# Calculus 3 Workbook Solutions

Vector functions and space curves

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## DOMAIN OF A VECTOR FUNCTION

■ 1. Find the domain of the vector function.

$$\overrightarrow{F}(t,s)=\left\langle \sqrt{ts}, \frac{t}{s}, e^{t^2+s^2} \right\rangle$$

*Solution:*

The domain of the vector function is the intersection of the domains of all its components.

$$\text{dom } \overrightarrow{F} = \text{dom } F_1 \cap \text{dom } F_2 \cap \text{dom } F_3$$

Find the domain of the first component.

$$F_1(t,s)=\sqrt{ts}$$

$$ts \geq 0$$

$[t \geq 0, s \geq 0]$ or $[t \leq 0, s \leq 0]$

Find the domain of the second component.

$$F_2(t,s)=\frac{t}{s}$$

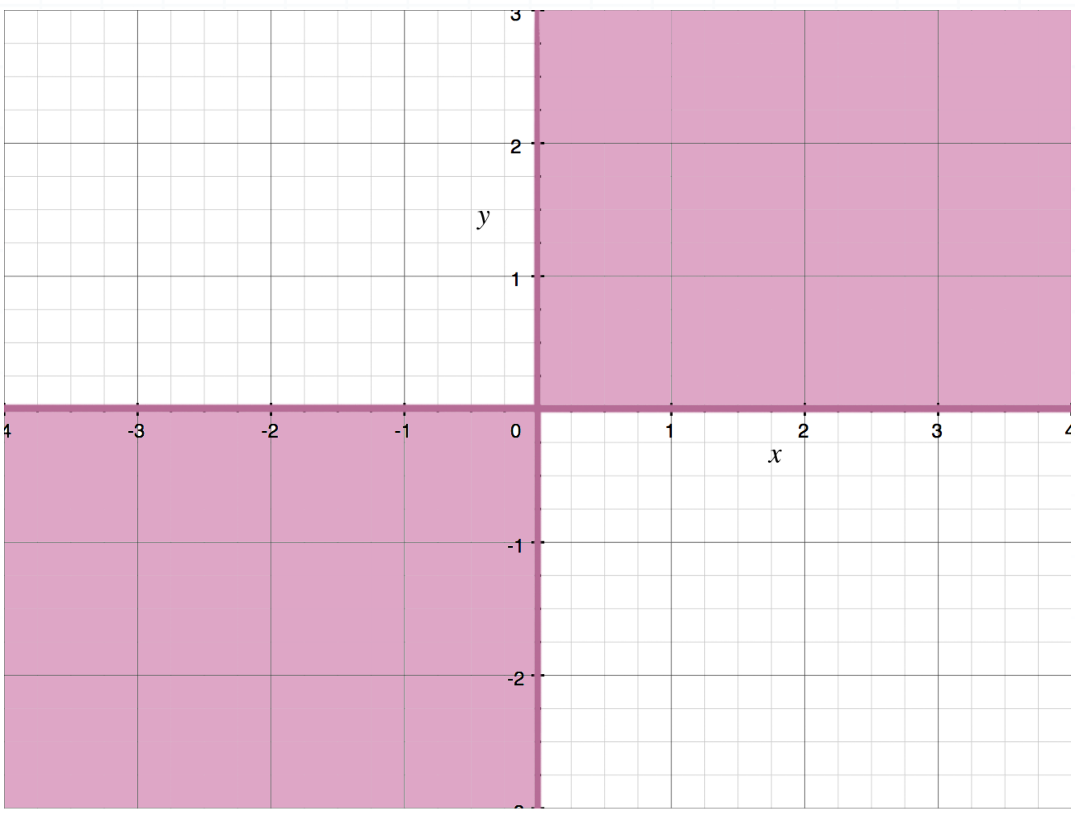
$t$ is any real number, $s \neq 0$

Find the domain of the third component.

$$F_3(t, s) = e^{t^2+s^2}$$

$s$ , $t$ are any real numbers

So the intersection of the domains is $[t \geq 0, s \geq 0]$ or $[t \leq 0, s \leq 0]$, intersected with $[s \neq 0]$. Therefore, $[t \geq 0, s > 0]$ or $[t \leq 0, s < 0]$. So the domain of the vector function is all of the points within the first and third quadrants including the $t$-axis, but excluding the $s$-axis and the origin.




■ 2. Find the domain of the vector function.

$$\vec{F}(x, y) = \ln(x + y - 3) \cdot \mathbf{i} + \sqrt{2x - 2} \cdot \mathbf{j} + \sqrt{6 - y} \cdot \mathbf{k}$$

*Solution:*

The domain of the vector function is the intersection of the domains of all its components.

$$\text{dom } \vec{F} = \text{dom } F_1 \cap \text{dom } F_2 \cap \text{dom } F_3$$

Find the domain of the first component.

$$F_1(x, y) = \ln(x + y - 3)$$

$$x + y - 3 > 0$$

$$y > -x + 3$$

So the domain of $F_1(x, y)$ is all of the points above the line $y = -x + 3$ (excluding the line).

Find the domain of the second component.

$$F_2(x, y) = \sqrt{2x - 2}$$

$$2x - 2 \geq 0$$

$$x \geq 1$$

So the domain of $F_2(x, y)$ is all of the points to the right of the vertical line $x = 1$ (including the line).

Find the domain of the third component.

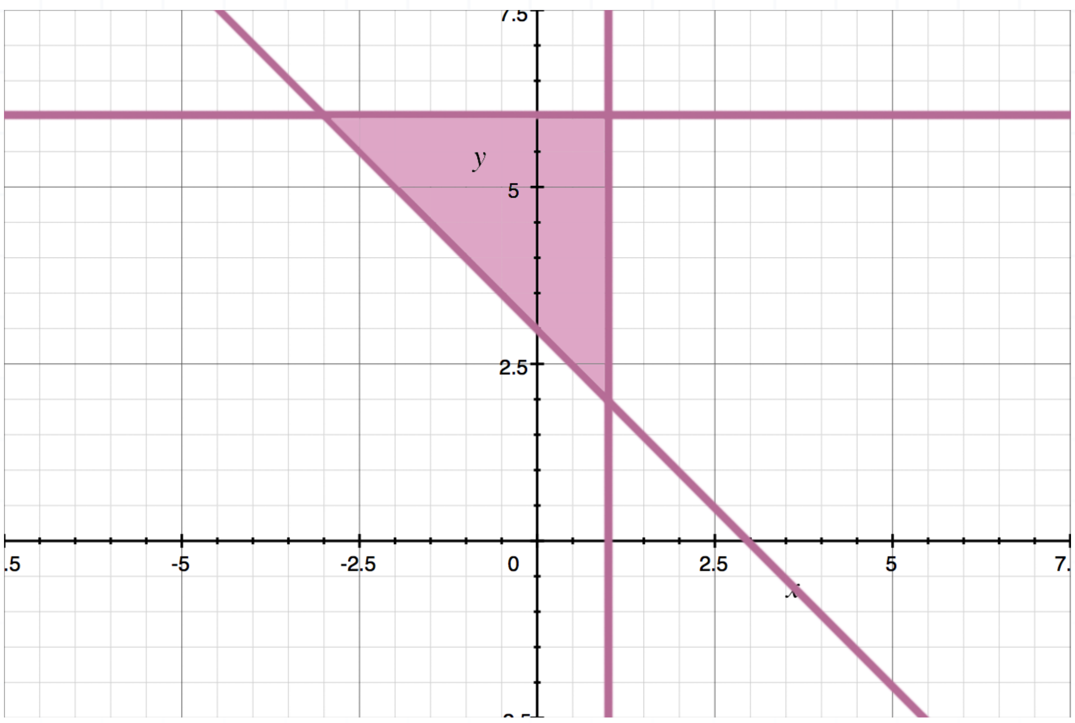
$$F_3(x, y) = \sqrt{6 - y}$$

$$6 - y \geq 0$$

$$y \leq 6$$

So the domain of $F_3(x,y)$ is all of the points below the horizontal line $y=6$ (including the line).

So the intersection of the domains is all of the points within the triangle bounded by the lines $y=-x+3$ (excluding), $x=1$ (including), and $y=6$ (including).




■ 3. Find the domain of the vector function.

$$\vec{F}(x,y,z)=\sqrt{4-x^2-y^2-z^2}\cdot\mathbf{i}+\frac{2x-y}{x+y+z-4}\cdot\mathbf{j}$$

*Solution:*

The domain of the vector function is the intersection of the domains of all its components.

$$\text{dom } \vec{F} = \text{dom } F_1 \cap \text{dom } F_2$$

Find the domain of the first component.

$$F_1(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$$

$$4 - x^2 - y^2 - z^2 \geq 0$$

$$x^2 + y^2 + z^2 \leq 4$$

So the domain of $F_1(x, y, z)$ is the set of interior points of the sphere with center at the origin and radius $2$ (including sphere).

Find the domain of the second component.

$$F_2(x, y, z) = \frac{2x - y}{x + y + z - 4}$$

$$x + y + z - 4 \neq 0$$

So the domain of $F_2(x, y, z)$ is all of the points in space except the plane $x + y + z - 4 = 0$.

