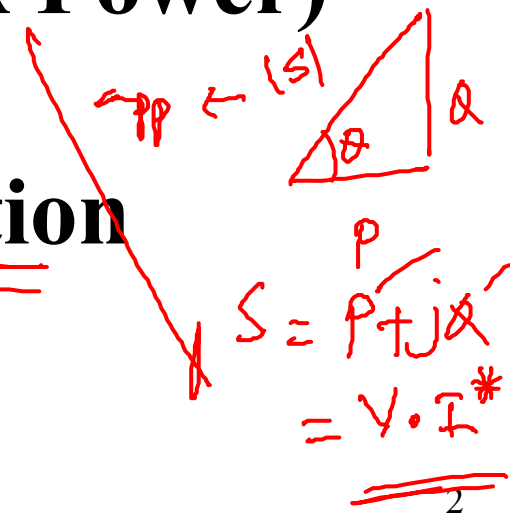
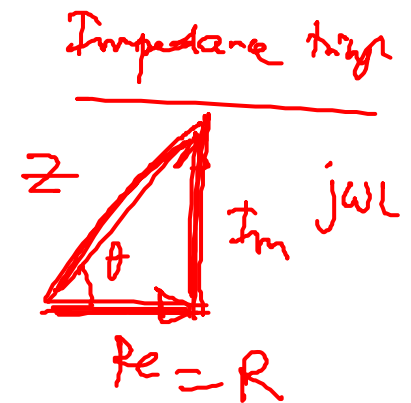


EE2022

Electrical Energy Systems

Lecture 4: AC Power

- What is power in AC circuits for R, L, C?
- Power in Generic AC Load
- Real, Reactive and Apparent Power
- Power Triangle (Complex Power)
- Power Factor $\cos \theta$.
- Power Factor Compensation



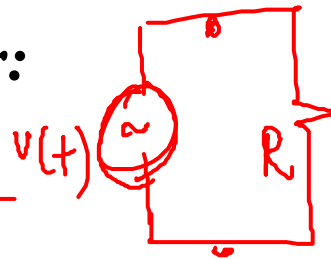
Power in a Resistor

$$v(t) = V_m \cos \omega t$$

$$i(t) = I_m \cos \omega t$$

(Instantaneous) power:

$$p(t) = V_m I_m \cos^2 \omega t$$



Mechanical power

$$= T \times \omega$$

$$I_m = \frac{V_m}{R}$$

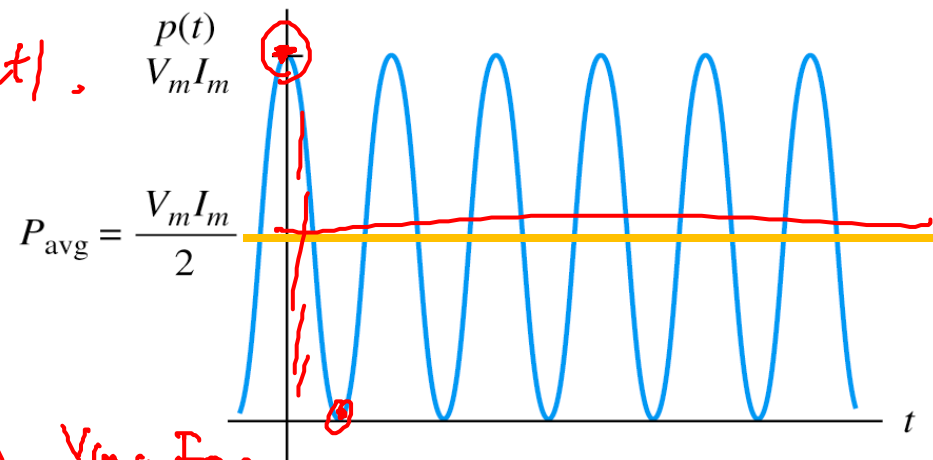
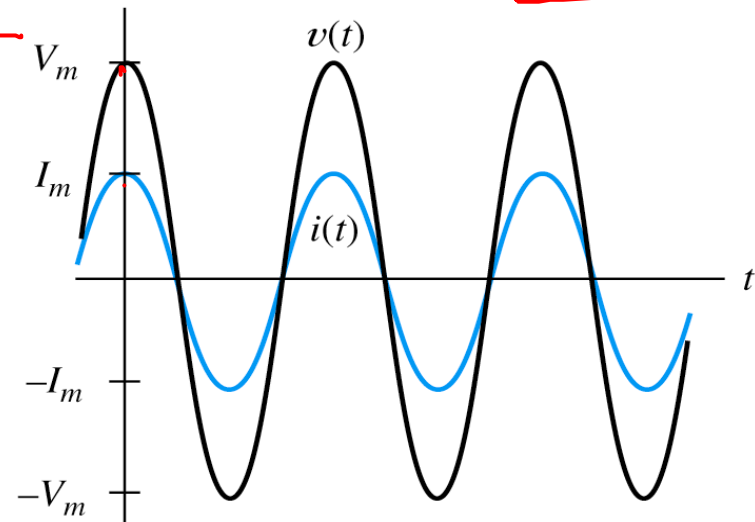
↑ Fixed
Oscillating like the $p(t)$.

$$P_{avg} = \frac{1}{T} \int_0^T p(t) \cdot dt$$

$$\cos^2 \omega t = \frac{1}{2} (1 + \cos 2\omega t)$$

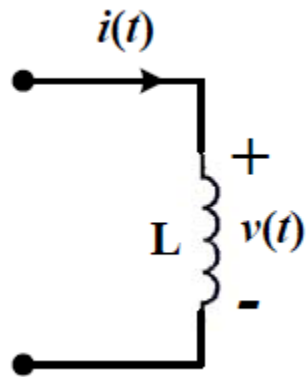
$$= \frac{1}{2}$$

$$P_{avg} = V_m \cdot I_m \cdot \frac{1}{2} = \left(\frac{V_m}{\sqrt{2}} \right) \left(\frac{I_m}{\sqrt{2}} \right) = V_{rms} \cdot I_{rms}$$



Current, voltage, and power versus time for a purely resistive load

Average Power in an Inductor



$$(I_m) = \frac{V_m}{\omega L}$$

$$I_{rms} = \frac{V_{rms}}{\omega L}$$

$$v(t) = V_m \cos(\omega t)$$

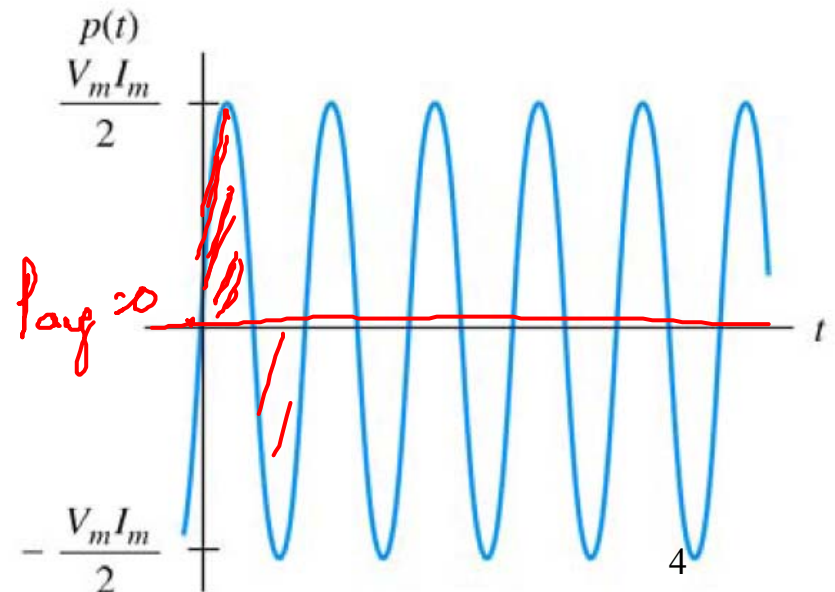
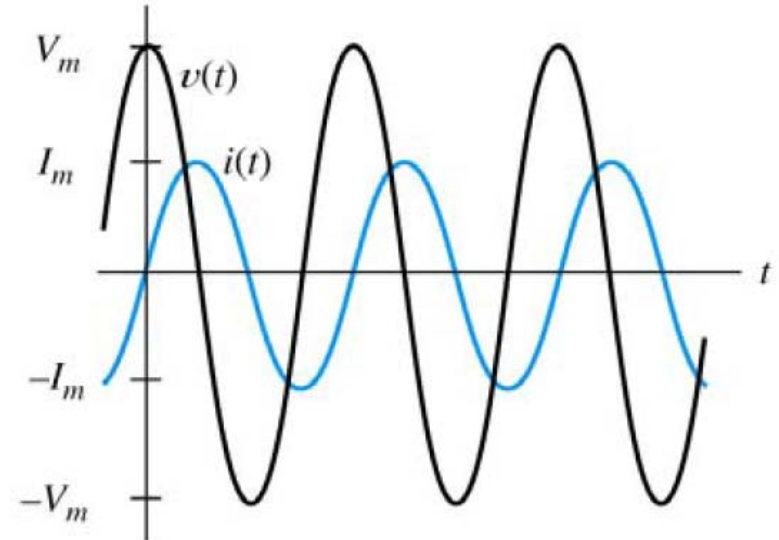
$$i(t) = I_m \cos(\omega t - 90^\circ) = I_m \sin(\omega t)$$

$$p(t) = V_{rms} I_{rms} \sin(2\omega t)$$

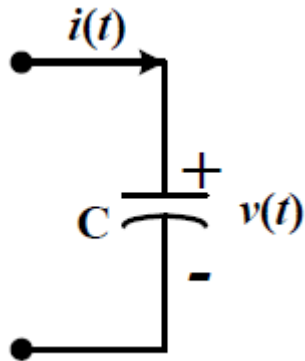
$$\begin{aligned} p(t) &= V_m \cos \omega t \times I_m \sin \omega t \\ &= \frac{V_m \cdot I_m}{2} (2 \sin \omega t \cdot \cos \omega t) \\ &= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \sin(2\omega t) \end{aligned}$$

$$\begin{aligned} p(t) &= V_{rms} \cdot I_{rms} \sin(2\omega t) \\ &= \left(\frac{V_{rms}}{\omega L} \right) \sin(2\omega t) \end{aligned}$$

