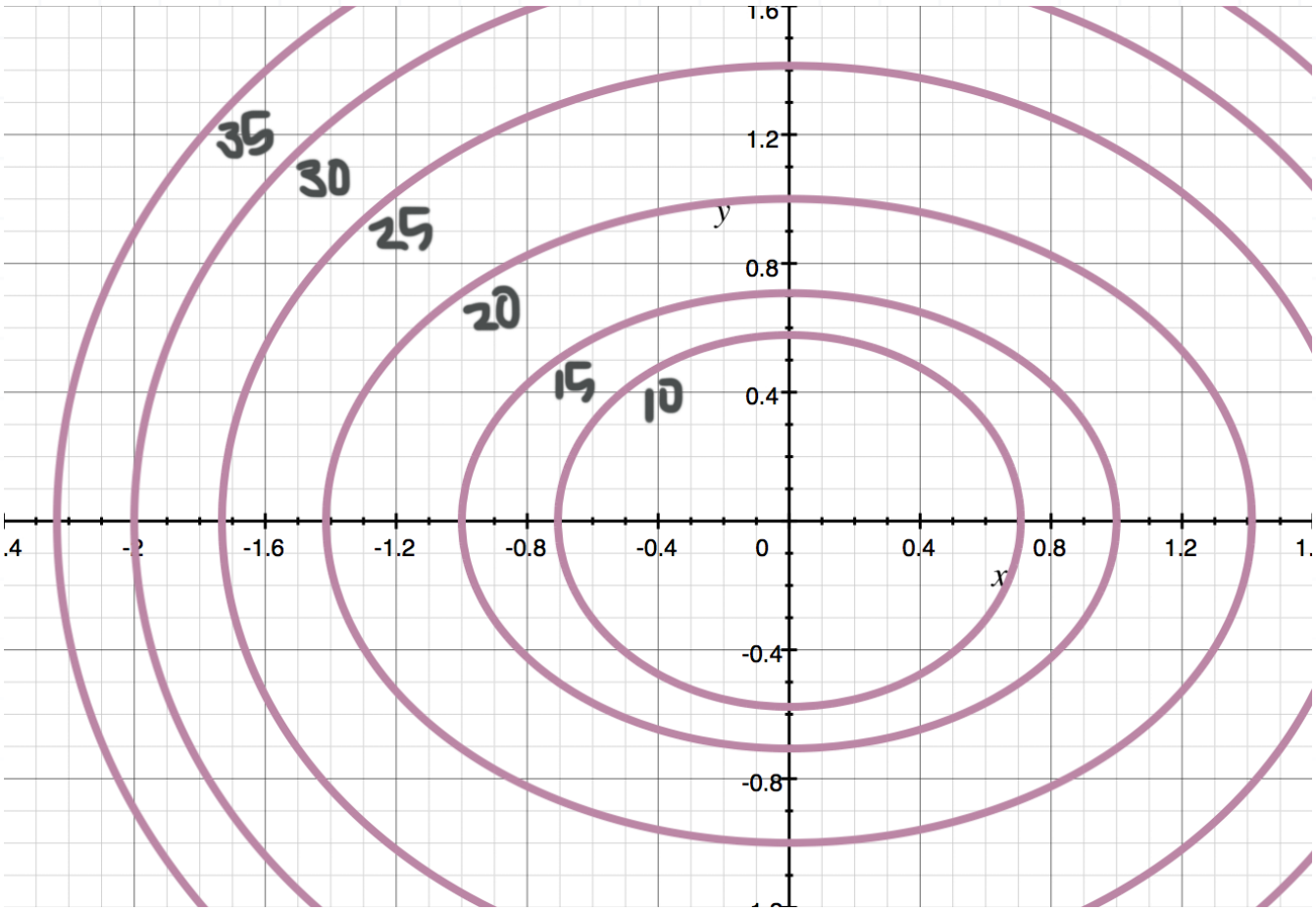
Calculus 3 Workbook

Double integrals

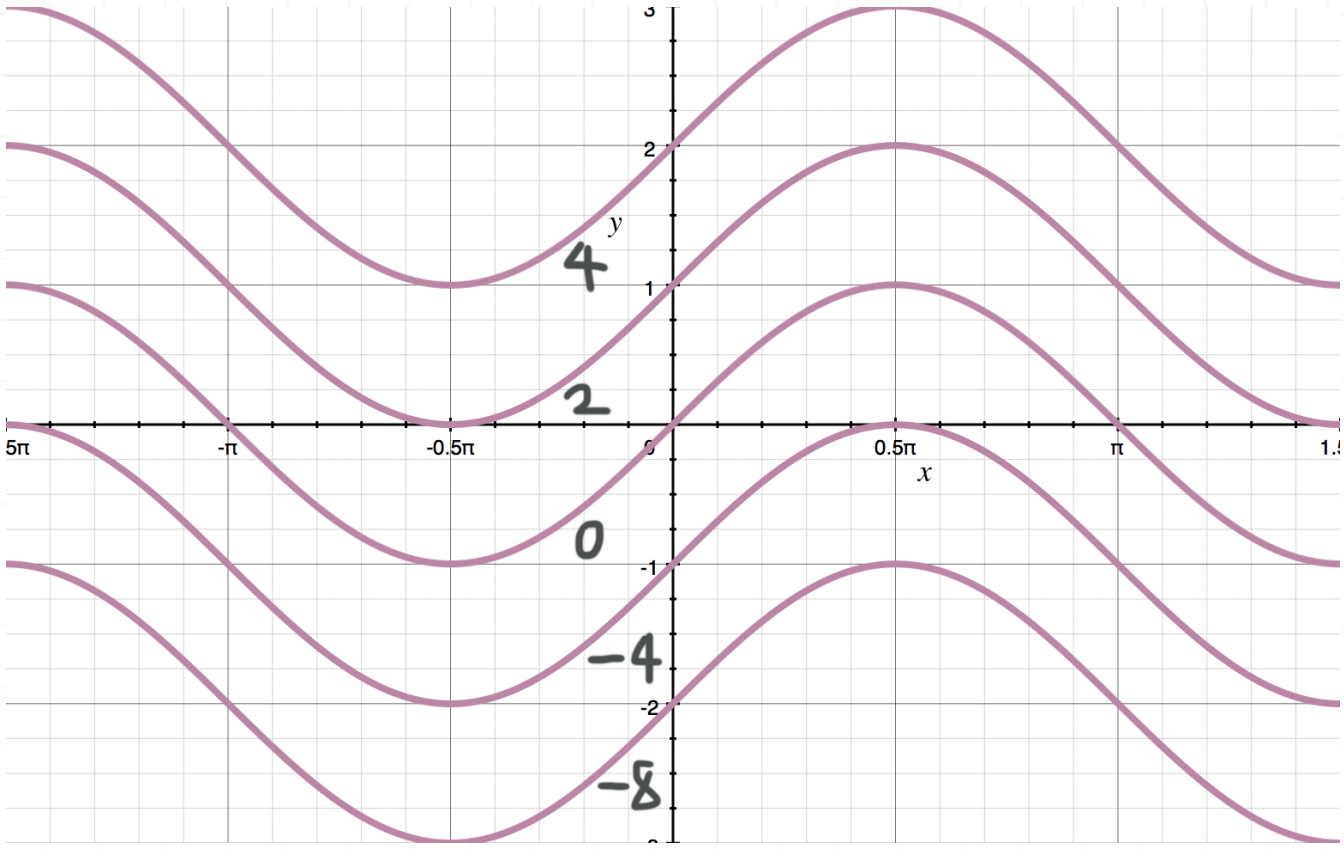
krista king
MATH

AVERAGE VALUE

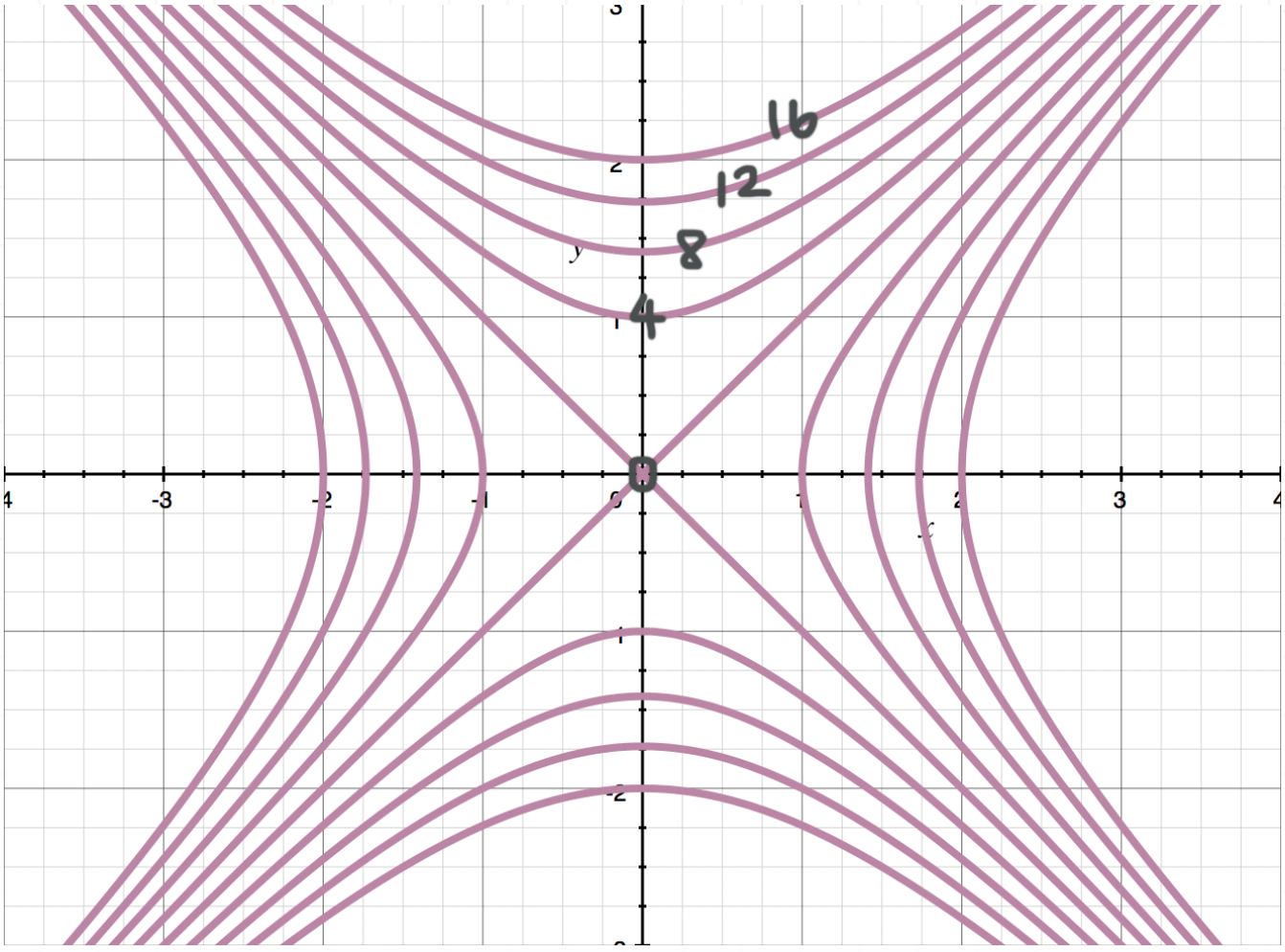
■ 1. Use midpoints of squares with side lengths 1 to estimate the average value of the region $R = [-2,1] \times [-2,2]$, given the sketch of level curves.



■ 2. Use midpoints of rectangles with dimensions $\pi \times 1$ to estimate the average value of the region $R = [-\pi, \pi] \times [-2,2]$, given the sketch of level curves.



3. Use midpoints of rectangles with dimensions 2×1 to estimate the average value of the region $R = [-2,2] \times [-2,2]$, given the sketch of level curves.



