NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II: 2010/2011)

EE2012 – ANALYTICAL METHODS IN ELECTRICAL & COMPUTER ENGINEERING

April/May 2011 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions comprises **FIVE** (5) printed pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. This is an OPEN BOOK examination. You are allowed to bring into the examination hall any material that you wish and use it during the examination.
5. Programmable calculators may be used during the examination.

Q.1 (a) The methodology in Applied Statistics consists of three parts: modeling, data collection, and information extraction from the data. In a recent survey, it was reported that,

"More than 2/3rd Singaporeans prefer EPL over SPL."

Describe the methodology with all the relevant details adopted by the reporter for arriving at this statement.

(6 marks)

(b) Is it possible that another reporter uses the same data to arrive at the conclusion, "More than 80% Singaporeans prefer EPL over SPL?" Justify your answer.

(4 marks)

(c) Did the reporters use maximum likelihood method for estimation? Was it efficient? Biased? Consistent?

(4 marks)

(d) The conclusion must now be reached with much higher level of confidence and lower margin of error. How would you achieve that? Which of the three parts in the methodology would be done differently? Justify your answer.

(4 marks)

(e) It is known that the average life span (ALS) in a Country C is a Gaussian RV X with variance equal to 25 years^2. During the period 1920-1990, the ALS was found to be 50 years. It is being claimed in 2010 that due to eradication of disease D, ALS has changed to 75 years. The variance is known to remain unchanged. Assume a sample size of 100.

(i) Describe a test for testing the claim such that P(type I error) = 2 × P(type II error).

(ii) Sketch the relevant pdfs under the two hypotheses. Mark the region for type II errors.

(7 marks)

Q.2 (a) SH is an Internet Service Provider (ISP) in a country S. It serves 20,000 subscribers from its central office that has a capacity of 201 Gbps. Quality of service (QoS) for ISP is given by

QoS = p(demand is met for all subscribers).

The demand for a subscriber is modeled as a rv X with pdf N(μ, σ^2), μ = 10 Mbps and σ^2 = 9 Mbps^2.

(i) What is the present value of QoS? Formulate the expression that can be solved to obtain the largest number of subscribers that SH can serve for QoS = 0.9999?

(ii) If SH sells a subscription for S$ 10 per Mbps per month, estimate the monthly income I for SH, where I is to be computed using

P(Actual income > I) = 0.999.

State all assumptions clearly.

(6 marks)

(b) It is proposed that the central office be split into two offices A and B with 10,000 subscribers and a capacity of 100.5 Gbps each. You are tasked to analyze the proposal from the standpoint of QoS. Would you recommend such a proposal? Provide detailed justification for your recommendation.

(6 marks)

(c) The proposal in 2(b) is not implemented. SH wants to recruit new subscribers while keeping the same capacity. What is the effect on QoS if there is a 0.5% increase in number of subscribers?

(5 marks)

(d) A coin Q1 is tossed to generate a sequence **I** = $I1, I2, \ldots$ where

IJ = 1 if H occurs at J-th toss;
IJ = 0 if T occurs at J-th toss; J = 1, 2, ….

It is known that p(H) = 0.1 for Q1. Similarly, a fair coin Q is tossed to generate a sequence **C**. Sequences **I** and **C** are used to create a sequence **D** = $D1, D2, \ldots$, where

DJ = 1 if (IJ = 0 and CJ = 1) or (IJ = 1 and CJ = 0)
DJ = 0 if (IJ = 0 and CJ = 0) or (IJ = 1 and CJ = 1); J = 1, 2, ...

(i) Does the sequence **D** correspond to Bernoulli trials? Hence or otherwise, write the pdf of the rv X defined as

X = Number of '1' in **D** of length 1,000.

(ii) Can X be approximated by a Gaussian rv or a Poisson rv? If yes, then state the relevant parameters. State all your assumptions. Justify all answers.

(8 marks)

Q.3 (a) Find the derivative of $f(z) = \mathrm{Im}(z)$ if existent. Justify your answer otherwise.

(5 marks)

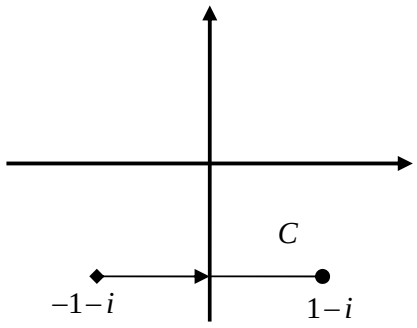
(b) Compute $\oint_{|z|=1} \overline{z} \cdot dz$, where $\overline{z}$ is the complex conjugate of *z*.

(5 marks)

(c) Compute $\oint_{|z|=\pi} z^2 e^{1/z} dz$

(5 marks)

(d) Compute $\int_C \mathrm{Re}(z)\, dz$, where C is a straight line connecting point $(-1-i)$ to point $(1-i)$, as showed in the figure below.



(5 marks)

(e) Compute $\int_0^{\infty} \frac{\cos(\pi x)}{4x^2+1} dx$

(5 marks)

Q.4 The C-gate Technology has hired three workers, W_1, W_2 and W_3, to produce two kinds of hard disk drives, D_1 and D_2, the profit being \$3 and \$2, respectively.

- Worker W_1 prepares all parts for D_1 in 4 minutes and for D_2 in 3 minutes. W_1 works for 8 hours daily.
- Worker W_2 assembles D_1 in 6 minutes and D_2 in 5 minutes. W_2 works for 8 hours daily.
- Worker W_3 packs both D_1 and D_2 in 1 minute each. W_3 only works for 4 hours daily in the afternoon.

Use the Simplex Method or any other technique to determine production figures that maximize the daily (8 working hours) profit.

(25 marks)

END OF PAPER