$P = \frac{V^2}{R}$



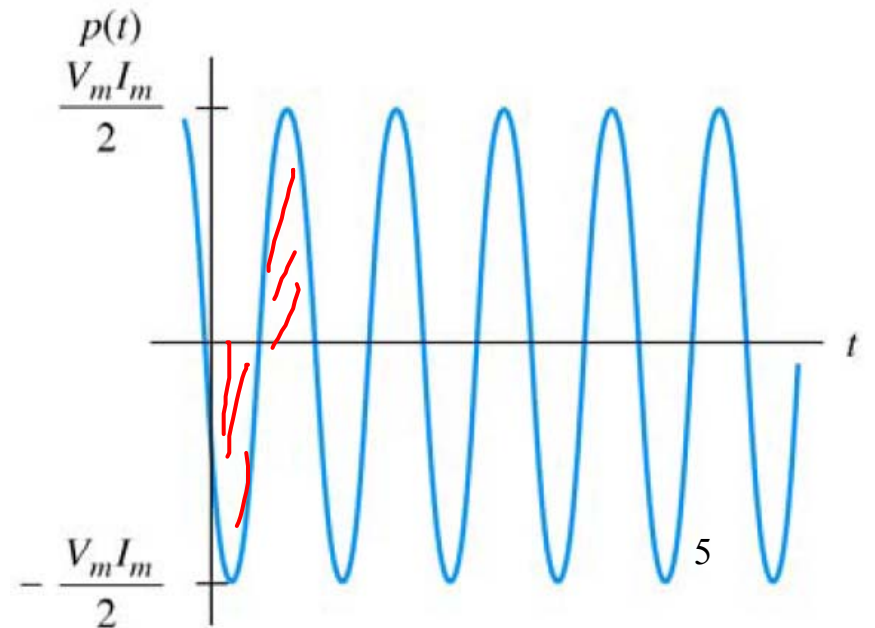
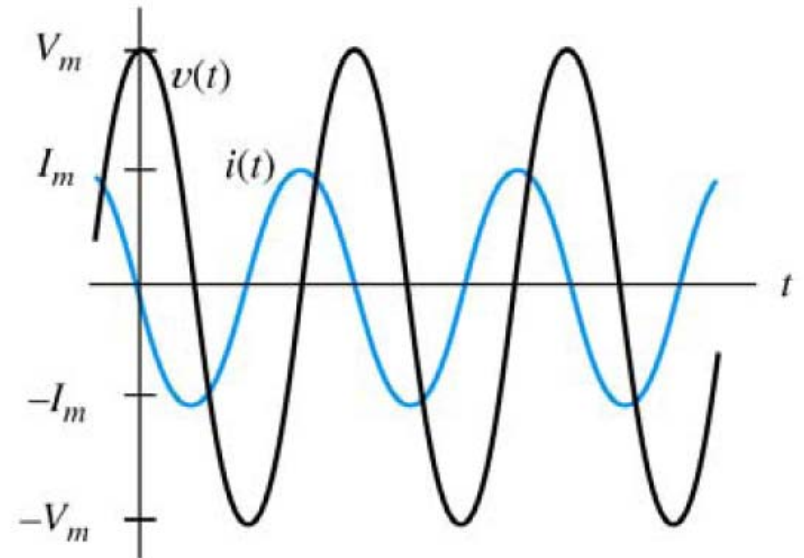
Power in a Capacitor



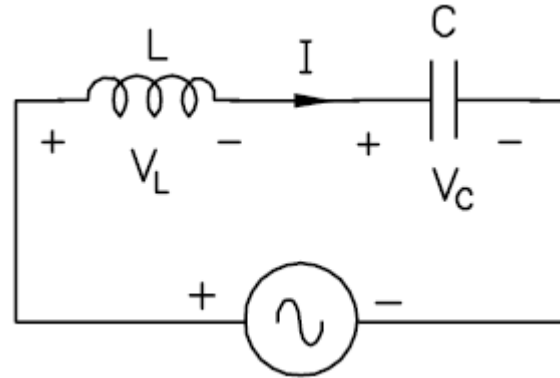
$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t + 90) = -I_m \sin \omega t$$

$$p(t) = -V_{rms} I_{rms} \sin(2\omega t)$$

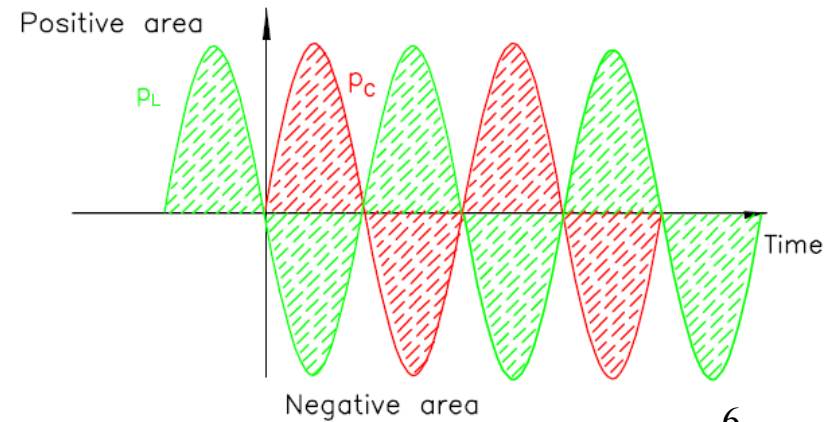
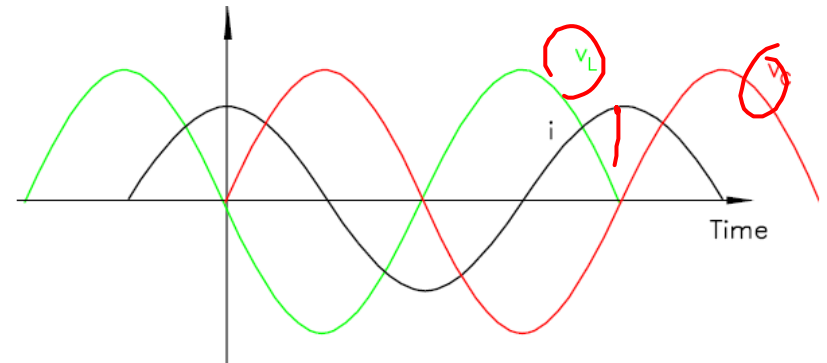


Power In L-C Series Circuit



When $p_L(t)$ is positive, $p_C(t)$ is negative.

Both have: zero average

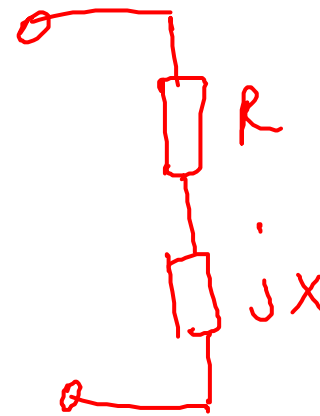
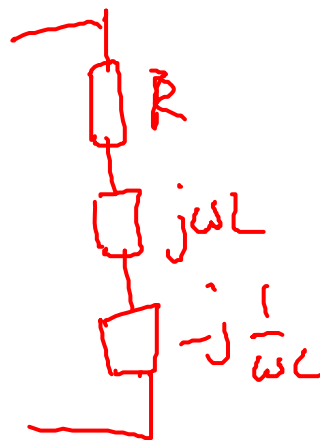
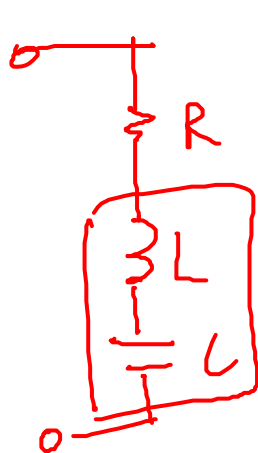


Power in Generic AC Load

R : $P(t) = V_{rms} \cdot I_{rms} [1 + \cos 2\omega t]$, $P_{avg} = V_{rms} I_{rms}$

