Scalar and vector projections

Scalar projections

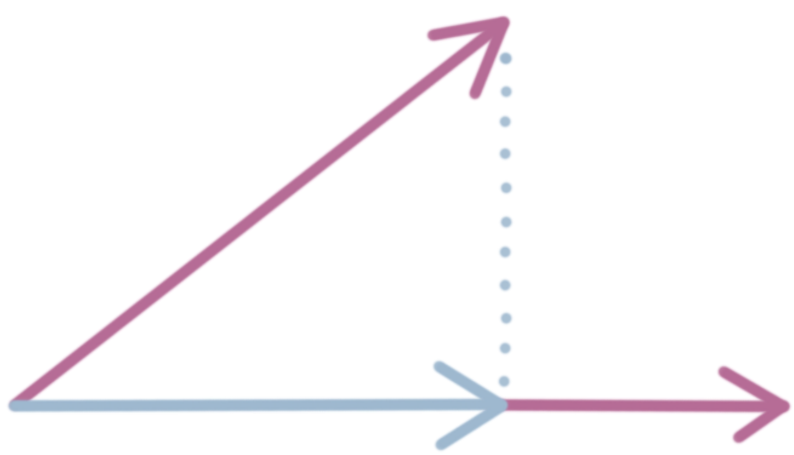
The scalar projection of one vector onto another (also called the component of one vector along another), is

$$\text{comp}_a b = \frac{a \cdot b}{|a|}$$

where $a \cdot b$ is the dot product of the vectors a and b, and $|a|$ is the length of a (also called the magnitude of a).

Vector projections

The vector projection of one vector onto another is like a shadow that one vector casts on another vector. For example, the projection of the top purple vector onto the bottom purple vector is the blue vector:



$$\text{proj}_a b = \left(\frac{a \cdot b}{|a|}\right)\frac{a}{|a|}$$

where $a \cdot b$ is the dot product of the vectors a and b, and $|a|$ is the length of a (also called the magnitude of a).

Example

Find the scalar and vector projections of b onto a.

$$a = i + 2j - 3k$$

$$b = 6i + j$$

Since we use the value of the scalar projection in the formula for the vector projection, we'll start by finding the scalar projection. We'll need the dot product of a and b and the magnitude of a.

We'll convert the given vector equations into the form

$$a = \langle 1,2,-3 \rangle$$

$$b = \langle 6,1,0 \rangle$$

We'll take the dot product.

$$a \cdot b = (1)(6) + (2)(1) + (-3)(0)$$

$$a \cdot b = 6 + 2 + 0$$

$$a \cdot b = 8$$

We'll find the magnitude (length) of a using the distance formula. Remember, the terminal point of a is $(1,2,-3)$ and the initial point of a is $(0,0,0)$.

$$|a|=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

$$|a|=\sqrt{(1-0)^2+(2-0)^2+(-3-0)^2}$$

$$|a|=\sqrt{1+4+9}$$

$$|a|=\sqrt{14}$$

We'll plug $a \cdot b$ and $|a|$ into the formula for the scalar projection.

$$\text{comp}_a b=\frac{8}{\sqrt{14}}$$

Since we have the scalar projection, we already have everything we need to find the vector projection.

$$\text{proj}_a b=\left(\frac{8}{\sqrt{14}}\right)\frac{i+2j-3k}{\sqrt{14}}$$

$$\text{proj}_a b=\frac{8i+16j-24k}{14}$$

$$\text{proj}_a b=\frac{4i+8j-12k}{7}$$

$$\text{proj}_a b=\frac{4}{7}i+\frac{8}{7}j-\frac{12}{7}k$$

To summarize our findings, we'll say that

the scalar projection of b onto a is

$$\text{comp}_a b = \frac{8}{\sqrt{14}}$$

the vector projection of b onto a is

$$\text{proj}_a b = \frac{4}{7}i + \frac{8}{7}j - \frac{12}{7}k$$