EE1002
Introduction to Circuits and Systems

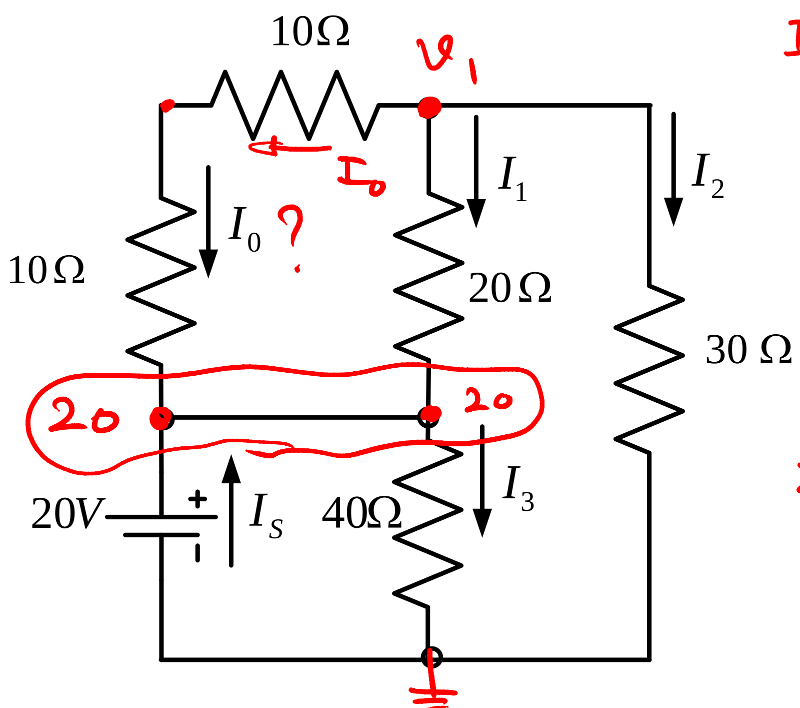
Part 1 : Lecture 5

Superposition Principle

Equivalent Circuits

Example 1

Find the currents shown in the circuit.



KCL at S. N.

$$-I_S - I_0 - I_1 + I_3 = 0$$

$$I_0 = \frac{v_1 - 20}{10+10}$$

KCL at node v_1:

$$\frac{v_1 - 20}{20} + \frac{v_1 - 20}{20} + \frac{v_1}{30} = 0$$

$$3v_1 - 60 + 3v_1 - 60 + 2v_1 = 0$$

$$8v_1 = 120 \Rightarrow v_1 = 15\ V.$$

$$I_0 = \frac{15-20}{20} = -0.25\ A$$

$$I_1 = \frac{15-20}{20} = -0.25\ A.$$

$$I_2 = \frac{15}{30} = 0.5\ A.$$

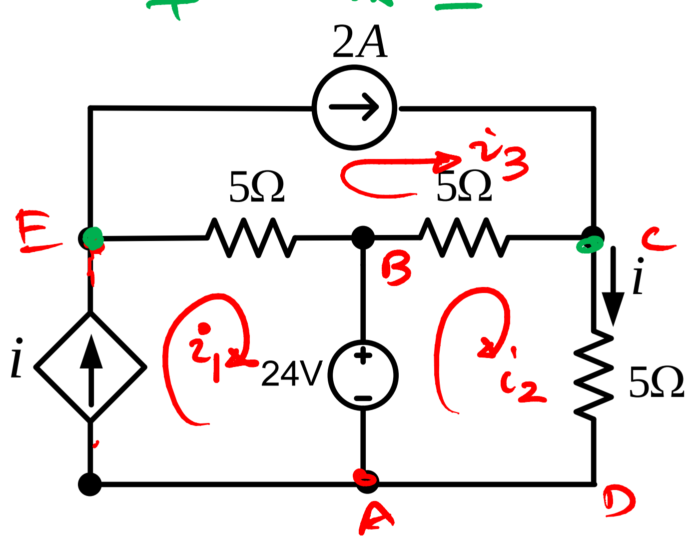
$$I_3 = \frac{20}{40} = 0.5\ A.$$

$$\Rightarrow I_3 = -I_0 - I_1 + I_3 = 0.25 + 0.25 + 0.5$$
$$= 1\ A.$$

Example 2

P.S.C.
V_{2A}

$P_{2A} = V_{2A} \times 2 = 14 \times 2 = 28\,W$ (absorbed)



Use **Mesh Current Analysis** to find the **power associated with the 2A current source** in the circuit in Figure Q2. Is the power **delivered or absorbed** by the current source?

