Calculus 3 Workbook Solutions

Velocity and acceleration

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MATH

VELOCITY AND ACCELERATION VECTORS

1. Find the value of t such that the velocity of the vector function $\vec{r}(t)$ is 0.

$$\vec{r}(t) = \langle 4t^3 - 5t^2 - 28t, 2e^{t-1} + e^{-2t+5}, \cos(\pi t)\rangle$$

Solution:

Since the velocity $\vec{r}\,'(t)$ of the vector function is 0, we know $\vec{r}\,'(t) = \vec{O}$.

Find derivatives for each component individually.

$$r_1'(t) = 12t^2 - 10t - 28$$

$$r_2'(t) = 2e^{t-1} - 2e^{-2t+5}$$

$$r_3'(t) = -\pi \sin(\pi t)$$

Solve the system of equations.

$$12t^2 - 10t - 28 = 0$$

$$2e^{t-1} - 2e^{-2t+5} = 0$$

$$-\pi \sin(\pi t) = 0$$

Since we have three equations and one variable, we can find the value of t from any equation, and then check to see whether the other equations hold.

From the first equation

$$6t^2 - 5t - 14 = 0$$

$$(t-2)(6t+7) = 0$$

$$t = 2 \text{ or } t = -\frac{7}{6}$$

Plug $t = 2$ into the second and third equations.

$$2e^{2-1} - 2e^{-2(2)+5} = 2e - 2e = 0$$

$$-\pi\sin(\pi \cdot 2) = -\pi \cdot 0 = 0$$

So $t = 2$ is a solution to the system. Plug $t = -7/6$ into the second equation.

$$2e^{(-7/6)-1} - 2e^{-2(-7/6)+5} = 2e^{-13/6} - 2e^{22/3} \neq 0$$

So $t = -7/6$ is not a solution.

■ 2. Find the point on the curve such that the velocity along the x-axis reaches its maximum value.

$$\vec{r}(t) = \left\langle \frac{t}{t^2+2}, 3\tan(3t^2), \ln 2t \right\rangle$$

Solution:

Find the velocity along the x-axis.

$$r_1(t) = \frac{t}{t^2+2}$$

$$r_1'(t) = \left(\frac{t}{t^2+2}\right)' = \frac{1 \cdot (t^2+2) - t \cdot 2t}{(t^2+2)^2} = \frac{2-t^2}{(t^2+2)^2}$$

Find the critical points of the function.

$$r_1'(t) = 0$$

$$\frac{2-t^2}{(t^2+2)^2} = 0$$

$$2 - t^2 = 0$$

$$t = \pm\sqrt{2}$$

Check the sign of the derivative.

$r_1'(t) > 0$ for t between $-\sqrt{2}$ and $\sqrt{2}$

$r_1'(t) < 0$ for $t \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

So $t = \sqrt{2}$ is the local and global maximum. To find the point on the curve, plug $t = \sqrt{2}$ into the vector function.

$$\vec{r}(\sqrt{2}) = \left\langle \frac{\sqrt{2}}{(\sqrt{2})^2+2}, 3\tan(3(\sqrt{2})^2), \ln 2(\sqrt{2}) \right\rangle$$

$$\vec{r}(\sqrt{2}) = \left\langle \frac{\sqrt{2}}{2+2}, 3\tan 6, \ln 2^{3/2} \right\rangle$$

$$\vec{r}(\sqrt{2}) = \left\langle \frac{\sqrt{2}}{4}, 3\tan 6, \frac{3}{2}\ln 2 \right\rangle$$

■ 3. Find the values of the parameters p and q such that the absolute value of acceleration of the non-constant function $\vec{r}(t) = \langle p\sin 3t, 4\cos qt \rangle$ is a constant for any value of t.

Solution:

To find the acceleration, compute the second-order derivative of the vector function for each component individually.

$$r_1(t) = p\sin 3t$$

$$r_1'(t) = 3p\cos 3t$$

$$r_1''(t) = -9p\sin 3t$$

and

$$r_2(t) = 4\cos qt$$

$$r_2'(t) = -4q\sin qt$$

$$r_2''(t) = -4q^2\cos qt$$

The absolute value (magnitude) of acceleration is

$$|\vec{r}''(t)| = \sqrt{(r_1''(t))^2 + (r_2''(t))^2}$$

$$|\vec{r}''(t)| = \sqrt{(-9p\sin 3t)^2 + (-4q^2\cos qt)^2}$$

$$|\vec{r}''(t)| = \sqrt{81p^2\sin^2 3t + 16q^4\cos^2 qt}$$

Since the absolute value of acceleration of $\vec{r}(t)$ is a constant for any value of t,

