Topic: Orthogonal, parallel or neither

Question: Say whether the vectors are orthogonal, parallel, or neither.

$$a = \langle 1,2,-1 \rangle$$

$$b = \langle 2,1,4 \rangle$$

Answer choices:

A Orthogonal

B Parallel

C Neither

D Impossible to know

Solution: A

Two vectors are orthogonal to one another (set at $90°$ from each other) when their dot product is 0.

Two vectors are parallel they can be in different directions and have different magnitudes, but their direction numbers will be equal to each other, or multiples of one another.

Two vectors that don't meet either of the above criteria will be neither orthogonal nor parallel.

We'll test to see whether or not the vectors are orthogonal by calculating their dot product.

$$a \cdot b = (1)(2) + (2)(1) + (-1)(4)$$

$$a \cdot b = 2 + 2 - 4$$

$$a \cdot b = 0$$

Since the dot product is 0, the vectors are orthogonal.

Topic: Orthogonal, parallel or neither

Question: Say whether the vectors are orthogonal, parallel, or neither.

$$a = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

$$b = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$$

Answer choices:

A Orthogonal

B Parallel

C Neither

D Impossible to know

Solution: B

Two vectors are orthogonal to one another (set at $90°$ from each other) when their dot product is 0.

Two vectors are parallel they can be in different directions and have different magnitudes, but their direction numbers will be equal to each other, or multiples of one another.

Two vectors that don't meet either of the above criteria will be neither orthogonal nor parallel.

We'll test to see whether or not the vectors are orthogonal by calculating their dot product.

$$a \cdot b = (1)(2) + (1)(2) + (2)(4)$$

$$a \cdot b = 2 + 2 + 8$$

$$a \cdot b = 12$$

Since the dot product is not 0, the vectors are not orthogonal.

Next we'll check to see whether or not the vectors are parallel. If we divide b by a factor of 2, we'll get

$$b = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$$

$$b = \frac{2}{2}\mathbf{i} + \frac{2}{2}\mathbf{j} + \frac{4}{2}\mathbf{k}$$

$$b = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

If we compare this to a, we can see that both a and b have the direction numbers $a = b = \langle 1,1,2 \rangle$. Therefore the vectors are parallel.

Topic: Orthogonal, parallel or neither

Question: Say whether the vectors are orthogonal, parallel, or neither.

$$a = -2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$$

$$b = 11\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$$

Answer choices:

A Orthogonal

B Parallel

C Neither

D Impossible to know

Solution: C

Two vectors are orthogonal to one another (set at $90°$ from each other) when their dot product is 0.

Two vectors are parallel they can be in different directions and have different magnitudes, but their direction numbers will be equal to each other, or multiples of one another.

Two vectors that don't meet either of the above criteria will be neither orthogonal nor parallel.

We'll test to see whether or not the vectors are orthogonal by calculating their dot product.

$$a \cdot b = (-2)(11) + (5)(-3) + (7)(-7)$$

$$a \cdot b = -22 - 15 - 49$$

$$a \cdot b = -86$$

Since the dot product is not 0, the vectors are not orthogonal.

Next we'll check to see whether or not the vectors are parallel. Neither vector has any factor that we can remove that would make the two base vectors equal, so we can say that the vectors are not parallel. Therefore the vectors are neither orthogonal nor parallel.