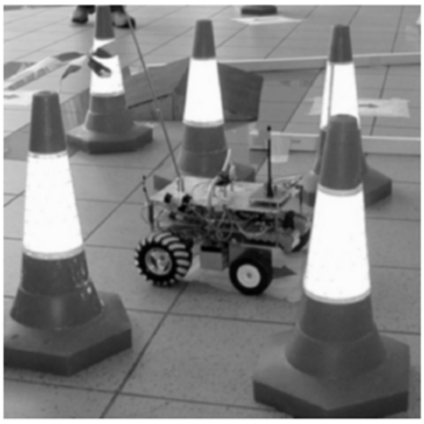
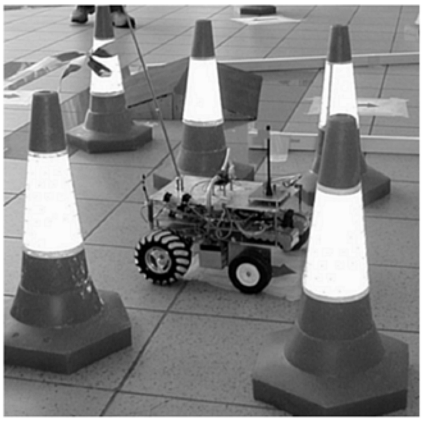
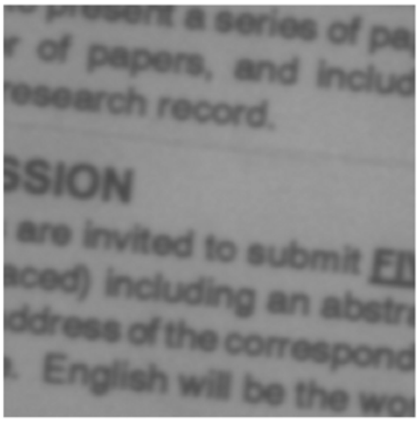
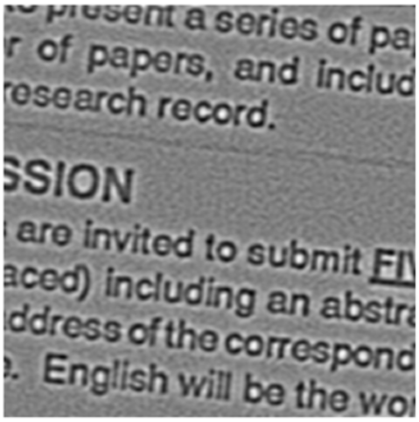
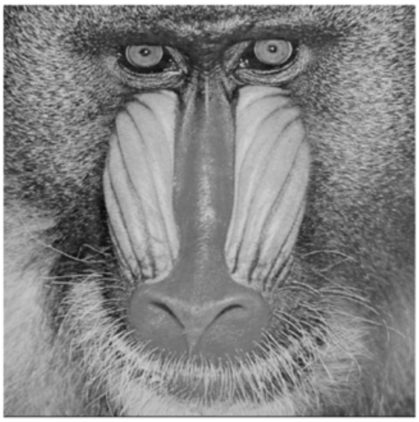
# 5 – IMAGE ENHANCEMENT (B)

## IMAGE SHARPENING

Sharpening techniques are used primarily as enhancement tools to enhance blurred images or to highlight fine details.


A sharpening operation is similar to highpass filtering in that edges, i.e., high-frequency components, are boosted and low-frequency components are suppressed. The ouput image is

