EE2023 SIGNALS & SYSTEMS PAST-YEAR EXAM ARCHIVE

Semester II : 2012/2013

w/ Numeric Answers appended

SECTION A : Answer ALL questions in this section

Q1. The impulse response of a linear time invariant (LTI) system is given by

$$g(t) = K\delta(t-\lambda)$$

where K and λ are unknown constants and $\delta(\cdot)$ represents the Dirac-delta function. The LTI system has a transfer function, $G(s)$, with magnitude (dB) and phase (radians) response given by Figure Q1 below.

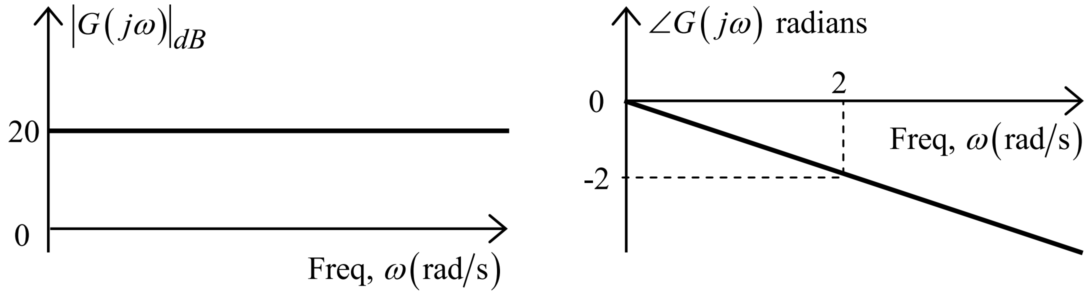


Figure Q1: Magnitude and Phase Response of $G(j\omega)$

(a) Find the transfer function of the LTI system. (3 marks)

(b) Find K and λ. (3 marks)

(c) Sketch the unit step response of the system. What type of system is $G(s)$? (4 marks)

Q2. The signal $x(t) = 5\cos(10\pi t)$ is modified by the signal $y(t)$ shown in Figure Q2 to obtain the signal $z(t) = x(t)y(t)$.

(a) Sketch the waveform for $z(t)$. (4 marks)

(b) Derive the Fourier transform of $z(t)$. (6 marks)

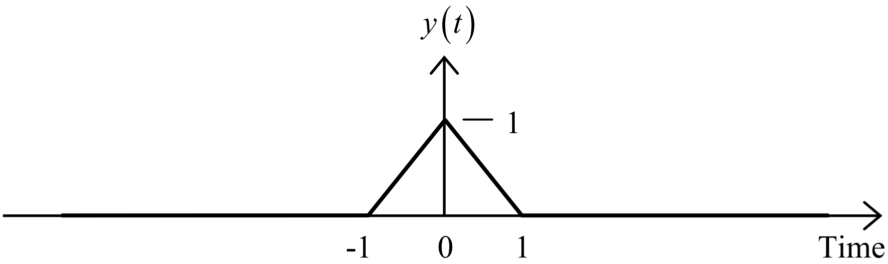
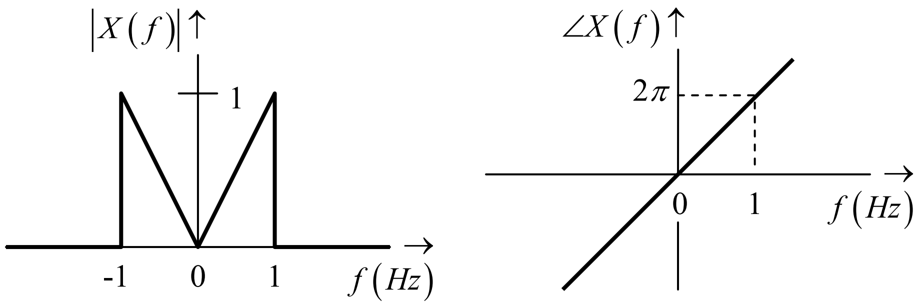


Figure Q2

Q3. The magnitude and phase spectra of a signal $x(t)$ are shown in Figure Q3.

