Engineering Electromagnetics

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Plane Wave Propagation in Lossy Medium

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1 Plane Waves in Lossy Media

In a lossy medium, conduction current density $\mathbf{J} = \sigma\mathbf{E}$

σ : Conductivity is a measure of how easily electrons can travel through the material under the influence of an externally applied electric field.

The work (or energy) expended by the electric field in moving electrons is converted to heat loss, known as Joule's law.

Ampere's Law:

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D}$$
$$= \sigma\mathbf{E} + j\omega\varepsilon\mathbf{E}$$
$$= j\omega\left(\frac{\sigma}{j\omega} + \varepsilon\right)\mathbf{E}$$
$$= j\omega\varepsilon_c\mathbf{E}$$

where we have defined the complex permittivity

$$\boxed{\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}, \text{ which can be written as } \varepsilon' - j\varepsilon''}$$

$$\boxed{\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma/\omega}{\varepsilon} = \frac{\sigma}{\omega\varepsilon}: \quad \text{defined as loss tangent}}$$

>> 1 for good conductor,
<< 1 for good dielectric

Note: a function of ω

, relative permittivity is defined through $\varepsilon = \varepsilon_r \varepsilon_0$, for complex permittivity we define complex relative permittivity:

$$\boxed{\varepsilon_{rc} = \frac{\varepsilon_c}{\varepsilon_0} = \varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0}, \ \text{which can be written as } \varepsilon_r' - j\varepsilon_r''}$$

Current terms:

Also a ratio of conduction current density to displacement current density

$\sigma\mathbf{E}$: conduction current density

$j\omega\mathbf{D} = j\omega\varepsilon\mathbf{E}$: displacement current density

Note that conduction current and displacement current are out of phase by π/2.

By replacing ε with ε_c, all the previous results derived for lossless media are applicable to lossy media.

Helmholtz's equation:

$$\nabla^2 \mathbf{E} + k_c^2 \mathbf{E} = 0$$

$$\boxed{k_c = \omega\sqrt{\mu\varepsilon_c}}$$

a complex number

The solutions for the Helmholtz equation in lossy media are same as those for lossless media if k is replaced by k_c.

We denote the complex number k_c by

$$k_c = \beta - j\alpha,$$

where $\alpha > 0$ and $\beta > 0$

$\alpha =$ attenuation constant (Np/m)

$\beta =$ propagation constant (rad/m)

The value of α and β can be calculated by:

(1) Use your calculator to directly calculate the square root of a complex number

(2) Change complex number to polar form and then calculate its square root

(3) Use the table in Slide 18

Consider the wave propagating in the +z-direction:

$$E_x(z) = E_0 e^{-jk_c z}$$

$$= E_0 e^{-\alpha z} e^{-j\beta z}$$

Here (in this module), the general form of a plane wave in a lossy medium is given (similar to the lossless case):

$$\mathbf{E} = \mathbf{E}_0 e^{-j\mathbf{k}_c \cdot \mathbf{r}}$$

$$\mathbf{H} = \mathbf{H}_0 e^{-j\mathbf{k}_c \cdot \mathbf{r}}$$

$$\mathbf{k}_c = k_{cx}\hat{\mathbf{x}} + k_{cy}\hat{\mathbf{y}} + k_{cz}\hat{\mathbf{z}} = k_c\hat{\mathbf{k}}$$

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

$\hat{\mathbf{k}}$ (propagation direction): real quantity

$$\mathbf{E} \perp \mathbf{H} \perp \hat{\mathbf{k}}$$

$$\mathbf{H} = \frac{1}{\eta_c}\hat{\mathbf{k}} \times \mathbf{E}, \qquad \mathbf{E} = \eta_c \mathbf{H} \times \hat{\mathbf{k}}$$

$$\hat{\mathbf{k}} = \hat{\mathbf{e}} \times \hat{\mathbf{h}}$$

$$\hat{\mathbf{h}} = \hat{\mathbf{k}} \times \hat{\mathbf{e}}$$

$$\hat{\mathbf{e}} = \hat{\mathbf{h}} \times \hat{\mathbf{k}}$$

η_c = complex intrinsic impedance

$$= \sqrt{\frac{\mu}{\varepsilon_c}} = |\eta_c| e^{j\theta_\eta}$$

Explicitly:

