EE1002
Introduction to Circuits and Systems

Part 1 : Lecture 10

AC Steady-state Analysis and Summary of Part 1

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Part 1 : Lecture 9

AC Steady-state Analysis

Example

$V_R \to v_R(t) = |V_R| \cdot \sin(300t + \theta_R)$

- Given sinusoidal source voltage, find the expression for the voltage across the resistor.

$\omega = 300$ rad/sec

$v_s(t) = 200 \sin(300t + 45^\circ)$

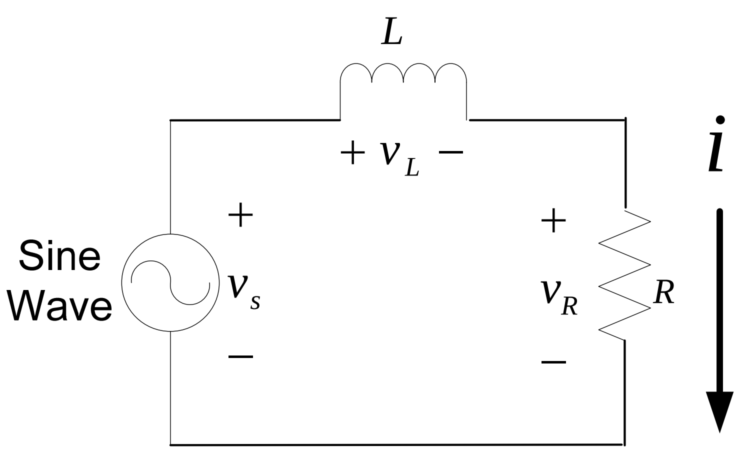
↓ Phasor $V_s = 200 \angle 45^\circ$

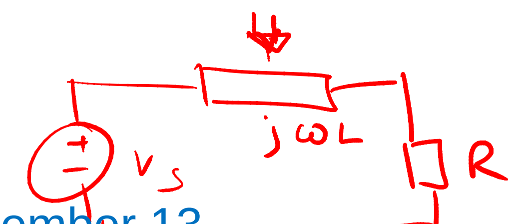
$\omega = 2\pi \cdot f \qquad f$ - Hz

$L \to Z_L = j\omega L$

$R \to Z_R = R$

$C \to Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$





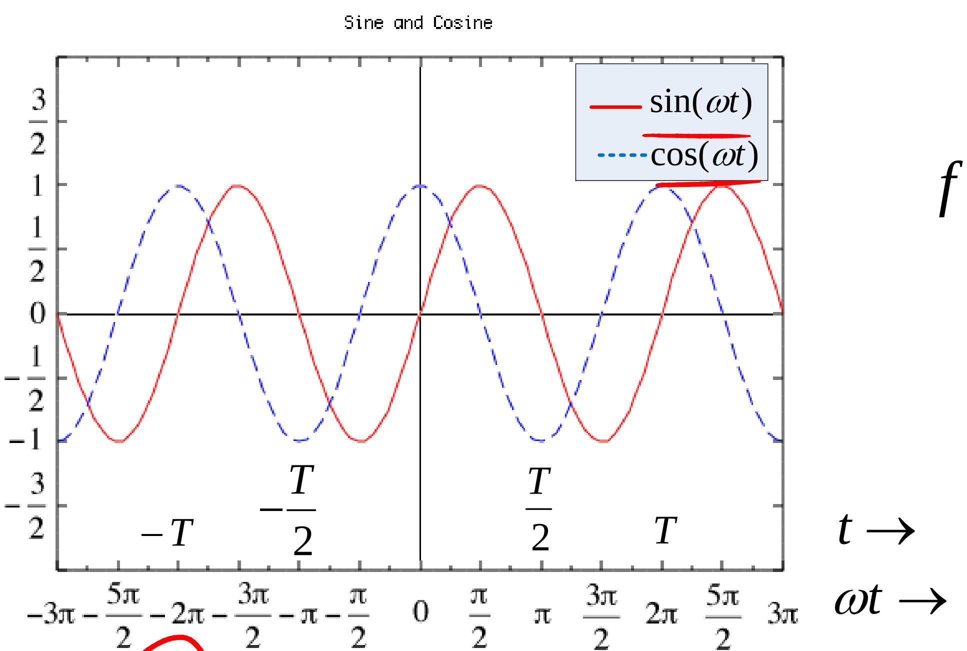
$j^2 = -1$

$V_R = V_s \times \frac{R}{R + j\omega L} = |V_R| \angle \theta_R$

AC Steady-state Analysis

- Time varying periodic voltage and current
- Complex number
- Sinusoidal sources : Peak value, frequency (f, ω), phase angle (θ)
- Phasors for sinusoidals
- Impedances for R,L and C
- AC Circuit Analysis with phasors and Impedances
- Examples of AC Circuit analysis

Sinusoidal AC signals



$$f = \frac{1}{T} \qquad \omega = \frac{2\pi}{T} = 2\pi f$$

→ $\sin\theta = \cos(\theta - 90^\circ)$

→ $\cos(\theta) = \sin(\theta + 90^\circ)$

$$v(t) = V_m \cos(\omega t + \theta),$$

V_m is the peak value of voltage

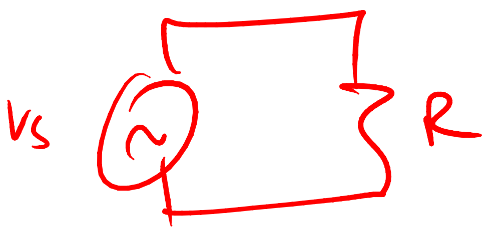
ω is the angular frequency

θ is the phase angle

RMS value for AC Sinusoidal

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t+\theta)dt}$$

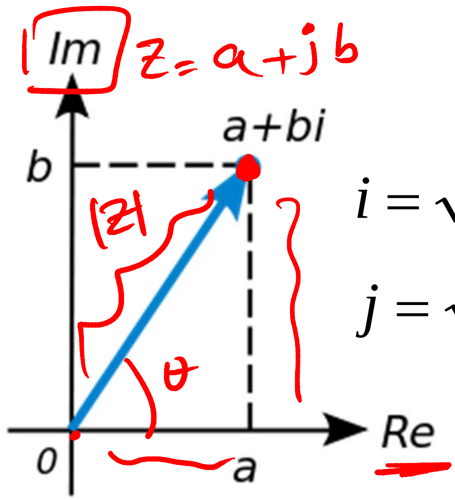
$$= \sqrt{\frac{1}{T}\frac{V_m^2}{2}\int_0^T (1+\cos 2(\omega t+\theta))dt} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$



$P_R(t) = i^2(t) \cdot R$

$P_{R,avg} = \frac{V_{Rms}^2}{R}$

Complex number – Euler's Formula



$$i = \sqrt{-1}$$

$$j = \sqrt{-1}$$

- Addition/Subtraction
  - Done in rectangular form
- Multiplication/Division
  - Done in polar form

