Calculus 3 Workbook

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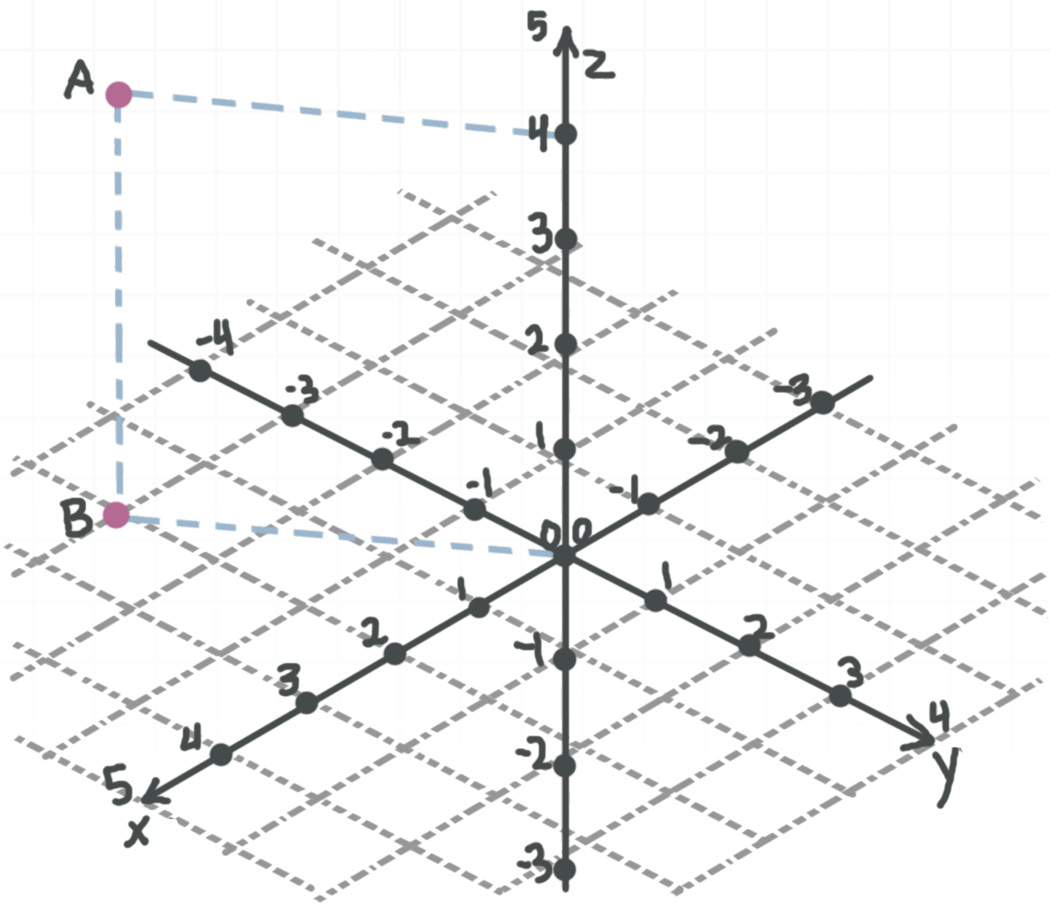
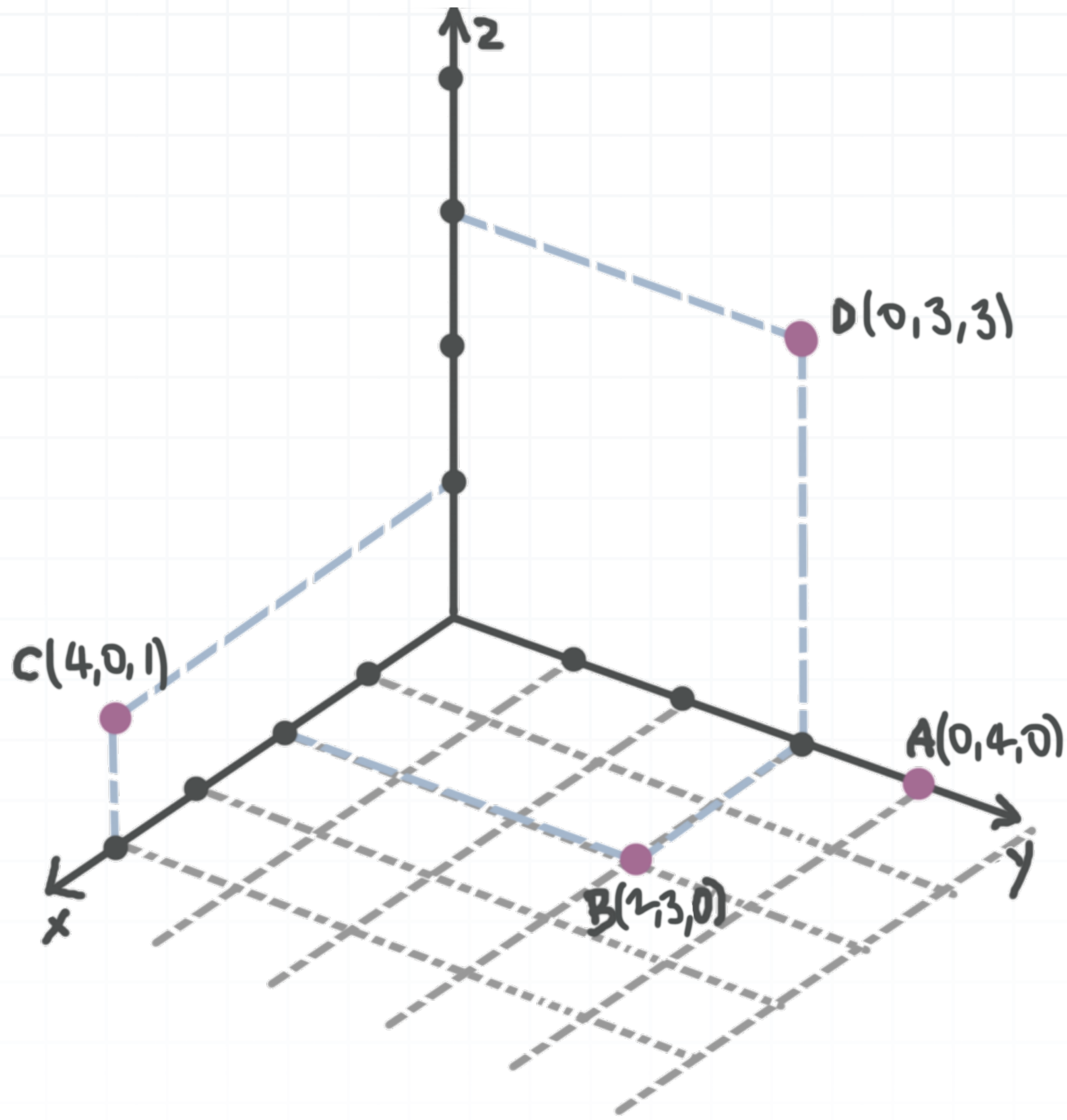
MATH

PLOTTING POINTS IN THREE DIMENSIONS

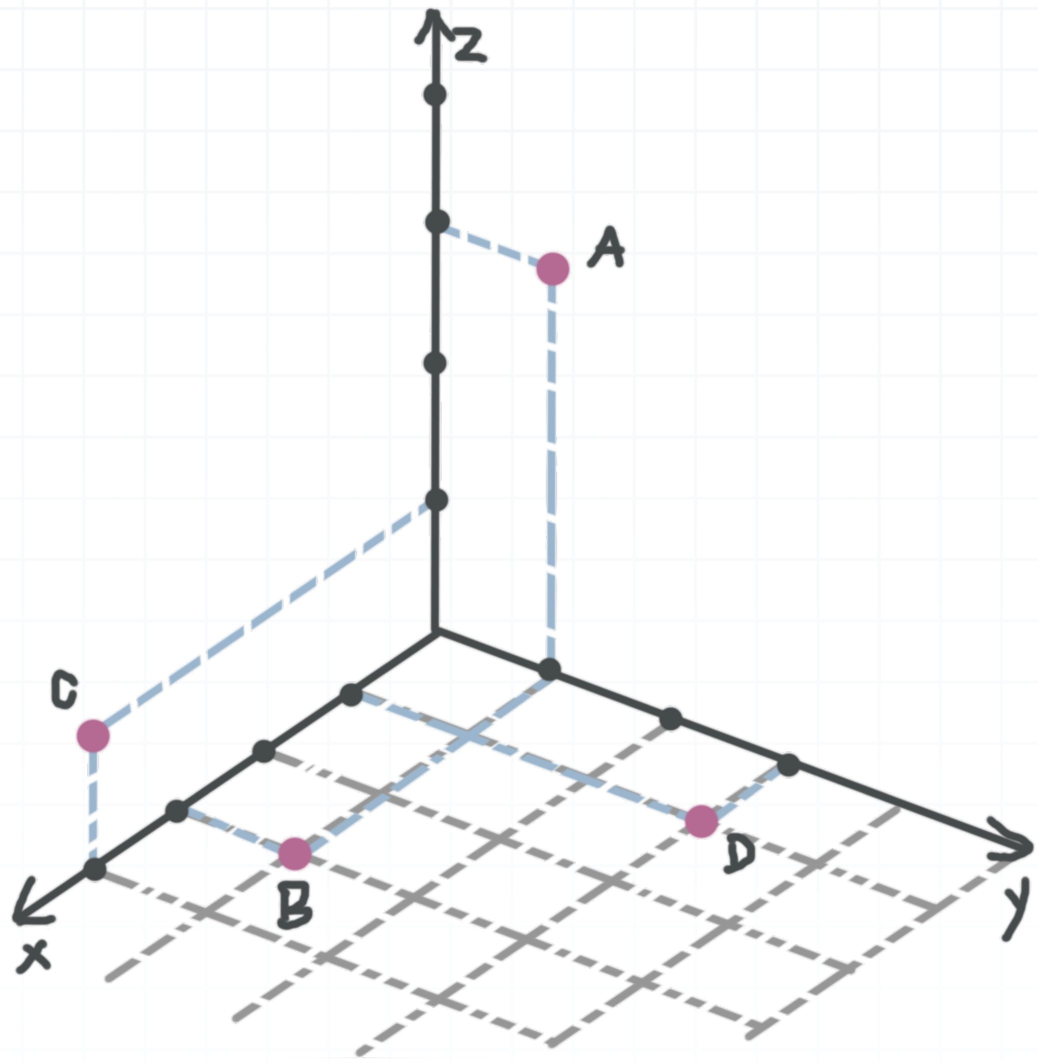
■ 1. What are the coordinates of point A?



■ 2. Which of the points A, B, C, and D lie on the xz-plane?



■ 3. Which of the points A, B, C, and D has coordinates $(3,1,0)$?


z
A
C
D
B
x
y

DISTANCE BETWEEN POINTS IN THREE DIMENSIONS

■ 1. Find the perimeter P of the triangle ABC given $A(1,0,0)$, $B(2,3,0)$, and $C(3,3,-3)$.

■ 2. Given $A(-1,0,0)$, $B(-2,0,2)$, and $C(0,1,0)$, find the measure of angle BAC in degrees using the law of cosines:

$$\cos(BAC)=\frac{AB^2+AC^2-BC^2}{2\cdot AB\cdot AC}$$

■ 3. Find the point on the x-axis that's equidistant from $A(-1,1,0)$ and $B(-2,1,-1)$.

CENTER, RADIUS, AND EQUATION OF THE SPHERE

■ 1. Find the equation of a sphere with center $(1,3,-2)$ and y-intercept 1.

■ 2. Of the points $A(-4,-1,7)$, $B(-5,1,5)$, $C(-6,-6,5)$, and $D(-7,0,3)$, which one does not lie in the interior of the sphere?

$$(x+5)^2+(y+3)^2+(z-4)^2=16$$

■ 3. The endpoints of the diameter of a sphere are $A(2,4,-3)$ and $B(6,0,-1)$. Find the equation of this sphere.

DESCRIBING A REGION IN THREE DIMENSIONAL SPACE

■ 1. Describe the surface in three-dimensional space.

$$x^2 + 2x + z^2 = 0$$

■ 2. Describe the surface in three-dimensional space.

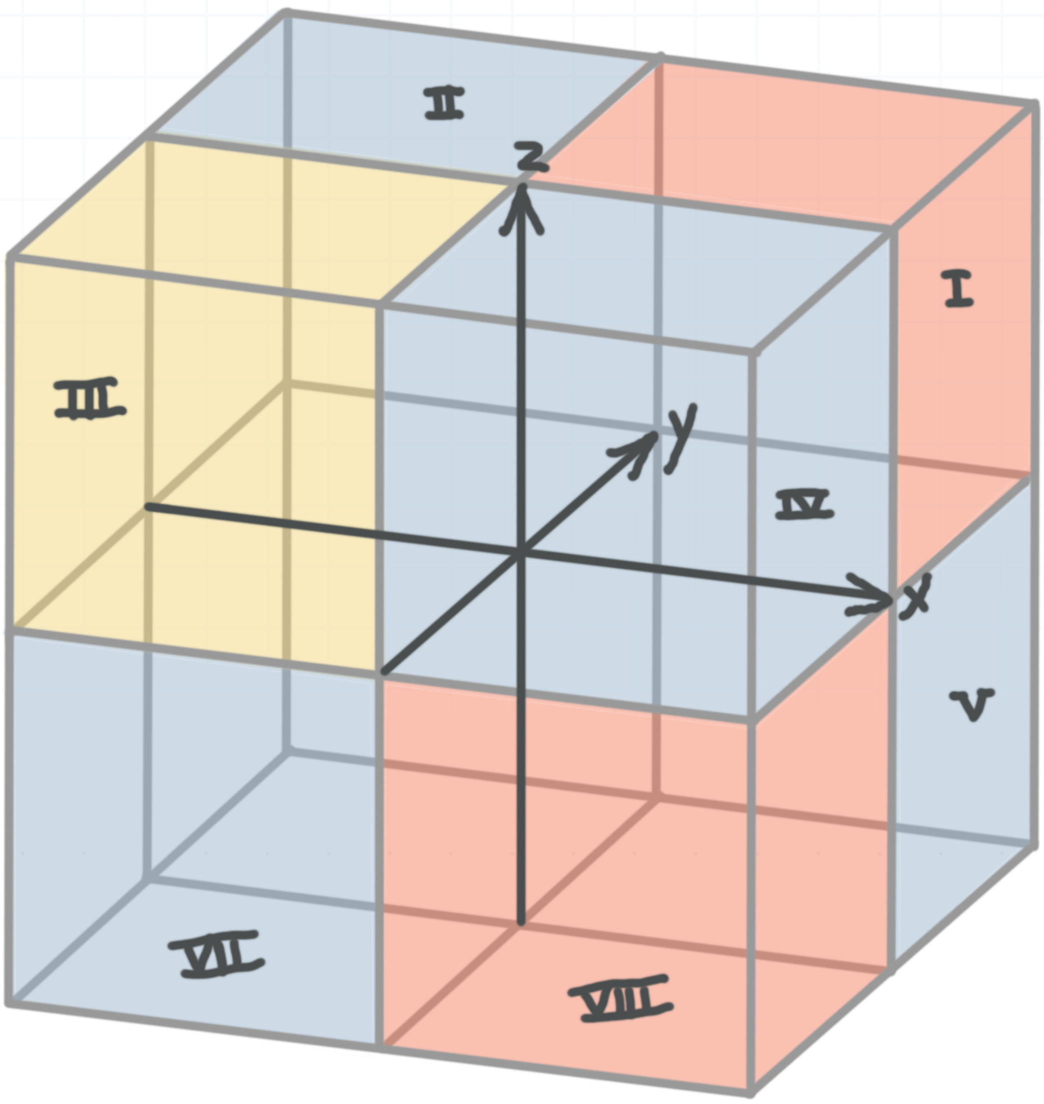
$$z = 7$$

■ 3. Describe the surface in three-dimensional space.

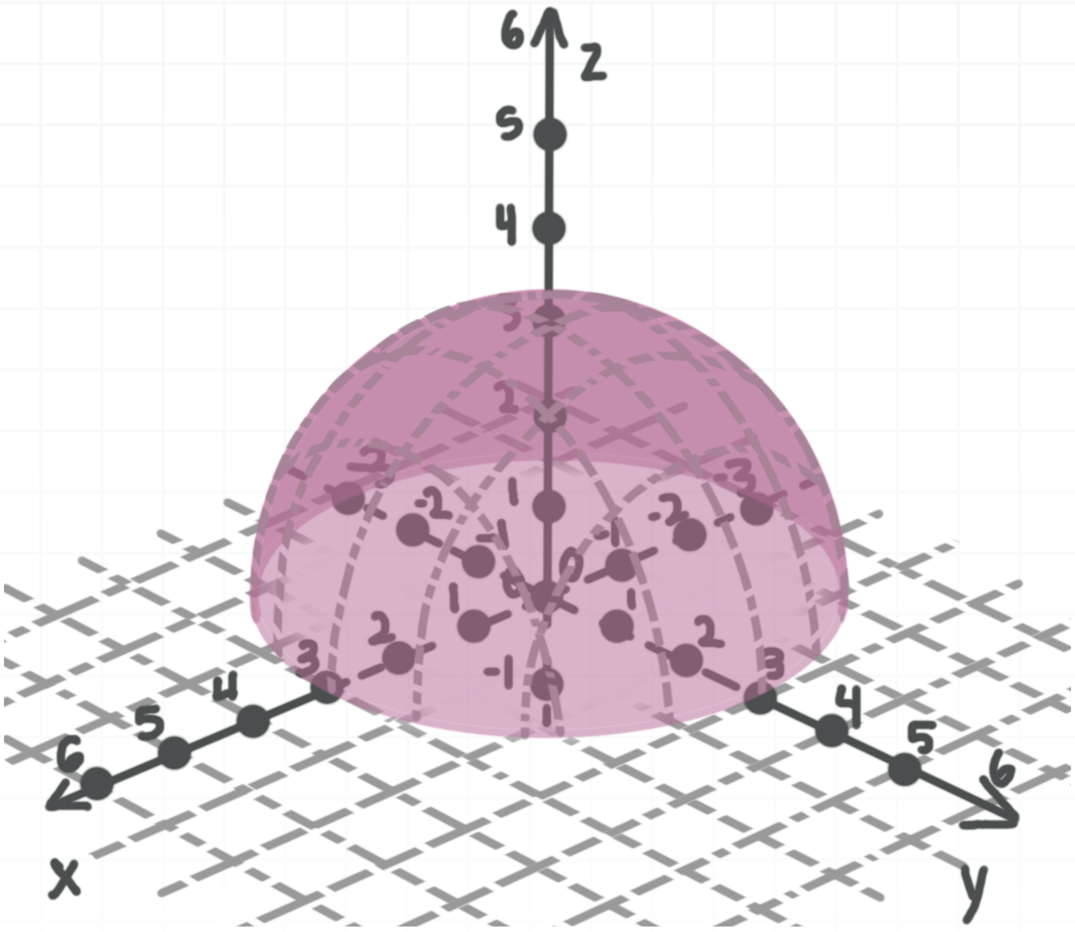
$$xy = 0$$

USING INEQUALITIES TO DESCRIBE THE REGION

■ 1. What set of inequalities describes Octant III? Remember that an "octant" is one of the eight spaces that make up the three-dimensional coordinate system.



■ 2. What set of inequalities describes the region consisting of all points inside the hemisphere, if the base of the sphere is centered at $(0,0,0)$?



■ 3. What set of inequalities describes the region consisting of all points which lie at most 5 units from the yz-plane?

SKETCHING GRAPHS OF MULTIVARIABLE FUNCTIONS

■ 1. Find the range of the function.

$$f(x, y) = x^2 + 2y^2 - 3$$

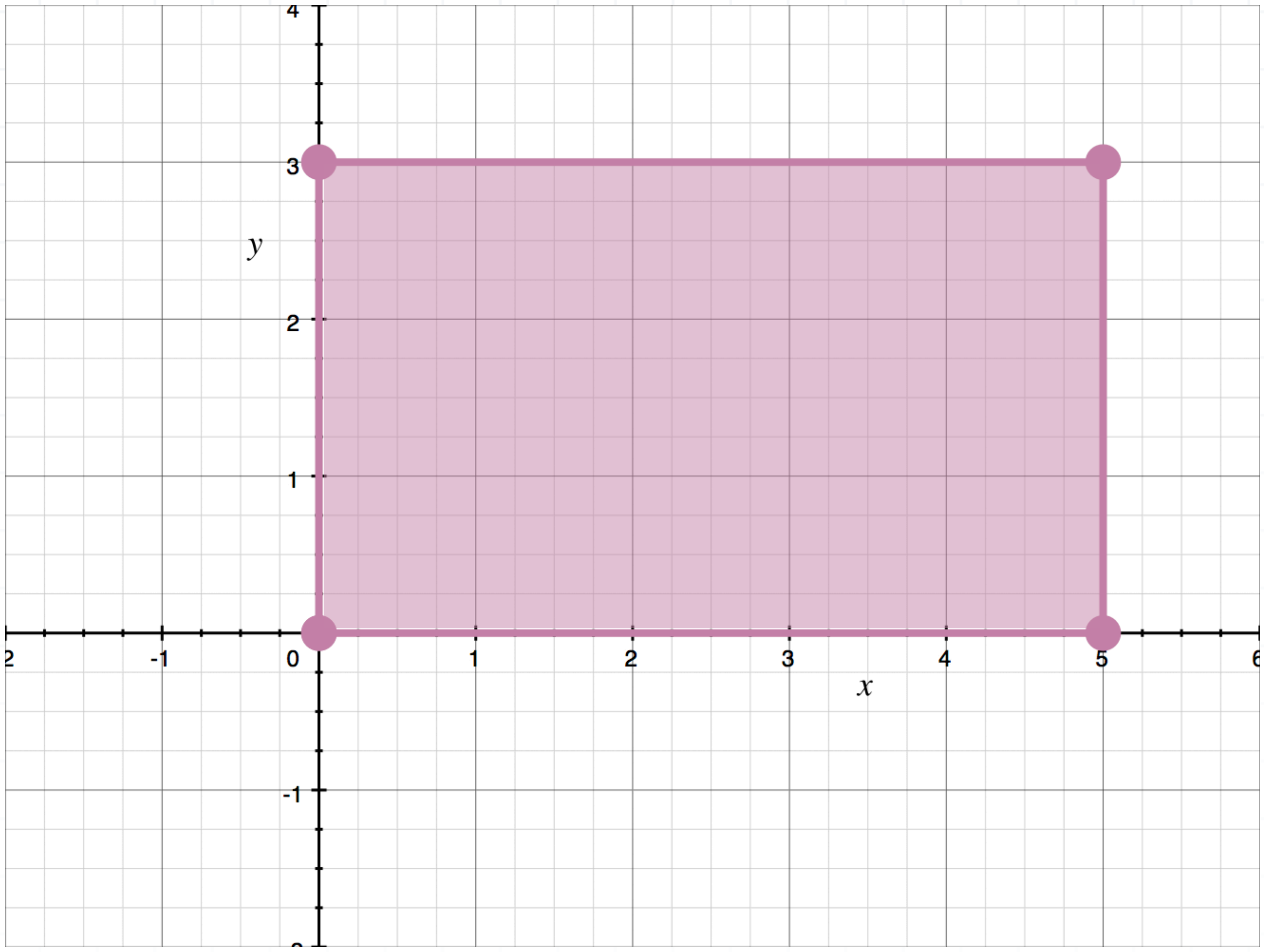
■ 2. Which function's domain is given by the graph, if the left and right sides of the rectangle are included in the domain, but the top and bottom sides are not?

A $\quad f(x, y) = 3y\sqrt{3x - x^2} + 4x\ln(5y - y^2)$

B $\quad f(x, y) = 3y\sqrt{5x - x^2} + 4x\ln(3y - y^2)$

C $\quad f(x, y) = 3x\sqrt{x^2 - 3x} + 4y\ln(y^2 - 5y)$

D $\quad f(x, y) = 3x\sqrt{x - 5x^2} + 4y\ln(y - 3y^2)$



■ 3. Find the value of the constant a for which $(2, -1, 0)$ lies on the graph of the function.

$$f(x, y) = x^2 + 2axy + y^2 - 1$$

■ 4. Find the intersection point of the function and the y-axis.

$$f(x, y) = \sqrt{x^2 - 5y + 15}$$

■ 5. Write the equation of the function $f(x, y)$ shifted in a positive direction along the x-axis by 2 units.

$$f(x,y) = x^2y^2 - 2xy - 4y^2 - 4y + 4x$$

■ 6. Which function A, B, C, or D is a reflection of $f(x,y)$ over the xz-plane? Hint: Use the even identity $\cos(-t) = \cos t$ to simplify.

$$f(x,y) = \cos(x^2 - y^2 + 2xy)$$

$$A(x,y) = \cos(-x^2 + y^2 + 2xy)$$

$$B(x,y) = \cos(x^2 - y^2 + 2xy)$$

$$C(x,y) = \cos(x^2 + y^2 - 2xy)$$

$$D(x,y) = \cos(-x^2 - y^2 - 2xy)$$

■ 7. Find the absolute maximum of the function.

$$f(x,y) = 5 - 2x^2 - 7y^2$$

SKETCHING LEVEL CURVES OF MULTIVARIABLE FUNCTIONS

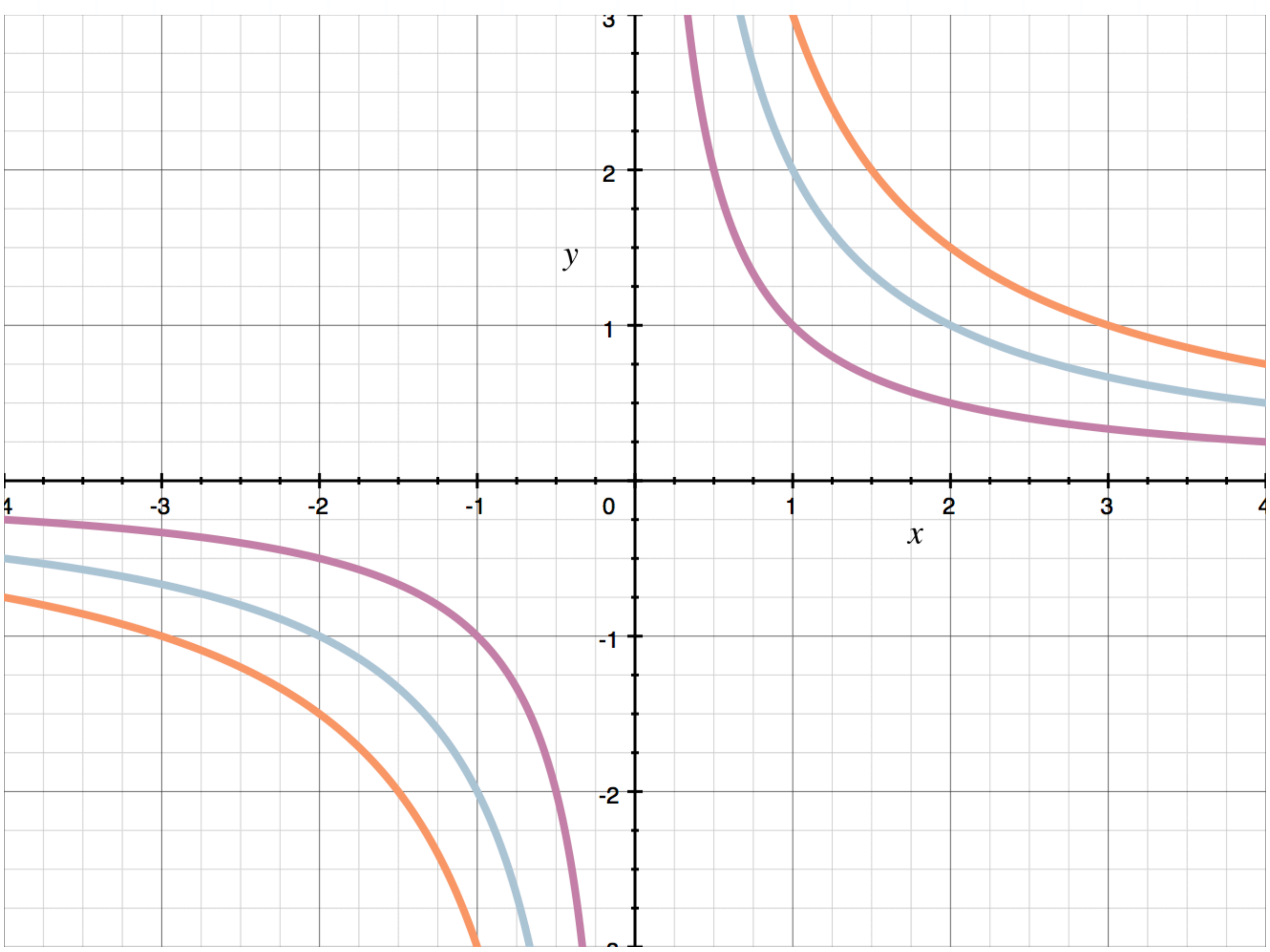
■ 1. Find the level curve of $f(x, y)$ when $z = 5$.

$$f(x, y) = x^2 - 2xy + 6y - 4$$

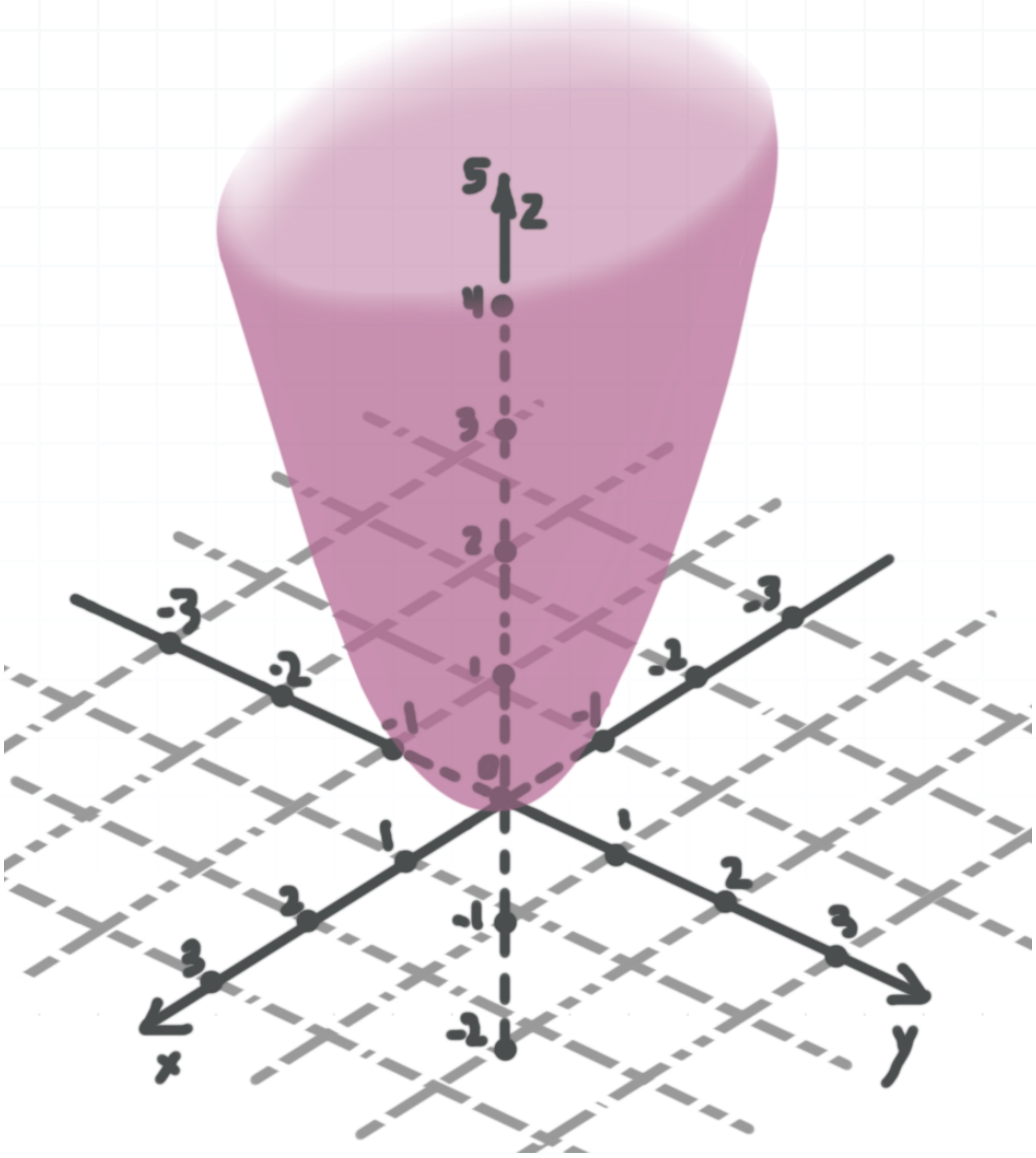
■ 2. Find the level curve of $f(x, y)$ which passes through $(0,1,z)$.

$$f(x, y) = 2x^2 - y + 2$$

■ 3. The graph shows level curves of $f(x, y) = 4xy$. Find the value of z that corresponds to the light blue curve.



■ 4. Think about the shape of the level curves of the graph of the elliptic paraboloid. Are they lines, ellipses, parabolas, or hyperbolas?



MATCHING THE FUNCTION WITH THE GRAPH AND LEVEL CURVES

■ 1. Which statement is true for the graph of the function?

$$x^2 - 2y^2 + z^2 - 8y - 6z = 0$$

A The graph is the hyperboloid centered at $(0,2,-3)$.

B The graph is the hyperboloid centered at $(0,-2,3)$.

C The graph is the ellipsoid centered at $(0,2,-3)$.

D The graph is the ellipsoid centered at $(0,-2,3)$.

■ 2. Find the equation of ellipsoid centered at $(2,0,2)$ that has the level curve $(x-2)^2 + 4y^2 = 0.75$ for $z = 1.5$.

■ 3. Which of the surfaces has the same level curves for any z?

A The plane $2x + 3y + z = 1$

B The ellipsoid $x^2 + 2y^2 + 4z^2 - 4x - 2y = 1$

C The cylinder $2x^2 + y^2 - 5x + 7y = 1$

D The elliptic cone $x^2 + 3y^2 - z^2 = 0$

VECTOR, PARAMETRIC, AND SYMMETRIC EQUATIONS OF A LINE

■ 1. Find the vector equation of the line that passes through $A(1,0,-2)$ and is parallel to the symmetric equation.

$$\frac{x-3}{2}=-\frac{y}{2}=z+1$$

■ 2. Find the parametric equation of the line that passes through $A(2,3,-2)$ and $B(0,-1,5)$.

■ 3. Which line passes through $A(1,0,-4)$?

A $x=1-2t,\ y=-5t,\ z=3-4t$

B $x=2-5t,\ y=5t,\ z=2-4t$

C $x=6-t,\ y=5-t,\ z=6-2t$

D $x=6+t,\ y=5+t,\ z=6+t$

PARALLEL, INTERSECTING, SKEW AND PERPENDICULAR LINES

■ 1. For $A(1,0,-1)$, $B(1,3,0)$, $C(0,0,2)$, and $D(-1,-2,3)$, are lines AB and CD parallel, intersecting, skew, or perpendicular?

■ 2. Find the line L_2 that passes through $A(1,0,1)$ and is perpendicular to L_1.

$$L_1: \quad x = 2 - 2t,\ y = t,\ z = 3 + t$$

■ 3. Which line is perpendicular to L_1?

$$L_1: \quad x = 2t,\ y = 21 - t,\ z = 6 - t$$

A $\quad x = 2 - 3t,\ y = 7 + 5t,\ z = 2t$

B $\quad x = 2 + 3t,\ y = 7 + 5t,\ z = t$

C $\quad x = 2 + 3t,\ y = 7 - 5t,\ z = -t$

D $\quad x = 2 - 3t,\ y = 5 - 5t,\ z = -t$

EQUATION OF A PLANE

■ 1. Find the equation of a plane that passes through $A(1,4,-2)$ and is perpendicular to the line.

$$r = \langle 1,3,3\rangle + t\langle -2,3,1\rangle$$

■ 2. Find the equation of a plane that passes through $A(1,4,-2)$ and the line given by the parametric equation.

$$x = 2 - 4t,\ y = 3t,\ z = 1 + t$$

■ 3. Which of the lines lie in the plane $2x - y + 3z = 1$? Choose as many of the answer choices as are correct.

A $\quad x = 1 + 2t,\ y = 1 - 3t,\ z = -5t$

B $\quad x = 1 - 2t,\ y = 1 - 5t,\ z = 4t$

C $\quad x = 1 + 4t,\ y = 1 + t,\ z = -3t$

D $\quad x = 1 + 2t,\ y = 1 + t,\ z = -t$

■ 4. Find the equation of a plane that passes through the intersecting lines L_1 and L_2.

$$L_1: \quad \frac{x-2}{2} = \frac{y+3}{3} = \frac{z}{2}$$

$$L_2: \quad \frac{x-2}{-1} = \frac{y+3}{2} = \frac{z}{5}$$

■ 5. Find the equation of a plane that passes through the parallel lines L_1 and L_2.

$$L_1: \quad r = \langle 1,2,-4\rangle + t\langle 0,1,-1\rangle$$

$$L_2: \quad r = \langle 2,-3,0\rangle + t\langle 0,1,-1\rangle$$

INTERSECTION OF A LINE AND A PLANE

■ 1. Find the x-, y-, and z-intercepts of the plane.

$$2x - 3y + z = 6$$

■ 2. Find the intersection of AB and the plane $x + 2y + 4z = 12$, where A and B are the points $A(0,1,-2)$ and $B(-1,2,0)$, or determine that the line segment and the plane do no intersect.

■ 3. Find the value of the constant p for which the line doesn't intersect the plane.

The line $\dfrac{x+1}{2} = \dfrac{y}{3} = z - 1$

The plane $px + 2y + z = 4$

PARALLEL, PERPENDICULAR, AND ANGLE BETWEEN PLANES

■ 1. Find the equation of the plane that passes through the points $A(1,0,-1)$ and $B(0,1,-1)$ and is perpendicular to the plane $x+2y+3z=6$.

■ 2. Find the equation of the plane that passes through $A(3,2,-4)$ and is parallel to the plane $-x+3y-2z=4$.

■ 3. Find the equation of the plane a that passes through the point $A(1,-2,-3)$ and form equal angles with all of the coordinate planes, xy, yz, and xz.

PARAMETRIC EQUATIONS FOR THE LINE OF INTERSECTION OF TWO PLANES

■ 1. Find the parametric equations for the line of intersection of the planes with normal vectors $a = \langle 2,0,-1 \rangle$ and $b = \langle 1,2,-3 \rangle$, and that have the common point $A(1,2,2)$.

■ 2. Find the parametric equations for the line of intersection of the plane $2x - 3y - 4z = 2$ with xz-plane.

■ 3. Find the equations of a plane a that's perpendicular to the plane b, which is $x - 3y + z = 2$, and intersects b along the line given by the parametric equation.

$$x = 2t$$

$$y = 1 + t$$

$$z = 2 - t$$

SYMMETRIC EQUATIONS FOR THE LINE OF INTERSECTION OF TWO PLANES

■ 1. Find the symmetric equations for the line of intersection of the plane a, which is $2x - y + 3z = 12$, and the plane a' that's symmetric to the plane a with respect to the xz-plane.

■ 2. Find the symmetric equations for the line of intersection of the plane $6x - 5y + z = 10$ with yz-plane.

■ 3. Find the equations of a plane a that's perpendicular to the plane b, which is $2x - y - z = 3$, and intersects b along the line

$$\frac{x-1}{3} = \frac{y+2}{2} = \frac{z}{2}$$

DISTANCE BETWEEN A POINT AND A LINE

■ 1. Determine the length of the height of triangle ABC, that's perpendicular to BC, if $A(2,0,-1)$, $B(4,5,2)$, and $C(4,3,0)$.

■ 2. Find the sum of distances from $A(3,3,-1)$ to all of the coordinate axes, x, y, and z.

■ 3. Find the distance between $A(0,-2,1)$, and the line.

$$\frac{x+1}{2}=\frac{y-2}{-1}=\frac{z}{2}$$

DISTANCE BETWEEN A POINT AND A PLANE

■ 1. Determine the length of the height of tetrahedron $ABCD$, that's perpendicular to the plane BCD, if $A(1,0,-1)$, $B(0,1,0)$, $C(2,3,4)$, and $D(2,2,2)$.

■ 2. Find the sum of distances from the point $A(2,3,-5)$ to all of the coordinate planes, xy, yz, and xz.

■ 3. Find the points on the line L_1 that lie at a distance of 6 from plane a.

$$L_1: \quad x = 1 + t,\ y = 2t,\ z = 3 - 2t$$

$$a: \quad x + 2y - 2z = 4$$

DISTANCE BETWEEN PARALLEL PLANES

■ 1. Determine the length of the height of triangular prism $ABCA_1B_1C_1$ that's perpendicular to the plane ABC if $A(2,0,1)$, $B(1,0,0)$, $C(2,2,3)$, $A_1(3,2,5)$, $B_1(2,2,4)$, and $C_1(3,4,7)$.

■ 2. Find the equations of the two planes that are parallel to the given plane and that lie at a distance of 2 from it.

$$2x-2y-z=3$$

■ 3. Find the equations of the two parallel planes a and b that pass through $A(2,-6,3)$ and $B(7,10,11)$ respectively, and have the largest possible distance between them.

REDUCING EQUATIONS TO STANDARD FORM

1. What is the standard form and identity of the quadratic surface?

$$16x^2 + 49y^2 + 784z^2 + 128x - 294y - 87 = 0$$

2. What is the standard form and identity of the quadratic surface?

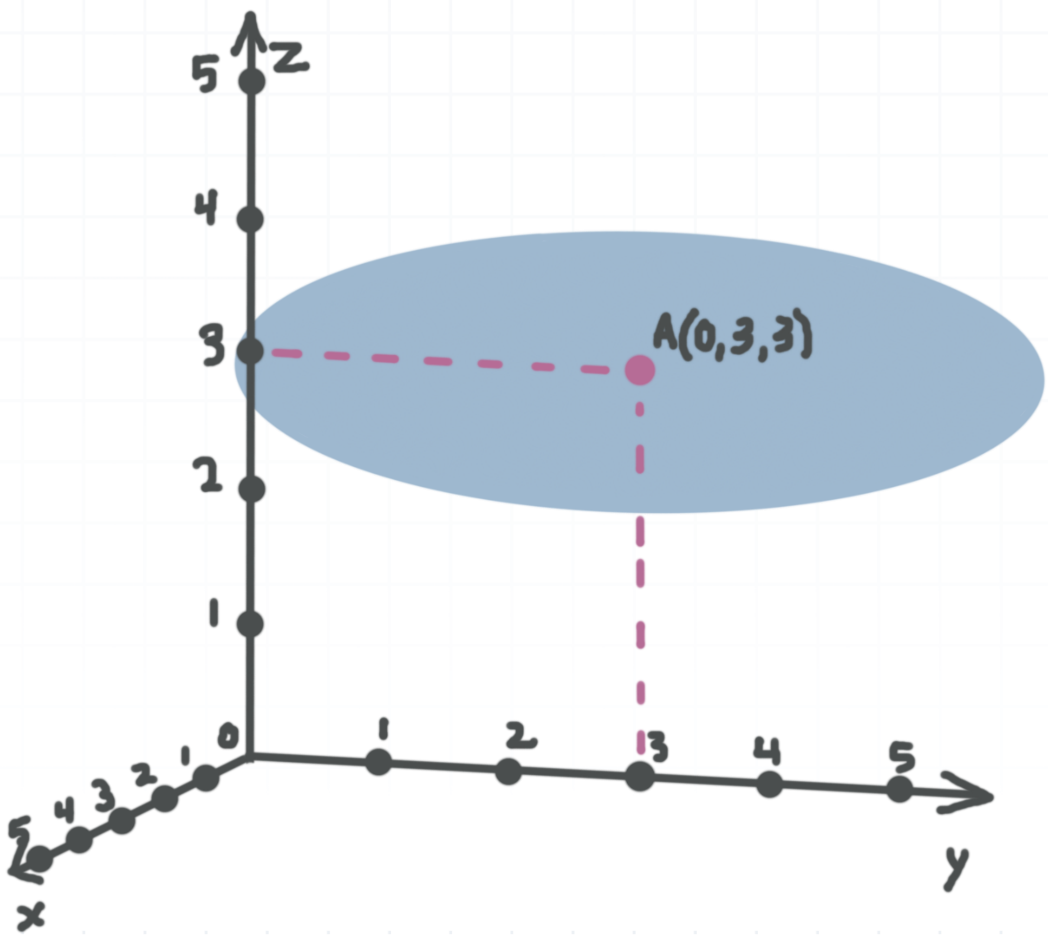
$$25y^2 + 9z^2 - 50y + -36z - 225x - 839 = 0$$

3. What is the standard form and identity of the quadratic surface?

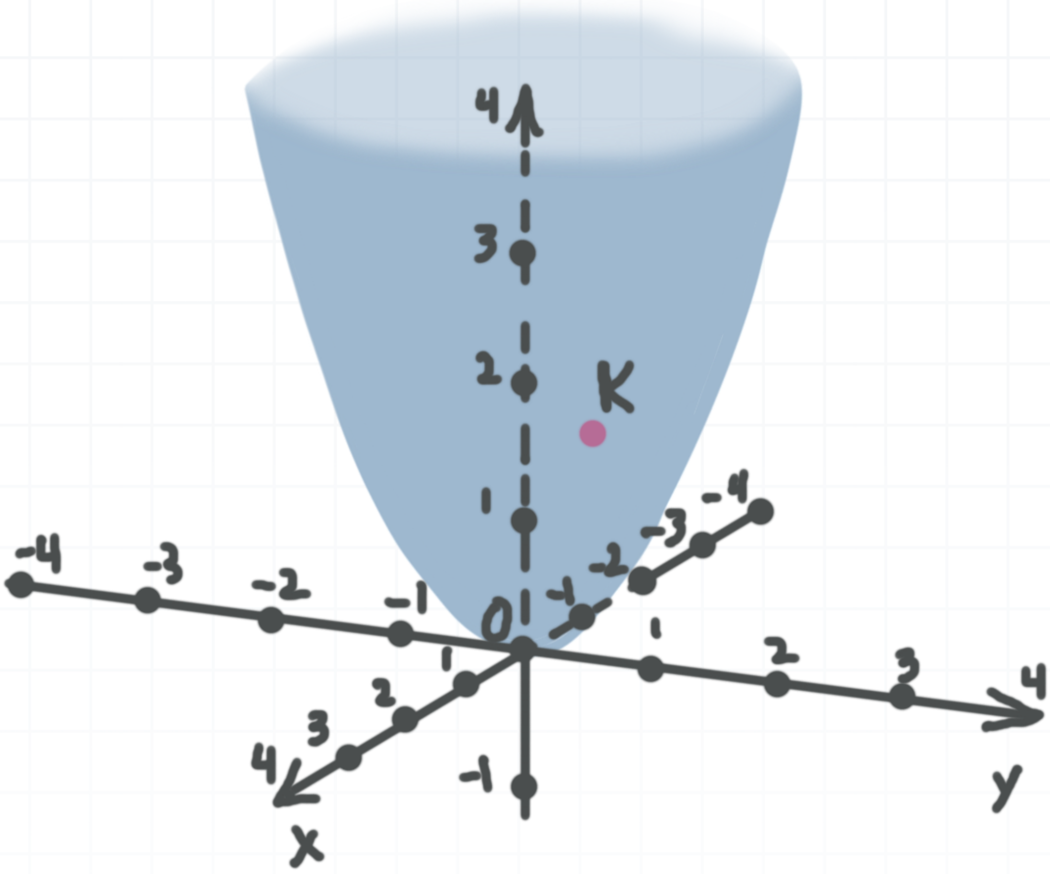
$$9x^2 - 9y^2 + 4z^2 + 18x - 36y - 8z = 23$$

SKETCHING THE SURFACE

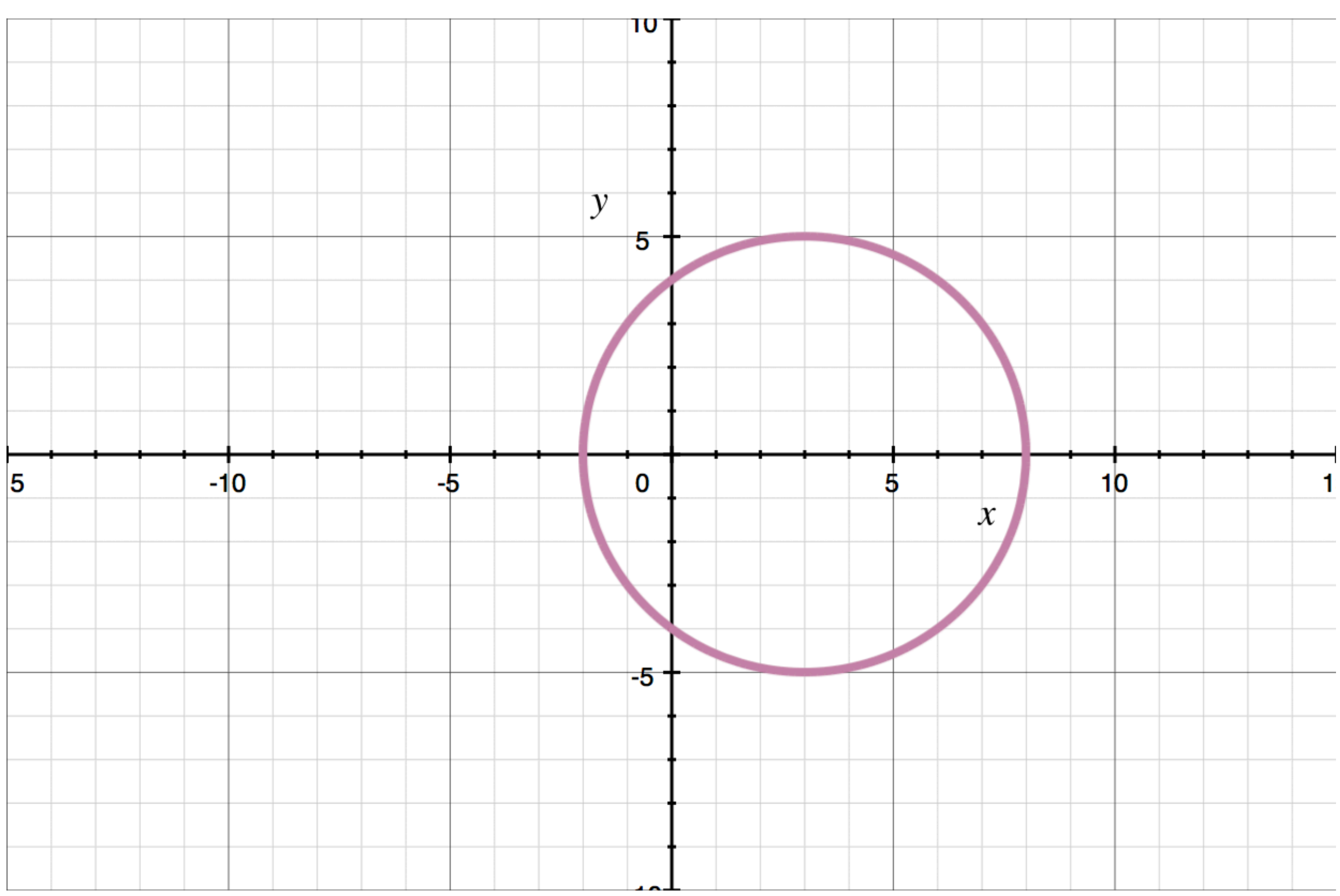
■ 1. Find the equation of the surface if its x- and z- principal axes have length 4 and 2 respectively.



