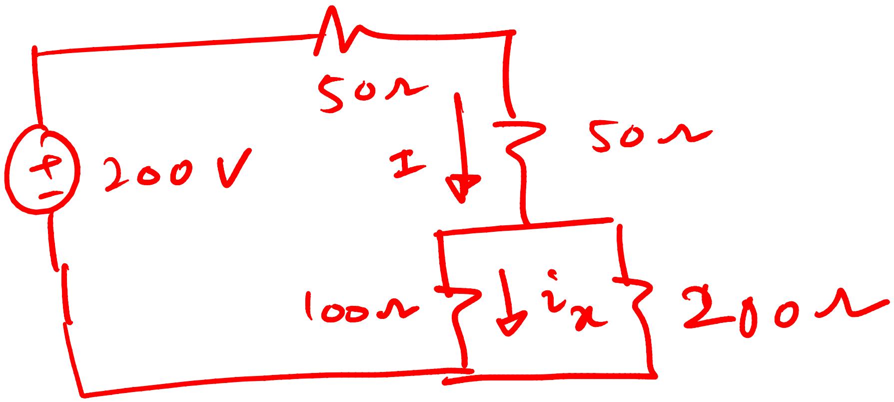
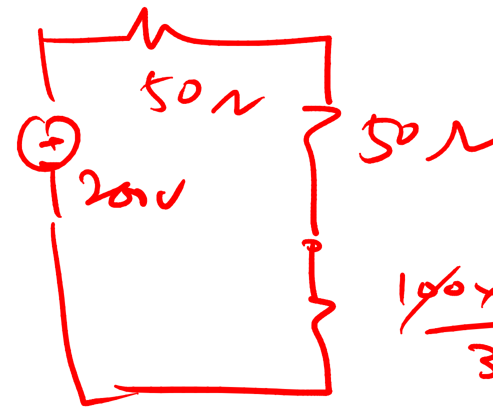
EE1002
Introduction to Circuits and Systems

Part 1 : Lecture 4

Error in last lecture : Example





$$\frac{100\times200}{300} = \frac{200}{3}\ \Omega$$

$$I = \frac{200}{50+50+\frac{200}{3}}\ \mathrm{A}$$

$$i_x = \cancel{I} \times \frac{200}{100+200} \times \frac{200}{100+\frac{200}{3}}$$

Node Voltage Analysis

Mesh Current Analysis

Steps of Node Voltage Analysis method

1. Select a reference node (Usually the ground of a voltage source.)

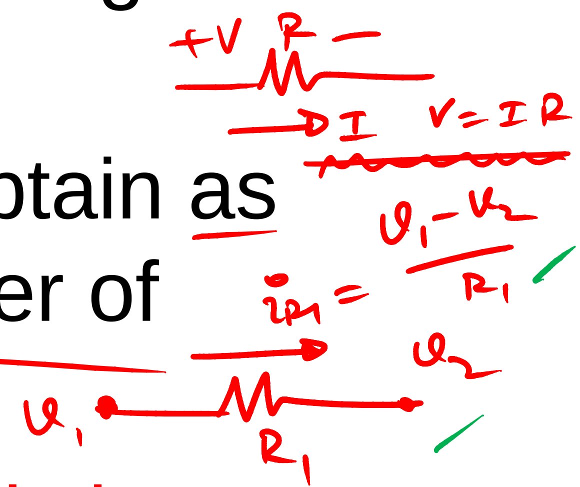
→ −ve of voltage source as ref

2. For each voltage source connected to the reference node, the other end is a known constant.

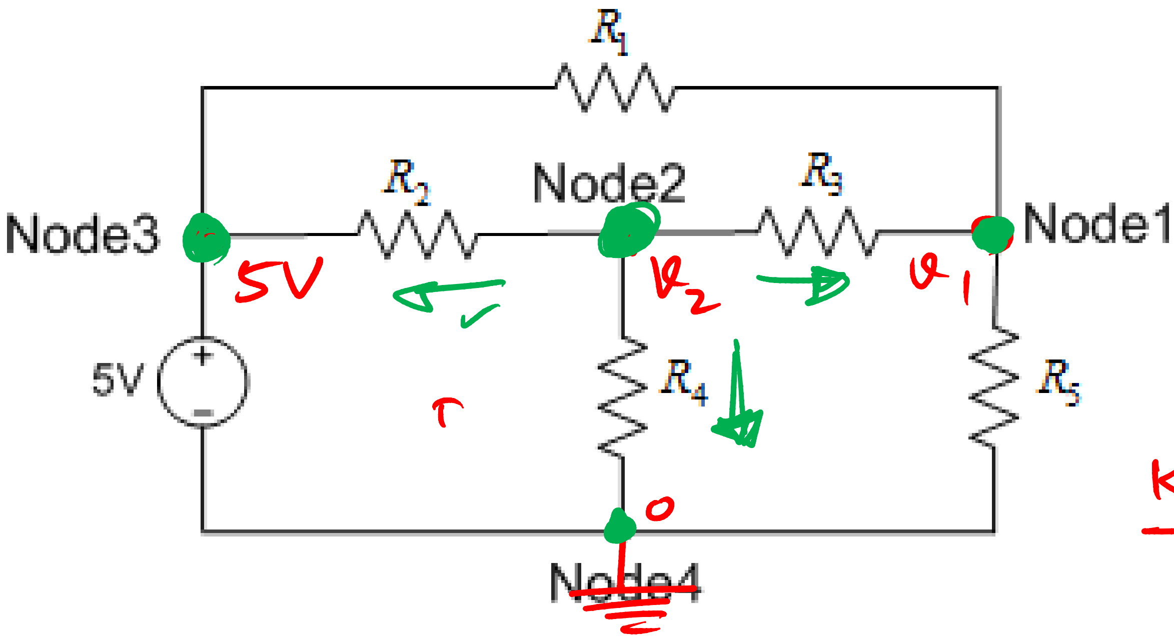
3. For all other voltage sources, one end is tied to the other. So only one unknown variable for each such voltage source.

