NATIONAL UNIVERSITY OF SINGAPORE
Department of Electrical and Computer Engineering

EE2021 : Tutorial 1 : Solutions

1. There are two samples of silicon, A and B. Each of them is uniformly doped with
    <u>both</u> donor and acceptor impurities. Assume that the silicon samples are at
    thermal equilibrium at 300 K, that all the dopants are ionized, and that the
    intrinsic carrier concentration, $n_i = 1.5 \times 10^{10}$ cm^{-3}. Complete the table below by
    filling in the missing values of N_D, N_A and n_0.

    Is Sample A n-type, p-type or intrinsic? Is Sample B n-type, p-type or intrinsic?

    Hints: Apply the Law of Mass Action and consider compensation doping.

Silicon Sample	Donor conc. N_D (cm^{-3})	Acceptor conc. N_A (cm^{-3})	Equilibrium electron conc. n_0 (cm^{-3})	Equilibrium hole conc. p_0 (cm^{-3})
Sample A	1×10^{17}			4.5×10^4
Sample B		8×10^{15}		1.5×10^{10}

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1. Since the silicon samples are at thermal equilibrium, we can find the equilibrium
    electron concentration, n_0, using the formula

    $$n_0 = \frac{n_i^2}{p_0}.$$

    For sample A, $n_0 = \dfrac{\left(1.5 \times 10^{10}\right)^2}{4.5 \times 10^4}$ cm^{-3} $= \underline{5 \times 10^{15}}$ cm^{-3}.

    Using the charge neutrality condition, and given that all dopants are ionized,

    $n_0 + N_A = p_0 + N_D$ hence,

    $N_A = N_D - n_0 + p_0 = 1 \times 10^{17} - 5 \times 10^{15} + 4.5 \times 10^4 + = \underline{9.5 \times 10^{16}}$ cm^{-3}.

    Note that we could have obtained the same result for N_A by neglecting p_0 in the
    above equation, since $p_0 \ll n_0$.

    Since the majority carriers in Sample A are electrons, it is n-type.

    For Sample B, $n_0 = \dfrac{\left(1.5 \times 10^{10}\right)^2}{1.5 \times 10^{10}} = 1.5 \times 10^{10}$ cm^{-3}.

    $N_D = N_A + n_0 - p_0 = 8 \times 10^{15} + 1.5 \times 10^{10} - 1.5 \times 10^{10} = \underline{8 \times 10^{15}}$ cm^{-3}.

Since the concentrations of the electrons and holes are equal in Sample B, it is intrinsic.

Note : As the silicon samples are doped with <u>both</u> donors and acceptors, there is compensation doping. We <u>cannot</u> simply equate the electron concentration to the donor concentration, and the hole concentration to the acceptor concentration.

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2. A silicon bar is in the shape of a circular cylinder with a cross-sectional area of 10^{-4} cm^2, and a length of 0.1 cm. The silicon bar is uniformly doped with phosphorus (Group V element) at a concentration of 2 x 10^{16} cm^{-3}. At this doping concentration, the electron mobility is 1100 cm^2 V^{-1} s^{-1} and the hole mobility is 400 cm^2 V^{-1} s^{-1}. The silicon bar is at thermal equilibrium at $T = 300$ K. The intrinsic carrier concentration of silicon at this temperature, $n_i = 1.5$ x 10^{10} cm^{-3}.

(a) Calculate the electron and hole concentrations in the silicon bar. Calculate also the conductivity and resistance of the silicon bar between the two ends.

(b) Boron (Group III element) is subsequently added uniformly to the silicon bar, at a concentration of 6 x 10^{16} cm^{-3}. The electron and hole mobilities are now 700 cm^2 V^{-1} s^{-1} and 350 cm^2 V^{-1} s^{-1} respectively. Calculate the electron and hole concentrations in the silicon bar. Calculate also the conductivity and resistance of the silicon bar. <u>Note that the mobilities of the electrons and holes are both reduced compared to the original mobilities.</u>

(c) Comment on the difference in the conductivities of the silicon bar in parts (a) and (b).

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2. (a) Phosphorus is a Group V element. It acts as a donor dopant in silicon. There is no acceptor doping (yet) in the silicon. The silicon bar is n-type.

$N_D = 2$ x 10^{16} cm^{-3}, $N_A = 0$ cm^{-3}.