Actual $\mathbf{E}$ field: $\hat{\mathbf{e}}|E_0|e^{-\alpha\hat{\mathbf{k}}\cdot\mathbf{r}}\cos(\omega t+\phi-\beta\hat{\mathbf{k}}\cdot\mathbf{r})$

$$\mathbf{E}=\mathbf{E}_0 e^{-j\mathbf{k}_c\cdot\mathbf{r}}=\hat{\mathbf{e}}|E_0|e^{j\phi}e^{-\alpha(\hat{\mathbf{k}}\cdot\mathbf{r})}e^{-j\beta(\hat{\mathbf{k}}\cdot\mathbf{r})}$$

$$\mathbf{H}=\mathbf{H}_0 e^{-j\mathbf{k}_c\cdot\mathbf{r}}=\hat{\mathbf{h}}\frac{|E_0|}{|\eta_c|}e^{-j\theta_\eta}e^{j\phi}e^{-\alpha(\hat{\mathbf{k}}\cdot\mathbf{r})}e^{-j\beta(\hat{\mathbf{k}}\cdot\mathbf{r})}$$

$$\hat{\mathbf{k}}=\hat{\mathbf{e}}\times\hat{\mathbf{h}}$$
$$\hat{\mathbf{h}}=\hat{\mathbf{k}}\times\hat{\mathbf{e}}$$
$$\hat{\mathbf{e}}=\hat{\mathbf{h}}\times\hat{\mathbf{k}}$$

α (attenuation constant) magnitude decays by $e^{-\alpha d}$ for a travel distance of d

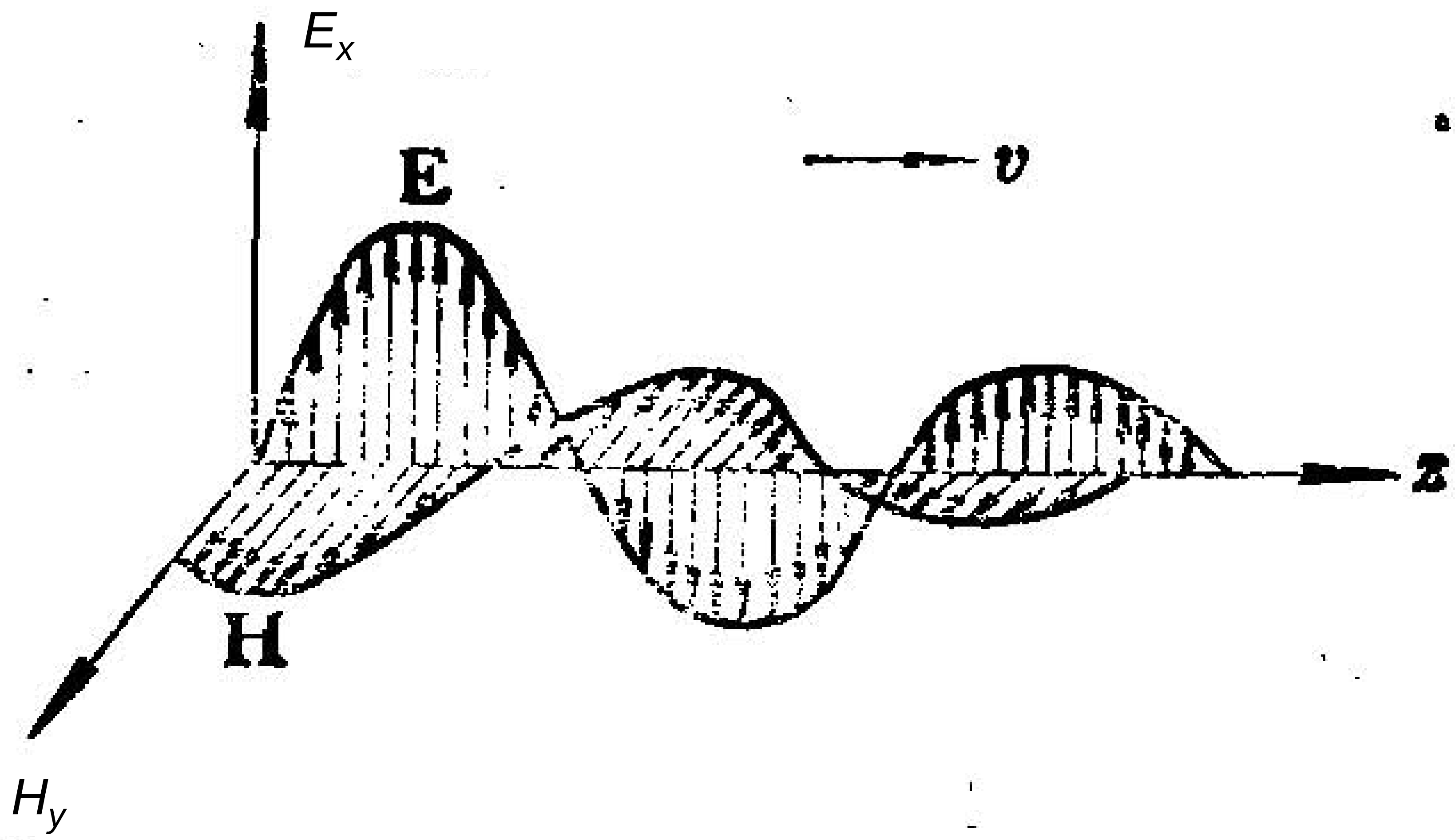
β (propagation constant): phase decreases by βd for a travel distance of d