Let's find the intersection of the domains. It seems that the plane doesn't intersect the sphere. To check this let's find the distance from the plane to the center of sphere (which is at the origin).

The distance from the point $(x_0, y_0, z_0)$ to the plane $Ax + By + Cz + D = 0$ is given by

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Plug in $(x_0, y_0, z_0) = (0,0,0)$, $A = B = C = 1$, and $D = -4$.

$$d = \frac{|1 \cdot 0 + 1 \cdot 0 + 1 \cdot 0 - 4|}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$d = \frac{4}{\sqrt{3}} \approx 2.3$$

Since the distance from the center of the sphere to the plane is larger than the radius, $2.3 > 2$, the sphere and the plane does not intersect.

Therefore, the domain of the vector function $\vec{F}(x, y, z)$ is the set of interior points of the sphere with center at the origin and radius $2$ (including the sphere itself).

## LIMIT OF A VECTOR FUNCTION

■ 1. Find the limit of the vector function.

$$\lim_{t\to 0, s\to 1} \vec{F}(t,s)$$

$$\vec{F}(t,s) = \left\langle \sqrt{s^2 - t^2}, \frac{\sin 3t}{t} + 3s^2, \frac{(t^2 - 2t - 3)(s^2 - 1)}{s - 1} \right\rangle$$

*Solution:*

Let's find the limit of each of the function's component separately, then evaluate at $t = 0$, $s = 1$.

$$\lim_{t\to 0, s\to 1} F_1(t,s) = \lim_{t\to 0, s\to 1} \sqrt{s^2 - t^2}$$

$$= \sqrt{1^2 - 0^2} = 1$$

$$\lim_{t\to 0, s\to 1} F_2(t,s) = \lim_{t\to 0, s\to 1} \left( \frac{\sin 3t}{t} + 3s^2 \right)$$

$$= \lim_{t\to 0, s\to 1} \frac{\sin 3t}{t} + \lim_{t\to 0, s\to 1} 3s^2$$

$$= \lim_{t\to 0} \frac{\sin 3t}{t} + \lim_{s\to 1} 3s^2$$

$$= \lim_{t\to 0} \frac{\sin 3t}{t} + 3$$

$$= 3 + 3 = 6$$

$$\lim_{t \to 0, s \to 1} F_3(t, s) = \lim_{t \to 0, s \to 1} \frac{(t^2 - 2t - 3)(s^2 - 1)}{s - 1}$$

$$= \lim_{t \to 0, s \to 1} \frac{(t^2 - 2t - 3)(s - 1)(s + 1)}{s - 1}$$

$$= \lim_{t \to 0, s \to 1} (t^2 - 2t - 3)(s + 1)$$

$$= (0^2 - 2 \cdot 0 - 3)(1 + 1) = -6$$

So the limit is $\langle 1, 6, -6 \rangle$.

## 2. Find the limit of the vector function.

$$\lim_{x \to \infty, y \to \infty} \vec{F}(t, s)$$

$$\vec{F}(x, y) = xye^{-(x^2+y^2)} \cdot \mathbf{i} + \frac{\sin(x + y)}{x + y} \cdot \mathbf{j} + \frac{x}{y^4} \cdot \mathbf{k}$$

*Solution:*

Find the limit for each of the function’s component separately.

$$\lim_{x \to \infty, y \to \infty} F_1(x, y) = \lim_{x \to \infty, y \to \infty} xye^{-(x^2+y^2)}$$

$$= \lim_{x \to \infty} xe^{-x^2} \cdot \lim_{y \to \infty} ye^{-y^2}$$

$$= 0 \cdot 0 = 0$$

For the second component, consider

$$\lim_{x \to \infty, y \to \infty} |F_2(x, y)| = \lim_{x \to \infty, y \to \infty} \left| \frac{\sin(x + y)}{x + y} \right| \leq \lim_{x \to \infty, y \to \infty} \left| \frac{1}{x + y} \right| = 0$$

Since $\lim_{x \to \infty, y \to \infty} |F_2(x, y)|$ exists and is $0$, $\lim_{x \to \infty, y \to \infty} F_2(x, y)$ also exists and is $0$.

For the third component,

$$\lim_{x \to \infty, y \to \infty} F_3(x, y) = \lim_{x \to \infty, y \to \infty} \frac{x}{y^4}$$

The limit does not exist. For example, approach $(\infty, \infty)$ along the curve $y = x$. In this case,

$$\lim_{x \to \infty, y \to \infty} \frac{x}{y^4} = \lim_{x \to \infty} \frac{x}{x^4} = \lim_{x \to \infty} \frac{1}{x^3} = 0$$

Next, approach $(\infty, \infty)$ along the curve $y = x^{1/8}$. In this case,

$$\lim_{x \to \infty, y \to \infty} \frac{x}{y^4} = \lim_{x \to \infty} \frac{x}{x^{4/8}} = \lim_{x \to \infty} \sqrt{x} = \infty$$

Since the function approaches different values, the limit does not exist.

Therefore, although the limits of the first two components of the function exist, the overall limit does not exist because the limit of the third component does not exist.

■ 3. Find the limit of the vector function.

$$\lim_{x\to 3,\, y\to\infty,\, z\to 1} \vec{F}(x,y,z)$$

$$\vec{F}(x,y,z) = (x^2y - 3xyz + z^2 - x + 3y - 3z + 5)\mathbf{i} + \ln\frac{x+y}{z+y}\mathbf{j}$$

*Solution:*

Find the limit for each of the function's component separately.

$$\lim_{x\to 3,\, y\to\infty,\, z\to 1} F_1(x,y,z) = \lim_{x\to 3,\, y\to\infty,\, z\to 1} (x^2y - 3xyz + z^2 - x + 3y - 3z + 5)$$

$$= \lim_{x\to 3,\, y\to\infty,\, z\to 1} (y(x^2 - 3xz + 3) + (z^2 - x - 3z + 5))$$

$$= \lim_{x\to 3,\, y\to\infty,\, z\to 1} F_1(x,y,z) = \infty$$

$$\lim_{x\to 3,\, y\to\infty,\, z\to 1} F_1(x,y,z) = \lim_{x\to 3,\, y\to\infty,\, z\to 1} \ln\frac{x+y}{z+y}$$

$$= \lim_{x\to 3,\, y\to\infty,\, z\to 1} \ln\frac{1+\frac{x}{y}}{1+\frac{z}{y}}$$

$$= \ln\frac{1+0}{1+0} = \ln(1) = 0$$

## SKETCHING THE VECTOR EQUATION

■ 1. Identify and sketch the curve that represents $\vec{r}(t) = \langle 3 - 5t, 2t + 1, -3t \rangle$.

*Solution:*

Write the vector function in parametric form.

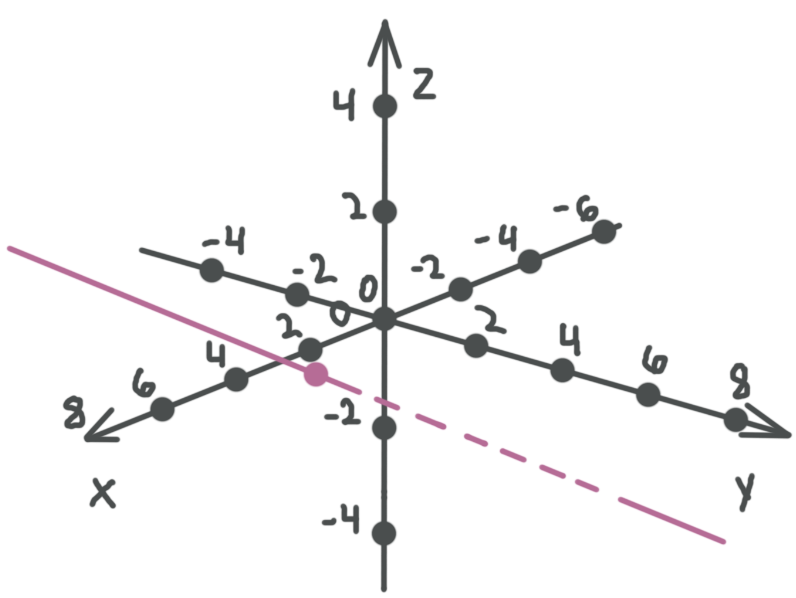
$$x(t) = 3 - 5t$$

$$y(t) = 1 + 2t$$

$$z(t) = -3t$$

These equations represent the line in three dimensions that passes through the point $(3,1,0)$ and has a direction vector of $\langle -5, 2, -3 \rangle$, so we can rewrite the equation of the line in vector form.

$$\vec{r}(t) = \langle 3,1,0 \rangle + t \langle -5, 2, -3 \rangle$$



2. Identify and sketch the curve representing the graph of the vector function $\vec{r}(t) = \langle 5\sin t, 3\cos t, -2 \rangle$.