Figure Q3



(a) Magnitude Spectrum (b) Phase Spectrum

(a) Express $|X(f)|$ in terms of the $\text{rect}(\cdot)$ and $\text{tri}(\cdot)$ functions. (2 marks)

(b) Write down the mathematical expression for $\angle X(f)$. (1 mark)

(c) Using the results of Parts (a) and (b), or otherwise, find $x(t)$. (4 marks)

(d) Find the total energy of $x(t)$. (3 marks)

Q4. Figure Q4 shows the Bode magnitude plot of a system with the following transfer function structure.

$$G(s) = \frac{K\left(s+a\right)\left(\frac{s}{b}+1\right)}{\left(\frac{s}{c}+1\right)\left(\frac{s}{d}+1\right)}$$

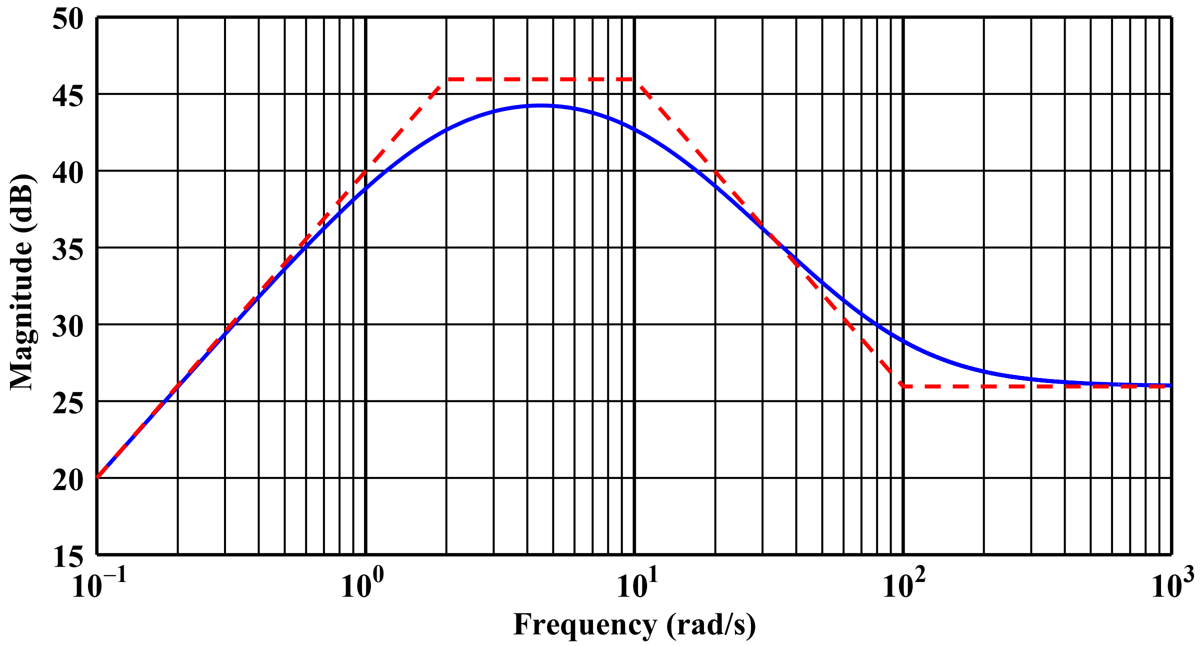


Figure Q4: Bode magnitude plot

(a) Using Figure Q4, identify the values of K, a, b, c and d. (6 marks)

(b) What values will the low frequency and high frequency asymptotes of the Bode phase plot for $G(s)$ converge to?

(4 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5 A standard second order system has a transfer function given by

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}.$$

(a) Sketch the region in the complex plane where the poles of $G(s)$ will satisfy the following specifications :

- Damping ratio, $\varsigma \geq 0.5$
- Undamped natural frequency, $1 \leq \omega_n \leq 4$

(4 marks)

(b) Find the values of ς and ω_n if the poles of $G(s)$ are at $s_{1,2} = -\frac{3}{2} \pm j\frac{3\sqrt{3}}{2}$.

(4 marks)

(c) Find the values of ς and ω_n if $G(s)$ is critically damped with poles at $s_{1,2} = -2, -2$.

(3 marks)

(d) In the sketched region in Part (a) above, find the poles of $G(s)$ which will result in the fastest non-oscillatory step response.

(3 marks)

(e) Based on the region in Part (a) above, sketch the unit step response of $G(s)$ which has the highest possible <u>undamped</u> natural frequency and lowest damping ratio. State the values of the corresponding damping ratio and <u>damped</u> natural frequency.

