# ALGORITHMIC STATE MACHINES

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# Overview...

#### **Clocked synchronous state machines**

State machine structures – Mealy and Moore types.

Analysis of state machines.

Algorithmic state machine (ASM) chart notation.

Synthesis of state machines from ASM Chart

Traditional, Multiplexer, One-hot methods.

Practical Design Considerations (clock skew, input signal sync)

Design examples – a few, diverse examples illustrating the top-down digital circuit design process.

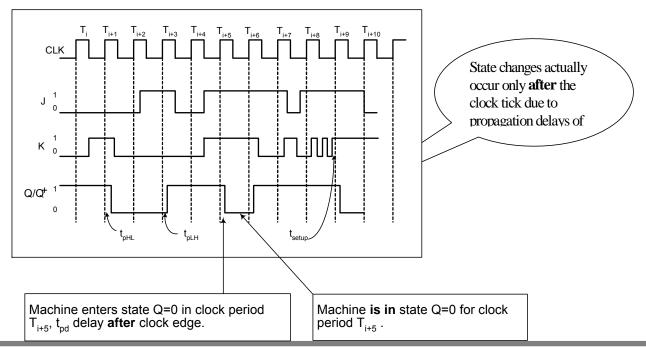
○ Modular design of state machines – example.

# **Clocked synchronous state machines...**

## **Meaning:**

- Clocked synchronous: clock signal is directly connected to all components with clock inputs.
- State machine: a circuit with sequential (& combinational) circuit elements.

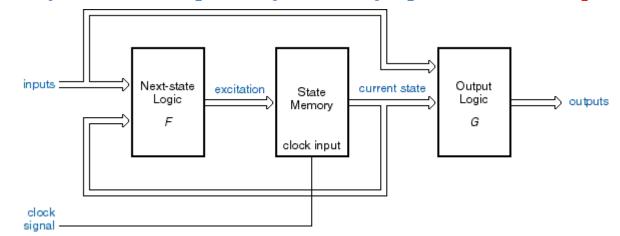
#### State machines change state only at active clock transition.



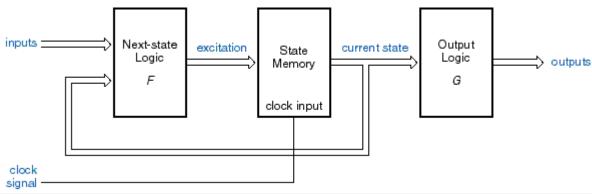
# **State machine structure...**

#### Two types: Mealy and Moore machines.

o Mealy machine: output is a function of a present state & inputs.



o Moore machine: output is a function of a present state only.



# State machine structure... cont'd

#### **State memory**

- o set of *n* FFs store current state of machine; up to  $2^n$  states.
- FFs can be J-K or D, but D FFs preferred as they are simpler (1 input vs. 2 inputs for J-K FFs)

#### Next state logic

o a combinational circuit which decides the next state of the machine based on current state and inputs:

Next state = f (inputs, current state)

# **Output logic for Mealy machines**

o outputs depend on the current state as well as the inputs.

outputs = g (inputs, current state)

Moore & Mealy machines differ only in the output logic; in Moore machines, outputs are functions of current state only.

outputs = g (current state)

# Analysis of state machines... Goals..

Characterize as Mealy or Moore machine

Determine next-state as function of inputs & current state

- o as in design of synchronous counters...
- $\circ$  set of *n* FFs store current state of machine; up to  $2^n$  states.

Determine output as function of current state (Moore) or as function of current state & current inputs (Mealy).

o decode state bits to identify particular states & generate outputs

Express as machine behavior as state/output table or as state diagram.

Describe machine behavior.

# **Algorithmic state machine ASM chart notation**

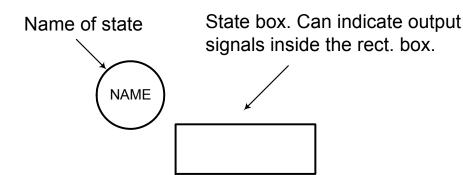
This is notation for describing synchronous state machines.

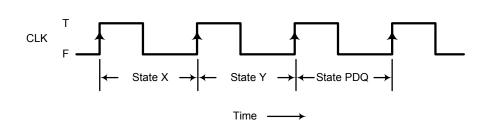
ASM charts bear a superficial resemblance to flow charts:

- o difference is that ASM charts have the concept of a sequence of time intervals built in.
- o flow charts describe only the sequence of events, not their duration.

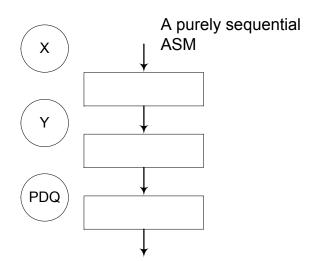
In ASMs, machine changes from present state to next state at the *active clock transition* (ACT) & remains there for duration of clock cycle.

# **ASM.. notations**



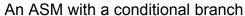


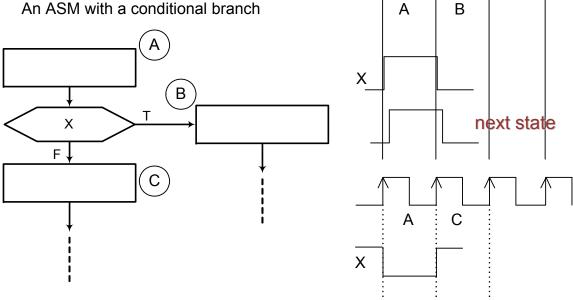
The waveform shows the transition from one state to the next at the clock tick. The ASM notation at right is <u>equivalent</u> to the waveform notation.



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# ASM.. with a conditional branch



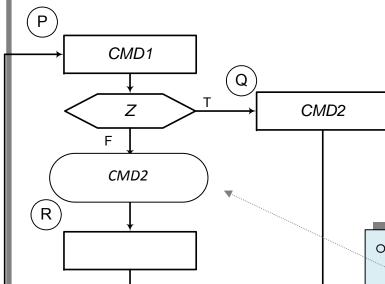


X is an input signal whose value decides what the next state of the machine should be when it is currently in state A.

