EE2023 SIGNALS & SYSTEMS
PAST-YEAR EXAM ARCHIVE

Semester I : 2012/2013

w/ Numeric Answers appended

SECTION A : Answer ALL questions in this section

Q1. For the system shown in Figure Q1-1 below, the transfer function, $G(s)$, is given by: $G(s)=\dfrac{K}{s+a}$.

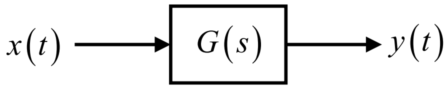


Figure Q1-1: System $G(s)$

(a) If $x(t)=2\cos(4t)$ and the steady state output, $y(t)=5\cos(4t-45°)$, find the parameters K and a.

(4 marks)

(b) Suppose $K=2$ and $a=1$ in $G(s)$ in Figure Q1-1. Sketch the output response, $y(t)$, when the input, $x(t)$, is as shown in Figure Q1-2 below. You may assume zero initial conditions. Label your sketch clearly.

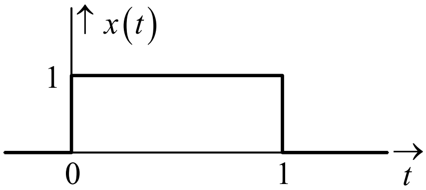


Figure Q1-2: Input signal, $x(t)$

(6 marks)

Q2. The signal $x(t)=\text{sinc}(2t)$ is sampled at 6 Hz to produce the signal $x_s(t)$.

(a) Derive the Fourier transform of the sampled signal $x_s(t)$. (6 marks)

(b) Sketch the spectrum of the sampled signal $x_s(t)$. (4 marks)

Q3. The Fourier transform of an energy signal $x(t)=\alpha^2 t\exp(-2\pi\alpha^2 t^2)$ has the form

$$X(f)=-j\frac{f}{\alpha}\exp(-0.5f^2)$$

where α is a positive real constant.

(a) Draw a labeled sketch of the phase spectrum of $x(t)$. (3 marks)

(b) Find the DC value of $x(t)$. (2 marks)

(c) The Rayleigh Energy Theorem states that the energy, E, of $x(t)$ can be computed in the time domain as well as in the frequency domain, namely,

$$E=\underbrace{\int_{-\infty}^{\infty}|x(t)|^2\,dt}_{\text{time domain}}=\underbrace{\int_{-\infty}^{\infty}|X(f)|^2\,df}_{\text{frequency domain}}.$$

Use this relationship to determine the value of α. [*Hint: There is no need to solve any integral.*]

(5 marks)

Q4. Figure Q4-1 shows the Bode magnitude plot of a system with the following transfer function

$$G(s)=\frac{K(s+a)}{\left(\frac{s}{b}+1\right)\left(s^2+2\varsigma\omega_n s+\omega_n^2\right)}.$$

(a) Using Figure Q4-1, identify the values of a, b, ω_n and K. (6 marks)

(b) Identify the feature of the Bode magnitude plot that indicates the damping ratio must lie in the range $0<\varsigma<d$. Without performing any calculations, what is the constraint on the value of d?

(2 marks)

(c) What value will the high frequency asymptote of the Bode phase plot of $G(s)$ converge to?

(2 marks)

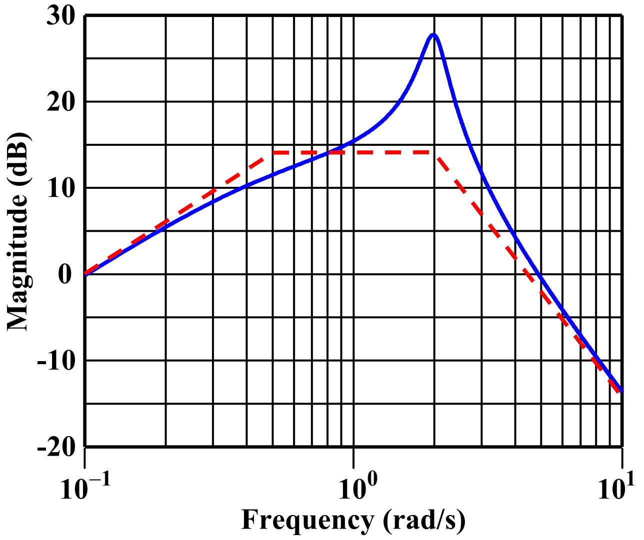


Figure Q4-1: Bode magnitude plot

SECTION B : Answer 3 out of the 4 questions in this section

Q5 (a) The system, *SYS*, shown in Figure Q5-1 below, consists of two sub-systems, *SYS1* and *SYS2*. The impulse responses of *SYS1* and *SYS2*, are given by $g_1(t)$ and $g_2(t)$, respectively, where $g_1(t) = \exp(-at)$ and $g_2(t) = \exp(-bt)$; $a, b > 0$.

