Formulae Sheet for EE1002: Introduction to Circuits and Systems

Part 1

$$\text{Resistos in series}: R_{eq} = R_1 + R_2 + R_3;\ \text{Resistors in parallel}: \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\text{Capacitors in series}: \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3};\quad \text{Capacitors in parallel}: C_{eq} = C_1 + C_2 + C_3$$

$$\text{Indctuors in series}: L_{eq} = L_1 + L_2 + L_3;\ \text{Inductors in parallel}: \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$\text{Charge in capacitor}: Q = CV, \text{Energy stored in Capacitor}: E_C = \frac{1}{2}CV^2$$

$$\text{Flux - linkage for inductor}: \lambda = LI, \text{Energy stored in Inductor}: E_L = \frac{1}{2}LI^2$$

$$\text{Capacitor voltage transient in an RC circuit}: v_c(t) = v_c(0)e^{-\frac{t}{\tau}} + v_c(\infty)\left(1 - e^{-\frac{t}{\tau}}\right), \tau = RC$$

$$\text{Inductor Current transient in an RL circuit}: i_L(t) = i_L(0)e^{-\frac{t}{\tau}} + i_L(\infty)\left(1 - e^{-\frac{t}{\tau}}\right), \tau = \frac{L}{R}$$

AC:

$$\text{Impedance for R, L, C}: Z_C = \frac{1}{jC\omega}, Z_L = j\omega L, Z_R = R$$

$$\text{where } \omega = 2\pi f = 2\pi\frac{1}{T}, V_m = \sqrt{2}V_{RMS}, I_m = \sqrt{2}I_{RMS}$$

AC Power:

$$\text{AC supply connected to AC load}: Z = R + jX = \sqrt{R^2 + X^2}\angle\phi,\ \phi = \tan^{-1}\left(\frac{X}{R}\right), X = \omega L \text{ or } X = -\frac{1}{\omega C}$$

Given $v(t) = V_m\cos(\omega t), i(t) = I_m\cos(\omega t - \phi)$,

Apparent power $= V_{RMS}I_{RMS}$

Average power (Real power) $= V_{RMS}I_{RMS}\cos\phi$

Imaginary power $= V_{RMS}I_{RMS}\sin\phi$

Power factor $= \cos\phi$,

(lagging power factor if current lags voltage, else leading power factor if current leads voltage)

Part 2

Chapter 1: Magnetic Circuits and transformers	
Magnetic flux	$B = \frac{\phi}{A}$ (1.2a)
Flux-linkage	$\lambda = N \times \phi$ (1.3)
Faraday's Law	$e = N\frac{d\phi}{dt} = \frac{d(N\phi)}{dt} = \frac{d\lambda}{dt}$ (1.4a)
Voltage Induced in Field-cutting Conductors	$e = \frac{d\lambda}{dt} = Bl\frac{dx}{dt} = Bl(u = \frac{dx}{dt}) = Blu$ (1.5)
Voltage Induced by Time-varying Flux	$\phi = \phi_m \, sin(\omega t)$, $e = N\frac{d\phi}{dt} = N\phi_m\omega\cos(\omega t)\, volts$ (1.6)
Magnetic Field Intensity and Ampere's Law	$B = \mu H = \mu_r\mu_0 H$ (1.7) $\oint \mathrm{H}.dl = \sum i$ (1.8)
Magnetic Field around a long Straight Wire	$Hl = \sum i = H \times (2\pi r) = I \Rightarrow H = \frac{I}{2\pi r} \Rightarrow B = \mu H = \frac{\mu I}{2\pi r}$ (1.9)
Magnetic Field in a Toroidal Core	$Hl = \sum i = H \times (2\pi R) = NI \Rightarrow H = \frac{NI}{2\pi R} \Rightarrow B = \mu H = \frac{\mu NI}{2\pi R} \Rightarrow$ $\phi = B \times A = \frac{\mu NI}{2\pi R} \times (\pi r^2) = \frac{\mu NIr^2}{2R} \Rightarrow \lambda = N\phi = \frac{\mu N^2 Ir^2}{2R}$ (1.9a)
Magnetic Circuits, MMF	$\mathcal{F} = N \times i \quad A - turns$ (1.10)
Reluctance of magnetic path	$\mathcal{R} = \frac{l}{\mu A}$ (1.11)
MMF	$\mathcal{F} = \mathcal{R} \times \phi$ (1.12)
Self-Inductance	$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N(\frac{Ni}{\mathcal{R}})}{i} = \frac{N^2}{\mathcal{R}}$ (1.13)
Voltage induced in the self-inductance	$e = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt} = v_L$ (1.14)
Self-inductances of coils	$L_1 = \frac{\lambda_{11}}{i_1}$ and $L_2 = \frac{\lambda_{22}}{i_2}$ (1.15)
Mutual inductances between coils	$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$ (1.16)
Total flux-linkage	$\lambda_1 = \lambda_{11} \pm \lambda_{12} = L_1 i_1 \pm M i_2 and \; \lambda_2 = \pm M i_1 + L_2 i_2$ (1.17)
Transformer voltage relationship	$v_2(t) = \frac{N_2}{N_1} v_1(t)$ (1.22)
Transformer current relationship	$\mathcal{F} = N_1 i_1(t) - N_2 i_2(t) = 0 \Rightarrow i_2(t) = \frac{N_1}{N_2} i_1(t)$ (1.23)

