Contents

Fundamental SI units

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	Kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Luminous intensity	Candela	cd

Prefixes for units

Prefix	Abbreviation	Scale Factor (as power of 10)
giga-	G	9
meg- or mega-	M	6
kilo-	k	3
milli-	m	-3
micro-	µ	-6
nano-	n	-9
pico-	p	-12
femto-	f	-15

Fundamentals of Electric Circuits

Learning Objectives

1. Be familiar with some electrical quantities like charge (electron), voltage, current and power
2. Learn about Voltage Source and Current Source.
3. Apply the Passive Sign Convention.
4. Learn about Resistance, Resistor and Ohm's law.
5. Calculate the Power Dissipated by a Resistor.

Definitions

Charge

- The fundamental electrical quantity is Charge. The unit of charge is Coulomb (C).
- Smallest amount of charge that exists is the charge carried by an electron which equals to:

$$q_e = -1.602 \times 10^{-19} C$$

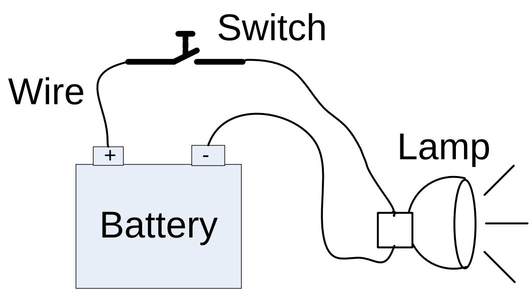
- Electrons are the most common charge carriers. In a conductor, a large number of electrons move under the influence of an electric field giving rise to current.
- The other charge carriers are holes (absence of electrons) or ions.

Circuit

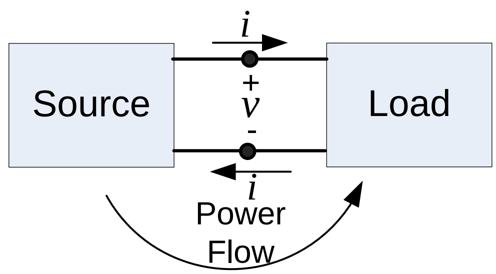
- In EE, a Circuit is a closed path where charge carriers move around, to carry useful information or energy from one place to another.

Example:

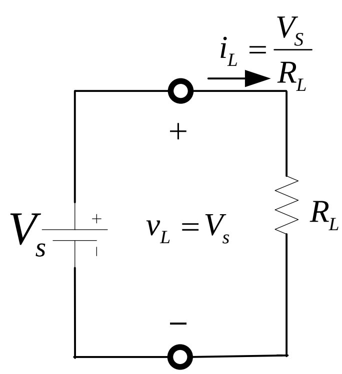
In a torch light, the circuit consists of the battery, the switch and the lamp along with the conductor wires. When switched is closed, the circuit is provides a closed path for the charge carriers to move energy from the battery to the lamp, where it is converted to light as well as some unwanted heat.



(a) Physical system



(b) Conceptual representation



(c) Symbolic representation

Electric Current

- Electrical current is the time rate of flow of electrical charge through a circuit.
- The SI unit current is ampere (A), which is equivalent of coulomb/sec (C/S). It is measured as the net flow of charge across a cross-section.

$$i(t) = \frac{dq(t)}{dt}$$

$$q(t) = q(t_0) + \int_{t_0}^{t} i(t)dt$$

where *i(t)* is the current in the conductor, $q(t_0)$ is the initial charge stored at time t_0 . Current in any branch of a circuit is specified by both magnitude and direction.

Direct current (DC) and Alternating current (AC):

- DC current has constant magnitude and direction.
- AC current reverses direction periodically. AC current has time average of zero.
- There can be current having both a DC component and an AC component.
- When current is neither constant with time, nor periodically reversing (i.e. it neither satisfies the definition of DC nor AC), then it is said to be in transient.

Double-subscript notation for currents

The two ends of a circuit element be named as *a* and *b*. Then the current in the element is either from *a* to *b* as $I_{ab,}$ or from *b* to *a* as I_{ba} and $I_{ab,}$ =- I_{ba}.

Voltage

- Voltage (Potential difference) is a measure of the energy transferred per unit charge when charge moves from one point in an electrical circuit to a second point.
- The units of voltage are volts (V), which is equivalent to Joules/Coulomb (J/C).
- Voltage is measured *across* an electrical element (i.e. between two ends of an element), where are current is a measure of the rate of charge flow *through* the element.
- Voltage has polarity (- and +). Positive terminal is at a higher energy level than the negative.

If positive charge enters the positive polarity and exits the negative polarity, then the charge looses energy in the element (the element absorbs energy). When positive charge enters the negative terminal and exits at the positive terminal, then the charge absorbs energy from the element.

