Differential Equations Final Exam

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This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** Solve the initial value problem, given $y(0)=3/2$.

$$xy'+4x^2y=2x^2$$

A $y=1+e^{x^2}$

B $y=e^{-2x^2}+\frac{1}{2}$

C $y=e^x+\frac{1}{2}$

D $y=\frac{3}{2}+e^{x^2}$

E $y=e^{2x^2}+\frac{1}{2}$

2. **(5 pts)** Find the solution to the differential equation.

$$xy'+y=y^2\ln x$$

A $y=\frac{1}{\ln x+C}$

B $y=Cx+\ln x+1$

C $y=\frac{1}{\ln x-1+Cx}$

D $y=3\ln x+2-Cx$

E $y=\frac{1}{\ln x+1+Cx}$

3. **(5 pts)** Solve the initial value problem, given $y(0) = 1/6$ and $y'(0) = 0$.

$$y'' - 2y' - 3y = 3t - 5\sin t$$

A $\quad y(t) = -t + \frac{2}{3} - \frac{1}{2}\cos t + \sin t$

B $\quad y(t) = -t + \frac{1}{2}\cos t - \sin t$

C $\quad y(t) = 3t^2 - \sin t$

D $\quad y(t) = t + \frac{1}{2}\sin t - \cos t$

E $\quad y(t) = 3e^{3t} - e^{-t} + \sin t$

4. **(5 pts)** Use variation of parameters and Wronskian integrals to find the general solution to the nonhomogeneous equation.

$$y'' - y' - 6y = \frac{1}{e^{2x}}$$

A $y(x) = c_1 e^{2x} + c_2 e^{3x} + xe^{-2x}$

B $y(x) = c_1 e^{-3x} + c_2 e^{2x} - \frac{1}{5}xe^{-2x}$

C $y(x) = c_1 e^{-2x} + c_2 e^{3x} - \frac{1}{5}xe^{-2x} - \frac{1}{25}e^{-2x}$

D $y(x) = c_1 \cos 2x + c_2 \sin 2x - 3e^{-2x}$

E $y(x) = c_1 e^{-2x} + c_2 e^{3x} + \frac{1}{5}xe^{2x}$

5. **(5 pts)** Use Euler's method to approximate $y(1)$, using a step size of 0.2, and given $y(0) = 1$.

$$y' = 2y + t$$

A $y(1) \approx 7.35$

B $y(1) \approx 5.97$

C $y(1) \approx 4.92$

D $y(1) \approx 8.91$

E $y(1) \approx 6.75$

6. **(5 pts)** A population of insects is observed to be 20 at time $t = 0$, where t is measured in minutes. After 2 hours, the population doubles. Assuming exponential growth, how long would it take for the population to grow to 100?

A 4 hours 39 minutes

B 4 hours 30 minutes

C 5 hours

D 10 hours

E 7 hours 37 minutes

7. **(5 pts)** Solve the system of differential equations.

$$x_1' = x_1 + 3x_2$$

$$x_2' = 2x_2$$

A $\quad \vec{x} = c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{2t}$

B $\quad \vec{x} = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{2t}$

C $\quad \vec{x} = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{2t}$

D $\quad \vec{x} = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{-2t}$

E $\quad \vec{x} = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{2t}$

8. **(5 pts)** Rewrite the boundary conditions for the product solution $u(x, y) = v(x)w(y)$.

$$\frac{\partial^2 u}{\partial y^2} - 3\frac{\partial u}{\partial x} = 0$$

$$u(x,0) = 0,\ u(0,y) = 0,\ \frac{\partial u}{\partial y}(x,0) = 0$$

A $w(0) = 0,\ w'(0) = 0,\ v'(0) = 0$

B $w(0) = 0,\ v(0) = 0,\ v'(0) = 0$

C $w(0) = 0,\ v(0) = 0,\ v'(\pi) = 0$

D $w'(0) = 0,\ v(0) = 0,\ v'(0) = 0$

E $w(0) = 0,\ w'(0) = 0,\ v(0) = 0$

9. **(15 pts)** Rewrite the differential equation as a system, then solve it.

$$y''' - 3y'' + 2y' = e^{-t} - 3\sin 3t$$

10. **(15 pts)** Find the Fourier series representation of the piecewise function on $-L \leq x \leq L$.

$$f(x) = \begin{cases} 2x + 2 & -L \leq x \leq 0 \\ 4x - 3x^2 & 0 \leq x \leq L \end{cases}$$

11. **(15 pts)** Find a solution to the partial differential equation.

$$\frac{\partial u}{\partial x} - 16\frac{\partial^2 u}{\partial y^2} = 0$$

$$u(0,y) = 3\sin\left(\frac{3y}{2}\right),\ u(x,0) = u_y(x,\pi) = 0$$

12. **(15 pts)** Find a power series around $x_0 = 0$ and solve the differential equation.

$$y'' - y' = 0$$