EE2022 Electrical Energy Systems

Per Unit Analysis: Single Phase

Lecturer : Dr. Sangit Sasidhar (elesang)
Slides prepared by Dr. Panida Jirutitijaroen
Department of Electrical and Computer Engineering

Detailed Syllabus

Topic 1	**Transformer**: Principle of transformer. Ideal transformer. Reflected load. Impedance matching. Practical transformer. Examples
Topic 2	**Renewable Energy Sources:** Sustainable and clean energy sources; Solar Photovoltaic, Wind Energy; Examples
Topic 3	**Per unit analysis**: Single-phase per unit analysis. Three-phase transformer, Three-phase per unit analysis. Examples.
Topic 4	**Generator**: Simple generator concept. Equivalent circuit of synchronous generators. Operating consideration of synchronous generators, i.e. excitation voltage control, real power control, and loading capability. Principle of asynchronous generators. Examples.
Topic 5	Electric energy market operation; Cost of Electricity
Topic 6	**Distributed Generation:** Concept of distributed energy generation and utility interfacing; Energy Storage

Learning Outcomes

- Apply the concepts of ***per-phase analysis*** and **per-unit analysis** to solve three-phase balanced circuit problems in power engineering.

Per Unit Quantity

Base Value

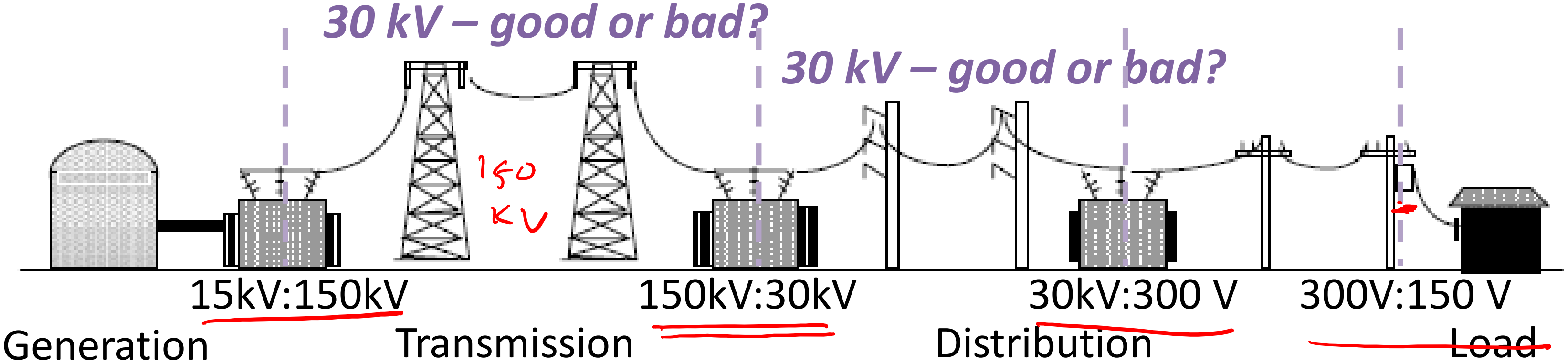
Change of Base

Steps of Calculation

SINGLE PHASE PER UNIT ANALYSIS

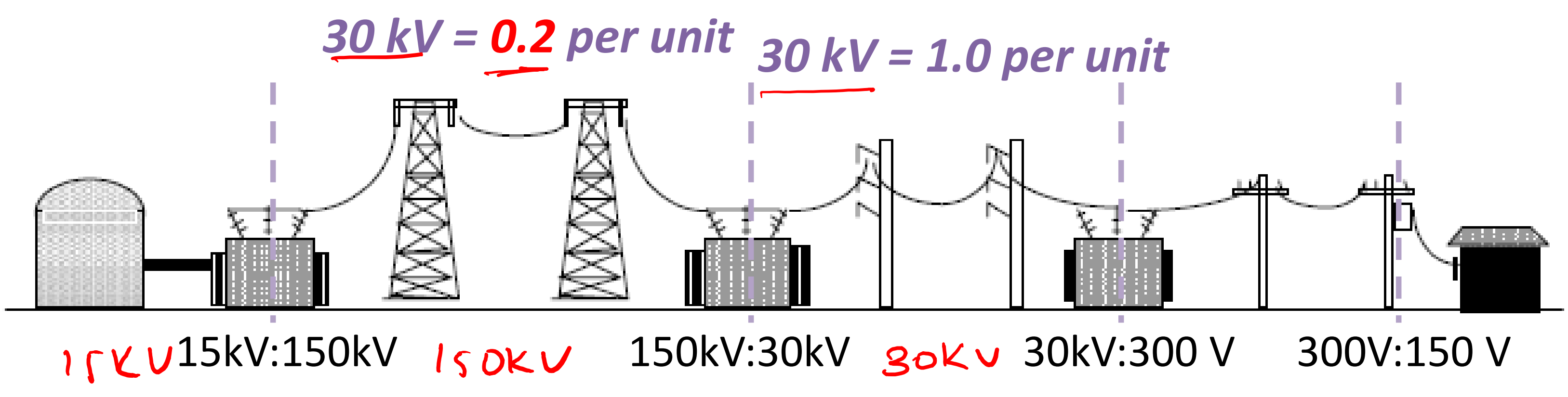
Motivations

- Transformer introduces various voltage levels.
- So far we can only reflect the load from one side of the transformer to another. Still we need to use turns ratio to find voltage and current at each side of the transformer.
- It is *difficult to calculate voltage and current* of the system at various points.
- It is **even more difficult** for system operators **to observe the current situation of the system**.



Per Unit System

- Per unit system is when we **normalize** the voltage and current at each location.
- The normalization typically follows transformer ratings.
- This usually makes the per unit value of both voltage and current to be around 1.0 per unit.
- Per unit system allows system operators to overlook abnormalities in the system easily.



Per Unit Quantity

- The per unit quantity of voltage, current, power and impedance is found from dividing the actual quantity by a base value of that quantity.

$$\text{per}-\text{unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

- Per unit value is denoted by 'p.u.'.
- <u>All base values are **real numbers**</u>, denoted by subscript '*B*'.
- The base value is *used only to normalize the quantity*.

Base Value for Voltage

- Transformers separate overall circuit to different zones with different voltage levels.
- We typically set the base value quantity for voltage following transformers' voltage ratio.