■ 2. Find the equation of the circular paraboloid that passes through $K(1,1,2)$.

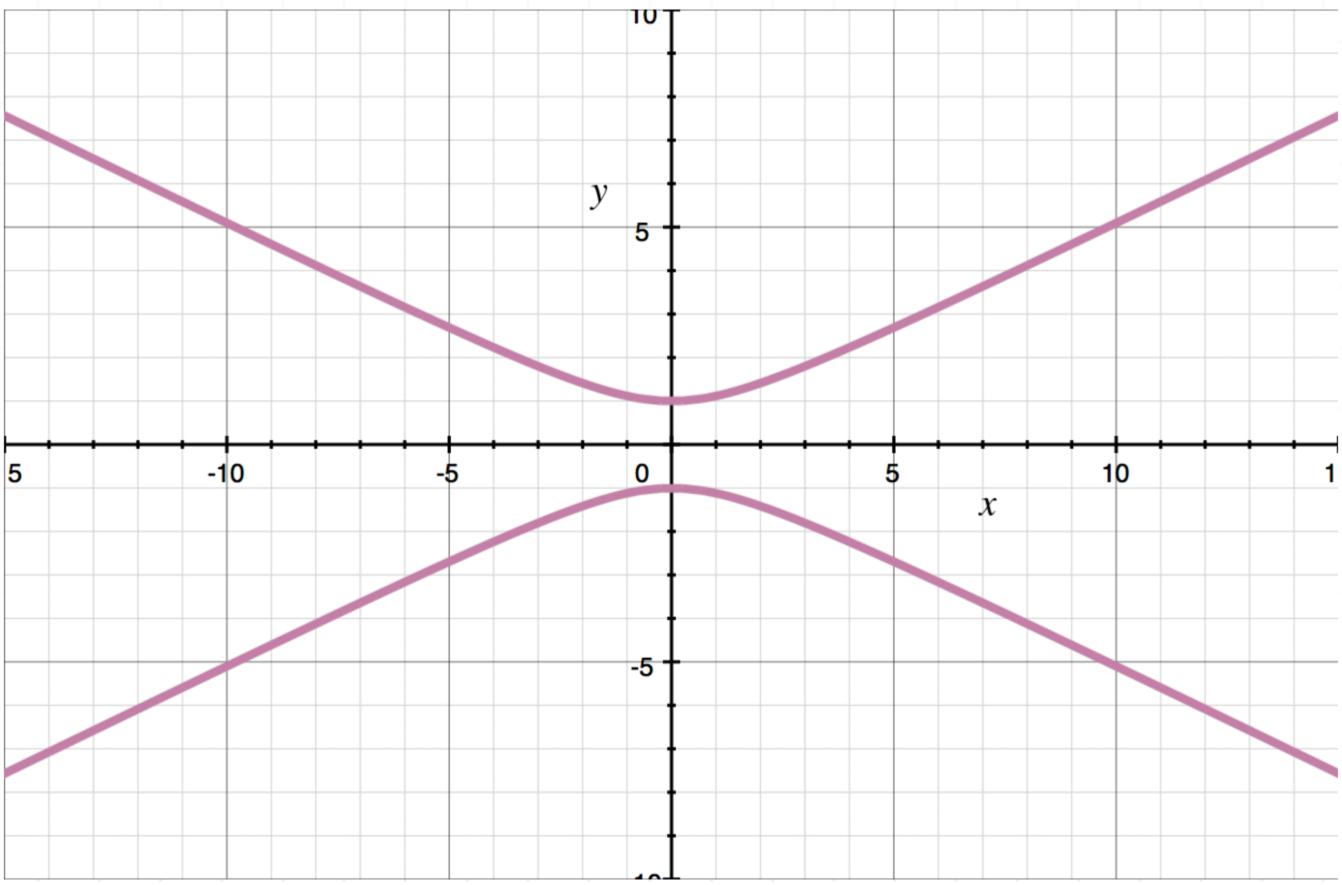


■ 3. Find the equation of the surface obtained by rotating the circle about the x-axis.



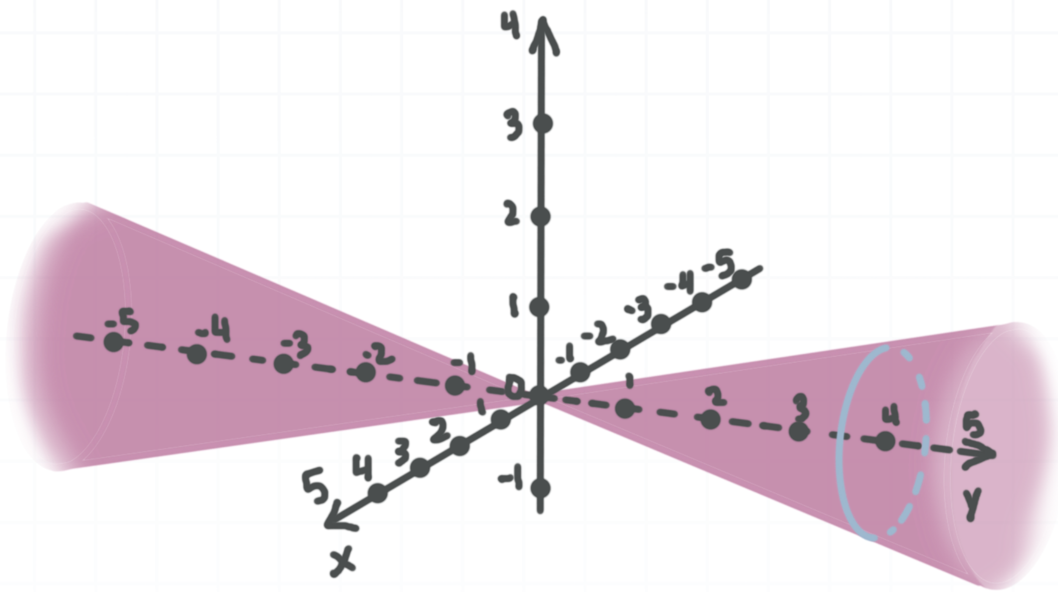
■ 4. Determine the identity and the equation of the surface obtained by rotating the hyperbola about the x-axis.

$$\frac{x^2}{2^2} - z^2 = -1$$



TRACES TO SKETCH AND IDENTIFY THE SURFACE

■ 1. Find the identity and the equation of the surface that has a trace $x^2 + z^2 = 1$ for $y = 4$.



■ 2. Find the trace of the surface in the plane $y = 7$ and identify it.

$$\frac{(x+5)^2}{81} + \frac{(y-3)^2}{4} - \frac{(z+8)^2}{49} = 1$$

■ 3. Find the traces of the surface in the planes $x = -2$, $y = 8$, and $z = -4$ and use them to identify the surface.

$$\frac{(x+2)^2}{49} + \frac{(y-8)^2}{16} = z + 5$$

DOMAIN OF A MULTIVARIABLE FUNCTION

■ 1. Find the domain of the multivariable function.

$$f(x, y) = \sqrt{\sin(2x + y)}$$

■ 2. Find the domain of the multivariable function.

$$f(x, y) = (x^2 - y^2)\tan(2x)\cot(y + \pi)$$

■ 3. Find the domain of the multivariable function.

$$f(x, y) = \sin(3x + y)\log_{x-y}(x^2)$$

■ 4. Find the set of points that lie within the domain of the multivariable function.

$$f(x, y) = 3\sqrt{x^2 + 2x + y^2 - 4y - 4}$$

■ 5. Find the set of points that lie within the domain of the multivariable function.

$$f(x, y) = (2xy)^{-\frac{3}{4}}$$

LIMIT OF A MULTIVARIABLE FUNCTION

■ 1. If the limit exists, find its value.

$$\lim_{(x,y)\to(0,0)} \ln(2x + 3ey + e^2)$$

■ 2. If the limit exists, find its value.

$$\lim_{(x,y)\to(\pi,\frac{\pi}{2})} \frac{\sin(3x + y)}{\cos(x - 2y)}$$

■ 3. If the limit exists, find its value.

$$\lim_{(x,y)\to(-\infty,-\infty)} (x^3 + 4y)(\sin(x^2 + 2y) + 3)$$

■ 4. If the limit exists, find its value.

$$\lim_{(x,y)\to 0,0)} \frac{4x^4 - y^4}{2x^2 + y^2}$$

■ 5. If the limit exists, find its value.

$$\lim_{(x,y)\to(\infty,\infty)} 2^y - x^2$$

■ 6. If the limit exists, find its value.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + 2x^2y^2 - xy}{2x^3 + y^2}$$

PRECISE DEFINITION OF THE LIMIT FOR MULTIVARIABLE FUNCTIONS

■ 1. Which value of δ can be used to apply the precise definition of the limit to $f(x,y)$ with $\epsilon = 0.002$ at the point $(0,0)$?

$$f(x,y) = (x^2 + y^2)(3 - xy)$$

■ 2. Which value of δ can be used to apply the precise definition of the limit to $f(x,y)$ with $\epsilon = 0.001$ at the point $(0,0)$? Hint: Use the polar form of the function.

$$f(x,y) = \frac{5x^2y}{x^2 + y^2}$$

■ 3. We know that $f(x,y)$ is a continuous function, and that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that $\sqrt{(x-4)^2 + (y+3)^2} < \delta$ implies $|f(x,y) - 7| < \epsilon$. If the limit exists, find its value.

$$\lim_{(x,y)\to(4,-3)} (f(x,y))^2$$

■ 4. We know that $f(x,y)$ and $g(x,y)$ are continuous functions, and that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that $\sqrt{(x-2)^2 + y^2} < \delta$ implies $|f(x,y) + 3| + |g(x,y) - 5| < \epsilon$. If the limit exists, find its value

$$\lim_{(x,y)\to(2,0)} (3f(x,y) - 2g(x,y))$$

5. We know that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that

for $x > 0$, $\sqrt{x^2 + y^2} < \delta$ implies $|f(x,y) - 4| < \epsilon$

for $x \le 0$, $\sqrt{x^2 + y^2} < \delta$ implies $|f(x,y) + 4| < \epsilon$

If the limit exists, find its value.

$$\lim_{(x,y)\to(0,0)} 3^{f(x,y)}$$

6. We know that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that

$\sqrt{(x+1)^2 + (y-12)^2} < \delta$ implies $f(x,y) > \epsilon$. If the limit exists, find its value.

$$\lim_{(x,y)\to(-1,12)} (f(x,y) - 13)$$

DISCONTINUITIES OF MULTIVARIABLE FUNCTIONS

1. Find any discontinuities of the function.

$$f(x, y) = 3^{x^2 - 2y^2 + \sqrt{x^2 + 5y^2 - x + 1}}$$

2. Find any discontinuities of the function.

$$f(x, y) = \sqrt{\sin x \cos y + \sin y \cos x}$$

3. Find any discontinuities of the function.

$$f(x, y) = \begin{cases} \frac{4x^2 - y^2}{2x - y} & y \neq 2x \\ 0 & y = 2x \end{cases}$$

4. Find and classify any discontinuities of the function.

$$f(x, y) = \frac{7x - y}{4x^2 + y^2 - 4x + 1}$$

5. Find and classify any discontinuities of the function.

$$f(x,y)=\frac{x^2-9y^2-2x+1}{|x-1|+|3y|}$$

COMPOSITIONS OF MULTIVARIABLE FUNCTIONS

■ 1. Find $f(g(x,y))$.

$$f(t)=\ln(3t)$$

$$g(x,y)=\frac{x+1}{y+2}$$

■ 2. Find $f(x(t),y(t))$.

$$f(x,y)=x^2-y^2+3$$

$$x(t)=\sqrt{t-5}$$

$$y(t)=2^{t+2}$$

■ 3. Find $f(u(x,y),v(x,y))$.

$$f(u,v)=u^2+v^2+\frac{u-v}{\sqrt{2}}$$

$$u(x,y)=\sin(x+y)$$

$$v(x,y)=\cos(x+y)$$

PARTIAL DERIVATIVES

1. Find $f_x + f_y$.

$$f(x, y) = \sqrt{\sin(x + y)}$$

2. Find f_r and f_θ.

$$f(r, \theta) = r^2(\sin 2\theta - \cos 2\theta)$$

3. Find u_s and u_t.

$$u(t, s) = 2^{\frac{t}{s}}$$

4. Find the point (x, y) where $f_x = f_y = 0$.

$$f(x, y) = 3x^2 - 2xy + 3y^2 - 4x + 2y - 1$$

PARTIAL DERIVATIVES IN THREE OR MORE VARIABLES

■ 1. Find $f_x^2 + f_y^2 + f_z^2$.

$$f(x, y, z) = \tan(x^2 + y^2 + z^2)$$

■ 2. Find f_u, f_v, and f_w.

$$f(u, v, w) = u^{v^w}$$

■ 3. Find the point (a, b, c, d) where $f_a = f_b = f_c = f_d = 0$.

$$f(a, b, c, d) = a^2 + b^2 - c^2 - d^2 + 4ab - 4cd - 6a + 6c + 8 = 0$$

HIGHER ORDER PARTIAL DERIVATIVES

■ 1. Find f_{uvw}.

$$f(u, v, w) = \sqrt{u^2 + v^2 + w^2}$$

■ 2. Find and identify the curve for the set of the points (x, y) where $f_{xx} = f_{yy}$.

$$f(x, y) = 3x^3 - 4x^2y + y^3 - x^2 + 5y + 7$$

■ 3. Find and identify the curve(s) for the set of the points (x, y) where $f_{xx} = f_{yy}$.

$$f(x, y) = \sin(x^2 + y^2)$$

■ 4. Find all four second-order partial derivatives for the function. Is $f_{ts} = f_{st}$?

$$f(t, s) = e^{ts}$$

■ 5. Find the nth-order partial derivatives $\partial^n/\partial x^n$ and $\partial^n/\partial y^n$ by looking for patterns in the partial derivatives with respect to x and y.

$$f(x, y) = 2^{2x+4y}$$

DIFFERENTIAL OF A MULTIVARIABLE FUNCTION

■ 1. Find the differential of the multivariable function.

$$f(r,\theta)=\frac{r^2}{\sin 2\theta+\cos 2\theta}$$

■ 2. Find the differential of the multivariable function.

$$U(u,v,w)=\frac{(2u+1)^2(3v+4)}{\sqrt{w-2}}$$

■ 3. Find the differential of the multivariable function at $(-6,2)$.

$$f(x,y)=4\log_2(x^2-2xy+y^2)$$

■ 4. Find the point(s) where the differential of the multivariable function is equal to 0 (i.e., find the critical points of the function).

$$f(s,t)=3t^4-2t^2s-s^2+16t+5$$

■ 5. Find and identify the set of point(s) where the differential of the multivariable function $f(x,y)$ doesn't depend on dy.

$$f(x, y) = \cos(e^{x^2+y})$$

CHAIN RULE FOR MULTIVARIABLE FUNCTIONS

■ 1. If $x = e^t$, $y = t^2 - 3$, and $z = 2t + 1$, use chain rule to find df/dt.

$$f(x, y, z) = xy^2z^3$$

■ 2. If $r = \phi^2$ and $\theta = \phi + \pi$, use chain rule to find $dz/d\phi$ at $\phi = \pi/4$.

$$z(r, \theta) = r^2 \sin \theta$$

■ 3. If $u = \ln(3t)$ and $v = \ln t$ with $t > 0$, use chain rule to find the global maximum of the function.

$$f(u, v) = 3u - 2v^2$$

CHAIN RULE FOR MULTIVARIABLE FUNCTIONS AND TREE DIAGRAMS

■ 1. If $x = \sin(t + s)$, $y = 2ts$, and $z = 2t - 5s$, use chain rule to find the partial derivatives f_t and f_s.

$$f(x, y, z) = 7x + 2y^2z$$

■ 2. If $x = \log_2(ts)$ and $y = \log_3(2t + s)$, use chain rule to find partial derivatives f_s and f_t at $(1,1)$.

$$f(x, y) = x^2 - 2xy - y^2 + x + 3y - 4$$

■ 3. If $x = 2t - s$ and $y = t + 2s$, use chain rule to find the point (s, t) where $f_t = f_s = 0$.

$$f(x, y) = 2x^2 - 3xy + y^2 + y + 9$$

IMPLICIT DIFFERENTIATION

■ 1. Use implicit differentiation to find the partial derivative dy/dx.

$$\sin(x+y)=x+y$$

■ 2. Use implicit differentiation to find the partial derivative $\partial z/\partial x$ of the multivariable function.

$$y\ln z=2x-3y+2z$$

■ 3. Use implicit differentiation to find the partial derivative $\partial z/\partial y$ of the multivariable function.

$$e^z=x^2+y+z$$

DIRECTIONAL DERIVATIVES

■ 1. Find the directional derivative in the direction of $\vec{v} = \langle 2,2,1 \rangle$.

$$f(x, y, z) = \cos(2x + 3y + z)$$

■ 2. Find the directional derivative in the direction of $\vec{v} = \langle 0, -3, -4 \rangle$.

$$f(x, y, z) = x^2 \ln(y - z)$$

■ 3. Find the directional derivative in the direction of $\vec{v} = \langle 3, -6,2 \rangle$ at the point $A(\pi/2,1/2,\pi)$.

$$f(x, y, z) = x \sin(yz)$$

LINEAR APPROXIMATION IN TWO VARIABLES

■ 1. Find the linear approximation of the function at $(1,1)$ and use it to approximate $f(0.99,0.99)$. Compare it to the exact value of $f(0.99,0.99)$.

$$f(t,s)=\sqrt{3t^2+s^2}$$

■ 2. Calculate the percentage error of the linear approximation of the function at $f(0.9e,0.81)$. Use the initial point $(e,1)$.

$$f(x,y)=\ln\left(\frac{x^2}{y}\right)$$

■ 3. Find the linear approximation of the function at $(0,0)$ and use it to approximate $f(0.2,0.01)$. Round to two decimal places.

$$f(u,v)=3e^{2u-7v}$$

■ 4. Find the values of a and b and write down the linear approximation of the function at (a,b), given that $f_x(a,b)=-5$ and $f_y(a,b)=11$.

$$f(x,y)=x^2-3y^2-7x-y$$

5. Find $f_x(1,2)$ and $f_y(1,2)$, given that $f(1,2) = 5$, $L(1,2.1) = 5.5$, and $L(1.1,1.95) = 5.4$, where $L(x, y)$ is the linear approximation of $f(x, y)$ at $(1,2)$.

LINEARIZATION OF A MULTIVARIABLE FUNCTION

■ 1. Find the percentage error of the linear approximation of the function at $(3.2,-1.1,0.8)$, if the initial point is $(3,-1,1)$.

$$f(x,y,z)=2xy^2z^3$$

■ 2. Find the linear approximation of the function at $(2,\pi/6,-\pi/6)$ and use it to approximate $R(2,0.5,-0.5)$. Round to two decimal places.

$$R(r,\phi,\theta)=r^2\sin(2\phi)\cos(\theta+\pi)$$

■ 3. Find the values of the first order partial derivatives of $f(x,y,z)$ at $(3,4,-8)$, where $L(x,y,z)$ is the linear approximation of the function $f(x,y,z)$ at $(3,4,-8)$, and $f(3,4,-8)=3$.

$$L(3.1,4.2,-8.1)=3$$

$$L(3.2,3.9,-7.8)=3.4$$

$$L(2.9,4.3,-8.1)=2.8$$

GRADIENT VECTORS

■ 1. Find the gradient vector ∇f at $(\sqrt{\pi},0,0)$.

$$f(x,y,z)=\sin(x^2+2y^2-z^2-2xyz)$$

■ 2. Find unit gradient vector of the function f at $(-2,1)$.

$$f(t,s)=\frac{4t-st^4}{t^2s}$$

■ 3. Find the point where the gradient vector of the function f is equal to the zero vector.

$$f(x,y)=\ln\frac{(x-2)^2y}{x-y}$$

■ 4. Find and identify the set of points where the magnitude of the gradient vector of the function f is equal to 1.

$$f(x,y)=x^2+4y^2-2x+8y-5$$

■ 5. Find the directional derivative of the function f in the direction $m=3\mathbf{i}-4\mathbf{j}$ at $(0,4)$.

$$f(x,y) = 2^x(y^2 - 1)$$

GRADIENT VECTORS AND THE TANGENT PLANE

■ 1. Use the gradient vector to find the tangent line to the curve $(y-2)^2 - e^x = 0$ at $(0,3)$.

■ 2. Use the gradient vector to find the tangent plane to the surface $(r+1)\sin(\phi+\pi)\tan(\theta) = 0$ at $(2,\pi/6,\pi/4)$.

■ 3. Use the gradient vector to find the tangent plane(s) to the surface $x^2 - 2xy - 3y^2 + z^2 + 4x + 4y - 2z - 3 = 0$ that are parallel to the xy-plane.

MAXIMUM RATE OF CHANGE AND ITS DIRECTION

■ 1. Find the point where the maximum rate of change of the function f is equal to 0.

$$f(x, y) = 6x^2 - 4xy + y^2 - 12x - 6y + 4$$

■ 2. Find the maximum rate of change and its direction for the function f at $(3, -\pi/2, 0)$.

$$f(x, y, z) = x^2(2z - 1)\sin^2 y$$

■ 3. Find the minimum rate of change and its direction for the function f at $(2, 1, 4)$.

$$f(u, v, w) = \sqrt{2u - 4v + 6w + 1}$$

EQUATION OF THE TANGENT PLANE

■ 1. Find a tangent plane to the surface $f(u, v, w) = 0$ at $(3, -1, 5)$.

$$f(u, v, w) = \ln \frac{u^2 + 1}{v^2 w^5}$$

■ 2. Find any tangent planes to the surface $f(x, y, z) = 0$ that are parallel to the plane $5x - 4y + 2z + 5 = 0$.

$$f(x, y, z) = x^3 - 4y^2 + z^2 + 2x + 12y + 5$$

■ 3. Find a line of intersection of the xy-plane and tangent plane to the surface $f(x, y, z) = 0$ at $(\pi, -1, \sqrt{6})$.

$$f(x, y, z) = 2\cos(x + \pi)(y^2 + y + 5) - 3z^3$$

■ 4. Find and identify the set of the points where the tangent plane to the surface $f(x, y, z) = 0$ is parallel to z-axis.

$$f(x, y, z) = x^2 + 4y^2 + z^2 + 2x - 8y + 8z + 17 = 0$$

NORMAL LINE TO THE SURFACE

■ 1. Use the gradient vector to find a symmetric equation of the normal line to the curve $f(s,t)=0$ at $(-3,3)$, where

$$f(s,t)=t2^{2t+s-3}$$

■ 2. Use the gradient vector to find a vector equation of the normal line to the surface $f(x,y,z)=0$ at $(0,-5,1)$.

$$f(x,y,z)=\frac{2x-y^2}{z}$$

■ 3. Use the gradient vector to find a parametric equation of the normal line to the surface $f(x,y,z)=0$ at $(2,-3,0)$.

$$f(x,y,z)=3x^3-2xyz-2y^2+5yz+1$$

■ 4. Use the gradient vector to find a parametric equation of the normal line to the surface $f(x,y,z)=0$ that's parallel to the line $r=\langle 2,17,-6\rangle+t\langle 9,1,-6\rangle$.

$$f(x,y,z)=2x^2+y^2+3z^2-3x-5y+5$$

5. Use the gradient vector to find a vector equation of the normal line to the surface $f(x, y, z) = 0$ that's perpendicular to the plane $x + 4y - 8z + 12 = 0$.

$$f(x, y, z) = ye^{2x+6} + z^2 - 5$$

CRITICAL POINTS

1. Find a set of critical points for $f(t, s)$.

$$f(t, s) = \ln \frac{2t^3 - 12t^2 + 1}{s^2 + 6s + 11}$$

2. Find and identify a set of critical points for $f(x, y, z)$.

$$f(x, y, z) = x^2 \cos(y + z)$$

3. Find a set of critical points for $f(x_1, x_2, x_3, x_4)$.

$$f(x_1, x_2, x_3, x_4) = x_1^2 - 2x_1x_2 + 2x_2^2 - 4x_3^2 + 4x_1 + 5x_4^2 - 10x_4 + 6$$

SECOND DERIVATIVE TEST

1. Use the second derivative test to classify the critical points of $f(t, s)$.

$$f(t, s) = \frac{t^5 s - 3t + 6s^2}{s^2 t^5}$$

2. Use the second derivative test to classify the critical points of $f(r, \theta)$.

$$f(r, \theta) = (r^2 - 2r - 3)\sin\theta$$

3. Find the set of all possible values of a for which $f(x, y)$ has only one local minimum.

$$f(x, y) = x^2 + ay^2 - 4x + 8y - 6$$

LOCAL EXTREMA AND SADDLE POINTS

■ 1. Find the local extrema of $f(t,s)$.

$$f(t,s)=\frac{t^3+s^3+1}{ts}$$

■ 2. Find the local extrema of $f(x,y)$.

$$f(x,y)=\sin(0.5x)\cos(0.25y)$$

■ 3. Find the equation(s) of the tangent plane to $f(x,y)$ at the function's local maximum.

$$f(x,y)=-x^2-2y^2+4x-12y-9$$

■ 4. Find the values of a and b where $f(x,y)$ has a local minimum at $(5,-3)$.

$$f(x,y)=4x^2+2y^4-ax-by+5$$

■ 5. Find and identify the set of local maxima of $f(x,y)$.

$$f(x,y)=e^{-4(x^2+y^2)}(x^2+y^2)$$

6. Find and identify the saddle points and local extrema of $f(x, y)$.

$$f(x, y) = (x - 4)^8 - (y + 7)^{12}$$

GLOBAL EXTREMA

■ 1. Find the global extrema of $f(t, s)$ over R^2.

$$f(t, s) = \frac{4s}{t^2 + 2s^2 + 2}$$

■ 2. Find the global extrema of $f(t, s)$ over R^2.

$$f(t, s) = t^2s^2 - 4t^2 - 4s^2 - 4s + 1$$

■ 3. Find the global extrema of $f(x, y)$ over R^2.

$$f(x, y) = \frac{\sin(3x)}{x^2 + 3y^2}$$

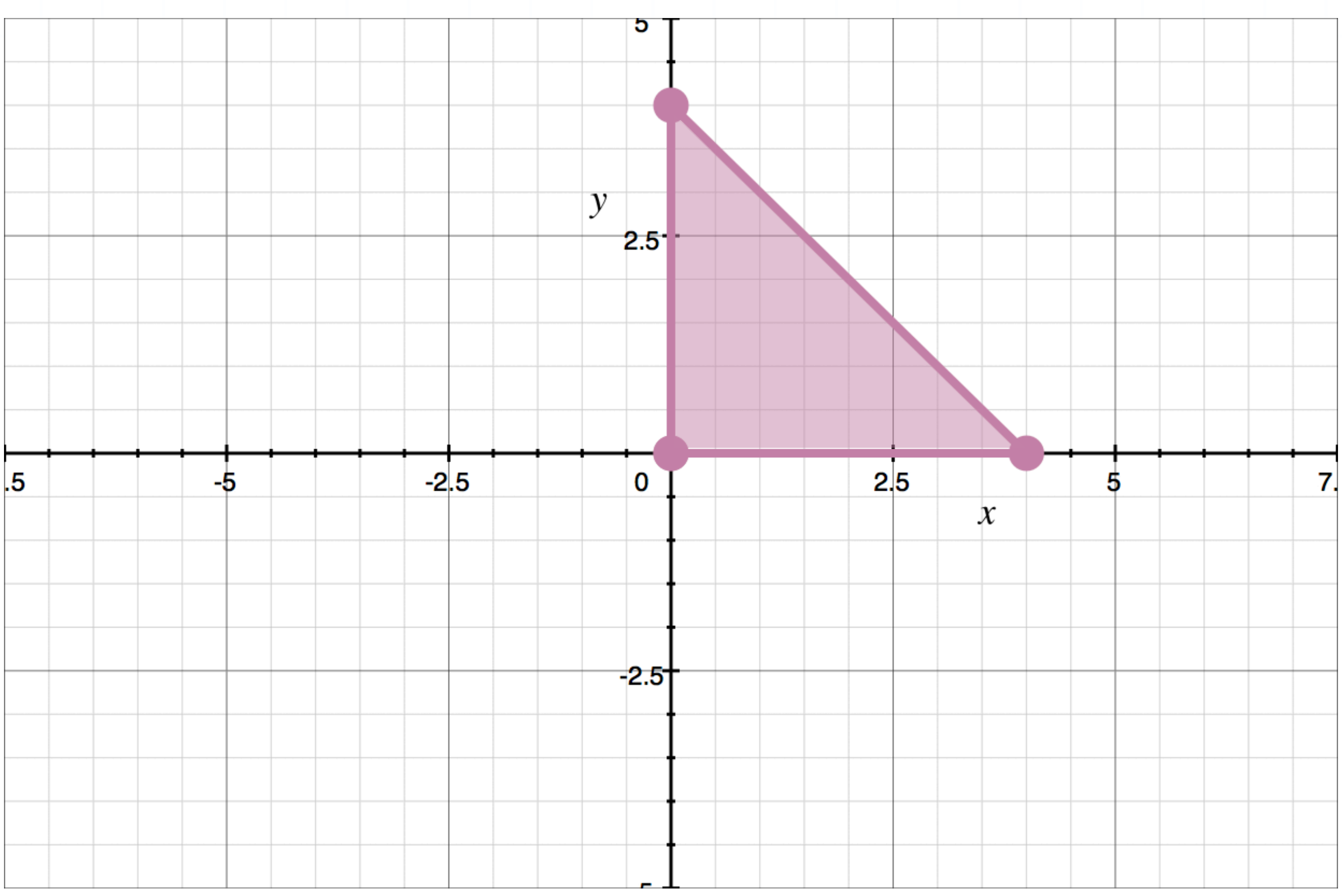
EXTREME VALUE THEOREM

■ 1. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of $f(x,y)$ on the closed rectangle $-1 \leq x \leq 3$, $0 \leq y \leq 3$.

$$f(x,y) = 2x^2 - 2xy + y^2 - 4x - 1$$

■ 2. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of $f(x,y)$ on a closed triangle bounded by $x = 0$, $y = 0$, and $x + y - 4 = 0$.

$$f(x,y) = \ln(x^2 + y^2 - 2y - x + 2)$$



3. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of $f(x, y)$ on the closed rectangle $-\pi \le x \le \pi$, $-1 \le y \le 3$.

$$f(x, y) = y^2 \tan(2x)$$

4. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of the function $f(x, y)$ on the closed rectangle $-\pi \le x \le \pi$, $-2 \le y \le 2$.

$$f(x, y) = (y^2 + 2y + 3)\tan\left(\frac{x}{4}\right)$$

5. Determine whether the Extreme Value Theorem applies. If the theorem applies, find the global extrema of the function $f(x, y)$ on the closed circle with center at the origin and radius 1.

$$f(x, y) = x^2 + y^2 - 2x + 2\sqrt{3}y - 3$$

APPLIED OPTIMIZATION

■ 1. Find the maximum volume of a rectangular box inscribed in a hemisphere with radius 4.

■ 2. Find the minimum distance from $(2,2,-1)$ to the plane $8x-4y+z+11=0$.

■ 3. Find the minimum distance from $(-4,4,0)$ to the cone $3x^2+y^2=z^2$.

TWO DIMENSIONS, ONE CONSTRAINT

■ 1. Use Lagrange multipliers to find the extrema of the function, subject to the given constraint.

$$f(x, y) = x + y - 5$$

$$xy = 1$$

■ 2. Use Lagrange multipliers to find the extrema of the function, subject to the given constraint.

$$f(x, y) = e^{2x+y}$$

$$x^2 + y^2 = 5$$

■ 3. Use Lagrange multipliers to find the extrema of the function, subject to the given constraint.

$$f(x, y) = x^2 + y^2 + 3$$

$$\sin(x + y) = 0$$

■ 4. Use Lagrange multipliers to find the extrema of the function, subject to the given constraint.

$$f(x, y) = \ln \frac{2x - 4}{y^2}$$

$$4x + 8y - 15 = 0$$

THREE DIMENSIONS, ONE CONSTRAINT

1. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraint.

$$f(x, y, z) = x^5 - 160y + 160z$$

$$x + y^2 + z^2 = 0$$

2. Use Lagrange multipliers to find the extrema of the function, subject to the given constraint.

$$f(x, y, z) = x + 2y^2 - 3z^2 - 4$$

$$e^x + y - 3z = -\frac{1}{4}$$

3. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraint.

$$f(x, y, z) = |x^3y^7z^5|$$

$$3x + 7y + 5z = 60$$

4. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraint.

$$f(x, y, z) = x + yz + 2y$$

$$y^2 + xyz = 2$$

■ 5. Use Lagrange multipliers to find the extrema of the function, subject to the given constraint.

$$f(x, y, z) = \ln \frac{xy + 3y}{z - 2}$$

$$x + y + 3z = 6$$

■ 6. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraint.

$$f(x, y, z) = \sin^2 x \cdot \sin 2y \cdot \sin z$$

$2x + 2y + z = \frac{2}{3}\pi$ with $x > 0$, $y > 0$, $z > 0$

THREE DIMENSIONS, TWO CONSTRAINTS

■ 1. Use Lagrange multipliers to find the shortest distance from the vertex of the elliptic paraboloid $(x-2)^2+2(y+1)^2=3z+6$ to the line that's the intersection of the planes $x+3y+5z=18$ and $3x+5y+z=28$.

■ 2. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraints.

$$f(x,y,z)=x^2+2y-3z^2$$

$4x-y=0$ and $y+8z=0$

■ 3. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraints.

$$f(x,y,z)=z$$

$4x+2y+3z=-2$ and $3x^2+y^2-z^2=5$

■ 4. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraints.

$$f(x,y,z)=2\ln x+z$$

$$2x + y + z = 4 \text{ and } y^2 + z^2 = 2$$

■ 5. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraints.

$$f(x, y, z) = 2\ln x - \ln y^4 + z^2 + 5$$

$$2x + 3z^2 = 12 \text{ and } 4y + z^2 = 4$$

■ 6. Use Lagrange multipliers to find the local extrema of the function, subject to the given constraints.

$$f(x, y, z) = 4x^3 + 2y^3 - 4z + 1$$

$$3y^2 - 4z = 12 \text{ and } 6x^2 + 4z = 15$$

APPROXIMATING DOUBLE INTEGRALS WITH RECTANGLES

■ 1. Estimate the volume between the surface $z = 3(x - 2)e^{y-2}$ and the xy-plane, on the region $-1 \le x \le 1$ and $0 \le y \le 3$. Use lower-left corners and 1×1 squares, then 0.5×0.5. Round the answers for volume to the nearest tenth. Which square size gives the better approximation if exact volume is 31.

■ 2. Estimate the volume below the surface $z = 3 + x + y^2$, over the triangular region bounded by the x- and y-axes and the line $2x + 3y = 6$. Use lower-left corners and 1×1 squares. For squares that lie partially within the region, divide area by 2.

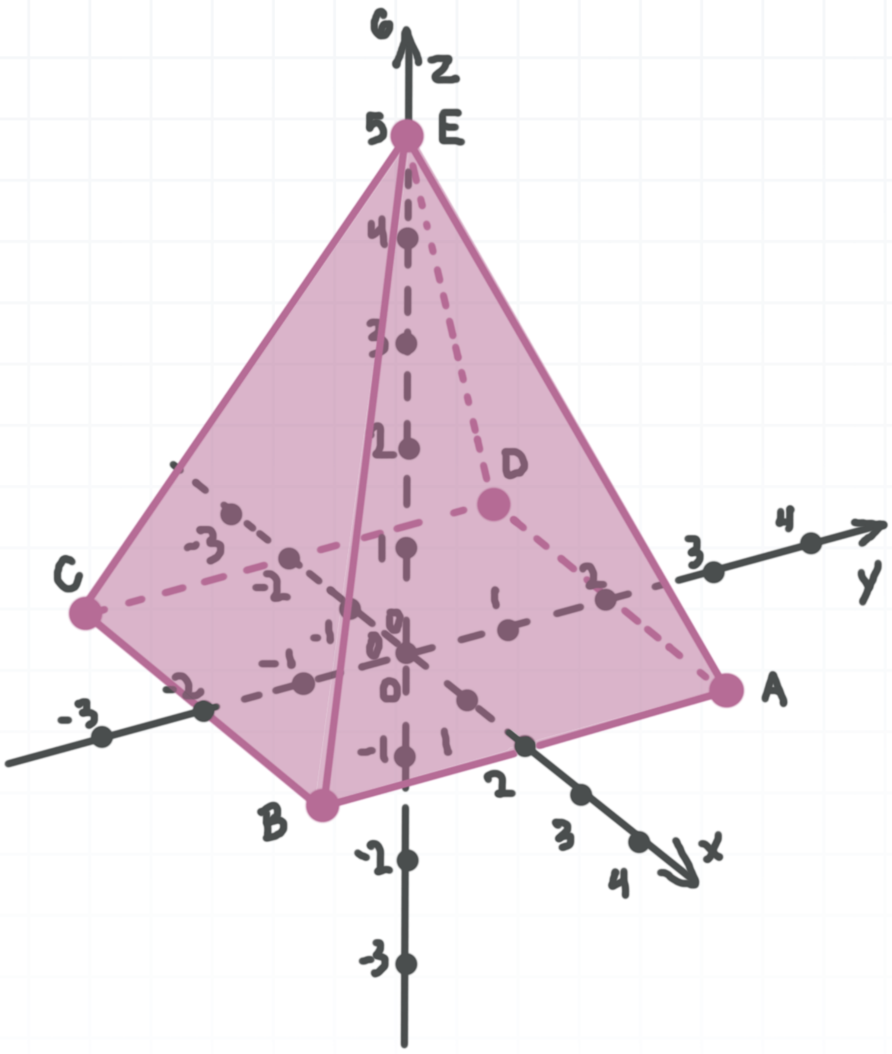
■ 3. Assume the base of a right circular cylinder with radius 3 and height 5 lies in the xy-plane with its center at the origin. Use 1×1 squares and lower-left corners to estimate the volume of the cylinder. If an approximating square lies only partially within the circle, divide its area in half as part of the estimation. Calculate exact volume of the cylinder using $V = \pi r^2 h$, then find the percentage error of the approximation.

MIDPOINT RULE FOR DOUBLE INTEGRALS

■ 1. Assuming the integral's exact value is 12, say which estimation is more accurate if we estimate volume of the integral below using Midpoint Rule and rectangles of dimensions $\pi/2 \times 1$, and then rectangles of dimensions $\pi/3 \times 2/3$.

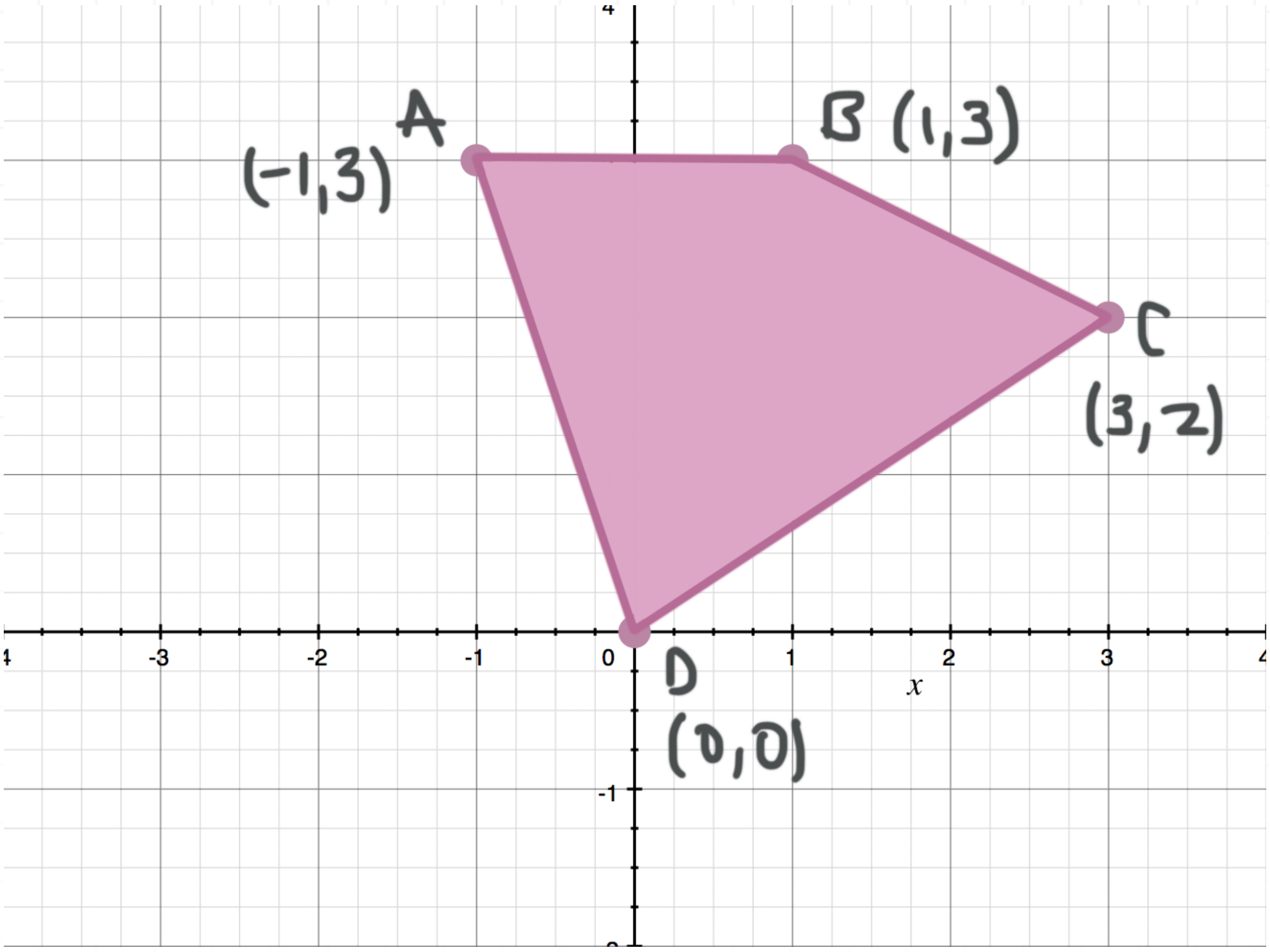
$$\int_0^\pi \int_{-1}^1 (y+3)\sin x \ dy \ dx$$

■ 2. Use Midpoint Rule and 1×1 squares to estimate the volume of the right square pyramid $ABCDE$ with base side length 4 and height 5, assuming its base lies in the xy-plane with its sides parallel to the major coordinate axes, and the vertex of the pyramid lies on z-axis. If the pyramid's exact volume is $V = 80/3$, find the percentage error of the approximation.



■ 3. Use Midpoint Rule with 1×1 squares to estimate the value of the given integral, where $ABCD$ is the quadrilateral shown in the xy-plane below. For any square that lie only partially inside the region, divide the area in half, then round the final approximation to the nearest tenth.

$$\iint_{ABCD} x + \sqrt{y+3}\ dy\ dx$$


A
(-1,3)
B (1,3)
C
(3,2)
D
(0,0)
x

RIEMANN SUMS FOR DOUBLE INTEGRALS

■ 1. Use Riemann sums to approximate the integral, using squares with sides of length 1, and the upper-left corner of the squares. If the square partially lies within the domain, then divide its area in half.

$$\int_0^2 \int_x^{x^2+2} x^2 + 2x - 3y + 1 \ dy \ dx$$

■ 2. Use Riemann sums to approximate the integral over the rectangle $R = [1,3] \times [-1,2]$, using squares with sides of length 1, and the upper-right corners of the squares. Round your answer to the nearest tenth.

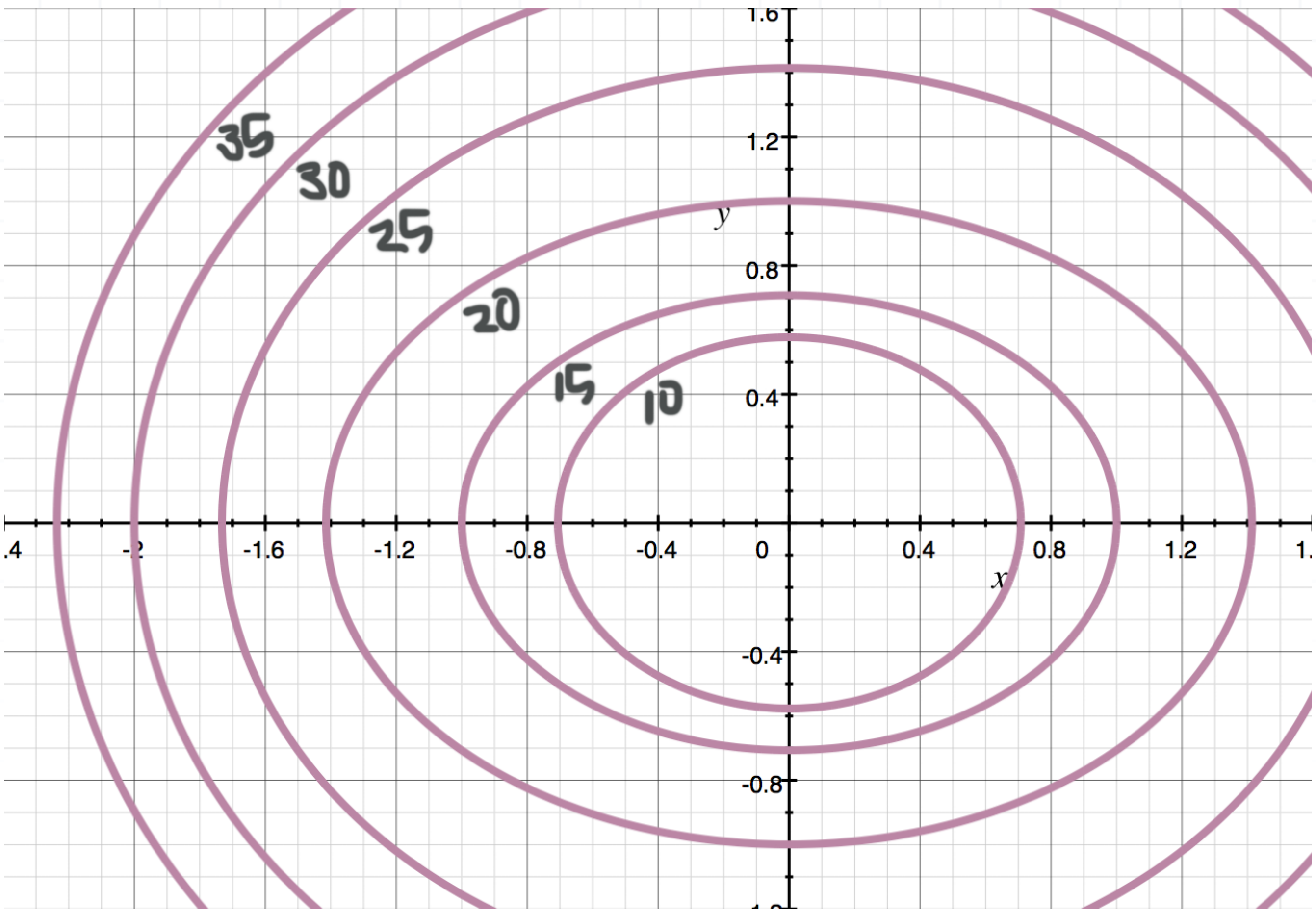
$$\iint_R \ln(x^2 + y + 2) \ dy \ dx$$

■ 3. Use Riemann sums to approximate the integral, using rectangles with sides $1 \times \pi/2$, and the upper-right corners of the rectangles. If the exact value is 14π, find the percentage error of the approximation.

