EE1002
Introduction to Circuits and Systems

Part 1 : Lecture 6

Equivalent Circuits

Nonlinear Elements

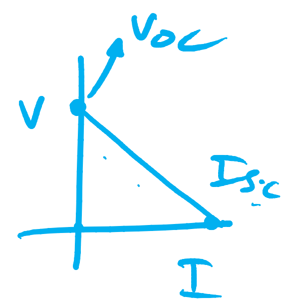
Maximum Power

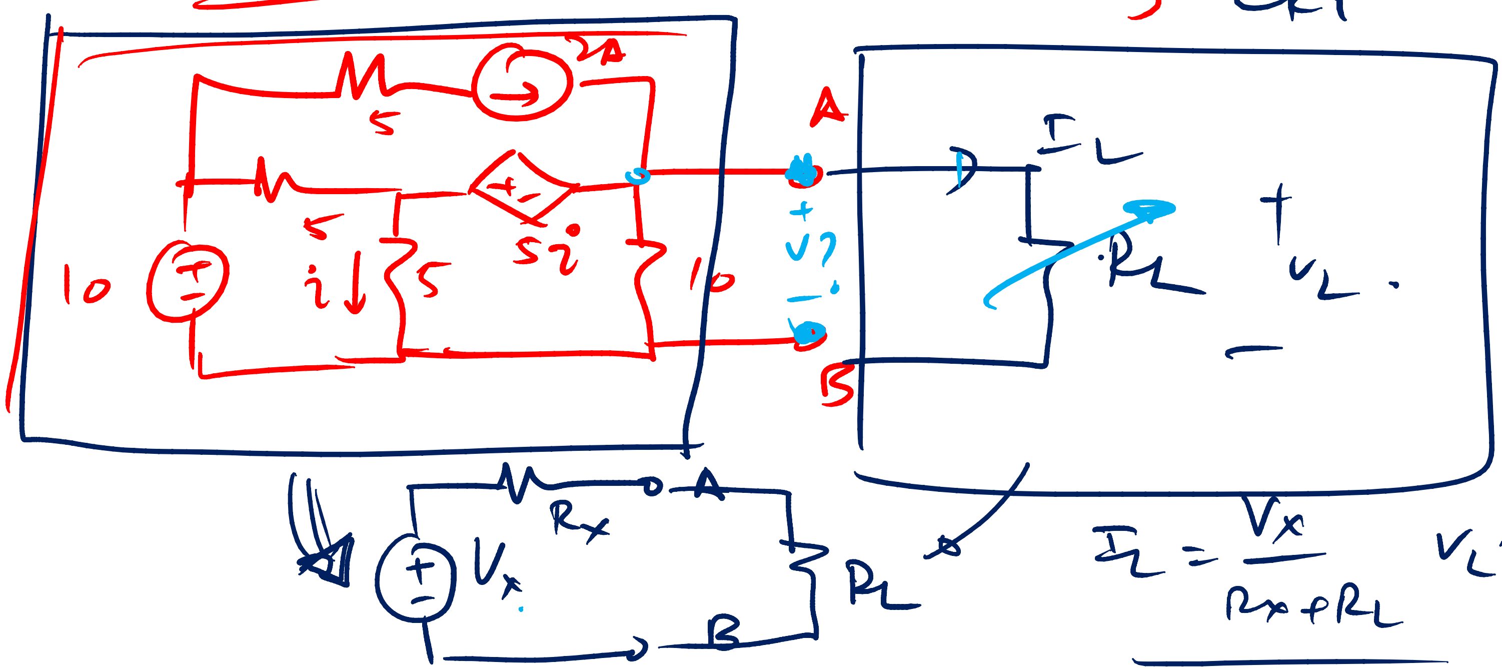
Equivalent Circuits

- One-port networks and equivalent circuits
  - Thevenin's equivalent circuit
  - Norton's equivalent circuit

→ linear circuits

ckt.





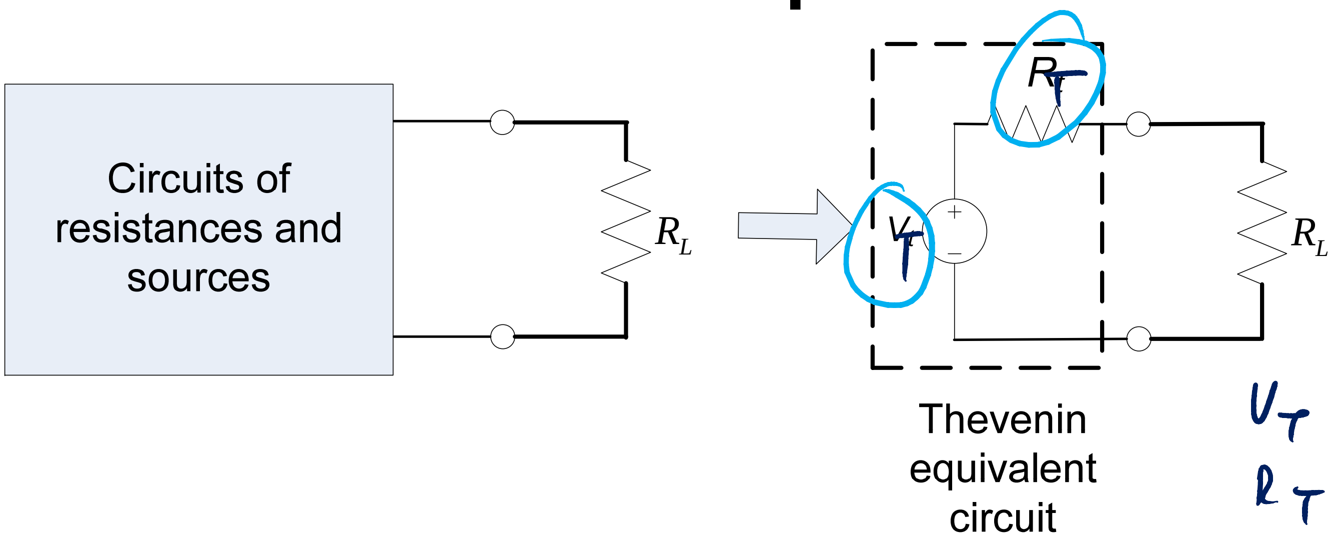
$$I_L = \frac{V_x}{R_x + R_L}$$

$$V_L = V_x \cdot \frac{R_L}{R_x + R_L}$$

One-port networks and equivalent circuits

- Two-terminal circuits can be replaced by an equivalent circuit consisting of a source and a resistance.
- A voltage source with a series resistance (Thevenin equivalent circuit)
- A current source with a parallel resistance (Norton equivalent circuit)

