

APPENDIX

The following information may be used where applicable:

Elementary charge	q	$= 1.602 \times 10^{-19} \text{ C}$
Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-14} \text{ F cm}^{-1}$
Room temperature		$= 300 \text{ K}$
Thermal voltage at 300 K	V_T	$= kT/q = 0.025 \text{ V} = 25 \text{ mV}$
Intrinsic carrier concentration of silicon at 300 K	n_i	$= 1.5 \times 10^{10} \text{ cm}^{-3}$
Relative permittivity of silicon	$\epsilon_r (\text{Si})$	$= 11.7$
Relative permittivity of silicon dioxide	$\epsilon_r (\text{SiO}_2)$	$= 3.9$

Summary of Equations

I. Semiconductor Physics

<u>Quantity</u>	<u>Equation</u>	<u>Remarks</u>
Law of mass action	$p_0 n_0 = n_i^2$	$n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ for Si at $T = 300 \text{ K}$
Charge neutrality	$p + N_D = n + N_A$	
Carrier concentrations in p -type semiconductor at thermal equilibrium (cm^{-3})	$p_{p0} = N_A$ $n_{p0} = \frac{n_i^2}{N_A}$	$T = 300 \text{ K}$ N_A = net p -type doping concentration
Carrier concentrations in n -type semiconductor at thermal equilibrium (cm^{-3})	$n_{n0} = N_D$ $p_{n0} = \frac{n_i^2}{N_D}$	$T = 300 \text{ K}$ N_D = net n -type doping concentration
Drift current density (A cm^{-2})	$J_{drift} = J_{p,drift} + J_{n,drift}$ $J_{p,drift} = qp\mu_p E$ $J_{n,drift} = qn\mu_n E$ $J_{drift} = q(p\mu_p + n\mu_n)E = \sigma E$	σ is conductivity, E is electric field
Conductivity ($\Omega \text{ cm}$) ⁻¹	$\sigma = q(p\mu_p + n\mu_n) = \frac{1}{\rho}$	ρ is resistivity
Diffusion current density (A cm^{-2})	$J_{diff} = J_{p,diff} + J_{n,diff}$ $J_{p,diff} = -qD_p \frac{dp}{dx}$ $J_{n,diff} = qD_n \frac{dn}{dx}$ $J_{diff} = -qD_p \frac{dp}{dx} + qD_n \frac{dn}{dx}$	
Einstein Relation	$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{q} = V_T$	$k = 1.381 \times 10^{-23} \text{ J/K}$ $= 8.62 \times 10^{-5} \text{ eV/K}$ $q = 1.602 \times 10^{-19} \text{ C}$ $V_T \approx 0.025 \text{ V}$ at $T = 300 \text{ K}$.

II. pn Junction

Quantity	Equation	Remarks
Junction built-in voltage (V)	$V_0 = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right)$	$V_0 \cong 0.6 - 0.8$ V for Si at $T = 300$ K. N_D is the doping concentration of the n -region. N_A is the doping concentration of the p -region.
Width of depletion region (cm or μm)	$W_{dep} = x_p + x_n$ $= \sqrt{\frac{2\epsilon_s}{q} \left[\frac{1}{N_A} + \frac{1}{N_D} \right] (V_0 - V)}$ $x_p = \frac{N_D}{N_D + N_A} W_{dep}, \quad x_n = \frac{N_A}{N_D + N_A} W_{dep}$	V is positive for forward bias and negative for reverse bias $\epsilon_s = 11.7 \times \epsilon_0$ for Si $\epsilon_0 = 8.854 \times 10^{-14}$ F cm^{-1} . x_p is the depletion width in the p -region. x_n is the depletion width in the n -region.
Minority carrier concentration at the edge of depletion region (cm^{-3})	$p\text{-side: } n_p(-x_p) = n_{p0} e^{V/V_T}$ $n\text{-side: } p_n(x_n) = p_{n0} e^{V/V_T}$	= thermal equilibrium minority carrier concentration $\times e^{V/V_T}$
Minority carrier concentration distributions in the neutral n - and p -regions (cm^{-3})	$n\text{-side:}$ $p_n(x) = p_{n0} + [p_n(x_n) - p_{n0}] e^{-(x-x_n)/L_p}$ $p\text{-side:}$ $n_p(x) = n_{p0} + [n_p(-x_p) - n_{p0}] e^{(x+x_p)/L_n}$	L_p, L_n are the diffusion lengths of the minority carriers in the n -side and p -side respectively. In the equations, $x > x_n$ on the n -side, and $x < -x_p$ on the p -side.
Current (A)	$I = I_p + I_n = I_s (e^{V/nV_T} - 1)$ $I_p = A q n_i^2 \frac{D_p}{L_p N_D} (e^{V/nV_T} - 1)$ $I_n = A q n_i^2 \frac{D_n}{L_n N_A} (e^{V/nV_T} - 1)$	n is a value between 1 and 2. I_p is the hole current injected into the neutral n -region. I_n is the electron current injected into the neutral p -region.
Saturation current (A)	$I_s = A q n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$	
Amount of charge stored in (one side) of the depletion region (C)	$q_j = q x_p N_A A = q x_n N_D A$ $= q \left(\frac{N_A N_D}{N_A + N_D} \right) W_{dep} A$	
Depletion or junction capacitance (F)	$C_j = \frac{\epsilon_s A}{W_{dep}}$	m is a value the ranges between 1/3 to 1/2.

