Calculus 3 Workbook

Double integrals in polar coordinates

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CHANGING ITERATED INTEGRALS TO POLAR COORDINATES

■ 1. Convert the iterated integral to polar coordinates, then find its value.

$$\int_{-5}^{0}\int_{0}^{\sqrt{25-x^2}} xy\ dy\ dx$$

■ 2. Convert the sum of iterated integrals to polar coordinates, then find its value.

$$\int_{-4}^{-2}\int_{0}^{\sqrt{16-x^2}} x-y\ dy\ dx+\int_{-2}^{2}\int_{\sqrt{4-x^2}}^{\sqrt{16-x^2}} x-y\ dy\ dx+\int_{2}^{4}\int_{0}^{\sqrt{16-x^2}} x-y\ dy\ dx$$

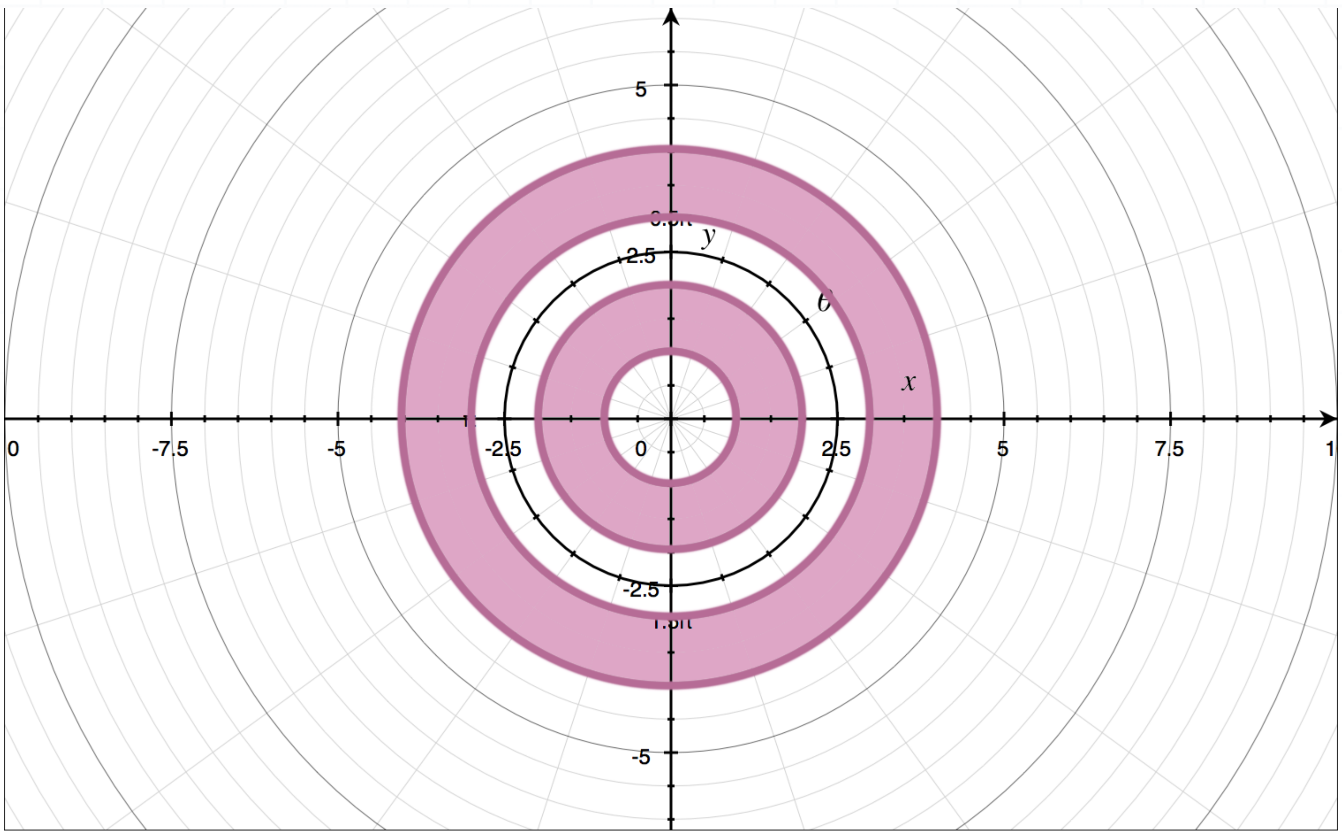
■ 3. Convert the iterated integral to polar coordinates, then find its value.