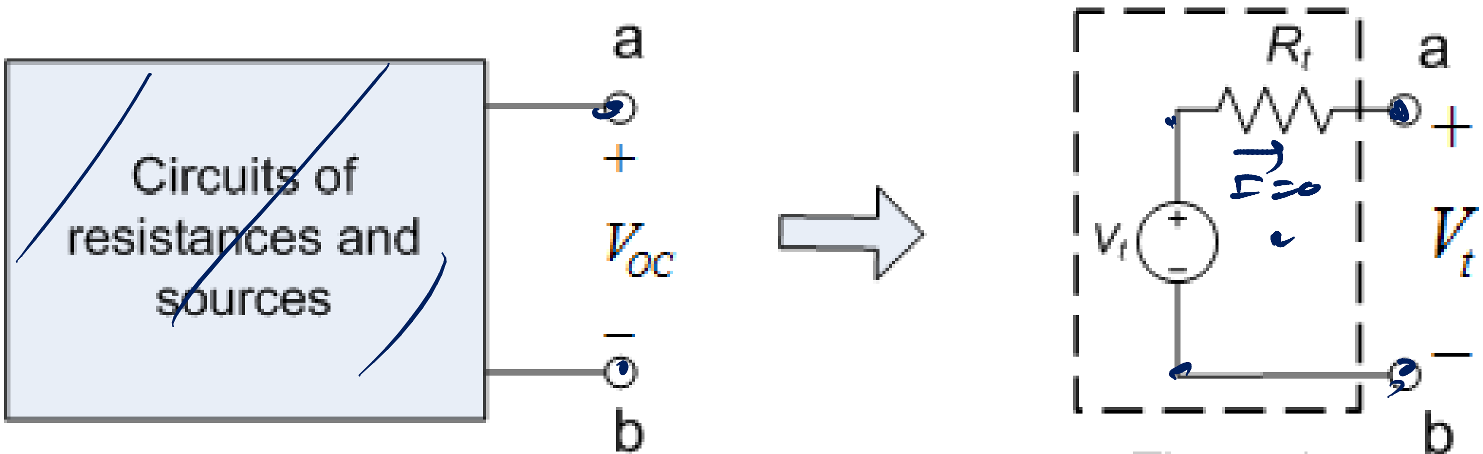
Thevenin equivalent



- A voltage source in series with a resistance
- The voltage source is called Thevenin's voltage
- The series resistance is called Thevenin's resistance

Thevenin Voltage

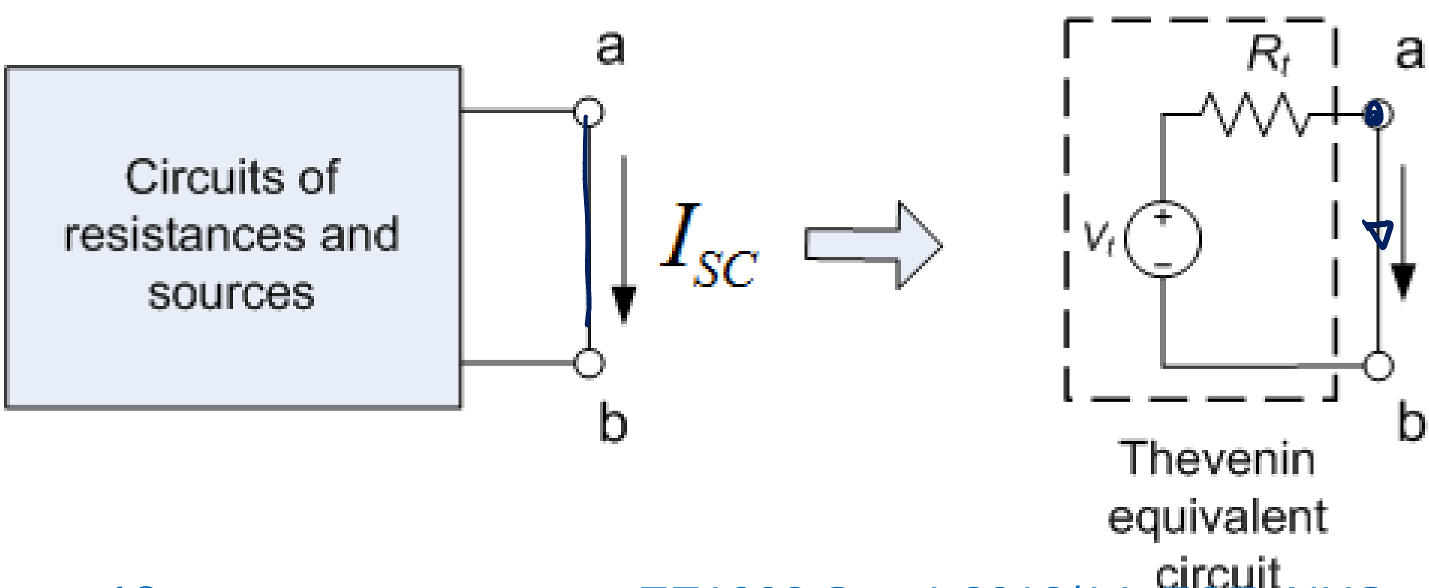
- Value of the voltage source is the open circuit voltage between the two terminals
- Called the Thevenin voltage



Thevenin equivalent circuit

Thevenin Resistance

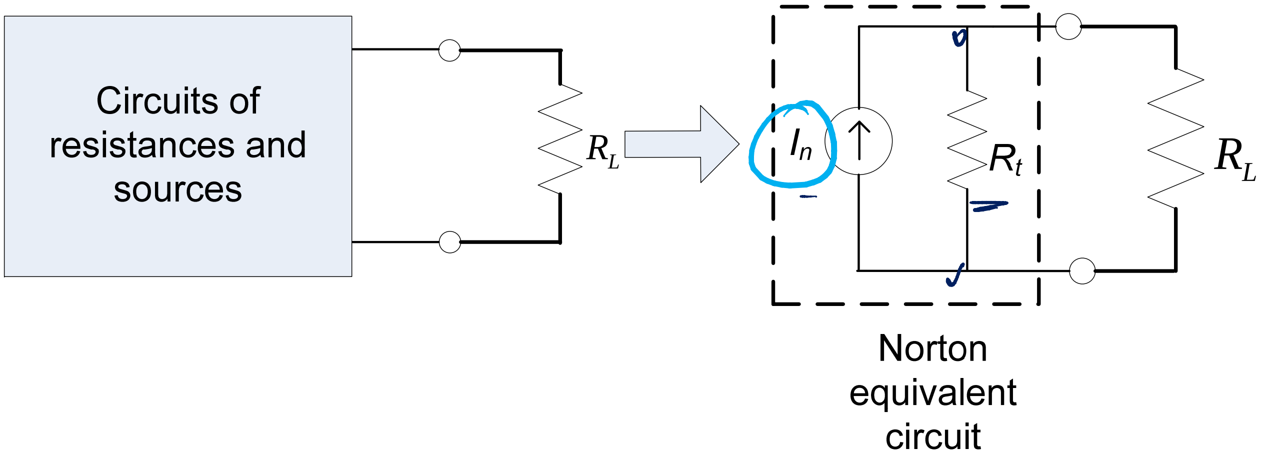
- Find the short circuit current between the two terminals
- Calculate the Thevenin resistance