Mesh i_1:

$i_1 = i_2$ — ①

$i = i_2$

Mesh i_2 : ABCDA

KVL : $-24 + 5\times(i_2 - i_3) + 5\times i_2 = 0$

② ⇒ $-24 + 5(i_2 - 2) + 5i_2 = 0$ — ②

Mesh i_3:

$i_3 = 2\,A$ — ③

$\Rightarrow i_2 = 3.4\,A.$
$= i_1$

V_{EC} = Find a path from E to C and add all voltage drops along the path

$= 5\times(i_1 - i_3) + 5(i_2 - i_3)$

$= 5(3.4 - 2) + 5(3.4 - 2)$

$= 14\,V$

The principle of Superposition

- The total response in a linear circuit is the sum of responses to each of the independent sources acting individually.

$$r_T = r_1 + r_2 + ... + r_N$$

Circuit has N sources.

r_T - total response

r_n - the response due to the n^{th} source, acting individually.

Linearity

$y = mx + c$?

Ans: NO

① $f(x_1 + x_2) = f(x_1) + f(x_2)$ Additivity

② $f(\alpha \times x) = \alpha \times f(x)$ Homogeneity

$\boxed{y = kx}$

$y(x_1) = k \cdot x_1$

$y(x_2) = k \cdot x_2$

$$v = f(i)$$

$\boxed{v = R \cdot i}$ $\boxed{i = \frac{1}{R} \cdot v}$

① $y(x_1 + x_2) = k \cdot (x_1 + x_2)$
$= k x_1 + k x_2$
$= y(x_1) + y(x_2)$

- Resistor, inductor, capacitor are linear elements. ? ?

② $y(\alpha \cdot x) = k \cdot \alpha x$
$= \alpha \cdot kx$
$= \alpha \cdot y(x)$

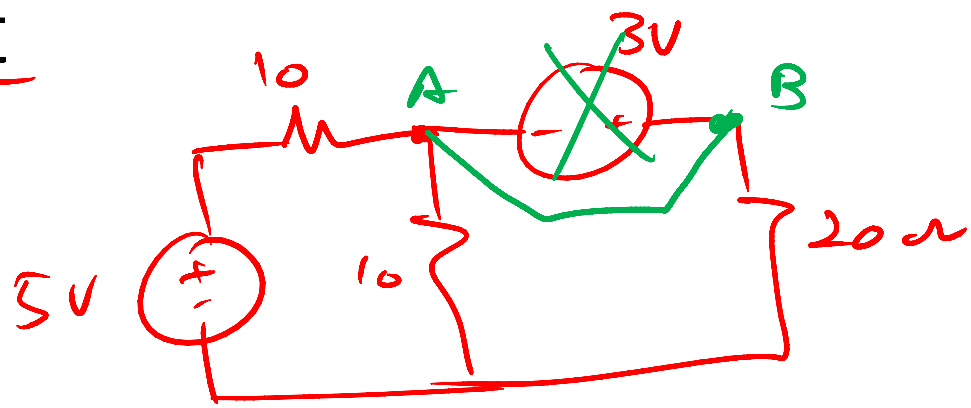
- Diode, MOSFET are nonlinear elements.

Superposition to solve circuits

- ‘Kill’ all the other independent sources but one. DO NOT kill dependent sources.
- This should result in a simpler circuit
- Use network analysis to find the required response (voltage/current)
- Do this one by one for all independent sources
- Add up the responses to find the total response

