Topic: Changing the order of integration

Question: Change the order of integration for the iterated integral.

$$\int_1^2 \int_1^{3-y} x^2 \, dx \, dy$$

Answer choices:

A $\int_1^2 \int_1^{3-x} x^2 \, dy \, dx$

B $\int_1^{3+x} \int_1^2 x^2 \, dy \, dx$

C $\int_1^2 \int_1^{3+x} x^2 \, dy \, dx$

D $\int_1^{3-x} \int_1^2 x^2 \, dy \, dx$

Solution: A

To change the order of integration of a double integral, we cannot simply reverse the two integrals. The outer integral must have limits that are constants.

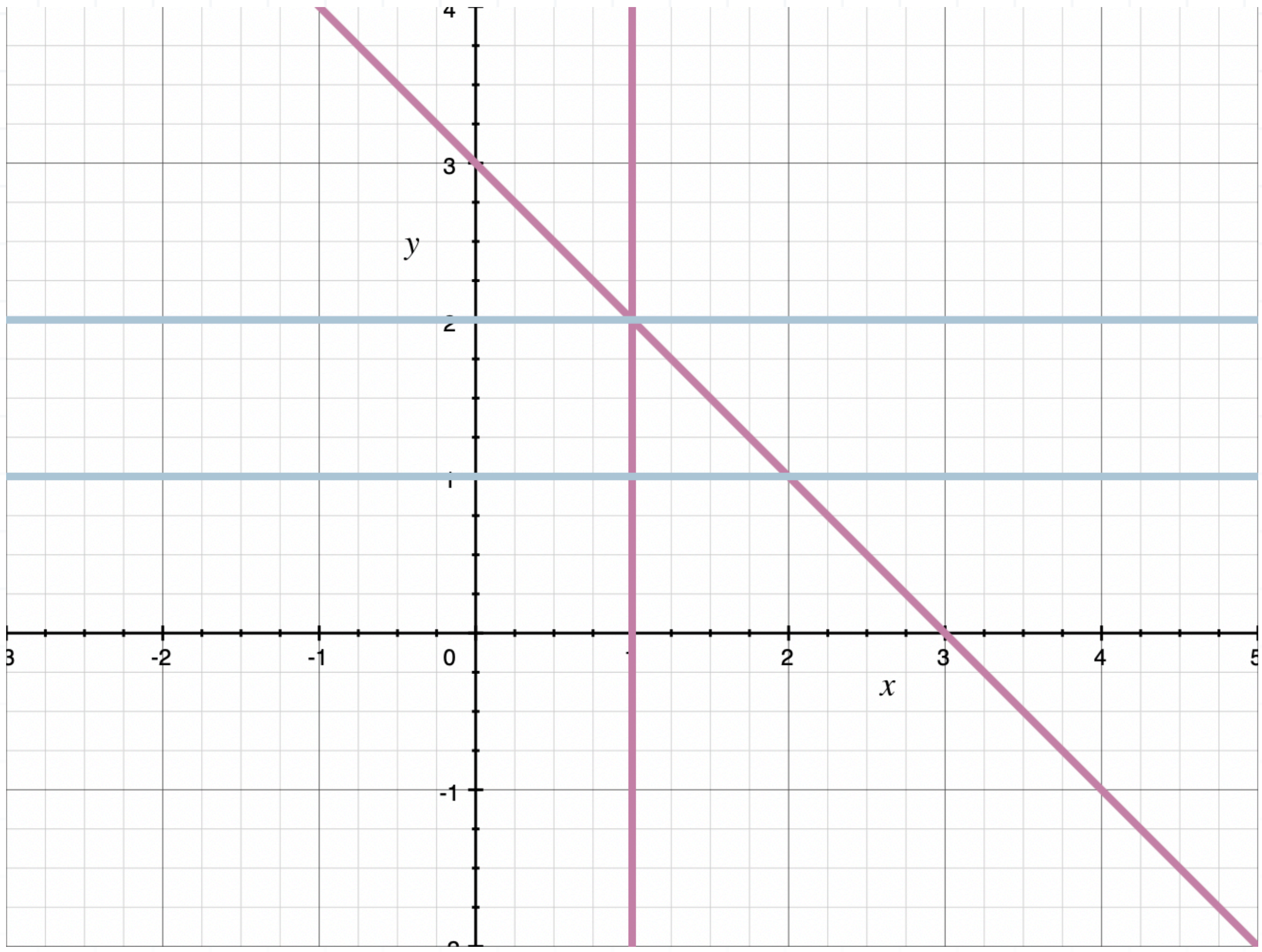
The integral we've been given has $dx\ dy$ on the end of it, so dx is on the inside and dy is on the outside. Which means we've been told to integrate first with respect to x, and then with respect to y. And since we've been asked to switch the order of integration, it means we'll need to change the iterated integral to $dy\ dx$, where we integrate first with respect to y, then x.