ITERATED INTEGRALS

■ 1. Evaluate the iterated integral.

$$\int_2^4 \int_1^2 \log_2 \frac{y^2}{x^4} \, dx \, dy$$

■ 2. Evaluate the iterated integral.

$$\int_{-5}^5 \int_0^\pi (3x^2 - 4x + 10)\sin(y + \pi) \, dy \, dx$$

■ 3. Evaluate the iterated integral.

$$\int_{-1}^1 \int_0^2 xe^{x^2-3y+1} \, dx \, dy$$

DOUBLE INTEGRALS

■ 1. Evaluate the double integral, where R is the rectangle $[0,\pi] \times [0,1]$.

$$\iint_R \cos(x - \pi y)\ dx\ dy$$

■ 2. Evaluate the double integral, where R is the rectangle $[1,3] \times [1,5]$.

$$\iint_R \frac{1}{(x+y)^2}\ dx\ dy$$

■ 3. Evaluate the double integral, where R is the rectangle $[x, y] = [-\pi/2,\pi/2] \times [0,\pi]$.

$$\iint_R \cos(x+y) - x\sin(x+y)\ dx\ dy$$

TYPE I AND II REGIONS

■ 1. Evaluate the double integral if D is the circle centered at the origin with radius 4.

$$\iint_D 4x^2y + 3 \; dA$$

■ 2. Evaluate the double integral if D is the region bounded by the curves $y + x^2 - 4 = 0$ and $y + 2x^2 - 8 = 0$.

$$\iint_D 462y\sqrt{x+2} \; dA$$

■ 3. Evaluate the double integral if D is the region bounded by $x - \sin y = 0$, $x - \sin y = 5$, $y = 0$, and $y = \pi$.

$$\iint_D 2x \; dA$$

FINDING SURFACE AREA

■ 1. Find area of the surface $z = \sqrt{3}x + y^2 + 1$ inside the rectangle $-1 \leq x \leq 1, 0 \leq y \leq 1$.

■ 2. Find area of the surface $z = \ln(\sin(3x)) + 2\sqrt{2}y - 5$ inside the rectangle $\pi/6 \leq x \leq \pi/4, 0 \leq y \leq 1$.

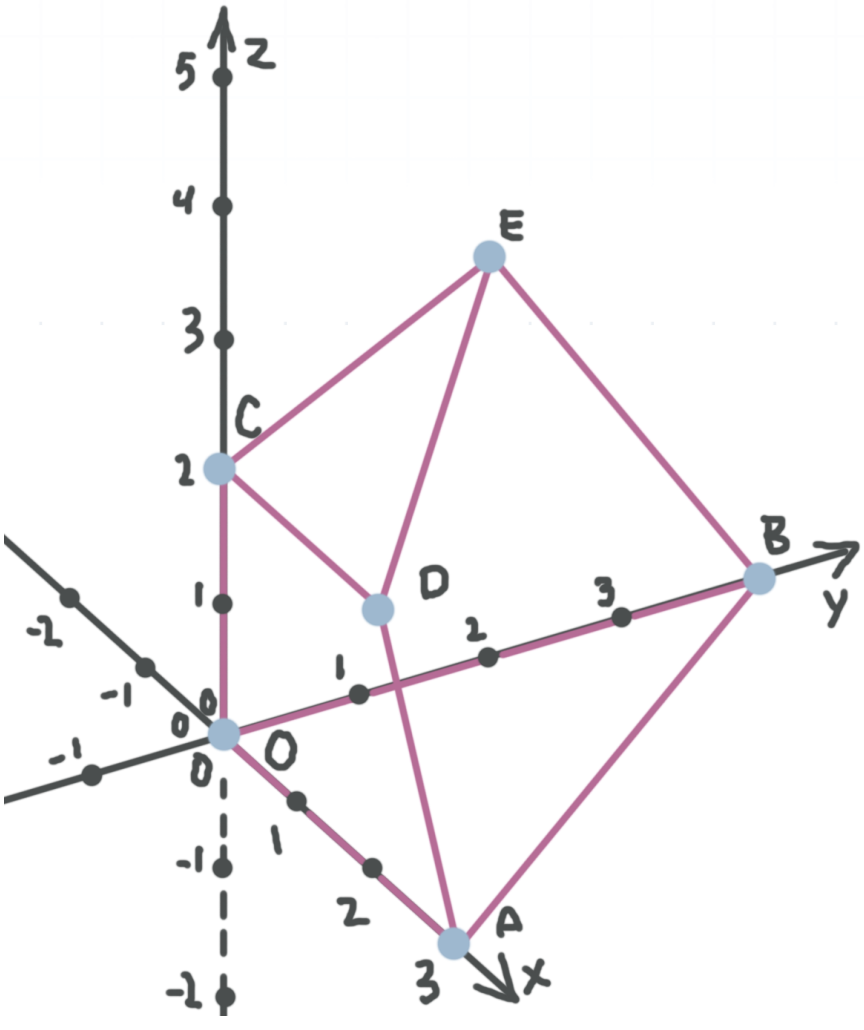
■ 3. Find area of the surface $z = 2(x+3)^{3/2} + 5^{3/2}y - 6$ inside the triangle OAB, if O is the origin and A and B are at $A(3,0)$ and $B(2,2)$.

