Bipolar Junction Transistor (BJT)

1. Introduction to the BJT ✓
2. Principles of Operation of the BJT ✓
3. Analysis of the BJT ✓
4. Large Signal Model of the BJT ⬅
5. Operating Point of BJT Circuits
6. Signal Amplification in Amplifiers
7. Small Signal (Simplified Hybrid-π) Model of the BJT
8. Full Hybrid-π Model

4. Large Signal Model of the BJT

For the npn BJT in the forward active region of operation:

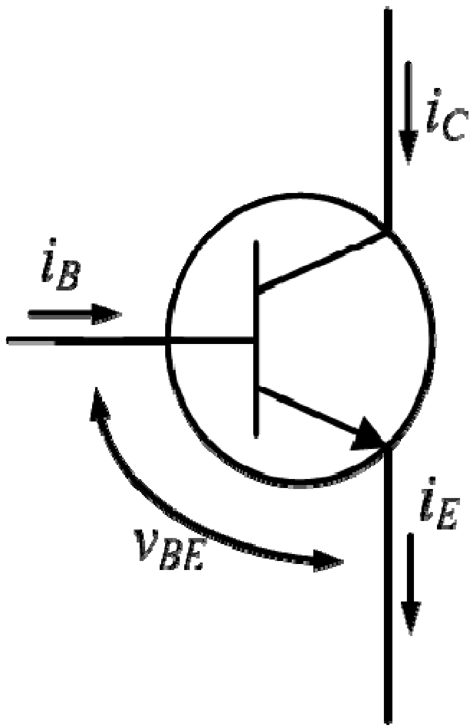
- Collector current $$i_C = I_S e^{v_{BE}/V_T} \qquad (3.7)$$

  where $$I_S = qA \frac{D_n}{w_B} \frac{n_i^2}{N_{AB}} \qquad (3.8)$$

- Base current $$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T} \qquad (3.13)$$

- $$i_C = \beta\, i_B \qquad (3.14)$$

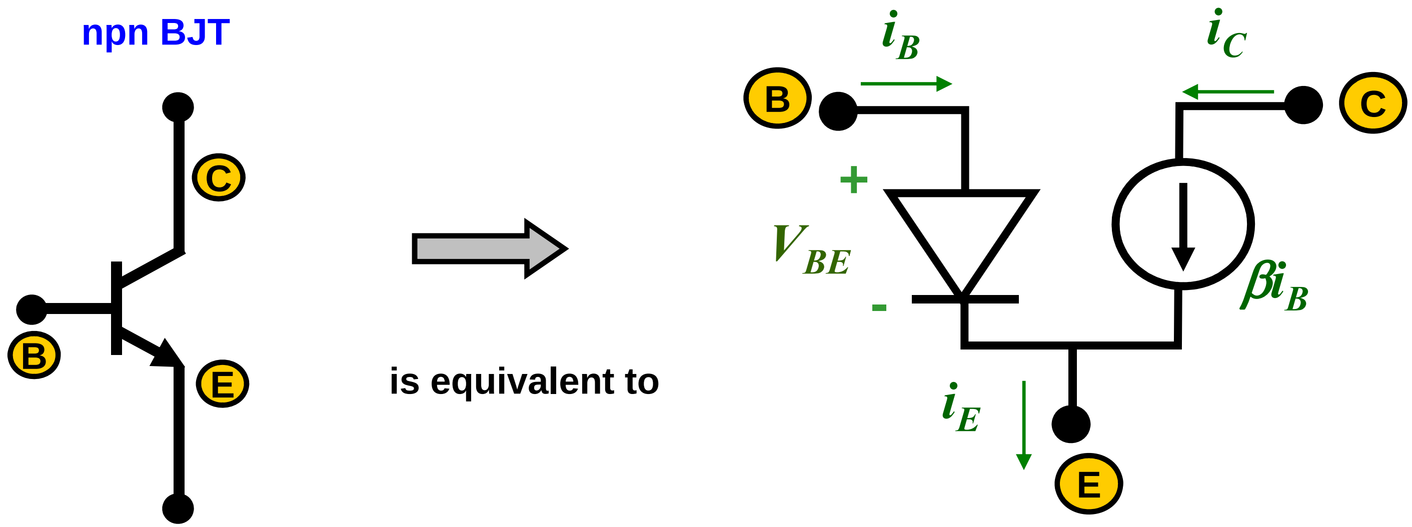
  where $$\beta = \frac{D_n}{D_p} \frac{L_p}{w_B} \frac{N_{DE}}{N_{AB}} \qquad (3.11)$$



4.1 npn BJT

In forward active operation, we model the npn BJT using the following equivalent circuit :

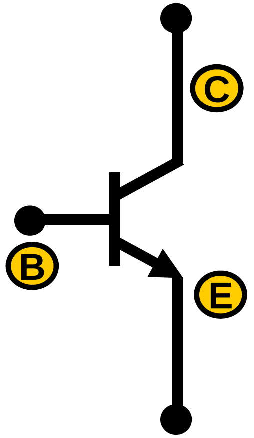
- $$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$$ (3.13) ⇒ diode.
- $$i_C = \beta i_B$$ (3.15) ⇒ dependent current source



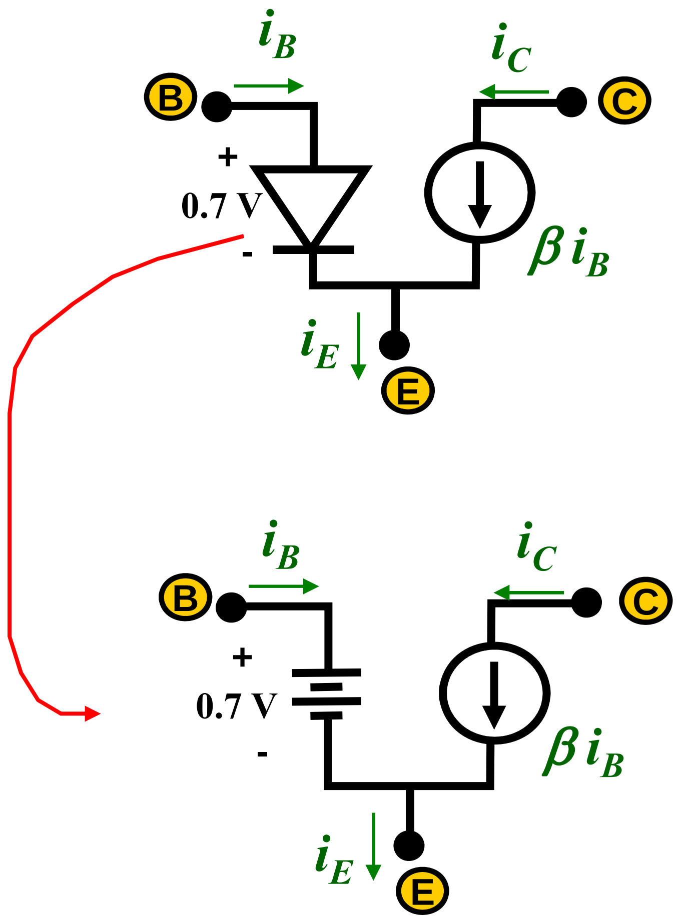
4.1 npn BJT

- The forward biased p-n base-emitter junction of the npn bjt has a voltage drop of ~0.7 V.

npn BJT



is equivalent to

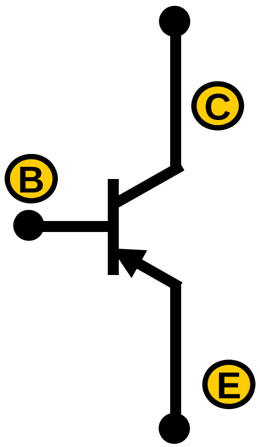


The forward-biased base-emitter junction is modeled by this constant-voltage-drop of 0.7 V.

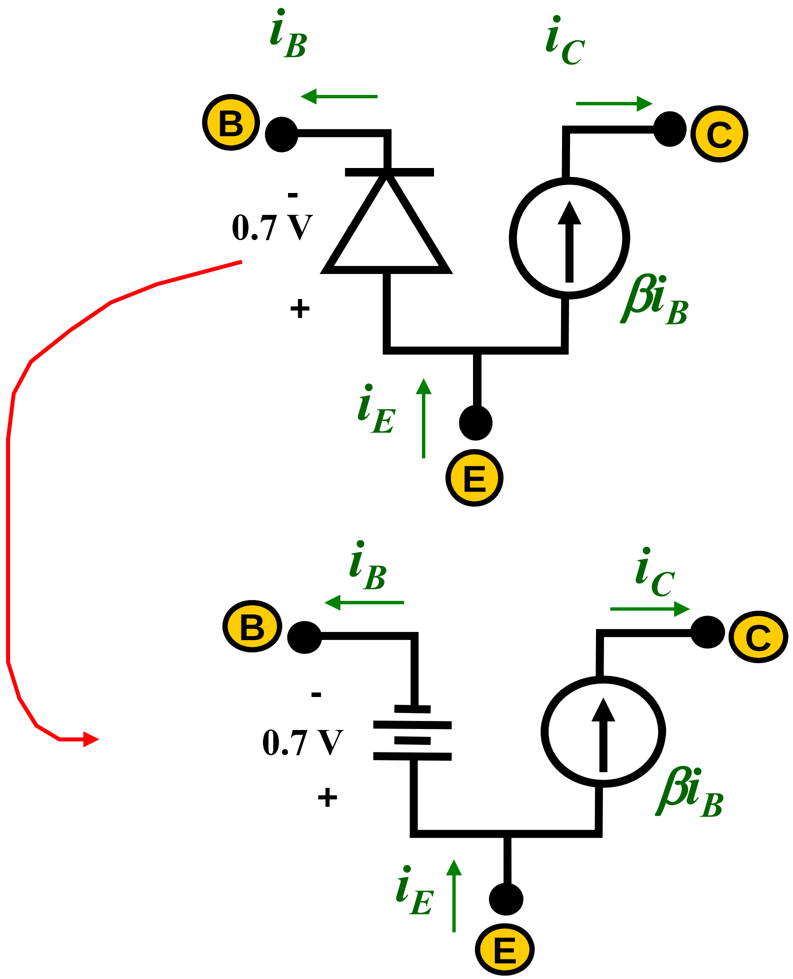
4.2 pnp BJT

- Equivalent circuit model of the pnp bipolar junction transistor :
- The polarities of the voltages, and the directions of the currents, are opposite to those of the npn bjt.

pnp BJT



is equivalent to



The forward-biased emitter-base junction is modeled by this constant-voltage-drop of 0.7 V.

