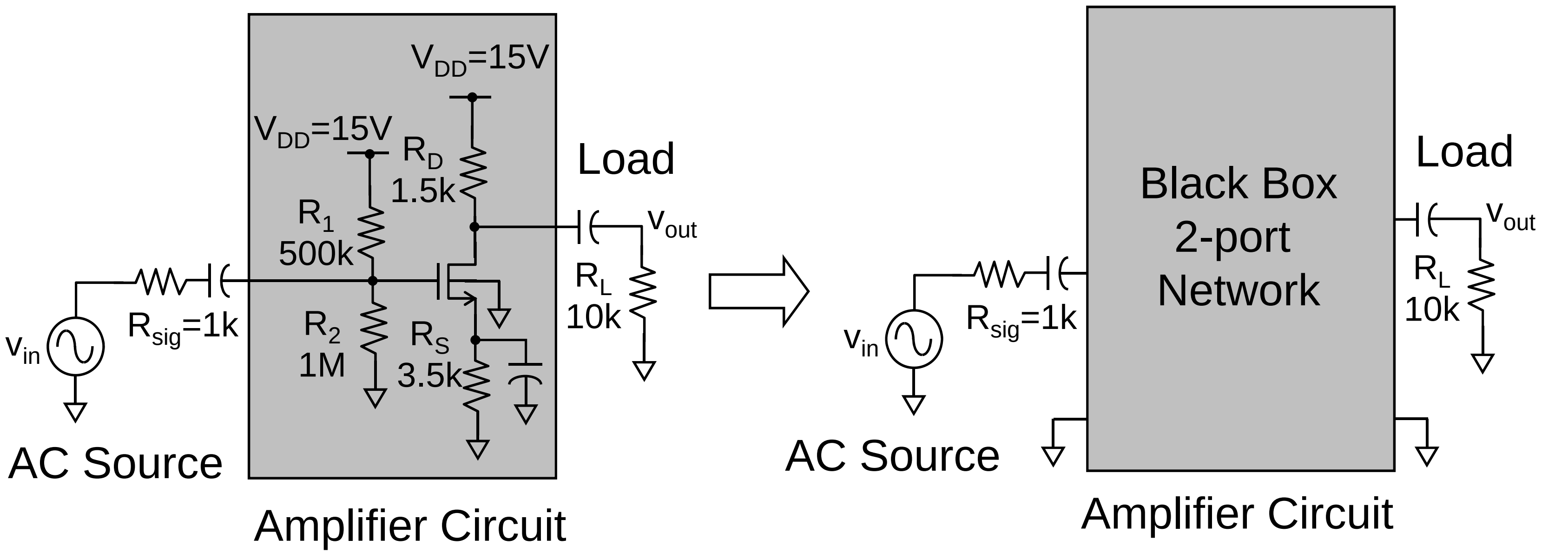
EE2021
Devices and Circuits

Two-Port Network,
Single Transistor Amplifiers -
CE, CS, CB, CG

Lecture Outline

- **2-Port Network (Voltage and Transconductance)**
  Important parameters that characterize 2-port;
- **Single Transistor Amplifiers,**
  CE/CS, CB/CG.

Modeling Amplifier Circuit using 2-Port Network

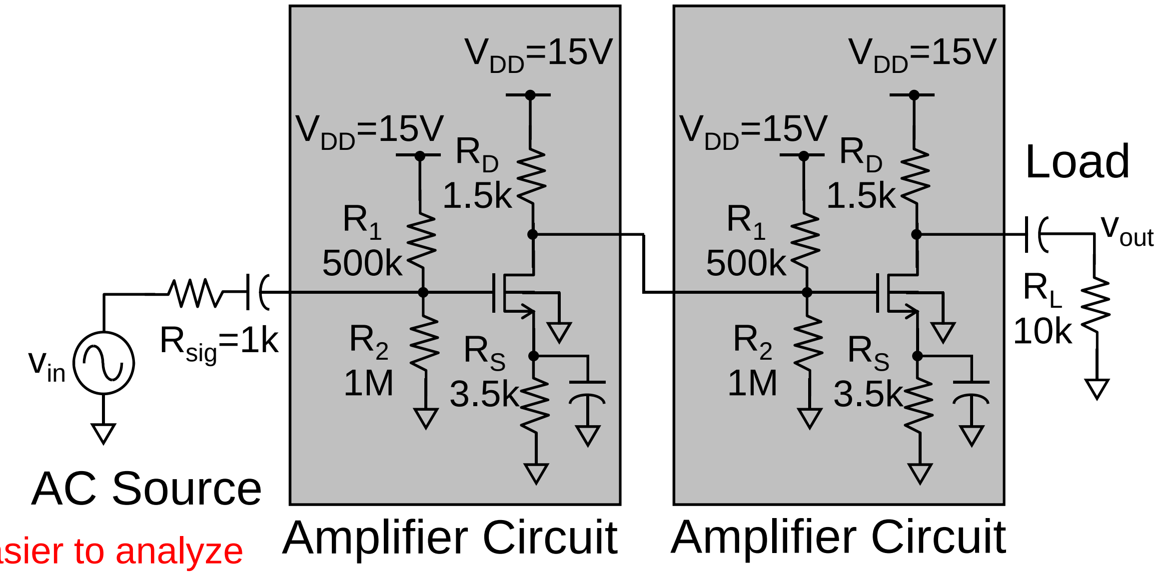


Can we fully characterize the complicated amplifier circuit using simple black box (2-port network) with limited set of parameters?
For example, similar to a complicated linear network replaced by Thevenin Equivalent with only one Thevenin Voltage source and one Thevenin equivalent resistor, i.e. only 2 parameters.

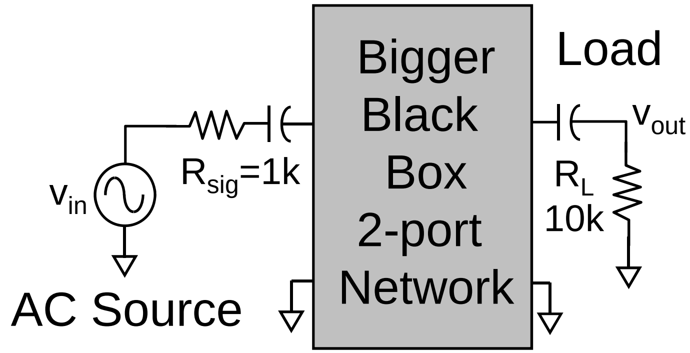
Analogy of Two-Port Network to Investment Scheme

- We are only interested in the relationship between output and input
- We don’t care about what happens in between

Why Use 2-port Network?

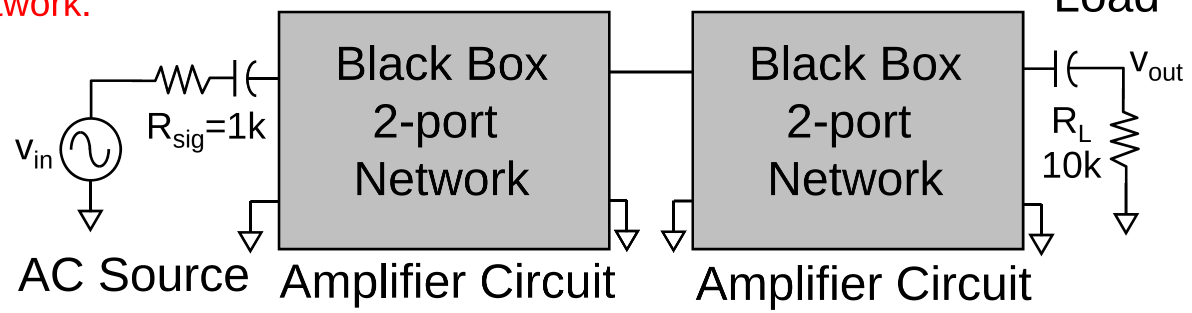


Cascaded 2-port networks can be combined into a bigger 2-port network.

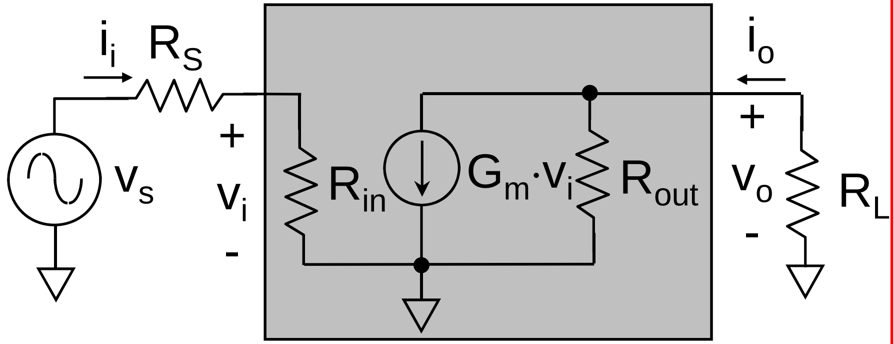


Easier to analyze complicated circuit (e.g., multi-stage amplifier) as each amplifier is simplified to its equivalent 2-port network.

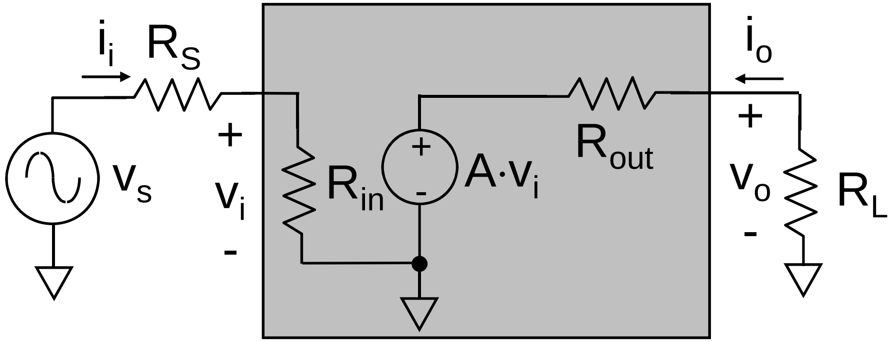
Similar concept as Thevenin equivalent or Norton equivalent.



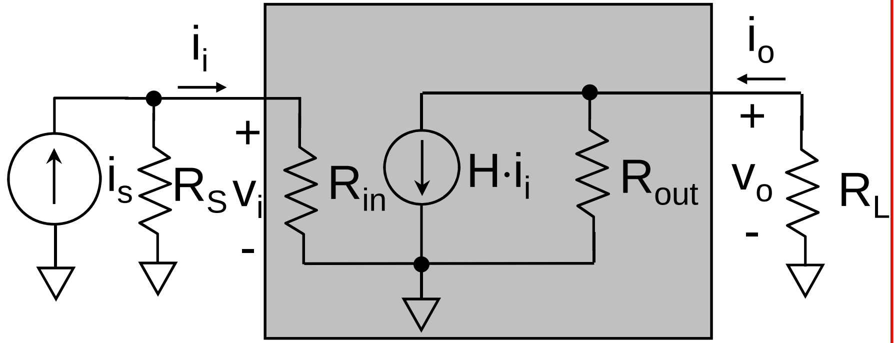
Different 2-Port Amplifier Models



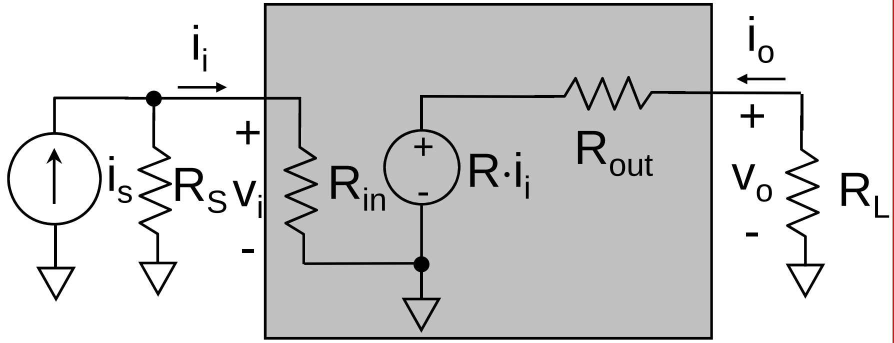
2-Port Transconductance Amplifier



2-Port Voltage Amplifier



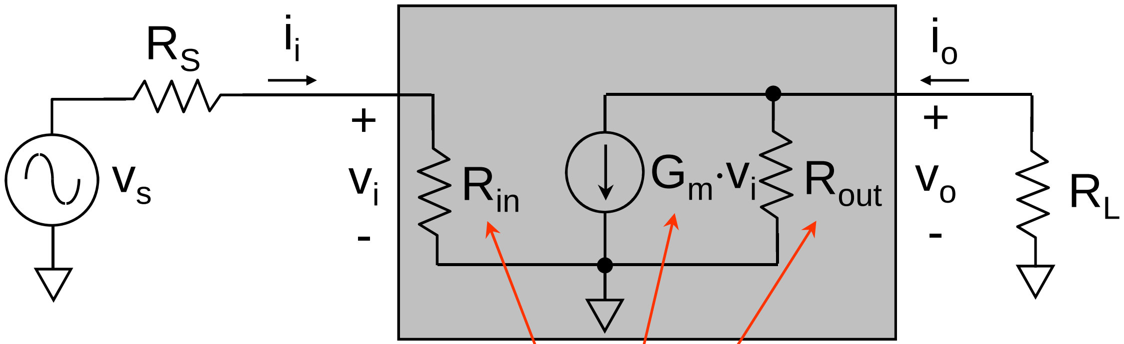
2-Port Current Amplifier



2-Port Trans-resistance Amplifier

- They all can be used to represent same amplifier circuit.
- Same concept as Thevenin equivalent versus Norton equivalent.
- In some scenario, it is easier to consider the amplifier as 2-port transconductance amplifier to simplify the analysis. In some cases, it is easier to consider the amplifier as 2-port voltage amplifier to simplify the analysis. Same reasoning applies to 2-port current and trans-resistance amplifier.
- In this module, only 2-port transconductance and 2-port voltage amplifiers are considered as they simplify the analysis. (Based on experience from circuit designers)

Two Port Network –
Transconductance Amplifier



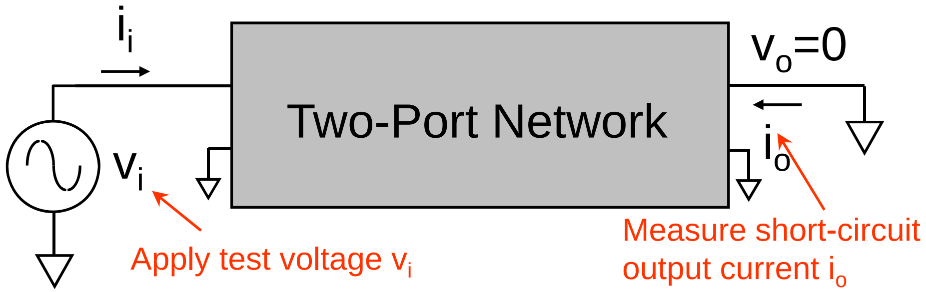
R_{in}, G_m and R_{out} characterize the whole two ports network used to represent the transconductance amplifier

- After converting amplifier circuit to 2-ports network, it is easier to cascade multiple of them together and analyze (Refer to 6A-5)

Characteristic:

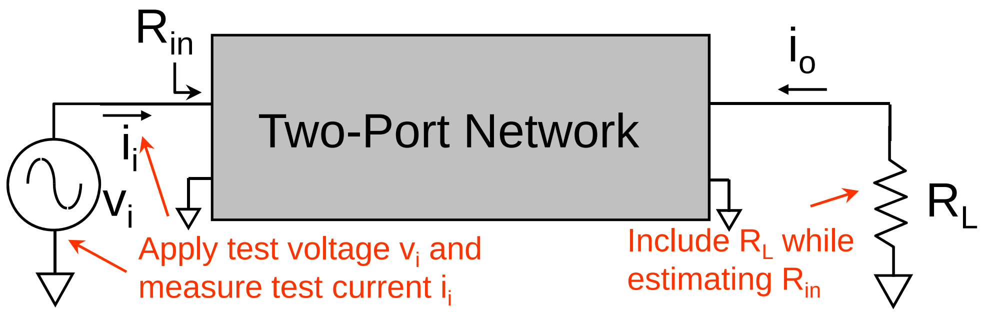
- High input resistance (R_{in})
- High output resistance (R_{out})
- Transconductance amplifier (Voltage-to-current : $G_m v_i$)

