Contents

Resistive Network Analysis

Learning Objectives:

1. Compute solutions to circuits using *Node Voltage Analysis Method*.
2. Compute solutions to circuits using *Mesh Current Analysis Method*.

Node-Voltage Analysis Method

It is the most general method for analysis of electric circuits. This method is based on defining the voltage (with respect to one node chosen as the reference node whose voltage is taken as zero) at each node as an independent variable. Then, each branch current is expressed in terms of one or more node voltages. Once current in each branch is defined in terms of the node voltages, Kirchoff's current law is applied at each node except at the reference node.

$$\sum i = 0$$

For a circuit with N nodes, this results in set of (N-1) linear equations.

Steps of Node Voltage Analysis method

1. Select a reference node (voltage = 0). Choose negative terminal of a voltage source as the reference node.
2. For each voltage source in the circuit which is connected to the reference node, the non-ground node voltage is a known constant.
3. For all other voltage sources (not connected to the reference node), one end is an independent variable and the other end node voltage can be written in terms of this independent variable.
4. Mark all the node voltages this way, and determine the number of independent variables.
5. Apply KCL at the nodes to obtain as many independent equations as the number of independent variables. Express current in each resistive branch in terms of the adjacent node voltages.
6. If one of the branches connected to a node is a voltage source, then use the concept of super node by encircling the voltage source.
7. Solve the linear equations to determine the node voltages.
8. All other branch voltages and currents can then be calculated from the node voltages.

Case 1: With an Ideal Voltage Source with one end Grounded

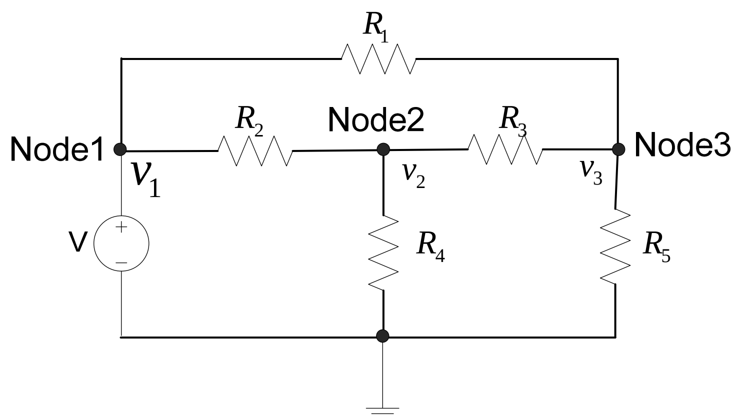


Fig. 1 Simple case of node analysis

Selecting the Reference Node

Any node can be taken as the reference node. However, the negative end of a voltage source is taken as the reference node.

Assigning Node Voltages

Node 1 voltage will be V (same as the voltage source).
Label the other node voltages as v_2, v_3.

Writing KCL equations in terms of the Node Voltages

We need two independent equations as we have two unknowns. These are obtained by applying KCL at nodes 2 and 3.

KCL states that the algebraic sum of currents leaving a node is equal to zero.

To find the current leaving the node *n* through each resistor connected to the node *k,* we subtract the voltage at node *k* from the voltage at node *n* and divide the difference by the resistance.

$$I_{nk} = \frac{v_n - v_k}{R_{nk}}$$

Applying KCL at node 2,

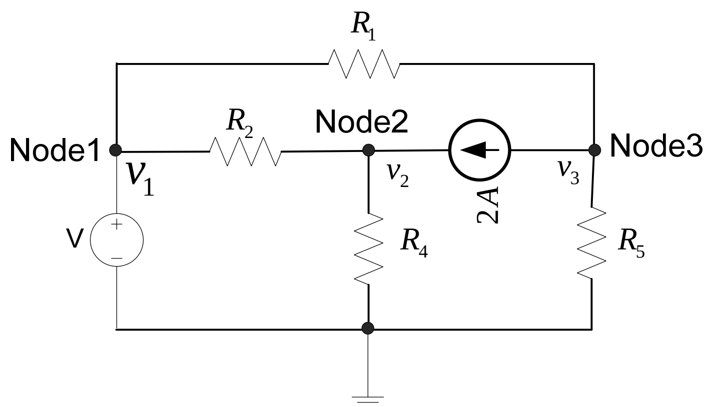
$$\frac{v_2 - V}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0 \qquad (1)$$

Applying KCL at node 3,

$$\frac{v_3 - V}{R_1} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_5} = 0 \qquad (2)$$

These two equations can be solved to determine the two unknowns v_2 and v_3.

