Calculus 3 Workbook

Triple integrals in spherical coordinates

krista king MATH

SPHERICAL COORDINATES

1. Evaluate the triple integral given in the spherical coordinates, where $f(\rho, \theta, \phi) = 2\rho \sin\theta \cos\phi$.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{3}} \int_{1}^{5} f(\rho, \theta, \phi)\ \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$$

2. Identify the solid given by the iterated integral in spherical coordinates.

$$\int_{0}^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_{0}^{2} f(\rho, \theta, \phi)\ \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$$

3. Identify the solid given by the iterated improper integral in the spherical coordinates.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2\pi} \int_{\pi}^{\infty} f(\rho, \theta, \phi)\ \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$$

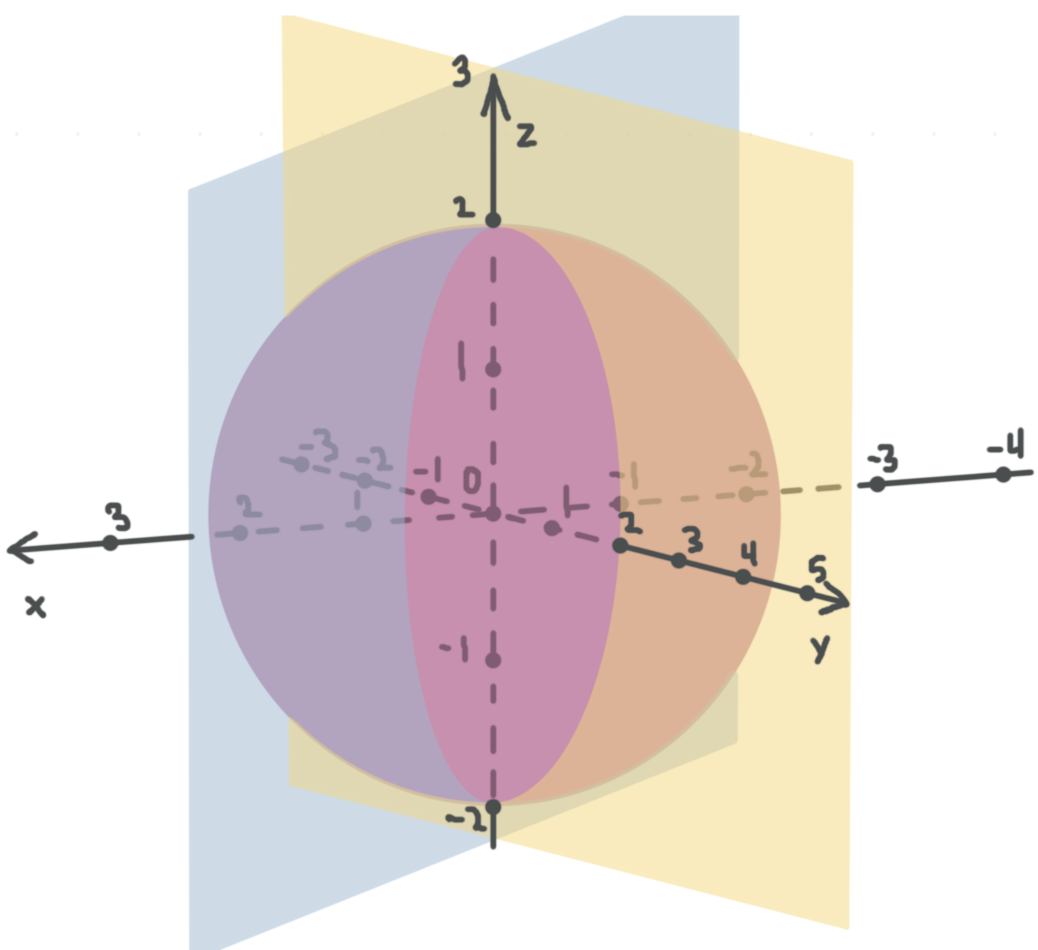
CHANGING TRIPLE INTEGRALS TO SPHERICAL COORDINATES

■ 1. Evaluate the triple integral by changing it to spherical coordinates, if E is the sphere with center at the origin and radius 3.