$$G(u,v) \;=\; H_{\mathrm{HP}}(u,v) \times F(u,v) \tag{1}$$

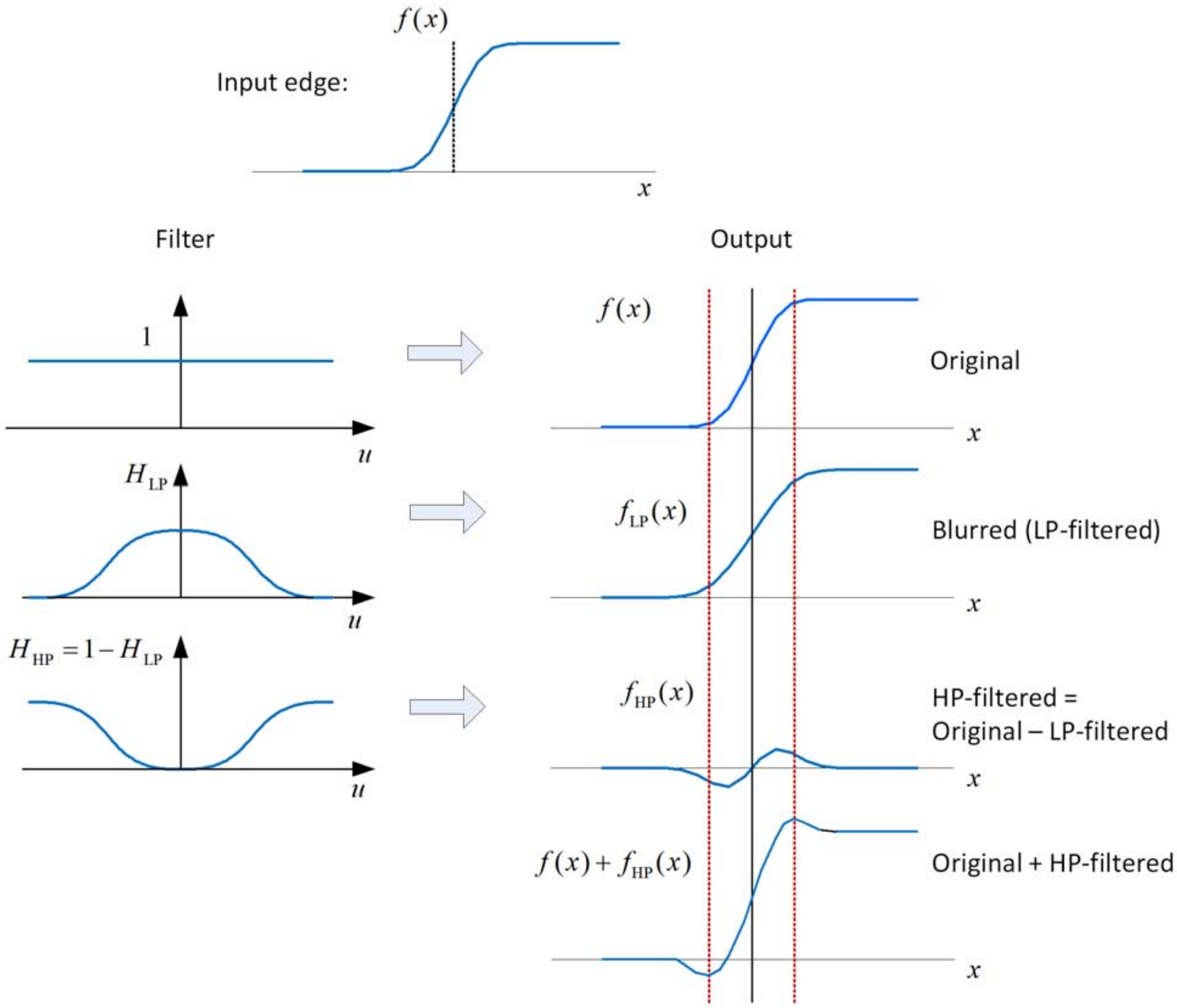
A highpass filter is commonly be obtained from a given lowpass filter:

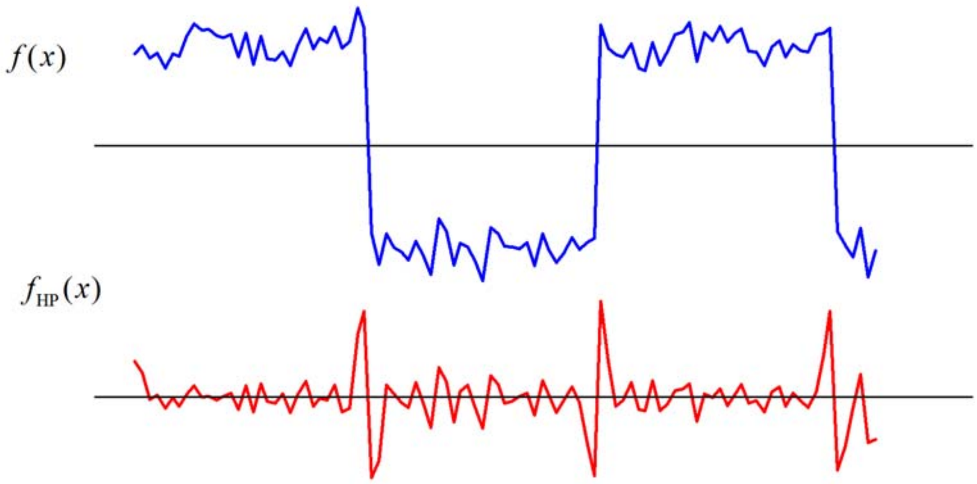
$$H_{\mathrm{HP}}(u,v) \;=\; 1 - H_{\mathrm{LP}}(u,v) \tag{2}$$

Thus, we have

$$G(u,v) \;=\; F(u,v) - H_{\mathrm{LP}}(u,v) \times F(u,v) \tag{3}$$

$$\text{HP-filtered image} \;=\; \text{Original image} - \text{LP-filtered image} \tag{4}$$




$f(x)$
$f_{\mathrm{HP}}(x)$


Input
HP filtered + 128
( 3 x HP filtered ) + 128

A *high-boost* or *high-frequency-emphasis* filter is obtained by adding a multiple of the highpass-filtered signal to the original:

$$\begin{aligned}
\text{High boost} &= \text{Original} + k \times \text{Highpass} \qquad (5)\\
&= \text{Original} + k \times (\text{Original} - \text{Lowpass}) \qquad (6)\\
&= (1+k) \times (\text{Original}) - k \times \text{Lowpass} \qquad (7)
\end{aligned}$$

