Magnitude and angle of the resultant force

When we're given two vectors with the same initial point, and they're different lengths and pointing in different directions, we can think about each of them as a force. The longer the vector, the more force it pulls in its direction.

Oftentimes we want to be able to find the net force of the two vectors, which will be a third vector that counterbalances the force and direction of the first two. Think about the resultant vector as representing the amount and direction of force that cancels out the first two vectors, leaving the whole system in balance.

In order to define this third vector, we need to find

its **magnitude** (its length), which will be force, in Newtons N, and

its **angle**, from the positive direction of the x-axis.

To find the magnitude and angle of a resultant force, we

create vector equations for each of the given forces

add the vector equations together to get the vector equation of the resultant force

find **magnitude** of the resultant force using the new vector equation and the distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

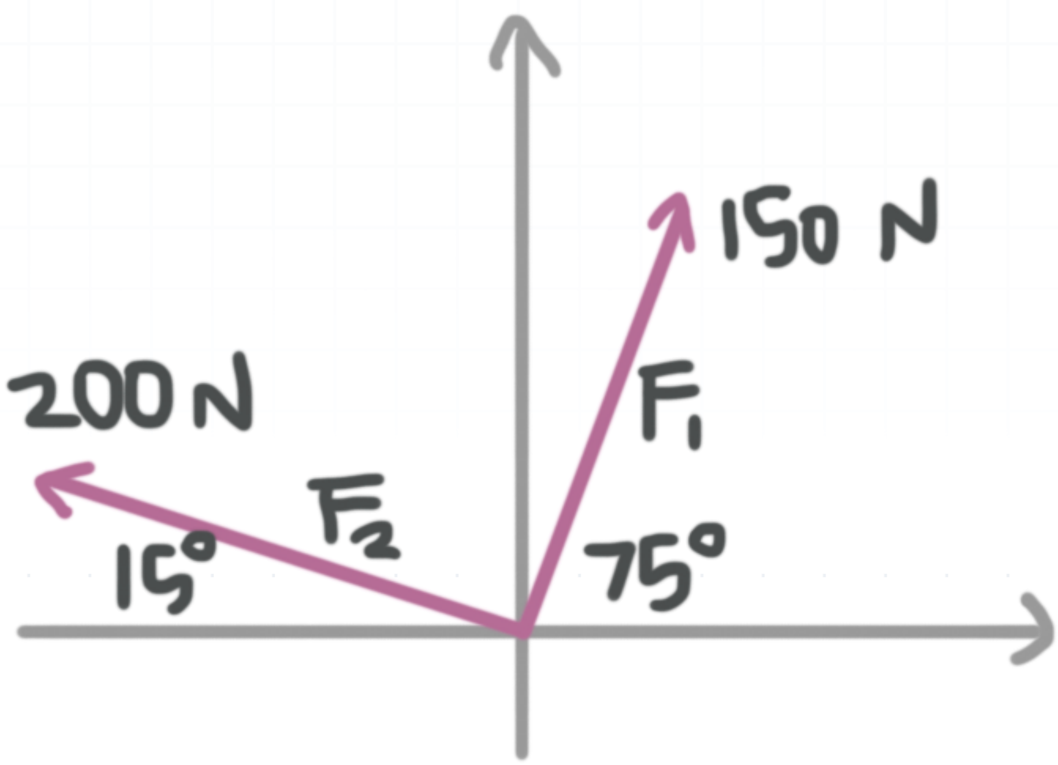
find the **angle** of the resultant force using the new vector equation and the formula

$$\theta_R = 180^\circ - \arctan \frac{|y|}{|x|}$$

Let's do an example in which we're given two forces in the xy-plane.

Example

Find the magnitude and angle of the resultant force.



We'll use the forces and the angles to find vector equations for F_1 and F_2.

$$F_1 = 150 \cos 75^\circ \ \mathbf{i} + 150 \sin 75^\circ \ \mathbf{j}$$

$$F_1 = 38.82 \ \mathbf{i} + 144.89 \ \mathbf{j}$$

$$F_1 = \langle 38.82, \ 144.89 \rangle$$

and

$$F_2 = -200\cos 15^\circ\ \mathbf{i} + 200\sin 15^\circ\ \mathbf{j}$$

$$F_2 = -193.19\ \mathbf{i} + 51.76\ \mathbf{j}$$

$$F_2 = \langle -193.19,\ 51.76 \rangle$$

We'll add our forces together to find the vector equation of the resultant force.

$$F_R = F_1 + F_2$$

$$F_R = 38.82\ \mathbf{i} + 144.89\ \mathbf{j} - 193.19\ \mathbf{i} + 51.76\ \mathbf{j}$$

$$F_R = -154.37\ \mathbf{i} + 196.65\ \mathbf{j}$$

$$F_R = \langle -154.37,\ 196.65 \rangle$$

Now we can plug the vector equation into the distance formula to find the length of the resultant force vector. Because both F_1 and F_2 have their initial point at the origin, (x_1, y_1) will be $(0,0)$.

