Calculus 3 Workbook

Triple integrals

krista king MATH

ITERATED INTEGRALS

1. Evaluate the iterated integral.

$$\int_{-2}^{3}\int_{0}^{\pi}\int_{-4}^{-2}\frac{2x^3}{x^2+1}\sin y(3z^2-4z+3)\ dz\ dy\ dx$$

2. Evaluate the iterated improper integral.

$$\int_{0}^{\infty}\int_{0}^{\infty}\int_{1}^{\infty}\frac{1}{(x+2y+z)^5}\ dz\ dy\ dx$$

3. Evaluate the iterated integral.

$$\int_{0}^{\frac{\pi}{2}}\int_{x}^{\frac{\pi}{4}}\int_{2y}^{x+\frac{\pi}{2}}\cos(x-2y+z)\ dz\ dy\ dx$$

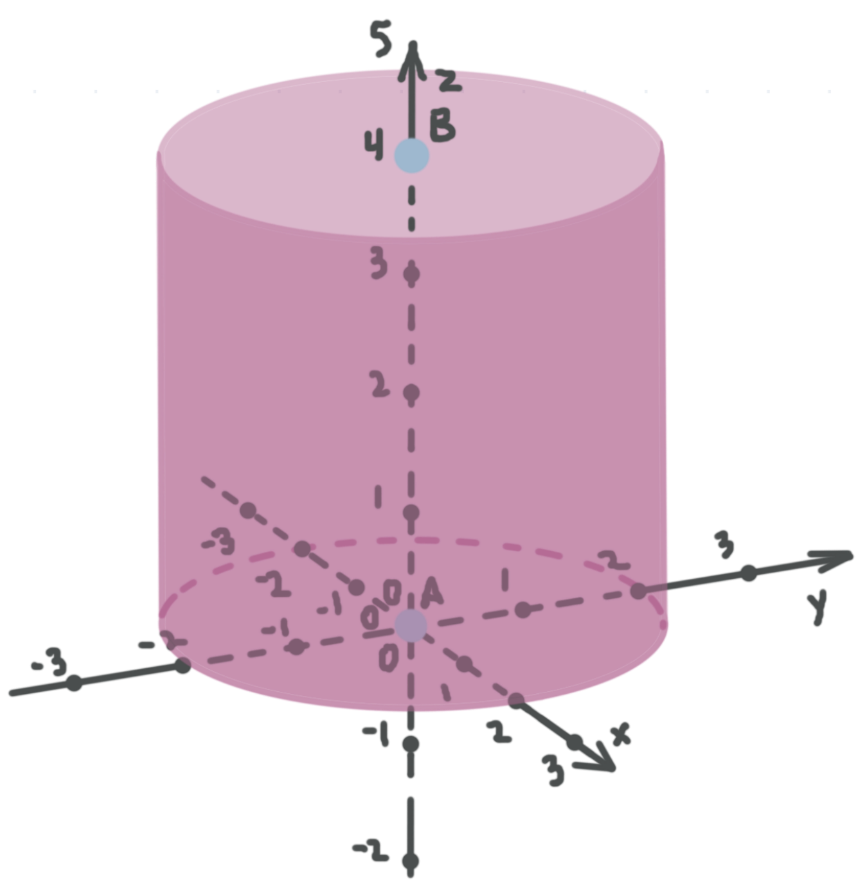
TRIPLE INTEGRALS

■ 1. Evaluate the triple integral, where D is the box with opposite corners $(5,0,1)$ and $(14,2,10)$.