Currently, the limits of integration are

$$1 \le x \le 3 - y$$

$$1 \le y \le 2$$

Ideally, we want to sketch the limits. Our sketch will help us confirm what the limits should be when we try to switch them.



To change the order of integration, we want to integrate first with respect to y. That means we need to put our limits of integration for y in terms of x.

The top of the region can be defined by the slanted line $x = 3 - y$. If we solve that for y in terms of x, we get

$$x + y = 3$$

$$y = 3 - x$$

And the lower bound for y can be given by $y = 1$. So the new limits of integration for y are

$$1 \leq y \leq 3 - x$$

The largest value for which the region is defined for x is $x = 2$, and the lower bound for x is $x = 1$. So the limits of integration for x are

$$1 \le x \le 2$$

So when we switch the order of integration, we get

$$\int_1^2 \int_1^{3-y} x^2 \ dx \ dy = \int_1^2 \int_1^{3-x} x^2 \ dy \ dx$$

Topic: Changing the order of integration

Question: Change the order of integration for the iterated integral.

$$\int_{-2}^{4}\int_{1}^{x+3} 4x^2y\ dy\ dx$$

Answer choices:

A $\displaystyle\int_{1}^{7}\int_{y+3}^{4} 4x^2y\ dx\ dy$

B $\displaystyle\int_{1}^{7}\int_{4}^{y+3} 4x^2y\ dx\ dy$

C $\displaystyle\int_{1}^{7}\int_{y-3}^{4} 4x^2y\ dx\ dy$

D $\displaystyle\int_{1}^{7}\int_{4}^{y-3} 4x^2y\ dx\ dy$

Solution: C

To change the order of integration of a double integral, we cannot simply reverse the two integrals. The outer integral must have limits that are constants.

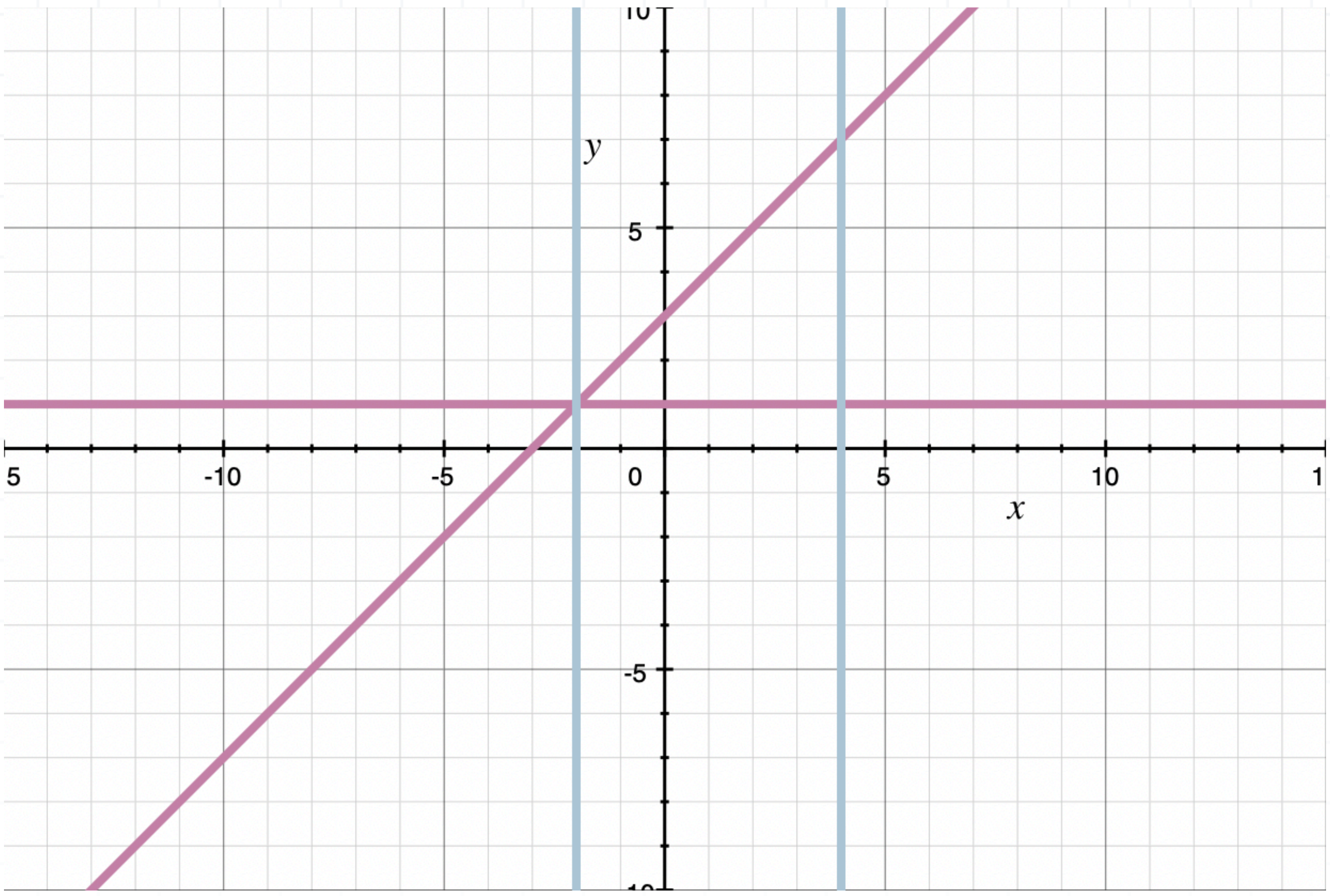
The integral we've been given has $dy\ dx$ on the end of it, so dy is on the inside and dx is on the outside. Which means we've been told to integrate first with respect to y, and then with respect to x. And since we've been asked to switch the order of integration, it means we'll need to change the iterated integral to $dx\ dy$, where we integrate first with respect to x, then y.

