Name : | **Group** : | **Matric #** :

Due date : 13 November 2014 (Thursday)

National University of Singapore
Department of Electrical & Computer Engineering
EE2023 Signal & Systems

Consider the system in Figure 1 which shows a crane hoisting and moving a load which is attached to the end of a rope. The cart is supposed to deliver the load from one place to another. The objective is to move the cart in a way which does not cause the rope and its load to swing too much. This assignment explores the modelling of such a system and leads up to the design of a control system for the cart such that the load does not swing wildly.

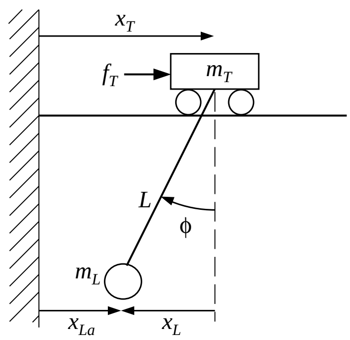

Figure 1: Schematic of crane hoisting a load.

Although the actual system is highly nonlinear, if the rope is considered to be stiff with a fixed length L, the system can be modelled using the following differential equations :

$$\begin{aligned}\ddot{x}_{La} &= g\phi \\ m_T\ddot{x}_T &= f_T - m_L g\phi \\ x_{La} &= x_T - x_L \\ x_L &= L\phi\end{aligned}$$

where m_L is the mass of the load, m_T is the mass of the cart, x_T and x_L are displacements as defined in Figure 1, ϕ is the rope angle with respect to the vertical, and f_T is the force applied to the cart. In this problem, the mass of the cart, m_T can be considered a constant. For simplicity, you may choose $m_T = 1$ unit mass while the gravitational constant can be approximated as $g = 10\ m/s^2$. The mass of the load, m_L and the length of the rope, L are parameters which vary for each hoisting problem.

In this assignment, generate m_L and L using your matriculation number as follows :

- $m_L = (C + 10)$ unit mass
- $L = (D + 1)$ unit length

where C and D may be derived by reading the last 4 digits of your student matriculation number. Equate the first two digits to C and the remaining two to D. For example, if the last 4 digits of your matriculation number is 1234, then choose $C = 12$ and $D = 34$. With the values of m_L and L determined in this manner, you may proceed to answer the following questions.

(1) Obtain the transfer function, $\dfrac{\Phi(s)}{V_T(s)}$ where $V_T(s) = \mathcal{L}\{v_T(t)\}$ and $\Phi(s) = \mathcal{L}\{\phi(t)\}$ are the Laplace transforms of the cart velocity, $v_T(t)$ and the rope angle, $\phi(t)$ respectively.

(3 marks)

(2) Assume that the cart is driven at a constant velocity, $v_0 = 10$ m/s. Derive and sketch the angle of sway of the rope as the cart moves. (5 marks)

(3) Under the condition in part (2) above, show that the load will sway with a frequency of $\omega_0 = \sqrt{\frac{g}{L}}$ rad/s. (2 marks)

(4) Find the transfer function, $\dfrac{V_T(s)}{F_T(s)}$, from the applied force to the cart's velocity where $V_T(s) = \mathcal{L}\{v_T(t)\}$ and $F_T(s) = \mathcal{L}\{f_T(t)\}$ are the Laplace transforms of the cart's velocity, $v_T(t)$ and the force, $f(t)$ respectively.

(3 marks)

(5) Analyze what happens to the velocity of the cart if a constant force is applied to it. Sketch the trajectory of the velocity. You may assume the constant force to be 1 unit. (5 marks)

(6) Suppose a crane controller is designed as shown in Figure 2, to automatically modulate the cart velocity in a manner which ensures that the load does not swing wildly when the cart is driven at the constant velocity, v_0 i.e. $V_T(s) = \mathcal{L}\{v_0 U(t)\}$ where $U(t)$ is the unit step function. K is the controller parameter to be tuned in order to achieve the control objective.

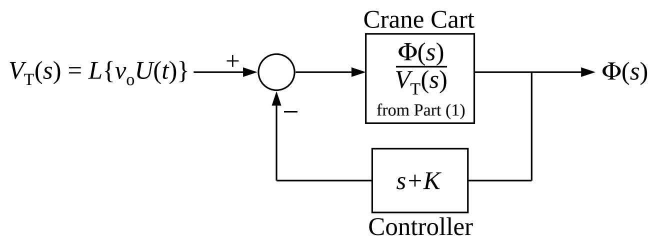

Figure 2: Control Configuration for the Crane.

- Derive the final transfer function $\frac{\Phi(s)}{V_T(s)}$, of the crane cart plus controller shown in Figure 2.
- Design a suitable value of K such that the resulting response of the rope sway is non-oscillatory.
- Sketch the trajectory of the rope sway when the cart is driven at a constant velocity of $v_0 = 2$ m/s. You may assume that the initial angle of the rope is zero.
- Explain qualitatively how the control system works. How does the control parameter K affect the quality of control?

(12 marks)