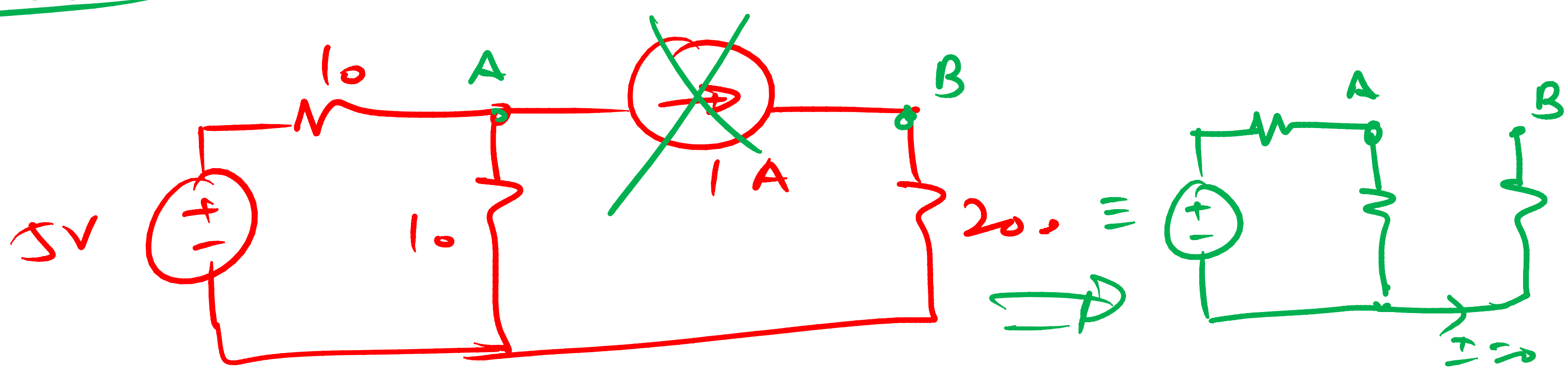
Killing a voltage source

- To kill a voltage source, we make its output voltage equal to zero.
- Replace the voltage source with a short circuit

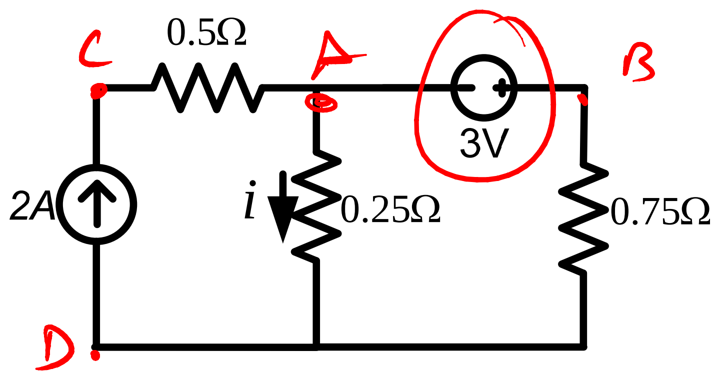


Killing a current source

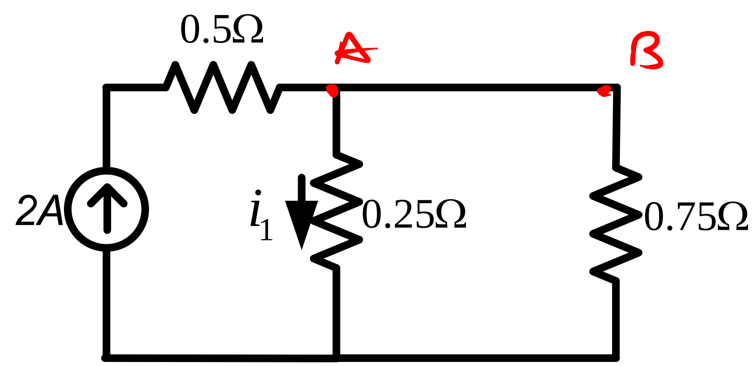
- To kill a current source, we make its output current equal to zero.
- replace the current source with an open circuit



