EE1002

Chapter 5: Introduction to DC Machines

National University of Singapore

Learning Objectives

- Understanding of the basic principles of operation of linear and rotating electrical machines.
- Understanding of classification of DC machines.
- Analyze DC generators under steady-state operation.
- Analyze DC motors under steady-state and dynamic operation.
- Understanding of torque-speed characteristics of separately-excited and permanent DC motors.
- Understanding of speed control of DC motors.

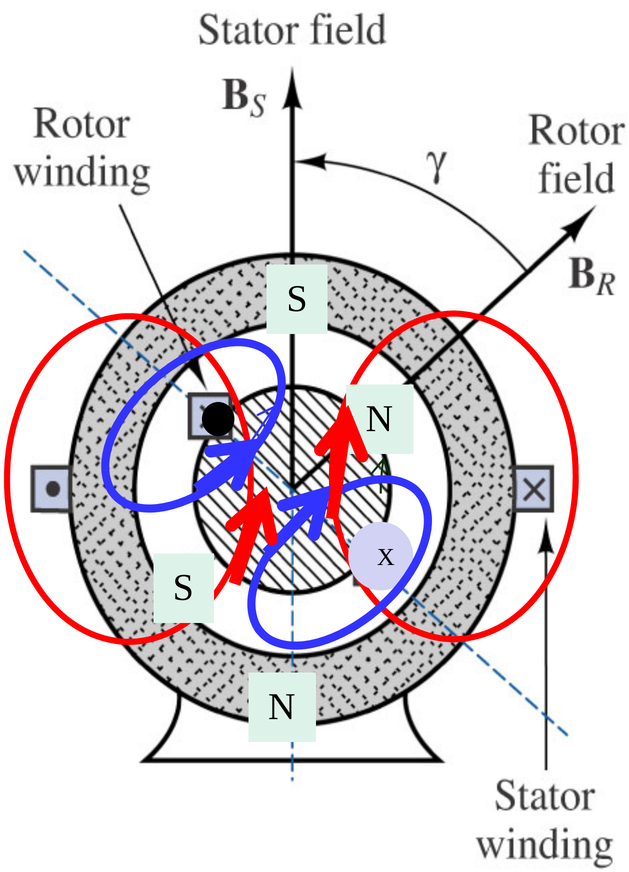
Introduction to Rotating Electrical Machines

- For the autonomous vehicle to move around following a track you need electric motors to provide mechanical energy to propel the vehicle.
- In this part of the lecture you would be introduced how to make use of DC motors to provide mechanical energy and also pulse-width-modulated circuit to control the speed of motor.
- Electric motors are used in various power ratings from micro-watt to mega-watt range to provide mechanical energy to improve our standard of living.
- In our day-to-day life, we come across hundreds of motors everyday at home, in office environment, in industrial environment and for that matter almost everywhere.

Rotating Electrical Machines

- Any rotating electrical machine has two basic components, namely, the stator and the rotor as shown in Fig. 5.1 and the rotor rotates inside the stationary part i.e. stator and is separated through a small air-gap.
- Depending on the construction and type of rotating machine, either the stator or the rotor or both contain current carrying conductors configured in the form of coils.
- Currents in these stator and/or rotor coils set-up magnetic fields $\mathbf{B}_s$ and $\mathbf{B}_r$ as in Fig.3.1 and they interact with each other to produce torque in the machine.

$$T_{ind} = (\mathrm{B_S} \times \mathrm{B_R}) = k\,|\mathrm{B_S}|\,|\mathrm{B_R}|\sin(\gamma) \quad (5.1)$$



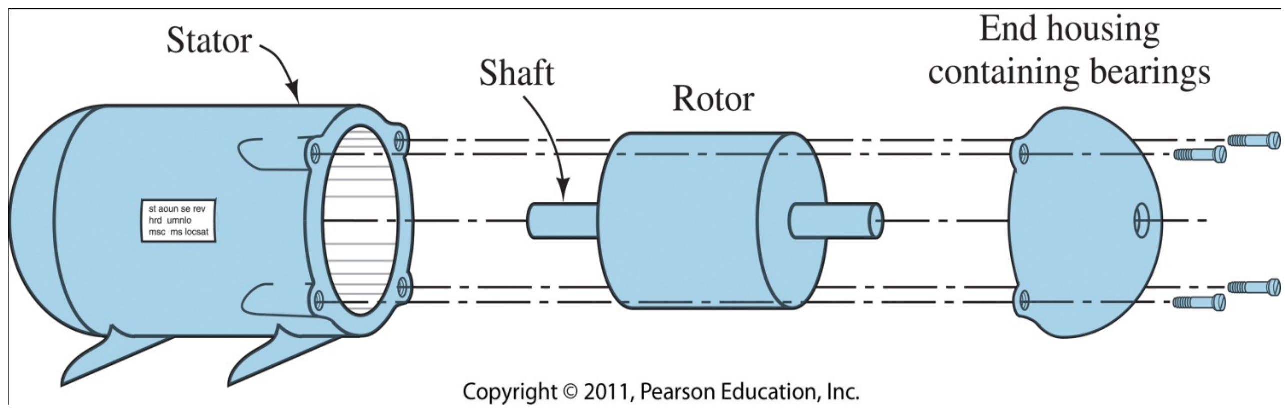


Fig. 5.1 (a) (b) An electrical motor consists of a cylindrical rotor that spins inside the stator.

- Stator or rotor or both contain current carrying conductors configured into coils or sometimes permanent magnets to set-up the magnetic fields and they interact with each other to produce torque.
- Stators and rotors are made up of iron to intensify the magnetic field. The magnetic fields alternates in direction through the iron with time and therefore the iron must be laminated to limit the power losses in the core due to eddy currents.

Linear DC Machines at Starting

- We introduce the basic principles of DC machines by considering the idealized linear DC machine as shown in Fig. 5.2.
- The operation of the linear DC machine can be explained from the following four basic equations:

$$1.\ \mathbf{F} = i(\mathbf{l} \times \mathbf{B}) \qquad (5.2)$$

$$2.\ e_{ind} = (\mathbf{u} \times \mathbf{B}) \bullet \mathbf{l} \qquad (5.3)$$

$$3.\ \text{KVL}: V_T - i_A R_A - e_{ind} = 0 \qquad (5.4)$$

$$4.\ \text{Netwon's Law}: \mathbf{F}_{\text{net}} = m \times a \qquad (5.5)$$

$\mathbf{F}$ – force on conductor, $\mathbf{l}$ – length of conductor, $\mathbf{B}$ – flux density, m – mass of conductor, a – acceleration, $\mathbf{F}_{\mathbf{net}}$–net force, e_{ind} –induced emf,

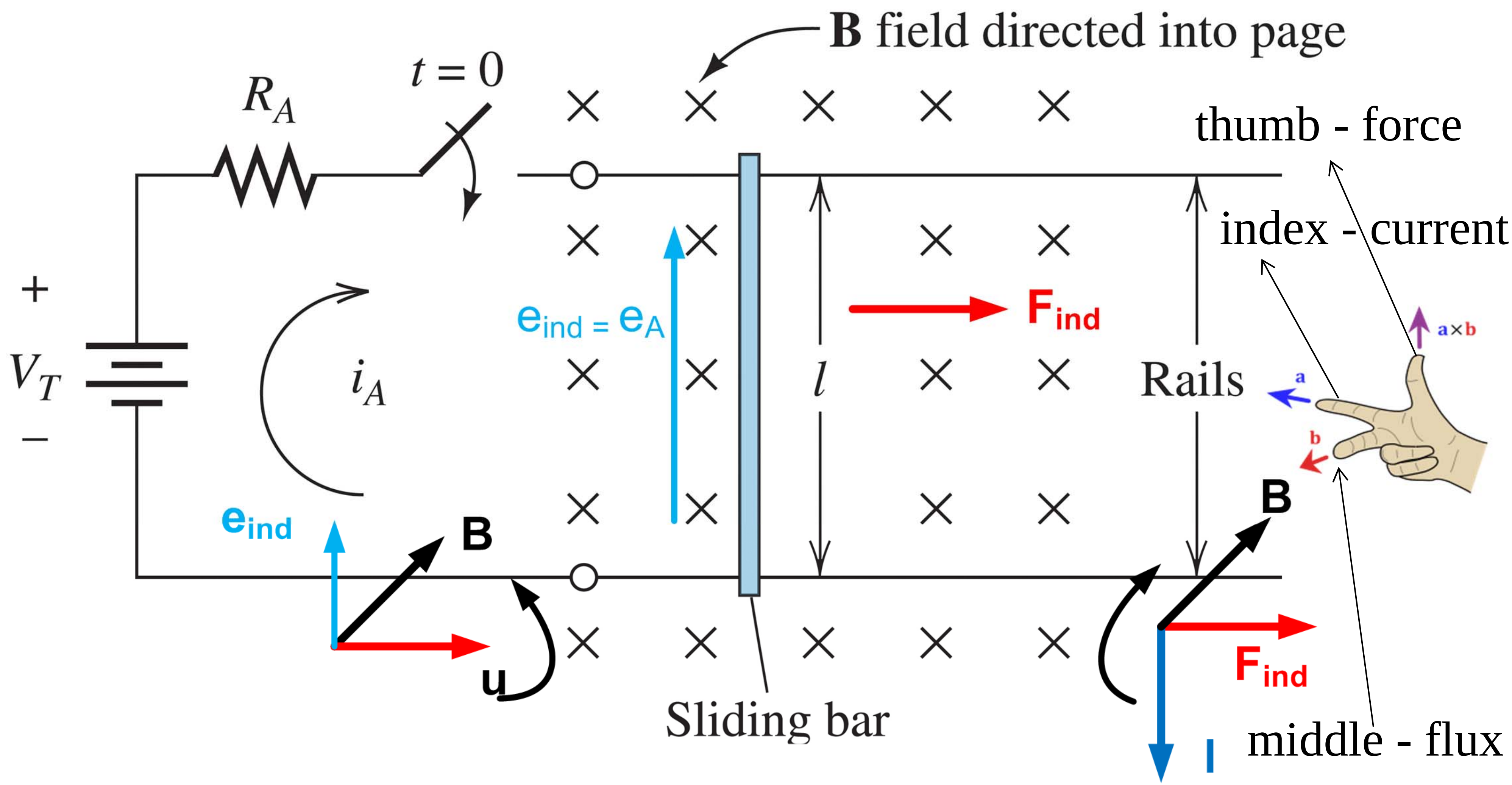


Fig. 5.2: A simple dc machine consisting of a conducting bar sliding on conducting rails.

- When the switch is closed at (t = 0^+), current i_A flows through the machine.

$$i_A = \frac{V_T - e_{ind}}{R_A} \quad (5.6)$$

- The current i_A flowing through the conductor placed in a magnetic field **B** (x - into the page of the paper) would induce a force on the conductor.

$$\mathrm{F}_{ind} = Bi_A l \text{ - to the right} \qquad (5.7)$$

- According to Newton's Law, the bar would now accelerate to the right. As the bar velocity begins to increase it would induce a voltage e_{ind}.

$$e_{ind} = uBl \text{ - positive upward} \quad (5.8)$$

- The induced voltage e_{ind} reduces the current according to KVL:

$$i_A \downarrow = \frac{V_T - e_{ind} \uparrow}{R_A} \qquad (5.9)$$

- As u increases increasing $e_{ind,}$ the current i_A reduces reducing the induced force on the conductor, eventually reaching a steady-state speed, u_{ss} where the net force on the bar is zero.
- The induced voltage would rise to a value when it is equal to the supply voltage and at that point the bar attains steady-state speed.

$$V_T = e_{ind} = \mathrm{u}_{ss} Bl \Rightarrow \mathrm{u}_{ss} = \frac{V_T}{Bl} \qquad (5.10)$$

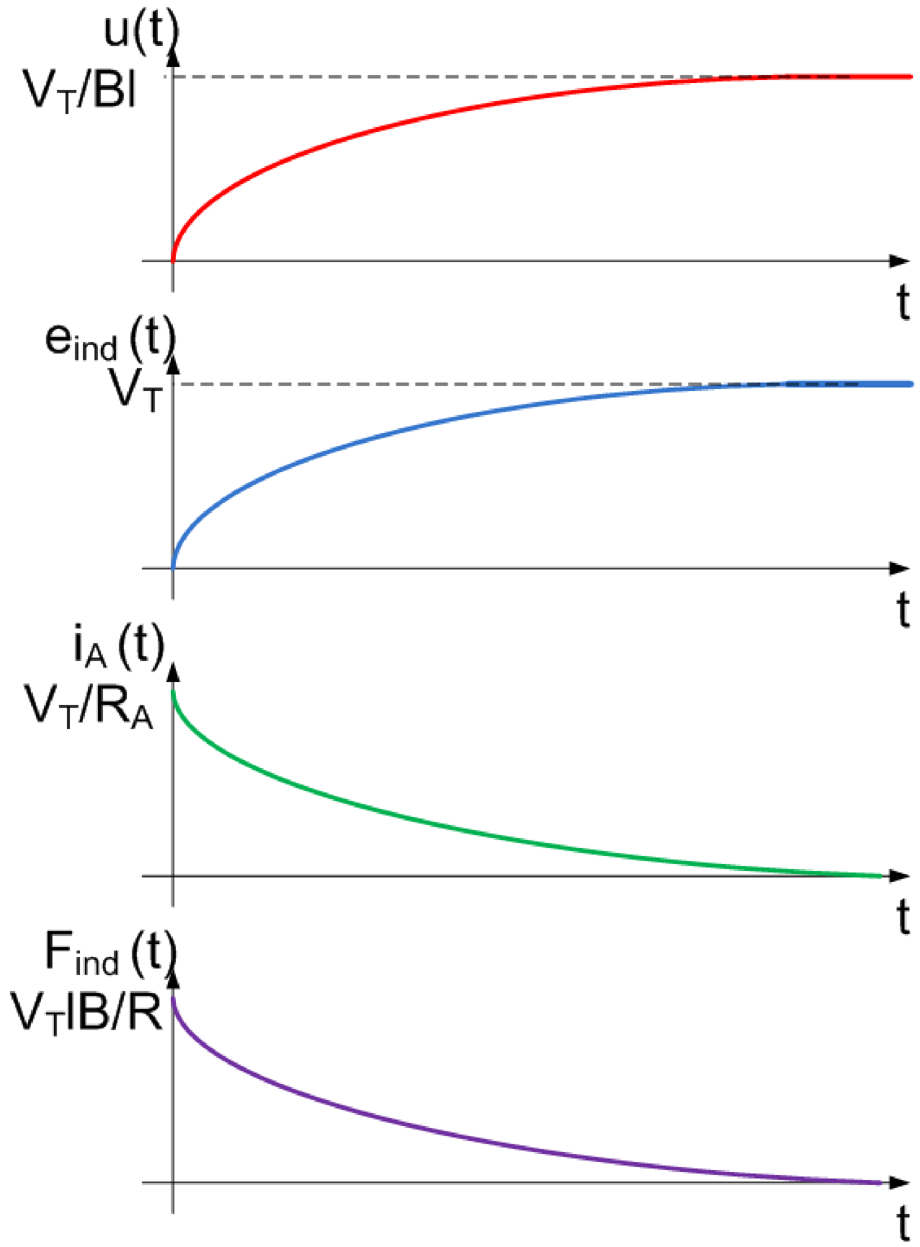


Fig. 5.4 The linear DC machine on starting.