η_c (complex intrinsic impedance) $=|\eta_c|e^{j\theta_\eta}$:

$|\eta_c|$: Ratio of magnitude of $\mathbf{E}$-field to magnitude of $\mathbf{H}$-field

θ_η : Phase difference of $\mathbf{E}$-field and $\mathbf{H}$-field

Lossless medium is a special case where $\alpha=0$, $\theta_\eta=0$



H and **E** inside a lossy medium (spatial point of view)

2 Approximation Analysis for Two Special Cases

The formulas for k_c and η_c are complicated. For some special cases depending on the loss tangent $\sigma/\omega\varepsilon$, approximation formulas can be derived. We study two special cases in which the loss tangent is either much greater than 1 or much smaller than 1.

$$\frac{\sigma}{\omega\varepsilon} >> 1 \quad \Rightarrow \quad \text{a good conductor}$$

$$\frac{\sigma}{\omega\varepsilon} << 1 \quad \Rightarrow \quad \text{a low-loss dielectric}$$

Most important Section in this Chapter.

2.1 Good conductors

Condition:

$$\boxed{\text{loss tangent} = \frac{\sigma}{\omega\varepsilon} >> 1} \qquad \left(\text{for example } \frac{\sigma}{\omega\varepsilon} > 100\right)$$

Dropping off small terms in k_c and η_c :

