Calculus 3 Workbook

Lines and planes

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MATH

VECTOR, PARAMETRIC, AND SYMMETRIC EQUATIONS OF A LINE

■ 1. Find the vector equation of the line that passes through $A(1,0,-2)$ and is parallel to the symmetric equation.

$$\frac{x-3}{2} = -\frac{y}{2} = z+1$$

■ 2. Find the parametric equation of the line that passes through $A(2,3,-2)$ and $B(0,-1,5)$.

■ 3. Which line passes through $A(1,0,-4)$?

A $x = 1-2t,\ y = -5t,\ z = 3-4t$

B $x = 2-5t,\ y = 5t,\ z = 2-4t$

C $x = 6-t,\ y = 5-t,\ z = 6-2t$

D $x = 6+t,\ y = 5+t,\ z = 6+t$

PARALLEL, INTERSECTING, SKEW AND PERPENDICULAR LINES

■ 1. For $A(1,0,-1)$, $B(1,3,0)$, $C(0,0,2)$, and $D(-1,-2,3)$, are lines AB and CD parallel, intersecting, skew, or perpendicular?

■ 2. Find the line L_2 that passes through $A(1,0,1)$ and is perpendicular to L_1.

$$L_1: \quad x = 2 - 2t,\ y = t,\ z = 3 + t$$

■ 3. Which line is perpendicular to L_1?

$$L_1: \quad x = 2t,\ y = 21 - t,\ z = 6 - t$$

A $\quad x = 2 - 3t,\ y = 7 + 5t,\ z = 2t$

B $\quad x = 2 + 3t,\ y = 7 + 5t,\ z = t$

C $\quad x = 2 + 3t,\ y = 7 - 5t,\ z = -t$

D $\quad x = 2 - 3t,\ y = 5 - 5t,\ z = -t$

EQUATION OF A PLANE

■ 1. Find the equation of a plane that passes through $A(1,4,-2)$ and is perpendicular to the line.

$$r = \langle 1,3,3 \rangle + t\langle -2,3,1 \rangle$$

■ 2. Find the equation of a plane that passes through $A(1,4,-2)$ and the line given by the parametric equation.

$$x = 2 - 4t,\ y = 3t,\ z = 1 + t$$

■ 3. Which of the lines lie in the plane $2x - y + 3z = 1$? Choose as many of the answer choices as are correct.

A $x = 1 + 2t,\ y = 1 - 3t,\ z = -5t$

B $x = 1 - 2t,\ y = 1 - 5t,\ z = 4t$

C $x = 1 + 4t,\ y = 1 + t,\ z = -3t$

D $x = 1 + 2t,\ y = 1 + t,\ z = -t$

■ 4. Find the equation of a plane that passes through the intersecting lines L_1 and L_2.

$$L_1: \quad \frac{x-2}{2} = \frac{y+3}{3} = \frac{z}{2}$$

$$L_2: \quad \frac{x-2}{-1} = \frac{y+3}{2} = \frac{z}{5}$$

■ 5. Find the equation of a plane that passes through the parallel lines L_1 and L_2.

$$L_1: \quad r = \langle 1,2,-4 \rangle + t\langle 0,1,-1 \rangle$$

$$L_2: \quad r = \langle 2,-3,0 \rangle + t\langle 0,1,-1 \rangle$$

INTERSECTION OF A LINE AND A PLANE

■ 1. Find the x-, y-, and z-intercepts of the plane.

$$2x - 3y + z = 6$$

■ 2. Find the intersection of AB and the plane $x + 2y + 4z = 12$, where A and B are the points $A(0,1,-2)$ and $B(-1,2,0)$, or determine that the line segment and the plane do no intersect.

■ 3. Find the value of the constant p for which the line doesn't intersect the plane.

The line $\dfrac{x+1}{2} = \dfrac{y}{3} = z - 1$

The plane $px + 2y + z = 4$

PARALLEL, PERPENDICULAR, AND ANGLE BETWEEN PLANES

■ 1. Find the equation of the plane that passes through the points $A(1,0,-1)$ and $B(0,1,-1)$ and is perpendicular to the plane $x+2y+3z=6$.

■ 2. Find the equation of the plane that passes through $A(3,2,-4)$ and is parallel to the plane $-x+3y-2z=4$.

■ 3. Find the equation of the plane a that passes through the point $A(1,-2,-3)$ and form equal angles with all of the coordinate planes, xy, yz, and xz.

PARAMETRIC EQUATIONS FOR THE LINE OF INTERSECTION OF TWO PLANES

■ 1. Find the parametric equations for the line of intersection of the planes with normal vectors $a = \langle 2,0,-1 \rangle$ and $b = \langle 1,2,-3 \rangle$, and that have the common point $A(1,2,2)$.

■ 2. Find the parametric equations for the line of intersection of the plane $2x - 3y - 4z = 2$ with xz-plane.

■ 3. Find the equations of a plane a that's perpendicular to the plane b, which is $x - 3y + z = 2$, and intersects b along the line given by the parametric equation.

$$x = 2t$$

$$y = 1 + t$$

$$z = 2 - t$$

SYMMETRIC EQUATIONS FOR THE LINE OF INTERSECTION OF TWO PLANES

■ 1. Find the symmetric equations for the line of intersection of the plane a, which is $2x - y + 3z = 12$, and the plane a' that's symmetric to the plane a with respect to the xz-plane.

■ 2. Find the symmetric equations for the line of intersection of the plane $6x - 5y + z = 10$ with yz-plane.

■ 3. Find the equations of a plane a that's perpendicular to the plane b, which is $2x - y - z = 3$, and intersects b along the line

$$\frac{x-1}{3} = \frac{y+2}{2} = \frac{z}{2}$$

DISTANCE BETWEEN A POINT AND A LINE

■ 1. Determine the length of the height of triangle ABC, that's perpendicular to BC, if $A(2,0,-1)$, $B(4,5,2)$, and $C(4,3,0)$.

