



Calculus 3 Workbook

Tangent planes and normal lines

EQUATION OF THE TANGENT PLANE

- 1. Find a tangent plane to the surface $f(u, v, w) = 0$ at $(3, -1, 5)$.

$$f(u, v, w) = \ln \frac{u^2 + 1}{v^2 w^5}$$

- 2. Find any tangent planes to the surface $f(x, y, z) = 0$ that are parallel to the plane $5x - 4y + 2z + 5 = 0$.

$$f(x, y, z) = x^3 - 4y^2 + z^2 + 2x + 12y + 5$$

- 3. Find a line of intersection of the xy -plane and tangent plane to the surface $f(x, y, z) = 0$ at $(\pi, -1, \sqrt{6})$.

$$f(x, y, z) = 2 \cos(x + \pi)(y^2 + y + 5) - 3z^3$$

- 4. Find and identify the set of the points where the tangent plane to the surface $f(x, y, z) = 0$ is parallel to z -axis.

$$f(x, y, z) = x^2 + 4y^2 + z^2 + 2x - 8y + 8z + 17 = 0$$



NORMAL LINE TO THE SURFACE

- 1. Use the gradient vector to find a symmetric equation of the normal line to the curve $f(s, t) = 0$ at $(-3, 3)$, where

$$f(s, t) = t2^{2t+s-3}$$

- 2. Use the gradient vector to find a vector equation of the normal line to the surface $f(x, y, z) = 0$ at $(0, -5, 1)$.

$$f(x, y, z) = \frac{2x - y^2}{z}$$

- 3. Use the gradient vector to find a parametric equation of the normal line to the surface $f(x, y, z) = 0$ at $(2, -3, 0)$.

$$f(x, y, z) = 3x^3 - 2xyz - 2y^2 + 5yz + 1$$

- 4. Use the gradient vector to find a parametric equation of the normal line to the surface $f(x, y, z) = 0$ that's parallel to the line $r = \langle 2, 17, -6 \rangle + t \langle 9, 1, -6 \rangle$.

$$f(x, y, z) = 2x^2 + y^2 + 3z^2 - 3x - 5y + 5$$



■ 5. Use the gradient vector to find a vector equation of the normal line to the surface $f(x, y, z) = 0$ that's perpendicular to the plane $x + 4y - 8z + 12 = 0$.

$$f(x, y, z) = ye^{2x+6} + z^2 - 5$$



