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The principle of Superposition

In a linear circuit containing N sources, each branch voltage and current is the sum of N voltages and currents, each of which may be computed by setting all but one source equal to zero and solving the circuit containing that single source.

The superposition principle states that the total response is the sum of responses to each of the independent sources acting individually.

$$r_T = r_1 + r_2 + ... + r_n$$

where r_T is the total response and r_n is the response due to the n^{th} source acting individually. Here, the voltage (or current) in the branches of the linear circuit is called as 'response'. The response (voltage or current) is found for the active source alone.

To find the individual response, we **'kill'** all the other independent sources. An independent **voltage source is killed by replacing it with a short circuit (zero voltage across it)** and a **current source is killed by replacing it with an open circuit (zero current through it)** between its two terminals.

Superposition principle is not a commonly used analysis technique, but rather a conceptual aid to visualize the behavior of the linear circuits containing multiple sources. Though sometimes, the analysis of a circuit is simplified by applying superposition principle i.e. by considering each independent source separately.

Finally, the total response is obtained by adding the individual responses from all the independent sources together.

When all the independent sources are killed at the same time (i.e. keeping the dependent sources only in the circuit) then, the response becomes zero. Thus, dependent sources do not contribute to a separate term to the total response. We must not kill dependent source while applying superposition principle.

Linearity

In mathematics, a linear function has the following two properties:

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$
$$f(\alpha \times x) = \alpha \times f(x)$$

The first property is called additive property (superposition principle) and the second property is called homogeneity (multiplicative property).

The function, $y = kx$ has these two properties and hence is a linear function.

However, the function, $y = mx + c$ is not a linear function (although it represents the equation of a straight line).

An ideal resistance follows Ohm's law as voltage is current times its resistance

$v = iR \Rightarrow i = \dfrac{v}{R}$.

For the ideal resistor, voltage is said to be a linear function of current. Any such element, where the current through the element is a linear function of voltage across it is called a linear element. Superposition principle holds only circuits having linear elements and linear controlled sources if any.

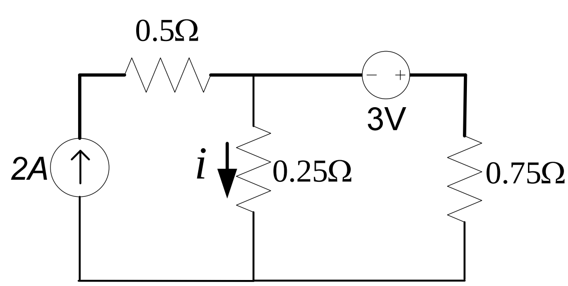
If a dependent source is described by $i_{cs} = Ki_x$ i.e. current in the controlled source is a constant times the control variable. Hence, it is said to be a linear controlled source (linear dependent source).

On the other hand if an element has voltage current relationship as $v = Ki^2$, which is a nonlinear function, then the element is said to be a nonlinear element.

Resistor, inductor, capacitor are linear elements. Semiconductor devices like diode, MOSFET are nonlinear elements.

Example:

Given the circuit, find the current in the 0.25 Ohm resistor,

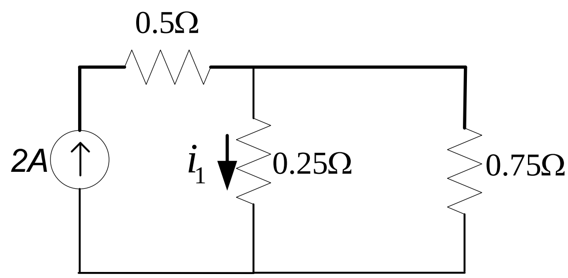


There are two independent sources in the circuit: one 2A current source and one 3V voltage source.

To apply Superposition method, we keep one source at a time by killing the other source. We find the current in each case and then add them up to find the total current.

Step1:

Keeping the current source only and killing the voltage source, the circuit will be



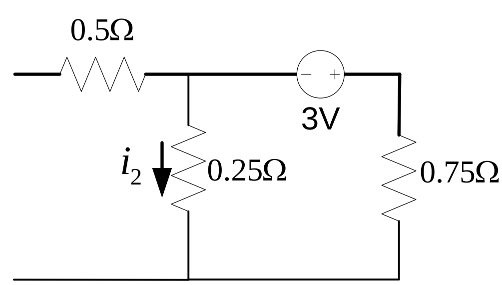
The current i_1 can be obtained current divider rule.

