Partial derivatives in three or more variables

Sometimes we need to find partial derivatives for functions with three or more variables, and we'll do it the same way we found partial derivatives for functions in two variables.

We'll take the derivative of the function with respect to each variable separately, which means we'll end up with one partial derivative for each of our variables.

When we take the derivative with respect to one variable, we'll treat all the other variables as constants.

Example

Find the partial derivatives of the function.

$$f(x, y, z) = 3x^5y^2z^3$$

With three variables x, y, and z, we need to find three partial derivatives. When we take the partial derivative with respect to one variable, we'll hold all others constant.

$$\frac{\partial f}{\partial x} = 3\left(5x^4\right)y^2z^3$$

$$\frac{\partial f}{\partial x} = 15x^4y^2z^3$$

and

$$\frac{\partial f}{\partial y} = 3x^5(2y)z^3$$

$$\frac{\partial f}{\partial y} = 6x^5yz^3$$

and

$$\frac{\partial f}{\partial z} = 3x^5y^2\left(3z^2\right)$$

$$\frac{\partial f}{\partial z} = 9x^5y^2z^2$$

$\partial f/\partial x$ is the partial derivative of the function f with respect to x, $\partial f/\partial y$ is the partial derivative of the function f with respect to y, and $\partial f/\partial z$ is the partial derivative of the function f with respect to z.

Let's try a more complex example with more than three variables.

Example

Find the partial derivatives of the function.

$$f(w, x, y, z) = 2wx^4 - \frac{3y^3}{7z^2} + \sin 2x$$

With four variables w, x, y, and z, we need to find four partial derivatives. When we take the partial derivative with respect to one variable, we'll hold all others constant.

$$\frac{\partial f}{\partial w} = 2x^4$$

and

$$\frac{\partial f}{\partial x} = 2w\left(4x^3\right) + (2)\cos 2x$$

$$\frac{\partial f}{\partial x} = 8wx^3 + 2\cos 2x$$

and

$$\frac{\partial f}{\partial y} = -\frac{3\left(3y^2\right)}{7z^2}$$

$$\frac{\partial f}{\partial y} = -\frac{9y^2}{7z^2}$$

and

$$\frac{\partial f}{\partial z} = -\frac{3y^3}{7}(-2)z^{-3}$$

$$\frac{\partial f}{\partial z} = \frac{6y^3}{7z^3}$$

$\partial f/\partial w$ is the partial derivative of the function f with respect to w, $\partial f/\partial x$ is the partial derivative of the function f with respect to x, $\partial f/\partial y$ is the partial

derivative of the function f with respect to y, and $\partial f/\partial z$ is the partial derivative of the function f with respect to z.
