Topic: Changing double integrals to polar coordinates

Question: Convert the double integral to polar coordinates.

$$\iint_D x^2 + y^2 \ dA$$

D is bounded by $y = \pm\sqrt{1 - x^2}$

Answer choices:

A $\quad \int_0^{2\pi} \int_0^1 r^3 \ dr \ d\theta$

B $\quad \int_0^{2\pi} \int_0^1 r^2 \ dr \ d\theta$

C $\quad \int_0^{2\pi} \int_{-1}^1 r^2 \ dr \ d\theta$

D $\quad \int_0^{2\pi} \int_{-1}^1 r^3 \ dr \ d\theta$

Solution: A

When we convert from rectangular coordinates to polar coordinates, we use the following conversion formulas.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$dy\ dx = r\ dr\ d\theta$$

The region D is bounded by $y = \pm\sqrt{1 - x^2}$, which can be rewritten as

$$y = \pm\sqrt{1 - x^2}$$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

So the region D is the circle centered at the origin with radius 1. Which means we can evaluate it as a type I region, and that we'll integrate first with respect to y and then with respect to x. The limits of integration for y will be what we've already been given, and the limits of integration for x will simply be $[-1,1]$.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2\ dy\ dx$$

If we use the conversion formulas above to convert the integrand, but we leave the limits of integration alone, we get

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2\ dy\ dx = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} r^2 r\ dr\ d\theta$$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2\ dy\ dx = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} r^3\ dr\ d\theta$$

Next we need to convert the limits of integration. Since we're talking about the circle with radius 1, and since in polar coordinates r represents radius, the bounds for r have to be $[0,1]$. Similarly, because we're dealing with the entire circle, the limits of integration for the angle θ have to be $[0,2\pi]$. Therefore, the converted integral is

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2 dydx = \int_{0}^{2\pi}\int_{0}^{1} r^3\ dr\ d\theta$$

Topic: Changing double integrals to polar coordinates

Question: Convert the double integral to polar coordinates.

$$\iint_D e^{x^2+y^2}\ dA$$

where D is bounded by $y = \pm\sqrt{25 - x^2}$

Answer choices:

A $\quad \int_0^{2\pi}\int_0^{25} e^{r^2}\ dr\ d\theta$

B $\quad \int_0^{2\pi}\int_0^{5} e^{r^2}\ dr\ d\theta$

C $\quad \int_0^{2\pi}\int_0^{25} re^{r^2}\ dr\ d\theta$

D $\quad \int_0^{2\pi}\int_0^{5} re^{r^2}\ dr\ d\theta$

Solution: D

When we convert from rectangular coordinates to polar coordinates, we use the following conversion formulas.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$dy\ dx = r\ dr\ d\theta$$

The region D is bounded by $y = \pm\sqrt{25 - x^2}$, which can be rewritten as

$$y = \pm\sqrt{25 - x^2}$$

$$y^2 = 25 - x^2$$

$$x^2 + y^2 = 25$$

So the region D is the circle centered at the origin with radius 5. Which means we can evaluate it as a type I region, and that we'll integrate first with respect to y and then with respect to x. The limits of integration for y will be what we've already been given, and the limits of integration for x will simply be $[-5,5]$.

$$\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} e^{x^2+y^2}\ dy\ dx$$

If we use the conversion formulas above to convert the integrand, but we leave the limits of integration alone, we get

$$\int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} e^{x^2+y^2} \ dy \ dx = \int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} e^{r^2} r \ dr \ d\theta$$

$$\int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} e^{x^2+y^2} \ dy \ dx = \int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} re^{r^2} \ dr \ d\theta$$

Next we need to convert the limits of integration. Since we're talking about the circle with radius 5, and since in polar coordinates r represents radius, the bounds for r have to be $[0,5]$. Similarly, because we're dealing with the entire circle, the limits of integration for the angle θ have to be $[0,2\pi]$. Therefore, the converted integral is

$$\int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} e^{x^2+y^2} \ dy \ dx = \int_{0}^{2\pi} \int_{0}^{5} re^{r^2} \ dr \ d\theta$$

Topic: Changing double integrals to polar coordinates

Question: Convert the double integral to polar coordinates.

$$\int\int_D \sin\left(x^2+y^2\right)\ dA$$

where D is bounded by $y=\pm\sqrt{4-x^2}$

Answer choices:

A $\int_0^{2\pi}\int_0^4 r\sin r^2\ dr\ d\theta$

B $\int_0^{2\pi}\int_0^2 \sin r^2\ dr\ d\theta$

C $\int_0^{2\pi}\int_0^2 r\sin r^2\ dr\ d\theta$

D $\int_0^{2\pi}\int_0^4 \sin r^2\ dr\ d\theta$

Solution: C

When we convert from rectangular coordinates to polar coordinates, we use the following conversion formulas.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$dy\ dx = r\ dr\ d\theta$$

The region D is bounded by $y = \pm\sqrt{4 - x^2}$, which can be rewritten as

$$y = \pm\sqrt{4 - x^2}$$

$$y^2 = 4 - x^2$$

$$x^2 + y^2 = 4$$

So the region D is the circle centered at the origin with radius 2. Which means we can evaluate it as a type I region, and that we'll integrate first with respect to y and then with respect to x. The limits of integration for y will be what we've already been given, and the limits of integration for x will simply be $[-2,2]$.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sin\left(x^2 + y^2\right)\ dy\ dx$$

If we use the conversion formulas above to convert the integrand, but we leave the limits of integration alone, we get

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right)\ dy\ dx = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} r\sin r^2\ dr\ d\theta$$

Next we need to convert the limits of integration. Since we're talking about the circle with radius 2, and since in polar coordinates r represents radius, the bounds for r have to be $[0,2]$. Similarly, because we're dealing with the entire circle, the limits of integration for the angle θ have to be $[0,2\pi]$. Therefore, the converted integral is

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right)\ dy\ dx = \int_{0}^{2\pi}\int_{0}^{2} r\sin r^2\ dr\ d\theta$$