Topic: Partial derivatives in three or more variables

Question: Find the partial derivative(s).

Find f_x, f_y, and f_z

for $f(x, y, z) = 2x^3e^{2y}\ln z$

Answer choices:

A $\quad f_x = 12x^2e^{2y}\ln z \qquad f_y = 4x^3e^{2y}\ln z \qquad f_z = \frac{12x^2e^{2y}}{z}$

B $\quad f_x = \frac{4x^3e^{2y}}{z} \qquad f_y = \frac{6x^2e^{2y}}{z} \qquad f_z = 12x^2e^{2y}\ln z$

C $\quad f_x = 6x^2e^{2y}\ln z \qquad f_y = 4x^3e^{2y}\ln z \qquad f_z = \frac{2x^3e^{2y}}{z}$

D $\quad f_x = \frac{12x^2e^{2y}}{z} \qquad f_y = \frac{12x^2e^{2y}}{z} \qquad f_z = \frac{12x^2e^{2y}}{z}$

Solution: C

To find the partial derivative f_x, we want to treat y and z as constants when we differentiate $f(x, y, z)$ with respect to x. Remember that if we treat y and z as constants, then e^{2y} and $\ln z$ are also constants. Therefore, we don't need to use the product rule here, since we're just differentiating a function of x multiplied by a constant.

$$f(x, y, z) = 2x^3e^{2y}\ln z$$

$$f(x, y, z) = \left(2e^{2y}\ln z\right)x^3$$

$$f_x = \left(2e^{2y}\ln z\right)3x^2$$

$$f_x = 6x^2e^{2y}\ln z$$

Similarly, to find f_y we treat x and z (and therefore x^3 and $\ln z$) as constants, while differentiating $f(x, y, z)$ with respect to y. Note that in this case we need to use the chain rule to differentiate e^{2y}.

$$f(x, y, z) = 2x^3e^{2y}\ln z$$

$$f(x, y, z) = \left(2x^3\ln z\right)e^{2y}$$

$$f_y = \left(2x^3\ln z\right)2e^{2y}$$

$$f_y = 4x^3e^{2y}\ln z$$

Finally, to find f_z, treat x and y as constants while differentiating $f(x, y, z)$ with respect to z.

$$f(x, y, z) = 2x^3e^{2y}\ln z$$

$$f(x, y, z) = \left(2x^3e^{2y}\right) \ln z$$

$$f_z = \left(2x^3e^{2y}\right) \frac{1}{z}$$

$$f_z = \frac{2x^3e^{2y}}{z}$$

Topic: Partial derivatives in three or more variables

Question: Find the partial derivative(s).

Find f_z

for $f(x, y, z) = 2x^2yz^3 \ln z$

Answer choices:

A $\quad f_z = 12xz^3 \ln z$

B $\quad f_z = \dfrac{12x}{z}$

C $\quad f_z = 4x^2yz$

D $\quad f_z = 2x^2yz^2 + 6x^2yz^2 \ln z$

Solution: D

To find the partial derivative f_z, we want to treat x and y as constants, while differentiating $f(x, y, z)$ with respect to z. Remember that if we treat x and y as constants, then x^2y is also a constant, so we can rearrange the function like this:

$$f(x, y, z) = 2x^2yz^3 \ln z$$

$$f(x, y, z) = \left(2x^2y\right) z^3 \ln z$$

This shows us that the function we are differentiating is a product of two expressions containing our variable z, along with a constant multiplier ($2x^2y$). We must therefore use the product rule to find f_z, and we get

$$f(x, y, z) = \left(2x^2y\right) z^3 \ln z$$

$$f_z = \left(2x^2y\right) \left(z^3 \cdot \frac{1}{z} + 3z^2 \cdot \ln z\right)$$

$$f_z = \left(2x^2y\right) \left(z^2 + 3z^2 \ln z\right)$$

$$f_z = 2x^2yz^2 + 6x^2yz^2 \ln z$$

Topic: Partial derivatives in three or more variables

Question: Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ for $f(x,y,z)=x^3y+x^2y^2z+2yz^2$.

Answer choices:

A $\quad \frac{\partial f}{\partial x}=2x^2y+2xy^2z+2yz^2 \qquad \frac{\partial f}{\partial y}=x^3+2x^2yz+2z^2 \qquad \frac{\partial f}{\partial z}=x^3y+x^2y^2z+4yz$

B $\quad \frac{\partial f}{\partial x}=3x^2y+2xy^2z \qquad \frac{\partial f}{\partial y}=x^3+2x^2yz+2z^2 \qquad \frac{\partial f}{\partial z}=x^2y^2+4yz$

C $\quad \frac{\partial f}{\partial x}=x^3+2x^2y+4z \qquad \frac{\partial f}{\partial y}=2x^2y+2xy^2+4z \qquad \frac{\partial f}{\partial z}=3x^2+4xy+2z^2$

D $\quad \frac{\partial f}{\partial x}=2x^2+2xy^2 \qquad \frac{\partial f}{\partial y}=x^3+2x^2y+2z^2 \qquad \frac{\partial f}{\partial z}=x^2y^2+4z$

Solution: B

To find the partial derivative $\partial f/\partial x$, we want to treat y and z as constants, while differentiating $f(x, y, z)$ with respect to x. Remember that if we treat y and z as constants, then y^2 and z^2 are also constants, so we can rearrange and differentiate the function as follows:

$$f(x, y, z) = x^3y + x^2y^2z + 2yz^2$$

$$f(x, y, z) = (y)x^3 + \left(y^2z\right)x^2 + \left(2yz^2\right)$$

$$\frac{\partial f}{\partial x} = (y)3x^2 + \left(y^2z\right)2x + \left(2yz^2\right)0$$

$$\frac{\partial f}{\partial x} = 3x^2y + 2xy^2z$$

Then to find $\partial f/\partial y$, treat x and z as constants while differentiating $f(x, y, z)$ with respect to y.

$$f(x, y, z) = x^3y + x^2y^2z + 2yz^2$$

$$f(x, y, z) = \left(x^3\right)y + \left(x^2z\right)y^2 + \left(2z^2\right)y$$

$$\frac{\partial f}{\partial y} = \left(x^3\right)1 + \left(x^2z\right)2y + \left(2z^2\right)1$$

$$\frac{\partial f}{\partial y} = x^3 + 2x^2yz + 2z^2$$

To find $\partial f/\partial z$, treat x and y as constants while differentiating $f(x, y, z)$ with respect to z.

$$f(x, y, z) = x^3y + x^2y^2z + 2yz^2$$

$$f(x, y, z) = \left(x^3y\right) + \left(x^2y^2\right)z + (2y)z^2$$

$$\frac{\partial f}{\partial z} = \left(x^3y\right)0 + \left(x^2y^2\right)1 + (2y)2z$$

$$\frac{\partial f}{\partial z} = x^2y^2 + 4yz$$