EE2023 Signals & Systems Quiz
Semester 2 AY2012/13

Date : 7 March 2013 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. No programmable or graphic calculator is allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ______________________________

Matric # : ____________________________

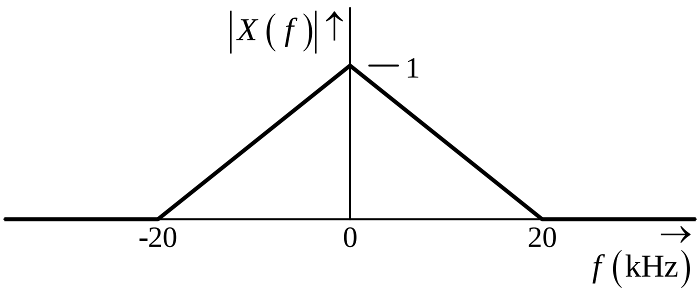
Lecture Group # : _______________________

Question #	**Marks**
1	
2	
3	
4	
Total Marks	

For your information :

Group 1 : A/Prof Loh Ai Poh
Group 2 : A/Prof Ng Chun Sum
Group 3 : A/Prof Tan Woei Wan
Group 4 : Prof Lawrence Wong

Q.1 The signal $x(t)$ has an amplitude spectrum given in Figure Q.1 below.



(a) What is the minimum sampling frequency that will allow $x(t)$ to be recovered from its samples?

(b) Suppose that $x(t)$ is sampled at a rate of $f_s = 38\ \mathrm{kHz}$. What part of $X(f)$ can be recovered without error from the samples? Explain your answer with an illustration of the appropriate spectrum.

(c) Is $x(t)$ a power or energy signal? Explain your answer.

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q2. Consider the complex signal $x(t) = \text{sinc}(t) + j\ \text{sinc}(t)$.

(a) Find the energy spectral density of $x(t)$. Hence, or otherwise, determine the total energy of $x(t)$.

(b) Suppose $y(t) = |x(t)|^2$. Find the 3dB bandwidth of $y(t)$.

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 The complex exponential Fourier Series representation of a periodic signal, $x(t)$, is

$$\sum_{k=-\infty}^{\infty} c_k e^{jkt}$$

where the Fourier Series coefficients are $c_k = \begin{cases} 0.49-0.1j & : \quad k=-2 \\ 1.5j\ e^{j0.1} & : \quad k=-1 \\ 2 & : \quad k=0 \\ -1.5j\ e^{-j0.1} & : \quad k=1 \\ 0.49+0.1j & : \quad k=2 \\ 0 & : \quad otherwise \end{cases}$

(a) What is the fundamental period (seconds) and fundamental frequency (Hz) of the signal, $x(t)$?

(b) Write an equation for the continuous frequency spectrum, $X(f)$, of the signal.

(c) Express $x(t)$ as a function of real sinusoids and constant.

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4 The signal $x(t)$ is shown in Figure Q.4(a).

(a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$.

(b) Using the replication property of the Dirac-δ function, obtain an expression for the periodic signal $x_p(t)$ shown in Figure Q.4(b) in terms of $x(t)$.

(c) Derive the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$.

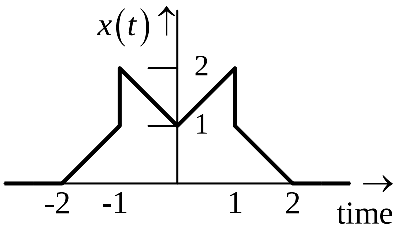


Figure Q.4(a)

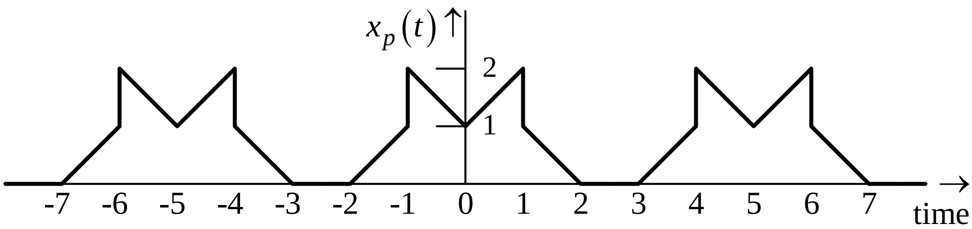


Figure Q.4(b)

Q.4 ANSWER

Q.4 ANSWER ~ continued

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference. Anything written on this page will not be graded.

Fourier Series: $$\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \displaystyle\sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform: $$\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	Frequency-domain
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi ft_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = \frac{1}{2}\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = \frac{1}{j2}\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = \frac{1}{2}\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = \frac{1}{2}\left[1 + \cos(2\theta)\right]$	$\mathbf{C}\cos(\theta) - \mathbf{S}\sin(\theta) = \sqrt{\mathbf{C}^2 + \mathbf{S}^2}\cos\left[\theta + \tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$