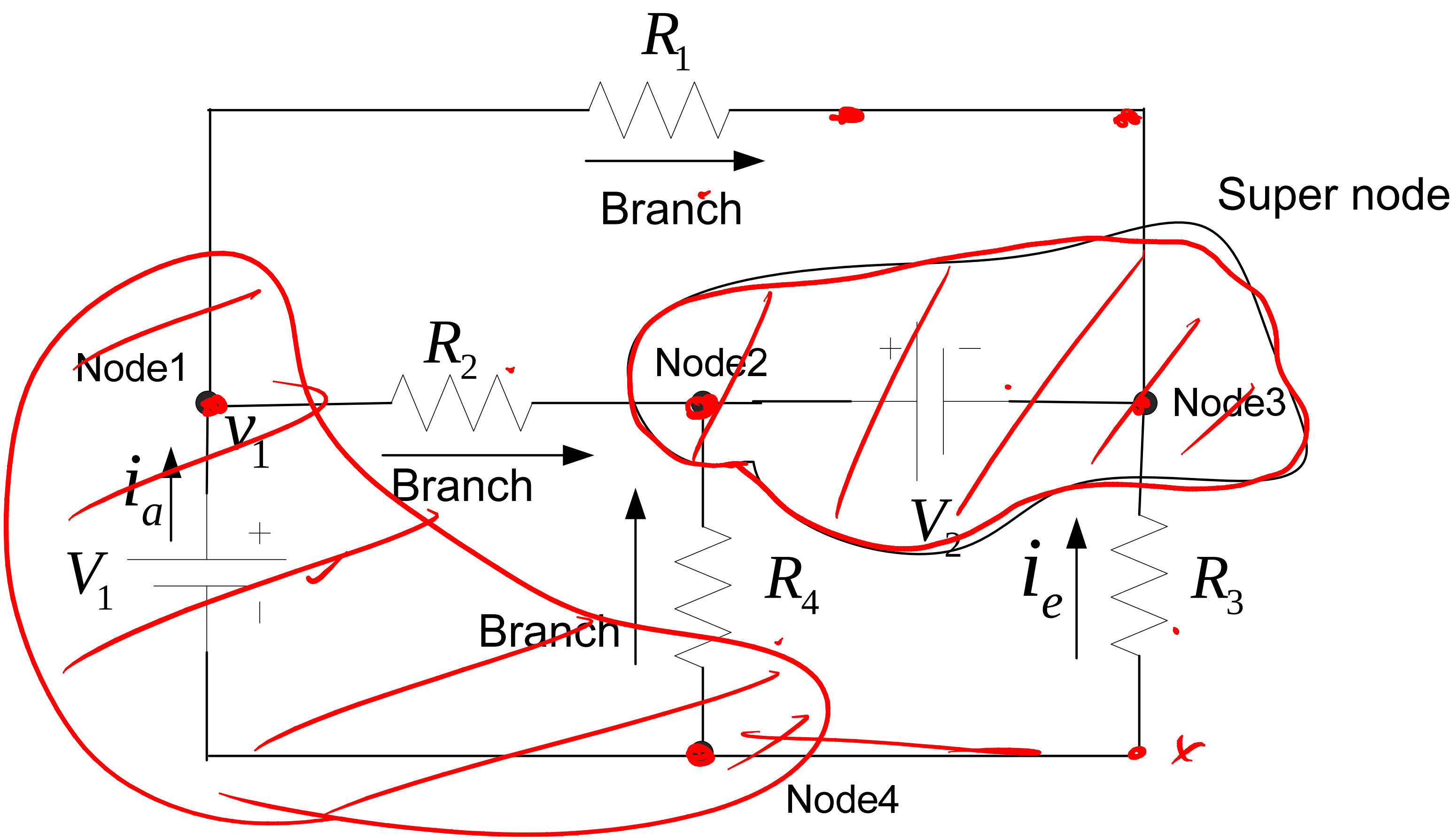
EE1002
Introduction to Circuits and Systems

Part 1 : Lecture 3

Node, branch, etc. defined

- A node in an electrical circuit is a point at which two or more elements are joined together.
- A super node is a closed surface enclosing part of a circuit. It may contain some sources and other nodes.

