EE1002
Introduction to Circuits and Systems

Part 1 : Lecture 7

Energy Storage Elements:

Capacitors, Inductors

Capacitors and Inductors

- **Energy storage elements**
  - **Capacitance**
  - **Inductance**
- **i,v relationship, series/parallel combination**

- **DC steady-state behavior**

- **DC Transients**

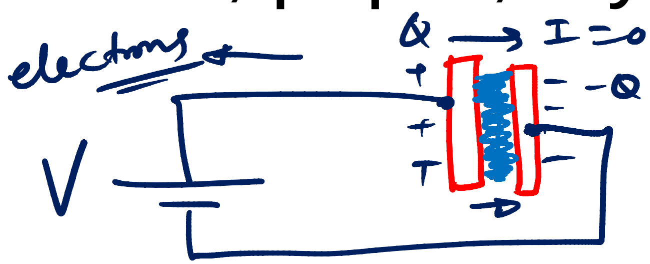
Capacitor and Inductor

Capacitor

Inductor

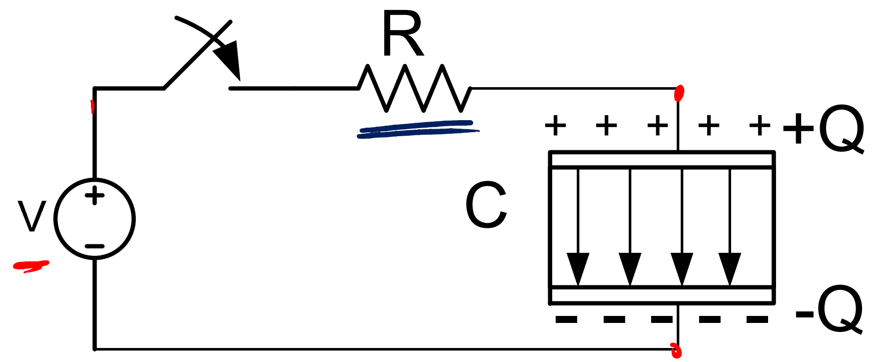
Capacitance

- Capacitors are constructed by separating two sheets of conductors by a thin layer of insulating material like air, paper, Mylar, polyester etc.



- When capacitor is connected to a DC voltage source, opposite charges develop on the two conductors.

Charge stored in Capacitance



$$Q = CV$$

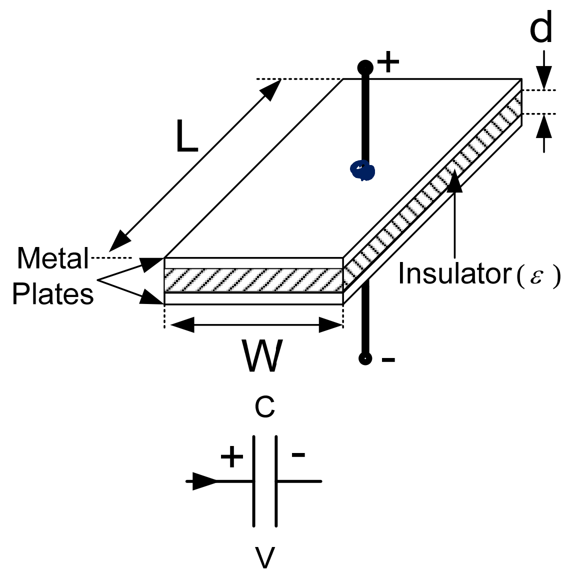
C = capacitance

$i = \frac{dQ}{dt} = \frac{d}{dt}(CV)$

$\boxed{i = C \cdot \frac{dV}{dt}}$

- Unit of capacitance is farad (F).
- One farad is equivalent to coulomb per volt which is a large value.
- Practical capacitors have values only a few pico farad (10^{-12}) to a few micro farad (10^{-6}).