	$C_j = A \left[\frac{q\epsilon_s}{2} \left(\frac{N_A N_D}{N_A + N_D} \right) \frac{1}{(V_0 - V)} \right]^m$ $C_j = A \times C_{j0} / \left(1 - \frac{V}{V_0} \right)^m$ $C_{j0} = \left[\frac{q\epsilon_s}{2} \left(\frac{N_A N_D}{N_A + N_D} \right) \frac{1}{V_0} \right]^m$	
Small signal (or incremental) resistance of the pn junction diode (Ω)	$r_d = \frac{nV_T}{I_D}$	I_D is the (d.c.) diode current at the operating point, n is a value between 1 and 2.

III. Bipolar Junction Transistor (BJT)

<u>Quantity</u>	<u>Equation</u>	<u>Remarks</u>
Magnitude of the diffusion current of the minority carriers (electrons) in the base of an npn BJT (A)	$ i_{En} = \left qAD_n \frac{n_p(0) - n_p(w_B)}{0 - w_B} \right $	D_n is the diffusivity of the electrons in the base, w_B is the width of the neutral part of the base, $n_p(0)$ and $n_p(w_B)$ are the concentrations of the electrons on the base side of the edges of the E-B and C-B space charge regions respectively.
Collector current of an npn BJT (A)	$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$	
Collector saturation current of an npn BJT (A)	$I_S = qA \frac{D_n}{w_B} \frac{n_i^2}{N_{AB}}$	Note that the symbol I_S is also the symbol for the saturation current of a p-n junction diode, but the expression is different.
Base current of an npn BJT (A)	$i_B = \frac{I_S}{\beta} \exp\left(\frac{v_{BE}}{V_T}\right)$ $\text{where } \beta = \frac{D_n}{D_p} \frac{L_p}{w_B} \frac{N_{DE}}{N_{AB}}$	The expression for β here is for an npn BJT with a long emitter.
Common emitter current gain of a BJT (general definition)	$\beta = \frac{i_C}{i_B}$	
Common base current gain of a BJT (general definition)	$\alpha = \frac{i_C}{i_E}$	
Relationship between α and β	$\beta = \frac{\alpha}{1 - \alpha}$	
Collector current of an npn BJT, with Early effect taken into account (A)	$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right) \left(1 + \frac{v_{CE}}{V_A} \right)$	V_A is the Early voltage.

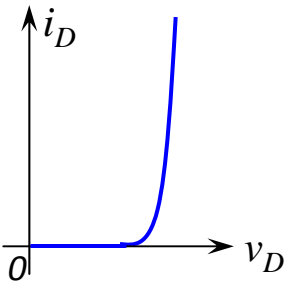
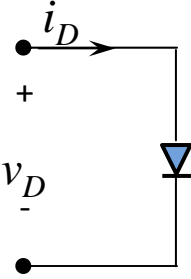
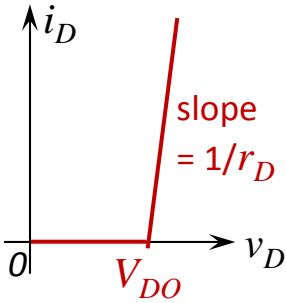
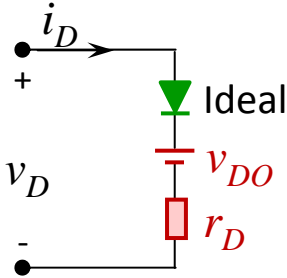
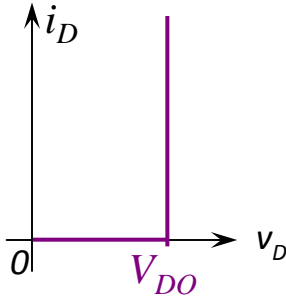
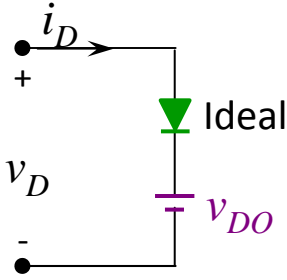
Transconductance in the small signal equivalent circuit of a BJT (Ω^{-1})	$g_m = \frac{I_C}{V_T} \Big _{\text{at bias point}}$	I_C is the value of the collector current at the d.c. operating (bias) point.
Input resistance in the small signal equivalent circuit of a BJT (Ω)	$r_\pi = \frac{\beta}{g_m}$	
Output resistance in the small signal equivalent circuit of a BJT (Ω)	$r_o \approx \frac{V_A}{I_C}$	I_C is the value of the collector current at the d.c. operating (bias) point.

