Contents

Magnetic Circuits, Transformers and DC Power Supply

Magnetic Field

Magnetic fields are produced due to the movement of electrical charge. They exist around permanent magnets where the fields due to spinning electrons in atoms aid each other. The magnetic field is created by current-carrying wire, which is greatly intensified if the wire is wound in a multi-turn coil form around an iron core.

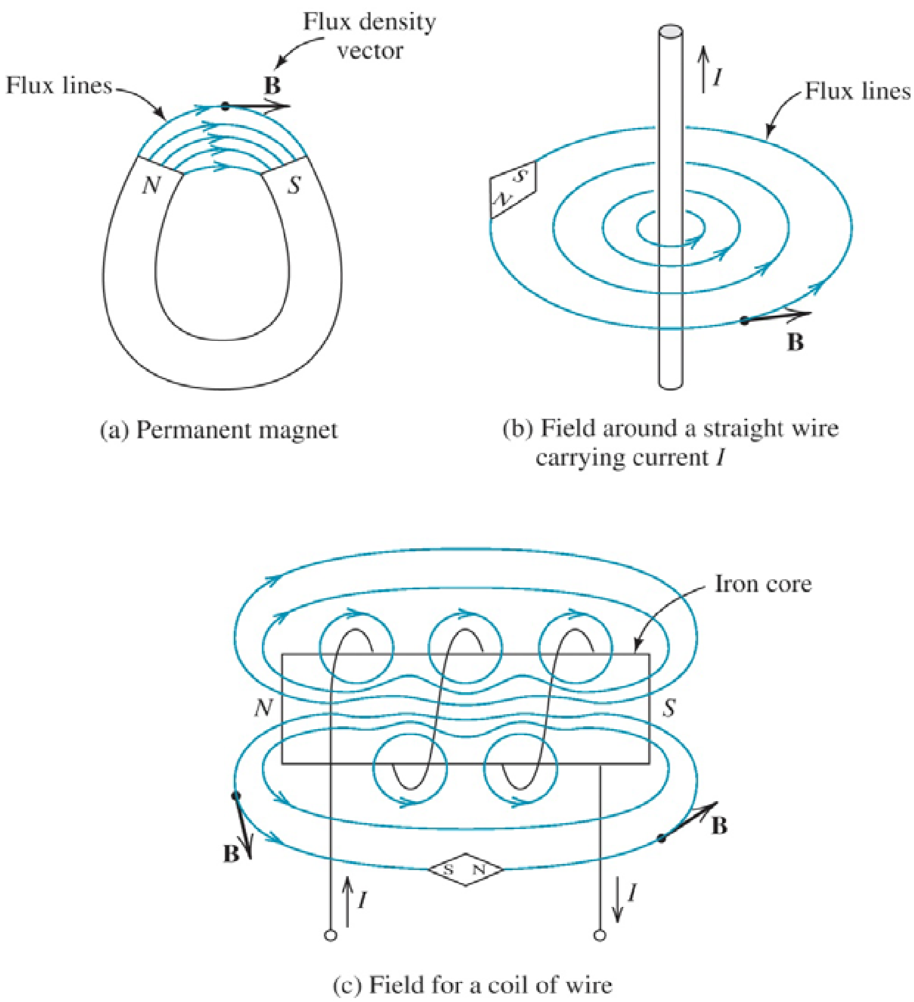


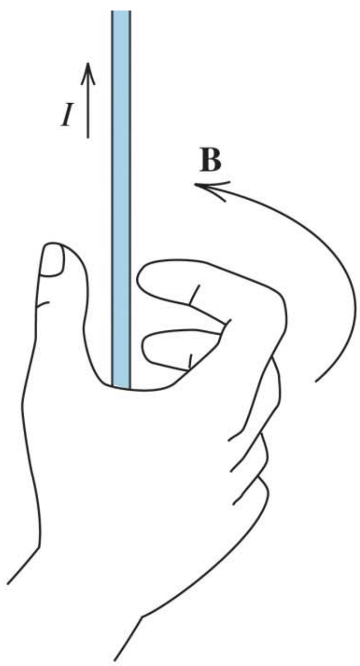
Figure Magnetic fields created by (a) permanent magnet, (b) single-conductor carrying current and (c) coil of wire wound on an iron core.

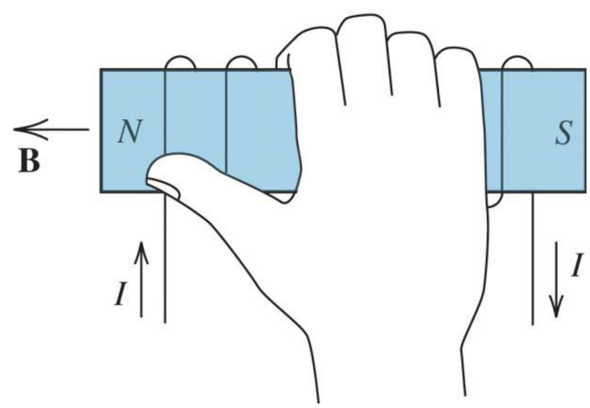
Magnetic field can be visualized as line of magnetic flux that form closed paths. By convention, magnetic flux lines start from the North pole and end in the South pole.

The magnetic field forms the essential intermediate link between the energy conversion from electrical to mechanical or vice-versa in electrical machines. The magnetic fields form the basis for the operation of transformers, electrical motors and generators.

Direction of Magnetic field

Right-hand Rule: If the current carrying conductor is grabbed by right hand with the thumb in the direction of current, then the finger encircling the wire also point in the direction of the magnetic field. Alternatively, if finger are wrapped around a coil in the direction of current flow, then the thumb points in the direction of the magnetic field inside the coil.





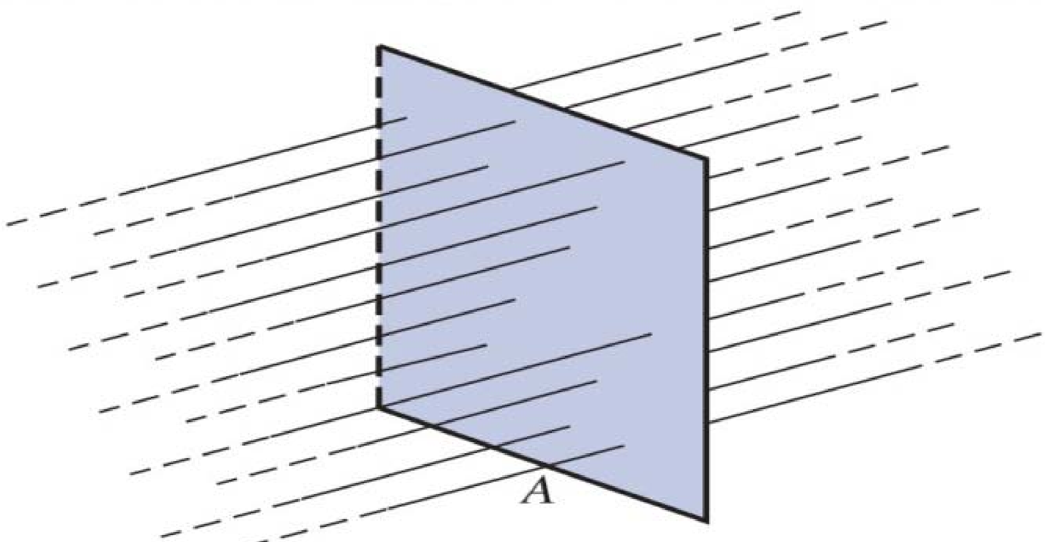
(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Magnetic Flux, Flux density and Flux-linkage

Magnetic flux passing through a surface inside the magnetic field is denoted by ϕ. The unit of flux is *webers* (Wb).

