Quiz Solution : EE2023 Signals & Systems

Semester 1 AY2011/12

Question 1

Consider the periodic signal $x(t)$ given by the expression

$$x(t) = (2+2j)e^{-j3t} - 3je^{-2t} + 5 + 3je^{j2t} + (2-2j)e^{j3t}$$

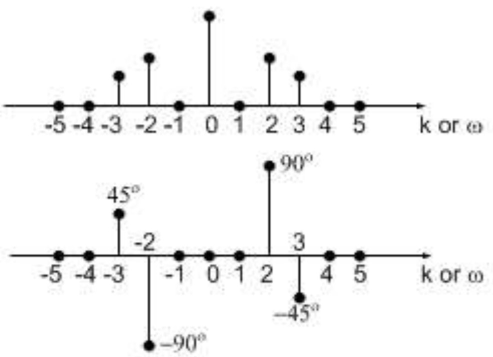
a) What is the fundamental period and fundamental frequency of $x(t)$?
b) Sketch the amplitude and phase spectra of $x(t)$.
c) Is $x(t)$ a real signal ? Justify your answer.
d) What is the power of $x(t)$?

Answer

(a) Fundamental frequency of $x(t)$ = HCF{2,3} = 1 rad/s

Fundamental period of $x(t)$ = $\frac{2\pi}{1} = 2\pi$ seconds.

(b) Amplitude and Phase Spectrum



(c) $x(t)$ is a real signal because the amplitude spectra is even, and the phase spectrum is odd.

The Fourier Series coefficients satisfy the condition that $c_{-k} = c_k^*$

(d) Using the Parseval's Theorem, power of $x(t)$ is

$$\sum_{k=-\infty}^{\infty} |c_k|^2 = |2+2j|^2 + |-3j|^2 + |5|^2 + |3j|^2 + |2-2j|^2 = 2(8+9) + 25 = 59$$

Question 2

(a) Derive the Fourier transform of the signal $x(t)$ shown in Figure Q2-1.

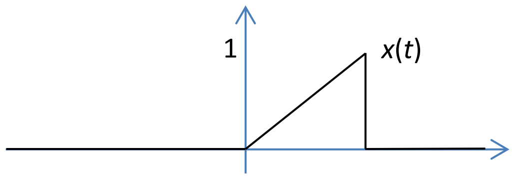


Figure Q2-1

[Hint: $\text{rect}(t/T) \Leftrightarrow T.\text{sinc}(fT)$ and $u(t) \Leftrightarrow 0.5[\delta(f) + 1/(j\pi f)]$]

(b) Derive the Fourier transform of the periodic signal $y(t)$ shown in Figure Q2-2.

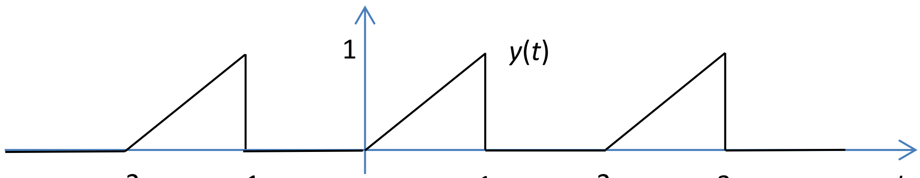


Figure Q2-2

Answer

(a) The linear ramp pulse can be obtained as follows:

$$x(t) = \int_{-\infty}^{t} \text{rect}(\tau - 0.5)d\tau - u(t)$$

Hence the Fourier transform is:

$$\begin{aligned} X(f) &= \left[\frac{1}{j2\pi f}\text{sinc}(f) + \pi X(0)\delta(2\pi f)\right]\exp(-j\pi f) - \frac{1}{2}\left[\delta(f) + \frac{1}{j\pi f}\right] \\ &= \left[\frac{1}{j2\pi f}\text{sinc}(f) + \pi\delta(2\pi f)\right]\exp(-j\pi f) - \frac{1}{2}\left[\delta(f) + \frac{1}{j\pi f}\right] \end{aligned}$$

(b) The Fourier transform of $y(t)$ is:

$$\begin{aligned} Y(f) &= \sum_{k=-\infty}^{\infty} c_k \delta\left(f - \frac{k}{2}\right) \\ &= \sum_{k=-\infty}^{\infty} \frac{1}{2}\left\{\left[\frac{1}{j2\pi(k/2)}\text{sinc}\left(\frac{k}{2}\right) + \pi\delta\left(2\pi\frac{k}{2}\right)\right]\exp\left(-j\pi\frac{k}{2}\right) - \frac{1}{2}\left[\delta\left(\frac{k}{2}\right) + \frac{1}{j\pi(k/2)}\right]\right\}\delta\left(f - \frac{k}{2}\right) \\ &= \sum_{k=-\infty}^{\infty} \frac{1}{2}\left\{\left[\frac{1}{j\pi k}\text{sinc}\left(\frac{k}{2}\right) + \pi\delta(\pi k)\right]\exp\left(-j\pi\frac{k}{2}\right) - \frac{1}{2}\left[\delta\left(\frac{k}{2}\right) + \frac{2}{j\pi k}\right]\right\}\delta\left(f - \frac{k}{2}\right) \end{aligned}$$

Question 3

A signal is modeled by $x(t)=2rect\left(\frac{t}{2}\right)\ast rect\left(\frac{t}{2}\right)$ where $\ast$ denotes convolution. Determine the spectrum, $X(f)$, of $x(t)$. Sketch and label the Energy Spectral Density (ESD) and Power Spectral Density (PSD) of $x(t)$ for frequencies between -2Hz and 2Hz.