IV. Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

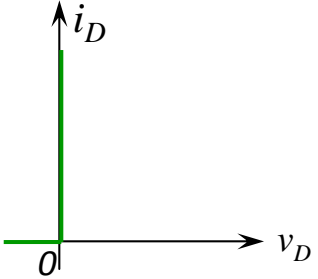
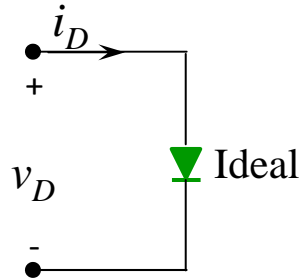
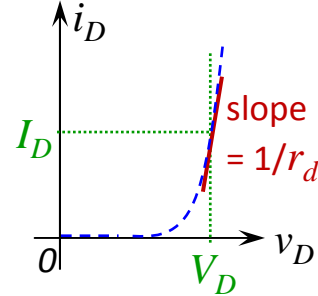
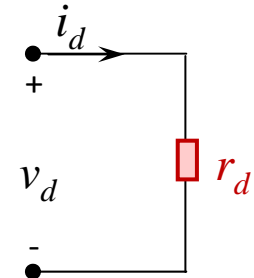
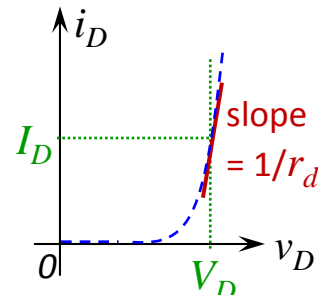
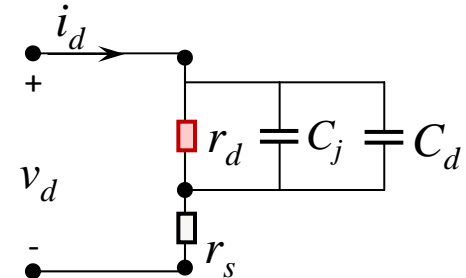
<u>Quantity</u>	<u>Equation</u>	<u>Remarks</u>
Drain current of an n-MOSFET operating in the linear region	$i_D = \mu_n \frac{W}{L} C_{ox} \left[(v_{GS} - V_{TH}) v_{DS} - \frac{1}{2} v_{DS}^2 \right]$ $= 2K_n \left[(v_{GS} - V_{TH}) v_{DS} - \frac{1}{2} v_{DS}^2 \right]$	Valid for $v_{DS} \leq v_{GS} - V_{TH}$
Drain current of an n-MOSFET operating in the saturation region	$i_D = i_{Dsat} = \frac{1}{2} \mu_n \frac{W}{L} C_{ox} (v_{GS} - V_{TH})^2$ $= K_n (v_{GS} - V_{TH})^2$	Valid for $v_{DS} \geq v_{GS} - V_{TH}$
Transconductance parameter of an n-MOSFET	$K_n = \frac{1}{2} \mu_n \frac{W}{L} C_{ox}$	
Drain current of a p-MOSFET operating in the linear region	$ i_D = \mu_p \frac{W}{L} C_{ox} \left[(v_{GS} - V_{TH}) v_{DS} - \frac{1}{2} v_{DS} ^2 \right]$ $= 2K_p \left[(v_{GS} - V_{TH}) v_{DS} - \frac{1}{2} v_{DS} ^2 \right]$	Valid for $ v_{DS} \leq v_{GS} - V_{TH} $
Drain current of a p-MOSFET operating in the saturation region	$ i_D = i_{Dsat} = \frac{1}{2} \mu_p \frac{W}{L} C_{ox} (v_{GS} - V_{TH})^2$ $= K_p (v_{GS} - V_{TH})^2$	Valid for $ v_{DS} \geq v_{GS} - V_{TH} $
Transconductance parameter of a p-MOSFET	$K_p = \frac{1}{2} \mu_p \frac{W}{L} C_{ox}$	
Gate oxide capacitance (per unit gate area) of a MOSFET	$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$	
Transconductance in the small signal equivalent circuit of an n-MOSFET operating in the saturation region	$g_m = 2K_n (V_{GS} - V_{TH}) = 2\sqrt{K_n I_D}$	I_D and V_{GS} are the magnitudes of the drain current and gate-to-source voltage, respectively, at the d.c. operating point.

Transconductance in the small signal equivalent circuit of a p-MOSFET operating in the saturation region	$g_m = 2K_p(V_{GS} - V_{TH}) = 2\sqrt{K_p I_D }$	I_D and V_{GS} are the magnitudes of the drain current and gate-to-source voltage, respectively, at the d.c. operating point.
Drain-to-source resistance in the large signal model of the n-MOSFET operating in the linear region	$R_{DS} = \frac{1}{2K_n(V_{GS} - V_{TH})}$	V_{GS} is the magnitude of the gate-to-source voltage, at the d.c. operating point.
Drain-to-source resistance in the large signal model of the p-MOSFET operating in the linear region	$R_{DS} = \frac{1}{2K_p(V_{GS} - V_{TH})}$	V_{GS} is the magnitude of the gate-to-source voltage, at the d.c. operating point.
Output resistance in the small signal equivalent circuit of a MOSFET operating in the saturation region	$r_o \approx \frac{V_A}{I_D} = \frac{1}{\lambda I_D}$	I_D is the magnitude of the drain current at the d.c. operating point.
Drain current of an n-MOSFET operating in the saturation region, including channel length modulation effect	$i_D = K_n(v_{GS} - V_{TH})^2(1 + \lambda v_{DS})$	Valid for $v_{DS} \geq v_{GS} - V_{TH}$
Drain current of an p-MOSFET operating in the saturation region, including channel length modulation effect	$ i_D = K_p(v_{GS} - V_{TH})^2(1 + \lambda v_{DS})$	Valid for $ v_{DS} \geq v_{GS} - V_{TH} $

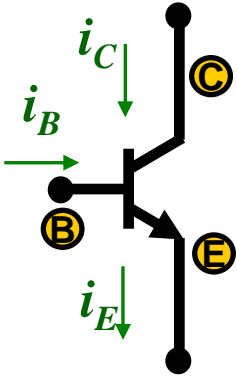
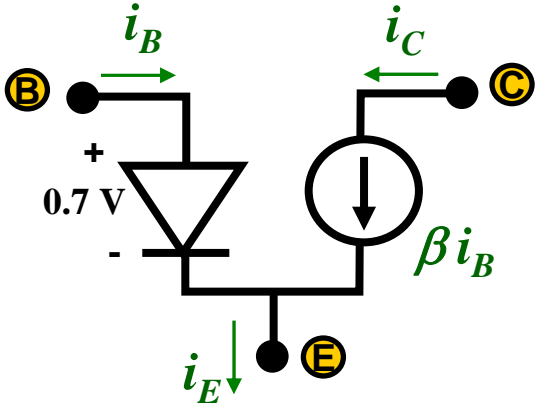
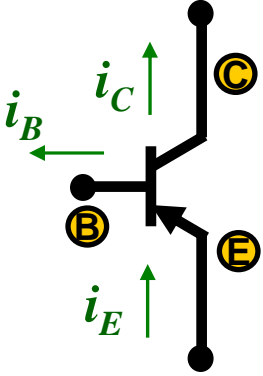
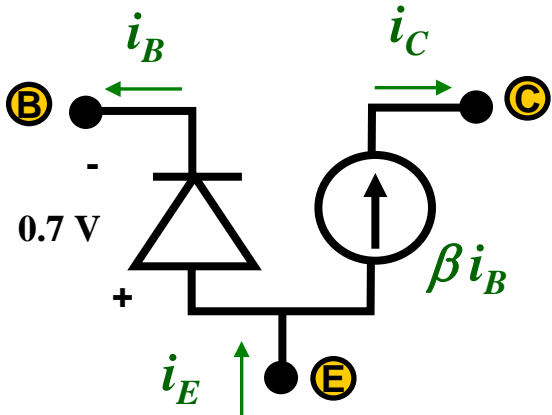
Summary of *pn* junction models (Forward Bias)

