EE2021
Devices and Circuits

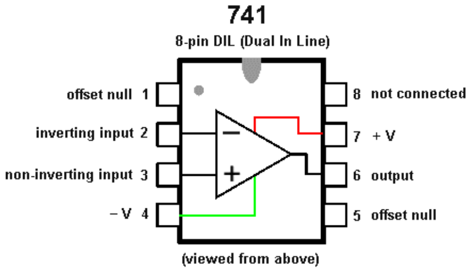
Opamp and Opamp based circuits

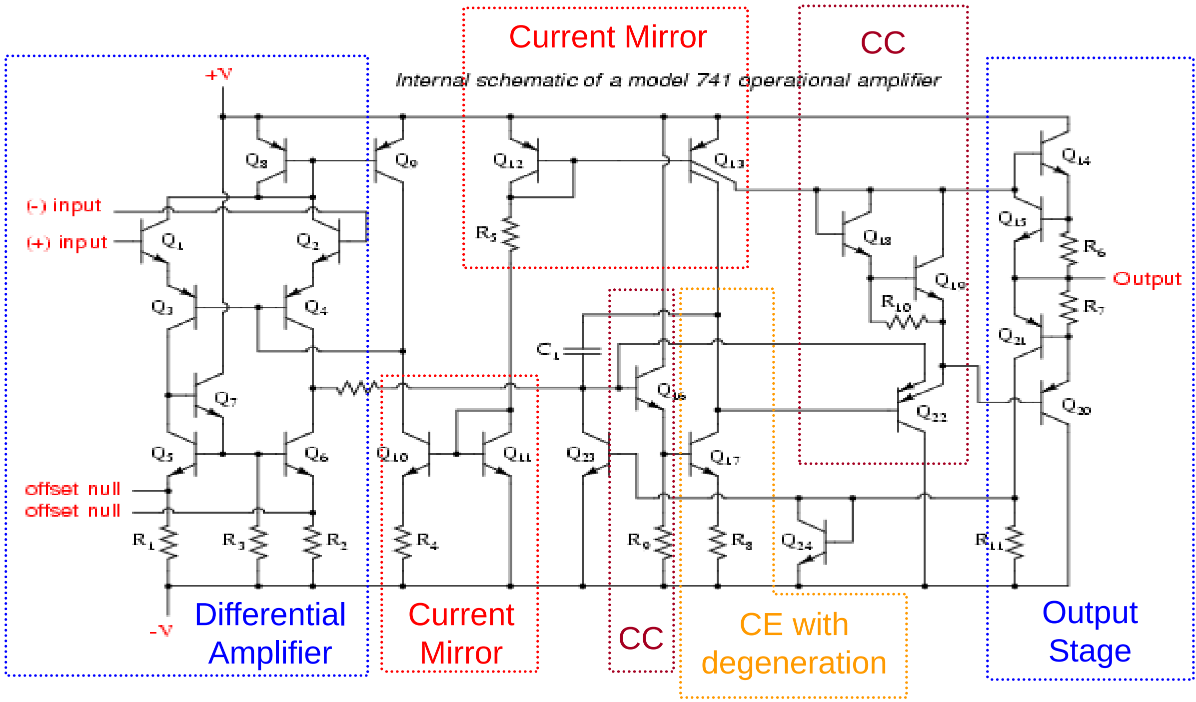
Lecture Outline

- **Different Opamp Transfer function**

  Inverting amplifier, non-inverting amplifier, summer, buffer, logarithm amplifier, super-diode, SK-filter, comparator, comparator with hysterisis

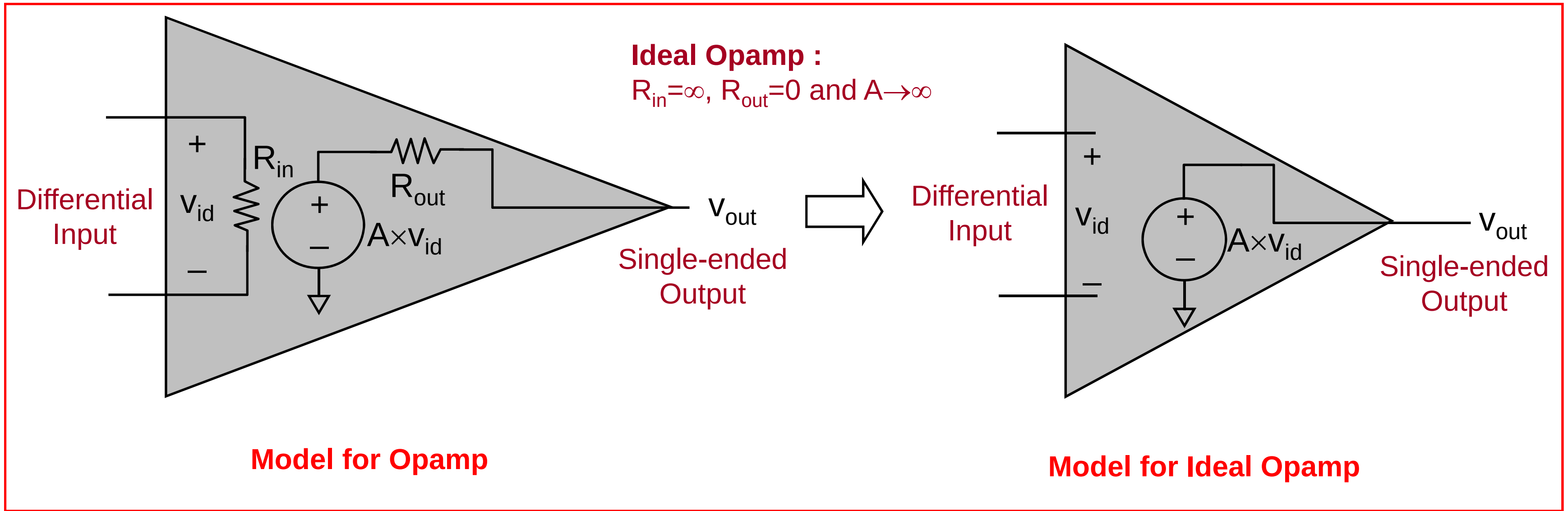
Opamp Schematic





- Multistage Amplifier Analysis
- All the techniques learnt earlier in amplifier analysis will help you understand and design an opamp eventually

Opamp Applications



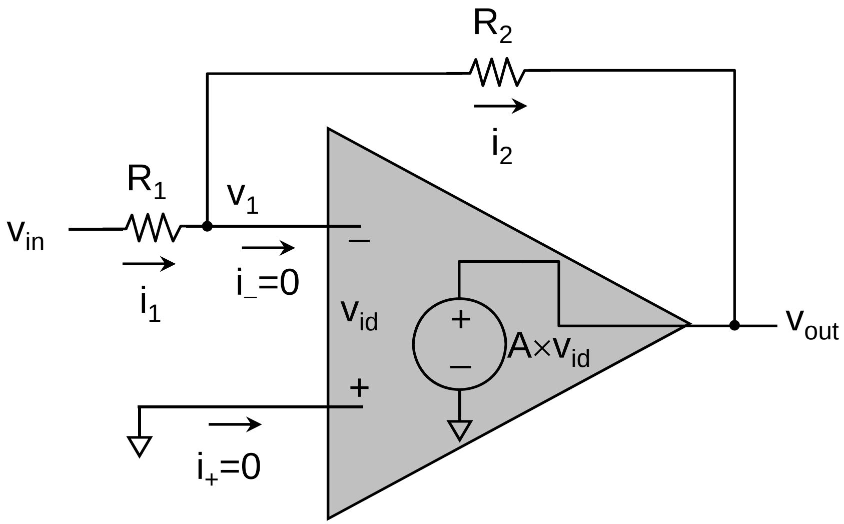
Possible Applications :

- Feedback Amplifier
- Integrator, Differentiator, Active Filter
- Simulated Inductor
- Switched-Capacitor Filter
- Oscillator
- A/D Converter

Connect some electronic components (Resistors, capacitors, transistors, diodes) surrounding the opamp will result in interesting applications

For simplicity, we always analyze opamp based circuits with ideal opamp model

How to Derive Transfer Function for Opamp Circuit?



Important Observation :

- No current flowing into the opamp, i.e. $i_-=i_+=0$
- If A is large and v_{out} is finite $\Rightarrow v_{id}=-v_1\approx 0 \Rightarrow$ v+≈v- $\Rightarrow$ Virtually short
- A real short, current can potentially flow through between two nodes. A virtual short, only the voltage appears to be zero, but there is no current flowing through between the two nodes.
- Since V+ is connected to ground, we can say v_1 is virtual ground

Transfer Function for Opamp Circuit

$$v_{id} = -v_1$$

$$v_{out} = Av_{id} = A(-v_1)$$

$$\Rightarrow v_1 = -\frac{v_{out}}{A}$$

$$\boxed{\frac{v_{in}-v_1}{R_1} = \frac{v_1 - v_{out}}{R_2}}$$

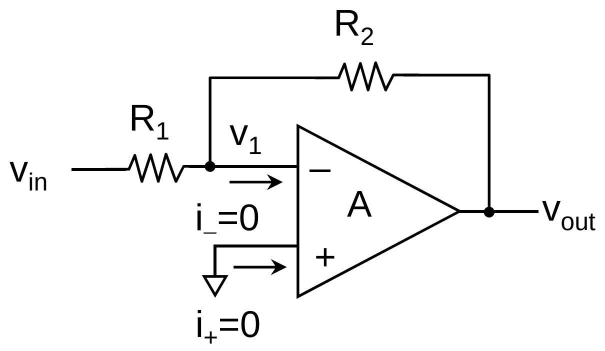
KCL: $i_-=0 \Rightarrow i_1=i_2$

$$\Rightarrow \frac{v_{in}+\frac{v_{out}}{A}}{R_1} = \frac{-\frac{v_{out}}{A}-v_{out}}{R_2}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}\frac{1}{\frac{R_1+R_2}{AR_1}+1}$$

$$\approx -\frac{R_2}{R_1} \quad if \quad A \to \infty$$

Inverting Amplifier



Transfer Function for Inverting Amplifier

$$v_1 = v_- \approx v_+ = 0 \quad [\because Virtually \ \ short \ \ to \ \ ground]$$

$$\frac{v_{in} - v_1}{R_1} = \frac{v_1 - v_{out}}{R_2}$$

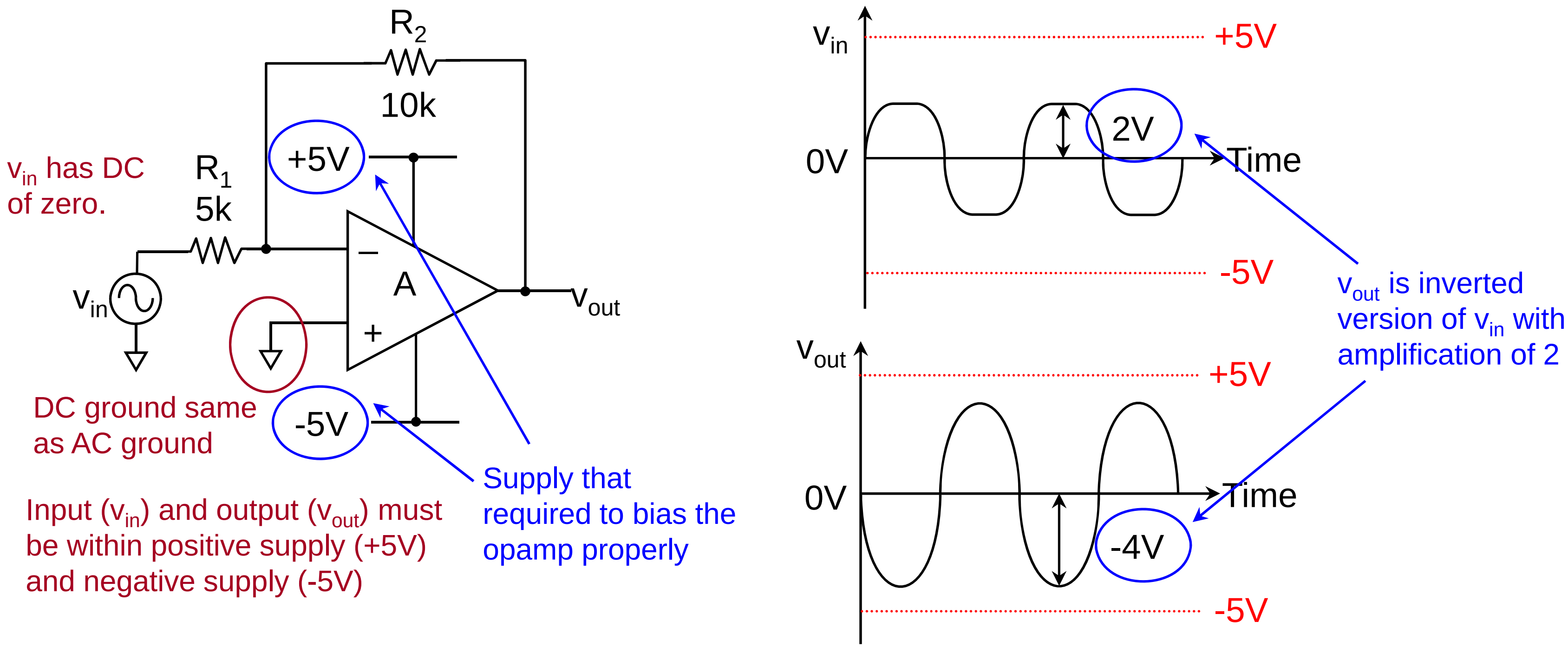
$$\frac{v_{in}}{R_1} = \frac{-v_{out}}{R_2}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1} \cdots\cdots(1)$$