The current of 2A, is divided between 0.25Ω and 0.75Ω. So the current in the 0.25Ω is:

$$i_1 = 2\times\frac{0.75}{0.25+0.75} = 1.5A$$

Step2:

Keeping the voltage source only and killing the current source, the circuit will be



As the resultant circuit is one with a single voltage source with two resistances in series, we can find the current easily.

$$i_2 = \frac{-3}{0.25+0.75} = -3A$$

Step3:

Find the total current, $i = i_1 + i_2 = 1.5 - 3 = -1.5A$

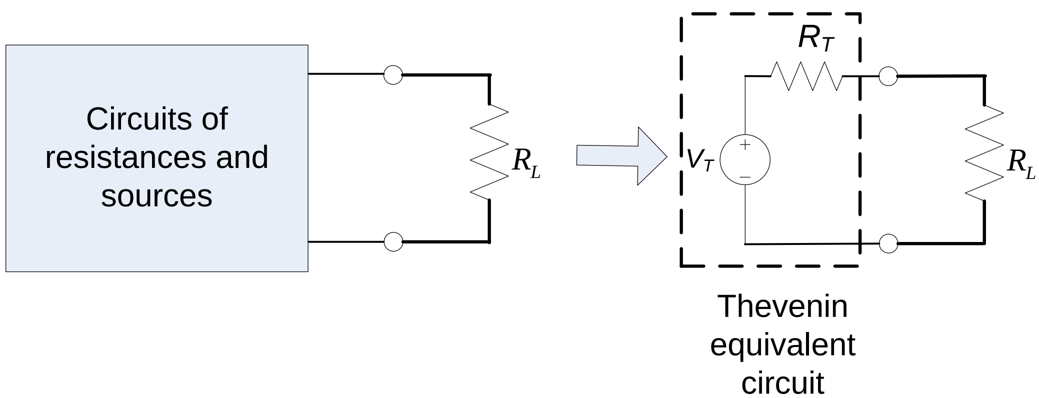
It can be observed that, applying superposition method resulted in simple circuits and ease of analysis. However, this may not be the case always. Hence, superposition method is seldom used for circuit analysis.

One-port Networks and Equivalent Circuits

While building complex circuits, sometimes, part of the circuit is interfaced with another circuit. At the point of interface, each sub-circuit is seen as a two-terminal circuit (one-port circuit).

Such two-terminal circuits can be replaced by an equivalent circuit consisting of a source and a resistance. This can be a voltage source with a series resistance or current source with a parallel resistance. The two equivalent circuits are known as Thevenin and Norton Equivalent Circuits respectively.

Thevenin's Equivalent



The Thevenin equivalent of a two-terminal circuit is represented by a voltage source in series with a resistor.

The value of the voltage source is the **open circuit voltage** of the circuit between the two terminals. Looking at the Thevenin's equivalen circuit, if nothing is connected between the two terminals, there will be no current flowing. Hence voltage drop across the resistor R_T will be zero. The voltage across the terminals then is the Thevenin voltage for the circuit.

$V_T = V_{oc}$

So to obtain the Thevenin equivalent for a given two-terminal circuit, we first obtain the open circuit voltage and the short circuit current, using circuit analysis methods like node voltage analysis or mesh current analysis. From these, we obtain the voltage and the resistance of the Thevenin's equivalent circuit.

Finding the Thevenin's resistance R_T

If there are only independent sources in the circuit, then R_T can be found be killing all the sources (short the terminals of an independent voltage source and open the end of an independent current source) and doing series parallel manipulations to find the equivalent resistance.

Short-circuit current method:

In the Thevenin's equivalent circuit, if the two terminals of the circuit are shorted, then the current flowing in the shorted branch is called the short circuit current. This will be equal to $I_{sc} = \frac{V_T}{R_T}$.

Using this, we can find the Thevenin resistance for a circuit from its Thevenin's voltage and short circuit current I_{sc} : $R_T = \frac{V_T}{I_{sc}}$

We need to determine the short circuit current between the two terminals (between which Thevenin equivalent is asked). This can be done by shorting the two terminals and then solving the circuit to find the current in the shorted branch.

