EE2012 2014/15 Problem Set 8

Functions of a Random Variable

1. Let X be a geometric random variable with $E[X] = 1/p$. Find the PMF of the following functions of X:

   (a) $Y = X^2$;

   (b) $Z = \ln(X)$;

   (c) $V = e^X$.

2. The number X is drawn at random from the unit interval $[0,1]$, and Y is defined as X rounded to the nearest tenth, i.e. if $X = 0.12$ then $Y = 0.1$, if $X = 0.87$ then $Y = 0.9$, etc. Find the PMF of Y.

3. A wire has length X, an exponential random variable with mean 5π cm. The wire is cut to make rings of diameter 1 cm. Let Y be the number of complete rings made from the wire. Find the PMF of Y.

4. Let X be uniformly distributed in $(0,1]$. Sketch the function $g(x)$ in the following cases, and then find the PDF of $Y = g(X)$.

   (a) $g(x) = x^2$;

   (b) $g(x) = e^{-x}$;

   (c) $g(x) = \cos 2\pi x$.

5. Let $X \sim \mathcal{N}(2,4)$, and
$$Y = (X)^+ = \begin{cases} X & X > 0 \\ 0 & X \leq 0 \end{cases}$$
   Find the CDF and hence the PDF of Y.

6. Let X be a Rayleigh random variable with PDF
$$f_X(x) = xe^{-x^2/2}, \quad x > 0.$$
   Find the PDF of $Z = X^2$ by (i) first finding the CDF, and (ii) finding the PDF directly.

7. A random variable Y is said to be log-normally distributed if $X = \ln Y$ is an $\mathcal{N}(\mu, \sigma^2)$ random variable. Find the PDF of Y, and hence show that $E[Y] = \exp\left[\mu + \frac{\sigma^2}{2}\right]$.

8. The input to a full-wave rectifier is X and its output is $Y = |X|$. Find the PDF of Y if $X \sim \mathcal{N}(0, \sigma^2)$.