Topic: Riemann sums for double integrals

Question: Use Riemann sums to approximate the double integral.

$$\iint_R x + y^2 \ dA$$

$$m = n = 2$$

$$R = [0,2] \times [0,2]$$

Answer choices:

A 8

B 16

C 32

D 48

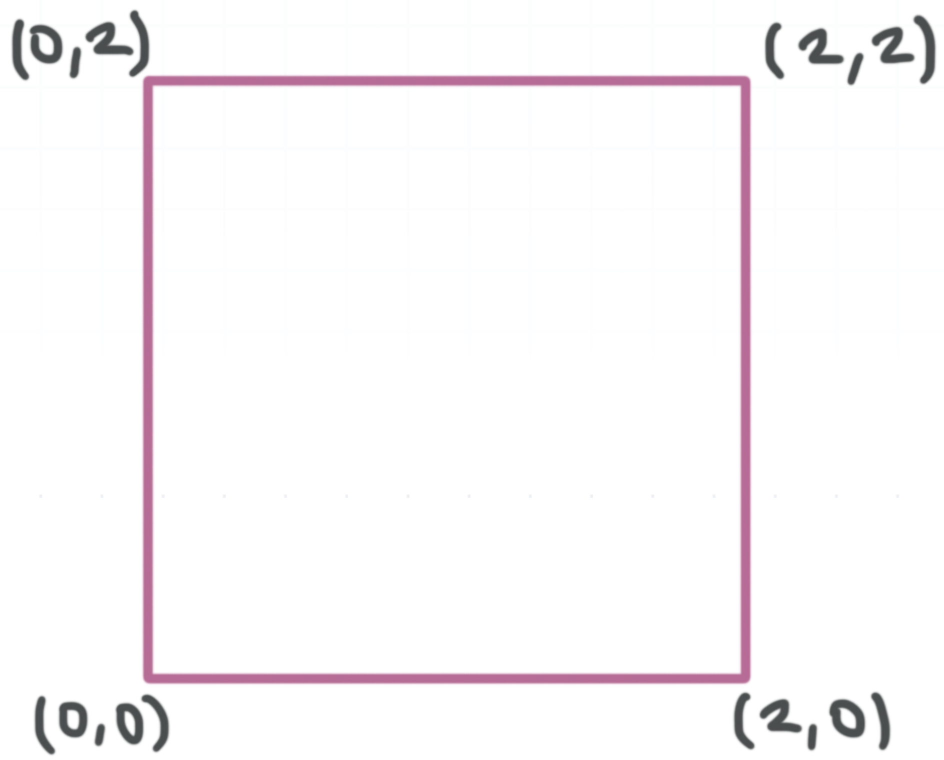
Solution: B

The question is asking us to use Riemann sums to approximate a double integral so we will need to use the formula

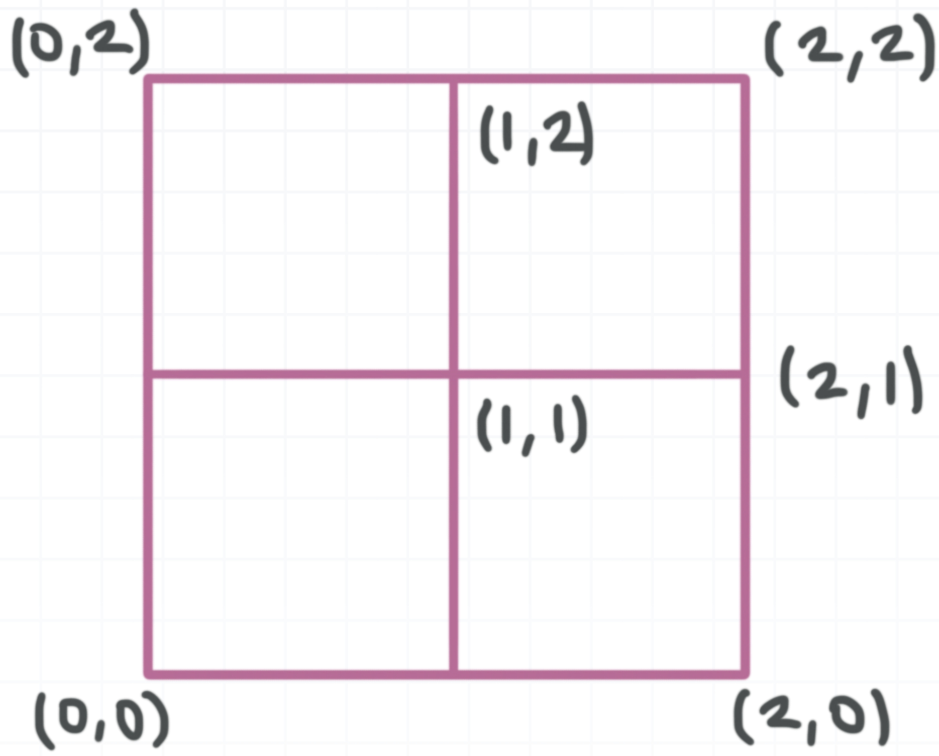
$$\iint_R f(x,y)\ dA = \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}, y_{ij})\Delta A$$

