Topic: Finding area

Question: Find the area given by the double polar integral.

$$\int_0^{2\pi} \int_0^2 r \ dr \ d\theta$$

Answer choices:

A $\frac{\pi}{4}$

B 2π

C 4π

D $\frac{\pi}{2}$

Solution: C

To find area, we just need to evaluate the double integral. We always integrate from the inside out, which means we'll integrate first with respect to r.

$$\int_0^{2\pi}\int_0^2 r\ dr\ d\theta$$

$$\int_0^{2\pi}\int_0^2 r\ dr\ d\theta = \int_0^{2\pi} \frac{1}{2}r^2\Big|_0^2\ d\theta$$

$$\int_0^{2\pi}\int_0^2 r\ dr\ d\theta = \int_0^{2\pi} \frac{1}{2}(2)^2 - \frac{1}{2}(0)^2\ d\theta$$

$$\int_0^{2\pi}\int_0^2 r\ dr\ d\theta = \int_0^{2\pi} 2\ d\theta$$

Integrate with respect to θ.

$$\int_0^{2\pi}\int_0^2 r\ dr\ d\theta = 2\theta\Big|_0^{2\pi}$$

$$\int_0^{2\pi}\int_0^2 r\ dr\ d\theta = 2(2\pi) - 2(0)$$

$$\int_0^{2\pi}\int_0^2 r\ dr\ d\theta = 4\pi$$

This is the area given by the double integral.

Topic: Finding area

Question: Find the area given by the double polar integral.

$$\int_0^{\frac{\pi}{2}} \int_0^4 e^r \, dr \, d\theta$$

Answer choices:

A $\quad \dfrac{\pi}{2}(e^4 - 1)$

B $\quad e^3\pi$

C $\quad e^4\pi$

D $\quad \dfrac{\pi}{2}(e^3 - 1)$

Solution: A

To find area, we just need to evaluate the double integral. We always integrate from the inside out, which means we'll integrate first with respect to r.

$$\int_0^{\frac{\pi}{2}}\int_0^4 e^r\ dr\ d\theta$$

$$\int_0^{\frac{\pi}{2}}\int_0^4 e^r\ dr\ d\theta=\int_0^{\frac{\pi}{2}} e^r\Big|_0^4\ d\theta$$

$$\int_0^{\frac{\pi}{2}}\int_0^4 e^r\ dr\ d\theta=\int_0^{\frac{\pi}{2}} e^4-e^0\ d\theta$$

$$\int_0^{\frac{\pi}{2}}\int_0^4 e^r\ dr\ d\theta=\int_0^{\frac{\pi}{2}} e^4-1\ d\theta$$

Integrate with respect to θ.

$$\int_0^{\frac{\pi}{2}}\int_0^4 e^r\ dr\ d\theta=e^4\theta-\theta\Big|_0^{\frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}}\int_0^4 e^r\ dr\ d\theta=e^4\left(\frac{\pi}{2}\right)-\frac{\pi}{2}-\left(e^4(0)-(0)\right)$$

$$\int_0^{\frac{\pi}{2}}\int_0^4 e^r\ dr\ d\theta=\frac{\pi}{2}(e^4-1)$$

This is the area given by the double integral.

Topic: Finding area

Question: Find the area of the region.

The region D where D is bounded by $y = \pm \sqrt{1 - x^2}$

$$\iint_D x^2 + y^2 \ dA$$

Answer choices:

A 2π

B $\frac{\pi}{4}$

C π

D $\frac{\pi}{2}$

Solution: D

We need to turn the given integral

$$\iint_D x^2 + y^2 \ dA$$

into an iterated integral.

We've been told that the region D is bounded by $y = \pm\sqrt{1 - x^2}$. If we rearrange D, we get

$$y = \pm\sqrt{1 - x^2}$$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

This is the circle centered at the origin with radius 1, which means we can define it as a type I region. For a type I region, we'll integrate first with respect to y and then with respect to x. Therefore, the integral will be

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2 \ dy \ dx$$

This integral will be easier to handle in polar form, so we'll convert it, remembering that$r^2 = x^2 + y^2$ and that $dy \ dx = r \ dr \ d\theta$. We'll need to remember to change the limits of integration as well. We know that the region D is everything inside the circle with radius 1, so the bounds for r become $[0,1]$. Since we're dealing with the entire circle, the bounds for θ will be $[0,2\pi]$. The integral becomes

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2\ dy\ dx=\int_0^{2\pi}\int_0^1 r^2(r)\ dr\ d\theta$$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2\ dy\ dx=\int_0^{2\pi}\int_0^1 r^3\ dr\ d\theta$$

Now we'll integrate with respect to r.

$$\int_0^{2\pi}\int_0^1 r^3\ dr\ d\theta=\int_0^{2\pi}\frac{1}{4}r^4\Big|_0^1\ d\theta$$

$$\int_0^{2\pi}\int_0^1 r^3\ dr\ d\theta=\int_0^{2\pi}\frac{1}{4}(1)^4-\frac{1}{4}(0)^4\ d\theta$$

$$\int_0^{2\pi}\int_0^1 r^3\ dr\ d\theta=\int_0^{2\pi}\frac{1}{4}\ d\theta$$

Integrate with respect to θ.

$$\int_0^{2\pi}\int_0^1 r^3\ dr\ d\theta=\frac{1}{4}\theta\Big|_0^{2\pi}$$

$$\int_0^{2\pi}\int_0^1 r^3\ dr\ d\theta=\frac{1}{4}(2\pi)-\frac{1}{4}(0)$$

$$\int_0^{2\pi}\int_0^1 r^3\ dr\ d\theta=\frac{\pi}{2}$$

This is the area given by the double integral.