Capacitance of the parallel-plate capacitor



$$C = \frac{\varepsilon \boxed{A}}{\boxed{d}}$$

$\to$ Area of the plate

$$A = WxL$$

$\to$ distance between the plates

$$\varepsilon = \varepsilon_r \varepsilon_0$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\, F/m$$

ε_r – relative permittivity (unitless qty)

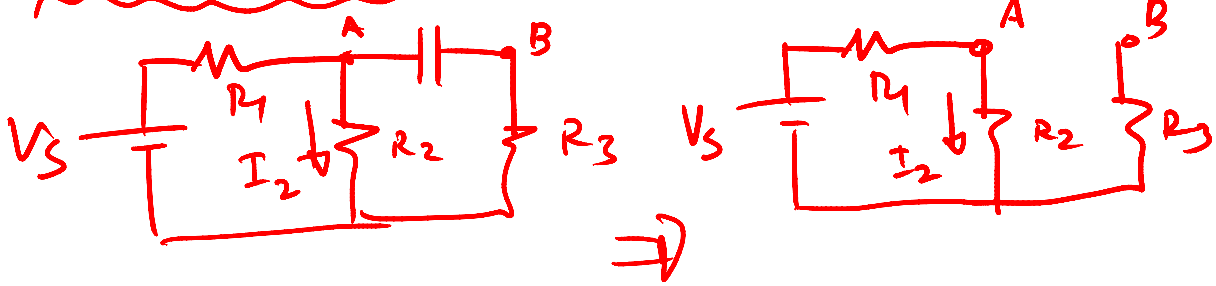
ε_0 – permittivity of free space/air

Table : Relative permittivity of selected materials

Material	Relative permittivity
Diamond	5.5
Mica	7.0
Polyester	3.4
Quartz	4.3
Silicon dioxide	3.9

Capacitor current in terms of voltage

- In capacitor current is present only when charge is building up.
- At steady state, when the voltage is stable, the capacitor current will be zero.
- Hence, capacitors in DC circuits behave as open circuited in steady state.



Capacitor voltage in terms of current and DC Steady-state

$\boxed{q} = Cv$

$$\frac{dq}{dt} = C\frac{dv}{dt} \Rightarrow \boxed{i = C\frac{dv}{dt}}$$

$$\frac{dv(t)}{dt} = \frac{1}{C} i(t)$$

$$v(t) - v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(t)dt$$

initial value

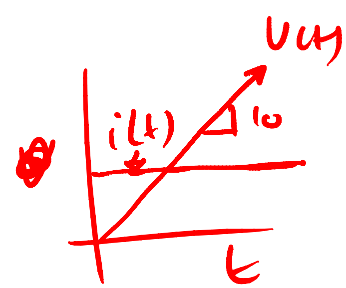
$$\underline{v(t)} = \frac{q_0}{C} + \frac{1}{C}\int_{t_0}^{t} i(t)dt$$

~~DC steady state~~:

$C = 1\,\mu F$

$v(t) = \underline{10.t}$, t time in sec

Find $i(t) = C \cdot \frac{dv}{dt}$



$= 1\times10^{-6} \times 10$

$= 10^{-5}\ A$.

DC steady-state: $\underline{x}$

$\rightarrow \frac{dx}{dt} = 0$.

Energy Stored in Capacitor

power = $\frac{d}{dt}$ (energy).

$$p(t) = v(t)i(t) = v(t)C\frac{dv(t)}{dt} = Cv(t)\frac{dv(t)}{dt}$$

$$e = \int_{t_0}^{t} p(t)dt = \int_{t_0}^{t} Cv(t)\frac{dv}{dt}dt = \int_{v_0}^{v} Cvdv = \frac{1}{2}C\left(v^2 - v_0^2\right)$$

