Topic: Reparametrizing the curve

Question: Reparametrize the curve of the vector function from $t=0$ in the direction of increasing t.

$$r(t)=3t\mathbf{i}-t\mathbf{j}+(3+t)\mathbf{k}$$

Answer choices:

A $\quad r(t(s))=\frac{3}{\sqrt{5}}s\mathbf{i}-\frac{1}{\sqrt{5}}s\mathbf{j}+\left(3+\frac{1}{\sqrt{5}}s\right)\mathbf{k}$

B $\quad r(t(s))=-\frac{3}{\sqrt{5}}s\mathbf{i}+\frac{1}{\sqrt{5}}s\mathbf{j}-\left(3+\frac{1}{\sqrt{5}}s\right)\mathbf{k}$

C $\quad r(t(s))=\frac{3}{\sqrt{11}}s\mathbf{i}-\frac{1}{\sqrt{11}}s\mathbf{j}+\left(3+\frac{1}{\sqrt{11}}s\right)\mathbf{k}$

D $\quad r(t(s))=-\frac{3}{\sqrt{11}}s\mathbf{i}+\frac{1}{\sqrt{11}}s\mathbf{j}-\left(3+\frac{1}{\sqrt{11}}s\right)\mathbf{k}$

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Solution: C

First we'll turn the vector equation into parametric equations.

$r(t) = 3t\mathbf{i} - t\mathbf{j} + (3+t)\mathbf{k}$ becomes

$$x = 3t$$

$$y = -t$$

$$z = 3 + t$$

Then we'll take the derivative of these.

$$\frac{dx}{dt} = 3$$

$$\frac{dy}{dt} = -1$$

$$\frac{dz}{dt} = 1$$

Since we're told that we'll start at $t = 0$ and move in the direction of increasing t, the limits of integration are given by $[0,t]$. Now we can plug everything we have into the arc length formula and integrate.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

$$L = \int_0^t \sqrt{(3)^2 + (-1)^2 + (1)^2}\ dt$$

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$$L=\int_0^t \sqrt{9+1+1}\ dt$$

$$L=\int_0^t \sqrt{11}\ dt$$

$$L=\sqrt{11}t\Big|_0^t$$

Evaluate over the interval.

$$L=\sqrt{11}(t)-\sqrt{11}(0)$$

$$L=\sqrt{11}t$$

Now we can set $L=s$ and then solve for t.

$$s=\sqrt{11}t$$

$$t=\frac{1}{\sqrt{11}}s$$

Now we can finally reparametrize the curve by substituting this value for t into the vector function.

$$r(t(s))=3\left(\frac{1}{\sqrt{11}}s\right)\mathbf{i}-\left(\frac{1}{\sqrt{11}}s\right)\mathbf{j}+\left[3+\left(\frac{1}{\sqrt{11}}s\right)\right]\mathbf{k}$$

$$r(t(s))=\frac{3}{\sqrt{11}}s\mathbf{i}-\frac{1}{\sqrt{11}}s\mathbf{j}+\left(3+\frac{1}{\sqrt{11}}s\right)\mathbf{k}$$

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Topic: Reparametrizing the curve

Question: Reparametrize the curve of the vector function from $t = 0$ in the direction of increasing t.

$$r(t) = (1 - 3t)\mathbf{i} + 6t\mathbf{j} + (4 - 5t)\mathbf{k}$$

Answer choices:

A $$r(t(s)) = \left(1 + \frac{3}{\sqrt{70}}s\right)\mathbf{i} + \frac{6}{\sqrt{70}}s\mathbf{j} + \left(4 + \frac{5}{\sqrt{70}}s\right)\mathbf{k}$$

B $$r(t(s)) = \left(1 - \frac{3}{\sqrt{70}}s\right)\mathbf{i} + \frac{6}{\sqrt{70}}s\mathbf{j} + \left(4 - \frac{5}{\sqrt{70}}s\right)\mathbf{k}$$

C $$r(t(s)) = \left(1 + \frac{3}{\sqrt{14}}s\right)\mathbf{i} + \frac{6}{\sqrt{14}}s\mathbf{j} + \left(4 + \frac{5}{\sqrt{14}}s\right)\mathbf{k}$$

D $$r(t(s)) = \left(1 - \frac{3}{\sqrt{14}}s\right)\mathbf{i} + \frac{6}{\sqrt{14}}s\mathbf{j} + \left(4 - \frac{5}{\sqrt{14}}s\right)\mathbf{k}$$

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Solution: B

First we'll turn the vector equation into parametric equations.