Eq. (7) may be implemented by spatial filtering with the $3 \times 3$ mask

$$\begin{aligned}
M &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & (1+k) & 0 \\ 0 & 0 & 0 \end{bmatrix} - k \times \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\
&= \frac{k}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & w & -1 \\ -1 & -1 & -1 \end{bmatrix}
\end{aligned}$$

with

$$w = 8 + \frac{9}{k} \qquad (k > 0)$$

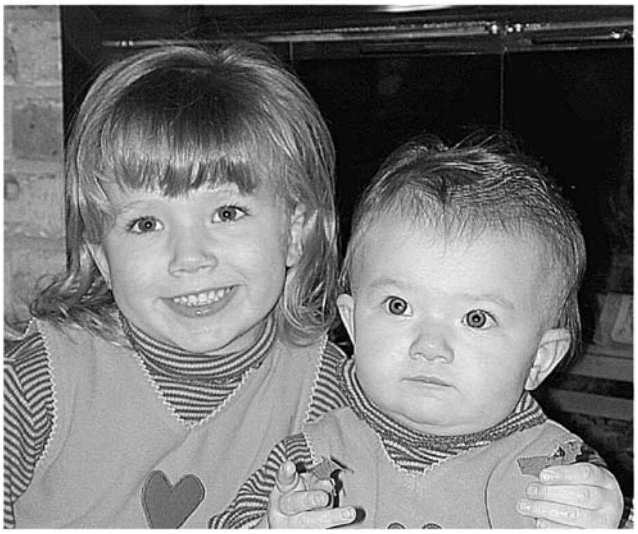
The value of $k$ determines the amount of high-frequency emphasis. For example, if $k = 9$, then

$$M = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

## Example

Original

$k = 5$



$k = 15$

# FILTERING IN THE FREQUENCY DOMAIN

## Lowpass Filtering

Smoothing in achieved in the frequency domain by attenuating a specified range of high-frequency components. We have

$$G(u, v) = H(u, v)F(u, v) \tag{8}$$

The problem is to select a filter function $H(u, v)$ that yields the desired $G(u, v)$. The inverse transform of $G(u, v)$ is then the output image. We consider transfer functions that affect the real and imaginary parts of $F(u, v)$ in exactly the same manner (i.e., $\angle H(u, v) = 0$. Such filters are referred to as zero-phase-shift filters.

### Ideal Filter

A 2-D ideal low-pass filter (ILPF) has the transfer function

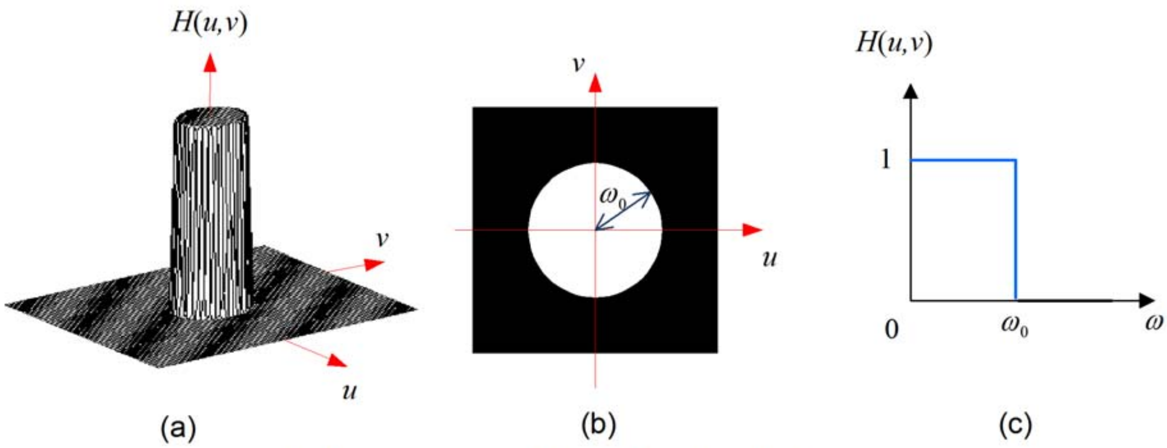
$$H(u, v) = \begin{cases} 1 & \text{if } \omega \leq \omega_0 \\ 0 & \text{if } \omega > \omega_0 \end{cases} \tag{9}$$

where

$$\omega = (u^2 + v^2)^{1/2} \tag{10}$$

and $\omega_0 > 0$.

The filters considered here are radially symmetrical about the origin. For an ideal LPF cross section, the point of transition between $H(u, v) = 1$ and $H(u, v) = 0$ is called the cutoff frequency.