Steps of Node Voltage Analysis method

4. Define the remaining node voltages as unknown variables.
5. Apply KCL at the nodes, to obtain as many equations as the number of unknown variables.
6. Express each current in a resistive branch in terms of the adjacent node voltages.
7. Solve the linear system of equations.



Case 1: Node analysis with one Ideal Voltage Source



KCL at node 1

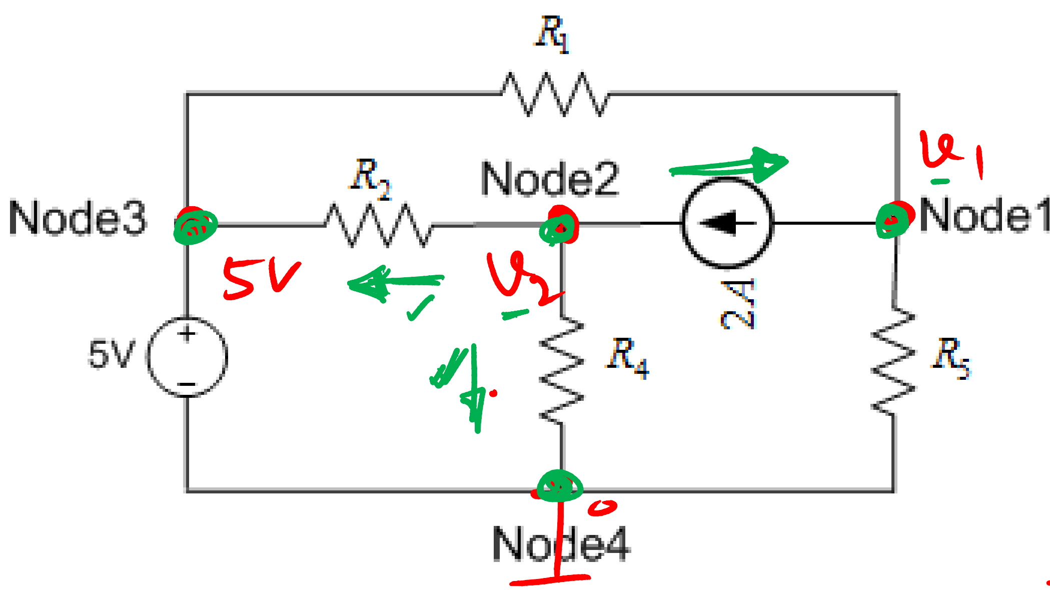
$$\frac{v_1-5}{R_1} + \frac{v_1-v_2}{R_3} + \frac{v_1}{R_5} = 0 \quad \text{—} \; (1)$$

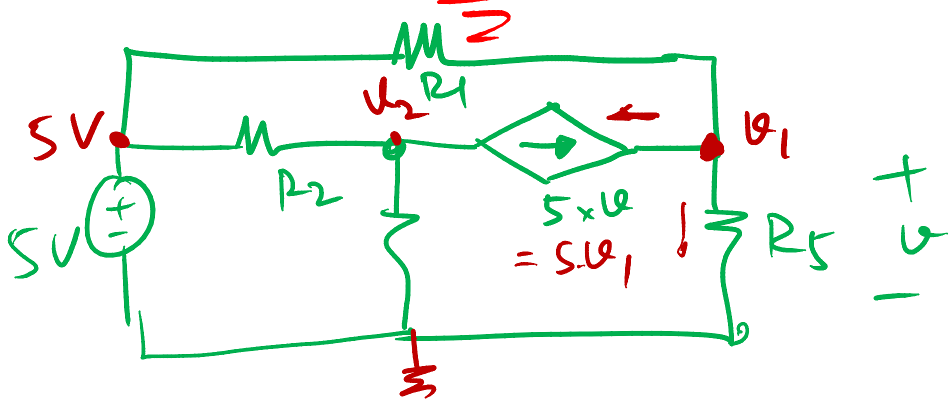
KCL at node 2

$$\frac{v_2-5}{R_2} + \frac{v_2-v_1}{R_3} + \frac{v_2}{R_4} = 0 \quad \text{—} \; (2)$$

- Choose reference node (GND)
- Identify unknown node voltages as variables
- Express each current in a resistive branch in terms of the adjacent node voltages.

