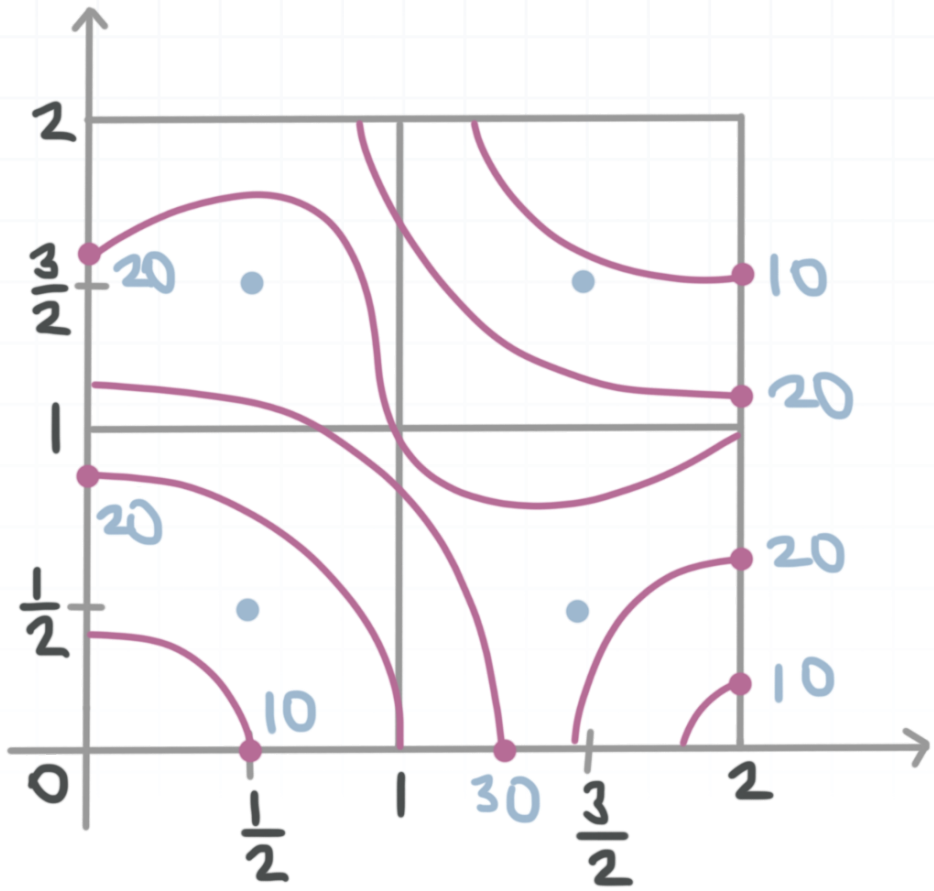
Topic: Average value

Question: Use midpoints to estimate the average value of the region $R = [0,2] \times [0,2]$ with $m = n = 2$ given the sketch of level curves.



Answer choices:

A 30

B 12

C 18

D 25

Solution: C

To approximate the average value over the region, we'll use the average value formula

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x, y) \Delta A$$

where $A(R)$ is the area of the larger rectangle, and ΔA is the area of one of the smaller rectangles.

Because the larger rectangle has a width of 2 and a height of 2, the area is

$$A(R) = (2)(2)$$

$$A(R) = 4$$

The area of a smaller rectangle ΔA, is given by

$$\Delta A = (1)(1)$$

$$\Delta A = 1$$

The midpoints of the smaller rectangles are

$$\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{3}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{3}{2}\right) \text{ and } \left(\frac{3}{2}, \frac{3}{2}\right)$$

Since we don't have the equation of the function, we can use the level curves to estimate the value of the function at each midpoint.

For the point $\left(\frac{1}{2}, \frac{1}{2}\right)$, the function is approximately 15, since it's halfway between level curves with values of 10 and 15.

For the point $\left(\frac{3}{2}, \frac{1}{2}\right)$, the function is approximately 21, since it's just past a level curve with a value of 20, towards a level curve with a value of 30.