$$I_{SC} = \frac{V_T}{R_T}$$

$$R_T = \frac{V_T}{I_{SC}}$$

Norton Equivalent



$I_N = I_{sc}$

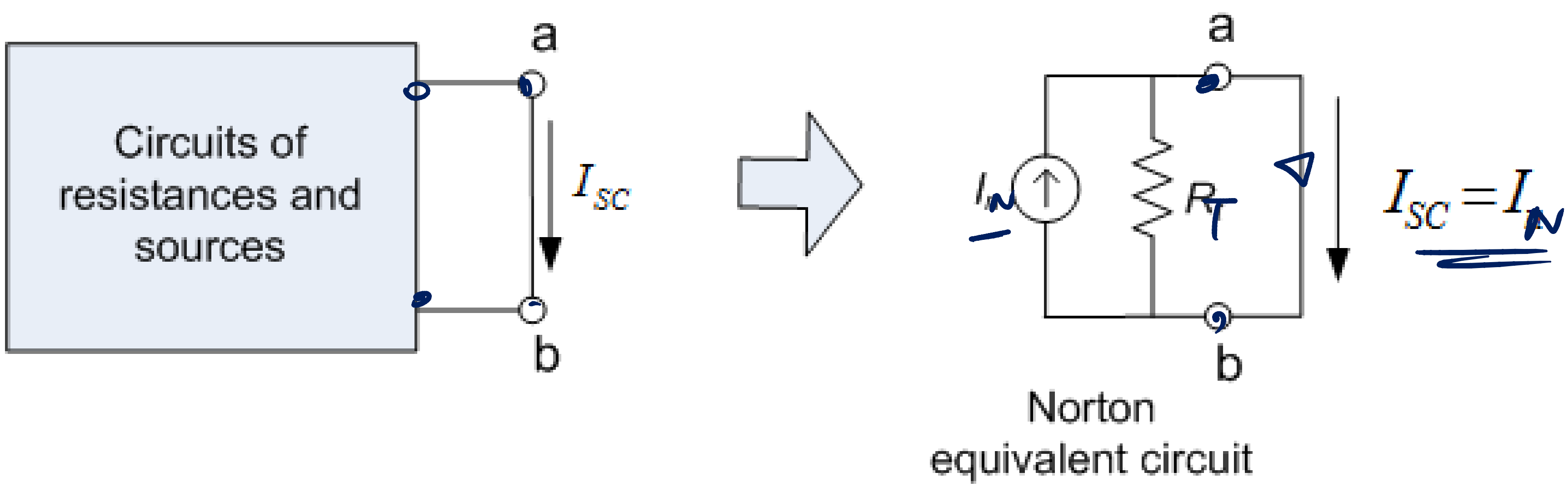
R_T

- A current source in parallel with a resistance
- The current source is called Norton's current
- The series resistance is called Thevenin's resistance

Norton's current

- Value of the current source is the short circuit current between the two terminals
- Called the Norton's current

$I_{sc} = I_N \times \frac{R_T}{R_T + 0}$



Steps to find the equivalent circuits

- Obtain the open circuit voltage between the two terminals – Thevenin's voltage
- Obtain the short circuit current between the two terminals – Norton's current
- Calculate the Thevenin's resistance as

$$\boxed{R_T = \frac{V_{OC}}{I_{SC}}}$$

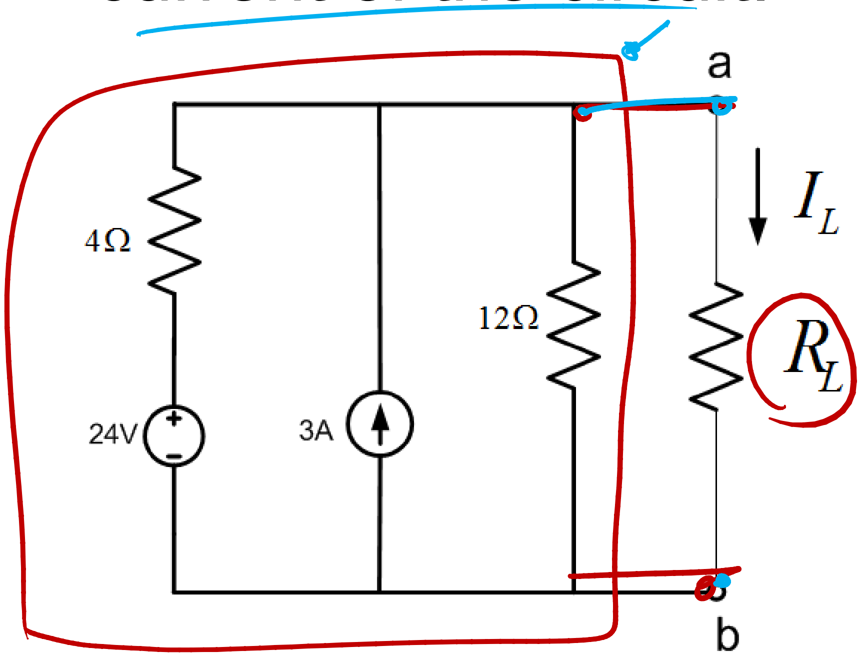
KCL at node a:

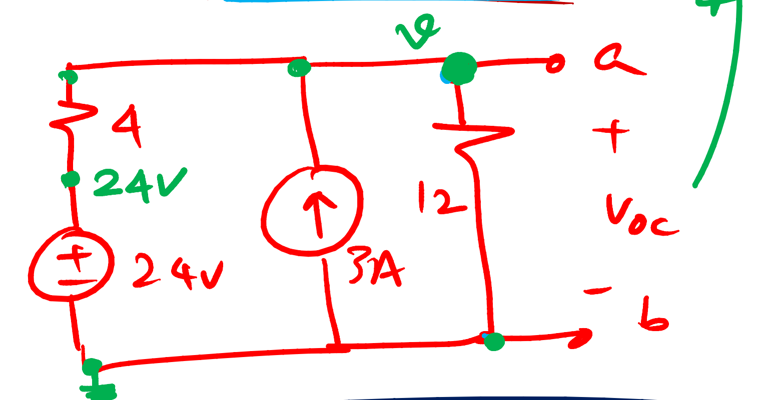
$$\frac{v-24}{4} - 3 + \frac{v}{12} = 0$$

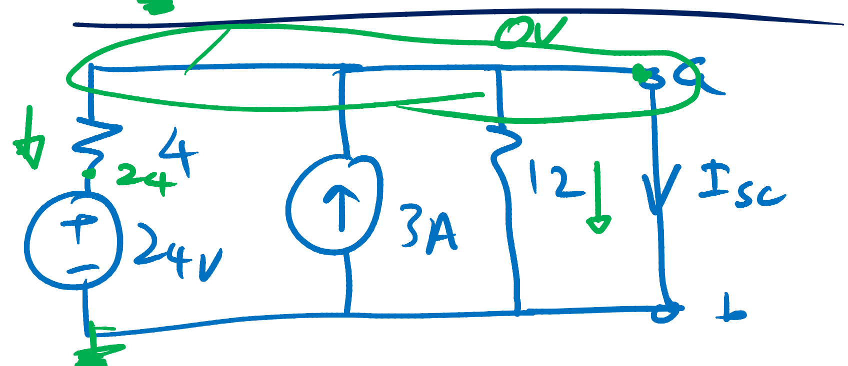
$$\Rightarrow 3(v-24) - 36 + v = 0 \Rightarrow 4v = 72+36 = 108 \Rightarrow v = 27V = V_{oc}$$

Example2

To find open circuit voltage and short circuit current of the circuit.







KCL: $\frac{0-24}{4} + (-3) + 0 + I_{sc} = 0 \Rightarrow I_{sc} = 9A.$

$$R_T = \frac{V_{oc}}{I_{sc}} = \frac{27}{9} = 3\Omega.$$

~~Example2 Contd.~~

1) Ohm's law, P.S.C.
2). KCL & KVL
3). Node voltage Analysis, Mesh Current Analysis.
4). Superposition – kill Sources.
5). Equivalent ckts. Thev., Norton
6). Nonlinear elements – Graphical Analysis. (Load line)
7). Maximum Power Transfer

→ Mid term. 1 syllabus.

