EE2023 SIGNALS & SYSTEMS
PAST-YEAR EXAM ARCHIVE

Semester II : 2011/2012

w/ Numeric Answers appended

SECTION A : Answer ALL questions in this section

Q1. Consider the circuit shown in Figure Q1-1 below.

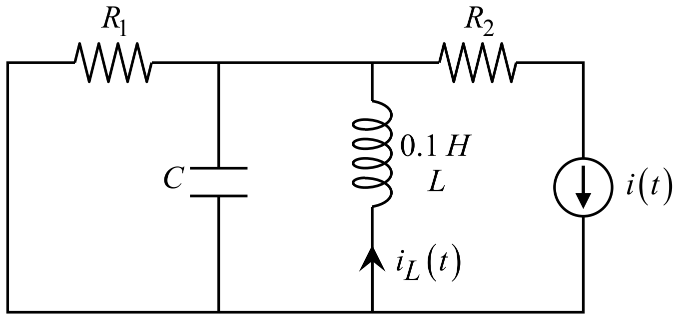

Figure Q1-1 : Electrical Circuit

(a) Show that the transfer function relating the current flowing through the inductor, $i_L(t)$, and the current source, $i(t)$, is given by

$$\frac{I_L(s)}{I(s)} = \frac{R_1}{R_1 LCs^2 + Ls + R_1}.$$ (3 marks)

(b) Find the values of R_1 and C such that the circuit is critically damped with poles at $s_{1,2} = -10^3, -10^3$. (5 marks)

(c) What is the natural frequency of the circuit for the values of the circuit components which you have obtained in part (b) above? (2 marks)

Q2. Figure Q2-1 shows the signal

$$x(t) = \mathrm{tri}\left(\frac{t-\alpha}{2}\right) - \mathrm{tri}\left(\frac{t+\alpha}{2}\right).$$

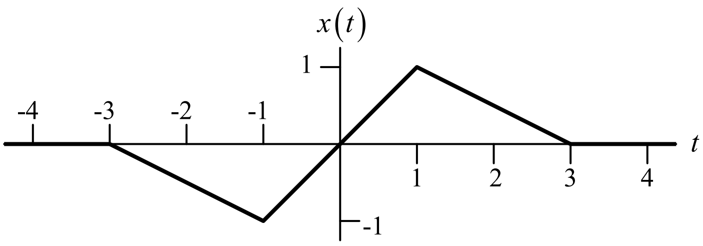

Figure Q2-1

(a) Find the value of α. Hence, determine the Fourier transform of $x(t)$. (6 marks)

(b) If $x(t)$ is sampled using the Dirac comb function, $y(t)$, given by:

$$y(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$$

where n is an integer, derive the Fourier transform of the resultant sampled signal.

(4 marks)

Q3. The spectrum of a signal $x(t)$ is given by $X(f) = \exp(-|f|)$.

(a) Compute the total energy of $x(t)$. (5 marks)

(b) Determine the magnitude and phase spectra of $y(t) = \frac{d}{dt}x(0.5t)$. (5 marks)

Q4. The pole-zero map for a linear time invariant system is shown in Figure Q4-1.

(a) What is the order of the system? (1 mark)

(b) Is the system stable, marginally stable or stable ? Justify your answer. (2 marks)

(c) Suppose the unit impulse response of the system is known to approach 8 as $t \to \infty$. Derive the transfer function of the system using the pole-zero map. (4 marks)

(d) Which of the following functions are terms in the step response of the system ?