For the point $\left(\frac{1}{2}, \frac{3}{2}\right)$, the function is approximately 24, since it's about halfway between level curves with values of 20 and 30.

For the point $\left(\frac{3}{2}, \frac{3}{2}\right)$, the function is approximately 12, since it's just past a level curve with a value of 10, towards a level curve with a value of 20.

Now we're ready to estimate the average value.

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x, y) \Delta A$$

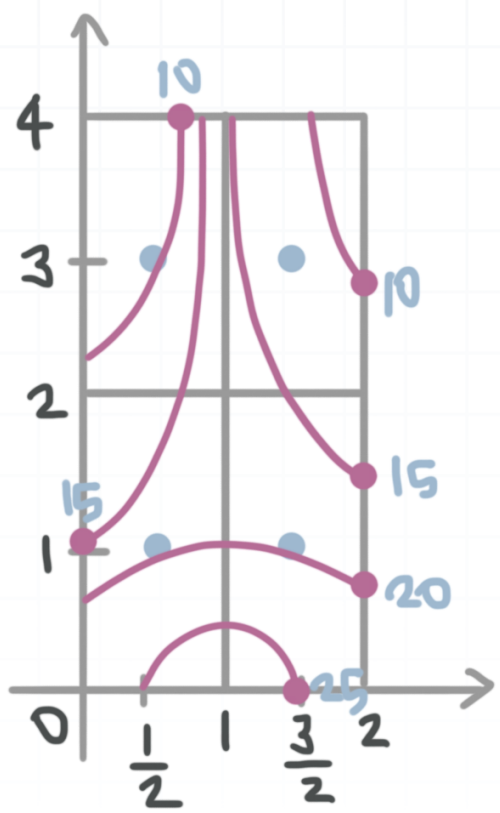
$$f_{avg} = \frac{1}{4} \left(15 + 21 + 24 + 12\right)(1)$$

$$f_{avg} = \frac{1}{4}(72)$$

$$f_{avg} = 18$$

Topic: Average value

Question: Use midpoints to estimate the average value of the region $R = [0,2] \times [0,4]$ with $m = n = 2$ given the sketch of level curves.



Answer choices:

A 16

B 64

C 8

D 32

Solution: A

To approximate the average value over the region, we'll use the average value formula

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x, y) \Delta A$$

where $A(R)$ is the area of the larger rectangle, and ΔA is the area of one of the smaller rectangles.

Because the larger rectangle has a width of 2 and a height of 4, the area is

$$A(R) = (2)(4)$$

$$A(R) = 8$$

The area of a smaller rectangle ΔA, is given by

$$\Delta A = (1)(2)$$

$$\Delta A = 2$$

The midpoints of the smaller rectangles are

$$\left(\frac{1}{2},1\right), \left(\frac{3}{2},1\right), \left(\frac{1}{2},3\right) \text{ and } \left(\frac{3}{2},3\right)$$

Since we don't have the equation of the function, we can use the level curves to estimate the value of the function at each midpoint.

For the point $\left(\frac{1}{2},1\right)$, the function is approximately 20, since it's right next to a level curve with a value of 20.

For the point $\left(\frac{3}{2},1\right)$, the function is approximately 20, since it's right next to a level curve with a value of 20.

For the point $\left(\frac{1}{2},3\right)$, the function is approximately 10, since it's right next to a level curve with a value of 10.

For the point $\left(\frac{3}{2},3\right)$, the function is approximately 15, since it's halfway between level curves with values of 10 and 15.

Now we're ready to estimate the average value.