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-y^2}} \ln(x^2+y^2)\ dx\ dy$$

CHANGING DOUBLE INTEGRALS TO POLAR COORDINATES

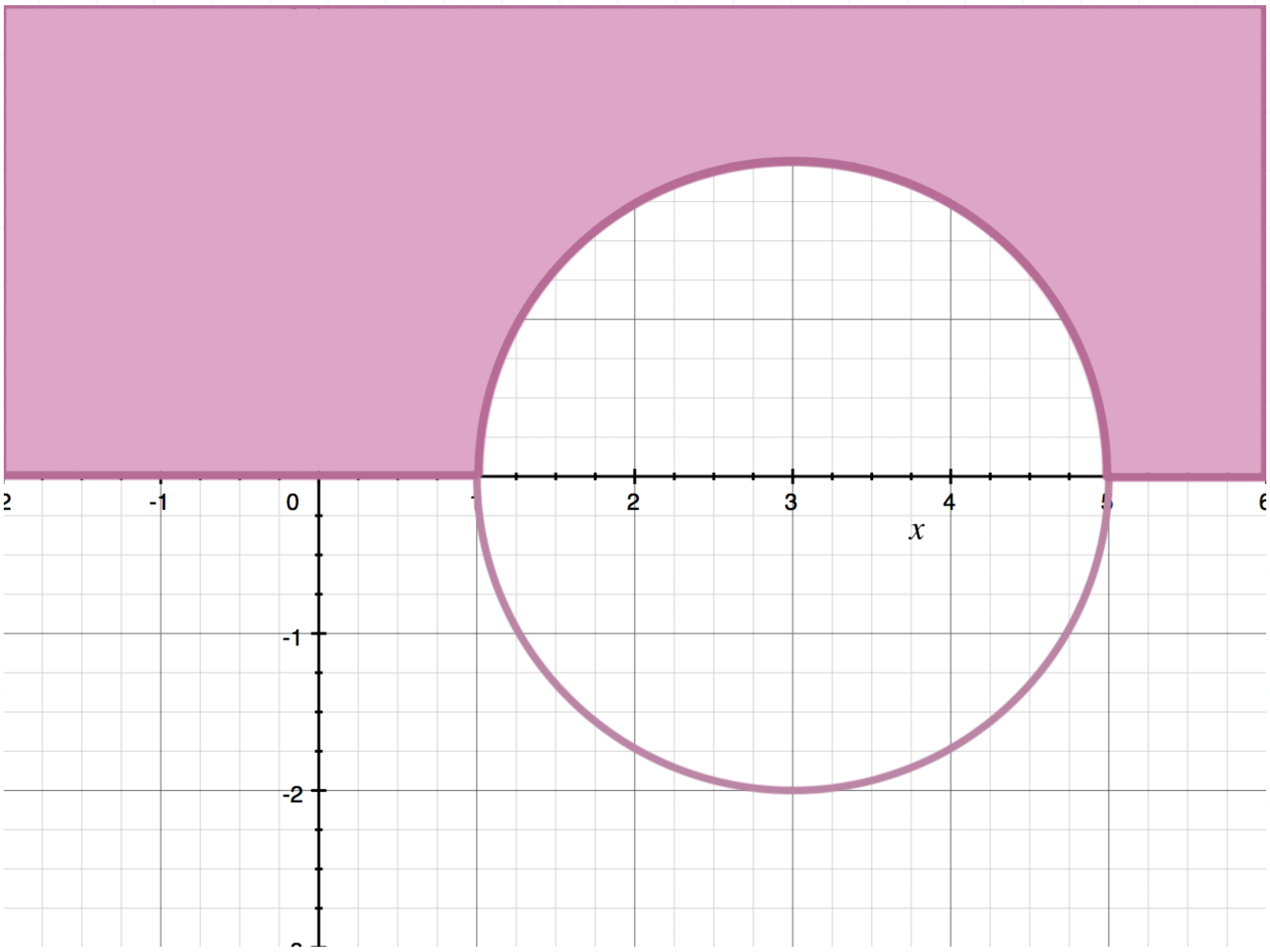
■ 1. The region D consists of two rings centered at the origin, where the inner ring is defined on $r=[1,2]$ and the outer ring is defined on $r=[3,4]$. Convert the double integral to polar coordinates, then find its value.

