Topic: Parallel, intersecting, skew and perpendicular lines

Question: Are the lines parallel, intersecting, skew, or perpendicular?

L_1: $x_1 = 1 - t$ $\quad y_1 = 1 + 2t$ $\quad z_1 = 1 + t$

L_2: $x_2 = 2 - 2s$ $\quad y_2 = 2 + s$ $\quad z_2 = 3 - s$

Answer choices:

A Parallel

B Intersecting

C Intersecting and perpendicular

D Skew

Solution: B

If we match the lines we've been given to

$$L_1: \quad x_1 = a_1 + b_1 t \qquad y_1 = c_1 + d_1 t \qquad z_1 = e_1 + f_1 t$$

$$L_2: \quad x_2 = a_2 + b_2 s \qquad y_2 = c_2 + d_2 s \qquad z_2 = e_2 + f_2 s$$

we can plug into the ratio equality to get

$$\frac{b_1}{b_2} = \frac{d_1}{d_2} = \frac{f_1}{f_2}$$

$$\frac{-1}{-2} = \frac{2}{1} = \frac{1}{-1}$$

$$\frac{1}{2} = 2 = -1$$

Because this equation is false, we know that the lines are not parallel. So we'll test to see if the lines are intersecting. In order to do so, we'll need to treat the lines as a system of equations. Because we have two equations for x, we can set them equal to each other. We can do the same for y and z.

[1] $1 - t = 2 - 2s$

[2] $1 + 2t = 2 + s$

[3] $1 + t = 3 - s$

We'll solve [1] for t and then plug it into [2].

[1] $1 - t = 2 - 2s$

$$-t = 1 - 2s$$

[4] $t = -1 + 2s$

Plugging **[4]** into **[2]** gives

$$1 + 2(-1 + 2s) = 2 + s$$

$$1 - 2 + 4s = 2 + s$$

$$3s = 3$$

[5] $s = 1$

Now we'll plug **[5]** into **[4]** to find a value for t.

$$t = -1 + 2(1)$$

$$t = -1 + 2$$

[6] $t = 1$

Plugging **[5]** and **[6]** into **[3]**, the only equation we haven't used yet, gives

[3] $1 + t = 3 - s$

$$1 + 1 = 3 - 1$$

$$2 = 2$$

Because this equation is true, we know that the lines are intersecting. If this equation was false, we would have shown that the lines were not parallel, and not intersecting, which would prove that they must be skew.

To see whether or not these intersecting lines are perpendicular, we'll take their dot product.

$$a \cdot b = (-1)(-2) + (2)(1) + (1)(-1)$$

$$a \cdot b = 2 + 2 - 1$$

$$a \cdot b = 3$$

Because the dot product isn't 0, the lines are intersecting, but not perpendicular.

Topic: Parallel, intersecting, skew and perpendicular lines

Question: Are the lines parallel, intersecting, skew, or perpendicular?

L_1: $x_1 = 3 + 4t$ $\quad y_1 = -2 + 3t$ $\quad z_1 = 1 - t$

L_2: $x_2 = -2 + 8s$ $\quad y_2 = 4 + 6s$ $\quad z_2 = 1 - 2s$

Answer choices:

A Parallel

B Intersecting

C Intersecting and perpendicular

D Skew

Solution: A

If we match the lines we've been given to

$$L_1: \quad x_1 = a_1 + b_1 t \quad y_1 = c_1 + d_1 t \quad z_1 = e_1 + f_1 t$$

$$L_2: \quad x_2 = a_2 + b_2 s \quad y_2 = c_2 + d_2 s \quad z_2 = e_2 + f_2 s$$

we can plug into the ratio equality to get

$$\frac{b_1}{b_2} = \frac{d_1}{d_2} = \frac{f_1}{f_2}$$

$$\frac{4}{8} = \frac{3}{6} = \frac{-1}{-2}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

Because this equation is true, we know that the lines are parallel.

Topic: Parallel, intersecting, skew and perpendicular lines

Question: Are the lines parallel, intersecting, skew, or perpendicular?

L_1: $x_1 = 1 - 3t$ $y_1 = -3 + 5t$ $z_1 = 4 + t$

L_2: $x_2 = 2 - 4s$ $y_2 = 6 + 4s$ $z_2 = 3 + 8s$

Answer choices:

A Parallel

B Intersecting

C Intersecting and perpendicular

D Skew

Solution: D

If we match the lines we've been given to

$$L_1: \quad x_1 = a_1 + b_1 t \qquad y_1 = c_1 + d_1 t \qquad z_1 = e_1 + f_1 t$$

$$L_2: \quad x_2 = a_2 + b_2 s \qquad y_2 = c_2 + d_2 s \qquad z_2 = e_2 + f_2 s$$

we can plug into the ratio equality to get

$$\frac{b_1}{b_2} = \frac{d_1}{d_2} = \frac{f_1}{f_2}$$

$$\frac{-3}{-4} = \frac{5}{4} = \frac{1}{8}$$

$$\frac{3}{4} = \frac{5}{4} = \frac{1}{8}$$

Because this equation is false, we know that the lines are not parallel. So we'll test to see if the lines are intersecting. In order to do so, we'll need to treat the lines as a system of equations. Because we have two equations for x, we can set them equal to each other. We can do the same for y and z.

[1] $1 - 3t = 2 - 4s$

[2] $-3 + 5t = 6 + 4s$

[3] $4 + t = 3 + 8s$

We'll solve [1] for t and then plug it into [2].

[1] $1 - 3t = 2 - 4s$

$$-3t = 1 - 4s$$

$$[4] \quad t = -\frac{1}{3} + \frac{4}{3}s$$

Plugging [4] into [2] gives

$$-3 + 5\left(-\frac{1}{3} + \frac{4}{3}s\right) = 6 + 4s$$

$$-3 - \frac{5}{3} + \frac{20}{3}s = 6 + 4s$$

$$\frac{20}{3}s - 4s = 6 + 3 + \frac{5}{3}$$

$$20s - 12s = 18 + 9 + 5$$

$$8s = 32$$

$$[5] \quad s = 4$$

Now we'll plug [5] into [4] to find a value for t.

$$t = -\frac{1}{3} + \frac{4}{3}(4)$$

$$t = \frac{15}{3}$$

$$[6] \quad t = 5$$

Plugging [5] and [6] into [3], the only equation we haven't used yet, gives

$$[3] \quad 4 + t = 3 + 8s$$

$$4 + 5 = 3 + 8(4)$$

$$9 = 35$$

Because this equation is false, we know that the lines are not intersecting. Since they are not parallel, and not intersecting, they must be skew.