Topic: Equation of a plane

Question: Find the equation of the plane that includes the points.

$P(1,-1,1)$

$Q(2,-2,0)$

$R(3,3,-1)$

Answer choices:

A $x-y+z=12$

B $x+z=2$

C $x-z=2$

D $-x+y-z=12$

Solution: B

We'll start by turning the three points we've been given into two vectors.

$$\overrightarrow{PQ} = \left\langle Q_1 - P_1, Q_2 - P_2, Q_3 - P_3 \right\rangle$$

$$\overrightarrow{PQ} = \langle 2 - 1, -2 - (-1), 0 - 1 \rangle$$

$$\overrightarrow{PQ} = \langle 1, -1, -1 \rangle$$

and

$$\overrightarrow{PR} = \left\langle R_1 - P_1, R_2 - P_2, R_3 - P_3 \right\rangle$$

$$\overrightarrow{PR} = \langle 3 - 1, 3 - (-1), -1 - 1 \rangle$$

$$\overrightarrow{PR} = \langle 2, 4, -2 \rangle$$

Now we'll take the cross product of $\overrightarrow{PQ} = \langle 1, -1, -1 \rangle$ and $\overrightarrow{PR} = \langle 2, 4, -2 \rangle$ in order to find the normal vector to the plane.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ PQ_1 & PQ_2 & PQ_3 \\ PR_1 & PR_2 & PR_3 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \mathbf{i} \begin{vmatrix} PQ_2 & PQ_3 \\ PR_2 & PR_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} PQ_1 & PQ_3 \\ PR_1 & PR_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} PQ_1 & PQ_2 \\ PR_1 & PR_2 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \mathbf{i}\left(PQ_2PR_3 - PQ_3PR_2\right) - \mathbf{j}\left(PQ_1PR_3 - PQ_3PR_1\right) + \mathbf{k}\left(PQ_1PR_2 - PQ_2PR_1\right)$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \left[(-1)(-2) - (-1)(4)\right]\mathbf{i} - \left[(1)(-2) - (-1)(2)\right]\mathbf{j} + \left[(1)(4) - (-1)(2)\right]\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (2 + 4)\mathbf{i} - (-2 + 2)\mathbf{j} + (4 + 2)\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = 6\mathbf{i} - 0\mathbf{j} + 6\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle 6,0,6 \rangle$$

Now we can plug this normal vector and either of the points we were given into the equation of the plane. We'll get a, b and c from the normal vector, and (x_1, y_1, z_1) from $P(1, -1,1)$, and this will give us the equation of the plane.

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$6(x - 1) + 0\left[y - (-1)\right] + 6(z - 1) = 0$$

$$6x - 6 + 6z - 6 = 0$$

$$6x + 6z = 12$$

$$x + z = 2$$

Topic: Equation of a plane

Question: Find the equation of the plane that includes the points.

$$P(3,4,-5)$$

$$Q(5,-4,11)$$

$$R(10,-3,-7)$$

Answer choices:

A $64x-58y-21z=-319$

B $-64x+58y-21z=319$

C $64x-58y+21z=-319$

D $64x+58y+21z=319$

Solution: D

We'll start by turning the three points we've been given into two vectors.

$$\overrightarrow{PQ}=\left\langle Q_1-P_1,Q_2-P_2,Q_3-P_3\right\rangle$$

$$\overrightarrow{PQ}=\langle 5-3,-4-4,11-(-5)\rangle$$

$$\overrightarrow{PQ}=\langle 2,-8,16\rangle$$

and

$$\overrightarrow{PR}=\left\langle R_1-P_1,R_2-P_2,R_3-P_3\right\rangle$$

$$\overrightarrow{PR}=\langle 10-3,-3-4,-7-(-5)\rangle$$

$$\overrightarrow{PR}=\langle 7,-7,-2\rangle$$

Now we'll take the cross product of $\overrightarrow{PQ}=\langle 2,-8,16\rangle$ and $\overrightarrow{PR}=\langle 7,-7,-2\rangle$ in order to find the normal vector to the plane.

