EE2023 Signals & Systems Quiz
Semester 2 AY2012/13

Date : 7 March 2013 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. No programmable or graphic calculator is allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ______________________________

Matric # : ____________________________

Lecture Group # : ______________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

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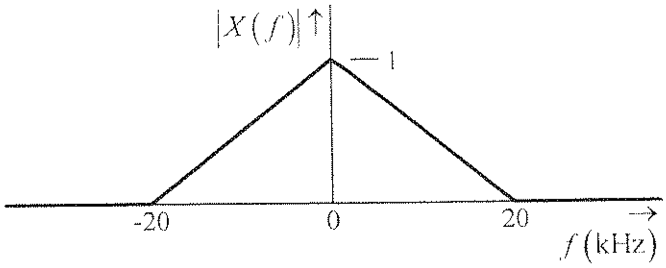
Group 2 : A/Prof Ng Chun Sum
Group 3 : A/Prof Tan Woei Wan
Group 4 : Prof Lawrence Wong

Q.1 The signal $x(t)$ has an amplitude spectrum given in Figure Q.1 below.



(a) What is the minimum sampling frequency that will allow $x(t)$ to be recovered from its samples?

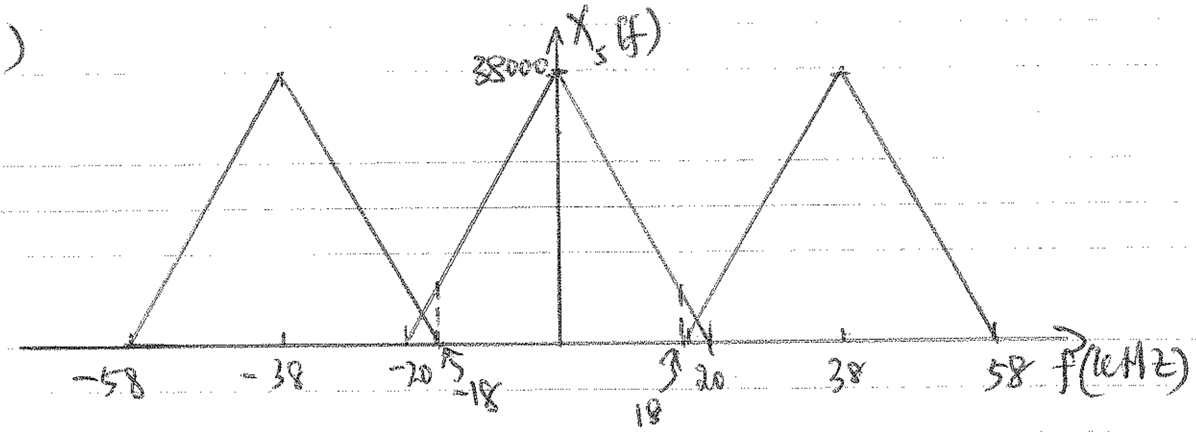
(b) Suppose that $x(t)$ is sampled at a rate of $f_s = 38$ kHz. What part of $X(f)$ can be recovered without error from the samples? Explain your answer with an illustration of the appropriate spectrum.

(c) Is $x(t)$ a power or energy signal? Explain your answer.

Q.1 ANSWER

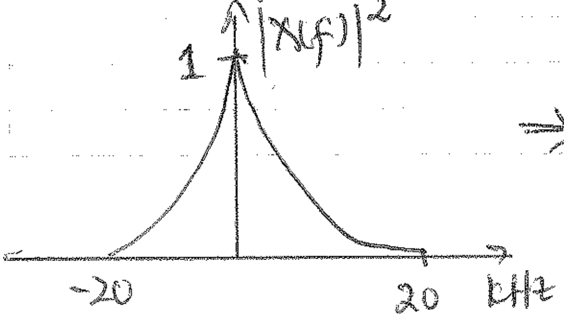
(a) minimum sampling frequency = 40 kHz.

(b)



part that can be recovered with LPF is between frequencies -18 kHz to 18 kHz

(c) Energy ~~power~~ spectral density : $E_s(f) = |X(f)|^2$



$\Rightarrow$ clear that $E = \int_{-\infty}^{\infty} |X(f)|^2 df < \infty$

$\therefore$ Energy signal

Q2. Consider the complex signal $x(t) = \text{sinc}(t) + j\ \text{sinc}(t)$.

(a) Find the energy spectral density of $x(t)$. Hence, or otherwise, determine the total energy of $x(t)$.

(b) Suppose $y(t) = |x(t)|^2$. Find the 3dB bandwidth of $y(t)$.