$$\iiint_E 5x^2 - 2\ dV$$

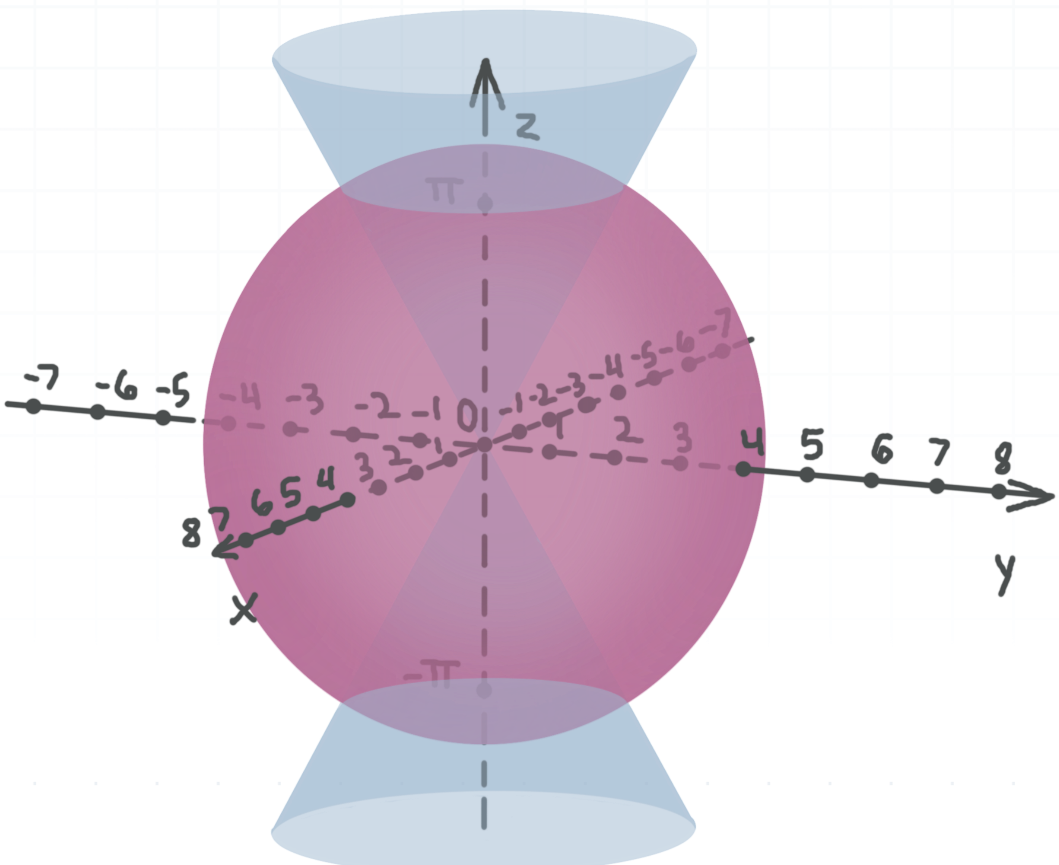
■ 2. Write down the triple integral by converting it to spherical coordinates, if E is the part of the sphere with center at the origin, radius 2, that lies between the planes $x = 0$ and $y = x$, and in the space $y > 0$.