Ideal Source

- An Ideal Current Source forces a specified Current to flow through itself, irrespective of the voltage across it.
- An Ideal Voltage Source maintains a specified Voltage across its terminals irrespective of the current flowing through it.

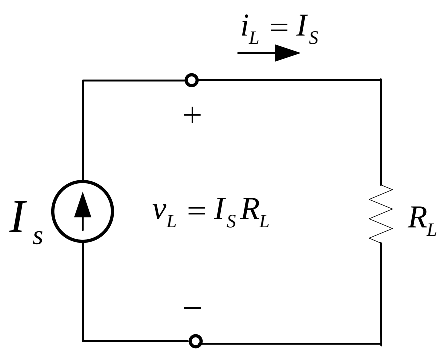


Fig. Ideal Current source

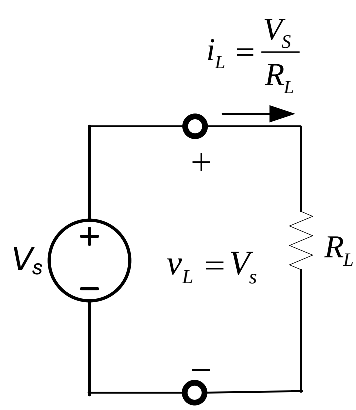


Fig. Ideal Voltage source

Dependent Source

- A dependent current source forces a certain current to flow from it, but the magnitude of the current is controlled by another element in the circuit.
- A dependent voltage source maintains a certain voltage across it, but the magnitude of the voltage is is controlled by another element in the circuit.

The magnitude of the dependent source depends on (i.e. a function of) the voltage or current (x) in another element in the circuit.

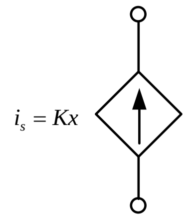


Fig. Symbol for dependent Current source

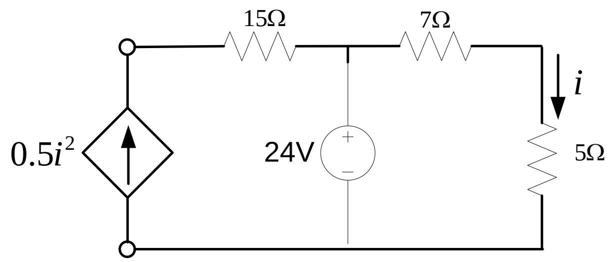


Fig. Circuit showing a Dependent Current Source

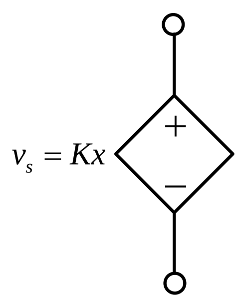
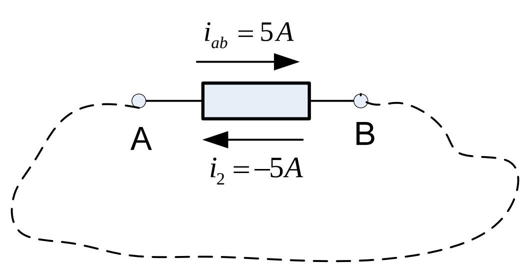


Fig. Symbol for dependent Voltage Source

Reference direction for current:

In analyzing circuits, before actual currents are known, we assign reference direction for current in each branch. After solving the circuit, the current value may be negative or positive. If positive, then the actual current is in the same direction as the reference direction and if negative, then the actual current will be in the opposite direction of the reference direction.



As given in the figure, the current in the branch between A and B is same. However, i_{ab} and i_2 are two current variables. The reference direction of the two variables are opposite of each other. As they represent the same current, their magnitude is same but the signs are different.

Reference polarity for voltage

Similar to the reference direction for current, there is the concept of reference polarity for voltages. The actual direction of voltage can be obtained after solving the circuit.

Similar to the double-subscript notation for current, there is double-subscript notation for voltage as well. Another way to show voltage across an element is to use an arrow, with the arrowhead being equivalent to the positive polarity of the voltage.

Electric Power and Sign Convention

Because current is rate of flow of charge and voltage is the measure of the energy transferred per unit of charge, product of current and voltage is the rate of energy transfer (power).

Power = Voltage x Current: $p = vi$

The physical units of voltage times current is :
volts x amperes = (joules/coulomb)x(coulomb/sec)=joules/sec=watts (W).

Energy absorbed by the element is:

$w = \int_{t_1}^{t_2} p(t)dt$ which is in joules.

