Topic: Reducing equations to standard form

Question: What is the standard form and identity of the figure?

$$y^2 + z^2 = -x^2 + 1$$

Answer choices:

A $x^2 + y^2 + z^2 = 1$ is an elliptic paraboloid

B $x^2 + y^2 + z^2 = 1$ is an elliptic cone

C $x^2 + y^2 + z^2 = 1$ is a cylinder

D $x^2 + y^2 + z^2 = 1$ is a sphere

Solution: D

The first thing to notice is the presence of the 1 on the right-hand side of the equation. We always like to have that constant alone on one side of the equation by itself, so we'll rearrange the equation.

$$y^2 + z^2 = -x^2 + 1$$

$$x^2 + y^2 + z^2 = 1$$

When all three variables are positive on one side of the equation, and the constant on the other side is 1, the equation $x^2 + y^2 + z^2 = 1$ is a sphere.

Topic: Reducing equations to standard form

Question: What is the standard form and identity of the figure?

$$\frac{x}{2} + y^2 + z^2 = 0$$

Answer choices:

A $\quad y^2 + z^2 = -\frac{x}{2}$ is an elliptic cone

B $\quad y^2 + z^2 = -\frac{x}{2}$ is an elliptic paraboloid

C $\quad y^2 + z^2 = -\frac{x}{2}$ is a hyperbolic paraboloid

D $\quad y^2 + z^2 = -\frac{x}{2}$ is an ellipsoid

Solution: B

The first thing to notice is the presence of the 0 on the right-hand side of the equation. We never want to have a zero value on either side, so we'll rearrange the equation. Since we have two squared variables and one linear variable, we'll move the linear variable to the opposite side of the equation by itself.

$$\frac{x}{2} + y^2 + z^2 = 0$$

$$y^2 + z^2 = -\frac{x}{2}$$

For a standard form where two of the variables are squared, the third variable is not squared, the equation represents an elliptic paraboloid.

Topic: Reducing equations to standard form

Question: What is the standard form and identity of the figure?

$$x^2 + 4x - y + 4 - z^2 + 2z = 0$$

Answer choices:

A $(x+2)^2 + (z-1)^2 = y - 1$ is a hyperbolic paraboloid centered at $(-2,1,1)$

B $(x+2)^2 - (z-1)^2 = y - 1$ is an elliptic paraboloid centered at $(-2,1,1)$

C $(x+2)^2 - (z-1)^2 = y - 1$ is a hyperbolic paraboloid centered at $(-2,1,1)$

D $(x+2)^2 + (z-1)^2 = y - 1$ is an elliptic paraboloid centered at $(-2,1,1)$

Solution: C

The first thing to notice is that two of the variables have both squared and non-squared appearances. This indicates that we'll need to complete the square for these variables.

$$\left(x^2+4x\right)-y+4+\left(-z^2+2z\right)=0$$

$$\left(x^2+4x\right)-y+4-\left(z^2-2z\right)=0$$

Complete the squares.

$$\left[x^2+4x+\left(\frac{4}{2}\right)^2\right]-y+4-\left[z^2-2z+\left(\frac{-2}{2}\right)^2\right]-\left(\frac{4}{2}\right)^2+\left(\frac{-2}{2}\right)^2=0$$

$$\left(x^2+4x+4\right)-y+4-\left(z^2-2z+1\right)-4+1=0$$

$$(x+2)^2-y+4-(z-1)^2-4+1=0$$

$$(x+2)^2-(z-1)^2-y+1=0$$

Looking at our reference sheet, we can see that the two squared elements should stay together and the non-squared variable and the number element should be moved to the other side of the equation.

$$(x+2)^2-(z-1)^2=y-1$$

An equation in this form is a hyperbolic paraboloid with center $(-2,1,1)$.