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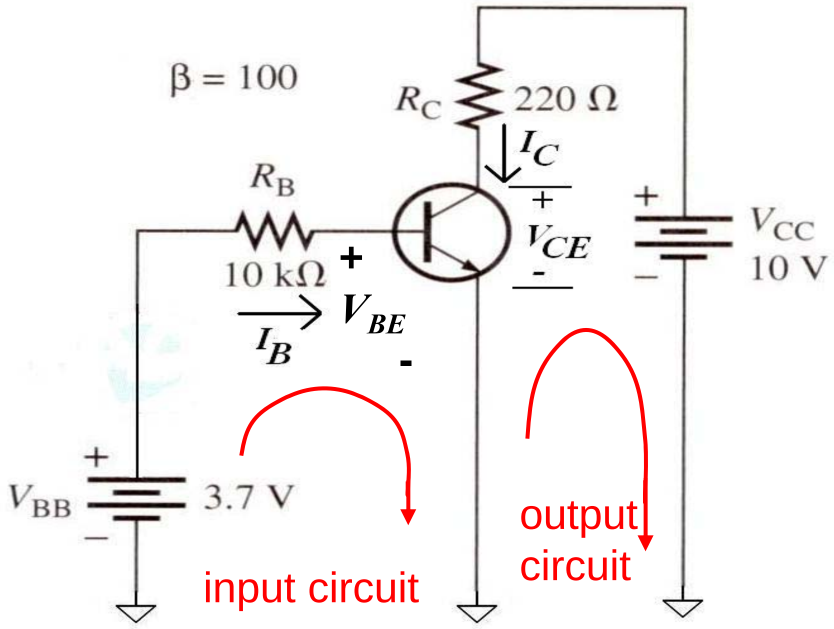
5. Operating Point of a BJT circuit



Assume that V_{BE} = 0.7 V

- Input circuit, KVL :

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{(3.7 - 0.7)\,\text{V}}{10\ \text{k}\Omega} = 0.3\ \text{mA} = 300\mu\text{A}. \qquad (3.17)$$

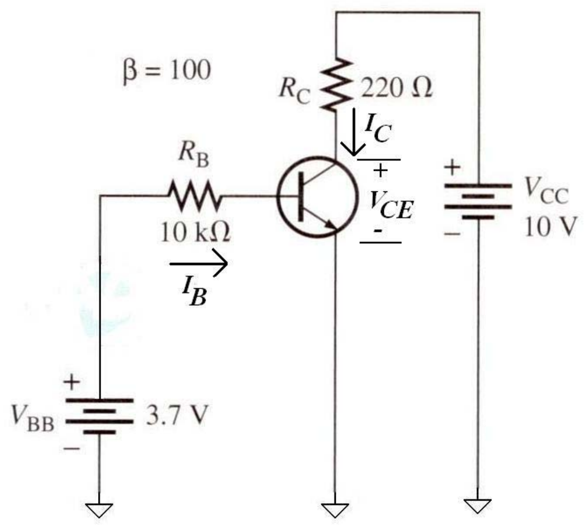
- Output circuit, KVL :

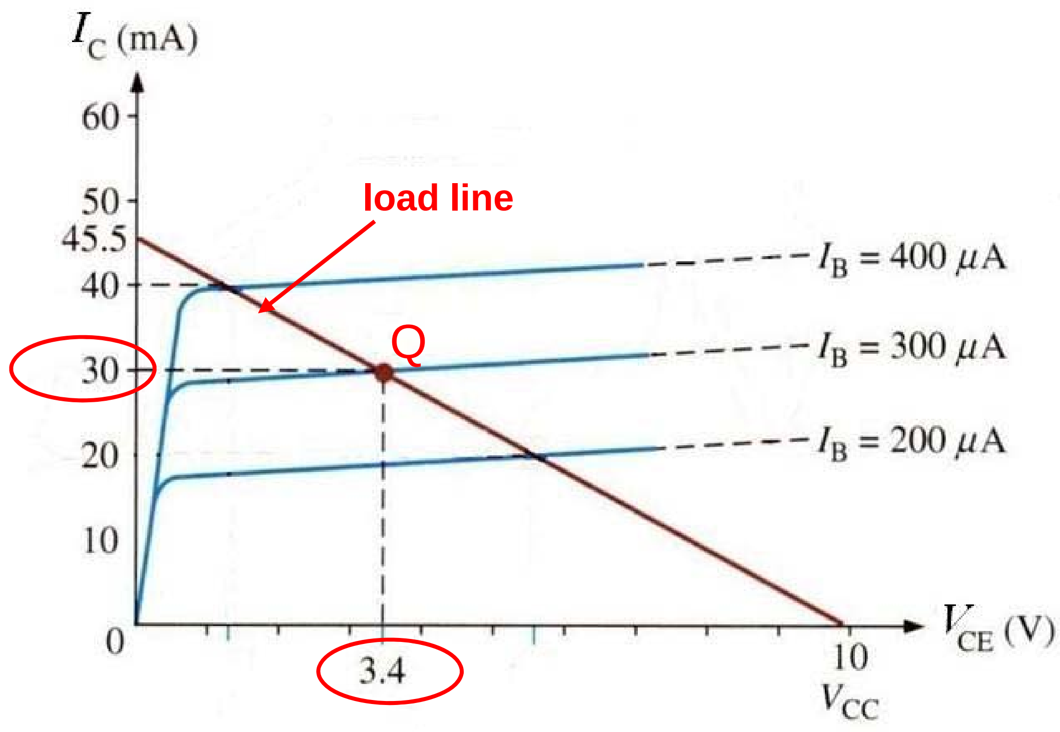
$$V_{CC} = I_C R_C + V_{CE} \qquad (3.18)$$

Symbol for d.c. values (example): I_B

capital letter capital subscript

5. Operating Point of a BJT circuit





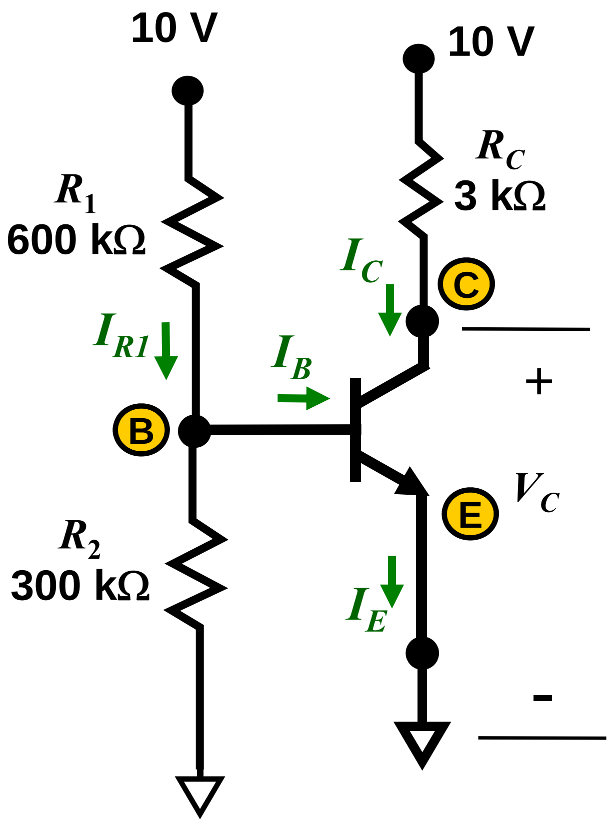
- By re-writing eqn (3.18), we have $I_C = \dfrac{V_{CC} - V_{CE}}{R_C}$. (3.19)

- At the operating point "Q", I_C = 30 mA, V_{CE} = 3.4 V.

Calculation of Operating Point of BJT Circuits

Example 1 (Thevenin equivalent method).

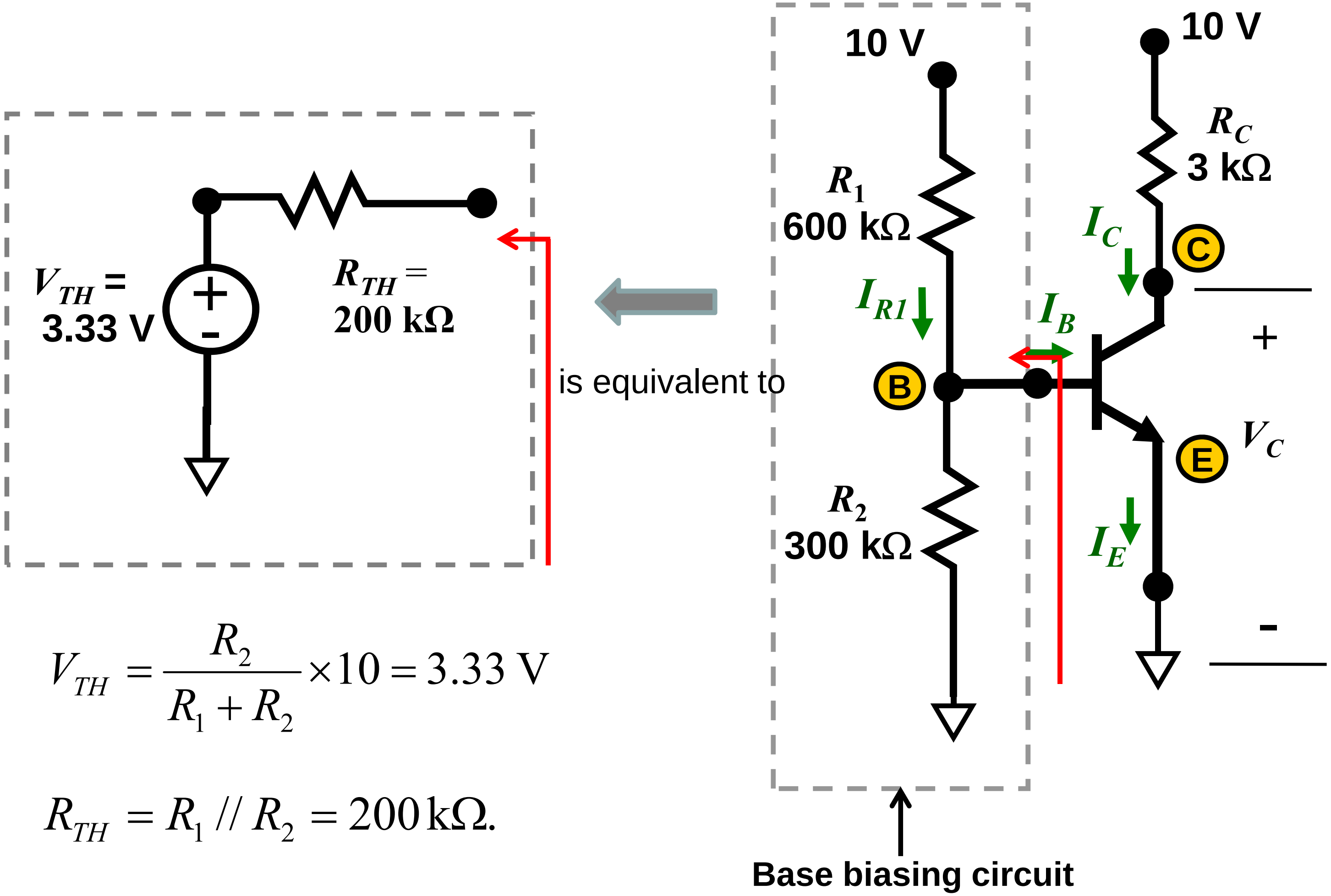
Find the currents I_C, I_B, and I_E , and the voltage V_C using an appropriate large signal model for the BJT, assuming that it is operating in the forward active region and has the parameter $\beta = 100$.



Calculation of Operating Point of BJT Circuits

Example 1 (Thevenin equivalent method) (contd.)