- To summarize at starting the linear DC machine behaves as follows:
  1. Closing the switch produces a current flow $i_A = \frac{V_T}{R_A}$.
  2. The current flow produces a force on the bar $F = Bil$.
  3. The bar accelerates to the right producing an induced voltage e_{ind} as it speeds up. $e_{ind} \uparrow = \uparrow uBl$
  4. The induced voltage reduces the current flow. $i_A \downarrow = \frac{V_T - e_{ind} \uparrow}{R_A}$
  5. The induced force is thus decreased until eventually

$$F_{ind} = 0 \Rightarrow u_{ss} = \frac{V_T}{Bl}$$

Linear DC Machine as Motor

- Assume that the LDCM is initially running at no-load steady-state speed, u_{ss}.
- Now an external load F_{load} is applied opposite to the direction in which motor (conductor) is moving.
- As initially the bar is at steady-state speed with F_{ind} = 0, with the applied external load, F_{load} will make, F_{net} = 0 – F_{load} = - F_{load}.
- The effect is that the bar speed would slow down and as soon as the bar slows down, the induced voltage on the bar drops.

$$i_A \uparrow = \frac{V_T - e_{ind} \downarrow}{R_A} \quad (5.12)$$

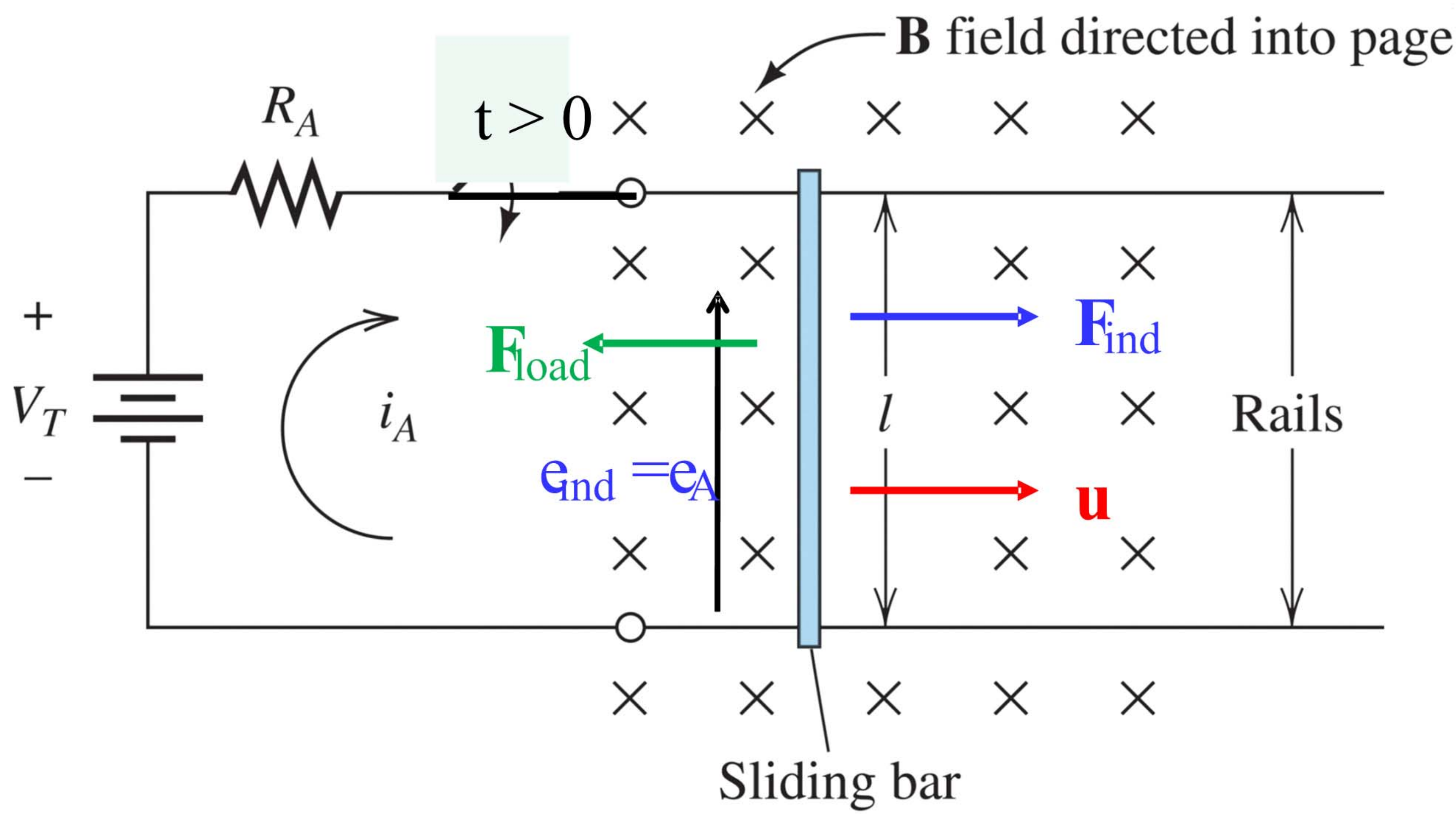


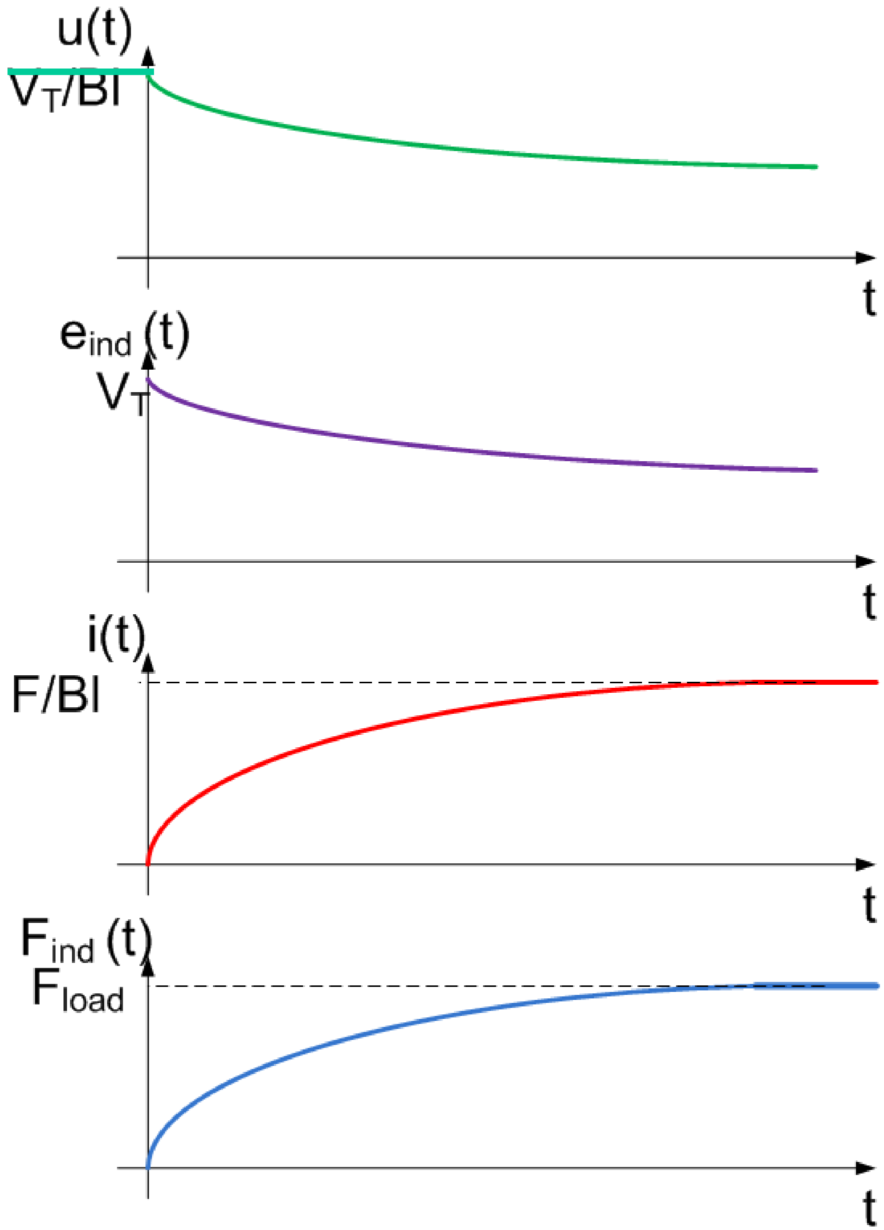
Fig. 5.5 A simple linear dc machine operating as a motor.

- The increase in bar current would increase the induced force and it would rise until the $F_{ind} = F_{load}$ and the bar again attains a steady-state but at a lower speed that the no-load speed, u_{ss}.
- Thus, there is an induced force in the direction of motion and *power is converted from electrical to mechanical form* to keep the bar moving.

$$P_{conv} = e_{ind} \times i_A = F_{ind} \times u \quad (5.13)$$

- Thus, we say that the LDCM is operating as a motor.

Fig. 5.6 The linear DC machine operating at no-load condition and then loaded to operate as a motor.



- To summarize the linear DC machine operating as a motor behavior as follows:

1. A force F_{load} is applied to opposite to the direction of motion, which causes a net force F_{net} opposite to the direction of motion.
2. The resulting acceleration $a = F_{net}/m$ is negative and therefore the bar slows down ($u\downarrow$).
3. The induced voltage $e_{ind} = u\downarrow Bl$ falls, and so $i_A = (V_T - e_{ind}\downarrow)/R_A$ increases.
4. The induced force $F_{Ind} = i\uparrow Bl$ increases until $|F_{Ind}| = |F_{load}|$ at a lower speed u.
5. An amount of electrical power equal to $e_{ind} \times i_A$ is converted to mechanical power $F_{Ind} \times u$ and machine operates as a motor.

Linear DC Machine as Generator

- Assume that the LDCM is initially running at no-load steady-state speed, u_{ss}.
- Now if an external load $\mathbf{F}_{load}$ (= $\mathbf{F}_{app}$) is applied in the same direction in which machine (conductor) is moving and see what happens.
- The applied force $\mathbf{F}_{load}$ would cause the bar to accelerate in the same direction of motion and the velocity of the bar u would increase.
- If $e_{ind} > V_T$ the current would reverse direction and is given by

$$i_A \uparrow = \frac{e_{ind} \uparrow - V_T}{R_A} \quad (5.14)$$

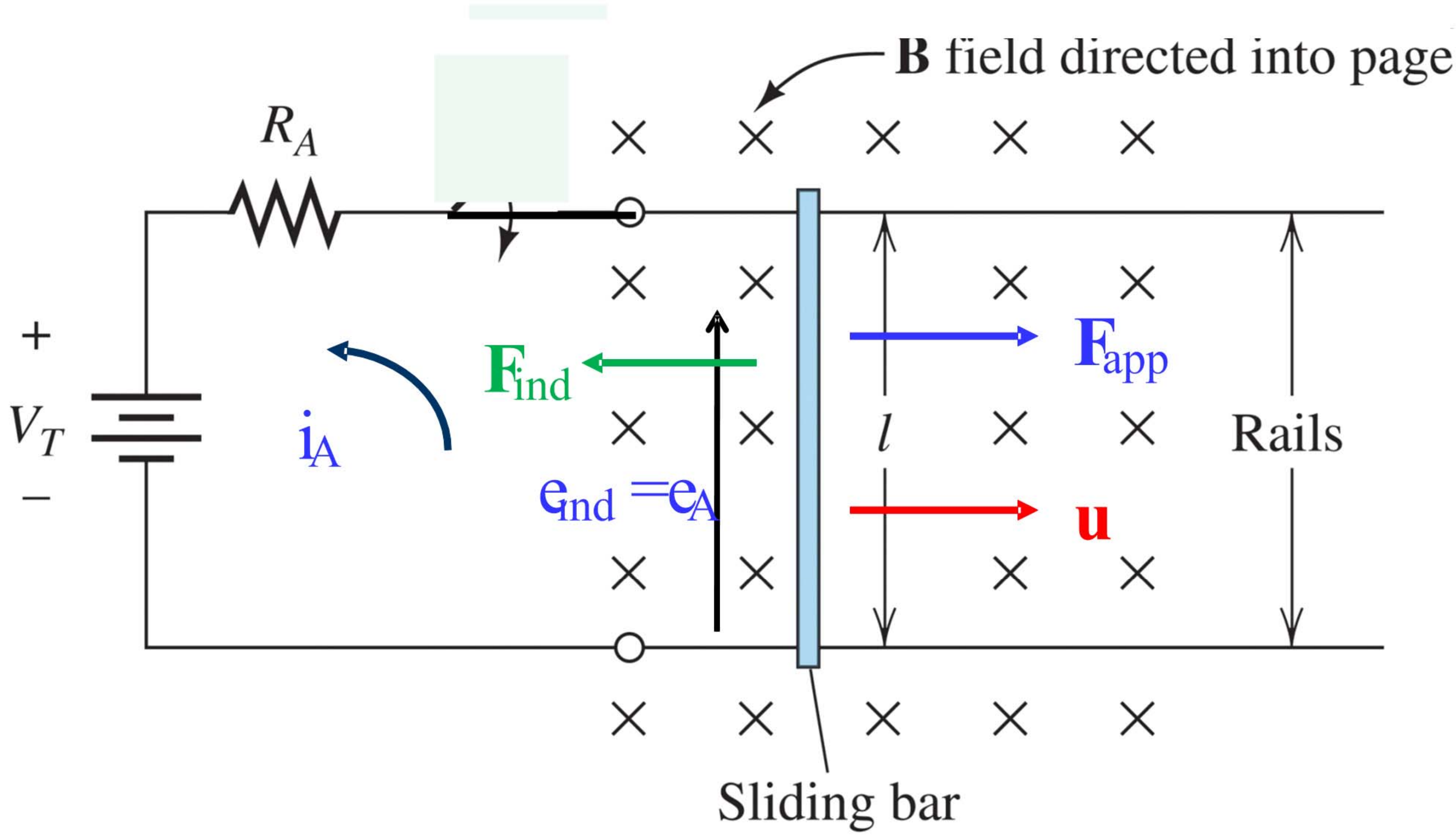


Fig. 5.7 A simple linear dc machine operating as a generator.

