Topic: Changing iterated integrals to polar coordinates

Question: Convert the iterated integral to polar coordinates.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2\ dy\ dx$$

Answer choices:

A $\int_{0}^{2\pi}\int_{-1}^{1} r^2\ dr\ d\theta$

B $\int_{0}^{2\pi}\int_{0}^{1} r^3\ dr\ d\theta$

C $\int_{0}^{2\pi}\int_{-1}^{1} r^3\ dr\ d\theta$

D $\int_{0}^{2\pi}\int_{0}^{1} r^2\ dr\ d\theta$

Solution: B

When we convert from rectangular coordinates to polar coordinates, we use the following conversion formulas.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$dy\ dx = r\ dr\ d\theta$$

We were given the double integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2\ dy\ dx$$

We'll convert the integrand first, leaving the limits of integration.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2\ dy\ dx = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} r^2 r\ dr\ d\theta$$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2\ dy\ dx = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} r^3\ dr\ d\theta$$

Next we need to convert the limits of integration. We know that the limits of integration with respect to y are $y = \pm\sqrt{1-x^2}$. We can rewrite those as

$$y = \pm\sqrt{1-x^2}$$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

Since we can now see that we're talking about the circle with radius 1, and since in polar coordinates r represents radius, the bounds for r have to be $[0,1]$. Similarly, because we're dealing with the entire circle, the limits of integration for the angle θ have to be $[0,2\pi]$. Therefore, the converted integral is

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2 \ dy \ dx = \int_{0}^{2\pi} \int_{0}^{1} r^3 \ dr \ d\theta$$

Topic: Changing iterated integrals to polar coordinates

Question: Convert the iterated integral to polar coordinates.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right)\ dy\ dx$$

Answer choices:

A $\int_0^{2\pi}\int_0^2 \sin r^2\ dr\ d\theta$

B $\int_0^{2\pi}\int_0^2 r\cos r^2\ dr\ d\theta$

C $\int_0^{2\pi}\int_0^2 \cos r^2\ dr\ d\theta$

D $\int_0^{2\pi}\int_0^2 r\sin r^2\ dr\ d\theta$

Solution: D

When we convert from rectangular coordinates to polar coordinates, we use the following conversion formulas.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$dy\ dx = r\ dr\ d\theta$$

We were given the double integral

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right)\ dy\ dx$$

We'll convert the integrand first, leaving the limits of integration.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right)\ dy\ dx = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} r\sin r^2\ dr\ d\theta$$

Next we need to convert the limits of integration. We know that the limits of integration with respect to y are $y = \pm\sqrt{4-x^2}$. We can rewrite those as

$$y = \pm\sqrt{4-x^2}$$

$$y^2 = 4 - x^2$$

$$x^2 + y^2 = 4$$

Since we can now see that we're talking about the circle with radius 2, and since in polar coordinates r represents radius, the bounds for r have to be $[0,2]$. Similarly, because we're dealing with the entire circle, the limits of integration for the angle θ have to be $[0,2\pi]$. Therefore, the converted integral is

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right)\ dy\ dx = \int_{0}^{2\pi}\int_{0}^{2} r\sin r^2\ dr\ d\theta$$

Topic: Changing iterated integrals to polar coordinates

Question: Convert the iterated integral to polar coordinates.

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \ln\left(x^2+y^2\right)\ dy\ dx$$

Answer choices:

A $\quad \displaystyle\int_{0}^{2\pi}\int_{0}^{4} e^{r^2}\ dr\ d\theta$

B $\quad \displaystyle\int_{0}^{2\pi}\int_{0}^{4} \ln r^2\ dr\ d\theta$

C $\quad \displaystyle\int_{0}^{2\pi}\int_{0}^{4} r\ln r^2\ dr\ d\theta$

D $\quad \displaystyle\int_{0}^{2\pi}\int_{0}^{4} re^{r^2}\ dr\ d\theta$

Solution: C

When we convert from rectangular coordinates to polar coordinates, we use the following conversion formulas.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$dy\ dx = r\ dr\ d\theta$$

We were given the double integral

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \ln\left(x^2+y^2\right)\ dy\ dx$$

We'll convert the integrand first, leaving the limits of integration.

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \ln\left(x^2+y^2\right)\ dy\ dx = \int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} r\ln r^2\ dr\ d\theta$$

Next we need to convert the limits of integration. We know that the limits of integration with respect to y are $y = \pm\sqrt{16-x^2}$. We can rewrite those as

$$y = \pm\sqrt{16-x^2}$$

$$y^2 = 16 - x^2$$

$$x^2 + y^2 = 16$$

Since we can now see that we're talking about the circle with radius 4, and since in polar coordinates r represents radius, the bounds for r have to be $[0,4]$. Similarly, because we're dealing with the entire circle, the limits of integration for the angle θ have to be $[0,2\pi]$. Therefore, the converted integral is

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \ln\left(x^2+y^2\right)\ dy\ dx = \int_{0}^{2\pi}\int_{0}^{4} r\ln r^2\ dr\ d\theta$$