Topic: Vector function for the curve of intersection of two surfaces

Question: Find the vector function for the curve of intersection of the surfaces.

Sphere: $z=\sqrt{x^2+y^2-25}$

Plane: $z=1+x$

Answer choices:

A $\quad r(t)=\left(\frac{1}{2}t^2+13\right)\mathbf{i}+t\mathbf{j}+\left(\frac{1}{2}t^2+12\right)\mathbf{k}$

B $\quad r(t)=\left(\frac{1}{2}t^2+13\right)\mathbf{i}+t\mathbf{j}+\left(\frac{1}{2}t^2-12\right)\mathbf{k}$

C $\quad r(t)=\left(\frac{1}{2}t^2-13\right)\mathbf{i}+t\mathbf{j}+\left(\frac{1}{2}t^2-12\right)\mathbf{k}$

D $\quad r(t)=\left(\frac{1}{2}t^2-13\right)\mathbf{i}+t\mathbf{j}+\left(\frac{1}{2}t^2+12\right)\mathbf{k}$

Solution: C

First, we'll solve both equations for the same variable. The equations we've been given in this problem are both already solved for z, so we can go ahead and set them equal to each other, and then solve for x.

$$1 + x = \sqrt{x^2 + y^2 - 25}$$

$$(1 + x)^2 = x^2 + y^2 - 25$$

$$x^2 + 2x + 1 = x^2 + y^2 - 25$$

$$2x + 1 = y^2 - 25$$

$$2x = y^2 - 26$$

$$x = \frac{1}{2}y^2 - 13$$

Set $y = t$ in this equation.

$$x = \frac{1}{2}t^2 - 13$$

Plug this value of x into $z = 1 + x$.

$$z = 1 + \frac{1}{2}t^2 - 13$$

$$z = \frac{1}{2}t^2 - 12$$

We set $y = t$ originally and used that to find values for x and z in terms of t, and so our vector function is

$$r(t) = \left(\frac{1}{2}t^2 - 13\right)\mathbf{i} + t\mathbf{j} + \left(\frac{1}{2}t^2 - 12\right)\mathbf{k}$$

Topic: Vector function for the curve of intersection of two surfaces

Question: Find the vector function for the curve of intersection of the surfaces.

Cone: $z=\sqrt{x^2+y^2}$

Plane: $z=y-1$

Answer choices:

A $\quad r(t)=t\mathbf{i}+\left(-\frac{1}{2}t^2+\frac{1}{2}\right)\mathbf{j}+\left(-\frac{1}{2}t^2-\frac{1}{2}\right)\mathbf{k}$

B $\quad r(t)=t\mathbf{i}+\left(-\frac{1}{2}t^2+\frac{1}{2}\right)\mathbf{j}+\left(-\frac{1}{2}t^2+\frac{1}{2}\right)\mathbf{k}$

C $\quad r(t)=t\mathbf{i}+\left(\frac{1}{2}t^2+\frac{1}{2}\right)\mathbf{j}+\left(\frac{1}{2}t^2-\frac{1}{2}\right)\mathbf{k}$

D $\quad r(t)=t\mathbf{i}+\left(\frac{1}{2}t^2+\frac{1}{2}\right)\mathbf{j}+\left(\frac{1}{2}t^2+\frac{1}{2}\right)\mathbf{k}$

Solution: A

First, we'll solve both equations for the same variable. The equations we've been given in this problem are both already solved for z, so we can go ahead and set them equal to each other, and then solve for y.

$$y - 1 = \sqrt{x^2 + y^2}$$

$$(y - 1)^2 = x^2 + y^2$$

$$y^2 - 2y + 1 = x^2 + y^2$$

$$-2y + 1 = x^2$$

$$-2y = x^2 - 1$$

$$y = -\frac{1}{2}x^2 + \frac{1}{2}$$

Set $x = t$ in this equation.

$$y = -\frac{1}{2}t^2 + \frac{1}{2}$$

Plug this value of y into $z = y - 1$.

$$z = -\frac{1}{2}t^2 + \frac{1}{2} - 1$$

$$z = -\frac{1}{2}t^2 - \frac{1}{2}$$

We set $x = t$ originally and used that to find values for y and z in terms of t, and so our vector function is

$$r(t) = t\mathbf{i} + \left(-\frac{1}{2}t^2 + \frac{1}{2}\right)\mathbf{j} + \left(-\frac{1}{2}t^2 - \frac{1}{2}\right)\mathbf{k}$$

Topic: Vector function for the curve of intersection of two surfaces

Question: Find the vector function for the curve of intersection of the surfaces.

Ellipsoid: $z=\sqrt{x^2+\frac{1}{3}y^2-4}$

Plane: $z=x+2$

Answer choices:

A $\quad r(t)=\left(\frac{1}{12}t^2+2\right)\mathbf{i}+t\mathbf{j}+\left(\frac{1}{12}t^2+4\right)\mathbf{k}$

B $\quad r(t)=\left(\frac{1}{12}t^2-2\right)\mathbf{i}+t\mathbf{j}+\frac{1}{12}t^2\mathbf{k}$

C $\quad r(t)=\left(-\frac{1}{12}t^2-2\right)\mathbf{i}+t\mathbf{j}+\frac{1}{12}t^2\mathbf{k}$

D $\quad r(t)=\left(\frac{1}{12}t^2+2\right)\mathbf{i}+t\mathbf{j}+\left(\frac{1}{12}t^2-4\right)\mathbf{k}$

Solution: B

First, we'll solve both equations for the same variable. The equations we've been given in this problem are both already solved for z, so we can go ahead and set them equal to each other, and then solve for x.

$$x+2=\sqrt{x^2+\frac{1}{3}y^2-4}$$

$$(x+2)^2=x^2+\frac{1}{3}y^2-4$$

$$x^2+4x+4=x^2+\frac{1}{3}y^2-4$$

$$4x+4=\frac{1}{3}y^2-4$$

$$4x=\frac{1}{3}y^2-8$$

$$x=\frac{1}{12}y^2-2$$

Set $y=t$ in this equation.

$$x=\frac{1}{12}t^2-2$$

Plug this value of x into $z=x+2$.

$$z=\frac{1}{12}t^2-2+2$$

$$z = \frac{1}{12}t^2$$

We set $y = t$ originally and used that to find values for x and z in terms of t, and so our vector function is

$$r(t) = \left(\frac{1}{12}t^2 - 2\right)\mathbf{i} + t\mathbf{j} + \frac{1}{12}t^2\mathbf{k}$$