Chapter1: Power in AC Circuits

Learning Objectives:

- Understanding the power in AC circuits
- Solving steady-state ac circuit power using phasors and complex impedances
- Compute power for steady-state ac circuits
- Understanding of real or active power, reactive power and apparent power in ac circuits
- Understanding the concept of power factor and power factor improvement using capacitors

1.1: Power in AC Circuits

Consider the electric circuit as shown in Fig. 1.1, consisting of a voltage source $v(t) = V_m \cos(\omega t)$ that is applied to an electrical network composed of resistance, *R* and inductance and capacitance with reactive impedance, *Z* in series. The phasor for the voltage source is $\mathbf{V} = V_m \angle 0^0$ and the equivalent network impedance is $Z = |Z|\angle\theta = R + jX$.

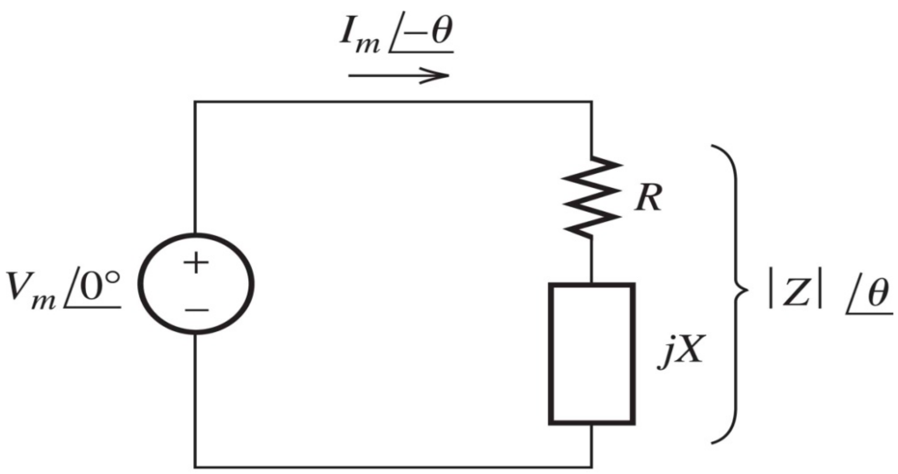


Figure 1.1 A voltage source delivering power to a load impedance *Z* = *R* + *jX*.

The phasor current is

$$\mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{V_m \angle 0^0}{|Z|\angle\theta} = I_m \angle -\theta \qquad (1.1)$$

Before proceeding to find out the power delivered to a general R-L-C type of load, let us first consider the power delivered to a pure *resistive, inductive* and *capacitive load.*

1.1.1 Current, Voltage and Power for a Resistive Load

Let us consider that the load network is pure resistive then $|Z| = R \; and \; \angle\theta = 0$

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t)$$

$$p(t) = v(t)i(t) = V_m \cos(\omega t) \times I_m \cos(\omega t) = V_m I_m \cos(\omega t)^{\mathbf{2}} \qquad (1.2)$$

The plot of voltage, current and power are shown in Fig. 1.2. The current is in phase with the voltage due to the resistive nature of the load. The power, *p(t)* is positive all the time i.e. during both positive as well as negative cycles. We conclude from this that *electrical energy* flows continuously *from the voltage source to the load* and is *converted and dissipated as heat* in the load resister, *R*. The instantaneous power changes with rise and fall in the voltage and current correspondingly.

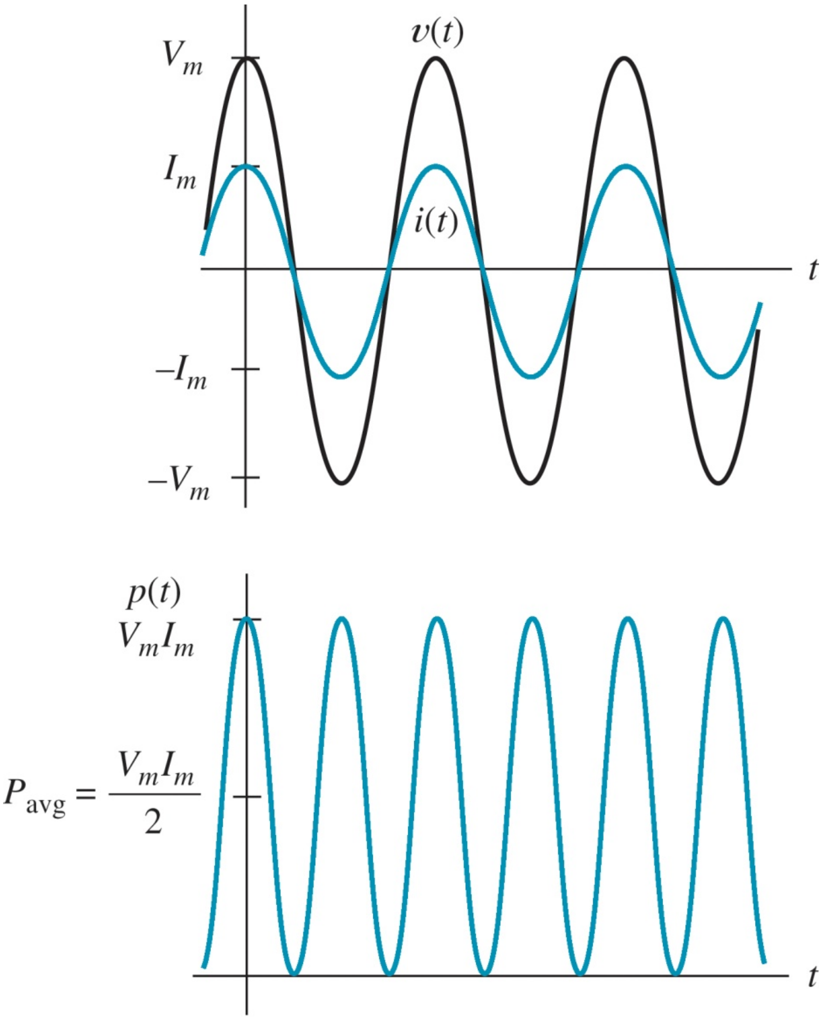


Figure 1.2: Current, voltage, and power versus time for a purely resistive load.

1.1.2: Current, Voltage and Power for an Inductive Load

Let us consider that the load network is pure inductive then $|Z| = \omega L \text{ and } \angle\theta = 90^0$

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - 90^0) = I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = V_m \cos(\omega t) \times I_m \sin(\omega t) = V_m I_m \sin(\omega t)\ \cos(\omega t) = \frac{V_m I_m}{2} \sin(2\omega t) \qquad (1.3)$$

The plot of voltage, current and power are shown in Fig. 1.3(a). The current lags the voltage by 90^0 due to the pure inductive nature of the load. The power, *p(t)* is positive half of the time and the electrical energy is transferred from the source to the inductance where it is stored in the magnetic field. For the rest of the half time, power is negative, indicating that the energy stored in the inductance is returned back to the electrical source. The inductor can only store the energy and transfer it back to the electrical source but can't dissipate it unlike the resistor. **Note that the average power over an electrical cycle is zero i.e. a pure inductor does not consume any electrical power from the source over a cycle.**

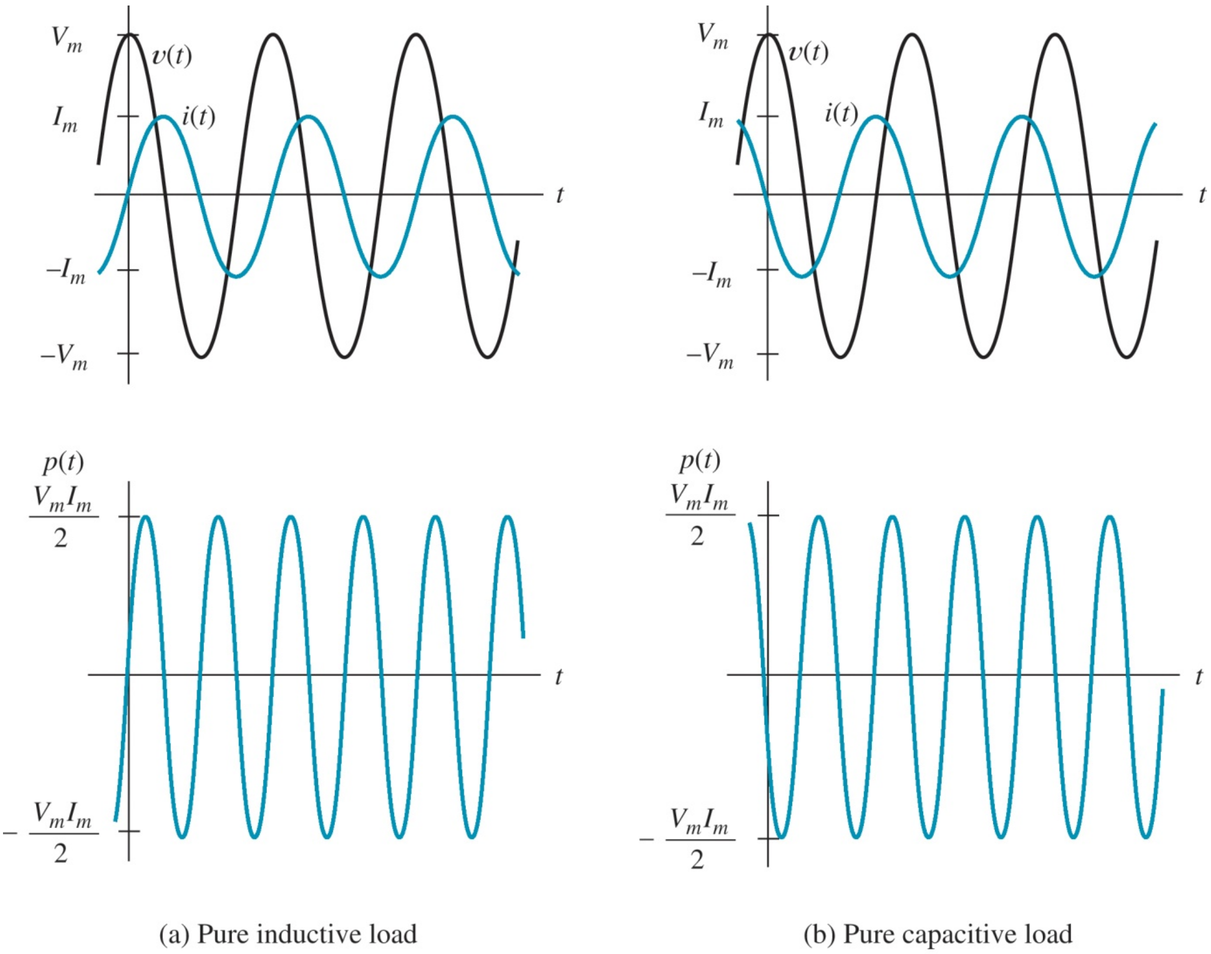


Figure 1.3: Current, voltage, and power versus time for pure energy-storage elements.

1.1.3: Current, Voltage and Power for a capacitive Load

Let us consider that the load network is pure inductive then $|Z| = \frac{1}{\omega C}$ $and\ \angle\theta = -90^0$

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t + 90^0) = -I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = V_m \cos(\omega t) \times (-I_m \sin(\omega t)) = -V_m I_m \sin(\omega t)\ \cos(\omega t)$$
$$= -\frac{V_m I_m}{2} \sin(2\omega t)\ (1.4)$$

The plot of voltage, current and power are shown in Fig. 1.3(b). The current leads the voltage by 90^0 due to the pure capacitive nature of the load. The power, *p(t)* is positive half of the time and the electrical energy is transferred from the source to the capacitance where it is stored. For the rest of the half time, power is negative, indicating that the energy stored in the capacitance is returned back to the electrical source. The capacitor can only store the energy and transfer it back to the electrical source but can't dissipate it unlike the resistor. **Note that the average power over an electrical cycle is zero i.e. a pure capacitor does not consume any electrical power from the source over a cycle.**

1.1.4: Reactive Power and Its Importance

Even if there is no average active power is consumed by the pure energy storage elements there is a continuous reactive power flow either from the source to the load or from the load to the source and that is of concerns to the Power Systems engineers because they have to design the transmission lines, transformers, fuses etc. that must be able to withstand the reactive power flow and the currents associated with the reactive power. If the loads have energy storage elements then they may draw large amount of currents requiring the heavy-duty wiring, even though they consume little average power. Thus the power supply companies charge the customers for their reactive power consumption.

1.1.5: Power Calculations for a General Load

Let us consider the case of a general R-L-C load for which the phase angle of the complex load can vary and have any value from -90^0 to $+90^0$.

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - \theta)$$

$$\begin{aligned} p(t) = v(t)i(t) &= V_m \cos(\omega t) \times I_m\, cos(\omega t - \theta) \\ &= V_m I_m \cos(\omega t)\, [\cos(\omega t) \cos(\theta) + \sin(\omega t) \sin(\theta)] \\ &= V_m I_m \cos(\theta) \cos(\omega t)^2 + V_m I_m \sin(\theta)\ \sin(\omega t) \cos(\omega t) \\ &= V_m I_m \cos(\theta) \left\{\frac{1}{2} + \frac{1}{2}\cos(2\omega t)\right\} + V_m I_m \sin(\theta)\ \frac{1}{2}\sin(2\omega t) \\ &= \left[\frac{V_m I_m}{2}\cos(\theta)\right] + \left[\frac{V_m I_m}{2}\cos(\theta)\cos(2\omega t)\right] + \left[\frac{V_m I_m}{2}\sin(\theta)\sin(2\omega t)\right] (1.5) \end{aligned}$$

Note that the first term in the eqn(1.5) has a constant value for a given ac source voltage and for a given load and represents the average power consumed by the load, where as the 2nd and 3rd terms in the eqn. involving sine and cosine terms that vary with time and therefore would have zero average values.

$$P = \frac{V_m I_m}{2}\cos(\theta) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta) = V_{rms} I_{rms}\cos(\theta) \quad Watts(W) \quad (1.6)$$

1.1.6: Power Factor

The term $\cos(\theta)$ is called the **power factor**, it is the cosine of the phase angle between the source voltage *v(t)* and the current through the load, *i(t).* In this case, we have assumed that the source voltage has zero phase angle but in some cases the source phase angle might be zero and in that case we have the general eqn. for the phase angle difference between the supply voltage and load current would be

$$\theta = \theta_v - \theta_i, \qquad \cos(\theta) = \cos(\theta_v - \theta_i) \quad (1.7)$$

Sometimes, θ is also called as the **power angle**. The power factor varies between 0 and 1 and it can be said as *leading* or *lagging* depending on whether the current is *lagging* the supply voltage (in case of resistive-inductive load) and leading the supply voltage (in case of resistive-capacitive load).

1.1.7: Reactive Power

We have seen that in AC circuits, energy flows into and out of the energy storage elements (inductances and capacitances) as the voltage magnitude across a capacitance increases then energy flows into the capacitor and when the voltage magnitude decreases, energy flows out of the capacitor. Similarly, when the current through the inductance increases the energy is transferred to the inductor and when it decreases then energy flows out of the inductance. Although the instantaneous power can be large and therefore the current to handle would be large but the net energy transferred over a cycle *is zero only for ideal energy storage elements*.

The power associated with the energy storage elements contained in a general load is called as **reactive power** and is given by

$$Q = V_{rms} I_{rms}\sin(\theta) \quad Volt\ Ampere\ Reactive(VARs) \quad (1.8)$$

Note that in the case of a resistive element, *R*, θ = 0 and thus *Q = 0,* whereas in case of pure inductive load element, *L,* θ = +90^0 and thus $Q = +V_{rms} I_{rms}$, and in case of pure capacitive load element, *C,* θ= -90^0 and thus $Q = -V_{rms} I_{rms}$.

1.1.8: Apparent Power

Another quantity of interest is the **apparent power,** which is defined as the product of the rms values of the voltage and the current and is given by

$$\text{apparent power} = V_{rms} I_{rms} \quad Volt\ Ampere\ (VAs) \quad (1.9)$$

Using eqns. (1.6) and (1.8) we have

$$P^2 + Q^2 = (V_{rms} I_{rms}\cos(\theta))^2 + (V_{rms} I_{rms}\sin(\theta))^2 = (V_{rms} I_{rms})^2 \quad (1.10)$$

1.1.9: Units of Power

The units for active power, P is (W), reactive power, Q is (VARs) and that for apparent power is (VAs). When we say, it is a 5-kW load, that means P = 5 kW. On the other hand, if we say the load is 1 kVA, that means the $V_{rms}I_{rms} = 1kVA$. Similarly, if we say the load absorbs 3 kVAR, then, we mean Q = 3 kVAR.

1.1.9: Power Triangle

The relationships between, active power, *P,* reactive power, *Q*, apparent power, $V_{rms}I_{rms}$ and the power angle, θ can be represented by the **power triangle** as shown in Fig. 1. 4(a) for an inductive load, in which θ and *Q* are both positive. Similarly, for a capacitive load, in which θ and *Q* are both negative are shown in Fig. 1.4(b).

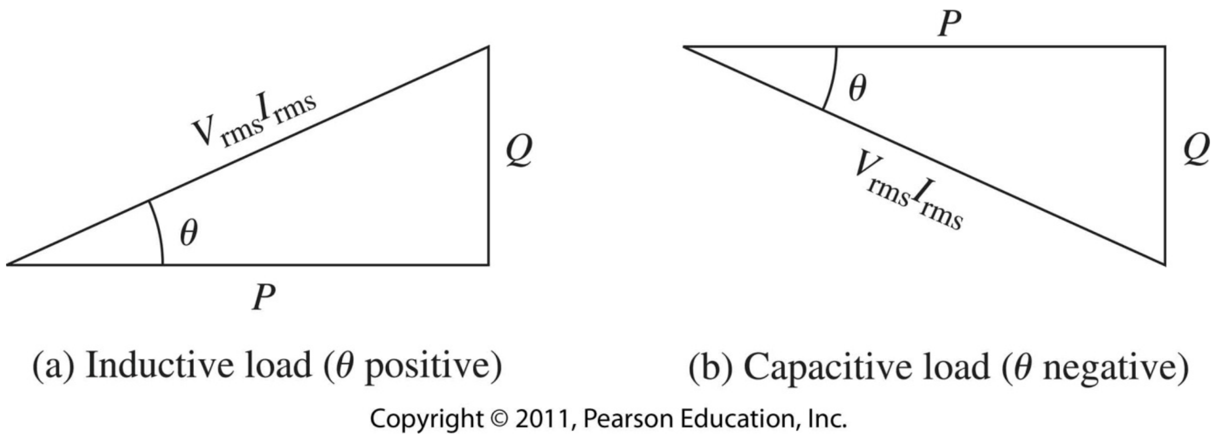


Figure 1.4 Power triangles for inductive and capacitive loads.

1.1.10: Additional Power Relationships

The impedance, *Z,* in which *R* is the resistive element and *X* is the reactive element is as shown in Fig. 1.5.

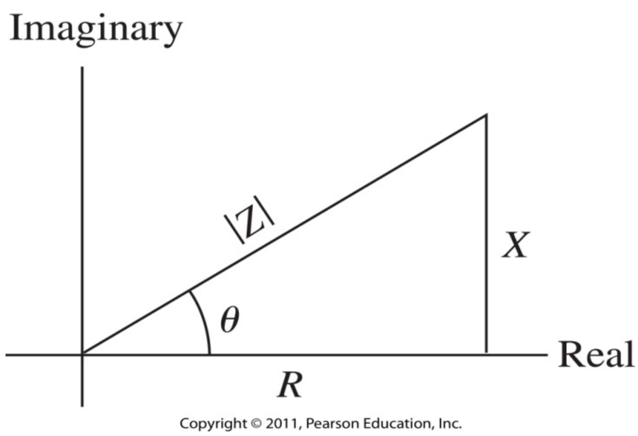


Figure 1.5 The load impedance in the complex plane.

$$Z = |Z|\angle\theta = R + JX, \qquad \cos(\theta) = \frac{R}{|Z|} \ (1.11), \qquad \sin(\theta) = \frac{X}{|Z|} \ (1.12)$$

Substituting, eqn. (1.11) is eqn. (1.6) we have,

$$P = \frac{V_m I_m}{2}\cos(\theta) = \frac{V_m I_m}{2} \times \frac{R}{|Z|} = \frac{I_m^{\,2}}{2} R = \frac{I_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times R = I_{rms}^2 \times R = \frac{V_{rms}^2}{R^2} \times R = \frac{V_{rms}^2}{R} \, (1.13)$$

In a similar way, it can be shown that

$$Q = \frac{V_m I_m}{2}\sin\,(\theta) = \frac{V_m I_m}{2} \times \frac{X}{|Z|} = \frac{I_m^{\,2}}{2} X = \frac{I_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times X = I_{rms}^2 \times X = \frac{V_{X,rms}^2}{X^2} \times X = \frac{V_{X,rms}^2}{X} \, (1.13)$$

where $V_{X,rms}$ is the rms value of the *voltage across the reactance*, *X*. Also, note that *X* is positive for an inductance and negative for a capacitance.

1.1.11: Complex Power

Consider the electric circuit as shown in Fig. 1.6. The **complex power, S** delivered to the load is defined as one half of the product of the phasor voltage, **V** and teh complex conjugate of the phasor current, $\mathbf{I}^*$.

$$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = \frac{1}{2}(V_m \angle \theta_v) \times (I_m \angle{-\theta_i}) = \frac{V_m I_m}{2}\angle(\theta_v - \theta_i = \frac{V_m I_m}{2}\angle\theta \;\; (1.14)$$

$$\mathbf{S} = \frac{V_m I_m}{2} \cos(\theta) + j\frac{V_m I_m}{2} sin(\theta) \;\; (1.15)$$

$$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = P + j\,Q \; (1.16)$$

If we know the complex power, **S** then we can compute the active power, *P*, reactive power, *Q* and apparent power as follows.

$$P = \mathrm{Re}(\mathbf{S}) = \mathrm{Re}\left(\frac{1}{2}\mathbf{VI}^*\right) \quad (1.17)$$

$$Q = \mathrm{Im}(\mathbf{S}) = \mathrm{Im}\left(\frac{1}{2}\mathbf{VI}^*\right) \quad (1.18)$$

$$\text{apparent power} = |\mathbf{S}| = \left|\frac{1}{2}\mathbf{VI}^*\right| \quad (1.19)$$

where Re(**S)** denotes the real part of **S** and Im(**S**) denotes the imaginary part of **S.**

Example 1.1

Consider the circuit as shown in Fig. 1. 6. Find the phasor current through each element. Compute the active and reactive power delivered to each element in the circuit.

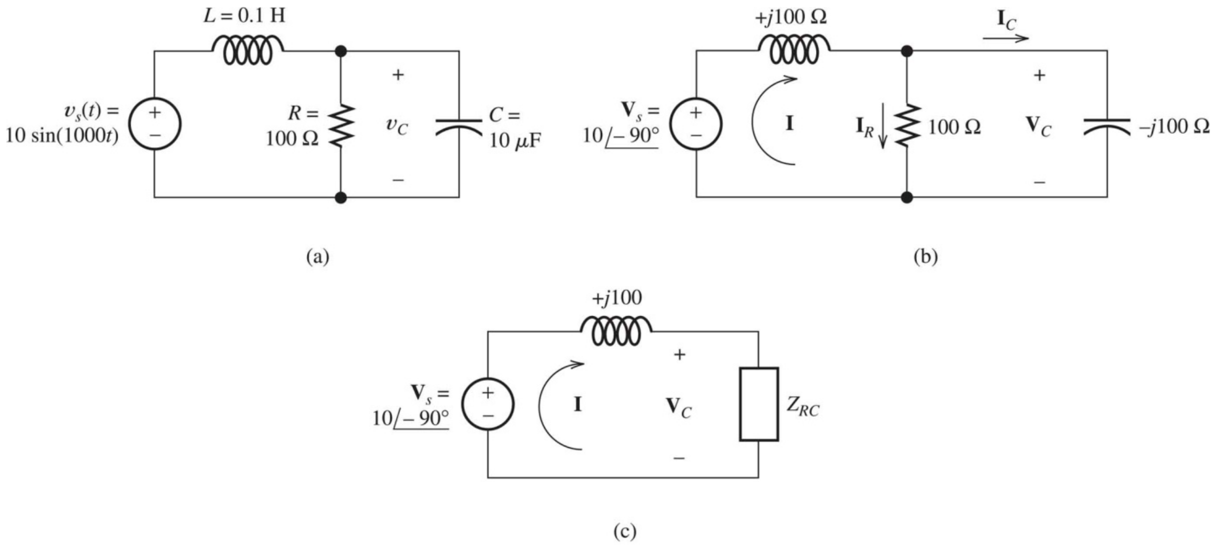


Figure 1.6: Circuit for Example 1.1.

The phasor voltage of the source is $\mathbf{V_s} = 10\angle-90^0$.

$$Z_L = j\omega L = j1000 \times 0.1 = j100\ \Omega$$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{1000 \times 10 \times 10^{-6}} = -j100\ \Omega$$

$$Z_{RC} = \frac{1}{\frac{1}{R} + 1/Z_c} = \frac{1}{\frac{1}{100} + 1/(-j100)} = \frac{1}{0.01 + j0.01} = \frac{1\angle0^0}{0.01414\angle45^0} = 70.71\angle-45^0$$
$$= 50 - \mathrm{j}\ 50\ \Omega$$

$$\mathbf{V}_C = \mathbf{V}_S \frac{Z_{RC}}{Z_L + Z_{RC}} = 10\angle-90^0 \frac{70.71\angle-45^0}{j100+50-j50} = 10\angle-90^0 \frac{70.71\angle-45^0}{50+j50} = 10\angle-90^0 \frac{70.71\angle-45^0}{70.71\angle45^0} =$$
$$10\angle-180^0 = 10\cos(1000t - 180^0)$$

$$\mathbf{I} = \frac{\mathbf{V}_S}{Z_L + Z_{RC}} = \frac{10\angle-90^0}{50 + j50} = \frac{10\angle-90^0}{70.71\angle45^0} = 0.1414\angle-135^0$$

$$\mathbf{I}_R = \frac{\mathbf{V}_c}{R} = \frac{10\angle-180^0}{100} = 0.1\angle-180^0$$

$$\mathbf{I}_C = \frac{\mathbf{V}_c}{Z_C} = \frac{10\angle-180^0}{-j100} = \frac{10\angle-180^0}{100\angle-90^0} = 0.1\angle-90^0$$

The power angle, θ is given by

$$\theta = \theta_v - \theta_i = -90^0 - (-135^0) = 45^0$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071\ V$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{0.1414}{\sqrt{2}} = 0.1\ A$$

$$P = V_{rms} I_{rms} \cos(\theta) = 7.071 \times 0.1 \times \cos(45^0) = 0.5\ Watts(W)$$

$$Q = V_{rms} I_{rms} \sin(\theta) = 7.071 \times 0.1 \times \sin(45^0) = 0.5\ (VARs)$$

Alternatively, from the complex power calculations we have

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}(10\angle -90^0)(\,0.1414\angle -135^0) = 0.707\angle -135^0 = 0.5 + j0.5 = P + j\,Q$$

$$P = \mathrm{Re}(\mathbf{S}) = 0.5\ \mathrm{W}$$

$$Q = \mathrm{Im}(\mathbf{S}) = 0.5\ \mathrm{VAR}$$

$$Q_L = I_{rms}^2 \times X_L = (0.1)^2 \times (100) = 1\ VAR$$

$$Q_C = I_{C,rms}^2 \times X_C = \left(\frac{0.1}{\sqrt{2}}\right)^2 \times (-100) = 0.5\ VAR$$

$$Q = Q_L + Q_C = 1 - 0.5 = 0.5\ VAR$$

$$P_R = I_{rms}^2 \times R = \left(\frac{0.1}{\sqrt{2}}\right)^2 \times 100 = 0.5\ W\,, \qquad P_L = P_C = 0$$

Example 1.2

Consider the circuit as shown in Fig. 1. 7. The voltage source delivers power to two loads connected in parallel. Find the active power, reactive power and power factor for the source. Also find the phasor current, **I.**

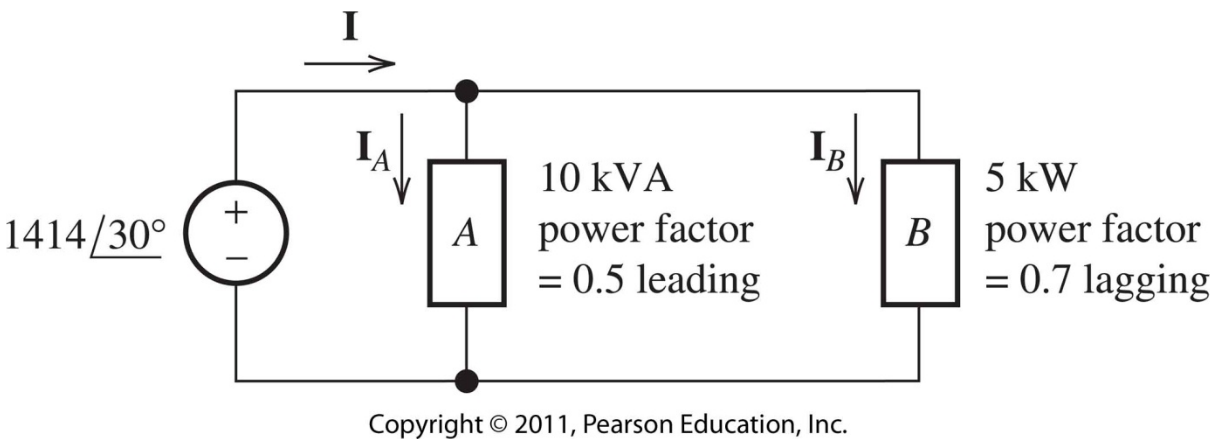


Figure 1.7: Circuit for Example 1.2.

For load-A, we have

$$P_A = 10\ kVA\ \times 0.5 = 5\ kW,$$

$$Q_A = -10\ kVA\ \times \sin\left((cos^{-1}(0.5))\right) = -8.660\ kVAR$$

For load-B, we have

$$P_B = 5\ kW,$$

$$Q_B = \mathbf{S}_\mathrm{B} \times \sin(\theta) = \frac{5\ kW}{0.7} \times \sin\left(\left(cos^{-1}(0.7)\right)\right) = 5.101\ kVAR$$

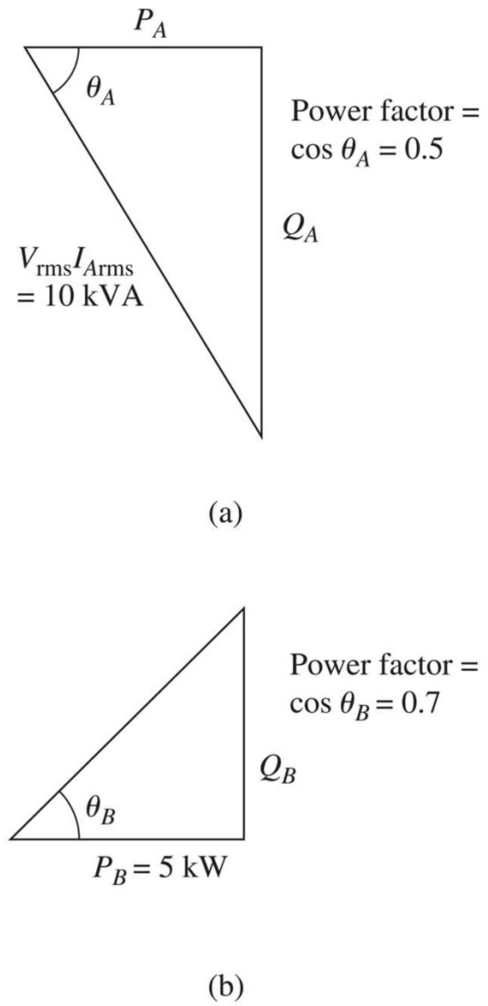


Figure 1.8 Power triangles for loads *A* and *B* of Example 1.2

Thus, the total load active power is given by

$$P = P_A + P_B = 5\ kW = 5\ kW = 10\ kW$$

Thus, the total load reactive power is given by

$$Q = Q_A + Q_B = -8.66\ kVAR + 5.101\ kVAR = -3.559\ kVAR$$

Thus, we have the total load power given by

$$\mathbf{S} = P + jQ = 10\ kW - j3.559\ kVAR = 10.61 \angle -19.59^0\ kVA$$

Thus, we have

$$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = \frac{1}{2}(1414\angle 30^0)\mathbf{I}^* = P + j\,Q = 10.61\angle -19.59^0\ kVA \quad \Rightarrow \mathbf{I}^* = 15.0\angle{-49.59^0}$$

$$\Rightarrow \qquad \mathbf{I} = 15.0\angle 49.59^0$$

The corresponding phasor diagram is as shown in Fig. 1.9, in which the current leads the voltage by an angle of 19.59^0.

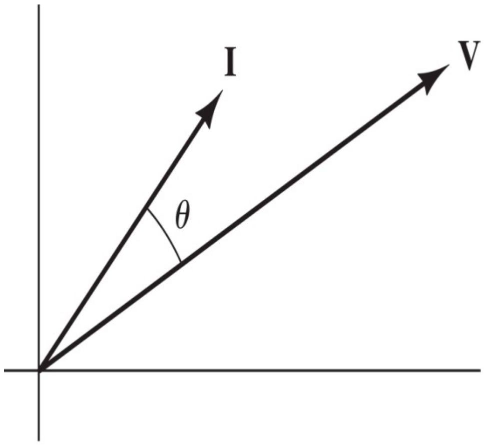


Figure 1.9 Phasor diagram for Example 1.2.

1.1.12: Power-Factor Correction

We have seen earlier that with energy storage elements in AC circuits, large amount of current flows through the circuit without average power being delivered. In heavy industries most of the loads are partly inductive in nature and large amount of reactive power flow takes place. The presence of reactive power increases the apparent power and therefore the current drawn from the mains supply in the power distribution networks. Thus, the transmission lines as well as the distribution transformers have higher ratings than that would be necessary to deliver the same average active power to a resistive load.

Typically, for heavy industries the energy rates are charged not only on the average kWhr of energy used by the consumer but also on the power factor at which they draw the active power with higher energy cost for operating at lower power factor. For this reason, the customers put power factor correction circuits by using capacitors to deliver the inductive kVAR of the load so that the power factor is improved and less kVAR is drawn from the source.

Example 1.3

A 50-kW load operates from 1 50 Hz, 10 KV rms line with a power factor of 60 percent lagging. Compute the capacitance that must be placed in parallel with the load to achieve a 90 percent lagging power factor.