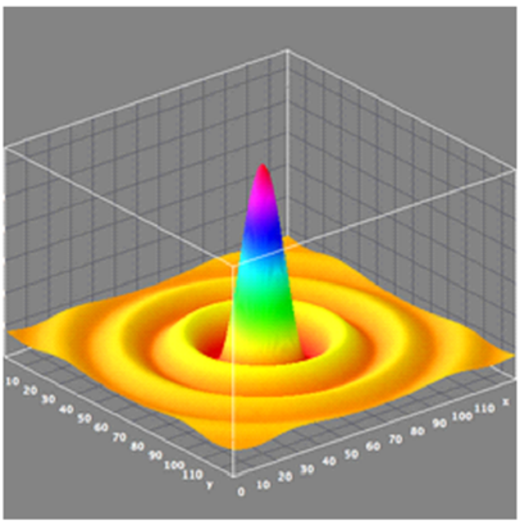

(a) Perspective plot of an ILPF transfer function;
(b) filter displayed as an image; (c) radial cross section.

As in the 1D case, applying the ILPF results in blurring and ringing. This can be explained by first noting that the output image is

$$g(x, y) = h(x, y) \star f(x, y) \tag{11}$$

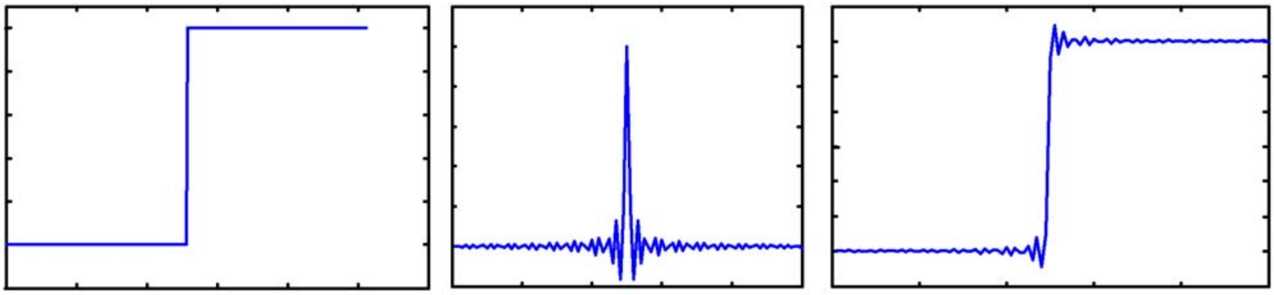
where $h(x, y)$ is the inverse Fourier transform of the filter transfer function $H(u, v)$.

For an ILPF, $h(x, y)$ has the general form of a 2D "jinc" function, which is the circular analogue of the 2D sinc function. Like the sinc function, the jinc function is an oscillatory function; thus convolving the input image with $h(x, y)$ will result in ringing which is more noticeable around prominent bright points or lines.