$$\iiint_D y \log\left(\frac{z^4}{(x-4)^2 \cdot 10^{y^2}}\right)\, dV$$

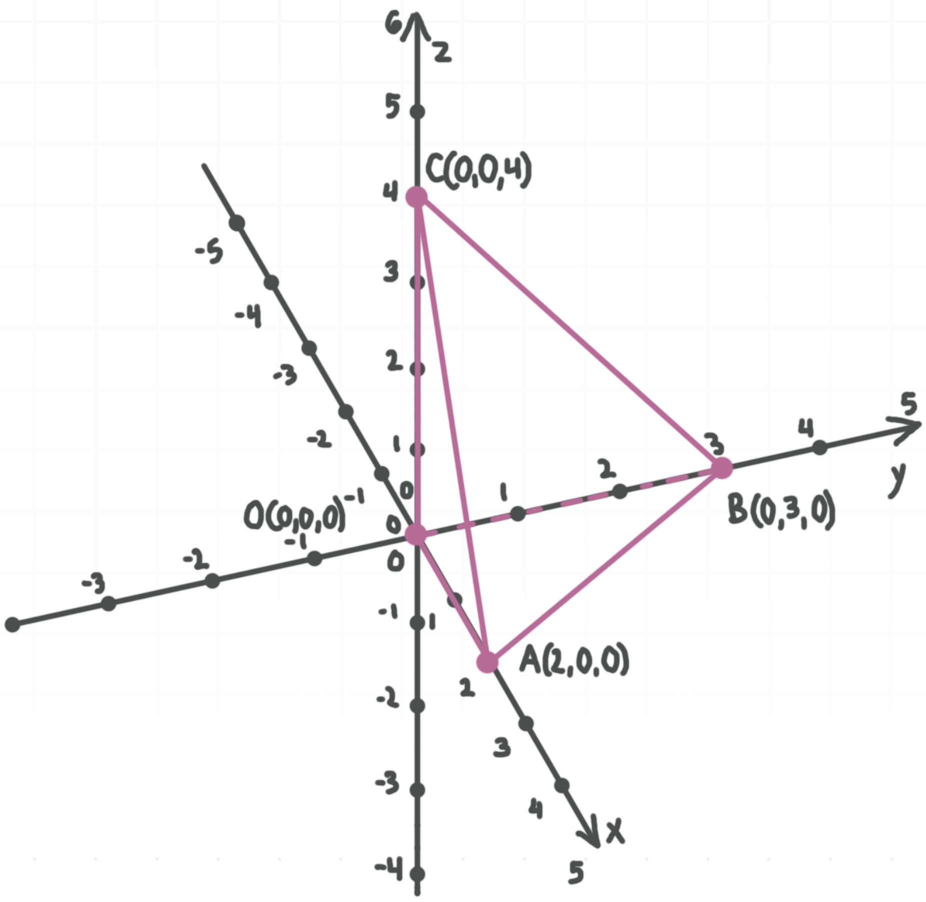
■ 2. Evaluate the triple integral, where D is the right circular cylinder with radius 2, height 4, and a base that lies in the xy-plane with center at the origin.

$$\iiint_D e^{0.5z}\sqrt{x^2+y^2}\, dV$$



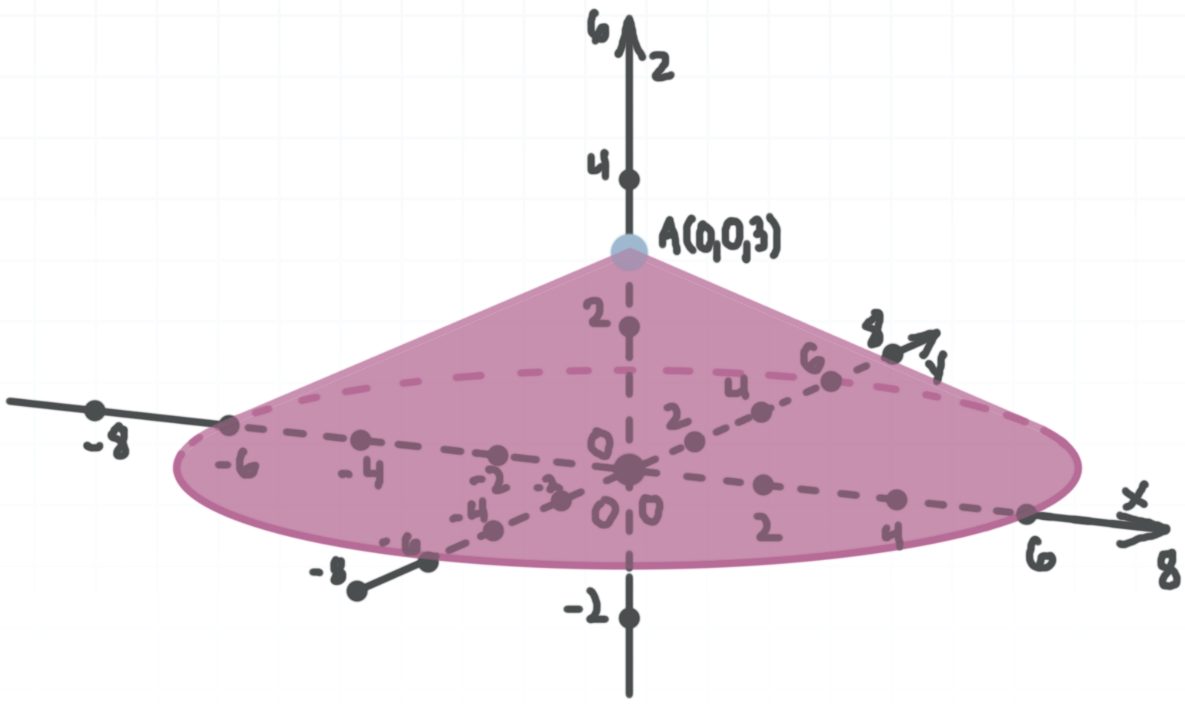
■ 3. Evaluate the triple integral, where $ABCO$ is the irregular pyramid such that O is the origin and the vertices are $A(2,0,0)$, $B(0,3,0)$, and $C(0,0,4)$.