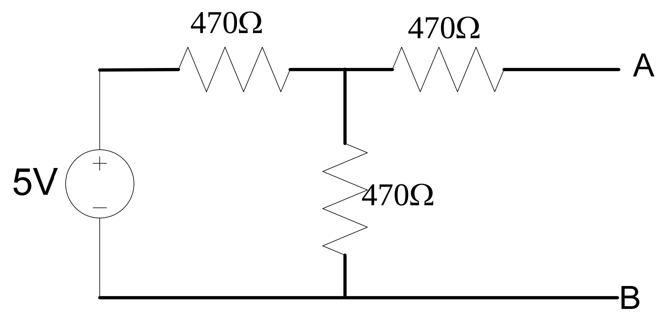
Test source method:

Test source method can also be used to find the Thevenin resistance.

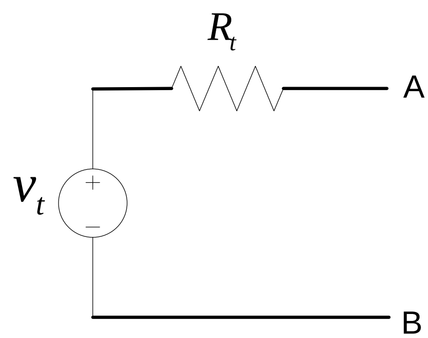
1) Kill all the independent sources. **Do not kill the dependent sources**
2) Connect a source (either voltage or current source) of known value across the two terminals.
3) Do the circuit analysis to find the current (if test source is a voltage source) from it, or the voltage across it (if the test source is a current source).
4) Divide the voltage across the terminals by the current into the terminals to find the Thevenin's resistance.

Test source method may have some advantages over the short circuit current method, if the circuit has many sources.

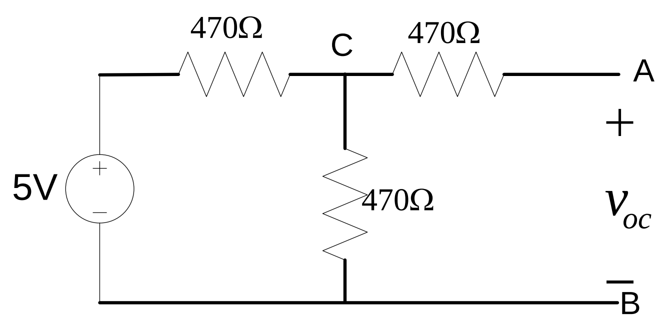
Example: Finding the Thevenin's equivalent circuit for the circuit between AB, where it is connected to another circuit.



To find the Thevenin equivalent:



Step1: To find the open-circuit voltage between points AB.



After opening the branch AB, there will be no current through the branch CA and the voltage between CB will be same as the voltage between AB.

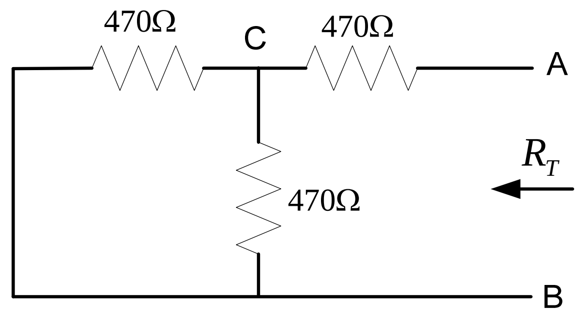
Voltage between CB can be obtained from voltage divider principle as:

$$v_{CB} = \frac{470}{470+470} 5 = 2.5V$$

So Thevenin voltage $V_t = v_{oc} = 2.5V$.

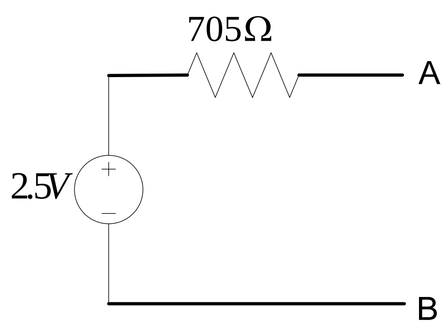
Step2: To find the Thevenin resistance

As the circuit only contains independent sources, we can kill them to reduce the circuit to a purely resistive circuit.



