



Calculus 3 Workbook

Sketching graphs and level curves

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MATH

SKETCHING GRAPHS OF MULTIVARIABLE FUNCTIONS

- 1. Find the range of the function.

$$f(x, y) = x^2 + 2y^2 - 3$$

- 2. Which function's domain is given by the graph, if the left and right sides of the rectangle are included in the domain, but the top and bottom sides are not?

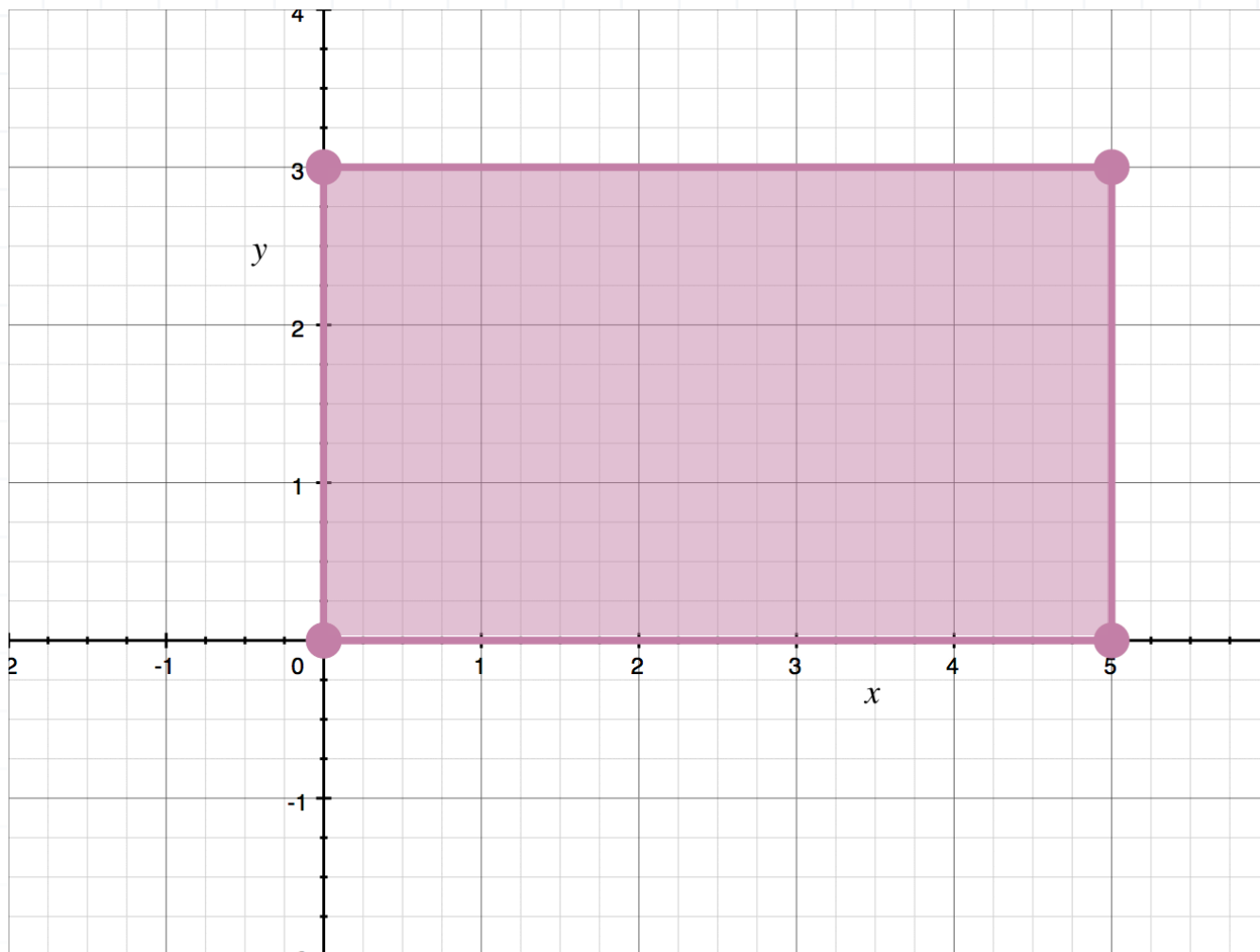
A $f(x, y) = 3y\sqrt{3x - x^2} + 4x \ln(5y - y^2)$

B $f(x, y) = 3y\sqrt{5x - x^2} + 4x \ln(3y - y^2)$

C $f(x, y) = 3x\sqrt{x^2 - 3x} + 4y \ln(y^2 - 5y)$

D $f(x, y) = 3x\sqrt{x - 5x^2} + 4y \ln(y - 3y^2)$





- 3. Find the value of the constant a for which $(2, -1, 0)$ lies on the graph of the function.

$$f(x, y) = x^2 + 2axy + y^2 - 1$$

- 4. Find the intersection point of the function and the y -axis.

$$f(x, y) = \sqrt{x^2 - 5y + 15}$$

- 5. Write the equation of the function $f(x, y)$ shifted in a positive direction along the x -axis by 2 units.