$$\iint_D x+2y\ dA$$



■ 2. The region D consists of all the points in the first and second quadrants outside the circle centered at $(3,0)$ with radius $r=2$. Convert the double integral to polar coordinates, using the conversion formulas $x=x_0+r\cos\theta$ and $y=y_0+r\sin\theta$ for a circle shifted off the origin, then find its value.

$$\iint_D \frac{1}{((x-3)^2+y^2)^2}\ dA$$



■ 3. The region D consists of all the points inside the circle centered at $(-2,2)$ with radius $r=1$. Convert the double integral to polar coordinates, using the conversion formulas $x=x_0+r\cos\theta$ and $y=y_0+r\sin\theta$ for a circle shifted off the origin, then find its value.

$$\iint_D x^2+y^2\ dA$$

SKETCHING AREA

■ 1. Identify the region of integration given by the double integral.

$$\int_0^{2\pi}\int_0^{\frac{3}{\sqrt{1+1.25\cos^2\theta}}} f(r,\theta)\ dr\ d\theta$$

■ 2. Identify the region of integration given by the double integral.

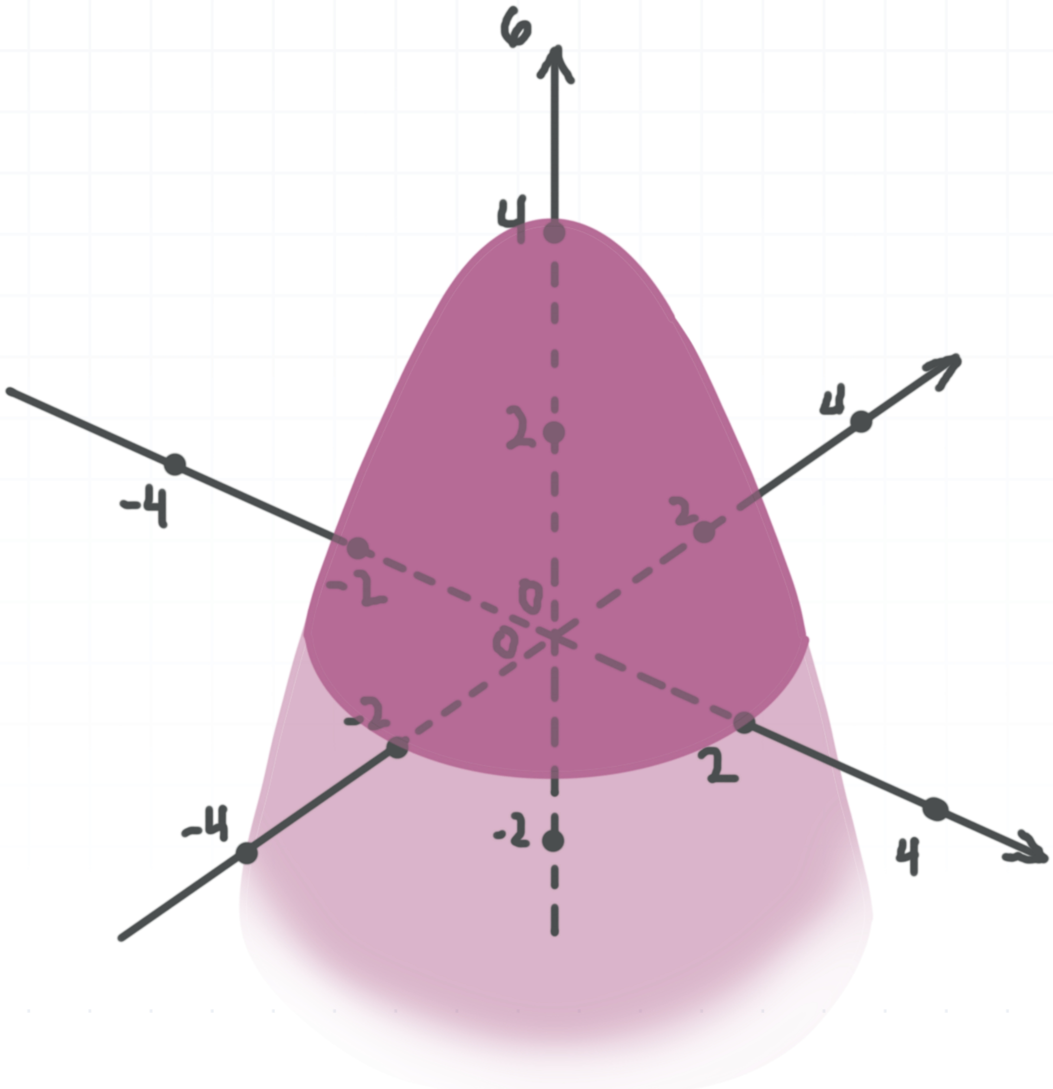
$$\int_0^{5}\int_0^{\frac{1}{3}\cos^{-1}\left(\frac{r}{5}\right)} f(r,\theta)\ d\theta\ dr$$