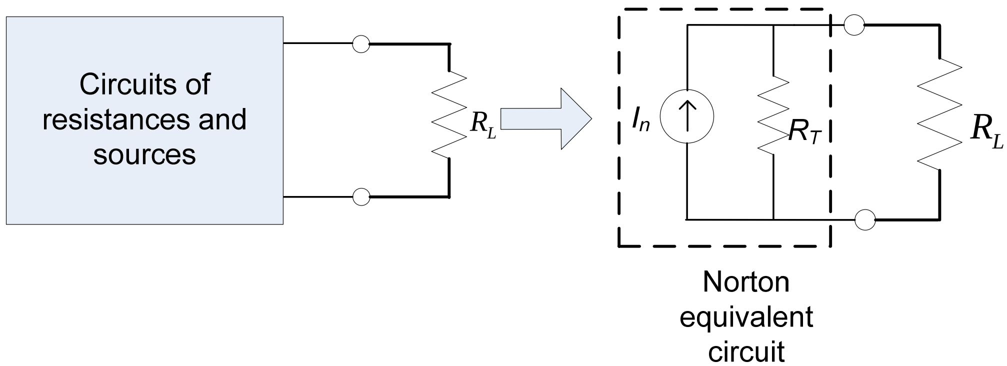
Thevein resistance $R_T = 470 + 470 \| 470 = 470 + 235 = 705\Omega$.

We have the Thevenin equivalent for the circuit.



Norton Equivalent

Norton equivalent circuit consists of a current source with a resistance in parallel.



Looking at the Norton equivalent circuit, if we short the two terminals, then the current through the shorting path would be same as the Norton current I_N.

Thus, $I_N = I_{SC}$.

And if we leave the two terminals open and measure the voltage across them, then it would be $I_N R_N$.

Thus,

$$V_{oc} = I_N R_N \Rightarrow R_N = \frac{V_{oc}}{I_N} = \frac{V_{oc}}{I_{sc}} = R_T$$

The parallel resistance in Norotn equivalent will be same as the Thevenin resistance.

If Thevenin's equivalent is already known, then Norton equivalent can be directly obtained by doing conversion from the voltage source to the current source.

$$R_N = R_T$$

$$I_N = I_{sc} = \frac{V_T}{R_T}$$

For finding Norton equivalent directly, we need to first determine the short-circuit current and then determine the Thevenin's resistance as shown earlier.

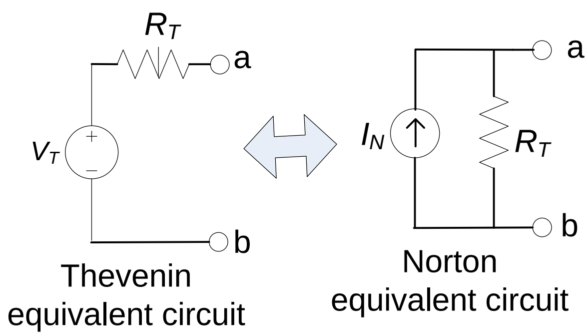
Source conversion:

Voltage source with series resistance can be converted to equivalent current source with the same resistance in parallel. Similarly, a current source with a parallel resistance can be converted to an equivalent voltage source in series with the resistance.

The relationship between the voltage and the current source is:

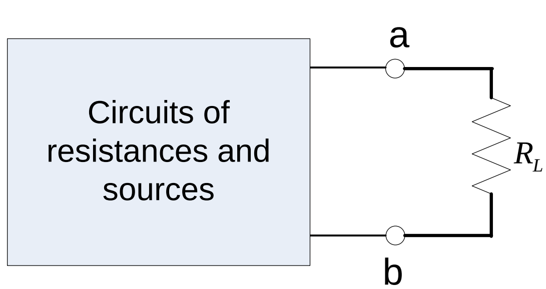
$$I_s = \frac{V_s}{R}$$

$$V_s = I_s R$$

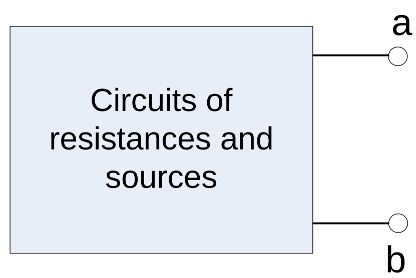


The relationship between the reference current direction and the voltage polarity should be maintained. This means that the positive of the voltage source is closer to terminal a, then the current should be entering into terminal a.