Currently, the limits of integration are

$$1 \leq y \leq x + 3$$

$$-2 \leq x \leq 4$$

Ideally, we want to sketch the limits. Our sketch will help us confirm what the limits should be when we try to switch them.



To change the order of integration, we want to integrate first with respect to x. That means we need to put our limits of integration for x in terms of y.

The right side of the region can be defined by the line $x = 4$. And the left side of the region is given by the slanted line $y = x + 3$. If we solve that for x, we get

$$x = y - 3$$

So the new limits of integration for x are

$$y - 3 \le x \le 4$$

The largest value for which the region is defined for y is $y = 7$, and the lower bound for y is $y = 1$. So the limits of integration for y are

$$1 \le y \le 7$$

So when we switch the order of integration, we get

$$\int_{-2}^{4}\int_{1}^{x+3} 4x^2y\ dy\ dx = \int_{1}^{7}\int_{y-3}^{4} 4x^2y\ dx\ dy$$

Topic: Changing the order of integration

Question: Change the order of integration for the iterated integral.

$$\int_1^3 \int_1^{-y+4} 3e^{xy}\ dx\ dy$$

Answer choices:

A $\int_1^3 \int_1^{-x-4} 3e^{xy}\ dy\ dx$

B $\int_1^3 \int_1^{x+4} 3e^{xy}\ dy\ dx$

C $\int_1^3 \int_1^{x-4} 3e^{xy}\ dy\ dx$

D $\int_1^3 \int_1^{4-x} 3e^{xy}\ dy\ dx$

Solution: D

To change the order of integration of a double integral, we cannot simply reverse the two integrals. The outer integral must have limits that are constants.