Solution:

The load power angle is

$$\theta_L = cos^{-1}(0.6) = 53.13^0$$

Using the power-triangle concept, we have

$$Q_L = P_L \tan(\theta_L) = 50\ kW\ \times \tan(53.13^0) = 66.67\ kVAR$$

After adding the capacitor in parallel the power factor is improved to 0.9 lag, so the ne load power angle is

$$\theta_{L,new} = cos^{-1}(0.9) = 25.84^0$$

$$Q_{L,new} = P_L \tan(\theta_{L,new}) = 50\ kW\ \times \tan(25.84^0) = 24.22\ kVAR$$

Thus, the reactive power of the capacitance must be

$$Q_C = Q_{L,new} - Q_L = 24.22 - 66.67\ kVAR = -42.45\ kVAR$$

$$Q_C = \frac{V_{C,rms}^2}{X_C} \Rightarrow X_C = \frac{V_{C,rms}^2}{|Q_C|} = \frac{(10^4)^2}{42.45\ \times 1000} = 2356\ \Omega$$

$$C = \frac{1}{\omega|X_C|} = \frac{1}{(2\pi 50) \times 2356} = 1.351\ \mu F$$

Summary

- A sinusoidal voltage is given by $v(t) = V_m \cos(\omega t + \theta)$, where V_m is the peak values of the voltage, ω is the angular frequency in rad/sec, and θ is the phase angle. The frequency in hertz is $f = \frac{1}{T}$, where T is the period. Furthermore, $\omega = 2\pi f$.
- The root-mean-square (rms) value of a periodic voltage $v(t)$ is $V_{rms} = \sqrt{\frac{1}{T}\int_o^T v^2\,(t)dt}$. The average power delivered to a resistance by $v(t)$ is $P_{avg} = \frac{v_{rms}^2}{R}$. Similarly, for a current $i(t)$ is $I_{rms} = \sqrt{\frac{1}{T}\int_o^T i^2\,(t)dt}$. The average power delivered to a resistance by $i(t)$ is $P_{avg} = i_{rms}^2 R$.
- For a sinusoidal signal the rms value is the peak value divided by $\sqrt{2}$.
- We can represent sinusoids with phasors. The magnitude of the phasor is the peak value of the sinusoid and the phase angle of the phasor is the same as that of the sinusoid.
- When a sinusoidal current flows through a sinusoidal voltage, the average power delivered is $P = V_{rms} I_{rms} \cos(\theta)$.
- Reactive power is the flow of the energy back and forth between the source and the energy-storage element. We define reactive power to be positive for an inductance and negative for a capacitance. Net energy transferred per cycle by reactive power is zero.
- Apparent power is the product of rms voltage and rms current.

References

1. Allan R. Hambley, Electrical Engineering: Principles and Applications, Fifth Edition, Pearson, 2011. Chapter 5.

2. Giorgio Rizzoni, Principles and Applications of Electrical Engineering, Fifth Edition, McGraw-Hill, 2007. Chapters 7.

Chapter2: Introduction to Digital Logic Circuits, Operational Amplifiers, Light Emitting Diodes (LEDs) and Light Dependant Resistors (LDRs)

Learning Objectives:

- Understanding the advantage of digital over analog technology
- Understanding of terminology of digital circuits
- Understanding of binary arithmetic operations used in computers and other digital systems
- Interconnections of logic gates of various types to implement a given logic function
- Formulation of truth table
- De Morgan's Laws
- Sum-of-products (SPO) and Products-of-sum (POS) in logical expressions
- Karnaugh maps
- Ideal and real operational amplifiers, inverting and non-inverting amplifier circuits
- LED drive circuit
- LDR as sensor

2.1: Logic Circuits

All the signals that we have considered so far are analog in nature i.e. an analog signal is an electrical signal whose value varies in correspondence with a physical quantity e.g. (temperature, light intensity, force etc.). An analog signal $f(t)$ as shown in Fig. 2.1 (a) has a single value for each value of time, t that it can take a series of values in a given range – for example the voltage proportional to measured variable pressure in an internal combustion engine (ICE).

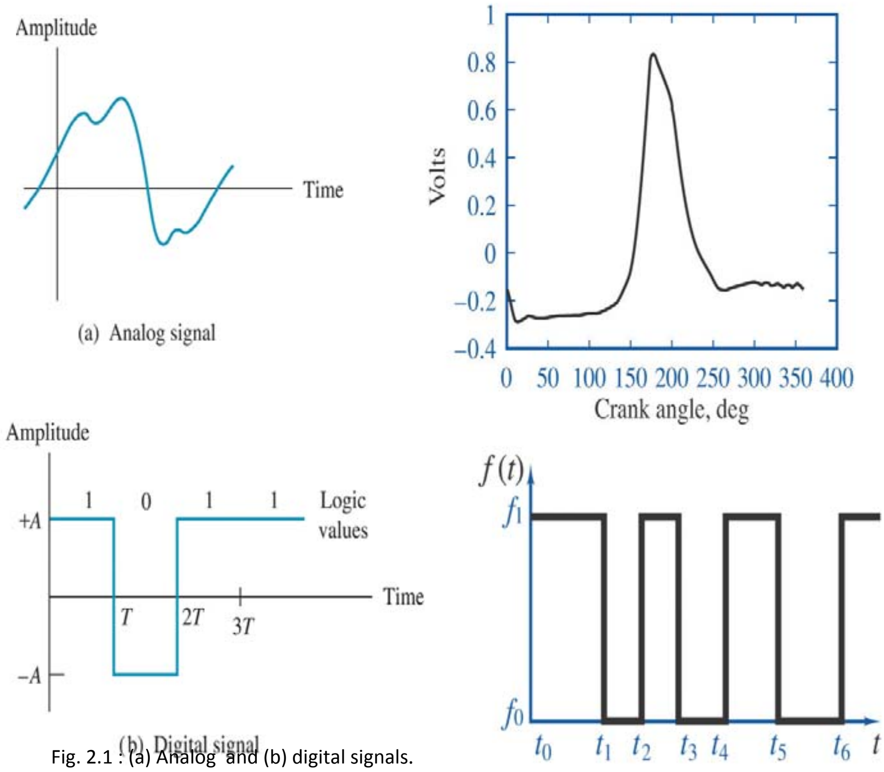


Fig. 2.1 : (a) Analog and (b) digital signals.

For a digital signal, it can take *only a finite number of values* and each value in a given range has the same significance e.g. the binary signal can only take two values i.e. logic values “1” or “0” as shown in Fig. 2.1(b). Computers are example of digital circuits.

2.1.1 Basic Logic Circuit Concepts

We often come across analog signals in various physical systems e.g. current, voltage, speed and torque of a DC motor. These analog signals can be converted into digital form that contains more or less the same information and the signals in digital form can be fed to the computers to do some mathematical processing for subsequent usage. In many applications, we have a choice between analog and digital approaches.

2.1.1.1 Advantages of the Digital Approach

Digital signals have several important advantages over analog signals. When noise is added to analog signal then it becomes difficult to determine the actual amplitude of the original signal. On the other hand, when noise is added to digital signal it is still possible to determine the logic values – provided the noise signal is not that high as shown in Fig. 2.2.

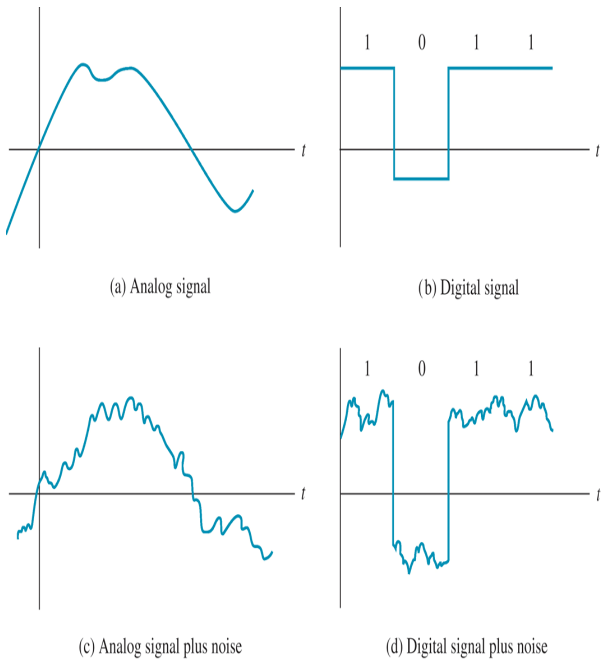


Fig. 4.2: Analog and digital signals with noises.

Analog electronic circuits demand precision components that would be difficult to manufacture whereas it is much easier to produce digital circuits and even if the voltage fluctuates due to addition of noise still they would be around the logic values as shown in Fig. 4.6(d). Thus, the importance of using digital system in the past two/three decades. Moreover, as computers operate on digital signal the controllers are also implemented in digital form and then converted into analog form to interface with the analog physical world.

Higher amplitude in a binary system represents “1” and the lower-amplitude range represents “0” and it is referred to as positive logic. Logic “1” is also referred to as **high**, **true** or **on** and logic “0” is also referred to ***low, false*** or ***off***.

Logic circuits are designed such that they accept logic high or low for a range of input and output voltages. The largest input voltage that is considered as logic “0” is denoted as V_{IL} and the smallest input voltage accepted as logic “1” is denoted as V_{IH} as shown in Fig. 2.3.

The output voltages fall into narrow ranges than the inputs – V_{OL} highest logic - 0 output voltage and V_{OH} highest logic -1 output voltage. Outputs have narrower ranges than the acceptable inputs as noise can be added at interconnections between inputs and outputs.

The differences are called *noise margins*:

$$NM_L = V_{IL} - V_{OL} \qquad NM_H = V_{OH} - V_{IH} \qquad (2.1)$$

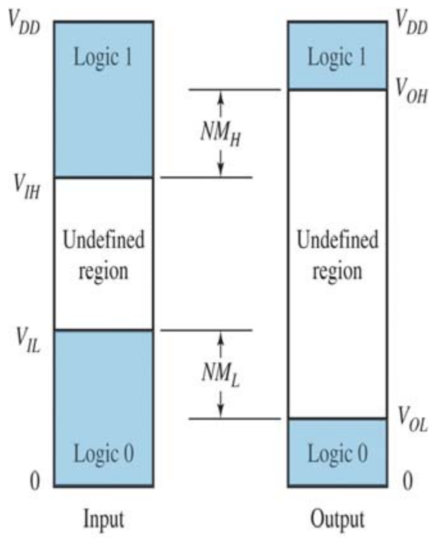


Fig. 2.3 : Voltage ranges for logic-circuit inputs and outputs.

Digital Words

A single binary digit is called a "**bit**" and can take either a 0 or a 1. To represent more information, we can use group of logic variables called *digital words*. A "**byte**" is a digital word consisting of 8 bits and a "**nibble**" is a four-bit digital word.

Transmission of Digital Information

For parallel transmission of a *n*-bit digital word, *n+1* number of wires are needed (n for n bit data and one for GND signal). For serial transmission of *n*-bit word the successive bits of the word are transmitted one after another on a single-pair of wire.

Parallel transmission is *faster but expensive* and therefore used for short-distance transmission whereas *serial-transmission* is *slower but cheap* and typically used for long distances.

2.1.1.2 Representation of Numerical Data in Binary Form

Digital words can represent numerical data. With three bits, we can have 2^3 distinct words.

000	–	0
001	–	1
010	–	2
011	–	3
100	–	4
101	–	5
110	–	6
111	–	7

Similarly, a four-bit word has 16 different combinations that represent integers 0 through 15.

4.2 Boolean Algebra

The mathematics associated with binary number systems is called Boolean – in honor of English Mathematician George Boole (1854). Boolean algebra is useful in the analysis and synthesis of logical expressions. The value of the logical expression boils down to any one of the two logical outcomes i.e. **TRUE** or **FALSE**.

In Boolean algebra, a variable *x* may take any one of the two possible values 1 or 0.

$$x = 1 \; or \; x = 0 \qquad (2.2)$$

which may represent (a) True or False of a statement, (b) ON or OFF state of a switch or (c) HIGH (5 V) or LOW (0V) of a voltage level.

Electrical circuits used to represent logical expressions are known as *logic circuits*. Such logic circuits are used in many industrial processes, computers, communication devices etc. to make logical decisions.

Logic circuits are usually represented by logic operations involving Boolean variables. There are three basic logic operations:

- **OR** operation
- **AND** operation
- **NOT** operation

The **OR** operation represents the following logical statement:

$$\text{If either } A \text{ or } B \text{ is true } (1) \text{ then } C \text{ is true } (1) \qquad \text{Logical OR} \quad (2.3)$$

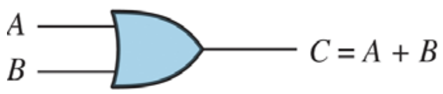
The associate truth table of the OR operation and the symbol of the OR gate are as shown in Fig. 2.4(a).

A	B	$C = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

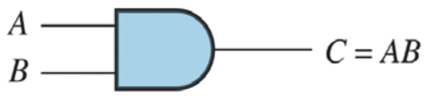
(a) Truth table

A	B	$C = AB$
0	0	0
0	1	0
1	0	0
1	1	1

(a) Truth table



(b) Symbol for two-input OR gate



(b) Symbol for two-input AND gate

Fig. 2.4 : Symbol and truth-table for (a) OR gate and (b) AND gates.

The AND gate represents the following logical statement:

$$\text{If both } A \text{ and } B \text{ are true (1) then } C \text{ is true (1)} \quad \text{Logical AND} \quad (2.4)$$

The associated truth table of the AND operation and the symbol of the AND gate are as shown in Fig. 2.4(b).

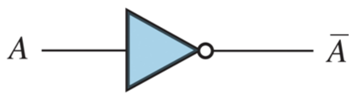
The NOT gate represents the following logical statement:

$$\text{If } A \text{ is true (1) then } \overline{\mathrm{A}} \text{ is false (0)} \qquad \text{Logical NOT} \quad (2.5)$$

The associated truth table of the NOT operation and the symbol are as shown in Fig. 2.5.

A	$\overline{A}$
0	1
1	0

(a) Truth table



(b) Symbol for an inverter

Figure 2.5: Truth table and symbol of NOT gate.

Some of the basic laws for the Boolean algebra are shown in Fig. 2. 6.

$$
\begin{array}{ll}
\text{for AND operation} & \text{for OR operation} \\
AA = A & A + A = A \\
A1 = A & A + 1 = 1 \\
A0 = 0 & A + 0 = A \\
AB = BA & A(B+C) = AB + AC \\
A(BC) = (AB)C = ABC & (A+B)+C = A+(B+C) = A+B+C \\
\text{for NOT operation} & \\
A\overline{A} = 0 & \overline{\overline{A}} = A
\end{array}
$$

Figure 2.6: Basic laws of Boolean algebra for AND, OR and NOT operations.

2.2.1 Implementation of Boolean Expressions

Boolean algebra expressions can be implemented using AND, OR gates and inverters. For example, the logic expression as given in eqn. 2.6 can be implemented by using logic circuit as shown in Fig. 2.7 using AND, OR and NOT gates.

$$
\begin{aligned}
F &= A\overline{B}C + ABC + (C+D)(\overline{D}+E) && (2.6) \\
&= A\overline{B}C + ABC + C\overline{D} + CE + D\overline{D} + DE \\
&= AC(\overline{B}+B) + C\overline{D} + CE + DE && (\because D\overline{D} = 0) \\
&= AC + C\overline{D} + CE + DE && (\because B + \overline{B} = 1) \\
&= C(A + \overline{D} + E) + DE && (2.6a)
\end{aligned}
$$

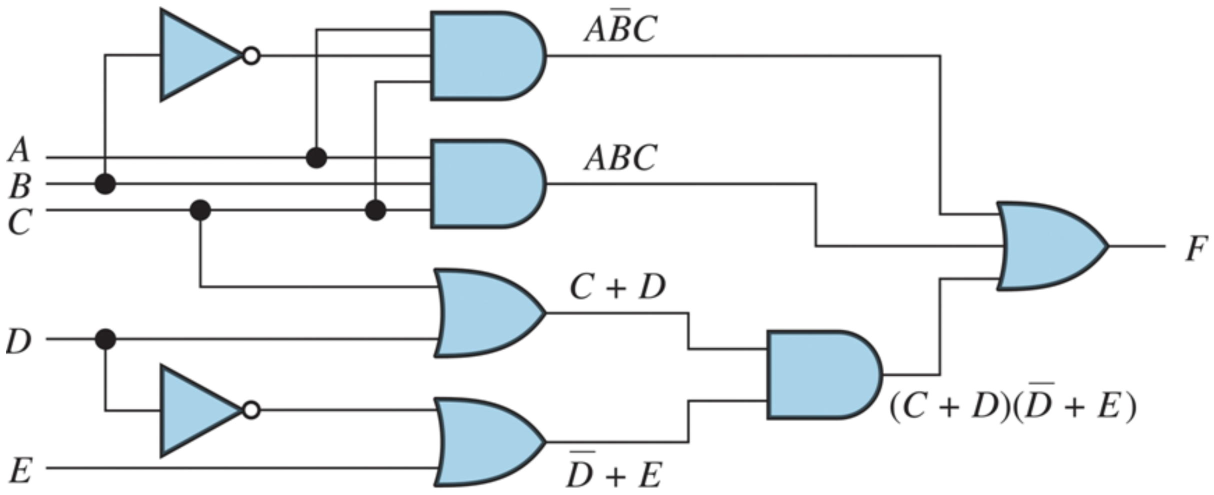


Figure 2.7: Logic circuit that implements the logic expression *F* as shown in eqn. (2.6).

Sometimes, it is possible to manipulate the logic expression so that a simpler logic expression can be obtained that is much easier and cheaper to implement. For example, the logic expression of eqn. (2.6) can be simplified to result in a simpler expression as shown by eqn. (2.6a) and the corresponding logic circuit is as shown in Fig. 2.8.

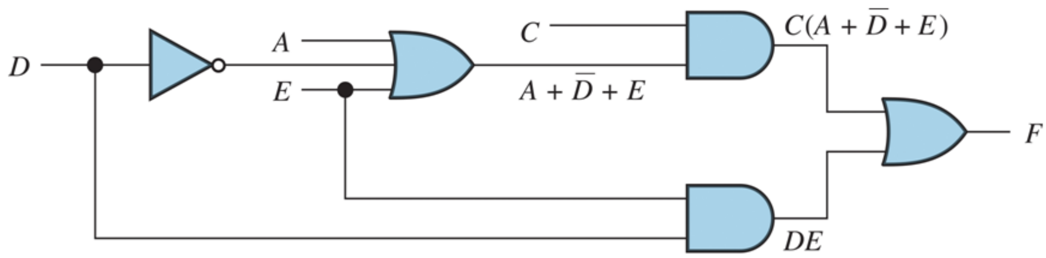


Figure 2.8: Logic circuit that implements the logic expression *F* in a simpler circuit as shown in eqn. (2.6a).

2.2.2 De Morgan's Laws

Two important results of the Boolean algebra are the De Morgan's laws given by

$$ABC = \overline{\overline{A}+\overline{B}+\overline{C}} \qquad (2.7)$$

$$(A+B+C) = \overline{\overline{A}\,\overline{B}\,\overline{C}} \qquad (2.8)$$

In another way, the De Morgan's Laws can be stated as: If the variables in a logic expression are replaced by their *inverses*, then the AND operation in the original expression is replaced by OR operation and the OR operation is replaced by AND operation and the entire expression is inverted, resulting the same logic expression that yields the same value as the original one before the changes are incurred.

Any logic function can then be implemented by using ONLY OR and NOT gates (2.7) or AND and NOT gates (2.8). For example, the De Morgan's Laws using two variables *X* and *Y* are shown in Fig. 2.9 where the logic function *Z* has been realized using either AND and NOT gates or OR and NOT gates.

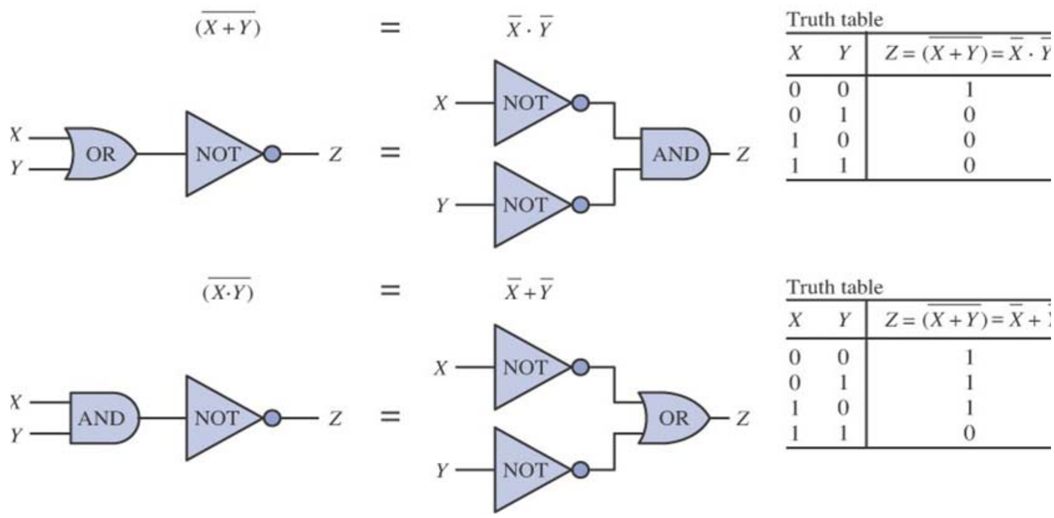


Figure 2.9: De Morgan's Laws using two variables.

The importance of De Morgan's laws lies in the statement of the *duality* that exists between AND and OR operations; any function can be realized by just one of the two basic operations i.e. AND or OR operation, plus the complement i.e. NOT operation.

This gives rise to families of logic functions, *sum of products* (SOP) and *products of sums* (POS). Any logical expression can be reduced to one of the two basic forms. Although the two forms are equivalent but one of the two would be simpler from implementation point of view with fewer gates.

Example 2.1

Fail-safe Autopilot Logic: A fail-safe autopilot system in a commercial aircraft requires that, prior to initiating a take-off or landing manoeuvres, the following check must be passed i.e. two of three possible pilots' i.e. the *pilot,* the *co-pilot* and the *auto-pilot* must be available. Imagine there are switches installed on the pilot and co-pilot's seats so that if they are on their respective seats (due to their weight, the switch would have been closed) then the switches would provide a HIGH or 5V and if they are not on their seats then the switches would provide a LOW or 0 V signal. Similarly, the electronic circuit would provide a 5 V or 0 V if the auto-pilot circuit is working properly or not. Let the three logic variables X – represent the pilot, Y – represent the co-pilot and Z – represents the auto-pilot.

Solution:

Since we wish two of the conditions out of the three must be active before the aircraft manoeuvre can be initiated, the logic function corresponding to the "system ready" is given by

$$f = X \cdot Y + X \cdot Z + Y \cdot Z$$

The corresponding truth table is given by

Pilot (X)	Co-pilot (Y)	Auto-pilot (Z)	System ready
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

The logic function f defined above truth table is based on the notation of *positive check*, i.e. it indicates when the system is ready.

Now applying the De Morgan's Laws to the function f which is the *sum-of-products* (SOP) we have

$$\bar{f} = g = \overline{X \cdot Y + X \cdot Z + Y \cdot Z} = (\bar{X} + \bar{Y}) \cdot (\bar{X} + \bar{Z}) \cdot (\bar{Y} + \bar{Z})$$

The function g on the other hand is in the *product-of-sums* (POS) form and conveys exactly the same information as the function f but it performs a *negative check* i.e. it verifies when the *system is not in* *ready condition*. It is up to the user to choose one or the other and use it for implementing the logic function.

2.2.3 Synthesis of Logic Circuits

Sometimes the specifications for the output are given in terms of the inputs in natural language. This can be translated into a truth table and from the truth table the Boolean logic expression can be written to implement the logic circuit.

Consider the truth table as shown in Table 2.1.

Table 2.1: Truth Table used to illustrate SOP Logical Expression

Rows	A	B	C	D	
0	0	0	0	1	$(\overline{A}\,\overline{B}\,\overline{C})$
1	0	0	1	0	
2	0	1	0	1	$(\overline{A}B\overline{C})$
3	0	1	1	0	
4	1	0	0	0	
5	1	0	1	0	
6	1	1	0	1	$(AB\overline{C})$
7	1	1	1	1	(ABC)

$D = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + AB\overline{C} + ABC$ (SOP)

A, B and *C* are the three logic input variables and *D* is the logic variable that is the desired output. Suppose we want to implement a logic circuit that would produce the desired output, *D* then one way to write the logic expression for *D* is to concentrate on the rows on the truth table where the output variable *D* is 1. These are the rows numbered as 0, 2, 6 and 7 then we write logical product of the input logic variables or their inverses such that equals 1 for each on these selected rows. Each input variable or its inverse is included in each product. Finally, we write the logical expression for the output variable as a logical sum of each product terms as shown in eqn. 2. 9.

$$D = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} + ABC \quad (2.9)$$

This expression is called a ***sum-of-products*** (SOP). The logic circuit for the implementation of eqn. (2.9) is shown in Fig. 2. 10.

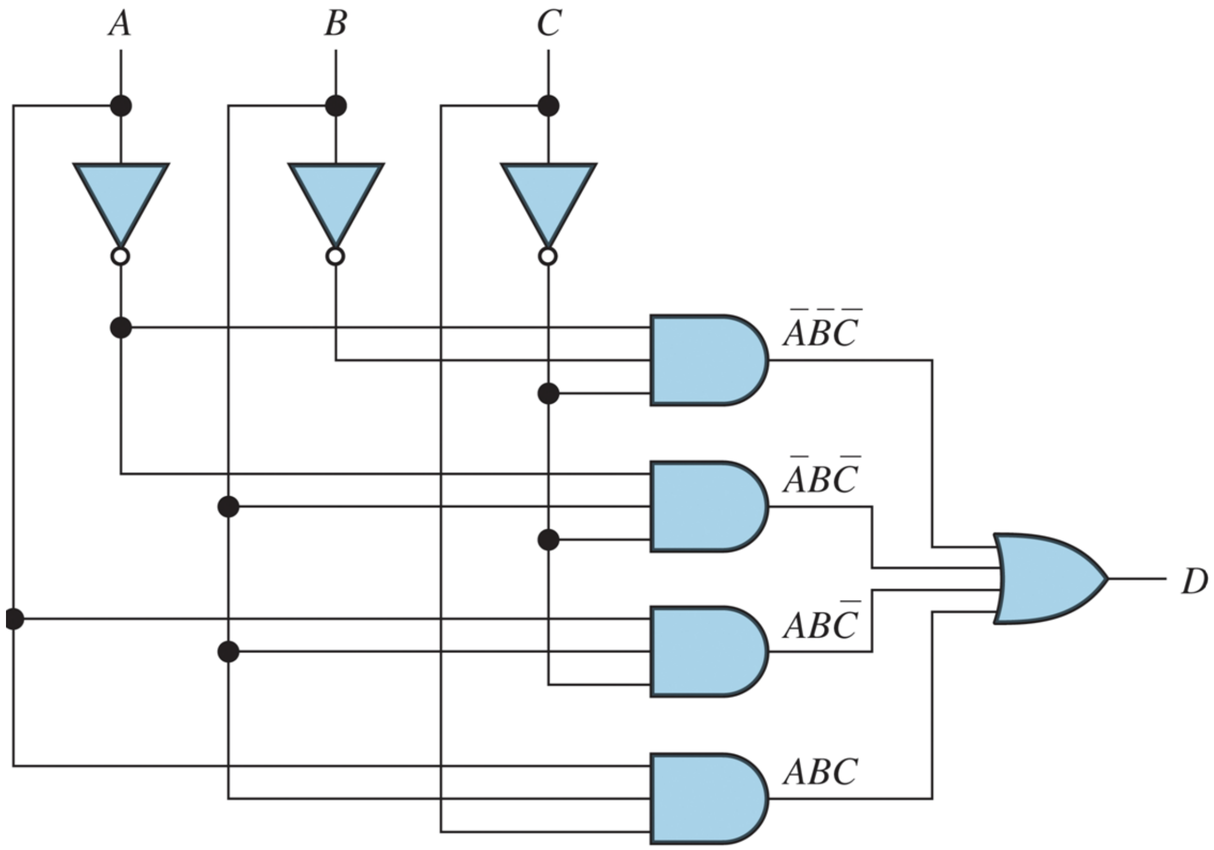


Figure 2.10: SPO logic circuit for the truth Table 2.1.

Another way to write a logic expression for *D* as shown in Table 2.1. is to concentrate on the rows of the truth table for which *D* is 0. For example, in Table 2.1, the corresponding rows are numbered as 1, 3, 4 and 5. Then we write a logical sum that equals 0 for each of these rows. Each input variable or their inverses is included in each sum. While writing the logic variables that are 1 in that row e.g. for row nuber 1, we have logic varoable $A = 0$, $B = 0$ and $C = 1$ so we inverse logic variable *C* to result a 0, so the term for row 1 is $(A + B + \bar{C})$ as shown in truth table Table 2.2.

Table 2.2. - Truth Table used to illustrate POS Logical Expression

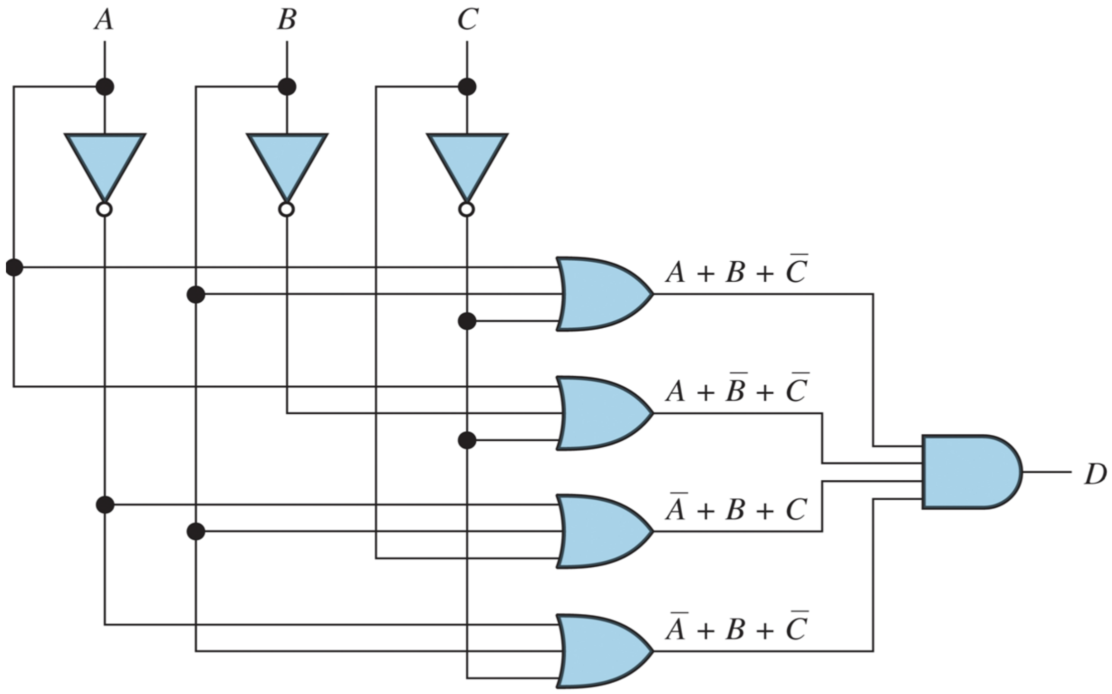
Rows	A	B	C	D	
0	0	0	0	1	
1	0	0	1	0	$(A+B+\overline{C})$
2	0	1	0	1	
3	0	1	1	0	$(A+\overline{B}+\overline{C})$
4	1	0	0	0	$(\overline{A}+B+C)$
5	1	0	1	0	$(\overline{A}+B+\overline{C})$
6	1	1	0	1	
7	1	1	1	1	

$$D=(A+B+\overline{C})(A+\overline{B}+\overline{C})(\overline{A}+B+C)(\overline{A}+B+\overline{C}) \quad \text{(POS)}$$

Finally, we write the logical expression for the output variable, *D* as a logical product of each sum terms as shown in eqn. 2. 10.

$$D = (A + B + \bar{C}) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + C) \cdot (\bar{A} + B + \bar{C}) \quad (2.10)$$

This expression is called a ***products -of -sum*** (POS). The logic circuit for the implementation of eqn. (2.10) is shown in Fig. 2. 11.



$$D = (A + B + \overline{C})(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C}) \ \textbf{(POS)}$$

Figure 4.11: POS logic circuit for the truth Table 2.1/Table2.2.

2.2.4 Minimization of Logic Circuits

We have seen that any logic function can be implemented in either POS or SOP form but such direct implementation may not yield the best circuits in terms of minimizing the number of gates required. For example the logic expression given below

$$F = \bar{A}\,\bar{B}D + \bar{A}\,BD + BCD + ABC \qquad (2.11)$$

can be implemented using two NOT gates and four AND gates and one OR gate.

$$F = \bar{A}\,\bar{B}D + \bar{A}\,BD + BCD + ABC = (\bar{A}\,\bar{B}D + \bar{A}\,BD) + BCD + ABC$$

$$= \bar{A}D\,(\bar{B} + B) + BCD + ABC = \bar{A}D + BCD + ABC \ [\because \ (\bar{B} + B) = 1] = \bar{A}D + ABC$$

The term *BCD* is redundant as it can be 1 only when *B* = 1, *C* = 1 and *D* = 1 and in that case the same can be realized through term $\bar{A}D$ if $\bar{A}$ = 1 or through the term *ABC* if *A* = 1. Thus, we need one inverter, two AND gates and one OR gate to realize the logic expression.

2.2.5 Karnaugh Maps

We have shown in the previous Section that logic expressions can sometimes be simplified. The algebraic manipulations needed to simplify the given expression are not that apparent. However, a graphical approach known as *Karnaugh map* can be used to minimize the number of terms in a logic expression.