Example3

KCL at v:

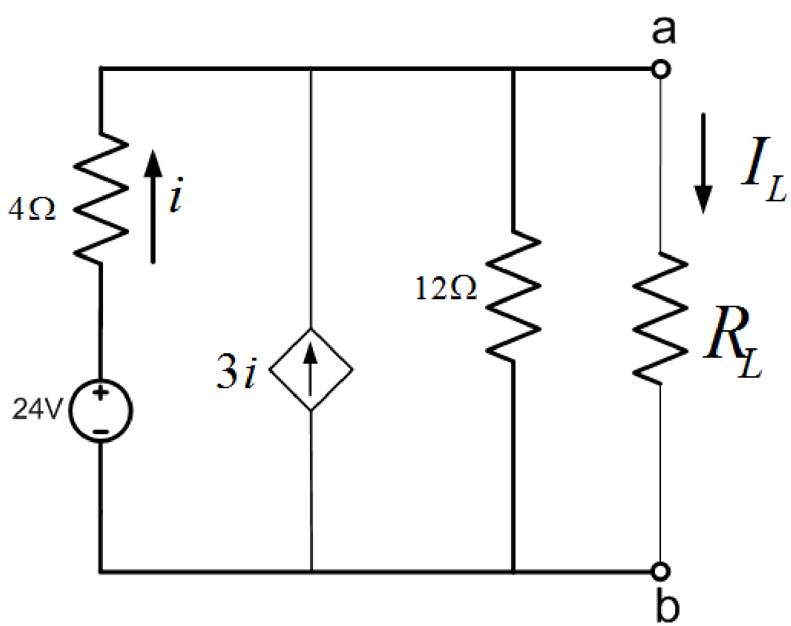
$i = \frac{24-v}{4}$

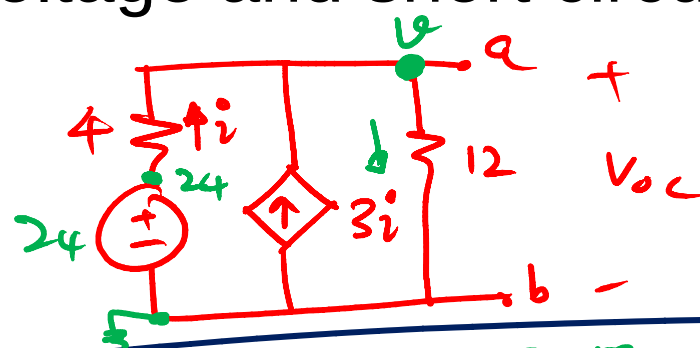
$-i - 3i + \frac{v}{12} = 0$, $-4 \times \frac{24-v}{4} + \frac{v}{12} = 0$, $\frac{v}{12} = 24 - v$

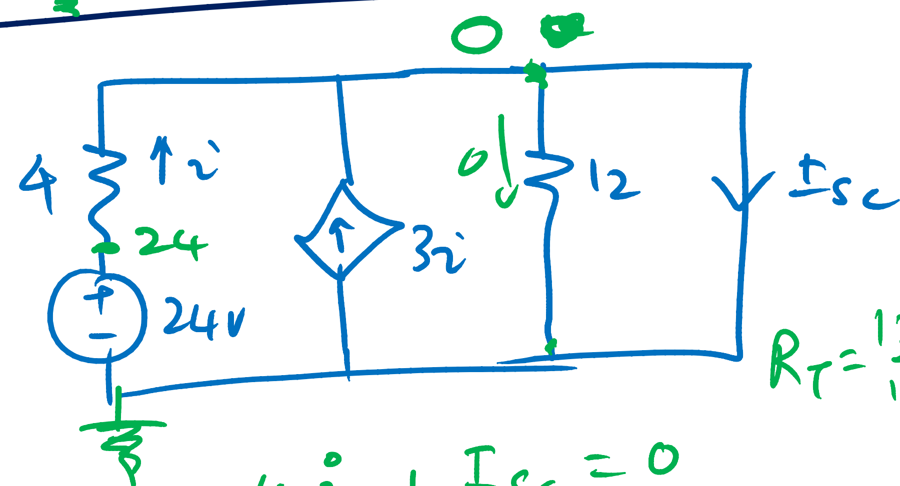
$13v = 12 \times 24$

$v = \frac{12 \times 24}{13}$ ✓ $= V_{oc}$

To find open circuit voltage and short circuit current of the circuit







$R_T = \frac{12}{13}\Omega$

KCL:

$i = \frac{24-0}{4} = 6A$; $-4i + I_{sc} = 0$

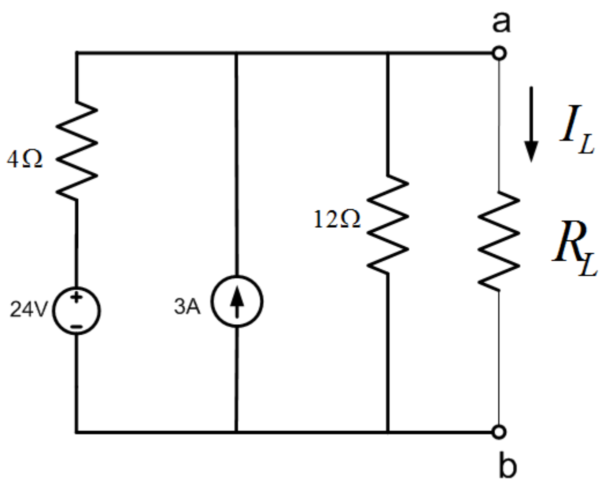
$\Rightarrow I_{sc} = 24A.$

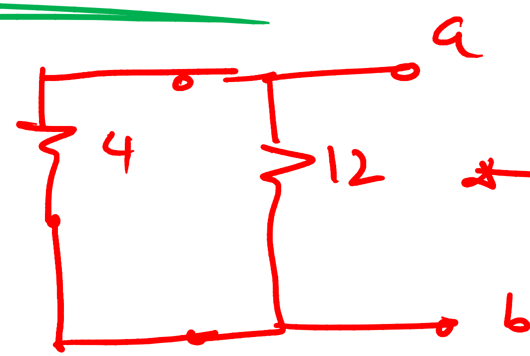
Other ways to find Thevenin Resistance

- If the circuit consists of independent sources only (can not use this if there are dependent sources) and resistances,
  - Kill all the sources. This would result in a purely resistive network
  - Find the equivalent resistance between the two terminals, by repeated application of series parallel rule
  - Thevenin resistance is equal to the equivalent resistance

Example1

Finding Thevenin resistance





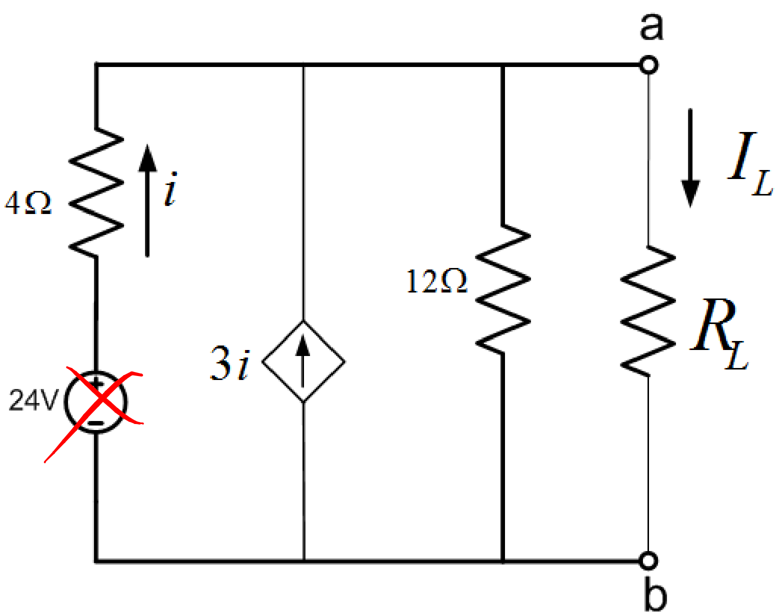
$$R_{eq} = \frac{4\times12}{4+12} = \frac{4\times12}{16} = 3\Omega.$$

