Topic: Implicit differentiation for multivariable functions

Question: Use implicit differentiation to find the derivative of the multivariable function.

$$x^2e^y = 3x^2 + 4y^2$$

Answer choices:

A $\quad \dfrac{dy}{dx} = \dfrac{6x + 2xe^y}{8y + x^2e^y}$

B $\quad \dfrac{dy}{dx} = \dfrac{-6x + 2xe^y}{-8y + x^2e^y}$

C $\quad \dfrac{dy}{dx} = \dfrac{-6x + 2xe^y}{8y - x^2e^y}$

D $\quad \dfrac{dy}{dx} = \dfrac{6x - 2xe^y}{8y - x^2e^y}$

Solution: C

We can use

$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

to find the derivative of a multivariable function, which means we'll need to find the partial derivatives of F with respect to x and y.

First though, we'll collect all of our terms on one side of the equation so that we can set the function equal to $F(x, y)$.

$$x^2e^y = 3x^2 + 4y^2$$

$$0 = 3x^2 + 4y^2 - x^2e^y$$

$$F(x, y) = 3x^2 + 4y^2 - x^2e^y$$

Next we'll find the partial derivatives.

$$\frac{\partial F}{\partial x} = 6x - 2xe^y$$

and

$$\frac{\partial F}{\partial y} = 8y - x^2e^y$$

Plugging these values into the formula for the derivative, we get

$$\frac{dy}{dx} = -\frac{6x - 2xe^y}{8y - x^2e^y}$$

$$\frac{dy}{dx} = \frac{-6x + 2xe^y}{8y - x^2e^y}$$

This is the derivative of the multivariable function.

Topic: Implicit differentiation for multivariable functions

Question: Use implicit differentiation to find the partial derivatives of the multivariable function.

$$2yz^2 = 4xz + 3y^3$$

Answer choices:

A $\quad \frac{\partial z}{\partial x} = -\frac{4z}{4x - 4yz} \qquad \frac{\partial z}{\partial y} = \frac{9y^2 - 2z^2}{4x - 4yz}$

B $\quad \frac{\partial z}{\partial x} = -\frac{4z}{4x - 4yz} \qquad \frac{\partial z}{\partial y} = -\frac{9y^2 - 2z^2}{4x - 4yz}$

C $\quad \frac{\partial z}{\partial x} = \frac{4z}{4x - 4yz} \qquad \frac{\partial z}{\partial y} = -\frac{9y^2 - 2z^2}{4x - 4yz}$

D $\quad \frac{\partial z}{\partial x} = \frac{4z}{4x - 4yz} \qquad \frac{\partial z}{\partial y} = \frac{9y^2 - 2z^2}{4x - 4yz}$

Solution: B

Normally we'd be able to use

$$\frac{dy}{dx}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

to find the derivative of a multivariable function. In this case though, we have a function in terms of three variables, which means we'll have to find partial derivatives, instead of just one derivative. We'll use the formulas

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

and

$$\frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$

So we'll need to find the partial derivatives of F with respect to x, y, and z.

First though, we'll collect all of our terms on one side of the equation so that we can set the function equal to $F(x,y,z)$.

$$2yz^2=4xz+3y^3$$

$$0=4xz+3y^3-2yz^2$$

$$F(x,y,z)=4xz+3y^3-2yz^2$$

Next we'll find the partial derivatives.

$$\frac{\partial F}{\partial x} = 4z$$

$$\frac{\partial F}{\partial y} = 9y^2 - 2z^2$$

$$\frac{\partial F}{\partial z} = 4x - 4yz$$

Plugging these values into the formulas for the partial derivatives, we get

$$\frac{\partial z}{\partial x} = -\frac{4z}{4x - 4yz}$$

and

$$\frac{\partial z}{\partial y} = -\frac{9y^2 - 2z^2}{4x - 4yz}$$

These are the partial derivatives of the multivariable function.

Topic: Implicit differentiation for multivariable functions

Question: Use implicit differentiation to find the partial derivatives of the multivariable function.

$$-8y^2\ln(z) = 4e^y + 6x\cos(2z)$$

Answer choices:

A $\quad \dfrac{\partial z}{\partial x} = \dfrac{6\cos(2z)}{12x\sin(2z) - \frac{8y^2}{z}} \qquad \dfrac{\partial z}{\partial y} = \dfrac{4e^y + 16y\ln(z)}{12x\sin(2z) - \frac{8y^2}{z}}$

B $\quad \dfrac{\partial z}{\partial x} = -\dfrac{4e^y + 16y\ln(z)}{12x\sin(2z) + \frac{8y^2}{z}} \qquad \dfrac{\partial z}{\partial y} = -\dfrac{6\cos(2z)}{12x\sin(2z) + \frac{8y^2}{z}}$

C $\quad \dfrac{\partial z}{\partial x} = \dfrac{6\cos(2z)}{12x\sin(2z) + \frac{8y^2}{z}} \qquad \dfrac{\partial z}{\partial y} = \dfrac{4e^y + 16y\ln(z)}{12x\sin(2z) + \frac{8y^2}{z}}$

D $\quad \dfrac{\partial z}{\partial x} = \dfrac{4e^y + 16y\ln(z)}{12x\sin(2z) + \frac{8y^2}{z}} \qquad \dfrac{\partial z}{\partial y} = \dfrac{6\cos(2z)}{12x\sin(2z) + \frac{8y^2}{z}}$

Solution: A

Normally we'd be able to use

$$\frac{dy}{dx}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

to find the derivative of a multivariable function. In this case though, we have a function in terms of three variables, which means we'll have to find partial derivatives, instead of just one derivative. We'll use the formulas

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

and

$$\frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$

So we'll need to find the partial derivatives of F with respect to x, y, and z.

First though, we'll collect all of our terms on one side of the equation so that we can set the function equal to $F(x, y, z)$.

$$-8y^2\ln(z)=4e^y+6x\cos(2z)$$

$$0=4e^y+6x\cos(2z)+8y^2\ln(z)$$

$$F(x,y,z)=4e^y+6x\cos(2z)+8y^2\ln(z)$$

Next we'll find the partial derivatives.

$$\frac{\partial F}{\partial x} = 6\cos(2z)$$

and

$$\frac{\partial F}{\partial y} = 4e^y + 16y\ln(z)$$

and

$$\frac{\partial F}{\partial z} = 6x(2)\left[-\sin(2z)\right] + 8y^2\left(\frac{1}{z}\right)$$

$$\frac{\partial F}{\partial z} = -12x\sin(2z) + \frac{8y^2}{z}$$

Plugging these values into the formulas for the partial derivatives, we get

$$\frac{\partial z}{\partial x} = -\frac{6\cos(2z)}{-12x\sin(2z) + \frac{8y^2}{z}}$$

$$\frac{\partial z}{\partial x} = \frac{6\cos(2z)}{12x\sin(2z) - \frac{8y^2}{z}}$$

and

$$\frac{\partial z}{\partial y} = -\frac{4e^y + 16y\ln(z)}{-12x\sin(2z) + \frac{8y^2}{z}}$$

$$\frac{\partial z}{\partial y} = \frac{4e^y + 16y\ln(z)}{12x\sin(2z) - \frac{8y^2}{z}}$$

These are the partial derivatives of the multivariable function.