*Solution:*

Write the vector function in parametric form.

$$x(t) = 5\sin t$$

$$y(t) = 3\cos t$$

$$z(t) = -2$$

Use the trigonometric identity $\sin^2\phi + \cos^2\phi = 1$ to relate $x$ and $y$.

$$(3x)^2 + (5y)^2 = (3 \cdot 5\sin t)^2 + (5 \cdot 3\cos t)^2$$

$$15^2\sin^2 t + 15^2\cos^2 t$$

$$15^2(\sin^2 t + \cos^2 t)$$

$$15^2$$

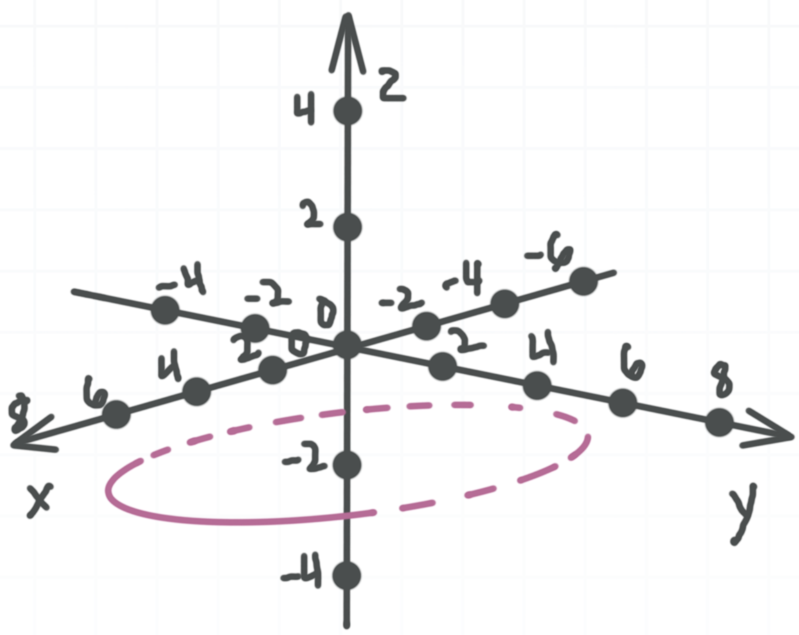
So

$$(3x)^2 + (5y)^2 = 15^2$$

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$$

$$z = -2$$

So the curve is the ellipse that lies in the plane $z = -2$, with center at the point $(0,0,-2)$, semi-axis of $5$ in the $x$-direction, and semi-axis of $3$ in the $y$-direction.




■ 3. Identify and sketch the surface representing the graph of the vector function.

$$\vec{r}(t,s) = \langle 4\sin t\cos s, 4\sin t\sin s, 4\cos t\rangle$$

*Solution:*

Write the vector function in parametric form.

$$x(t,s) = 4\sin t\cos s$$

$$y(t,s) = 4\sin t\sin s$$

$$z(t, s) = 4\cos t$$

These equations are really similar to the formulas we use to convert to spherical coordinates, $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, and $z = \rho\cos\phi$, but in our equations, it looks like $\rho = 4$, $\phi = t$, and $\theta = s$. Use the trigonometric identity $\sin^2\alpha + \cos^2\alpha = 1$ to build a relationship between $x$, $y$, and $z$.

$$x^2 + y^2 + z^2 = (4\sin t\cos s)^2 + (4\sin t\sin s)^2 + (4\cos t)^2$$

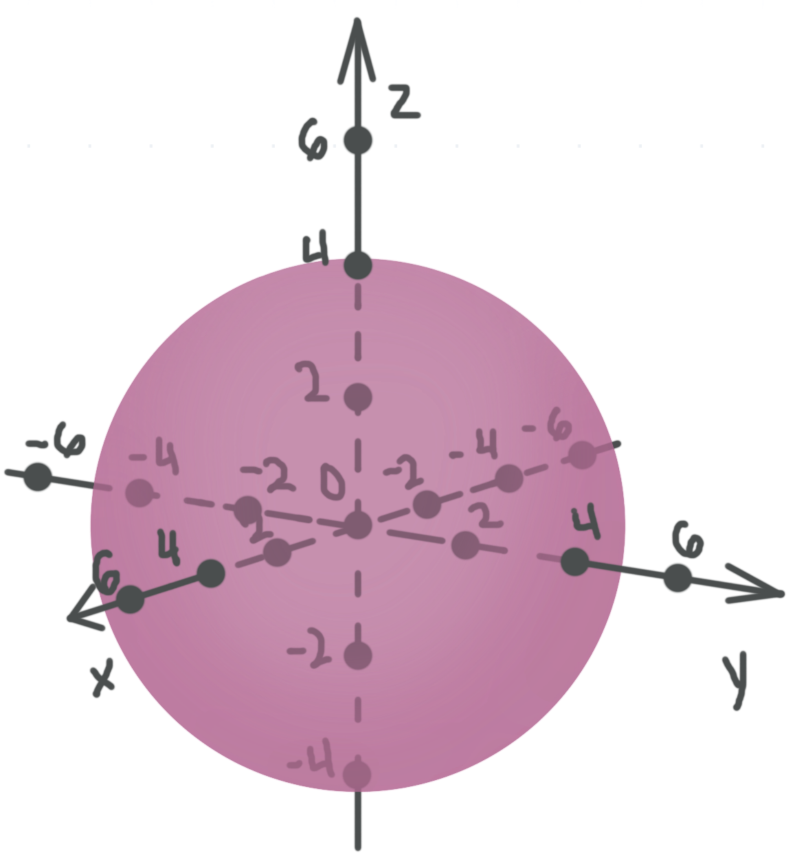
$$x^2 + y^2 + z^2 = 4^2(\sin^2 t\cos^2 s + \sin^2 t\sin^2 s + \cos^2 t)$$

$$x^2 + y^2 + z^2 = 4^2(\sin^2 t(\cos^2 s + \sin^2 s) + \cos^2 t)$$

$$x^2 + y^2 + z^2 = 4^2(\sin^2 t + \cos^2 t)$$

$$x^2 + y^2 + z^2 = 4^2$$

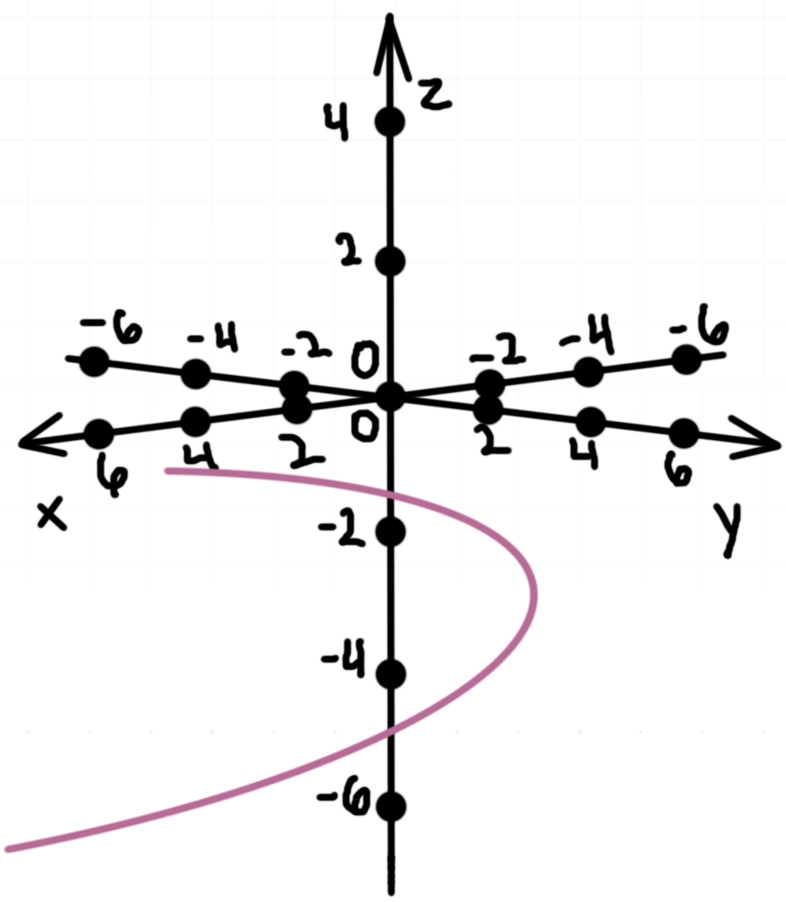
So the surface is the sphere with center at the origin and radius 4.