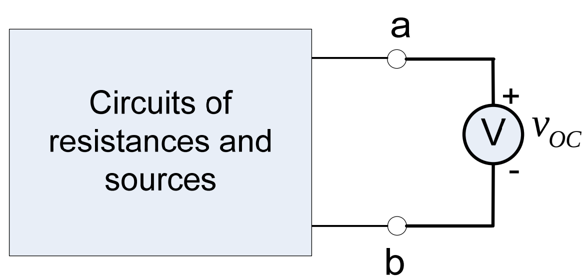
Experimental determination of Thevenin and Norton equivalents



1) Make sure the load (or the circuit to be interfaced) is disconnected.

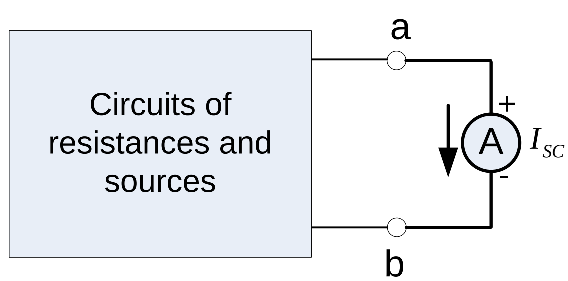


2) When the circuit is powered up, measure the voltage between the terminals using a voltmeter. This will be the open circuit voltage or Thevenin voltage.



$V_t = v_{OC}$

3) Power down the circuit. Connect an ammeter between the two terminals and power up the circuit to measure the current. This will be short circuit current or Norton current.



$I_n = I_{SC}$

4) Thevenin resistance can then be obtained by dividing the Thevenin voltage by the Norton current.

$$R_T = \frac{V_T}{I_N}$$

5) These will be good approximations for the parameters of Thevenin and Norton equivalent circuits as the practical voltmeter and the ammeter will have a finite resistances as shown in the figure.

Thevenin and Norton equivalent can be used for circuit analysis. Suppose we are asked to find the current in any branch in the circuit, we treat the concerned branch as a load and then find the Thevenin or Norton equivalent of the remaining circuit. This results in a simpler circuit, and the current in the concerned branch can be obtained easily.

Maximum power transfer

Representation of circuits in their Thevenin and Norton equivalents shows that part of the power supplied to the load will be lost in the Thevenin resistance.

$$i_L = \frac{V_T}{r_L + R_T}$$

$$p_L = i_L^2 r_L = \frac{V_T^2}{(r_L + R_T)^2} r_L$$

The power transfer between the source and the load will be dependent on the load resistance.

The value of load resistance for which the load power will be maximum, $\frac{dp_L}{dr_L} = 0$.

$$\frac{dp_L}{dr_L} = \frac{v_T^2 (r_L + R_T)^2 - 2v_T^2 r_L (r_L + R_T)}{(r_L + R_T)^4} = 0$$

$$v_T^2 (r_L + R_T)^2 - 2v_T^2 r_L (r_L + R_T) = 0$$

$$r_L = R_T$$

This implies that the power transfer to the load will be maximum when the load resistance is equal to the Thevenin resistance.

Energy efficiency $\eta = \frac{P_L}{P_{source}}$, i.e. the ratio power absorbed by the load to the power delivered by the source.

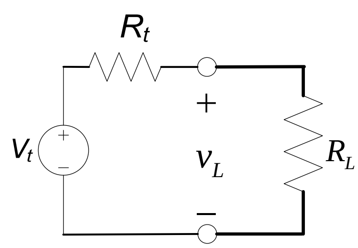
$$\eta = \frac{p_L}{p_{source}} = \frac{i_L^2 r_L}{V_T i_L} = \frac{i_L r_L}{V_T} = \frac{V_T}{r_L + R_T} \frac{r_L}{V_T} = \frac{r_L}{r_L + R_T}$$

It can be observed that the efficiency of the system is only 50% when the maximum power transfer occurs. Hence, maximum power transfer is not desirable for high power systems as considerable power would be wasted. For high power systems, the efficiency is vey important and hence, it is desirable to have minimum (zero would make an ideal source) internal resistance for the source.