- Decision to jump to state B or C from state A is made during state A.
- Jump occurs at next ACT.
- Test does NOT require a separate clock period it is done in state A. Hence, decision diamond is part of parent state A.

# ASM.. with conditional (& unconditional) outputs

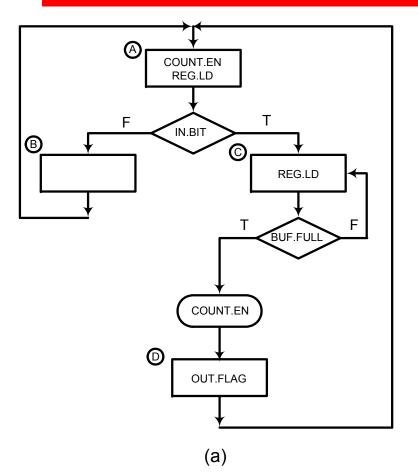
An ASM with a conditional output

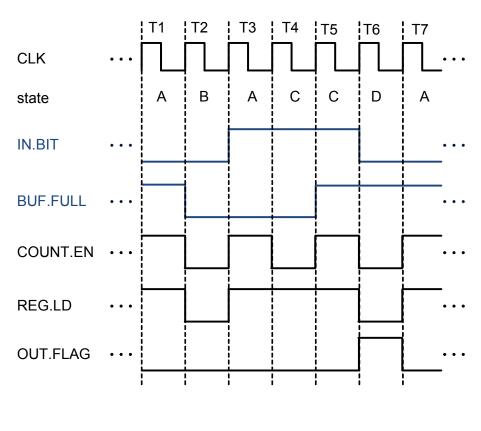


P CMD1 Z CMD2

- A signal written within a state rectangle is 1 whenever the controller is in that state. These are called unconditional outputs.
- o If an output signal also depends on the value of an input signal, it is called a *conditional output*. This is indicated by writing the signal in an oval box.
- In this example, CMD2 is unconditional in state Q, and conditional in state P.

# Analysis of state machines... An example...



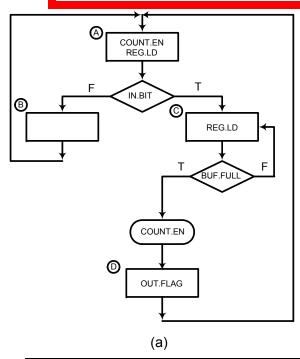


Above **ASM chart** shows how the **state machine** makes state transitions as a function of current states & inputs.

ASM charts, next-state table and timing diagrams are all equivalent descriptions. Each can be inferred from the other.

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# **Analysis of state machines... Next state table...**



	IN.BIT, BUF.FULL			
S	0 0	0 1	11	10
Α	В	В	С	С
В	Α	Α	Α	Α
D	Α	Α	Α	Α
С	С	D	D	С

Next state table is inferred from ASM chart:
 o given a current state A,B,C or D and inputs IN.BIT & BUF.FULL ▶ next state

For realization of state machine, **state code bits** C1 C0 used to represent states:

$$A=\overline{C1C0}$$
  $B=\overline{C1}C0$   $C=\overline{C1}C0$   $D=C1C0$ 

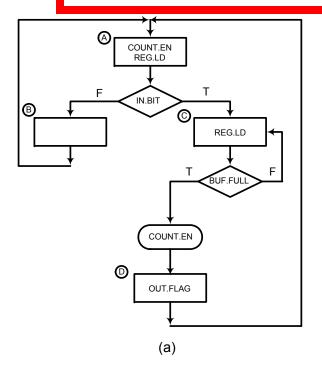
But next tables below do not completely represent the state machine. *Why?* 

	IN.BIT, BUF.FULL			
C1, C0	0 0	0 1	1 <b>1</b>	1 <b>0</b>
0 0	0 1	0 1	10	10
0 1	0 0	0 0	0 0	0 0
11	0 0	0 0	0 0	0 0
1 0	10	11	11	10

Next state table in terms of state names.

Next state table in terms of state code bits...

# State machine structure... System I

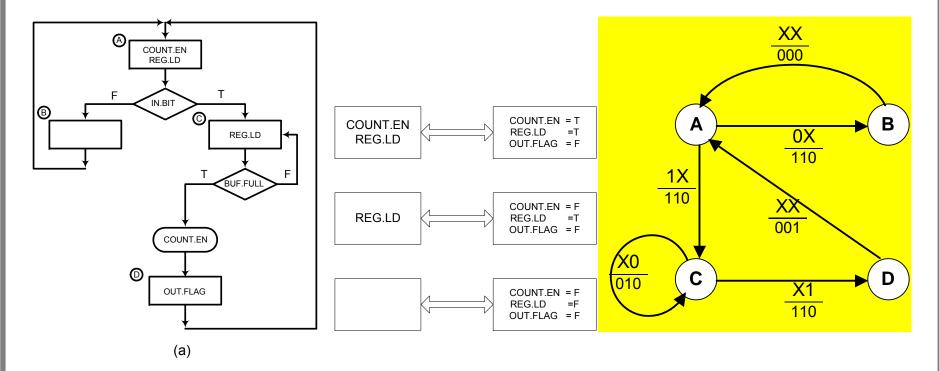


	IN.BIT, BUF.FULL			
S	0 0	0 1	11	1 0
Α	B	B	C	C
	(1,0,1)	(1,0,1)	(1,0,1)	(1,0,1)
В	A	A	A	A
	(0,0,0)	(0,0,0)	(0,0,0)	(0,0, <mark>0</mark> )
D	A	A	A	A
	(0,1,0)	(0,1,0)	(0,1,0)	(0,1, <mark>0</mark> )
С	C	D	D	C
	(1,0,0)	(1,0,1)	(1,0,1)	(1,0,0)

Next state table **and** current outputs (REG.LD, OUT.FLAG, COUNT.EN) in terms of state names.