$$|\vec{r}''(t)| = \sqrt{81p^2\sin^2 3t + 16q^4\cos^2 qt} = \text{constant}$$

Therefore

$$81p^2\sin^2 3t + 16q^4\cos^2 qt = \text{constant}$$

So

$$(81p^2\sin^2 3t + 16q^4\cos^2 qt)' = 0$$

$$486p^2\sin 3t\cos 3t - 32q^5\sin qt\cos qt = 0$$

$$243p^2\sin 6t - 16q^5\sin 2qt = 0$$

$$243p^2\sin 6t = 16q^5\sin 2qt$$

Since p and q can't both be 0 (otherwise $\vec{r}$ is a constant vector), we get a system of equations for p and q.

$$243p^2 = 16q^5$$

$$6 = 2q$$

Solve the system.

$$q = 3$$

$$243p^2 = 16(3)^5$$

$$p^2 = 16$$

$$p = \pm 4$$

Therefore, we have two possible pairs of the parameters p and q such that the absolute value of acceleration is a constant, namely $p = -4$ and $q = 3$, or $p = 4$ and $q = 3$.

VELOCITY, ACCELERATION, AND SPEED, GIVEN POSITION

1. Find the point where the speed is 0, given the position function.

$$\vec{r}(t)=\left\langle \ln(2t^2+8t+50), t^4+32t+17, \arctan t-\frac{t}{5}\right\rangle$$

Solution:

Since the speed $|\vec{r}\,'(t)|$ of the vector function is 0, we have the equation for t.

$$\sqrt{(r_1'(t))^2+(r_2'(t))^2+(r_3'(t))^2}=0$$

The sum of squares is 0 if and only if each term is 0, so

$$r_1'(t)=0,\ r_2'(t)=0,\ r_3'(t)=0$$

Find derivatives.

$$r_1'(t)=\frac{(2t^2+8t+50)'}{2t^2+8t+50}=\frac{4t+8}{2t^2+8t+50}=0$$

$$r_2'(t)=4t^3+32=0$$

$$r_3'(t)=\frac{1}{t^2+1}-\frac{1}{5}=0$$

Since we have three equations and one variable, we can find the value of t from any equation, and then verify that the other equations hold. From the first equation, we get

$$4t + 8 = 0$$

$$t = -2$$

Plug $t = -2$ into the second and third equations.

$$4(-2)^3 + 32 = 0$$

$$\frac{1}{(-2)^2 + 1} - \frac{1}{5} = 0$$

So $t = -2$ is a solution of the system. Plug $t = -2$ into $\vec{r}(t)$ to find the point where the speed is 0.

$$\vec{r}(-2) = \left\langle \ln(2(-2)^2 + 8(-2) + 50),\ (-2)^4 + 32(-2) + 17,\ \arctan(-2) - \frac{(-2)}{5} \right\rangle$$

$$\vec{r}(-2) = \left\langle \ln(42), -31, \frac{2}{5} - \arctan(2) \right\rangle$$

■ 2. Find the interval(s) of t values where the acceleration along the z-axis is negative for the position function.

$$\vec{r}(t) = \langle 2\sin(2t), e^{t^2+1}, t^4 - 10t^3 - 36t^2 - 5t + 45 \rangle$$

Solution:

Since the acceleration along the z-axis, $r_3''(t)$, is negative, we need to solve $r_3''(t) < 0$. First, find the derivatives.

$$r_3'(t) = 4t^3 - 30t^2 - 72t - 5$$

$$r_3''(t) = 12t^2 - 60t - 72$$

Solve the inequality.

$$12t^2 - 60t - 72 < 0$$

$$12(t - 6)(t + 1) < 0$$

$$t \in (-1,6)$$

■ 3. Find the velocity, speed, and acceleration of the position function at the point(s) where the trajectory intersects the xy-plane.

$\vec{r}(t) = \langle \sin 4t, 2\cos(t + \pi), 2 + 2\sin t \rangle$, where $t \in [0,2\pi]$

Solution:

In order to find the point(s) where the trajectory $\vec{r}(t)$ intersects the xy-plane, solve $r_3(t) = 0$ for t over the interval $[0,2\pi]$.

$$2 + 2\sin t = 0$$

$$\sin t = -1$$

$$t = -\frac{\pi}{2} + 2\pi k, \text{ where } k \text{ is any integer}$$

Therefore, inside the interval $[0,2\pi]$, the only possible value of t is $t = 3\pi/2$.