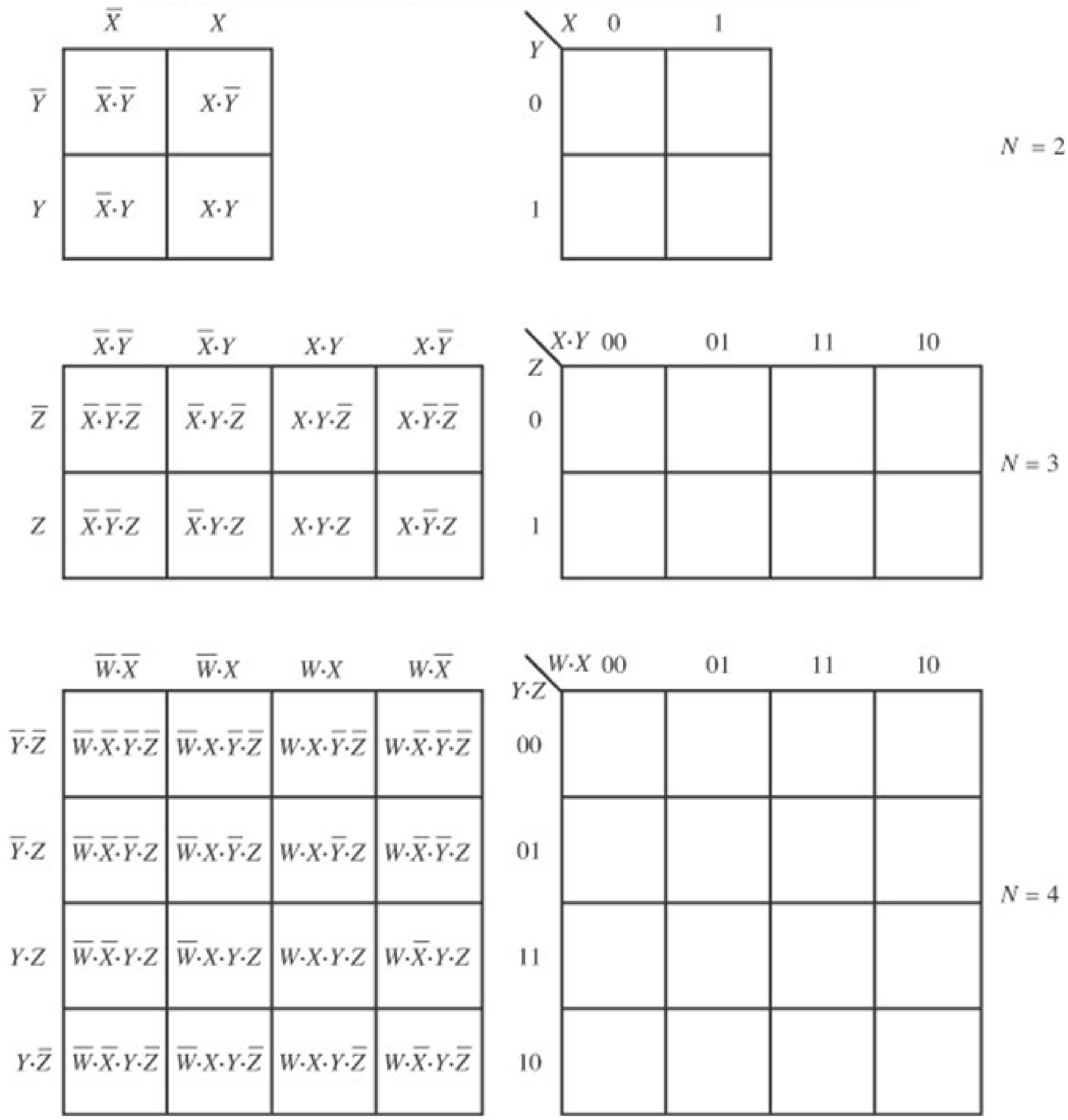


Figure 2.12: Two-, three- and four-variable Karnaugh maps.

Figure 2.12 shows the Karnaugh maps for two-, three- and four logic variables. As can be seen, the rows and columns are arranged in such a way that all adjacent terms change by only one bit e.g. for the three variable map, the column next to the column 01 are columns 00 and 11. Also note that each map consists of 2^N cells where *N* is the number of logic variables.

Consider the following truth table

A	*B*	*X*
0	0	1
0	1	0
1	0	0
1	1	1

The logical expression *X* can be written from the truth table as given by eqn.

$$X = \bar{A} \cdot \bar{B} + A \cdot B \qquad (2.12)$$

The corresponding Karnaugh map for this expression is represented as above. No further simplification can be done.

	A	$\bar{A}$
B	1	0
$\bar{B}$	0	1

Figure 2.13: Karnaugh map for logical expression as shown in eqn. (2.12).

Now consider the logical expression *Y* as given by eqn.

$$Y = \bar{A} \cdot B + A \cdot B \qquad (2.13)$$

The corresponding Karnaugh map for this expression is shown in Fig. 2.14.

	A	$\bar{A}$
B	1	1
$\bar{B}$	0	0

Figure 2.14: Karnaugh map for logical expression as shown in eqn. (2.12).

The two adjacent squares may be combined together as shown by the loop. It simply means that we are combining the two terms of the expression for *Y* to yield a simpler expression as follows.

$$Y = \bar{A} \cdot B + A \cdot B = (\bar{A} + A) \cdot B = 1 \cdot B = B$$

Now consider the logical expression *Z* as given by eqn.

$$Z = \bar{A} \cdot B + A \cdot B + \bar{B} \cdot A \qquad (2.14)$$

The corresponding Karnaugh map for the expression in eqn. (2.14) is shown in Fig. 4.15.

	A	$\bar{A}$
B	1	1
$\bar{B}$	1	0

Figure 4.15: Karnaugh map for logical expression as shown in eqn. (2.14).

The two adjacent squares may be combined together as shown by the loop. It simply means that we are combining the two terms of the expression for *Z* to yield a simpler expression as follows.

$$Z = \bar{A}\cdot B + A\cdot B + \bar{B}\cdot A \;= \bar{A}\cdot B + A\cdot B + A\cdot B + \bar{B}\cdot A \;=\; (\bar{A} + A)\cdot B + A\cdot(\bar{B} + B)$$
$$= 1\cdot B + A\cdot 1 = B + A$$

Combining two adjacent squares in Karnaugh map eliminates one variable form the resulting Boolean expression of the corresponding squares.

Now consider the following truth table of three logic variables.

A	*B*	*C*	*X*
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

The logical expression *X* is given by

$$X = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + \bar{A}\cdot B\cdot\bar{C} + A\cdot B\cdot\bar{C} \quad (2.15)$$

The corresponding Karnaugh map for the expression in eqn. (2.15) is shown in Fig. 2.16.

	AB	$A\bar{B}$	$\bar{A}\bar{B}$	$\bar{A}B$
C	0	0	1	0
$\bar{C}$	1	0	1	1

Figure 2.16: Karnaugh map for logical expression as shown in eqn. (2.15).

The simplified expression of *X* is given by:

$$X = \bar{A}\cdot\bar{B} + B\cdot\bar{C} \quad (2.16)$$

2.3 Operational Amplifiers

A major function of the electronic instrumentation circuit is the amplification of low level electrical signal to a higher-level signal is of prime importance in many engineering applications. For example, the low-level signal from a music player to be amplified to a level suitable for driving high-wattage speaker or a low-level signal picked up from a sensor such as LDR in your project and use it to amplify to higher level so that it can detect whether the vehicle is on track or not and then take necessary action to drive the motor to move towards the track if not already on it. Also amplifiers are needed to amplify the low-level signal from transducers such as temperature sensor, light sensor etc. to high-level so that it can be used for other function in an electronic circuit.

We have seen *discrete* semiconductor devices like bipolar-junction-transistors (BJTs) and MOSFETs but rather than using discrete devices it is possible to make use of a circuit that consists of numbers of BJTs, MOSFETs, resistors and capacitors all integrated in one chip for performing wide range of engineering instrumentation and that is known as *operational amplifiers.* These components for a given circuit can be manufactured on a single silicon crystal known as chip. Circuits manufactured in this way are called *integrated circuits* (ICs).

2.3.1 Ideal Amplifiers

Simplest model of an amplifier is shown in Fig. 2.17(a), where $v_S(t)$ is the input signal, $v_L(t)$ is the output signal, R_S is the source resistance, R_L is the load resistance and A is the gain of the ideal amplifier.

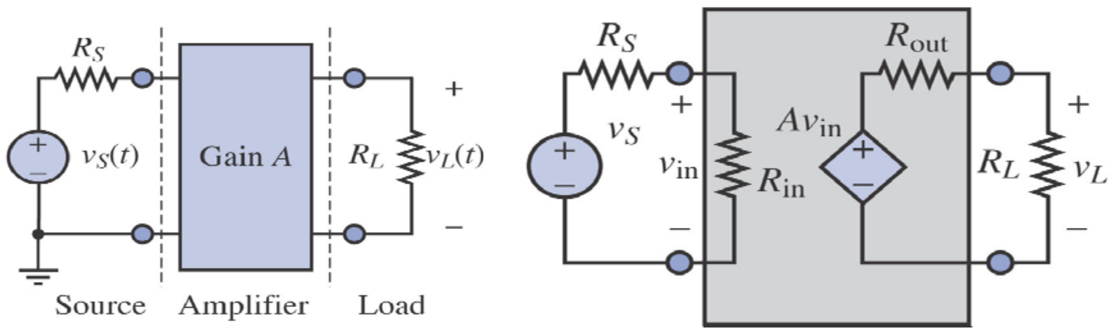


Fig. 2.17: (a) A voltage amplifier and (b) simple voltage amplifier model.

Ideally, the load voltage should be

$$v_L(t) = A \times v_S(t) \qquad (2.17)$$

Source has been modeled as Thevenin's equivalent circuit and the load as an equivalent resistance, R_L. The amplifier can act as an equivalent load to the source and an equivalent source to the load as per the Thevenin's theorem. The equivalent circuit of the voltage amplifier circuit is shown in Fig. 2.17(b) where $v_S(t)$ - input signal, $v_L(t)$ - output signal, R_S - source resistance, R_L - load resistance, A - gain of the ideal amplifier, $v_{in}(t)$ - input signal, R_{in} - input resistance, Av_{in} - controlled voltage source, and R_{out} -internal resistance.

$$v_{in} = \frac{R_{in}}{R_S + R_{in}} v_S \qquad v_L = A v_{in} \frac{R_L}{R_{out} + R_L} \qquad (2.18)$$

$$v_L = \left(A \frac{R_{in}}{R_S + R_{in}} \times \frac{R_L}{R_{out} + R_L} \right) v_S$$

v_L depends on R_S, R_L, R_{in} and R_{out} - not desirable

If $R_{in} >> R_S$ and $R_L >> R_{out}$ we have

$$v_L = A v_S \qquad (2.19)$$

Thus, the two desirable characteristics for a general-purpose amplifier circuit are: (1) *large input impedance*, R_{in} and (2) *very small output impedance*, R_{out}.

The ideal operational-amplifier (op-amp) behaves like an ideal *difference amplifier* i.e. an amplifier that amplifies the difference between the two input voltages as shown in Fig. 2.18. The input voltage at the +ve terminal is *the non-inverting input voltage*, v^+ and the same at the –ve terminal is called *the inverting input voltage*, v^-, and $A_{V(OL)}$ is the *open-loop voltage gain* and is quite large of the order of 10^5 - 10^7.

The simplified op-amp symbol is as shown in Fig. 2.18(b) which has two input thermals known as inverting and non-inverting terminals, one output terminal and two terminals for the power supply voltages. For analysis of op-amp we assume that input current, i_{in} ~ 0, which is realistic as R_{in} is almost infinite.

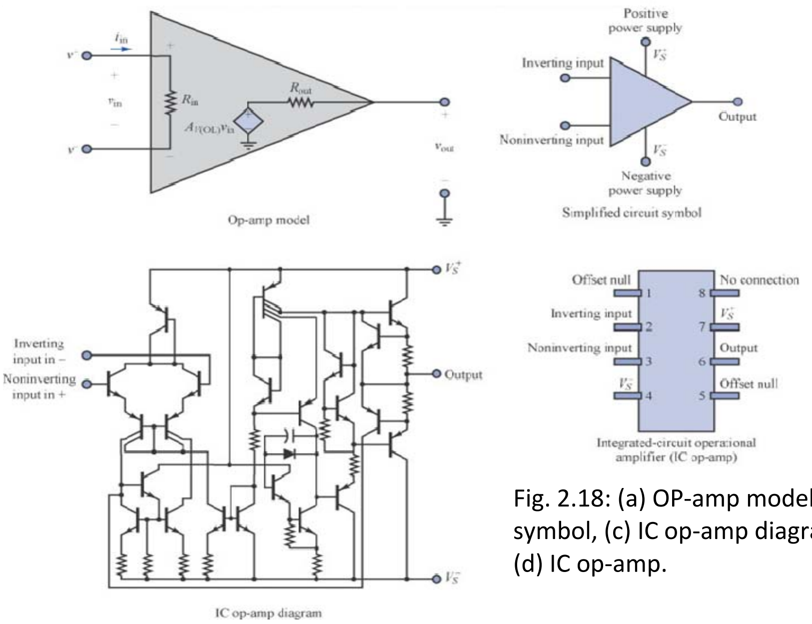


Fig. 2.18: (a) OP-amp model, (b) symbol, (c) IC op-amp diagram and (d) IC op-amp.

2.3.1.1 The Inverting Amplifier

One of the most widely used amplifier circuit is the so called *inverting amplifier*. The signal to be amplified is connected to the inverting terminal, v^- while the non-inverting terminal, v^+ is grounded as shown in Fig. 2.19.

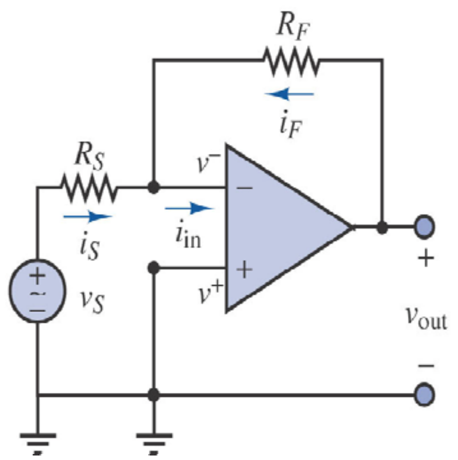


Fig. 2.19: Inverting amplifier.

It can be shown that the voltage gain of the amplifier can be set at any arbitrary value by choosing the ratio of two resistances R_S and R_F.

The effect of the feedback connection from output to inverting input terminal is to force the voltage at the inverting input to be equal to that at the non-inverting input. This condition at the op-amp input terminals is called as a *virtual short circuit*.

$$i_S + i_F = i_{in} \qquad (2.20)$$

$$i_S = \frac{v_S - v^-}{R_S} \qquad (2.21)$$

$$i_F = \frac{v_{out} - v^-}{R_F}, \quad i_{in} = 0 \quad (2.22)$$

$$v_{out} = A_{V(OL)}\left(v^+ - v^-\right) = A_{V(OL)}\left(0 - v^-\right) = -A_{V(OL)}v^- \Rightarrow v^- = -\frac{v_{out}}{A_{V(OL)}} \qquad (2.23)$$

$$i_S = -i_F \Rightarrow i_S = \frac{v_S - v^-}{R_S} = \frac{v_S - \left(-\frac{v_{out}}{A_{V(OL)}}\right)}{R_S} = \frac{v_S}{R_S} + \frac{v_{out}}{A_{V(OL)}R_S}$$

$$-i_F = -\frac{v_{out} - v^-}{R_F} = -\frac{v_{out} - \left(-\frac{v_{out}}{A_{V(OL)}}\right)}{R_F} = -\frac{v_{out}}{R_F} - \frac{v_{out}}{A_{V(OL)}R_F}$$

$$\frac{v_S}{R_S} + \frac{v_{out}}{A_{V(OL)}R_S} = -\frac{v_{out}}{R_F} - \frac{v_{out}}{A_{V(OL)}R_F} \Rightarrow \frac{v_S}{R_S} = -\frac{v_{out}}{R_F} - \frac{v_{out}}{A_{V(OL)}R_F} - \frac{v_{out}}{A_{V(OL)}R_S}$$

$$v_S = -v_{out}\left(\frac{R_S}{R_F} + \frac{R_S}{A_{V(OL)}R_F} + \frac{R_S}{A_{V(OL)}R_S}\right)\left(\because A_{V(OL)} = 10^5 - 10^7\right)$$

$$v_S = -v_{out}\left(\frac{R_S}{R_F}\right) \Rightarrow v_{out} = -\left(\frac{R_F}{R_S}\right)v_S \qquad (2.24)$$

$$v^- = -\frac{v_{out}}{A_{V(OL)}} \approx 0 \Rightarrow v^- \approx v^+$$

Example 2.2

Determine the voltage gain and output voltage for the inverting amplifier circuit as shown in Fig. 2.20. Assume that R_s = 1 kΩ, R_F = 10 kΩ, $v_s(t)$ = 0.015 cos(50t) and the operational amplifier is ideal.

Solution:

The voltage gain of the inverting amplifier circuit is given by

$$A_v = \frac{R_F}{R_s} = \frac{10k\Omega}{1k\Omega} = 10, \qquad v_{out} = -\frac{R_F}{R_s} v_s = -10 \times 0.015\cos(50t) = -0.15\cos(50t)$$

The corresponding input and output voltage waveforms are as shown in Fig. 4. 20. Note that v_S and v_{out} are out of phase due to the –ve sign.

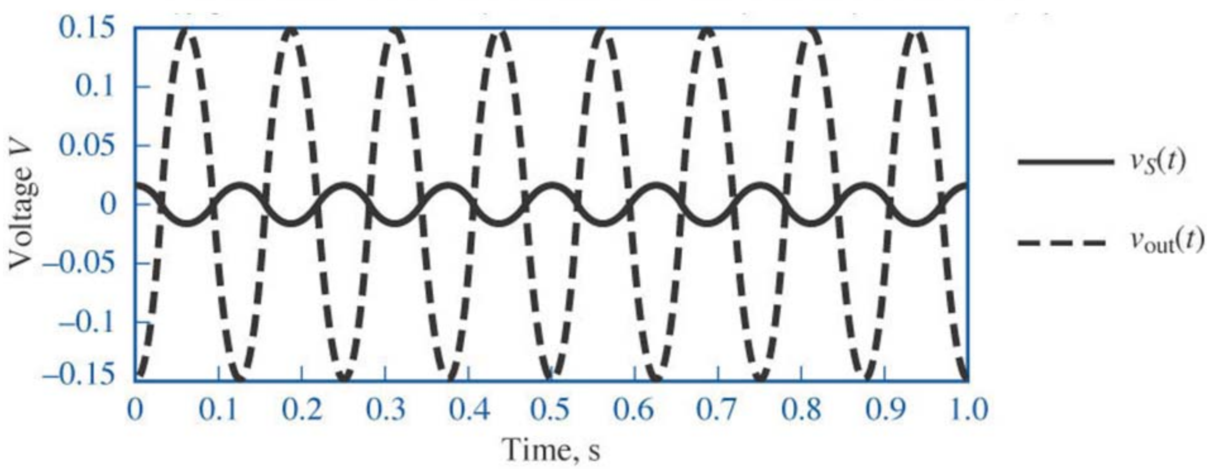


Figure 4.20: Plot of $v_s(t)$ and $v_o(t)$

A useful op-amp circuit that is based on the inverting amplifier is the *summing amplifier* as shown in Fig. 4.21 that is used to add different signal sources.

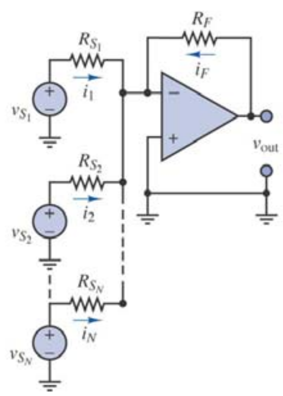


Figure 4.21: Summing amplifier

Applying KCL at the inverting terminal we have

$$i_1 + i_2 + i_3 + i_4 + \cdots\cdots\cdots + i_N = -i_F$$

$$\sum_{n=1}^{N} \frac{v_{s_n}}{R_{s_n}} = -\frac{v_{out}}{R_F}$$

$$v_{out} = -\sum_{n=1}^{N} \frac{R_F}{R_{s_n}} v_{s_n} \quad (2.25)$$

Example 2.3

Find an expression for the output voltage of the circuit as shown in Fig. 2.22.

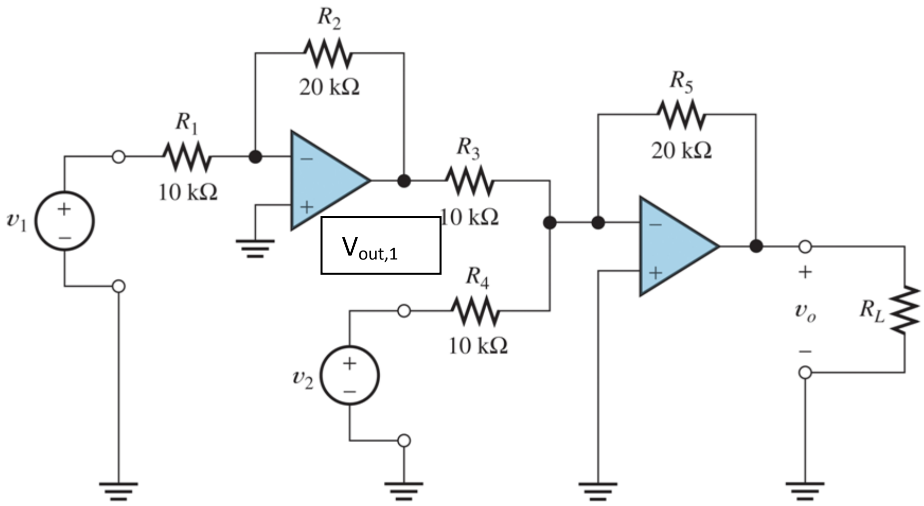


Figure 2.22: Circuit for Example 2.3

$$v_{out,1} = -\frac{R_2}{R_1} v_1 = -2v_1$$

$$v_o = -\left(\frac{R_5}{R_3} v_{out,1} + \frac{R_5}{R_4} v_2\right) = -\left(\frac{20}{10}(-2v_1) + \frac{20}{10} v_2\right) = 4v_1 - 2v_2$$

2.3.1.2 The Non-inverting Amplifier

To avoid the –ve sign i.e. phase inversion introduced by the inverting amplifier a *non-inverting* amplifier configuration as shown in Fig. 4.23 is shown. Please take note that the input signal is applied at the non-inverting terminal of the op-amp.

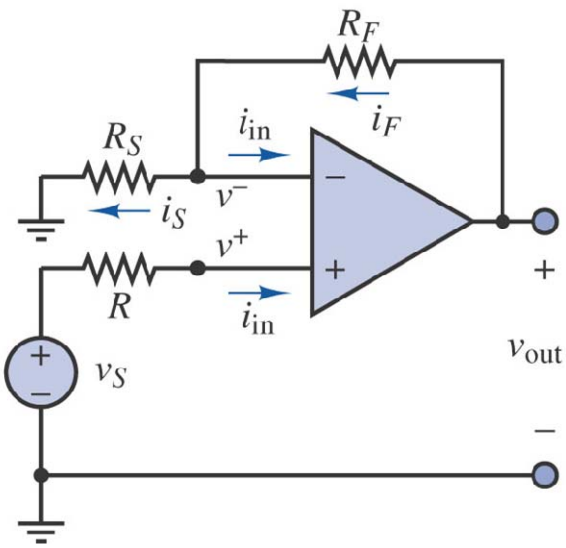


Fig. 4.23: Non-inverting amplifier.

$$i_F = i_S + i_{in} \approx i_S,$$

$$i_S = \frac{v^-}{R_S},\ i_F = \frac{v_{out} - v^-}{R_F},$$

$$i_{in} = 0 \Rightarrow v^+ = v_S = v^-$$

$$i_F = i_S \Rightarrow (i_F =)\frac{v_{out} - v^-}{R_F} = (i_S =)\frac{v^-}{R_S} \Rightarrow$$

$$\frac{v_{out} - v_S}{R_F} = \frac{v_S}{R_S} \Rightarrow \frac{v_{out}}{v_S} = \left(1 + \frac{R_F}{R_S}\right) \qquad (2.26)$$

Thus, by constructing an non-ideal amplifier with a very large gain and near infinite input resistance, it is possible to design amplifiers that have near-ideal performance and can provide a variable range of gains, that can be easily controlled by the selection of external resistors such as R_F and $R_{S.}$

The –ve feedback mechanism allows the near-ideal performance of an op-amp to happen and unless and otherwise stated it is reasonable and sufficient to assume that:

1. i_{in} = 0 and
2. v^- = v^+

Consider the non-inverting op-amp circuit as shown in Fig. 4.24 in which case R_F = 0 and R_s = very large, then according to eqn. (2.26) we have

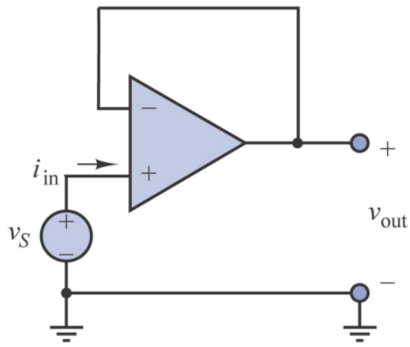


Figure 2.24: Voltage follower

$$\frac{v_{out}}{v_s} = \left(1 + \frac{R_F}{R_s}\right) = \left(1 + \frac{0}{\infty}\right) = 1$$

$$\Rightarrow v_{out} = v_s \qquad (2.27)$$

Thus the output voltage follows the input voltage and that is why the circuit is known as *voltage follower*. The input resistance of the voltage follower circuit is

$$R_{in} = \frac{v_s}{i_{in}} = \frac{v_s}{0} = \infty$$

4.3.1.3 The Differential Amplifier

It is a combination of the inverting and non-inverting amplifiers and finds its application where the difference between the two signals needs to be amplified. The basic differential amplifier circuit is as shown in Fig. 2.25.

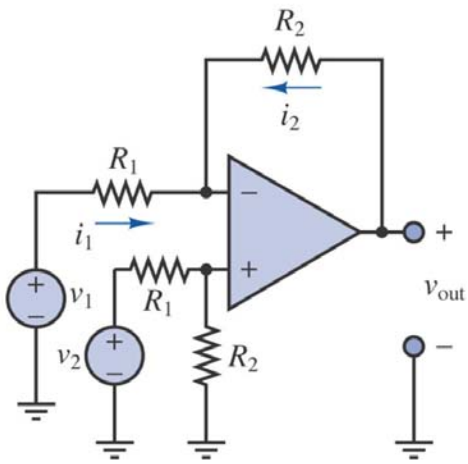


Figure 2.25: Differential amplifier

As in an ideal amplifier no current flows into the op-amp, we have

$$v^+ = \frac{R_2}{R_1 + R_2} v_2$$

$$i_1 = \frac{v_1 - v^+}{R_1}, \quad i_2 = \frac{v_{out} - v^+}{R_2}$$

$$i_2 = -i_1 \quad \Rightarrow \frac{v_{out} - \frac{R_2}{R_1 + R_2} v_2}{R_2} = -\frac{v_1 - \frac{R_2}{R_1 + R_2} v_2}{R_1}$$

$$v_{out} = R_2 \left[\frac{-v_1}{R_1} + \frac{1}{R_1 + R_2} v_2 + \frac{R_2}{R_1(R_1 + R_2)} v_2 \right]$$

$$= \frac{R_2}{R_1}(v_2 - v_1) \quad (2.28)$$

Thus, the differential amplifier amplifies the difference between the two input signals by the closed-loop gain of R_2/R_1.

4.4 Light-emitting Diodes

A light emitting diode (LED) is a special kind of diode which emits light when forward biased. The circuit symbol of a LED is as shown in Fig. 2.26(a). They exhibit a forward voltage drop of the order of 1 – 2 V. The drive circuit for the LED is as shown in Fig. 2.26(b) and the corresponding voltage-current characteristic is also shown in Fig. 2.26 (c) which is similar to that of a diode except that the forward voltage drop is slightly larger.

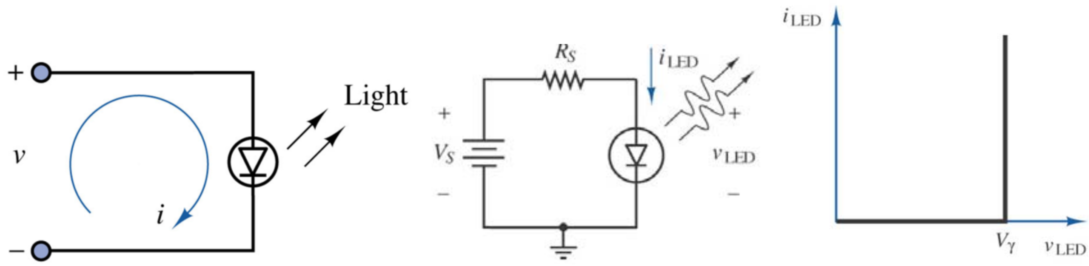


Figure 2.26: Light emitting diode (a) symbol, (b) drive circuit and (c) *i-v* characteristic

Example 2.4

For the LED circuit as shown in Fig. 2.26(b) determine (a) the LED power consumption, (b) the value of the resistance R_s and (c) the power delivered by the source. Assume that V_S = 5 V, V_{LED} = 1.7 V and i_{LED} = 40 mA.

Solution:

(a)

$$P_{LED} = V_{LED} \times I_{LED} = 1.7\,V \times 40\,mA = 68\,mW$$

(b)

$$R_s = \frac{V_S - V_{LED}}{I_{LED}} = \frac{5V - 1.7V}{40\,mA} = 82.5\,\Omega$$

(c)

$$P_s = V_s \times I_{LED} = 5\,V \times 40\,mA = 200\,mW$$

2.5 Light-dependant Resistor (LDR)

A *photo-resistor* or *light dependent resistor* (LDR) is a resistor whose resistance decreases with increasing incident light intensity. If the light falling on the LDR is of high then it lowers the resistance of the LDR and the variable resistance in an electric circuit can be used as a sensor to activate the motor to move in a direction so that it would follow the track.

The LDR output is compared with some reference voltage and the comparator output would be driving the motor in the desired direction.

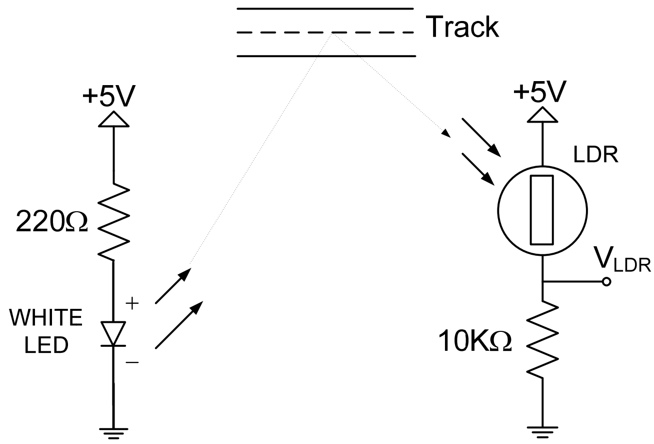


Figure 2.27: Light emitting diode and light dependent resistor are used to monitor the track

Summary

- Understanding the concepts of analog and digital signals.
- Writing truth tables and realizing logic functions from truth tables using logic gates.
- Sum-of-products (SOP) and products-of-sum (POS) for logical expressions.
- De Morgan's Laws for simplifying logical expressions.
- Karnaugh map to simplify logical expressions
- Ideal, real inverting and non-inverting operational amplifiers
- LED drive circuit
- LDR as a sensor

References

3. Allan R. Hambley, Electrical Engineering: Principles and Applications, Fifth Edition, Pearson, 2011. Chapters 7 and 14.

4. Giorgio Rizzoni, Principles and Applications of Electrical Engineering, Fifth Edition, McGraw-Hill, 2007. Chapters 9 and 13.

Chapter3: Magnetic Circuits and Transformers

Learning Objectives:

- Review of basic principles of electricity and magnetism
- Understanding of magnetic field and their interactions with moving charges
- Use of right-hand-rule to determine the direction of magnetic field around a current carrying wire
- Calculate force induced on a current carrying wire placed in a magnetic field
- Calculate voltage induced in a coil by changing magnetic flux and also in a conductor cutting through a magnetic field
- Use of Lenz's law to determine the polarities of the induced voltage
- Use of concepts of reluctance and magnetic circuit equivalents to compute magnetic flux and currents in simple magnetic structure
- Use of magnetic circuit models to analyze transformers
- Understand ideal transformers and solve circuits involving transformer

3.1. Introduction to Magnetic Circuits and Transformers

In this EE1002 module, you would be building an autonomous vehicle that would be able to move around following a track marked on the ground. In the project, you would be using components such as DC power supply and DC electric motors to propel the autonomous vehicle. In this and following lectures, we introduce the basic laws of electromagnetism and electromagnetic induction that would help you understand the basic principles of operation of a transformer, how to design DC power supply and also how to make use of DC motors to provide mechanical energy and pulse-width-modulated circuit to control the speed of motor that would be used to propel the vehicle.

Magnetic fields are created by electrical charges in motion. When charges move through magnetic fields, they experience forces and similarly, changing magnetic fields induce voltages in stationary conductors. In this chapter, we would review the basic magnetic-field concepts, the relationships between magnetic fields and inductance including mutual inductance. This would lead to the basic principles of operation of transformer. Transformers are commonly used in applications where we need to step-up or step-down AC voltages from one level to another. Power transformers are used in transmission and distribution networks for delivering electrical power to the users.

Magnetic fields also form the basis of most of the practical devices for converting electrical energy to mechanical energy and we would study basic principles of operation of electromechanical energy conversion devices. Electromechanical transducers are widely used in the design of various industrial and residential control systems as well as in biomedical applications, and form the basis of many common appliances.

3.1.1 Magnetic Field

Magnetic fields are produced due to the movement of electrical charge and they exist around permanent magnets and also around wire that carry current known as *electromagnets* as shown in Fig. 3.1(c).

Magnetic field can be visualized as lines of magnetic flux lines that emanate from the North-pole and enters at the South-pole to form a closed path as shown in Fig. 1.1 (a).

