8 – REPRESENTATION AND DESCRIPTION (B)

IMAGE MOMENTS

When a region is given in terms of its interior points, we can describe it by a set of moments. These moment values can be used to characterise the shape of a region and also the spatial distribution of the pixel intensities within the region. Moments that are invariant to translation, rotation and scale change can be defined.

Given a two-dimensional continuous function $f(x, y)$, we define the moment of order $(p + q)$ by the relation

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y)\, dxdy \quad p, q = 0, 1, 2, \ldots \tag{1}$$

For a digital image, we have

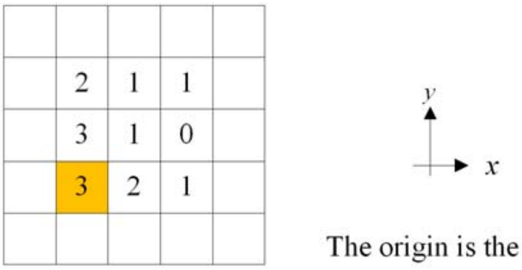
$$m_{pq} = \sum_x \sum_y x^p y^q f(x, y) \tag{2}$$

where the summation is taken over all spatial coordinates (x, y) of points in the region.

The centroid is the point $(\bar{x}, \bar{y})$ defined by

$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}. \tag{3}$$

Consider the image below.



The origin is the shaded pixel

The first few moments are

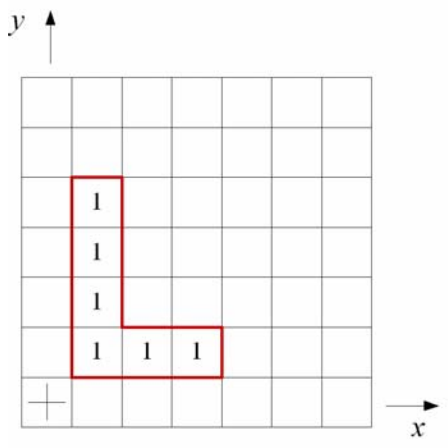
p	0	1	0	2	1	0
q	0	0	1	0	1	2
m_{pq}	14	8	12	12	7	20

Example

$$\begin{aligned}
m_{00} &= \sum_x \sum_y f(x,y) \\
&= \sum_{x=0}^{2} \sum_{y=0}^{2} f(x,y) \\
&= [f(0,0)+f(0,1)+f(0,2)]+ \\
&\quad\ [f(1,0)+f(1,1)+f(1,2)]+ \\
&\quad\ [f(2,0)+f(2,1)+f(2,2)] \\
&= [3+3+2]+ \\
&\quad\ [2+1+1]+ \\
&\quad\ [1+0+1] \\
&= 14
\end{aligned}$$

$$\begin{aligned}
m_{11} &= \sum_x \sum_y xyf(x,y) \\
&= \sum_{x=0}^{2} x\left[\sum_{y=0}^{2} yf(x,y)\right] \\
&= 0[0\times f(0,0)+1\times f(0,1)+2\times f(0,2)]+ \\
&\quad\ 1[0\times f(1,0)+1\times f(1,1)+2\times f(1,2)]+ \\
&\quad\ 2[0\times f(2,0)+1\times f(2,1)+2\times f(2,2)] \\
&= 0+ \\
&\quad\ 1[0\times 2+1\times 1+2\times 1]+ \\
&\quad\ 2[0\times 1+1\times 0+2\times 1] \\
&= 7
\end{aligned}$$

Example



$$m_{00} = 6 \qquad \bar{x} = \frac{9}{6} = 1.5$$
$$m_{10} = 9$$
$$m_{01} = 12 \qquad \bar{y} = \frac{12}{6} = 2$$
$$m_{11} = 15$$

The *central moment* of order $(p+q)$ defined by

$$\mu_{pq} = \int\int (x-\bar{x})^p (y-\bar{y})^q f(x,y)\, dx\, dy \tag{4}$$

is translation invariant.

Proof

Let the translated image be

$$f'(x,y) = f(x-x_0, y-y_0)$$

Its centroid is at

$$(\bar{x}', \bar{y}') = (\bar{x}+x_0, \bar{y}+y_0)$$

where $(\bar{x}, \bar{y})$ is the centroid of $f(x,y)$. The central moment of $f'(x,y)$ is

$$\begin{aligned}\mu'_{pq} &= \int\int (x-\bar{x}')^p (y-\bar{y}')^q f'(x,y)\, dx\, dy \\ &= \int\int (x-(\bar{x}+x_0))^p (y-(\bar{y}+y_0))^q f(x-x_0, y-y_0)\, dx\, dy\end{aligned}$$

By substituting

$$\alpha = x - x_0, \qquad \beta = y - y_0$$

we obtain

$$\begin{aligned}\mu'_{pq} &= \int\int (\alpha-\bar{x})^p (\beta-\bar{y})^q f(\alpha,\beta)\, d\alpha\, d\beta \\ &= \mu_{pq}\end{aligned}$$

The central moments of order 3 and below are:

$$\begin{aligned}
\mu_{00} &= m_{00} \\
\mu_{10} &= 0 \\
\mu_{01} &= 0 \\
\mu_{11} &= m_{11} - \bar{y} m_{10} \\
\mu_{20} &= m_{20} - \bar{x} m_{10} \\
\mu_{02} &= m_{02} - \bar{y} m_{01} \\
\mu_{30} &= m_{30} - 3\bar{x} m_{20} + 2 m_{10} \bar{x}^2 \\
\mu_{12} &= m_{12} - 2\bar{y} m_{11} - \bar{x} m_{02} + 2\bar{y}^2 m_{10} \\
\mu_{21} &= m_{21} - 2\bar{x} m_{11} - \bar{y} m_{20} + 2\bar{x}^2 m_{01} \\
\mu_{03} &= m_{03} - 3\bar{y} m_{02} + 2\bar{y}^2 m_{01}
\end{aligned}$$

Example

$$\begin{aligned}
\mu_{20} &= \sum\sum (x - \bar{x})^2 f(x, y) \\
&= \sum\sum (x^2 - 2x\bar{x} + \bar{x}^2) f(x, y) \\
&= \sum\sum x^2 f(x, y) - 2\bar{x} \sum\sum x f(x, y) + \bar{x}^2 \sum\sum f(x, y) \\
&= m_{20} - 2\bar{x} m_{10} + \bar{x} \frac{m_{10}}{m_{00}} m_{00} \\
&= m_{20} - 2\bar{x} m_{10} + \bar{x} m_{10} \\
&= m_{20} - \bar{x} m_{10}
\end{aligned}$$

The *normalized central moments* of order $(p+q)$ defined by

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \tag{5}$$

where

$$\gamma = \frac{p+q}{2} + 1 \qquad \text{for } p+q = 2, 3, \ldots$$

are invariant to scale change.

