Topic: Double integrals

Question: Evaluate the double integral.

$$\iint_R y^2 \sin x + y^2 \cos(2x)\ dA$$

$$R = \left\{ (x, y)\ \middle|\ 0 \le x \le \frac{\pi}{2}, -2 \le y \le 1 \right\}$$

Answer choices:

A -3

B 1

C 3

D 0

Solution: C

In this problem, we haven't been given the order of integration inside the integral as $dy\ dx$ or $dx\ dy$, so we can pick either order. We'll integrate first with respect to y, and then with respect to x.

$$\iint_R y^2 \sin x + y^2 \cos(2x)\ dA$$

$$\int_0^{\frac{\pi}{2}} \int_{-2}^{1} y^2 \sin x + y^2 \cos(2x)\ dy\ dx$$

When we integrate with respect to y, we have to treat x like a constant.

$$\int_0^{\frac{\pi}{2}} \frac{1}{3}y^3 \sin x + \frac{1}{3}y^3 \cos(2x) \Big|_{y=-2}^{y=1}\ dx$$

Now we can evaluate over the interval $[-2,1]$.

$$\int_0^{\frac{\pi}{2}} \frac{1}{3}(1)^3 \sin x + \frac{1}{3}(1)^3 \cos(2x) - \left[\frac{1}{3}(-2)^3 \sin x + \frac{1}{3}(-2)^3 \cos(2x)\right]\ dx$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{3} \sin x + \frac{1}{3} \cos(2x) - \left[-\frac{8}{3} \sin x - \frac{8}{3} \cos(2x)\right]\ dx$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{3} \sin x + \frac{1}{3} \cos(2x) + \frac{8}{3} \sin x + \frac{8}{3} \cos(2x)\ dx$$

$$\int_0^{\frac{\pi}{2}} \frac{9}{3} \sin x + \frac{9}{3} \cos(2x)\ dx$$

$$\int_0^{\frac{\pi}{2}} 3\sin x + 3\cos(2x)\ dx$$

Now we'll integrate with respect to x and evaluate over the interval $\left[0,\frac{\pi}{2}\right]$.

$$-3\cos x + \frac{3}{2}\sin(2x)\Big|_0^{\frac{\pi}{2}}$$

$$-3\cos\frac{\pi}{2} + \frac{3}{2}\sin\left(2\frac{\pi}{2}\right) - \left[-3\cos 0 + \frac{3}{2}\sin(2(0))\right]$$

$$-3\cos\frac{\pi}{2} + \frac{3}{2}\sin\pi + 3\cos 0 - \frac{3}{2}\sin 0$$

$$-3(0) + \frac{3}{2}(0) + 3(1) - \frac{3}{2}(0)$$

$$3$$

This is the volume given by the iterated integral.

Topic: Double integrals

Question: Evaluate the double integral.

$$\iint_R 3y - 2xy \, dx \, dy$$

where R is the rectangle on the interval

$$2 \le x \le 3$$

$$0 \le y \le 2$$

Answer choices:

A 16

B 11

C -6

D -4

Solution: D

First, we'll apply the given interval to the double integral, to turn it into an iterated integral.

$$\int\int_R 3y - 2xy \ dx \ dy$$

$$\int_0^2 \int_2^3 3y - 2xy \ dx \ dy$$

Then integrate with respect to x, and evaluate over the interval.

$$\int_0^2 3xy - x^2y \Big|_{x=2}^{x=3} \ dy$$

$$\int_0^2 3(3)y - (3)^2y - \left(3(2)y - (2)^2y\right) \ dy$$

$$\int_0^2 9y - 9y - \left(6y - 4y\right) \ dy$$

$$\int_0^2 -2y \ dy$$

Integrate with respect to y, and evaluate over the interval.

$$-y^2\Big|_0^2$$

$$-(2)^2 - \left(-(0)^2\right)$$

$$-4$$

Topic: Double integrals

Question: Which double integral is equal to $1/2$?

Answer choices:

A $\quad L = \int_0^1 dy \int_x^1 x - y \ dx$

B $\quad L = \int_0^1 dy \int_x^1 x + y \ dx$

C $\quad L = \int_0^1 dx \int_x^1 x + y \ dy$

D $\quad L = \int_0^1 dx \int_x^1 x - y \ dy$

Solution: C

Starting with answer choice C,

$$L = \int_0^1 dx \int_x^1 x + y \ dy$$

integrate first with respect to y, then evaluate over the interval.

$$L = \int_0^1 xy + \frac{1}{2}y^2 \Big|_{y=x}^{y=1} dx$$

$$L = \int_0^1 x(1) + \frac{1}{2}(1)^2 - \left(x(x) + \frac{1}{2}(x)^2\right) \ dx$$

$$L = \int_0^1 x + \frac{1}{2} - x^2 - \frac{1}{2}x^2 \ dx$$

$$L = \int_0^1 -\frac{3}{2}x^2 + x + \frac{1}{2} \ dx$$

Integrate with respect to x, then evaluate over the interval.

$$L = -\frac{1}{2}x^3 + \frac{1}{2}x^2 + \frac{1}{2}x \Big|_0^1$$

$$L = -\frac{1}{2}(1)^3 + \frac{1}{2}(1)^2 + \frac{1}{2}(1) - \left(-\frac{1}{2}(0)^3 + \frac{1}{2}(0)^2 + \frac{1}{2}(0)\right)$$

$$L = \frac{1}{2}$$