EE2023 Signals & Systems Quiz
Semester 1 AY2012/13

Date : 4 October 2012 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. No programmable or graphic calculator is allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ______________________________

Matric # : ____________________________

Lecture Group # : ______________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

Group 1 : A/Prof Loh Ai Poh
Group 2 : A/Prof Ng Chun Sum
Group 3 : A/Prof Tan Woei Wan
Group 4 : Prof Lawrence Wong

Q.1 (a) Determine the appropriate sampling frequency for the signal $x(t)$ given by:

$$x(t) = 2\sin(2\pi t)\sin(10\pi t) + 6\cos(6\pi t),$$

in order to be completely recoverable from its samples.

(b) If $x(t)$ above is sampled at 20π rad/s, sketch the amplitude spectrum of the sampled signal. Can $x(t)$ be recovered from the samples? Explain your answer.

(c) Is $x(t)$ a power or energy signal? Explain your answer.

Q.1 ANSWER

(a) $x(t) = 2\sin 2\pi t \sin 10\pi t + 6\cos 6\pi t$

$= \cos(2\pi - 10\pi)t - \cos(2\pi + 10\pi)t + 6\cos 6\pi t$

$= \cos 8\pi t - \cos 12\pi t + 6\cos 6\pi t$

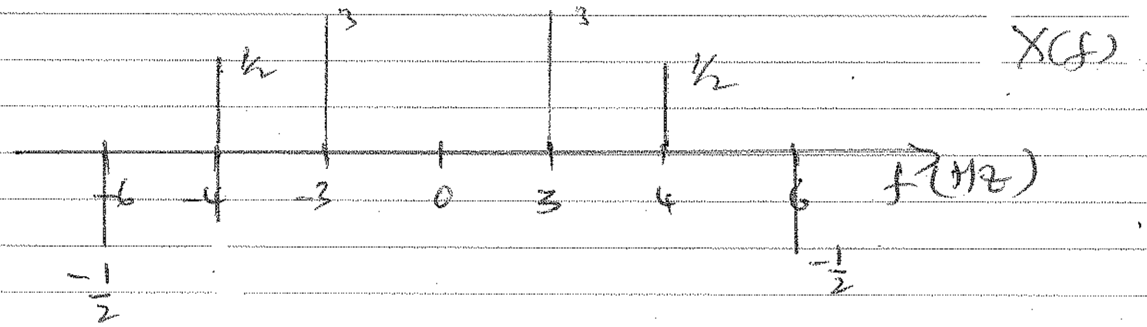
freq components are -8π, 12π & 6π rad/s.

Highest freq component = 12π rad/s

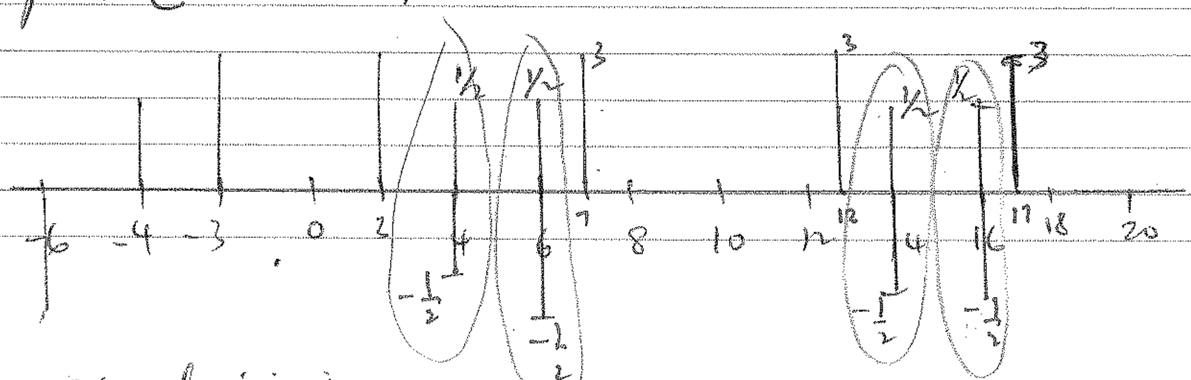
(b) If sampled at 20π rad/s < 24π rad/s

⇒ cannot recover.