Next state table above completely describes state machine behavior as in the **Algorithmic State Machine** (ASM) chart which shows state transitions as well as outputs.

# Comparing ASM chart vs. State Transition Graph



Note: ASM chart uses minimal notation – no clutter.

Inputs: IN.BIT, BUF.FULL

Outputs: COUNT.EN, REG.LD, OUT.FLAG

# Synthesis of state machines from ASM Chart

Previously, we saw how state machine operation could be inferred from circuit diagram.

Now let's consider the reverse: given an ASM chart or next state/output table or state transition graph: how to realize the state machine circuit (synthesis).

The first task is to represent controller states numerically. This is called state assignment.

Previous ASM chart had states A,B,C,D; each of these states must be assigned a code number to be used in a digital circuit. For example (there are 4 states, so need two bits to represent them):

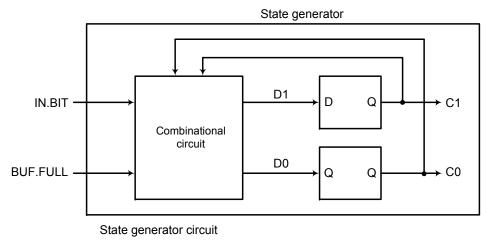
State Name	State number	State Bits C1 C0
Α	0	0 0
В	1	0 1
С	2	1 0
D	3	1 1

# Synthesis from ASM Chart ... continued

Thus, controller states can be stored in FFs and in this example, 2 FFs (D, J-K, etc.) are needed.

Next, given an ASM chart for which state assignment has been done, we must first realize a state generator circuit which will be responsible for *moving* the machine from one state to another depending on inputs & the current state. (Note: this is similar to designing synchronous counters).

For the example ASM chart, the functional block diagram of the state generator circuit:



# Synthesis from ASM Chart

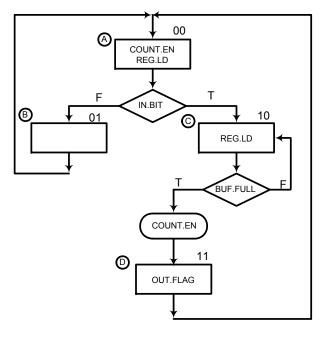
This basically involves the design of the *combinational circuit* which will use the inputs & *current state* of machine to move it into required *next state* (next state logic). (like designing synchronous counters).

Following this, the output logic circuit is (easily) realized and there are several methods to realize the next state logic:

- Traditional method (with SSI chips)
- o PLA/PAL based method (LSI)
- Multiplexer based method (MSI)
- One-hot method.

Depending on problem, one of these methods can be chosen.

# ASM Synthesis...: 1. Traditional method



Example ASM chart with state assignment

Equations for outputs:

REG.LD = A + C

OUT.FLAG = D

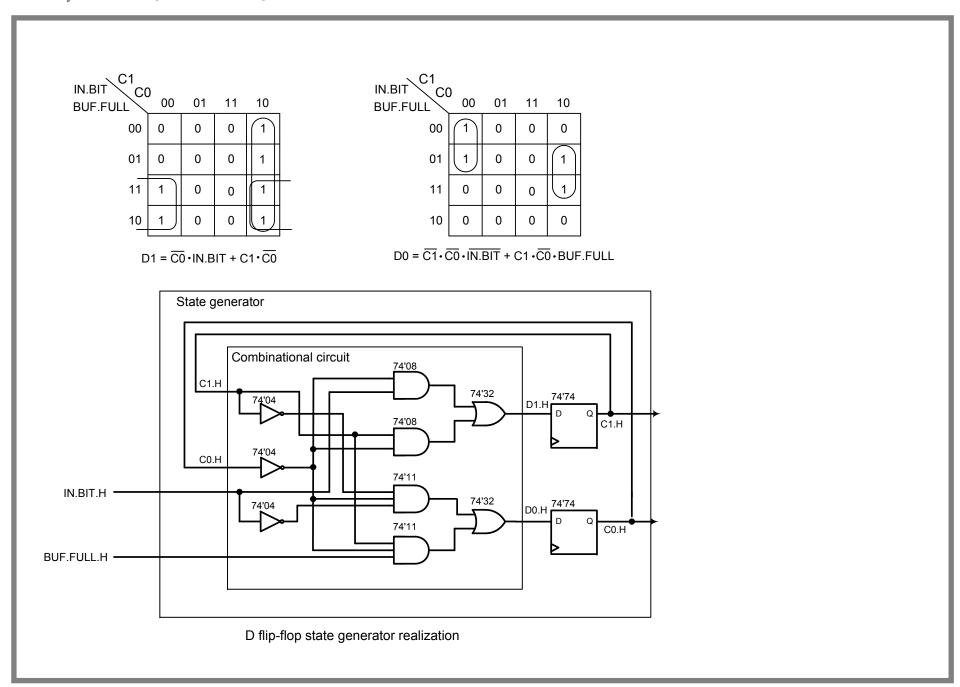
COUNT.EN = A + C BUF.FUL

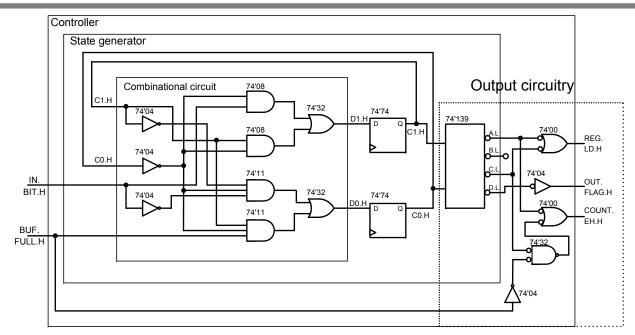
	IN.BIT, BUF.FULL			
C1, C0	0 0	0 1	1 <b>1</b>	1 <b>0</b>
0 0	0 1	0 1	1 0	10
0 1	0 0	0 0	0 0	0 0
11	0 0	0 0	0 0	0 0
1 0	10	11	11	10

Next state table for C1+, C0+ in terms of state bits.