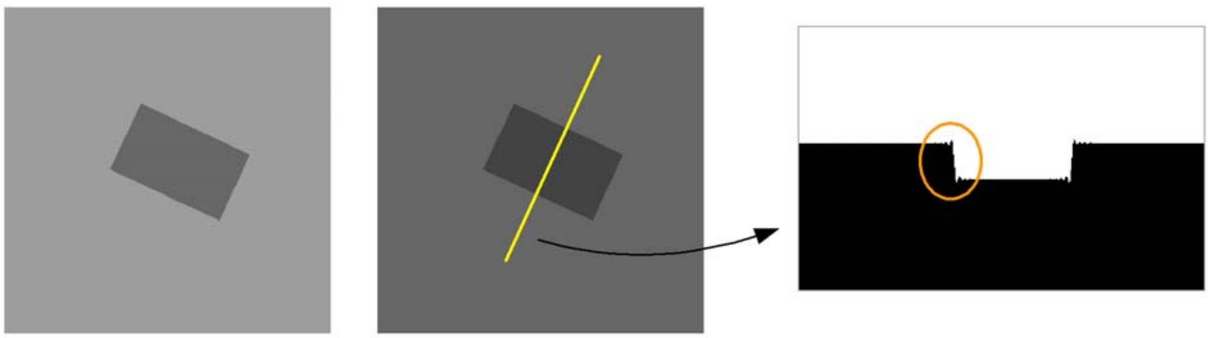


Jinc function

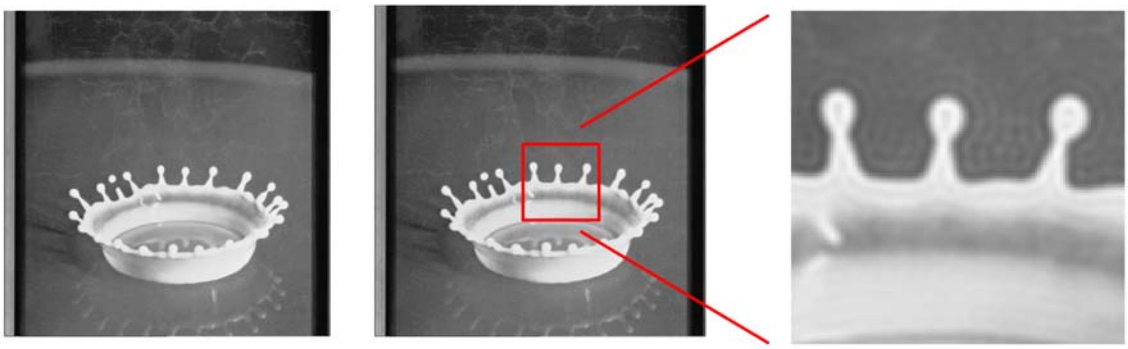
## Examples



$f(x)$ $\quad$ $h(x)$ $\quad$ $g(x) = f(x) * h(x)$



Original image $\quad$ After ideal LPF



Original image $\quad$ After ideal LPF

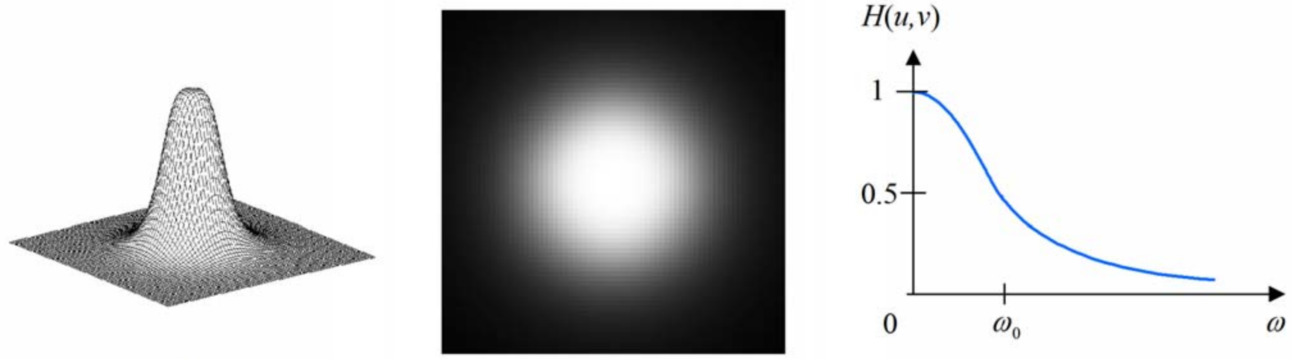
## Butterworth Filter

The transfer function of the Butterworth LPF (BPLF) of order $n$ and with cutoff frequency a distance $D_0$ from the origin is defined by the relation

$$H(\omega) = \frac{1}{1 + (\omega/\omega_0)^{2n}} \tag{12}$$

$H(\omega)$ has value 0.5 when $\omega = \omega_0$.

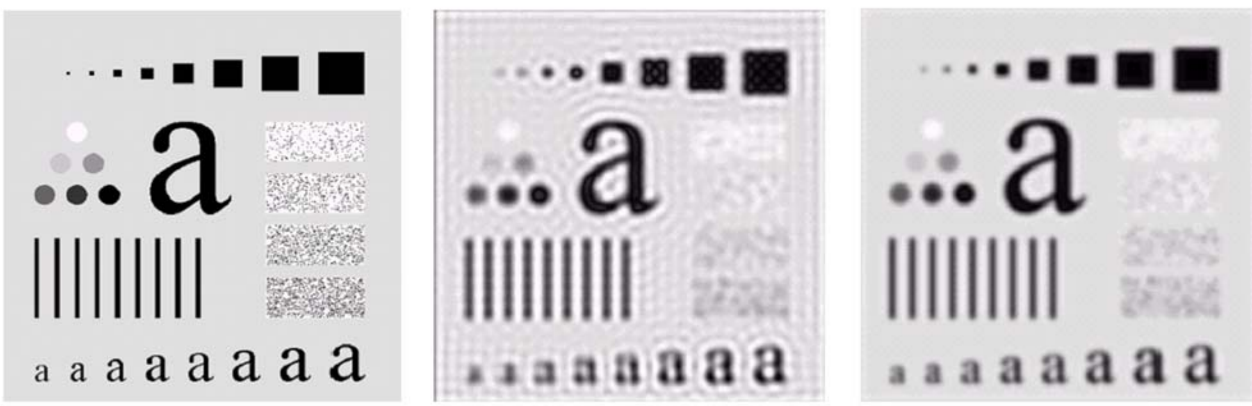
The BLPF does not have a sharp discontinuity that establishes a clear cutoff between passed and filtered frequencies. The filtered image will not exhibit any ringing.




(a) (b) (c)

(a) Perspective plot of a Butterworth LPF transfer function;
(b) filter displayed as an image; (c) radial cross section for n = 1.

