Vector and parametric equations of a line segment

Sometimes we need to find the equation of a line segment when we only have the endpoints of the line segment.

The **vector equation** of the line segment is given by

$$r(t) = (1 - t)r_0 + tr_1$$

where $0 \le t \le 1$ and r_0 and r_1 are the vector equivalents of the endpoints.

The **parametric equations** of the line segment are given by

$$x = r(t)_1$$

$$y = r(t)_2$$

$$z = r(t)_3$$

where $r(t)_1$, $r(t)_2$ and $r(t)_3$ come from the vector function

$$r(t) = r(t)_1\mathbf{i} + r(t)_2\mathbf{j} + r(t)_3\mathbf{k}$$

$$r(t) = \langle r(t)_1, r(t)_2, r(t)_3\rangle$$

Example

Find the vector and parametric equations of the line segment defined by its endpoints.

$$P(1,2, -1)$$

$Q(1,0,3)$

To find the vector equation of the line segment, we'll convert its endpoints to their vector equivalents.

$P(1,2,-1)$ becomes $r_0 = \langle 1,2,-1 \rangle$

$Q(1,0,3)$ becomes $r_1 = \langle 1,0,3 \rangle$

Plugging these into the vector formula for the equation of the line segment gives

$$r(t) = (1-t)\langle 1,2,-1 \rangle + t\langle 1,0,3 \rangle$$

$$r(t) = \langle 1-t, 2-2t, -1+t \rangle + \langle t, 0, 3t \rangle$$

$$r(t) = \langle 1-t+t, 2-2t+0, -1+t+3t \rangle$$

$$r(t) = \langle 1, 2-2t, -1+4t \rangle$$

We can also write the vector equation as

$$r(t) = 1\mathbf{i} + (2-2t)\mathbf{j} + (-1+4t)\mathbf{k}$$

$$r(t) = \mathbf{i} + (2-2t)\mathbf{j} + (-1+4t)\mathbf{k}$$

Now that we have the vector equation of the line segment, we just take its direction numbers, or the coefficients on $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ to get the parametric equations of the line segment.

$$x = 1$$

$$y = 2 - 2t$$

$$z = -1 + 4t$$

We'll summarize our findings.

Vector equation	$r(t) = \langle 1, 2 - 2t, -1 + 4t \rangle$
Parametric equations	$x = 1$, $y = 2 - 2t$ and $z = -1 + 4t$