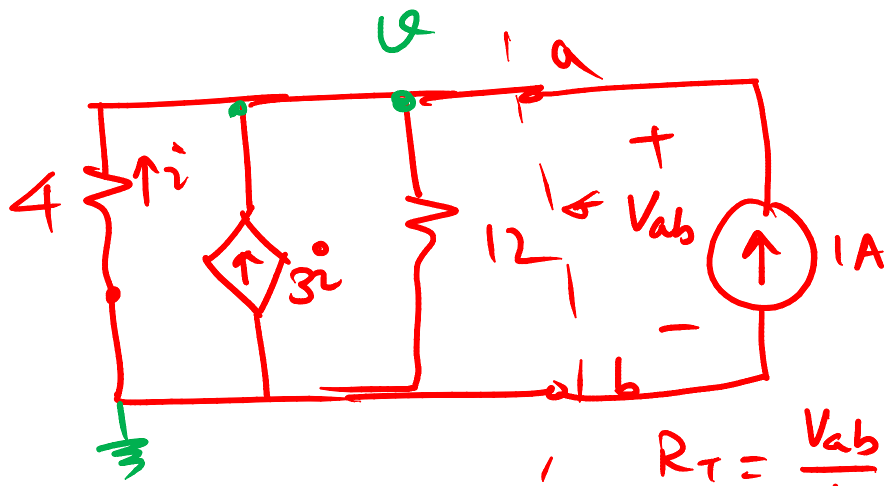
Other ways to find Thevenin Resistance

- Test source method is used if the circuit contains dependent sources
  1. Kill the independent sources
     (Do not kill the dependent sources)
  2. Apply a known source (either a voltage or current source) between the two terminals
  3. Find the voltage and current across the terminals by solving the circuit
  4. Then calculate Thevenin's resistance as voltage divided by current.

Example2

Finding Thevenin resistance – Test source method





$R_T = \frac{V_{ab}}{1}$

KCL at node v:

$i = \frac{0-v}{4} = -\frac{v}{4}$.

$-i - 3i + \frac{v}{12} - 1 = 0$

$+\cancel{4} \times \left(\frac{+v}{\cancel{4}}\right) + \frac{v}{12} - 1 = 0$

$13\frac{v}{12} = 1$

$v = \frac{12}{13}$ V.

$V_{ab} = \frac{12}{13}$ V.

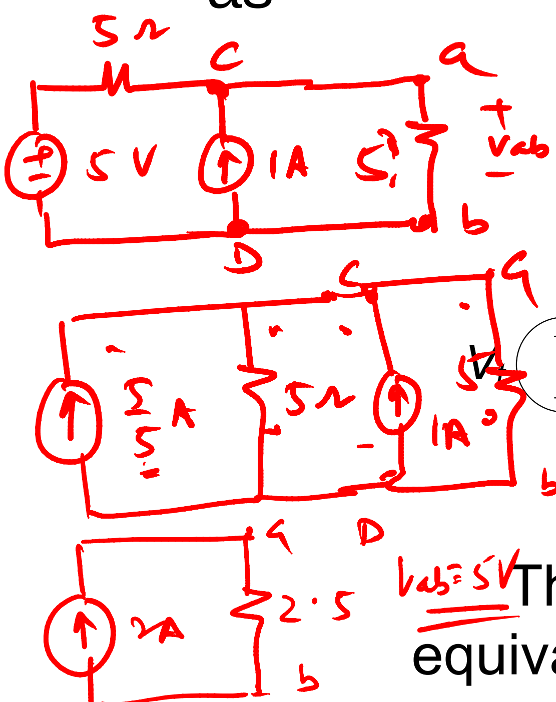
$R_T = \frac{12}{13}\,\Omega$.

Source Conversion

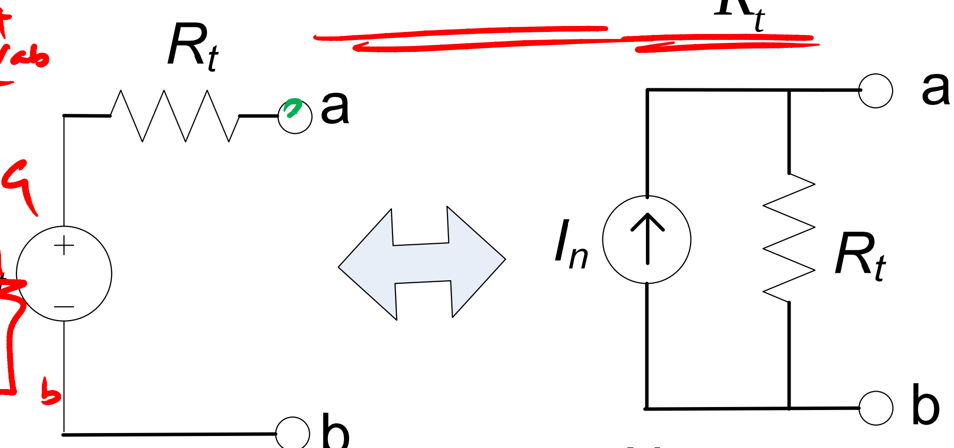
Mid-term will be in 6th week Thursday

- A voltage source with a series resistance is equivalent to a current source with the resistance in parallel.
- The values of the voltage and current source are given as

