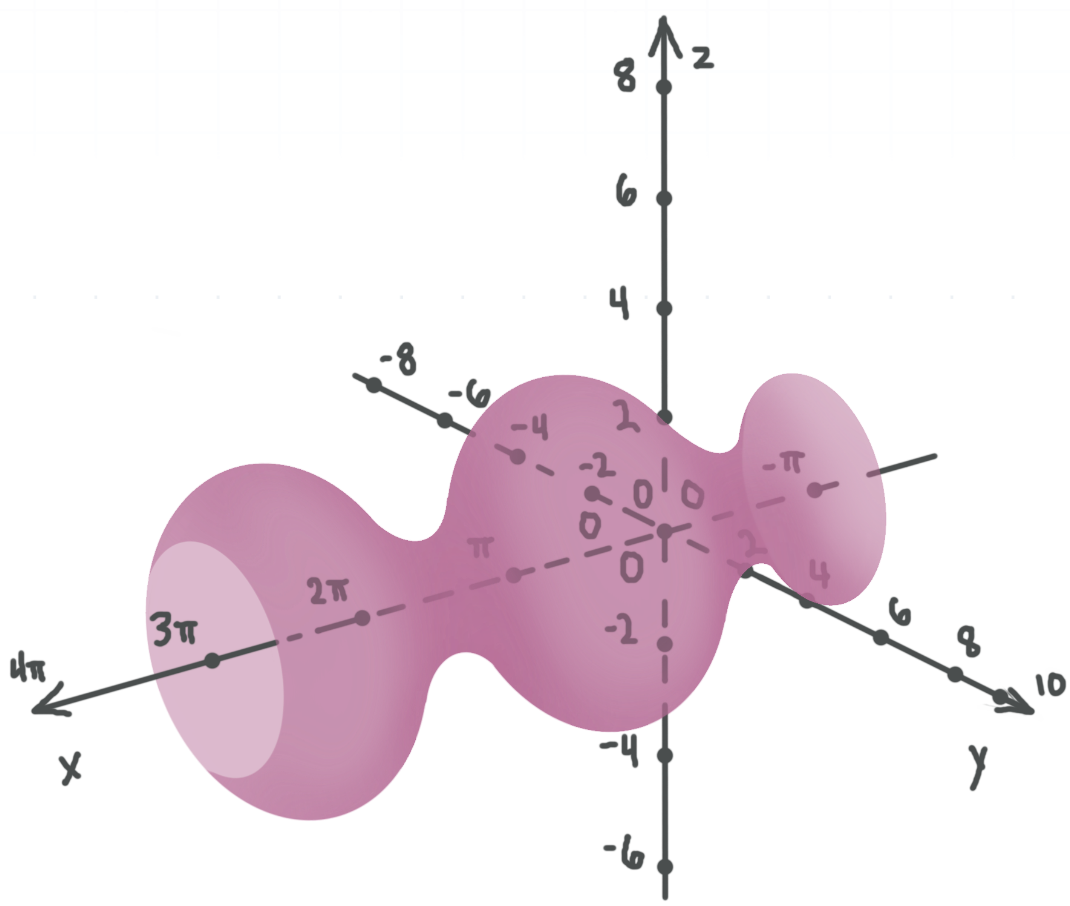

z
4
3
2
1
-3
-2
-1
0
0
0
1
2
3
x
-4
-3
-2
-1
1
2
3
y

FINDING VOLUME

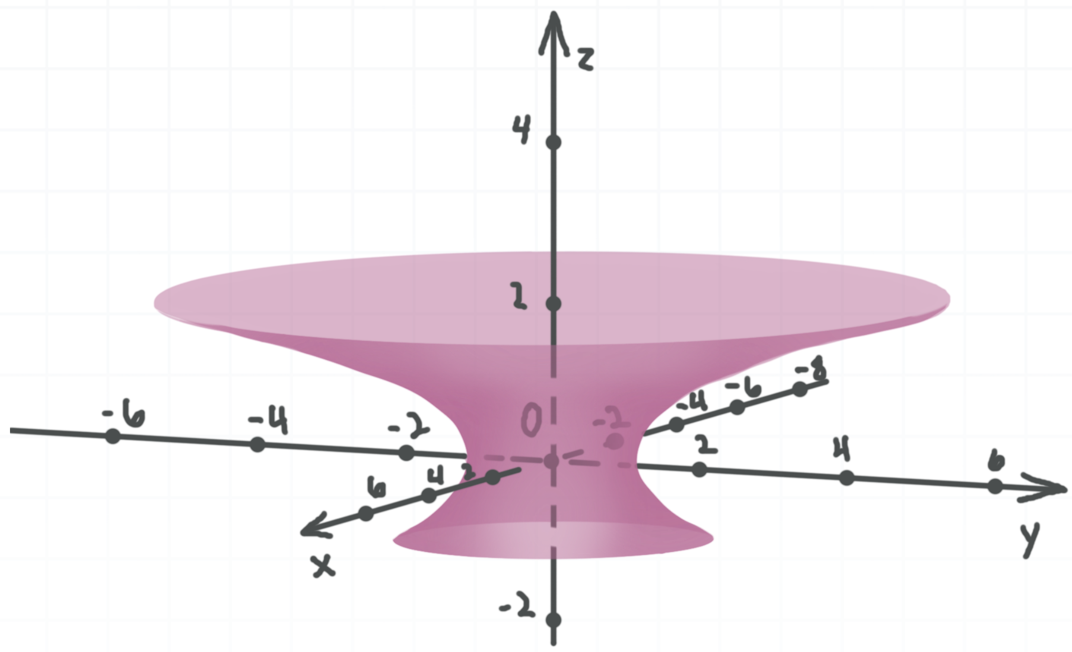
■ 1. Evaluate the integral.