- As the current i_A moves up along the bar, the force induced in the conductor would be to the left and given by the RHR and is shown in Fig. 5.7.
- The direction of F_{ind} is given by the RHR hand and opposes the applied force F_{load}.

$$F_{ind} = ilB \qquad (5.15)$$

- Finally, the induced force F_{ind} becomes equal to the applied load force F_{load} and machine operates at a higher speed than before.
- Note that the battery source is now charging and mechanical power input ($P_{mech} = F_{load} \times u$) is converted to electrical power and machine operates as a generator.

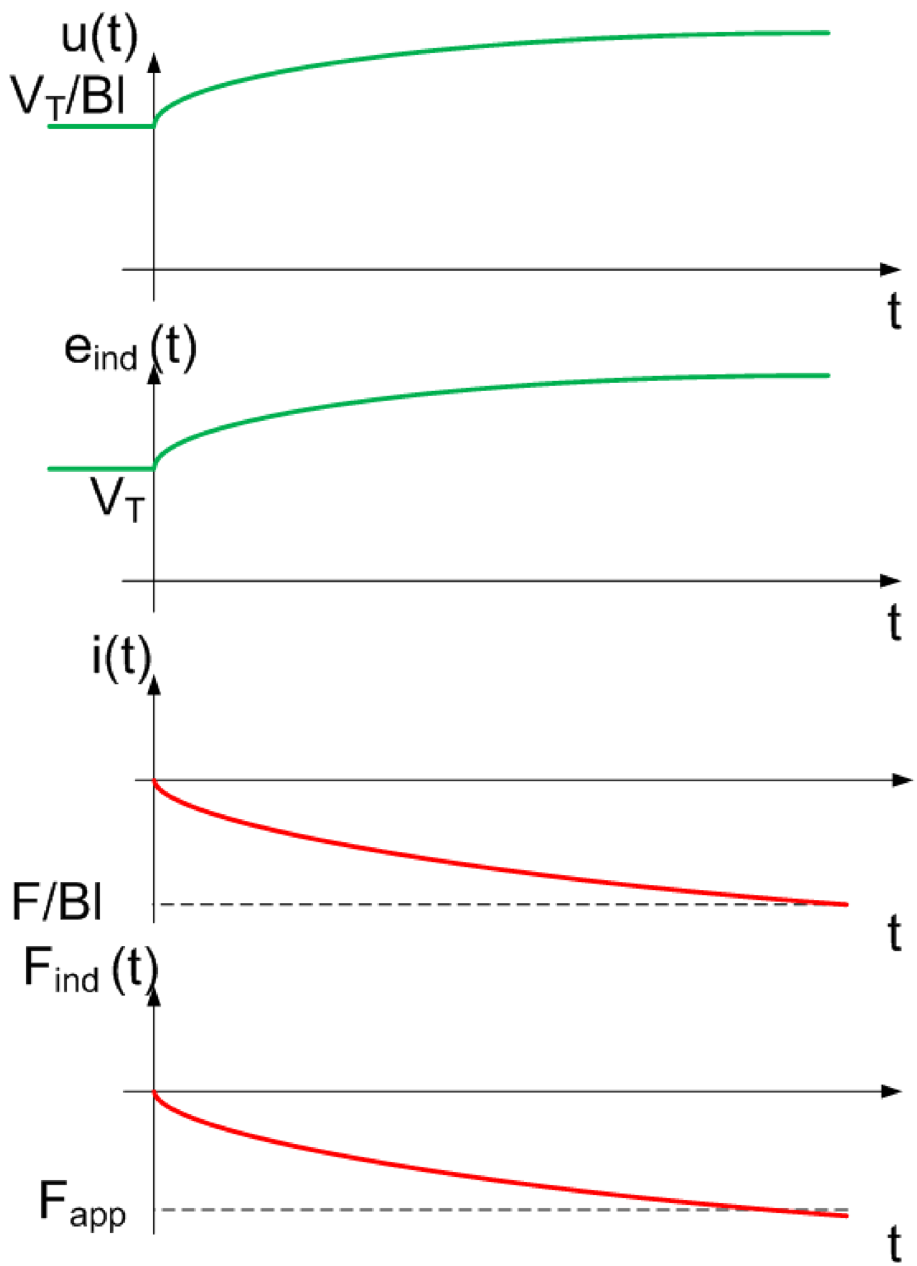


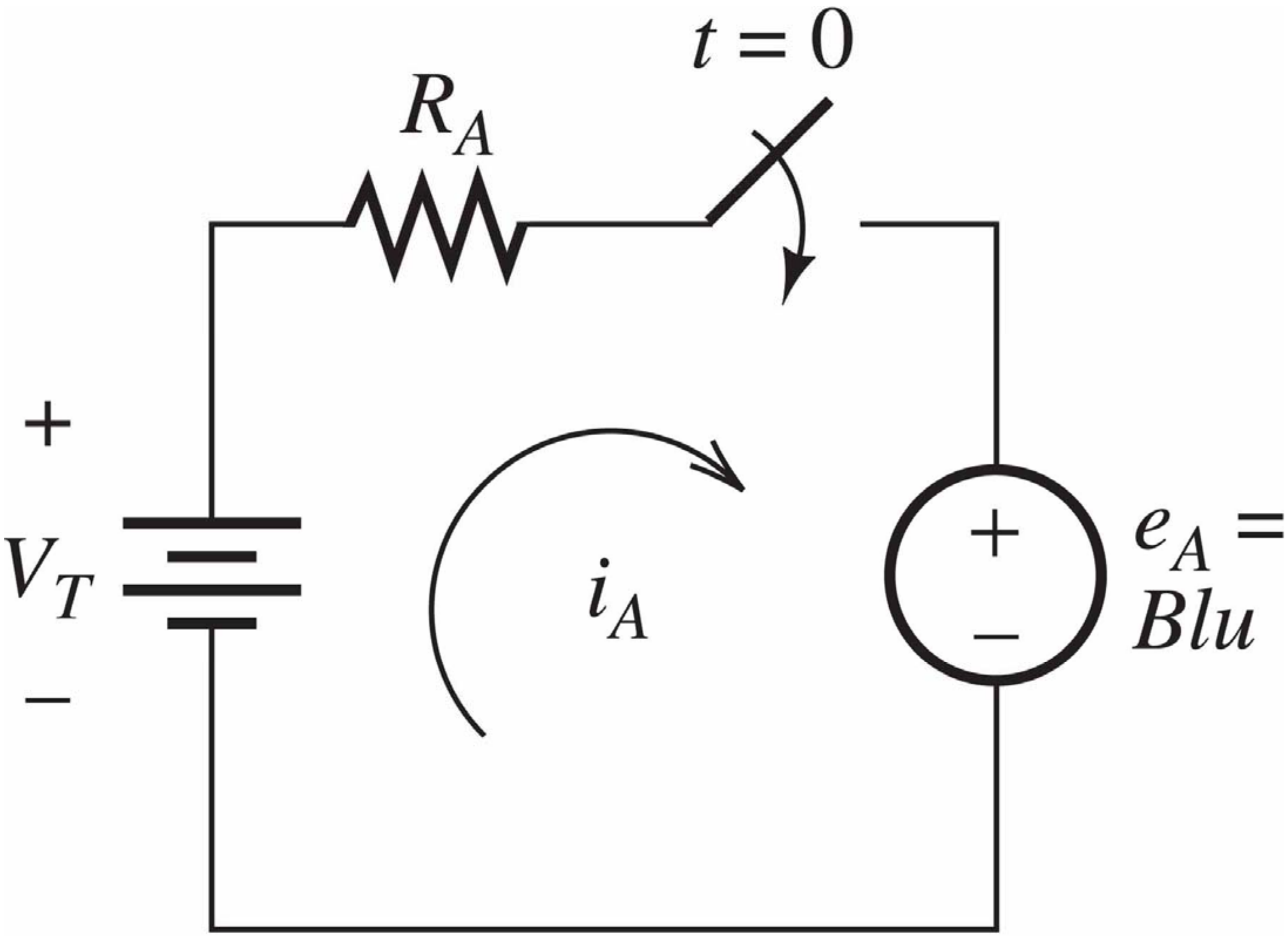
Fig. 5.8 The linear DC machine operating at no-load condition and then loaded to operate as a generator.

- To summarize the linear DC machine operating as a generator behavior as follows:
  1. A force $\mathbf{F}_{app}$ is applied in the same direction of motion, which causes a net force $\mathbf{F}_{net}$ in the direction of motion.
  2. The resulting acceleration $a = \mathbf{F}_{net}/m$ is positive and therefore the bar speeds up ($u\uparrow$).
  3. The induced voltage $e_{ind} = u\uparrow Bl$ increases, and so $i_A = (e_{ind}\uparrow - V_T)/R_A$ increases (in the reverse direction).
  4. The induced force $\mathbf{F}_{Ind} = i_A\uparrow Bl$ increases until $|\mathbf{F}_{Ind}| = |\mathbf{F}_{app}|$ at a higher speed u.
  5. An amount of mechanical power equal to $\mathbf{F}_{Ind} \times u$ is converted to electrical power $e_{ind} \times i_A$ and the machine operates as a generator.

- It is interesting to see that the same machine operates both as motor as well as generator depending on what is input and what is output.
- The only difference is that if the external applied force is in the direction of motion the machine operates as a (generator) or if the applied force is in the direction opposite to motion then the machine operates as a (motor).
- Electrically, when $e_{ind} > V_T$ the machine operates as a generator and when $e_{ind} < V_T$ the machine operates as a motor.
- When the machine operates at a higher speed it operates as a generator and when it moves slowly it operates as a motor.

Example 5.1: Suppose that for a linear machine as shown in Fig. 5.2, we have $B = 1$ Wb/m^2, $l = 0.3$ m, $V_T = 2$ V, and $R_A = 0.05\ \Omega$.

1. Assuming that the bar is stationary at $t = 0$, compute the initial current and initial force on the bar. Also determine the final steady-state speed assuming that no mechanical load is applied on the bar.
2. Now, suppose that a mechanical load of 4 N directed to the left is applied to the moving bar. In steady-state, determine the speed at which the bar is moving, the power delivered by the electrical source, the power delivered to the mechanical load, the power lost as heat in the resistance, R_A, and the efficiency of the electromechanical system.
3. Now, suppose that a mechanical pulling force of 2 N directed to the right is applied to the moving bar. In steady-state, determine the speed, the power taken from the mechanical source, the power delivered to the battery, the power lost as heat in the resistance, R_A, and the efficiency.



$t = 0$
R_A
+
V_T
–
i_A
+
–
$e_A =$
Blu

(a) Initially, we have **u** = 0, and thus, $e_{ind} = e_A = 0$, and thus the initial current is given by

$$i_A(0^+) = \frac{V_T - e_A}{R_A} = \frac{2-0}{0.05} = 40\ A$$

The resulting induced force on the bar is given by

$$F_{ind}(0^+) = Bli_A(0^+) = 1\frac{Wb}{m^2} \times 0.3\ m \times 40A = 12\ N$$

In steady-state with no mechanical force applied

$$e_A = Blu = V_T \Rightarrow u_{ss} = \frac{V_T}{B \times l} = \frac{2}{1 \times 0.3} = 6.667\ m/s$$

(b) As the mechanical force applied opposes the motion of the bar, the machine operates as a motor. At steady-state the net force acting on the bar is zero i.e. $F_{ind} = F_{load}$.

$$F_{load} = F_{ind} = Bli_A \Rightarrow i_A = \frac{F_{load}}{B \times l} = \frac{4\,N}{1\frac{Wb}{m^2} \times 0.3m} = 13.33\,A$$

From the circuit KVL equation we have

$$e_A = V_T - i_A \times R_A = 2\,V - 13.33\,A \times 0.05\,\Omega = 1.333\,V$$

Thus, the steady-state speed is:

$$u = \frac{e_A}{B \times l} = \frac{1.33\,V}{1\frac{Wb}{m^2} \times 0.3m} = 4.444\,m/s$$

The mechanical power delivered to the load is given by

$$p_m = F_{load} \times u = 4N \times 4.44\frac{m}{s} = 17.77\ W$$

The electrical power drawn from the battery source is given by

$$p_{in} = V_T \times i_A = 2V \times 13.33\ A = 26.67\ W$$

The power dissipated as heat in the resistance

$$p_R = i_A^2 \times R_A = (13.33\ A)^2 \times 0.05\ \Omega = 8.889\ W$$

- The efficiency of the linear DC machine is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{17.77\ W}{26.67\ W} \times 100\% = 66.67\ \%$$

(c) When the pulling force, F_{app} is applied to the bar it helps to speed up the bar and the corresponding induced voltage, e_A exceeds the supply voltage, V_T and therefore the current flows out of the bar rather than flowing into the bar.