Using virtual short concept, the analysis is much simpler

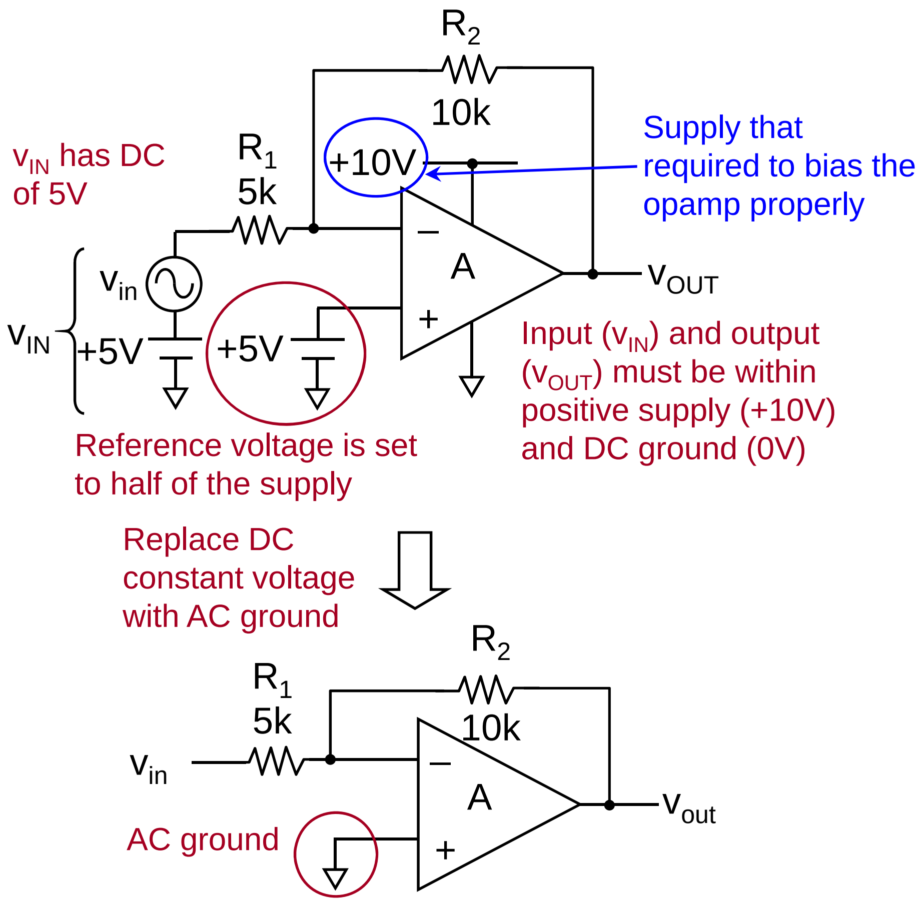
- The overall gain ($A_V = v_{out}/v_{in}$) is controlled by the ratio of the resistor (R_2/R_1)
- The overall gain is independent of the opamp gain (A)
- It is a feedback amplifier
- There is polarity inversion between v_{out} and v_{in}

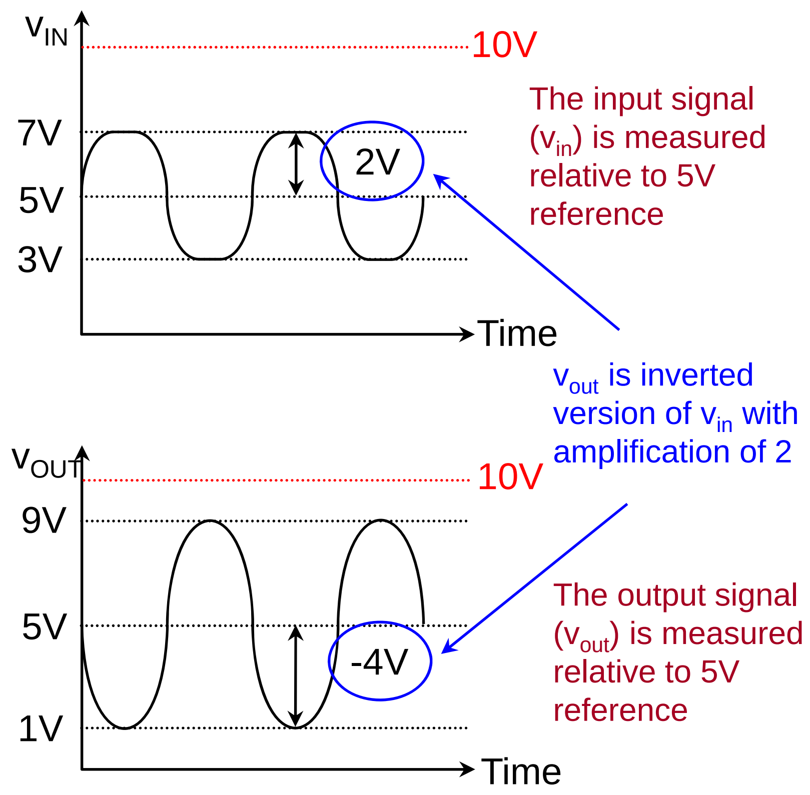
Opamp Biasing – Old Days



- Luxury of dual supply (+5V and -5V)
- DC ground and AC ground are the same

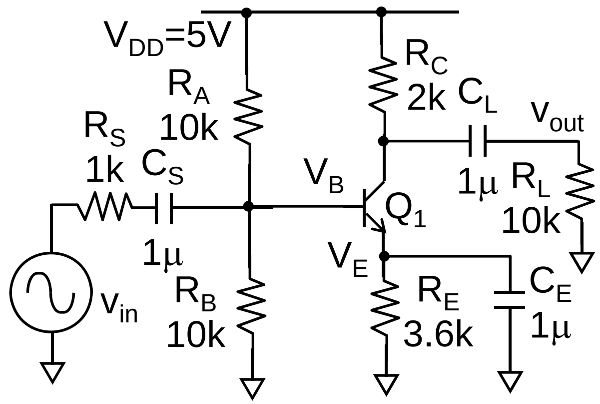
Opamp Biasing – Nowadays





- Single supply only (+10V)
- The reference voltage is usually implemented using simple resistor divider
- DC ground and AC ground are not the same
- For analysis simplicity, we will always ignore the DC portion and perform the AC analysis.

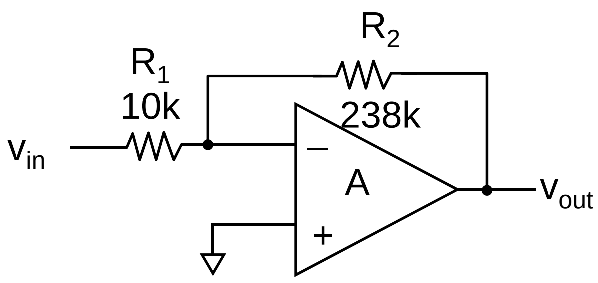
Why Feedback Amplifier?



$$A_V = \frac{v_{out}}{v_{in}} = -\frac{R_{in}}{R_{in}+R_S} g_{m1}(R_C // R_L) = -23.8$$

Open Loop Amplifier (CE)

VS

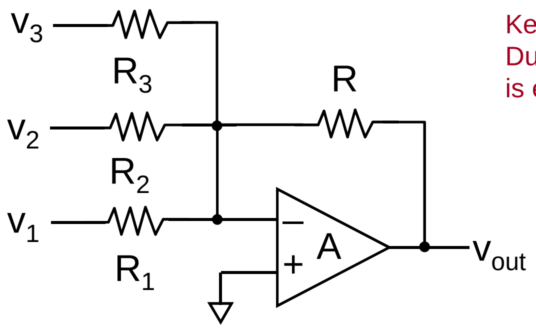


$$A_V = \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1} = -23.8$$

Feedback Amplifier

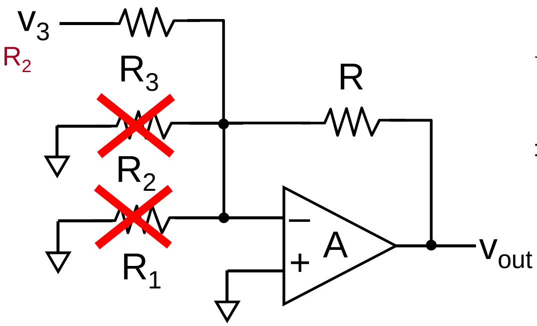
	Open Loop Amplifier	Feedback Amplifier
Gain	Not accurate (Difficult to control transistor parameters)	Accurate gain (Gain only depends on the ratio of resistors, which can be accurately controlled)
Linearity	Poor Linearity (More distortion)	High Linearity (Less distortion)
Complexity	Simple and consume less power	Opamp is complicated and consume more power

Summing Amplifier



1) Estimate output due to individual voltage source
2) Apply linear superposition to find the total contribution
3) While estimating contribution from one voltage source, kill all the other voltage source

Keep v_3 and kill v_1, v_2
Due to virtual ground, R_1 and R_2 is effectively removed

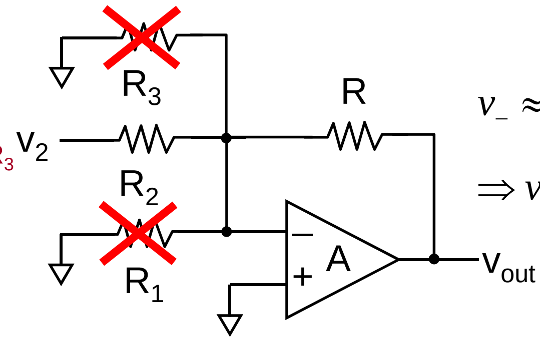


$$v_- \approx v_+ = 0$$

$$\Rightarrow v_{out,v3} = -\frac{R}{R_3} v_3$$

Equation (1) from slide 8-6

Keep v_2 and kill v_1, v_3
Due to virtual ground, R_1 and R_3 is effectively removed

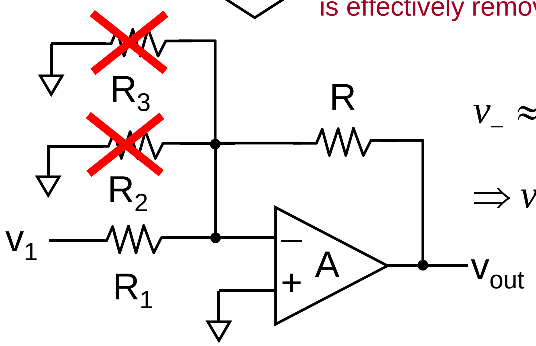


$$v_- \approx v_+ = 0$$

$$\Rightarrow v_{out,v2} = -\frac{R}{R_2} v_2$$

Equation (1) from slide 8-6

Keep v_1 and kill v_2, v_3
Due to virtual ground, R_2 and R_3 is effectively removed



$$v_- \approx v_+ = 0$$

$$\Rightarrow v_{out,v1} = -\frac{R}{R_1} v_1$$

Equation (1) from slide 8-6

Transfer Function for Summing Amplifier

$$\boxed{v_{out} = v_{out,v_1} + v_{out,v_2} + v_{out,v_3}} \quad \text{Superposition}$$

$$= -\left(\frac{R}{R_1} v_1 + \frac{R}{R_2} v_2 + \frac{R}{R_3} v_3\right)$$

$$= -(v_1 + v_2 + v_3) \quad if \quad R_1 = R_2 = R_3 = R$$

Steps for Opamp Circuit Analysis

Opamp Circuit Analysis

⇓

Assume virtual short
$v_+ \approx v_-$

⇓

Assume no input current
$i_+ = i_- = 0$

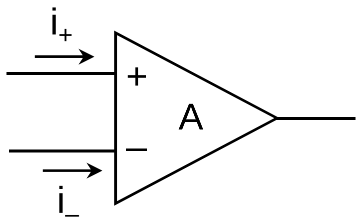
⇓

Apply linear superposition for multiple input voltage source

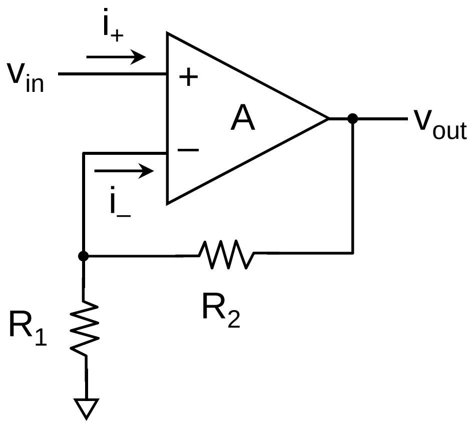
⇓

Derive $A_V = v_{out}/v_{in}$ using KCL and KVL

Valid only if there is negative feedback



Non-Inverting Amplifier



What is the limitation on the gain of non-inverting amplifier?

Transfer Function for Non-Inverting Amplifier

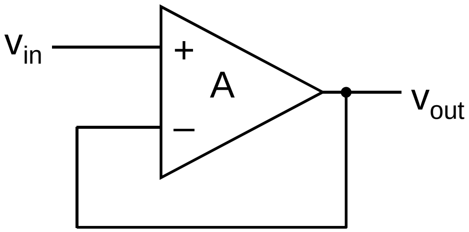
$$v_- \approx v_+ = v_{in} \quad [\because Virtual \;\; Short]$$

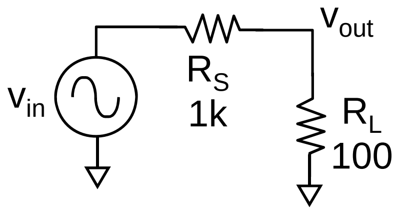
$$v_- = v_{out} \times \frac{R_1}{R_1 + R_2} = v_{in} \quad [\because i_+ = i_- = 0]$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \left(1 + \frac{R_2}{R_1}\right) \cdots\cdots(2)$$

- The overall gain ($A_V = v_{out}/v_{in}$) is controlled by the ratio of the resistor ($1+R_2/R_1$)
- The overall gain is independent of the opamp gain (A)
- It is a feedback amplifier
- There is no polarity inversion between v_{out} and v_{in}

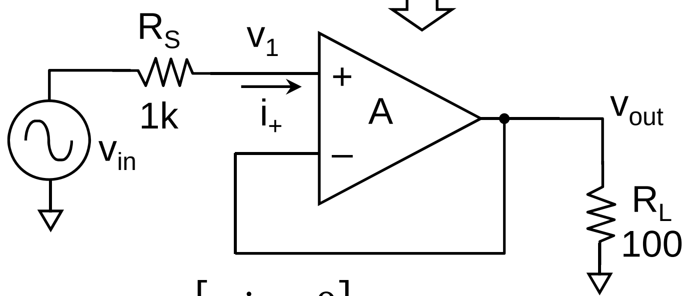
Unity Gain Buffer (Source Follower)





Due to loading effect (R_L<R_S) $\Rightarrow A_V = v_{out}/v_{in} < 1$

$$\frac{v_{out}}{v_{in}} = \frac{R_L}{R_L + R_S} = 0.09$$



Transfer Function for Unity Gain Buffer

$$v_{out} = v_- \approx v_+ = v_{in} \quad [\because Virtual \;\; Short]$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = 1$$

$$R_{in} = \infty$$

$$R_{out} \to 0$$

$$v_1 = v_{in} [\because i_+ = 0]$$

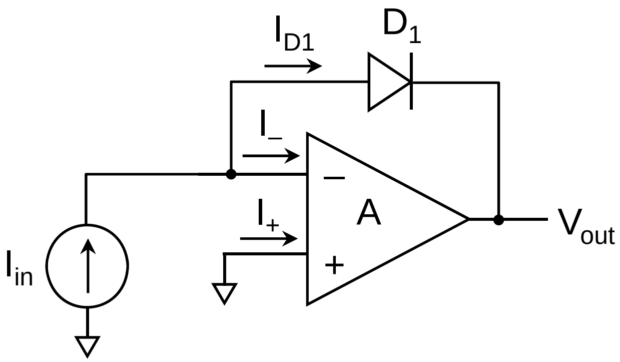
$$\Rightarrow v_{out} = v_- \approx v_+ = v_1 = v_{in}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = 1$$

The unity gain buffer presents high R_{in} to the source and is able to drive low impedance load

- Ideal for driving low impedance load

Logarithmic Amplifier



Because the signals we are dealing with consist of DC and AC components, we use upper case (V_{out}, I_{in}) rather than lower case (v_{out}, i_{in})

