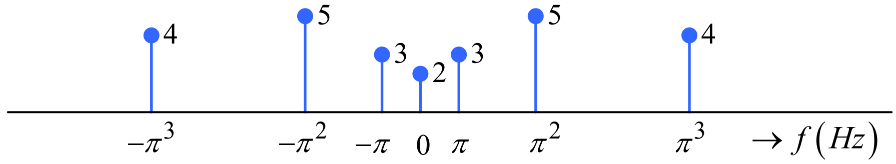
EE2023 TUTORIAL 2 (PROBLEMS)

Q.1 The discrete-frequency spectrum of a signal $x(t)$ is shown in Fig.Q.1. Classify $x(t)$ based on inferences drawn from Fig.Q.1 alone. What is the Fourier series expansion of $x(t)$?

Fig.Q.1



Q.2 Determine the Fourier series coefficients of each of the following periodic signals.

(a) $x(t) = 6\sin(12\pi t) + 4\exp\big(j(8\pi t + \pi/4)\big) + 2$

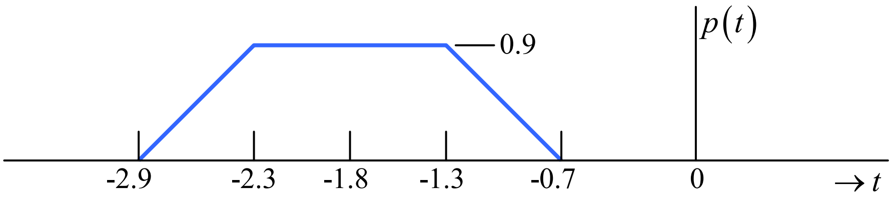
(b) $x(t) = 0.5\big(|\sin(\pi t)| + \sin(\pi t)\big)$

Q.3 Determine the Fourier series coefficients of

$$x(t) = \sum_{n=-\infty}^{\infty} 2p(t - 1.6n)$$

where $p(t)$ is given in Fig.Q.3.

Fig.Q.3



Q.4 A Fourier series analysis-synthesis system is given in Fig.Q.4.

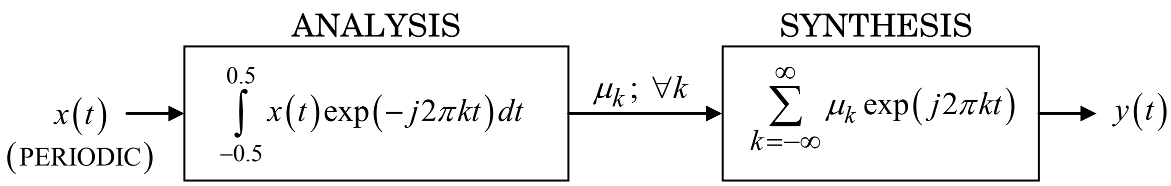


Fig.Q.4

(a) What does the analysis subsystem do?

(b) What does the synthesis subsystem do?

(c) Let $x(t) = \cos(3\pi t)$. Simply based on your understanding of the Fourier series, sketch $y(t)$ without performing any computation.

Supplementary Problems

These problems will not be discussed in class.

S.1 Consider a rectified sine wave signal $x(t)$ defined by

$$x(t) = |\sin(\pi t)|.$$

(a) Sketch $x(t)$ and find its fundamental period.

(b) Find the complex exponential Fourier series of $x(t)$.

(c) Find the trigonometric Fourier series of $x(t)$.

Answer: *(a)* $period = 1$ *(b)* $x(t) = -\frac{2}{\pi}\sum_{k=-\infty}^{\infty}\frac{1}{4k^2-1}\exp(j2\pi kt)$

(c) $x(t) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{4k^2-1}\cos(2\pi kt)$

S.2 Find the complex exponential Fourier series of a periodic signal $x(t)$ defined by

$$x(t) = t^2; \quad -\pi < t < \pi \quad \text{and} \quad x(t+2\pi) = x(t).$$

Answer: $x(t) = \frac{\pi^2}{3} + 4\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos(kt)$

S.3 The harmonic form Fourier series of a *real* periodic signal $x(t)$ with fundamental period T_0 is given by

$$x(t) = C_0 + \sum_{k=1}^{\infty} C_k \cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$$

where C_0 is known as the dc component, and the term $C_k \cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$ is referred to as the *kth-harmonic component* of $x(t)$. Express C_0, C_k and θ_k in terms of the complex exponential Fourier series coefficients X_k of $x(t)$.

Answer: $C_0 = X_0$, $C_k = 2|X_k|$, $\theta_k = -\tan^{-1}\left(\frac{\mathrm{Im}[X_k]}{\mathrm{Re}[X_k]}\right)$

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*Below is a list of solved problems selected from **Chapter 5** of **Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition**.*

Selected solved-problems: 5.4-to-5.13

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.
