EE2021
Devices and Circuits

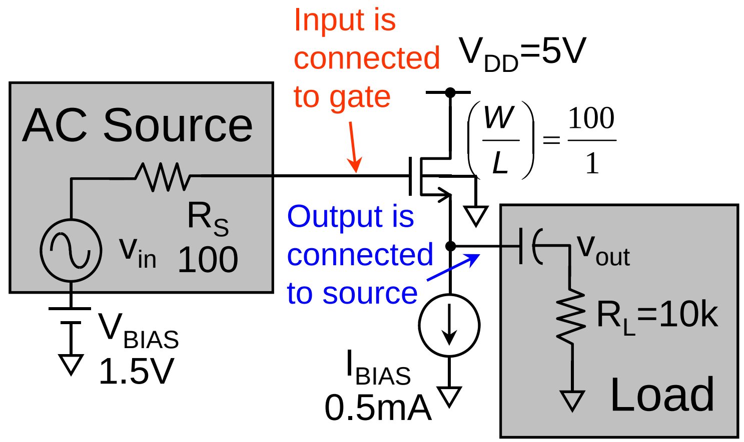
Single Transistor Amplifier – CC/CD, CE/CS with Degeneration, Current Mirror

Lecture Outline

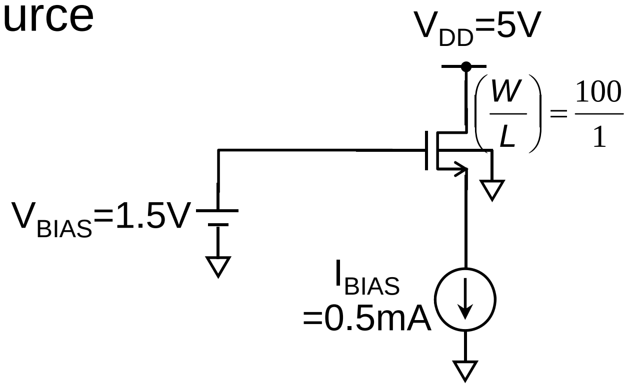
- **Single Transistor Amplifiers,**
  CC/CD, CE/CS with degeneration
- **Current Mirror**

Common Drain (CD)

$\mu_n C_{ox}$=80μA/V^2
V_{THN}=0.7V
λ_n=0.02V^{-1}



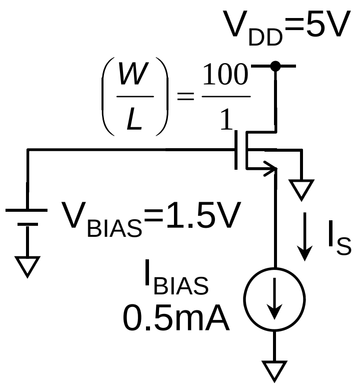
For DC analysis, remove AC source and load



- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Gate, output connected to Source, Drain connected to neither input nor output $\Rightarrow$ Common Drain (CD)

DC Analysis

- Remove source/load section when doing DC analysis



$\mu_n C_{ox}$=80μA/V^2
V_{THN}=0.7V
λ_n=0.02V^{-1}

Determine DC biasing

$$I_D = I_S = I_{BIAS} = 0.5mA$$

Determine AC
small signal parameter

$$\left(\frac{W}{L}\right)=\frac{100}{1}$$

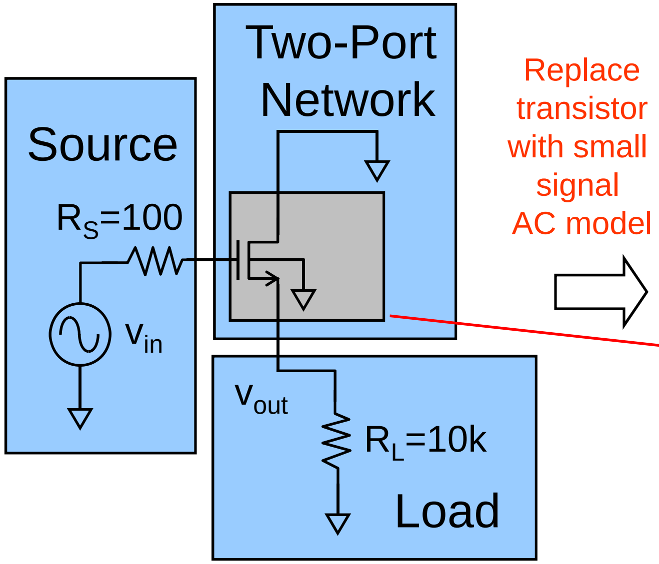
$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D} = 2.83mA/V$$

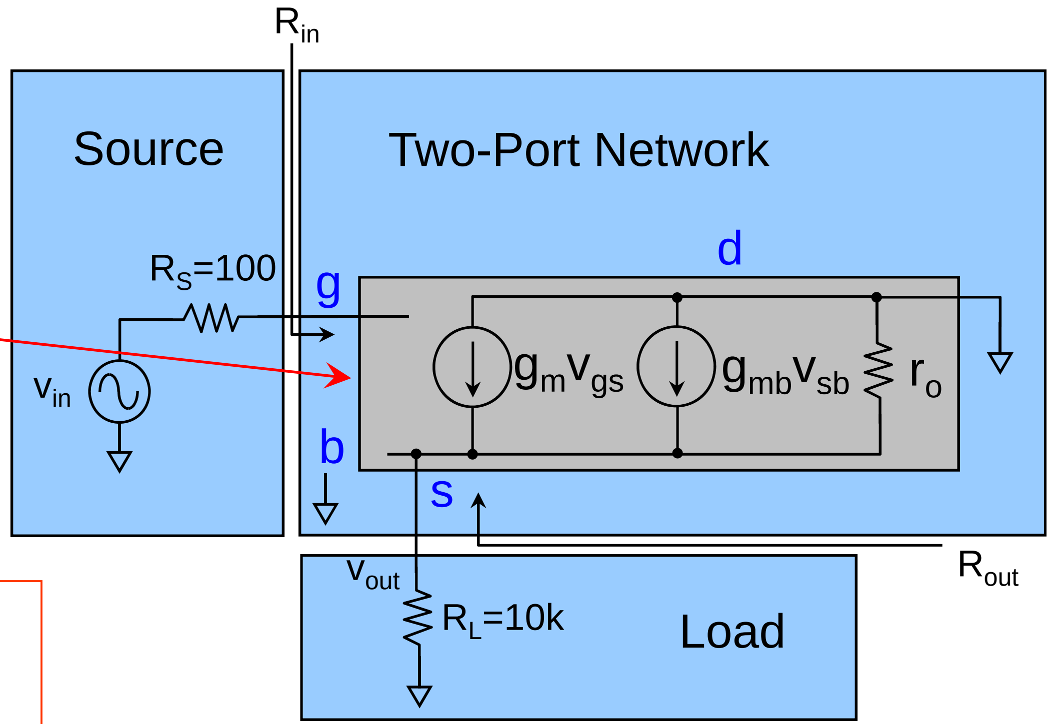
$$g_{mb} \approx -\frac{g_m}{4} = -0.71mA/V$$

$$r_i = \infty$$

$$r_o = \frac{1}{\lambda_n I_D} = 100k\Omega$$

AC Analysis





Step 1 :

Identify source/load

Step 2 :

Group the remaining component into two-port network

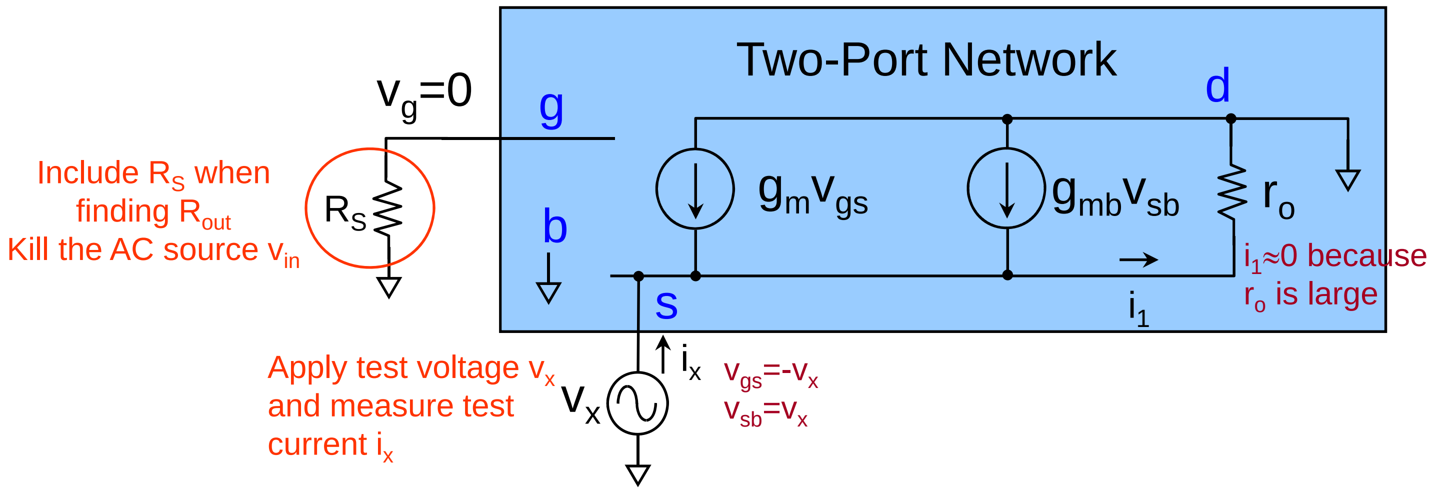
Step 3 :

Replace transistor with small signal AC model

Interested in finding out R_{in}, R_{out} and A for the two ports network (Voltage Amplifier)

$$R_{in} = \infty$$

CD – Two-Port Network (R_{out})

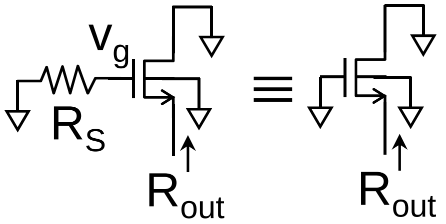


$$i_x \approx -g_m v_{gs} - g_{mb} v_{sb} = (g_m - g_{mb}) v_x$$

$$\Rightarrow R_{out} \approx \frac{1}{g_m - g_{mb}}$$

CD – Two-Port Network (R_{out})

Since no current flow through R_S, v_g is same as AC ground



$$R_{out} \approx \frac{1}{g_m - g_{mb}}$$

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the source (R_{out}) is directly given by the formula. **No need to rederive.**

Example :

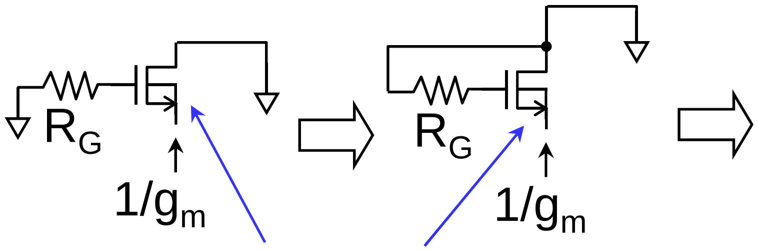
$$R_S = 100 \quad r_o = 100k$$

$$g_m = 2.83m \quad g_{mb} = -0.71m$$

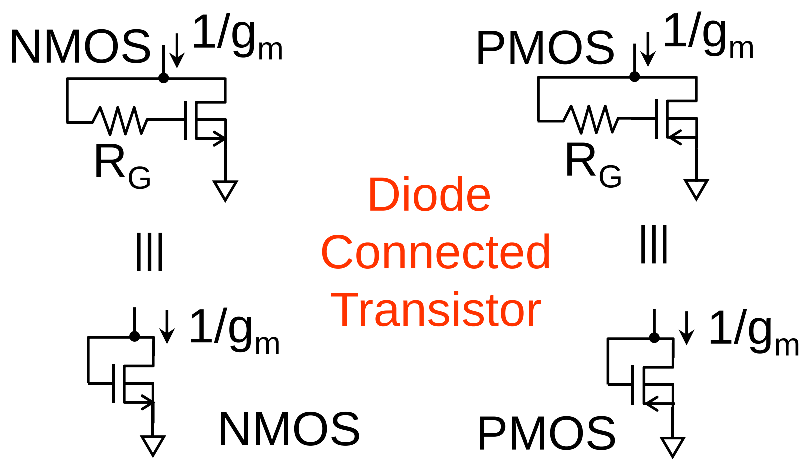
$$\Rightarrow R_{out} \approx 283$$

- The output impedance does not depend on R_S

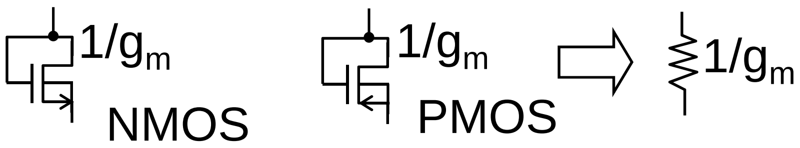
MOS –Equivalent Resistance



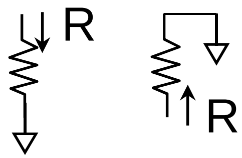
When body and source connected together, i.e. v_{sb}=0
⇒there is no body effect
⇒ $g_{mb}v_{sb}$=0 Eliminate g_{mb}



Diode Connected Transistor



Since no current flow through R_G, there is no voltage drop across R_G, R_G is effectively short circuit

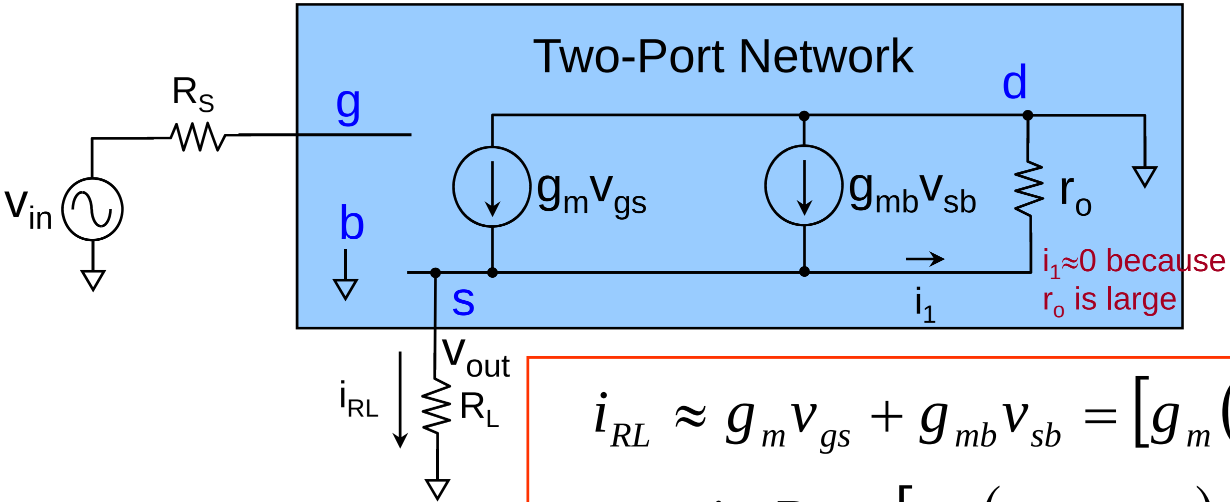


A resistor, whether you look from top or bottom, always is a resistor with resistance R

Only APPLICABLE for AC analysis

- When the drain and gate are connected together, the transistor is called diode connected transistor
- Diode connected transistor has equivalent resistance of $1/g_m$ if there is no body effect

CD – Two-Port Network (A_V)



$$v_{gs} = v_{in} - v_{out}$$
$$v_{sb} = v_{out}$$

$$i_{RL} \approx g_m v_{gs} + g_{mb} v_{sb} = \left[g_m (v_{in} - v_{out}) + g_{mb} v_{out}\right]$$

$$v_{out} = i_{RL} R_L \approx \left[g_m (v_{in} - v_{out}) + g_{mb} v_{out}\right] R_L$$

$$\Rightarrow \left[1 + (g_m - g_{mb}) R_L\right] v_{out} = g_m R_L v_{in}$$

$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = \frac{g_m R_L}{1 + (g_m - g_{mb}) R_L} \approx \frac{g_m}{g_m - g_{mb}} \approx 0.8$$

If $(g_m - g_{mb}) R_L >> 1$

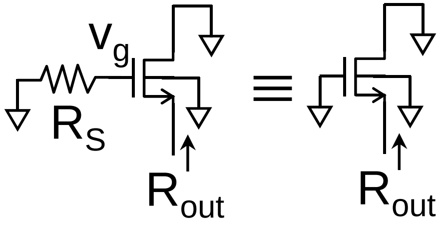
No body effect means source and body are connected together
$\Rightarrow$ g_{mb} can be eliminated in the equation

