EE2023 SIGNALS & SYSTEMS PAST-YEAR **EXAM** ARCHIVE

Semester I : 2013/2014

w/ Numeric Answers appended

SECTION A : Answer ALL questions in this section

Q1. The dynamic model of a system is given by

$$\frac{d^2x}{dt^2} + K\frac{dx}{dt} + \alpha x(t) = T(t).$$

where $T(t)$ and $x(t)$ are the input and output respectively, and $K, \alpha > 0$.

(a) Find the transfer function $\left(G(s) = \dfrac{X(s)}{T(s)} \right)$ of the system in terms of K and α.

(2 marks)

(b) Let $K = 2$ and $\alpha = 5$. What happens to the steady state gain of the system if K is doubled?

(2 marks)

(c) For the same values of K and α in part (b), show what happens to the poles of $G(s)$ if K is doubled. Show the effect in relation to the position of the poles on the complex plane.

(3 marks)

(d) For the same values of K and α in part (b), show what happens to the damping ratio of $G(s)$ if α is doubled while keeping K constant. Explain the effect on the response to a unit step input.

(3 marks)

Q2. Given the periodic signal $x(t) = 5\cos(4t) + 2\cos\left(6t + \dfrac{\pi}{6}\right) + 10$.

(a) Determine the complex Fourier series coefficients of $x(t)$. (4 marks)

(b) Determine the Fourier transform of $x(t)$. (3 marks)

(c) Determine the average power of $x(t)$. (3 marks)

Q3. Consider the signal $x(t) = t\exp(-t)u(t)$ where $u(t)$ is the unit step function. The spectrum of $x(t)$ is given by $X(f) = \dfrac{1}{(1+j2\pi f)^2}$.

(a) Let $y(t) = 2t\exp(-0.5t)u(t)$.

(i) Express $y(t)$ in terms of $x(t)$.

$$\left[Hint:\ u(\alpha t) = \begin{cases} u(t); & \alpha > 0 \\ u(-t); & \alpha < 0 \\ 1; & \alpha = 0 \end{cases} \right]$$

(3 marks)

(ii) Based on the result obtained in Part (i), or otherwise, which of the two signals, $x(t)$ or $y(t)$, would you expect to have a larger bandwidth, and why?

(3 marks)

(b) Determine the spectrum of $z(t) = \begin{cases} x(t); & t \geq 0 \\ x(-t); & t < 0 \end{cases}$.

(4 marks)

Q4. The unit impulse response of a standard second order system, $G(s)$, is shown in Figure Q4

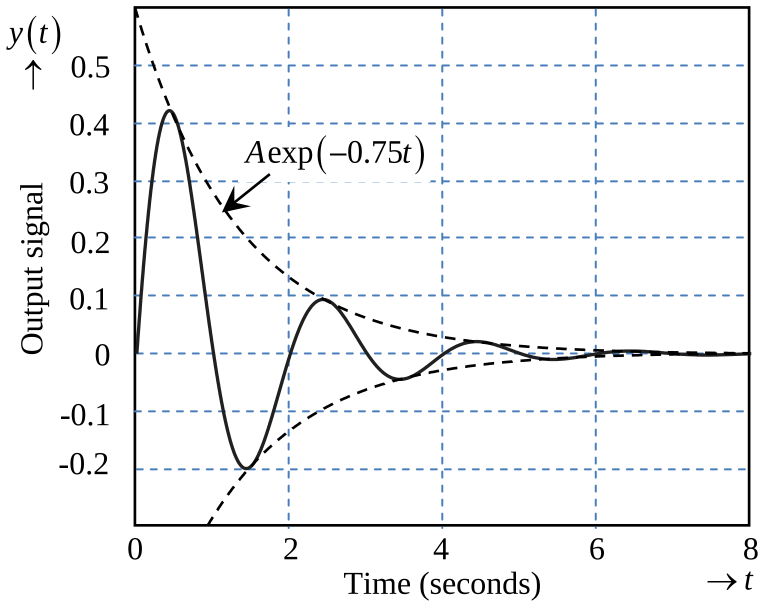


Figure Q4: Unit Impulse Response of $G(s)$

(a) Determine the poles of $G(s)$.