The passive sign convention

When positive charge enters an element at the positive voltage terminal and leaves the negative voltage terminal, then the positive charge loses energy. In this case, the element is said to have absorbed the energy. Such elements are said to be passive elements. The current always enters the positive voltage terminal for passive elements.

While solving circuits, we need to assign voltage and current variables for each element, sometimes without any prior information about them. In such cases, we can assume them to be passive elements and assign the current variable to be entering into the positive terminal of the voltage variable. This assignment is said to follow the 'passive sign convention'. If this convention is followed, then power in passive elements (also called load) will be positive and power in active elements (also called source) will be negative.

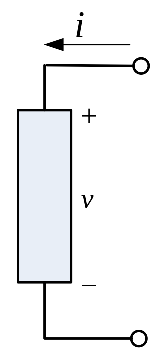


Fig. Passive Sign convention

Circuit Elements and their i-v characteristics

The relationship between current (i) and voltage (v) at the terminals of a circuit element defines the behavior of that element within the circuit. The current flowing through the element and the voltage across it, form a unique pair of values. Thus, varying the voltage across the element, one can obtain a set of such pairs, which can be plotted to give the i-v characteristics of the element.

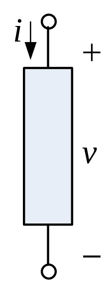


Fig. Current and Voltage associated with an element

The i-v characteristics can be expressed in functional form as

$$i = f(v)$$
$$v = g(i)$$

The i-v pairs can can also be plotted to obtain the i-v curve of the element.

Resistance and Ohm's Law

When electric current flows through a metal wire or other circuit elements, it encounters some resistance, depending on the properties of the material. The voltage (*v*) across an ideal resistor is proportional to the current (*i*) through it. The constant of proportionality is the resistance *R*.

$$v = iR \qquad \textbf{Ohm's law}$$

Ohm's law is an empirical relationship. It is only an approximation and does not hold at very high voltage and current or at very low voltage and current.

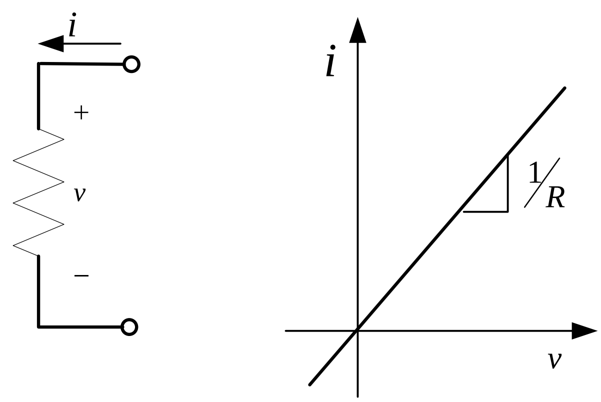
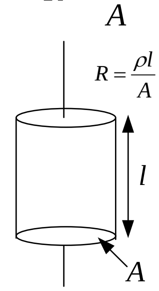


Fig. Graphical representation of Ohm's Law for resistance

Resistance depends on the material as well as the geometry of the conductor:

$$R = \frac{\rho l}{A}$$



where ρ is the resistivity of the material of the resistor; l, A are the length and cross-sectional area of the resistor.

Conductors have very low resistivity in the order of 10^{-6} ohm-meter (Ωm) whereas insulators have very high resistivity in the order of 10^{10} ohm-meter (Ωm) or more.

Conductance

$$i = \frac{1}{R} v$$

$$G = \frac{1}{R}$$

G is the reciprocal of resistance, and is called the conductance of the conductor. It has units of Mho (ohm in reverse order).

Resistivity of common materials at room temperature

Material	**Resistivity (Ohm-meter)**
Silver	1.629×10^{-8}
Copper	1.725×10^{-8}
Gold	2.271×10^{-8}
Aluminium	2.733×10^{-8}
Iron	9.98×10^{-8}
Carbon	3.5×10^{-5}

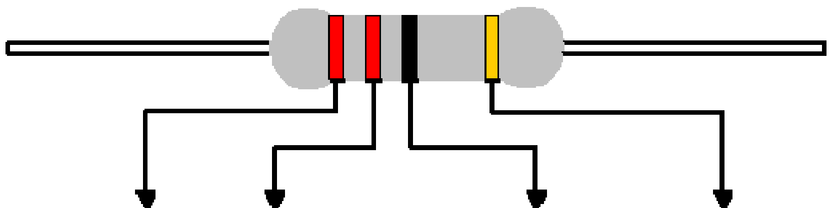
Difference between resistor and resistance

Resistor is an electrical element made up of conducting material and resistance is the property of this element where the voltage is proportional to the current.