Transconductance Amplifier – Parameter Characterizations



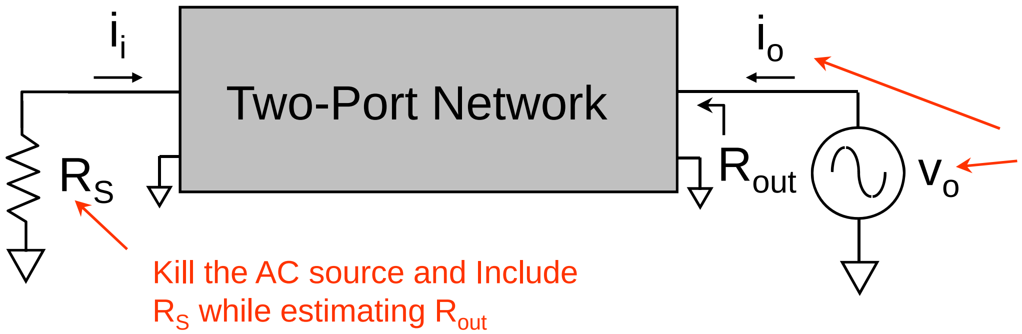
$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0}$$

By short-circuit the output, current through R_{out} would be zero, and the measured current i_o would be $G_m v_i$.



$$R_{in} = \frac{v_i}{i_i}$$

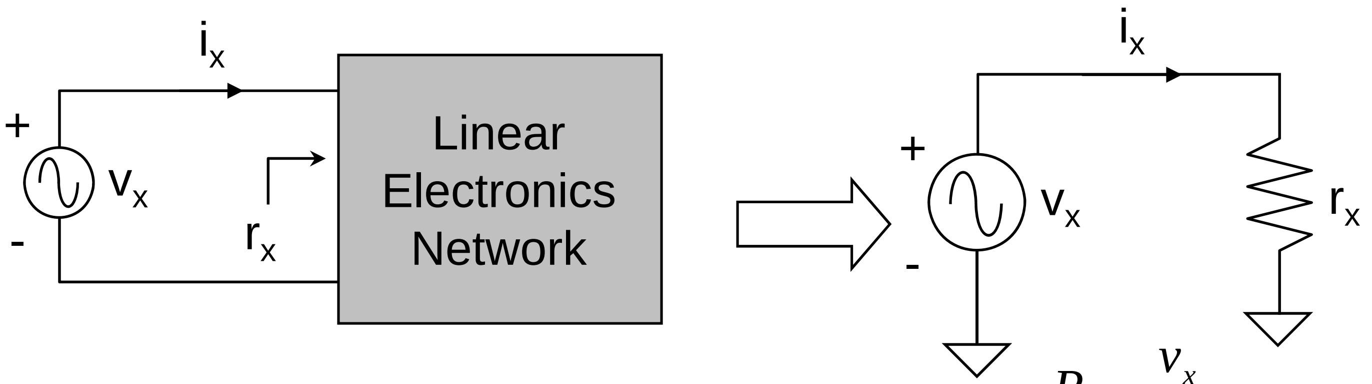
R_{in} is the equivalent resistance looking into the input port, R_L needs to be included for the estimation of R_{in}.



$$R_{out} = \frac{v_o}{i_o}$$

R_{out} is the equivalent resistance looking into the output port, R_S needs to be included for the estimation of R_{out}.

Equivalent Resistance

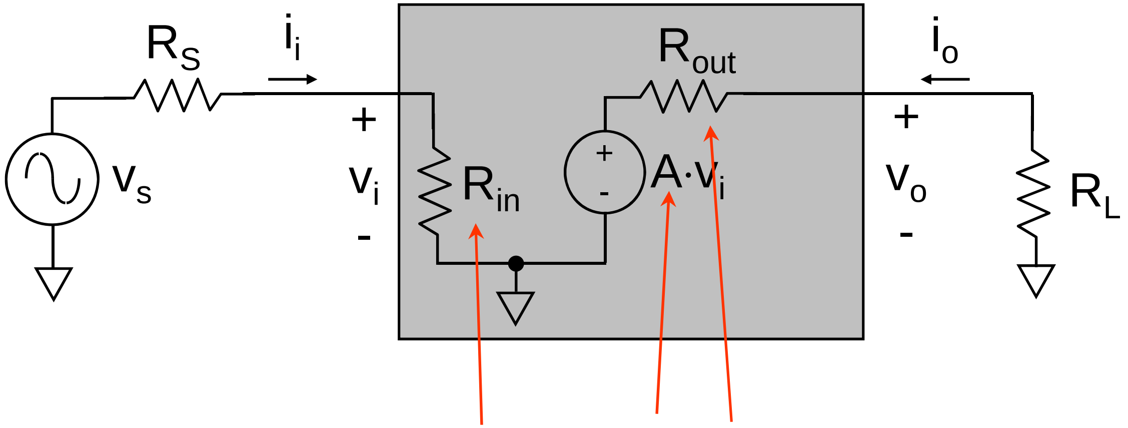


$$R_x = \frac{v_x}{i_x}$$

If the linear network draw current of i_x under apply test voltage of v_x, the linear network behave similarly to a simple resistor R_x obeying the Ohm's law

- Similar concept as Thevenin equivalent
- For a complicated LINEAR network, the current-voltage relationship can be modeled as a simple equivalent resistor

Two-Port Network – Voltage Amplifier



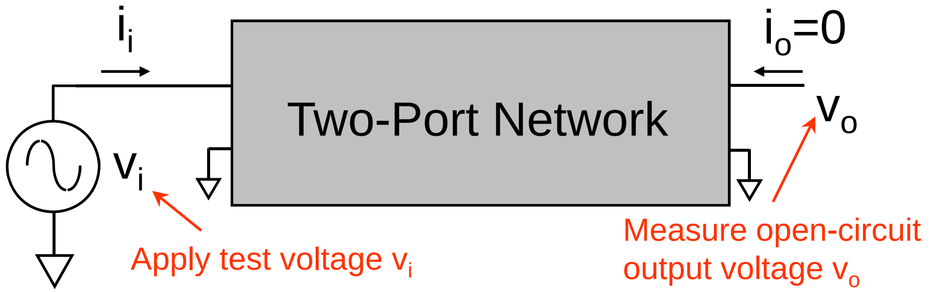
R_{in}, A and R_{out} characterize the whole two ports network used to represent the voltage amplifier

- After converting amplifier circuit to 2-port network, it is easier to cascade multiple of them together and analyze (Refer to 6A-5)

Characteristics:

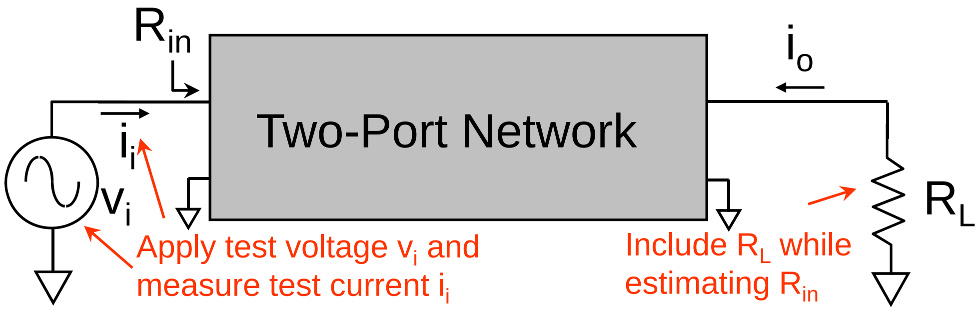
- High input resistance (R_{in})
- Low output resistance (R_{out})
- Voltage amplifier (A)

Two-Port Network – Voltage Amplifier



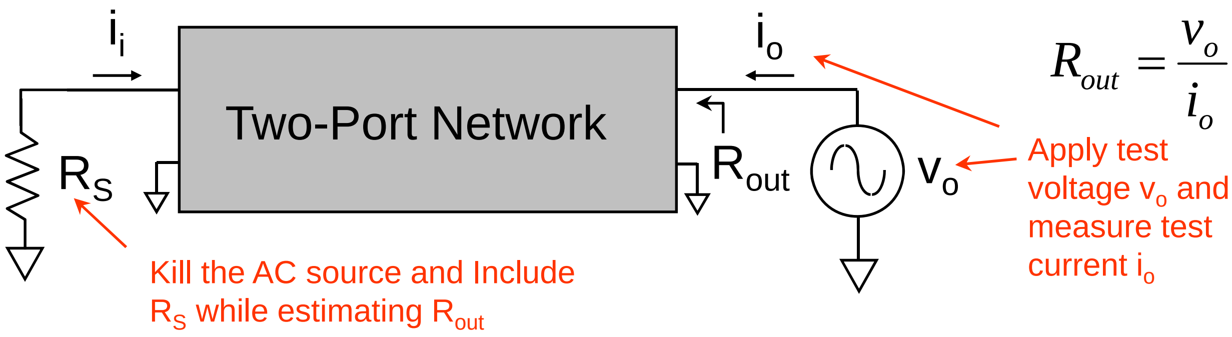
$$A = \left. \frac{v_o}{v_i} \right|_{i_o = 0}$$

By open-circuit the output, current i_o would be zero. Thus the voltage drop across R_{out} would be zero, and v_o would be same as Av_i.



$$R_{in} = \frac{v_i}{i_i}$$

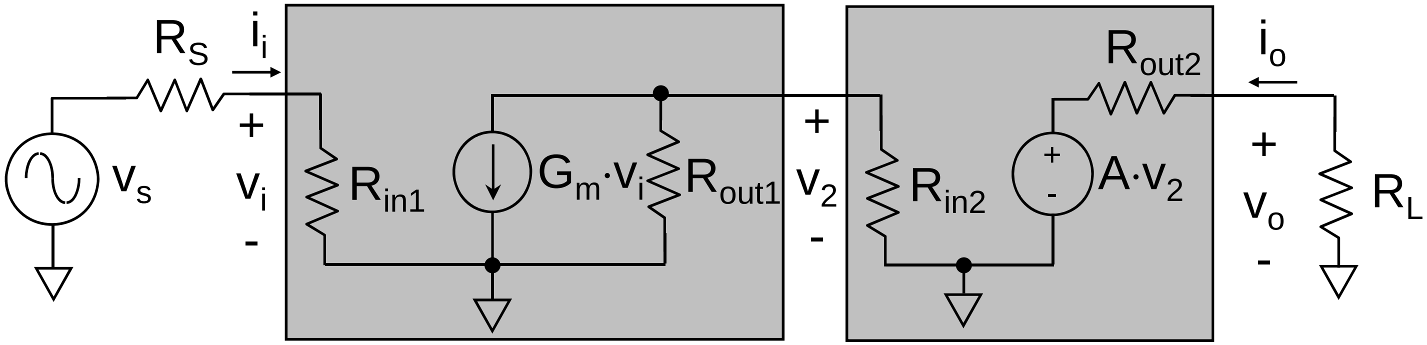
R_{in} is the equivalent resistance looking into the input port, R_L needs to be included for the estimation of R_{in}.



$$R_{out} = \frac{v_o}{i_o}$$

R_{out} is the equivalent resistance looking into the output port, R_S needs to be included for the estimation of R_{out}.

Example on Cascade Two-Port



$$v_i = \frac{R_{in1}}{R_{in1}+R_S} \times v_s$$

$$v_2 = -G_m v_i \times \left(R_{out1} // R_{in2}\right)$$

$$= -G_m \times \frac{R_{in1}}{R_{in1}+R_S} \times v_s \times \left(R_{out1} // R_{in2}\right)$$

$$v_o = A v_2 \times \frac{R_L}{R_{out2}+R_L}$$

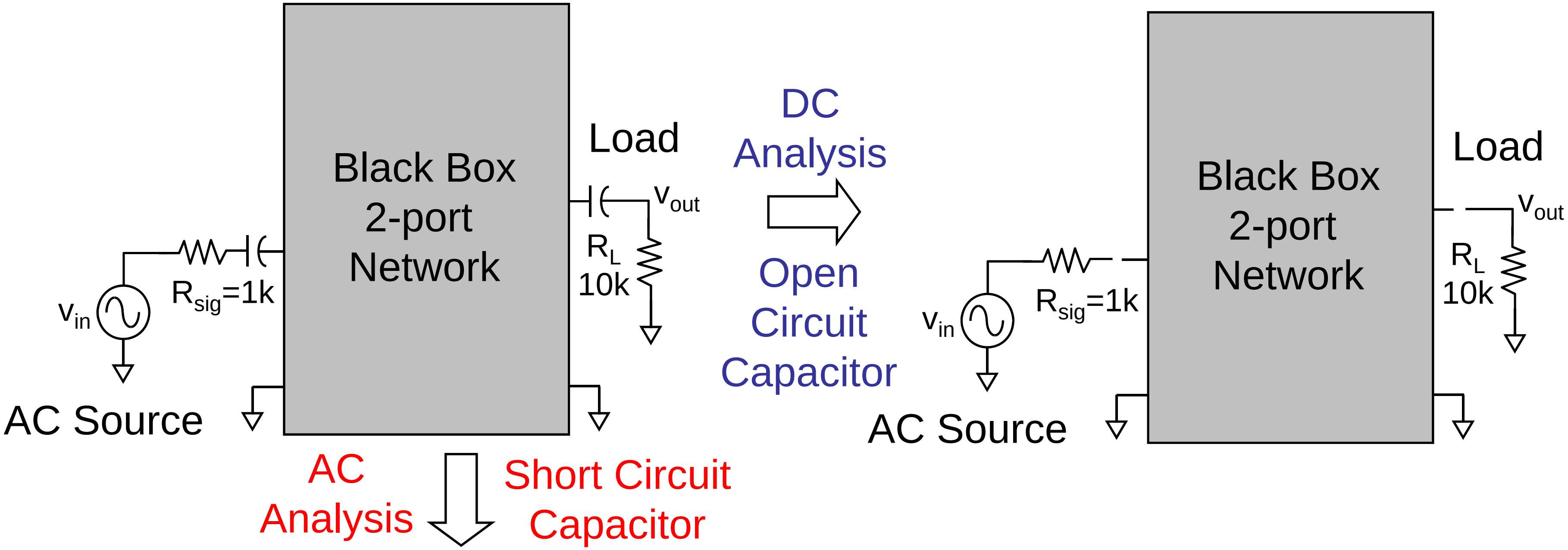
$$= \frac{AR_L}{R_{out2}+R_L}\left[-G_m \times \frac{R_{in1}}{R_{in1}+R_S} \times v_s \times \left(R_{out1} // R_{in2}\right)\right]$$

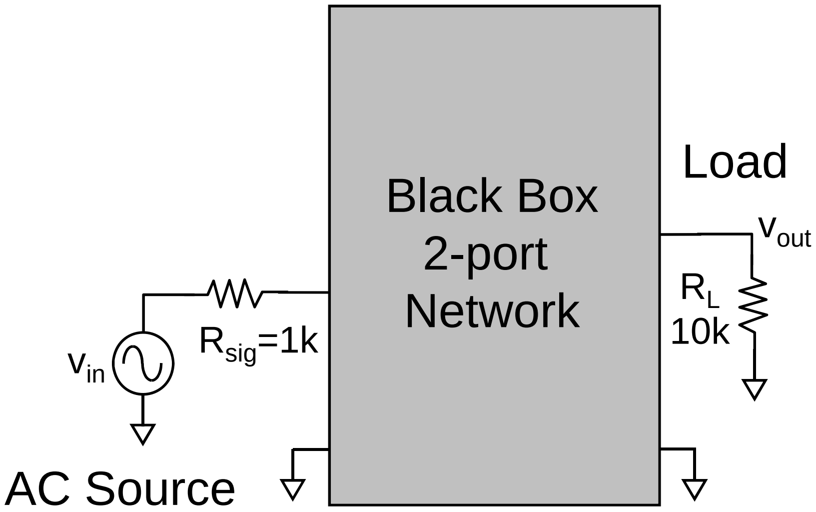
$$Gain = \frac{v_o}{v_s} = -\underbrace{\frac{R_{in1}}{R_{in1}+R_S}}_{1^{st}} \times \underbrace{G_m\left(R_{out1} // R_{in2}\right)}_{2^{nd}} \times \underbrace{A \times \frac{R_L}{R_{out2}+R_L}}_{3^{rd}} = \frac{v_i}{v_s} \times \frac{v_2}{v_i} \times \frac{v_o}{v_2}$$

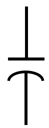
In amplifier design,

- How should you choose R_{in1} with respect to R_S?
- How should you choose R_{out2} with respect to R_L?
- Why?
- What if the source and load are antenna or equipment with long coaxial cable?

