Topic: Critical points

Question: For which values a and b is the critical point of the function $(-8,-5)$?

$$f(x,y)=x^2+y^2-ax+by$$

Answer choices:

A	$a=-18$	$b=-10$
B	$a=-16$	$b=5$
C	$a=-16$	$b=10$
D	$a=16$	$b=10$

Solution: C

Take partial derivatives of $f(x, y)$.

$$\frac{\partial f}{\partial x} = \frac{\partial f(x^2 + y^2 - ax + by)}{\partial x}$$

$$\frac{\partial f}{\partial x} = 2x - a$$

and

$$\frac{\partial f}{\partial y} = \frac{\partial f(x^2 + y^2 - ax + by)}{\partial x}$$

$$\frac{\partial f}{\partial y} = 2y + b$$

Setting these equal to 0 gives the system

$$2x - a = 0$$

$$2y + b = 0$$

Since we already know the critical point is $(-8, -5)$, we can plug $x = -8$ and $y = -5$ into the system to solve for a and b.

$$2(-8) - a = 0$$

$$-16 - a = 0$$

$$a = -16$$

and

$$2(-5) + b = 0$$

$$-10 + b = 0$$

$$b = 10$$

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Question: Which function has a critical point that's equidistant from all the three major axes?

Answer choices:

A $f(x, y, z) = x^2 + y^2 + z^2 - x - y - z$

B $f(x, y, z) = x^2 + 2y^2 + 4z^2 + x - y - z$

C $f(x, y, z) = 2x^2 + y^2 + 4z^2 - x - y + z$

D $f(x, y, z) = 2x^2 + 4y^2 + z^2 - x + y - z$

Solution: A

To test each system, start by taking partial derivatives of $f(x,y,z)$. These are the partial derivatives of $f(x,y,z)=x^2+y^2+z^2-x-y-z$ from answer choice A:

$$\frac{\partial f}{\partial x}=\frac{\partial f(x^2+y^2+z^2-x-y-z)}{\partial x}$$

$$\frac{\partial f}{\partial x}=2x-1$$

and

$$\frac{\partial f}{\partial y}=\frac{\partial f(x^2+y^2+z^2-x-y-z)}{\partial y}$$

$$\frac{\partial f}{\partial y}=2y-1$$

and

$$\frac{\partial f}{\partial z}=\frac{\partial f(x^2+y^2+z^2-x-y-z)}{\partial z}$$

$$\frac{\partial f}{\partial z}=2z-1$$

Setting these equal to 0 gives

$$2x-1=0 \qquad 2y-1=0 \qquad 2z-1=0$$

$$x=\frac{1}{2} \qquad y=\frac{1}{2} \qquad z=\frac{1}{2}$$

Because $x = y = z$, this critical point is equidistant from all three major axes.

Topic: Critical points

Question: Which two functions have the same critical points?

$$f(x, y) = x^2 + y^2 - 4x - 8y$$

$$g(x, y) = x^2 + 8y^2 - 6x + 15y$$

$$h(x, y) = 2x^2 - 4y^2 - 8x + 32y$$

$$k(x, y) = x^2 - 9y^2 - 5x + 4y$$

Answer choices:

A $f(x, y)$ and $g(x, y)$

B $f(x, y)$ and $h(x, y)$

C $g(x, y)$ and $h(x, y)$

D $g(x, y)$ and $k(x, y)$

Solution: B

To find the critical point of each function, take its partial derivatives. For $f(x, y)$:

$$\frac{\partial f}{\partial x} = \frac{\partial(x^2 + y^2 - 4x - 8y)}{\partial x}$$

$$\frac{\partial f}{\partial x} = 2x - 4$$

and

$$\frac{\partial f}{\partial y} = \frac{\partial(x^2 + y^2 - 4x - 8y)}{\partial x}$$

$$\frac{\partial f}{\partial y} = 2y - 8$$

Setting these equations equal to 0 gives us the critical point $(2,4)$. For $h(x, y)$:

$$\frac{\partial h}{\partial x} = \frac{\partial(2x^2 - 4y^2 - 8x + 32y)}{\partial x}$$

$$\frac{\partial h}{\partial x} = 4x - 8$$

and

$$\frac{\partial h}{\partial y} = \frac{\partial(2x^2 - 4y^2 - 8x + 32y)}{\partial y}$$

$$\frac{\partial h}{\partial y} = -8y + 32$$

Setting these equations equal to 0 gives us the critical point $(2,4)$. Therefore, $f(x, y)$ and $h(x, y)$ have the same critical point.