Parametric equations of the tangent line

The parametric equations of the tangent line of a vector function $r(t) = \langle r(t)_1, r(t)_2, r(t)_3 \rangle$ are

$$x = x_1 + r'(t_0)_1 t$$

$$y = y_1 + r'(t_0)_2 t$$

$$z = z_1 + r'(t_0)_3 t$$

x_1, y_1 and z_1 come from the point $P(x_1, y_1, z_1)$, which is the point of tangency.

You find $r'(t)_1$, $r'(t)_2$ and $r'(t)_3$ by taking the derivative of the vector $r(t) = \langle r(t)_1, r(t)_2, r(t)_3 \rangle$ or $r(t) = r(t)_1 \mathbf{i} + r(t)_2 \mathbf{j} + r(t)_3 \mathbf{k}$.

You find t_0 by plugging $P(x_1, y_1, z_1)$ into the vector function.

Then you find $r'(t_0)_1$, $r'(t_0)_2$ and $r'(t_0)_3$ by plugging t_0 into the derivative of the vector function.

Example

Find the parametric equations of the tangent line to the vector at the point P.

$$x = e^t$$

$$y = -t \cos t$$

$$z = \sin t$$

at $P(1,0,0)$

First, since the point of tangency is $P(1,0,0)$, we can plug that point into the formulas for the parametric equation of the tangent line from above, and they become

$$x = 1 + r'(t_0)_1 t$$

and

$$y = 0 + r'(t_0)_2 t$$

$$y = r'(t_0)_2 t$$

and

$$z = 0 + r'(t_0)_3 t$$

$$z = r'(t_0)_3 t$$

Now we'll find a value for t_0. We'll use $x = e^t$, change t to t_0 and plug $x = 1$ (from $P(1,0,0)$) into the equation and get

$$1 = e^{t_0}$$

$$\ln 1 = \ln e^{t_0}$$

$$t_0 = 0$$

Plugging $t_0 = 0$ and $y = 0$ (from $P(1,0,0)$) into $y = -t\cos t$ and get

$$0 = -0\cos 0$$

$$0 = 0$$

Since this equation is true, $t_0 = 0$ works for $y = -t\cos t$ as well as $x = e^t$. Now we'll plug $t_0 = 0$ and $z = 0$ (from $P(1,0,0)$) into $z = \sin t$ and get

$$0 = \sin 0$$

$$0 = 0$$

Since this equation is true, we've now shown that $t_0 = 0$ satisfies $x = e^t$, $y = -t\cos t$ and $z = \sin t$, so 0 is the value we want to use for t_0. Therefore, the parametric equations of the tangent line become

$$x = 1 + r'(0)_1 t$$

$$y = r'(0)_2 t$$

$$z = r'(0)_3 t$$

Next we need to find the derivative of the vector function. The original function is

$$r(t) = \left\langle e^t, -t\cos t, \sin t \right\rangle$$

so its derivative is

$$r'(t) = \left\langle e^t, (-1)(\cos t) + (-t)(-\sin t), \cos(t) \right\rangle$$

$$r'(t) = \left\langle e^t, -\cos t + t\sin t, \cos t \right\rangle$$

$$r'(t) = \left\langle e^t, t\sin t - \cos t, \cos t \right\rangle$$

Plugging $t_0 = 0$ into the derivative, we get

$$r'(0) = \left\langle e^0, 0 \sin 0 - \cos 0, \cos 0 \right\rangle$$

$$r'(0) = \langle 1, 0 - 1, 1 \rangle$$

$$r'(0) = \langle 1, -1, 1 \rangle$$

We’ll take these three values, plug them into our parametric equations, and the parametric equations become

$$x = 1 + 1t$$

$$y = -1t$$

$$z = 1t$$

and these simplify to

$$x = 1 + t$$

$$y = -t$$

$$z = t$$