Tan Kim Seg $25\frac{1}{2}$

EE2023 Signals & Systems Quiz
Semester 1 AY2013/14

Date : 8 October 2013 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. No programmable or graphic calculator is allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ____________

Matric # : ____________

Class Group # : ____________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

Group 1 : A/Prof Loh Ai Poh
Group 2 : A/Prof Ng Chun Sum
Group 3 : A/Prof Tan Woei Wan
Group 4 : Prof Lawrence Wong

Q.1 A signal $x(t)$ has a spectrum shown in Figure Q.1 below. You may assume that only signals with frequencies below 3 Hz are important.

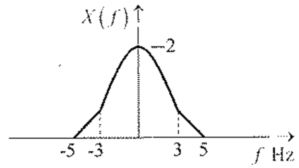


Figure Q.1: Spectrum of $x(t)$

(a) Without using any anti-aliasing filter, design a sampling frequency to sample $x(t)$ such that the sampled signal has no frequency aliasing effects.

(b) What is the lowest sampling frequency which can be used to sample $x(t)$ in such a way that there is no distortion to the important frequency components? Show how you arrive at your choice of the sampling frequency.

(c) If $x(t)$ is sampled at 6 Hz, can the important part of $x(t)$ be reconstructed from the sampled signal? Justify your answer.

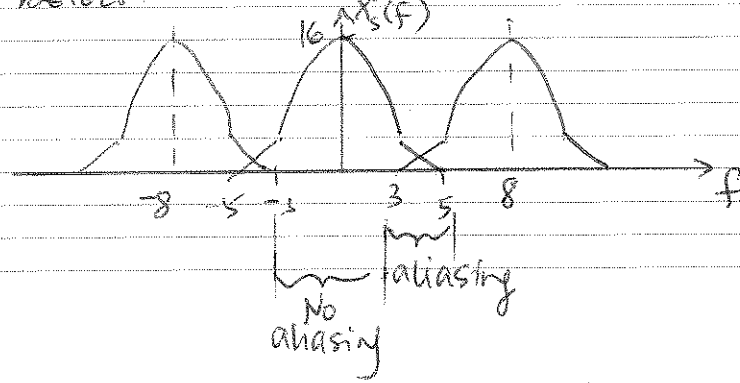
(d) Suppose another signal $y(t) = x(t)\cos(20\pi t)$ is sampled at 20Hz. Sketch the spectrum of the sampled $y(t)$.

Q.1 ANSWER

(a) lowest f_s = 10 Hz.

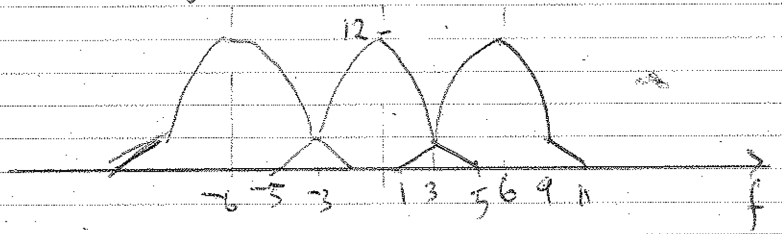
(b) If using an anti-aliasing filter, with ideal characteristics & b/w of 3Hz, then min. sampling freq = 6Hz.

If not using anti-aliasing filter, then min. f_s = 8Hz. See sampled spectrum below.



Q.1 ANSWER ~ continued

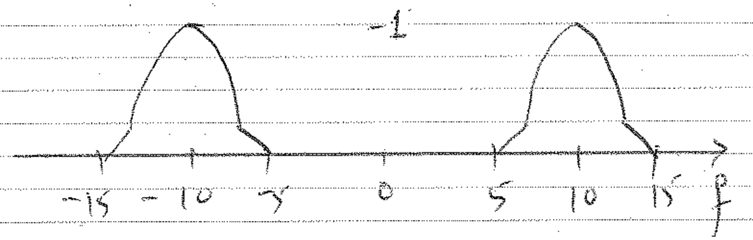
(c) If sampled at 6 Hz < 8 Hz, there will be aliasing, & signal cannot be recovered accurately.



