EE2012 2013/14 Quiz 1

February 17, 2014

Instructions

- Write your student number, your name as it appears in IVLE, and your tutorial group at the top of your answer sheet.
- You have 20 minutes. Answer all questions.
- No books, notes or other written or printed material are allowed.
- The only electronic device you can use is a non-programmable calculator, that functions only as a calculator.
- All communicating devices must be turned off prior to starting the quiz.

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1. A bag contains four balls numbered 1 through 4. Pick a ball at random from the bag, and if ball k is picked, toss a fair coin k times.

   (a) (1 mark) Find the probability of $A =$ "coin is tossed two or more times".
   Ans: Event A is equivalent to picking ball 2, 3 or 4. Since each ball is equally likely to be picked, we have $P[A] = 3/4$.

   (b) (2 marks) Let $B =$ "2 heads are seen". Find $P[B]$, using the total probability theorem.
   Ans: Let A_k be the event "k-th ball is picked". Then

$$\begin{aligned} P[B|A_1] &= 0 \\ P[B|A_2] &= \frac{1}{4} \\ P[B|A_3] &= \binom{3}{2}\frac{1}{8} = \frac{3}{8} \\ P[B|A_4] &= \binom{4}{2}\frac{1}{16} = \frac{3}{8}. \end{aligned}$$

   Therefore, using total probability, we have

$$\begin{aligned} P[B] &= \sum_{k=1}^{4} P[B|A_k]P[A_k] \\ &= \frac{1}{4}\left(0 + \frac{1}{4} + \frac{3}{8} + \frac{3}{8}\right) \\ &= \frac{1}{4}. \end{aligned}$$

2. (2 marks) Consider two events A and B, such that

$$\begin{aligned} P[A \cap B] &= 0.75 \qquad P[A \cap B^c] = 0.05 \\ P[A^c \cap B] &= 0.05 \qquad P[A^c \cap B^c] = 0.15. \end{aligned}$$

Find $P[B|A]$ and $P[A^c|B]$.

Ans: From first principles, $P[A] = P[A \cap B] + P[A \cap B^c] = 0.8$, and $P[B] = P[B \cap A] + P[B \cap A^c] = 0.8$. Therefore,

$$\begin{aligned} P[B|A] &= \frac{P[A \cap B]}{P[A]} = \frac{0.75}{0.8} \\ &= \frac{15}{16}, \\ P[A^c|B] &= \frac{P[A^c \cap B]}{P[B]} = \frac{0.05}{0.8} \\ &= \frac{1}{16}. \end{aligned}$$