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ PQ_1&PQ_2&PQ_3\\ PR_1&PR_2&PR_3\end{vmatrix}$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\mathbf{i}\begin{vmatrix}PQ_2&PQ_3\\ PR_2&PR_3\end{vmatrix}-\mathbf{j}\begin{vmatrix}PQ_1&PQ_3\\ PR_1&PR_3\end{vmatrix}+\mathbf{k}\begin{vmatrix}PQ_1&PQ_2\\ PR_1&PR_2\end{vmatrix}$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\mathbf{i}\left(PQ_2PR_3-PQ_3PR_2\right)-\mathbf{j}\left(PQ_1PR_3-PQ_3PR_1\right)+\mathbf{k}\left(PQ_1PR_2-PQ_2PR_1\right)$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\left[(-8)(-2)-(16)(-7)\right]\mathbf{i}-\left[(2)(-2)-(16)(7)\right]\mathbf{j}+\left[(2)(-7)-(-8)(7)\right]\mathbf{k}$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=(16+112)\mathbf{i}-(-4-112)\mathbf{j}+(-14+56)\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = 128\mathbf{i} + 116\mathbf{j} + 42\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle 128,116,42 \rangle$$

Now we can plug this normal vector and either of the points we were given into the equation of the plane. We'll get a, b and c from the normal vector, and (x_1, y_1, z_1) from $P(3,4,-5)$, and this will give us the equation of the plane.

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$128(x - 3) + 116(y - 4) + 42\left[z - (-5)\right] = 0$$

$$128(x - 3) + 116(y - 4) + 42(z + 5) = 0$$

$$128x - 384 + 116y - 464 + 42z + 210 = 0$$

$$128x + 116y + 42z - 638 = 0$$

$$128x + 116y + 42z = 638$$

$$64x + 58y + 21z = 319$$

Topic: Equation of a plane

Question: Find the equation of the plane that includes the points.

$$P(-2,-2,-2)$$

$$Q(4,4,4)$$

$$R(15,-15,-15)$$

Answer choices:

A $x+y-z=0$

B $x-y+z=0$

C $y-z=0$

D $y-z=-8$

Solution: C

We'll start by turning the three points we've been given into two vectors.

$$\overrightarrow{PQ}=\left\langle Q_1-P_1,Q_2-P_2,Q_3-P_3\right\rangle$$

$$\overrightarrow{PQ}=\langle 4-(-2),4-(-2),4-(-2)\rangle$$

$$\overrightarrow{PQ}=\langle 6,6,6\rangle$$

and

$$\overrightarrow{PR}=\left\langle R_1-P_1,R_2-P_2,R_3-P_3\right\rangle$$

$$\overrightarrow{PR}=\langle 15-(-2),-15-(-2),-15-(-2)\rangle$$

$$\overrightarrow{PR}=\langle 17,-13,-13\rangle$$

Now we'll take the cross product of $\overrightarrow{PQ}=\langle 6,6,6\rangle$ and $\overrightarrow{PR}=\langle 17,-13,-13\rangle$ in order to find the normal vector to the plane.

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ PQ_1 & PQ_2 & PQ_3\\ PR_1 & PR_2 & PR_3\end{vmatrix}$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\mathbf{i}\begin{vmatrix}PQ_2 & PQ_3\\ PR_2 & PR_3\end{vmatrix}-\mathbf{j}\begin{vmatrix}PQ_1 & PQ_3\\ PR_1 & PR_3\end{vmatrix}+\mathbf{k}\begin{vmatrix}PQ_1 & PQ_2\\ PR_1 & PR_2\end{vmatrix}$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\mathbf{i}\left(PQ_2PR_3-PQ_3PR_2\right)-\mathbf{j}\left(PQ_1PR_3-PQ_3PR_1\right)+\mathbf{k}\left(PQ_1PR_2-PQ_2PR_1\right)$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\left[(6)(-13)-(6)(-13)\right]\mathbf{i}-\left[(6)(-13)-(6)(17)\right]\mathbf{j}+\left[(6)(-13)-(6)(17)\right]\mathbf{k}$$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=(-78+78)\mathbf{i}-(-78-102)\mathbf{j}+(-78-102)\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = 0\mathbf{i} + 180\mathbf{j} - 180\mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle 0,180,-180 \rangle$$

Now we can plug this normal vector and either of the points we were given into the equation of the plane. We'll get a, b and c from the normal vector, and (x_1, y_1, z_1) from $Q(4,4,4)$, and this will give us the equation of the plane.

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$(0)(x - 4) + (180)(y - 4) + (-180)(z - 4) = 0$$

$$0(x - 4) + 180(y - 4) - 180(z - 4) = 0$$

$$180y - 720 - 180z + 720 = 0$$

$$180y - 180z = 0$$

$$y - z = 0$$