Transfer Function for Logarithmic Amplifier

$$V_- \approx V_+ = 0 \quad [\because Virtual\ \ Short]$$

$$V_{D1} = V_- - V_{out} \approx -V_{out}$$

$$\boxed{I_{D1} = I_S\left(e^{V_{D1}/V_T} - 1\right) \approx I_S e^{-V_{out}/V_T}}$$

Diode characteristic equation

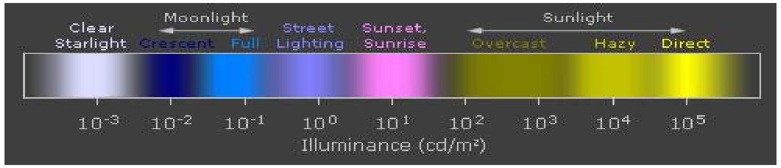
$$I_{D1} = I_{in} \approx I_S e^{-V_{out}/V_T} \quad [\because i_+ = i_- = 0]$$

$$\boxed{\Rightarrow V_{out} = -V_T \ln\left(\frac{I_{in}}{I_S}\right)}$$

Output is natural log of input

- Suitable for application where the input has very wide dynamic range

Logarithmic Amplifier (Camera)

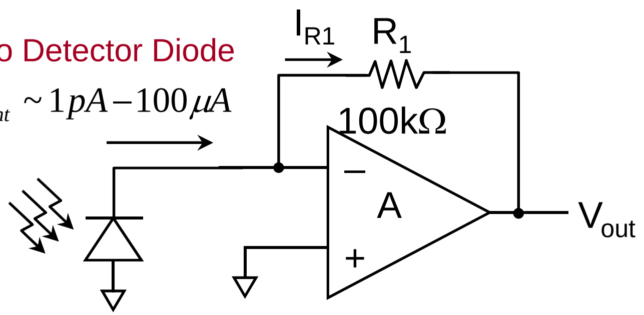


I_{light} 1pA 10pA 100pA 1nA 10nA 100n 1µA 10µA 100µA

I_{light} changes with light intensity
Light intensity changes by order of magnitude $\Rightarrow$ I_{light} also changes by order of magnitude

Photo Detector Diode

$I_{light} \sim 1pA - 100\mu A$



$$V_{out} = -I_{light} \times R_1$$
$$\sim \underbrace{-0.1\mu V}_{\text{Clear Starlight}} \; to \; \underbrace{-10V}_{\text{Direct Sunlight}}$$

Output changes linearly with input
$\Rightarrow$ Difficult to detect and differentiate the input signal

V_{out}<1mV cannot be easily measured
$\Rightarrow$ Can only detect sunset/sunrise and above

$I_S = 10^{15}$ $\quad V_T = 25mV$

$$\boxed{I_{light} \sim 1pA - 100\mu A}$$

Input changes by eight order of magnitude (10^{-12} – 10^{-4})

$I_{light} \sim 1pA - 100\mu A$

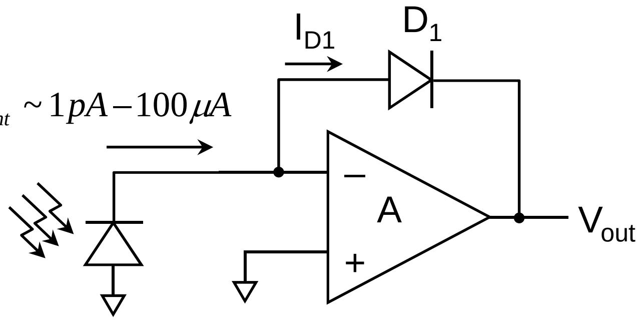


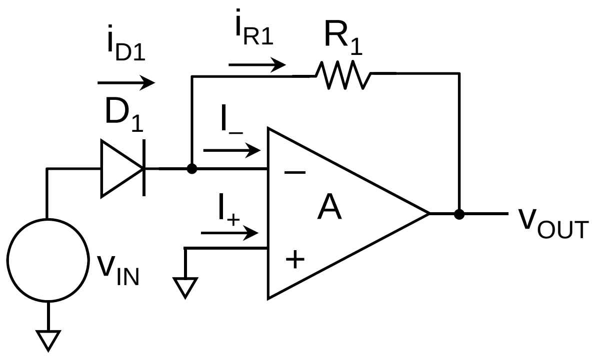
Photo Detector Diode

$$V_{out} = -V_T \ln\left(\frac{I_{light}}{I_S}\right) \sim \underbrace{-0.17V}_{\text{Clear Starlight}} \; to \; \underbrace{-0.63V}_{\text{Direct Sunlight}}$$

Output changes in log scale with input
$\Rightarrow$ Easy to detect and differentiate the input signal

-0.17V and -0.63V can be easily measured
$\Rightarrow$ All light conditions can be detected

Exponential Amplifier



Because the signals we are dealing with consist of DC and AC components, we use lower case variable with upper case subscript (v_{OUT}, v_{IN})

Transfer Function for Exponential Amplifier

$$V_- \approx V_+ = 0 \quad [\because \text{Virtual Short}]$$

$$v_{D1} = v_{IN} - V_- \approx v_{IN}$$

$$\boxed{i_{D1} = I_S\left(e^{v_{IN}/V_T} - 1\right) \approx I_S e^{v_{IN}/V_T}}$$

Diode characteristic equation

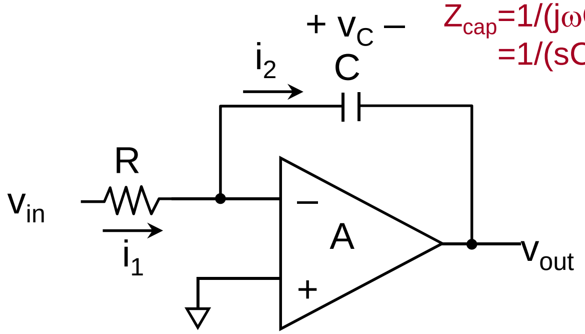
$$i_{D1} = i_{R1} = \frac{V_- - v_{OUT}}{R_1} \approx \frac{-v_{OUT}}{R_1} \quad [\because i_+ = i_- = 0]$$

$$\boxed{\Rightarrow v_{OUT} = -R_1 I_S e^{v_{IN}/V_T}}$$

Output is exponential of input

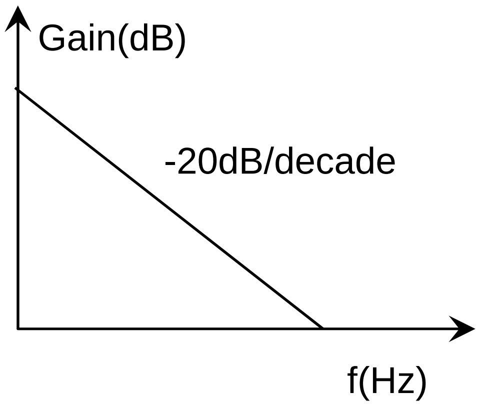
- Suitable for application where the input has very narrow dynamic range and you want to expand it for easier differentiation

Integrator



$Z_{cap}=1/(j\omega C)$
$=1/(sC)$

The circuit performs analog integration



Transfer Function for Integrator

$$v_- \approx v_+ = 0 \qquad [\because Virtual\ \ Short]$$

$$v_C(t) = v_- - v_{out}(t) \approx -v_{out}(t)$$

$$i_1(t) = \frac{v_{in}(t)}{R} \qquad i_2(t) = C\frac{dv_C(t)}{dt} \approx -C\frac{dv_{out}(t)}{dt}$$

$$i_1(t) = i_2(t) \qquad [\because i_+ = i_- = 0]$$

$$\Rightarrow -C\frac{dv_{out}(t)}{dt} = \frac{v_{in}(t)}{R}$$

$$\Rightarrow v_{out}(t) = -\frac{1}{RC}\int_0^t v_{in}(t)dt$$

Alternatively based on inverting amplifier

$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{Z_{cap}}{R} = -\frac{1}{j\omega CR} = -\frac{1}{sCR} = H(s)}$$

Slide 8-6 equation 1

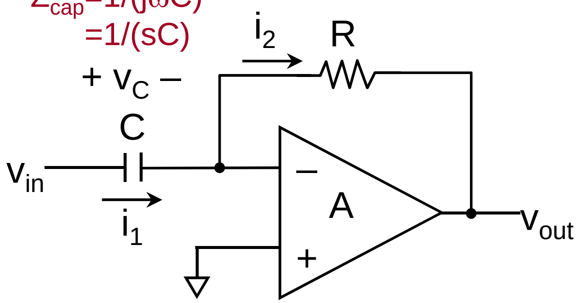
$$\boxed{\frac{1}{j\omega} = \frac{1}{s} \rightarrow \int(\cdot)dt}$$

S-Transform (Learnt in EE2023, Signals and Systems)

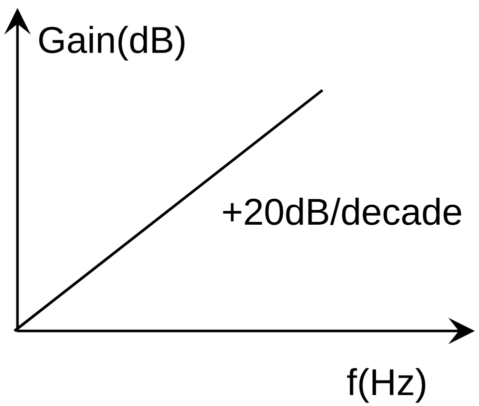
- $H(j\omega)$ reduces with increasing ω ($\omega=2\pi f$)
- $dB(H(j\omega))=20\times\log_{10}(|H(j\omega)|)$
- -20dB/decade means $H(j\omega)$ decreases by 20dB when the frequency (f or ω) increases by 10 times.

Differentiator

Z_{cap}=1/(jωC)
=1/(sC)



The circuit performs
analog differentiation



Transfer Function for Differentiator

$$v_- \approx v_+ = 0 \qquad [\because Virtual \;\; Short]$$

$$v_C(t) = v_{in}(t) - v_- \approx v_{in}(t)$$

$$i_1(t) = C\frac{dv_C(t)}{dt} \approx C\frac{dv_{in}(t)}{dt} \qquad i_2(t) = \frac{v_- - v_{out}(t)}{R} \approx -\frac{v_{out}(t)}{R}$$

$$i_1(t) = i_2(t) \qquad [\because i_+ = i_- = 0]$$

$$\Rightarrow -\frac{v_{out}(t)}{R} = C\frac{dv_{in}(t)}{dt}$$

$$\Rightarrow v_{out}(t) = -RC\frac{dv_{in}(t)}{dt}$$

Alternatively based on inverting amplifier

$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{R}{Z_{cap}} = -j\omega CR = -sCR = H(s)}$$

Slide 8-6
equation 1

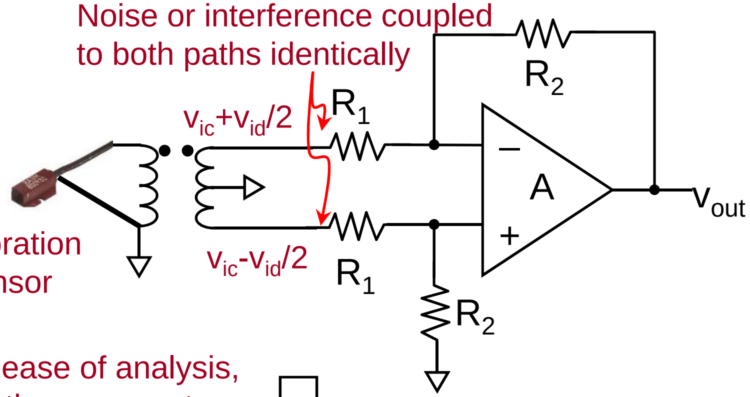
$$\boxed{\Rightarrow \;\; j\omega = s \rightarrow \frac{d(\cdot)}{dt}}$$

S-Transform (Learnt in EE2023,
Signals and Systems)

- H(jω) increases with increasing ω (ω=2πf)
- dB(H(jω))=20×$\log_{10}$(|H(jω)|)
- +20dB/decade means H(jω) increases by 20dB when the frequency (f or ω) increases by 10 times.

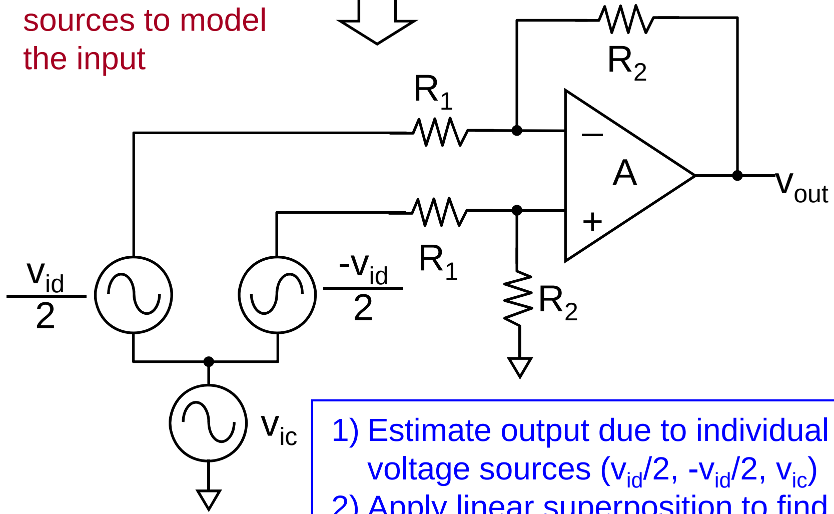
Instrumentation Amplifier

Noise or interference coupled to both paths identically



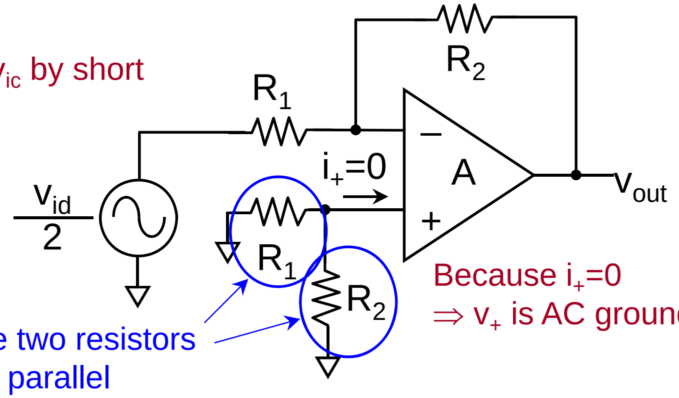
Vibration sensor

For ease of analysis, use three separate sources to model the input



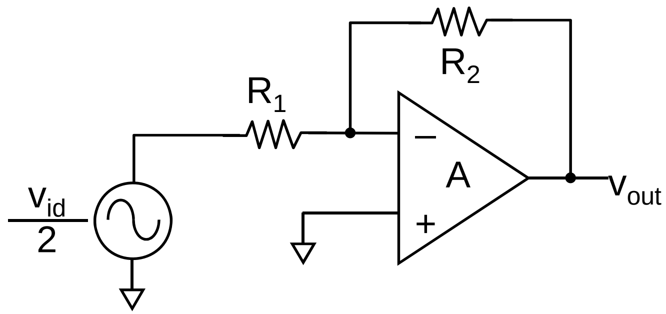
1) Estimate output due to individual voltage sources ($v_{id}/2$, $-v_{id}/2$, v_{ic})
2) Apply linear superposition to find the total contribution
3) While estimating contribution from one voltage source, kill all the other voltage sources