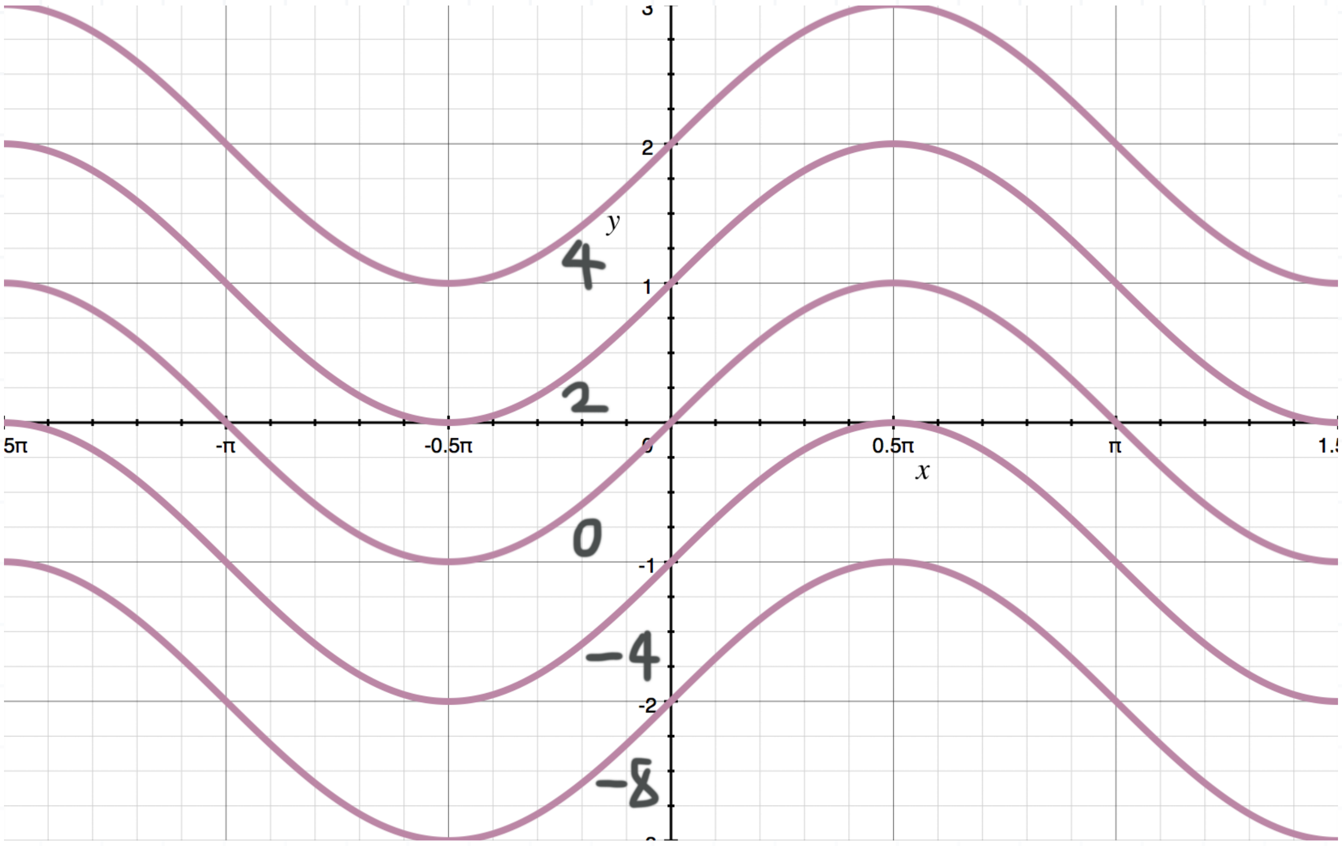
$$\int_{-\pi}^{\pi} \int_0^2 2x + \sin^2 y + 1 \ dx \ dy$$

AVERAGE VALUE

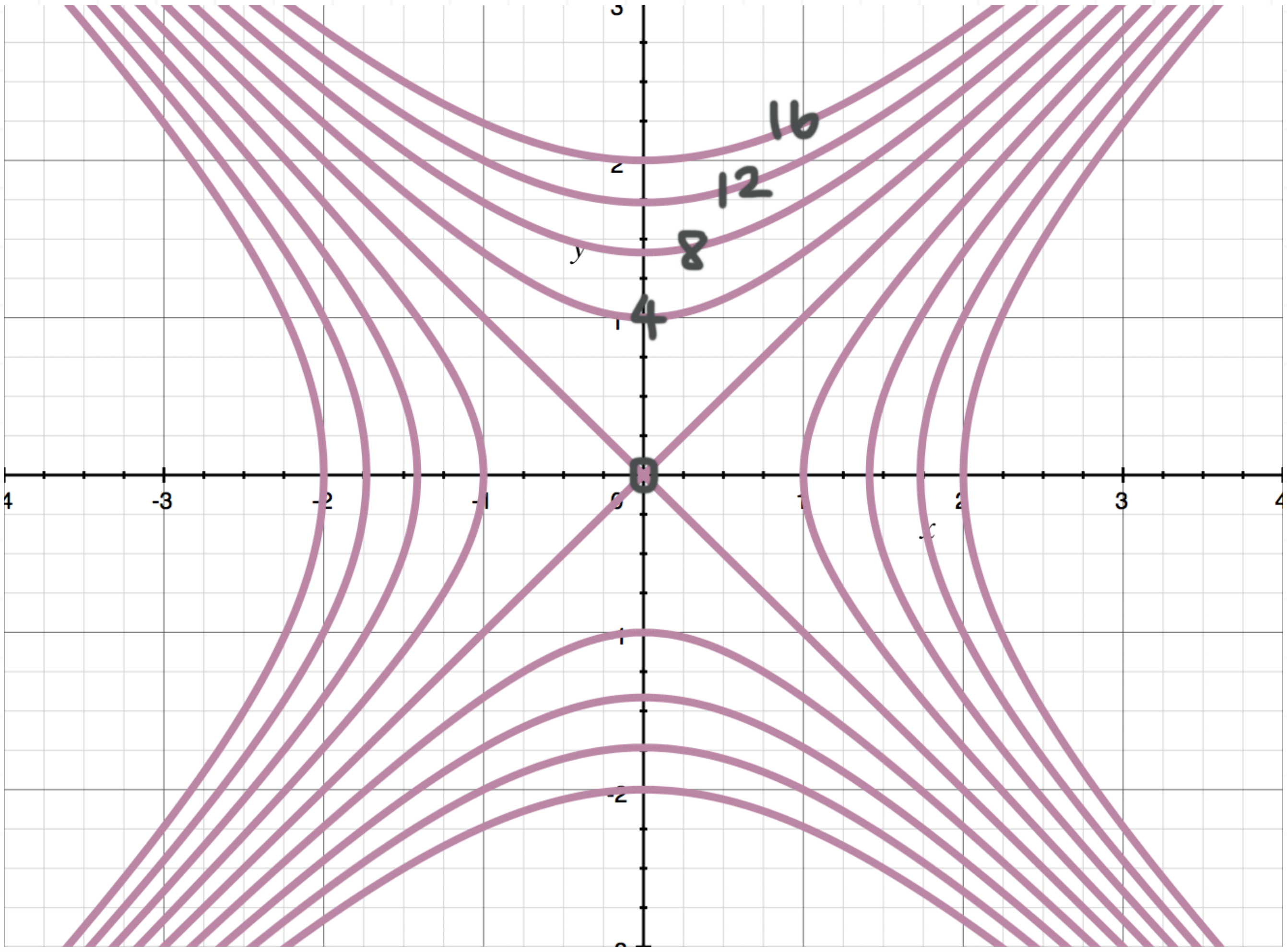
■ 1. Use midpoints of squares with side lengths 1 to estimate the average value of the region $R = [-2,1] \times [-2,2]$, given the sketch of level curves.



■ 2. Use midpoints of rectangles with dimensions $\pi \times 1$ to estimate the average value of the region $R = [-\pi, \pi] \times [-2,2]$, given the sketch of level curves.



■ 3. Use midpoints of rectangles with dimensions 2×1 to estimate the average value of the region $R = [-2,2] \times [-2,2]$, given the sketch of level curves.


16
12
8
4
0
-3
-2
-1
1
2
3
y
x

ITERATED INTEGRALS

■ 1. Evaluate the iterated integral.

$$\int_{2}^{4}\int_{1}^{2}\log_2\frac{y^2}{x^4}\ dx\ dy$$

■ 2. Evaluate the iterated integral.

$$\int_{-5}^{5}\int_{0}^{\pi}(3x^2-4x+10)\sin(y+\pi)\ dy\ dx$$

■ 3. Evaluate the iterated integral.

$$\int_{-1}^{1}\int_{0}^{2}xe^{x^2-3y+1}\ dx\ dy$$

DOUBLE INTEGRALS

■ 1. Evaluate the double integral, where R is the rectangle $[0,\pi] \times [0,1]$.

$$\iint_R \cos(x - \pi y)\ dx\ dy$$

■ 2. Evaluate the double integral, where R is the rectangle $[1,3] \times [1,5]$.

$$\iint_R \frac{1}{(x+y)^2}\ dx\ dy$$

■ 3. Evaluate the double integral, where R is the rectangle $[x, y] = [-\pi/2,\pi/2] \times [0,\pi]$.

$$\iint_R \cos(x+y) - x\sin(x+y)\ dx\ dy$$

TYPE I AND II REGIONS

■ 1. Evaluate the double integral if D is the circle centered at the origin with radius 4.

$$\iint_D 4x^2y + 3 \ dA$$

■ 2. Evaluate the double integral if D is the region bounded by the curves $y + x^2 - 4 = 0$ and $y + 2x^2 - 8 = 0$.

$$\iint_D 462y\sqrt{x+2} \ dA$$

■ 3. Evaluate the double integral if D is the region bounded by $x - \sin y = 0$, $x - \sin y = 5$, $y = 0$, and $y = \pi$.

$$\iint_D 2x \ dA$$

FINDING SURFACE AREA

■ 1. Find area of the surface $z = \sqrt{3}x + y^2 + 1$ inside the rectangle $-1 \le x \le 1$, $0 \le y \le 1$.

■ 2. Find area of the surface $z = \ln(\sin(3x)) + 2\sqrt{2}y - 5$ inside the rectangle $\pi/6 \le x \le \pi/4$, $0 \le y \le 1$.

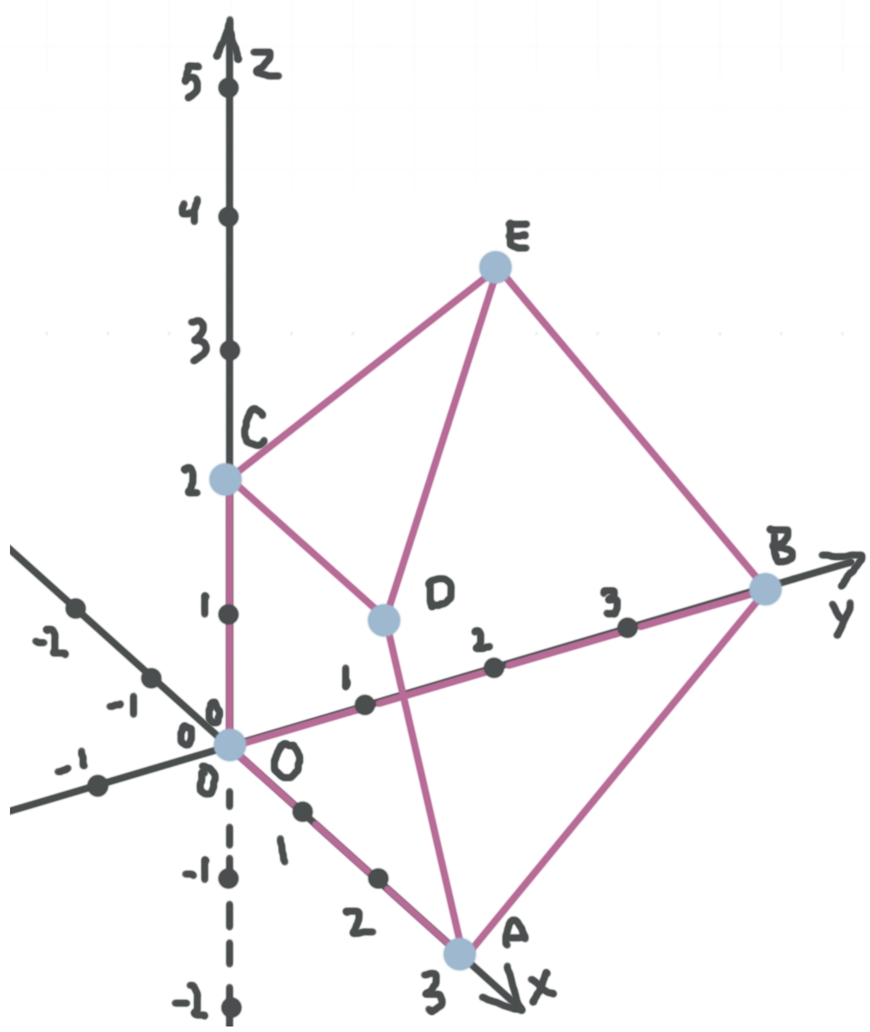
■ 3. Find area of the surface $z = 2(x+3)^{3/2} + 5^{3/2}y - 6$ inside the triangle OAB, if O is the origin and A and B are at $A(3,0)$ and $B(2,2)$.

FINDING VOLUME

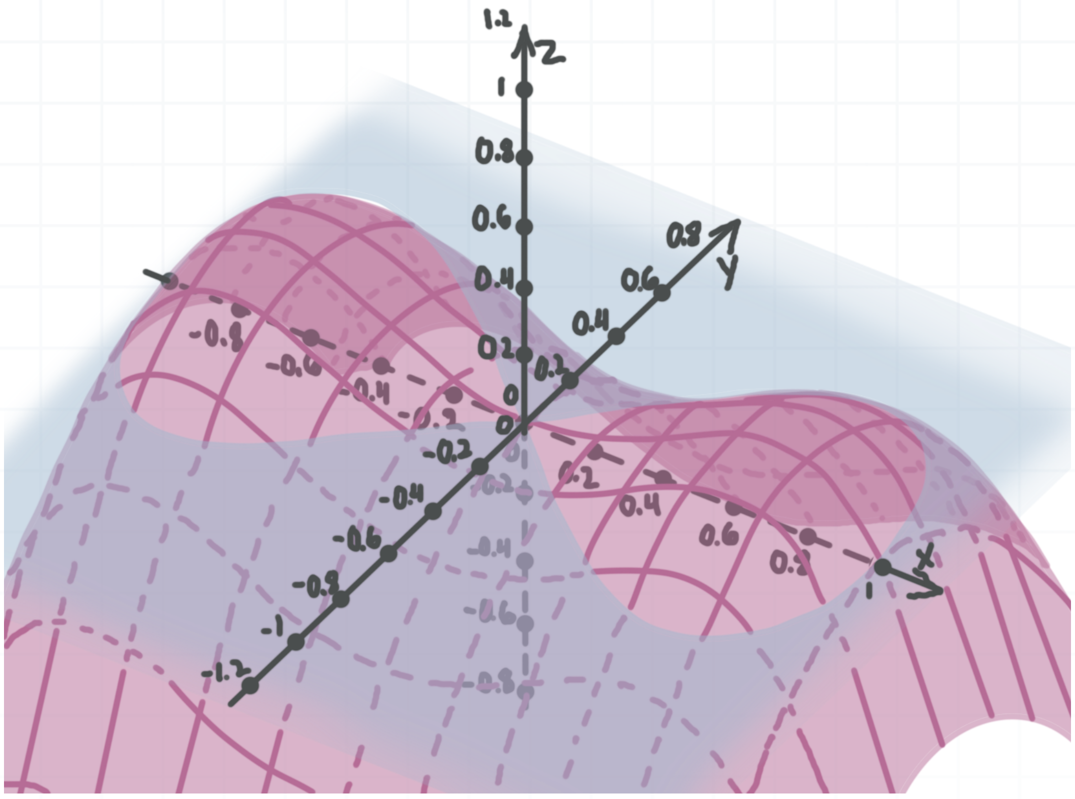
■ 1. Use a double integral to find the volume of the solid that's bounded by the surface and the xy-plane, on $0 \le x \le 2$ and $0 \le y \le \pi/2$.

$$z = \frac{\sin(2y)}{(x+1)^2}$$

■ 2. Use a double integral to find the volume of the irregular hexagon $OABCDE$, where O is the origin, and the hexagon's other vertices are $A(3,0,0)$, $B(0,4,0)$, $C(0,0,2)$, $D(2,0,2)$, and $E(0,2,3)$.



■ 3. Use a double integral to find the volume of the solid that's bounded by the surface $z = -x^4 + x^2 - y^2$ and the xy-plane.



CHANGING THE ORDER OF INTEGRATION

1. Change the order of integration of the iterated integral.

$$\int_{-3}^{0}\int_{-\frac{2}{3}\sqrt{9-x^2}}^{\frac{2}{3}\sqrt{9-x^2}} z(x,y)\ dy\ dx$$

2. Change the order of integration of the iterated integral.

$$\int_{3}^{5}\int_{2}^{e^{x-3}+1} z(x,y)\ dy\ dx$$

3. Change the order of integration of the iterated integral.

$$\int_{-2}^{2}\int_{\frac{1}{4}x^4-x^2}^{0} z(x,y)\ dy\ dx$$

CHANGING ITERATED INTEGRALS TO POLAR COORDINATES

■ 1. Convert the iterated integral to polar coordinates, then find its value.

$$\int_{-5}^{0}\int_{0}^{\sqrt{25-x^2}} xy\ dy\ dx$$

■ 2. Convert the sum of iterated integrals to polar coordinates, then find its value.

$$\int_{-4}^{-2}\int_{0}^{\sqrt{16-x^2}} x-y\ dy\ dx+\int_{-2}^{2}\int_{\sqrt{4-x^2}}^{\sqrt{16-x^2}} x-y\ dy\ dx+\int_{2}^{4}\int_{0}^{\sqrt{16-x^2}} x-y\ dy\ dx$$

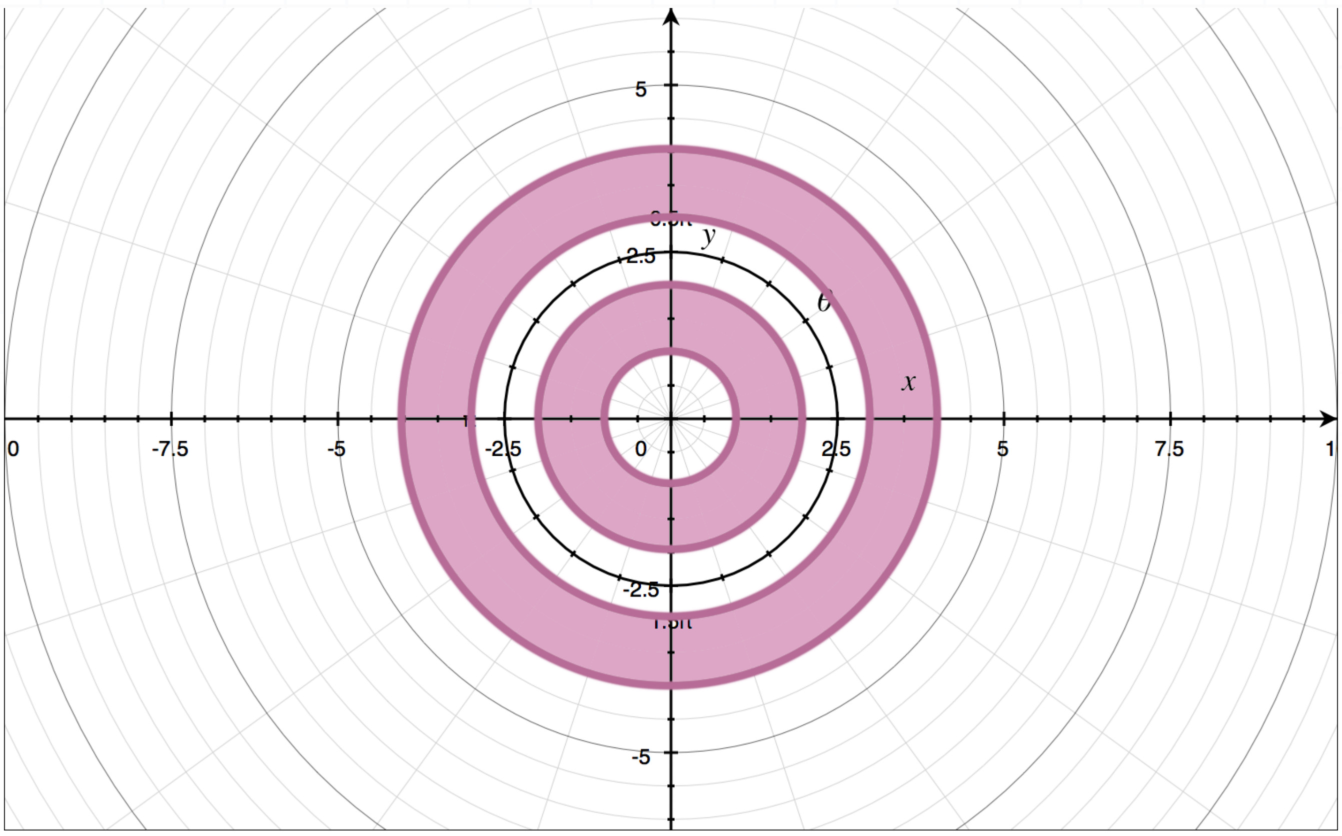
■ 3. Convert the iterated integral to polar coordinates, then find its value.

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-y^2}} \ln(x^2+y^2)\ dx\ dy$$

CHANGING DOUBLE INTEGRALS TO POLAR COORDINATES

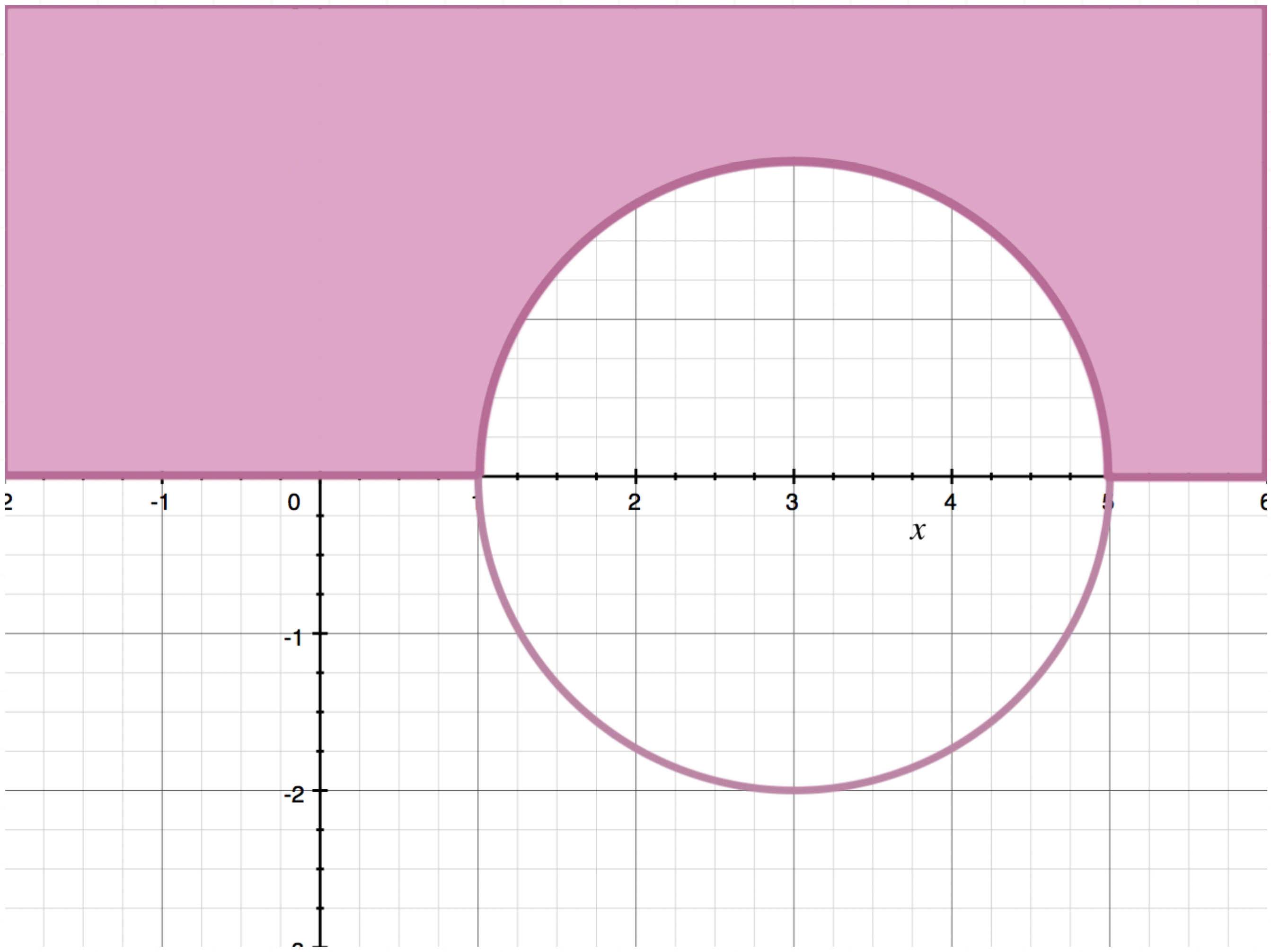
■ 1. The region D consists of two rings centered at the origin, where the inner ring is defined on $r = [1,2]$ and the outer ring is defined on $r = [3,4]$. Convert the double integral to polar coordinates, then find its value.

$$\int\int_D x + 2y \ dA$$



■ 2. The region D consists of all the points in the first and second quadrants outside the circle centered at $(3,0)$ with radius $r = 2$. Convert the double integral to polar coordinates, using the conversion formulas $x = x_0 + r\cos\theta$ and $y = y_0 + r\sin\theta$ for a circle shifted off the origin, then find its value.

$$\iint_D \frac{1}{((x-3)^2+y^2)^2}\ dA$$



■ 3. The region D consists of all the points inside the circle centered at $(-2,2)$ with radius $r=1$. Convert the double integral to polar coordinates, using the conversion formulas $x=x_0+r\cos\theta$ and $y=y_0+r\sin\theta$ for a circle shifted off the origin, then find its value.

$$\iint_D x^2+y^2\ dA$$

SKETCHING AREA

■ 1. Identify the region of integration given by the double integral.

$$\int_{0}^{2\pi}\int_{0}^{\frac{3}{\sqrt{1+1.25\cos^2\theta}}} f(r,\theta)\ dr\ d\theta$$

■ 2. Identify the region of integration given by the double integral.

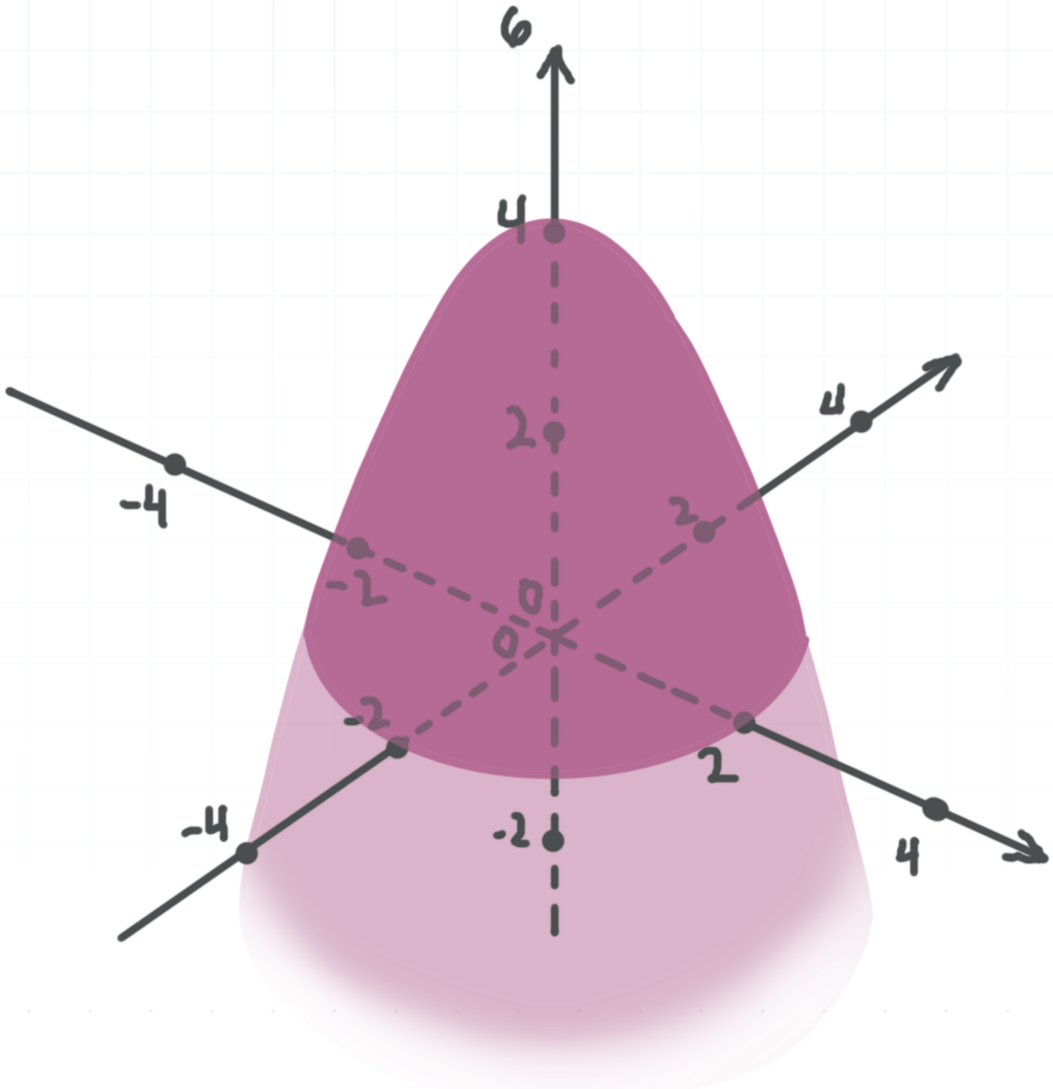
$$\int_{0}^{5}\int_{0}^{\frac{1}{3}\cos^{-1}\left(\frac{r}{5}\right)} f(r,\theta)\ d\theta\ dr$$

■ 3. Identify the region of integration given by the double integral.

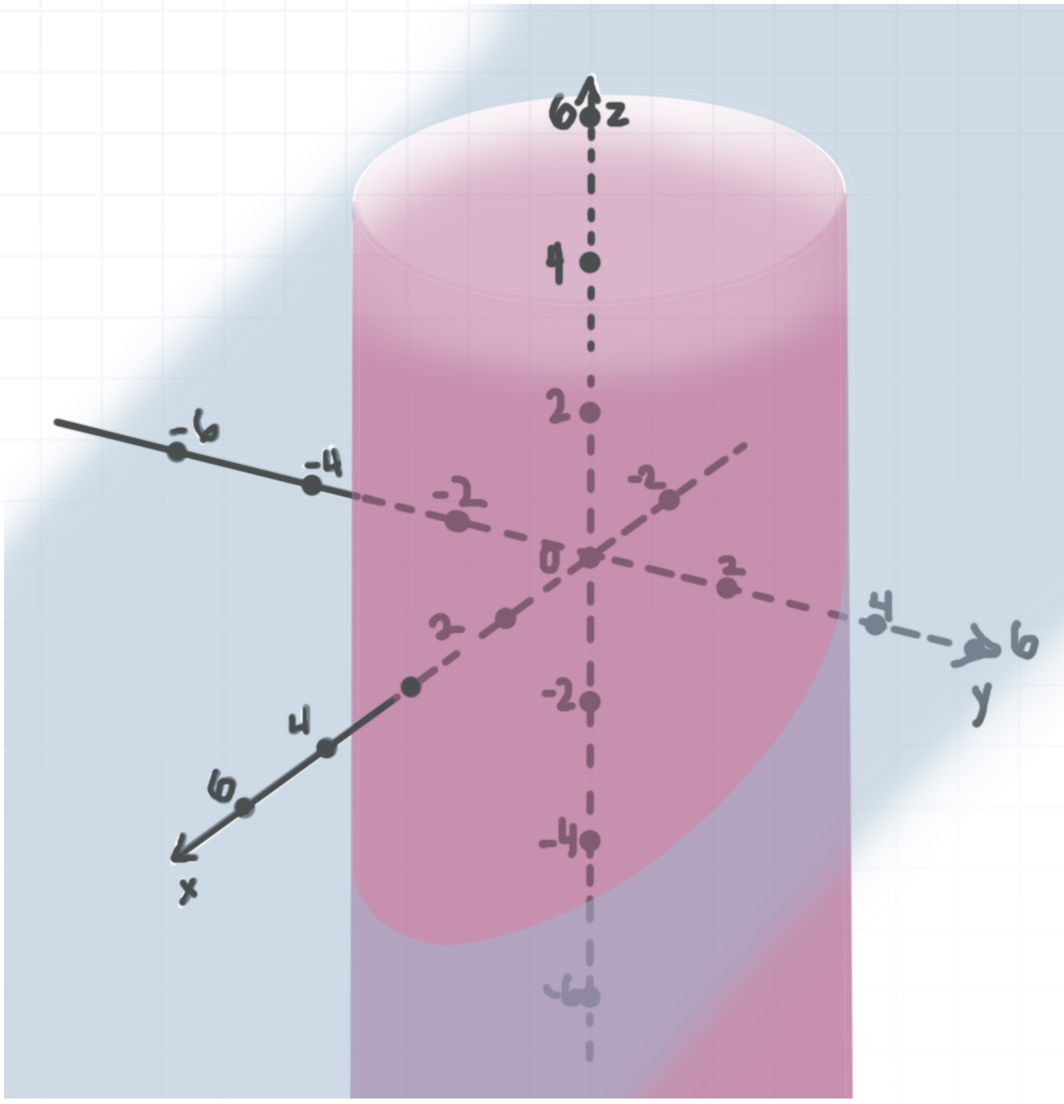
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{2\cos\theta}^{4\cos\theta} f(r,\theta)\ dr\ d\theta$$

FINDING AREA

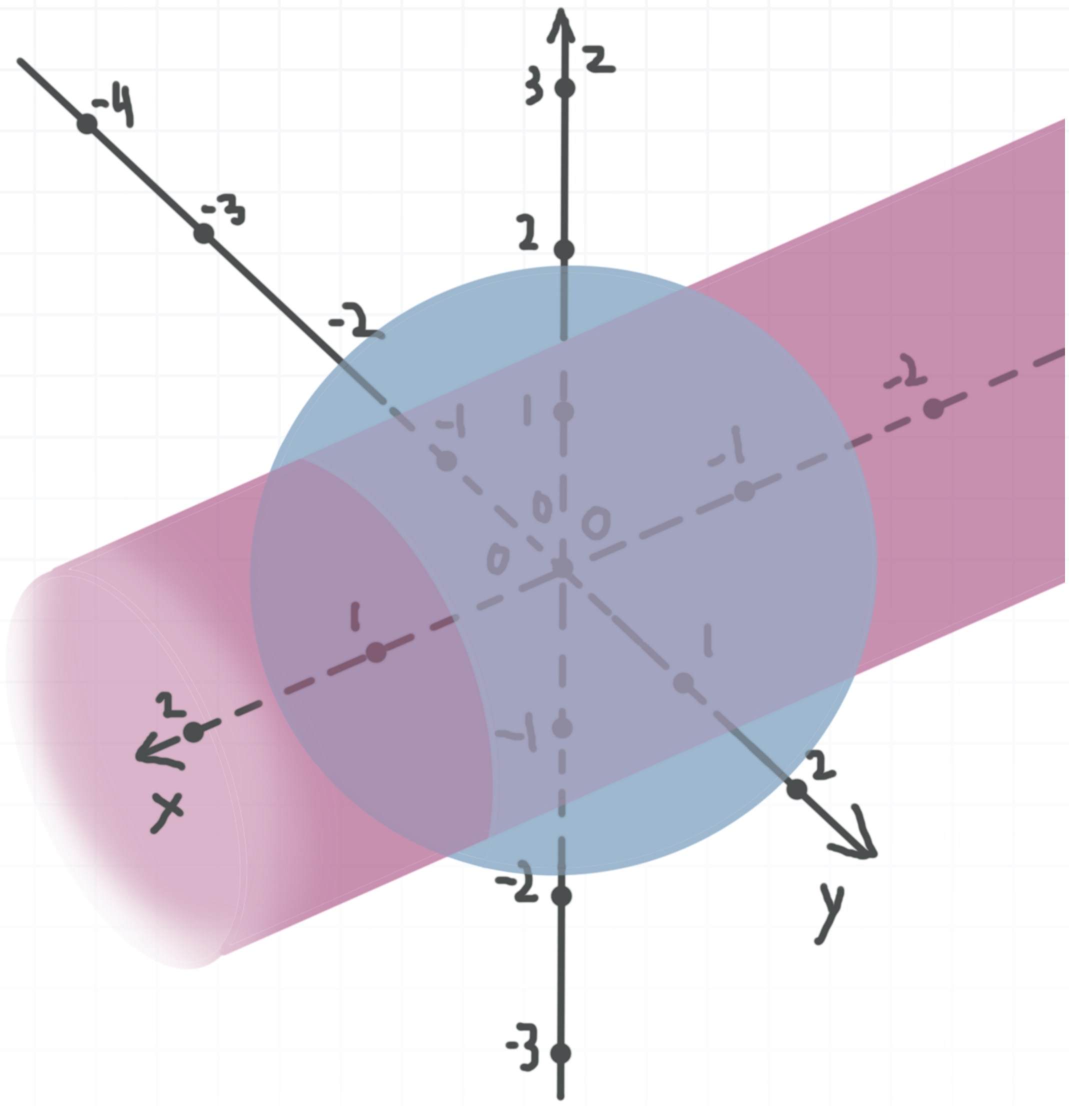
■ 1. Find area of the surface $x^2 + y^2 + z - 4 = 0$ above the xy-plane.



■ 2. Find area of the part of the plane $2x - y + 3z - 3 = 0$ that lies within the cylinder $(x - 3)^2 + (y - 2)^2 = 3^2$.

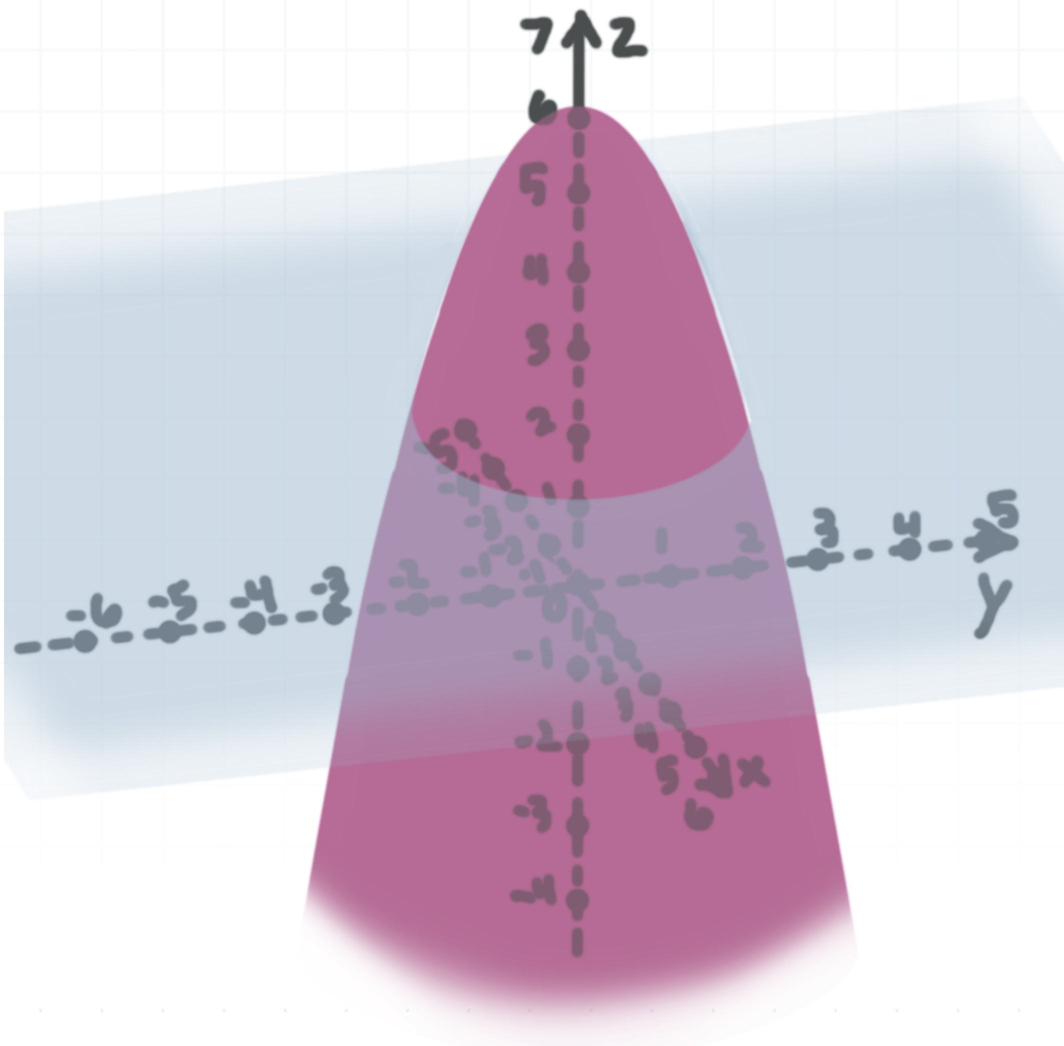


■ 3. Find area of the sphere $x^2 + y^2 + z^2 - 2 = 0$ that lies within the cylinder $y^2 + z^2 = 1$.

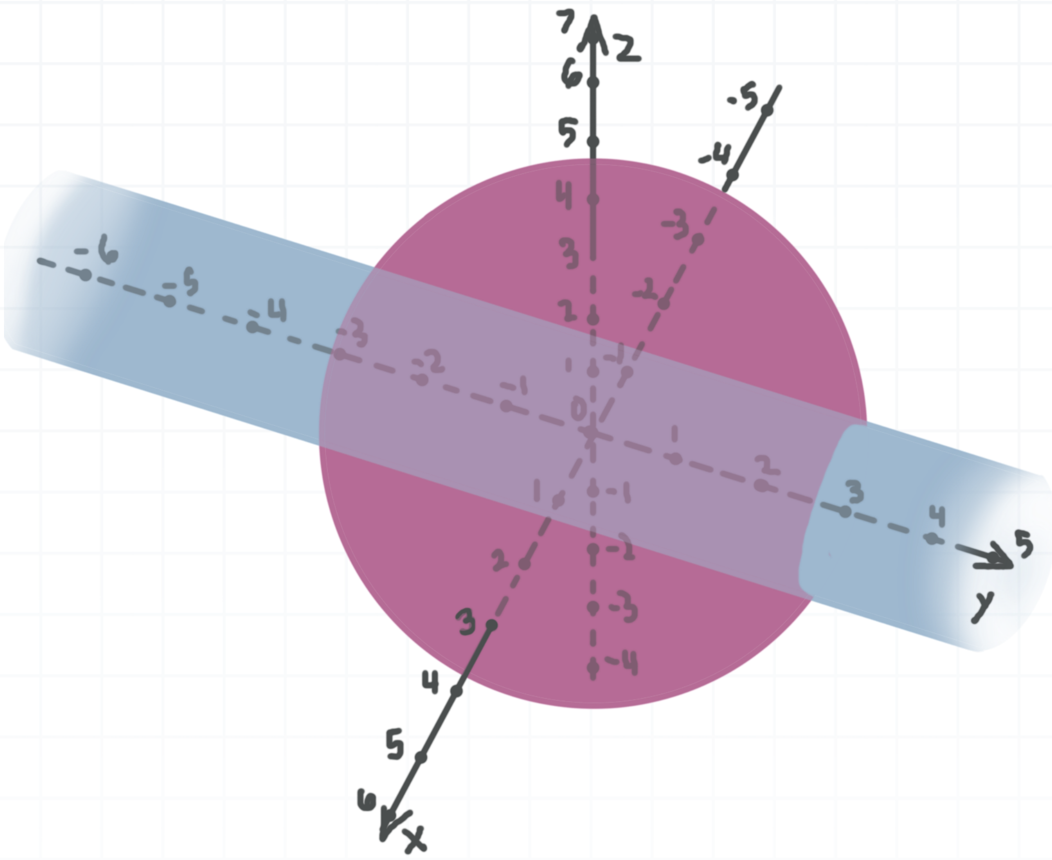

z
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
y
2
1
0
-1
-2
x

FINDING VOLUME

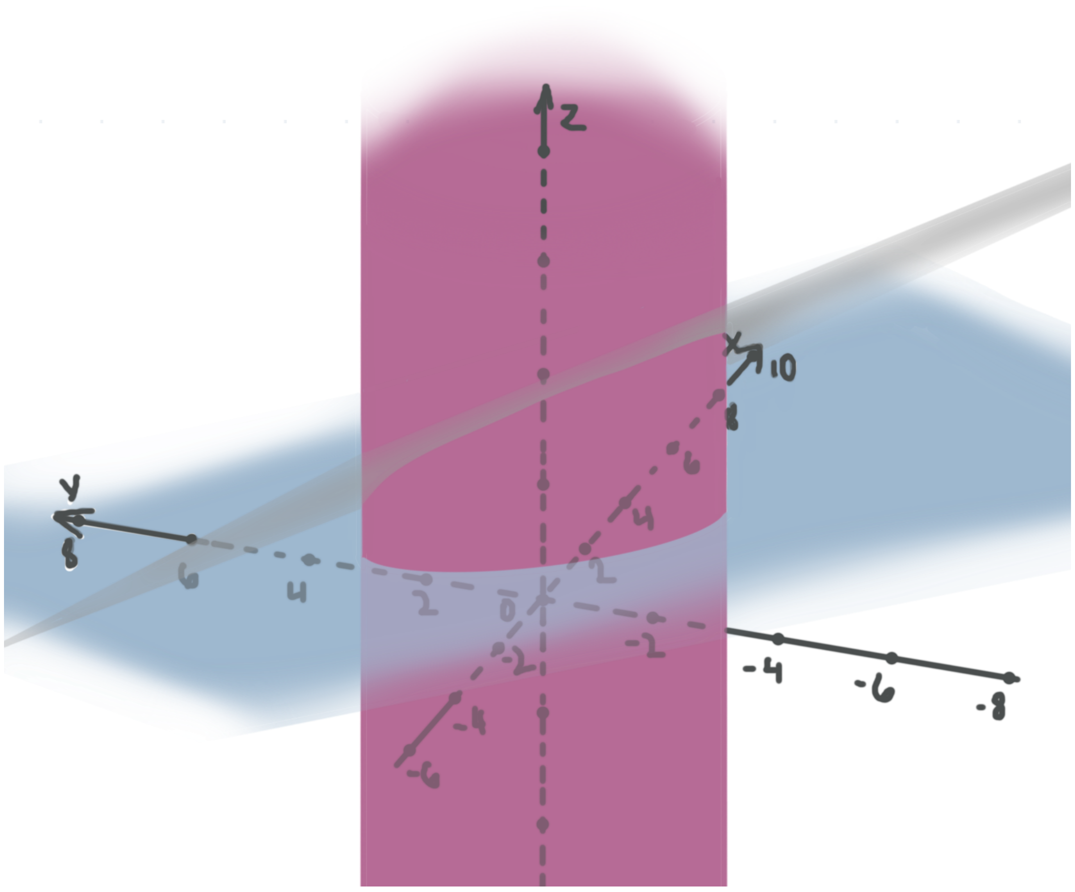
■ 1. Find the volume of the region bounded $x^2 + y^2 + z - 6 = 0$ and $z = 2$.



■ 2. Find volume of the sphere $x^2 + y^2 + z^2 - 9 = 0$ that lies within the cylinder $x^2 + z^2 = 1$.



3. Find the volume bounded by the cylinder $x^2 + y^2 = 9$, the plane $x + y + 5z - 6 = 0$, and the plane $x + 2y + 3z - 10 = 0$.



