EE1002
Introduction to Circuits and Systems

Part 1 : Lecture 8

DC Transients

Comparison of capacitor and inductor

Cap

Energy due to static charge

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

DC-steadystate behaviour:

$i_c = 0$ as $\frac{dv_c}{dt} = 0$

↳ open ckt

Series. $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

parallel $C_{eq} = C_1 + C_2 + C_3$

Energy $= \frac{1}{2} \cdot C \cdot V^2$

Ind

Energy due to moving charge

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$V_L = 0$ as $\frac{di_L}{dt} = 0$

↳ Short circuit.

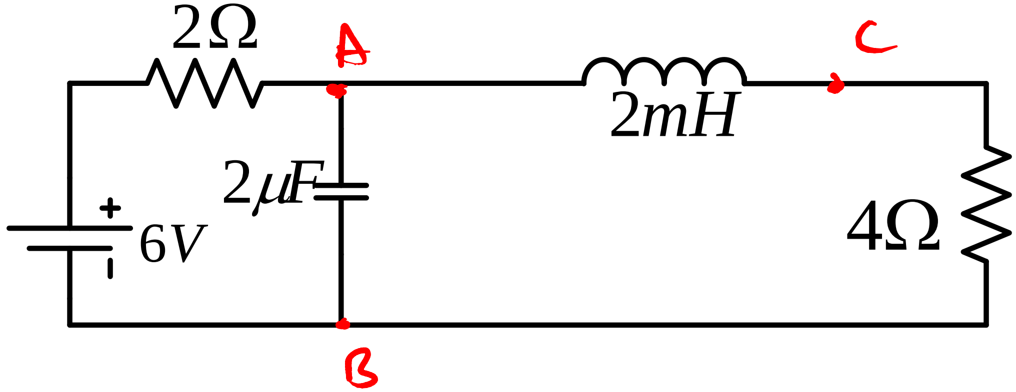
Series $L_{eq} = L_1 + L_2 + L_3$

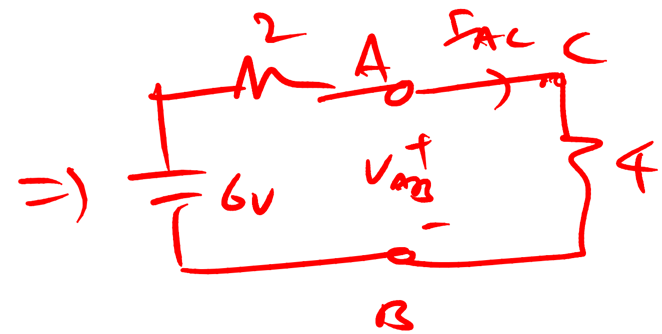
parallel $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

Eng $= \frac{1}{2} L \cdot I^2$.

DC Steady-state of circuits containing L and C

Find the energy stored in the inductor and the capacitor.