Proof

Let the scaled image be $f'(x, y) = f(sx, sy)$ where s is the scale factor, and its centroid be $(\bar{x}', \bar{y}')$. We have

$$\begin{aligned} m'_{10} &= \int\int x f(sx, sy) dx\, dy \\ &= \frac{1}{s^3} m_{10} \\ m'_{00} &= \int\int f(sx, sy) dx\, dy \\ &= \frac{1}{s^2} m_{00} \end{aligned}$$

Hence

$$\bar{x}' = \frac{1}{s}\bar{x}$$

Similarly,

$$\bar{y}' = \frac{1}{s}\bar{y}$$

Now,

$$\begin{aligned} \mu'_{pq} &= \int\int (x - \bar{x}')^p (y - \bar{y}')^q f(sx, sy)\, dx\, dy \\ &= \int\int (x - \frac{1}{s}\bar{x})^p (y - \frac{1}{s}\bar{y})^q f(sx, sy)\, dx\, dy \\ &= \frac{1}{s^{p+q}} \int\int (sx - \bar{x})^p (sy - \bar{y})^q f(sx, sy)\, dx\, dy \end{aligned}$$

Using the substitutions $\alpha = sx, \beta = sy$,

$$\begin{aligned} \mu'_{pq} &= \frac{1}{s^{p+q+2}} \int\int (\alpha - \bar{x})^p (\beta - \bar{y})^q f(\alpha, \beta)\, dx\, dy \\ &= \frac{1}{s^{p+q+2}} \mu_{pq} \\ &= \frac{1}{s^{2\gamma}} \mu_{pq} \quad \text{where } \gamma = \frac{p+q}{2} + 1 \end{aligned}$$

We also have

$$\mu'_{00} = \frac{1}{s^2}\mu_{00}$$

Hence

$$\begin{aligned} \eta'_{pq} &= \frac{\mu'_{pq}}{(\mu'_{00})^\gamma} \\ &= \left(\frac{\mu_{pq}}{s^{2\gamma}}\right) \Big/ \left(\frac{1}{s^{2\gamma}}\mu_{00}^\gamma\right) \\ &= \frac{\mu_{pq}}{\mu_{00}^\gamma} \\ &= \eta_{pq} \end{aligned}$$

Example

$\mu_{11} = 3.29 \times 10^6$, $\mu_{20} = 2.71 \times 10^7$

$\eta_{11} = 2.02 \times 10^{-2}$, $\eta_{20} = 1.69 \times 10^{-1}$

$\mu_{11} = 1.62 \times 10^7$, $\mu_{20} = 1.36 \times 10^8$

$\eta_{11} = 2.06 \times 10^{-2}$, $\eta_{20} = 1.70 \times 10^{-1}$

From the second and third moments, a set of seven *invariant moments* can be derived. It can be shown that they are invariant to rotation, scale change, and translation.

$$
\begin{aligned}
\phi_1 &= \eta_{20} + \eta_{02} \\
\phi_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\
\phi_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
\phi_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\
\phi_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\
&\quad +(3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
\phi_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
&\quad +4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\
\phi_7 &= (3\eta_{21} - \eta_{30})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\
&\quad +(3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]
\end{aligned}
$$

The moment invariants are useful features for shape analyis. They have been used in distinguishing between shapes of aircraft, character recognition, and scene-matching applications.

For a digital image, the invariant moments

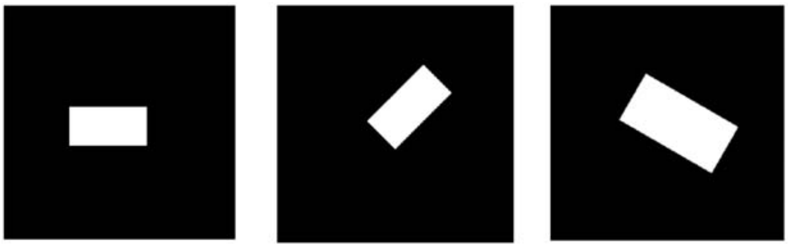
$$\phi_1, \phi_2, \ldots, \phi_7$$

are invariant under translation, but not strictly invariant under image rotation and scale change.

It can be shown that:

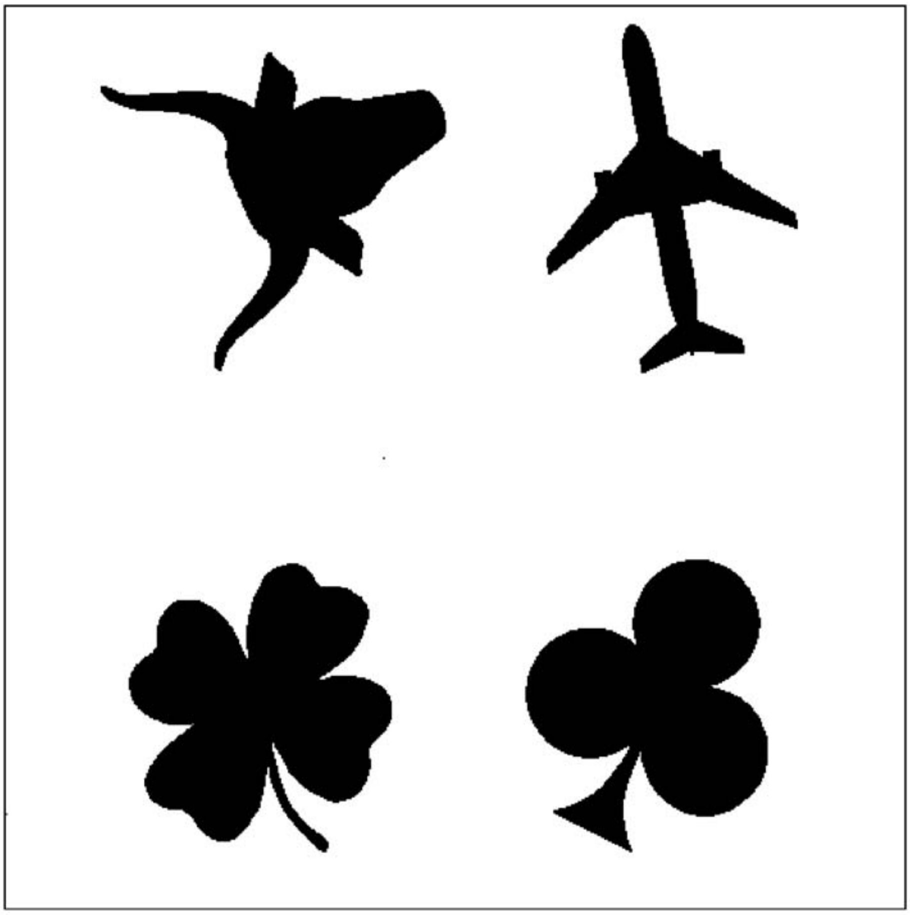
- A ratio of two moments that have the same value of $p+q$, e.g., m_{01}/m_{10}, is invariant under scale change.
- Moments for which p, q, or $p+q$ is odd can be used as measures of asymmetry about the y-axis, x-axis, and the origin, respectively.
  If $f(x,y)$ is symmetric about the y-axis, $m_{pq} = 0$ for p odd.
  If $f(x,y)$ is symmetric about the x-axis, $m_{pq} = 0$ for q odd.
  If $f(x,y)$ is symmetric about the origin, $m_{pq} = 0$ for $p+q$ odd.