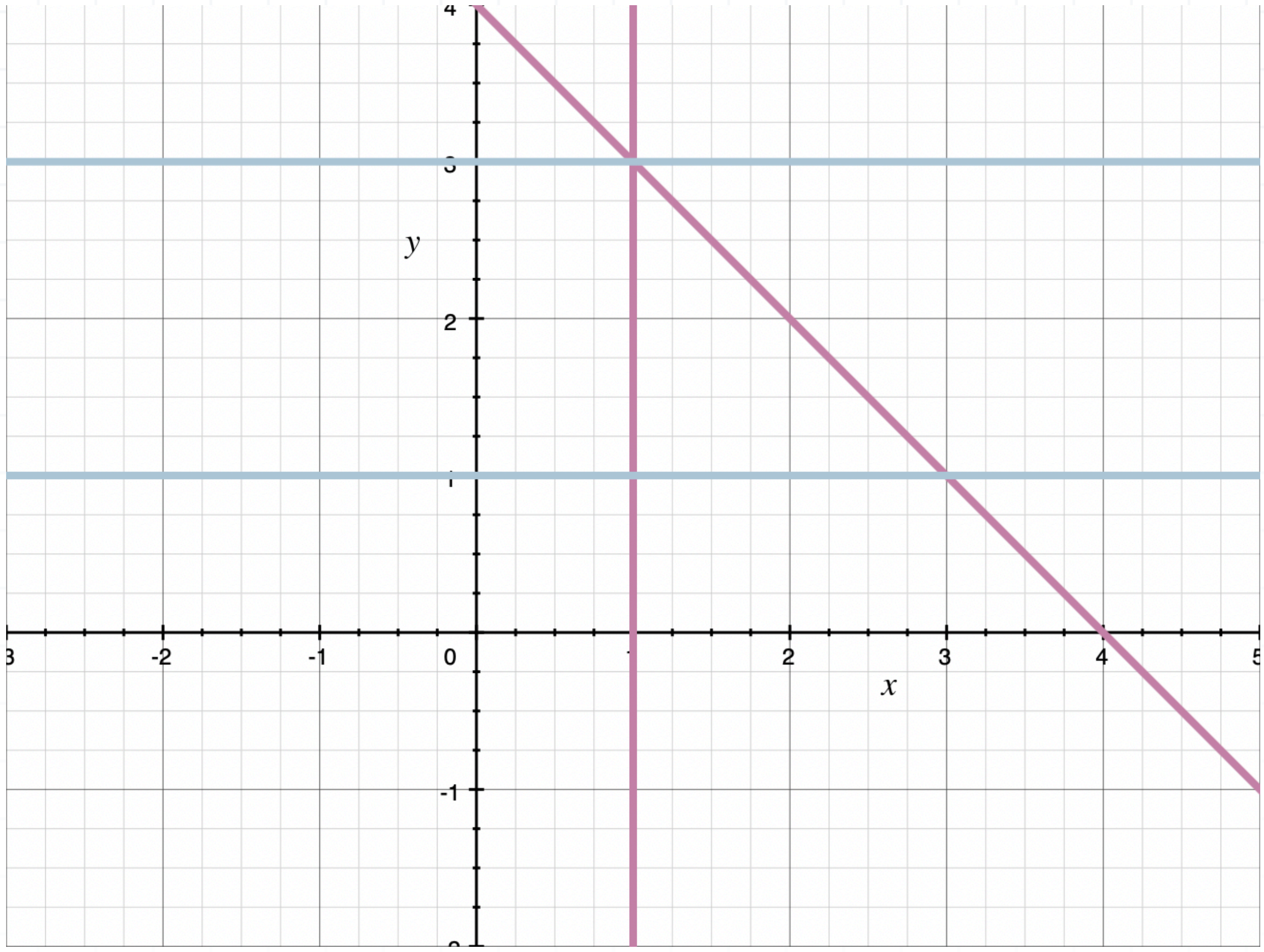
The integral we've been given has $dx\ dy$ on the end of it, so dx is on the inside and dy is on the outside. Which means we've been told to integrate first with respect to x, and then with respect to y. And since we've been asked to switch the order of integration, it means we'll need to change the iterated integral to $dy\ dx$, where we integrate first with respect to y, then x.

Currently, the limits of integration are

$$1 \le x \le -y + 4$$

$$1 \le y \le 3$$

Ideally, we want to sketch the limits. Our sketch will help us confirm what the limits should be when we try to switch them.



To change the order of integration, we want to integrate first with respect to y. That means we need to put our limits of integration for y in terms of x.

The top of the region can be defined by the slanted line $x = -y + 4$. If we solve that for y in terms of x, we get

$$x + y = 4$$

$$y = 4 - x$$

And the lower bound for y can be given by $y = 1$. So the new limits of integration for y are

$$1 \leq y \leq 4 - x$$

The largest value for which the region is defined for x is $x = 3$, and the lower bound for x is $x = 1$. So the limits of integration for x are

$$1 \leq x \leq 3$$

So when we switch the order of integration, we get

$$\int_{1}^{3} \int_{1}^{-y+4} 3e^{xy}\ dx\ dy = \int_{1}^{3} \int_{1}^{-x+4} 3e^{xy}\ dy\ dx$$