Cross product of two vectors

To take the cross product of two vectors

$$a\langle a_1, a_2, a_3\rangle$$

$$b\langle b_1, b_2, b_3\rangle$$

we'll create a matrix in the form

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

As always, we'll use the sign matrix

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

to determine the signs for our top row. We'll expand the matrix to

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= \mathbf{i}\left(a_2b_3 - a_3b_2\right) - \mathbf{j}\left(a_1b_3 - a_3b_1\right) + \mathbf{k}\left(a_1b_2 - a_2b_1\right)$$

and then take the coefficients on $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ to form the cross product vector $c\langle c_1, c_2, c_3\rangle$, where

$$c_1 = a_2b_3 - a_3b_2$$

© Krista King Math

$$c_2 = a_1b_3 - a_3b_1$$

$$c_3 = a_1b_2 - a_2b_1$$

If you can remember the formula for

$$\mathbf{i}\left(a_2b_3 - a_3b_2\right) - \mathbf{j}\left(a_1b_3 - a_3b_1\right) + \mathbf{k}\left(a_1b_2 - a_2b_1\right)$$

then you can skip the matrices and go straight to this step. If not, just use the matrix approach.

Example

Find the cross product of the vectors.

$$a\langle 2, -4, 1\rangle$$

$$b\langle -2, 5, 7\rangle$$

For the sake of this example, we'll assume we can't remember the formula for

$$\mathbf{i}\left(a_2b_3 - a_3b_2\right) - \mathbf{j}\left(a_1b_3 - a_3b_1\right) + \mathbf{k}\left(a_1b_2 - a_2b_1\right)$$

and use the matrix. Plugging the values from the given vectors into our 3×3 matrix, we get

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 1 \\ -2 & 5 & 7 \end{vmatrix}$$

© Krista King Math

$$\vec{a} \times \vec{b} = \mathbf{i}\begin{vmatrix} -4 & 1 \\ 5 & 7 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 2 & 1 \\ -2 & 7 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 2 & -4 \\ -2 & 5 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \mathbf{i}\left[(-4)(7) - (1)(5)\right] - \mathbf{j}\left[(2)(7) - (1)(-2)\right] + \mathbf{k}\left[(2)(5) - (-4)(-2)\right]$$

$$\vec{a} \times \vec{b} = \mathbf{i}(-28 - 5) - \mathbf{j}(14 + 2) + \mathbf{k}(10 - 8)$$

$$\vec{a} \times \vec{b} = -33\mathbf{i} - 16\mathbf{j} + 2\mathbf{k}$$

$$\vec{a} \times \vec{b} = \langle -33, -16, 2 \rangle$$

This is the cross product of the vectors a and b.

© Krista King Math