Topic: Unit tangent vector

Question: Find the unit tangent vector.

$$r(t) = 3t^2\mathbf{i} - 4\mathbf{j} - t^3\mathbf{k}$$

at $t = 2$

Answer choices:

A $\quad T(2) = \dfrac{6}{\sqrt{6}}\mathbf{i} - \dfrac{6}{\sqrt{6}}\mathbf{k}$

B $\quad T(2) = \dfrac{1}{\sqrt{2}}\mathbf{i} - \dfrac{1}{\sqrt{2}}\mathbf{k}$

C $\quad T(2) = \dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{k}$

D $\quad T(2) = \dfrac{6}{\sqrt{6}}\mathbf{i} + \dfrac{6}{\sqrt{6}}\mathbf{k}$

Solution: B

First find the derivative of the vector function with respect to t.

$$r'(t) = 6t\mathbf{i} - 0\mathbf{j} - 3t^2\mathbf{k}$$

$$r'(t) = 6t\mathbf{i} - 3t^2\mathbf{k}$$

Now we'll plug $t = 2$ into the derivative.

$$r'(2) = 6(2)\mathbf{i} - 3(2)^2\mathbf{k}$$

$$r'(2) = 12\mathbf{i} - 12\mathbf{k}$$

Next we'll find the magnitude of the derivate at $t = 2$.

$$\left| r'(t) \right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left| r'(2) \right| = \sqrt{\left[r'(2)_1\right]^2 + \left[r'(2)_2\right]^2 + \left[r'(2)_3\right]^2}$$

$$\left| r'(2) \right| = \sqrt{(12)^2 + (0)^2 + (-12)^2}$$

$$\left| r'(2) \right| = \sqrt{144 + 144}$$

$$\left| r'(2) \right| = \sqrt{288}$$

$$\left| r'(2) \right| = 12\sqrt{2}$$

Now we can use everything we just found to find the unit tangent vector at $t = 2$.

$$T(t)=\frac{r'(t)}{\left|r'(t)\right|}$$

$$T(2)=\frac{r'(2)}{\left|r'(2)\right|}$$

$$T(2)=\frac{12\mathbf{i}-12\mathbf{k}}{12\sqrt{2}}$$

$$T(2)=\frac{\mathbf{i}-\mathbf{k}}{\sqrt{2}}$$

$$T(2)=\frac{1}{\sqrt{2}}\mathbf{i}-\frac{1}{\sqrt{2}}\mathbf{k}$$

This is the unit tangent vector at $t=2$.

Topic: Unit tangent vector

Question: Find the unit tangent vector.

$$r(t) = -t^2\mathbf{i} + t\mathbf{j} + 2\ln(3t)\mathbf{k}$$

at $t = 1$

Answer choices:

A $\quad T(1) = \frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j} + \frac{2}{\sqrt{5}}\mathbf{k}$

B $\quad T(1) = \frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

C $\quad T(1) = -\frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j} + \frac{2}{\sqrt{5}}\mathbf{k}$

D $\quad T(1) = -\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$

Solution: D

First find the derivative of the vector function with respect to t.

$$r'(t) = -2t\mathbf{i} + (1)\mathbf{j} + 2\left(\frac{1}{3t}\right)(3)\mathbf{k}$$

$$r'(t) = -2t\mathbf{i} + \mathbf{j} + \frac{2}{t}\mathbf{k}$$

Now we'll plug $t = 1$ into the derivative.

$$r'(1) = -2(1)\mathbf{i} + \mathbf{j} + \frac{2}{(1)}\mathbf{k}$$

$$r'(1) = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

Next we'll find the magnitude of the derivate at $t = 1$.

$$\left|r'(t)\right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left|r'(1)\right| = \sqrt{\left[r'(1)_1\right]^2 + \left[r'(1)_2\right]^2 + \left[r'(1)_3\right]^2}$$

$$\left|r'(1)\right| = \sqrt{(-2)^2 + (1)^2 + (2)^2}$$

$$\left|r'(1)\right| = \sqrt{4 + 1 + 4}$$

$$\left|r'(1)\right| = \sqrt{9}$$

$$\left|r'(1)\right| = \sqrt{3}$$

Now we can use everything we just found to find the unit tangent vector at $t = 1$.

$$T(t) = \frac{r'(t)}{\left| r'(t) \right|}$$

$$T(1) = \frac{r'(1)}{\left| r'(1) \right|}$$

$$T(1) = \frac{-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}}{\sqrt{3}}$$

$$T(1) = -\frac{2}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{2}{\sqrt{3}}\mathbf{k}$$

This is the unit tangent vector at $t = 1$.

Topic: Unit tangent vector

Question: Find the unit tangent vector.

$$r(t) = 2\sin(3t)\mathbf{i} - \cos(4t)\mathbf{j} + 4t\mathbf{k}$$

at $t = 0$

Answer choices:

A $\quad T(0) = -\frac{6}{\sqrt{10}}\mathbf{i} - \frac{4}{\sqrt{10}}\mathbf{k}$

B $\quad T(0) = \frac{6}{\sqrt{10}}\mathbf{i} + \frac{4}{\sqrt{10}}\mathbf{k}$

C $\quad T(0) = \frac{3}{\sqrt{13}}\mathbf{i} + \frac{2}{\sqrt{13}}\mathbf{k}$

D $\quad T(0) = -\frac{3}{\sqrt{13}}\mathbf{i} - \frac{2}{\sqrt{13}}\mathbf{k}$

Solution: C

First find the derivative of the vector function with respect to t.

$$r'(t) = 2\cos(3t)(3)\mathbf{i} + \sin(4t)(4)\mathbf{j} + 4\mathbf{k}$$

$$r'(t) = 6\cos(3t)\mathbf{i} + 4\sin(4t)\mathbf{j} + 4\mathbf{k}$$

Now we'll plug $t = 0$ into the derivative.

$$r'(0) = 6\cos(3(0))\mathbf{i} + 4\sin(4(0))\mathbf{j} + 4\mathbf{k}$$

$$r'(0) = 6(1)\mathbf{i} + 4(0)\mathbf{j} + 4\mathbf{k}$$

$$r'(0) = 6\mathbf{i} + 4\mathbf{k}$$

Next we'll find the magnitude of the derivate at $t = 0$.

$$\left|r'(t)\right| = \sqrt{\left[r'(t)_1\right]^2 + \left[r'(t)_2\right]^2 + \left[r'(t)_3\right]^2}$$

$$\left|r'(0)\right| = \sqrt{\left[r'(0)_1\right]^2 + \left[r'(0)_2\right]^2 + \left[r'(0)_3\right]^2}$$

$$\left|r'(0)\right| = \sqrt{(6)^2 + (0)^2 + (4)^2}$$

$$\left|r'(0)\right| = \sqrt{36 + 0 + 16}$$

$$\left|r'(0)\right| = \sqrt{52}$$

$$\left|r'(0)\right| = 2\sqrt{13}$$

Now we can use everything we just found to find the unit tangent vector at $t = 0$.

$$T(t) = \frac{r'(t)}{\left| r'(t) \right|}$$

$$T(0) = \frac{r'(0)}{\left| r'(0) \right|}$$

$$T(0) = \frac{6\mathbf{i} + 4\mathbf{k}}{2\sqrt{13}}$$

$$T(0) = \frac{6}{2\sqrt{13}}\mathbf{i} + \frac{4}{2\sqrt{13}}\mathbf{k}$$

$$T(0) = \frac{3}{\sqrt{13}}\mathbf{i} + \frac{2}{\sqrt{13}}\mathbf{k}$$

This is the unit tangent vector at $t = 0$.