Rewrite the position function in parametric form.

$$r_1(t) = \sin 4t$$

$$r_2(t) = 2\cos(t+\pi)$$

$$r_3(t) = 2 + 2\sin t$$

To find velocity, find the first-order derivatives of the position function for each component individually.

$$r_1'(t) = 4\cos 4t$$

$$r_2'(t) = -2\sin(t+\pi)$$

$$r_3'(t) = 2\cos t$$

Plug $t = 3\pi/2$ into the derivative equations.

$$r_1'\left(\frac{3\pi}{2}\right) = 4\cos\left(4\cdot\frac{3\pi}{2}\right) = 4$$

$$r_2'\left(\frac{3\pi}{2}\right) = -2\sin\left(\frac{3\pi}{2}+\pi\right) = -2$$

$$r_3'\left(\frac{3\pi}{2}\right) = 2\cos\left(\frac{3\pi}{2}\right) = 0$$

Find the speed at $t = 3\pi/2$.

$$\sqrt{\left[r_1'\left(\frac{3\pi}{2}\right)\right]^2+\left[r_2'\left(\frac{3\pi}{2}\right)\right]^2+\left[r_3'\left(\frac{3\pi}{2}\right)\right]^2}$$

$$\sqrt{4^2+(-2)^2+0^2}$$

$$\sqrt{20}$$

$$2\sqrt{5}$$

To find the acceleration, find the second-order derivatives of the position function for each component individually.

$$r_1''(t) = -16\sin 4t$$

$$r_2''(t) = -2\cos(t+\pi)$$

$$r_3''(t) = -2\sin t$$

Evaluate the second derivatives at $t = 3\pi/2$.

$$r_1''\left(\frac{3\pi}{2}\right) = -16\sin\left(4\cdot\frac{3\pi}{2}\right) = 0$$

$$r_2''\left(\frac{3\pi}{2}\right) = -2\cos\left(\frac{3\pi}{2}+\pi\right) = 0$$

$$r_3''\left(\frac{3\pi}{2}\right) = -2\sin\left(\frac{3\pi}{2}\right) = 2$$

VELOCITY AND POSITION GIVEN ACCELERATION AND INITIAL CONDITIONS

■ 1. Find the velocity and position of the acceleration function $\vec{a}(t) = \langle 4\sin^2 t, -\cos t\rangle$ if $\vec{r}(\pi) = \langle -2,1\rangle$, and $\vec{v}(\pi) = \langle 0,0\rangle$.

Solution:

Rewrite the acceleration function in parametric form.

$$a_1(t) = 4\sin^2 t$$

$$a_2(t) = -\cos t$$

To find the velocity, integrate each component of the acceleration function with respect to t.

$$v_1(t) = \int a_1(t)\ dt = \int 4\sin^2 t\ dt = \int 2 - 2\cos 2t\ dt = 2t - \sin 2t + C_1$$

$$v_2(t) = \int a_2(t)\ dt = \int -\cos t\ dt = -\sin t + C_2$$

Use the initial condition $\vec{v}(\pi) = \langle 0,0\rangle$ to find the values of the constants C_1 and C_2.

$$v_1(\pi) = 2\pi - \sin 2\pi + C_1 = 0$$

$$v_2(\pi) = -\sin\pi + C_2 = 0$$

$$C_1 = -2\pi + \sin 2\pi = -2\pi$$

$$C_2 = \sin \pi = 0$$

So the velocity function is

$$\vec{v}(t) = \langle 2t - \sin 2t - 2\pi, \ -\sin t \rangle$$

To find the position function, integrate each component of the velocity function with respect to t.

$$r_1(t) = \int v_1(t)\ dt = \int (2t - \sin 2t - 2\pi)\ dt = t^2 + \frac{1}{2}\cos 2t - 2\pi t + C_3$$

$$r_2(t) = \int v_2(t)\ dt = \int -\sin t\ dt = \cos t + C_4$$

Use the initial condition $\vec{r}(\pi) = \langle -2,1 \rangle$ to find the values of the constants C_3 and C_4.

$$r_1(\pi) = \pi^2 + \frac{1}{2}\cos 2\pi - 2\pi \cdot \pi + C_3 = -2$$

$$r_2(\pi) = \cos \pi + C_4 = 1$$

$$C_3 = -2 - \pi^2 - \frac{1}{2}\cos 2\pi + 2\pi^2 = \pi^2 - \frac{5}{2}$$

$$C_4 = 1 - \cos \pi = 2$$

So the position function is

$$\vec{r}(t) = \left\langle t^2 + \frac{1}{2}\cos 2t - 2\pi t + \pi^2 - \frac{5}{2}, \cos t + 2 \right\rangle$$

■ 2. Find the speed function given the acceleration function $\vec{a}(t) = \langle 4t^3 - 1, 6t^2 + 2t, 2e^{2t} \rangle$ if $\vec{v}(0) = \langle 1, -3, 1 \rangle$.

