# Tutorial 1

1. Use Kirchoff's current law to determine the unknown currents in the circuit Figure 1. Assume that $I_0$=-2A, $I_1$=-4A, $I_s$=8A and $V_s$=12V.

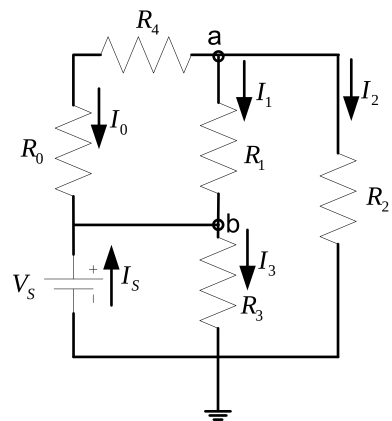



**Figure 1**

Analysis:
KCL can be used to find one unknown current if all the other currents into and out of a node are known.

Solution:

Applying KCL at node (a), sum of currents leaving node (a)=0

$$I_0 + I_1 + I_2 = 0 \Rightarrow I_2 = -(I_0 + I_1) = -(-2-4) = 6A$$

Applying KCL at super node (b), sum of currents leaving = 0

$$-I_0 - I_s - I_1 + I_3 = 0 \Rightarrow I_3 = I_0 + I_s + I_1 = -2-4+8 = 2A$$

2. Apply KVL to find the voltages v1 and v2 in Figure 2.

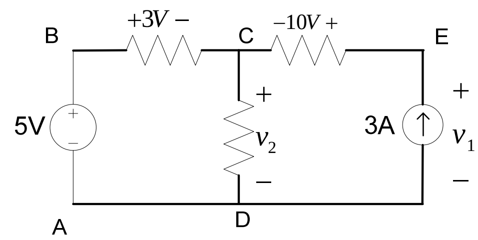



**Figure 2**

Analysis:

According to KVL, the algebraic sum of voltage rises (or voltage drops) around a closed loop is zero.

Solution:

Applying sum of voltage rises around the loop ABCDA, we get

$$5-3-v_2=0 \Rightarrow v_2=2V$$

Applying sum of voltage falls (just to show as an alternate way) around the loop ABCEDA:

$$-5+3-10+v_1=0 \Rightarrow v_1=12V$$

3. For the circuit given in Figure 3,
   a. Determine which components are absorbing power and which are delivering power.
   b. Is conservation of power satisfied?

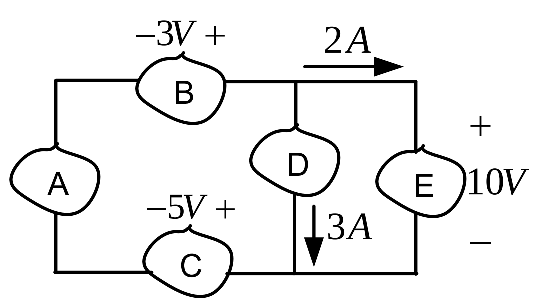


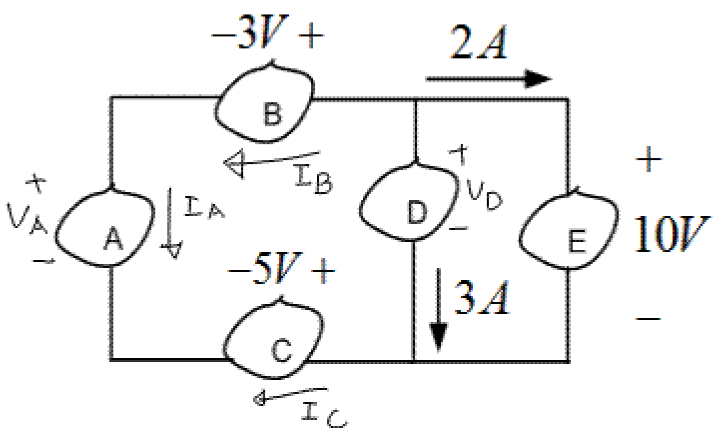

**Figure 3**

Analysis:
We can do circuit analysis to find the voltage and current associated with all the elements in the circuit. Then we can find the power delivered each element.

We can label the unknown voltages and currents for the circuit following passive sign convention (current entering into the positive reference terminal for voltage).

Solution:

If we consider the loop containing ABEC, and apply KVL (sum of voltage rises):




$V_A + 3 - 10 - 5 = 0 \Rightarrow V_A = 12V$

$V_D = V_E = 10V$

Applying KCL at the junction (node) of B, D and E; sum of outgoing currents =0:

$I_B + 2 + 3 = 0 \Rightarrow I_B = -5A$

As elements A, B and C are in series,

$I_A = I_B = -5A$

$I_C = -I_A = 5A$

With voltages and currents known for all the elements, following the passive sign convention.

We can calculate the power as product of voltage and current:

$P_A = V_A I_A = 12*(-5) = -60W$ i.e. element A is delivering power.

$P_B = V_B I_B = 3*(-5) = -15W$ i.e. element B is delivering power.

$P_C = V_C I_C = 5*5 = 25W$ i.e. element C is absorbing power.

$P_D = V_D I_D = 10*3 = 30W$ i.e. element D is absorbing power.

$P_E = V_E I_E = 10*2 = 20W$ i.e. element E is absorbing power.

Total power supplied = 60+15=75W

Total power absorbed = 25+30+20=75W

Conservation of power is proved.

4. In the circuit given in Figure 4, the power absorbed by the 15-Ohm resistor is 15W. Find R.

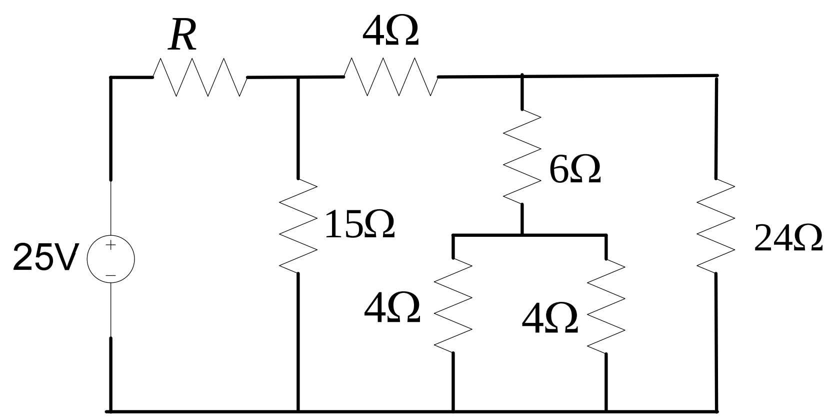



**Figure 4**

Analysis:

We can apply series/parallel equivalent for resistors to find the equivalent resistance parallel to the  We can then apply current divider principle to find the current in the branch parallel to the 15Ohm branch. We can also find the total current through the unknown resistor.

As the voltage drop across the 15 Ohm resistor is known, from KVL, we can find the voltage drop across the unknown resistor R.

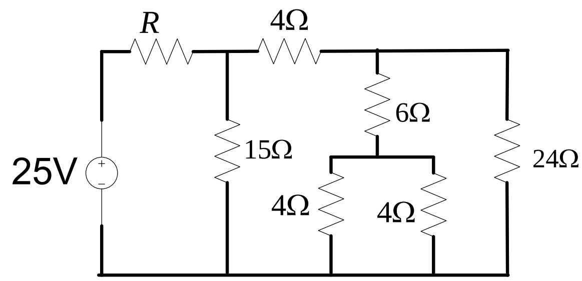
Then we can determine the unknown resistor R.

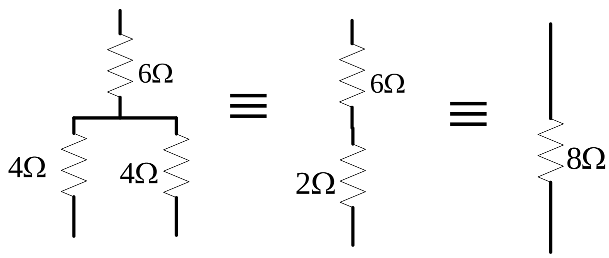
Solution:

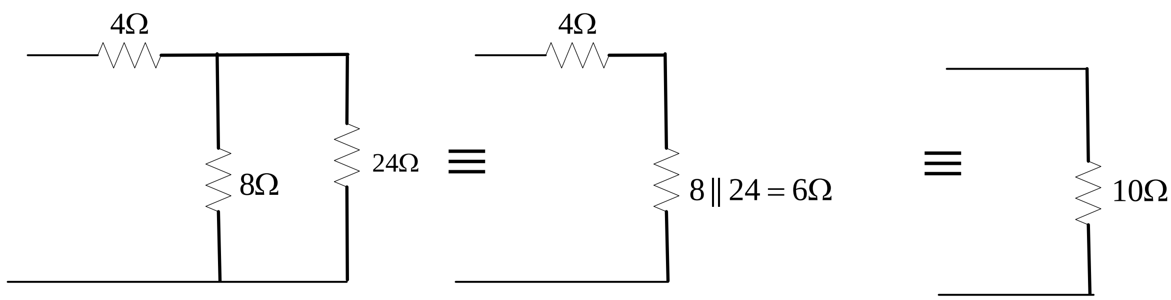
$$R = 15\Omega, P = 15W$$

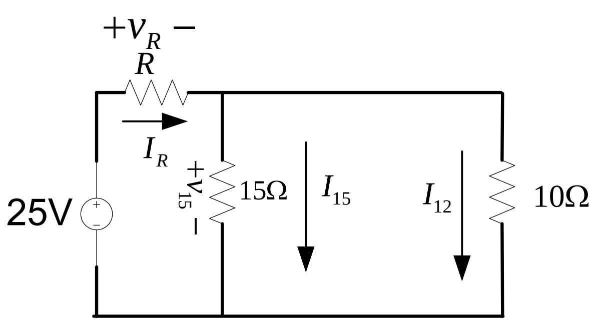
$$P = I^2 R \Rightarrow I = \sqrt{\frac{P}{R}} = \sqrt{\frac{15}{15}} = 1A$$