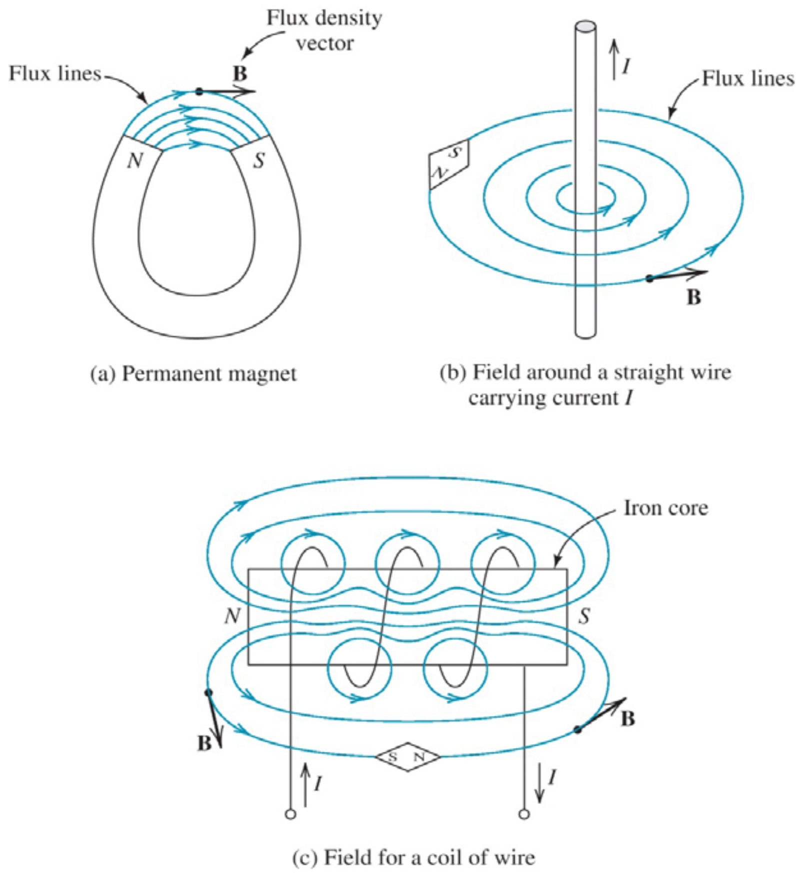


Figure 3.1 Magnetic fields created by (a) permanent magnet, (b) single-conductor carrying current and (c) coil of wire wound on an iron core.

If the coil is formed by having many numbers of turns rather than a single wire carrying current, then the net magnetic field is stronger. Moreover, if the coil is wound around an iron core, the magnetic field strength can be further enhanced as shown in Fig. 3.1(c).

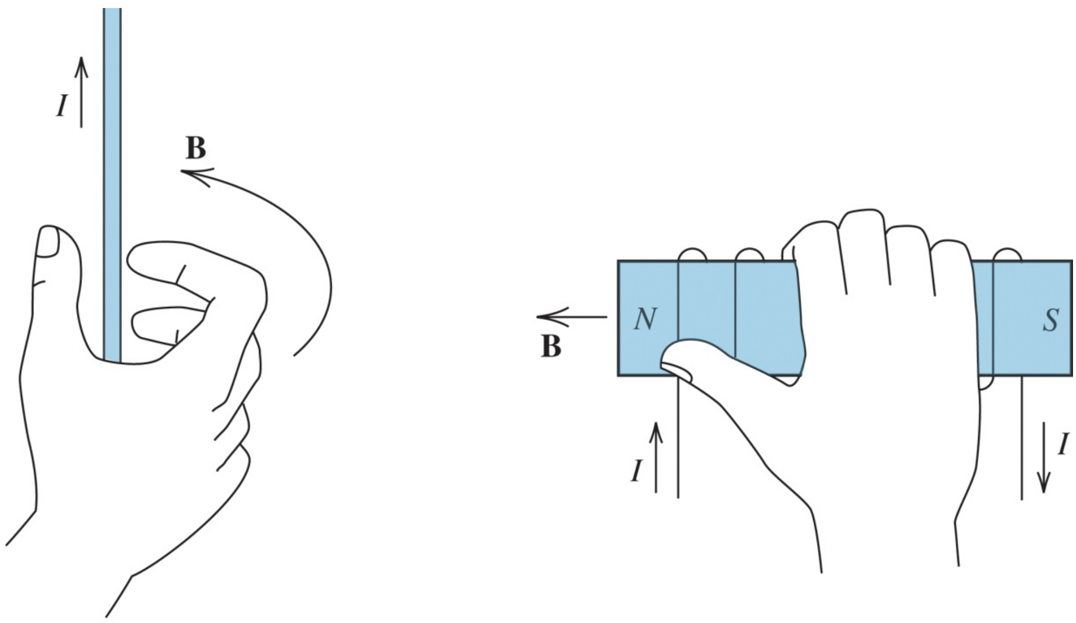
The magnetic field strength in the case of a permanent magnet is constant. However, in many applications it is intended to vary the magnetic field strength which can be done using electromagnets and varying the current through the coil.

The magnetic field forms the essential intermediate link between the energy conversion from electrical to mechanical or vice-versa in electrical machines.

The magnetic fields form the basis for the operation of transformers, electrical motors and generators.

3.1.1.1 Direction of the Magnetic Field

The direction of the magnetic field can be determined by using the *right-hand-rule*, which states that if you point the thumb of your right hand in the direction of the current, your fingers encircling the wire would point in the direction of the magnetic field as shown in Fig. 1.2(a). Alternatively, if the fingers are wrapped around a coil wound around a core in the direction of the current flow, the thumb points the direction of the magnetic field as shown in Fig. 1.2(b).



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Figure 3.2 Illustration of the right hand rule.

3.2 Magnetic Flux, Flux-density and Flux-linkage

Magnetic fields can be visualized as lines of *magnetic flux,* ϕ that form closed paths as shown in Fig. 3.1. Magnetic flux starts from the North Pole and returns through the South Pole as shown in Fig. 3.1(a).

Magnetic flux describes the total amount of magnetic field in a given region and is unnoticeable by the five senses of the human being and therefore hard to describe. It is only known through its effects. The unit for flux, ϕ is webers (Wb).

The magnetic flux, ϕ is defined as the integral of the flux-density over some surface area, *A* and if the magnetic flux-lines are perpendicular to the cross-sectional area, *A* the flux is given by

$$\phi = \int_A B\, dA \qquad (3.1)$$

in Webers.

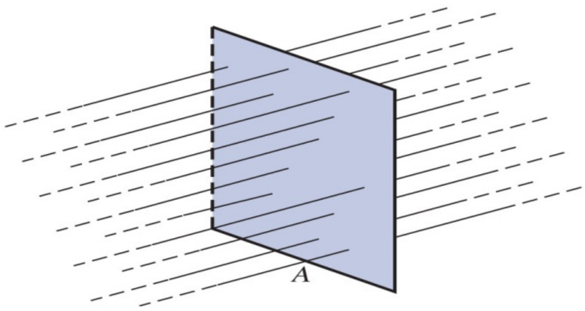


Figure 3.3 Magnetic flux-lines and flux-density.

If the flux were to be uniform over the cross-sectional area, *A* the above integral can be simplified to

$$\phi = B \cdot A \qquad (3.2)$$

The magnetic flux-density, **B** is defined as the magnetic flux per cross-sectional area, *A*

$$B = \frac{\phi}{A} \qquad (3.2a)$$

The unit for flux-density, **B** is Wb/m^2 or Tesla(T). The magnetic flux-density, **B** is a vector quantity.

The flux passing through the surface bounded by a coil is said to be linking the coil as shown in Fig. 3.1(c). If the coil has *N* number of turns, then the total flux-linkage for the coil is given by

$$\lambda = N \times \phi \qquad (3.3)$$

The unit for flux-linkage, λ is Weber-turns. If the *N*-turn coil is tightly wound and linking a certain amount of magnetic flux, ϕ as shown in Fig. 3.4 then it is not unreasonable to assume that each turn of the coil links the same flux, ϕ.

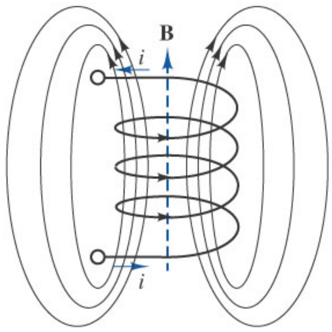


Figure 3.4 Concept of flux-linkage.

1.3 Faraday's Law

The Faraday's law of magnetic induction states that a voltage is induced in a coil whenever there is a change in the flux-linkage. The flux-linkage can change either due to the *time varying magnetic field and therefore the flux* or the *magnetic field is constant but the coil is moving with respect to the magnetic field*.

The voltage induced in a single coil is given by

$$e = \frac{d\phi}{dt} \qquad (3.4)$$

For a coil with *N* number of turns, the total induced voltage can be determined by the summation of voltages induced in each turn in the coil,

$$e = N\frac{d\phi}{dt} = \frac{d(N\phi)}{dt} = \frac{d\lambda}{dt} \qquad (3.4a)$$

The induced emf, *e* would cause a current to flow (provided the coil is connected through an external resistor as shown in Fig. 3.5) in such a direction that it would produce a reaction flux density, B_i that would oppose the original flux density, *B*. This is known as **Lenz's law**.

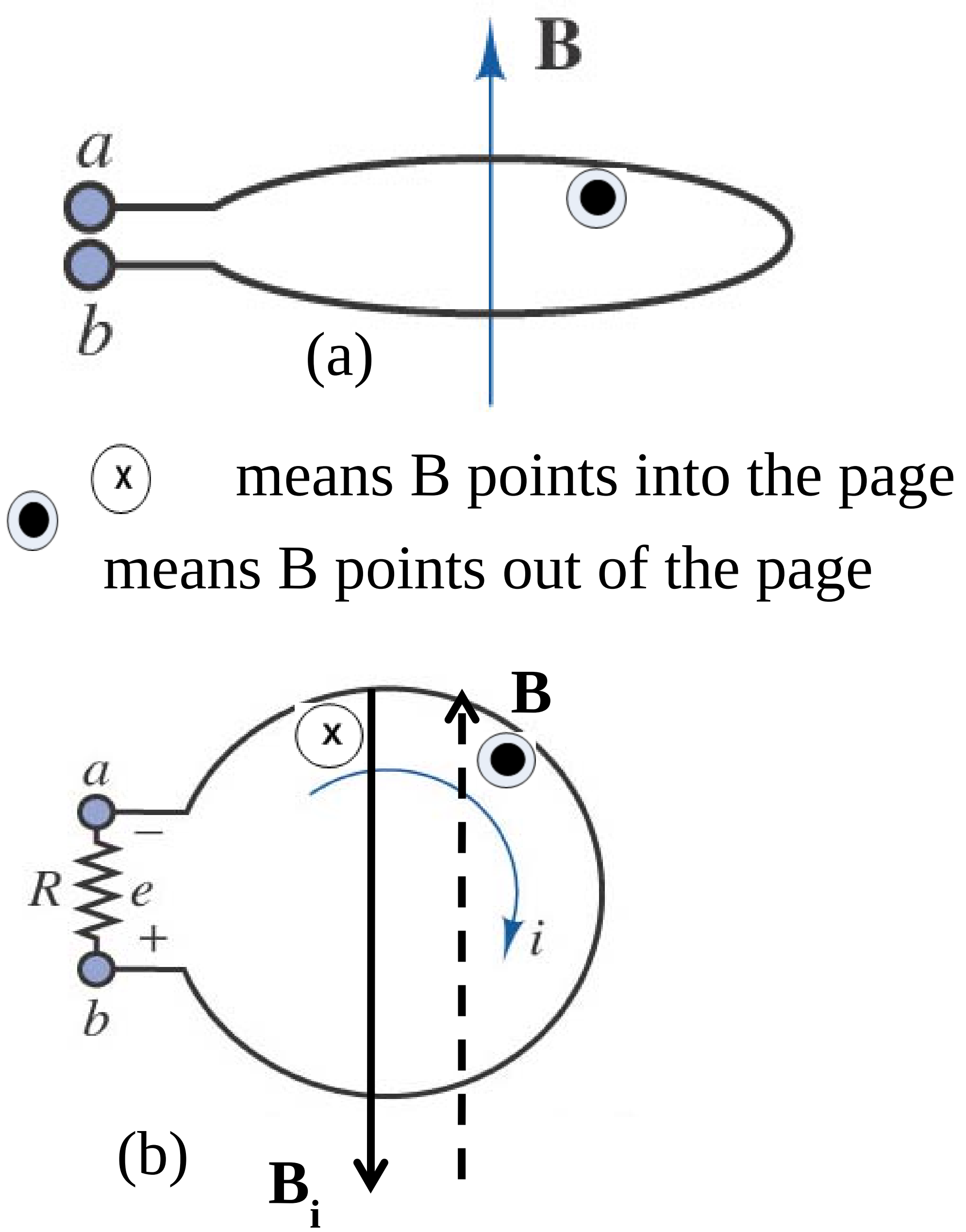


Figure 3.5 (a) Magnetic flux-density, *B* linking the coil and (b) current, *i* due to *e* generating a reaction flux-density, B_i to oppose the main flux-density, *B*.

There are two ways to create the magnetic flux to change and therefore to induce a voltage in the coil (1) moving a permanent magnet in the vicinity of a coil i.e. the magnetic field is constant but the coil linking the flux-linkage to change and therefore producing a time varying flux and (2) the magnetic field is produced by an ac electric current flowing through the coil and the current and hence the flux is time varying. The voltage induced by moving conductor in the magnetic field is called *motional voltage* and that generated by time-varying magnetic field is termed as *transformer voltage*.

3.3.1 Voltage Induced in Field-cutting Conductors

Voltage is also induced in a conductor when moving through a magnetic field in a direction such that these magnetic lines of flux are cut as shown in Fig. 3.6.

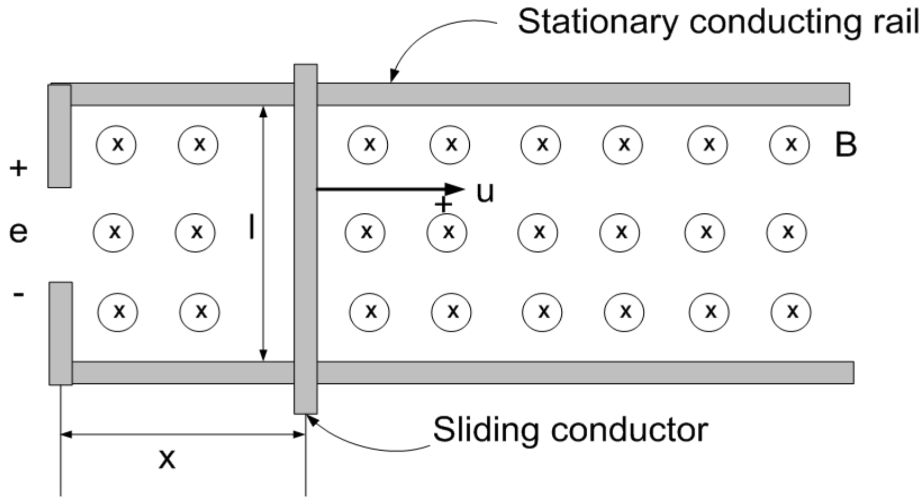


Figure 3.6 Voltage induced in a conductor moving so as to cut through magnetic flux lines.

The sliding conductor and the stationary rails form a single loop having an area of $A = l\,x$.

$$\lambda = 1 \times \phi = B \times A = B \times (lx) \Rightarrow e = \frac{d\lambda}{dt} = Bl\frac{dx}{dt} = Bl(u = \frac{dx}{dt}) = Blu \quad (3.5)$$

Eqn.(3.5) can be used to compute the voltage induced across the ends of a straight conductor moving in a uniform magnetic field, provided the velocity of the conductor and the magnetic flux density vector are mutually perpendicular.

3.3.2 Voltage Induced by Time-varying Flux

Let us assume a coil of N turns and the flux produced by the current flowing through the coil is sinusoidal. Then the voltage induced in the coil is given by

$$\phi = \phi_m\, sin(\omega t), \qquad e = N\frac{d\phi}{dt} = N\phi_m\omega\cos(\omega t)\; volts \qquad (3.6)$$

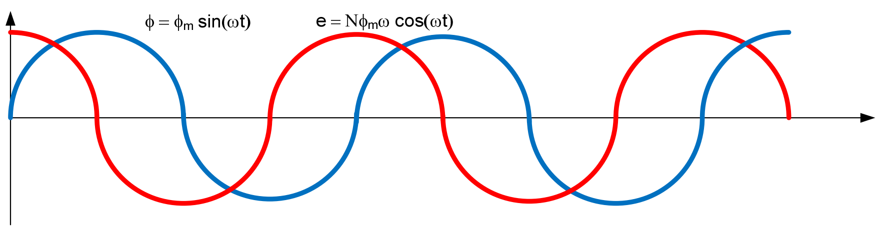


Figure 3.7 Voltage induced in a conductor due to sinusoidally varying magnetic flux.

Example 3.1

A conductor in a typical dc generator rated for 1 kW has a length of 0.2m, the conductor is moving at a velocity of 12 m/s, and cuts through the magnetic field of 0.5 Wb/m^2 pointed into the plane of the paper. Determine the corresponding voltage induced in the conductor.

Solution:

Using eqn. 3.5 we have,

$$e = \frac{d\lambda}{dt} = Blu = 0.5Wb/m^2 \times 0.2m \times 12m/s = 1.2V$$

Example 3.2

A 10-turn circular coil has a radius of 5 cm.

(a) A flux-density of 0.5 wb/m^2 is directed perpendicular to the plane of the coil. Determine the corresponding flux linking the coil and the flux-linkage.

(b) Assume that the flux is reduced to zero at a uniform rate during an interval of 1 ms. Determine the corresponding voltage induced in the coil.

Solution:

(a) Using eqn. 3.2 we have

$$\phi = B \times A = 0.5 \times (\pi \times (0.05)^2) = 3.927\ mWb,$$

$$\lambda = N \times \phi = 10 \times 0.003927 = 39.27\ mWb - turns$$

(b) Using eqn. 3.4a, we have

$$e = \frac{d\lambda}{dt} = \frac{0 - 0.03972}{0.001} = -39.72\ V$$

The –ve sign is due to the reason that the flux-linkage is reducing from a positive value of 39.27 mWb-turns to zero.

3.4 Magnetic Field Intensity and Ampere's Law

Let us introduce another field vector, known as the **magnetic field intensity**, ***H*** and see how it can be established.

The magnetic field intensity, *H* and the magnetic flux-density, *B* are related by

$$B = \mu H = \mu_r \mu_0 H \quad (3.7)$$

where μ - permeability of the magnetic material, μ_r - relative permeability of the magnetic material and $\mu_0 = 4\pi \times 10^{-7}$ Wb/A-turn - permeability of free space i.e. air.

Ampere's law states that the line integral of the magnetic field intensity around a closed path is equal to the algebraic sum of the currents flowing through the area enclosed by the path and is given by

$$\oint \mathrm{H}.\, d\mathrm{l} = \sum i \qquad (3.8)$$

d**l** vector element of length having its direction tangent to the path of the integration.

If ***H*** is constant and points in the same direction as the incremental length d**l** along the path, we have Ampere's Law as:

$$Hl = \sum i \qquad (3.8a)$$

A given magnetic field intensity, ***H*** would give rise to different flux densities ***B*** in different magnetic materials.

It would be good to define the *source* of the magnetic energy in terms of magnetic field intensity, ***H*** so that different magnetic materials can be evaluated for a given source. This source would be termed as *magnotomotive force* (MMF), $\mathcal{F}$.

3.4.1 Magnetic Field around a long Straight Wire

Consider a long straight wire carrying current *i* out of the page of the slide as shown in Fig. 3.8 and of a circular path of radius, *r* surrounding the wire.

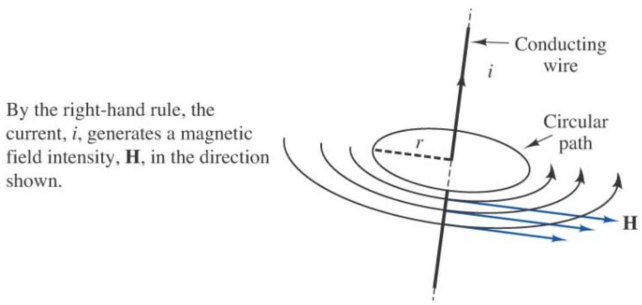


Figure 3.8 Illustration of Ampere's Law.

The magnetic field intensity, *H* and flux-density, *B* around the wire is given by

$$Hl = \sum i = H \times (2\pi r) = I \Rightarrow H = \frac{I}{2\pi r} \Rightarrow B = \mu H = \frac{\mu I}{2\pi r} \quad (3.9)$$

3.4.2 Magnetic Field in a Toroidal Core

Consider a toroidal core as shown in Fig. 3.8 and assume that there are *N* number of turns in the coil wound around the core and a DC current, *I* is flowing through it. Applying Ampere's law we have

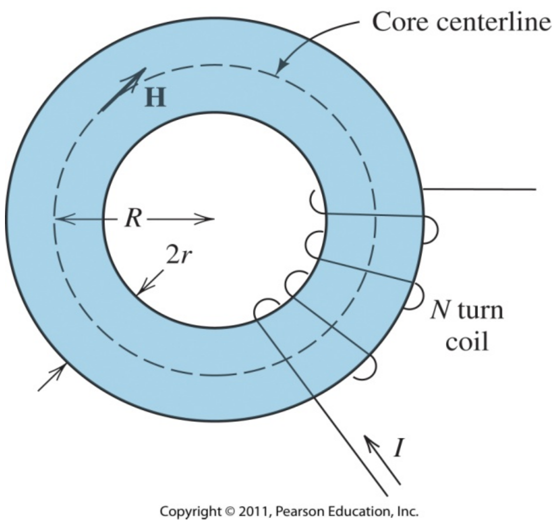


Figure 3.8a Toroidal core.

$$Hl = \sum i = H \times (2\pi R) = NI \Rightarrow H = \frac{NI}{2\pi R} \Rightarrow B = \mu H = \frac{\mu NI}{2\pi R} \Rightarrow$$

$$\phi = B \times A = \frac{\mu NI}{2\pi R} \times (\pi r^2) = \frac{\mu NIr^2}{2R} \Rightarrow \lambda = N\phi = \frac{\mu N^2 Ir^2}{2R} (3.9a)$$

Example 3.3

Assume that we have a toroidal core with μ_r = 5000, R = 10 cm, r = 2 cm and N = 100. The current flowing through the coil is given by

$$i(t) = 2\sin(200\pi t)$$

(a) Compute the flux and the flux-linkage in the toroidal core.

(b) Using Faraday's law of induction, determine the voltage induced in the coil due to the time varying flux.

Solution:

The magnetic permeability of the core material is given by

$$\mu = \mu_0 \times \mu_r = 4\pi \times 10^{-7} \times 5000$$

(a) Using eqn. (1.9a) we have,

$$\phi = \frac{\mu NIr^2}{2R} = \frac{4\pi \times 10^{-7} \times 5000 \times 100 \times 2\sin(200\pi t) \times (0.02)^2}{2 \times 0.1}$$
$$= (2.531 \times 10^{-3})\sin(200\pi t)\, Wb$$

$$\lambda = N\phi = 100 \times (2.531 \times 10^{-3})\sin(200\pi t) = 0.2531\sin(200\pi t)\, Wb - turns$$

(b) Using Faraday's law eqn. (3.4(a)) we have

$$e = \frac{d\lambda}{dt} = 0.2531 \times 200\pi \cos(200\pi t) = 157.9 \cos(200\pi t)\ V$$

3.5 Magnetic Circuits

Many electromagnetic devices such as transformers, motors and generators contain coils wound on iron cores. We will study how to calculate the magnetic fields in these devices using magnetic circuit principles.

Flux lines around a conductor would be far greater in the presence of a magnetic material than surrounded by air.

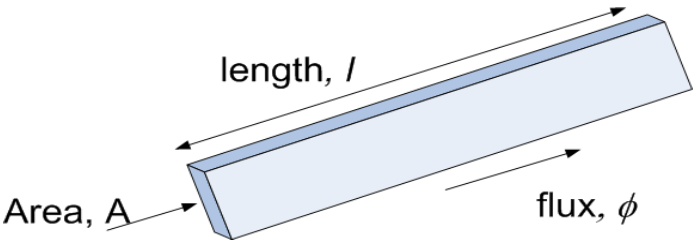
Moreover, a single conductor carrying current is not that strong; a tightly wound coil with many turns as shown in Fig. 3.8 would greatly increase the strength of the magnetic field.

For a coil with *N* turns carrying a current of *i* A, the flux-lines is increased by *N* -fold.

The product (*N* x *i)* is a useful quantity in electromagnetic circuits and is called *magnotomotive force (MMF),* $\mathcal{F}$ and is given by

$$\mathcal{F} = N \times i \qquad A - turns \qquad (1.10)$$

A current carrying coil in a magnetic circuit is analogous to voltage source in an electric circuit.



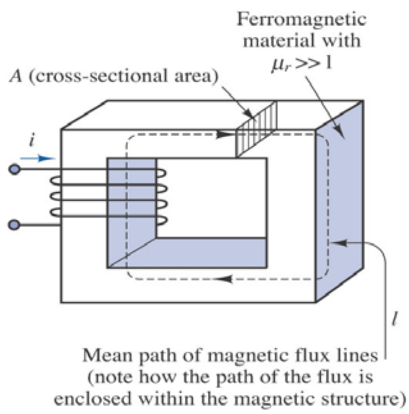


Fig. 3.9 The reluctance $\Re$ of a magnetic path

The reluctance, $\Re$ of the magnetic flux path such as a bar of iron as shown in Fig. 3.9 is given by

$$\mathcal{R} = \frac{l}{\mu A} \qquad (3.11)$$

where, *l* is the length of the magnetic path , *A* is the cross sectional area of the magnetic path. Reluctance in magnetic circuit is analogous to resistance is electrical circuit.

Magnetic flux, ϕ in a magnetic circuit is analogous to the current, *i* in an electric circuit. Magnetic flux, reluctance, and mmf are related by

$$\mathcal{F} = \mathcal{R} \times \phi \qquad (3.12)$$

which is analogous to Ohm's law ($V = iR$) in an electric circuit.

The analogy between the magnetic and electrical circuit is as shown below.

Analogy between electric and magnetic circuits	
Electrical quantity	Magnetic quantity
Electric field intensity E, V/m	Magnetic field intensity, **H** A-turns/m
Voltage, v, V	Magnetic force, $\mathcal{F}$, A-turns
Current, i, A	Magnetic flux, ϕ, Wb
Current density, J, A/m^2	Magnetic flux density, **B**, Wb/m^2
Resistance, R, Ω	Reluctance, $\mathfrak{R}$, A-turns/Wb
Conductivity, σ, 1/Ω.m	Permeability, μ, Wb/A.m

The advantage of magnetic circuit approach is that it can be applied to unsymmetrical magnetic cores with multiple coils. Coils are basically sources for magnetic circuits and they can be manipulated as voltage sources in an electric circuit. Reluctances can be added in series and parallel just as electrical resistances in electric circuits. Magnetic fluxes are analogous to currents. Although magnetic-circuit approach is not accurate for determining magnetic fields but is sufficiently accurate for many engineering applications.

Example 3.4

The iron core as shown in Fig. 1.10 has a cross section of 2cm by 2 cm and a relative permeability of 1000. The coil has 500 turns and carries a current of 2 A. Determine the flux-density in each air-gap.

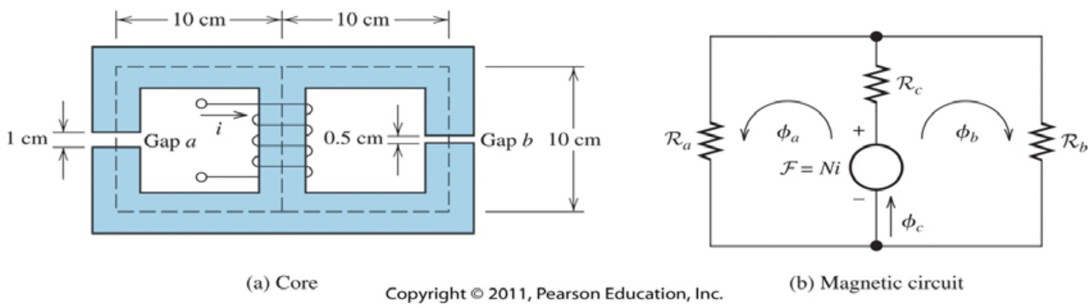


Figure 1.10: Magnetic circuit of Example 3.4.

Solution:

The equivalent magnetic circuit is shown in Fig. 3.10(b).

The reluctance for the centre path is given by

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A_{core}} = \frac{10 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 1.989 \times 10^5 \ A.turns/Wb$$

The flux-lines tend to bow out in the air-gap as shown in the Fig. 1.10a. This is called ***fringing****. Thus, the effective area of the air-gap is larger than that of the iron core. We take this into account by adding the length of the air-gap to each of the dimensions of the air-gap cross section.*

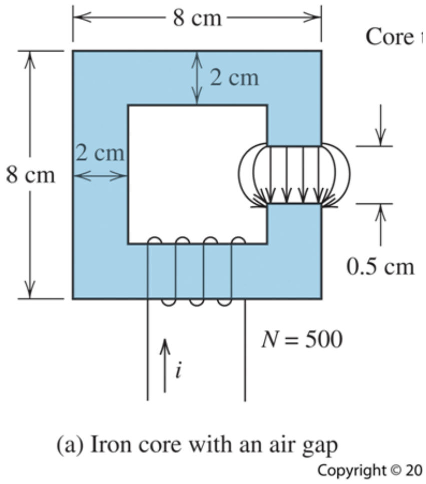


Figure 3.10a: Flux fringing effect.

The reluctance for the left air-gap path, *a* taking into account the *fringing effect* by adding the gap length to its length and depth in computing the area of the gap is given by

$$\mathcal{R}_a = \frac{l_a}{\mu_0 A_{gap,a}} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times ((2+1) \times (2+1)) \times 10^{-4}} = 8.842 \times 10^6 \ A.turns/Wb$$

The reluctance for the core for the left-hand magnetic path is given by

$$\mathcal{R}_{core,LH} = \frac{l_{c,mean}}{\mu_0 \mu_r A_{core}} = \frac{(10+9+10) \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 0.5769 \times 10^6 \ A.turns/Wb$$

Thus, the total reluctance of left-hand (LH) magnetic path is

$$\mathcal{R}_{LH} = \mathcal{R}_a + \mathcal{R}_{core,LH} = 8.848 \times 10^6 + 0.5769 \times 10^6 = 9.420 \times 10^6 \ A.turns/Wb$$

The reluctance for the right air-gap path, *b* taking into account the fringing effect by adding the gap length to its length and depth in computing the area of the gap is given by

$$\mathcal{R}_b = \frac{l_b}{\mu_0 A_{gap,b}} = \frac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times ((2+0.5) \times (2+0.5)) \times 10^{-4}} = 6.369 \times 10^6 \ A.turns/Wb$$

The reluctance for the core for the right-hand (RH) magnetic path is given by

$$\mathcal{R}_{core,RH} = \frac{l_c}{\mu_0 \mu_r A_{core}} = \frac{(10 + 9.5 + 10) \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 0.5869 \times 10^6 \, A.turns/Wb$$

Thus, the total reluctance of right-hand magnetic path is

$$\mathcal{R}_{RH} = \mathcal{R}_b + \mathcal{R}_{core,RH} = 6.369 \times 10^6 + 0.5869 \times 10^6 = 6.9559 \times 10^6 \, A.turns/Wb$$

We can combine the two reluctances of the LH side and RH side in parallel and that is in series with the reluctance of the centre core:

$$\mathcal{R}_{total} = \mathcal{R}_c + \frac{1}{{}^{1}/_{\mathcal{R}_a} + {}^{1}/_{\mathcal{R}_b}} = 1.989 \times 10^5 + \frac{1}{{}^{1}/_{9.420 \times 10^6} + {}^{1}/_{6.9559 \times 10^6}}$$
$$= 4.199 \times 10^6 A.turns/Wb$$

The magnetic flux in the centre leg of the coil is

$$\phi_c = \frac{Ni}{\mathcal{R}_{total}} = \frac{500 \times 2}{4.199 \times 10^6 A.turns/Wb} = 238.15 \, \mu Wb$$

Fluxes are analogous to current so we can use the current division rule to compute the flux in the LH and RH paths as follows:

$$\phi_a = \phi_c \times \frac{\mathcal{R}_b}{\mathcal{R}_a + \mathcal{R}_b} = 238.15 \times 10^{-6} \times \frac{6.9559 \times 10^6}{9.420 \times 10^6 + 6.9559 \times 10^6} = 101.1 \, \mu Wb$$

Similarly, for the air-gap *b* we have

$$\phi_b = \phi_c \times \frac{\mathcal{R}_a}{\mathcal{R}_a + \mathcal{R}_b} = 238.15 \times 10^{-6} \times \frac{9.420 \times 10^6}{9.420 \times 10^6 + 6.9559 \times 10^6} = 137.0 \, \mu Wb$$

The flux-densities in the air-gaps are:

$$B_a = \frac{\phi_a}{A_a} = \frac{101.1 \, \mu Wb}{((2+1) \times (2+1)) \times 10^{-4}} = 0.1123 \, Wb/m^2$$

$$B_b = \frac{\phi_b}{A_b} = \frac{137.0 \, \mu Wb}{((2+0.5) \times (2+0.5)) \times 10^{-4}} = 0.2192 \, Wb/m^2$$

If the magnetic circuit consists of iron core and air-gap then the reluctance of the iron core is almost negligible to that of the air-gaps. If the precise value of the permeability of the iron core is unknown then it can be assumed that the reluctance of the iron core is zero just like the zero resistance of the wire in electric circuits.

Assumptions and simplifications made in analyzing magnetic structures:

- All the magnetic flux is linked by all the turns of the coil.
- The flux is confined exclusively within the magnetic core.
- The flux-density is uniform across the cross-sectional area of the core.

3.6: Self and Mutual Inductances

Consider a coil having N number of turns, carrying a current, i that sets up a flux ϕ linking the coil, the inductance, L of the coil can be defined as flux-linkage divided by the current:

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N(\frac{Ni}{\mathcal{R}})}{i} = \frac{N^2}{\mathcal{R}} \qquad (3.13)$$

Thus, the inductance of a coil depends on the number of turns, the core dimensions i.e. length, l and area, A and the core material.

According to the Faraday's law, the voltage induced in the coil having an inductance, L when the flux-linking the coil changes is given by

$$e = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt} = v_L \qquad (3.14)$$

For a coil wound around a core the inductance, L remains constant with time.

When two coils are wound on the same core, some of the flux produced by one coil links the other coil as shown in Fig. 3.11.

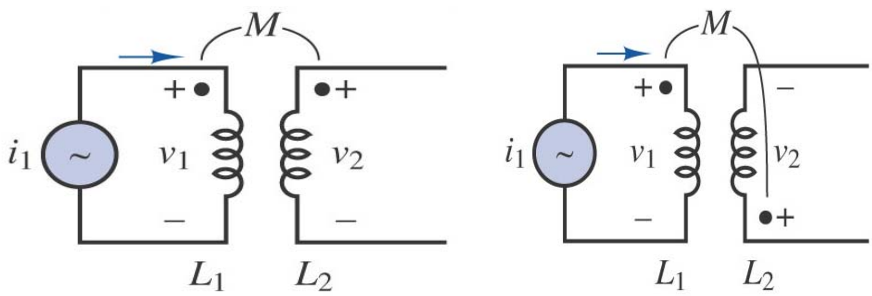


Figure 3.11: Self- and mutual inductances.