Case 2: With an Ideal Current Source.



Applying KCL at node 2 (sum of currents leaving node2 equals zero), current going from node2 to node3 is **-2A**.

$$\frac{v_2 - V}{R_2} + \frac{v_2}{R_4} - 2 = 0 \qquad (1)$$

Applying KCL at node 3, sum of currents leaving the node equals zero.

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + 2 = 0 \qquad (2)$$

These two equations can be solved to determine the two unknowns v_2 and v_3.

Case 3: With an Independent Voltage Source connected between two nodes.

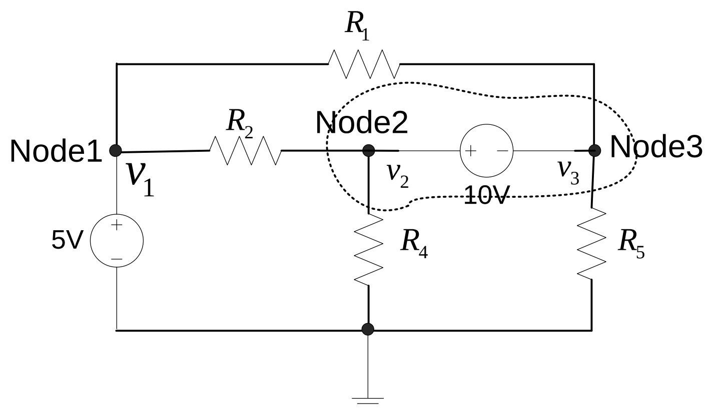


Fig. 2 Case with a voltage source between two nodes

As shown in the Figure 2, the 5V source is connected between ground and node 1. Hence, node 1 voltage is known as 5V.

The 10V source is connected between two nodes (node2 and node3), neither of which is the reference node.

If we take the node 2 voltage as unknown variable v_2, then node3 voltage is $v_2 - 10$. In this problem, we have only one independent variable (v_2) and hence we need only one equation to solve this.

We cannot express the current through the voltage source in terms of the node voltages. Hence, we cannot write the usual KCL at node 2 and node 3.

To solve this problem, we identify a super node (marked by the dotted line) around the voltage source as shown in Figure 2. We can then find the currents in the branches (R1, R2, R4, R5) associated with the super node in terms of the node voltage variables. We can apply the KCL to the super node to find the desired equation.

The KCL equation will be,

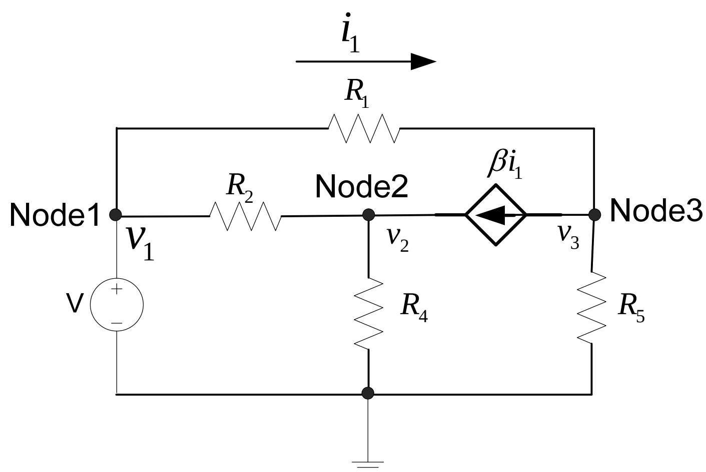
$$\frac{v_2 - 5}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - 10}{R_5} + \frac{v_2 - 10 - 5}{R_1} = 0 \qquad (1)$$

We can solve above equation to find v_2.