Since $| N_D - N_A| >> n_i$. we can calculate the majority carrier concentration, i.e.,

electron concentration, $\underline{n_0 = N_D - N_A = 2 \times 10^{16} \, \text{cm}^{-3}}$.

Minority carrier concentration,

hole concentration, $\underline{p_0 = \dfrac{n_i^2}{n_0} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{2 \times 10^{16}} = 1.125 \times 10^4 \, \text{cm}^{-3}}$.

[See the lecture notes on Semiconductor Physics, slides 1.40, 1.41.]

Conductivity of the silicon bar

$$\sigma = qn_0\mu_n + qp_0\mu_p = 1.602 \times 10^{-19} \times \left(2 \times 10^{16} \times 1100 + 1.125 \times 10^4 \times 400\right)$$
$$= 3.52 \, (\Omega \, cm)^{-1}.$$

Note that as the hole concentration is many orders of magnitude smaller than the electron concentration, we could neglect the contribution of the holes (minority carriers) in the calculation of the conductivity of the silicon bar.

Resistivity, $\rho = \dfrac{1}{\sigma} = \dfrac{1}{3.52} = 0.284 \, \Omega \, cm$.

The resistance of the silicon bar, $R = \dfrac{\rho l}{A} = \dfrac{0.284 \times 0.1}{1 \times 10^{-4}} = 284 \, \Omega$.

(b) Boron, which is an acceptor dopant, is added uniformly to the silicon bar. As the boron concentration added is greater than the original phosphorus concentration, there is over-compensation, and the silicon becomes p-type, and the majority carriers are holes.

hole concentration, $p_0 = N_A - N_D = 6 \times 10^{16} - 2 \times 10^{16} = 4 \times 10^{16} \, cm^{-3}$.

electron concentration, $n_0 = \dfrac{n_i^2}{p_0} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{4 \times 10^{16}} = 5.625 \times 10^3 \, cm^{-3}$.

Conductivity of the silicon bar

$$\sigma = qn_0\mu_n + qp_0\mu_p = 1.602 \times 10^{-19} \times \left(5.625 \times 10^3 \times 700 + 4 \times 10^{16} \times 350\right)$$
$$= 2.24 \, (\Omega \, cm)^{-1}.$$

The resistance of the silicon bar, $R = \dfrac{l}{\sigma A} = \dfrac{0.1}{2.24 \times 1 \times 10^{-4}} = 446 \, \Omega$.

(c) It is noted that the conductivity of the silicon bar in part (b) is lower than that in part (a), even though the majority carrier concentration (which contributes predominantly to the conductivity of the silicon bar) of the silicon in part (b) is twice that in part (a). There are two reasons for this. Conductivity is a function of the carrier concentrations and their mobilities. Firstly, in part (a), the majority carriers are electrons while in part (b), the majority carriers are holes due to over-compensation by acceptor dopants. Holes have lower mobilities than electrons. Secondly, the total (donor + acceptor) concentration of the dopants in the silicon part (b) is higher. This causes more scattering of the carriers (i.e. electrons and holes) as they drift through the silicon bar. The mobilities of both the electrons and holes (as given in part (b)) are therefore lower than those in part (a). Both these effects cause the conductivity of the silicon bar in part (b) to be lower than that in part (a).

3. The concept of drift is applicable to metals (e.g., aluminium) as well as semiconductors.

   The concentration of (conduction) electrons in an aluminium wire is 2×10^{23} cm^{-3}. The electrical resistivity of aluminium at 300 K is 2.5 $\mu\Omega$ cm.

   (a) Calculate the mobility of the (conduction) electrons in the aluminium wire.

   (b) An aluminium wire is 100 cm long and has a cross-sectional area of 10^{-4} cm^2. A voltage of 1 V is applied across its two ends. Calculate the drift velocity of the electrons in the wire. Hence find the time it takes for an electron to drift from one end of the wire to the other end.

   (c) Compare the conductivity of aluminium with that of the silicon bar in Q2(a) in terms of their carrier concentrations and mobilities.

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3. Aluminium is a metal. The charge carriers are (conduction) electrons.

   (a) Resistivity of the aluminium wire, $\rho = \dfrac{1}{\sigma} = \dfrac{1}{qn\mu_n}$.