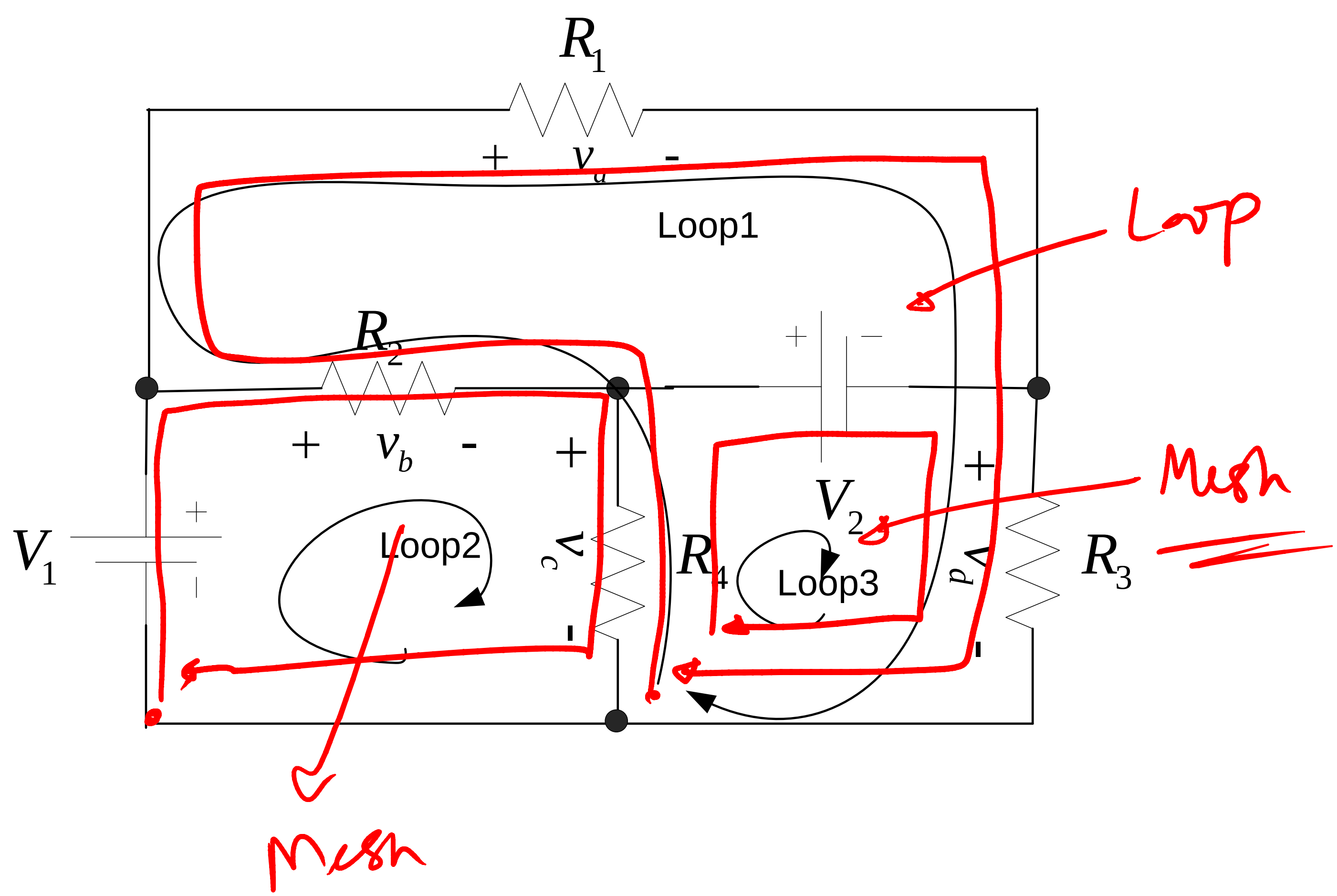
Terms of Circuit/Network Analysis



Mesh, Loop

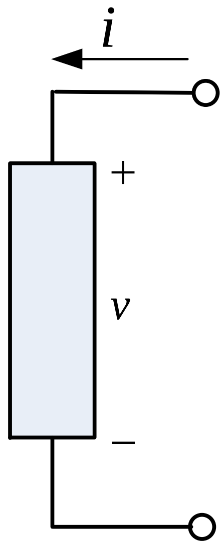
- A mesh is a closed path in the circuit.
- A loop is also a closed path in the circuit.
- A mesh cannot have any other mesh or loop inside it.
- A loop can have meshes inside it.

Mesh and Loop



Passive sign convention

- Reference direction (positive current direction) for current is to enter into the positive voltage terminal of the element
- If power is positive, the element is passive (dissipating energy)
- We shall follow the passive sign convention while solving circuits



Kirchoff’s Laws

Kirchoff’s Voltage Law (KVL)

- Net voltage fall around a closed path is zero
  - Voltage rises when we go from negative polarity to positive polarity
  - Voltage falls when we go from positive polarity to negative polarity
- We shall use KVL in Mesh Current Analysis Method

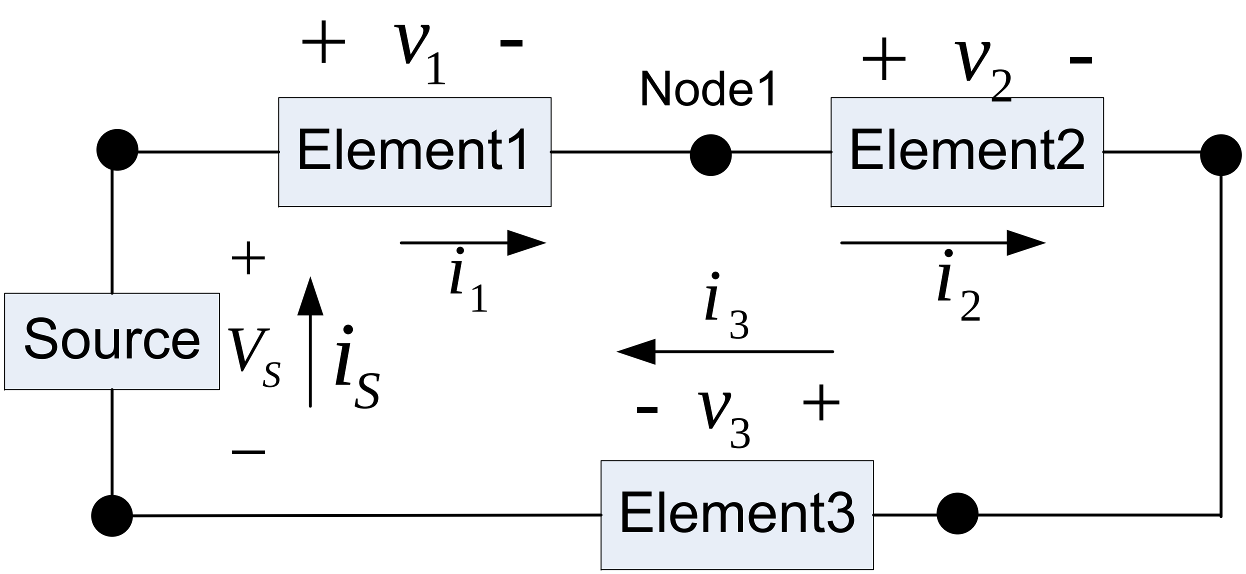
Kirchoff’s Current Law (KCL)

