EE2023 Signals & Systems Quiz
Semester 1 AY2012/13

Date : 4 October 2012 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. No programmable or graphic calculator is allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ______________________________

Matric # : ____________________________

Lecture Group # : ______________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

Group 1 : A/Prof Loh Ai Poh
Group 2 : A/Prof Ng Chun Sum
Group 3 : A/Prof Tan Woei Wan
Group 4 : Prof Lawrence Wong

Q.1 (a) Determine the appropriate sampling frequency for the signal $x(t)$ given by:

$$x(t) = 2\sin(2\pi t)\sin(10\pi t) + 6\cos(6\pi t),$$

in order to be completely recoverable from its samples.

(b) If $x(t)$ above is sampled at 20π rad/s, sketch the amplitude spectrum of the sampled signal. Can $x(t)$ be recovered from the samples? Explain your answer.

(c) Is $x(t)$ a power or energy signal? Explain your answer.

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 Let $x(t) = \operatorname{sgn}(t)\sin(2\pi f_0 t)$ and $y(t) = \operatorname{sinc}(f_0 t)$ where $\operatorname{sgn}(t) = \begin{cases} +1; & t \geq 0 \\ -1; & t < 0 \end{cases}$ and f_0 is a non-zero positive constant.

(a) Find the Fourier transforms of $x(t)$ and $y(t)$. Your final answers <u>should not</u> contain any unsolved integrals and/or the convolution operator '$\ast$'.

(b) With $x(t)$ and $y(t)$, we form the signal $z(t) = \int_{-\infty}^{\infty} x(\varsigma)\, y(t-\varsigma)\, d\varsigma$. If $z(t)$ is an energy signal, find its energy spectral density. If $z(t)$ is a power signal, find its power spectral density. If $z(t)$ is neither an energy nor a power signal, simply say so.

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 Consider the signal $x(t) = 5 + x_1(t) + x_2(3t) + e^{j2t}$, where $x_1(t) = 7\cos(8t)$ and $x_2(t) = \sin(2t)$.

(a) The discrete frequency amplitude spectrum for $x(t)$ shown in Figure Q.3 contains errors. Identify any two unique/different errors in the plot and correct them.

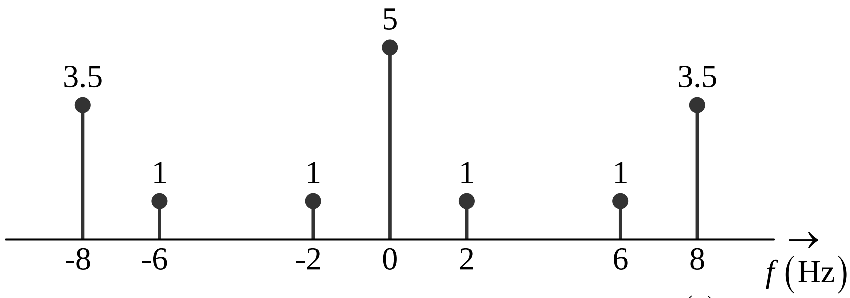

Figure Q.3: Erroneous amplitude spectrum for $x(t)$

(b) Does $x(t)$ contain a 2^{nd} harmonic component? Explain how you arrived at the conclusion.

(c) What is the relationship between the amplitude spectrum of $x(t)$ and $x(t-0.05)$?

(d) Sketch the phase (in radians) versus the Fourier Series index, *k*, graph of $x(t-0.05)$.

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4. (a) Determine the Fourier transform of the signal $x(t)$ shown in Figure Q.4(a).

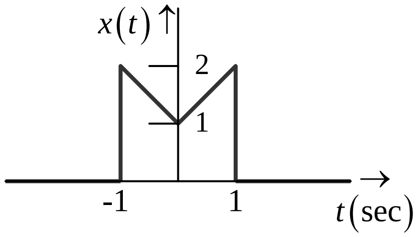

Figure Q.4(a)

(b) Using the Dirac-δ replication property, write the expression that shows the relationship between $x(t)$ and the periodic signal $x_p(t)$ shown in Figure Q.4(b).

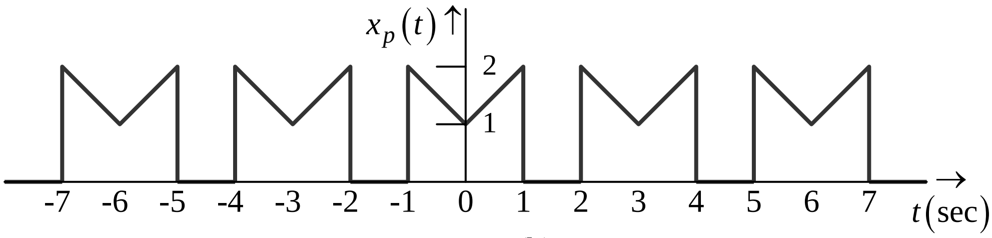

Figure Q.4(b)

(c) Determine the Fourier transform of the periodic signal $x_p(t)$.

Q.4 ANSWER

Q.4 ANSWER ~ continued

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference. Anything written on this page will not be graded.

Fourier Series: $\begin{cases} X_k = \dfrac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$

Fourier Transform: $\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	Frequency-domain
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi ft_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = \frac{1}{2}\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = \frac{1}{j2}\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = \frac{1}{2}\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = \frac{1}{2}\left[1 + \cos(2\theta)\right]$	$\mathbf{C}\cos(\theta) - \mathbf{S}\sin(\theta) = \sqrt{\mathbf{C}^2 + \mathbf{S}^2}\cos\left[\theta + \tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$