$|z| \cdot \angle\theta$

$z_1 + z_2 = 2 + i4 + 3 + i5$

$= 5 + i9$

Phase Difference

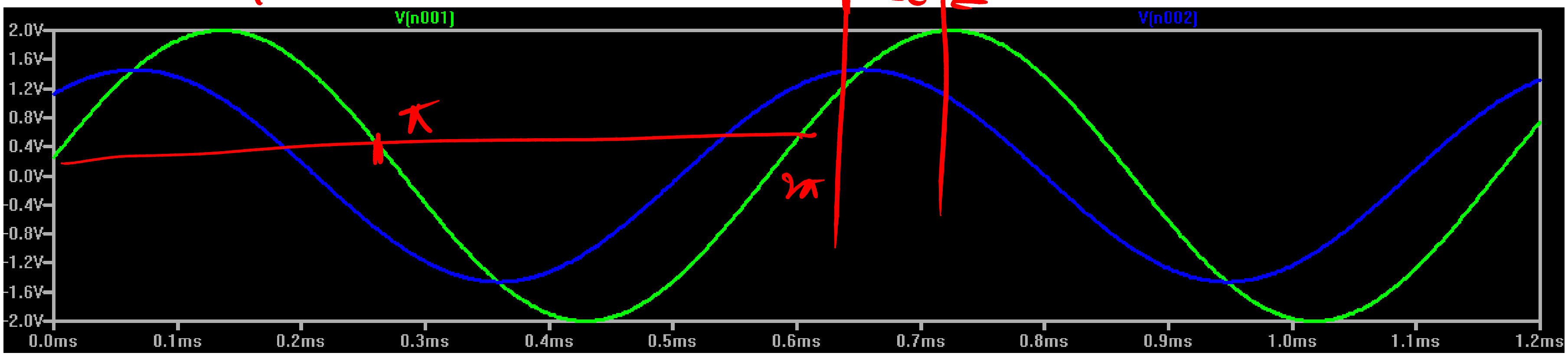
$\Delta\theta = \frac{\Delta t}{T} \times 2\pi$

$= \frac{\Delta t}{T} \times 360°$

Blue

Blue leads Green

Δt Green

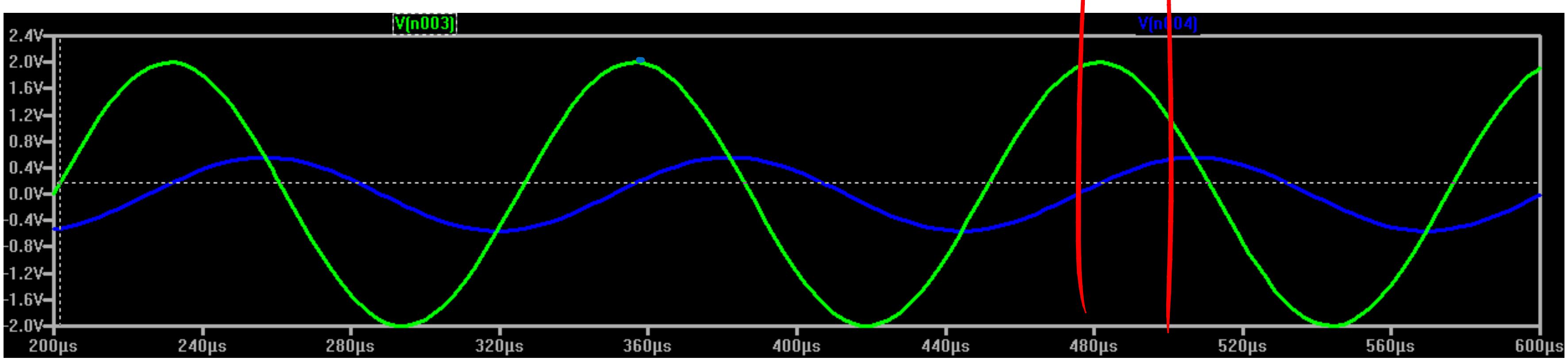


$t_1 < t_2$

t_1

t_2

Green leads Blue



Phasor Definition

The frequencies of all sinusoidals must be same.

- For a sinusoidal voltage of

$$v_1(t) = \boxed{V_m} \cos(\omega t + \theta)$$

- Phasor : $V_m \angle \theta$
  - Phasor is a complex number
  - Absolute value equal to the peak
  - Argument same as the phase angle
- Example: $v(t) = 200 \cos(300t + 45^\circ) \Rightarrow 200 \angle 45^\circ$

  $i(t) = 10 \cos(300t - 60^\circ) \Rightarrow 10 \angle -60^\circ$

Phasor diagram

$v_1(t) = 3\cos(\omega t + \overset{\theta_1}{40^0})$: $3\angle 40^\circ$

$v_2(t) = 4\cos(\omega t - \overset{\theta_2}{20^0})$: $4\angle -20^\circ$

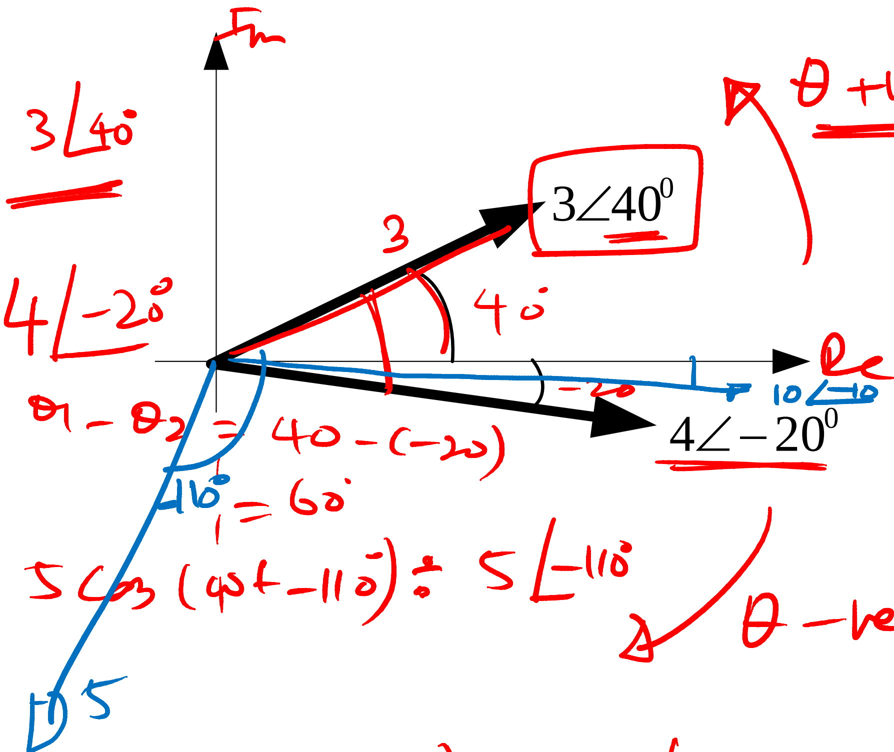
$\theta_1 - \theta_2 = 40 - (-20) = 60^\circ$

$v_3(t) = 5\sin(\omega t - 20^0)$

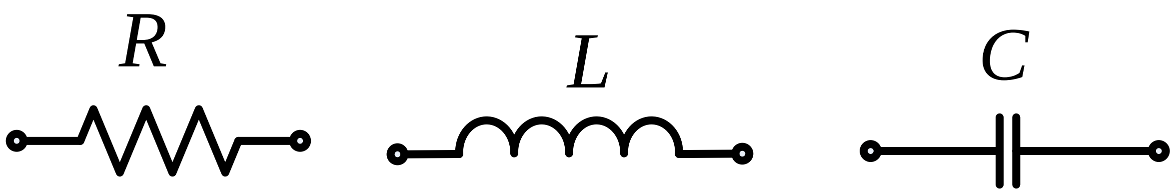
$= 5\cos(\omega t - 20 - 90^\circ) = 5\cos(\omega t - 110^\circ) : 5\angle -110^\circ$