Model	Graph	Equations	Circuit	Comments
Exponential		$i_D \cong I_S e^{\frac{v_D}{nV_T}}$ $v_D = nV_T \ln(i_D / I_S)$ $v_{D2} - v_{D1} = nV_T \ln(i_{D2} / i_{D1})$		Physically based and accurate model. Useful for accurate analysis.
Piece-wise linear		<p>For $v_D \leq V_{DO}$:</p> $i_D = 0$ <p>For $v_D \geq V_{DO}$:</p> $i_D = (v_D - V_{DO}) / r_D$		Choice of V_{DO} and r_D is determined by current range over which model is used.
Constant-voltage drop		<p>For $v_D \leq V_{DO}$:</p> $i_D = 0$ <p>For $v_D \geq V_{DO}$:</p> $i_D = \infty$		Easy to use model and very popular for quick analysis. Typically $V_{DO} = 0.7$ V.

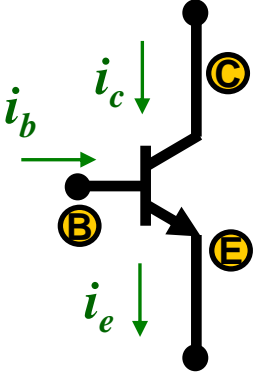
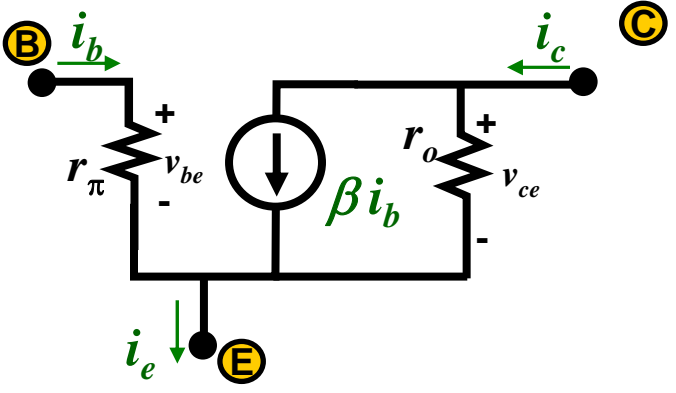
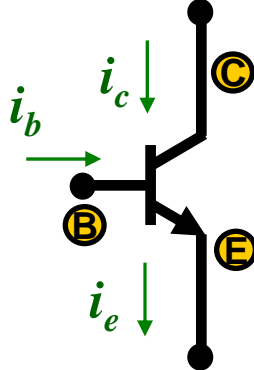
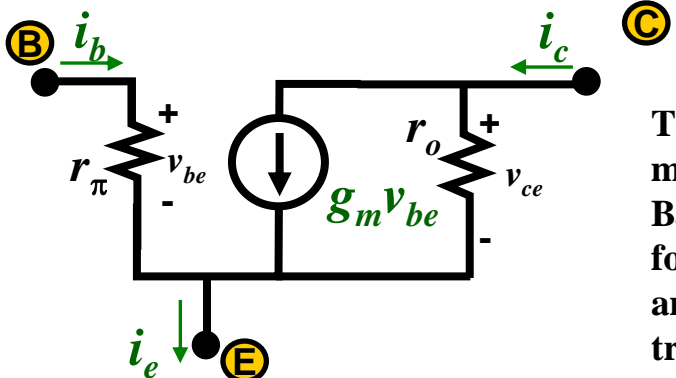
Summary of *pn* junction models (Forward Bias)

Model	Graph	Equations	Circuit	Comments
Ideal-diode		<p>For $v_D < 0$:</p> $i_D = 0.$ <p>For $v_D = 0$:</p> $i_D > 0.$		
Small-signal		<p>Around dc biasing (I_D, V_D), small-signal i_d and v_d:</p> $i_d \cong v_d / r_d$ $r_d = nV_T / I_D$		For analysis of small-signal superimposed on dc biasing point (I_D, V_D)
Small-signal (with capacitive & series resistance effects)				

Summary of *BJT* models

Model	Circuit Symbol	Equations	Circuit	Comments
Large signal model of the npn BJT		$i_C = I_S e^{v_{BE}/V_T}$ $i_C = \beta i_B$		
Large signal model of the pnp BJT		$i_C = I_S e^{v_{EB}/V_T}$ $i_C = \beta i_B$		

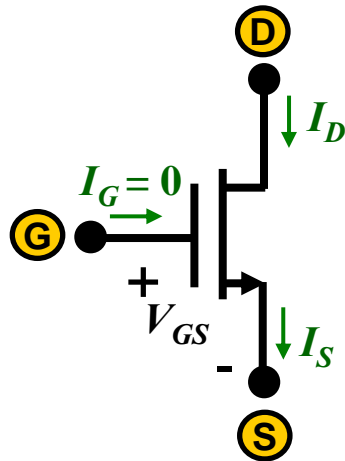
Summary of BJT models

Model	Circuit Symbol	Equations	Circuit	Comments
Small signal (or hybrid- π) model of the BJT (current-controlled version)		$g_m = \frac{I_C}{V_T}$ $r_\pi = \frac{\beta}{g_m}$ $r_o = \frac{V_A}{I_C}$		I_C is the collector current at the d.c. bias point. V_T is the thermal voltage and V_A is the Early voltage.
Small signal (or hybrid- π) model of the BJT (voltage-controlled version)				The small signal model of the BJT is the same for both npn and pnp transistors.