Examples



$\phi_1 = 0.208$ $\phi_1 = 0.207$ $\phi_1 = 0.208$

$\phi_1 = 0.408$ $\phi_1 = 0.406$ $\phi_1 = 0.407$

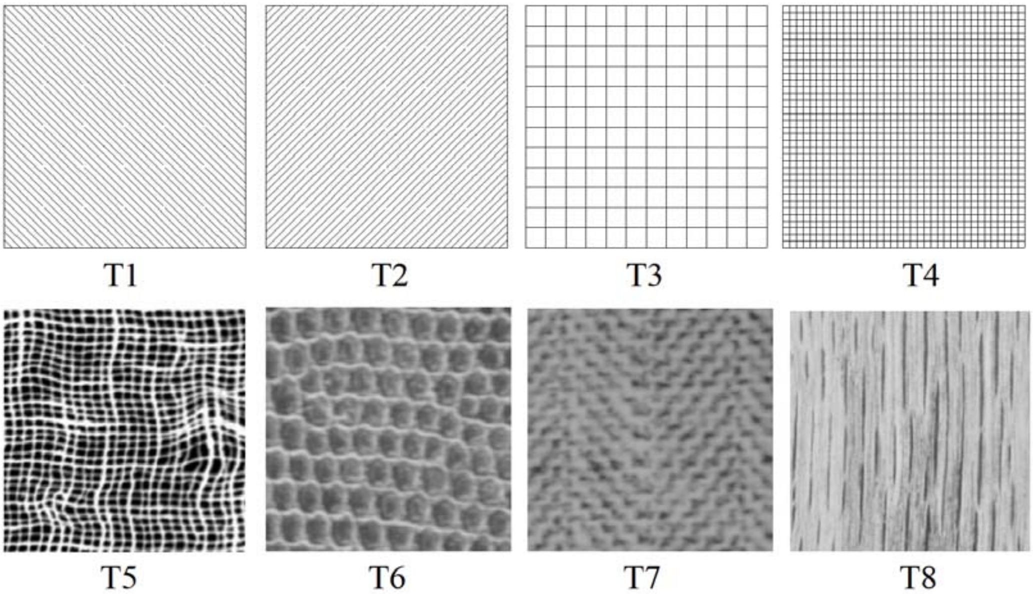


	μ_{11}		ϕ_1	
buffalo	-2.79×10^6	-40	0.250	62
plane	5.47×10^6	78	0.406	100
club	-6.99×10^6	-100	0.178	44
leaf	-6.55×10^6	-94	0.188	46
	(raw values)	(norm.)	(raw values)	(norm.)

TEXTURE

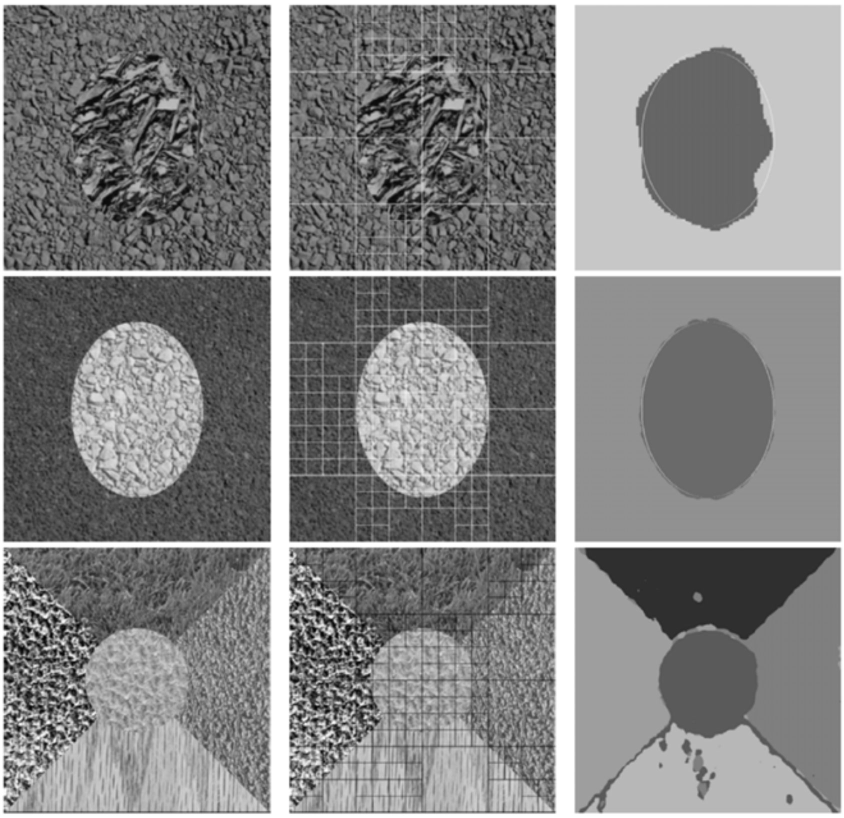
Texture provides a measure of such properties as smoothness, coarseness and regularity. It is a loosely defined term that refers to the spatial variation or arrangment in gray value within a small region. Since texture is a neighbourhood property of an image point, texture measures are dependent on the size of the measurement neighbourhood (window). It is measured over the windowed region as the window is moved from one pixel to the next.