   Mobility of the electrons,
   $$\mu_n = \frac{1}{qn\rho} = \frac{1}{1.602 \times 10^{-19} \times 2 \times 10^{23} \times 2.5 \times 10^{-6}} = 12.5 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}.$$

   (b) A voltage of 1 V applied uniformly across the aluminium wire of length 100 cm produces and electric field, E, where

   $$E = \frac{1 \text{ V}}{100 \text{ cm}} = 10^{-2} \text{ V cm}^{-1}.$$

   The drift velocity of the electrons in the aluminium wire,

   $$v_{drift} = \mu_n E = 12.5 \times 10^{-2} = 0.125 \text{ cm s}^{-1}.$$

   Time taken for an electron to drift from one end of the wire to the other end

   $$t = \frac{100 \text{ cm}}{0.125 \text{ cm s}^{-1}} = 800 \text{ s}.$$

   (c) The mobility of the electrons in metals such as aluminium is usually lower than that in semiconductors such as silicon. However, because the concentration of the electrons in metals is much higher than the concentration of the charge carriers (electrons and holes) in semiconductors, metals generally still have a higher conductivity.

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4. Figure Q.4 below shows the structure of a silicon n-p-n bipolar transistor with the regions and <u>majority</u> carrier concentrations as indicated. Starting with a silicon wafer (substrate) doped with donors at a concentration $N_D = 10^{15}$ cm^{-3}, the different regions (collector, base, emitter) are made by successively introducing dopants into selected regions by ion-implantation through the top surface. Assume, for simplicity, that uniform doping concentrations can be achieved in each of the regions.

In the figure, n and p are the electron and hole concentrations respectively, the subscript 0 indicates thermal equilibrium condition, and the subscripts E, B and C refer to the emitter, base and collector of the bipolar transistor, respectively.

Determine the doping sequence, and the type and concentration of the dopant to be added in each of the sequential steps.

What is the total concentration of dopants (i.e., $N_D + N_A$) in the emitter? Would the property of the emitter region be the same or different from a piece of silicon doped only with donors of concentration, $N_D = 10^{18}$ cm^{-3}?

Emitter

$n_{E0} = 10^{18}$ cm^{-3} Base

$p_{B0} = 2 \times 10^{17}$ cm^{-3}

$n_{C0} = 10^{16}$ cm^{-3} Collector
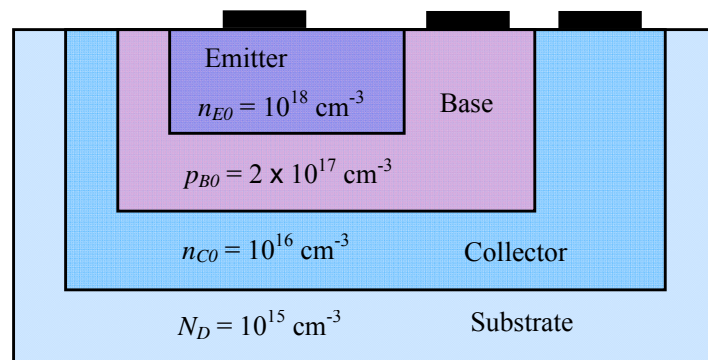
$N_D = 10^{15}$ cm^{-3} Substrate

Fig. Q.4

4. Formation of collector region :

We make use of the charge neutrality equation :

$$n_0 + N_A = N_D + p_0 ,$$

where we assume that all the donors and acceptors are ionised. As the electron concentration needed for the collector is higher than the (n-type) doping of the substrate, we need to implant additional <u>donors</u>, at a concentration of N_{D1}, where

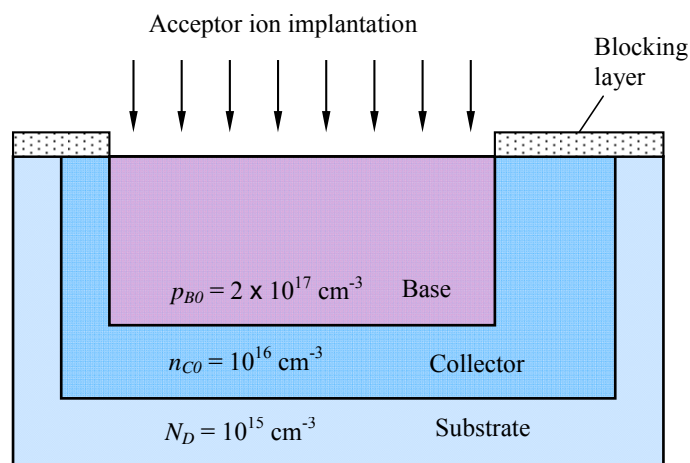$$n_{C0} = N_D + N_{D1} , \text{ (note: } N_A = 0 \text{ and } p_{C0} \ll n_{C0}.)$$