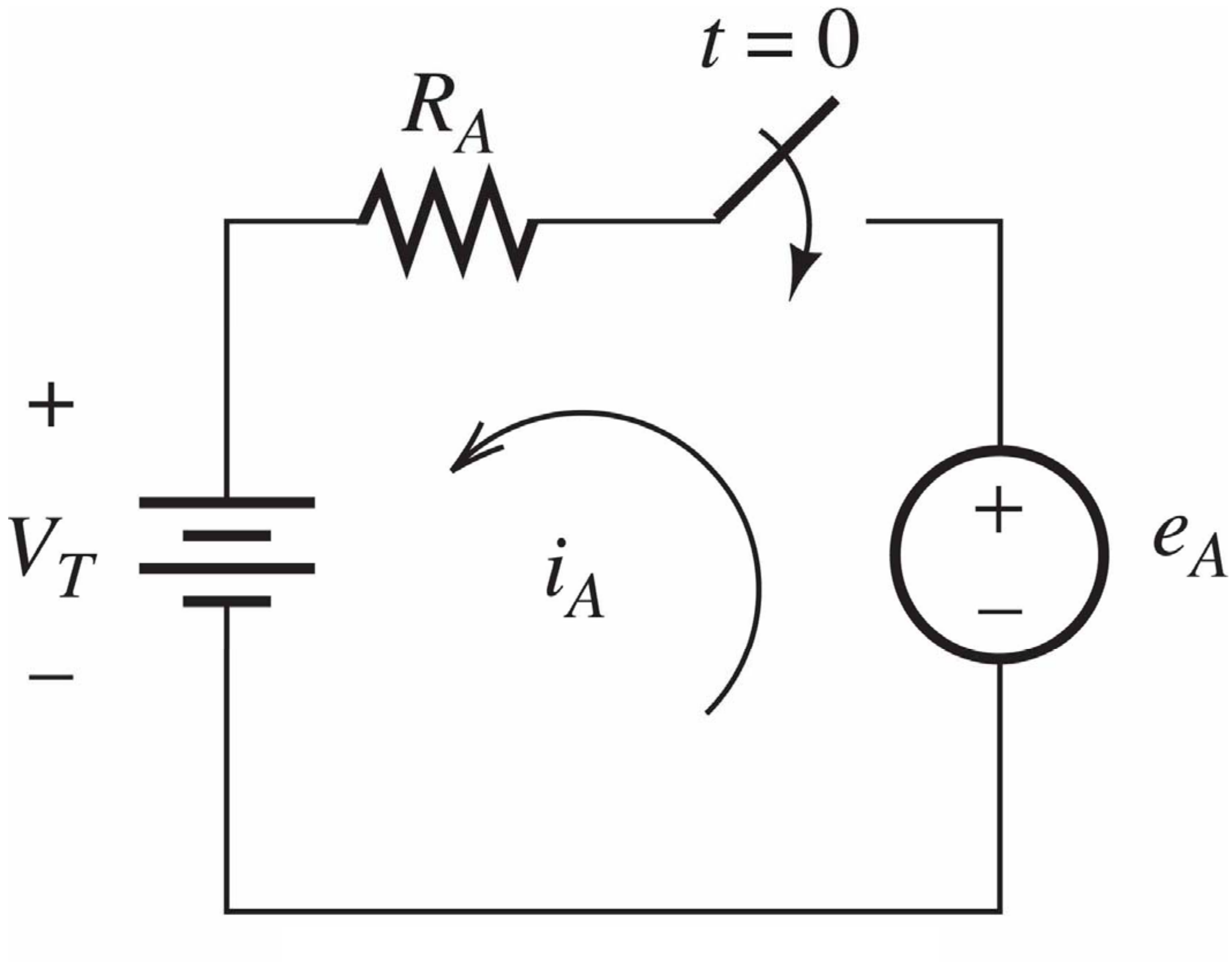
$$F_{app} = F_{ind} = Bi_A l \Rightarrow i_A = \frac{F_{app}}{B \times l} = \frac{2\,N}{1\frac{Wb}{m^2} \times 0.3m} = 6.667\,A$$

From the circuit KVL equation we have

$$e_A = V_T + i_A \times R_A = 2\,V + 6.667\,A \times 0.05\,\Omega = 2.333\,V$$

The new steady-state speed is given by

$$u = \frac{e_A}{B \times l} = \frac{2.333\,V}{1\frac{Wb}{m^2} \times 0.3m} = 7.778\,m/s$$



$t = 0$
R_A
+
V_T
–
i_A
+
–
e_A

- The mechanical power supplied by the source is given by

$$p_m = F_{app} \times u = 2N \times 7.778 \frac{m}{s} = 15.56\ W$$

- The electrical power absorbed by the battery is given by

$$p_{out} = V_T \times i_A = 2V \times 6.667\ A = 13.33\ W$$

- The power dissipated as heat in the resistance

$$p_R = i_A^2 \times R_A = (6.667\ A)^2 \times 0.05\ \Omega = 2.222\ W$$

- The efficiency of the linear DC machine is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{13.33\ W}{15.56\ W} \times 100\% = 85.67\ \%$$

Rotating Electrical Machines

- A rotating electrical machine may contain several sets of windings and they can be classified as: either a field winding or an armature winding.
- The field winding sets up the magnetic field (**B**) in the machine and is independent of the mechanical load imposed on the machine.
- On the other hand, the armature winding carries a current that is dependent on the mechanical load delivered by the machine. The armature current is more or less directly proportional to the mechanical load on the machine.

Structure of DC Machine

- DC machine contain a cylindrical stator with an even number of magnetic poles established by the permanent magnets or field windings as shown in Fig. 5. 10.
- Inside the stator is a rotor consisting of laminated cylindrical iron mounted on the shaft that is supported by bearings such that it can rotate.
- Rotor with slots for armature conductors are shown in Fig. 5.9.
- The cross sectional view of a 2-pole machine with the flux-lines in the air-gap is shown in Fig. 5.10.

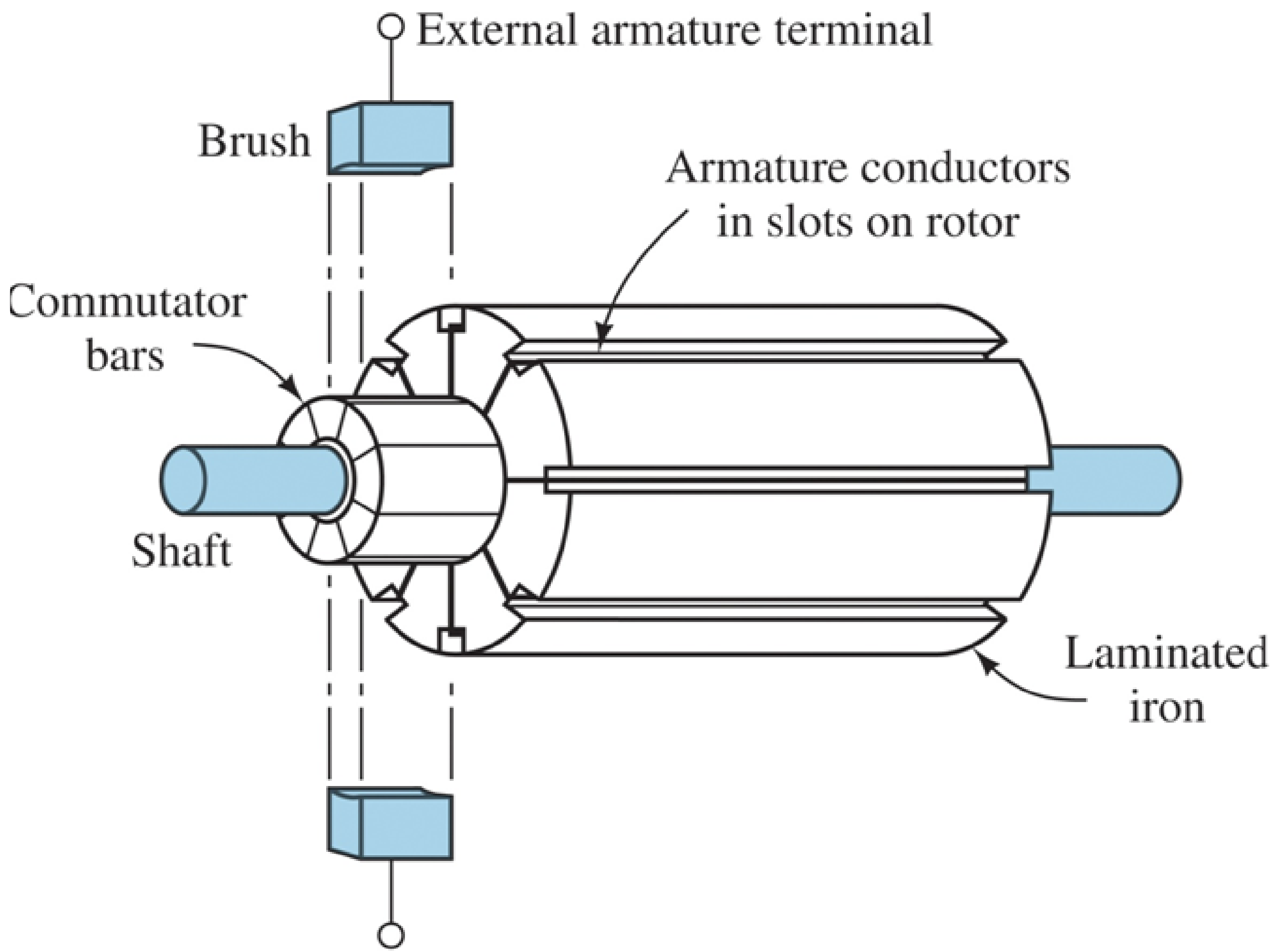


Fig. 5.9 Rotor assembly of a DC machine.

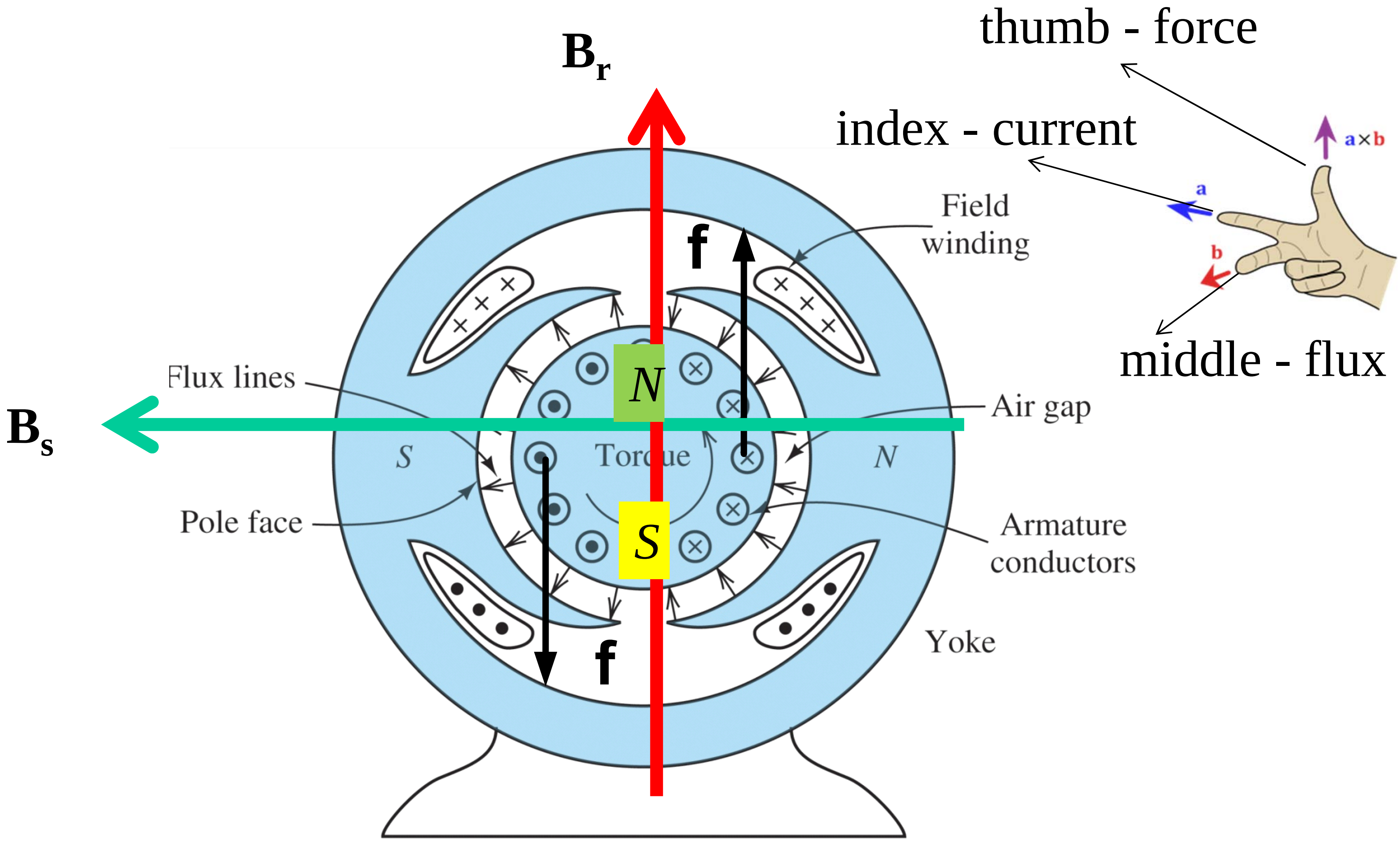


Fig. 5.10 Cross section of a 2-pole DC machine.

- Magnetic flux takes the path of least reluctance in the air-gap and therefore they are perpendicular to the surface of the rotor iron.
- Moreover, the flux-density is more or less constant magnitude over the surface of each pole face as shown in Fig. 5.10.
- External electrical sources provide currents to both field as well as armature windings.
- Based on the magnetic flux direction and the armature conductors current direction the torque generated is counter clockwise according to $f = i(l \times B)$.

Induced emf and Commutation

- As the conductor moves through the magnetic field produced by the stator, an emf is induced in the conductor.
- Under the pole face the conductor, the magnetic field and the direction of motion are mutually perpendicular to each other and an emf is produced.
- The induced emf remains constant under the pole face as the conductor moves along, however, as the conductor moves out of the magnetic pole face the flux-lines cutting the conductor reduces and thereby the emf induced as shown in Fig. 5.11. As the conductor moves under the opposite pole a negative voltage is induced.

- A mechanical switch known as commutator reverses the connections to the conductor as they move between the poles so the polarity of the induced emf is seen from the external terminal remains the same i.e. we get a rectified dc voltage as shown in Fig. 5.11.