Solution:

Rewrite the acceleration function in parametric form.

$$a_1(t) = 4t^3 - 1$$

$$a_2(t) = 6t^2 + 2t$$

$$a_3(t) = 2e^{2t}$$

To find the velocity, integrate each component of the acceleration function with respect to t.

$$v_1(t) = \int a_1(t)\ dt = \int 4t^3 - 1\ dt = t^4 - t + C_1$$

$$v_2(t) = \int a_2(t)\ dt = \int 6t^2 + 2t\ dt = 2t^3 + t^2 + C_2$$

$$v_3(t) = \int a_3(t)\ dt = \int 2e^{2t}\ dt = e^{2t} + C_3$$

Use the initial condition $\vec{v}(0) = \langle 1, -3, 1 \rangle$ to find the values of the constants C_1, C_2, and C_3.

$$v_1(0) = 0^4 - 0 + C_1 = 1$$

$$v_2(0) = 2 \cdot 0^3 + 0^2 + C_2 = -3$$

$$v_3(0) = e^{2 \cdot 0} + C_3 = 1$$

So $C_1 = 1$, $C_2 = -3$, and $C_3 = 1 - e^0 = 0$, which means the velocity function is

$$\vec{v}(t) = \langle t^4 - t + 1, 2t^3 + t^2 - 3, e^{2t} \rangle$$

The speed function is given by

$$s(t) = \sqrt{(v_1(t))^2 + (v_2(t))^2 + (v_3(t))^2}$$

$$s(t) = \sqrt{(t^4 - t + 1)^2 + (2t^3 + t^2 - 3)^2 + (e^{2t})^2}$$

$$s(t) = \sqrt{t^8 + 4t^6 + 2t^5 + 3t^4 - 12t^3 - 5t^2 - 2t + e^{4t} + 10}$$

■ 3. Find the distance travelled by a particle during the first 10 seconds, given its acceleration function $\vec{a}(t) = \langle 2\sin t, 2\cos t \rangle$, where t is the time in seconds and the initial velocity is $\vec{v}(0) = \langle -2,0 \rangle$.

Solution:

Rewrite the acceleration function in parametric form.

$$a_1(t) = 2\sin t$$

$$a_2(t) = 2\cos t$$

To find the velocity, integrate each component of the acceleration function with respect to t.

$$v_1(t) = \int a_1(t)\ dt = \int 2\sin t\ dt = -2\cos t + C_1$$

$$v_2(t) = \int a_2(t)\ dt = \int 2\cos t\ dt = 2\sin t + C_2$$

Use the initial condition $\vec{v}(0) = \langle -2,0\rangle$ to find the values of the constants C_1 and C_2.

$$v_1(0) = -2\cos 0 + C_1 = -2$$

$$v_2(0) = 2\sin 0 + C_2 = 0$$

We get $C_1 = 0$ and $C_2 = 0$, so the velocity function is

$$\vec{v}(t) = \langle -2\cos t, 2\sin t\rangle$$

We don't need the position function to find the arc length, which is equal to the distance travelled by the particle. The arc length is given by

$$\int_a^b \sqrt{(r_1'(t))^2 + (r_2'(t))^2}\ dt$$

Plug in $a = 0$, $b = 10$, and $\vec{r}\,'(t) = \vec{v}(t)$.

$$\int_0^{10} \sqrt{(-2\cos t)^2 + (2\sin t)^2}\ dt$$

$$\int_0^{10} \sqrt{4\cos^2 t + 4\sin^2 t}\ dt$$

$$\int_0^{10} \sqrt{4}\ dt$$

$$2\int_0^{10} dt$$

$$2 \cdot 10$$

$$20$$

TANGENTIAL AND NORMAL COMPONENTS OF ACCELERATION

■ 1. Find the tangential and normal components of acceleration for the vector function $\overrightarrow{r}(t) = \langle e^{2t} + 1, e^{-2t} - 1, t - 4 \rangle$ at the point $(2,0,-4)$.