Case 2: Having an additional Ideal Current source in the circuit





KCL at node 1

$$\frac{v_1-5}{R_1} + \frac{v_1}{R_5} + 2 = 0 \quad — (1)$$

KCL at node 2

$$\frac{v_2-5}{R_2} + \frac{v_2}{R_4} - 2 = 0 \quad — (2)$$

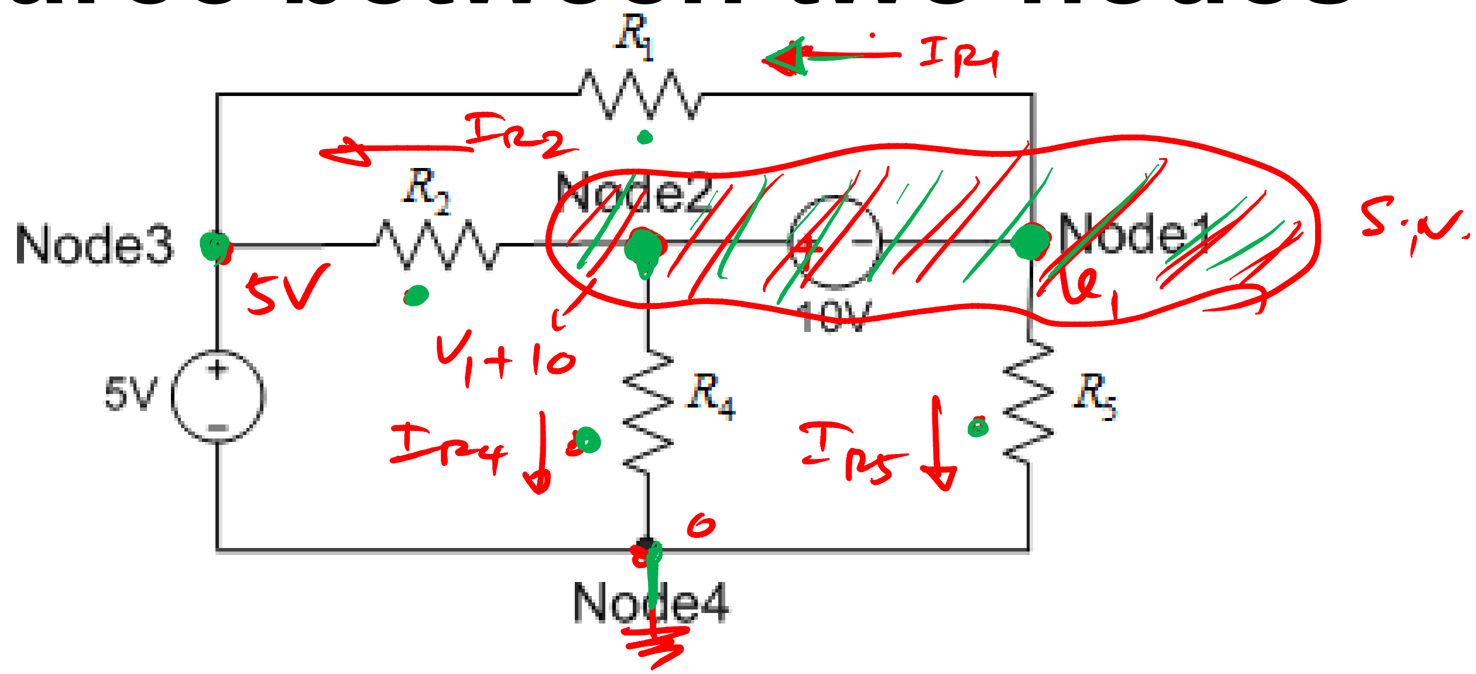
KCL at node v_1

$v = v_1$

$$\frac{v_1-5}{R_1} - 5v_1 + \frac{v_1}{R_5} = 0 \quad — (1)$$

- - - - (2)

Case 3: With an Ideal Voltage source between two nodes

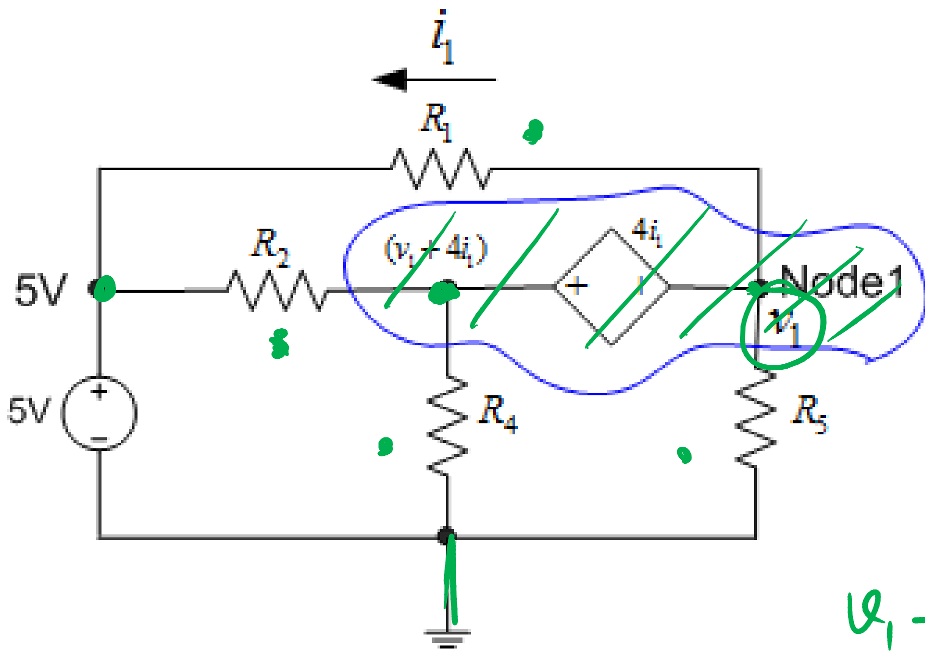


- Out of the two nodes across the voltage source, only one node voltage is an unknown variable.