$$\boxed{\alpha = \beta \approx \sqrt{\pi f \mu \sigma}}$$

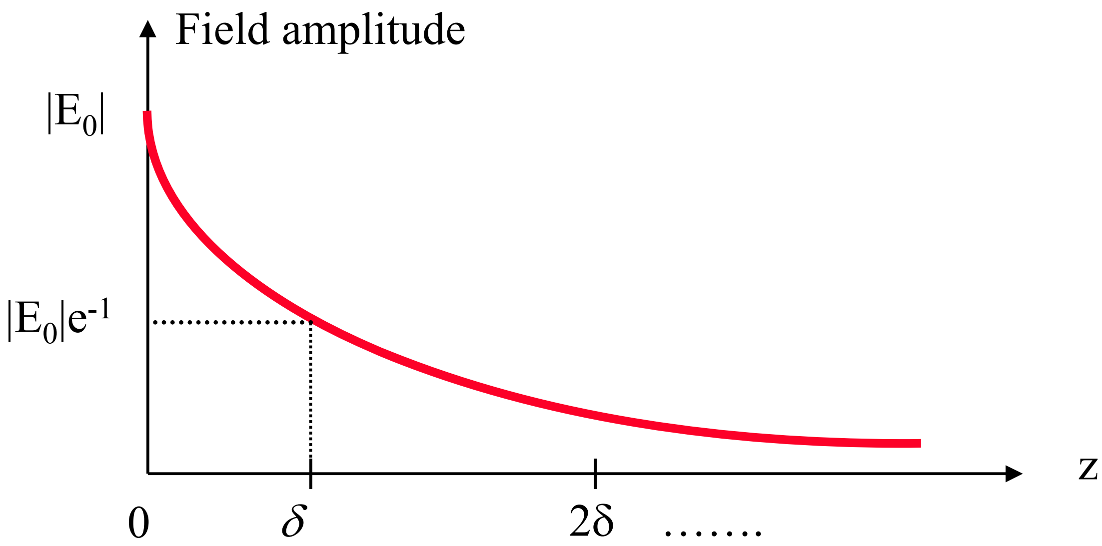
$$\boxed{\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} \approx (1+j)\frac{\alpha}{\sigma} = \sqrt{2}\,\frac{\alpha}{\sigma} e^{j\pi/4}}$$

In a good conductor, the intrinsic impedance η_c is a complex number, meaning that the electric and magnetic fields are not in phase as in the case of a lossless medium.

$$\boxed{u_p = \text{phase velocity} = \frac{\omega}{\beta} \approx \sqrt{\frac{2\omega}{\mu\sigma}}}$$

$$\boxed{\lambda = \text{wavelength} = \frac{2\pi}{\beta} \approx \frac{2\pi}{\sqrt{\pi f \mu\sigma}} = 2\sqrt{\frac{\pi}{f\mu\sigma}}}$$

In a good conductor, because of the attenuation constant α, the wave amplitude becomes smaller when it propagates. The distance δ through which the amplitude of a travelling plane wave decreases by a factor of $e^{-1} = 0.368 = 36.8\%$ is called the **skin depth** or **depth of penetration** of the conductor.



$$\frac{\left|E_x\left(z=\delta\right)\right|}{\left|E_x\left(z=0\right)\right|}=\frac{\left|E_0 e^{-\alpha\delta}e^{-j\beta\delta}\right|}{\left|E_0\right|}=e^{-\alpha\delta}=e^{-1}$$

For a r = 1mm wire, R per meter = 1/σA = 1/(σπr²) = 5.5 mΩ

$$\alpha\delta=1$$

$$\boxed{\delta=\frac{1}{\alpha}}$$

For the same wire at 1 MHz, R per meter = 1/(σ2πrδ) = 41.5 mΩ

For copper, σ = 5.8 × 10^7 S/m and μ_r = 1.

$$\delta=\frac{1}{\alpha}=\sqrt{\frac{1}{\pi f\mu\sigma}}=\frac{6.61\times10^{-2}}{\sqrt{f}}$$

at 60Hz, δ = 8.5 × 10^{-3} m
at 1MHz, δ =6.6 × 10^{-5} m
at 30GHz, δ = 3.8 × 10^{-7} m

Skin depth of some common materials

Material	Properties	δ		
		f = 60 Hz	f = 1 MHz	f = 1 GHz
Silver	$\sigma = 6.17 \times 10^{7}$ S/m	8.27 mm	0.064 mm	0.002 mm
Copper	$\sigma = 5.80 \times 10^{7}$ S/m	8.53 mm	0.066 mm	0.0021 mm
Gold	$\sigma = 4.10 \times 10^{7}$ S/m	10.14 mm	0.079 mm	0.0025 mm
Aluminium	$\sigma = 3.54 \times 10^{7}$ S/m	10.92 mm	0.084 mm	0.0027 mm
Iron	$\sigma = 1.00 \times 10^{7}$ S/m $\mu_r \approx 10^3$	0.65 mm	0.005 mm	0.00016 mm
Seawater	$\sigma = 4$ S/m $\varepsilon_r = 72$	32 m	0.25 m	12.37 mm

For perfect conductors, $\sigma \to \infty$

Thus, E and H fields are sero in the perfect conductor. J and I are only on the surface.