$$\iiint_E x^2 + y^2 + 2z\ dV$$



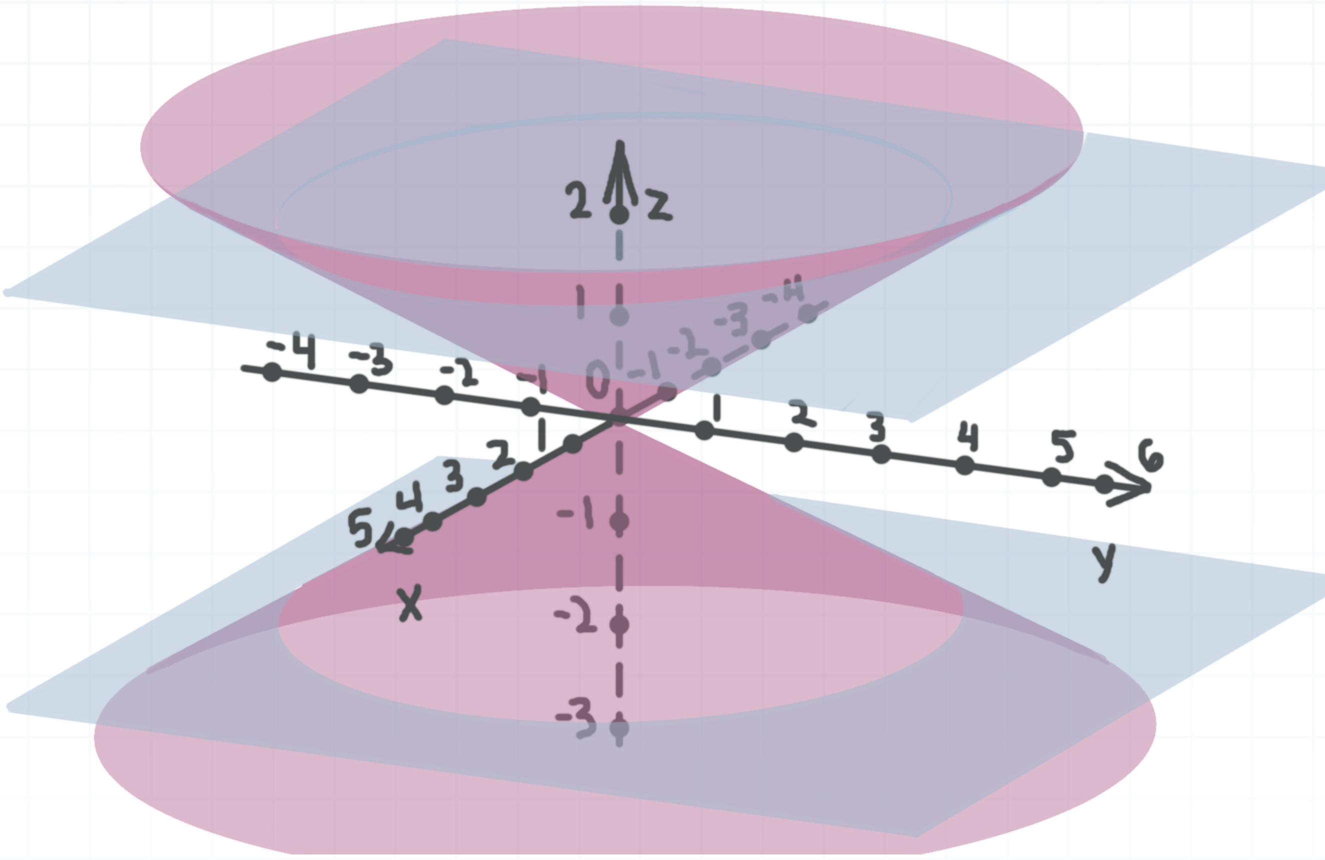
■ 3. Convert the triple integral to spherical coordinates, where E is the solid bounded by the sphere $x^2+y^2+z^2=16$ and the cone $x^2+y^2=z^2/3$, that lies in the half-space $z>0$.

$$\iiint_E \ln(x^2y^2z^2+1)\ dV$$



■ 4. If E is the solid bounded by the cone $x^2+y^2=3z^2$ and the planes $z=2$ and $z=-2$, evaluate the triple integral by changing it to spherical coordinates.