Keep $v_{id}/2$
Kill $-v_{id}/2$ and v_{ic} by short circuit them



These two resistors are in parallel

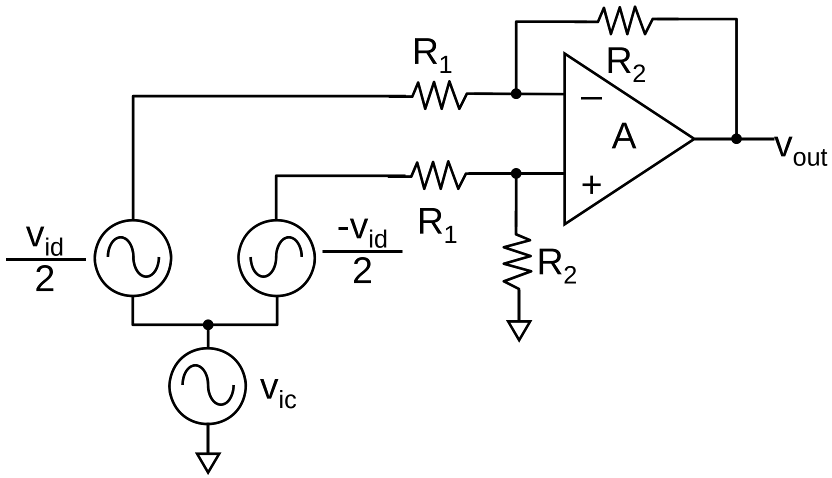
Because $i_+=0$
$\Rightarrow v_+$ is AC ground



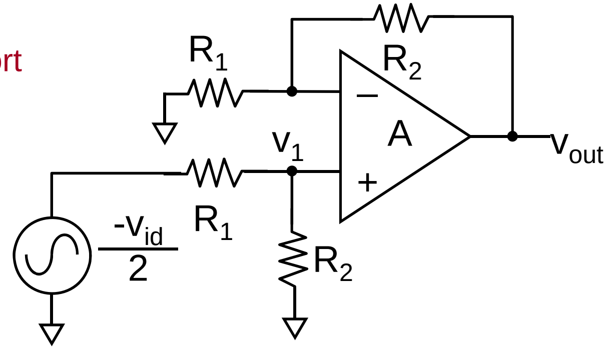
$$\underbrace{v_{out,\frac{v_{id}}{2}}}_{v_{out}\text{ due to }v_{id}/2} = -\frac{R_2}{R_1}\left(\frac{v_{id}}{2}\right)\cdots\cdots(1)$$

Equation (1) from slide 8-6

Instrumentation Amplifier



Keep $-v_{id}/2$
Kill $v_{id}/2$ and v_{ic} by short circuit them



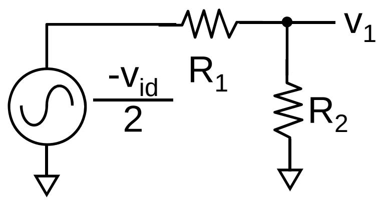
$$v_{1,-\frac{v_{id}}{2}} = \frac{R_2}{R_1+R_2}\left(\frac{-v_{id}}{2}\right)$$

$$\boxed{v_{out,-\frac{v_{id}}{2}} = \left(1+\frac{R_2}{R_1}\right)v_{1,-\frac{v_{id}}{2}}}$$

Equation (2) from slide 8-12

$$= \left(1+\frac{R_2}{R_1}\right)\frac{R_2}{R_1+R_2}\left(\frac{-v_{id}}{2}\right)$$

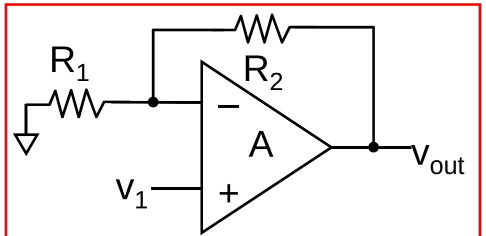
$$\Rightarrow \underbrace{v_{out,-\frac{v_{id}}{2}}}_{v_{out}\text{ due to } -v_{id}/2} = \frac{R_2}{R_1}\left(\frac{-v_{id}}{2}\right)\cdots\cdots(2)$$



Configuration A

Simple resistor divider

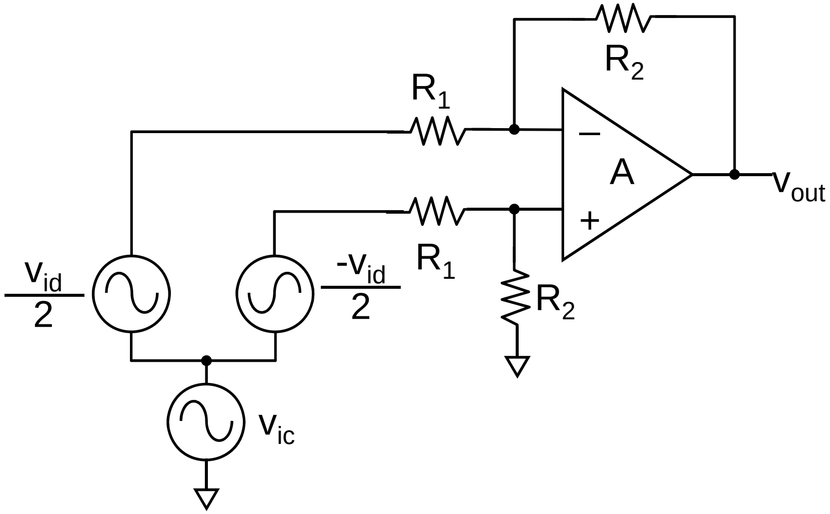
$\times$



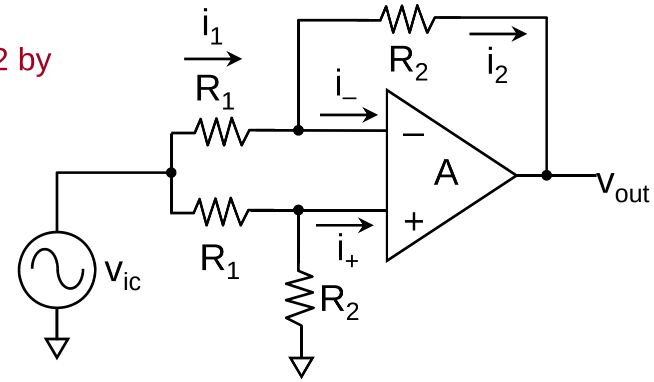
Configuration B

Non-inverting Amplifier
Equation (2) from slide 8-12

Instrumentation Amplifier



Keep v_{ic}
Kill $v_{id}/2$ and $-v_{id}/2$ by short circuit them



$$v_{+} = v_{ic} \times \frac{R_2}{R_1 + R_2} \approx v_{-} \left[\because Virtual \;\; short\right]$$

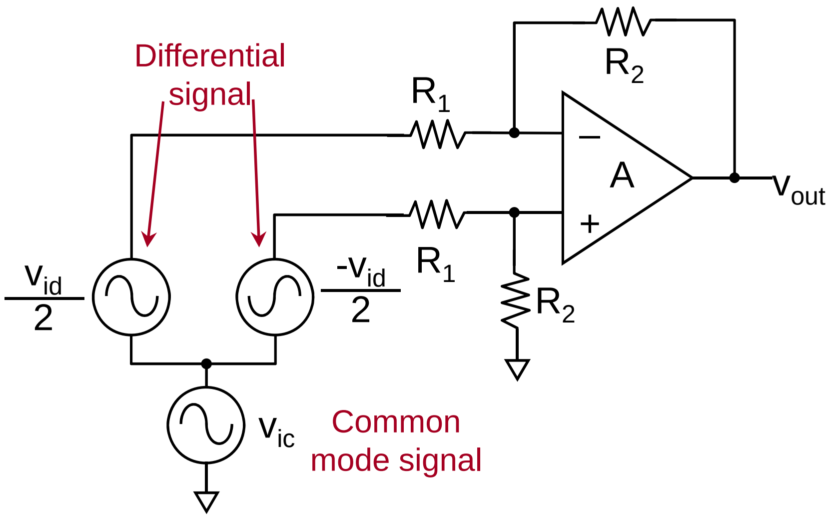
$$i_1 = i_2 \left[\because i_{+} = i_{-} = 0\right]$$

$$\Rightarrow \frac{v_{ic} - v_{-}}{R_1} = \frac{v_{-} - v_{out,v_{ic}}}{R_2}$$

$$R_2 v_{ic} - v_{ic} \times \frac{R_2^2}{R_1 + R_2} = v_{ic} \times \frac{R_1 R_2}{R_1 + R_2} - v_{out,v_{ic}}$$

$$\Rightarrow \underbrace{v_{out,v_{ic}}}_{v_{out} \text{ due to } v_{ic}} = 0 \cdots\cdots (3)$$

Instrumentation Amplifier



Transfer Function for Instrumentation Amplifier

$$\boxed{v_{out} = v_{out,\frac{v_{id}}{2}} + v_{out,-\frac{v_{id}}{2}} + v_{out,v_{ic}}}$$

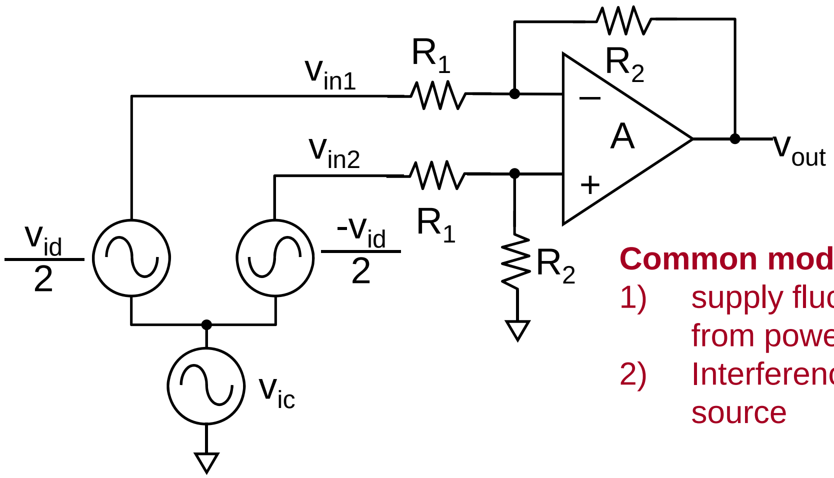
Superposition : Combine eqn (1), (2) and (3) from previous slides

$$= -\frac{R_2}{R_1}\left(\frac{v_{id}}{2}\right) + \frac{R_2}{R_1}\left(-\frac{v_{id}}{2}\right) + 0$$

$$= -\frac{R_2}{R_1}(v_{id})$$

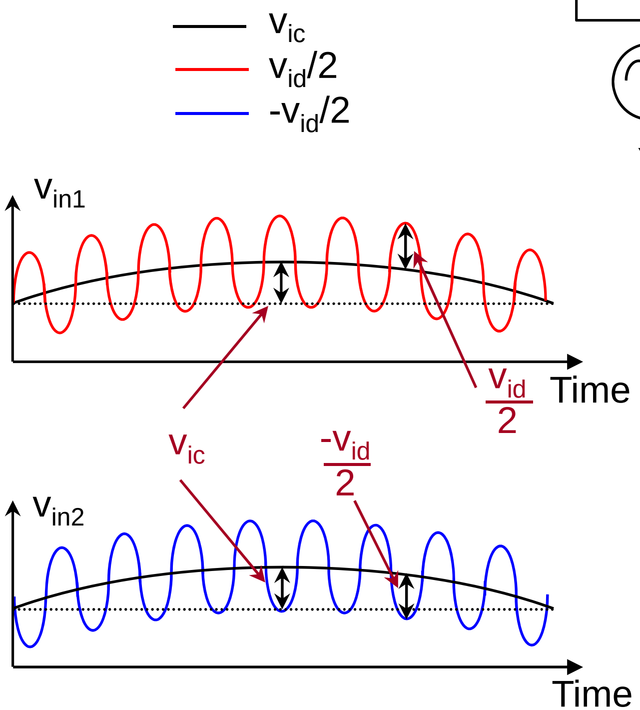
- It rejects common mode signal and only amplifies differential signal
- Good rejection for common mode interference and noise
- Required for measurement instrument
- Limited by the matching properties of the resistors

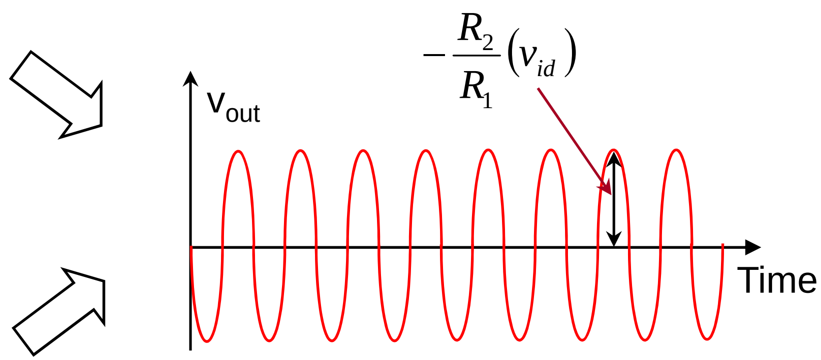
Common Mode Interference Rejection



Common mode interference (v_{ic}) :

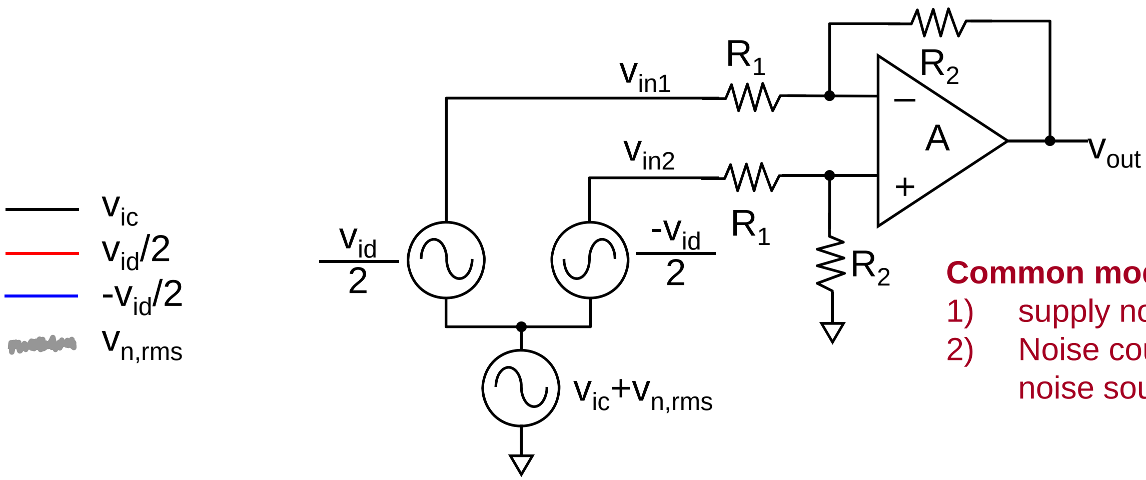
1) supply fluctuation (50~60Hz from power plant)
2) Interference from other signal source





