**Topic**: Vector orthogonal to the plane

**Question**: Find the vector orthogonal to the plane that includes the given vectors.

$$\overrightarrow{AB} = \langle 2,3,2 \rangle$$

$$\overrightarrow{AC} = \langle -2,1,2 \rangle$$

**Answer choices**:

A $\langle 1, -2, 2 \rangle$

B $\langle 1, 2, 2 \rangle$

C $\langle 1, -2, -2 \rangle$

D $\langle 1, 2, -2 \rangle$

**Solution**: A

Given two vectors that lie in the plane, we can find the vector orthogonal to the plane by taking the cross product of the vectors $\overrightarrow{AB} = \langle AB_1, AB_2, AB_3 \rangle$ and $\overrightarrow{AC} = \langle AC_1, AC_2, AC_3 \rangle$.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ AB_1 & AB_2 & AB_3 \\ AC_1 & AC_2 & AC_3 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i} \begin{vmatrix} AB_2 & AB_3 \\ AC_2 & AC_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} AB_1 & AB_3 \\ AC_1 & AC_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} AB_1 & AB_2 \\ AC_1 & AC_2 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i} \left(AB_2AC_3 - AB_3AC_2\right) - \mathbf{j} \left(AB_1AC_3 - AB_3AC_1\right) + \mathbf{k} \left(AB_1AC_2 - AB_2AC_1\right)$$

For the given vectors $\overrightarrow{AB} = \langle 2,3,2 \rangle$ and $\overrightarrow{AC} = \langle -2,1,2 \rangle$, we get

$$\overrightarrow{AB} \times \overrightarrow{AC} = \left[(3)(2) - (2)(1)\right] \mathbf{i} - \left[(2)(2) - (2)(-2)\right] \mathbf{j} + \left[(2)(1) - (3)(-2)\right] \mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = (6 - 2)\mathbf{i} - (4 + 4)\mathbf{j} + (2 + 6)\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = 4\mathbf{i} - 8\mathbf{j} + 8\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = 4(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$$

We'll convert this into standard vector form to get the vector orthogonal to the plane.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \langle 1, -2,2 \rangle$$

**Topic**: Vector orthogonal to the plane

**Question**: Find the vector orthogonal to the plane that includes the given points.

$$A(1,-1,1)$$

$$B(-2,2,-2)$$

$$C(3,1,-2)$$

**Answer choices**:

A $\langle -1,5,-4\rangle$

B $\langle 1,-5,-4\rangle$

C $\langle 1,5,4\rangle$

D $\langle -1,-5,4\rangle$

**Solution**: C

Since we've been given three points $A(a_1, a_2, a_3)$, $B(b_1, b_2, b_3)$ and $C(c_1, c_2, c_3)$, we'll use the points to generate two vectors, $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$$\overrightarrow{AB} = (b_1 - a_1)\mathbf{i} + (b_2 - a_2)\mathbf{j} + (b_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AB} = (AB_1)\mathbf{i} + (AB_2)\mathbf{j} + (AB_3)\mathbf{k}$$

and

$$\overrightarrow{AC} = (c_1 - a_1)\mathbf{i} + (c_2 - a_2)\mathbf{j} + (c_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AC} = (AC_1)\mathbf{i} + (AC_2)\mathbf{j} + (AC_3)\mathbf{k}$$

Plugging in the points we've been given, we get

$$\overrightarrow{AB} = (b_1 - a_1)\mathbf{i} + (b_2 - a_2)\mathbf{j} + (b_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AB} = (-2 - 1)\mathbf{i} + \left[2 - (-1)\right]\mathbf{j} + (-2 - 1)\mathbf{k}$$

$$\overrightarrow{AB} = -3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$$

and

$$\overrightarrow{AC} = (c_1 - a_1)\mathbf{i} + (c_2 - a_2)\mathbf{j} + (c_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AC} = (3 - 1)\mathbf{i} + \left[1 - (-1)\right]\mathbf{j} + (-2 - 1)\mathbf{k}$$

$$\overrightarrow{AC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$$

If we convert these into standard vector form, we get

$$AB\langle AB_1, AB_2, AB_3\rangle = \langle -3, 3, -3\rangle$$

$$AC\langle AC_1, AC_2, AC_3\rangle = \langle 2,2,-3\rangle$$

Now that we've converted the points into vectors, we'll take the cross product of $\overrightarrow{AB} = \langle AB_1, AB_2, AB_3\rangle$ and $\overrightarrow{AC} = \langle AC_1, AC_2, AC_3\rangle$.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ AB_1 & AB_2 & AB_3 \\ AC_1 & AC_2 & AC_3 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}\begin{vmatrix} AB_2 & AB_3 \\ AC_2 & AC_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} AB_1 & AB_3 \\ AC_1 & AC_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} AB_1 & AB_2 \\ AC_1 & AC_2 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}\left(AB_2AC_3 - AB_3AC_2\right) - \mathbf{j}\left(AB_1AC_3 - AB_3AC_1\right) + \mathbf{k}\left(AB_1AC_2 - AB_2AC_1\right)$$

