Topic: Parametric equations of the tangent line

Question: Find the parametric equations of the tangent line.

$$x = 3t^3, \ y = t^2, \ z = 2t$$

at $P(3,1,2)$

Answer choices:

A	$x = 3 + 9t$	$y = 1 + 2t$	$z = 2 + 2t$
B	$x = 3 - 9t$	$y = 1 - 2t$	$z = 2 - 2t$
C	$x = 9 + 3t$	$y = 2 + t$	$z = 2 + 2t$
D	$x = 9 - 3t$	$y = 2 - t$	$z = 2 - 2t$

Solution: A

The first thing we need to do is find the value of the parameter t that corresponds with the given point $P(3,1,2)$. We'll plug the value of x from the point into the parametric equation for x.

$$x = 3t^3$$

$$3 = 3t^3$$

$$1 = t^3$$

$$t = 1$$

If we try $t = 1$ in $y = t^2$ and $z = 2t$, along with $y = 1$ and $z = 2$ from the given point $P(3,1,2)$, we get

$$y = t^2$$

$$1 = (1)^2$$

$$1 = 1$$

and

$$z = 2t$$

$$2 = 2(1)$$

$$2 = 2$$

Because all of these equations are true, we can conclude that the value of the parameter is $t = 1$.

Now we'll convert the given parametric equations into vector form. So $x = 3t^3$, $y = t^2$, and $z = 2t$ become

$$r(t) = \left\langle 3t^3, t^2, 2t \right\rangle$$

Take the derivative of this vector.

$$r'(t) = \left\langle 9t^2, 2t, 2 \right\rangle$$

Plug the parameter $t = 1$ into this derivative vector.

$$r'(1) = \left\langle 9(1)^2, 2(1), 2 \right\rangle$$

$$r'(1) = \langle 9,2,2 \rangle$$

Now we can use everything we just found to get the parametric equations of the tangent line. x_1, y_1 and z_1 will come from $P(3,1,2)$, and $r'(t)_1$, $r'(t)_2$ and $r'(t)_3$ will come from $r'(1) = \langle 9,2,2 \rangle$. We get

$$x = x_1 + r'(t)_1 t$$

$$x = 3 + 9t$$

and

$$y = y_1 + r'(t)_2 t$$

$$y = 1 + 2t$$

and

$$z = z_1 + r'(t)_3 t$$

$$z = 2 + 2t$$

These are the parametric equations of the tangent line.

Topic: Parametric equations of the tangent line

Question: Find the parametric equations of the tangent line.

$$x = e^t,\ y = 4t^2,\ z = 2e^t \sin(2t)$$

at $P(1,0,0)$

Answer choices:

A	$x = 1 - t$	$y = 0$	$z = 4$
B	$x = 1 + t$	$y = 0$	$z = 4$
C	$x = 1 + t$	$y = 0$	$z = 4t$
D	$x = 1 - t$	$y = 0$	$z = -4t$

Solution: C

The first thing we need to do is find the value of the parameter t that corresponds with the given point $P(1,0,0)$. We'll plug the value of x from the point into the parametric equation for x.

$$x = e^t$$

$$1 = e^t$$

$$\ln 1 = \ln(e^t)$$

$$t = 0$$

If we try $t = 0$ in $y = 4t^2$ and $z = 2e^t \sin(2t)$, along with $y = 0$ and $z = 0$ from the given point $P(1,0,0)$, we get

$$y = 4t^2$$

$$0 = 4(0)^2$$

$$0 = 0$$

and

$$z = 2e^t \sin(2t)$$

$$0 = 2e^{(0)} \sin(2(0))$$

$$0 = 2(1)(0)$$

$$0 = 0$$

Because all of these equations are true, we can conclude that the value of the parameter is $t = 0$.

