Topic: Distance between parallel planes

Question: Find the distance between the parallel planes.

$$3x + 2y - z = 3$$

$$9x + 6y - 3z = 2$$

Answer choices:

A $\frac{1}{3\sqrt{2}}$

B $-\frac{7}{3\sqrt{14}}$

C $\frac{7}{3\sqrt{14}}$

D $-\frac{1}{3\sqrt{2}}$

Solution: C

First we'll confirm that the planes

$$3x + 2y - z = 3$$

$$9x + 6y - 3z = 2$$

are parallel. To test whether the planes are parallel, we'll take the ratio of the components of the normal vectors to each plane.

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

where the planes are given in the form

$$a_1x + a_2y + a_3z = c$$

$$b_1x + b_2y + b_3z = d$$

If the ratios are the same, then the planes are parallel.

This means the two normal vectors are $a\langle a_1, a_2, a_3 \rangle$ and $b\langle b_1, b_2, b_3 \rangle$. First we can determine our normal vectors. For the plane $3x + 2y - z = 3$, we'll get the normal vector $a\langle 3,2, -1 \rangle$. For the plane $9x + 6y - 3z = 2$, we'll get the normal vector $b\langle 9,6, -3 \rangle$. Now we can set up the ratio

$$\frac{3}{9} = \frac{2}{6} = \frac{-1}{-3}$$

$$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

We can see that these ratios are all equal, which means that the planes are parallel.

Next we can find a point on one of the planes. We can take the plane $3x + 2y - z = 3$ and set $y = 0$ and $z = 0$.

$$3x + 2(0) - (0) = 3$$

$$3x = 3$$

$$x = 1$$

This means a point on the plane is $(1,0,0)$.

Now we can find the distance from the point to a plane using the distance formula

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

where the point is (x_1, y_1, z_1) and the plane is $ax + by + cz = -d$.

The point $(1,0,0)$ will give us $x_1 = 1$, $y_1 = 0$, and $z_1 = 0$. The plane $9x + 6y - 3z = 2$ will give us $a = 9$, $b = 6$, $c = -3$, and $d = -2$.

$$d = \frac{|(9)(1) + (6)(0) + (-3)(0) + (-2)|}{\sqrt{(9)^2 + (6)^2 + (-3)^2}}$$

$$d = \frac{|9 + 0 + 0 - 2|}{\sqrt{81 + 36 + 9}}$$

$$d = \frac{|7|}{\sqrt{126}}$$

$$d = \frac{7}{3\sqrt{14}}$$

This is the distance between the planes.

Topic: Distance between parallel planes

Question: Find the distance between the parallel planes.

$$-2x + 1y - 2z = 6$$

$$-8x + 4y - 8z = -3$$

Answer choices:

A $\frac{9}{4}$

B $\frac{7}{4}$

C $\frac{3}{2}$

D $\frac{7}{2}$

Solution: A

First we'll confirm that the planes

$$-2x + 1y - 2z = 6$$

$$-8x + 4y - 8z = -3$$

are parallel. To test whether the planes are parallel, we'll take the ratio of the components of the normal vectors to each plane.

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

where the planes are given in the form

$$a_1x + a_2y + a_3z = c$$

$$b_1x + b_2y + b_3z = d$$

If the ratios are the same, then the planes are parallel.

This means the two normal vectors are $a\langle a_1, a_2, a_3\rangle$ and $b\langle b_1, b_2, b_3\rangle$. First we can determine our normal vectors. For the plane $-2x + 1y - 2z = 6$, we'll get the normal vector $a\langle -2,1,-2\rangle$. For the plane $-8x + 4y - 8z = -3$, we'll get the normal vector $b\langle -8,4,-8\rangle$. Now we can set up the ratio

$$\frac{-2}{-8} = \frac{1}{4} = \frac{-2}{-8}$$

$$\frac{1}{4} = \frac{1}{4} = \frac{1}{4}$$

We can see that these ratios are all equal, which means that the planes are parallel.

Next we can find a point on one of the planes. We can take the plane $-2x+1y-2z=6$ and set $y=0$ and $z=0$.

$$-2x+1(0)-2(0)=6$$

$$-2x=6$$

$$x=-3$$

This means a point on the plane is $(-3,0,0)$.

Now we can find the distance from the point to a plane using the distance formula

$$d=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

where the point is (x_1,y_1,z_1) and the plane is $ax+by+cz=-d$.

The point $(-3,0,0)$ will give us $x_1=-3$, $y_1=0$, and $z_1=0$. The plane $-8x+4y-8z=-3$ will give us $a=-8$, $b=4$, $c=-8$, and $d=3$.

$$d=\frac{|(-8)(-3)+(4)(0)+(-8)(0)+(3)|}{\sqrt{(-8)^2+(4)^2+(-8)^2}}$$

$$d=\frac{|24+0+0+3|}{\sqrt{64+16+64}}$$

$$d = \frac{|27|}{\sqrt{144}}$$

$$d = \frac{27}{12}$$

$$d = \frac{9}{4}$$

This is the distance between the planes.

Topic: Distance between parallel planes

Question: Find the distance between the parallel planes.

$$-6x + 2y + 4z = -12$$

$$-9x + 3y + 6z = 2$$

Answer choices:

A $-\dfrac{10}{3\sqrt{7}}$

B $-\dfrac{20}{3\sqrt{14}}$

C $\dfrac{10}{3\sqrt{7}}$

D $\dfrac{20}{3\sqrt{14}}$

Solution: D

First we'll confirm that the planes

$$-6x + 2y + 4z = -12$$

$$-9x + 3y + 6z = 2$$

are parallel. To test whether the planes are parallel, we'll take the ratio of the components of the normal vectors to each plane.

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

where the planes are given in the form

$$a_1x + a_2y + a_3z = c$$

$$b_1x + b_2y + b_3z = d$$

If the ratios are the same, then the planes are parallel.

This means the two normal vectors are $a\langle a_1, a_2, a_3\rangle$ and $b\langle b_1, b_2, b_3\rangle$. First we can determine our normal vectors. For the plane $-6x + 2y + 4z = -12$, we'll get the normal vector $a\langle -6,2,4\rangle$. For the plane $-9x + 3y + 6z = 2$, we'll get the normal vector $b\langle -9,3,6\rangle$. Now we can set up the ratio

$$\frac{-6}{-9} = \frac{2}{3} = \frac{4}{6}$$

$$\frac{2}{3} = \frac{2}{3} = \frac{2}{3}$$

We can see that these ratios are all equal, which means that the planes are parallel.

Next we can find a point on one of the planes. We can take the plane $-6x+2y+4z=-12$ and set $y=0$ and $z=0$.

$$-6x+2(0)+4(0)=-12$$

$$-6x=-12$$

$$x=2$$

This means a point on the plane is $(2,0,0)$.

Now we can find the distance from the point to a plane using the distance formula

$$d=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

where the point is (x_1,y_1,z_1) and the plane is $ax+by+cz=-d$.

The point $(2,0,0)$ will give us $x_1=2$, $y_1=0$, and $z_1=0$. The plane $-9x+3y+6z=2$ will give us $a=-9$, $b=3$, $c=6$, and $d=-2$.

$$d=\frac{|(-9)(2)+(3)(0)+(6)(0)+(-2)|}{\sqrt{(-9)^2+(3)^2+(6)^2}}$$

$$d=\frac{|-18+0+0-2|}{\sqrt{81+9+36}}$$

$$d = \frac{|-20|}{\sqrt{126}}$$

$$d = \frac{20}{3\sqrt{14}}$$

This is the distance between the planes.