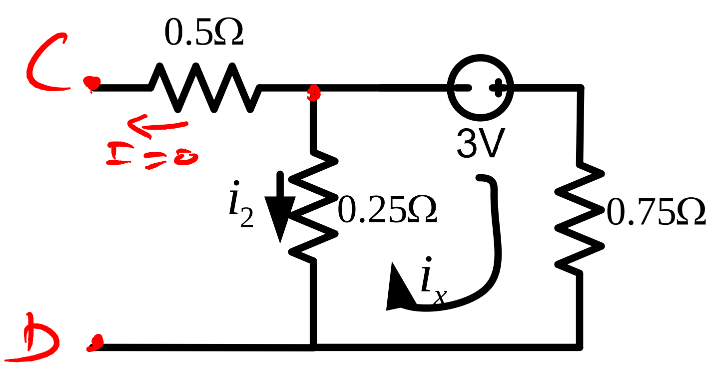
Example 1



Circuit with two independent sources



Circuit with only the current source



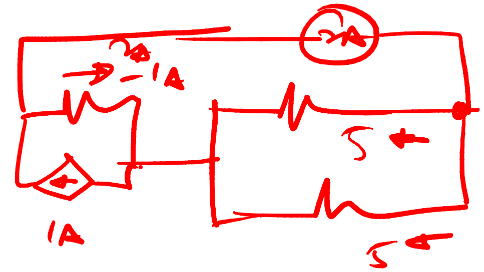
Circuit with only the voltage source

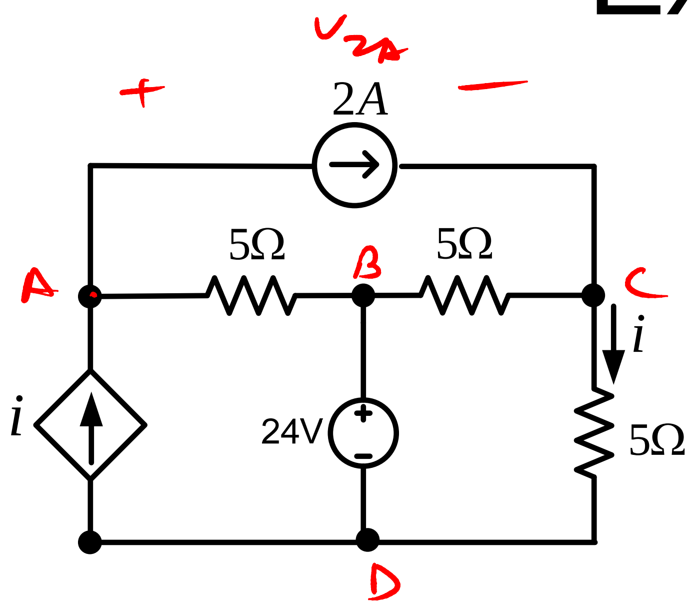
$$i_1 = 2 \times \frac{0.75}{0.25+0.75} = 1.5\,A$$

$$i_2 = -\frac{3}{0.25+0.75} = -3\,A.$$

$$i = i_1 + i_2 = 1.5 - 3 = \underline{\underline{-1.5\,A}}.$$

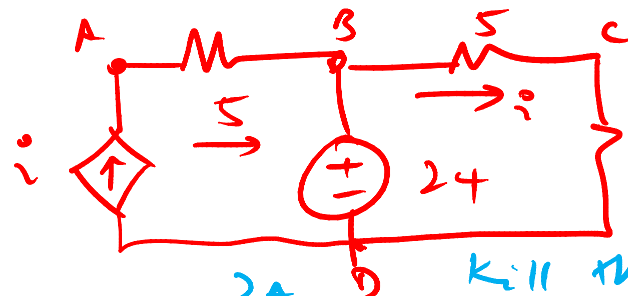
Example 2





Use **Superposition principle** to find the **power associated with the 2A current source** in the circuit in Figure Q2. Is the power **delivered or absorbed** by the current source?

Kill the current source.



