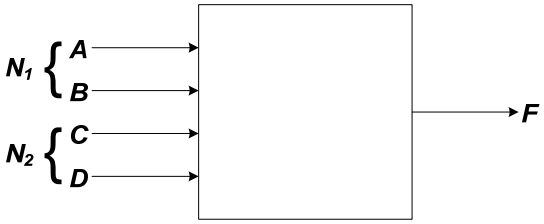
Tutorial 4 - Questions (Part 1)

Boolean algebra and minimization

1. A switching circuit has four inputs as shown. *A* and *B* represent the first and second bits of a binary number N_1. *C* and *D* represent the first and second bits of a binary number N_2. The output is to be 1 only if the product N_1 x N_2 is less than or equal to 2.
   (a) Write the truth table for the system.
   (b) Write the canonical SOP and POS expressions for *F*.
   (c) Draw a Karnaugh-map (K-map) for the function *F*.
   (d) From the K-map, derive a simplified SOP expression for *F*. *(Hint: Use groups of only size 4.)*



2. A bank vault has three locks with a different key for each lock. Each key is owned by a different person. To open the door, at least two people must insert their keys into the assigned locks. The signal lines *A*, *B* and *C* are 1 if there is a key inserted into lock 1, 2 or 3, respectively. Write an equation for the variable *Z* which is 1 if and only if the door should open.
   (a) Write the truth table for the system.
   (b) Write the canonical SOP and POS expressions for *Z*.
   (c) Draw a Karnaugh-map (K-map) for the function *Z*.
   (d) From the K-map, derive a simplified SOP expression for *Z*.

3. Use K-maps to obtain an MSOP and an MPOS for each of the following functions:
   (a) $Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + AB\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}\overline{C}\overline{D}$ with don't care for $ABCD$ = 1010
   (b) $Z = \left(\overline{A} + B + \overline{C}\right)\left(A + B + \overline{C}\right)$ with don't cares for ABC = 111 and 110
   (c) $f\left(x_1, \cdots, x_4\right) = \sum m(0,4,5,6,7) + D(1,12,13,14,15)$, where $m(\ )$ is the set of minterms for which f = 1 and $D(\ )$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4$ = 0010 (this alternate shorthand notation is often used to express SOPs).

4. A combinational circuit has four inputs A, B, C and D and an output Z. The output is asserted whenever three or more of the inputs are asserted, otherwise the output is de-asserted. Find an MSOP expression for Z. Design combinational circuits using only 74'04 inverters, 74'00 2-input NAND gates and 74'10 3-input NAND gates. Assume that A, B and Z are active high signals, while C and D are active low signals. Use alternate gate representations for clarity of circuit diagrams.