

# EE2023 SIGNALS & SYSTEMS PAST-YEAR **EXAM** ARCHIVE

**Semester II : 2013/2014**

*w/ Numeric Answers appended*

## SECTION A : Answer ALL questions in this section

Q1. Ah Kow has been given the task of determining the model of a circuit which has been placed in a black box. The black box has two external connections: one at the input to connect the signal source and the other at the output to measure the output signal. It is known that the circuit is a linear time invariant system, consisting of only inductors, capacitors and resistors.

- (a) Ah Kow suspects that the black box contains a circuit which can be modeled as a first order transfer function.
- Write down the general form of the first order transfer function. State the parameters in this transfer function.  
(1 mark)
  - Suggest a simple test which he can use to determine this transfer function. Your answer should include the type of test (input) signal which he should use. For this test signal which is used, you should also sketch the output signal and explain how the parameters of the first order transfer function can be obtained.  
(3 marks)
- (b) Sometime later, Ah Kow suspects that the black box contains a circuit which can be modeled as an underdamped second order system.
- Write down the general form of this second order transfer function and give the appropriate range of the damping ratio.  
(2 mark)
  - Suggest a simple test which he can use to determine this transfer function. Your answer should include the type of test (input) signal which he should use. For this test signal, you should also sketch the output signal and explain how the parameters of the underdamped second order transfer function can be obtained.  
(4 marks)

Q2. Given the periodic signal  $x(t) = e^{j(2t+\pi/4)} + 2\cos(6t + \pi/6) - 3\sin(14t)$ .

- (a) Determine the complex Fourier series coefficients of  $x(t)$ .  
(6 marks)
- (b) Sketch the magnitude spectrum of  $x(t)$ .  
(2 marks)
- (c) Sketch the phase spectrum of  $x(t)$ .  
(2 marks)

Q3. The energy spectral density of a signal  $x(t)$  is given by

$$E_x(f) = \cos^2(\pi f) \cdot \text{rect}(f).$$

(a) Draw a labeled sketch of  $E_x(f)$ .

(2 marks)

(b) Find the 3 dB bandwidth of  $x(t)$ .

(3 marks)

(c) If  $x(t)$  has a phase spectrum given by  $\angle X(f) = -\pi f$ , find  $x(t)$  and compute its value at  $t = 1/6$  to 4 decimal places..

(5 marks)

Q4. The unit impulse responses of 3 systems are shown in Figure Q4. You may assume that the systems do not have zeros.

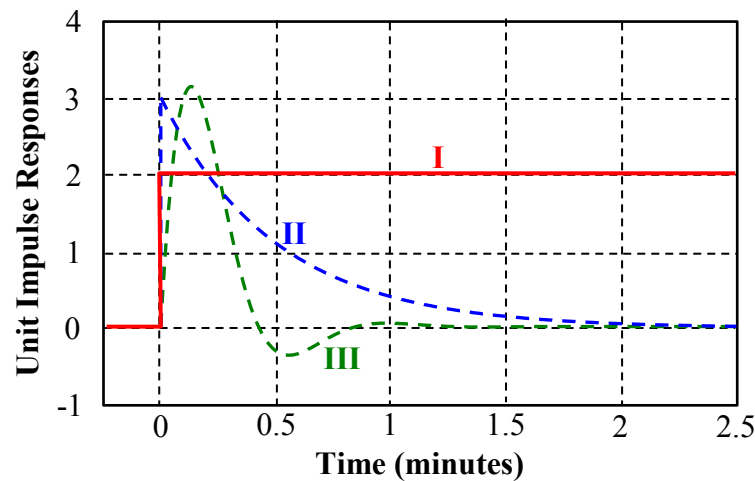


Figure Q4: Unit Impulse Responses

(a) Determine the transfer function for System II given that its unit impulse response has a value of 0.406 at  $t = 1$  minute.

