Calculus 3 Workbook

Limits and continuity

krista king
MATH

DOMAIN OF A MULTIVARIABLE FUNCTION

■ 1. Find the domain of the multivariable function.

$$f(x, y) = \sqrt{\sin(2x + y)}$$

■ 2. Find the domain of the multivariable function.

$$f(x, y) = (x^2 - y^2)\tan(2x)\cot(y + \pi)$$

■ 3. Find the domain of the multivariable function.

$$f(x, y) = \sin(3x + y)\log_{x-y}(x^2)$$

■ 4. Find the set of points that lie within the domain of the multivariable function.

$$f(x, y) = 3\sqrt{x^2 + 2x + y^2 - 4y - 4}$$

■ 5. Find the set of points that lie within the domain of the multivariable function.

$$f(x, y) = (2xy)^{-\frac{3}{4}}$$

LIMIT OF A MULTIVARIABLE FUNCTION

■ 1. If the limit exists, find its value.

$$\lim_{(x,y)\to(0,0)} \ln(2x+3ey+e^2)$$

■ 2. If the limit exists, find its value.

$$\lim_{(x,y)\to(\pi,\frac{\pi}{2})} \frac{\sin(3x+y)}{\cos(x-2y)}$$

■ 3. If the limit exists, find its value.

$$\lim_{(x,y)\to(-\infty,-\infty)} (x^3+4y)(\sin(x^2+2y)+3)$$

■ 4. If the limit exists, find its value.

$$\lim_{(x,y)\to 0,0)} \frac{4x^4-y^4}{2x^2+y^2}$$

■ 5. If the limit exists, find its value.

$$\lim_{(x,y)\to(\infty,\infty)} 2^y - x^2$$

■ 6. If the limit exists, find its value.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + 2x^2y^2 - xy}{2x^3 + y^2}$$

PRECISE DEFINITION OF THE LIMIT FOR MULTIVARIABLE FUNCTIONS

■ 1. Which value of δ can be used to apply the precise definition of the limit to $f(x,y)$ with $\epsilon = 0.002$ at the point $(0,0)$?

$$f(x,y) = (x^2 + y^2)(3 - xy)$$

■ 2. Which value of δ can be used to apply the precise definition of the limit to $f(x,y)$ with $\epsilon = 0.001$ at the point $(0,0)$? Hint: Use the polar form of the function.

$$f(x,y) = \frac{5x^2y}{x^2 + y^2}$$

■ 3. We know that $f(x,y)$ is a continuous function, and that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that $\sqrt{(x-4)^2 + (y+3)^2} < \delta$ implies $|f(x,y) - 7| < \epsilon$. If the limit exists, find its value.

$$\lim_{(x,y)\to(4,-3)} (f(x,y))^2$$

■ 4. We know that $f(x,y)$ and $g(x,y)$ are continuous functions, and that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that $\sqrt{(x-2)^2 + y^2} < \delta$ implies $|f(x,y) + 3| + |g(x,y) - 5| < \epsilon$. If the limit exists, find its value

$$\lim_{(x,y)\to(2,0)} (3f(x,y) - 2g(x,y))$$

■ 5. We know that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that

for $x > 0$, $\sqrt{x^2 + y^2} < \delta$ implies $|f(x,y) - 4| < \epsilon$

for $x \le 0$, $\sqrt{x^2 + y^2} < \delta$ implies $|f(x,y) + 4| < \epsilon$

If the limit exists, find its value.

$$\lim_{(x,y)\to(0,0)} 3^{f(x,y)}$$

■ 6. We know that for any real $\epsilon > 0$, there exists a $\delta > 0$ such that

$\sqrt{(x+1)^2 + (y-12)^2} < \delta$ implies $f(x,y) > \epsilon$. If the limit exists, find its value.

$$\lim_{(x,y)\to(-1,12)} (f(x,y) - 13)$$

DISCONTINUITIES OF MULTIVARIABLE FUNCTIONS

■ 1. Find any discontinuities of the function.

$$f(x,y) = 3^{x^2-2y^2+\sqrt{x^2+5y^2-x+1}}$$

■ 2. Find any discontinuities of the function.

$$f(x,y) = \sqrt{\sin x \cos y + \sin y \cos x}$$

■ 3. Find any discontinuities of the function.

$$f(x,y) = \begin{cases} \frac{4x^2-y^2}{2x-y} & y \neq 2x \\ 0 & y = 2x \end{cases}$$

■ 4. Find and classify any discontinuities of the function.

$$f(x,y) = \frac{7x-y}{4x^2+y^2-4x+1}$$

■ 5. Find and classify any discontinuities of the function.

$$f(x, y) = \frac{x^2 - 9y^2 - 2x + 1}{|x - 1| + |3y|}$$

COMPOSITIONS OF MULTIVARIABLE FUNCTIONS

■ 1. Find $f(g(x,y))$.

$$f(t)=\ln(3t)$$

$$g(x,y)=\frac{x+1}{y+2}$$

■ 2. Find $f(x(t),y(t))$.

$$f(x,y)=x^2-y^2+3$$

$$x(t)=\sqrt{t-5}$$

$$y(t)=2^{t+2}$$

■ 3. Find $f(u(x,y),v(x,y))$.

$$f(u,v)=u^2+v^2+\frac{u-v}{\sqrt{2}}$$

$$u(x,y)=\sin(x+y)$$

$$v(x,y)=\cos(x+y)$$

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