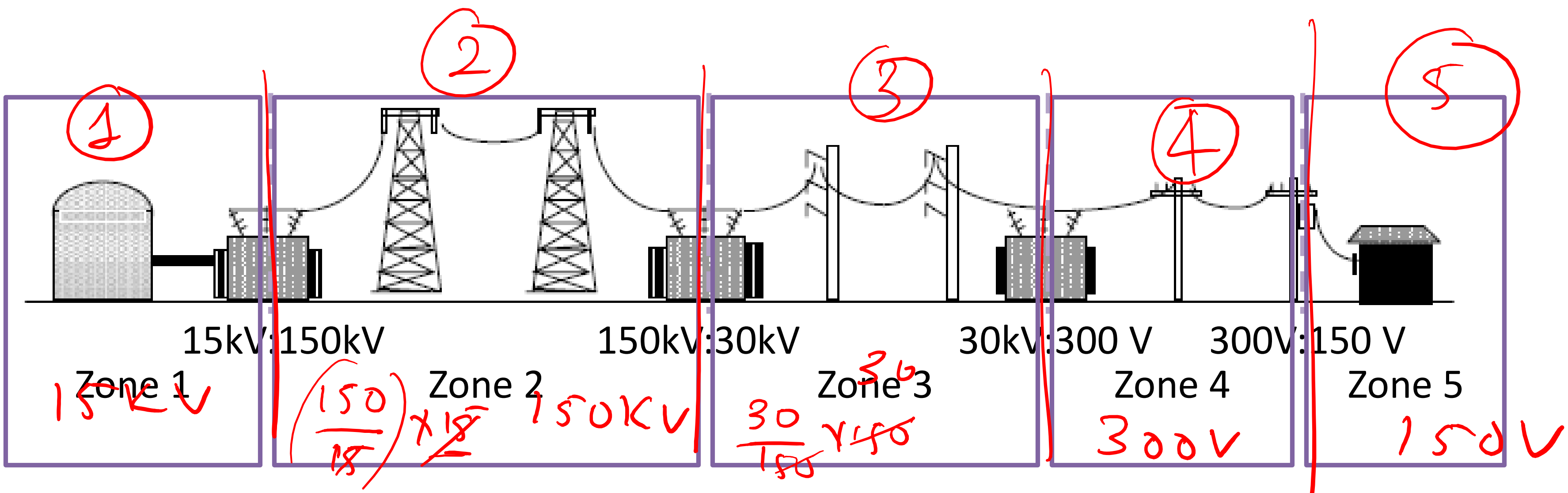
15kV : 150kV

$$\frac{15}{150}$$

Example 1

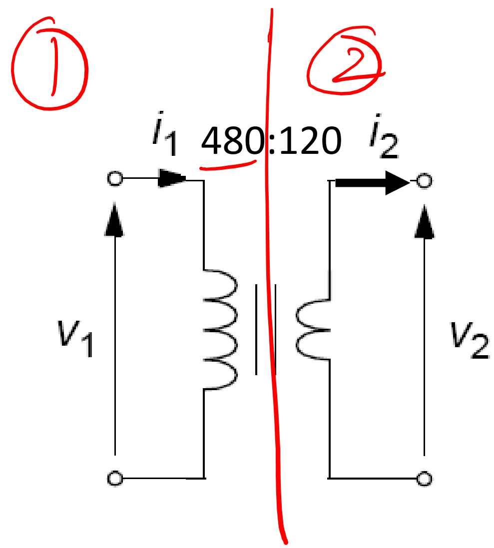
$$\boxed{\frac{V_{1B}}{V_{2B}} = \frac{N_1}{N_2}}$$

- Consider the following electrical energy system.
  - How many zones (with different voltage level) does the system has?
  - Find the base value of the voltage at each zone.



Example 2

- Consider a single phase 480/120 V transformer.
  - Choose the base value of voltage on the primary side to be 480 and that of the secondary side to be 120.
  - If the voltage at primary is measured to be 432 V, which is 0.9 per unit, the voltage at secondary side is 108 V.
  - What is the per unit quantity on the primary and secondary side?