$$V_t = I_n R_t, \quad I_n = \frac{V_t}{R_t}$$



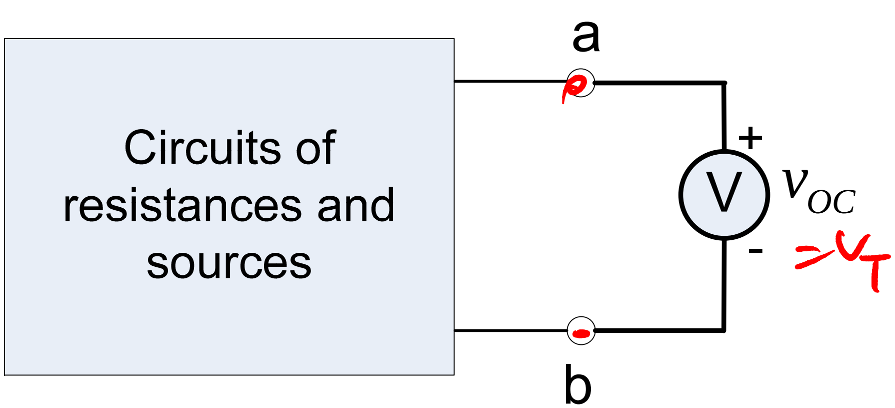
Vab = 5V

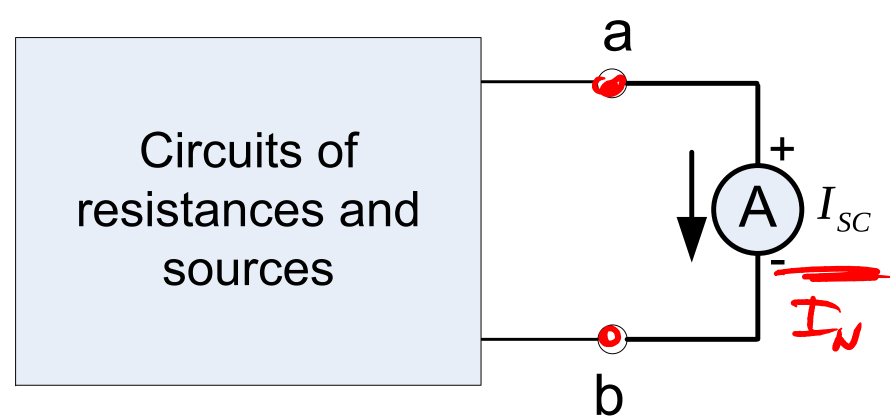


Thevenin equivalent circuit

Norton equivalent circuit

Experimental determination of Thevenin and Norton equivalents

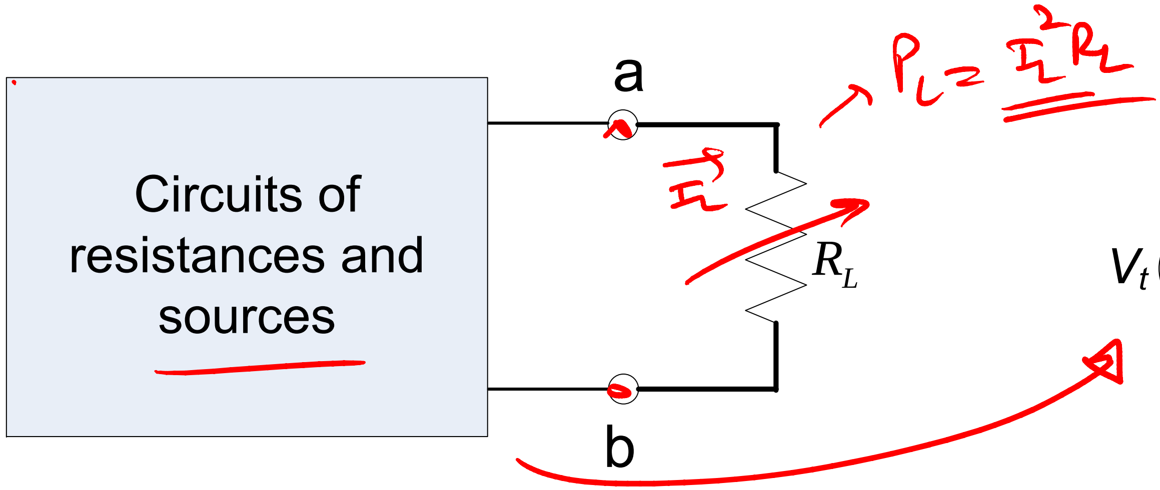


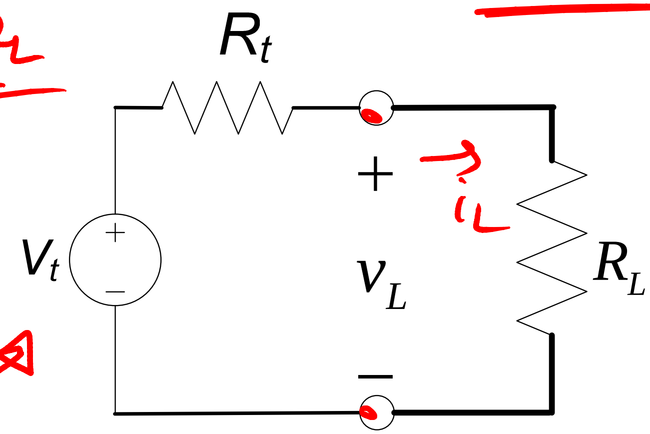


$$R_T = \frac{V_{oc}}{I_{sc}}$$

Maximum power transfer

Theorem





$$i_L = \frac{v_T}{R_L + R_T}$$

$$P_L = i_L^2 R_L = \frac{v_T^2}{(R_L + R_T)^2} R_L$$

$P_L(R_L)$ $f = y(x)$

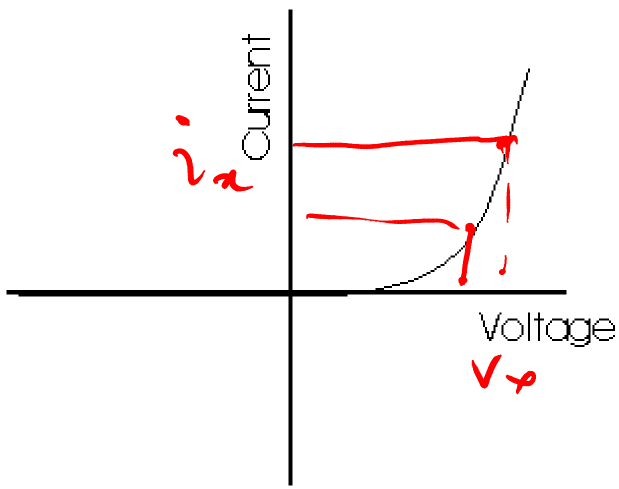
$$\frac{dP_L}{dR_L} = \frac{v_T^2 (R_L + R_T)^2 - 2v_T^2 R_L (R_L + R_T)}{(R_L + R_T)^4} = 0$$

$$v_T^2 (R_L + R_T)^2 - 2v_T^2 R_L (R_L + R_T) = 0$$

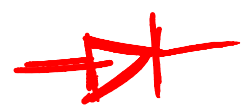
$$\boxed{R_L = R_T}$$

Nonlinear elements

- Nonlinear elements may not have an analytical function
- How to solve a linear circuit with one nonlinear element?



