Topic: Approximating double integrals with rectangles

Question: The rectangle R is defined on $0 \le x \le 6$ and $0 \le y \le 9$. A solid volume is defined above this rectangle and below $z = 3x + y^2$. Which value most closely approximates the volume if you divide the rectangle into 3×3 squares and use a Riemann sum to approximate the volume?

Answer choices:

A $1{,}200$

B $1{,}150$

C $2{,}800$

D $3{,}000$

Solution: D

The rectangle below the volume is bounded by the lines $x = 0$, $x = 6$, $y = 0$, and $y = 9$. The surface that defines the top of the volume is $z = 3x + y^2$. If we divide the area of the rectangle into 3×3 squares, then $\Delta A = 9$.

If x is defined from 0 to 6, that means we'll need $(6 - 0)/3$, or 2 squares across, and if y is defined from 0 to 9, that means we'll need $(9 - 0)/3$, or 3 squares down.

Therefore, using upper-right-hand corners in a Riemann sum, the estimate for the volume is given by

$$V \approx \sum_{i=1}^{2} \sum_{j=1}^{3} f(x_i, y_j)\Delta A = \Delta A \left[f(3,3) + f(3,6) + f(3,9) + f(6,3) + f(6,6) + f(6,9) \right]$$

If we plug in the values we know, we get

$$V \approx 9 \left[\left(3(3) + 3^2\right) + \left(3(3) + 6^2\right) + \left(3(3) + 9^2\right) + \left(3(6) + 3^2\right) + \left(3(6) + 6^2\right) + \left(3(6) + 9^2\right) \right]$$

$$V \approx 9 \left[(9 + 9) + \left(9 + 36\right) + (9 + 81) + (18 + 9) + \left(18 + 36\right) + (18 + 81) \right]$$

$$V \approx 9(18 + 45 + 90 + 27 + 54 + 99)$$

$$V \approx 2{,}997$$

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Question: The rectangle R is defined on the boundary set given in one of the answer choices. A solid volume is defined above this rectangle and below $z = x^2 + xy$. The approximate volume of the region is 33, if you divide the rectangle into 1×1 squares and use a Riemann sum to approximate the volume. Which are the boundaries of the rectangle R?

Answer choices:

A	$0 \le x \le 2$	and	$0 \le y \le 3$
B	$0 \le x \le 4$	and	$0 \le y \le 6$
C	$0 \le x \le 3$	and	$0 \le y \le 5$
D	$0 \le x \le 1$	and	$0 \le y \le 6$

Solution: A

Since we don't know the bounds yet, let's say that the rectangle below the volume is bounded by the lines $x = a$, $x = b$, $y = c$, and $y = d$. The surface that defines the top of the volume is $z = x^2 + xy$. If we divide the area of the rectangle into 1×1 squares, then $\Delta A = 1$.

If x is defined from a to b, that means we'll need $(b - a)/1$, or $b - a$ squares across, and if y is defined from c to d, that means we'll need $(d - c)/1$, or $d - c$ squares down.

Therefore, using upper-right-hand corners in a Riemann sum, the estimate for the volume is given by

$$V \approx \sum_{i=1}^{b-a} \sum_{j=1}^{d-c} f(x_i, y_j)\Delta A$$

$$V \approx \sum_{i=1}^{b-a} \sum_{j=1}^{d-c} f(x_i, y_j)(1)$$

$$V \approx \sum_{i=1}^{b-a} \sum_{j=1}^{d-c} f(x_i, y_j)$$

Assume that the rectangle is bounded by the interval given in answer choice A, $0 \le x \le 2$ and $0 \le y \le 3$. Then the volume is given by

$$V \approx \sum_{i=1}^{2-0} \sum_{j=1}^{3-0} f(x_i, y_j)$$

$$V \approx \sum_{i=1}^{2} \sum_{j=1}^{3} f(x_i, y_j)$$

Because each square is 1×1, plugging in the upper-right-hand corners gives

$$V \approx \sum_{i=1}^{2} \sum_{j=1}^{3} f(x_i, y_j) = f(1,1) + f(1,2) + f(1,3) + f(2,1) + f(2,2) + f(2,3)$$

Plug in all the other values we know.

$$V \approx \left(1^2 + (1)(1)\right) + \left(1^2 + (1)(2)\right) + \left(1^2 + (1)(3)\right) + \left(2^2 + (2)(1)\right) + \left(2^2 + (2)(2)\right) + \left(2^2 + (2)(3)\right)$$

$$V \approx (1 + 1) + (1 + 2) + (1 + 3) + (4 + 2) + (4 + 4) + (4 + 6)$$

$$V \approx 2 + 3 + 4 + 6 + 8 + 10$$

$$V \approx 33$$

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Question: A solid is defined above the rectangle R and below the surface S. S is defined on the rectangle $0 \leq x \leq 10$ and $0 \leq y \leq 15$. Divide the rectangle into 5×5 squares. If using a Riemann sum approximates the volume to be $12{,}750$, then which of the following functions is S?

Answer choices:

A $\quad z = xy + x$

B $\quad z = xy + y$

C $\quad z = xy + 2y$

D $\quad z = xy + 2x$

Solution: B

The rectangle R below the volume is bounded by the lines $x=0$, $x=10$, $y=0$, and $y=15$. The equation of the surface S that defines the top of the volume is unknown. If we divide the area of the rectangle into 5×5 squares, then $\Delta A=25$.

If x is defined from 0 to 10, that means we'll need $(10-0)/5$, or 2 squares across, and if y is defined from 0 to 15, that means we'll need $(15-0)/5$, or 3 squares down.

Therefore, using upper-right-hand corners in a Riemann sum, the estimate for the volume is given by

$$V\approx\sum_{i=1}^{2}\sum_{j=1}^{3}f(x_i,y_j)\Delta A=\Delta A\left[f(5,5)+f(5,10)+f(5,15)+f(10,5)+f(10,10)+f(10,15)\right]$$

If we plug in the values we know, we get

$$V\approx 25\left[f(5,5)+f(5,10)+f(5,15)+f(10,5)+f(10,10)+f(10,15)\right]$$

If we assume that the function f is given by the surface from answer choice B, $z=xy+y$, then we get

$$V\approx 25\left[((5)(5)+5)+((5)(10)+10)+((5)(15)+15)+((10)(5)+5)+((10)(10)+10)+((10)(15)+15)\right]$$

$$V\approx 25\left[(25+5)+(50+10)+(75+15)+(50+5)+(100+10)+(150+15)\right]$$

$$V\approx 25(30+60+90+55+110+165)$$

$$V\approx 12{,}750$$