Topic: Volume of the parallelepiped from adjacent edges

Question: Find the volume of the parallelepiped.

$\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$ are adjacent

$P(1,1,-1)$

$Q(2,0,-2)$

$R(1,2,-3)$

$S(-1,5,-2)$

Answer choices:

A 11

B 1

C 15

D 27

Solution: B

First we'll find the vectors PQ, PR and PS. To find a vector from two points, remember that we just subtract the initial point from the terminal point. So, for example, to find PQ, we'll subtract P from Q.

$$\overrightarrow{PQ} = \langle 2-1, 0-1, -2-(-1) \rangle$$

$$\overrightarrow{PQ} = \langle 1, -1, -1 \rangle$$

and

$$\overrightarrow{PR} = \langle 1-1, 2-1, -3-(-1) \rangle$$

$$\overrightarrow{PR} = \langle 0, 1, -2 \rangle$$

and

$$\overrightarrow{PS} = \langle -1-1, 5-1, -2-(-1) \rangle$$

$$\overrightarrow{PS} = \langle -2, 4, -1 \rangle$$

Next we'll find the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ PQ_1 & PQ_2 & PQ_3 \\ PR_1 & PR_2 & PR_3 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \mathbf{i} \begin{vmatrix} PQ_2 & PQ_3 \\ PR_2 & PR_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} PQ_1 & PQ_3 \\ PR_1 & PR_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} PQ_1 & PQ_2 \\ PR_1 & PR_2 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \left(PQ_2PR_3 - PQ_3PR_2\right)\mathbf{i} - \left(PQ_1PR_3 - PQ_3PR_1\right)\mathbf{j} + \left(PQ_1PR_2 - PQ_2PR_1\right)\mathbf{k}$$

Since $\overrightarrow{PQ} = \langle 1, -1, -1 \rangle$ and $\overrightarrow{PR} = \langle 0,1, -2 \rangle$, we get

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \left[(-1)(-2) - (-1)(1)\right] \mathbf{i} - \left[(1)(-2) - (-1)(0)\right] \mathbf{j} + \left[(1)(1) - (-1)(0)\right] \mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (2 + 1)\mathbf{i} - (-2 + 0)\mathbf{j} + (1 + 0)\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle 3,2,1 \rangle$$

Finally, we'll take the dot product of $\overrightarrow{PS} = \langle -2,4, -1 \rangle$ and the cross product we just found, which will give us the volume of the parallelepiped.

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = \left| (-2)(3) + (4)(2) + (-1)(1) \right|$$

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = \left| -6 + 8 - 1 \right|$$

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = \left| 1 \right|$$

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = 1$$

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$P(2,3,0)$

$Q(1,-3,4)$

$R(-2,6,-2)$

$S(7,5,-2)$

Answer choices:

A $\quad$ 13

B $\quad$ 23

C $\quad$ 90

D $\quad$ 18

Solution: D

First we'll find the vectors PQ, PR and PS. To find a vector from two points, remember that we just subtract the initial point from the terminal point. So, for example, to find PQ, we'll subtract P from Q.

$$\overrightarrow{PQ} = \langle 1-2, -3-3, 4-0 \rangle$$

$$\overrightarrow{PQ} = \langle -1, -6, 4 \rangle$$

and

$$\overrightarrow{PR} = \langle -2-2, 6-3, -2-0 \rangle$$

$$\overrightarrow{PR} = \langle -4, 3, -2 \rangle$$

and

$$\overrightarrow{PS} = \langle 7-2, 5-3, -2-0 \rangle$$

$$\overrightarrow{PS} = \langle 5, 2, -2 \rangle$$

Next we'll find the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ PQ_1 & PQ_2 & PQ_3 \\ PR_1 & PR_2 & PR_3 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \mathbf{i} \begin{vmatrix} PQ_2 & PQ_3 \\ PR_2 & PR_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} PQ_1 & PQ_3 \\ PR_1 & PR_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} PQ_1 & PQ_2 \\ PR_1 & PR_2 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \left(PQ_2PR_3 - PQ_3PR_2\right)\mathbf{i} - \left(PQ_1PR_3 - PQ_3PR_1\right)\mathbf{j} + \left(PQ_1PR_2 - PQ_2PR_1\right)\mathbf{k}$$