$$\iiint_{ABCO} 72xy \ dV$$

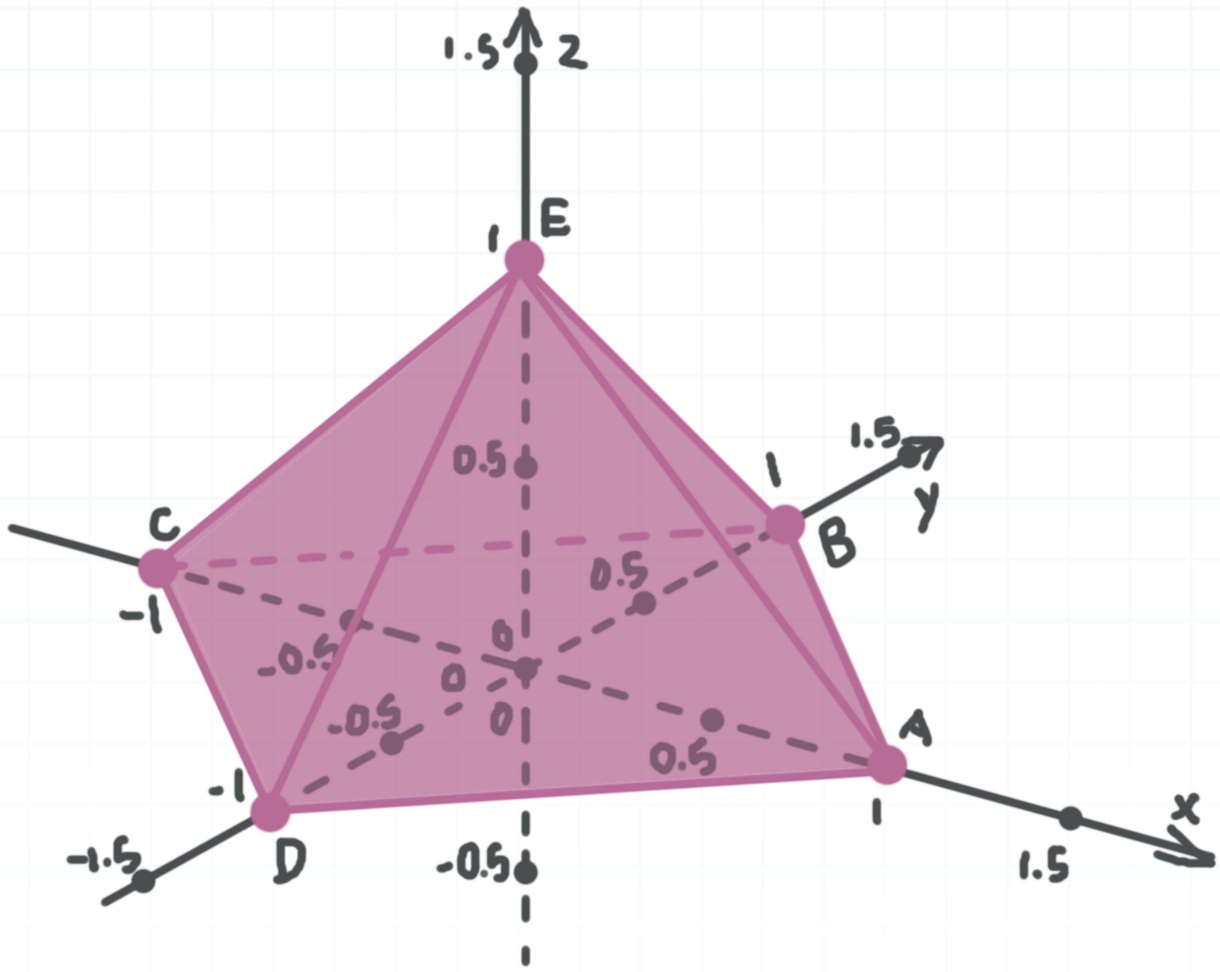


AVERAGE VALUE

■ 1. Use triple integrals to find the average value of the function $f(x, y, z) = x^2 + y^2 + z^2$ over a right circular cone with radius $R = 6$, height $h = 3$, and a base that lies in the xy-plane with center at the origin.



■ 2. Use triple integrals to find the average value of the function $f(x, y, z) = |xyz|$ over a regular pyramid $ABCDE$, where $A(1,0,0)$, $B(0,1,0)$, $C(-1,0,0)$, $D(0,-1,0)$, and $E(0,0,1)$.