## PROJECTIONS OF THE CURVE

1. Identify and sketch the projections of the curve onto each of the major coordinate planes.

$$\vec{r}(t) = \left\langle t^2 - 1, \frac{t+4}{2}, t - 3 \right\rangle$$




*Solution:*

Write the vector function in parametric form.

$$x(t) = t^2 - 1$$

$$y(t) = \frac{t+4}{2}$$

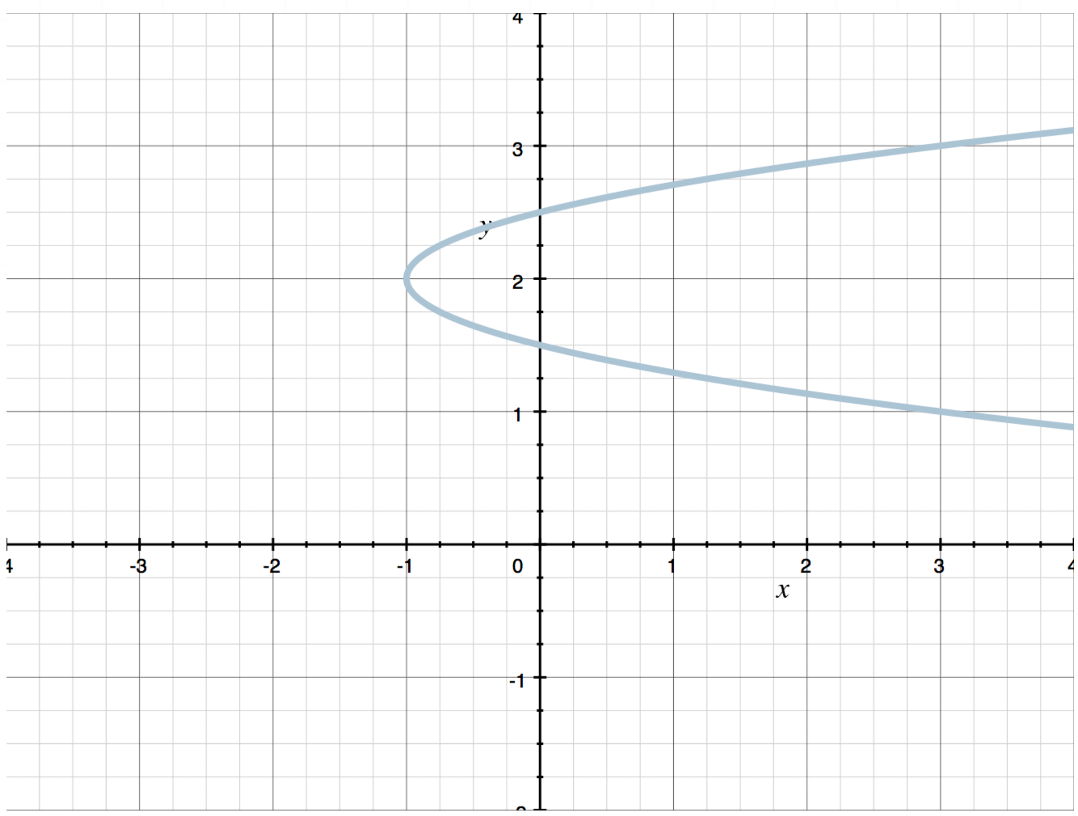
$$z(t) = t - 3$$

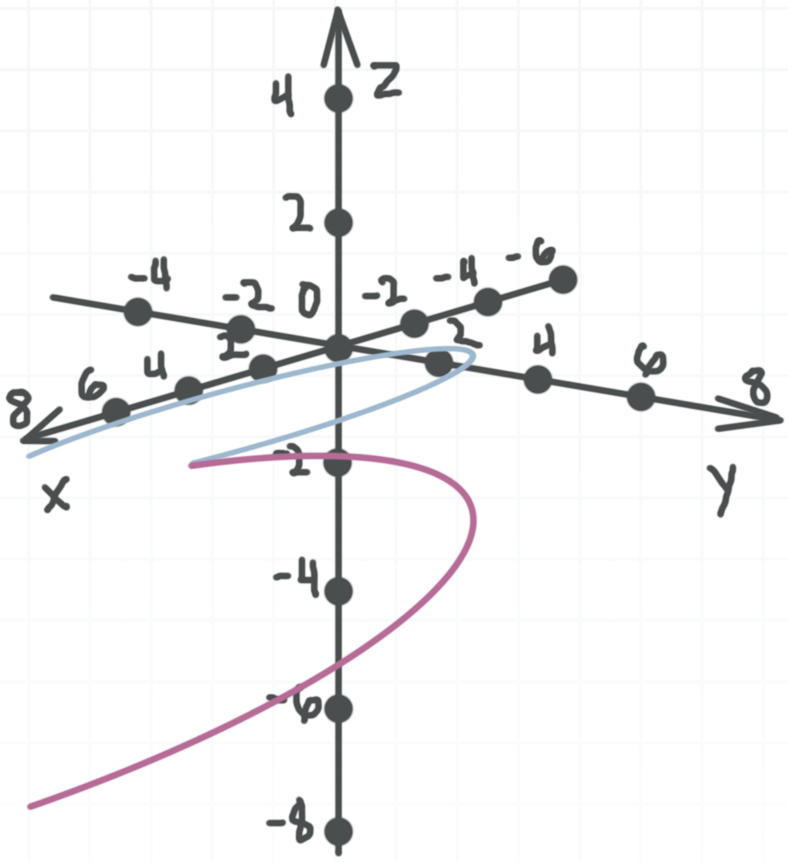
To get the projection onto the $xy$-plane, solve $y(t)$ for $t$, then plug the result into $x(t)$.

$$t = 2y - 4$$

$$x = (2y-4)^2 - 1 = 4(y-2)^2 - 1$$

So the curve's projection onto the $xy$-plane is the parabola $x = 4(y-2)^2 - 1$ that has its vertex at $(-1,2,0)$.



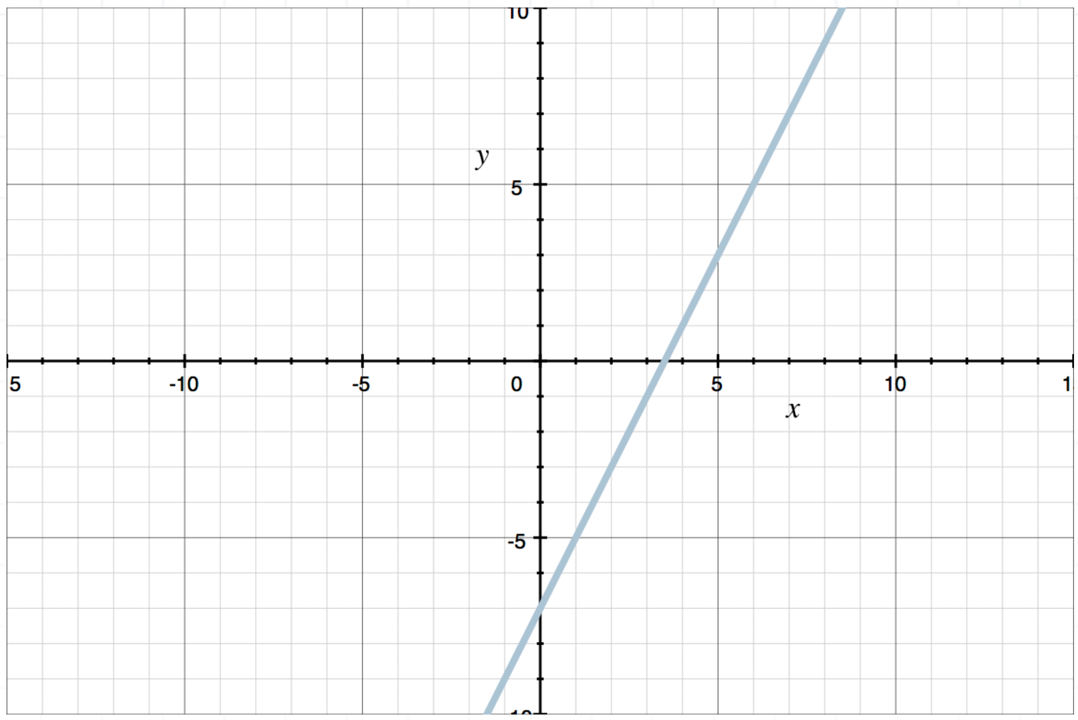


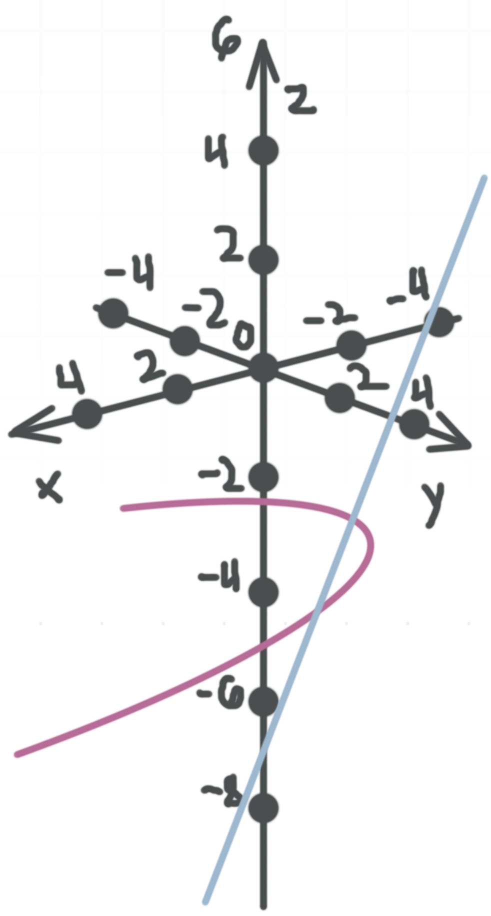

To get the projection onto the $yz$-plane, solve $y(t)$ for $t$, then plug the result into $z(t)$.

$$t = 2y - 4$$

$$z = 2y - 4 - 3 = 2y - 7$$

So the curve's projection onto the $yz$-plane is the line $z = 2y - 7$.



