EE1002: Introduction to Circuits and Systems

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Lecture Venue – <u>Engineering Auditorium (Monday and Thursday)</u>

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Learning Objectives

- Review of basic principles of electricity and magnetism.
- Understanding of magnetic fields and their interactions with moving charges.
- Use of **right-hand-rule** to determine the direction of magnetic field around a current carrying wire.
- Calculate **force induced** on current carrying wire placed in a magnetic field.
- Calculate **voltage induced in a coil** by changing magnetic flux and also in a moving conductor cutting through a static-magnetic field.

Learning Objectives (cntd.)

- Use of **Lenz's law** to determine the polarity of the induced voltage in a coil or conductor.
- Use of **concepts of reluctance and magnetic circuit equivalents** to compute magnetic flux and currents in simple magnetic structure.
- Use of **magnetic circuit models** to analyse transformers.
- Understand **ideal transformers** and solve circuits involving transformer.

Introduction to Magnetic Circuits and Transformers

- You would be building an autonomous vehicle that would be able to move around following a track.
- In the project you would use DC power supply and DC electric motors as components to propel the autonomous vehicle.
- In this and following lectures, we introduce the basic laws of electromagnetism and electromagnetic induction that would help you understand the basic principles of operation of:
  - a transformer;
  - how to design DC power supply;
  - how to make use of DC motors to provide mechanical energy and
  - pulse-width-modulated (PWM) circuit to control the speed of DC motor.

Magnetic Field

- Magnetic fields exist around permanent magnets and also around wires that carry current.
- Basic source of the existence of magnetic field is the electric charge in motion.
- Magnetic field can be visualized as lines of magnetic flux that form a closed path as shown in Fig. 3.1.

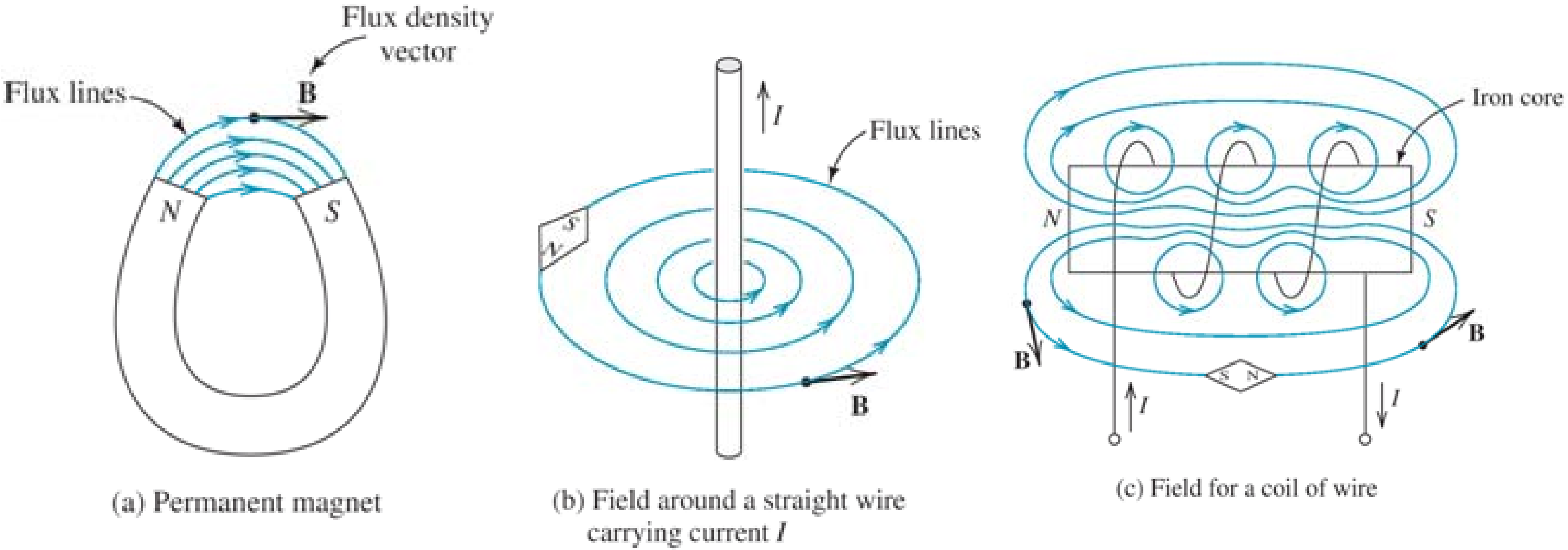


Fig. 3.1 (a) Permanent magnet, (b) field around a straight wire carrying current I and (c) field for a coil carrying current

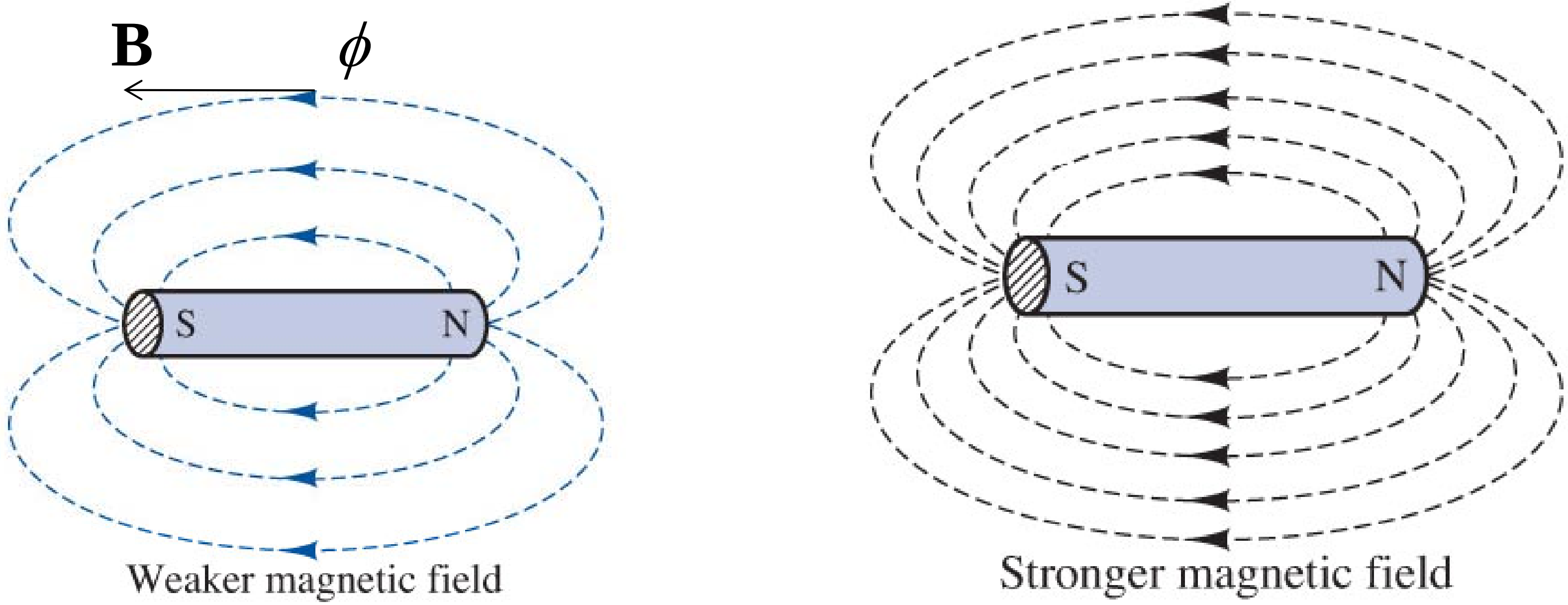
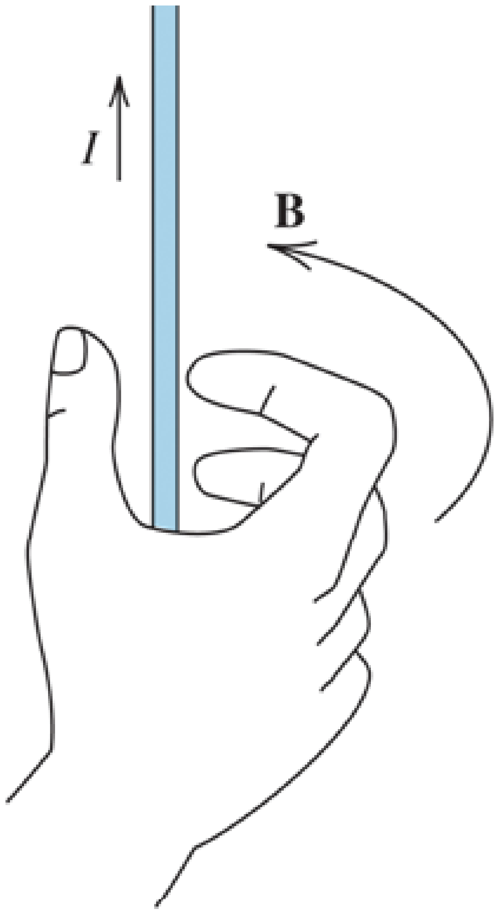


Fig. 3.2 Permanent magnet: (a) weaker magnetic field and (b) stronger field.

Right-Hand Rule

- The direction of the magnetic field produced by a current in a conductor can be determined by the **right-hand-rule** (RHR).
- If the wire grasped with the thumb pointing the direction of current then the finger encircling the wire would represent the direction of the magnetic field as shown in Fig. 3.3.



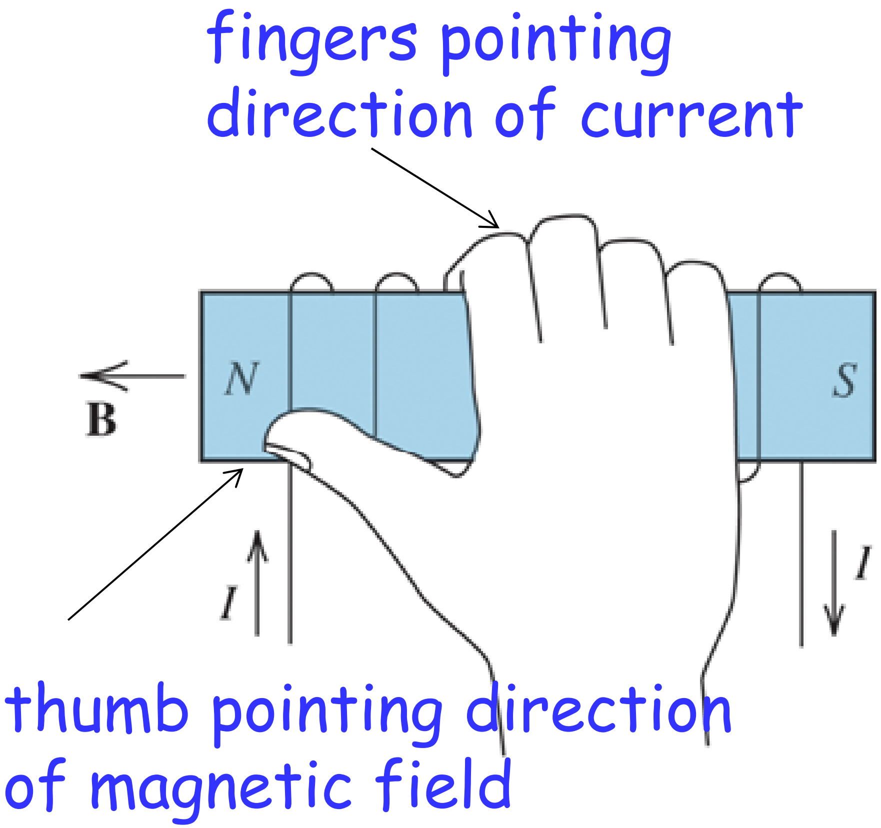


Fig. 3.3 (a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

ⓧ means current, *i* points into the page

⦿ means current, *i* points out of the page

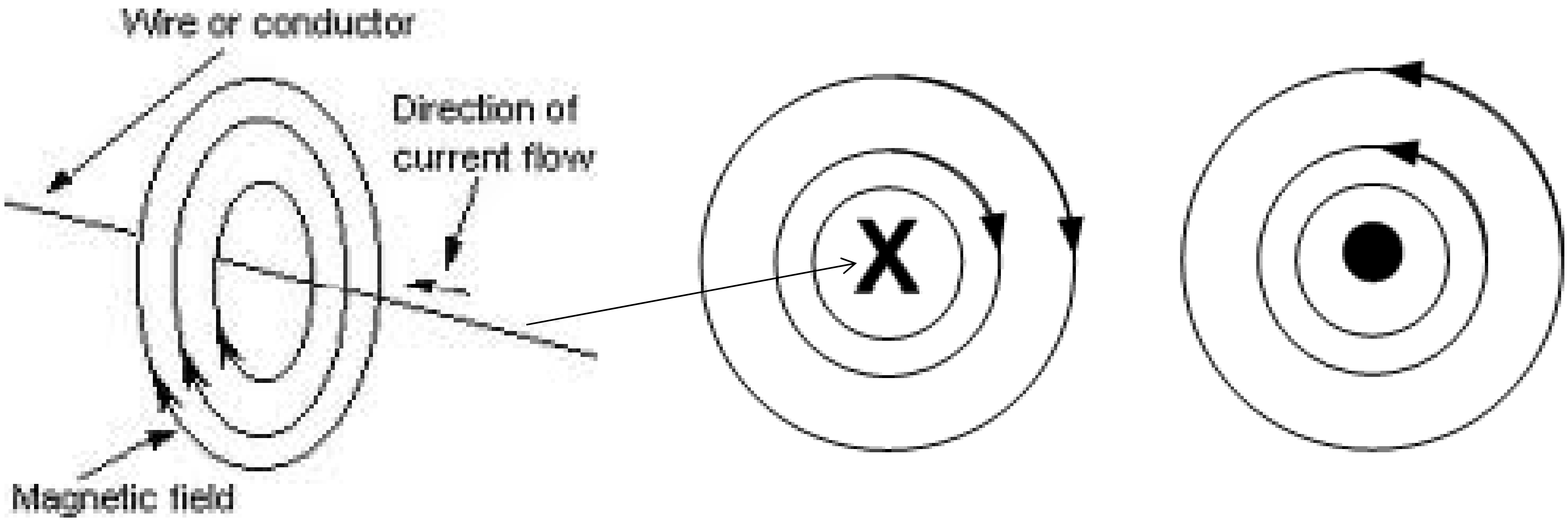


Fig. 3.3c

Flux-linkages, Faraday's Law and Lenz's Law

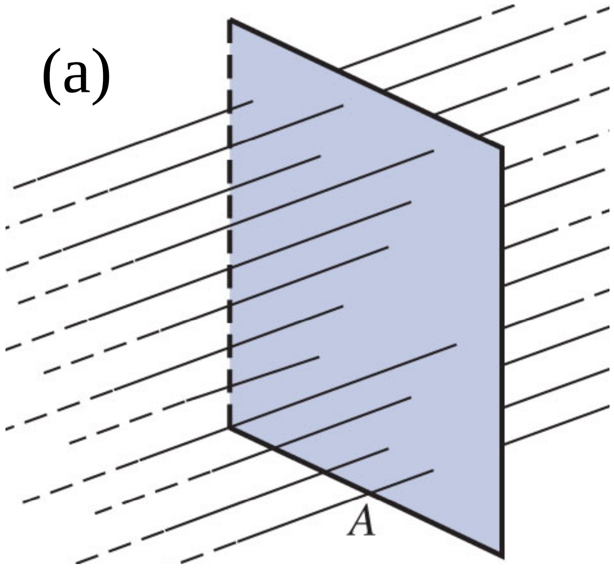
- The magnetic flux passing through a surface area, *A* as shown in Fig. 3.4(a) is given by eqn. 3.1.

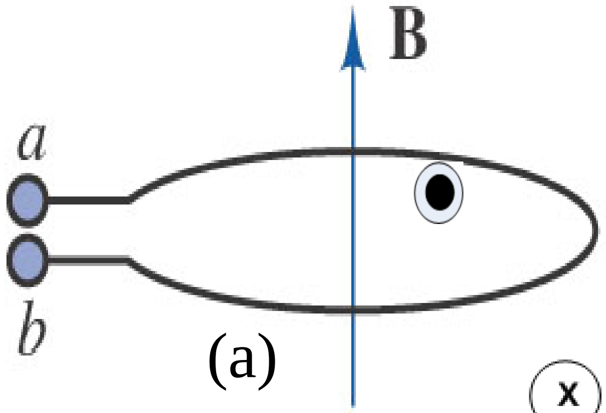
$$\phi = \int_A \mathrm{B}\ d\mathrm{A}, \quad (3.1)$$

$$\Rightarrow \phi = B \times A \ \ (3.2) \Rightarrow B = \frac{\phi}{A}\ (3.2a)$$

- If, the magnetic flux density, **B is constant** and perpendicular to the surface area, *A* as shown in Fig. 3.4(a), eqn. 3.1 gets reduced to eqn. 3.2a.