Since D-FFs are used, D1= C1+ and D2=C0+.

The **Karnaugh maps** for D1 & D2 follow from the above *next state* table.





Controller realization with D flip-flops for the ASM chart

#### ASM Synthesis...: 1. Traditional method : Some points to note!

- In traditional method, either J-K or D flip-flops can be used.
  - o **J-K FFs** lead to somewhat more compact designs because J-K FFs are more flexible than D's but more design work needed to find the logic for 2 inputs.
- No obvious correspondence between circuit and ASM chart.
- Minor changes in algorithm require a fresh design of next-state combinational circuit.
- o Lacks clarity of design.

# ASM Synthesis...: 2. PAL/PLA based method

Derive next-state table for state generator (as before).

#### Determine input equations of FFs composed of minterms

from truth table, D1 =  $\overline{C1} \cdot \overline{C0} \cdot \underline{IN} \cdot \underline{BUF} \cdot \overline{FULL} + \overline{C1} \cdot \overline{C0} \cdot \underline{IN} \cdot \underline{BIT} \cdot \underline{BUF} \cdot \underline{FULL}$ 

 $+ C1 \cdot \overline{C0} \cdot \overline{IN.BIT} \cdot \overline{BUF.FULL} + C1 \cdot \overline{C0} \cdot \overline{IN.BIT} \cdot BUF.FULL \\ + C1 \cdot \overline{C0} \cdot \overline{IN.BIT} \cdot \overline{BUF.FULL} + C1 \cdot \overline{C0} \cdot \overline{IN.BIT} \cdot BUF.FULL$ 

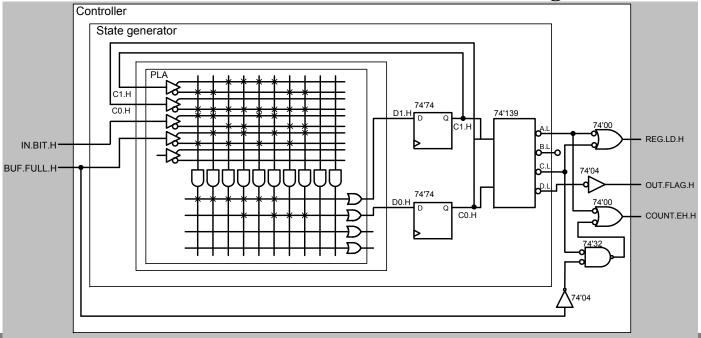
 $\mathbf{D0} = \overline{C1} \cdot \overline{C0} \cdot \overline{\mathbf{IN.BIT}} \cdot \overline{\mathbf{BUF.FULL}} + \overline{C1} \cdot \overline{\mathbf{C0}} \cdot \overline{\mathbf{IN.BIT}} \cdot \overline{\mathbf{BUF.FULL}}$ 

+  $C1 \cdot \overline{C0} \cdot \overline{IN.BIT} \cdot BUF.FULL$  +  $C1 \cdot \overline{C0} \cdot \overline{IN.BIT} \cdot BUF.FULL$ 

## **,**

	IN.BIT, BUF.FULL			
C1, C0	00	0 1	11	10
0 0	0 1	0 1	10	10
0 1	0 0	0 0	0 0	0 0
11	0 0	0 0	0 0	0 0
10	10	11	1 <b>1</b>	10

#### This leads to 2-level AND/OR circuit realization using PLA/PAL.



Controller with PLA state generator realization for the ASM chart

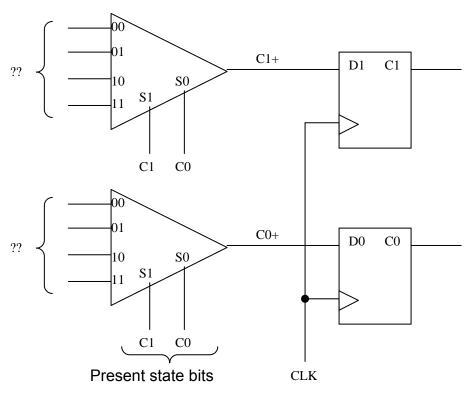
# ASM Synthesis...: 2. PAL/PLA based method

#### Some points to note

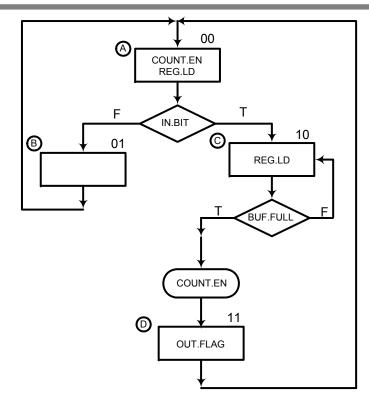
- Obvious advantage of this method over traditional method is replacement of several IC's with one.
- o Dramatic reduction in package count for complex controllers.
- Method becomes even more attractive when PLD contains on-board FFs possible to realize entire controller with one chip.
- With PLD devices, it makes sense to use D FFs instead of J-K FFs need to provide logic for only one D input rather 2 for J-K FFs.

# ASM Synthesis...: 3. Multiplexer based method

Here the combinational circuit required in the state generator is realized using multiplexers (to "look up" the next state).



- One MUX is used to compute the next state for each flip-flop.
- Each MUX must be wide enough to have an input for each ASM state.
- The present states of the FFs provide the selection control inputs to the MUX.
- So now the problem can be looked at as follows: given the present state, what must be the signal at the corresponding MUX input to drive the machine to the correct state at the next ACT.
- o This can be done by inspection!
- Consider the inputs of one MUX at a time,
   e.g. first the MUX who's output is C1+, and
   then the MUX with output C0+.