DOUBLE INTEGRALS TO FIND MASS AND CENTER OF MASS

■ 1. The circular disk with radius 12 has density $\delta = 1/(r+4)$, where r is the distance to the center of disk. Find the mass and center of mass of the disk.

■ 2. The rectangular plate with length 4 m and width 2 m has density $\delta = 2d$ kg/m^2, where d is the distance from its left 2 m side. Find the mass and center of mass of the plate.

■ 3. Some gas is distributed above the line with density $\delta = e^{-ad^2}$, where d is the distance to point A on the line, and a is a constant. Find the total mass of the gas and its center of mass.

MIDPOINT RULE FOR TRIPLE INTEGRALS

■ 1. Use the midpoint rule to approximate the value of the triple integral, using boxes with sides $2 \times 2 \times \pi$.

$$\int_{-2}^{2}\int_{0}^{4}\int_{-2\pi}^{2\pi} x^2 y \cos z \ dz \ dy \ dx$$

■ 2. Use the midpoint rule to approximate the value of the triple integral, where D is the cube with opposite corners $(0,1,-1)$ and $(4,5,3)$. Use cubes with side length 2.

$$\iiint_D \log_2((x+1)^5 y^2 (z+2)) \ dV$$

■ 3. Use the midpoint rule to approximate the value of the improper triple integral. Use cubes with side length 1.

$$\int_{0}^{1}\int_{0}^{1}\int_{0}^{\infty} \log_4(x)\frac{(y-1)^3}{z^2} \ dz \ dy \ dx$$

ITERATED INTEGRALS

1. Evaluate the iterated integral.

$$\int_{-2}^{3}\int_{0}^{\pi}\int_{-4}^{-2}\frac{2x^3}{x^2+1}\sin y(3z^2-4z+3)\ dz\ dy\ dx$$

2. Evaluate the iterated improper integral.

$$\int_{0}^{\infty}\int_{0}^{\infty}\int_{1}^{\infty}\frac{1}{(x+2y+z)^5}\ dz\ dy\ dx$$

3. Evaluate the iterated integral.

$$\int_{0}^{\frac{\pi}{2}}\int_{x}^{\frac{\pi}{4}}\int_{2y}^{x+\frac{\pi}{2}}\cos(x-2y+z)\ dz\ dy\ dx$$

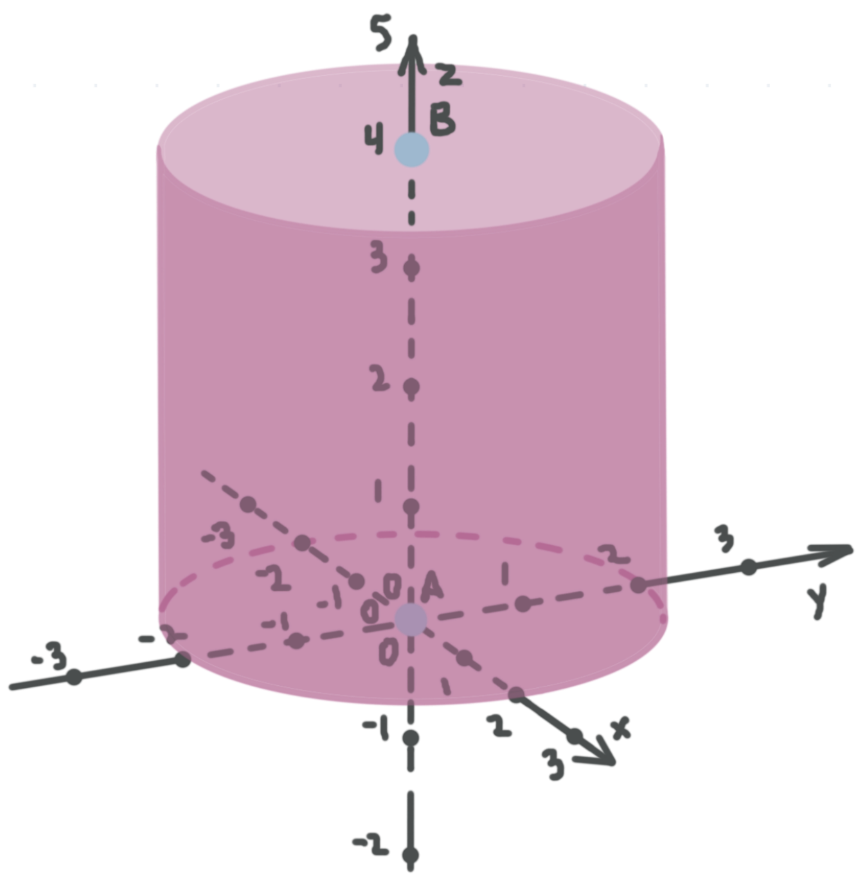
TRIPLE INTEGRALS

■ 1. Evaluate the triple integral, where D is the box with opposite corners $(5,0,1)$ and $(14,2,10)$.

$$\iiint_D y \log\left(\frac{z^4}{(x-4)^2 \cdot 10^{y^2}}\right)\ dV$$

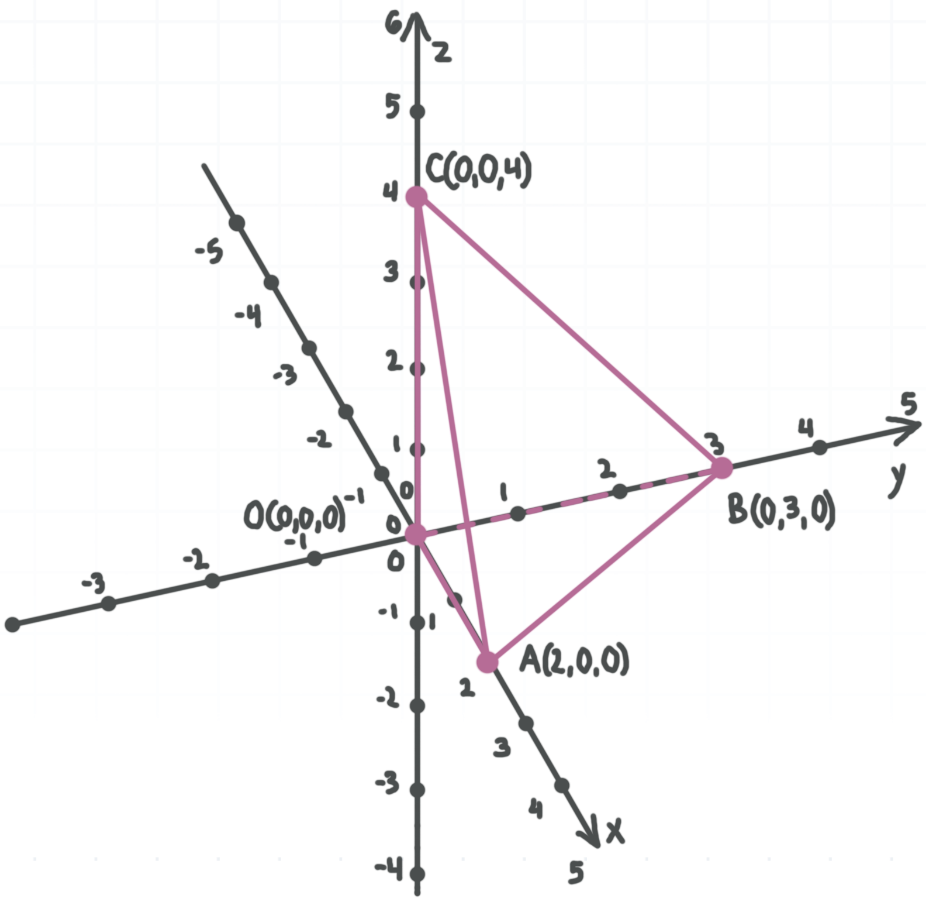
■ 2. Evaluate the triple integral, where D is the right circular cylinder with radius 2, height 4, and a base that lies in the xy-plane with center at the origin.

$$\iiint_D e^{0.5z}\sqrt{x^2+y^2}\ dV$$



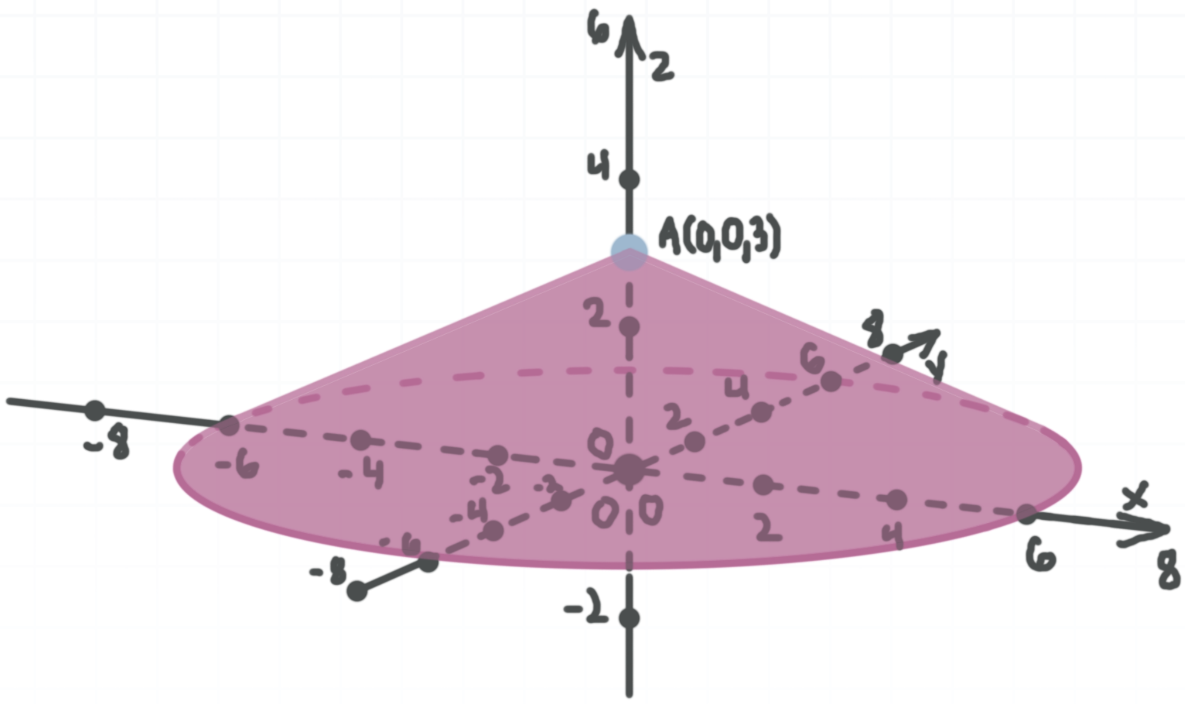
3. Evaluate the triple integral, where $ABCO$ is the irregular pyramid such that O is the origin and the vertices are $A(2,0,0)$, $B(0,3,0)$, and $C(0,0,4)$.

$$\iiint_{ABCO} 72xy\ dV$$

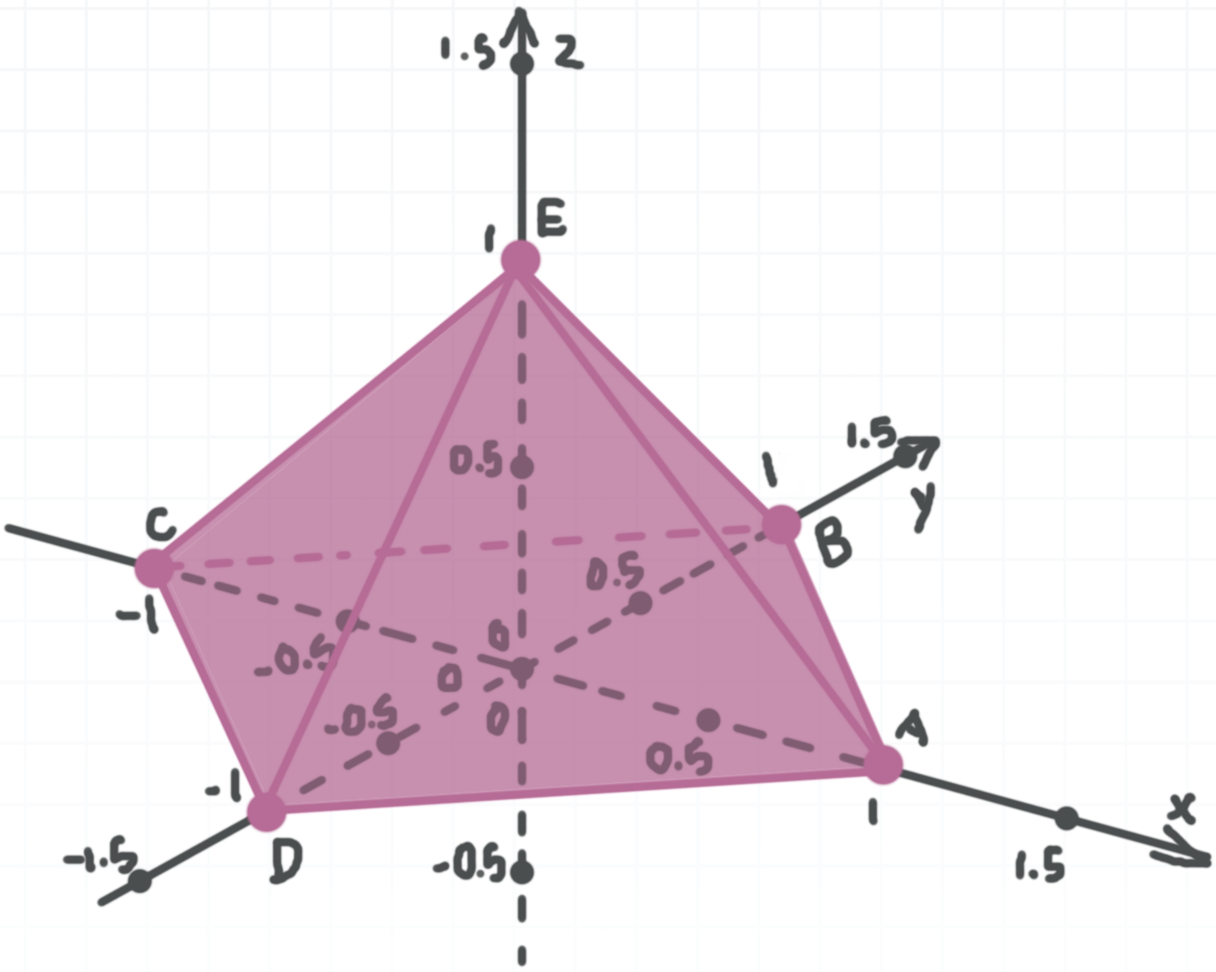


AVERAGE VALUE

■ 1. Use triple integrals to find the average value of the function $f(x, y, z) = x^2 + y^2 + z^2$ over a right circular cone with radius $R = 6$, height $h = 3$, and a base that lies in the xy-plane with center at the origin.



■ 2. Use triple integrals to find the average value of the function $f(x, y, z) = |xyz|$ over a regular pyramid $ABCDE$, where $A(1,0,0)$, $B(0,1,0)$, $C(-1,0,0)$, $D(0,-1,0)$, and $E(0,0,1)$.



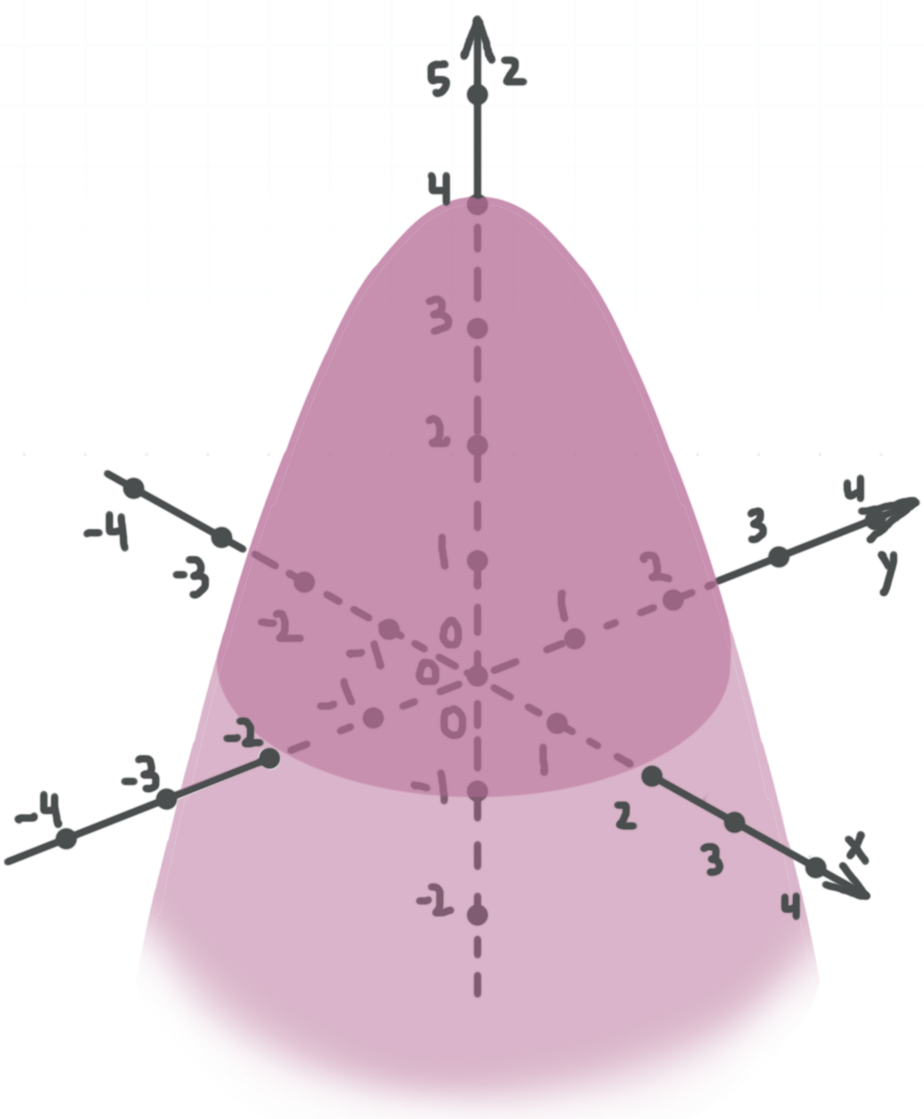
■ 3. Use triple integrals to find the average value of the function $f(x, y, z) = 2x - 3y + z$ over a layer bounded by the planes $z = 2$ and $z = 4$.

FINDING VOLUME

1. Find the volume given by the triple integral.

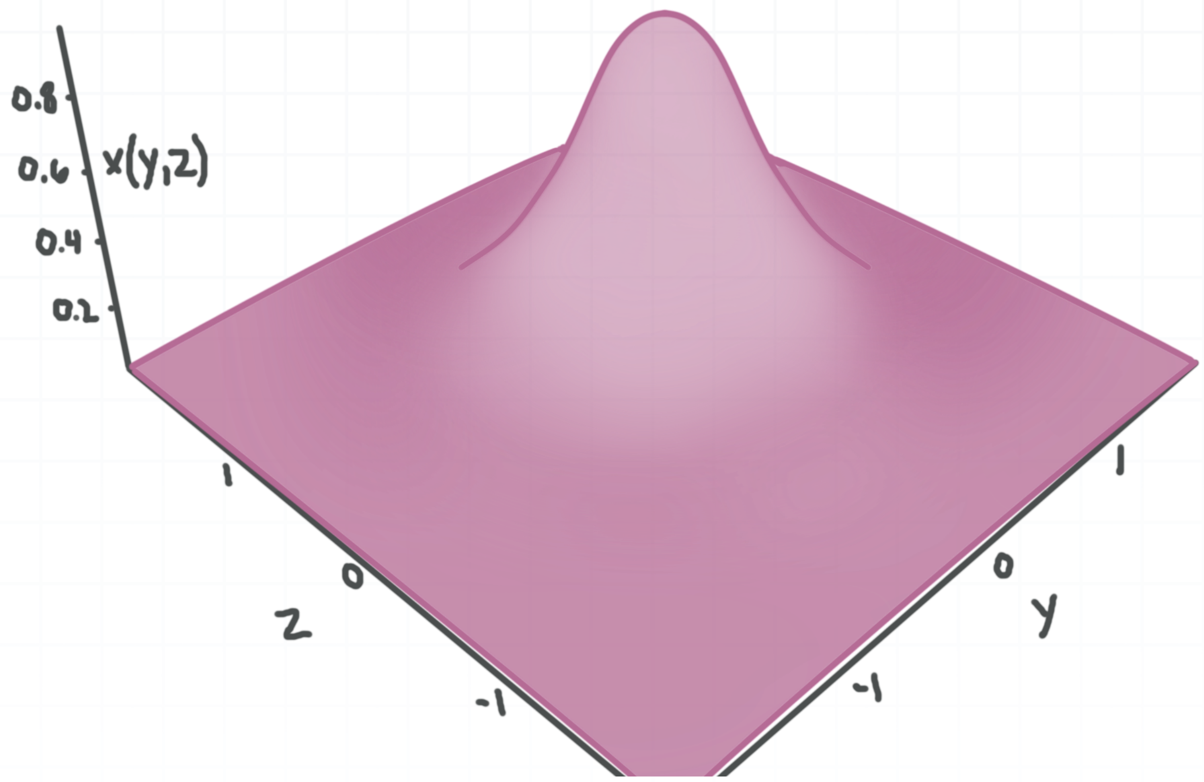
$$\int_{-4}^{6}\int_{3-2x^2}^{10}\int_{2x-y}^{12-y} dz\ dy\ dx$$

2. Use a triple integral to find the volume of the solid bounded by the circular paraboloid $4 - x^2 - y^2 - z = 0$ and the xy-plane.



■ 3. Use a triple integral to find the volume of the solid bounded by the surface $x=g(y,z)$ and the yz-plane.

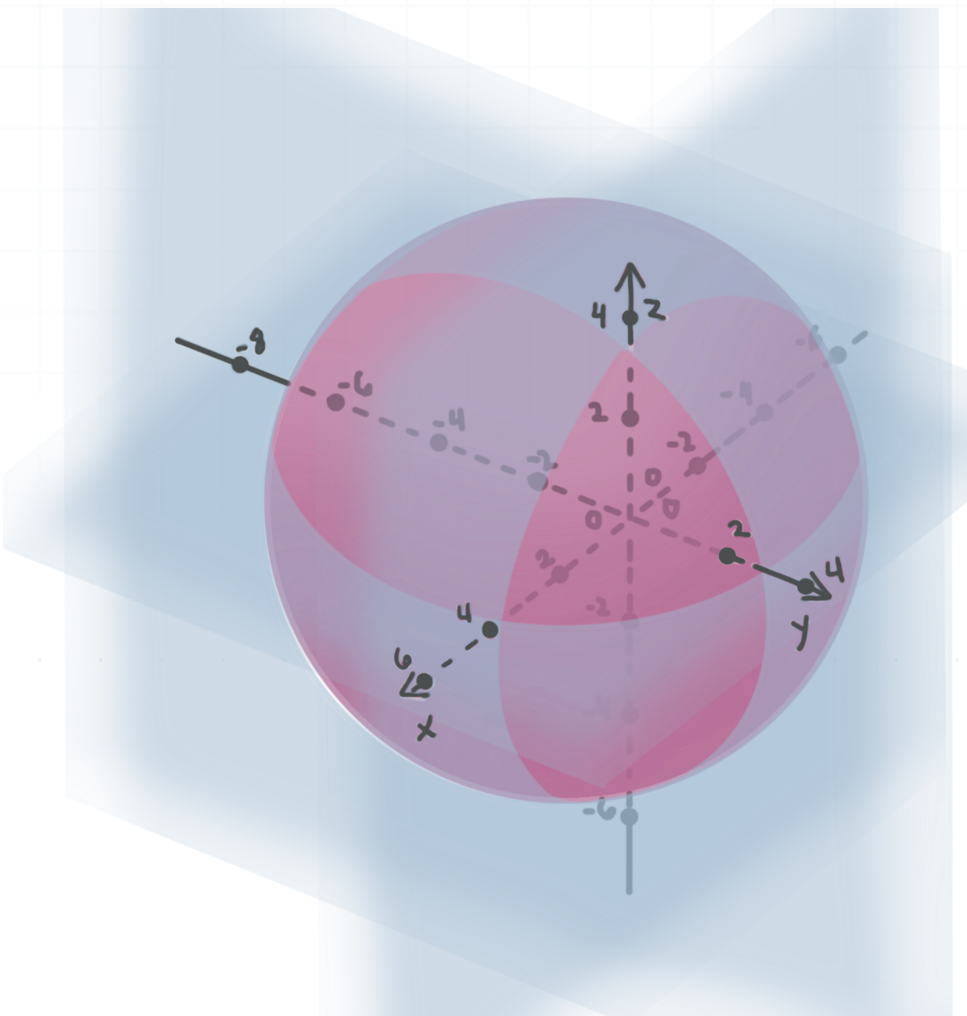
$$x=\frac{1}{(y^2+z^2+1)^2}$$



EXPRESSING THE INTEGRAL SIX WAYS

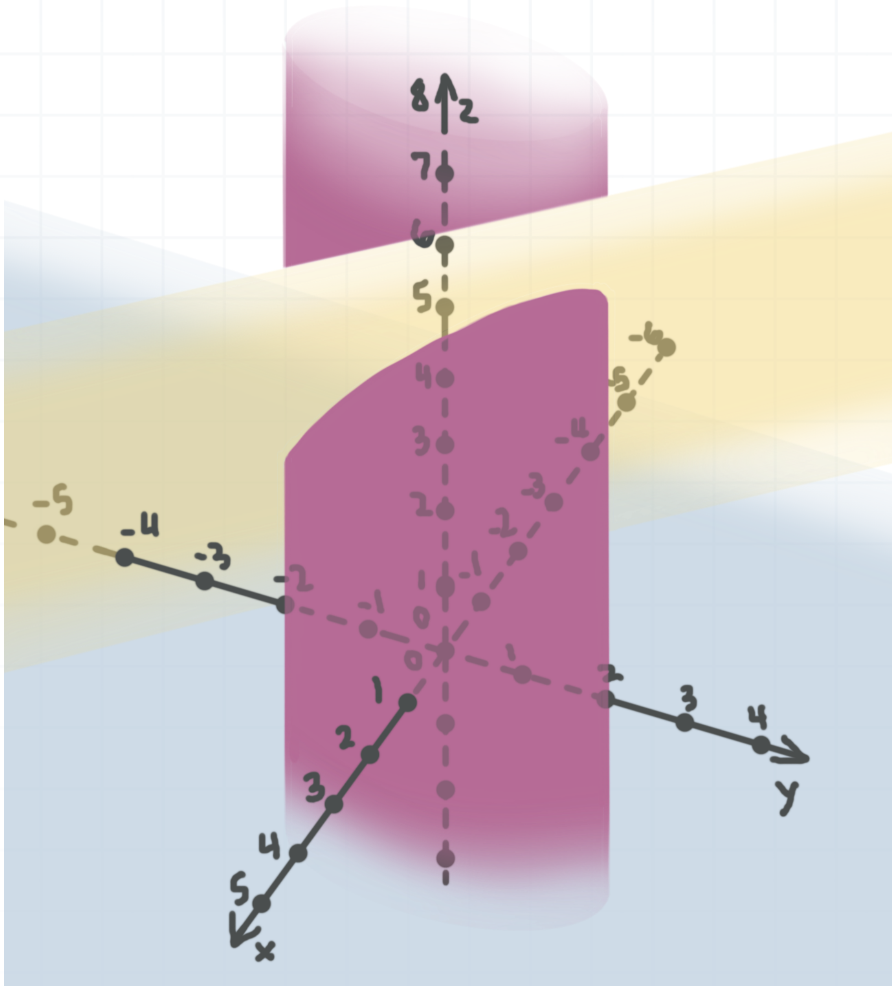
■ 1. Represent the triple integral as an iterated integral in which the order of integration is $dx\ dz\ dy$, where E is the part of the sphere with center at $(-1,-2,-1)$ and radius 25, lying in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

$$\iiint_E f(x,y,z)\ dV$$



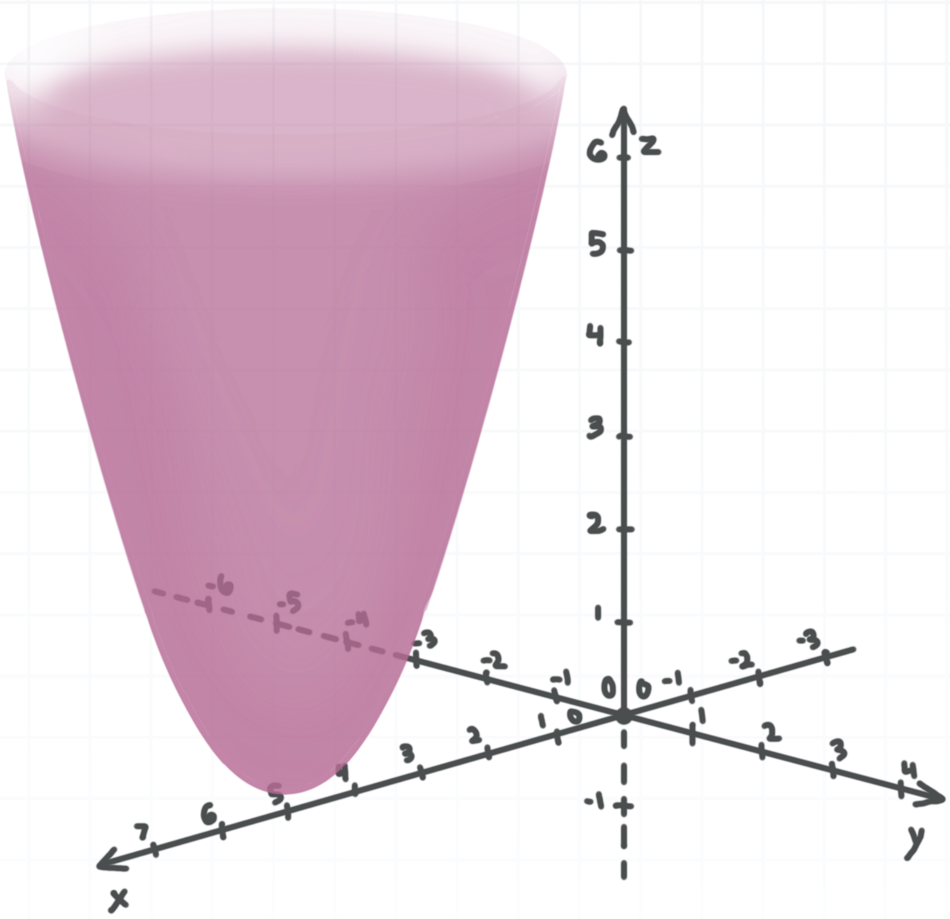
■ 2. Represent the triple integral as an iterated integral using the order of integration $dz\ dy\ dx$, where E is the part of the cylinder $4x^2+y^2=4$, between the planes $z=-3$ and $x+y-z+4=0$.

$$\iiint_E f(x, y, z) \, dV$$



■ 3. Represent the triple integral as an improper iterated integral using the order $dx \ dy \ dz$, where E is interior of the circular paraboloid $x^2 - 4x + y^2 + 6y - z + 12 = 0$.

$$\iiint_E f(x, y, z) \, dV$$

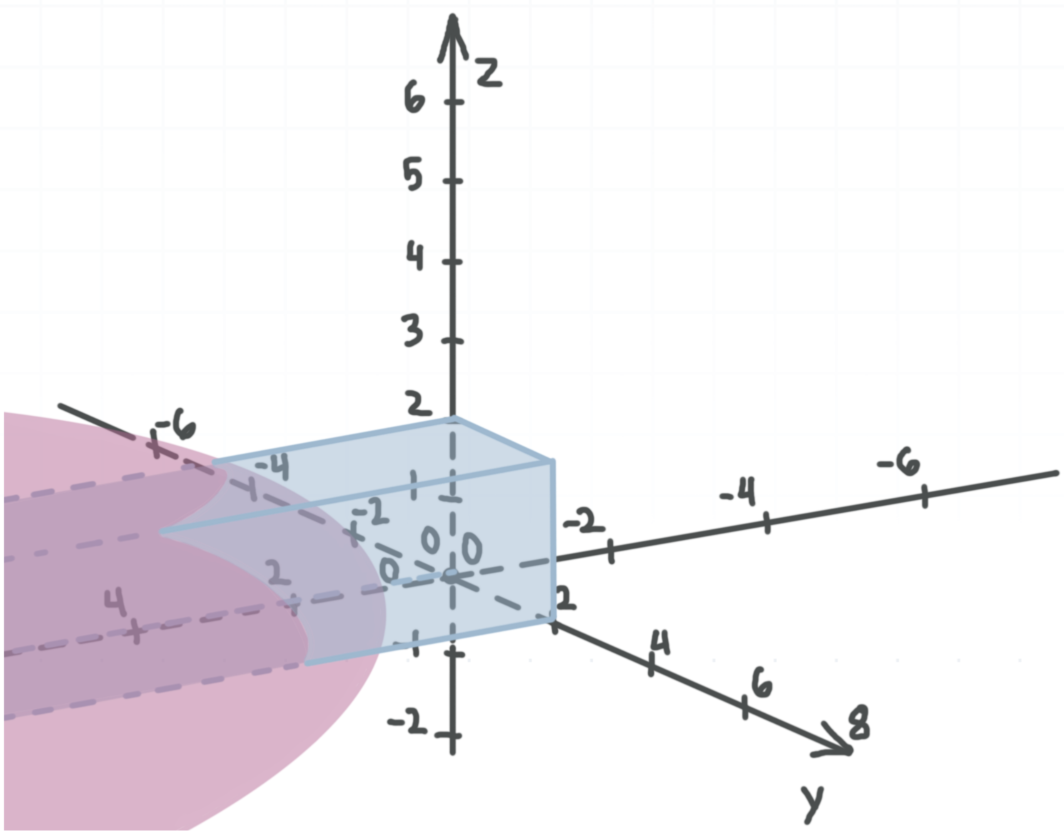


z
x
y

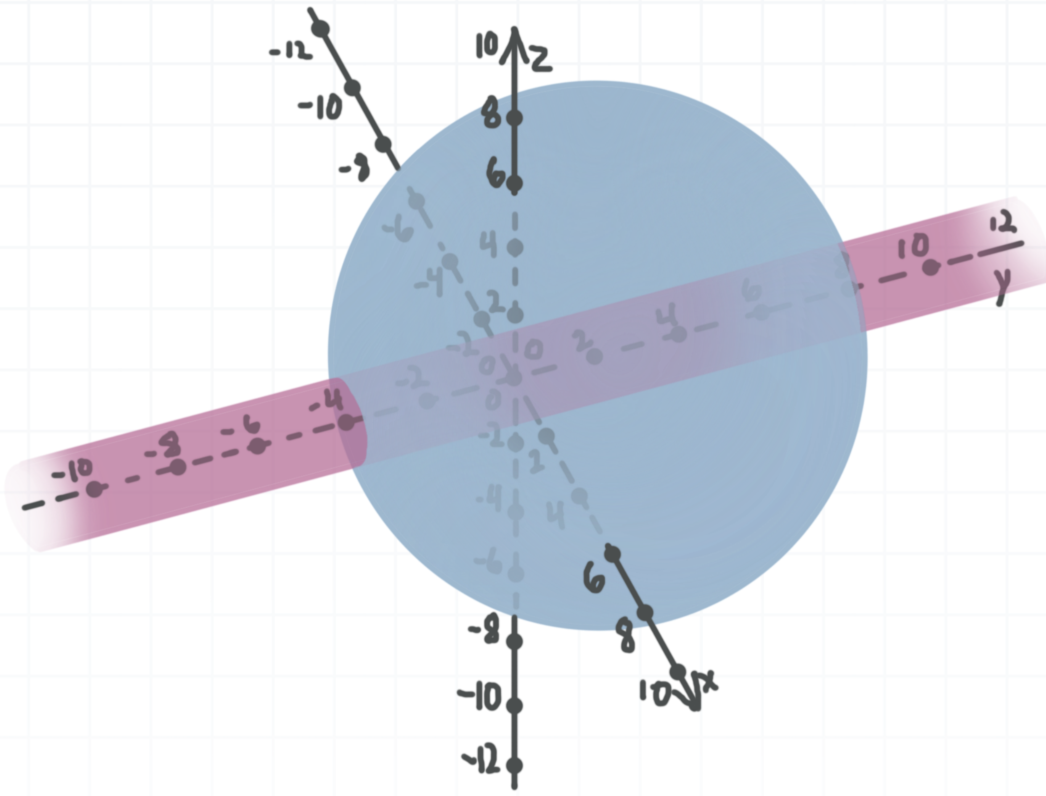
TYPE I, II, AND III REGIONS

■ 1. Evaluate the triple integral, where E is the region that lies in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$), and is bounded by the surfaces $y = 2$, $z = 2$, and $x - 0.5y^2 - 0.5z^2 - 1 = 0$.

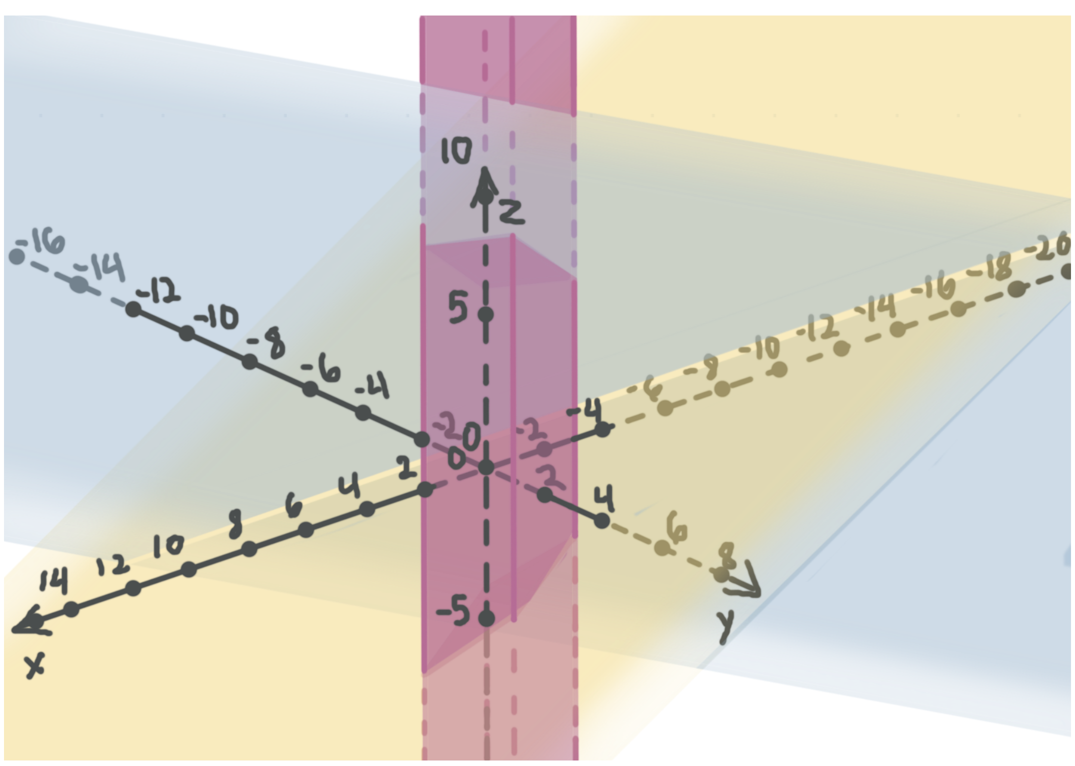
$$\iiint_E 4x + 2y - 2z \ dV$$



■ 2. Use a triple integral to find the volume of the region E that's bounded by the cylinder $x^2 + z^2 = 1$ and the sphere $x^2 + (y - 2)^2 + z^2 = 36$.



■ 3. Use a triple integral to find the volume of the region E that lies in the first and fifth octants ($x \geq 0$, $y \geq 0$), and is bounded by the planes $x = 2$, $y = 3$, $2x + y - 2z + 12 = 0$, and $x - y + z + 4 = 0$.



CYLINDRICAL COORDINATES

■ 1. Evaluate the triple integral given in cylindrical coordinates, where $f(r, \theta, z) = (3r - 12z^2)\cos\theta$.

$$\int_{-1}^{1} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{2}^{3} f(r, \theta, z)\ r\ dr\ d\theta\ dz$$

■ 2. Identify the solid given by the following iterated integral in cylindrical coordinates.

$$\int_{-4}^{6} \int_{0}^{\pi} \int_{0}^{2} f(r, \theta, x)\ r\ dr\ d\theta\ dx$$

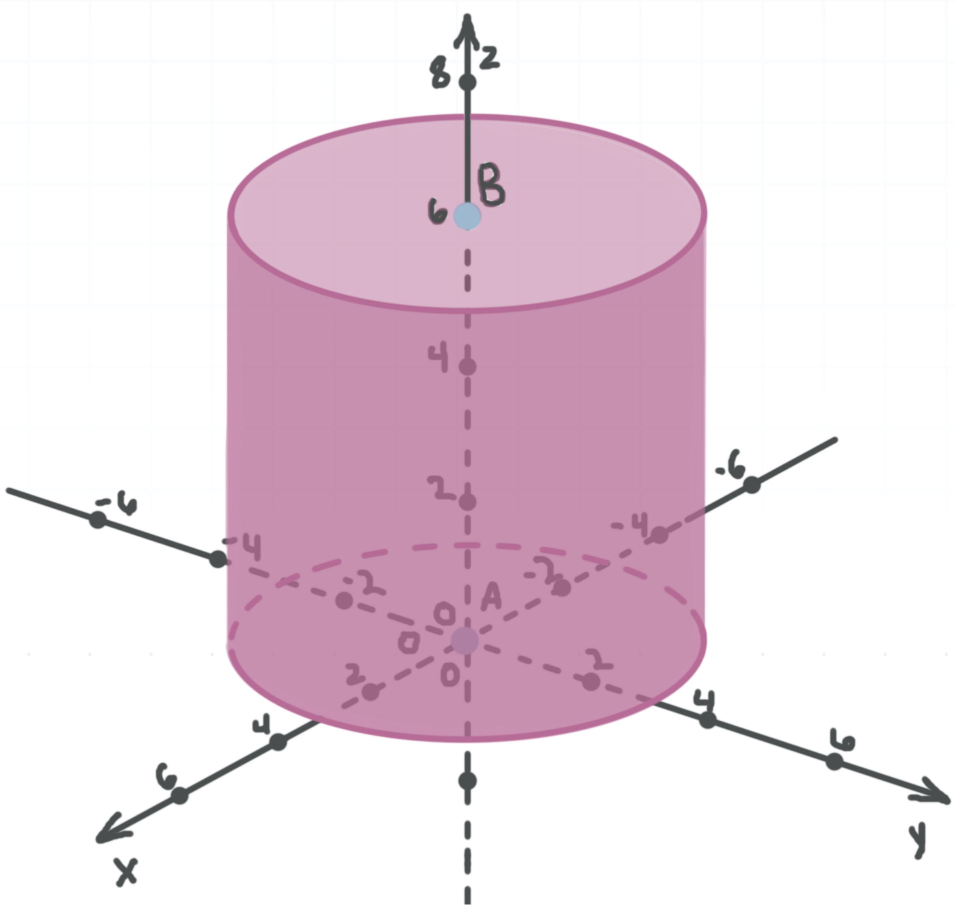
■ 3. Identify the solid given by the iterated improper integral in cylindrical coordinates.

$$\int_{2}^{\infty} \int_{0}^{2\pi} \int_{0}^{\sqrt{2y-4}} f(r, \theta, y)\ r\ dr\ d\theta\ dy$$

CHANGING TRIPLE INTEGRALS TO CYLINDRICAL COORDINATES

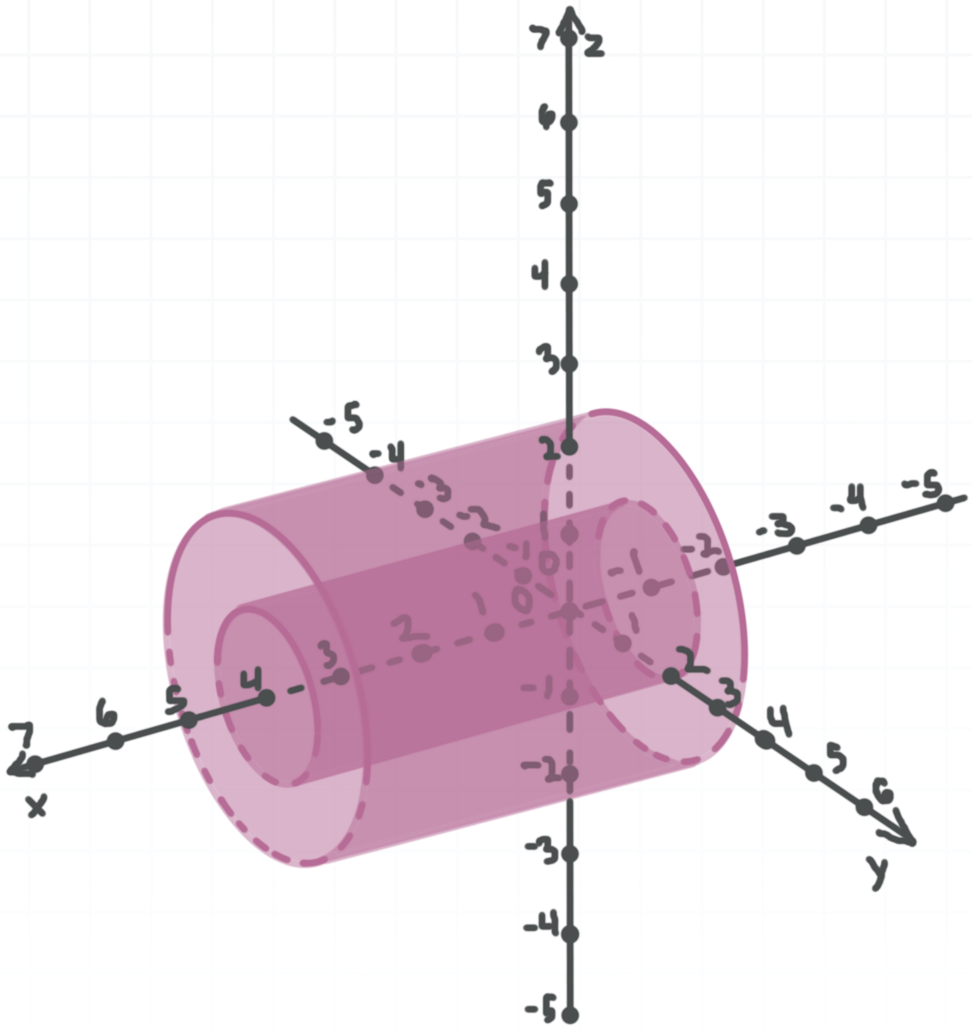
■ 1. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the right circular cylinder with radius 3, height 6, and a base that lies in the xy-plane with center at the origin.

$$\iiint_E (x^2+y^2)2^z \ dV$$



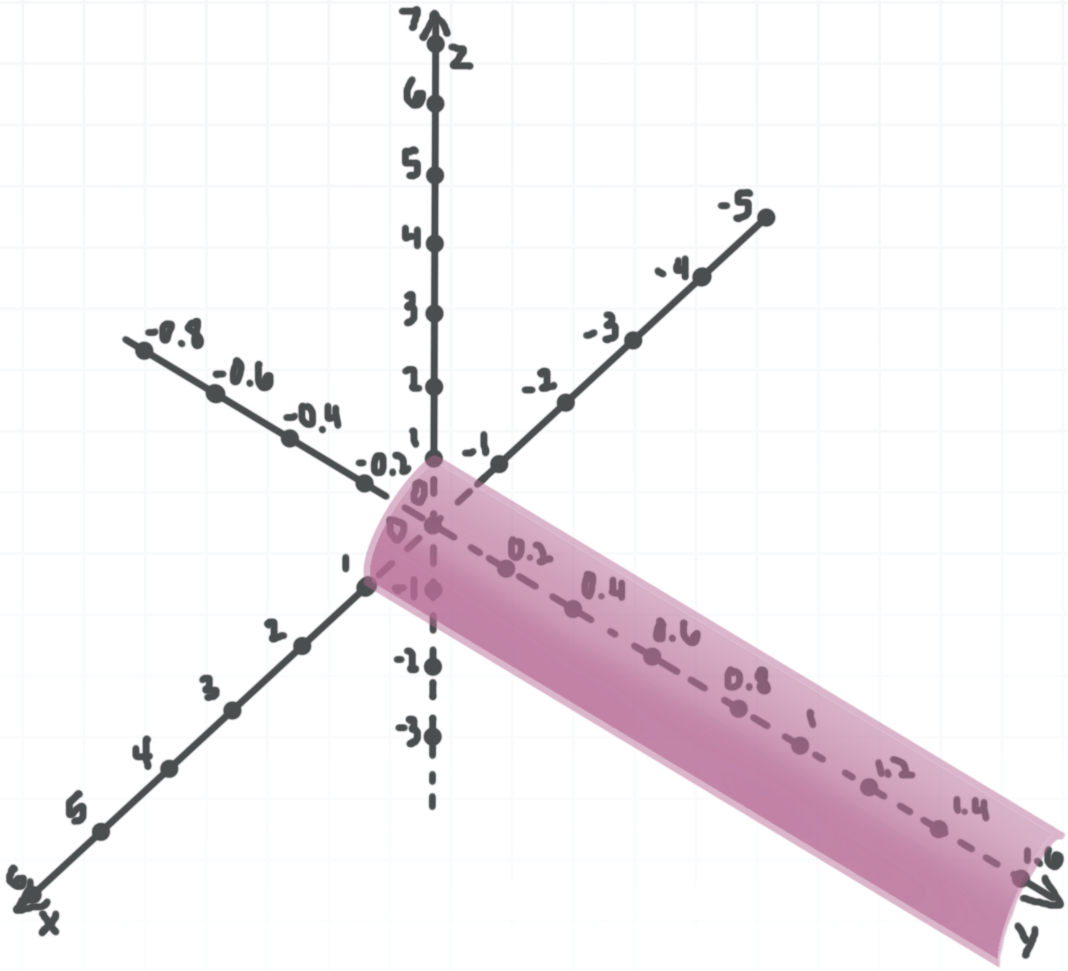
■ 2. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the set of points between right circular cylinders with radius 1 and 2, height 5, cylinder axes are x -axis, and bases that lie in the planes $x=-1$ and $x=4$.