$$= \Delta A\left[f(x_1,y_1) + f(x_2,y_2) + +f(x_3,y_3) + f(x_4,y_4) + \ldots + f(x_n,y_n)\right]$$

To use this formula, we need to find the upper right corner points. The rectangle $R = [0,2] \times [0,2]$ gives us the x interval $[0,2]$ and the y interval $[0,2]$.



Because $m = n = 2$, we'll divide this larger rectangle $R = [0,2] \times [0,2]$ into two parts in the x direction and two parts in the y direction.



Then the upper right corners of the smaller rectangles are given by

$(1,1)$, $(2,1)$, $(1,2)$ and $(2,2)$

Next, we need to solve for ΔA. We'll use the dimensions of one of the smaller rectangles to find ΔA.

$$\Delta A = (\text{length of small rectangle})(\text{width of small rectangle})$$

$$\Delta A = (1)(1)$$

$$\Delta A = 1$$

Now we can plug everything we've found into our Riemann sum formula.

$$\iint_R f(x,y)\ dA = \Delta A\left[f(x_1,y_1) + f(x_2,y_2) + + f(x_3,y_3) + f(x_4,y_4) + \ldots + f(x_n,y_n)\right]$$

$$\iint_R f(x,y)\ dA = \Delta A\left[f(1,1) + f(2,1) + + f(1,2) + f(2,2)\right]$$

$$\iint_R x + y^2 \, dA = (1)\left[\left(1 + (1)^2\right) + \left(2 + (1)^2\right) + \left(1 + (2)^2\right) + \left(2 + (2)^2\right)\right]$$

$$\iint_R x + y^2 \, dA = (1 + 1) + (2 + 1) + (1 + 4) + (2 + 4)$$

$$\iint_R x + y^2 \, dA = 2 + 3 + 5 + 6$$

$$\iint_R x + y^2 \, dA = 16$$

The approximate volume of the double integral is 16.

Topic: Riemann sums for double integrals

Question: Use Riemann sums to approximate the double integral.

$$\iint_R e^{xy}\, dA$$

$$m = n = 2$$

$$R = [0,2] \times [0,2]$$

Answer choices:

