Calculus 3 Final Exam

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MATH

Calculus 3 Final Exam

This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** Find the differential of the multivariable function.

$$z = 2x^2 \cos(3y) - 4 \ln x$$

A $dz = 4x \cos(3y)\, dx - \frac{4}{x}\, dx - 6x^2 \sin(3y)\, dy$

B $dz = 4x \cos(3y)\, dx$

C $dz = 4x \cos(3y)\, dx - \frac{4}{x}\, dx - 2x^2 \sin(3y)\, dy$

D $dz = -12x \sin(3y)\, dx - \frac{4}{x}\, dx$

E $dz = 4x \cos(3y)\, dx - \frac{4}{x}\, dx - \frac{2}{3}x^2 \sin(3y)\, dy$

2. **(5 pts)** Find the linear approximation of $f(x, y) = 4y^3 + x^2y^2$ at $(0, -2)$.

A $L(x, y) = 48y + 128$

B $L(x, y) = x + 48y + 128$

C $L(x, y) = 48y + 64$

D $L(x, y) = x + 48y - 128$

E $L(x, y) = 48y - 128$

3. **(5 pts)** For which values a and b is the critical point of the function equal to $(-3,2)$?

$$f(x,y) = 3x^2 - 2y^2 - ax + 2by$$

A	$a = -3$	$b = 2$
B	$a = -18$	$b = 4$
C	$a = 3$	$b = -2$
D	$a = -18$	$b = -4$
E	$a = 18$	$b = -4$

4. **(5 pts)** Evaluate the double integral over $R = \{(x,y) \mid 0 \le x \le \ln 3, 1 \le y \le \ln 5\}$.

$$\iint_R 2e^{2x+y}\, dA$$

A	$40 + 10e$	C	$40 + 8e$	E	$40 - 10e$
B	$40 - 8e$	D	$45 - e$		

5. **(5 pts)** Convert the double integral to polar coordinates, where D is bounded by $y = \pm\sqrt{9 - x^2}$.

$$\iint_D \cos(3x^2 + 3y^2)\, dA$$

A $\int_0^{2\pi} \int_0^3 r\cos(3r^2)\, dr\, d\theta$

B $\int_0^{2\pi} \int_0^{\frac{3}{2}} \cos(3r^2)\, dr\, d\theta$

C $\int_0^{2\pi} \int_0^3 r\cos r^2\, dr\, d\theta$

D $\int_0^{2\pi} \int_0^{\frac{3}{2}} r\cos r^2\, dr\, d\theta$

E $\int_0^{2\pi} \int_0^3 \cos(3r^2)\, dr\, d\theta$

6. **(5 pts)** Find the vector function for the curve of intersection of the sphere and the plane.

$$z = \sqrt{x^2 + (y-2)^2 - 16}$$

$$z = 1 - y$$

A $r(t) = t\mathbf{i} + \left(\frac{1}{2}t^2 - \frac{13}{2}\right)\mathbf{j} + \left(-\frac{1}{2}t^2 + \frac{13}{2}\right)\mathbf{k}$

B $r(t) = \left(\frac{1}{2}t^2 - \frac{13}{2}\right)\mathbf{i} + t\mathbf{j} + \left(-\frac{1}{2}t^2 + \frac{15}{2}\right)\mathbf{k}$

C $r(t) = t\mathbf{i} + \left(\frac{1}{2}t^2 - \frac{13}{2}\right)\mathbf{j} + \left(\frac{1}{2}t^2 - \frac{11}{2}\right)\mathbf{k}$

D $r(t) = \left(\frac{1}{2}t^2 - \frac{13}{2}\right)\mathbf{i} + t\mathbf{j} + \left(\frac{1}{2}t^2 - \frac{11}{2}\right)\mathbf{k}$

E $r(t) = t\mathbf{i} + \left(\frac{1}{2}t^2 - \frac{13}{2}\right)\mathbf{j} + \left(-\frac{1}{2}t^2 + \frac{15}{2}\right)\mathbf{k}$

7. **(5 pts)** Find the unit normal vector of the vector function at $t = \pi$.

$$r(t) = 6\sin t\mathbf{i} - 6\cos t\mathbf{j} - 8t\mathbf{k}$$

A $N(\pi) = \mathbf{i}$

B $N(\pi) = \mathbf{j}$

C $N(\pi) = -\mathbf{i}$

D $N(\pi) = -\mathbf{j}$

E $N(\pi) = -\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}$

8. **(5 pts)** Find the normal component of acceleration.

$$r(t) = 2t^2\mathbf{i} - t^2\mathbf{j} + 3t\mathbf{k}$$

A $a_N = \frac{2\sqrt{109(20t^2+9)}}{20t^2+9}$

B $a_N = \frac{2\sqrt{109}}{t\sqrt{29}}$

C $a_N = \frac{6\sqrt{5}}{t\sqrt{21}}$

D $a_N = \frac{6\sqrt{5}}{t\sqrt{29}}$

E $a_N = \frac{6\sqrt{5(20t^2+9)}}{20t^2+9}$

9. **(15 pts)** Find the symmetric equations for the line of intersection of the planes $x - 2y + z = 5$ and $-2x - y + 3z = 5$.

10. **(15 pts)** Find three positive numbers with maximum product if their sum is 45.

11. **(15 pts)** Use spherical coordinates to find the volume of the triple integral, where B is the upper half of the sphere $x^2 + y^2 + z^2 = 16$.

$$\iiint_B 8z \; dV$$

12. **(15 pts)** Find the point where the function $f(x) = 2 \ln x$ has maximum curvature. Use the following curvature formula.

$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}}$$