$$\iiint_E \sqrt{x^2+y^2+z^2}\ dV$$

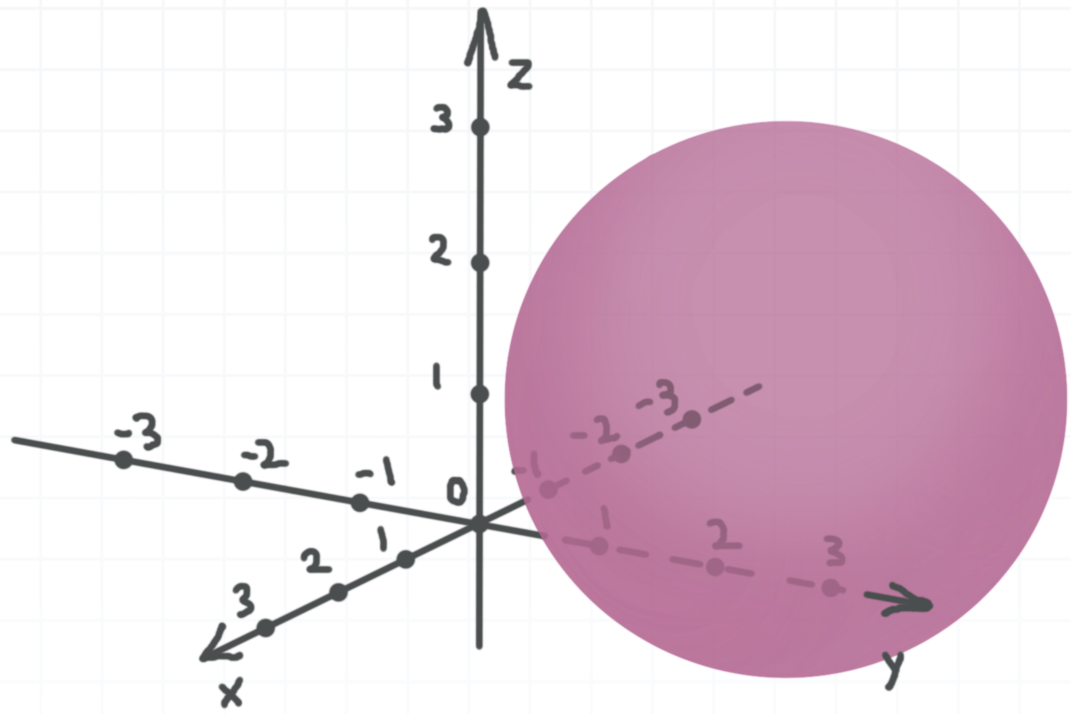


5. If E is the set of outer points of the sphere with center at the origin and radius 5, evaluate the improper triple integral by changing it to spherical coordinates.

$$\iiint_E \frac{1}{(x^2+y^2+z^2)^3}\ dV$$

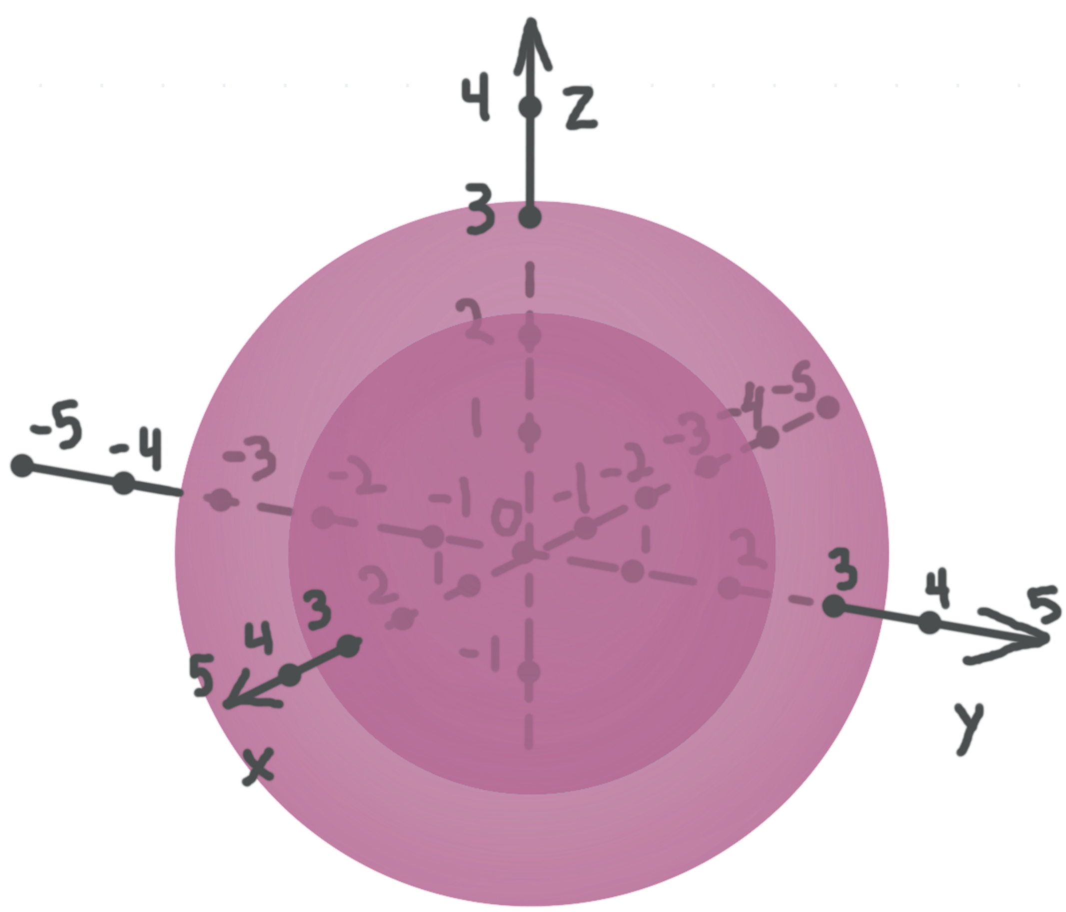
6. Evaluate the triple integral by changing it to spherical coordinates, where E is the sphere with center at the point $(-1,2,1)$ and radius 2.