$$e = \frac{1}{2}Cv^2$$

if $v_0 = 0$ initial voltage

$v = V$

energy stored in the capacitor $\boxed{e = \frac{1}{2} \cdot C \cdot V^2}$

eg $C = 10 \times 10^{-6}$

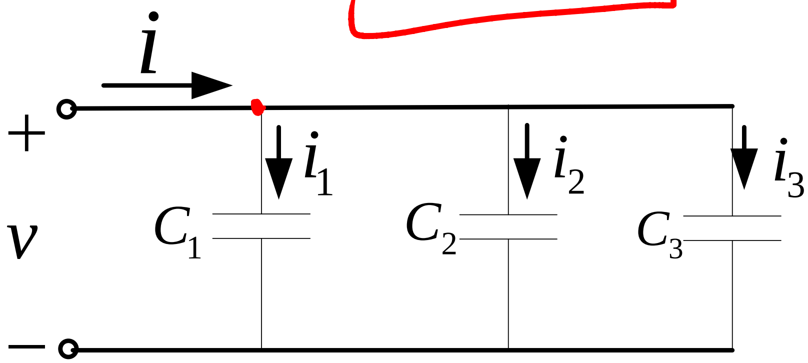
$V = 65v$

$e = \frac{1}{2} \times \cancel{10} 5 \times 10^{-6} \times 65 \times 65 \approx \frac{1}{2} mv^2$

$= 21125$ μJ (Joule).

Capacitances in Parallel

$$\boxed{i = C\frac{dv}{dt}}$$

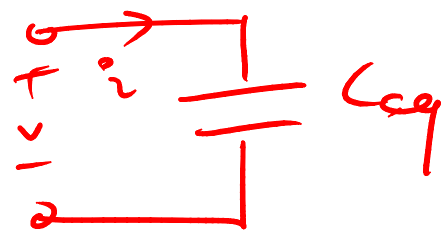


$$C_{eq} = C_1 + C_2 + C_3$$

$i = i_1 + i_2 + i_3$

$v_1 = v_2 = v_3 = v$

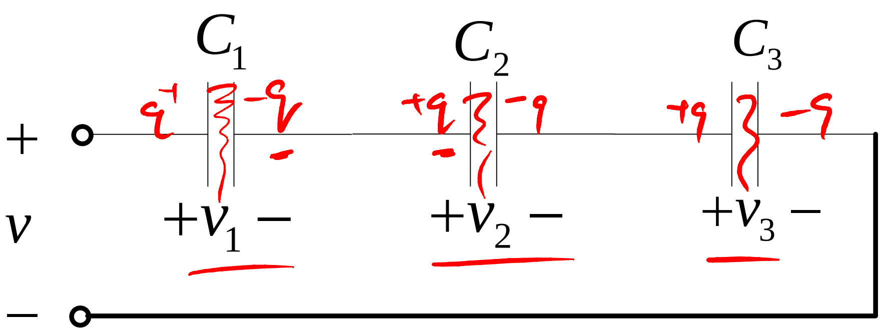
$\equiv$



$$\begin{aligned} i &= i_1 + i_2 + i_3 \\ &= C_1 \cdot \frac{dv_1}{dt} + C_2 \cdot \frac{dv_2}{dt} + C_3 \cdot \frac{dv_3}{dt} \\ &= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} \\ &= (C_1 + C_2 + C_3) \cdot \frac{dv}{dt} \\ &= C_{eq} \cdot dv/dt \end{aligned}$$

Capacitances in series

$$\boxed{Q = C \cdot V}$$



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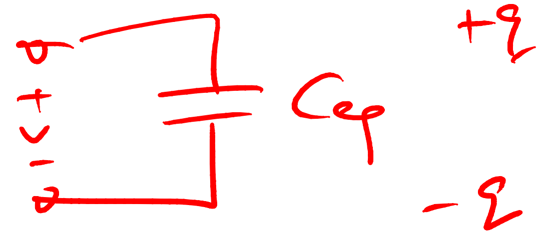