(6 marks)

Q.6 The output of a pulse-amplitude modulation (PAM) system, $x_{PAM}(t)$, is modeled by $x_{PAM}(t) = x(t)x_p(t)$, where $x(t)$ is the input signal and $x_p(t)$ is a rectangular pulse train with a pulse height of 1, pulse width of a and period of T, as shown in Figure Q6-1.

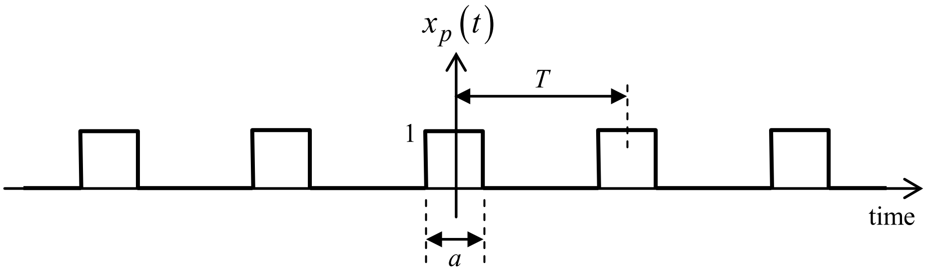


Figure Q6-1

(a) Write an expression for $x_p(t)$. (3 marks)

(b) Derive the Fourier transform of $x_{PAM}(t)$ in terms of $X(f)$ which is the Fourier transform of $x(t)$.

(7 marks)

(c) If $x(t)$ has a finite bandwidth of 3000Hz with a spectrum shown in Figure Q6-2, and $x_p(t)$ has a period of $T = 50\mu s$ with a pulse width of $a = 10\mu s$, sketch the spectrum of $x_{PAM}(t)$.

(7 marks)

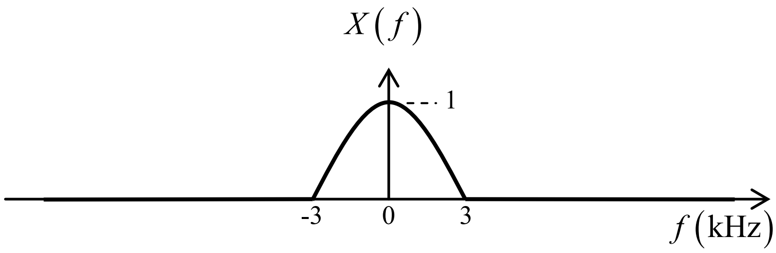


Figure Q6-2

(d) Is it possible to reconstruct the signal $x(t)$ from $x_{PAM}(t)$ with an ideal low pass filter? If yes, specify the characteristics of the low pass filter, otherwise explain why not.

(3 marks)

Q7. A bandpass signal $x(t) = m(t)\cos(120000\pi t)$ is sampled to produce

$$x_s(t) = x(t) \cdot \sum_n \delta(t - nT_s)$$

where $f_s = 1/T_s$ is the sampling frequency. Let the spectra of $m(t)$, $x(t)$ and $x_s(t)$ be denoted by $M(f)$, $X(f)$ and $X_s(f)$, respectively. A sketch of $M(f)$ is shown in Figure Q7.

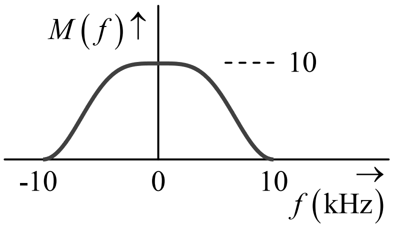


Figure Q7: Spectrum of $m(t)$

(a) Draw a labeled sketch of $X(f)$.

(5 marks)

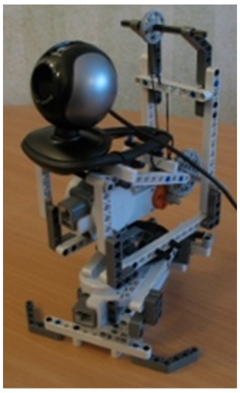
(b) Suppose $f_s = 40$ kHz. Can $x(t)$ be recovered by passing $x_s(t)$ through an ideal filter? If 'YES', specify the filter and state two advantages of sampling at $f_s = 40$ kHz compared to sampling at Nyquist rate. If 'NO', explain why?