We can use series/parallel rules for resistors to find the new equivalent circuit:




Applying current division principle:

$$I_{15} = I_R \frac{10}{10+15} = I_R \frac{10}{25}$$
$$I_{10} = I_R \frac{15}{10+15} = I_R \frac{15}{25}$$
$$\frac{I_{10}}{I_{15}} = \frac{15}{10} \Rightarrow I_{10} = I_{15} \frac{15}{10} = 1.5A$$
$$I_R = I_{15} \frac{25}{10} = 2.5A$$

Applying KVL around the loop containing the power supply, R and 15 Ohm resistor:

$$v_{15} = 15V$$
$$25 - v_R - v_{15} = 0 \Rightarrow v_R = 25 - 15 = 10V$$

We can now calculate the resistance R:

$$R = \frac{v_R}{I_R} = \frac{10}{2.5} = 4\Omega$$

5. For the circuit shown in the Figure 5, find:
   i) The currents $i_1$ and $i_2$.
   ii) The power delivered by the *3A* current source and by the *12V* voltage source.
   iii) The total power dissipated by the circuit.

   R1=25 ohm, R2= 10ohm, R3=5 ohm, R4=7 ohm.

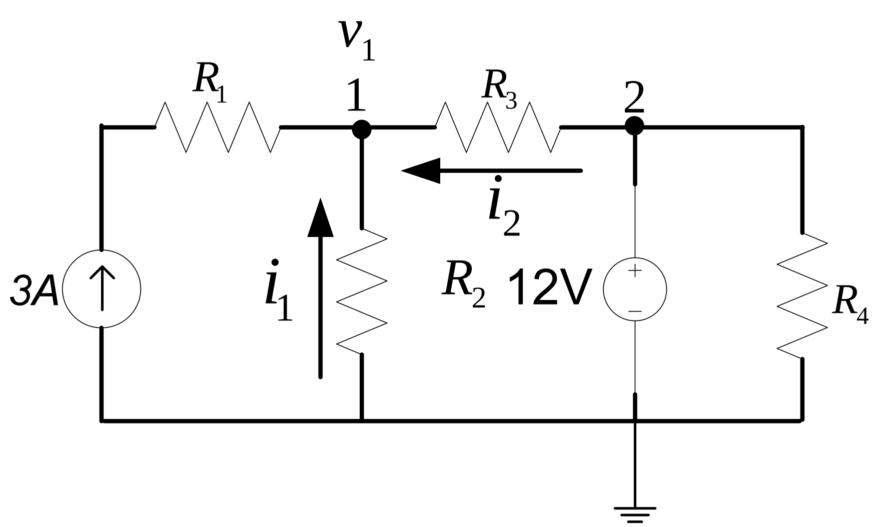



**Figure 5**

Solution:

We can find the currents if we know the voltage at node1.

Let the voltage at node 1 as $v_1$.

$$i_1 = -\frac{v_1}{R_2}$$

$$i_2 = \frac{12 - v_1}{R_3}$$

i)

Applying the KCL at node 1, algebraic sum of currents entering =0.

$$3+\frac{0-v_1}{R_2}+\frac{12-v_1}{R_3}=0$$

$$\frac{3R_2R_3-v_1R_3+12R_2-v_1R_2}{R_2R_3}=0 \Rightarrow v_1=3\frac{(4+R_3)R_2}{R_2+R_3}$$

Putting the values of the resistances, $v_1=18V$ .

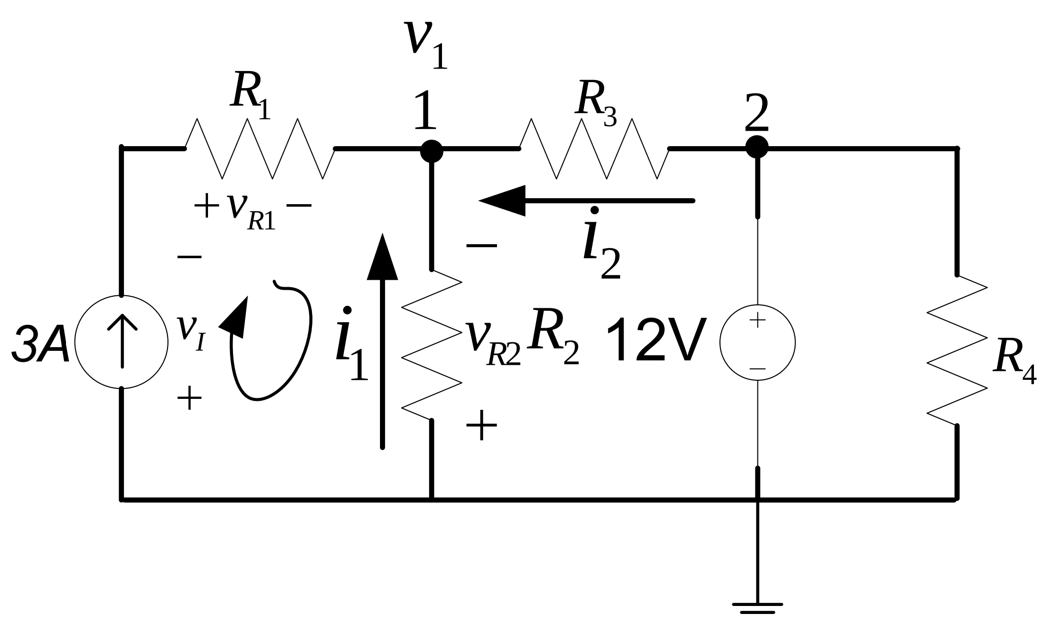
Therefore,

$$i_1=-\frac{18}{10}=-1.8A$$

$$i_2=\frac{12-v_1}{R_3}=\frac{12-18}{5}=-1.2A$$

ii)

To find the power delivered by the current source, we need to know the votlage across it. We have labeled the voltages across the elements in the mesh shown in the figure. We can apply KVL around the loop to find this voltage.




Starting from node1 going clock-wise along the mesh, the algebraic sum of voltage rises around the mesh:

$v_{R2} - v_I - v_{R1} = 0$

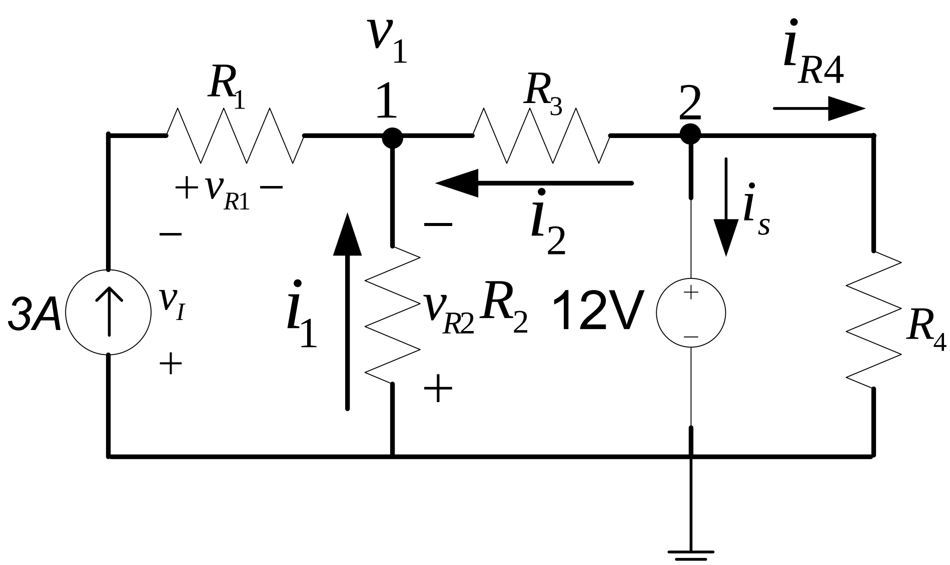
$i_1 R_2 - v_I - 3R_1 = 0$

$v_I = -1.8 \times 10 - 3 \times 25 = -18 - 75 = -93V$

$p = vi = -93 \times 3 = -279W$

We have labeled the voltage and current reference directions according to the passive sign convention. Thus negative power implies that power is delivered by the current source.

