Calculus 3 Workbook Solutions

Introduction to vectors

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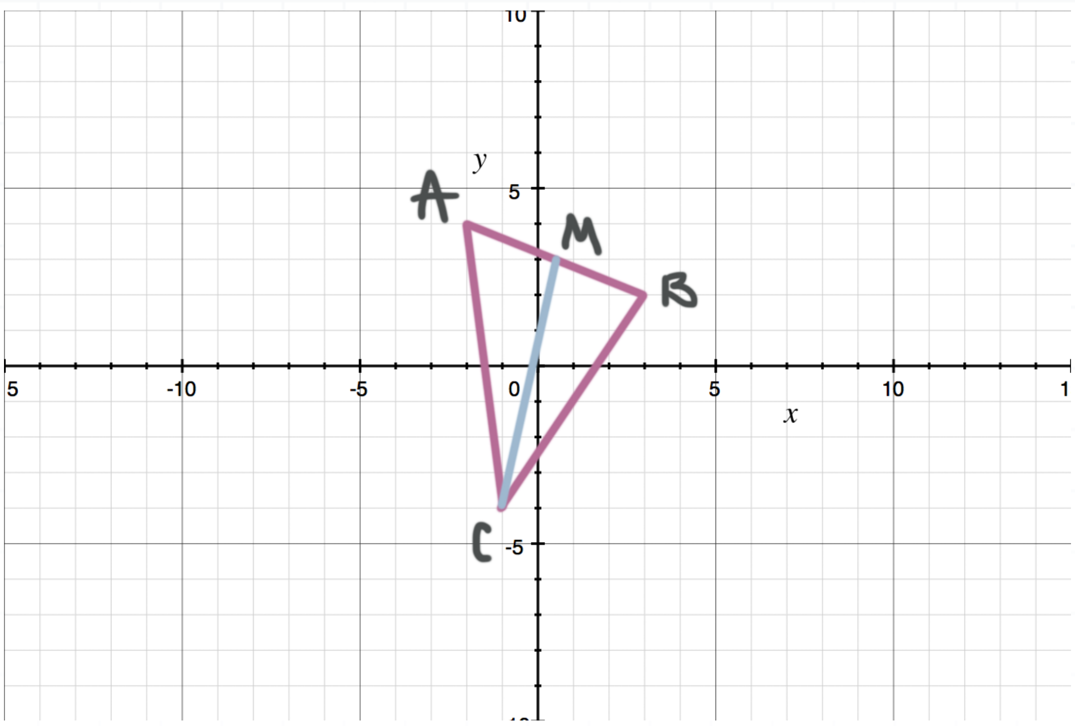
VECTOR FROM TWO POINTS

■ 1. Find the vector $\overrightarrow{CM}$, if M is the midpoint of $\overline{AB}$.



Solution:

The coordinates of the vertices are $A(-2,4)$, $B(3,2)$, and $C(-1,-4)$. Since M is the midpoint of AB, it has coordinates

$$x_M = \frac{x_A + x_B}{2} = \frac{-2+3}{2} = 0.5$$

$$y_M = \frac{y_A + y_B}{2} = \frac{4+2}{2} = 3$$

Since C is the initial point and M is the terminal point of the vector $\overrightarrow{CM}$,

$$\overrightarrow{CM} = \langle x_M - x_C, y_M - y_C \rangle$$

$$\overrightarrow{CM} = \langle 0.5 - (-1), 3 - (-4) \rangle$$

$$\overrightarrow{CM} = \langle 1.5, 7 \rangle$$

■ 2. Find the coordinates of the point P, given $Q(-\sqrt{2}, 0, \sqrt{2})$ and $\overrightarrow{PQ} = \langle \sqrt{2}, 4, \sqrt{2} \rangle$.

Solution:

Since P is the initial point and Q is the terminal point of the vector $\overrightarrow{PQ}$,

$$x_{\overrightarrow{PQ}} = x_Q - x_P$$

$$y_{\overrightarrow{PQ}} = y_Q - y_P$$

$$z_{\overrightarrow{PQ}} = z_Q - z_P$$

So

$$x_P = x_Q - x_{\overrightarrow{PQ}} = -\sqrt{2} - \sqrt{2} = -2\sqrt{2}$$

$$y_P = y_Q - y_{\overrightarrow{PQ}} = 0 - 4 = -4$$

$$z_P = z_Q - z_{\overrightarrow{PQ}} = \sqrt{2} - \sqrt{2} = 0$$

3. Find the coordinates of the point C, given the coordinates of the point $A(-2,3,4)$, and the vectors $\overrightarrow{AB} = \langle 0,5,0 \rangle$ and $\overrightarrow{BC} = \langle 2, -3,6 \rangle$.

Solution:

Since A is the initial point of the vector $\overrightarrow{AB}$, and B is the terminal point,

$$x_{\overrightarrow{AB}} = x_B - x_A$$

$$y_{\overrightarrow{AB}} = y_B - y_A$$

$$z_{\overrightarrow{AB}} = z_B - z_A$$

So

$$x_B = x_A + x_{\overrightarrow{AB}} = -2 + 0 = -2$$

$$y_B = y_A + y_{\overrightarrow{AB}} = 3 + 5 = 8$$

$$z_B = z_A + z_{\overrightarrow{AB}} = 4 + 0 = 4$$

Therefore, the point B has coordinates $(-2,8,4)$. Similarly, since B is the initial point of the vector $\overrightarrow{BC}$, and C is the terminal point,

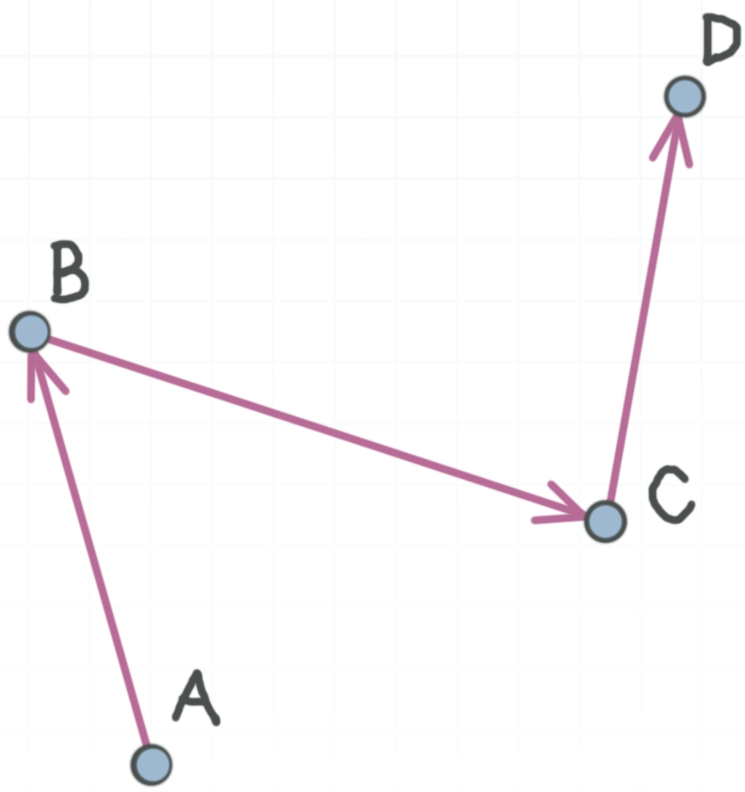
$$x_C = x_B + x_{\overrightarrow{BC}} = -2 + 2 = 0$$

$$y_C = y_B + y_{\overrightarrow{BC}} = 8 - 3 = 5$$

$$z_C = z_B + z_{\overrightarrow{BC}} = 4 + 6 = 10$$

COMBINATIONS OF VECTORS

1. Find the combination $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$.



Solution:

The combination of two vectors is the vector that connects the initial point of the first with the terminal point of the second.