$$D_R = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$$

$$D_R = \sqrt{(-154.37 - 0)^2 + (196.65 - 0)^2}$$

$$D_R = \sqrt{23{,}830.10 + 38{,}671.22}$$

$$D_R = 250.00$$

To find the angle of the resultant force, we'll use the formula

$$\theta_R = 180^\circ - \arctan\frac{|y|}{|x|}$$

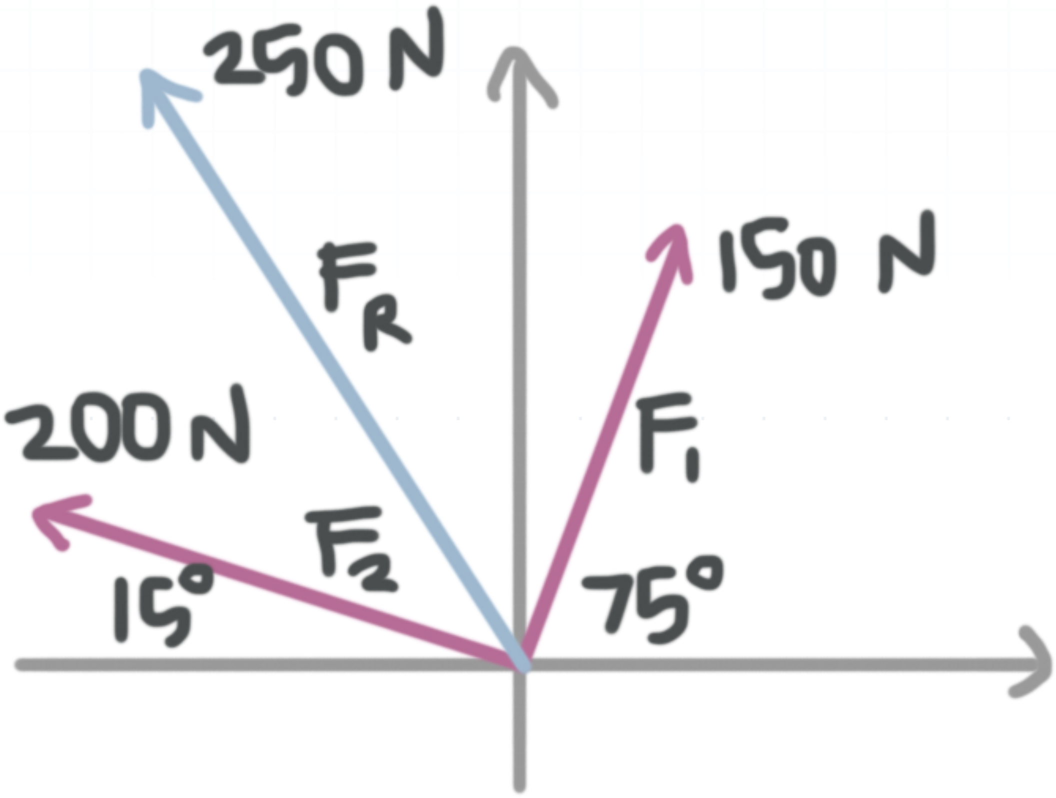
where $F_R = \langle x, y \rangle$. Plugging in x and y from the resultant force, we get

$$\theta_R = 180^\circ - \arctan\frac{196.65}{154.37}$$

$$\theta_R = 180^\circ - 51.87^\circ$$

$$\theta_R = 128.13^\circ$$

The magnitude of the resultant force is 250 N and the angle of the resultant force is 128.13°.



Notice how we built the vector equations for F_1 and F_2 in this last example.

When we measure the angle of the vector, we always measure it from the horizontal axis, which means we'll measure the angles of vectors in the first and fourth quadrants from the positive direction of the horizontal axis, but measure the angles of vectors in the second and third quadrants from the negative direction of the horizontal axis.

Because F_1 fell in the first quadrant, we measured its angle from the positive horizontal axis as 75°. But F_2 fell in the second quadrant, which means we measured its angle from the negative direction of the horizontal axis as 15°.

And we'll always treat the angle between the vector and the horizontal axis as a positive angle. So even for vectors in the third and fourth quadrants, we'll still measure a positive angle from the horizontal axis.

So, while we always keep the angles positive, we do need to change the signs of the coefficients on $\mathbf{i}$ and $\mathbf{j}$, depending on the quadrant of the vector. Consider a generic vector,

$$F_V = F_{V_x} \cos\theta \ \mathbf{i} + F_{V_y} \sin\theta \ \mathbf{j}$$

The signs we use for F_{V_x} and F_{V_y} depend on the quadrant.

In the first quadrant, F_{V_x} is positive, and F_{V_y} is positive

In the second quadrant, F_{V_x} is negative, and F_{V_y} is positive

In the third quadrant, F_{V_x} is negative, and F_{V_y} is negative

In the fourth quadrant, F_{V_x} is positive, and F_{V_y} is negative

That's why, in the previous example, F_1 in the first quadrant got two positive signs, $F_1 = 150\cos 75^\circ\ \mathbf{i} + 150\sin 75^\circ\ \mathbf{j}$, and F_2 in the second quadrant got a negative sign on the first term and a positive sign on the second term, $F_2 = -200\cos 15^\circ\ \mathbf{i} + 200\sin 15^\circ\ \mathbf{j}$.