$$\frac{v_1-5}{R_1}+\frac{v_1+10-5}{R_2}+\frac{v_1+10}{R_4}+\frac{v_1}{R_5}=0 \quad \text{— } (1)$$

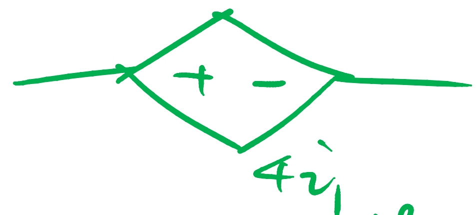
✓ I_{R1} ✓ I_{R2} ✓

Case 4: With dependent sources



Writing the equations

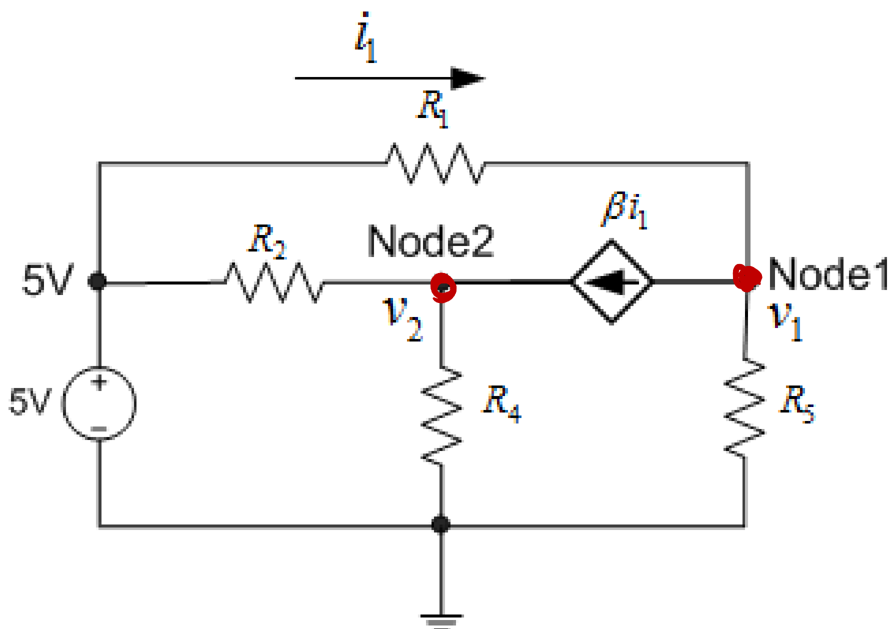
- Replace the control variable in terms of the node voltages



$$v_1 + 4i_1 = v_1 + 4 \times \frac{v_1 - 5}{R_1}$$

$$\frac{v_1 - 5}{R_1} + \frac{1}{R_2}\left[v_1 + 4\,\frac{v_1 - 5}{R_1} - 5\right] + \left(v_1 + 4\cdot\frac{v_1 - 5}{R_1}\right) \times \frac{1}{R_4} + \frac{v_1}{R_5} = 0$$

Case 4: With dependent sources



Writing the equations

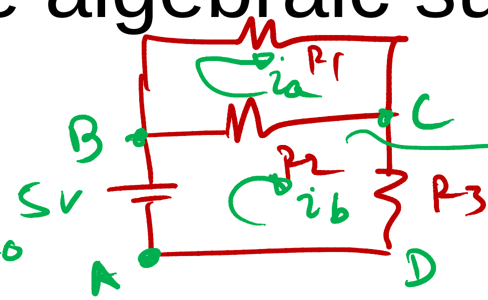
- Replace the control variable in terms of the node voltages

$$\beta i_1 = \beta\left(\frac{5-v_1}{R_1}\right)$$

✓

Steps of mesh current analysis

1. Identify all the mesh currents in the circuit
2. Write KVL equations around the closed paths to obtain required equations.
3. If a branch is a resistor, there is a voltage fall in the direction of current.
4. If a resistor is common to two meshes, resistor current will be algebraic sum of the mesh currents.



$I_{BC} = i_b - i_a$

ABCDA : KVL

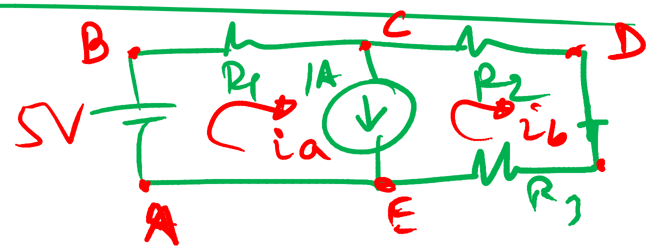
$-5 + R_2 \times (i_b - i_a) + R_3 \times i_b = 0$

Steps of mesh current analysis

5. If a branch is a voltage source, voltage fall is from the positive terminal to the negative terminal.
6. Avoid paths where a branch is a current source, as you cannot express voltage across them in terms of mesh currents.
7. For branches which are current sources, relate them to the mesh currents.