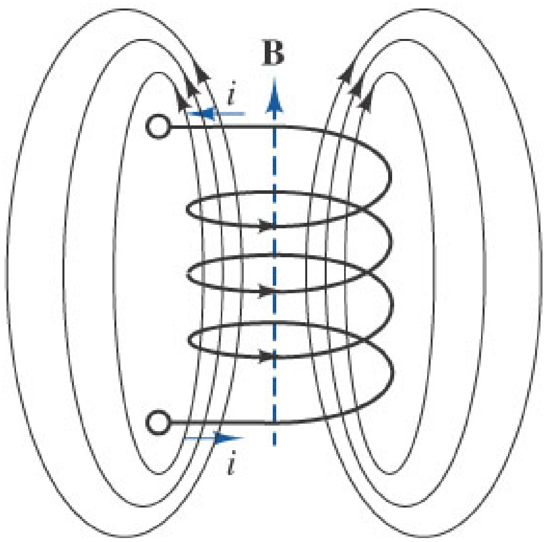
The **magnetic flux density** is defined by $B = \frac{\phi}{A}$ where A is the area of the surface perpendicular to the flux lines. It is a vector and has units as $webers/m^2$ or *tesla* (T).



It is said that the flux passing through the surface bounded by the coil **links** the coil.

If the coil has N turns, the total **flux linkages** are given by $\lambda = N\phi$.

The unit of flux-linkages is *weber-turns.*



Flux lines

Faraday's Law of Magnetic Induction

A voltage is induced in a coil whenever the flux-linkage is changing. $e = \frac{d\lambda}{dt}$

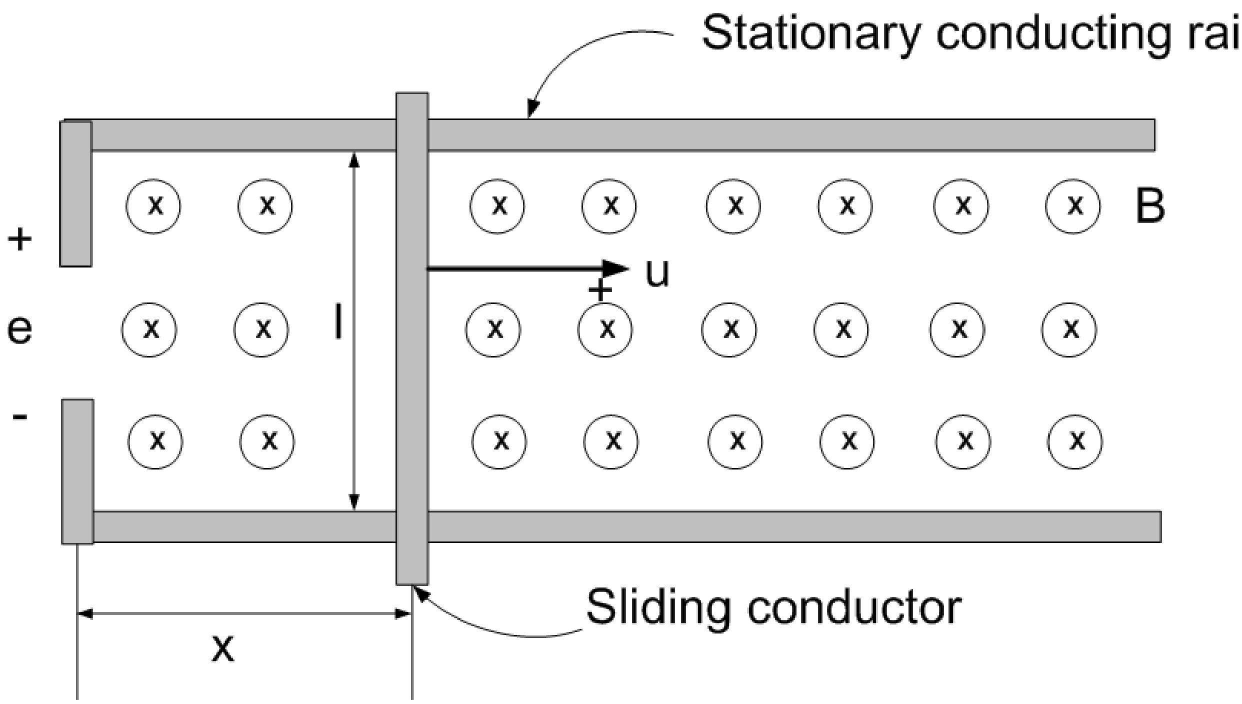
This can occur either when the magnetic field is changing with time or when the coil is moving inside the magnetic field.

The polarity of the induced voltage will be such that, when a resistor is connected across the coil, a current would flow in the coil to oppose the original change in flux linkages. This is known as **Lenz's law**.

Voltage induced by field cutting conductors

When a conductor moves inside a uniform magnetic field, as shown in the figure, a voltage will be induced in it. The magnitude of the voltage can be obtained by imagining a coil made up of the moving conductor and two conducting rails on both ends of the conductor. As the conductor moves, the area of this imaginary coil will be changing and hence the flux will be changing. From Faraday's law:

$$e = \frac{d\lambda}{dt} = \frac{d\phi}{dt} = \frac{d}{dt}(B.A) = \frac{d}{dt}(B.l.x) = Bl\frac{dx}{dt} = Blu$$



Voltage induced by time-varying flux

If the flux linking a coil is changing with time, e.g. $\phi = \phi_m \sin \omega t$, then the voltage induced will be:

$$e = \frac{d\lambda}{dt} = \frac{d}{dt}(N\phi) = \frac{d}{dt}(N\phi_m \sin \omega t) = N\omega\phi_m \cos \omega t \ .$$

Magnetic Field Intensity

Magnetic fields are set up by currents flowing in coils. Magnetic flux density **B** depends on **magnetic field intensity H**,

$B = \mu H$ where μ is the magnetic permeability of the material. The units of H are amperes/meter (A/m), and the units of μ are webers/ampere-meter (Wb/Am).

For free space, $\mu_0 = 4\pi \times 10^{-7}$ Wb/Am

For other materials, $\mu = \mu_r \mu_0$

μ_r is the relative permeability of the material. Materials like iron have high magnetic permeability.

