# Calculus 3 Workbook

## Surface integrals

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MATH

## SURFACE INTEGRALS

■ 1. Evaluate the surface integral of the scalar vector field $f(x, y, z) = \ln(x + y + z)$ over the surface $\vec{r} = \langle 3u - 7v + 1, u + 5v + 2, -3u + v - 1 \rangle$, where $u$ changes from $0$ to $4$ and $v$ changes from $-1$ to $1$.

■ 2. Evaluate the surface integral of the scalar vector field $f(x, y, z) = x^2 + y^2 + 4z^2$ over the part of the cylinder $x^2 + y^2 = 9$, where $-2 \leq z \leq 5$.

■ 3. Evaluate the surface integral of the scalar vector field $f(x, y, z) = x^2 + y^2 + z + 1$ over the sphere centered at $(2, -1, -3)$ with radius $2$.

## SURFACE INTEGRALS OF ORIENTED SURFACES

■ 1. Evaluate the surface integral of the vector field $\vec{F} = \langle x^2, y^2, x + y + z \rangle$ over $S$, where $S$ is the surface of the cube $[0,2] \times [0,2] \times [0,2]$. Assume that $S$ has a positive orientation.

■ 2. Evaluate the surface integral of the vector field $\vec{F} = \langle x + y, y + z, x + z \rangle$ over the surface $S$ which is the part of the right elliptic cylinder with an axis that coincides with the $y$-axis, an $x$-semi-axis of $3$, a $z$-semi-axis of $9$, and $-3 \leq y \leq 3$. Assume that $S$ has a positive orientation.

■ 3. Evaluate the surface integral of the vector field $\vec{F} = \langle x - 2, y + 1, z - 3 \rangle$ over the surface $S$, where $S$ is the surface of revolution generated by rotating the function $y = x^2 + 1$ around the $x$-axis for $-2 \leq x \leq 2$. Assume that $S$ has a negative orientation.

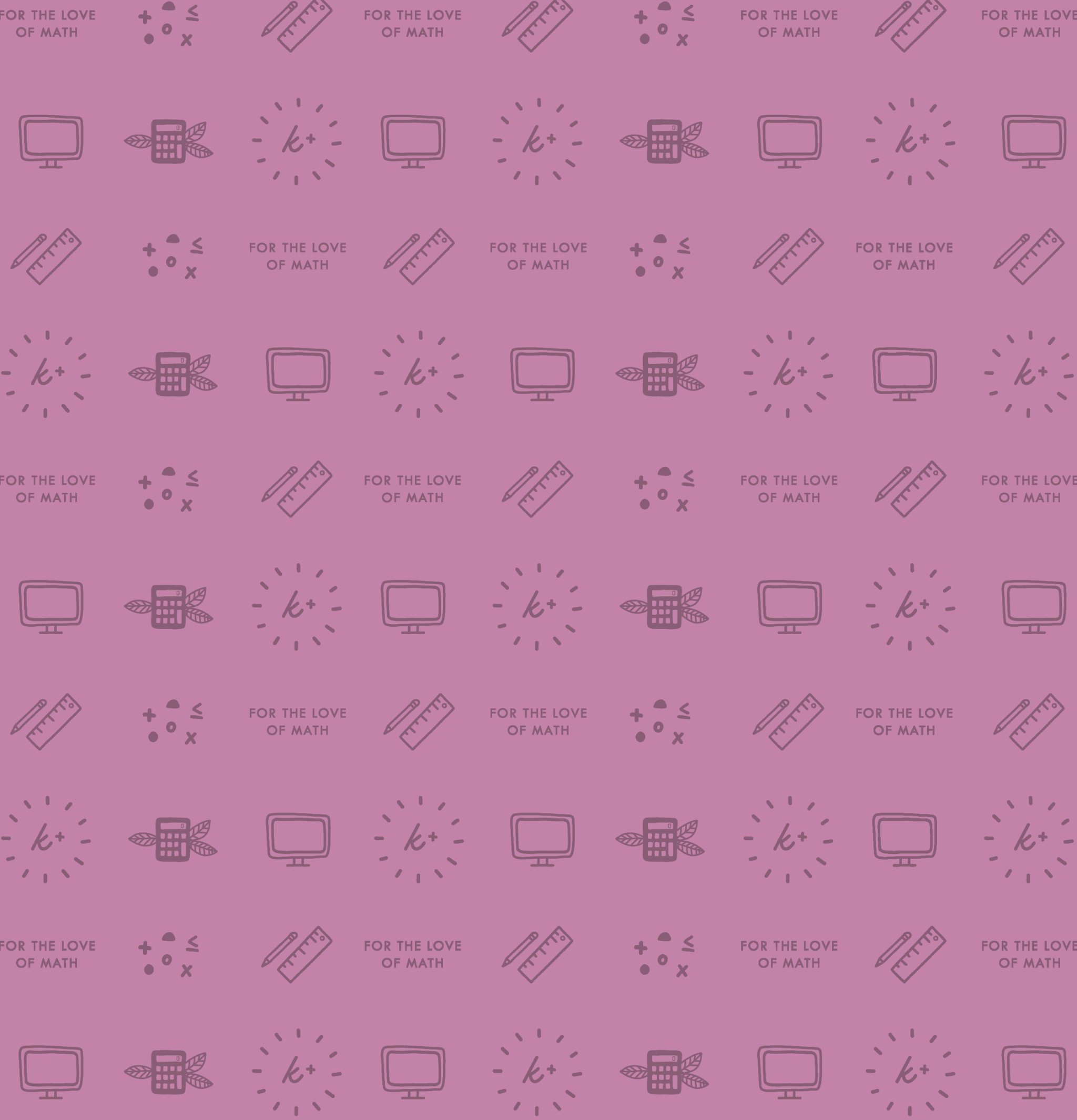
## FLUX ACROSS THE SURFACE

■ 1. Find the flux of the vector field $\vec{F}$ across the part the plane $x + y + z - 2 = 0$ that lies within the rectangle defined by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. Assume that $S$ has an upward orientation.

$$\vec{F} = \left\langle \frac{1}{x^2 + 4}, \frac{1}{4y^2 + 1}, 0 \right\rangle$$

■ 2. Find the flux of the vector field $\vec{F} = \langle x^2 + y^2 + z^2, 3y, 3 \rangle$ across the sphere with radius $4$ and center at the origin. Assume that $S$ has a positive orientation.

■ 3. Suppose the velocity of a fluid in three-dimensional space is described by the vector field $\vec{F} = \langle x^2 + 1, y^2 + 1, z^2 + 1 \rangle$. Find the volume of fluid crossing the disk $S$ defined by $(x + 1)^2 + (y - 2)^2 \leq 4$ in the $xy$-plane per $10$ units of time. Assume that $S$ has an upward orientation.

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