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{1}{\pi f \mu \sigma}} \to 0$$

2.2 Low-loss dielectrics

$$k_c = \omega\sqrt{\mu\varepsilon}\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)^{1/2}$$

Condition:

$$\boxed{\text{loss tangent} = \frac{\sigma}{\omega\varepsilon} << 1}$$

$$\left(\text{for example } \frac{\sigma}{\omega\varepsilon} \leq 0.01\right)$$

Binomial expansion (when $x << 1$):

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n-1)x^2 + \cdots$$

Then,

$$k_c \approx \omega\sqrt{\mu\varepsilon}\left[1 - j\frac{1}{2}\frac{\sigma}{\omega\varepsilon} + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]$$

$$\approx \omega\sqrt{\mu\varepsilon} - j\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}},$$

Hence,

$$\boxed{\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}, \qquad \beta \approx \omega\sqrt{\mu\varepsilon}}$$

$$\boxed{\eta_c \approx \sqrt{\frac{\mu}{\varepsilon}}}$$

$$\boxed{u_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{\mu\varepsilon}}}$$

$$\boxed{\lambda = \frac{2\pi}{\beta} \approx \frac{1}{f\sqrt{\mu\varepsilon}}}$$

$$\boxed{\delta = \frac{1}{\alpha} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$

Expressions for α, β, η_c, u_p, and λ for various types of media.

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega \left[\frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right] \right]^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f \mu \sigma}$	(Np/m)
$\beta =$	$\omega \left[\frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1 \right] \right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f \mu \sigma}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	(Ω)
$u_p =$	ω/β	$1/\sqrt{\mu\varepsilon}$	$1/\sqrt{\mu\varepsilon}$	$\sqrt{4\pi f/\mu\sigma}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Notes: $\varepsilon' = \varepsilon$; $\varepsilon'' = \sigma/\omega$; in free space, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$; in practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' > 100$.

Example 1

The electric field intensity of a linearly polarised uniform plane wave propagating in the $+z$ direction in seawater is $\mathbf{E} = \hat{\mathbf{x}}\,100\cos(10^7 \pi t)$ at $z = 0$. The constitutive parameters of seawater are $\varepsilon_r = 72$, $\mu_r = 1$, and $\sigma = 4$ S/m.

(a) Determine the attenuation constant, intrinsic impedance, phase velocity, wavelength, and skin depth.

(b) Write expressions for $\mathbf{H}(z,t)$ and $\mathbf{E}(z,t)$.

(c) Find the distance z_1 at which the amplitude of the electric field is 1% of its value at $z = 0$.

(d) Compute the skin depth at a frequency of 1 GHz.

Solutions

(a) $\omega = 10^7 \pi$ rad/s $\Rightarrow$ $f = 5\times10^6$ Hz

Here $\dfrac{\sigma}{\omega\varepsilon} = 200$ **>>1** We may therefore approximate seawater as a good conductor at this frequency.

$$\alpha = \beta = \sqrt{\pi f \mu_r \mu_0 \sigma} = 8.89 \text{ Np/m or rad/m}$$

$$\eta_c = (1+j)\sqrt{\frac{\pi f \mu_r \mu_0}{\sigma}} = \frac{\pi}{\sqrt{2}}(1+j) = \pi e^{j\pi/4} \;\; \Omega$$

$$u_p = \frac{\omega}{\beta} = 3.53\times10^6 \text{ m/s} \qquad \lambda = \frac{2\pi}{\beta} = 0.707 \text{ m}$$

$$\delta = 1/\alpha = 0.112 \text{ m}$$

(b) **Phasor fields:**

$\hat{\mathbf{e}}$ = unit vector of $\mathbf{E}(\mathbf{r})$

In a lossy medium, $$\mathbf{E}(\mathbf{r}) = \hat{\mathbf{e}} E_0 e^{j\phi} e^{-jk_c(\hat{\mathbf{k}}\cdot\mathbf{r})}$$

$$= \hat{\mathbf{e}} E_0 e^{j\phi} e^{-\alpha\,(\hat{\mathbf{k}}\cdot\mathbf{r})} e^{-j\beta\,(\hat{\mathbf{k}}\cdot\mathbf{r})}$$