$$\iiint_E 5x+3y-2z\ dV$$



■ 7. Evaluate the triple integral by changing it to spherical coordinates, if E is the set of points between the spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 4$.

$$\iiint_E 15y^2 + 4y \ dV$$

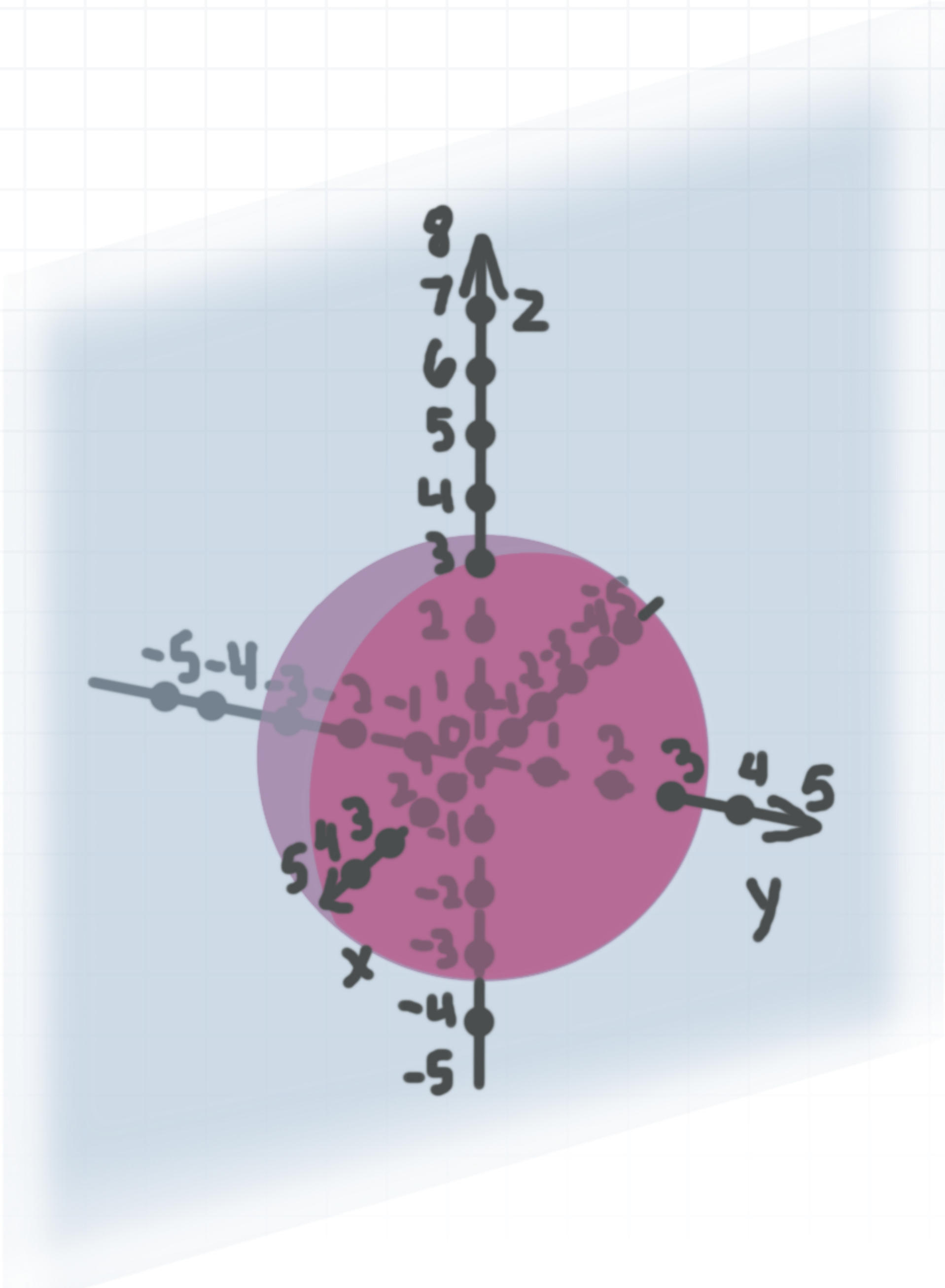


■ 8. Evaluate the improper triple integral by changing it to spherical coordinates, where E is the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

