Topic: Differential of a multivariable function

Question: Find the differential of the multivariable function.

$$z = 4x^3 + 2\ln y$$

Answer choices:

A $\quad dz = 12x^2\ dx - \frac{2}{y}\ dy$

B $\quad dz = 6x^2\ dx + \frac{1}{y}\ dy$

C $\quad dz = 12x^2\ dx + \frac{2}{y}\ dy$

D $\quad dz = 6x^2\ dx - \frac{1}{y}\ dy$

Solution: C

The differential of a multivariable function is given by

$$dz = \frac{\partial z}{\partial x}\ dx + \frac{\partial z}{\partial y}\ dy$$

so we'll need to find the partial derivatives of z with respect to x and y. If $z = 4x^3 + 2\ln y$, then

$$\frac{\partial z}{\partial x} = 12x^2$$

and

$$\frac{\partial z}{\partial y} = 2\left(\frac{1}{y}\right)$$

$$\frac{\partial z}{\partial y} = \frac{2}{y}$$

Plugging these values into the formula for the differential, we get

$$dz = \frac{\partial z}{\partial x}\ dx + \frac{\partial z}{\partial y}\ dy$$

$$dz = \left(12x^2\right)\ dx + \left(\frac{2}{y}\right)\ dy$$

$$dz = 12x^2\ dx + \frac{2}{y}\ dy$$

This is the differential of the multivariable function.

Topic: Differential of a multivariable function

Question: Find the differential of the multivariable function.

$$z = x\sin(2y) - 13y^2$$

Answer choices:

A $dz = \sin(2y)\ dx + 2x\cos(2y)\ dy - 26y\ dy$

B $dz = \sin y\ dx + 2x\cos y\ dy - 26y\ dy$

C $dz = \sin(2y)\ dx - 2x\cos(2y)\ dy - 26y\ dy$

D $dz = \sin y\ dx - 2x\cos y\ dy - 26y\ dy$

Solution: A

The differential of a multivariable function is given by

$$dz = \frac{\partial z}{\partial x}\ dx + \frac{\partial z}{\partial y}\ dy$$

so we'll need to find the partial derivatives of z with respect to x and y. If $z = x\sin(2y) - 13y^2$, then

$$\frac{\partial z}{\partial x} = \sin(2y)$$

and

$$\frac{\partial z}{\partial y} = x(2)\cos(2y) - 26y$$

$$\frac{\partial z}{\partial y} = 2x\cos(2y) - 26y$$

Plugging these values into the formula for the differential, we get

$$dz = \left[\sin(2y)\right]\ dx + \left[2x\cos(2y) - 26y\right]\ dy$$

$$dz = \sin(2y)\ dx + 2x\cos(2y)\ dy - 26y\ dy$$

This is the differential of the multivariable function.

Topic: Differential of a multivariable function

Question: Find the differential of the multivariable function.

$$z = 6x^2 \ln(3y) + y^2 \sec(4x)$$

Answer choices:

A $\quad dz = 6x \ln(3y)\ dx + 2y^2 \sec(4x)\tan(4x)\ dx + \frac{3x^2}{y}\ dy + y \sec(4x)\ dy$

B $\quad dz = 12x \ln(3y)\ dx + 4y^2 \sec x \tan x\ dx + \frac{6x^2}{y}\ dy + 2y \sec(4x)\ dy$

C $\quad dz = 6x \ln(3y)\ dx + 2y^2 \sec(4x)\csc(4x)\ dx + \frac{3x^2}{y}\ dy + y \sec(4x)\ dy$

D $\quad dz = 12x \ln(3y)\ dx + 4y^2 \sec(4x)\tan(4x)\ dx + \frac{6x^2}{y}\ dy + 2y \sec(4x)\ dy$

Solution: D

The differential of a multivariable function is given by

$$dz = \frac{\partial z}{\partial x}\ dx + \frac{\partial z}{\partial y}\ dy$$

so we'll need to find the partial derivatives of z with respect to x and y. If $z = 6x^2\ln(3y) + y^2\sec(4x)$, then

$$\frac{\partial z}{\partial x} = 6(2)x\ln(3y) + (4)\sec(4x)\tan(4x)y^2$$

$$\frac{\partial z}{\partial x} = 12x\ln(3y) + 4y^2\sec(4x)\tan(4x)$$

and

$$\frac{\partial z}{\partial y} = 6x^2(3)\left(\frac{1}{3y}\right) + 2y\sec(4x)$$

$$\frac{\partial z}{\partial y} = \frac{6x^2}{y} + 2y\sec(4x)$$

Plugging these values into the formula for the differential, we get

$$dz = \left[12x\ln(3y) + 4y^2\sec(4x)\tan(4x)\right]\ dx + \left[\frac{6x^2}{y} + 2y\sec(4x)\right]\ dy$$

$$dz = 12x\ln(3y)\ dx + 4y^2\sec(4x)\tan(4x)\ dx + \frac{6x^2}{y}\ dy + 2y\sec(4x)\ dy$$

This is the differential of the multivariable function.