Since $\overrightarrow{PQ} = \langle -1, -6, 4 \rangle$ and $\overrightarrow{PR} = \langle -4, 3, -2 \rangle$, we get

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \left[(-6)(-2) - (4)(3)\right] \mathbf{i} - \left[(-1)(-2) - (4)(-4)\right] \mathbf{j} + \left[(-1)(3) - (-6)(-4)\right] \mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (12 - 12)\mathbf{i} - (2 + 16)\mathbf{j} + (-3 - 24)\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = -18\mathbf{j} - 27\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle 0, -18, -27 \rangle$$

Finally, we'll take the dot product of $\overrightarrow{PS} = \langle 5, 2, -2 \rangle$ and the cross product we just found, which will give us the volume of the parallelepiped.

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = \left| (5)(0) + (2)(-18) + (-2)(-27) \right|$$

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = \left| 0 - 36 + 54 \right|$$

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = \left| 18 \right|$$

$$\left| \overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR} \right) \right| = 18$$

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$P(-2,-4,5)$

$Q(8,-3,7)$

$R(11,3,8)$

$S(3,7,6)$

Answer choices:

A $\quad 143$

B $\quad -29$

C $\quad 42$

D $\quad 123$

Solution: C

First we'll find the vectors PQ, PR and PS. To find a vector from two points, remember that we just subtract the initial point from the terminal point. So, for example, to find PQ, we'll subtract P from Q.

$$\overrightarrow{PQ} = \langle 8-(-2), -3-(-4), 7-5 \rangle$$

$$\overrightarrow{PQ} = \langle 10,1,2 \rangle$$

and

$$\overrightarrow{PR} = \langle 11-(-2), 3-(-4), 8-5 \rangle$$

$$\overrightarrow{PR} = \langle 13,7,3 \rangle$$

and

$$\overrightarrow{PS} = \langle 3-(-2), 7-(-4), 6-5 \rangle$$

$$\overrightarrow{PS} = \langle 5,11,1 \rangle$$

Next we'll find the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ PQ_1 & PQ_2 & PQ_3 \\ PR_1 & PR_2 & PR_3 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \mathbf{i} \begin{vmatrix} PQ_2 & PQ_3 \\ PR_2 & PR_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} PQ_1 & PQ_3 \\ PR_1 & PR_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} PQ_1 & PQ_2 \\ PR_1 & PR_2 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \left(PQ_2PR_3 - PQ_3PR_2\right)\mathbf{i} - \left(PQ_1PR_3 - PQ_3PR_1\right)\mathbf{j} + \left(PQ_1PR_2 - PQ_2PR_1\right)\mathbf{k}$$

Since $\overrightarrow{PQ} = \langle 10,1,2\rangle$ and $\overrightarrow{PR} = \langle 13,7,3\rangle$, we get

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \left[(1)(3) - (2)(7)\right] \mathbf{i} - \left[(10)(3) - (2)(13)\right] \mathbf{j} + \left[(10)(7) - (1)(13)\right] \mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (3 - 14)\mathbf{i} - (30 - 26)\mathbf{j} + (70 - 13)\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = -11\mathbf{i} - 4\mathbf{j} + 57\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -11, -4,57\rangle$$

Finally, we'll take the dot product of $\overrightarrow{PS} = \langle 5,11,1\rangle$ and the cross product we just found, which will give us the volume of the parallelepiped.

$$\left|\overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR}\right)\right| = \left|(5)(-11) + (11)(-4) + (1)(57)\right|$$

$$\left|\overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR}\right)\right| = \left|-55 - 44 + 57\right|$$

$$\left|\overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR}\right)\right| = \left|-42\right|$$

$$\left|\overrightarrow{PS} \cdot \left(\overrightarrow{PQ} \times \overrightarrow{PR}\right)\right| = 42$$