So $\overrightarrow{AC}$ is the combination of $\overrightarrow{AB}$ and $\overrightarrow{BC}$, or

$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$$

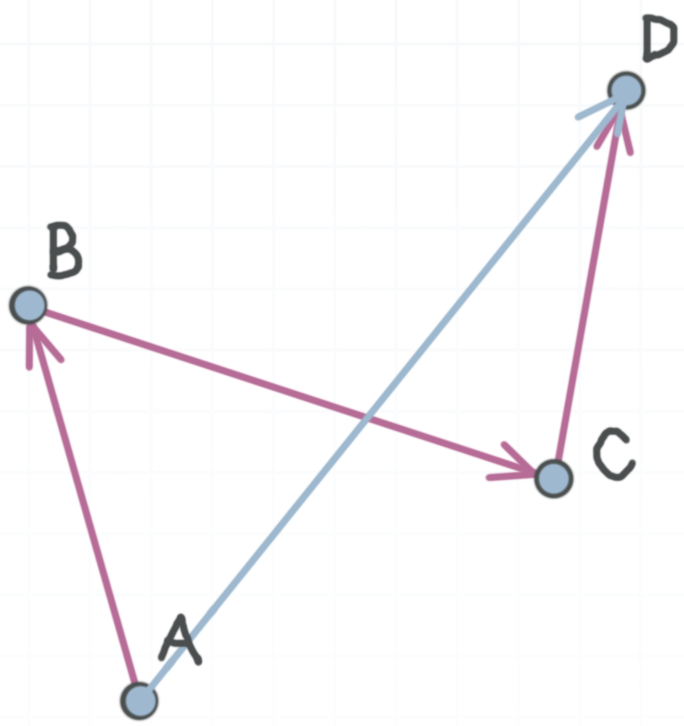
Similarly, $\overrightarrow{AD}$ is the combination of $\overrightarrow{AC}$ and $\overrightarrow{CD}$, or

$$\overrightarrow{AD}=\overrightarrow{AC}+\overrightarrow{CD}$$

Substitute the value of $\overrightarrow{AC}$.

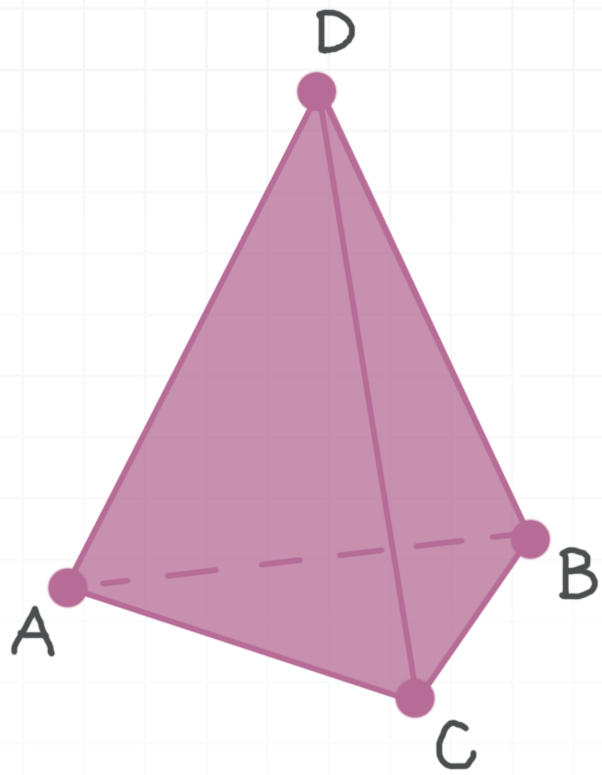
$$\overrightarrow{AD} = (\overrightarrow{AB} + \overrightarrow{BC}) + \overrightarrow{CD}$$

$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$$



We can generalize the problem and conclude that combination of a connected sequence of vectors, where the initial point of the next vector is the same as the terminal point of the previous, is the vector that connects the initial point of the first with the terminal point of the last.

■ 2. In the tetrahedron $ABCD$, find the resulting vector $\overrightarrow{DA} - \overrightarrow{DB} - \overrightarrow{BC}$.



Solution:

The combination of two vectors is the vector that connects the initial point of the first with the terminal point of the second. Consider the triangle ABD. Here, $\overrightarrow{DA}$ is the combination of $\overrightarrow{DB}$ and $\overrightarrow{BA}$, or

$$\overrightarrow{DA} = \overrightarrow{DB} + \overrightarrow{BA}$$

Subtract $\overrightarrow{DB}$ from each side

$$\overrightarrow{DA} - \overrightarrow{DB} = \overrightarrow{BA}$$

So

$$\overrightarrow{DA} - \overrightarrow{DB} - \overrightarrow{BC} = \overrightarrow{BA} - \overrightarrow{BC}$$

Similarly, consider the triangle ABC. Here, $\overrightarrow{BA}$ is the combination of $\overrightarrow{BC}$ and $\overrightarrow{CA}$, or

$$\overrightarrow{BA} = \overrightarrow{BC} + \overrightarrow{CA}$$

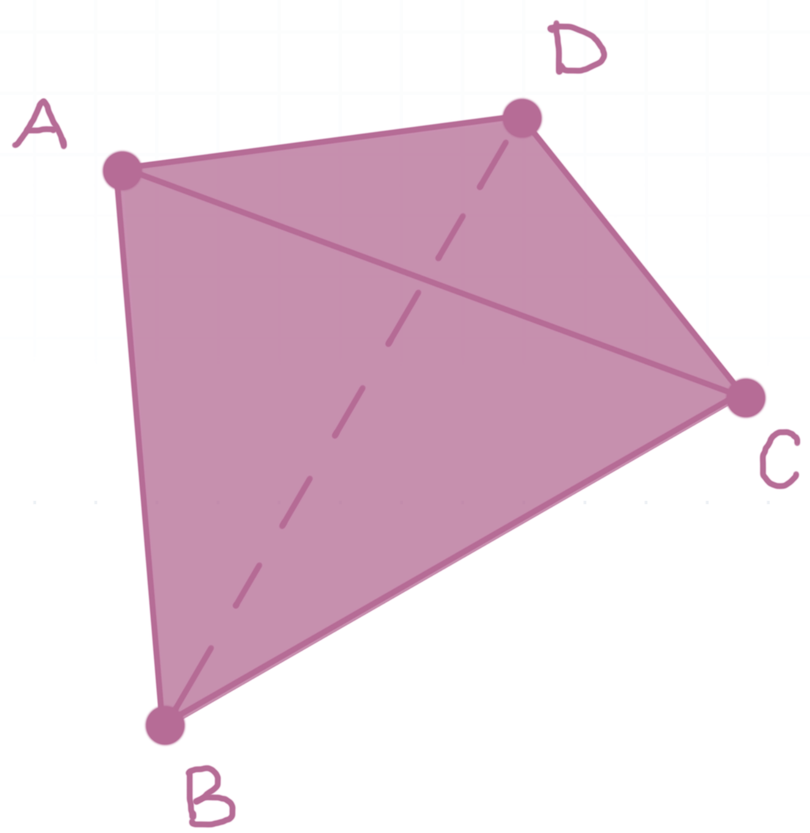
Subtract $\overrightarrow{BC}$ from each side

$$\overrightarrow{BA} - \overrightarrow{BC} = \overrightarrow{CA}$$

So

$$\overrightarrow{DA} - \overrightarrow{DB} - \overrightarrow{BC} = \overrightarrow{CA}$$

■ 3. In tetrahedron $ABCD$, find the vector $\overrightarrow{AB} + \overrightarrow{DC} + \overrightarrow{BD} - \overrightarrow{BC}$.