Summary of *MOSFET* models

Model	Circuit Symbol	Equations	Circuit	Comments
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Large signal model of the n-channel MOSFET operating in the saturation region

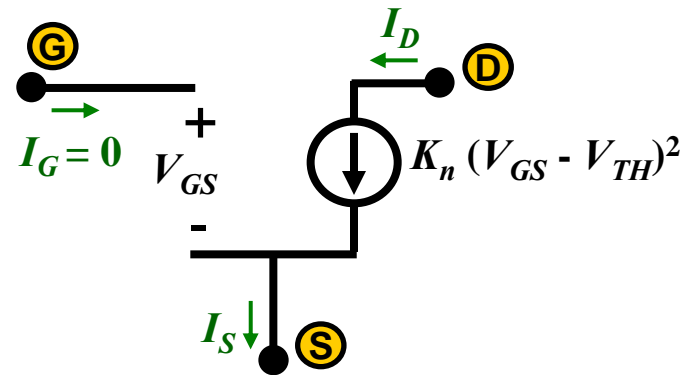


Under d.c. condition,

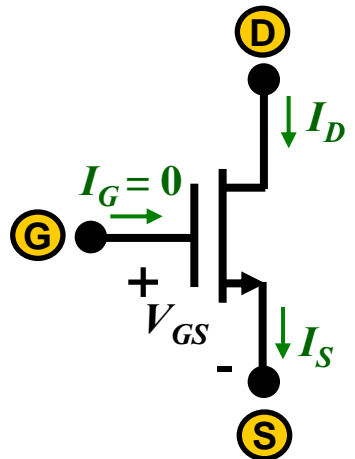
$$V_{GS} > V_{TH}$$

$$V_{DS} \geq V_{GS} - V_{TH}$$

$$I_{Dsat} = K_n (V_{GS} - V_{TH})^2$$



Large signal model of the n-channel MOSFET operating in the linear region



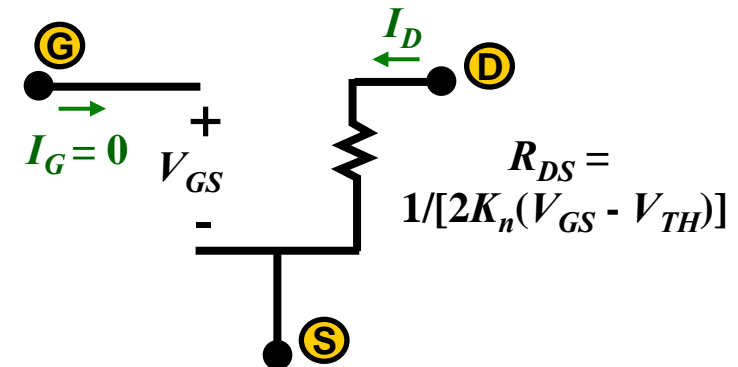
Under d.c. condition,

$$V_{GS} > V_{TH}$$

$$V_{DS} < V_{GS} - V_{TH}$$

For small V_D ,

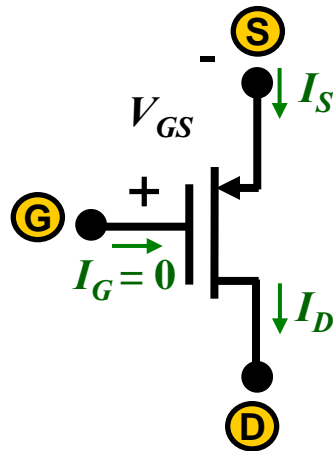
$$I_D \approx 2K_n (V_{GS} - V_{TH}) V_{DS}$$



Summary of *MOSFET* models

Model	Circuit Symbol	Equations	Circuit	Comments
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Large signal model of the p-channel MOSFET operating in the saturation region