$$E_{2\mu F} = \frac{1}{2} \times 2\times 10^{-6} \times 4^2 \text{ J.}$$

$$E_{2mH} = \frac{1}{2} \times 2\times 10^{-3} \times 1^2 \text{ J.}$$

$$V_{AB} = 6 \times \frac{4}{2+4} = 4V.$$

$$I_{AC} = \frac{6}{2+4} = 1A.$$

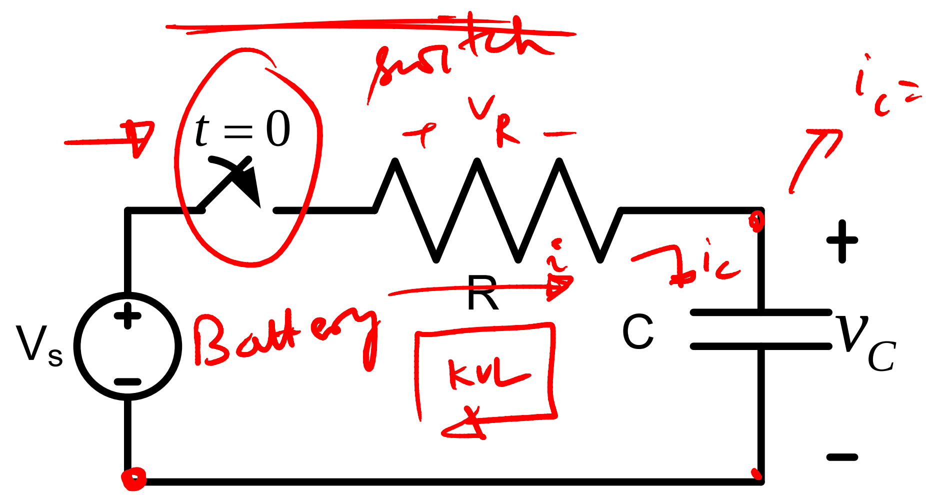
DC Transients

- Understanding DC transients
- Developing the time-function for the transients
- Solving First-order transients in circuits
- Preparing for Lab 4 and 5

DC Transients

- The time-varying voltages and currents resulting from the adding or removing voltage and current source to circuits containing energy storage elements, are called **transients**.

RC Circuit with a DC source



$i_c = C \cdot \frac{dv_c}{dt}$

$$-V_s + v_R + v_C = 0$$

$$v_R = iR = RC\frac{dv_c}{dt}$$

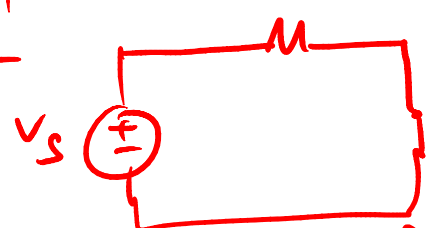
$$-V_s + RC\frac{dv_c}{dt} + v_c = 0$$

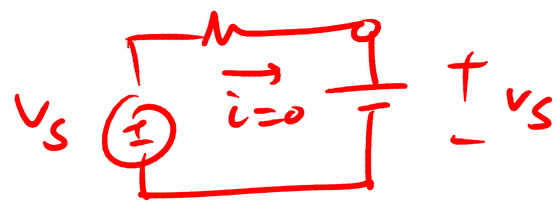
$$\boxed{RC\frac{dv_c}{dt} + v_c = V_s}$$

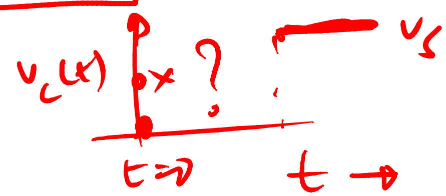
First order ODE

$v_c(t)$

At $t = 0^+$







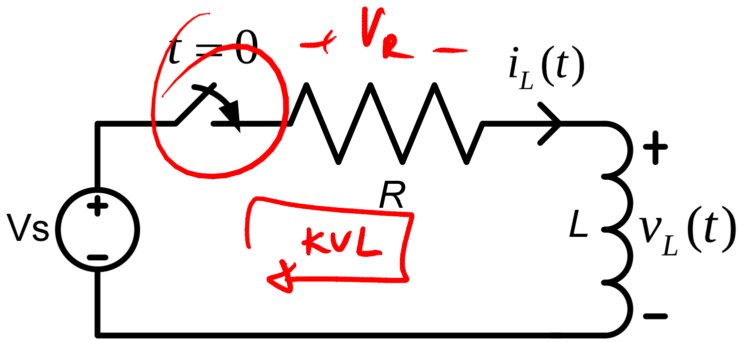
$V_s = 5\,V.$

$R = 100\,\Omega$

$C = 4700 \times 10^{-6}\,F$

$5 \times RC = 5 \times 100 \times 4700 \times 10^{-6}$

RL Circuit with DC source



$-V_s + V_R + V_L = 0$

$$-V_s + iR + L\frac{di}{dt} = 0$$

$i = i_L(t)$

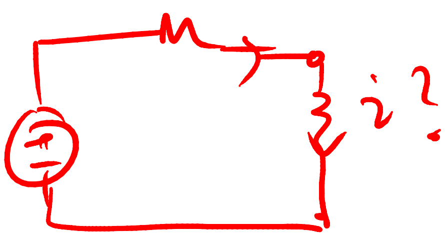
$$\boxed{\frac{L}{R}\frac{di}{dt} + i = \frac{V_s}{R}}$$

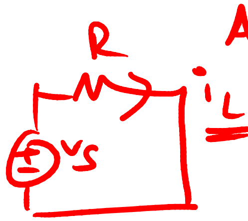
Rearrange eq.