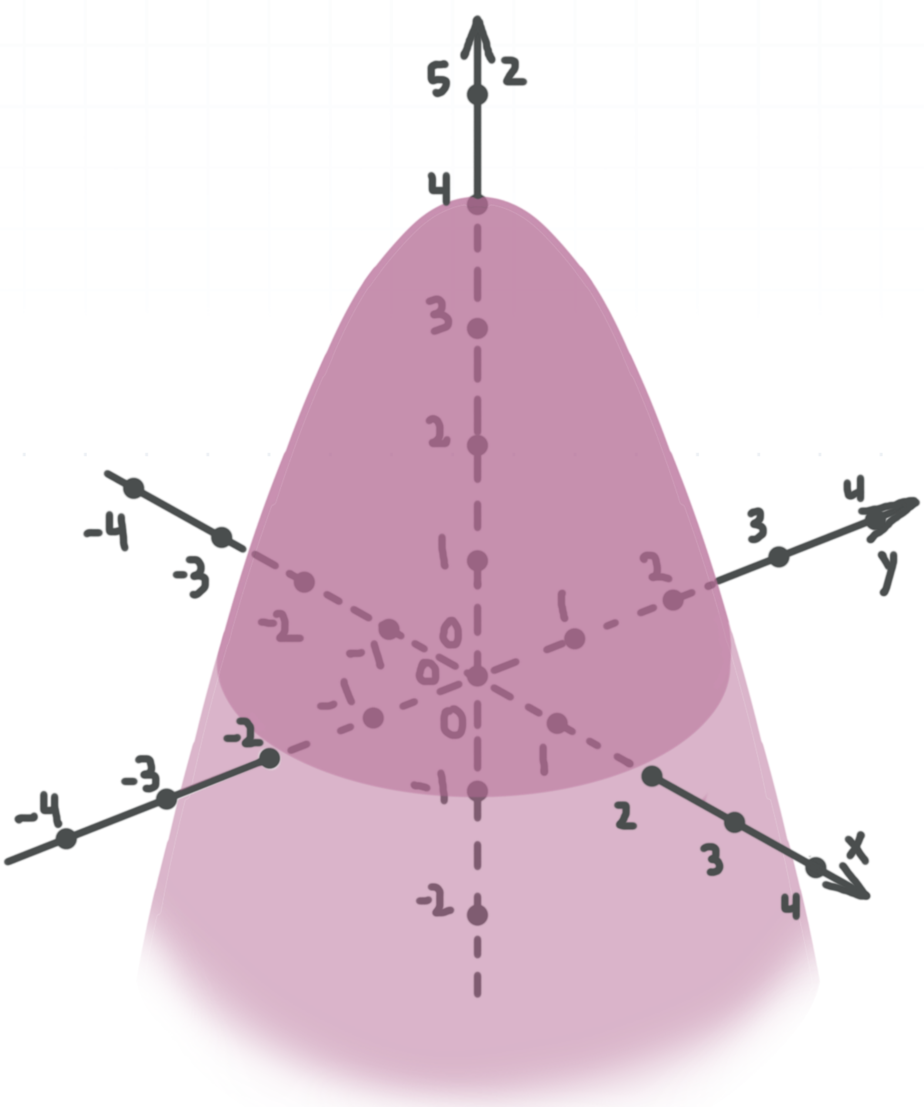
3. Use triple integrals to find the average value of the function $f(x, y, z) = 2x - 3y + z$ over a layer bounded by the planes $z = 2$ and $z = 4$.

FINDING VOLUME

1. Find the volume given by the triple integral.

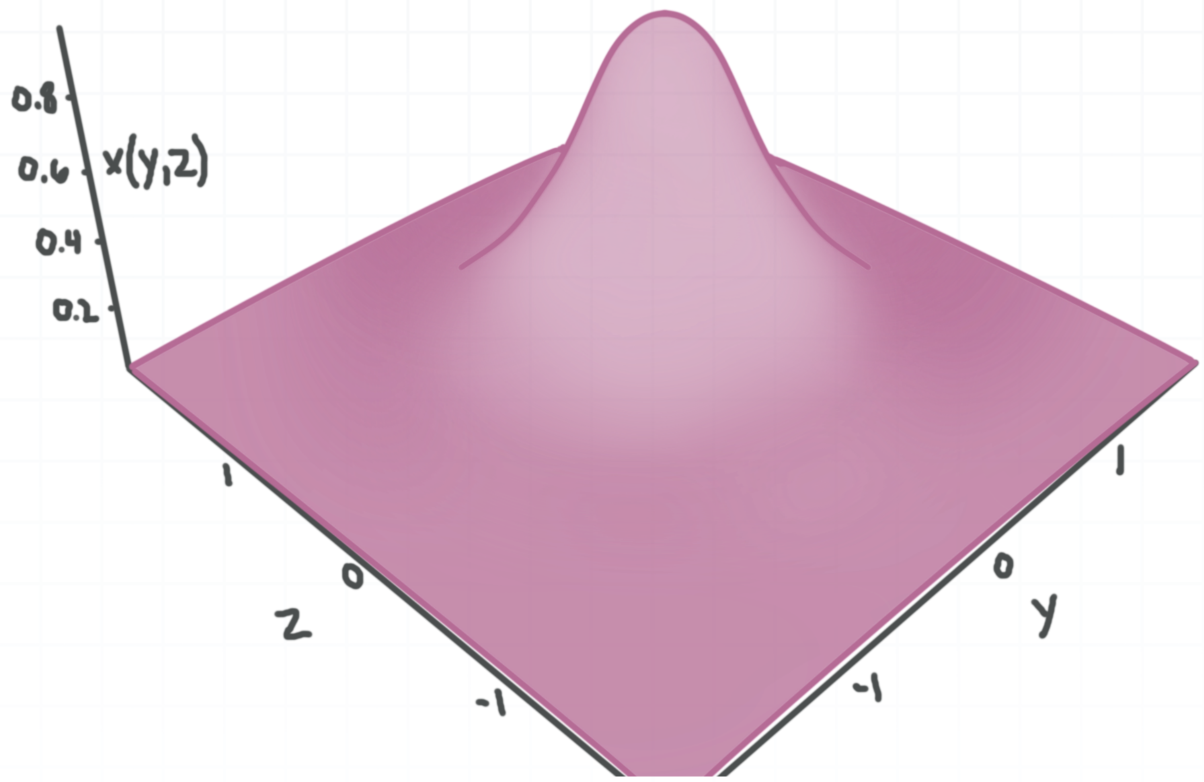
$$\int_{-4}^{6}\int_{3-2x^2}^{10}\int_{2x-y}^{12-y} dz\ dy\ dx$$