$X(f) = \frac{1}{2}\{\delta(f-4)+\delta(f+4)\} - \frac{1}{2}\{\delta(f-6)+\delta(f+6)\}$
$+ 3\{\delta(f-3)+\delta(f+3)\}$



Sampled @ 10 rad/s



(c) power signal ∵ it is periodic

Q.2 Let $x(t)=\text{sgn}(t)\sin(2\pi f_0 t)$ and $y(t)=\text{sinc}(f_0 t)$ where $\text{sgn}(t)=\begin{cases}+1; & t\geq 0\\ -1; & t<0\end{cases}$ and f_0 is a non-zero positive constant.

(a) Find the Fourier transforms of $x(t)$ and $y(t)$. Your final answers <u>should not</u> contain any unsolved integrals and/or the convolution operator '$*$'.

(b) With $x(t)$ and $y(t)$, we form the signal $z(t)=\int_{-\infty}^{\infty}x(\varsigma)y(t-\varsigma)d\varsigma$. If $z(t)$ is an energy signal, find its energy spectral density. If $z(t)$ is a power signal, find its power spectral density. If $z(t)$ is neither an energy nor a power signal, simply say so.

Q.2 ANSWER

$x(t) = \text{sgn}(t)\sin 2\pi f_0 t$ $\qquad\qquad \mathcal{F}\{\text{sgn}(t)\} = \frac{1}{j\pi f}$

$X(f) = \mathcal{F}\{\text{sgn}(t)\} * \mathcal{F}\{\sin 2\pi f_0 t\}$

$= \frac{1}{j\pi f} * \frac{1}{2j}\{\delta(f-f_0)-\delta(f+f_0)\}$

$= -\frac{1}{2\pi}\left\{\frac{1}{f-f_0} - \frac{1}{f+f_0}\right\} = \frac{f_0}{\pi(f_0^2-f^2)}$

$y(t) = \text{sinc}(f_0 t)$

$\text{rect}(t/T) \longleftrightarrow T\,\text{sinc}(fT)$

set $\quad T = f_0$

$\text{rect}(f/f_0) \longleftrightarrow f_0\,\text{sinc}(f_0 t)$

$1/f_0\,\text{rect}(f/f_0) \longleftrightarrow \text{sinc}(f_0 t)$

Q.2 ANSWER ~ continued

(b) $z(t) = \int_{-\infty}^{\infty} x(\xi)\, y(t-\xi)\, d\xi$

$= x(t) * y(t)$

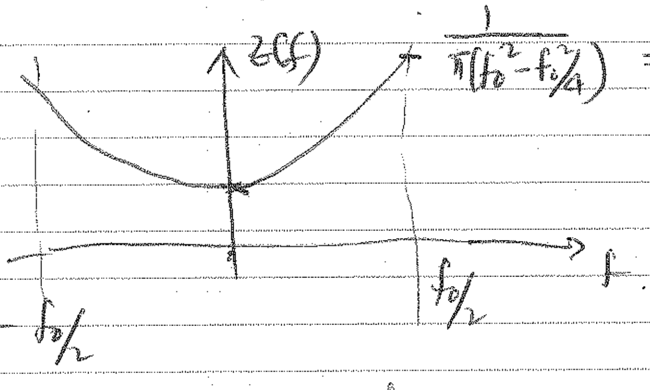
$Z(f) = X(f)\, Y(f)$

$= \dfrac{f_0}{\pi(f_0^2 - f^2)} \, \dfrac{1}{f_0} \, \mathrm{rect}(f/f_0)$

$= \dfrac{1}{\pi(f_0^2 - f^2)} \, \mathrm{rect}(f/f_0)$

$= \begin{cases} \dfrac{1}{\pi(f_0^2 - f^2)} & -\dfrac{f_0}{2} \le f < f_0/2 \\ 0 & \text{elsewhere} \end{cases}$

$\dfrac{1}{\pi(f_0^2 - f_0^2/4)} = \dfrac{4}{\pi\, 3 f_0^2}$



$E = \int_{-f_0/2}^{f_0/2} |Z(f)|^2\, df < \infty$.

because the integration limits are finite.