Ampere's Law

Amperes' law states that the line integral of the magnetic field intensity around a closed path is equal to the algebraic sum of the current flowing through the area enclosed by the path.

$$\oint H.dl = \sum i$$

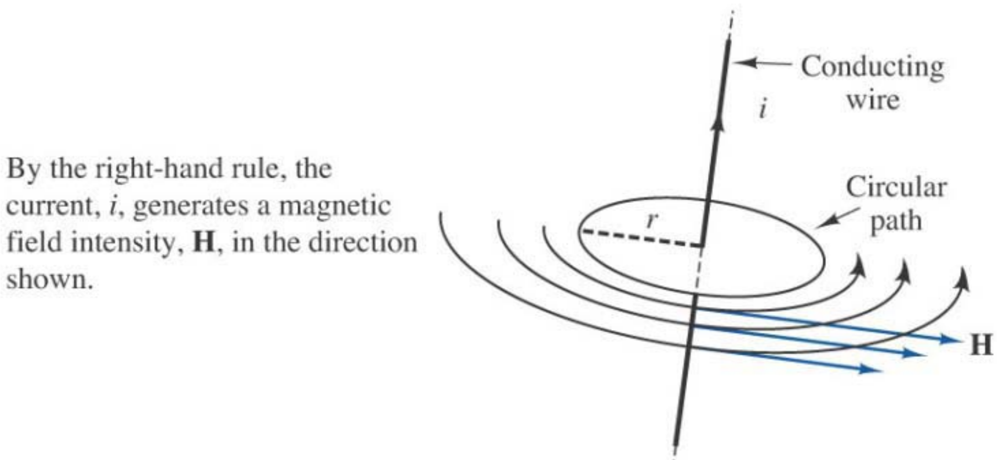
If H is constant around the path, then $\oint H.dl = H\oint dl = Hl$.

$$Hl = \sum i$$

Magnetic field around a long straight wire

Consider a long straight wire carrying current *i* out of the page of the slide as shown below. The magnetic field along a circular path of radius, *r* surrounding the wire will be constant:

$Hl = \sum i \Rightarrow H \times 2\pi r = i \Rightarrow H = \dfrac{i}{2\pi r}$. We can then calculate the magnetic flux density around the wire as $B = \mu_0 H = \dfrac{\mu_0 i}{2\pi r}$



Magnetic field in a Toroidal core

A wire is a wound around a toroidal core into a coil with N turns, symmetrically distributed around the core. If $R >> r$, the magnetic flux lines will be uniformly spaced around core i.e. the flux density is constant throughout the core. Thus, the magnetic field intensity is constant around the core.

The magnetic field intensity inside core can be found from Ampere's law, applying around a closed path at the center of the core:

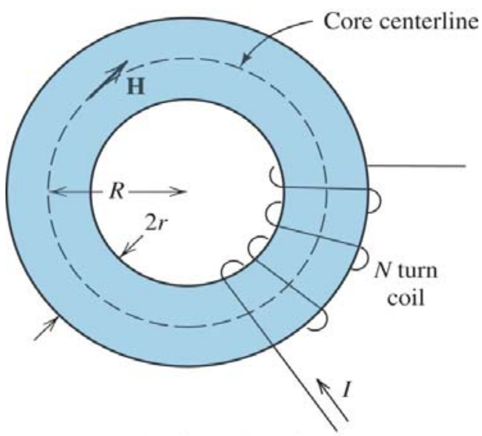
$$Hl = \sum i \Rightarrow H \times 2\pi R = NI \Rightarrow H = \frac{NI}{2\pi R}$$

$$B = \mu H = \frac{\mu NI}{2\pi R}$$

$$\phi = BA = \frac{\mu NI}{2\pi R}\pi r^2 = \frac{\mu NIr^2}{2R}$$

$$\lambda = N\phi = \frac{\mu N^2 Ir^2}{2R}$$

Thus, the flux created per unit current, is increased if a wire is wound into a coil around a core made of magnetic material.



Magnetic Circuits

Many useful devices like transformers, motors and generators contain coils wound on iron cores. Calculating the magnetic field in these devices where the cores are of asymmetrical shapes require the concept of magnetic circuits, analogous to the analysis of electrical circuits.

First, we assume that there exists a **mean path** for the magnetic flux and the flux density is approximately constant over the cross-sectional area of the magnetic structure.

$$B = \frac{\phi}{A} \Rightarrow H = \frac{B}{\mu} = \frac{\phi}{\mu A}$$

Then, we can relate to the Ampere's law as $N \cdot i = H \cdot l$ where l represents length of the magnetic mean path.

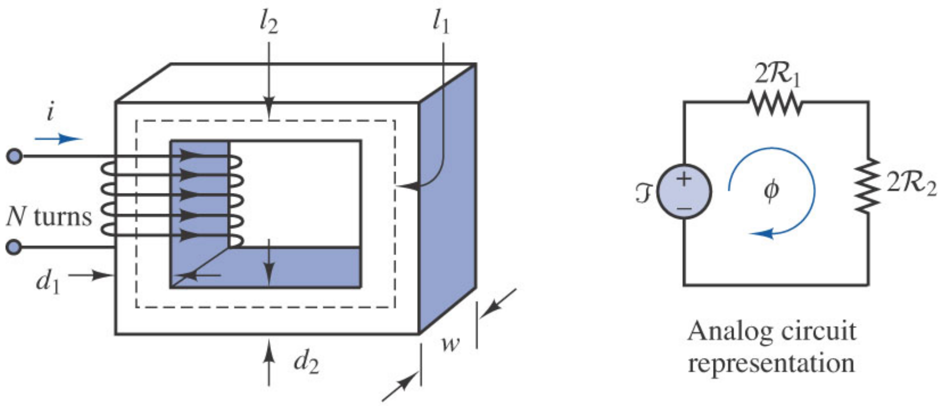
$$N \cdot i = H \cdot l = \frac{\phi}{\mu A} \cdot l = \phi \cdot \frac{l}{\mu A}$$

In magnetic circuits, a current carrying coil is analogous to the voltage source and we define **magneto-motive-force** $\Im = Ni$ having units equal to A-turns.

Magnetic flux ϕ in the core is analogous to the current in electrical circuit.

Analogous to resistance in electrical circuits, there is a term called **reluctance** associated with the magnetic path defined as $\Re = \dfrac{l}{\mu A}$ where l is the length of the mean path, A is the cross-sectional area and μ is the permeability of the material.

Then, for magnetic circuits, $\phi = \dfrac{\Im}{\Re}$. Similar to $I = \dfrac{V}{R}$.



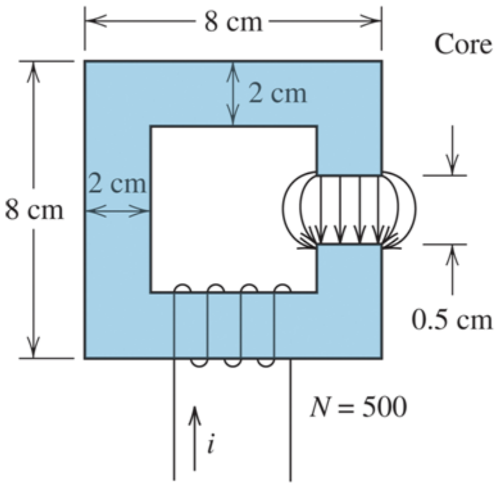
A magnetic structure excited by a magnetomotive force $\mathcal{F} = Ni$

Analog circuit representation

- A complex magnetic circuit with multiple coils and branches in the magnetic core, is akin to a resistive circuit with multiple voltage sources and resistive branches.