(a)





ⓧ means $\mathbf{B_i}$ points into the page

● means $\mathbf{B}$ points out of the page

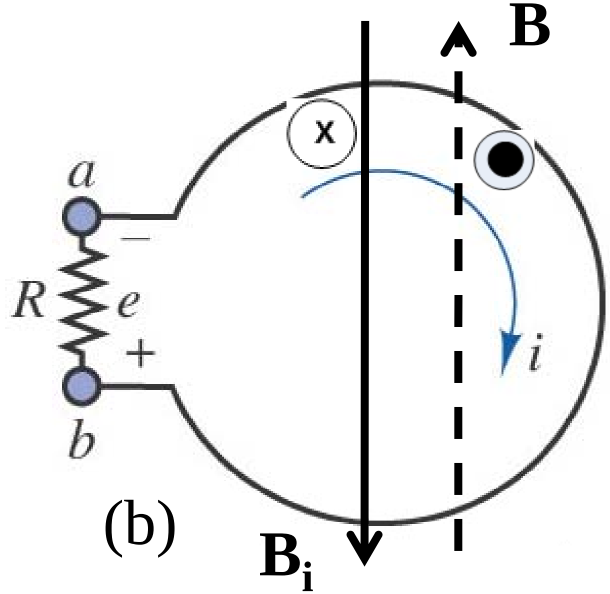


Fig. 3.4 (a) Magnetic flux lines passing through a surface, (b) magnetic field, B linking the coil and (c) current, *i* generating a reaction flux to oppose the main flux, B.

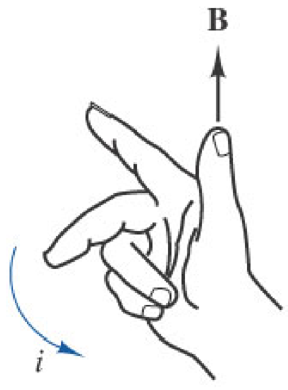
- Faraday's law states that a time varying flux causes an induced electromotive force or emf, *e* in a coil as follows:

$$e = -\frac{d\phi}{dt} \qquad (3.3)$$

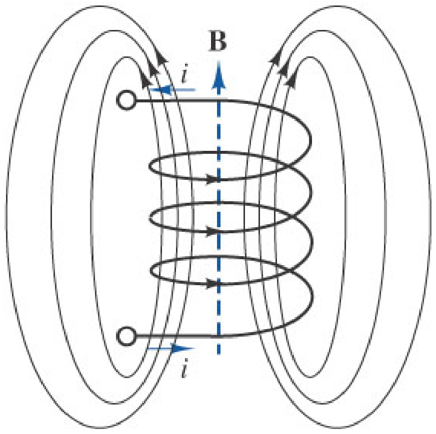
- The induced emf, *e* would cause a current to flow in such a direction that it would produce a reaction flux, ***B**ᵢ* that would oppose the original flux, ***B***. This is known as Lenz's law.
- The polarity of the induced emf can be determined from the physical considerations and leaving the -ve sign out of eqn. 3.3.

- The magnitude of the voltage induced due to magnetic field can be increased if the number of coils, N can be increased by having coils tightly wound.
- It would be good to define the term flux-linkage, λ as

$$\lambda = N \times \phi \qquad (3.4) \Rightarrow e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} \qquad (3.4a)$$



Right-hand rule



Flux lines

Fig. 3.5 (a) RHR, (b) coils tightly wound.

Voltage Induced in Field-Cutting Conductor

- Voltage is also induced in a conductor when it is moving through a magnetic field in a direction such that the magnetic lines of flux are cut as shown in Fig. 3.6.
- The sliding conductor of length, *l* , moving in the direction of, *x* and the stationary rails form a single loop having an area of $A = l\,x$.

$$\lambda = (N = 1) \times \phi = 1 \times (B \times A) = B \times (l \times x) = Blx$$

$$\Rightarrow e = \frac{d\lambda}{dt} = \frac{d(Blx)}{dt} = Bl\frac{dx}{dt} = Bl \times \left(\frac{dx}{dt} = u\right) = Blu \quad (3.5)$$

- Eqn.3.5 forms the basis of electric generator operation.

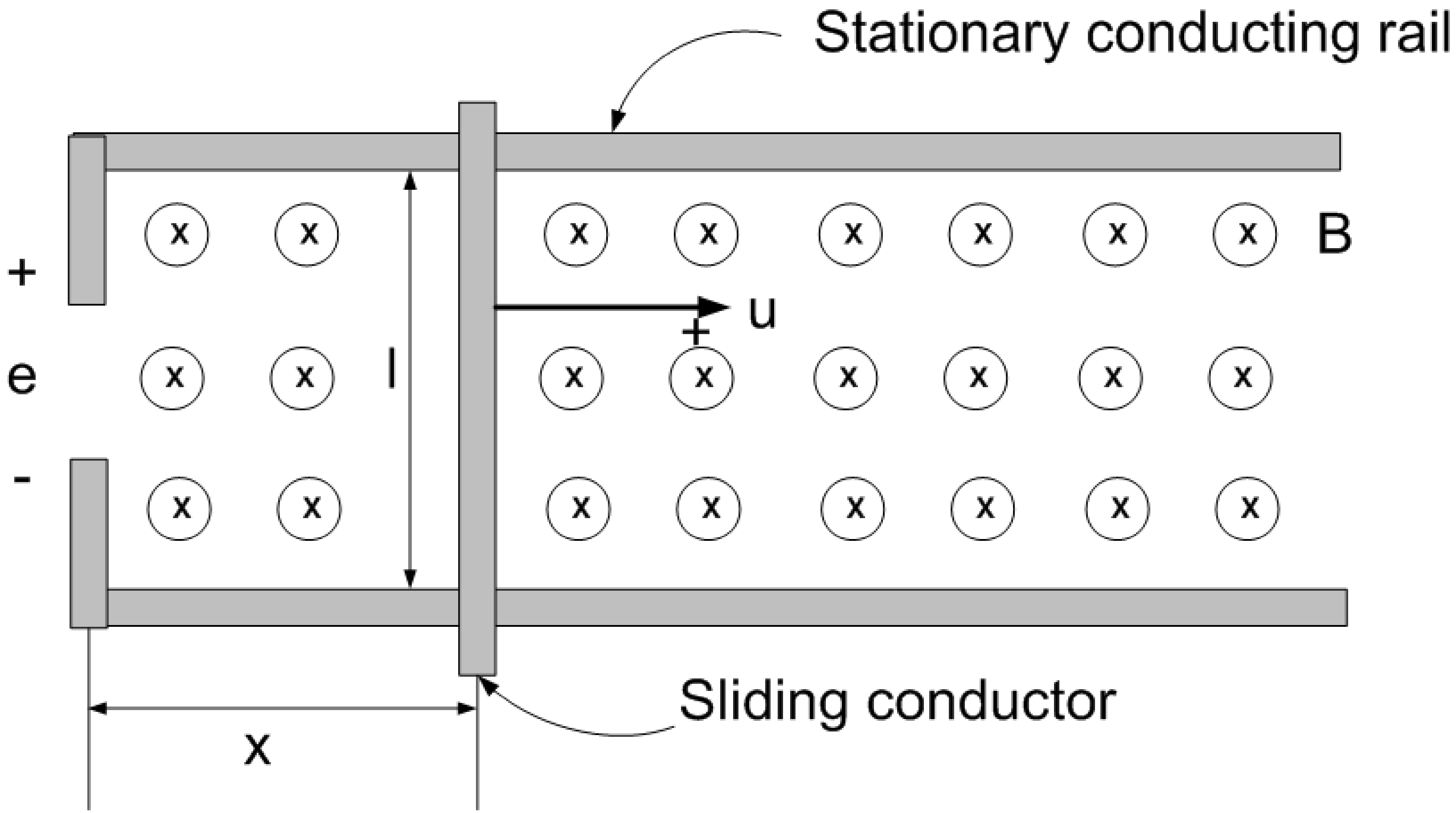


Fig. 3.6 Voltage induced in a conductor moving so as to cut through magnetic flux lines.

Example1: A conductor in a typical dc generator rated for 1 kW has a length of 0.2 m, the conductor is moving at a velocity of 12 m/s, and cuts through the magnetic field of 0.5 Wb/m^2 pointed into the plane of the paper. Determine the corresponding voltage induced in the conductor.

Solution: Using eqn. 3.5 we have,

$$e = \frac{d\lambda}{dt} = Blu = \left(0.5Wb/m^2\right)\times\left(0.2m\right)\times\left(12m/s\right) = 1.2\ V$$

Voltage Induced by Time-varying Flux

- Consider a N - turn coil in which the flux-linking the coil is time varying then a voltage would be induced in the coil as:

$$\phi = \phi_m \sin(\omega t), \qquad e = N\frac{d\phi}{dt} = N\phi_m \omega \cos(\omega t)\, volts\ (1.6)$$

- Eqn. (3.6) forms the basis of transformer operation.

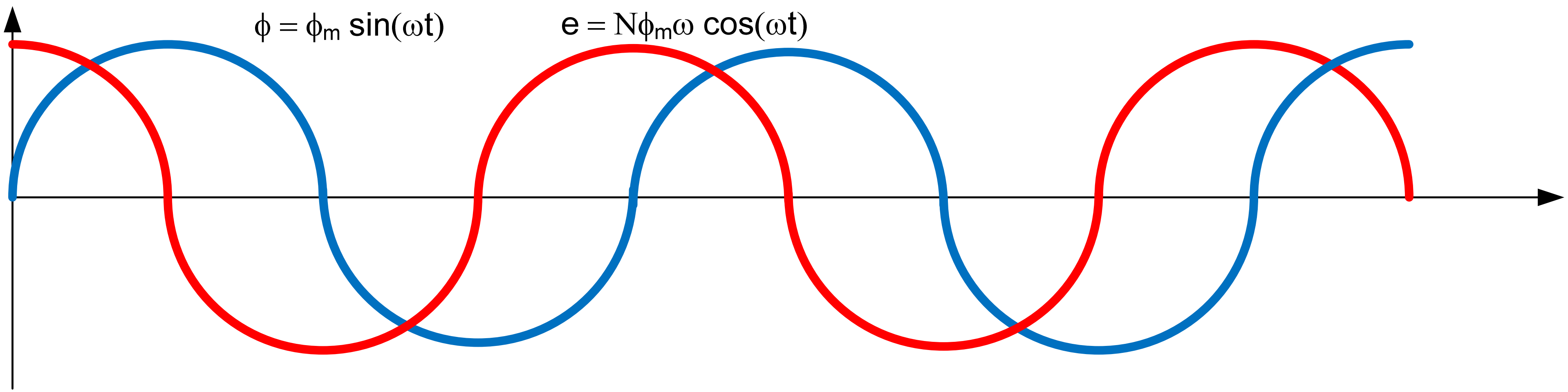


Fig. 3.7 Voltage induced in a stationary coil due to time varying flux.

Example2: A 10-turn circular coil has a radius of 5 cm.

(a) A flux-density of 0.5 Wb/m^2 is directed perpendicular to the plane of the coil. Determine the corresponding flux linking the coil and the flux-linkage.

(b) Assume that the flux is reduced to zero at a uniform rate during an interval of 1 ms. Determine the corresponding voltage induced in the coil.

Solution: (a) Using eqn. 3.2 we have

$$\phi = B \times A = 0.5 \times (\pi \times (0.05)^2) = 3.927\ mWb$$

$$\lambda = N \times \phi = 10 \times 0.003927 = 39.27\ mWb - turns$$

Solution: (b) Using eqn. 3.4a, we have

$$e = \frac{d\lambda}{dt} = \frac{0 - 0.03972}{0.001} = -39.72\,V$$

The -ve sign is due to the reason that the flux-linkage is reducing from a positive value of 39.27 mWb-turns to zero.

Magnetic Field Intensity and Ampere's Law

- Let us introduce another field vector, known as the magnetic field intensity, **H** and how it can be established.
- The magnetic field intensity, **H** and the magnetic flux-density, **B** are related by

$$\mathbf{B} = \mu \mathbf{H} = \mu_r \mu_0 \mathbf{H} \qquad (3.7)$$

μ – **magnetic permeability of the material,**

μ_r – **relative magnetic permeability of the material**

$\mu_0 = 4\pi \times 10^{-7}$ ***Wb / Am*** – **magnetic permeability of free – space,**

- Ampere's law states that the line integral of the magnetic field intensity around a closed path is equal to the algebraic sum of the currents flowing through the area enclosed by the path and is given by

$$\oint \mathbf{H} \cdot d\,\mathbf{l} = \sum i \quad (3.8)$$

- d**l** vector element of length having its direction tangent to the path of the integration.
- If **H** is constant and points in the same direction as the incremental length d**l** along the path, we have Ampere's Law as: $Hl = \sum i$ (3.8a)

- A given magnetic field intensity, **H** would give rise to different flux density, **B** in different magnetic materials.
- It would be good to define the *source* of the magnetic energy in terms of magnetic field intensity, **H** so that different magnetic materials can be evaluated for a given source.
- This source would be termed as magnotomotive force (mmf), $\mathcal{F}$.

Magnetic Field around a long Straight Wire

- Consider a wire carrying current, *i* , causing magnetic field, **B** around it.

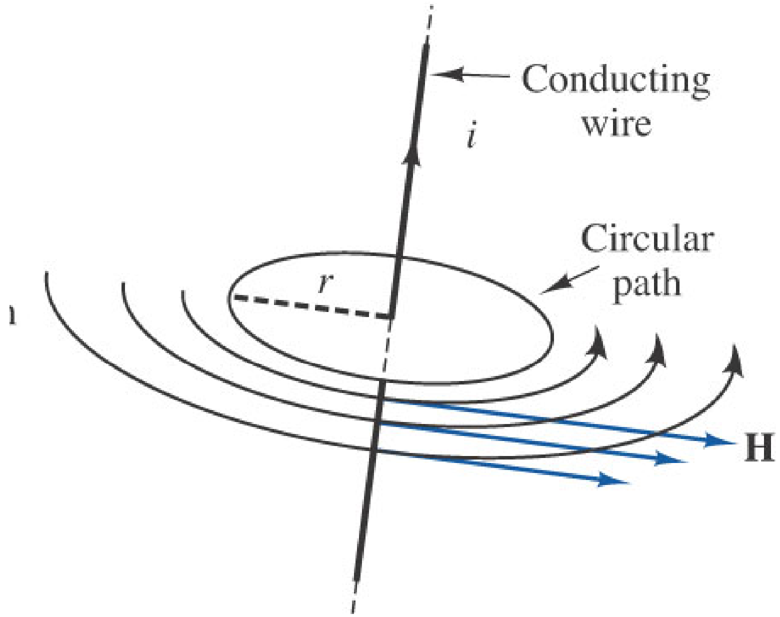


Fig. 3.8 Illustration of Ampere's Law

$$\boldsymbol{Hl} = \sum \boldsymbol{i} \Rightarrow \boldsymbol{H}\,2\pi\boldsymbol{r} = \boldsymbol{i} \Rightarrow \boldsymbol{H} = \frac{\boldsymbol{i}}{2\pi\boldsymbol{r}}$$

$$\Rightarrow \boldsymbol{B} = \mu\boldsymbol{H} = \mu \times \frac{\boldsymbol{i}}{2\pi\boldsymbol{r}} = \frac{\mu\boldsymbol{i}}{2\pi\boldsymbol{r}} \quad (3.9)$$

- Flux lines around a conductor would be far greater in the presence of a magnetic material than surrounded by air.
- Moreover, a single conductor carrying current is not that strong; a tightly wound coil with many turns as shown in Fig. 3.9 would greatly increase the strength of the magnetic field.
- For a coil with N turns carrying a current of i A, the flux-lines is increased by N-fold.
- The product $N \times i$ is a useful quantity in electromagnetic circuits and is called magnotomotive force, F.

