NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I : 2014/2015)

EE2023 – SIGNALS & SYSTEMS

Nov/Dec 2014 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **EIGHT (8)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
3. This is a **CLOSED BOOK** examination.
4. Programmable calculators are not allowed.
5. Tables of Fourier Transforms, Laplace Transforms and trigonometric identities are provided in Pages 9, 10 and 11, respectively.

SECTION A : Answer ALL questions in this section

Q1. Consider the system in Figure Q1-1 whose transfer function is given by $G(s) = \frac{1}{s}e^{-3s}$.

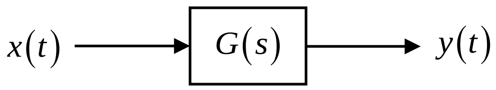

Figure Q1-1 : System, $G(s)$

(a) Derive the response of $G(s)$ to a unit impulse, $x(t) = \delta(t)$. Sketch the resulting impulse response.

(4 marks)

(b) <u>Derive</u> and <u>sketch</u> the response of $G(s)$ to an input, $x(t)$ given in Figure Q1-2. Label your sketch clearly.

(6 marks)

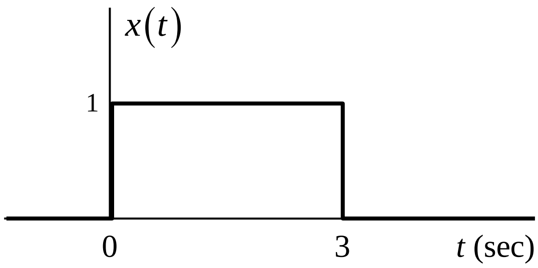

Figure Q1-2 : Input to $G(s)$

Part (a) was quite well done. However, for part (b), many students did not know how to write a mathematical expression for the signal $x(t)$ and hence could not proceed very well. Quite a number of students wrote the signal as a rect(.) function which is not useful for this problem. Those who tried to apply the Laplace transform formula to try to find X(s) did get the expression correct.

Many students also did not know how to interpret $G(s)$ as an integrator with a time delay. If students are aware of the interpretation for $G(s)$, the problem could have been solved by finding the area under the curve of $x(t)$ and the answer would be equally valid.

Q2. The signal $x(t) = \text{sinc}^2(2t)$ is sampled at 8 Hz to produce the sampled signal $x_s(t)$.

(a) Derive the expression for $x_s(t)$.

(3 marks)

(b) Derive the Fourier transform of $x_s(t)$.

(3 marks)

(c) Sketch the spectrum of $x_s(t)$.

(4 marks)

Parts (a) and (b) were quite well done. However, for Part (c) quite a few students were not able to sketch the spectrum. Where the students struggled was when the base signal $x(t)$ is meant to multiply with the Dirac-δ or to convolve with it.

Q3. An energy signal $x(t)$ is given by

$$x(t) = \exp\left(-\pi t^2\right).$$

(a) Determine the energy spectral density, $E_x(f)$, of $x(t)$.

(4 marks)

(b) Find the 3 dB bandwidth of $x(t)$.

(3 marks)

(c) In computing the total energy of $x(t)$, would the time-domain or frequency-domain approach lead to a simpler solution, and why?

(3 marks)

One common error made by students in this question was treating energy spectral density as equal to average power. Also many students failed to realize that the Rayleigh energy theorem was the first step in the solution to Part (c). Recognizing this, the solution to Part (c) can be easily deduced from the result of Part (a), which many had succeeded in solving.

Q4. The pole-zero diagrams for 2 systems (I and II) are shown in Figure Q4.

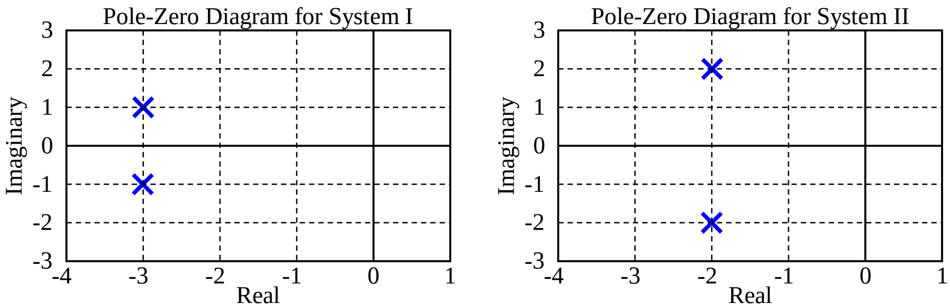

Figure Q4 : Pole-zero diagrams

(a) What is the damped natural frequency, undamped natural frequency and damping ratio of System I?

