Calculus 3 Workbook Solutions

Directional derivatives

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DIRECTIONAL DERIVATIVES

1. Find the directional derivative in the direction of $\vec{v} = \langle 2,2,1 \rangle$.

$$f(x, y, z) = \cos(2x + 3y + z)$$

Solution:

Convert the vector to its unit vector form.

$$\vec{u} = \left\langle \frac{2}{\sqrt{2^2 + 2^2 + 1^2}}, \frac{2}{\sqrt{2^2 + 2^2 + 1^2}}, \frac{1}{\sqrt{2^2 + 2^2 + 1^2}} \right\rangle$$

$$\vec{u} = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$$

Find the partial derivatives of f.

$$\frac{\partial f}{\partial x} = -2\sin(2x + 3y + z)$$

$$\frac{\partial f}{\partial y} = -3\sin(2x + 3y + z)$$

$$\frac{\partial f}{\partial z} = -\sin(2x + 3y + z)$$

Then the directional derivative is

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$$D_u f(x,y,z) = -\frac{2}{3} \cdot 2\sin(2x+3y+z) - \frac{2}{3} \cdot 3\sin(2x+3y+z)$$

$$-\frac{1}{3} \cdot \sin(2x+3y+z)$$

$$D_u f(x,y,z) = -\frac{\sin(2x+3y+z)}{3}(4+6+1)$$

$$D_u f(x,y,z) = -\frac{11\sin(2x+3y+z)}{3}$$

■ 2. Find the directional derivative in the direction of $\vec{v} = \langle 0, -3, -4 \rangle$.

$$f(x,y,z) = x^2 \ln(y-z)$$

Solution:

Convert the vector to its unit vector form.

$$\vec{u} = \left\langle \frac{0}{\sqrt{0^2+(-3)^2+(-4)^2}}, \frac{-3}{\sqrt{0^2+(-3)^2+(-4)^2}}, \frac{-4}{\sqrt{0^2+(-3)^2+(-4)^2}} \right\rangle$$

$$\vec{u} = \left\langle 0, -\frac{3}{5}, -\frac{4}{5} \right\rangle$$

Find the partial derivatives of f.

$$\frac{\partial f}{\partial x} = 2x\ln(y-z)$$

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$$\frac{\partial f}{\partial y} = \frac{x^2}{y - z}$$

$$\frac{\partial f}{\partial z} = -\frac{x^2}{y - z}$$

Then the directional derivative is

$$D_u f(x, y, z) = 0 \cdot 2x \ln(y - z) - \frac{3x^2}{5(y - z)} + \frac{4x^2}{5(y - z)}$$

$$D_u f(x, y, z) = -\frac{3x^2}{5(y - z)} + \frac{4x^2}{5(y - z)}$$

$$D_u f(x, y, z) = \frac{4x^2 - 3x^2}{5(y - z)}$$

$$D_u f(x, y, z) = \frac{x^2}{5(y - z)}$$

■ 3. Find the directional derivative in the direction of $\vec{v} = \langle 3, -6, 2 \rangle$ at the point $A(\pi/2, 1/2, \pi)$.

$$f(x, y, z) = x \sin(yz)$$

Solution:

We'll start by converting the given vector to its unit vector form.

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$$\vec{u} = \left\langle \frac{3}{\sqrt{3^2 + (-6)^2 + 2^2}}, \frac{-6}{\sqrt{3^2 + (-6)^2 + 2^2}}, \frac{2}{\sqrt{3^2 + (-6)^2 + 2^2}} \right\rangle$$

$$\vec{u} = \left\langle \frac{3}{7}, -\frac{6}{7}, \frac{2}{7} \right\rangle$$

Find the partial derivatives of f.

$$\frac{\partial f}{\partial x} = \sin(yz)$$

$$\frac{\partial f}{\partial y} = xz\cos(yz)$$

$$\frac{\partial f}{\partial z} = xy\cos(yz)$$

Then the directional derivative is

$$D_u f(x, y, z) = \frac{3}{7} \cdot \sin(yz) - \frac{6}{7} \cdot xz\cos(yz) + \frac{2}{7} \cdot xy\cos(yz)$$

$$D_u f\left(\frac{\pi}{2}, \frac{1}{2}, \pi\right) = \frac{3}{7} \cdot \sin\frac{\pi}{2} - \frac{6}{7} \cdot \pi^2 \cos\frac{\pi}{2} + \frac{2}{7} \cdot \frac{\pi}{4} \cos\frac{\pi}{2}$$

$$D_u f\left(\frac{\pi}{2}, \frac{1}{2}, \pi\right) = \frac{3}{7}$$

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