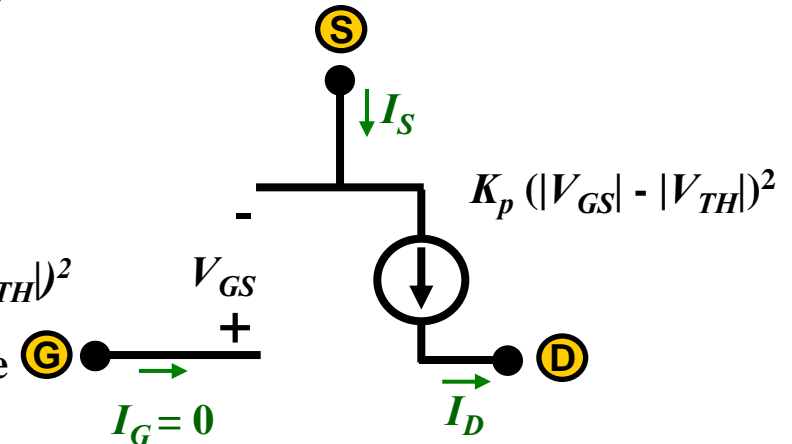
Under d.c. condition,

$$|V_{GS}| > |V_{TH}|$$

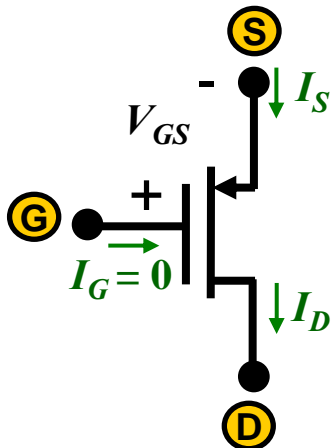
$$|V_{DS}| \geq |V_{GS}| - |V_{TH}|$$

$$|I_{Dsat}| = K_p (|V_{GS}| - |V_{TH}|)^2$$

V_{GS} , V_{TH} are negative



Large signal model of the p-channel MOSFET operating in the linear region



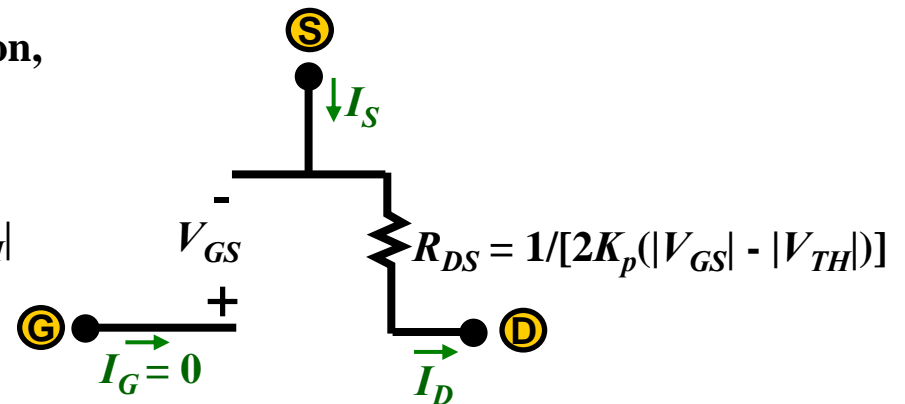
Under d.c. condition,

$$|V_{GS}| > |V_{TH}|$$

$$|V_{DS}| < |V_{GS}| - |V_{TH}|$$

For small V_D ,

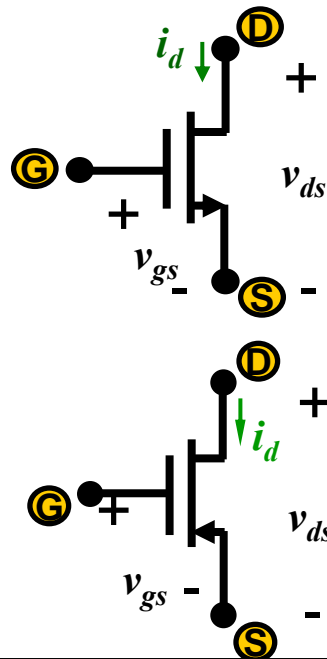
$$|I_D| \approx 2K_p (|V_{GS}| - |V_{TH}|) |V_{DS}|$$



Summary of *MOSFET* models

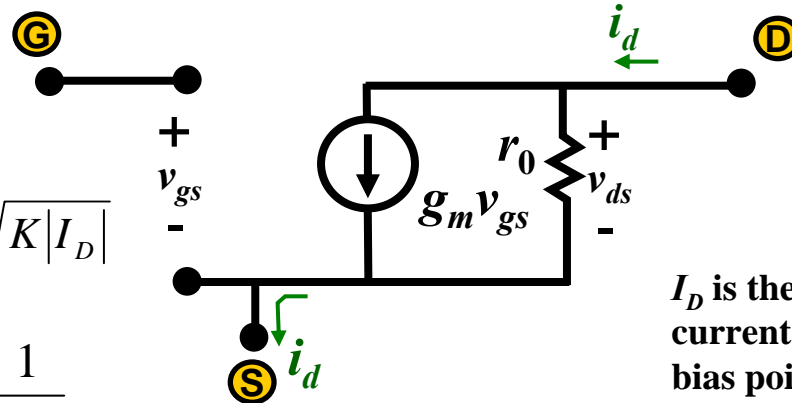
Model	Circuit Symbol	Equations	Circuit	Comments
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Small signal model of the MOSFET (both n-channel and p-channel)



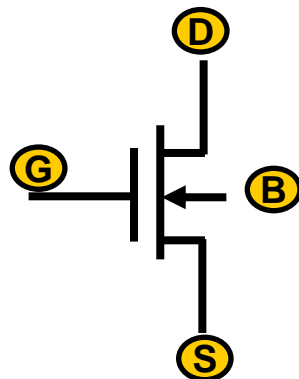
$$g_m = 2\sqrt{K|I_D|}$$

$$r_0 = \frac{1}{\lambda I_D}$$

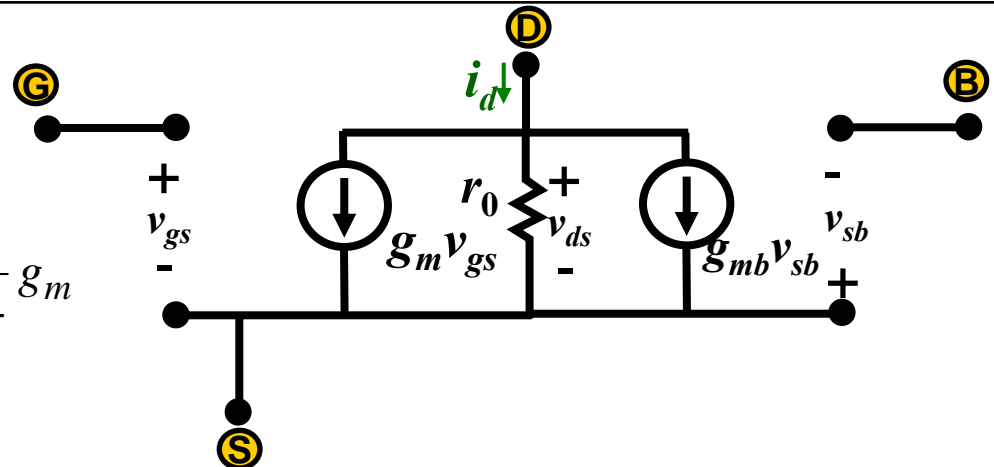


I_D is the drain current at the d.c. bias point, K is the transconductance factor, λ is the channel length modulation factor.