At $t = 0^-$: Before sw is closed

$t = 0^+$ After sw is closed

$i_L(0^-) = i_L(0^+)$



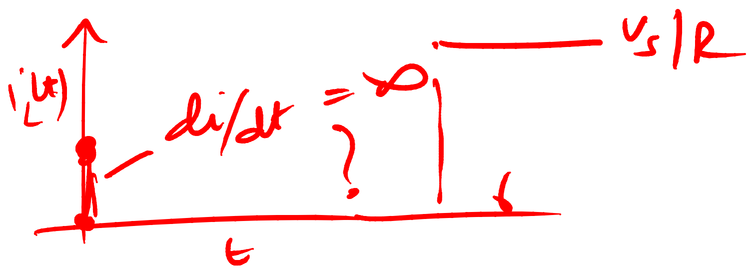


Finite number

At $t = 0^+$ $i_L = 0$

$v_L = L\cdot\frac{di_L}{dt}$ ← $\boxed{\frac{di_L}{dt}} = \frac{v_L}{L}$

$i_L(t)$



First order circuits with DC excitation

Time constant

$$\boxed{\tau}\frac{dx}{dt} + x = \boxed{K}$$

K : is a constant.

x(t)
DC steady-state value of x ?
$\frac{dx}{dt} = 0 \Rightarrow x_{ss} = K$.

Solution of the differential equation:

