Calculus 3 Workbook

Optimization

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MATH

CRITICAL POINTS

■ 1. Find a set of critical points for $f(t, s)$.

$$f(t, s) = \ln \frac{2t^3 - 12t^2 + 1}{s^2 + 6s + 11}$$

■ 2. Find and identify a set of critical points for $f(x, y, z)$.

$$f(x, y, z) = x^2 \cos(y + z)$$

■ 3. Find a set of critical points for $f(x_1, x_2, x_3, x_4)$.

$$f(x_1, x_2, x_3, x_4) = x_1^2 - 2x_1x_2 + 2x_2^2 - 4x_3^2 + 4x_1 + 5x_4^2 - 10x_4 + 6$$

SECOND DERIVATIVE TEST

1. Use the second derivative test to classify the critical points of $f(t, s)$.

$$f(t, s) = \frac{t^5 s - 3t + 6s^2}{s^2 t^5}$$

2. Use the second derivative test to classify the critical points of $f(r, \theta)$.

$$f(r, \theta) = (r^2 - 2r - 3)\sin\theta$$

3. Find the set of all possible values of a for which $f(x, y)$ has only one local minimum.

$$f(x, y) = x^2 + ay^2 - 4x + 8y - 6$$

LOCAL EXTREMA AND SADDLE POINTS

■ 1. Find the local extrema of $f(t, s)$.

$$f(t, s) = \frac{t^3 + s^3 + 1}{ts}$$

■ 2. Find the local extrema of $f(x, y)$.

$$f(x, y) = \sin(0.5x)\cos(0.25y)$$

■ 3. Find the equation(s) of the tangent plane to $f(x, y)$ at the function's local maximum.

$$f(x, y) = -x^2 - 2y^2 + 4x - 12y - 9$$

■ 4. Find the values of a and b where $f(x, y)$ has a local minimum at $(5, -3)$.

$$f(x, y) = 4x^2 + 2y^4 - ax - by + 5$$

■ 5. Find and identify the set of local maxima of $f(x, y)$.

$$f(x, y) = e^{-4(x^2+y^2)}(x^2 + y^2)$$

■ 6. Find and identify the saddle points and local extrema of $f(x, y)$.

$$f(x, y) = (x-4)^8 - (y+7)^{12}$$

GLOBAL EXTREMA

■ 1. Find the global extrema of $f(t,s)$ over R^2.

$$f(t,s)=\frac{4s}{t^2+2s^2+2}$$

■ 2. Find the global extrema of $f(t,s)$ over R^2.

$$f(t,s)=t^2s^2-4t^2-4s^2-4s+1$$

■ 3. Find the global extrema of $f(x,y)$ over R^2.