Hence $z(t)$ is an energy signal

Q.3 Consider the signal $x(t) = 5 + x_1(t) + x_2(3t) + e^{j2t}$, where $x_1(t) = 7\cos(8t)$ and $x_2(t) = \sin(2t)$.

(a) The discrete frequency amplitude spectrum for $x(t)$ shown in Figure Q.3 contains errors. Identify any two unique/different errors in the plot and correct them.

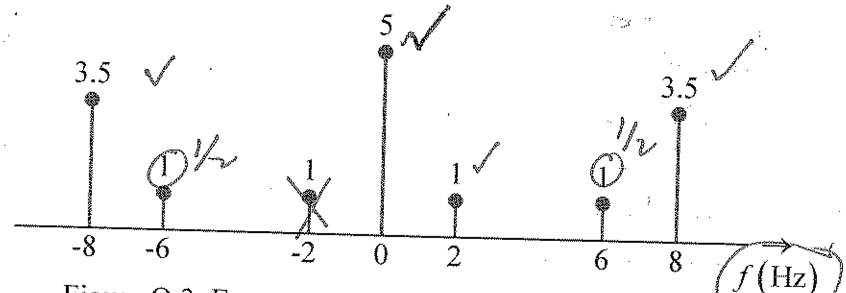


Figure Q.3: Erroneous amplitude spectrum for $x(t)$

rad/s

(b) Does $x(t)$ contain a 2nd harmonic component? Explain how you arrived at the conclusion.

(c) What is the relationship between the amplitude spectrum of $x(t)$ and $x(t-0.05)$?

(d) Sketch the phase (in radians) versus the Fourier Series index, *k*, graph of $x(t-0.05)$.

Q.3 ANSWER

$2\pi f$

(a) $x(t) = 5 + 7\cos 8t + \sin 6t + e^{j2t}$ $\quad e^{j2\pi f_0 t} \leftrightarrow \delta(f - f_0)$

$X(f) = 5\delta(f) + 7\left(\frac{1}{2}\right)\left\{\delta\left(f - \frac{4}{\pi}\right) + \delta\left(f + \frac{4}{\pi}\right)\right\} +$

$\frac{1}{2j}\left\{\delta\left(f - \frac{3}{\pi}\right) - \delta\left(f + \frac{3}{\pi}\right)\right] + \delta\left(f - \frac{1}{\pi}\right)$

Expect components @ $f = 0$, $f = \pm 4/\pi$, $\pm 6/\pi$, $\frac{1}{\pi}$

or ω @ 0, 2, 6, 8 rad/s

(b) HCF (2, 6, 8) = 2 rad/s

2nd harmonic = 4 rad/s.

∴ No component @ 4 rad/s.

~~$f = 2$~~ $\quad \omega = 2 = 2\pi f$

$f = \frac{1}{\pi}$.

Q.3 ANSWER ~ continued

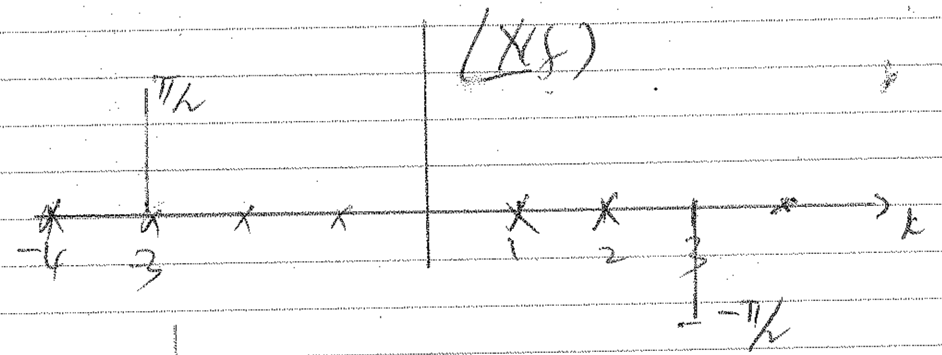
(c) $\mathcal{F}\{x(t-0.05)\} = X(f)e^{-j2\pi f(0.05)}$

