Calculus 3 Workbook

Gradient vectors

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GRADIENT VECTORS

1. Find the gradient vector ∇f at $(\sqrt{\pi},0,0)$.

$$f(x,y,z)=\sin(x^2+2y^2-z^2-2xyz)$$

2. Find unit gradient vector of the function f at $(-2,1)$.

$$f(t,s)=\frac{4t-st^4}{t^2s}$$

3. Find the point where the gradient vector of the function f is equal to the zero vector.

$$f(x,y)=\ln\frac{(x-2)^2y}{x-y}$$

4. Find and identify the set of points where the magnitude of the gradient vector of the function f is equal to 1.

$$f(x,y)=x^2+4y^2-2x+8y-5$$

5. Find the directional derivative of the function f in the direction $m=3\mathbf{i}-4\mathbf{j}$ at $(0,4)$.

$$f(x, y) = 2^x(y^2 - 1)$$

GRADIENT VECTORS AND THE TANGENT PLANE

■ 1. Use the gradient vector to find the tangent line to the curve $(y-2)^2 - e^x = 0$ at $(0,3)$.

■ 2. Use the gradient vector to find the tangent plane to the surface $(r+1)\sin(\phi+\pi)\tan(\theta) = 0$ at $(2,\pi/6,\pi/4)$.

■ 3. Use the gradient vector to find the tangent plane(s) to the surface $x^2 - 2xy - 3y^2 + z^2 + 4x + 4y - 2z - 3 = 0$ that are parallel to the xy-plane.

MAXIMUM RATE OF CHANGE AND ITS DIRECTION

■ 1. Find the point where the maximum rate of change of the function f is equal to 0.

$$f(x, y) = 6x^2 - 4xy + y^2 - 12x - 6y + 4$$

■ 2. Find the maximum rate of change and its direction for the function f at $(3, -\pi/2, 0)$.

$$f(x, y, z) = x^2(2z - 1)\sin^2 y$$

■ 3. Find the minimum rate of change and its direction for the function f at $(2, 1, 4)$.

$$f(u, v, w) = \sqrt{2u - 4v + 6w + 1}$$

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