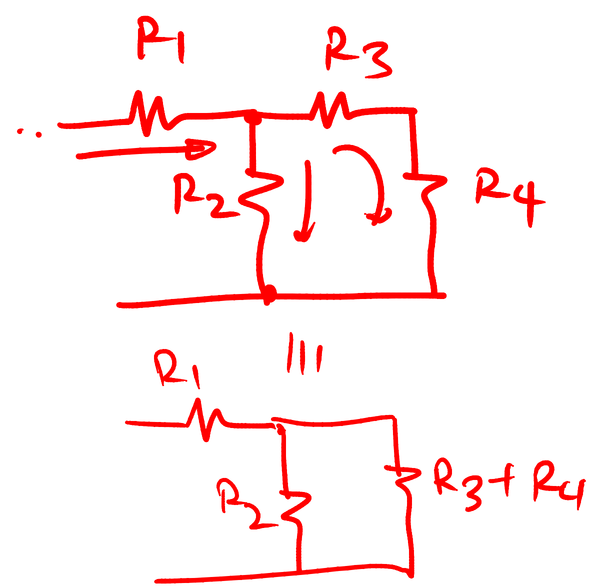
- Net current leaving a node/supernode is zero

- We shall use KCL in Node Voltage Analysis Method

Series Conection

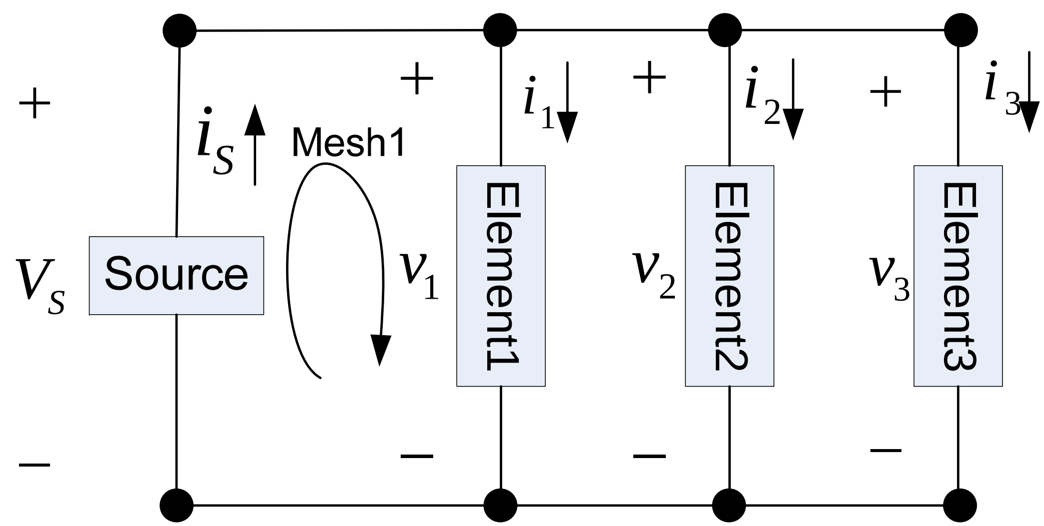
- When elements are connected end to end
- they carry the same current
  - Apply KCL at any node to find this



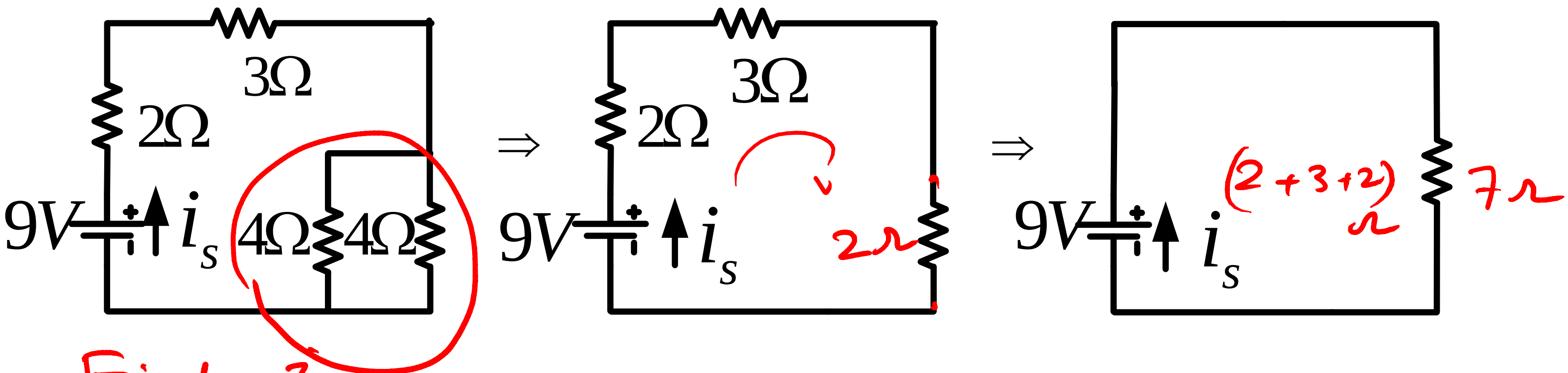


Parallel Circuit

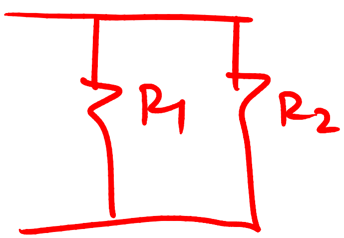
- If both ends of one elements are connected to the corresponding ends of the other element
- Voltages across all the elements are identical



Network analysis by series and parallel rules for resistance



Find i_s.