### Example - Low-pass filtering



Original | Filtered using ideal LPF | Filtered using Butterworth LPF

## Highpass Filtering

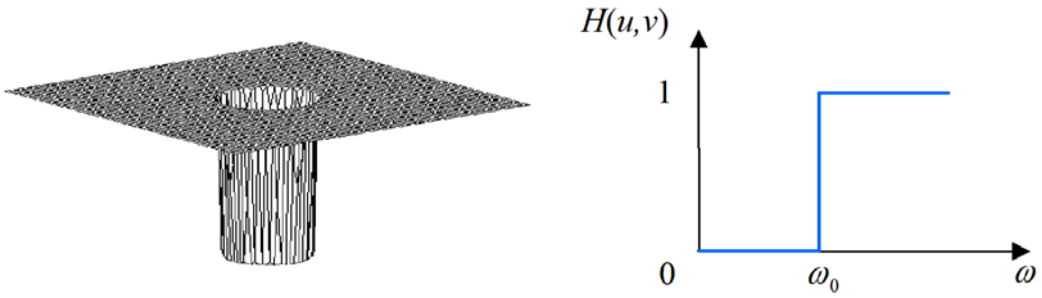
The ideal high-pass filter (IHPF) is defined by the transfer function

$$H(\omega) = \begin{cases} 0 & \text{if } \omega \leq \omega_0 \\ 1 & \text{if } \omega > \omega_0 \end{cases} \tag{13}$$

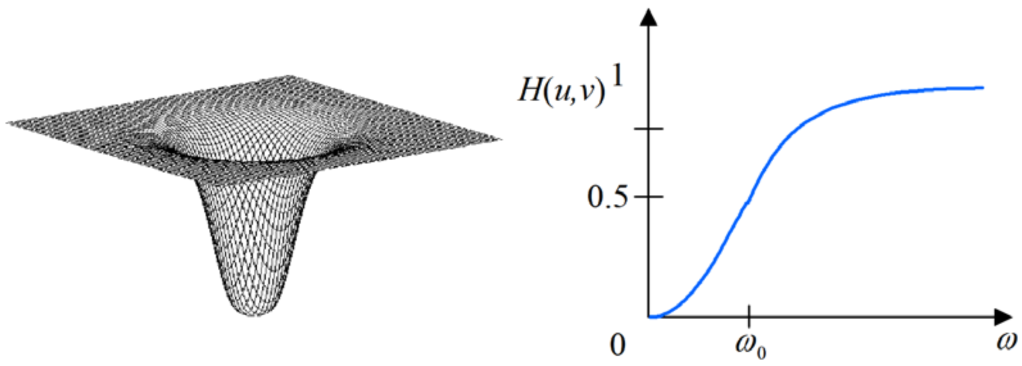
This filter completely attenuates all frequencies inside a circle of radius $\omega_0$ while passing, without attenuation, all frequencies outside the circle.

The transfer function of the Butterworth high-pass filter (BHPF) of order $n$ and cutoff frequency a distance $\omega_0$ from the origin is

$$H(\omega) = \frac{1}{1 + (\omega_0/\omega)^{2n}} \tag{14}$$




Perspective plot and radial cross section of ideal highpass filter.

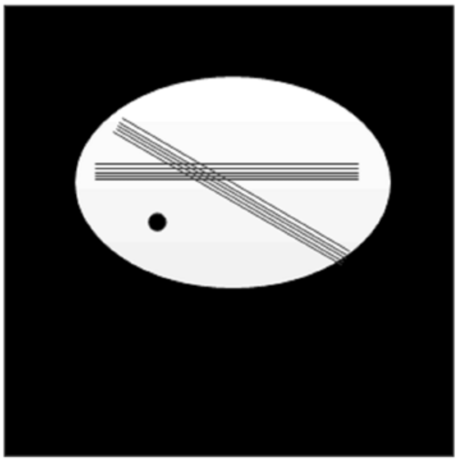



Perspective plot and radial cross section of Butterworth highpass filter for n = 1