Let us denote the flux-linkage of coil 2 due to current in coil 1 as λ_{21} , and the flux-linkage of coil 1 produced due to current in coil 1 as λ_{11}. Similarly, current in coil 2 produces flux linkages λ_{22} in coil 2 and λ_{12} in coil 1.

The self inductances of the coils are defined as

$$L_1 = \frac{\lambda_{11}}{i_1} \; and \; L_2 = \frac{\lambda_{22}}{i_2} \qquad (3.15)$$

The mutual inductance between the coils is given by

$$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2} \qquad (3.16)$$

The total fluxes linking the coils are:

$$\lambda_1 = \lambda_{11} \pm \lambda_{12} = L_1 i_1 \pm M i_2 and\ \lambda_2 = \pm M i_1 + L_2 i_2 \quad (3.17)$$

The + sign applies if the fluxes are aiding each other and the –ve sign applies if they are opposing each other.

3.6.1: Dot Convention

It is normal practice to place a dot on one end of each coil as shown in Fig. 3.12 to indicate how the fluxes interact. The dots are placed in such as way that currents entering the dotted terminals produce aiding fluxes. If one current enters the dotted terminal and the other leaves the dotted terminals then these two fluxes oppose each other and the mutual flux linkages carry a –ve sign.

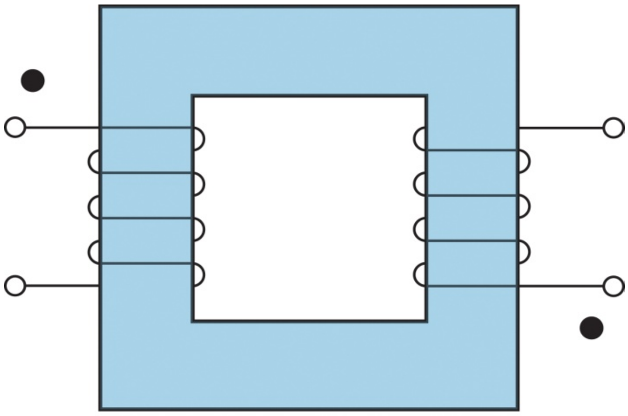


Figure 3.12: Dot convention, currents entering the dotted terminals produce aiding fluxes.

Applying Faraday's law to find voltage induced in the coils, we have

$$e_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}\ and\ e_2 = \frac{d\lambda_2}{dt} = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \quad (3.18)$$

Example 3.5

The magnetic circuit a shown in Fig. 3.12 has the following dimensions: the cross sectional, A_c = 12 cm^2 and mean-length of the magnetic flux-path, l_c = 50 cm. The magnetic core has a relative permeability μ_r = 20,000. The first coil has 500 turns and the second coil has 1000 turns.

(a) The first coil is supplied with a current i_1 = 10 A, while the second coil is left un-energized. Calculate the self-inductance L_{11} of coil 1 and the mutual inductance, M between the two coils.

(b) The first coil is de-energized (i_1 = 0 A), while the second coil is connected to a source from which it draws a current i_2 = 8 A. Calculate the self-inductance L_{22} of coil 2 and the mutual inductance, M between the two coils.

Solution:

(a) The reluctance of the magnetic circuit is given by

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A_{core}} = \frac{50 \times 10^{-2}}{20{,}000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 16.58 \times 10^3\ A.turns/Wb$$

The magnetic flux, ϕ_1 due to the current in coil 1 is given by

$$\phi_1 = \frac{N_1 i_1}{\mathcal{R}_c} = \frac{500 \times 10}{16.58 \times 10^3 A.turns/Wb} = 0.30\ Wb$$

The flux-linkages of the two coils are given by

$$\lambda_{11} = N_1 \phi_1 = 500 \times 0.30 = 150\ Wb - turns$$

$$\lambda_{21} = N_2 \phi_1 = 1000 \times 0.30 = 300\ Wb - turns$$

Thus, the self-inductance of coil 1 is given by

$$L_1 = \frac{\lambda_{11}}{i_1} = \frac{150}{10} = 15\ H$$

The mutual-inductance is given by

$$M = L_{21} = \frac{\lambda_{21}}{i_1} = \frac{300}{10} = 30\ H$$

The magnetic flux, ϕ_2 due to the current in coil 2 is given by

$$\phi_2 = \frac{N_2 i_2}{\mathcal{R}_c} = \frac{1000 \times 8}{16.58 \times 10^3 A.turns/Wb} = 0.48\ Wb$$

The flux-linkages of the two coils are given by

$$\lambda_{12} = N_1 \phi_2 = 500 \times 0.48 = 240\ Wb - turns$$

$$\lambda_{22} = N_2 \phi_2 = 1000 \times 0.48 = 480\ Wb - turns$$

Thus, the self-inductance of coil 2 is given by

$$L_{22} = \frac{\lambda_{22}}{i_2} = \frac{480}{8} = 60\ H$$

The mutual-inductance is given by

$$M = L_{12} = \frac{\lambda_{12}}{i_2} = \frac{240}{8} = 30\ H = L_{21}$$

3.7 Magnetic materials

In eqn. 3.7 ($B = \mu H$), we assume that the permeability of the magnetic core material is constant. However, in reality it is not constant but a function of the magnetic field intensity, H and the relationship between B and H is non-linear.

Fig. 3.13 shows a coil that is used to apply magnetic field intensity, *H* to a magnetic core and the corresponding B-H curve. Initially both *H* and *B* are zero and when *H* is increased by applying a current to the coil, the magnetic fields in the core material domain tends to align themselves in the direction of the applied field. As *H* is increased *B* initially increases linearly (between 2 and 3) but when sufficient high field *H* is applied then all the domains are aligned with the applied field the flux density saturates. For typical iron core material, saturation occurs in the range of 1 – 2 Wb/m^2.

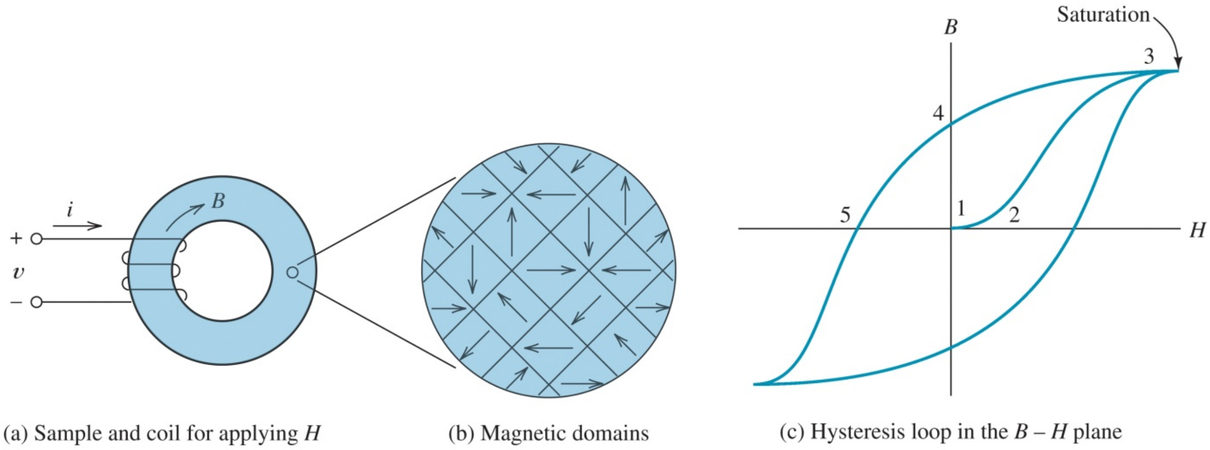


Figure 3.13: Materials such as iron display a *B*–*H* relationship with hysteresis and saturation.

If the magnetic field intensity, *H* is reduced starting from the saturated value at point 3 then the residual flux-density, *B* remains in the core when *H = 0* at point 4 and the corresponding flux-density is called as *remanent flux-density*. This occurs because the magnetic domains continue to point to the magnetic field direction applied earlier. However, if –ve *H* is applied then *B = 0* at point 5. Eventually with further increase in *H* saturation takes place in the reverse direction. If an ac current is applied to the coil then a hysteresis loop is traced on the *B-H* plane as shown in Fig. 3.13(c).

Energy Considerations

The energy, *W* delivered to the coil is given by

$$W = \int_0^t vi\,dt = \int_0^t N\frac{d\phi}{dt}idt = \int_0^t Ni\,d\phi = \int_0^t (Ni = Hl)\,(d\phi = AdB) = \int_0^t (Al)H\,dB \quad (3.19)$$

$$W_v = \frac{W}{Al} = \int_0^t H\,dB \qquad (3.20)$$

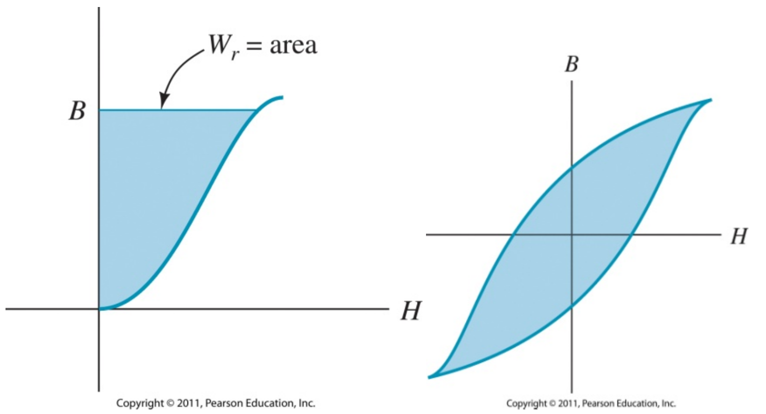


Figure 3.14: (a) The area between the *B–H* curve and the *B* axis represents the volumetric energy supplied to the core and (b) the area of the hysteresis loop is the volumetric energy converted to heat per cycle.

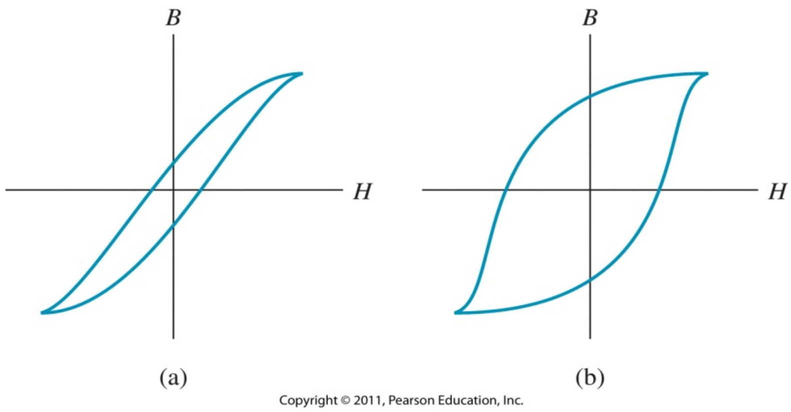


Figure 3.15: When we want to minimize core loss (as in a transformer or motor), we choose a material having a thin hysteresis loop. On the other hand, for a permanent magnet, we should choose a material with a wide loop

The term, W_v represents the energy per unit volume of the core. The volumetric energy delivered to the coil is the area between *B-H* curve and the *B*-axis as shown in Fig. 3.14(a). Part of this energy is returned back to the circuit when *H* is reduced to zero, part of it remains stored in the residual field and part of it is converted into heat in the process of magnetizing the core.

Core Loss

When ac current is applied to a coil having an iron core, more energy is put into the coil in each cycle than is returned back to the circuit and part of the energy is converted into heat in reversing the direction of the magnetic domains. The volumetric energy converted to heat per cycle is equal to the area of the hysteresis loop as shown in Fig. 3.14(b). This energy loss is called *core loss* and is proportional to the frequency.

In electrical machines and transformers, the energy loss as heat is undesirable and therefore, would choose a magnetic material that has a thin hysteresis loop as shown in Fig. 3.15(a), while with permanent magnet we would choose a material having large residual field as shown in Fig. 3.15(b).

Eddy-current Loss

As magnetic field changes due to the application of the ac current in the coil, voltages are induced in the core, causing currents, known as *eddy current*s be developed and circulate in the core material resulting power loss in the core. By laminating the core with thin sheets of iron, they are electrically insulated from one another and interrupt the flow of currents thereby reducing the eddy-current loss to a large extent.

3.8 Ideal Transformers

One of the common magnetic structures that we see in everyday applications is the transformer that is used to step-up or step-down AC voltages. Transformers find applications widely in electric power transmission and distribution (T & D) networks. For power T & D networks it is desirable to transfer active power, *P* at much higher voltages typically at kV, because for a given power rating, if the voltage is higher, current would be lower and therefore the transmission line losses (i_{rms}^2R) can be reduced increasing the efficiency of the power transmission.

Consider a transformer as shown in Fig. 3.16, it has a primary coil with N_1 number of turns and a secondary coil with N_2 number of turns. When the primary coil is connected to an ac source of voltage $v_1(t)$, then it would cause a current $i_1(t)$, to flow and produce a magnetic flux, $\phi(t)$ to flow through the core. When the flux links the secondary coil, it would induce voltage which then delivers power to the load. Depending on the turns ratio, N_2/N_1 the secondary coil rms voltage can be greater or less than the rms primary voltage.

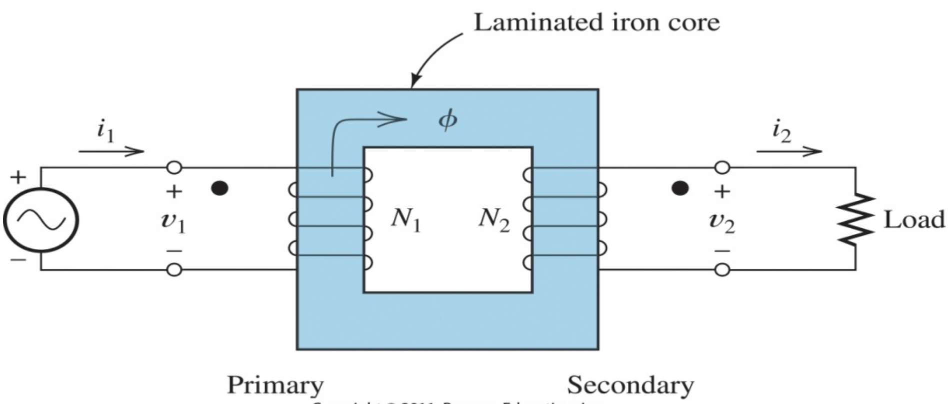


Figure 3.16: A transformer consists of several coils wound on a common core

Assume that the *resistances of the coils are neglected, core loss is neglected and the reluctance of the core is very small so that all the flux links both the coils*.

The primary voltage is given by

$$v_1(t) = V_{1m}\cos(\omega t) = N_1\frac{d\phi}{dt} \Rightarrow \phi(t) = \frac{V_{1m}}{N_1\omega}\sin(\omega t) \quad (3.21)$$

Assuming the flux links all the turns, the secondary voltage is given by

$$v_2(t) = N_2 \frac{d\phi}{dt} = N_2 \frac{V_{1m}}{N_1 \omega} \cos(\omega t)\ \omega = \frac{N_2}{N_1} V_{1m} \cos(\omega t) = \frac{N_2}{N_1}\ v_1(t) \quad (3.22)$$

Note that the voltage across the coil is proportional to the number of turns.

Lenz's law states that the voltage induced in the secondary coil would produce a current when secondary coil is externally closed that would oppose the magnetic field produced by primary coil current, i.e. current leaves the coil at the dotted point in the secondary coil as shown in Fig. 3.16. In other words, polarities of $v_1(t)$ and $v_2(t)$ are such that the two voltages are in phase with each other.

1.8.1 Current Ratio

Note that currents $i_1(t)$ and $i_2(t)$ produce magnetic field that oppose each other, thus the total MMF of the core that is being considered to be ideal is given by

$$\mathcal{F} = N_1 i_1(t) - N_2 i_2(t) = 0 \Rightarrow i_2(t) = \frac{N_1}{N_2} i_1(t) \quad (3.23)$$

Comparing eqns. (3.22) and (3.23) we see that if the voltage is stepped-up ($N_2/N_1 > 1$) then the current is stepped-down and vice-versa.

3.8.2 Power in an Ideal Transformer

Power delivered to the load is given by

$$p_2(t) = v_2(t) \times i_2(t) = \frac{N_2}{N_1} v_1(t)\ \times\ \frac{N_1}{N_2} i_1(t) = v_1(t) \times i_1(t) = p_1(t) \quad (3.24)$$

Thus, in an ideal transformer, the power drawn by the primary winding from the source is the delivered to the load by the secondary winding and therefore no power loss takes place.

Summary of an ideal transformer:

1. All of the flux links all of the windings of both the coils and that the resistances of the coils are zero – leads to:
$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$
2. Reluctance of the core is negligible, so the total MMF of both the coils is zero – leads to:
$$i_2(t) = \frac{N_1}{N_2} i_1(t)$$
3. In an ideal transformer, power delivered by the source is directly delivered to the load with 100% efficiency.

The transformer rating: 10 kVA, 1100/240V, 50 Hz indicates rated volt-ampere rating of 10 kVA, rated voltages of the two windings as 1100 V and 240 V respectively and the frequency of operation of 50 Hz.

Example 3.6

Consider the source, transformer, and load as shown in Fig. 3.17. Determine the rms values of the currents and voltages (a) with the switch open and (b) with the switch closed.

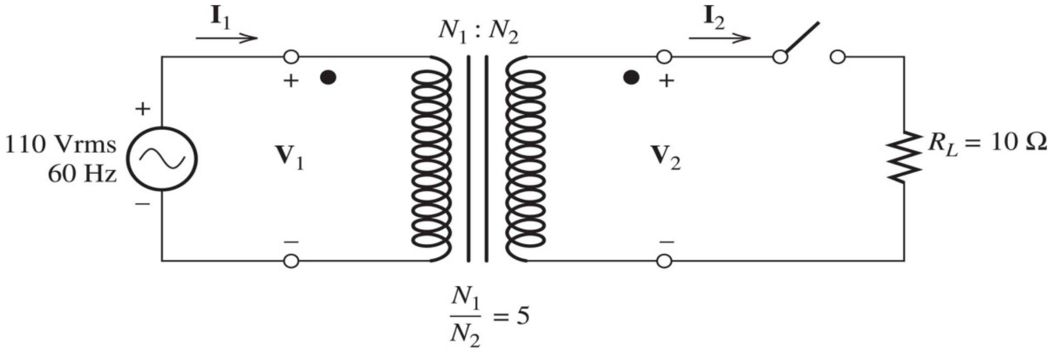


Figure 3.17: Circuit for Example6

Solution:

$$V_{2rms} = \frac{N_2}{N_1} V_{1rms} = \frac{1}{5} 110 = 22\ V$$

(a) When the switch is opened, we have

$$I_{2rms} = 0, \Rightarrow \quad I_{1rms} = \frac{N_2}{N_1} I_{2rms} = 0$$

(b) When the switch is closed, we have

$$I_{2rms} = \frac{V_{2rms}}{R_L} = \frac{22\ V}{10\ \Omega} = 2.2\ A, \Rightarrow \quad I_{1rms} = \frac{N_2}{N_1} I_{2rms} = \frac{1}{5} 2.2\ A = 0.44\ A$$

- When the switch is opened and the voltage is applied at the primary, a very small current is drawn from the source to set-up the magnetic flux.
- When the switch is closed, current $i_2(t)$ flows that would oppose the flux and therefore more current is drawn from the source by the primary coil to offset the MMF in the secondary coil and maintain flux in the core.

3.8.3 Impedance Transformations

Consider the transformer circuit as shown in Fig. 3.18. The impedance, Z_L based on the phasors, $\mathbf{V_2}$ and $\mathbf{I_2}$ is given by

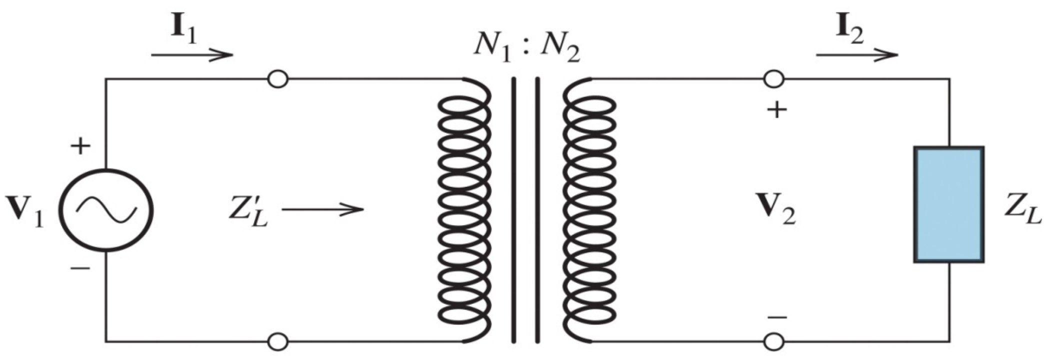


Figure 1.18: Circuit for Example6

$$Z_L = \frac{\mathbf{V_2}}{\boldsymbol{I_2}} = \frac{\left(N_2/N_1\right)V_1}{\left(N_1/N_2\right)I_1} = \left(\frac{N_2}{N_1}\right)^2 \frac{V_1}{I_1} \Rightarrow Z_L' = \frac{\mathbf{V_1}}{\boldsymbol{I_1}} = \left(\frac{N_1}{N_2}\right)^2 Z_L \qquad (3.25)$$

Thus, the secondary side impedance Z_L as seen from the primary or source side as $Z_L{}'$ is given by eqn. (3.25). We say that the load impedance when reflected to the source side it has to be multiplied by the square of the turns ratio.

Example 3.7

Consider the circuit as shown in Fig. 3.19(a). Determine the phasor currents and voltages and also the power delivered to the load.

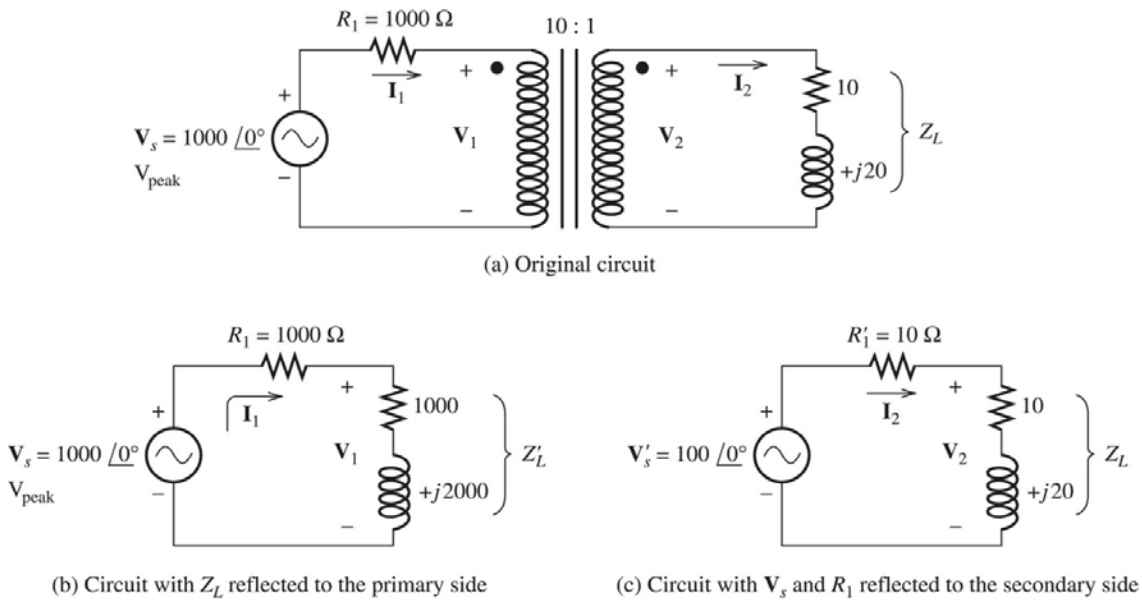


Figure 3.19: Circuit for Example3.7

Solution

$$Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L = (10)^2(10 + j20) = 1000 + j2000\ \Omega = 2236.1\angle 63.43^0$$

The total impedance seen by the source is

$$Z_s = R_1 + Z_L' = 1000 + (1000 + j2000) = 2000 + j2000 = 2828.43\angle 45^0$$

$$I_1 = \frac{V_s}{Z_s} = \frac{1000\ \angle 0^0}{2828.43\ \angle 45^0} = 0.3536\angle - 45^0\ \ A, peak$$

$$V_1 = I_1 Z_L' = 0.3536\angle - 45^0 \times 2236.1\angle 63.43^0 = 790.7\angle 18.43^0\ \mathrm{V, peak}$$

$$I_2 = \frac{N_1}{N_2} I_1 = 10\ \times 0.3536\angle - 45^0\ = 3.5336\ \angle - 45^0\ \ A, peak$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{1}{10} \times 790.7\angle 18.43^0\ = 79.07\angle 18.43^0\ \mathrm{V, peak}$$

The power loss is given by

$$P_L = I_{2,rms}^2 \times R_L = \left(\frac{3.5336}{\sqrt{2}}\right)^2 \times 10 = 62.43\ W$$

We can also transfer voltage and current from one side to the other side of the transformer such as

$$V_s' = \frac{N_2}{N_1} V_s = \frac{1}{10} \times 1000\angle 0^0 = 100\angle 0^0\ V, peak$$

$$R_1' = \left(\frac{N_2}{N_1}\right)^2 R_1 = \left(\frac{1}{10}\right)^2 \times 1000 = 10\ \Omega$$

The circuit with $\mathbf{V}_s$ and R_1 transferred to the secondary circuit is as shown in Fig. 3.19(c).

3.9 Real Transformer

We had made certain assumptions while discussing ideal transformer but in reality even if a good designed transformer do not meet those assumptions. In reality, there are imperfections in the transformer and there is a need to understand those imperfections and have a better equivalent circuit for a real transformer.

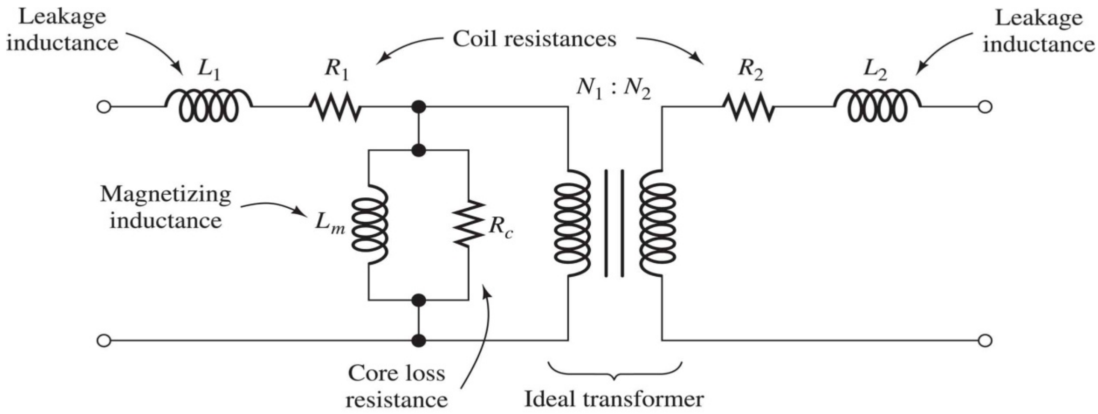


Figure 3.20: The equivalent circuit of a real transformer

The equivalent circuit of a real transformer is as shown in Fig. 3.20. The resistances R_1 and R_2 represent the windings of the transformer primary and secondary coils. For an ideal transformer, we had assumed that all the flux links both the coils. However, in reality, some of the flux produced by the coil leaves the core and does not link the other coil and this flux is called *leakage flux* and represented by leakage inductances L_1 and L_2 as shown in Fig. 3.20.

Also, we had assumed that the magnetic core reluctance was zero and ignored the core loss. However, in reality, the core has some reluctance and that is represented by the magnetizing inductance, L_m. Similarly, the core loss is represented by the resistance, R_c which accounts for the *hysteresis and eddy current losses* in the magnetic core.

Table 3.1 compares the values of transformer equivalent circuit parameters of a real-transformer with an ideal transformer.

Table 15.1. Circuit Values of a 60-Hz 20-kVA 2400/240-V Transformer Compared with Those of an Ideal Transformer

Element Name	*Symbol*	*Ideal*	*Real*
Primary resistance	R_1	0	3.0 Ω
Secondary resistance	R_2	0	0.03 Ω
Primary leakage reactance	$X_1 = \omega L_1$	0	6.5 Ω
Secondary leakage reactance	$X_2 = \omega L_2$	0	0.07 Ω
Magnetizing reactance	$X_m = \omega L_m$	∞	15 kΩ
Core-loss resistance	R_c	∞	100 kΩ

3.9.1 Regulation and Efficiency

Because of the presence of the resistances and leakage inductances, the output voltage delivered to the load is not constant and is a function of the load current. This is not desirable and the drop in the output voltage is known as *regulation* of the transformer and is defined as

$$\text{percent regulation} = \frac{V_{no-load} - V_{load}}{V_{load}} \times 100\% \qquad (3.26)$$

where, $V_{no\text{-}load}$ and V_{load} are the rms voltages at the output when there is no-load and under loaded conditions.

Also due to the resistances in the equivalent circuit, all the electrical power provided at the input is not delivered to the load. We define the *power efficiency* of the transformer as

$$\text{power efficiency} = \frac{P_{out}}{P_{in}}\times 100\% = \left(\frac{P_{in} - P_{loss}}{P_{in}}\right)\times 100\% = \left(1 - \frac{P_{loss}}{P_{in}}\right)\times 100\% \quad (3.27)$$

where, P_{in} is the input power, P_{out} is the output power and P_{loss} is the loss in the transformer.

Example 3.8

For the transformer specifications as given in Table15.1, determine the percentage regulation and power efficiency for a rated load with a lagging power factor of 0.8.

Solution:

Let us assume that the load voltage is taken as zero phase reference. We take the phasors for voltage and current as rms values.

$$\mathbf{V}_{load} = 240\angle 0^0 \; V\, rms$$

For rated load of 20 kVA, the load current is given by

$$I_2 = \frac{20\; kVA}{240\; V} = 83.33\; A\, rms$$

For a load power factor of 0.8 lagging, we have

$$\theta = \cos^{-1}(0.8) = 36.87^0$$

Thus the phasor load current is given by

$$\mathbf{I}_2 = 83.33\angle{-36.87^0}\; A\, rms$$

where the –ve phase angle is due to the lagging power factor.

The primary current is given by

$$\mathbf{I}_1 = \frac{N_2}{N_1}\mathbf{I}_2 = \frac{1}{10} \times 83.33\angle{-36.87^0} \; = 8.333 \angle - 36.87^0 \;\; A, rms$$

We can compute the voltages as:

$$\begin{aligned}\mathbf{V}_2 = \mathbf{V}_{load} + \mathbf{I}_2 \times (R_2 + jX_2) &= 240\angle 0^0 + 83.33\angle{-36.87^0} \times (0.03 + j0.07) \\ &= 245.50 + j3.166\; V\end{aligned}$$

The primary voltage is given by

$$\mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2 = 10 \times (245.50 + j3.166) = 2455.0 + j31.66\; V, rms$$

The source voltage is given by

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}_1 \times (R_1 + jX_1) = 2455.0 + j31.66 + 8.333 \angle -36.87^0 \times (3 + j6.5)$$
$$= 2508.2 \angle 1.37^0 \; V\, rms$$

The power loss in the transformer is given by

$$P_{loss} = \frac{V_s^2}{R_c} + I_1^2 R_1 + I_2^2 R_2 = \frac{2508.2^2}{100 \times 10^3} + 8.33^2 \times 3 + 83.33^2 \times 0.03 = 479.5\, W$$

The power delivered to the load is given by

$$P_{out} = 20\, kVA \times 0.8 = 16{,}000\, W$$

The input power is given by

$$P_{in} = P_{out} + P_{loss} = 16{,}000 + 479.5 = 16{,}479.5\, W$$

$$\text{power efficiency} = \frac{P_{out}}{P_{in}} \times 100\% = \frac{16{,}000}{16{,}479.5} \times 100\% = 97.09\%$$

$$V_{no,load} = V_2 = \frac{N_2}{N_1} V_s = \frac{1}{10} \times 2508.2 = 250.8\, V$$

The percentage regulation is

$$\text{percent regulation} = \frac{250.82 - 240}{240} \times 100\% = 4.51\%$$

Summary

1. The right-hand rule is used to determine the direction of the magnetic field produced by a current.

2. According to Faraday's law of induction, voltage is induced in a coil when the magnetic flux linkages change with time. Similarly, voltages are induced in moving conductors that cut through magnetic flux lines. The polarity of the induced voltage is determined by Lenz's law and it states that the induced voltage has a polarity that always opposes the main effect that induces the voltage.

3. Magnetic flux density, **B** and the magnetic field intensity**, H** are related by **B** = μ**H.**

4. Ampere's law states that the line integral of **H** around a closed path is equal to the algebraic sum of the currents flowing through the area bounded by the path.

5. Magnetic devices can be approximately analyzed using circuit concepts. MMFs are analogous to voltage sources, reluctance is analogous to resistances, and flux is analogous to current.

6. The self- and mutual- inductances of coils can be computed knowing the physical dimensions of the coils and the core on which they are wound.

7. The *B*-*H* relationship for iron takes the form of a hysteresis loop, and the flux-density saturates at around 1 – 2 Wb/m^2. The area of the hysteresis loop represents the energy loss as heat per cycle. Eddy currents are another cause of core loss. Energy can be stored in the magnetic field. In a magnetic circuit consisting of iron core and air-gap, most of the energy is stored in the air-gap.

8. Magnetic materials are characterized by a number of non-ideal properties, which must be considered in the detailed analysis. The most important phenomenon are saturation, eddy currents, and hysteresis.

9. In an ideal transformer, the voltage across each coil is proportional to the number of turns, the net MMF is zero, regulation is 0% and power efficiency is 100%.

10. Efficiency and regulation are important performance indices of transformer operation.

References

1. Allan R. Hambley, Electrical Engineering: Principles and Applications, Fifth Edition, Pearson, 2011. Chapter 15.

2. Giorgio Rizzoni, Principles and Applications of Electrical Engineering, Fifth Edition, McGraw-Hill, 2007. Chapter 18.

Chapter 4: Power Semiconductor Diodes, Rectifiers and DC Power Supplies

4.0 Introduction:

Semiconductor diodes are active devices those play important roles in various electronic as well as electrical circuits. Semiconductor diodes can be considered as *active non-linear circuit elements* with *non-linear voltage-current characteristics*. The diodes are used in power applications to regulate the flow of electrical power only in one direction while blocking the power flow in the reverse direction and are commonly exploited in various power related applications. The other major application of diodes is in rectifier circuits to convert AC power into regulated DC power.