(3 marks)

Most students are able to arrive at the final answer.
A number of students attempted to evaluate ζ and ω_n via the erroneous expression $\frac{\sqrt{(2\zeta\omega_n)^2 - 4\omega_n^{\ 2}}}{2} = 1$, which is obtained by applying the formula for finding the roots of a standard second order polynomial to $s^2 + 2\zeta\omega_n s + \omega_n^{\ 2} = 0$ and equating to the imaginary value of the poles for System I. The equation is incorrect because $(2\zeta\omega_n)^2 - 4\omega_n^{\ 2} = 4\omega_n^{\ 2}(\zeta^2 - 1) < 0$ in the case of an underdamped 2nd order system so $\frac{\sqrt{(2\zeta\omega_n)^2 - 4\omega_n^{\ 2}}}{2}$ works out to be a complex number and should not be equated to a real number.

(b) Following a step change of magnitude 3, the steady state output of System II is 15. Derive the transfer function of System II using the pole-zero diagram and the steady-state information.

(4 marks)

Students are generally able to use the information in the pole-zero diagram to derive the polynomial in the denominator of the transfer function, $G(s)$. The constant term in the numerator of the transfer function should be evaluated using the concept of DC gain, $G(0)$, and the information that the steady state output is 15 following a step change of magnitude 3 i.e. $G(0) = \frac{15}{3} = 5$.

(c) Suppose Systems I and II have the same DC gain. Will System I or System II exhibit a larger overshoot following a step change in the input signal? Justify your answer.

Hint : Overshoot is governed by the damping ratio of the system.

(3 marks)

The most serious conceptual error is to attempt to explain/analyze using resonance. Resonance is the tendency of a system to oscillate with greater amplitude at some frequencies, and is a frequency domain concept i.e. the input signal is a sinusoid. Overshoot refers to the difference between the maximum peak value and the steady-state value of the step response. The resonance peak and the overshoot are both dependent on the damping ratio i.e. the resonance peak and the overshoot will be larger if there is less damping.

SECTION B : Answer 3 out of the 4 questions in this section

Q5. The space booster in Figure Q5 has a transfer function, $G(s)$, given by

$$\frac{\Phi(s)}{F(s)} = G(s) = \frac{1}{s^2 - 0.04}$$

where $\Phi(s)$ and $F(s)$ are Laplace transforms of $\phi(t)$ and $f(t)$, respectively.

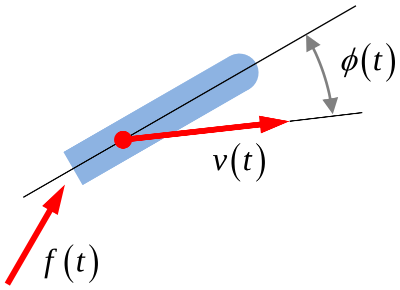

Figure Q5 : A Space Booster

(a) Analyze and describe what happens to the space booster when it is fired up with $f(t) = \delta(t)$, where $\delta(t)$ is a unit impulse function. Sketch the impulse response, $\phi(t)$.

(4 marks)

Some students do not know how to factorize $(s^2 - 0.04)$! Some thought that the inverse Laplace of $G(s)$ was a sinusoidal function.

(b) A control system is then developed for the space booster such that the overall closed loop control system has a transfer function given by

$$G_{cl}(s) = \frac{K}{s^2 + K_D s + K_p - 0.04}.$$

i. If $K_D = 0$, what is the minimum value of K_p required for the closed loop system to have bounded outputs? Justify your answer.

(4 marks)

Those who did not get this question correct did not know how to interpret “bounded outputs”. In this case, the system can only be either marginally stable or unstable when $K_D = 0$. Hence the only option is to choose K_p such that the system is marginally stable and this requires a minimum value of 0.04.

ii. If $K_p = 0$, why is K_D alone not able to stabilize the closed loop system?