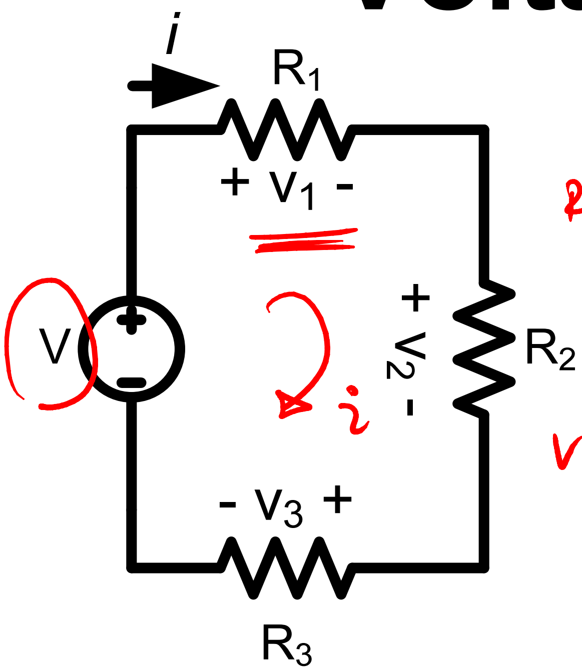


$\equiv$ $R_{eq} = \dfrac{R_1 \cdot R_2}{R_1 + R_2}$

$$i_s = \frac{9}{7}\ A.$$

- Voltage divider rule
- Current divider rule

Voltage Divider rule



$R_1 = 100\,\Omega$
$R_2 = 5\,\Omega$
$R_3 = 5\,\Omega$
$V = 110\,V$

$$V_1 = iR_1 = \frac{R_1}{R_1 + R_2 + R_3} V$$

$$V_2 = iR_2 = \frac{R_2}{R_1 + R_2 + R_3} V$$

$$V_3 = iR_3 = \frac{R_3}{R_1 + R_2 + R_3} V$$

$i = \frac{V}{R_1 + R_2 + R_3}$

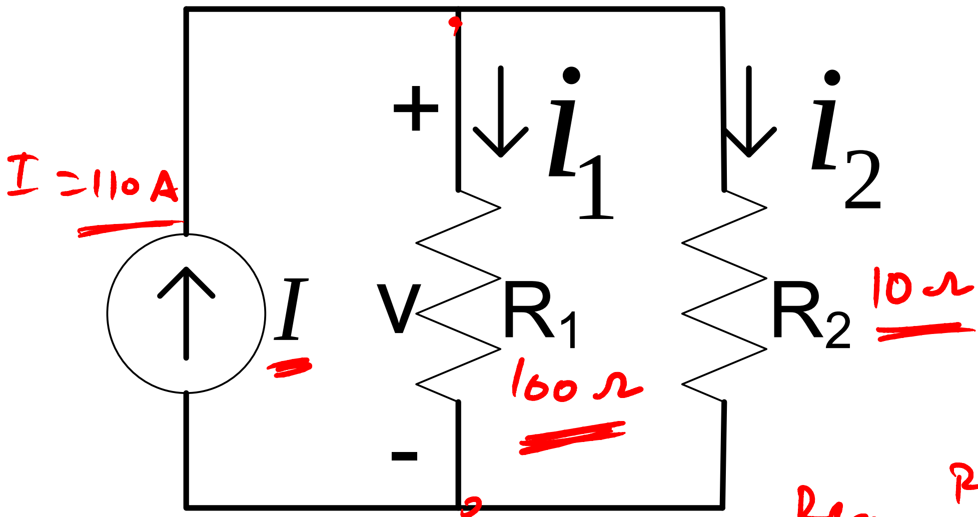
$V_{R_1} = 100V$

$V_{R_2} = 5V$

$V_{R_3} = 5V$

In a series circuit, the voltage a across each resistance is a fraction of the total voltage, which is equal to the ratio of the concerned resistance to the total resistance.

Current Divider Rule



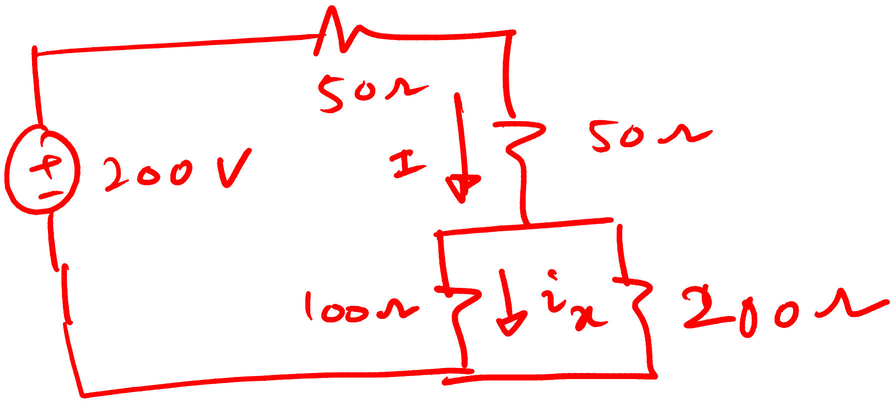
$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} I = 10\,A$$

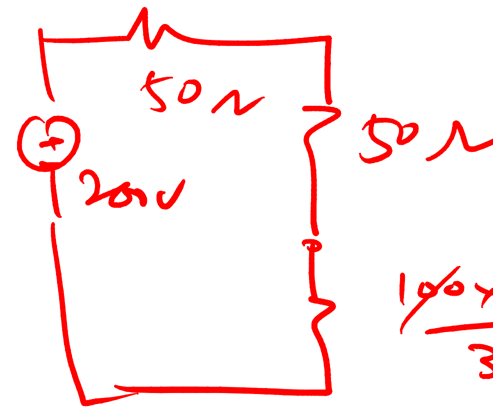
$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} I = 100\,A$$

$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$, $v = I \cdot R_{eq}$

For the two resistances in parallel, the current flowing in each resistance is a fraction of the total current equal to the ratio of the other resistor to the sum of both the resistors.

