Domain of a vector function

To find the domain of a vector function,

$$r(t) = \langle a, b, c \rangle$$

or

$$r(t) = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

we'll need to find the domain of the individual components a, b and c. Then the domain of the vector function is the values for which the domains of a, b and c overlap.

Example

Find the domain of the vector function.

$$r(t) = \left\langle \ln(t-1), t^2, \frac{1}{9-t^2} \right\rangle$$

We need to find the domain of each component individually.

The domain of $\ln(t-1)$ is

$$t - 1 > 0$$

$$t > 1$$

The domain of t^2 is

all real numbers

The domain of $\dfrac{1}{9-t^2}$ is

$$9 - t^2 \neq 0$$

$$-t^2 \neq -9$$

$$t^2 \neq 9$$

$$t \neq \pm 3$$

The first component restricts the domain of the entire vector function to $t > 1$. The second component doesn't restrict the domain at all. The third component tells us that the domain can't include $t = \pm 3$. $t = -3$ is outside of the domain $t > 1$, so we can ignore it. But $t = 3$ is inside the domain $t > 1$, so we have to include it.

The domain of the vector function is

$$t > 1 \text{ but } t \neq 3$$