To find the power associated with the voltage source, we need to know the current through the voltage source. For this, we assign the current in resistance R4 and the source as shown.




Applying KCL at node2:

Algebraic sum of currents entering the node =0.

$-i_s - i_2 - i_{R4} = 0$

$$i_s = -i_2 - i_{R4} = 1.2 - \frac{12}{R_4} = -514.3mA$$

Power associated with the voltage source = $12 \times (-0.5143) = -6.17W$ i.e. power is delivered by the source.

ii)

Total power dissipated by the circuit can be obtained from the conservation of energy principle, by equating with the total power delivered by the two sources.

Thus, total power dissipated = 279+6.17=285.7W.

6. Given the circuit of Figure 6:
   i) Determine the power delivered by the dependent current source.
   ii) Determine the power delivered by the voltage source.

   Ans. 0W

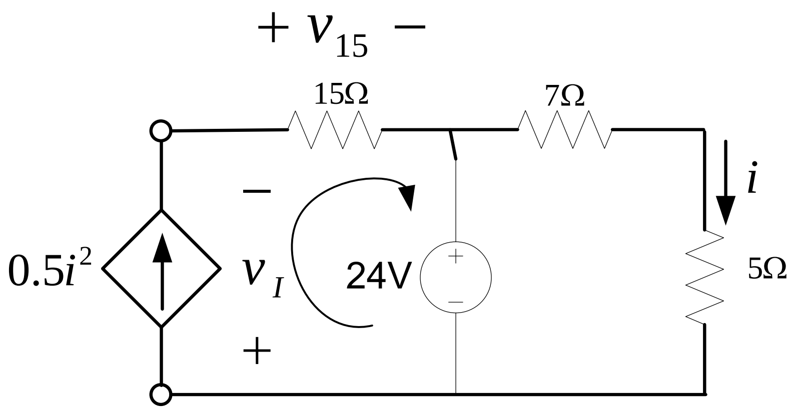



**Figure 6**

Analysis:
To solve the voltage and current values for the dependent source.

We can find the current $i$ from the circuit directly. Then we can find the value of the dependent current source. Then, we can apply KVL in the left mesh and find the voltage across the dependent current source.

Solution:

$$i = \frac{24}{7+5} = 2A$$

Value of the current source = $0.5 \times 4 = 2A$

Current through the 15 Ohm resistor = 2A

Applying KVL around the left mesh:
We have labeled the voltages according to passive sign convention.
Starting with the negative terminal of the voltage source, summing the voltage rises:

$$-v_I - v_{15} - 24 = 0$$

$$-v_I - 2\times 15 - 24 = 0 \Rightarrow v_I = -54V$$

Power associated with the dependent current source = $v_I \times 2 = -54 \times 2 = -108W$ i.e power is delivered by the dependent current source.

Please note that there is current through the 24V source and hence no power delivered by the voltage source.

7. Consider NiMH hobbyist batteries shown in the circuit of Figure 7:
   i) If $V_1$=12.0V, $R_1$=0.15 ohm, $R_L$=2.55 ohm, find the load current $I_L$ and the power dissipated by the load.
   ii) If we connect a second battery in parallel with battery 1 that has voltage V2=12V and R2=0.28 ohm, will the load current $I_L$ increase or decrease? By how much?

   Use mesh current analysis method.

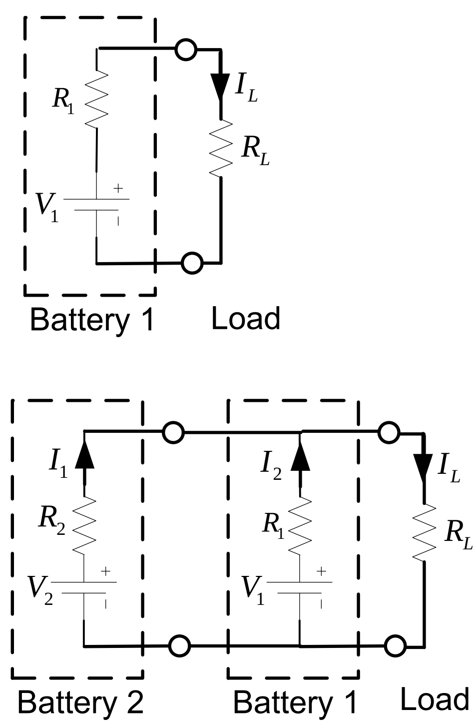



**Figure 7**

Analysis:
We can find the current with one battery easily.

When the second battery is connected, we can apply mesh current analysis to find the load current.

Solution:

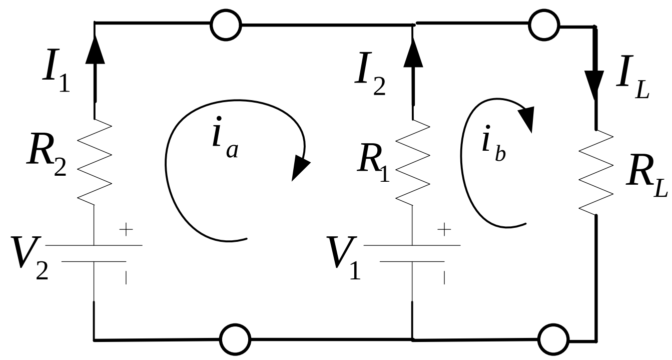
i)

With the first battery, the load current: $i_L = \dfrac{V_1}{R_1 + R_L} = \dfrac{12}{0.15 + 2.55} = \dfrac{12}{2.70} = 4.44A$.

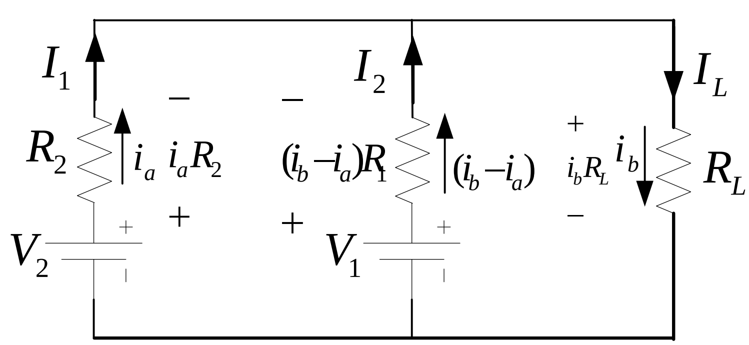
Power in the load = $i_L^2 R = 4.44 \times 4.44 \times 2.55 = 50.4W$

ii)

We can consider two meshes as shown in Figure:




Then, we can find the branch currents and voltages in terms of the mesh currents:




Then, we apply the KVL around mesh a and mesh b to obtain the two independent equations.

$$V_2 - i_a R_2 + (i_b - i_a)R_1 - V_1 = 0$$
$$V_1 - (i_b - i_a)R_1 - i_b R_L = 0$$

Rearranging the two equations:

$$-i_a(R_1 + R_2) + i_b R_1 = V_1 - V_2 \qquad (1)$$

$$i_a R_1 - i_b(R_1 + R_L) = -V_1 \qquad (2)$$

Putting the values of the resistors and voltages we get:

$$eqn(1): -0.43 i_a + 0.15 i_b = 0 \Rightarrow i_a = \frac{0.15}{0.43} i_b$$
$$eqn(2): 0.15 i_a - 2.70 i_b = -12 \Rightarrow i_b = 4.53A, i_a = 1.58A$$

The new load current = 4.53 A

The load current increases by 4.53-4.44=0.09A

8. Using Node Voltage Analysis method, find the current i through the voltage source in the circuit of the Figure 8.

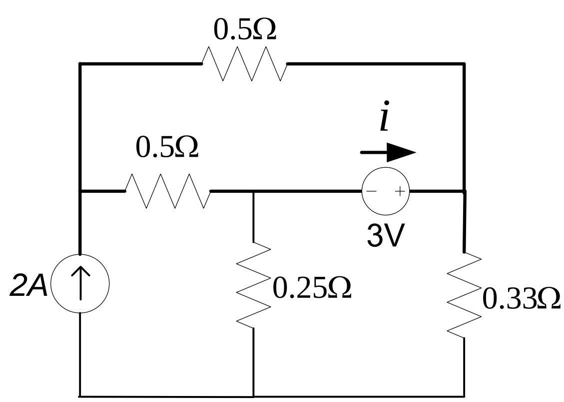
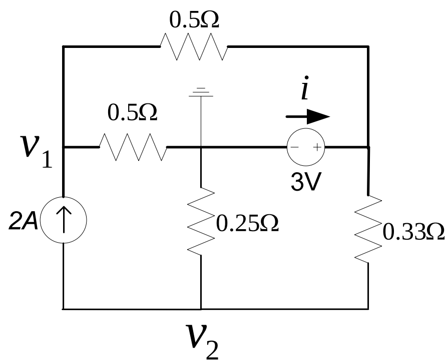



**Figure 8**

Solution:




The negative terminal of the voltage source is marked as the reference.

Two nodes are marked as node voltage $v_1, v_2$.