(4 mark)

(b) Is the system stable, marginally stable or unstable? Justify your answer.

(2 mark)

(c) Identify all the functions from the following list that may be terms in the output signal when the quadratic signal, $t^2u(t)$, is applied to the system?

- Decaying exponential function, $Ae^{-at}u(t)$
- Growing exponential function, $Be^{at}u(t)$
- Decaying complex exponential function, $e^{-at}(C_1 \sin \omega t + C_2 \cos \omega t)u(t)$
- Growing complex exponential function, $e^{at}(C_1 \sin \omega t + C_2 \cos \omega t)u(t)$
- Step function, $Du(t)$
- Ramp function, $Etu(t)$
- Quadratic function, $Ft^2u(t)$

$u(t)$ is the unit step function, A, B, C_1, C_2, D, E and F are constants.

(4 mark)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. Figure Q5 shows a Butterworth filter which can be designed using a resistor, an inductor and a capacitor.

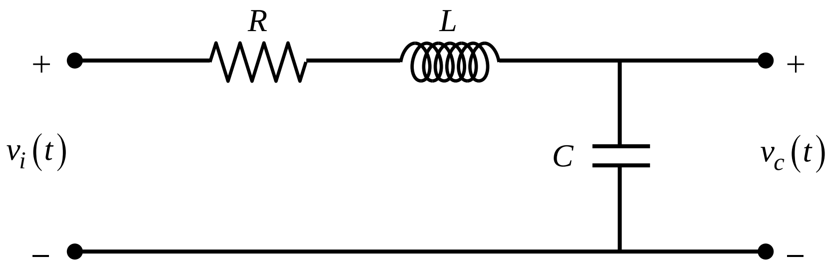


Figure Q5: A Butterworth Filter

Let the input and output of the filter be $v_i(t)$ and $v_c(t)$, respectively.

(a) Show that the transfer function of the filter is given by

$$G(s) = \frac{V_c(s)}{V_i(s)} = \frac{1}{s^2LC + sRC + 1}$$

where $V_i(s)$ and $V_c(s)$ are the Laplace transforms of $v_i(t)$ and $v_c(t)$, respectively.

(3 marks)

(b) Derive the relationship between R, L and C if the squared magnitude response of the filter has the following characteristics :

$$|G(j\omega)|^2 = \frac{1}{1+\omega^4 L^2 C^2}.$$

(4 marks)

(c) Assume $R = \sqrt{2L/C}$.

i. Find the damping ratio of the filter.

(4 marks)

ii. Suppose the 3 dB bandwidth of the filter is ω_c rad/s. Find ω_c in terms of L and C.

(3 marks)

iii. Design a Butterworth filter with a 3 dB bandwidth of 10^6 rad/s and R = 1 kΩ.

(6 marks)

Q.6 (a) Figure Q6(a) shows an emergency signaling system. The transmitter transmits a signal, $m(t) = x(t)c(t)$, where

$$x(t) = \cos(10\pi t)$$

is the emergency tone and

$$c(t) = \cos(1600\pi t)$$

is the carrier wave. The transmitter and receiver are connected by an ideal communication channel.

