Topic: Vector and parametric equations of a line segment

Question: Find the vector equation of the line segment.

$P(1,0,1)$

$Q(1,1,1)$

Answer choices:

A $r(t) = \langle 1,t,1 \rangle$ where $0 \leq t \leq 1$

B $r(t) = \langle 0,t,0 \rangle$ where $0 \leq t \leq 1$

C $r(t) = \langle t,1,t \rangle$ where $0 \leq t \leq 1$

D $r(t) = \langle t,0,t \rangle$ where $0 \leq t \leq 1$

Solution: A

First we'll change the given points $P(1,0,1)$ and $Q(1,1,1)$ to their vector equivalents. Using the origin $(0,0,0)$ as the initial point, and the given point as the terminal point of the vector, then they become

$$r_0\langle 1,0,1\rangle$$

$$r_1\langle 1,1,1\rangle$$

Now we can use the vector equation of a line segment

$$r(t) = (1-t)r_0 + tr_1$$

where $0 \leq t \leq 1$

In this equation, r_0 is the vector from the first point $r_0\langle 1,0,1\rangle$ and r_1 is the vector from the second point $r_1\langle 1,1,1\rangle$.

$$r(t) = (1-t)\langle 1,0,1\rangle + t\langle 1,1,1\rangle$$

$$r(t) = \langle 1-t,0,1-t\rangle + \langle t,t,t\rangle$$

$$r(t) = \langle 1,t,1\rangle$$

This is the vector equation of the line segment.

Topic: Vector and parametric equations of a line segment

Question: Find the parametric equations of the line segment.

$P(3,2,4)$

$Q(1,0,-3)$

Answer choices:

A $x_{r(t)} = 3 - 4t$ $y_{r(t)} = 2 - 2t$ $z_{r(t)} = 4 - t$

B $x_{r(t)} = 3 - 2t$ $y_{r(t)} = 2 - 2t$ $z_{r(t)} = 4 - 7t$

C $x_{r(t)} = 3 + 4t$ $y_{r(t)} = 2 + 2t$ $z_{r(t)} = 4 + t$

D $x_{r(t)} = 3 + 2t$ $y_{r(t)} = 2 + 2t$ $z_{r(t)} = 4 + 7t$

Solution: B

First we'll change the given points $P(3,2,4)$ and $Q(1,0,-3)$ to their vector equivalents. Using the origin $(0,0,0)$ as the initial point, and the given point as the terminal point of the vector, then they become

$$r_0\langle 3,2,4\rangle$$

$$r_1\langle 1,0,-3\rangle$$

Now we can use the vector equation of a line segment

$$r(t) = (1-t)r_0 + tr_1$$

where $0 \leq t \leq 1$

In this equation, r_0 is the vector from the first point $r_0\langle 3,2,4\rangle$ and r_1 is the vector from the second point $r_1\langle 1,0,-3\rangle$.

$$r(t) = (1-t)\langle 3,2,4\rangle + t\langle 1,0,-3\rangle$$

$$r(t) = \langle 3-3t,2-2t,4-4t\rangle + \langle t,0,-3t\rangle$$

$$r(t) = \langle 3-2t,2-2t,4-7t\rangle$$

This is the vector equation of the line segment. Next we can find the parametric equations of the line segment $r(t) = \langle 3-2t,2-2t,4-7t\rangle$ remembering that

$$x = r_{t_1}$$

$$y = r_{t_2}$$

$$z = r_{t_3}$$

This will give us

$$x = 3 - 2t$$

$$y = 2 - 2t$$

$$z = 4 - 7t$$

These are the parametric equations of the line segment.

Topic: Vector and parametric equations of a line segment

Question: Find the parametric equations of the line segment.

$P(-2,-4,0)$

$Q(2,3,5)$

Answer choices:

A $x_{r(t)} = 2$ $y_{r(t)} = 4 - t$ $z_{r(t)} = 5t$

B $x_{r(t)} = -2$ $y_{r(t)} = -4 + t$ $z_{r(t)} = -5t$

C $x_{r(t)} = 2 - 4t$ $y_{r(t)} = 4 - 7t$ $z_{r(t)} = -5t$

D $x_{r(t)} = -2 + 4t$ $y_{r(t)} = -4 + 7t$ $z_{r(t)} = 5t$

Solution: D

First we'll change the given points $P(-2,-4,0)$ and $Q(2,3,5)$ to their vector equivalents. Using the origin $(0,0,0)$ as the initial point, and the given point as the terminal point of the vector, then they become

$$r_0\langle -2,-4,0\rangle$$

$$r_1\langle 2,3,5\rangle$$

Now we can use the vector equation of a line segment

$$r(t) = (1-t)r_0 + tr_1$$

where $0 \leq t \leq 1$

In this equation, r_0 is the vector from the first point $r_0\langle -2,-4,0\rangle$ and r_1 is the vector from the second point $r_1\langle 2,3,5\rangle$.

$$r(t) = (1-t)\langle -2,-4,0\rangle + t\langle 2,3,5\rangle$$

$$r(t) = \langle -2+2t,-4+4t,0\rangle + \langle 2t,3t,5t\rangle$$

$$r(t) = \langle -2+4t,-4+7t,5t\rangle$$

This is the vector equation of the line segment. Next we can find the parametric equations of the line segment $r(t) = \langle -2+4t,-4+7t,5t\rangle$ remembering that

$$x = r_{t_1}$$

$$y = r_{t_2}$$

$$z = r_{t_3}$$

This will give us

$$x = -2 + 4t$$

$$y = -4 + 7t$$

$$z = 5t$$

These are the parametric equations of the line segment.