(4 marks)

(b) Draw a pole-zero diagram that shows the pole(s) location(s) of the 3 systems. *Numerical values of the poles need not be given but their relative positions must be clear.*

(4 marks)

(c) Can the Final Value Theorem be used to predict the steady-state step response value for System I? Justify your answer.

(2 marks)

## SECTION B : Answer 3 out of the 4 questions in this section

Q5. A mass-damper system has a transfer function given by

$$G(s) = \frac{K}{2s^2 + 3s + C}$$

where  $K$  and  $C$  are uncertain parameters of the system.

- (a) If the natural frequency of the system is 1 rad/s, find the value of  $C$ .  
(2 marks)
- (b) Assuming that  $C$  is uncertain, find the range of values of  $C$  for which  $G(s)$  has complex poles. Sketch the corresponding region of the complex plane where the complex poles will lie for this range of  $C$ .  
(5 marks)
- (c) Find the range of values of  $C$  for which  $G(s)$  has real poles. Sketch the corresponding region of the complex plane where the real poles will lie for this range of  $C$ .  
(4 marks)
- (d) For what value of  $C$  will  $G(s)$  be critically damped?  
(3 marks)
- (e) Design the values of  $K$  and  $C$  such that the output of  $G(s)$  is underdamped and the gain to a unit step input function is 2. Sketch and label the output response of  $G(s)$  to the unit step function. Explain the effect of  $C$  on the frequency of the oscillations.  
(6 marks)

Q.6 The periodic signal  $x_p(t)$  is composed of repetitions of the pulse  $x(t)$  at a period of 3 seconds, where  $x(t)$  is described by

$$x(t) = \begin{cases} e^{3t}; & t \leq 0 \\ 0; & t > 0 \end{cases}$$

and for  $t \leq -3$  seconds, we can assume that  $x(t) \approx 0$ .

- (a) Determine the Fourier transform of  $x_p(t)$ .  
(6 marks)
- (b) Determine the Fourier series coefficients of  $x_p(t)$ .  
(3 marks)
- (c) Determine the average power of  $x_p(t)$ .  
(4 marks)
- (d) If  $x_p(t)$  is sampled at a frequency 100 Hz to give the sampled signal  $x_s(t)$ :
  - i. Obtain an expression for the sampled signal  $x_s(t)$  in terms of  $x(t)$ .  
(3 marks)
  - ii. Determine the Fourier transform of  $x_s(t)$ .  
(4 marks)

Q7. The radio frequency pulse used in a radar ranging system is modeled by

$$x(t) = \cos(2\pi f_o t) \cdot \text{rect}\left(\frac{t}{T}\right)$$

where  $f_o > 0$  is the pulse frequency and  $T > 0$  is the pulse width. The parameters  $f_o$  and  $T$  are chosen such that  $f_o T \gg 1$ .

The pulse,  $x(t)$ , transmitted by the radar is reflected back to the radar by a stationary metallic object located at some distance away. The echo received by the radar has the form

$$y(t) = \beta \cdot \cos(2\pi f_o (t - \tau)) \cdot \text{rect}\left(\frac{t - \tau}{T}\right)$$

where  $\beta$  and  $\tau$  each takes on a positive value that depends on the distance between the radar and the metallic object.

At the radar,  $y(t)$  is applied to the input of a filter which produces  $z(t) = x(t) * y(t)$  at its output. The symbol “\*” denotes “convolution”.

Both the transmitted and reflected pulses travel at a speed of  $3 \times 10^8$  m/s.