■ 3. Identify the region of integration given by the double integral.

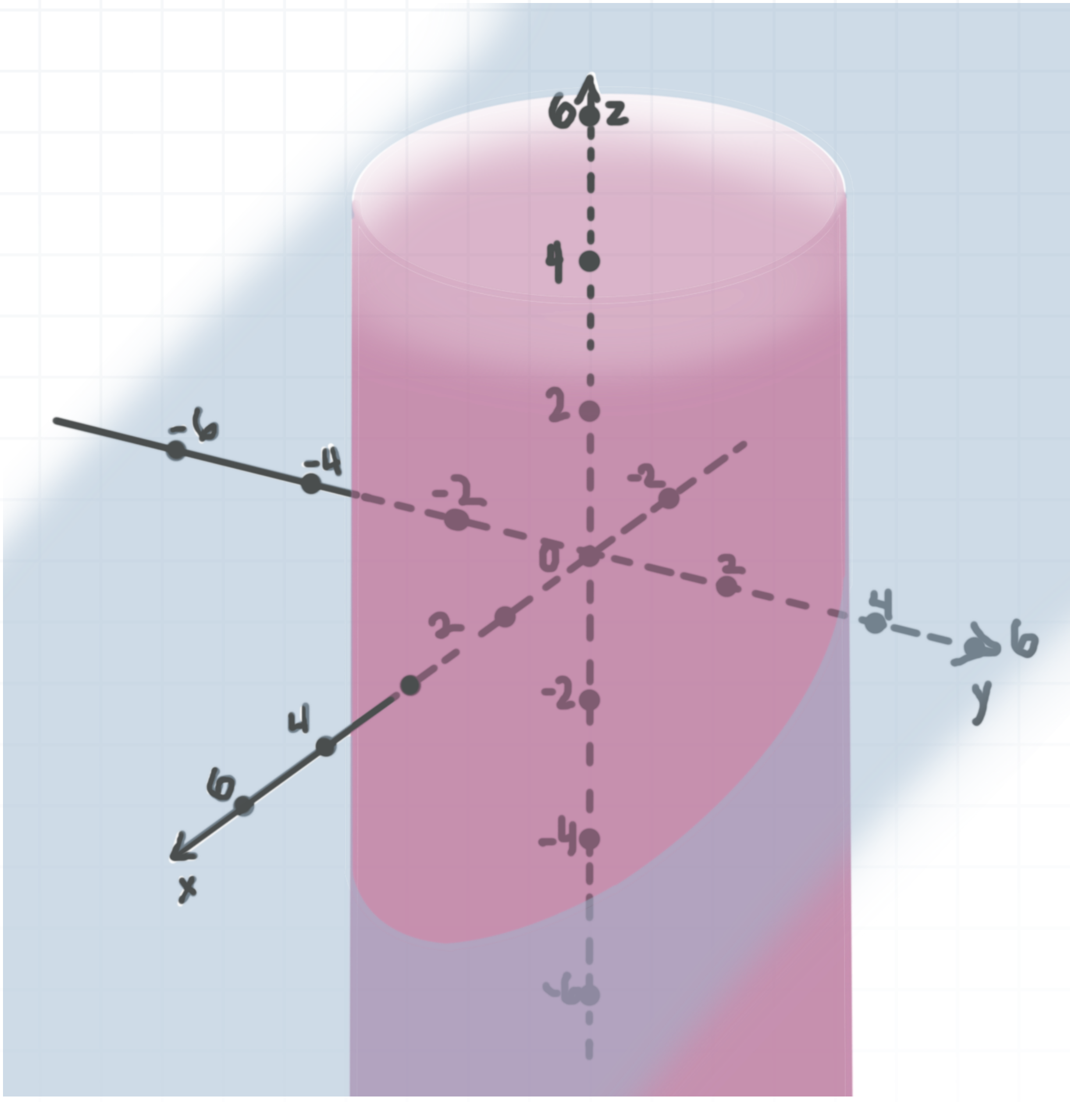
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{2\cos\theta}^{4\cos\theta} f(r,\theta)\ dr\ d\theta$$

