Topic: Center, radius and equation of a sphere

Question: Find the center and radius of the sphere.

$$(x-9)^2+(y-2)^2+(z+1)^2=16$$

Answer choices:

A $(-9,-2,1)$ and $r=4$

B $(-9,-2,1)$ and $r=16$

C $(9,2,-1)$ and $r=4$

D $(9,2,-1)$ and $r=16$

Solution: C

The standard equation of a sphere is

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

where (h,k,l) is the center of the sphere

where r is the radius of the sphere

If we match the equation we've been given to the standard equation

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

$$(x-9)^2+(y-2)^2+(z+1)^2=16$$

we can see that

$$h=9$$

$$k=2$$

$$l=-1$$

So the center of the sphere is $(9,2,-1)$.

If we set the right side of the given equation equal to the right side of the standard equation, we can say that the radius is

$$r^2=16$$

$$r=4$$

In summary, the center of the sphere is $(9,2,-1)$ and its radius is $r=4$.

Topic: Center, radius and equation of a sphere

Question: Find the center and radius of the sphere.

$$x^2 - 2x + y^2 + 4y + z^2 - 6z = 11$$

Answer choices:

A $(-1,2,-3)$ and $r = 5$

B $(1,-2,3)$ and $r = 5$

C $(-1,2,-3)$ and $r = \sqrt{11}$

D $(1,-2,3)$ and $r = \sqrt{11}$

Solution: B

The standard equation of a sphere is

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

where (h, k, l) is the center of the sphere

where r is the radius of the sphere

In order to get the given equation into standard form, we'll need to complete the square with respect to each variable. Remember that in order to complete the square, we'll take the coefficient on the first-degree term, divide it by 2, square the result, and then add that final value into the quadratic expression (and subtract it back out so that we don't change the value of the equation).

$$x^2-2x+\left(\frac{-2}{2}\right)^2-\left(\frac{-2}{2}\right)^2+y^2+4y+\left(\frac{4}{2}\right)^2-\left(\frac{4}{2}\right)^2$$

$$+z^2-6z+\left(\frac{-6}{2}\right)^2-\left(\frac{-6}{2}\right)^2=11$$

$$x^2-2x+(-1)^2-(-1)^2+y^2+4y+(2)^2-(2)^2+z^2-6z+(-3)^2-(-3)^2=11$$

$$\left[x^2-2x+(-1)^2\right]-(-1)^2+\left[y^2+4y+(2)^2\right]-(2)^2+\left[z^2-6z+(-3)^2\right]-(-3)^2=11$$

$$\left[x^2-2x+(-1)^2\right]+\left[y^2+4y+(2)^2\right]+\left[z^2-6z+(-3)^2\right]=11+(-1)^2+(2)^2+(-3)^2$$

$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)+\left(z^2-6z+9\right)=11+1+4+9$$

$$(x-1)^2+(y+2)^2+(z-3)^2=25$$

If we match this transformed equation to the standard equation

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

$$(x-1)^2+(y+2)^2+(z-3)^2=25$$

we can see that

$$h=1$$

$$k=-2$$

$$l=3$$

So the center of the sphere is $(1,-2,3)$.

If we set the right side of the transformed equation equal to the right side of the standard equation, we can say that the radius is

$$r^2=25$$

$$r=5$$

In summary, the center of the sphere is $(1,-2,3)$ and its radius is $r=5$.

Topic: Center, radius and equation of a sphere

Question: Find the equation of the sphere.

Passing through $(-2,2,0)$

Center at $(-1,1,-1)$

Answer choices:

A $(x-1)^2+(y+1)^2+(z-1)^2=3$

B $(x+1)^2+(y-1)^2+(z+1)^2=\sqrt{3}$

C $(x+1)^2+(y-1)^2+(z+1)^2=3$

D $(x-1)^2+(y+1)^2+(z-1)^2=\sqrt{3}$

Solution: C

The standard equation of a sphere is

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

where (h,k,l) is the center of the sphere

where r is the radius of the sphere

If we don't know the radius, but we have the center and a point on the sphere, then we can calculate the radius as the distance between them, using

$$r=D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

where (x_1,y_1,z_1) is one point on the surface of the sphere

where (x_2,y_2,z_2) is the center point of the sphere

Since we've been given the center of the sphere already as $(-1,1,-1)$, we can plug it into the standard equation of a sphere.

$$\left[x-(-1)\right]^2+(y-1)^2+\left[z-(-1)\right]^2=r^2$$

$$(x+1)^2+(y-1)^2+(z+1)^2=r^2$$

The only value left to find is the radius, so we'll plug the center $(-1,1,-1)$ and the point $(-2,2,0)$ on the surface of the sphere into the distance formula.

$$r=\sqrt{\left[-1-(-2)\right]^2+(1-2)^2+(-1-0)^2}$$

$$r=\sqrt{(1)^2+(-1)^2+(-1)^2}$$

$$r=\sqrt{1+1+1}$$

$$r=\sqrt{3}$$

Remember that we just found r, but we need r^2 for the equation of the sphere.

$$r=\sqrt{3}$$

$$r^2=3$$

Making this substitution into the equation of the sphere gives

$$(x+1)^2+(y-1)^2+(z+1)^2=3$$

This is the equation of the sphere with center $(-1,1,-1)$ that passes through the $(-2,2,0)$.