$$x(t) = \left(x(0) - K\right)e^{-\frac{t}{\tau}} + K$$

$v_S = V_{TH}$ $x(0) = x(t=0)$

$\frac{v_S}{R} = \frac{V_{TH}}{R_{TH}}$ initial value

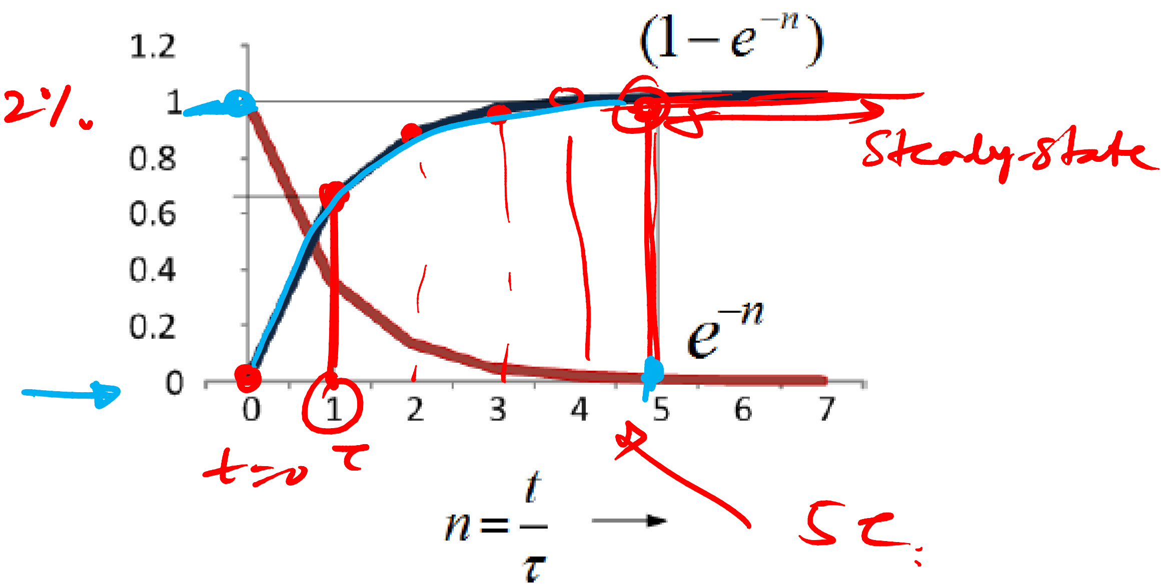
$\tau = RC, L/R$

Shape of First-order transient

$$x(t) = K + \left(x(0) - K\right) \times e^{-t/\tau}$$

t	$(1-e^{-t/\tau})$
τ	0.632121
2τ	0.864665
3τ	0.950213
4τ	0.981684
5τ	0.993262

63.2%



RC and RL comparing to the general form

$$\tau\frac{dx}{dt}+x=K$$ $\Rightarrow$ Solution: $$x(t)=\left(x(0)-K\right)e^{-\frac{t}{\tau}}+K$$

$$\boxed{RC\frac{dv_c}{dt}+v_c=V_s}$$

$x(t) = v_c(t)$

$\tau = R.C. \rightarrow$ Time constant $\rightarrow$

$K = V_s$

$v_c(t) = \left[V_c(0) - V_s\right] \times e^{-\frac{t}{RC}} + V_s$

$$\boxed{\frac{L}{R}\frac{di_L}{dt}+i_L=\frac{V_s}{R}}$$

$x(t) = i_L(t)$

$\tau = \frac{L}{R}$

$K = \frac{V_s}{R}$.

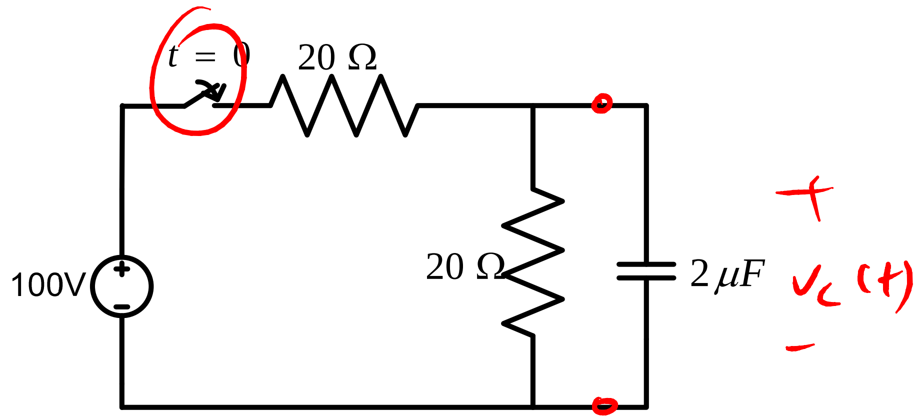
$i_L(t) = \left[i_L(0) - \frac{V_s}{R}\right] \cdot e^{-t/L/R} + \frac{V_s}{R}$.

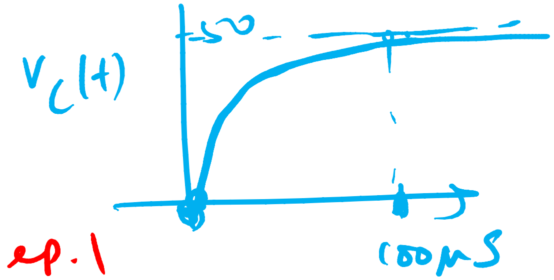
Steps for solving RC and RL circuits

1. - Use DC steady-state analysis of the circuit before the transient starts to find initial value
    - $v_c(0^-) = v_c(0^+)$, $i_L(0^-) = i_L(0^+)$.
2. - Use Thevenin's equivalent circuit to reduce any circuit to the standard form
3. - Find the time constant and DC voltage from the Thevein's equivalent

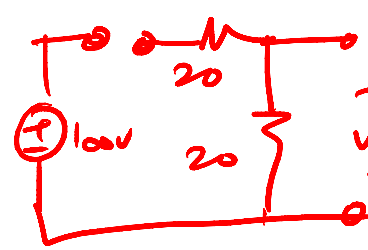
Example

Find the capacitor voltage as a function of time.