How to Treat Capacitors?







How to treat capacitors?
$Z_C = 1/j\omega C$
At DC (DC analysis), i.e. $\omega=0$, $Z_C \rightarrow \infty$
(open circuit)
At frequency of interest (AC analysis), C can always be chosen large enough such that $Z_C \rightarrow 0$ (short circuit)

Steps for Circuit Analysis

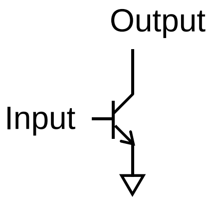
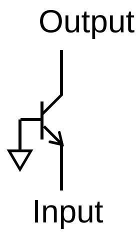
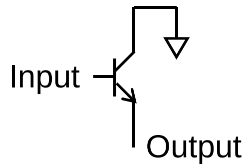
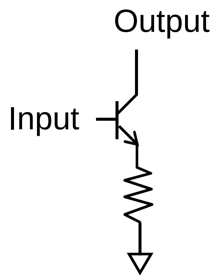
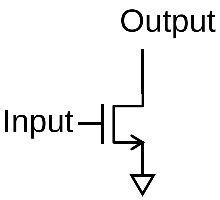
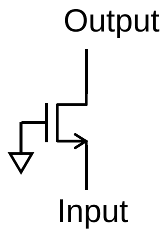
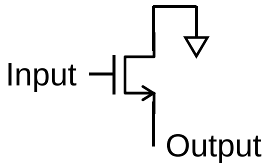
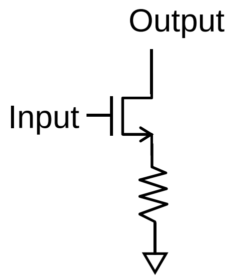
DC Analysis

1. Remove AC source and load section
2. Replace capacitor with open circuit
3. Obtain DC biasing
4. Obtain AC small signal parameter, such as g_m, r_π, r_o

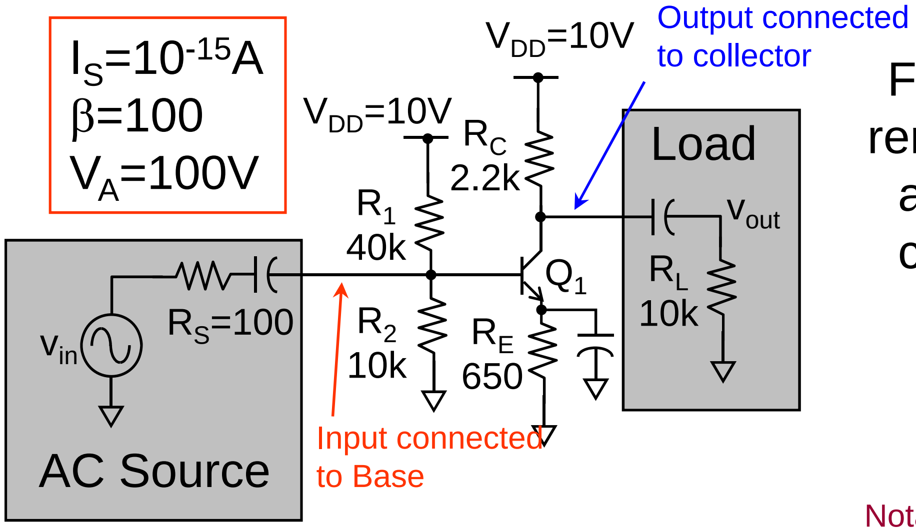
AC Analysis

1. DC voltage source $\Rightarrow$ short circuit to AC ground
   DC current source $\Rightarrow$ open circuit
2. Replace DC block capacitor with short circuit
3. Replace transistor with small signal equivalent circuit, such as hybrid-π
4. Finding the two-port parameter equivalent, such as R_{in}, R_{out}, G_m and A

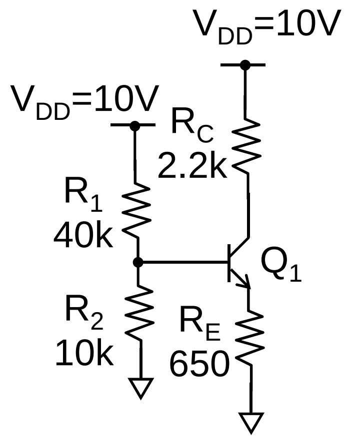
8 Amplifier Configurations

 Input – Base Output – Collector Neither – Emitter ⇒ CE	 Input – Emitter Output – Collector Neither – Base ⇒ CB	 Input – Base Output – Emitter Neither – Collector ⇒ CC	 Input – Base Output – Collector Neither – Emitter (with resistor) ⇒ CE with degeneration
 Input – Gate Output – Drain Neither – Source ⇒ CS	 Input – Source Output – Drain Neither – Gate ⇒ CG	 Input – Gate Output – Source Neither – Drain ⇒ CD	 Input – Gate Output – Drain Neither – Source (with resistor) ⇒ CS with degeneration

Common Emitter (CE) Amplifier



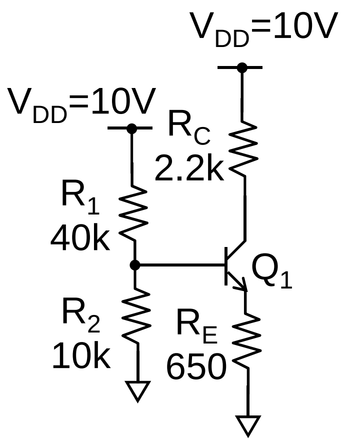
For DC analysis, remove AC source and load, open-circuit capacitor



Notation:
$V_{B,Q1}$: Base voltage of Q_1
$V_{E,Q1}$: Emitter voltage of Q_1
$V_{C,Q1}$: Collector voltage of Q_1

- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Base, output connected to Collector, Emitter connected to neither input nor output $\Rightarrow$ Common Emitter (CE)
- DC Analysis refer to “BJT Lecture Notes” slide 29

DC Analysis for CE Amplifier



$I_S=10^{-15}$A
β=100
V_A=100V

Determine DC biasing

$$\textit{Assume} \quad I_{B,Q1} << I_{R1}, I_{R2}$$

$$\Rightarrow \textit{voltage divider method}$$

$$V_{B,Q1} = \frac{10k}{40k + 10k} \times 10 = 2V$$

$$V_{E,Q1} = 2V - 0.7V = 1.3V$$

$$I_{C,Q1} \approx I_{E,Q1} = \frac{1.3V}{650} = 2mA$$

$$I_{B,Q1} \approx 20\mu A$$

$$I_{R1} \approx I_{R2} \approx 200\mu A >> I_{B,Q1}$$

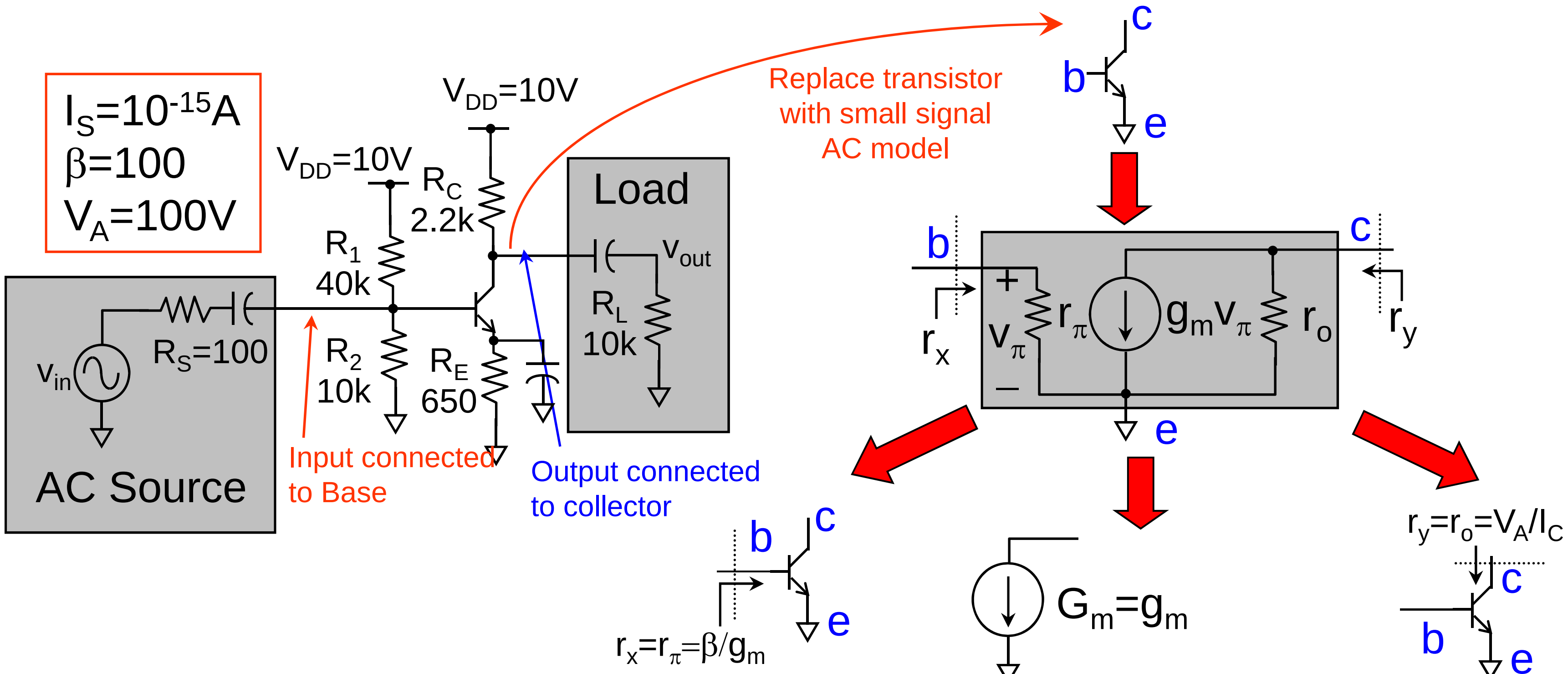
Determine AC small signal parameter

$$g_m = \frac{I_C}{V_T} = 80mA/V$$

$$r_\pi = \frac{\beta}{g_m} = 1.25k\Omega$$

$$r_o = \frac{V_A}{I_C} = 50k\Omega$$

Common Emitter (CE)

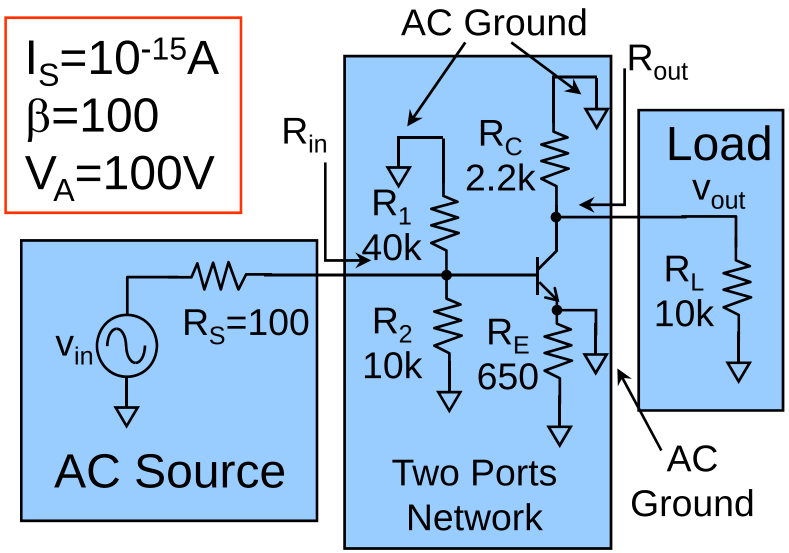


Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the base (r_x) is directly given by r_π. **No need to rederive.**

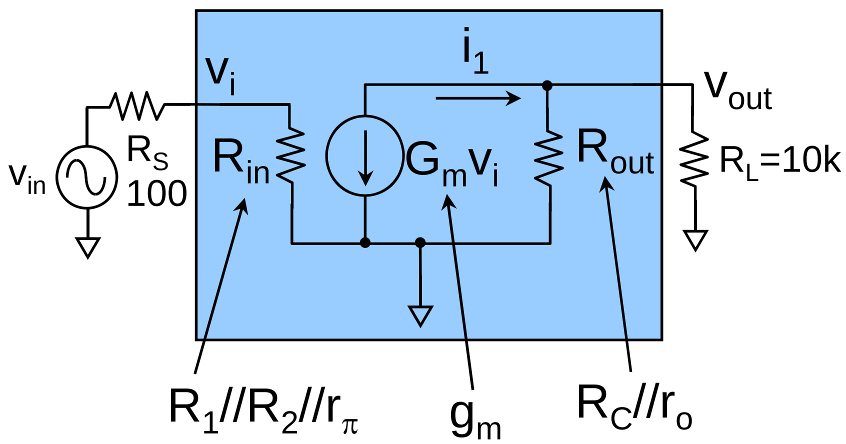
Important Result:
For CE, G_m is directly given by g_m. **No need to rederive.**

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the collector (r_y) is directly given by r_o. **No need to rederive.**

AC Analysis for CE



2-port Transconductance Amplifier



Wire split means resistors in parallel

g_m=80mA/V
r_π=1.25k
r_o=50k

$$v_i = \frac{R_{in}}{R_S + R_{in}} v_{in} = \frac{R_1 // R_2 // r_\pi}{R_S + R_1 // R_2 // r_\pi} v_{in} \approx v_{in}$$

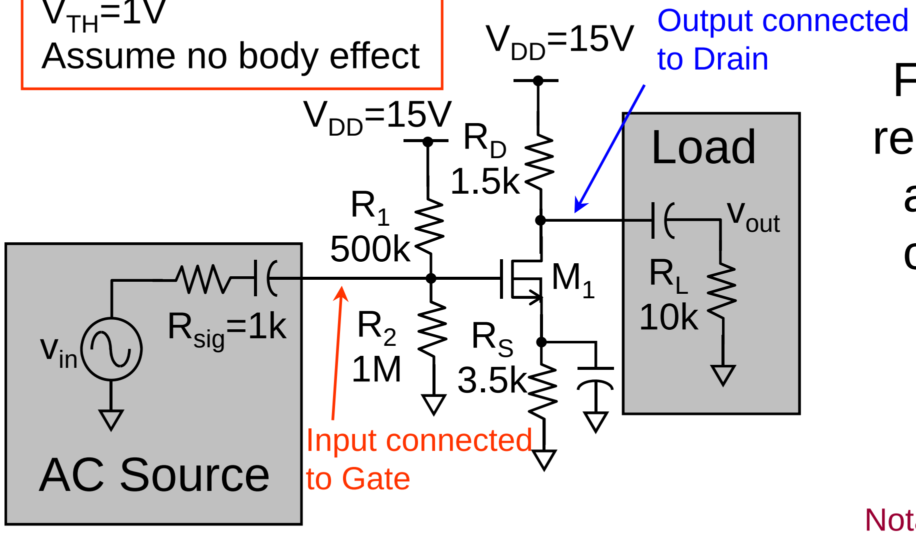
$$i_1 = -g_m v_i \approx -g_m v_{in}$$

$$v_{out} = i_1 \times \left[R_{out} // R_L\right] = i_1 \times \left[(R_C // r_o) // R_L\right] = -g_m (R_C // r_o // R_L) v_{in}$$

