Calculus 3 Workbook

Approximating double integrals

krista king MATH

APPROXIMATING DOUBLE INTEGRALS WITH RECTANGLES

■ 1. Estimate the volume between the surface $z = 3(x - 2)e^{y-2}$ and the xy-plane, on the region $-1 \leq x \leq 1$ and $0 \leq y \leq 3$. Use lower-left corners and 1×1 squares, then 0.5×0.5. Round the answers for volume to the nearest tenth. Which square size gives the better approximation if exact volume is 31.

■ 2. Estimate the volume below the surface $z = 3 + x + y^2$, over the triangular region bounded by the x- and y-axes and the line $2x + 3y = 6$. Use lower-left corners and 1×1 squares. For squares that lie partially within the region, divide area by 2.

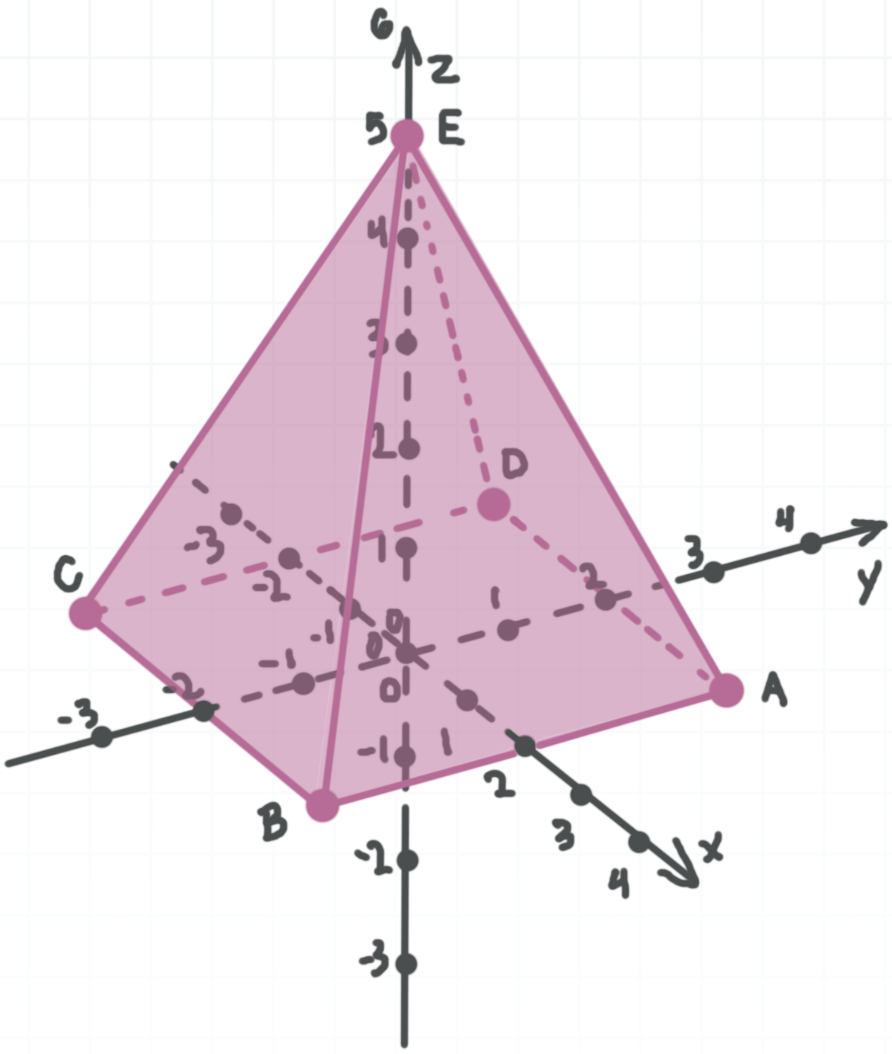
■ 3. Assume the base of a right circular cylinder with radius 3 and height 5 lies in the xy-plane with its center at the origin. Use 1×1 squares and lower-left corners to estimate the volume of the cylinder. If an approximating square lies only partially within the circle, divide its area in half as part of the estimation. Calculate exact volume of the cylinder using $V = \pi r^2 h$, then find the percentage error of the approximation.

MIDPOINT RULE FOR DOUBLE INTEGRALS

■ 1. Assuming the integral's exact value is 12, say which estimation is more accurate if we estimate volume of the integral below using Midpoint Rule and rectangles of dimensions $\pi/2 \times 1$, and then rectangles of dimensions $\pi/3 \times 2/3$.