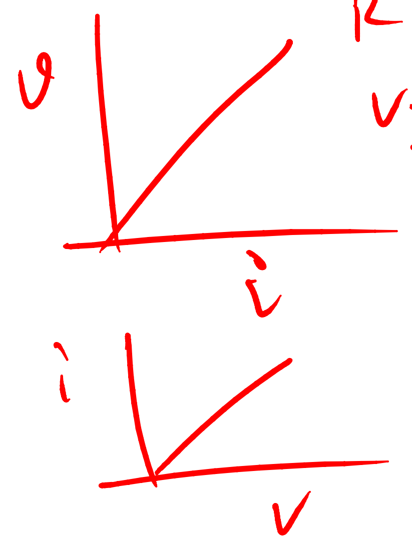
Diode



$i = f(v)$

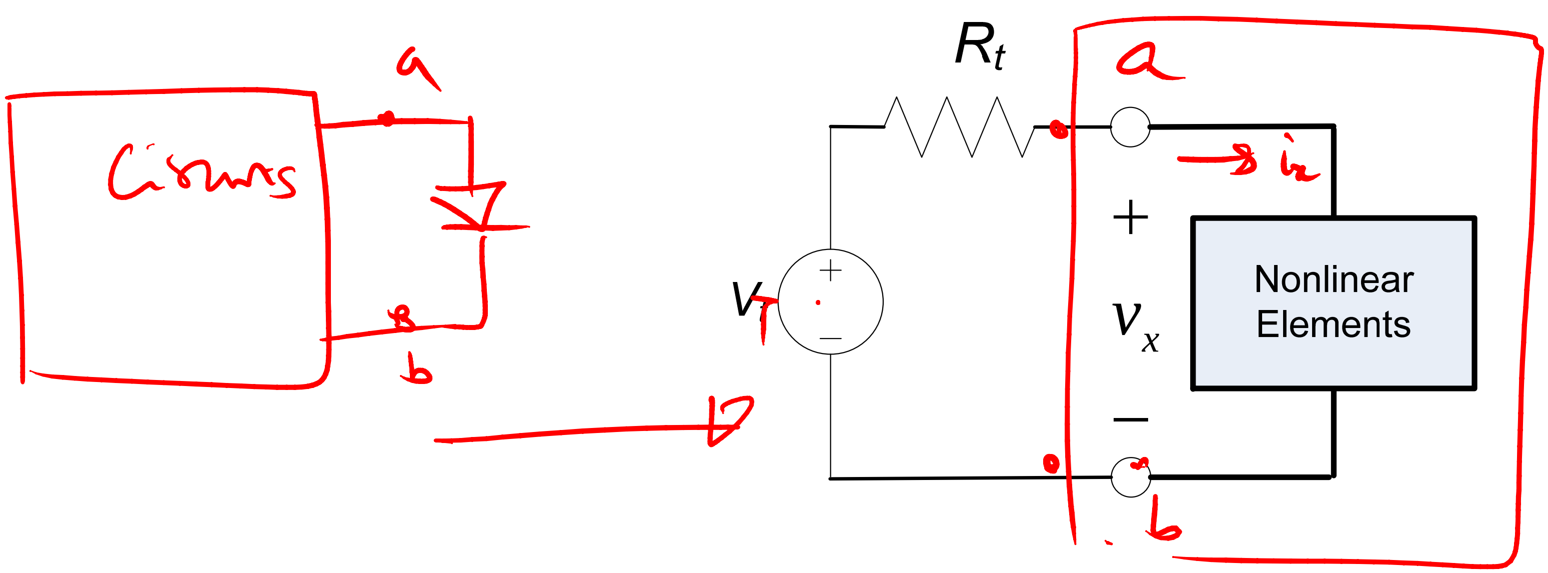
R

$v = iR$

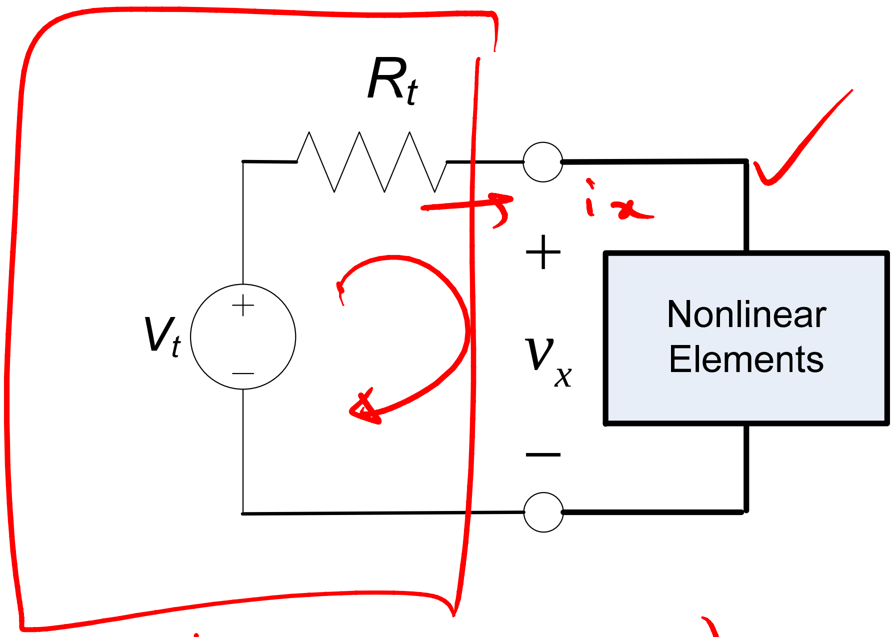


Solving circuits with one Nonlinear element

- Replace the circuit by its Thevenin's equivalent considering the nonlinear element as the load
- Use graphical analysis technique

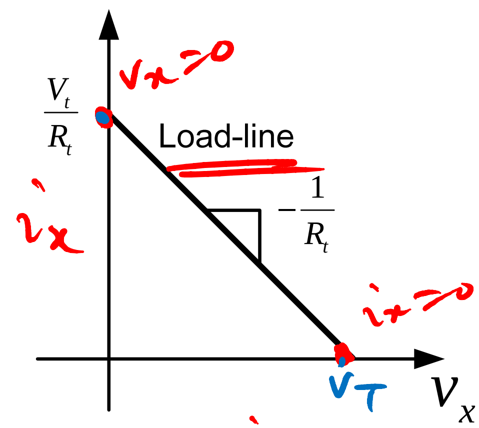


Treating the nonlinear elements as load



linear ckt

KVL:



$-V_T + i_x \cdot R_T + V_x = 0$

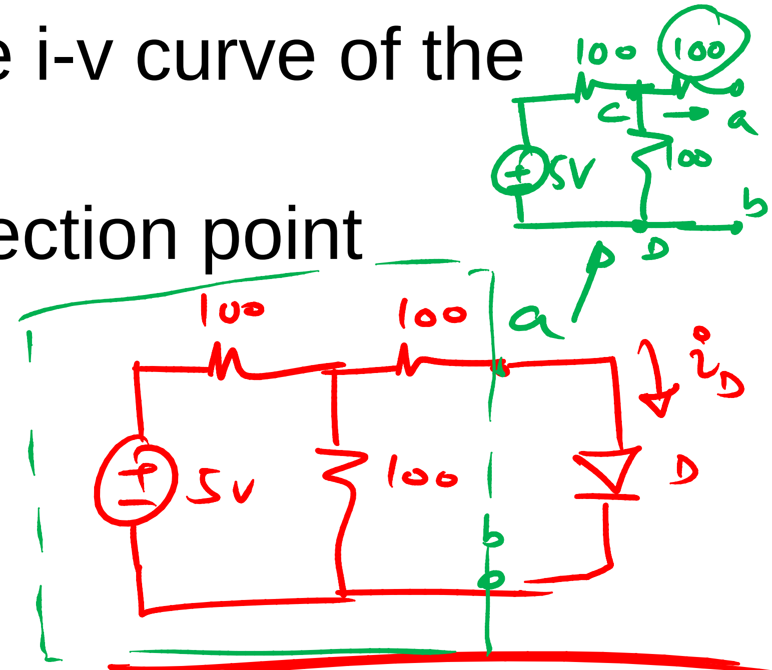
$$v_T = i_x R_T + v_x$$

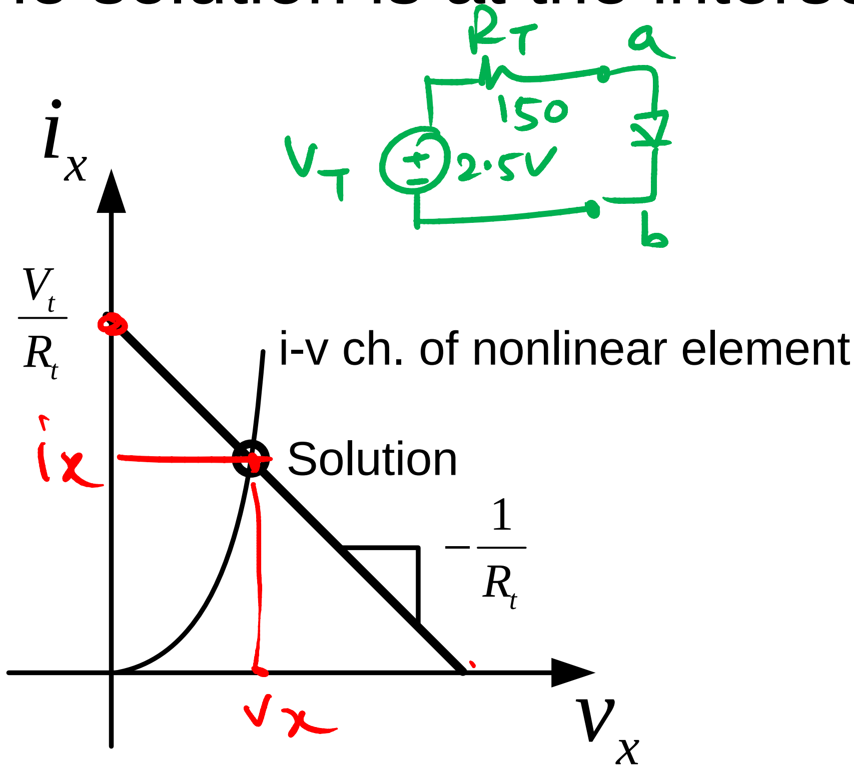
$$i_x = \left(-\frac{1}{R_T}\right) v_x + \left(\frac{v_T}{R_T}\right)$$