$$f(x, y) = x^2y^2 - 2xy - 4y^2 - 4y + 4x$$

- 6. Which function A , B , C , or D is a reflection of $f(x, y)$ over the xz -plane?
Hint: Use the even identity $\cos(-t) = \cos t$ to simplify.

$$f(x, y) = \cos(x^2 - y^2 + 2xy)$$

$$A(x, y) = \cos(-x^2 + y^2 + 2xy)$$

$$B(x, y) = \cos(x^2 - y^2 + 2xy)$$

$$C(x, y) = \cos(x^2 + y^2 - 2xy)$$

$$D(x, y) = \cos(-x^2 - y^2 - 2xy)$$

- 7. Find the absolute maximum of the function.

$$f(x, y) = 5 - 2x^2 - 7y^2$$



SKETCHING LEVEL CURVES OF MULTIVARIABLE FUNCTIONS

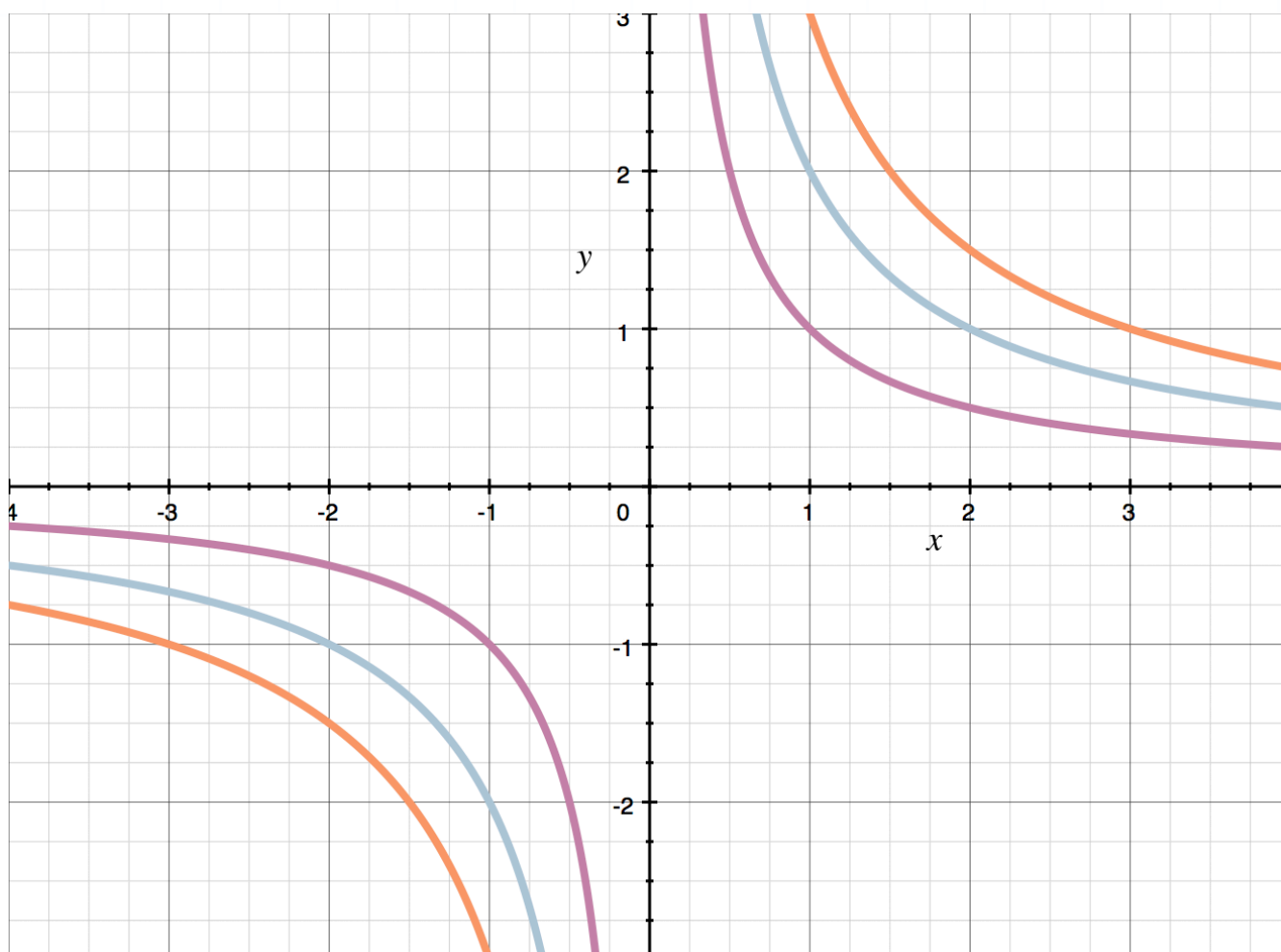
- 1. Find the level curve of $f(x, y)$ when $z = 5$.