Flux fringing in the air gap

The flux-lines tend to bow out in the air-gap as shown in the figure. This is called fringing. Thus, the effective area of the air-gap is larger than that of the iron core. We take this into account **by adding the length of the air-gap to each of the dimensions** of the air-gap cross section.

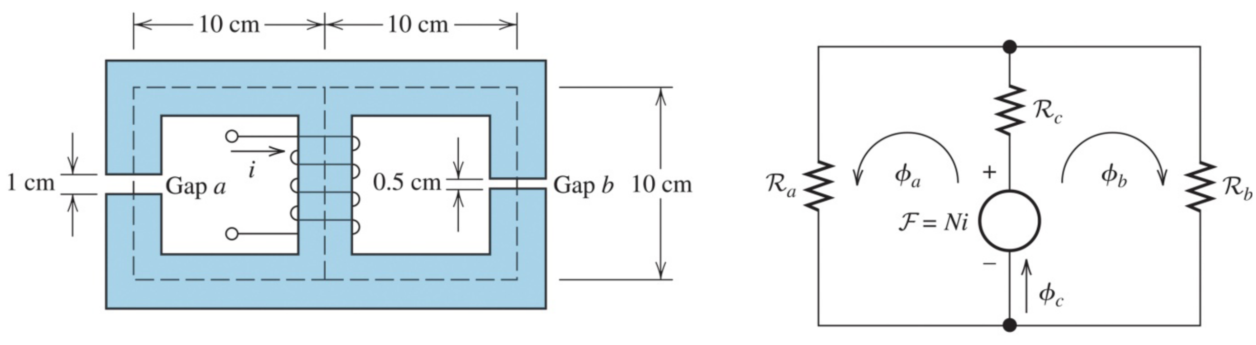


(a) Iron core with an air gap

$$R_{gap} = \frac{l_{gap}}{\mu_0 A_{gap}} = \frac{0.5\times 10^{-2}}{4\pi\times 10^{-2}\times(2+1)\times(2+1)\times 10^{-4}} = 8.842\times 10^{6}\,A.turns/Wb$$

Example:

The iron core as shown in Fig. 1.10 has a cross section of 2cm by 2 cm and a relative permeability of 1000. The coil has 500 turns and carries a current of 2 A. Determine the flux-linkage for the coil.



(a) Core

(b) Magnetic circuit

Reluctance of the air gap in the left arm (considering fringing effect):

$$R_{gap} = \frac{l_{gap}}{\mu_0 A_{gap}} = \frac{1\times 10^{-2}}{4\pi\times 10^{-7}\times(2+1)\times(2+1)\times 10^{-4}} = 8.842\times 10^{6}\,A.turns/Wb$$

Reluctance of the core in the left arm:

$$R_{core}=\frac{l_{mean}}{\mu_r\mu_0 A_{core}}=\frac{(10+9+10)\times10^{-2}}{1000\times4\pi\times10^{-7}\times2\times2\times10^{-4}}=576.937\times10^{3}\,A.turns/Wb$$

$$R_a=8.842\times10^{6}+576.937\times10^{3}=9.419\times10^{6}\,A.turns/Wb$$

Reluctance of the air gap in the right arm (considering fringing effect):

$$R_{gap}=\frac{l_{gap}}{\mu_0 A_{gap}}=\frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times(2+0.5)\times(2+0.5)\times10^{-4}}=6.366\times10^{6}\,A.turns/Wb$$

Reluctance of the core in the right arm:

$$R_{core}=\frac{l_{mean}}{\mu_r\mu_0 A_{core}}=\frac{(10+9.5+10)\times10^{-2}}{1000\times4\pi\times10^{-7}\times2\times2\times10^{-4}}=586.88\times10^{3}\,A.turns/Wb$$

$$R_b=6.366\times10^{6}+586.88\times10^{3}=6.953\times10^{6}\,A.turns/Wb$$

Reluctance of the core in the center:

$$R_C=\frac{l_{mean}}{\mu_r\mu_0 A_{core}}=\frac{10\times10^{-2}}{1000\times4\pi\times10^{-7}\times2\times2\times10^{-4}}=198.943\times10^{3}\,A.turns/Wb$$

$$R_a=9.419\times10^{6}\,A.turns/Wb$$

$$R_b=6.953\times10^{6}\,A.turns/Wb$$

$$R_c=0.199\times10^{6}\,A.turns/Wb$$

$$R_a\,||\,R_b=\frac{R_a\times R_b}{R_a+R_b}=\frac{9.419\times6.953}{9.419+6.953}=4\times10^{6}\,A.turns/Wb$$

$$\phi_c=\frac{Ni}{R_c+R_a\,||\,R_b}=\frac{500\times2}{0.199+4}\times10^{6}=0.238\times10^{-3}Wb$$

$$\lambda_c=N\times\phi_c=500\times0.238\times10^{-3}=0.119Wb.turns$$

Self and Mutual Inductance

Whenever a coil carries a current, magnetic flux will be produced that links the coil. The flux-linkage λ in the coil per unit current is called its inductance i.e.

$$L=\frac{\lambda}{i}=\frac{N\phi}{i}=\frac{N}{i}\frac{Ni}{\Re}=\frac{N^2}{\Re}$$

This is also called self-inductance.

$$e=\frac{d\lambda}{dt}=\frac{d(Li)}{dt}=L\frac{di}{dt}$$

When the coil is in the vicinity of another coil (like two coils wound on the same core), then the flux produced due to current in one winding will link the second winding as well.

Flux-linkage of coil 1 caused by current in coil 1: $\lambda_{11} = L_1 i_1$

Flux-linkage of coil 1 caused by current in coil 2: $\lambda_{12} = M i_2$

Flux-linkage of coil 2 caused by current in coil 1: $\lambda_{21} = M i_1$

Flux-linkage of coil 2 caused by current in coil 2: $\lambda_{22} = L i_2$

The total flux-linkage for the coils are

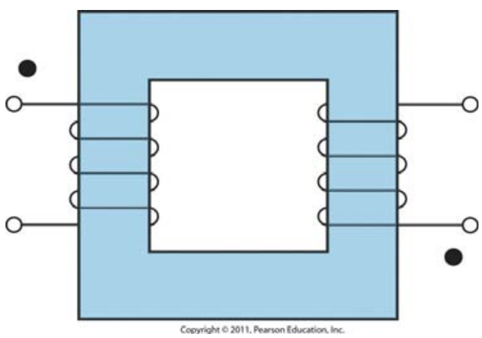
$\lambda_1 = \lambda_{11} \pm \lambda_{12} = L_1 i_1 \pm M i_2$

$\lambda_2 = \pm \lambda_{21} + \lambda_{22} = \pm M i_2 + L_2 i_2$

The sign is +ve when flux due to the coils aid each other and is –ve when the flux oppose each other.

