Calculus 3 Workbook

Stokes' and divergence theorem

krista king MATH

STOKES' THEOREM

■ 1. Use Stokes' theorem to evaluate the surface integral where S is the part of the elliptic paraboloid $z+x^2+y^2-3=0$ above the plane $z=-1$. Assume that S has a positive orientation.

$$\iint_S \text{curl } \vec{F} \cdot d\vec{S}$$

$$\vec{F} = \langle y+2, -z^2, 2xy \rangle$$

■ 2. Use Stokes' theorem to evaluate the line integral, where C is the rectangle $KMNO$ with vertices $K(0,0,0)$, $M(0,6,0)$, $N(3,6,0)$ and $O(3,0,0)$. Assume that C has a clockwise orientation as viewed from the positive z-axis.

$$\int_C \vec{F} \cdot d\vec{r}$$

$$\vec{F} = \langle 2xyz, x^2+y^2, 2xyz \rangle$$

■ 3. Use Stokes' theorem to evaluate the line integral, where C is the boundary curve of the semicircle centered at the origin with radius 4 that lies in the xz-plane, and with $z \geq 0$. Assume that C has a counterclockwise orientation as viewed from the positive y-axis.

$$\int_C \vec{F} \cdot d\vec{r}$$

$$\vec{F} = \langle x + 3y - z + 2, x - 5y + 9z - 7, -5x - y + 2z + 6 \rangle$$

DIVERGENCE THEOREM

■ 1. Use the Divergence theorem to evaluate the surface integral, where S is the boundary surface of the box $[-3,4] \times [3,5] \times [-3,0]$. Assume that S has a negative orientation.

$$\iint_S \vec{F} \cdot d\vec{S}$$

$$\vec{F} = \langle x + e^{z^2-y^2}, \ln y + y + x^4, z^2 - \arcsin(x+y) \rangle$$

■ 2. Use the Divergence theorem to evaluate the surface integral where S is the boundary surface of the part of the cylinder $y^2 + z^2 = 25$ with $-2 \le x \le 4$. Assume that S has a positive orientation.

$$\iint_S \vec{F} \cdot d\vec{S}$$

$$\vec{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$$

■ 3. Use the Divergence theorem to evaluate the triple integral where E is the sphere centered at the origin with radius 4.

$$\iiint_E \text{div } \vec{F} \, dV$$

$$\vec{F} = \left\langle \frac{x^2 + y^2 + z^2}{4}, -6y, 6 \right\rangle$$

WWW.KRISTAKINGMATH.COM