Solution:

The combination of two vectors is the vector that connects the initial point of the first with the terminal point of the second.

Notice that we can't find the combination $\overrightarrow{AB}+\overrightarrow{DC}$ directly, so change the order of the summation of the first three terms.

$$\overrightarrow{AB}+\overrightarrow{DC}+\overrightarrow{BD}=\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{DC}$$

Since the combination of the connected sequence of vectors is the vector that connects the initial point of the first with the terminal point of the last.

$$\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{DC}=\overrightarrow{AC}$$

So

$$\overrightarrow{AB}+\overrightarrow{DC}+\overrightarrow{BD}-\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{BC}$$

Consider the triangle ABC. Here $\overrightarrow{AC}$ is the combination of $\overrightarrow{AB}$ and $\overrightarrow{BC}$, or

$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$$

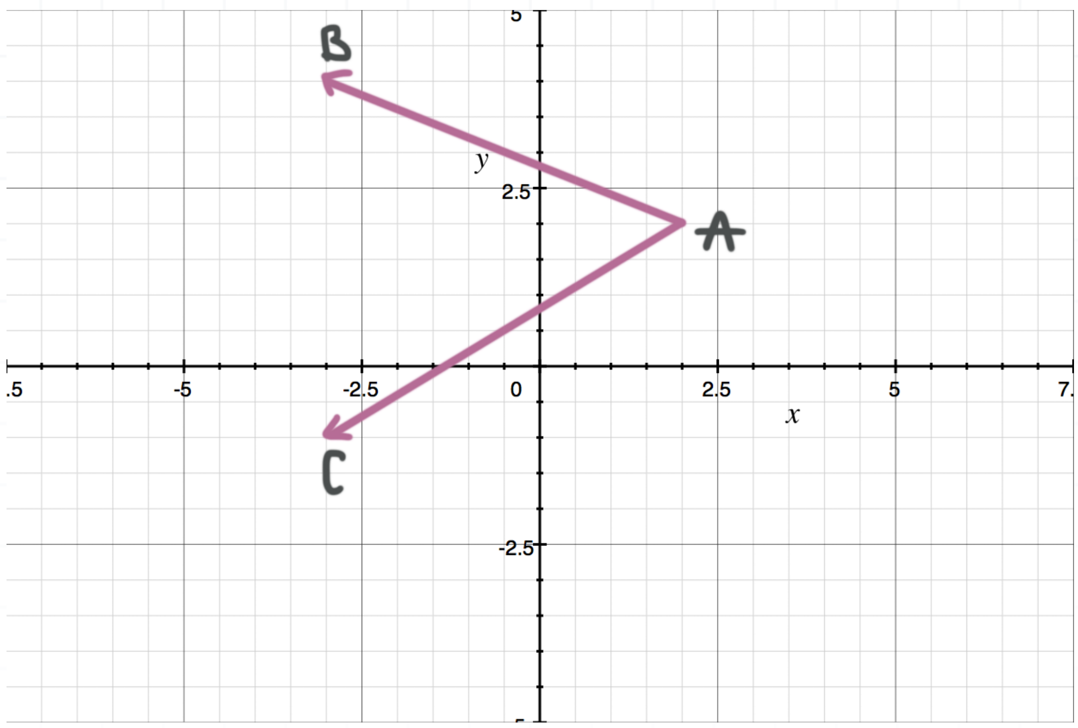
$$\overrightarrow{AC}-\overrightarrow{BC}=\overrightarrow{AB}$$

So

$$\overrightarrow{AB}+\overrightarrow{DC}+\overrightarrow{BD}-\overrightarrow{BC}=\overrightarrow{AB}$$

SUM OF TWO VECTORS

■ 1. Find the sum $\overrightarrow{AB} + \overrightarrow{AC}$.



Solution:

The coordinates of the three points are $A(2,2)$, $B(-3,4)$, and $C(-3,-1)$.

Since A is the initial point and B is the terminal point of the vector $\overrightarrow{AB}$,

$$\overrightarrow{AB} = \langle x_B - x_A, y_B - y_A \rangle$$

$$\overrightarrow{AB} = \langle -3 - 2, 4 - 2 \rangle$$

$$\overrightarrow{AB} = \langle -5, 2 \rangle$$

Similarly, since A is the initial point and C is the terminal point of the vector $\overrightarrow{AC}$,

$$\overrightarrow{AC} = \langle x_C - x_A, y_C - y_A \rangle$$

$$\overrightarrow{AC} = \langle -3 - 2, -1 - 2 \rangle$$

$$\overrightarrow{AC} = \langle -5, -3 \rangle$$

Then the sum of the vectors is

$$\overrightarrow{AB} + \overrightarrow{AC} = \langle x_{\overrightarrow{AB}} + x_{\overrightarrow{AC}}, y_{\overrightarrow{AB}} + y_{\overrightarrow{AC}} \rangle$$

$$\overrightarrow{AB} + \overrightarrow{AC} = \langle -5 - 5, 2 - 3 \rangle$$

$$\overrightarrow{AB} + \overrightarrow{AC} = \langle -10, -1 \rangle$$

■ 2. Find the vector $\overrightarrow{a} - \overrightarrow{b} + 2\overrightarrow{c}$, if $\overrightarrow{a} = \langle 0,4,5 \rangle$, $\overrightarrow{b} = \langle -3,2,1 \rangle$, and $\overrightarrow{c} = \langle 6,0,2 \rangle$.