$$f(x, y) = x^2 - 2xy + 6y - 4$$

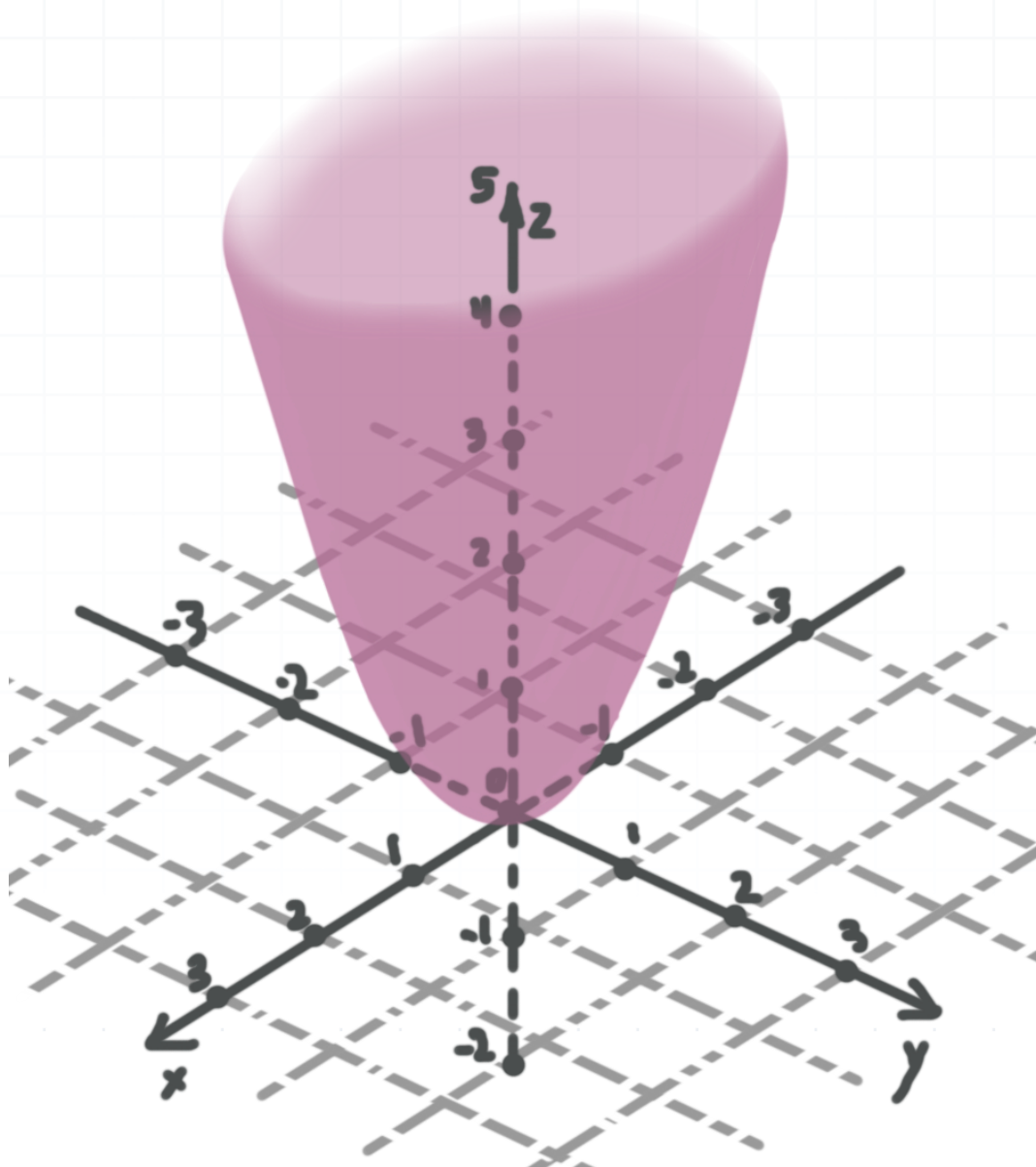
- 2. Find the level curve of $f(x, y)$ which passes through $(0, 1, z)$.

$$f(x, y) = 2x^2 - y + 2$$

- 3. The graph shows level curves of $f(x, y) = 4xy$. Find the value of z that corresponds to the light blue curve.



- 4. Think about the shape of the level curves of the graph of the elliptic paraboloid. Are they lines, ellipses, parabolas, or hyperbolas?



MATCHING THE FUNCTION WITH THE GRAPH AND LEVEL CURVES

- 1. Which statement is true for the graph of the function?

$$x^2 - 2y^2 + z^2 - 8y - 6z = 0$$

- A The graph is the hyperboloid centered at $(0, 2, -3)$.
- B The graph is the hyperboloid centered at $(0, -2, 3)$.
- C The graph is the ellipsoid centered at $(0, 2, -3)$.
- D The graph is the ellipsoid centered at $(0, -2, 3)$.

- 2. Find the equation of ellipsoid centered at $(2, 0, 2)$ that has the level curve $(x - 2)^2 + 4y^2 = 0.75$ for $z = 1.5$.

- 3. Which of the surfaces has the same level curves for any z ?

- A The plane $2x + 3y + z = 1$
- B The ellipsoid $x^2 + 2y^2 + 4z^2 - 4x - 2y = 1$
- C The cylinder $2x^2 + y^2 - 5x + 7y = 1$
- D The elliptic cone $x^2 + 3y^2 - z^2 = 0$