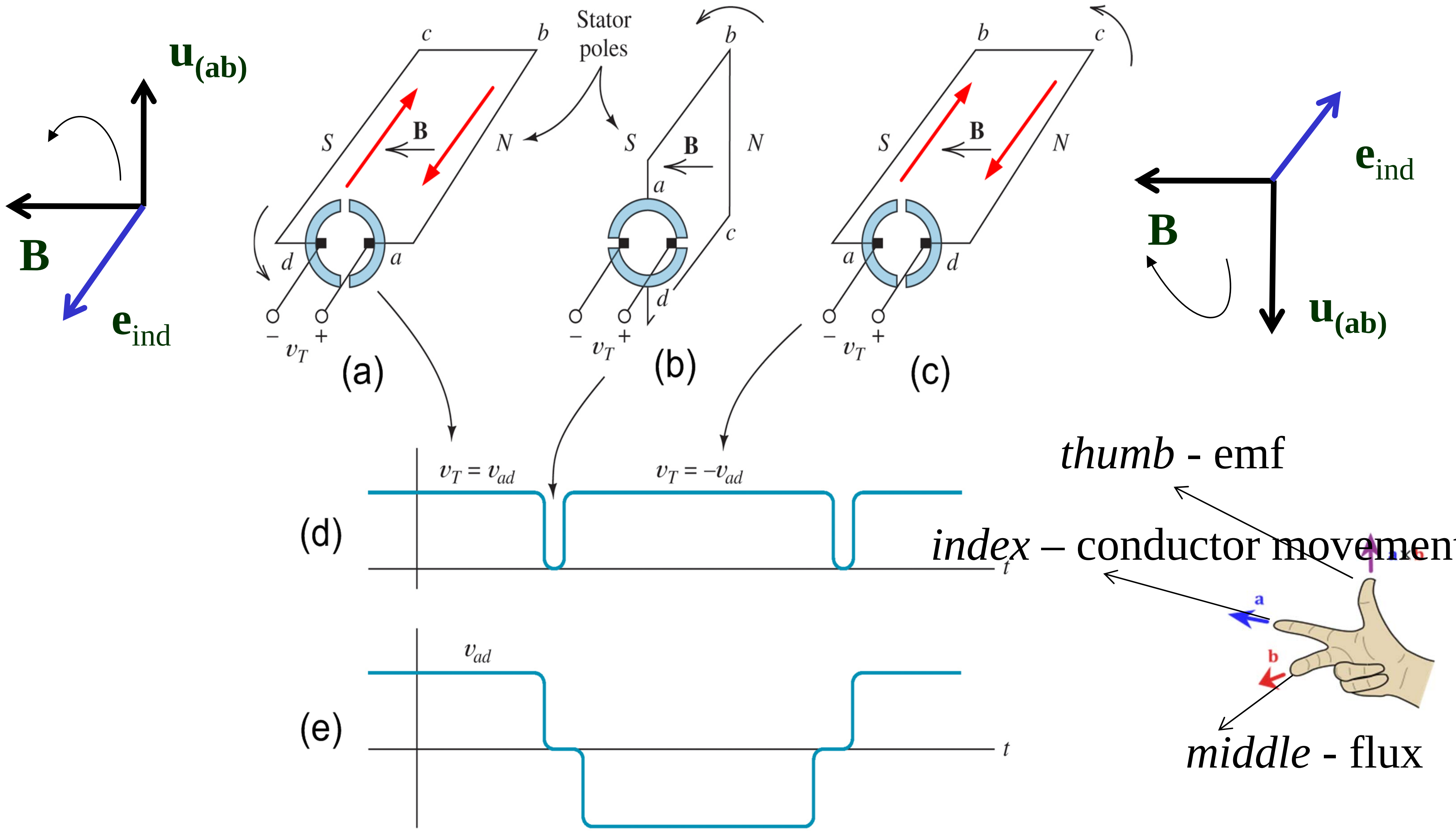


Fig. 5.11 Commutation for a single armature winding.

Configuration of DC Machines

- Depending on the various ways the field and armature windings are energized from different electrical sources, the classifications of DC machines can be carried out as follows:
  - Separately-excited (separate voltage sources for field and armature windings)
  - Shunt-excited (single voltage source and field winding is connected in parallel to armature winding)
  - Series-excited (single voltage source and field and armature windings are connected in series)
  - Compound-excited (have both series and shunt field windings)

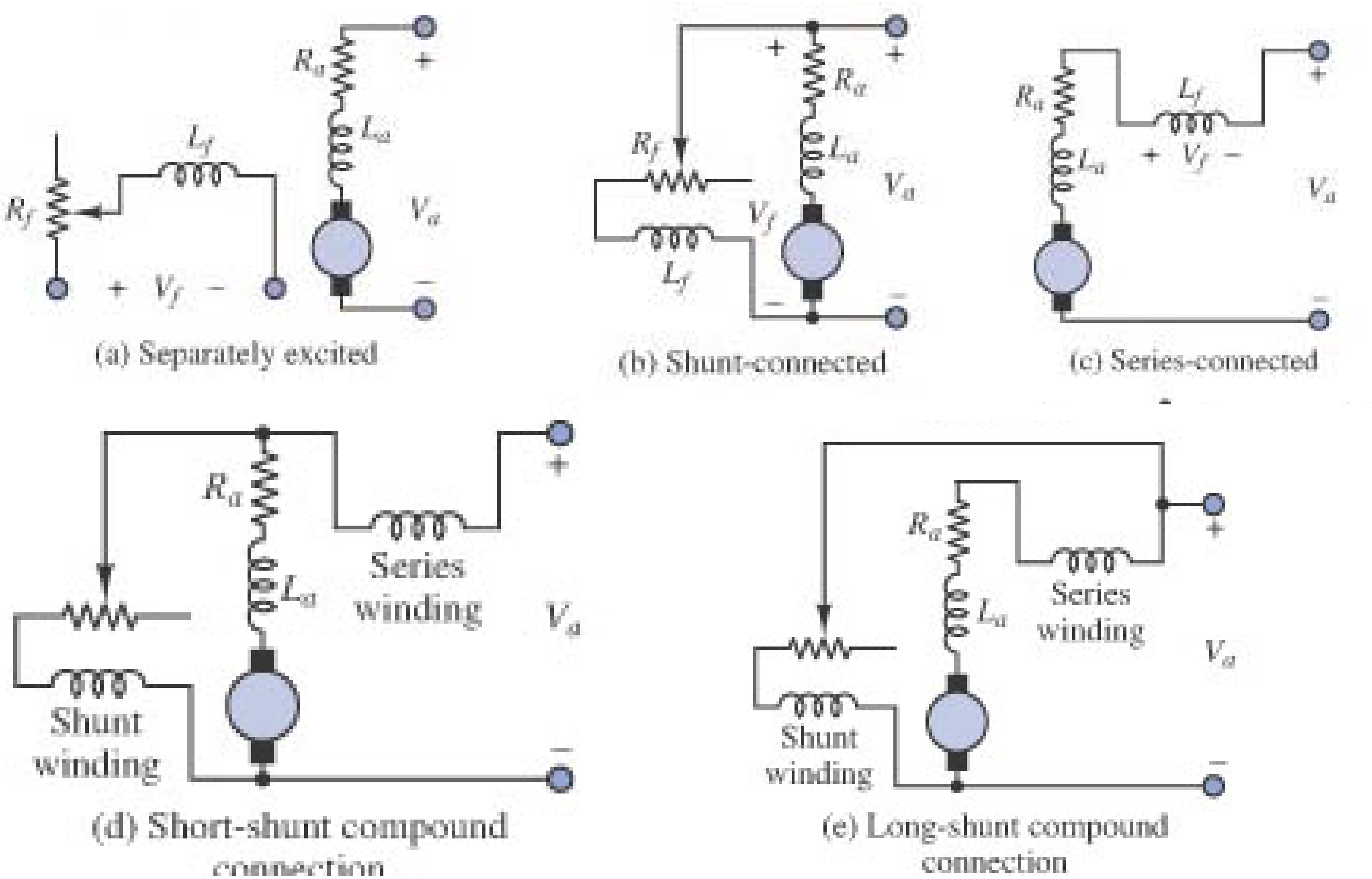


Fig. 5.12 DC machine classification.

DC Machine Equivalent Circuit

- The equivalent circuit of a DC machine is shown in Fig. 5.13. The field circuit is represented by R_F and L_F in series.
- For steady-state operation with dc quantities we have

$$V_F = R_F I_F \qquad (5.16)$$

- In the armature circuit, for steady-state operation with dc quantities we have E_A that represents the voltage induced in the armature conductors due to the rotation of the rotor in the presence of a magnetic field.
- It is known as the back-emf as it opposes the external armature voltage source, V_T or V_A.

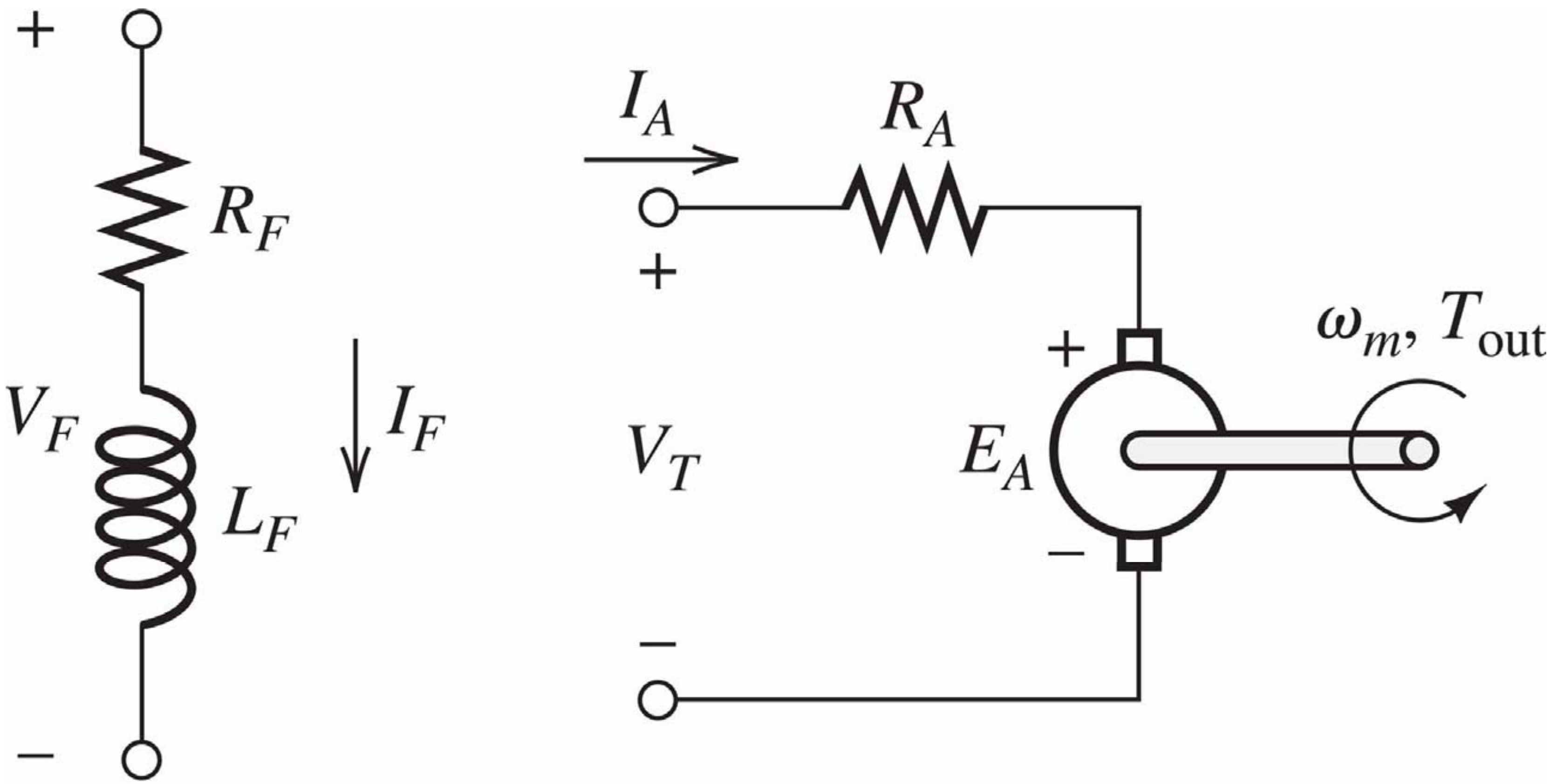


Fig. 5.13 Equivalent circuit of a DC machine.

- The armature winding is represented by the resistance R_A and inductance L_A.
- The induced emf is given by

$$E_A = [N\{2\times(B=\frac{\phi}{A})l(u=r\omega_m)\}] = \left[\left(\frac{2\times N\times l\times r}{A}\right)\times\phi\times\omega_m\right] = K_A\phi\omega_m \quad (5.17)$$

K_A - machine back-emf constant, ϕ - magnetic flux

ω_m – angular velocity of the rotor

- The torque developed is given by

$$T_{dev} = \left\{2N\left(f=\left(B=\frac{\phi}{A}\right)\right)il\right\}\times r = \left(\frac{2\times N\times l\times r}{A}\right)\phi I_A = K_T\phi I_A \qquad (5.18)$$

K_T - machine torque constant, ϕ - magnetic flux

I_A – armature current

- Note that output torque, T_{out} of DC machine is lower than T_{dev} due to friction and windage losses.