FINDING AREA

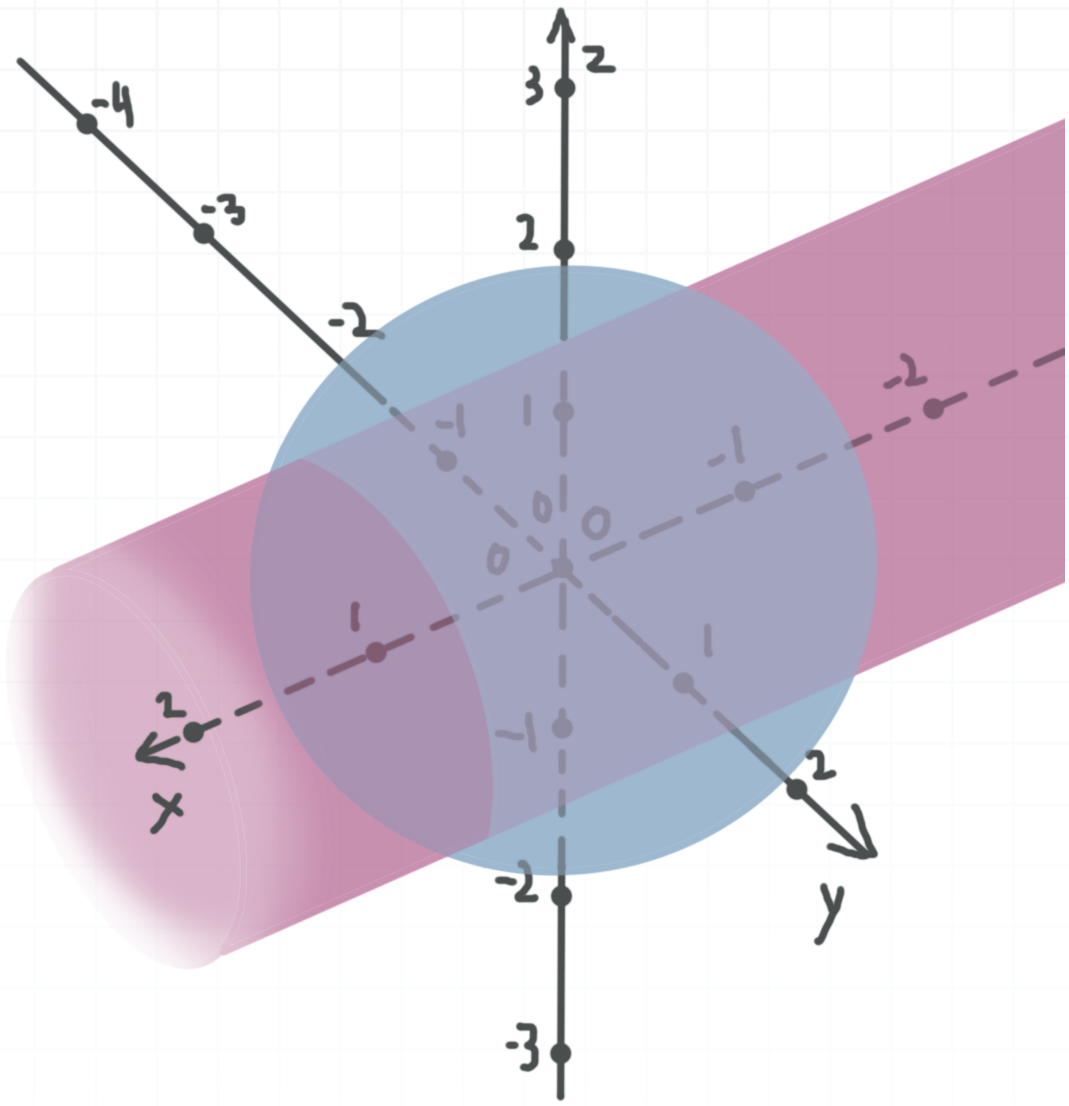
■ 1. Find area of the surface $x^2 + y^2 + z - 4 = 0$ above the xy-plane.