- Without body effect, $g_{mb} v_{sb} = 0 \Rightarrow A_V \approx 1$

CD – Important Results

1

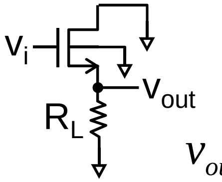
Since no current flow through R_S, v_g is same as AC ground



$$R_{out} \approx \frac{1}{g_m - g_{mb}}$$

2

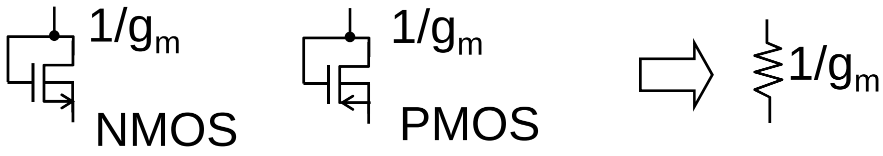
Relationship between v_{in} and v_{out} is defined as follows:



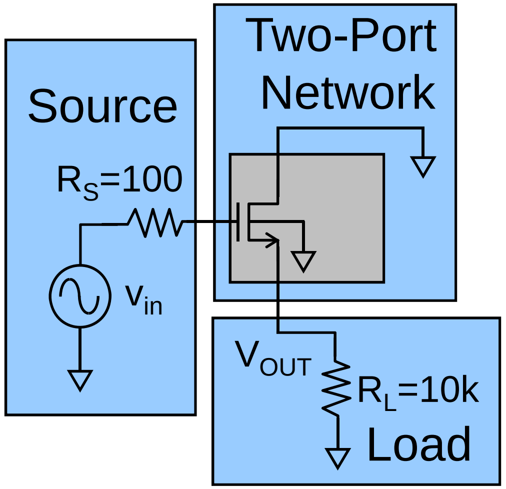
$$v_{out} = \frac{g_m R_L}{1 + (g_m - g_{mb}) R_L} v_i$$

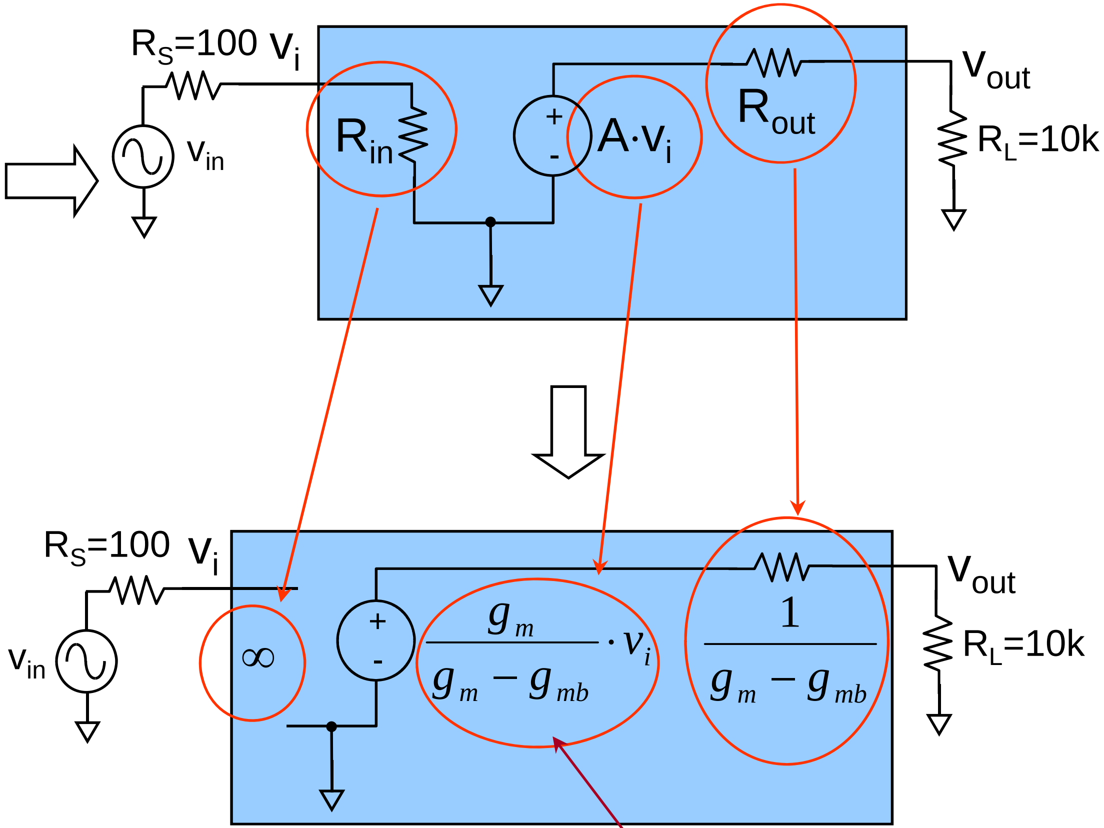
3

Diode Connected Transistor



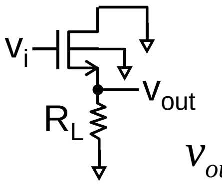
CD – Two-Port Network





Transform the CD amplifier into two-port voltage amplifier

Relationship between v_{in} and v_{out} is defined as follows:

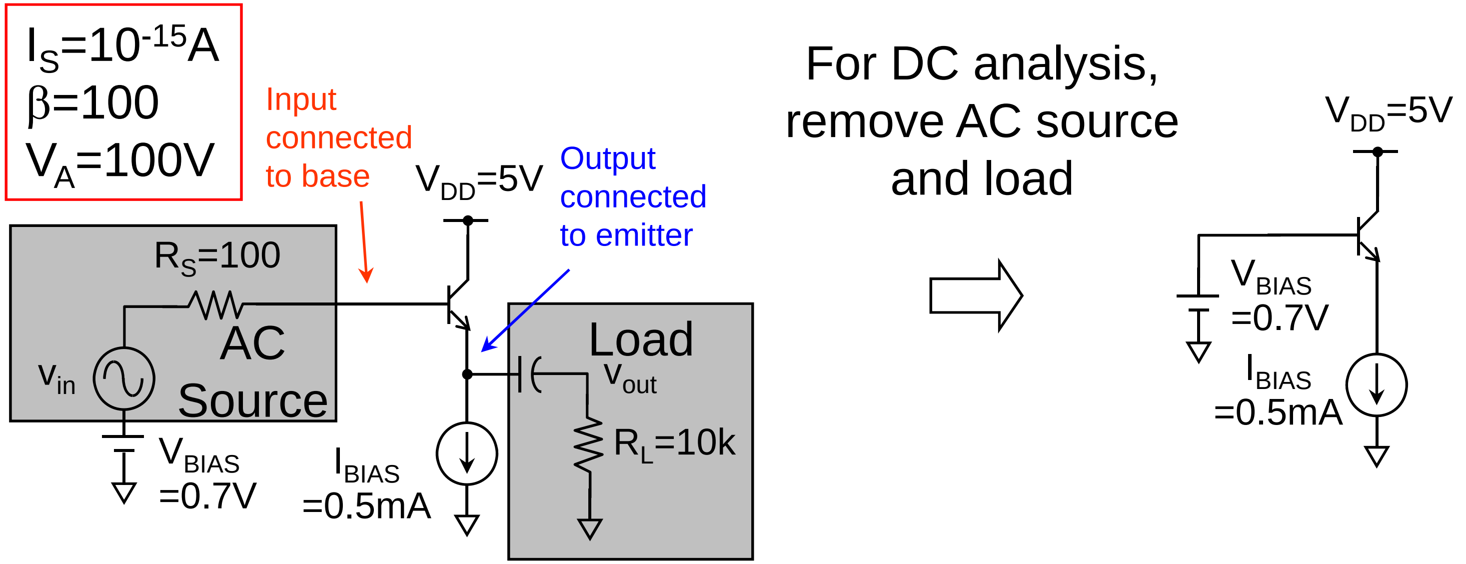


$$v_{out} = \frac{g_m R_L}{1 + (g_m - g_{mb}) R_L} v_i$$

It is simpler to remember A_v rather than A

A can be found based on the A_V.

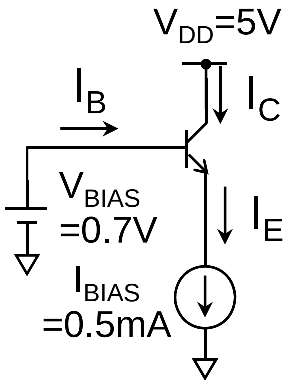
Common Collector (CC)



- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Base, output connected to Emitter, Collector connected to neither input nor output ⇒ Common Collector (CC)

DC Analysis (Self Reading)

- Remove source/load section when doing DC analysis



$I_S=10^{-15}A$
$\beta=100$
$V_A=100V$

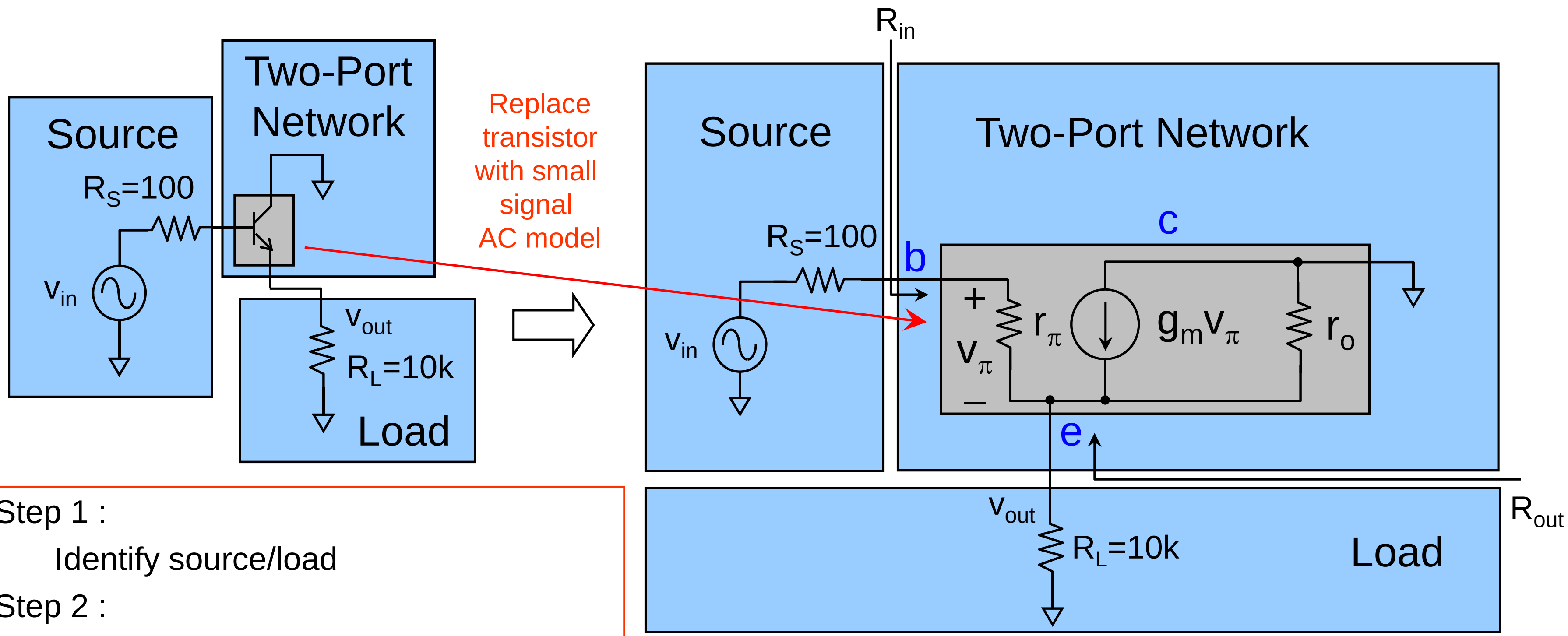
Determine DC biasing

$$V_T = 26mV$$
$$I_E = I_{BIAS} = 0.5mA$$
$$I_C = \frac{\beta}{\beta+1} I_E = 0.495mA$$
$$I_B = 4.95\mu A$$

Determine AC small signal parameter

$$g_m = \frac{I_C}{V_T} = 19mA/V$$
$$r_\pi = \frac{\beta}{g_m} = 5.26k\Omega$$
$$r_o = \frac{V_A}{I_C} = 202k\Omega$$

AC Analysis (Self Reading)



Step 1 :

Identify source/load

Step 2 :

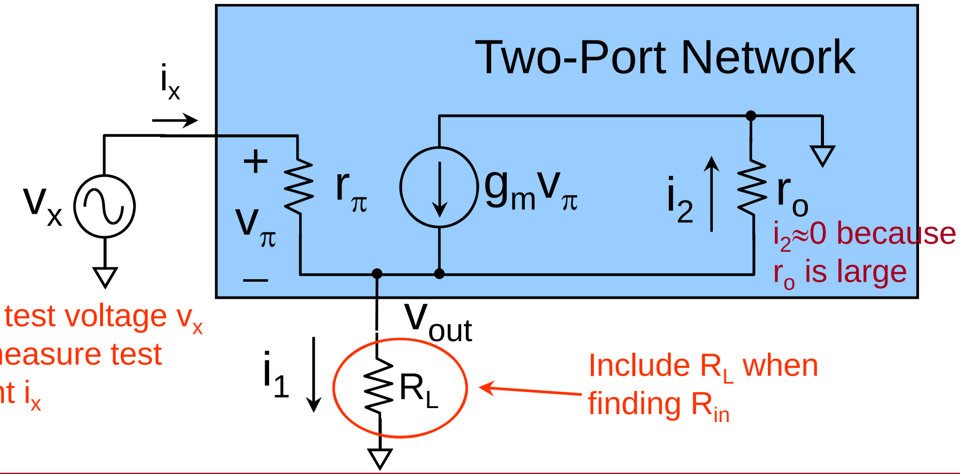
Group the remaining component into two-port network

Step 3 :

Replace transistor with small signal AC model

Interested in finding out R_{in}, R_{out} and A for the two ports network (Voltage Amplifier)

CC – Two-Port Network (R_{in}) (Self Reading)



Apply test voltage v_x and measure test current i_x

Include R_L when finding R_{in}

$$v_\pi = v_x - v_{out}$$

$$i_1 = \frac{v_{out}}{R_L}$$

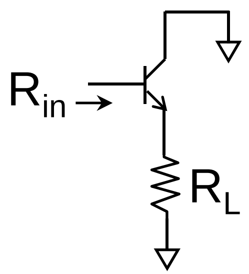
Eliminate v_{out} and keep v_x and i_x

$$\begin{cases} v_x - v_{out} = i_x r_\pi \\ i_x = i_1 + \cancel{i_2} - g_m v_\pi \\ \quad = \dfrac{v_{out}}{R_L} - g_m\left(v_x - v_{out}\right) \end{cases}$$

$$\Rightarrow i_x = \frac{v_x - i_x r_\pi}{R_L} - g_m i_x r_\pi$$

$$\begin{aligned} \Rightarrow R_{in} &= r_\pi + g_m r_\pi R_L + R_L \\ &= r_\pi + \left(\beta + 1\right) R_L \\ &\approx r_\pi \left(1 + g_m R_L\right) \end{aligned}$$

CC – Two-Port Network (R_{in}) (Self Reading)



$$R_{in} \approx r_\pi + (\beta+1)R_L$$
$$\approx r_\pi\left[1+\frac{(\beta+1)}{r_\pi}R_L\right]$$
$$\approx r_\pi\left[1+g_m R_L\right]$$

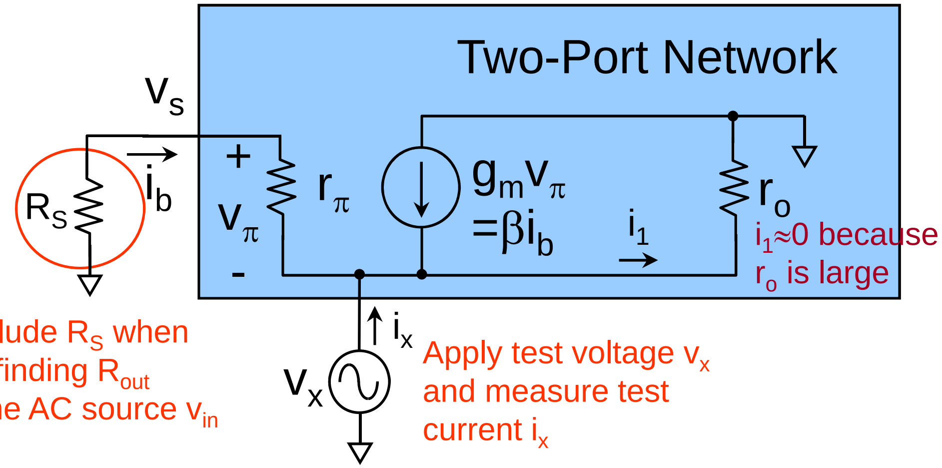
Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the base (R_{in}) is directly given by the formula. **No need to rederive.**