- (a) Express the spectrum,  $Y(f)$ , of  $y(t)$  in terms of the spectrum,  $X(f)$ , of  $x(t)$ .  
(4 marks)
- (b) Find the spectrum,  $Z(f)$ , of  $z(t)$ . Express your answer in terms of sine cardinal  $[\text{sinc}(\cdot)]$  and exponential  $[\exp(\cdot)]$  functions  
(6 marks)
- (c) Owing to  $f_o T \gg 1$ , we may assume that  $\text{sinc}((f - f_o)T) \text{sinc}((f + f_o)T) = 0$ . Applying this assumption to the  $Z(f)$  found in Part (b), find  $z(t)$ .  
(6 marks)
- (d) From the result of Part (c), it is noted that  $z(t)$  peaks at  $t = \tau$ . If  $\tau = 8 \times 10^{-5}$  seconds, what is the distance between the radar and the metallic object?  
(4 marks)

Q8. Figure Q8 shows the Bode magnitude plot of  $G(s) = \frac{\mathcal{L}\{y(t)\}}{\mathcal{L}\{x(t)\}}$ , a stable minimum phase system with no transportation lag.

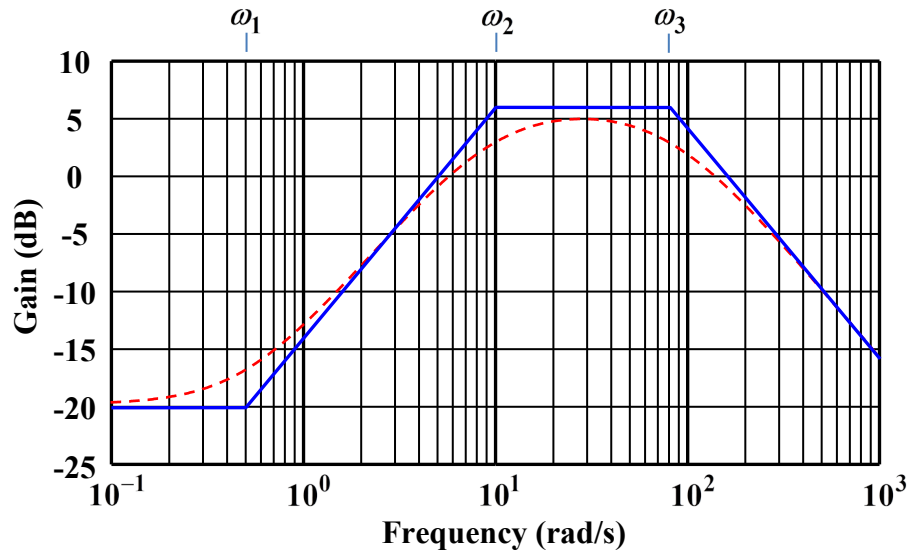


Figure Q8: Bode magnitude plot of  $G(s)$

- (a) Identify the transfer function,  $G(s)$ . (8 marks)
- (b) Following a breakdown, the system is refurbished. The following observations are made after analyzing data from experiments conducted on the refurbished system,  $G_{new}(s)$ :
- A step response test indicates that the DC gain of the new system is 0.15.
  - The first two corner frequencies,  $\omega_1$  and  $\omega_2$ , of the Bode magnitude plot are unchanged but the third corner frequency,  $\omega_3$ , has shifted.
- i. What is the low frequency asymptote of  $|G_{new}(j\omega)|$ ? (2 marks)
- ii. When the input signal is  $x(t) = 7 \sin\left(100t + \frac{\pi}{6}\right)$ , the steady-state output signal generated by  $G_{new}(s)$  is  $14 \sin(100t - 0.07\pi)$ . Determine the third corner frequency,  $\omega_3$ , of the refurbished system,  $G_{new}(s)$ . (5 marks)
- iii. The steady-state output of  $G(s)$  is  $2.22 \cos\left(t - \frac{5.7\pi}{18}\right)$  when  $x(t) = A \cos(t)$  is the input signal. Estimate the steady-state output of  $G_{new}(s)$  when the same input signal,  $x(t) = A \cos(t)$ , is applied. (5 marks)

**END OF QUESTIONS**

# NUMERIC ANSWERS

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## Section A

Q.1 (a) First order transfer function:

$$G(s) = \frac{K}{sT + 1} \quad \dots \quad \begin{cases} K \text{ is the DC or steady-state gain} \\ T \text{ is the time constant} \end{cases}$$

