Topic: Equation of the tangent plane

Question: Find the equation of the tangent plane.

$$z = 2x^2y^2 - 4xy^2 - 3y$$

at $(1, -3, -9)$

Answer choices:

A $z = 72x + 9y$

B $z = 9y + 18$

C $z = 72x + 9y - 54$

D $z = 9y - 36$

Solution: B

The formula for the equation of the tangent plane to the surface $z = f(x, y)$ at (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

In this problem,

$$z = f(x, y) = 2x^2y^2 - 4xy^2 - 3y$$

and

$$(x_0, y_0, z_0) = (1, -3, -9)$$

To find $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$, we'll take first-order partial derivatives of our function,

for f_x:

$$f(x, y) = 2x^2y^2 - 4xy^2 - 3y$$

$$f(x, y) = \left(2y^2\right)x^2 - \left(4y^2\right)x - 3y$$

$$f_x(x, y) = \left(2y^2\right)2x - \left(4y^2\right)1 - 0$$

$$f_x(x, y) = 4xy^2 - 4y^2$$

for f_y:

$$f(x, y) = 2x^2y^2 - 4xy^2 - 3y$$

$$f(x, y) = \left(2x^2\right)y^2 - (4x)y^2 - 3y$$

$$f_y(x,y) = \left(2x^2\right)2y - (4x)2y - 3$$

$$f_y(x,y) = 4x^2y - 8xy - 3$$

and then evaluate them at $(1,-3,-9)$, and we'll get

$$f_x(x,y) = 4xy^2 - 4y^2$$

$$f_x(1,-3) = 4(1)(-3)^2 - 4(-3)^2$$

$$f_x(1,-3) = 36 - 36 = 0$$

and

$$f_y(x,y) = 4x^2y - 8xy - 3$$

$$f_y(1,-3) = 4(1)^2(-3) - 8(1)(-3) - 3$$

$$f_y(1,-3) = -12 + 24 - 3 = 9$$

Now we'll plug these values and the given point into the formula for the equation of the tangent plane.

$$z + 9 = f_x(1,-3)(x-1) + f_y(1,-3)(y+3)$$

$$z + 9 = 0(x-1) + 9(y+3)$$

$$z = 0 + 9y + 27 - 9$$

$$z = 9y + 18$$

Topic: Equation of the tangent plane

Question: Find the equation of the tangent plane.

$$z = 2\cos x \sin y + 1$$

at $(0,0,1)$

Answer choices:

A $z = 2x + 1$

B $z = -2x + 1$

C $z = 2y + 1$

D $z = -2y + 1$

Solution: C

The formula for the equation of the tangent plane to the surface $z = f(x, y)$ at (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

In this problem,

$$z = f(x, y) = 2\cos x \sin y + 1$$

and

$$(x_0, y_0, z_0) = (0,0,1)$$

To find $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$, we'll take first-order partial derivatives of our function,

for f_x:

$$f(x, y) = 2\cos x \sin y + 1$$

$$f(x, y) = (2\sin y)(\cos x) + 1$$

$$f_x(x, y) = (2\sin y)(-\sin x) + 0$$

$$f_x(x, y) = -2\sin x \sin y$$

for f_y:

$$f(x, y) = 2\cos x \sin y + 1$$

$$f(x, y) = (2\cos x)(\sin y) + 1$$

$$f_y(x,y) = (2\cos x)(\cos y) + 0$$

$$f_y(x,y) = 2\cos x\cos y$$

and then evaluate them at $(0,0,1)$, and we'll get

$$f_x(x,y) = -2\sin x\sin y$$

$$f_x(0,0) = -2\sin 0\sin 0$$

$$f_x(0,0) = -2(0)(0) = 0$$

and

$$f_y(x,y) = 2\cos x\cos y$$

$$f_y(0,0) = 2\cos 0\cos 0$$

$$f_y(0,0) = 2(1)(1) = 2$$

Now we'll plug these values and the given point into the formula for the equation of the tangent plane.

$$z - 1 = f_x(0,0)(x-0) + f_y(0,0)(y-0)$$

$$z - 1 = 0(x-0) + 2(y-0)$$

$$z = 2y - 2(0) + 1$$

$$z = 2y + 1$$

Topic: Equation of the tangent plane

Question: Find the equation of the tangent plane.

$$z = x^3 - y^3$$

at $(2,1,7)$

Answer choices:

A $z = 11x + 5y - 20$

B $z = 9x + 9y - 20$

C $z = 8x - y - 8$

D $z = 12x - 3y - 14$

Solution: D

The formula for the equation of the tangent plane to the surface $z = f(x, y)$ at (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

In this problem,

$$z = f(x, y) = x^3 - y^3$$

and

$$(x_0, y_0, z_0) = (2,1,7)$$

To find $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$, we'll take first-order partial derivatives of our function,

for f_x:

$$f(x, y) = x^3 - y^3$$

$$f_x(x, y) = 3x^2 - 0$$

$$f_x(x, y) = 3x^2$$

for f_y:

$$f(x, y) = x^3 - y^3$$

$$f_y(x, y) = 0 - 3y^2$$

$$f_y(x, y) = -3y^2$$

and then evaluate them at $(2,1,7)$, and we'll get

$$f_x(x, y) = 3x^2$$

$$f_x(2,1) = 3(2)^2 = 12$$

and

$$f_y(x, y) = -3y^2$$

$$f_y(2,1) = -3(1)^2 = -3$$

Now we'll plug these values and the given point into the formula for the equation of the tangent plane.

$$z - 7 = f_x(2,1)(x - 2) + f_y(2,1)(y - 1)$$

$$z - 7 = 12(x - 2) - 3(y - 1)$$

$$z = 12x - 24 - 3y + 3 + 7$$

$$z = 12x - 3y - 14$$