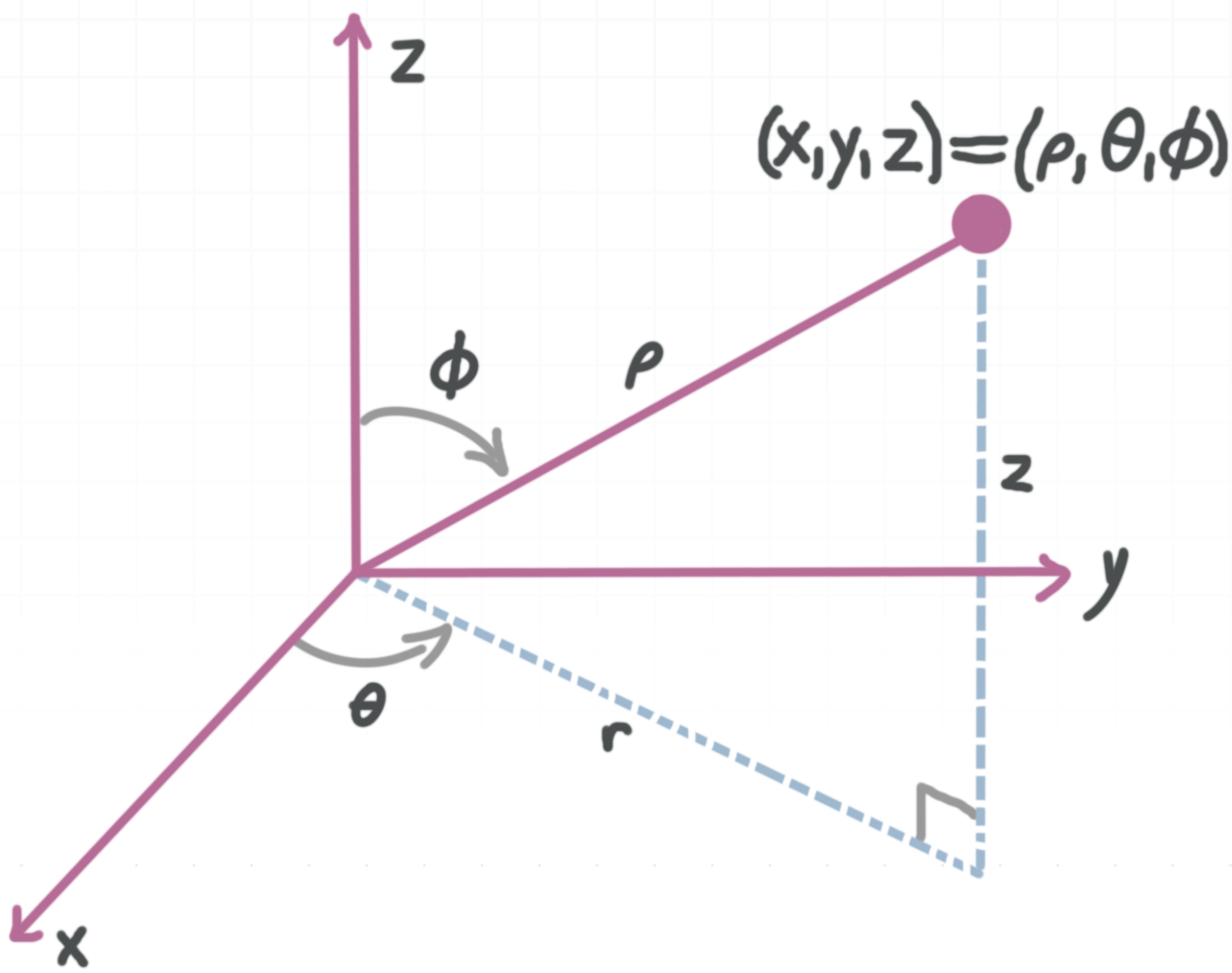
# Spherical coordinates

Like cartesian (or rectangular) coordinates and polar coordinates, spherical coordinates are just another way to describe points in three-dimensional space.




**Rectangular** coordinates are given as $(x, y, z)$

where $x$ is the distance of $(x, y, z)$ from the origin along the $x$-axis

$y$ is the distance of $(x, y, z)$ from the origin along the $y$-axis

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$z$ is the distance of $(x, y, z)$ from the origin along the $z$-axis

**Spherical** coordinates are given as $(\rho, \theta, \phi)$

where $\rho$ is the distance of $(\rho, \theta, \phi)$ from the origin, $p \geq 0$

$\theta$ is the angle between $r$ (the shadow of the line connecting $(\rho, \theta, \phi)$ to the origin) and the positive direction of the $x$-axis

$\phi$ is the angle between the line connecting $(\rho, \theta, \phi)$ to the origin and the positive direction of the $z$-axis, $0 \leq \phi \leq \pi$

To convert between spherical coordinates and rectangular coordinates, we will need to use the formulas

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

and

$$\rho^2 = x^2 + y^2 + z^2$$

Let's try an example where we convert rectangular coordinates to spherical coordinates.

**Example**

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Convert the rectangular coordinate point to a spherical coordinate point.

$$(0,0,1)$$

We'll start by plugging $(0,0,1)$ into $\rho^2 = x^2 + y^2 + z^2$.

$$\rho^2 = 0^2 + 0^2 + 1^2$$

$$\rho^2 = 1$$

Since $p \geq 0$, $\rho = 1$

We'll plug $(0,0,1)$ and $\rho = 1$ into $z = \rho \cos \phi$ to solve for $\phi$.

$$z = \rho \cos \phi$$

$$1 = (1)\cos \phi$$

$$\cos \phi = 1$$

$$\phi = 0,\ 2\pi$$

Since $0 \leq \phi \leq \pi$, $\phi = 0$

We'll plug $(0,0,1)$, $\rho = 1$, and $\phi = 0$ into $y = \rho \sin \phi \sin \theta$ to solve for $\theta$.

$$y = \rho \sin \phi \sin \theta$$

$$0 = (1)\sin(0)\sin \theta$$

$$0 = (1)(0)\sin \theta$$

Since we have $0$ on the right-hand side, $\theta$ could be any value and the equation would still be true. This makes sense, since the given point is on

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the $z$-axis, and $\theta$ is the angle between $r$ (the shadow of the line connecting $(\rho, \theta, \phi)$ to the origin) and the positive direction of the $x$-axis.

Since it can be any value, let's just choose $\theta = 0$.

Putting these values together, we can say that the spherical coordinate $(1,0,0)$ is the same as the rectangular coordinate $(0,0,1)$.

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Let's try an example where we convert spherical coordinates to rectangular coordinates.

**Example**

Convert the spherical coordinate point to a rectangular coordinate point.

$$\left(1,\pi,\frac{\pi}{2}\right)$$

We know that

$$\rho = 1$$

$$\phi = \frac{\pi}{2}$$

$$\theta = \pi$$

Plugging these into the conversion formulas, we get

$$x = \rho \sin \phi \cos \theta$$

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$$x = (1)\sin\frac{\pi}{2}\cos\pi$$

$$x = (1)(1)(-1)$$

$$x = -1$$

and

$$y = \rho\sin\phi\sin\theta$$

$$y = (1)\sin\frac{\pi}{2}\sin\pi$$

$$y = (1)(1)(0)$$

$$y = 0$$

and

$$z = \rho\cos\phi$$

$$z = (1)\cos\frac{\pi}{2}$$

$$z = (1)(0)$$

$$z = 0$$

The rectangular coordinate $(-1,0,0)$ is the same as the spherical coordinate $\left(1,\pi,\frac{\pi}{2}\right)$.

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