Maximum power transfer is desirable in the field of wireless communication where the power of the electric signal received by the antenna is not much and the only concern is to design the processing circuit such that the signal to noise ratio of the system is maximized by maximizing the power transfer at the carrier frequency by matching the load resistance (impedance) with that of the antenna.

Source loading

It can be noted from the Thevenin equivalent circuit that the Thevenin resistance causes a voltage drop and the load sees a reduced voltage. The amount of voltage reduction depends on the load current, more the load current lesser is the voltage at the load.



$$v_L = v_T - i_L R_T$$

This phenomenon is called source loading.

This is typical of practical voltage sources as most real-life voltage sources like batteries can be more accurately modeled as an ideal voltage source in series with a resistance called as the internal resistance. The terminal voltage of the battery will be maximum when the load is zero or it is open circuited.

Nonlinear elements

What if a circuit has all linear elements except for one which has a i-v characteristics that cannot be represented by a linear function or worse still cannot have an analytical function. Devices like diode or MOSFET are used in circuits along with voltage sources and resistors. How can we find the current and voltage for such devices?

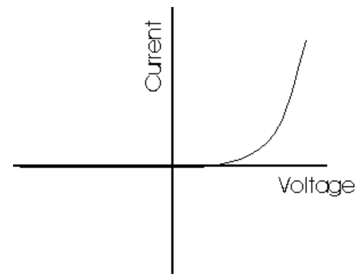


Fig. Diode characteristics

To solve these cases, a two-step method can be used.

1) Obtain the Thevenin equivalent of the circuit treating the nonlinear device as the load.
2) Use graphical (load-line) analysis.

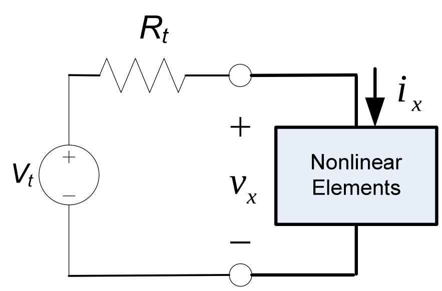


Fig. Linear circuit with one nonlinear element

Graphical (Load-line) analysis

Consider the voltage across the nonlinear device to be v_x and the current be i_x. Then using KVL,

$$v_T = i_x R_T + v_x$$

$$i_x = -\frac{1}{R_T} v_x + \frac{v_T}{R_T}$$

This is a straight line on $(i_x \text{ vs } v_x)$ plane with a slope of $-\frac{1}{R_T}$ and intercept of $\frac{v_T}{R_T}$.

This equation is known as the load-line equation which depends on the Thevenin voltage and Thevenin resistance. The actual values of current and voltage for the device will be the point of intersection between the load line and the i-v characteristics of the nonlinear device. This is known as graphical-analysis method and is useful when the device characteristic is highly nonlinear and no analytical formula is available.

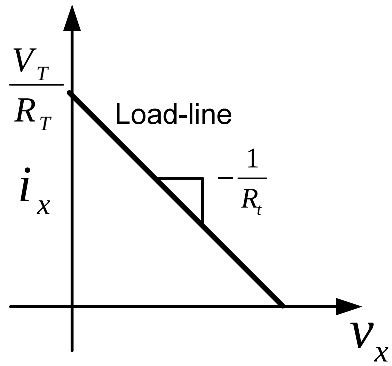


Fig. Load-line characteristics

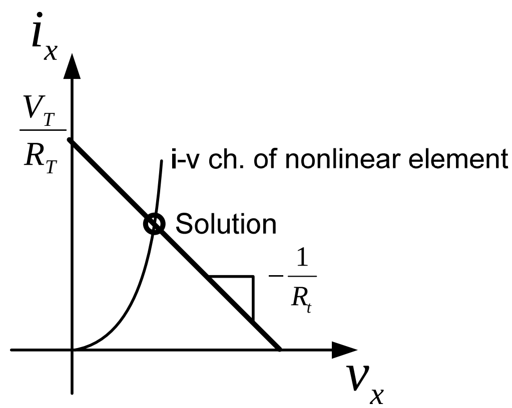


Fig. Graphical (load-line) analysis