L : $P(t) = V_{rms} \cdot I_{rms} \sin 2\omega t$: $P_{avg} = 0$

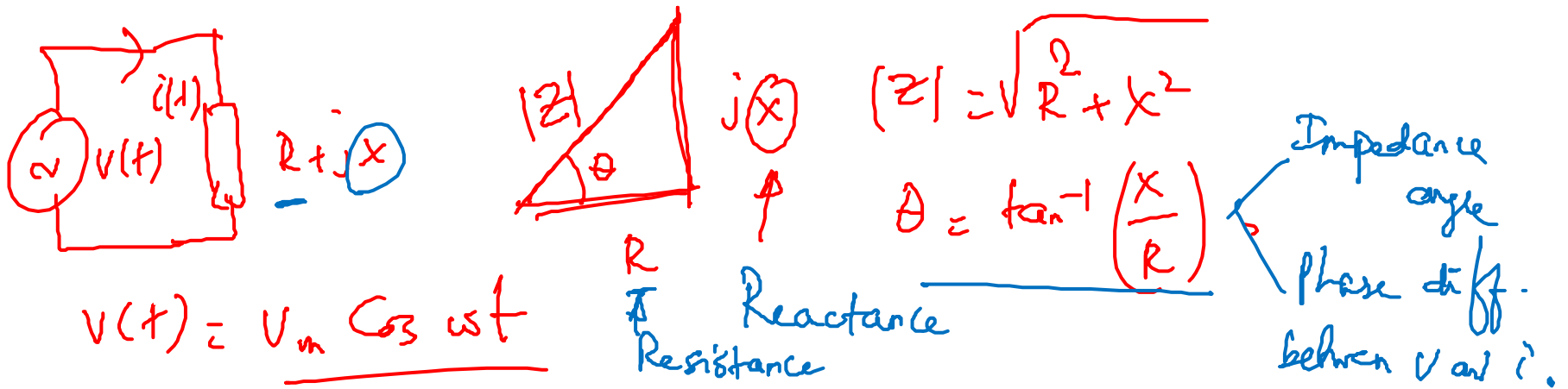
C : $P(t) = -V_{rms} \cdot I_{rms} \sin 2\omega t$: $P_{avg} = 0$



$$Z = R + jX$$

$$X = \left(\omega L - \frac{1}{\omega C} \right)$$

+ve = inductive
 -ve = Capacitive



$P(t) = V(t) \cdot i(t) = V_m \cos \omega t \cdot \frac{V_m}{|Z|} \cos(\omega t - \theta)$

$P_{avg} = \frac{V_m^2}{2|Z|} \cos \theta$

$= \frac{V_m^2}{|Z|} \cos \omega t \cdot \cos(\omega t - \theta)$

$= \frac{V_m^2}{|Z|} \cdot \frac{1}{2} \cdot [\cos(\omega t + \omega t - \theta) + \cos(\cancel{\omega t} - \cancel{\omega t} + \theta)]$

$= \frac{V_m^2}{|Z|} \cdot \frac{1}{2} \cdot [\cos \theta + \cos(2\omega t - \theta)]$

$\rightarrow \text{avg} =$

$$P = \frac{V_m^2}{2|Z|} \cos \theta = \left(\frac{V_m}{\sqrt{2}} \right) \cdot \left(\frac{V_m}{\sqrt{2}} \right) \cdot \frac{1}{|Z|} \cdot \cos \theta$$

Real power

P_{avg}

$$= V_{rms} \cdot V_{rms} \cdot \frac{1}{|Z|} \cdot \cos \theta$$

$$= \underline{V_{rms} \cdot I_{rms} \cdot \cos \theta}$$

$$R \Rightarrow \theta = 0 \quad P = V_{rms} \cdot I_{rms}$$

$$L \Rightarrow \theta = +90^\circ \quad P = 0$$

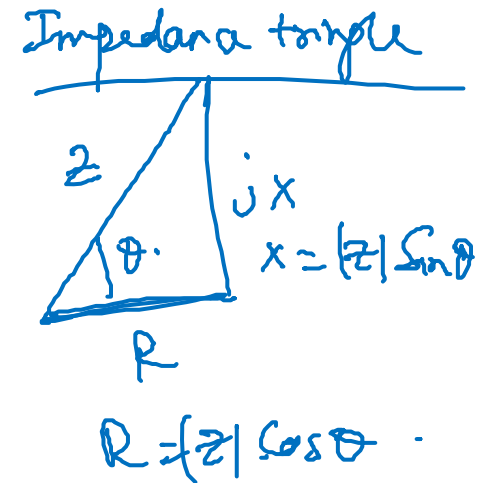
$$C \Rightarrow \theta = -90^\circ \quad P = 0$$

$$P = \underline{V_{rms} \cdot I_{rms} \cdot \cos \theta}$$

$$= I_{rms} \cdot |Z| \cdot I_{rms} \cdot \cos \theta$$

$$= I_{rms}^2 \cdot \underline{|Z| \cdot \cos \theta}$$

$$= I_{rms}^2 \cdot \underline{R}$$

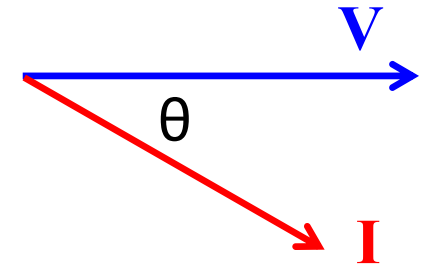


P ✓
 X ✓
 $\text{Total } V \cdot I$ ✓

Real, Reactive and Apparent Power

Apparent Power

We express power in d.c. and a.c. circuits as follows:



$$P_{dc} = V_{dc} I_{dc} \text{ Watts}$$

$$P_{ac} = V_{rms} I_{rms} \cos \theta \text{ Watts}$$

Power factor

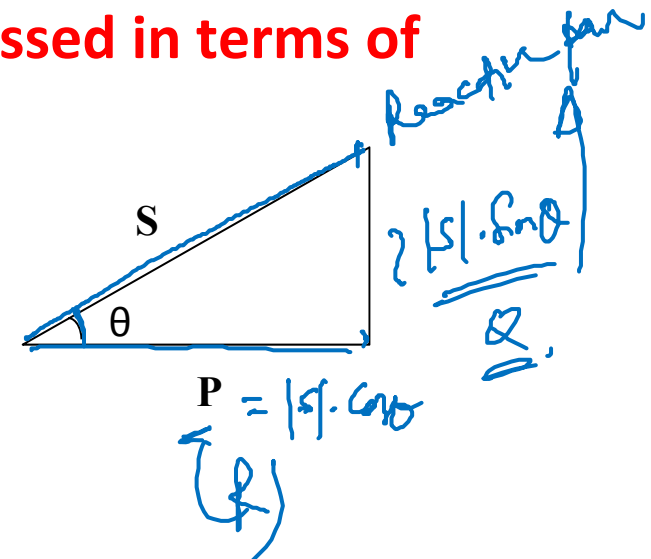
Define $(V_{rms} I_{rms})$ as Apparent Power, i.e.

$$|S| = V_{rms} I_{rms} \text{ (product of voltage and current magnitude)}$$

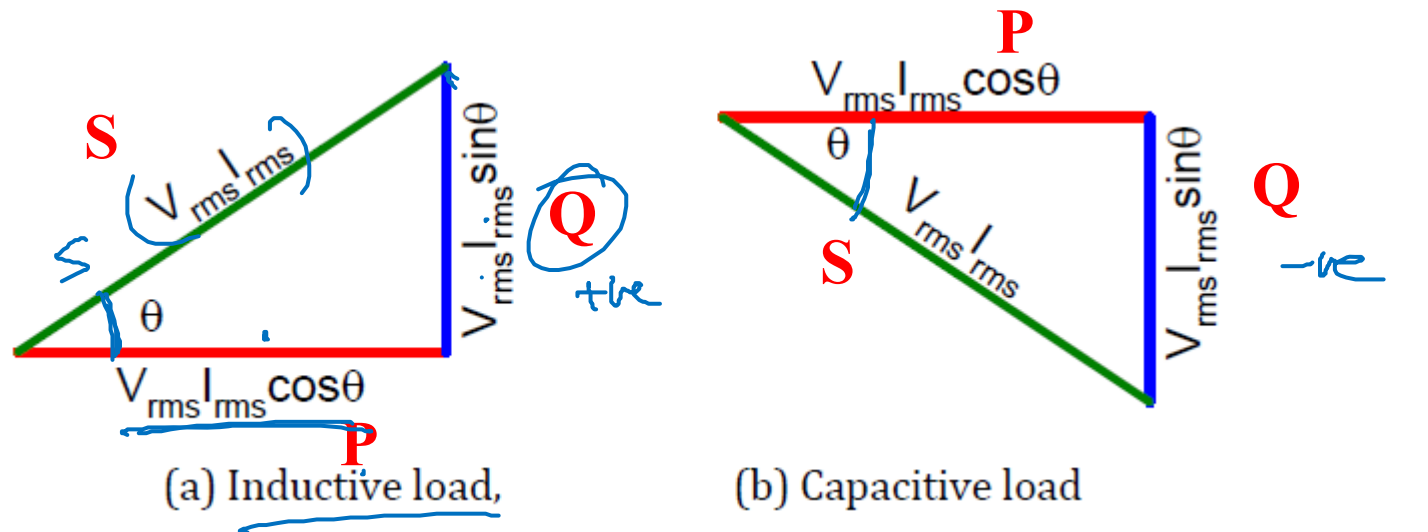
Unit of apparent power = VA (volt-amperes)

So, the power in a.c. circuit may also be expressed in terms of apparent power as follows:

$$P = |S| \cos \theta$$



Power Triangle



P – Real (or active) power; power consumed in the resistive part of the circuit

Q – Reactive power consumed by the device, due to inductor or capacitor

Complex Power: $S = V I^*$

We can show that,

$$\text{Complex power, } S = \underbrace{V_{\text{rms}} I_{\text{rms}} \cos \theta}_P + j \underbrace{V_{\text{rms}} I_{\text{rms}} \sin \theta}_Q$$

$$\text{Or, } S = |S| \cos \theta + j |S| \sin \theta = \underline{\underline{P + jQ}}$$

voltage: $V \angle \alpha$, current: $I \angle \beta$

$$V = V_{\text{rms}}, I = I_{\text{rms}}$$

$$P = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos \theta, \quad Q = \theta_v - \theta_i$$

$$= V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos(\alpha - \beta) = \alpha - \beta$$