$$\iiint_E 2^{-\sqrt{(x^2+y^2+z^2)^3}}\ dV$$

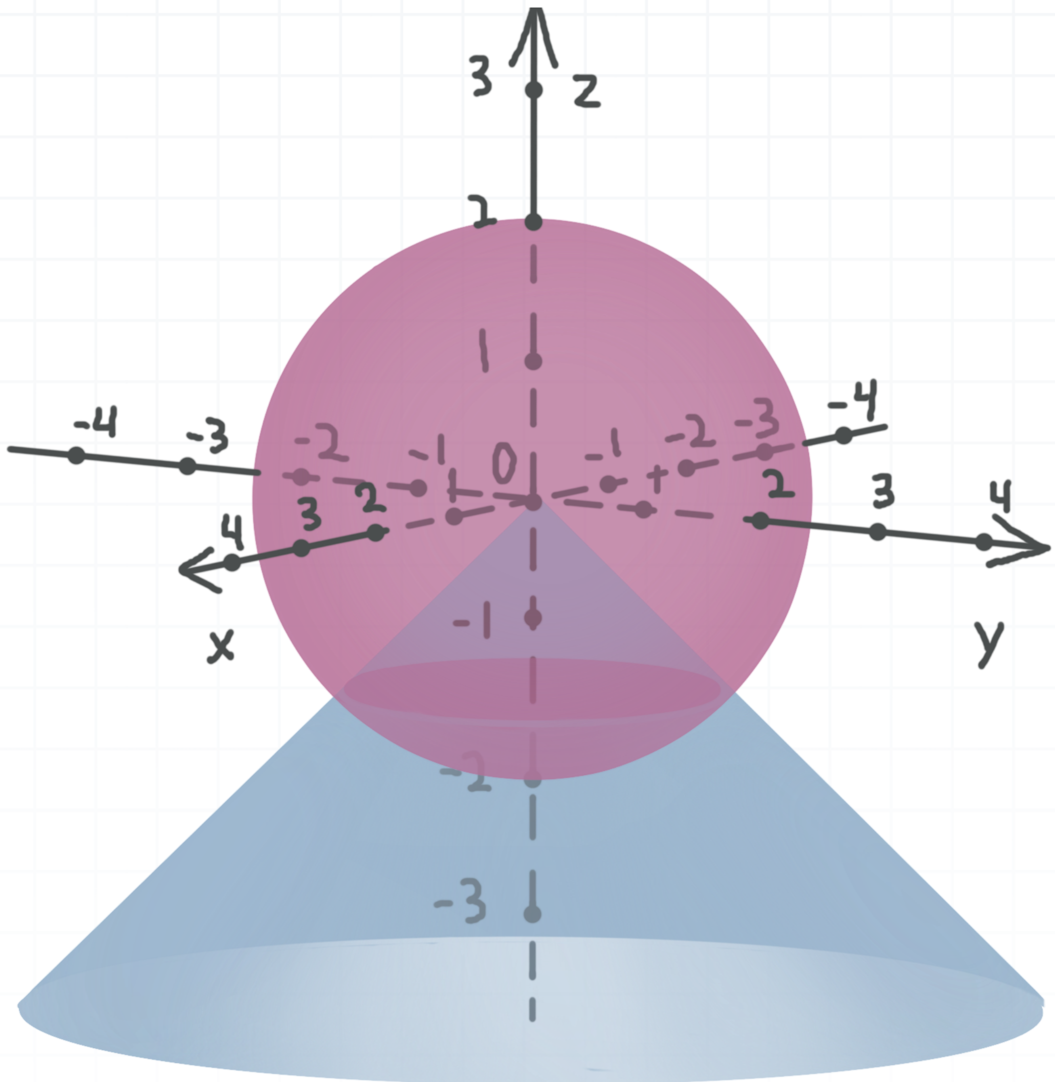
■ 9. Evaluate the triple integral by changing it to spherical coordinates, if E is the solid bounded by the sphere with center at the origin and radius 3, and the plane $x+\sqrt{3}y=0$. Consider the hemisphere that includes the points from the first octant.

