Topic: Directional derivatives in the direction of the angle

Question: Find the directional derivative.

$$D_u f(1,1)$$

$$f(x, y) = 2x^2y - 2y^2$$

$\vec{u}$ is the unit vector toward $\theta = \frac{\pi}{4}$

Answer choices:

A $\quad D_u f(1,1) = 3\sqrt{2}$

B $\quad D_u f(1,1) = \frac{3\sqrt{2}}{2}$

C $\quad D_u f(1,1) = \frac{\sqrt{2}}{2}$

D $\quad D_u f(1,1) = \sqrt{2}$

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Solution: D

We need to change the unit vector so that it's in the direction of a specific point, instead of in the direction of an angle. To do that we'll use the formula

$$\vec{u} = \langle \cos\theta, \sin\theta \rangle$$

Plugging in the angle we've been given, we get

$$\vec{u} = \left\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4} \right\rangle$$

$$\vec{u} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

To get the directional derivative, we'll use

$$D_u f(x, y) = a\left(\frac{\partial F}{\partial x}\right) + b\left(\frac{\partial F}{\partial y}\right)$$

where a and b come from the unit vector $\vec{u} = \langle a, b \rangle$ we found earlier. All we need now are the first order partial derivatives.

$$\frac{\partial F}{\partial x} = 4xy$$

and

$$\frac{\partial F}{\partial y} = 2x^2 - 4y$$

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Since we were asked to find $D_u f(1,1)$, we need to evaluate the partial derivatives at $(1,1)$.

$$\frac{\partial F}{\partial x}(1,1) = 4(1)(1)$$

$$\frac{\partial F}{\partial x}(1,1) = 4$$

and

$$\frac{\partial F}{\partial y}(1,1) = 2(1)^2 - 4(1)$$

$$\frac{\partial F}{\partial y}(1,1) = -2$$

Plugging everything into the formula for the directional derivative, we get

$$D_u f(1,1) = \frac{\sqrt{2}}{2}(4) + \frac{\sqrt{2}}{2}(-2)$$

$$D_u f(1,1) = 2\sqrt{2} - \sqrt{2}$$

$$D_u f(1,1) = \sqrt{2}$$

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Topic: Directional derivatives in the direction of the angle

Question: Find the directional derivative.

$$D_u f(-1,0)$$

$$f(x, y) = 2e^{xy} - 3x^2y^3$$

$\vec{u}$ is the unit vector toward $\theta = \dfrac{2\pi}{3}$

Answer choices:

A $\quad D_u f(1,1) = -\dfrac{\sqrt{3}}{2}$

B $\quad D_u f(-1,0) = -\sqrt{3}$

C $\quad D_u f(1,1) = \dfrac{\sqrt{3}}{2}$

D $\quad D_u f(-1,0) = \sqrt{3}$

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Solution: B

We need to change the unit vector so that it's in the direction of a specific point, instead of in the direction of an angle. To do that we'll use the formula

$$\vec{u} = \langle \cos\theta, \sin\theta \rangle$$

Plugging in the angle we've been given, we get

$$\vec{u} = \left\langle \cos\frac{2\pi}{3}, \sin\frac{2\pi}{3} \right\rangle$$

$$\vec{u} = \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

To get the directional derivative, we'll use

$$D_u f(x, y) = a\left(\frac{\partial F}{\partial x}\right) + b\left(\frac{\partial F}{\partial y}\right)$$

where a and b come from the unit vector $\vec{u} = \langle a, b \rangle$ we found earlier. All we need now are the first order partial derivatives.

$$\frac{\partial F}{\partial x} = 2ye^{xy} - 6xy^3$$

and

$$\frac{\partial F}{\partial y} = 2xe^{xy} - 9x^2y^2$$

Since we were asked to find $D_u f(-1,0)$, we need to evaluate the partial derivatives at $(-1,0)$.

$$\frac{\partial F}{\partial x}(-1,0) = 2(0)e^{(-1)(0)} - 6(-1)(0)^3$$

$$\frac{\partial F}{\partial x}(-1,0) = 0$$

and

$$\frac{\partial F}{\partial y}(-1,0) = 2(-1)e^{(-1)(0)} - 9(-1)^2(0)^2$$

$$\frac{\partial F}{\partial y}(-1,0) = -2$$

Plugging everything into the formula for the directional derivative, we get

$$D_u f(-1,0) = -\frac{1}{2}(0) + \frac{\sqrt{3}}{2}(-2)$$

$$D_u f(-1,0) = -\sqrt{3}$$

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Topic: Directional derivatives in the direction of the angle

Question: Find the directional derivative.

$$D_u f(0,2)$$

$$f(x, y) = 4x^2e^{2y} - 2xy^4 + 5e^x$$

$\vec{u}$ is the unit vector toward $\theta = \dfrac{\pi}{4}$

Answer choices:

A $\quad D_u f(0,2) = -\dfrac{27\sqrt{2}}{2}$

B $\quad D_u f(0,2) = -27\sqrt{2}$

C $\quad D_u f(0,2) = 27\sqrt{2}$

D $\quad D_u f(0,2) = \dfrac{27\sqrt{2}}{2}$

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Solution: A

We need to change the unit vector so that it's in the direction of a specific point, instead of in the direction of an angle. To do that we'll use the formula

$$\vec{u} = \langle \cos\theta, \sin\theta \rangle$$

Plugging in the angle we've been given, we get

$$\vec{u} = \left\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4} \right\rangle$$

$$\vec{u} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

To get the directional derivative, we'll use

$$D_u f(x, y) = a\left(\frac{\partial F}{\partial x}\right) + b\left(\frac{\partial F}{\partial y}\right)$$

where a and b come from the unit vector $\vec{u} = \langle a, b \rangle$ we found earlier. All we need now are the first order partial derivatives.

$$\frac{\partial F}{\partial x} = 8xe^{2y} - 2y^4 + 5e^x$$

and

$$\frac{\partial F}{\partial y} = 8x^2e^{2y} - 8xy^3$$

Since we were asked to find $D_uf(0,2)$, we need to evaluate the partial derivatives at $(0,2)$.

$$\frac{\partial F}{\partial x}(0,2) = 8(0)e^{2(2)} - 2(2)^4 + 5e^0$$

$$\frac{\partial F}{\partial x}(0,2) = -27$$

and

$$\frac{\partial F}{\partial y}(0,2) = 8(0)^2e^{2(2)} - 8(0)(2)^3$$

$$\frac{\partial F}{\partial y}(0,2) = 0$$

Plugging everything into the formula for the directional derivative, we get

$$D_uf(0,2) = \frac{\sqrt{2}}{2}(-27) + \frac{\sqrt{2}}{2}(0)$$

$$D_uf(0,2) = -\frac{27\sqrt{2}}{2}$$

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