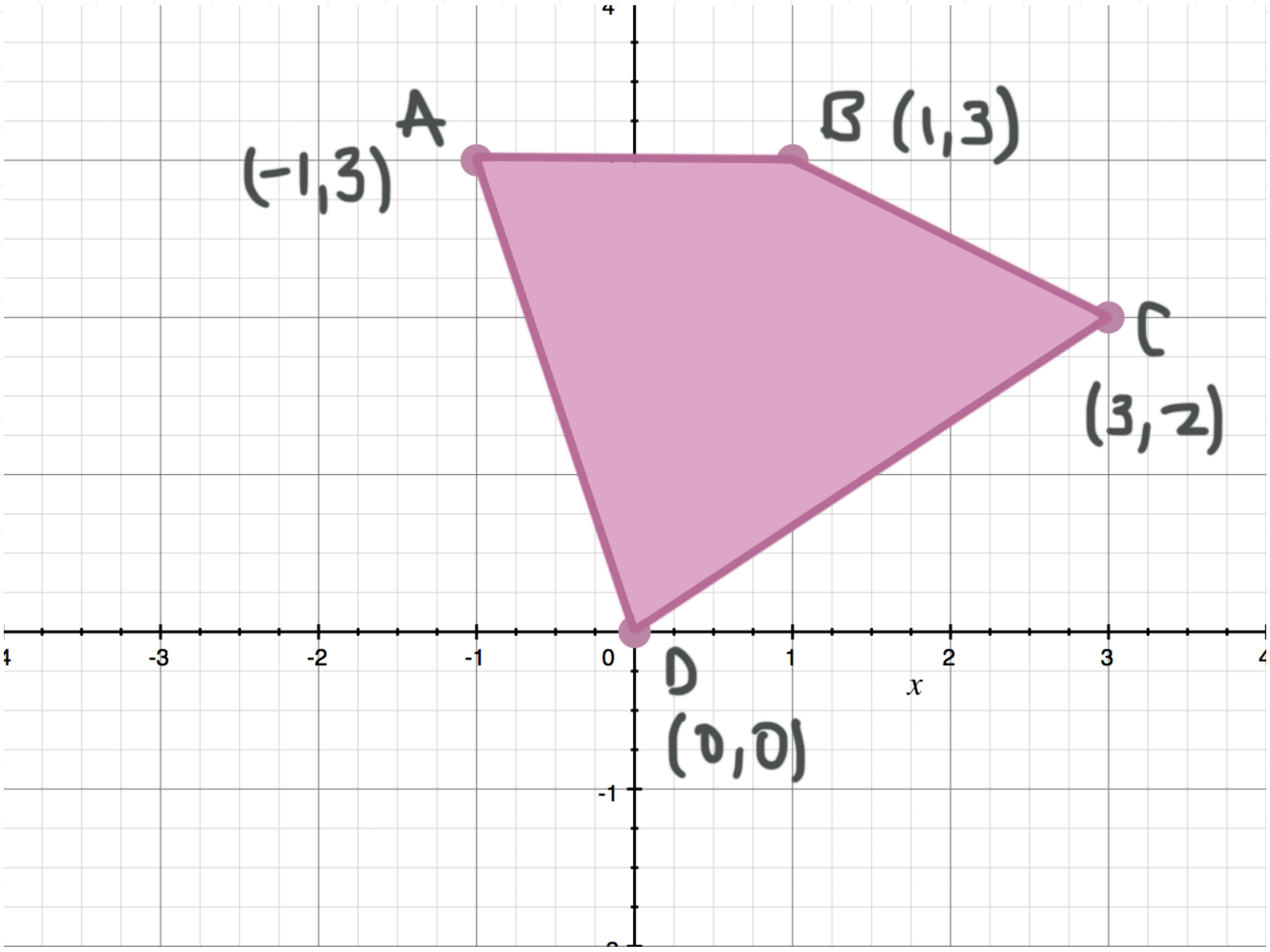
$$\int_0^{\pi} \int_{-1}^{1} (y+3)\sin x \ dy \ dx$$

■ 2. Use Midpoint Rule and 1×1 squares to estimate the volume of the right square pyramid $ABCDE$ with base side length 4 and height 5, assuming its base lies in the xy-plane with its sides parallel to the major coordinate axes, and the vertex of the pyramid lies on z-axis. If the pyramid's exact volume is $V = 80/3$, find the percentage error of the approximation.



3. Use Midpoint Rule with 1×1 squares to estimate the value of the given integral, where $ABCD$ is the quadrilateral shown in the xy-plane below. For any square that lie only partially inside the region, divide the area in half, then round the final approximation to the nearest tenth.

$$\iint_{ABCD} x + \sqrt{y+3}\ dy\ dx$$


A
(-1,3)
B (1,3)
C
(3,2)
D
(0,0)
-3
-2
-1
0
1
2
3
x
-1

RIEMANN SUMS FOR DOUBLE INTEGRALS

■ 1. Use Riemann sums to approximate the integral, using squares with sides of length 1, and the upper-left corner of the squares. If the square partially lies within the domain, then divide its area in half.

$$\int_{0}^{2}\int_{x}^{x^2+2} x^2+2x-3y+1\ dy\ dx$$

■ 2. Use Riemann sums to approximate the integral over the rectangle $R=[1,3]\times[-1,2]$, using squares with sides of length 1, and the upper-right corners of the squares. Round your answer to the nearest tenth.

$$\iint_{R} \ln(x^2+y+2)\ dy\ dx$$

■ 3. Use Riemann sums to approximate the integral, using rectangles with sides $1\times\pi/2$, and the upper-right corners of the rectangles. If the exact value is 14π, find the percentage error of the approximation.

$$\int_{-\pi}^{\pi}\int_{0}^{2} 2x+\sin^2 y+1\ dx\ dy$$

W W W . K R I S T A K I N G M A T H . C O M