Example ASM chart with state assignment

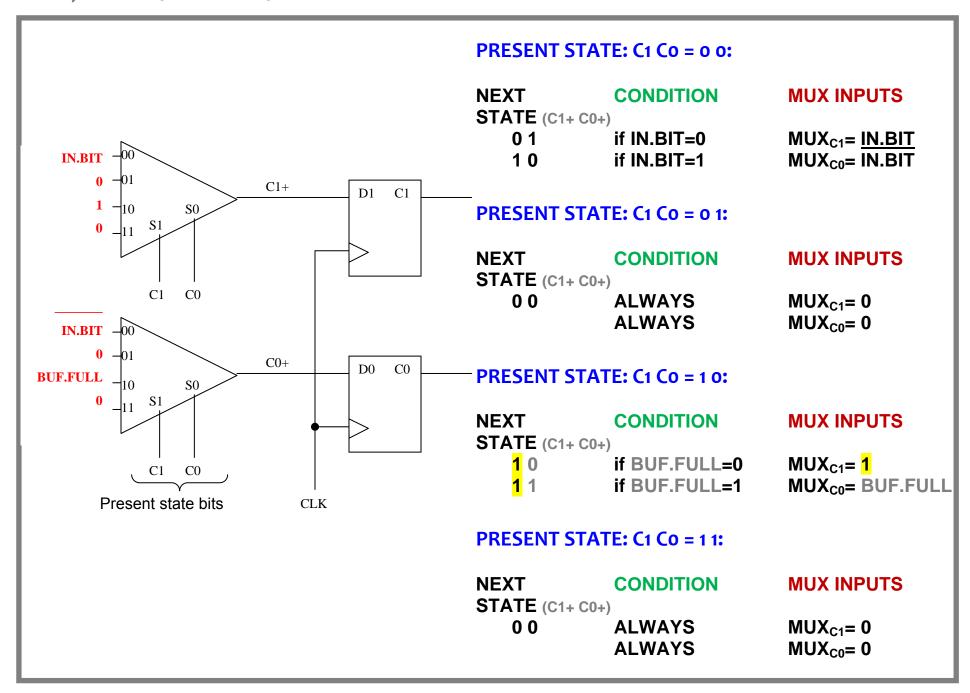
#### PRESENT STATE: C1 C0 = 0 0:

#### PRESENT STATE: C1 C0 = 0 1:

#### PRESENT STATE: C1 C0 = 1 0:

#### PRESENT STATE: C1 C0 = 11:

NEXT CONDITION MUX INPUTS STATE (C1+ C0+)  $0 \ 0 \ ALWAYS \ ALWAYS \ MUX_{c0} = 0$ 



#### ASM Synthesis...: 3.MUX method : Some points to note

- ASM controller outputs formed as before.
- Since MUX inputs obtained directly from ASM charts, greater clarity.
- Size of MUX increases linearly with number of states.

No. of states	No. of MUXs	MUX input size
5-8	3	8
9-16	4	16
17-32	5	32

Method attractive up to about 16 states, unwieldy after that. For greater than 16 states, the one-hot method may be better.

#### Dealing with unused states: minimum cost vs minimum risk.

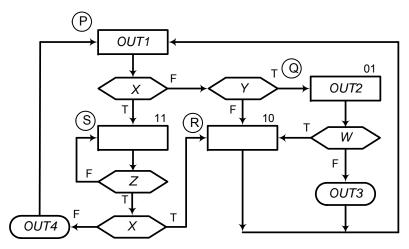
- o Minimum cost assumes that a machine will never enter an unused state.
  - o Then the next state in the next state table is marked as don't care. This can simplify excitation logic and hence, cost.
  - But what happens if machine does get into an unused state, e.g. during power up or due to noise? Behaviour could be weird.
- Minimum risk designs explicitly guard against this:
  - o If machine ever goes into an unused state, must provide a way to get back to the main algorithm, e.g. choose to go to a particular state (*arbitrarily can be chosen*).

# ASM Synthesis...: 4. One Hot method

One FF used per state  $\rightarrow$  no state assignments are necessary.

The design is such that only one state FF is 1 during each clock period – hence the name "one hot".





#### Equations for FF inputs from the table:

next state P: 
$$P(D) = QW + R + SZX$$

next state Q: Q(D) = PXY

next state R: R(D) = PXY + QW + SXZ

next state S: S(D) = PX + SZ

#### STATE TRANSITION DATA FOR A ONE-HOT IMPLEMENTATION OF THE ASM

Next state	Present state	Condition for transition
Р	Q	$\overline{W}$
	R	T
	S	$Z^{\bullet}\overline{X}$
Q	Р	<b></b> ₹• Y
R	Р	$\overline{X} \cdot \overline{Y}$
	Q	W
	S	X•Z
S	Р	X
	S	Z

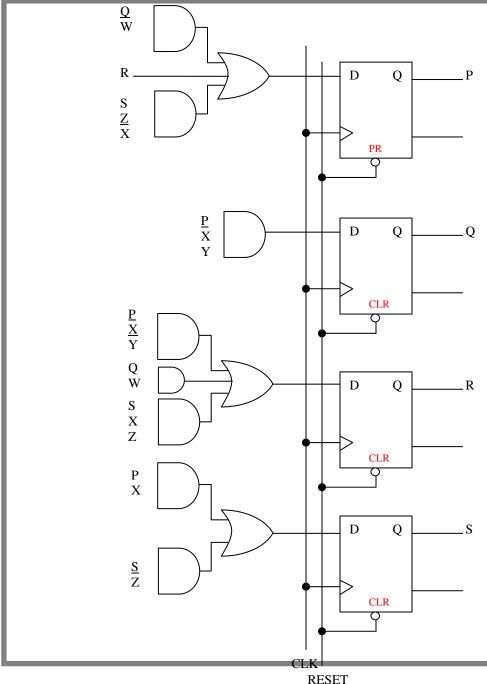
#### **Controller Outputs:**

OUT1 = P

OUT2 = Q

OUT3 = QW

OUT4 = SZX



- Generation of controller outputs, not shown here (but which must be done to complete the design), is simple.
- The **one-hot** controller must be initialized properly when powered up, i.e., exactly one FF representing the state is 1, and all others are 0.
- Once properly initialized, controller will sequence through all states correctly.
- Initialization can be done by using a master reset (asynchronous) signal on power up.
- Use of this idea is shown in previous diagram, where it is assumed that P is the starting state. (Note clever use of mixed logic notation).

