Topic: Higher order partial derivatives

Question: Find the partial derivative(s).

Find all four second-order partial derivatives.

$$f(x, y) = \sin x \cos 2y$$

Answer choices:

A $f_{xx} = -\sin x \cos 2y$ $f_{xy} = -2\cos x \sin 2y$

$f_{yy} = -4\sin x \cos 2y$ $f_{yx} = -2\cos x \sin 2y$

B $f_{xx} = -4\sin x \cos 2y$ $f_{xy} = -2\cos x \sin 2y$

$f_{yy} = -\sin x \cos 2y$ $f_{yx} = -2\cos x \sin 2y$

C $f_{xx} = -2\cos x \sin 2y$ $f_{xy} = -\sin x \cos 2y$

$f_{yy} = -2\cos x \sin 2y$ $f_{yx} = -4\sin x \cos 2y$

D $f_{xx} = -2\cos x \sin 2y$ $f_{xy} = -4\sin x \cos 2y$

$f_{yy} = -2\cos x \sin 2y$ $f_{yx} = -\sin x \cos 2y$

Solution: A

To find the second-order partial derivatives, first we want to find the first-order partial derivatives f_x (by treating y as a constant and differentiating $f(x, y)$ with respect to x), and f_y (by treating x as a constant and differentiating $f(x, y)$ with respect to y).

$$f(x, y) = \sin x \cos 2y$$

$$f_x = (\cos 2y)(\cos x)$$

$$f_x = \cos x \cos 2y$$

$$f_y = (\sin x)(-2 \sin 2y)$$

$$f_y = -2 \sin x \sin 2y$$

Then, to find the second-order partial derivative f_{xx}, treat y as a constant while differentiating f_x with respect to x.

$$f_x = \cos x \cos 2y$$

$$f_{xx} = (\cos 2y)(-\sin x) = -\sin x \cos 2y$$

And find f_{xy} by treating x as a constant while differentiating f_x with respect to y.

$$f_x = \cos x \cos 2y$$

$$f_{xy} = (\cos x)(-2 \sin 2y) = -2 \cos x \sin 2y$$

Finally, find f_{yy} and f_{yx} by differentiating f_y with respect to y (while holding x constant), and with respect to x (while holding y constant), respectively:

$$f_y = -2\sin x \sin 2y$$

$$f_{yy} = (-2\sin x)2\cos 2y = -4\sin x \cos 2y$$

$$f_{yx} = (-2\sin 2y)(\cos x) = -2\cos x \sin 2y$$

Topic: Higher order partial derivatives

Question: Find the partial derivative(s).

Find f_{xzy}

for $f(x, y, z) = \sin\left(xy + z^2\right)$

Answer choices:

A $f_{xzy} = \cos\left(xy + z^2\right) + 2z\cos\left(xy + z^2\right)$

B $f_{xzy} = -2xz\cos\left(xy + z^2\right)$

C $f_{xzy} = -4xz^2\cos\left(xy + z^2\right) + \sin\left(xy + z^2\right)$

D $f_{xzy} = -2z\sin\left(xy + z^2\right) - 2xyz\cos\left(xy + z^2\right)$

Solution: D

To find the third-order partial derivative f_{xzy}, first we want to find the first-order partial derivative f_x (by treating y and z as constants and differentiating $f(x, y, z)$ with respect to x), using the chain rule.

$$f(x, y, z) = \sin\left(xy + z^2\right)$$

$$f_x = \cos\left(xy + z^2\right) \cdot \frac{\partial}{\partial x}\left(xy + z^2\right)$$

$$f_x = \cos\left(xy + z^2\right) \cdot y = y\cos\left(xy + z^2\right)$$

Next, we can find the second-order partial derivative f_{xz} by treating x and y as constants while differentiating f_x with respect to z (again, using the chain rule).

$$f_x = y\cos\left(xy + z^2\right)$$

$$f_{xz} = y\left[-\sin\left(xy + z^2\right)\right] \cdot \frac{\partial}{\partial z}\left(xy + z^2\right)$$

$$f_{xz} = y\left[-\sin\left(xy + z^2\right)\right] \cdot 2z = -2yz\sin\left(xy + z^2\right)$$

Now we can find the third-order partial derivative f_{xzy} by treating x and z as constants while differentiating f_{xz} with respect to y (we'll use product rule and chain rule since f_{xz} is the product of two expressions containing y):

$$f_{xz} = -2yz\sin\left(xy + z^2\right)$$

$$f_{xzy} = (-2z)\left[\sin\left(xy + z^2\right)\right] + (-2yz)\left[\cos\left(xy + z^2\right)(x)\right]$$

$$f_{xzy} = -2z\sin\left(xy+z^2\right) - 2xyz\cos\left(xy+z^2\right)$$

Topic: Higher order partial derivatives

Question: Find f_{xyx} and f_{yxy} for $f(x, y) = x^3y^2 + xy^3$.

Answer choices:

A $f_{xyx} = 6x^2 + 3y^2$ and $f_{yxy} = 6x^2 + 3y^2$

B $f_{xyx} = 12xy$ and $f_{yxy} = 6x^2 + 6y$

C $f_{xyx} = 6y^2$ and $f_{yxy} = 4x$

D $f_{xyx} = 12x + 6y$ and $f_{yxy} = 12x + 6y$

Solution: B

To find the third-order partial derivative f_{xyx}, first we want to find the first-order partial derivative f_x by treating y as a constant and differentiating $f(x,y)$ with respect to x.

$$f(x,y) = x^3y^2 + xy^3$$

$$f_x = \left(3x^2\right)y^2 + (1)y^3 = 3x^2y^2 + y^3$$

Next, we can find the second-order partial derivative f_{xy} by treating x as a constant while differentiating f_x with respect to y.

$$f_x = 3x^2y^2 + y^3$$

$$f_{xy} = 3x^2(2y) + \left(3y^2\right) = 6x^2y + 3y^2$$

Now we can find the third-order partial derivative f_{xyx} by again treating y as a constant, this time while differentiating f_{xy} with respect to x.

$$f_{xy} = 6x^2y + 3y^2$$

$$f_{xyx} = 6(2x)y + 3y^2(0) = 12xy$$

Similarly, to find f_{yxy}, we must first find f_y (by treating x as a constant and differentiating $f(x,y)$ with respect to y) and then find f_{yx} (by treating y as a constant while differentiating f_y with respect to x). Then, we can find f_{yxy} by again treating x as a constant, while differentiating f_{yx} with respect to y.

$$f(x,y) = x^3y^2 + xy^3$$

$$f_y = x^3(2y) + x\left(3y^2\right) = 2x^3y + 3xy^2$$

$$f_{yx} = 2\left(3x^2\right)y + 3(1)y^2 = 6x^2y + 3y^2$$

$$f_{yxy} = 6x^2(1) + 3(2y) = 6x^2 + 6y$$