The KCL at the two nodes will be:

$$\frac{v_1 - 3}{0.5} + \frac{v_1}{0.5} - 2 = 0 \quad (1)$$

$$\frac{v_2}{0.25} + \frac{v_2 - 3}{0.33} + 2 = 0 \quad (2)$$

Eqn(1)and (2) give:

$$v_1 = 2V$$

$$v_2 = \frac{-2*0.25*0.33 + 3*0.25}{0.25 + 0.33} = 1.01V$$

The current through the voltage source, by apying KCL:

$$i = \frac{3 - v_1}{0.5} + \frac{3 - v_2}{0.33} = 2 + 6.03 = 8.03A$$

9. Using KCL, perform node analysis n the circuit shown in Figure 9 and determine voltage across R4. Note that one source is a controlled voltage source! Let Vs=5V, Av=70, R1=2.2kohm, R2=1.8kohm, R3=6.8kohm, R4=220ohm.

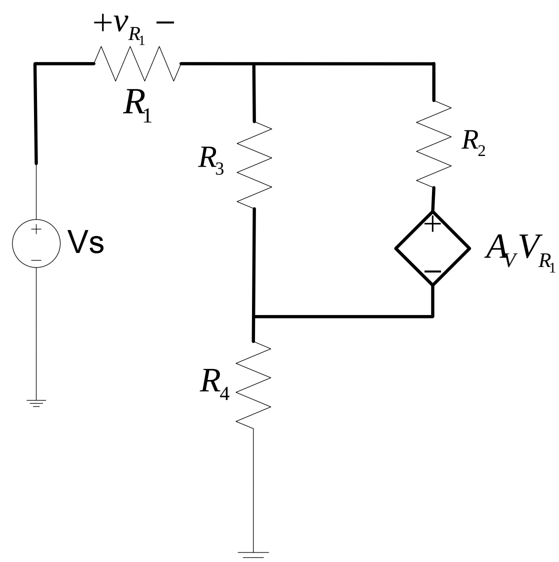
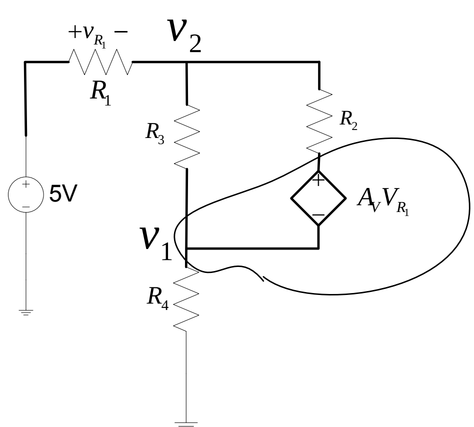



**Figure 9**

Solution:

There are two unknown node voltages as shwon in the figure:




The dependent source voltage can be written in terms of the node voltages:

$A_v V_{R1} = A_v(5 - v_2)$

Writing the KCL at the two nodes:

At the super node:

$$\frac{v_1}{R_4}+\frac{v_1-v_2}{R_3}+\frac{v_1+A_v(5-v_2)-v_2}{R_2}=0$$

$$v_1\left(\frac{1}{R_4}+\frac{1}{R_3}+\frac{1}{R_2}\right)+v_2\left(-\frac{1}{R_3}+\frac{-A_v-1}{R_2}\right)=\frac{-5A_v}{R_2}$$

At node2:

$$\frac{v_2-5}{R_1}+\frac{v_2-v_1}{R_3}+\frac{v_2-(v_1+A_v(5-v_2))}{R_2}=0$$

$$v_1\left(-\frac{1}{R_3}-\frac{1}{R_2}\right)+v_2\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1+A_v}{R_2}\right)=\frac{5A_v}{R_2}+\frac{5}{R_1}$$

Putting the values (factoring our $10^{-3}$), we get:

$$5.2481v_1-39.592v_2=-194.44$$

$$-0.7026v_1+40.046v_2=196.717$$

Solving these equaiton swe get:

$$v_1=8.7572mV$$

$$v_2=4.9124V$$

# EE1002/CG1108 Tutorial 3

10. Determine, using superposition, the voltage $v$ across $R$ in the circuit of Figure 10.

$$I_B = 3A, R_B = 1\Omega, V_G = 15V, R_G = 1\Omega, R = 2\Omega$$

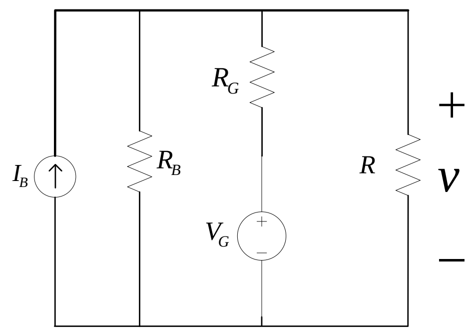


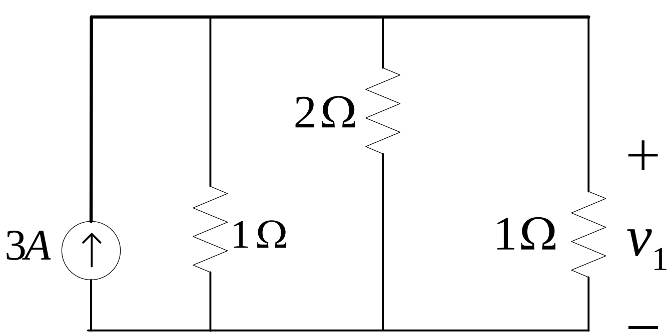

**Figure 10**

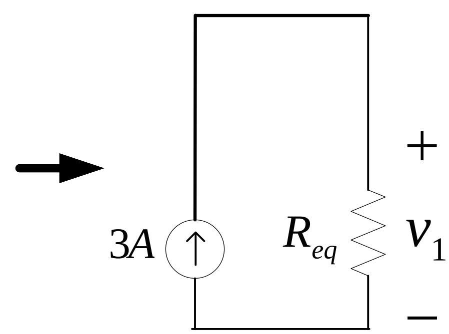
Analysis:
To evaluate the voltage with only one source at a time. Finally add up the two voltages.

Kill the voltage source by replacing it with a short. Kill the current source by replacing it with an open circuit.

Solution:

Kill the voltage source by shorting it as shown here. Obtain the voltage due to the current source alone.



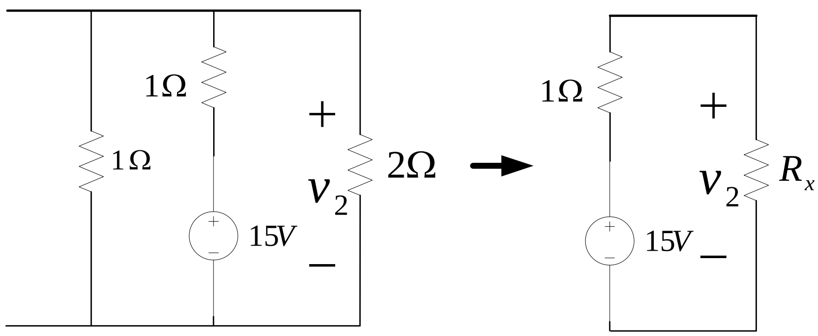




The three resistors in parallel can be reduced to their equivalent resistance:

$$\frac{1}{R_{eq}}=\frac{1}{1}+\frac{1}{2}+\frac{1}{1}=2.5\Rightarrow R_{eq}=\frac{2}{5}\Omega$$

$$v_1=3\times\frac{2}{5}=\frac{6}{5}=1.2V$$

Then, kill the current source by opening it as shown here. Obtain the voltage due to the current source alone.




Then, the two resistors $(1\Omega, 2\Omega)$ in parallel can be reduced to its equivalent resistance *R*$R_x$.

$$\frac{1}{R_x}=\frac{1}{1}+\frac{1}{2}=\frac{3}{2}\Rightarrow R_x=\frac{2}{3}\Omega$$

Then, voltage divider principle can be used to find the voltage across $R_x$.

$$v_2=15\times\tfrac{R_x}{R_x+1}=15\times\frac{2/3}{2/3+1}=6V$$

Finally, the original voltage in question can be obtained as $v=v_1+v_2=1.2+6=7.2V$ .