■ 2. Find area of the part of the plane $2x - y + 3z - 3 = 0$ that lies within the cylinder $(x - 3)^2 + (y - 2)^2 = 3^2$.

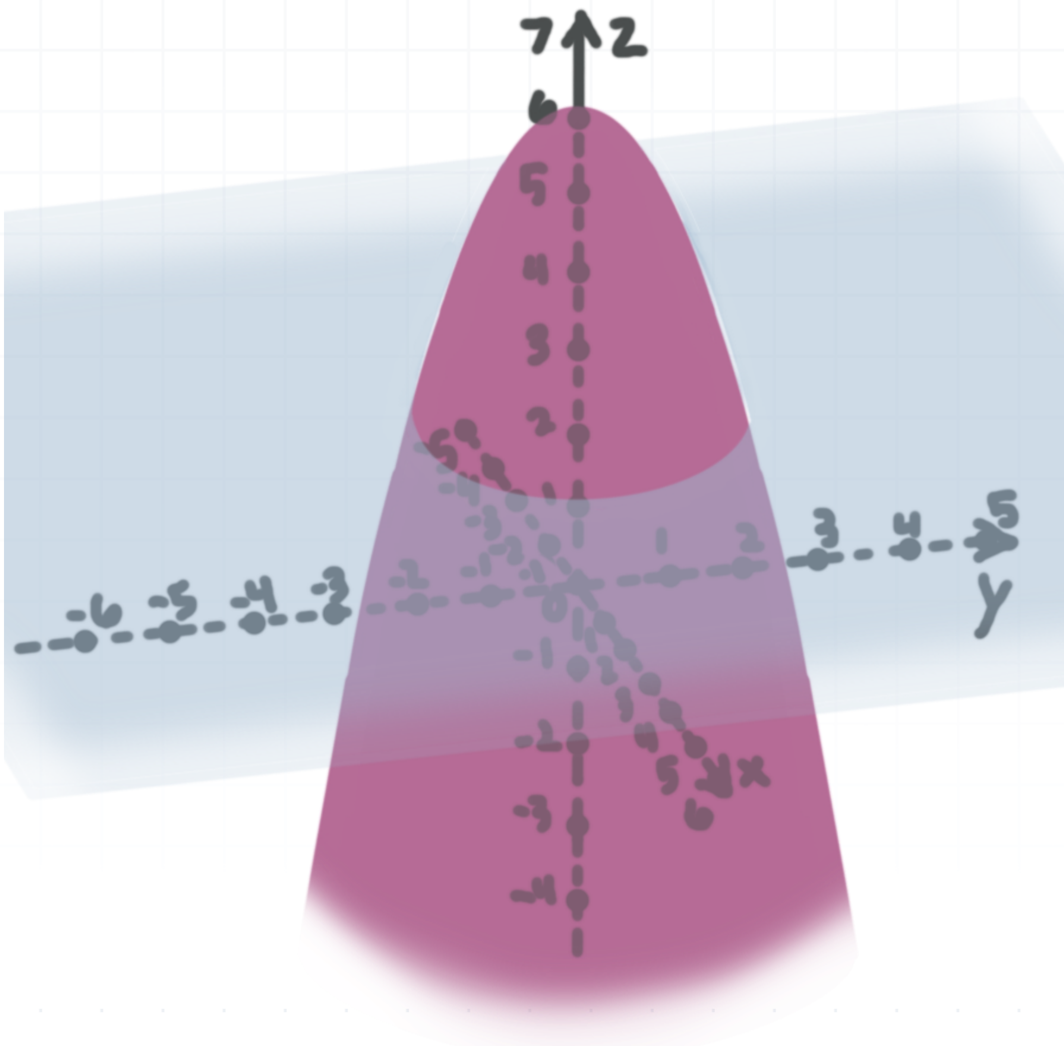


3. Find area of the sphere $x^2 + y^2 + z^2 - 2 = 0$ that lies within the cylinder $y^2 + z^2 = 1$.