For the vectors $\overrightarrow{AB} = \langle -3,3,-3\rangle$ and $\overrightarrow{AC} = \langle 2,2,-3\rangle$, we get

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}\left[(3)(-3) - (-3)(2)\right] - \mathbf{j}\left[(-3)(-3) - (-3)(2)\right] + \mathbf{k}\left[(-3)(2) - (3)(2)\right]$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}\left[-9 - (-6)\right] - \mathbf{j}\left[9 - (-6)\right] + \mathbf{k}\left[-6 - 6\right]$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = -3\mathbf{i} - 15\mathbf{j} - 12\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = -3\left(\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}\right)$$

We'll convert this into standard vector form to get the vector orthogonal to the plane.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \langle 1,5,4\rangle$$

**Topic**: Vector orthogonal to the plane

**Question**: Find the vector orthogonal to the plane that includes the given points.

$$A(4,-5,-4)$$

$$B(7,-5,0)$$

$$C(11,6,8)$$

**Answer choices**:

A $\langle -44,8,33\rangle$

B $\langle -44,-8,33\rangle$

C $\langle 7,11,12\rangle$

D $\langle 7,11,-12\rangle$

**Solution**: B

Since we've been given three points $A(a_1, a_2, a_3)$, $B(b_1, b_2, b_3)$ and $C(c_1, c_2, c_3)$, we'll use the points to generate two vectors, $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$$\overrightarrow{AB} = (b_1 - a_1)\mathbf{i} + (b_2 - a_2)\mathbf{j} + (b_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AB} = (AB_1)\mathbf{i} + (AB_2)\mathbf{j} + (AB_3)\mathbf{k}$$

and

$$\overrightarrow{AC} = (c_1 - a_1)\mathbf{i} + (c_2 - a_2)\mathbf{j} + (c_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AC} = (AC_1)\mathbf{i} + (AC_2)\mathbf{j} + (AC_3)\mathbf{k}$$

Plugging in the points we've been given, we get

$$\overrightarrow{AB} = (b_1 - a_1)\mathbf{i} + (b_2 - a_2)\mathbf{j} + (b_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AB} = (7 - 4)\mathbf{i} + \left[-5 - (-5)\right]\mathbf{j} + \left[0 - (-4)\right]\mathbf{k}$$

$$\overrightarrow{AB} = 3\mathbf{i} + 0\mathbf{j} + 4\mathbf{k}$$

and

$$\overrightarrow{AC} = (c_1 - a_1)\mathbf{i} + (c_2 - a_2)\mathbf{j} + (c_3 - a_3)\mathbf{k}$$

$$\overrightarrow{AC} = (11 - 4)\mathbf{i} + \left[6 - (-5)\right]\mathbf{j} + \left[8 - (-4)\right]\mathbf{k}$$

$$\overrightarrow{AC} = 7\mathbf{i} + 11\mathbf{j} + 12\mathbf{k}$$

If we convert these into standard vector form, we get

$$AB\langle AB_1, AB_2, AB_3\rangle = \langle 3,0,4\rangle$$

$$AC\langle AC_1, AC_2, AC_3\rangle = \langle 7,11,12\rangle$$

Now that we've converted the points into vectors, we'll take the cross product of $\overrightarrow{AB} = \langle AB_1, AB_2, AB_3\rangle$ and $\overrightarrow{AC} = \langle AC_1, AC_2, AC_3\rangle$.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ AB_1 & AB_2 & AB_3 \\ AC_1 & AC_2 & AC_3 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}\begin{vmatrix} AB_2 & AB_3 \\ AC_2 & AC_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} AB_1 & AB_3 \\ AC_1 & AC_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} AB_1 & AB_2 \\ AC_1 & AC_2 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}\left(AB_2AC_3 - AB_3AC_2\right) - \mathbf{j}\left(AB_1AC_3 - AB_3AC_1\right) + \mathbf{k}\left(AB_1AC_2 - AB_2AC_1\right)$$

For the vectors $\overrightarrow{AB} = \langle 3,0,4\rangle$ and $\overrightarrow{AC} = \langle 7,11,12\rangle$, we get

$$\overrightarrow{AB} \times \overrightarrow{AC} = \left[(0)(12) - (4)(11)\right]\mathbf{i} - \left[(3)(12) - (4)(7)\right]\mathbf{j} + \left[(3)(11) - (0)(7)\right]\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = (0 - 44)\mathbf{i} - (36 - 28)\mathbf{j} + (33 - 0)\mathbf{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = -44\mathbf{i} - 8\mathbf{j} + 33\mathbf{k}$$

We'll convert this into standard vector form to get the vector orthogonal to the plane.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \langle -44, -8, 33\rangle$$