Example





$$\frac{100\times 200}{300} = \frac{200}{3}\,\Omega$$

$$I = \frac{200}{50+50+\frac{200}{3}}\ \text{A}.$$

$$i_x = I \times \frac{200}{100+200} \times \frac{200}{100+\frac{200}{3}}$$

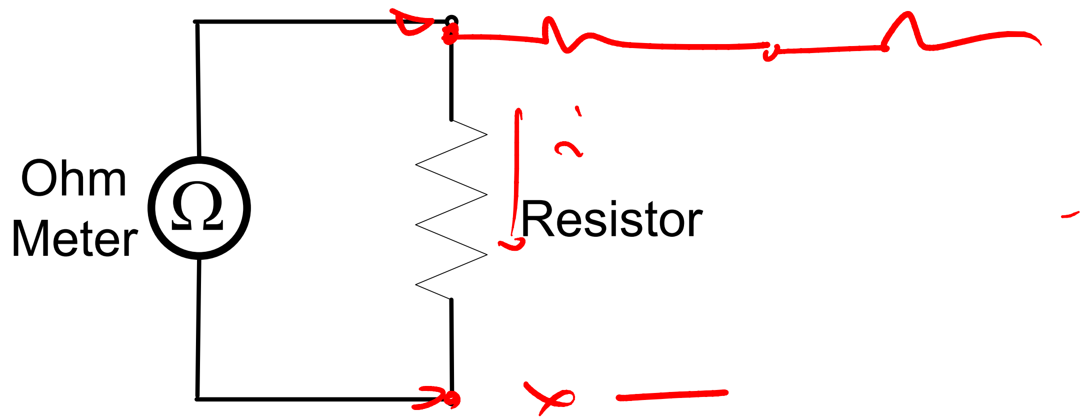
- Measuring Electrical quantities

Measuring Devices

- Ohm meter – for resistance measurement
- ✓ Ammeter – for current measurement
- Voltmeter – for voltage measurement

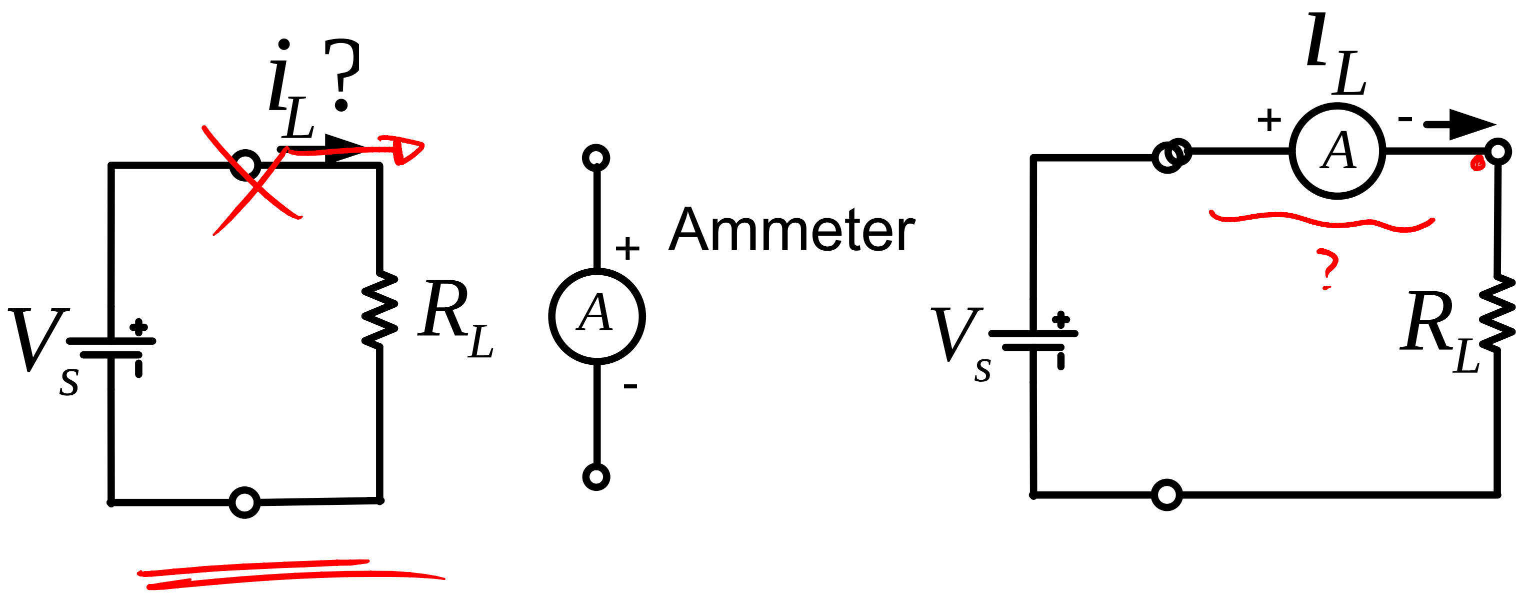
✓ Multi-meter

Ohmmeter



The element should be removed from the circuit when resistance is measured (at least one end of the element should be removed from the circuit and the circuit should be de-energised.)