In this chapter, we would discuss diodes, and basic principles of operation, the voltage-current characteristics and their applications in rectifier circuits for DC power supplies. Although there are many different types of rectifier circuits, we would mainly discuss the **half-wave**, and **full-wave bridge rectifier** circuits. We would also discuss *voltage ripples* and how capacitive filter circuit can be used to regulate the dc voltage and reduce the peak-to-peak ripple voltages at the output.

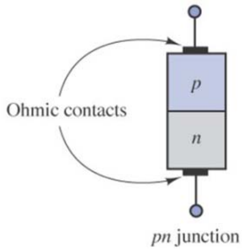
The Learning Objectives for this chapter are:

- Understanding of voltage-current characteristics of a semiconductor diode
- Understanding of half-wave and full-wave bridge-type rectifier circuits
- Understanding of voltage and currents in rectifier circuits
- Analysis of the operation of the rectifier circuits
- Analysis of the bridge rectifier circuit with capacitive filter
- Design of DC power supplies

4.1 Diode

Diodes allow current to flow through *only in one direction* and it can be considered to be the electrical counterpart of mechanical valves those used to regulate the flow of fluid through the valves.

The diode is a two terminal semiconductor device as shown in Fig. 4.1. When the **anode** (A) terminal is made positive potential with respect to the **cathode** (K) terminal, we say that the diode is *forward biased* and *conducts* i.e. current flows from the anode to the cathode. The diode almost acts like short-circuit with a *forward voltage drop* of 0.7 V for silicon based diodes. Alternatively, if the anode is made –ve potential with respect to the cathode terminal then we say that the diode is *reverse biased* and does not conduct i.e. no current flows through the diode. The diode almost acts like an open-circuit under this mode of operation.



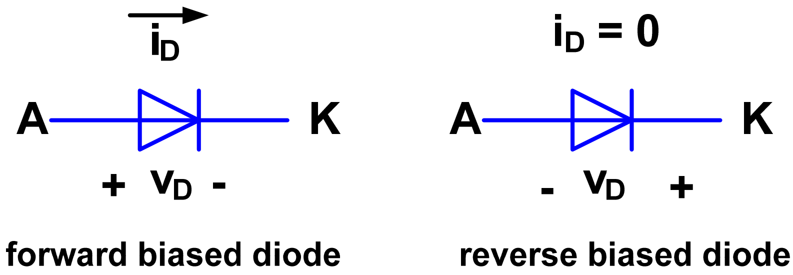


Figure 4.1: Diode operation (a) forward-biased and (b) reverse-biased.

The terminal voltage-current (V-I) characteristic of a diode is as shown in Fig. 4.2. When the diode is forward biased by making the anode voltage, v_{AK} more positive potential w.r.t. the cathode, the diode conducts and the current, i_D through the circuit is controlled by external circuit element as the forward voltage-drop, v_D of the diode is very small of the order of 0.7 V for silicon based signal diodes and of the order of 1.0 – 2.0 V for power diodes. The operating point is in quadrant-I of the V-I characteristic as shown in Fig. 4.2.

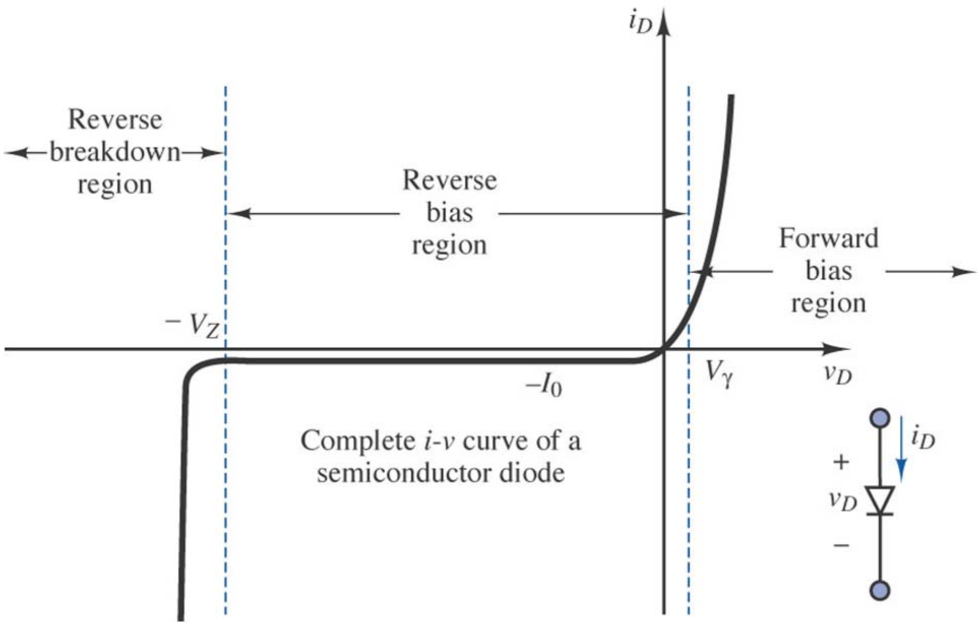


Figure 4.2: V-I characteristics of diode.

When the anode is made –ve potential w.r.t. the cathode terminal, we say that the diode is reverse biased and does not conduct. A very small amount of current, I_0 flows through the diode and is called as leakage current of the order of few hundreds of μA. As the anode-to-cathode voltage, v_{AK} is increased beyond the *reverse breakdown voltage* the diode breaks down and large amount of current starts to flow through the diode. Normally, the diode is not supposed to be operated under this mode of operation. The diode's normal regions of operation are*: forward conducting region* or *reverse blocking region*.

4.1.1 Forward Voltage Drop

The diode uses very little amount of electrical power to regulate the flow of large amount of electrical power. The forward voltage drop, v_D is typically about 0.7 V for silicon based signal diodes and is maintained constant. However, for power diodes, v_D is a function of the current through the diode, i_D and is typically of the order of 1 – 2 V for power diodes.

4.1.2 Reverse Breakdown Voltage

We know that when the diode is reverse biased it does not conduct but there is a maximum reverse voltage, V_z that can be applied to a diode safely without causing any damage to the diode. This *peak inverse voltage* (PIV) that can be safely applied to the diode is as shown in Fig. 4.2. If this PIV is increased beyond the maximum allowable limit then the diode fails and large current flows in the reverse direction. This maximum voltage is called the reverse breakdown voltage and can be typically of the order of 2000 V for power diodes.

4.1.3 Ideal Diode

In most of the applications the supply voltage is much larger than the voltage drop across the diode during conducting state and therefore can be neglected. The V-I characteristics of an ideal diode (approximate thick lines) is as shown in Fig. 4.3.

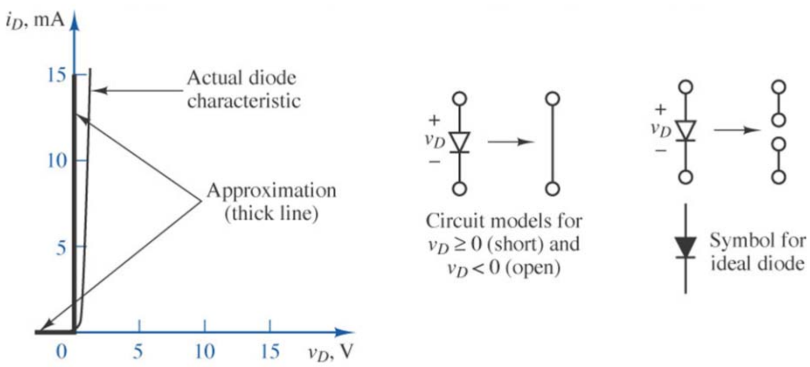


Figure 4.3: Ideal diode model.

The ideal diode acts as short-circuit as shown in Fig. 4.3 when conducting and as open circuit when reverse biased.

4.3 Diode Rectifier Circuits

One of the important applications of power diodes is in rectifiers, in which AC power is converted into DC power and widely used as DC power supplies.

There are different types of rectifier circuits and the most commonly used ones are:

- Half-wave rectifier
- Full-wave rectifier
    - Bridge type
    - Centre-type

4.3.1 Half-wave Rectifier

The simplest type of rectifier circuit is the half-wave rectifier circuit as shown in Fig. 2.4(a). As the source voltage, v_s is AC in nature and the diode is forward biased only when the anode is made positive w.r.t. the cathode, the diode would conduct the current only during the positive half-cycle. The diode being forward biased would close the circuit and link the source voltage to the resistive load. We say that the diode acts as a switch and the switch is closed during the positive half-cycle.

However, during the –ve half-cycle the anode of the diode is made –ve w.r.t. the cathode and therefore the diode is reverse biased and won't conduct. Thus, the diode as a switch is open. No current flows through the diode and the load is disconnected from the source. The corresponding output voltage is zero as shown in Fig. 4.4(c). The voltage at the output of the rectifier circuit is of one polarity and hence said to be *rectified*. The corresponding source voltage, v_s and output voltage, v_o are shown in Fig. 4.4.

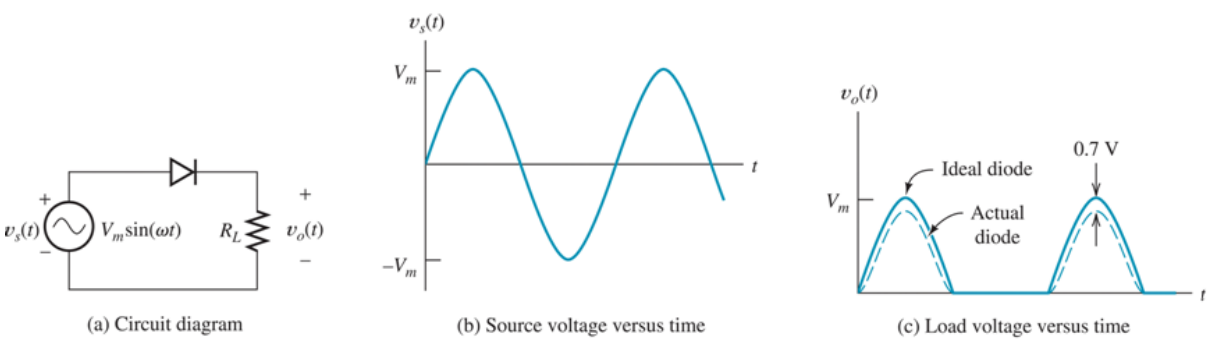


Figure 4.4: Half-wave rectifier with resistive load.

Note that the output voltage instantaneous value varies and has a maximum value of V_m and a minimum value of 0 V. This variation of the output DC voltage is called "**ripple**" and the corresponding voltage is called as *peak-to-peak ripple voltage*, V_{pk-pk}.

Average Load Voltage

If a DC volt meter is connected to measure the output voltage then it would read the average output voltage and is given by

$$V_{o,avg} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t) + \int_{\pi}^{2\pi} 0\, d(\omega t)\right] = \frac{V_m}{\pi} \quad (4.1)$$

The output voltage can be seen to have a DC component whose value is given by eqn. 4.1 and an AC component and can be seen from Fig. 4.4 the AC component has a peak-to-peak magnitude of V_m.

Average Load Current

Being a resistive load the average load current is given by

$$I_{L,avg} = \frac{V_{o,avg}}{R_L} \quad (4.2)$$

Peak Inverse Voltage

The PIV for the diode is given

$$PIV = V_m \quad (4.3)$$

It is called a half-wave circuit as it preserves only half-of the AC voltage waveform. This is not very efficient way of rectifying the AC signal as the –ve half-cycle energy of the AC signal is not recovered. By making use of a full-wave rectifier circuit it would be possible to recover the –ve half-cycle as well and would be explained in the next section.

Half-wave Rectifier with Smoothing Capacitor

Many a times, we want to convert the AC voltage signal to get a ripple free DC voltage that can be used for electronic circuits. However, there exists significant amount of ripple voltage in the output. One way to eliminate this ripple voltage is to make use of a large smoothing capacitor as shown in Fig. 4.5(a). The corresponding output voltage and current waveforms are shown in Fig. 4.5 (b) and (c). When the input source voltage reaches to its peak value then the capacitor is charged to its peak source voltage assuming the diode to be ideal. Once the instantaneous source voltage magnitude drops below the peak value then the capacitor voltage is maintained at its peak value and therefore the diode becomes reverse biased and the turns-off and no current flows through the diode as can be seen in Fig. 4.5(c). The capacitor now continues to provide the load current and discharges slowly until the next +ve half-cycle of the source voltage appears and when the source voltage magnitude is higher than that of the capacitor voltage the diode becomes forward biased and starts to conduct and current flows through the diode to charge the capacitor as shown in Fig. 4.5 (c). Because of the capacitor charge and discharge cycles the output voltage contains a smaller amount of ac ripple voltage in comparison with that when there is no capacitor at the output. Thus the quality of the output DC voltage is improved.

The capacitor discharges for *almost* one electrical cycle (approximation used) and the charge removed from the capacitor is given by

$$Q \cong I_L \times T \quad (4.4)$$

where, Q is the charge, I_L is the average load current and T is the period of one electrical cycle of the AC voltage.

The charge removed from the capacitor can be expressed as the product of change in the output voltage and the capacitance, we have

$$Q = V_{pk-pk} \times C \quad (4.5)$$

Equating eqns. (4.4) and (4.5) we have

$$I_L \times T = V_{pk-pk} \times C \quad \Rightarrow C = \frac{I_L T}{V_{pk-pk}} = \frac{I_L}{V_{pk-pk} \times f} \quad (4.6)$$

In reality, eqn. (4.6) is just an approximation as the load current varies rather than being a constant value and the capacitor does not discharge for one complete electrical cycle. However, it gives a good initial approximation for calculating the value of the capacitance required to minimize the ripple voltage based on a given specification.

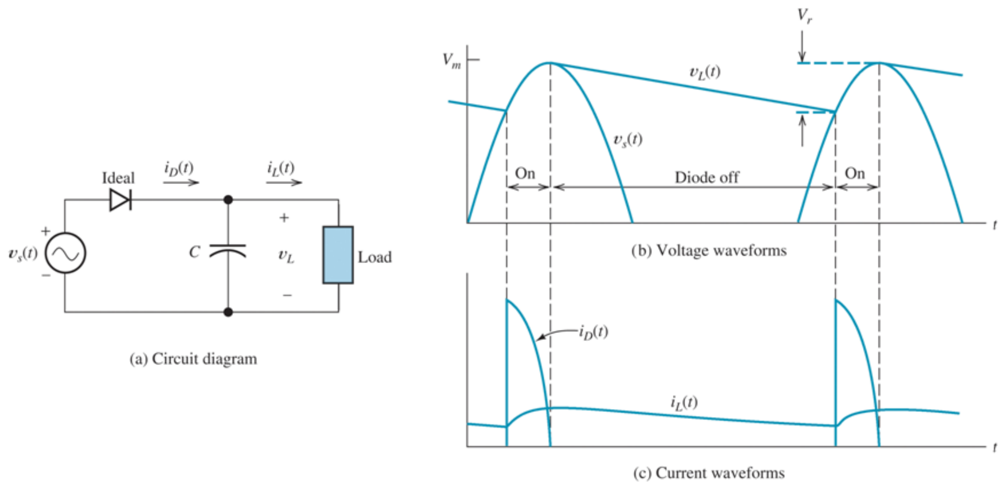


Figure 4.5: Half-wave rectifier with smoothing capacitor.

Example 4.1:

A half-wave rectifier circuit as shown in Fig. 4.4(a) is used to provide a DC supply to a 50 Ω resistive load. If the AC source voltage is 230 V rms, find the peak and average current in the load. Also determine the percentage of the peak-to-peak ripple in the output voltage. Assume the diode to be ideal.

Solution:

The peak load current is given by

$$i_{L,pk} = \frac{V_m}{R_L} == \frac{230\sqrt{2}}{50} = 6.5\ A$$

The average load current is given by

$$i_{L,avg} = \frac{V_{o,avg}}{R_L} = \frac{V_m/_\pi}{50} = \frac{230\sqrt{2}}{50 \times \pi} = 2.1\ A$$

The peak-to-peak ripple voltage at the output is given by

$$V_{pk-pk} = V_m = \sqrt{2} \times V_{s,rms} = \sqrt{2} \times 230 = 325.3\ V,$$

$$\text{Percentage ripple} = \frac{V_{pk-pk}}{V_{o,avg}} \times 100\% = \frac{325.3}{103.5} \times 100\% = 314.2\%$$

4.3.2 Full-wave Diode Bridge Rectifier

The full-wave rectifier provides substantial improvement in efficiency over the half-wave rectifier and therefore widely accepted in the industry. Consider the full-wave bridge type of rectifier circuit as shown in Fig. 4.6.

During the +ve half-cycle of the AC source voltage, $v_s(t)$ the diodes D_1 and D_3 conduct while during the –ve half-cycle the diode pair D_2 and D_4 conduct. Due to the structure of the bridge, the current flow through the load is always unidirectional and from terminal *c* to *d* during both the half-cycles and thus, the full-wave rectification of the AC source waveform. The source voltage, $v_s(t)$ and the rectified output voltages are shown in Fig. 4.7.

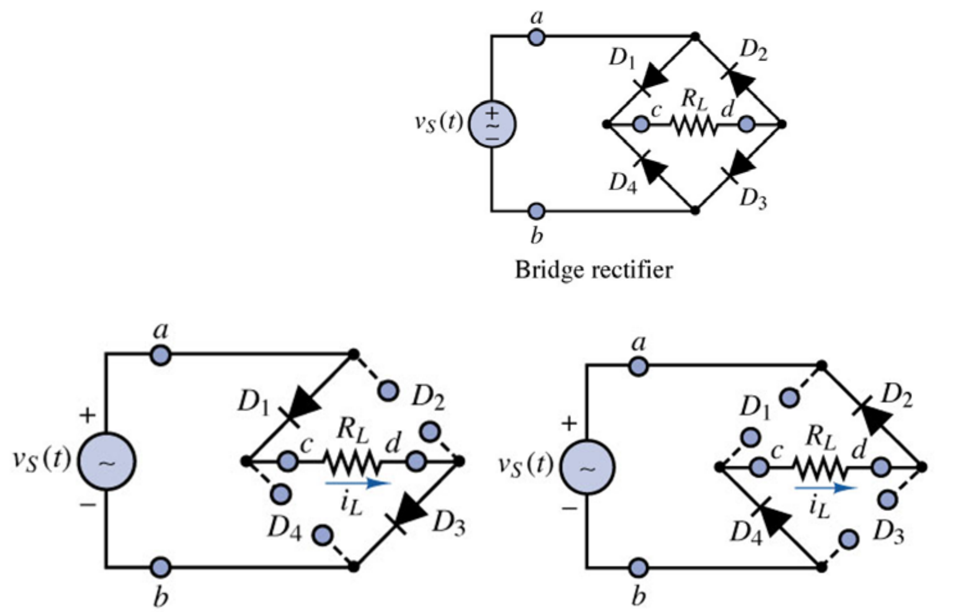


Figure 4.6: Full-wave bridge rectifier circuit with resistive load.

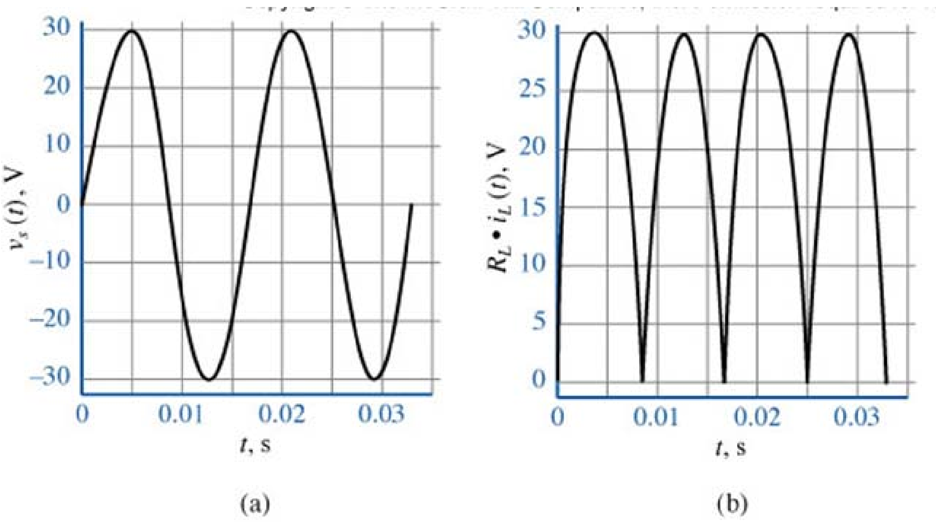


Figure 4.7: (a) Source voltage and (b) rectified output voltage (assuming ideal diode).

Average Load Voltage

The average output voltage and is given by

$$V_{o,avg} = \frac{1}{\pi}\left[\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t)\right] = \frac{2V_m}{\pi} \qquad (4.7)$$

The output voltage can be seen to have a DC component whose value is given by eqn. 4.7 and an AC component and as can be seen from Fig. 4.7(b) the AC component has a peak-to-peak magnitude of V_m. We can see the average output voltage of a full-wave rectifier circuit is twice that of the half-wave rectifier circuit.

Although the full-wave bridge rectifier circuit produces a DC output voltage from an AC source voltage, the output voltage is not smooth and rather fluctuating as shown in Fig. 4.7 (b), whereas, it is necessary to have a smooth DC output voltage. The *ripple* i.e. the fluctuation in the output voltage is the characteristics of a rectifier circuit and is undesirable. The ripple frequency of the full-wave rectifier circuit is $2f_s$ where f_s is the frequency of the source voltage, $v_s(t)$. This is because we have two pulses in the output voltage in one electrical cycle of the supply voltage.

Full-wave Bridge Rectifier with Smoothing Capacitor

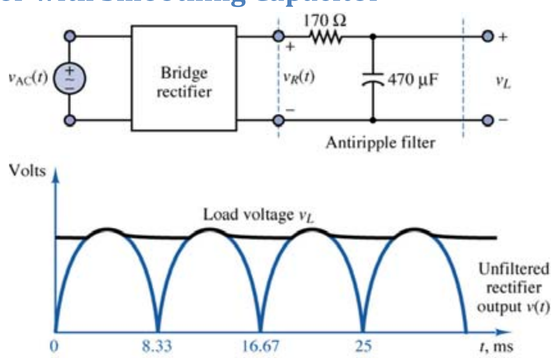


Figure 4.8: Full-wave bridge rectifier with filter circuit

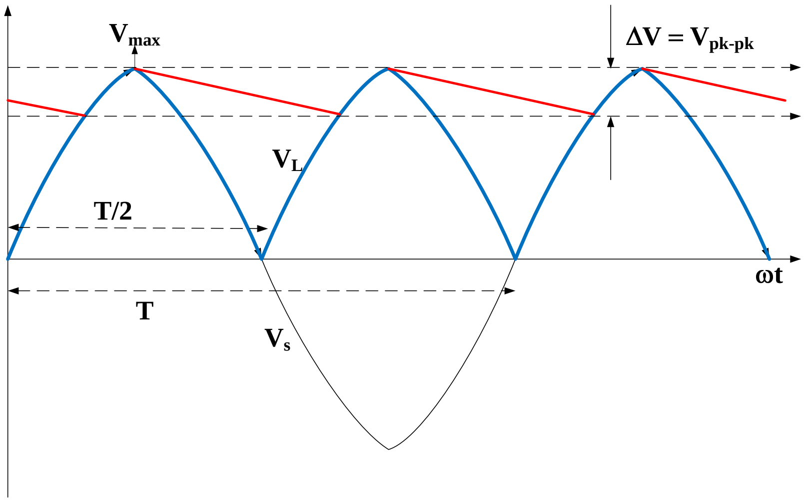


Figure 4.9: Bridge rectifier with filter capacitive filter waveforms.

A simple yet effective way to eliminate the ripples in the output voltage is to make use of a capacitive filter circuit. Just as we analyzed the half-wave rectifier with a smoothing capacitor, we can extend the same analysis for the full-wave rectifier except that the charge removed from the capacitor is for a period of half-cycle, *T/2* unlike for the full-cycle, *T.*

Thus we have,

$$Q \cong I_L \times \frac{T}{2} = V_{pk-pk} \times C \Rightarrow C = \frac{I_L \times T}{2 \times V_{pk-pk}} = \frac{I_L}{2 \times f_s \times V_{pk-pk}} \quad (4.8)$$

From eqn. (4.8) it can be seen that if we want to reduce the V_{pk-pk} ripple to zero then we must use a very large capacitor. Typically DC power supply would have a ripple voltage of less than 1%.

Example 4.2:

A DC power-supply circuit is needed to deliver 0.1 A with an average voltage of 15 V. The ac source has a frequency of 50 Hz. Assume that a full-wave rectifier circuit is used. The peak-to-peak ripple voltage is to be less than 0.4 V. Instead of assuming ideal diode, allow a forward voltage drop of 0.7 V across the diode. Determine the peak value of the source voltage needed and also the smoothing capacitor.

Solution:

The peak value of the voltage is given by

$$V_m = V_{o,avg} + 2 \times v_D + \frac{V_{pk-pk}}{2} = 15 + 2 \times 0.7 + \frac{0.4}{2} = 16.6\ V$$

The approximate value of the capacitor is given by

$$C = \frac{I_L}{2 \times f_s \times V_{pk-pk}} = \frac{0.1\ A}{2 \times 50\ Hz \times 0.4\ V} = 2500\ \mu F$$

4.4 DC Power Supplies and Voltage Regulation

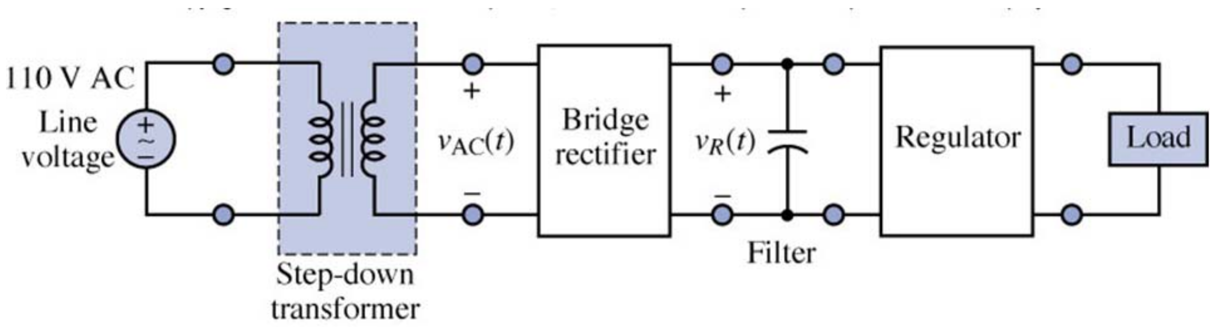


Figure 4.10: Regulated DC power supply.

The main application of the rectifier circuit is the conversion of AC to DC power. A circuit that accomplishes this function is called a **DC power supply**. A typical DC power supply circuit is as shown in Fig. 4.10 and consists of components such as step-down transformer, bridge rectifier circuit, filter circuit and voltage regulator that is used to maintain the output voltage constant at the desired value irrespective of the voltage variations from the source side or load side. Finally, the load is connected at the output. In your project, you would be using the IC7805 as the voltage regulator to provide 5 V DC to the electronic circuit.

Summary

- A p-n-junction diode is a two terminal device that conducts current easily in one direction i.e. from anode to cathode, but not in the opposite direction.
- The V-I characteristic has three regions of operation: forward bias, reverse biased and reverse breakdown regions.
- The rectifier circuits can be either half-wave or full-wave rectifiers. A full-wave rectifier is more desirable than half-wave rectifiers as it has two pulses per cycle and therefore more efficiency as compared to half-wave rectifier circuit.
- The output voltage of the rectifier circuit consists of a DC component and also and an AC component, this is referred to as ripples in output voltage.
- The ripples can be minimized by using a capacitive filter circuit.

References

1. Allan R. Hambley, Electrical Engineering: Principles and Applications, Fifth Edition, Pearson, 2011. Chapter 10.
2. Giorgio Rizzoni, Principles and Applications of Electrical Engineering, Fifth Edition, McGraw-Hill, 2007. Chapter 9.

Chapter 5: Introduction to Electromechanical Energy Converters

Learning Objectives:

- Understanding of the basic principles of operation of linear and rotating DC machines.
- Understanding of principle of operation of electric machines operating as motor and generator.
- Understanding of basic classifications of DC machines.
- Analyze DC motors under steady-state and dynamic operation.
- Analyze DC generators under steady-state operation.
- Understanding of the torque-speed characteristics of separately-excited and permanent DC motors.
- Understanding of speed control of DC motors.

5.0 Introduction to Electromechanical Energy Converters

For the autonomous vehicle in your EE1002 Project to move around following a track you need electric motors to provide mechanical energy to propel the vehicle. In this part of the lecture notes, you would be introduced how to make use of DC motors to provide mechanical energy and also pulse-width-modulated electronic circuit to control the speed of the motor.

Electric motors are used in various power ratings from micro-watt to mega-watt range to provide us mechanical energy to do useful work for us and thereby improve our standard of living. In our day-to-day life, we come across hundreds of motors everyday at home, in office environment, in industrial environment and for that matter almost everywhere e.g. computer HDD, refrigerator, printer, fax machine, washing machine, food mixer, air-conditioner etc. just to name a few. In transportation applications, electric motors are used in MRT, LRT and now we are talking about hybrid-electric-vehicle (HEV) or plug-in-hybrid-EV (PHEV) and full electric-vehicle (EV), even for modern ship-propulsion in war-ships, electric motors are being used. Almost 70% of the electrical energy generated in this world is converted back into mechanical energy by using electric motors.

In view of these technological development, it is important for us to understand what are these electric motors, their principles of operation, how they can be controlled so that we have a better appreciation of these electromechanical devices.

5.1 Rotating Electrical Machines

Any rotating electrical machine has two basic components, namely, the *stator* and the *rotor* as shown in Fig. 3.1 and the rotor rotates inside the stationary part i.e. stator and is separated through a small air-gap. The shaft on the rotor couples the machine to the mechanical load. The shaft is supported to the stator part through bearings so that they can rotate freely as illustrated in Fig. 3.1.

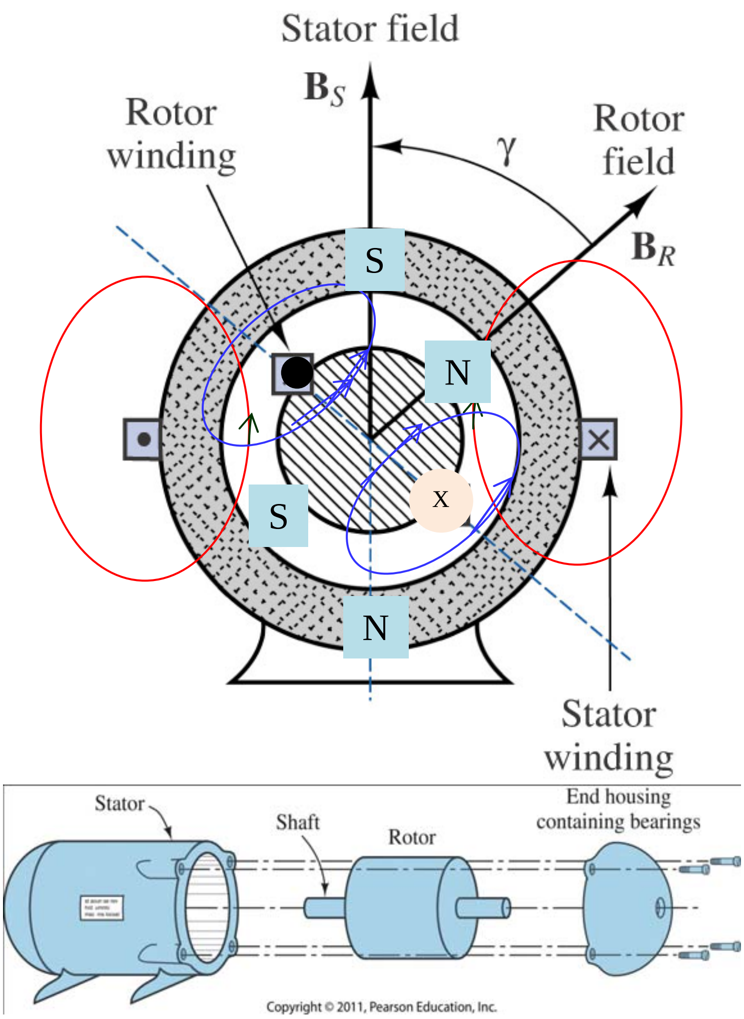


Figure 5.1: (a) rotating electrical machine cross sectional view and (b) An electrical motor consists of a cylindrical rotor that spins inside the stator.

Depending on the construction and type of rotating machine, either the stator or the rotor contain current carrying conductors configured in the form of coils or some may have coils while the other may have permanent magnets. Stators and rotors are made up of iron to intensify the magnetic field. The magnetic field alternates in direction through the iron core with time and therefore the iron must be laminated to limit the power losses in the core due to eddy currents.

Currents in these stator and/or rotor coils set-up magnetic fields $\mathbf{B_s}$ and $\mathbf{B_r}$ as in Fig.5.1 and they interact with each other to produce torque in the machine. Referring to Fig. 5.1, the stator winding carries a current with the polarity of the current shown as dot (•) means current flowing out of the page and cross (×) means current flowing into the page sets up magnetic flux-lines around it (for dot

(•) sign the flux-lines are counter-clockwise and for and cross (×) sign the flux-lines are clockwise according to the RHR).

With the polarity of the currents as shown in Fig. 5.1 (a) in the stator windings the flux-lines moves upward as shown in the figure and sets up the principal magnetic axis, showing the direction of the stator magnetic field, $\mathbf{B_s}$. In the same way, the current in the rotor coil produces the rotor magnetic field, $\mathbf{B_r}$ and these two stator and rotor magnetic fields interact with each other to produce the torque in the electrical machine which causes the rotor to rotate.

These two magnetic fields are produced by different ways (by using permanent magnet or electromagnet) in different electrical machines (DC machines and AC machines) but the presence of these two magnetic fields is what causes the electrical machines to rotate and either produce electrical power (in case of generator) or mechanical power (in case of a motor). It can be seen that the *N-pole* of the rotor field, $\mathbf{B_r}$ would be attracted towards the *S-pole* of the stator field, $\mathbf{B_s}$ and that would cause the stator magnetic field to drag the rotor magnetic field and therefore the rotor would rotate in the counter-clockwise direction in this case.

The torque produced in an electrical machine due to the interaction of the stator and rotor magnetic fields is given by

$$T = k(\mathbf{B_s} \times \mathbf{B_r}) = k|B_s||B_r|\sin(\gamma) \quad (5.2)$$

where k – constant and depends on the geometry of the machine, γ - angle between $\mathbf{B_s}$ and $\mathbf{B_r}$.

