



Calculus 3 Workbook

Derivatives and integrals of vector functions

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MATH

DERIVATIVE OF A VECTOR FUNCTION

- 1. Find the second order derivative of the vector function.

$$\vec{r}(t) = \left\langle \sqrt{t}, \frac{2}{t}, e^{t+3} \right\rangle$$

- 2. Find the Jacobian matrix of the vector function at $(u, v) = (1, 2)$.

$$\vec{r}(u, v) = \langle 2uv + 1, u^2 + v^2 \rangle$$

- 3. Find the Jacobian matrix for the vector function.

$$\vec{r}(t, s) = \langle \ln(ts), 3t + 2s - 1, \sin(t + s) \rangle$$



UNIT TANGENT VECTOR

- 1. Find the unit tangent vector to the function that that sits at a 30° angle.

$$\vec{r}(t) = \langle t^2 + 4, 2t^3 - 3 \rangle$$

- 2. Find the tangent vector at the point $(-1, 0, 1)$.

$$\vec{r}(t) = \langle 2t^3 - 3t^2 + 5t - 5, \sin(\pi t), e^{t-1} \rangle$$

- 3. Find the point(s) where the unit tangent vector to the curve is orthogonal to the xz -plane

$$\vec{r}(t) = \langle t^3 + 2, 5t^2 - 3t + 8, t^2 + 5 \rangle$$



PARAMETRIC EQUATIONS OF THE TANGENT LINE

- 1. Find the parametric equation of the tangent line to $\vec{r}(u)$ at $u = -2$.

$$\vec{r}(u) = \langle e^{u+3}, \ln(1-u) \rangle$$

- 2. Find the parametric equation(s) of the tangent line to the function $\vec{r}(t)$ that passes through the origin.

$$\vec{r}(t) = \langle 2t^2, 3t + 3, t + 1 \rangle$$

- 3. Find the equation of the tangent plane to the surface $\vec{r}(t, s)$ at the point $t = 1$ and $s = 4$.

$$\vec{r}(t, s) = \langle t^2 + s^2, -3t + 5, 2s + 1 \rangle$$



INTEGRAL OF A VECTOR FUNCTION

- 1. Find the integral of the vector function.

$$\int \langle e^{3u-2}, e^{5-u}, \sin^2(u - \pi) \rangle du$$

- 2. Find the improper integral of the vector function.

$$\int_2^{\infty} \left\langle \frac{t-2}{t^3-8}, 2^{-t+1} \right\rangle dt$$

- 3. Find the double integral of the vector function, where R is the square $[0, \pi] \times [0, \pi]$.

$$\iint_R \langle ts, \sin(t-s) \rangle dA$$