Example :

$$R_L = 10k \quad r_o = 202k$$
$$\Rightarrow R_{in} = r_\pi + (\beta+1)R_L$$
$$= 1.01M$$

- Emitter side resistor helps boost up the input resistance looking into the base

CC – Two-Port Network (R_{out}) (Self Reading)



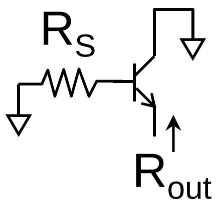
Eliminate i_b and keep v_x and i_x

$$\Rightarrow \begin{cases} i_x \approx -(\beta+1)i_b \\ i_b = -\dfrac{v_x}{R_S + r_\pi} \end{cases}$$

$$\Rightarrow R_{out} = \frac{R_S + r_\pi}{\beta+1}$$

$$\approx \frac{R_S}{\beta+1} + \frac{1}{g_m}$$

CC – Two-Port Network (R_{out}) (Self Reading)



$$R_{out} \approx \frac{R_S}{\beta+1} + \frac{1}{g_m}$$

$$\approx \frac{1}{g_m} \quad [if \quad R_S \quad is \quad small]$$

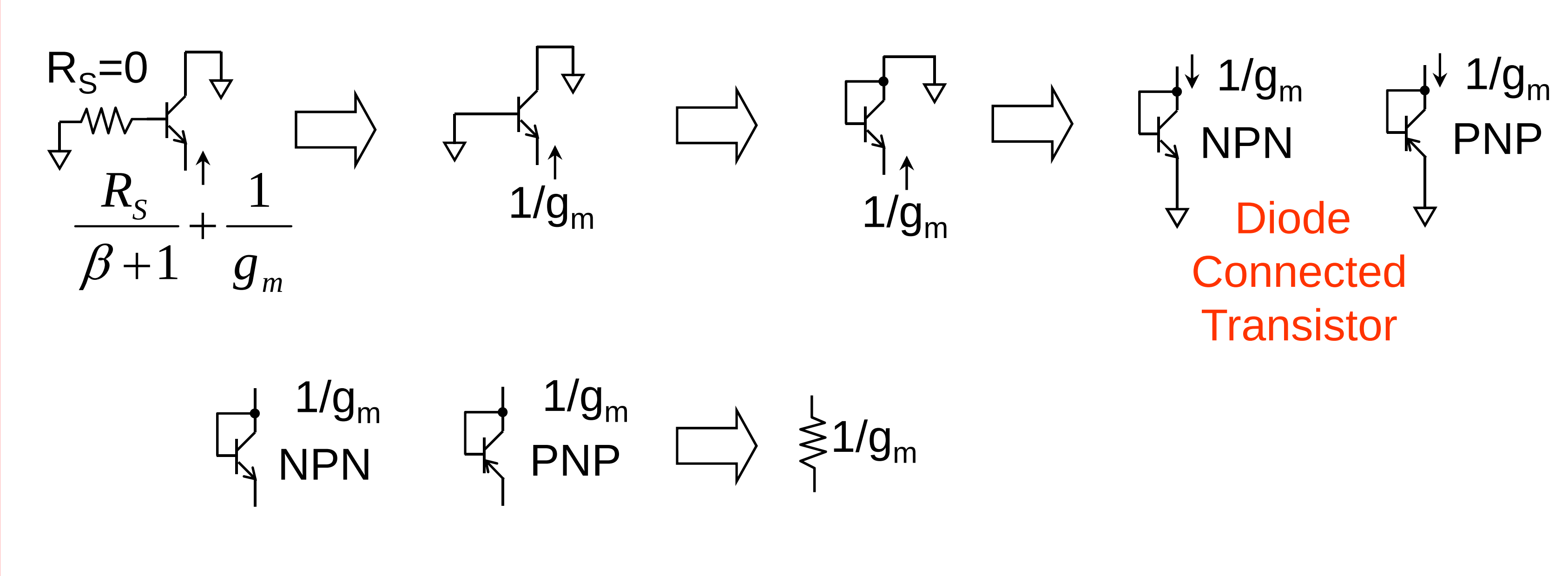
Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the emitter (R_{out}) is directly given by the formula. **No need to rederive.**

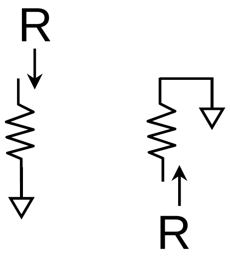
Example :

$$R_S = 100 \quad r_\pi = 5.26k$$
$$r_o = 202k \quad g_m = 19m$$
$$\Rightarrow R_{out} \approx 54$$

- If R_S is small, the output impedance is just the inverse of the transconductance

BJT – Diode Connected Transistor Equivalent Resistance (Self Reading)



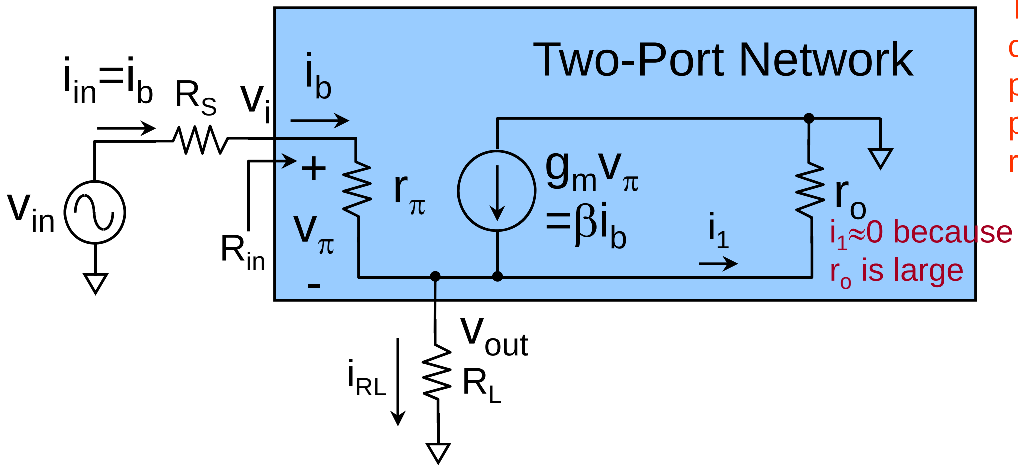


A resistor, whether you look from top or bottom, always is a resistor with resistance R

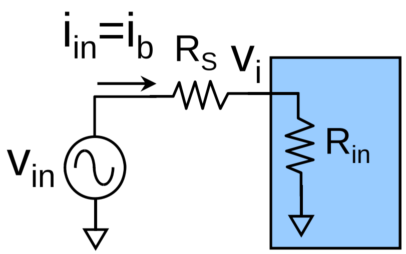
Only APPLICABLE for AC analysis

- When the collector and base are connected together, the transistor is called diode connected transistor
- Diode connected transistor has equivalent resistance of $1/g_m$

CC – Two-Port Network (A_V) (Self Reading)



To determine i_b, we can assume the two ports network present an input resistance R_{in}



$$i_b = \frac{v_{in}}{R_{in} + R_S}$$

$$R_{in} = r_\pi + (\beta + 1)R_L$$

Eliminate i_b and keep v_{out} and v_{in}

$$i_{RL} = (\beta + 1)i_b - \cancel{i_1}$$

$$v_{out} = i_{RL} \times R_L$$

$$= (\beta + 1)i_b \times R_L$$

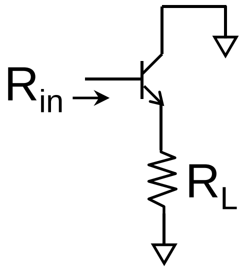
$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = \frac{(\beta + 1)i_b \times R_L}{[R_S + r_\pi + (\beta + 1)R_L]i_b}$$

$$= \frac{(\beta + 1)R_L}{\cancel{R_S} + r_\pi + (\beta + 1)R_L}$$

$$= \frac{g_m R_L}{1 + g_m R_L} \approx 1 \quad [If \quad g_m R_L >> 1]$$

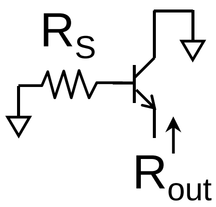
CC – Important Results

1



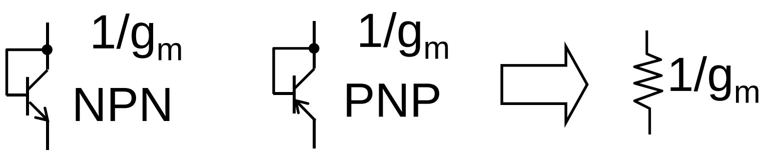
$$R_{in} \approx r_\pi + (\beta+1)R_L$$
$$\approx r_\pi\left[1+\frac{(\beta+1)}{r_\pi}R_L\right]$$
$$\approx r_\pi[1+g_mR_L]$$

2



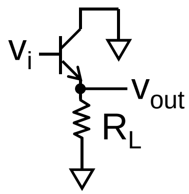
$$R_{out} \approx \frac{R_S}{\beta+1}+\frac{1}{g_m}$$
$$\approx \frac{1}{g_m} \quad [if \quad R_S \quad is \quad small]$$

3



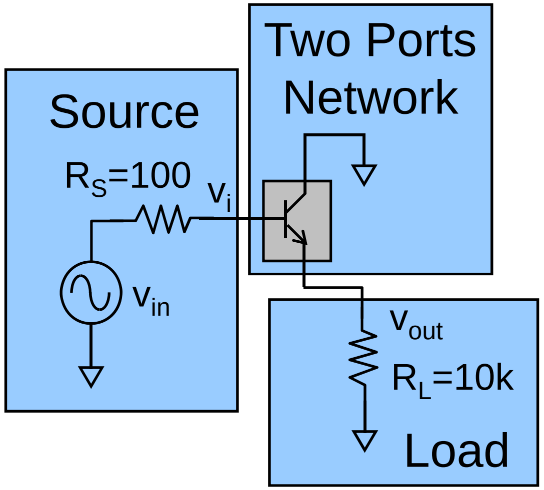
4

Relationship between v_i and v_{out} is defined as follows:

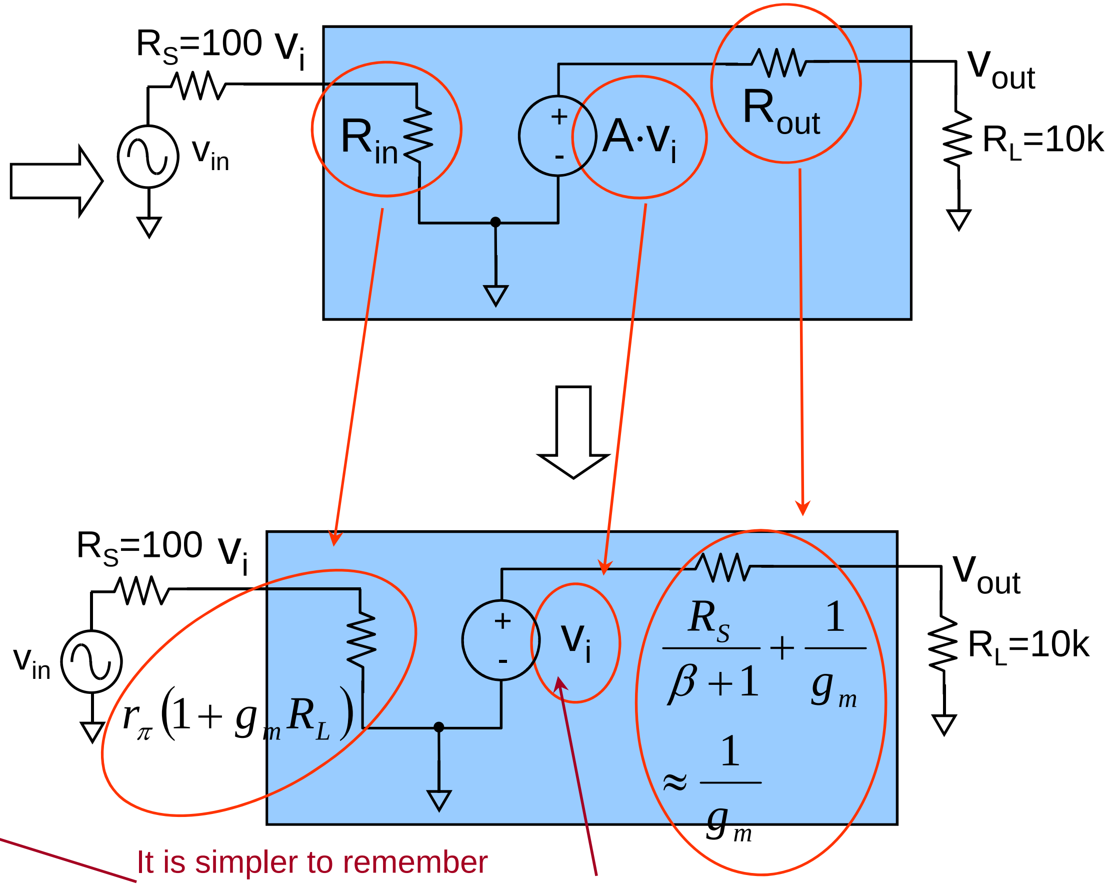


$$v_{out} = \frac{g_mR_L}{1+g_mR_L}v_i$$

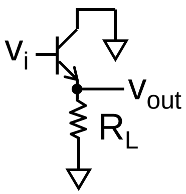
CC – Two-Port Network



Transform the CC amplifier into two-port voltage amplifier



Relationship between v_i and v_{out} is defined as follows:



$$v_{out} = \frac{g_m R_L}{1+g_m R_L} v_i$$

It is simpler to remember A_v rather than A

A can be found based on A_v

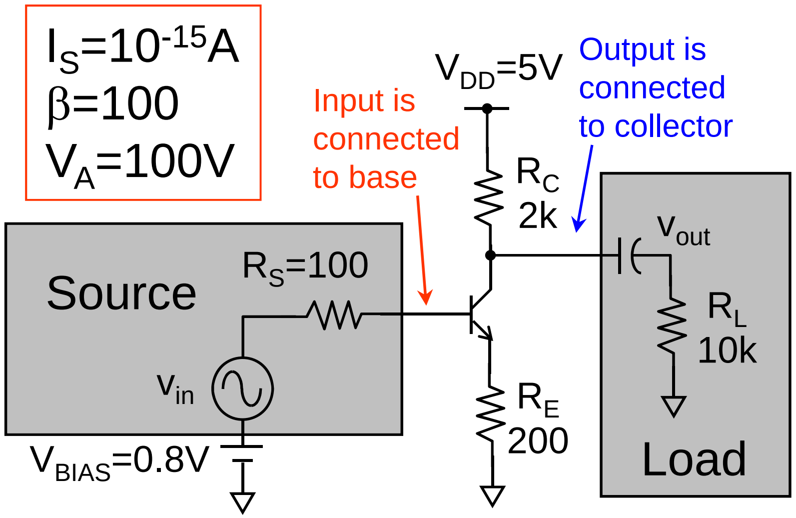
Characteristic of CC/CD

- High input resistance
- Low output resistance
- Close to unity gain
- No polarity inversion
- Ideal buffer (Emitter/Source follower)
- BJT provides larger g_m than MOS (g_m-g_{mb})

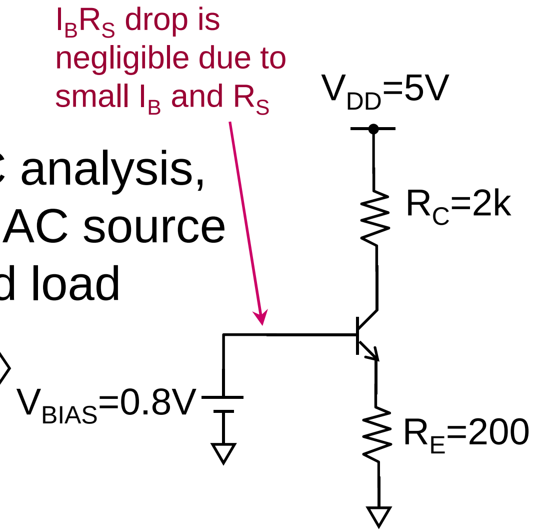
  $\Rightarrow$ Lower output resistance

  $\Rightarrow$ Better buffer

CE with Emitter Degeneration



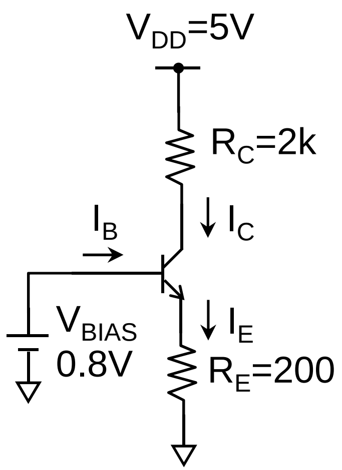
For DC analysis, remove AC source and load



- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Base, output connected to Collector, Emitter connected to neither input nor output and is connected to a resistor $\Rightarrow$ CE with Emitter Degeneration

DC Analysis

- Remove source/load section when doing DC analysis



$I_S = 10^{-15}A$
$\beta = 100$
$V_A = 100V$

Determine DC biasing

$$V_T = 26mV$$

$$Assume \quad V_{BE} \approx 0.7V$$

$$I_E = \frac{V_{BIAS} - V_{BE}}{R_E} = 0.5mA$$

$$I_C = \frac{\beta}{\beta+1} I_E = 0.495mA$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) \approx 0.7001$$

$$\therefore Valid \quad Assumption$$

$$I_B = 4.95\mu A$$

Determine AC small signal parameter

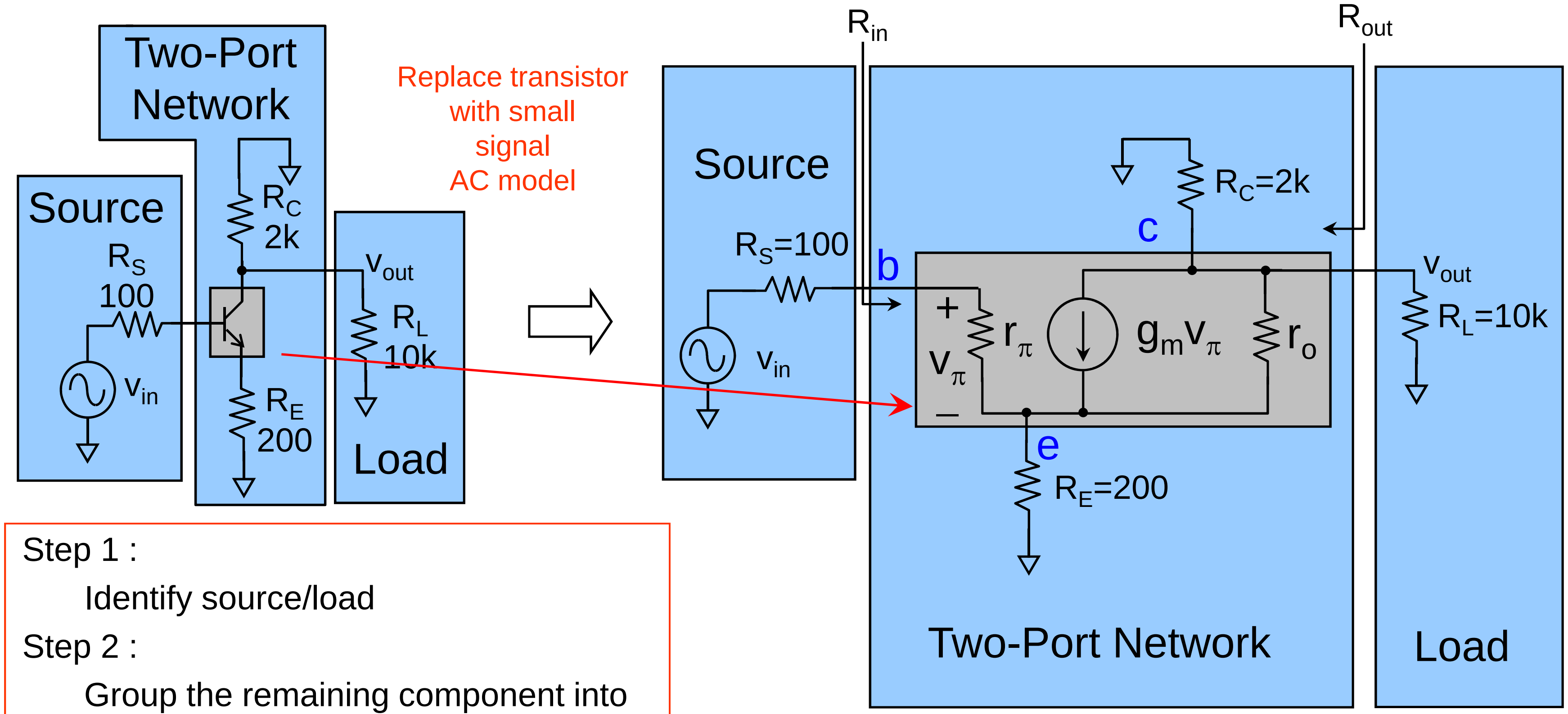
$$g_m = \frac{I_C}{V_T} = 19mA/V$$

$$r_\pi = \frac{\beta}{g_m} = 5.26k\Omega$$

$$r_o = \frac{V_A}{I_C} = 202k\Omega$$

If the assumption is invalid, use the updated V_{BE}, re-estimate I_C, a few iterations might be needed

AC Analysis



Step 1 :

Identify source/load

Step 2 :

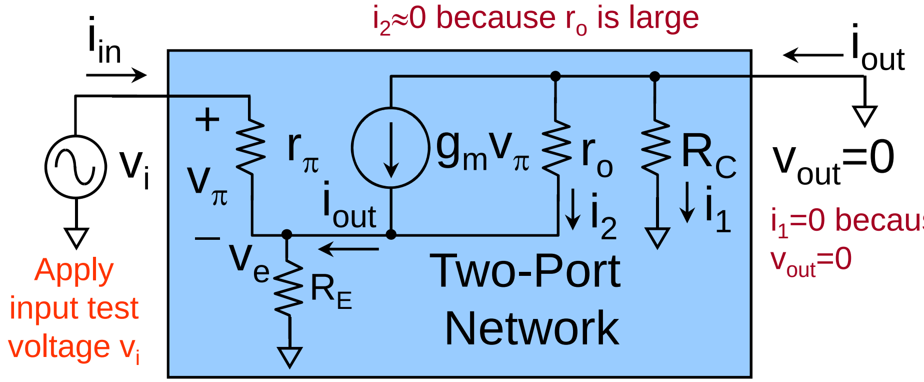
Group the remaining component into two-port network

Step 3 :

Replace transistor with small signal AC model

Interested in finding out R_{in}, R_{out} and G_m for the two ports network (Transconductance Amplifier)

CE with Emitter Degeneration – Two-Port Network (G_m)



Measure short-circuit output current i_{out}

$i_1=0$ because $v_{out}=0$

Important Result :
Transconductance (G_m) is just $g_m/(1+g_m R_E)$

$$v_\pi = v_i - v_e \qquad i_{in} = \frac{v_i - v_e}{r_\pi}$$

$$\begin{cases} i_{in} + i_{out}\big|_{v_{out}=0} = v_e / R_E \\ i_{out}\big|_{v_{out}=0} \approx g_m v_\pi = g_m (v_i - v_e) \end{cases}$$

$$\Rightarrow v_e = v_i - \frac{i_{out}\big|_{v_{out}=0}}{g_m}$$

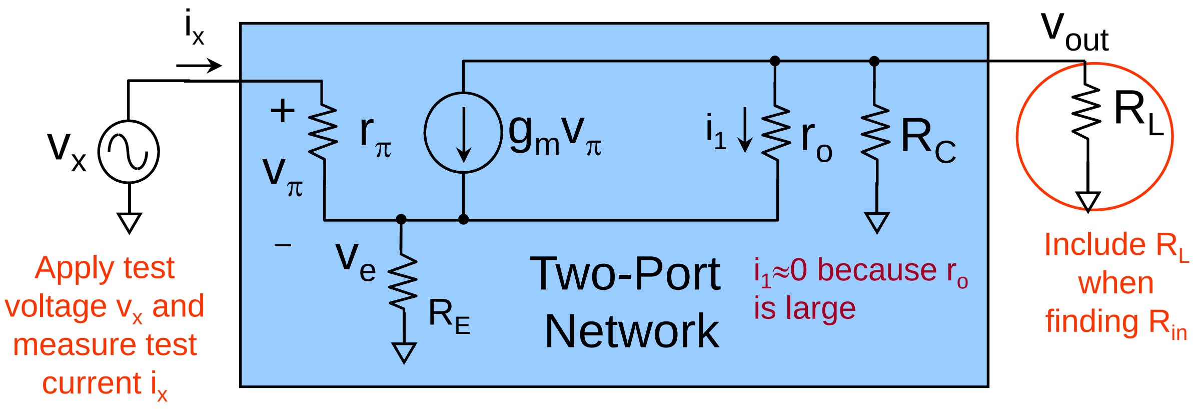
$$\frac{i_{out}\big|_{v_{out}=0}}{g_m r_\pi} + i_{out}\big|_{v_{out}=0} = \frac{v_i}{R_E} - \frac{i_{out}\big|_{v_{out}=0}}{g_m R_E}$$

$$\Rightarrow G_m = \left.\frac{i_{out}}{v_i}\right|_{v_{out}=0} = \frac{g_m r_\pi}{r_\pi + (1 + g_m r_\pi) R_E}$$

$$\approx \frac{g_m}{1 + g_m R_E} = 4mA/V$$

Eliminate i_{in}, v_e and keep v_i and i_{out}

CE with Emitter Degeneration – Two-Port Network (R_{in})



$$\boxed{\begin{aligned} v_\pi &= v_x - v_e \\ &= i_x r_\pi \end{aligned}}$$

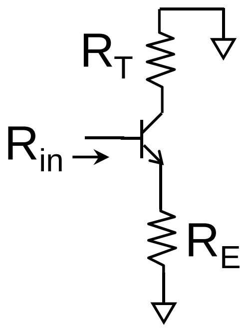
$$\begin{cases} i_x = \dfrac{v_e}{R_E} - g_m v_\pi = \dfrac{v_e}{R_E} - g_m\left(v_x - v_e\right) \\ v_e = v_x - i_x r_\pi \end{cases}$$

Eliminate v_e and keep v_x and i_x

$$\Rightarrow i_x = \frac{v_x}{R_E} - \frac{i_x r_\pi}{R_E} - g_m i_x r_\pi$$

$$\begin{aligned} \Rightarrow R_{in} &= r_\pi + g_m r_\pi R_E + R_E \\ &\approx r_\pi + (\beta + 1) R_E \\ &\approx r_\pi \left[1 + g_m R_E\right] \end{aligned}$$

CE with Emitter Degeneration – Two-Port Network (R_{in})



$$R_{in} \approx r_\pi + (\beta+1)R_E$$
$$\approx r_\pi\left[1+g_m R_E\right]$$

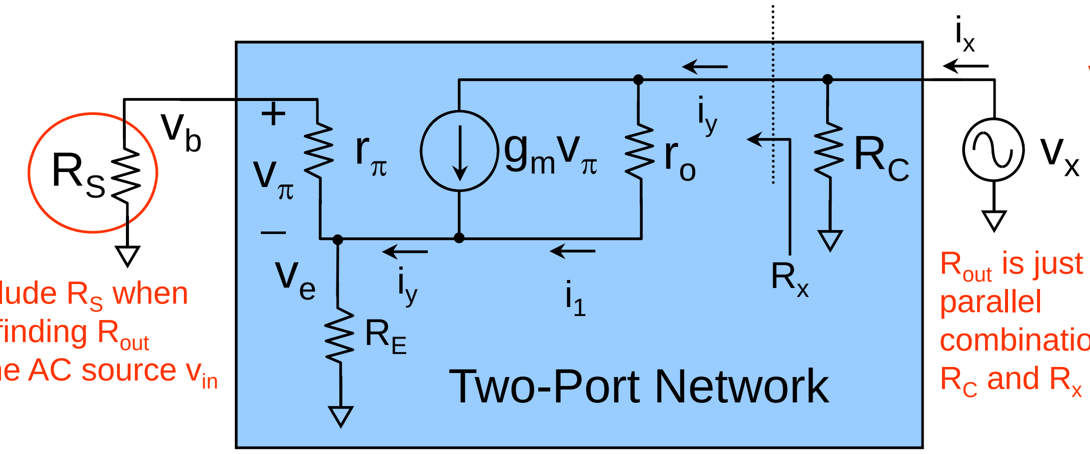
Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the base (R_{in}) is directly given by the formula. **No need to rederive.**

Example :

$$R_E = 200 \quad r_o = 202k$$
$$\Rightarrow R_{in} = r_\pi + (\beta+1)R_E$$
$$= 25.5k$$

- Emitter side resistor helps boost up the input resistance looking into the base, same formula as CC

CE with Emitter Degeneration – Two-Port Network (R_{out})



Apply test voltage v_x and measure test current i_x

Include R_S when finding R_{out}
Kill the AC source v_{in}

R_{out} is just parallel combination of R_C and R_x

$$v_\pi = v_b - v_e$$

$$i_1 = i_y - g_m v_\pi$$
$$= i_y + g_m (v_e - v_b)$$

$$\Rightarrow \begin{cases} v_x \approx v_e + i_1 r_o = v_e + \left[i_y + g_m (v_e - v_b)\right] r_o \\ v_e = i_y \left[(R_S + r_\pi) // R_E\right] \\ v_e - v_b = \dfrac{r_\pi}{R_S + r_\pi} v_e \end{cases}$$

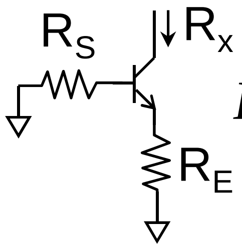
Eliminate v_e, v_b and keep v_x and i_y

$$\Rightarrow R_x \approx r_o \left\{1 + g_m \left[(R_S + r_\pi) // R_E\right] \left(\frac{r_\pi}{R_S + r_\pi}\right)\right\}$$

$$R_{out} = R_x // R_C \approx R_C$$

$$\Rightarrow v_x \approx i_y \left[(R_S + r_\pi) // R_E\right] + i_y r_o + g_m i_y \left[(R_S + r_\pi) // R_E\right] \left(\frac{r_\pi}{R_S + r_\pi}\right) r_o$$

CE with Emitter Degeneration – Two-Port Network (R_{out})



$$R_x \approx r_o\left\{1 + g_m\left[\left(R_S + r_\pi\right) // R_E\right]\left(\frac{r_\pi}{R_S + r_\pi}\right)\right\}$$

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the collector (R_x) is directly given by the formula.
No need to rederive.

Example :

$R_S = 100 \quad r_\pi = 5.26k$

$R_E = 200 \quad r_o = 202k$

$g_m = 19m$

$\Rightarrow R_x \approx 928k$

$\Rightarrow R_{out} \approx R_C = 2k$

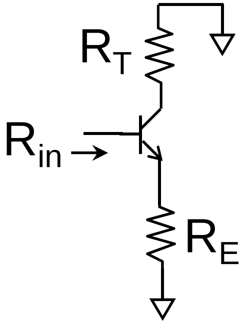
- Emitter side resistor helps boost up the transistor output resistance (R_x)
- Very similar to CB configuration. Same formula as CB if R_S=0.