Case 4: With Dependent Sources

Such situations arise in study of transistor amplifiers. Method for handling such situations:

1) Write the equations using the value of the controlled source as it is.
2) Then use the **constraint equations** (which relates the dependent source to one of the other voltage and currents) to substitute the value of controlled source.
3) Then, the values of the controlling variables are replaced in terms of the node voltages



Applying KCL at node 2 (sum of currents leaving node2 equals zero), current going from node2 to node3 is $-\beta i_1$.

$$\frac{v_2 - V}{R_2} + \frac{v_2}{R_4} - \beta i_1 = 0 \qquad (1)$$

We have to next replace the current $i_1 = \dfrac{V - v_3}{R_1}$ in above equation to get the equation in terms of the node voltages as desired:

$$\frac{v_2 - V}{R_2} + \frac{v_2}{R_4} - \beta \frac{V - v_3}{R_1} = 0 \qquad (1)$$

We can apply KCL at node 3 and obtain the second equation in similar fashion. This way, we end up having the all the independent equations in terms of the node voltage variables.

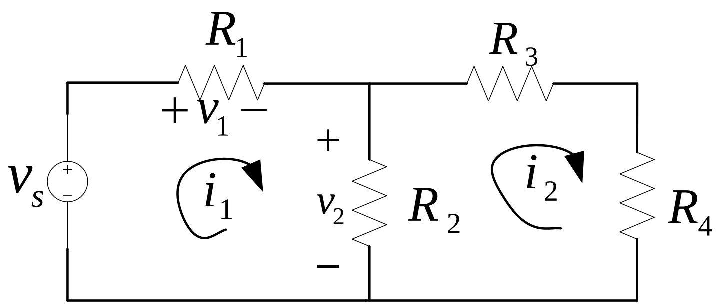
Mesh current analysis method

The mesh current method employs mesh currents as the independent variables. From the mesh currents, the current and voltage for each branch is obtained using Ohm's law. Subsequent application of Kirchoff's voltage law (KVL) around closed paths provides the desired system of equations.

Mesh 1: KVL requires that

$v_s - v_1 - v_2 = 0$

where $v_1 = i_1 R_1, v_2 = (i_1 - i_2) R_2$



Steps of Mesh Current Analysis Method

1. Identify **all** the mesh currents in the circuit.
2. Write KVL equations to obtain linear equations of same number as the number of mesh currents.
3. While applying KVL, we have to do an algebraic sum of the voltages along the path. It is often convenient to take voltage drop along the path as positive.
4. If the branch is a resistor, the voltage drop along the direction of motion is equal to the resistance multiplied by the net resistor current in that direction. If the resistor is common to two meshes, then the resistor current will be an algebraic sum of the two mesh currents: the mesh current in the direction of motion will be of positive sign and the mesh current in the opposite direction will be of negative sign.
5. If the branch is a voltage source, there will be a voltage drop when we go from positive terminal to negative terminal. However, if we go from the negative terminal to the positive terminal, then there will be a voltage rise.
6. If the branch is a current source, we cannot know the voltage across it. Hence, we have to avoid that branch and take a detour to complete the closed path.

Example: Mesh current analysis to find the voltages and currents for all the resistances in the circuit.

Case 1: With a single Independent Voltage Source

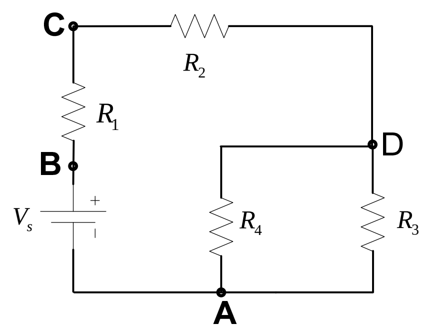


Fig. Given circuit to be solved

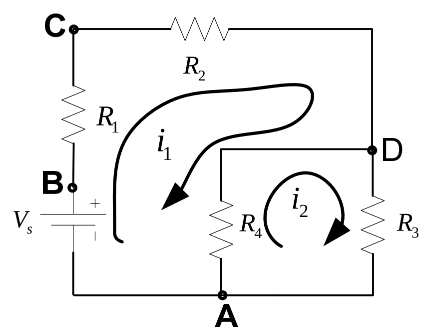


Fig. Identifying all the mesh currents

For path ABCDA, applying KVL:

$$-V_S + R_1 i_1 + R_2 i_1 + R_4(i_1 - i_2) = 0 \qquad (1)$$

For path ADA, applying KVL:

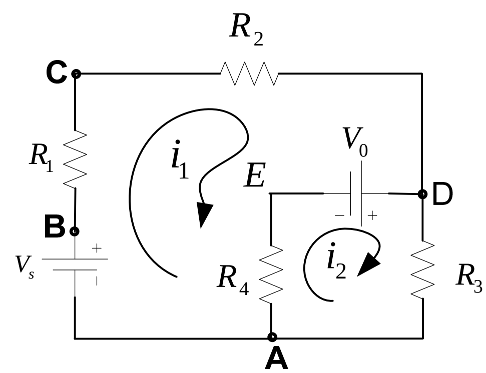
$$R_4(i_2 - i_1) + R_3 i_2 = 0 \qquad (2)$$

Solving the two equations above, one can get the values of i_1 and i_2.

