NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2010/2011)

EE2012 – ANALYTICAL METHODS IN ELECTRICAL & COMPUTER ENGINEERING

November/December 2010 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions comprises **FIVE** (5) printed pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. This is an OPEN BOOK examination. You are allowed to bring into the examination hall any material that you wish and use it during the examination.

Q1. (a) In recent surveys conducted by two EE2012 students, it is reported that

Report I: “45% Singaporeans prefer to eat durian and 55% prefer to eat mango”;

Report II: “42% Singaporeans prefer to eat durian and 58% prefer to eat mango”.

Describe the complete methodology adopted by the students for conducting the surveys. Explain why the numbers are not the same in both surveys.

(7 marks)

(b) Did the students use maximum likelihood method for estimation? Was it biased? Consistent?

(5 marks)

(c) A few days later, another student argued that a better method would be to combine the two reports into one report. Describe how you would do it. How does this influence the results? Justify your answer.

(5 marks)

(d) It is known that the price of HBD apartment in a Country S is a Gaussian RV X with variance equal to 90,000\$. During slowdown in 2007-2009, the average price was found to be 250,000\$. It is being claimed in 2010 that due to economic recovery, the average price has gone up to 350,000\$. The variance is known to remain unchanged.

(i) Describe a test for testing the above claim such that P(type I error) = 0.01.

(ii) Sketch the relevant pdfs under the two hypotheses. Mark the region for type II errors.

(iii) Show that we cannot make P(type II error) and P(type I error) equal to 0 simultaneously.

(8 marks)

Q2. (a) Emails that arrive at NUS Computer Centre (NCC) are two types: Type A that have attachments and Type B that have no attachments. The number of Type A and B emails that arrive at NCC are Poisson RVs A and B with parameters λ_1 and λ_2, respectively. The size of any Type A email is a Gaussian RV $N(\mu_1, \sigma_1^2)$, and the size of any Type B email is a Gaussian RV $N(\mu_2, \sigma_2^2)$. Let N be the total number of emails that arrive at NCC and W be their total size. Make and state any reasonable assumptions.

(i) Write the PDF of N.

(ii) The system support person at NCC counts that NCC has '$A = a$' Type A emails and '$B = b$' Type B emails. Write an expression for

$$P(w < W \leq w + dw \mid A = a, B = b)$$

in this case.

(iii) Hence or otherwise, write an expression for $P(w < W \leq w + dw \mid A = a)$.

(iv) Hence or otherwise, write an expression for $P(w < W \leq w + dw)$.

(v) It is claimed that the RV W is Gaussian. Do you agree? Justify.

(vi) Write an expression for probability of the event "the NCC buffer is full", if the NCC buffer has a capacity of 50 emails.

(18 marks)

(b) We are given a series of jointly Gaussian RVs X_0, X_1, X_2, … each with 0 mean and variance 1, $\text{cov}(X_J, X_K) = 0.9^{|K-J|}$. Define another series of RVs Y_0, Y_1, Y_2, …, defined as $Y_I = X_I - a\, X_{I-2}$.

(i) Write an expression for $\text{Var}(Y_I)$.

(ii) Find the smallest value for $\text{Var}(Y_I)$ and the associated value of 'a'.

(7 marks)

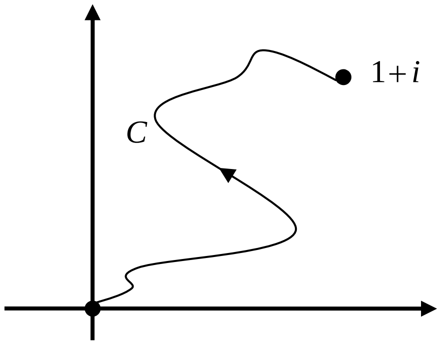
Q.3 (a) Express $(1+i)^{400}$ in the form of $x+iy$.

(5 marks)

(b) Give the Taylor series expansion of $\frac{1}{z}$ about $z_0 = i$. Show the first four terms and give the radius of convergence.

(5 marks)

(c) Compute $\int_C z^2\, dz$, where C is a curve connecting point the origin to point $(1+i)$, as showed in the figure below.



(5 marks)

(d) Compute $\oint_{|z|=\pi} \sin\left(\frac{1}{z}\right) dz$ (5 marks)

(e) Compute $\int_{-\infty}^{\infty} \sin x \cdot \frac{x+\sin 1}{x^2+1} dx$ (5 marks)

Q.4 (a) What is the difference between the Cauchy's method of steepest descent and the simplex method?

(5 marks)

(b) Consider an objective function $f(x_1, x_2) = 29x_1^2 + 45x_2^2$. Suggest an optimization technique that can be used to determine its minimum. Give the iterative procedure (algorithm) for computing the minimum.

(***Note that you do not need to compute the minimum***)

(10 marks)

(c) Consider a function $f(x_1, x_2) = 29x_1^2 + 45x_2^2$ with constraints $2x_1^2 + 8x_2^2 \leq 60$ and $4x_1^2 + 4x_2^2 \leq 60$. Can we use linear programming techniques, such as the simplex method, to find its maximum? Why or why not? State clearly your reasoning.

(***Note that you do not need to compute the maximum***)

(10 marks)

END OF PAPER