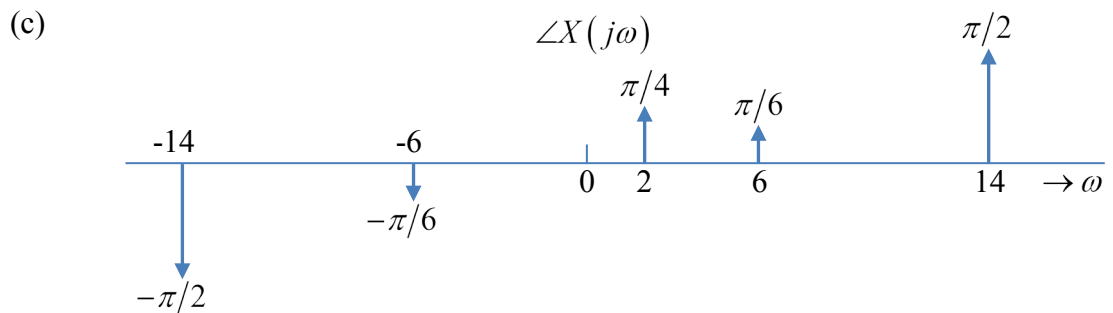
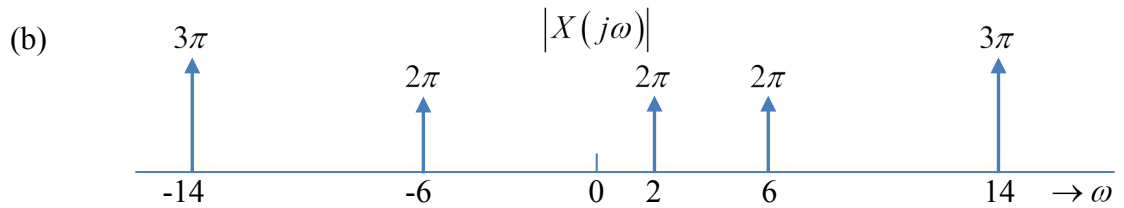
$K$  and  $T$  can be extracted from the unit step response of the system.

(b) Second order transfer function:

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \dots \quad \begin{cases} K \text{ is the DC or steady-state gain} \\ \zeta \text{ is the damping ratio} \\ \omega_n \text{ is the undamped natural frequency} \end{cases}$$

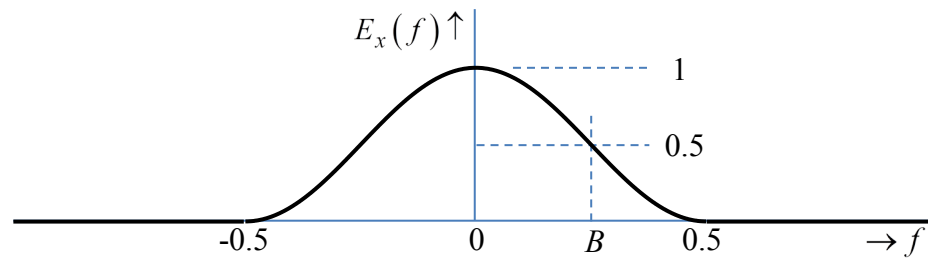
$K$ ,  $\zeta$  and  $\omega_n$  can be extracted from the unit step response of the system.