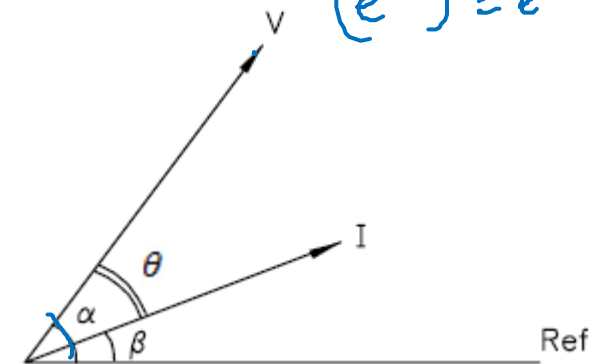
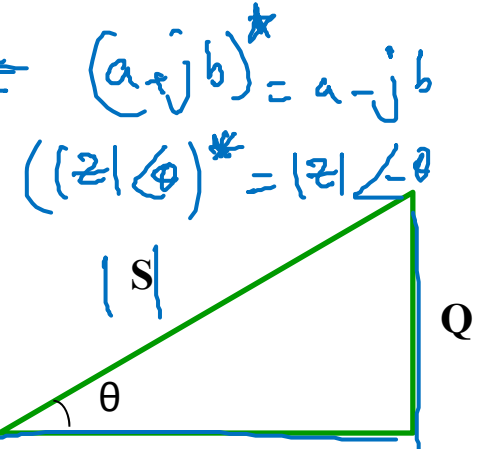
$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\alpha - \beta)$$

$$S = P + jQ = V_{\text{rms}} \cdot I_{\text{rms}} \cdot [\cos(\alpha - \beta) + j \sin(\alpha - \beta)] = V_{\text{rms}} I_{\text{rms}} \cdot e^{j(\alpha - \beta)}$$

(Euler's formula $e^{j\theta} = \cos \theta + j \sin \theta$)

$$= (V_{\text{rms}} \cdot e^{j\alpha}) (I_{\text{rms}} \cdot e^{-j\beta})$$

$$= V_{\text{phasor}} \cdot I_{\text{phasor}}^*$$



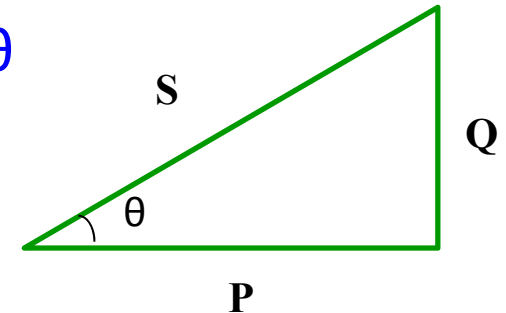
Complex Power

We can show that,

$$\text{Complex power } S = V I^*$$

$$\text{Complex power, } S = V_{\text{rms}} I_{\text{rms}} \cos \theta + j V_{\text{rms}} I_{\text{rms}} \sin \theta$$

$$\text{Or, } S = |S| \cos \theta + j |S| \sin \theta = P + jQ$$



S	Complex power ✓	VA	$S = S \angle \theta = P + jQ = VI^* =$ $= V I \angle \theta = V I (\cos \theta + j \sin \theta)$
$ S $	Apparent power ✓	VA	$ S = V I = \sqrt{P^2 + Q^2}$
P	Active power Average power, Real power	W	$P = \text{Re}(S) = S \cos(\theta) = V I \cos(\theta)$
Q	Reactive power ✓	var	$Q = \text{Im}(S) = S \sin(\theta) = V I \sin(\theta)$

Complex Power of Series and Parallel-connected loads

Series connected load: If two loads Z_1 and Z_2 are connected in **series** across a voltage source,

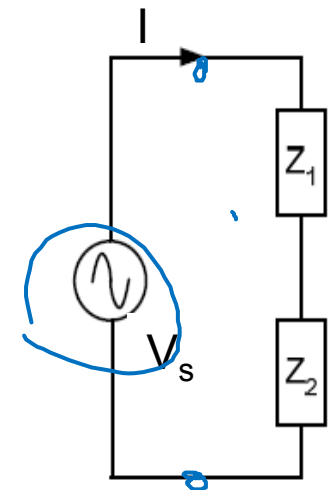
For load 1: $S_1 = V_1 I^* = P_1 + jQ_1$

For load 2: $S_2 = V_2 I^* = P_2 + jQ_2$

Total complex power $S = V_s I^* = (V_1 + V_2) I^* = V_1 I^* + V_2 I^*$

Or, $S = P_1 + jQ_1 + P_2 + jQ_2 = (P_1 + P_2) + j(Q_1 + Q_2)$

Handwritten notes:
 $|S_1| = \sqrt{13} \text{ VA} \leftarrow S_1 = 2 + j3 \text{ VA}$
 $|S_2| = 5 \text{ VA} \leftarrow S_2 = 3 + j4 \text{ VA}$
 $|S| = |S_1| + |S_2|$ (circled in red)
 $S = S_1 + S_2 = 5 + j7 \text{ VA}$



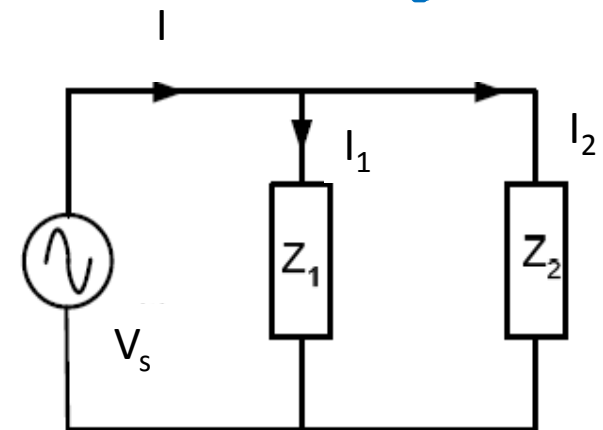
Parallel connected load: If two loads Z_1 and Z_2 are connected in **parallel**,

For load 1: $S_1 = V_s I_1^* = P_1 + jQ_1$

For load 2: $S_2 = V_s I_2^* = P_2 + jQ_2$

Total complex power $S = V_s I^* = V_s (I_1^* + I_2^*)$

Or, $S = P_1 + jQ_1 + P_2 + jQ_2 = (P_1 + P_2) + j(Q_1 + Q_2)$

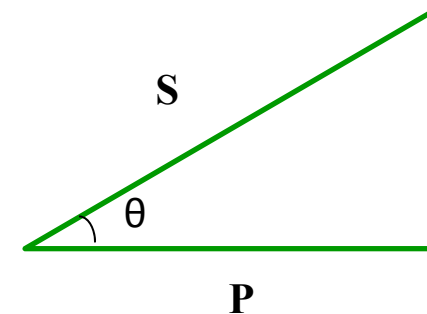


- **Complex power of a network of interconnected loads is equal to the sum of complex powers of individual loads.**
- **Real (or Reactive) power of a network of interconnected loads is equal to the sum of real (or reactive) powers of individual loads.**

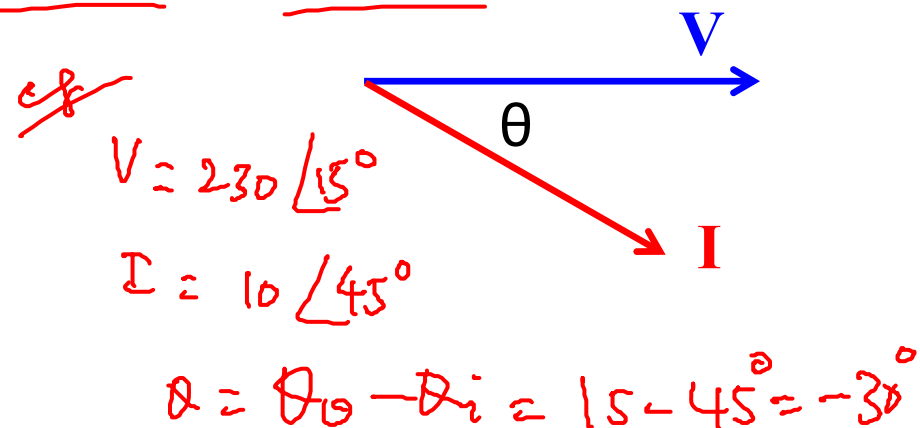
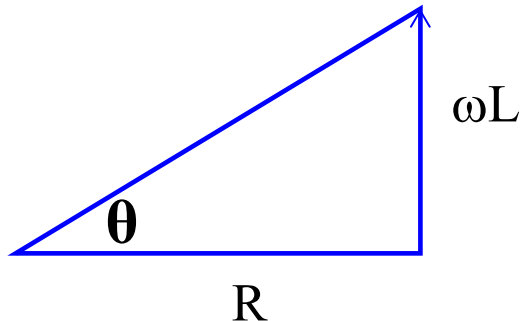
Power Factor of an a.c. circuit

Now, we will define Power Factor of an a.c. circuit as the ratio of real power to the apparent power.

$$\text{Power Factor} = \frac{P}{|S|} = \frac{P}{|V||I|} = \cos \theta$$



Power factor angle (θ) is the same in power triangle, impedance triangle, and the angle between voltage and current.



