Contents

DC motor

In the autonomous vehicle project to move around following a track you need electric motors to propel the vehicle. In this part of the lecture notes, you would be introduced how to make use of DC motors to provide mechanical energy and also pulse-width-modulation (PWM) control the speed of the motor.

Linear DC motor

The basic principles of DC machines are explained by considering an idealized linear DC machine. Assuming there is a uniform magnetic field, *B* directed into the page of the paper. The sliding bar is the conductor that is placed on the two rails and moves from left of the page to the right.

- When a current flows in the conductor inside a magnetic field, it will experience a force.
- When the conductor moves inside a magnetic field, an emf will be induced in it

The equations governing these two are given below:

$$F_{ind} = i(l \times B)$$
$$e_{ind} = (u \times B) \bullet l$$

where *F* – force induced on the conductor in newton, *i* – length of the conductor in meter, *B* – flux-density of the magnetic field in Wb/m^2, *e* – induced emf on the conductor in V, *u* – velocity of the moving conductor in m/s.

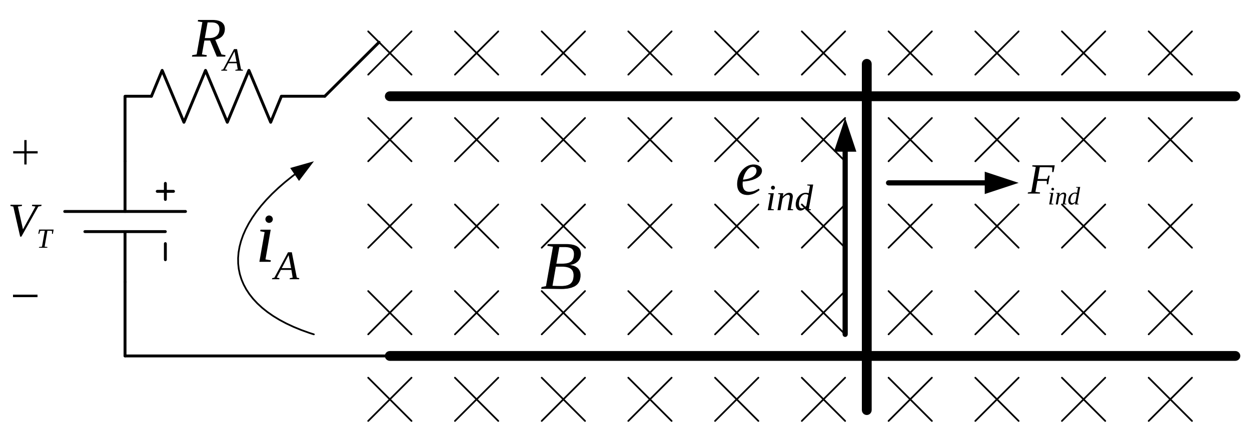
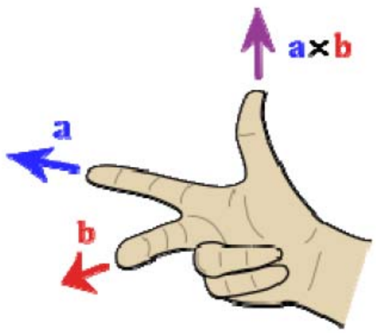
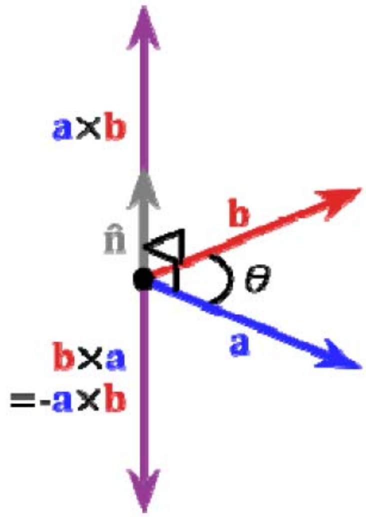


Figure : A simple linear dc machine consisting of a conducting bar sliding on conducting rails.

The direction of the induced force is given by the right-hand-rule (RHR). The *thumb* gives the direction of force, the *index-finger* gives the direction of the current in the conductor and the *middle-finger* gives the direction of the magnetic field, **B**.