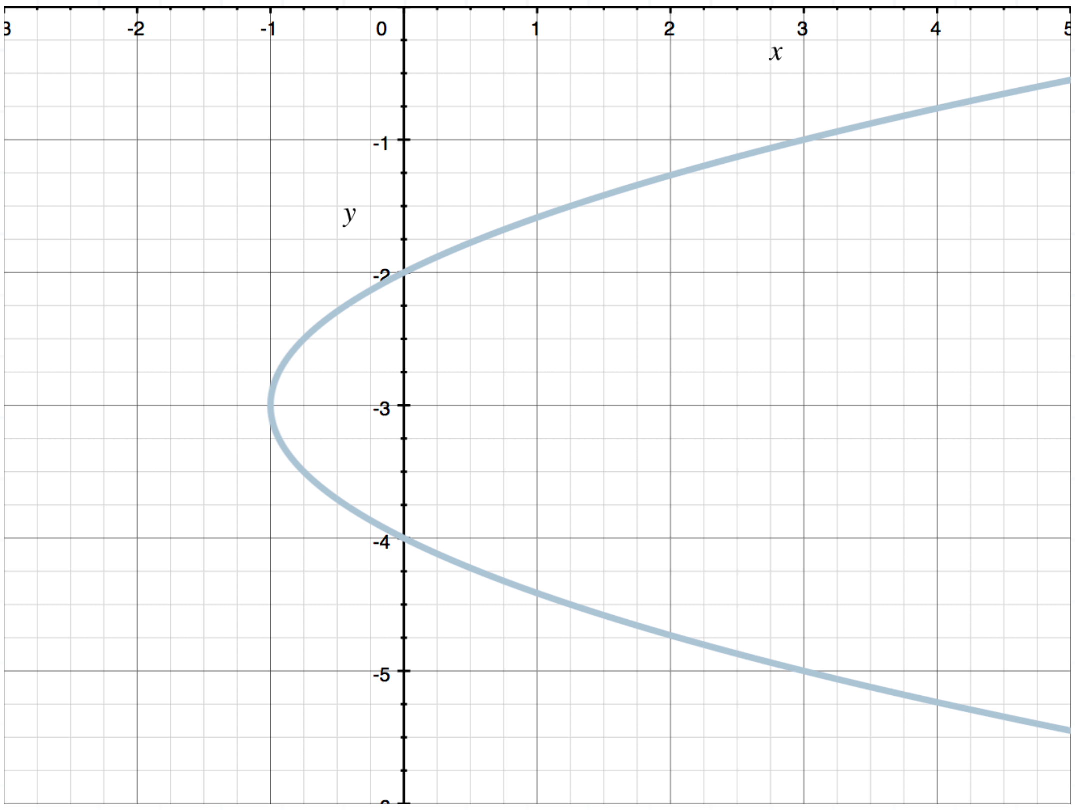



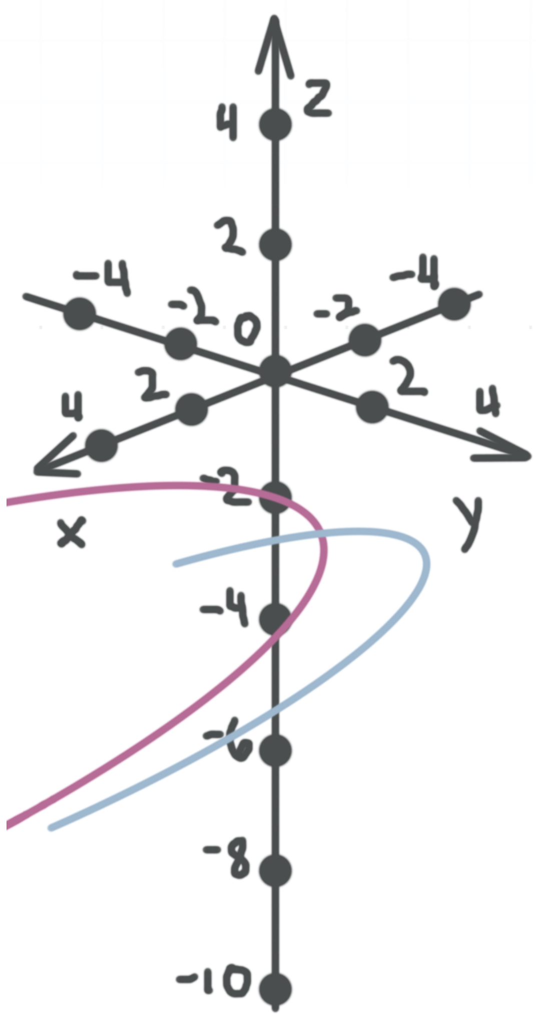

Finally, to get the projection onto the $xz$-plane, solve $z(t)$ for $t$, then plug the result into $x(t)$.

$$t = z + 3$$

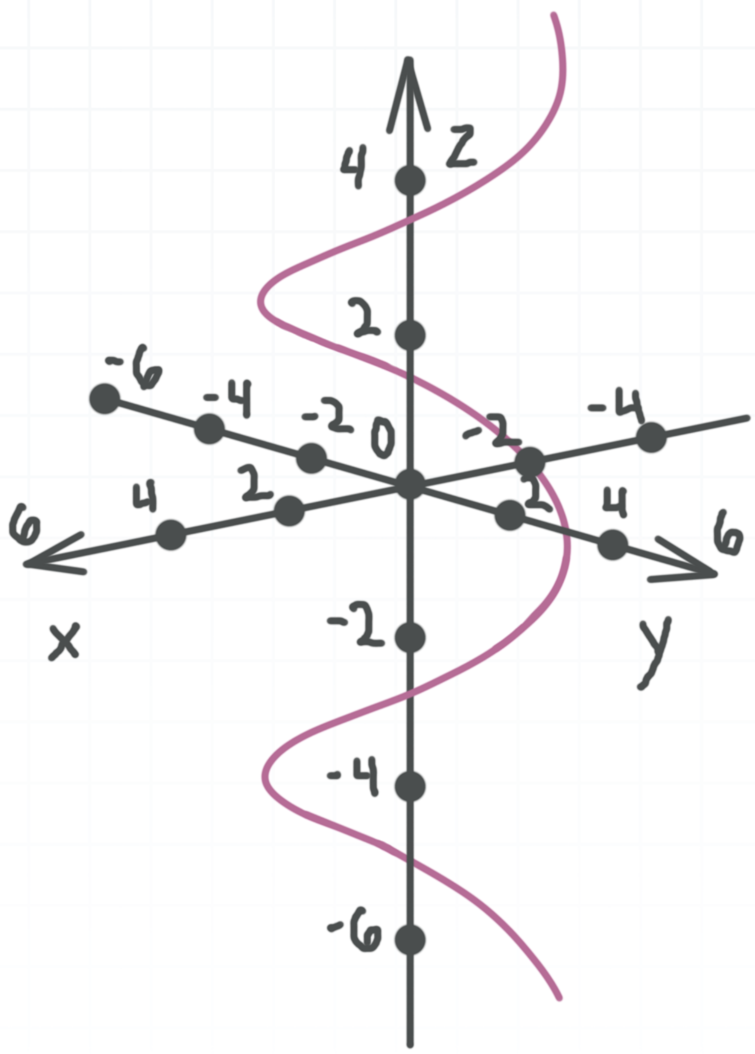
$$x = (z + 3)^2 - 1$$

So the curve projection onto the $xz$-plane is the parabola $x = (z + 3)^2 - 1$ that has its vertex at $(-1,0,-3)$.





■ 2. Identify and sketch the projections of the curve $\vec{r}(t)=\langle 2\cos t, 2\sin t, t+\pi\rangle$ onto each of the coordinate planes.




