EE2012 2014/15 PROBLEM SET 1

Set Theory

1. For each of the following sets, write down two subsets:
   (a) $\mathbb{R}$, the set of real numbers.
   (b) $\mathbb{C}$, the set of complex numbers.
   (c) $\mathbb{Z}$, the set of integers.
   (d) $S = [0, 2)$, the set of all real numbers from 0 to 2, including 0, excluding 2.
   (e) $T = \{\text{Black, White, Grey}\}$
   (f) $U = \{x : x \in \mathbb{R}, x \geq 2\}$

2. Identify the sets in problem 1 that are countable. Find the cardinality of each of the countable sets.

3. Are the following statements true or false? Why?
   (a) $\{0\} = \emptyset$
   (b) $\{2\} = 2$
   (c) $[2, 3) \cap \{1, 2, 3, 4\} = \{2, 3\}$
   (d) $\emptyset \cup A = A$ for any set A.
   (e) $x \in A \Rightarrow -x \in A^c$.
   (f) $\{a, b, c\} \cup \{b, c, d\} = \{a, b, c, d\}$.
   (g) $A \cup A^c = S$, the universal set.
   (h) $x \in A \Leftrightarrow \{x\} \subset A$.

4. Using a Venn diagram, prove de Morgan's rules:

$$(A \cup B)^c = A^c \cap B^c \quad (1)$$
$$(A \cap B)^c = A^c \cup B^c. \quad (2)$$

5. On a Venn diagram, illustrate the distributive property $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (This is not a rigorous proof, which would require the consideration of all possible relationships between A, B and C with regards to whether they are mutually exclusive or not.)

Probability Space

1. For each of the following experiments, write down the sample space and state if it is countable or uncountable.

   (a) Measure the time it takes to travel from Clementi to Jurong East stations on the MRT.

   (b) Count the number of vehicles that pass under the overhead bridge across Clementi Road between 5.00pm and 5.05pm.

   (c) Take a chest X-ray of someone picked randomly, and note if it is normal.

   (d) Throw a dart at a dartboard and note the (x, y) coordinates of where it lands.

2. For each of the sample spaces in the previous question, write down one possible event, both in words and as a subset of S.

3. If $P(A) = 0.6$, show using the Axioms of Probability that $P(A^c)$ must be 0.4, i.e. we are not free to assign values to both $P(A)$ and $P(A^c)$.

4. Roll a die and note the number showing up on top. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. Write down two valid probability assignments (i.e. they satisfy the axioms of probability) for the elementary events of S. Which of the two (if any) are useful for modeling the rolling of a fair die? Can you see that a model need not be useful even if it is mathematically valid?

5. Let $S = \{a, b, c, d\}$ be the sample space of some experiment, and let $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d\}$ be three events (i.e. they are each in the event field $\mathcal{F}$).

   (a) Write down the sets representing the composite events "A and B occur", "neither A nor C occurs", and "B occurs but not C".

   (b) Let the probabilities of the elementary events be p_a, p_b, p_c and p_d respectively. Find the probabilities of the events of part (a) in terms of these probabilities.

6. Consider the sample space $S = \{a, b, c, d, e, f\}$.

   (a) If we are only interested in whether event $A = \{a, d, f\}$ occurs, write down the event field.

   (b) If we are interested in all subsets of S, find the total number of events in $\mathcal{F}$. Explain your answer.

   (c) If we know the probabilities of all elementary events, i.e. $p_a = P[\{a\}]$, $p_b = P[\{b\}]$, etc. are all known, what are the probability mappings (P_1 and P_2) in parts (a) and (b)?