CE with Degeneration – Important Results

1

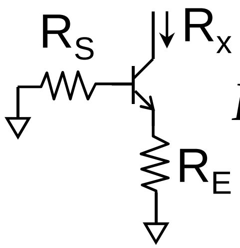
Important Result :
Transconductance (G_m) is just $g_m/(1+g_mR_E)$

2



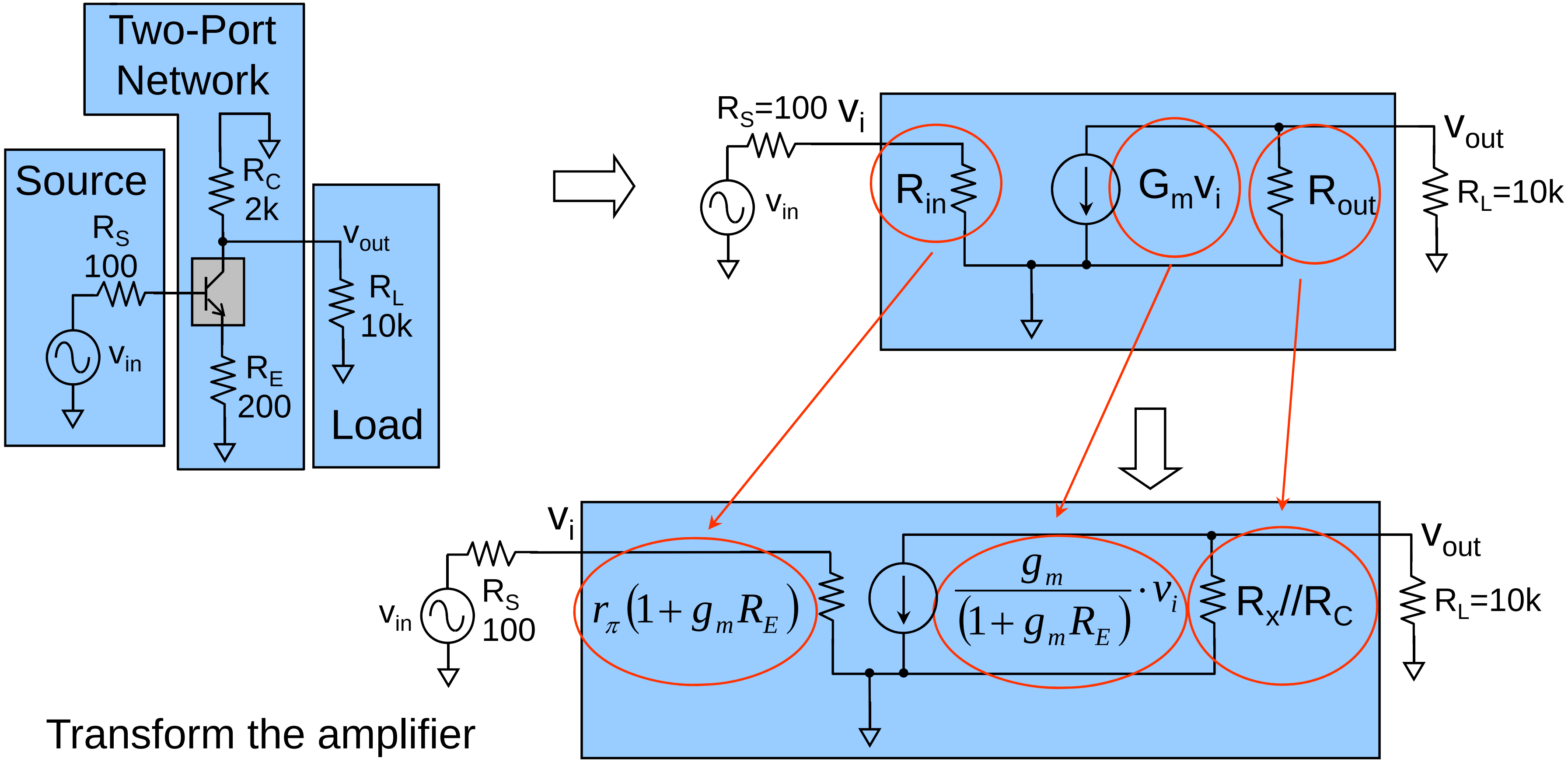
$$R_{in} \approx r_\pi + (\beta+1)R_E$$
$$\approx r_\pi[1+g_mR_E]$$

3



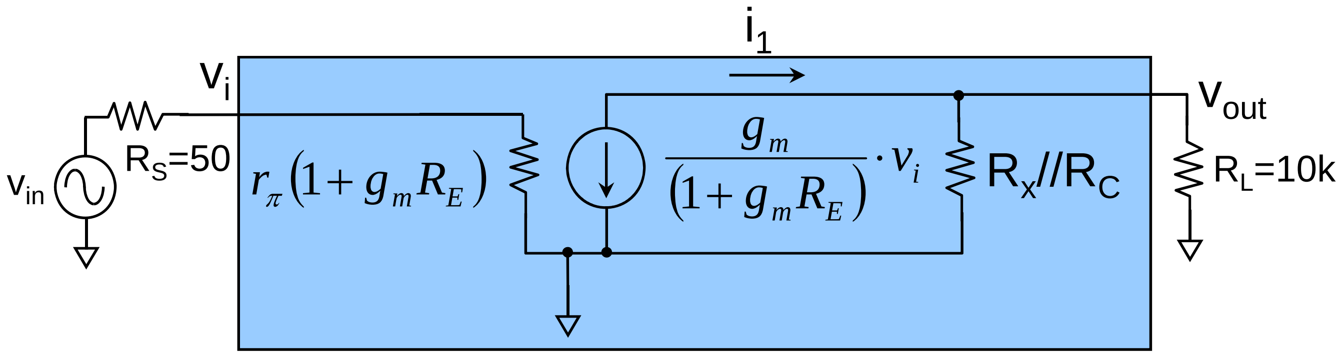
$$R_x \approx r_o\left\{1+g_m[(R_S+r_\pi)//R_E]\left(\frac{r_\pi}{R_S+r_\pi}\right)\right\}$$

CE with Emitter Degeneration – Two-Port Network



Transform the amplifier into two-port transconductance amplifier

CE with Emitter Degeneration – Two-Port Network (A_V)



$$v_i = \frac{r_\pi(1+g_m R_E)}{R_S + r_\pi(1+g_m R_E)} v_{in} \qquad i_1 = -\frac{g_m}{1+g_m R_E} v_i$$

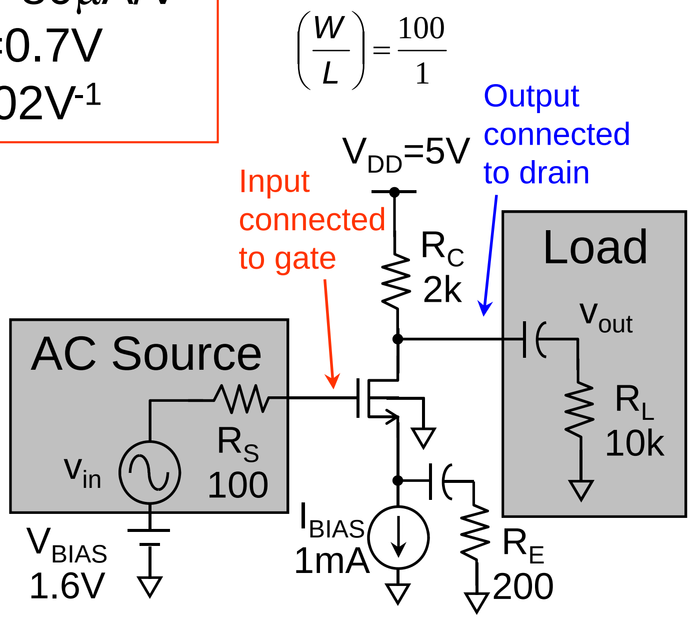
$$v_{out} = i_1 \times \left[(R_x // R_C) // R_L\right]$$

$$= -\frac{g_m}{1+g_m R_E} v_i \times \left[(R_x // R_C) // R_L\right]$$

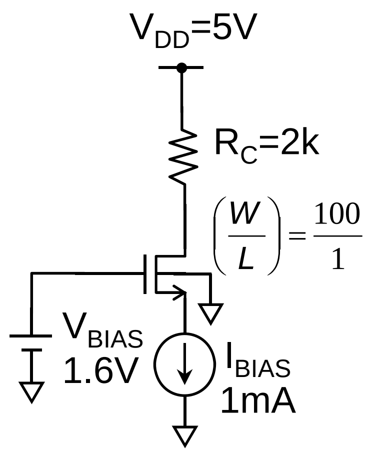
$$A_V = \frac{v_{out}}{v_{in}} = -\frac{r_\pi(1+g_m R_E)}{R_S + r_\pi(1+g_m R_E)} \times \frac{g_m}{1+g_m R_E} \times \left[(R_x // R_C) // R_L\right]$$

CS with Source Degeneration

$\mu_n C_{ox}=80\mu A/V^2$
$V_{THN}=0.7V$
$\lambda_n=0.02V^{-1}$



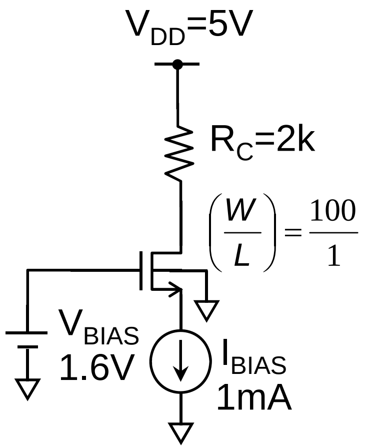
For DC analysis, remove AC source and load



- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Gate, output connected to Drain, Source connected to neither input nor output and is connected to a resistor $\Rightarrow$ CS with Source Degeneration

DC Analysis (Self Reading)

- Remove source/load section when doing DC analysis



$\mu_n C_{ox}$=80μA/V^2
V_{THN}=0.7V
λ_n=0.02V^{-1}

Determine DC biasing

$$I_D = I_S = I_{BIAS} = 1mA$$

$$\left(\frac{W}{L}\right) = \frac{100}{1}$$

Good approximation, no need to go through detailed calculations

Determine AC small signal parameter

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}$$

$$= 4mA/V$$

$$\boxed{g_{mb} \approx -\frac{g_m}{4} = -1mA/V}$$

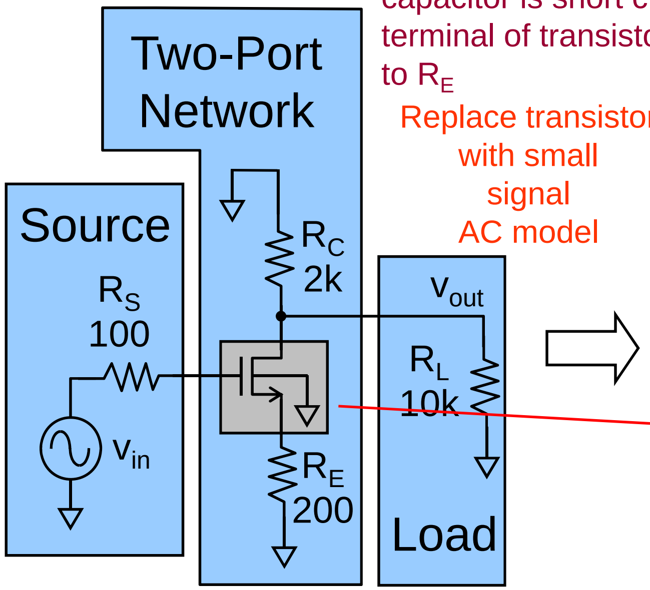
$$r_i = \infty$$

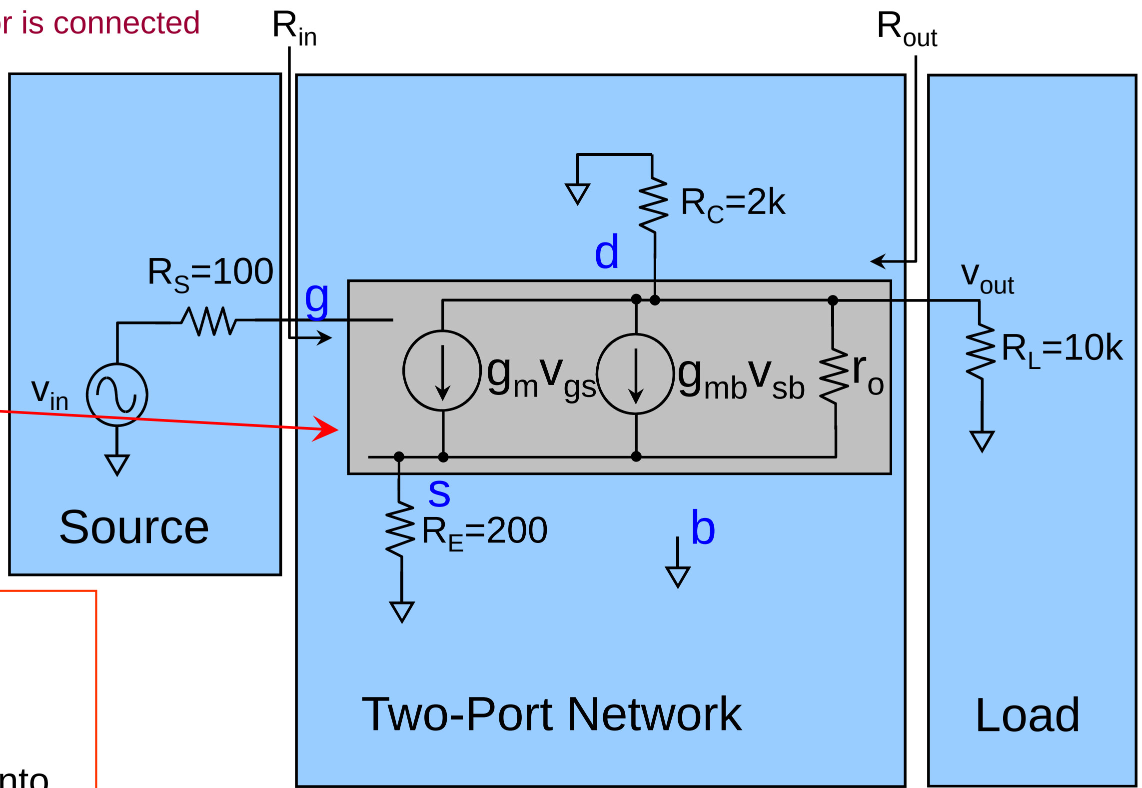
$$r_o = \frac{1}{\lambda_n I_D} = 50k\Omega$$

AC Analysis (Self Reading)

DC current source is open circuit, capacitor is short circuit⇒Source terminal of transistor is connected to R_E

Replace transistor with small signal AC model





Step 1 :

Identify source/load

Step 2 :

Group the remaining component into two-port network

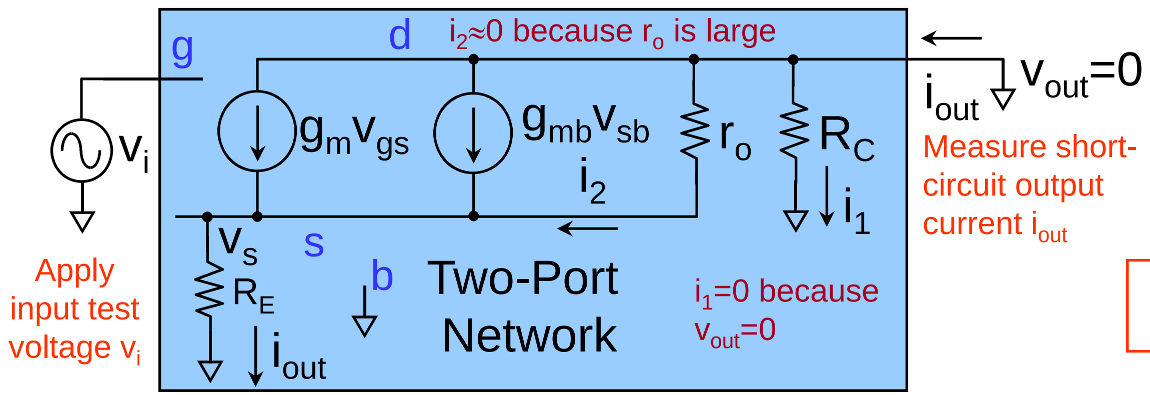
Step 3 :

Replace transistor with small signal AC model

Interested in finding out R_{in}, R_{out} and G_m for the two ports network (Transconductance Amplifier)

$$R_{in} = \infty$$

CS with Source Degeneration – Two-Port Network (G_m) (Self Reading)



Important Result : Transconductance (G_m) is just $g_m/[1+(g_m-g_{mb})R_E]$

$$v_{gs} = v_i - v_s \qquad v_{sb} = v_s$$

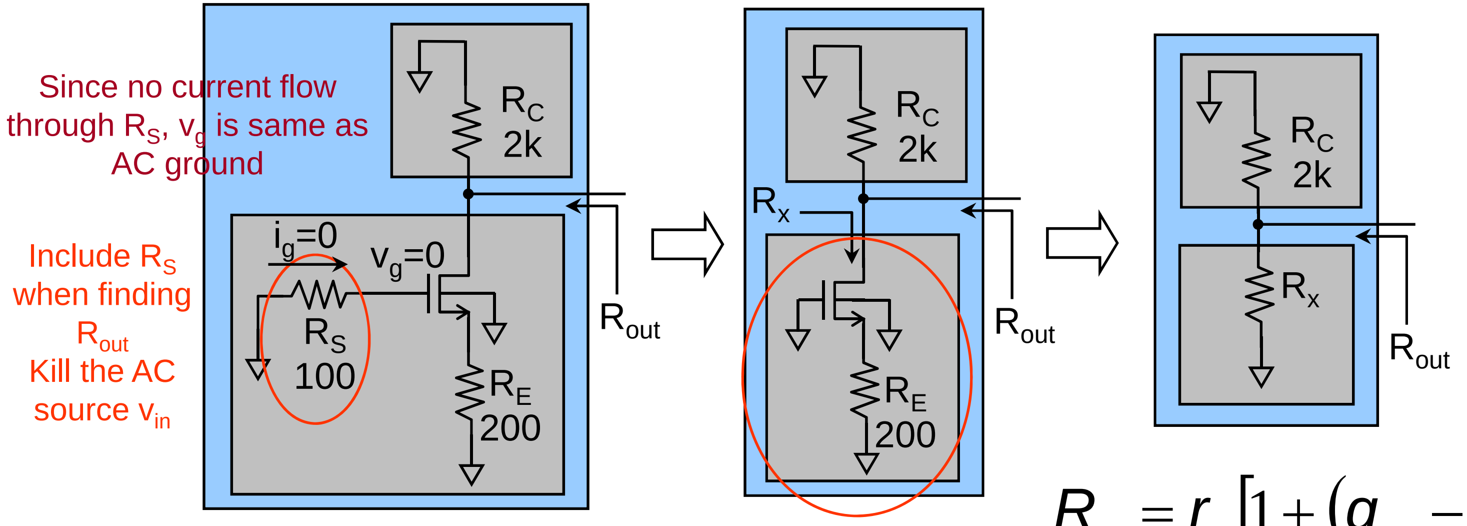
Eliminate v_s and keep v_i and i_{out}

$$\begin{cases} v_s \approx i_{out}\big|_{v_{out}=0} \times R_E \\ i_{out}\big|_{v_{out}=0} \approx g_m v_{gs} + g_{mb} v_{sb} \\ \qquad \approx g_m (v_i - v_s) + g_{mb} v_s \end{cases}$$

$$\Rightarrow i_{out}\big|_{v_{out}=0} \approx g_m v_i - (g_m - g_{mb}) i_{out}\big|_{v_{out}=0} \times R_E$$

$$\Rightarrow G_m = \left.\frac{i_{out}}{v_i}\right|_{v_{out}=0} = \frac{g_m}{1+(g_m - g_{mb})R_E} = 2mA/V$$