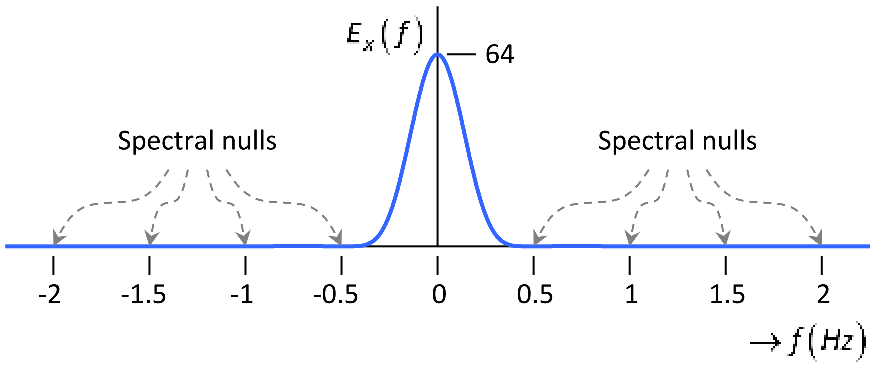
Answer

Applying $\Im\left\{\text{rect}\left(\frac{t}{T}\right)\right\}=T\text{sinc}(fT)$ and the "Convolution in time-domain" property of the Fourier transform, we get

$X(f)=2\left(2\text{sinc}(2f)\cdot 2\text{sinc}(2f)\right)=8\text{sinc}^2(2f)$.

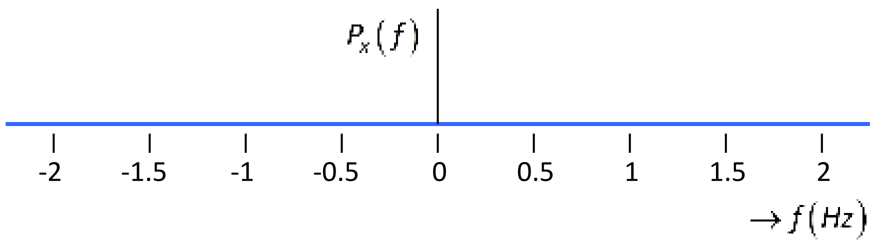
(3 marks)

ESD: $E_x(f)=\left|X(f)\right|^2=64\text{sinc}^4(2f)$



(4 marks)

PSD: Since the energy of $x(t)$ is finite, i.e. $0<\int_{-\infty}^{\infty}E_x(f)df<\infty$, its average power must be equal to zero, or $\int_{-\infty}^{\infty}P_x(f)df=0$. This together with the fact that $P_x(f)\geq 0$ implies that $P_x(f)=0;\ \forall f$.



(3 marks)

Question 4

Consider 2 signals, $x_1(t) = \sin 4\pi t, x_2(t) = 2\cos 8\pi t$. Suppose $y(t) = x_1(t)x_2(t)$. Write down the Fourier Transforms, $X_1(f)$, $X_2(f)$ and $Y(f)$ where $X_1(f) \Leftrightarrow x_1(t)$, $X_2(f) \Leftrightarrow x_2(t)$ and $Y(f) \Leftrightarrow y(t)$. Sketch their amplitude spectra.

Assume that $y(t)$ is sampled with a sampling frequency of $f_s = 10$ Hz. Sketch the amplitude spectrum of the sampled signal. Can $y(t)$ be reconstructed completely from the sampled signal?

Answer

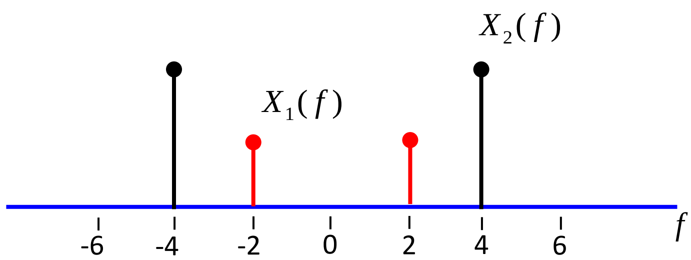
$$x_1(t) = \frac{1}{2j}\left(e^{j4\pi t} - e^{-j4\pi t}\right) \Leftrightarrow X_1(f) = \frac{1}{2j}\left(\left(\delta(f-2) - \delta(f+2)\right)\right.$$

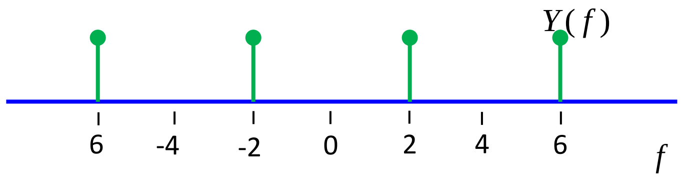
$$x_2(t) = \left(e^{j8\pi t} + e^{-j8\pi t}\right) \Leftrightarrow X_2(f) = \delta(f-4) + \delta(f+4)$$

$$y(t) = x_1(t)x_2(t) \Leftrightarrow Y(f) = X_1(f) * X_2(f)$$

$$Y(f) = \frac{1}{2j}[\delta(f-2) - \delta(f+2)] * [\delta(f-4) + \delta(f+4)]$$

$$= \frac{1}{2j}[\delta(f-6) - \delta(f-2) + \delta(f+2) - \delta(f+6)]$$





The original signal cannot be reconstructed completely because of aliaising. Nyquist freq = 12 Hz.