

**National University of Singapore**  
**Department of Electrical & Computer Engineering**  
**EE-1002: Introduction to Circuits and Systems**  
**Tutorial - 6 (Power for AC Circuits)**  
**Year 2013-14**

Q.1 The supply voltage phasor is given by

$$\mathbf{V} = 1000\sqrt{2}\angle 0^\circ$$

The phasor for the phase current through the resistive branch is given by

$$I_R = \frac{\mathbf{V}}{\mathbf{R}} = \frac{1000\sqrt{2}\angle 0^\circ}{100\angle 0^\circ} = 10\sqrt{2}\angle 0^\circ$$

The phasor for the phase current through the resistive branch is given by

$$I_L = \frac{\mathbf{V}}{X_L} = \frac{1000\sqrt{2}\angle 0^\circ}{377 \times 0.5\angle 90^\circ} = 5.3\sqrt{2}\angle -90^\circ$$

Thus, the total current phasor is given by

$$\mathbf{I} = I_R + I_L = 10\sqrt{2}\angle 0^\circ + 5.3\sqrt{2}\angle -90^\circ = 11.3\sqrt{2}\angle -27.92^\circ$$

The active power is given by

$$P = V_{rms}I_{rms}\cos(\theta) = 1000 \times 11.3\cos(27.92) = 10 \text{ kW}$$

The reactive power is given by

$$Q = V_{rms}I_{rms}\sin(\theta) = 1000 \times 11.3\sin(27.92) = 5.29 \text{ kVAR}$$

The apparent power is given by

$$S = V_{rms}I_{rms} = 1000 \times 11.3 = 11.3 \text{ kVA}$$

The power factor is given by

$$\cos\theta = \cos(27.92) = 0.88 \text{ lagging}$$

Q.2 The phasor for the current is given by

$$\mathbf{I} = \frac{\mathbf{V}_A - \mathbf{V}_B}{\mathbf{Z}} = \frac{200\sqrt{2}\angle 50^\circ - 220\sqrt{2}\angle 30^\circ}{5 + j12} = 7.1\sqrt{2}\angle 37.3^\circ$$

The active power  $P_A$  supplied by source-A is given by

$$P_A = V_{rms}I_{rms}\cos(\theta) = 200 \times 7.1\cos(50 - 37.3) = 1.798 \text{ kW}$$

The reactive power  $Q_A$  supplied by source-A is given by

$$Q_A = V_{rms}I_{rms}\sin(\theta) = 200 \times 7.1\sin(50 - 37.3) = 0.405 \text{ kVAR}$$

The active power  $P_B$  consumed by source-B is given by

$$P_B = V_{rms}I_{rms}\cos(\theta) = 200 \times 7.1\cos(30 - 37.3) = 1.547 \text{ kW}$$

The reactive power  $Q_B$  consumed by source-B is given by

$$Q_B = V_{rms}I_{rms}\sin(\theta) = 200 \times 7.1\sin(30 - 37.3) = -0.198 \text{ kVAR}$$

The active power  $P_R$  consumed by the resistor,  $R$  is given by

$$P_R = I_{rms}^2 \text{ times } R = 7.1^2 \times 5 = 0.251 \text{ kW}$$

The reactive power  $Q_{X_L}$  consumed by the resistor,  $X_L$  is given by

$$Q_{X_L} = I_{rms}^2 \text{ times } X_L = 7.1^2 \times 12 = 0.604 \text{ kVAR}$$

Q.3 The active power  $P_A$  consumed by load-A is given by

$$P_A = 10.0 \text{ kW}$$

The reactive power  $Q_A$  consumed by load-A is given by

$$Q_B = \frac{P_A}{0.9} \times \sin(\cos^{-1}(0.9)) = \frac{10\text{kW}}{0.9} \times \sin(\cos^{-1}(0.9)) = 4.84 \text{ kVAR}$$

The active power  $P_B$  consumed by load-B is given by

$$P_B = S_B \times \cos(\theta_B) = 15\text{kVA} \times 0.8 = 12 \text{ kW}$$

The reactive power  $Q_B$  consumed by load-B is given by

$$Q_B = S_B \times \sin(\cos^{-1}(0.8)) = 15\text{kVA} \times (0.6) = 9 \text{ kVAR}$$

The total active power,  $P_S$  supplied by the source is

$$P_S = P_A + P_B = 10 \text{ kW} + 12 \text{ kW} = 22 \text{ kW}$$

The total reactive power,  $Q_S$  supplied by the source is

$$Q_S = Q_A + Q_B = 4.84 \text{ kVAR} + 9 \text{ kVAR} = 13.84 \text{ kVAR}$$

The total apparent power supplied by the source, is given by

$$S_S = \sqrt{P_S^2 + Q_S^2} = \sqrt{22 \text{ kW}^2 + 13.84 \text{ kVAR}^2} = 26 \text{ kVA}$$

The power factor of the over all load is given by

$$\cos\theta = \tan^{-1}\left(\frac{Q_S}{P_S}\right) = \tan^{-1}\left(\frac{13.84 \text{ kVAR}}{22 \text{ kW}}\right) = 0.846 \text{ lagging}$$

Q.4 The load power is 100 kW and the phase angle is given by

$$\theta = \cos^{-1}(0.25) = 75.5^\circ$$

The rms value of the load current is given by

$$I_{rms} = \frac{P_{load}}{V_{rms} \times \cos(\theta)} = \frac{100 \text{ kW}}{1000 \text{ V} \times 0.25} = 400 \text{ A}$$

Thus, the current phasor is given

$$\mathbf{I} = 400\sqrt{2}\angle -75.5^\circ$$

The reactive power consumed by the load is given by

$$Q = P \times \sin(\theta) = 100 \text{ kW} \times \sin(75.5) = 387.3 \text{ kVAR}$$

Thus, the value of the reactive power supplied by the capacitor should be 387.3 kVAR in order to make the overall load power factor unity.

Thus, the value of the capacitance required to make the reactive power supplied by the capacitor to be 387.3 kVAR is

$$X_C = \frac{V_{rms}^2}{Q_C} = \frac{1000^2}{387.3 \text{ kVAR}} = 2.582 \Omega \Rightarrow C = 1.02 \text{ mF}$$

Thus, the new current phasor is given

$$\mathbf{I} = \frac{P_{Load}}{V_{rms}} = \frac{100 \text{ kW}}{1000 \text{ V}} = 100\angle 0^\circ$$

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