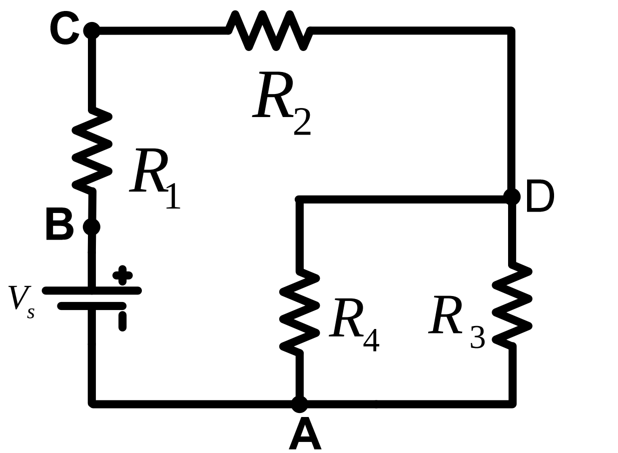
ABCEA : ~~KVL~~

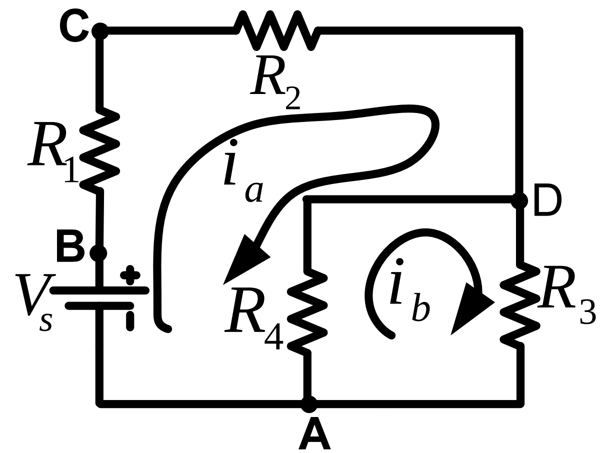
ABCDEA : KVL (1)

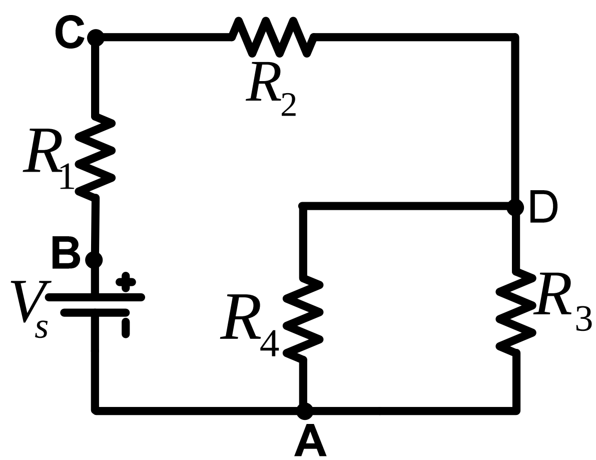


$1 = i_a - i_b$ (2)