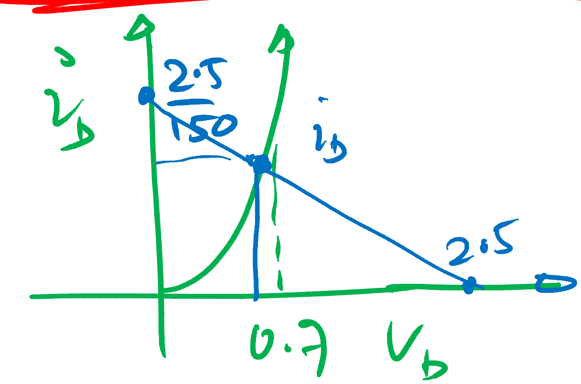
$y = mx + c$

Graphical (Load-line) analysis

- Merge the load line onto the i-v curve of the nonlinear element
- The solution is at the intersection point

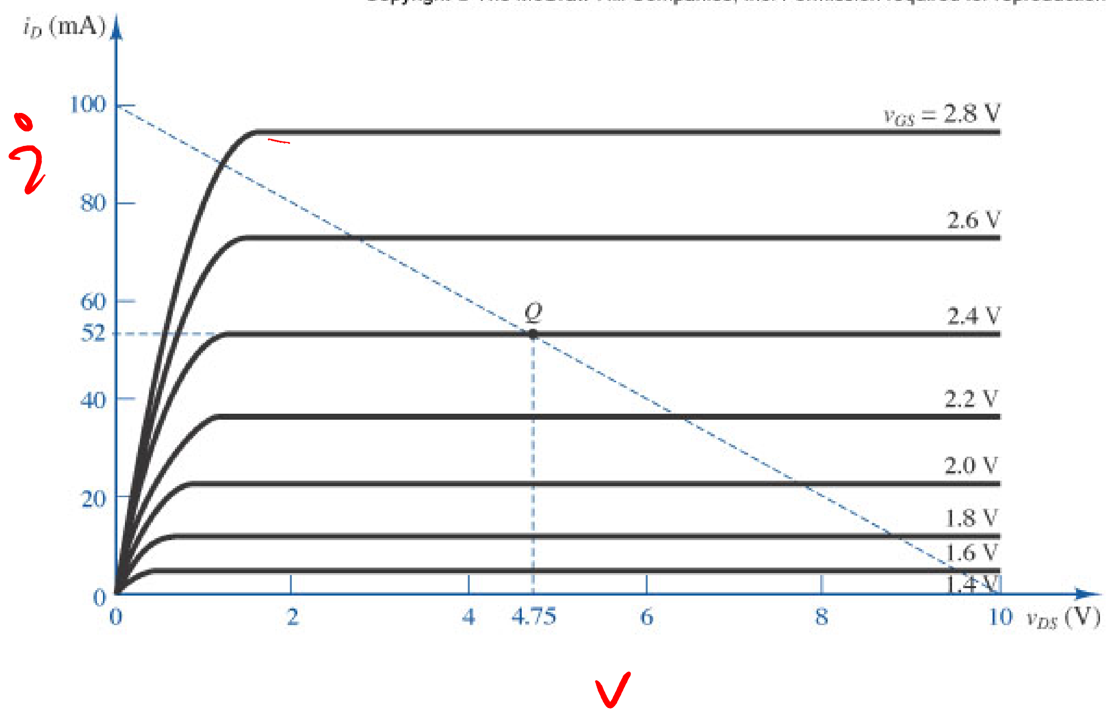


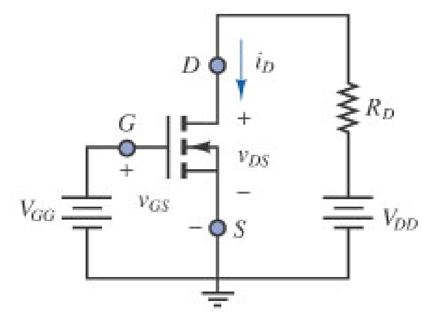




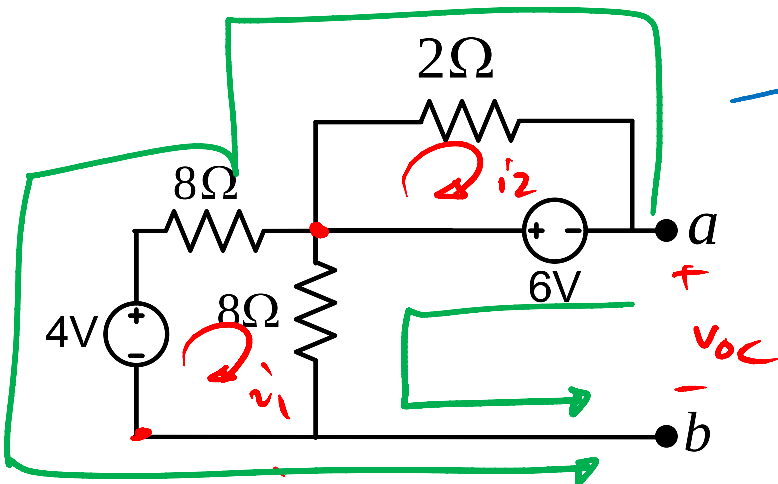
Practical Example

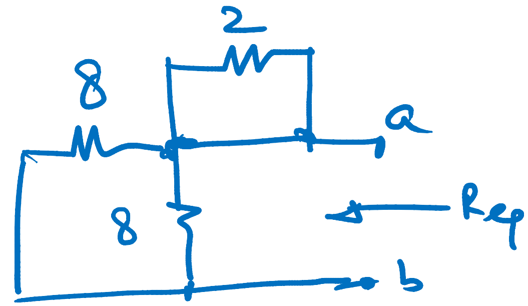
Transistor





Example 1





Find the Thevenin's equivalent between nodes 'a' and 'b'.

$R_{eq} = 4\Omega$

Mesh i_1 : KVL : $-4 + 8i_1 + 8i_1 = 0 \Rightarrow i_1 = 0.25$ A.

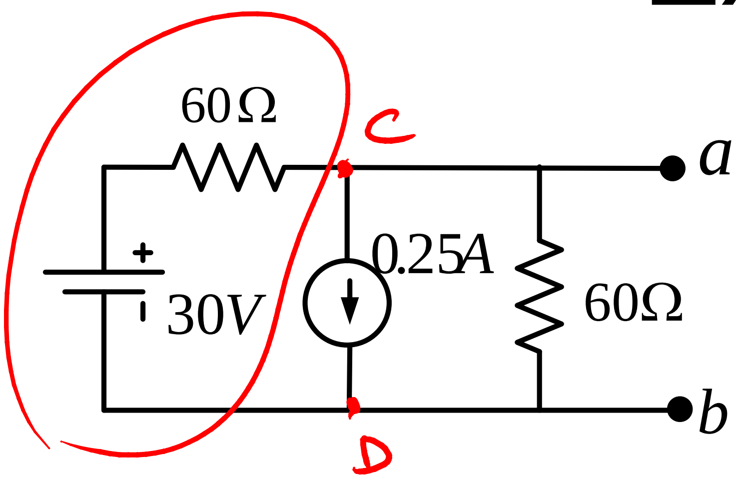
Mesh i_2 KVL : $2i_2 - 6 = 0 \Rightarrow i_2 = 3$ A.

$V_{oc} = V_{ab} = -6 + 8i_1 = -6 + 8 \times 0.25 = -4V = V_T$.

$V_{ab} = -2 \cdot i_2 - 8i_1 + 4 = -2 \times 3 - 8 \times 0.25 + 4$

$= -6 - 2 + 4 = -4V = V_T$

Example 2



Find the Norton equivalent of the circuit between nodes 'a' and 'b'.