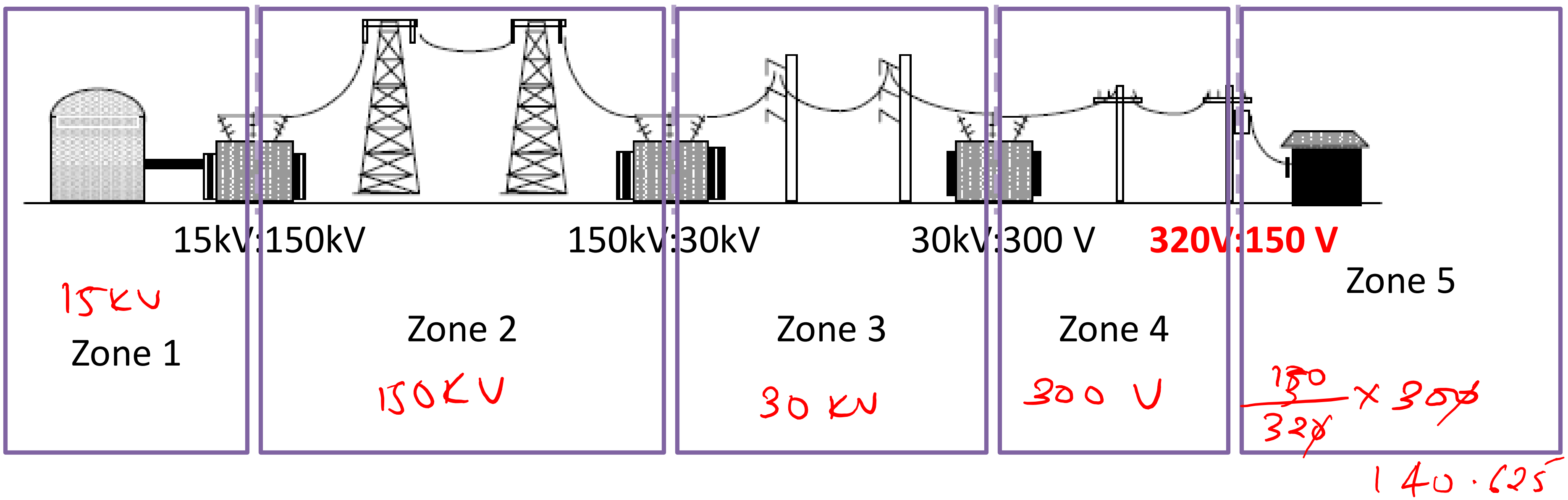
$V_{1B} = 480\ V$ $V_1 = 432$

$V_{2B} = 120\ V$ $V_2 = 108\ V$

$V_{1pu} = \frac{432}{480} = 0.9$ $V_{2pu} = \frac{108}{120} = 0.9$

Example 3

- Consider the following electrical energy system.
  - Find the base value of the voltage at each zone.



Base Value for Complex Power

- First, choose voltage base values following transformer voltage ratings.
- Select only **single base complex power** $S_B^{1\Phi}$ in the system.
- The base value of power is *used to* ***normalize*** *the quantity*. Thus, the base values of real power, reactive power, and complex power are all the same real number.

$$P_B^{1\Phi} = Q_B^{1\Phi} = S_B^{1\Phi}$$

Base Value for Current and Impedance

- Current base values are calculated from the base power and base voltage.

$$I_B = \frac{S_B}{V_B}$$

- Impedance base values (same value for impedance, resistance, or reactance) are calculated from voltage and current.

$$Z_B = \frac{V_B}{I_B} = \frac{(V_B)^2}{S_B} = \boxed{\frac{(V_B)^2}{S_B}} \qquad Z_B = R_B = X_B$$

KVL, KCL, Complex Power Calculation

- We can still apply KVL, KCL, complex power calculation to the per unit value.
- The <u>actual quantity</u> is simply found from **multiplying the *per unit quantity*** (normalized quantity) **with the *base value***.

$$S_B = V_B I_B^*$$

$$V_B = Z_B I_B$$

Think of Base value as 'Normalization'.

$$S_{pu} = V_{pu} I_{pu}^*$$

$$V_{pu} = I_{pu} Z_{pu}$$

$$S_{pu} = V_{pu} I_{pu}^{*}$$

$$\frac{S_{actual}}{\cancel{S_{Base}}} = \frac{V_{actual}}{\cancel{V_{Base}}} \times \left(\frac{I_{actual}}{\cancel{I_{Base}}}\right)^{*}$$

$$S_{actual} = V_{actual} \times (I_{actual}^{*})$$

$$V_{pu} = I_{pu} \times Z_{pu}$$

$$\frac{V_{actual}}{\cancel{(V_{Base})}} = \frac{I_{actual} \times Z_{actual}}{\cancel{(I_B \times Z_B)}} \Rightarrow V_{actual} = I_{act} \times Z_{actual}$$

Example 4: Per Unit Value

- A single-phase 20kVA, (480/120 V), 60 Hz transformer has an equivalent leakage impedance referred to 120-volt winding of $Z_{eq2} = 0.0525\angle 78.13°\ \Omega$. Using the transformer rating as base values, find per-unit leakage impedance.

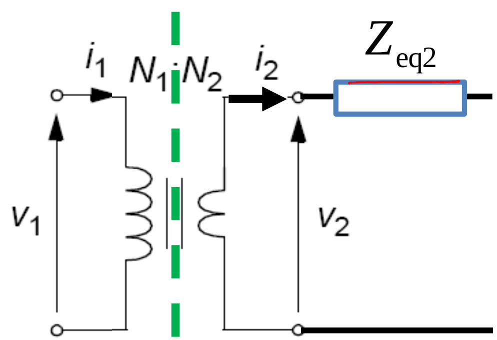
Z_{pu}

(1) $\qquad S_B = 20kVA \qquad$ (2)