11. Find the Thevenin equivalent circuit that the load ( $R_L$ ) sees for the circuit of Figure 11.

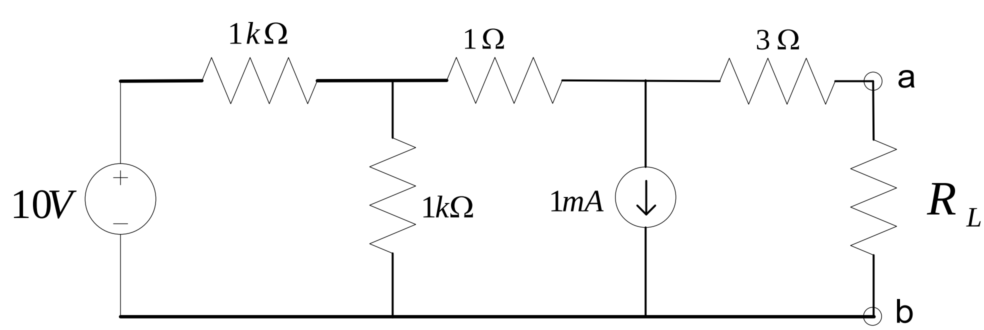


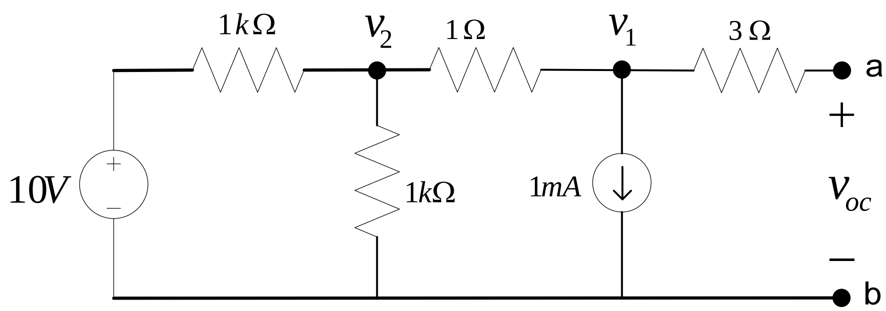

**Figure 11**

Analysis:

We need to find the open circuit voltage between points a and b, which will be the Thevenin's voltage.

Then we need to find the Thevenin resistance.

Solution




Let us apply node voltage analysis method. Note the reference node and the two unknown node voltages, $v_1, v_2$.

Writing KCL at $v_1$:

$$\frac{v_1 - v_2}{1} + 0.001 = 0 \Rightarrow v_1 - v_2 = -0.001 \qquad (1)$$

Writing KCL at node $v_2$:

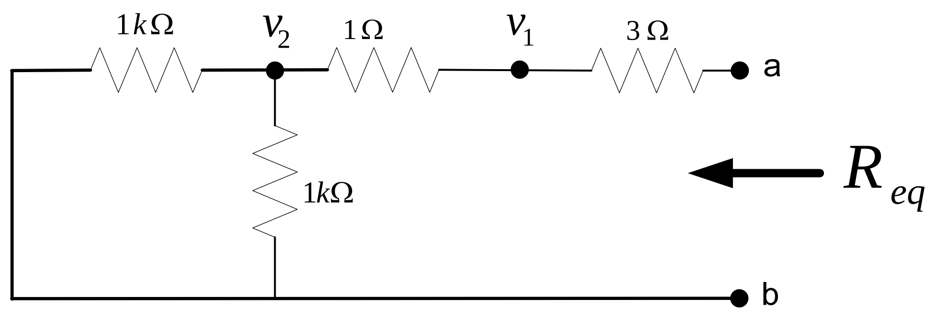
$$\frac{v_2 - 10}{1000} + \frac{v_2}{1000} + \frac{v_2 - v_1}{1} = 0 \qquad (2)$$

Putting the eqn(1) in eqn(2), and multiplying both sides by 1000, we get:

$$v_2 - 10 + v_2 + 1 = 0 \Rightarrow v_2 = \frac{9}{2} = 4.5V$$

Putting value of $v_2$ in eqn(1), we get $v_1 = v_2 - 0.001 = 4.499V$.
To find the value of the Thevenin resistance:

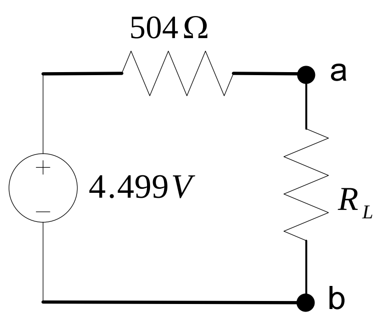
As the circuit contains independent sources only, we can kill the sources and calculate the equivalent resistance between the points $a$ and $b$.




The equivalent resistance can be calculated as:

$$R_{eq} = 3 + 1 + \frac{1000 \times 1000}{1000 + 1000} = 504\Omega \ .$$

The Thevenin equivalent circuit is:




12. For the circuit given in Figure 12:

   i) Obtain the Thevenin's equivalent for the circuit which contains a dependent voltage source.

   ii) What should be the optimum value of a load resistor $R_L$ to be connected between **a** and **b** so that the power delivered to it by the network is maximum?

   iii) What is the maximum power?

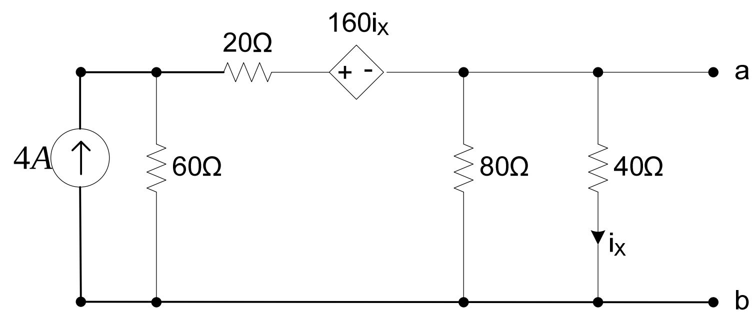


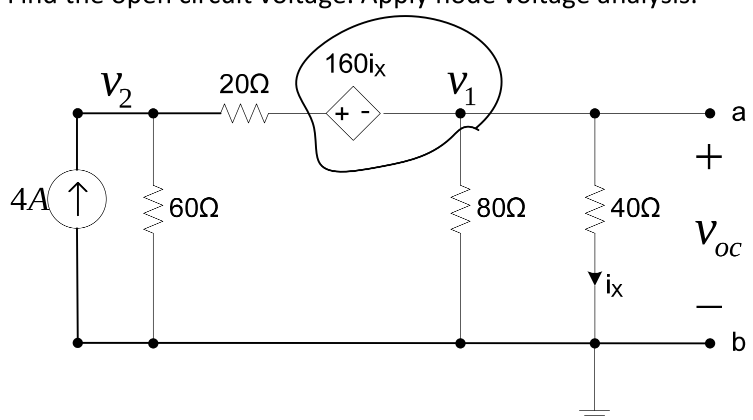

**Figure 12**

To find :
Thevenin's equivalent and then obtain the load at which power transfer is maximum.

Analysis:
We can apply node voltage analysis to find the Thevenin's voltage.
As there is a dependent source, we can use test source method to obtain the Thevenin's resistance.

Solution:
i)

Find the open circuit voltage. Apply node voltage analysis.




Note the choice of the reference node and the unknown node voltages $v_1, v_2$.

We can write the current $i_x$ as $i_x = \dfrac{v_1}{40}$.

Then the dependent voltage source becomes $160 i_x = 160 \times \dfrac{v_1}{40} = 4v_1$.

Applying KCL at the super node surrounding $v_1$ :

$$\frac{v_1}{40} + \frac{v_1}{80} + \frac{(v_1 + 160 i_x) - v_2}{20} = 0 \qquad (1)$$

Replacing $i_x$ we get $v_1 + 160 i_x = v_1 + 160 \times \dfrac{v_1}{40} = 5v_1$

Rewriting equation (1), we get:

$$v_1\left(\frac{1}{40}+\frac{1}{80}+\frac{5}{20}\right)-\frac{v_2}{20}=0 \Rightarrow v_1\frac{23}{4}-v_2=0 \qquad (1)$$

Applying KCL at node $v_2$:

$$-4+\frac{v_2}{60}+\frac{v_2-(v_1+160i_x)}{20}=0$$

Replacing $(v_1+160i_x)$ by $5v_1$:

$$-\frac{5}{20}v_1+v_2\left(\frac{1}{60}+\frac{1}{20}\right)=4 \Rightarrow -\frac{v_1}{4}+\frac{v_2}{15}=4 \qquad (2)$$

$(1)+(2)\times 23$:

$$-v_2+v_2.\frac{23}{15}=4\times 23 \Rightarrow v_2=\frac{23\times 15}{2}$$

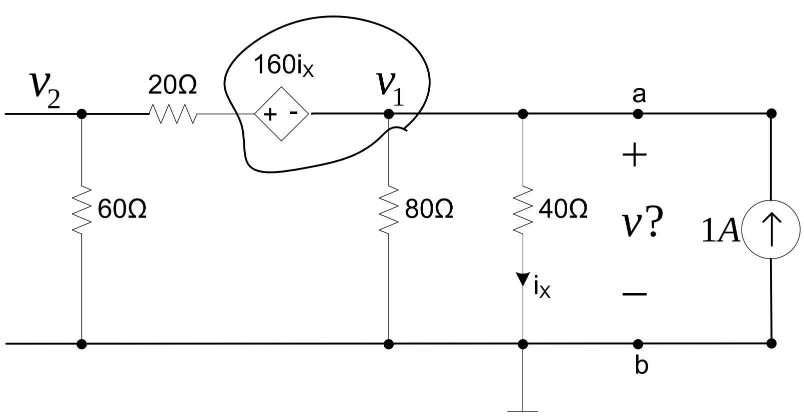
Putting the value of $v_2$ in eqn (1), we get:

$$v_1=v_2\times\frac{4}{23}=30V$$

Thus the open circuit voltage is $v_{oc}=v_1=30V$ .

**Find the Thevenin resistance using 'Test source method'.**

We shall kill the independent current source of 4A and connect a current source of 1A to the terminals and then calculate the voltage across them.