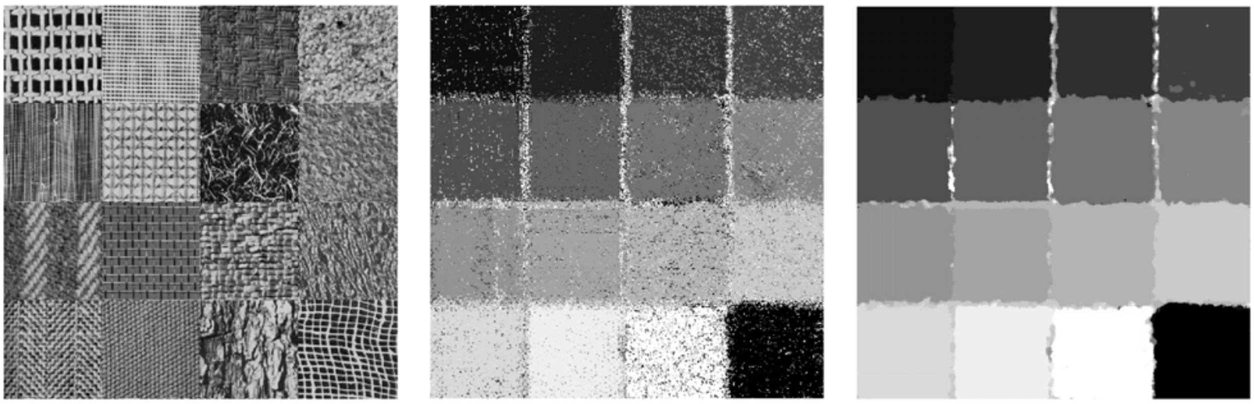
Texture information can be used for image segmentation and region classification.



- T1-T4 are examples of artificial (computer-generated) textures.
- T1 and T2 illustrate different directional textures.
- T3 and T4 illustrate coarse and fine (or busy) textures.
- T5-T8 are examples of natural textures.

Examples of texture segmentation





Texture from Gray-level Histogram Statistics

Let z be a random variable denoting image intensity and $p(z_i)$, $i = 0, 1, 2, \ldots, L-1$, be the corresponding histogram, where L is the number of distinct gray levels. The nth moment of z about the mean m is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i) \tag{6}$$

where

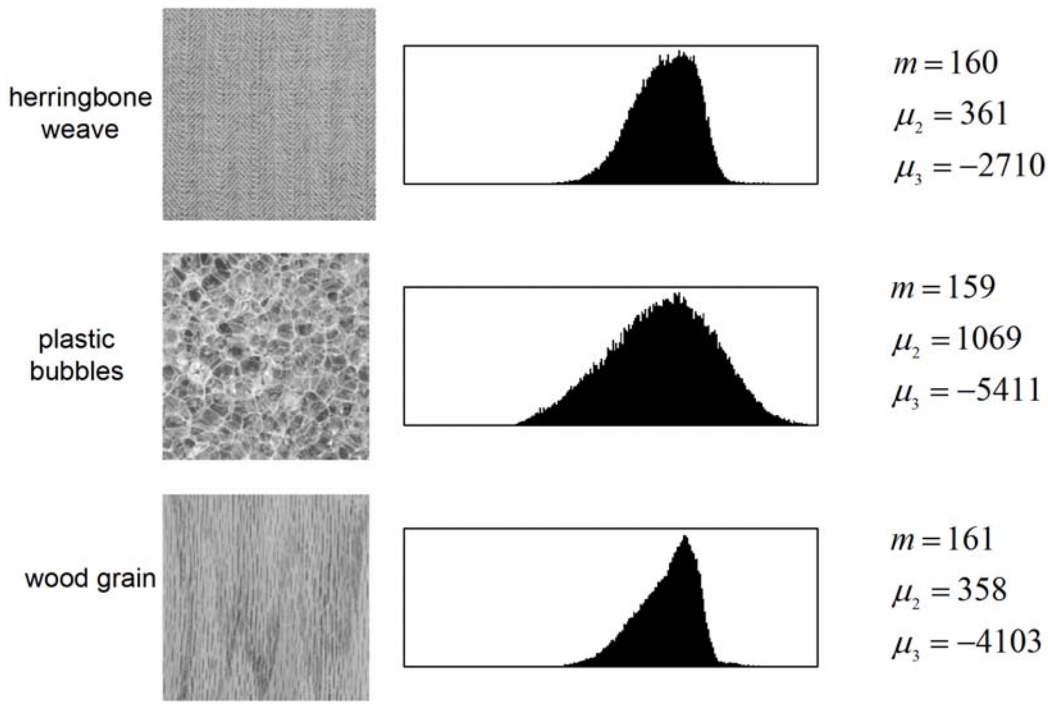
$$m = \sum_{i=0}^{L-1} z_i p(z_i). \tag{7}$$

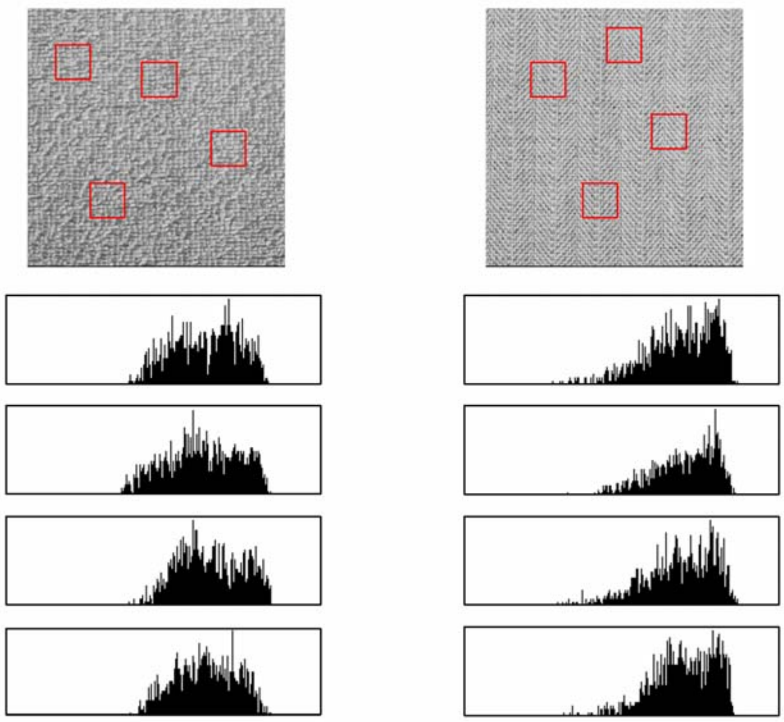
We see that $\mu_0 = 1$ and $\mu_1 = 0$.

- The mean is a measure of the overall lightness/darkness of the image. The median is another such measure.
- The second moment, μ_2 is also called the variance, σ^2. The variance and the standard deviation, σ, are measures of the overall contrast. If they are small the gray levels of the image are all close to the mean, while if they are large the image has a large range of gray levels. A descriptor of relative smoothness is

  $$R = 1 - \frac{1}{1 + \sigma^2(z)}. \tag{8}$$

  It is 0 for areas of constant intensity and approaches 1 for large values of $\sigma^2(z)$.
- μ_3 is called skewness and μ_4 kurtosis.