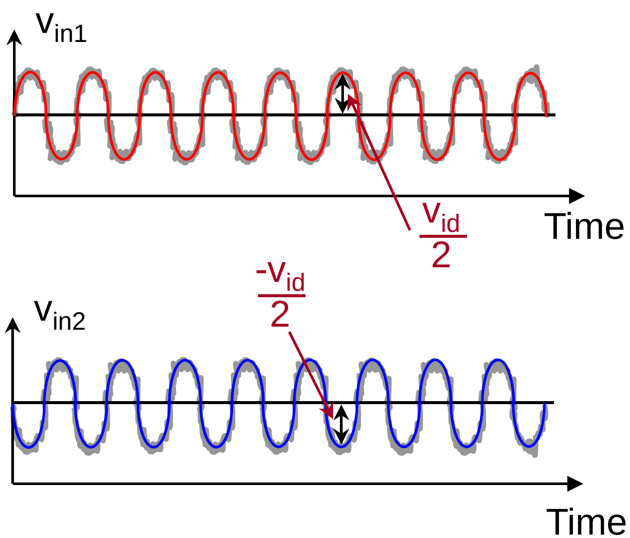
Free of common mode interference

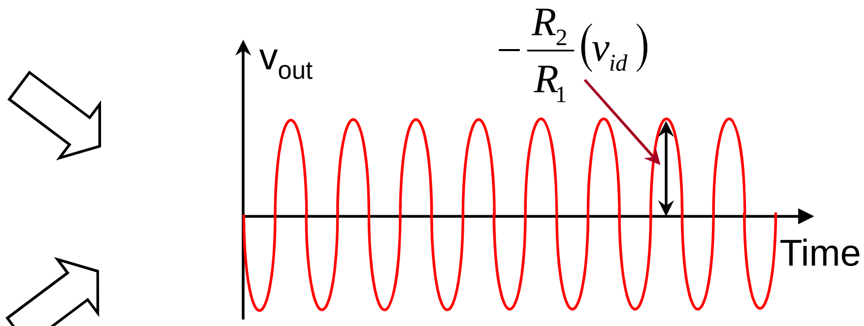
Common Mode Noise Rejection



Common mode noise ($v_{n,rms}$) :

1) supply noise
2) Noise coupling from other noise source

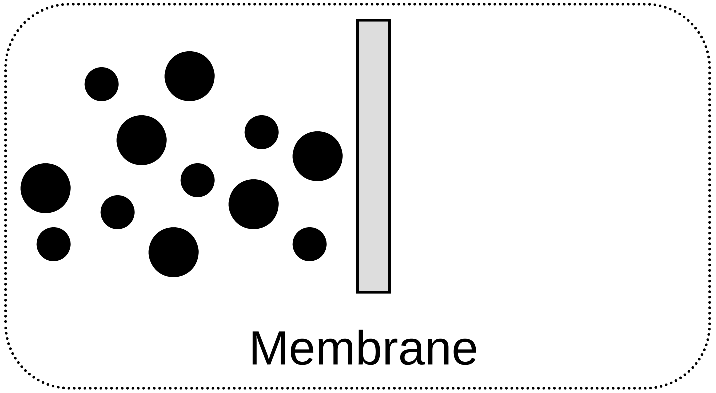




Free of common mode noise

What is Filter?

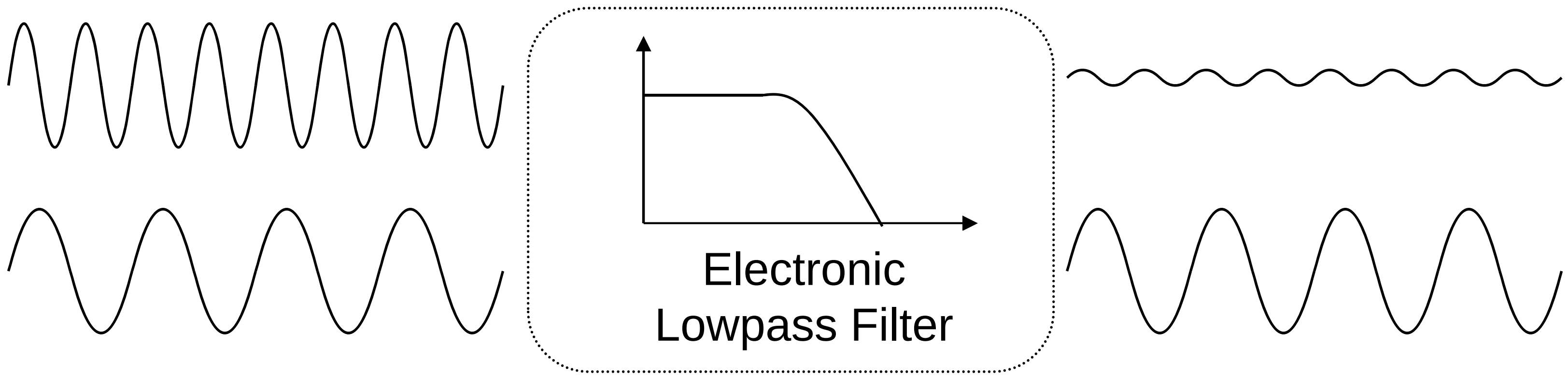
Filtering consist of the following factors:
1)Filtering criteria
2)Filtering action



The membrane behaves like a filter which allows small particle to get through and block out the big particle

Selection Criteria	Action
Size	Small size goes through

What is Electronic Filter?



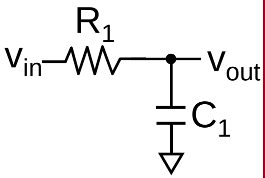
The electronic lowpass filter only allows low frequency signal to go through and suppress the high frequency signal

Selection Criteria	Action
Frequency	Pass low frequency Reject high frequency

Simple Lowpass

Passive 1st Order Lowpass Filter

Simple resistor divider



$$\boxed{H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}+R_1}=\frac{1}{j\omega C_1 R_1+1}}$$

- At low frequency ($\omega\rightarrow 0$), ignore 1st term of denominator ($j\omega C_1R_1$)$\Rightarrow H(j\omega)\approx 1\Rightarrow$Unity gain
- At high frequency ($\omega\rightarrow\infty$), ignore 2nd term of denominator (1)$\Rightarrow H(j\omega)\approx 1/(j\omega C_1R_1)\Rightarrow$Gain reduces with increasing frequency

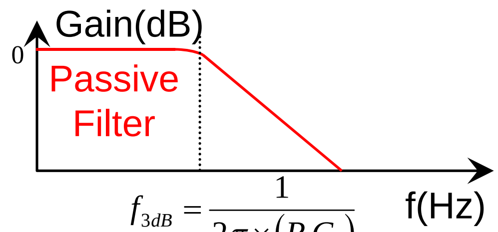
$$\boxed{\left|H(j\omega_{3dB})\right|=\frac{1}{\sqrt{2}}}$$

Interested in finding frequency (ω_{3dB}) where the power drop by half (Power is square of voltage)

$$\Rightarrow \frac{1}{\sqrt{(\omega_{3dB}R_1C_1)^2+1}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega_{3dB}=\frac{1}{R_1C_1} \quad or \quad f_{3dB}=\frac{1}{2\pi R_1C_1}$$

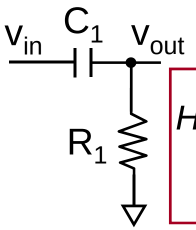
When the gain reduced by √2, is equivalent to reduce by 3dB



$$f_{3dB}=\frac{1}{2\pi\times(R_1C_1)}$$

Simple Highpass

Passive 1st Order Highpass Filter



Simple resistor divider

$$\boxed{H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{R_1}{\frac{1}{j\omega C_1}+R_1}=\frac{j\omega C_1R_1}{j\omega C_1R_1+1}=\frac{j\omega}{j\omega+\frac{1}{C_1R_1}}}$$

- At low frequency ($\omega\rightarrow 0$), ignore 1st term of denominator $(j\omega)\Rightarrow H(j\omega)\approx j\omega C_1R_1\Rightarrow$ Gain increases with increasing frequency
- At high frequency ($\omega\rightarrow\infty$), ignore 2nd term of denominator $(1/(C_1R_1))$ $\Rightarrow H(j\omega)\approx 1\Rightarrow$ Unity gain

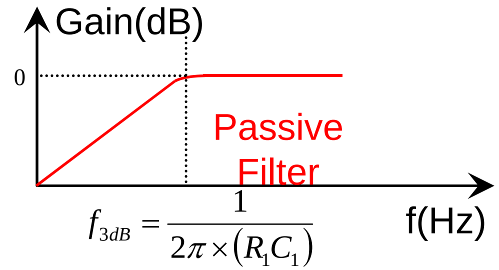
$$\boxed{\left|H(j\omega_{3dB})\right|=\frac{1}{\sqrt{2}}}$$

Interested in finding frequency (ω_{3dB}) where the power drop by half (Power is square of voltage)

$$\Rightarrow \frac{1}{\sqrt{\left(\frac{1}{\omega_{3dB}R_1C_1}\right)^2+1}}=\frac{1}{\sqrt{2}}$$

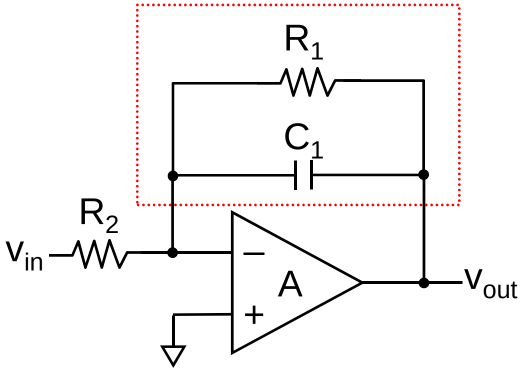
$$\Rightarrow \omega_{3dB}=\frac{1}{R_1C_1}\quad or\quad f_{3dB}=\frac{1}{2\pi R_1C_1}$$

When the gain reduced by $\sqrt{2}$, is equivalent to reduce by 3dB

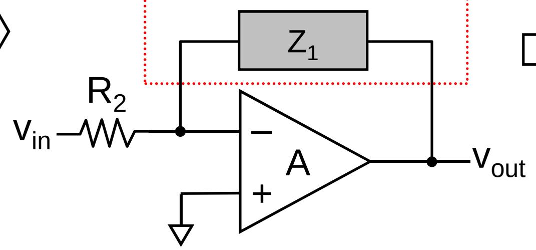


First Order Lowpass

Active 1st order Lowpass Filter



$$Z_1 = \left(\frac{1}{j\omega C_1} // R_1\right)$$

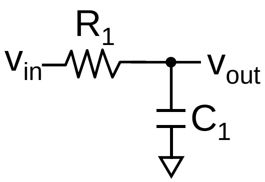


Equation (1) from slide 8-6

$$H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{Z_1}{R_2} = -\frac{\left(\frac{1}{j\omega C_1} // R_1\right)}{R_2}$$

$$= -\frac{\dfrac{1}{j\omega C_1 + 1/R_1}}{R_2} = -\underbrace{\frac{R_1}{R_2}}_{\text{Gain term}} \times \frac{1}{j\omega C_1 R_1 + 1}$$

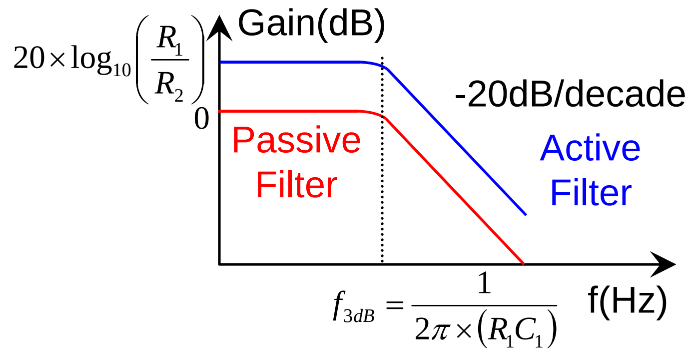
Passive 1st Order Lowpass Filter



Simple resistor divider

$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{1}{j\omega C_1 R_1 + 1}$$

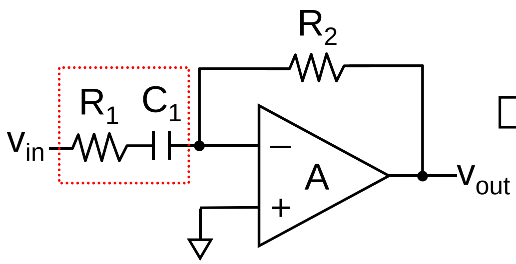
Unity gain



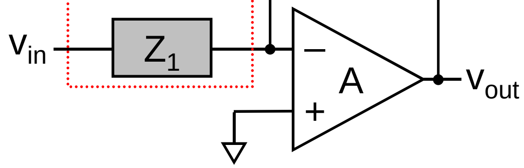
- Active 1st order lowpass has gain of R_1/R_2 versus gain of 1 for passive 1st order lowpass

First Order Highpass

Active 1st order Highpass Filter



$$Z_1 = \frac{1}{j\omega C_1} + R_1$$

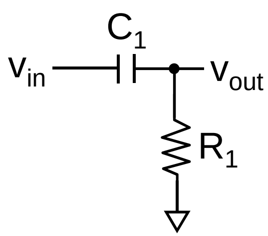


Equation (1) from slide 8-6

$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{R_2}{Z_1} = -\frac{R_2}{\frac{1}{j\omega C_1} + R_1}}$$

$$= -\frac{j\omega C_1 R_2}{j\omega R_1 C_1 + 1} = \underbrace{-\frac{R_2}{R_1}}_{\text{Gain term}} \times \frac{j\omega}{j\omega + \frac{1}{R_1 C_1}}$$