(4 marks)

For this part, some students have difficulty analyzing the roots of the quadratic equation and determining whether some of the roots will be in the right half plane. Main issue is due to the term $\sqrt{K_D^2 + 4(0.04)}$ which will always be bigger than K_D. Thus since the roots are $-K_D \pm \sqrt{K_D^2 + 4(0.04)}$, there will always be one root which is positive and hence without K_p, the system cannot be stabilized.

iii Design K_p and K_D so that the closed loop system has poles at $s_{1,2} = -0.2 \pm 0.3j$.

(4 marks)

Most students got this correct. No issue.

iv Find the damping ratio and the undamped natural frequency of the closed loop system with poles at $s_{1,2} = -0.2 \pm 0.3j$. Sketch the impulse response of this closed loop system.

(4 marks)

Also no major problems with this part but not all were able to sketch the impulse response correctly. Quite a number of students sketched the response which looked like a step response rather than impulse response. What is the main difference? For impulse response, since this is not a marginally stable system (because of the controller and because the poles are where they are), the impulse response should go to zero after some time. A step response will not have a steady state which is zero! Instead it will reach a non-zero DC value.

Q.6 Consider the periodic signal $x(t)$ shown in Figure Q6 which comprises periodic Gaussian pulses, where

$$x(t)=\sum_{k=-\infty}^{\infty} e^{-(t-4k)^2/0.25}.$$

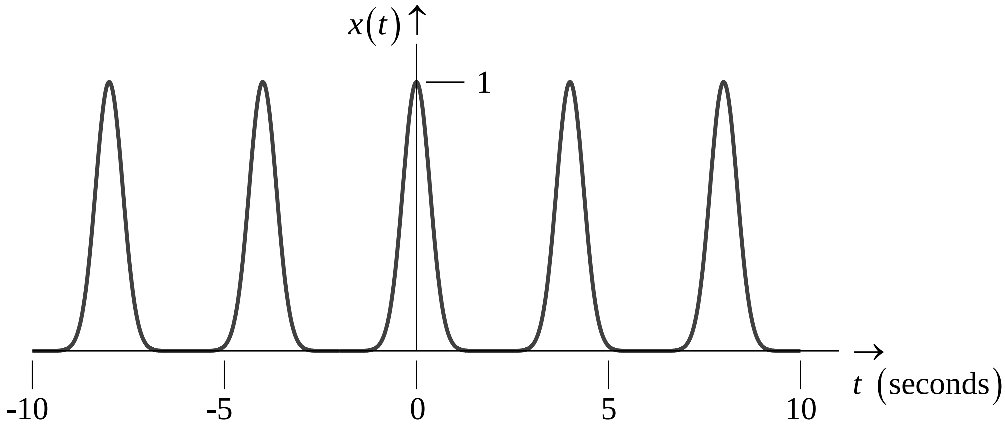

Figure Q6

(a) Derive the Fourier transform, $X(f)$, of $x(t)$.

(7 marks)

(b) Derive the Fourier series coefficient, X_k, of $x(t)$.

(3 marks)

(c) Derive an expression for the average power of $x(t)$.

(3 marks)

(d) Let the M^{th} harmonic of $x(t)$ be the harmonic that is closest to the 98% containment bandwidth of $x(t)$. Show and explain how M should be found.

(7 marks)

Most students were able to deal with part (a) which was relatively straightforward. Some students didn't see the relationship between the Fourier transform and Fourier series coefficients and went about deriving the Fourier series coefficients in part (b) from first principles, often making careless mistakes along the way. As a result this affect their ability to answer part (c) where, in fact, the average power was just the summation of the magnitude squared of the Fourier series coefficients.

For part (d) the many of the students were not able to derive the inequality that would provide the 98% containment bandwidth. Many did not even use the intended approach in part (c) which would have led to a relatively straightforward solution.

Q7. Radio station X transmits the signal $x(t) = 10 \cdot \text{sinc}(10t) \cdot \cos(2000\pi t)$, and radio station Y transmits the signal

$$y(t) = m(t) \cdot \cos(2\pi f_c t)$$

where the spectrum of $m(t)$ is given by $M(f) = \text{tri}\left(\frac{f}{B}\right)$, and $f_c >> 2B$. Radio interference between the two stations will occur if the spectra of their transmissions overlap.

(a) Sketch of the spectrum, $X(f)$, of $x(t)$. Show all the important dimensions in your sketch.

(5 marks)

(b) Suppose $B = 8$. What is the range of f_c values that should be avoided by radio station Y so as to avoid radio interference between the two stations?