- First, we obtain the Thevenin's Equivalent Circuit of the base-biasing circuit, shown in the dashed grey box:



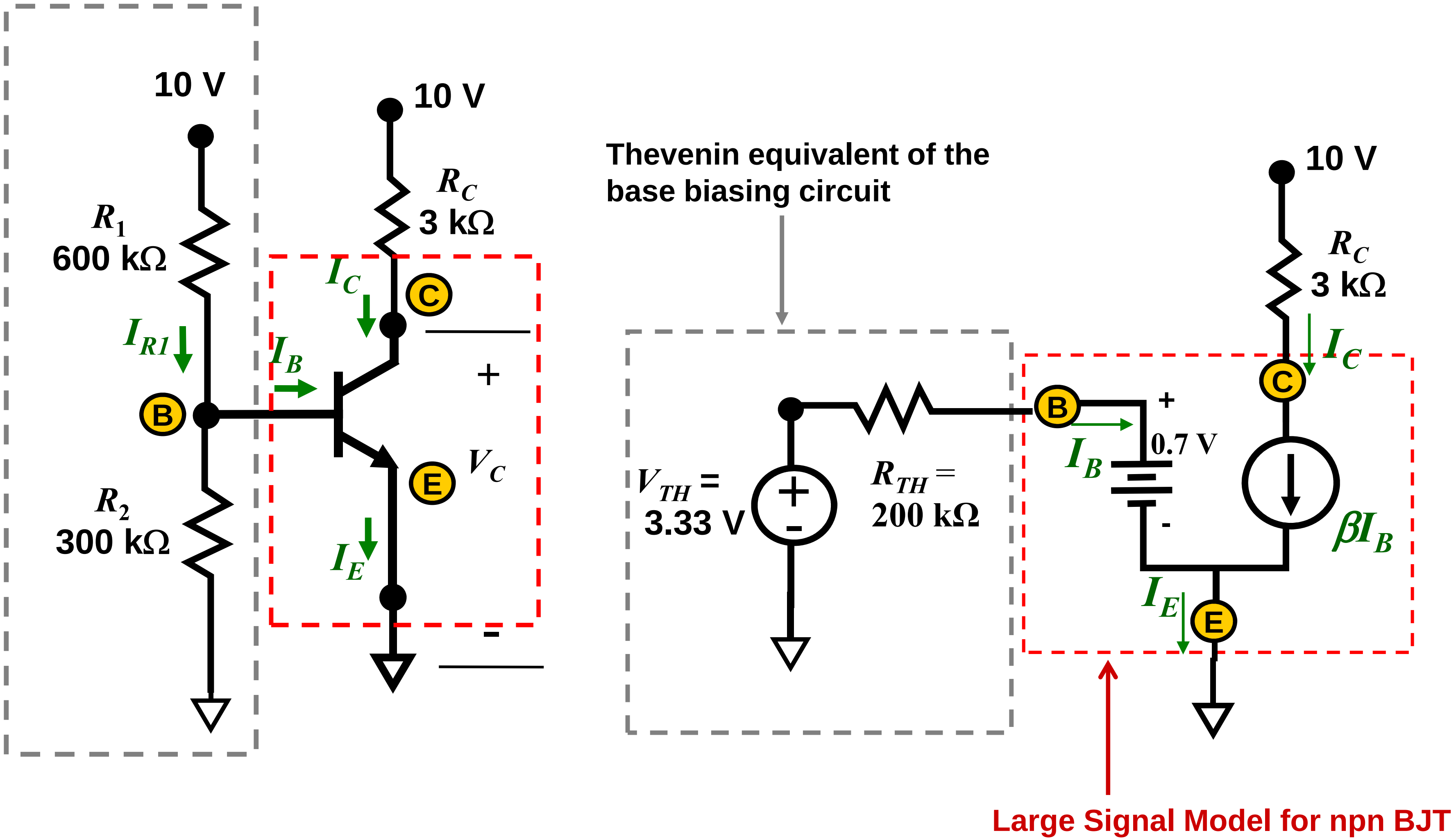
$$V_{TH} = \frac{R_2}{R_1 + R_2} \times 10 = 3.33\ \text{V}$$

$$R_{TH} = R_1 // R_2 = 200\,\text{k}\Omega.$$

Calculation of Operating Point of BJT Circuits

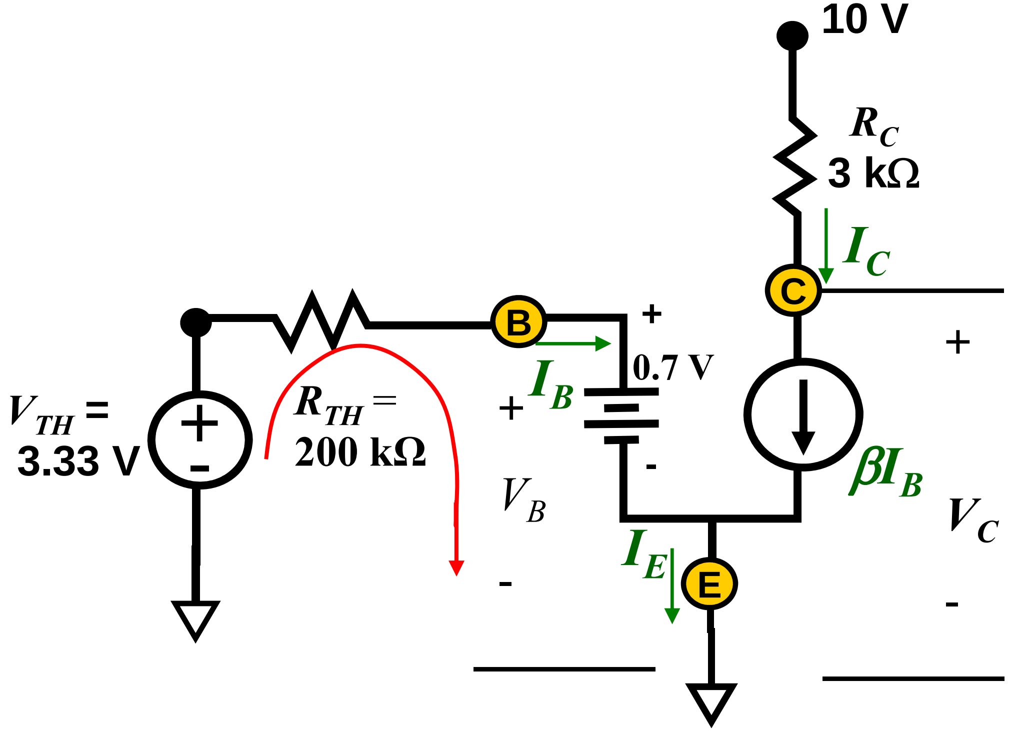
Example 1 (Thevenin equivalent method) (contd.)

- Next, we replace the npn BJT with its large signal model.



Calculation of Operating Point of BJT Circuits

Example 1 (Thevenin equivalent method) (contd.)



$$I_B = \frac{3.33\text{ V} - 0.7\text{ V}}{200\text{ k}\Omega} = \mathbf{0.0132\text{ mA.}}$$

$$I_C = \beta I_B = 100 \times 0.0132\text{ mA} = \mathbf{1.32\text{ mA}}.$$

$$I_E = (\beta + 1)I_B = 101 \times 0.0132\text{ mA} = \mathbf{1.33\text{ mA}}.$$

$$V_C = 10\text{ V} - 3\text{ k}\Omega \times (1.32\text{ mA}) = \mathbf{6.04\text{ V.}}$$

Check: Base voltage, $V_B = V_{BE} = 0.7$ V.

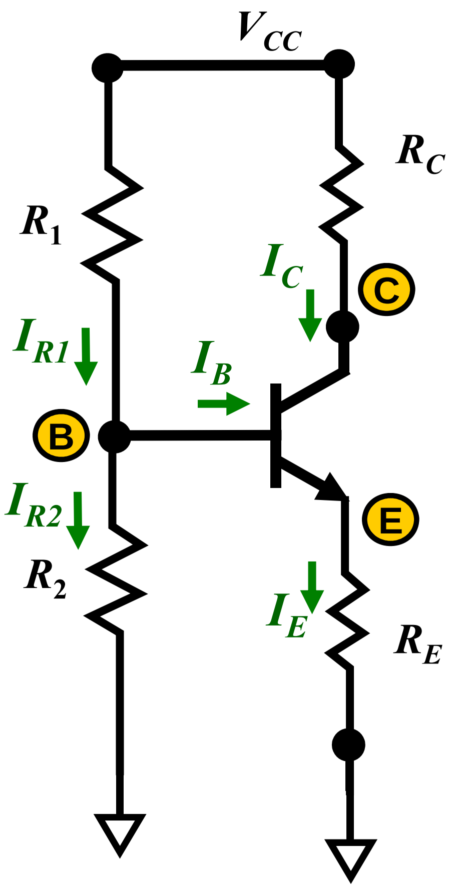
Base-collector voltage $V_{BC} = V_B - V_C = 0.7\text{ V} - 6.04\text{ V} = -5.34\text{ V}$,

Since the base-emitter pn junction is forward biased, and the base-collector pn junction is reverse biased, the npn BJT is in operating in the forward active region.

Calculation of Operating Point of BJT Circuits

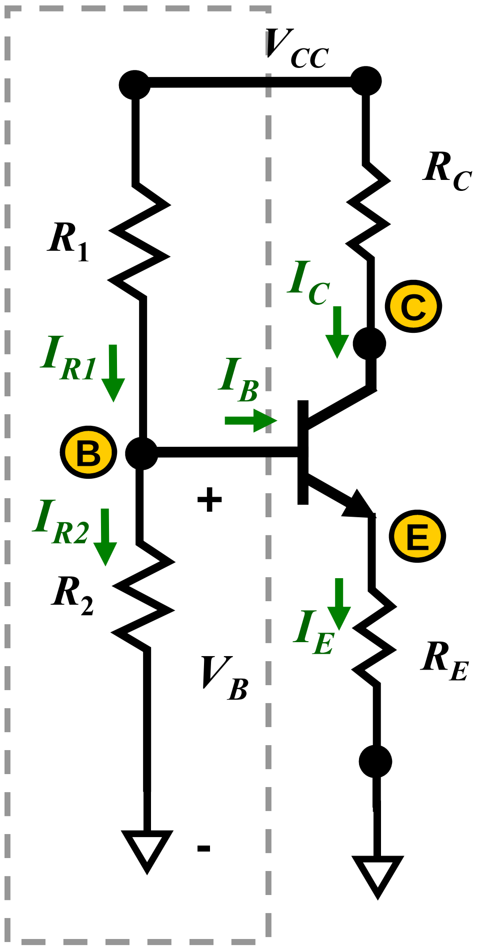
Example 2 (Voltage divider method).

The circuit on the left has a V_{CC} = 20 V, R_C = 5 kΩ, R_E = 1 kΩ, R_1 = 20 kΩ, and R_2 = 3 kΩ. The value of β for the BJT is 100. Determine the values of I_C and I_B.



Operating Point of a BJT Circuit (Voltage Divider)

Example 2 (Voltage divider method).



Given : V_{CC} = 20 V, R_C = 5 kΩ, R_E = 1 kΩ, R_1 = 20 kΩ, and R_2 = 3 kΩ.