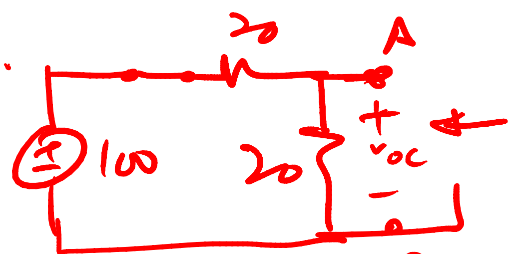
Step.1



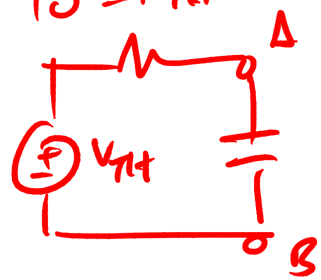
$v_c(0) = 0$

$v_c(0^-) = v_c(0^+) = 0$

Step.2



$\equiv$ 50



$\tau = R_{TH} C$

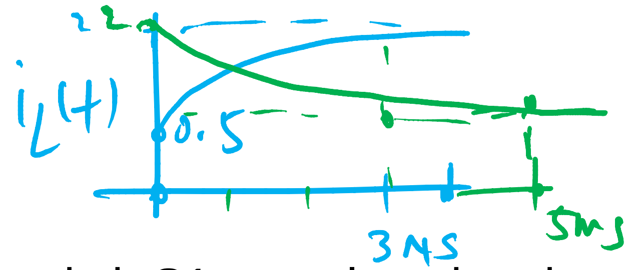
$= 10 \times 2\mu F$

$= 20 \mu S.$

Step.3

$$v_c(t) = [v_c(0) - V_{TH}] * e^{-t/\tau} + V_{TH} \quad V.$$

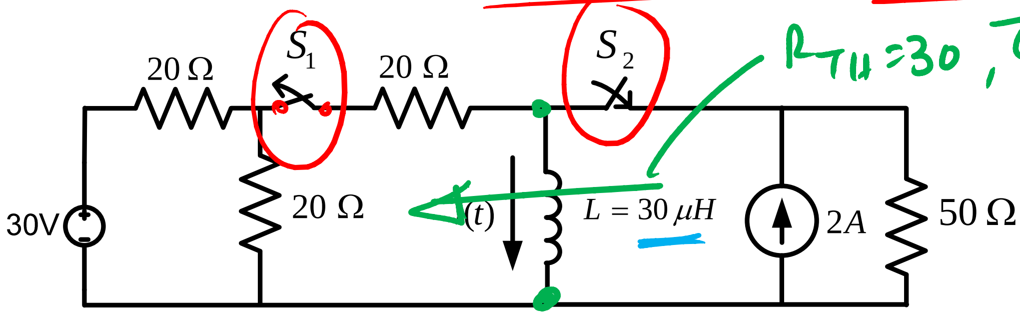
Example



$i_L(t)$

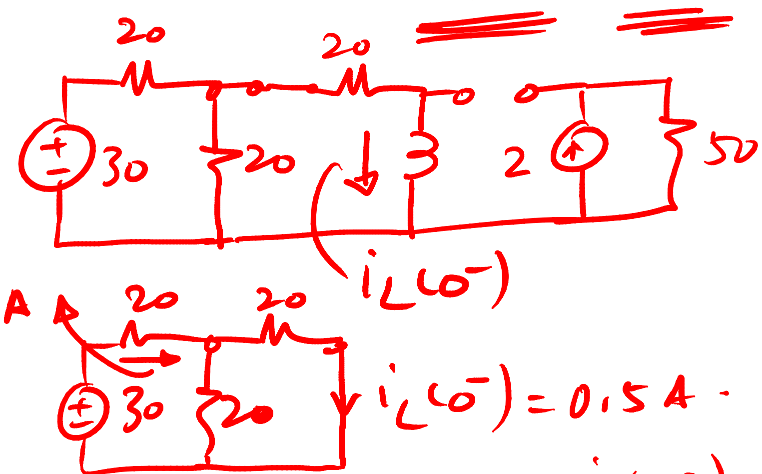
Find the inductor current as a function of time. Before t=0, switch S1 was closed and S2 was open. At t=0, S1 is opened and S2 is closed. Find the current i(t) after t=0.