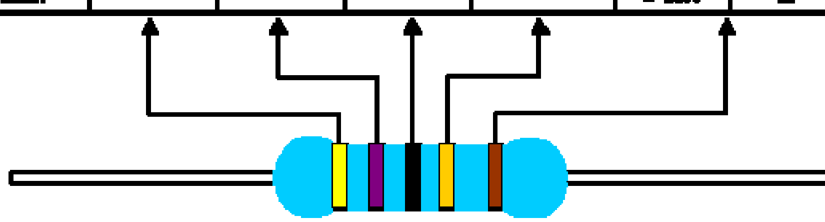
Resistor color code

Color bands are placed on commonly used resistors which can be used for inferring their values. The first three bands closest to one end of the resistors are used to determine the resistance. The first two bands represent the numerical value of the resistor and third band represents the power-of-10 multiplier. The fourth band represents the tolerance of the resistor (gold-5%, silver-10% and plain-20%).

Resistance value = (band1) (band2) x $10^{(band3)}$



COLOR	1ST BAND	2ND BAND	3TH BAND	MULTIPLIER	TOLERANCE	
	0	0	0	1		
	1	1	1	10	± 1%	F
	2	2	2	100	± 2%	G
ORANGE	3	3	3	1K		
YELLOW	4	4	4	10K		
	5	5	5	100K	± 0.5%	D
	6	6	6	1M	± 0.25%	C
	7	7	7	10M	± 0.10%	B
	8	8	8		± 0.05%	A
WHITE	9	9	9			
GOLD				0.1	± 5%	J
SILVER				0.01	± 10%	K
PLAIN					± 20%	M



Example resistor color bands:

band1 – Brown (1)
band2 – Black (0)
band3 – Red (2)
band4 – Gold (5%)

R = 10×10^2 (5%)=10x100 =1000 Ohm (with 5% tolerance)

Tolerance of 5% here means, the actual value of the resistor will be 1000x(1 ± 0.05) i.e its value should be anywhere between 950 Ohm to 1050 Ohm.

Power in resistors

Power is product of voltage and current.

$$p = vi$$

With $v = iR$, $p = i^2 R$

With $i = \frac{v}{R}$, $p = \frac{v^2}{R}$

Thus, power in resistor is always positive (energy is always absorbed by the resistor irrespective of the sign of current or voltage). This energy is either dissipated as heat or is converted to light as in an incandescent lamp.

Each resistor will have a power rating, beyond which the resistor may get over heated and may get burnt. The commonly used resistors in most electronic circuits are rated as ¼ Watt.

Open and Short Circuits

A circuit element with resistance approaching zero is called a **short circuit**. Formally, a short circuit is defined as a circuit element across which the voltage is zero, regardless of the current flowing through it. Physically, as a wire or conductor of appropriate size depending on the current will have a very small voltage drop across it and hence can be approximated as a short circuit.

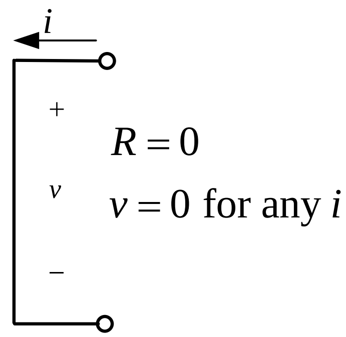


Fig. Short circuit

A circuit element whose resistance approaches infinity is called an **open circuit**. Again, an open circuit would mean the current flowing through it will be zero regardless of the voltage applied across it. Physically, any break in the conducting path of a circuit is called an open circuit. This is an idealization only and the insulation between two points will break down at sufficiently high voltages. If insulation is air, then there may be arcing very high voltages (this principle is used in spark plug of internal combustion engine).

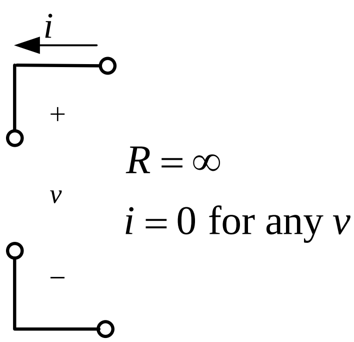


Fig. Open Circuit

Conductors

The voltage between the ends of an ideal conductor is zero regardless of the currents in them.

When two points in a circuit are connected through an ideal conductor, they are said to be shorted together and the voltage between them will be zero. In fact, they will be considered to be same point electrically.

A piece of copper wire is used as the conductor to connect two elements in a circuit. Its resistance will be very small (but not zero).

Circuits (Networks) Analysis

An electrical circuit contains a source (voltage or current) and other elements, with closed paths for the electrical current to move. Given a circuit, it is often required to obtain the value of the voltage, current and power in any of the elements. This is called circuit analysis.