If the magnetic field and the direction of current are perpendicular to each other, the formula for force induced is : *F=ilB*

Similarly, if the velocity of the conductor is at perpendicular to the magnetic field, then induced emf formula will be: $e_{ind}=Blu$

From KVL: $V_T = i_A R_A + e_{ind} = i_A R_A + Blu$

Newtons' law: $F_{net} = ma \Rightarrow a = \frac{F_{net}}{m} \Rightarrow a = \frac{Bli_A}{m}$

V_T – supply voltage in V, i_A – current through the conductor in A, R_A – resistance of the conductor, $\mathbf{F}_{net}$ – net force acting on the conductor in N, m – mass of the conductor in Kg and a – acceleration in m/s^2.

According to Newton's law of motion, the bar would now accelerate to the right. As the bar velocity begins to increase, it would induce a voltage e_{ind} and the induced emf will have a polarity that so that it will oppose the main source that produces the current i.e. V_T. Thus, it follows Lenz's law.

Initially, the bar is at rest and the circuit is opened. When the switch is closed, the induced voltage at that instant would be 0.

$$i_A = \frac{V_T - e_{ind}}{R_A} = \frac{V_T}{R_A}$$

The current in the bar would cause force $F_{ind} = \frac{V_T}{R_A} lB$ to the right of the bar. The bar would accelerate.

As the velocity of the bar increases, the induced voltage will increase, which would decrease the current through the bar and thereby reduce the force induced. The steady-state velocity would be reached when the induced force equal the opposing force.

However, if there is no opposing force, then steady steady-state speed would be reached when the current through the conductor becomes zero again i.e.

$$V_T = e_{ind} \Rightarrow V_T = Blu_{ss}$$
$$u_{ss} = \frac{V_T}{Bl}$$

is the steady-state velocity under no load.

Example:

Suppose that for a linear machine, we have B = 1 Wb/m[2], l = 0.3 m, V_T = 2 V, and R = 0.05 ohm.

a) Assuming that the bar is stationary at t = 0, compute the initial current and initial force on the bar. Also determine the final steady-state speed assuming that no mechanical load is applied on the bar.

b) Now, suppose that a mechanical load of 4 N directed to the left is applied to the moving bar. In steady-state, determine the speed at which the bar is moving, the power delivered by the electrical source, the power delivered to the mechanical load, the power lost as heat in the resistance, R_A, and the efficiency of the electromechanical system.

Solution:

a)

As the bar is at rest initially, $e_{ind} = 0$.

Initial current $i(0+) = \frac{V_T - e_{ind}}{R_A} = \frac{2-0}{0.05} = 40A$

Initial force = $F = ilB = 40 \times 0.3 \times 1 = 12N$

At steady-state speed $i = 0$. $V_T = e_{ind} \Rightarrow V_T = Blu_{ss} \Rightarrow u_{ss} = \frac{V_T}{Bl} = \frac{2}{1 \times 0.3} = 6.67 m/s$

b)

When 4N force is applied to the left, the bar will begin to accelerate.

At steady-state speed, the force induced will be same as the applied force.

$$F_{ind} = IlB = 4 \Rightarrow I = \frac{4}{0.3 \times 1} = 13.33A$$

$$I = \frac{V_T - e_{ind}}{R_A} \Rightarrow e_{ind} = 2 - 13.33 \times 0.05 = 1.333V$$

$$u_{ss} = \frac{e_{ind}}{Bl} = \frac{1.33}{1 \times 0.3} = 4.43m/s$$

Mechanical power delivered $P_{out} = Fu = 4 \times 4.43 = 17.72W$

Electrical power input to the motor $P_{in} = V_T \times I = 2 \times 13.33 = 26.66W$

Power dissipated in the resistance $P_R = I^2 R = (13.33)^2 \times 0.05 = 8.89W$

Efficiency of the linear DC motor $\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{17.72}{26.66} \times 100 = 66.47\%$

Rotating DC Motors

The rotary motor consists of the stator and the rotor. A rotating DC motor can be constructed by rolling up the linear DC motor. Field windings or permanent magnet poles are embedded in the stator which produce the uniform magnetic field. The conductor bars are embedded in the rotor as shown in the figure and are connected together to form the armature winding. The armature conductors are brought out to the commutator segments. Current is applied to the armature winding from external supply through the carbon brushes which are pressed against the commutator segments as shown in Figure.
The cross sectional view of a 2-pole machine shows the flux-lines in the air-gap.