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x,y)\Delta A$$

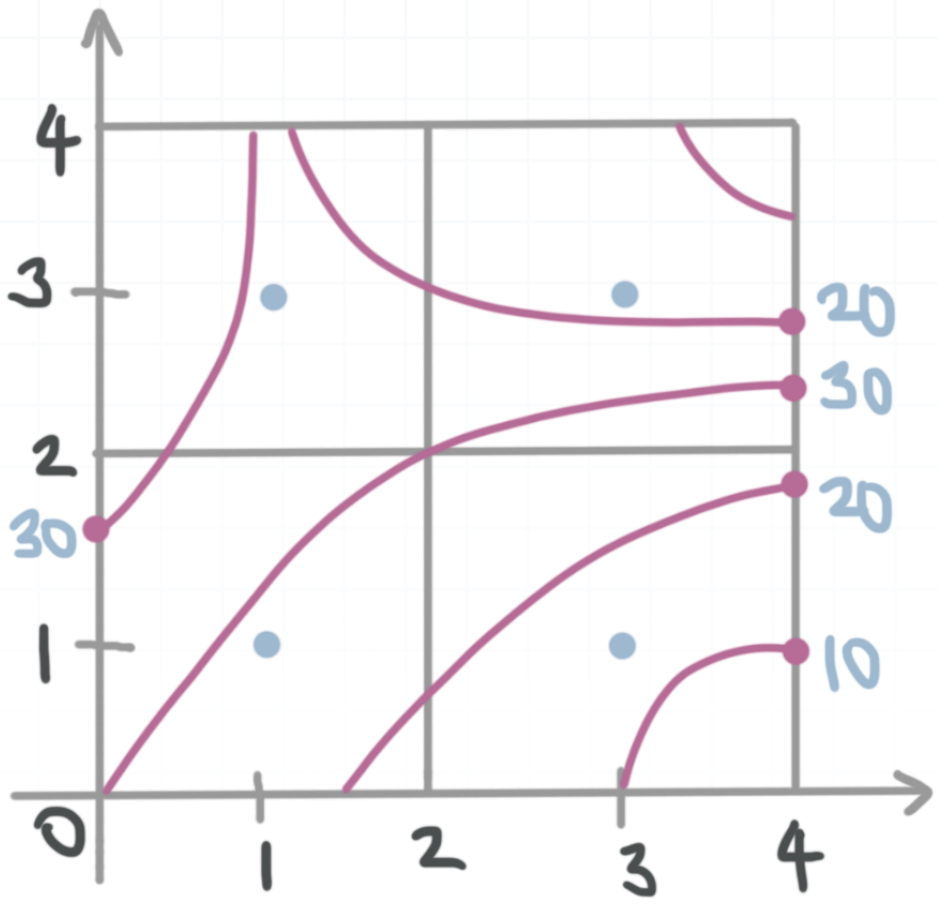
$$f_{avg} = \frac{1}{8}\left(20+20+10+15\right)(2)$$

$$f_{avg} = \frac{1}{4}\left(65\right)$$

$$f_{avg} = 16.25$$

Topic: Average value

Question: Use midpoints to estimate the average value of the region $R = [0,4] \times [0,4]$ with $m = n = 2$ given the sketch of level curves.



Answer choices:

A 88

B 11

C 44

D 21

Solution: D

To approximate the average value over the region, we'll use the average value formula

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x, y) \Delta A$$

where $A(R)$ is the area of the larger rectangle, and ΔA is the area of one of the smaller rectangles.

Because the larger rectangle has a width of 2 and a height of 2, the area is

$$A(R) = (4)(4)$$

$$A(R) = 16$$

The area of a smaller rectangle ΔA, is given by

$$\Delta A = (2)(2)$$

$$\Delta A = 4$$

The midpoints of the smaller rectangles are

$(1,1)$, $(3,1)$, $(1,3)$ and $(3,3)$

Since we don't have the equation of the function, we can use the level curves to estimate the value of the function at each midpoint.

For the point $(1,1)$, the function is approximately 29, since it's right next to a level curve with a value of 30, towards a level curve with a value of 20.

For the point $(3,1)$, the function is approximately 15, since it's halfway between level curves with values of 10 and 20.

For the point $(1,3)$, the function is approximately 29, since it's right next to a level curve with a value of 30, towards a level curve with a value of 20.

For the point $(3,3)$, the function is approximately 10, since it's right next to a level curve with a value of 10.

Now we're ready to estimate the average value.

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x,y) \Delta A$$

$$f_{avg} = \frac{1}{16} \big(29 + 15 + 29 + 10\big)(4)$$

$$f_{avg} = \frac{1}{4} (83)$$

$$f_{avg} = 20.75$$