Topic: Partial derivatives in two variables

Question: Find the partial derivative(s).

Find f_y

for $f(x,y) = 4x^3y^2 + 2x^2y^2 + xy + 3x$

Answer choices:

A $\quad f_y = 24x^2y + 8xy + 1$

B $\quad f_y = 12x^2y^2 + 4xy^2 + y + 3$

C $\quad f_y = 8x^3y + 4x^2y + x$

D $\quad f_y = 12x^2y^2 + 8x^3y + 4x^2y + 4xy^2 + x + y + 3$

Solution: C

To find the partial derivative f_y, we want to treat x as a constant, while differentiating $f(x, y)$ with respect to y. Remember that if we treat x as a constant, then x^2 and x^3 are also constants.

Therefore,

$$f(x, y) = 4x^3y^2 + 2x^2y^2 + xy + 3x$$

$$f(x, y) = \left(4x^3\right)y^2 + \left(2x^2\right)y^2 + (x)y + (3x)$$

$$f_y = \left(4x^3\right)(2y) + \left(2x^2\right)(2y) + (x)(1) + (3x)(0)$$

$$f_y = 8x^3y + 4x^2y + x$$

Topic: Partial derivatives in two variables

Question: Find the partial derivative(s).

Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

for $f(x, y) = \sin\left(2x^2y\right)$

Answer choices:

A $\quad \frac{\partial f}{\partial x} = 4xy\cos\left(2x^2y\right)$ and $\frac{\partial f}{\partial y} = 2x^2\cos\left(2x^2y\right)$

B $\quad \frac{\partial f}{\partial x} = \cos(4xy)$ and $\frac{\partial f}{\partial y} = \cos\left(2x^2\right)$

C $\quad \frac{\partial f}{\partial x} = 4xy\cos(4xy)$ and $\frac{\partial f}{\partial y} = 2x^2\cos\left(2x^2\right)$

D $\quad \frac{\partial f}{\partial x} = 2x^2y\cos(4xy)$ and $\frac{\partial f}{\partial y} = 2x^2y\cos\left(2x^2\right)$

Solution: A

To find $\partial f/\partial x$, we want to treat y as a constant, while differentiating $f(x,y)$ with respect to x. Since our function $f(x,y)=\sin\left(2x^2y\right)$ involves a function within a function, we need to use the chain rule to differentiate it, just like we would for a single variable function.

$$\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\left[\sin\left(2x^2y\right)\right]$$

$$\frac{\partial f}{\partial x}=\cos\left(2x^2y\right)\cdot\frac{\partial}{\partial x}\left(2x^2y\right)$$

$$\frac{\partial f}{\partial x}=\cos\left(2x^2y\right)\cdot(2)(2x)(y)$$

$$\frac{\partial f}{\partial x}=4xy\cos\left(2x^2y\right)$$

To find $\partial f/\partial y$, we follow a similar process, but this time we hold x as a constant, while differentiating $f(x,y)$ with respect to y.

$$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\left[\sin\left(2x^2y\right)\right]$$

$$\frac{\partial f}{\partial y}=\cos\left(2x^2y\right)\cdot\frac{\partial}{\partial y}\left(2x^2y\right)$$

$$\frac{\partial f}{\partial y}=\cos\left(2x^2y\right)\cdot\left(2x^2\right)(1)$$

$$\frac{\partial f}{\partial y}=2x^2\cos\left(2x^2y\right)$$

Topic: Partial derivatives in two variables

Question: Find f_x and f_y for $f(x, y) = 2x^3 \cos y$.

Answer choices:

A	$f_x = -12x \sin y$	and	$f_y = -6x^2 \cos y$
B	$f_x = 6x^2 \cos y$	and	$f_y = -2x^3 \sin y$
C	$f_x = -6x^2 \sin y + 12x \cos y$	and	$f_y = -2x^3 \cos y - 6x^2 \sin y$
D	$f_x = -6x^2 \sin y$	and	$f_y = -6x^2 \sin y$

Solution: B

Looking at the function $f(x, y) = 2x^3 \cos y$, it seems like we might need to use the product rule to differentiate it, since it's the product of two functions ($2x^2$ and $\cos y$). However, since finding partial derivatives involves differentiating with respect to only one variable at a time (while holding the other constant), and since x and y each only appear in one of these functions, there will effectively only be one variable function present in each case, so we can avoid use of the product rule.

For example, to find the partial derivative f_x, we want to treat y as a constant, while differentiating $f(x, y)$ with respect to x. If we treat y as a constant, then $\cos y$ is also effectively a constant, leaving x^3 as the only portion of the original function that needs to be differentiated.

$$f(x, y) = 2x^3 \cos y$$

$$f(x, y) = (2 \cos y)x^3$$

$$f_x = (2 \cos y)3x^2$$

$$f_x = 6x^2 \cos y$$

Similarly, to find f_y, we can treat x (and therefore x^3) as a constant, and differentiate $f(x, y)$ with respect to y.

$$f(x, y) = 2x^3 \cos y$$

$$f(x, y) = \left(2x^3\right) \cos y$$

$$f_y = \left(2x^3\right)(-\sin y)$$

$$f_y = -2x^3 \sin y$$