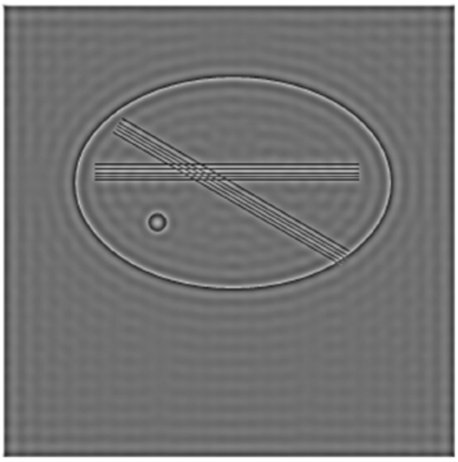
## Example - High-pass filtering

Original

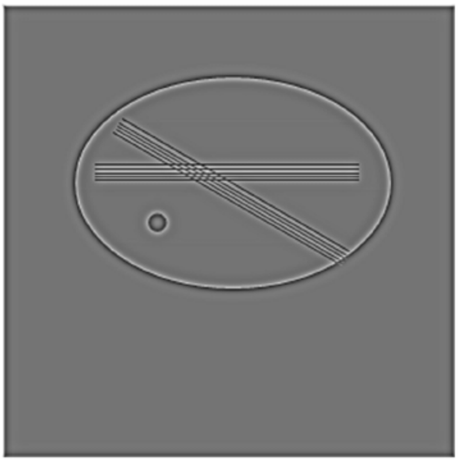
Filtered using ideal HPF



Original



Filtered using ideal HPF



Filtered using Butterworth HPF

# Gaussian Filtering

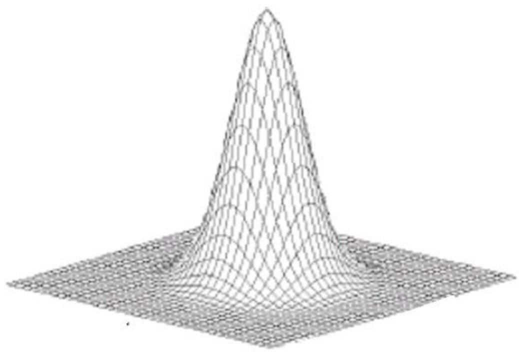
Filters based on the Gaussian function may also be used.

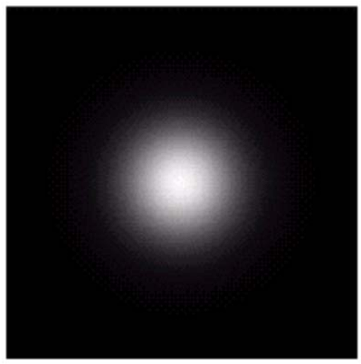
Gaussian low-pass filter:

$$H(u,v) = e^{-\omega^2/2\sigma^2}$$

Gaussian high-pass filter:

$$H(u,v) = 1 - e^{-\omega^2/2\sigma^2}$$





## Other Uses of Filtering

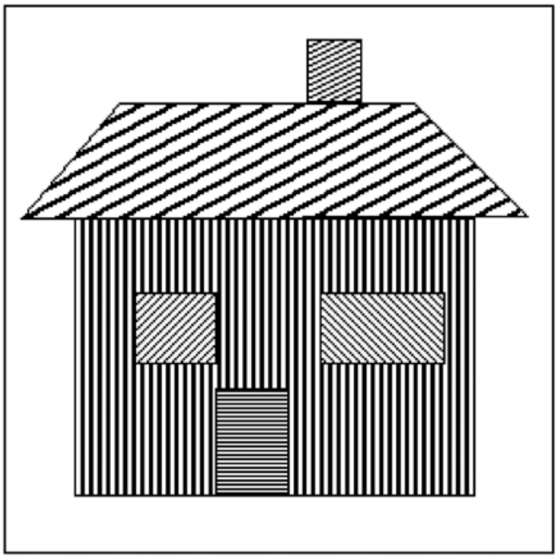
Frequency filters are quite useful when processing parts of an image which can be associated with certain frequencies. For example, in the figure, each part of the house is made of stripes of a different frequency and orientation.

From the Fourier transform, we can see the main peaks in the image corresponding to the periodic patterns in the spatial domain image which now can be accessed separately. For example, we can smooth the vertical stripes (i.e. those components which make up the wall in the spatial image) by multiplying the Fourier image with an appropriate filter (as shown).

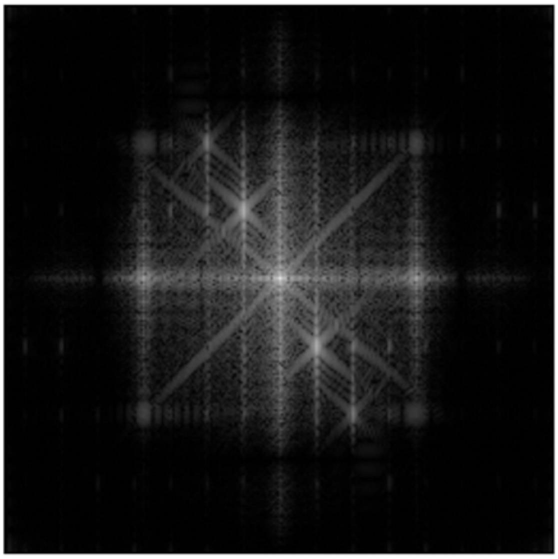
The effect is that all frequencies within the black rectangle are set to zero, the others remain unchanged. Applying the inverse Fourier Transform and normalizing the resulting image gives the result shown.

Although the image shows some regular patterns in the formerly constant background, the vertical stripes are almost totally removed whereas the other patterns remained mostly unchanged.