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$V = v_1 + v_2 + v_3$

$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$

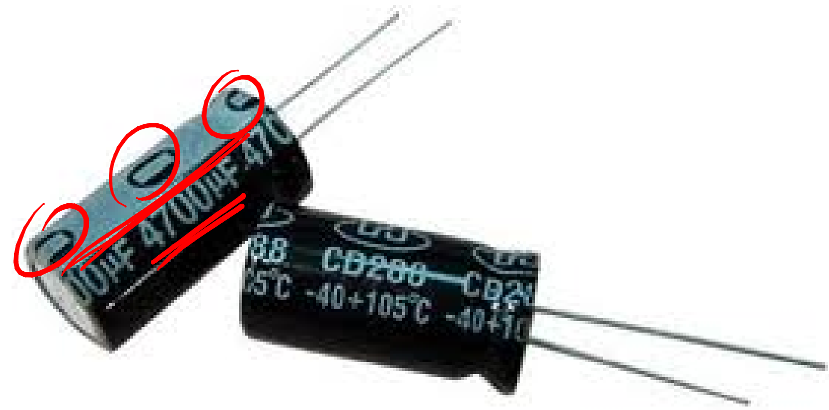
$q = C_{eq} \cdot V$

$V = \frac{q}{C_{eq}}$

Resistors in parallel

– **Practical capacitors**

Electrolytic Capacitors



- In such capacitors, one of the plates is metallic aluminium or tantalum, the dielectric is an oxide layer on the surface of the metal and the other ‘plate’ is an electrolytic solution.

Electrolytic Capacitors

- High capacitance per volume.
- These capacitors have polarity.
- If voltage of the opposite polarity is applied, capacitor may fail at high voltage.
- Commonly used in DC voltage systems as the bulk capacitor to filter out voltage ripples from the DC supply.
- These capacitors cannot be used where voltage polarity reverses.

Ceramic Capacitor value

154: 15×10^4 pF

474: 47×10^4 pF

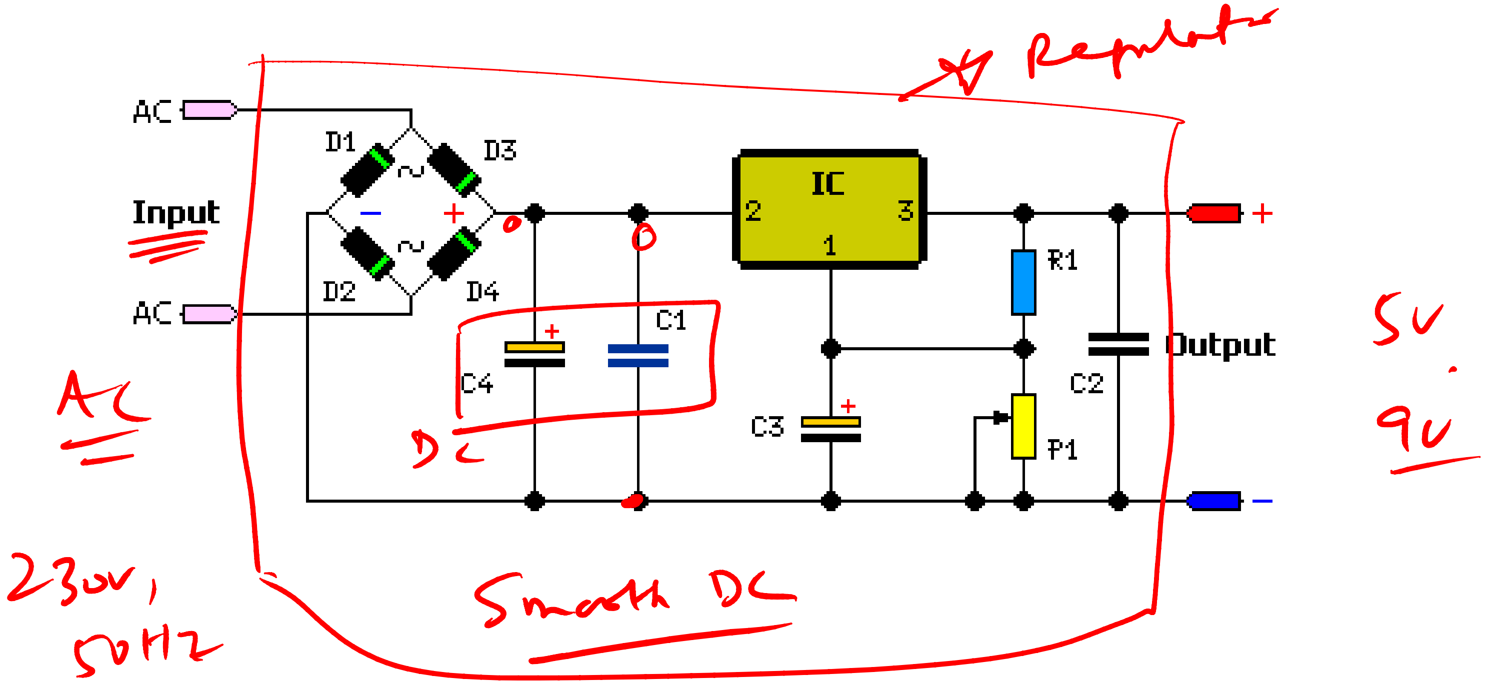
- There is a three digit code printed on a ceramic capacitor specifying its value. The first two digits are the two significant figures and the third digit is a base 10 multiplier. The value is given in picofarads (pF).