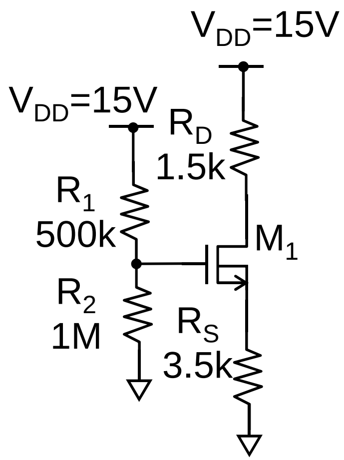
$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = -g_m (R_C // r_o // R_L) = -144.3$$

Common Source (CS) Amplifier

K_n=500μA/V^2
V_{TH}=1V
Assume no body effect



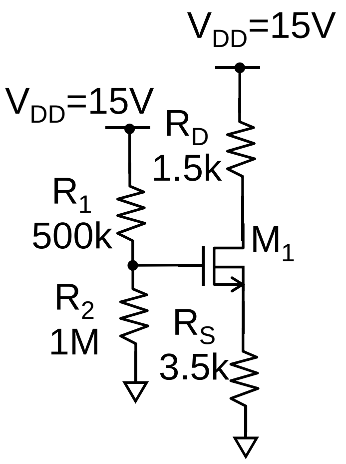
For DC analysis, remove AC source and load, open-circuit capacitor



Notation:
$V_{G,M1}$: Drain voltage of M_1
$V_{S,M1}$: Source voltage of M_1
$V_{D,M1}$: Drain voltage of M_1
$V_{B,M1}$: Body voltage of M1

- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Gate, output connected to Drain, Source connected to neither input nor output ⇒ Common Source (CS)
- DC Analysis refer to “MOSFET Lecture Notes” slide 29

DC Analysis for CS Amplifier



K_n=500μA/V^2
V_{TH}=1V
Assume no body effect

Determine DC biasing

$$V_{G,M1} = \frac{1M}{500k + 1M} \times 15 = 10V$$

Square Law :

$$I_{D,M1} = K_n (V_{GS} - V_{TH})^2$$
$$= K_n (V_{G,M1} - V_{S,M1} - V_{TH})^2$$
$$= 500\mu (9 - V_{S,M1})^2 \cdots (1)$$

Ohm's Law :

$$I_{D,M1} = I_{S,M1} = \frac{V_{S,M1}}{R_S} = \frac{V_{S,M1}}{3.5k} \cdots (2)$$

$$\Rightarrow \frac{V_{S,M1}}{3.5k} = 500\mu (9 - V_{S,M1})^2$$

$$1.75 V_{S,M1}^2 - 32.5 V_{S,M1} + 141.75 = 0$$

$$\Rightarrow V_{S,M1} = 6.94 \ \ or \ \ 11.63$$

$$11.63 > V_{G,M1} \Rightarrow invalid$$

$$I_{D,M1} = 2mA$$

$$V_{DS} = V_{DD} - I_{D,M1} R_D - V_{S,M1} = 5.06$$

$$V_{GS} - V_{TH} = 2.06$$

$$\Rightarrow V_{DS} > V_{GS} - V_{TH}$$

Device operates at saturation region

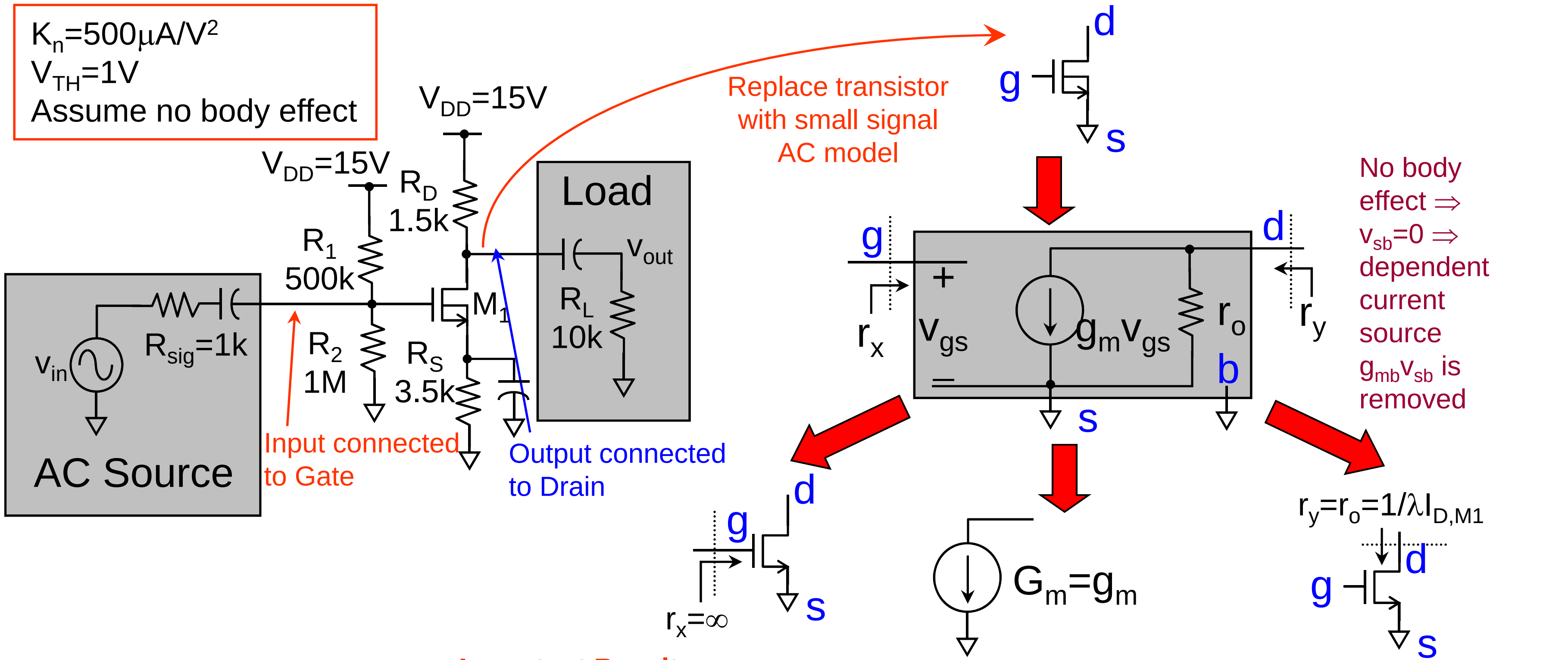
Determine AC small signal parameter

$$g_m = 2\sqrt{K_n I_{D,M1}} = 2mA/V$$

$$r_o = \frac{1}{\lambda I_{D,M1}} = \infty$$

Need to check for device operation region to ensure Square Law formula can be applied.

Common Source (CS)



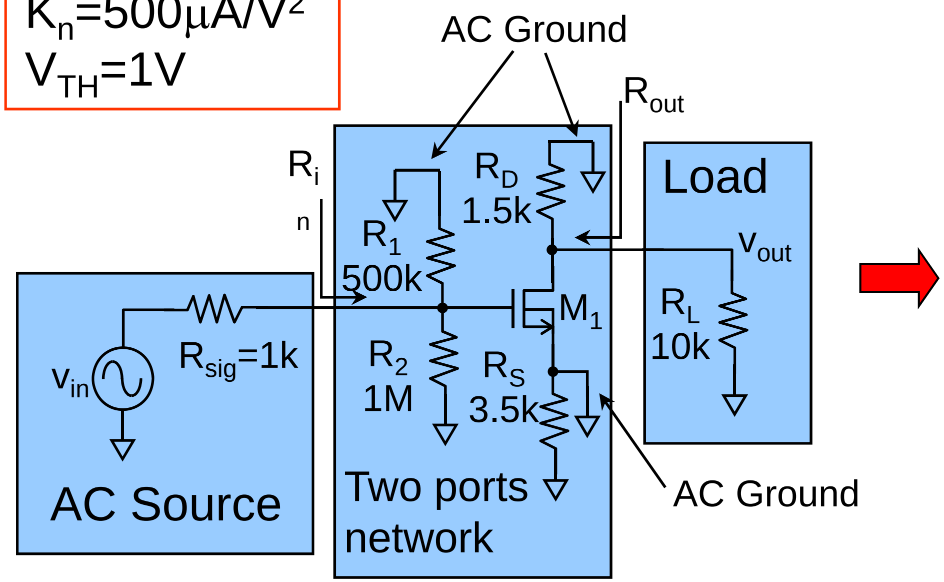
Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the gate (r_x) is open circuit.
No need to rederive.

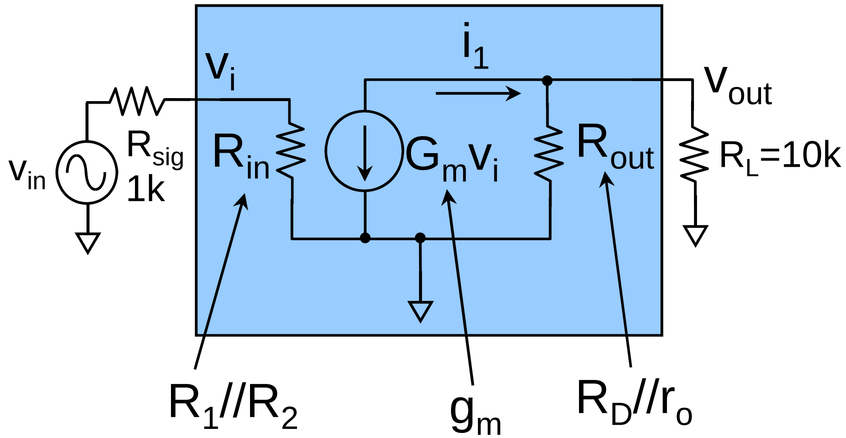
Important Result:
For CS, G_m is directly given by g_m. **No need to rederive.**

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the drain (r_y) is directly given by r_o. **No need to rederive.**

AC Analysis for CS

K_n=500μA/V^2
V_{TH}=1V





Wire split means resistors in parallel

g_m=2mA/V
r_o=∞

$$v_i = \frac{R_{in}}{R_{sig} + R_{in}} v_{in} = \frac{R_1 // R_2}{R_{sig} + R_1 // R_2} v_{in} \approx v_{in}$$

$$i_1 = -g_m v_i \approx -g_m v_{in}$$

$$v_{out} = i_1 \times [R_{out} // R_L] = i_1 \times [(R_D // r_o) // R_L] = -g_m (R_D // r_o // R_L) v_{in}$$

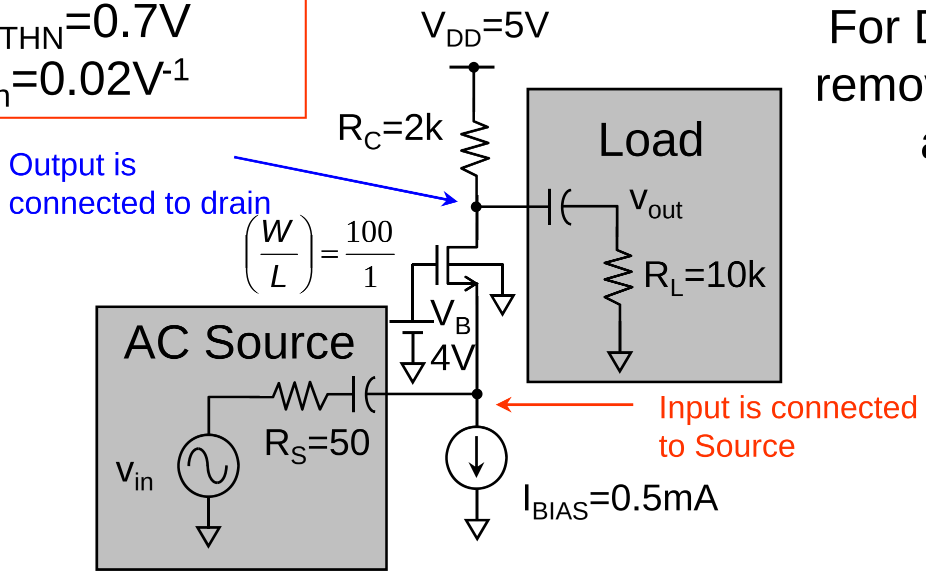
$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = -g_m (R_D // r_o // R_L) = -2.61$$

Characteristic of CE/CS

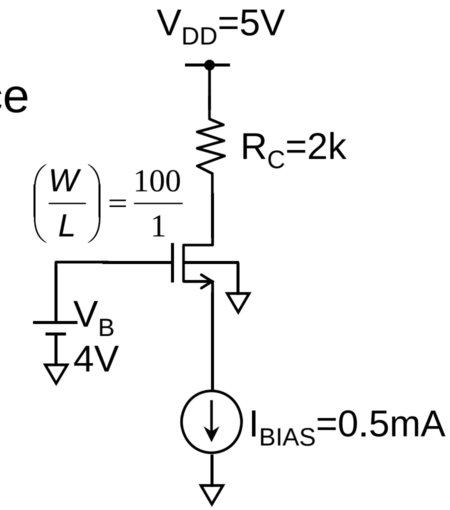
- High input resistance
- High output resistance
- Medium gain
- Polarity inversion, i.e. v_{out} and v_{in} has opposite sign
- The higher the G_m and the total output resistance, the higher the gain (A_V)
- BJT provides larger g_m than MOS

Common Gate (CG)

$\mu_n C_{ox}=80\mu A/V^2$
$V_{THN}=0.7V$
$\lambda_n=0.02V^{-1}$



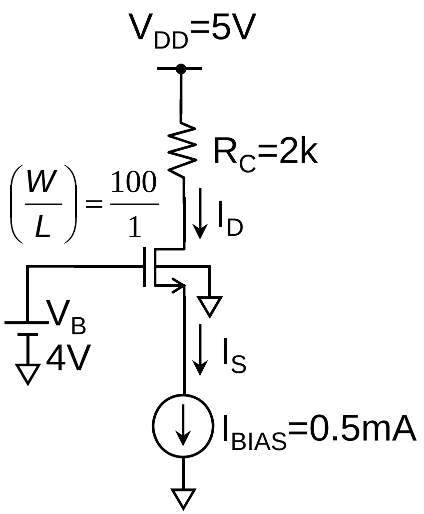
For DC analysis, remove AC source and load



- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Source, output connected to Drain, Gate connected to neither input nor output $\Rightarrow$ Common Gate (CG)

