Topic: Unit tangent and unit normal vectors

Question: Find the unit tangent vector.

$$r(t) = 2t\mathbf{i} - \sin t\mathbf{j} + \cos t\mathbf{k}$$

Answer choices:

A $\quad T(t) = -\frac{2}{\sqrt{5}}\mathbf{i} + \frac{\sin t}{\sqrt{5}}\mathbf{j} + \frac{\cos t}{\sqrt{5}}\mathbf{k}$

B $\quad T(t) = \frac{2}{\sqrt{5}}\mathbf{i} - \frac{\cos t}{\sqrt{5}}\mathbf{j} - \frac{\sin t}{\sqrt{5}}\mathbf{k}$

C $\quad T(t) = \frac{2}{\sqrt{5}}\mathbf{i} - \frac{\sin t}{\sqrt{5}}\mathbf{j} - \frac{\cos t}{\sqrt{5}}\mathbf{k}$

D $\quad T(t) = -\frac{2}{\sqrt{5}}\mathbf{i} + \frac{\cos t}{\sqrt{5}}\mathbf{j} + \frac{\sin t}{\sqrt{5}}\mathbf{k}$

Solution: B

We'll first find the derivative of the vector function.

$$r(t) = 2t\mathbf{i} - \sin t\mathbf{j} + \cos t\mathbf{k}$$

$$r'(t) = 2\mathbf{i} - \cos t\mathbf{j} - \sin t\mathbf{k}$$

Then we'll find the magnitude of the derivative.

$$\left| r'(t) \right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left| r'(t) \right| = \sqrt{(2)^2 + (-\cos t)^2 + (-\sin t)^2}$$

$$\left| r'(t) \right| = \sqrt{4 + \cos^2 t + \sin^2 t}$$

$$\left| r'(t) \right| = \sqrt{4 + 1}$$

$$\left| r'(t) \right| = \sqrt{5}$$

Now we'll use everything we just found to solve for the unit tangent vector.

$$T(t) = \frac{r'(t)}{\left| r'(t) \right|}$$

$$T(t) = \frac{2\mathbf{i} - \cos t\mathbf{j} - \sin t\mathbf{k}}{\sqrt{5}}$$

$$T(t) = \frac{2}{\sqrt{5}}\mathbf{i} - \frac{\cos t}{\sqrt{5}}\mathbf{j} - \frac{\sin t}{\sqrt{5}}\mathbf{k}$$

This is the unit tangent vector.

Topic: Unit tangent and unit normal vectors

Question: Find the unit tangent and unit normal vectors.

$$r(t) = 5\mathbf{i} + 4\sin t\mathbf{j} + 4\cos t\mathbf{k}$$

Answer choices:

A $\quad T(t) = -\cos t\mathbf{j} + \sin t\mathbf{k} \qquad N(t) = \sin t\mathbf{j} + \cos t\mathbf{k}$

B $\quad T(t) = \sin t\mathbf{j} - \cos t\mathbf{k} \qquad N(t) = -\cos t\mathbf{j} - \sin t\mathbf{k}$

C $\quad T(t) = \cos t\mathbf{j} - \sin t\mathbf{k} \qquad N(t) = -\sin t\mathbf{j} - \cos t\mathbf{k}$

D $\quad T(t) = -\sin t\mathbf{j} + \cos t\mathbf{k} \qquad N(t) = \cos t\mathbf{j} + \sin t\mathbf{k}$

Solution: C

We'll first find the derivative of the vector function.

$$r(t) = 5\mathbf{i} + 4\sin t\mathbf{j} + 4\cos t\mathbf{k}$$

$$r'(t) = 0\mathbf{i} + 4\cos t\mathbf{j} - 4\sin t\mathbf{k}$$

Then we'll find the magnitude of the derivative.

$$\left|r'(t)\right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left|r'(t)\right| = \sqrt{(0)^2 + (4\cos t)^2 + (-4\sin t)^2}$$

$$\left|r'(t)\right| = \sqrt{16\cos^2 t + 16\sin^2 t}$$

$$\left|r'(t)\right| = \sqrt{16\left(\cos^2 t + \sin^2 t\right)}$$

$$\left|r'(t)\right| = \sqrt{16(1)}$$

$$\left|r'(t)\right| = 4$$

Now we'll use everything we just found to solve for the unit tangent vector.

$$T(t) = \frac{r'(t)}{\left|r'(t)\right|}$$

$$T(t) = \frac{0\mathbf{i} + 4\cos t\mathbf{j} - 4\sin t\mathbf{k}}{4}$$

$$T(t) = \frac{0}{4}\mathbf{i} + \frac{4\cos t}{4}\mathbf{j} - \frac{4\sin t}{4}\mathbf{k}$$

$$T(t) = \cos t\mathbf{j} - \sin t\mathbf{k}$$

This is the unit tangent vector, and now we need to find the unit normal vector. We'll take the derivative of the unit tangent vector.

$$T'(t) = -\sin t\mathbf{j} - \cos t\mathbf{k}$$

Then we have to find the magnitude of this derivative.