$$\iiint_E \frac{x+y+z}{y^2+z^2}\, dV$$



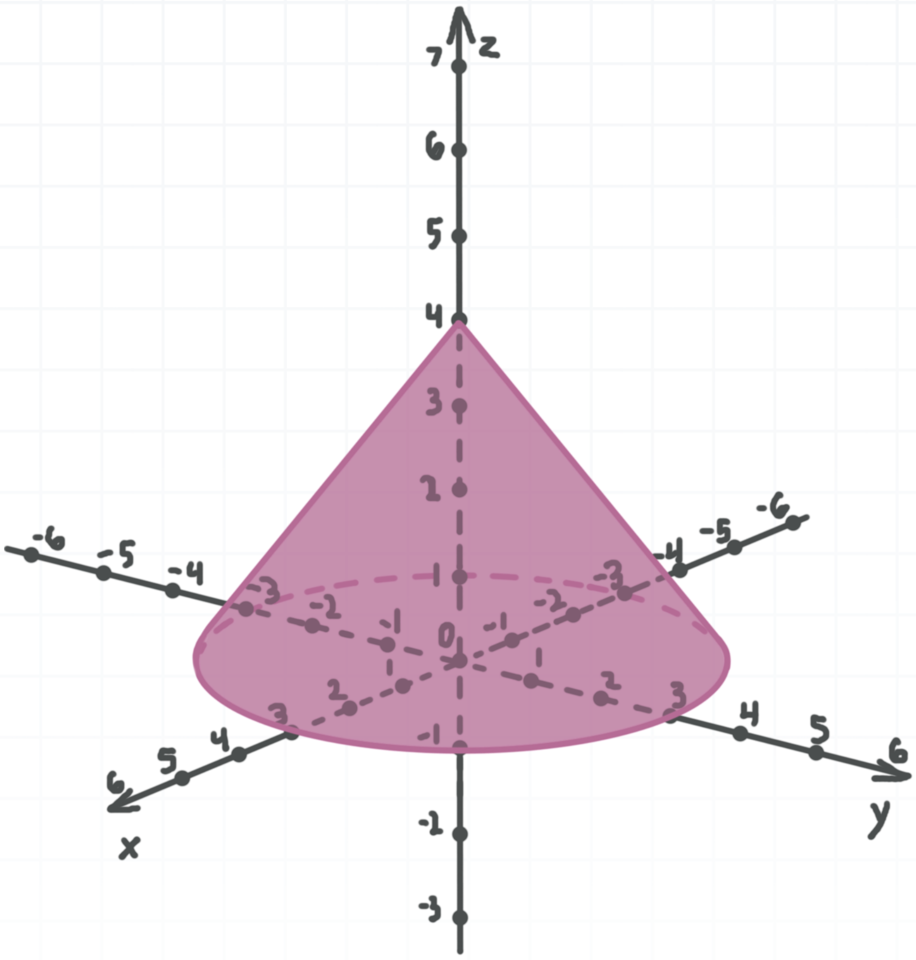
■ 3. Evaluate the improper triple integral by changing it to cylindrical coordinates, where E is part of the cylinder $x^2+z^2=1$ that lies in the first octant.

$$\iiint_E 2e^{-x^2-y^2-z^2}\, dV$$



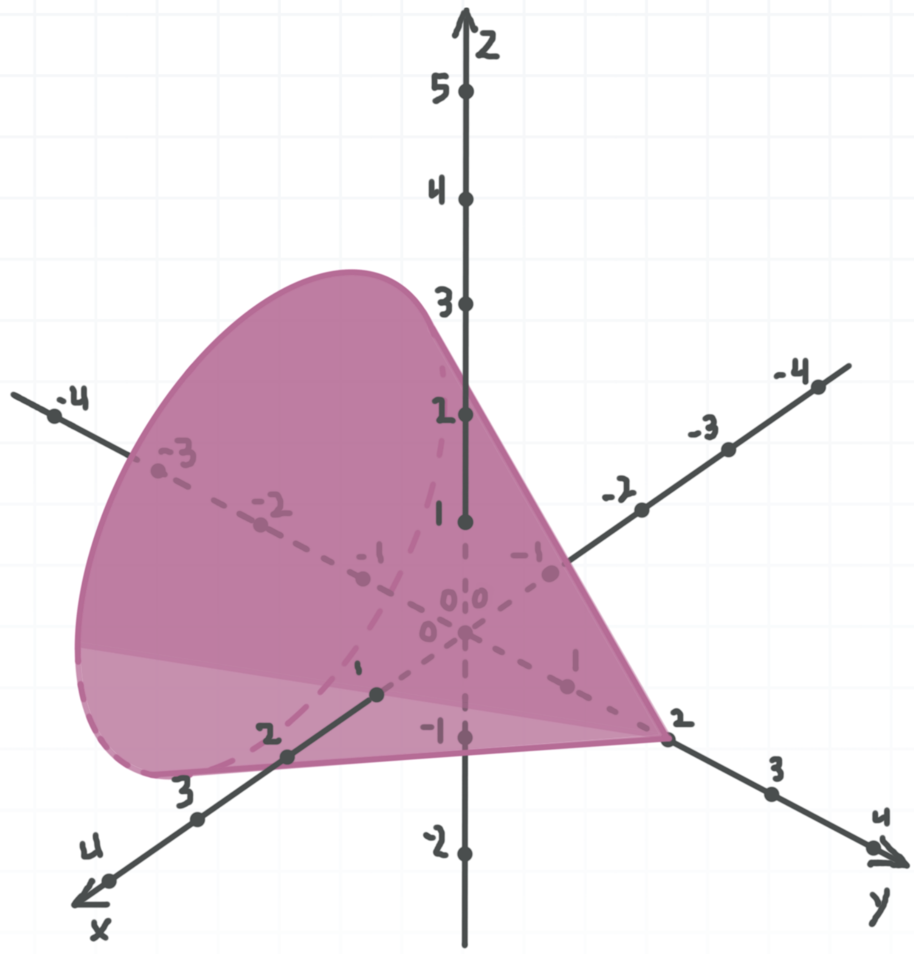
■ 4. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the right circular cone with radius 3, vertex at the point $(0,0,4)$, and a base that lies in the xy-plane with its center at the origin.

$$\iiint_E 48x^2y^2(z+2)\ dV$$



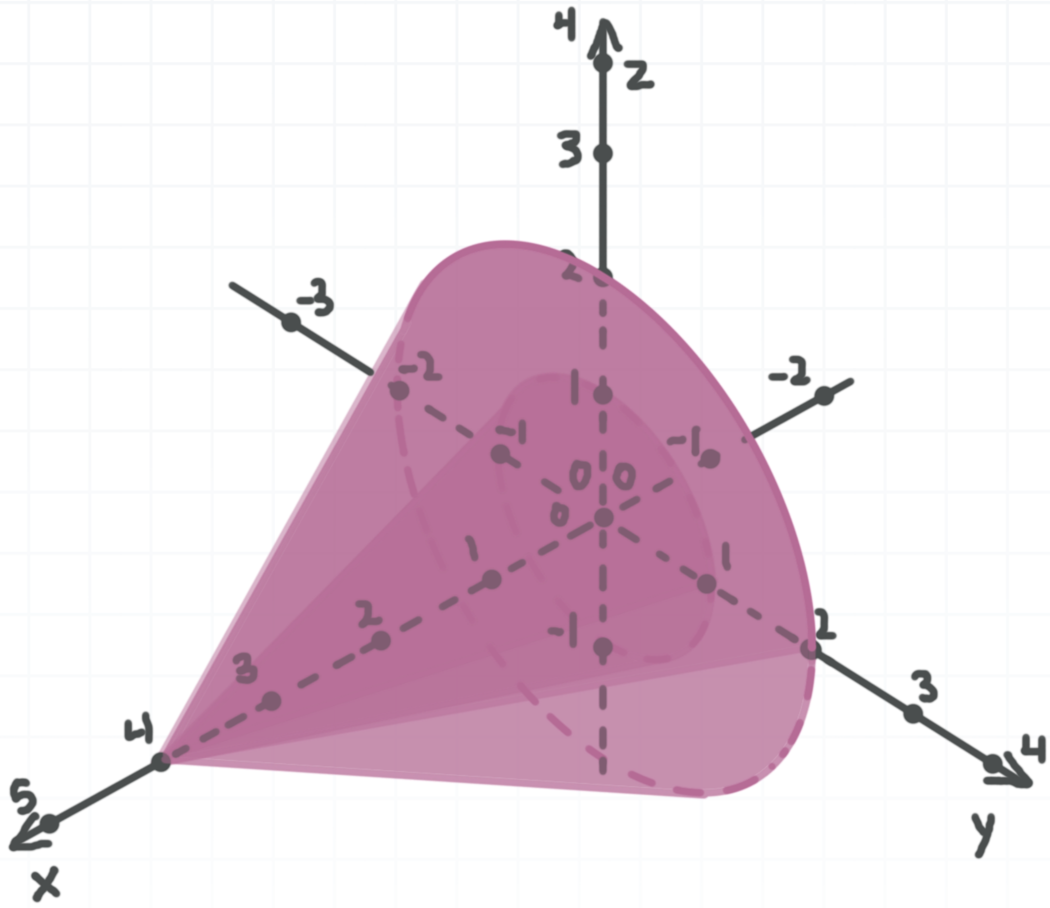
■ 5. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the right circular cone with radius 2, vertex at the point $(0,2,0)$, and a base that lies in the plane $y=-2$ with its center at the point $(0,-2,0)$.

$$\iiint_E \frac{(x+z)^2}{(y+4)}\ dV$$



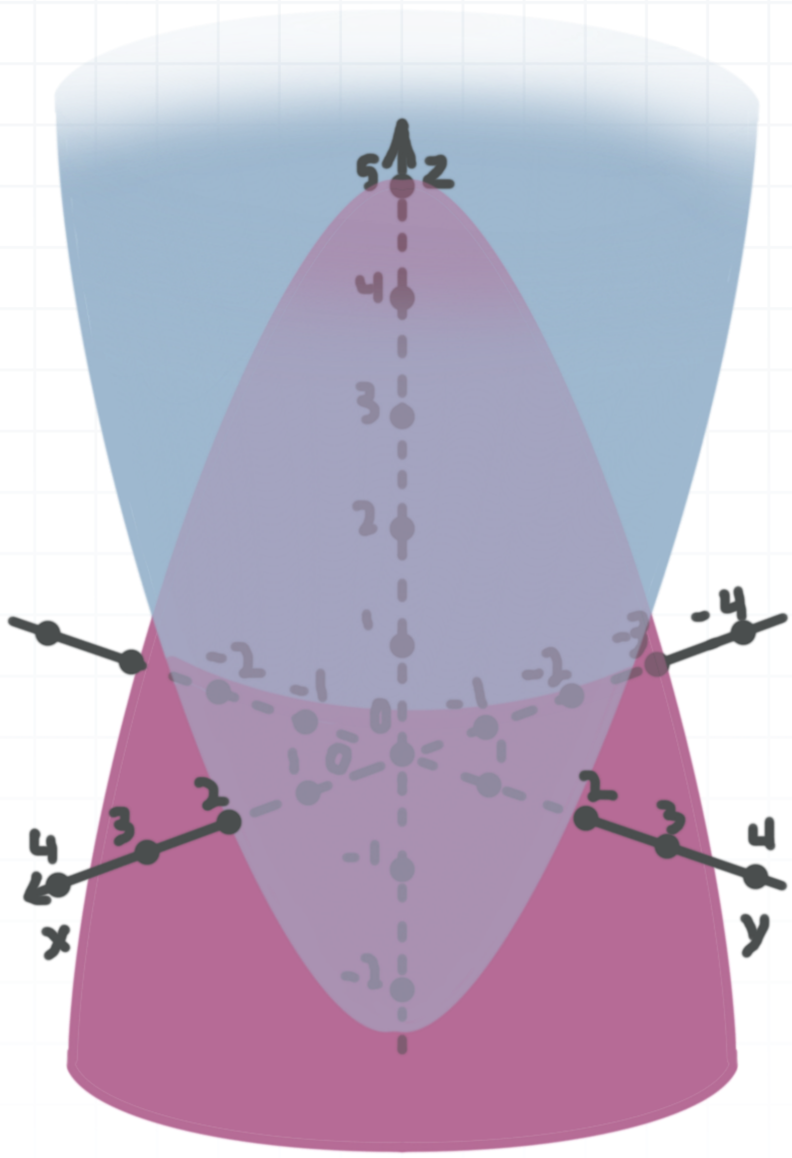
■ 6. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the set of points between two right circular cones with radii 1 and 2, vertexes at the point $(4,0,0)$, and bases that lie in the yz-plane with center at the origin.

$$\iiint_E x^2 - y^2 - z^2 \, dV$$



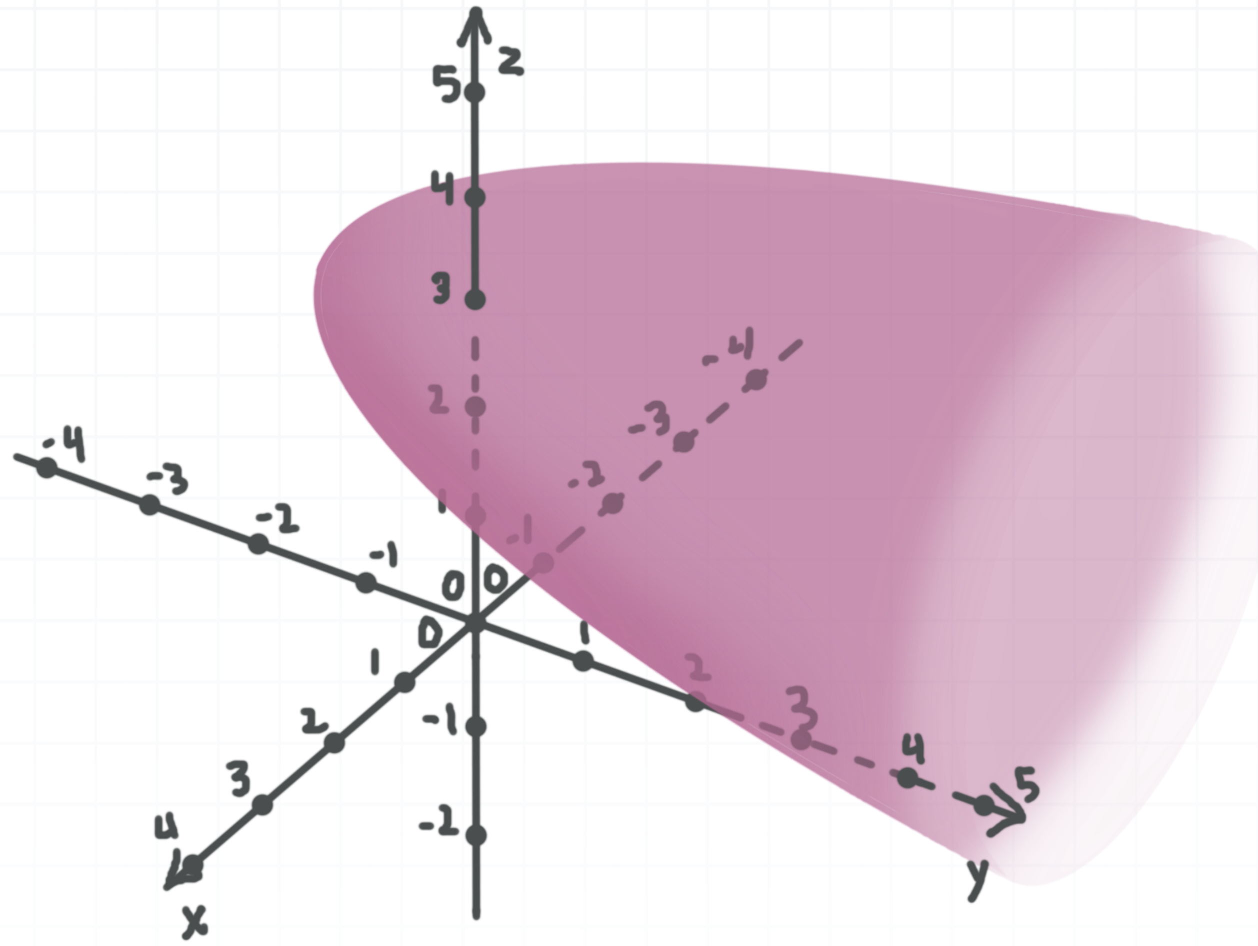
■ 7. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the solid bounded by the surfaces $x^2+y^2+z-5=0$ and $x^2+y^2-z-3=0$.

$$\iiint_E 3\sqrt{x^2+y^2}\ dV$$



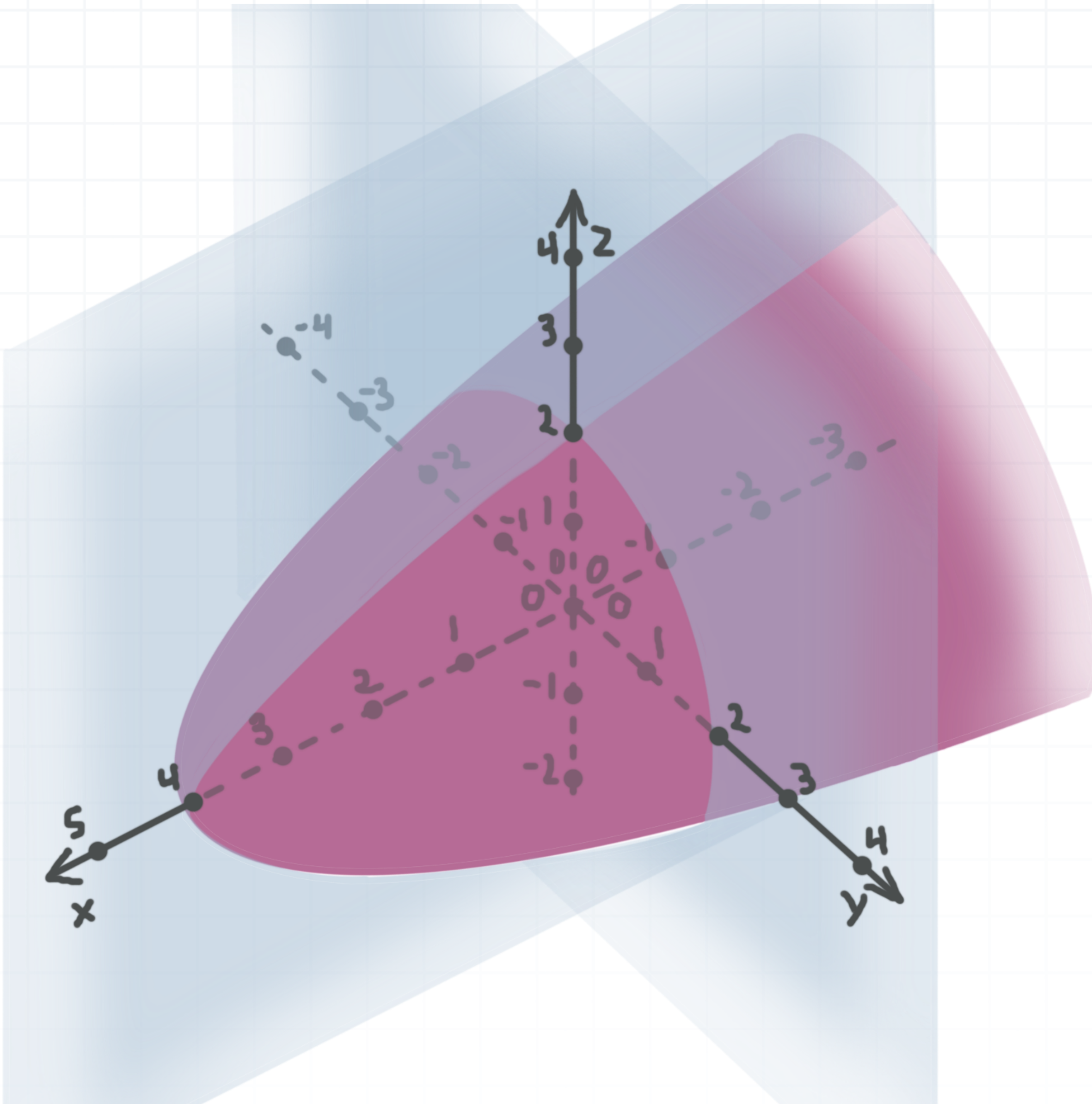
■ 8. Evaluate the triple improper integral by changing it to cylindrical coordinates, where E is the interior of the surface $(x+1)^2 + (z-2)^2 = y+2$.

$$\iiint_E xz10^{-y}\ dV$$



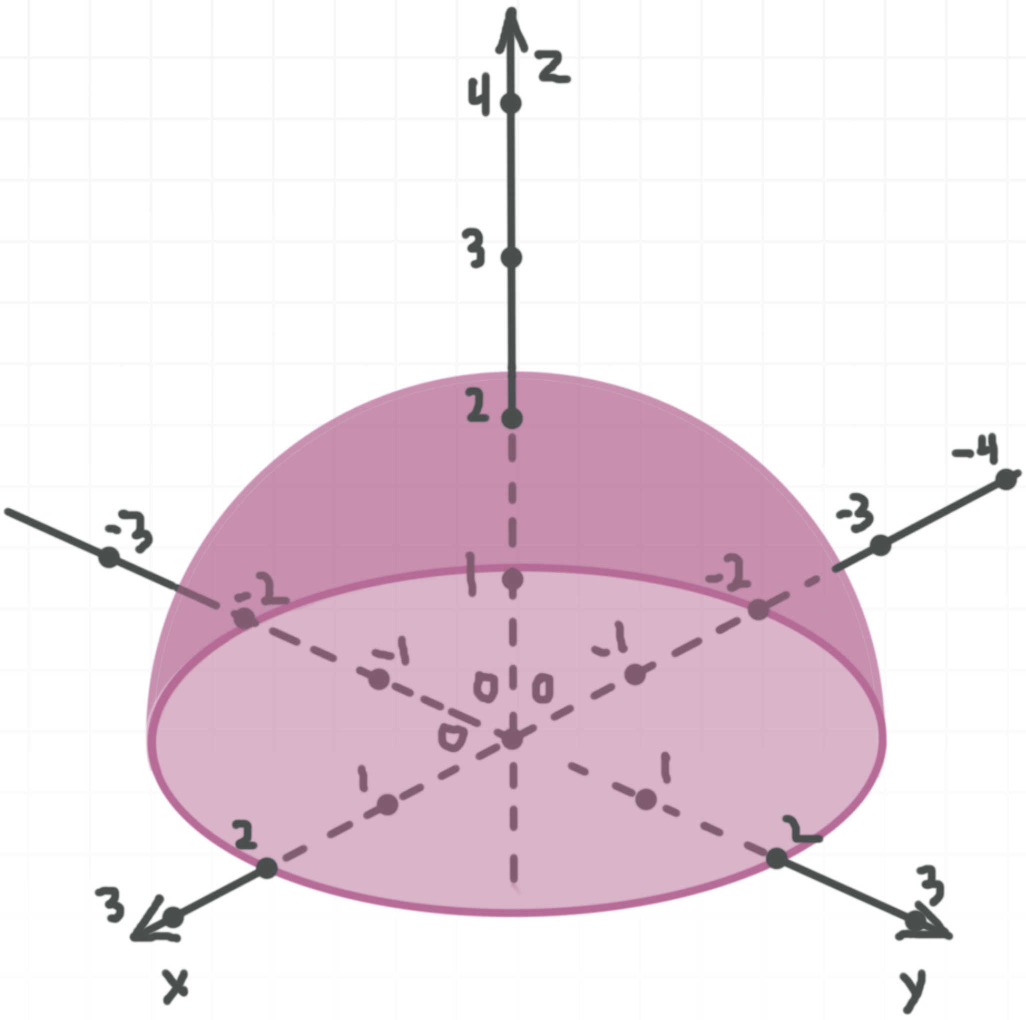
■ 9. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the interior of the surface $y^2 + z^2 + x - 4 = 0$ that lies within the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

$$\iiint_E x + 8yz \ dV$$



■ 10. Evaluate the triple integral by changing it to cylindrical coordinates, where E is the hemisphere with center at the origin, radius 2, and $z \geq 0$.

$$\iiint_E 4x^2 + 4y^2 - 12z^2 \ dV$$

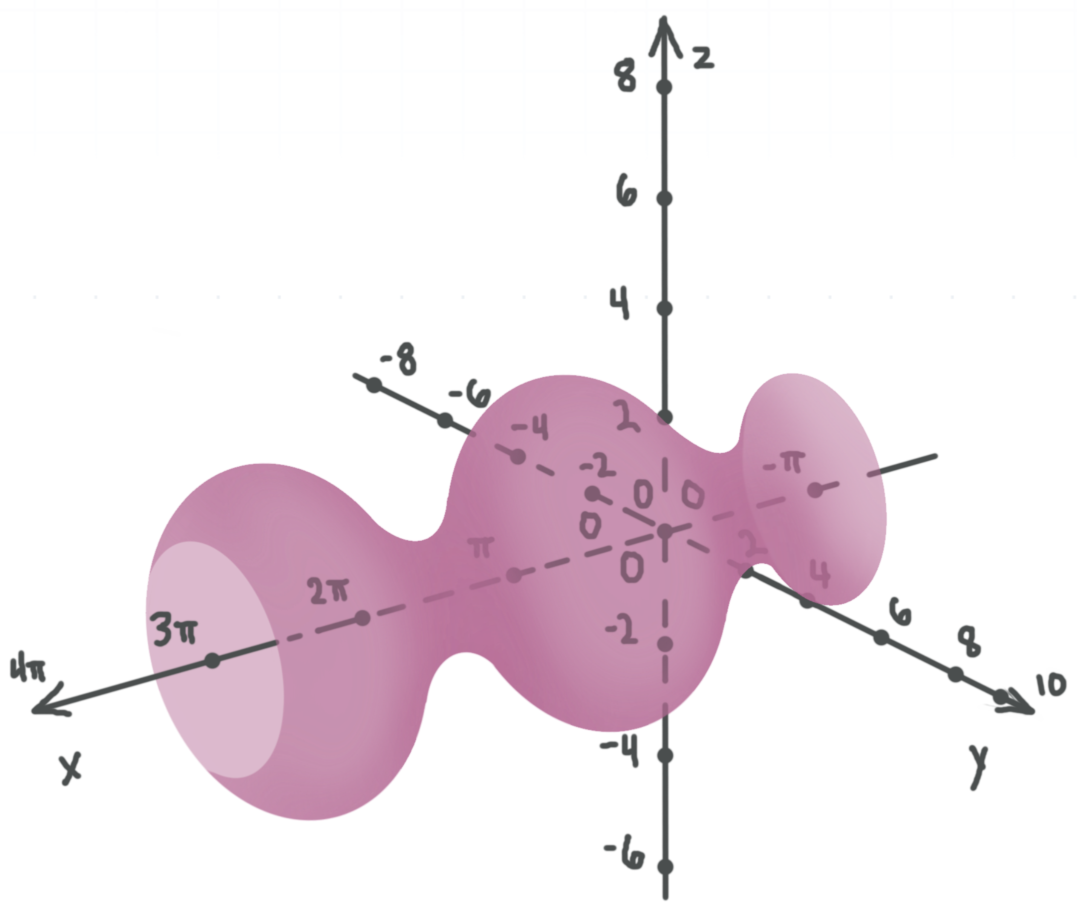

z
4
3
2
1
0
-1
-2
-3
-4
x
y

FINDING VOLUME

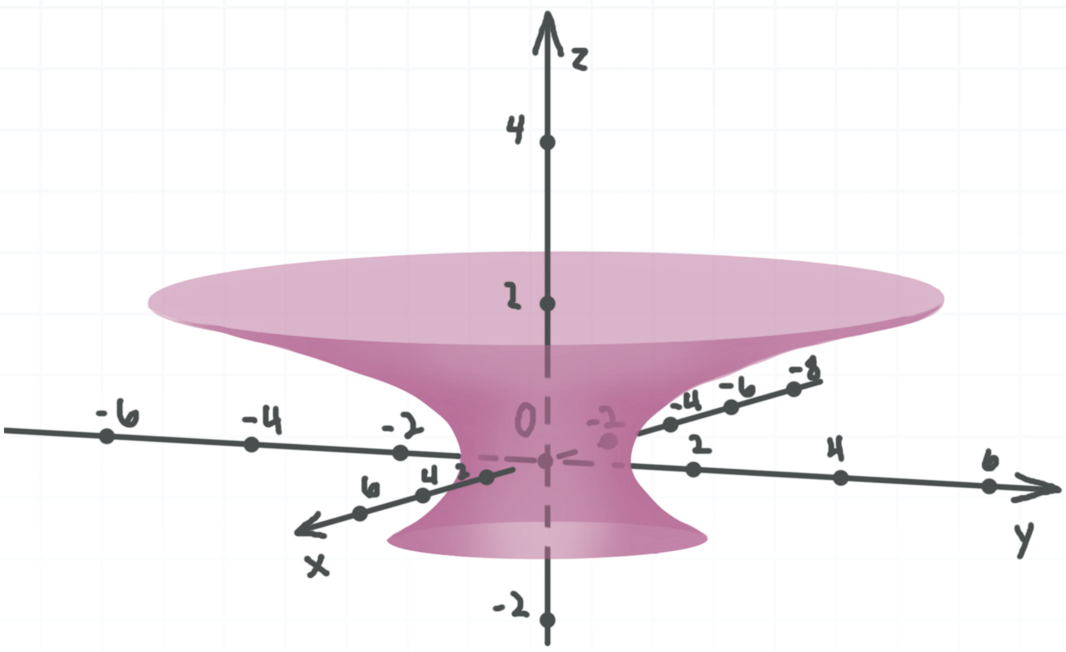
■ 1. Evaluate the integral.

$$\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} 4rz\ dz\ dr\ d\theta$$

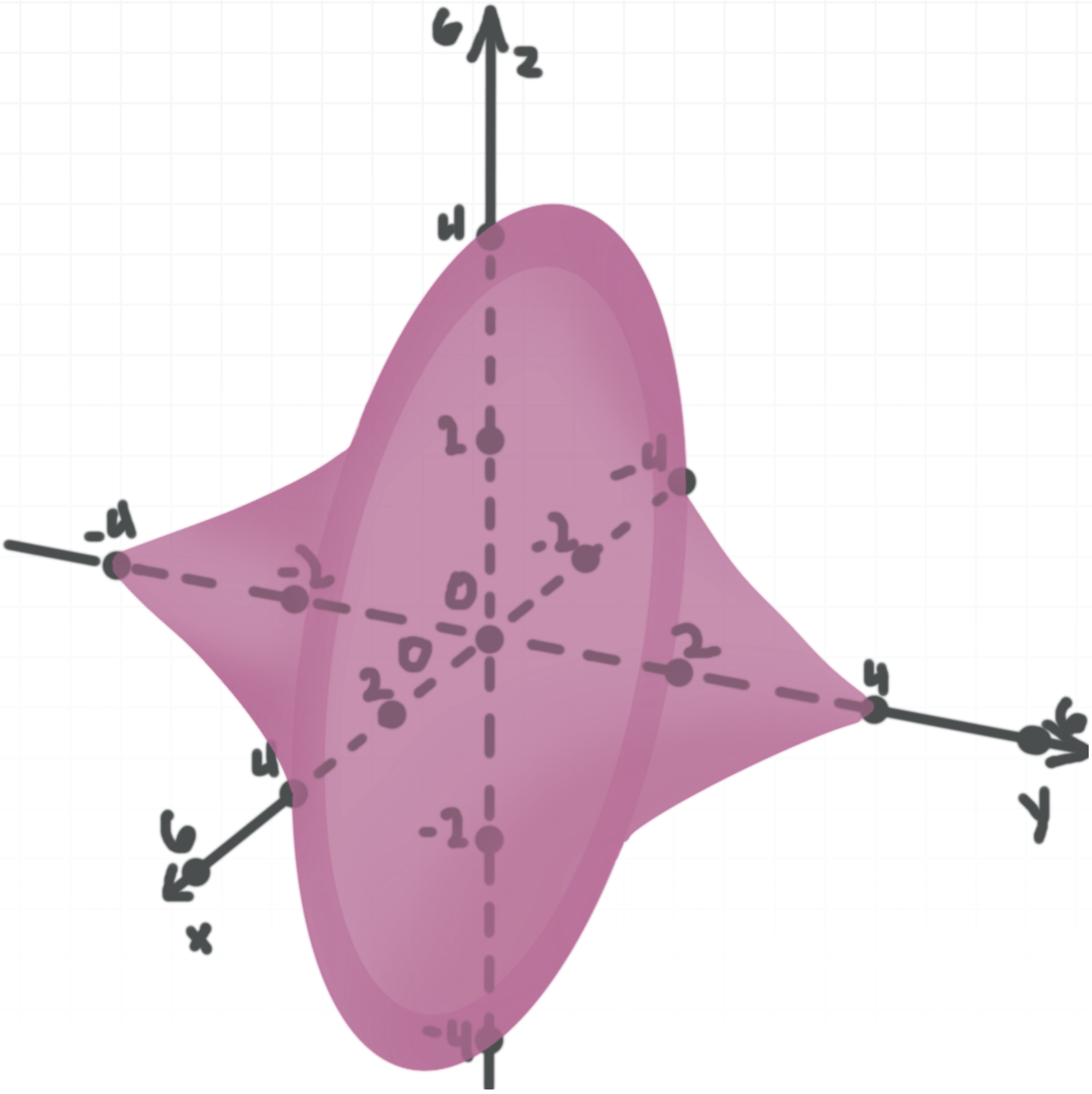
■ 2. Use a triple integral in cylindrical coordinates to find the volume of the solid E, where E is the set of points within the surface of revolution created by rotating the curve $z=2+\sin x$ around the x-axis, and bounded by the planes $x=-\pi$ and $x=3\pi$.



■ 3. Use a triple integral in cylindrical coordinates to find the volume of the solid E, where E is the set of points within the surface of revolution created by rotating the curve $x = z^2 + 1$ around the z-axis, and bounded by the planes $z = -1$ and $z = 2$.



■ 4. Use a triple integral in cylindrical coordinates to find the volume of the solid E, where E is the set of the points within the surface of revolution created by rotating the curve $x = 4 - 2\sqrt{|y|}$ around the y-axis, and bounded by the planes $y = -4$ and $y = 4$.


6
z
4
2
-4
-2
0
-4
-2
0
2
2
4
4
6
y
6
x
-2
-4

SPHERICAL COORDINATES

■ 1. Evaluate the triple integral given in the spherical coordinates, where $f(\rho, \theta, \phi) = 2\rho \sin\theta \cos\phi$.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{3}} \int_{1}^{5} f(\rho, \theta, \phi)\ \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$$

■ 2. Identify the solid given by the iterated integral in spherical coordinates.

$$\int_{0}^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_{0}^{2} f(\rho, \theta, \phi)\ \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$$

■ 3. Identify the solid given by the iterated improper integral in the spherical coordinates.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2\pi} \int_{\pi}^{\infty} f(\rho, \theta, \phi)\ \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$$

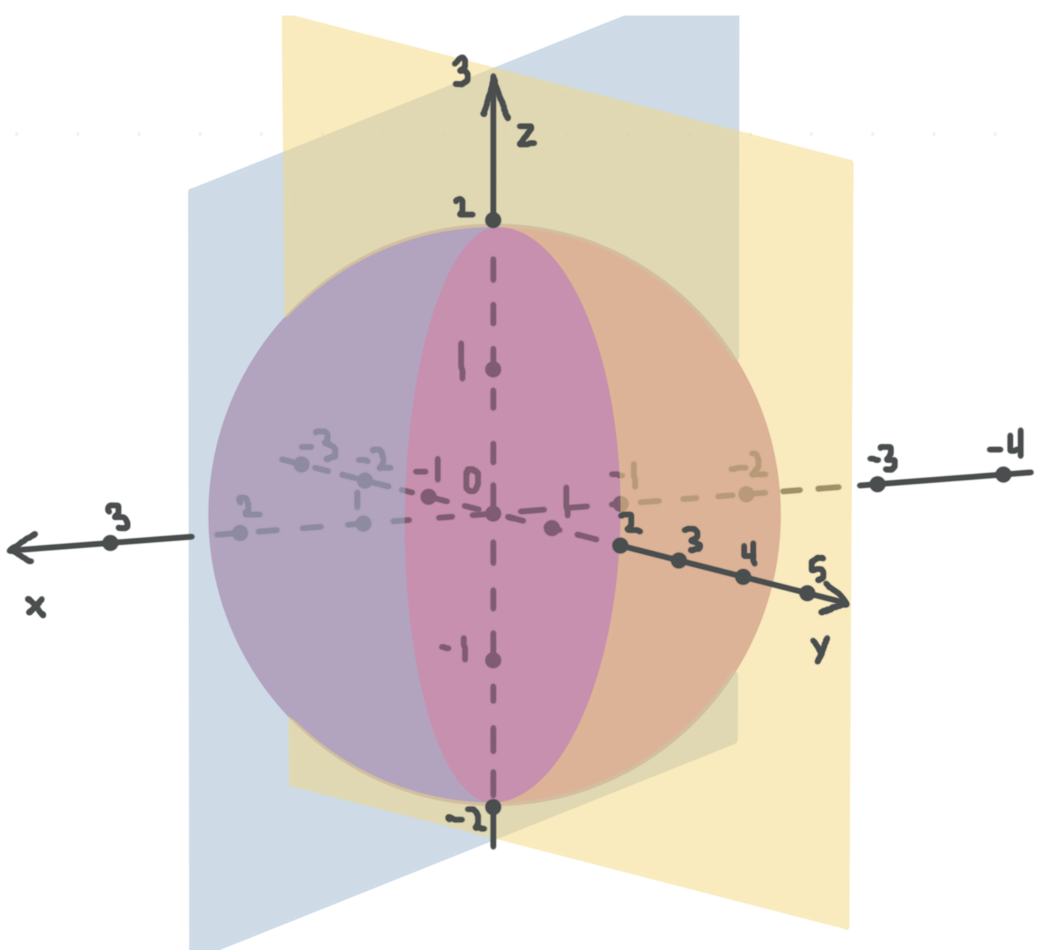
CHANGING TRIPLE INTEGRALS TO SPHERICAL COORDINATES

■ 1. Evaluate the triple integral by changing it to spherical coordinates, if E is the sphere with center at the origin and radius 3.

$$\iiint_E 5x^2 - 2\ dV$$

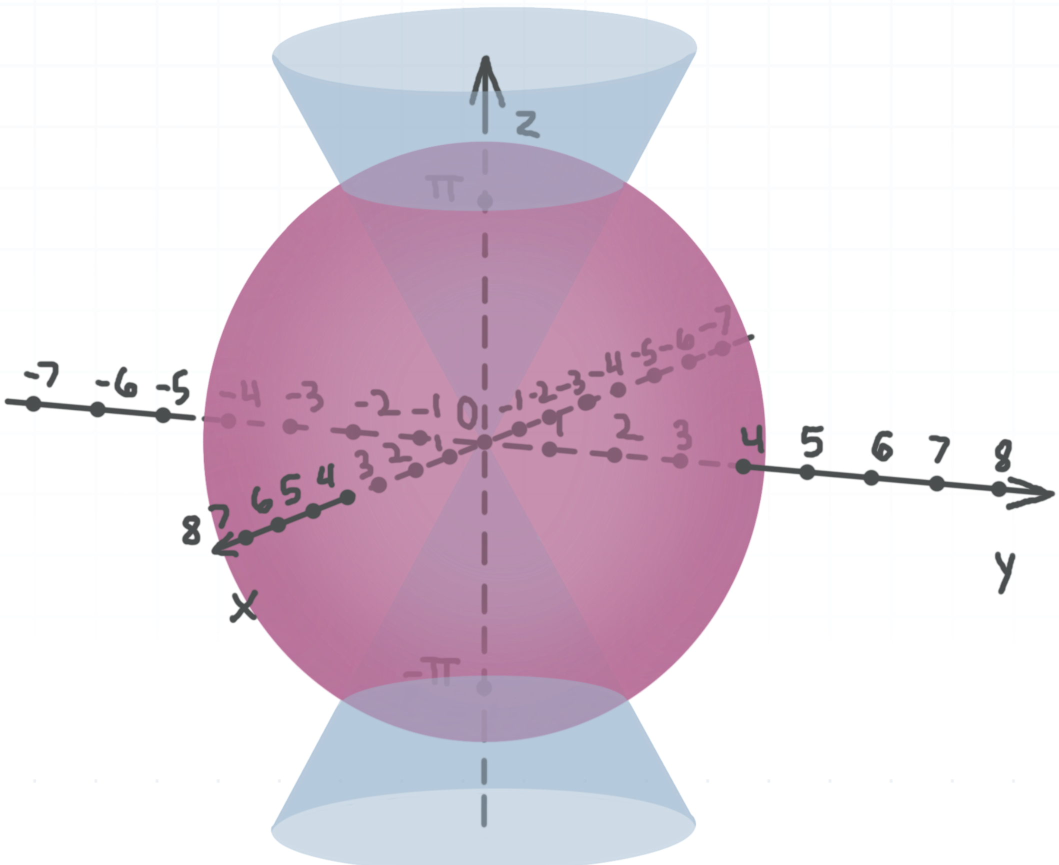
■ 2. Write down the triple integral by converting it to spherical coordinates, if E is the part of the sphere with center at the origin, radius 2, that lies between the planes $x = 0$ and $y = x$, and in the space $y > 0$.

$$\iiint_E x^2 + y^2 + 2z\ dV$$



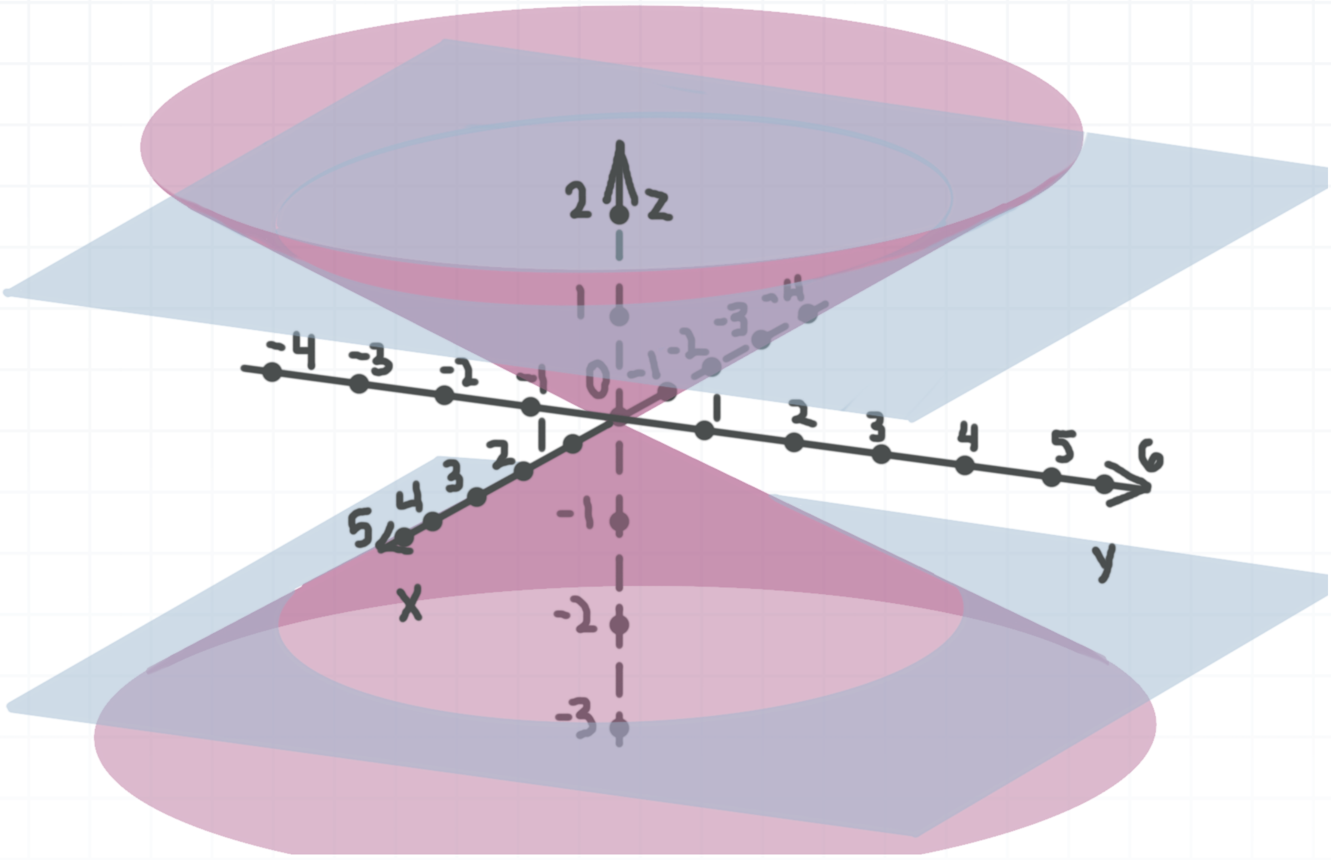
■ 3. Convert the triple integral to spherical coordinates, where E is the solid bounded by the sphere $x^2 + y^2 + z^2 = 16$ and the cone $x^2 + y^2 = z^2/3$, that lies in the half-space $z > 0$.

$$\iiint_E \ln(x^2y^2z^2 + 1)\ dV$$



■ 4. If E is the solid bounded by the cone $x^2 + y^2 = 3z^2$ and the planes $z = 2$ and $z = -2$, evaluate the triple integral by changing it to spherical coordinates.

$$\iiint_E \sqrt{x^2 + y^2 + z^2}\ dV$$

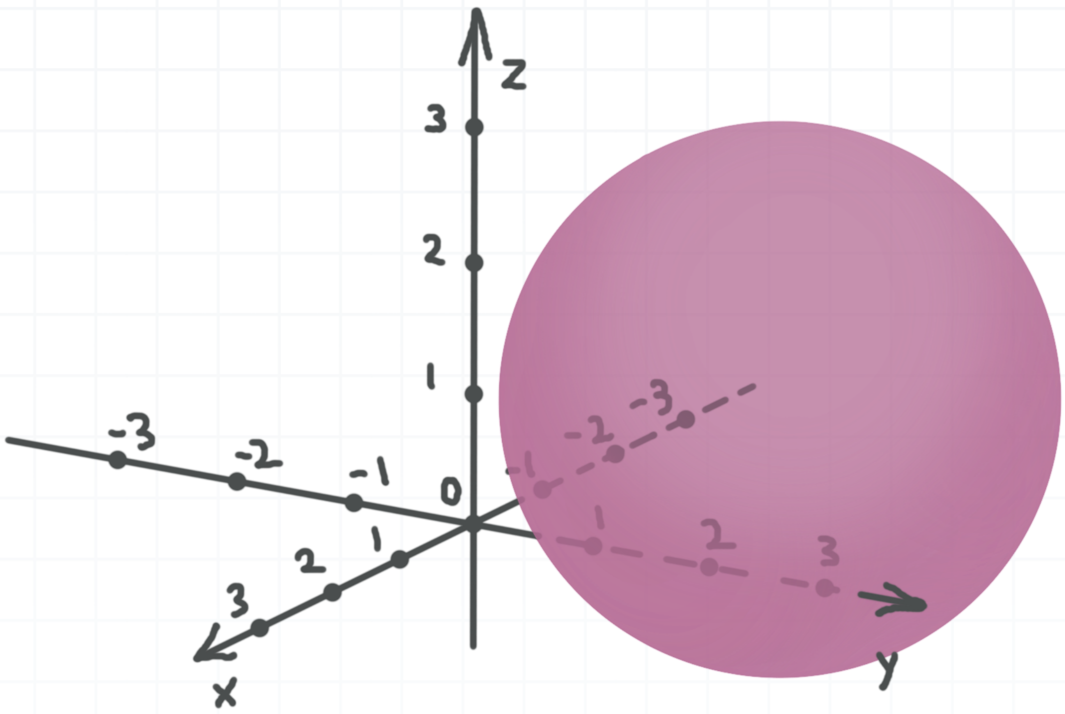


■ 5. If E is the set of outer points of the sphere with center at the origin and radius 5, evaluate the improper triple integral by changing it to spherical coordinates.