DC Analysis for CG

- Remove source/load section when doing DC analysis



$\mu_n C_{ox}$=80μA/V^2
V_{THN}=0.7V
λ_n=0.02V^{-1}

Determine DC biasing

$$I_D = I_S = I_{BIAS} = 0.5mA$$

$$\left(\frac{W}{L}\right) = \frac{100}{1}$$

Good approximation, no need to go through detailed calculations

Determine AC small signal parameter

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}$$

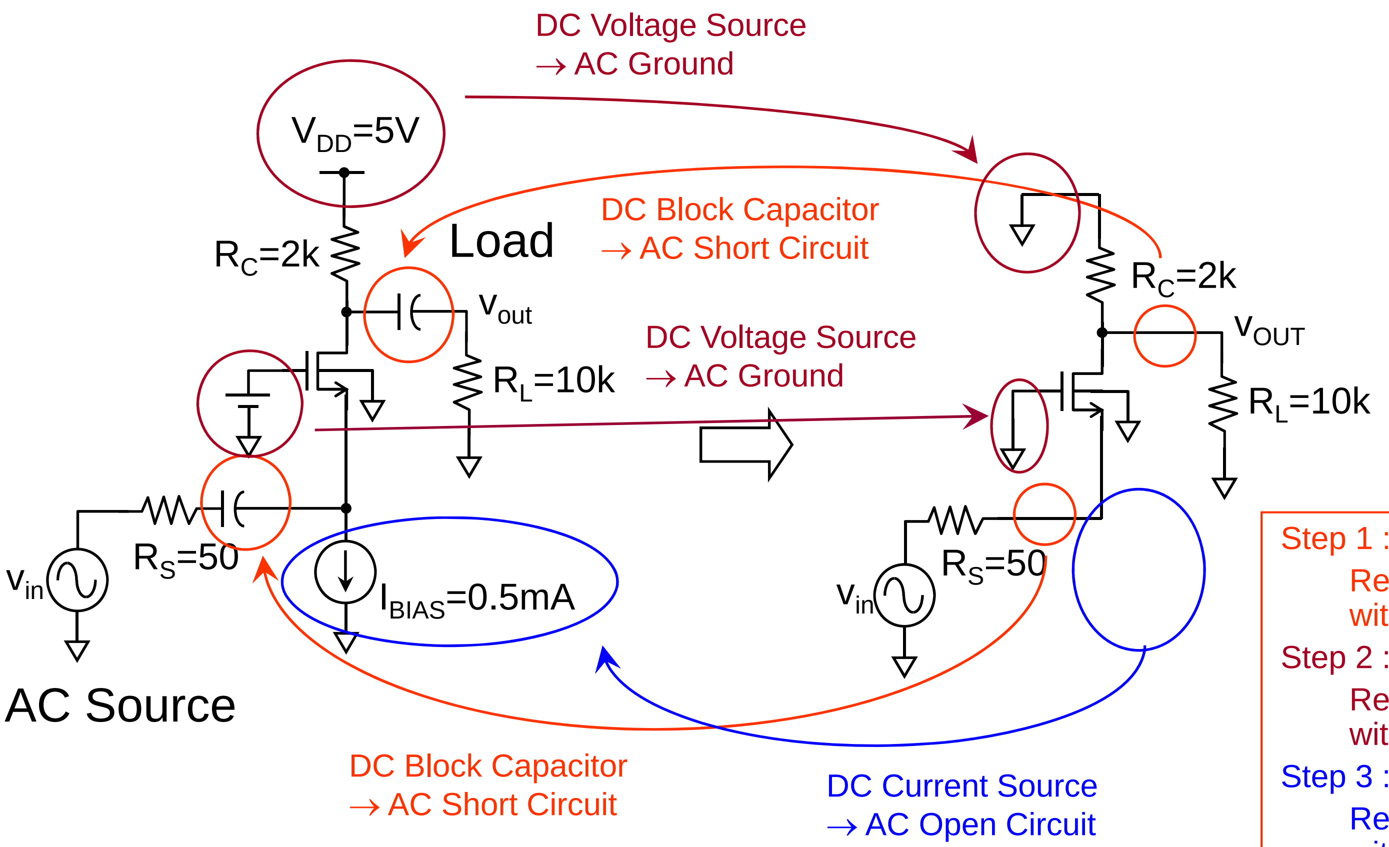
$$= 2.83mA/V$$

$$\boxed{g_{mb} \approx -\frac{g_m}{4} = -0.71mA/V}$$

$$r_i = \infty$$

$$r_o = \frac{1}{\lambda_n I_D} = 100k\Omega$$

AC Analysis for CG

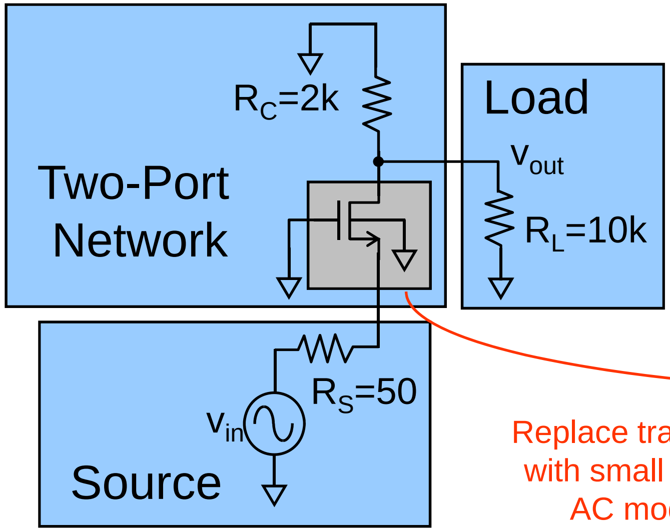


Step 1 :
Replace DC block capacitor with AC short circuit

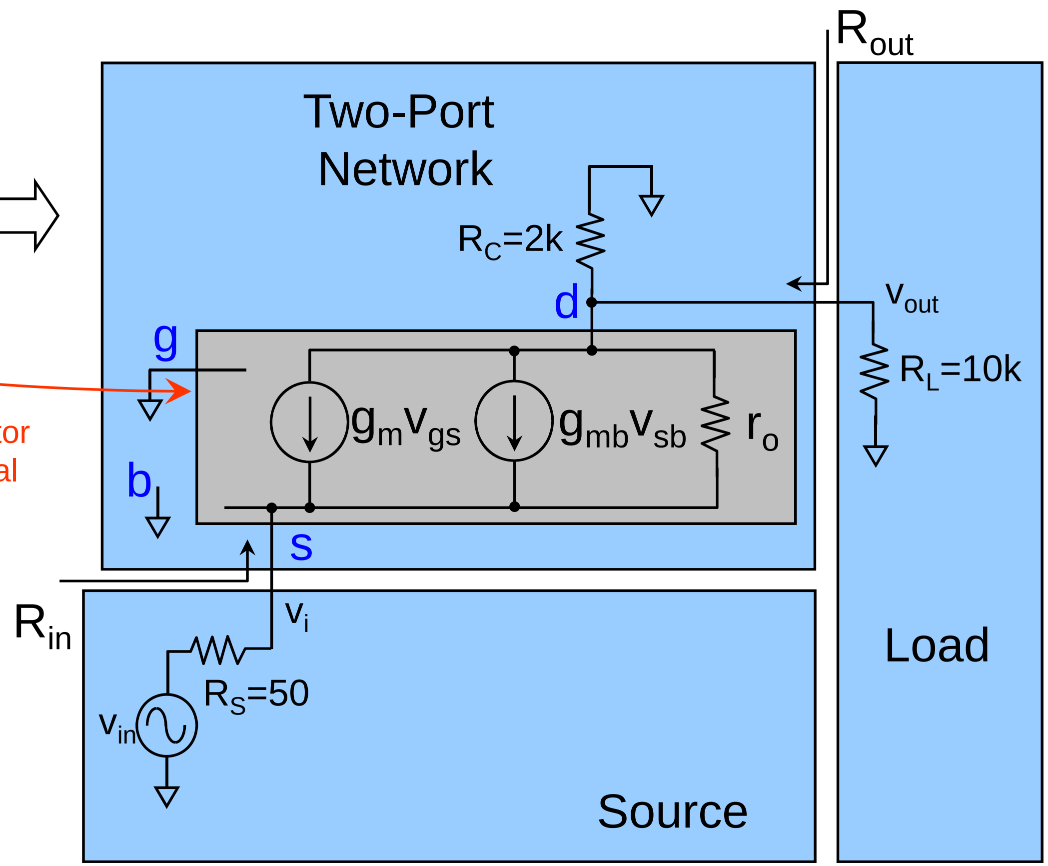
Step 2 :
Replace DC voltage source with AC ground

Step 3 :
Replace DC current source with AC open circuit

AC Analysis for CG



Replace transistor with small signal AC model



Step 1 :

Identify source/load

Step 2 :

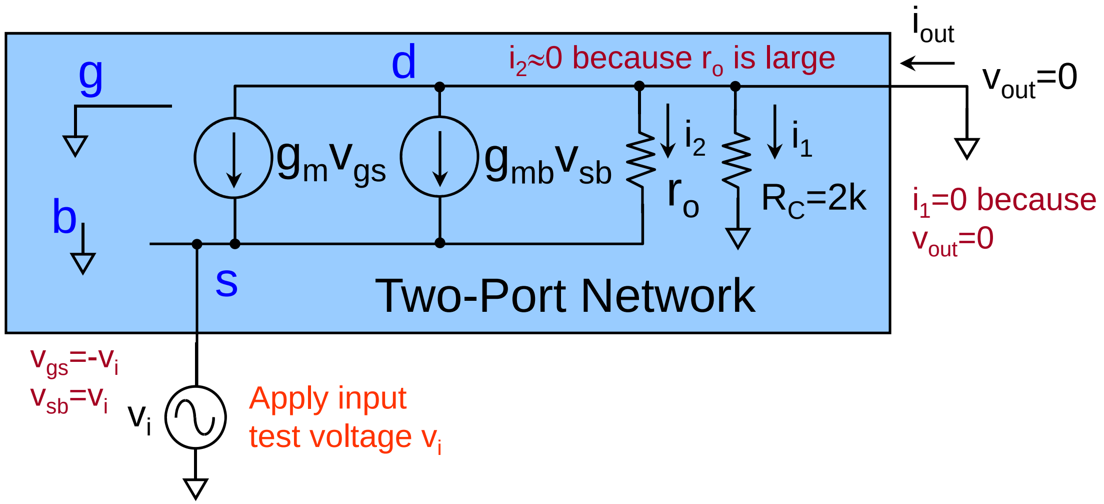
Group the remaining component into two-port network

Step 3 :

Replace transistor with small signal AC model

Interested in finding out R_{in}, R_{out} and G_m for the two-port network (Transconductance Amplifier)

CG – Two-Port Network (G_m)



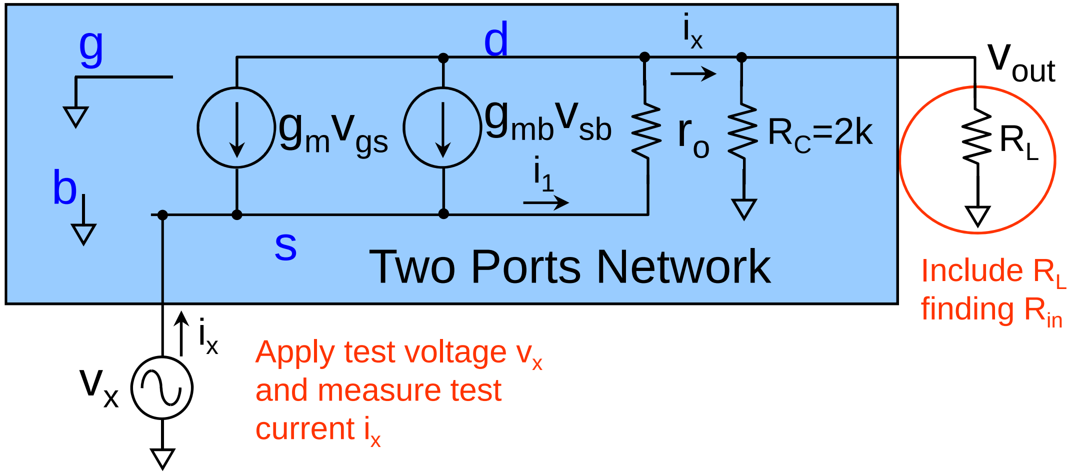
Measure short-circuit output current i_{out}

Important Result :
Transconductance (G_m) for CG is just –(g_m-g_{mb})

$$i_{out}\Big|_{v_{out}=0} = g_m v_{gs} + g_{mb} v_{sb} + \cancel{i_2} \approx -(g_m - g_{mb})v_i$$

$$G_m = \frac{i_{out}}{v_i}\Bigg|_{v_{out}=0} \approx -(g_m - g_{mb}) = -3.54mA/V$$

CG – Two-Port Network (R_{in})



Include R_L when finding R_{in}

Apply test voltage v_x and measure test current i_x

$$v_{gs} = -v_x \quad v_{sb} = v_x$$

$$i_1 = \frac{v_x - v_{out}}{r_o}$$

$$\begin{cases} i_x = -g_m v_{gs} - g_{mb} v_{sb} + i_1 \\ \quad = (g_m - g_{mb}) v_x + \dfrac{v_x - v_{out}}{r_o} \\ v_{out} = i_x (R_C // R_L) \end{cases}$$

Eliminate v_{out} and keep v_x and i_x

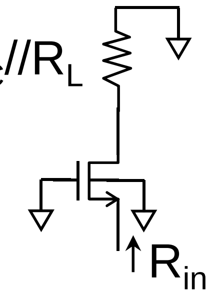
$$\Rightarrow i_x = (g_m - g_{mb}) v_x + \frac{v_x}{r_o} - i_x \frac{(R_C // R_L)}{r_o}$$

g_m-g_{mb}>>1/r_o

$$\Rightarrow R_{in} = \frac{1 + \dfrac{(R_C // R_L)}{r_o}}{(g_m - g_{mb}) + \cancel{\dfrac{1}{r_o}}} \approx \frac{1}{g_m - g_{mb}} \cdot \frac{r_o + (R_C // R_L)}{r_o}$$

CG – Two-Port Network (R_{in})

$R_T = R_C // R_L$



$$R_{in} \approx \frac{1}{g_m - g_{mb}} \cdot \frac{r_o + R_T}{r_o}$$

$$\approx \frac{1}{g_m - g_{mb}} \quad [If \quad R_T << r_o]$$

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the source (R_{in}) is directly given by the formula. **No need to rederive.**

Example :

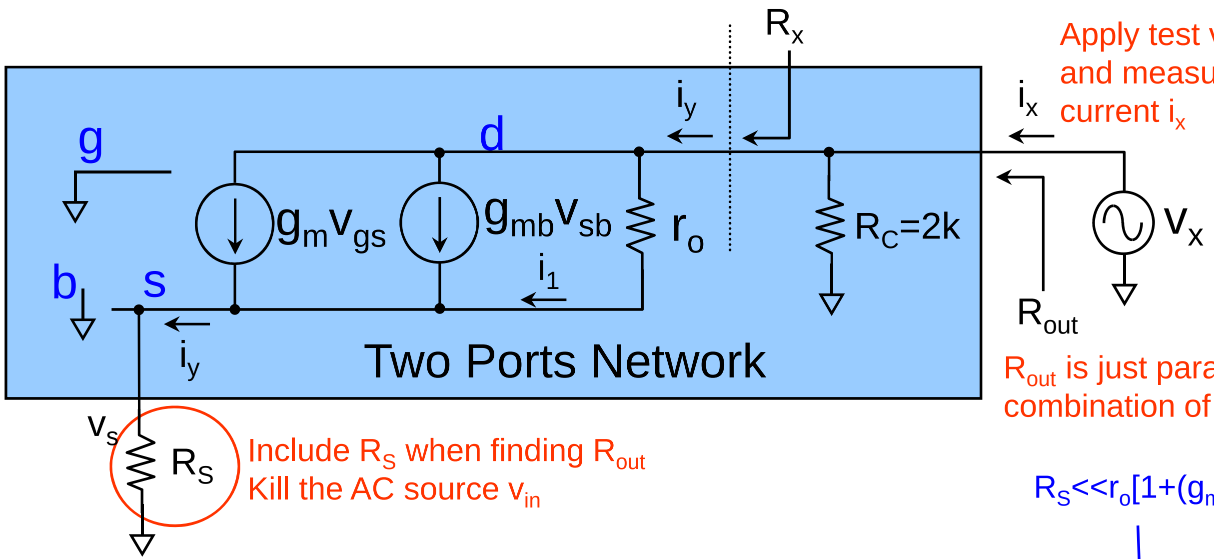
$R_C = 2k \quad R_L = 10k \quad r_o = 100k$

$\Rightarrow R_T = (R_C // R_L) << r_o$

$$\Rightarrow R_{in} = \frac{1}{g_m - g_{mb}} = 282$$

- If $R_C//R_L$ is negligible compared to r_o, the input resistance (R_{in}) reduced to the inverse of the transconductance $[1/(g_m - g_{mb})]$

CG – Two-Port Network (R_{out})