$$\left|T'(t)\right| = \sqrt{\left[T'(t)_1\right]^2 + \left[T'(t)_2\right]^2 + \left[T'(t)_3\right]^2}$$

$$\left|T'(t)\right| = \sqrt{(0)^2 + (-\sin t)^2 + (-\cos t)^2}$$

$$\left|T'(t)\right| = \sqrt{\sin^2 t + \cos^2 t}$$

$$\left|T'(t)\right| = \sqrt{1}$$

$$\left|T'(t)\right| = 1$$

Now we can use everything we just found to solve for the unit normal vector.

$$N(t) = \frac{T'(t)}{\left|T'(t)\right|}$$

$$N(t) = \frac{-\sin t\mathbf{j} - \cos t\mathbf{k}}{1}$$

$$N(t) = -\sin t\mathbf{j} - \cos t\mathbf{k}$$

This is the unit normal vector.

Topic: Unit tangent and unit normal vectors

Question: Find the unit tangent and unit normal vectors.

$$r(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 4t\mathbf{k}$$

Answer choices:

A $\quad T(t) = \frac{3}{5}\cos t\mathbf{i} - \frac{3}{5}\sin t\mathbf{j} - \frac{4}{5}\mathbf{k} \qquad N(t) = \sin t\mathbf{i} + \cos t\mathbf{j}$

B $\quad T(t) = -\frac{3}{5}\cos t\mathbf{i} + \frac{3}{5}\sin t\mathbf{j} + \frac{4}{5}\mathbf{k} \qquad N(t) = -\sin t\mathbf{i} - \cos t\mathbf{j}$

C $\quad T(t) = \frac{3}{5}\sin t\mathbf{i} - \frac{3}{5}\cos t\mathbf{j} - \frac{4}{5}\mathbf{k} \qquad N(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$

D $\quad T(t) = -\frac{3}{5}\sin t\mathbf{i} + \frac{3}{5}\cos t\mathbf{j} + \frac{4}{5}\mathbf{k} \qquad N(t) = -\cos t\mathbf{i} - \sin t\mathbf{j}$

Solution: D

We'll first find the derivative of the vector function.

$$r(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 4t\mathbf{k}$$

$$r'(t) = -3\sin t\mathbf{i} + 3\cos t\mathbf{j} + 4\mathbf{k}$$

Then we'll find the magnitude of the derivative.

$$\left| r'(t) \right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left| r'(t) \right| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (4)^2}$$

$$\left| r'(t) \right| = \sqrt{9\sin^2 t + 9\cos^2 t + 16}$$

$$\left| r'(t) \right| = \sqrt{9\left(\sin^2 t + \cos^2 t\right) + 16}$$

$$\left| r'(t) \right| = \sqrt{9(1) + 16}$$

$$\left| r'(t) \right| = \sqrt{25}$$

$$\left| r'(t) \right| = 5$$

Now we'll use everything we just found to solve for the unit tangent vector.

$$T(t) = \frac{r'(t)}{\left| r'(t) \right|}$$

$$T(t) = \frac{-3\sin t\mathbf{i} + 3\cos t\mathbf{j} + 4\mathbf{k}}{5}$$

$$T(t) = -\frac{3\sin t}{5}\mathbf{i} + \frac{3\cos t}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$$

$$T(t) = -\frac{3}{5}\sin t\mathbf{i} + \frac{3}{5}\cos t\mathbf{j} + \frac{4}{5}\mathbf{k}$$

This is the unit tangent vector, and now we need to find the unit normal vector. We'll take the derivative of the unit tangent vector.

$$T'(t) = -\frac{3}{5}\cos t\mathbf{i} - \frac{3}{5}\sin t\mathbf{j} + 0\mathbf{k}$$

Then we have to find the magnitude of this derivative.

$$\left|T'(t)\right| = \sqrt{\left[T'(t)_1\right]^2 + \left[T'(t)_2\right]^2 + \left[T'(t)_3\right]^2}$$

$$\left|T'(t)\right| = \sqrt{\left(-\frac{3}{5}\cos t\right)^2 + \left(-\frac{3}{5}\sin t\right)^2 + (0)^2}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}\cos^2 t + \frac{9}{25}\sin^2 t}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}\left(\cos^2 t + \sin^2 t\right)}$$

$$\left|T'(t)\right| = \sqrt{\frac{9}{25}(1)}$$

$$\left|T'(t)\right| = \frac{3}{5}$$

Now we can use everything we just found to solve for the unit normal vector.

$$N(t) = \frac{T'(t)}{\left| T'(t) \right|}$$

$$N(t) = \frac{-\frac{3}{5}\cos t\mathbf{i} - \frac{3}{5}\sin t\mathbf{j} + 0\mathbf{k}}{\frac{3}{5}}$$

$$N(t) = \frac{-\frac{3}{5}\cos t\mathbf{i} - \frac{3}{5}\sin t\mathbf{j}}{\frac{3}{5}}$$

$$N(t) = -\cos t\mathbf{i} - \sin t\mathbf{j}$$

This is the unit normal vector.