(5 marks)

(c) Suppose $f_c = 1020$. Find the maximum value of B that can be used by radio station Y without causing radio interference between the two stations.

(5 marks)

(d) Suppose $y(t)$ is sampled to form $y_s(t)$. Suggest a sampling frequency, f_s, so that $m(t)$ can be recovered without distortion by passing $y_s(t)$ through a suitably designed ideal lowpass filter. Explain your answer.

(5 marks)

Through Parts (a), (b) and (c), most students had demonstrated adequate competency in the application of Fourier transform and its properties. The key issue in Part (d) was finding a sampling frequency such that m(t) can be extracted from $y_s(t)$ by low-pass filtering. Many students indiscriminately apply the Nyquist sampling frequency to sample y(t), failing to recognize that by doing so the spectrum of the sampled signal $y_s(t)$ will not contain the spectrum of m(t).

Q8. Most loudspeakers are not capable of reproducing the entire audio spectrum with negligible distortion. Consequently, most hi-fi speaker systems use a combination of loudspeakers, each catering to a different frequency band. Filter systems are used to split the audio signal into frequency bands that can be separately routed to loudspeakers optimized for those bands i.e. a lowpass filter is used to isolate signals for the woofer loudspeaker and the output signal of a highpass filter drives the tweeter loudspeaker.

(a) Derive the transfer functions of the two filter circuits shown in Figure Q8.

(8 marks)

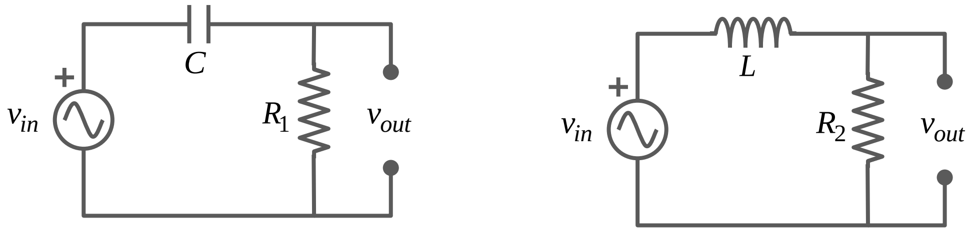

Figure Q8 : Series resistor-capacitor and resistor-inductor filtering circuits

Most students who attempted are able to derive the transfer functions. A small number of students incorrectly derived transfer functions that related the input voltage to the voltages across the capacitor/inductor, instead of the resistor.

(b) Sketch the straight line asymptotic Bode Magnitude diagrams of the resistor-capacitor (R_1C) and resistor-inductor (R_2L) filtering circuits, clearly labelling the corner frequencies and the slope of the asymptotes. Hence, or otherwise, determine if the output signal of the R_1C circuit should be used to drive the woofer or the tweeter loudspeaker?

(5 marks)

The R_2L circuit is a standard first order system, so most students are able to sketch its Bode diagram. Some students did not know how to handle the differentiator term in the R_1C circuit. Another issue is the slopes of the asymptotes are not labelled properly.

(c) A bandpass filtering system for generating audio signals in the mid-frequency range may be constructed by cascading the R_1C and R_2L filtering circuits shown in Figure Q8, and ensuring that $R_1C > \dfrac{L}{R_2}$.

A small number of students who attempted this part did not understand that “cascading” means the output of the R_1C circuit is fed into the R_2L circuit as the input signal. In the s-domain, the transfer function of the bandpass filter is the product of the answers in part (a).

i. Sketch the straight line asymptotic Bode Magnitude diagram of the bandpass filter, clearly labelling the corner frequencies.

(3 marks)

A common mistake is to swap the values of the corner frequencies due to mistakes in the earlier parts or not noticing the $R_1 C > \frac{L}{R_2}$ constraint.

ii. Suppose $R_1 = R_2 = 8\Omega$ and the bandpass system should not distort signals between 800 Hz and 3000 Hz. Design suitable values for C and L.

(4 marks)

The x-axis of Bode diagrams are expressed in angular frequency (rad/s), while the pass band in the question is given in cyclic frequency (Hz). Hence, a common error is the failure to convert from Hz to rad/s before solving for C and L.

END OF QUESTIONS

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference.