A 18.024

B 27.094

C 72.094

D 36.047

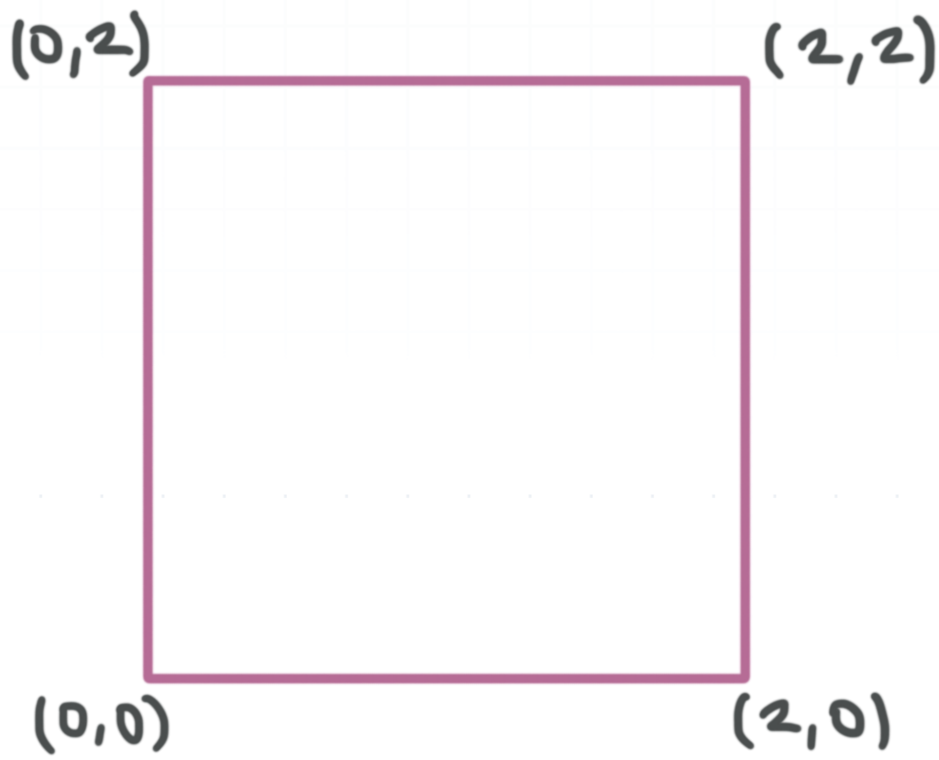
Solution: C

The question is asking us to use Riemann sums to approximate a double integral so we will need to use the formula

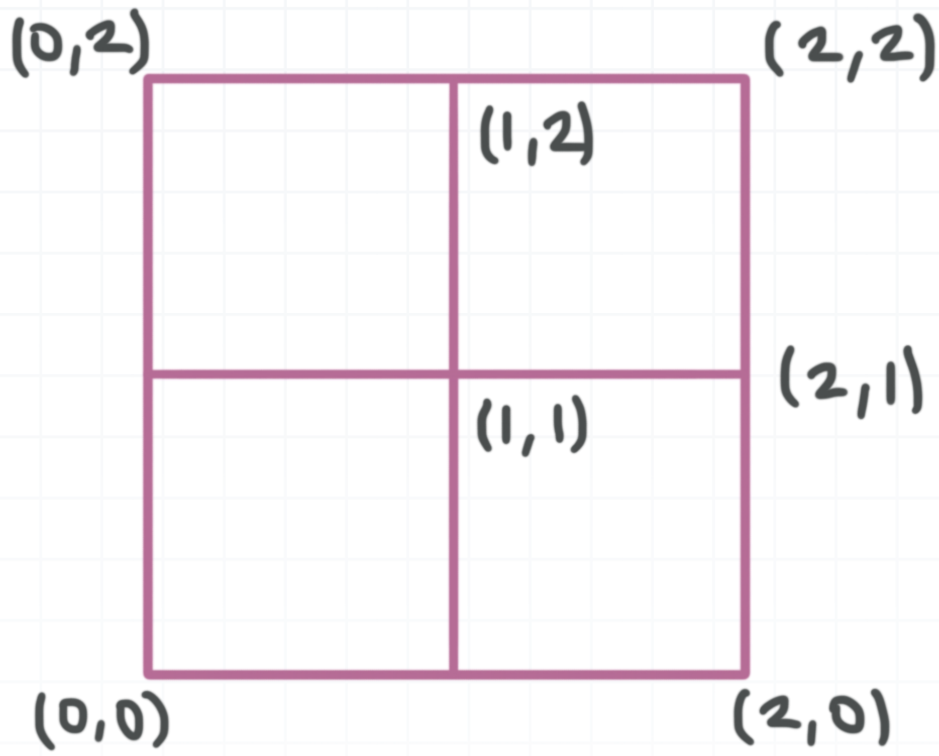
$$\int\int_R f(x,y)\ dA = \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij},y_{ij})\Delta A$$

$$= \Delta A\left[f(x_1,y_1)+f(x_2,y_2)++f(x_3,y_3)+f(x_4,y_4)+\ldots+f(x_n,y_n)\right]$$

To use this formula, we need to find the upper right corner points. The rectangle $R=[0,2]\times[0,2]$ gives us the x interval $[0,2]$ and the y interval $[0,2]$.



Because $m=n=2$, we'll divide this larger rectangle $R=[0,2]\times[0,2]$ into two parts in the x direction and two parts in the y direction.