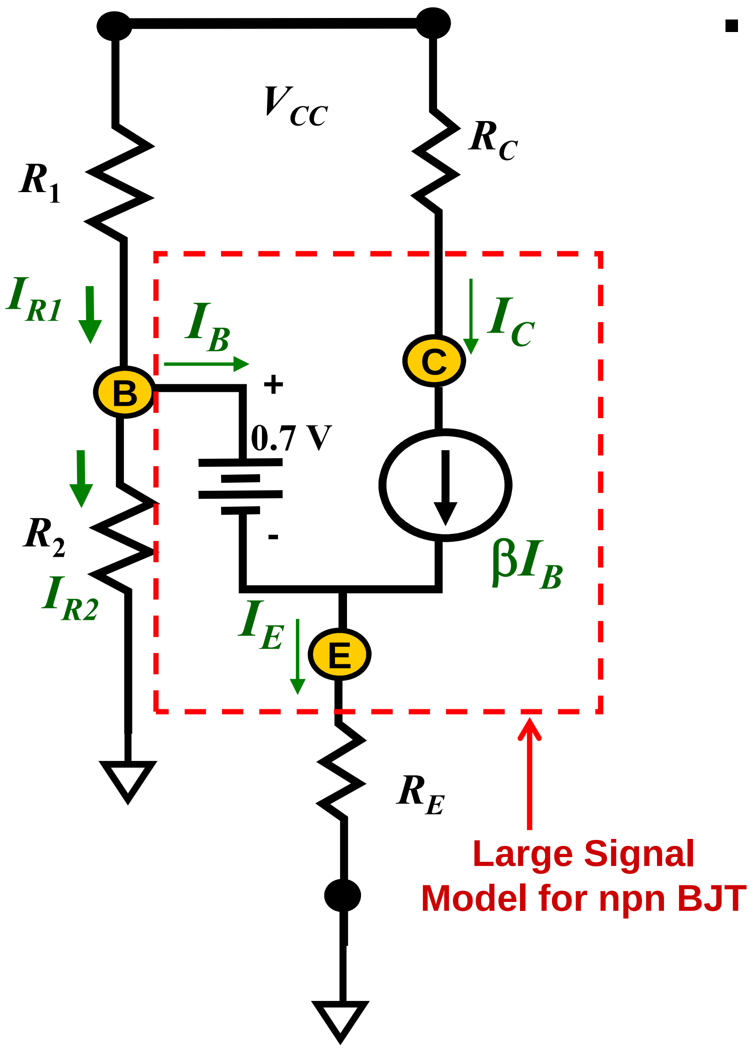
- **Assuming** that I_B is small compared to I_{R1} and I_{R2},

  we apply the voltage division rule to obtain

$$I_{R1} \approx I_{R2} \approx \frac{V_{CC}}{R_1 + R_2} = \frac{20\text{ V}}{20\text{ k}\Omega + 3\text{ k}\Omega} = 0.87\text{ mA}$$

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{3\text{ k}\Omega}{(20+3)\text{k}\Omega} 20\text{ V} = 2.61\text{ V}$$

Operating Point of a BJT Circuit (Voltage Divider)



- Replace the npn BJT with its large signal model.

$V_E = V_B - 0.7\text{ V} = 2.61 - 0.7 = 1.91\text{ V}.$

Thus, $I_E = V_E / 1\text{k}\Omega = 1.91\text{ mA}.$

$I_C = [\beta / (\beta + 1)]I_E = [100/101] \times 1.91\text{ mA} = \mathbf{1.89\text{ mA}}.$

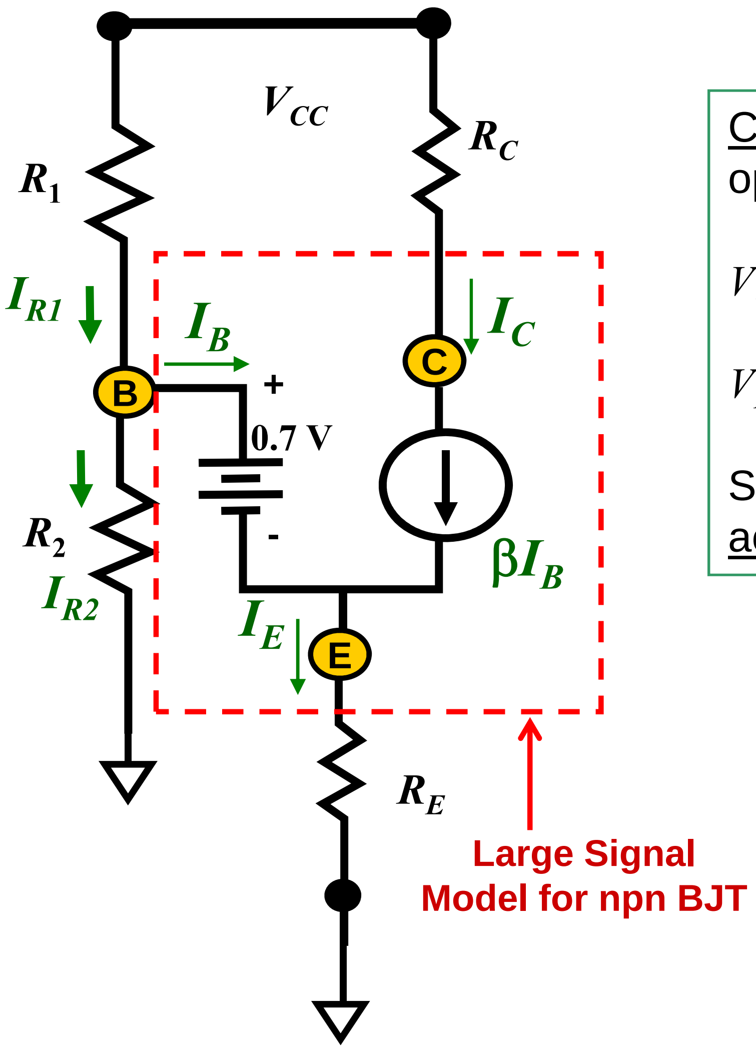
$I_B = I_C / \beta = \mathbf{0.0189\text{ mA} = 18.9\ \mu A}$

<u>Check</u> : I_B (0.0189 mA), is indeed much smaller than I_{R1} and I_{R2} (0.87 mA). Assumption is valid.

<u>Note:</u> If I_B is not small compared to the currents flowing through R_1 and R_2, the voltage divider rule cannot be applied.

We would then to use the Thevenin's equivalent method.

Operating Point of a BJT Circuit (Voltage Divider)



Check that the BJT is in the forward-active region of operation :

$V_C = V_{CC} - I_C R_C$ = 20 – 1.89 mA × 5 kΩ = 10.55 V.

$V_{BC} = V_B - V_C$ = 2.61 – 10.55 = - 7.94 V.

Since $V_{BC} < 0$ and $V_{BE} > 0$, the BJT is in the forward active region of operation.

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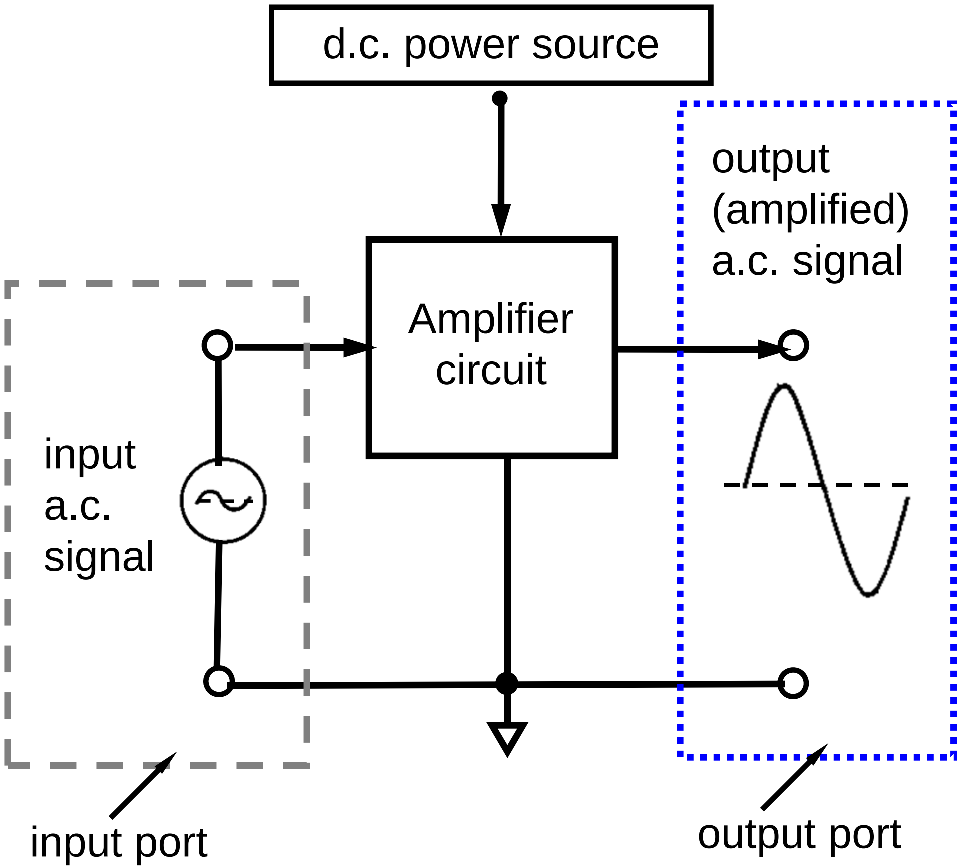
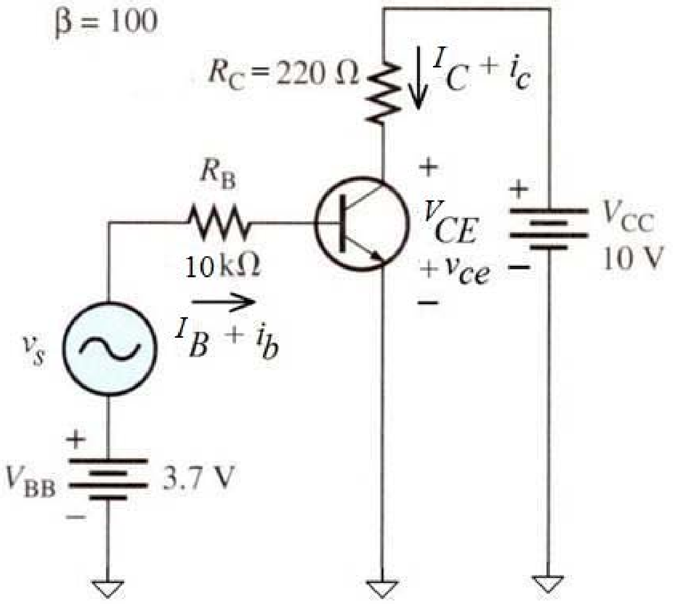
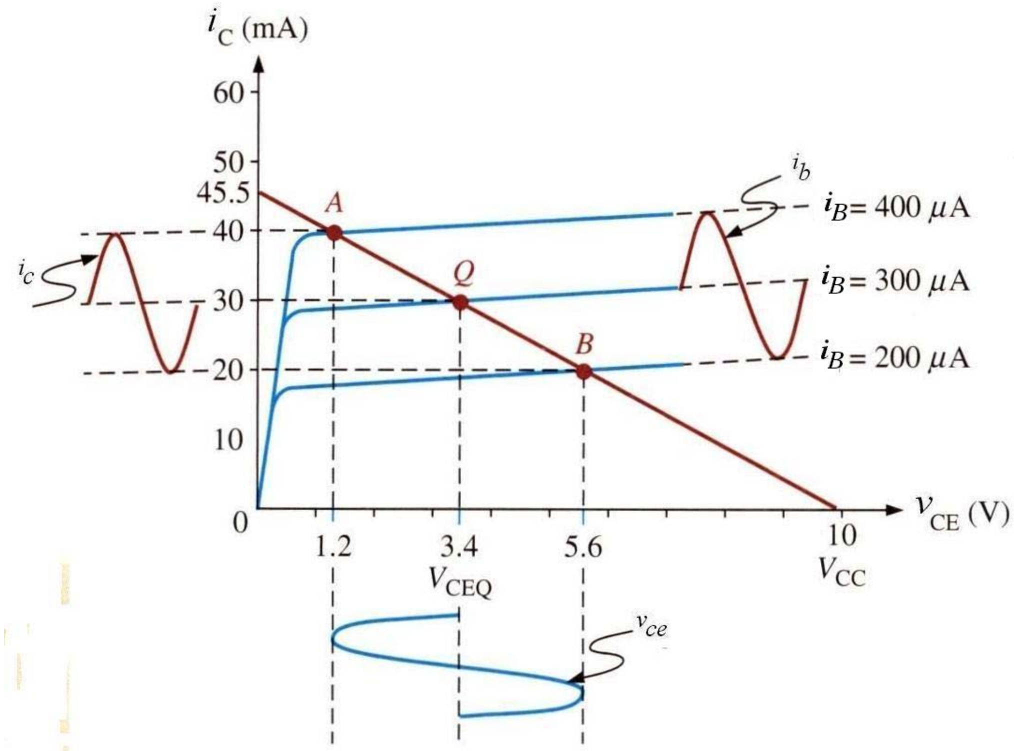
6. Signal Amplification in Amplifiers



- The d.c. source supplies power to the amplifier, while the a.c. signal is applied to the input port .
- The transistor in the amplifier circuit acts as a control device, which controls the flow of power from the d.c. source, to produce an enhanced a.c. signal at the output port .
- The output a.c. signal is of the same waveform as the input a.c. signal, but is of larger amplitude (in current, voltage, or power).