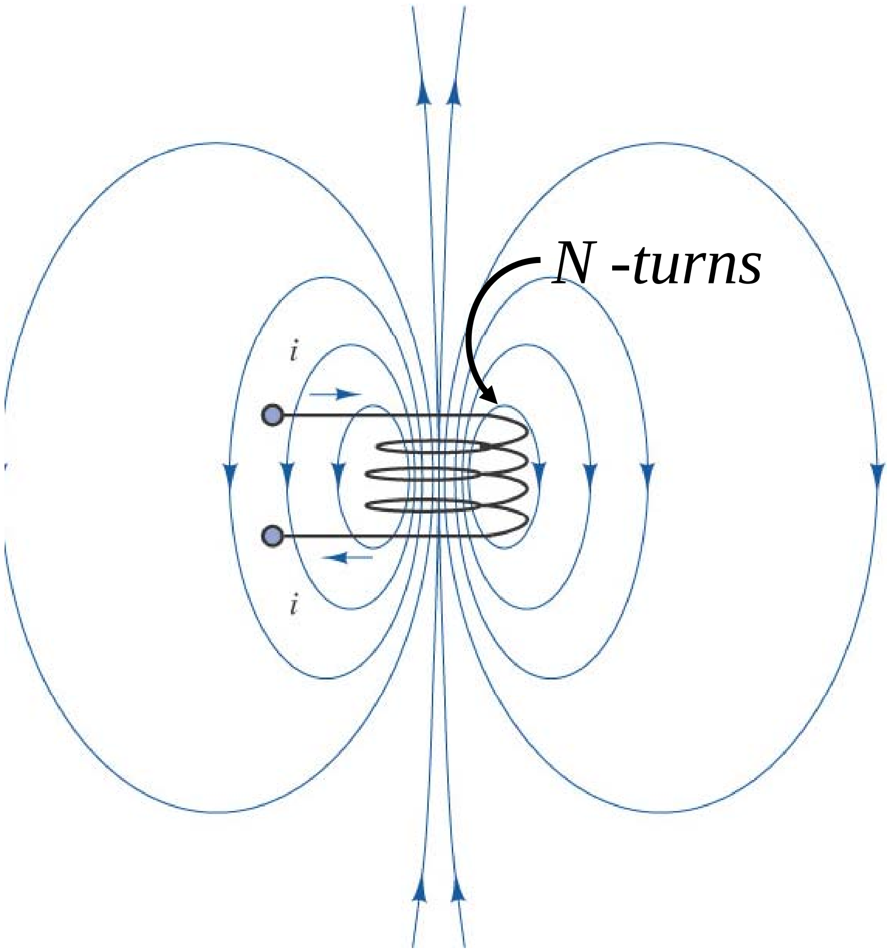
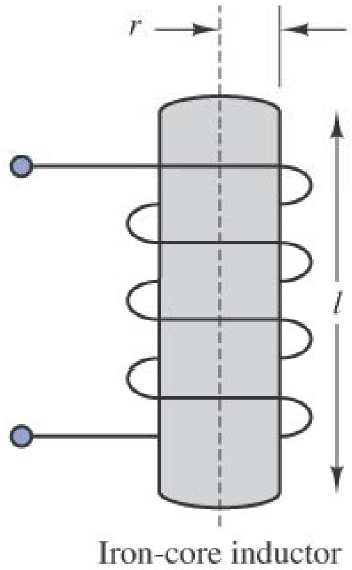


Fig. 3.9 Magnetic flux-lines in the vicinity of a current-carrying coil



$$H = \frac{N \times i}{l}$$

$$B = \mu\left[\frac{(Ni)}{l}\right]$$

- Winding a coil around a ferromagnetic material: (1) forces magnetic flux to be concentrated within the coil and (2) with appropriate shape forces the closed path for the flux-lines to be entirely enclosed within the magnetic material e.g. iron-core inductor and toroidal core.

Magnetic Field in a Toroidal Core

- Consider a toroidal core with, N number of turns coil and a dc current, I flowing through it.

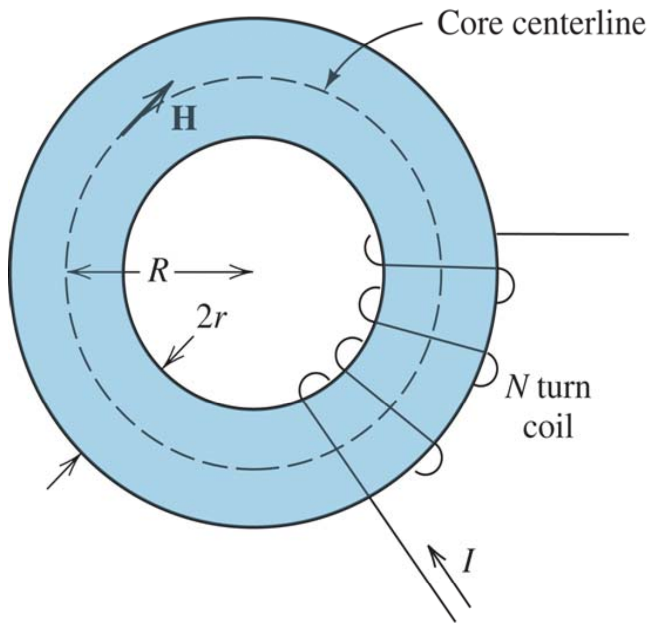


Fig. 3.10 Flux-density in toroidal core

$$Hl = \sum i = H \times (2\pi R) = NI \Rightarrow H = \frac{NI}{2\pi R} \Rightarrow B = \mu H = \frac{\mu NI}{2\pi R} \Rightarrow$$

$$\phi = B \times A = \frac{\mu NI}{2\pi R} \times (\pi r^2) = \frac{\mu NIr^2}{2R} \Rightarrow \lambda = N\phi = \frac{\mu N^2 Ir^2}{2R} \quad (1.9a)$$

Magnetic variables and units		
Variables	Symbol	Units
Current	i	A
Magnetic flux density	**B**	Wb/m^2 or Tesla
Magnetic flux	ϕ	Wb
Magnetic field intensity	**H**	A/m
Electromotive force	e	V
Magnotomotive force	F	A-turns
Flux-linkage	λ	Wb-turns

Relative permeability for common materials	
Material	Relative permeability, μ_r
Air	1
Permalloy	100,000
Cast steel	1,000
Iron	5,195

Example 3: Assume that we have a toroidal core with μ_r = 5000, R = 10 cm, r = 2 cm and N = 100.

The current flowing through the coil is given by

$$i(t) = 2\sin(200\pi t)$$

(a) Compute the flux and the flux-linkage in the toroidal core.

(b) Using Faraday's law of induction, determine the voltage induced in the coil due to the time varying flux.

Solution: The magnetic permeability of the core material is given by

$$\mu = \mu_0 \times \mu_r = 4\pi \times 10^{-7} \times 5000$$

- (a) Using eqn. (3.9a) we have,

$$\phi = \frac{\mu N I r^2}{2R} = \frac{4\pi \times 10^{-7} \times 5000 \times 100 \times 2\sin(200\pi t) \times (0.02)^2}{2 \times 0.1} = (2.531 \times 10^{-3})\sin(200\pi t)\ Wb$$

$$\lambda = N\phi = 100 \times (2.531 \times 10^{-3})\sin(200\pi t) = 0.2531\sin(200\pi t)\ Wb - turns$$

- (b) Using Faraday's law, we have,

$$e = \frac{d\lambda}{dt} = 0.2531 \times 200\pi\cos(200\pi t) = 157.9\cos(200\pi t)\ V$$

- Note that we have ignored the –ve sign for the voltage induced.

Magnetic Circuits

- Many electromagnetic devices such as transformers, motors and generators contain coils wound on iron cores.
- We will study how to calculate the magnetic fields in these devices using magnetic circuit principles.
- The mmf of a N-turn current carrying coil is

$$\mathcal{F} = N \times i \qquad A - turns \qquad (1.10)$$

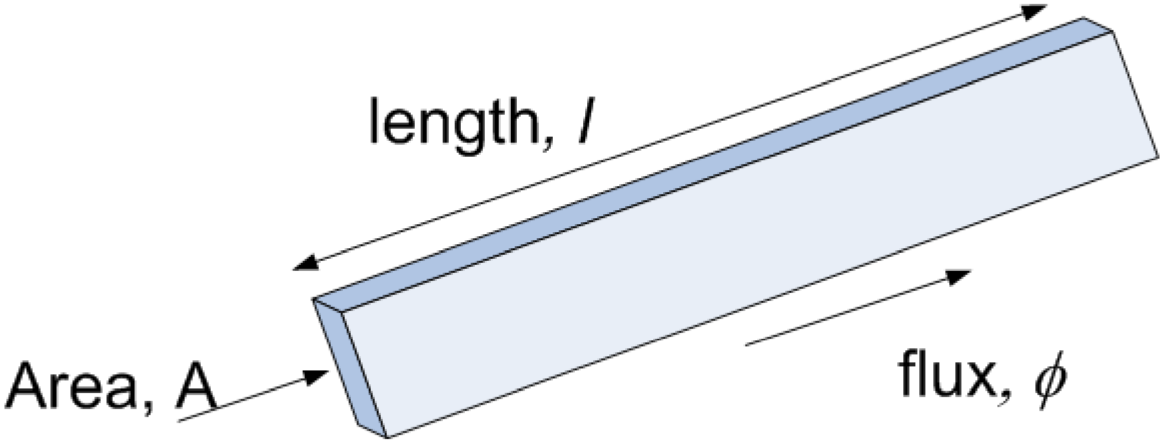
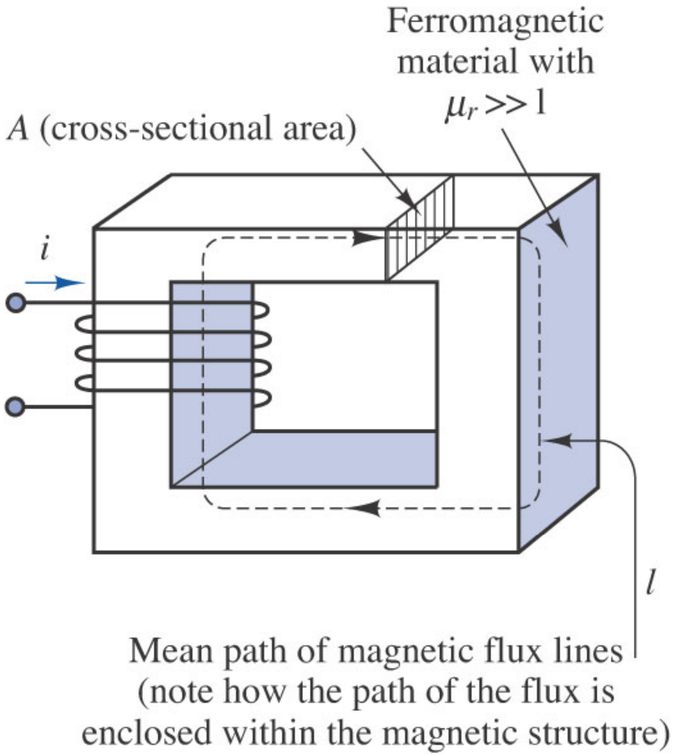


Fig. 3.11 The reluctance $\Re$ of a magnetic path



- The reluctance, $\mathcal{R}$ of a path for magnetic flux, is given by:

$$\mathcal{R} = \frac{l}{\mu A} \qquad (1.11)$$

where l - length of the path in the direction of magnetic flux, A - cross-sectional area, and μ permeability of the material.

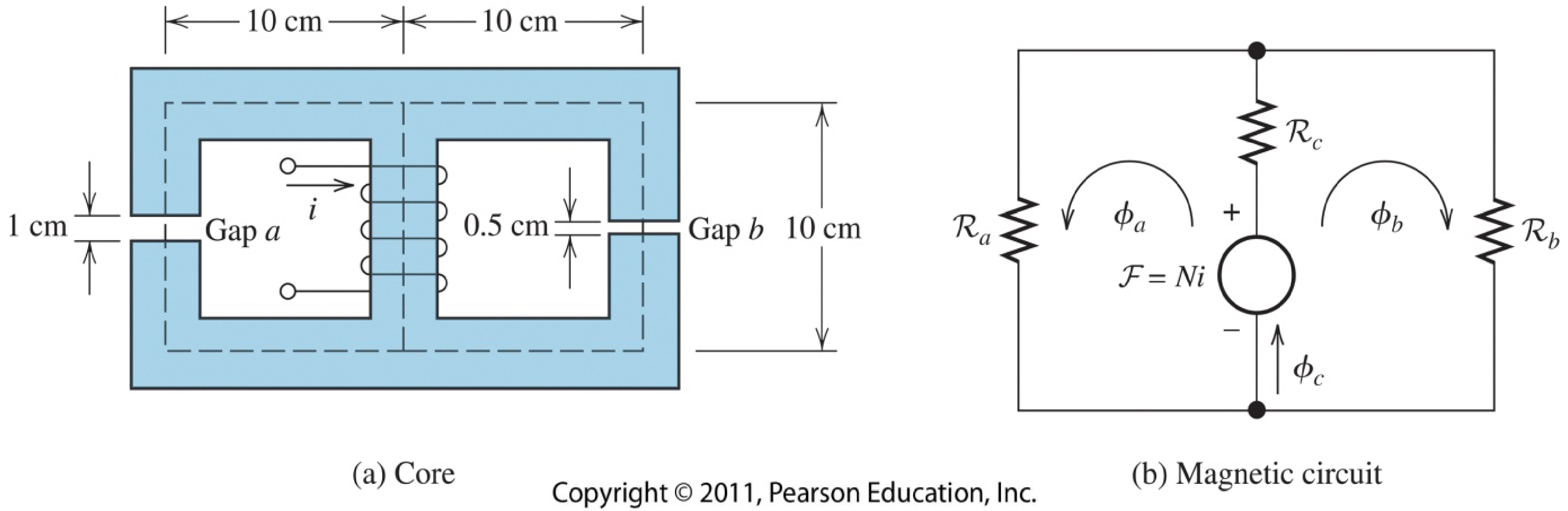
- Magnetic flux, ϕ in a magnetic circuit is analogous to current, i in an electrical circuit.
- Magnetic flux, reluctance, and mmf are related by

$$\mathcal{F} = \mathcal{R} \times \phi \qquad (1.12)$$

the counterpart of Ohm's law ($V = R \times I$) in electric circuits.

Analogy between electric and magnetic circuits	
Electrical quantity	Magnetic quantity
Electric field intensity E, V/m	Magnetic field intensity, **H** A-turns/m
Voltage, v, V	Magnetic force, , A-turns
Current, i, A	Magnetic flux, ϕ, Wb
Current density, J, A/m^2	Magnetic flux density, **B**, Wb/m^2
Resistance, R, Ω	Reluctance, $\Re$, A-turns/Wb
Conductivity, σ, 1/Ω.m	Permeability, μ, Wb/A.m

Example 4: The iron core as shown below has a cross section of 2 cm by 2 cm and a relative permeability of 1000. The coil has 500 turns and carries a current of 2 A. Determine the flux-density in each air-gap.

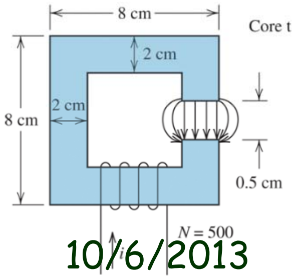


Solution: The equivalent magnetic circuit is shown. The reluctance for the central magnetic path is given by

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A_{core}} = \frac{10 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 1.989 \times 10^5 \; A.turns/Wb$$

The reluctance for the left air-gap path, *a* taking into account the *fringing effect* by adding the gap length to its length and depth in computing the area

$$\mathcal{R}_a = \frac{l_a}{\mu_0 A_{gap,a}} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times ((2+1) \times (2+1)) \times 10^{-4}} = 8.842 \times 10^6 \; A.turns/Wb$$



- The reluctance for the core for the left-hand magnetic path is given by

$$\mathcal{R}_{core,LH} = \frac{l_{c,mean}}{\mu_0 \mu_r A_{core}} = \frac{(10+9+10)\times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 0.5769 \times 10^{6}\ A.turns/Wb$$

- Thus, the total reluctance of left-hand (L-H) magnetic path is

$$\mathcal{R}_{LH} = \mathcal{R}_a + \mathcal{R}_{core,LH} = 8.848 \times 10^{6} + 0.5769 \times 10^{6} = 9.420 \times 10^{6}\ A.turns/Wb$$

- The reluctance for the right air-gap path, *b* is given by

$$\mathcal{R}_b = \frac{l_b}{\mu_0 A_{gap,b}} = \frac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times ((2+0.5)\times(2+0.5)) \times 10^{-4}} = 6.369 \times 10^{6}\ A.turns/Wb$$

- The reluctance for the core for the right-hand (R-H) magnetic path is given by

$$\mathcal{R}_{core,RH} = \frac{l_c}{\mu_0 \mu_r A_{core}} = \frac{(10 + 9.5 + 10) \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 0.5869 \times 10^6 \, A.turns/Wb$$

- Thus, the total reluctance of right-hand magnetic path is

$$\mathcal{R}_{RH} = \mathcal{R}_b + \mathcal{R}_{core,RH} = 6.369 \times 10^6 + 0.5869 \times 10^6 = 6.9559 \times 10^6 \, A.turns/Wb$$

- We can combine the two reluctances of the LH side and RH side as:

$$\mathcal{R}_{total} = \mathcal{R}_c + \frac{1}{1/\mathcal{R}_a + 1/\mathcal{R}_b} = 1.989 \times 10^5 + \frac{1}{1/9.420 \times 10^6 + 1/6.9559 \times 10^6}$$
$$= 4.199 \times 10^6 A.turns/Wb$$

- The magnetic flux in the centre leg of the coil is

$$\phi_c = \frac{Ni}{\mathcal{R}_{total}} = \frac{500 \times 2}{4.199 \times 10^6 A.turns/Wb} = 238.15\ \mu Wb$$

- Fluxes are analogous to current so we can use the current division rule to compute the flux in the LH and RH paths as follows:

$$\phi_a = \phi_c \times \frac{\mathcal{R}_b}{\mathcal{R}_a + \mathcal{R}_b} = 238.15 \times 10^{-6} \times \frac{6.9559 \times 10^6}{9.420 \times 10^6 + 6.9559 \times 10^6} = 101.1\ \mu Wb$$

$$\phi_b = \phi_c \times \frac{\mathcal{R}_a}{\mathcal{R}_a + \mathcal{R}_b} = 238.15 \times 10^{-6} \times \frac{9.420 \times 10^6}{9.420 \times 10^6 + 6.9559 \times 10^6} = 137.0\ \mu Wb$$

- The flux-densities in the air-gaps are:

$$B_a = \frac{\phi_a}{A_a} = \frac{101.1\ \mu Wb}{((2+1) \times (2+1)) \times 10^{-4}} = 0.1123\ Wb/m^2$$

$$B_b = \frac{\phi_b}{A_b} = \frac{137.0\ \mu Wb}{((2+0.5) \times (2+0.5)) \times 10^{-4}} = 0.2192\ Wb/m^2$$

- Assumptions and simplifications made in analyzing magnetic structures:
  1. All the magnetic flux is linked by all the turns of the coil.
  2. The flux is confined exclusively within the magnetic core.
  3. The flux-density is uniform across the cross-sectional area of the core.