$v_4(t) = 10\sin(\omega t + 80^0)$

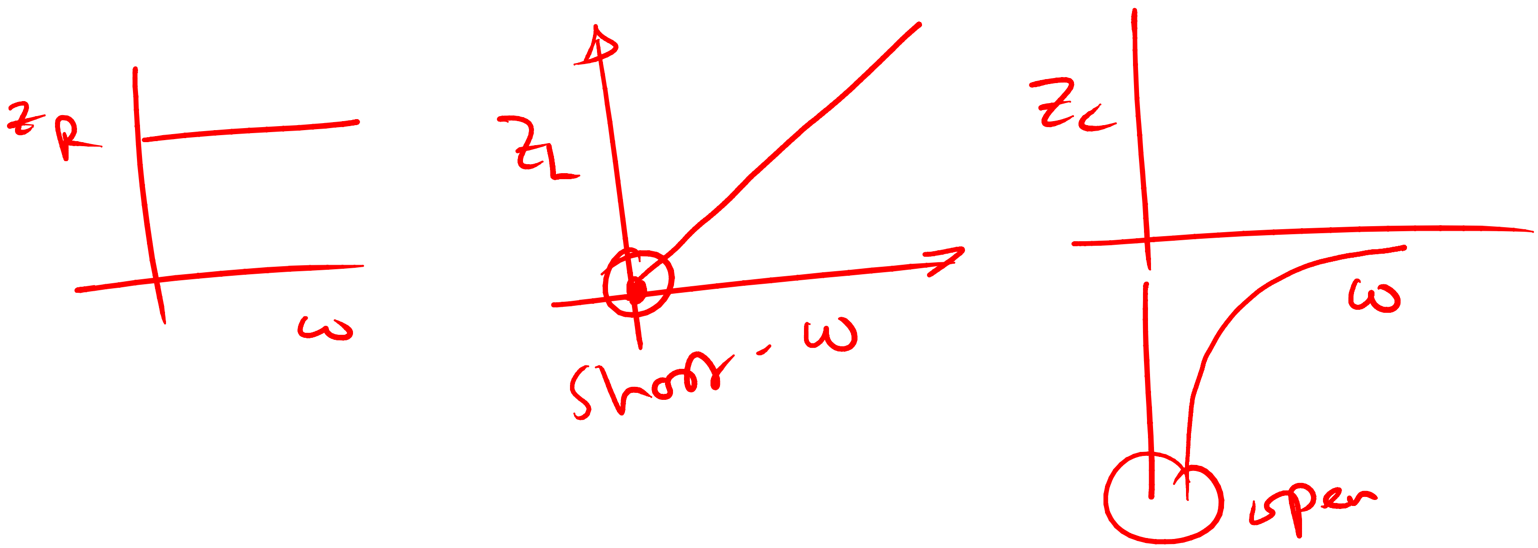
$= 10\cos(\omega t + 80 - 90^\circ) = 10\cos(\omega t - 10^\circ) : 10\angle -10^\circ$



Impedances for R, L and C



$$Z_R = R \qquad Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$$



Circuit Analysis with phasors and Complex Impedances

① • Replace the sinusoidal voltage and current sources with the corresponding phasors.

② • Replace the resistance, inductance and capacitance with their respective impedances.

③ • Analyze the circuits as before with DC sources and resistances only.

④ • Convert phasors back to the sinusoidals.

EE1002 Part 1 Review

Circuit Analysis

1. DC Analysis (Sources are constant in time)

$\frac{dx}{dt}=0$

- Steady-state analysis (constant in time)
- Transient analysis (time-varying) : C, L - energy storage elements.

2. AC Analysis (Sources are Sinusoidals)

$\cancel{\frac{dx}{dt}=0}$

- Steady-state analysis (constant in time)

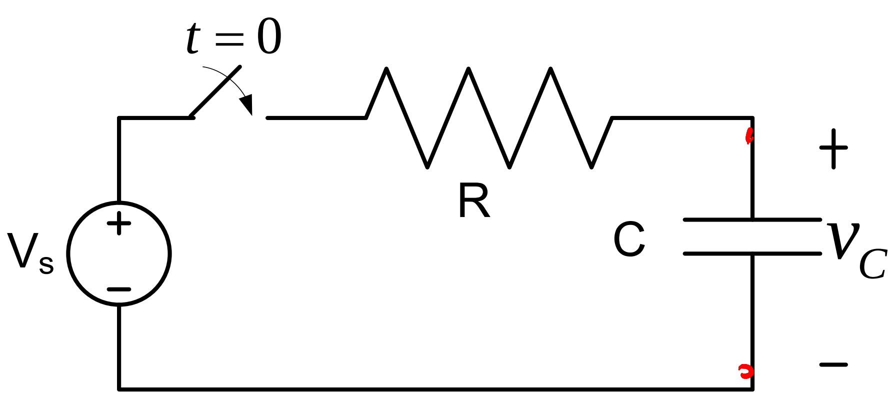
if the peak value, frequency

DC Transients

Transients

- The time-varying voltages and currents resulting from the adding or removing voltage and current source to circuits containing energy storage elements, are called **transients**.

RC Cricruit with a DC source



$$-V_s + v_R + v_C = 0$$

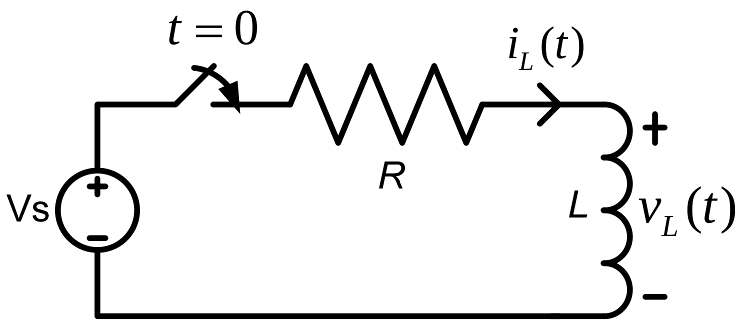
$$v_R = iR = RC\frac{dv_c}{dt}$$

$$-V_s + RC\frac{dv_c}{dt} + v_c = 0$$

$$RC\frac{dv_c}{dt} + v_c = V_s$$

$v_c(t)$

RL circuit with DC source



$$-V_s + iR + L\frac{di}{dt} = 0$$

$$\frac{L}{R}\frac{di}{dt} + i = \frac{V_s}{R}$$

$i_L(t)$

RC and RL comparing to the general form

$dx/dt = 0$

$x(\infty) = K$

$$\boxed{\tau \frac{dx}{dt} + x = K}$$

Solution: $x(t) = \left(x(0) - K\right)e^{-\frac{t}{\tau}} + K$

$$RC\frac{dv_c}{dt} + v_c = V_s$$

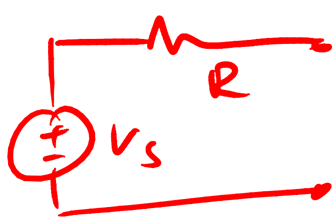
$$\frac{L}{R}\frac{di}{dt} + i = \frac{V_s}{R}$$