CS with Source Degeneration – Two-Port Network (R_{out}) (Self Reading)



Any complicated network can be reduced to an equivalent resistance (AC analysis only)

Slide 6A-32
CG

$$R_x = r_o\left[1+(g_m - g_{mb})R_E\right]$$
$$\approx 100k$$
$$R_{out} = R_C // R_x \approx R_C = 2k$$

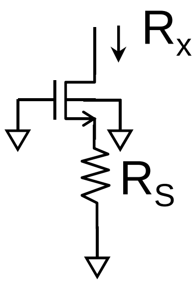
- Source side resistance helps boost up the transistor output resistance (R_x)
- Same as CG

CS with Degeneration – Important Results

1

Important Result :
Transconductance (G_m) is just $g_m/[1+(g_m-g_{mb})R_E]$

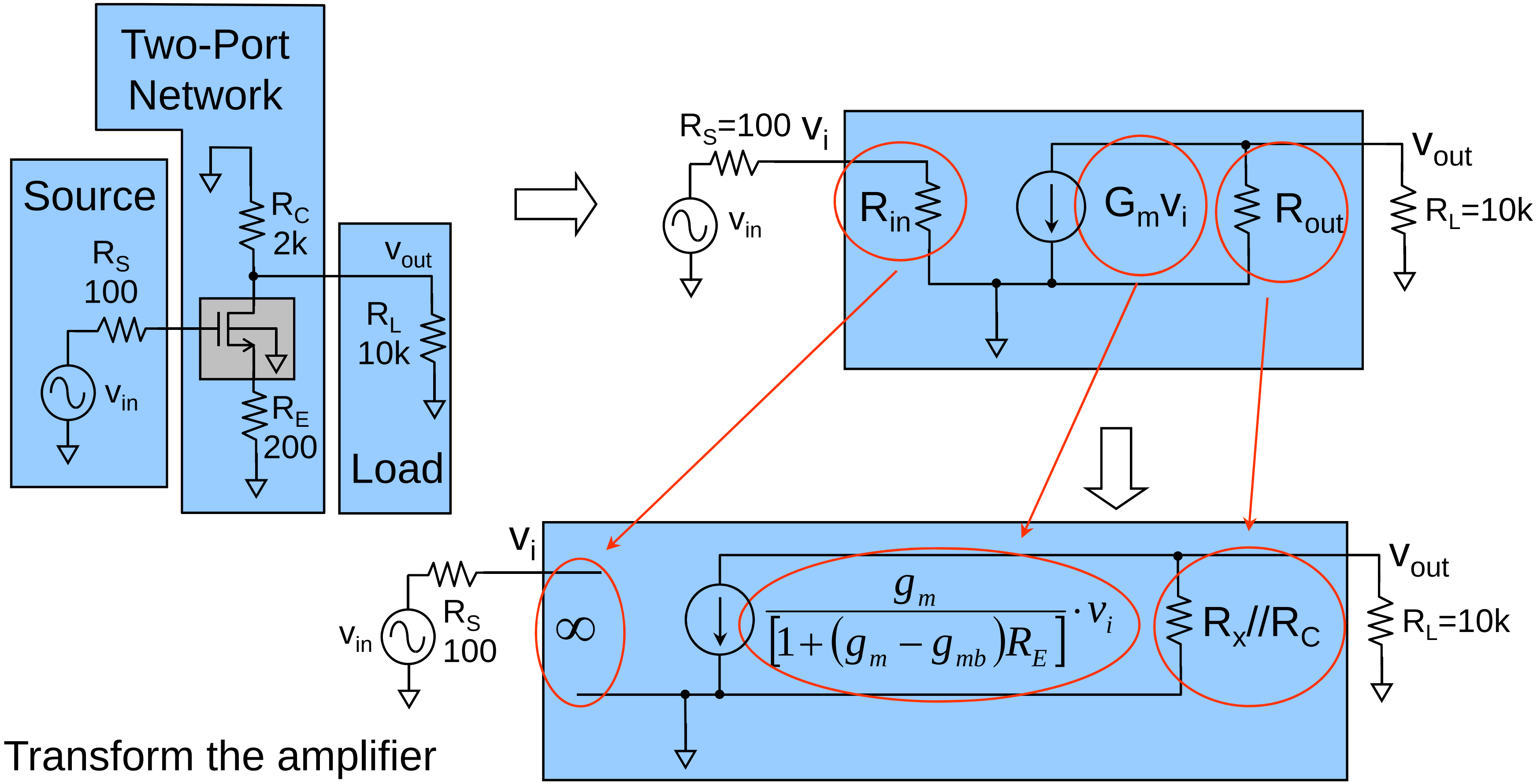
2



$$R_x \approx r_o\left[1+\left(g_m-g_{mb}\right)R_S\right]$$

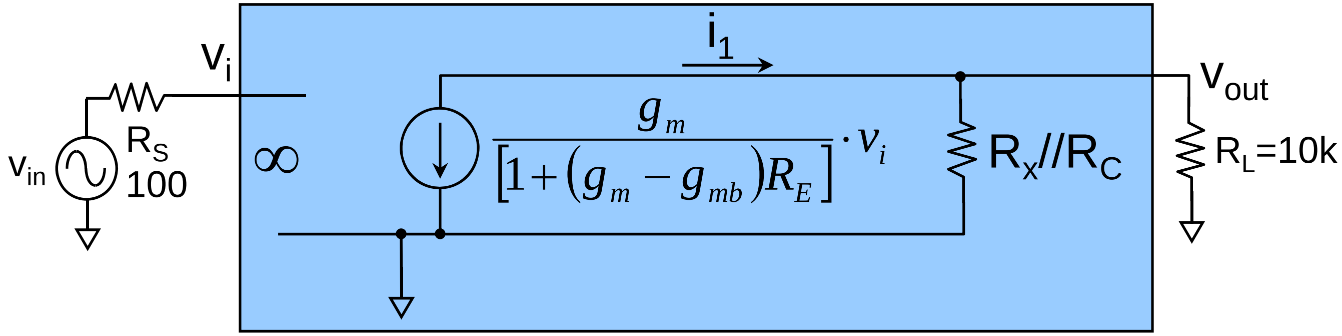
Same result as CG

CS with Source Degeneration – Two-Port Network



Transform the amplifier into two-port transconductance amplifier

CS with Source Degeneration – Two-Port Network (A_V)



$$v_i = v_{in} \qquad i_1 = -\frac{g_m}{1+(g_m-g_{mb})R_E}\times v_i$$

$$v_{out} = i_1 \times (R_{out} // R_L) = -\frac{g_m}{1+(g_m-g_{mb})R_E}\times v_i \times (R_{out} // R_L)$$

$$\approx -v_{in} \times \frac{g_m}{1+(g_m-g_{mb})R_E}\times [(R_C // R_x) // R_L]$$

$$\Rightarrow A_V \approx -\frac{g_m}{1+(g_m-g_{mb})R_E}\times (R_C // R_L)$$

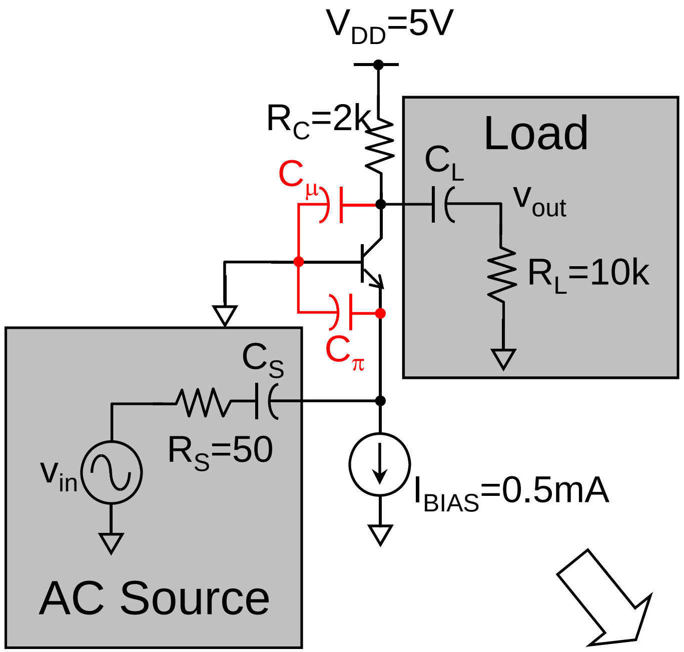
Characteristic of CE/CS with Emitter/Source Degeneration

- High input resistance
- High output resistance
- Lower Gain than CE and CS
- Polarity inversion

Effect of Capacitors

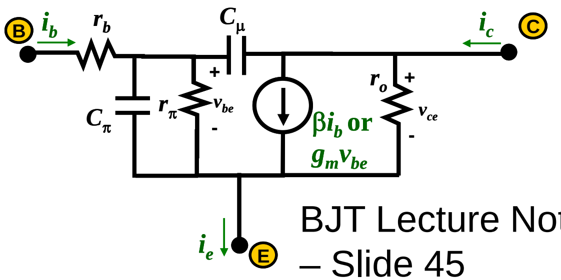
Consider CB amplifier example (Slide 6A-49):

- C_L and C_S are external capacitors added to the circuit, it is usually large (~µF range)
- C_μ and C_π are parasitic capacitor inherent to the BJT device, it is usually small (~pF range)

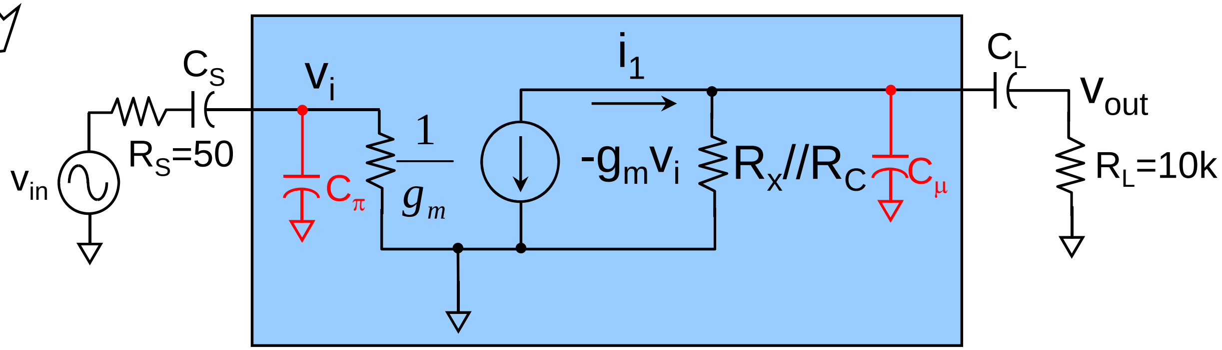


	Low freq (→DC)	Mid freq	High freq
C_S, C_L	Open	Short	Short
C_μ, C_π	Open	Open	Short

Audio : <20kHz
Video : <6MHz
RF : ~GHz

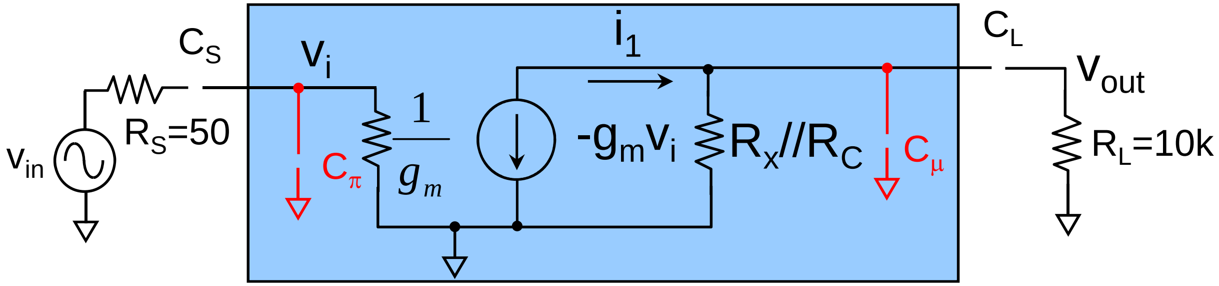


BJT Lecture Notes – Slide 45

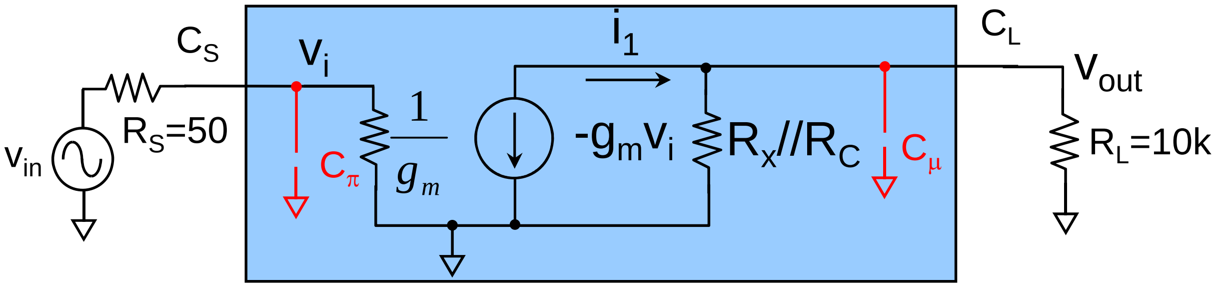


Effect of Capacitors

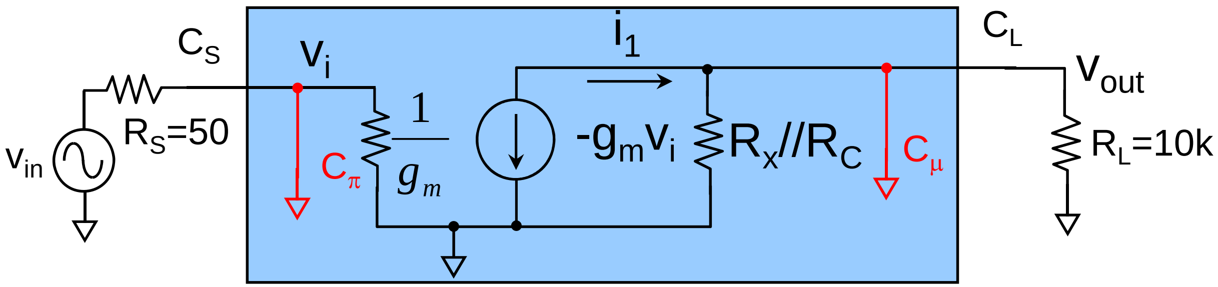
At low frequency,
C_S, C_L, C_μ, $C_\pi \rightarrow$ open circuit
v_{out} isolated from $v_{in} \Rightarrow v_{out}=0$



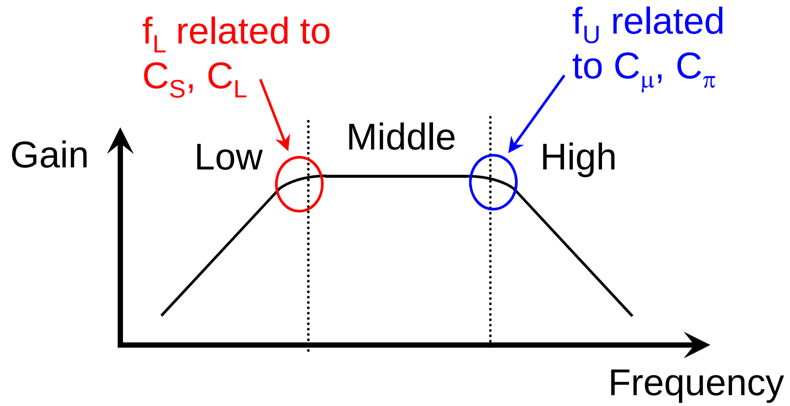
At middle frequency,
C_S, $C_L \rightarrow$ short circuit
C_μ, $C_\pi \rightarrow$ open circuit
2-port network $\Rightarrow v_{out}=A_V \times v_{in}$



At high frequency,
C_S, $C_L \rightarrow$ short circuit
C_μ, $C_\pi \rightarrow$ short circuit
v_{in} and v_{out} short to ground
$\Rightarrow v_{out}=0$



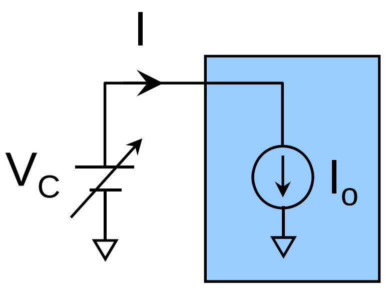
Effect of Capacitors



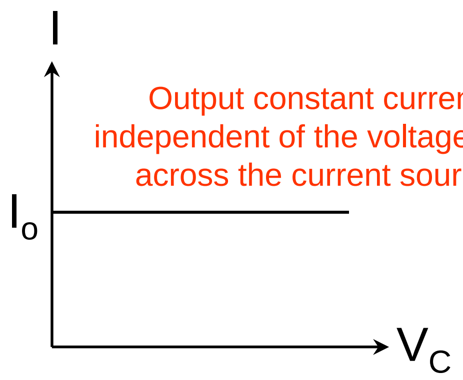
- C_S and C_L cause the gain to drop at low frequency due to open circuit at low frequency.
- C_μ and C_π cause the gain to drop at high frequency due to short circuit at high frequency.
- At the middle frequency, C_S and C_L are short circuit whereas C_μ and C_π are open circuit. The amplifier behaves like 2-port network with gain.
- The lower cut-off point (f_L), i.e. the frequency below which the gain starts to drop are related to RC time constant related to C_S and C_L.
- The upper cut-off point (f_U), i.e. the frequency above which the gain starts to drop are related to RC time constant related to C_μ and C_π.