Q.2 ANSWER

(a) $x(t) = \text{sinc}(t) + j\,\text{sinc}(t)$

$A\,\text{rect}(t/T) \xleftrightarrow{F.T.} AT\,\text{sinc}(fT)$

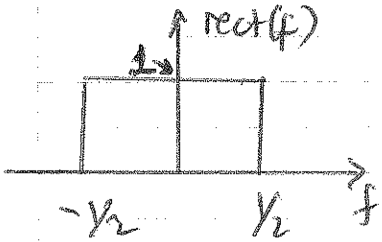
$A\,\text{rect}(f/T) \xleftrightarrow{F.T.} AT\,\text{sinc}(tT)$ → By duality

$\text{rect}(f) \xleftrightarrow{F.T.} \text{sinc}(t)$ $A = 1,\ T = 1.$

$\therefore X(f) = \text{rect}(f) + j\,\text{rect}(f)$

$|X(f)|^2 = \text{rect}^2(f) + \text{rect}^2(f) = 2\,\text{rect}^2(f)$ # ← ESD

Total energy $= \int_{-\infty}^{\infty} |X(f)|^2 df.$



$= \int_{-\infty}^{\infty} 2\,\text{rect}^2(f)\,df$

$= 2\int_{-\infty}^{\infty} \text{rect}^2(f)\,df = 2\int_{-1/2}^{1/2} 1\,df.$

$= 2$ #

(b) $y(t) = |x(t)|^2$

$= \text{sinc}^2(t) + \text{sinc}^2(t)$

$= 2\,\text{sinc}^2(t)$

$\text{tri}(t/T) \longleftrightarrow T\,\text{sinc}^2(fT)$

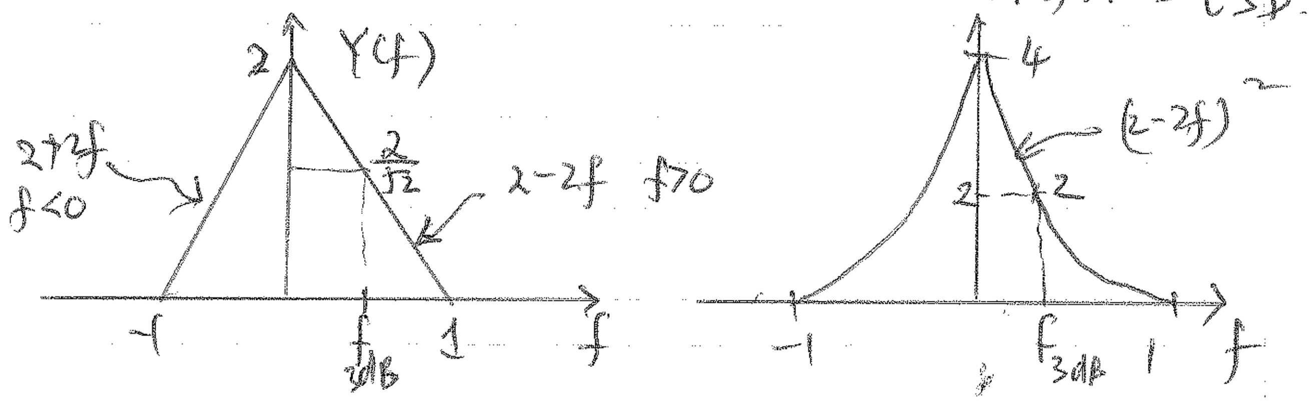
$\text{tri}(f/T) \longleftrightarrow T\,\text{sinc}^2(tT)$

$2\,\text{tri}(f) \longleftrightarrow 2\,\text{sinc}^2(t)$

Q.2 ANSWER ~ continued

$$\therefore Y(f) = 2\,\mathrm{tri}(f)$$

$$|Y^2(f)| = 4\,\mathrm{tri}^2(f)$$



3-dB bandwidth can be determined from either the plot of $Y(f)$ or $|Y(f)|^2$. Easier to see it from $Y(f)$ because you deal with a straight line.

At $f = f_{3dB}$: $Y(f) = \frac{2}{\sqrt{2}}$

$$\therefore 2 - 2f_{3dB} = \frac{2}{\sqrt{2}}.$$

$$1 - f_{3dB} = \frac{1}{\sqrt{2}}.$$

$$f_{3dB} = 1 - \frac{1}{\sqrt{2}} \quad \#$$

Q.3 The complex exponential Fourier Series representation of a periodic signal, $x(t)$, is

$$\sum_{k=-\infty}^{\infty} c_k e^{jkt}$$

where the Fourier Series coefficients are $c_k = \begin{cases} 0.49-0.1j & : \quad k=-2 \\ 1.5j\, e^{j0.1} & : \quad k=-1 \\ 2 & : \quad k=0 \\ -1.5j\, e^{-j0.1} & : \quad k=1 \\ 0.49+0.1j & : \quad k=2 \\ 0 & : \quad otherwise \end{cases}$

(a) What is the fundamental period (seconds) and fundamental frequency (Hz) of the signal, $x(t)$?

(b) Write an equation for the continuous frequency spectrum, $X(f)$, of the signal.

(c) Express $x(t)$ as a function of real sinusoids and constant.