Case 1: With Single Independent Voltage Source







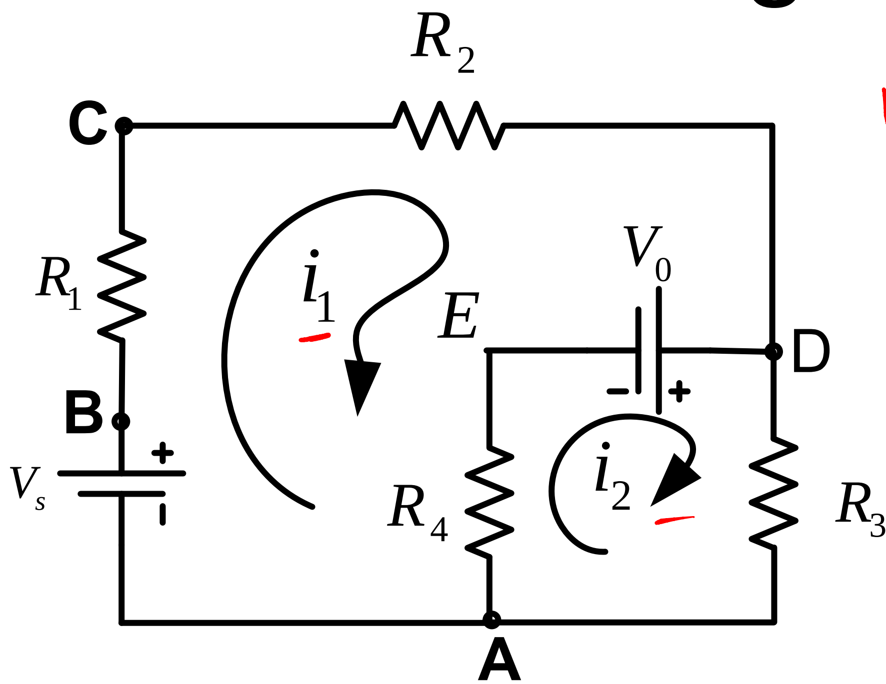
KVL for mesh i_a: ABCDA

$-V_s + R_1 \times i_a + R_2 \times i_a + R_4 \times (i_a - i_b) = 0$ — ①

KVL for mesh i_b: ADA

$R_4 \times (i_b - i_a) + R_3 \times i_b = 0$ — ②

Case 2: With two Independent Voltage source



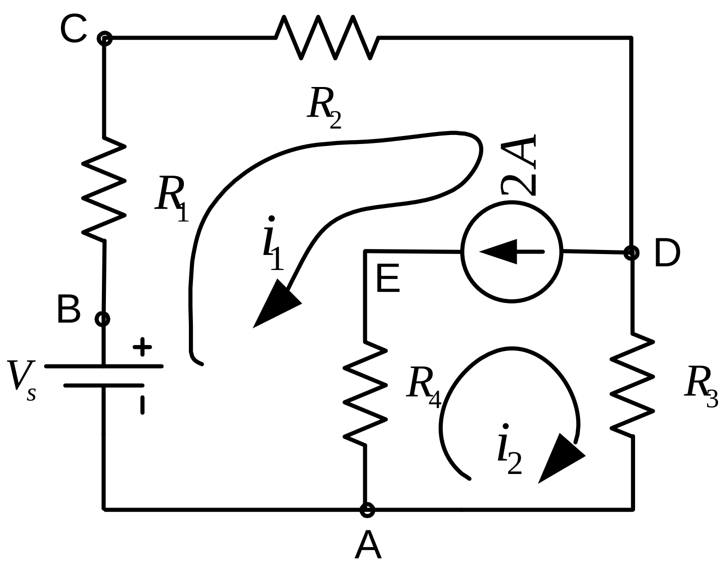
KVL for ~~AB~~ i_1: ABCDEA

$$-V_s + R_1 \times i_1 + R_2 \times i_1 + V_0 + R_4 \times (i_1 - i_2) = 0 \quad \text{— (1)}$$

KVL for i_2: AEDA

$$R_4 \times (i_2 - i_1) - V_0 + R_3 \times i_2 = 0 \quad \text{— (2)}$$

Case 3: With both independent Voltage and Current Source



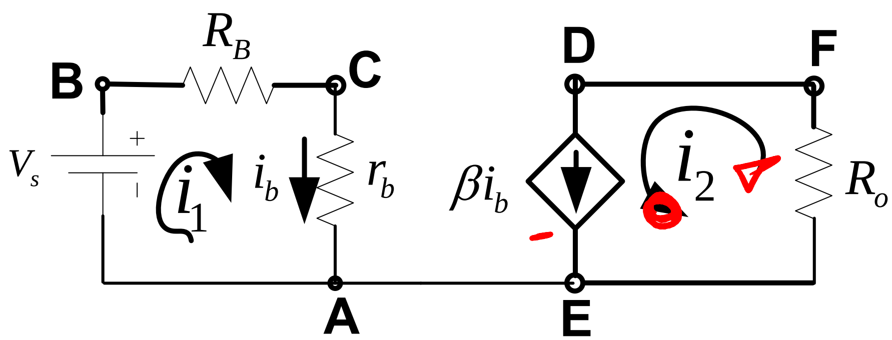
ABCDA : KVL

$$-V_s + R_1 \times i_1 + R_2 \times i_1 + R_3 \times i_2 = 0 \quad \text{—— ①}$$

$$2 = i_1 - i_2 \quad \text{—— ②}$$

- Can NOT write the voltage across a Current Source, in terms of the mesh currents
- Avoid this path for writing KVL
- Relate the current source to the mesh currents.

Case 4: With dependent current source



- Replace the control variable in terms of the mesh currents

$\beta i_b = \beta \cdot i_1$

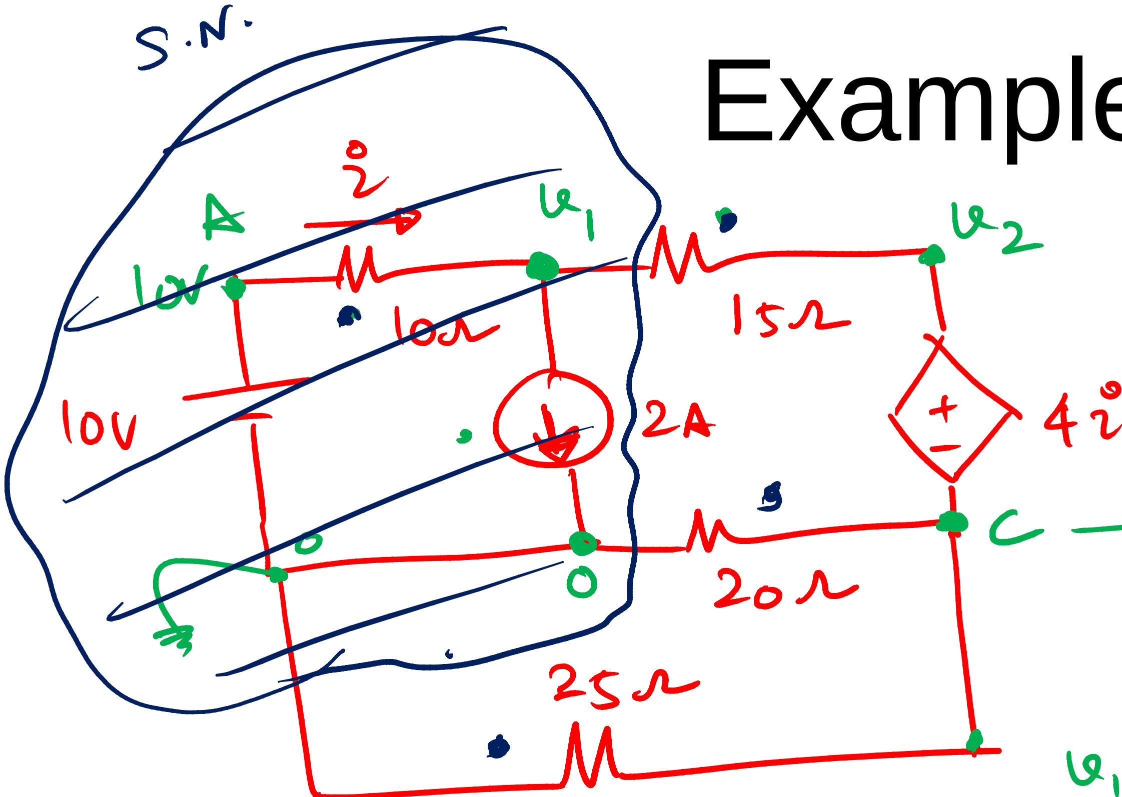
KVL for ABCA:

$-V_s + R_B \times i_1 + r_b \times i_1 = 0$ — (1)

KVL for EDFE? No.

