Topic: Directional derivatives in the direction of the vector

Question: Find the directional derivative.

$$D_u f(-1,1)$$

$$f(x, y) = 4xy + 7y^2$$

$\vec{u}$ is the unit vector toward $\vec{v} = \langle 1,1 \rangle$

Answer choices:

A $D_u f(-1,1) = 3\sqrt{2}$

B $D_u f(-1,1) = 7\sqrt{2}$

C $D_u f(-1,1) = -3\sqrt{2}$

D $D_u f(-1,1) = -7\sqrt{2}$

Solution: B

We need to convert the vector $\vec{v} = \langle c, d \rangle$ into a unit vector using the formula

$$\vec{u} = \left\langle \frac{c}{\sqrt{c^2 + d^2}}, \frac{d}{\sqrt{c^2 + d^2}} \right\rangle$$

Since in this problem $\vec{v} = \langle 1,1 \rangle$, we get

$$\vec{u} = \left\langle \frac{1}{\sqrt{1^2 + 1^2}}, \frac{1}{\sqrt{1^2 + 1^2}} \right\rangle$$

$$\vec{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$\vec{u} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

To get the directional derivative, we'll use

$$D_u f(x, y) = a \left(\frac{\partial F}{\partial x} \right) + b \left(\frac{\partial F}{\partial y} \right)$$

where a and b come from the unit vector $\vec{u} = \langle a, b \rangle$ we found earlier. All we need now are the first order partial derivatives.

$$\frac{\partial F}{\partial x} = 4y$$

and

$$\frac{\partial F}{\partial y} = 4x + 14y$$

Since we were asked to find $D_u f(-1,1)$, we need to evaluate the partial derivatives at $(-1,1)$.

$$\frac{\partial F}{\partial x}(-1,1) = 4(1)$$

$$\frac{\partial F}{\partial x}(-1,1) = 4$$

and

$$\frac{\partial F}{\partial y}(-1,1) = 4(-1) + 14(1)$$

$$\frac{\partial F}{\partial y}(-1,1) = 10$$

Plugging everything into the formula for the directional derivative, we get

$$D_u f(-1,1) = \frac{\sqrt{2}}{2}(4) + \frac{\sqrt{2}}{2}(10)$$

$$D_u f(-1,1) = 2\sqrt{2} + 5\sqrt{2}$$

$$D_u f(-1,1) = 7\sqrt{2}$$

Topic: Directional derivatives in the direction of the vector

Question: Find the directional derivative.

$$D_uf(2,0)$$

$$f(x,y)=-3xe^{2y}+2x^3$$

$\vec{u}$ is the unit vector toward $\vec{v}=\langle -1,-1\rangle$

Answer choices:

A $\quad D_uf(2,0)=-\frac{33\sqrt{2}}{2}$

B $\quad D_uf(2,0)=\frac{9\sqrt{2}}{2}$

C $\quad D_uf(2,0)=-\frac{9\sqrt{2}}{2}$

D $\quad D_uf(2,0)=\frac{33\sqrt{2}}{2}$

Solution: C

We need to convert the vector $\vec{v}=\langle c,d\rangle$ into a unit vector using the formula

$$\vec{u}=\left\langle \frac{c}{\sqrt{c^2+d^2}},\frac{d}{\sqrt{c^2+d^2}}\right\rangle$$

Since in this problem $\vec{v}=\langle -1,-1\rangle$, we get

$$\vec{u}=\left\langle \frac{-1}{\sqrt{(-1)^2+(-1)^2}},\frac{-1}{\sqrt{(-1)^2+(-1)^2}}\right\rangle$$

$$\vec{u}=\left\langle -\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right\rangle$$

$$\vec{u}=\left\langle -\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right\rangle$$

To get the directional derivative, we'll use

$$D_uf(x,y)=a\left(\frac{\partial F}{\partial x}\right)+b\left(\frac{\partial F}{\partial y}\right)$$

where a and b come from the unit vector $\vec{u}=\langle a,b\rangle$ we found earlier. All we need now are the first order partial derivatives.

$$\frac{\partial F}{\partial x}=-3e^{2y}+6x^2$$

and

$$\frac{\partial F}{\partial y} = -6xe^{2y}$$

Since we were asked to find $D_u f(2,0)$, we need to evaluate the partial derivatives at $(2,0)$.

