Topic: Chain rule for multivariable functions

Question: If $x = 1 + t$ and $y = 2 + t^2$, use chain rule to find dz/dt.

$$z = x^2y + x$$

Answer choices:

A $\dfrac{dz}{dt} = t^4 + 2t^3 + 3t^2 + 5t + 3$

B $\dfrac{dz}{dt} = 2t + 1$

C $\dfrac{dz}{dt} = 4t^3 + 6t^2 + 6t + 5$

D $\dfrac{dz}{dt} = 3t^3 + 5t^2 + 4t + 3$

Solution: C

The chain rule tells us that

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

So in order to find dz/dt, we need to find all the pieces from the right-hand side of the formula above. First, let's find the derivatives dx/dt and dy/dt.

$$x = 1 + t$$

$$\frac{dx}{dt} = 1$$

and

$$y = 2 + t^2$$

$$\frac{dy}{dt} = 2t$$

Now we'll find the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$.

$$z = x^2y + x$$

$$\frac{\partial z}{\partial x} = 2xy + 1$$

$$\frac{\partial z}{\partial y} = x^2$$

Plugging these pieces back into our chain rule formula, we get

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = (2xy + 1)(1) + \left(x^2\right)(2t)$$

$$\frac{dz}{dt} = 2tx^2 + 2xy + 1$$

We want our answer in terms of t only, so we'll substitute for x and y.

$$\frac{dz}{dt} = 2t(1 + t)^2 + 2(1 + t)\left(2 + t^2\right) + 1$$

$$\frac{dz}{dt} = 2t\left(t^2 + 2t + 1\right) + 2\left(t^3 + t^2 + 2t + 2\right) + 1$$

$$\frac{dz}{dt} = 2t^3 + 4t^2 + 2t + 2t^3 + 2t^2 + 4t + 4 + 1$$

$$\frac{dz}{dt} = 4t^3 + 6t^2 + 6t + 5$$

Topic: Chain rule for multivariable functions

Question: If $x = -\cos t$ and $y = \sin 2t$, use chain rule to find dz/dt.

$$z = x^2 - y^2$$

Answer choices:

A $\quad \dfrac{dz}{dt} = \cos^2 t - \sin^2 2t$

B $\quad \dfrac{dz}{dt} = -2\cos t - 2\sin 2t$

C $\quad \dfrac{dz}{dt} = \sin^2 t - 4\cos^2 2t$

D $\quad \dfrac{dz}{dt} = -2\cos t \sin t - 4\sin 2t \cos 2t$

Solution: D

The chain rule tells us that

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

So in order to find dz/dt, we need to find all the pieces from the right-hand side of the formula above. First, let's find the derivatives dx/dt and dy/dt.

$$x = -\cos t$$

$$\frac{dx}{dt} = \sin t$$

and

$$y = \sin 2t$$

$$\frac{dy}{dt} = 2\cos 2t$$

Now we'll find the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$.

$$z = x^2 - y^2$$

$$\frac{\partial z}{\partial x} = 2x$$

$$\frac{\partial z}{\partial y} = -2y$$

Plugging these pieces back into our chain rule formula, we get

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = (2x)(\sin t) + (-2y)(2\cos 2t)$$

$$\frac{dz}{dt} = 2x \sin t - 4y \cos 2t$$

We want our answer in terms of t only, so we'll substitute for x and y.

$$\frac{dz}{dt} = 2(-\cos t)\sin t - 4(\sin 2t)\cos 2t$$

$$\frac{dz}{dt} = -2\cos t \sin t - 4\sin 2t \cos 2t$$

Topic: Chain rule for multivariable functions

Question: If $x = t^2 + 2$ and $y = -t - 3$, use chain rule to find dz/dt.

$$z = \ln\left(x^2 + y\right)$$

Answer choices:

A $\frac{dz}{dt} = \frac{4t^3 + 8t - 1}{t^4 + 4t^2 - t + 1}$

B $\frac{dz}{dt} = \frac{4t^2 + 8t - 1}{t^2 + 3t + 1}$

C $\frac{dz}{dt} = \frac{1}{t^4 + 4t^2 - t + 1}$

D $\frac{dz}{dt} = \frac{1}{2t^2 + 4t + 1}$

Solution: A

The chain rule tells us that

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

So in order to find dz/dt, we need to find all the pieces from the right-hand side of the formula above. First, let's find the derivatives dx/dt and dy/dt.

$$x=t^2+2$$

$$\frac{dx}{dt}=2t$$

and

$$y=-t-3$$

$$\frac{dy}{dt}=-1$$

Now we'll find the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$.

$$z=\ln\left(x^2+y\right)$$

$$\frac{\partial z}{\partial x}=\frac{1}{x^2+y}\cdot 2x=\frac{2x}{x^2+y}$$

$$\frac{\partial z}{\partial y}=\frac{1}{x^2+y}\cdot 1=\frac{1}{x^2+y}$$

Plugging these pieces back into our chain rule formula, we get

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{2x}{x^2 + y} \cdot 2t + \frac{1}{x^2 + y} \cdot -1 = \frac{4xt - 1}{x^2 + y}$$

We want our answer in terms of t only, so we'll substitute for x and y.

$$\frac{dz}{dt} = \frac{4\left(t^2 + 2\right)t - 1}{\left(t^2 + 2\right)^2 + (-t - 3)}$$

$$\frac{dz}{dt} = \frac{4t^3 + 8t - 1}{t^4 + 4t^2 - t + 1}$$