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Distance between points in three dimensions

Given two points A and B in three-dimensional space,

$$A(x_1, y_1, z_1)$$

$$B(x_2, y_2, z_2)$$

we can calculate the distance between them using the distance formula.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

It doesn't matter which point is A and which point is B. The fact that the square the differences inside the square root means that all of our values will be positive, which means we'll get a positive value for the distance between the points.

Example

Use the distance formula to

- 1. Find the distance between (0,1,3) and (-1,4,5).
- 2. Say which of (0,1,3) and (-1,4,5) lies in the yz-plane.
- 3. Say which of (0,1,3) and (-1,4,5) is closer to the xy-plane.

For the first part of the question, we'll use the distance formula to calculate the distance between the points.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$D = \sqrt{(-1-0)^2 + (4-1)^2 + (5-3)^2}$$

$$D = \sqrt{1+9+4}$$

$$D = \sqrt{14}$$

For the second part of the question, we need to realize that in order for a point to be in the yz-plane, its x-coordinate must be 0. With that in mind, we can say that (0,1,3) lies on the yz-plane, and that (-1,4,5) does not lie in the yz-plane.

For the third part of the question, we need to realize that the z-value of the coordinate point will tell us how far the point is from the xy-plane. So if we just take the absolute value of the z-coordinate for each of our points, we'll be able to say which one is closer.

Point
$$(0,1,3)$$
 has $|z| = |3| = 3$

Point
$$(-1,4,5)$$
 has $|z| = |5| = 5$

Since the absolute value of z in the point (0,1,3) is less than the absolute value of z in the point (-1,4,5), we can say that (0,1,3) is closer to the xy-plane.