Q.3 ANSWER

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jkt}$$

(a) fundamental component is obtained from the frequency of $x(t)$ for $k=1$. This is because the general expression for Fourier series is given by $x(t) = \sum c_k e^{j2\pi f_0 k t}$

When $k=1$, component has freq $2\pi f_0$. This happens to be 1 in the expression $\sum c_k e^{jkt}$

$\therefore \; 2\pi f_0 = 1$.

$f_0 = \frac{1}{2\pi}$ Hz #

$T_0 = \frac{1}{f_0} = 2\pi$ sec #

Q.3 ANSWER ~ continued

(b) The continuous freq spectrum is the Fourier transform of a periodic signal.

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk2\pi f_0 t}$$

$$\Big\Updownarrow \text{F.T.} \quad \text{continuous freq spectrum}$$

$$X(f) = \sum c_k \delta(f - kf_0)$$

$$\therefore X(f) = \sum_{k=-\infty}^{\infty} c_k \delta\left(f - \frac{k}{2\pi}\right)$$

(c) $x(t) = (0.49 - j0.1)e^{-j2t} + (0.49 + j0.1)e^{j2t} + 1.5je^{j0.1}e^{-jt} + (-1.5je^{-j0.1})e^{jt} + 2$

$= 0.98\cos(2t) - 0.2\sin(2t) + 3\sin(t - 0.1) + 2$

Q.4 The signal $x(t)$ is shown in Figure Q.4(a).

(a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$.

(b) Using the replication property of the Dirac-δ function, obtain an expression for the periodic signal $x_p(t)$ shown in Figure Q.4(b) in terms of $x(t)$.

(c) Derive the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$.

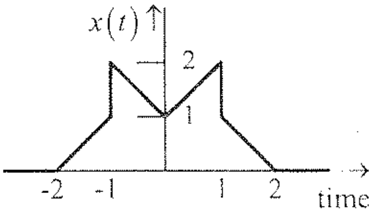


Figure Q.4(a)

period 5 sec
fundamental freq $= \frac{1}{5}$

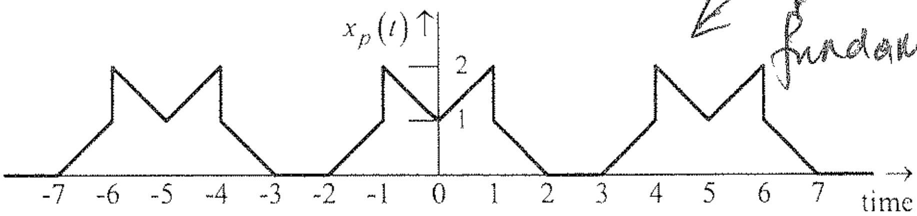


Figure Q.4(b)

Q.4 ANSWER

(a) $x(t) = 2\,\mathrm{tri}(t/2) + \mathrm{rect}(t/2) - 2\,\mathrm{tri}(t)$

$$X(f) = 4\,\mathrm{sinc}^2(2f) + 2\,\mathrm{sinc}(2f) - 2\,\mathrm{sinc}^2(f)$$

(b) $$x_p(t) = x(t) \circledast \sum_{k=-\infty}^{\infty} \delta(t - 5k)$$

$$= \sum_{k=-\infty}^{\infty} x(t - 5k)$$

(c) Recall from tutorial 4 that

$$X_p(f) = \sum X_k\, \delta(f - k/f)$$

where $X_k = \frac{1}{T_p} X(f_k) = \frac{1}{5} X(k f_p) = \frac{1}{5} X\left(\frac{k}{5}\right)$

fourier coefficients of $X_p(f)$

Q.4 ANSWER ~ continued

$$X(f) = 4\,\mathrm{sinc}^2(2f) + 2\,\mathrm{sinc}(2f) - 2\,\mathrm{sinc}^2(f)$$

$$\therefore X(k/5) = 4\,\mathrm{sinc}^2\left(\frac{2k}{5}\right) + 2\,\mathrm{sinc}\left(2\frac{k}{5}\right) - 2\,\mathrm{sinc}^2\left(\frac{k}{5}\right)$$

$$\therefore X_p(f) = \frac{1}{5}\sum_{k=-\infty}^{\infty}\left[4\,\mathrm{sinc}^2\left(\frac{2k}{5}\right) + 2\,\mathrm{sinc}\left(\frac{2k}{5}\right) - 2\,\mathrm{sinc}^2\left(\frac{k}{5}\right)\right]\delta\left(f-\frac{k}{5}\right)$$

Key point here is that $x(t)$ is the generating function for $X_p(f)$. The Fourier coefficients of $X_p(f)$ is a sampling of the F.T. of $x(t)$

Proof: $x_p(t) = x(t) \circledast \sum \delta(t - T_p k)$

$$X_p(f) = X(f)\frac{1}{T_p}\sum \delta(f - k/T_p)$$

$$= \frac{1}{T_p}\sum \underbrace{X(kf_p)}_{\text{sampling of } X(f)}\delta(f - kf_p)$$