Dot convention

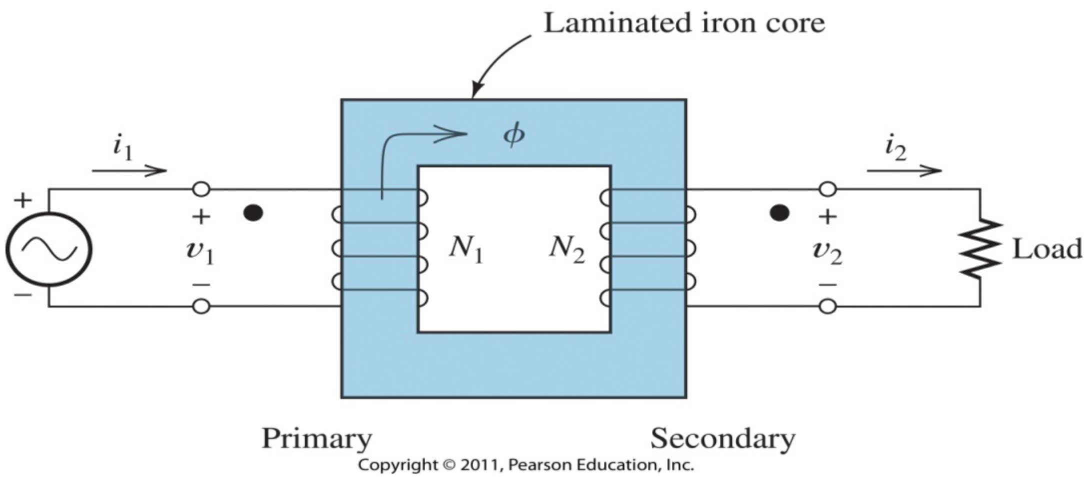
A dot is placed on one end of each coil in the circuit diagram to indicate how the coils interact. When the currents enter the dotted terminal they produce aiding flux. When current enters into the dotted terminal in one coil but exits the dotted terminal of the other coil then the flux will be opposing.



Ideal Transformers

A transformer is used to step up or step down the AC voltages. In transporting of electric power over large voltages, the voltage is stepped up to very large values. This reduces the current and the I^2R losses for transmission will be reduced.

Transformer consists of two or more coils wound on a common core. When an AC voltage is connected to one coil, an AC current will flow. This will create a time-varying flux in the core. The other coil will have time-varying flux-linkage and according to Faraday's law, a voltage will be induced in the second coil.



Voltage ratio

The voltage connected to the first coil: $v_1(t) = V_{1m}\cos(\omega t)$

The time-varying flux established in the core be $\phi(t)$, then $v_1(t) = N_1 \frac{d\phi}{dt}$

$$N_1 \frac{d\phi}{dt} = V_{1m}\cos(\omega t) \Rightarrow \phi(t) = \frac{1}{N_1}\int V_{1m}\cos(\omega t)dt = \frac{V_{1m}}{N_1\omega}\sin(\omega t)$$

Voltage induced in the second coil:

$$v_2(t) = N_2\frac{d\phi}{dt} = N_2\frac{d}{dt}\left(\frac{V_{1m}}{N_1\omega}\sin(\omega t)\right) = N_2\frac{V_{1m}}{N_1\omega}\omega\cos(\omega t) = \frac{N_2}{N_1}V_{1m}\cos(\omega t)$$

So $v_2(t) = \frac{N_2}{N_1}v_1(t)$

Also, $V_{2,RMS} = \frac{N_2}{N_1}V_{1,RMS}$

Voltage across each coil is proportional to its number of turns.

- Polarities of the voltages at the doted terminals are same.

Current ratio

It can be noted that the current enters the dotted terminal in coil 1 but leaves the dotted terminal in coil 2. Thus, they create opposing magnetic fields.

In an ideal transformer, the reluctance of the core is 0 i.e. mmf required to establish a flux in the core is zero.

Net mmf in the core will be i.e. $N_1 i_1(t) - N_2 i_2(t) = 0 \Rightarrow \dfrac{i_1(t)}{i_2(t)} = \dfrac{N_2}{N_1}$

Similarly, $I_{2,RMS} = \dfrac{N_1}{N_2} I_{1,RMS}$.

Power in Ideal Transformers

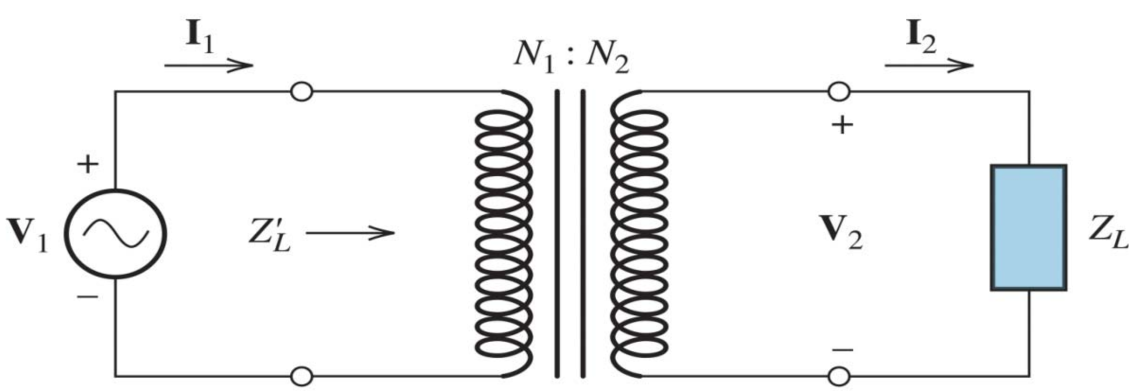
$$p_2(t) = v_2(t) \times i_2(t) = \frac{N_2}{N_1} \times v_1(t) \times i_1(t) \times \frac{N_1}{N_2} = v_1(t) \times i_1(t) = p_1(t)$$

Thus, in an ideal transformer, the power received from the input is delivered to the secondary side. Only the voltage and current levels change, while the power remains the same.

Impedance Transformation

A load connected on secondary side can be represented by connecting an equivalent impedance to the primary side.

This represents the load impedance being seen from the primary side. In both the representations, the primary current will be same.



$$I_2 = \frac{V_2}{Z_L}$$

$$I_1 = \frac{N_2}{N_1} I_2 = \frac{N_2}{N_1}\frac{V_2}{Z_L} = \frac{N_2}{N_1} V_1 \times \frac{N_2}{N_1} \times \frac{1}{Z_L} = \left(\frac{N_2}{N_1}\right)^2 \times \frac{V_1}{Z_L} = \frac{V_1}{Z_L^{'}}$$

$$Z_L^{'} = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

Diodes, Rectifiers and DC Power Supplies

Introduction

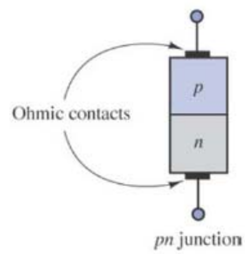
Diodes are semiconductor devices those play important roles in various electronic as well as electrical circuits. Semiconductor diodes can be considered as *non-linear circuit elements* with *non-linear voltage-current characteristics*. The diodes are used in power applications to regulate the flow of electrical power only in one direction while blocking the power flow in the reverse direction and are commonly exploited in various power related applications. The other major application of diodes is in rectifier circuits to convert AC power into regulated DC power.