Solution:

In order to find the value of t which corresponds to the point $(2,0,-4)$, solve the system of equations for t.

$$e^{2t} + 1 = 2$$

$$e^{-2t} - 1 = 0$$

$$t - 4 = -4$$

From the third equation, we get $t = 0$. Check to see whether the other equations hold.

$$e^{2(0)} + 1 = 2$$

$$e^{-2(0)} - 1 = 0$$

So the value of $t = 0$ is the solution of the system.

The tangential component of acceleration is given by

$$a_T = \frac{\overrightarrow{r}'(t) \cdot \overrightarrow{r}''(t)}{|\overrightarrow{r}'(t)|}$$

$$a_T = \frac{\vec{v}(t) \cdot \vec{a}(t)}{s(t)}$$

where $\vec{v}(t)$ is the velocity, $\vec{a}(t)$ is the acceleration, and $s(t)$ is the speed.

Rewrite the vector function in parametric form.

$$r_1(t) = e^{2t} + 1$$

$$r_2(t) = e^{-2t} - 1$$

$$r_3(t) = t - 4$$

To find the velocity, take the first-order derivatives.

$$v_1(t) = 2e^{2t}$$

$$v_2(t) = -2e^{-2t}$$

$$v_3(t) = 1$$

Plug in $t = 0$.

$$v_1(0) = 2e^0 = 2$$

$$v_2(0) = -2e^0 = -2$$

$$v_3(0) = 1$$

Find the speed at $t = 0$.

$$s(0) = \sqrt{[v_1(0)]^2 + [v_2(0)]^2 + [v_3(0)]^2} = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$$

To find the acceleration, take the second-order derivative.

$$a_1(t) = 4e^{2t}$$

$$a_2(t) = 4e^{-2t}$$

$$a_3(t) = 0$$

Plug in $t = 0$.

$$a_1(0) = 4e^0 = 4$$

$$a_2(0) = 4e^0 = 4$$

$$a_3(0) = 0$$

Plug in the values we've found to the tangential component of acceleration formula.

$$a_T(0) = \frac{\vec{v}(0) \cdot \vec{a}(0)}{s(0)}$$

$$a_T(0) = \frac{\langle 2, -2, 1 \rangle \cdot \langle 4, 4, 0 \rangle}{3}$$

$$a_T(0) = \frac{2 \cdot 4 + (-2) \cdot 4 + 1 \cdot 0}{3} = 0$$

The normal component of acceleration function is given by

$$a_N = |\vec{r}'(t) \times \vec{r}''(t)|$$

$$a_N = |\vec{v}(t) \times \vec{a}(t)|$$

where $\vec{v}(t)$ is the velocity and $\vec{a}(t)$ is the acceleration.

$$a_N(0) = |\vec{v}(0) \times \vec{a}(0)|$$

$$a_N(0) = |\langle 2, -2, 1\rangle \times \langle 4,4,0\rangle|$$

The cross product of two vectors $\vec{v}$ and $\vec{a}$ is given by

$$\vec{v} \times \vec{a} = \mathbf{i}(v_2a_3 - v_3a_2) - \mathbf{j}(v_1a_3 - v_3a_1) + \mathbf{k}(v_1a_2 - v_2a_1)$$

Plug in $\langle v_1, v_2, v_3\rangle = \langle 2, -2, 1\rangle$ and $\langle a_1, a_2, a_3\rangle = \langle 4,4,0\rangle$.

$$\vec{v} \times \vec{a} = \mathbf{i}(-2 \cdot 0 - 1 \cdot 4) - \mathbf{j}(2 \cdot 0 - 1 \cdot 4) + \mathbf{k}(2 \cdot 4 + 2 \cdot 4)$$

$$\vec{v} \times \vec{a} = -4\mathbf{i} + 4\mathbf{j} + 16\mathbf{k}$$

$$a_N(0) = |\langle -4,4,16\rangle|$$

$$a_N(0) = \sqrt{(-4)^2 + 4^2 + 16^2} = \sqrt{288} = 12\sqrt{2}$$

■ 2. Find the point(s) where the tangential component of acceleration for the vector function $\vec{r}(t) = \langle 2\cos t - 2, 3\sin t + 5, 4t - 1\rangle$ is 0.