$$\iiint_E \frac{1}{(x^2+y^2+z^2)^3}\ dV$$

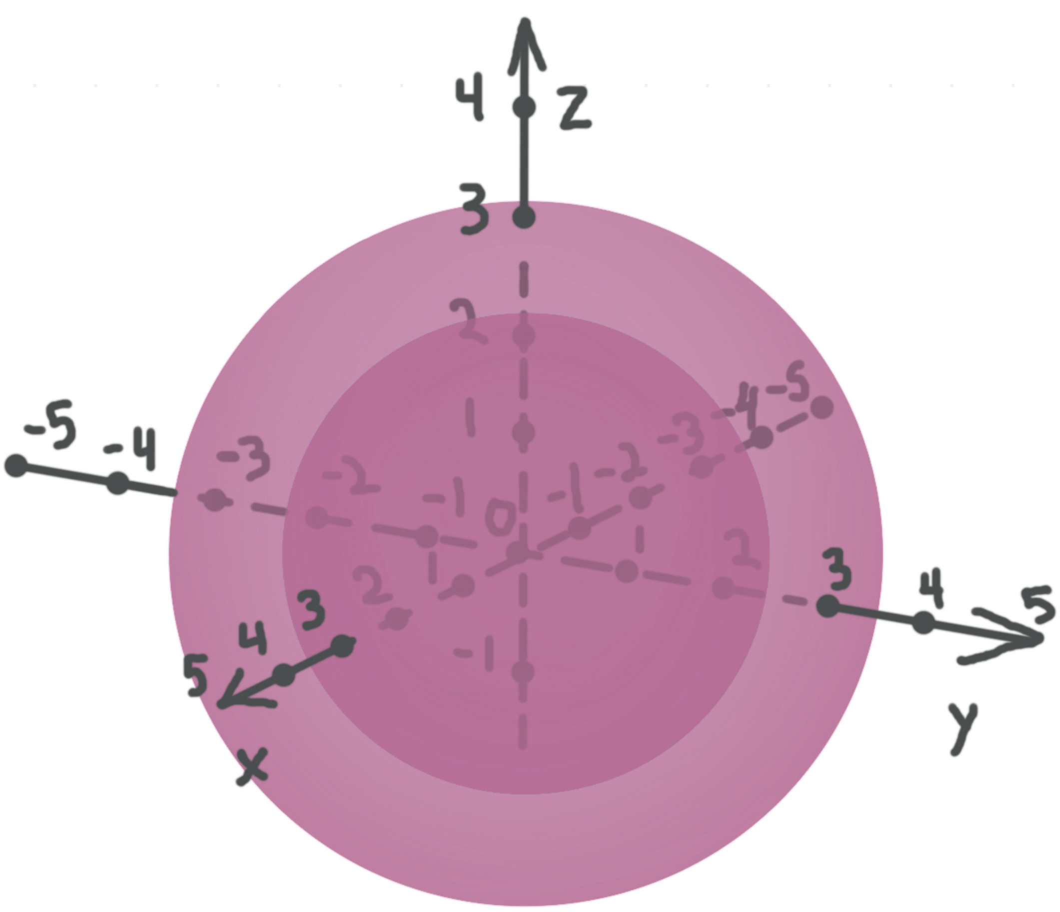
■ 6. Evaluate the triple integral by changing it to spherical coordinates, where E is the sphere with center at the point $(-1,2,1)$ and radius 2.

$$\iiint_E 5x+3y-2z\ dV$$



■ 7. Evaluate the triple integral by changing it to spherical coordinates, if E is the set of points between the spheres $x^2+y^2+z^2=9$ and $x^2+y^2+z^2=4$.

$$\iiint_E 15y^2+4y\ dV$$

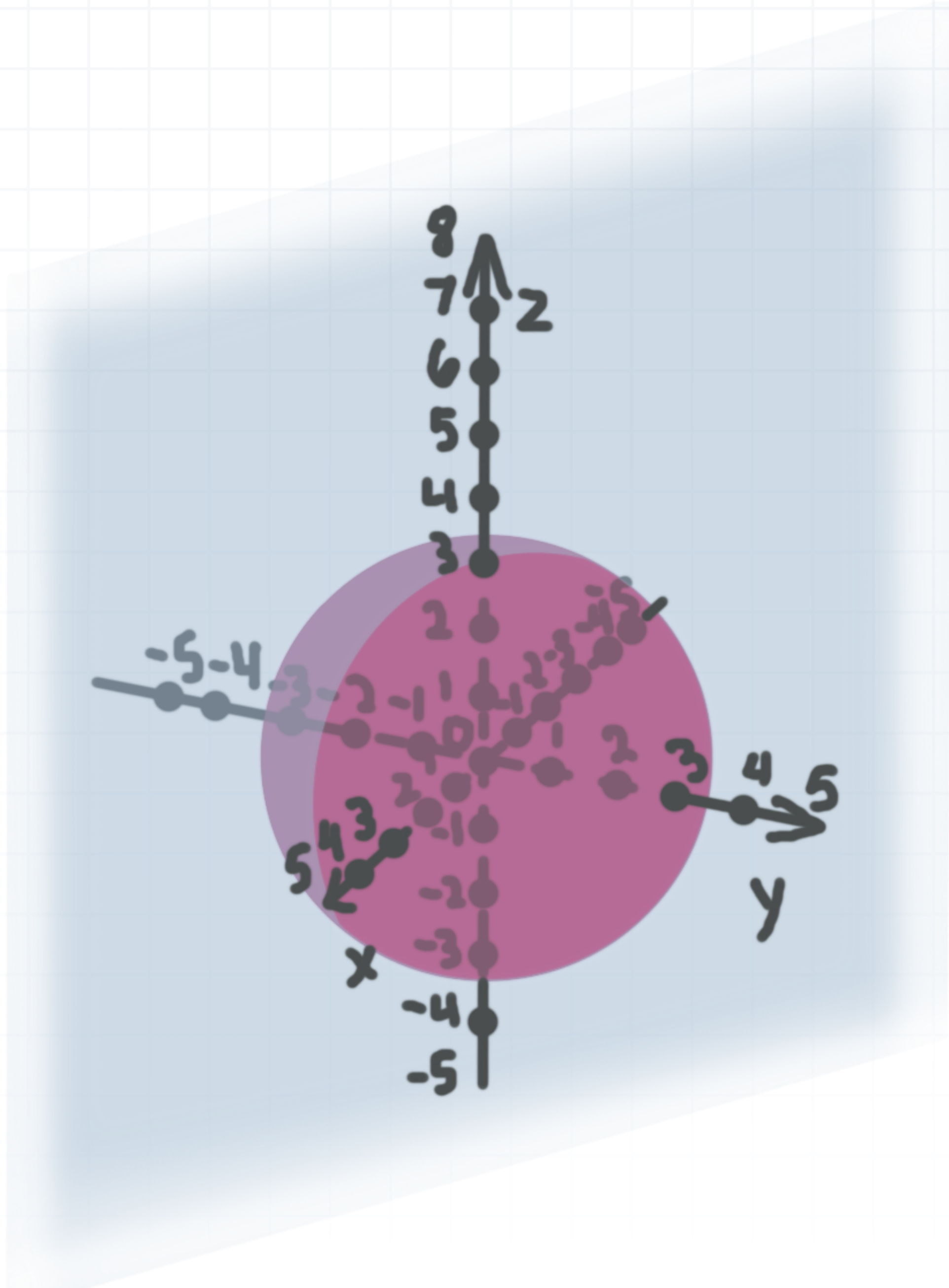


■ 8. Evaluate the improper triple integral by changing it to spherical coordinates, where E is the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

$$\iiint_E 2^{-\sqrt{(x^2+y^2+z^2)^3}}\ dV$$

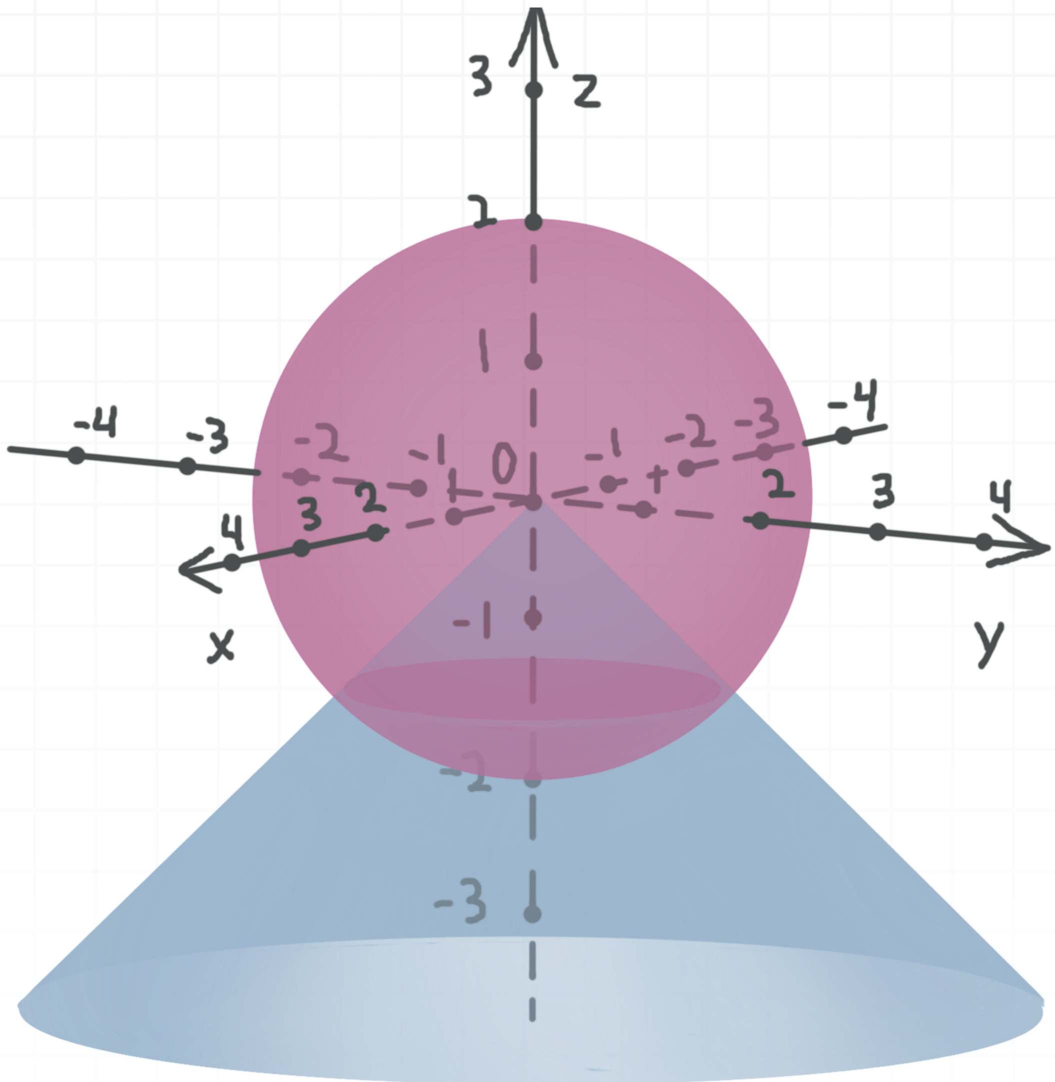
■ 9. Evaluate the triple integral by changing it to spherical coordinates, if E is the solid bounded by the sphere with center at the origin and radius 3, and the plane $x + \sqrt{3}y = 0$. Consider the hemisphere that includes the points from the first octant.

$$\iiint_E x^4 + y^4 + z^4 + 2x^2y^2 + 2x^2z^2 + 2y^2z^2\ dV$$



■ 10. Evaluate the improper triple integral by changing it to spherical coordinates, where E is the region that consists of the points inside the cone $x^2 + y^2 = z^2$ and outside the sphere $x^2 + y^2 + z^2 = 4$, that lie in the half-space $z < 0$.

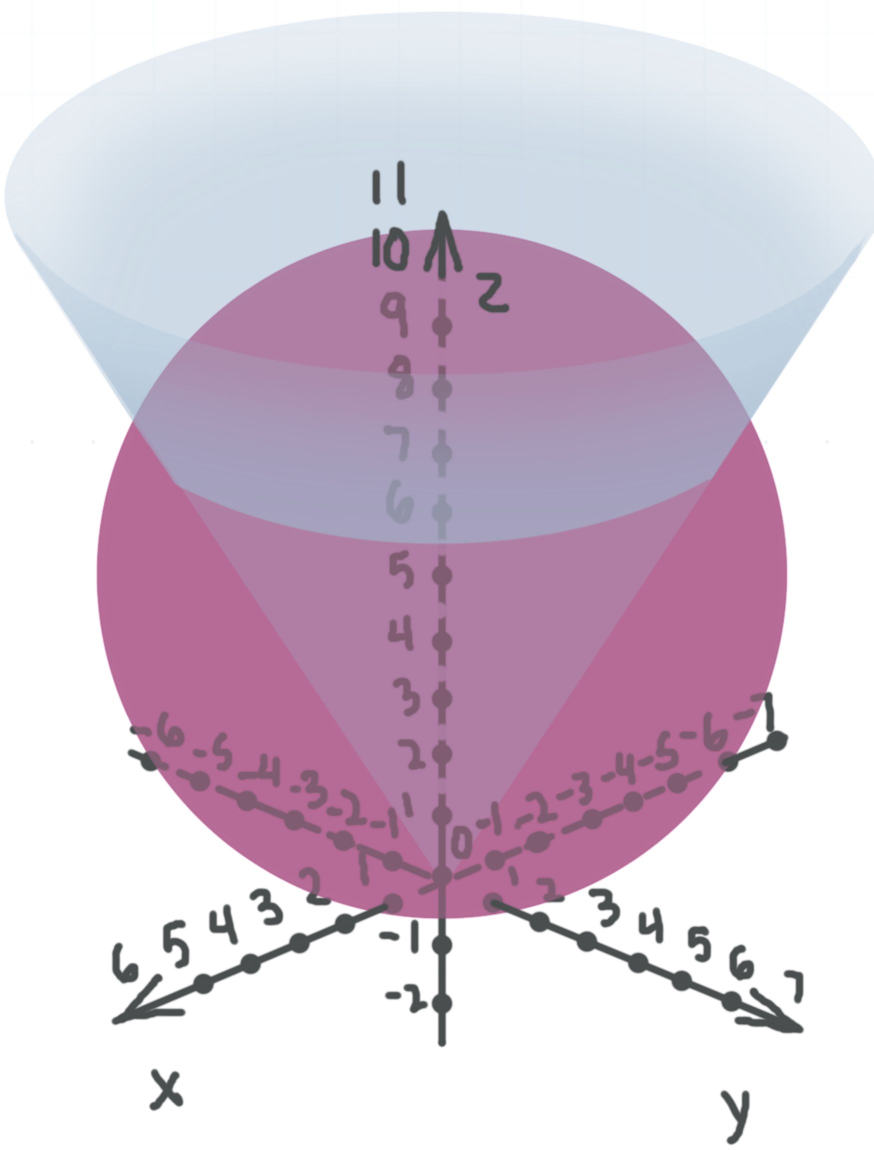
$$\iiint_E \frac{4z + 10}{(x^2 + y^2 + z^2)^4} \, dV$$


3
z
2
1
-4
-3
-2
-1
0
-1
-2
-3
-4
4
3
2
1
1
2
3
4
x
y
-1
-2
-3

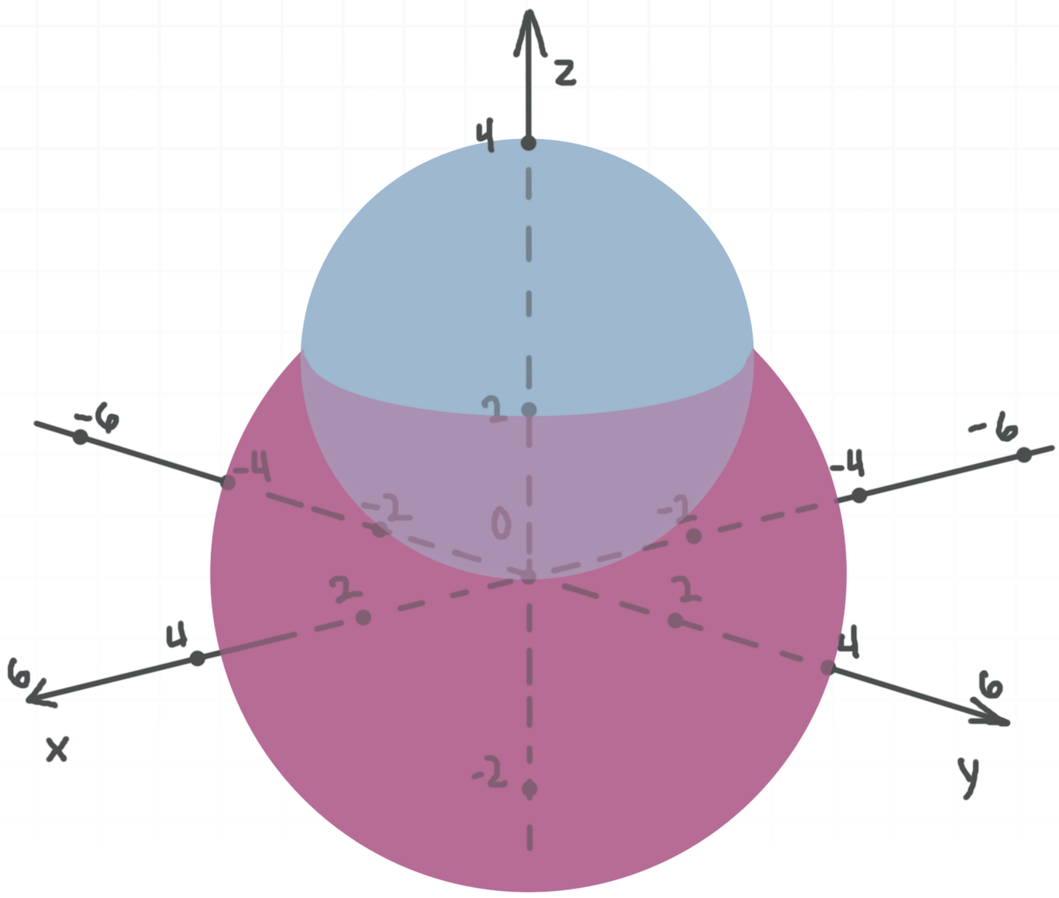
FINDING VOLUME

■ 1. Use a triple integral in spherical coordinates to find the volume of the region E that consists of the points inside the sphere $x^2+y^2+z^2=6$ and outside the cone $y^2+z^2=3x^2$.

■ 2. Use a triple integral in spherical coordinates to find the volume of an ice cream cone formed by the points common to the cone $3x^2+3y^2=z^2$ and the sphere $x^2+y^2+z^2-10z=0$.

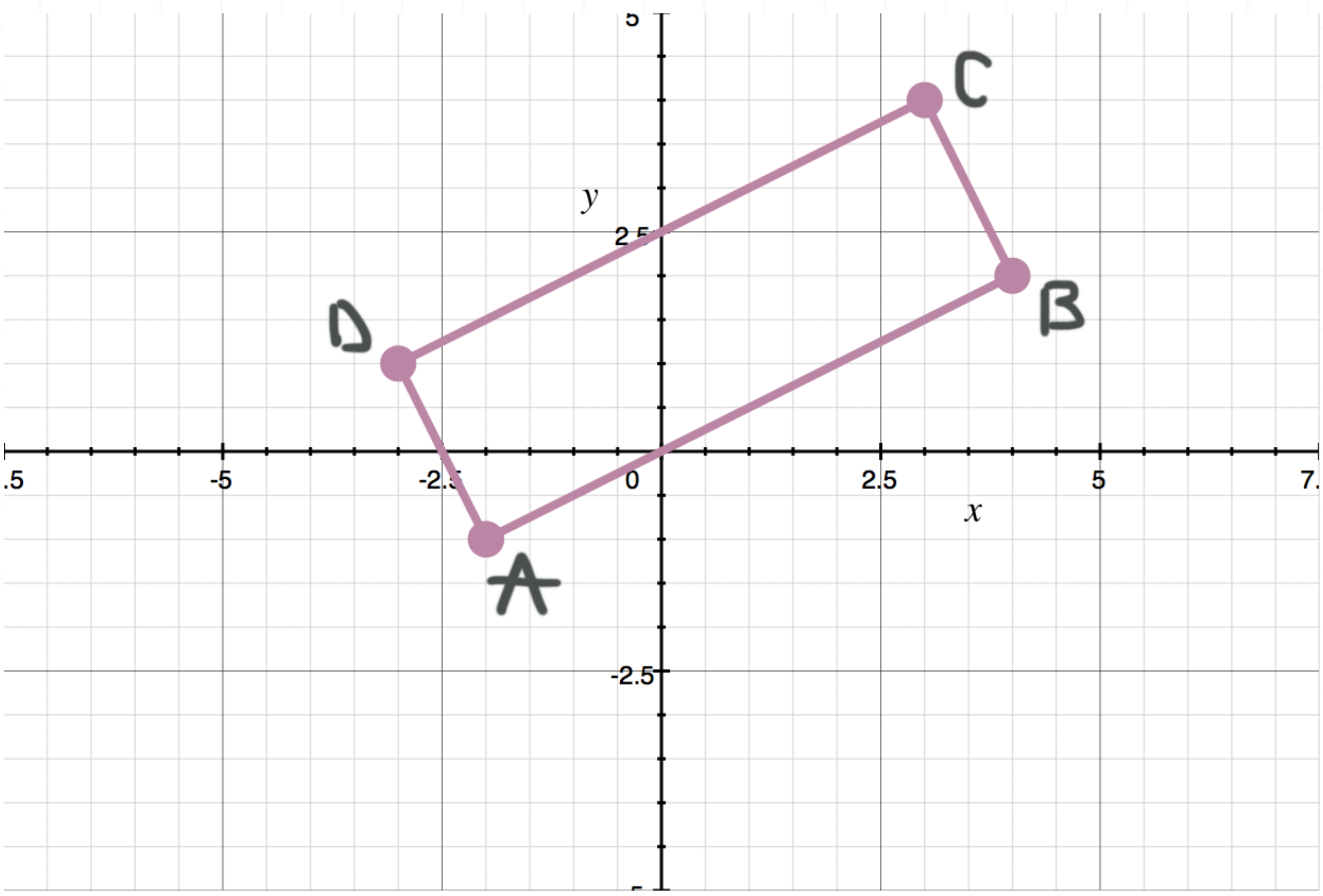


■ 3. Use a triple integral in spherical coordinates to find the volume of the three-dimensional lens common to the two spheres $x^2+y^2+z^2-8=0$ and $x^2+y^2+z^2-4z=0$.



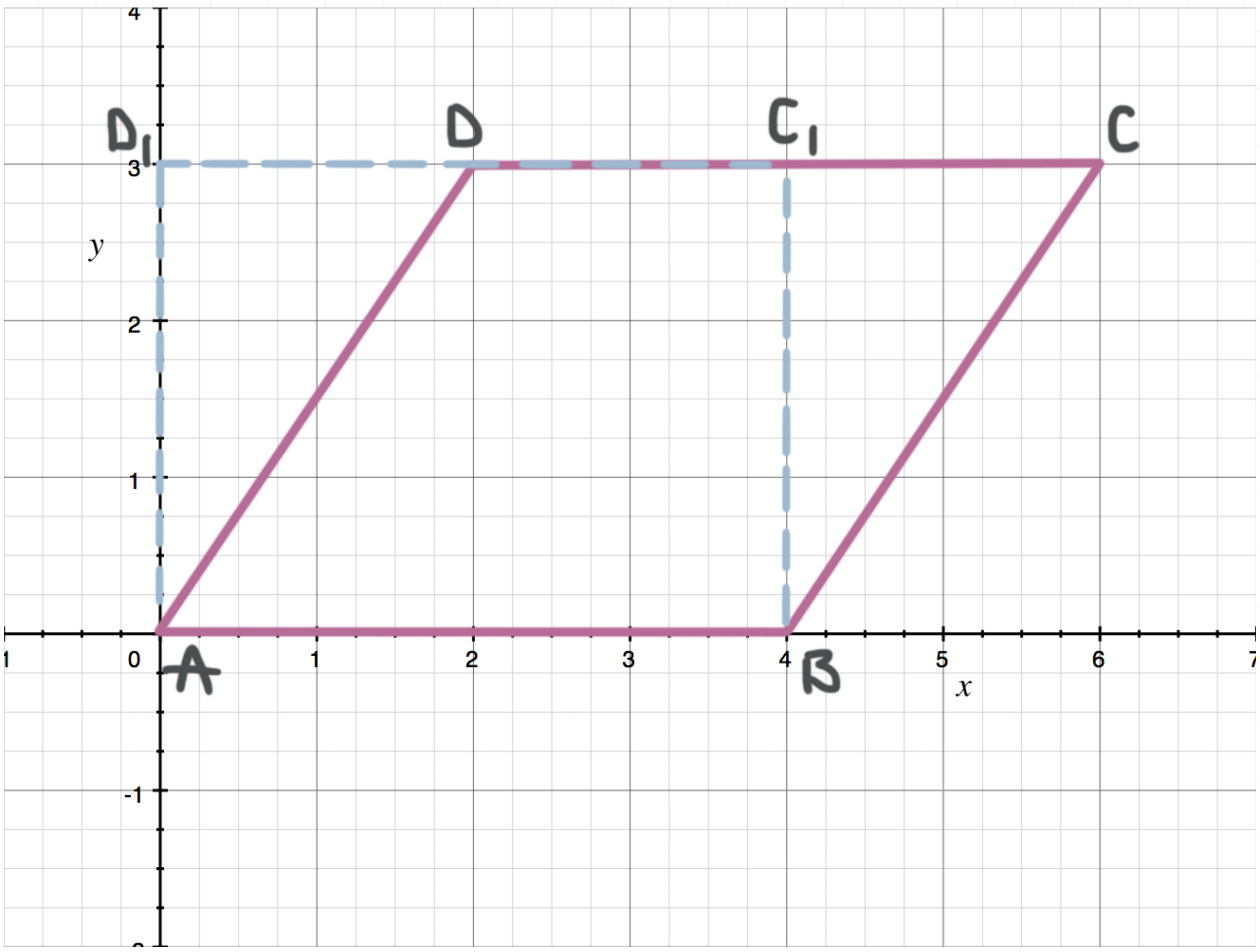
JACOBIAN FOR TWO VARIABLES

■ 1. Find the Jacobian of the transformation that rotates the rectangle $ABCD$, given by $A(-2,-1)$, $B(4,2)$, $C(3,4)$, and $D(-3,1)$, clockwise about the origin in such a way that AB will lie on the x-axis.



■ 2. Find the Jacobian of the transformation which converts the ellipse $5\sqrt{2}x^2+6\sqrt{2}xy+8x+5\sqrt{2}y^2-8y=0$ into the ellipse with center at the origin, and x- and $y$semi-axes 2 and 1 respectively. Use a rotation counterclockwise by $\pi/4$, and then move it by 2 to the positive direction of the x-axis.

■ 3. Find the Jacobian of the linear transformation that converts the parallelogram $ABCD$, given by $A(0,0)$, $B(4,0)$, $C(6,3)$, $D(2,3)$, into the rectangle ABC_1D_1 with the same base and height.



JACOBIAN FOR THREE VARIABLES

■ 1. Find the Jacobian of the transformation that rotates the space clockwise about the y-axis by $\pi/6$.

■ 2. Find the Jacobian of the transformation to spherical coordinates that converts the ellipsoid to the unit sphere (a sphere with center at the origin and radius 1).

$$\frac{x^2}{4}+\frac{y^2}{25}+\frac{z^2}{9}=1$$

■ 3. Find the following transformations:

1. the transformation that converts the given cylinder to a circular cylinder with radius 1 and an axis parallel to the z-axis

$$\frac{x^2}{5}+\frac{z^2}{4}=1$$

2. the transformation that converts to cylindrical coordinates

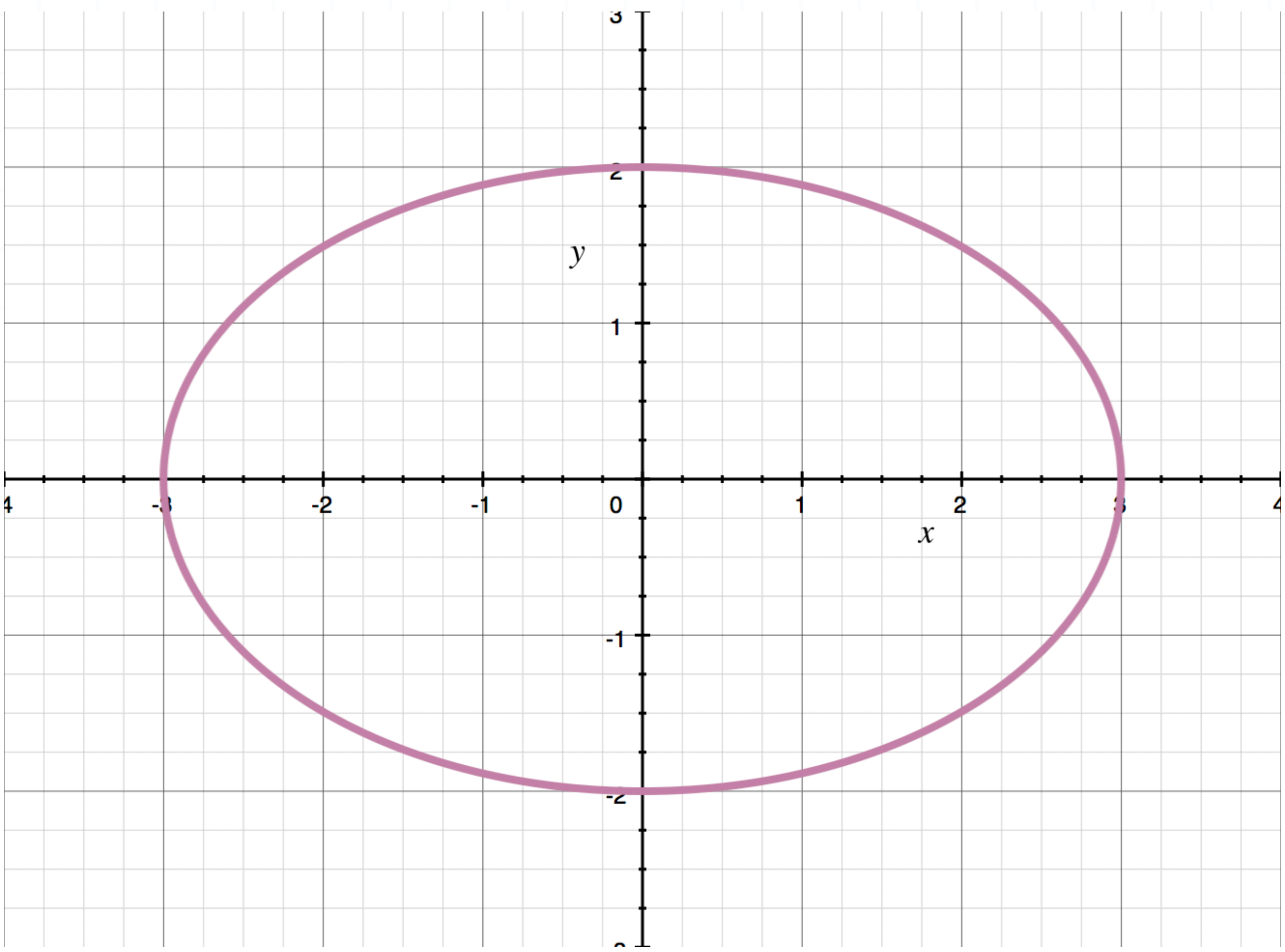
Then write down the composition of these two transformations and find its Jacobian.

EVALUATING DOUBLE INTEGRALS

■ 1. Find the Jacobian of the transformation and use it to find the area of the ellipse with center at the origin, semi-major axis along the x-axis with length 3, and semi-minor axis along the y-axis with length 2.

$$x = 3r\cos\phi$$

$$y = 2r\sin\phi$$



■ 2. Find the Jacobian of the transformation and use it to find the double integral of the function $f(x, y) = x^2 + y^2$ over the circle with center at $(-2,3)$ and radius 2.

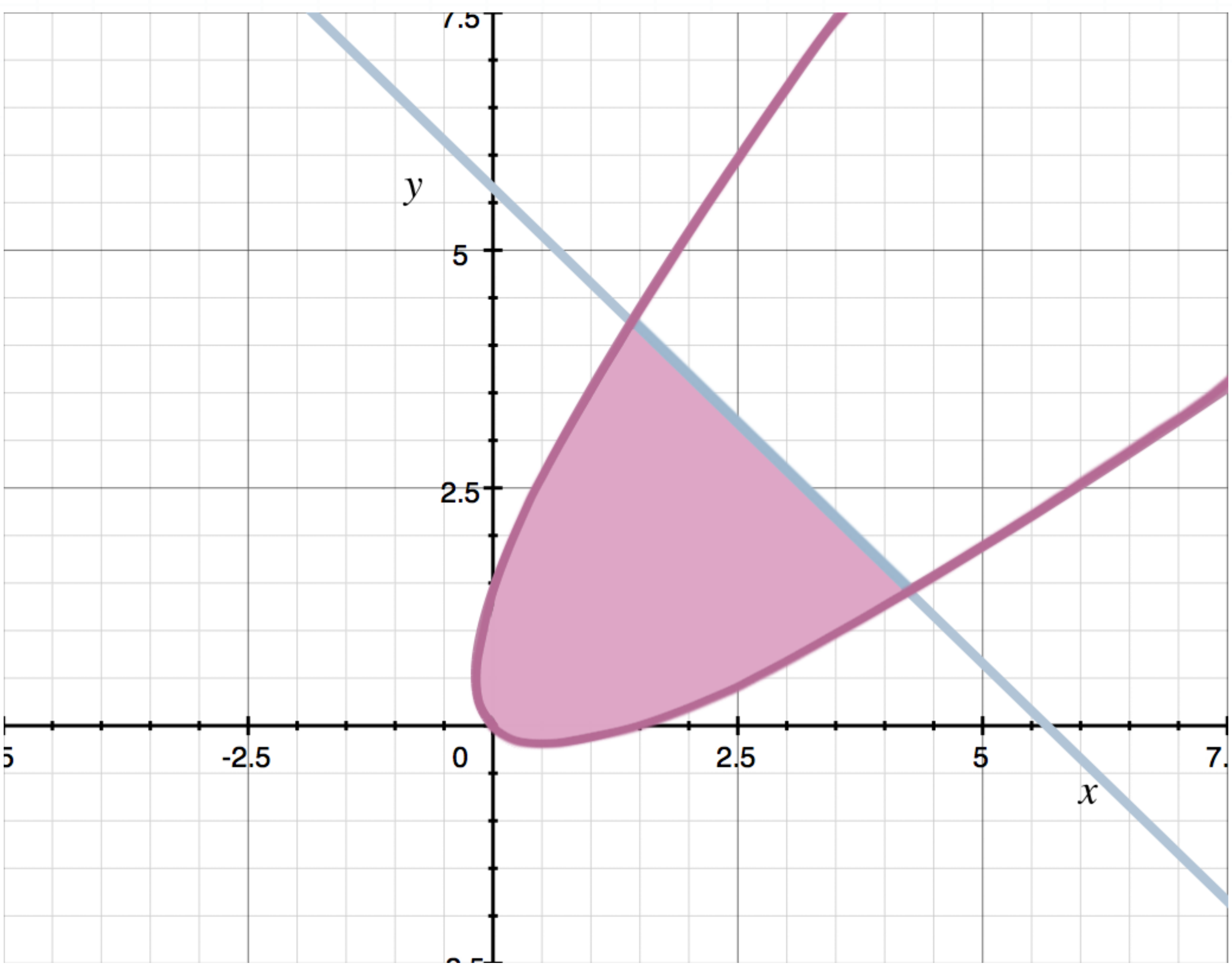
$$x = -2 + r\cos\phi$$

$$y = 3 + r\sin\phi$$

■ 3. Find the Jacobian of the transformation, and use it to find the area bounded by the curves $x^2 - 2xy + y^2 - \sqrt{2}x - \sqrt{2}y = 0$ and $x + y = 4\sqrt{2}$.

$$x = \frac{\sqrt{2}}{2}(u + v)$$

$$y = \frac{\sqrt{2}}{2}(v - u)$$



EQUATIONS OF THE TRANSFORMATION

■ 1. Identify the equation obtained from $f(x, y, z) = x^2 + y^2 + z^2 - 2$ by applying the transformation.

$$x = \sin u + \cos u$$

$$y = \sin u - \cos u$$

$$z = \sqrt{u + v + w}$$

■ 2. Find the inverse transformation and determine its Jacobian.

$$u = x - 2y + 1$$

$$v = -3x + y + 2$$

■ 3. Find the inverse transformation $x(r, \phi)$ and $y(r, \phi)$ and determine its Jacobian.

$$r = \sqrt{\frac{x^2 + y^2}{4}}$$

$\phi = \arctan \dfrac{y}{x}$ for $x \neq 0$

IMAGE OF THE SET UNDER THE TRANSFORMATION

■ 1. Identify the surface obtained from the unit sphere (a sphere with center at the origin and radius 1 by applying the transformation.

$$x = u - 2v + 2$$

$$y = u + v + w$$

$$z = u + v - w + 4$$

■ 2. Identify the shape obtained from the parallelogram $ABCD$, where $A(-2,2)$, $B(-2,5)$, $C(-3,6)$, and $D(-3,3)$, by applying the transformation.

$$u = -x - 2$$

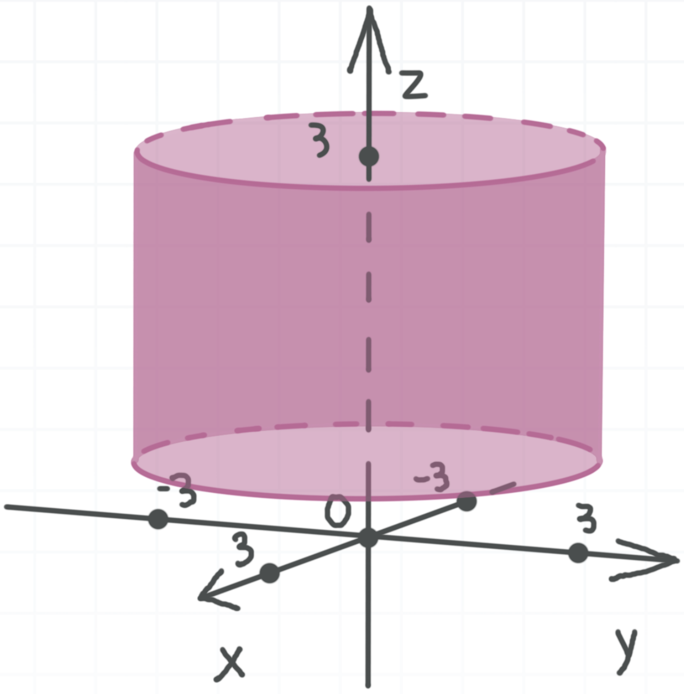
$$v = \frac{x + y}{3}$$

■ 3. Identify the solid obtained from the set of interior points of the circular cylinder $x^2 + y^2 = 9$, with $1 \le z \le 5$, when the following transformation is applied.

$$u = \sqrt{x^2 + y^2}$$

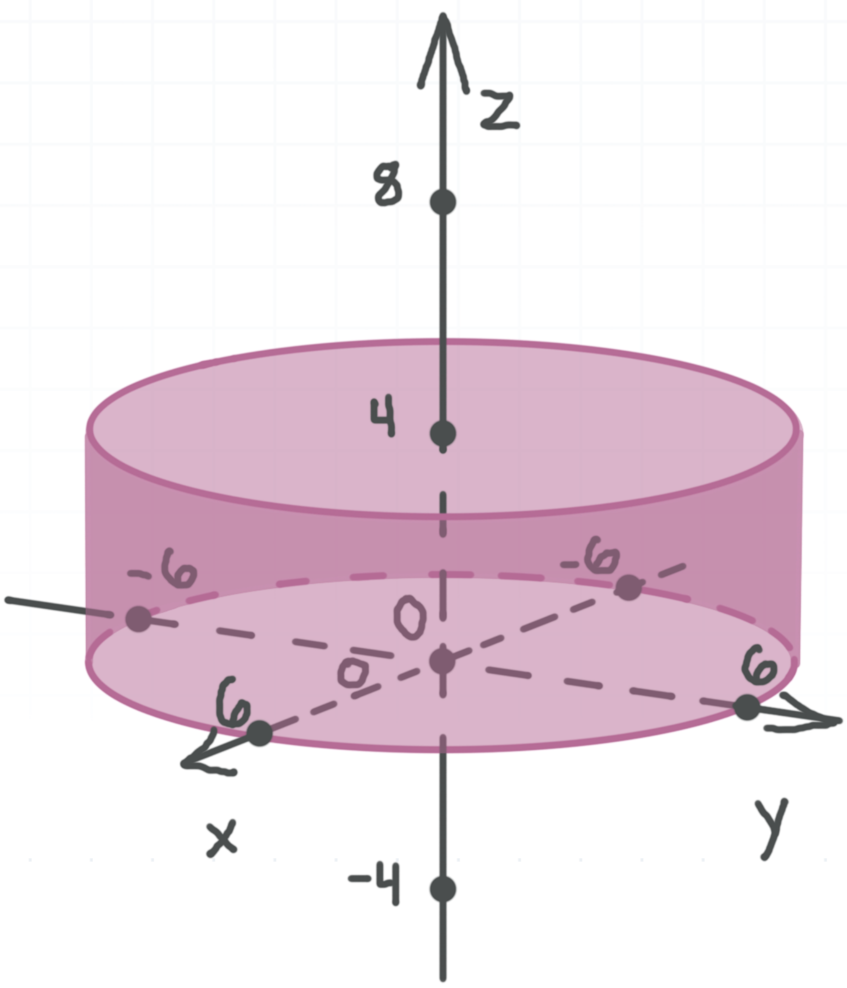
$$v = \arctan \frac{y}{x}$$

$w = z - 1$

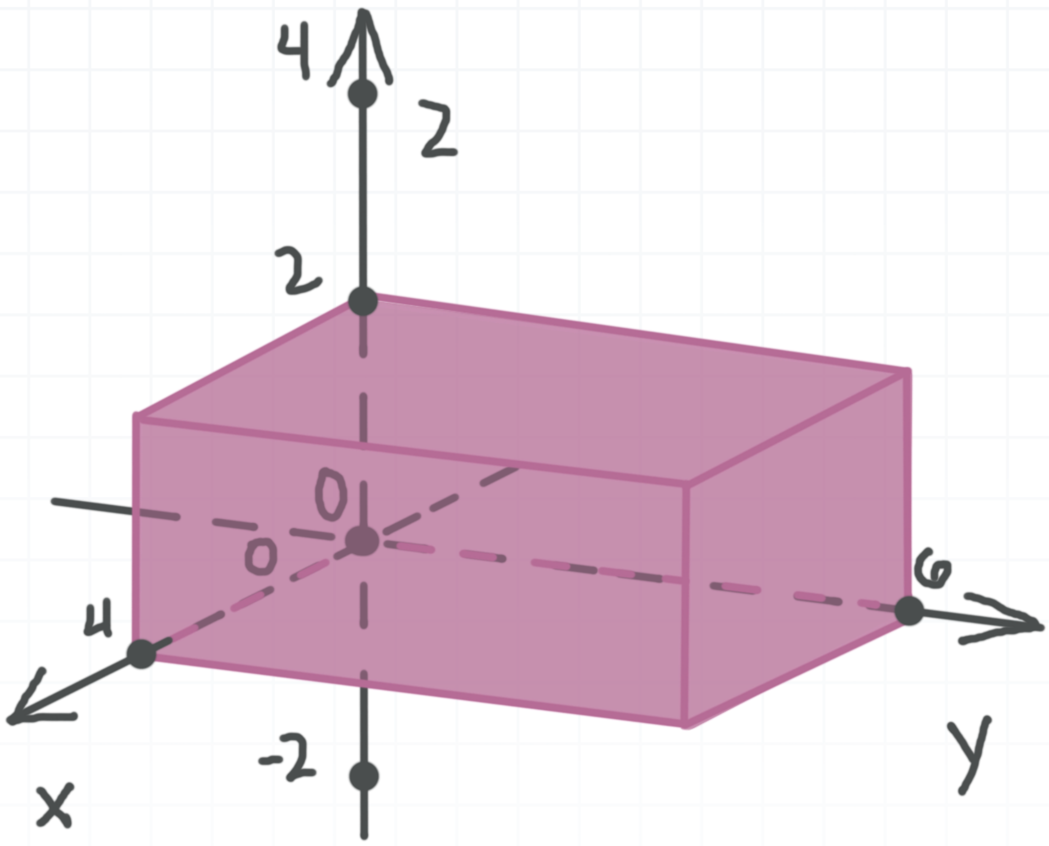


TRIPLE INTEGRALS TO FIND MASS AND CENTER OF MASS

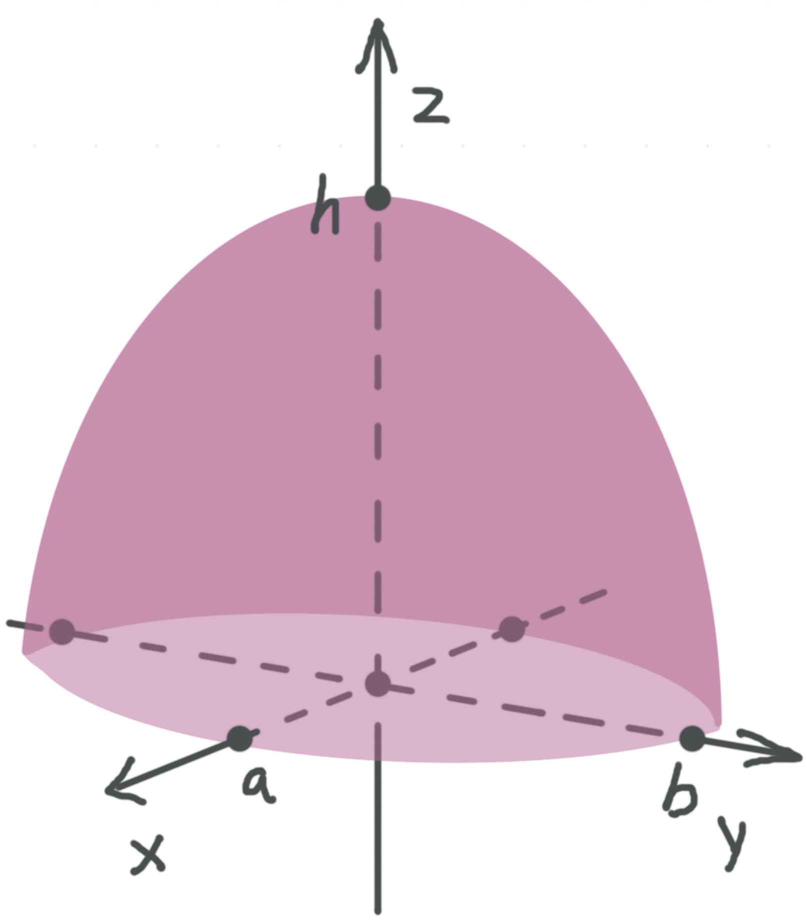
■ 1. The disk with radius 6 and height 4 has density $\delta = 1/(d+2)$, where d is the distance to the central axis of the disk. Find the mass and center of mass of the disk.



■ 2. The rectangular plate with base dimensions 4×5 m and height 2 m has density $\delta = 4d$ kg/m^2, where d is the distance from its 4×2 m left face. Find the mass and center of mass of the plate.



■ 3. The half ellipsoid has a base with semi-axes a and b, height h, and constant density δ. Find its mass and center of mass.