Solution:

The difference $\overrightarrow{a} - \overrightarrow{b}$ is

$$\overrightarrow{a} - \overrightarrow{b} = \langle x_a - x_b, y_a - y_b, z_a - z_b \rangle$$

$$\overrightarrow{a} - \overrightarrow{b} = \langle 0 - (-3), 4 - 2, 5 - 1 \rangle$$

$$\overrightarrow{a} - \overrightarrow{b} = \langle 3,2,4 \rangle$$

The scaled vector $2\overrightarrow{c}$ is

$$2\overrightarrow{c} = \langle 2x_c, 2y_c, 2z_c \rangle$$

$$2\vec{c} = \langle 2(6), 2(0), 2(2) \rangle$$

$$2\vec{c} = \langle 12, 0, 4 \rangle$$

Finally, the sum $\vec{a} - \vec{b} + 2\vec{c}$ is

$$\vec{a} - \vec{b} + 2\vec{c} = \langle 3, 2, 4 \rangle + \langle 12, 0, 4 \rangle$$

$$\vec{a} - \vec{b} + 2\vec{c} = \langle 3 + 12, 2 + 0, 4 + 4 \rangle$$

$$\vec{a} - \vec{b} + 2\vec{c} = \langle 15, 2, 8 \rangle$$

■ 3. Find the sum of the vectors.

$$\sum_{k=1}^{100} \langle 5, k \rangle$$

Solution:

The sum of x_k coordinates is

$$x_1 + x_2 + \ldots + x_{100} = 5 + 5 + \ldots + 5 = 100(5) = 500$$

The coordinates y_k form the arithmetic progression

$$y_1 + y_2 + \ldots + y_{100} = 1 + 2 + 3 + \ldots + 100$$

Its sum is given by the formula

$$S_n = \frac{n(a_1 + a_n)}{2}$$

So

$$y_1 + y_2 + \ldots + y_{100} = S_{100} = 1 + 2 + 3 + \ldots + 100$$

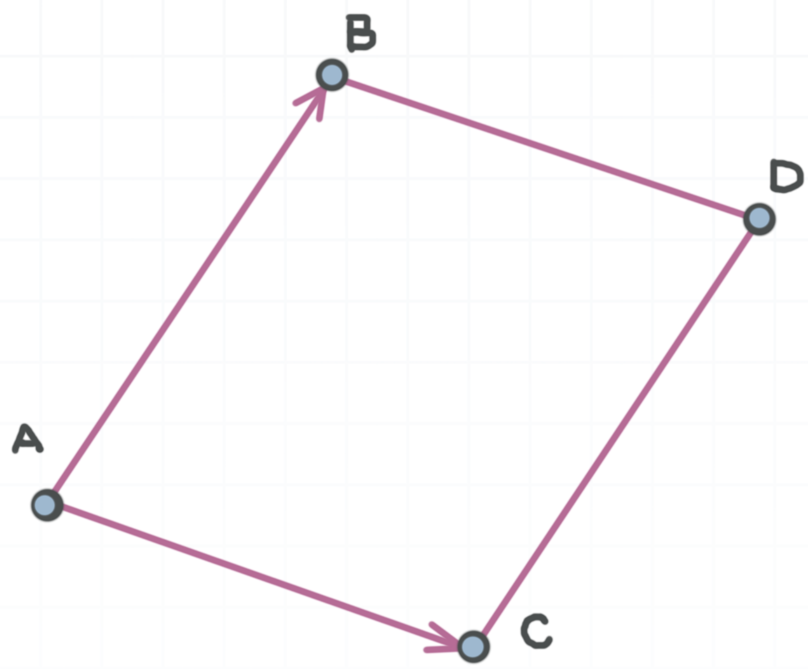
$$y_1 + y_2 + \ldots + y_{100} = S_{100} = \frac{100(1 + 100)}{2} = 50(101) = 5{,}050$$

Therefore,

$$\sum_{k=1}^{100} \langle 5,k \rangle = \langle 500,5{,}050 \rangle$$

COPYING VECTORS AND USING THEM TO DRAW COMBINATIONS

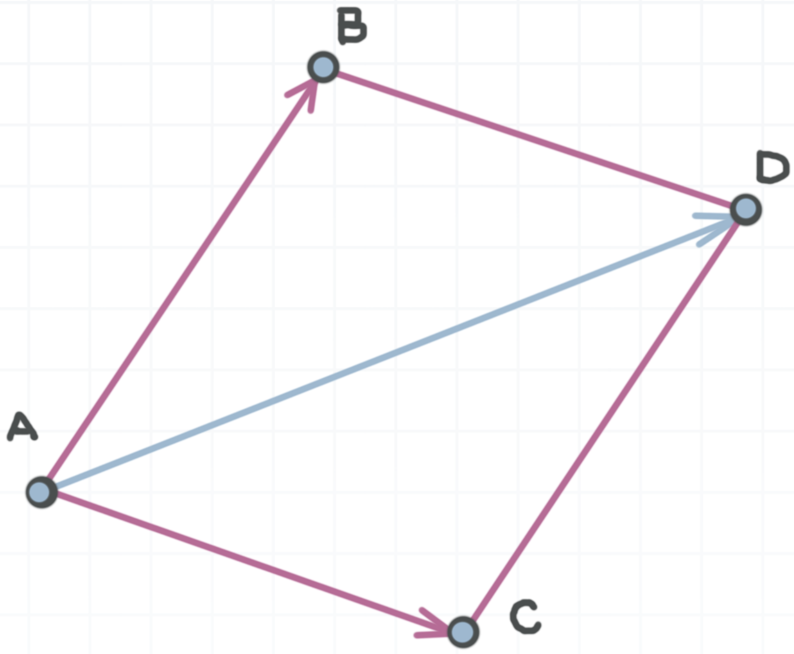
1. In parallelogram $ABDC$, find the combination $\overrightarrow{AB}+\overrightarrow{AC}$



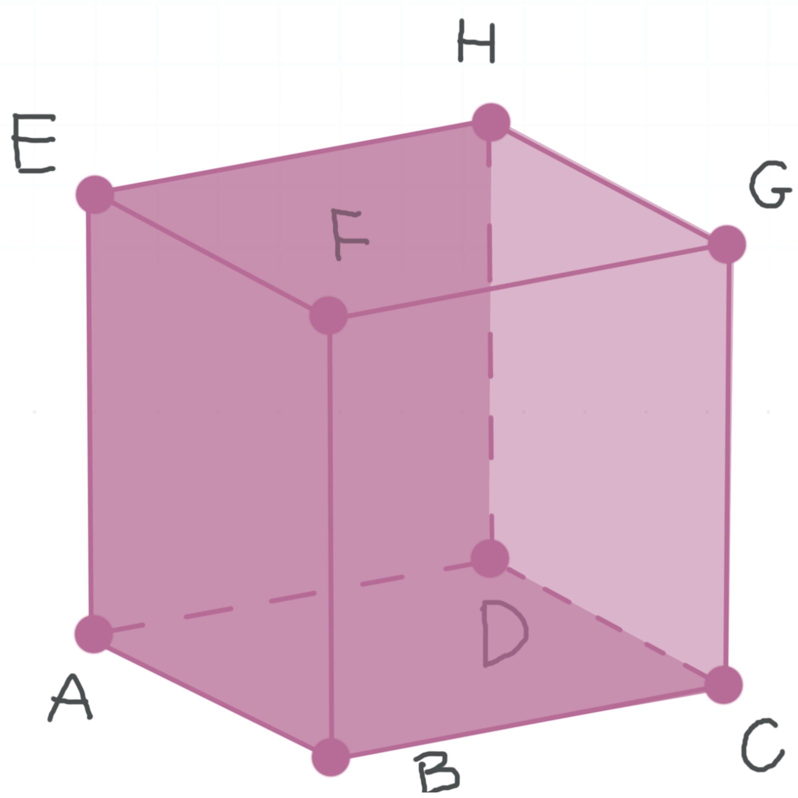
Solution:

Since the vectors $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are parallel and have the same magnitude, they are equal. So we can connect the initial point A of the vector $\overrightarrow{AC}$ to the terminal point B of the vector $\overrightarrow{AB}$. So

$$\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BD}=\overrightarrow{AD}$$



■ 2. In the cube $ABCDEFGH$, find the combination $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AE}$.



