APPENDIX

The following information may be used where applicable:

Elementary charge	q	$= 1.602 \times 10^{-19}$ C
Permittivity of free space	ε_0	$= 8.854 \times 10^{-14}$ F cm^{-1}
Room temperature		= 300 K
Thermal voltage at 300 K	V_T	$= kT/q = 0.025$ V = 25 mV
Intrinsic carrier concentration of silicon at 300 K	n_i	$= 1.5 \times 10^{10}$ cm^{-3}
Relative permittivity of silicon	$\varepsilon_{r,}$ (Si)	= 11.7
Relative permittivity of silicon dioxide	$\varepsilon_{r,}$ (SiO_2)	= 3.9

Summary of Equations

I. Semiconductor Physics

Quantity	**Equation**	**Remarks**
Law of mass action	$p_0 n_0 = n_i^2$	$n_i = 1.5 \times 10^{10}$ cm^{-3} for Si at $T = 300$ K
Charge neutrality	$p + N_D = n + N_A$	
Carrier concentrations in *p*-type semi-conductor at thermal equilibrium (cm^{-3})	$p_{p0} = N_A$ $n_{p0} = \dfrac{n_i^2}{N_A}$	$T = 300$ K N_A = net *p*-type doping concentration
Carrier concentrations in *n*-type semi-conductor at thermal equilibrium (cm^{-3})	$n_{n0} = N_D$ $p_{n0} = \dfrac{n_i^2}{N_D}$	$T = 300$ K N_D = net *n*-type doping concentration
Drift current density (A cm^{-2})	$J_{drift} = J_{p,drift} + J_{n,drift}$ $J_{p,drift} = qp\mu_p E$ $J_{n,drift} = qn\mu_n E$ $J_{drift} = q(p\mu_p + n\mu_n)E = \sigma E$	σ is conductivity, E is electric field
Conductivity (Ω cm)$^{-1}$	$\sigma = q(p\mu_p + n\mu_n) = \dfrac{1}{\rho}$	ρ is resistivity
Diffusion current density (A cm^{-2})	$J_{diff} = J_{p,diff} + J_{n,diff}$ $J_{p,diff} = -qD_p \dfrac{dp}{dx}$ $J_{n,diff} = qD_n \dfrac{dn}{dx}$ $J_{diff} = -qD_p \dfrac{dp}{dx} + qD_n \dfrac{dn}{dx}$	
Einstein Relation	$\dfrac{D_p}{\mu_p} = \dfrac{D_n}{\mu_n} = \dfrac{kT}{q} = V_T$	$k = 1.381 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K $q = 1.602 \times 10^{-19}$ C $V_T \approx 0.025$ V at $T = 300$ K.

II. pn Junction

Quantity	Equation	Remarks
Junction built-in (or barrier) voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	$V_0 \cong 0.6$ - 0.8 V for Si at $T = 300$ K. N_D is the doping concentration of the *n*-region. N_A is the doping concentration of the *p*-region.
Width of depletion region (cm or µm)	$W_{dep} = x_p + x_n$ $= \sqrt{\frac{2\varepsilon_s}{q}\left[\frac{1}{N_A} + \frac{1}{N_D}\right](V_0 - V)}$ $x_p = \frac{N_D}{N_D + N_A} W_{dep}$, $x_n = \frac{N_A}{N_D + N_A} W_{dep}$	V is positive for forward bias and negative for reverse bias $\varepsilon_s = 11.7 \times \varepsilon_0$ for Si $\varepsilon_0 = 8.854 \times 10^{-14}$ F cm^{-1}. x_p is the depletion width in the *p*-region. x_n is the depletion width in the *n*-region.
Minority carrier concentration at the edge of depletion region (cm^{-3})	*p*-side: $n_p(-x_p) = n_{p0}\, e^{V/V_T}$ *n*-side: $p_n(x_n) = p_{n0}\, e^{V/V_T}$	= thermal equilibrium minority carrier concentration x e^{V/V_T}
Minority carrier concentration distributions in the neutral n- and p-regions (cm^{-3})	*n*-side : $p_n(x) = p_{n0} + [p_n(x_n) - p_{n0}]\, e^{-(x - x_n)/L_p}$ *p*-side : $n_p(x) = n_{p0} + [n_p(-x_p) - n_{p0}]\, e^{(x + x_p)/L_n}$	L_p, L_n are the diffusion lengths of the minority carriers in the *n*-side and *p*-side respectively. In the equations, $x > x_n$ on the *n*-side, and $x < -x_p$ on the *p*-side.
Current (A)	$I = I_p + I_n = I_s\left(e^{V/nV_T} - 1\right)$ $I_p = Aqn_i^2 \frac{D_p}{L_p N_D}\left(e^{V/nV_T} - 1\right)$ $I_n = Aqn_i^2 \frac{D_n}{L_n N_A}\left(e^{V/nV_T} - 1\right)$	n is a value between 1 and 2. I_P is the hole current injected into the neutral *n*-region. I_n is the electron current injected into the neutral *p*-region.
Saturation current (A)	$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$	
Amount of charge stored in (one side) of the depletion region (C)	$q_j = qx_p N_A A = qx_n N_D A$ $= q\left(\frac{N_A N_D}{N_A + N_D}\right) W_{dep} A$	