Applying node voltage analysis method. Note the choice of the reference node and the unknown node voltages.

We can write the current $i_x$ as $i_x=\frac{v_1}{40}$.

Then the dependent voltage source becomes $160i_x = 160 \times \frac{v_1}{40} = 4v_1$.

Applying KCL at the super node surrounding $v_1$:

$$-1+\frac{v_1}{40}+\frac{v_1}{80}+\frac{(v_1+160i_x)-v_2}{20}=0 \Rightarrow v_1\left(\frac{1}{40}+\frac{1}{80}+\frac{5}{20}\right)-\frac{v_2}{20}=1 \Rightarrow v_1\frac{23}{4}-v_2=20 \qquad (1)$$

Applying KCL at $v_2$:

$$\frac{v_2}{60}+\frac{v_2-(v_1+160i_x)}{20}=0$$

Replacing $(v_1+160i_x)$ by $5v_1$:

$$-\frac{5}{20}v_1+v_2\left(\frac{1}{60}+\frac{1}{20}\right)=0 \Rightarrow -\frac{v_1}{4}+\frac{v_2}{15}=0 \qquad (2)$$

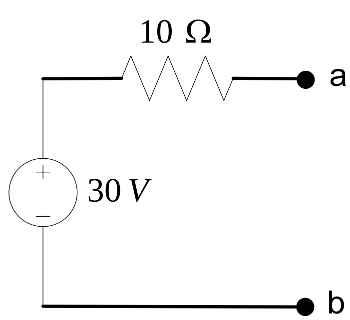
(1)+(2)X23=>

$$-v_2+v_2\frac{23}{15}=20 \Rightarrow v_2=\frac{20\times 15}{8}$$

Putting the value of $v_2$ in eqn(2), we get:

$$-\frac{v_1}{4}+\frac{20\times 15}{15\times 8}=0 \Rightarrow v_1=10V$$

Thevenin resistance is $\frac{v}{1}=10\Omega$. Thevenin equivalent is:




ii)

According to maximum power transfer theorem, the load resistance for maximum power transfer would be same as the soruce resistance (Thevenin resistance).

$$R_L = 10\Omega$$

iii)

Maximum power delivered to the load at $R_L = 10\Omega$:

$$P_{L(\max)}=\left(\frac{30}{10+10}\right)^2 10=\frac{9}{4}\times 10=22.5W$$

# EE1002/CG1008 Tutorial 4

13. If the switch in the circuit of Figure 13 is closed at t=0,
    i) Determine the current flow through the resistors and the capacitor when t=0+.
    ii) What will be the current flow under steady state condition?
    iii) Determine the voltage across the capacitor under steady state condition.
    iv) Find an expression for the capacitor voltage as a function of time t>0.

    Assume that the capacitor is initially uncharged.

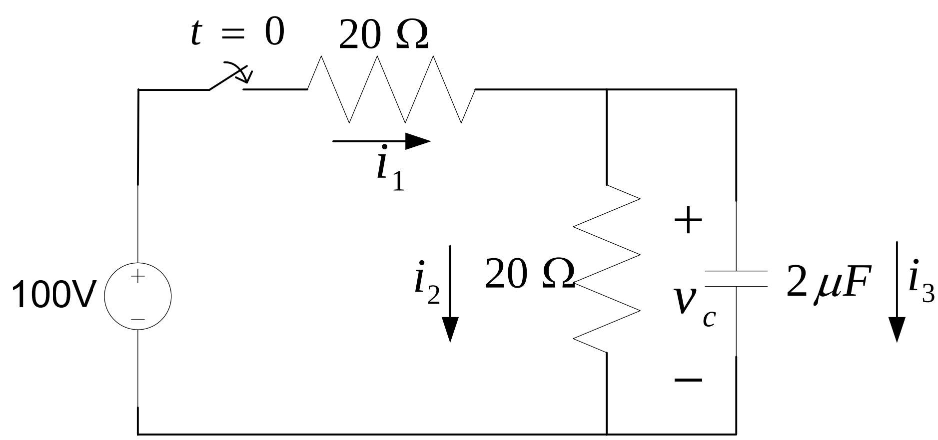



**Figure 13**

Solution:

As capacitor voltage cannot change instantaneously, $v_c(0-) = v_c(0+)$, i.e. the voltage immediately before the switch is closed will be same as the voltage immediately after the switch is closed.

$v_c(0-)$ can be obtained by the DC analysis before switch is closed.

As can be seen, the capacitor will be discharged before switch is closed i.e. $v_c(0-) = 0$.

Thus, $v_c(0+) = v_c(0-) = 0$.

i)

Current through the resistors immediately after switch is closed:

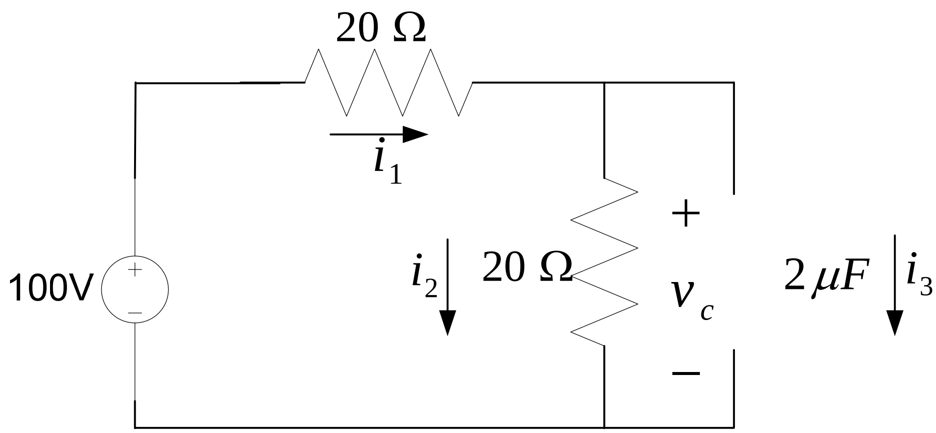
$$i_1(0+) = \frac{100-0}{20} = 5A$$

$$i_2(0+) = \frac{0}{20} = 0A$$

$$i_3(0+) = 5-0 = 5A$$

ii)

At steady-state, the capacitor will be open circuited.




$$i_3(\infty) = 0$$

$$i_1 = i_2 = \frac{100}{20+20} = 2.5A$$

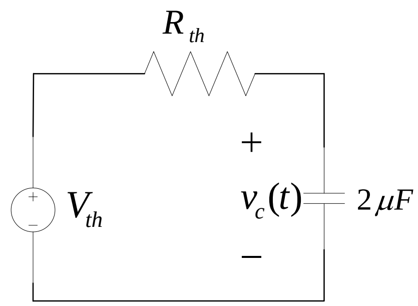
iii)

At steady-state , the capacitor voltage can be obtained using voltage divider principle:

$$v_c(\infty) = 100 \times \frac{20}{20+20} = 50V$$

iv)

To find the capacitor voltage as a function of time $t$ i.e. $v_c(t)$ at $t = 0+$, we shall put the circuit in a standard form i.e. the Thevenin's equivalent of rest of the circuit seen from the capacitor.




$R_{th} = 10\Omega, V_{th} = 50V$

Time constant of the circuit $\tau = R_{th}C = 10 \times 2 \times 10^{-6} = 20\mu S$

$v_c(0+) = v_c(0-) = 0V$

$v_c(\infty) = V_{th} = 50V$

$$v_c(t) = v_c(0+)e^{-\frac{t}{\tau}} + v_c(\infty)\left(1 - e^{-\frac{t}{\tau}}\right) = 50\left(1 - e^{-\frac{t}{20\times10^{-6}}}\right)V$$

14. For the circuit shown in Figure 14, assume that switch S1 was closed and switch S2 was opened for a long time. Then, at time t=0, switch S1 is opened and switch S2 is closed.

 i) Find the capacitor voltage vc(t) at t=0+.
 ii) Find the time constant $\tau$ for t>=0.
 iii) Find an expression for vc(t), and sketch the function.
 iv) Find vc(t) for each of the following values of t zero, the time constant, twice the time constant, five times the time constant and ten times the time constant.