*Solution:*

Write the vector function in parametric form.

$$x(t)=2\cos t$$

$$y(t)=2\sin t$$

$$z(t)=t+\pi$$

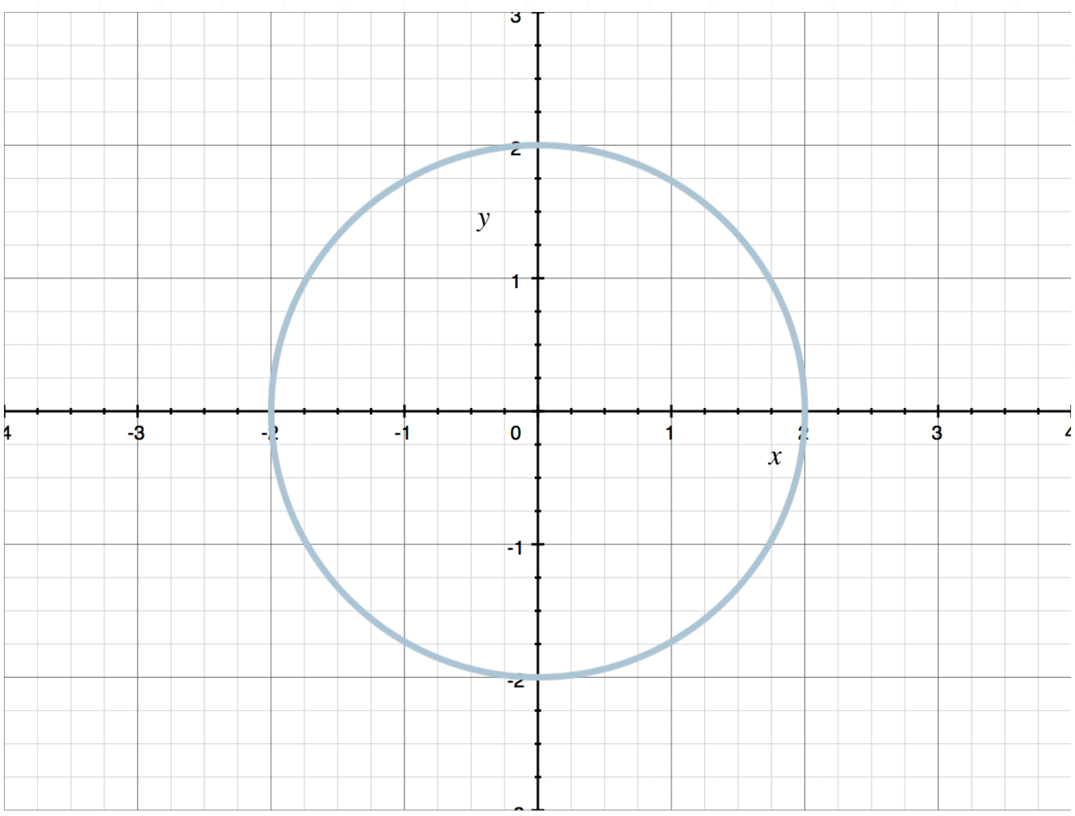
To get the projection onto the $xy$-plane, use $x(t)$ and $y(t)$ with the trigonometric identity $\sin^2\phi + \cos^2\phi = 1$ to get a relationship between $x$ and $y$.

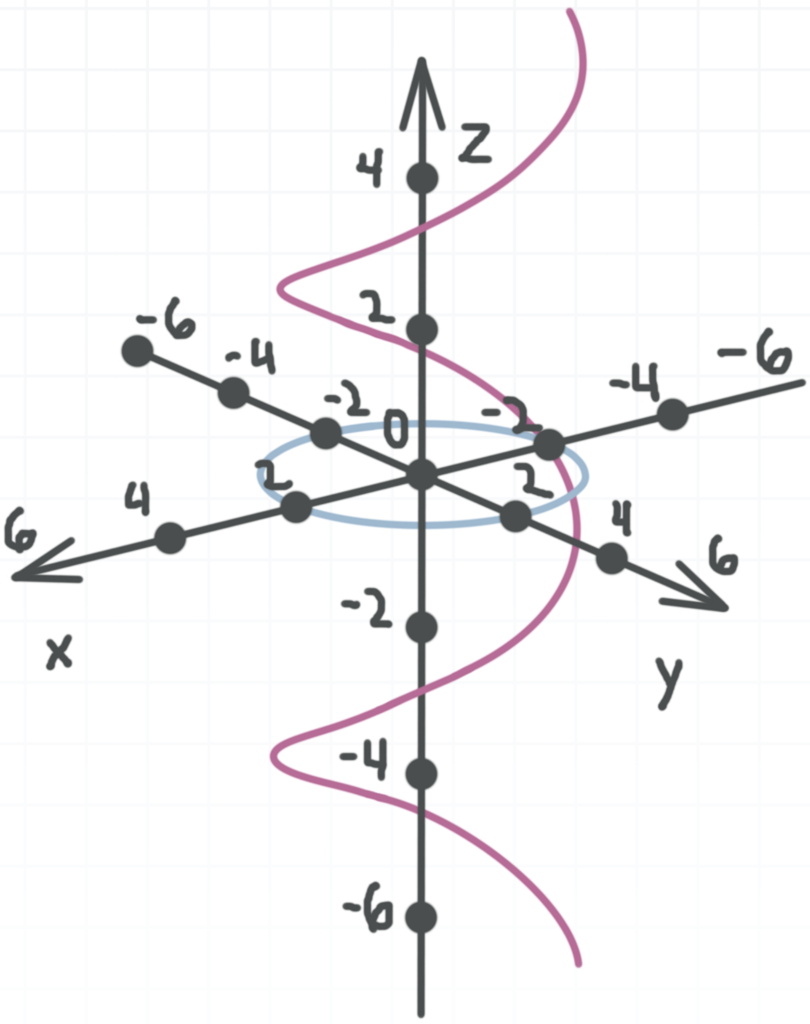
$$x^2 + y^2 = (2\cos t)^2 + (2\sin t)^2$$

$$x^2 + y^2 = 2^2(\cos^2 t + \sin^2 t)$$

$$x^2 + y^2 = 2^2$$

Therefore, the curve's projection onto the $xy$-plane is the circle with center at the origin and radius $2$.



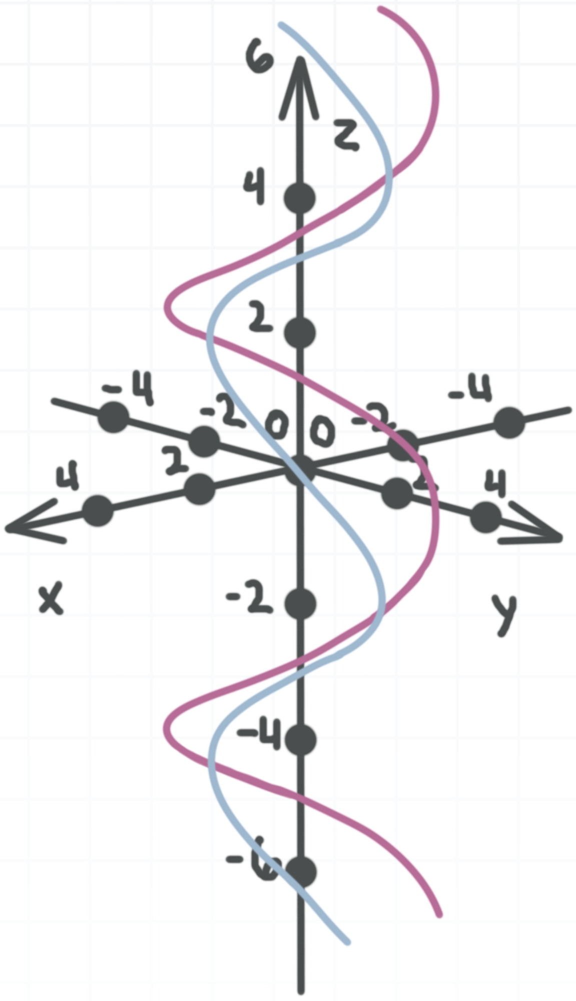



To get the projection onto the $yz$-plane, solve $z(t)$ for $t$, then plug the result into $y(t)$.

$$t = z - \pi$$

$$y = 2\sin(z - \pi) = -2\sin z$$

So the curve's projection onto the $yz$-plane is the sinusoid $y = -2\sin z$.

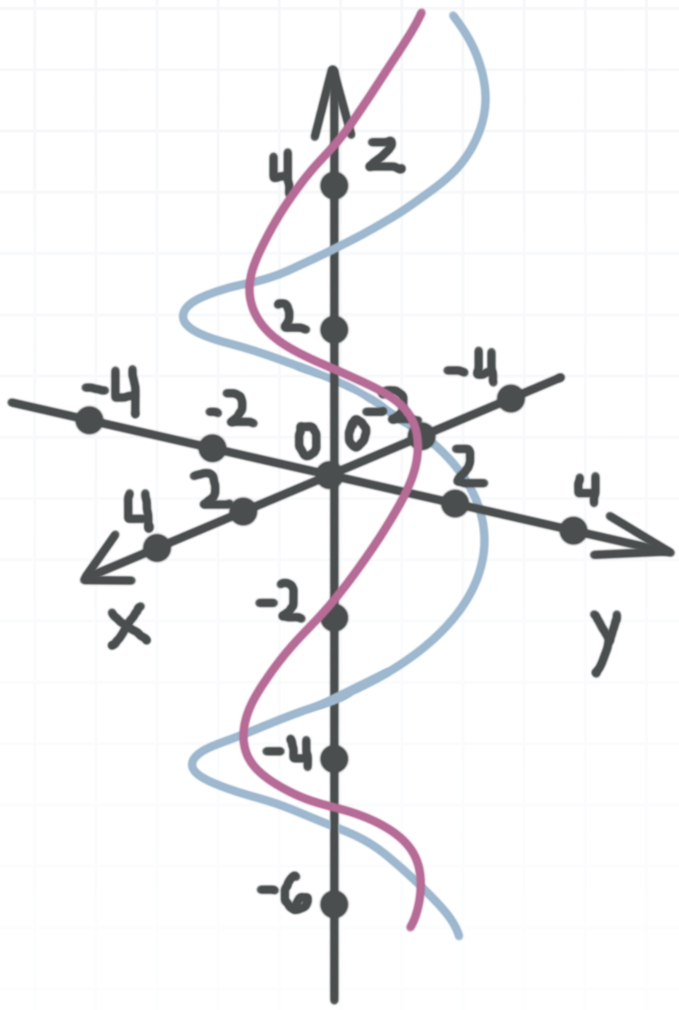



Finally, to get the projection onto the $xz$-plane, solve $z(t)$ for $t$, then plug the result into $x(t)$.

$$t = z - \pi$$

$$x = 2\cos(z - \pi) = -2\cos z$$

So the curve's projection onto the $yz$-plane is the cosinusoid $y = -2\cos z$.