Signal Amplification in amplifiers





<u>Notations :</u>

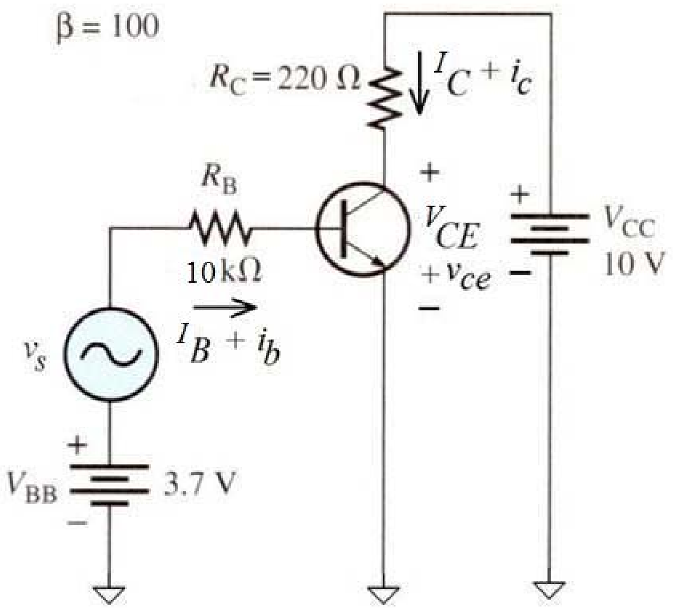
D.C. quantities, e.g., I_B : capital letter, capital subscript.

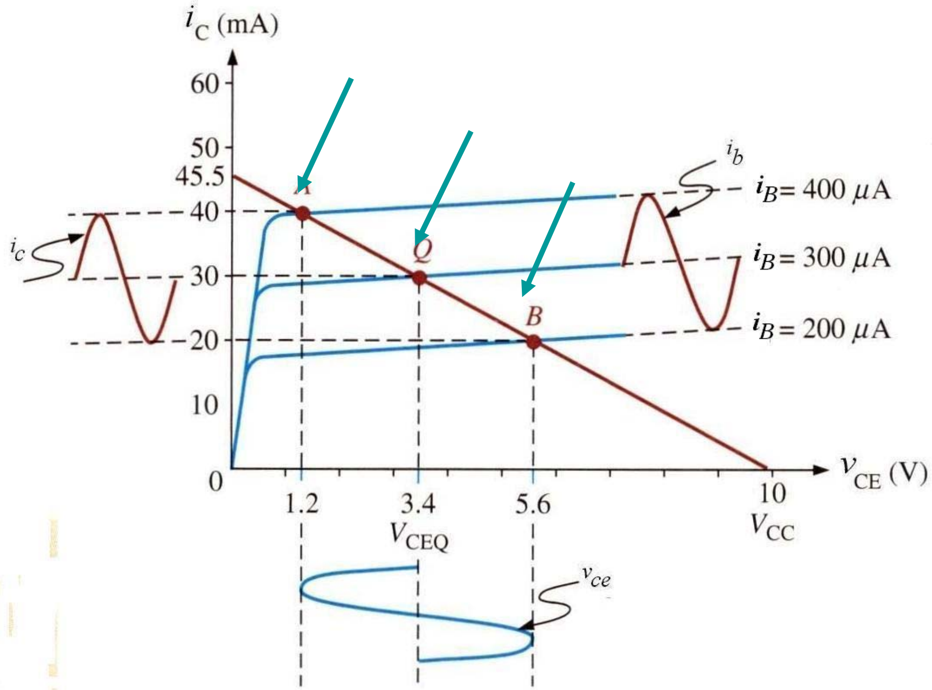
Small signal A.C. quantities, e.g., i_b : lower case letter, lower case subscript.

Total (instantaneous) quantities, e.g., i_B : lower case letter, capital subscript

<u>Example :</u> $$i_B = I_B + i_b$$

Signal Amplification in amplifiers



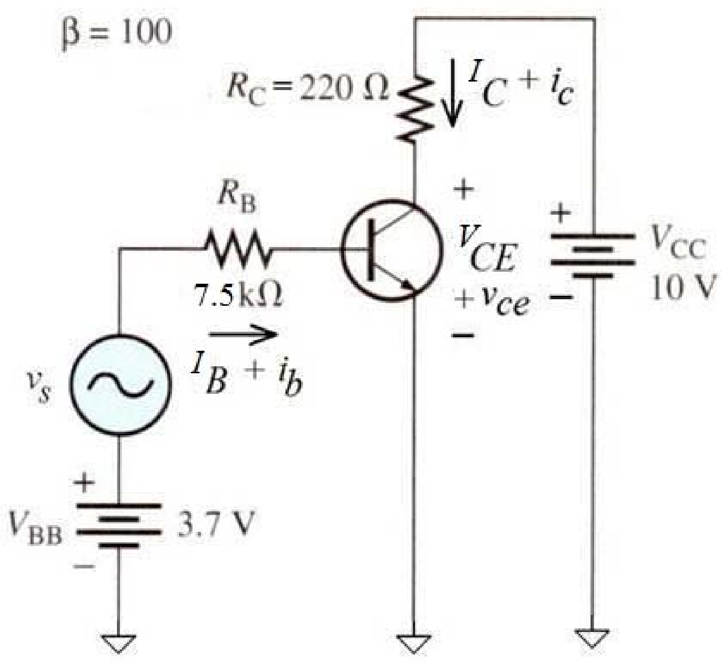


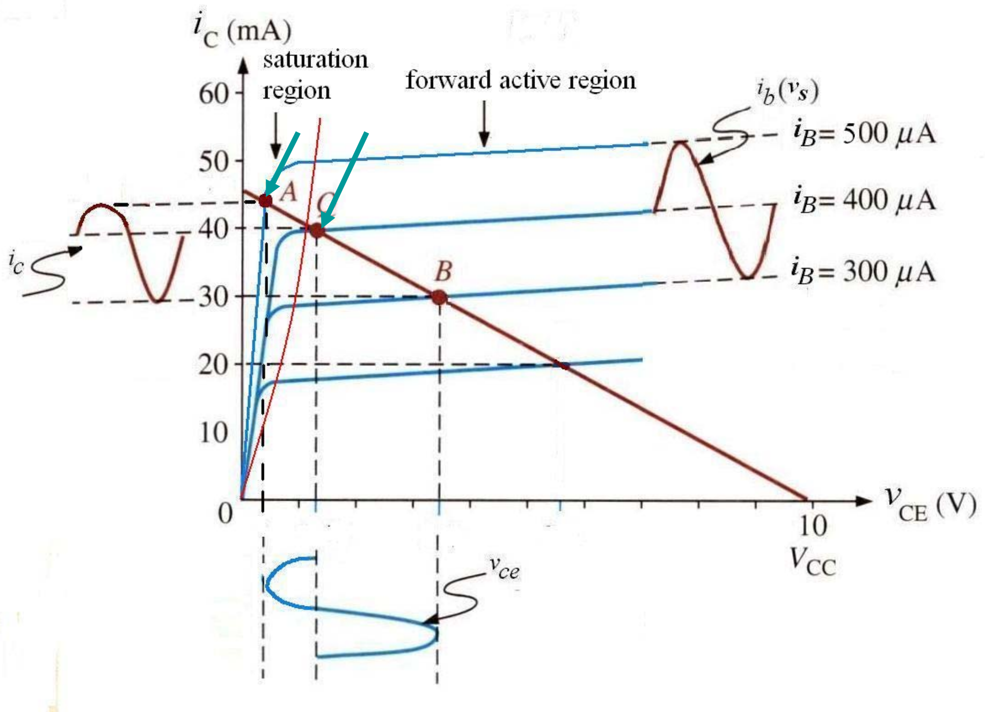
- When i_b = 0 μA, $i_B = I_B + i_b$= 300 μA, i_C = 30 mA, and v_{CE} = 3.4 V. (Point Q)
- When i_b = 100 μA, $i_B = I_B + i_b$= 300 μA +100 μA = 400 μA.

  i_C = 40 mA, and v_{CE} = 1.2 V. (Point A)
- When i_b = -100 μA, i_B = 300 μA -100 μA = 200 μA,

  i_C = 20 mA, and v_{CE} = 5.6 V. (Point B)

Signal Amplification in amplifiers

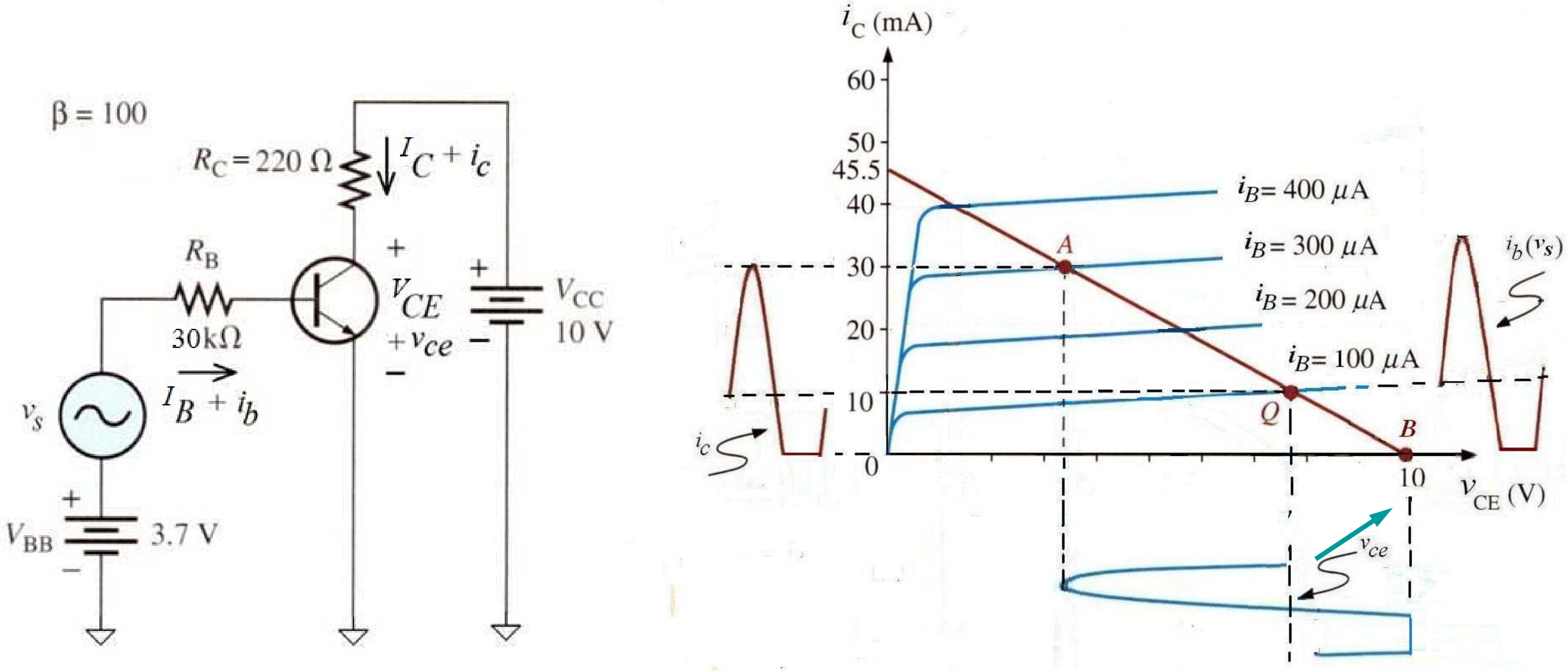




- D.C. base current $I_B = (V_{BB} - V_{BE}) / R_B = 400\ \mu A$. Operating point at "Q"
- When $i_b = 100\ \mu A$, $i_B = I_B + i_b = 500\ \mu A$,

  The intersection point between the load line and the BJT characteristics move to point "A", where the BJT is no longer operating in the forward active region.
- The a.c. signal waveforms of i_c and v_{ce} are distorted.