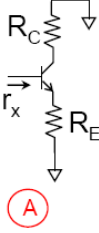
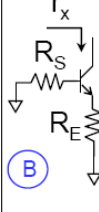
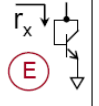
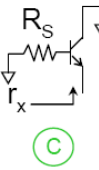
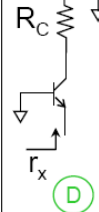
Small signal model of the MOSFET, including body effect (body not tied to source)



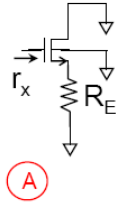
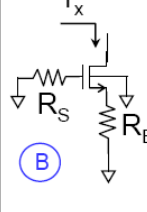
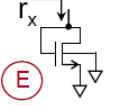

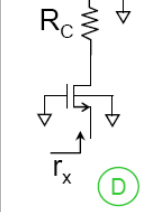
$$g_{mb} \approx -\frac{1}{4}g_m$$



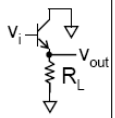
BJT Equivalent Resistance Summary (Table 1)

Conf	r_x	Conf	r_x	Conf	r_x
 (A)	$r_\pi + (1 + \beta)R_E$ $\approx r_\pi(1 + g_m R_E)$	 (B)	$r_o \left\{ 1 + g_m \left[(r_\pi + R_S) // R_E \right] \left(\frac{r_\pi}{r_\pi + R_S} \right) \right\}$ If $R_S = 0$ and $r_\pi \ll R_E$ $\Rightarrow r_{x, \max} = r_o(\beta + 1)$	 (E)	$\frac{1}{g_m}$
 (C)	$\frac{R_S + r_\pi}{1 + \beta} // r_o$ $\approx \frac{R_S}{1 + \beta} + \frac{1}{g_m}$	 (D)	$\frac{1}{g_m} \times \frac{r_o + R_C}{r_o + R_C / \beta}$		

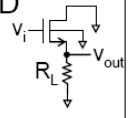
MOS Equivalent Resistance Summary (Table 2)

Conf	r_x	Conf	r_x	Conf	r_x
 (A)	∞	 (B)	$r_o [1 + (g_m - g_{mb})R_E]$	 (E)	$\frac{1}{g_m}$
 (C)	$\frac{1}{g_m - g_{mb}}$	 (D)	$\frac{1}{g_m - g_{mb}} \times \frac{r_o + R_C}{r_o}$		

BJT Amplifier Configurations (Table 3)

BJT	G_m	A_v
CE (A)	g_m	Derive Based on 2-ports Network
CB (B)	$-g_m$	Derive Based on 2-ports Network
CC (C) 	Too Complex To Be Useful	$\frac{g_m R_L}{1 + g_m R_L}$
CE with Emitter Degeneration (D)	$\frac{g_m}{1 + g_m R_E}$	Derive Based on 2-ports Network

MOS Amplifier Configurations (Table 4)

MOS	G_m	A_v
CS (A)	g_m	Derive Based on 2-ports Network
CG (B)	$-(g_m - g_{mb})$ <i>Drop g_{mb} if no body effect</i>	Derive Based on 2-ports Network
CD (C) 	Too Complex To Be Useful	$\frac{g_m R_L}{1 + (g_m - g_{mb}) R_L} \approx \frac{g_m}{g_m - g_{mb}}$ <i>Drop g_{mb} if no body effect</i>
CS with R_E (D)	$\frac{g_m}{1 + (g_m - g_{mb}) R_E}$ <i>Drop g_{mb} if no body effect</i>	Derive Based on 2-ports Network

1) Logic Gates:

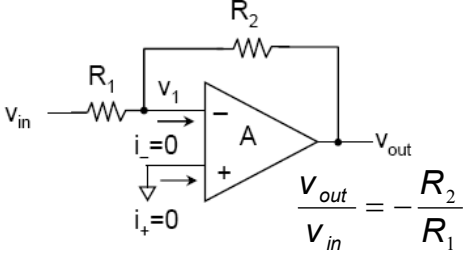
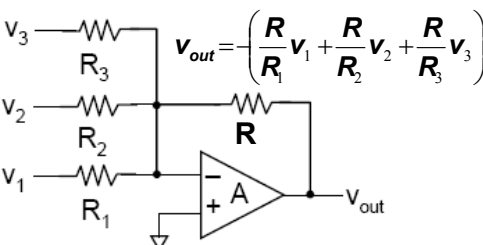
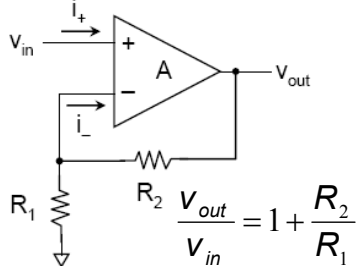
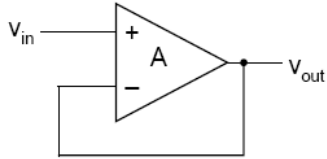
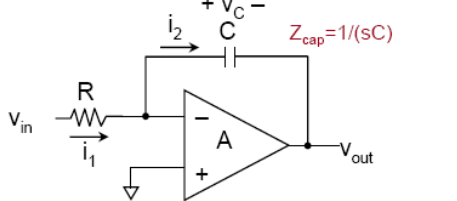
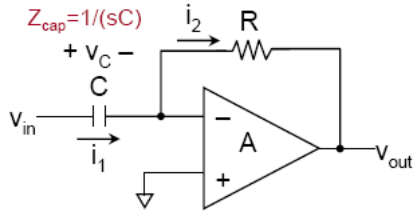
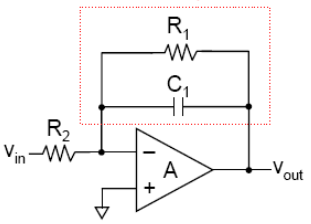
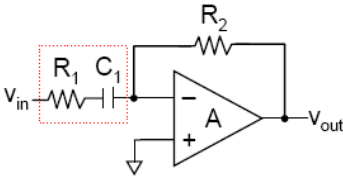
For K transistors in series:

$$\left(\frac{W}{L}\right)_{eq} = \left[\left(\frac{W}{L}\right)_1^{-1} + \left(\frac{W}{L}\right)_2^{-1} + \cdots + \left(\frac{W}{L}\right)_K^{-1} \right]^{-1}$$

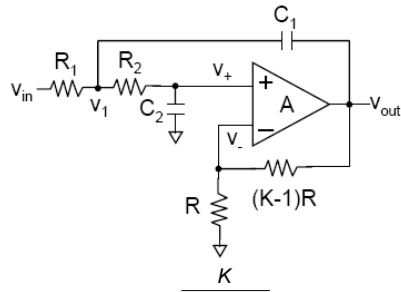
For K transistors in parallel:

$$\left(\frac{W}{L}\right)_{eq} = \left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2 + \cdots + \left(\frac{W}{L}\right)_K$$

2) Opamp Circuits:

<p>Inverting Amplifier</p>  $\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$	<p>Summing Amplifier</p>  $v_{out} = -\left(\frac{R}{R_1}v_1 + \frac{R}{R_2}v_2 + \frac{R}{R_3}v_3\right)$
<p>Non-inverting Amplifier</p>  $\frac{v_{out}}{v_{in}} = 1 + \frac{R_2}{R_1}$	<p>Buffer</p>  $\frac{v_{out}}{v_{in}} = 1$
<p>Integrator</p>  $\frac{v_{out}}{v_{in}} = -\frac{1}{sCR}$ $v_{out}(t) = -\frac{1}{RC} \int v_{in}(t) dt$	<p>Differentiator</p>  $\frac{v_{out}}{v_{in}} = -sCR$ $v_{out}(t) = -CR \frac{dv_{in}(t)}{dt}$
<p>First Order Lowpass Filter</p>  $\frac{v_{out}}{v_{in}} = -\frac{R_1}{R_2} \times \frac{1}{sC_1R_1 + 1}$	<p>First Order Highpass Filter</p>  $\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1} \times \frac{s}{s + \frac{1}{C_1R_1}}$

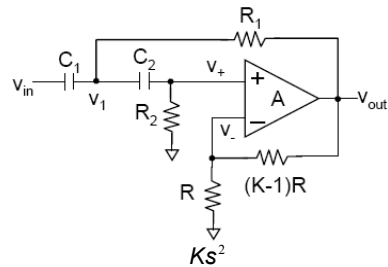
SK Lowpass Filter



$$\frac{V_{out}}{V_{in}} = \frac{\frac{K}{C_1 C_2 R_1 R_2}}{s^2 + s \left(\frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_1 R_2} - \frac{K}{C_2 R_2} \right) + \frac{1}{C_1 C_2 R_1 R_2}}$$

$$= \frac{H_o \omega_o^2}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$

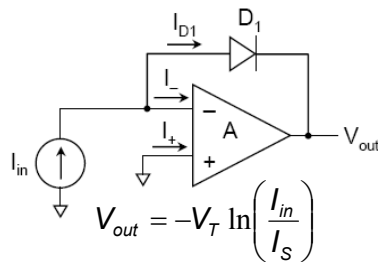
SK Highpass Filter



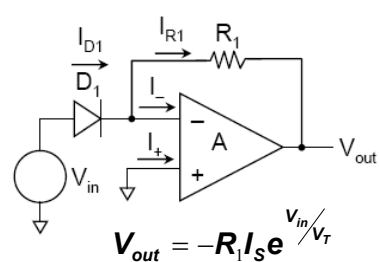
$$\frac{V_{out}}{V_{in}} = \frac{K s^2}{s^2 + s \left(\frac{1}{C_2 R_2} + \frac{1}{C_1 R_2} + \frac{1}{C_1 R_1} - \frac{K}{C_1 R_1} \right) + \frac{1}{C_1 C_2 R_1 R_2}}$$

$$= \frac{H_o s^2}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$

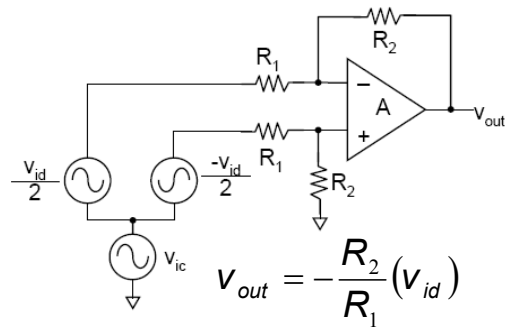
Logarithm Amplifier



Exponential Amplifier



Instrumentation Amplifier



Super Diode