2. Use a triple integral to find the volume of the solid bounded by the circular paraboloid $4 - x^2 - y^2 - z = 0$ and the xy-plane.



■ 3. Use a triple integral to find the volume of the solid bounded by the surface $x = g(y, z)$ and the yz-plane.

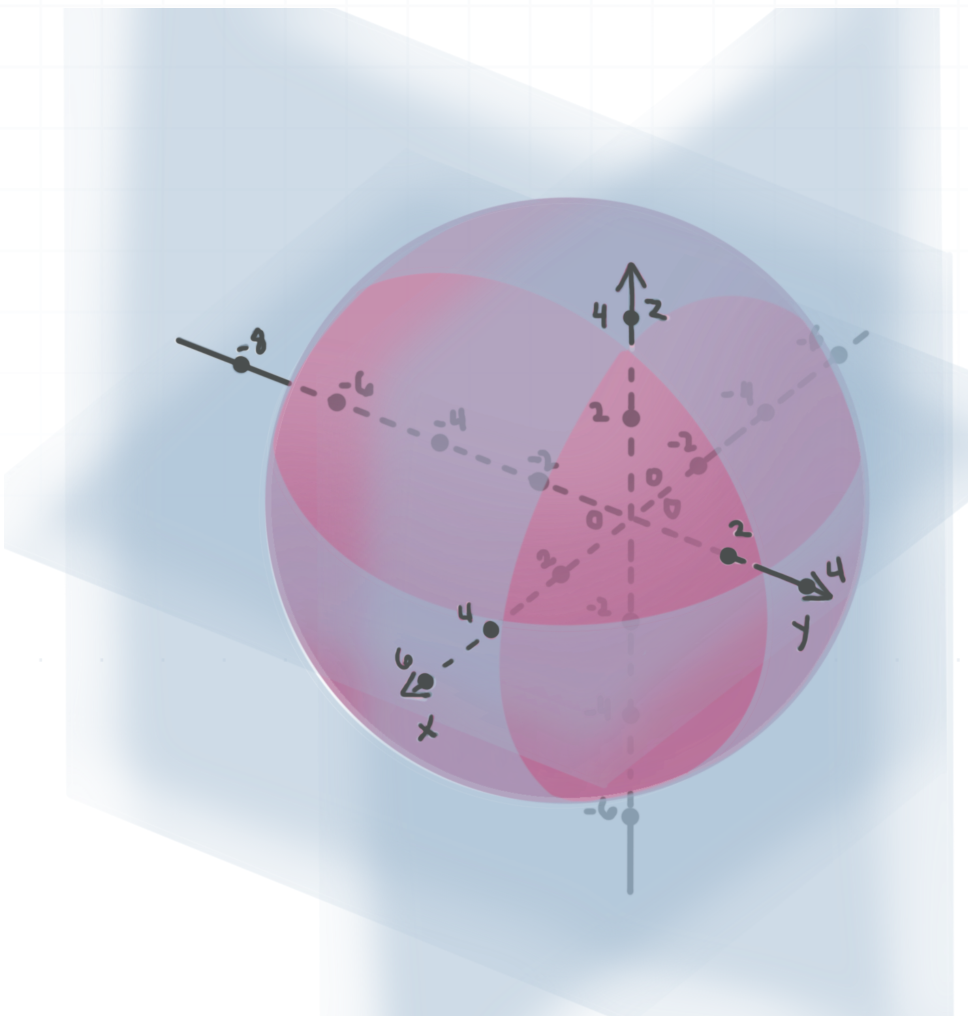
$$x = \frac{1}{(y^2 + z^2 + 1)^2}$$



EXPRESSING THE INTEGRAL SIX WAYS

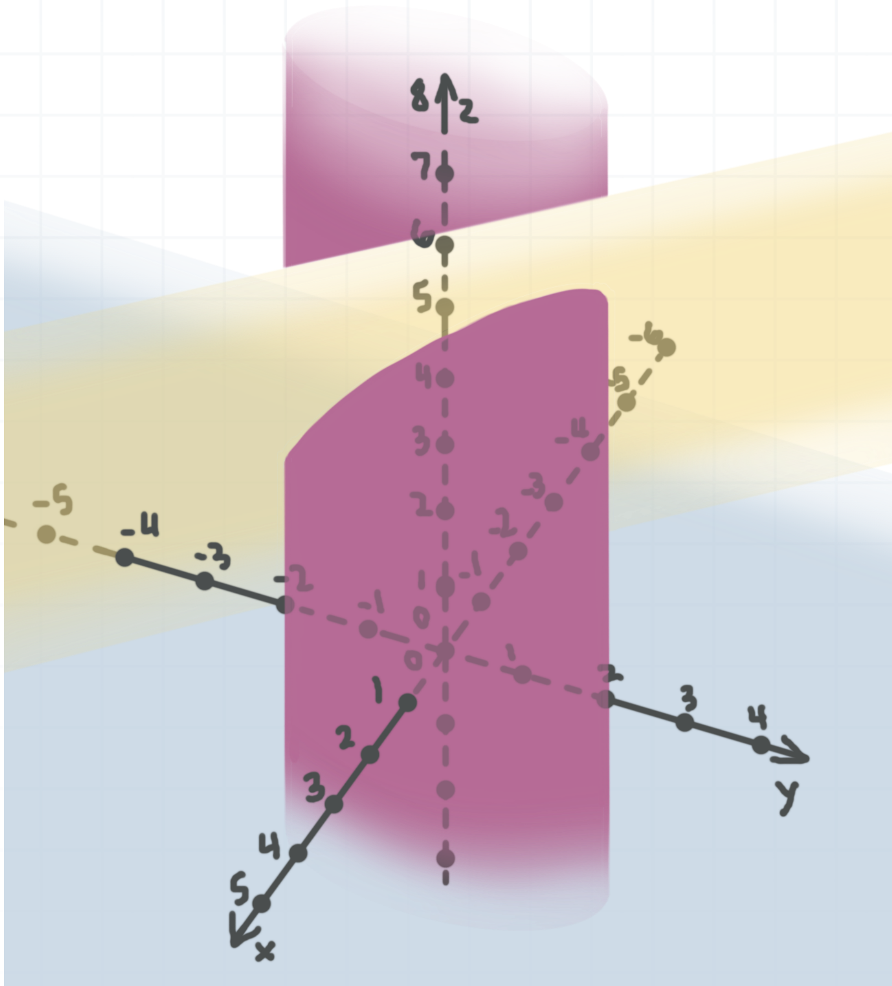