Learning objectives:

1. Identify the principal elements of electric circuits: nodes, loops, meshes, branches, and voltage and current sources.
2. Apply Kirchoff's laws to simple electric circuits and derive the basic circuit equations.
3. Apply voltage divider and current divider laws to find the value of unknown variables in simple series, parallel, and series-parallel circuits.
4. Understand the rules for connecting electrical measuring instruments to electric circuits for the measurement of voltage, current, and power.

Definitions

Branch

Any portion of the circuit with two terminals connected to it is known as branch. A branch may consist of one or more circuit elements. The current will be same throughout the branch.

Node/Super node

A node in an electrical circuit is a point at which two or more elements are joined together.

A super node is a closed surface enclosing part of a circuit. It may contain some sources and other nodes.

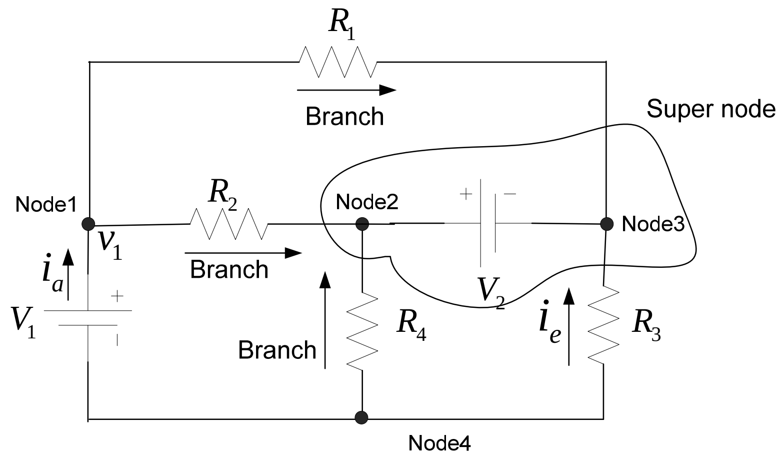


Fig. Circuit showing Branch, Node and Super node

Mesh and loop

A mesh is a closed path in the circuit. A loop is also a closed path in the circuit. The difference between loop and mesh is that a loop may contain other meshes or loops inside it, where as a mesh cannot have any other mesh or loop inside it. In the figure, loop2 and loop3 are meshes, where as loop1 is not a mesh. There are many other loops but only one more mesh is possible in this circuit.

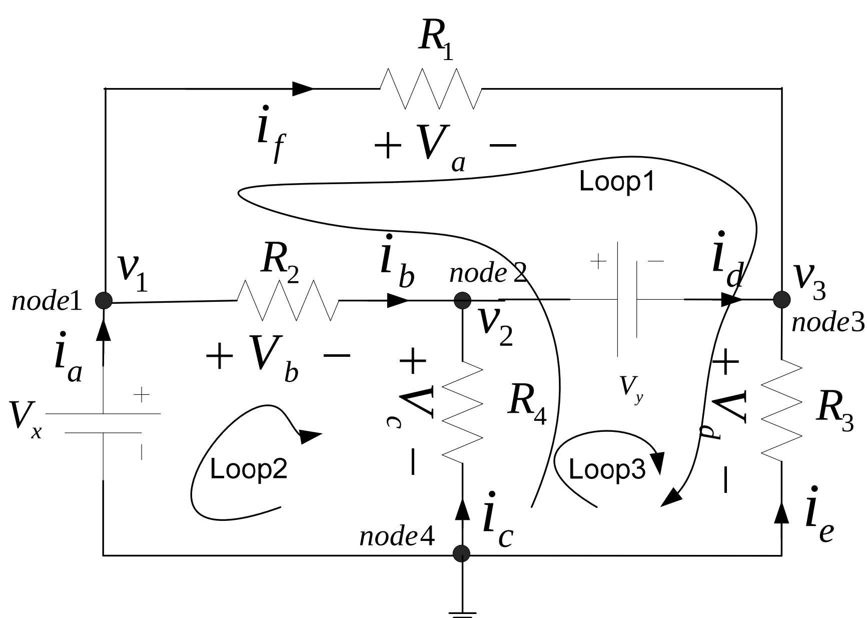


Fig. Mesh and Loop

Kirchoff's Current Law (KCL)

Kirchoff's current law states that the net current entering a node is zero (It is also equivalent to state that the net current leaving a node is equal to zero).

Net current entering is found by adding the currents entering and by subtracting the currents leaving. If only two elements are connected to the node then their currents must be equal in magnitude.

If KCL is violated, then there would be continuous accumulation of charge. Such an accumulation of charge would have its opposite charge accumulating at another point in the circuit. There would be a large force of attraction between these two opposite charges and disturb the circuit physically.