i.e., $\qquad\qquad N_{D1} = n_{C0} - N_D = 10^{16} - 10^{15} = \underline{9 \times 10^{15} \text{ cm}^{-3}}.$
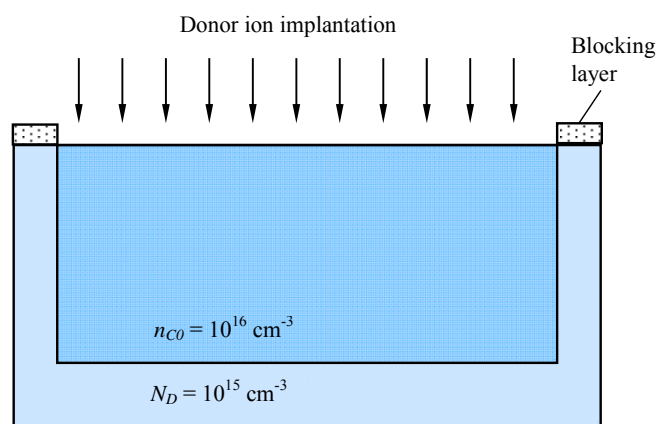
Formation of base region :

We need to over-compensate the doping of part of the collector region in order to form the base region. The <u>acceptor</u> doping that is needed, N_{A2}, is given by
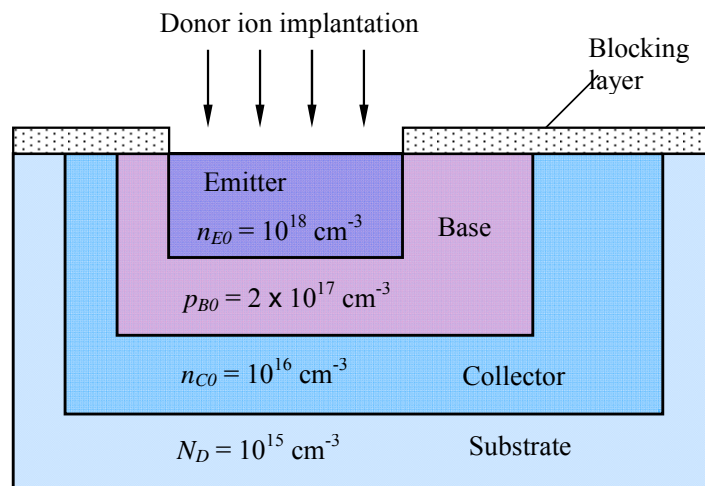
$$N_{A2} = p_{B0} + N_{D1} + N_D = 2 \times 10^{17} + 9 \times 10^{15} + 10^{15} = \underline{2.1 \times 10^{17} \text{ cm}^{-3}}.$$

Formation of the emitter region:

We need to over-compensate the doping of part of the base region in order to form the emitter region. The <u>donor</u> doping that is needed, N_{D3}, is given by

$N_{D3} = n_{E0} + N_{A2} - N_{D1} - N_D = 10^{18} + 2.1 \times 10^{17} - 9 \times 10^{15} - 10^{15} = \underline{1.2 \times 10^{18}}$ cm^{-3}.

The emitter region has undergone 3 rounds of compensation doping. The <u>total</u> dopant concentration in the emitter region

$$= N_D + N_{D1} + N_{A2} + N_{D3}$$
$$= 10^{15} + 9 \times 10^{15} + 2.1 \times 10^{17} + 1.2 \times 10^{18} = 1.42 \times 10^{18} \text{ cm}^{-3}.$$

The total dopant concentration in the emitter is 42% higher than that of a piece of silicon doped only with donors of concentration, $N_D = 10^{18}$ cm^{-3}. This would reduce the mobilities of the charge carriers in the emitter, and may also have an effect on the quality of the silicon crystal.