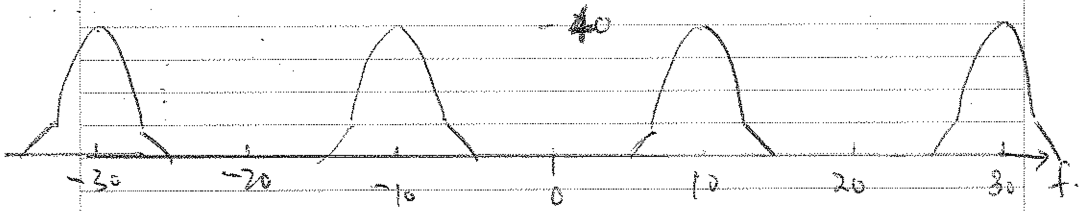
d) $y(t) = x(t) \cos 20\pi t$

$y(t)$ is a modulated signal.

$Y(f) = X(f) \otimes \frac{1}{2}\{\delta(f-10) + \delta(f+10)\}$.



Sampled $y_s(t)$:



Q.2 Let $x(t) = \text{sinc}^2\left(t - \frac{1}{2\pi}\right)$.

(a) Determine the phase spectrum and energy spectral density of $x(t)$. Draw a labeled sketch for each.

(b) Calculate the amount of energy contained within the 3dB bandwidth of $x(t)$. Round your answer to 3 decimal places.

Q.2 ANSWER

(a) Consider $x'(t) = \text{sinc}^2(t)$

$X'(f) = \text{tri}(f)$

For $x(t) = \text{sinc}^2(t - \frac{1}{2\pi})$

Then $X(f) = e^{-j2\pi f \frac{1}{2\pi}} \, \text{tri}(f)$

$= e^{-jf} \, \text{tri}(f)$

Phase spectrum : $\angle X(f) = \angle e^{-jf} + \angle \text{tri}(f)$

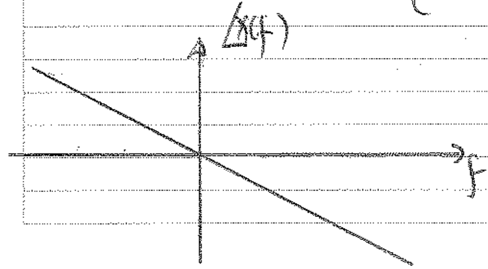
$\text{tri}(f)$ is real & +ve for all f.

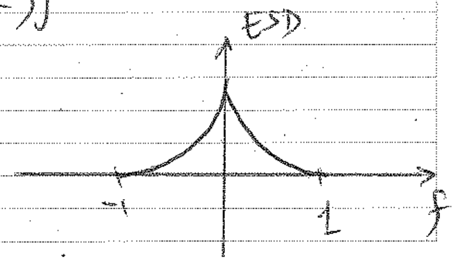
$\angle e^{-jf} = -f$ rad

$\therefore \angle X(f) = -f$

ESD of $X(t) = |X(f)|^2$

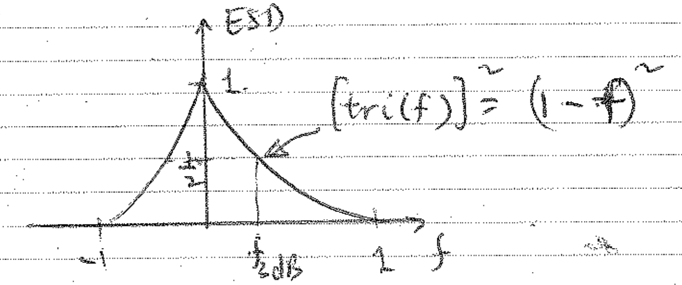
$= [\text{tri}(f)]^2$





Q.2 ANSWER ~ continued

(b)



$$(1-f_{3dB})^2 = \frac{1}{2}$$

$$1 - f_{3dB} = \frac{1}{\sqrt{2}}$$

$$f_{3dB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} \quad \#$$

Energy contained:

$$E = 2\int_0^{f_{3dB}} (1-f)^2 \, df$$

$$= 2\left(f - f^2 + f^3/3\right)_0^{f_{3dB}}$$

$$= 0.465$$

Q.3 Consider the periodic signal, $x(t)$, shown in Figure Q.3.

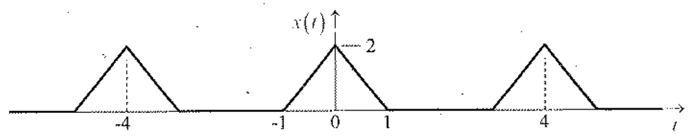


Figure Q.3

(a) What is the fundamental period, T_p, of $x(t)$?