$$f(x,y)=\frac{\sin(3x)}{x^2+3y^2}$$

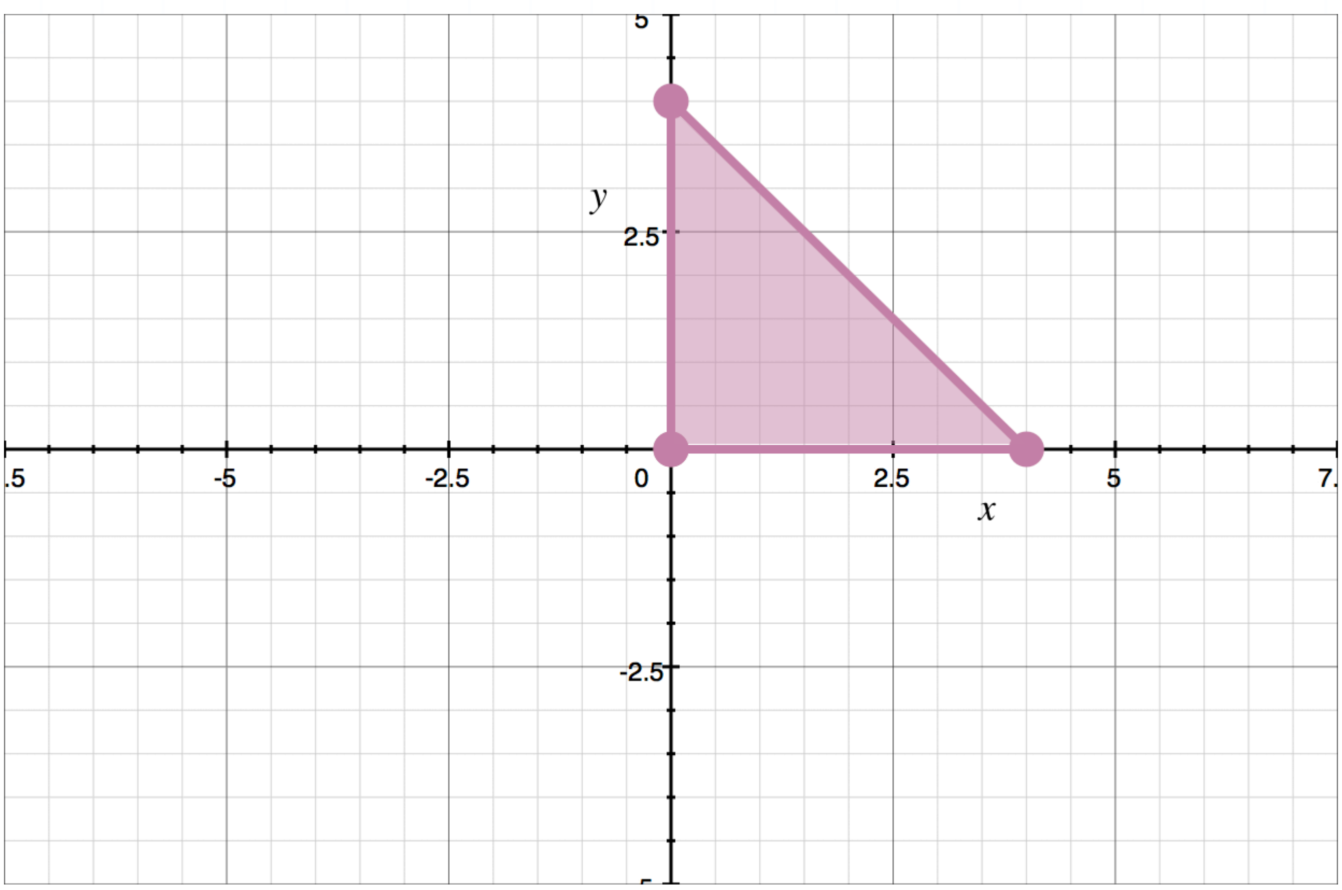
EXTREME VALUE THEOREM

■ 1. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of $f(x,y)$ on the closed rectangle $-1 \leq x \leq 3$, $0 \leq y \leq 3$.

$$f(x,y) = 2x^2 - 2xy + y^2 - 4x - 1$$

■ 2. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of $f(x,y)$ on a closed triangle bounded by $x = 0$, $y = 0$, and $x + y - 4 = 0$.

$$f(x,y) = \ln(x^2 + y^2 - 2y - x + 2)$$



■ 3. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of $f(x, y)$ on the closed rectangle $-\pi \leq x \leq \pi$, $-1 \leq y \leq 3$.

$$f(x, y) = y^2 \tan(2x)$$

■ 4. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of the function $f(x, y)$ on the closed rectangle $-\pi \leq x \leq \pi$, $-2 \leq y \leq 2$.

$$f(x, y) = (y^2 + 2y + 3)\tan\left(\frac{x}{4}\right)$$

■ 5. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of the function $f(x, y)$ on the closed circle with center at the origin and radius 1.

$$f(x, y) = x^2 + y^2 - 2x + 2\sqrt{3}y - 3$$

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