Lagging Power Factor

Consider an inductive device or load, the impedance of which is given by

$$Z = R + j\omega L = R + jX_L = |Z| \angle \theta^{+ve}$$

Taking voltage V as reference phasor, we have

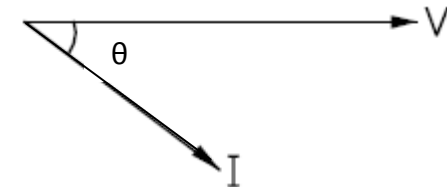
$$V = |V| \angle 0^\circ$$

$$I = \frac{|V| \angle 0^\circ}{|Z| \angle \theta} = \frac{|V|}{|Z|} \angle -\theta = |I| \angle -\theta$$

\rightarrow ve θ \rightarrow $\cos \theta = \cos(-\theta)$
inductance

\uparrow
+ve θ : $\cos \theta$ lags
voltage

As the impedance angle is positive, the current is lagging behind the voltage as shown in the phasor diagram below:



The real power in the device is

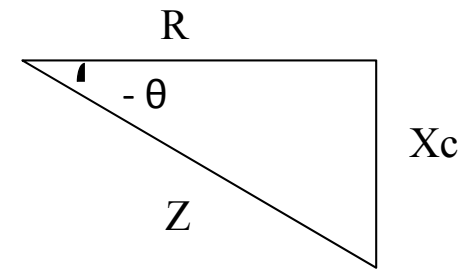
$$P = |V| \cdot |I| \cos \theta$$

Power factor is said to be lagging, if the current lags behind the voltage. In other words, p.f. is lagging when the impedance angle θ is positive, i.e. When the load or device is inductive in nature.

Leading Power Factor

Consider a capacitive device or load, the impedance of which is given by

$$Z = R - j\frac{1}{\omega C} = R - jX_C = |Z| \angle -\theta$$



Taking the voltage V as reference phasor,

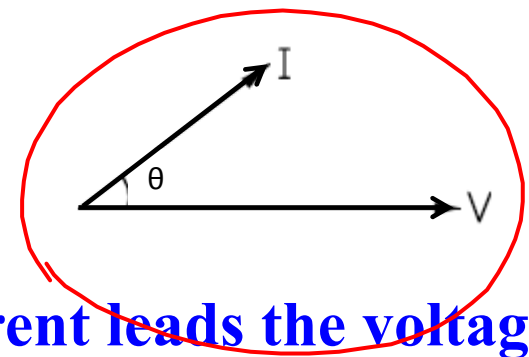
$$V = |V| \angle 0^\circ$$

$$I = \frac{|V| \angle 0^\circ}{|Z| \angle -\theta} = \frac{|V|}{|Z|} \angle \theta = |I| \angle \theta$$

As the impedance angle is negative, the current is leading the voltage as shown in the phasor diagram below:

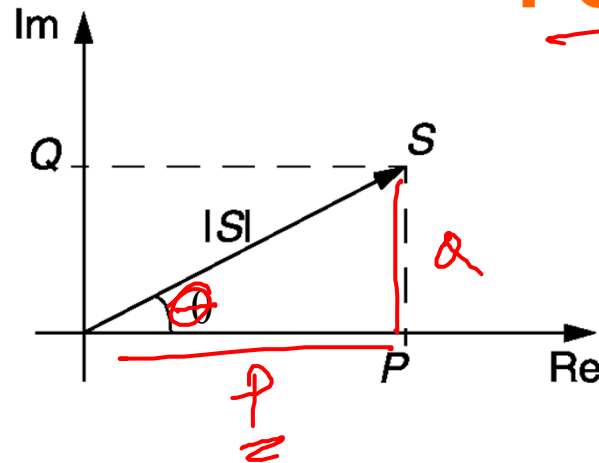
The real power in the device is

$$P = |V| \cdot |I| \cos \theta$$



Power factor is said to be **leading**, if the current leads the voltage. In other words, p.f. is leading when the impedance angle θ is negative, i.e. when the load or device is capacitive in nature.

Power Factor



$$\rightarrow P = |S| \cos(\theta) = |V||I| \cos(\theta)$$

$$\rightarrow Q = |S| \sin \theta$$

$$(|S| \Rightarrow \text{VA})$$

$$\text{eg. } P = 1000 \text{ W}$$

$$Q = 500 \text{ VAR}$$

$$\cos(\theta) = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}}$$

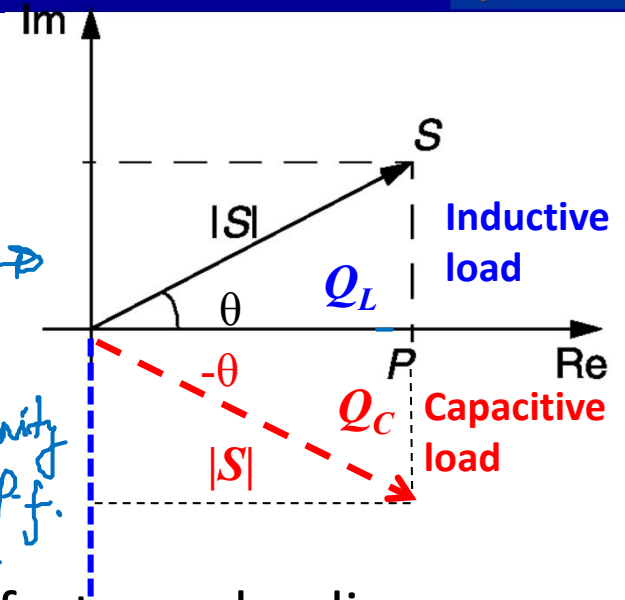
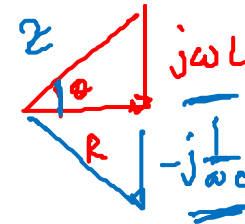
$$\cos(\theta) = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$$

P.F.

Power Factor

$$P = |S| \cos(\theta) = |V||I| \cos(\theta)$$

$$0 \leq \theta \leq 90^\circ$$



Hence, power factor lies between: $0 \leq \text{p.f.} \leq 1$ unity
low p.f. \rightarrow high p.f.

Since $\cos(+\theta) = \cos(-\theta)$, we identify the power factor as leading or lagging.

For inductive load: Current lags behind voltage

Hence, power factor is lagging p.f.

For capacitive load: Current leads the voltage

Hence, power factor is leading p.f.

$$Z_{\text{load}} = 10 \angle 25^\circ$$

$\cos 25^\circ$

For example: If the load is $10 \angle 25^\circ$ the power factor is 0.906 lagging

If the load is $20 \angle -30^\circ$ the power factor is 0.87 leading

cap.

cos 30°

Power Factor Correction

Loads are usually connected to a fixed voltage supply, e.g. 220V, 50 Hz in Singapore, hence V_{rms} is given.

$$p.f. = \frac{P}{V_{rms} I_{rms}}$$

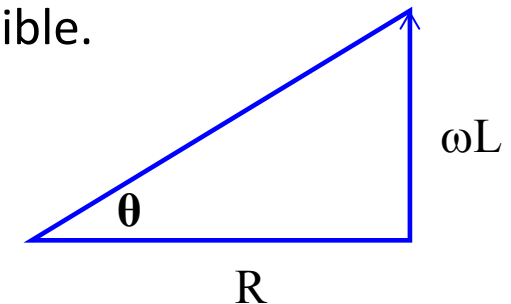
$$\text{Or, } I_{rms} = \frac{P}{V_{rms} \times \cos \theta}$$

Useful work

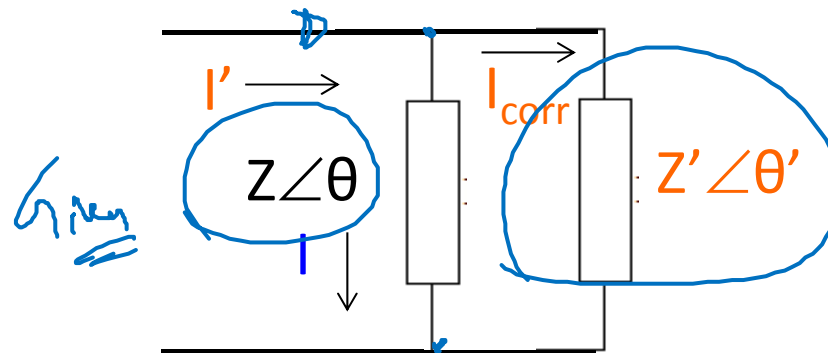
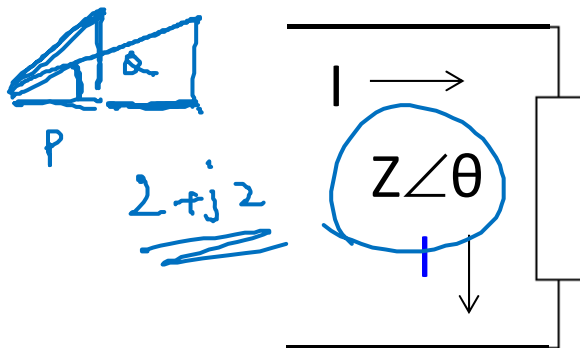
To deliver a certain amount of power to the load, current will be larger if power factor is smaller.

Hence it is desirable that power factor be as close to 1 as possible.
i.e. θ should be as small as possible.

Once the load is connected, its θ cannot be changed.



Another reactive element can be added parallel to the load to improve power factor.