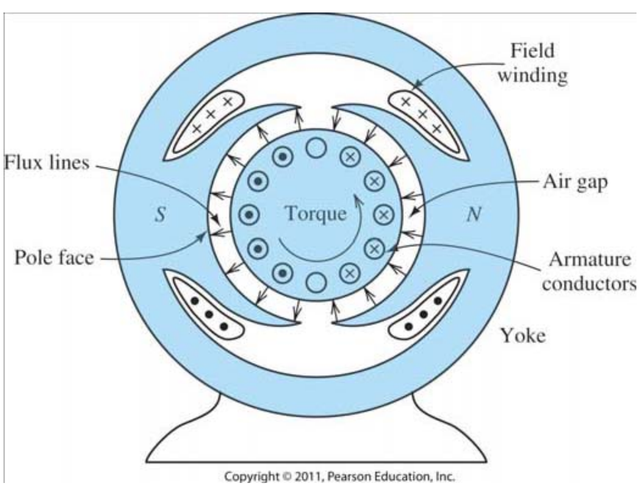


Figure: Cross sectional view of a 2-pole DC machine.

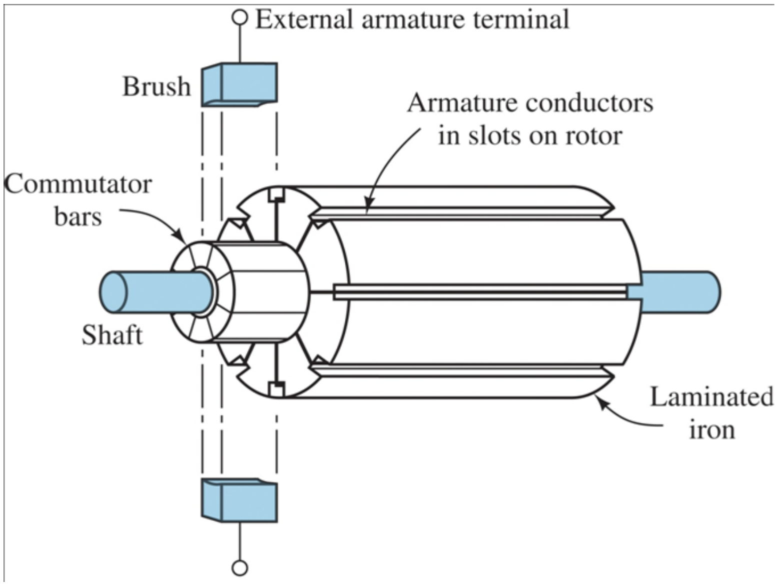


Figure : Side view of the rotor in DC Motor

The rotor consists of laminated cylindrical iron mounted on the shaft that is supported by bearings such that it can rotate.

Magnetic flux lines take the path of least reluctance in the air-gap and therefore they are perpendicular to the surface of the rotor iron. Moreover, the flux-density is more or less constant magnitude over the surface of each pole face.

With the current directions as shown in the field windings, the flux lines leave from the right-hand side of the stator under the north-pole cross the air-gap and enter the rotor and then cross the air-gap again and reach the south-pole on the left-hand side and thereby complete the magnetic circuit.

Induced EMF and Commutation

As rotor rotates, the conductor bars will be cutting the magnetic flux and emf will be induced in them. The single-turn coil consists of conductor segments, *d-c*, *c-b* and *b-a.* Under the pole face the conductor, the magnetic field and the direction of motion are mutually perpendicular to each other and an emf is produced in the conductors according to the equation: $e=Blu$.

Consider the position of the coil as shown in Fig. 3.11 (a). The conductor *a-b* is under the north-pole and the conductor *c-d* under the south-pole. The emf induced in the conductors c-d and a-b will have such polarity that voltage at the terminals would be sum of them. The induced emf remains constant under the pole face as the conductor moves along in counterclockwise rotation. However, as the conductor moves out of the magnetic pole face the flux-lines cutting the conductor reduces and thereby the emf induced reduces to zero where the turn is at right-angle to the original position. In this position, no flux-lines cuts the conductors *a-b* and *c-d* and therefore the voltage induced is zero. As the conductor *a-b* moves under the opposite south-pole, a negative voltage is induced in it. During that time, the conductor *c-d* will be under the north-pole and positive voltage will be induced. However, a mechanical switch known as *commutator* reverses the connections to the conductor as they move between the poles so the polarity of the induced emf as seen from the external terminal remains the same i.e. we get a rectified dc voltage as shown below.