$\tau = R.C.$

$\tau = \frac{L}{R}$

$K = v_c(\infty) = V_s$

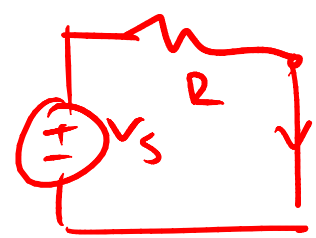
$K = i_L(\infty) = \frac{V_s}{R}$



$+$

$v_c(\infty) = V_s$

$-$



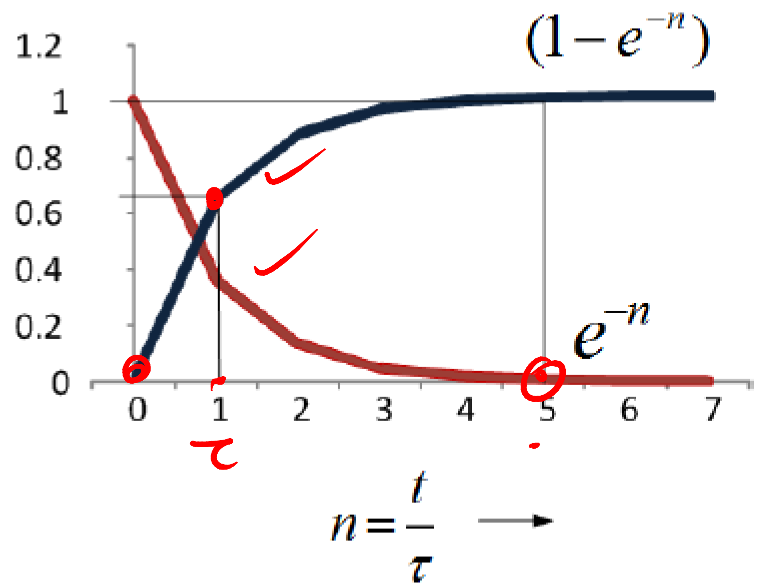
$i_L(\infty) = \frac{V_s}{R}$

Shape of First-order transient

t	$(1-e^{-t/\tau})$
τ	0.632121
2τ	0.864665
3τ	0.950213
4τ	0.981684
5τ	0.993262

63.2%

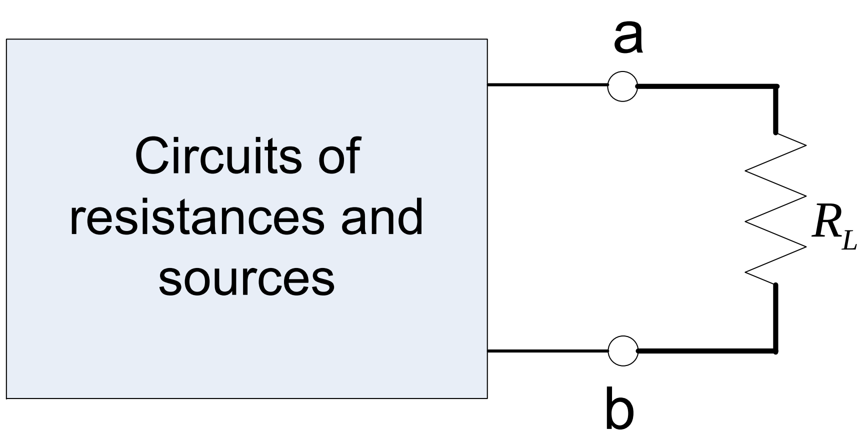
$$x(t) = K + \left(x(0) - K\right) \times e^{-t/\tau}$$

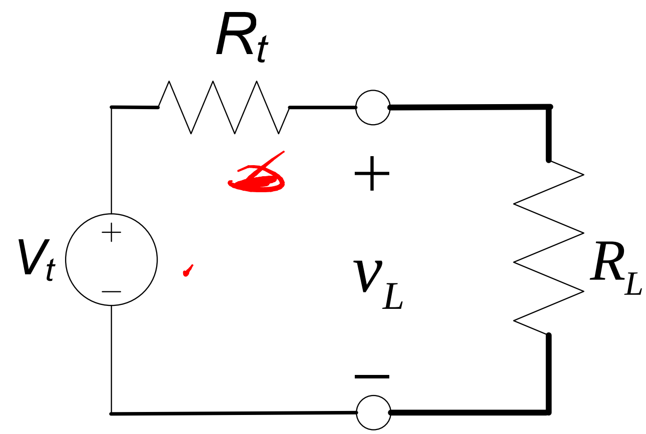


Steps for solving RC and RL circuits

- Use DC steady-state behavior of circuits before the transient starts to find initial value
- Use: $v_c(0^-) = v_c(0^+)$, $i_L(0^-) = i_L(0^+)$.
- Use Thevenin's equivalent circuit to reduce any circuit to the standard form
- Find the time constant and DC voltage from the Thevein's equivalent

Maximum power transfer





$$i_L = \frac{v_T}{R_L + R_T}$$

$$P_L = i_L^2 R_L = \frac{v_T^2}{\left(R_L + R_T\right)^2} R_L$$

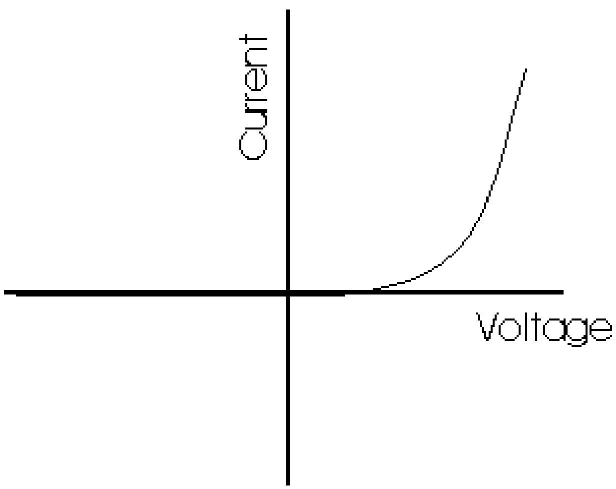
$$\frac{dP_L}{dR_L} = \frac{v_T^2 (R_L + R_T)^2 - 2v_T^2 R_L (R_L + R_T)}{(R_L + R_T)^4} = 0$$

$$v_T^2 (R_L + R_T)^2 - 2v_T^2 R_L (R_L + R_T) = 0$$

$$R_L = R_T$$

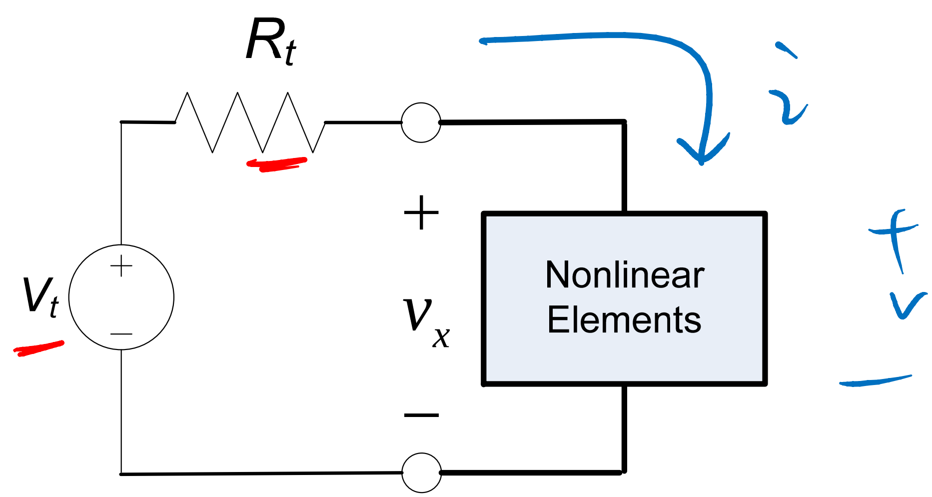
Nonlinear elements

- How to solve a linear circuit with one nonlinear element?
- Nonlinear elements may not have an analytical function



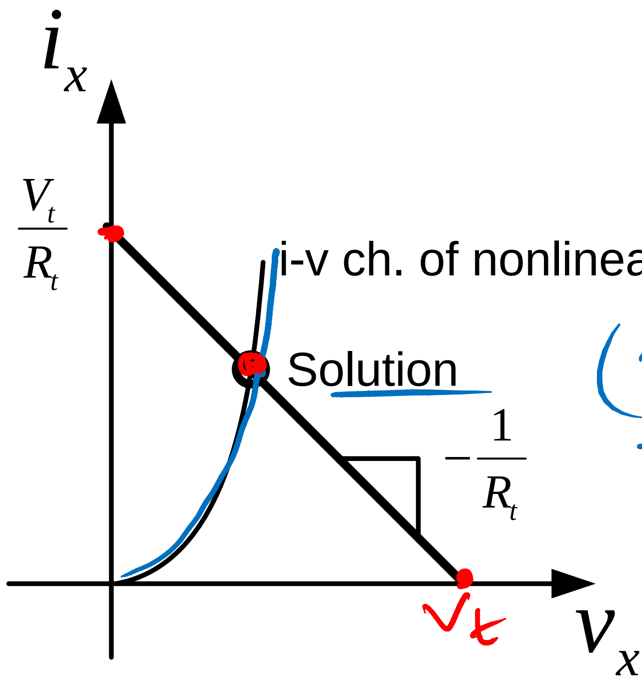
Solving circuits with one Nonlinear element