If the load is originally inductive, choose a capacitor

If the load is originally capacitive, choose an inductor as correcting device

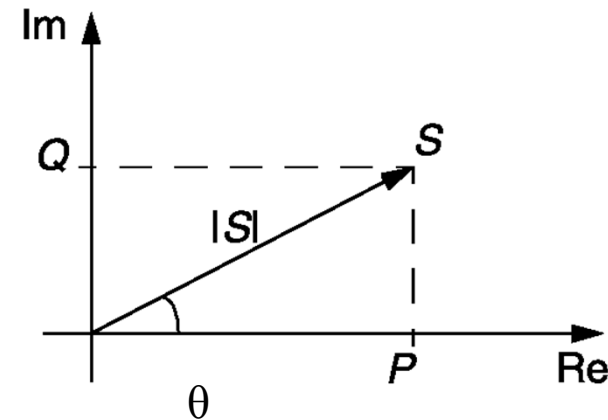
Power Factor Correction

If the power factor is poor (i.e., very low), current drawn by the load will be high.

This will result in

- Poor voltage regulation at the load
- Heavy transmission lines losses
- High operating cost

Transmission
line



$$P = |S| \cos(\theta) = |V| |I| \cos(\theta)$$

$$|I| = P / |V| \cos(\theta)$$

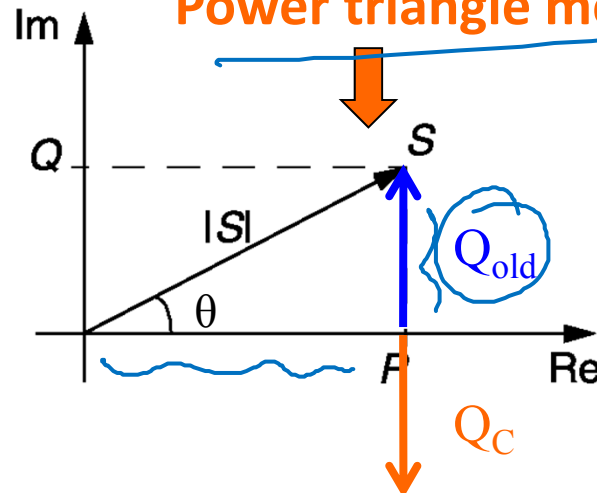
A capacitor can be added in parallel to the load for power factor correction.

There are various ways to find the value of capacitor:

- Impedance triangle method
- Power triangle method
- Power equations

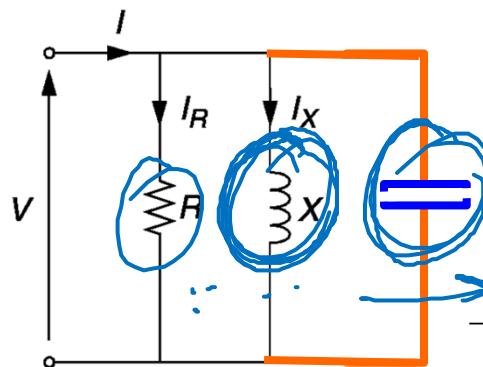
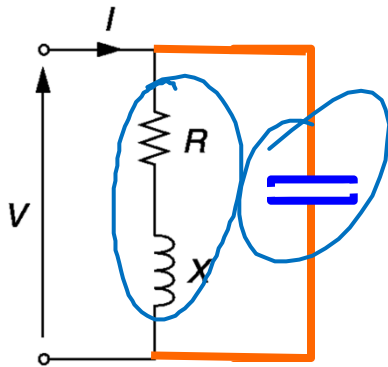
Power Factor Correction

Power triangle method

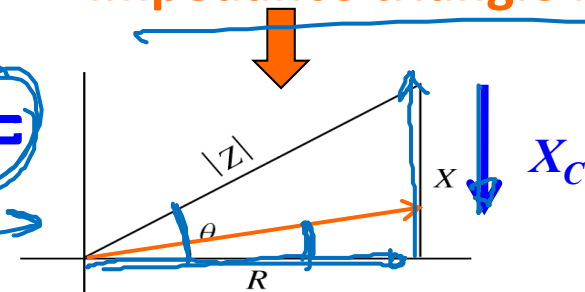


$$P = |S| \cos(\theta) = |V| |I| \cos(\theta)$$

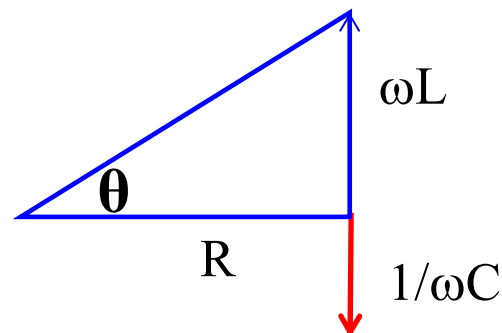
$$|I| = P / |V| \cos(\theta)$$



Impedance triangle method

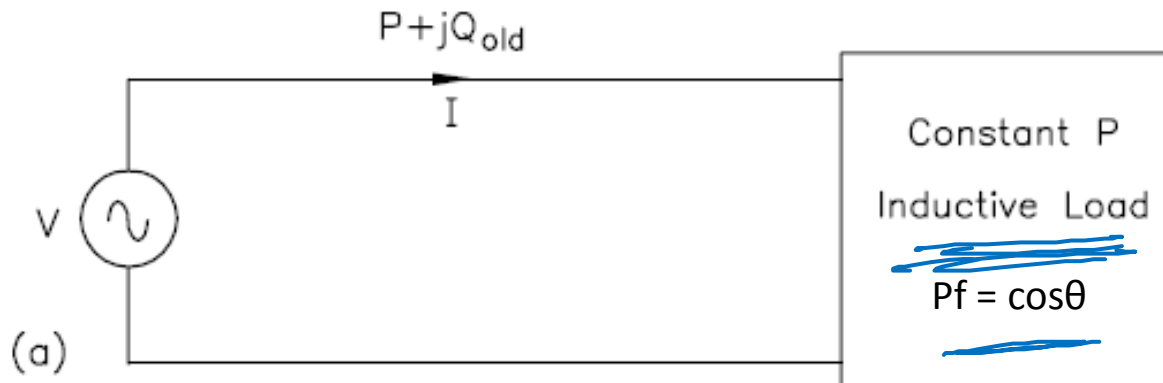


The load impedance in the complex plane.



Power Factor Correction

If P is the active power consumption of the load, the current drawn by the load can be calculated as:



$$P = |V| \cdot |I| \cos\theta$$

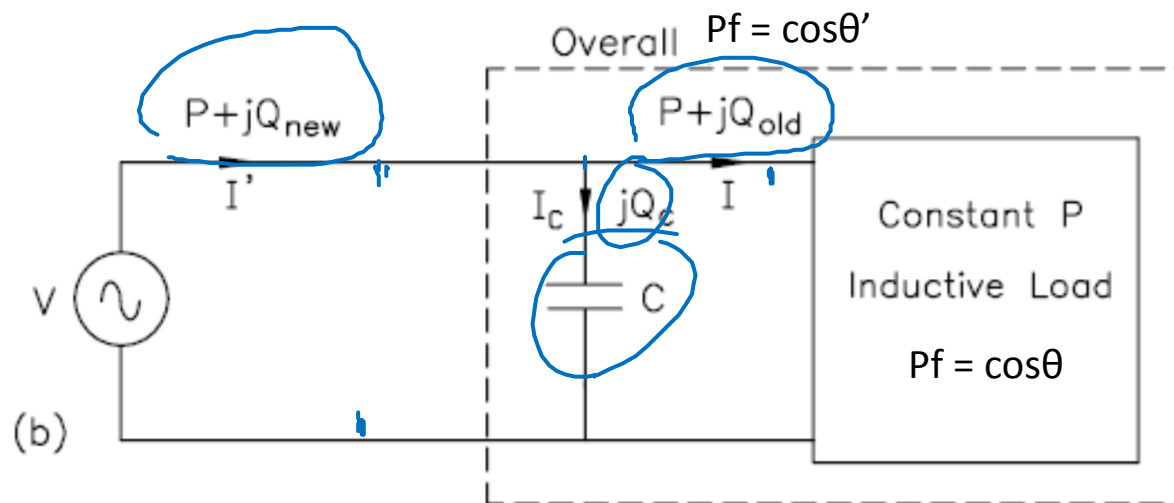
$$|I| = \frac{P}{|V| \cos\theta}$$

$$I = |I| \angle -\theta$$

The apparent power:

$$S_{old} = VI^* = P + jQ_{old}$$

When a capacitor is connected to improve the power factor, the current drawn by this capacitor: $I_c = j\omega CV = \omega CV \angle 90^\circ$



Applying KCL, the total current drawn by the load is:

$$I' = I + I_c$$

$$P = |V| \cdot |I'| \cos\theta'$$

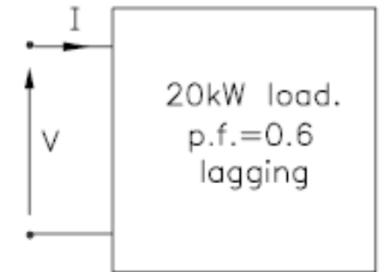
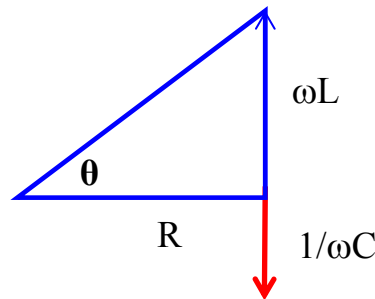
$$S_{new} = VI'^* = P + jQ_{new}$$

