Topic: Using inequalities to describe the region

Question: What inequality describes the region consisting of all points between (but not on) two spheres of radius r and R, both centered at the origin, where $r < R$?

Answer choices:

A $r^2 \leq x^2 + y^2 + z^2 \leq R^2$

B $r^2 \geq x^2 + y^2 + z^2 \geq R^2$

C $r^2 < x^2 + y^2 + z^2 < R^2$

D $r^2 > x^2 + y^2 + z^2 > R^2$

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Solution: C

The first thing we need to remember is the base equation for a sphere $(x-h)^2+(y-k)^2+(z-l)^2=r^2$ where (h,k,l) is the point at the center of the sphere. Both of our spheres are centered at the origin $(0,0,0)$ which will make the equations for our two spheres

$$x^2+y^2+z^2=r^2$$

$$x^2+y^2+z^2=R^2$$

Now we need to remember that we're looking for the points which exist between the sphere of radius r (which is smaller than the other sphere so exists completely within the larger sphere) and the sphere of radius R but not on either surface. The points that exist within the sphere of radius R but not on either surface are

$$x^2+y^2+z^2<R^2$$

Remember $x^2+y^2+z^2\le R^2$ would be correct only if we also wanted to include the points on the surface as well as the points inside the sphere and $x^2+y^2+z^2=R^2$ would be correct if we only wanted the points on the surface of the sphere.

Next we need to remove the points that are within $x^2+y^2+z^2=r^2$, which are shown by

$$x^2+y^2+z^2>r^2$$

When we put our two inequalities together to describe the region, we get

$$r^2<x^2+y^2+z^2<R^2$$

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Topic: Using inequalities to describe the region

Question: What inequality describes the region consisting of all points between (but not on) two spheres of radius r and R, both centered at $(1,-4,2)$, where $r<R$?

Answer choices:

A $\quad r^2 \le (x+1)^2+(y-4)^2+(z+2)^2 \le R^2$

B $\quad r^2 < (x-1)^2+(y+4)^2+(z-2)^2 < R^2$

C $\quad r^2 < (x+1)^2+(y-4)^2+(z+2)^2 < R^2$

D $\quad r^2 \le (x-1)^2+(y+4)^2+(z-2)^2 \le R^2$

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Solution: B

To describe a region in three dimensional space, you will need to analyze how the region presents itself. The basic equations for the objects used will help define the region. Remember that these equations include the points on the surface of the object as well as inside the object.

The first thing we need to remember is the base equation for a sphere $(x-h)^2+(y-k)^2+(z-l)^2=r^2$ where (h,k,l) is the point at the center of the sphere. Both of our spheres are centered at the point $(1,-4,2)$ which will make the equations for our two spheres

$$(x-1)^2+(y+4)^2+(z-2)^2=r^2$$

$$(x-1)^2+(y+4)^2+(z-2)^2=R^2$$

Now we need to remember that we're looking for the points which exist between the sphere of radius r (which is smaller than the other sphere so exits completely within the larger sphere) and the sphere of radius R but not on either surface. The points that exist within the sphere of radius R but not on either surface are

$$(x-1)^2+(y+4)^2+(z-2)^2<R^2$$

Remember $(x-1)^2+(y+4)^2+(z-2)^2 \le R^2$ would be correct only if we also wanted to include the points on the surface as well as the points inside the sphere and $(x-1)^2+(y+4)^2+(z-2)^2=R^2$ would be correct if we only wanted the points on the surface of the sphere.

Next we need to remove the points that are within $(x-1)^2+(y+4)^2+(z-2)^2=r^2$ which are

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$$(x-1)^2 + (y+4)^2 + (z-2)^2 > r^2$$

When we put our two inequalities together to describe the region, we get

$$r^2 < (x-1)^2 + (y+4)^2 + (z-2)^2 < R^2$$

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Topic: Using inequalities to describe the region

Question: What inequality describes the region consisting of all points between two spheres of radius r and R, both centered at $(0,2,-1)$, where $r < R$, and the surfaces of both spheres are included in the region?

Answer choices:

A $\quad r^2 < x^2 + (y-2)^2 + (z+1)^2 < R^2$

B $\quad r^2 < x^2 + (y+2)^2 + (z-1)^2 < R^2$

C $\quad r^2 \leq x^2 + (y+2)^2 + (z-1)^2 \leq R^2$

D $\quad r^2 \leq x^2 + (y-2)^2 + (z+1)^2 \leq R^2$

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Solution: D

The first thing we need to remember is the base equation for a sphere $(x-h)^2+(y-k)^2+(z-l)^2=r^2$ where (h,k,l) is the point at the center of the sphere. Both of our spheres are centered at the point $(0,2,-1)$ which will make the equations for our two spheres

$$(x-0)^2+(y-2)^2+(z+1)^2=r^2$$

$$x^2+(y-2)^2+(z+1)^2=r^2$$

and

$$(x-0)^2+(y-2)^2+(z+1)^2=R^2$$

$$x^2+(y-2)^2+(z+1)^2=R^2$$

Now we need to remember that we're looking for the points which exist between the sphere of radius r (which is smaller than the other sphere so exits completely within the larger sphere) and the sphere of radius R including both surfaces. The points that exist within the sphere of radius R and on its surface are

$$x^2+(y-2)^2+(z+1)^2\le R^2$$

Remember $x^2+(y-2)^2+(z+1)^2<R^2$ would be correct only if we wanted to exclude the points on the surface but use the points inside the sphere and $x^2+(y-2)^2+(z+1)^2=R^2$ would be correct if we only wanted the points on the surface of the sphere.

Next we need to remove the points that are within $x^2+(y-2)^2+(z+1)^2=r^2$ which are

$$x^2 + (y-2)^2 + (z+1)^2 \ge r^2$$

When we put our two inequalities together to describe the region, we get

$$r^2 \le x^2 + (y-2)^2 + (z+1)^2 \le R^2$$

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