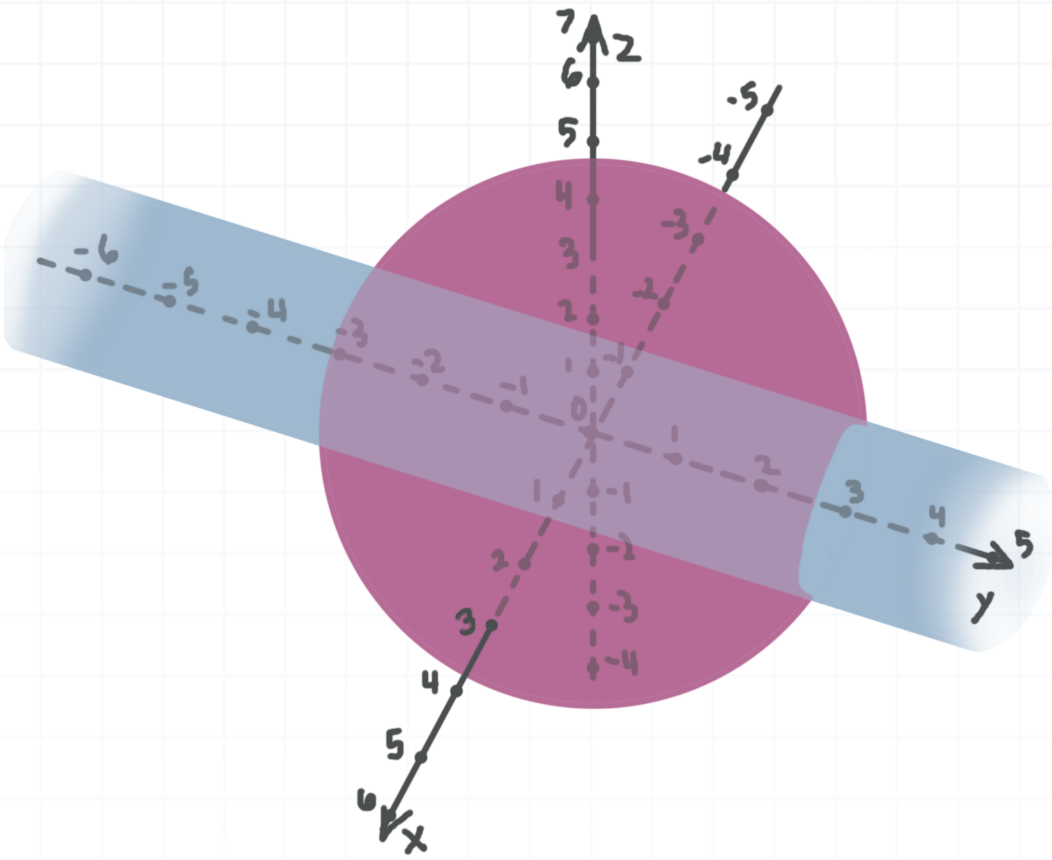

z
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
y
-2
-1
0
1
2
x

FINDING VOLUME

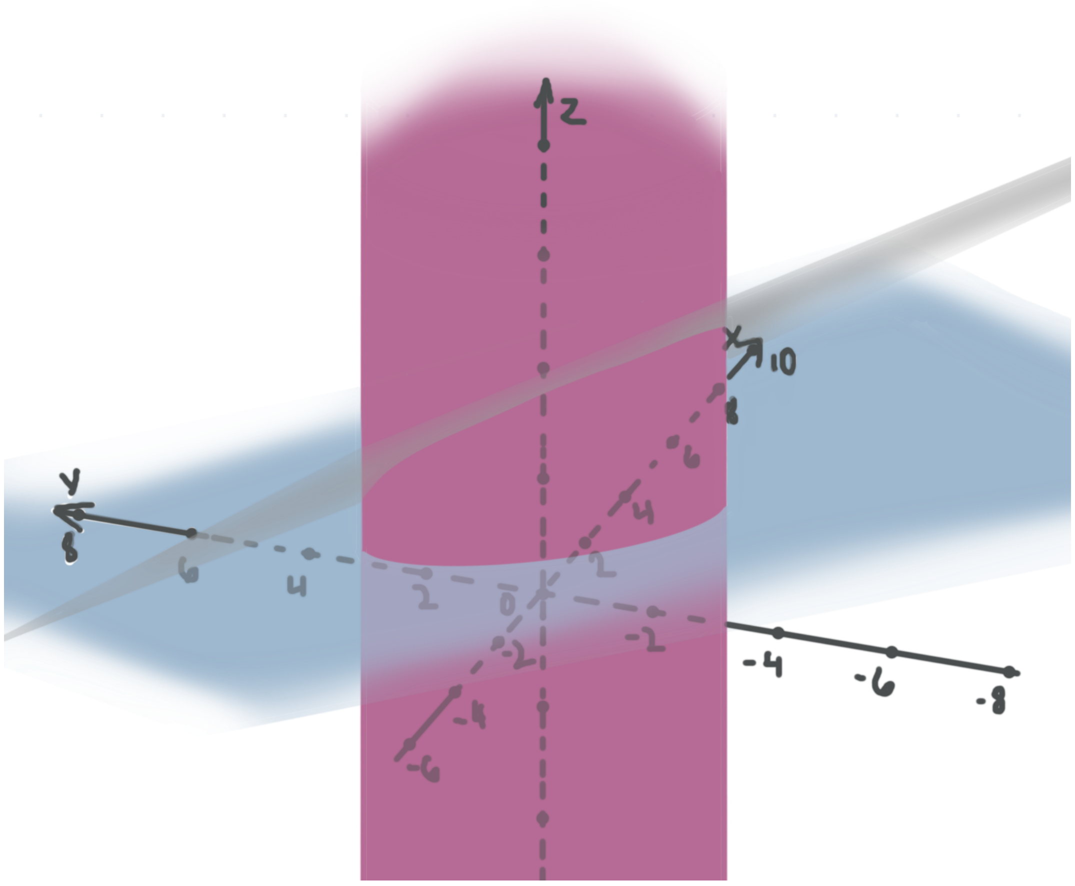
■ 1. Find the volume of the region bounded $x^2 + y^2 + z - 6 = 0$ and $z = 2$.



■ 2. Find volume of the sphere $x^2 + y^2 + z^2 - 9 = 0$ that lies within the cylinder $x^2 + z^2 = 1$.



3. Find the volume bounded by the cylinder $x^2 + y^2 = 9$, the plane $x + y + 5z - 6 = 0$, and the plane $x + 2y + 3z - 10 = 0$.



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