Apply test voltage v_x and measure test current i_x

R_{out} is just parallel combination of R_C and R_x

Include R_S when finding R_{out}
Kill the AC source v_{in}

$$v_{gs} = -v_s \quad v_{sb} = v_s$$

$$i_1 = i_y - g_m v_{gs} - g_{mb} v_{sb}$$
$$= i_y + (g_m - g_{mb}) v_s$$

R_S<<$r_o[1+(g_m-g_{mb})R_S)]$

$$R_x = \frac{v_x}{i_y}$$

Eliminate v_s and keep v_x and i_y

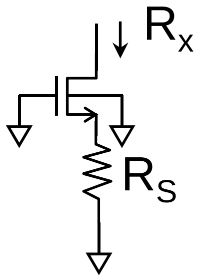
$$\begin{cases} v_s = i_y R_S \\ v_x = v_s + i_1 r_o \\ \quad = v_s + [i_y + (g_m - g_{mb}) v_s] r_o \end{cases}$$

$$\Rightarrow v_x = i_y R_S + i_y r_o + i_y (g_m - g_{mb}) R_S r_o$$

$$\Rightarrow R_x = \frac{v_x}{i_y} = \cancel{R_S} + r_o [1 + (g_m - g_{mb}) R_S]$$

$$R_{out} = R_C // R_x$$
$$\approx R_C // \{r_o [1 + (g_m - g_{mb}) R_S]\} \approx R_C$$

CG – Two-Port Network (R_{out})



$$R_x \approx r_o\left[1+\left(g_m - g_{mb}\right)R_S\right]$$

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the drain (R_x) is directly given by the formula. **No need to rederive.**

Example :

$$R_S = 50 \quad r_o = 100k$$
$$g_m = 2.83m \quad g_{mb} = -0.71m$$
$$\Rightarrow R_x \approx 118k >> R_C$$
$$\Rightarrow R_{out} \approx R_C = 2k$$

- Source side resistor help boost up the output resistance of the transistor (R_x)

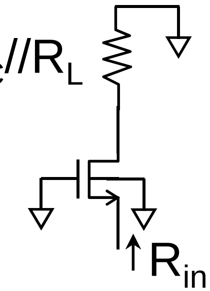
CG – Important Results

1

Important Result :
Transconductance (G_m) for CG is just $-(g_m - g_{mb})$

2

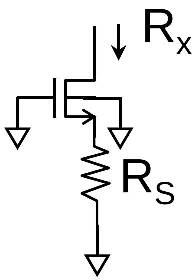
$R_T = R_C // R_L$



$$R_{in} \approx \frac{1}{g_m - g_{mb}} \cdot \frac{r_o + R_T}{r_o}$$

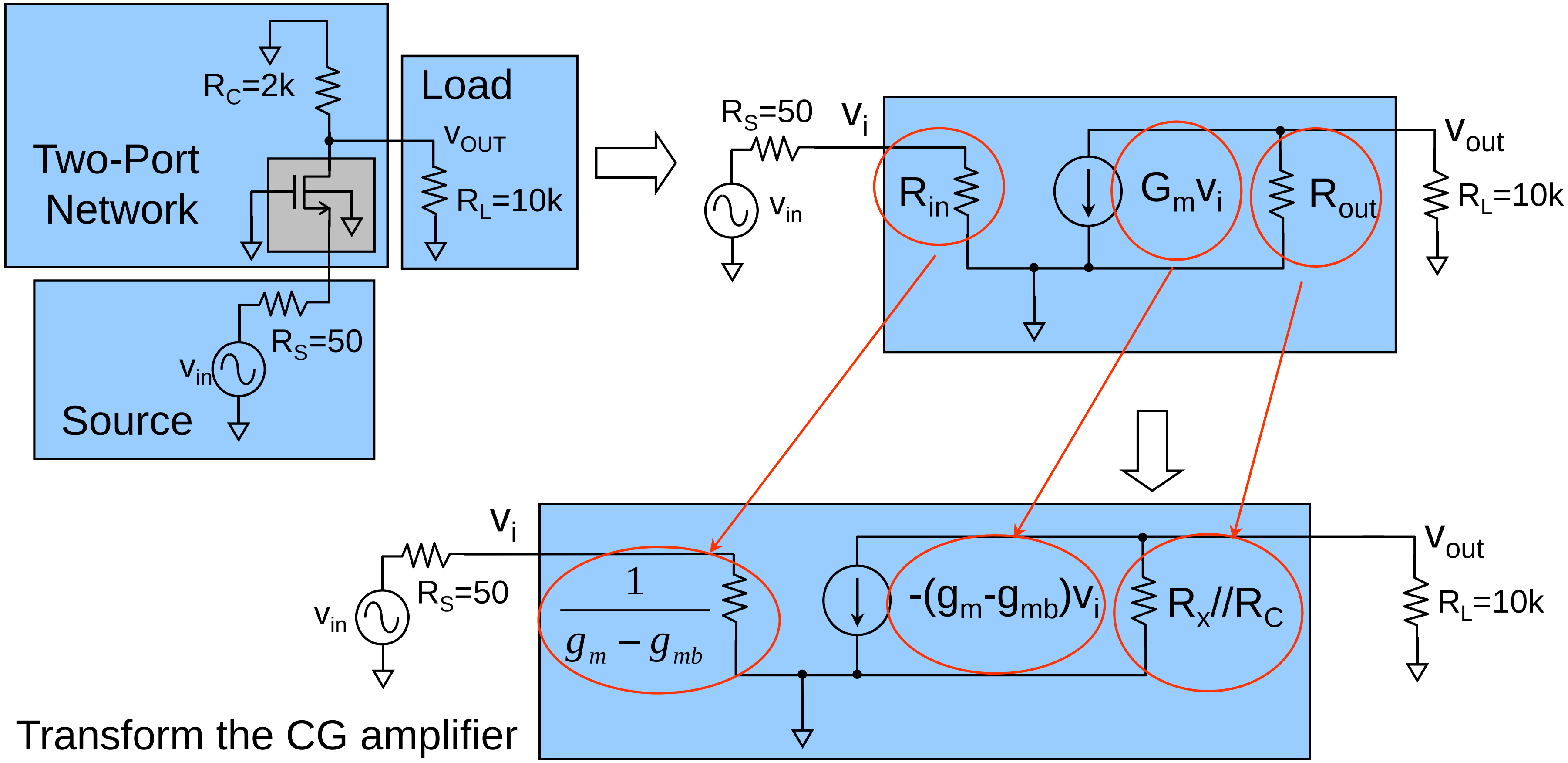
$$\approx \frac{1}{g_m - g_{mb}} \quad [If \quad R_T << r_o]$$

3



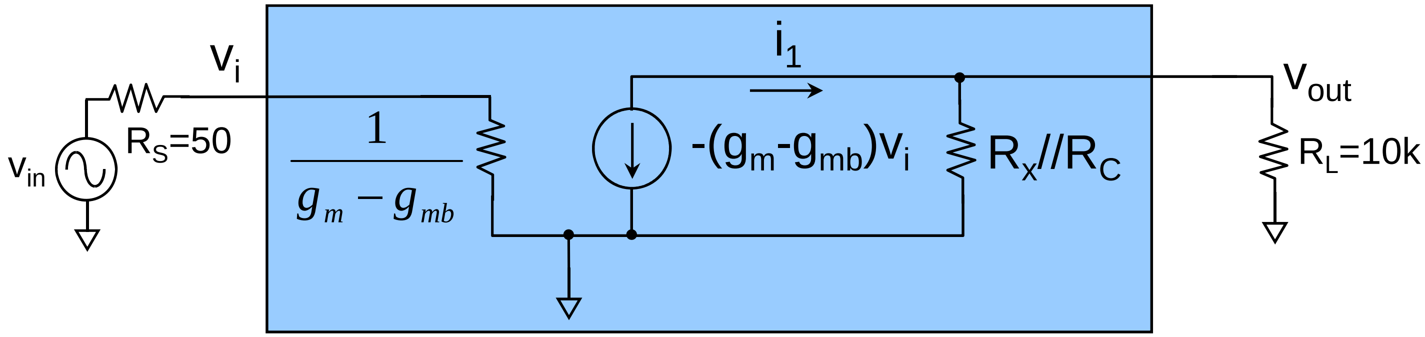
$$R_x \approx r_o\left[1 + (g_m - g_{mb})R_S\right]$$

CG – Two-Port Network



Transform the CG amplifier into two-port transconductance amplifier

CG – Two-Port Network (A_V)



$$v_i = v_{in} \times \frac{R_{in}}{R_S + R_{in}}$$

$$R_{in} = \frac{1}{g_m - g_{mb}}$$

$$i_1 = -\left[-\left(g_m - g_{mb}\right)v_i\right]$$
$$= \left(g_m - g_{mb}\right)v_i$$

$$v_{out} = i_1 \times \left[\left(\cancel{R_x} // R_C\right) // R_L\right]$$

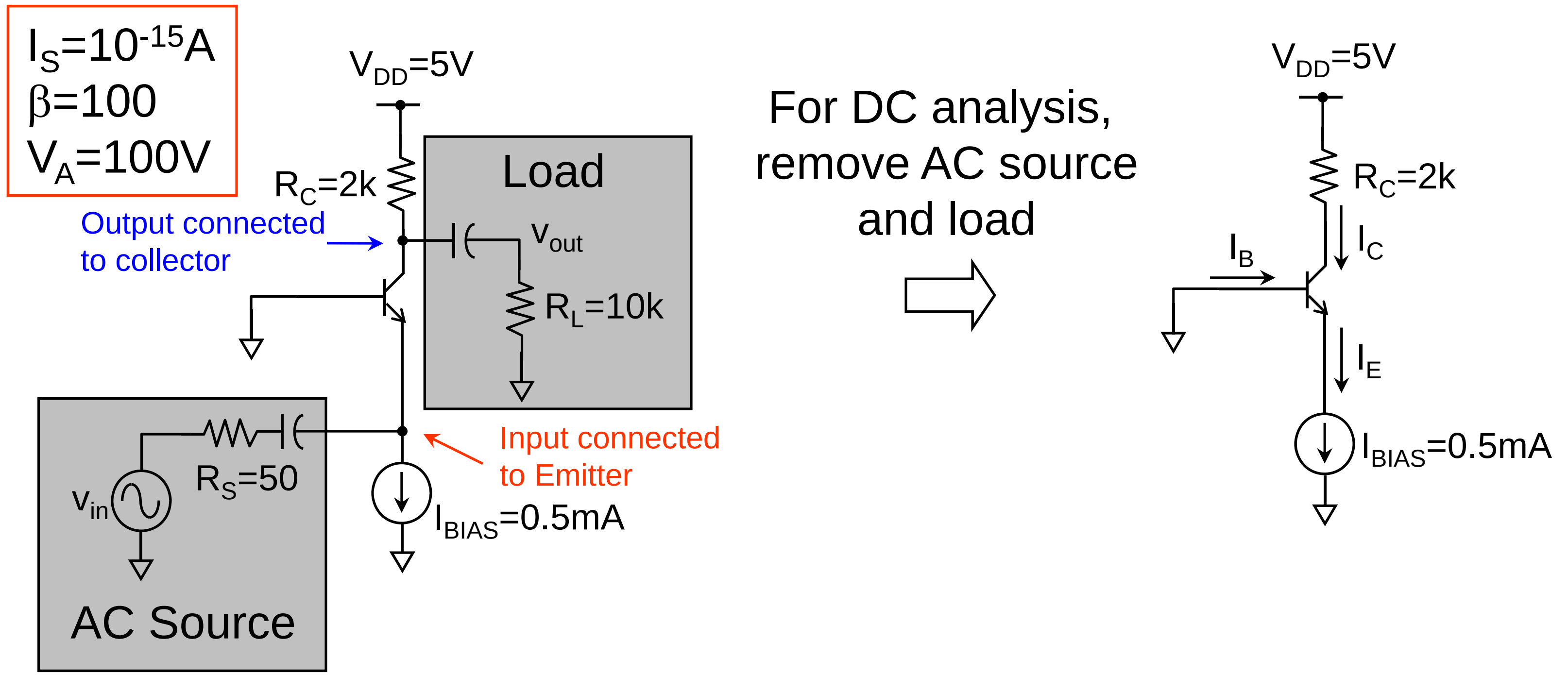
R_x is too big

$$\approx \left(g_m - g_{mb}\right) \times v_i \times \left(R_C // R_L\right)$$

$$\approx \left(g_m - g_{mb}\right) \times \left(v_{in} \times \frac{R_{in}}{R_S + R_{in}}\right) \times \left(R_C // R_L\right)$$

$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_S + R_{in}}\left(g_m - g_{mb}\right)\left(R_C // R_L\right)$$

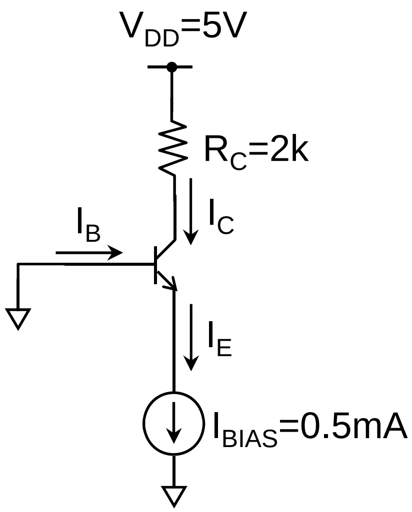
Common Base (CB)



- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuits, i.e. short circuit capacitors
- Input connected to Emitter, output connected to Collector, Base connected to neither input nor output $\Rightarrow$ Common Base (CB)

DC Analysis for CB (Self Reading)

- Remove AC source/load section when doing DC analysis



$I_S=10^{-15}A$
$\beta=100$
$V_A=100V$

Determine DC biasing

$$V_T = \frac{kT}{q} = 26mV$$

$$I_E = I_{BIAS} = 0.5mA$$

$$I_C = \frac{\beta}{\beta+1} I_E = 0.495mA$$

$$I_B = \frac{I_C}{\beta} = 4.95\mu A$$

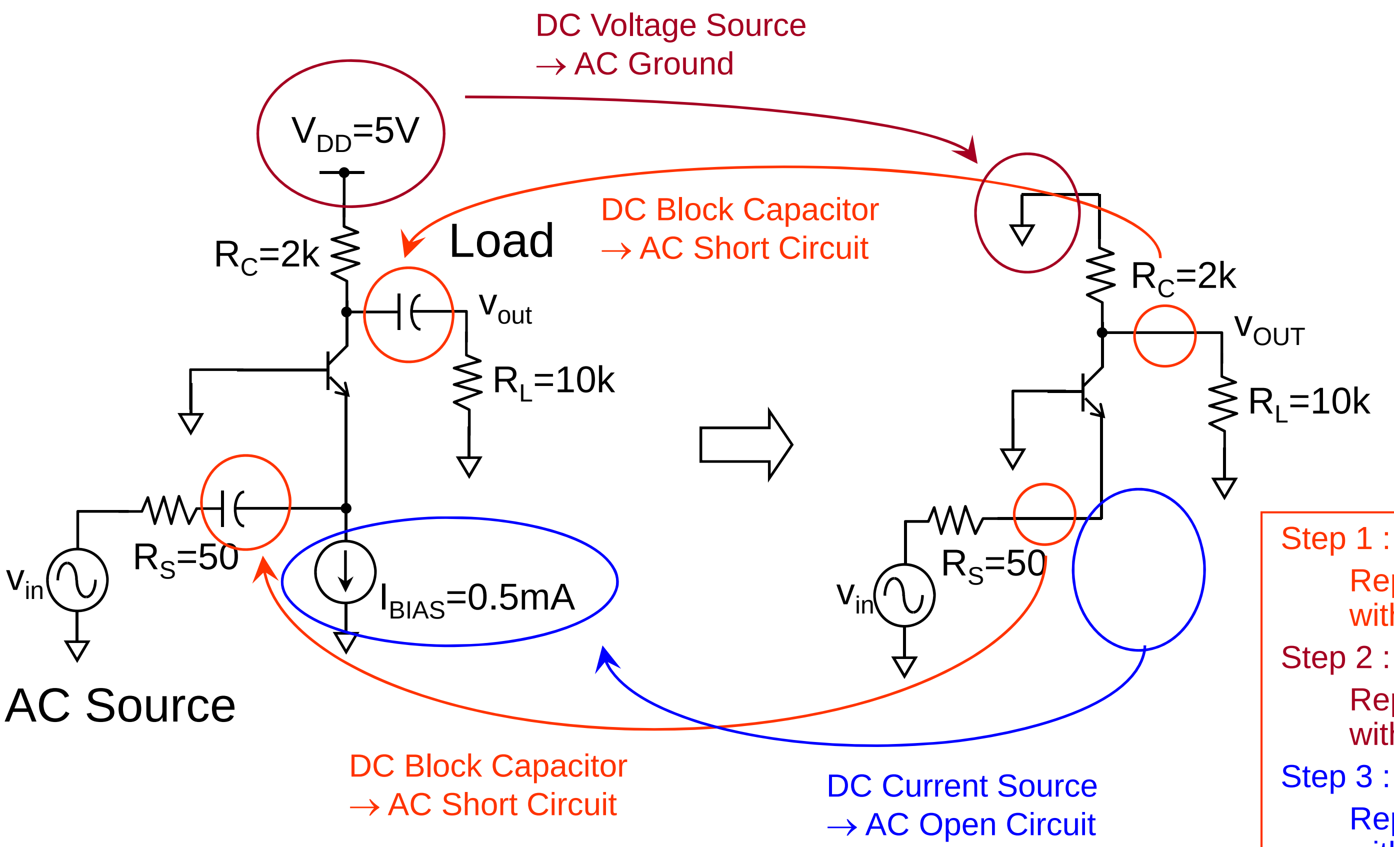
Determine AC small signal parameter

$$g_m = \frac{I_C}{V_T} = 19mA/V$$

$$r_\pi = \frac{\beta}{g_m} = 5.26k\Omega$$

$$r_o = \frac{V_A}{I_C} = 202k\Omega$$

AC Analysis for CB (Self Reading)