- Decaying exponential function, $Ae^{-2t}U(t)$
- Decaying exponential function, $Be^{-\frac{t}{2}}U(t)$
- Growing exponential function, $Ce^{2t}U(t)$
- Step function, $DU(t)$
- Ramp function, $EtU(t)$
- Quadratic function, $Ft^2U(t)$

$U(t)$ is the unit step function, A, B, C, D, E and F are constants. (3 marks)

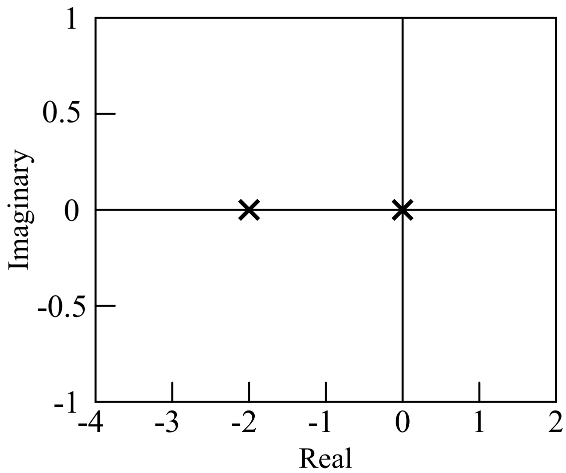

Figure Q4-1: Pole-Zero Map

SECTION B : Answer 3 out of the 4 questions in this section

Q5. An engineer was asked to identify the model for a system which is known only to have a second order transfer function as follows :

$$G(s) = \frac{4}{ms^2 + as + b}$$

where m, a and b are unknown parameters. In order to determine these parameters, he decides to inject a constant signal of amplitude 2 into the system and measure the output by probing some points in the system. Unfortunately, in his first trial, he placed a probe at one position where the signal recorded is as shown in Figure Q5-1 below. Based on his observation, he concluded that $G(s)$ is a first order system and then proceeded to determine m, a and b.

(a) Based on his observation of the output signal shown in Figure Q5-1, what values of m, a and b do you think he would have obtained? (6 marks)

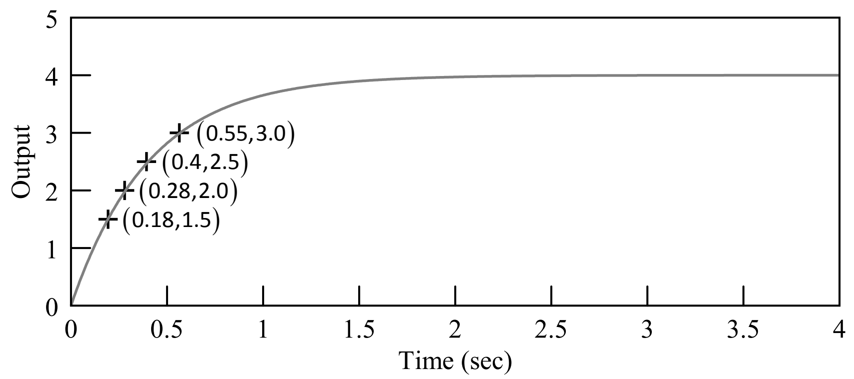

Figure Q5-1 : Output Response in First Trial

(b) Later in the day, he discovered that he has made a mistake in placing the probe. He then decided to place the probe at another point and this time, the results he recorded are shown in Figure Q5-2. What values of m, a and b do you think he would have obtained in his second trial? (10 marks)

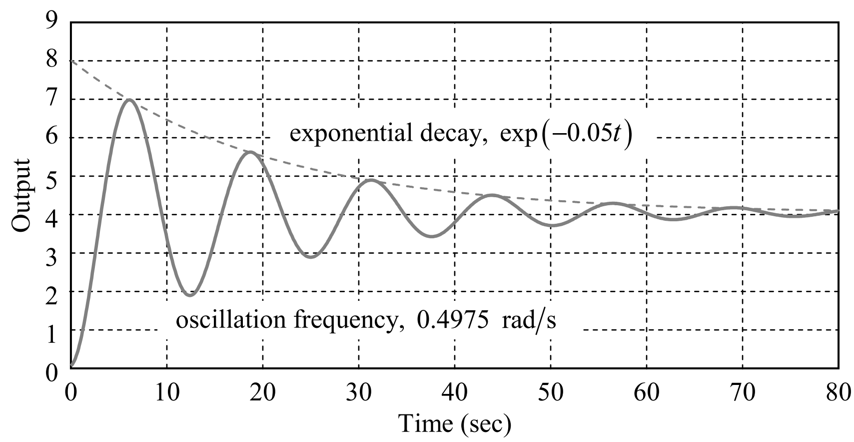

Figure Q5-2 : Output Response in Second Trial

(c) Find the poles of the second order system. (4 marks)

Q6. The signal $x(t) = \exp(-4t).u(t)$, where $u(t)$ is the unit step function, is applied to a low pass filter with frequency response $H(f)$ as shown in Figure Q6-1.