$\beta i_1 = -i_2$ — (2)

Example



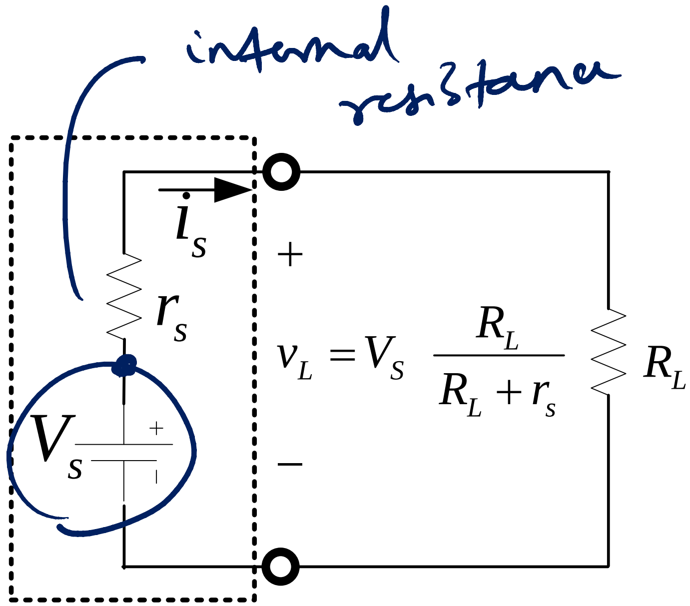
$C \to v_2 - 4i = v_2 - 4 \times \frac{10 - v_1}{10}$

Node voltage Analysis Method

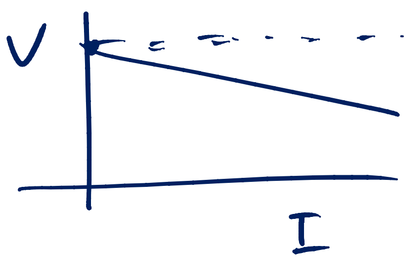
KCL at v_1: $\frac{v_1 - 10}{10} + 2 + \frac{v_1 - v_2}{15} = 0$ — ①

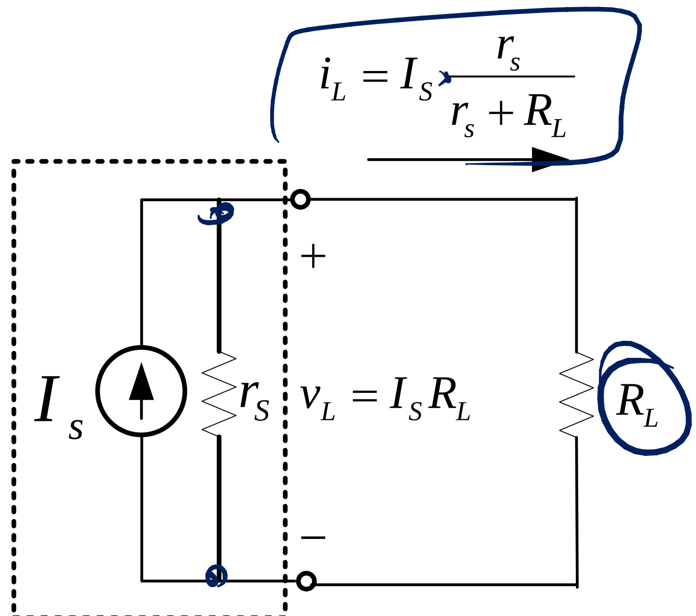
KCL at S.N.: $\frac{v_1 - v_2}{15} + \underbrace{\left[0 - \left(v_2 - 4\,\frac{10 - v_1}{10}\right)\right] \times \left[\frac{1}{20} + \frac{1}{25}\right]} = 0$ — ②