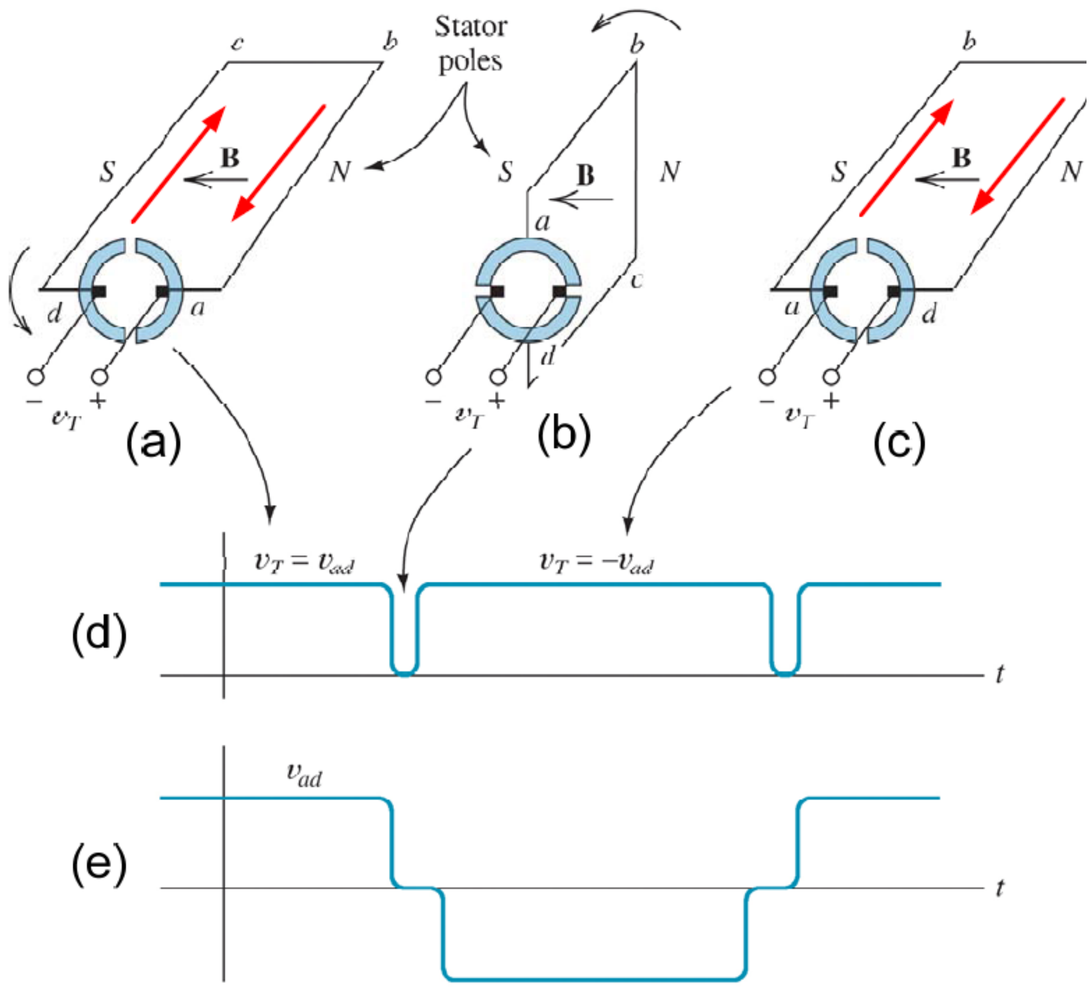
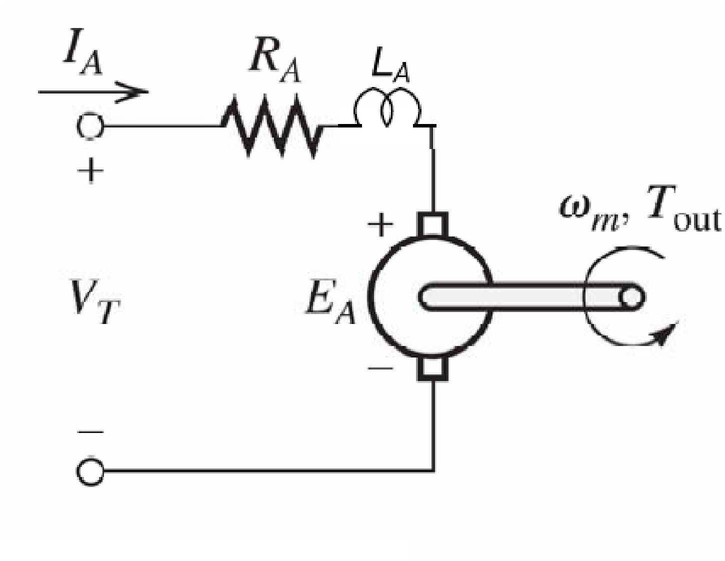