Solution:

Since the vectors $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are parallel and have the same magnitude, they are equal. So we can connect the initial point A of the vector $\overrightarrow{AD}$ to the terminal point B of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

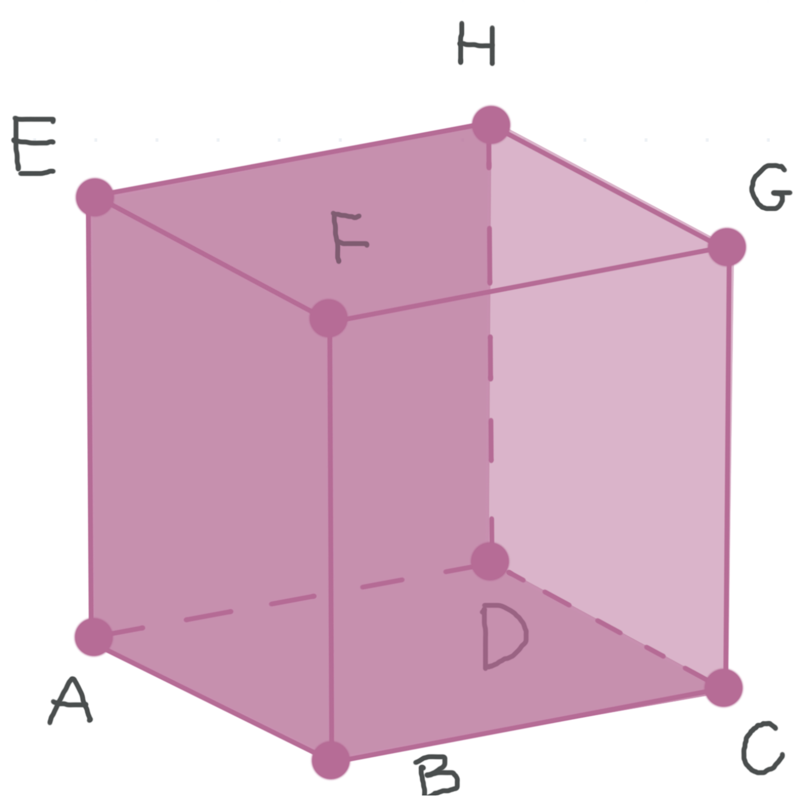
So

$$\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{AE}$$

Similarly, since the vectors $\overrightarrow{AE}$ and $\overrightarrow{CG}$ are parallel and have the same magnitude, they are equal. So

$$\overrightarrow{AC} + \overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CG} = \overrightarrow{AG}$$

■ 3. In the cube $ABCDEFGH$, find the combination $\overrightarrow{AB} - \overrightarrow{AD} - \overrightarrow{AE}$.



Solution:

Consider the triangle ABD. Here, $\overrightarrow{AB}$ is the combination of $\overrightarrow{AD}$ and $\overrightarrow{DB}$.

$$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$$

$$\overrightarrow{AB} - \overrightarrow{AD} = \overrightarrow{DB}$$

So

$$\overrightarrow{AB} - \overrightarrow{AD} - \overrightarrow{AE} = \overrightarrow{DB} - \overrightarrow{AE}$$

Since the vectors $\overrightarrow{AE}$ and $\overrightarrow{DH}$ are parallel and have the same magnitude, they are equal. So

$$\overrightarrow{DB} - \overrightarrow{AE} = \overrightarrow{DB} - \overrightarrow{DH}$$

Consider the triangle BDH. Here, $\overrightarrow{DB}$ is the combination of $\overrightarrow{DH}$ and $\overrightarrow{HB}$.

$$\overrightarrow{DB} = \overrightarrow{DH} + \overrightarrow{HB}$$

$$\overrightarrow{DB} - \overrightarrow{DH} = \overrightarrow{HB}$$

So

$$\overrightarrow{AB} - \overrightarrow{AD} - \overrightarrow{AE} = \overrightarrow{HB}$$

UNIT VECTOR IN THE DIRECTION OF THE GIVEN VECTOR

■ 1. Find the unit vector in the direction of the combination $\vec{a}+\vec{b}$, where $\vec{a}=\langle -2,-7\rangle$ and $\vec{b}=\langle 5,3\rangle$.

Solution:

Let $\vec{c}$ be the combination $\vec{c}=\vec{a}+\vec{b}$.

$$\vec{c}=\langle x_a+x_b, y_a+y_b\rangle$$

$$\vec{c}=\langle -2+5,-7+3\rangle$$

$$\vec{c}=\langle 3,-4\rangle$$

The magnitude of $\vec{c}$ is

$$|\vec{c}|=\sqrt{x_c^2+y_c^2}=\sqrt{(3)^2+(-4)^2}=\sqrt{25}=5$$

The unit vector in the direction of $\vec{c}$ is

$$\vec{u}_c=\frac{\vec{c}}{|\vec{c}|}$$

$$\vec{u}_c=\frac{\langle 3,-4\rangle}{5}$$

$$\vec{u}_c=\left\langle \frac{3}{5},-\frac{4}{5}\right\rangle$$

■ 2. The magnitude of the vector $\vec{a}$ is three times larger than the unit vector in the same direction. Find the vector $\vec{a}$.

$$\vec{u}_a = \left\langle \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \right\rangle$$

Solution:

Since magnitude of $\vec{u}_a$ is 1, and since $\vec{a}$ three times larger,

$$|\vec{a}| = 3$$

The unit vector in the direction of $\vec{a}$ is

$$\vec{u}_a = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{3}$$

So

$$\vec{a} = 3\vec{u}_a$$

$$\vec{a} = 3\left\langle \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \right\rangle$$

