Calculus 3 Final Exam

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This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** Find f_{zxy} for $f(x, y, z) = x^4y^2z + xy^2$.

A $f_{zxy} = 8xy$

B $f_{zxy} = 8x^3y$

C $f_{zxy} = 4x^3y$

D $f_{zxy} = 8x^3y + 2xy$

E $f_{zxy} = 8x^3y + 2y$

2. **(5 pts)** Find the directional derivative $D_uf(0,-2)$ of the function $f(x, y) = -2ye^{3x} + x^2y$, $\vec{u}$ is the unit vector toward $\vec{v} = \langle 1, -1 \rangle$.

A $D_uf(0,-2) = 5\sqrt{2}$

B $D_uf(0,-2) = 4\sqrt{2}$

C $D_uf(0,-2) = 7\sqrt{2}$

D $D_uf(0,-2) = -5\sqrt{2}$

E $D_uf(0,-2) = 0$

3. **(5 pts)** Find $\nabla(fg)$ at $(-1,3)$, if $f(x, y) = 2x^2y - y^2$ and $g(x, y) = 2xy^2 + y^2$.

A $\nabla(fg) = -162\mathbf{i} + 126\mathbf{j}$

B $\nabla(fg) = 522\mathbf{i} - 150\mathbf{j}$

C $\nabla(fg) = -378\mathbf{i} - 90\mathbf{j}$

D $\nabla(fg) = 54\mathbf{i} + 54\mathbf{j}$

E $\nabla(fg) = -126\mathbf{i} + 66\mathbf{j}$

4. **(5 pts)** Evaluate the double polar integral.

$$\int_0^{\pi}\int_0^2 r^3\ dr\ d\theta$$

A 12π

C 8π

E 4π

B π

D 2π

5. **(5 pts)** Find the Jacobian of the transformation.

$$x = r^2 \cos\theta$$

$$y = r^2 \sin\theta$$

A 0

B $-2r^2 \sin\theta \cos\theta$

C $4r^2 \sin\theta \cos\theta + r^4 \sin\theta \cos\theta$

D $2r^3$

E $2r^2 \sin\theta \cos\theta$

6. **(5 pts)** Given $A(-1,2,5)$, $B(3,6,7)$, $C(2,9,6)$ and $D(1,4,-5)$, find the volume of the parallelepiped, if $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$ are adjacent.

A 176

B 156

C 144

D 196

E 184

7. **(5 pts)** Reparametrize the curve with respect to arc length from $t = 0$ in the direction of decreasing t.

$$r(t) = \left(2 - \frac{5}{2}t\right)\mathbf{i} - (1 - 2t)\mathbf{j} + \sqrt{2}t\mathbf{k}$$

A $r(t(s)) = \left(2 - \frac{5}{7}s\right)\mathbf{i} - \left(1 - \frac{4}{7}s\right)\mathbf{j} + \frac{2\sqrt{2}}{7}s\mathbf{k}$

B $r(t(s)) = \left(2 - \frac{35}{4}s\right)\mathbf{i} - (1 - 7s)\mathbf{j} + \frac{7\sqrt{2}}{2}s\mathbf{k}$

C $r(t(s)) = \left(2 - \frac{5\sqrt{61}}{4}s\right)\mathbf{i} - (1 - \sqrt{61}s)\mathbf{j} + \frac{\sqrt{122}}{2}s\mathbf{k}$

D $r(t(s)) = \left(2 - \frac{5}{14}s\right)\mathbf{i} - \left(1 - \frac{2}{7}s\right)\mathbf{j} + \frac{\sqrt{2}}{7}s\mathbf{k}$

E $r(t(s)) = \left(2 - \frac{5}{\sqrt{61}}s\right)\mathbf{i} - \left(1 - \frac{4}{\sqrt{61}}s\right)\mathbf{j} + \frac{2\sqrt{2}}{\sqrt{61}}s\mathbf{k}$

8. **(5 pts)** Find the line integral when $F = \nabla f$, where c is the line segment from $(1,3,2)$ to $(4,2,5)$.

$$F(x, y, z) = 2xy^2z^3\mathbf{i} + 2x^2yz^3\mathbf{j} + 3x^2y^2z^2\mathbf{k}$$

A	8,072	C	7,928	E	1,528
B	1,564	D	1,636		

9. **(15 pts)** Find the distance between the point $(-1,-3,5)$ and the line given by the parametric equation.

$$x = -2 + t$$

$$y = 1 - 2t$$

$$z = -5t + 3$$

10. **(15 pts)** Find the critical points of the function.

$$f(x,y) = x^2y + x^2 + 2y^2 - 4$$

11. **(15 pts)** A solid is bounded by the paraboloid $z = -2x^2 - 4y^2 + 25$ and the xy-plane. If the volume of the solid is defined by the single integral below, then what are the bounds for y if a double integral is used to calculate the volume of the solid?

$$V = \int_{-1}^{1} \frac{2}{3}(\sqrt{25 - 2x^2})^3 \ dx$$

12. **(15 pts)** What is the value of the integral for the region bounded by $(0,0)$, $(2,4)$, and $(6,0)$?

$$\oint_C 2\sin x + y^2 \ dx + 3x + e^{2y} \ dy$$