Mutual- and Self- Inductances

- Consider a coil having N turns and carrying a current, i that sets up a flux ϕ linking the coil, the self-inductance of the coil can be defined as:

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N\left(\frac{Ni}{\Re}\right)}{i} = \frac{N^2}{\Re} \quad (3.13)$$

$$e = N\frac{d\phi}{dt} = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt} = v_L \quad (3.14)$$

- Self-inductance of the coil, L is independent of the excitation current, i and dependent only on the geometry, permeability and number of turns.

- When two coils are wound on the same core, some of the flux produced by one coil links the other coil.
- Let us denote the flux linkage of coil 2 due to currents in coil 1 as λ_{21} , and the flux-linkages of coil 1 produced due to current in coil 1 as λ_{11}.
- Similarly, current in coil 2 produces flux linkages λ_{22} in coil 2 and λ_{12} in coil 1.

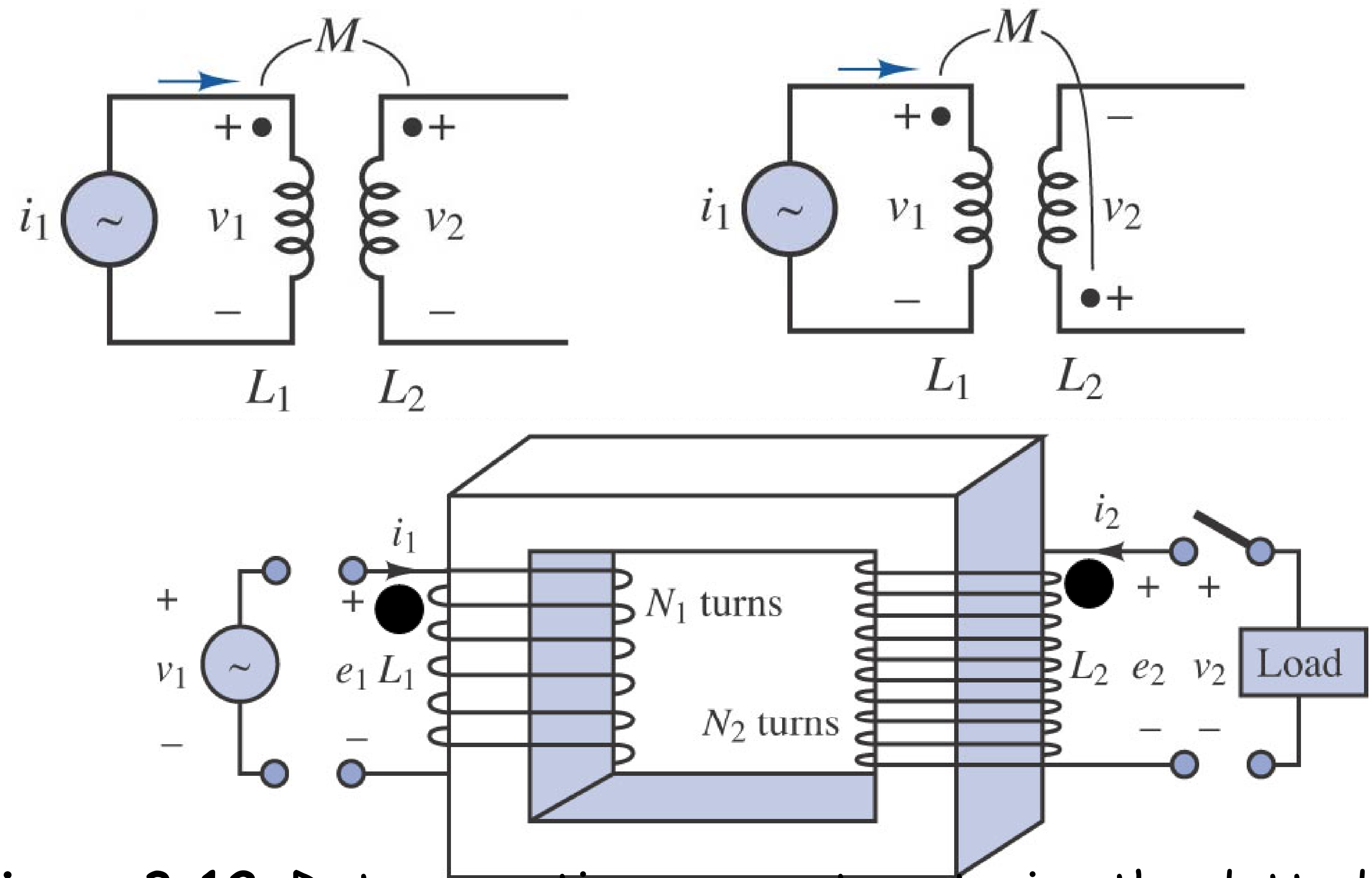


Figure 3.12: Dot convention, currents entering the dotted terminals produce aiding fluxes.

- The self, (L_1 and L_2) and mutual, (M) inductances are given by

$$\boldsymbol{L}_1 = \frac{\lambda_{11}}{\boldsymbol{i}_1}, \ \boldsymbol{L}_2 = \frac{\lambda_{22}}{\boldsymbol{i}_2}, (3.15) \quad \boldsymbol{M} = \frac{\lambda_{21}}{\boldsymbol{i}_1} = \frac{\lambda_{12}}{\boldsymbol{i}_2} \ (3.16)$$

$$\lambda_1 = \lambda_{11} \pm \lambda_{12}, \quad \lambda_2 = \pm\lambda_{21} + \lambda_{22} \ (3.17)$$

where + sign applies if fluxes are additive and -ve sign if fluxes are opposing.

- Whether the fluxes are additive or subtractive can be determined by dot convention.

Dot Convention

- It is normal practice to place a dot on one end of each coil to indicate how the fluxes interact.
- The dots are placed in such as way that currents entering the dotted terminals produce aiding fluxes.
- If one current enters the dotted terminal and the other leaves the dotted terminal the mutual flux linkages carry a -ve sign.

$$\lambda_1 = L_1 i_1 \pm M i_2, \qquad \lambda_2 = \pm M i_1 + L_2 i_2$$

$$e_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}, \; e_2 = \frac{d\lambda_2}{dt} = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \quad (3.18)$$

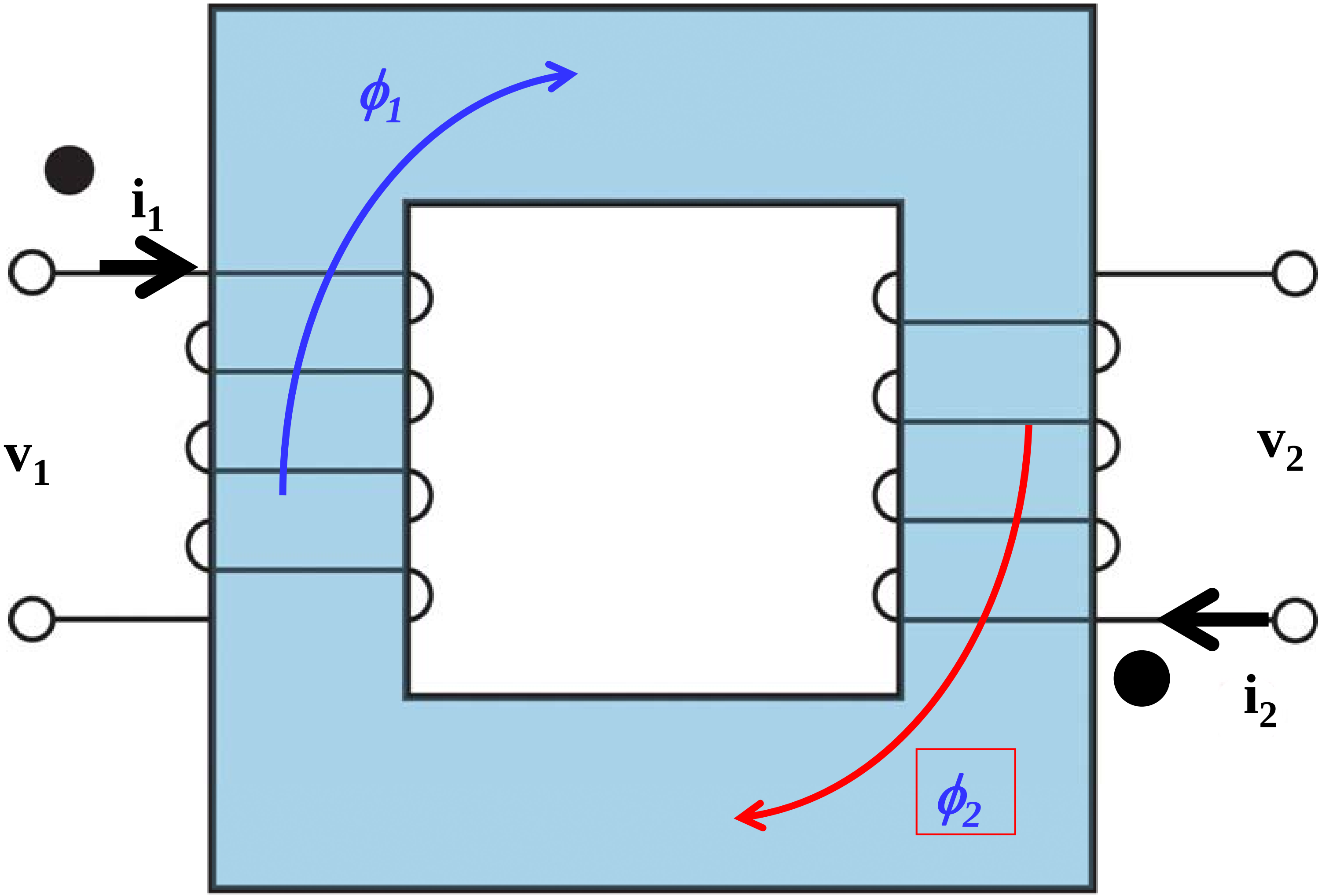


Fig. 3.12a: Dot convention, currents entering the dotted terminals produce aiding fluxes.

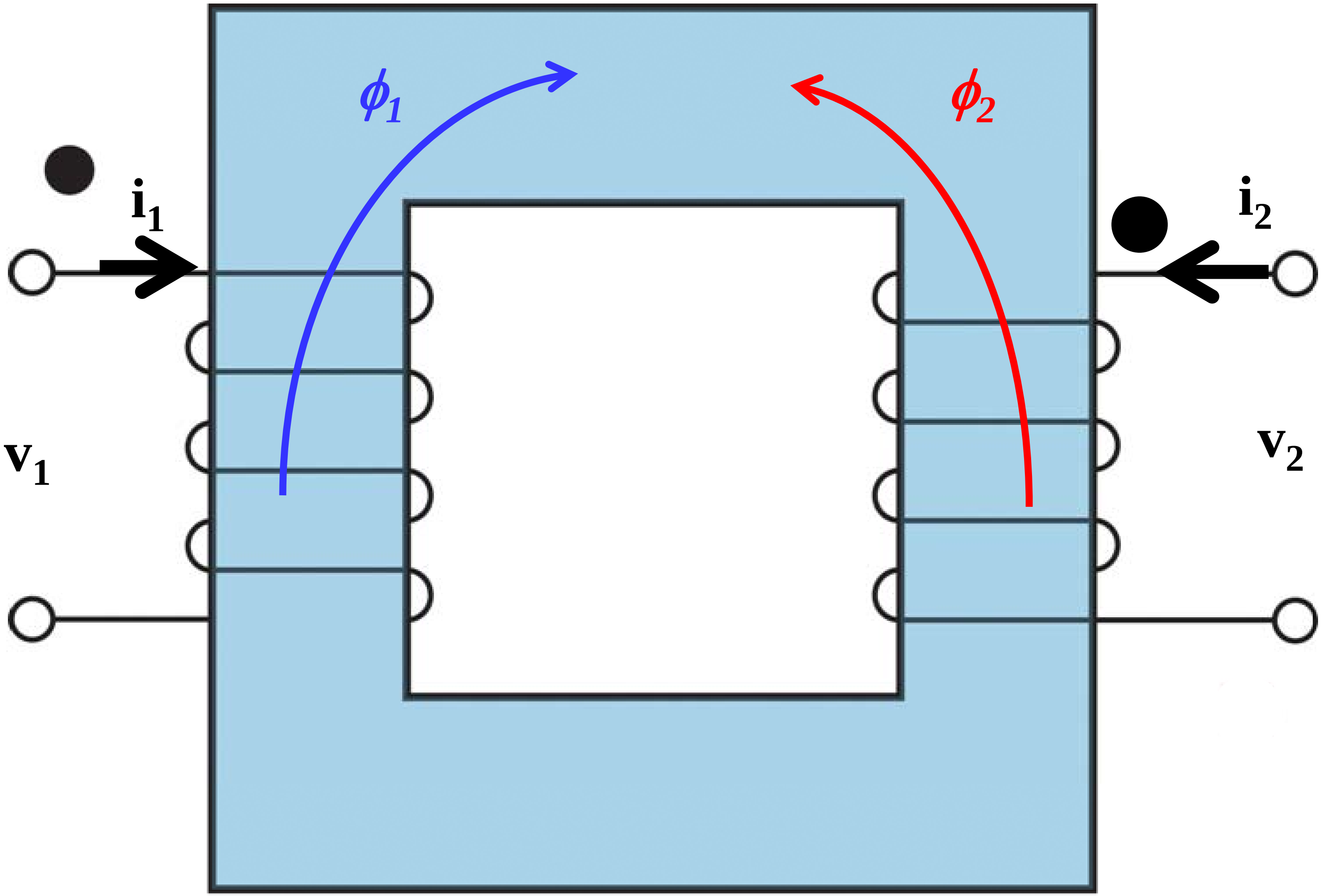


Fig. 3.12b: Dot convention, currents entering the dotted terminals produce opposing fluxes.

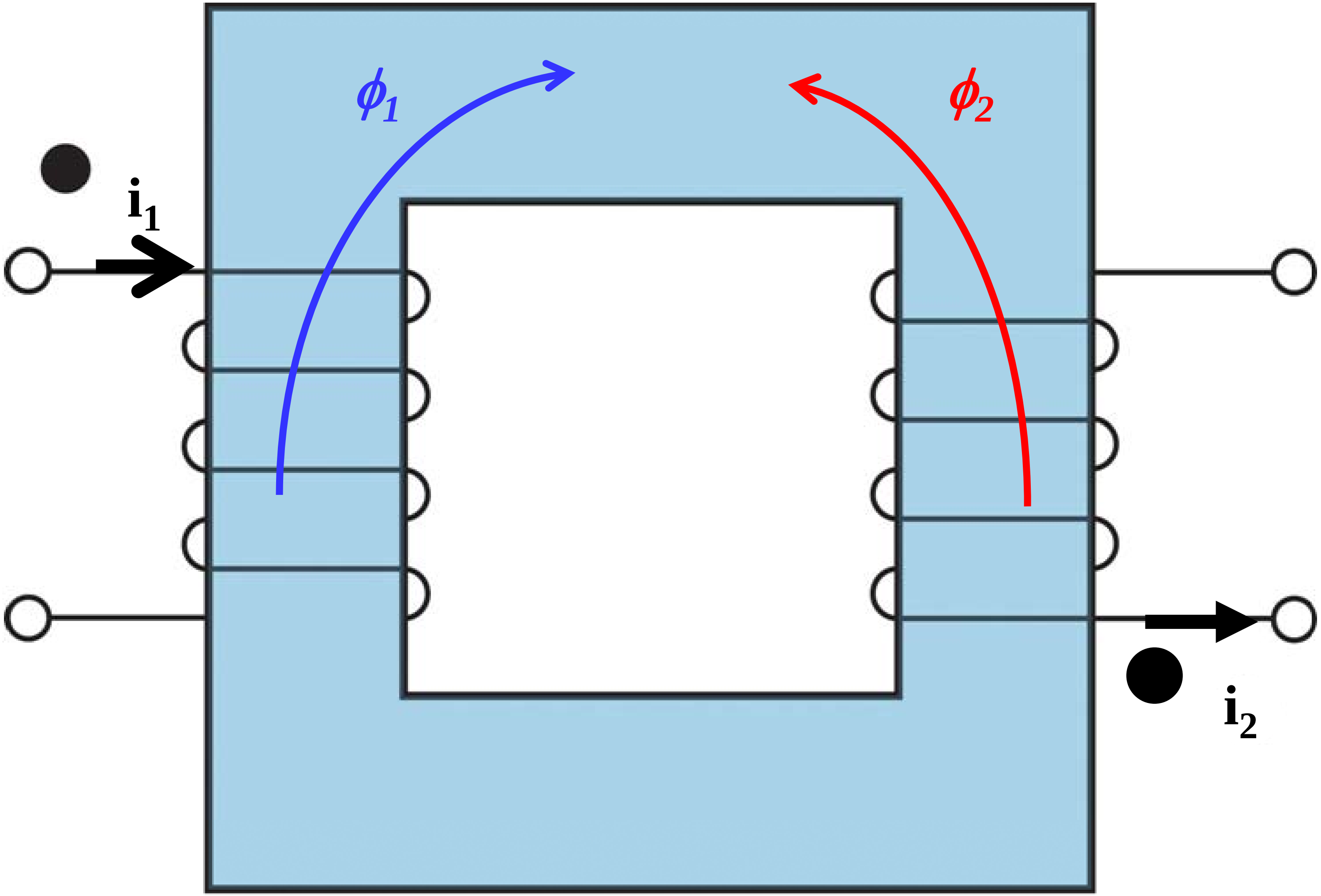


Fig. 3.12b1: Dot convention, currents entering primary winding and leaving at the secondary winding at the dotted terminals produce **opposing** fluxes.