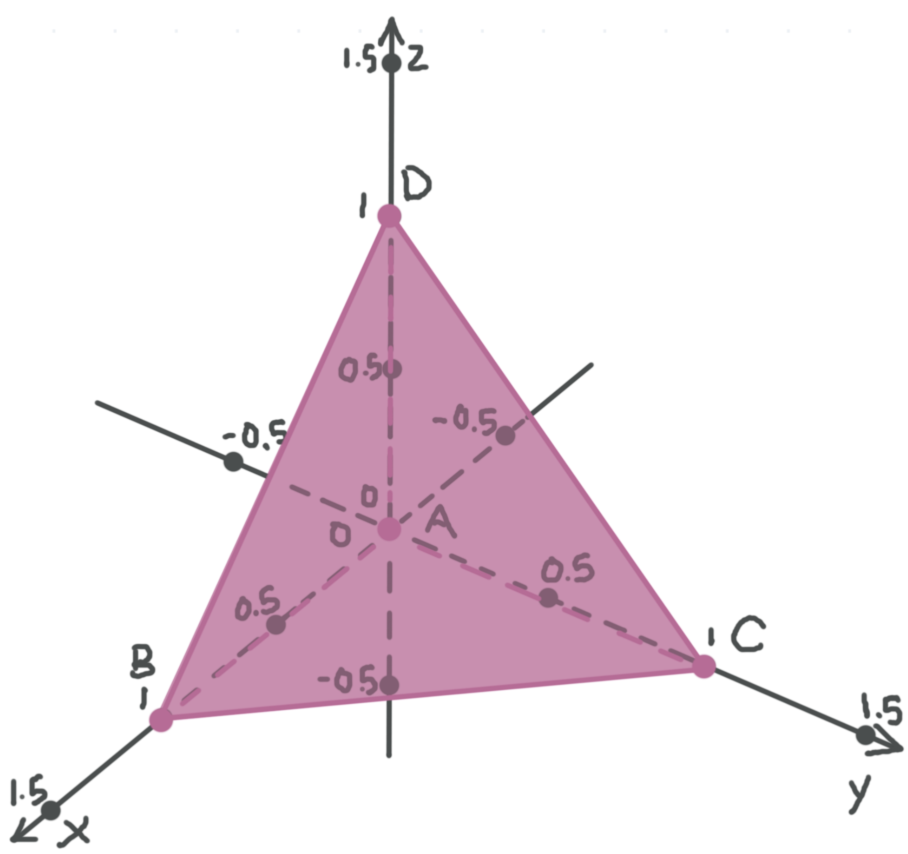


MOMENTS OF INERTIA

■ 1. The spherical solid object with radius 5 has density $\delta = d^2$, where d is the distance to the center of the sphere. Find the moment of inertia of the object about any line that passes through its center.

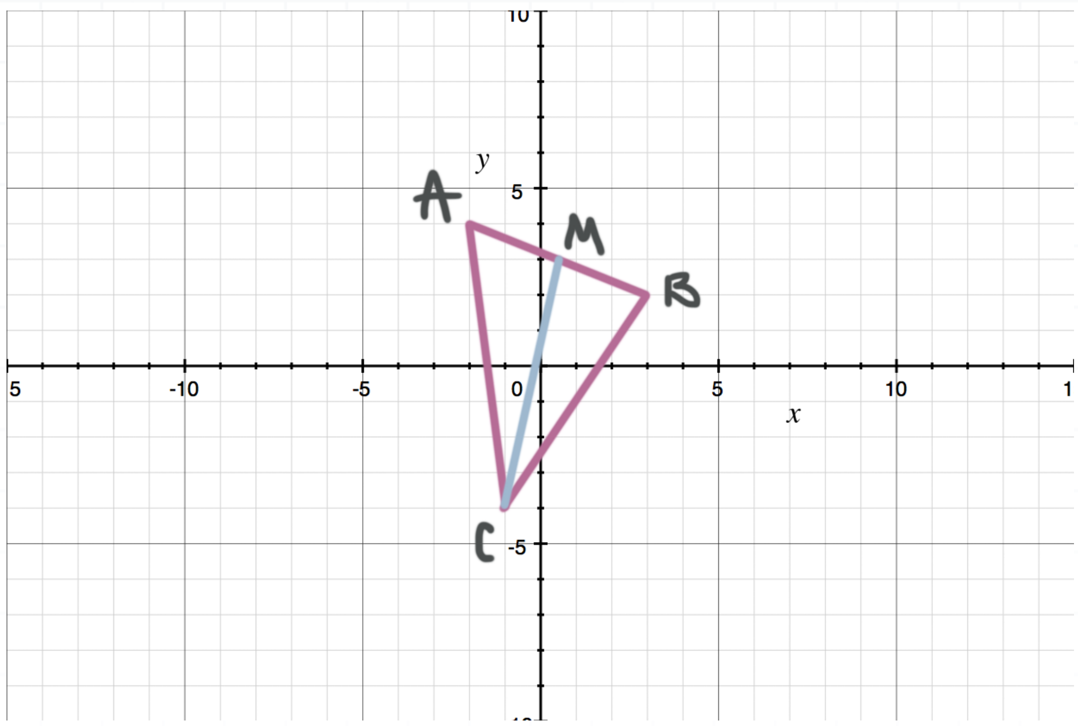
■ 2. A box (rectangular cuboid) has length 6, width 4, height 2, and constant density δ. Find the moment of inertia of the box about all of its edges.

■ 3. The tetrahedron $ABCD$ has constant density δ. Find the moment of inertia of the solid about the line AB, where $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$, and $D(0,0,1)$.



VECTOR FROM TWO POINTS

■ 1. Find the vector $\overrightarrow{CM}$, if M is the midpoint of $\overline{AB}$.

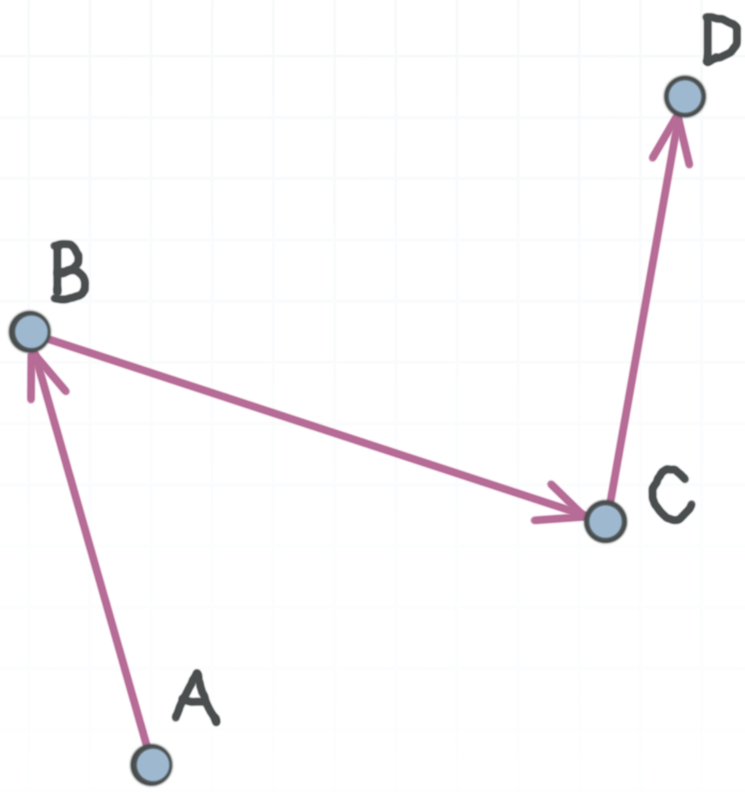


■ 2. Find the coordinates of the point P, given $Q(-\sqrt{2},0,\sqrt{2})$ and $\overrightarrow{PQ}=\langle\sqrt{2},4,\sqrt{2}\rangle$.

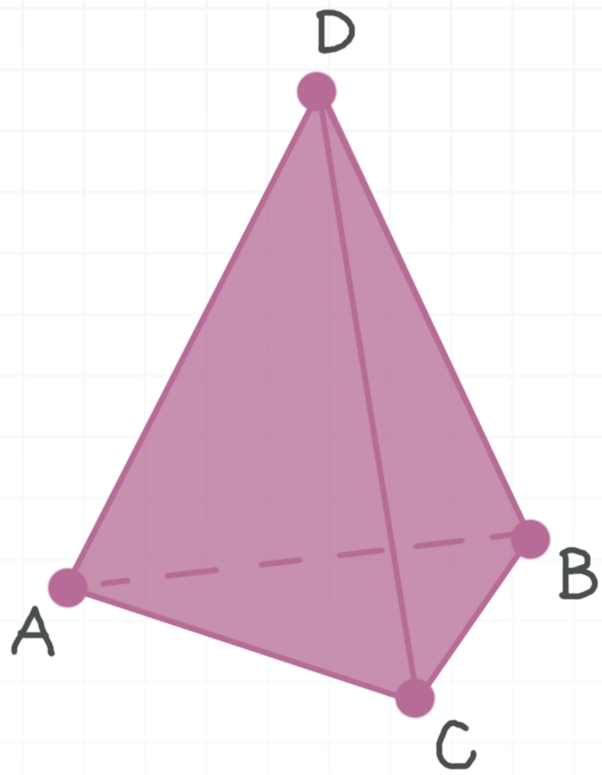
■ 3. Find the coordinates of the point C, given the coordinates of the point $A(-2,3,4)$, and the vectors $\overrightarrow{AB}=\langle 0,5,0\rangle$ and $\overrightarrow{BC}=\langle 2,-3,6\rangle$.

COMBINATIONS OF VECTORS

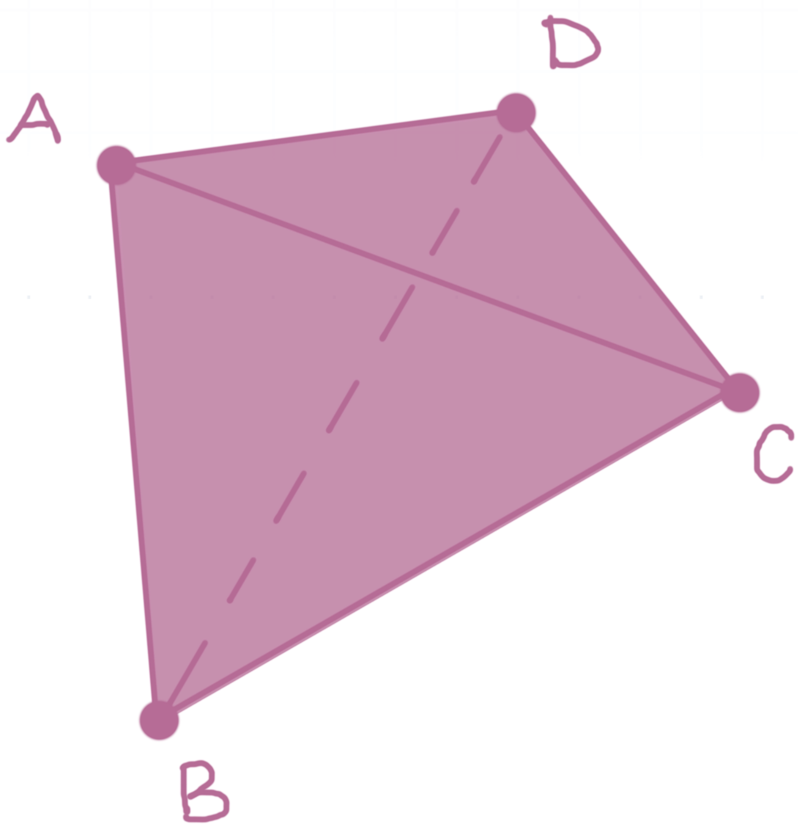
■ 1. Find the combination $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$.



■ 2. In the tetrahedron $ABCD$, find the resulting vector $\overrightarrow{DA}-\overrightarrow{DB}-\overrightarrow{BC}$.

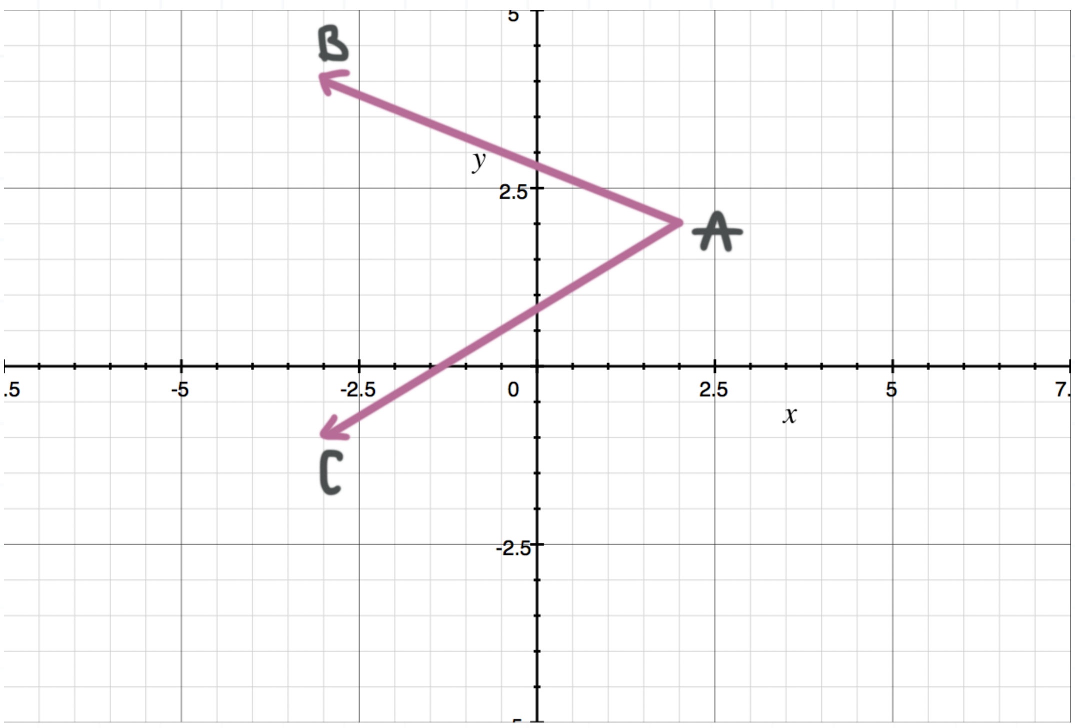


3. In tetrahedron $ABCD$, find the vector $\overrightarrow{AB} + \overrightarrow{DC} + \overrightarrow{BD} - \overrightarrow{BC}$.



SUM OF TWO VECTORS

■ 1. Find the sum $\overrightarrow{AB}+\overrightarrow{AC}$.



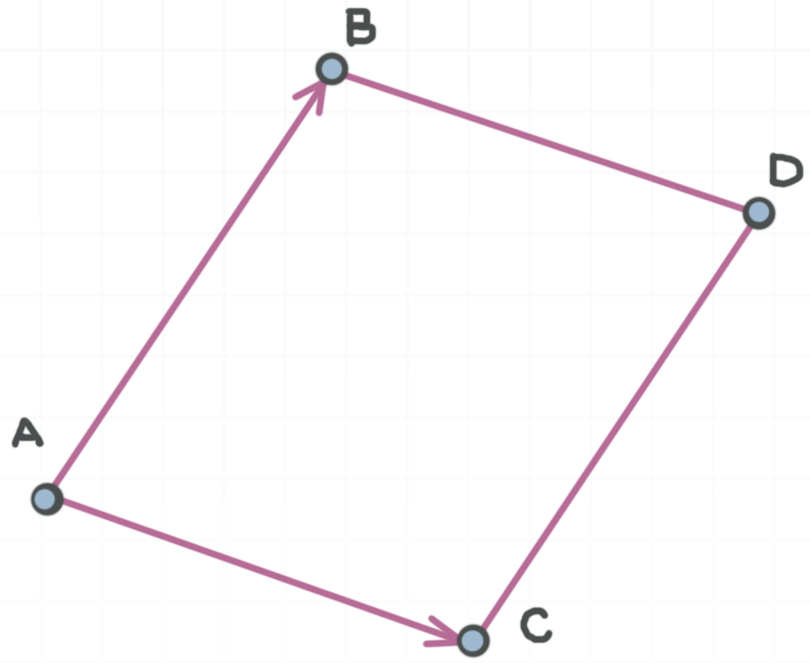
■ 2. Find the vector $\overrightarrow{a}-\overrightarrow{b}+2\overrightarrow{c}$, if $\overrightarrow{a}=\langle 0,4,5\rangle$, $\overrightarrow{b}=\langle -3,2,1\rangle$, and $\overrightarrow{c}=\langle 6,0,2\rangle$.

■ 3. Find the sum of the vectors.

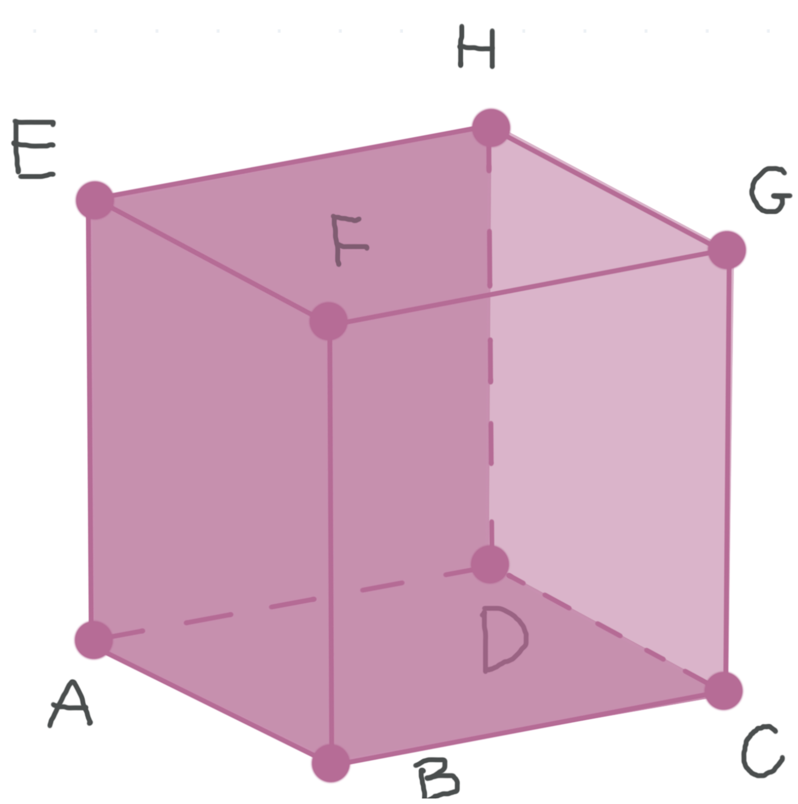
$$\sum_{k=1}^{100}\langle 5,k\rangle$$

COPYING VECTORS AND USING THEM TO DRAW COMBINATIONS

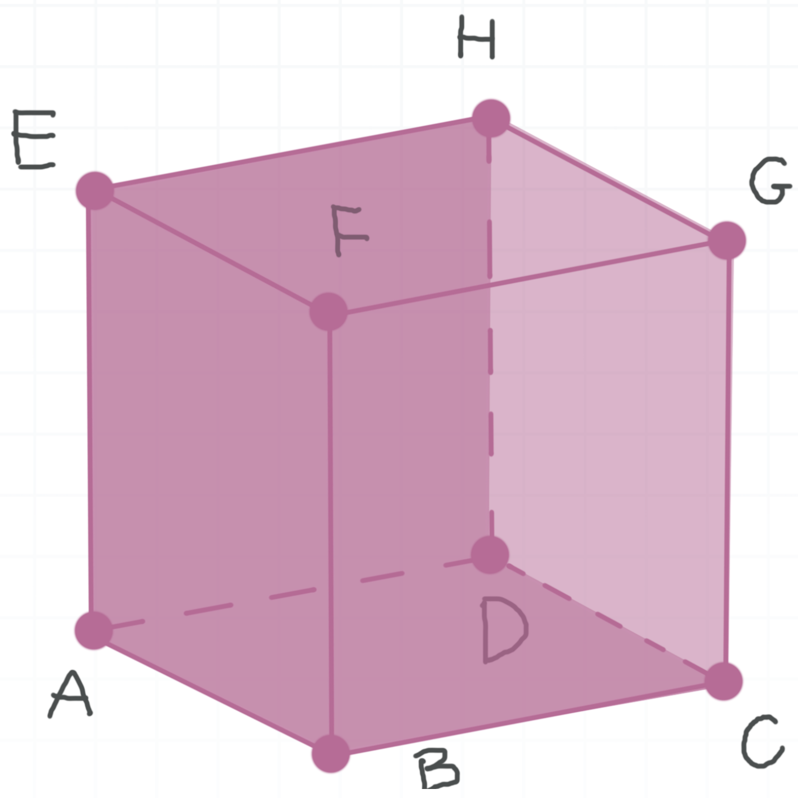
■ 1. In parallelogram $ABDC$, find the combination $\overrightarrow{AB}+\overrightarrow{AC}$



■ 2. In the cube $ABCDEFGH$, find the combination $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{AE}$.



■ 3. In the cube $ABCDEFGH$, find the combination $\overrightarrow{AB} - \overrightarrow{AD} - \overrightarrow{AE}$.



UNIT VECTOR IN THE DIRECTION OF THE GIVEN VECTOR

■ 1. Find the unit vector in the direction of the combination $\overrightarrow{a}+\overrightarrow{b}$, where $\overrightarrow{a}=\langle -2,-7\rangle$ and $\overrightarrow{b}=\langle 5,3\rangle$.

■ 2. The magnitude of the vector $\overrightarrow{a}$ is three times larger than the unit vector in the same direction. Find the vector $\overrightarrow{a}$.

$$\overrightarrow{u_a}=\left\langle \frac{1}{3},-\frac{2}{3},-\frac{2}{3}\right\rangle$$

■ 3. Find the unit vector in the direction of $\overrightarrow{AC}$ in the rectangle $ABCD$, if $A(4,1)$, $B(1,4)$, and $D(9,6)$.

ANGLE BETWEEN A VECTOR AND THE X-AXIS

■ 1. Find the clockwise angle in radians between the vector $\overrightarrow{a} = \langle \sqrt{3}, -1 \rangle$ and the negative direction of the x-axis.

■ 2. Find the angle between the vector $\overrightarrow{OA} = \langle 4, -4, 2 \rangle$ and the positive direction of the x-axis.

■ 3. Find the angle between the vectors $\overrightarrow{a} = \langle 3, 4 \rangle$ and $\overrightarrow{b} = \langle -5, 12 \rangle$.

MAGNITUDE AND ANGLE OF THE RESULTANT FORCE

■ 1. Find the magnitude and angle of the resultant force $\vec{f}$ of the vectors $\overrightarrow{a} = \langle 2, -1 \rangle$, $\overrightarrow{b} = \langle 5,1 \rangle$, and $\overrightarrow{c} = \langle -3,3 \rangle$.

■ 2. Find the magnitude of the resultant force $\vec{f}$ of the vectors $\overrightarrow{a} = \langle 4,0,0 \rangle$, $\overrightarrow{b} = \langle 0,4,0 \rangle$, and $\overrightarrow{c} = \langle 0,0,2 \rangle$, then find the angles between $\vec{f}$ and each of the major coordinate axes.

■ 3. The resultant force $\vec{f}$ of the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ has a magnitude of 12 and an angle of $2\pi/3$. Find vector $\overrightarrow{b}$, if $\overrightarrow{a} = \langle -8,5\sqrt{3} \rangle$.

DOT PRODUCT OF TWO VECTORS

■ 1. Find the dot product $\vec{a} \cdot \vec{b}$, where the vectors $\vec{a}$ and $\vec{b}$ have opposite directions, and $\vec{b}$ has a magnitude two times larger than $\vec{a} = \langle 2, -3, 5 \rangle$.

■ 2. Find the value(s) of the parameter p such that the dot product of the vectors $\vec{a} = \langle p, 2p+1, 3 \rangle$ and $\vec{b} = \langle p-2, 5, -4 \rangle$ is 2.

■ 3. Find the unit vector(s) $\vec{u}$ such that the dot product $\vec{a} \cdot \vec{u}$ reaches its maximum value, if $\vec{a} = \langle 2, 2 \rangle$.

ANGLE BETWEEN TWO VECTORS

■ 1. Use dot products to find the angles between the vector $\vec{a} = \langle -2,4,-4 \rangle$ and the positive direction of each major coordinate axis.

■ 2. Find the angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, if $\vec{a} = \langle 3,-4,4 \rangle$ and $\vec{b} = \langle -6,2,-1 \rangle$.

■ 3. Find the two vectors $\vec{b_1}$ and $\vec{b_2}$ with magnitude 5 that each have an angle of 30° with $\vec{a} = \langle -2,1 \rangle$.

ORTHOGONAL, PARALLEL, OR NEITHER

■ 1. Find the terminal point B of the vector $\overrightarrow{AB}$ that has initial point $A(2,0,-1)$, magnitude 24, and is parallel to the vector $\overrightarrow{c} = \langle -2,4,4 \rangle$.

■ 2. Find two vectors $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ with magnitude 2, that are orthogonal to $\overrightarrow{a} = \langle 3,-1 \rangle$.

■ 3. Find value(s) of the parameter p, such that the vectors $\overrightarrow{a} = \langle p, p+3, 6-p \rangle$ and $\overrightarrow{b} = \langle p-1, 4, 2 \rangle$ are (a) parallel, and (b) orthogonal.

ACUTE ANGLE BETWEEN THE LINES

1. Find the acute angle between the lines.

Line 1: $x = 2t + 1$, $y = t - 4$, $z = 6$

Line 2: $\dfrac{x - 1}{4} = \dfrac{y + 1}{5} = z$

2. Find the acute angle between the line and the plane.

Line: $x = t + 7$, $y = -2t - 5$, $z = 3t + 6$

Plane: $3x - y - 4z + 15 = 0$

3. Find the acute angle between the planes.

Plane 1: $x - 2y + 1 = 0$

Plane 2: $x + y + 2z + 4 = 0$

ACUTE ANGLES BETWEEN THE CURVES

■ 1. Find the acute angle(s) between the curves.

$$x^2 + y^2 = 4$$

$$x^2 + 4y^2 = 4$$

■ 2. Find the acute angle(s) between the curves given in parametric form.

$$x = t^2 + 1,\ y = 2t^2 + t - 3,\ z = t - 1$$

$$x = 2s^2 - 7,\ y = s - 5,\ z = s - 3$$

■ 3. Find the value of the parameter p such that $f(x) = e^x$ and $g(x) = e^{-x} + 2p$ are orthogonal at the point(s) of intersection.

DIRECTION COSINES AND DIRECTION ANGLES

■ 1. Find the direction angles of the linear combination $\vec{c}=2\vec{a}-3\vec{b}$, where $\vec{a}=\langle 3,1,-3\rangle$ and $\vec{b}=\langle 0,-2,-2\rangle$.

■ 2. Find the vector $\vec{a}$ with magnitude 6 that has direction angles 120°, 45°, and 135° with respect to x, y, and z-axes, respectively.

■ 3. Find the vector $\vec{a}$ that has an x-coordinate of 2, y-coordinate of -1, and direction angle with respect to the z-axis of $\pi/3$.

SCALAR EQUATION OF A LINE

■ 1. Find the parametric scalar equations of the line that pass through the points $A(5,4,-3)$ and $B(1,0,3)$.

■ 2. Find the parametric scalar equations of the line that passes through the point $A(4,-1,0)$ and is orthogonal to the plane $x+2y-z=7$.

■ 3. Find the parametric scalar equations of the line that forms the intersection of the planes $2x+3y-z=1$ and $x-y+4z=-4$.

SCALAR EQUATION OF A PLANE

■ 1. Find the scalar equations of the plane, given its vector equation.

$$\langle 1,2,-1\rangle \cdot (\vec{r} - \langle 0,5,-4\rangle) = 0$$

■ 2. Find the scalar equations of the plane that passes through the points $A(2,0,1)$, $B(-1,3,2)$, and $C(1,1,-4)$.

■ 3. Find the scalar equation of a plane(s) that's 6 units from, and parallel to, the plane $x-2y+2z-2=0$.

SCALAR AND VECTOR PROJECTIONS

■ 1. Find the vector sum of projections of the vector $\vec{a} = \langle 13, -8, 9 \rangle$ onto the three coordinate axes.

■ 2. Find the projection of the vector $\vec{a} = \langle 4, 3, -1 \rangle$ onto the plane Q, which is given by $2x - y + 2z - 7 = 0$.

■ 3. Find the vector $\vec{a}$ if its scalar projections onto the vectors $\vec{b} = \langle 4, -3 \rangle$ and $\vec{c} = \langle 0, 2 \rangle$ are both 3.

CROSS PRODUCT OF TWO VECTORS

■ 1. Find the vector $\vec{a}$ given that $\vec{a} \times \vec{b} = \vec{c}$, where $\vec{a} = \langle 1, a_2, a_3 \rangle$, $\vec{b} = \langle 3,1,1 \rangle$, and $\vec{c} = \langle 1,2,-5 \rangle$.

■ 2. Find the cross product $\vec{a} \times \vec{a}$ for an arbitrary vector $\vec{a}$.

■ 3. Find the cross product $\vec{a} \times \text{proj}_{xy}\vec{a}$, where $\vec{a} = \langle 4,5,-3 \rangle$ and $\text{proj}_{xy}\vec{a}$ is the vector projection of the vector $\vec{a}$ onto the xy-plane.

VECTOR ORTHOGONAL TO THE PLANE

■ 1. Find the vector orthogonal to the plane which passes through the point $A(2,3,1)$ and the z-axis.

■ 2. Find the equation of the plane that passes through the point D and is parallel to the plane ABC, if $A(1,2,-2)$, $B(1,4,3)$, $C(-5,3,-1)$, and $D(2,-4,7)$.

■ 3. Find the equation of the line that passes through the point $A(-2,3,4)$ and is orthogonal to the plane that includes the vectors $\vec{a}=\langle 2,4,0\rangle$ and $\vec{b}=\langle -1,1,2\rangle$.

VOLUME OF THE PARALLELEPIPED FROM VECTORS

■ 1. Find the height of the parallelepiped given that its volume is 670, and that the vectors $\vec{a}=\langle 1,0,-1\rangle$ and $\vec{b}=\langle 2,3,5\rangle$ are the adjacent edges of its base.

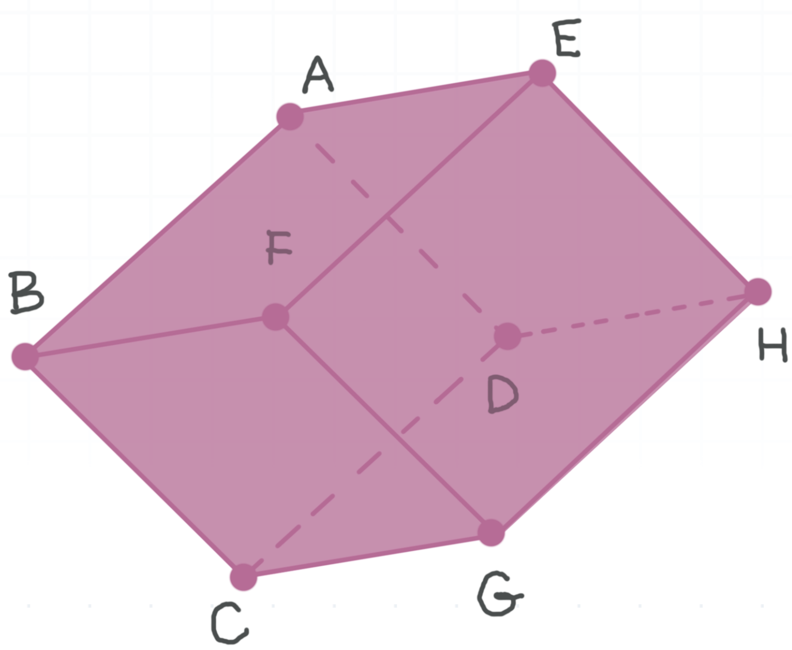
■ 2. Find the volume of the tetrahedron whose adjacent edges are the vectors $\vec{a}=\langle 0,0,3\rangle$, $\vec{b}=\langle 2,1,4\rangle$, and $\vec{c}=\langle -1,-2,1\rangle$.

■ 3. Find the value of p such that the volume of the parallelepiped with adjacent edges $\vec{a}=\langle 0,2,3\rangle$, $\vec{b}=\langle 1,-2,1\rangle$, and $\vec{c}=\langle p,p,p\rangle$ is equal to 63.

VOLUME OF THE PARALLELEPIPED FROM ADJACENT EDGES

■ 1. Find the volume of tetrahedron $ABCD$, given $A(2,0,3)$, $B(-1,1,3)$, $C(4,5,-2)$, and $D(2,2,3)$.

■ 2. Find the volume of parallelepiped $ABCDEFGH$, given $A(1,2,2)$, $B(-1,-2,0)$, $F(4,3,-1)$, and $G(5,6,-4)$.



■ 3. Find the volume of the parallelepiped with base $ABCD$ and height 5, if $A(3,3,3)$, $B(0,-2,-2)$, and $C(-3,1,0)$.

SCALAR TRIPLE PRODUCT TO PROVE VECTORS ARE COPLANAR

■ 1. Find the value of the parameter p such that the vectors $\vec{a}=\langle 1,3,-1\rangle$, $\vec{b}=\langle 2,2,2\rangle$, and $\vec{c}=\langle 0,-1,p\rangle$ are coplanar.

■ 2. Check if the vectors $\vec{a}=\langle 1,1,0\rangle$, $\vec{b}=\langle 0,1,1\rangle$, and $\vec{c}=\langle 1,0,-1\rangle$ are coplanar. If they are, find the equation of the plane, assuming that the initial point of the vectors is the origin.

■ 3. Check if the points $A(0,0,1)$, $B(2,0,3)$, $C(2,3,0)$, and $D(3,2,2)$ lie in the same plane.

DOMAIN OF A VECTOR FUNCTION

■ 1. Find the domain of the vector function.

$$\vec{F}(t, s) = \left\langle \sqrt{ts}, \frac{t}{s}, e^{t^2+s^2} \right\rangle$$

■ 2. Find the domain of the vector function.

$$\vec{F}(x, y) = \ln(x + y - 3) \cdot \mathbf{i} + \sqrt{2x - 2} \cdot \mathbf{j} + \sqrt{6 - y} \cdot \mathbf{k}$$

■ 3. Find the domain of the vector function.

$$\vec{F}(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2} \cdot \mathbf{i} + \frac{2x - y}{x + y + z - 4} \cdot \mathbf{j}$$

LIMIT OF A VECTOR FUNCTION

■ 1. Find the limit of the vector function.

$$\lim_{t \to 0,\, s \to 1} \vec{F}(t, s)$$

$$\vec{F}(t, s) = \left\langle \sqrt{s^2 - t^2}, \frac{\sin 3t}{t} + 3s^2, \frac{(t^2 - 2t - 3)(s^2 - 1)}{s - 1} \right\rangle$$

■ 2. Find the limit of the vector function.

$$\lim_{x \to \infty,\, y \to \infty} \vec{F}(t, s)$$

$$\vec{F}(x, y) = xy\, e^{-(x^2+y^2)} \cdot \mathbf{i} + \frac{\sin(x + y)}{x + y} \cdot \mathbf{j} + \frac{x}{y^4} \cdot \mathbf{k}$$

■ 3. Find the limit of the vector function.

$$\lim_{x \to 3,\, y \to \infty,\, z \to 1} \vec{F}(x, y, z)$$

$$\vec{F}(x, y, z) = (x^2 y - 3xyz + z^2 - x + 3y - 3z + 5)\mathbf{i} + \ln \frac{x + y}{z + y}\mathbf{j}$$

SKETCHING THE VECTOR EQUATION

■ 1. Identify and sketch the curve that represents $\vec{r}(t)=\langle 3-5t,2t+1,-3t\rangle$.

■ 2. Identify and sketch the curve representing the graph of the vector function $\vec{r}(t)=\langle 5\sin t,3\cos t,-2\rangle$.

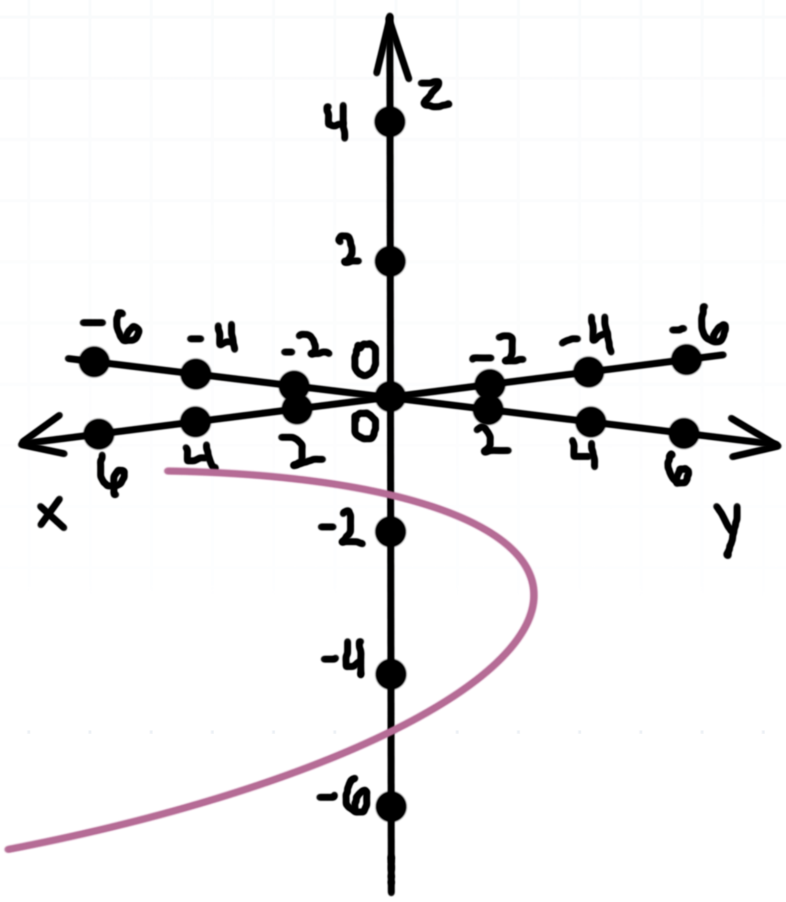
■ 3. Identify and sketch the surface representing the graph of the vector function.

$$\vec{r}(t,s)=\langle 4\sin t\cos s,4\sin t\sin s,4\cos t\rangle$$

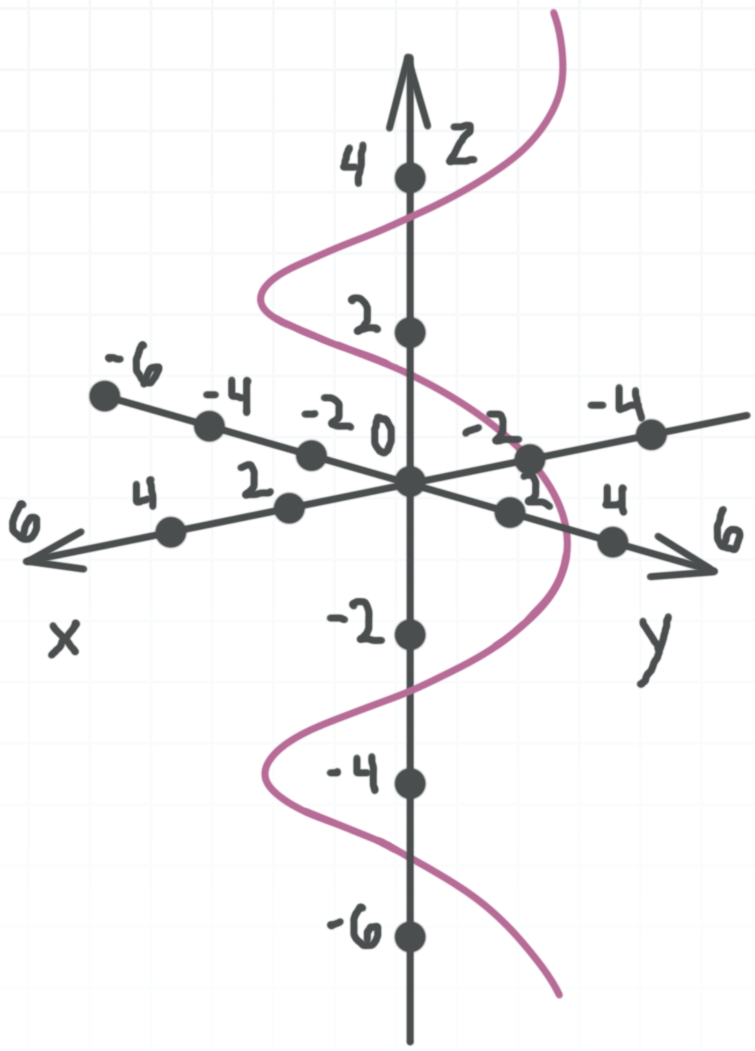
PROJECTIONS OF THE CURVE

■ 1. Identify and sketch the projections of the curve onto each of the major coordinate planes.

$$\vec{r}(t) = \left\langle t^2 - 1, \frac{t+4}{2}, t - 3 \right\rangle$$



■ 2. Identify and sketch the projections of the curve $\vec{r}(t) = \langle 2\cos t, 2\sin t, t + \pi \rangle$ onto each of the coordinate planes.



■ 3. Identify and sketch the projections of the surface onto each of the coordinate planes. Using the projections, identify the surface.

$$\vec{r}(u,v)=\left\langle 3\cos u, 3\sin u, \frac{v}{2}\right\rangle$$

VECTOR AND PARAMETRIC EQUATIONS OF A LINE SEGMENT

■ 1. Find the vector and parametric equation of the line segment AB, given $A(-4,2)$ and $B(1,5)$.

■ 2. Find the vector equation of the line segment AB, if $A(2,-1,3)$, $\overrightarrow{AB}$ is parallel to $\langle -2,2,1 \rangle$, and B is the intersection point of the line AB with the xz-plane.

■ 3. Find the endpoints, midpoint, and the length of the line segment for $\vec{r}(t) = \langle 2-3t, 4+t, 2-5t \rangle$ with $0 \le t \le 1$.

VECTOR FUNCTION FOR THE CURVE OF INTERSECTION OF TWO SURFACES

■ 1. Find the vector function for the line of intersection of the two planes.

$$2x - y + 3z - 5 = 0$$

$$x + y - 2z + 1 = 0$$

■ 2. Find the vector function for the curve of intersection of two spheres.

$$x^2 + y^2 + z^2 = 5^2$$

$$(x - 3)^2 + y^2 + z^2 = 4^2$$

■ 3. Find the vector function for the curve of intersection of the elliptic cylinder and the plane.

$$\frac{(x-2)^2}{3^2} + \frac{(y+1)^2}{4^2} = 1$$

$$2x - 3y - z - 4 = 0$$

DERIVATIVE OF A VECTOR FUNCTION

■ 1. Find the second order derivative of the vector function.

$$\vec{r}(t)=\left\langle \sqrt{t},\frac{2}{t},e^{t+3}\right\rangle$$

■ 2. Find the Jacobian matrix of the vector function at $(u,v)=(1,2)$.

$$\vec{r}(u,v)=\langle 2uv+1,u^2+v^2\rangle$$

■ 3. Find the Jacobian matrix for the vector function.

$$\vec{r}(t,s)=\langle \ln(ts),3t+2s-1,\ \sin(t+s)\rangle$$

UNIT TANGENT VECTOR

■ 1. Find the unit tangent vector to the function that that sits at a 30° angle.

$$\vec{r}(t)=\langle t^2+4,2t^3-3\rangle$$

■ 2. Find the tangent vector at the point $(-1,0,1)$.

$$\vec{r}(t)=\langle 2t^3-3t^2+5t-5,\ \sin(\pi t),e^{t-1}\rangle$$

■ 3. Find the point(s) where the unit tangent vector to the curve is orthogonal to the xz-plane

$$\vec{r}(t)=\langle t^3+2,5t^2-3t+8,t^2+5\rangle$$

PARAMETRIC EQUATIONS OF THE TANGENT LINE

■ 1. Find the parametric equation of the tangent line to $\vec{r}(u)$ at $u = -2$.

$$\vec{r}(u) = \langle e^{u+3}, \ln(1 - u) \rangle$$

■ 2. Find the parametric equation(s) of the tangent line to the function $\vec{r}(t)$ that passes through the origin.

$$\vec{r}(t) = \langle 2t^2, 3t + 3, t + 1 \rangle$$

■ 3. Find the equation of the tangent plane to the surface $\vec{r}(t, s)$ at the point $t = 1$ and $s = 4$.

$$\vec{r}(t, s) = \langle t^2 + s^2, -3t + 5, 2s + 1 \rangle$$

INTEGRAL OF A VECTOR FUNCTION

1. Find the integral of the vector function.

$$\int \langle e^{3u-2}, e^{5-u}, \sin^2(u-\pi)\rangle \ du$$

2. Find the improper integral of the vector function.

$$\int_2^\infty \left\langle \frac{t-2}{t^3-8}, 2^{-t+1} \right\rangle \ dt$$

3. Find the double integral of the vector function, where R is the square $[0,\pi] \times [0,\pi]$.

$$\int\int_R \langle ts, \sin(t-s)\rangle \ dA$$

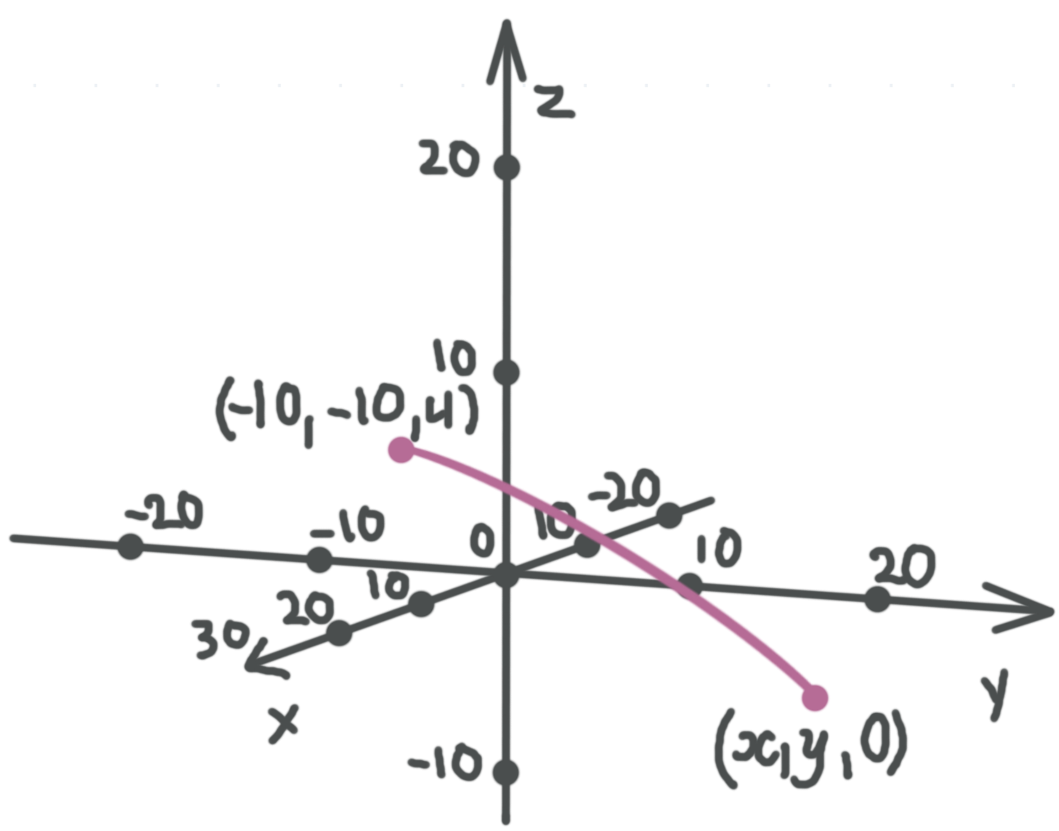
ARC LENGTH OF A VECTOR FUNCTION

■ 1. Confirm the formula for the arc length $L = 2\pi R$ around the circle by considering the circle's equation as the vector function in polar coordinates, where R is the radius of the circle.

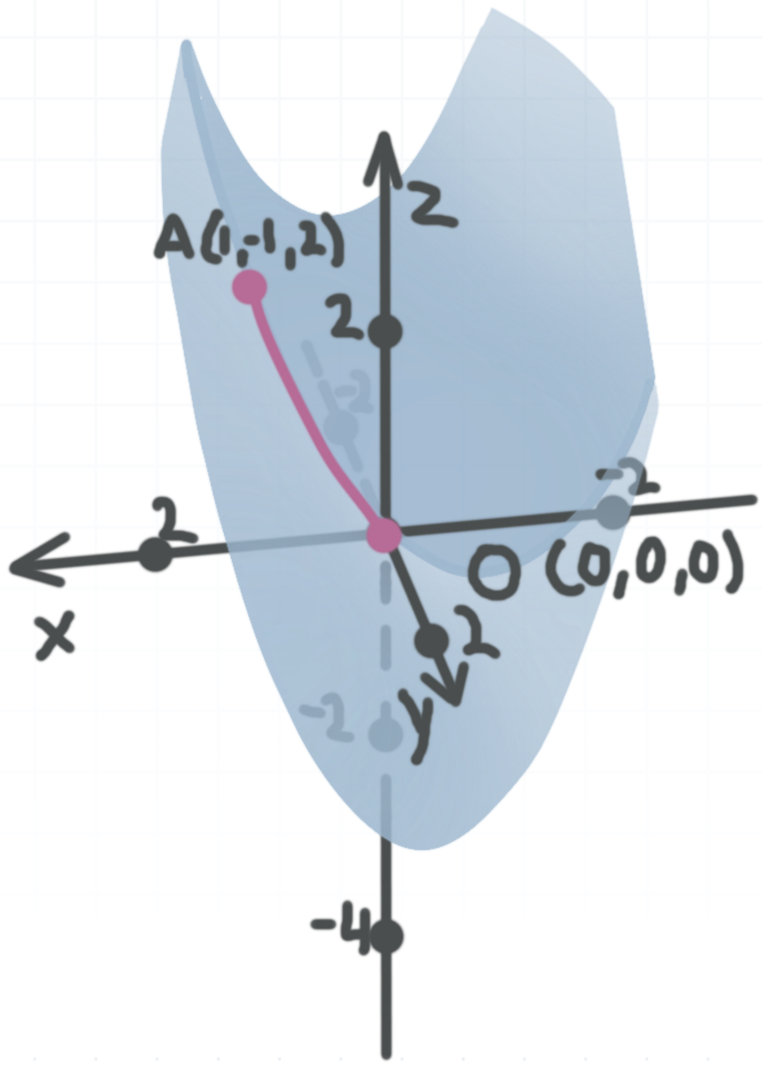
$$\vec{r}(\phi) = \langle R\cos\phi, R\sin\phi \rangle \text{ with } 0 \le \phi \le 2\pi$$

■ 2. A cannon ball is shot from the point $A(-10,-10,4)$. Its trajectory can be modeled by the vector function, where $t \ge 0$ is the time. Find the arc length of the ball's trajectory before it hits the ground $z = 0$.

$$\vec{r}(t) = \left\langle t - 10, t - 10, \frac{-t^2 + 20t + 800}{200} \right\rangle$$



■ 3. Find the arc length of the curve that's the intersection of the cylinder $x^2 - y - z = 0$ and the plane $x + y = 0$, between $O(0,0,0)$ and $A(1, -1,2)$.