#### 4. One-Hot: Some points to note

- o Design is easy, and circuits easily relate to ASM chart.
- Size of state generator circuit grows linearly with number of states (not bad).
- Since many FFs are available on one chip. package count compares favorably with other methods.

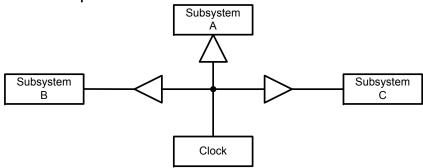
# ASM Synthesis. Practical Design Considerations

#### Assumptions made about the systems:

- (a) State machines are synchronous, actions governed by a master clock.
- (b) At time of state change, all inputs to ASM are stable, i.e. inputs change sync. with system clock.
- (c) Clock edge reaches each circuit element simultaneously.

Violation of (a) or (b) will lead to serious problems; they are the basis of synchronous design. (c) Can be enforced by good construction practices.

- ◆ Do **NOT** gate the clock!
- ♦ Use either all +ve edge triggered FFs or all –ve edge triggered FFS.
- ♦ Beware different lengths of clock paths. Use:



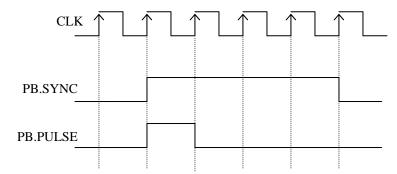
A buffered radial distribution system for the master clock

- Asynchronous inputs cause problems, and must be avoided. A common example of an asynchronous input is operator controlled push button switches these are not synchronized to the clock.
- Two kinds of problems can occur with asynchronous signals, transition races, and output races.

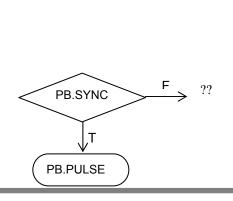
# Design example1 : Single pulser circuit.

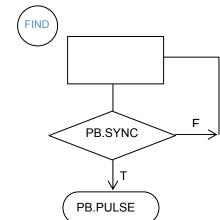
Design a circuit using a debounced push button which produces a clock synchronized signal PB.SYNC (i.e., satisfies set-up & hold times) and operates as follows:

- (a) Asserts an output PB.PULSE for one clock period when debounced push button switch is pressed.
- (b) Disallows additional assertions of output until PB is released.



• Translate understanding of problem into an ASM chart, next state/output table, etc.





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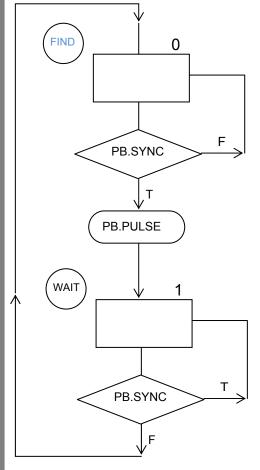
# Design example1: Single pulser circuit.

Final ASM chart with state assignments shown.

State variable: A

A=1: WAIT



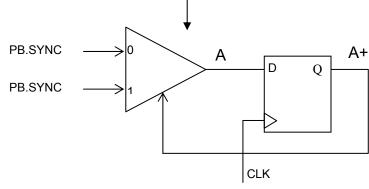


#### Output:

PB.PULSE = FIND . PB.SYNC=  $\overline{A} . PB.SYNC$ 

#### Realization:

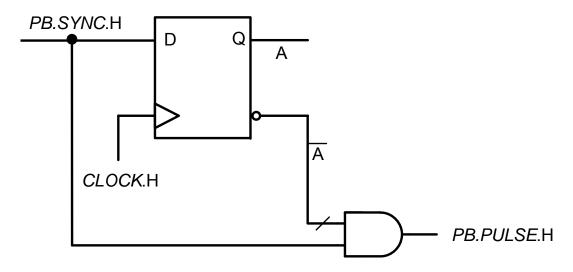
MUX method seems like a reasonable way – need one D FF and one 2-input MUX.



→ No need for MUX!

# Design example1 : Single pulser circuit.

Complete single pulser circuit with synchronizing D FF:

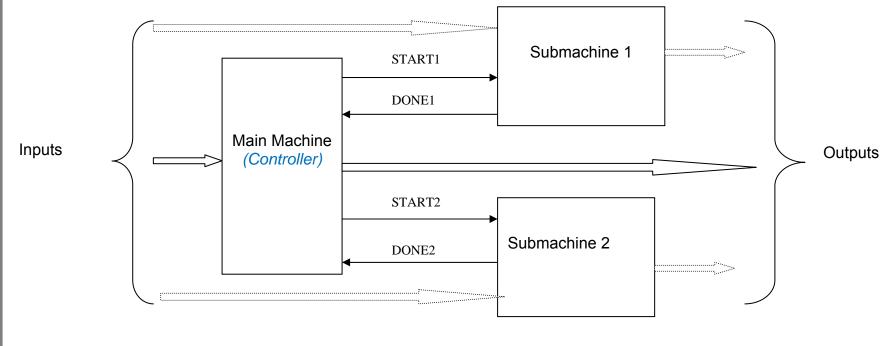


A single-pulser circuit

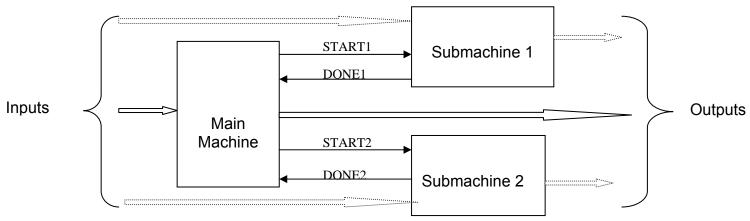
# ASM Synthesis: Modular design of state machines

# Modular design of state machines

- Large state machines are difficult to conceptualize, design and debug.
- Hence, useful to solve problem with a collection of smaller state machines.