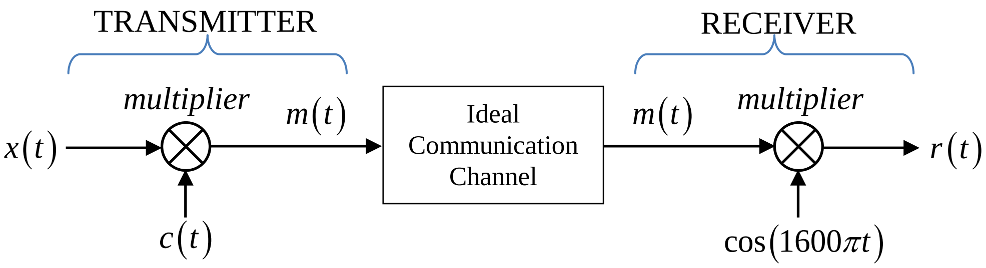


Figure Q6(a): Emergency Signaling System

i. Sketch the spectrum of $m(t)$.

(4 marks)

ii. Derive the expression for the receiver output, $r(t)$.

(4 marks)

iii. Can $x(t)$ be recovered from $r(t)$ by passing $r(t)$ through a lowpass filter? If 'YES', specify the filter. If 'NO', explain why?

(2 marks)

(b) Derive the Fourier transform of the periodic signal shown in Figure Q6(b) of which the double-hump generating function is a full-wave rectified sine pulse.

(10 marks)

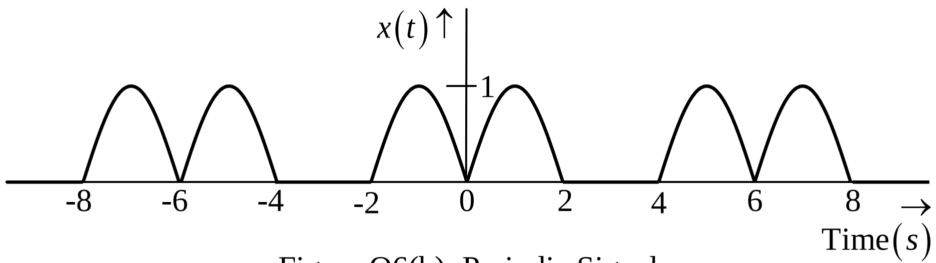


Figure Q6(b): Periodic Signal

Q7. Figure Q7(a) shows the block diagram of a signal generator where $x(t)$ is the source signal, $h(t)$ is the impulse response of the filter, and $y(t)$ is the desired signal.

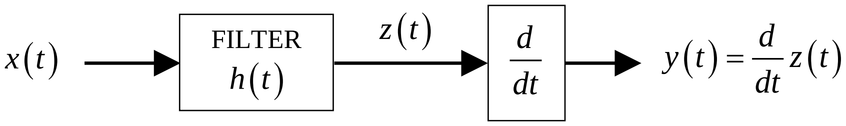


Figure Q7(a): Signal Generator

The impulse response of the filter is given by

$$h(t) = \frac{3}{4}\operatorname{sinc}(150t)\cos(2\pi f_o t),$$

where $100 \text{ Hz} \leq f_o \leq 1000 \text{ Hz}$, and the spectrum of $x(t)$ is shown in Figure Q7(b).

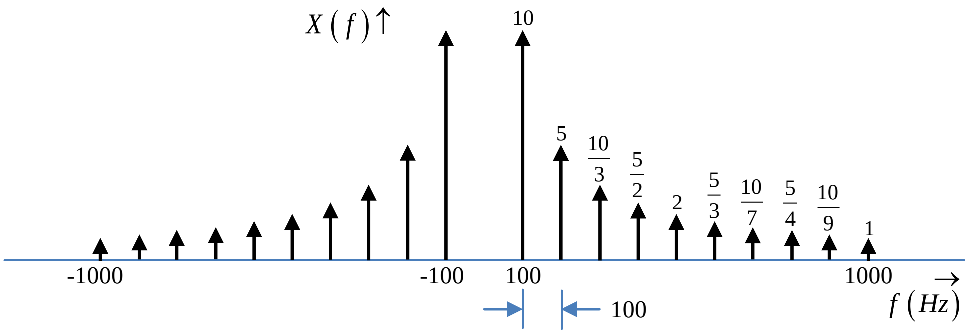


Figure Q7(b): Spectrum of $x(t)$

(a) (i) Is $x(t)$ an energy or power signal?