5.2 Principles of Linear DC Machine

We introduce the basic principles of DC machines by considering an idealized linear DC machine as shown in Fig. 5.2.

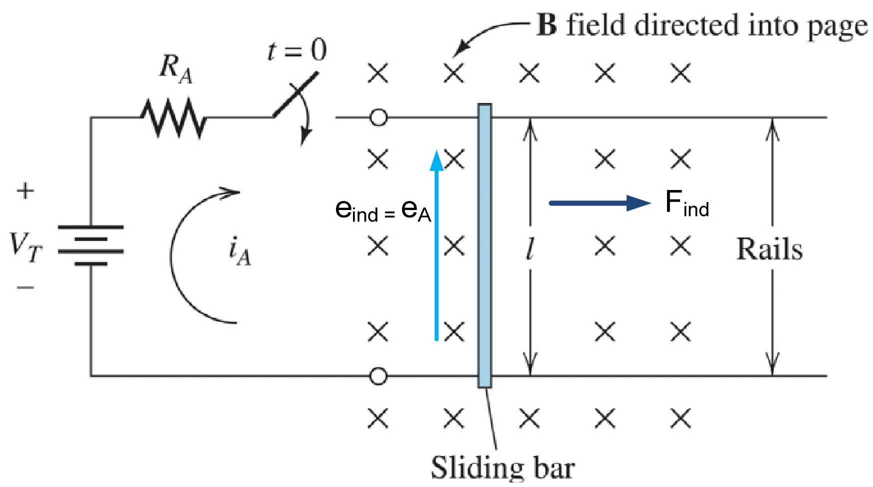


Figure 5.2: A simple dc machine consisting of a conducting bar sliding on conducting rails.

We have the uniform magnetic field, **B** directed into the page of the paper. The sliding bar is the conductor that is placed on the two rails and moves from left of the page to the right. The operation of the linear DC machine can be best explained by using the four equations as given in eqns. 5.3 to 5.6.

$$\mathrm{F_{ind}} = i(\mathrm{l} \times \mathrm{B}) \qquad (3.3)$$
$$e_{ind} = (\mathrm{u} \times \mathrm{B}) \bullet \mathrm{l} \qquad (3.4)$$
$$\mathbf{KVL}: V_T - i_A R_A - e_{ind} = 0 \qquad (3.5)$$
$$\mathbf{Netwon's\ Law:} \mathrm{F_{net}} = m \times a \qquad (3.6)$$

where F_{ind} – force induced on the conductor in N, **l** – length of the conductor in m, **B** – flux-density of the magnetic field in Wb/m^2, e_{ind} – induced emf on the conductor in V, **u** – velocity of the moving conductor in m/s, V_T – supply voltage in V, i_A – current through the conductor in A, R_A – resistance of the conductor in Ω, $\mathbf{F}_{net}$ – net force acting on the conductor in N, m – mass of the conductor in Kg and a – acceleration due to gravity in m/s^2.

When the switch is closed at ($t = 0^+$), current i_A flows through the machine and is given by

$$i_A = \frac{V_T - e_{ind}}{R_A} \qquad \mathbf{(3.7)}$$

The current i_A flowing through the conductor placed in a magnetic field **B** (x - into the page of the paper) would induce a force on the conductor and is given by

$$\mathrm{F}_{ind} = Bi_A l \ \textbf{- to the right} \qquad \mathbf{(3.8)}$$

The direction of the induced force is given by the right-hand-rule (RHR). The *thumb* gives the direction of force, the *index-finger* gives the direction of the current and therefore the conductor and the *middle-finger* gives the direction of the magnetic field, **B**.

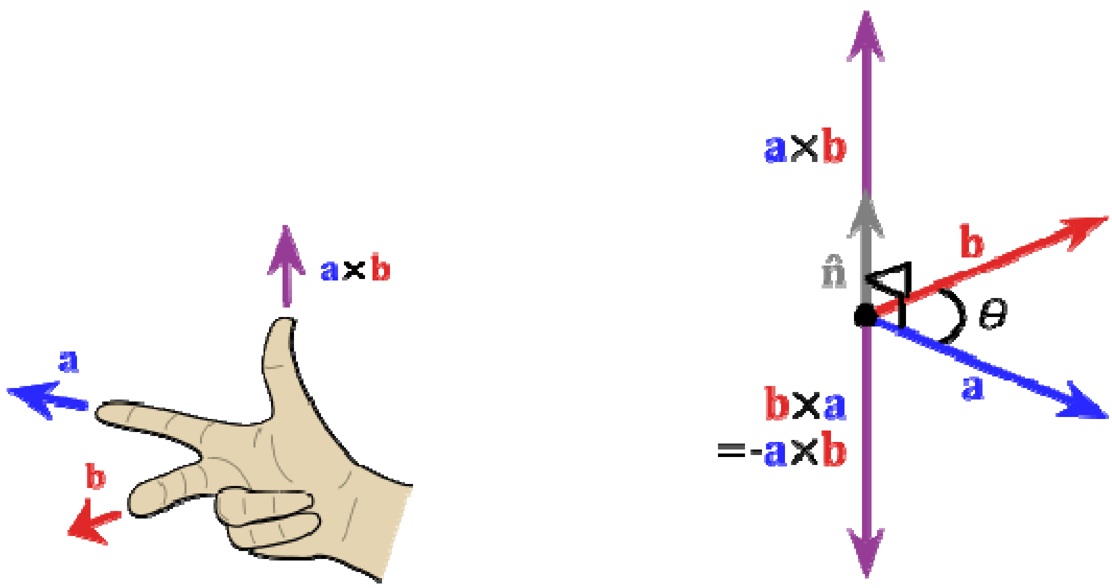


Figure 5.3: Right-hand-rule (RHR) showing the direction of force for motor operation.

According to Newton's law of motion, the bar would now accelerate to the right. As the bar velocity begins to increase it would induce a voltage e_{ind} as shown in Fig. 5.2 and note that the induced emf has a polarity that is such that it would oppose the main source that produces it i.e. V_T. Thus, it follows Lenz's law.

$$e_{ind} = Blu \text{ - positive upward} \quad (3.9)$$

The induced voltage, e_{ind} reduces the current according to KVL:

$$i_A \downarrow = \frac{V_T - e_{ind} \uparrow}{R_A} \qquad (3.10)$$

As u increases increasing e_{ind} the current, i_A reduces reducing the induced force, F_{ind} on the conductor, eventually reaching a steady-state speed where the net force, F_{net} acting on the bar is zero. The induced voltage, e_{ind} would rise to a value when it is equal to the supply voltage, V_T and at that point the bar attains steady-state speed, $\mathbf{u_{ss}}$ and is given by

$$V_T = e_{ind} = Blu_{ss} \Rightarrow u_{ss} = \frac{V_T}{Bl} \qquad (3.11)$$

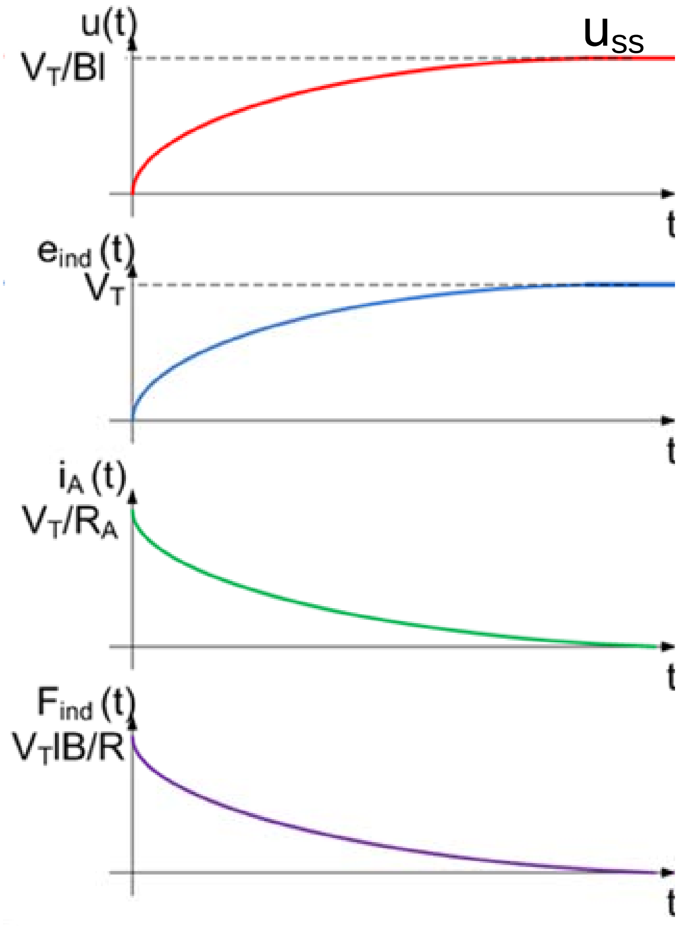


Figure 5.4: The linear DC machine on starting.

To summarize at starting the linear DC machine (LDCM) behaves as follows:

1. Closing the switch produces a current flow in the machine, $i_A = V_T / R_A$.

2. The current flow produces a force on the bar, $F_{ind} = Bi_A l$.

3. The bar accelerates to the right producing an induced voltage e_{ind} as it speeds up.
   $e_{ind} \uparrow = \uparrow uBl$

4. The induced voltage reduces the current flow: $i_A \downarrow = \frac{V_T - e_{ind} \uparrow}{R_A}$

5. The induced force is thus decreased until eventually, $F_{ind} = 0$

6. With induced force, the current reduces to zero leading to steady-state speed, $u_{ss} = \frac{V_T}{Bl}$

5.3.1 Linear DC Machine Operating as a Motor

Assume that the LDCM is initially running at no-load steady-state speed of u_{ss} as shown in Fig. 5.4.

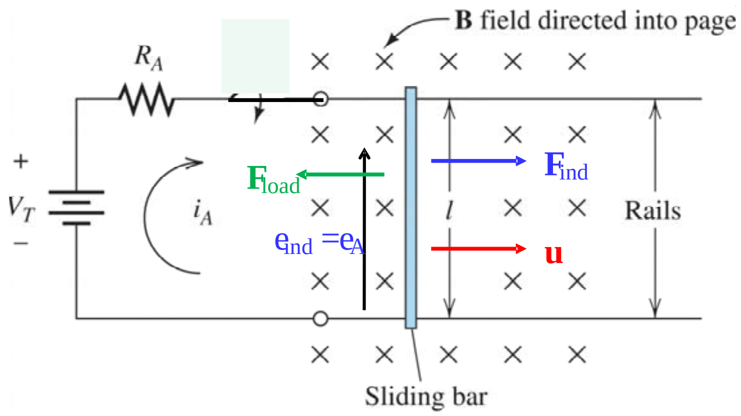


Figure 5.5: The linear DC machine operating as a motor.

Now if an external load, F_{load} is applied opposite to the direction in which motor (conductor) is moving.

As initially the bar is at steady-state speed with $F_{ind} = 0$, with the applied external load, $F_{net} = 0 - F_{load} = - F_{load}$.

The effect of F_{net} is that the bar speed, u would slow down and as soon as the bar slows down, the induced voltage in the bar drops.

$$i_A \uparrow = \frac{V_T - e_{ind} \downarrow}{R_A} \qquad (3.12)$$

The drop in induced voltage increases the bar current, i_A which increases the induced force, F_{ind} and it would rise until the $F_{ind} = F_{load}$ and the bar again attains a new steady-state but at a lower speed, **u** than that of the no-load speed.

Thus, there is an induced force in the direction of motion and *energy/power is converted from electrical to mechanical form* to keep the bar moving.

$$P_{conv} = e_{ind} \times i_A = F_{ind} \times u \qquad (3.13)$$

Thus, we say that the LDCM is operating as a *motor*.

The various variables of the linear DC machines are plotted in Figure 5.6.

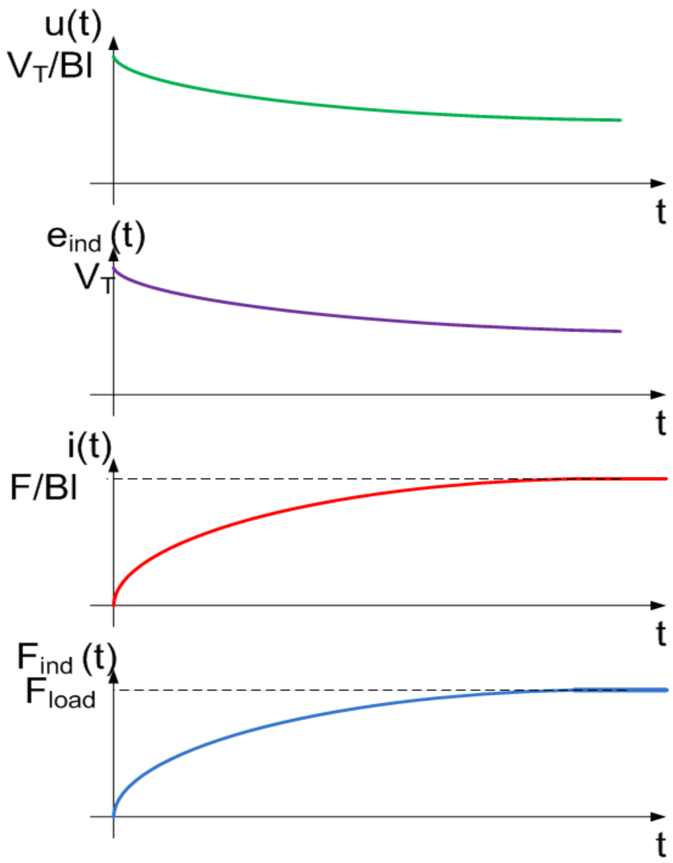


Figure 5.6: The linear DC machine operating at no-load condition and then loaded to operate as a motor.

- To summarize the linear DC machine operating as a *motor* behavior as follows:

    1. A force $\boldsymbol{F}_{load}$ is applied opposite to the direction of motion, which causes a net force $\boldsymbol{F}_{net}$ opposite to the direction of motion.

    2. The resulting acceleration $a = \boldsymbol{F}_{net}/m$ is negative and therefore the bar slows down (u↓).

    3. The induced voltage $e_{ind} = u \downarrow Bl$ falls, and so $i_A = (V_T - e_{ind} \downarrow)/R_A$ increases.

    4. The induced force $\boldsymbol{F}_{Ind} = i \uparrow Bl$ increases until $|\boldsymbol{F}_{Ind}| = |\boldsymbol{F}_{load}|$ at a lower speed u.

    5. An amount of electrical power equal to $e_{ind} \times i_A$ is converted to mechanical power $\boldsymbol{F}_{Ind} \times u$ and machine operates as a motor.

5.3.2 Linear DC Machine as a Generator

Assume that the LDCM is initially running at no-load steady-state speed as shown in Fig. 5.7.

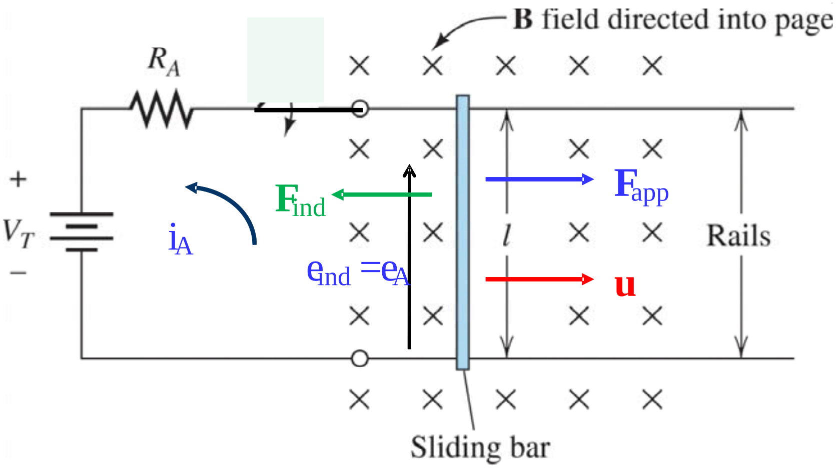


Figure 5.7: The linear DC machine operating as a generator.

Now if an external load $\boldsymbol{F_{load}}$ **(=** $\boldsymbol{F_{app}}$**)** is applied in the same direction in which machine (conductor) is moving and see what happens.

The applied force, $\boldsymbol{F_{load}}$ would cause the bar to accelerate in the same direction of motion and therefore the velocity, **u** increases increasing the induced voltage, $e_{ind.}$

If $e_{ind} > V_T$ the current, i_A would reverse direction and is given by

$$i_A \uparrow = \frac{e_{ind} \uparrow - V_T}{R_A} \qquad (3.14)$$

As the current i_A moves up along the bar, the force induced in the conductor would be to the left and given by the RHR and is shown in Fig. 5.7.

The direction of F_{ind} is given by the RHR hand opposes the applied force $F_{load.}$

$$F_{ind} = i_A l B \qquad (3.15)$$

Finally, the induced force, F_{ind} becomes equal to the applied load force, F_{load} and machine operates at a higher speed than before. Note that the battery source is now *charging* and mechanical power input ($P_{mech.} = F_{load} \times u$) is *converted to electrical power* ($P_{elec.} = e_{ind} \times i_A$) and machine is said to be operating as a *generator*.

The corresponding waveforms of various variables are as shown in Fig. 5. 8.

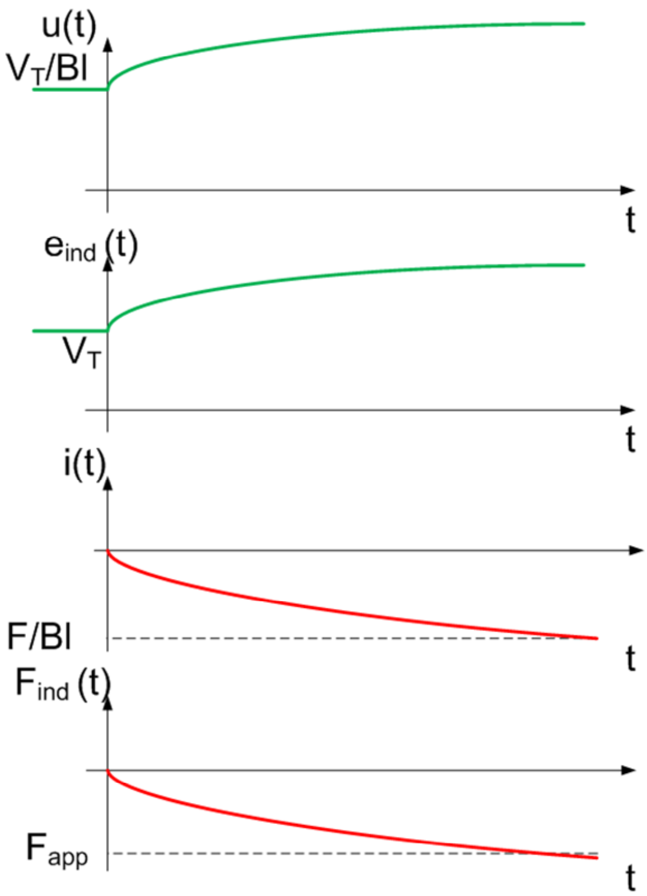


Figure 5.8: The linear DC machine operating at no-load condition and then loaded to operate as a generator

- To summarize the linear DC machine operating as a generator behavior as follows:

  1. A force $\boldsymbol{F}_{app}$ is applied in the same direction of motion, which causes a net force $\boldsymbol{F}_{net}$ in the direction of motion.
  2. The resulting acceleration $a = \boldsymbol{F}_{net}/m$ is positive and therefore the bar speeds up ($u\uparrow$).
  3. The induced voltage $e_{ind} = (u\uparrow)Bl$ increases, and so $i_A = ((e_{ind}\uparrow)-V_T)/R_A$ increases in the reverse direction i.e. -ve.
  4. The induced force $\boldsymbol{F}_{Ind} = (i_A\uparrow)Bl$ increases until $|\boldsymbol{F}_{Ind}| = |\boldsymbol{F}_{app}|$ at a higher speed, u.
  5. An amount of mechanical power equal to ($\boldsymbol{F}_{Ind}$ x u) is converted to electrical power (e_{ind} x i_A) and the machine operates as a generator.

It is interesting to see that the *same machine* operates both as *motor* as well as *generator* depending on what is input i.e. whether mechanical power or electrical power. If input is electrical energy and the output is mechanical energy then we say that the machine is operating as a motor. Alternatively, if the input is mechanical energy and the output is electrical energy then we say that the machine is operating as a generator.

The only difference is that if the external applied force is in the direction of motion the machine operates as a (generator) or if the applied force is in the direction opposite to motion then the machine operates as a (motor).

Electrically, when $e_{ind} > V_T$ the machine operates as a generator and when $e_{ind} < V_T$ the machine operates as a motor.

When the machine operates at a higher speed it operates as a generator and when it moves slowly it operates as a motor.

Example 5.1:

Suppose that for a linear machine as shown in Fig. 5.2, we have B = 1 Wb/m^2, l = 0.3 m, V_T = 2 V, and R = 0.05 Ω.

a. Assuming that the bar is stationary at t = 0, compute the initial current and initial force on the bar. Also determine the final steady-state speed assuming that no mechanical load is applied on the bar.

b. Now, suppose that a mechanical load of 4 N directed to the left is applied to the moving bar. In steady-state, determine the speed at which the bar is moving, the power delivered by the electrical source, the power delivered to the mechanical load, the power lost as heat in the resistance, R_A, and the efficiency of the electromechanical system.

c. Now, suppose that a mechanical pulling force of 2 N directed to the right is applied to the moving bar. In steady-state, determine the speed, the power taken from the mechanical source, the power delivered to the battery, the power lost as heat in the resistance, R_A, and the efficiency.

Solution:

(a) Initially, we have **u** = 0, and thus, $e_{ind} = e_A = 0$, and thus the initial current is given by

$$i_A(0^+) = \frac{V_T - e_A}{R_A} = \frac{2-0}{0.05} = 40\ A$$

The resulting induced force on the bar is given by

$$F_{ind}(0^+) = Bli_A(0^+) = 1\frac{Wb}{m^2} \times 0.3\ m \times 40A = 12\ N$$

In steady-state with no mechanical force we have

$$e_A = Blu = V_T \Rightarrow u_{ss} = \frac{V_T}{B \times l} = \frac{2}{1 \times 0.3} = 6.667\ m/s$$

(b) As the mechanical force applied opposes the motion of the bar, the machine operates as a motor. At steady-state the net force acting on the bar is zero i.e. $F_{ind} = F_{load}$.

$$F_{load} = F_{ind} = Bli_A \Longrightarrow i_A = \frac{F_{load}}{B \times l} = \frac{4\ N}{1\dfrac{Wb}{m^2} \times 0.3m} = 13.33\ A$$

From the circuit KVL equation we have

$$e_A = V_T - i_A \times R_A = 2\ V - 13.33\ A\ \times 0.05\ \Omega = 1.333\ V$$

Thus, the steady-state speed is:

$$u = \frac{e_A}{B \times l} = \frac{1.33\ V}{1\frac{Wb}{m^2} \times 0.3m} = 4.444\ m/s$$

The mechanical power delivered to the load is given by

$$p_m = F_{load} \times u = 4N\ \times 4.44\frac{m}{s} = 17.77\ W$$

The electrical power drawn from the battery source is given by

$$p_{in} = V_T \times i_A = 2V \times 13.33\ A = 26.67\ W$$

The power dissipated as heat in the resistance

$$p_R = i_A^2 \times R_A = (13.33\ A)^2 \times 0.05\ \Omega = 8.889\ W$$

The efficiency of the linear DC machine is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{17.77\ W}{26.67\ W} \times 100\% = 66.67\ \%$$

(c) When the pulling force, F_{app} is applied to the bar it helps to speed up the bar and the corresponding induced voltage, e_A exceeds the supply voltage, V_T and therefore the current flows out of the bar rather than flowing into the bar. We say that the machine operates as a generator. This current in the counter-clockwise direction would induce a force on the conductor that is pointed to the left and under steady state condition this induced force is equal in magnitude but opposite to the pulling force.

$$F_{app} = F_{ind} = Bi_A l \Longrightarrow i_A = \frac{F_{app}}{B \times l} = \frac{2\ N}{1\frac{Wb}{m^2} \times 0.3m} = 6.667\ A$$

From the circuit KVL equation we have

$$e_A = V_T + i_A \times R_A = 2\ V + 6.667\ A\ \times 0.05\ \Omega = 2.333\ V$$

The new steady-state speed is given by

$$u = \frac{e_A}{B \times l} = \frac{2.333\ V}{1\frac{Wb}{m^2} \times 0.3m} = 7.778\ m/s$$

The mechanical power supplied by the source is given by

$$p_m = F_{app} \times u = 2N\ \times 7.778\frac{m}{s} = 15.56\ W$$

The electrical power absorbed by the battery is given by

$$p_{out} = V_T \times i_A = 2V \times 6.667\ A = 13.33\ W$$

The power dissipated as heat in the resistance

$$p_R = i_A^2 \times R_A = (6.667\ A)^2 \times 0.05\ \Omega = 2.222\ W$$

The efficiency of the linear DC machine is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{13.33\ W}{15.56\ W} \times 100\% = 85.67\ \%$$

5.4 Rotating DC Machine

A rotating electrical machine may contain several sets of windings and they can be classified as: either a *field* winding or an *armature* winding.

The field winding sets up the magnetic field (**B**) in the machine and is independent of the mechanical load imposed on the machine.

On the other hand, the armature winding carries a current that is dependent on the mechanical load produced by the machine. The armature current is more or less directly proportional to the mechanical load on the machine.

5.4.1 Structure of the Rotor and Stator

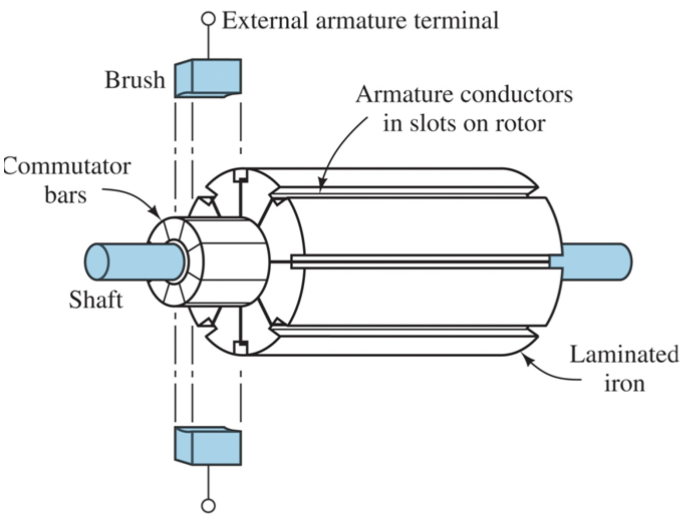


Figure 5.9: Rotor assembly in a DC machine

DC machine contains a cylindrical stator with an even number of magnetic poles established by the permanent magnets or field windings and the poles alternate between the north and south poles around the periphery of the stator.

Inside the stator, the rotor consists of laminated cylindrical iron mounted on the shaft that is supported by bearings such that it can rotate. Rotor with slots for armature conductors is shown in Fig. 5.9. The armature conductors are placed on the slots on the rotor and are brought out to the commutator segments. Currents are applied to the armature conductors form external supply through the carbon brushes those are pressed against the commutator segments as shown in Fig. 5.9

The cross sectional view of a 2-pole machine with the flux-lines in the air-gap is shown in Fig. 5.10.

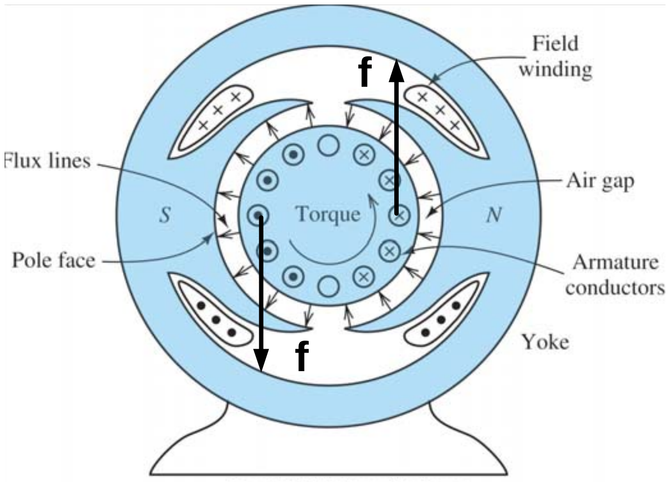


Figure 5.10: Cross sectional view of a DC machine.

Magnetic flux lines take the path of least reluctance in the air-gap and therefore they are perpendicular to the surface of the rotor iron. Moreover, the flux-density is more or less constant magnitude over the surface of each pole face as shown in Fig. 5.10.

External electrical sources provide currents to both field as well as armature windings. With the current directions as shown in the field windings, the flux lines leave from the right-hand side of the stator under the north-pole cross the air-gap and enter the rotor and then cross the air-gap again and reach the south-pole on the left-hand side and thereby complete the magnetic circuit.

Based on the magnetic flux direction i.e. from left to the right on the plane of the paper and the armature conductors current direction the torque generated is counter-clockwise according to **f** = i(**l** x **B**).

5.4.2 Induced EMF and Commutation

As the conductor moves through the magnetic field produced by the stator, an emf is induced in the single turn coil conductor, as shown in Fig. 5.11. The single-turn coil consists of conductor segments, *d-c*, *c-b* and *b-a.* Under the pole face the conductor, the magnetic field and the direction of motion are mutually perpendicular to each other and an emf is produced in the conductors according to eqn. 5.4

$$e_{ind} = (\mathrm{u} \times \mathrm{B}) \bullet \mathrm{l} \qquad (3.4)$$

Consider the position of the coil as shown in Fig. 5.11 (a). The conductor *a-b* is under the north-pole and the conductor *c-d* under the south-pole. The emf induced in the conductors c-d and a-b are in series and the corresponding polarities of the induced emf at the terminal are as shown in Fig. 5.11 (d). Thus, the voltage, v_T appearing at the terminal is of the polarity as shown in Fig. 5.11 (d). The induced emf remains constant under the pole face as the conductor moves along i.e. counter-clockwise rotation. However, as the conductor moves out of the magnetic pole face the flux-lines cutting the conductor reduces and thereby the emf induced reduces to zero as shown in Fig. 5.11(b) where the turn is at right-angle to the orginal position Fig. 5.11(a). In this position no flux-lines cuts the conductors *a-b* and *c-d* and therefore the voltage induced is zero as shown in Fig. 5.11 (d). As the conductor *a-b* moves under the opposite south-pole a negative voltage is induced in the conductor *a-b* as shown in Fig. 5.11(c) and the same is true for the conductor *c-d* as shown in Fig. 5.11(c). Thus, the output voltage at the end of the coil a-d in this position is –ve as shown in Fig. 5.11(e).

However, a mechanical switch known as *commutator* reverses the connections to the conductor as they move between the poles so the polarity of the induced emf as seen from the external terminal remains the same i.e. we get a rectified dc voltage as shown in Fig. 5.11.

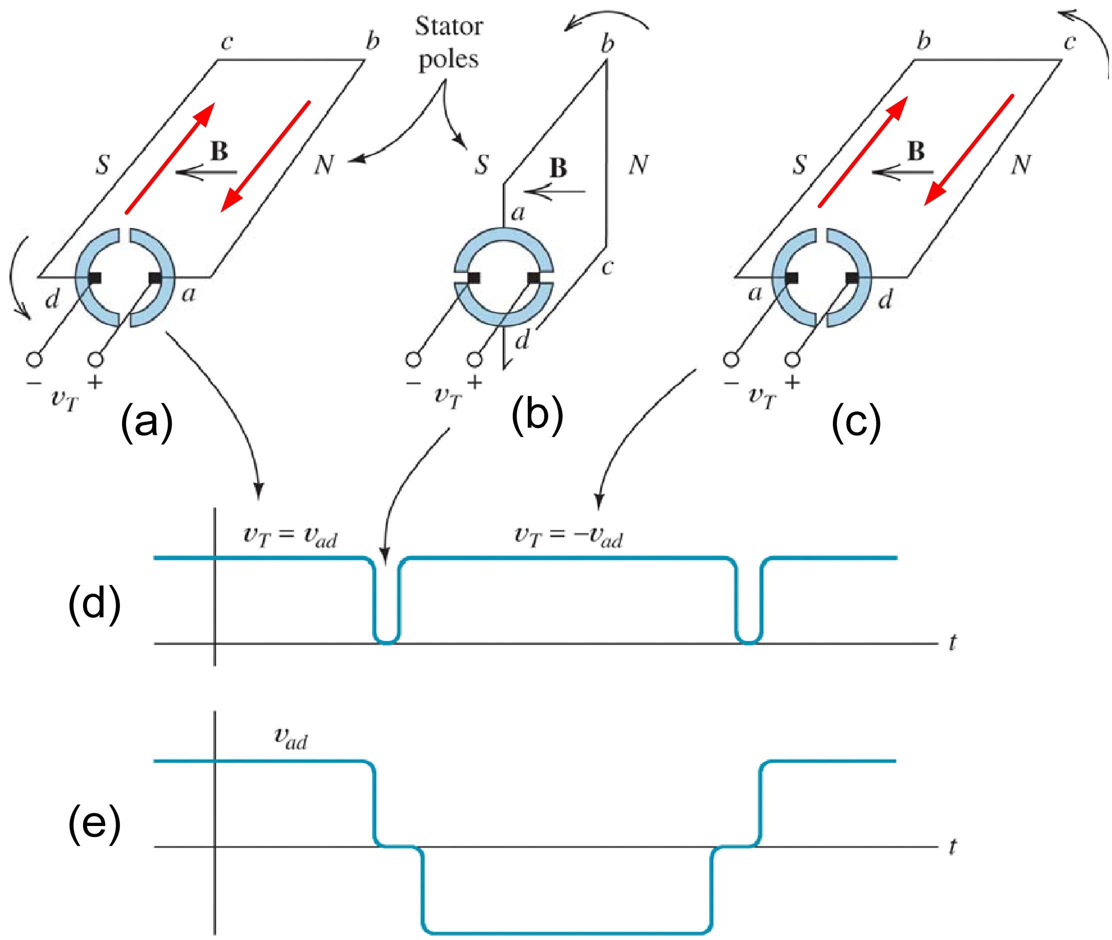


Figure 5.11: Commutation for a single armature winding.