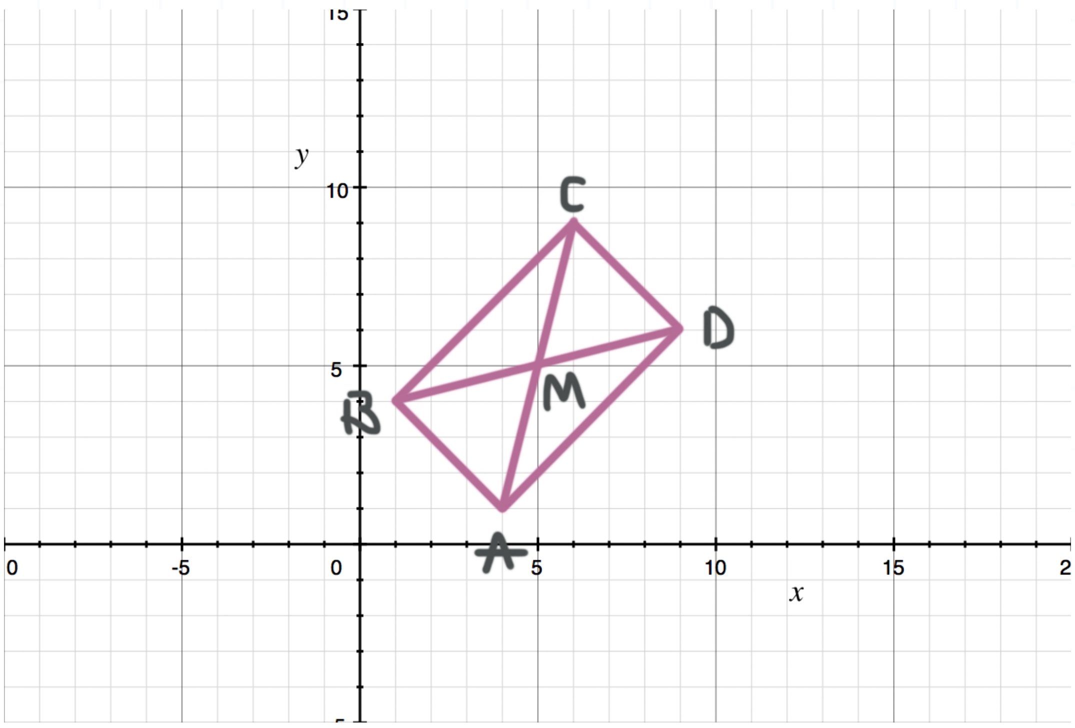
$$\vec{a} = \langle 1, -2, -2 \rangle$$

■ 3. Find the unit vector in the direction of $\overrightarrow{AC}$ in the rectangle $ABCD$, if $A(4,1)$, $B(1,4)$, and $D(9,6)$.

Solution:

We don't know the coordinates of the point C. Use the fact that in a rectangle its diagonal passes through the midpoint of the other diagonal.

Let M be the midpoint of BD. Since $\overrightarrow{AC}$ and $\overrightarrow{AM}$ have the same direction, they have the same unit vector $\overrightarrow{u}$.



Since M is a midpoint of BD, it has coordinates

$$x_M = \frac{x_B + x_D}{2} = \frac{1+9}{2} = 5$$

$$y_M = \frac{y_B + y_D}{2} = \frac{4+6}{2} = 5$$

Since A is the initial point of the vector $\overrightarrow{AM}$, and M is the terminal point,

$$\overrightarrow{AM} = \langle x_M - x_A, y_M - y_A \rangle$$

$$\overrightarrow{AM} = \langle 5 - 4, 5 - 1 \rangle$$

$$\overrightarrow{AM} = \langle 1, 4 \rangle$$

The unit vector in the direction of $\overrightarrow{AM}$ is

$$\vec{u} = \frac{\overrightarrow{AM}}{|\overrightarrow{AM}|}$$

where

$$|\overrightarrow{AM}| = \sqrt{\left(x_{\overrightarrow{AM}}\right)^2 + \left(y_{\overrightarrow{AM}}\right)^2} = \sqrt{(1)^2 + (4)^2} = \sqrt{17}$$

So

$$\vec{u} = \frac{\langle 1, 4 \rangle}{\sqrt{17}}$$

$$\vec{u} = \left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$$

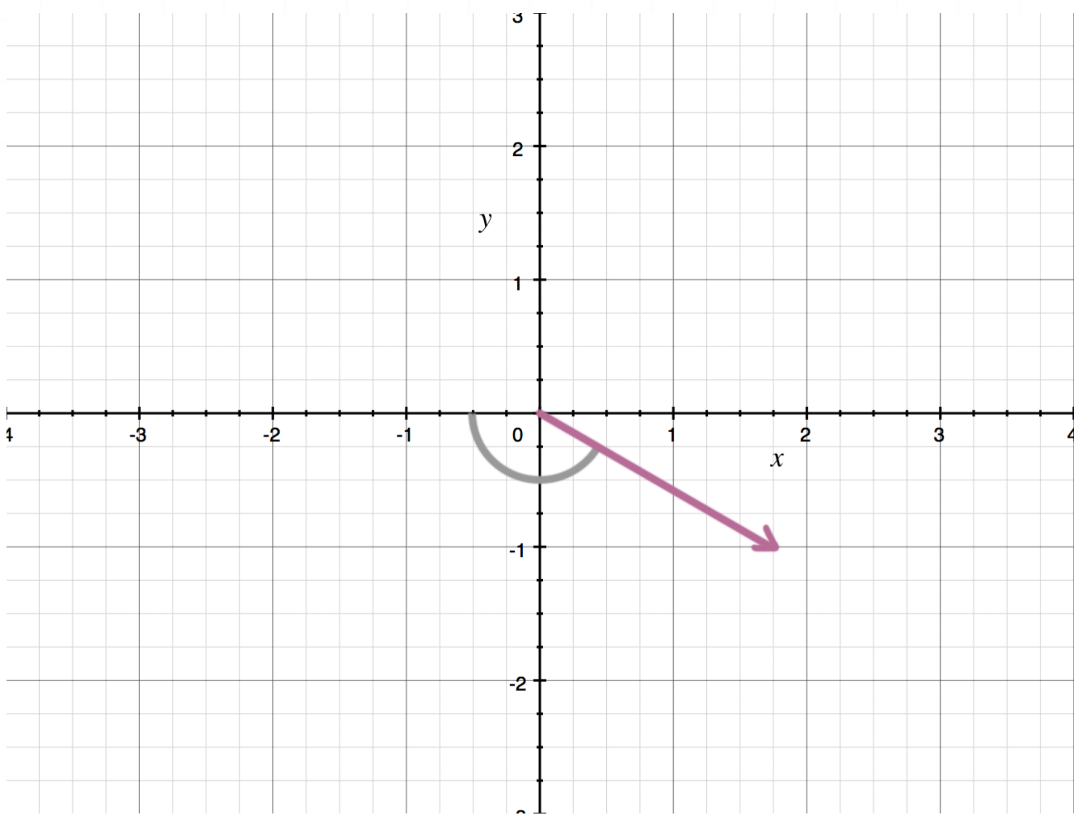
ANGLE BETWEEN A VECTOR AND THE X-AXIS

■ 1. Find the clockwise angle in radians between the vector $\overrightarrow{a} = \langle \sqrt{3}, -1 \rangle$ and the negative direction of the x-axis.

Solution:

Sketch the vector and its angle.

$$\overrightarrow{a} = \langle \sqrt{3}, -1 \rangle \approx \langle 1.7, -1 \rangle$$



Let α be the angle between $\overrightarrow{a}$ and the positive direction of the x-axis. Then the clockwise angle between $\overrightarrow{a}$ and the negative direction of the x-axis is $\pi - \alpha$. Use the formula for the angle.