$$\iiint_E x^4+y^4+z^4+2x^2y^2+2x^2z^2+2y^2z^2\ dV$$



■ 10. Evaluate the improper triple integral by changing it to spherical coordinates, where E is the region that consists of the points inside the cone $x^2 + y^2 = z^2$ and outside the sphere $x^2 + y^2 + z^2 = 4$, that lie in the half-space $z < 0$.

$$\iiint_E \frac{4z + 10}{(x^2 + y^2 + z^2)^4} \, dV$$

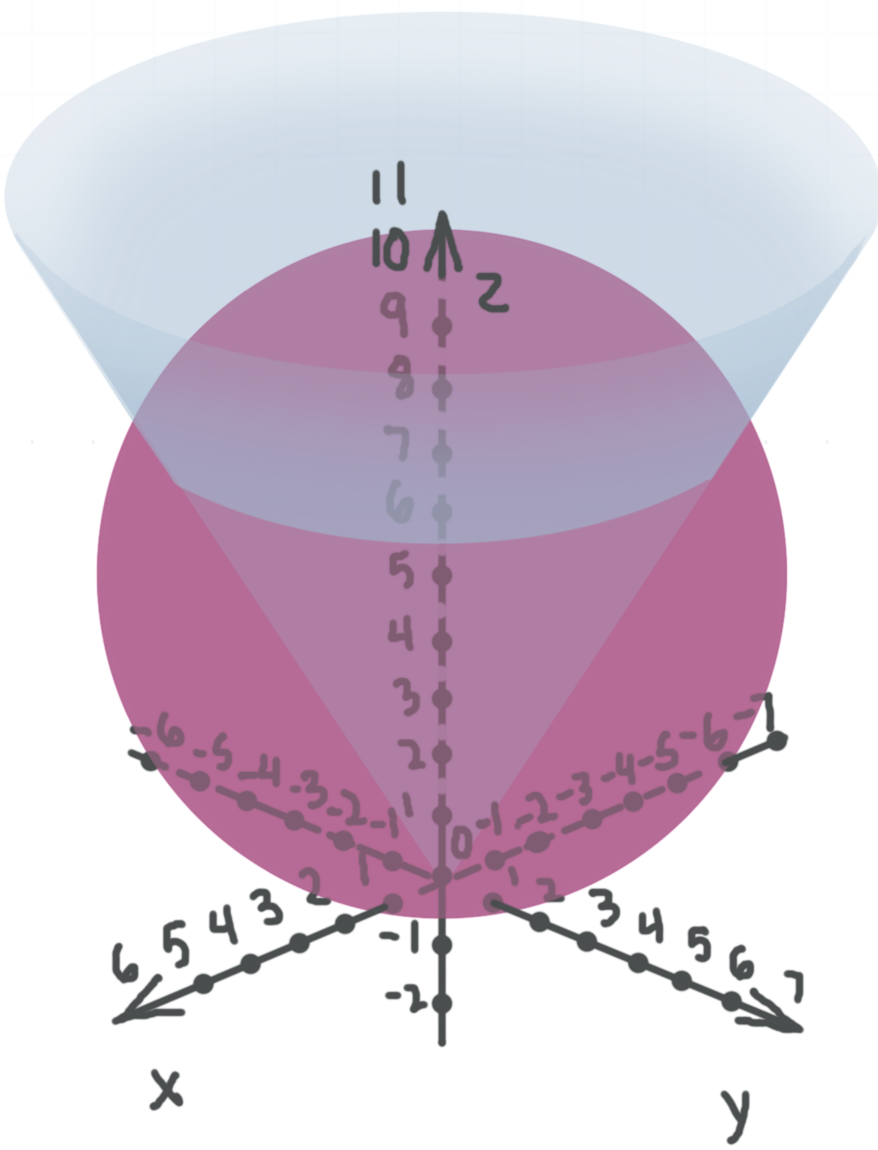


z
3
2
1
-1
-2
-3
-4
-3
-2
-1
0
-1
-2
-3
-4
4
3
2
2
3
4
x
y

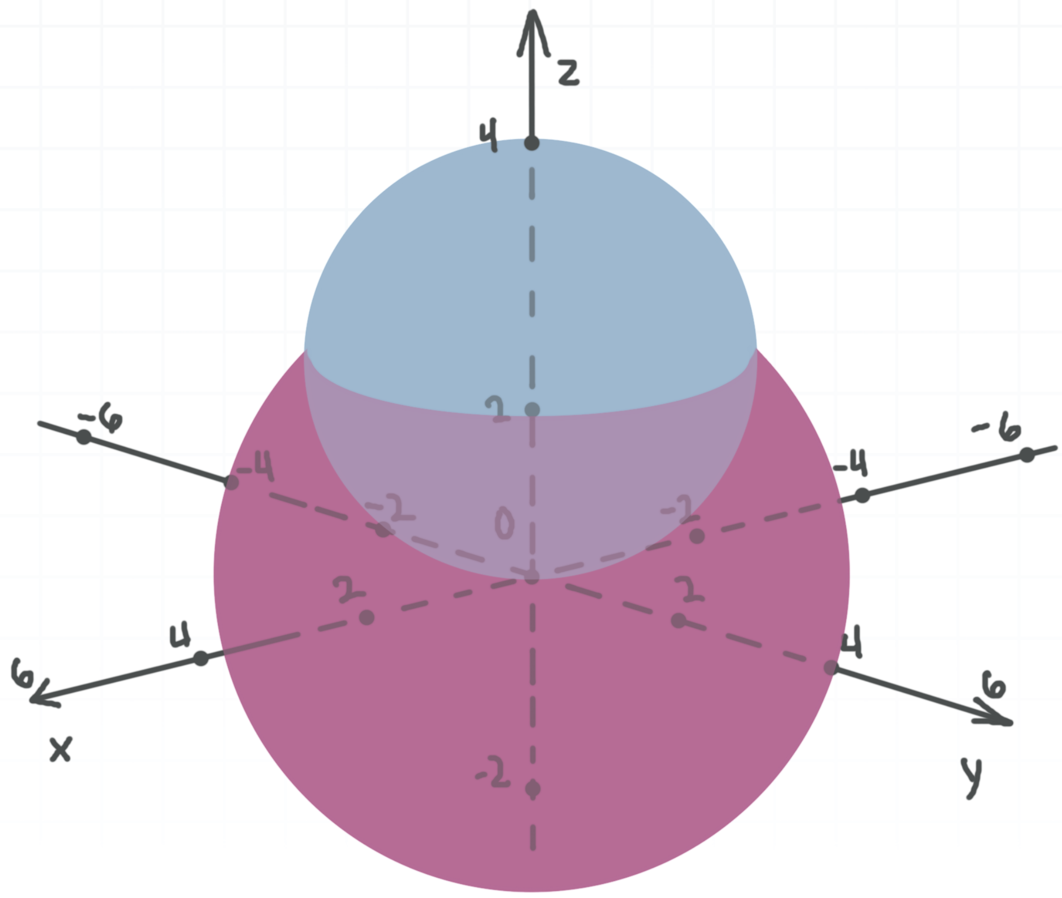
FINDING VOLUME

■ 1. Use a triple integral in spherical coordinates to find the volume of the region E that consists of the points inside the sphere $x^2 + y^2 + z^2 = 6$ and outside the cone $y^2 + z^2 = 3x^2$.

■ 2. Use a triple integral in spherical coordinates to find the volume of an ice cream cone formed by the points common to the cone $3x^2 + 3y^2 = z^2$ and the sphere $x^2 + y^2 + z^2 - 10z = 0$.



■ 3. Use a triple integral in spherical coordinates to find the volume of the three-dimensional lens common to the two spheres $x^2 + y^2 + z^2 - 8 = 0$ and $x^2 + y^2 + z^2 - 4z = 0$.



WWW.KRISTAKINGMATH.COM