Topic: Parametric and symmetric equations of the line

Question: Find the symmetric equation of the line.

Passing through $a(1,-3,-1)$

Perpendicular to $\mathbf{i}+4\mathbf{j}-2\mathbf{k}=3$

Answer choices:

A $\quad x-1=\dfrac{y+3}{4}=-\dfrac{z+1}{2}$

B $\quad -x-1=-\dfrac{y-3}{4}=\dfrac{z-1}{2}$

C $\quad -x+1=-\dfrac{y+3}{4}=\dfrac{z+1}{2}$

D $\quad x+1=\dfrac{y-3}{4}=-\dfrac{z-1}{2}$

Solution: A

The symmetric equation of a line is given by

$$\frac{x-a_1}{v_1}=\frac{y-a_2}{v_2}=\frac{z-a_3}{v_3}$$

where $a(a_1,a_2,a_3)$ is a point on the line, and where $v\langle v_1,v_2,v_3\rangle$ is a vector parallel to the line. Since we already know the lines passes through $a(1,-3,-1)$, we can plug this into the formula to get

$$\frac{x-1}{v_1}=\frac{y-(-3)}{v_2}=\frac{z-(-1)}{v_3}$$

$$\frac{x-1}{v_1}=\frac{y+3}{v_2}=\frac{z+1}{v_3}$$

Since we want the line that's perpendicular to $\mathbf{i}+4\mathbf{j}-2\mathbf{k}=3$, we could also say we're looking for the line that's parallel to the normal vector of $\mathbf{i}+4\mathbf{j}-2\mathbf{k}=3$. The normal vector is given by the coefficients $\langle 1,4,-2\rangle$, so $\langle 1,4,-2\rangle$ is parallel to the line we need. Therefore we can find the symmetric equation of the line.

$$\frac{x-1}{1}=\frac{y+3}{4}=\frac{z+1}{-2}$$

$$x-1=\frac{y+3}{4}=-\frac{z+1}{2}$$

Topic: Parametric and symmetric equations of the line

Question: Find the parametric and symmetric equations of the line.

Passes through $a(-1, -1, -1)$

Perpendicular to $-\mathbf{i} - 3\mathbf{j} - 2\mathbf{k} = 5$

Answer choices:

A $\quad x = -1 + t \quad y = -1 + 3t \quad z = -1 + 2t \quad x - 1 = \frac{y-1}{3} = \frac{z-1}{2}$

B $\quad x = -1 - t \quad y = -1 - 3t \quad z = -1 - 2t \quad x - 1 = \frac{y-1}{3} = \frac{z-1}{2}$

C $\quad x = -1 - t \quad y = -1 - 3t \quad z = -1 - 2t \quad -x - 1 = -\frac{y+1}{3} = -\frac{z+1}{2}$

D $\quad x = -1 + t \quad y = -1 + 3t \quad z = -1 + 2t \quad x + 1 = \frac{y+1}{3} = \frac{z+1}{2}$

Solution: C

The symmetric equation of a line is given by

$$\frac{x-a_1}{v_1}=\frac{y-a_2}{v_2}=\frac{z-a_3}{v_3}$$

where $a(a_1, a_2, a_3)$ is a point on the line, and where $v\langle v_1, v_2, v_3\rangle$ is a vector parallel to the line. Since we already know the lines passes through $a(-1,-1,-1)$, we can plug this into the formula to get

$$\frac{x-(-1)}{v_1}=\frac{y-(-1)}{v_2}=\frac{z-(-1)}{v_3}$$

$$\frac{x+1}{v_1}=\frac{y+1}{v_2}=\frac{z+1}{v_3}$$

Since we want the line that's perpendicular to $-\mathbf{i}-3\mathbf{j}-2\mathbf{k}=5$, we could also say we're looking for the line that's parallel to the normal vector of $-\mathbf{i}-3\mathbf{j}-2\mathbf{k}=5$. The normal vector is given by the coefficients $\langle -1,-3,-2\rangle$, so $\langle -1,-3,-2\rangle$ is parallel to the line we need. Therefore we can find the symmetric equation of the line.

$$\frac{x+1}{-1}=\frac{y+1}{-3}=\frac{z+1}{-2}$$

$$-x-1=-\frac{y+1}{3}=-\frac{z+1}{2}$$

Now in order to find the parametric equations, we have to first find the vector equation of the line, which is given by

$$r=r_0+tv$$

where r_0 is a point on the line and where v is a vector that's parallel to the line. The line we want passes through $a(-1, -1, -1)$, which we can rewrite as $-\mathbf{i} - \mathbf{j} - \mathbf{k}$.

Since we want the line that's perpendicular to $-\mathbf{i} - 3\mathbf{j} - 2\mathbf{k} = 5$, we could also say we're looking for the line that's parallel to the normal vector of $-\mathbf{i} - 3\mathbf{j} - 2\mathbf{k} = 5$. The normal vector is given by the coefficients $\langle -1, -3, -2 \rangle$, so $\langle -1, -3, -2 \rangle$ is parallel to the line we need. We can rewrite this as $-\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$.