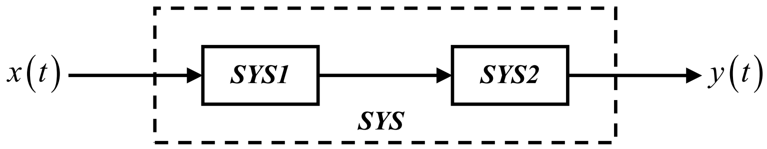


Figure Q5-1: System, *SYS*, consisting of *SYS1* and *SYS2*

Find the overall impulse response, $g(t)$, of the system, *SYS*, in terms of a and b.

(4 marks)

(b) For the RC-circuit shown in Figure Q5-2 below, find the value of the capacitance, C, such that the impulse response, $g(t)$, from $v(t)$ to $v_c(t)$ is given by $g(t) = \exp(-t)$.

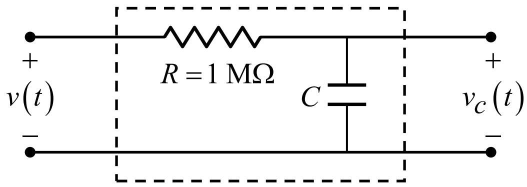


Figure Q5-2: RC-Circuit

(4 marks)

(c) Design the filter circuit (by finding suitable values of K, R_2 and C_2) in Figure Q5-3 such that the transfer function of the overall system, $G(s)$, is given by:

$$G(s) = \frac{V_0(s)}{V(s)} = \frac{1}{(s+1)(s+3)}$$

where $V(s) = \mathcal{L}\{v(t)\}$ and $V_0(s) = \mathcal{L}\{v_0(t)\}$. Assume $R_1 = 0.5\ \text{M}\Omega$ and $C_1 = 2\ \mu F$.
[*Hint: You may consider the filter circuit to consists of 3 connected subsystems.*]

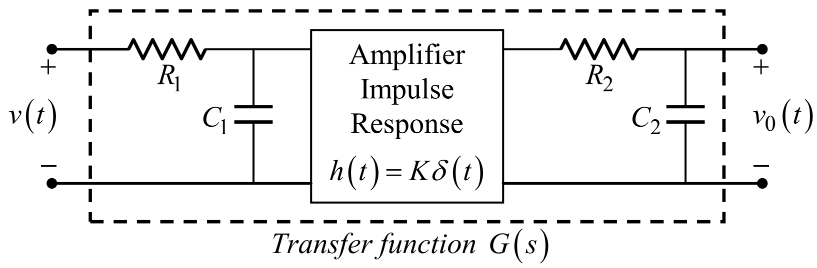


Figure Q5-3: Filter Circuit

The 3-dB bandwidth of a system is defined to be the frequency where the magnitude of the frequency response drops by $\frac{1}{\sqrt{2}}$ of the DC gain. Find the 3-dB bandwidth of the system, $G(s)$.

(12 marks)

Q.6 Consider the periodic signal $x(t)$ shown in Figure Q6.

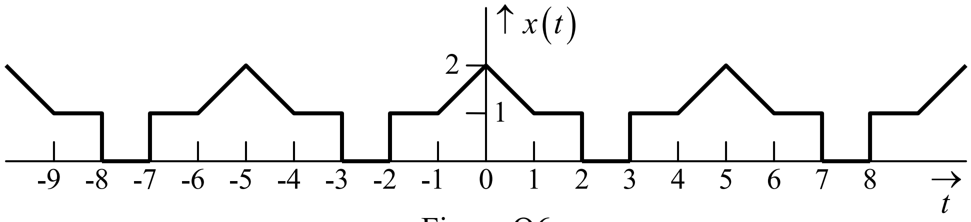


Figure Q6

(a) Derive the Fourier transform of $x(t)$. (6 marks)

(b) Derive the Fourier series coefficients of $x(t)$. (4 marks)

(c) Evaluate the average power of $x(t)$. (10 marks)

Q7. Figure Q7-1 shows the demodulator of a radio receiver where $x(t)$ is the transmitted message and $\hat{x}(t)$ is the demodulated message. The spectrum of $x(t)$ is shown in Figure Q7-2, and the lowpass filter has a frequency response given by

$$H(f) = \alpha \cdot \mathrm{rect}\left(\frac{f}{\beta}\right)\exp(-j\gamma f)$$

where α, β and γ are constants.