- The developed power is the electrical power that is converted into mechanical form and is given by

$$P_{dev} = T_{dev}\omega_m = E_A I_A \quad (5.19/5.20)$$

- Looking at the armature and field circuits and writing KVL eqn. at steady-state we have

$$V_T \text{ or } V_A = E_A + I_A R_A \quad \text{armature circuit } (5.21)$$

$$V_F = I_F R_F \quad \text{field circuit} (5.22)$$

- Looking at the armature and field circuits and writing KVL eqns. under dynamic operation we have

$$v_T(\mathrm{t}) \text{ or } v_A(t) = e_A(t) + i_A(t)R_A + L_a \frac{di_A}{dt} \quad \text{(armature circuit) } (5.23)$$

$$v_F = i_F(t)R_F + L_f \frac{di_f}{dt} \quad \text{(field circuit) } (5.24)$$

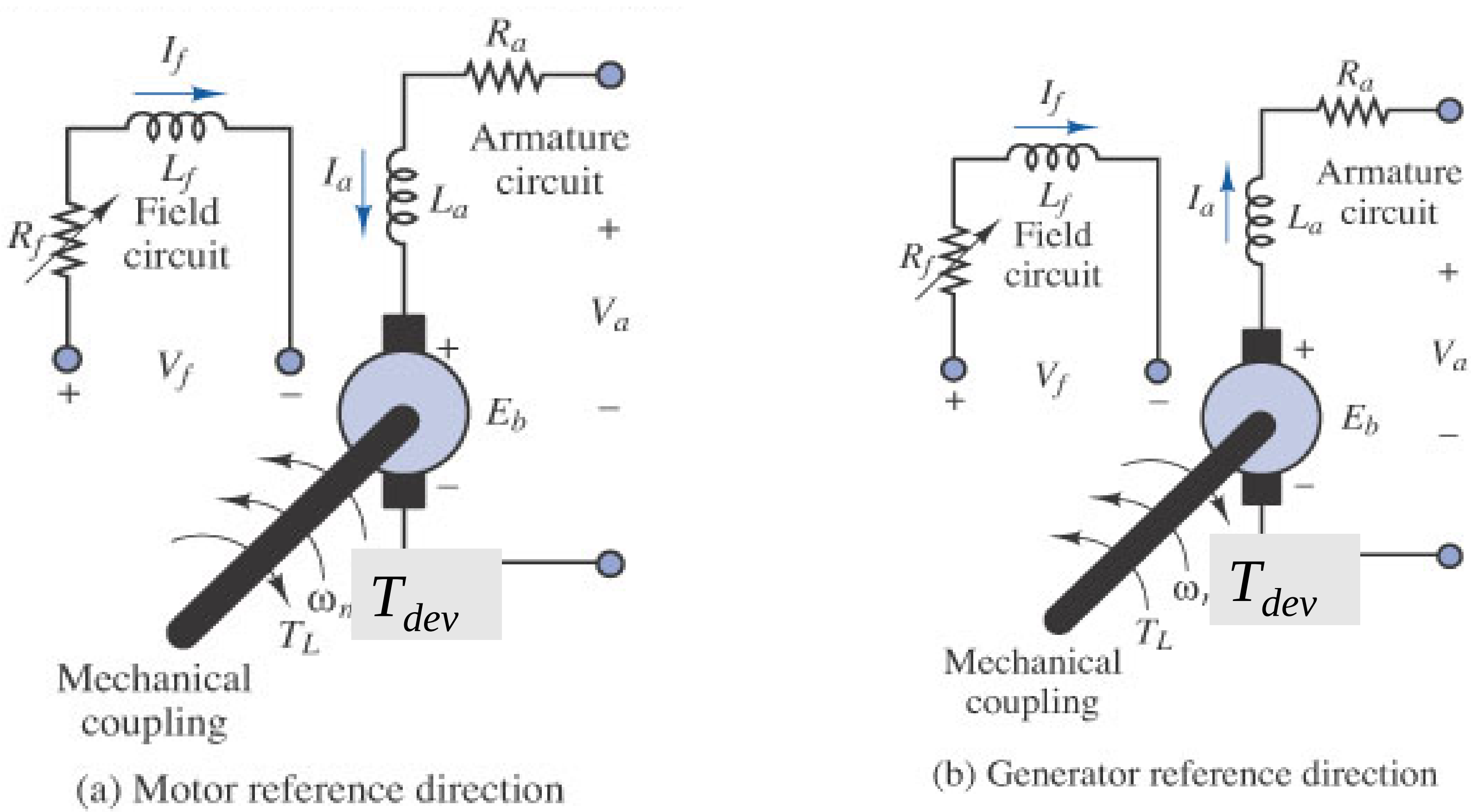


Fig. 5.14 Electrical equivalent circuit of a separately-excited DC machine.

- If an external mechanical load is connected to the motor shaft as shown in Fig. 5.14., we have the equation of motion as

$$T_{dev}(t) = k_T \phi(t) i_A(t) = T_L + b(t)\omega_m + J\frac{d\omega_m}{dt} \quad (5.25)$$

T_L - mechanical load torque, b - frictional coefficient

J - moment of inertia of the rotating masses

Magnetization Curve for DC Machines

- The magnetization curve of a dc machine is a graph between back-emf, E_A and the field current, I_F, as shown in Fig. 5. 15.
- As E_A is directly proportional to ϕ the shape of the magnetization curve is similar to the B-H curve of the magnetic circuit.
- If flux is maintained constant then according to eqn. 5.17, if the back-emf is E_{A1} at a speed of n_1 and E_{A2} at a speed of n_2 we have

$$\frac{E_{A1}}{E_{A2}} = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} \quad (5.26)$$

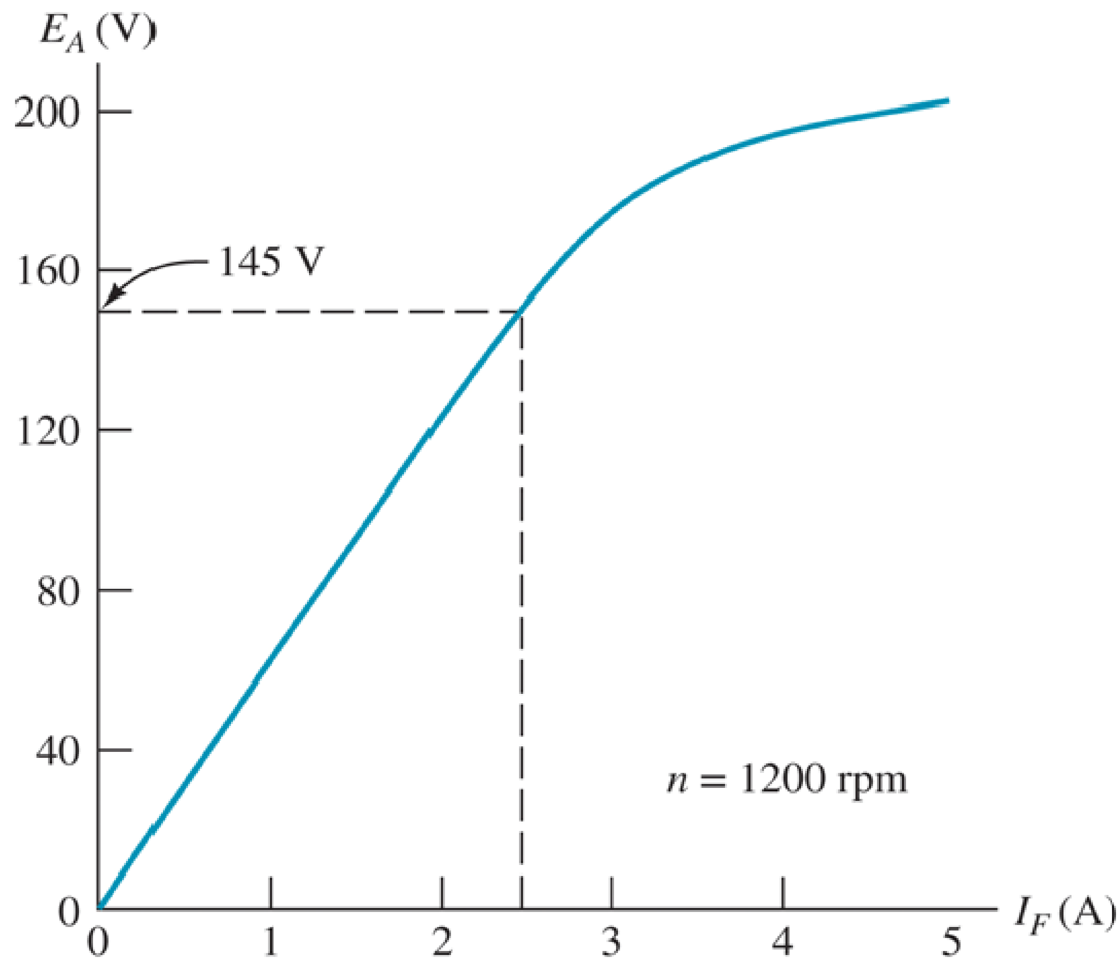


Fig. 5.15 Magnetization curve for a 200 V, 10-hp DC machine.

Example 5.2: The machine having a magnetization curve as shown in Fig. 5.15 is operating as a motor at a speed of 800 rpm with I_A = 30 A and I_F = 2.5 A. The armature resistance is 0.3 Ω and the field circuit resistance is 50 Ω. Find the voltage, V_F applied to the field circuit, the voltage, V_T applied to the armature, the developed torque, and the developed power.

Solution: According to eqn. 5.22 we have the field winding voltage given as

$$V_F = I_F R_F = 2.5\,A \times 50\ \Omega = 125\ \ V$$

From the magnetization curve, E_A is about 145 V for I_F = 2.5 A at a speed of 1200 rpm. Thus, the induced emf at a speed of 800 rpm is given by

$$E_{A2} = \frac{n_2}{n_1} \times E_{A1} = \frac{800}{1200} \times 145 = 96.67\ V$$

- The machine speed in rad/s is given by

$$\omega = n_2 \times \frac{2\pi}{60} = 800\ rpm \times \frac{2\pi}{60} = 83.78\ rad/s$$

The back-emf constant can be obtained as

$$K\phi = \frac{E_A}{\omega_m} = \frac{96.67\ V}{83.78\ rad/s} = 1.154\ V/(\frac{rad}{s})$$

The developed torque is given by eqn. (5.18) as

$$T_{dev} = K\phi I_A = 1.154\ \times 30\ A = 34.62\ N.m$$

The developed power is given by eqn. (5.19)

$$P_{dev} = T_{dev} \times \omega_m = 34.62\ \times 83.78 = 2900\ W$$

As a check P_{dev} as given by eqn. (5.20) as

$$P_{dev} = E_A \times I_A = 96.67 \times 30 = 2900\ W$$

Applying KVL to the armature circuit eqn. (5.21)

$$V_A = E_A + I_A \times R_A = 96.67\ V + 30\ A \times 0.3\ \Omega = 105.67\ V$$

Shunt-connected DC Motor

- As shown in Fig. 5.16, in the shunt connected DC motor the field and the armature windings are connected in parallel and both are supplied from a single source.
- The field circuit has an additional variable resistance (rehostat), R_{adj} that is connected in series with the field winding.
- This variable resistance in the field circuit can be used to vary the field current, I_F and therefore the torque-speed characteristics of the DC shunt motor.

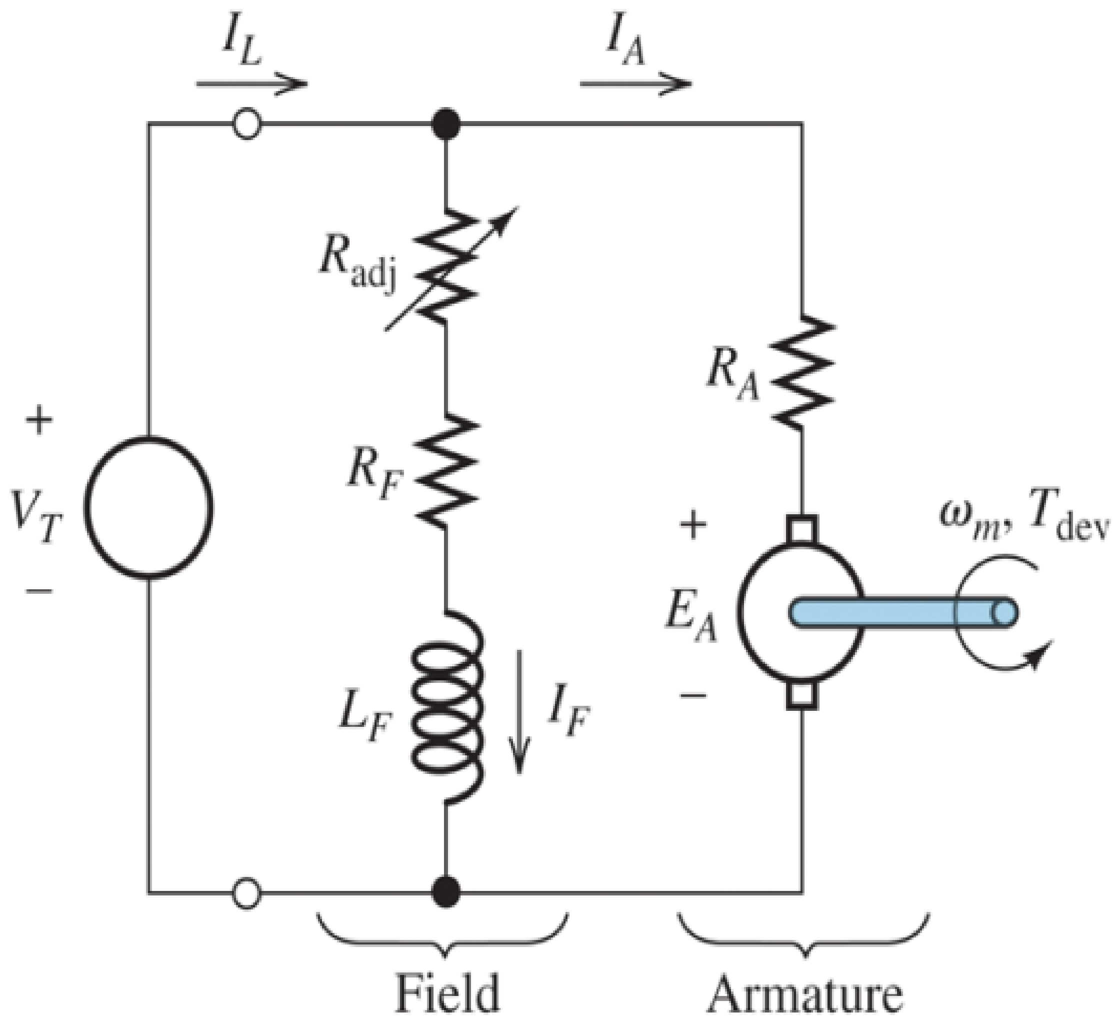


Fig. 5.16 Equivalent circuit of a shunt-connected DC motor.

Power Flow in Shunt-Connected DC Motor

- Fig. 5.17 shows the power flow in a shunt connected DC motor.

$$P_{in} = V_T I_L \tag{5.30}$$

$$P_{field-loss} = \frac{V_T^2}{R_F + R_{adj}} = V_T I_F \tag{5.31}$$

$$P_{arm-loss} = I_A^2 R_A \tag{5.32}$$

$$P_{dev} = E_A I_A = T_{dev} \omega_m \tag{5.33}$$

- The output power, P_{out} and output torque, T_{out} are less than that developed because of the rotational losses due to friction, windage, eddy currents and core losses.