5.5 Classification of DC Machines

Depending on the various ways the field and armature windings are energized from different electrical sources, the DC machines can be classified as follows:

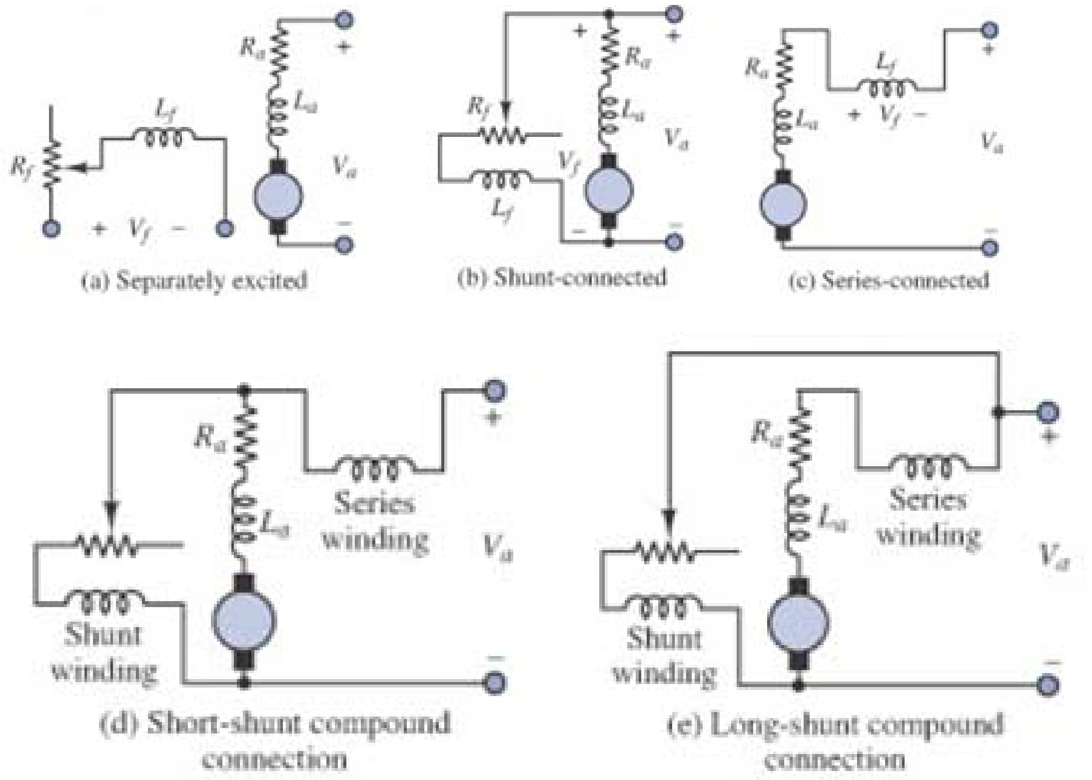


Figure 5.12: Classification of DC machines.

- (a) Separately-excited (separate voltage sources)
- (b) Shunt-excited (single voltage source)
- (c) Series-excited (single voltage source)
- (d) and (e) Compound-excited (single voltage source)

5.6 DC Machine Equivalent Circuit

The equivalent circuit of a DC machine is shown in Fig. 5.13. The field circuit is represented by R_F and L_F in series and the corresponding source voltage is represented by V_F and the corresponding DC current flowing through it is represented as I_F. Take note that the field current sets-up the magnetic flux in the DC machine.

For steady-state operation we deal with dc quantities only and neglecting the field inductance, L_F we have

$$V_F = R_F I_F \qquad (3.16)$$

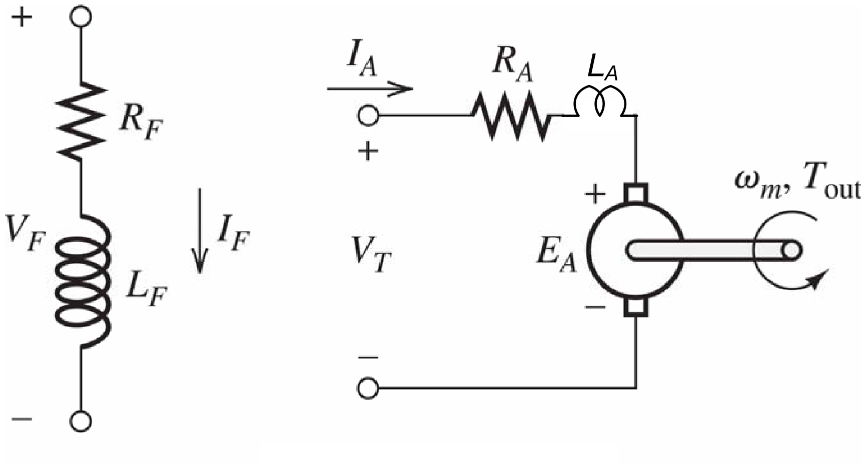


Figure 5.13: Equivalent circuit of rotating DC machine.

The armature winding is represented by the resistance R_A (including the brush resistance) and inductance $L_{A.}$ In the armature circuit, for steady-state operation, we have E_A that represents the voltage induced in the armature conductors due to the rotation of the rotor in the presence of a magnetic field. It is known as the *back-emf* as it opposes the external armature voltage source, V_T *or*V_A. Note that Lez's law states that the voltage induced has a polarity that opposes the source.

The induced emf on the armature circuit is given by

$$E_A = K_A \phi \omega_m \quad (3.17)$$
$$K_A \text{ - machine back - emf constant, } \phi \text{ - magnetic flux}$$
$$\omega_{\mathrm{m}} - \text{angular velocity of the rotor}$$

The torque developed in the machine is given by

$$T_{dev} = K_T \phi I_A \quad (3.18)$$
$$K_T \text{ - machine torque constant, } \phi \text{ - magnetic flux}$$
$$I_{\mathrm{A}} - \text{armature current}$$

Note that output torque of the DC machine is lower than T_{dev} inside the machine due to friction loss at the ball-bearing through which the rotor is supported to the stator and also windage loss (the wind in the air opposes the rotor motion).

The developed power is the electrical power that is converted into mechanical form and is given by

$$P_{dev} = T_{dev} \omega_m \quad (3.19)$$

The power delivered to the induced armature voltage, E_A that is converted into mechanical power and is also given by

$$P_{dev} = E_A I_A \quad (3.20)$$

Considering the armature and field circuits at *steady-state* and writing KVL eqn. we have

$$V_T \text{ or } V_A = E_A + I_A R_A \quad \text{armature circuit (3.21)}$$
$$V_F = I_F R_F \quad \text{field circuit(3.22)}$$

Considering the armature and field circuits and writing KVL eqns. under dynamic operation we have

$$v_T(\mathrm{t}) \text{ or } v_A(t) = e_A(t) + i_A(t)R_A + L_a \frac{di_A}{dt} \text{ (armature circuit)} \qquad (3.23)$$

$$v_F = i_F(t)R_F + L_f \frac{di_F}{dt} \text{ (field circuit)} \qquad (3.24)$$

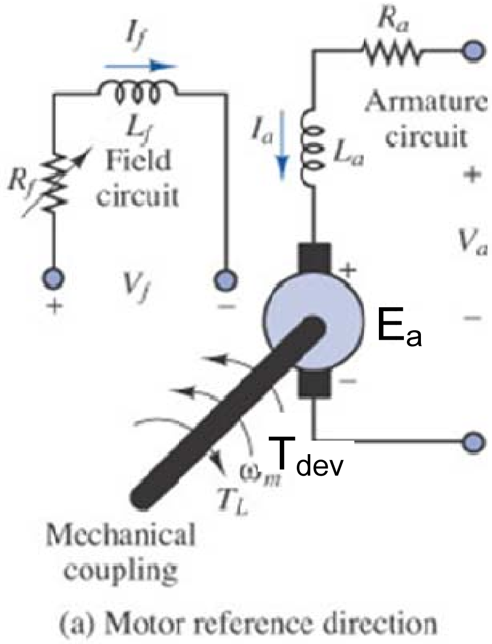


Figure 5.13a: Equivalent circuit of a separately-excited DC machine.

If an external mechanical load is connected to the motor shaft as shown in Fig. 5.14., we have the equation of motion as

$$T_{dev}(t) = k_T\phi(t)i_A(t) = T_L + b(t)\omega_m + J\frac{d\omega_m}{dt} \qquad (3.25)$$

T_L - mechanical load torque , b - frictional coefficient

J - moment of inertia of the rotating masses

5.7 Magnetization Curve for a DC Machine

The magnetization curve of a dc machine is a graph between back-emf,E_A and the field current, I_F. as shown in Fig. 5. 14. As E_A is directly proportional to ϕ the shape of the magnetization curve is similar to the B-H curve of the magnetic circuit. The magnetization curve shows that the induced voltage is directly proportional to the field current, I_F initially but as the field current is increased beyond a certain point the iron core saturates and therefore the induced voltage saturates.

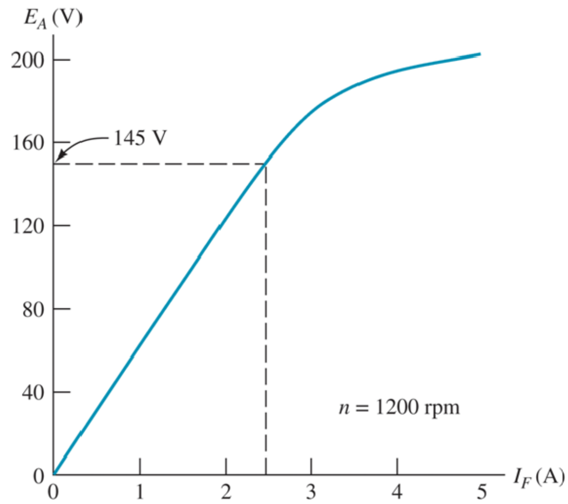


Figure 5.14: Magnetization curve for a 200 V, 7.46 kW DC machine.

If the flux is maintained constant then according to eqn. 5.17, if the back-emf is E_{A1} at a speed of n_1 and E_{A2} at a speed of n_2 we have

$$\frac{E_{A1}}{E_{A2}} = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} \qquad (3.26)$$

Example 5.2:

The machine having a magnetization curve as shown in Fig. 5.14 is operating as a motor at a speed of 800 rpm with I_A = 30 A and I_F = 2.5 A. The armature resistance is 0.3 Ω and the field circuit resistance is 50 Ω. Find the voltage, V_F applied to the field circuit, the voltage, V_T applied to the armature, the developed torque, and the developed power.

Solution:

According to eqn. 5.22 we have the field winding voltage given as

$$V_F = I_F R_F = 2.5A \times 50\Omega = 125\,V$$

From the magnetization curve, we can see that the induced emf E_A is about 145 V for a field current I_F = 2.5 A at a speed of 1200 rpm. Thus, the induced emf at a speed of 800 rpm is given by

$$E_{A2} = \frac{n_2}{n_1} \times E_{A1} = \frac{800}{1200} \times 145 = 96.67\,V$$

The machine speed in rad/s is given by

$$\omega = n_2 \times \frac{2\pi}{60} = 800\,rpm \times \frac{2\pi}{60} = 83.78\,rad/s$$

The back-emf constant can be obtained as

$$K\phi = \frac{E_A}{\omega_m} = \frac{96.67\,V}{83.78\,rad/s} = 1.154\,V/(\frac{rad}{s})$$

The developed torque is given by eqn. (5.18) as

$$T_{dev} = K\phi I_A = 1.154 \ \times 30\ A = 34.62\ N.m$$

The developed power is given by eqn. (5.19) as

$$P_{dev} = T_{dev} \times \omega_m = 34.62 \ \times 83.78 = 2900\ W$$

As a check we can also compute developed power as given by eqn. (5.20) as

$$P_{dev} = E_A \times I_A = 96.67 \times 30 = 2900\ W$$

Applying KVL to the armature circuit we have according to eqn. (5.21)

$$V_A = E_A + I_A \times R_A = 96.67\ V + 30\ A \times 0.3\ \Omega = 105.67\ \text{V}$$

5.8 Torque-Speed Characteristics of Separately-excited DC Machine

Applying KVL to the equivalent circuit of a separately excited DC machine as shown in Fig. 5.13, we have

$$V_T = I_A R_A + E_A \qquad (3.21)$$

$$I_A = \frac{T_{dev}}{K_T\phi} \qquad (3.18)$$

$$P_{dev} = T_{dev}\omega_m = (K_T\phi I_A)\times\left(\frac{E_A}{K_a\phi}\right) = \left(\frac{K_T}{K_a}\right)(E_A \times I_A) = (E_A \times I_A) \Rightarrow K_A = K_T = K (3.27)$$

$$V_T = I_A R_A + E_A = \frac{T_{dev}}{K_T\phi} R_A + K_A\phi\omega_m \qquad (3.28)$$

$$\omega_m = \frac{V_T}{K_A\phi} - \frac{R_A T_{dev}}{(K_T\phi)(K_A\phi)} = \frac{V_T}{K\ \phi} - \frac{R_A}{(K\ \phi)^2} T_{dev} \ (3.29)$$

From eqn. 5.29, we can see that the torque-speed characteristics is a straight-line (y = mx + c) and shown Fig. 5.15.

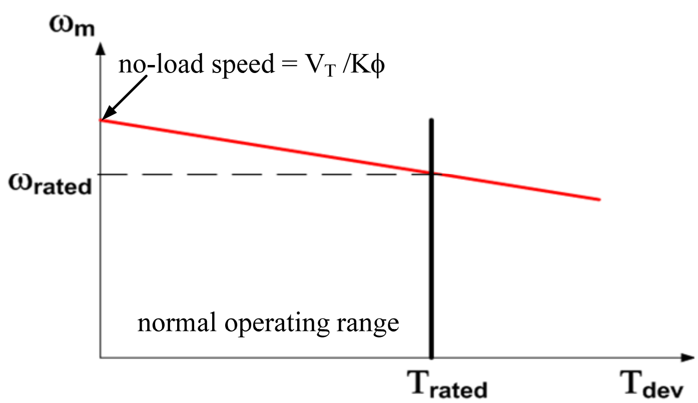
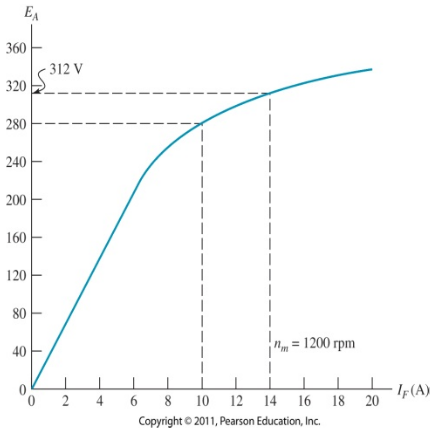
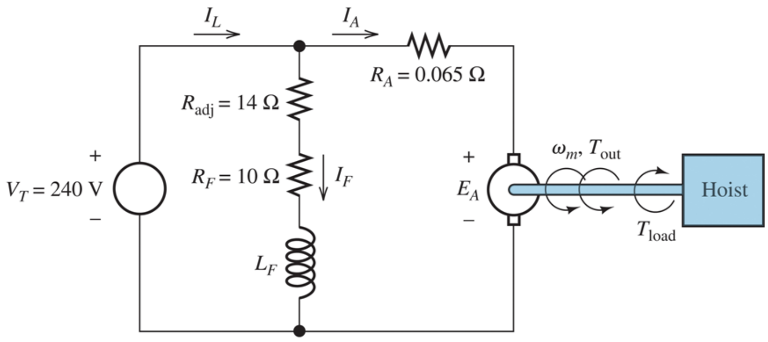


Figure 5.15: Torque-speed characteristic of a separately-excited DC machine.

Example 5.3:

A 50 horse-power shunt-connected dc motor has the magnetization curve shown in Fig. 5. Ex3. The dc supply voltage is 240 V, the armature resistance is 0.065 Ω, the field circuit resistance is 10 Ω and the adjustable resistance in the field circuit is 14 Ω. At a speed of 1200 rpm, the rotational loss due to friction and windage is 1450 W. If this motor drives a hoist that demands a torque of 250 N.m. independent of the speed, determine the motor speed and efficiency.





Solution:

From the equivalent circuit as shown above, the field current is given by

$$I_F = \frac{V_T}{R_F + R_{adj}} = \frac{240\ V}{10\ \Omega + 14\ \Omega} = 10\ A$$

From the magnetization curve, we have the induced voltage at a field current of 10 A to be 280 V.

Thus, the back emf constant can be obtained as

$$K\phi = \frac{E_A}{\omega_m} = \frac{280\ V}{1200(2\pi/60)\ rad/s} = 2.228\ V/(\frac{rad}{s})$$

The rotational torque loss is given by

$$T_{rot} = \frac{P_{rot}}{\omega_m} = \frac{1450}{1200(2\pi/60)\ rad/s} = 11.54\ N.m$$

Thus, the developed torque is given by

$$T_{dev} = T_{out} + T_{rot} = 250 + 11.54 = 261.54\ N.m$$

The armature current is given by

$$i_A = \frac{T_{dev}}{K\phi} = \frac{261.54}{2.228} = 117.4\ A$$

The back-emf is given by

$$E_A = V_T - I_A \times R_A = 240 - 117.4 \times 0.065 = 232.4\ V$$

The motor speed is given by

$$\omega_m = \frac{E_A}{K\phi} = \frac{232.4}{2.228} = 104.3\frac{rad}{s} \Longrightarrow n_m = \omega_m \times \left(\frac{60}{2\pi}\right) = 104.3 \times \left(\frac{60}{2\pi}\right) = 996\; rpm$$

The output power is given by

$$P_{dev} = T_{out} \times \omega_m = 250\;\times 104.3 = 26.08\; kW$$

The input power is given by

$$P_{in} = V_T(I_A + I_F) = 240 \times (117.4 + 10) = 30.58\; kW$$

The efficiency is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{26.08}{30.58} \times 100\% = 85.3\;\%$$

5.9 Power Flow in DC Motors

In a shunt-connected dc machine, the field circuit is in parallel with the armature circuit as shown in Fig. 5.16. The field circuit consists of a variable resistance connected in series with the field circuit and is denoted as R_{adj}.

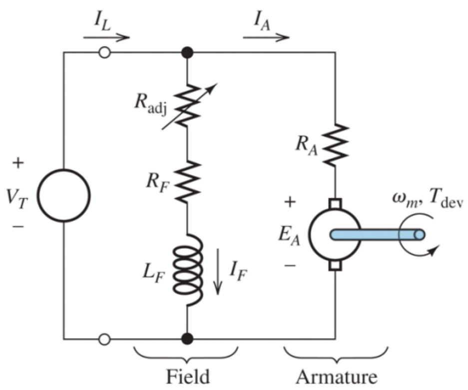


Figure 5.16: Equivalent circuit of a shunt-connected dc motor.

The input power is given by

$$P_{in} = V_T \times I_L \quad (5.30)$$

Some of the input power is used to establish the field flux and the power loss in the field circuit is given by

$$P_{field-loss} = \frac{V_T^2}{R_F + R_{adj}} = V_T \times I_F \quad (5.31)$$

Some power is lost in the armature circuit as copper loss and is given by

$$P_{arm-loss} = I_A^2 \times R_A \;\; (5.32)$$

The power delivered to the armature is converted into mechanical power and is also called as the developed power given by

$$P_{dev} = E_A \times I_A = T_{dev} \times \omega_m \quad (5.27)$$

The output power as well as the torque are less than the developed power and torque due to rotational losses, which include friction loss, windage, hysteresis and eddy current loss.Rorational power loss is approximately proportional to the speed.

5.10 Speed Control of DC Motors

For many applications it is necessary to control the speed of the DC motor e.g. the autonomous vehicle that you are going to design you need to control the speed of the DC motor driving this vehicle so that sometimes it can move faster and some other time it can move slower.

From eqn. 5.29, we can see that the speed of a DC motor, ω_m can be controlled using three different methods:

1. Armature voltage, V_T control while maintaining the field current and therefore flux ϕ constant.
2. Vary the field current, I_F and therefore flux ϕ in the machine while maintaining armature voltage V_T constant.
3. Inserting series resistance $R_{A,add}$ in the armature circuit.

The armature voltage control is the most desirable form of speed control of DC motor and therefore we would be using it.

5.10.1: Speed Control using Armature Voltage Control

$$\omega_m = \frac{V_T}{K_A\phi} - \frac{R_A T_{dev}}{(K_T\phi)(K_A\phi)} = \frac{V_T}{K\ \phi} - \frac{R_A}{(K\ \phi)^2} T_{dev} \quad (3.29)$$

$$\omega_m \approx \frac{V_T}{K\ \phi} \Rightarrow \omega_m \propto V_T \quad (3.29a)$$

Thus, the motor speed is directly proportional to the armature voltage, V_T and thus motor speed can be controlled by controlling V_T while maintaining flux ϕ and R_A constant.

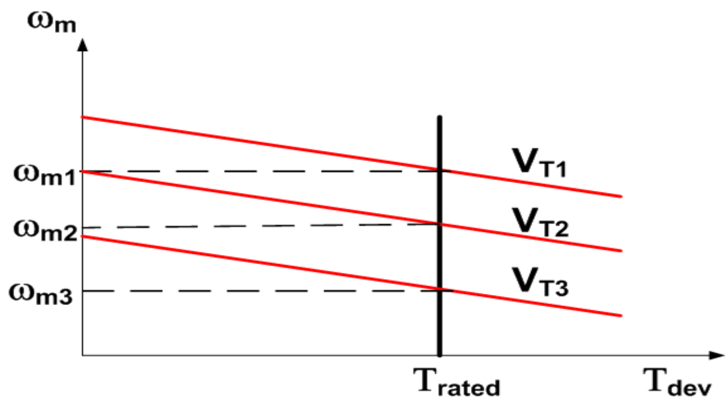


Figure 5.17: Torque-speed characteristics of separately-excited DC motor for varying armature voltage.

5.10.2 Variable DC Voltage Source

A dc voltage source can be obtained by using either a battery or if it is AC supply then using a diode-bridge rectifier followed by a smoothing filter capacitor as we have seen in Lab. 6 Manual.

Once, a constant dc voltage source is obtained, an electronic switching circuit as shown in Fig. 5.18 can be used to control the average dc voltage delivered to the load.

The switch periodically opens and closes with a time period T, closed for a period of T_{on} and opened for the rest of the period T_{off} (= $T - T_{on}$). This process is known as *pulse-width-modulation* (PWM) in power electronics field.

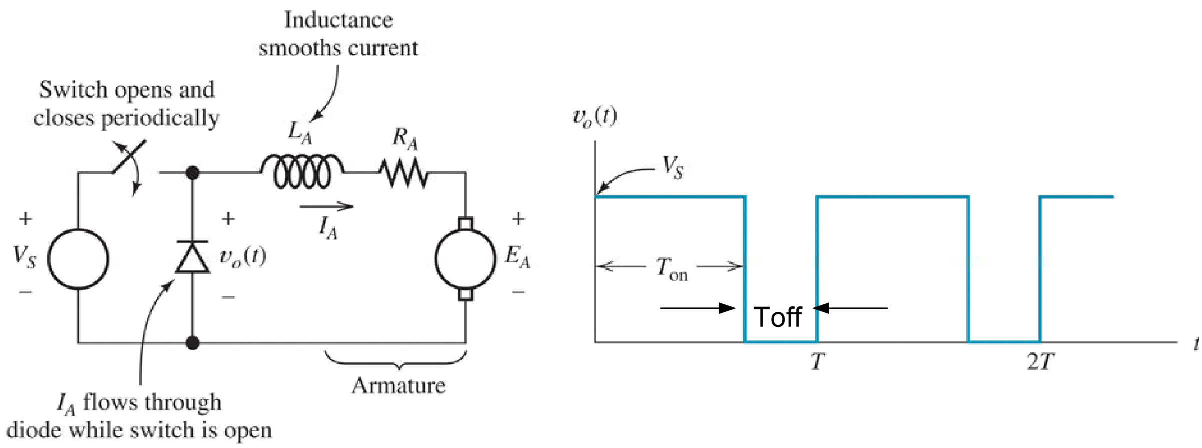


Figure 5.18: An electronic switch that opens and closes periodically can efficiently supply a variable dc voltage to a motor from a fixed dc supply voltage.

The average output voltage across the armature circuit is given by

$$V_o = \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{T}\left[\int_0^T v_o(t)\,dt\right] = \frac{1}{T}\left[\int_0^{T_{on}} V_s\ dt + \int_{T_{on}}^{T} V_s\ dt\right] = \frac{1}{T}[V_s \times T_{on}] = V_s \times \frac{T_{on}}{T}$$
$$= V_s \times D \quad (5.33)$$

where D – duty cycle of the electronic switch.

The presence of the free-wheeling diode across the armature circuit is very essential as when the switch is opened and the armature circuit current does not find a path to flow through (as the switch is opened during the turn-off period), and drops to zero abruptly then it induces a very large voltage across the armature circuit.

Consider that the armature circuit inductance is 1 mH, a current of 10 A is flowing through the circuit and the current is interrupted in 1 μs (semiconductor switch can be opened very fast) then we have

$$v_{L_A} = L_A \times \frac{di_A}{dt} = 10^{-3} \times \frac{10}{10^{-6}} = 100 \ \times 10^3 = 10\ kV$$

Such a large voltage would appear across the semiconductor switch and blow it up so the freewheeling diode is provided so that current can flow through the diode and therefore it is not interrupted abruptly.

When the switch is closed the supply voltage reverse biases the freewheeling diode and therefore does not operate but when the switch is opened it becomes forward biased and conducts.

5.11 Introduction to MOSFET and Its Operation as a Switch

We know that we can control the speed of a DC motor by varying the armature voltage. One of the ways, the variable armature voltage that can be obtained from a fixed DC voltage source is by making use of the PWM scheme. As explained in Section 5.10.2, a power semiconductor device can be used as a *switch* to apply the full DC source voltage or zero voltage to the load. The power semiconductor switch can be of different types but the *metal-semiconductor-field-effect-transistor* (MOSFET) is the most commonly used device especially at low-power (< 2 – 5 kW) level. In our autonomous vehicle, we make use of the power MOSFET as a switch to vary the armature voltage applied to the permanent-magnet DC motor to vary the speed of the motor.

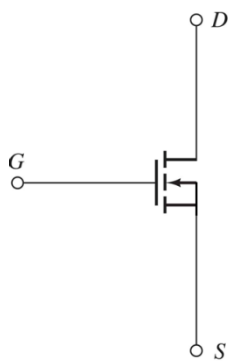


Fig. 5.19 Circuit symbol for an n-channel MOSFET.

A MOSFET is a three terminal *voltage-controlled* semiconductor device as shown in Fig. 5.19. It is an n-channel MOSFET and the three terminals are *gate* (G), *source*(S) and *drain* (D).

When a +ve drian-to-source voltage, v_{DS} is applied as shown in circuit Fig. 5.20 and the gate-to-source v_{GS} is made zero then no current flows from the drain-to-source and therefore, $i_D = 0$ and we say that the MOSFET switch is in OFF mode i.e. it acts as an open-circuit condition. In this mode, the full-supply voltage is blocked by the drain-to-source of the MOSFET and the operating point is in the *cut-off region* on the terminal characteristic as shown in Fig. 5.22.

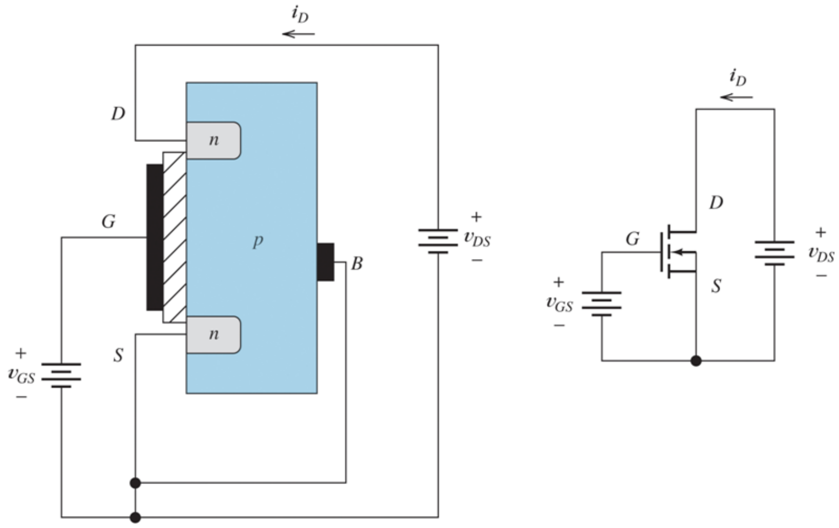


Fig. 5.20 For $v_{GS} < v_{TH}$ the MOSFET is in OFF state and $i_D = 0$.

When the gate-source voltage, v_{GS} is increased and made above a certain *threshold voltage*, v_{TH} it induces a channel between the source and the drain terminals as shown in Fig. 5.21 and current starts to flow from the drain to the source and the MOSFET is said to be ON and closes the circuit. During this stage the operating point is in the triode region as shown in Fig. 5.22. The on-stage voltage drop across the MOSFET is v_{DS} and typically about 2 V approximately.

The terminal characteristics of the MOSFET is a plot between the drain current, i_D and the drain-to-source voltage, v_{DS} as shown in Fig. 5.22. The voltage-current characteristics can be sub-divided into three different regions but when the MOSFET operates as a switch as is the case the operating point is either in the *cut-off region* when it is in *OFF state* or in the *triode region* when it is in the *ON state*.

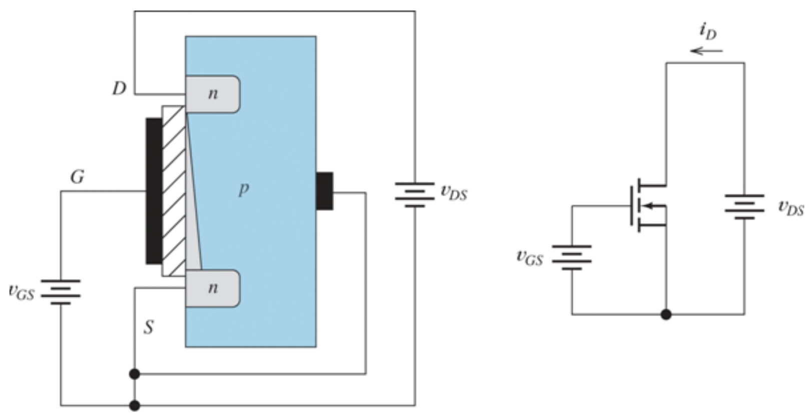


Fig. 5.21 For $v_{GS} > v_{TH}$ the MOSFET is in ON state and i_D flows from the drain to source.

The reason why the MOSFET is operated as a switch in power electronics applications is to minimize the losses across the MOSFET. During the OFF state, v_{DS} is high and may be of the order of few 100's of volts but the drain current, i_D is very small and may be of the order of few tens of µA so the power loss i.e. $v_{DS} \times i_D$ is of the order of mW only. Similarly, when the MOSFET switch is in the ON state, the on-stage voltage drop, v_{DS} is of the order of 2 V and the drain current, i_D is dependent on the load and may be of the order of 10 – 100's of Ampere so the on-state power loss is of the order of 20 – 200's W. This is still much smaller in comparison to having the operating point in the *saturation region* when both the on-stage voltage drop, v_{DS} as well as the drain current, i_D can be high leading to power loss of the order of kW (100 V x 50 A = 5 kW) level.

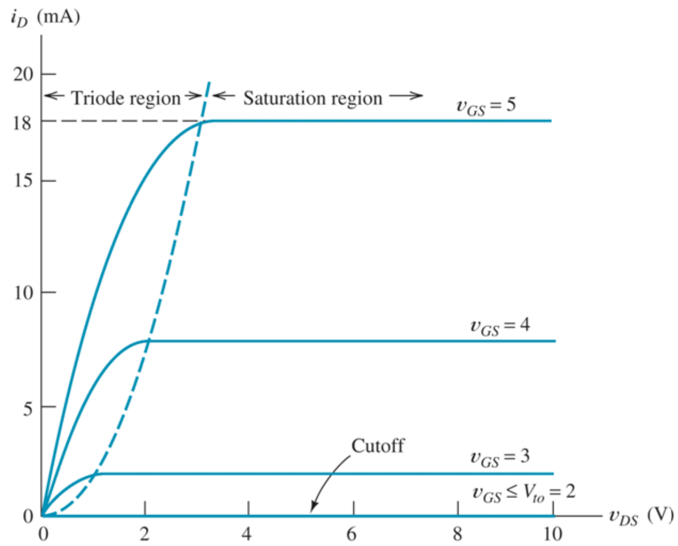


Fig. 5.22: Voltage-current characteristic of an n-channel MOSFET.

The MOSFET can be used as a switch to obtain variable DC voltage from a fixed DC voltage using the chopper circuit as shown in Fig. 5.23 reproduced below.

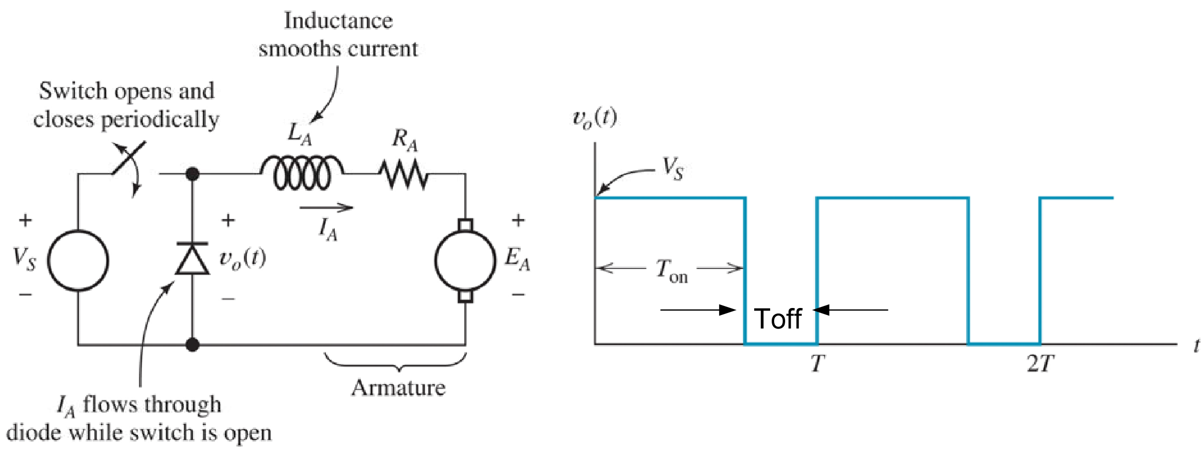


Figure 5.23: An electronic switch that opens and closes periodically can efficiently supply a variable dc voltage to a motor from a fixed dc supply voltage.

Summary

- Basic principles of operation of linear DC motor and generators
- Structure of rotating electrical machines
- Basic principles of operation of rotating DC machines
- Configuration of DC machines
- DC Machine equivalent circuit
- DC machine magnetization curve
- Torque-speed characteristics of DC shunt and separately excited motors
- Speed control of DC motor

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