■ 1. Represent the triple integral as an iterated integral in which the order of integration is $dx\ dz\ dy$, where E is the part of the sphere with center at $(-1,-2,-1)$ and radius 25, lying in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

$$\iiint_E f(x,y,z)\ dV$$



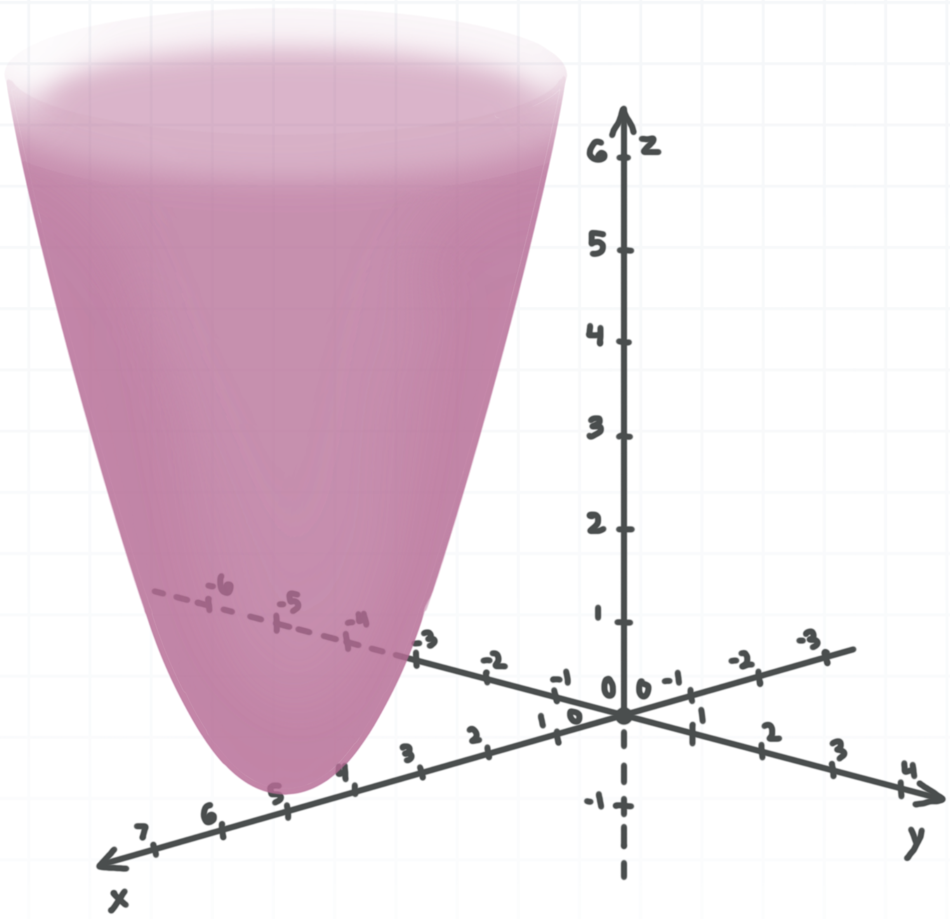
■ 2. Represent the triple integral as an iterated integral using the order of integration $dz\ dy\ dx$, where E is the part of the cylinder $4x^2+y^2=4$, between the planes $z=-3$ and $x+y-z+4=0$.