Signal Amplification in amplifiers



- D.C. base current $I_B = (V_{BB} - V_{BE}) / R_B$ = 100 μA . Operating point at "Q"
- Assume that the amplitude of the a.c. base current i_b is 200 μA.
- At some point during part of the negative half-cycle of the a.c .base current, the intersection point between the load line and the BJT characteristics is at point "B
- The waveform of i_C is clipped as it cannot be negative.
- Similarly, the waveform of v_{CE} is clipped at 10 V as it cannot exceed the power supply voltage.

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7. Small Signal (Simplified Hybrid-π) Model of the BJT

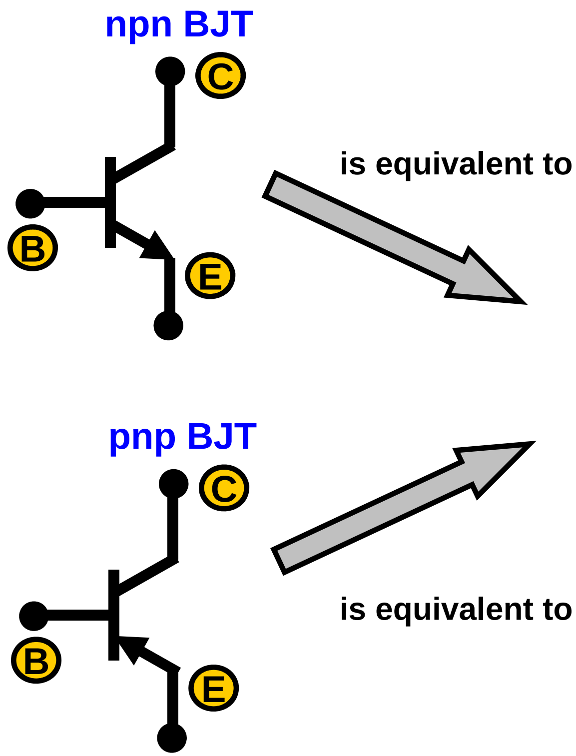
- To develop a small signal model of the BJT under **forward active region of operation**.

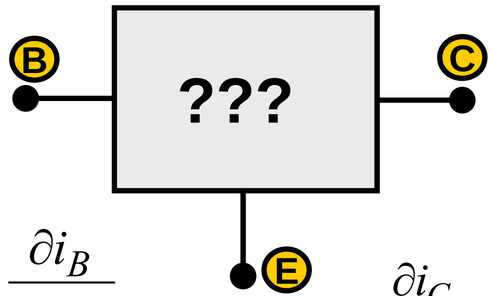
- We seek <u>linear relationships</u> among the **small signal a.c. components** of the base current, the collector current, the base-emitter voltage and the collector-emitter voltage.

$$i_C = i_C(v_{BE}, v_{CE})$$

$$\Delta i_C = i_c \approx [\partial i_C / \partial v_{BE}]\,\Delta v_{BE} + [\partial i_C / \partial v_{CE}]\,\Delta v_{CE}$$

$$i_c = [\partial i_C / \partial v_{BE}]\, v_{be} + [\partial i_C / \partial v_{CE}]\, v_{ce}$$





$$\frac{\partial i_B}{\partial v_{BE}}$$

$$\frac{\partial i_C}{\partial v_{BE}}, \frac{\partial i_C}{\partial v_{CE}}$$

Transconductance g_m

- It has been shown previously* that the collector current, $i_C = I_S\, e^{v_{BE}/V_T}$

- At the d.c. operating point, the d.c. collector current, $I_C = I_S\, e^{V_{BE}/V_T}$

- A linear relationship of small variations of the collector current, Δi_C, and the emitter-base voltage Δv_{BE}, around the d.c. operating point can be found by linearlizing the I-V characteristic.

- We approximate a small change in i_C with respect to a small change in v_{BE}, by the derivative of i_C with respect to v_{BE}:

$$\left.\frac{\Delta i_C}{\Delta v_{BE}}\right|_{V_{BE}} \approx \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{V_{BE}} = \left.\frac{i_c}{v_{be}}\right|_{v_{be}\to 0} = \frac{I_S\, e^{V_{BE}/V_T}}{V_T} = \frac{I_C}{V_T} \qquad (3.20)$$

* Refer to slide 3.22, eqn (3.12).

Transconductance g_m

- Transconductance $$g_m = \frac{I_C}{V_T} \qquad (3.21)$$

  where the value of I_C is calculated at the d.c. operating point.

- The transconductance models a small change in the collector current, i_c, caused by a small change in the base-emitter voltage, v_{be}.

Change in Collector current.

Change in Base-Emitter voltage.

$$i_c = g_m v_{be} \qquad (3.22)$$

Input Resistance r_π

- It has been shown previously* that the base current, $i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$
- At the d.c. operating point, the d.c.base current, $I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T}$
- We approximate a small change in i_B with respect to a small change in v_{BE}, by the derivative of i_C with respect to v_{BE}:

$$\left.\frac{\Delta i_B}{\Delta v_{BE}}\right|_{V_{BE}} \approx \left.\frac{\partial i_B}{\partial v_{BE}}\right|_{V_{BE}} = \left.\frac{i_b}{v_{be}}\right|_{v_{be}\to 0} = \frac{I_S\, e^{V_{BE}/V_T}}{\beta\, V_T} = \frac{I_C}{\beta\, V_T} = g_\pi \qquad (3.23)$$

* Refer to slide 3.22, eqn (3.13).

Input Resistance r_π

- Taking reciprocal of g_π, we have, the following equation :

$$r_\pi = \frac{1}{g_\pi} = \frac{\beta V_T}{I_C} = \frac{\beta}{g_m} \qquad (3.24)$$

- Hence, $$\beta = g_m r_\pi \qquad (3.25)$$

- We use r_π to model the relationship between small signal values of v_{be} and i_b of the base-emitter pn junction of the BJT.

Change in Base-Emitter voltage.

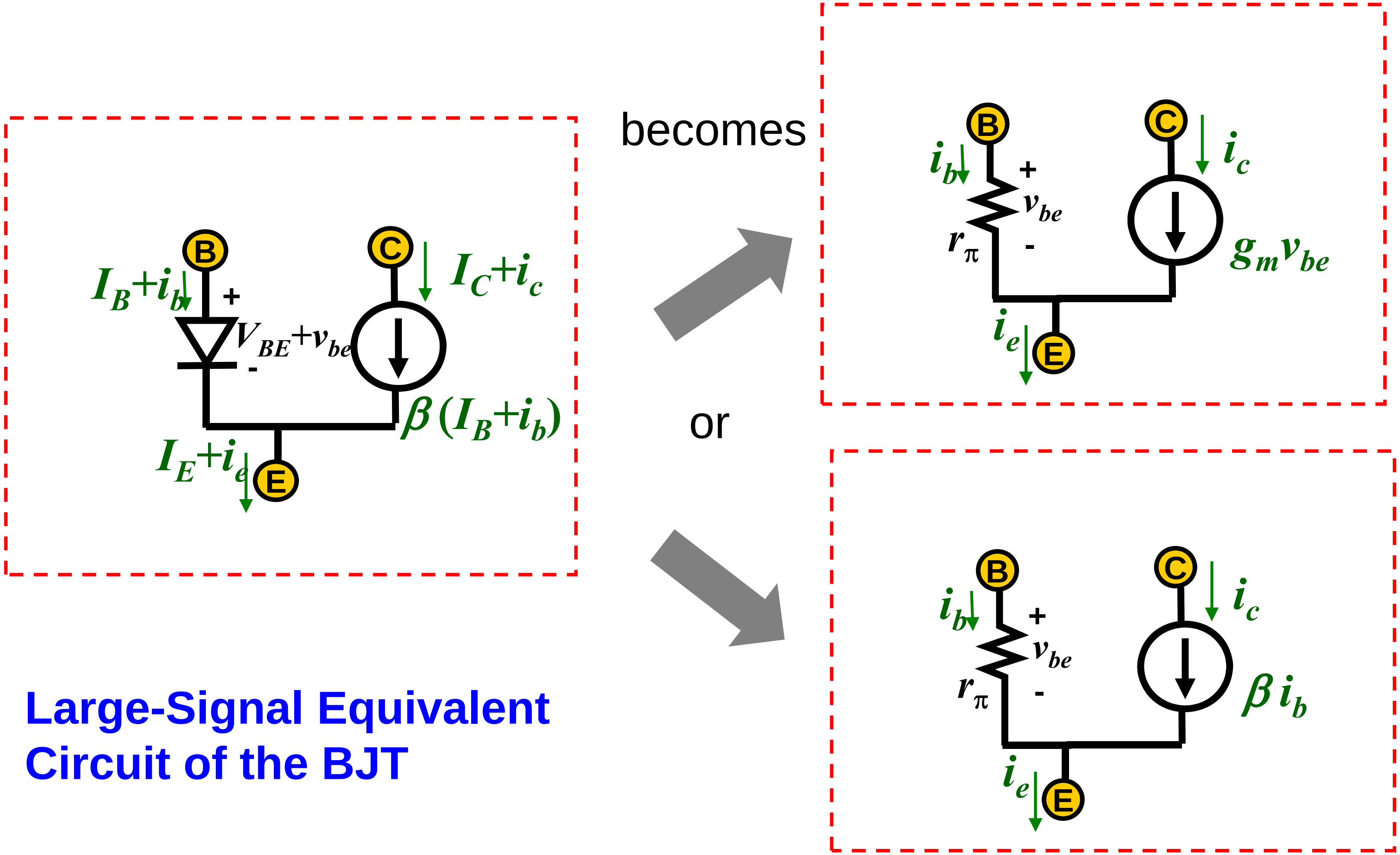
Change in Base current.