(ii) Is $x(t)$ a real, imaginary or complex signal?

(iii) Is $x(t)$ an odd or even function of t?

(iv) Is $x(t)$ periodic? If 'YES', what is the period?

(v) What is the DC value of $x(t)$?

(5 marks)

(b) Determine the frequency response, $H(f) = \Im\{h(t)\}$, of the filter.

(5 marks)

(c) Suppose $f_o = 450 \text{ Hz}$. Using the result of Part (b), or otherwise, find $y(t)$.

$[\mathit{Hint: Find}\ z(t)\ \mathit{first}]$

(6 marks)

(d) How many different signals can the signal generator produce if the value of f_o can be continuously adjusted between 100 Hz and 1000 Hz?

(4 marks)

Q8. Consider a system, which may be approximated by a first order plus dead-time factor $G_a(s) = \dfrac{Ke^{-sL}}{(\tau s+1)}$.

(a) The following observations were obtained from experiments conducted to identify the first order plus dead-time factor, $G_a(s)$:

- The steady-state output value of the system is 26 if the input signal is $2u(t)$ where $u(t)$ is the unit step function.
- When the input signal is $x(t) = 7\cos(5t+30°)$, the steady-state output signal is $\lim_{t\to\infty} y(t) = \dfrac{91}{\sqrt{2}}\cos(5t-40°)$.

Using this information, determine *K*, τ and *L*, the parameters of $G_a(s)$.

(10 marks)

(b) Further experiments revealed that the system should be modelled as $G(s) = G_a(s)\cdot G_b(s)$. The Bode magnitude plot of $G(s)$ is shown in Figure Q8.

- Using results from part (a), show that the value of *M* and the first corner frequency, ω_1, in the Bode magnitude plot are 22.3 dB and 5 rad/s respectively.

  [*Hint: The contribution of* $G_b(s)$ *to* M *and* ω_1 *is negligible compared to that of* $G_a(s)$]

- What is the transfer function $G(s)$?

(10 marks)

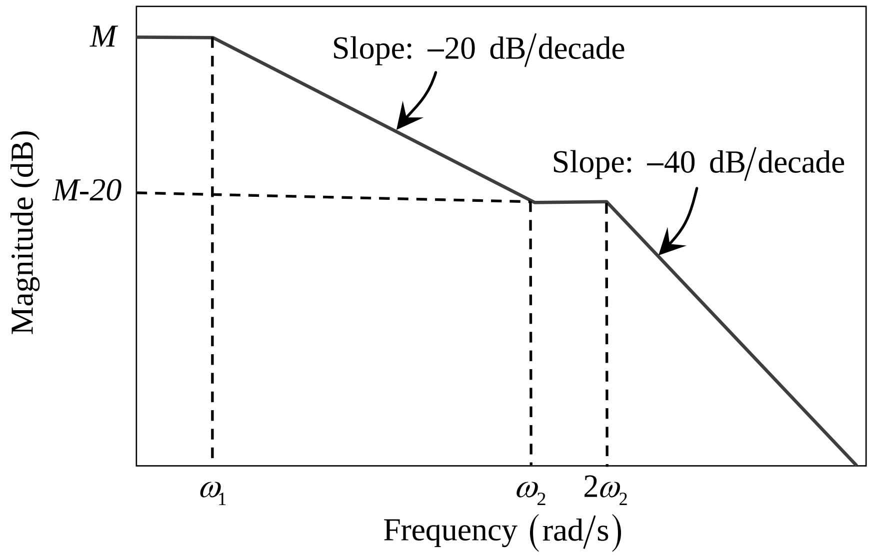


Figure Q8: Bode Magnitude Plot of $G(s)$

END OF QUESTIONS

NUMERIC ANSWERS

Section A

Q.1 (a) $G(s)=\dfrac{1}{s^2+Ks+\alpha}$

(b) Steady state gain does not change

(c) Poles move further to the left of the complex plane and transient response decays faster