Example 6 - Power Factor Correction

A load connected across a 230V, 50Hz line draws 20kW at 0.6 p.f. lagging. Determine the current drawn by the load. If a capacitor of $800\mu\text{F}$ is connected in parallel with the load, what will be the current drawn from the source? Also determine the overall power factor of the system (load and capacitor connected in parallel) as seen by the source.

$$V = 230\angle 0^\circ$$

$$|I| = \frac{20000}{230 \times 0.6} = 144.93 \text{ A}$$



As $\cos \theta = 0.6$ lagging, the impedance angle θ is

$$\theta = \cos^{-1} 0.6 = 53.13^\circ$$

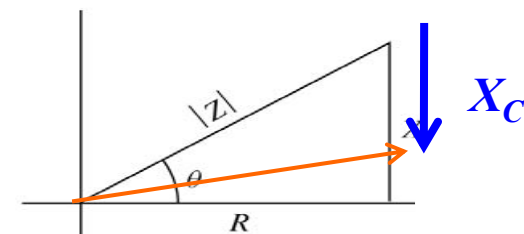
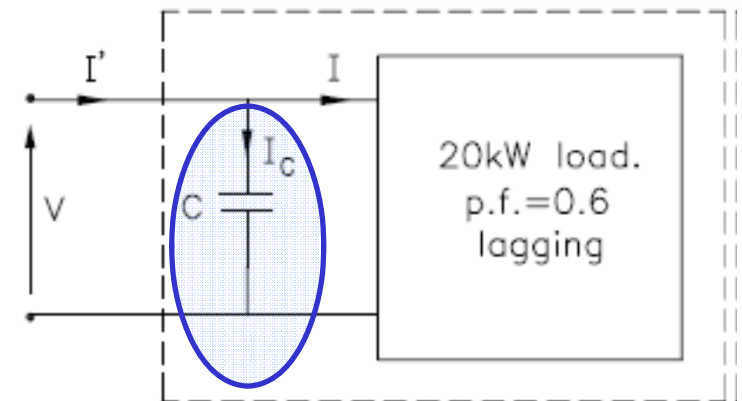
$$I = 144.93 \angle -53.13 = 86.96 - j 115.94 \text{ A}$$

$$I_c = j\omega CV = j 314 \times 800 \times 10^{-6} \times 230 = j57.78$$

$$I' = I + I_c = 86.96 - j 115.94 + j57.78$$

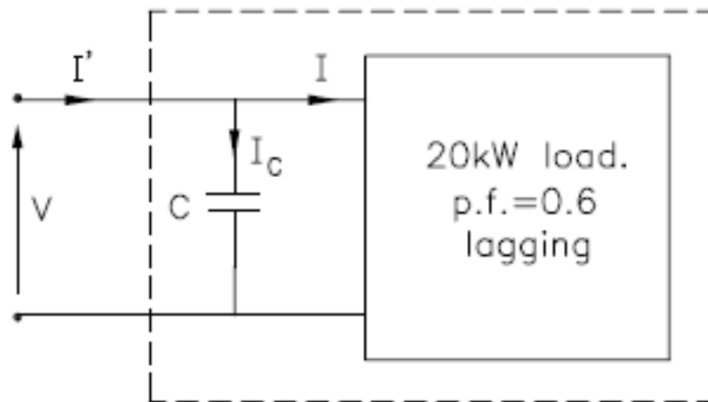
$$= 104.62 \angle -33.78^\circ$$

$$\text{Power factor} = \cos 33.78^\circ = 0.83 \text{ lagging}$$



The load impedance in the complex plane.

How to Determine C, given $\cos\theta'$?



$$S_{\text{new}} = P + jQ_{\text{new}} = P + jQ_{\text{old}} + jQ_c$$

$$Q_{\text{new}} = Q_{\text{old}} + Q_c$$

$$\text{Or, } Q_c = Q_{\text{new}} - Q_{\text{old}}$$

So, the capacitance C needed to improve the power factor of the load can be calculated as

$$Q = I_{\text{ms}}^2 (X) = I_{\text{ms}}^2 \left(\frac{1}{\omega C} \right)$$

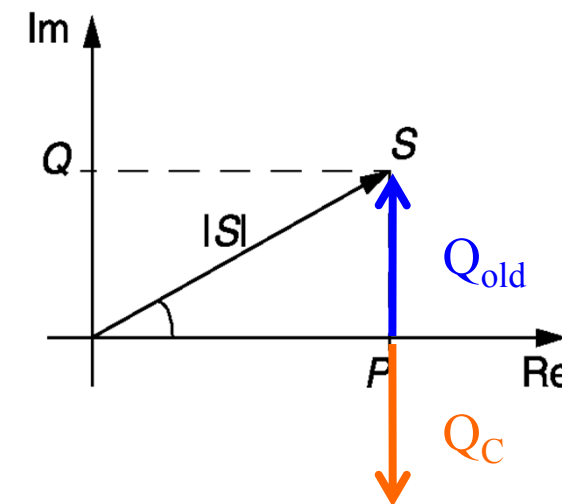
$$Q_c = -\omega C |V|^2$$

If the overall power factor is improved to unity,

$$\cos\theta' = 1 = \left(\frac{V_{\text{ms}}}{V_{\text{ms}}} \right) \left(-\frac{1}{\omega C} \right) = -\omega C \cdot V^2$$

$$S_{\text{new}} = |V| \cdot |I'| \cos\theta' + j |V| \cdot |I'| \sin\theta' = P + jQ_{\text{new}}$$

$$Q_{\text{new}} = |V| \cdot |I'| \sin\theta' = 0$$



Then capacitor supplies 100% of the reactive power required by the load.

Example 2 - Power Factor Correction

A load connected across a 200 V, 50Hz line draws 10 kW at 0.5 power factor lagging. Determine the current drawn by the load. A capacitor C is now connected in parallel with the load to improve the power factor. What must be the value of C to make the overall power factor

(i) 0.9 lagging, (ii) unity and (iii) 0.8 leading?

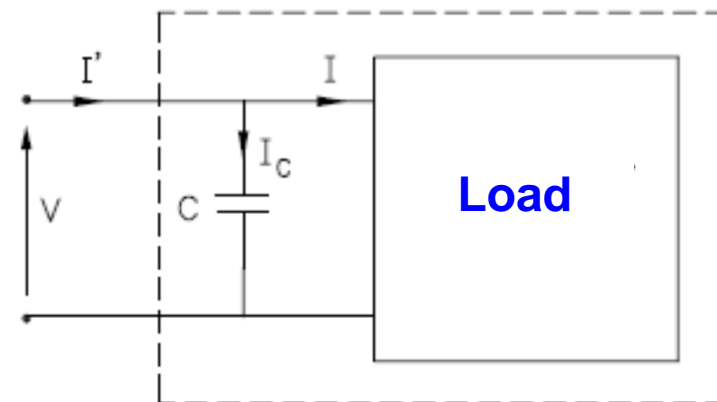
Solution: Take source voltage as reference. So, current drawn at 0.5 p.f. lagging is

$$V = 200 \angle 0^\circ$$

$$|I| = \frac{P}{|V| \cos \theta} = \frac{10000}{200 \times 0.5} = 100 \text{ A}$$

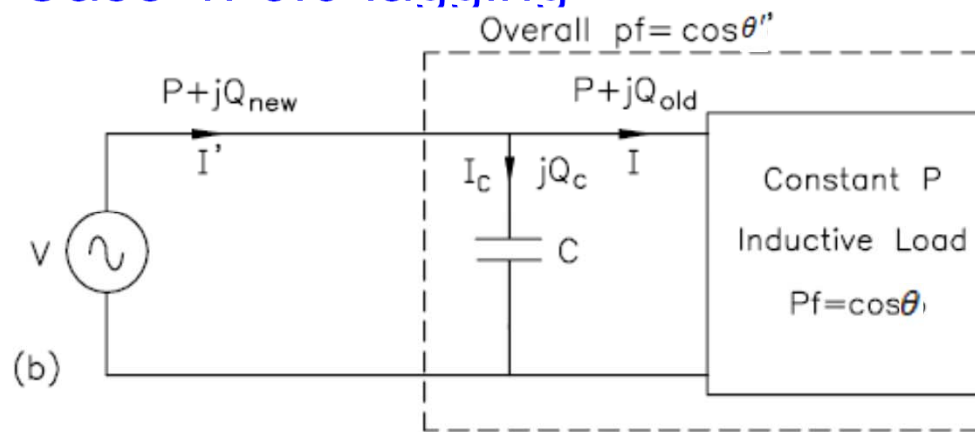
$$\theta = \cos^{-1} 0.5 = 60^\circ$$

$$I = 100 \angle -60^\circ$$



$$S_{old} = P + jQ_{old} = V I^* = 200 \times 100 \angle 60^\circ = 10000 + j17320.5$$