Then the upper right corners of the smaller rectangles are given by

$(1,1)$, $(2,1)$, $(1,2)$ and $(2,2)$

Next, we need to solve for ΔA. We'll use the dimensions of one of the smaller rectangles to find ΔA.

$$\Delta A = (\text{length of small rectangle})(\text{width of small rectangle})$$

$$\Delta A = (1)(1)$$

$$\Delta A = 1$$

Now we can plug everything we've found into our Riemann sum formula.

$$\iint_R f(x,y)\ dA = \Delta A\left[f(x_1,y_1) + f(x_2,y_2) + +f(x_3,y_3) + f(x_4,y_4) + \ldots + f(x_n,y_n)\right]$$

$$\iint_R f(x,y)\ dA = \Delta A\left[f(1,1) + f(2,1) + +f(1,2) + f(2,2)\right]$$

$$\iint_R e^{xy}\ dA = (1)\left[\left(e^{(1)(1)}\right) + \left(e^{(2)(1)}\right) + \left(e^{(1)(2)}\right) + \left(e^{(2)(2)}\right)\right]$$

$$\iint_R e^{xy}\ dA = e^1 + e^2 + e^2 + e^4$$

$$\iint_R e^{xy}\ dA = e + e^2 + e^2 + e^4$$

We could leave the answer this way, or we could calculate a decimal value for the approximate volume of the double integral. If we do that, then we can say that approximate volume is 72.094.

Topic: Riemann sums for double integrals

Question: Use Riemann sums to approximate the double integral.

$$\iint_R 3ye^x \, dA$$

$$m = n = 2$$

$$R = [0,3] \times [0,2]$$

Answer choices:

A 110.553

B 663.316

C 221.105

D 331.658

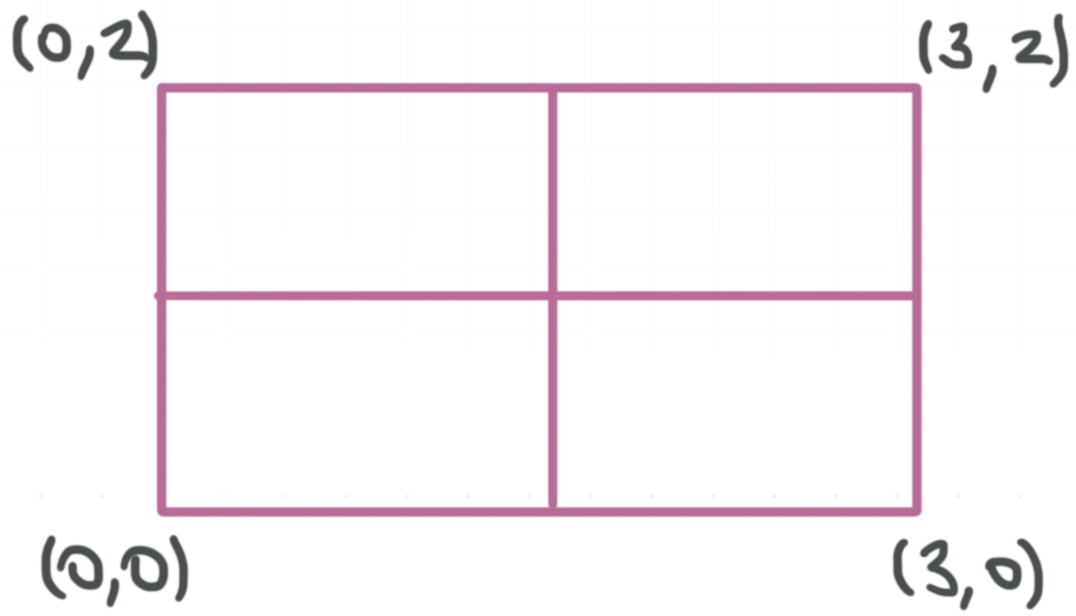
Solution: D

The question is asking us to use Riemann sums to approximate a double integral so we will need to use the formula

