Topic: Intersection of a line and a plane

Question: Find the point where the line intersects the plane.

Line $x = -t$ $y = 1 + 2t$ $z = 4$

Plane $x + y + z = 4$

Answer choices:

A $(1, -1, 4)$

B $(1, 1, 4)$

C $(-1, -1, -4)$

D $(-1, 1, -4)$

Solution: A

To find the coordinate point at which the line intersects the plane, we'll take the parametric equations of the line, and plug them into the equation of the plane. With $x = -t$, $y = 1 + 2t$ and $z = 4$, we get

$$x + y + z = 4$$

$$(-t) + (1 + 2t) + (4) = 4$$

$$-t + 1 + 2t + 4 = 4$$

$$t + 5 = 4$$

$$t = -1$$

Now we'll plug $t = -1$ back into the parametric equations.

$$x = -t$$

$$x = -(-1)$$

$$x = 1$$

and

$$y = 1 + 2t$$

$$y = 1 + 2(-1)$$

$$y = -1$$

and

$$z = 4$$

Putting these values together tells us that the intersection point is $(1, -1, 4)$. Let's make sure that we've got the right point by making sure it still satisfies the equation of the plane.

$$x + y + z = 4$$

$$(1) + (-1) + (4) = 4$$

$$1 - 1 + 4 = 4$$

$$4 = 4$$

The equation is true, so $(1, -1, 4)$ is the point where the line intersects the plane.

Topic: Intersection of a line and a plane

Question: Find the point where the line intersects the plane.

Line $x = 2 + 2t$ $y = 4 - 5t$ $z = -3 + 2t$

Plane $4x + y - 3z = 6$

Answer choices:

A $(12,21,7)$

B $(-12,21,-7)$

C $(-12,-21,-7)$

D $(12,-21,7)$

Solution: D

To find the coordinate point at which the line intersects the plane, we'll take the parametric equations of the line, and plug them into the equation of the plane. With $x = 2 + 2t$, $y = 4 - 5t$ and $z = -3 + 2t$, we get

$$4x + y - 3z = 6$$

$$4(2 + 2t) + (4 - 5t) - 3(-3 + 2t) = 6$$

$$8 + 8t + 4 - 5t + 9 - 6t = 6$$

$$-3t + 21 = 6$$

$$-3t = -15$$

$$t = 5$$

Now we'll plug $t = 5$ back into the parametric equations.

$$x = 2 + 2t$$

$$x = 2 + 2(5)$$

$$x = 12$$

and

$$y = 4 - 5t$$

$$y = 4 - 5(5)$$

$$y = -21$$

and

$$z = -3 + 2t$$

$$z = -3 + 2(5)$$

$$z = 7$$

Putting these values together tells us that the intersection point is $(12, -21, 7)$. Let's make sure that we've got the right point by making sure it still satisfies the equation of the plane.

$$4x + y - 3z = 6$$

$$4(12) + (-21) - 3(7) = 6$$

$$48 - 21 - 21 = 6$$

$$6 = 6$$

The equation is true, so $(12, -21, 7)$ is the point where the line intersects the plane.

Topic: Intersection of a line and a plane

Question: Find the point where the line intersects the plane.

Line $x = 11 - 5t$ $y = -7 + 9t$ $z = -7 - 6t$

Plane $-5x - 7y + 5z = 12$

Answer choices:

A $\left(-\frac{1{,}013}{68}, \frac{953}{68}, \frac{158}{68}\right)$

B $\left(-\frac{1{,}013}{68}, -\frac{953}{68}, -\frac{158}{68}\right)$

C $\left(\frac{1{,}013}{68}, -\frac{953}{68}, -\frac{158}{68}\right)$

D $\left(\frac{1{,}013}{68}, \frac{953}{68}, \frac{158}{68}\right)$

Solution: C

To find the coordinate point at which the line intersects the plane, we'll take the parametric equations of the line, and plug them into the equation of the plane. With $x = 11 - 5t$, $y = -7 + 9t$ and $z = -7 - 6t$, we get

$$-5x - 7y + 5z = 12$$

$$-5(11 - 5t) - 7(-7 + 9t) + 5(-7 - 6t) = 12$$

$$-55 + 25t + 49 - 63t - 35 - 30t = 12$$

$$-68t - 41 = 12$$

$$-68t = 53$$

$$t = -\frac{53}{68}$$

Now we'll plug the value we just found for t back into the parametric equations.

$$x = 11 - 5t$$

$$x = 11 - 5\left(-\frac{53}{68}\right)$$

$$x = \frac{748}{68} + \frac{265}{68}$$

$$x = \frac{1{,}013}{68}$$

and

$$y = -7 + 9t$$

$$y = -7 + 9\left(-\frac{53}{68}\right)$$

$$y = -\frac{476}{68} - \frac{477}{68}$$

$$y = -\frac{953}{68}$$

and

$$z = -7 - 6t$$

$$z = -7 - 6\left(-\frac{53}{68}\right)$$

$$z = -\frac{476}{68} + \frac{318}{68}$$

$$z = -\frac{158}{68}$$

Putting these values together tells us that the intersection point is

$$\left(\frac{1{,}013}{68}, -\frac{953}{68}, -\frac{158}{68}\right)$$

Let's make sure that we've got the right point by making sure it still satisfies the equation of the plane.

$$-5x - 7y + 5z = 12$$

$$-5\left(\frac{1{,}013}{68}\right)-7\left(-\frac{953}{68}\right)+5\left(-\frac{158}{68}\right)=12$$

$$-\frac{5{,}065}{68}+\frac{6{,}671}{68}-\frac{790}{68}=12$$

$$\frac{816}{68}=12$$

$$12=12$$

The equation is true, so

$$\left(\frac{1{,}013}{68},-\frac{953}{68},-\frac{158}{68}\right)$$

is the point where the line intersects the plane.