$$\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{4-r^2}} 4rz \ dz \ dr \ d\theta$$

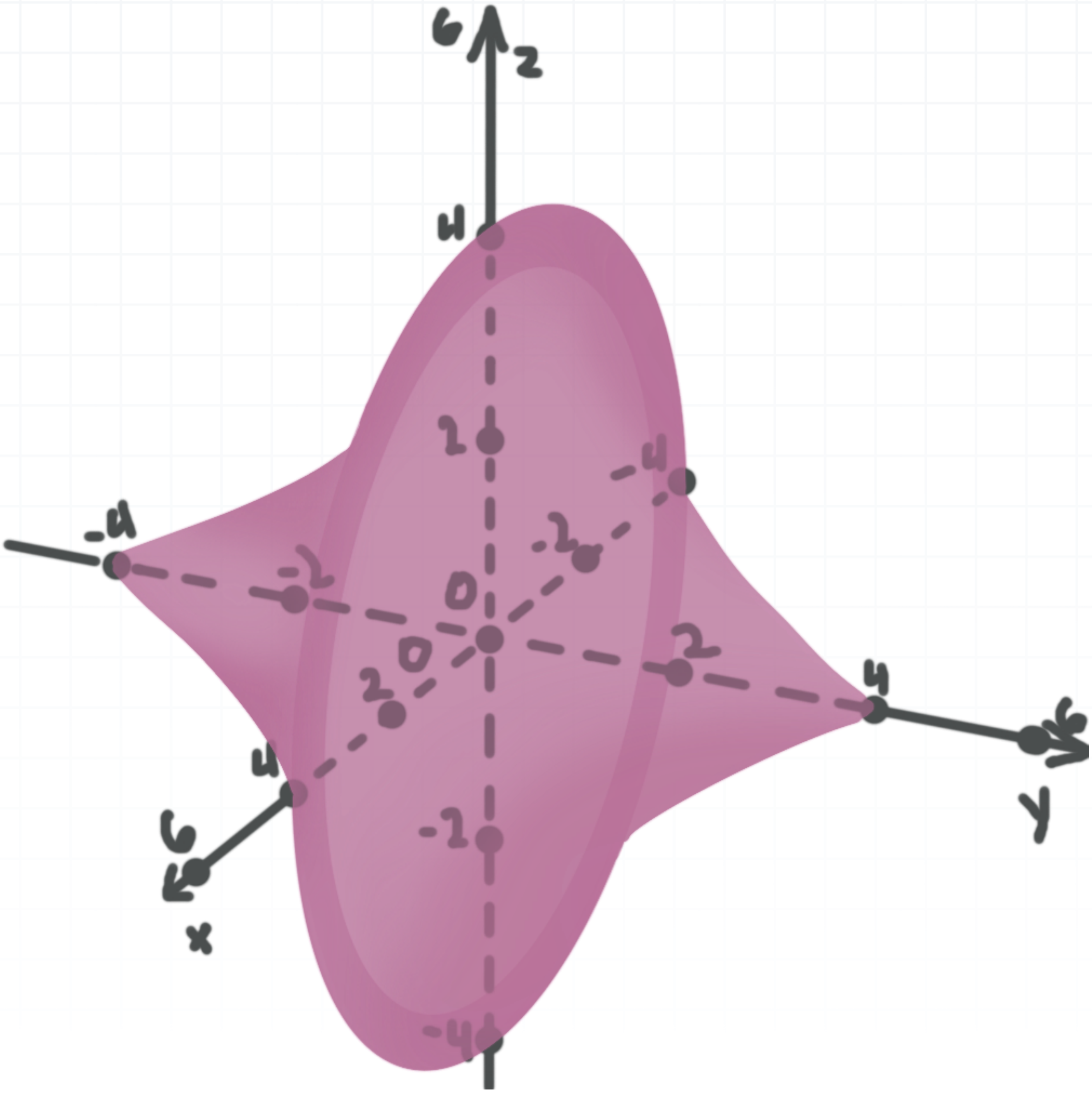
■ 2. Use a triple integral in cylindrical coordinates to find the volume of the solid E, where E is the set of points within the surface of revolution created by rotating the curve $z = 2 + \sin x$ around the x-axis, and bounded by the planes $x = -\pi$ and $x = 3\pi$.



■ 3. Use a triple integral in cylindrical coordinates to find the volume of the solid E, where E is the set of points within the surface of revolution created by rotating the curve $x = z^2 + 1$ around the z-axis, and bounded by the planes $z = -1$ and $z = 2$.



■ 4. Use a triple integral in cylindrical coordinates to find the volume of the solid E, where E is the set of the points within the surface of revolution created by rotating the curve $x = 4 - 2\sqrt{|y|}$ around the y-axis, and bounded by the planes $y = -4$ and $y = 4$.


6
z
4
2
-4
-2
-4
-2
0
0
2
2
4
6
y
4
6
x
-2
-4

W W W . K R I S T A K I N G M A T H . C O M