Ceramic Capacitor value

- These capacitors do not have polarity
- Ceramic capacitors are suitable for moderately high-frequency work
- Often used as a decoupling capacitor

  (to supply small high frequency current at the point of demand)

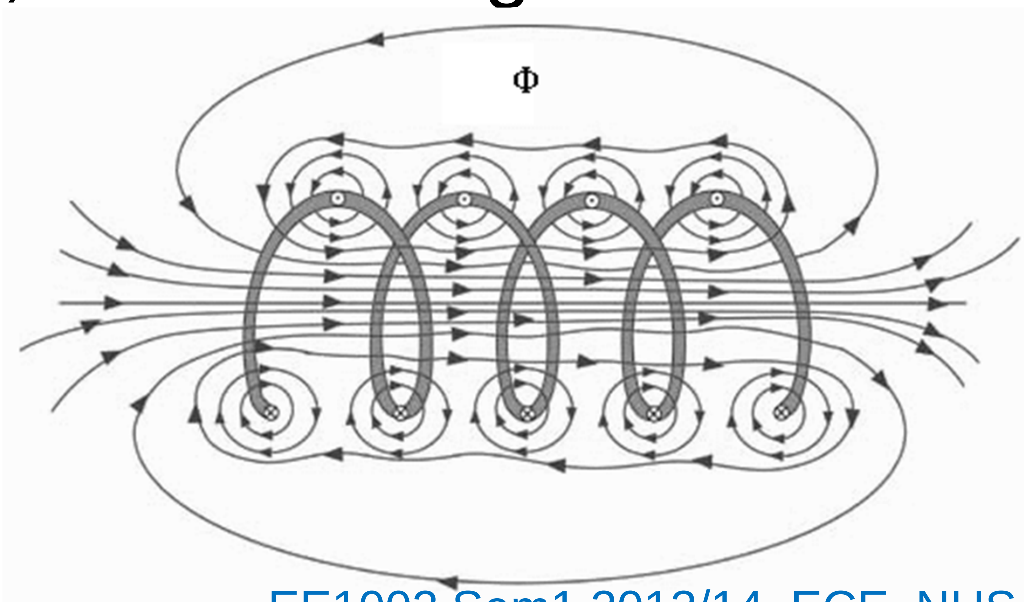
Use of Capacitor



Inductance

Inductance

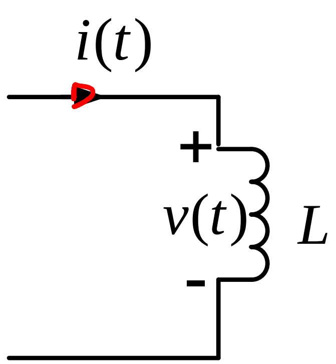
- An inductor is constructed by coiling a wire around some type of form. When current flows in the coil, a magnetic field is produced, with the magnetic flux linking the coil.