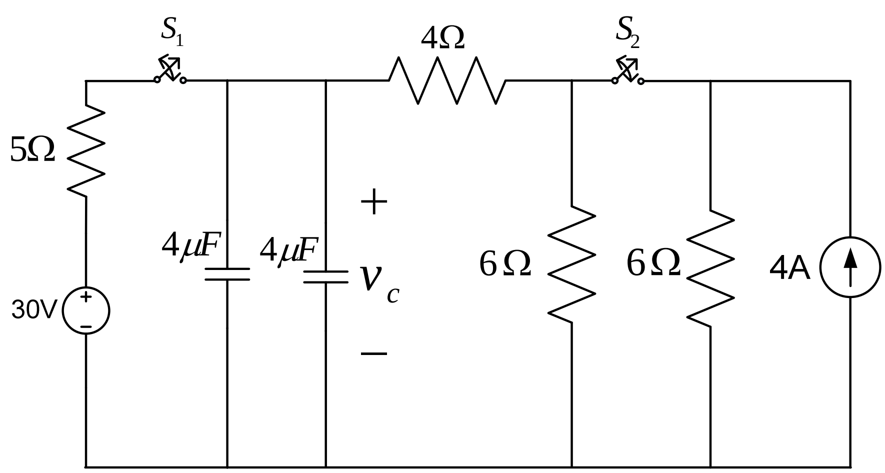



**Figure 14**

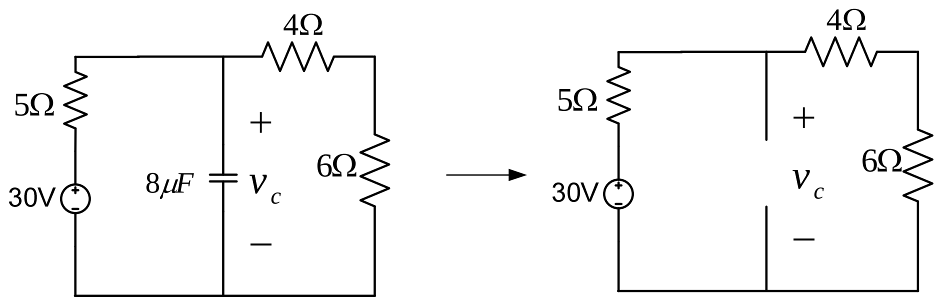
Analysis:

First, we need to find the capacitor voltage $v_c(0-)$ before time t=0, which will be same as $v_c(0+)$.

Then, we need to find the Thevenin equivalent of the circuit after time t=0, so that we can use the standard formula for capacitor voltage transient.

Solution:

With only S1 closed, and S2 open, the circuit is reduced to:

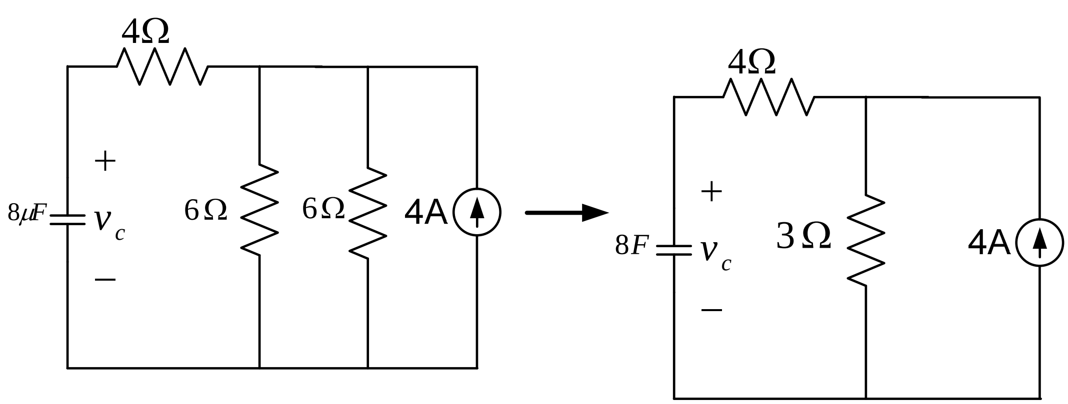



The two capacitors in parallel are put as their equivalent.

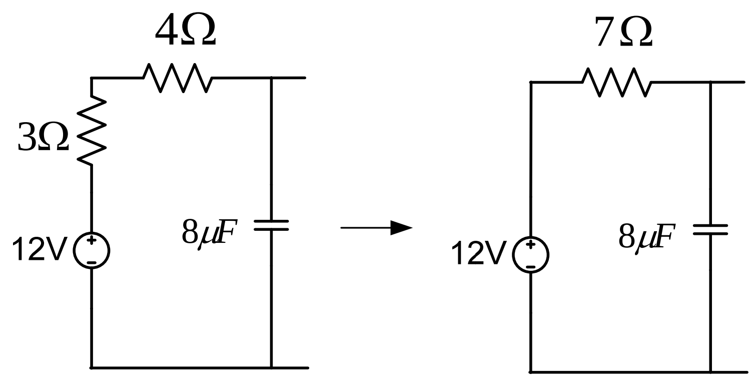
As the capacitor behaves like an open circuit in steady-state, we can use voltage divider principle:

$$v_c(0-) = 30 \times \frac{10}{15} = 20V = v_c(0+)$$

Next when switch S1 is opened and S2 is closed, the circuit becomes:




We can use source transformation, to replace the 4A current source with $3\Omega$ parallel resistor by a voltage source of 12V and series resistance of $3\Omega$.




Time constant of the circuit after $t \geq 0$, $\tau = RC = 7 \times 8\mu S = 56\mu S$.

$v_c(\infty) = 12V$

$$v_c(t) = v_c(0)e^{-\frac{t}{\tau}} + v_c(\infty)\left(1 - e^{-\frac{t}{\tau}}\right) = v_c(\infty) + \left(v_c(0+) - v_c(\infty)\right)e^{-\frac{t}{\tau}} =$$

$$= 12 + (20 - 12)e^{-\frac{t}{56\times10^{-6}}} = 12 + 8e^{-\frac{t}{56\times10^{-6}}}$$

15. For the circuit given in Figure 15, switch S2 was closed for a long time before t=0. At t=0, the switch S1 is closed and S2 is opened.
    i) Find the inductor current i(t) at t=0+.
    ii) Find the time constant $\tau$ for t>=0.
    iii) Find an expression for i(t).
    iv) Find i(t) for each of the following values, the time constant, twice the time constant, five times the time constant and ten times the time constant. Sketch the function

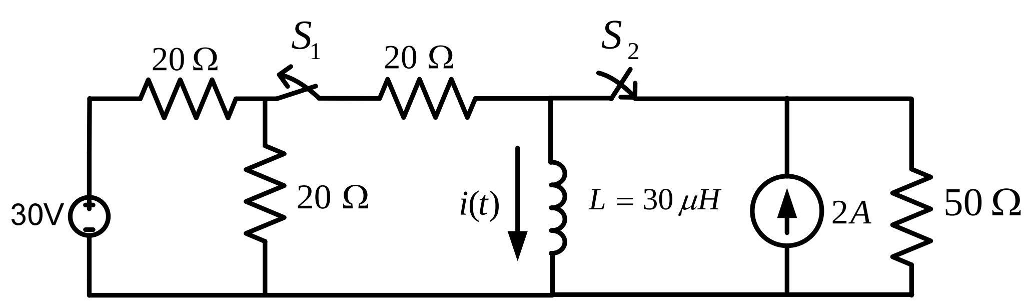



**Figure 15**

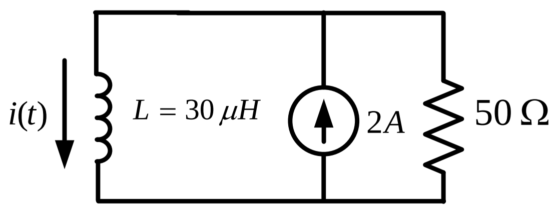
Analysis:
When switch S2 is closed for a long time, the circuit should be in steady-state. We can consider the inductor as a short-circuit and then determine the current in it.

Also, inductor current cannot change instantaneously and hence, the current immediately before t=0 will be same as immediately after t=0.

Solution:

i)

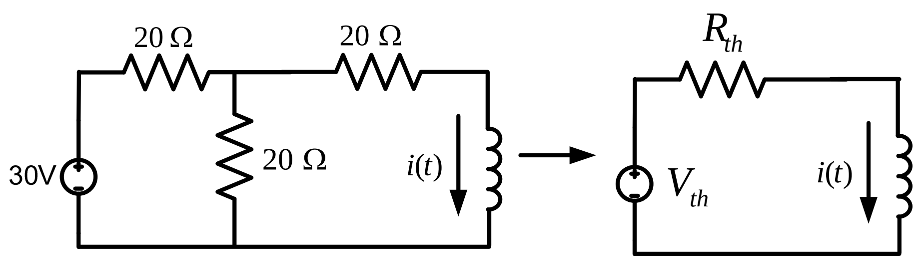
When S1 is open and S2 is closed, the circuit would be like:




The inductor would act as a short circuit, and hence the current through the inductor will be 2A.