$i = \frac{24}{10}$

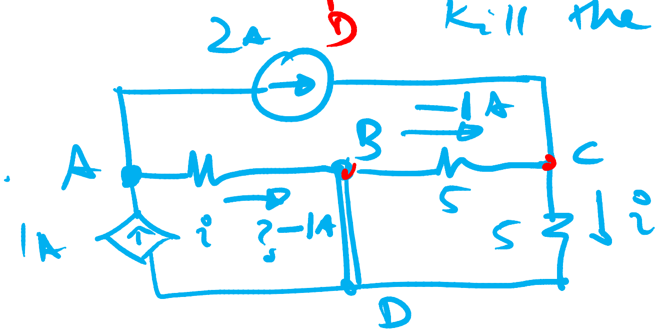
$= 2.4 A$

$v_{2A1} = v_{AC}$

$= 5 \times 2.4 + 5 \times 2.4$

$= 24V.$

Kill the voltage source



$i = 1A.$

$V_{2A2} = 5 \times (-1) + 5(-1)$

$= -10V.$

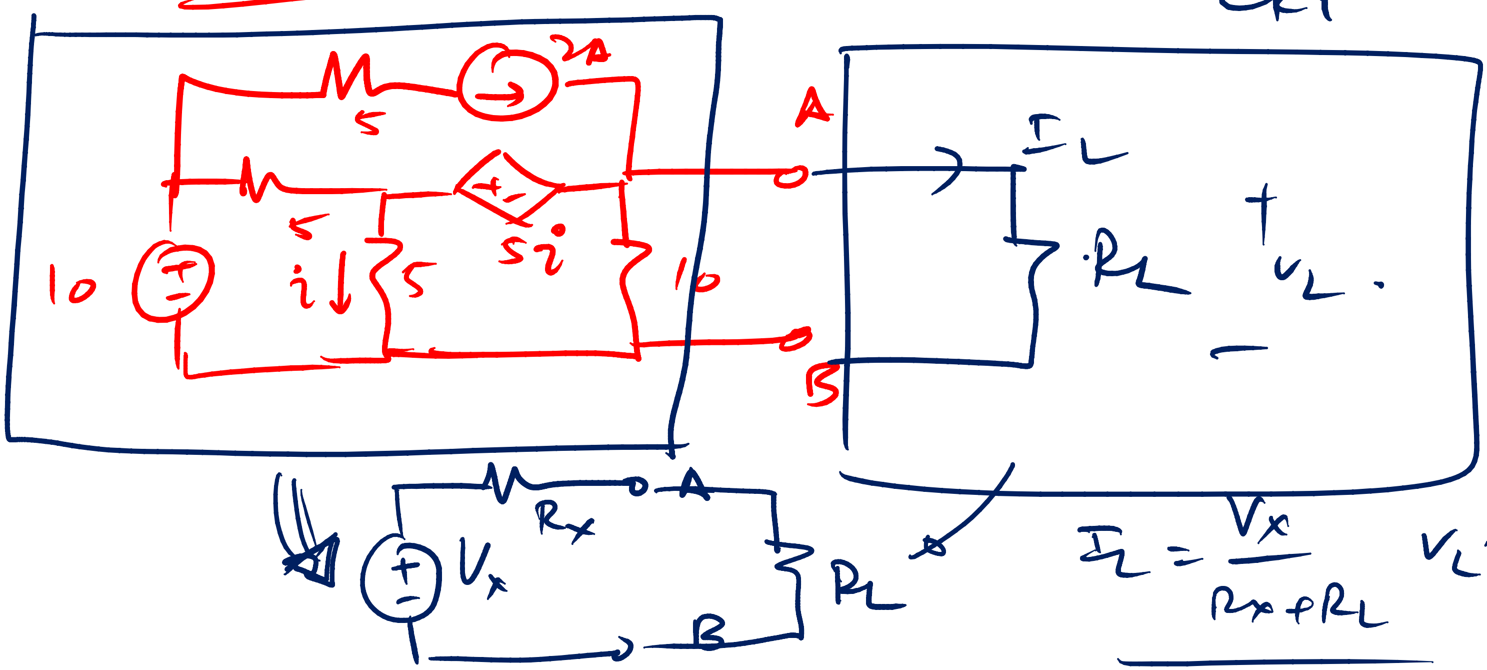
$V_{2A} = V_{2A1} + V_{2A2}$

$= 14V.$

Equivalent Circuits

- One-port networks and equivalent circuits
  - Thevenin's equivalent circuit
  - Norton's equivalent circuit

ckt.



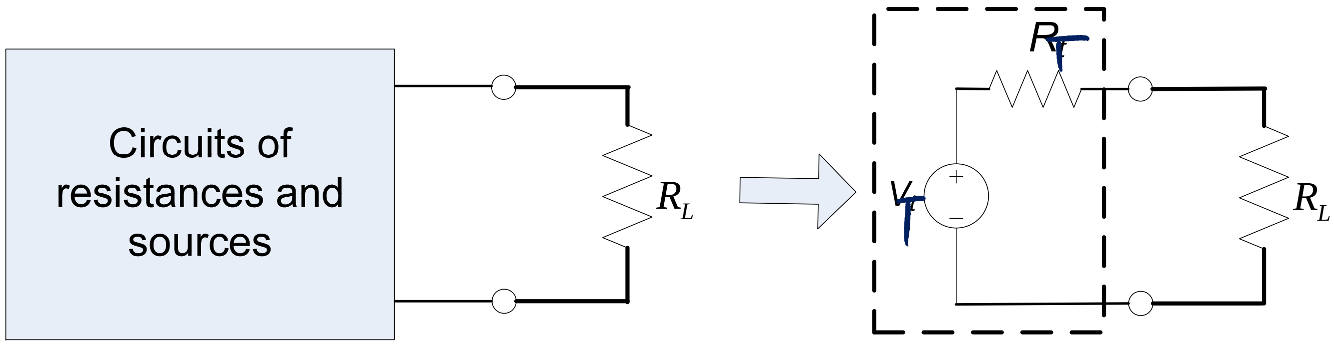
$I_L = \dfrac{V_x}{R_x + R_L}$

$V_L = V_x \cdot \dfrac{R_L}{R_x + R_L}$

One-port networks and equivalent circuits

- Two-terminal circuits can be replaced by an equivalent circuit consisting of a source and a resistance.
- A voltage source with a series resistance (Thevenin equivalent circuit)
- A current source with a parallel resistance (Norton equivalent circuit)