■ 2. Find the sum of distances from $A(3,3,-1)$ to all of the coordinate axes, x, y, and z.

■ 3. Find the distance between $A(0,-2,1)$, and the line.

$$\frac{x+1}{2} = \frac{y-2}{-1} = \frac{z}{2}$$

DISTANCE BETWEEN A POINT AND A PLANE

■ 1. Determine the length of the height of tetrahedron $ABCD$, that's perpendicular to the plane BCD, if $A(1,0,-1)$, $B(0,1,0)$, $C(2,3,4)$, and $D(2,2,2)$.

■ 2. Find the sum of distances from the point $A(2,3,-5)$ to all of the coordinate planes, xy, yz, and xz.

■ 3. Find the points on the line L_1 that lie at a distance of 6 from plane a.

$$L_1: \quad x = 1 + t,\ y = 2t,\ z = 3 - 2t$$

$$a: \quad x + 2y - 2z = 4$$

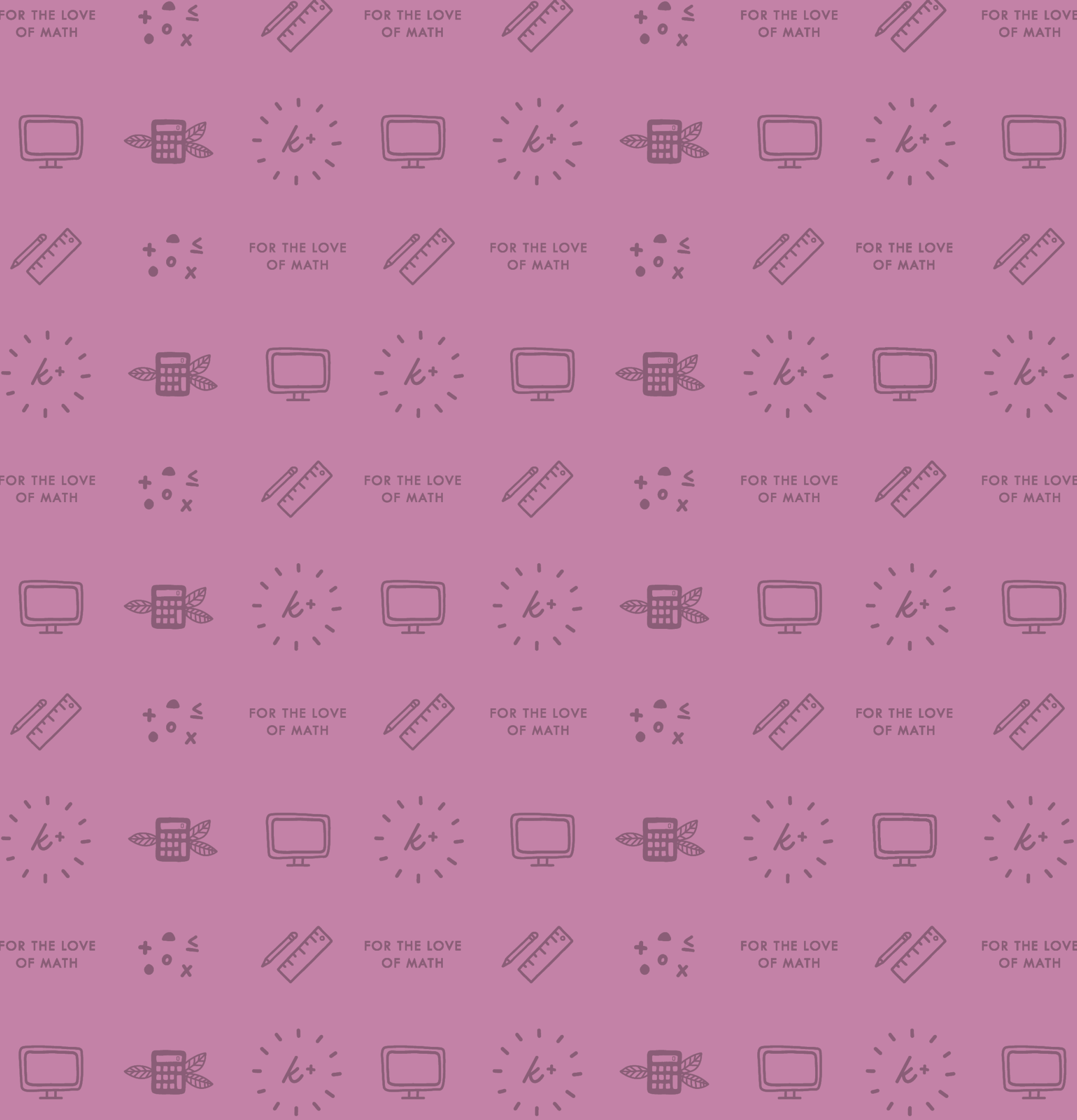
DISTANCE BETWEEN PARALLEL PLANES

■ 1. Determine the length of the height of triangular prism $ABCA_1B_1C_1$ that's perpendicular to the plane ABC if $A(2,0,1)$, $B(1,0,0)$, $C(2,2,3)$, $A_1(3,2,5)$, $B_1(2,2,4)$, and $C_1(3,4,7)$.

■ 2. Find the equations of the two planes that are parallel to the given plane and that lie at a distance of 2 from it.

$$2x - 2y - z = 3$$

■ 3. Find the equations of the two parallel planes a and b that pass through $A(2, -6,3)$ and $B(7,10,11)$ respectively, and have the largest possible distance between them.



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