NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II: 2011/2012)

EE2012 – ANALYTICAL METHODS IN ELECTRICAL & COMPUTER ENGINEERING

April/May 2012 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions comprises **FIVE** (5) printed pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. This is a closed book examination. You are allowed to bring into the examination hall a size A3 cheat-sheet that you may use during the examination.
5. Programmable calculators are not allowed for this examination.

Q.1

(a) In a recent news-paper article, it was reported that,

"More than 60% Singaporeans live in HDB."

You are approached by a neighbor and asked how this could be deduced. Describe your reply to the neighbor including all the pertinent concepts such as maximum likelihood method for estimation, biased vs unbiased, efficiency, consistency.

(10 marks)

(b) Not satisfied, your neighbor borrows your statistical tables and using same data as in Q.1(a) arrives at the conclusion: "More than 65% Singaporeans live in HDB." Is that possible? Explain your answer.

(5 marks)

(c) Describe the steps you took to ensure that the samples taken in Experiment II were random. How would your results change if the samples drawn are non-random?

(5 marks)

(d) Historically, % of property transactions in a Country S by foreigners is Gaussian rv X with mean $\mu = 15$, standard deviation $\sigma = 3$. It is reported that the mean has declined to 10 due to recent changes in taxes for foreigners. The variance is known to remain unchanged. In the past 5 quarters the % of property transactions by foreigners is 8, 9, 12, 15, 9. Describe a test for testing the claim such that P(type I error) = 0.1 by first sketching the pdfs of rvs $\hat{\mu}|H_0$ and $\hat{\mu}|H_1$.

(5 marks)

Q.2 This problem deals with design of land-line and mobile phones.

(a) It is known that humans can hear signals ranging from 20 Hz to 20,000 Hz in frequency. The land line phones are designed based on the assumption that frequency components above 3,400 Hz are not needed for voice communications. Describe how this assumption could be tested in the lab.

(6 marks)

(b) "Human voice is a stationary Gaussian random process $\boldsymbol{X}(t)$." Do you agree? Give reasoning in support of your answer.

(6 marks)

(c) Voice is modeled as stationary low pass Gaussian random process $\boldsymbol{X}(t)$ having psd

$$S_X(f) = 1 \text{ mW / Hz for } |\, f \,| < 3.4 \text{ KHz.}$$

$S_X(f) = 0$ for all other frequencies. Voice is sampled at a rate of 8,000 samples / sec for an acceptable quality of telephony. Further, it is known that each sample $X(t)$ must be digitized using b bits where

$$2^b = K \times \mathrm{Var}(X(t)).$$

Determine K given that $b = 8$ for landline phones.

(8 marks)

(d) Let two adjacent voice samples of $\boldsymbol{X}(t)$ taken at time $t = 0$ and $t = 125$ micro-sec be $X0$ and $X1$. For mobile telephony, a processed sample Y is digitized where

$$Y = X0 - 0.2 \times X1.$$

"Y requires fewer bits than 8 for digitization." Do you agree? Justify your answer.

(5 marks)

Q.3 One of two coins is selected in a random manner. For coin C1, P(H | C1) = 1 / 5 and for coin C2, P(H | C2) = 4 / 5.

(a) Suppose that the selected coin is tossed twice. Find P(H in 1^{st} toss) and P(HH). Do these two tosses constitute Bernoulli trials?

(7 marks)

(b) Suppose that the selected coin is tossed N times. Define a rv X as

$$X = \#\text{H in } N \text{ tosses.}$$

Find the pdf of rv X. Does the rv X have binomial pdf?

(6 marks)

(c) Compute E(X) and Var(X) for the rv X defined in Q3(b).

(6 marks)

(d) Define a random experiment as “toss selected coin till H occurs for the first time.” Let Y be the number of such tosses. Given that Y = 10, identify the coin more likely to be the coin selected for these tosses.

(6 marks)

Q.4

(a) (multichannel transmission for mobile comms). A fair coin is tossed and the outcome is transmitted over two channels C1 and C2 using an information signal of S volts for 1 sec such that

$S = +1$ for H, $S = -1$ for T.

The received signal for the two channels is given by

$R1 = S + N1, \qquad R2 = S + N2;$

respectively. Noise signals are SI rvs with pdf $N1 \sim \mathrm{N}(0, \sigma_1^2)$, $N2 \sim \mathrm{N}(0, \sigma_2^2)$. It is claimed that the 'optimal' way to process $R1$ and $R2$ is to take a linear combination

$R = R1 + b\, R2.$

Show that received signal R can be written as a sum of two signals, an information signal and a noise signal. Write an expression for SNR (Signal to Noise Ratio) defined as

SNR = Av power in information signal in R / Av power in noise signal in R.

Find the value of 'b' that maximizes SNR.

(10 marks)

(b) A coin with $P(H) = p = 10^{-3}$ was tossed 1,000,000 times and H occurred 990 times. It is now claimed that $p < 10^{-3}$. Is this claim valid at 0.01 level of significance? 0.001 level of significance?

(7 marks)

(c) In PCM (pulse-code-modulation), each analog amplitude X, modeled as a uniformly distributed rv in the interval $[-A, +A]$, is digitized to a value X_D represented by b bits as follows:

Partition the interval $[-A, +A]$ into 2^b equal sub-intervals.
Associate each sub-interval with unique b-bits that represent its mid-value.
If X belongs to i-th sub-interval, digitize X to b-bits for i-th sub-interval.

Let $E = X - X_D$ denote digitization error. Plot the pdf of E and compute (i) mean and (ii) variance of the rv E.

(8 marks)

END OF PAPER