Example 5: The magnetic circuit a shown in Fig. 3.12 has the following dimensions: the cross sectional, A_c = 12 cm^2 and mean-length of the magnetic flux-path, l_c = 50 cm. The magnetic core has a relative permeability μ_r = 20,000. The first coil has 500 turns and the second coil has 1000 turns.

(a) The first coil is supplied with a current i_1 = 10 A, while the second coil is left un-energized. Calculate the self-inductance L_1 of coil 1 and the mutual inductance, M between the two coils.

Solution: The reluctance of the magnetic path is given by

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A_{core}} = \frac{50 \times 10^{-2}}{20{,}000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 16.58 \times 10^3 \; A.turns/Wb$$

The flux, ϕ_1 due to current, i_1 in coil-1 is given by

$$\phi_1 = \frac{N_1 i_1}{\mathcal{R}_c} = \frac{500 \times 10}{16.58 \times 10^3 A.turns/Wb} = 0.30 \; Wb$$

The flux-linkages of the two coils are given by

$$\lambda_{11} = N_1 \phi_1 = 500 \times 0.30 = 150 \; Wb - turns$$

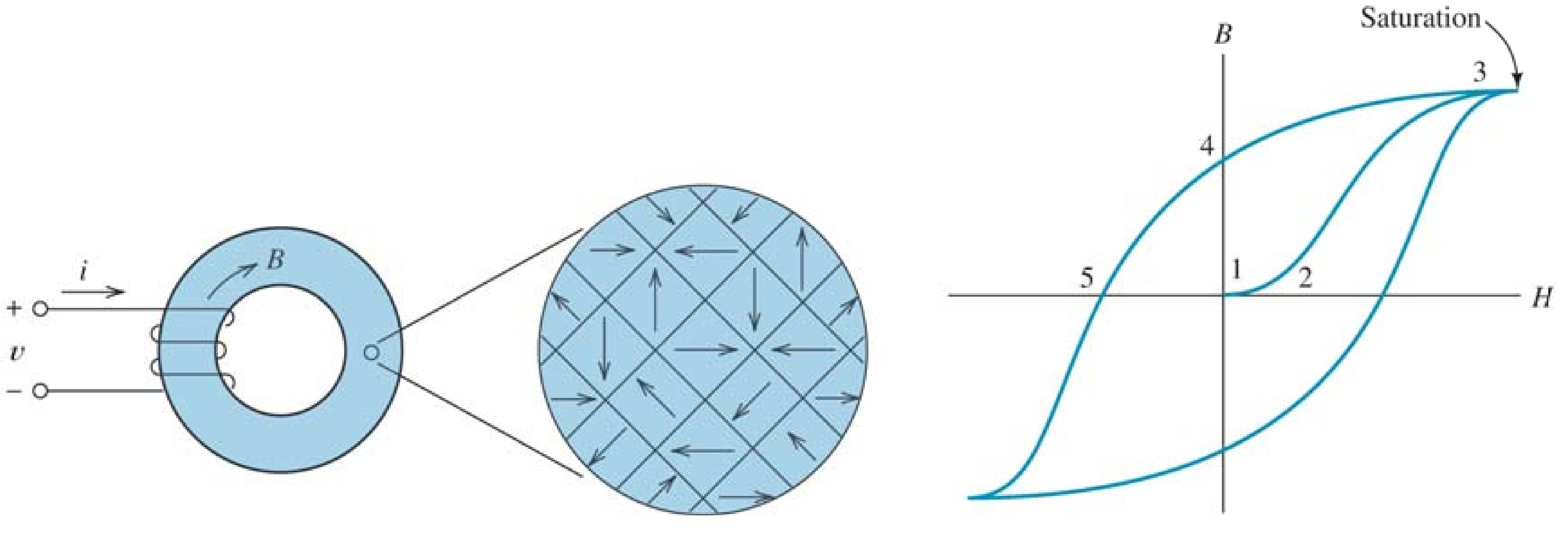
$$\lambda_{21} = N_2 \phi_1 = 1000 \times 0.30 = 300 \; Wb - turns$$

The self- and mutual inductances are given by

$$L_1 = \frac{\lambda_{11}}{i_1} = \frac{150}{10} = 15 \; H \qquad M = L_{21} = \frac{\lambda_{21}}{i_1} = \frac{300}{10} = 30 \; H$$

Magnetic Materials and B-H Curve

- Fig. 3.13 shows a coil that is used to apply magnetic field intensity, **H** to a magnetic core and the corresponding B - H curve.



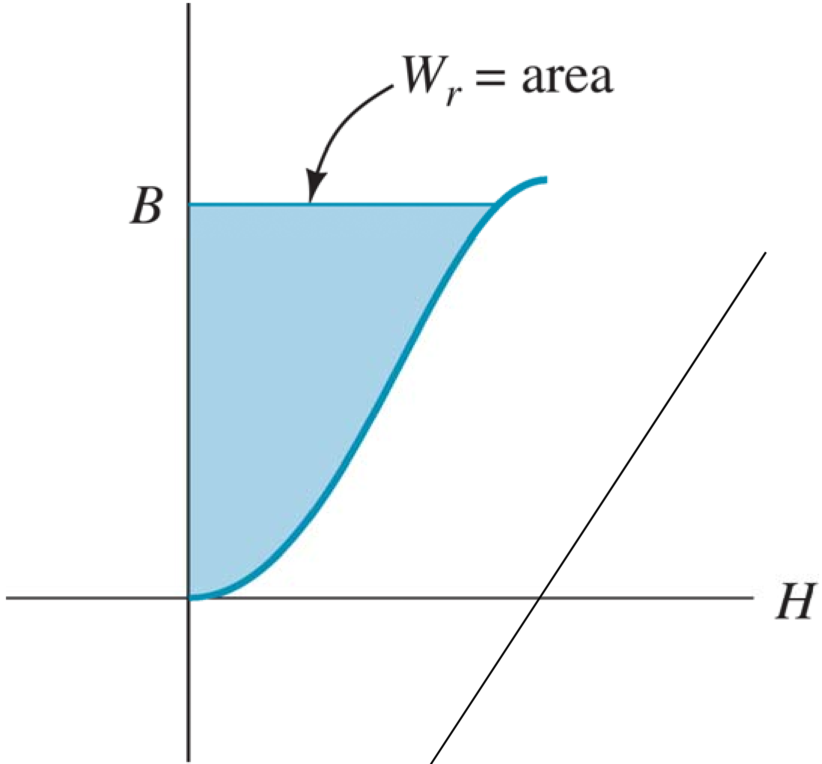
(a) Sample and coil for applying *H* (b) Magnetic domains (c) Hysteresis loop in the *B* – *H* plane

Figure 3.13: Materials such as iron display a *B*-*H* relationship with hysteresis and saturation.

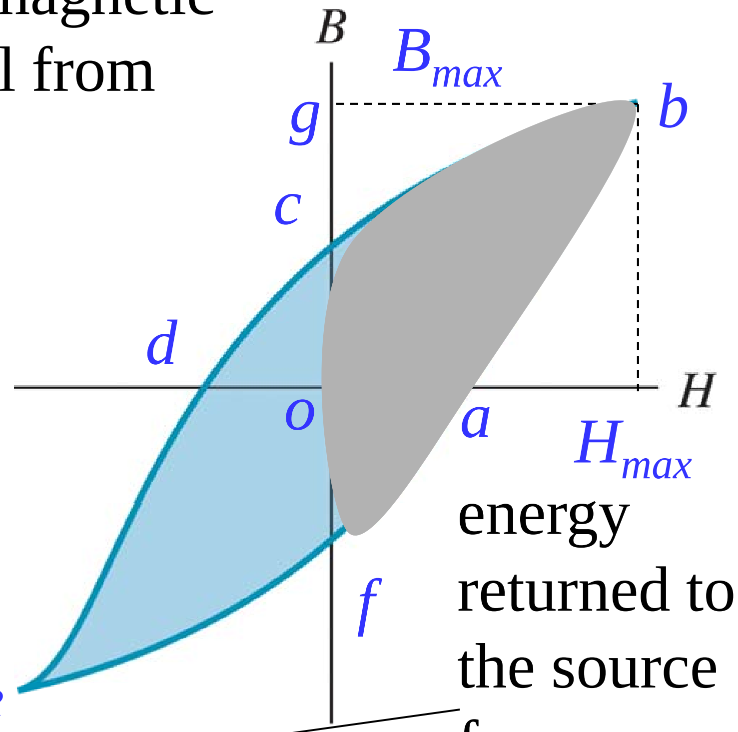
- The energy, *W* delivered to the coil is given by

$$W = \int_0^t ei\,dt = \int_0^t N\frac{d\emptyset}{dt}i\,dt = \int_0^\emptyset Ni\,d\emptyset = \int_0^\emptyset (Ni = Hl) \times (d\emptyset$$
$$= A\,dB) = \int_0^B (Al)H\,dB \quad (1.19) \Rightarrow W_v = \frac{W}{Al} = \int_0^B (Al)H\,dB \quad (1.20)$$

- The term, W_v represents the energy per unit volume of the core.
- The energy delivered to the coil from the electrical source as shown in Fig. 3.13(a) is the area between *B-H* curve and the *B*-axis as shown in Fig. 3.14(a).



energy absorbed by the magnetic material from source



energy returned to the source from source the magnetic material

$$\int_{-B_f}^{B_{max}} H\,dB = area\ fabgf$$

$$\int_{B_{max}}^{B_c} H\,dB = area\ cbgc$$

Fig. 3.14: (a) The area between the B – H curve and the B axis represents the volumetric energy supplied to the core and (b) the area of the hysteresis loop is the volumetric energy converted to heat per cycle.

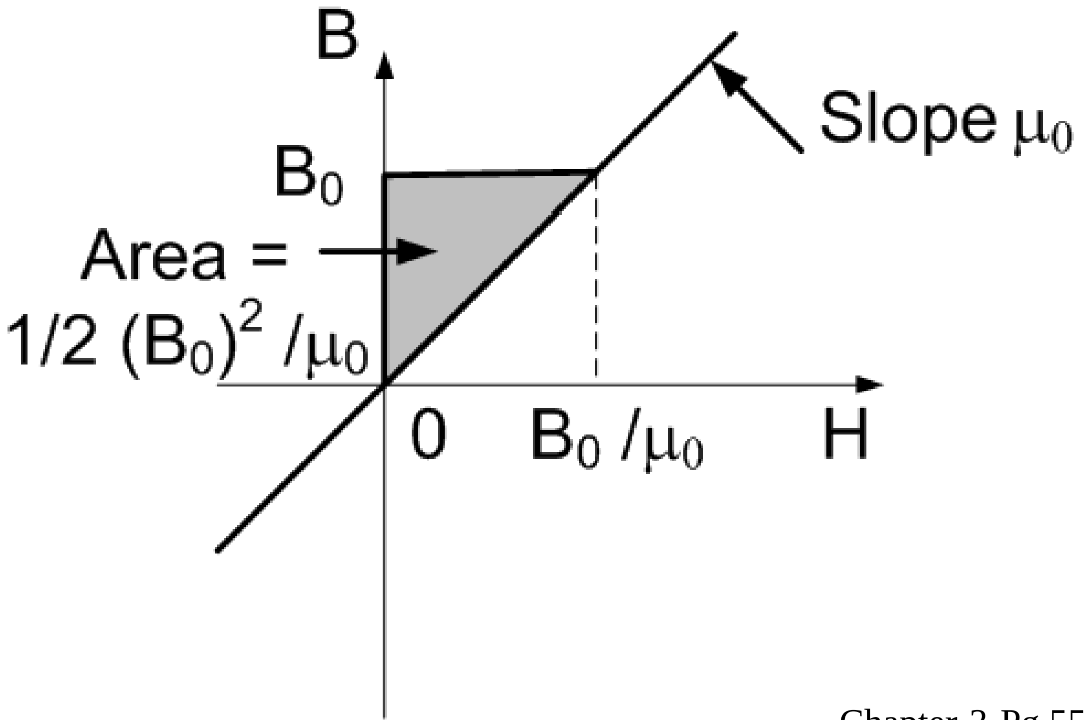
$$v = Ri + \frac{d\lambda}{dt} = Ri + NA\frac{dB}{dt}\ \mathrm{V}$$

$$p = vi = Ri^2 + NAi\frac{dB}{dt} = p_R + p_B\ \mathrm{W}$$

$$p_B = NAi\frac{dB}{dt} = HlA\frac{dB}{dt} = (Al)H\frac{dB}{dt}\ \mathrm{W}$$

$$W_B = \int_0^t p_B\,dt = \int_0^B AlH\,dB = \int_0^B \frac{AlB}{\mu_0}\,dB = \frac{AlB^2}{2\mu_0}\ \mathrm{J}$$

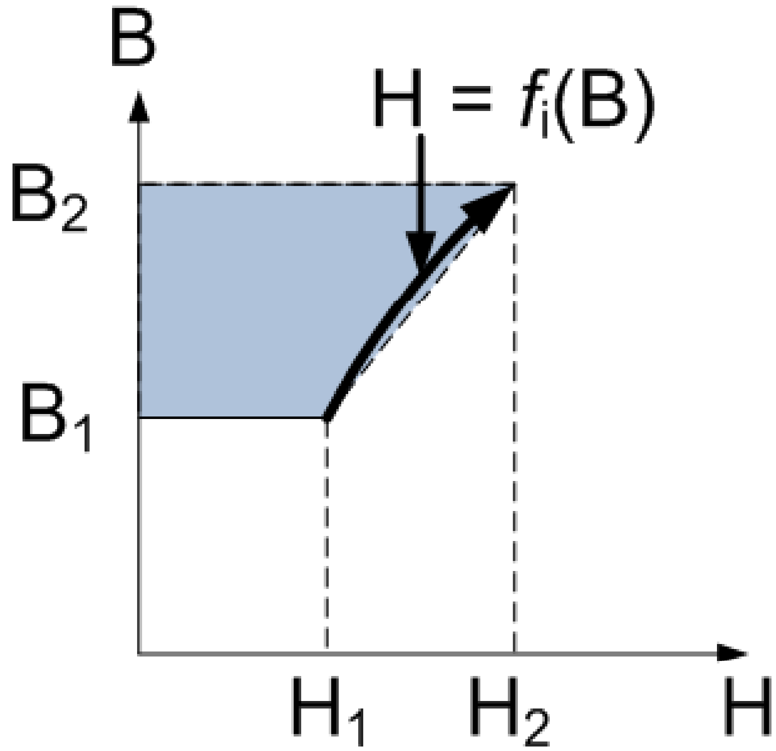
$$W_{B,v} = \frac{W_B}{V=Al} = \frac{1}{2}\frac{B^2}{\mu_0}\ \mathrm{J}$$

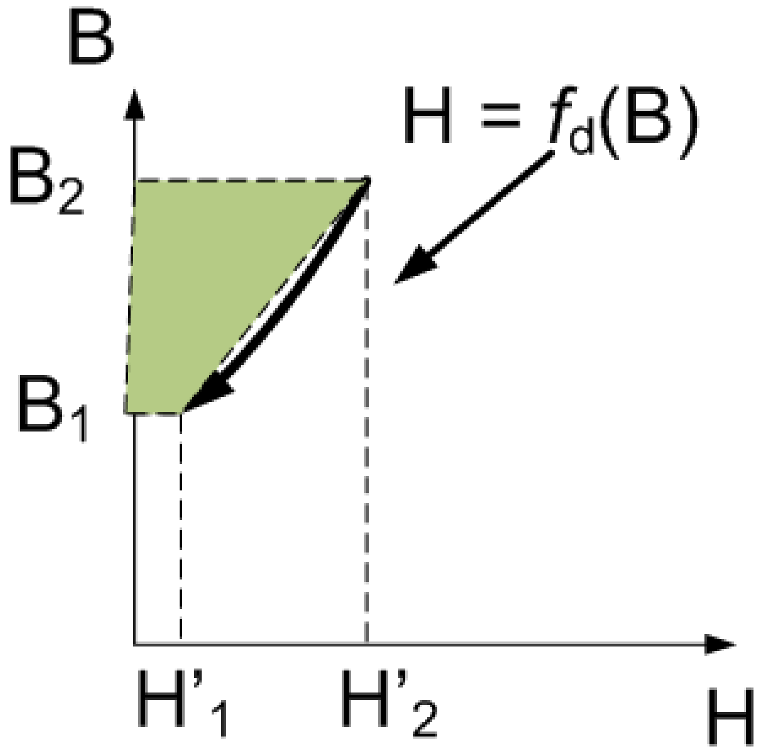


$$H = f_i(B) \ \mathrm{A/m}$$

$$\Delta W_{B,increase} = \int_{B_1}^{B_2} H \, dB \ \ J/m^3$$

$$\Delta W_{B,decrease} = \int_{B_2}^{B_1} H \, dB \ \ J/m^3$$





- Part of this energy is returned back to the circuit when **H** is reduced to zero, part of it remains stored in the residual field, B_c and part of it is converted into heat in the process of magnetizing the core.
- The volumetric energy converted to heat per cycle is equal to the area of the hysteresis loop as shown in Fig. 3.14(b).
- This energy loss is called *core loss* and is proportional to the frequency.

$$P_h = k_h f B_{max}{}^n V \ \ Watt$$

k_h - constant of core material, *f* – frequency, *V* – volume, *n* - constant

- In electrical machines and transformers, the energy loss as heat is undesirable and therefore, would choose a magnetic material that has a thin hysteresis loop as shown in Fig. 3.15(a).
- While with permanent magnet we would choose a material having large residual field as shown in Fig. 3.15(b) to have a higher value of residual flux-density, **B**.