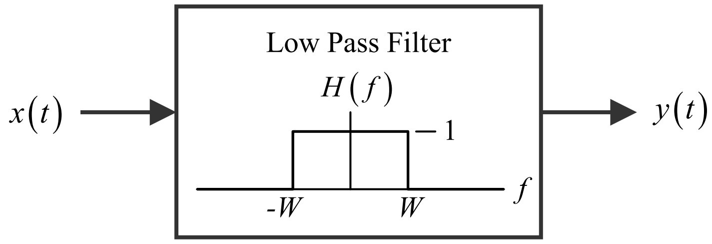

Figure Q6-1

(a) Derive the Fourier transform $X(f)$ of the signal $x(t)$. (5 marks)

(b) Sketch the magnitude spectrum of $X(f)$. (5 marks)

(c) What is the value of W such that the energy of the output signal $y(t)$ is half of the energy of its input signal $x(t)$? $\left[Hint: \int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \right]$ (10 marks)

Q7. Let X_k be the Fourier series coefficients and $X(f)$ be the Fourier transform of a bounded periodic signal $x(t)$ of period T.

(a) Express $X(f)$ in terms of X_k and T. (4 marks)

(b) Let $Y(f)$ be the Fourier transform of $y(t) = x(t) \ast \mathrm{rect}(t/T)$ where $\ast$ denotes convolution. Find $Y(f)$ and $y(t)$, each in terms of X_k and T. (8 marks)

(c) Let $z(t) = x(\alpha t)$ where α is a positive constant. Find the relationship between Z_k and X_k, where Z_k are the Fourier series coefficients of $z(t)$. (8 marks)

Q8. Figure Q8-1 shows the Bode magnitude plot of a stable system, whose transfer function is $G(s)$.