FINDING VOLUME

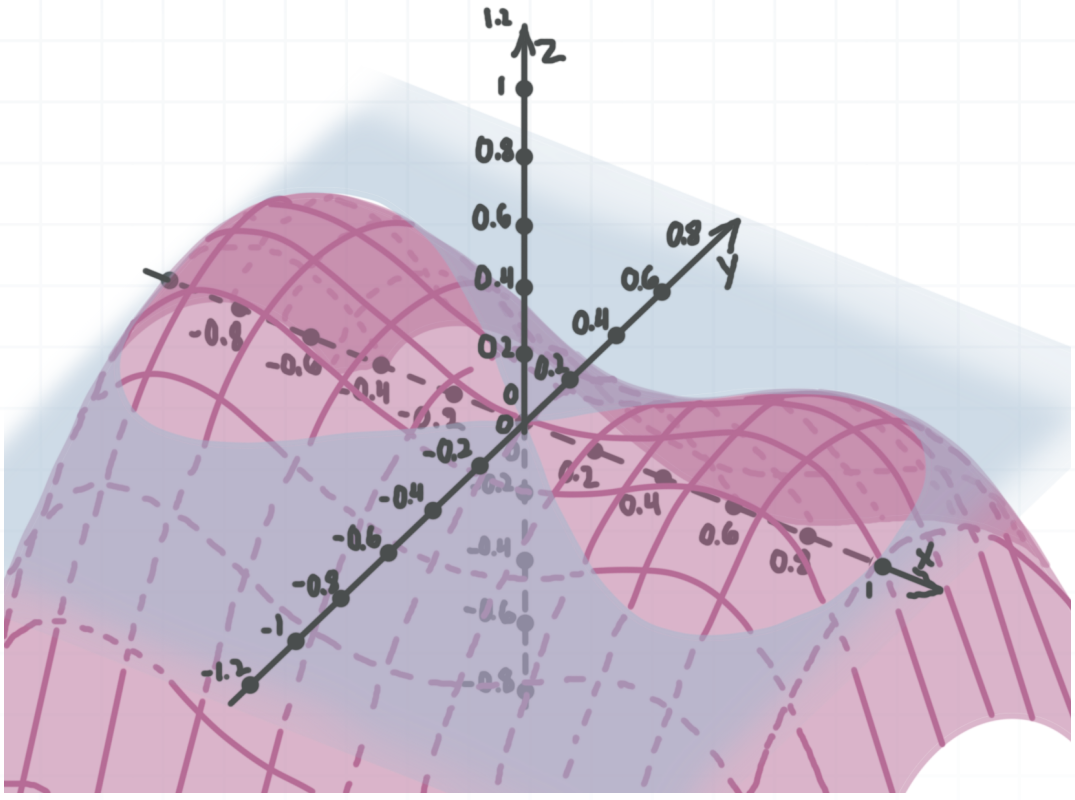
■ 1. Use a double integral to find the volume of the solid that's bounded by the surface and the xy-plane, on $0 \leq x \leq 2$ and $0 \leq y \leq \pi/2$.

$$z = \frac{\sin(2y)}{(x+1)^2}$$

■ 2. Use a double integral to find the volume of the irregular hexagon $OABCDE$, where O is the origin, and the hexagon's other vertices are $A(3,0,0)$, $B(0,4,0)$, $C(0,0,2)$, $D(2,0,2)$, and $E(0,2,3)$.



3. Use a double integral to find the volume of the solid that's bounded by the surface $z = -x^4 + x^2 - y^2$ and the xy-plane.



CHANGING THE ORDER OF INTEGRATION

■ 1. Change the order of integration of the iterated integral.

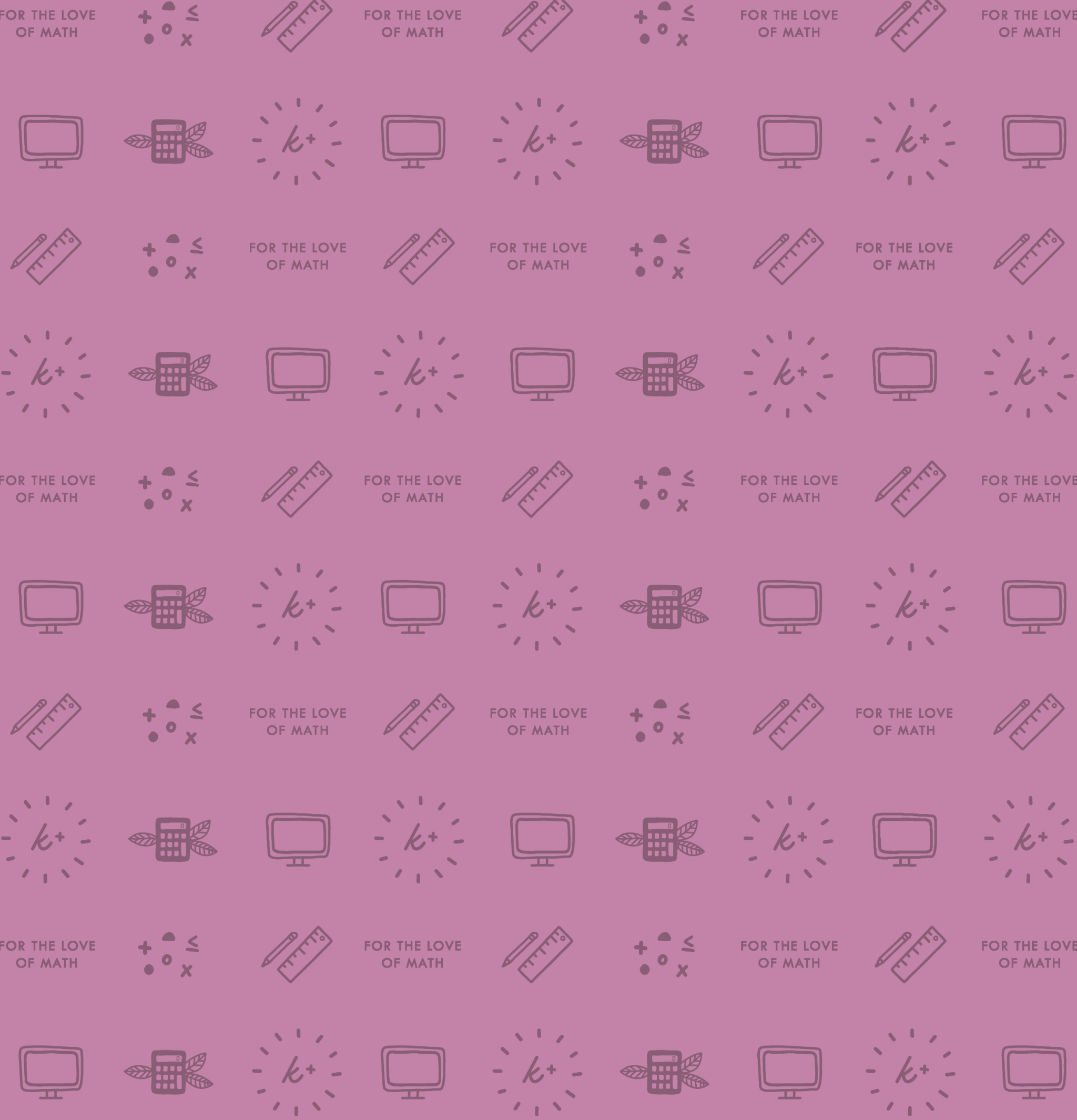
$$\int_{-3}^{0} \int_{-\frac{2}{3}\sqrt{9-x^2}}^{\frac{2}{3}\sqrt{9-x^2}} z(x,y)\ dy\ dx$$

■ 2. Change the order of integration of the iterated integral.

$$\int_{3}^{5} \int_{2}^{e^{x-3}+1} z(x,y)\ dy\ dx$$

■ 3. Change the order of integration of the iterated integral.

$$\int_{-2}^{2} \int_{\frac{1}{4}x^4-x^2}^{0} z(x,y)\ dy\ dx$$



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