$$\alpha = \arctan\left|\frac{y_a}{x_a}\right|$$

$$\alpha = \arctan\left|\frac{1}{-\sqrt{3}}\right|$$

$$\alpha = \arctan\frac{1}{\sqrt{3}}$$

$$\alpha = \frac{\pi}{6}$$

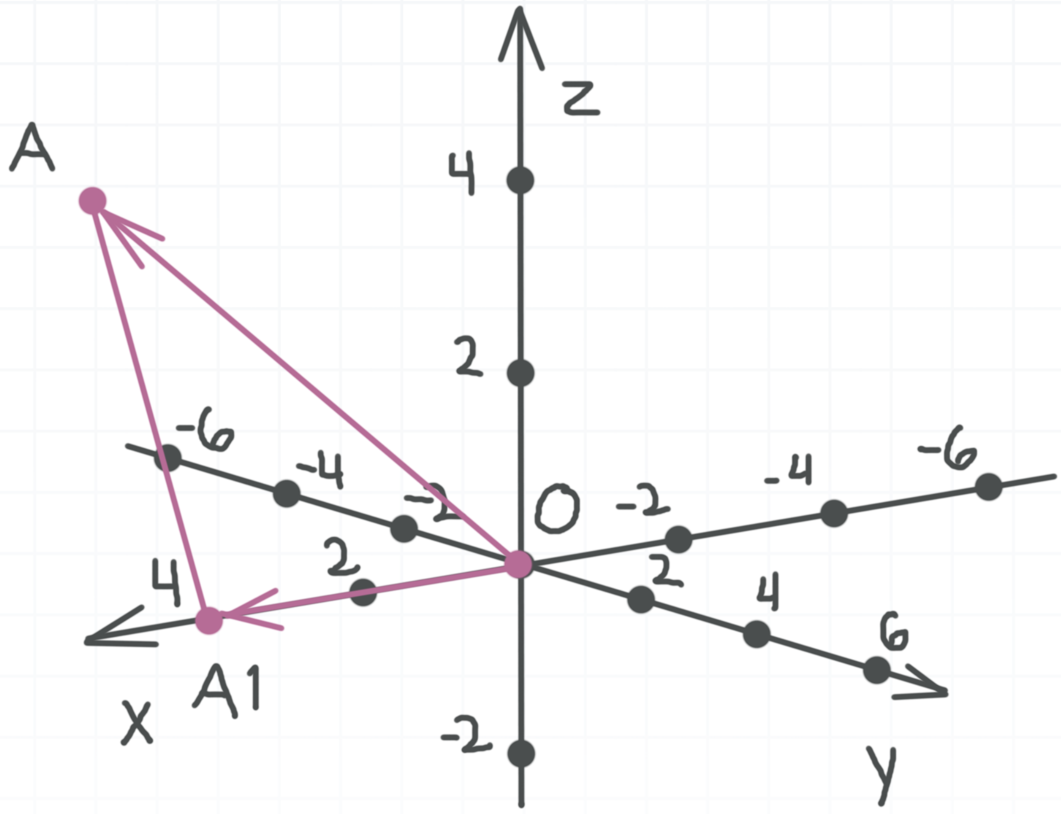
So

$$\pi - \alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

■ 2. Find the angle between the vector $\overrightarrow{OA} = \langle 4, -4, 2 \rangle$ and the positive direction of the x-axis.

Solution:

Let $A_1(4,0,0)$ be the projection of the point A onto the x-axis.



In the right triangle OAA_1 we need to find the angle AOA_1. Cosine of this angle is

$$\cos AOA_1 = \frac{OA_1}{OA}$$

Find magnitudes.

$$OA = \sqrt{(4)^2 + (-4)^2 + (2)^2} = \sqrt{36} = 6$$

$$OA_1 = \sqrt{(4)^2 + (0)^2 + (0)^2} = \sqrt{16} = 4$$

Then

$$\cos AOA_1 = \frac{4}{6} = \frac{2}{3}$$

$$AOA_1 = \arccos\frac{2}{3} \approx 0.84$$

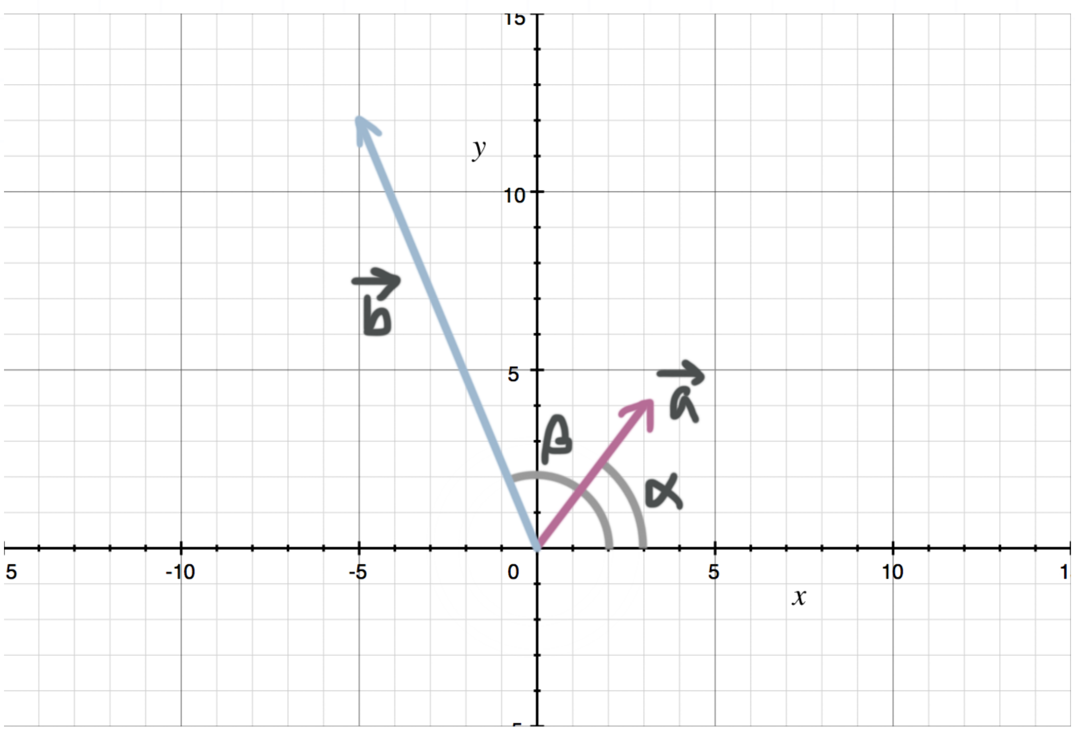
Convert to degrees.

$$\frac{0.84 \cdot 180^\circ}{\pi} \approx 48^\circ$$

■ 3. Find the angle between the vectors $\overrightarrow{a} = \langle 3,4 \rangle$ and $\overrightarrow{b} = \langle -5,12 \rangle$.

Solution:

The main idea is to find the angles α and β between the positive direction of the x-axis, and the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ respectively, and then compute the difference $\beta - \alpha$.



Use the formula for the angle.

$$\alpha = \arctan \left| \frac{y_a}{x_a} \right|$$

$$\alpha = \arctan\left|\frac{4}{3}\right|$$

$$\alpha = \arctan\frac{4}{3}$$

Find the angle between β and the negative direction of the x-axis, then subtract it from π.

$$\beta = \pi - \arctan\left|\frac{y_b}{x_b}\right|$$

$$\beta = \pi - \arctan\left|\frac{12}{-5}\right|$$

$$\beta = \pi - \arctan\frac{12}{5}$$

Compute the difference.

$$\beta - \alpha = \pi - \arctan\frac{12}{5} - \arctan\frac{4}{3} \approx 1.038$$

Convert to degrees.

$$\frac{1.038 \cdot 180^\circ}{\pi} \approx 59.5^\circ$$

MAGNITUDE AND ANGLE OF THE RESULTANT FORCE

1. Find the magnitude and angle of the resultant force $\vec{f}$ of the vectors $\overrightarrow{a} = \langle 2, -1 \rangle$, $\overrightarrow{b} = \langle 5,1 \rangle$, and $\overrightarrow{c} = \langle -3,3 \rangle$.