Figure 3.11: Commutation for a single armature winding.

DC Motor equivalent circuit



The armature winding is represented by the resistance R_A (including the brush resistance) and inductance L_A. For steady-state operation, E_A represents the voltage induced in the armature conductors due to the rotation of the rotor in the presence of a magnetic field. It is known as the *back-emf* as it opposes the external armature voltage source, V_T *or* V_A. Note that Lez's law states that the voltage induced has a polarity that opposes the source.

The induced emf on the armature circuit is given by: $E_A = K_A \phi \omega_m$

K_A – Motor back-emf constant, ϕ -Magnetic flux, ω_m – Rotor angular velocity.

The torque developed in the machine is given by: $T_{dev} = K_T \phi I_A$

K_T – Motor torque constant, ϕ -Magnetic flux, I_A – Rotor angular velocity.

The mechanical power at the rotor shaft : $P_{mech} = T_{dev} \omega_m$

The electric power that is converted to mechanical power will be equal to the power input to the motor minus the copper loss in the winding : $P_{elec} = V_T I_A - I_A^2 R_A = (V_T - I_A R_A) I_A = E_A I_A$.

Equating $P_{elec} = P_{mech} \Rightarrow E_A I_A = T_{dev} \omega_m \Rightarrow K_A \phi \omega_m I_A = K_T \phi I_A \omega_m \Rightarrow K_A = K_T$ i.e the motor back-emf constant equals the motor torque constant. We can have the common term $K_A \phi = K_T \phi = K$.

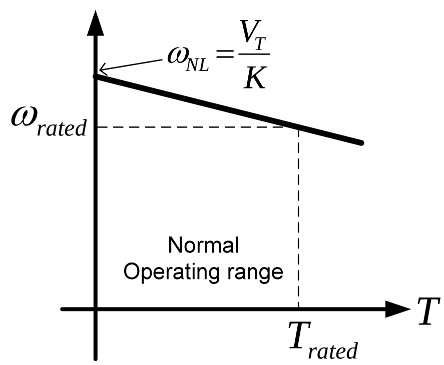
For PMDC motors, the magnetic flux is produced by permanent magnets and is constant. Then back-emf in the PMDC motor $E_A = K\omega_m$, torque developed in the PMDC motor $T_{dev} = KI_A$.

Torque speed characteristic of PM DC motor

$$V_T = E_A + I_A R_A = K\omega_m + I_A R_A \Rightarrow \omega_m = \frac{V_T}{K} - \frac{R_A}{K} I_A$$

$$T_{dev} = KI_A \Rightarrow I_A = \frac{T_{dev}}{K}$$

$$\omega_m = \frac{V_T}{K} - \frac{R_A}{K}\frac{T_{dev}}{K}$$



For fixed armature voltage, the no-load speed will be: $\omega_{NL} = \frac{V_T}{K}$

When the load is increased, the motor speed will drop along a straight line.