$$\iiint_E f(x, y, z)\, dV$$



■ 3. Represent the triple integral as an improper iterated integral using the order $dx\ dy\ dz$, where E is interior of the circular paraboloid $x^2 - 4x + y^2 + 6y - z + 12 = 0$.

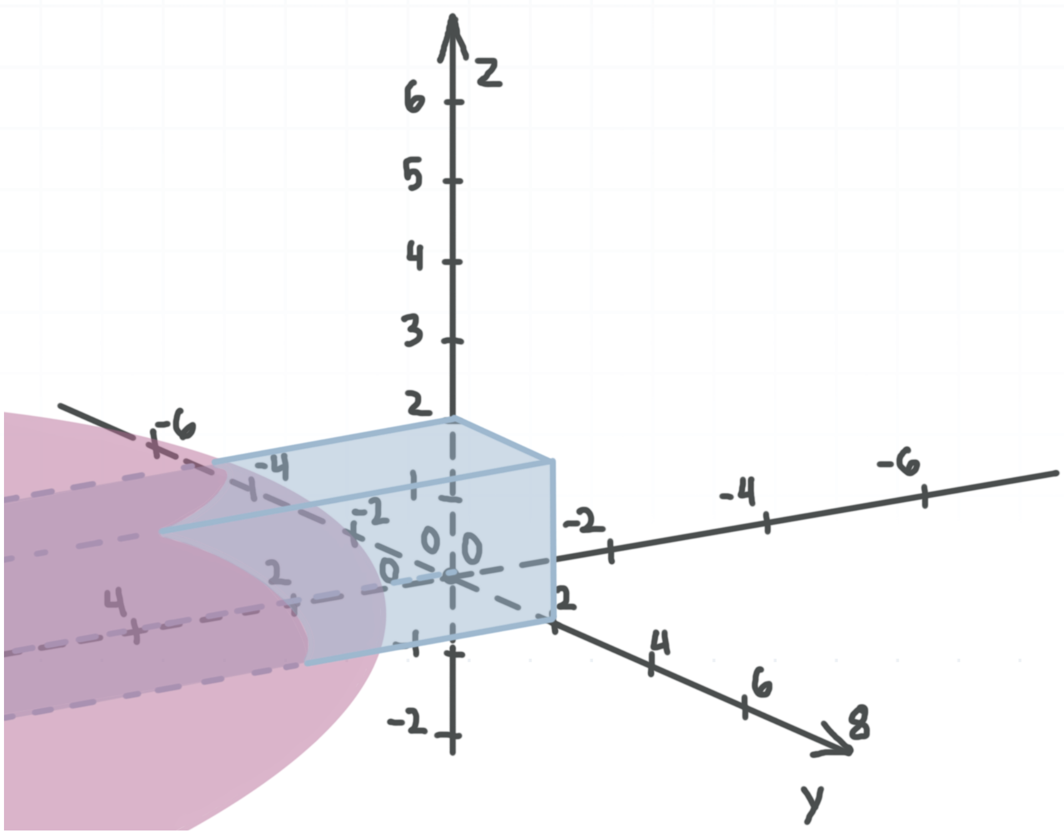
$$\iiint_E f(x, y, z)\, dV$$


z
6
5
4
3
2
1
-1
x
y

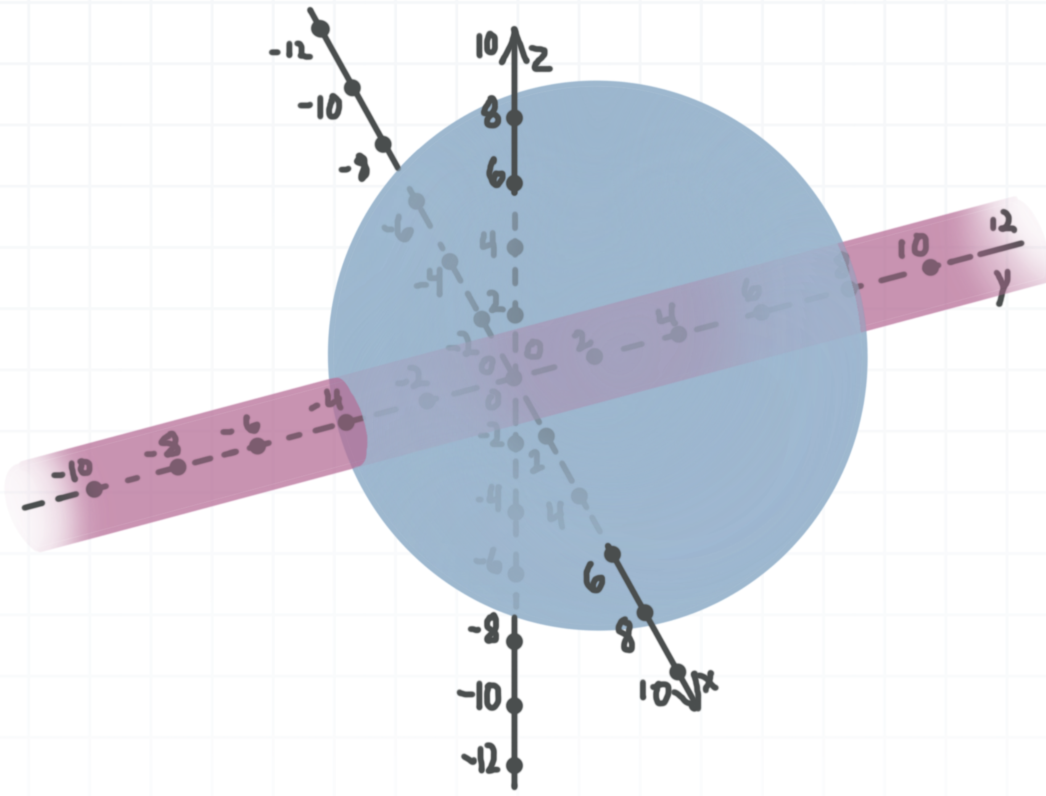
TYPE I, II, AND III REGIONS

■ 1. Evaluate the triple integral, where E is the region that lies in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$), and is bounded by the surfaces $y = 2$, $z = 2$, and $x - 0.5y^2 - 0.5z^2 - 1 = 0$.