Thevenin equivalent



Thevenin equivalent circuit

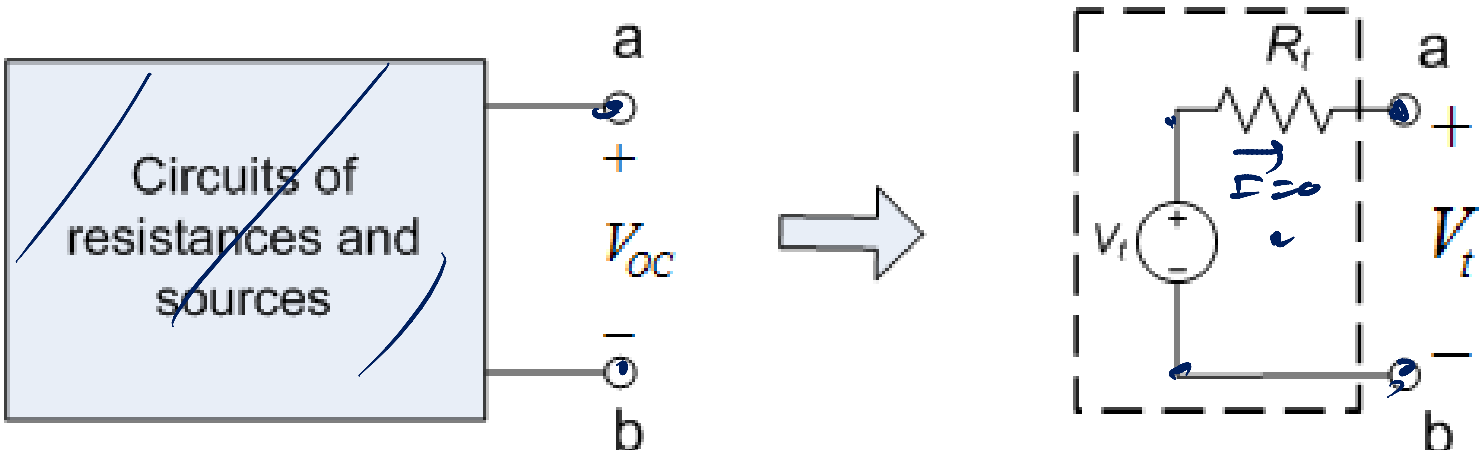
V_T

R_T

- A voltage source in series with a resistance
- The voltage source is called Thevenin's voltage
- The series resistance is called Thevenin's resistance

Thevenin Voltage

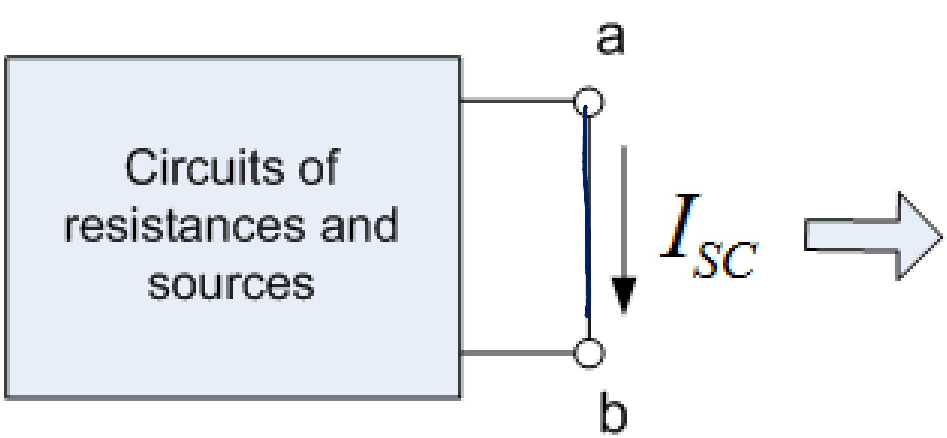
- Value of the voltage source is the open circuit voltage between the two terminals
- Called the Thevenin voltage

