

National University of Singapore
Department of Electrical & Computer Engineering
EE1002: Circuits and Systems
Tutorial - 8 Solution (Transformers and Rectifiers)
Year 2013-14

Q.1 The parameters given are:

$$N_{HV} = N_1 = 800, N_{LV} = N_2 = 100, V_1 = 240 \text{ V}, Z_L = 3 \Omega$$

(a) $V_2 = \left(\frac{N_2}{N_1}\right) \times V_1 = \left(\frac{100}{800}\right) \times 240 = 30 \text{ V}, I_2 = \frac{V_2}{Z_L} = \frac{30 \text{ V}}{3 \Omega} = 10 \text{ A}$

(b) $I_1 = \left(\frac{N_2}{N_1}\right) \times I_2 = \left(\frac{100}{800}\right) \times 10 \text{ A} = 1.25 \text{ A}$

(c) $Z_1 = \frac{V_1}{I_1} = \frac{240 \text{ V}}{1.25 \text{ A}} = 192 \Omega$

(d) $Z_{L'} = \left(\frac{N_1}{N_2}\right)^2 \times Z_L = \left(\frac{800}{100}\right)^2 \times 3 = 192 \Omega$

Q.2 The parameters given are:

$$V_1 = 100 \angle 0^\circ \text{ V}, \left(\frac{N_2}{N_1}\right) = 1/2, \left(\frac{N_3}{N_1}\right) = 1, Z_2 = 5 \Omega, Z_3 = 10 \Omega$$

(a) The dots are placed at top ends of coils 2 and 3 so that they would be opposing the MMF produced by coil 1.

(b) $V_2 = \left(\frac{N_2}{N_1}\right) \times V_1 = \left(\frac{1}{2}\right) \times 100 \angle 0^\circ \text{ V} = 50 \angle 0^\circ \text{ V}, V_3 = \left(\frac{N_3}{N_1}\right) \times V_1 = \left(\frac{1}{1}\right) \times 100 \angle 0^\circ \text{ V} = 100 \angle 0^\circ \text{ V}, I_2 = \frac{V_2}{Z_2} = \frac{50 \angle 0^\circ \text{ V}}{5 \Omega} = 10 \angle 0^\circ \text{ A}, I_3 = \frac{V_3}{Z_3} = \frac{100 \angle 0^\circ \text{ V}}{10 \Omega} = 10 \angle 0^\circ \text{ A},$

(c) $N_1 \times I_1 = N_2 \times I_2 + N_3 \times I_3 \Rightarrow I_1 = \frac{N_2 \times I_2 + N_3 \times I_3}{N_1} = \frac{\frac{1}{2} \times N_1 \times I_2 + N_1 \times I_3}{N_1} = \frac{1}{2} \times 10 \angle 0^\circ + 10 \angle 0^\circ = 15 \angle 0^\circ$

Q.3 The induced voltages are given by:

$$v_1 = N_1 \frac{d\phi}{dt} \text{ and } v_2 = N_2 \frac{d\phi}{dt} \Rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2} \Rightarrow v_2 = \frac{N_2}{N_1} v_1 = \frac{100}{1000} 120 \angle 0^\circ = 12 \angle 0^\circ \text{ V}$$

From the circuit we have

$$I_1 = I_2 + I_3$$

As the net MMF required to set-up the flux is zero we have

$$(N_1 - N_2)I_1 + N_2 I_3 = (N_1 - N_2)I_1 + N_2(I_1 - I_2) = 0 \Rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

We have

$$I_2 = \frac{V_2}{Z_2} = \frac{12 \angle 0^\circ \text{ V}}{6 \angle 0^\circ} = 2 \angle 0^\circ \text{ A} \Rightarrow I_1 = \frac{N_2}{N_1} I_2 = \frac{100}{1000} 2 \angle 0^\circ \text{ A} = 0.2 \angle 0^\circ \text{ A}$$

$$I_3 = I_1 - I_2 = 0.2 \angle 0^\circ \text{ A} - 2.0 \angle 0^\circ \text{ A} = -1.8 \angle 0^\circ \text{ A}$$

Q.4 Assume that the primary side series-resistance is referred to the secondary side and let the combined resistances in the secondary side be R_{eq} and the corresponding output voltage and current on the secondary side be V_2 , I_2 and let the power-factor of the load $\cos \phi$ is maintained constant.

When the output power increases normally, V_2 is maintained constant and it is I_2 increases with increase in P_{out} . Then we have

$$P_{out} = V_2 \times I_2 \times \cos \phi, R_{eq} = a^2 R_1 + R_2, P_{cu-loss} = I_2^2 R_{eq}, P_{const-loss} = \frac{V_1^2}{R_c}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{out}}{P_{out} + P_{cu-loss} + P_{const-loss}} = \frac{V_2 \times I_2 \times \cos \phi}{V_2 \times I_2 \times \cos \phi + I_2^2 R_{eq} + P_{const-loss}}$$

Variable load means that it is I_2 that is changing as a function of load.

$$\frac{d\eta}{dI_2} = \frac{d}{dI_2} \left(\frac{V_2 \times I_2 \times \cos \phi}{V_2 \times I_2 \times \cos \phi + I_2^2 R_{eq} + P_{const-loss}} \right) = 0 \Rightarrow P_{cu-loss} = P_{const-loss}$$

Q.5 The parameters given are:

$$V_{rms} = 230 \text{ V}, f_s = 50 \text{ Hz}, R_L = 200 \Omega$$

The average output voltage is given by

$$V_{o,avg.} = \frac{2V_m}{\pi} = \frac{2 \times 230\sqrt{2}}{\pi} = 207.1 \text{ V}$$

The capacitance needed to reduce the peak-to-peak ripple voltage to 1% of the output voltage is given by

$$C = \frac{I_L}{2f_s V_{pk-to-pk}} = \frac{V_{o,avg}}{2f_s R_L V_{pk-to-pk}} = \frac{1}{2f_s R_L \left(\frac{V_{pk-to-pk}}{V_{o,avg}} \right)} = \frac{1}{2 \times 200 \times 50 \text{ Hz} \times 0.01} = 5000 \mu F$$

— END —