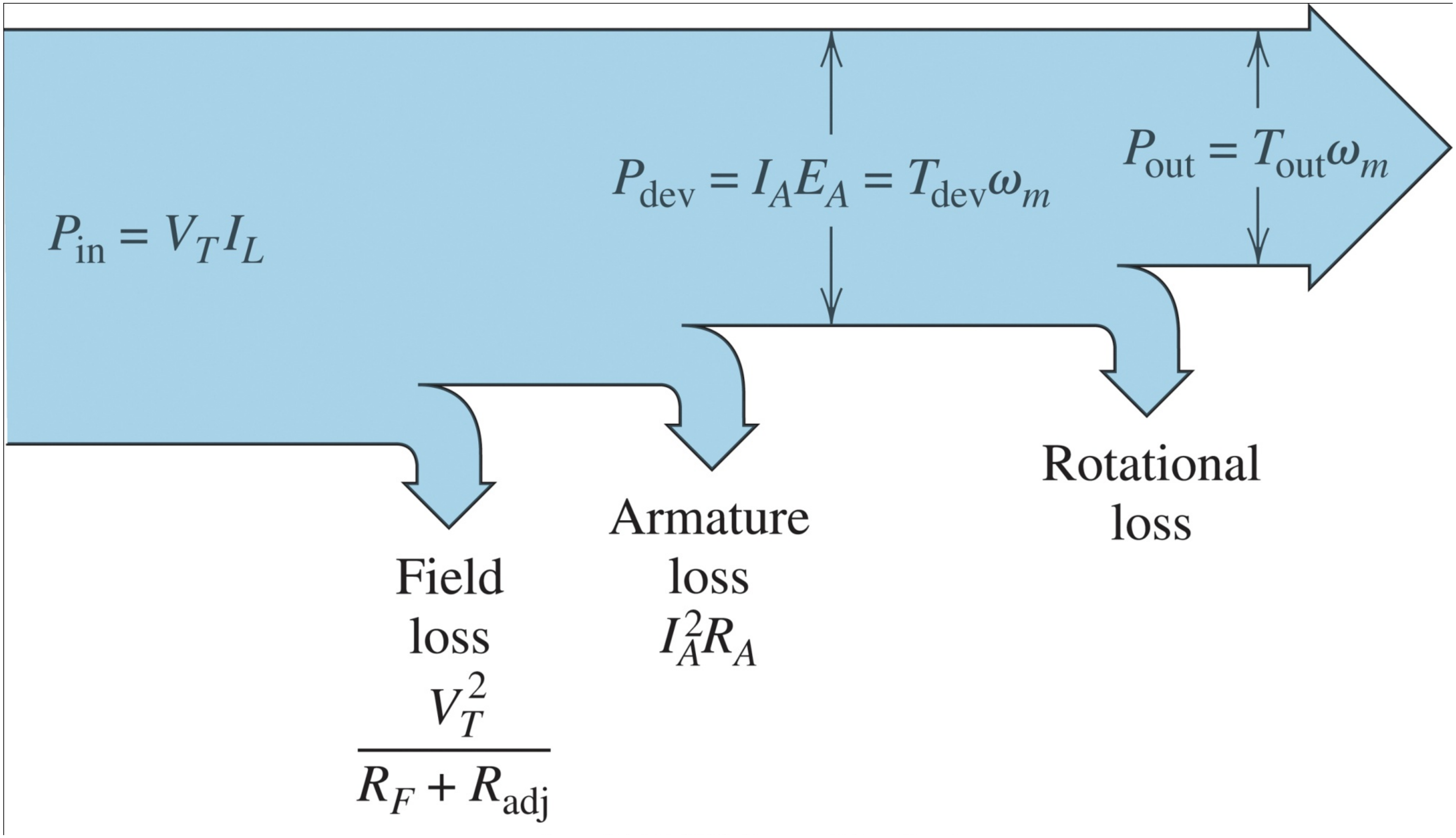


Fig. 5.17 Power flow in a shunt-connected DC motor.

Example 5.3: A 50 horse-power shunt-connected dc motor has the magnetization curve shown in Fig. Ex.3. The dc supply voltage is 240 V, the armature resistance is 0.065 Ω, the field circuit resistance is 10 Ω and the adjustable resistance in the field circuit is 14 Ω.

At a speed of 1200 rpm, the rotational loss due to friction and windage is 1450 W. If this motor drives a hoist that demands a torque of 250 N.m. independent of the speed, determine the motor speed and efficiency.

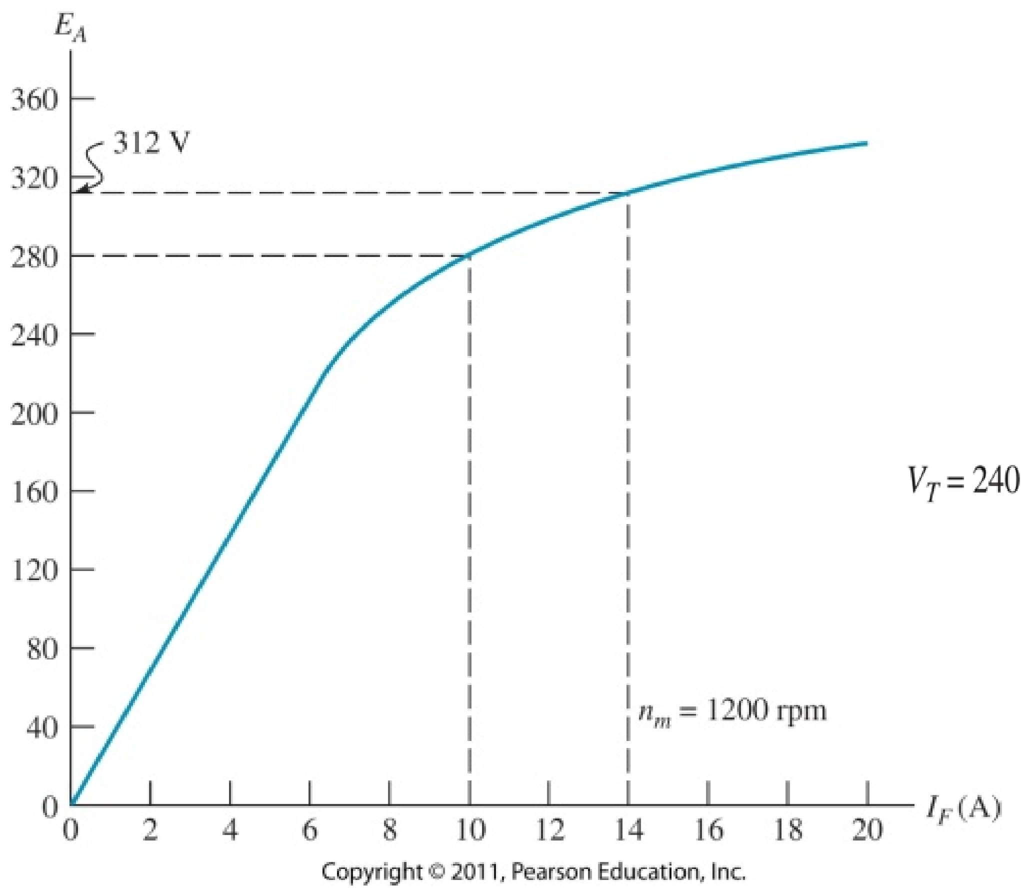


Fig. Ex3: (a) Magnetization curve at 1200 rpm and (b) shunt-connected DC motor equivalent circuit with hoist load.

Solution: From the equivalent circuit, the field current is given by

$$I_F = \frac{V_T}{R_F + R_{adj}} = \frac{240\ V}{10\ \Omega + 14\ \Omega} = 10\ A$$

- From the magnetization curve, we have the induced voltage at a field current of 10 A and 1200 rpm to be 280 V.
- Thus, the back emf constant can be obtained as

$$K\phi = \frac{E_A}{\omega_m} = \frac{280\ V}{1200(2\pi/60)\ rad/s} = 2.228\ V/(\frac{rad}{s})$$

- The rotational torque loss is given by

$$T_{rot} = \frac{P_{rot}}{\omega_m} = \frac{1450}{1200(2\pi/60)\ rad/s} = 11.54\ N.m$$

- Thus, the torque to be developed by the machine is:

$$T_{dev} = T_{out} + T_{rot} = 250 + 11.54 = 261.54\ N.m$$

- The armature current is given by:

$$i_A = \frac{T_{dev}}{K\phi} = \frac{261.54}{2.228} = 117.4\ A$$

- The developed back-emf is given by:

$$E_A = V_T - I_A \times R_A = 240 - 117.4 \times 0.065 = 232.4\ V$$

- The motor speed is given by:

$$\omega_m = \frac{E_A}{K\phi} = \frac{232.4}{2.228} = 104.3\frac{rad}{s} \Rightarrow n_m = \omega_m \times \left(\frac{60}{2\pi}\right) = 104.3 \times \left(\frac{60}{2\pi}\right) = 996\ rpm$$

- The output power and efficiency are given by:

$$P_{dev} = T_{out} \times \omega_m = 250 \times 104.3 = 26.08\ kW$$

$$P_{in} = V_T(I_A + I_F) = 240 \times (117.4 + 10) = 30.58\ kW$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{26.08}{30.58} \times 100\% = 85.3\ \%$$

Torque-Speed Characteristics

- Applying KVL to the equivalent circuit as shown in Fig. 5.13, we have

$$V_T = I_A R_A + E_A \quad \text{from eqn.(5.21)}$$

$$I_A = \frac{T_{dev}}{K_T \phi} \qquad \text{from eqn. (5.18)}$$

$$P_{dev} = T_{dev}\omega_m = (K_T \phi I_A)\times\left(\frac{E_A}{K_A \phi}\right) = \left(\frac{K_T}{K_A}\right)(E_A \times I_A) = (E_A \times I_A) \Rightarrow K_A = K_T = K (5.27)$$

$$V_T = I_A R_A + E_A = \frac{T_{dev}}{K_T \phi} R_A + K_A \phi \omega_m \qquad (5.28)$$

$$\omega_m = \frac{V_T}{K_A \phi} - \frac{R_A T_{dev}}{(K_T \phi)(K_A \phi)} = \frac{V_T}{K\ \phi} - \frac{R_A}{(K\ \phi)^2} T_{dev} \quad (5.29)$$

Speed Control of DC Motors

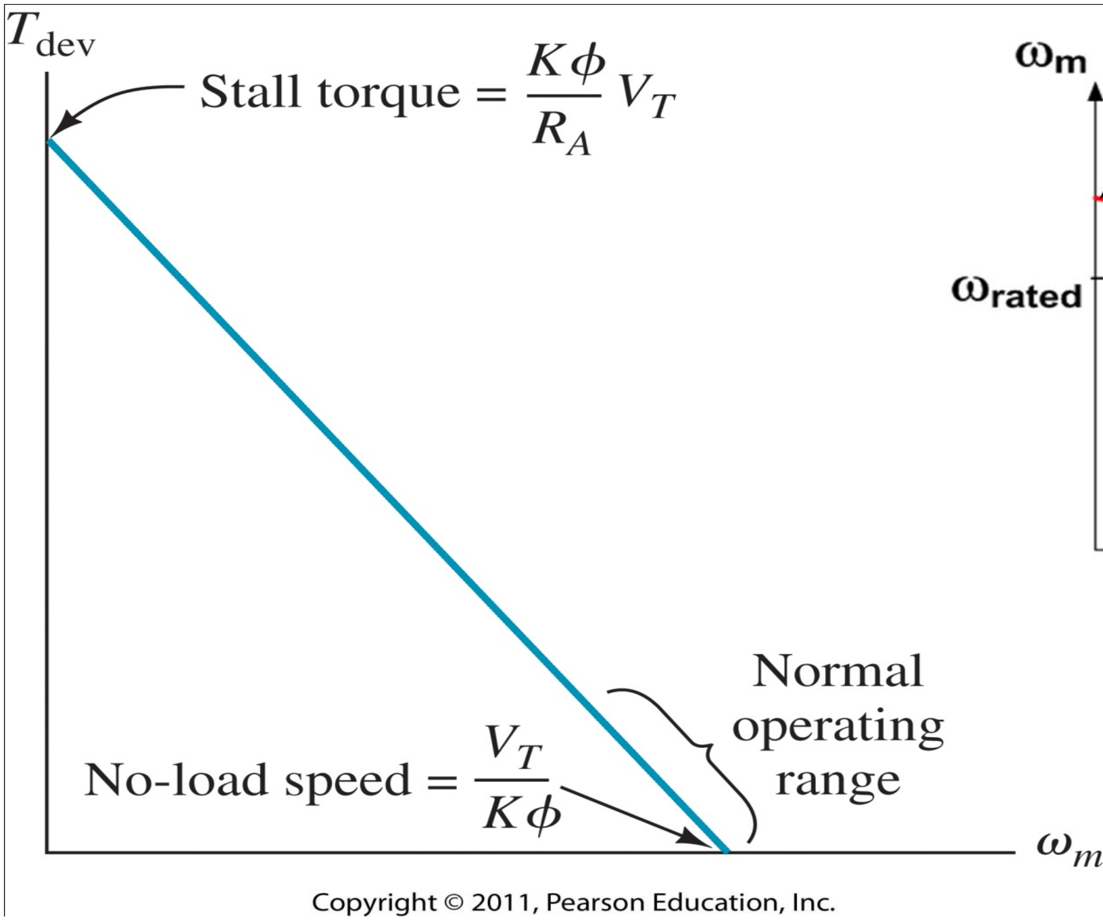
- From eqn. 5.29, we can see that the speed of a DC motor, ω_m can be controlled using three different methods:

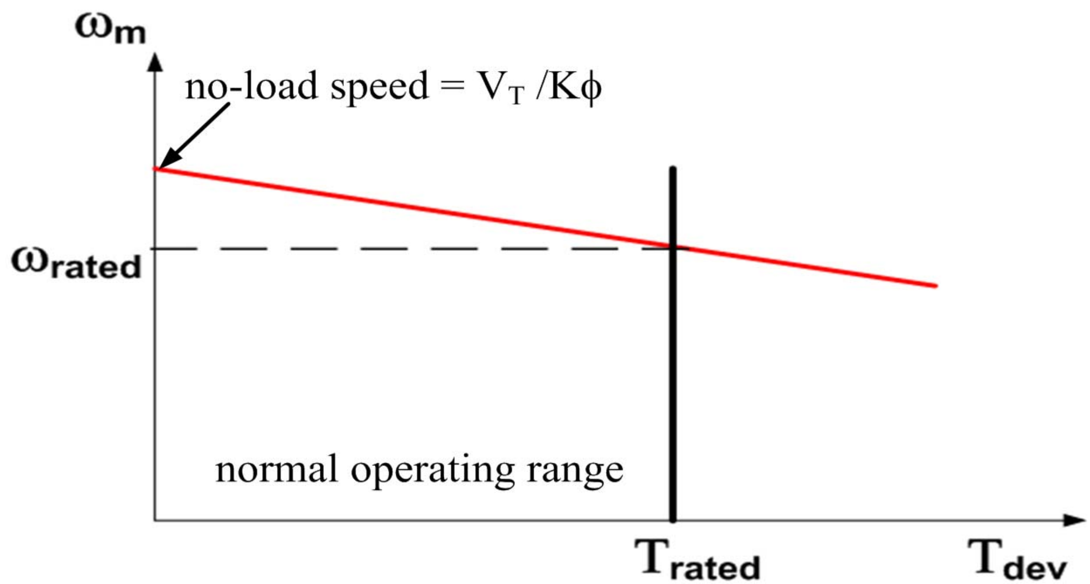
$$\omega_m = \frac{V_T}{K\,\phi} - \frac{R_A}{(K\,\phi)^2} T_{dev} \quad (5.29)$$

1. Armature voltage, V_T control while maintaining the field current and therefore flux ϕ constant.
2. Vary the field current, I_F and therefore flux ϕ in the machine while maintaining armature voltage V_T constant.
3. Inserting series resistance, $R_{A,add}$ in the armature circuit.