3. Identify and sketch the projections of the surface onto each of the coordinate planes. Using the projections, identify the surface.

$$\vec{r}(u,v)=\left\langle 3\cos u, 3\sin u, \frac{v}{2}\right\rangle$$

*Solution:*

Write the vector function in parametric form.

$$x(u,v)=3\cos u$$

$$y(u,v)=3\sin u$$

$$z(u,v) = \frac{v}{2}$$

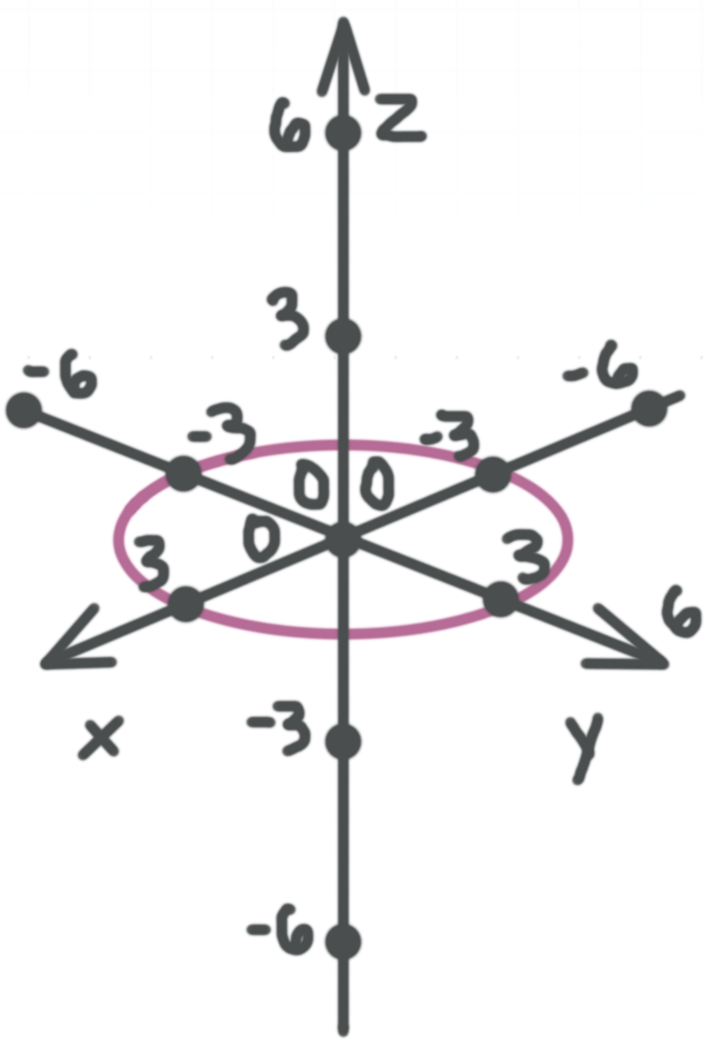
To get the projection onto the $xy$-plane, use $x(u,v)$ and $y(u,v)$ and the trigonometric identity $\sin^2\phi + \cos^2\phi = 1$ to get a relationship between $x$ and $y$.

$$x^2 + y^2 = (3\cos u)^2 + (3\sin u)^2$$

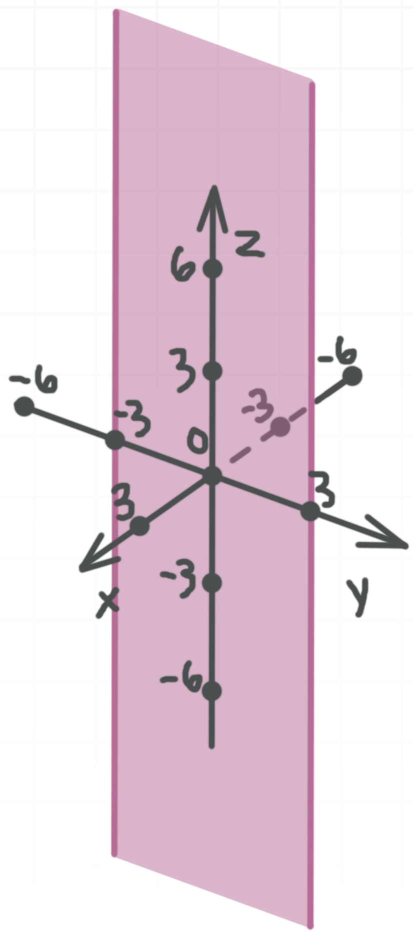
$$x^2 + y^2 = 3^2(\cos^2 u + \sin^2 u)$$

$$x^2 + y^2 = 3^2$$

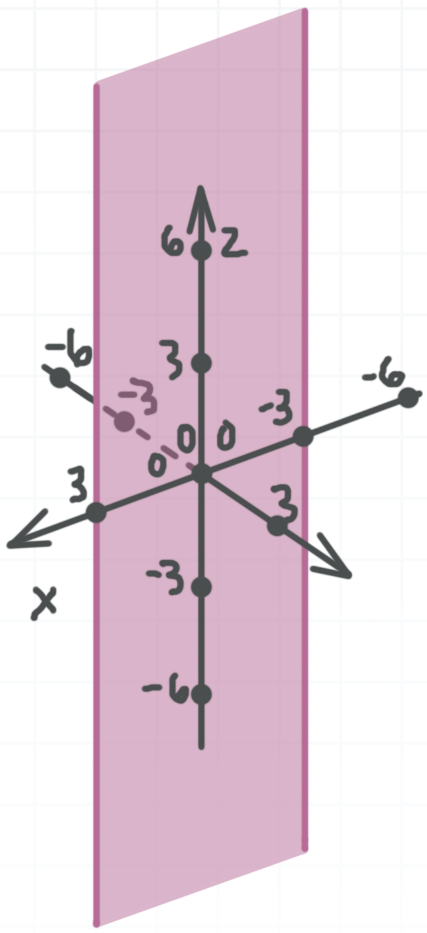
Therefore, the projection onto the $xy$-plane is the circle with center at the origin and radius $3$.



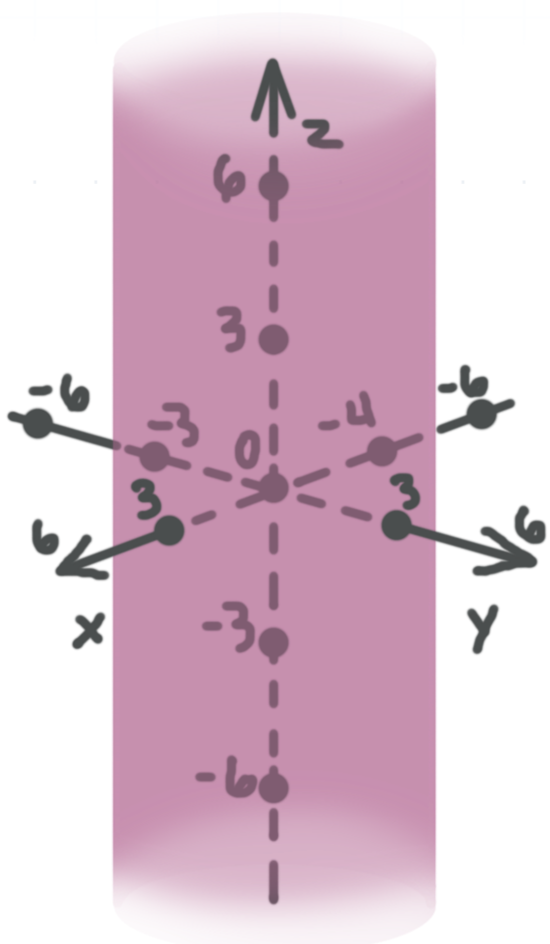
To get the projection onto the $yz$-plane, use $y(u,v)$ and $z(u,v)$. Since $u$ is any real number, $y$ changes from $-3$ to $3$. Since $v$ is the any real number, $z$ changes from $-\infty$ to $\infty$. So the projection onto the $yz$-plane is the infinite bar bounded by $-3 \le y \le 3$ and $-\infty < z < \infty$.




