Calculus 3 Workbook Solutions

Implicit differentiation

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IMPLICIT DIFFERENTIATION

1. Use implicit differentiation to find the partial derivative dy/dx.

$$\sin(x+y)=x+y$$

Solution:

Implicitly differentiate both sides.

$$\cos(x+y)\left(1+1\frac{dy}{dx}\right)=1+1\frac{dy}{dx}$$

Solve for dy/dx.

$$\cos(x+y)+\frac{dy}{dx}\cos(x+y)=1+\frac{dy}{dx}$$

$$\cos(x+y)-1=\frac{dy}{dx}-\frac{dy}{dx}\cos(x+y)$$

$$\cos(x+y)-1=\frac{dy}{dx}(1-\cos(x+y))$$

$$\frac{dy}{dx}=\frac{\cos(x+y)-1}{1-\cos(x+y)}$$

Simplify.

$$\frac{dy}{dx}=-\frac{1-\cos(x+y)}{1-\cos(x+y)}$$

$$\frac{dy}{dx} = -1$$

■ 2. Use implicit differentiation to find the partial derivative $\partial z/\partial x$ of the multivariable function.

$$y \ln z = 2x - 3y + 2z$$

Solution:

Rewrite the equation as

$$0 = 2x - 3y + 2z - y \ln z$$

$$F(x, y, z) = 2x - 3y + 2z - y \ln z$$

Then the partial derivatives of F are

$$\frac{\partial F}{\partial x} = 2$$

$$\frac{\partial F}{\partial z} = 2 - \frac{y}{z}$$

So the partial derivative $\partial z/\partial x$ is

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{2}{2 - \frac{y}{z}} = -\frac{2z}{2z - y} = \frac{2z}{y - 2z}$$

3. Use implicit differentiation to find the partial derivative $\partial z / \partial y$ of the multivariable function.

$$e^z = x^2 + y + z$$

Solution:

Rewrite the equation as

$$0 = x^2 + y + z - e^z$$

$$F(x, y, z) = x^2 + y + z - e^z$$

Then the partial derivatives of F are

$$\frac{\partial F}{\partial y} = 1$$

$$\frac{\partial F}{\partial y} = 1 - e^z$$

So the partial derivative $\partial z / \partial y$ is

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{1}{1 - e^z} = \frac{1}{e^z - 1}$$

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