$V_{B1} = 480\ V$

$Z_{B1} = \dfrac{V_{1B}^2}{S_B}$

$= \dfrac{480^2}{20\times10^3} = 11.52\,\Omega$



$V_{B2} = 120\ V$

$Z_{B2} = \dfrac{V_{2B}^2}{S_B} = \dfrac{Z_{B1}}{a^2}$

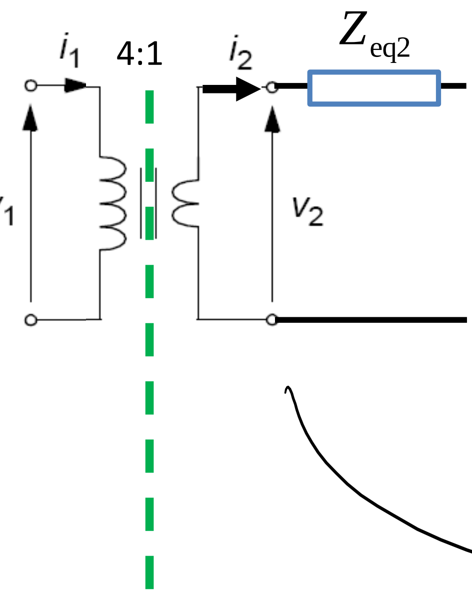
$= \dfrac{120^2}{20\times10^3} = 0.72\,\Omega$

$Z_{pu} = \dfrac{0.0525\angle 78.13}{.72} = .0729\angle 78.13$

Example 4: Per Unit Value

$V_{1B} = 480$

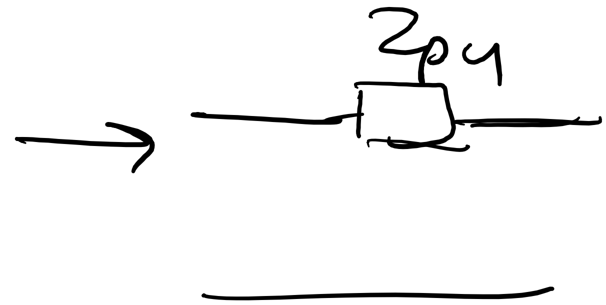
$Z_{1B} = 11.52$



$V_{2B} = 120$

$Z_{2B} = 0.722$

$Z_{pu} = 0.0729 \angle 78.13$

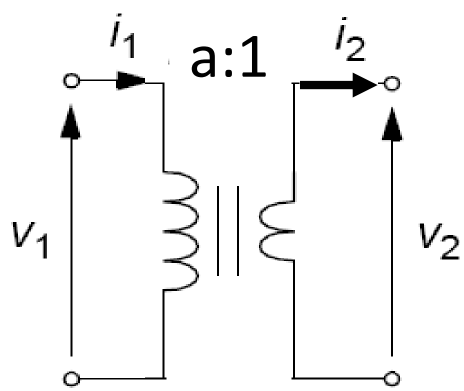


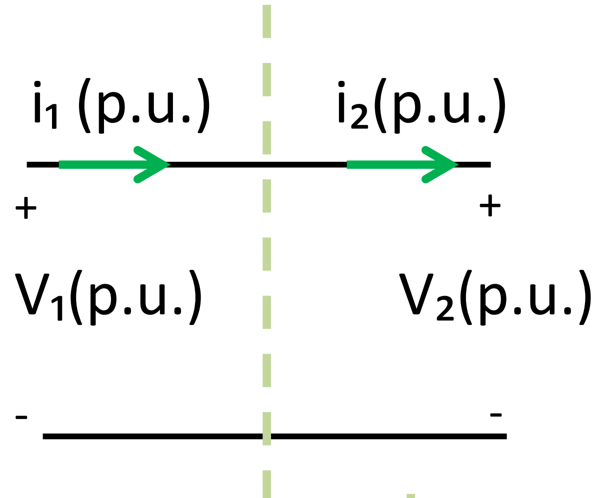
$Z_{eq1} = .0525 \times 16 \angle 78.13$

$$Z_{pu} = \frac{Z_{eq1}}{Z_{1B}} = \frac{0.0525 \times 16}{11.52} \angle 78.13 = .0729 \angle 78.13$$

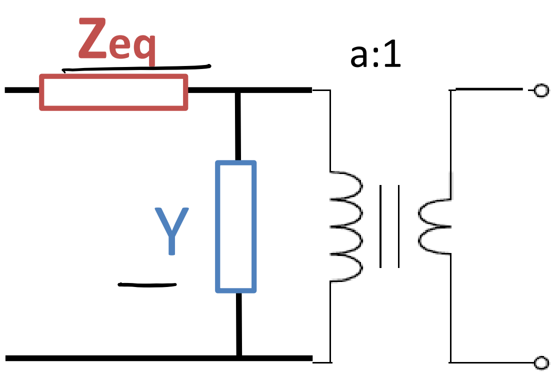
P.U. Equivalent Circuit of a Transformer