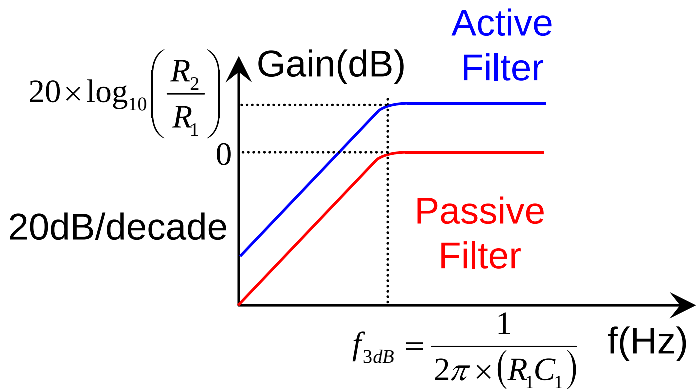
Passive 1st Order Highpass Filter



Simple resistor divider

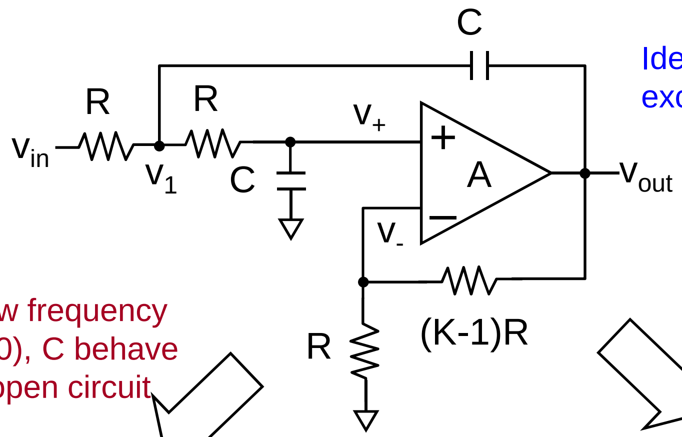
$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{j\omega}{j\omega + \frac{1}{R_1 C_1}}}$$

Unity gain



- Active 1st order highpass has gain of R_2/R_1 versus gain of 1 for passive 1st order highpass

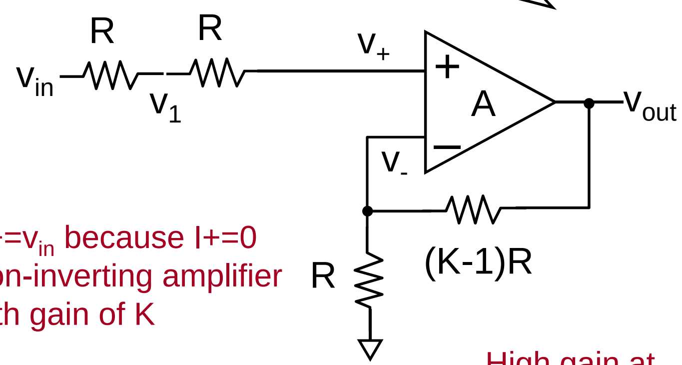
Simplified Sallen-Key Lowpass Filter



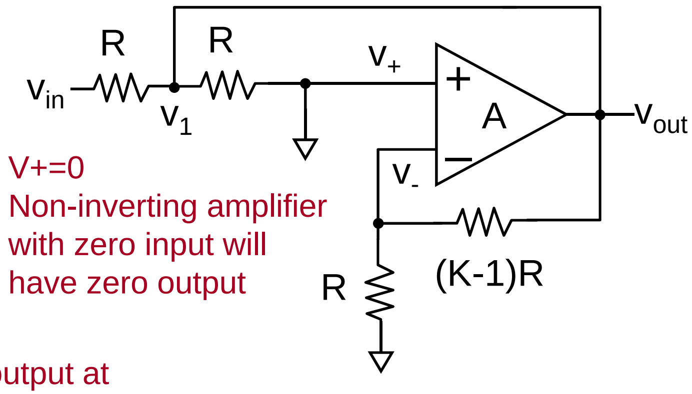
Identical R and C except for (K-1)R

At low frequency ($\omega \rightarrow 0$), C behave like open circuit

At high frequency ($\omega \rightarrow \infty$), C behave like short circuit



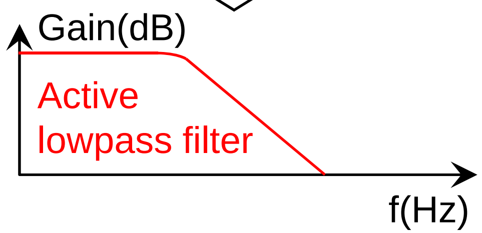
- V+=v_{in} because I+=0
- Non-inverting amplifier with gain of K



- V+=0
- Non-inverting amplifier with zero input will have zero output

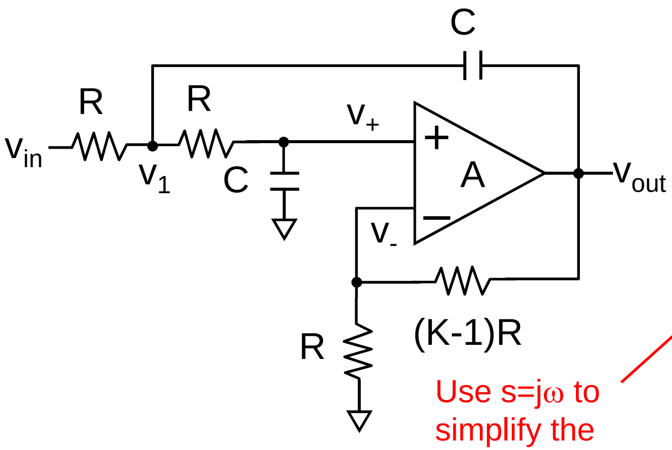
High gain at low frequency

Zero output at high frequency



Sallen-Key Lowpass Filter

Simplified Design Equation



$$H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{\frac{K}{(RC)^2}}{(j\omega)^2+(j\omega)\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\frac{H_o\omega_o^2}{(j\omega)^2+\frac{\omega_o}{Q}(j\omega)+\omega_o^2}$$

Filter function

Use s=jω to simplify the notation

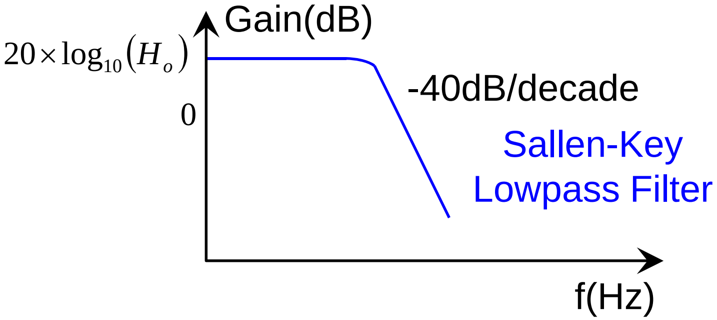
$$=\frac{\frac{K}{(RC)^2}}{s^2+s\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\frac{H_o\omega_o^2}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}$$

$$\Rightarrow \omega_o=\frac{1}{RC} \qquad \frac{1}{Q}=3-K \qquad H_o=K$$

Important design equation

Filter function is usually given by system designer, circuit designer only concerns about how to implement the desired filter function through opamp.

Why using 2nd order lowpass filter?



- At low frequency, ignore the 1st and 2nd terms of denominator of H(jω)⇒$H(j\omega)\approx H_o$
- At high frequency, ignore the 2nd and 3rd terms of denominator of H(jω)⇒$H(j\omega)\approx H_o\omega_o^2/(j\omega)^2$
- The gain reduces with ω^2⇒sharper roll-off of -40dB/decade compared to 1st order lowpass filter
- It can suppress unwanted high frequency component more compared to 1st order lowpass filter

Lowpass Design Example

Design a second order lowpass filter that has Q of 0.7071 and cut-off frequency (ω_o) at 1kHz

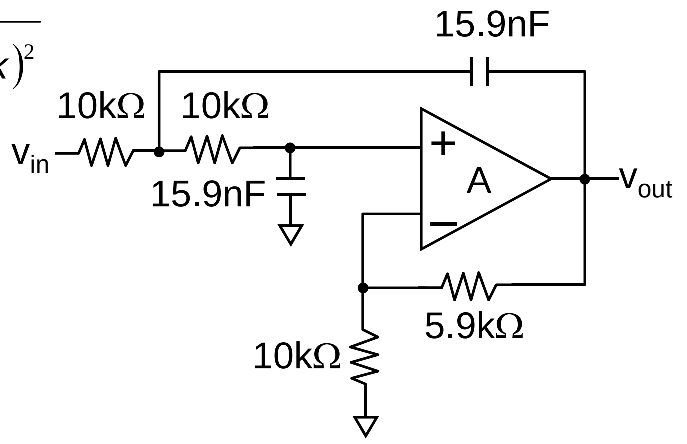
$$\frac{v_{out}}{v_{in}} = \frac{H_o\omega_o^2}{(j\omega)^2 + \frac{\omega_o}{Q}(j\omega) + \omega_o^2} = \frac{H_o(2\pi\times 1k)^2}{(j\omega)^2 + \frac{2\pi\times 1k}{0.7071}(j\omega) + (2\pi\times 1k)^2}$$

Using simplified design equation

$$\frac{1}{Q} = 3 - K = \frac{1}{0.7071} \Rightarrow K = 1.59 \qquad H_o = K = 1.59$$

$$\omega_o = \frac{1}{RC} = 2\pi\times 1k$$

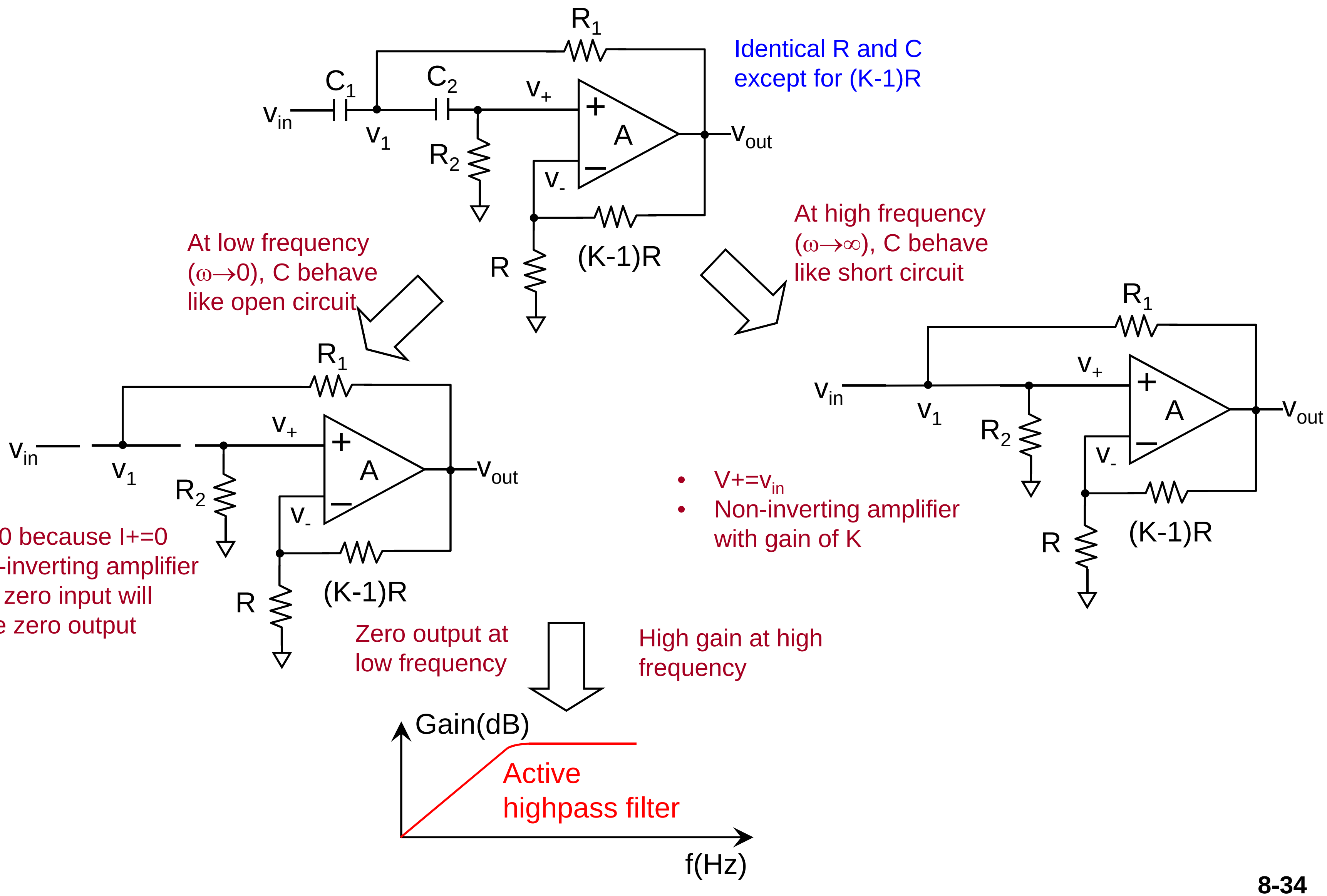
Choose $R = 10k\Omega \Rightarrow C = 15.9nF$



Important Notes on Sallen-Key Lowpass Filter :

1) Be able to identify the circuit topology as Sallen-Key Lowpass Filter
2) Be able to locate the design equations for Sallen-Key Lowpass Filter
3) Be able to use the design equations to design the desired filter

Simplified Sallen-Key Highpass Filter

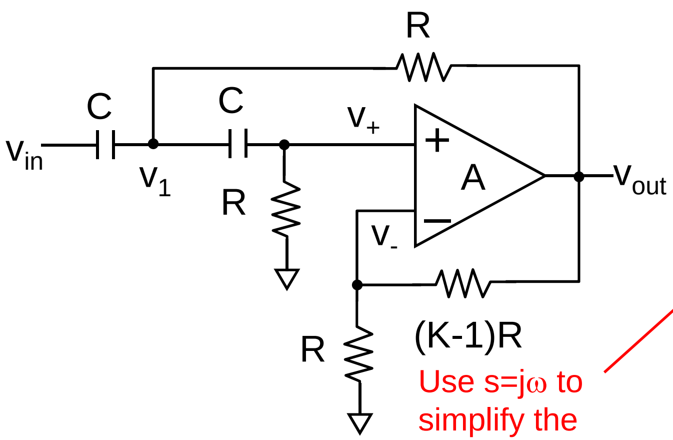


- V+=0 because I+=0
- Non-inverting amplifier with zero input will have zero output

- V+=v_{in}
- Non-inverting amplifier with gain of K

Sallen-Key Highpass Filter

Simplified Design Equation



Use s=jω to simplify the notation

Filter function

$$H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{K(j\omega)^2}{(j\omega)^2+(j\omega)\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\boxed{\frac{H_o(j\omega)^2}{(j\omega)^2+\frac{\omega_o}{Q}(j\omega)+\omega_o^2}}$$

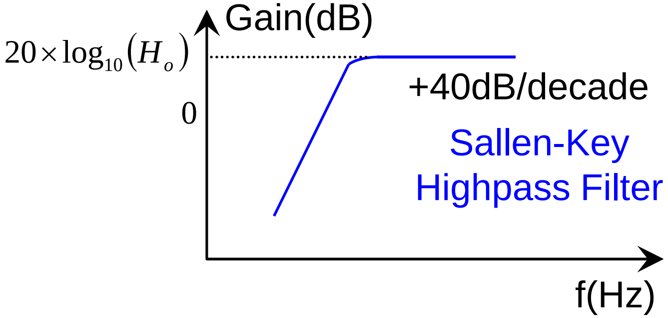
$$\boxed{=\frac{Ks^2}{s^2+s\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\frac{H_os^2}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}}$$

Filter function is usually given by system designer, circuit designer only concerns about how to implement the desired filter function through opamp.

$$\Rightarrow \boxed{\omega_o=\frac{1}{RC} \qquad \frac{1}{Q}=3-K \qquad H_o=K}$$

Important design equation

Why using 2nd order lowpass filter?