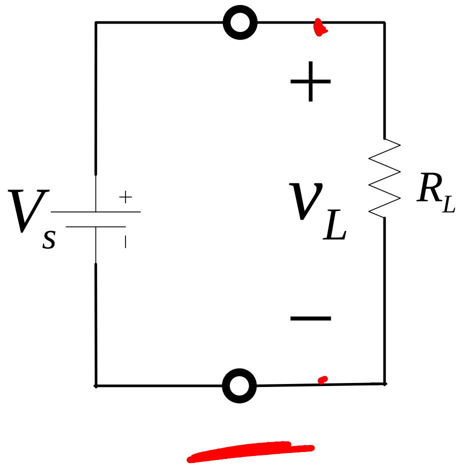
Ammeter

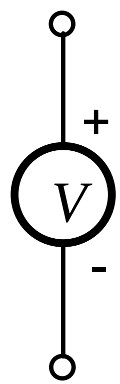


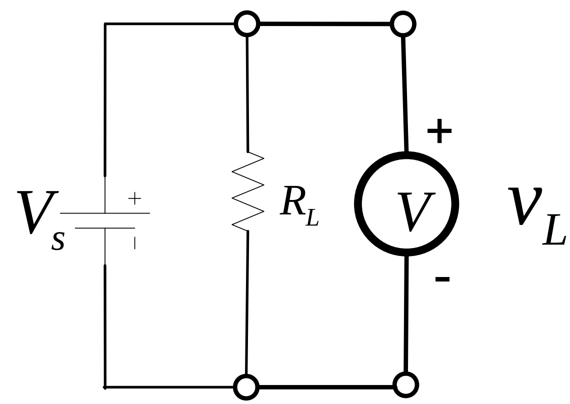
- Ammeter is connected in series with the element so that the same current passes through both the ammeter
- Ideal ammeter has zero resistance

Voltmeter

+ve – Red
–ve – Black





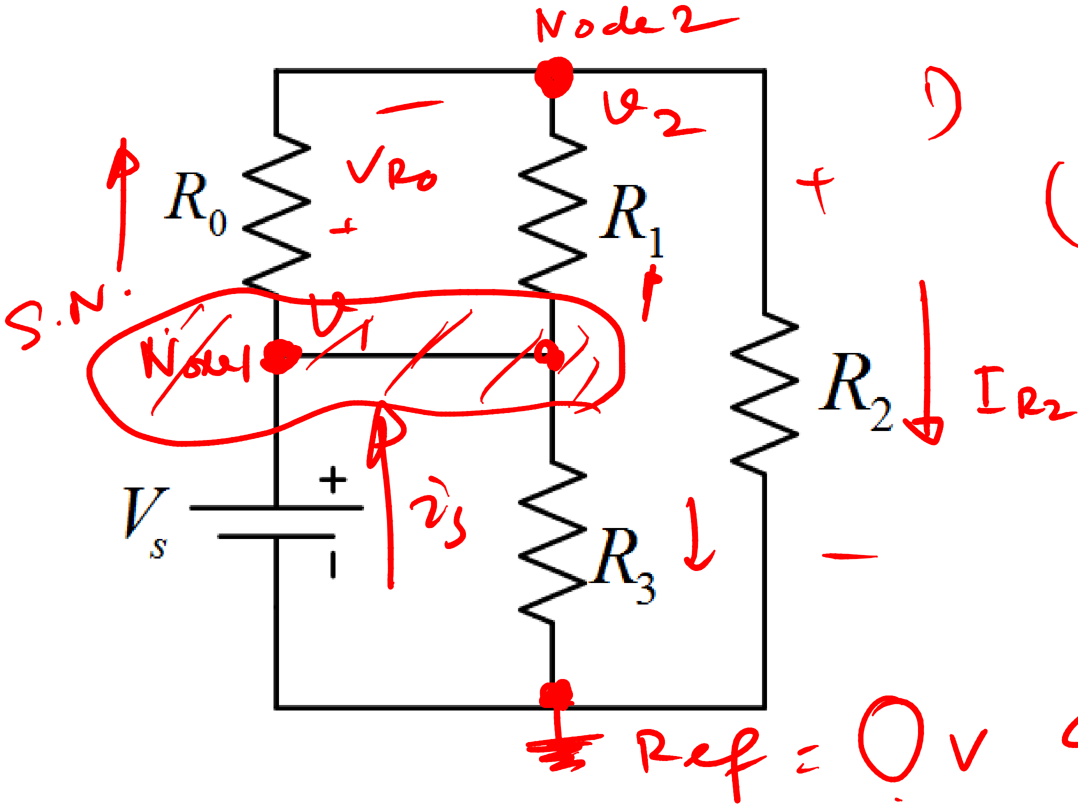


- Voltmeter is connected in parallel to the two points
- ideal voltmeter has infinite resistance