Faraday's law

- According to Faraday's law, a voltage is induced in a coil when the magnetic field linking it varies with time.

$$v = \frac{d\lambda}{dt}$$



$$\lambda = Li$$ – Flux linkage

$$v = \frac{d\lambda}{dt} = L\frac{di}{dt}$$

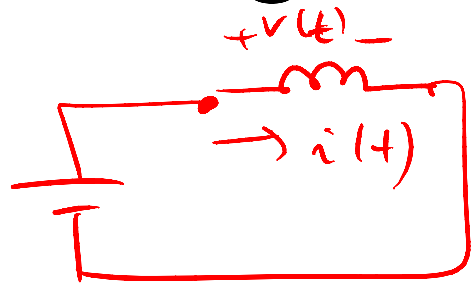
$$\boxed{v = L \cdot \frac{di}{dt}}$$

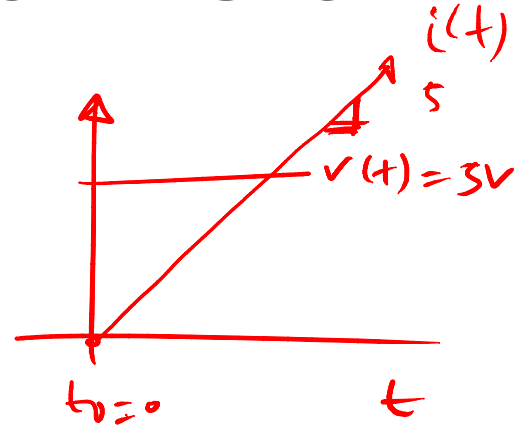
Capacitor $i_c = C \cdot \frac{dv_c}{dt}$

Inductor current in terms of voltage

$v = L\frac{di}{dt}$

$$di = \frac{1}{L} v(t)\,dt$$





$$\int_{i(t_0)}^{i(t)} di = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt \Rightarrow i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

↓ initial current

$= \frac{1}{L}\int_{t_0}^{t} 5 \cdot dt = \frac{1}{L} \times 5(t - t_0) = \frac{5t}{L}$

$t_0 = 0$

i(t) at t = 1 sec?

$\boxed{\frac{5 \times 1}{10^{-3}} = 5000A}$

L is in Henry

L → 1mH (typical value)

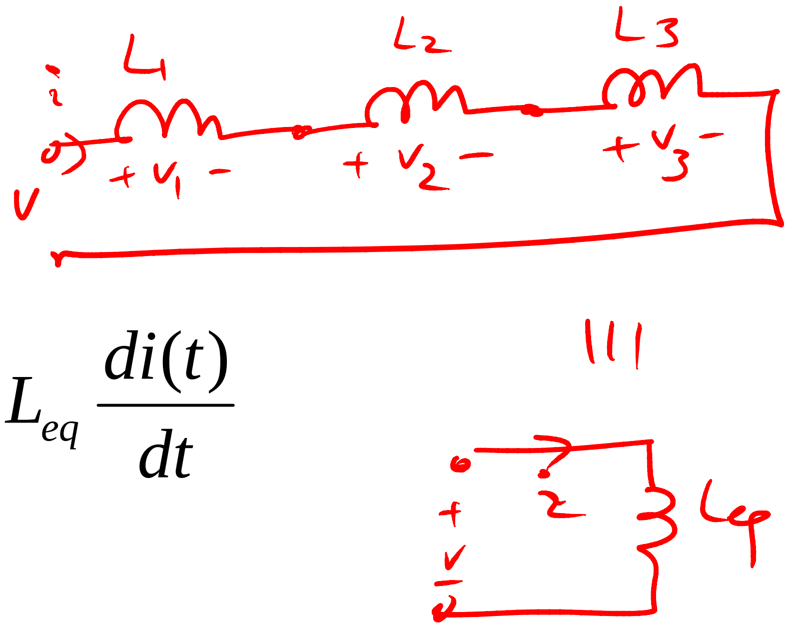
Inductors in series

$$i_1(t) = i_2(t) = i_3(t) = i(t)$$

$$v_1(t) + v_2(t) + v_3(t) = v(t)$$

$$L_1 \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} + L_3 \frac{di_3(t)}{dt} = L_{eq} \frac{di(t)}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + L_3}$$