Ideal versus Non-ideal Current Source

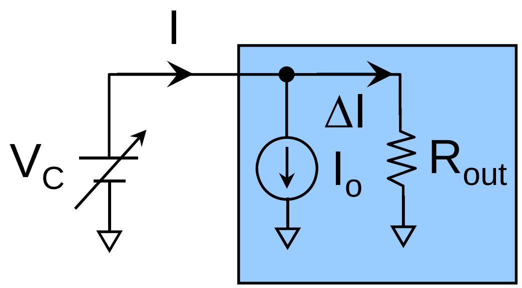
Ideal Current Source



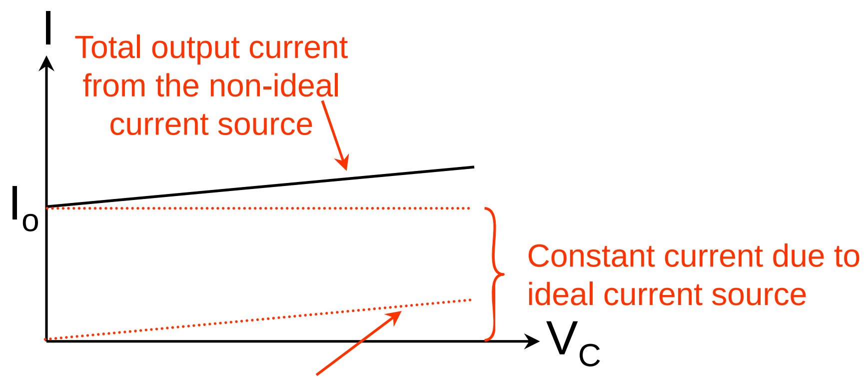
Ideal current source is simply a parallel combination of ideal current source (I_o) with infinite output resistance (∞)



Non-ideal Current Source



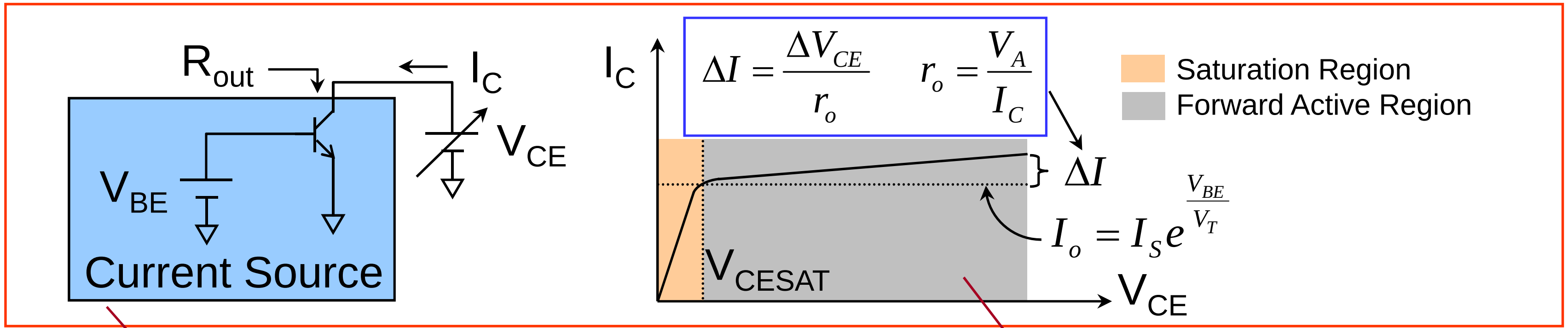
Non-ideal current source is simply a parallel combination of ideal current source (I_o) with finite output resistance (R_{out})

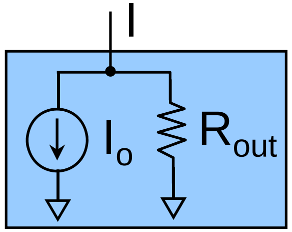


Small variable current (ΔI) due to finite output resistance (R_{out})

$$\Delta I = \frac{V_C}{R_{out}}$$

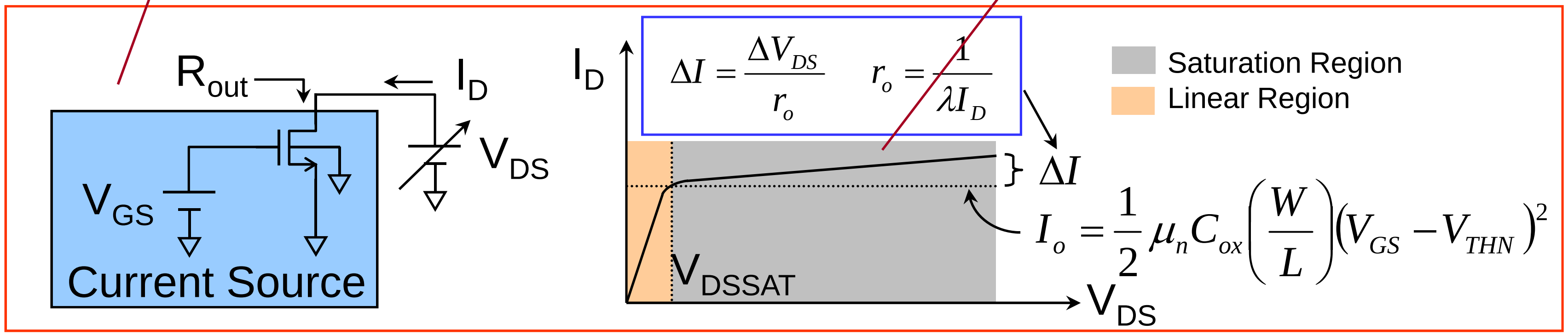
Transistor as Current Source



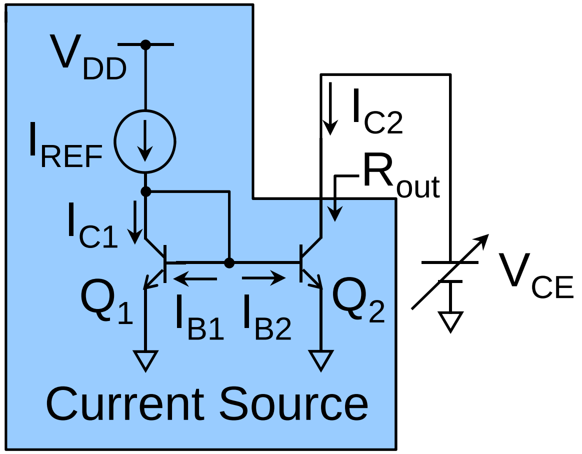


$R_{out}=V_A/I_C$ for BJT transistor current source
$R_{out}=1/(\lambda I_D)$ for MOS transistor current source

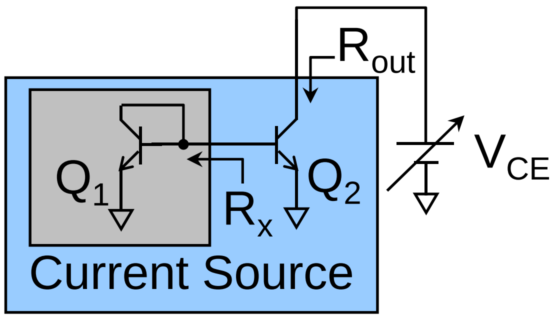
Transistor with voltage bias behave very similarly to non-ideal current source as long as they operate at **GRAY** region



BJT Current Mirror



AC Analysis



Determine DC Biasing

$$I_{REF} = I_{C1} + I_{B1} + I_{B2}$$

$$\because V_{BE1} = V_{BE2}$$

$$\Rightarrow I_{B1} = I_{B2} = \frac{I_{C1}}{\beta}$$

$$\Rightarrow \boxed{I_{C2} \approx I_{C1} \approx \frac{\beta}{\beta+2} I_{REF}}$$

I_{C2} not exactly equal to I_{REF} due to finite current gain β

Determine AC small signal parameter

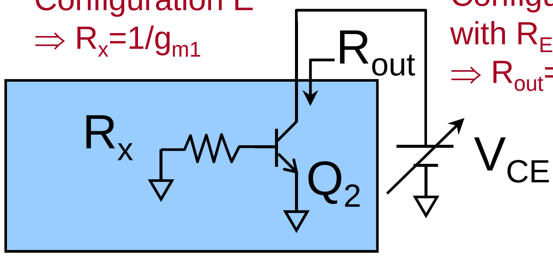
$$g_{m1} = g_{m2} = \frac{I_C}{V_T}$$

$$R_{out} = r_o = \frac{V_A}{I_C}$$

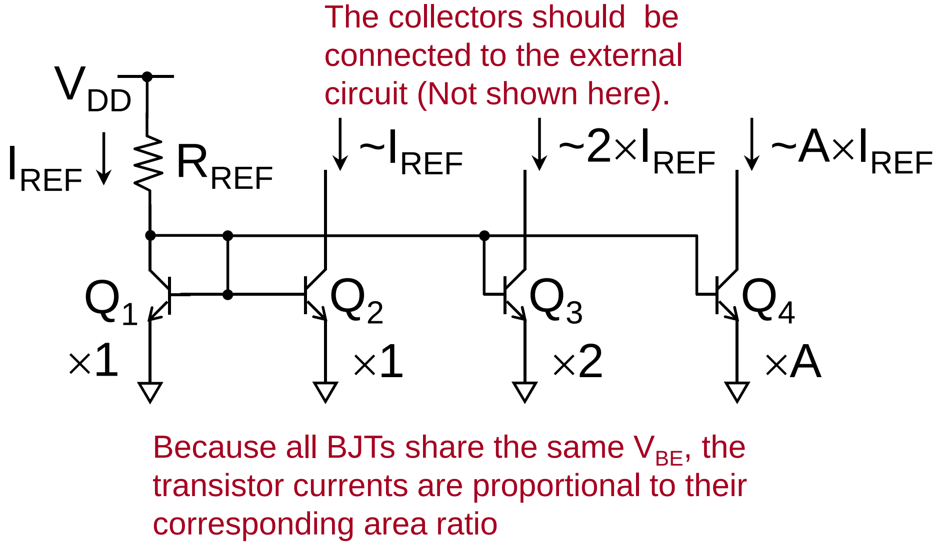
Look-up Table 1

Configuration E
$\Rightarrow R_x = 1/g_{m1}$

Configuration B with $R_E = 0$
$\Rightarrow R_{out} = V_A / I_{C2}$



BJT Current Mirror



$$I_{REF} = \frac{V_{DD} - V_{BE}}{R_{REF}}$$

I_{REF} can be obtained easily through a resistor

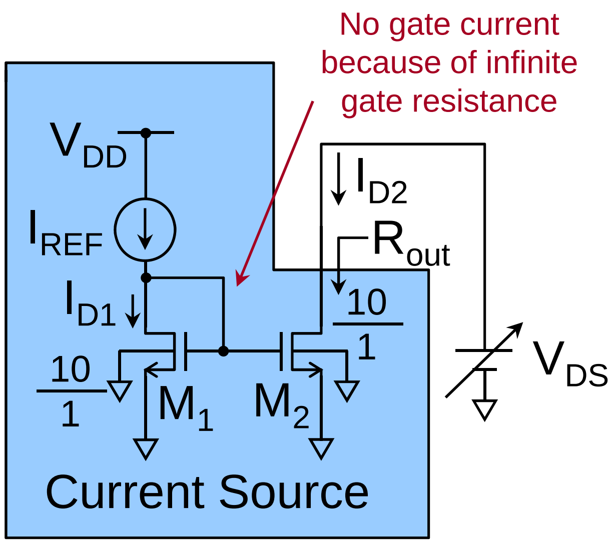
$$I_{C1} \approx I_{REF} = J_S e^{\frac{V_{BE}}{V_T}} \times Area$$

$$I_{C2} \approx I_{C1} \quad I_{C3} \approx 2 \times I_{C1}$$

$$I_{C4} \approx A \times I_{C1}$$

- Using same V_{BE} created from the reference branch, different current source can be created by laying out different area of BJT
- The current source is not identical to I_{REF} due to :
  - Finite current gain β
  - Finite output resistance r_o and different V_{CE} from the reference branch

MOS Current Mirror



Determine DC Biasing

$$I_{REF} = I_{D1}$$

$$\because V_{GS1} = V_{GS2}$$

$$\Rightarrow \boxed{I_{D1} = I_{D2} = I_{REF}}$$

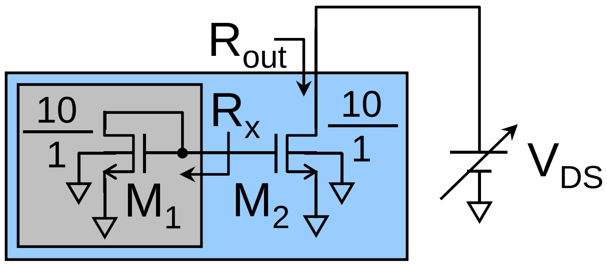
Determine AC small signal parameter

$$g_{m1} = g_{m2}$$

$$= \sqrt{2\mu_n C_{ox}\left(\frac{10}{1}\right) I_{D1}}$$

$$R_{out} = r_o = \frac{1}{\lambda I_{D2}}$$

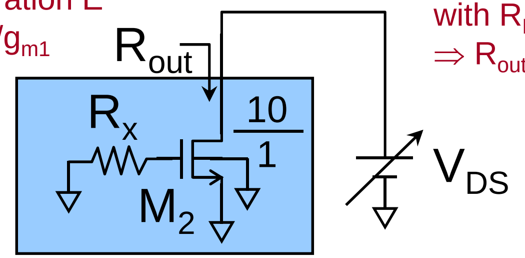
AC Analysis



Look-up Table 2

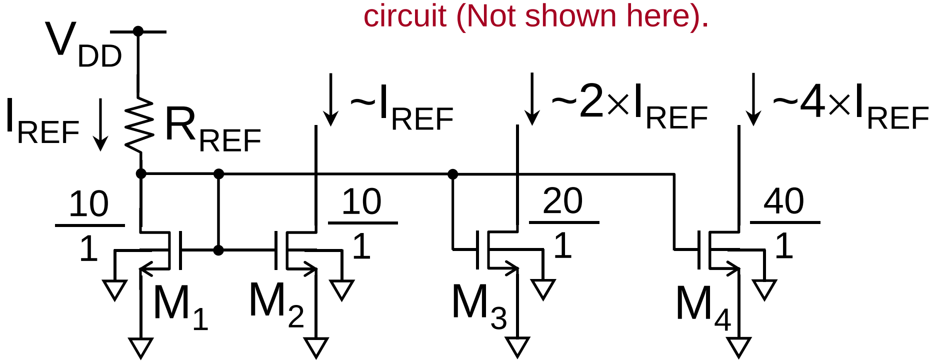
Configuration E
$\Rightarrow R_x=1/g_{m1}$

Configuration B with $R_E=0$
$\Rightarrow R_{out}=1/\lambda I_{D2}$



MOS Current Mirror

The drain should be connected to the external circuit (Not shown here).