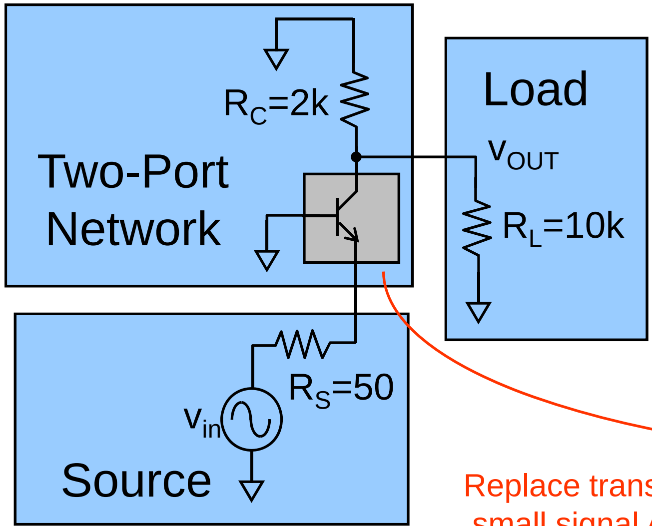


Step 1 :
Replace DC block capacitor with AC short circuit

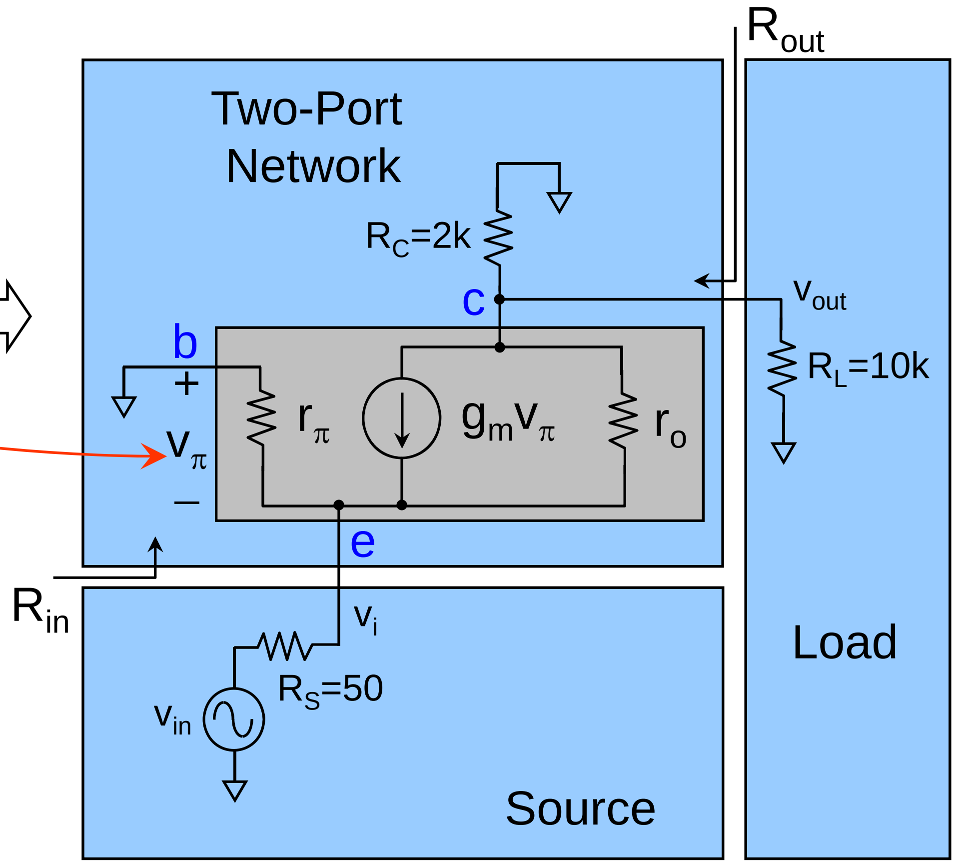
Step 2 :
Replace DC voltage source with AC ground

Step 3 :
Replace DC current source with AC open circuit

AC Analysis for CB (Self Reading)



Replace transistor with small signal AC model



Step 1 :

Identify source/load

Step 2 :

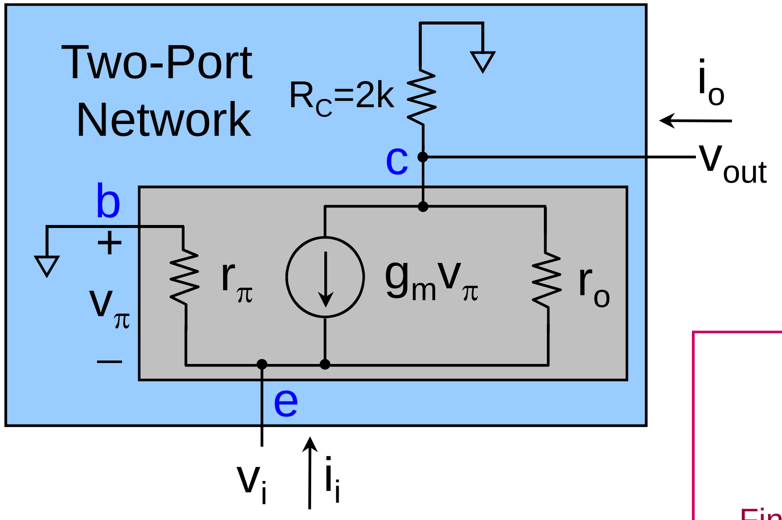
Group the remaining component into two-port network

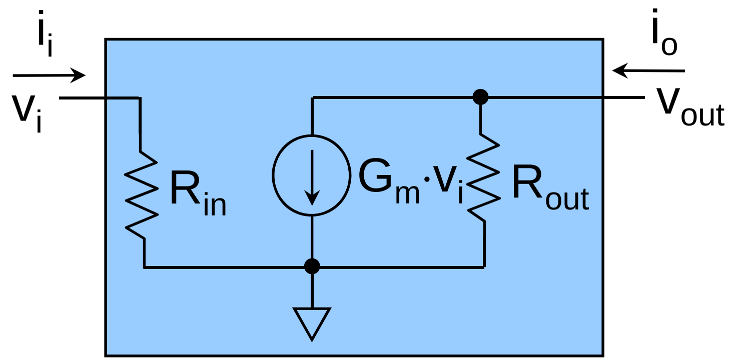
Step 3 :

Replace transistor with small signal AC model

Interested in finding out R_{in}, R_{out} and G_m for the two ports network (Transconductance Amplifier)

Mapping Two-Port Network for CB (Self Reading)



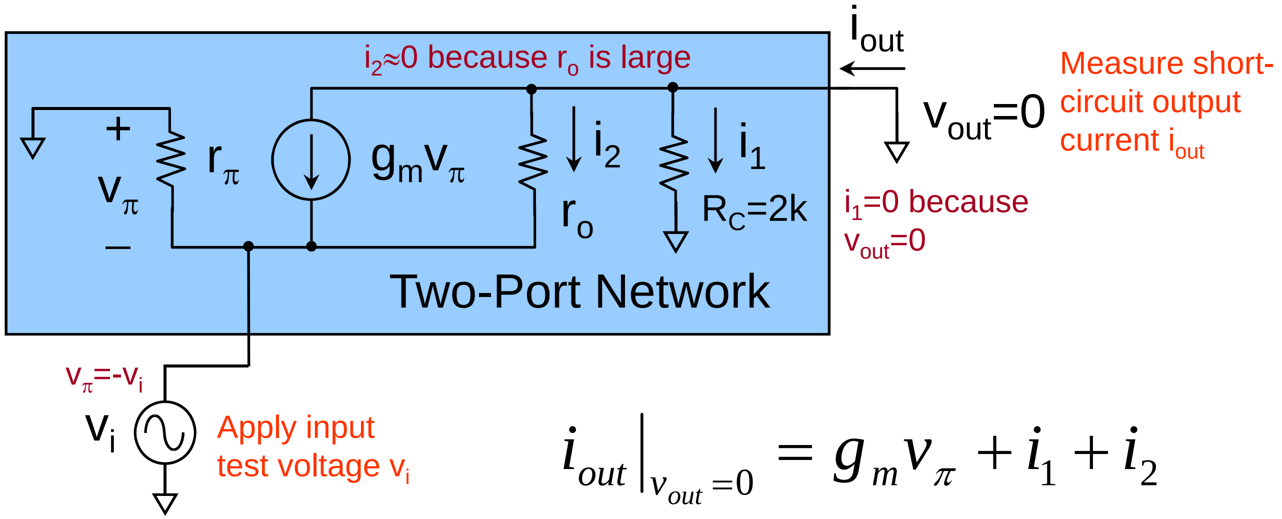


$$G_m = \left.\frac{i_o}{v_i}\right|_{v_{out}=0} \qquad R_{in} = \frac{v_i}{i_i} \qquad R_{out} = \frac{v_{out}}{i_o}$$

Find the 2-port parameters according to method described in 5A-9

- Similar concept as Thevenin equivalent
- Complicated two-port network can be represented by a two-port network characterized by only 3 parameters, R_{in}, R_{out} and G_m

CB – Finding G_m (Self Reading)



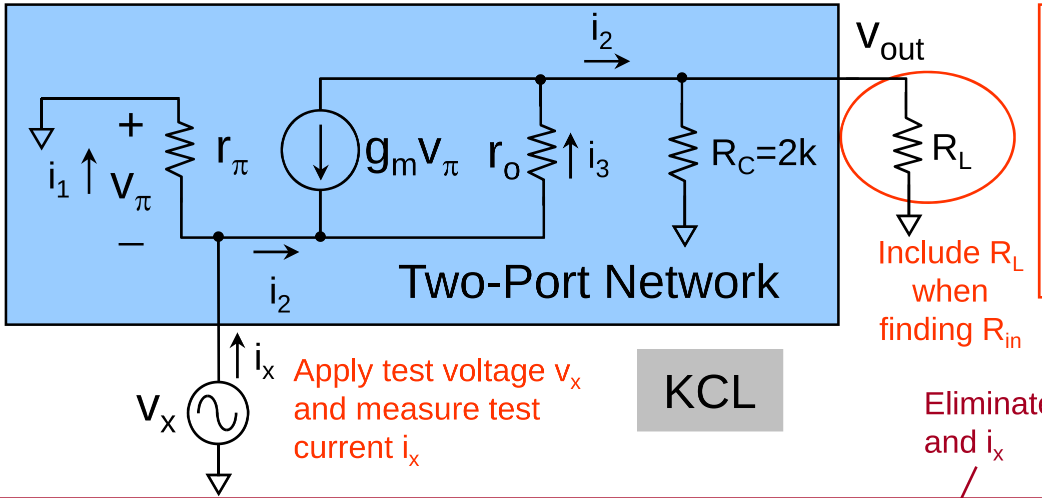
$$i_{out}\Big|_{v_{out}=0} = g_m v_\pi + i_1 + i_2$$

$$\approx -g_m v_i$$

$$G_m = \frac{i_{out}}{v_i}\Bigg|_{v_{out}=0} \approx -g_m = -19mA/V$$

Important Result :
Transconductance (G_m) for CB is just $-g_m$

CB – Finding R_{in} (Self Reading)



Include R_L when finding R_{in}

$$v_\pi = -v_x \quad i_1 = \frac{v_x}{r_\pi} \quad i_3 = \frac{v_x - v_{out}}{r_o}$$

$$i_2 = i_x - i_1 = i_x - \frac{v_x}{r_\pi}$$

Apply test voltage v_x and measure test current i_x

KCL

Eliminate v_{out} and keep v_x and i_x

g_m>>$1/r_\pi$ and $1/r_o$

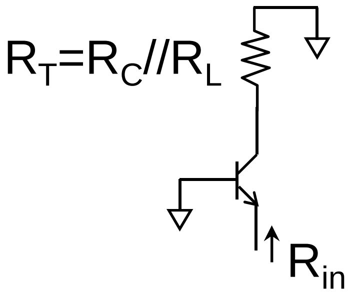
$$\begin{cases} i_x = i_1 - g_m v_\pi + i_3 = \dfrac{v_x}{r_\pi} + g_m v_x + \dfrac{v_x - v_{out}}{r_o} \\ v_{out} = i_2 \left(R_C // R_L\right) = \left(i_x - \dfrac{v_x}{r_\pi}\right)\left(R_C // R_L\right) \end{cases}$$

$$\Rightarrow i_x = \frac{v_x}{r_\pi} + g_m v_x + \frac{v_x}{r_o} - i_x \frac{\left(R_C // R_L\right)}{r_o} + \frac{v_x\left(R_C // R_L\right)}{r_\pi r_o}$$

$$\Rightarrow R_{in} = \frac{1 + \dfrac{\left(R_C // R_L\right)}{r_o}}{g_m + \dfrac{1}{r_\pi} + \dfrac{1}{r_o} + \dfrac{\left(R_C // R_L\right)}{r_\pi r_o}}$$

$$\approx \frac{1}{g_m} \cdot \frac{r_o + \left(R_C // R_L\right)}{r_o + \dfrac{\left(R_C // R_L\right)}{\beta}}$$

CB – Finding R_{in} (Self Reading)



$$R_{in} \approx \frac{1}{g_m} \cdot \frac{r_o + R_T}{r_o + \frac{R_T}{\beta}}$$

$$\approx \frac{1}{g_m} \quad \left[If \quad R_T << r_o \right]$$

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the emitter (R_{in}) is directly given by the formula. **No need to rederive.**

Example :

