Calculus 3 Workbook

Dot products

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MATH

DOT PRODUCT OF TWO VECTORS

■ 1. Find the dot product $\vec{a} \cdot \vec{b}$, where the vectors $\vec{a}$ and $\vec{b}$ have opposite directions, and $\vec{b}$ has a magnitude two times larger than $\vec{a} = \langle 2, -3, 5 \rangle$.

■ 2. Find the value(s) of the parameter p such that the dot product of the vectors $\vec{a} = \langle p, 2p+1, 3 \rangle$ and $\vec{b} = \langle p-2, 5, -4 \rangle$ is 2.

■ 3. Find the unit vector(s) $\vec{u}$ such that the dot product $\vec{a} \cdot \vec{u}$ reaches its maximum value, if $\vec{a} = \langle 2, 2 \rangle$.

ANGLE BETWEEN TWO VECTORS

■ 1. Use dot products to find the angles between the vector $\vec{a} = \langle -2,4,-4\rangle$ and the positive direction of each major coordinate axis.

■ 2. Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, if $\vec{a} = \langle 3,-4,4\rangle$ and $\vec{b} = \langle -6,2,-1\rangle$.

■ 3. Find the two vectors $\vec{b_1}$ and $\vec{b_2}$ with magnitude 5 that each have an angle of 30° with $\vec{a} = \langle -2,1\rangle$.

ORTHOGONAL, PARALLEL, OR NEITHER

■ 1. Find the terminal point B of the vector $\overrightarrow{AB}$ that has initial point $A(2,0,-1)$, magnitude 24, and is parallel to the vector $\overrightarrow{c} = \langle -2,4,4 \rangle$.

■ 2. Find two vectors $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ with magnitude 2, that are orthogonal to $\overrightarrow{a} = \langle 3,-1 \rangle$.

■ 3. Find value(s) of the parameter p, such that the vectors $\overrightarrow{a} = \langle p, p+3, 6-p \rangle$ and $\overrightarrow{b} = \langle p-1, 4, 2 \rangle$ are (a) parallel, and (b) orthogonal.

ACUTE ANGLE BETWEEN THE LINES

■ 1. Find the acute angle between the lines.

Line 1: $x = 2t + 1$, $y = t - 4$, $z = 6$

Line 2: $\dfrac{x-1}{4} = \dfrac{y+1}{5} = z$

■ 2. Find the acute angle between the line and the plane.

Line: $x = t + 7$, $y = -2t - 5$, $z = 3t + 6$

Plane: $3x - y - 4z + 15 = 0$

■ 3. Find the acute angle between the planes.

Plane 1: $x - 2y + 1 = 0$

Plane 2: $x + y + 2z + 4 = 0$

ACUTE ANGLES BETWEEN THE CURVES

■ 1. Find the acute angle(s) between the curves.

$$x^2 + y^2 = 4$$

$$x^2 + 4y^2 = 4$$

■ 2. Find the acute angle(s) between the curves given in parametric form.

$$x = t^2 + 1,\ y = 2t^2 + t - 3,\ z = t - 1$$

$$x = 2s^2 - 7,\ y = s - 5,\ z = s - 3$$

■ 3. Find the value of the parameter p such that $f(x) = e^x$ and $g(x) = e^{-x} + 2p$ are orthogonal at the point(s) of intersection.

DIRECTION COSINES AND DIRECTION ANGLES

■ 1. Find the direction angles of the linear combination $\vec{c} = 2\vec{a} - 3\vec{b}$, where $\vec{a} = \langle 3,1,-3 \rangle$ and $\vec{b} = \langle 0,-2,-2 \rangle$.

■ 2. Find the vector $\vec{a}$ with magnitude 6 that has direction angles 120°, 45°, and 135° with respect to x, y, and z-axes, respectively.

■ 3. Find the vector $\vec{a}$ that has an x-coordinate of 2, y-coordinate of -1, and direction angle with respect to the z-axis of $\pi/3$.

SCALAR EQUATION OF A LINE

■ 1. Find the parametric scalar equations of the line that pass through the points $A(5,4,-3)$ and $B(1,0,3)$.

■ 2. Find the parametric scalar equations of the line that passes through the point $A(4,-1,0)$ and is orthogonal to the plane $x+2y-z=7$.

■ 3. Find the parametric scalar equations of the line that forms the intersection of the planes $2x+3y-z=1$ and $x-y+4z=-4$.

SCALAR EQUATION OF A PLANE

■ 1. Find the scalar equations of the plane, given its vector equation.

$$\langle 1,2,-1\rangle \cdot (\vec{r} - \langle 0,5,-4\rangle) = 0$$

■ 2. Find the scalar equations of the plane that passes through the points $A(2,0,1)$, $B(-1,3,2)$, and $C(1,1,-4)$.

■ 3. Find the scalar equation of a plane(s) that's 6 units from, and parallel to, the plane $x - 2y + 2z - 2 = 0$.

SCALAR AND VECTOR PROJECTIONS

■ 1. Find the vector sum of projections of the vector $\vec{a} = \langle 13, -8, 9 \rangle$ onto the three coordinate axes.

■ 2. Find the projection of the vector $\vec{a} = \langle 4, 3, -1 \rangle$ onto the plane Q, which is given by $2x - y + 2z - 7 = 0$.

■ 3. Find the vector $\vec{a}$ if its scalar projections onto the vectors $\vec{b} = \langle 4, -3 \rangle$ and $\vec{c} = \langle 0, 2 \rangle$ are both 3.

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