(d) Damping ratio becomes smaller, transient response is more oscillatory

Q.2 (a) $c_{-3}=e^{-j\pi/6}$, $c_{-2}=5/2$, $c_0=10$, $c_2=5/2$, $c_3=e^{j\pi/6}$ and $c_k=0;\ |k|\neq 0,2,3$

(b) $\Big\{ X(f)=e^{-j\pi/6}\delta\left(f+\dfrac{3}{\pi}\right)+\dfrac{5}{2}\delta\left(f+\dfrac{2}{\pi}\right)+10+\dfrac{5}{2}e^{-j4}\delta\left(f-\dfrac{2}{\pi}\right)+e^{j\pi/6}\delta\left(f-\dfrac{3}{\pi}\right)$

(c) 114.5

Q.3 (a)(i) $y(t)=2t\exp(-0.5t)u(t)=4x(0.5t)$

(a)(ii) $x(t)$

(b) $Z(f)=\dfrac{2\left(1-4\pi^2f^2\right)}{\left(1+4\pi^2f^2\right)^2}$

Q.4 (a) $s=-0.75\pm j\pi$

(b) Stable

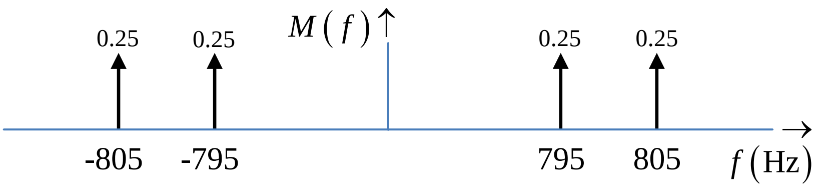
(c) $y(t)=\left[e^{-0.75t}\left(a\cos(\pi t)+b_1\sin(\pi t)\right)+ct^2+dt+e\right]u(t)$

Section B

Q.5 (a) - - - (b) $R=\sqrt{2L/C}$

(c)(i) $\varsigma=1/\sqrt{2}$ (c)(ii) $\omega_c=1/\sqrt{LC}$ (c)(iii) $\begin{cases} L=5\sqrt{2}\times 10^{-4} \\ C=\sqrt{2}\times 10^{-9} \end{cases}$

Q.6 (a)(i)



(a)(ii) $r(t)=0.5\cos(10\pi t)+0.5\cos(10\pi t)\cos(3200\pi t)$

(a)(iii) LPF with 5 Hz bandwidth

(b) $X_p(f)=\dfrac{1}{3}\sum_k\left[\mathrm{sinc}\left(\dfrac{k}{3}+\dfrac{1}{2}\right)\sin\left(\dfrac{k\pi}{3}+\dfrac{\pi}{2}\right)-\mathrm{sinc}\left(\dfrac{k}{3}-\dfrac{1}{2}\right)\sin\left(\dfrac{k\pi}{3}-\dfrac{\pi}{2}\right)\right]\delta\left(f-\dfrac{k}{6}\right)$

Q.7 (a) (i) Power signal.
(ii) Real signal.
(iii) Even function t.
(iv) Periodic with period = 0.01 sec
(v) DC value = 0

(b) $H(f)=\dfrac{1}{400}\mathrm{rect}\left(\dfrac{f-f_o}{150}\right)+\dfrac{1}{400}\mathrm{rect}\left(\dfrac{f+f_o}{150}\right)$

(c) $y(t)=-10\pi\left[\sin(800\pi t)+\sin(1000\pi t)\right]$

(d) 19

Q.8 (a) $K=13 \quad \tau=0.2 \quad L=\dfrac{\pi}{36}=0.087$

(b) $G(s)=\dfrac{13(0.02s+1)}{(0.2s+1)(0.01s+1)^2}e^{-s\frac{\pi}{36}}$