Kirchoff's Voltage Law (KVL)

A loop in an electrical circuit is a closed path, starting at a node and proceeding through the circuit elements, eventually returning to the starting node. In a typical circuit, several such loops can be found.

The algebraic sum of voltages equals zero for any closed path (loop) in an electric circuit. In other words, net voltage rise (or net voltage fall) around a loop is equal to zero. Net voltage rise is obtained by adding the voltage rises and subtracting the voltage falls.

A convenient convention is to add the voltage of the element if we first encounter the positive polarity mark and to subtract the voltage for the element if we first encounter the negative polarity mark. Travelling from positive terminal to the negative terminal implies voltage is dropping across the element. Travelling from the negative terminal to the positive terminal implies voltage is increasing along the path.

The voltages across each element in the closed path would be the energy lost or gained per unit charge travelling along the path. As we come back to the same point, the charge should not have gained or lost ay energy as otherwise it would violate the energy conservation principle.

Series circuits

When elements are connected end to end, they will carry the same current (applying KCL to each joint (node) of two ends) and are said to be in series.

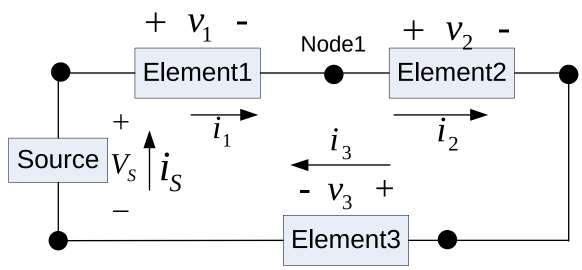


Fig. Series circuit

At node1, applying KCL:

$$i_1 - i_2 = 0 \Rightarrow i_1 = i_2$$

We can apply KCL at all the other nodes and show that the current is same in all elements:

$$i_s = i_1 = i_2 = i_3$$

Applying KVL we can show that

$$V_S = v_1 + v_2 + v_3$$

Parallel Circuits

Two elements are said to be connected in parallel if both ends of one elements are connected to the corresponding ends of the other element and voltages across both the elements are identical (i.e. equal in magnitude as well as polarity).

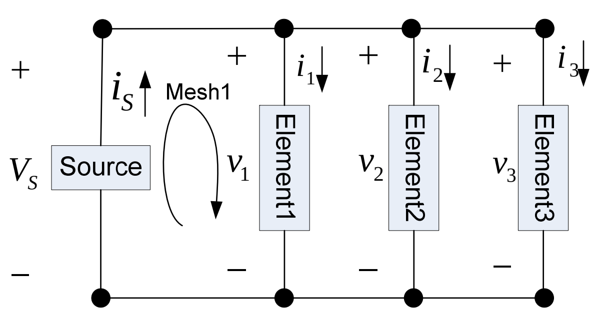


Fig. Parallel circuit

Applying KVL at mesh1:

$V_s - v_1 = 0 \Rightarrow v_1 = V_s$

It can be shown thus that:

$v_1 = v_2 = v_3 = V_s$

Applying KCL we can show that $i_S = i_1 + i_2 + i_3$

Resistances in series

Our aim is to find the equivalent resistance of a series connection of resistors to simplify the circuit.

From the KCL, we know that currents in all the resistors in series will be identical.

From KVL, the total voltage is the sum of the voltage drops in all the resistors.

$$v_1 = R_1 i, v_2 = R_2 i, v_3 = R_3 i$$
$$v = v_1 + v_2 + v_3 = \left(R_1 + R_2 + R_3\right) i = R_{eq} i$$
$$R_{eq} = R_1 + R_2 + R_3$$

Resistors in series lead to increased resistance.

N resistors of value *R* in series have equivalent resistance of *NR.*

Resistances in parallel

Our aim is to find the equivalent resistance of a parallel connection of resistors to simplify the circuit. The voltages across all the resistors are identical.

$v_1 = v_2 = v_3 = v$

From KCL, the sum of currents in the individual resistances is equal to the current in the equivalent resistance.

$$i_1 = \frac{v}{R_1}, i_2 = \frac{v}{R_2}, i_3 = \frac{v}{R_3}$$
$$i = i_1 + i_2 + i_3$$

$$\frac{v}{R_{eq}} = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistances in parallel reduce the equivalent resistance.