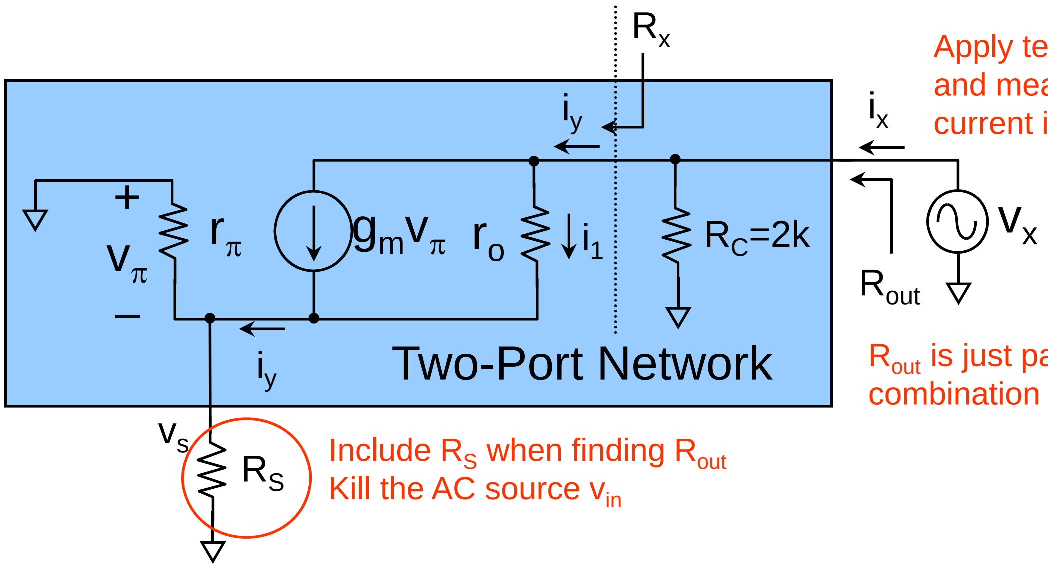
$$R_C = 2k \quad R_L = 10k \quad r_o = 202k$$

$$\Rightarrow R_T = \left(R_C \,//\, R_L\right) << r_o$$

$$\Rightarrow R_{in} = \frac{1}{g_m} = 53$$

- If $R_C//R_L$ is negligible compared to r_o, the input resistance (R_{in}) reduced to the inverse of the transconductance ($1/g_m$)

CB – Finding R_{out} (Self Reading)



Apply test voltage v_x and measure test current i_x

R_{out} is just parallel combination of R_C and R_x

Include R_S when finding R_{out}
Kill the AC source v_{in}

$$v_\pi = -v_s$$

$$i_1 = i_y - g_m v_\pi = i_y + g_m v_s$$

$r_\pi//R_S << r_o[1+g_m(r_\pi//R_S)]$

Eliminate v_s and keep v_x and i_y

$$R_x = \frac{v_x}{i_y}$$

$$\begin{cases} v_s = i_y(r_\pi // R_S) \\ v_x = v_s + i_1 r_o = v_s + (i_y + g_m v_s) r_o \end{cases}$$

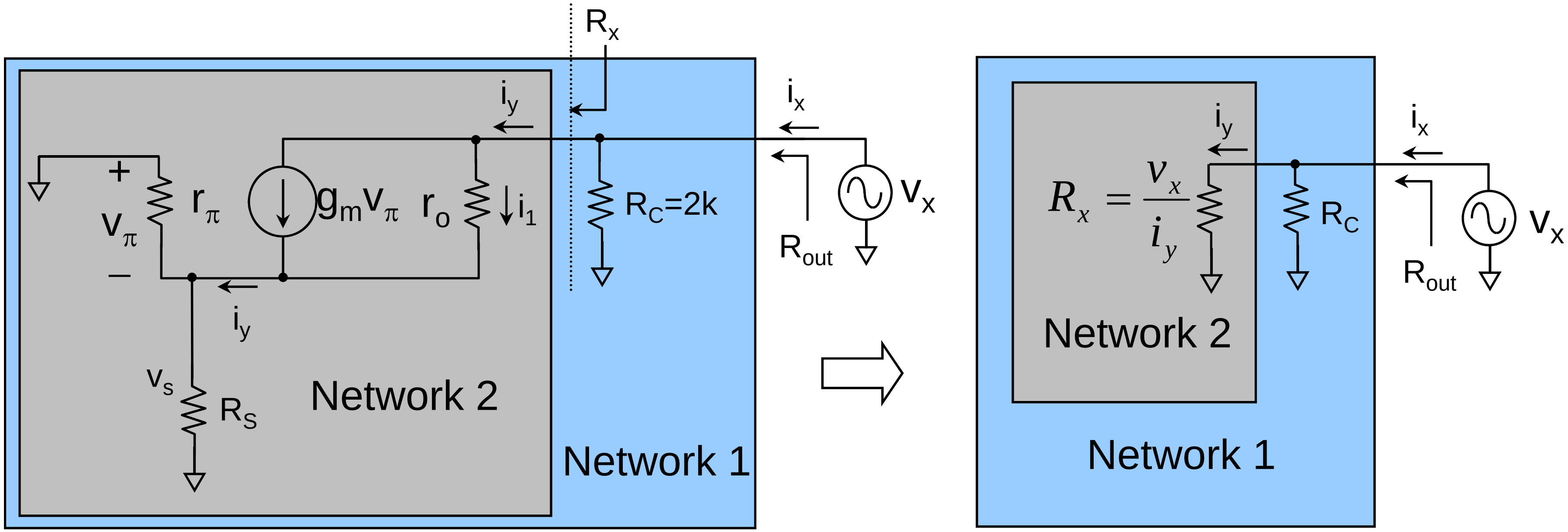
$$\Rightarrow v_x = i_y(r_\pi // R_S) + i_y r_o + i_y g_m (r_\pi // R_S) r_o$$

$$\Rightarrow R_x = \frac{v_x}{i_y} = \cancel{(r_\pi // R_S)} + r_o[1 + g_m(r_\pi // R_S)]$$

$$R_{out} = R_C // R_x$$

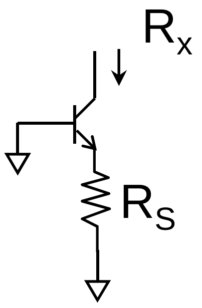
$$\approx R_C // \{r_o[1 + g_m(r_\pi // R_S)]\} \approx R_C$$

Equivalent Resistance (Self Reading)



- A complicated network can be reduced to an equivalent driving point resistance

CB – Finding R_{out} (Self Reading)



$$R_x \approx r_o\left[1+g_m\left(r_\pi // R_S\right)\right]$$

$$If \quad R_S << r_\pi \Rightarrow R_x = r_o\left(1+g_m R_S\right)$$

$$If \quad R_S >> r_\pi \Rightarrow R_x = r_o\left(1+g_m r_\pi\right) = r_o\left(1+\beta\right)$$

Important Result:
If you see the transistor connected in the similar fashion, the resistance looking into the collector (R_x) is directly given by the formula. **No need to rederive.**

Example :

$$R_S = 50 \quad r_\pi = 5.26k$$

$$r_o = 202k \quad g_m = 19m$$

$$\Rightarrow R_x \approx 394k >> R_C$$

$$\Rightarrow R_{out} \approx R_C = 2k$$

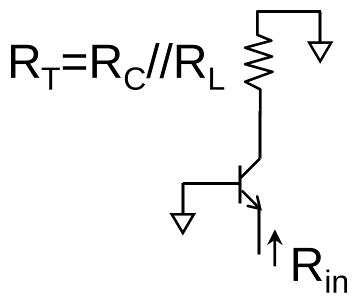
- Emitter side resistor help boost up the output resistance of the transistor (R_x)
- For BJT, the maximum achievable boost up is $(1+\beta)r_o$

CB – Important Results

1

Important Result :
Transconductance (G_m) for CB is just $-g_m$

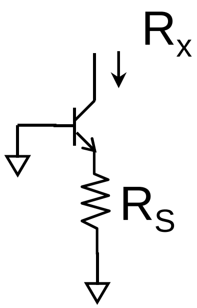
2



$$R_{in} \approx \frac{1}{g_m} \cdot \frac{r_o + R_T}{r_o + \frac{R_T}{\beta}}$$

$$\approx \frac{1}{g_m} \quad [If \quad R_T << r_o]$$

3



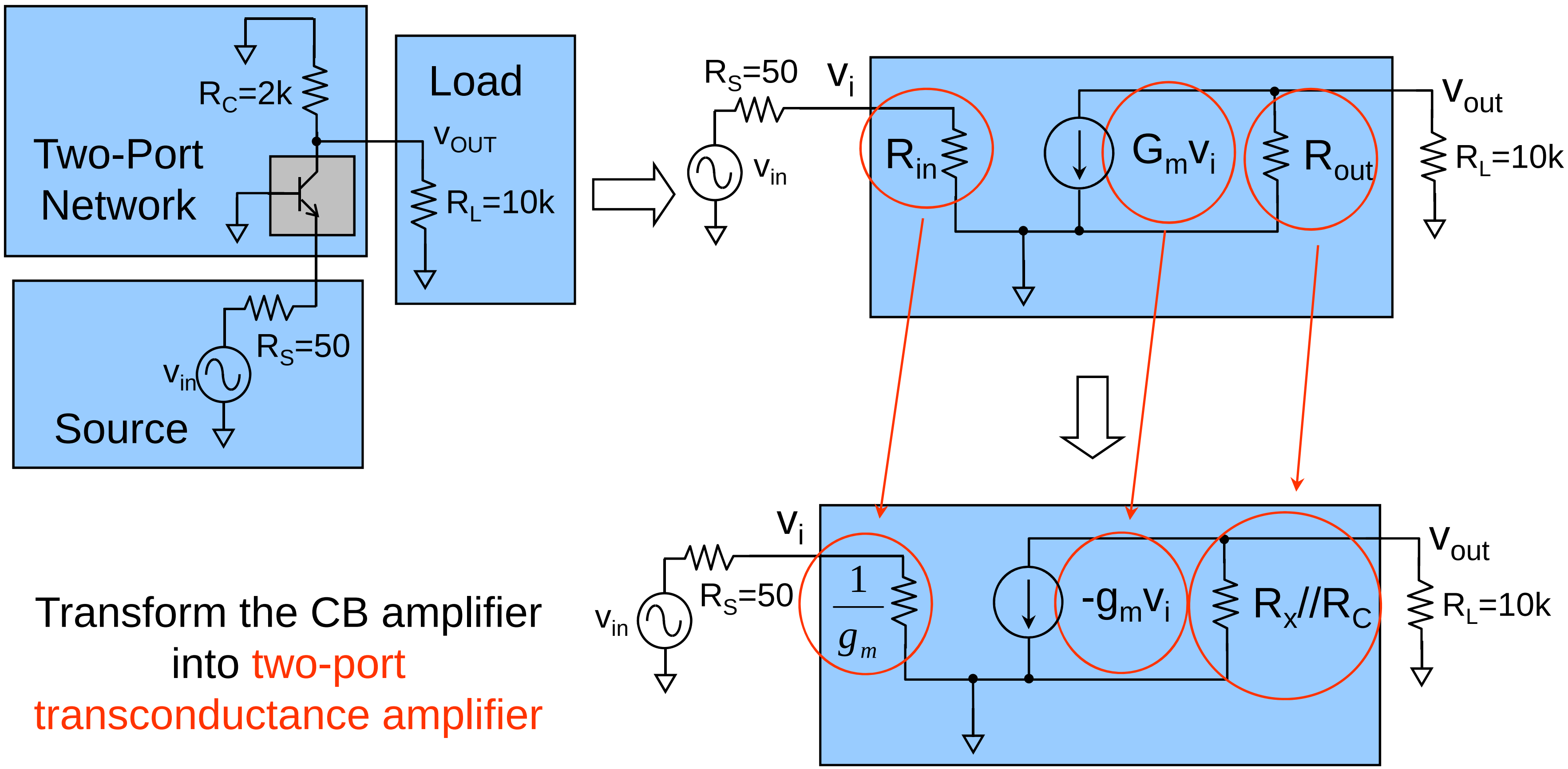
$$R_x \approx r_o\left[1 + g_m\left(r_\pi // R_S\right)\right]$$

$$If \quad R_S << r_\pi \Rightarrow R_x = r_o\left(1 + g_m R_S\right)$$

$$If \quad R_S >> r_\pi \Rightarrow R_x = r_o\left(1 + g_m r_\pi\right)$$

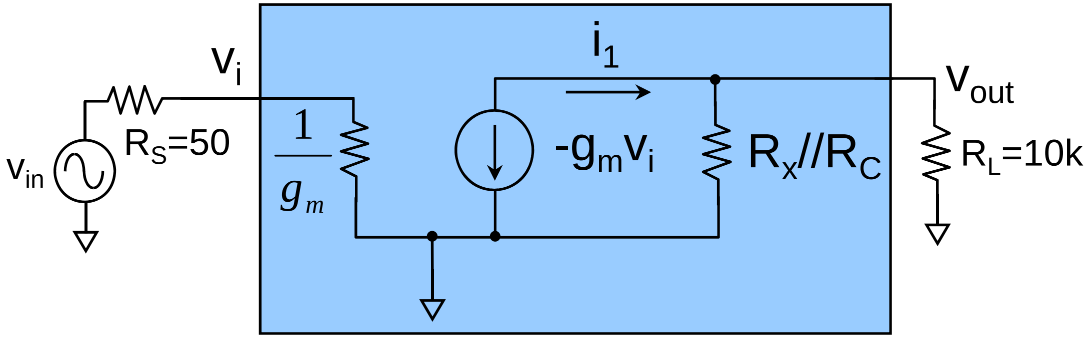
$$= r_o\left(1 + \beta\right)$$

CB – Two-Port Network



Transform the CB amplifier into two-port transconductance amplifier

CB – Two-Port Network (A_V)



$$v_i = v_{in} \times \frac{R_{in}}{R_S + R_{in}}$$

$$R_{in} = \frac{1}{g_m}$$

$$i_1 = -\left(-g_m v_i\right) = g_m v_i$$

$$\begin{aligned} v_{out} &= i_1 \times \left[\left(R_x // R_C\right) // R_L\right] \\ &= g_m \times v_i \times \left[\left(\cancel{R_x} // R_C\right) // R_L\right] \\ &\approx g_m \times \left(v_{in} \times \frac{R_{in}}{R_S + R_{in}}\right) \times \left(R_C // R_L\right) \end{aligned}$$

R_x is too big

$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_S + R_{in}} g_m \left(R_C // R_L\right)$$

Characteristic of CB/CG

- Low input resistance
- High output resistance
- Medium gain
- No polarity inversion
- The higher the G_m and the total output resistance, the higher the gain (A_V)
- BJT provides larger g_m than MOS (g_m-g_{mb})

Lecture Summary

- Introduced 2-port voltage and transconductance network.
- Analyze CE/CS amplifier.
- Analyze CB/CG amplifier.

Reading Assignment

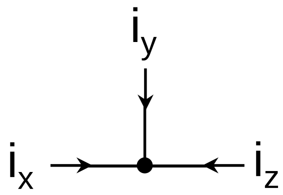
- **Reading: Reference Book (Sedra & Smith)**

  Chapter 1, pp. 23 – 31. (2-port networks)

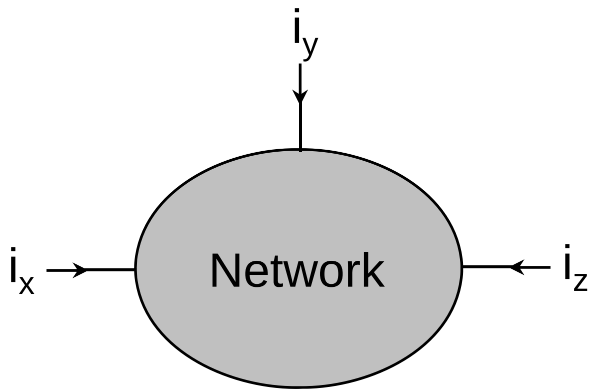
  Chapter 3, pp. 249 – 252, pp. 257 – 260. (CE, CB)

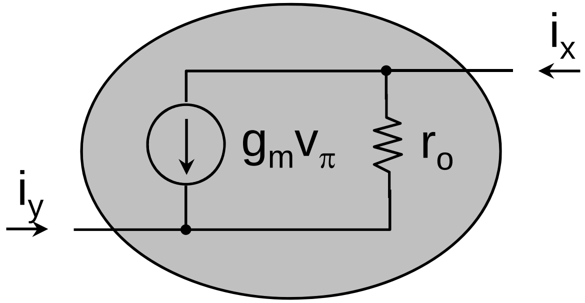
  Chapter 4, pp. 396 – 405. (CS, CG)

KCL



$$i_x + i_y + i_z = 0$$





Total current going into the network sums to zero

$$i_x + i_y = 0$$

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