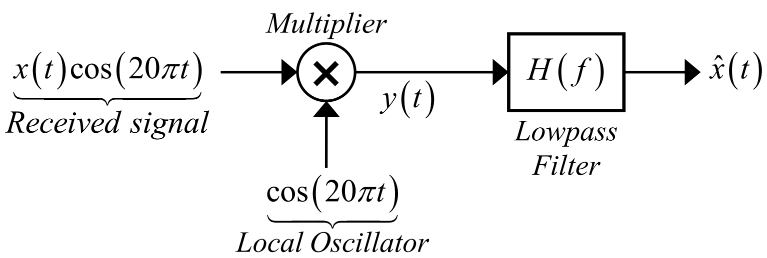


Figure Q7-1

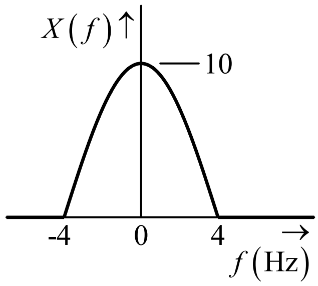


Figure Q7-2

(a) Draw a labeled sketch of the spectrum of $y(t)$. (6 marks)

(b) Find the values of α, β and γ so that $\hat{x}(t) = x(t)$. (6 marks)

(c) Based on the results in parts (a) and (b) above, explain how the requirements of $H(f)$ can be relaxed to reduce implementation cost while $\hat{x}(t) = x(t)$ can still be achieved. Provide a labeled sketch of your proposed $H(f)$.

(8 marks)

Q8. Consider a second order system, $G(s)$, that is excited by the input signal $x(t)$. The output signal of the system, $y(t)$, when $t \to \infty$ may be expressed as

$$\lim_{t\to\infty} y(t) = 7.2\sin(4t - \varphi).$$

(a) Suppose $G(s)$ is a critically damped standard second order system with unity steady-state (DC) gain and the input signal $x(t) = 20\sin(4t)$. Derive the transfer function of $G(s)$ and the value of φ.

(8 marks)

(b) For this part, assume that $G(s)$ is a unity steady-state (DC) gain second order system with a <u>zero</u> located at $s = \alpha$. Let the input signal be $x(t) = \delta(t)$, and the output signal, $y(t)$, when $t \to \infty$, be the expression given above.

i. Using the concepts of stability and steady-state (DC) gain, explain why the system transfer function may be written as

$$G(s) = \frac{\omega_o^2(-s+\alpha)}{\alpha(s^2+\omega_o^2)}.$$

(4 marks)

ii. Determine the values of ω_o, α and φ. (8 marks)

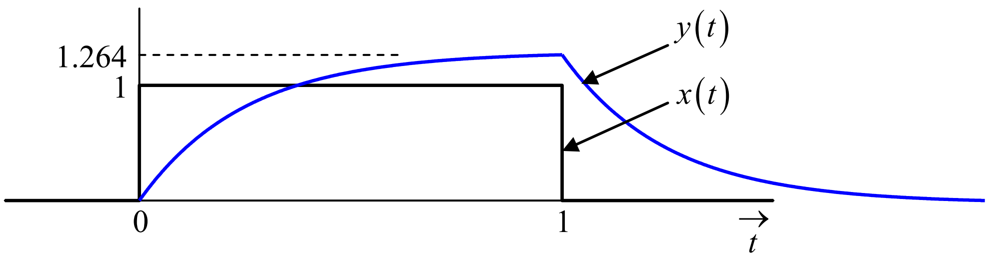
END OF QUESTIONS

NUMERIC ANSWERS

Section A

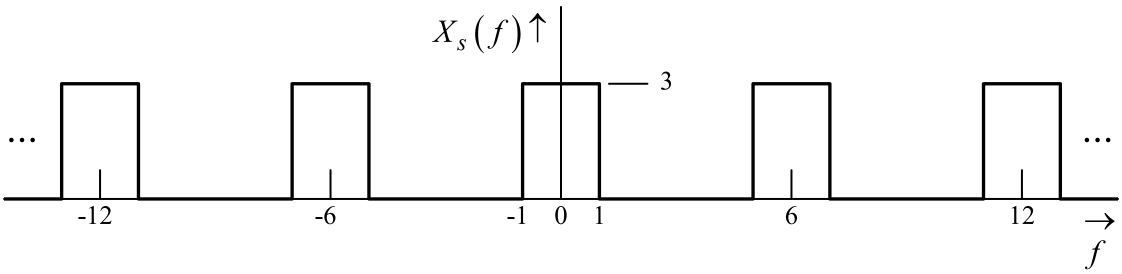
Q.1 (a) $a = 4,\ \ K = 10\sqrt{2}$

(b)