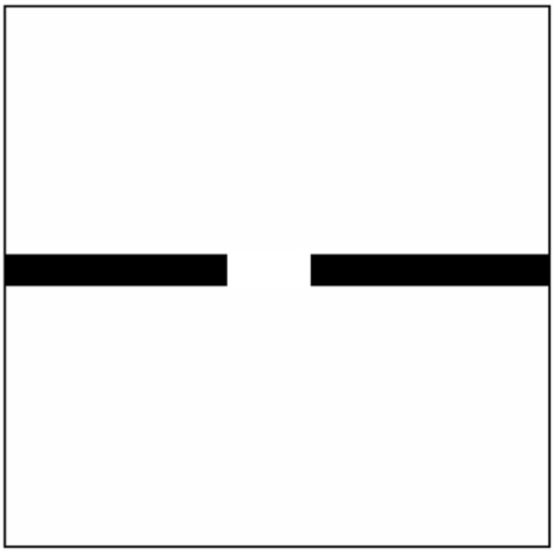
**Example**



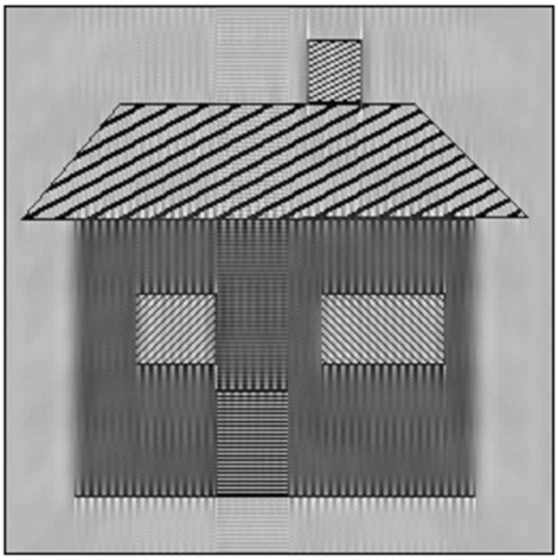
(a) Original



(b) Fourier transform magnitude



(c) Filter function



(d) After filtering (vertical lines have been removed)

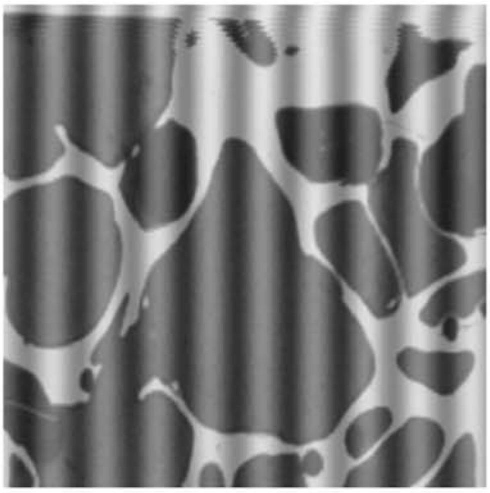
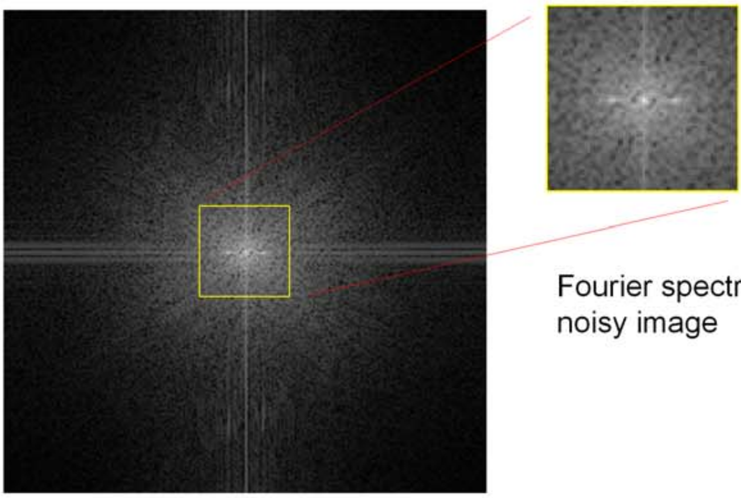


Image contaminated by sinusoidal noise pattern



Fourier spectrum of noisy image

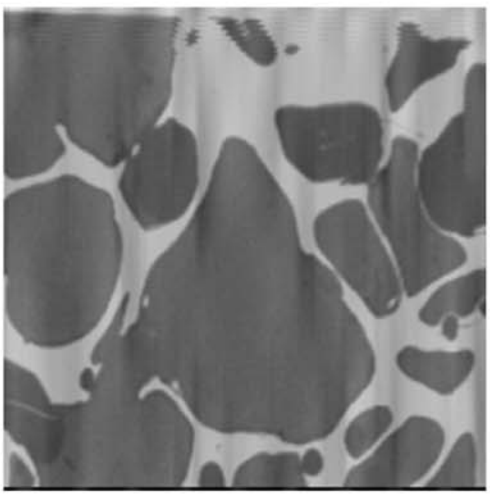


Image after filtering