$r(t) = (1 - 3t)\mathbf{i} + 6t\mathbf{j} + (4 - 5t)\mathbf{k}$ becomes

$$x = 1 - 3t$$

$$y = 6t$$

$$z = 4 - 5t$$

Then we'll take the derivative of these.

$$\frac{dx}{dt} = -3$$

$$\frac{dy}{dt} = 6$$

$$\frac{dz}{dt} = -5$$

Since we're told that we'll start at $t = 0$ and move in the direction of increasing t, the limits of integration are given by $[0,t]$. Now we can plug everything we have into the arc length formula and integrate.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

$$L = \int_0^t \sqrt{(-3)^2 + (6)^2 + (-5)^2}\ dt$$

$$L=\int_0^t \sqrt{9+36+25}\ dt$$

$$L=\int_0^t \sqrt{70}\ dt$$

$$L=\sqrt{70}t\Big|_0^t$$

Evaluate over the interval.

$$L=\sqrt{70}(t)-\sqrt{70}(0)$$

$$L=\sqrt{70}t$$

Now we can set $L=s$ and then solve for t.

$$s=\sqrt{70}t$$

$$t=\frac{1}{\sqrt{70}}s$$

Now we can finally reparametrize the curve by substituting this value for t into the vector function.

$$r(t(s))=\left[1-3\left(\frac{1}{\sqrt{70}}s\right)\right]\mathbf{i}+6\left(\frac{1}{\sqrt{70}}s\right)\mathbf{j}+\left[4-5\left(\frac{1}{\sqrt{70}}s\right)\right]\mathbf{k}$$

$$r(t(s))=\left(1-\frac{3}{\sqrt{70}}s\right)\mathbf{i}+\frac{6}{\sqrt{70}}s\mathbf{j}+\left(4-\frac{5}{\sqrt{70}}s\right)\mathbf{k}$$

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Topic: Reparametrizing the curve

Question: Reparametrize the curve of the vector function from $t=0$ in the direction of decreasing t.

$$r(t)=t\mathbf{i}+(t-4)\mathbf{j}+7t\mathbf{k}$$

Answer choices:

A $\quad r(t(s))=-\frac{1}{\sqrt{9}}s\mathbf{i}-\left(\frac{1}{\sqrt{9}}s-4\right)\mathbf{j}-\frac{7}{\sqrt{9}}s\mathbf{k}$

B $\quad r(t(s))=\frac{1}{\sqrt{9}}s\mathbf{i}+\left(\frac{1}{\sqrt{9}}s-4\right)\mathbf{j}+\frac{7}{\sqrt{9}}s\mathbf{k}$

C $\quad r(t(s))=\frac{1}{\sqrt{51}}s\mathbf{i}+\left(\frac{1}{\sqrt{51}}s-4\right)\mathbf{j}+\frac{7}{\sqrt{51}}s\mathbf{k}$

D $\quad r(t(s))=-\frac{1}{\sqrt{51}}s\mathbf{i}-\left(\frac{1}{\sqrt{51}}s+4\right)\mathbf{j}-\frac{7}{\sqrt{51}}s\mathbf{k}$

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Solution: D

First we'll turn the vector equation into parametric equations.

$r(t) = t\mathbf{i} + (t-4)\mathbf{j} + 7t\mathbf{k}$ becomes

$$x = t$$

$$y = t - 4$$

$$z = 7t$$

Then we'll take the derivative of these.

$$\frac{dx}{dt} = 1$$

$$\frac{dy}{dt} = 1$$

$$\frac{dz}{dt} = 7$$

Since we're told that we'll start at $t = 0$ and move in the direction of decreasing t, the limits of integration are given by $[t,0]$. Now we can plug everything we have into the arc length formula and integrate.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

$$L = \int_t^0 \sqrt{(1)^2 + (1)^2 + (7)^2}\ dt$$

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$$L = \int_t^0 \sqrt{1+1+49}\ dt$$

$$L = \int_t^0 \sqrt{51}\ dt$$

$$L = \sqrt{51}t\Big|_t^0$$

Evaluate over the interval.

$$L = \sqrt{51}(0) - \sqrt{51}(t)$$

$$L = -\sqrt{51}t$$

Now we can set $L = s$ and then solve for t.

$$s = -\sqrt{51}t$$

$$t = -\frac{1}{\sqrt{51}}s$$

Now we can finally reparametrize the curve by substituting this value for t into the vector function.

$$r(t(s)) = \left(-\frac{1}{\sqrt{51}}s\right)\mathbf{i} + \left[\left(-\frac{1}{\sqrt{51}}s\right) - 4\right]\mathbf{j} + 7\left(-\frac{1}{\sqrt{51}}s\right)\mathbf{k}$$

$$r(t(s)) = -\frac{1}{\sqrt{51}}s\mathbf{i} - \left(\frac{1}{\sqrt{51}}s + 4\right)\mathbf{j} - \frac{7}{\sqrt{51}}s\mathbf{k}$$

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