Inductors in parallel

$v = L\frac{di}{dt}$

$v_1 = L_1 \cdot \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = L_3 \frac{di_3}{dt}$

$= v \quad \text{— ①}$

$$v_1 = v_2 = v_3 = v$$

$$v\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) = \frac{v}{L_{eq}} \Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$v = L_{eq}\frac{di}{dt} \quad \text{— ②}$

① ⇒

$i = i_1 + i_2 + i_3$

$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} = \frac{v}{L_1} + \frac{v}{L_2} + \frac{v}{L_3} = \frac{v}{L_{eq}}$

Energy stored in energy

$$e(t) = \int_{t_0}^{t} \underline{p(t)dt} = \int_{t_0}^{t} v(t) \cdot i(t) \cdot dt = \int_{t_0}^{t} L \cdot \frac{di(t)}{dt} \cdot i(t) \cdot dt$$

$$e(t) = \int_{t_0}^{t} Li(t)\frac{di}{dt}dt = \int_{i_0}^{i(t)} \underline{Lidi} = \underline{\frac{1}{2}L\left(i^2 - i_0^2\right)}$$

$$= $$

$$e(t) = \frac{1}{2}Li^2(t) \Leftarrow \text{if } i_0 = 0$$

$$\boxed{e = \frac{1}{2}Li^2}$$

Comparison of capacitor and inductor

Cap

Energy due to static charge

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

DC-steady state behaviour:

$i_c = 0$ as $\frac{dv_c}{dt} = 0$

→ Open ckt

Series. $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

parallel $C_{eq} = C_1 + C_2 + C_3$

Energy $= \frac{1}{2} \cdot C \cdot V^2$

Ind

Energy due to moving charge

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$V_L = 0$ as $\frac{di_L}{dt} = 0$

→ Short circuit.