- Replace the circuit by its Thevenin's equivalent considering the nonlinear element as the load
- Use graphical analysis technique



Graphical (Load-line) analysis

- Merge the load line onto the i-v curve of the nonlinear element
- The solution is at the intersection point



(i, v)

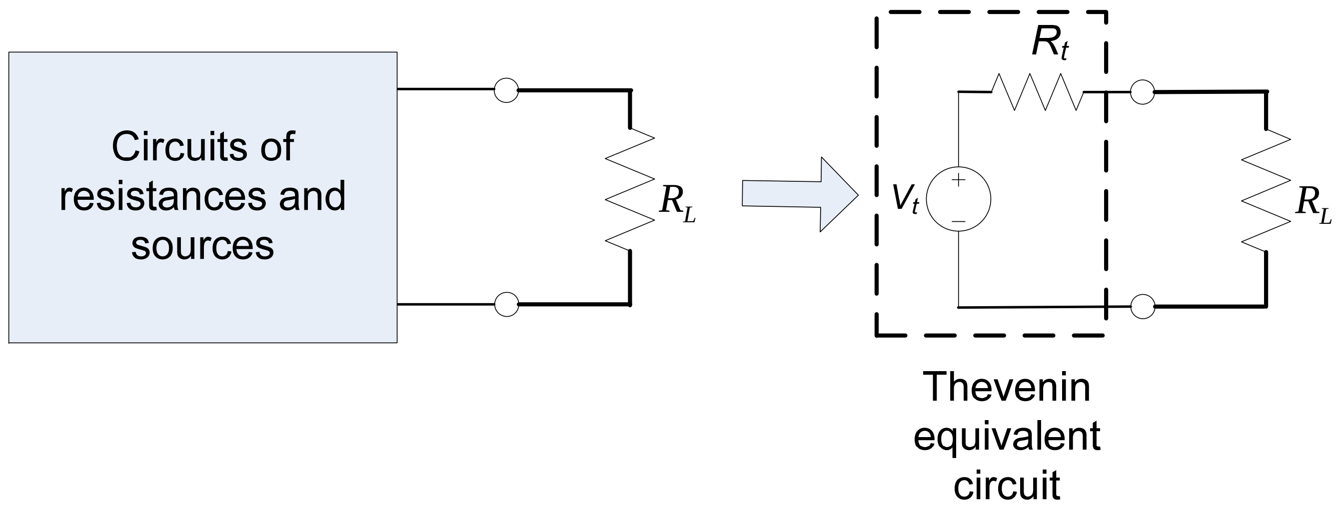
Equivalent Circuits

- **One-port networks and equivalent circuits**
  - Thevenin's equivalent circuit
  - Norton equivalent circuit

One-port networks and equivalent circuits

- Two-terminal circuits can be replaced by an equivalent circuit consisting of a source and a resistance.
- A voltage source with a series resistance (Thevenin equivalent circuit)
- A current source with a parallel resistance (Norton equivalent circuit)

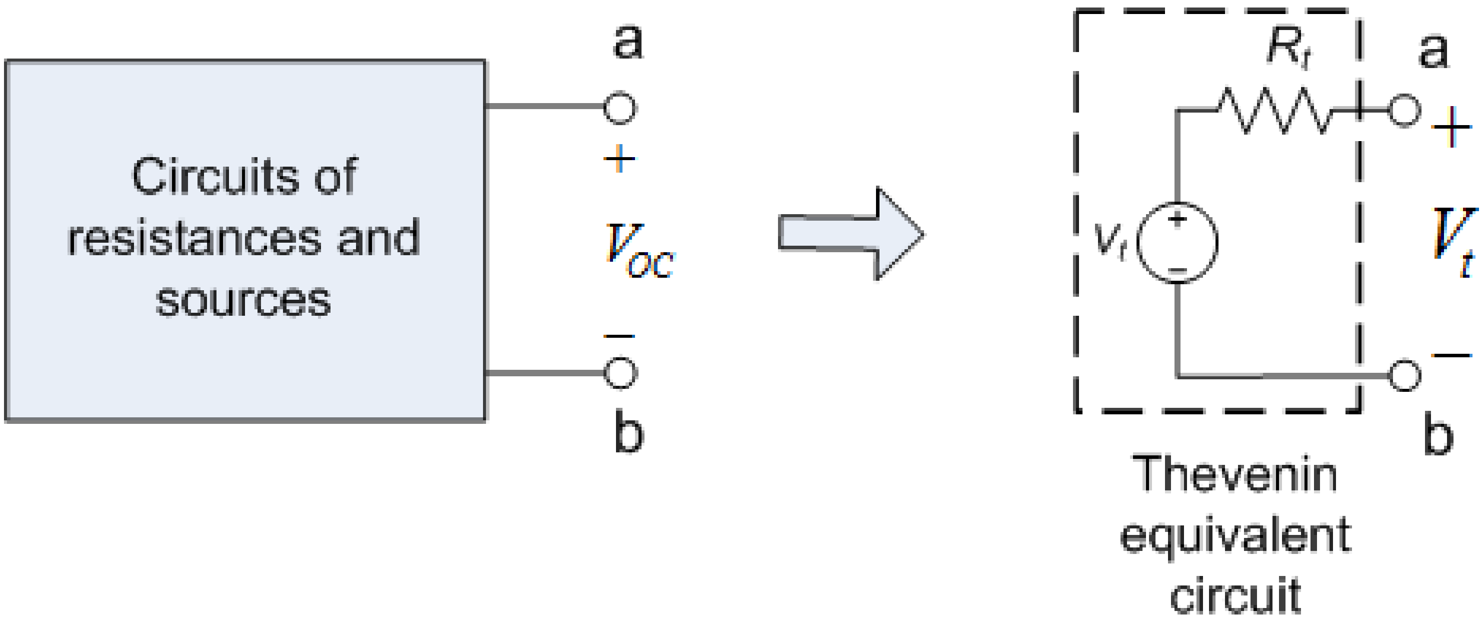
Thevenin equivalent



- A voltage source in series with a resistance
- The voltage source is called Thevenin's voltage
- The series resistance is called Thevenin's resistance