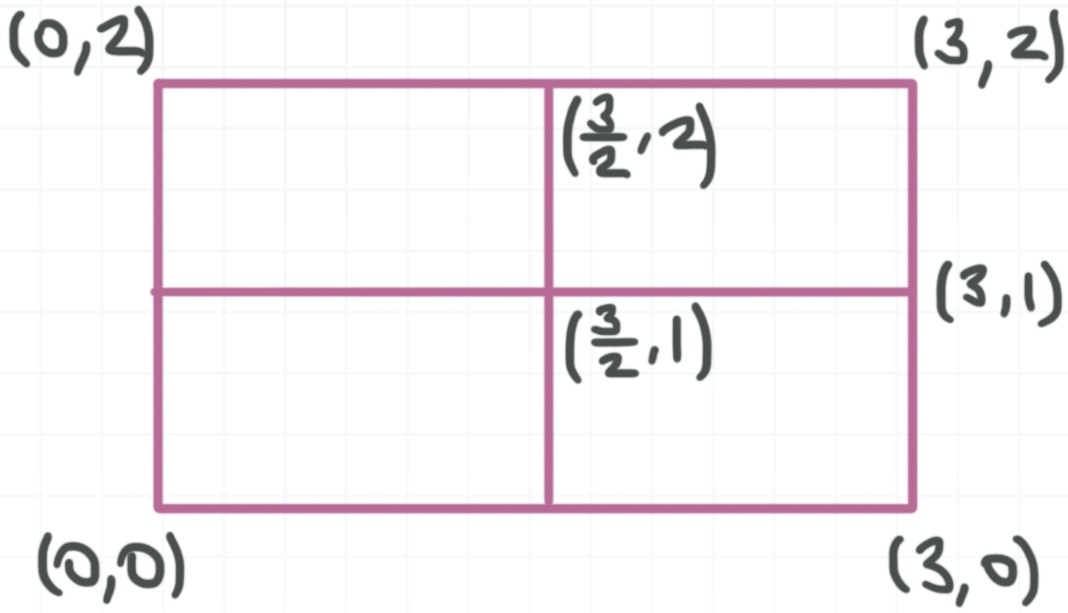
$$\iint_R f(x,y)\ dA = \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij},y_{ij})\Delta A$$

$$= \Delta A \left[f(x_1,y_1) + f(x_2,y_2) + +f(x_3,y_3) + f(x_4,y_4) + \dots + f(x_n,y_n)\right]$$

To use this formula, we need to find the upper right corner points. The rectangle $R = [0,3] \times [0,2]$ gives us the x interval $[0,3]$ and the y interval $[0,2]$.



Because $m = n = 2$, we'll divide this larger rectangle $R = [0,2] \times [0,2]$ into two parts in the x direction and two parts in the y direction.



Then the upper right corners of the smaller rectangles are given by

$$\left(\frac{3}{2},1\right),\ (3,1),\ \left(\frac{3}{2},2\right) \text{ and } (3,2)$$

Next, we need to solve for ΔA. We'll use the dimensions of one of the smaller rectangles to find ΔA.

$$\Delta A = (\text{length of small rectangle})(\text{width of small rectangle})$$

$$\Delta A = \left(\frac{3}{2}\right)(1)$$

$$\Delta A = \frac{3}{2}$$

Now we can plug everything we've found into our Riemann sum formula.

$$\iint_R f(x,y)\ dA = \Delta A\left[f(x_1,y_1) + f(x_2,y_2) + +f(x_3,y_3) + f(x_4,y_4) + \ldots + f(x_n,y_n)\right]$$

$$\iint_R f(x,y)\ dA = \Delta A\left[f\left(\frac{3}{2},1\right) + f(3,1) + +f\left(\frac{3}{2},2\right) + f(3,2)\right]$$

$$\iint_R 3ye^x \, dA = \frac{3}{2}\left[3(1)e^{\frac{3}{2}} + 3(1)e^3 + 3(2)e^{\frac{3}{2}} + 3(2)e^3\right]$$

$$\iint_R 3ye^x \, dA = \frac{3}{2}\left(3e^{\frac{3}{2}} + 3e^3 + 6e^{\frac{3}{2}} + 6e^3\right)$$

$$\iint_R 3ye^x \, dA = \frac{3}{2}\left(9e^{\frac{3}{2}} + 9e^3\right)$$

$$\iint_R 3ye^x \, dA = \frac{27}{2}e^{\frac{3}{2}}\left(1 + e^{\frac{3}{2}}\right)$$

We could leave the answer this way, or we could calculate a decimal value for the approximate volume of the double integral. If we do that, then we can say that approximate volume is 331.658.