$$v_{be} = r_\pi i_b \qquad (3.26)$$

- Combining eqns (3.25) and (3.26),

$$i_c = g_m v_{be} = g_m r_\pi i_b = \beta i_b \qquad (3.27)$$

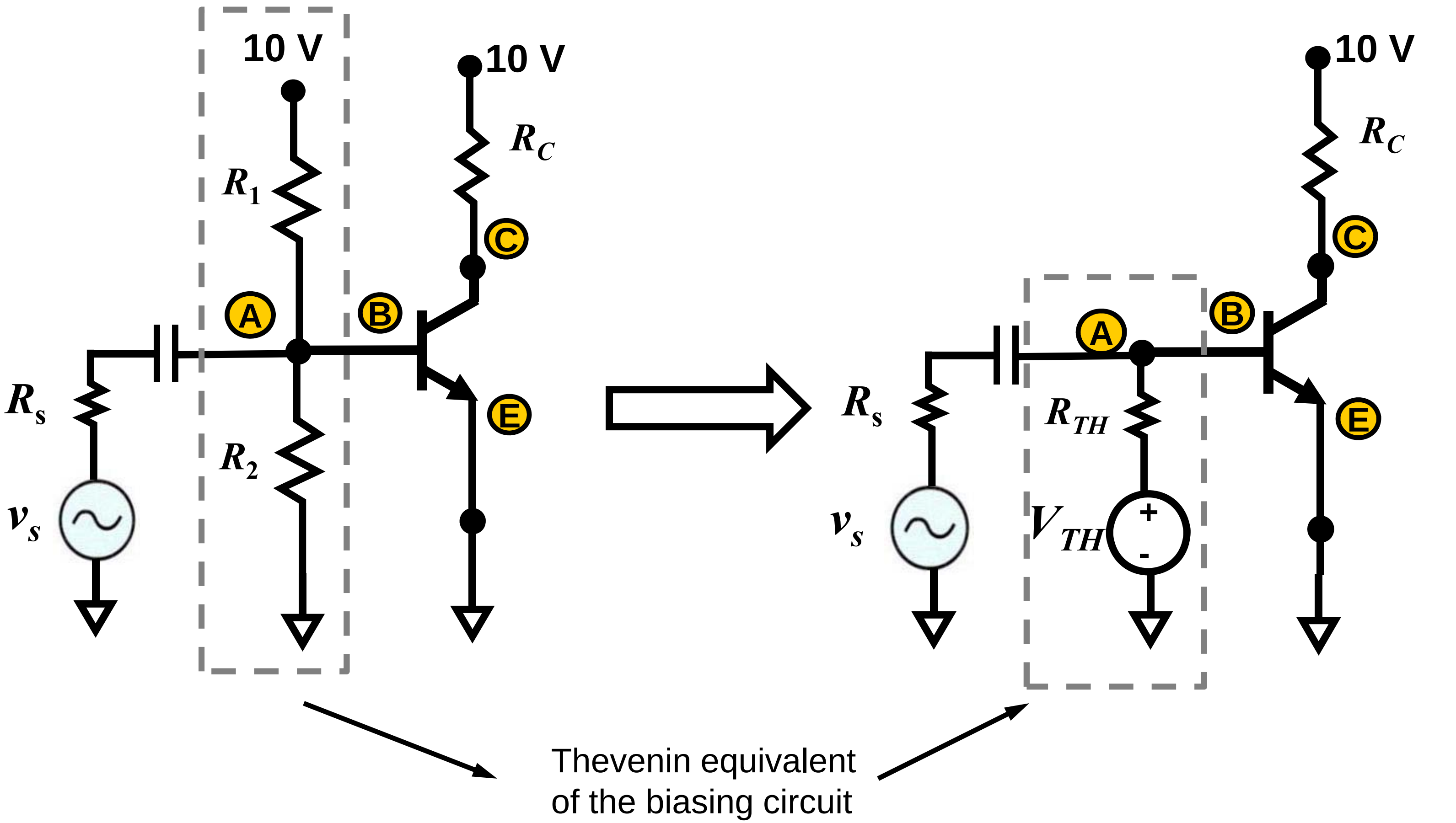
Small-Signal (Simplified Hybrid-π) Equivalent Circuit of the BJT



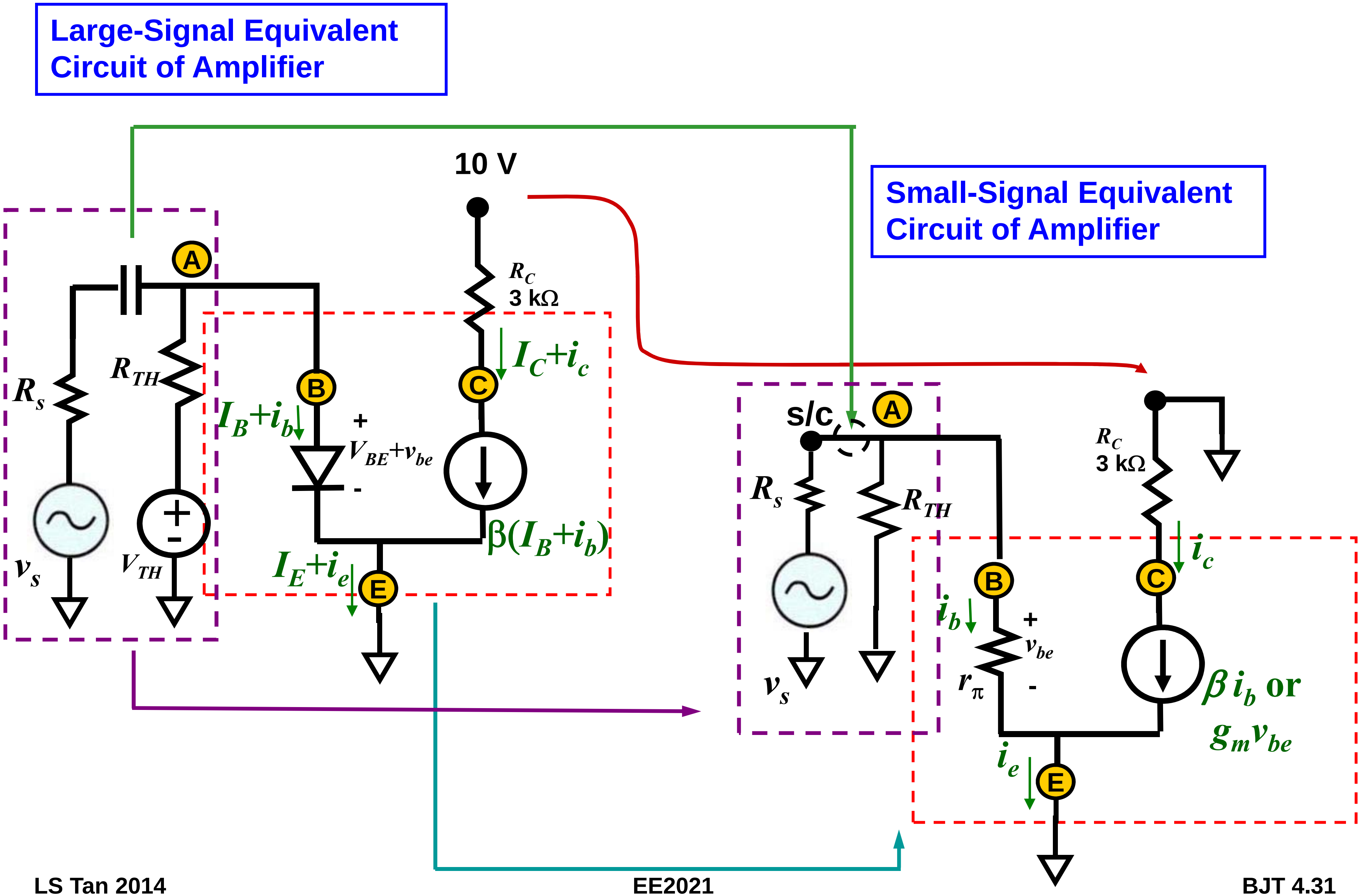
Large-Signal Equivalent Circuit of the BJT

Small-Signal (Simplified Hybrid-π) Equivalent Circuits of the BJT

Small-Signal Equivalent Circuit of a BJT Amplifier Circuit



Small-Signal Equivalent Circuit of a BJT Amplifier Circuit



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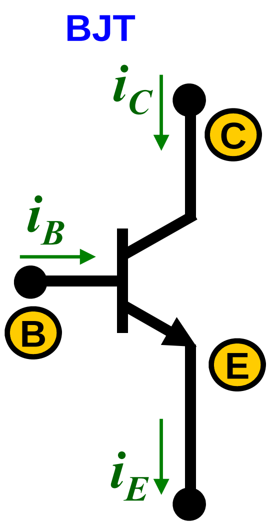
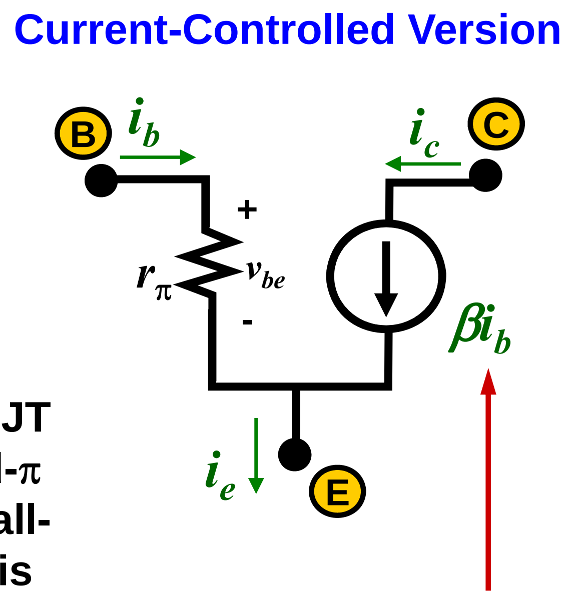
8. Full Hybrid-π Model of the BJT

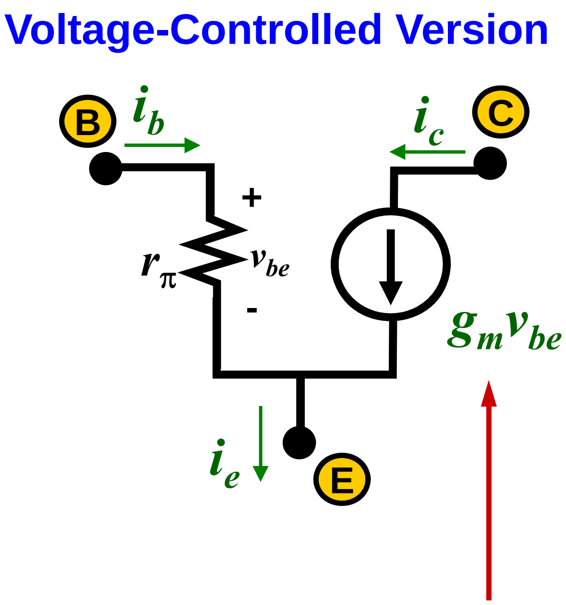
A simple Hybrid-π Model



Replace the BJT by this Hybrid-π Model for Small-Signal Analysis



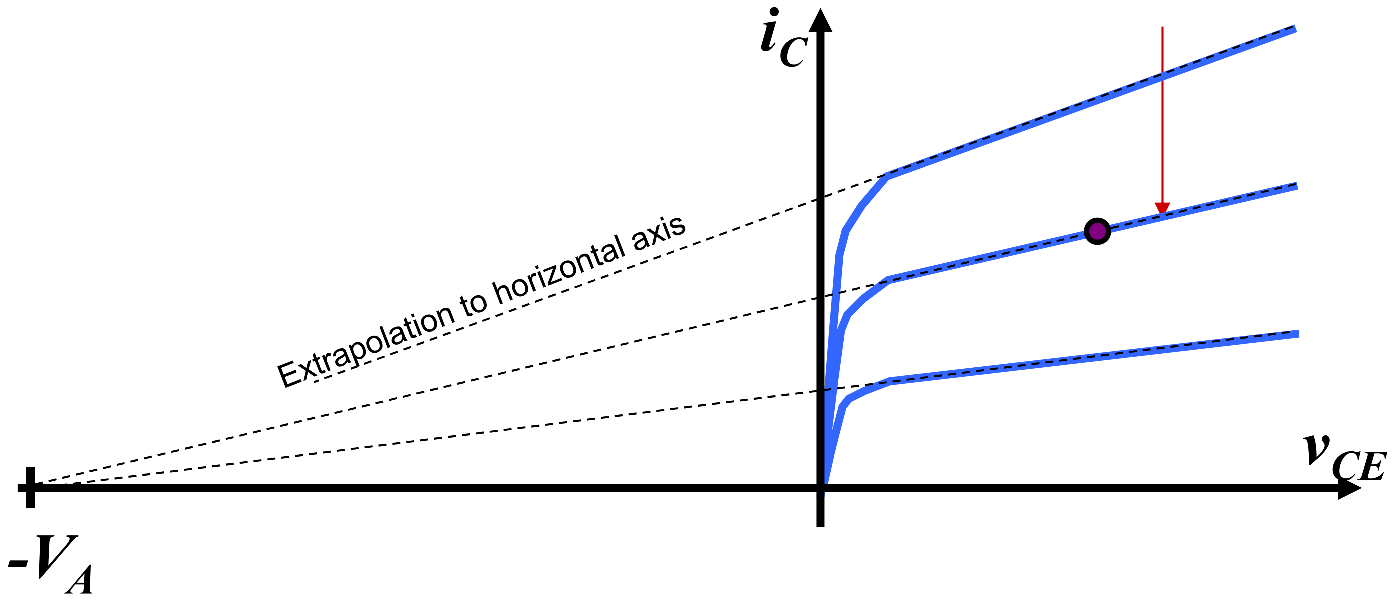
The value of i_c is β times i_b.
A current-dependent current source is used here.



