Topic: Distance between a point and a plane

Question: Find the distance between the point and the plane.

$$(1,1,1)$$

$$-x + 2y - z = 2$$

Answer choices:

A $-\frac{2}{\sqrt{6}}$

B $-\frac{1}{\sqrt{3}}$

C $\frac{2}{\sqrt{6}}$

D $\frac{1}{\sqrt{3}}$

Solution: C

The distance d between a point and a plane is given by the component of b along n, or the scalar projection of b along n. b is the vector connecting a point on the plane to the given point, and n is the normal vector to the plane.

$$d = \left|\text{comp}_n b\right| = \frac{|n \cdot b|}{|n|}$$

We can also write this formula as

$$d = \frac{\left|ax_1 + by_1 + cz_1 + d\right|}{\sqrt{a^2 + b^2 + c^2}}$$

where (x_1, y_1, z_1) is the given point, and where $ax + by + cz = -d$ is the equation of the plane. Since in this case the given point is $(1,1,1)$, we can plug this into the formula to get

$$d = \frac{\left|a(1) + b(1) + c(1) + d\right|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d = \frac{\left|a + b + c + d\right|}{\sqrt{a^2 + b^2 + c^2}}$$

The plane $-x + 2y - z = 2$ tells us that $a = -1$, $b = 2$, $c = -1$, and $d = -2$. Plugging these in and then simplifying gives the distance between the point and the plane.

$$d = \frac{\left| -1 + 2 - 1 - 2 \right|}{\sqrt{(-1)^2 + (2)^2 + (-1)^2}}$$

$$d = \frac{\left| -2 \right|}{\sqrt{1 + 4 + 1}}$$

$$d = \frac{2}{\sqrt{6}}$$

Topic: Distance between a point and a plane

Question: Find the distance between the point and the plane.

$$(0,3,-2)$$

$$2x+3y+z=-3$$

Answer choices:

A $\frac{5}{\sqrt{7}}$

B $\frac{10}{\sqrt{14}}$

C $-\frac{10}{\sqrt{14}}$

D $-\frac{5}{\sqrt{7}}$

Solution: B

The distance d between a point and a plane is given by the component of b along n, or the scalar projection of b along n. b is the vector connecting a point on the plane to the given point, and n is the normal vector to the plane.

$$d = \left|\text{comp}_n b\right| = \frac{|n \cdot b|}{|n|}$$

We can also write this formula as

$$d = \frac{\left|ax_1 + by_1 + cz_1 + d\right|}{\sqrt{a^2 + b^2 + c^2}}$$

where (x_1, y_1, z_1) is the given point, and where $ax + by + cz = -d$ is the equation of the plane. Since in this case the given point is $(0,3,-2)$, we can plug this into the formula to get

$$d = \frac{\left|a(0) + b(3) + c(-2) + d\right|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d = \frac{\left|3b - 2c + d\right|}{\sqrt{a^2 + b^2 + c^2}}$$

The plane $2x + 3y + z = -3$ tells us that $a = 2$, $b = 3$, $c = 1$, and $d = 3$. Plugging these in and then simplifying gives the distance between the point and the plane.

$$d = \frac{|3(3) - 2(1) + 3|}{\sqrt{(2)^2 + (3)^2 + (1)^2}}$$

$$d = \frac{|10|}{\sqrt{4 + 9 + 1}}$$

$$d = \frac{10}{\sqrt{14}}$$

Topic: Distance between a point and a plane

Question: Find the distance between the point and the plane.

$$(-2,-1,5)$$

$$x-4y-2z=-6$$

Answer choices:

A $\quad -\dfrac{2}{\sqrt{21}}$

B $\quad \dfrac{2}{\sqrt{7}}$

C $\quad -\dfrac{2}{\sqrt{7}}$

D $\quad \dfrac{2}{\sqrt{21}}$

Solution: D

The distance d between a point and a plane is given by the component of b along n, or the scalar projection of b along n. b is the vector connecting a point on the plane to the given point, and n is the normal vector to the plane.

$$d = \left| \text{comp}_n b \right| = \frac{|n \cdot b|}{|n|}$$

We can also write this formula as

$$d = \frac{\left| ax_1 + by_1 + cz_1 + d \right|}{\sqrt{a^2 + b^2 + c^2}}$$

where (x_1, y_1, z_1) is the given point, and where $ax + by + cz = -d$ is the equation of the plane. Since in this case the given point is $(-2, -1, 5)$, we can plug this into the formula to get

$$d = \frac{\left| a(-2) + b(-1) + c(5) + d \right|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d = \frac{\left| -2a - b + 5c + d \right|}{\sqrt{a^2 + b^2 + c^2}}$$

The plane $x - 4y - 2z = -6$ tells us that $a = 1$, $b = -4$, $c = -2$, and $d = 6$. Plugging these in and then simplifying gives the distance between the point and the plane.

$$d = \frac{\left|-2(1) - (-4) + 5(-2) + 6\right|}{\sqrt{(1)^2 + (-4)^2 + (-2)^2}}$$

$$d = \frac{\left|-2 + 4 - 10 + 6\right|}{\sqrt{1 + 16 + 4}}$$

$$d = \frac{2}{\sqrt{21}}$$