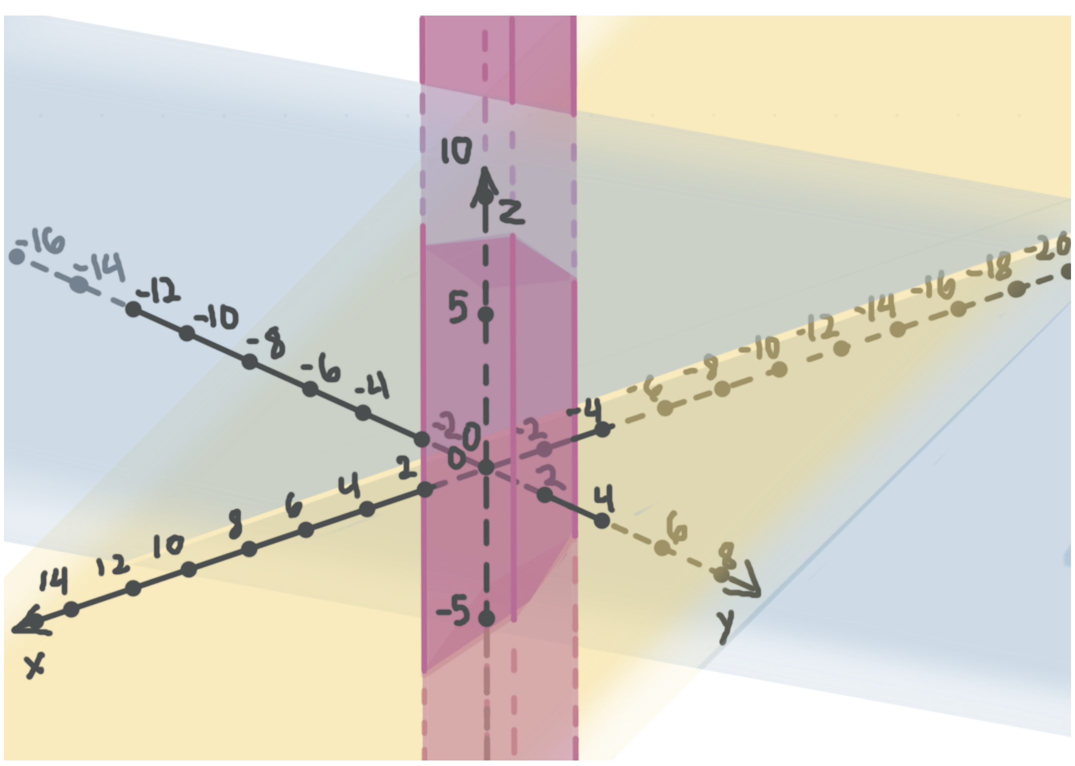
$$\iiint_E 4x + 2y - 2z \; dV$$



■ 2. Use a triple integral to find the volume of the region E that's bounded by the cylinder $x^2 + z^2 = 1$ and the sphere $x^2 + (y-2)^2 + z^2 = 36$.



■ 3. Use a triple integral to find the volume of the region E that lies in the first and fifth octants ($x \geq 0$, $y \geq 0$), and is bounded by the planes $x = 2$, $y = 3$, $2x + y - 2z + 12 = 0$, and $x - y + z + 4 = 0$.



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