Because all MOSFETs share the same V_{GS}, the transistor currents are proportional to their corresponding sizing ratio

$$I_{REF} = \frac{V_{DD} - V_{GS}}{R_{REF}}$$

I_{REF} can be obtained easily through a resistor

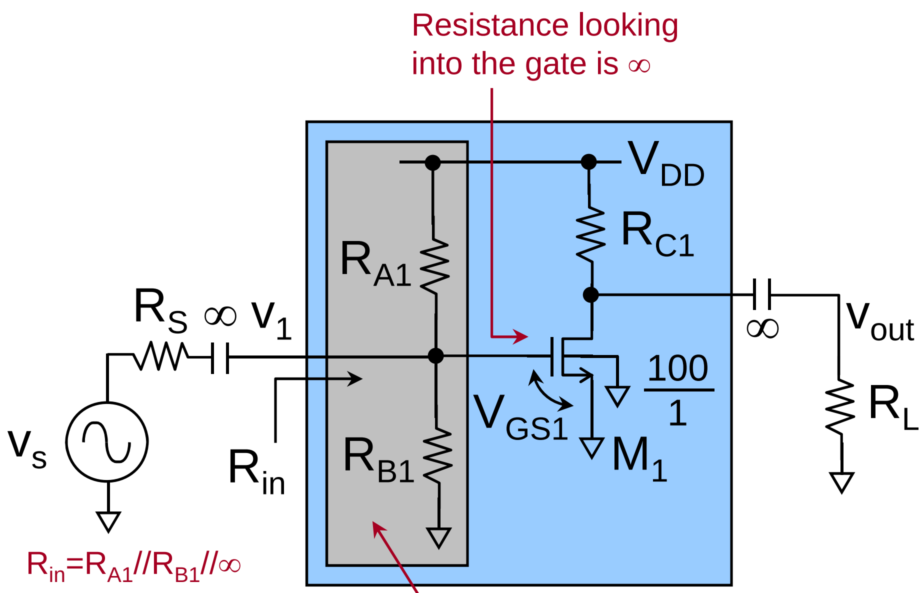
$$I_{D1} = I_{REF}$$

$$= \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)_{M1}\left(V_{GS} - V_{THN}\right)^2$$

$$I_{D2} = I_{D1} \quad I_{D3} = 2\times I_{D1} \quad I_{D4} = 4\times I_{D1}$$

- Using same V_{GS} created from the reference branch, different current source can be created by laying out different size of MOS transistors
- The current source is not identical to I_{REF} due to finite output resistance r_o and different V_{DS} from the reference branch

Conventional Resistor Biasing for CS



Determine DC Biasing

$$I_{D1} = K\left(V_{GS1} - V_{THN}\right)^2$$

In the design, the desired I_{D1} is usually given

$$\Rightarrow V_{GS1} \;\; can \;\; be \;\; determined$$

$$\because V_{GS1} = V_{DD} \times \frac{R_{B1}}{R_{A1} + R_{B1}}$$

$$R_{A1} \;\; and \;\; R_{B1} \;\; can \;\; be \;\; determined$$

Determine AC Small Signal Parameter

$$g_{m1} = \sqrt{4KI_{D1}}$$

$$r_{o1} = \frac{1}{\lambda I_{D1}}$$

Commonly used technique for discrete circuit design. Difficult to gauge the M_1 current due to device parameters variation.

Biasing Network:
The resistors (R_{A1}, R_{B1}) determine the V_{GS} biasing and the V_{GS} biasing in turns determines the drain current (I_{D1}) of the transistor (M_1). All AC small signal parameters are related to the drain current of the transistor (I_{D1}).

Current Mirror Biasing for CS

Determine DC Biasing

$$\because V_{GS1}=V_{GS2} \quad and \quad M_1=M_2$$

$$\Rightarrow \boxed{I_{D1}=I_{D2}=I_{REF}}$$

$$I_{REF}=I_{D2}=K\left(V_{GS2}-V_{THN}\right)^2$$

$$\Rightarrow V_{GS2} \quad can \ be \ determined$$

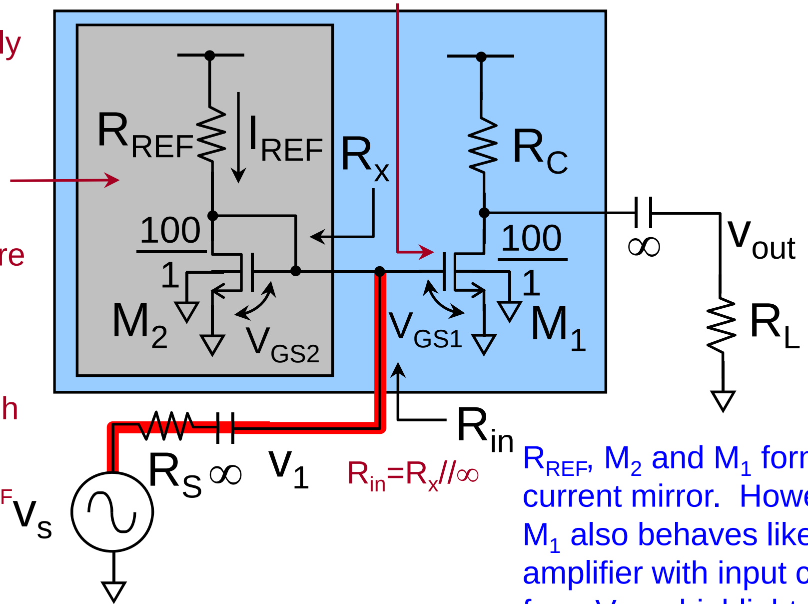
$$\because I_{REF}=\frac{V_{DD}-V_{GS2}}{R_{REF}}$$

$$\Rightarrow R_{REF} \quad can \ be \ determined$$

In the design, the desired I_{D1} is usually given

Biasing Network: The current mirror configuration ensure M_1 and M_2 has the same current ($I_{D1}=I_{D2}$). The current flow through M_2 ($I_{D2}=I_{REF}$) is determined by R_{REF}

Resistance looking into the gate is ∞



R_{REF}, M_2 and M_1 form a current mirror. However, M_1 also behaves like CS amplifier with input coming from V_s as highlighted in red.

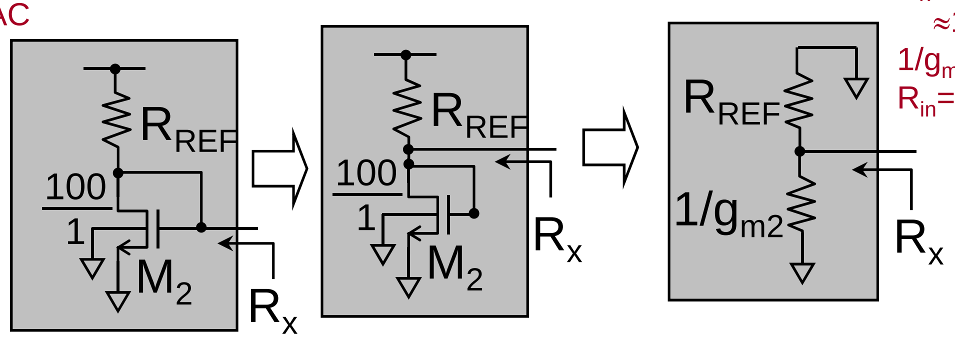
Determine AC Small Signal Parameter

$$g_{m1}=\sqrt{4KI_{D1}}$$

$$r_{o1}=1/\left(\lambda I_{D1}\right)$$

Replace the DC voltage with AC ground

Replace the diode-connected transistor M_2 with the equivalent resistor $1/g_{m2}$



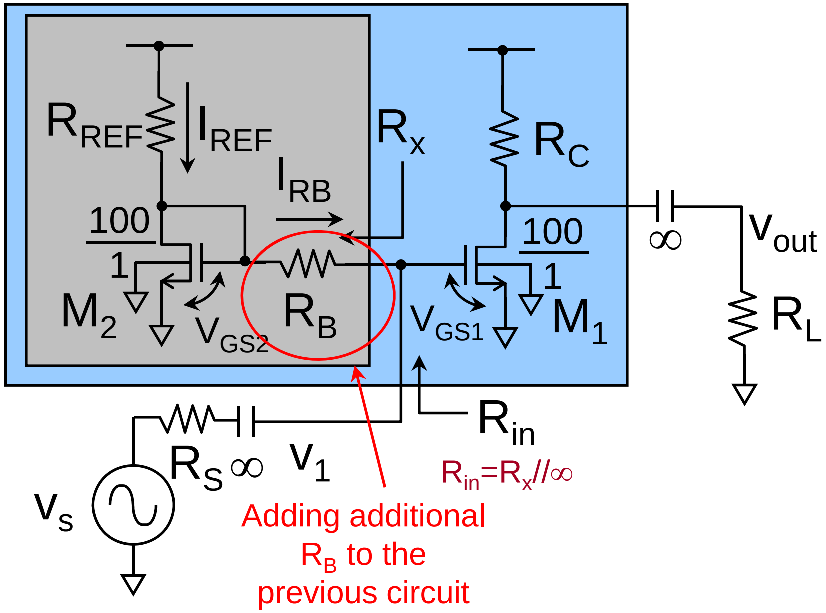
$R_x=(1/g_{m2})//R_{REF}$
$\approx 1/g_{m2}$
$1/g_{m2} \ll R_{REF}$
$R_{in}=R_x$

R_{in} too small

Current Mirror Biasing for CS

Because there is no current flowing through R_B (I_{RB}=0) ⇒ $V_{GS1}=V_{GS2}$, i.e. there is no change in DC biasing condition by adding additional R_B

Commonly used technique for integrated circuit design as R_{REF} is externally adjustable to give the desired current

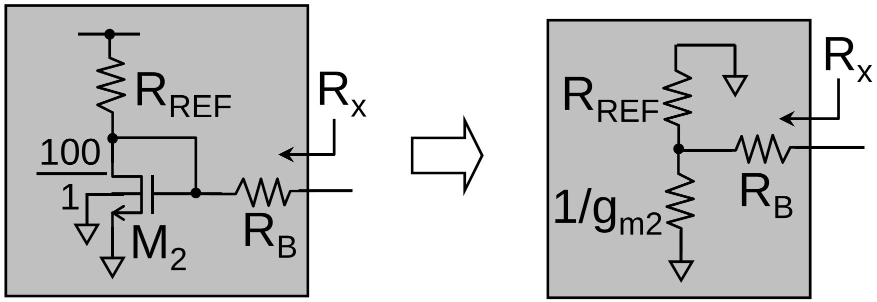


$R_{in}=R_x//\infty$

Adding additional R_B to the previous circuit

Replace the DC voltage with AC ground

Replace the diode-connected transistor M_2 with the equivalent resistor $1/g_{m2}$

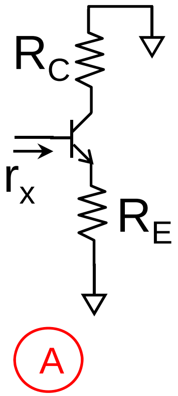
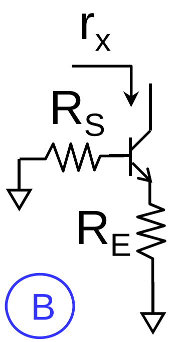
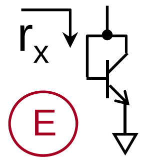
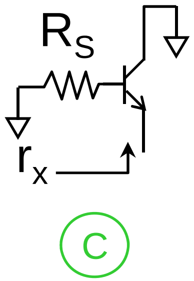
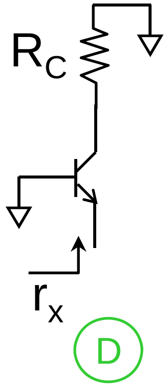


$R_x=R_B+(1/g_{m2})//R_{REF}$
$\approx R_B+1/g_{m2}$
$1/g_{m2} << R_{REF}$
$R_{in}=R_x$

R_{in} can be made large by choosing large R_B

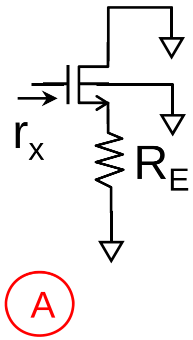
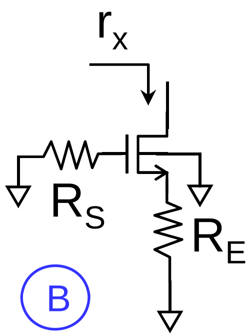
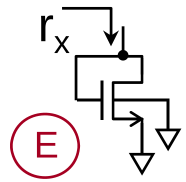
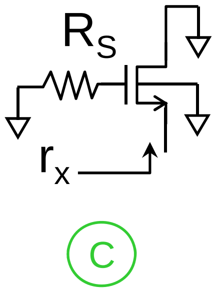
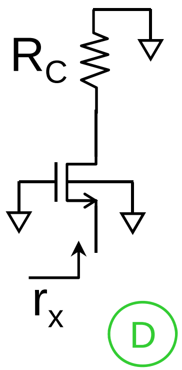
BJT Equivalent Resistance Summary (Table 1)

Blue: look into collector terminal   Red: look into base terminal   Green: look into emitter terminal

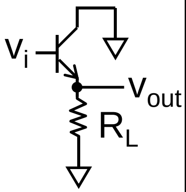
Conf	r_x	Conf	r_x	Conf	r_x
	$r_\pi + (1+\beta)R_E$ $\approx r_\pi(1+g_m R_E)$		$r_o\left\{1+g_m\left[(r_\pi+R_S)//R_E\right]\left(\frac{r_\pi}{r_\pi+R_S}\right)\right\}$ *If* $R_S=0$ *and* $r_\pi \ll R_E$ $\Rightarrow r_{x,\max}=r_o(\beta+1)$		$\frac{1}{g_m}$
	$\frac{R_S+r_\pi}{1+\beta}//r_o$ $\approx \frac{R_S}{1+\beta}+\frac{1}{g_m}$		$\frac{1}{g_m}\times\frac{r_o+R_C}{r_o+R_C/\beta}$		

MOS Equivalent Resistance Summary (Table 2)

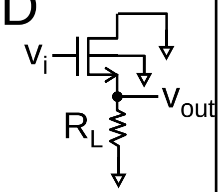
Blue: look into drain terminal Red: look into gate terminal Green: look into source terminal

Conf	r_x	Conf	r_x	Conf	r_x
	∞		$r_o\left[1+(g_m-g_{mb})R_E\right]$		$\dfrac{1}{g_m}$
	$\dfrac{1}{g_m-g_{mb}}$		$\dfrac{1}{g_m-g_{mb}}\times\dfrac{r_o+R_C}{r_o}$		

BJT Amplifier Configurations (Table 3)

BJT	G_m	A_V
CE A	g_m	Derive Based on 2-port Network
CB B	$-g_m$	Derive Based on 2-port Network
CC C 	Too Complex To Be Useful	$\dfrac{g_m R_L}{1+g_m R_L}$
CE with Emitter Degeneration D	$\dfrac{g_m}{1+g_m R_E}$	Derive Based on 2-port Network

MOS Amplifier Configurations (Table 4)

MOS	G_m	A_V
CS A	g_m	Derive Based on 2-port Network
CG B	$-(g_m - g_{mb})$ *Drop* g_{mb} *if no body effect*	Derive Based on 2-port Network
CD C 	Too Complex To Be Useful	$\dfrac{g_m R_L}{1+(g_m - g_{mb})R_L} \approx \dfrac{g_m}{g_m - g_{mb}}$ *Drop* g_{mb} *if no body effect*
CS with R_E D	$\dfrac{g_m}{1+(g_m - g_{mb})R_E}$ *Drop* g_{mb} *if no body effect*	Derive Based on 2-port Network

Lecture Summary

- **Analyze CC/CD, CS/CE with degeneration amplifiers.**
- **Analyze MOSFET/BJT current mirror**

Reading Assignment

- **Reading: Reference Book (Sedra & Smith)**

Chapter 4, pp. 401 – 405, pp. 405 – 408. (CD)

Chapter 3, pp. 260 – 265. (CC)

Chapter 3, pp. 252 – 257. (CE with degeneration)

Chapter 4, pp. 399 – 401. (CS with degeneration)

Chapter 6, pp. 587 – 595. (BJT and MOS current mirror)