Topic: Domain of the vector function

Question: Find the domain of the vector function.

$$r(t) = \left\langle 6t + 1,\ 2t^2,\ \frac{1}{25 - t^2} \right\rangle$$

Answer choices:

A $-5 \le t \le 5$

B $t \ne \pm 5$

C $[\infty, 5]$

D $(-5, 5)$

Solution: B

For the vector function

$$r(t) = \left\langle 6t + 1,\ 2t^2,\ \frac{1}{25 - t^2} \right\rangle$$

the parameter is t. We'll find the domain for each component of the vector function. The domain for the vector function $r(t)$ will include only the values of t that are included in the domain of all three components.

The domain of $6t + 1$ is the set of all real numbers.

The domain of $2t^2$ is the set of all real numbers.

For the rational function

$$\frac{1}{25 - t^2}$$

we know that the denominator cannot equal 0. So to find the limitations on the domain we can set the denominator equal to 0.

$$25 - t^2 = 0$$

$$t^2 = 25$$

$$t = \pm 5$$

The domain of the third component is $t \neq \pm 5$.

Finally we need to combine these domains. The first and second components have no restrictions on their domains so we can ignore them.

If we look at the third component, we can see that the domain will be

$t \neq \pm 5$

The domain of $r(t)$ is $t \neq \pm 5$.

Topic: Domain of the vector function

Question: Find the domain of the vector function.

$$r(t) = \left\langle t-2, \frac{1}{t^2-16}, \sqrt{t+3} \right\rangle$$

Answer choices:

A $[-3,4) \cup (4,\infty)$

B $(-3,4) \cup (4,\infty)$

C $(-3,4)$

D $(-4,3)$

Solution: A

For the vector function

$$r(t) = \left\langle t - 2, \frac{1}{t^2 - 16}, \sqrt{t + 3} \right\rangle$$

the parameter is t. We'll find the domain for each component of the vector function. The domain for the vector function $r(t)$ will include only the values of t that are included in the domain of all three components.

The domain of $t - 2$ is the set of all real numbers.

For the rational function

$$\frac{1}{t^2 - 16}$$

we know that the denominator cannot equal 0. So to find the limitations on the domain we can set the denominator equal to 0.

$$t^2 - 16 = 0$$

$$t^2 = 16$$

$$t = \pm 4$$

The domain of the second component is $t \neq \pm 4$.

For the radical function

$$\sqrt{t + 3}$$

we know that the value underneath the square root cannot be negative. So we'll restrict that value to only positive values and 0.

$$t + 3 \geq 0$$

$$t \geq -3$$

Finally we need to combine these domains. The first component has no restriction on its domain so we can ignore it. If we look at the second component, we can see that the domain will be $t \neq \pm 4$.

But if we look at the third component, we can see that $t \geq -3$. Which means that we don't have to consider $t = -4$ being excluded from the domain, since it's outside $t \geq -3$ anyway. We only need to exclude $t = 4$ from $t \geq -3$.

The domain of $r(t)$ is $[-3,4) \cup (4,\infty)$.

Topic: Domain of the vector function

Question: Find the domain of the vector function.

$$r(t) = \left\langle \ln(t-4), \sqrt{t^2+6}, \frac{1}{144-t^2} \right\rangle$$

Answer choices:

A $(4,12)$

B $(4,12]$

C $(4,12) \cup (12,\infty)$

D $-12 \leq t \leq 12$

Solution: C

For the vector function

$$r(t) = \left\langle \ln(t-4), \sqrt{t^2+6}, \frac{1}{144-t^2} \right\rangle$$

the parameter is t. We'll find the domain for each component of the vector function. The domain for the vector function $r(t)$ will include only the values of t that are included in the domain of all three components.

For the logarithmic function

$$\ln(t-4)$$

we know that the argument inside the log function must be greater than 0, since the log function is undefined when its argument is 0 or negative.

$$t-4>0$$

$$t>4$$

For the radical function

$$\sqrt{t^2+6}$$

we know that the value underneath the square root cannot be negative. But since t is squared beneath the root, it doesn't matter what value we plug in for t, t^2 will always be positive. And therefore t^2+6 will always be positive. So there's no way to make the value underneath the root positive, which means we can plug in any number we want to for t. So the domain for this component is all real numbers.

For the rational function

$$\frac{1}{144 - t^2}$$

we know that the denominator cannot equal 0. So to find the limitations on the domain we can set the denominator equal to 0.

$$144 - t^2 = 0$$

$$t^2 = 144$$

$$t = \pm 12$$

The domain of the third component is $t \neq \pm 12$.

Finally we need to combine these domains. The first component tells us that $t > 4$. There's no restriction on the domain of the second component, so we can ignore it. If we look at the third component, we can see that the domain can't include $t = \pm 12$. But we've already said that $t > 4$, and the only forbidden value in that interval is $t = 12$.

The domain of $r(t)$ is $(4,12) \cup (12,\infty)$.