Q.2 (a)  $x(t) = -1.5e^{j\pi/2}e^{-j14t} + e^{-j\pi/6}e^{-j6t} + e^{j\pi/4}e^{j2t} + e^{j\pi/6}e^{j6t} + 1.5e^{j\pi/2}e^{j14t}$   
 $c_{-7} = 1.5e^{-j\pi/2}$ ,  $c_{-3} = e^{-j\pi/6}$ ,  $c_1 = e^{j\pi/4}$ ,  $c_3 = e^{j\pi/6}$ ,  $c_7 = 1.5e^{j\pi/2}$





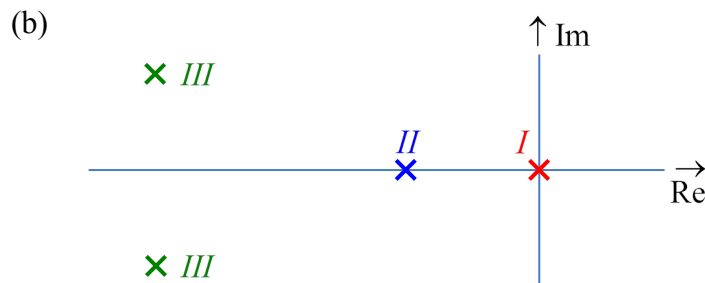
Q.3 (a)



(b)  $B_{3dB} = 0.25$

(c)  $x(t) = 0.5[\text{sinc}(t-1) + \text{sinc}(t)]$   
 $x\left(\frac{1}{6}\right) = \frac{9}{5\pi} \approx 0.573$

Q.4 (a) System II transfer function:  $G_{II}(s) = \frac{3}{s+2}$



(c) No, FVT is not applicable to step response of System I because the system is marginally stable with  $G_I(s) = \frac{2}{s}$

## Section B

Q.5 (a)  $C = 2$

(b)  $C > 1.125$

(c)  $C < 1.125$

(d)  $C = 1.125$

(e) If the gain to a unit step is 2, then it implies that  $\frac{K}{C} = 2$ . Since the system is to be designed such that it is underdamped, then we require  $C > 1.125$ . Choose  $C = 2$ , and therefore  $K = 4$ . The higher the value of  $C$ , the higher is the oscillation frequency.

$$\text{Q.6 (a)} \quad X_p(f) = \frac{1}{3 - j2\pi f} \cdot \frac{1}{3} \cdot \sum_k \delta\left(f - \frac{k}{3}\right) = \sum_k \frac{1}{9 - j2\pi k} \delta\left(f - \frac{k}{3}\right)$$

$$\text{(b)} \quad X_{p,k} = \frac{1}{9 - j2\pi k}$$

$$\text{(c)} \quad P = \frac{1}{3} \int_{-3}^0 e^{6t} dt = \frac{1}{18} [1 - e^{-18}] = 0.056$$

$$\text{(d)(i)} \quad x_s(t) = \sum_m \sum_n x\left(\frac{n}{100} - 3m\right) \delta\left(t - \frac{n}{100}\right).$$

$$\text{(d)(ii)} \quad X_s(f) = 100 \sum_l \sum_k \frac{1}{9 - j2\pi k} \delta\left(f - \frac{k}{3} - 100l\right)$$

$$\text{Q.7 (a)} \quad Y(f) = \beta X(f) \exp(-j2\pi f\tau)$$

$$\text{(b)} \quad Z(f) = \frac{\beta T^2}{4} \left[ \begin{array}{l} \text{sinc}^2((f - f_o)T) + \text{sinc}^2((f + f_o)T) \\ + 2\text{sinc}((f - f_o)T) \text{sinc}((f + f_o)T) \end{array} \right] \exp(-2\pi f\tau)$$

$$\text{(c)} \quad z(t) = \frac{\beta T}{2} \text{tri}\left(\frac{t - \tau}{T}\right) \cos(2\pi f_o(t - \tau))$$

$$\text{(d)} \quad 6000 \text{ km}$$

$$\text{Q.8 (a)} \quad G(s) = \frac{160(s + 0.5)}{(s + 10)(s + 80)}$$

$$\text{(b)(i)} \quad 20 \log_{10} 0.15 = -16.5 \text{ dB}$$

$$\text{(b)(ii)} \quad \omega_3 = 90.25 \text{ rad/s}$$

$$\text{(b)(iii)} \quad 3.33 \cos\left(t - \frac{5.7\pi}{18}\right)$$


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