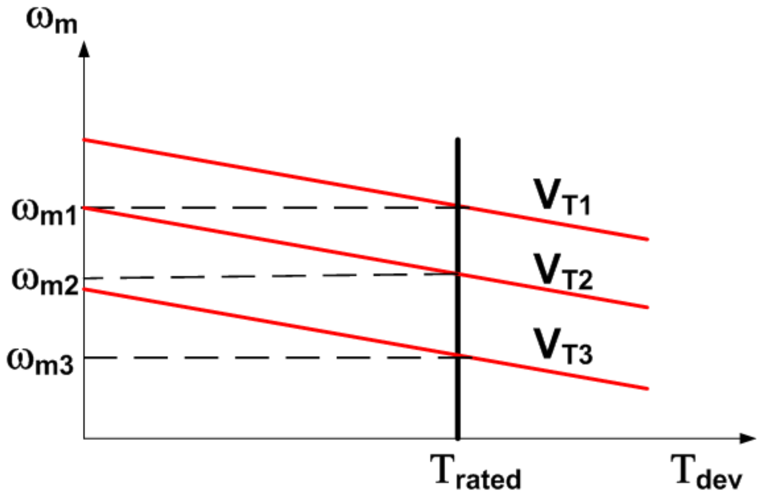
Speed Control of PMDC motor

For many applications it is necessary to control the speed of the DC motor e.g. in the autonomous vehicle project, you need to control the speed of the vehicle so that sometimes it can move faster and some other time it can move slower.

From the equation for speed-torque, it can be seen that motor speed can be varied in two ways:

1) by varying the armature voltage V_T

2) by inserting a resistor in series with the armature to increase the effective resistance R_A



The armature voltage control is the most desirable form of speed control of DC motor and therefore we would be using it.

Variable DC voltage source

A dc voltage source can be obtained by using either a battery or if it is AC supply then using a diode bridge rectifier followed by a smoothing filter capacitor. Once, a constant dc voltage source is obtained, an electronic switching circuit can be used to control the average dc voltage delivered to the load.

The switch periodically opens and closes with a time period T , closed for a period of T_{on} and opened for the rest of the period T_{off} (= $T - T_{on}$). This process is known as *pulse-width-modulation* (PWM) in power electronics field.

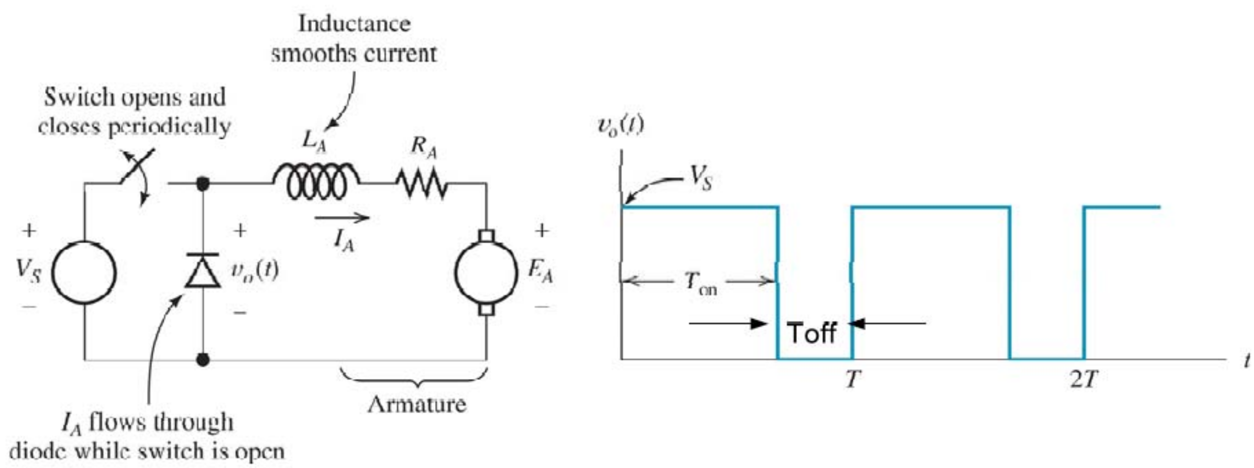


Figure showing use of a switch and a diode to obtain a variable voltage output

The average value of the output voltage is:

$$v_{o,avg} = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{T}V_S T_{on} = V_S \frac{T_{on}}{T} = V_S D$$

where D – duty cycle of the electronic switch.

The presence of the free-wheeling diode across the armature circuit is very essential as when the switch is opened and the armature circuit current does not find a path to flow through (as the switch is opened during the turn-off period), and drops to zero abruptly then it induces a very large voltage across the armature circuit.