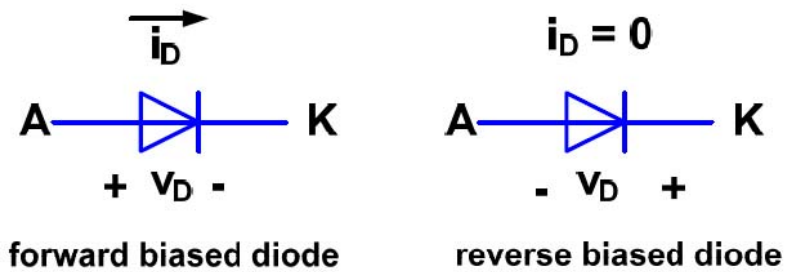
In this chapter, we would discuss about diodes, their basic principles of operation, the voltage-current characteristics and their applications in diode bridge rectifier circuits for DC power supplies. We would also discuss *voltage ripples* and how capacitive filter circuit can be used to regulate the dc voltage and reduce the peak-to-peak ripple voltages at the output.

Diode

Diodes allow current to flow through *only in one direction* and it can be considered to be the electrical counterpart of mechanical valves those used to regulate the flow of fluid through the valves.

The diode is a two terminal semiconductor device. When the **anode** (A) terminal is made positive potential with respect to the **cathode** (K) terminal, we say that the diode is *forward biased* and *conducts* i.e. current flows from the anode to the cathode. The diode almost acts like short-circuit with a *forward voltage drop* of 0.7 V for silicon based diodes. Alternatively, if the anode is made –ve potential with respect to the cathode terminal then we say that the diode is *reverse biased* and does not conduct i.e. no current flows through the diode. The diode almost acts like an open-circuit under this mode of operation.





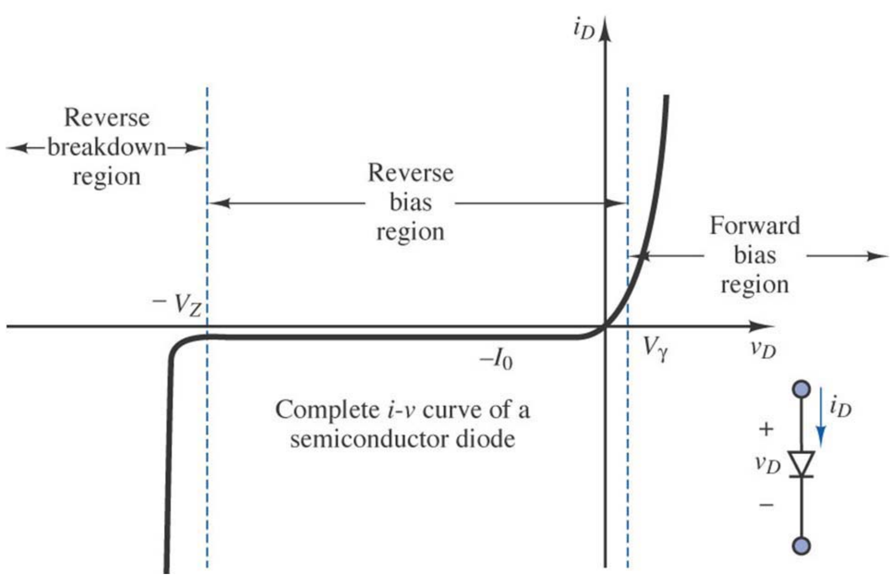


Figure : The terminal voltage-current (V-I) characteristic of a diode

When the anode is made –ve potential w.r.t. the cathode terminal, we say that the diode is reverse biased and does not conduct. A very small amount of current, I_0 flows through the diode and is called as leakage current of the order of few hundreds of μA. As the anode-to-cathode voltage, v_{AK} is increased beyond the *reverse breakdown voltage* the diode breaks down and large amount of current starts to flow through the diode. Normally, the diode is not supposed to be operated under this mode of operation. The diode's normal regions of operation are*: forward conducting region* or *reverse blocking region*.

Forward Voltage Drop

The diode uses very little amount of electrical power to regulate the flow of large amount of electrical power. The forward voltage drop, v_D is typically about 0.7 V for silicon based signal diodes and is maintained constant. However, for power diodes, v_D is a function of the current through the diode, i_D and is typically of the order of 1 – 2 V for power diodes.

Reverse Breakdown Voltage

We know that when the diode is reverse biased it does not conduct but there is a maximum reverse voltage, V_Z that can be applied to a diode safely without causing any damage to the diode. This *peak inverse voltage* (PIV) that can be safely applied to the diode is as shown in Fig. 2.2. If this PIV is increased beyond the maximum allowable limit then the diode fails and large current flows in the reverse direction. This maximum voltage is called the reverse breakdown voltage and can be typically of the order of 2000 V for power diodes.

Ideal Diode

In most of the applications the supply voltage is much larger than the voltage drop across the diode during conducting state and therefore can be neglected.

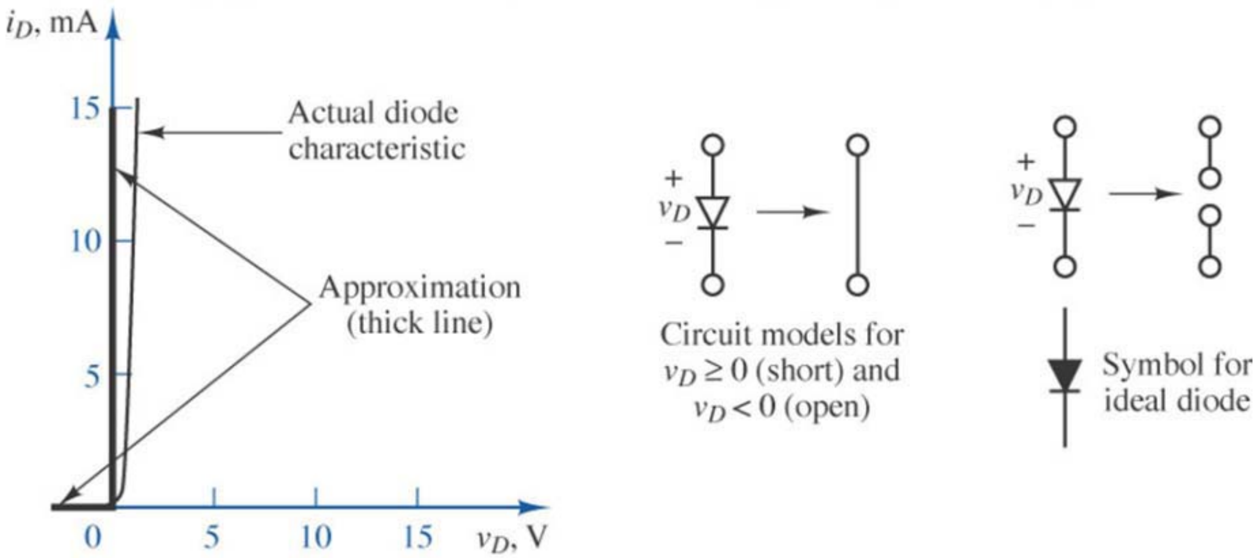
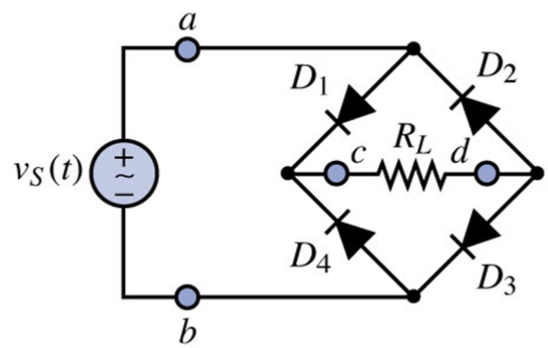