Solution:

The tangential component of acceleration is given by

$$a_T = \frac{\vec{r}\,'(t) \cdot \vec{r}\,''(t)}{|\vec{r}\,'(t)|}$$

$$a_T = \frac{\vec{v}(t) \cdot \vec{a}(t)}{s(t)}$$

where $\vec{v}(t)$ is velocity, $\vec{a}(t)$ is acceleration, and $s(t)$ is speed. Since $a_T = 0$, we have the equation for t.

$$\frac{\vec{v}(t) \cdot \vec{a}(t)}{s(t)} = 0$$

$$\vec{v}(t) \cdot \vec{a}(t) = 0$$

Rewrite the vector function in parametric form.

$$r_1(t) = 2\cos t - 2$$

$$r_2(t) = 3\sin t + 5$$

$$r_3(t) = 4t - 1$$

To find velocity, take the first-order derivative.

$$v_1(t) = -2\sin t$$

$$v_2(t) = 3\cos t$$

$$v_3(t) = 4$$

To find acceleration, take the second-order derivative.

$$a_1(t) = -2\cos t$$

$$a_2(t) = -3\sin t$$

$$a_3(t) = 0$$

So the equation is

$$\langle -2\sin t, 3\cos t, 4\rangle \cdot \langle -2\cos t, -3\sin t, 0\rangle = 0$$

$$(-2\sin t)(-2\cos t) + (3\cos t)(-3\sin t) + 4 \cdot 0 = 0$$

$$4\sin t\cos t - 9\sin t\cos t = 0$$

$$-5\sin t\cos t = 0$$

Use the trigonometric identity $\sin 2\phi = 2\sin\phi\cos\phi$.

$$-\frac{5}{2}\sin 2t = 0$$

$$\sin 2t = 0$$

So $2t = \pi n$, where n is any integer, or $t = (\pi/2)n$, where n is any integer.

■ 3. Find the values of parameters p and q, such that the normal components of acceleration for $\vec{r}(t) = \langle 2t^2, 3pt, t^2 - 4t + qt\rangle$ are 0 at the origin.

Solution:

The normal component of acceleration function is given by

$$a_N = |\vec{r}'(t) \times \vec{r}''(t)|$$

$$a_N = |\vec{v}(t) \times \vec{a}(t)|$$

where $\vec{v}(t)$ is velocity and $\vec{a}(t)$ is acceleration. The origin corresponds to the value of $t = 0$. Since $a_N = 0$ at $t = 0$, we can get an equation for p and q.

$$|\vec{v}(0) \times \vec{a}(0)| = 0$$

Rewrite the vector function in parametric form.

$$r_1(t) = 2t^2$$

$$r_2(t) = 3pt$$

$$r_3(t) = t^2 - 4t + qt$$

To find the velocity, take the first-order derivative.

$$v_1(t) = 4t$$

$$v_2(t) = 3p$$

$$v_3(t) = 2t - 4 + q$$

Evaluate these derivatives at $t = 0$.

$$v_1(0) = 0$$

$$v_2(0) = 3p$$

$$v_3(0) = -4 + q$$

To find the acceleration, take the second-order derivative.

$$a_1(t) = 4$$

$$a_2(t) = 0$$

$$a_3(t) = 2$$

So the equation is

$$|\langle 0, 3p, -4 + q\rangle \times \langle 4, 0, 2\rangle| = 0$$

The cross product of two vectors $\vec{v}$ and $\vec{a}$ is given by

$$\vec{v} \times \vec{a} = \mathbf{i}(v_2a_3 - v_3a_2) - \mathbf{j}(v_1a_3 - v_3a_1) + \mathbf{k}(v_1a_2 - v_2a_1)$$

Plug in $\langle v_1, v_2, v_3\rangle = \langle 0, 3p, -4 + q\rangle$ and $\langle a_1, a_2, a_3\rangle = \langle 4,0,2\rangle$.

$$\vec{v} \times \vec{a} = \mathbf{i}(3p \cdot 2 - (-4 + q) \cdot 0) - \mathbf{j}(0 \cdot 2 - (-4 + q) \cdot 4) + \mathbf{k}(0 \cdot 0 - 3p \cdot 4)$$

$$\vec{v} \times \vec{a} = 6p\mathbf{i} + (-16 + 4q)\mathbf{j} - 12p\mathbf{k}$$

$$|\langle 6p, -16 + 4q, -12p\rangle| = 0$$

The vector magnitude is 0 if and only if each of its component is 0. So we have the system of equations for p and q.

$$6p = 0$$

$$-16 + 4q = 0$$

$$-12p = 0$$

Since we get $p = 0$, we also find

$$4q = 16$$

$$q = 4$$

W W W . K R I S T A K I N G M A T H . C O M