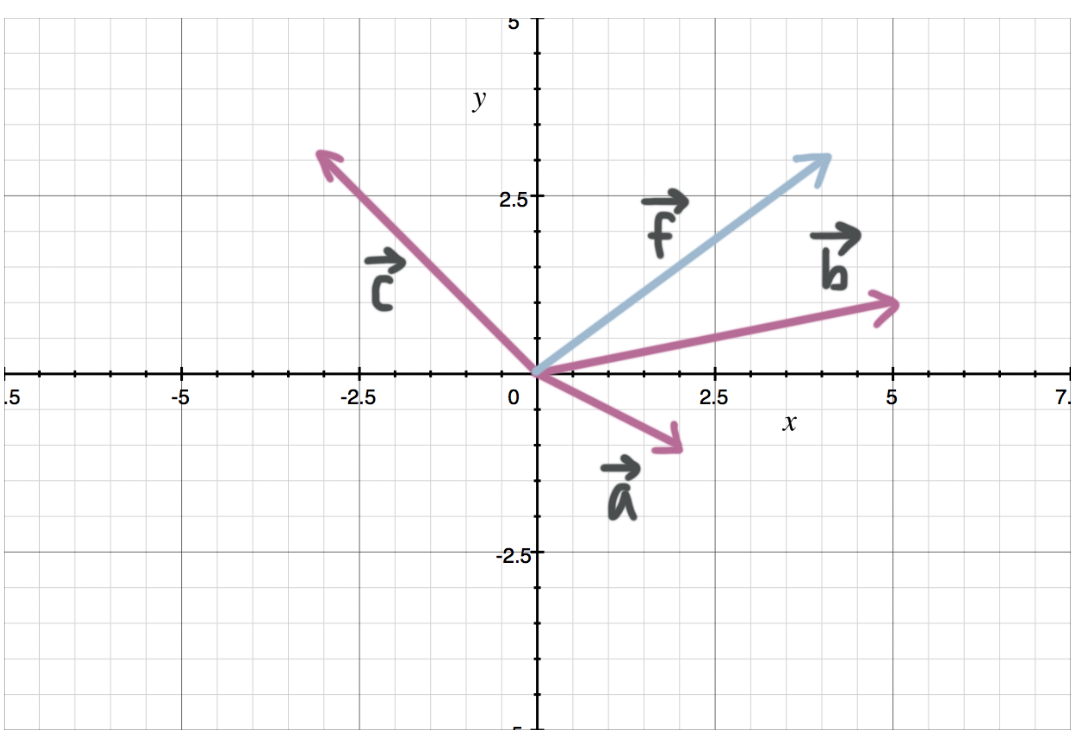
Solution:

Find the resultant force vector $\vec{f}$ as the summation of $\overrightarrow{a} = \langle 2, -1 \rangle$, $\overrightarrow{b} = \langle 5,1 \rangle$, and $\overrightarrow{c} = \langle -3,3 \rangle$.

$$\vec{f} = \langle x_a + x_b + x_c, y_a + y_b + y_c \rangle$$

$$\vec{f} = \langle 2 + 5 - 3, -1 + 1 + 3 \rangle$$

$$\vec{f} = \langle 4,3 \rangle$$



Then the magnitude is

$$|\vec{f}| = \sqrt{(x_f)^2 + (y_f)^2}$$

$$|\vec{f}| = \sqrt{4^2 + 3^2}$$

$$|\vec{f}| = 5$$

Use the formula for the angle of the resulting force vector.

$$\arctan\left|\frac{y_f}{x_f}\right| = \arctan\frac{3}{4} \approx 0.6435$$

Convert to degrees.

$$\frac{0.6435 \cdot 180^\circ}{\pi} \approx 36.9^\circ$$

■ 2. Find the magnitude of the resultant force $\vec{f}$ of the vectors $\vec{a} = \langle 4,0,0 \rangle$, $\vec{b} = \langle 0,4,0 \rangle$, and $\vec{c} = \langle 0,0,2 \rangle$, then find the angles between $\vec{f}$ and each of the major coordinate axes.

Solution:

Find the resultant force vector $\vec{f}$ by adding $\vec{a} = \langle 4,0,0 \rangle$, $\vec{b} = \langle 0,4,0 \rangle$, and $\vec{c} = \langle 0,0,2 \rangle$.

$$\vec{f} = \langle x_a + x_b + x_c, y_a + y_b + y_c, z_a + z_b + z_c \rangle$$

$$\vec{f} = \langle 4 + 0 + 0, 0 + 4 + 0, 0 + 0 + 2 \rangle$$

$$\vec{f} = \langle 4,4,2 \rangle$$

Then the magnitude is

$$|\vec{f}| = \sqrt{(x_f)^2 + (y_f)^2 + (z_f)^2}$$

$$|\vec{f}| = \sqrt{4^2 + 4^2 + 2^2}$$

$$|\vec{f}| = 6$$

Since $\vec{a}$ is the projection of $\vec{f}$ onto the x-axis,

$$\cos\alpha_x = \frac{|\vec{a}|}{|\vec{f}|}$$

$$\cos\alpha_x = \frac{4}{6} = \frac{2}{3}$$

$$\alpha_x = \arccos\frac{2}{3} \approx 0.84$$

$$\frac{0.84 \cdot 180^\circ}{\pi} \approx 48^\circ$$

Since $\vec{b}$ is the projection of $\vec{f}$ onto the y-axis,

$$\cos\alpha_y = \frac{|\vec{b}|}{|\vec{f}|}$$

$$\cos\alpha_y = \frac{4}{6} = \frac{2}{3}$$

$$\alpha_y = \arccos\frac{2}{3} \approx 0.84$$

$$\frac{0.84 \cdot 180^\circ}{\pi} \approx 48^\circ$$

Since $\vec{c}$ is the projection of $\vec{f}$ onto the z-axis,

$$\cos \alpha_z = \frac{|\vec{c}|}{|\vec{f}|}$$

$$\cos \alpha_z = \frac{2}{6} = \frac{1}{3}$$

$$\alpha_z = \arccos \frac{1}{3} \approx 1.23$$

$$\frac{1.23 \cdot 180^\circ}{\pi} \approx 70.5^\circ$$

■ 3. The resultant force $\vec{f}$ of the vectors $\vec{a}$ and $\vec{b}$ has a magnitude of 12 and an angle of $2\pi/3$. Find vector $\vec{b}$, if $\vec{a} = \langle -8, 5\sqrt{3} \rangle$.

Solution:

Find the coordinates of the vector $\vec{f}$.

$$\vec{f} = \langle |\vec{f}| \cdot \cos \alpha_f, |\vec{f}| \cdot \sin \alpha_f \rangle$$

$$\vec{f} = \left\langle 12 \cos \frac{2\pi}{3}, 12 \sin \frac{2\pi}{3} \right\rangle$$

$$\vec{f} = \left\langle 12\left(-\frac{1}{2}\right), 12\left(\frac{\sqrt{3}}{2}\right)\right\rangle$$

$$\vec{f} = \langle -6, 6\sqrt{3} \rangle$$

Since $\vec{f} = \vec{a} + \vec{b}$, then $\vec{b} = \vec{f} - \vec{a}$. So

$$\vec{b} = \langle x_f - x_a, y_f - y_a \rangle$$

$$\vec{b} = \langle -6 - (-8), 6\sqrt{3} - 5\sqrt{3} \rangle$$

$$\vec{b} = \langle 2, \sqrt{3} \rangle$$

W W W . K R I S T A K I N G M A T H . C O M