Depletion or junction capacitance (F)	$C_j = \frac{\varepsilon_s A}{W_{dep}}$ $C_j = A\left[\frac{q\varepsilon_s}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{(V_0 - V)}\right]^m$ $C_j = A \times C_{j0} / \left(1 - \frac{V}{V_0}\right)^m$ $C_{j0} = \left[\frac{q\varepsilon_s}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_0}\right]^m$	*m* is a value that ranges between 1/3 to 1/2. $\varepsilon_s = 11.7 \times \varepsilon_0$ for Si $\varepsilon_0 = 8.854 \times 10^{-14}$ F cm^{-1}.
Small signal (or incremental) resistance of the pn junction diode (Ω)	$r_d = \frac{nV_T}{I_D}$	I_D is the (d.c.) diode current at the operating point, *n* is a value between 1 and 2.

III. Bipolar Junction Transistor (BJT)

Quantity	**Equation**	**Remarks**
Magnitude of the diffusion current of the minority carriers (electrons) in the base of an npn BJT (A)	$\lvert i_{En} \rvert = \left\lvert qAD_n \frac{n_p(0) - n_p(w_B)}{0 - w_B} \right\rvert$	D_n is the diffusivity of the electrons in the base, w_B is the width of the neutral part of the base, $n_p(0)$ and $n_p(w_B)$ are the concentrations of the electrons on the base side of the edges of the E-B and C-B space charge regions respectively.
Collector current of an npn BJT (A)	$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$	
Collector saturation current of an npn BJT (A)	$I_S = qA\frac{D_n}{w_B}\frac{n_i^2}{N_{AB}}$	Note that the symbol I_S is also the symbol for the saturation current of a p-n junction diode, but the expression is different.
Base current of an npn BJT (A)	$i_B = \frac{I_S}{\beta}\exp\left(\frac{v_{BE}}{V_T}\right)$ where $\beta = \frac{D_n}{D_p}\frac{L_p}{w_B}\frac{N_{DE}}{N_{AB}}$	The expression for β here is for an npn BJT with a long emitter.
Common emitter current gain of a BJT (general definition)	$\beta = \frac{i_C}{i_B}$	
Common base current gain of a BJT (general definition)	$\alpha = \frac{i_C}{i_E}$	
Relationship between α and β	$\beta = \frac{\alpha}{1 - \alpha}$	

Collector current of an npn BJT, with Early effect taken into account (A)	$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)\left(1+\frac{v_{CE}}{V_A}\right)$	V_A is the Early voltage.
Transconductance in the small signal equivalent circuit of a BJT (Ω^{-1})	$g_m = \left.\frac{I_C}{V_T}\right\|_{\text{at bias point}}$	I_C is the value of the collector current at the d.c. operating (bias) point.
Input resistance in the small signal equivalent circuit of a BJT (Ω)	$r_\pi = \frac{\beta}{g_m}$	
Output resistance in the small signal equivalent circuit of a BJT (Ω)	$r_o \approx \frac{V_A}{I_C}$	I_C is the value of the collector current at the d.c. operating (bias) point.