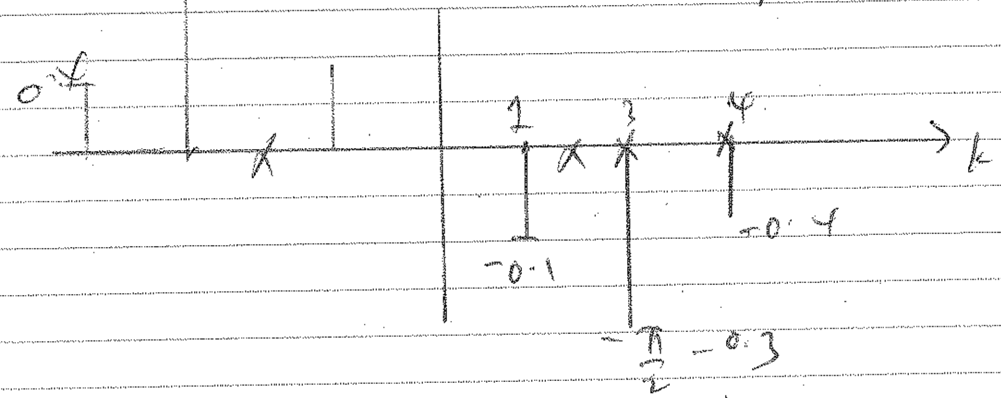
$|X(f)e^{-j2\pi f 0.05}| = |X(f)|$

No change in amplitude spectrum.

(d) $\angle X(f)e^{-j0.1\pi f} = \angle X(f) - 0.1\pi f = -0.1\pi\, k\frac{1}{\pi}$

$= -0.1k.$





fundamental ~~(i) harmonic~~, $f = \cancel{2\pi}\frac{\omega}{2\pi} = \frac{2}{2\pi} = \frac{1}{T_1}.$

3rd harmonic, $f = \frac{3}{T_1}$

Q.4. (a) Determine the Fourier transform of the signal $x(t)$ shown in Figure Q.4(a).

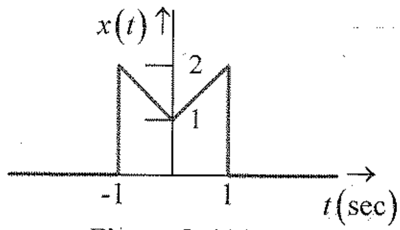


Figure Q.4(a)

(b) Using the Dirac-δ replication property, write the expression that shows the relationship between $x(t)$ and the periodic signal $x_p(t)$ shown in Figure Q.4(b).

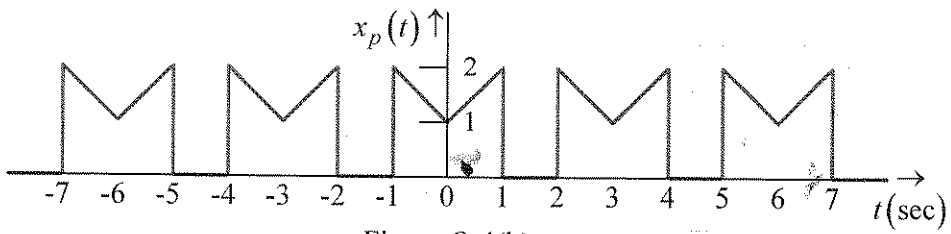


Figure Q.4(b)

(c) Determine the Fourier transform of the periodic signal $x_p(t)$.

Q.4 ANSWER

(a) $x(t) = 2\,\text{rect}(t/2) - \text{tri}(t)$

$X(f) = 4\,\text{sinc}(2f) - \text{sinc}^2(f)$

(b) $x_p(t) = x(t) * \sum_{n=-\infty}^{\infty} \delta(t - 3n)$

(c) $X(f) = X(f)\, \frac{1}{3} \sum_{n=-\infty}^{\infty} \delta(f - n/3)$

$= \frac{1}{3} \sum \left[4\,\text{sinc}(2n/3) - \text{sinc}^2(n/3)\right] \delta(f - n/3)$