Ideal transformer power relationship	$p_2(t) = v_2(t) \times i_2(t) = v_1(t) \times i_1(t) = p_1(t)$ (1.24)
Impedance Transformations	$Z_L = \frac{\mathbf{V_2}}{\boldsymbol{I_2}} \Rightarrow Z_L' = \frac{\mathbf{V_1}}{\boldsymbol{I_1}} = \left(\frac{N_1}{N_2}\right)^2 Z_L$ (1.25)
Real transformer	$P = V_{rms} I_{rms} \cos\phi$, $Q = V_{rms} I_{rms} \sin\phi$, $S = V_{rms} I_{rms}$
Voltage regulation of transformer	$\text{percent regulation} = \frac{V_{no-load} - V_{load}}{V_{load}} \times 100\%$ (1.26)
Efficiency of transformer	$\text{power efficiency} = \frac{P_{out}}{P_{in}} \times 100\%$ (1.27)
Chapter 2: Power Semiconductor Diodes and Rectifiers	
Half-wave rectifier average voltage	$V_{o,avg} = \frac{V_m}{\pi}$ (2.1)
Output filter capacitor for half-wave rectifier	$C = \frac{I_L T}{V_{pk-pk}} = \frac{I_L}{V_{pk-pk} \times f}$ (2.6)
Full-wave bridge rectifier average voltage	$V_{o,avg} = \frac{1}{\pi}\left[\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t)\right] = \frac{2V_m}{\pi}$ (2.7)
Output filter capacitor for full-wave bridge rectifier	$C = \frac{I_L \times T}{2 \times V_{pk-pk}} = \frac{I_L}{2 \times f_s \times V_{pk-pk}}$ (2.8)
Chapter 3: Introduction to Electromechanical Energy Converters	
Torque in an electrical machine	$T = k(\mathbf{B_s} \times \mathbf{B_r}) = k\lvert B_s\rvert\lvert B_r\rvert \sin(\gamma)$ (3.2)
Linear DC machine	$\mathrm{F_{ind}} = i(\mathrm{l} \times \mathrm{B})$ (3.3) $e_{ind} = (\mathrm{u} \times \mathrm{B}) \bullet \mathrm{l}$ (3.4) $\textbf{KVL}: V_T - i_A R_A - e_{ind} = 0$ (3.5) $\textbf{Netwon's Law}: \mathrm{F_{net}} = m \times a$ (3.6)
Steady-state linear velocity	$V_T = e_{ind} = Bl\mathrm{u_{ss}} \Rightarrow \mathrm{u_{ss}} = \frac{V_T}{Bl}$ (3.11)
Converted power in linear DC machine	$P_{conv} = e_{ind} \times i_A = F_{ind} \times u$ (3.13)
Field-circuit voltage in rotating DC machine	$V_F = R_F I_F$ (3.16)
Back-emf in DC machine	$E_A = K_A \phi \omega_m$ (3.17)
Motor torque in DC machine	$T_{dev} = K_T \phi I_A$ (3.18)
Converted power in rotating DC machine	$P_{dev} = T_{dev}\omega_m = E_A \times I_A$ (3.19)
Armature and field circuit voltages under steady-state condition	$V_T \text{ or } V_A = E_A + I_A R_A$ armature circuit (3.21) $V_F = I_F R_F$ field circuit(3.22)

Armature and field circuit voltages under dynamic condition	$v_T(\mathrm{t})$ or $v_A(t) = e_A(t) + i_A(t)R_A + L_a \dfrac{di_A}{dt}$ (armature circuit) (3.23) $v_F = i_F(t)R_F + L_f \dfrac{di_F}{dt}$ (field circuit) (3.24)
Equation of motion	$T_{dev}(t) = k_T\phi(t)i_A(t) = T_L + b(t)\omega_m + J\dfrac{d\omega_m}{dt}$ (3.25)
Generator voltage relationship with speed	$\dfrac{E_{A1}}{E_{A2}} = \dfrac{n_1}{n_2} = \dfrac{\omega_1}{\omega_2}$ (3.26)
Motor torque-speed characteristic	$\omega_m = \dfrac{V_T}{K_A\phi} - \dfrac{R_A T_{dev}}{(K_T\phi)(K_A\phi)} = \dfrac{V_T}{K\ \phi} - \dfrac{R_A}{(K\ \phi)^2}T_{dev}$ (3.29)
Chopper output voltage	$V_o = \dfrac{1}{T}\int_0^T v_o\ (t)\ dt = V_s \times \dfrac{T_{on}}{T} = V_s \times D$ (3.33)