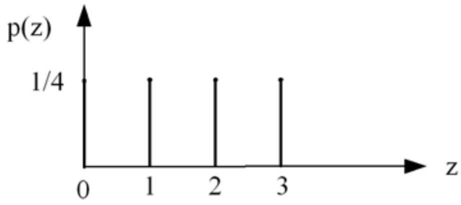


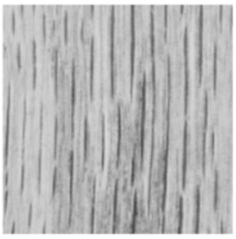
Histograms at different points in the image

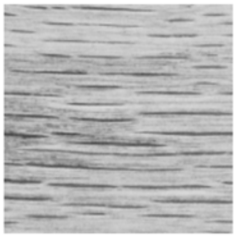
Statistics computed from the histogram are of limited value in describing the image since $p(z)$ remains the same no matter how the pixels of the image are distributed.

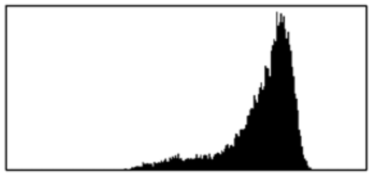
0	0	1	1
0	0	1	1
2	2	3	3
2	2	3	3

0	1	2	3
3	0	1	2
2	3	0	1
1	2	3	0









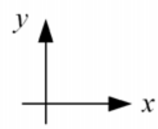
Second-order Gray Level Statistics

Let $\delta = (\Delta x, \Delta y)$ be a displacement in the xy plane and $\mathbf{A}$ a $k \times k$ matrix whose element a_{ij} is the number of times that a pixel with gray level z_i occurs (in the position specified by δ) relative to a pixel with gray level z_j, with $1 \leq i,\ j \leq k$. Suppose an image has three gray levels, $z_1 = 0$, $z_2 = 1$, and $z_3 = 2$. Then, the "A matrix" is given by:

$$\mathbf{A} \;=\; \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & \#(0,0) & \#(1,0) & \#(2,0) \\ 1 & \#(0,1) & \#(1,1) & \#(2,1) \\ 2 & \#(0,2) & \#(1,2) & \#(2,2) \end{array}$$

Given a 5×5 image

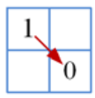
0	0	0	1	2
1	1	0	1	1
2	2	1	0	0
1	1	0	2	0
0	0	1	0	1



and $\delta = (1, -1) \searrow$, we obtain

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 1 \\ 1 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix} \quad \text{or} \quad \begin{array}{c|ccc} \delta_{(1,-1)} & 0 & 1 & 2 \\ \hline 0 & 4 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 0 & 2 & 0 \end{array}$$

Number of times this pattern occurs in the image:



The element a_{13} is the number of times that a pixel with level z_1 ($= 0$) appears one pixel location below and to the right of a pixel with gray level z_3 ($= 2$). The element a_{21} is the number of times that a pixel with level z_2 ($= 1$) appears one pixel location below and to the right of a pixel with gray level z_1 ($= 0$).

Since the size of $\mathbf{A}$ is determined by the number of distinct gray levels in the image, it is often necessary to requantize the image into a few gray-level bands.

Let n_p be the total number of pixel pairs in the image that satisfy δ, i.e., $n_p = \sum a_{ij}$. ($n_p = 16$ in the above example.) We define the *gray-level co-occurrence matrix* (GLCM):

$$\mathbf{C} = \frac{1}{n_p}\mathbf{A} \tag{9}$$

formed by dividing every element of $\mathbf{A}$ by n_p. c_{ij} is an estimate of the joint probability that a pair of pixels satisfying δ will have values (z_i, z_j).

It is possible to detect the presence of given texture patterns by choosing an appropriate position operator.

Examples

$$T_1 \quad \begin{array}{cccccc} & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 1 & 2 & 1 & 2 & \cdots \\ \cdots & 1 & 2 & 1 & 2 & \cdots \\ \cdots & 1 & 2 & 1 & 2 & \cdots \\ \cdots & 1 & 2 & 1 & 2 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \end{array}$$

$\delta_{1,0}$	0	1	2
0	0	0	0
1	0	0	8
2	0	8	0

$\delta_{0,1}$	0	1	2
0	0	0	0
1	0	8	0
2	0	0	8

$$T_2 \quad \begin{array}{cccccc} & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 1 & 1 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 2 & 2 & \cdots \\ \cdots & 1 & 1 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 2 & 2 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \end{array}$$

$\delta_{1,0}$	0	1	2
0	0	0	0
1	0	8	0
2	0	0	8

$\delta_{0,1}$	0	1	2
0	0	0	0
1	0	0	8
2	0	8	0

$$T_3 \quad \begin{array}{cccccc} & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 1 & 2 & 1 & 2 & \cdots \\ \cdots & 2 & 1 & 2 & 1 & \cdots \\ \cdots & 1 & 2 & 1 & 2 & \cdots \\ \cdots & 2 & 1 & 2 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \end{array}$$

$\delta_{1,0}$	0	1	2
0	0	0	0
1	0	0	8
2	0	8	0

$\delta_{0,1}$	0	1	2
0	0	0	0
1	0	0	8
2	0	8	0

Note:

- the x axis points to the right, y axis points up
- for each pixel pair, the start pixel must lie in the 3×3 sub-image, while the end pixel can lie outside the sub-image
- relatively large diagonal values are obtained when δ is aligned with texture orientation.

Descriptors can be defined to characterize the content of $\mathbf{C}$.

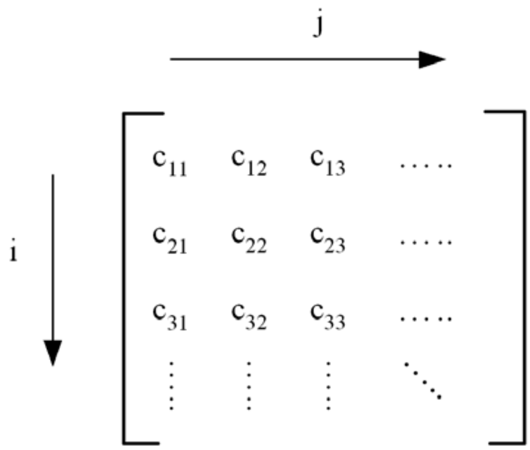
1. *Maximum probability*:
$$\max\ (c_{ij})$$
This property gives an indication of the strongest response to δ.