REPARAMETRIZING THE CURVE

■ 1. Reparametrize $\vec{r}(t) = \langle -3 + t, 2 + 2t, 6 - 2t \rangle$ in terms of the arc length measured from $(-3,2,6)$ in the direction of increasing t.

■ 2. Reparametrize $\vec{r}(t) = \langle 4\cos 3t, -2t, 4\sin 3t \rangle$ in terms of arc length, measured from $(-4,2\pi,0)$.

■ 3. Reparametrize the curve $\vec{r}(t) = \langle 2e^{2t}, e^{2t} \rangle$ in terms of arc length measured from $t = 0$. Use the parametrization to find the position after traveling 5 units.

CURVATURE

■ 1. Find the curvature of $f(x) = 2x^2 - 4$ at $x = 1$.

■ 2. Find the curvature of the vector function at $t = 0$.

$$\vec{r}(t) = \left\langle 2(2+t)^{3/2}, 6t, 2(2-t)^{3/2} \right\rangle$$

■ 3. Find the value(s) of t_0 such that the curvature of $\vec{r}(t) = \langle e^t + 5, 2e^t, -2e^t \rangle$ is 0 at $t = t_0$.

MAXIMUM CURVATURE

■ 1. Find the absolute maximum curvature $k(t)$ of $\vec{r}(t) = \langle 2 + \sin t, \cos(t + \pi) \rangle$ on the interval $[0,2\pi]$.

■ 2. Find the absolute minimum and maximum curvature $k(x)$ of the function $f(x) = \ln(6x)$ on the interval $(0,1]$.

■ 3. Find the absolute maximum curvature $k(t)$ of $\vec{r}(t) = \langle 3t + 1, 2.5t^2 - 3, 4 - 4t \rangle$ on the interval $(-\infty, \infty)$.

NORMAL AND OSCULATING PLANES

■ 1. Find the point(s) at which the normal plane to the curve $\vec{r}(t)$ is parallel to the y-axis, then find the equation(s) of the normal plane at each point.

$$\vec{r}(t) = \langle 3t^3 - 10t, t^3 - 6t^2 - 15t, 4t + 1 \rangle$$

■ 2. Find the equation of the osculating plane to $\vec{r}(t) = \langle 12 - 6t, 5t^2 - 10, 7 - 8t \rangle$ at the point $(0,10,-9)$.

■ 3. Use the binormal vector to prove that the graph of the vector function $\vec{r}(t)$ is a planar curve (a curve that lies in a single plane), then find the equation of the plane.

$$\vec{r}(t) = \langle 2\sin t - 2,\ \cos t + 1, 2\cos t + 5 \rangle$$

EQUATION OF THE OSCULATING CIRCLE

■ 1. Find the equation of the osculating circle to the curve $\vec{r}(t)=\langle 2+5\sin t, 5\cos t-1\rangle$ at an arbitrary point.

■ 2. Find the center and radius of the osculating circle to the curve $\vec{r}(t)$ at the point $(7,6)$.

$$\vec{r}(t)=\langle 4(5-t)^{5/2}+3, 24t-90\rangle$$

■ 3. Find the point(s) on the curve $\vec{r}(t)=\langle t^2+3, 2t-5\rangle$ where the osculating circle has a radius of 2.

VELOCITY AND ACCELERATION VECTORS

1. Find the value of t such that the velocity of the vector function $\vec{r}(t)$ is 0.

$$\vec{r}(t)=\langle 4t^3-5t^2-28t, 2e^{t-1}+e^{-2t+5}, \cos(\pi t)\rangle$$

2. Find the point on the curve such that the velocity along the x-axis reaches its maximum value.

$$\vec{r}(t)=\left\langle \frac{t}{t^2+2}, 3\tan(3t^2), \ln 2t\right\rangle$$

3. Find the values of the parameters p and q such that the absolute value of acceleration of the non-constant function $\vec{r}(t)=\langle p\sin 3t, 4\cos qt\rangle$ is a constant for any value of t.

VELOCITY, ACCELERATION, AND SPEED, GIVEN POSITION

■ 1. Find the point where the speed is 0, given the position function.

$$\vec{r}(t)=\left\langle \ln(2t^2+8t+50), t^4+32t+17,\ \arctan t-\frac{t}{5}\right\rangle$$

■ 2. Find the interval(s) of t values where the acceleration along the z-axis is negative for the position function.

$$\vec{r}(t)=\langle 2\sin(2t), e^{t^2+1}, t^4-10t^3-36t^2-5t+45\rangle$$

■ 3. Find the velocity, speed, and acceleration of the position function at the point(s) where the trajectory intersects the xy-plane.

$\vec{r}(t)=\langle \sin 4t, 2\cos(t+\pi), 2+2\sin t\rangle$, where $t\in[0,2\pi]$

VELOCITY AND POSITION GIVEN ACCELERATION AND INITIAL CONDITIONS

■ 1. Find the velocity and position of the acceleration function $\vec{a}(t) = \langle 4\sin^2 t, -\cos t \rangle$ if $\vec{r}(\pi) = \langle -2,1 \rangle$, and $\vec{v}(\pi) = \langle 0,0 \rangle$.

■ 2. Find the speed function given the acceleration function $\vec{a}(t) = \langle 4t^3 - 1, 6t^2 + 2t, 2e^{2t} \rangle$ if $\vec{v}(0) = \langle 1, -3,1 \rangle$.

■ 3. Find the distance travelled by a particle during the first 10 seconds, given its acceleration function $\vec{a}(t) = \langle 2\sin t, 2\cos t \rangle$, where t is the time in seconds and the initial velocity is $\vec{v}(0) = \langle -2,0 \rangle$.

TANGENTIAL AND NORMAL COMPONENTS OF ACCELERATION

■ 1. Find the tangential and normal components of acceleration for the vector function $\vec{r}(t) = \langle e^{2t} + 1, e^{-2t} - 1, t - 4 \rangle$ at the point $(2,0, - 4)$.

■ 2. Find the point(s) where the tangential component of acceleration for the vector function $\vec{r}(t) = \langle 2\cos t - 2, 3\sin t + 5, 4t - 1 \rangle$ is 0.

■ 3. Find the values of parameters p and q, such that the normal components of acceleration for $\vec{r}(t) = \langle 2t^2, 3pt, t^2 - 4t + qt \rangle$ are 0 at the origin.

SKETCHING THE VECTOR FIELD

■ 1. Find a two-dimensional vector field in which all of the vectors are orthogonal to $y = -2x$, then sketch the vector field.

■ 2. Find a two-dimensional vector field such that each vector is tangent to some circle centered at the origin, then sketch the vector field.

■ 3. Find a three-dimensional unit radial vector field (a vector field where all the vectors have magnitude 1, and point straight towards or away from the origin).

GRADIENT VECTOR FIELD

■ 1. Sketch the gradient vector field of $f(x,y)=x^2+y^2$.

■ 2. Sketch the gradient vector field of $f(x,y)=\ln x+\ln y$.

■ 3. Find the point(s) such that the gradient vector of the function $f(x,y,z)$ is equal to the zero vector $\vec{O}$.

$$f(x,y,z)=x^2+y^2+z^2-4xyz-2x-2y-2z+5$$

LINE INTEGRAL OF A CURVE

1. Calculate the line integral over c, where c is the circle that lies in the plane $z=3$, with center on the z-axis and radius 4.

$$\int_c x^2+y^2+z^2\ ds$$

2. Calculate the line integral P over c, where c is the part of the graph of the vector function $\vec{r}(t)$ between the points $(-2,6,-2)$ and $(4,9,1)$.

$$\vec{r}(t)=\langle 2t,t^2+5,t-1\rangle$$

$$P=\int_c (y-z^2)\sqrt{5+x^2}\ ds$$

3. Calculate the improper line integral over c, where c is the line of intersection of the surfaces $z-x^2-y^2+2y+1=0$ and $x-y-1=0$.

$$\int_c \frac{1}{(1+8(x-1)y)^2}\ ds$$

LINE INTEGRAL OF A VECTOR FUNCTION

■ 1. Calculate the line integral of the vector function $\vec{F}(x, y) = \langle x + y, x - y \rangle$ over the curve $\vec{r}(t) = \langle t^2 - 1, t^2 + 1 \rangle$ for $-2 \le t \le 3$.

■ 2. Calculate the line integral of the vector function $\vec{F}(x, y, z) = \langle xyz, -z, y \rangle$ over c, where c is the ellipse that lies in the plane $x = -4$ with the center on the x-axis, a semi-axis of 2 in the y-direction, and a semi-axis of 5 in the z-direction.

■ 3. Calculate the improper line integral of the vector function $\vec{F}(x, y, z)$ over the curve $\vec{r}(t) = \langle e^t, -e^{-t}, 2t \rangle$ for $t \ge 0$.

$$\vec{F}(x, y, z) = \left\langle y^2, \frac{3}{x^2}, 2xy^2z \right\rangle$$

POTENTIAL FUNCTION OF A CONSERVATIVE VECTOR FIELD

■ 1. Determine whether or not the vector field is conservative.

$$\vec{F}(x, y, z) = \left\langle \ln(2y + z), \frac{2x}{2y + z}, \frac{x}{2y + z} \right\rangle$$

■ 2. Find the potential function of the vector field.

$$\vec{F}(x, y) = \langle \cos(x - 3y) + 5, -3\cos(x - 3y) - 8 \rangle$$

■ 3. Find the potential function of the vector field.

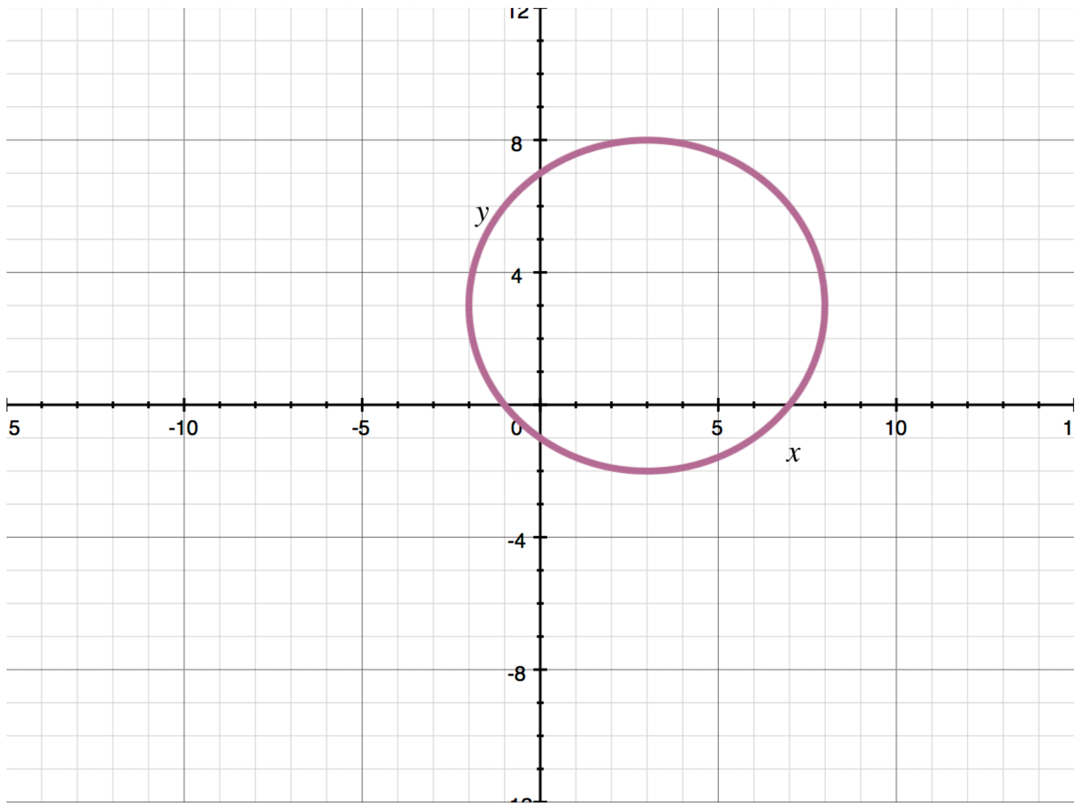
$$\vec{F}(x, y, z) = \langle z^2\, 2^{x+4y} \ln 2,\ z^2\, 2^{x+4y+2} \ln 2,\ z\, 2^{x+4y+1} - 6z^2 \rangle$$

POTENTIAL FUNCTION OF A CONSERVATIVE VECTOR FIELD TO EVALUATE A LINE INTEGRAL

■ 1. Calculate the line integral of the conservative vector field $\vec{F}(x, y)$ over the curve $\vec{r}(t) = \langle 9\arctan^2 t, t^4 - 2t^2 + 2\rangle$ between $(0,2)$ and $(\pi^2, 5)$.

$$\vec{F}(x, y) = \left\langle \frac{y}{\sqrt{x}}, 2(y + \sqrt{x}) \right\rangle$$

■ 2. Calculate the line integral of the conservative vector field $\vec{F}(x, y) = \langle x^2 + y^2, 2xy + 1\rangle$ over the part of the circle with center at $(3,3)$ and radius 5, that lies in the first quadrant, with clockwise rotation.



3. Calculate the line integral of the conservative vector field $\vec{F}(x,y,z) = \langle y^2, 2xy, (1+z)^{-1} \rangle$ over the curve $\vec{r}(t) = \langle \sin(\pi t^2), t^3 e^{t-1}, (t-2)^2 \rangle$ for $1 \le t \le 2$.

INDEPENDENCE OF PATH

■ 1. Check if the line integral of the vector field $\vec{F}(x, y)$ is independent of path for any curve connecting the points $(2,0)$ and $(0,2)$. If it *is* independent of path, then prove it. If not, give a counterexample.

$$\vec{F}(x, y) = \left\langle \frac{4y}{x^2 + y^2}, \frac{-4x}{x^2 + y^2} \right\rangle$$

■ 2. Check if the line integral of the vector field $\vec{F}(x, y)$ is independent of path for any curve that lies within the rectangle given by $1 < x < 5$ and $1 < y < 5$, and that connects the points $(2,4)$ and $(4,2)$.

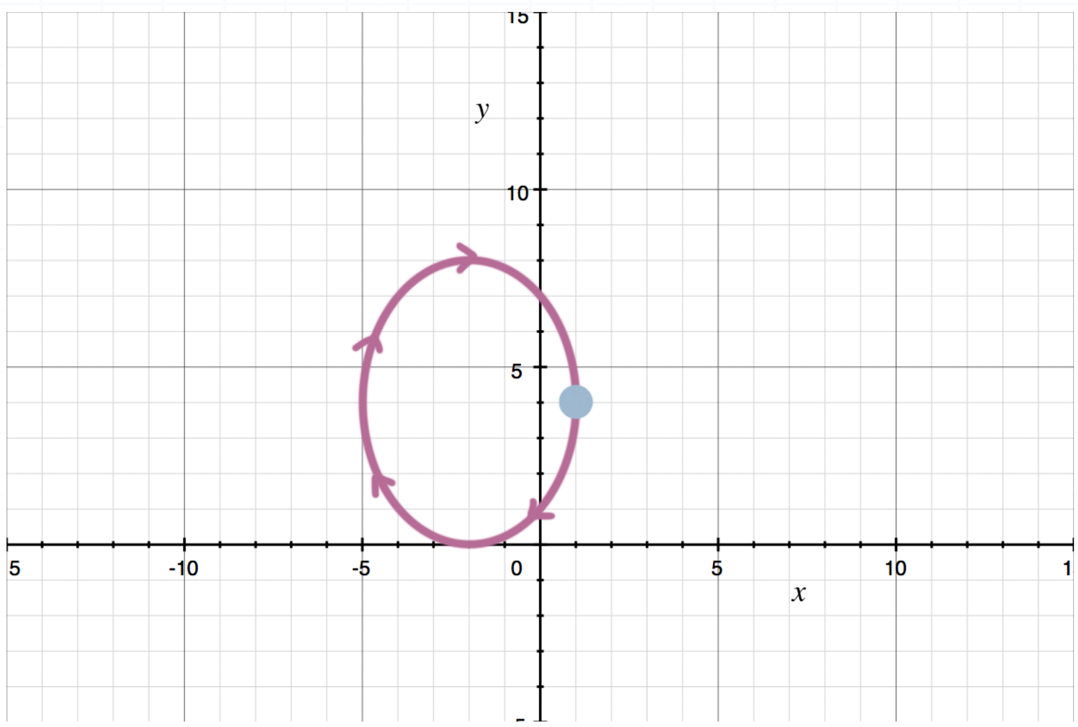
$$\vec{F}(x, y) = \left\langle \frac{2(x-1)}{(x^2 - 2x + y^2 + 1)^2}, \frac{2y}{(x^2 - 2x + y^2 + 1)^2} \right\rangle$$

■ 3. Determine whether the line integral of the vector field $\vec{F}(x, y, z)$ is independent of path for any curve that connects any two points within the vector field's domain.

$$\vec{F}(x, y, z) = \langle x\ln(x^2 + y^2 + z^2 - 9), y\ln(x^2 + y^2 + z^2 - 9), z\ln(x^2 + y^2 + z^2 - 9)\rangle$$

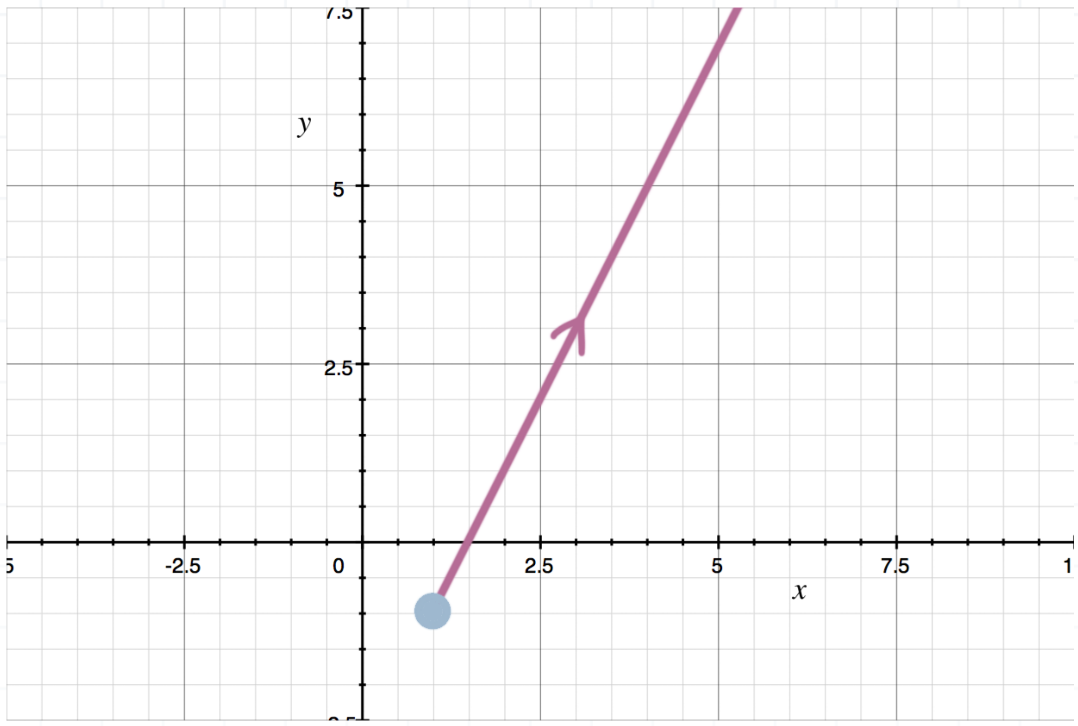
WORK DONE BY A FORCE FIELD

■ 1. Calculate the work done by the force field $\vec{F}(x,y) = \langle 25x^2 + 9y^2 + 1, x - y - 3 \rangle$ to move an object clockwise along the ellipse centered at $(-2,4)$ with semi-axis of 3 in the x-direction and semi-axis of 5 in the y-direction.



■ 2. Find the work done by the force field $\vec{F}(x,y)$ to move an object infinitely along the line $y = 2x - 3$, starting from $(1, -1)$, in the positive direction of x.

$$\vec{F}(x,y) = \left\langle xe^{-y}, \frac{y+2}{x^3} \right\rangle$$

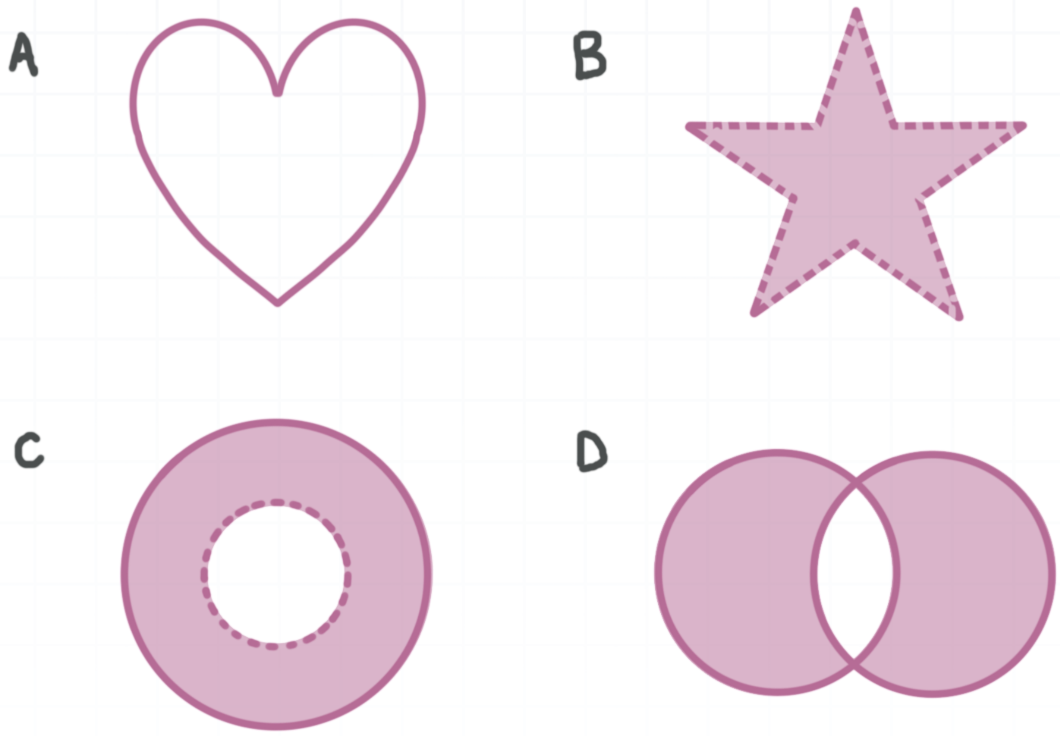


■ 3. Find the work done by the conservative force field $\vec{F}(x, y, z)$ to move an object between the four points $A(0,-1,2)$, $B(1,1,3)$, $C(2,3,0)$, and $D(0,2,1)$ (starting from A to B, then to C, and finally to D).

$$\vec{F}(x, y, z) = \langle 1 + 4x + yz + 3z^2, xz - 1, x(y + 6z) \rangle$$

OPEN, CONNECTED, AND SIMPLY CONNECTED

■ 1. Determine whether each set is open, closed, connected, or simply-connected.



■ 2. Find the domain D of the vector field $\vec{F}$, then determine whether it's open, closed, connected, or simply-connected.

$$\vec{F}(u,v)=\left\langle\sqrt{36-9u^2-4v^2},\ \log_2(uv-v)\right\rangle$$

■ 3. Find the domain D of the vector field $\vec{F}$, then determine whether it's open, closed, connected, or simply-connected.

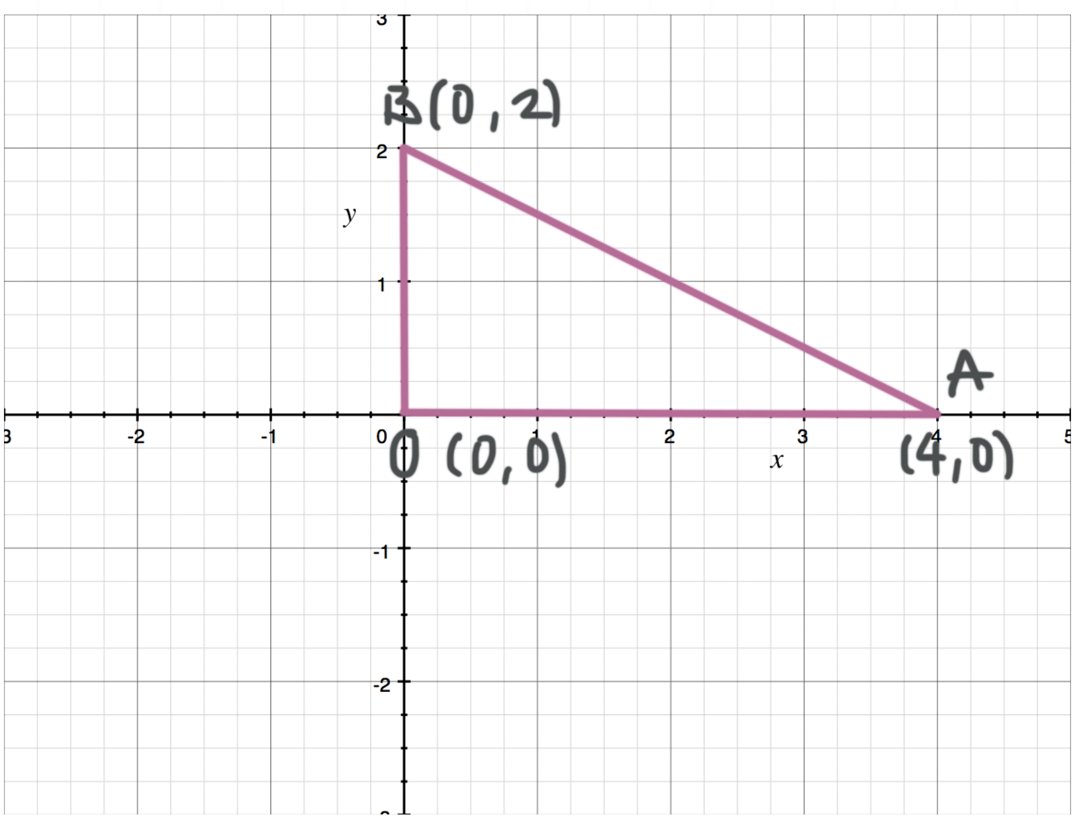
$$\vec{F}(x,y,z)=\left\langle\ln(4x-x^2-y^2-z^2),\frac{3x}{y^2+z^2},\frac{y}{x+8}\right\rangle$$

GREEN'S THEOREM FOR ONE REGION

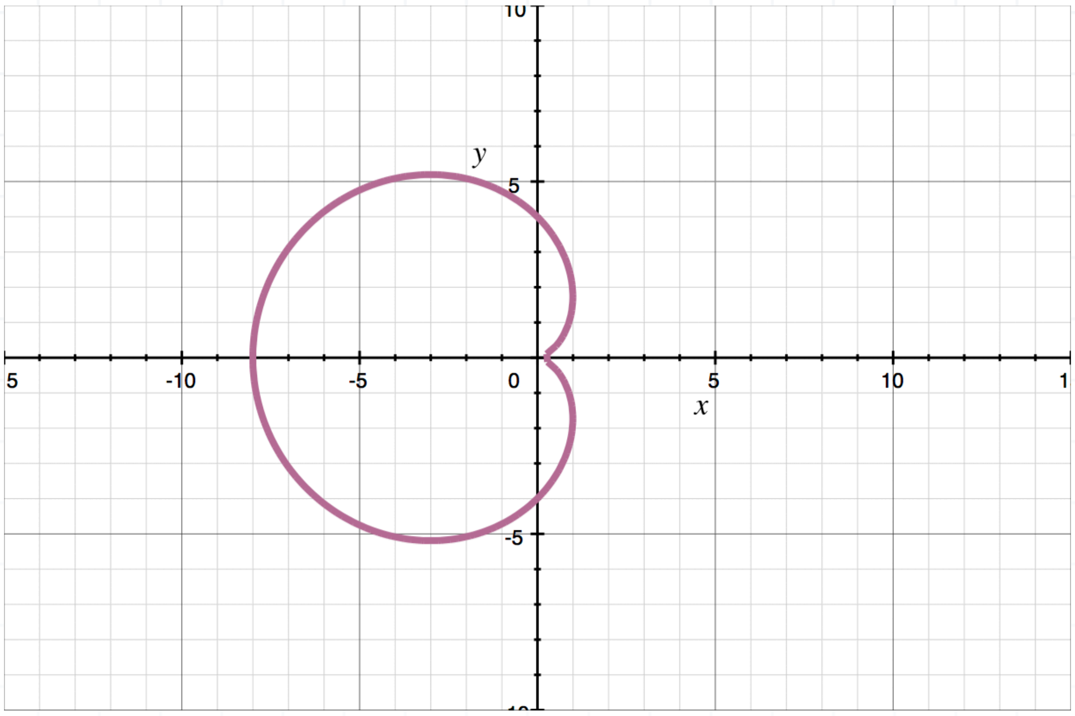
■ 1. Use Green's theorem to calculate the line integral of the vector field $\vec{F}(x,y)$ over the circle with the center at the origin and radius 4.

$$\vec{F}(x,y) = \left\langle \ln(x^2+y^2+20) - 2y - 3x, \sqrt{x^2+y^2+9} \right\rangle$$

■ 2. Use Green's theorem to calculate the line integral of the vector field $\vec{F}(x,y) = \langle y(y^2+\sin x), y^2 - \cos x \rangle$ over the triangle OAB, where $O(0,0)$, $A(4,0)$, and $B(0,2)$.

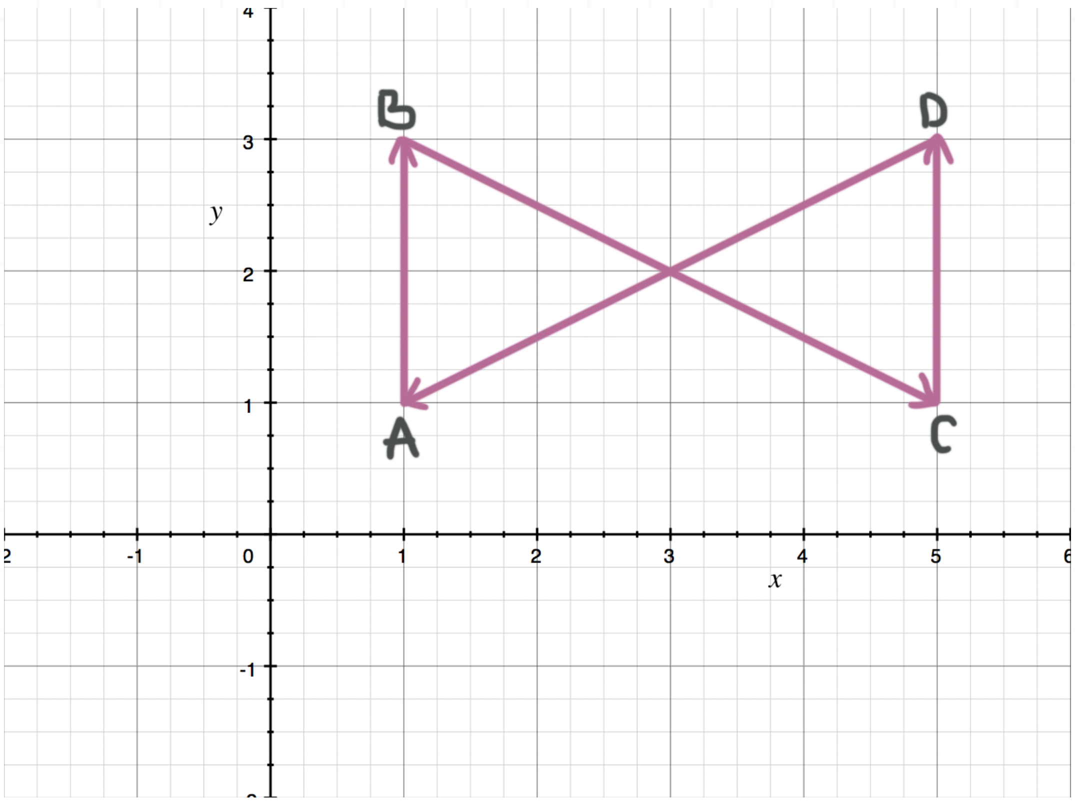


■ 3. Use Green's theorem to calculate the line integral of the vector field $\vec{F}(x,y) = \langle x^3 - y^3, x^3 + y^3 \rangle$ over the cardioid $(x^2+y^2)^2 + 8x(x^2+y^2) - 16y^2 = 0$.


y
5
-5
x
-10
-5
0
5
10

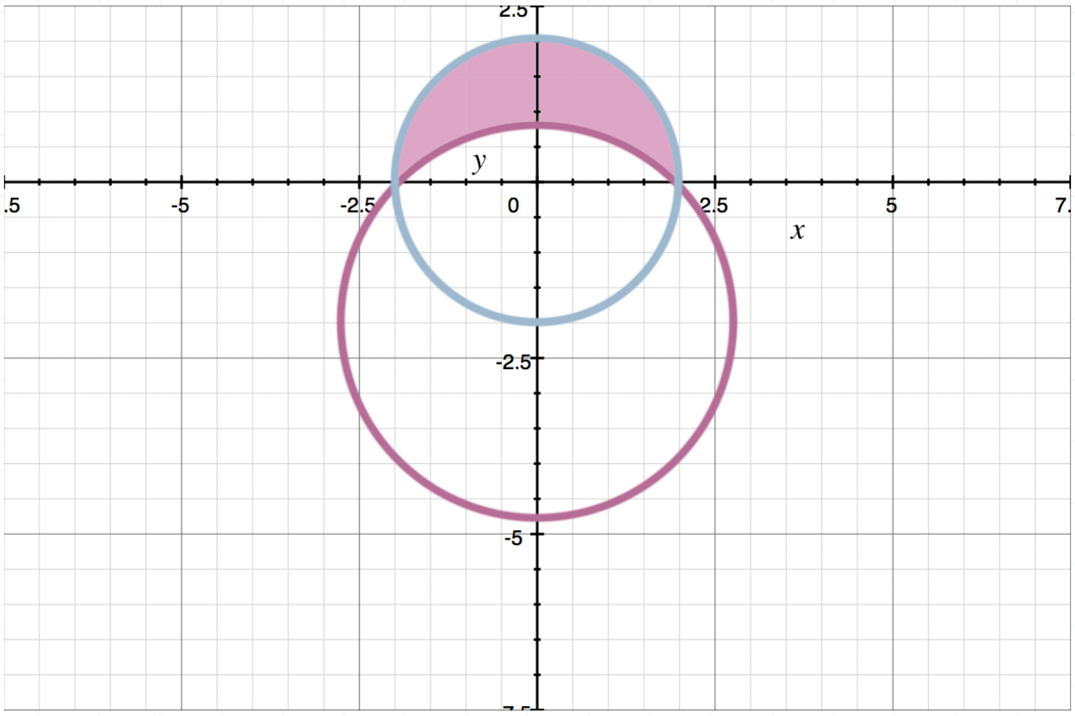
GREEN'S THEOREM FOR TWO REGIONS

■ 1. Use Green's theorem to calculate the line integral of the vector field $\vec{F}(x,y) = \langle x^2, x^3y \rangle$ over the piecewise linear closed curve $ABCDA$, where $A(1,1)$, $B(1,3)$, $C(5,1)$, and $D(5,3)$.

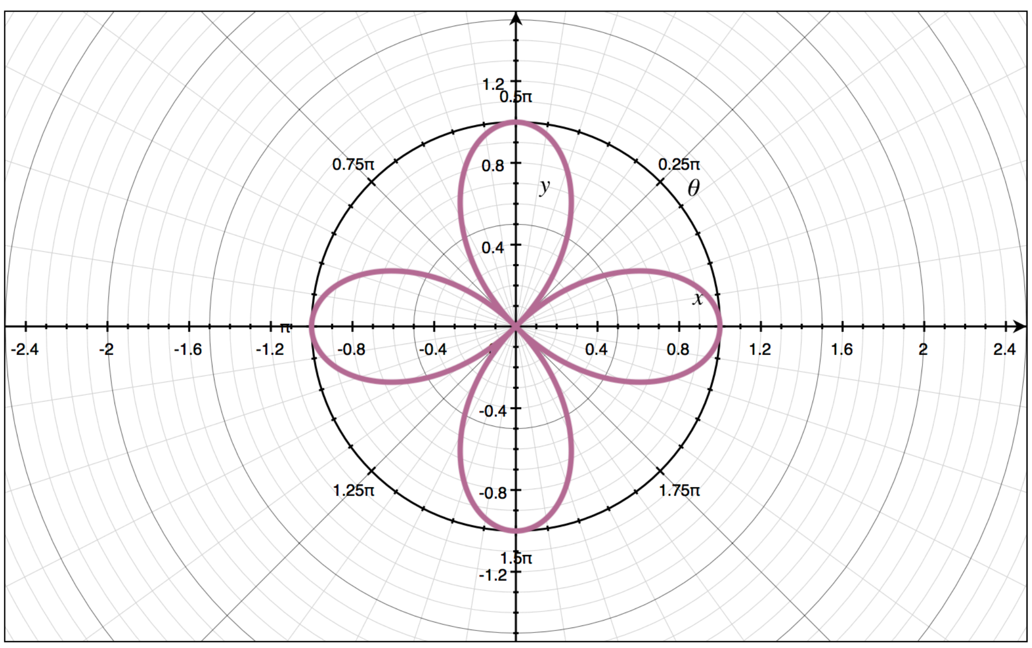


■ 2. Use Green's theorem (in reverse order) to calculate the double integral over the region D inside the circle $C_1: \ x^2 + y^2 = 4$, but outside the circle $C_2: \ x^2 + (y+2)^2 = 8$.

$$\iint_D 3x^2 \ dA$$



■ 3. Use Green's theorem to calculate the line integral of the vector field $\vec{F}(x,y)=\langle e^{x^2}-2y, y^2+2x\rangle$ over the four-petaled rose $r=\cos 2\phi$.



CURL AND DIVERGENCE OF A VECTOR FIELD

■ 1. Find the set of points in R^3 where the curl of the vector field $\vec{F}(x, y, z)$ is parallel to the vector $\vec{a} = \langle 2, 1, 2 \rangle$.

$$\vec{F}(x, y, z) = \left\langle \frac{z}{2}, \ln(xyz), z^2 \right\rangle$$

■ 2. Find the set of points in R^3, where the divergence of the vector field $\vec{F}(x, y, z) = \langle x^3 + 12xy, y^3 + 3z^2y - 9y, 3z^2 - 6xz \rangle$ is 0.

■ 3. Find the maximum value of the divergence of the vector field $\vec{F}(x, y, z)$.

$$\vec{F}(x, y, z) = \langle \ln(x^2 + 4), -e^{y+2}, -z e^{-y} - z^3 \rangle$$

POTENTIAL FUNCTION OF THE CONSERVATIVE VECTOR FIELD, THREE DIMENSIONS

■ 1. Find the potential function of the conservative vector field.

$$\vec{F}(x,y,z)=\left\langle \frac{2x}{z},\frac{1}{z},-\frac{x^2+y}{z^2}\right\rangle$$

■ 2. Find the value of a such that the vector field $\vec{F}$ has a potential function, then find that potential function.

$$\vec{F}(x,y,z)=\langle 4x^ay^3z^2, 3x^4y^2z^2, 2x^4y^3z\rangle$$

■ 3. Find a potential function of the conservative vector field $\vec{F}(x,y,z)$, then use this function to calculate the line integral of $\vec{F}$ over the curve $\vec{r}(t)$ between the parameter values $t=-2$ and $t=2$.

$$\vec{F}(x,y,z)=\langle 2(x+1), 2(z-y), 2(y-1)\rangle$$

$$\vec{r}(t)=\left\langle e^{t^2-4}, \sin\frac{\pi t}{4}, e^{-t^2+4}\right\rangle$$

POINTS ON THE SURFACE

■ 1. Find the points of the surface $\vec{r}(u,v)$ that lie on the z-axis.

$$\vec{r}(u,v)=\langle u^2-3v^2-1,4u^2-9v^2-7,e^{u+v}\rangle$$

■ 2. Find the intersection point(s) of the surface $\vec{r}(u,v)$ and the line $x=y+2=z-1$.

$$\vec{r}(u,v)=\langle \sin u+v,\cos u+v-3,2v+7+\sin u\rangle$$

■ 3. Identify the set of points of the surface $\vec{r}(u,v)=\langle u^2+2v^2,u,v+2\rangle$ that lie in the xy-plane.

SURFACE OF THE VECTOR EQUATION

■ 1. Identify the quadratic surface given as a vector function, where $u \in [0,2\pi]$ and $v \in (-\infty, \infty)$.

$$\vec{r}(u, v) = \langle 3 \sin u, 2v - 3, 5 \cos u \rangle$$

■ 2. Identify the quadratic surface given as a vector function, where $u \in [0,\pi]$ and $v \in [0,2\pi]$.

$$\vec{r}(u, v) = \langle -3 + 2 \cos u, 2 + 2 \sin u \cos v, 2 \sin u \sin v \rangle$$

■ 3. Identify the quadratic surface given as a vector function, where $u^2 + v^2 \leq 9$.

$$\vec{r}(u, v) = \langle v + 1, 5 + \sqrt{9 - u^2 - v^2}, u - 2 \rangle$$

PARAMETRIC REPRESENTATION OF THE SURFACE

■ 1. Consider the right circular cylinder with radius 5 and a cylindrical axis that's parallel to the z-axis and passes through $(2,-4,5)$. Find the parametrization of the part of the cylinder that lies above the xy-plane.

■ 2. Consider the plane $2x-3y+z-1=0$. Find the parametrization of the part of the plane that lies between the planes $y=-3$ and $y=3$.

■ 3. Consider the elliptic paraboloid $2(y+3)^2+4(z-2)^2-x-1=0$. Find the parametrization of the paraboloid for $x\leq 3$.

TANGENT PLANE TO THE PARAMETRIC SURFACE

■ 1. Find the equation of the tangent plane to the surface $\vec{r}(u,v)=\langle u+2\cos v, u-2\cos v, uv\rangle$ at the point $(4,0,\pi)$.

■ 2. Find the equation of the tangent plane(s) to the parametric surface $\vec{r}(u,v)=\langle u^2+2v, u-2v, uv+1\rangle$ such that its normal vector $\vec{n}$ is parallel to the y-axis.

■ 3. Find the equation of the tangent plane(s) to the parametric surface $\vec{r}(u,v)=\langle v^2, u-v+2, u^2-2\rangle$ such that it's parallel to $3x-24y+2z-1=0$.

AREA OF A SURFACE

■ 1. Find the area of the part of the surface $z = 2x + 2y - 1$ that lies within the rectangle given by $0 \le x \le \pi$ and $-1 \le y \le 1$.

■ 2. Find the area of the part of the surface $\vec{r}(u, v)$ that lies within the values of the parameters $-1 \le u \le 1$ and $0 \le v \le \sqrt{5}$.

$$\vec{r}(u, v) = \langle 2u - 3v + 1, 5u - v + 4, -u + 4v - 11 \rangle$$

■ 3. Find the area of the part of the surface $\vec{r}(u, v) = \langle 2\cos u, 5v + 3, 2\sin u \rangle$ that lies within the values of the parameters $\pi/6 \le u \le \pi/3$ and $0 \le v \le 3$.

SURFACE INTEGRALS

■ 1. Evaluate the surface integral of the scalar vector field $f(x, y, z) = \ln(x + y + z)$ over the surface $\vec{r} = \langle 3u - 7v + 1, u + 5v + 2, -3u + v - 1 \rangle$, where u changes from 0 to 4 and v changes from -1 to 1.

■ 2. Evaluate the surface integral of the scalar vector field $f(x, y, z) = x^2 + y^2 + 4z^2$ over the part of the cylinder $x^2 + y^2 = 9$, where $-2 \le z \le 5$.

■ 3. Evaluate the surface integral of the scalar vector field $f(x, y, z) = x^2 + y^2 + z + 1$ over the sphere centered at $(2, -1, -3)$ with radius 2.

SURFACE INTEGRALS OF ORIENTED SURFACES

■ 1. Evaluate the surface integral of the vector field $\vec{F} = \langle x^2, y^2, x + y + z \rangle$ over S, where S is the surface of the cube $[0,2] \times [0,2] \times [0,2]$. Assume that S has a positive orientation.

■ 2. Evaluate the surface integral of the vector field $\vec{F} = \langle x + y, y + z, x + z \rangle$ over the surface S which is the part of the right elliptic cylinder with an axis that coincides with the y-axis, an x-semi-axis of 3, a z-semi-axis of 9, and $-3 \le y \le 3$. Assume that S has a positive orientation.

■ 3. Evaluate the surface integral of the vector field $\vec{F} = \langle x - 2, y + 1, z - 3 \rangle$ over the surface S, where S is the surface of revolution generated by rotating the function $y = x^2 + 1$ around the x-axis for $-2 \le x \le 2$. Assume that S has a negative orientation.

FLUX ACROSS THE SURFACE

■ 1. Find the flux of the vector field $\vec{F}$ across the part the plane $x+y+z-2=0$ that lies within the rectangle defined by $-1 \le x \le 1$ and $-1 \le y \le 1$. Assume that S has an upward orientation.

$$\vec{F} = \left\langle \frac{1}{x^2+4}, \frac{1}{4y^2+1}, 0 \right\rangle$$

■ 2. Find the flux of the vector field $\vec{F} = \langle x^2+y^2+z^2, 3y, 3 \rangle$ across the sphere with radius 4 and center at the origin. Assume that S has a positive orientation.

■ 3. Suppose the velocity of a fluid in three-dimensional space is described by the vector field $\vec{F} = \langle x^2+1, y^2+1, z^2+1 \rangle$. Find the volume of fluid crossing the disk S defined by $(x+1)^2+(y-2)^2 \le 4$ in the xy-plane per 10 units of time. Assume that S has an upward orientation.

STOKES' THEOREM

1. Use Stokes' theorem to evaluate the surface integral where S is the part of the elliptic paraboloid $z + x^2 + y^2 - 3 = 0$ above the plane $z = -1$. Assume that S has a positive orientation.

$$\iint_S \text{curl } \vec{F} \cdot d\vec{S}$$

$$\vec{F} = \langle y + 2, -z^2, 2xy \rangle$$

2. Use Stokes' theorem to evaluate the line integral, where C is the rectangle $KMNO$ with vertices $K(0,0,0)$, $M(0,6,0)$, $N(3,6,0)$ and $O(3,0,0)$. Assume that C has a clockwise orientation as viewed from the positive z-axis.

$$\int_C \vec{F} \cdot d\vec{r}$$

$$\vec{F} = \langle 2xyz, x^2 + y^2, 2xyz \rangle$$

3. Use Stokes' theorem to evaluate the line integral, where C is the boundary curve of the semicircle centered at the origin with radius 4 that lies in the xz-plane, and with $z \geq 0$. Assume that C has a counterclockwise orientation as viewed from the positive y-axis.

$$\int_C \vec{F} \cdot d\vec{r}$$

$$\overrightarrow{F} = \langle x + 3y - z + 2, x - 5y + 9z - 7, -5x - y + 2z + 6 \rangle$$

DIVERGENCE THEOREM

■ 1. Use the Divergence theorem to evaluate the surface integral, where S is the boundary surface of the box $[-3,4] \times [3,5] \times [-3,0]$. Assume that S has a negative orientation.

$$\iint_S \vec{F} \cdot d\vec{S}$$

$$\vec{F} = \langle x + e^{z^2-y^2}, \ln y + y + x^4, z^2 - \arcsin(x+y) \rangle$$

■ 2. Use the Divergence theorem to evaluate the surface integral where S is the boundary surface of the part of the cylinder $y^2 + z^2 = 25$ with $-2 \leq x \leq 4$. Assume that S has a positive orientation.

$$\iint_S \vec{F} \cdot d\vec{S}$$

$$\vec{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$$

■ 3. Use the Divergence theorem to evaluate the triple integral where E is the sphere centered at the origin with radius 4.

$$\iiint_E \text{div } \vec{F} \, dV$$

$$\vec{F} = \left\langle \frac{x^2 + y^2 + z^2}{4}, -6y, 6 \right\rangle$$

W W W . K R I S T A K I N G M A T H . C O M