Case 1: 0.9 lagging



$$|I'| = \frac{P}{|V| \cos \theta'} = \frac{10000}{200 \times 0.9} = 55.56 \text{ A}$$

$$\theta' = \cos^{-1} 0.9 = 25.84^\circ$$

$$I' = 55.56 \angle -25.84^\circ$$

$$S_{\text{new}} = P + jQ_{\text{new}} = V I'^*$$

$$= 200 \times 55.56 \angle 25.84^\circ = 10000 + j4843.3$$

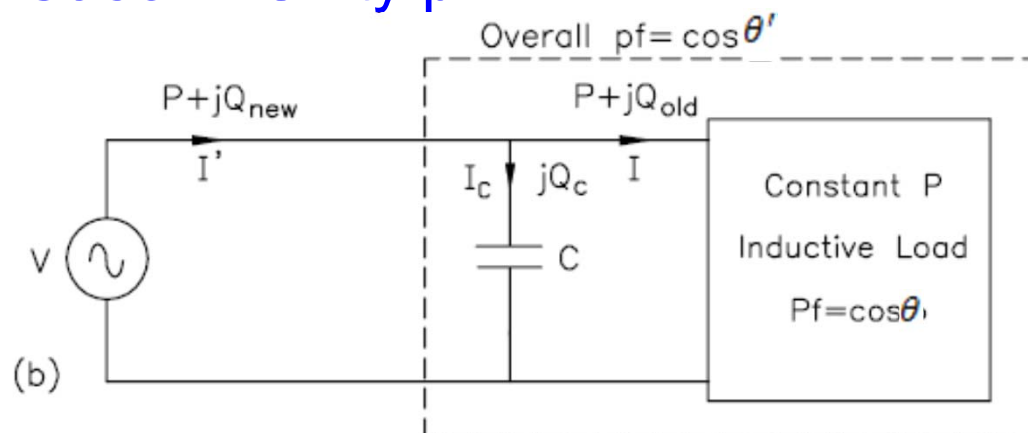
Hence, Q_C and C may be obtained from

$$Q_C = Q_{\text{new}} - Q_{\text{old}}$$

$$Q_C = -\omega C |V|^2 = -12476.7 \text{ VAR}$$

$$C = \frac{12476.7}{314 \times 200^2} = 993.4 \text{ } \mu\text{F}$$

Case 2: Unity p.f.



$$S_{new} = P + jQ_{new} = 10000 + j0$$

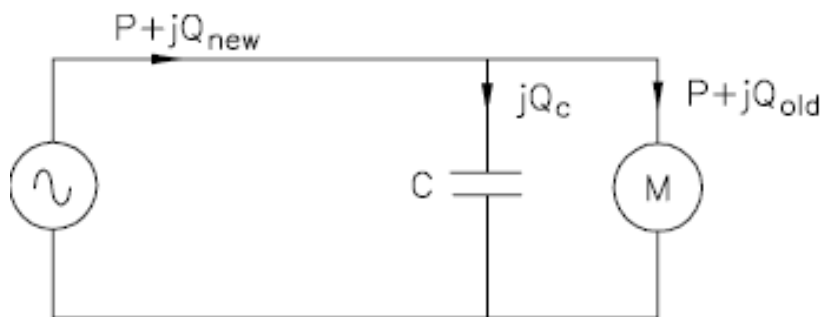
$$Q_{new} = 0$$

$$\begin{aligned} Q_C &= Q_{new} - Q_{old} = -Q_{old} \\ &= -\omega C |V|^2 \end{aligned}$$

$$C = \frac{17320.5}{314 \times 200^2} = 1378.3 \mu F$$

Example 3

A 5000 W electric motor is connected to a source of 230 V, 50 Hz and the result is a lagging power factor of 0.8. To correct the power factor to 0.95 lagging, a capacitor is placed in parallel with the motor. Calculate the current drawn from the source with and without capacitor. Determine the value of the capacitor required to make the correction.



$$|I| = \frac{P}{|V| \cos \theta} = \frac{5000}{230 \times 0.8} = 27.174 \text{ A}$$

$$\theta = \cos^{-1} 0.8 = 36.87^\circ$$

$$I = 27.174 \angle -36.87^\circ$$

$$S_{old} = V I^* = P + jQ_{old}$$

$$= 230 \times 27.174 \angle 36.87^\circ = 5000 + j3750$$

$$|I'| = \frac{P}{|V| \cos \theta'} = \frac{5000}{230 \times 0.95} = 22.883 \text{ A}$$

$$\theta' = \cos^{-1} 0.95 = 18.19^\circ$$

$$I' = 22.883 \angle -18.19^\circ$$

$$S_{new} = V I'^* = P + jQ_{new} = 230 \times 22.883 \angle 18.19^\circ = 5000 + j1643$$

$$Q_C = Q_{new} - Q_{old}$$

$$Q_C = -\omega C |V|^2 = -2107 \text{ VAR}$$

$$C = \frac{2107}{100\pi \times 230^2} = 126.85 \text{ } \mu\text{F}^{31}$$

Transmission Line Loss

Industrial load connected to a substation

V_s = Substation or sending end voltage.

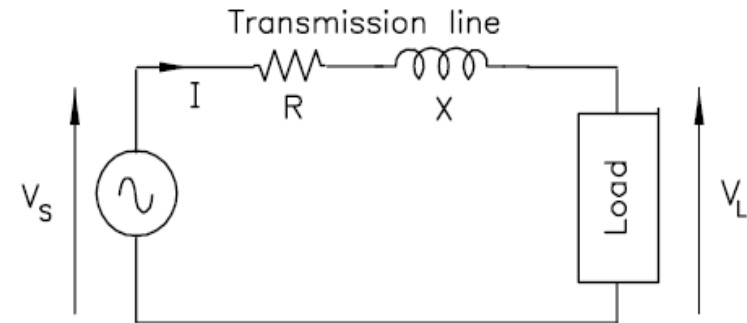
V_L = Voltage at the load.

I = Current drawn by the load.

R = Resistance of the transmission line.

X = Reactance of the transmission line.

The transmission line loss is given by: $P_{\text{loss}} = |I|^2 R$



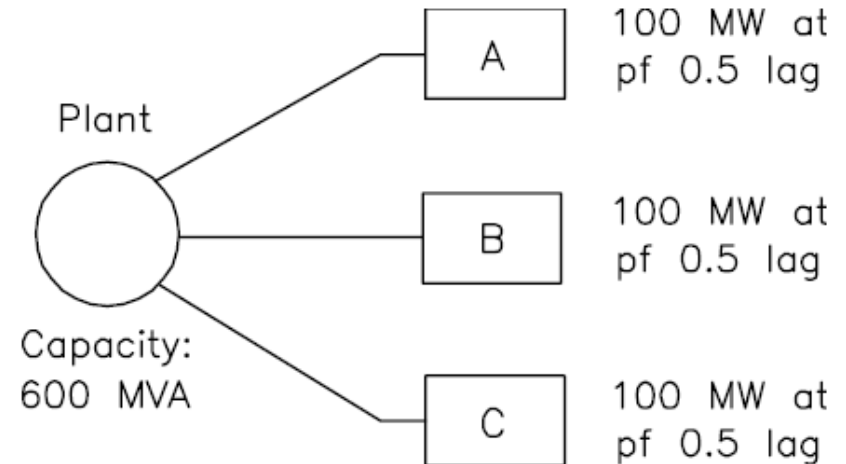
For the same real power demand, if the power factor of the load decreases, then $|I|$ increases as shown by the following equation. This results in heavy line losses.

$$|I| = \frac{P}{|V| \cos \theta}$$

A poor power factor results in higher current and hence higher power loss.

Capital Cost of Power Plants

Three industrial customers A, B and C are drawing power from the Power Plant. Let the load at each of the three industries be 100 MW at a p.f. of 0.5 lagging. The maximum demand of each consumer is



$$\text{Max. Demand} = |V||I| = \frac{P}{\cos \theta} = \frac{100}{0.5} = 200 \text{ MVA}$$

So the installed capacity of the plant should be

$$\sum \text{Max. Demand} = 600 \text{ MVA}$$

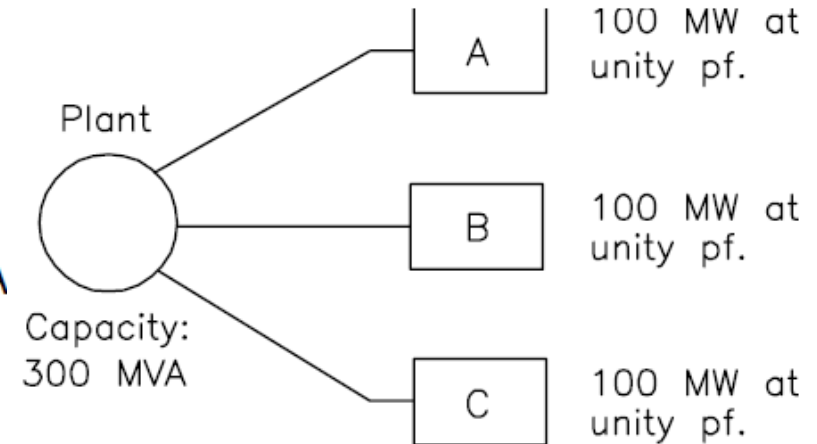
If \$X is the capital cost per annum per MVA of the plant, the total annual capital cost is

$$\text{Capital cost} = \$600X$$

The three consumers will share the capital cost equally in this case and each will pay \$200X.

If the load at each of the three industries is 100 MW at unity p.f., the maximum demand of each consumer is

$$\text{Max. Demand} = |V||I| = \frac{P}{\cos \theta} = \frac{100}{1.0} = 100 \text{ MVA}$$



So the installed capacity of the plant should be

$$\sum \text{Max. Demand} = 300 \text{ MVA}$$

As the plant capacity is only 300 MVA, the total annual capital cost in this case is

$$\text{Capital cost} = \$300X$$

The three consumers will share the capital cost equally in this case also and each will pay \$100X.

Two points become evident from this example:

- Plant capacity gets affected by the maximum demand of each consumer.
- Annual capital cost gets affected by the power factor of the load.

A poor power factor results in higher capital cost.