2. *Element-difference moment of order k*:
$$\sum_i \sum_j (i-j)^k c_{ij}$$
This descriptor has a relatively low value where the high values of $\mathbf{C}$ are near the main diagonal since the differences $(i-j)$ are smaller there.

3. *Inverse element-difference moment of order k*:
$$\sum_i \sum_j c_{ij}/(i-j)^k \quad i \neq j$$
This descriptor has the opposite effect from the second.

4. *Entropy*:
$$-\sum_i \sum_j c_{ij} \log c_{ij}$$
This descriptor is a measure of randomness, achieving its highest value when all elements of $\mathbf{C}$ are equal.

5. *Uniformity*:
$$\sum_i \sum_j c_{ij}^2$$
This descriptor is lowest when the c_{ij}'s are all equal.

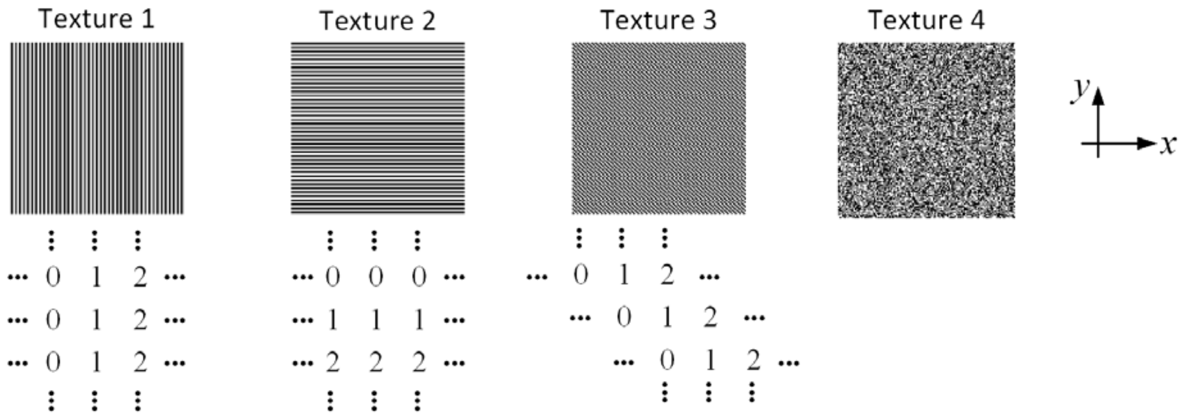


Example

$$
\begin{array}{cc}
 & \begin{array}{ccc} j=1 & j=2 & j=3 \end{array} \\
\begin{array}{c} i=1 \\ i=2 \\ i=3 \end{array} & \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}
\end{array}
$$

$$
\begin{aligned}
\sum_i \sum_j (i-j)^2 c_{ij} &= (1-1)^2 c_{11} + (1-2)^2 c_{12} + (1-3)^2 c_{13} \\
&+ (2-1)^2 c_{21} + (2-2)^2 c_{22} + (2-3)^2 c_{23} \\
&+ (3-1)^2 c_{31} + (3-2)^2 c_{32} + (3-3)^2 c_{33}
\end{aligned}
$$

Example



Computation of $\mathbf{C}$ for $\delta_1 = (0, -1) \downarrow$, values of descriptors D_1, D_2, D_3 are given in the last row.

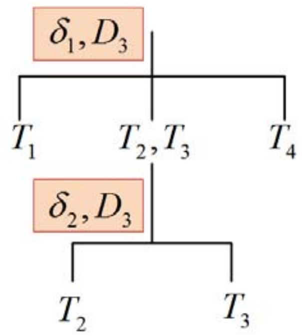
Texture 1			Texture 2			Texture 3			Texture 4		
0.33	0	0	0	0	0.33	0	0.33	0	0.11	0.11	0.11
0	0.33	0	0.33	0	0	0	0	0.33	0.11	0.11	0.11
0	0	0.33	0	0.33	0	0.33	0	0	0.11	0.11	0.11
0.33, 0, 0			0.33, 0, 1.98			0.33, 0, 1.98			0.11, 0, 1.34		

Computation of $\mathbf{C}$ for $\delta_2 = (1, -1) \searrow$, values of D_1, D_2, D_3 are given in the last row.

Texture 1			Texture 2			Texture 3			Texture 4		
0	0	0.33	0	0	0.33	0.33	0	0	0.11	0.11	0.11
0.33	0	0	0.33	0	0	0	0.33	0	0.11	0.11	0.11
0	0.33	0	0	0.33	0	0	0	0.33	0.11	0.11	0.11
0.33, 0, 1.98			0.33, 0, 1.98			0.33, 0, 0			0.11, 0 1.34		

$D_1 = \max(c_{ij}),\ D_2 = \sum\sum(i-j)^1\, c_{ij},\ D_3 = \sum\sum(i-j)^2\, c_{ij}$

The computed descriptors can be used to differentiate the textures, e.g.:



Local Property Statistics

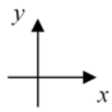
Another texture measure is to compute statistics of various local property values measured at the points of f. As an example, let

$$g(x,y) \equiv f(x+\Delta x, y) - f(x,y)$$

We denote the histogram of $g(x,y)$ by h_g.

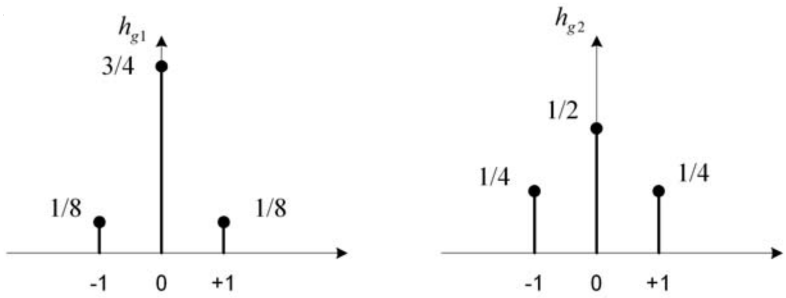
Consider the two textures below.

$$f_1 = \begin{array}{cccccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & \cdots \\ \cdots & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & \cdots \\ \cdots & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & \cdots \\ \cdots & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & \cdots \\ \cdots & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array} \qquad f_2 = \begin{array}{cccccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & \cdots \\ \cdots & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & \cdots \\ \cdots & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & \cdots \\ \cdots & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & \cdots \\ \cdots & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & \cdots \\ \cdots & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array}$$



$$g_1 = \begin{array}{cccccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & \cdots \\ \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & \cdots \\ \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & \cdots \\ \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & \cdots \\ \cdots & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & \cdots \\ \cdots & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & \cdots \\ \cdots & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & \cdots \\ \cdots & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array} \qquad g_2 = \begin{array}{cccccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & \cdots \\ \cdots & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & \cdots \\ \cdots & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & \cdots \\ \cdots & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & \cdots \\ \cdots & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & \cdots \\ \cdots & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & \cdots \\ \cdots & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & \cdots \\ \cdots & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array}$$

Histogram of ~·



With image f_1, h_{g1} will be concentrated near 0. With image f_2, the entries in h_{g2} will be more spread out. The concentration of h_g near 0 is a measure of the "coarseness" of f, or equivalently, the spread of h_g away from 0 is a measure of the "busyness" of f.