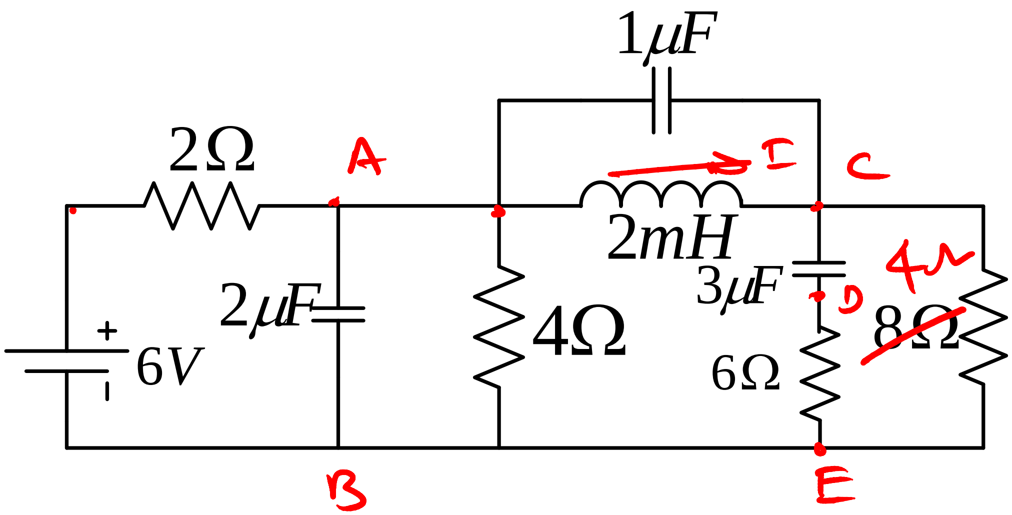
Series $L_{eq} = L_1 + L_2 + L_3$

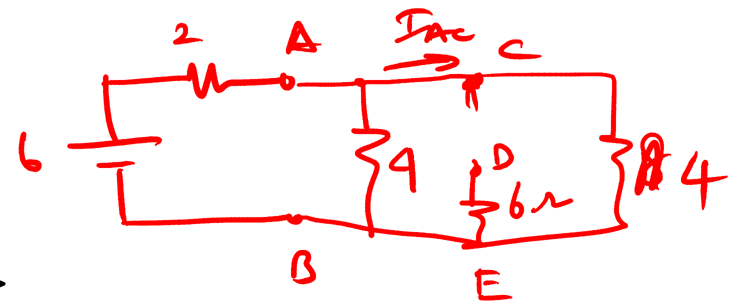
parallel $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

Eng : $\frac{1}{2} L \cdot I^2$.

DC Steady-state of circuits containing L and C

Find the energy stored in the inductor and the capacitors.





V_{AB} ✓, V_{AC} ✓, I_{AC}

V_{CD} ✓

$V_{AB} = 3V = V_{CD}$

$V_{AC} = 0$ ✓

$I_{AC} = \frac{3}{4}$ A.

$E_{2\mu F} = \frac{1}{2} \times 2\times10^{-6} \times 9$ J

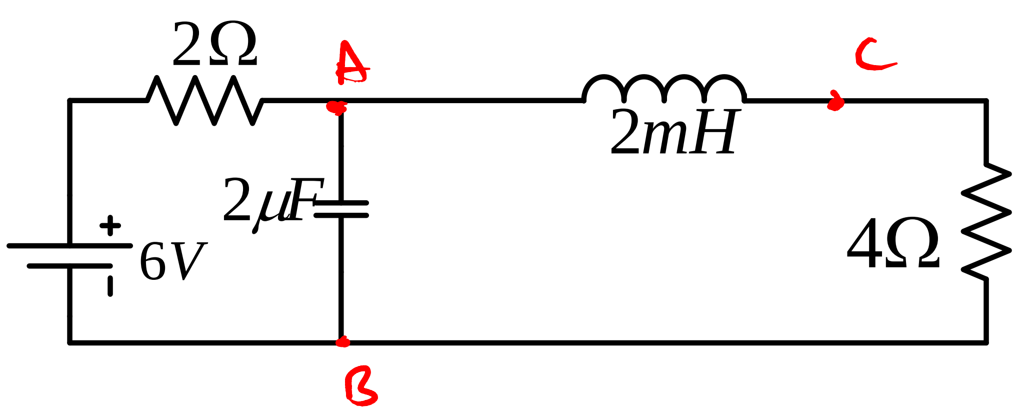
$E_{1\mu F} = \frac{1}{2} \times 1\times10^{-6} \times 0$ J

$E_{2mH} = \frac{1}{2} \times 2\times10^{-3} \times \left(\frac{3}{4}\right)^2$ J

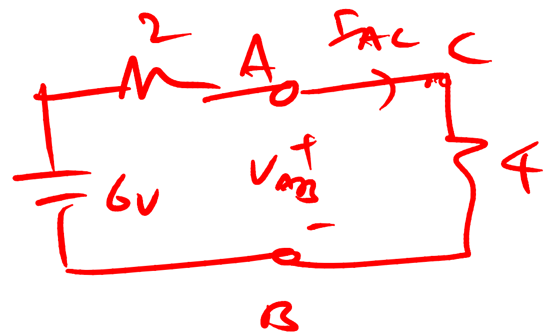
$E_{3\mu F} = \frac{1}{2} \times 3\times10^{-6} \times 9$ J

DC Steady-state of circuits containing L and C

Find the energy stored in the inductor and the capacitor.



$\Rightarrow$



$$V_{AB} = 6 \times \frac{4}{2+4} = 4V.$$

$$I_{AC} = \frac{6}{2+4} = 1A.$$

$$E_{2\mu F} = \frac{1}{2} \times 2 \times 10^{-6} \times 4^2 \ J.$$

$$E_{2mH} = \frac{1}{2} \times 2 \times 10^{-3} \times 1^2 \ J.$$