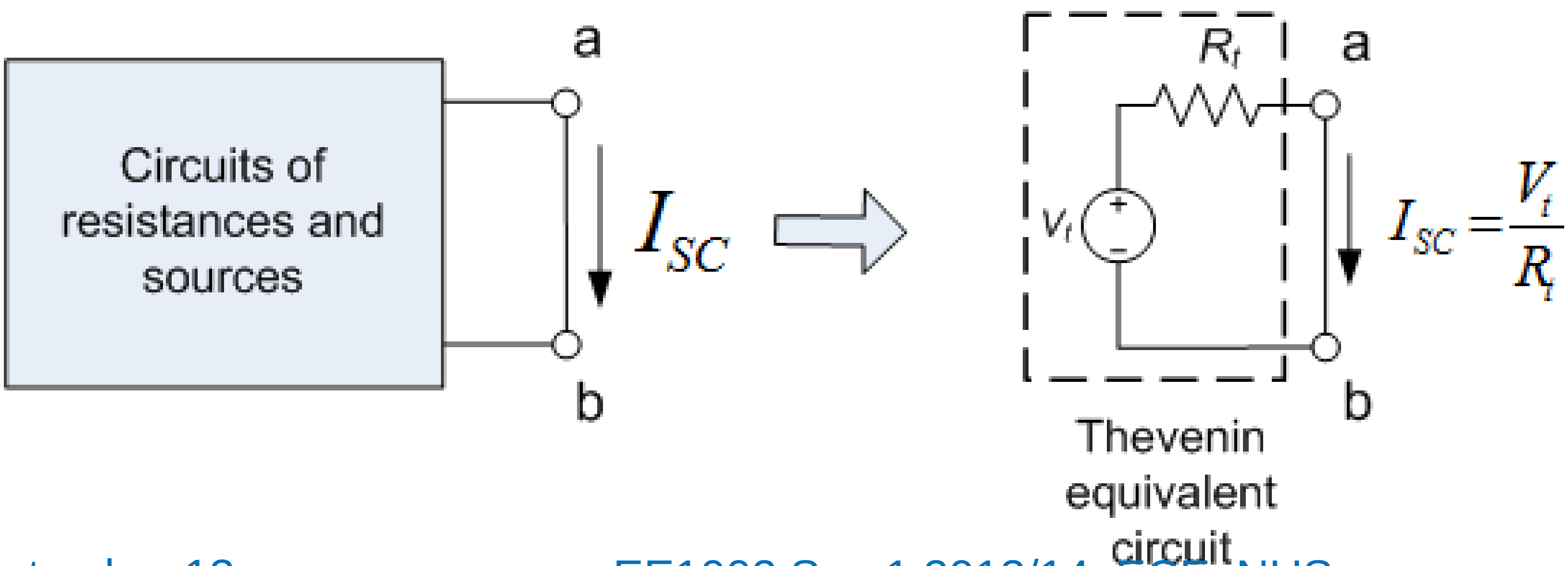
Thevenin Voltage

- Value of the voltage source is the open circuit voltage between the two terminals
- Called the Thevenin voltage

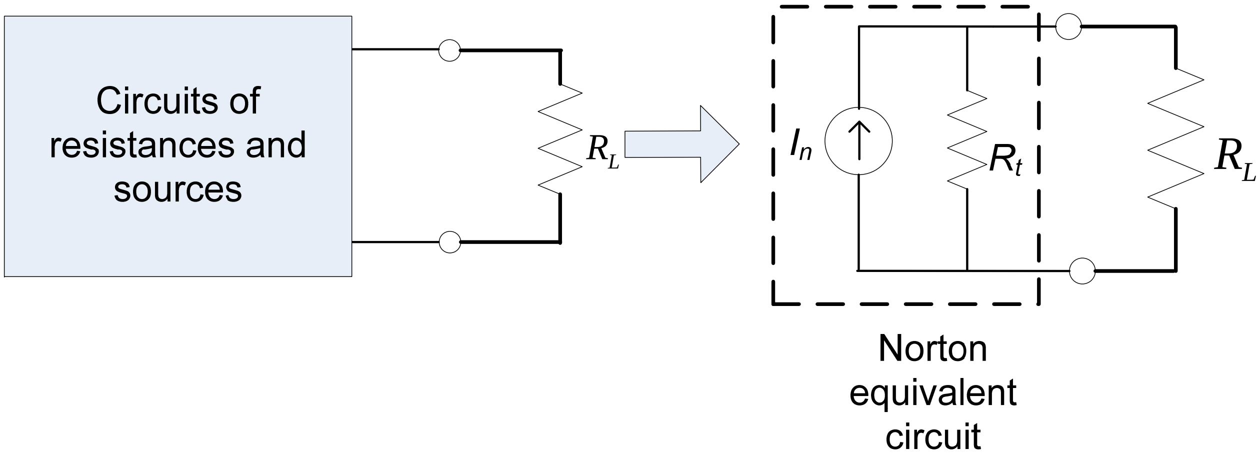


Thevenin Resistance

- Find the short circuit current between the two terminals
- Calculate the Thevenin resistance



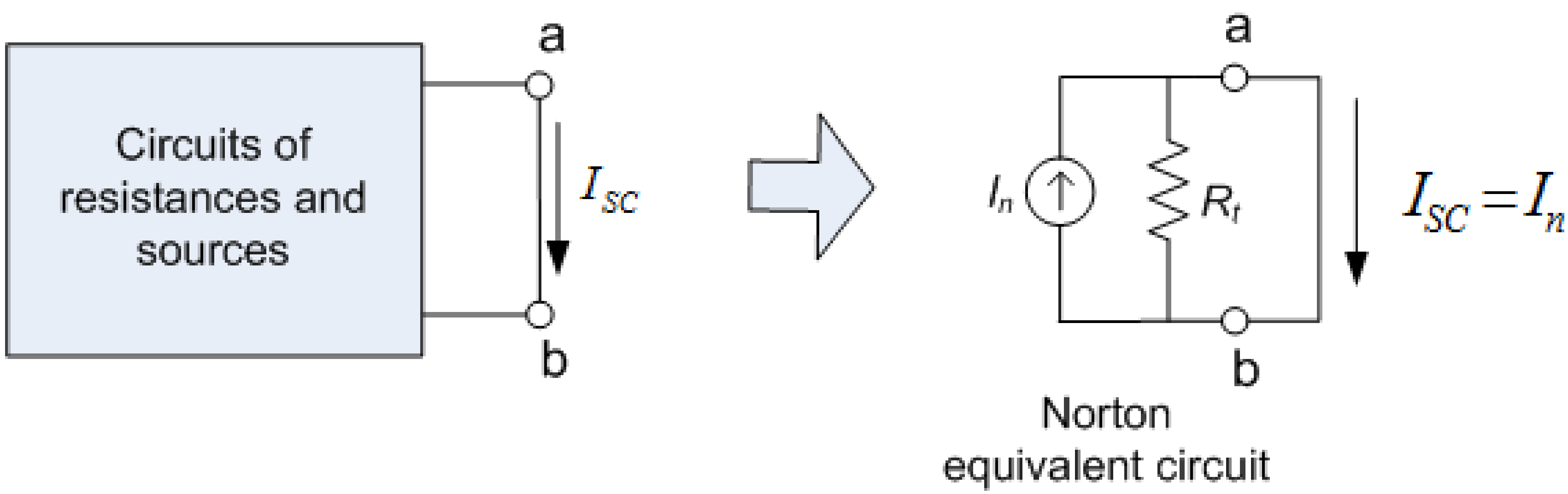
Norton Equivalent



- A current source in parallel with a resistance
- The current source is called Norton's current
- The series resistance is called Thevenin's resistance

Norton's current

- Value of the current source is the short circuit current between the two terminals
- Called the Norton's current



Steps to find the equivalent circuits

- Obtain the open circuit voltage between the two terminals – Thevenin's voltage
- Obtain the short circuit current between the two terminals – Norton's current
- Calculate the Thevenin's resistance as

$$R_t = \frac{V_{OC}}{I_{SC}}$$

Steps of Node Voltage Analysis method

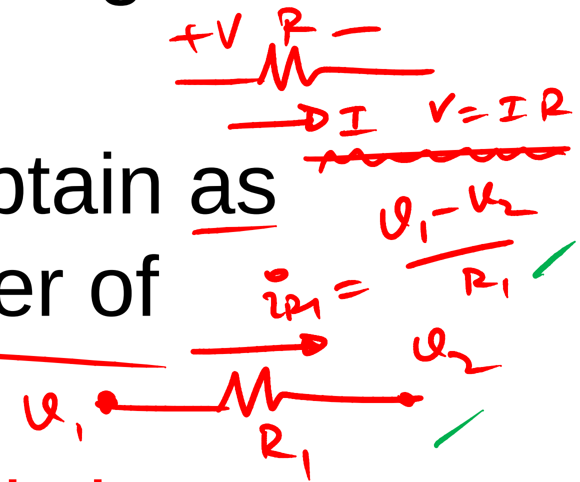
1. Select a reference node (Usually the ground of a voltage source.)
2. For each voltage source connected to the reference node, the other end is a known constant.