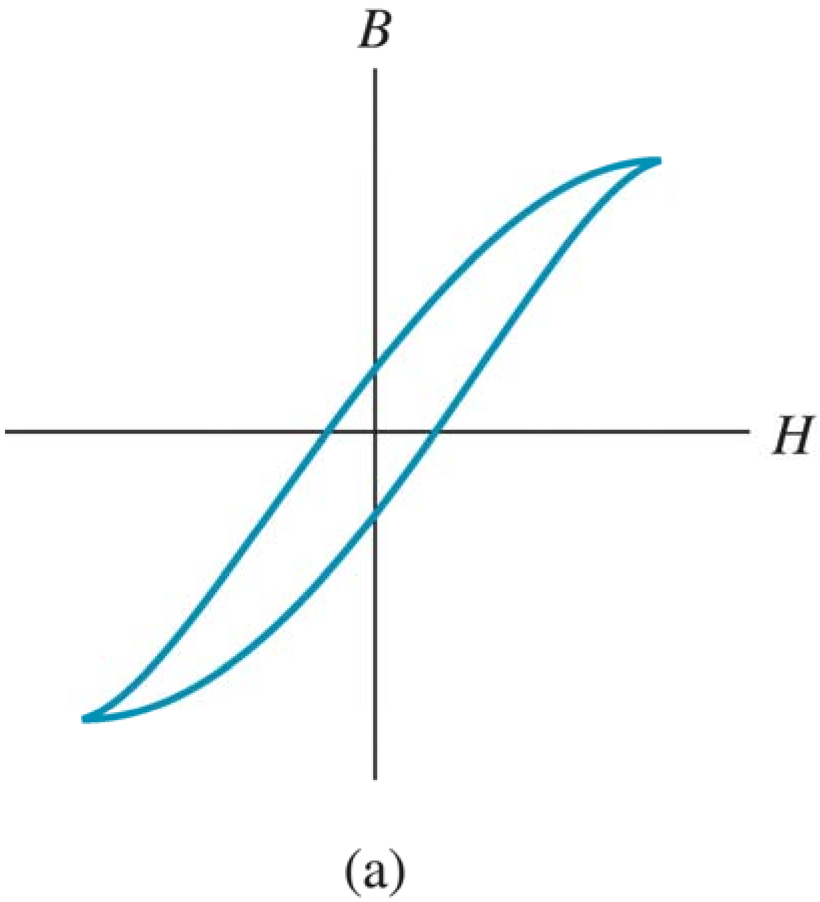
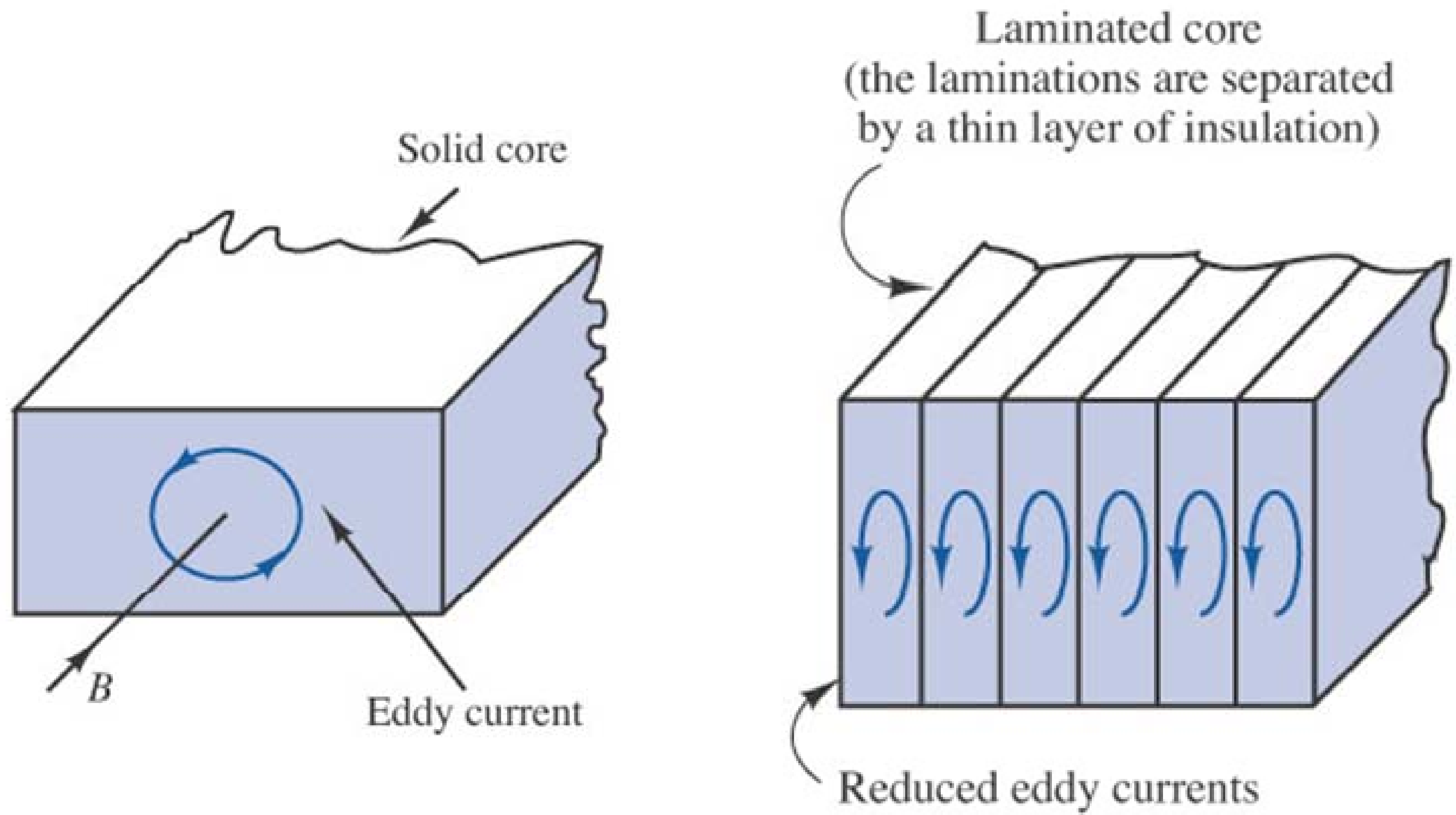


Fig. 3.15: When we want to minimize core loss (as in a transformer or motor), we choose a material having a thin hysteresis loop. On the other hand, for a permanent magnet, we should choose a material with a wide loop

Eddy Currents in Transformers

- As magnetic field changes due to the application of the ac current in the coil, voltages are induced in the core, causing currents, known as eddy currents be developed and circulate in the core material resulting power loss in the core.
- By laminating the core with thin sheets of iron, they are electrically insulated from one another and interrupt the flow of currents thereby reducing the eddy-current loss.

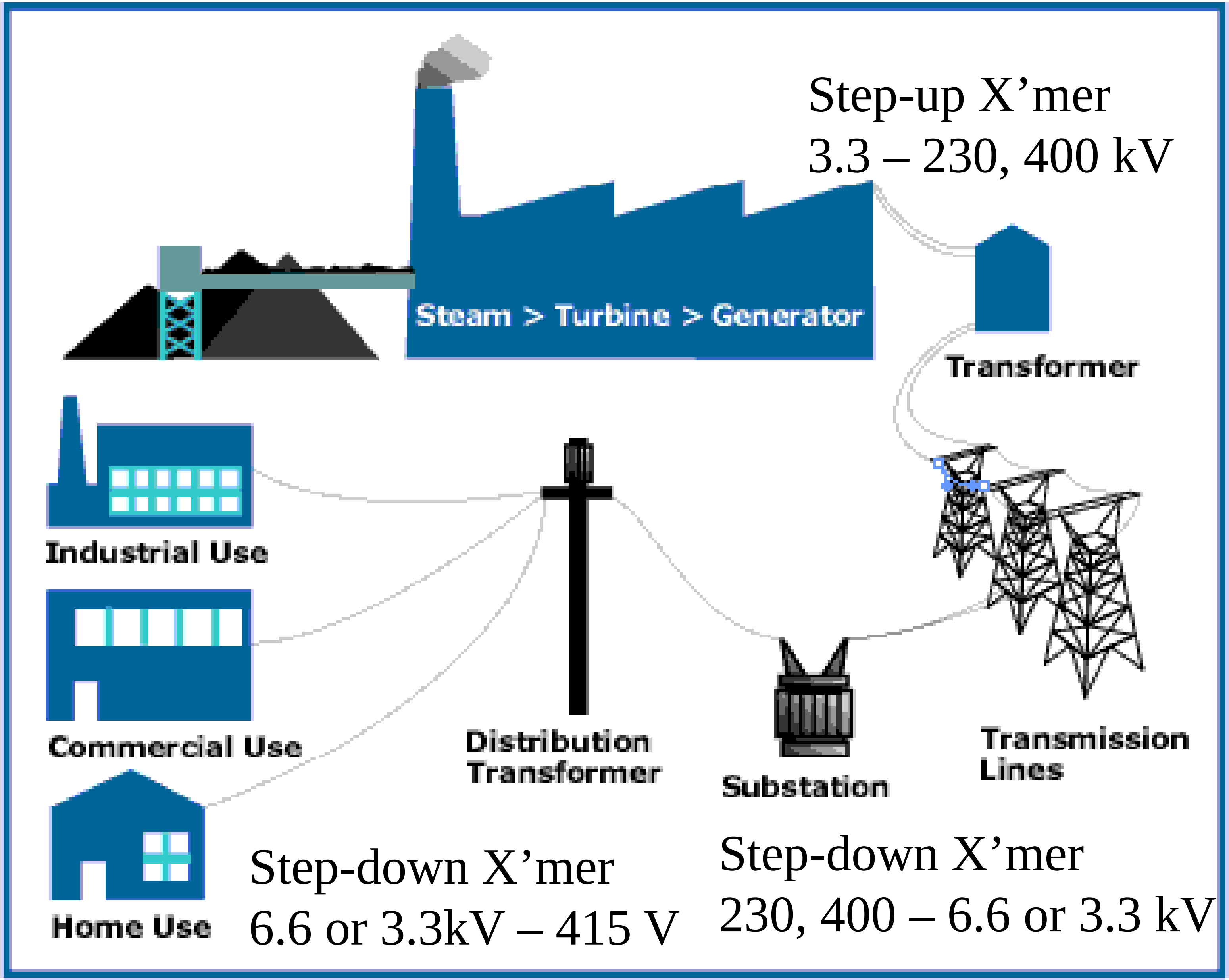


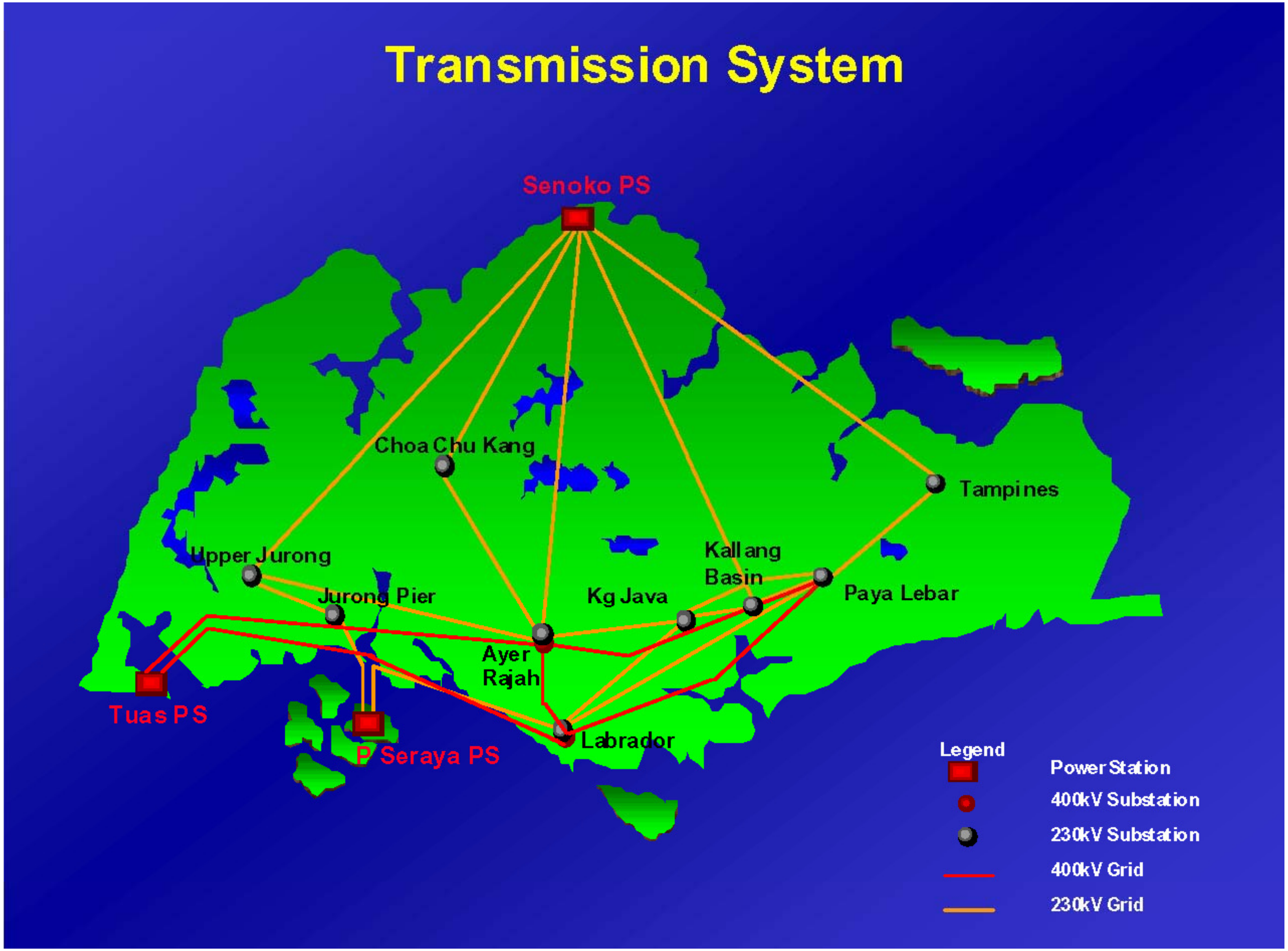
$$P_e = k_e f^2 B_{max}{}^2 V \;\; Watt$$

Ideal Transformers

- One of the common magnetic structures that we see in everyday application is the transformer that is used to step-up or step-down *AC* voltages.
- Transformers find applications widely in electric power Transmission and Distribution (T & D) networks.
- For power T & D networks it is desirable to transfer active power, *P* at much higher voltages typically at several kVs:

$$P = V_{rms} \uparrow I_{rms} \downarrow \cos(\theta) \ , P_{TL,loss} \downarrow = I_{rms}^{2} \downarrow \times R_{TL}$$


Step-up X'mer
3.3 – 230, 400 kV
Steam > Turbine > Generator
Transformer
Industrial Use
Commercial Use
Distribution Transformer
Substation
Transmission Lines
Home Use
Step-down X'mer
6.6 or 3.3kV – 415 V
Step-down X'mer
230, 400 – 6.6 or 3.3 kV


Transmission System
Senoko PS
Choa Chu Kang
Tampines
Upper Jurong
Kallang Basin
Jurong Pier
Kg Java
Paya Lebar
Ayer Rajah
Tuas PS
Labrador
P Seraya PS
Legend
Power Station
400kV Substation
230kV Substation
400kV Grid
230kV Grid