Here, $\hat{\mathbf{e}} = \hat{\mathbf{x}}$, $\hat{\mathbf{k}} = \hat{\mathbf{z}}$, $\alpha = \beta = 8.89$

Since $\mathbf{E} = \hat{\mathbf{x}}\, 100 \cos(10^7 \pi t)$ at z=0, we find $\phi = 0$, $E_0 = 100$

Therefore,

$$\mathbf{E}(z) = \hat{\mathbf{x}}\, 100\, e^{-8.89z}\; e^{-j8.89z}$$

$$\mathbf{H}(\mathbf{r}) = \frac{1}{\eta_c} \hat{\mathbf{k}} \times \mathbf{E}$$

$$\Rightarrow \mathbf{H}(z) = \hat{\mathbf{y}} \frac{100}{\pi\, e^{j\pi/4}} e^{-8.89z}\; e^{-j8.89z} = \hat{\mathbf{y}} \frac{100}{\pi} e^{-8.89z}\; e^{-j(8.89z + \pi/4)}$$

Instantaneous fields:

$$\mathbf{E}(z,t) = \mathrm{Re}\left[\mathbf{E}(z)\, e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\hat{\mathbf{x}}\, 100\ e^{-8.89z}\ \ e^{-j8.89z}\, e^{j10^7 \pi t}\right]$$

$$= \hat{\mathbf{x}}\, 100\, e^{-8.89z} \cos(10^7 \pi t - 8.89z)$$

$$\mathbf{H}(z,t) = \mathrm{Re}\left[\mathbf{H}(z)\, e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\hat{\mathbf{y}}\, \frac{100}{\pi}\, e^{-8.89z}\ \ e^{-j(8.89z+\pi/4)}\, e^{j10^7 \pi t}\right]$$

$$= \hat{\mathbf{y}}\, \frac{100}{\pi}\, e^{-8.89z} \cos(10^7 \pi t - 8.89z - \pi/4)$$

(c) $\exp\left(-\alpha z_1\right) = 0.01 \quad \Rightarrow \quad z_1 = 0.518 \text{ m}$

(d) At $f = 1$ GHz,

$$\omega = 2\pi \times 10^9, \quad \sigma = 4, \quad \mu = \mu_0 = 4\pi \times 10^{-7}, \quad \varepsilon = \varepsilon_0 \varepsilon_r = 8.854 \times 10^{-12} \times 72$$

$$\frac{\sigma}{\omega\varepsilon} = 0.9986 \approx 1.$$

The approximate formulas are thus not applicable in this case.

$$\varepsilon_{rc} = \varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0} = 72 - j71.9019$$

$$k_c = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_{rc}}$$

$$= \beta - j\alpha$$

$$= 195.347 - j80.837$$

Hence at 1 GHz, $\alpha = 80.837$ Np/m and $\delta = 1/\alpha = 12.37$ mm.

3 Poynting Vector In a lossy medium

$$\mathbf{E} = \mathbf{E}_0 e^{-j\mathbf{k}_c\cdot\mathbf{r}} = \hat{\mathbf{e}}\left|E_0\right| e^{j\phi} e^{-\alpha(\hat{\mathbf{k}}\cdot\mathbf{r})} e^{-j\beta(\hat{\mathbf{k}}\cdot\mathbf{r})}$$

$$\mathbf{H} = \mathbf{H}_0 e^{-j\mathbf{k}_c\cdot\mathbf{r}} = \hat{\mathbf{h}}\frac{\left|E_0\right|}{\eta_c} e^{j\phi} e^{-\alpha(\hat{\mathbf{k}}\cdot\mathbf{r})} e^{-j\beta(\hat{\mathbf{k}}\cdot\mathbf{r})}$$

$$\mathbf{S}_{\mathrm{av}} = \frac{1}{2}\mathrm{Re}\left\{\mathbf{E}\times\mathbf{H}^*\right\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{\hat{\mathbf{k}}\frac{\left|E_0\right|^2}{\eta_c^*} e^{-2\alpha(\hat{\mathbf{k}}\cdot\mathbf{r})}\right\}$$