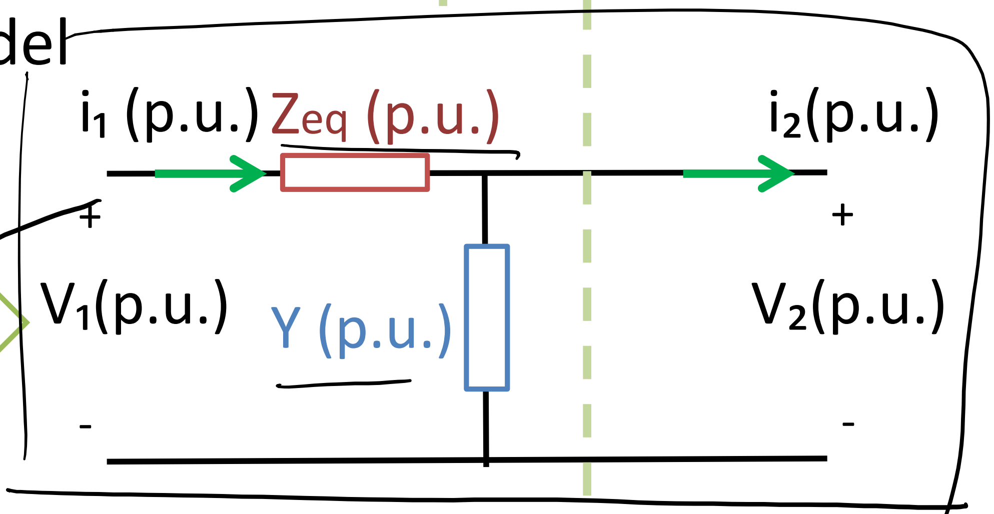
- Ideal transformer model





- Practical transformer model





Change of Base Value

- Manufacturers usually specify equipment impedances in per unit values together with voltage ratings (V) and apparent power rating (VA).

$$Z_{actual} = (Z_{Base} \times Z_{pu})_{old}$$

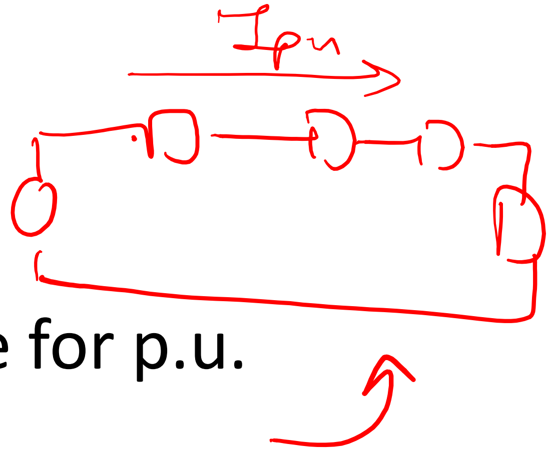
$$= (Z_{Base} \times Z_{pu})_{new}$$

$$Z_{pu\,new} = \left(\frac{Z_{Base\,old}}{Z_{Base\,new}}\right) \times Z_{pu\,old} = \frac{\frac{V_{B\,old}^2}{S_{B\,old}}}{\frac{V_{B\,new}^2}{S_{B\,new}}} \times Z_{pu\,old}$$

$$Z_{pu\,new} = \left(\frac{V_{B\,old}}{V_{B\,new}}\right)^2 \times \left(\frac{S_{B\,new}}{S_{B\,old}}\right) \times Z_{pu\,old}$$

Steps of Per Unit Analysis

1. Choose $S_B^{1\Phi}$ for the system.
2. Select V_B for different zones (usually follows transformer voltage ratings).
3. Calculate Z_B for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.



Summary

→ Per Unit ?

→ Voltage Base + S (Power) Base

→ Current Base

→ Z (Impedance Base)

→ Change the bases