   -ve of voltage source as ref.

3. For all other voltage sources, one end is tied to the other. So only one unknown variable for each such voltage source.

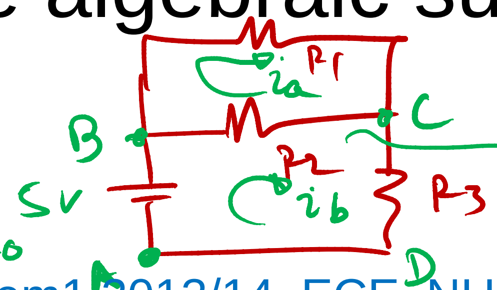
Steps of Node Voltage Analysis method

4. Define the remaining node voltages as unknown variables.
5. Apply KCL at the nodes, to obtain as many equations as the number of unknown variables.
6. Express each current in a resistive branch in terms of the adjacent node voltages.
7. Solve the linear system of equations.



Steps of mesh current analysis

1. Identify all the mesh currents in the circuit
2. Write KVL equations around the closed paths to obtain required equations.
3. If a branch is a resistor, there is a voltage fall in the direction of current.
4. If a resistor is common to two meshes, resistor current will be algebraic sum of the mesh currents.



ABCDA : KVL

$-5 + R_2 \times (i_b - i_a) + R_3 \times i_b = 0$

$B2 = i_b - i_a$

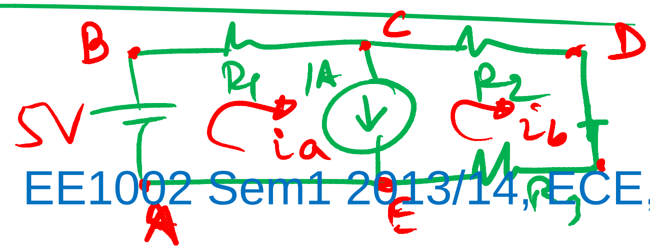
$I_{BC} =$

Steps of mesh current analysis

5. If a branch is a voltage source, voltage fall is from the positive terminal to the negative terminal.
6. Avoid paths where a branch is a current source, as you cannot express voltage across them in terms of mesh currents.
7. For branches which are current sources, relate them to the mesh currents.

ABCEA : ~~KVL~~

ABCDEA : KVL (1)



$1 = i_a - i_b$ — (2)

The principle of Superposition

- The total response in a $\boxed{\text{linear circuit}}$ is the sum of responses to each of the $\boxed{\text{independent sources}}$ acting individually.

$$r_T = r_1 + r_2 + \dots + r_N$$

→ Multiple sources

r_T - total response

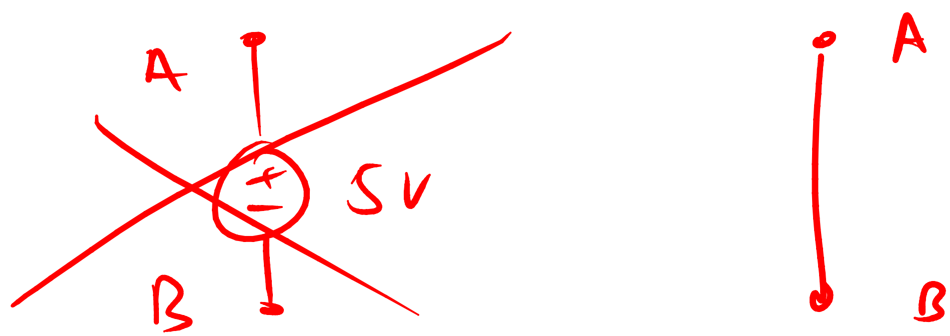
r_n - the response due to the n^{th} source, acting individually.

To find the individual response

- ‘Kill’ all the other independent sources but one
- This should result in a simpler circuit
- Use network analysis to find the required response
- Do this one by one for all independent sources
- Add up the responses to find the total response

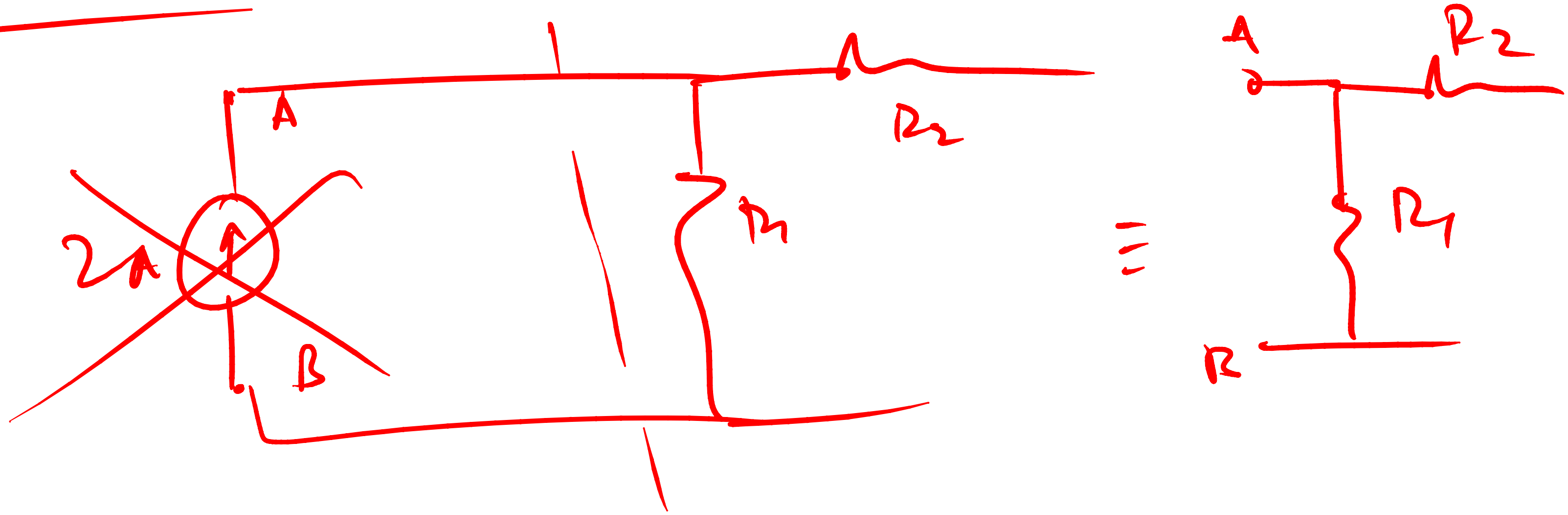
Killing a voltage source

- To kill a voltage source, we make its output voltage equal to zero.
- Replace the voltage source with a short circuit

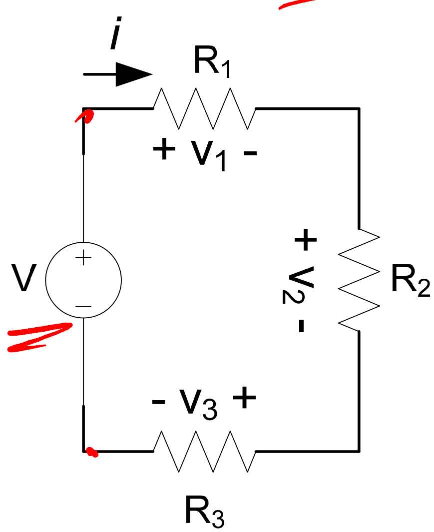


Killing a current source

- To kill a current source, we make its output current equal to zero.
- replace the current source with an open circuit



Voltage Divider rule



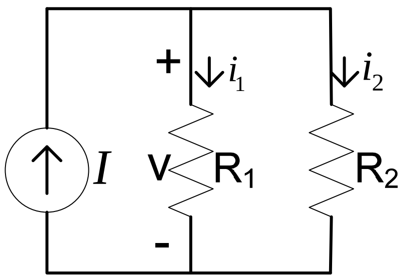
$$V_1 = iR_1 = \frac{R_1}{R_1 + R_2 + R_3} V$$

$$V_2 = iR_2 = \frac{R_2}{R_1 + R_2 + R_3} V$$

$$V_3 = iR_3 = \frac{R_3}{R_1 + R_2 + R_3} V$$

In a series circuit, the voltage a across each resistance is a fraction of the total voltage, which is equal to the ratio of the concerned resistance to the total resistance.