(10 marks)

(c) Suppose $f_s = 30$ kHz. Can $x(t)$ and $m(t)$ be recovered from $x_s(t)$ and why?

(5 marks)

Q8. Figure Q8-1 shows a remote monitoring system that comprises of a web-cam mounted on a frame and a motor which pans the frame right and left via a set of gears. Let $v(t)$ be the voltage applied to the motor, and the output signal, $\theta_p(t)$, is the orientation of the web-cam, in degrees.



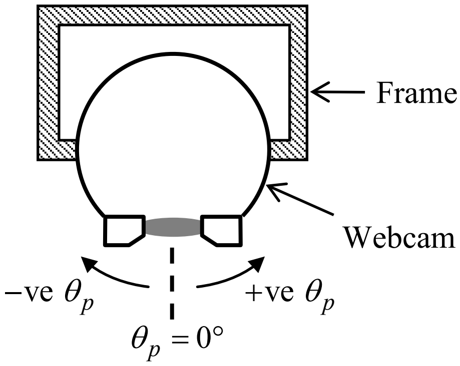


Figure Q8-1: Photo and top view of the remote monitoring system

(a) The unit step response of the monitoring system is shown in Figure Q8-2. Using Figure Q8-2 and the information that the area of the shaded region is 2.25, show that the first order transfer function relating the orientation of the camera, and the voltage applied to the motor has

- DC gain of 15 degrees/volt,
- Time constant of 0.1 seconds,
- Transportation delay of 0.05 seconds.

(12 marks)

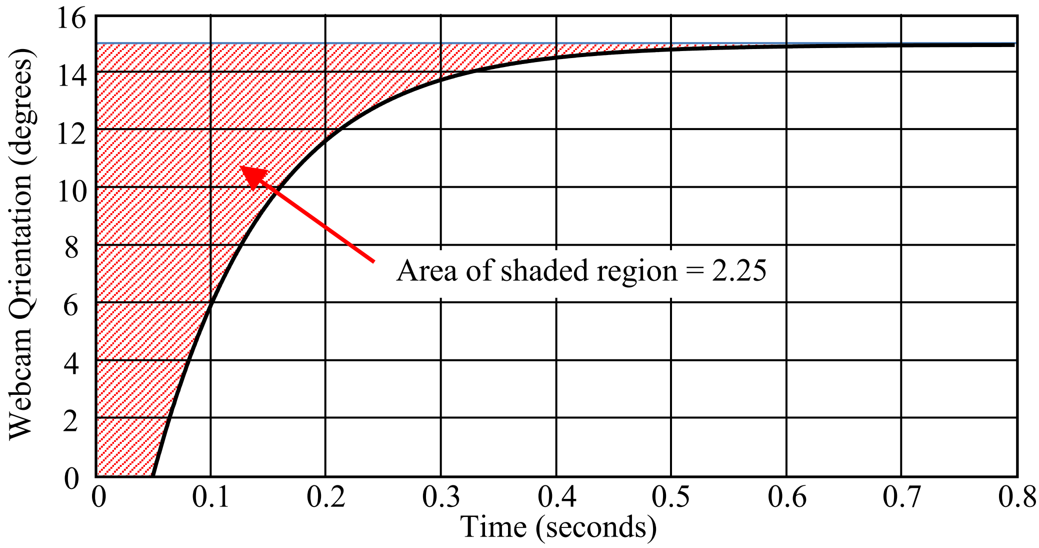


Figure Q8-2: Unit step response of remote monitoring system

Q8 continues on Page 9

(b) Suppose the system output, $\theta_p(t)$, at steady state is

$$\theta_{p,ss}(t) = 45\sin(8t).$$

i. Using the transfer function obtained from Part (a) above, derive the input voltage, $v(t)$, of the system.

ii. Suggest a practical scenario where it may be necessary to model the desired orientation of the camera, $\theta_p(t)$, as a sinusoidal signal.