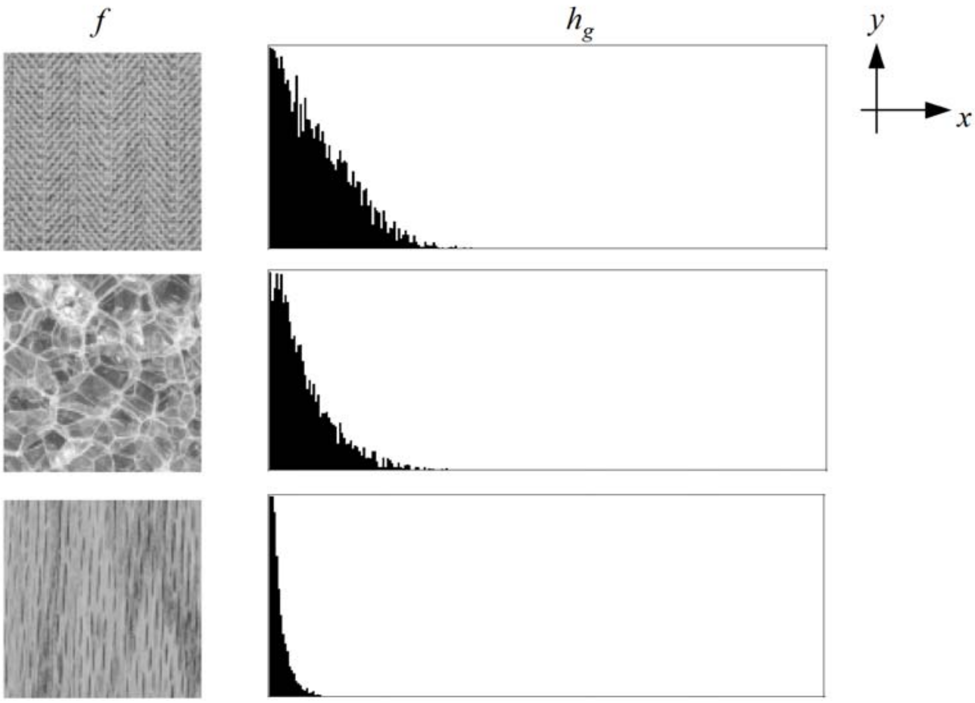
Various statistics can be used to describe h_g, including its mean, second moment, and so on. For example,

h_{g1}: $\sigma^2 = 0.25$
h_{g2}: $\sigma^2 = 0.50$

A wide variety of local properties can be used, e.g., gradient magnitude and Laplacian.

Example

$$g(x,y) = | f(x, y+\Delta y) - f(x,y) | \qquad \begin{bmatrix} +1 \\ -1 \end{bmatrix}$$

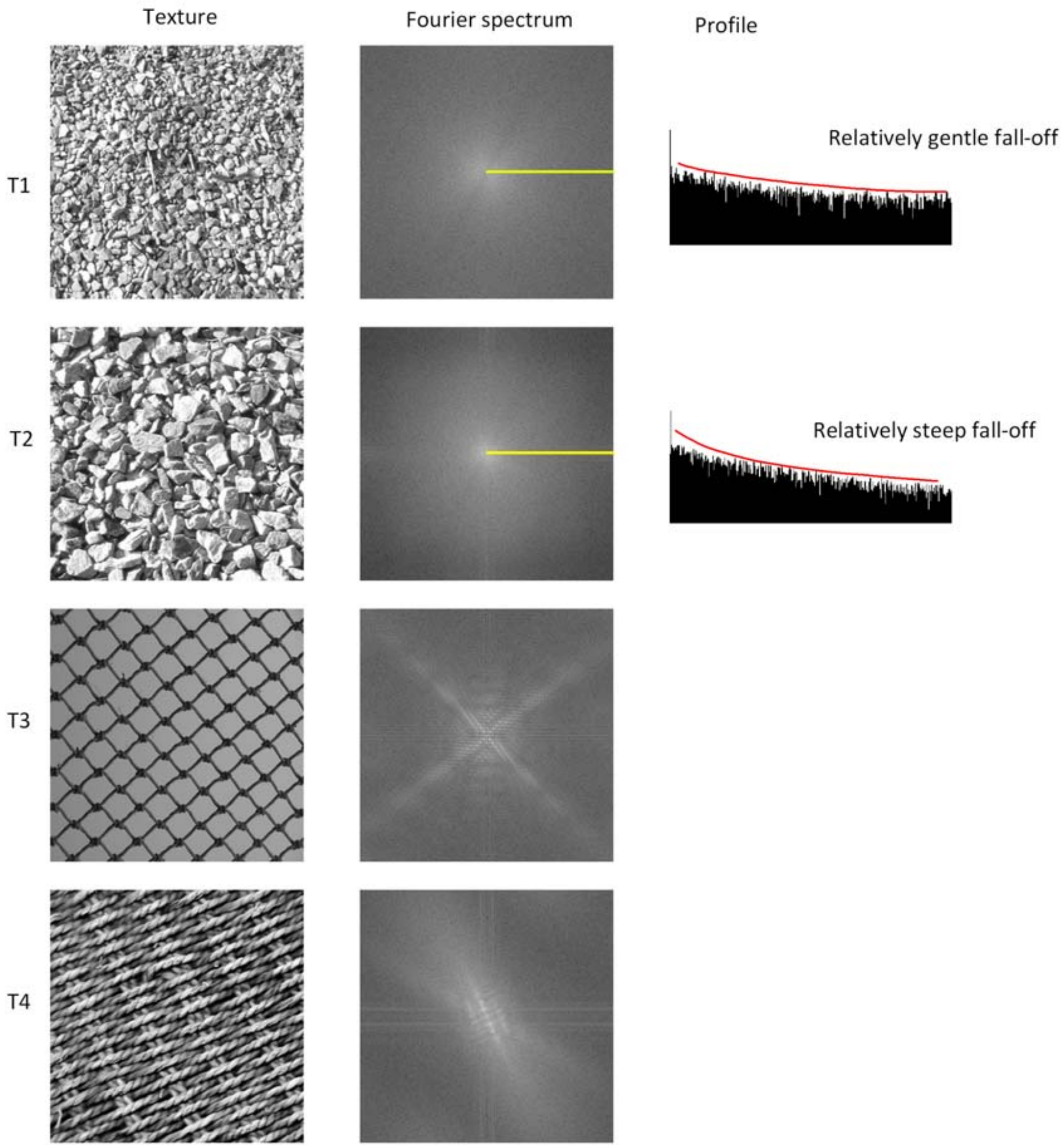


Fourier Spectra

The Fourier spectrum $|F|$ can be used as a texture descriptor.