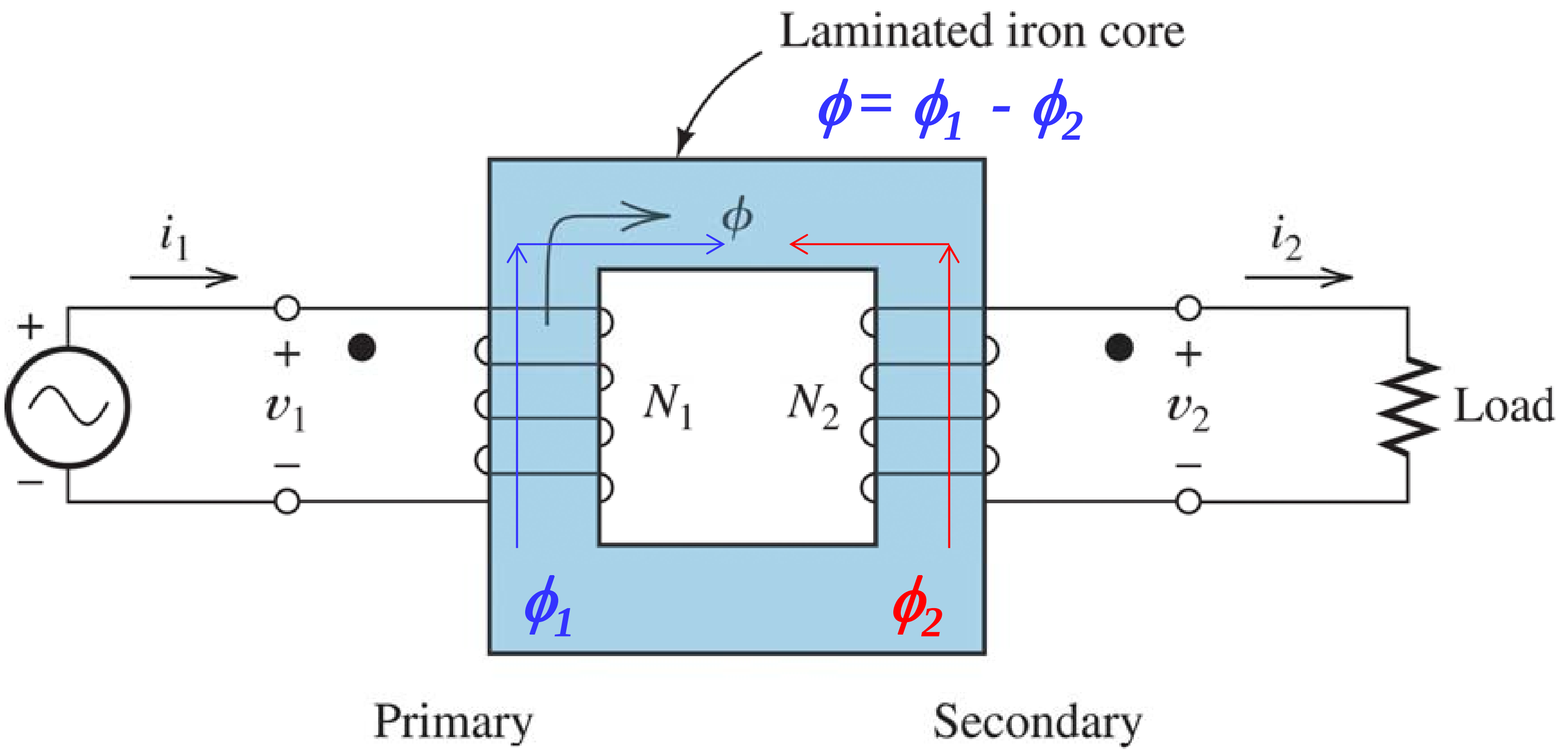


Fig. 3.16: A transformer consists of several coils wound on a common core

$$v_1(t) = V_{1m}\cos(\omega t) = N_1\frac{d\phi}{dt} \Rightarrow \phi(t) = \frac{V_{1m}}{N_1\omega}\sin(\omega t) \quad (3.21)$$

$$v_2(t) = N_2\frac{d\phi}{dt} = N_2\frac{V_{1m}}{N_1\omega}\frac{d}{dt}\left[\sin(\omega t)\right] = \frac{N_2}{N_1}V_{1m}\cos(\omega t) = \frac{N_2}{N_1}v_1(t) \quad (3.22$$

- Note that the voltage across each coil is proportional to its number of turns.
- The dots are placed such that if currents entered at the dotted terminals, they would produce aiding magnetic field.
- However, as current at secondary side leaves the terminal at the dotted point it produces flux that opposes the flux produced by coil, 1.
- Thus, in transformers, the polarities of the voltages at dotted terminals agree i.e. when a voltage, $v_1(t)$ appears across coil 1, the voltage $v_2(t)$ at coil2 is also positive at the dotted terminal i.e. <u>the two voltages are in phase</u>.

- Note that currents $i_1(t)$ and $i_2(t)$ produce opposing magnetic fields, the two MMFs oppose each other and in an ideal transformer the net MMF and therefore the reluctance of core is zero.

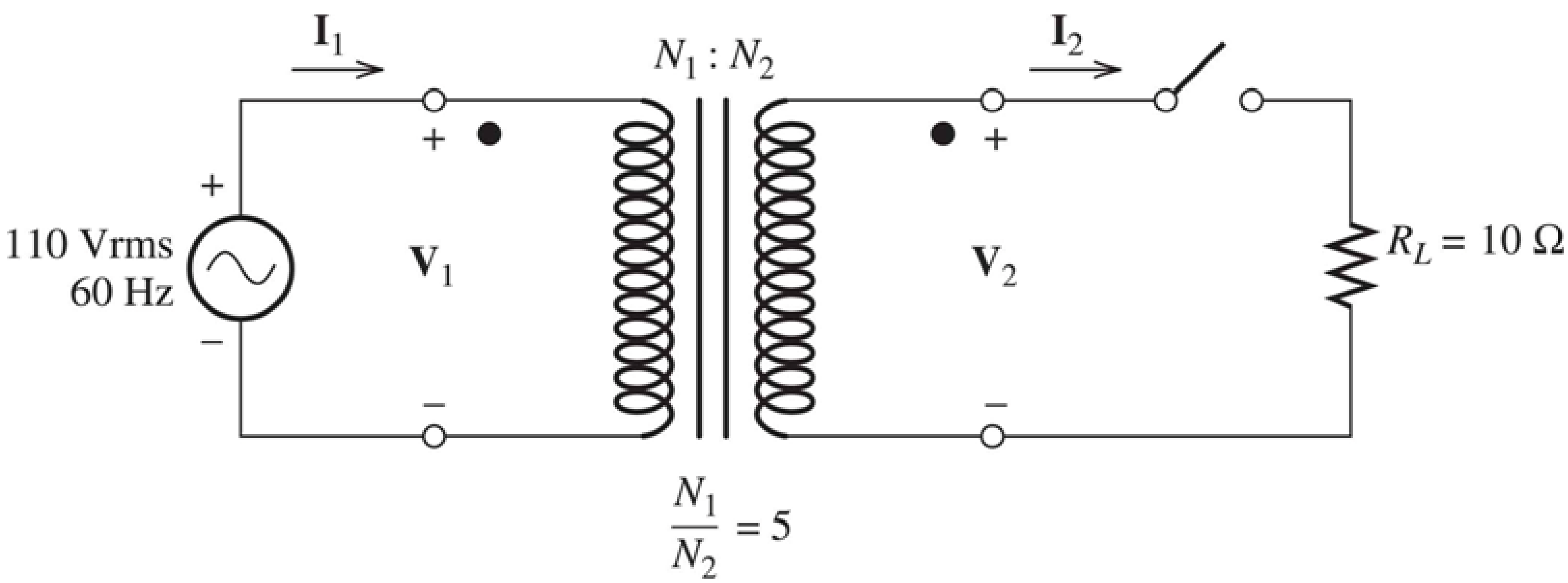
$$\mathcal{F} = N_1 i_1(t) - N_2 i_2(t) = 0 \Rightarrow i_2(t) = \frac{N_1}{N_2} i_1(t) \quad (1.23)$$

- Power delivered to the load is given by

$$p_2(t) = v_2(t) \times i_2(t) = \frac{N_2}{N_1} v_1(t) \times \frac{N_1}{N_2} i_1(t) = v_1(t) \times i_1(t) = p_1(t) \quad (1.24)$$

- Ideal transformer is <u>100% efficient</u> so $p_1(t) = p_2(t)$.

Example 6: Consider the voltage source, transformer, and load as shown below. Determine the rms values of the currents and voltages (a) with the switch opened and (b) with the switch closed.



Solutions: (a) When the switch is opened, we have

$$V_{2rms} = \frac{N_2}{N_1} V_{1rms} = \frac{1}{5} 110 = 22\, V$$

$$I_{2rms} = 0, \Rightarrow \quad I_{1rms} = \frac{N_2}{N_1} I_{2rms} = 0$$

(b) When the switch is closed, we have

$$I_{2rms} = \frac{V_{2rms}}{R_L} = \frac{22\, V}{10\, \Omega} = 2.2\, A, \Rightarrow \quad I_{1rms} = \frac{N_2}{N_1} I_{2rms} = \frac{1}{5} 2.2\, A = 0.44\, A$$

- In an ideal transformer, power delivered by the source to the primary winding is also the power delivered to the load by the secondary winding, thus power is neither generated nor lost.
- Summary of an ideal transformer:
  1. All of the flux links all of the windings of both the coils and that the resistance of the coils is zero – leads to:

$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

  2. Reluctance of the core is negligible, so the total mmf of both the coils combined is zero – leads to:

$$i_2(t) = \frac{N_1}{N_2} i_1(t),$$

  3. Power delivered by the source is directly delivered to the load with 100% efficiency.

Impedance transformation in a transformer

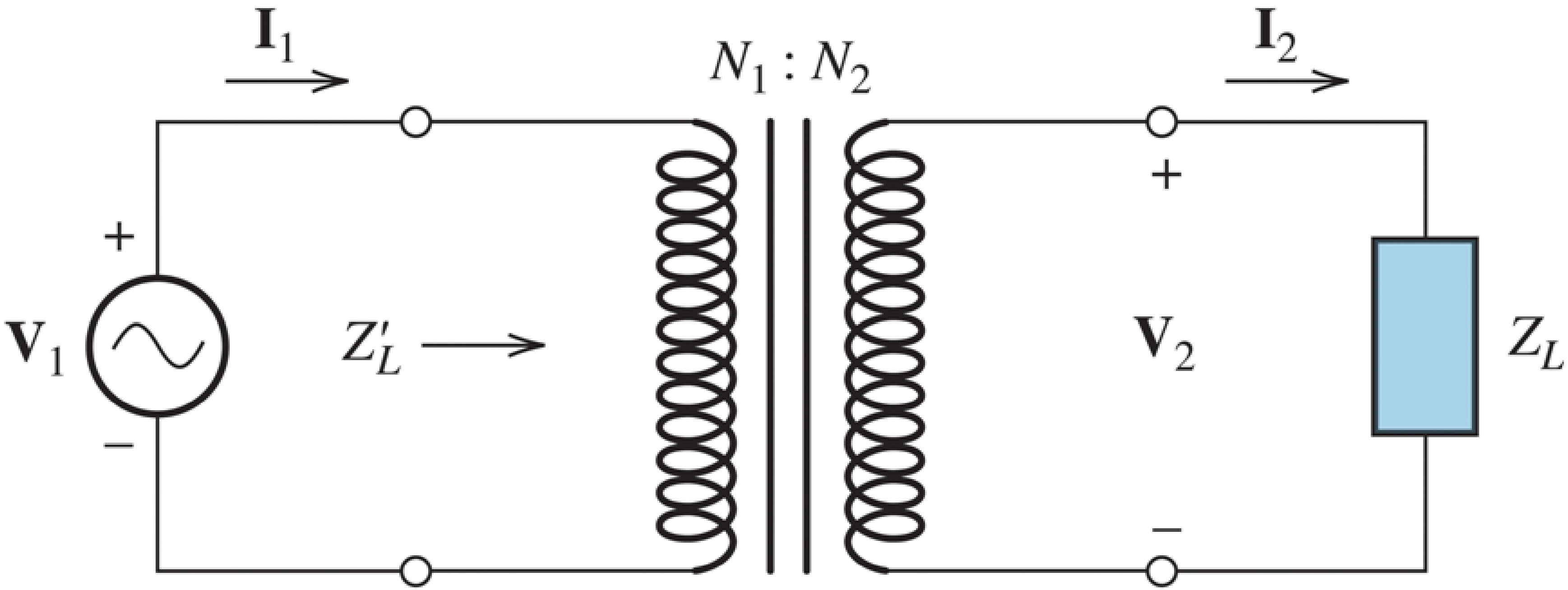
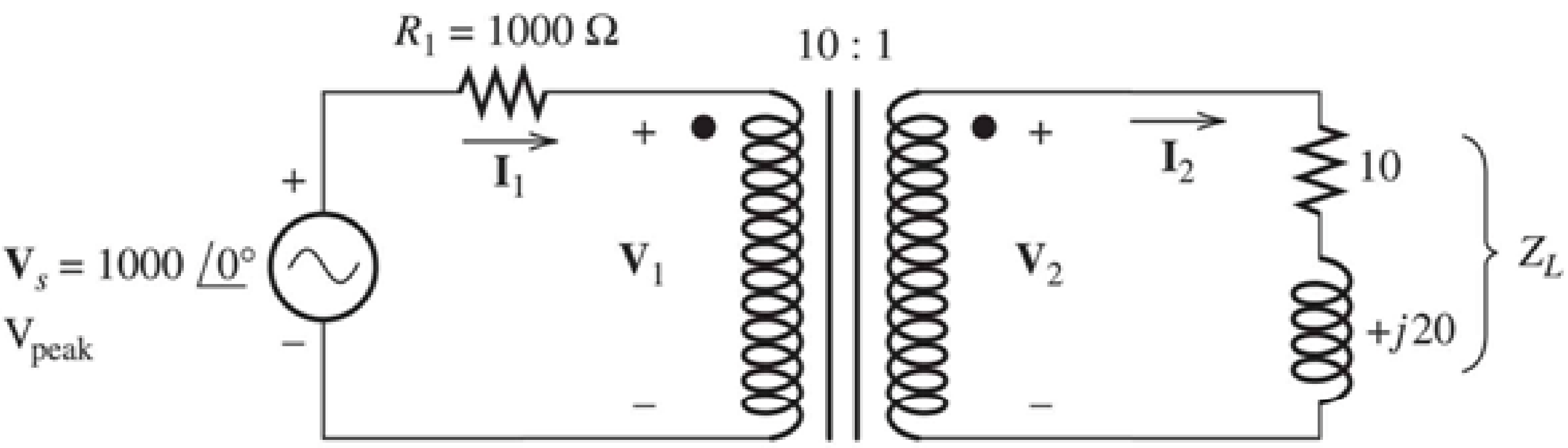


Fig. 3.17 The impedance seen looking at the primary

$$Z_L = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\left(N_2/N_1\right)V_1}{\left(N_1/N_2\right)I_1} = \left(\frac{N_2}{N_1}\right)^2 \frac{V_1}{I_1} \Rightarrow Z'_L = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \left(\frac{N_1}{N_2}\right)^2 Z_L \qquad (1.25)$$

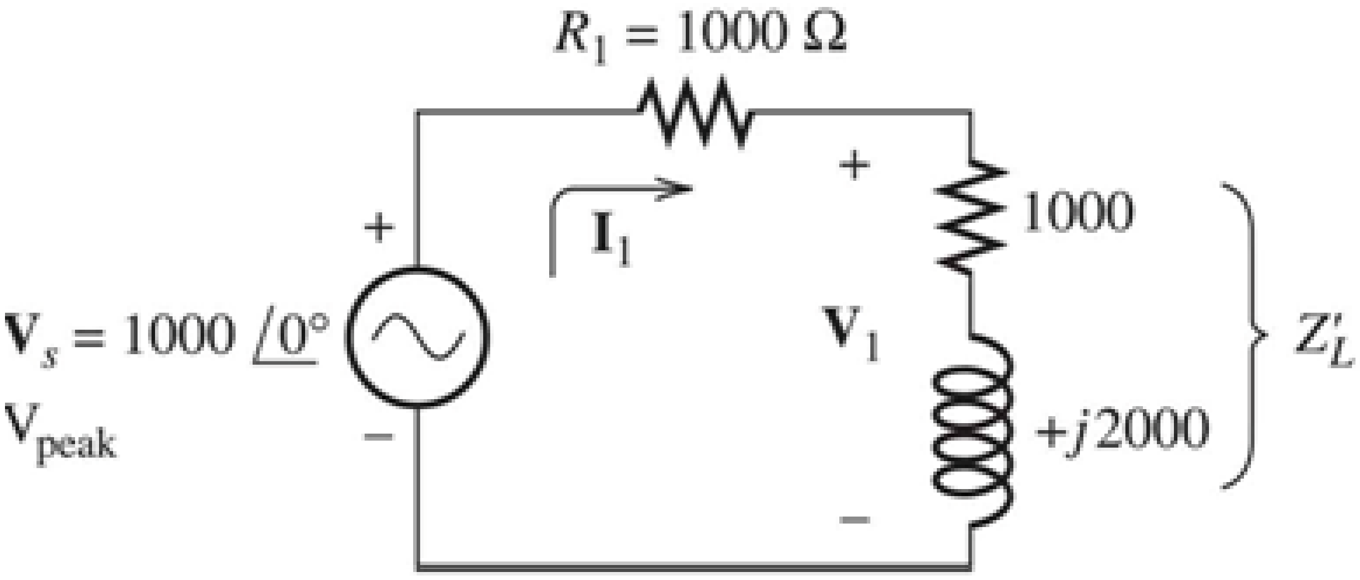
Example 7: Consider the circuit as shown below. Determine the phasor currents and voltages and also the power delivered to the load.



