Linear Algebra Final Exam

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This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** What is the reduced row-echelon form of the matrix?

$$A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ -2 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 \\ 3 & 1 & 2 & -2 \end{bmatrix}$$

A $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

E $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

C $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. **(5 pts)** What is the matrix product?

$$AB = \begin{bmatrix} -2 & 1 & 2 & 5 \\ 0 & -1 & 4 & 2 \\ 1 & -3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -5 & 0 & 2 \\ 2 & -3 & -1 \\ 2 & 0 & -1 \end{bmatrix}$$

A $AB = \begin{bmatrix} 7 & -8 & -9 \\ 17 & -12 & -8 \\ 16 & -2 & -4 \end{bmatrix}$

D $AB = \begin{bmatrix} 17 & -12 & -8 \\ 7 & -8 & -9 \\ 16 & -2 & -4 \end{bmatrix}$

B $AB = \begin{bmatrix} -8 & 7 & -9 \\ -12 & 17 & -8 \\ -2 & 16 & -4 \end{bmatrix}$

E $AB = \begin{bmatrix} 7 & -8 & -9 \\ 16 & -2 & -4 \\ 17 & -12 & -8 \end{bmatrix}$

C $AB = \begin{bmatrix} -9 & -8 & 7 \\ -8 & -12 & 17 \\ -4 & -2 & 16 \end{bmatrix}$

3. **(5 pts)** Find the vector sum $2\vec{a}-3\vec{b}+5\vec{c}-\vec{d}$, if $\vec{a}=(2,6,-1)$, $\vec{b}=(-3,1,1)$, $\vec{c}=(0,5,-2)$, and $\vec{d}=(1,4,-4)$.

A $(-12,-30,11)$

B $(6,-36,5)$

C $(12,30,-11)$

D $(14,38,-19)$

E $(-6,36,-5)$

4. **(5 pts)** What is the length of $\vec{x}=(4,-2,1,0)$?

A $||\vec{x}||=\sqrt{13}$

B $||\vec{x}||=\sqrt{21}$

C $||\vec{x}||=-\sqrt{13}$

D $||\vec{x}||=-\sqrt{21}$

E $||\vec{x}||=\sqrt{9}$

5. **(5 pts)** Find the equation of a plane with normal vector $\vec{n} = (2, -6, 1)$ that passes through $(5, 2, -3)$.

A $2x - 6y + z = -5$

B $2x - 6y + z = 5$

C $2x - 6y + z = 1$

D $2x - 6y + z = -1$

E $2x + 6y + z = -5$

6. **(5 pts)** Transform $\vec{x} = (4, -1)$ with $T \circ S$, if $S : \mathbb{R}^2 \to \mathbb{R}^2$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$.

$$S(\vec{x}) = \begin{bmatrix} -2x_1 + x_2 \\ 3x_2 \end{bmatrix}$$

$$T(\vec{x}) = \begin{bmatrix} x_1 - 4x_2 \\ -4x_2 \end{bmatrix}$$

A $(T \circ S)(\vec{x}) = (-12, -3)$

B $(T \circ S)(\vec{x}) = (-3, -12)$

C $(T \circ S)(\vec{x}) = (-3, 12)$

D $(T \circ S)(\vec{x}) = (12, 3)$

E $(T \circ S)(\vec{x}) = (3, 12)$

7. **(5 pts)** Find the transformation in the alternate basis, $[T(\vec{x})]_B$ of the vector $\vec{x} = (6, -1)$.

$$T(\vec{x}) = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \vec{x}$$

$$B = \text{Span}\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \end{bmatrix}\right)$$

A $[T(\vec{x})]_B = \begin{bmatrix} 28 \\ -\frac{64}{3} \end{bmatrix}$

B $[T(\vec{x})]_B = \begin{bmatrix} -28 \\ \frac{64}{3} \end{bmatrix}$

C $[T(\vec{x})]_B = \begin{bmatrix} -28 \\ -\frac{64}{3} \end{bmatrix}$

D $[T(\vec{x})]_B = \begin{bmatrix} \frac{64}{3} \\ 28 \end{bmatrix}$

E $[T(\vec{x})]_B = \begin{bmatrix} -\frac{64}{3} \\ -28 \end{bmatrix}$

8. **(5 pts)** Use the Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\right)$$

A $V = \text{Span}\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, -\frac{1}{\sqrt{34}}\begin{bmatrix} 4 \\ -3 \\ -3 \end{bmatrix}, \frac{1}{\sqrt{17}}\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}\right)$

B $V = \text{Span}\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{34}}\begin{bmatrix} 4 \\ -3 \\ -3 \end{bmatrix}, -\frac{1}{\sqrt{17}}\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}\right)$

C $V = \text{Span}\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{34}}\begin{bmatrix} 4 \\ -3 \\ -3 \end{bmatrix}, \frac{1}{\sqrt{17}}\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}\right)$

D $V = \text{Span}\left(-\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{34}}\begin{bmatrix} 4 \\ -3 \\ -3 \end{bmatrix}, \frac{1}{\sqrt{17}}\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}\right)$

E $V = \text{Span}\left(-\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, -\frac{1}{\sqrt{34}}\begin{bmatrix} 4 \\ -3 \\ -3 \end{bmatrix}, \frac{1}{\sqrt{17}}\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}\right)$

9. **(15 pts)** Solve $A\vec{x}=\vec{b}$, using values $b_1=-2$, $b_2=1$, and $b_3=1$.

$$A=\begin{bmatrix}1 & 1\\ 1 & 2\\ -2 & -3\end{bmatrix}$$

10. **(15 pts)** Find the four fundamental subspaces of A, including their spaces and dimensions.

$$A=\begin{bmatrix}1 & -2 & 0 & 1\\ -1 & 0 & 2 & 4\\ 0 & 1 & -1 & 2\end{bmatrix}$$

11. **(15 pts)** Find the orthogonal complement of V.

$$V = \text{Span}\left(\begin{bmatrix} 2 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 0 \end{bmatrix}\right)$$

12. **(15 pts)** Find the Eigenvalues and Eigenvectors of the matrix, then describe what's happening in the Eigenbases.

$$A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$