- At low frequency, ignore the 1st and 2nd terms of denominator of H(jω)⇒H(jω)≈$H_o(j\omega)^2/\omega_o^2$
- At high frequency, ignore the 2nd and 3rd terms of denominator of H(jω)⇒H(jω)≈H_o
- The gain increases with ω^2⇒sharper roll-off of +40dB/decade compared to 1st order highpass filter
- It can suppress unwanted low frequency component more compared to 1st order highpass filter

Highpass Design Example

Design a second order highpass filter that has Q of 0.7071 and cut-off frequency (ω_o) at 5kHz

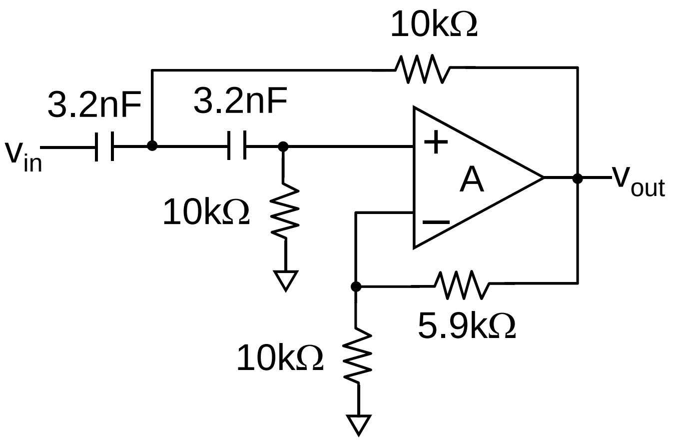
$$\frac{v_{out}}{v_{in}} = \frac{H_o s^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} = \frac{H_o s^2}{s^2 + \frac{2\pi\times 5k}{0.7071}s + (2\pi\times 5k)^2}$$

Using simplified design equation

$$\frac{1}{Q} = 3 - K = \frac{1}{0.7071} \Rightarrow K = 1.59 \qquad H_o = K = 1.59$$

$$\omega_o = \frac{1}{RC} = 2\pi\times 5k$$

$$Choose \quad R = 10k\Omega \Rightarrow C = 3.2nF$$



Important Notes on Sallen-Key Highpass Filter :

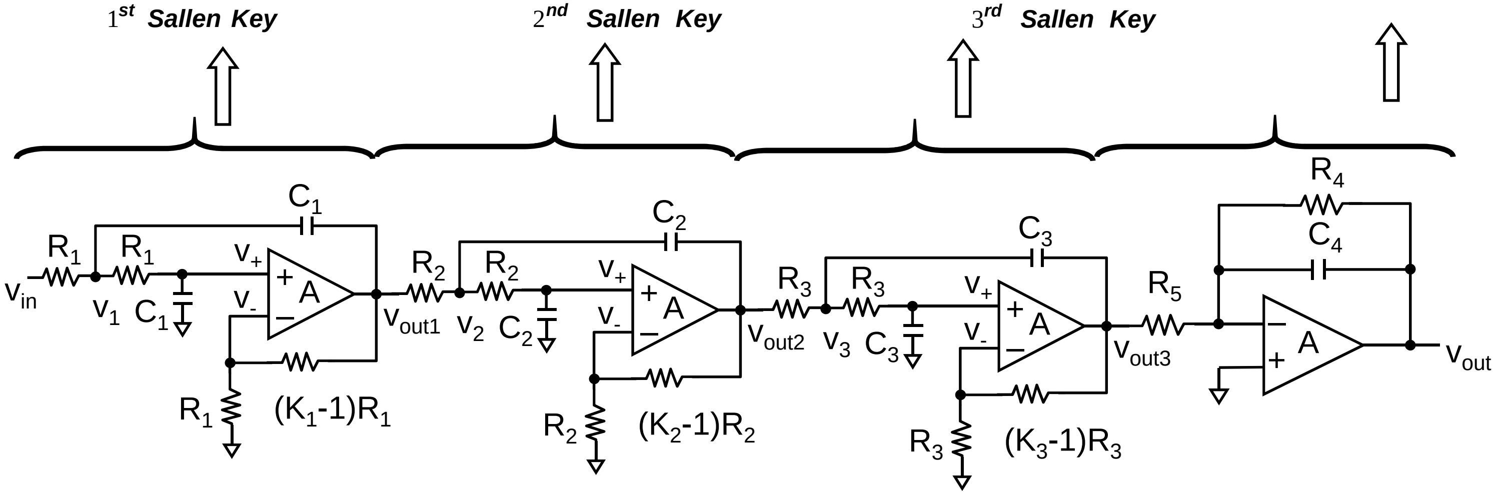
1) Be able to identify the circuit topology as Sallen-Key Highpass Filter
2) Be able to locate the design equations for Sallen-Key Highpass Filter
3) Be able to use the design equations to design the desired filter

How to Build Higher Order Filter?

7th Order Lowpass Filter:

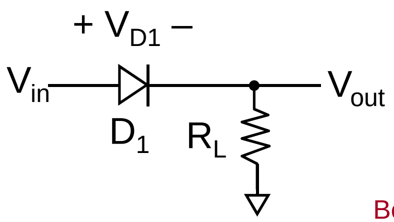
$$H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{H_o}{a_7(j\omega)^7+a_6(j\omega)^6+a_5(j\omega)^5+a_4(j\omega)^4+a_3(j\omega)^3+a_2(j\omega)^2+a_1(j\omega)^1+1}$$

$$=\underbrace{\frac{H_{o1}(\omega_{o1}^2)}{(j\omega)^2+\frac{\omega_{o1}}{Q_1}(j\omega)+\omega_{o1}^2}}_{1^{st}\ \text{Sallen Key}}\times\underbrace{\frac{H_{o2}(\omega_{o2}^2)}{(j\omega)^2+\frac{\omega_{o2}}{Q_2}(j\omega)+\omega_{o2}^2}}_{2^{nd}\ \text{Sallen Key}}\times\underbrace{\frac{H_{o3}(\omega_{o3}^2)}{(j\omega)^2+\frac{\omega_{o3}}{Q_3}(j\omega)+\omega_{o3}^2}}_{3^{rd}\ \text{Sallen Key}}\times\underbrace{\frac{H_{o4}(\omega_{o4})}{j\omega+\omega_{o4}}}_{4^{th}\ \text{First Order}}$$



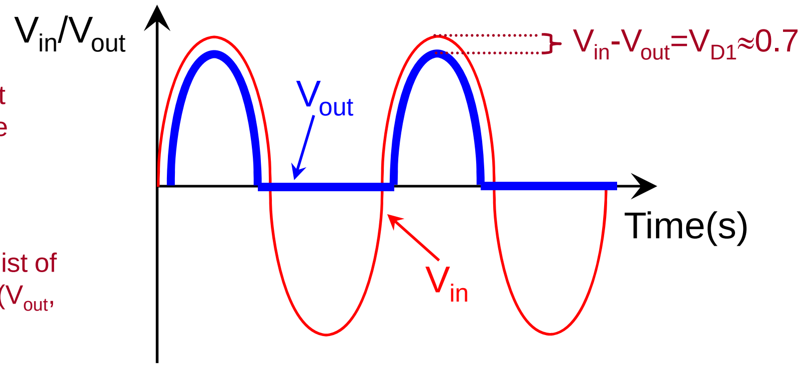
Superdiode

Conventional Rectifier

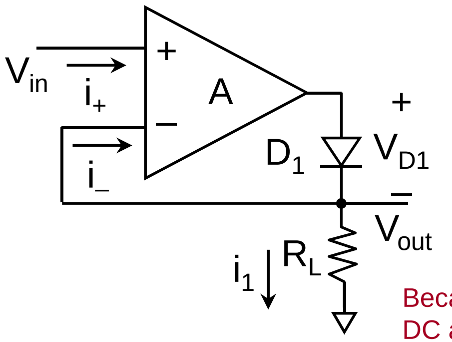


The rectifier circuit only rectifies input when the input (V_{in}) is bigger than the diode voltage (V_{D1}) by 0.7V

Because the signals we are dealing with consist of DC and AC components, we use upper case (V_{out}, V_{in}, V_{D1}) rather than lower case (v_{out}, v_{in}, v_{D1})

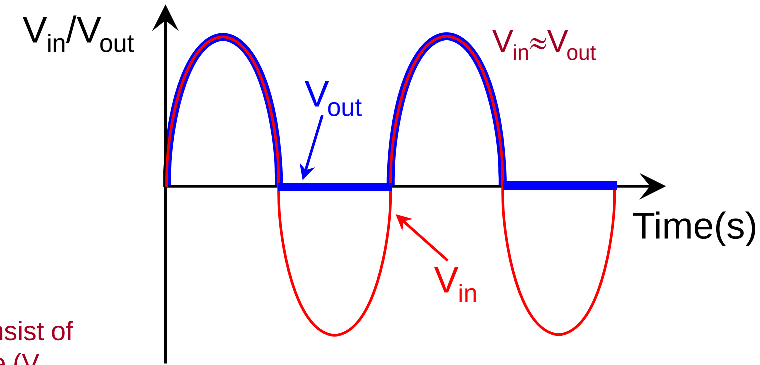


Superdiode Rectifier



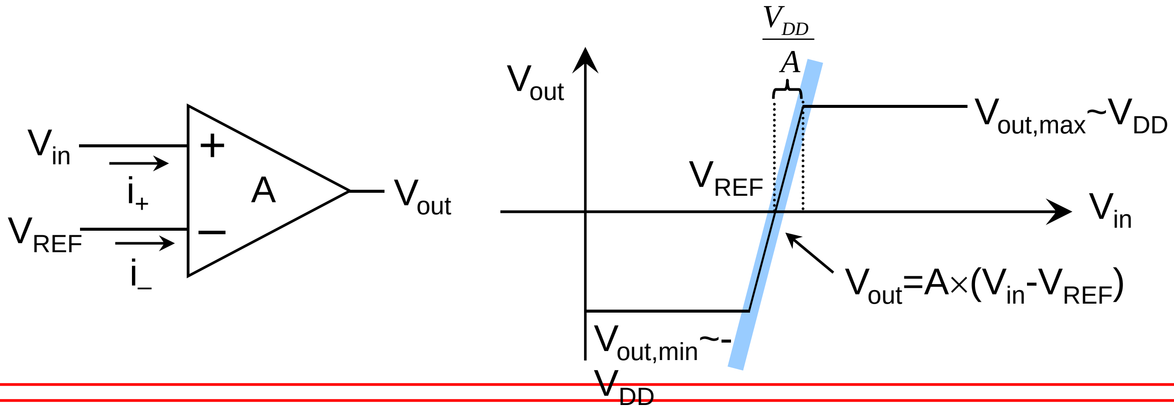
V_{in}>0 :
V_{D1}~0.7⇒Diode starts conducting
$V_+–V_–=(V_{out}+V_{D1})/'A\approx0$ if $A\to\infty$
$\Rightarrow V_{out}\approx V_{in}$
V_{in}<0 :
V_{D1}<<0⇒Diode stops conducting
$i_–=0$ and $i_{D1}\approx0\Rightarrow i_1=0$
$\Rightarrow V_{out}\approx0$

Because the signals we are dealing with consist of DC and AC components, we use upper case (V_{out}, V_{in}, V_{D1}) rather than lower case (v_{out}, v_{in}, v_{D1})

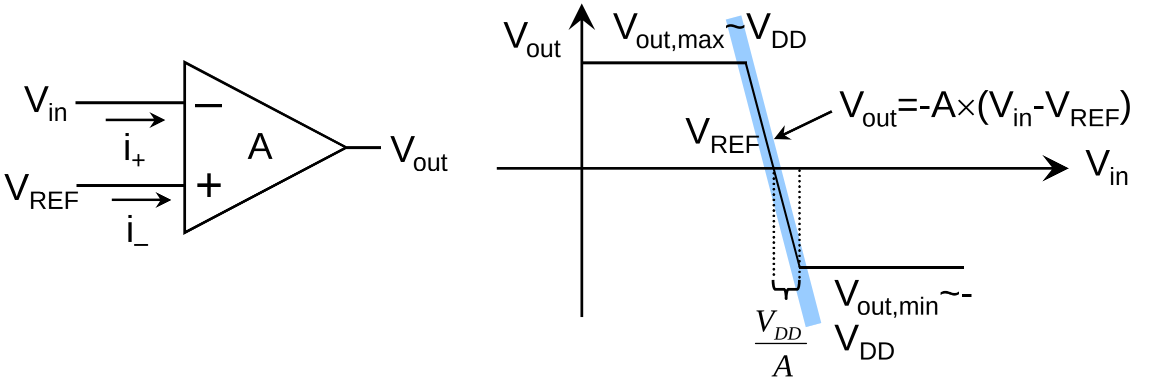


- Ideal rectifier circuit should rectify input when the input (V_{in}) is bigger than zero
- Useful for AM demodulator when signal is small

Opamp Used as Comparator



Ideally, $V_{out}=A\times(V_{in}-V_{REF})$. Due to the huge gain and limited supply voltage, the output grows rapidly and saturate at positive and negative supply voltage. If $A\rightarrow\infty$, the transfer characteristic behave like a step function center at V_{REF}.



Ideally, $V_{out}=-A\times(V_{in}-V_{REF})$. Due to the huge gain and limited supply voltage, the output grows rapidly and saturate at positive and negative supply voltage. If $A\rightarrow\infty$, the transfer characteristic behave like an inverted step function center at V_{REF}.

As an example, opamp with gain of 10000 and supply voltage of ±10V, when $|V_{in}-V_{ref}|>1mV$, V_{out} would saturate to ±10V.