(a) Original circuit

Note that V_1 opposes V_s as per the Lez's law.

Solution: Consider the load, Z_L transferred to the primary side as Z_L':



(b) Circuit with Z_L reflected to the primary side

$$Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L = (10)^2(10 + j20) = 1000 + j2000\ \Omega = 2236.1\angle 63.43^0$$

$$Z_s = R_1 + Z_L' = 1000 + (1000 + j2000) = 2000 + j2000 = 2828.43\angle 45^0$$

$$I_1 = \frac{V_s}{Z_s} = \frac{1000\angle 0^0}{2828.43\angle 45^0} = 0.3536\angle -45^0 \;\; A, peak$$

$$V_1 = I_1 Z_L' = 0.3536\angle -45^0 \times 2236.1\angle 63.43^0 = 790.7\angle 18.43^0 \text{ V, peak}$$

$$I_2 = \frac{N_1}{N_2} I_1 = 10 \times 0.3536\angle -45^0 = 3.5336\angle -45^0 \;\; A, peak$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{1}{10} \times 790.7\angle 18.43^0 = 79.07\angle 18.43^0 \text{ V, peak}$$

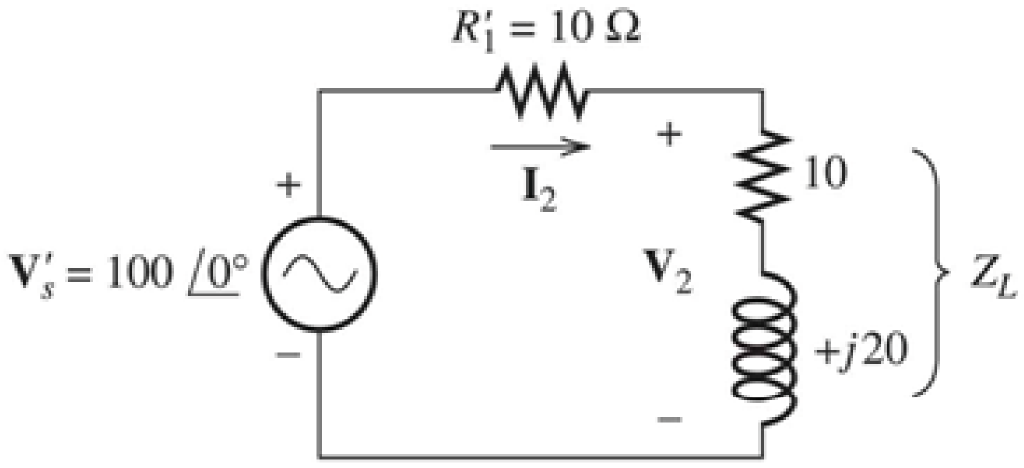
The power delivered to the load is given by:

$$P_L = I_{2,rms}^2 \times R_L = \left(\frac{3.5336}{\sqrt{2}}\right)^2 \times 10 = 62.43\ W$$

- We can also transfer voltage and current from primary side to the secondary side of the transformer such as

$$V_s' = \frac{N_2}{N_1} V_s = \frac{1}{10} \times 1000\angle 0^0 = 100\angle 0^0 \ V, peak$$

$$R_1' = \left(\frac{N_2}{N_1}\right)^2 R_1 = \left(\frac{1}{10}\right)^2 \times 1000 = 10 \ \Omega$$



(c) Circuit with $\mathbf{V}_s$ and R_1 reflected to the secondary side

Real-Transformer

- In real transformer, the coils have resistances (loss elements) and all the flux produced by the current in coil 1 might not link to coil 2 leading to leakage flux. Core reluctance is not zero leading to core losses and represented by R_c.

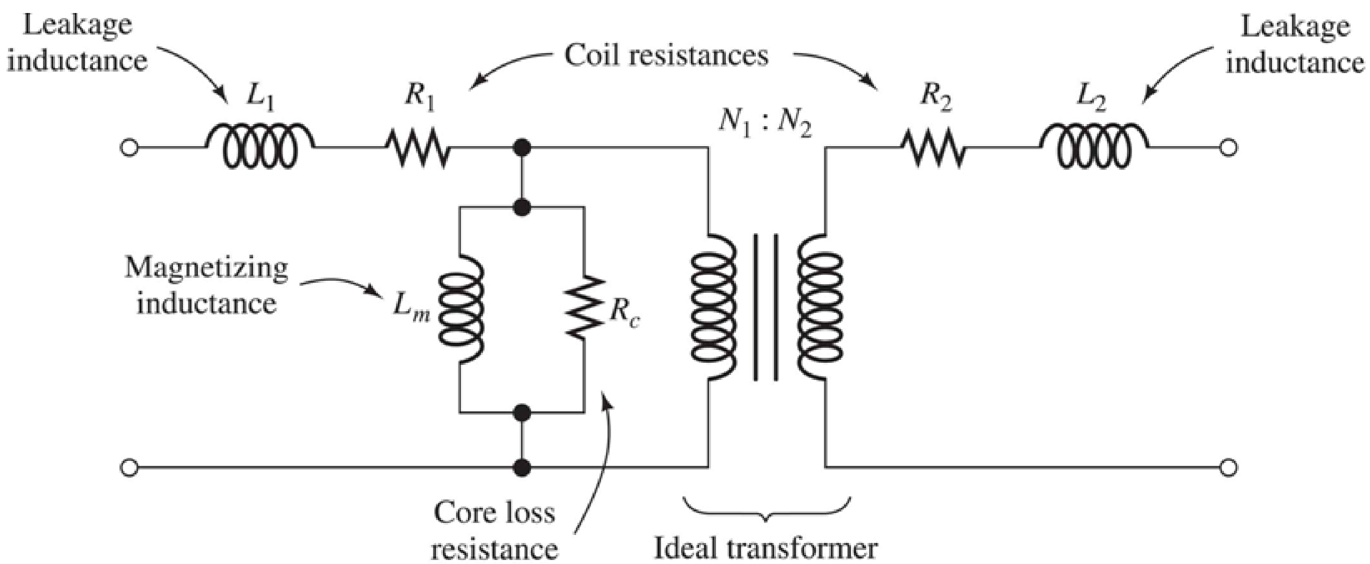


Fig. 3.18 Real transformer equivalent circuit

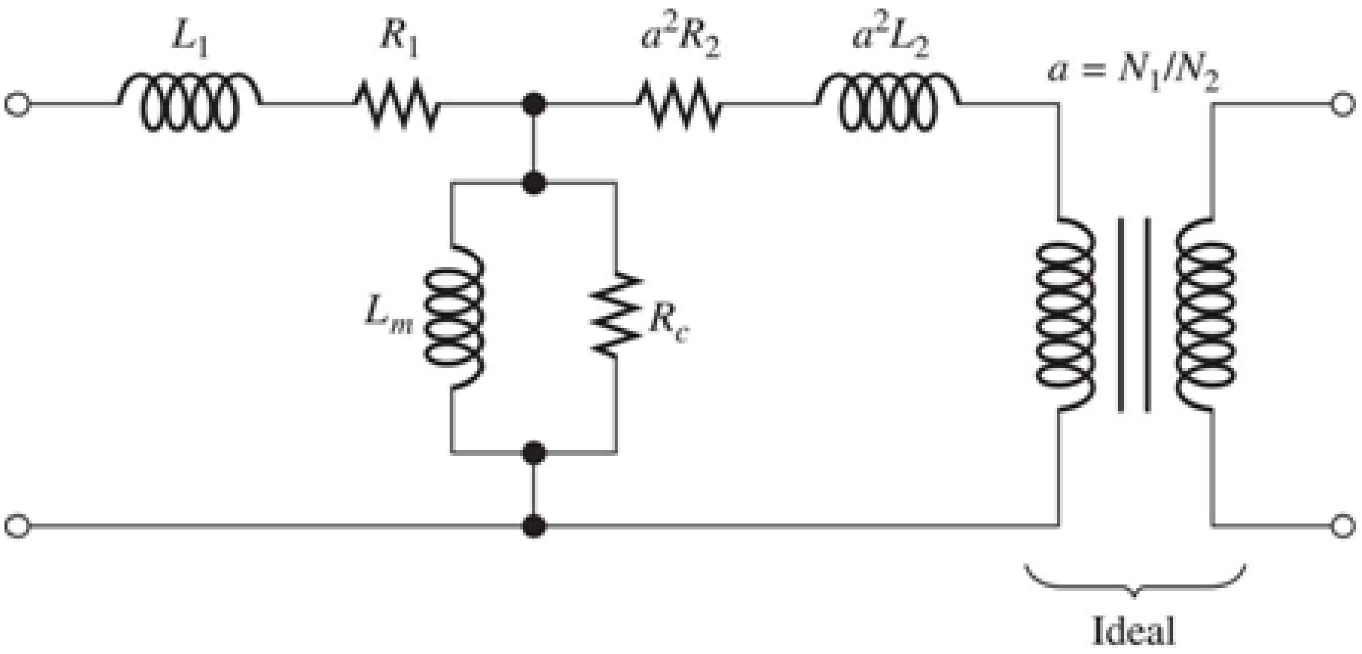


Fig. 3.18 (a) All elements referred to the primary side

- Transformer equivalent circuit with all the elements represented on the primary side.

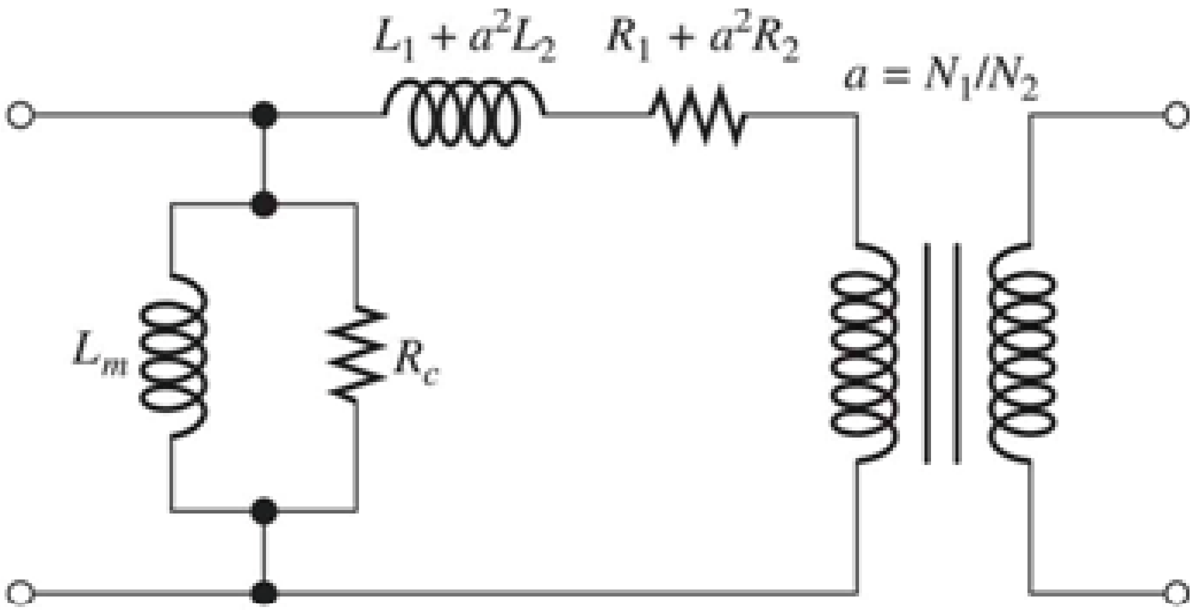


Fig. 3.18 (b) Approximate equivalent circuit that is sometimes more convenient to use than that of part (a)

- Approximate transformer equivalent circuit that is more convenient to use for computational purposes.

- Because of the presence of coil impedance, the voltage delivered to the load side varies with load current, which is an undesirable effect.
- This variation of load voltage is measured with a performance index called voltage regulation and is defined as

$$\textbf{percentage regulation} = \frac{V_{no-load} - V_{load}}{V_{load}} \times 100\% \quad (1.26)$$

- The power efficiency is defined as

$$\textbf{power efficiency} = \frac{P_{load}}{P_{in}} \times 100\% = \frac{P_{load}}{P_{load} + P_{loss}} \times 100\% \quad (1.27)$$

- Transformer rating: 10 kVA, 1100/240V, 50 Hz

Example 8: For the transformer with specifications 20 kVA, 2400/240 V, 60 Hz, R_1 = 3.0 Ω, X_1 = 6.5 Ω, R_2 = 0.03 Ω, X_2 = 0.07 Ω, R_c = 100.0 kΩ, X_m = 15.0 kΩ.

Determine the percentage regulation and power efficiency for a rated load with a lagging power factor of 0.8.

Solution: Load voltage is taken as phase reference.

$$\mathbf{V}_{load} = 240\angle 0^0\ V\ rms$$

For rated load of 20 kVA, we have rated load current

$$I_2 = \frac{20\ kVA}{240\ V} = 83.33\ A\ rms$$

- For a load power factor of 0.8 lagging, we have

$$\theta = \cos^{-1}(0.8) = 36.87^0$$

- Thus the phasor load current is given by

$$\mathbf{I}_2 = 83.33\angle{-36.87^0}\ A\ rms$$

- The primary current is given by

$$\mathbf{I}_1 = \frac{N_2}{N_1}\mathbf{I}_2 = \frac{1}{10} \times 83.33\angle{-36.87^0} = 8.333\angle - 36.87^0\ A, rms$$

- The voltage, V_2 is given by

$$\mathbf{V}_2 = \mathbf{V}_{load} + \mathbf{I}_2 \times (R_2 + jX_2) = 240\angle 0^0 + 83.33\angle{-36.87^0} \times (0.03 + j0.07) = 245.50 + j3.166\ V$$

- The primary voltage, V_1 is given by

$$\mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2 = 10 \times (245.50 + j3.166) = 2455.0 + j31.66\, V, rms$$

- Thus the source voltage, V_s is given by

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}_1 \times (R_1 + jX_1) = 2455.0 + j31.66 + 8.333\angle -36.87^0 \times (3 + j6.5)$$
$$= 2508.2\angle 1.37^0\, V\, rms$$

- The real power loss in the transformer is

$$P_{loss} = \frac{V_s^2}{R_c} + I_1^2 R_1 + I_2^2 R_2 = \frac{2508.2^2}{100 \times 10^3} + 8.33^2 \times 3 + 83.33^2 \times 0.03 = 479.5\, W$$

- The power delivered to the load is given by

$$P_{out} = 20\, kVA \times 0.8 = 16{,}000\, W$$

- The input power is given by is given by

$$P_{in} = P_{out} + P_{loss} = 16{,}000 + 479.5 = 16{,}479.5\ W$$

- Thus the power efficiency is given by

$$\text{power efficiency} = \frac{P_{out}}{P_{in}} \times 100\% = \frac{16{,}000}{16{,}479.5} \times 100\% = 97.09\%$$

- The no-load voltage is given by

$$V_{no,load} = V_2 = \frac{N_2}{N_1} V_s = \frac{1}{10} \times 2508.2 = 250.8\ V$$

- The voltage regulation is given by

$$\text{percent regulation} = \frac{250.82 - 240}{240} \times 100\% \quad = 4.51\%$$

Summary

- The right-hand rule is used to determine the direction of the magnetic field produced by a current.
- According to Faraday's law of induction, voltage is induced in a coil when the magnetic flux linkages change with time. Similarly, voltages are induced in moving conductors that cut through magnetic flux lines.
- The polarity of the induced voltage is determined by Lenz's law and it states that the induced voltage has a polarity that always opposes the main effect that induces the voltage.
- Magnetic flux density, **B** and the magnetic field intensity, **H** are related by **B** = μ**H**.

- Ampere's law states that the line integral of **H** around a closed path is equal to the algebraic sum of the currents flowing through the area bounded by the path.
- Magnetic devices can be approximately analyzed using circuit concepts. MMFs are analogous to voltage sources, reluctance is analogous to resistances, and flux is analogous to current.
- The self- and mutual- inductances of coils can be computed knowing the physical dimensions of the coils and the core on which they are wound.
- The *B-H* relationship for iron takes the form of a hysteresis loop, and the flux-density saturates at around 1 - 2 Wb/m^2.

- The area of the hysteresis loop represents the energy loss as heat per cycle.
- Eddy currents are another cause of core loss.
- Energy can be stored in the magnetic field.
- In a magnetic circuit consisting of iron core and air-gap, most of the energy is stored in the air-gap.
- Magnetic materials are characterized by a number of non-ideal properties, which must be considered in the detailed analysis. The most important phenomenon are saturation, eddy currents, and hysteresis.
- In an ideal transformer, the voltage across each coil is proportional to the number of turns, the net MMF is zero, regulation is 0% and power efficiency is 100%.
- Efficiency and regulation are important performance indices of transformer operation.

References

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2. "Principles and Applications of Electrical Engineering" – Giorgio Rizonni, Mc Graw Hill, 5th Edition 2007, Chapters 9 and 18.