Two resistances in parallel: $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$

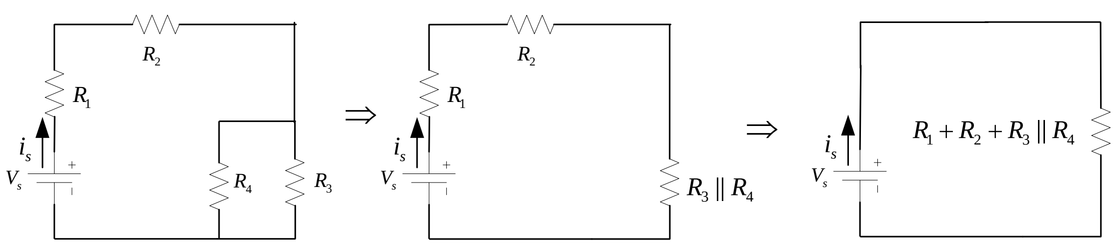
Two equal resistances in parallel $R_{eq} = \dfrac{R}{2}$

N resistances of equal values, $R_{eq} = \dfrac{R}{N}$

All the appliances at home are connected to the main supply in parallel, as all of them need the same voltage to operate. The total current drawn from the main supply is the sum of currents drawn by individual loads.

Network Analysis by Using Series and Parallel Equivalents

As can be seen in the figure, a resistive circuit can be solved by repeated simplification through application of series and parallel equivalents.



$$i_s = \frac{V_s}{R_1 + R_2 + R_3 \parallel R_4}$$

Fig. Network analysis by series and parallel equivalents

Voltage-Divider and Current-Divider Circuits

Voltage Division

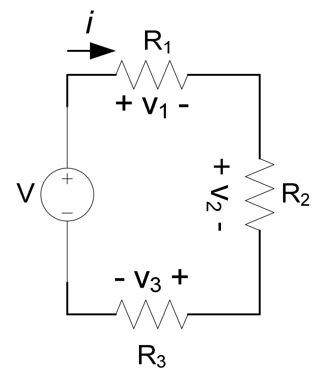


Fig. Series circuit

When voltage is applied to a series combination of resistances, a fraction of the voltage appears across each of the resistances.

The voltage drop across each resistance is the current times the resistance.

The current $i = \dfrac{V}{R_1 + R_2 + R_3}$

Hence,

$$V_1 = iR_1 = \frac{R_1}{R_1 + R_2 + R_3} V$$

$$V_2 = iR_2 = \frac{R_2}{R_1 + R_2 + R_3} V$$

$$V_3 = iR_3 = \frac{R_3}{R_1 + R_2 + R_3} V$$

Voltage division principle: In a series circuit, the voltage a across each resistance is a fraction of the total voltage, which is equal to the ratio of the concerned resistance to the total resistance.

Current Division

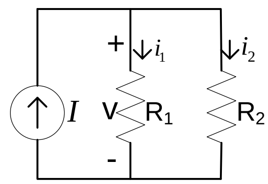


Fig. Parallel circuit

The current I is passing through the two resistors in parallel (its equivalent being $\frac{R_1 R_2}{R_1 + R_2}$) so voltage at the output of the current source $v = \frac{R_1 R_2}{R_1 + R_2} I$.

Then,

$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} I$$
$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} I$$

Current division principle: For the two resistances in parallel, the current flowing in a resistance is a fraction of the total current equal to the ratio of the other resistor to the sum of both the resistors.

This principle applies to only two resistances in parallel. For more resistances in parallel, we combine resistances to reduce to the two resistance case.

Practical Voltage and Current Sources

In practice, the voltage sources like batteries will have an internal resistance, across which there will be a voltage drop. The voltage output at the terminals of the supply will be dependent on the load, higher the load current, lesser will be the voltage output. Internal resistance will be quite small for most practical cases.

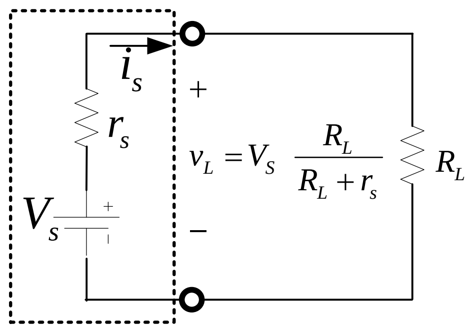


Fig. Practical voltage source

$$V_L = V_s - i_L r_s$$

The voltage at the output terminal can also be obtained by applying the voltage divider principle:

$$V_L = V_s \frac{R_L}{R_L + r_s}$$

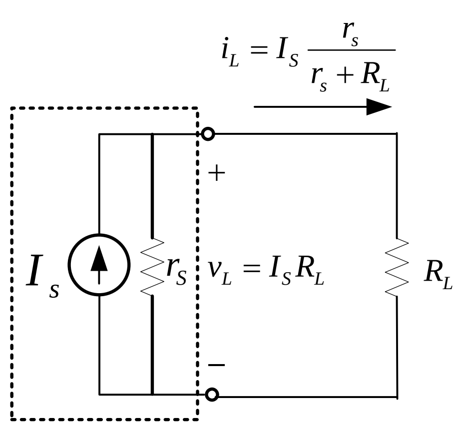


Fig. Practical current source

Similarly, the practical current source will have an internal resistor in parallel with an ideal current source. Hence, the current in the load will depend on the load resistance, as can be seen from the current divider principle.

$$i_L = i_s \frac{r_s}{r_s + R_L}$$

For practical cases, the internal resistance for the current source will be very large.