# ASM Synthesis: Modular design of state machines



- Main machine executes a main algorithm to control the submachines & get the job done. Sends command signals to submachines and gets feedback signals from submachines.
- **Submachines** respond to external inputs as well as commands from **main machine**. Can give outputs as well as feedback to the **main machine**.
- Common examples of submachines are counters, shift registers, etc.
- Sometimes the main machine is called the controller and the submachines are called controlled circuit elements or architectural elements.
- The trick here is to do the modularization appropriately, and pick natural components for the submachines. This can greatly simplify design problems.

# Modeling

State Machines

Using UADL

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# State machines modeling using VADL

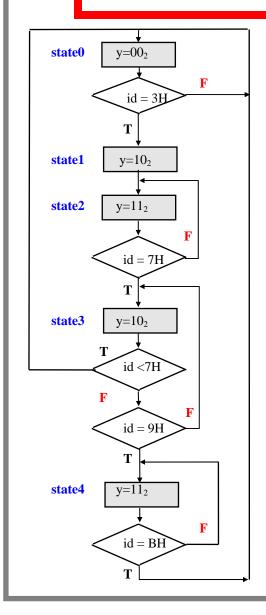
• State Machine triggered at rising or falling edge by:

```
clk'event and clk='l' -- rising edge triggered
clk'event and clk='0' -- falling edge triggered
OR
rising edge(clk) -- function
```

- Modeled using case statement in a process.
- Branches of case contain behavior for each state.
- State information is stored in a signal.
- Conditional check implemented using if statement

Z F Unconditional output

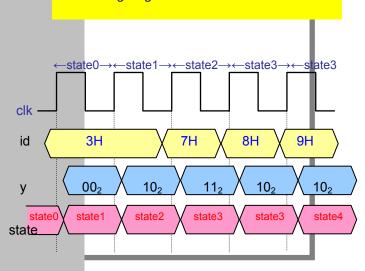
# State Machine m\_1 based on ASM chart

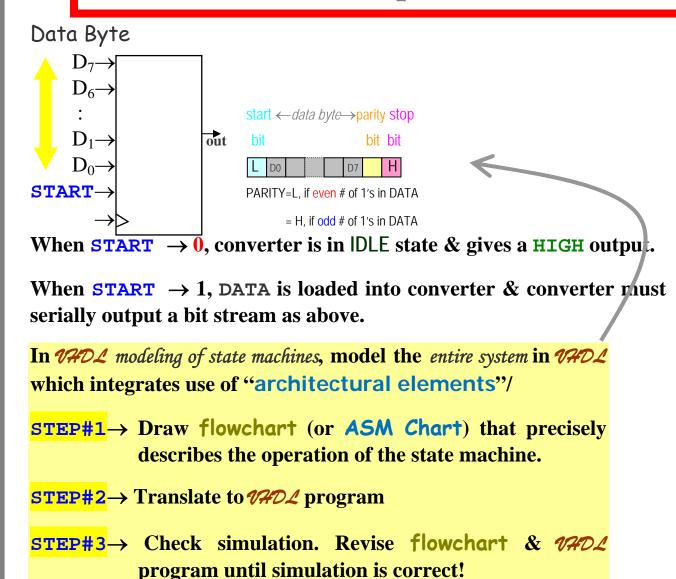


```
entity state m 1 is
          port(clk, rst:in std logic; id: in std logic vector(3 downto 0);
                  v: out std logic vector(1 downto O));
end state m 1:
architecture archstate m 1 of state m 1 is
        type states is (state0, state1, state2, state3, state4);
        signal state: states;
begin
        process (clk, rst)
         beain
         if rst='1' then state <= state0; -- asynchronous reset</pre>
                        v <= "00":
         elsif (clk'event and clk='l') then
                case state is
                 when state0 =>
                        v <= "00":
                        if id = x"3" then state <= state1:
                        end if:
                 when state1 =>
                        v <= "10";
                        state <= state2:
                 when state2 =>
                        v <= "11":
                        if id = x"7" then state <= state3:
                        end if:
                 when state3 =>
                        v <= "10";
                        if id < x"7" then state <= state0:
                        elsif id = x"9" then
                                 state <= state4:
                        end if:
                 when state4 =>
                        v <= "11":
                        if id = x"b" then state <= state0:
                        end if:
                end case:
           end if:
          end process;
```

end archstate m 1;

- states is an enumerated type
- state machine is described in a process
- outputs are specified for *every* state and *next state* determined in *parallel*
- CASE contains all of state transitions
- example demonstrates algorithmic & intuitive fashion that WDL permits in the description of state machines.
- A timing diagram is shown below:





# Data Byte $\begin{array}{c} D_7 \\ D_6 \\ \vdots \\ D_1 \\ D_0 \\ \end{array}$ $\begin{array}{c} \text{start} \leftarrow \textit{data byte} \rightarrow \text{parity stop} \\ \text{bit} \\ D_0 \\ \end{array}$ $\begin{array}{c} \text{PARITY=L, if even \# of 1's in DATA} \\ \end{array}$ $= \text{H. if odd \# of 1's in DATA} \\ \end{array}$

When START  $\rightarrow 0$ , converter is in IDLE state & gives a HIGH output.

When START  $\rightarrow$  1, DATA is loaded into converter & converter must serially output a bit stream as above starting with START-BIT.