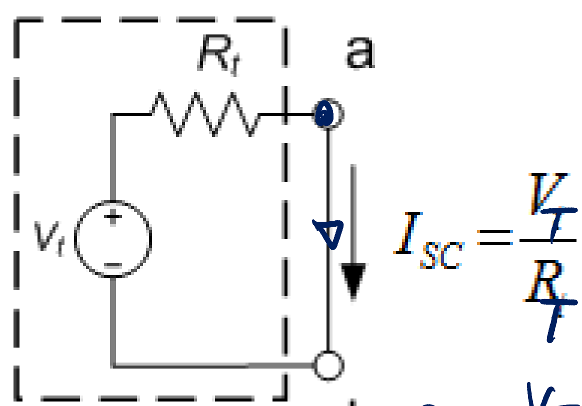


Thevenin equivalent circuit

Thevenin Resistance

- Find the short circuit current between the two terminals
- Calculate the Thevenin resistance

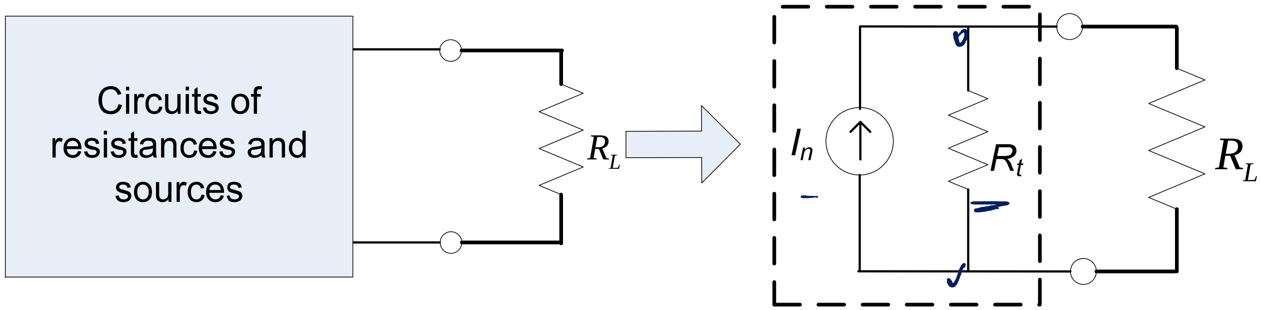




Thevenin equivalent circuit

$R_T = \frac{V_T}{I_{SC}}$

Norton Equivalent



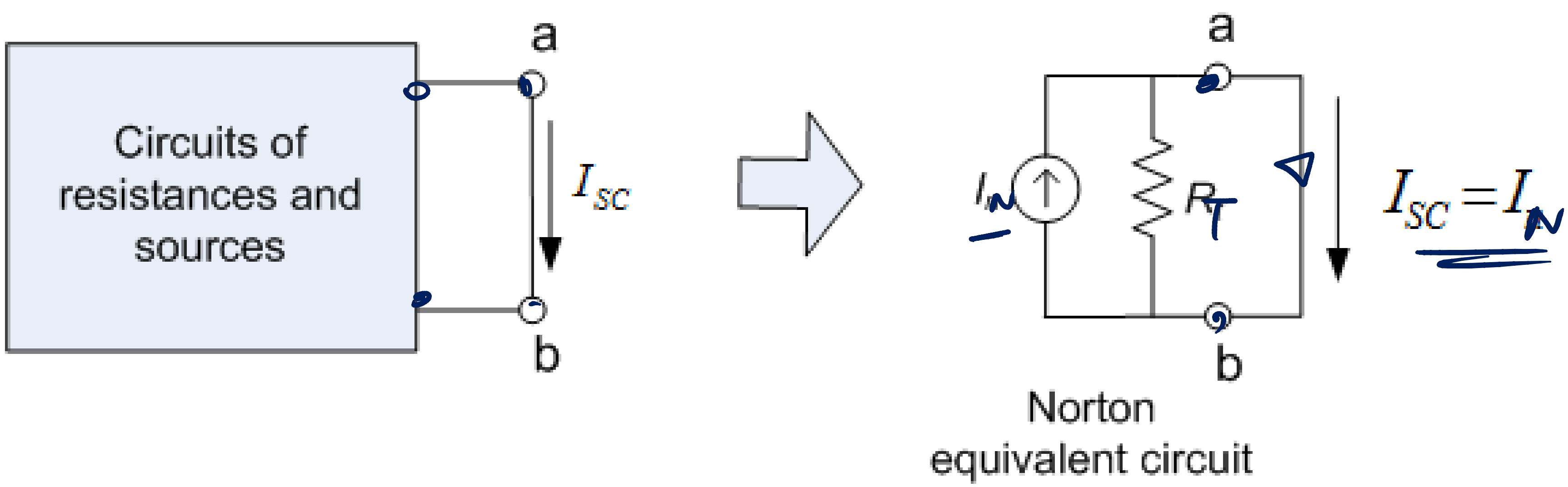
Norton equivalent circuit

- A current source in parallel with a resistance
- The current source is called Norton's current
- The series resistance is called Thevenin's resistance