(8 marks)

END OF QUESTIONS

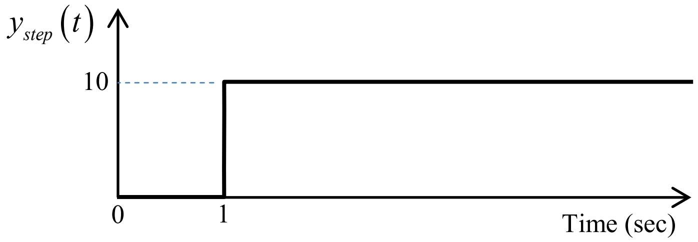
NUMERIC ANSWERS

Section A

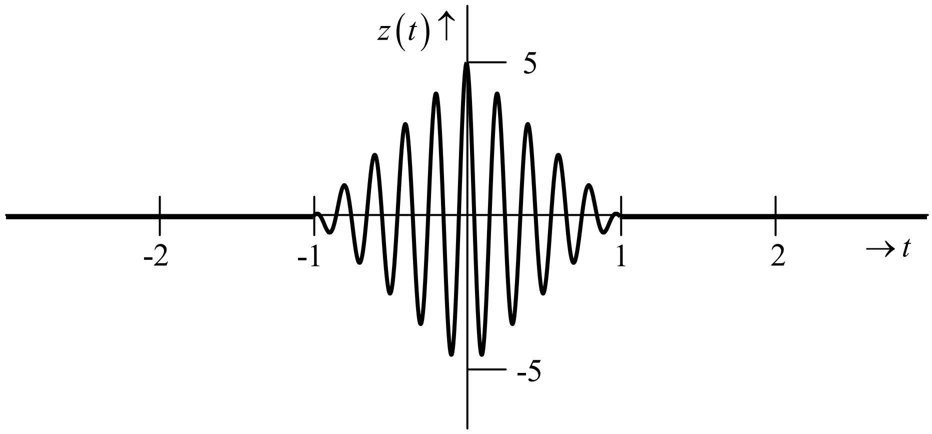
Q.1 (a) Transfer function: $G(s) = Ke^{-s\lambda}$

(b) $K = 10$ and $\lambda = 1$

(c)



Q.2 (a)



(b) $\Im\{z(t)\} = Z(f) = \frac{5}{2}\left[\mathrm{sinc}^2(f-5) + \mathrm{sinc}^2(f+5)\right]$

Q.3 (a) $|X(f)| = \mathrm{rect}\left(\frac{f}{2}\right) - \mathrm{tri}(f)$

(b) $\angle X(f) = 2\pi f$

(c) $x(t) = 2\mathrm{sinc}(2t+2) - \mathrm{sinc}^2(t+1)$

(d) $Energy = \frac{2}{3}$

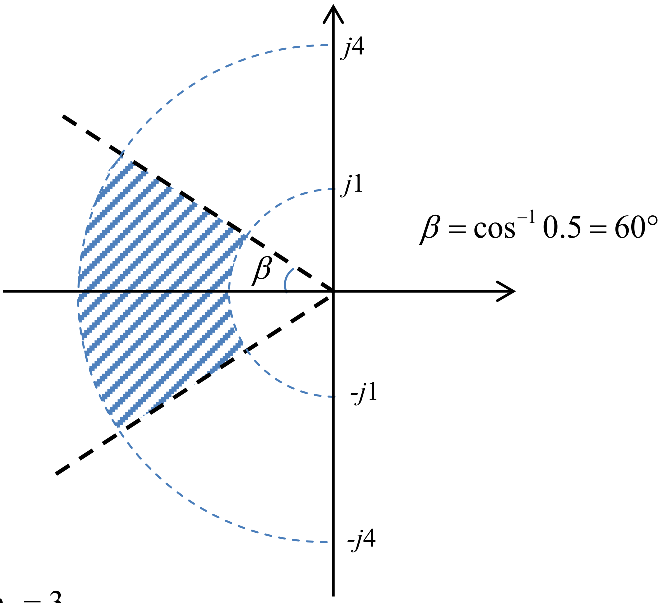
Q.4 (a) $K = 100, \ a = 0, \ b = 100, \ c = 2, \ d = 10$

(b) Low frequency asymptote of phase plot tends to 90 degrees and high frequency asymptote tends to 0 degree.

Section B

Q.5

(a)



(b) $\varsigma = 0.5$ and $\omega_n = 3$.

(c) $\varsigma = 1$ and $\omega_n = 2$.