Now we'll convert the given parametric equations into vector form. So $x = e^t$, $y = 4t^2$, and $z = 2e^t \sin(2t)$ become

$$r(t) = \left\langle e^t, 4t^2, 2e^t \sin(2t) \right\rangle$$

Take the derivative of this vector, using product rule to find the derivative of $2e^t \sin(2t)$.

$$r'(t) = \left\langle e^t, 8t, 2e^t \sin(2t) + 2e^t \cos(2t)(2) \right\rangle$$

$$r'(t) = \left\langle e^t, 8t, 2e^t \sin(2t) + 4e^t \cos(2t) \right\rangle$$

Plug the parameter $t = 0$ into this derivative vector.

$$r'(0) = \left\langle e^{(0)}, 8(0), 2e^{(0)} \sin(2(0)) + 4e^{(0)} \cos(2(0)) \right\rangle$$

$$r'(0) = \langle 1, 0, 2(1)(0) + 4(1)(1) \rangle$$

$$r'(0) = \langle 1, 0, 4 \rangle$$

Now we can use everything we just found to get the parametric equations of the tangent line. x_1, y_1 and z_1 will come from $P(1,0,0)$, and $r'(t)_1$, $r'(t)_2$ and $r'(t)_3$ will come from $r'(0) = \langle 1,0,4 \rangle$. We get

$$x = x_1 + r'(t)_1 t$$

$$x = 1 + 1t$$

$$x = 1 + t$$

and

$$y = y_1 + r'(t)_2 t$$

$$y = 0 + 0t$$

$$y = 0$$

and

$$z = z_1 + r'(t)_3 t$$

$$z = 0 + 4t$$

$$z = 4t$$

These are the parametric equations of the tangent line.

Topic: Parametric equations of the tangent line

Question: Find the parametric equations of the tangent line.

$$x = e^{-t}, y = e^{-t}\cos t, z = 4e^{-t}$$

at $P(1,1,4)$

Answer choices:

A	$x = 1 + t$	$y = 1 + t$	$z = 1 + t$
B	$x = 1 - t$	$y = 1 - t$	$z = 4 - 4t$
C	$x = 1 + t$	$y = 1 + t$	$z = 4 + 4t$
D	$x = 1 - t$	$y = 1 - t$	$z = 1 - t$

Solution: B

The first thing we need to do is find the value of the parameter t that corresponds with the given point $P(1,1,4)$. We'll plug the value of x from the point into the parametric equation for x.

$$x = e^{-t}$$

$$1 = e^{-t}$$

$$\ln 1 = \ln \left(e^{-t}\right)$$

$$0 = -t$$

$$t = 0$$

If we try $t = 0$ in $y = e^{-t} \cos t$ and $z = 4e^{-t}$, along with $y = 1$ and $z = 4$ from the given point $P(1,1,4)$, we get

$$y = e^{-t} \cos t$$

$$1 = e^{-(0)} \cos(0)$$

$$1 = (1)(1)$$

$$1 = 1$$

and

$$z = 4e^{-t}$$

$$4 = 4e^{-(0)}$$

$$4 = 4(1)$$

$$4 = 4$$

Because all of these equations are true, we can conclude that the value of the parameter is $t = 0$.

Now we'll convert the given parametric equations into vector form. So $x = e^{-t}$, $y = e^{-t}\cos t$, and $z = 4e^{-t}$ become

$$r(t) = \left\langle e^{-t}, e^{-t}\cos t, 4e^{-t} \right\rangle$$

Take the derivative of this vector, using product rule to find the derivative of $e^{-t}\cos t$.

$$r'(t) = \langle -e^{-t}, -e^{-t}\cos t + e^{-t}(-\sin t), -4e^{-t} \rangle$$

$$r'(t) = \langle -e^{-t}, -e^{-t}\cos t - e^{-t}\sin t, -4e^{-t} \rangle$$

Plug the parameter $t = 0$ into this derivative vector.

$$r'(0) = \left\langle -e^{-(0)}, -e^{-(0)}\cos(0) - e^{-(0)}\sin(0), -4e^{-(0)} \right\rangle$$

$$r'(0) = \langle -1, -(1)(1) - (1)(0), -4(1) \rangle$$

$$r'(0) = \langle -1, -1, -4 \rangle$$

Now we can use everything we just found to get the parametric equations of the tangent line. x_1, y_1 and z_1 will come from $P(1,1,4)$, and $r'(t)_1$, $r'(t)_2$ and $r'(t)_3$ will come from $r'(0) = \langle -1, -1, -4 \rangle$. We get

$$x = x_1 + r'(t)_1 t$$

$$x = 1 - 1t$$

$$x = 1 - t$$

and

$$y = y_1 + r'(t)_2 t$$

$$y = 1 - 1t$$

$$y = 1 - t$$

and

$$z = z_1 + r'(t)_3 t$$

$$z = 4 - 4t$$

These are the parametric equations of the tangent line.