The rate at which $|F|$ falls off as the spatial frequency (u, v) moves away from $(0, 0)$ is a measure of the coarseness of f. The fall-off is faster for a coarse texture and slower for a busy one, since fine details give rise to more energy at high spatial frequencies.

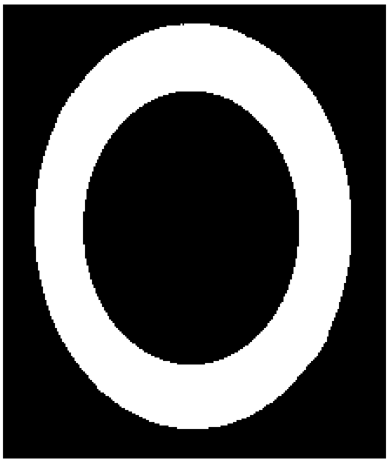
Texture orientation can be seen in the prominent lines present in $|F|$.



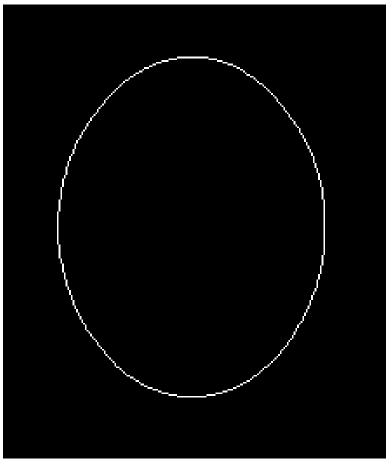
STRUCTURE

In many applications, the objects in a scene can be characterised by structures composed of line or arc patterns. Examples are handwritten and printed characters. This reduction may be accomplished by obtaining the *skeleton* of the region via thinning or skeletonising algorithms.

Original

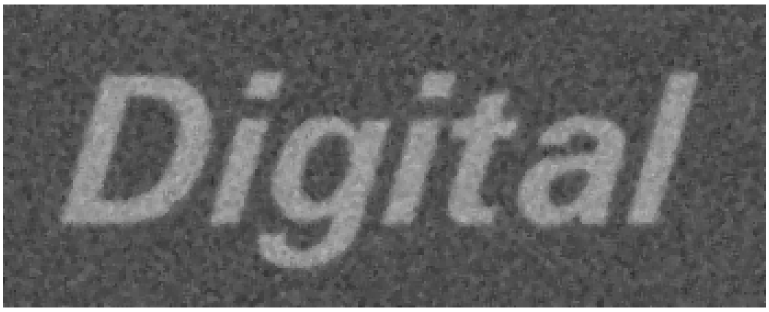


After thresholding



After skeletonisation

Original



After noise filtering

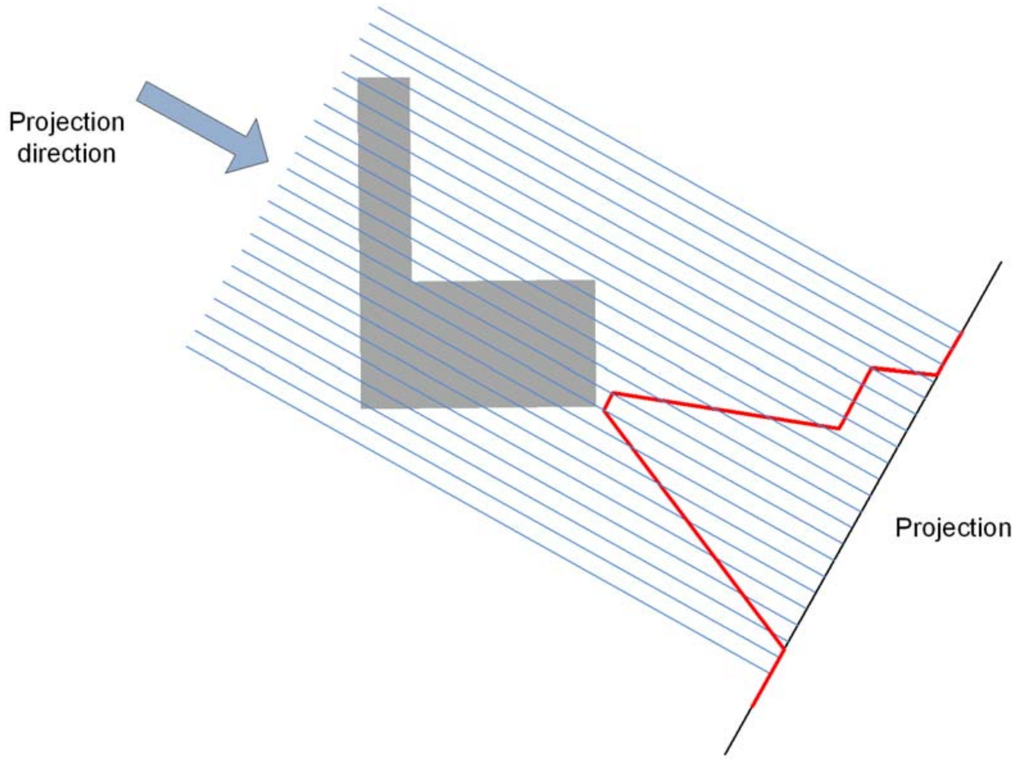
After thresholding

After skeletonisation

PROJECTIONS

The projection of a binary image onto a line may be obtained by partitioning the line into bins and finding the number of 1 pixels that are on lines perpendicular to each bin. Projections can be used as features for recognition of objects.

Projections are compact representations of images, since much useful information is retained in the projection. However, projections are not unique in the sense that more than one image may have the same projections.



Horizontal and vertical projections can easily be obtained by finding the number of 1 pixels for each bin along the horizontal and vertical directions, respectively. The projection $H(j)$ along the rows and the projection $V(i)$ along the columns of a binary image are given by

$$H(j) = \sum_{i=0}^{M-1} f(i,j) \qquad j = 0, 1, \ldots, N-1 \tag{10}$$

$$V(i) = \sum_{j=0}^{N-1} f(i,j) \qquad i = 0, 1, \ldots, M-1 \tag{11}$$

for an image of N rows and M columns.