Current Divider Rule

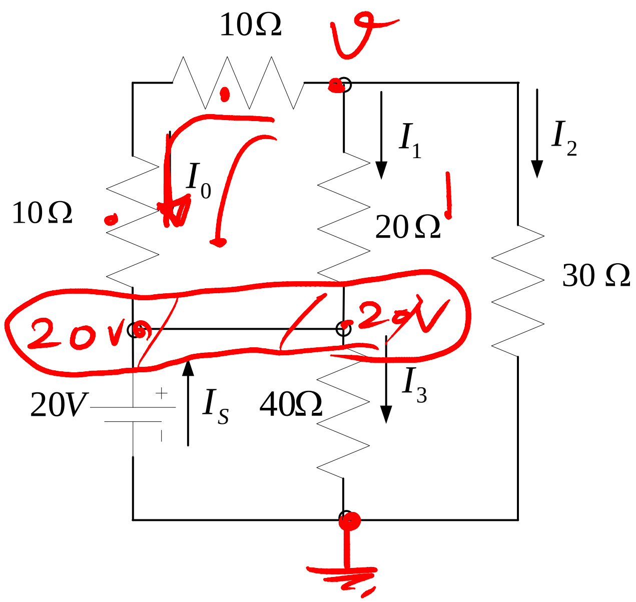


$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} I$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} I$$

For the two resistances in parallel, the current flowing in each resistance is a fraction of the total current equal to the ratio of the other resistor to the sum of both the resistors.

Midterm test prep - Q1



KCL at v

$$\frac{v-20}{10+10} + \frac{v-20}{20} + \frac{v}{30} = 0$$

$$3(v-20) + 3(v-20) + 2v = 0$$

$$8v = 120 \Rightarrow v = 15\,V.$$

$$I_0 = \frac{15-20}{20} = -0.25\,A$$

$$I_1 = \frac{15-20}{20} = -0.25\,A.$$

$$I_2 = \frac{15-0}{30} = 0.5\,A.$$

$$I_3 = \frac{20}{40} = 0.5\,A$$

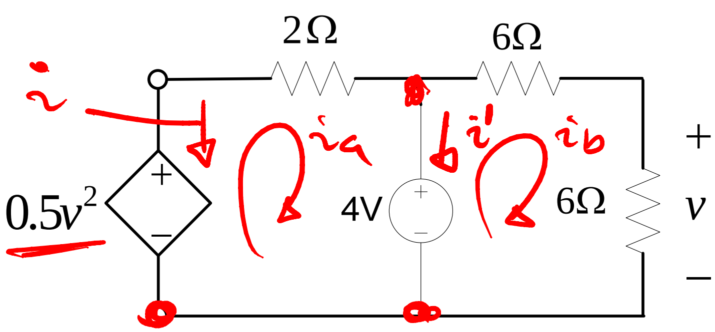
KCL at the s.n.

$$-I_S - I_0 - I_1 + I_3 = 0$$

$$I_S = -I_0 - I_1 + I_3 = 0.25 + 0.25 + 0.5 = 1\,A.$$

Midterm test Prep - Q2

Find the power in the sources.



$v = \frac{1}{2} \times 4 = 2V.$

$0.5v^2 = 2v.$

$v = 6 \times i_b \Rightarrow i_b = \frac{2}{6} = \frac{1}{3} A$

KVL around mesh for i_b:

$-2 + 2 \times i_a + 4 = 0$

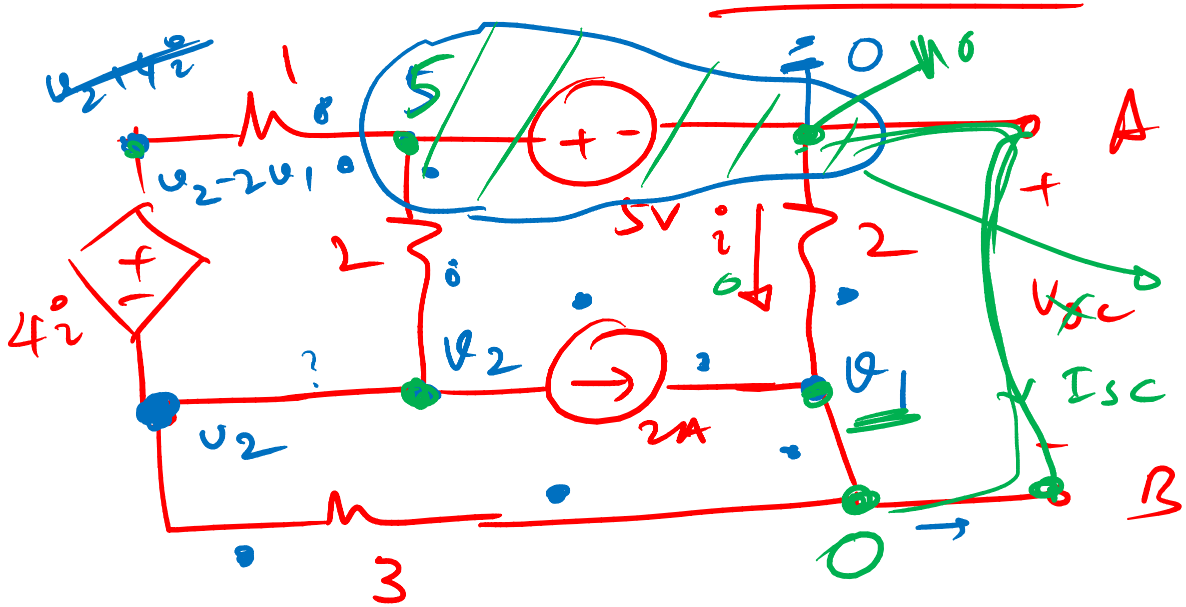
$i_a = -\frac{2}{2} = -1 A. \rightarrow$ find i_a, i_b

$P_{0.5v^2} = 0.5v^2 \times i = 2 \times (-i_a) = 2 \times 1 = 2W.$ (abs)

$P_{4V} = 4 \times i' = 4 \times (i_a - i_b) = 4(-1-0.33)$

$= -4 \times 1.33$ W. (Del)

Thevenin's



$$V_{oc} = 0 - V_1$$
$$= -v_1$$

$$5 - (v_2 - 2v_1) + \frac{5 - v_2}{2} + 0 + I_{sc} = 0$$

I_{sc}?

$$i = \frac{0 - v_1}{2} = -0.5 v_1 \qquad v_2 + 4i = v_2 + 4(-0.5 v_1)$$
$$= v_2 - 2v_1$$

$\boxed{V_2}$

KCL at v_1:

$$\frac{v_1}{2} - 2 + \frac{v_1 - V_2}{3} = 0 \quad (1)$$

KCL at S.N.:

$$\frac{5 - (V_2 - 2v_1)}{1} + \frac{5 - v_2}{2} + \frac{0 - V_1}{2} = 0 \quad (2)$$

$\Rightarrow V_1, V_2$