IV. Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

Quantity	**Equation**	**Remarks**
Drain current of an n-MOSFET operating in the linear region	$i_D = \mu_n \frac{W}{L} C_{ox}\left[(v_{GS}-V_{TH})\, v_{DS} - \frac{1}{2}v_{DS}^2\right]$ $= 2K_n\left[(v_{GS}-V_{TH})\, v_{DS} - \frac{1}{2}v_{DS}^2\right]$	Valid for $v_{DS} \le v_{GS} - V_{TH}$
Drain current of an n-MOSFET operating in the saturation region	$i_D = i_{Dsat} = \frac{1}{2}\mu_n \frac{W}{L} C_{ox}(v_{GS}-V_{TH})^2$ $= K_n(v_{GS}-V_{TH})^2$	Valid for $v_{DS} \ge v_{GS} - V_{TH}$
Transconductance parameter of an n-MOSFET	$K_n = \frac{1}{2}\mu_n \frac{W}{L} C_{ox}$	
Drain current of a p-MOSFET operating in the linear region	$\lvert i_D\rvert = \mu_p \frac{W}{L} C_{ox}\left[(\lvert v_{GS}\rvert - \lvert V_{TH}\rvert)\,\lvert v_{DS}\rvert - \frac{1}{2}\lvert v_{DS}\rvert^2\right]$ $= 2K_p\left[(\lvert v_{GS}\rvert - \lvert V_{TH}\rvert)\,\lvert v_{DS}\rvert - \frac{1}{2}\lvert v_{DS}\rvert^2\right]$	Valid for $\lvert v_{DS}\rvert \le \lvert v_{GS}\rvert - \lvert V_{TH}\rvert$
Drain current of a p-MOSFET operating in the saturation region	$\lvert i_D\rvert = \lvert i_{Dsat}\rvert = \frac{1}{2}\mu_p \frac{W}{L} C_{ox}(\lvert v_{GS}\rvert - \lvert V_{TH}\rvert)^2$ $= K_p(\lvert v_{GS}\rvert - \lvert V_{TH}\rvert)^2$	Valid for $\lvert v_{DS}\rvert \ge \lvert v_{GS}\rvert - \lvert V_{TH}\rvert$
Transconductance parameter of a p-MOSFET	$K_p = \frac{1}{2}\mu_p \frac{W}{L} C_{ox}$	
Gate oxide capacitance (per unit gate area) of a MOSFET	$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$	

Drain-to-source resistance in the large signal model of the n-MOSFET operating in the linear region	$R_{DS} = \dfrac{1}{2K_n\left(V_{GS} - V_{TH}\right)}$	V_{GS} is the magnitude of the gate-to-source voltage, at the d.c. operating point.
Drain-to-source resistance in the large signal model of the p-MOSFET operating in the linear region	$R_{DS} = \dfrac{1}{2K_p\left(\lvert V_{GS}\rvert - \lvert V_{TH}\rvert\right)}$	V_{GS} is the magnitude of the gate-to-source voltage, at the d.c. operating point.
Transconductance in the small signal equivalent circuit of an n-MOSFET operating in the saturation region	$g_m = 2K_n\left(V_{GS} - V_{TH}\right) = 2\sqrt{K_n I_D}$	I_D and V_{GS} are the magnitudes of the drain current and gate-to-source voltage, respectively, at the d.c. operating point.
Transconductance in the small signal equivalent circuit of a p-MOSFET operating in the saturation region	$g_m = 2K_p\left(\lvert V_{GS}\rvert - \lvert V_{TH}\rvert\right) = 2\sqrt{K_p \lvert I_D\rvert}$	I_D and V_{GS} are the magnitudes of the drain current and gate-to-source voltage, respectively, at the d.c. operating point.
Output resistance in the small signal equivalent circuit of a MOSFET operating in the saturation region	$r_o \approx \dfrac{V_A}{I_D} = \dfrac{1}{\lambda I_D}$	I_D is the magnitude of the drain current at the d.c. operating point.
Drain current of an n-MOSFET operating in the saturation region, including channel length modulation effect	$i_D = K_n(v_{GS} - V_{TH})^2(1 + \lambda v_{DS})$	Valid for $v_{DS} \ge v_{GS} - V_{TH}$ λ is the channel length modulation factor.
Drain current of an p-MOSFET operating in the saturation region, including channel length modulation effect	$\lvert i_D\rvert = K_p(\lvert v_{GS}\rvert - \lvert V_{TH}\rvert)^2(1 + \lambda \lvert v_{DS}\rvert)$	Valid for $\lvert v_{DS}\rvert \ge \lvert v_{GS}\rvert - \lvert V_{TH}\rvert$ λ is the channel length modulation factor.