Using the two mesh currents, we can find the current and voltages associated with all the resistors in the circuit.

Case 2: With two Independent Voltage Sources

Suppose there were an extra voltage source between in D and E as given in the Figure below.



Applying the KVL to ABCDEA:

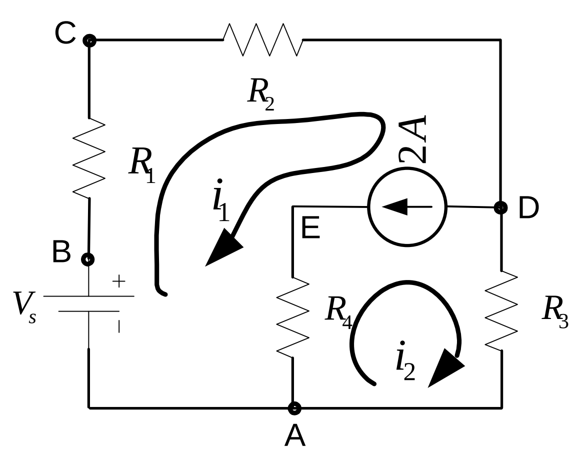
$$-V_s + R_1 i_1 + R_2 i_1 + V_0 + R_4(i_1 - i_2) = 0 \quad (1)$$

Applying the KVL to AEA:

$$R_4(i_2 - i_1) - V_0 + R_3 i_2 = 0 \quad (2)$$

Case 3: With both Independent Voltage and Current Source.

Suppose there was a current source between in D and E as given in the Figure below.



To apply the KVL to ABCDEA, we need the voltage across the current source (between DE). However, we cannot express the voltage across the current source. Hence, we should not choose ABCDEA as the closed path to write the KVL.

This problem is solved by applying KVL for the loop ABCDA:

$$-V_s + R_1 i_1 + R_2 i_1 + R_3 i_2 = 0 \qquad (1)$$

We can get the other equation from the current source:

$$i_1 - i_2 = 2 \qquad (2)$$

We can solve for both the mesh currents and from there determine the voltage and current in all the parts of the circuit.

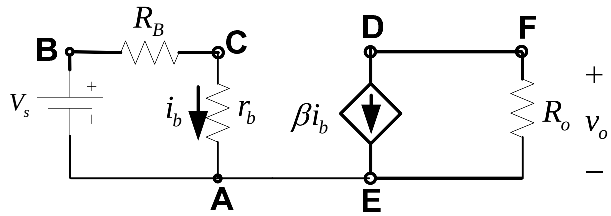
Case 4: With Dependent Sources

Such situations arise in study of transistor amplifiers.

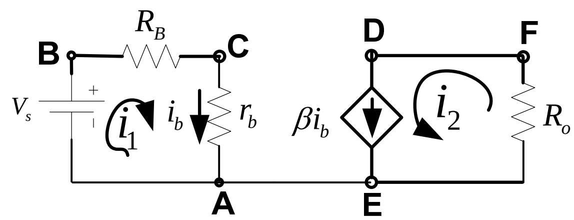
Method for handling such situations:

1) Write the equations as before, using the value of the dependent source.
2) Then use the constraint equations to substitute the controlled source in terms of the controlling voltage or current.
3) Substitute the controlling voltage or current in terms of the mesh currents.

Example: To find the output voltage v_o.



Identifying the mesh currents.



Applying KVL to ABCA:

$$-V_S + R_B i_1 + r_b i_1 = 0 \qquad (1)$$

However we cannot apply KVL to EDFE.

However, we can see that the dependent current source related to the current source:

$$i_2 = \beta i_b$$

Again, we can substitute $i_b = i_1$

$$i_2 = \beta i_1 \qquad (2)$$

Using these two equations, the two mesh currents can be solved.