Node Voltage Analysis

Mesh Current Analysis

Intro to node voltage analysis



1) $v_1 - v_2 = v_{R_0}$

$(v_1 - v_{ref}) - (v_2 - v_{ref})$

$= v_1 - v_2$

$I_{R_2} = \frac{\text{voltage across } R_2}{R_2}$

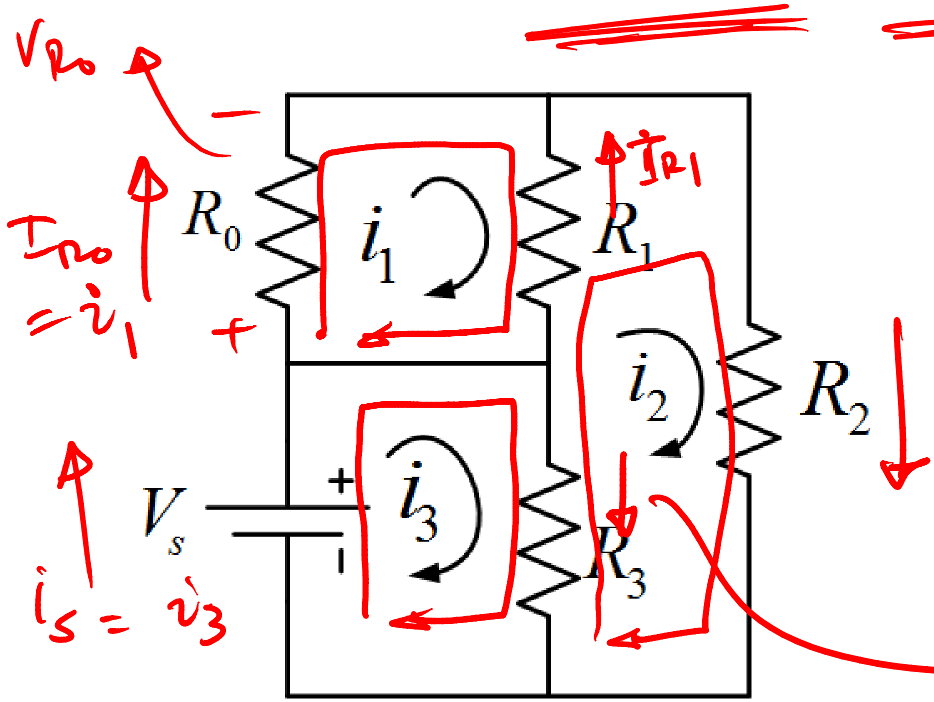
$= \frac{(v_2 - v_{ref}) - 0}{R_2}$

$= \frac{v_2}{R_2}$

Ground (GND)

- If voltage at all the nodes are known, then all the voltages and currents can be obtained

Intro to mesh current Analysis



Imaginary

$V_{R_0} = R_0 \times I_{R_0} = R_0 \times i_1$

$I_{R_2} = i_2$

$I_{R_1} = i_2 - i_1$

$I_{R_3} = i_3 - i_2$

- If currents in the meshes are known, then all the other currents and voltages can be obtained.

Steps of Node Voltage Analysis method

1. Select a reference node (Usually the ground of a voltage source.)

   ⊖ —ve of voltage source as ref.

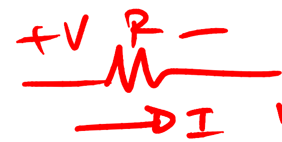
2. For each voltage source connected to the reference node, the other end is a known constant.

3. For all other voltage sources, one end is tied to the other. So only one unknown variable for each such voltage source.

Steps of Node Voltage Analysis method

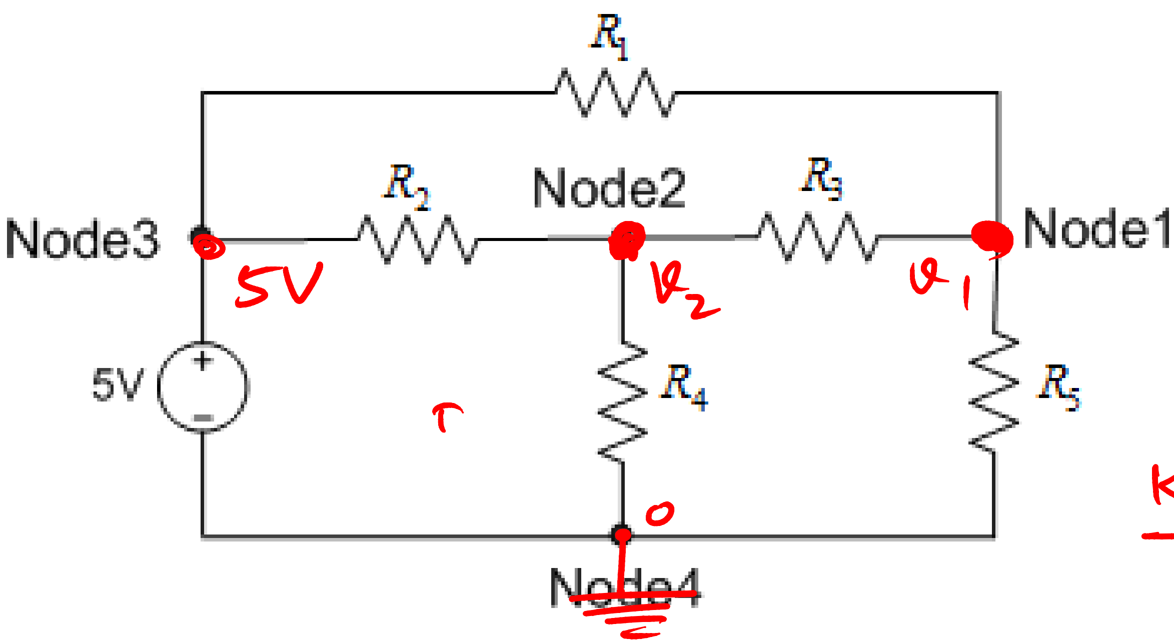
4. Define the remaining node voltages as unknown variables.
5. Apply KCL at the nodes, to obtain as many equations as the number of unknown variables.
6. Express each current in a resistive branch in terms of the adjacent node voltages.
7. Solve the linear system of equations.