(b) The signal $x(t)$ may be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} \alpha(t - nT_p)$$

where $\alpha(t) = \begin{cases} \alpha_1(t); & -1 \le t < 0 \\ \alpha_2(t); & 0 < t < 1 \\ 0; & \text{otherwise} \end{cases}$

Find $\alpha_1(t)$ and $\alpha_2(t)$.

(c) Derive the expression of a definite integral for the complex exponential Fourier Series coefficients, X_k, of $x(t)$.
Note : There is no need to evaluate the definite integral.

(d) Sketch the continuous-frequency spectrum of $x(t)$, clearly labelling the axes and important features of the graph.

Q.3 ANSWER

(a) fundamental period $T_p = 4$ sec.

(b) $\alpha_1(t) = 2 + 2t \quad -1 < t < 0$

$\alpha_2(t) = 2 - 2t \quad 0 < t < 1$

(c) $$X_k = \frac{1}{4}\int_{-2}^{2} \alpha(t)\, e^{-j2\pi k f t}\, dt$$

$$= \frac{1}{4}\left[\int_{-1}^{0} \alpha_1(t)\, e^{-j2\pi k \frac{1}{4} t}\, dt + \int_{0}^{1} \alpha_2(t)\, e^{-j2\pi k \frac{1}{4} t}\, dt\right]$$

Q.3 ANSWER ~ continued

(c). $x(t)$ is periodic.

Generation function of $x(t) = 2tri(t)$

$G(f) = 2\,\mathrm{sinc}^2(f)$

$\therefore$ F.T. of $x(t)$ is given by

$$X(f) = \sum_{k=-\infty}^{\infty} \frac{1}{4} G(k\tfrac{1}{4})\,\delta(f - k\tfrac{1}{4})$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{4}\, 2\,\mathrm{sinc}^2\left(\frac{k}{4}\right) \delta(f - k/4)$$

$$= \sum \frac{1}{2}\,\mathrm{sinc}^2(k/4)\,\delta(f - k/4)$$

Q.4. The signal $x(t)$ is shown in Figure Q.4(a).

(a) Write the expression for the single sinusoid pulse $x(t)$ shown in Figure Q.4(a).

(b) Determine the Fourier transform, $X(f)$, of the signal $x(t)$.

(c) Using the replication property of the Dirac-δ function, obtain an expression for the periodic signal $x_p(t)$ shown in Figure Q.4(b) in terms of $x(t)$.

(d) Derive the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$.

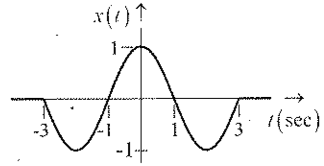


Figure Q.4(a)

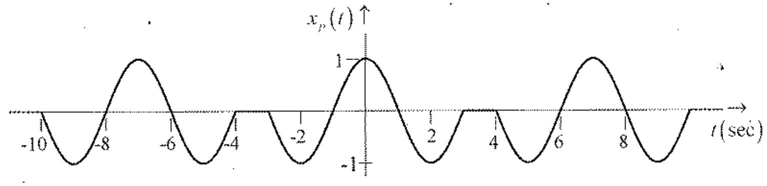


Figure Q.4(b)

Q.4 ANSWER

(a) $x(t) = \cos(0.5\pi t)\,\mathrm{rect}(t/6)$

(b) $X(f) = 6\,\mathrm{sinc}(f6) \otimes \frac{1}{2}\left[\delta(f - \tfrac{1}{4}) + \delta(f + \tfrac{1}{4})\right]$

$= 3\left[\mathrm{sinc}(6(f - \tfrac{1}{4})) + \mathrm{sinc}(6(f + \tfrac{1}{4}))\right]$

(c) $x_p(t) = \sum_{n=-\infty}^{\infty} x(t) \otimes \delta(t - 7n)$ $\quad T_p = 7;\ f_p = \frac{1}{7}$

(d) $X_p(f) = \sum_{n=-\infty}^{\infty} \frac{1}{7}\, 3\left[\mathrm{sinc}(6(n\tfrac{1}{7} - \tfrac{1}{4})) + \mathrm{sinc}(6(n\tfrac{1}{7} + \tfrac{1}{4}))\right] \delta(f - n/7)$

$= \sum_{n=-\infty}^{\infty} \frac{3}{7}\left[\mathrm{sinc}(\tfrac{6n}{7} - \tfrac{3}{2}) + \mathrm{sinc}(\tfrac{6n}{7} + \tfrac{3}{2})\right] \delta(f - \tfrac{n}{7})$