# Orthogonal, parallel, or neither

We say that two vectors $a$ and $b$ are

**orthogonal** if they are perpendicular (set at $90^\circ$ from each other)

$$a \cdot b = 0$$

**parallel** if they point in exactly the same or opposite directions, and never cross each other

after factoring out any common factors, the remaining direction numbers will be equal

**neither**

Since it's easy to take a dot product, it's a good idea to get in the habit of testing the vectors to see whether they're orthogonal, and then if they're not, testing to see whether they're parallel.

**Example**

Say whether the following vectors are orthogonal, parallel or neither.

$a = \langle 2,1 \rangle$ and $b = \langle -1,2 \rangle$

$a = 2i + 3j + 5k$ and $b = i + 4j - 2k$

$a = \langle 1, -2,3 \rangle$ and $b = \langle -2,4, -6 \rangle$

For $a = \langle 2,1 \rangle$ and $b = \langle -1,2 \rangle$:

We'll take the dot product of our vectors to see whether they're orthogonal to one another.

$$a \cdot b = (2)(-1) + (1)(2)$$

$$a \cdot b = -2 + 2$$

$$a \cdot b = 0$$

Since the dot product is $0$, we can say that $a = \langle 2,1 \rangle$ and $b = \langle -1,2 \rangle$ are orthogonal. If we know that they're orthogonal, then by definition they can't be parallel, so we're done with our testing.

For $a = 2i + 3j + 5k$ and $b = i + 4j - 2k$:

First we'll put the vectors in standard form.

$$a = 2i + 3j + 5k$$

$$a = \langle 2,3,5 \rangle$$

and

$$b = i + 4j - 2k$$

$$b = \langle 1,4,-2 \rangle$$

Now we'll take the dot product of our vectors to see whether they're orthogonal to one another.

$$a \cdot b = (2)(1) + (3)(4) + (5)(-2)$$

$$a \cdot b = 2 + 12 - 10$$

$$a \cdot b = 4$$

Since the dot product is not $0$, we can say that $a = 2i + 3j + 5k$ and $b = i + 4j - 2k$ are not orthogonal.

To say whether or not the vectors are parallel, we want to look for a common factor in the direction numbers of either vector, and pull it out until both vectors are irreducible.

$a = \langle 2,3,5 \rangle$ is already irreducible because $2$, $3$ and $5$ have no common factors. $b = \langle 1,4,-2 \rangle$ is also irreducible because $1$, $4$ and $-2$ have no common factors either.

Therefore, we can say that $a = 2i + 3j + 5k$ and $b = i + 4j - 2k$ are neither orthogonal nor parallel.

For $a = \langle 1,-2,3 \rangle$ and $b = \langle -2,4,-6 \rangle$:

We'll take the dot product of our vectors to see whether they're orthogonal to one another.

$$a \cdot b = (1)(-2) + (-2)(4) + (3)(-6)$$

$$a \cdot b = -2 - 8 - 18$$

$$a \cdot b = -28$$

Since the dot product is not $0$, we can say that $a = \langle 1,-2,3 \rangle$ and $b = \langle -2,4,-6 \rangle$ are not orthogonal.

To say whether or not the vectors are parallel, we want to look for a common factor in the direction numbers of either vector, and pull it out until both vectors are irreducible.

$a = \langle 1, -2, 3\rangle$ is already irreducible because $1$, $-2$ and $3$ have no common factors. On the other hand, $b = \langle -2, 4, -6\rangle$ has a common factor of $-2$ that can be factored out of the vector.

$$b = \langle -2, 4, -6\rangle$$

$$b = -2\langle 1, -2, 3\rangle$$

Now the direction numbers of $a$ and $b$ are equal, so we can say that $a$ and $b$ are parallel.