Measuring Devices

Ohmmeter

The Ohmmeter is used to measure the resistance of elements. The two probes of the Ohmmeter are connected across the two ends (terminals) of the element. The element should be removed from the circuit when resistance is measured (at least one end of the element should be removed from the circuit and the circuit should be de-energised.)

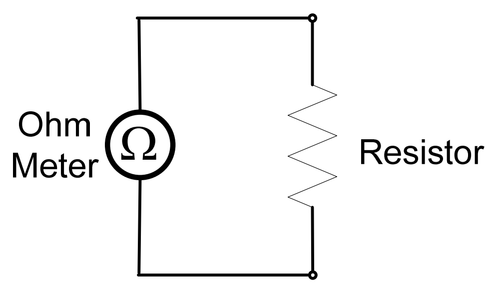


Fig. Measuring resistance with Ohmmeter

Ammeter

An ammeter is used to measure the current through an element. Hence, the ammeter is connected in series with the element so that the same current passes through both the ammeter and the element. The circuit has to be broken to insert the ammeter in the circuit. The ideal ammeter has zero resistance and hence does not affect the circuit.

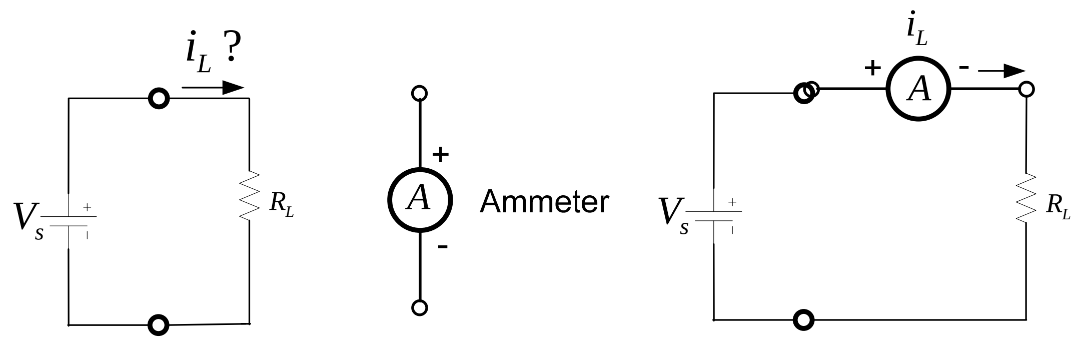


Fig. Measuring current using an ammeter

Voltmeter

A voltmeter measures the voltage between two points in a circuit (it could be two ends of an element in the circuit). Voltmeter is connected in parallel to the two points. It is not necessary to break the circuit for connecting the voltmeter. The ideal voltmeter has infinite resistance and connecting the voltmeter in parallel does not affect the circuit.

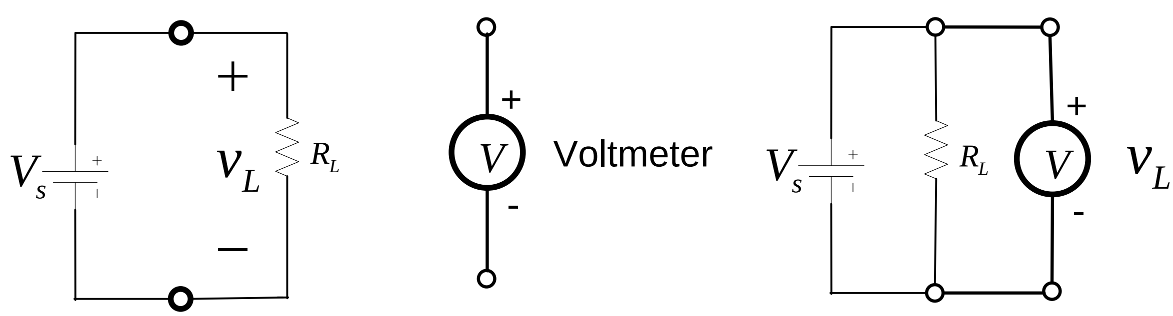


Fig. Measuring the voltage across an element

The digital multi-meter (DMM)

A multi-meter is a very commonly used device which can be operated in multiple modes to measure various electrical quantities like voltage, current, resistance etc.