

Probability Laws

1. Pick a real value in the range $[0, 1]$. Let

- $A = [0, 0.5]$;
- $B = (0.4, 0.8]$;
- $C = (0.6, 1.0]$.

- (a) If the distribution function is

$$F_1(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x < 1 \\ 1 & \text{otherwise} \end{cases}$$

find $P[A \cup B]$, $P[B \cap C^c]$ and $P[A^c \cup B \cup C]$.

- (b) Repeat part (a) if the distribution function is

$$F_2(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & \text{otherwise} \end{cases}.$$

- (c) Describe in words the random numbers generated using the two distribution functions above.
2. Consider the sample space $S = \{0, 1, 2, \dots, 9\}$. Suppose the probability of the outcome k is half of that of the outcome $k - 1$, for $k = 1, 2, \dots, 9$, with $P[\{0\}] = p_0$. Find p_0 .
 3. A fair coin is flipped five times. Find the probability of obtaining k heads, where $k \in \{0, 1, 2, 3, 4, 5\}$.
 4. Roll two dice and sum the number of dots on the two top faces. Find the probability of obtaining 12.

Counting Methods

1. A multiple choice test has 10 questions with 5 choices each. How many ways are there to answer the test? What is the probability that two papers have the same answer if students choose their answers strictly at random?
2. A student has five different T-shirts and three pairs of shorts.

- (a) How many days can the student dress without repeating the combination of shorts and T-shirt?
 - (b) How many days can the student dress without repeating the combination of shorts and T-shirt, and without wearing the same T-shirt on two consecutive days?
3. A classroom has 60 seats. In how many ways can 45 students occupy the seats in the room?
 4. Five balls are placed at random in five buckets. What is the probability that each bucket has a ball?
 5. A dinner party is attended by four men and four women. How many unique ways can the eight people sit around the table? If a man must sit beside a woman, how many unique seating arrangements are there now? (Assume that the table is round.)
 6. Find the probability that in a class of 28 students, exactly four were born in each of the seven days of the week.