If we plug both of these into the vector equation of the line, we get

$$r = r_0 + tv$$

$$r = (-\mathbf{i} - \mathbf{j} - \mathbf{k}) + t(-\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})$$

$$r = -\mathbf{i} - \mathbf{j} - \mathbf{k} - t\mathbf{i} - 3t\mathbf{j} - 2t\mathbf{k}$$

$$r = (-\mathbf{i} - t\mathbf{i}) + (-\mathbf{j} - 3t\mathbf{j}) + (-\mathbf{k} - 2t\mathbf{k})$$

$$r = (-1 - t)\mathbf{i} + (-1 - 3t)\mathbf{j} + (-1 - 2t)\mathbf{k}$$

If the vector equation is $r = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, then the parametric equations of the line are given by $x = a$, $y = b$, and $z = c$.

$$x = -1 - t$$

$$y = -1 - 3t$$

$$z = -1 - 2t$$

Topic: Parametric and symmetric equations of the line

Question: Find the parametric and symmetric equations of the line.

Passing through $a(4,6,-5)$

Perpendicular to $6\mathbf{i}-2\mathbf{j}+7\mathbf{k}=-2$

Answer choices:

A $\quad x=4-6t \quad y=6+2t \quad z=-5-7t \quad -\frac{x-4}{6}=\frac{y-6}{2}=-\frac{z+5}{7}$

B $\quad x=4+6t \quad y=6-2t \quad z=-5+7t \quad -\frac{x-4}{6}=\frac{y-6}{2}=-\frac{z+5}{7}$

C $\quad x=4-6t \quad y=6+2t \quad z=-5-7t \quad \frac{x-4}{6}=-\frac{y-6}{2}=\frac{z+5}{7}$

D $\quad x=4+6t \quad y=6-2t \quad z=-5+7t \quad \frac{x-4}{6}=-\frac{y-6}{2}=\frac{z+5}{7}$

Solution: D

The symmetric equation of a line is given by

$$\frac{x-a_1}{v_1}=\frac{y-a_2}{v_2}=\frac{z-a_3}{v_3}$$

where $a(a_1, a_2, a_3)$ is a point on the line, and where $v\langle v_1, v_2, v_3\rangle$ is a vector parallel to the line. Since we already know the lines passes through $a(4,6,-5)$, we can plug this into the formula to get

$$\frac{x-4}{v_1}=\frac{y-6}{v_2}=\frac{z-(-5)}{v_3}$$

$$\frac{x-4}{v_1}=\frac{y-6}{v_2}=\frac{z+5}{v_3}$$

Since we want the line that's perpendicular to $6\mathbf{i}-2\mathbf{j}+7\mathbf{k}=-2$, we could also say we're looking for the line that's parallel to the normal vector of $6\mathbf{i}-2\mathbf{j}+7\mathbf{k}=-2$. The normal vector is given by the coefficients $\langle 6,-2,7\rangle$, so $\langle 6,-2,7\rangle$ is parallel to the line we need. Therefore we can find the symmetric equation of the line.

$$\frac{x-4}{6}=\frac{y-6}{-2}=\frac{z+5}{7}$$

$$\frac{x-4}{6}=-\frac{y-6}{2}=\frac{z+5}{7}$$

Now in order to find the parametric equations, we have to first find the vector equation of the line, which is given by

$$r=r_0+tv$$

where r_0 is a point on the line and where v is a vector that's parallel to the line. The line we want passes through $a(4,6,-5)$, which we can rewrite as $4\mathbf{i}+6\mathbf{j}-5\mathbf{k}$.

Since we want the line that's perpendicular to $6\mathbf{i}-2\mathbf{j}+7\mathbf{k}=-2$, we could also say we're looking for the line that's parallel to the normal vector of $6\mathbf{i}-2\mathbf{j}+7\mathbf{k}=-2$. The normal vector is given by the coefficients $\langle 6,-2,7\rangle$, so $\langle 6,-2,7\rangle$ is parallel to the line we need. We can rewrite this as $6\mathbf{i}-2\mathbf{j}+7\mathbf{k}$.

If we plug both of these into the vector equation of the line, we get

$$r=r_0+tv$$

$$r=(4\mathbf{i}+6\mathbf{j}-5\mathbf{k})+t(6\mathbf{i}-2\mathbf{j}+7\mathbf{k})$$

$$r=4\mathbf{i}+6\mathbf{j}-5\mathbf{k}+6t\mathbf{i}-2t\mathbf{j}+7t\mathbf{k}$$

$$r=(4\mathbf{i}+6t\mathbf{i})+(6\mathbf{j}-2t\mathbf{j})+(-5\mathbf{k}+7t\mathbf{k})$$

$$r=(4+6t)\mathbf{i}+(6-2t)\mathbf{j}+(-5+7t)\mathbf{k}$$

If the vector equation is $r=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$, then the parametric equations of the line are given by $x=a$, $y=b$, and $z=c$.

$$x=4+6t$$

$$y=6-2t$$

$$z=-5+7t$$