In VADL modeling of state machines, model the entire system in VADL
which integrates use of "architectural elements"

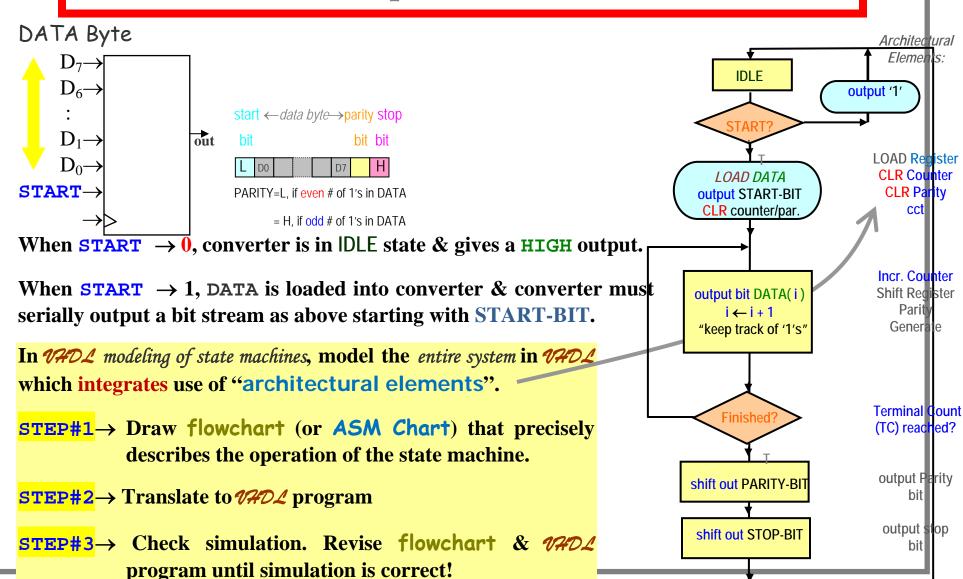
STEP#1→ Draw flowchart (or ASM Chart) that precisely
describes the operation of the state machine.

STEP#2→ Translate to VADL program

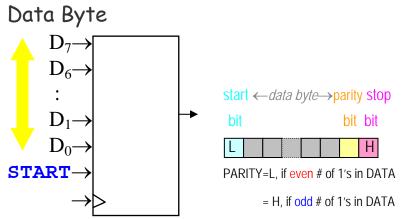
STEP#3→ Check simulation. Revise flowchart & VADL
program until simulation is correct!

#### **Architectural elements:**

- 1. **Shift Register** to load & shift the input data bits
- 2. **Counter** to keep track of number of bits being *shifted*.
- 3. Generate Parity.



DIGITAL DESIGN



When START  $\rightarrow$  0, converter is in an IDLE state & gives a HIGH output

When START  $\rightarrow$  1, DATA is loaded into converter & converter must serially output a bit stream as above.

```
databyte: 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0

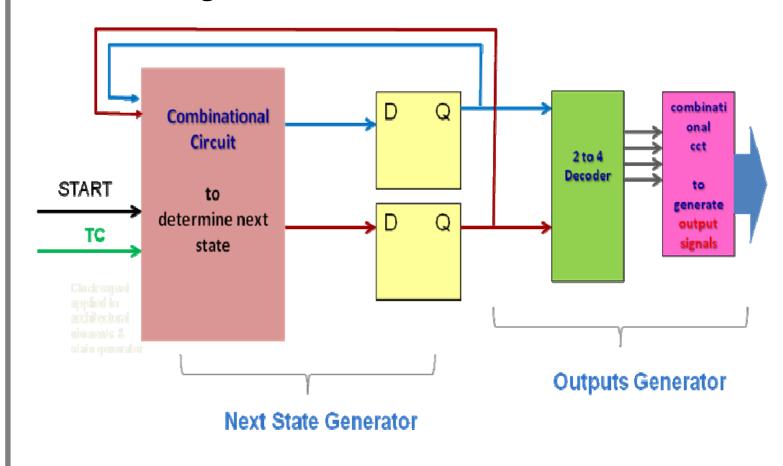
par 0 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0

databyte: 0 \rightarrow 1 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 0

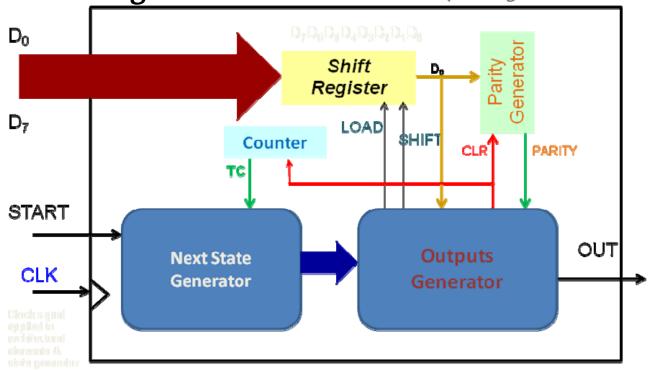
par 0 \rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 1
```

```
use ieee.std logic 1164.ALL;
use ieee.std logic arith.ALL;
entity serial sft is
 port( clk, rst, start :in std_logic; data: in std_logic_vector(7 downto 0);
       shift out: out std logic);
end serial sft;
architecture state m of serial sft is
        type states is (idle, shift, parity, stop);
        signal state: states;
begin
 process (clk, rst)
 variable tmp: std logic vector(7 downto 0);
 variable par: std logic;
 variable i : std logic vector (2 downto 0);
 beain
 if rst='1' then state <= idle; shift_out <= '1';-- asynchronous reset</pre>
  elsif (clk'event and clk='l') then
    case state is
         when idle =>
                                            -- STATEO
           if start= '1' then
                                               -- if START = '1'
             tmp := data; shift out<='0';
                                              -- load DATA, out start bit
            state <= shift; par :='0'; i:="000"; -- clear parity bit & cntr
           else shift out<='1'; end if;
                                               -- if START = '0'
        when shift=>
                                             -- STATE1
                shift out <= tmp(i);
                                             -- output DATA(I)
                par := par xor tmp(i);
                                             -- parity bit update
                i := i + 1:
                                             -- increment counter
                if i = "000" then state <= parity;
                end if:
        when parity => shift out<=par; state<=stop; -- STATE2
        when stop => shift out<='1'; state<=idle;
                                                      -- STATE3
        end case:
  end if:
end process:
end state m;
```

# Modular design of this state machine:



Modular design of this state machine: (showing the arch. elements)



#### **Shift Register**

Loads DATA when LOAD is true Shifts Right when SHIFT is true

#### **COUNTER (3-bit)**

Cleared when CLR is true
When Terminal Count : TC is true

#### PARITY GEN.

TOGGLE Flip-flop output cleared when GLR is true

For state machines that require architectural elements, lets consider **VADL** realizations next.