Consider that the armature circuit inductance id 1 mH, a current of 10 A is flowing through the circuit and the current is interrupted in 1 μs (semiconductor switch can be opened very fast) then we have

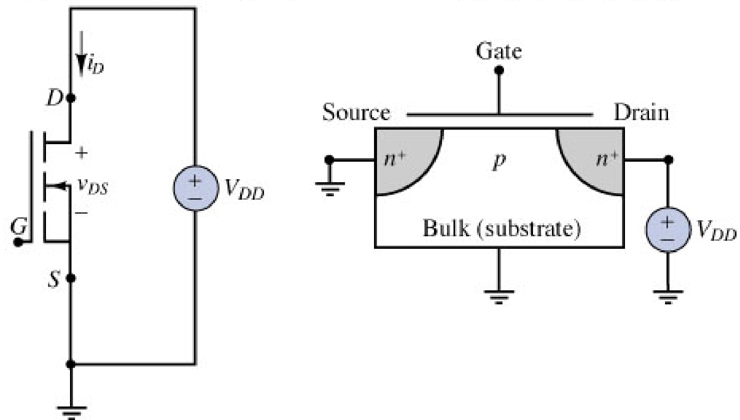
$$V_{L_A} = L_A \frac{di_A}{dt} = 10^{-3} \times \frac{10}{10^{-6}} = 10kV$$

Such a large voltage would appear across the semiconductor switch and blow it up. The freewheeling diode is provided so that current can flow through the diode and therefore it is not interrupted abruptly.

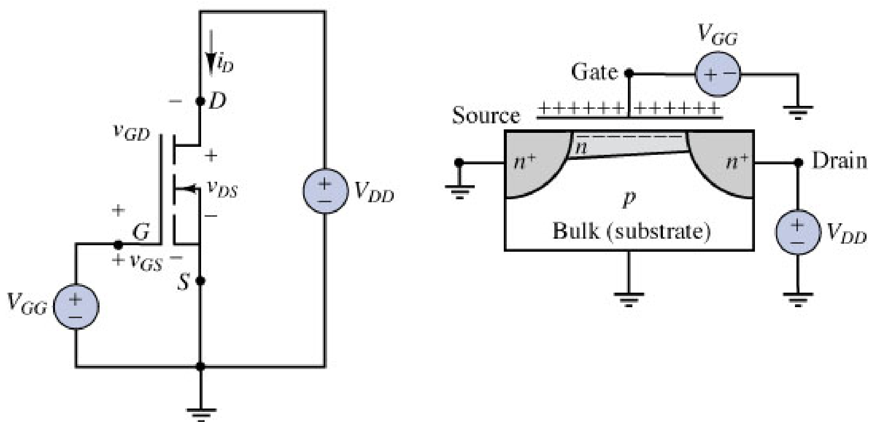
When the switch is closed, the supply voltage reverse biases the freewheeling diode but when the switch is opened it becomes forward biased and conducts.

MOSFET

MOSFET stands for **M**etal-**O**xide-**S**emiconductor **F**ield **E**ffect **T**ransistor. The basis of operation of FET is that an **external field is used to vary the conductivity of a channel**, which makes the FET behave either as a voltage-controlled resistor or a voltage-controlled current source.



(a)



(b)

Figure. Channel formation in NMOS transistor. (a) With no external gate voltage, no channel is formed and hence no conduction occurs (b) when gate voltage is applied, electrons are drawn between the source and drain regions to form the conducting channel

The device has three terminals: the **gate**, the **drain** and the **source**. The bulk (substrate) is electrically connected to the gate and hence does not come out as a separate terminal. The drain and source terminals are the main current carrying terminals whereas the gate is the control terminal.

The gate consists of a metal film layer, separated from the bulk by a thin oxide layer (non-conducting).

When the gate to source voltage is more than a voltage called **threshold voltage** V_T , then the channel is formed and the MOSFET is ready to conduct.

If $V_{GS} < V_T$ is then the channel is off and the MOSFET is said to be in **cut-off region**.

When $V_{GS} > V_T$ and V_{DS} is small, then the MOSFET is in **linear region**.

If $V_{GS} > V_T$ and V_{DS} is large, then the drain current is constant, irrespective of the drain-to-source voltage. This region of operation is called the **saturation region**.