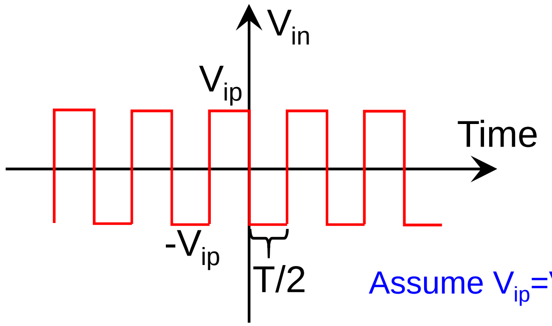
Important Notes :

When opamp operates in open loop, **virtual short ($v_+\approx v_-$) no longer applies.**

Applications Built using Opamp

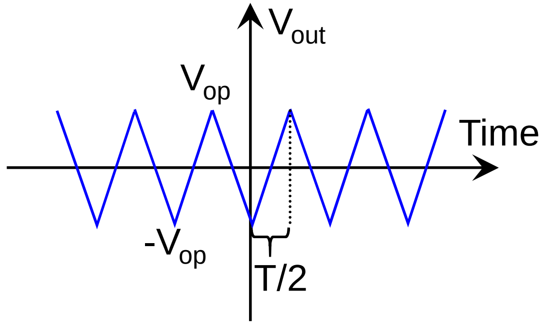
- Now we have a library of opamp-based circuits
- We can realize more complicated functions by combining a number of opamp circuits
- This will form sub-system
- The concept will be illustrated in the following few examples

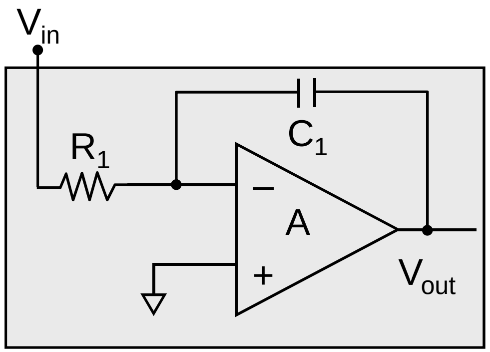
Triangular Wave



Integration of square wave results in triangle wave. Intuitively, integration is finding the area under square wave.

Assume $V_{ip}=V_{DD}/2$





$$V_{out}(t) = -\frac{1}{R_1C_1}\int V_{in}(t)\,dt$$

For $0 < t < \frac{T}{2}$:

$$V_{out}(t)\Big|_0^{\frac{T}{2}} = -\frac{1}{R_1C_1}\int_0^{\frac{T}{2}}(-V_{ip})\,dt$$

$$V_{out}\left(\frac{T}{2}\right) - V_{out}(0) = -\frac{1}{R_1C_1}\int_0^{\frac{T}{2}}\left(-\frac{V_{DD}}{2}\right)dt$$

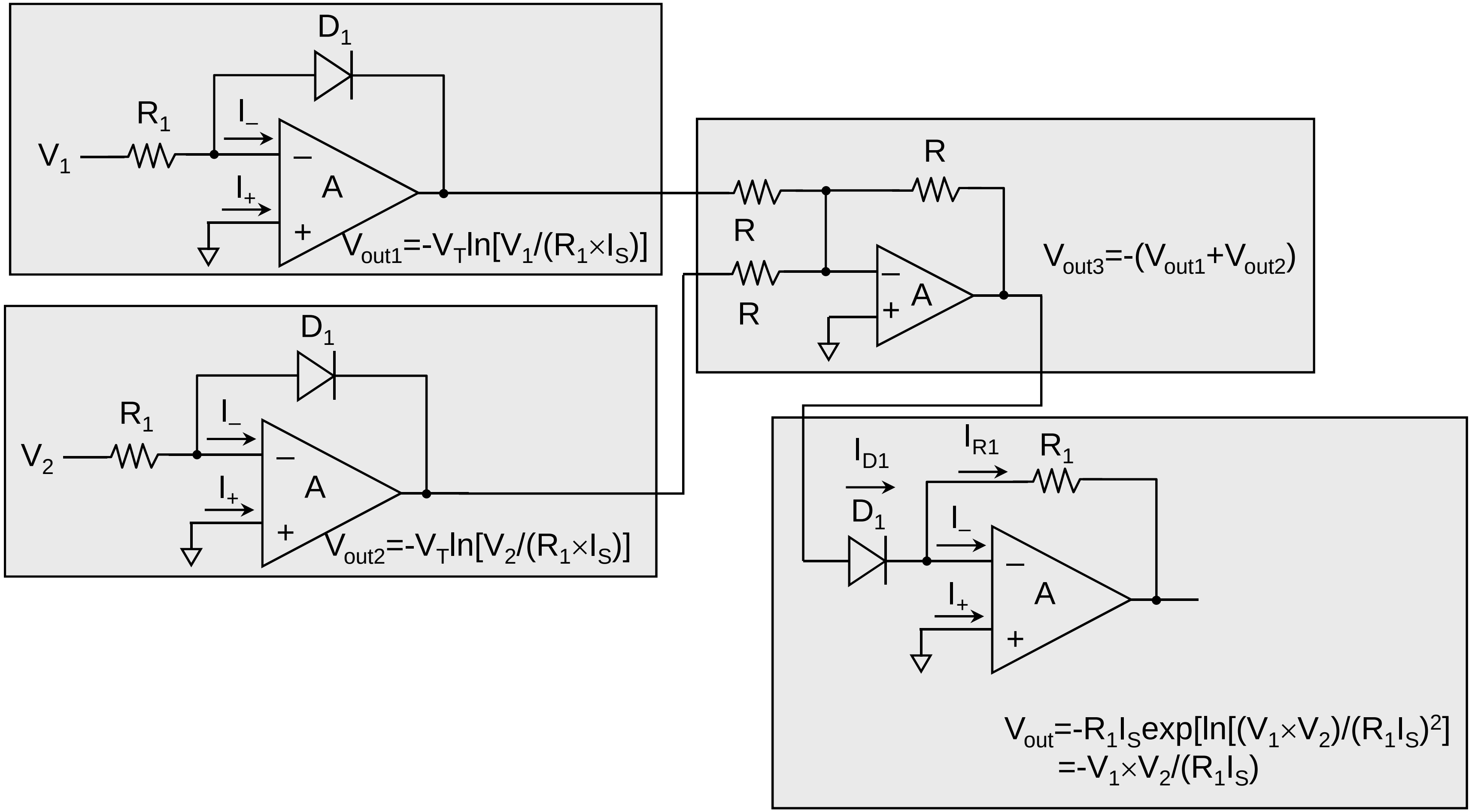
$$2V_{op} = -\frac{1}{R_1C_1}\left[-\frac{V_{DD}}{2}t\right]_{t=0}^{t=\frac{T}{2}}$$

$$2V_{op} = \frac{V_{DD}T}{4R_1C_1}$$

Multiplier

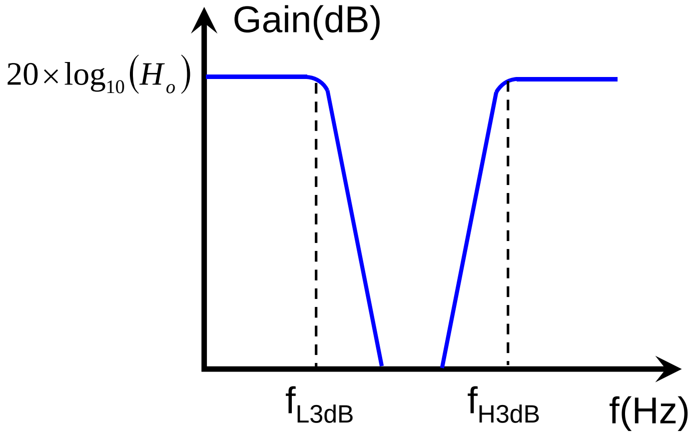
- How to create multiplier using existing opamp circuits covered so far, i.e. $V_{out}=kV_1\times V_2$?

  (Hints: Pre-university mathematics, $\log(A\times B)=\log(A)+\log(B)$ and $e^{\ln(X)}=X$)

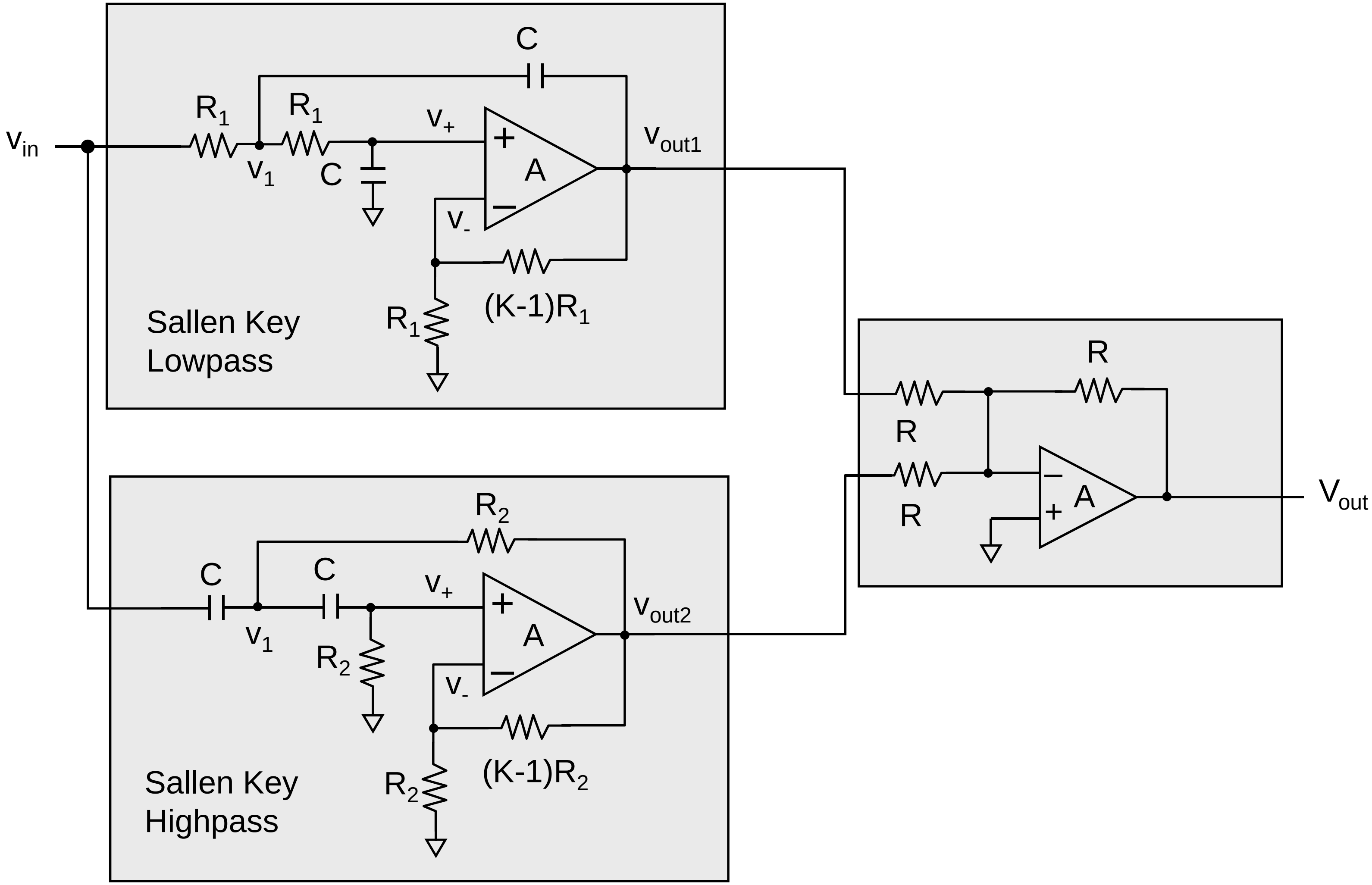


Bandstop Filter

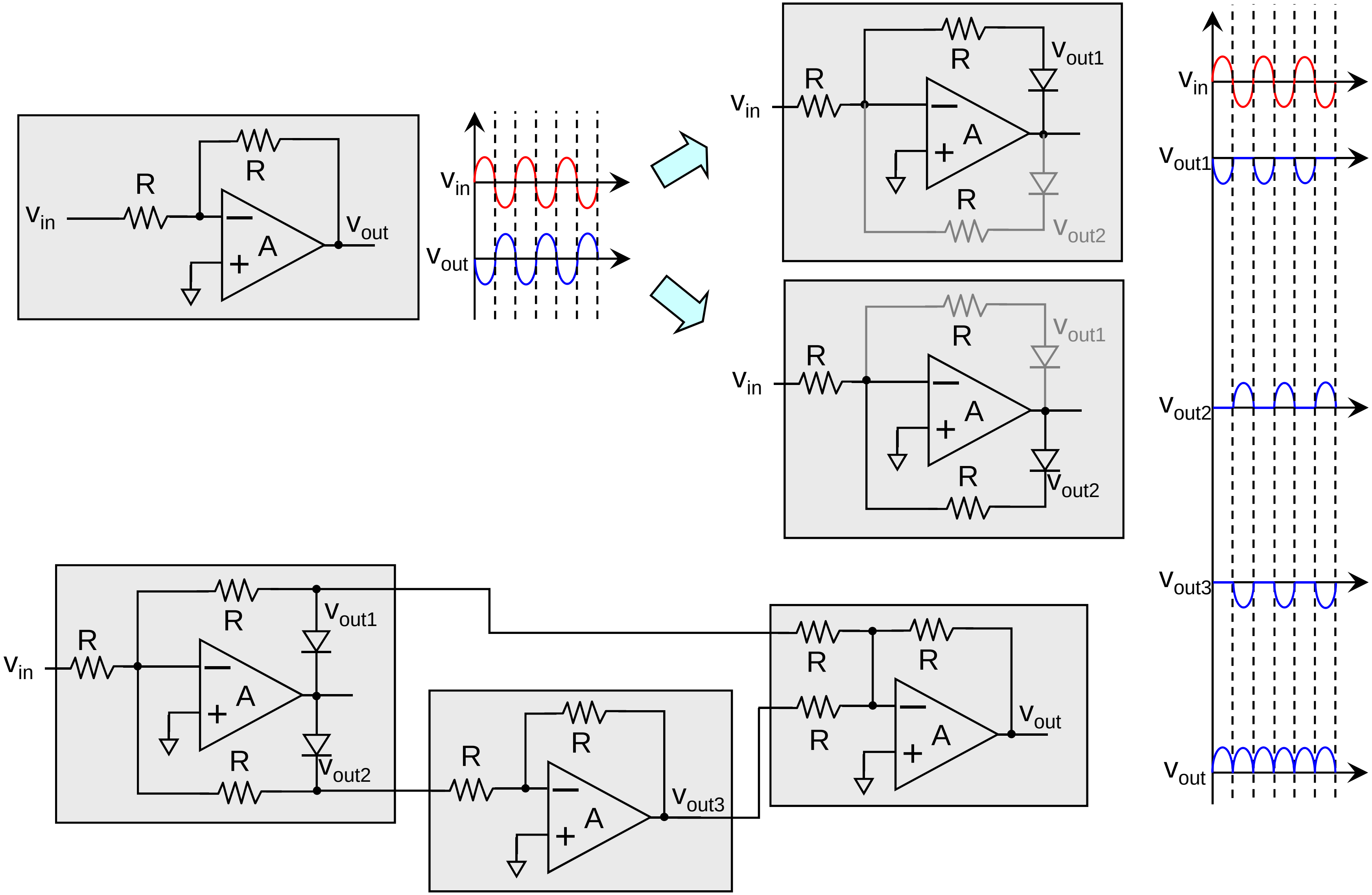
- How to create bandstop filter that has the following characteristic?
- What is the use of bandstop?
- Anti-jamming



Bandstop Filter



Full Wave Rectifier



Lecture Summary

- **Various opamp based circuit**
- **How to build sub-system using opamp based circuits**

Reading Assignment

- **Reading: Reference Book (Sedra & Smith)**
  Chapter 5, pp. 473 – 543. (Opamp)