$$= \frac{1}{2}\left|E_0\right|^2 e^{-2\alpha(\hat{\mathbf{k}}\cdot\mathbf{r})}\,\mathrm{Re}\left\{\frac{1}{\eta_c^*}\right\}\hat{\mathbf{k}} \quad (\mathrm{W/m^2})$$

$$= P_{av}\hat{\mathbf{k}} \qquad (\mathrm{W/m^2})$$

Complex conjugate opertor: can be dropped off since

$$\mathrm{Re}\left\{\frac{1}{\eta_c}\right\} = \frac{1}{\left|\eta_c\right|}\mathrm{Re}\left\{\frac{1}{e^{j\theta_\eta}}\right\}$$

$$= \frac{1}{\left|\eta_c\right|}\mathrm{Re}\left\{\cos\theta_\eta - j\sin\theta_\eta\right\}$$

$$= \frac{\cos\theta_\eta}{\left|\eta_c\right|}$$

and $\mathrm{Re}\left\{\frac{1}{\eta_c^*}\right\}$

$$= \frac{1}{\left|\eta_c\right|}\mathrm{Re}\left\{\cos\theta_\eta + j\sin\theta_\eta\right\}$$

$$= \frac{\cos\theta_\eta}{\left|\eta_c\right|}$$

Example 2

For the plane wave in Example 1, find the power densities at distances of skin depth $z = \delta$ and $z = 0$.

Solutions

From Example 1:

$$\mathbf{E}(z) = \hat{\mathbf{x}}\, 100\, e^{-8.89z}\ e^{-j8.89z}$$

$$\mathbf{H}(z) = \hat{\mathbf{y}}\, \frac{100}{\pi} e^{-8.89z}\ e^{-j(8.89z+\pi/4)}$$

$$\text{skin depth } \delta = 0.112 \text{ m}$$

$$\eta_c = \pi\, e^{j\pi/4}\ \ \Omega$$

$$\alpha = 8.89\ \ \text{Np/m}$$

Power density at skin depth δ:

δ

$$P_\delta = \frac{1}{2}\left|E_0\right|^2 e^{-2\alpha\left(\hat{\mathbf{k}}\cdot\mathbf{r}\right)} \operatorname{Re}\left\{\frac{1}{\eta_c}\right\}$$

$$= \frac{1}{2}\left|100\right|^2 e^{-2\times 8.89\times 0.112} \operatorname{Re}\left\{\frac{1}{\pi\, e^{j\pi/4}}\right\}$$

$$= 153.63 \quad (\mathrm{W/m^2})$$

= e^{-2}. At one skin depth, E field drops by e. Then power density drops by e^2.

Power density at $z = 0$:

$$P_0 = \frac{1}{2}\left|100\right|^2 \operatorname{Re}\left\{\frac{1}{\pi\, e^{j\pi/4}}\right\}$$

$$= 1125.4 \quad (\mathrm{W/m^2})$$

Example 3

For the plane wave in Example 1, assume that the average power density at z = 0.5 m is S_{av}. Find the location, i.e., the value of z, at which the average power density is $0.0001S_{av}$.

Solutions:

Let the location be $z=0.5+z_1$. Then,

$$e^{-2\alpha z_1} = \frac{0.0001\mathrm{S_{av}}}{\mathrm{S_{av}}} \quad \Rightarrow \quad e^{\alpha z_1} = 100 \quad \Rightarrow \quad z_1 = \frac{\ln 100}{\alpha} = 0.518 \;\; \mathrm{m}$$

which yields z=0.5+0.518 = 1.018 m.

- **Textbooks:**
  - *Fundamentals of Applied Electromagnetics*,

    F. T. Ulaby, E. Michielssen, U. Ravaioli,

    Pearson Education, 2010, 6th edition

Suggested reading [textbook]:

- Section 7-4: Plane-Wave Propagation in Lossy Media
- Section 7-6: Electromagnetic Power Density

Optional reading [textbook]:

- Section 7-5: Current Flow in a Good Conductor