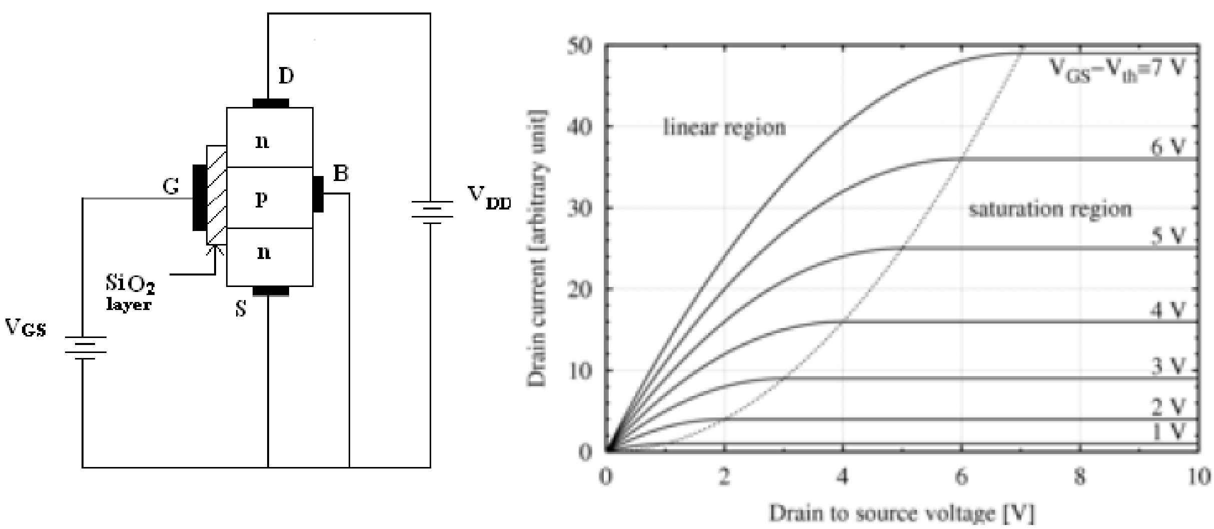


Figure: MOSFET I-V characteristics

MOSFETs can be used to build amplifier circuits. These can also be used as switches. In this module, we shall study the use of MOSFET as switches.

MOSFET as a switch

An ideal switch is either ON or OFF. When the switch is ON, the voltage across it will be zero irrespective of the current through it. Similarly when the switch is OFF, the current through it will be zero, irrespective of the voltage across it.

As can be seen from the $I-V$ characteristics of the MOSFET, if the V_{GS} is large, then it operates in the linear region. In the linear region, the voltage V_{DS} is small for a large range of values for drain current. It is similar to the ideal switch being ON.

Again when V_{GS} is zero, the MOSFET is in cutoff region. There, it is equivalent to the switch being OFF as drain current is very small irrespective of the voltage across it.

Thus, MOSFET can be operated as a switch by applying appropriate voltage to the gate terminal.
MOSFET is ON when $V_{GS} = 5V$ and MOSFET is OFF when $V_{GS} = 0V$.

The circuit for speed control of dc motor is given. As can be seen, when the PWM signal is at 5V, the MOSFET is ON (DS shorted) and DC motor sees 5V across it. When PWM signal is at 0V, the MOSFET is OFF (DS opened), DC motor current flows through the free-wheeling diode and hence the motor voltage is 0V. The average DC motor voltage will be proportional to the duty cycle of the PWM input.

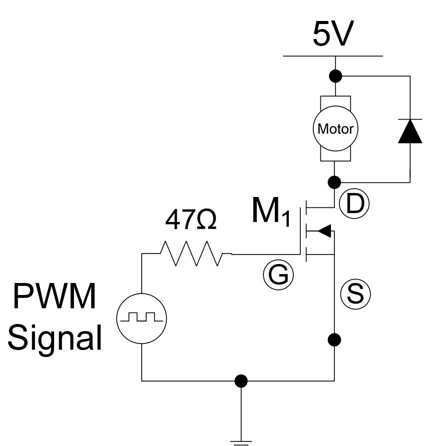


Figure: MOSFET used as a switch to drive a variable-speed DC Motor