- Methods 1 and 2 are more widely used than method 3.

- From eqn. 5.29, we can see that the torque-speed characteristics is a straight-line and shown below.





$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2} T_{dev} \quad (y = -mx + c) \quad (5.29)$$

Fig. 5.18 Torque-speed characteristics of shunt-connected DC motor.

Speed Control using Armature Voltage

- Assuming the DC motor is a separately excited DC motor or permanent-magnet DC motor, we have

$$\omega_m = \frac{V_T}{K_A\phi} - \frac{R_A T_{dev}}{(K_T\phi)(K_A\phi)} = \frac{V_T}{K\ \phi} - \frac{R_A}{(K\ \phi)^2} T_{dev} \quad (5.29)$$

$$\omega_m \approx \frac{V_T}{K\ \phi} \Rightarrow \omega_m \propto V_T \quad (5.29a)$$

- Thus, the motor speed is directly proportional to the armature voltage, V_T and thus motor speed can be controlled by controlling V_T while maintaining flux, ϕ and R_A constant.

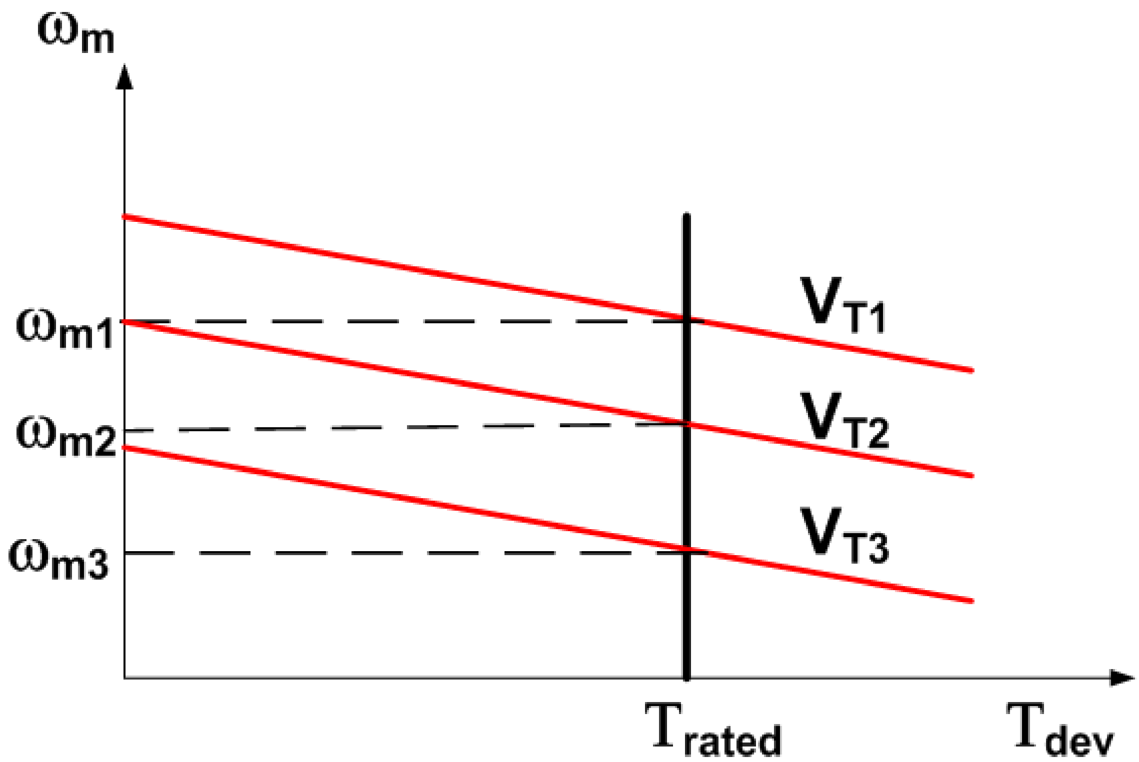


Fig. 5.19 Torque-speed characteristics of shunt-connected DC motor for varying armature voltage.

Variable DC Voltage Source - PWM

- A dc voltage source can be obtained by using either a battery or if it is AC supply then using a diode-bridge rectifier followed by a smoothing filter capacitor as we have seen in Lab. 7 Manual.
- Once, a constant dc voltage source is obtained, an electronic switching circuit as shown in Fig. 5.20 can be used to control the average dc voltage delivered to the load.
- The switch periodically opens and closes with a time period T, closed for a period of T_{on} and opened for the rest of the period T_{off} (= T - T_{on}). This process is known as pulse-width-modulation (PWM) in power electronics.

- The presence of armature inductance L_A opposes the change of current by creating a voltage drop across it (according to $v_{L,A} = L_A \frac{di_A}{dt}$) and thereby smoothes the armature current when the switch is open by allowing it to flow through the freewheeling diode D.
- The presence of the freewheeling diode is quite essential in maintaining a smooth armature current otherwise a large voltage would have induced across the semiconductor switch when the switch is opened and the armature current is abruptly interrupted.
- The average output voltage applied to the motor is

$$V_T = V_S \frac{T_{on}}{T} = d \times V_s \qquad d \text{ - duty cycle of the switch} \quad (5.30)$$

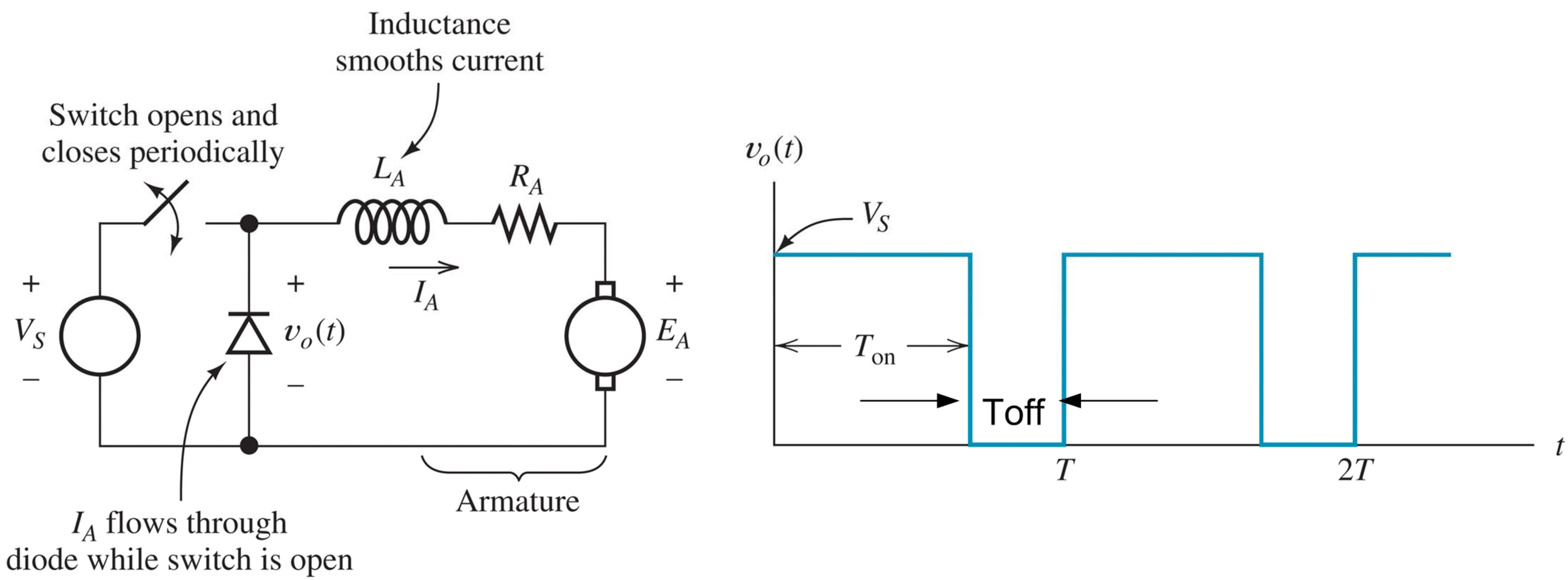


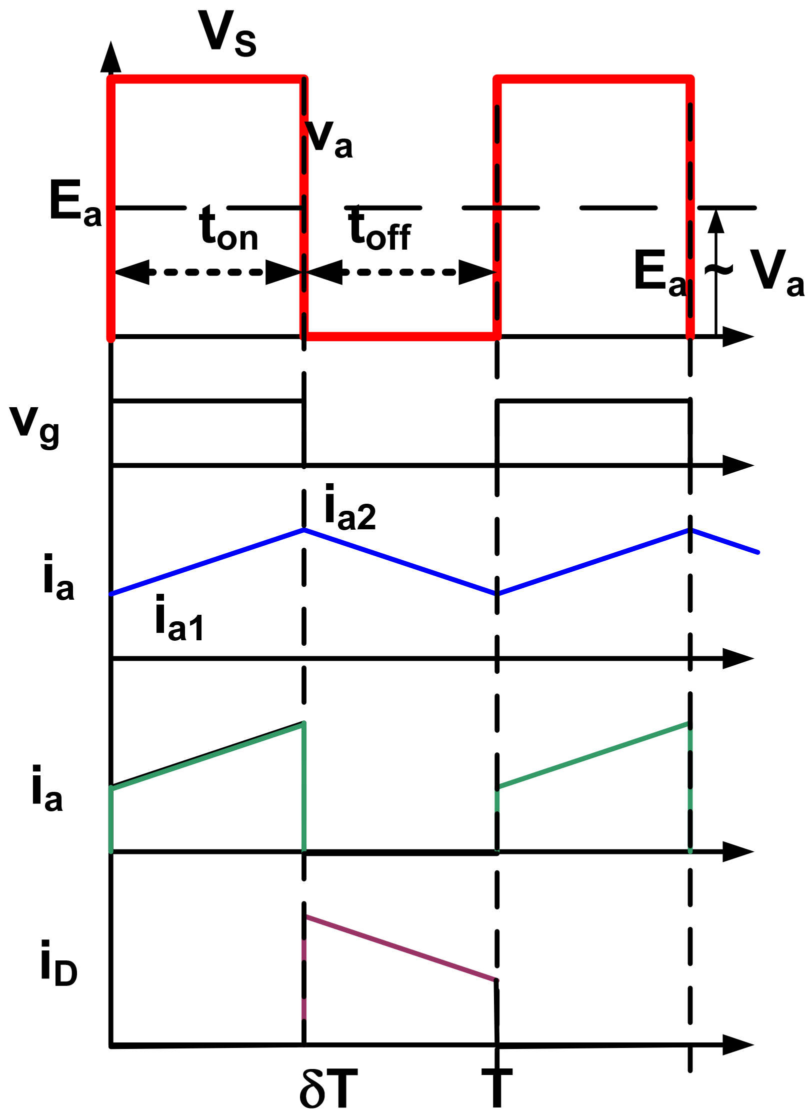
Fig. 5.20 An electronic switch that opens and closes periodically can efficiently supply a variable dc voltage to a motor from a fixed dc supply voltage.

- Consider that the armature circuit inductance is 1 mH, a current of 10 A is flowing through the circuit and the current is interrupted in 1 μs (semiconductor switch can be opened very fast) then we have

$$v_{L_A} = L_A \times \frac{di_A}{dt} = 10^{-3} \times \frac{10}{10^{-6}} = 100 \times 10^3 = 10\,kV$$

- Such a large voltage would appear across the semiconductor switch and blow it up so the freewheeling diode is provided so that current can flow through the diode and therefore it is not interrupted abruptly.
- By putting a freewheeling diode would slow down the rate at which current would drop and thereby reducing the voltage induced across the armature inductance, L_A

$$v_{L,A} = L_A \times \frac{di_A}{dt} = 10^{-3} \times \frac{10-9}{0.1 \times 10^{-3}} = 10\,V$$



V_S
v_a
E_a
t_{on}
t_{off}
$E_a \sim V_a$
v_g
i_{a2}
i_a
i_{a1}
i_a
i_D
δT
T

Summary

- Basic principles of operation of linear DC motor and generators.
- Structure of rotating electrical machines.
- Basic principles of operation of rotating DC machines.
- Configuration of DC machines.
- DC Machine equivalent circuit.
- DC machine magnetization curve.
- Torque-speed characteristics of DC shunt and separately excited motors.
- Speed control of DC motor.

References

1. "Principles and Applications of Electrical Engineering" – Giorgio Rizonni, Mc Graw Hill, 5th Edition 2007, Chapters 19.
2. "Electrical Engineering Principles and Applications", - Allan R. Hambley, Pearson – Prentice Hall, 5th Edition 2010, Chapter 16.