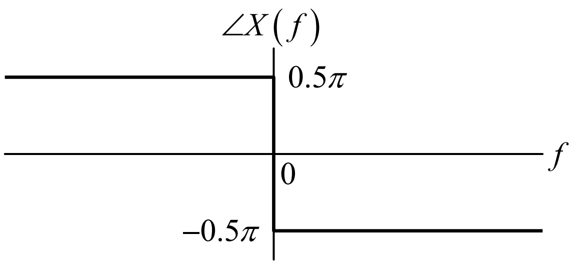


Q.2 (a) $X_s(f) = 3\sum_k \text{rect}\left(\frac{f-6k}{2}\right)$

(b)



Q.3 (a)



(b) DC value = 0

(c) $\alpha = 2\sqrt{\pi}$

Q.4 (a) $a = 0$, $b = 0.5$, $\omega_n = 2\,\text{rad/s}$, $K = 40$.

(b) $d = \frac{1}{\sqrt{2}}$

(c) $-180°$

Section B

Q.5 (a) $g(t) = \frac{1}{b-a}\left[\exp(-at) - \exp(-bt)\right]$

(b) $C = 1 \times 10^{-6} F$

(c) $K = \frac{1}{3}$. Pick any values of R_2 and C_2 which satisfy $R_2 C_2 = \frac{1}{3}$.

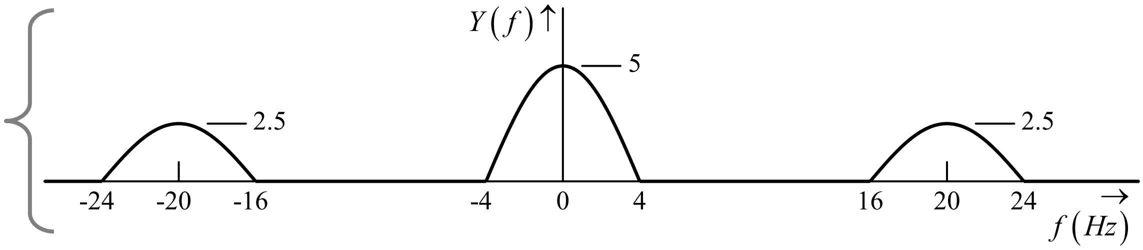
3-dB bandwidth is 0.91 rad/s

Q.6 (a) $X(f) = \frac{1}{5}\sum_k \left[4\,\mathrm{sinc}\left(\frac{4k}{5}\right) + \mathrm{sinc}^2\left(\frac{k}{5}\right)\right]\delta\left(f - \frac{k}{5}\right)$

(b) $X_k = \frac{1}{5}\left[4\,\mathrm{sinc}\left(\frac{4k}{5}\right) + \mathrm{sinc}^2\left(\frac{k}{5}\right)\right]$

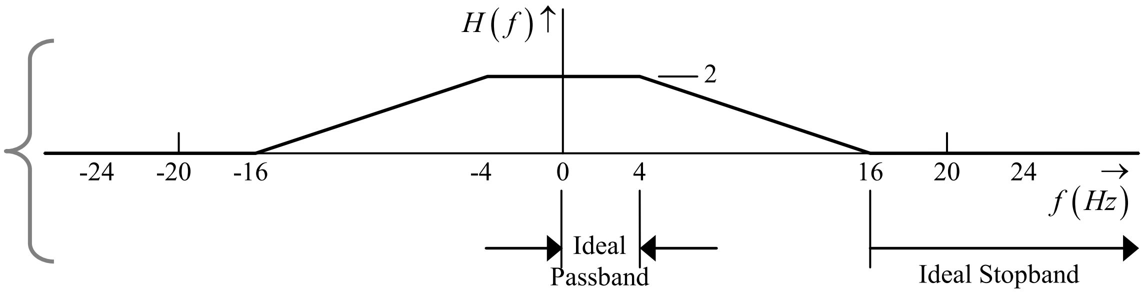
(c) $\frac{4}{3}$

Q.7 (a)



(b) $\alpha = 2$, $8 \le \beta \le 32$, and $\gamma = 0$

(c)



Q.8 (a) $G(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$ with $\omega_n = 3$

$\varphi = 106.26^o$ or 1.85 rad

(b) (i) ?

(b) (ii) $\omega_o = 4\,\mathrm{rad/s}$, $\alpha = 2.67$, $\varphi = 0.98$ rad or $56.25°$