Practical Sources

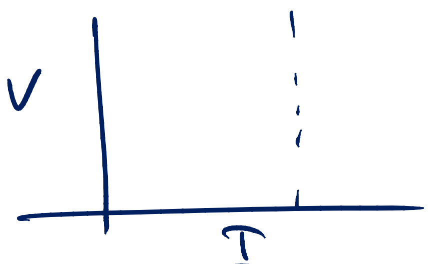


Practical Voltage Source



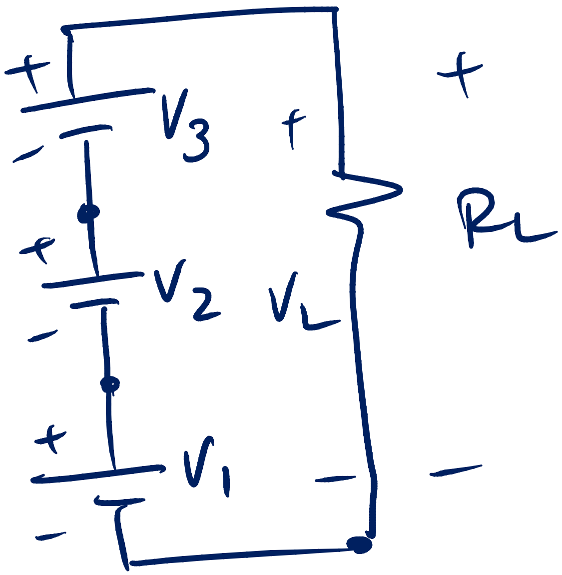


Practical Current Source

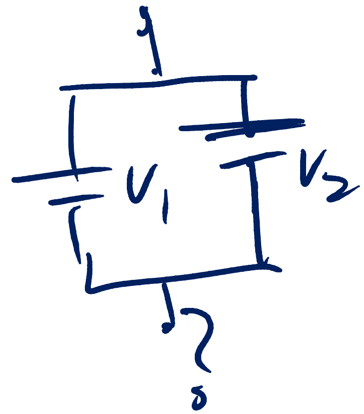


Source Combination

$V_L = V_1 + V_2 + V_3$

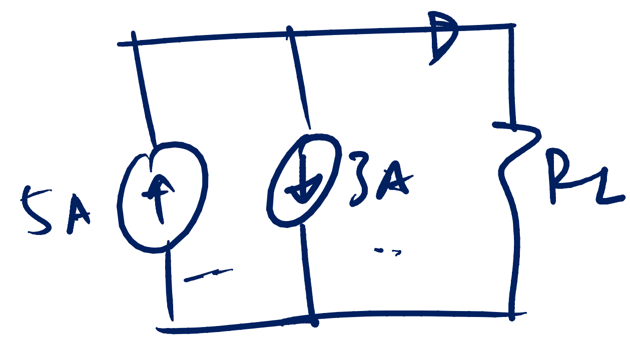


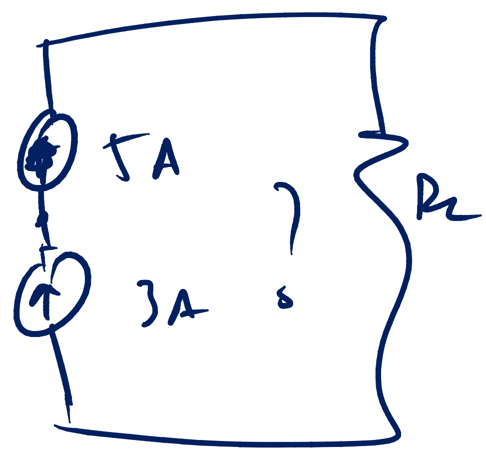
$0\text{–}30V,\ 0\text{–}30V,\ 5V$



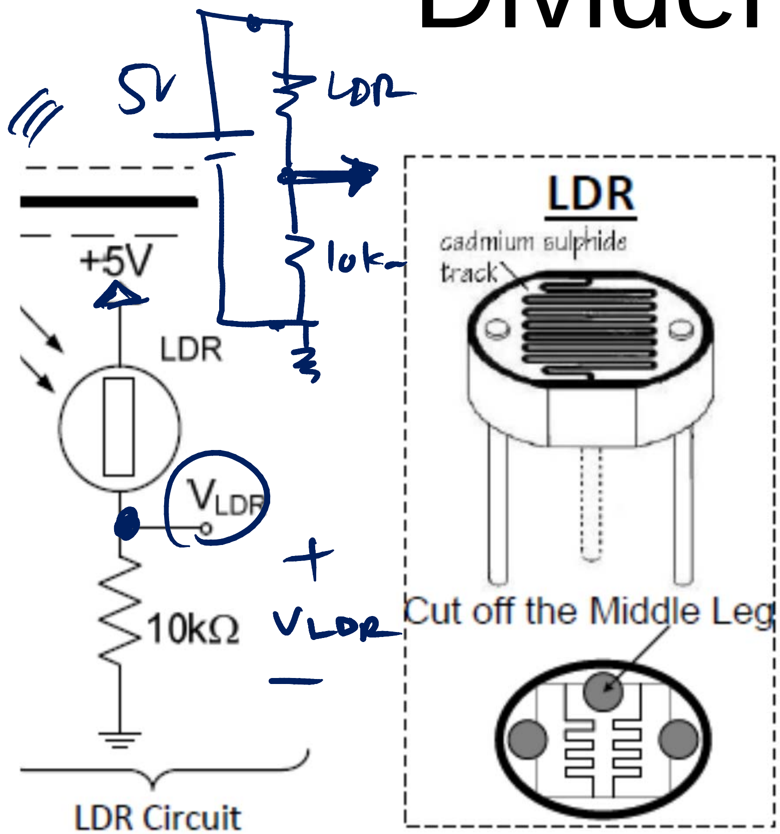
$V_1 = V_2$

$5 - 3 = I_L$





Light Sensor using Voltage Divider Principle



R_{LDR} is small when there is light.

- An LDR (Light Dependent Resistor) can be used for this purpose. As its name suggests, the LDR is just a resistor that has a resistance varying with light intensity.

V_{LDR} (light): Large

V_{LDR} (No light) = Small

$$V_{LDR} = 5 \times \frac{10K}{10K + R_{LDR}}$$