Differential Equations Final Exam

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MATH

Differential Equations Final Exam

This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** Solve the differential equation.

$$y' = -3t^2y$$

A $y = Ce^{-t}$

B $y = Ce^{-t^2}$

C $y = Ce^{t^2}$

D $y = Ce^{-t^3}$

E $y = Ce^{t^3}$

2. **(5 pts)** Find the solution to the Bernoulli differential equation, if $y(0) = 9$.

$$y^{\frac{1}{2}}\frac{dy}{dx} + y^{\frac{3}{2}} = 1$$

A $y = (1 + 25e^{-\frac{3}{2}x})^{\frac{2}{3}}$

B $y = (1 + 26e^{-\frac{3}{2}x})^{\frac{2}{3}}$

C $y = (1 + 27e^{-\frac{3}{2}x})^{\frac{2}{3}}$

D $y^{\frac{3}{2}} = 1 + Ce^{-\frac{3}{2}x}$

E $y = (1 + Ce^{-\frac{3}{2}x})^{\frac{2}{3}}$

3. **(5 pts)** Use any method to find the general solution to the differential equation.

$$y'' - 6y' + 9y = 3e^{3x} + \sin x$$

A $\quad y(x) = c_1e^{-3x} + c_2xe^{-3x} + \frac{3}{2}x^2e^{-3x} + \frac{2}{25}\sin x + \frac{3}{50}\cos x$

B $\quad y(x) = c_1e^{-3x} + c_2e^{-3x} + \frac{3}{2}x^2e^{3x} + \frac{2}{25}\sin x + \frac{3}{50}\cos x$

C $\quad y(x) = c_1e^{3x} + c_2xe^{3x} + \frac{3}{2}x^2e^{3x} + \frac{2}{25}\sin x + \frac{3}{50}\cos x$

D $\quad y(x) = c_1e^{3x} + c_2xe^{3x} + \frac{3}{2}x^2e^{3x} + \frac{2}{25}\sin x$

E $\quad y(x) = c_1e^{3x} + c_2xe^{3x} + \frac{3}{2}x^2e^{3x}\frac{3}{50}\cos x$

4. **(5 pts)** Use Euler's method to approximate $y(\pi)$, using $n=4$ steps and given $y(0)=0$.

$$y' = -4\sin t$$

A $y(\pi) \approx -4\sin\sqrt{\pi}$

B $y(\pi) \approx \sqrt{\pi}$

C $y(\pi) \approx -\sqrt{\pi}$

D $y(\pi) \approx \sqrt{2}\pi$

E $y(\pi) \approx -\dfrac{2+\sqrt{2}}{2}\pi$

5. **(5 pts)** Determine the stability of any equilibrium solutions of the autonomous equation.

$$\frac{dy}{dt} = y^2 - 7y + 12$$

A Stable solutions at $y=3$ and $y=4$

B Unstable solutions at $y=3$ and $y=4$

C Stable solution at $y=3$; unstable solution at $y=4$

D Unstable solution at $y=3$; stable solution at $y=4$

E No equilibrium solutions

6. **(5 pts)** Find a power series solution around $x_0 = 0$ to the differential equation.

$$y' = y$$

A $y = c_0 \sum_{k=0}^{\infty} k!x^{2k}$

B $y = c_0 \sum_{k=0}^{\infty} \frac{1}{k!}x^{2k}$

C $y = c_0 \sum_{k=0}^{\infty} \frac{1}{2k!}x^{2k}$

D $y = c_0 \sum_{k=0}^{\infty} \frac{1}{2k!}x^{k}$

E $y = c_0 \sum_{k=0}^{\infty} \frac{1}{k!}x^{k}$

7. **(5 pts)** Use a Laplace transform to find the solution to the differential equation, given $y(0)=0$ and $y'(0)=4$.

$$y''-7y'+10y=3t^2$$

A $\quad Y(s)=\frac{117}{500}+\frac{21}{50}t+\frac{3}{10}t^2-\frac{19}{12}e^{2t}+\frac{506}{375}e^{5t}$

B $\quad Y(s)=\frac{117}{500}+\frac{21}{50}t+\frac{3}{10}t^2+\frac{19}{12}e^{2t}-\frac{506}{375}e^{5t}$

C $\quad Y(s)=-\frac{117}{500}+\frac{21}{50}t+\frac{3}{10}t^2+\frac{19}{12}e^{2t}+\frac{506}{375}e^{5t}$

D $\quad Y(s)=\frac{117}{500}-\frac{21}{50}t+\frac{3}{10}t^2+\frac{19}{12}e^{2t}+\frac{506}{375}e^{5t}$

E $\quad Y(s)=\frac{117}{500}+\frac{21}{50}t-\frac{3}{10}t^2+\frac{19}{12}e^{2t}+\frac{506}{375}e^{5t}$

8. **(5 pts)** Use a convolution integral to find the general solution to the differential equation, if $y(0)=0$ and $y'(0)=0$.

$$y''+3y'+6y=g(t)$$

A $\quad y(t)=\frac{2\sqrt{15}}{15}\int_0^t e^{\frac{3}{2}\tau}\sin\left(\frac{\sqrt{15}}{2}\tau\right)\ d\tau$

B $\quad y(t)=\frac{2\sqrt{15}}{15}\int_0^t e^{-\frac{3}{2}\tau}\sin\left(\frac{\sqrt{15}}{2}\tau\right)g(\tau-t)\ d\tau$

C $\quad y(t)=\frac{\sqrt{15}}{2}\int_0^t e^{-\frac{3}{2}\tau}\sin\left(\frac{\sqrt{15}}{2}\tau\right)g(\tau-t)\ d\tau$

D $\quad y(t)=\frac{2\sqrt{15}}{15}\int_0^t e^{\frac{3}{2}\tau}\sin\left(\frac{\sqrt{15}}{2}\tau\right)g(t-\tau)\ d\tau$

E $\quad y(t)=\frac{\sqrt{15}}{2}\int_0^t e^{\frac{3}{2}\tau}\sin\left(\frac{\sqrt{15}}{2}\tau\right)g(t-\tau)\ d\tau$

9. **(15 pts)** Use undetermined coefficients to find the general solution to the system $\vec{x}' = A\vec{x} + F$.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & -3 \\ 0 & -3 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} -e^{2t} \\ \cos t \\ 3t \end{bmatrix}$$

10. **(15 pts)** Find the general solution to the homogeneous differential equation, given its associated characteristic equation.

$$r(r-2)(r+3)(r+3)(r^2+2r+7)^4 = 0$$

11. **(15 pts)** Find the Fourier series representation of $f(x) = x^4$ on $-L \leq x \leq L$.

12. **(15 pts)** Given a one-dimensional temperature distribution, if temperature at $x = 0$ is 0°, temperature at $x = L$ is 100°, and $u(x,0) = 25$, find an equation that models heat in the system.