Linear Algebra Final Exam

krista king MATH

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This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** Use Gauss-Jordan elimination to solve the system with a rref matrix.

$$x - 4y + z = 20$$

$$-x + z = 10$$

$$4x + y - 2z = -25$$

A $(x, y, z) = (1, -3, -9)$

B $(x, y, z) = (29,27,59)$

C $(x, y, z) = (-1, -3,9)$

D $(x, y, z) = (-29, -27, -59)$

E $(x, y, z) = (-85, -75, -195)$

2. **(5 pts)** Use the distributive property to find $(A-C)B$.

$$A=\begin{bmatrix}5 & -4\\ -3 & 9\\ 0 & 4\end{bmatrix} \qquad B=\begin{bmatrix}4 & -1 & 7\\ 3 & 7 & 8\end{bmatrix} \qquad C=\begin{bmatrix}3 & 2\\ -1 & 7\\ -3 & 3\end{bmatrix}$$

A $\begin{bmatrix}-10 & -44 & -34\\ -2 & 16 & 2\\ 15 & 4 & 29\end{bmatrix}$

D $\begin{bmatrix}-9 & -3\\ -46 & 16\end{bmatrix}$

B $\begin{bmatrix}2 & -16 & -2\\ -10 & 18 & -12\\ -9 & 10 & -12\end{bmatrix}$

E $\begin{bmatrix}31 & -19\\ 16 & 4\end{bmatrix}$

C $\begin{bmatrix}2 & -16 & -2\\ -2 & 16 & 2\\ 15 & 4 & 29\end{bmatrix}$

3. **(5 pts)** Find the unit vector in the direction of $\overrightarrow{u}=\overrightarrow{a}+2\overrightarrow{b}-3\overrightarrow{c}-\overrightarrow{d}$, if $\overrightarrow{a}=(-3,5,-1)$, $\overrightarrow{b}=(4,2,7)$, $\overrightarrow{c}=(0,2,1)$, and $\overrightarrow{d}=(-1,3,5)$.

A $\quad \overrightarrow{v}=\begin{bmatrix}6\\0\\-5\end{bmatrix}$

D $\quad \overrightarrow{v}=\begin{bmatrix}4\\6\\15\end{bmatrix}$

B $\quad \overrightarrow{v}=\begin{bmatrix}\frac{6}{\sqrt{11}}\\0\\-\frac{5}{\sqrt{11}}\end{bmatrix}$

E $\quad \overrightarrow{v}=\begin{bmatrix}\frac{6}{\sqrt{61}}\\0\\\frac{5}{\sqrt{61}}\end{bmatrix}$

C $\quad \overrightarrow{v}=\begin{bmatrix}\frac{6}{\sqrt{61}}\\0\\-\frac{5}{\sqrt{61}}\end{bmatrix}$

4. **(5 pts)** Find the angle between $\vec{u}$ and $\vec{v}$.

$$\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 0 \\ 5 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 12 \\ 3 \\ 8 \\ -3 \end{bmatrix}$$

A $\theta = 0^\circ$

B $\theta = 90^\circ$

C $\theta = 45^\circ$

D $\theta = 180^\circ$

E $\theta = 60^\circ$

5. **(5 pts)** Find the cross product of $\vec{a} = (-2,3,-1)$ and $\vec{b} = (0,-2,2)$.

A $\vec{a} \times \vec{b} = (3,-4,4)$

B $\vec{a} \times \vec{b} = (9,4,4)$

C $\vec{a} \times \vec{b} = (9,-4,4)$

D $\vec{a} \times \vec{b} = (3,-4,-4)$

E $\vec{a} \times \vec{b} = (4,4,4)$

6. **(5 pts)** Find the orthogonal complement of W, if W is a vector set in $\mathbb{R}^4$.

$$W = \begin{bmatrix} y+z \\ -2x+y \\ x-z \\ x \end{bmatrix}$$

A $W^{\perp} = \text{Span}\left(\begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}\right)$

B $W^{\perp} = \text{Span}\left(\begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix}\right)$

C $W^{\perp} = \text{Span}\left(\begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ -\frac{1}{3} \\ 1 \end{bmatrix}\right)$

D $W^{\perp} = \text{Span}\left(\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}\right)$

E $W^{\perp} = \text{Span}\left(\begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \\ 0 \end{bmatrix}\right)$

7. (5 pts) For the matrix A, which choice is incorrect?

$$A = \begin{bmatrix} -1 & -3 \\ 2 & 4 \\ 6 & 0 \end{bmatrix}$$

A $N(A^T) = \text{Span}\left(\begin{bmatrix} -12 \\ -9 \\ 1 \end{bmatrix}\right)$

B $\text{Dim}(C(A^T)) = 2$

C $\text{Dim}(N(A^T)) = 1$

D $C(A^T) = \text{Span}\left(\begin{bmatrix} -1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}\right)$

E $C(A^T) = \text{Span}\left(\begin{bmatrix} -1 \\ -3 \end{bmatrix}\right)$

8. **(5 pts)** Find the eigenvectors of the transformation matrix.

$$A = \begin{bmatrix} -5 & 0 \\ 3 & 1 \end{bmatrix}$$

A $\begin{bmatrix} -5 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

B $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

D $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

E $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

9. **(15 pts)** Find the general solution to $A\vec{x}=\vec{b}$.

$$A=\begin{bmatrix}1&-2&-5&-3\\3&-1&-5&-4\\0&-5&-10&-5\end{bmatrix}\text{ with }\vec{b}=\begin{bmatrix}1\\1\\2\end{bmatrix}$$

10. **(15 pts)** If $S:\mathbb{R}^3\to\mathbb{R}^3$ and $T:\mathbb{R}^3\to\mathbb{R}^3$, then what are $T(S(\vec{x}))$ and $S(T(\vec{x}))$?

$$S(\vec{x})=\begin{bmatrix}-x_2-3x_1\\x_2-x_3\\x_2\end{bmatrix}$$

$$T(\vec{x})=\begin{bmatrix}x_1-2x_2+x_3\\x_3-x_1\\2x_1+x_2-x_3\end{bmatrix}$$

$$\vec{x}=\begin{bmatrix}-2\\-1\\0\end{bmatrix}$$

11. **(15 pts)** If $\det(A) = 7$, find $\det(B)$.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} a & 3g & d \\ b & 3h & e \\ c & 3i & f \end{bmatrix} \qquad C = \begin{bmatrix} a & b & c \\ 3g & 3h & 3i \\ d & e & f \end{bmatrix}$$

12. **(15 pts)** The subspace V is a plane in $\mathbb{R}^4$. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ -1 \end{bmatrix}\right)$$