Topic: Parallel, perpendicular, and angle between planes

Question: Say whether the planes are parallel or perpendicular, otherwise find the angle between them.

$$2x + 3y - 5z = 3$$

$$4x + 6y - 10z = 17$$

Answer choices:

A Parallel

B Perpendicular

C $\theta = 23.4^\circ$

D $\theta = 66.6^\circ$

Solution: A

First we'll test to see if planes are parallel by taking the ratio of their components. Since the planes are $2x+3y-5z=3$ and $4x+6y-10z=17$, we get

$$\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$$

$$\frac{2}{4}=\frac{3}{6}=\frac{-5}{-10}$$

$$\frac{1}{2}=\frac{1}{2}=\frac{1}{2}$$

Because this equation is true, we can say that the planes are parallel.

Topic: Parallel, perpendicular, and angle between planes

Question: Say whether the planes are parallel or perpendicular, otherwise find the angle between them.

$$7x - 5y - 2z = 1$$

$$-2x - 6y + 5z = 7$$

Answer choices:

A Parallel

B Perpendicular

C $\theta = 4.8^\circ$

D $\theta = 85.2^\circ$

Solution: D

First we'll test to see if planes are parallel by taking the ratio of their components. Since the planes are $7x - 5y - 2z = 1$ and $-2x - 6y + 5z = 7$, we get

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

$$\frac{7}{-2} = \frac{-5}{-6} = \frac{-2}{5}$$

$$-\frac{7}{2} = \frac{5}{6} = -\frac{2}{5}$$

Because this equation is not true, the planes are not parallel. So we'll check to see if they're perpendicular by taking the dot product of the components.

$$a \cdot b = (7)(-2) + (-5)(-6) + (-2)(5)$$

$$a \cdot b = -14 + 30 - 10$$

$$a \cdot b = 6$$

Because the dot product does not equal 0, the planes are not perpendicular. And because we've shown that they're neither parallel nor perpendicular, it means they must be skew, so we'll find the angle between them. To find the angle between them, we'll need the magnitude of the normal vectors of each plane. The normal vectors are given by the components, so the normal vectors are $\langle 7, -5, -2 \rangle$ and $\langle -2, -6, 5 \rangle$. If we use the origin $(0,0,0)$ as (x_1, y_1, z_1), we get

$$|a| = \sqrt{(a_1 - x_1)^2 + (a_2 - y_1)^2 + (a_3 - z_1)^2}$$

$$|a| = \sqrt{(7 - 0)^2 + (-5 - 0)^2 + (-2 - 0)^2}$$

$$|a| = \sqrt{49 + 25 + 4}$$

$$|a| = \sqrt{78}$$

and

$$|b| = \sqrt{(b_1 - x_1)^2 + (b_2 - y_1)^2 + (b_3 - z_1)^2}$$

$$|b| = \sqrt{(-2 - 0)^2 + (-6 - 0)^2 + (5 - 0)^2}$$

$$|b| = \sqrt{4 + 36 + 25}$$

$$|b| = \sqrt{65}$$

Now we'll plug $a \cdot b = 6$, $|a| = \sqrt{78}$, and $|b| = \sqrt{65}$ into the formula for the angle between planes.

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\cos\theta = \frac{6}{\sqrt{78}\sqrt{65}}$$

$$\cos\theta = \frac{6}{\sqrt{5{,}070}}$$

$$\theta = \arccos \frac{6}{\sqrt{5{,}070}}$$

$$\theta = 85.2^\circ$$

Topic: Parallel, perpendicular, and angle between planes

Question: Say whether the planes are parallel or perpendicular, otherwise find the angle between them.

$$4x + y + 5z = 1$$

$$-2x + 3y + z = 8$$

Answer choices:

A Parallel

B Perpendicular

C $\theta = 11.9^\circ$

D $\theta = 78.1^\circ$

Solution: B

First we'll test to see if planes are parallel by taking the ratio of their components. Since the planes are $4x + y + 5z = 1$ and $-2x + 3y + z = 8$, we get

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

$$\frac{4}{-2} = \frac{1}{3} = \frac{5}{1}$$

$$-2 = \frac{1}{3} = 5$$

Because this equation is not true, the planes are not parallel. So we'll check to see if they're perpendicular by taking the dot product of the components.

$$a \cdot b = (4)(-2) + (1)(3) + (5)(1)$$

$$a \cdot b = -8 + 3 + 5$$

$$a \cdot b = 0$$

Because the dot product is 0, the planes are perpendicular.