The value of i_c is dependent on v_{be}. A voltage-dependent current source is used here.

Output Resistance r_o

For a given v_{BE}, see that when v_{CE} changes, i_C will change. How much i_C changes when v_{CE} changes is given by the slope of this line.



- From eqn (3.16) we have, $$i_C = I_S\, e^{v_{BE}/V_T}\left(1+\frac{v_{CE}}{V_A}\right)$$

$$g_o = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{V_{CE}} = \frac{I_S}{V_A}\, e^{V_{BE}/V_T} = \frac{I_C}{V_A}$$

- Output resistance, $$r_o = \frac{1}{g_o} = \frac{V_A}{I_C} \qquad (3.28)$$

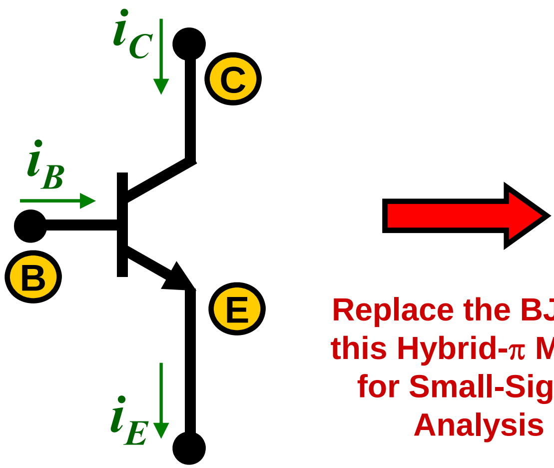
Hybrid-π Model with Output Resistance

- To model the increase in i_C with an increase in v_{CE}, a resistance r_o is included between nodes C and E in the Hybrid-π Model.

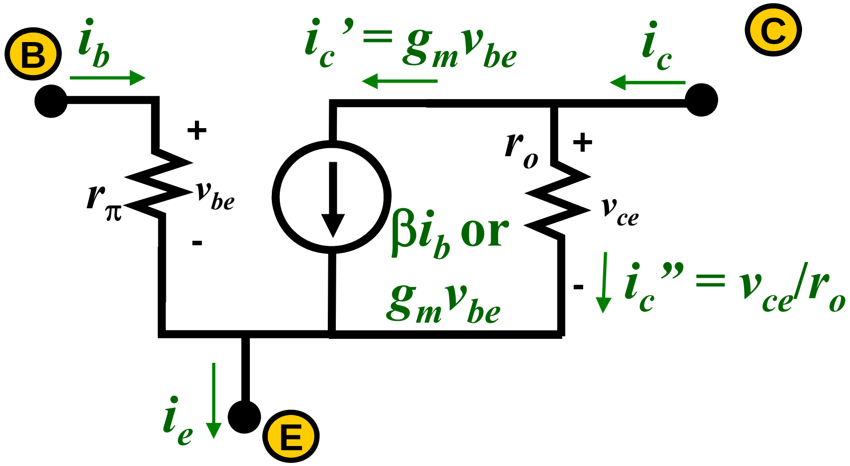
function of

$$i_C = i_C(v_{BE}, v_{CE})$$

$$\Delta i_C = i_c \approx \underbrace{[\partial i_C/\partial v_{BE}]}_{g_m}\ \underbrace{\Delta v_{BE}}_{v_{be}} + \underbrace{[\partial i_C/\partial v_{CE}]}_{1/r_o}\ \underbrace{\Delta v_{CE}}_{v_{ce}}$$



Replace the BJT by this Hybrid-π Model for Small-Signal Analysis



- See that i_c is given by

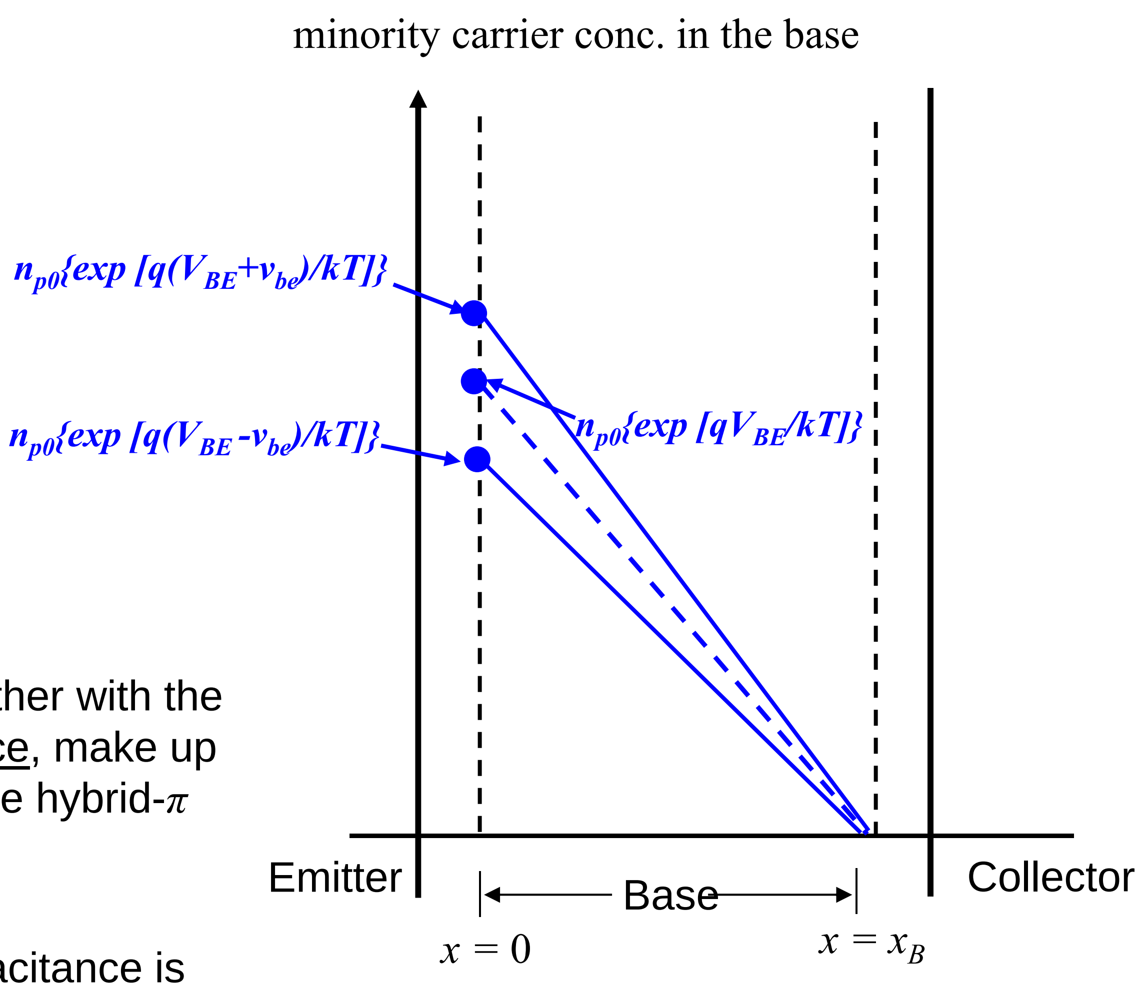
$$i_c = [\, g_m v_{be} + (v_{ce}/r_o)] \ \text{ or } \ [\, \beta i_b + \underbrace{(v_{ce}/r_o)}]$$

This term accounts for the dependence of i_c on v_{ce}.

Note: In EE2021, you can assume that r_o is infinite by default. However, if the Early voltage V_A is given, then r_o should be calculated and employed in the Small-Signal Analysis.

Capacitances in the BJT

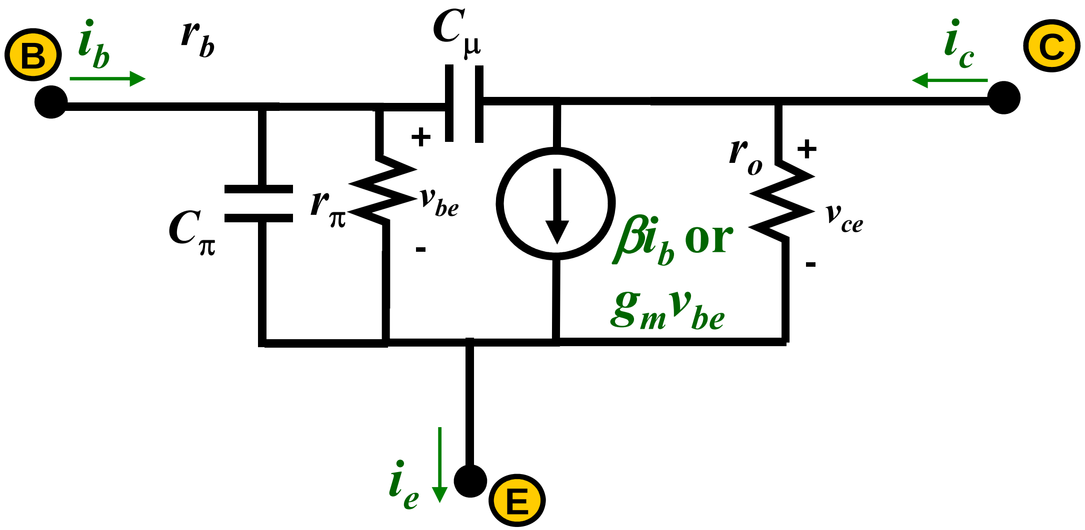
- When a small signal v_{be} is superimposed on the base-emitter (d.c.) forward bias V_{BE}, the concentration of the minority carriers at the base edge of the SCR is changed slightly.
- This results in a small change of the minority carrier charge in the base of the transistor, hence giving rise to a <u>diffusion capacitance.</u>
- This diffusion capacitance, together with the <u>emitter-base junction capacitance</u>, make up the <u>input capacitance</u> c_π in full the hybrid-π model of the bipolar transistor.
- The collector-base junction capacitance is denoted as c_μ in the hybrid-π model.



Note that v_{be} also causes a change in charge in the emitter. However, in the bjt, the minority carrier conc. in the emitter is much less than that in the base, and so this component of the diffusion capacitance is negligible.

Full Hybrid-π Model at High Frequency

- BJTs have capacitances associated with pn junctions and with charge storage.
- At high frequencies, two parasitic capacitors have to be included: C_μ and C_π.



Note: We introduce the existence of these capacitances C_μ and C_π here, and their inclusion in the Hybrid-π Model is only necessary when frequency response is discussed. We will ignore these capacitances (in the pF range) for low frequency signals.