Norton's current

- Value of the current source is the short circuit current between the two terminals
- Called the Norton's current

$I_{sc} = I_N \times \frac{R_T}{R_T + 0}$

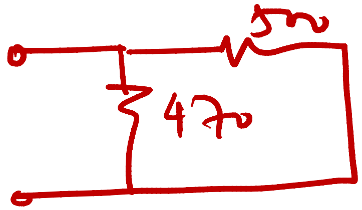


Steps to find the equivalent circuits

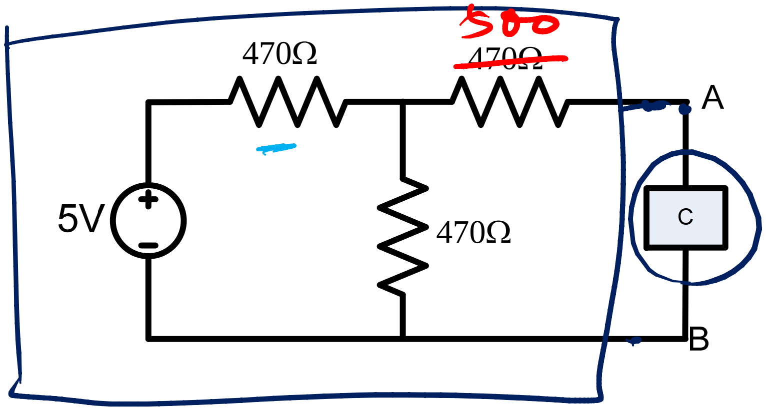
- Obtain the open circuit voltage between the two terminals – Thevenin's voltage
- Obtain the short circuit current between the two terminals – Norton's current
- Calculate the Thevenin's resistance as

$$R_T = \frac{V_{OC}}{I_{SC}}$$

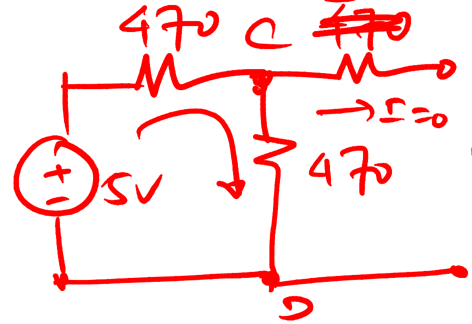
Example1



$= \frac{470\times500}{470+500}$



2nd stage



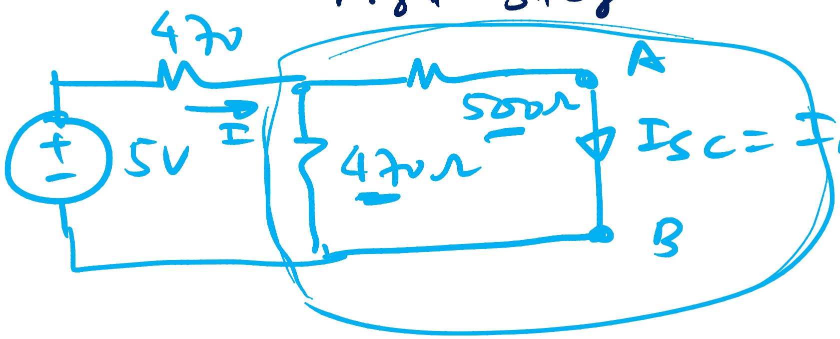
$V_{AB} = V_{oc}$

$V_{CD} = V_{AB}$

$V_{CD} = 5\times\frac{470}{2\times470} = 2.5V = V_T$

$= V_{oc}$

First stage



$I_{sc} = I_N$

$I = \frac{5}{470 + \frac{470\times500}{470+500}}$

$I_{sc} = I\times\frac{470}{470+500}$ A $= I_N$

To find open circuit voltage and short circuit current of the circuit

$R_T = \frac{V_{oc}}{I_{sc}} = \frac{V_T}{I_N}$

Example2

To find open circuit voltage and short circuit current of the circuit.