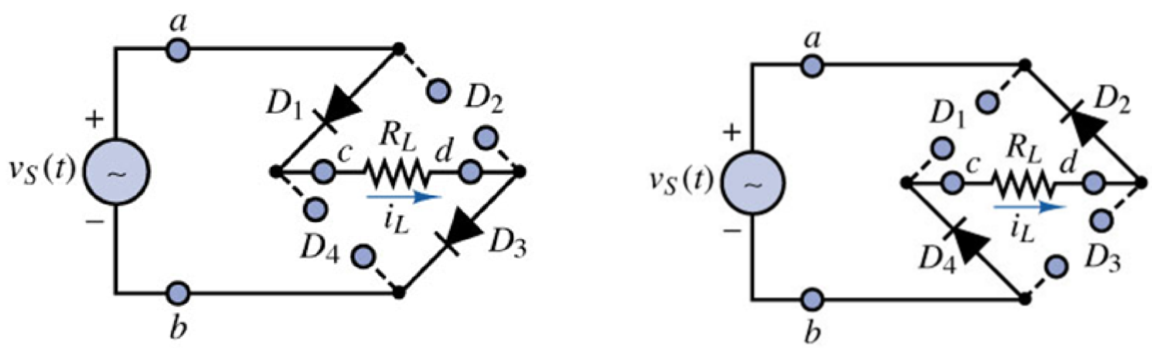
Figure: The V-I characteristics of an ideal diode (approximate thick lines)

Diode Bridge Rectifier

One of the important applications of power diodes is in rectifiers, in which AC power is converted into DC power and widely used as DC power supplies.



Bridge rectifier



Full-wave bridge type of rectifier circuit

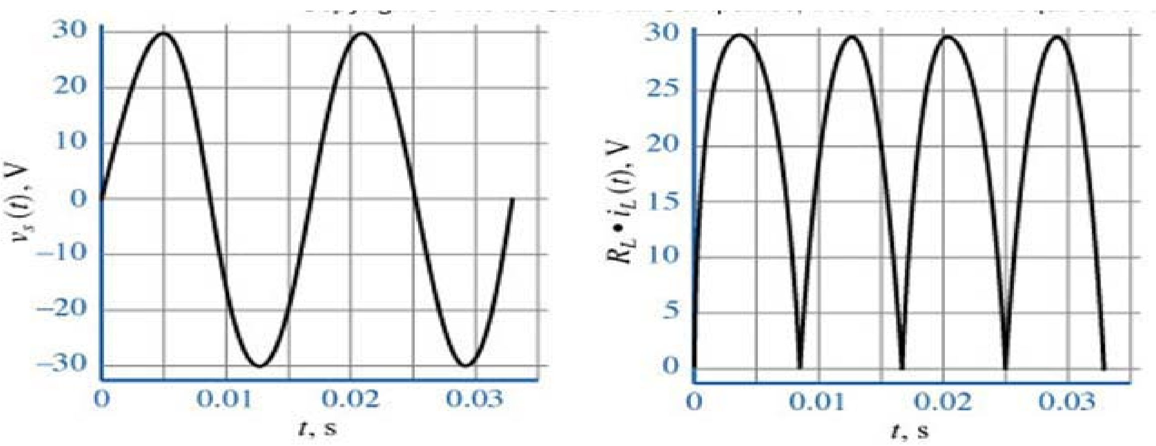
Basic principle of operation

During the +ve half-cycle of AC source voltage, $v_S(t)$ the diodes D_1 and D_3 conduct while during the –ve half-cycle the diode pair D_2 and D_4 conduct. Due to the structure of the bridge, the current flow through the load is always unidirectional and from terminal *c* to *d* during both the half-cycles and thus, the full-wave rectification of the AC source waveform. The output voltage is same during both positive and negative halves of the source voltage.

Let the source voltage be a sinusoidal given as $v_s(t) = V_m \sin \omega t$

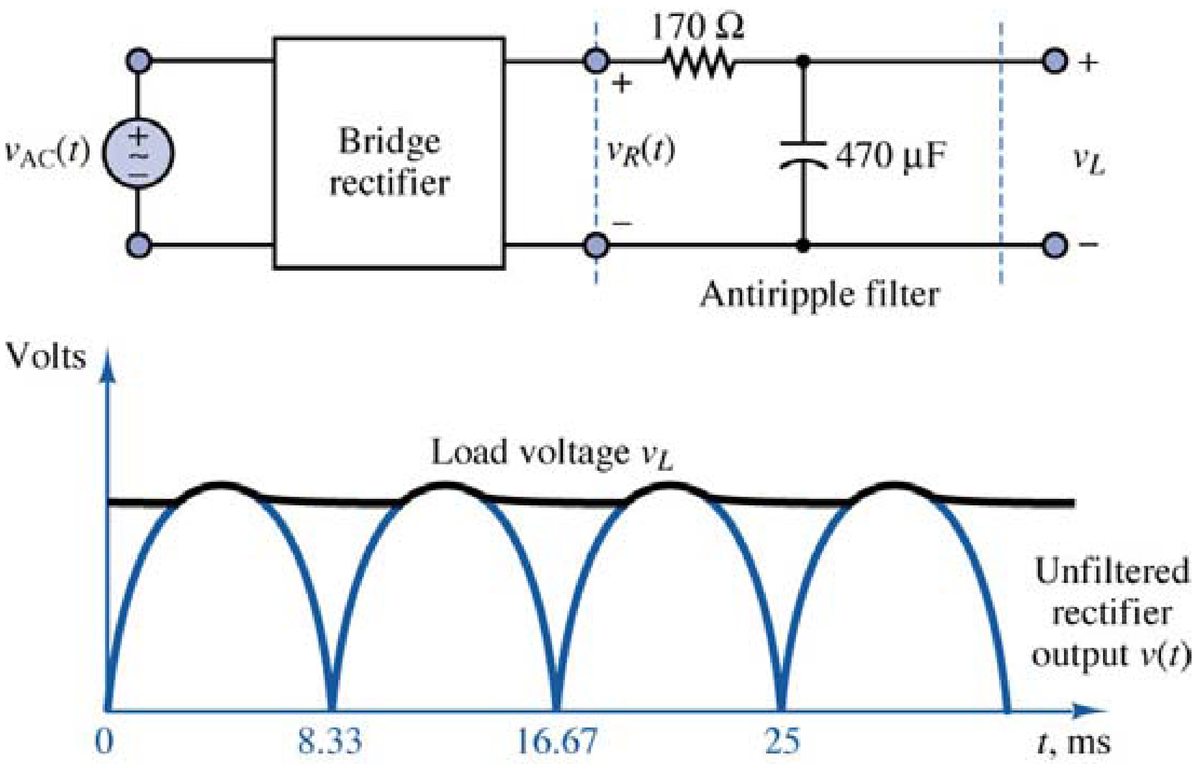
The average value of the output voltage will be:

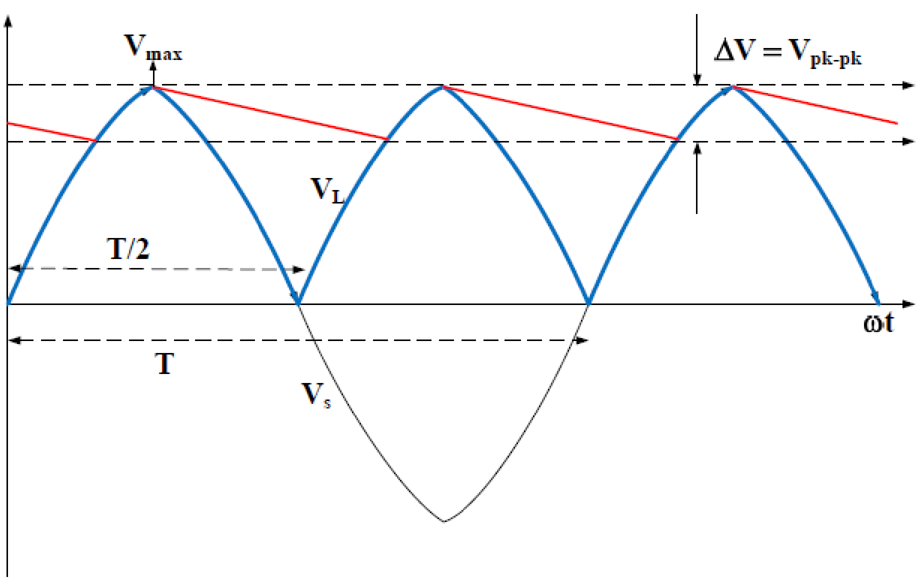
$$V_o = \frac{1}{2\pi}\int_0^{2\pi} v_s(t)d\omega t = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t d\omega t = \frac{1}{2\pi}\times 2\times\int_0^{\pi} V_m \sin\omega t d\omega t = \frac{V_m}{\pi}\left(-\cos\omega t\big|_0^{\pi}\right) = \frac{2V_m}{\pi}$$



(a) (b)

Although the full-wave bridge rectifier circuit produces a DC output voltage from an AC source voltage, the output voltage is not smooth and rather fluctuating, whereas, it is necessary to have a smooth DC output voltage. The *ripple* i.e. the fluctuation in the output voltage is the characteristics of a rectifier circuit and is undesirable. The ripple frequency of the full-wave rectifier circuit is $2f_s$ where f_s is the frequency of the source voltage, $v_s(t)$. This is because we have two pulses in the output voltage in one electrical cycle of the supply voltage.


170 Ω
$v_{AC}(t)$
Bridge rectifier
$v_R(t)$
470 µF
v_L
Antiripple filter
Volts
Load voltage v_L
Unfiltered rectifier output $v(t)$
0
8.33
16.67
25
t, ms


V_{max}
$\Delta V = V_{pk\text{-}pk}$
V_L
T/2
T
V_s
ωt

A simple yet effective way to eliminate the ripples in the output voltage is to connect a large capacitor at the output of the bridge rectifier. The capacitor holds the output voltage to a value close to the peak value of the voltage source.

Energy is transferred from the source to the capacitor when the source voltage value is more than the capacitor voltage. Thus, the capacitor is charged only for a short time near the peak of the source voltage. For the rest of the half-cycle, charge is removed from the capacitor through the load resistor. The load current can be taken to be constant.

Thus we have the peak to peak voltage ripple:

$$\Delta V_{pp} = \frac{\Delta Q}{C} = \frac{I_L \times (T/2)}{C}$$

$$I_L = \frac{V_o}{R_L}, T = \frac{1}{f_s} \Rightarrow \Delta V_{pp} = \frac{V_o}{R_L} \times \frac{1}{2 f_s} \times \frac{1}{C}$$

$$\frac{\Delta V_{pp}}{V_o} = \frac{1}{2 f_s R_L C}$$

Typically DC power supply would have a ripple voltage of less than 1%.

Example:

A DC power-supply circuit is needed to deliver 0.1 A with an average voltage of 15 V. The ac source has a frequency of 50 Hz. Assume that a bridge rectifier circuit is used. The peak-to-peak ripple voltage is to be less than 0.4 V. Assume that diodes are ideal, with no forward voltage drop. Determine the value of the smoothing capacitor.

Solution:

The average output voltage $V_o = 15V$, Load current $I_L = 0.1A = \frac{V_o}{R_L}$

Peak-to-peak voltage ripple $\Delta V = 0.4V$

$$\frac{\Delta V_{pp}}{V_o} = \frac{1}{2 f_s R_L C}$$

$$C = \frac{V_o}{R_L} \frac{1}{2 f_s \Delta V_{pp}}$$

Required filter capacitor $C = 0.1 \times \dfrac{1}{2 \times 50 \times 0.4} = 2500 \mu F$

DC Power Supplies and Voltage Regulation

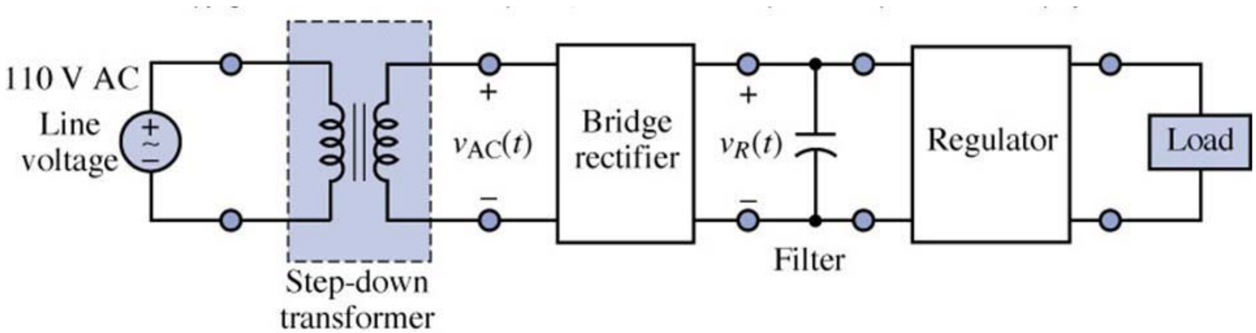


Figure: Regulated DC power supply.

The main application of the rectifier circuit is the conversion of AC to DC power. A circuit that accomplishes this function is called a **DC power supply**. A typical DC power supply consists of components such as step-down transformer, bridge rectifier, filter and voltage regulator. The rectifier output voltage is close to the value of the peak of the source voltage. A voltage regulator is used to maintain the output voltage constant at the desired value irrespective of the voltage variations from the source side or load side. Finally the load is connected at the output. In your project, you would be using the IC7805 as the voltage regulator to provide 5 V DC to the electronic circuit.