Finally, to get the projection onto the $xz$-plane, use $x(u,v)$ and $z(u,v)$. Similarly to the $yz$-plane, the surface projection onto the $xz$-plane is the infinite bar bounded by $-3 \le x \le 3$ and $-\infty < z < \infty$.




Using the projections, we can conclude that the surface is the cylinder $x^2 + y^2 = 3^2$.



## VECTOR AND PARAMETRIC EQUATIONS OF A LINE SEGMENT

■ 1. Find the vector and parametric equation of the line segment $AB$, given $A(-4,2)$ and $B(1,5)$.

*Solution:*

The vector equation of a line segment is given by

$$\vec{r}(t) = (1-t)\vec{r_0} + t\vec{r_1} \text{ with } 0 \le t \le 1$$

where $\vec{r_0}$ and $\vec{r_1}$ are vectors with initial points at the origin, and terminal points at the endpoints of the line segment.

Plug in $\vec{r_0} = \langle -4,2 \rangle$ and $\vec{r_1} = \langle 1,5 \rangle$.

$$\vec{r}(t) = (1-t)\langle -4,2 \rangle + t\langle 1,5 \rangle$$

$$\vec{r}(t) = \langle -4(1-t) + t, 2(1-t) + 5t \rangle$$

$$\vec{r}(t) = \langle -4 + 5t, 2 + 3t \rangle \text{ with } 0 \le t \le 1$$

To get parametric equations, write down each component of the vector equation.

$$x(t) = -4 + 5t$$

$$y(t) = 2 + 3t$$

$$0 \le t \le 1$$

■ 2. Find the vector equation of the line segment $AB$, if $A(2,-1,3)$, $\overrightarrow{AB}$ is parallel to $\langle -2,2,1\rangle$, and $B$ is the intersection point of the line $AB$ with the $xz$-plane.

*Solution:*

Find the coordinates of the point $B$. The vector equation of the line that passes through the point $A(2,-1,3)$ and has direction vector $\langle -2,2,1\rangle$ is

$$\vec{r}(t)=\langle 2,-1,3\rangle+t\langle -2,2,1\rangle$$

In parametric form, the $y$ component is

$$y(t)=-1+2t$$

For the intersection point of the line $AB$ with the $xz$-plane, we know $y=0$, which means

$$-1+2t=0$$

$$t=0.5$$

Plug $t=0.5$ into the vector equation of the line to get the coordinates of the point $B$.

$$\vec{r}(0.5)=\langle 2,-1,3\rangle+0.5\langle -2,2,1\rangle$$

$$\vec{r}(0.5)=\langle 2+0.5\cdot(-2),-1+0.5\cdot 2,3+0.5\cdot 1\rangle$$

$$\vec{r}(0.5) = \langle 1, 0, 3.5 \rangle$$

So we need to find the vector equation of the line segment connecting $A(2, -1, 3)$ to $B(1, 0, 3.5)$. The vector equation of a line segment is given by

$$\vec{r}(t) = (1 - t)\vec{r_0} + t\vec{r_1}, \ 0 \le t \le 1$$

Plug in $\overrightarrow{OA}$ for $\vec{r_0}$ and $\overrightarrow{OB}$ for $\vec{r_1}$.

$$\vec{r}(t) = (1 - t)\langle 2, -1, 3 \rangle + t\langle 1, 0, 3.5 \rangle$$

$$\vec{r}(t) = \langle 2(1 - t) + t, -1(1 - t) + 0 \cdot t, 3(1 - t) + 3.5 \cdot t \rangle$$

$$\vec{r}(t) = \langle 2 - t, -1 + t, 3 + 0.5t \rangle \text{ with } 0 \le t \le 1$$

■ 3. Find the endpoints, midpoint, and the length of the line segment for $\vec{r}(t) = \langle 2 - 3t, 4 + t, 2 - 5t \rangle$ with $0 \le t \le 1$.

*Solution:*

Let $A$ and $B$ be the endpoints of the line segment, and $M$ be the midpoint. The coordinates of $A$ correspond to the parameter value $t = 0$.

$$\vec{r}(0) = \langle 2 - 3 \cdot 0, 4 + 0, 2 - 5 \cdot 0 \rangle$$

$$\vec{r}(0) = \langle 2, 4, 2 \rangle$$

The coordinates of $B$ correspond to the parameter value $t = 1$.

$$\vec{r}(1) = \langle 2 - 3 \cdot 1, 4 + 1, 2 - 5 \cdot 1 \rangle$$

$$\vec{r}(1) = \langle -1, 5, -3 \rangle$$

The coordinates of $M$ correspond to the parameter value $t = 0.5$.

$$\vec{r}(0.5) = \langle 2 - 3 \cdot 0.5, 4 + 0.5, 2 - 5 \cdot 0.5 \rangle$$

$$\vec{r}(0.5) = \langle 0.5, 4.5, -0.5 \rangle$$

So the length of the line segment is the distance between $A$ and $B$.

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

$$AB = \sqrt{(-1 - 2)^2 + (5 - 4)^2 + (-3 - 2)^2}$$

$$AB = \sqrt{35}$$

# VECTOR FUNCTION FOR THE CURVE OF INTERSECTION OF TWO SURFACES

■ 1. Find the vector function for the line of intersection of the two planes.

$$2x - y + 3z - 5 = 0$$

$$x + y - 2z + 1 = 0$$

*Solution:*

Let $x = t$. Substitute $x = t$ into the system of equations and solve it for $y$ and $z$, treating $t$ as a constant.

$$2t - y + 3z - 5 = 0$$

$$t + y - 2z + 1 = 0$$

Add the equations.

$$3t + z - 4 = 0$$

$$z = 4 - 3t$$

Substitute this value into the second equation to find $y$.

$$t + y - 2(4 - 3t) + 1 = 0$$

$$t + y - 8 + 6t + 1 = 0$$

$$y = 7 - 7t$$

The parametric equation of the line is

$$x(t) = t$$

$$y(t) = 7 - 7t$$

$$z(t) = 4 - 3t$$

The vector equation is

$$\vec{r}(t) = \langle t, 7 - 7t, 4 - 3t \rangle$$

In standard form, the equation is

$$\vec{r}(t) = \langle 0,7,4 \rangle + t\langle 1, -7, -3 \rangle$$

■ 2. Find the vector function for the curve of intersection of two spheres.

$$x^2 + y^2 + z^2 = 5^2$$

$$(x - 3)^2 + y^2 + z^2 = 4^2$$

*Solution:*

Try to solve the system for $x$, $y$, and $z$. Subtract the equations

$$x^2 - (x - 3)^2 = 25 - 16$$

$$x^2 - x^2 + 6x - 9 = 9$$

$$6x - 18 = 0$$

$$x = 3$$

Substitute $x = 3$ back into the first equation.

$$3^2 + y^2 + z^2 = 5^2$$

$$y^2 + z^2 = 5^2 - 3^2 = 16 = 4^2$$

So far we have two equations:

$$x = 3$$

$$y^2 + z^2 = 4^2$$

The parametrization of the circle $u^2 + v^2 = R^2$ is given by the standard trigonometric substitution $u = R\cos t$ and $v = R\sin t$. So since $R = 4$,

$$x = 3$$

$$y = 4\cos t$$

$$z = 4\sin t$$

Verify that $y^2 + z^2 = 4^2$.

$$y^2 + z^2 = (4\cos t)^2 + (4\sin t)^2$$

$$y^2 + z^2 = 4^2(\cos^2 t + \sin^2 t)$$

$$y^2 + z^2 = 4^2$$

3. Find the vector function for the curve of intersection of the elliptic cylinder and the plane.

$$\frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{4^2}=1$$

$$2x-3y-z-4=0$$

*Solution:*

The parametrization of an ellipse in the form

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

is given by the standard trigonometric substitution $x(t)=x_0+a\cos t$ and $y(t)=y_0+b\sin t$. In this case $a=3$, $b=4$, and $(x_0,y_0)=(2,-1)$. So

$$x(t)=2+3\cos t$$

$$y(t)=-1+4\sin t$$

Verify that the initial equation holds.

$$\frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{4^2}=\frac{(2+3\cos t-2)^2}{3^2}+\frac{(-1+4\sin t+1)^2}{4^2}$$

$$\frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{4^2}=\frac{(3\cos t)^2}{3^2}+\frac{(4\sin t)^2}{4^2}$$

$$\frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{4^2}=\cos^2 t+\sin^2 t$$

$$\frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{4^2}=1$$

Substitute $x(t)$ and $y(t)$ into the equation of the plane, then solve it for $z$.

$$2x-3y-z-4=0$$

$$z=2x-3y-4$$

$$z=2(2+3\cos t)-3(-1+4\sin t)-4$$

$$z(t)=6\cos t-12\sin t+3$$

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