EE2012 2014/15 PROBLEM SET 7

Expected Values

1. Find the mean and variance of the following PDFs:

   (a) $f_X(x) = \frac{5}{8}(1 - x^4)$, $-1 < x \leq 1$.

   (b) $f_Y(y) = 6y(1 - y)$, $0 \leq y \leq 1$.

   (c) $f_Z(z) = 0.5\delta(z + 2) + 0.25[u(z) - u(z - 2)]$.

2. Let the function $g(x)$ be defined as

$$g(x) = \begin{cases} -a & x \leq -a \\ x & -a < x \leq a \\ a & x > a \end{cases}$$

   (a) Sketch $g(x)$.

   (b) Let $Y = g(X)$, where X is a random variable with the PDF

$$f_X(x) = \frac{1}{2}e^{-|x|}.$$

   Find $E[Y]$ and $\text{var}(Y)$.

3. Let a and b be constants such that $a < b$, and let X have the PDF

$$f_X(x) = \lambda e^{-\lambda(x-a)}, \quad x > a, \lambda > 0.$$

   Find the conditional PDF of X given $\{X \leq b\}$, and hence find $E[X|X \leq b]$.

4. If the first and second moments of X are $E[X] = 1$ and $E[X^2] = 2$, find

   (a) $E[(X - 2)^2]$;

   (b) the value of a that minimizes $E[(aX - 1)^2]$;

   (c) $\text{var}(X - 1)$.

Important Random Variables

1. An average of 50 pedestrians pass a given point on a sidewalk per hour in the day, and the average drops to 10 in the night. Assuming that the number of pedestrians per hour is Poisson, find the probability density function of the time between successive pedestrians passing this point.

2. The r-th percentile, $\pi(r)$, of a random variable X is defined by $P[X \leq \pi(r)] = r/100$.

(a) Find the 90th, 95th, and 99th percentiles of the exponential random variable with parameter λ.

(b) Repeat part (a) for the Gaussian random variable with parameters $\mu = 0$ and σ^2.

3. Let X be an exponential random variable with parameter λ.

   (a) For $d > 0$ and k a non-negative integer, find $P[kd < X \leq (k+1)d]$.

   (b) Segment the positive real line into four equi-probable disjoint intervals.

4. Let X be a Gaussian random variable with mean μ and variance σ^2.

   (a) Find $P[X \leq \mu]$.

   (b) Find $P[|X - \mu| < k\sigma]$, for $k = 1, 2, 3, 4$.

   (c) Find the value of k for which $P[X > \mu + k\sigma] = 10^{-j}$, for $j = 1, 2, 3, 4, 5, 6$.

5. Two chips are being considered for use in a certain system. The lifetime of chip 1 is modelled by a Gaussian RV with mean 20,000 hours and standard deviation 5,000 hours. The lifetime of chip 2 is also Gaussian, with mean 22,000 hours and standard deviation 1,000 hours. Which chip is preferred if the target lifetime is 20,000 hours? What if the target is 24,000 hours?