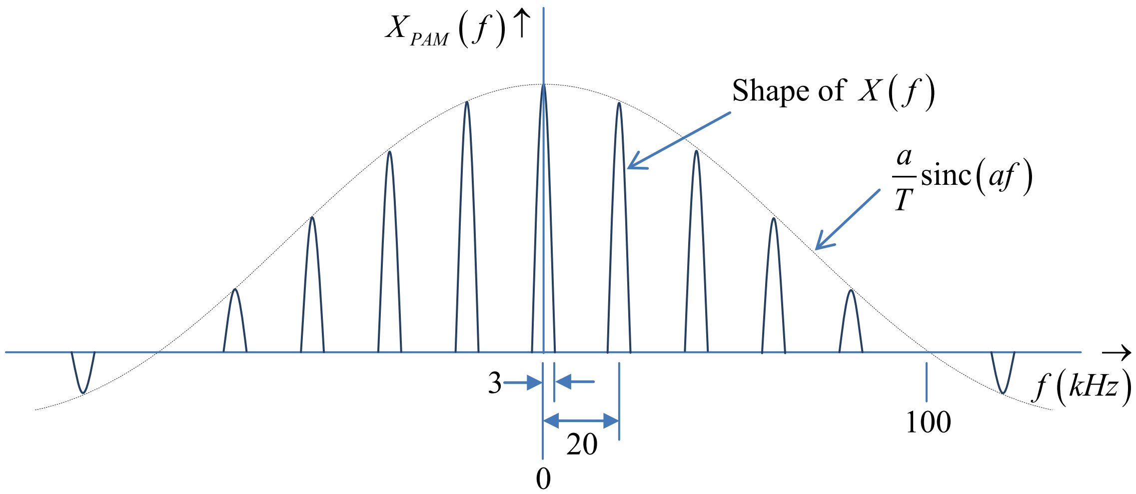
(d) $s_{1,2} = -4, -4$

(e) $\omega_{n,\max} = 4$, $\varsigma_{\min} = 0.5$, $\omega_d = 2\sqrt{3}$

Q.6 (a) $x_p(t) = \sum_k \mathrm{rect}\left(\frac{t-kT}{a}\right) = \mathrm{rect}\left(\frac{t}{a}\right) * \sum_k \delta(t-kT)$

(b) $X_{PAM}(f) = \frac{a}{T}\sum_k \mathrm{sinc}\left(\frac{ak}{T}\right) X\left(f - \frac{k}{T}\right)$

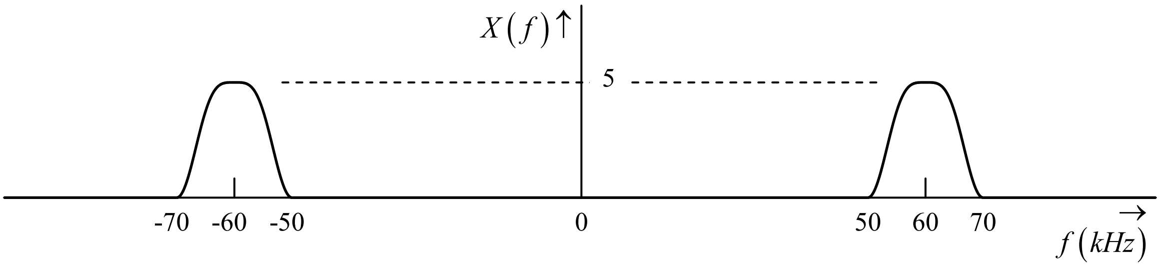
(c)



(d) Yes. Use a low pass filter with a cut-off frequency of 3 *kHz* and whose passband to stopband transition does not exceed (17 *kHz* – 3 *kHz*) = 14 *kHz*.

Q.7

(a)



(b) $f_s = 40$ kHz. YES, $x(t)$ can be recovered by passing $x_s(t)$ through an ideal filter with frequency response:

$$H(f) = \begin{cases} 0; & |f| \le 50 \text{ and } |f| \ge 70 \\ K = \dfrac{1}{2f_s}; & \text{elsewhere} \end{cases}$$

Advantages:

(1) Lower sampling frequency
(Reduces acquisition, computation and storage complexity)

(2) Requirement of sharp cut-off reconstruction filter can be relaxed due to the existence of a 20 kHz gap between adjacent spectral images.
(Reduces reconstruction filter complexity)

(c) $f_s = 30$ kHz. YES, spectrum of $x_s(t)$ contains undistorted spectra of $x(t)$ and $m(t)$.

Q.8 (a) No numeric answer.

(b) (i) $v(t) = 3.84\sin(t + 1.07)$ volts

(b) (ii) Practical scenario: Surveillance camera where the surveillance area is wider than the camera's field of view.