EE2012 2013/14 Quiz 2

March 31, 2014

Instructions

- Write your student number, your name as it appears in IVLE, and your tutorial group at the top of your answer sheet.
- You have 20 minutes. Answer all questions.
- No books, notes or other written or printed material are allowed.
- The only electronic device you can use is a non-programmable calculator, that functions only as a calculator.
- All communicating devices must be turned off prior to starting the quiz.

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1. A random variable X is generated by first tossing a coin that comes up Heads with probability 0.4. Then if the coin toss is Heads, $X = 0$ with probability 0.5, and $X = 2$ with probability 0.5. If the coin toss is Tails, X is Gaussian with mean 1 and variance 1.

   (a) Find the PDF of X. (4 marks)

   Ans: We have the two conditional PDFs

   $$f_X(x|H) = 0.5\delta(x) + 0.5\delta(x-2) \quad (1)$$

   $$f_X(x|T) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-1)^2}{2}\right) \quad (2)$$

   and therefore, by total probability,

   $$f_X(x) = 0.2\left[\delta(x) + \delta(x-2)\right] + \frac{0.6}{\sqrt{2\pi}} \exp\left(-\frac{(x-1)^2}{2}\right). \quad (3)$$

   (b) Find $E[X]$ and $\mathrm{var}(X)$. (4 marks)

   Ans: Due to the even symmetry of $f_X(x)$ around $x = 1$, $E[X] = 1$. We also have

   $$\begin{aligned} E[X^2] &= E[X^2|H]P[H] + E[X^2|T]P[T] \\ &= 2(0.4) + 2(0.6) = 2. \end{aligned}$$

   Therefore, $\mathrm{var}(X) = E[X^2] - E^2[X] = 2 - 1 = 1$. (Although in this case we have $\mathrm{var}(X) = \mathrm{var}(X|H)P(H) + \mathrm{var}(X|T)P(T)$, because $E[X|T] = E[X|H]$, variance cannot in general be computed directly from conditional variances.)

   (c) What type (continuous, mixed or discrete) of random variable is X? Explain your answer. (2 marks)

   Ans: Since the PDF of X is a sum of impulse functions and a continuous function, it is a mixed random variable.