Fourier Series:
$$\begin{cases} X_k = \dfrac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform:
$$\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Unilateral Laplace Transform: $X(s)=\int_{0^-}^{\infty} x(t)\exp(-st)\,dt$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$1/s$
Ramp	$t\,u(t)$	$1/s^2$
n[th] order Ramp	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
Damped Ramp	$t\exp(-\alpha t)u(t)$	$1/(s+\alpha)^2$
Exponential	$\exp(-\alpha t)u(t)$	$1/(s+\alpha)$
Cosine	$\cos(\omega_o t)u(t)$	$s/(s^2+\omega_o^2)$
Sine	$\sin(\omega_o t)u(t)$	$\omega_o/(s^2+\omega_o^2)$
Damped Cosine	$\exp(-\alpha t)\cos(\omega_o t)u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_o^2}$
Damped Sine	$\exp(-\alpha t)\sin(\omega_o t)u(t)$	$\frac{\omega_o}{(s+\alpha)^2+\omega_o^2}$

LAPLACE TRANSFORM PROPERTIES		
	Time-domain	***s*-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Time shifting	$x(t-t_o)u(t-t_o)$	$\exp(-st_o)X(s)$
Shifting in the s-domain	$\exp(s_o t)x(t)$	$X(s-s_o)$
Time scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^{t} x(\zeta)\,d\zeta$	$\frac{1}{s}X(s)$
Differentiation in the time-domain	$\frac{dx(t)}{dt}$	$sX(s)-x(0^-)$
	$\frac{d^n x(t)}{dt^n}$	$s^n X(s)-\sum_{k=0}^{n-1} s^{n-1-k}\left.\frac{d^k x(t)}{dt^k}\right\rvert_{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\frac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\frac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$	$X_1(s)X_2(s)$
Initial value theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$	
Final value theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$	

	$y_{step}(t)$	$Y_{step}(s)$	SYSTEM PARAMETERS
Step response of 1^{st} order system	$K\left[1-\exp\left(-\frac{t}{T}\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K}{(sT+1)}$	$\begin{pmatrix} T: & \text{System Time-constant} \\ K: & \text{System Steady-state (or DC) Gain} \end{pmatrix}$
Step response of 2^{nd} order <u>underdamped</u> system: $(0<\zeta<1)$	$K\left[1-\frac{\exp(-\omega_n\zeta t)}{(1-\zeta^2)^{0.5}}\sin\left(\omega_n(1-\zeta^2)^{0.5}t+\phi\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2}$	$\begin{pmatrix} \omega_n: & \text{System Undamped Natural Frequency} \\ \zeta: & \text{System Damping Factor} \\ \omega_d: & \text{System Damped Natural Frequency} \\ K: & \text{System Steady-state (or DC) Gain} \end{pmatrix}$ $\begin{pmatrix} \sigma=\omega_n\zeta \\ \omega_d^2=\omega_n^2(1-\zeta^2) \\ \omega_n^2=\sigma^2+\omega_d^2 \\ \tan(\phi)=\omega_d/\sigma \end{pmatrix}$
	$K\left[1-\left(\frac{\sigma^2+\omega_d^2}{\omega_d^2}\right)^{0.5}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$	$\frac{1}{s}\cdot\frac{K(\sigma^2+\omega_d{}^2)}{(s+\sigma)^2+\omega_d{}^2}$	
2^{nd} order system - RESONANCE - $(0\le\zeta<1/\sqrt{2})$	*RESONANCE FREQUENCY :* $\omega_r=\omega_n(1-2\zeta^2)^{0.5}$		*RESONANCE PEAK :* $M_r=\lvert H(j\omega_r)\rvert=\frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$
$\cos(\theta)=\frac{1}{2}\left[\exp(j\theta)+\exp(-j\theta)\right]$	$\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$
$\sin(\theta)=\frac{1}{j2}\left[\exp(j\theta)-\exp(-j\theta)\right]$	$\tan(\alpha\pm\beta)=\frac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$
$\sin^2(\theta)+\cos^2(\theta)=1$	
$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$
$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$	$\cos(\alpha)\cos(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$
$\sin^2(\theta)=\frac{1}{2}\left[1-\cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta)=\frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$
$\cos^2(\theta)=\frac{1}{2}\left[1+\cos(2\theta)\right]$	$\mathbf{C}\cos(\theta)-\mathbf{S}\sin(\theta)=\sqrt{\mathbf{C}^2+\mathbf{S}^2}\cos\left[\theta+\tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$