$$\frac{\partial F}{\partial x}(2,0) = -3e^{2(0)} + 6(2)^2$$

$$\frac{\partial F}{\partial x}(2,0) = 21$$

and

$$\frac{\partial F}{\partial y}(2,0) = -6(2)e^{2(0)}$$

$$\frac{\partial F}{\partial y}(2,0) = -12$$

Plugging everything into the formula for the directional derivative, we get

$$D_u f(2,0) = -\frac{\sqrt{2}}{2}(21) - \frac{\sqrt{2}}{2}(-12)$$

$$D_u f(2,0) = -\frac{21\sqrt{2}}{2} + \frac{12\sqrt{2}}{2}$$

$$D_u f(2,0) = -\frac{9\sqrt{2}}{2}$$

Topic: Directional derivatives in the direction of the vector

Question: Find the directional derivative.

$$D_u f(1,0,-1)$$

$$f(x,y,z) = 4x^2e^z - 7\cos y + 16y^2z^3$$

$\vec{u}$ is the unit vector toward $\vec{v} = \langle 1,1,-1 \rangle$

Answer choices:

A $\quad D_u f(1,0,-1) = \dfrac{4\sqrt{3}}{3e}$

B $\quad D_u f(1,0,-1) = -\dfrac{4e\sqrt{3}}{3}$

C $\quad D_u f(1,0,-1) = \dfrac{4e\sqrt{3}}{3}$

D $\quad D_u f(1,0,-1) = -\dfrac{4\sqrt{3}}{3e}$

Solution: A

We need to convert the vector $\overrightarrow{v} = \langle d, e, f \rangle$ into a unit vector using the formula

$$\overrightarrow{u} = \left\langle \frac{d}{\sqrt{d^2 + e^2 + f^2}}, \frac{e}{\sqrt{d^2 + e^2 + f^2}}, \frac{f}{\sqrt{d^2 + e^2 + f^2}} \right\rangle$$

Since in this problem $\overrightarrow{v} = \langle 1,1, -1 \rangle$, we get

$$\overrightarrow{u} = \left\langle \frac{1}{\sqrt{1^2 + 1^2 + (-1)^2}}, \frac{1}{\sqrt{1^2 + 1^2 + (-1)^2}}, \frac{-1}{\sqrt{1^2 + 1^2 + (-1)^2}} \right\rangle$$

$$\overrightarrow{u} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right\rangle$$

$$\overrightarrow{u} = \left\langle \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3} \right\rangle$$

To get the directional derivative, we'll use

$$D_u f(x, y, z) = a\left(\frac{\partial F}{\partial x}\right) + b\left(\frac{\partial F}{\partial y}\right) + c\left(\frac{\partial F}{\partial z}\right)$$

where a, b and c come from the unit vector $\overrightarrow{u} = \langle a, b, c \rangle$ we found earlier. All we need now are the first order partial derivatives.

$$\frac{\partial F}{\partial x} = 8xe^z$$

$$\frac{\partial F}{\partial y} = 7\sin y + 32yz^3$$

$$\frac{\partial F}{\partial z} = 4x^2e^z + 48y^2z^2$$

Since we were asked to find $D_uf(1,0,-1)$, we need to evaluate the partial derivatives at $(1,0,-1)$.

$$\frac{\partial F}{\partial x}(1,0,-1) = 8(1)e^{-1} = \frac{8}{e}$$

$$\frac{\partial F}{\partial y}(1,0,-1) = 7\sin 0 + 32(0)(-1)^3 = 0$$

$$\frac{\partial F}{\partial z}(1,0,-1) = 4(1)^2e^{-1} + 48(0)^2(-1)^2 = \frac{4}{e}$$

Plugging everything into the formula for the directional derivative, we get

$$D_uf(1,0,-1) = \frac{\sqrt{3}}{3}\left(\frac{8}{e}\right) + \frac{\sqrt{3}}{3}(0) - \frac{\sqrt{3}}{3}\left(\frac{4}{e}\right)$$

$$D_uf(1,0,-1) = \frac{8\sqrt{3}}{3e} - \frac{4\sqrt{3}}{3e}$$

$$D_uf(1,0,-1) = \frac{4\sqrt{3}}{3e}$$