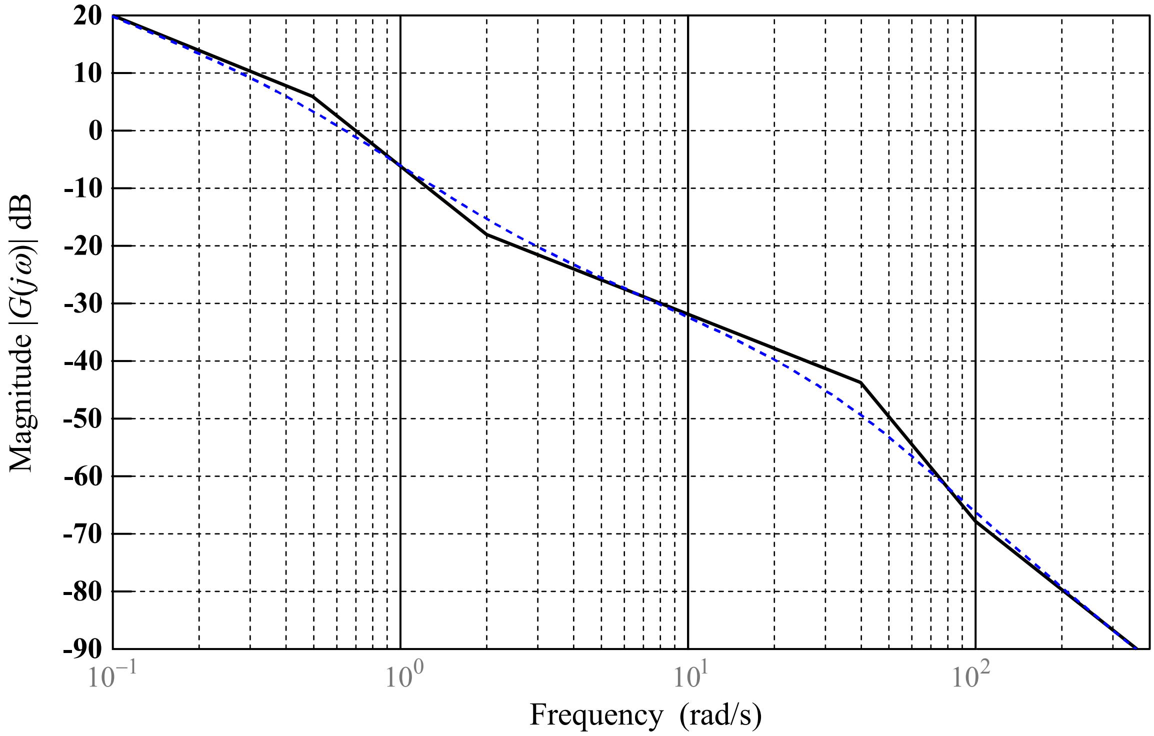

Figure Q8-1: Bode magnitude plot of $G(s)$

(a) Identify the transfer function $G(s)$. (8 marks)

(b) The system transfer function may be approximated as $G_a(s) = \dfrac{K(s+a)}{(s+b)(s+c)}$ for frequencies below 5 rad/s.

- Determine the values of a, b and c.
- Hence, derive K such that $\lim_{t\to\infty} h(t)$, where $h(t)$ is the unit impulse response, obtained using $G(s)$ and $G_a(s)$ are equal.

(6 marks)

(c) Suppose the steady-state output signal is $3\sin(0.8t)$. Using $G_a(s)$, or otherwise, estimate the input signal. (6 marks)

END OF QUESTIONS

NUMERIC ANSWERS

Section A

Q.1 (b) $R_1 = 50\,\Omega,\ C = 10\,\mu F$

(c) $\omega_n = 10^3\ rad/s$

Q.2 (a) $\alpha = 1,\ X(f) = -j4\text{sinc}^2(2f)\sin(2\pi f)$

(b) $X_s(f) = -j4\sum_{k=-\infty}^{\infty}\text{sinc}^2\big(2(f-k)\big)\sin\big(2\pi(f-k)\big)$

Q.3 (a) Total energy $= 1$

(b) $|Y(f)| = 4\pi|f|\exp(-|2f|),\ \angle Y(f) = 0.5\pi\,\text{sgn}(f)$

Q.4 (a) 2^{nd}-order

(b) Marginally stable (?)

(c) Transfer function: $G(s) = \dfrac{16}{s(s+2)}$

(d) $Ae^{-2t}U(t),\ DU(t),\ EtU(t)$

Section B

Q.5 (a) $m = 0,\ a = 0.8,\ b = 2$

(b) $m = 8,\ a = 0.8,\ b = 2$

(c) Poles: $s_{1,2} = -0.05 \pm j0.4975$

Q.6 (a) $X(f) = \dfrac{1}{4 + j2\pi f}$

(b) Sketch $|X(f)| = \left(\dfrac{1}{4^2 + (2\pi f)^2}\right)^{1/2}$

(c) $W = \dfrac{2}{\pi}$

Q.7 (a) $X(f) = \sum_{k=-\infty}^{\infty} X_k\delta\left(f - \dfrac{k}{T}\right)$

(b) $Y(f) = TX_0\delta(f),\ y(t) = TX_0$

(c) $Z_k = X_k$

Q.8 (a) $G(s) = \dfrac{4(s+2)(s+100)}{s(s+0.5)(s+40)^2}$

(b) $a = 2,\ b = 0,\ c = 0.5,\ K = 0.25$

(c) $4.2\sin(0.8t + 126.2^\circ)$