$V = IR$

$i_{R_1} = \frac{v_1 - v_2}{R_1}$

Case 1: Node analysis with one Ideal Voltage Source



KCL at node 1

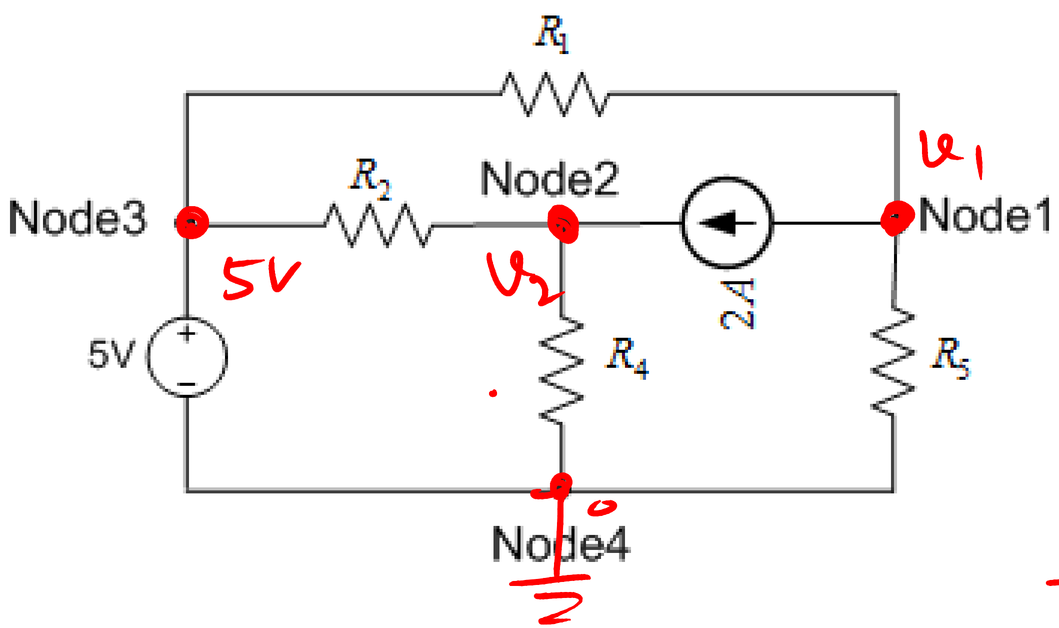
$$\frac{v_1-5}{R_1}+\frac{v_1-v_2}{R_3}+\frac{v_1}{R_5}=0 \quad \text{— (1)}$$

KCL at node 2

$$\frac{v_2-5}{R_2}+\frac{v_2-v_1}{R_3}+\frac{v_2}{R_4}=0 \quad \text{— (2)}$$

- Choose reference node (GND)
- Identify unknown node voltages as variables
- Express each current in a resistive branch in terms of the adjacent node voltages.

Case 2: Having an additional Ideal Current source in the circuit



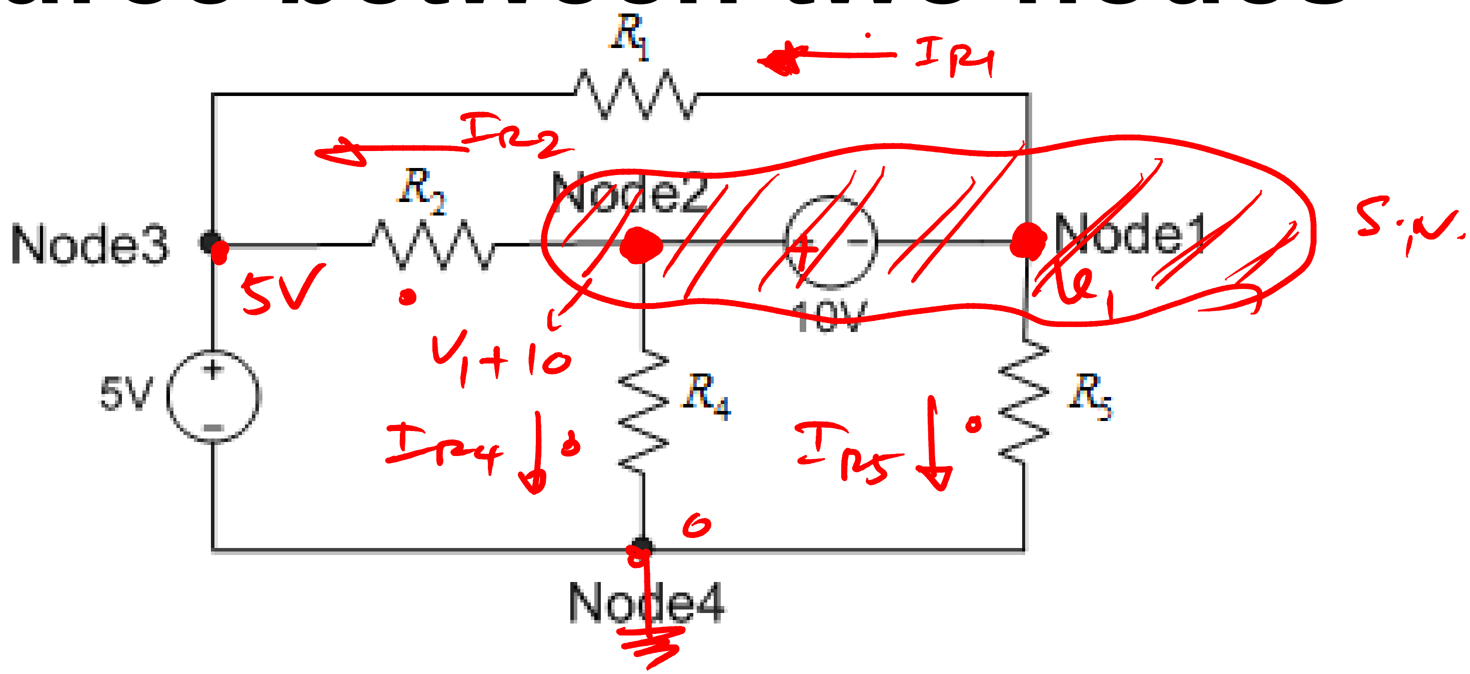
KCL at node 1

$$\frac{v_1-5}{R_1} + \frac{v_1}{R_5} + 2 = 0 \quad \text{— (1)}$$

KCL at node 2

$$\frac{v_2-5}{R_2} + \frac{v_2}{R_4} - 2 = 0 \quad \text{— (2)}$$

Case 3: With an Ideal Voltage source between two nodes



- Out of the two nodes across the voltage source, only one node voltage is an unknown variable.

$$\frac{v_1 - 5}{R_1} + \frac{v_1 + 10 - 5}{R_2} + \frac{v_1 + 10}{R_4} + \frac{v_1}{R_5} = 0 \quad (1)$$