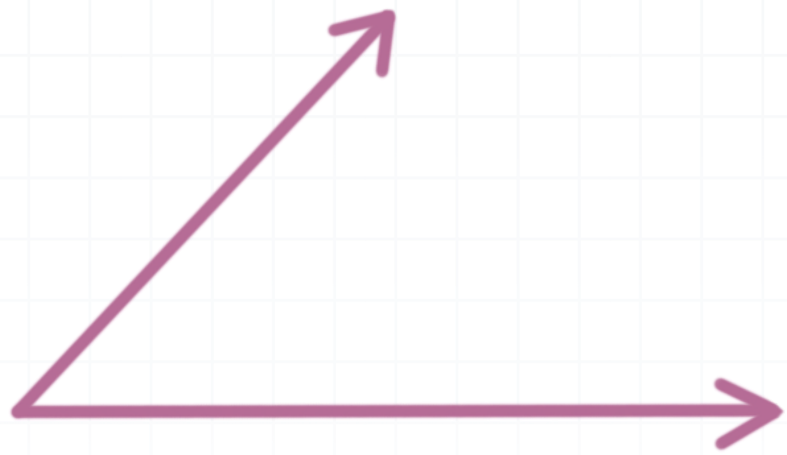
Acute angle between the lines

An acute angle is an angle that's less than 90°, like this:



If we want to find the acute angle between two lines, we can convert the lines to standard vector form and then use the formula

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

where a and b are the given vectors, $a \cdot b$ is the dot product of the vectors, $|a|$ is the magnitude of the vector a (its length) and $|b|$ is the magnitude of the vector b (its length). We can find the magnitude of both vectors using the distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

for a two-dimensional vector where the point (x_1, y_1) is the origin $(0,0)$.

If the formula above gives a result that's greater than 90°, then we've found the obtuse angle between the lines. To find the acute angle, we just subtract the obtuse angle from 180°, and we'll get the acute angle.

Example

Find the acute angle between the lines.

$$x + 3y = 2$$

$$3x - 6y = 5$$

First we'll convert the lines to standard vector form.

$$x + 3y = 2$$

$$a = \langle 1,3 \rangle$$

and

$$3x - 6y = 5$$

$$b = \langle 3, -6 \rangle$$

Before we can use our formula, we need to find the dot product of a and b.

$$a \cdot b = (1)(3) + (3)(-6)$$

$$a \cdot b = 3 - 18$$

$$a \cdot b = -15$$

Now we need to find the length of each vector using the distance formula.

$$|a| = \sqrt{(1-0)^2 + (3-0)^2}$$

$$|a| = \sqrt{1+9}$$

$$|a| = \sqrt{10}$$

and

$$|b| = \sqrt{(3-0)^2 + (-6-0)^2}$$

$$|b| = \sqrt{9+36}$$

$$|b| = \sqrt{45}$$

Plugging $a \cdot b = -15$, $|a| = \sqrt{10}$, and $|b| = \sqrt{45}$ into the formula, we get

$$\cos\theta = \frac{-15}{\sqrt{10}\sqrt{45}}$$

$$\cos\theta = \frac{-15}{\sqrt{450}}$$

$$\cos\theta = \frac{-15}{\sqrt{450}}$$

$$\cos\theta = \frac{-15}{\sqrt{225 \cdot 2}}$$

$$\cos\theta = \frac{-15}{15\sqrt{2}}$$

$$\cos\theta = \frac{-1}{\sqrt{2}}$$

Rationalize the denominator.

$$\cos\theta = \frac{-1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$\cos\theta = \frac{-\sqrt{2}}{2}$$

Looking at the top half of the unit circle, we can see that

$$\theta = \frac{3\pi}{4} = 135^\circ$$

Since the answer is greater than 90°, we've found the obtuse angle between the lines. To find the acute angle, we'll just subtract this value from 180°.

$$\theta = 180^\circ - 135^\circ$$

$$\theta = 45^\circ$$

The acute angle between the lines is 45°.