$i_L(t)$



$R_{TH} = 30$, $\tau_2 = 1\,\mu s$.

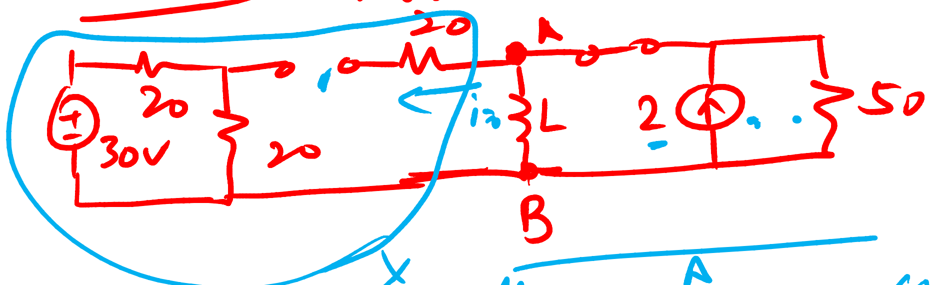
Step 1. Find $i_L(0^+) = i_L(0^-)$

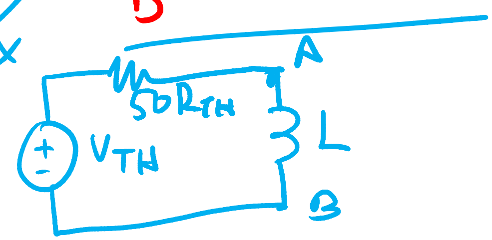


$i_L(0^-) = 0.5\,A$

$= i_L(0^+)$

Step 2. After t = 0





$\tau = \dfrac{L}{R_{TH}}$

$= \dfrac{30}{50}\,\mu s$

Step 3

$i_L(t) = \left[i_L(0) - \dfrac{V_{TH}}{R_{TH}}\right] \cdot e^{-t/\tau} + \dfrac{V_{TH}}{R_{TH}}$

$= [0.5 - 2] \cdot e^{-t/\tau} + 2$ A

Appendix

Solving the First order ordinary linear differential equation with a constant forcing function.

First order circuits with DC excitation

$$\tau \frac{dx}{dt} + x = K$$

K is a constant.

- Two parts of the general solution
  - Particular solution (forced solution)
  - Complementary solution (homogeneous eqn)

Particular solution

- The particular solution is obtained from the forcing function.
- It is normally of the same functional form as the forcing function and its derivatives.
- A table containing various forcing functions and their corresponding particular solutions are readily available.
- http://www.efunda.com/math/ode/linearode_undeterminedcoeff.cfm

When forcing function is DC

- The particular solution is a constant

$$\tau \frac{dx_p}{dt} + x_p = K$$

$$\text{Let } x_p = K^{'}.$$

$$\text{Then } 0 + K^{'} = K \Rightarrow K^{'} = K$$

$$\text{i.e. } x_p = K$$

Homogeneous equation

$$\tau \frac{dx_c(t)}{dt} + x_c(t) = 0$$

$$\frac{dx_c(t)/dt}{x_c(t)} = \frac{-1}{\tau}$$

$$\ln\left[x_c(t)\right] = \frac{-t}{\tau} + c$$

$$x_c(t) = e^c e^{-\frac{t}{\tau}} = K'' e^{-\frac{t}{\tau}}$$

Complete Solution

$$\tau \frac{dx}{dt} + x \;\; = K$$

$$x(t) = x_c + x_p = K^{''} e^{-\frac{t}{\tau}} + K$$

$$x(0) = K^{''} e^{-0} + K \Rightarrow K^{''} = x(0) - K$$

$$x(t) = \left(x(0) - K\right) e^{-\frac{t}{\tau}} + K$$