Independence of path

Independence of path is a property of conservative vector fields. If a conservative vector field contains the entire curve C, then the line integral over the curve C will be independent of path, because every line integral in a conservative vector field is independent of path, since all conservative vector fields are path independent.

We can state the following facts:

1. Conservative vector fields are independent of path

2. Vector fields that are independent of path are conservative

As a result, the value of the line integral depends only on the endpoints of the curve C, and not on the path taken by the integral between the endpoints.

No matter which path you follow between two points in a conservative vector field, whether it's a direct, straight line, or a curvy, winding path, or any other path, the value of the line integral will be the same if the endpoints are the same.

That fact that conservative vector fields are independent of path makes finding the line integral of the vector field easy. All we need is the potential function f of the vector field $\mathbf{F}$, such that

$$\mathbf{F} = \nabla f$$

Once we find f, we simply evaluate it over the interval defined by the endpoints of the curve C, and our answer will be the value of the line integral of the vector field $\mathbf{F}$.

In other words, if the endpoints of the curve C are a and b or (x_1, y_1) and (x_2, y_2), then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r}$$

$$= f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

$$= f(x_2, y_2) - f(x_1, y_1)$$