Topic: Symmetric equations of a line

Question: Find the symmetric equation of the line.

Passing through $a(0{,}6, -2)$

Perpendicular to $-\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} = -1$

Answer choices:

A $x = \dfrac{y-6}{2} = \dfrac{z+2}{4}$

B $-x = \dfrac{y-6}{2} = \dfrac{z+2}{4}$

C $x = \dfrac{y+6}{2} = \dfrac{z-2}{4}$

D $-x = \dfrac{y+6}{2} = \dfrac{z-2}{4}$

Solution: B

The symmetric equation of a line is given by

$$\frac{x-a_1}{v_1}=\frac{y-a_2}{v_2}=\frac{z-a_3}{v_3}$$

where $a(a_1,a_2,a_3)$ is a point on the line, and where v is a vector parallel to the line.

Since we want the line that's perpendicular to $-\mathbf{i}+2\mathbf{j}+4\mathbf{k}=-1$, we know we want the line that's parallel to the normal vector to $-\mathbf{i}+2\mathbf{j}+4\mathbf{k}=-1$. The normal vector is given by the coefficients, $\langle -1,2,4\rangle$, so the line we want is parallel to $\langle -1,2,4\rangle$.

Plugging all of this and $a(0,6,-2)$ into the formula for the symmetric equation gives

$$\frac{x-0}{-1}=\frac{y-6}{2}=\frac{z-(-2)}{4}$$

$$-x=\frac{y-6}{2}=\frac{z+2}{4}$$

Topic: Symmetric equations of a line

Question: Find the symmetric equation of the line.

Passing through $a(-3, -2, 4)$

Perpendicular to $4\mathbf{i} - 10\mathbf{j} - 3\mathbf{k} = 6$

Answer choices:

A $\dfrac{x+3}{4} = -\dfrac{y+2}{10} = -\dfrac{z-4}{3}$

B $\dfrac{x+3}{4} = \dfrac{y+2}{10} = \dfrac{z-4}{3}$

C $\dfrac{x-3}{4} = -\dfrac{y-2}{10} = -\dfrac{z+4}{3}$

D $\dfrac{x-3}{4} = \dfrac{y-2}{10} = \dfrac{z+4}{3}$

Solution: A

The symmetric equation of a line is given by

$$\frac{x-a_1}{v_1}=\frac{y-a_2}{v_2}=\frac{z-a_3}{v_3}$$

where $a(a_1, a_2, a_3)$ is a point on the line, and where v is a vector parallel to the line.

Since we want the line that's perpendicular to $4\mathbf{i}-10\mathbf{j}-3\mathbf{k}=6$, we know we want the line that's parallel to the normal vector to $4\mathbf{i}-10\mathbf{j}-3\mathbf{k}=6$. The normal vector is given by the coefficients, $\langle 4, -10, -3\rangle$, so the line we want is parallel to $\langle 4, -10, -3\rangle$.

Plugging all of this and $a(-3, -2, 4)$ into the formula for the symmetric equation gives

$$\frac{x-(-3)}{4}=\frac{y-(-2)}{-10}=\frac{z-4}{-3}$$

$$\frac{x+3}{4}=-\frac{y+2}{10}=-\frac{z-4}{3}$$

Topic: Symmetric equations of a line

Question: Find the symmetric equation of the line.

Passes through $a(-5, -8, -9)$

Perpendicular to $-5\mathbf{i} + 3\mathbf{j} - \mathbf{k} = -8$

Answer choices:

A $\dfrac{x-5}{5} = \dfrac{y-8}{3} = z - 9$

B $-\dfrac{x-5}{5} = \dfrac{y-8}{3} = -z + 9$

C $\dfrac{x+5}{5} = \dfrac{y+8}{3} = z + 9$

D $-\dfrac{x+5}{5} = \dfrac{y+8}{3} = -z - 9$

Solution: D

The symmetric equation of a line is given by

$$\frac{x-a_1}{v_1}=\frac{y-a_2}{v_2}=\frac{z-a_3}{v_3}$$

where $a(a_1, a_2, a_3)$ is a point on the line, and where v is a vector parallel to the line.

Since we want the line that's perpendicular to $-5\mathbf{i}+3\mathbf{j}-\mathbf{k}=-8$, we know we want the line that's parallel to the normal vector to $-5\mathbf{i}+3\mathbf{j}-\mathbf{k}=-8$. The normal vector is given by the coefficients, $\langle -5,3,-1\rangle$, so the line we want is parallel to $\langle -5,3,-1\rangle$.

Plugging all of this and $a(-5,-8,-9)$ into the formula for the symmetric equation gives

$$\frac{x-(-5)}{-5}=\frac{y-(-8)}{3}=\frac{z-(-9)}{-1}$$

$$-\frac{x+5}{5}=\frac{y+8}{3}=-z-9$$