$i(0-) = i(0+) = 2A$

At t=0, when S1 is closed and S2 is opened, the circuit is reduced which is then converted to its Thevenin equivalent form:




ii)

The Thevein voltage and Thevenin resistance for the circuit on the left can be obtained by inspection alone.

$$V_{th} = 30 \times \frac{20}{20+20} = 15V$$

$$R_{th} = 20 + \frac{20}{2} = 30\Omega$$

The time constant for t>0 is $\tau = \frac{L}{R} = \frac{30\mu H}{30\Omega} = 1\mu S$

iii)

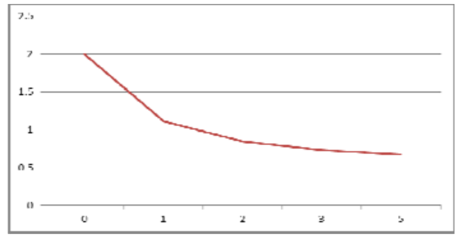
$$i(t) = i(0+) \times e^{-\frac{t}{\tau}} + i(\infty) \times \left(1 - e^{-\frac{t}{\tau}}\right) = i(\infty) + \left(i(0+) - i(\infty)\right) \times e^{-\frac{t}{\tau}} \text{ A}$$

As the inductor will behave like short circuit in steady-state, $i(\infty) = \frac{V_{th}}{R_{th}} = \frac{15}{30} = 0.5A$

So the current will be decaying exponentially from 2A to 0.5A.

iv)

$$i(t)_{t=\tau} = 0.5 + 1.5 \times e^{-1} = 1.0518A$$
$$i(t)_{t=2\tau} = 0.5 + 1.5 \times e^{-2} = 0.7030A$$
$$i(t)_{t=3\tau} = 0.5 + 1.5 \times e^{-3} = 0.5498A$$
$$i(t)_{t=5\tau} = 0.5 + 1.5 \times e^{-5} = 0.5101A$$




# EE1002/CG1108 Tutorial 5

16. For the circuit in Figure 16,
    - i) Find the expression for $v_R(t)$.
    - ii) If the sinusoidal has a frequency of 10 kHz, and the inductor is 1 $mH$, what is the value of R for phase difference between $v_s(t)$ and $v_R(t)$ to be 45 deg?
    - iii) Draw the phasor diagram showing the $v_s(t)$ and $v_R(t)$ for part (ii).

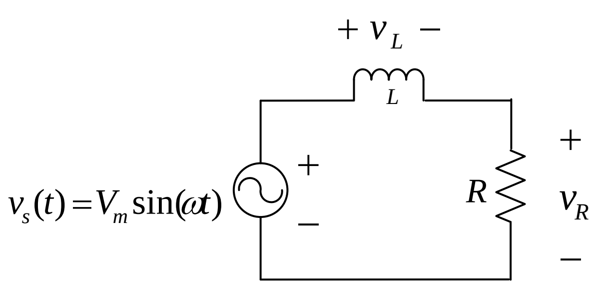



**Figure 16**

Analysis:
We need to replace the voltage source by its phasor and the circuit components by their impdances. Then we can use the DC circuit laws to solve the AC circuit.

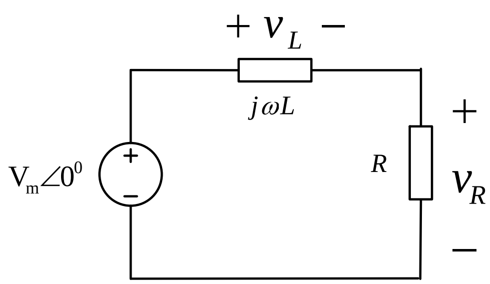
Solution.

i)

Given $v_s(t) = V_m \sin \omega t$ we can replace it by its phasor i.e. $\mathrm{V_m}\angle 0^0$.

Inductor L can be replaced by its impedance i.e. $Z_L = j\omega L = \omega L \angle 90^0$.

$Z_R = \mathrm{R} = \mathrm{R}\angle 0^0$




By applying voltage divider rule,

$$V_R = \frac{\mathrm{R}}{\mathrm{R} + j\omega L}\mathrm{V_m} = \frac{\mathrm{R}}{\sqrt{\mathrm{R}^2 + \omega^2 L^2}\angle\theta}\mathrm{V_m} = \frac{\mathrm{RV_m}}{\sqrt{\mathrm{R}^2 + \omega^2 L^2}}\angle -\theta$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$v_R(t) = \frac{\mathrm{RV_m}}{\sqrt{\mathrm{R}^2 + \omega^2 L^2}}\sin(\omega t - \theta)$$

ii)

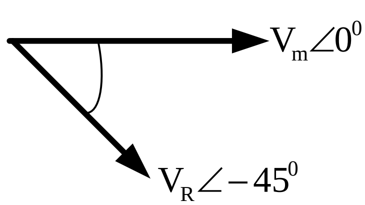
Phase difference between $v_s(t)$ and $v_R(t)$ is $\theta$.

$$f = 10000 \Rightarrow \omega = 2\pi f = 2 \times \pi \times 10000 = 62832$$

$$\theta = 45^0 \Rightarrow \tan\theta = 1 = \frac{\omega L}{R}$$

$$R = \omega L = 62832 * 1 \times 10^{-3} = 62.832\Omega$$

iii)



17. Determine the current i(t) in the circuit shown in Figure 17.

$$v_s(t) = 636\cos\left(3000t + \pi/12\right)$$
$$R_1 = 2.3k\Omega, R_2 = 1.1k\Omega$$
$$L = 190mH, C = 55nF$$

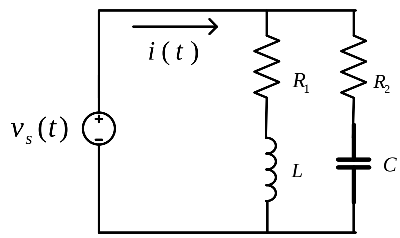
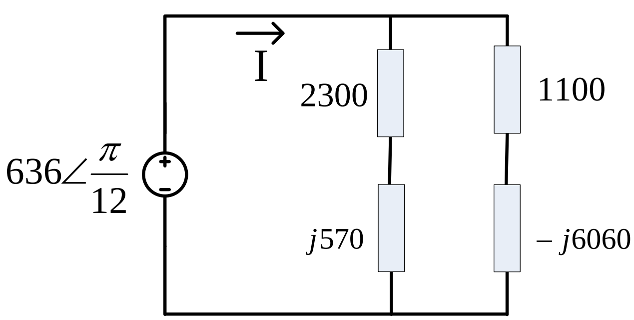



**Figure 17**

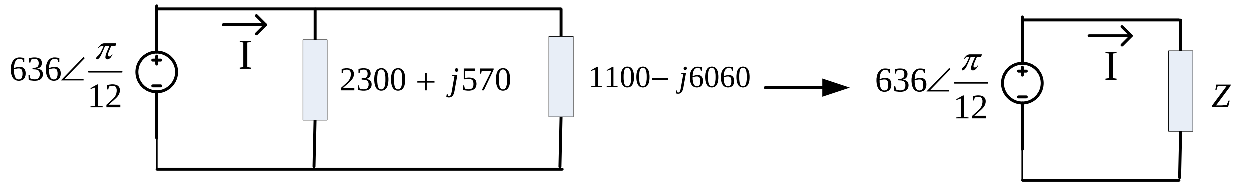
Solution:

Find the phasor for the voltage source and the impedances for the circuit components.

$$v_s(t) \Rightarrow 636\angle\frac{\pi}{12}$$
$$\omega = 3000$$
$$Z_L = j\omega L = j3000\times190\times10^{-3} = j570$$
$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{3000\times55\times10^{-9}} = -j6060$$
$$Z_{R1} = 2300$$
$$Z_{R2} = 1100$$




We can use the common techniques like series/parallel reduction for solving the circuit.




$$Z = \frac{(2300 + j570)\times(1100 - j6060)}{2300 + j570 + 1100 - j6060} = \frac{2369.6\angle 13.92^0 \times 6159\angle -79.7^0}{3400 - j5490} =$$

$= -$

$$Z = 2260\angle -7.56^0$$

$$\mathrm{I} = \frac{\mathrm{V_s}}{\mathrm{Z}} = \frac{636\angle 15^0}{2260\angle -7.56^0} = 0.2814\angle 22.56^0$$

$$i(t) = 0.2814\cos(3000t + 22.56^0)$$

18. Find the Thevenin equivalent of the circuit as seen from terminals a-b for the circuit shown in Figure 18.

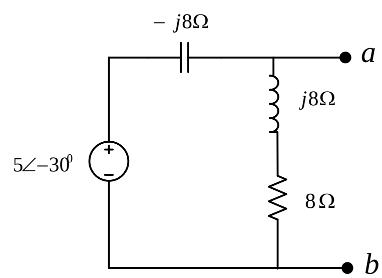



**Figure 18**

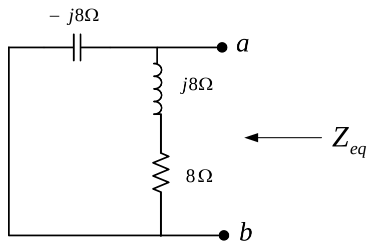
Analysis:
The circuit given ha the ac sources in phasor form and the circuit elements in their impedances. We can use the same techniques as DC circuit analysis, except that we have to deal with complex numbers.

Solution:
Open circuit voltage can be obtained using the voltage divider principle:

$$\mathrm{V_{th}} = \mathrm{V_{oc}} = 5\angle -30^0 \times \frac{8 + j8}{8 + j8 - j8} = 5\angle -30^0 \times \frac{8 + j8}{8} = 5\angle -30^0 \times (1 + j1) =$$

$$= 5\angle -30^0 \times \sqrt{2}\angle 45^0 = 5\sqrt{2}\angle 15^0$$

Thevenin impedance can be obtained by killing the voltage source and finding the equivalent impedance between *a* and *b*.




$$Z_{eq} = \frac{-j8\times(8+j8)}{8+j8-j8} = \frac{-j8\times(8+j8)}{8} = -j\times(8+j8) = 8-j8$$

The Thevenin equivalent of the circuit is: