EE2022 Electrical Energy Systems

Transmission Line – Problem Solving and Quiz

Quiz

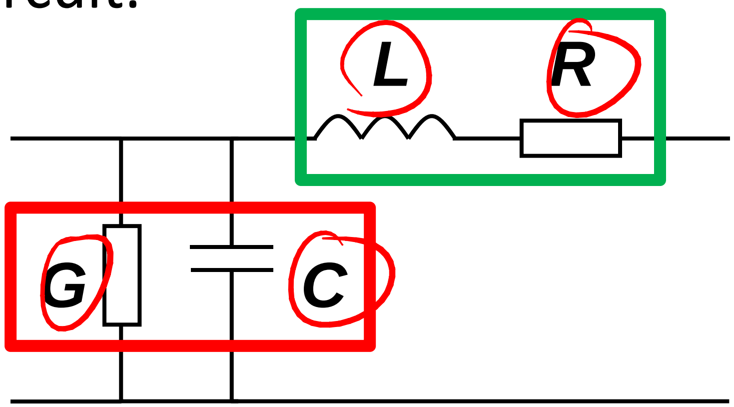
- A 3-phase load draws 1MW power at 11kV.
  - Find the current drawn at power factor of
    - 0.9 lagging
    - Unity
    - 0.9 leading
- The three-phase transmission line has a total series impedance of $Z = 5+j20$ ohm and a total shunt admittance of $Y = j133\times10^{-6}$ S.
- At 'Full Load', for all three power factors
  - **Find the voltage at the sending end.**
  - **Find the current at the sending end.**
  - **Find the voltage regulation.**
  - **Find the efficiency of the transmission line.**

Transmission Line Parameters

- **R** from Ohmic losses
  - Types, sizes of conductor determine resistance value.
- **G** from insulator leakage current and corona losses
  - Types, number of insulators determine conductance value.
- **L** from magnetic field and **C** from electric field
  - Conductor spacing, bundling, determines magnetic and electric field strength
- All these values can be measured.

Per Phase Conductor Model

- We can now use the per phase conductor model to describe the circuit model of each phase in three-phase circuit.



Series Impedance (Ω/m)

$$\boxed{z = r + j\omega l}$$

$$\boxed{y = g + j\omega c}$$

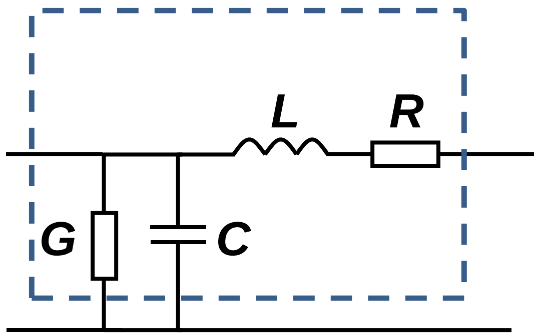
Shunt Admittance (S/m)

- These parameters are given as per unit length of the transmission line.
- We will use this information to derive an equivalent circuit of the transmission line.

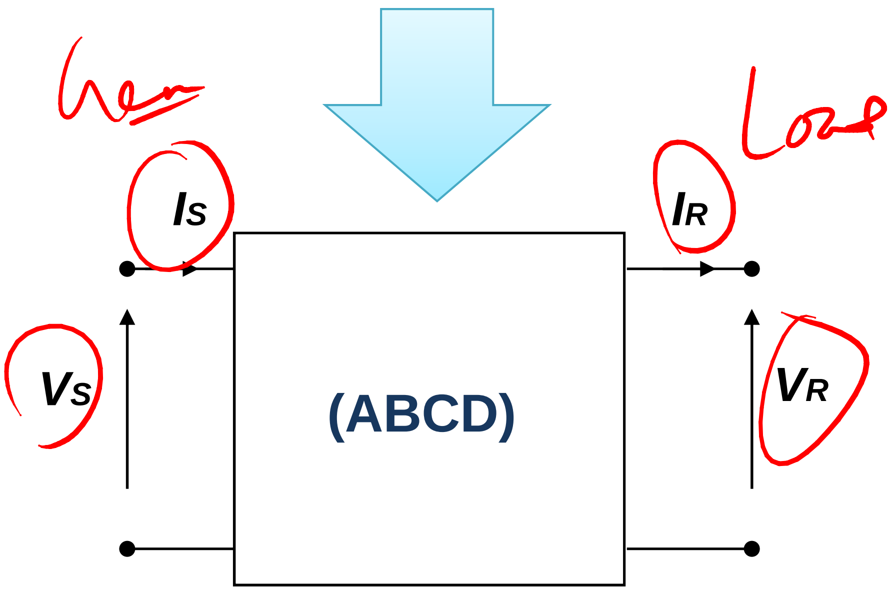
Outline

- Transmission line modeling
  - Short length model
  - Medium length model lumped
  - Long length model
- Transmission line operation
  - Voltage regulation
  - Line loadability
  - Transmission line efficiency

Equivalent Circuit of A Transmission Line : Matrix Representation



Conductor per phase model



$$V_S = AV_R + BI_R$$

$$I_S = CV_R + DI_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

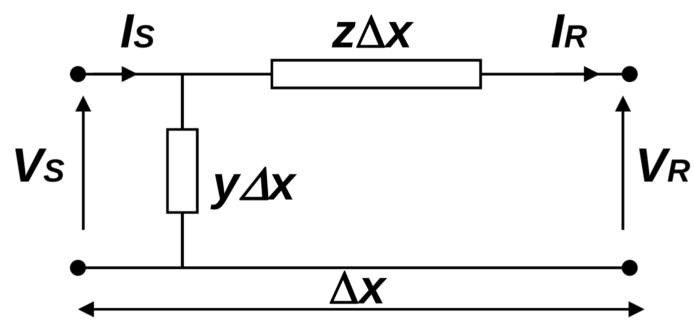
A, B, C, and D are parameters to be found.

Purposes of Equivalent Circuit

- To calculate the voltage at the receiving end when the sending end voltage is known or vice versa.
  - This is used to find the voltage difference between sending and receiving end.
- To find the amount of real and reactive power transfer in the line.
  - To make sure that the power does not exceed the heating limit by the lines.
  - For transmission line efficiency calculation.

Transmission Line Models

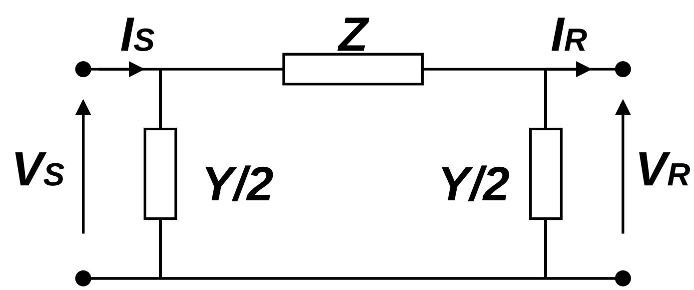
Long-Line Model :Distributed Model



z: Series Impedance (Ω/m)
y: Shunt Admittance (S/m)
Δx: distance (m)

Simplification

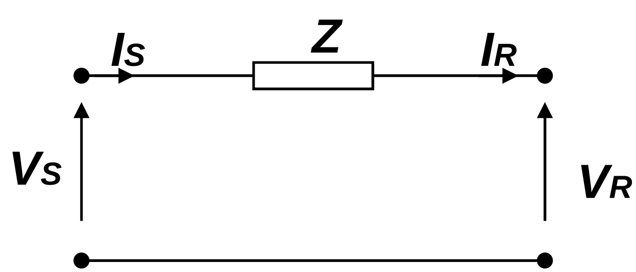
Medium-Line Model: Lumped Model



Z: Series Impedance (Ω) = zΔx
Y: Shunt Admittance (S) = yΔx

Simplification

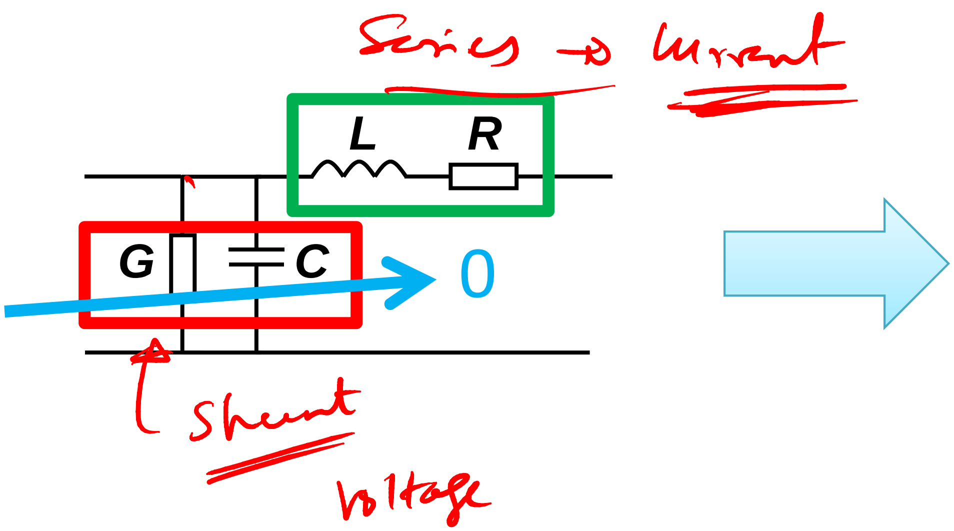
Short-Line Model

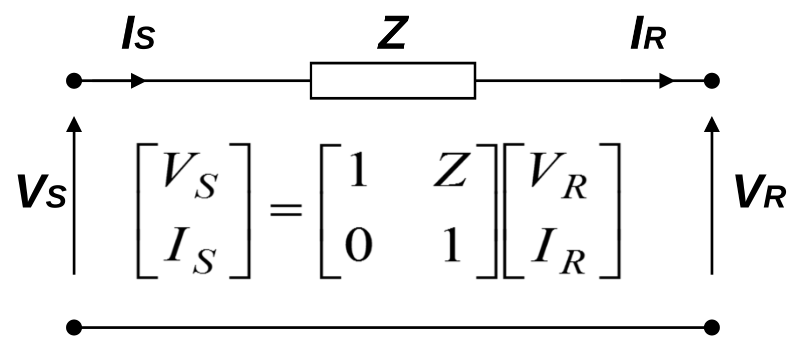


Z: Series Impedance (Ω) = zΔx
Y ≈ 0

Short Line: A Simplified Model

- In this model, we ignore the shunt admittance and only consider series impedance.

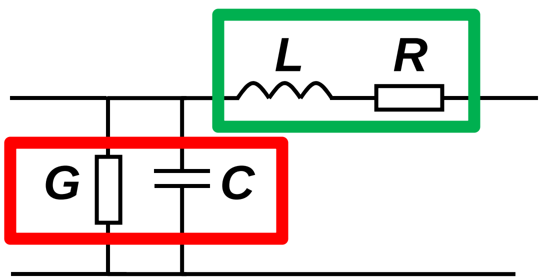


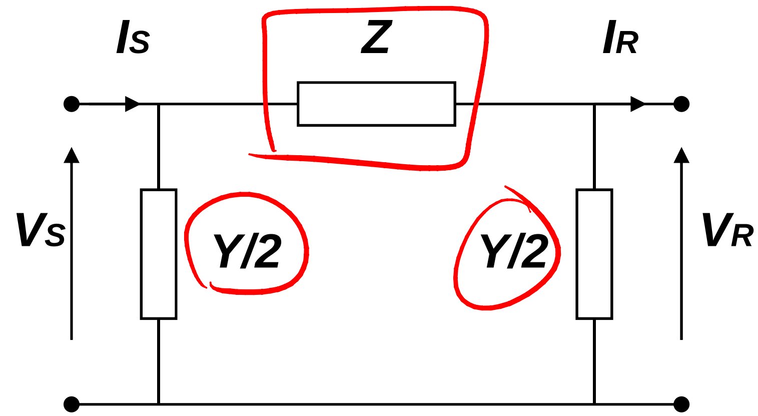


$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} A=1, & B=Z \\ C=0, & D=1. \end{bmatrix}$$

Medium Line: A Lumped Model 'nominal π circuit'.





$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y\left(1+\frac{ZY}{4}\right) & \frac{ZY}{2}+1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Equivalent Models: Summary

Short (<80 km)

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Medium (80 km..240 km)

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y\left(1+\frac{ZY}{4}\right) & \frac{ZY}{2}+1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Long (>240 km), 'l' is the length of the transmission line.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{\sinh(\gamma l)}{Z_c} & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The ranges of transmission length for each model are only suggestion! The selection of an appropriate model mainly depends on the conductor parameter and the application.

Example

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y\left(1+\frac{ZY}{4}\right) & \frac{ZY}{2}+1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

- A three-phase transmission line has a total series impedance of Z = 5+j20 ohm and a total shunt admittance of Y = j133×10^{-6} S.
  - **Find the ABCD parameters for both short and medium-length models.**

Full-Load to No-load

$V_{R,FL} = \frac{11\times 10^3}{\sqrt{3}}\angle 0^\circ$

(P.f.) → 1 MW, at 11 kV,

When we now disconnect the three-phase load at the receiving end (no load condition). If we fix the sending end voltage to be the same value computed from the previous 'Full Load' case, we find the receiving end voltage using three line models. $I_{R,FL} = ?$

$V_{S,FL} = A \cdot V_{R,FL} + B \cdot I_{R,FL}$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

No load

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ 0 \end{bmatrix}$$

$I_{S,FL} = C \cdot V_{R,FL} + D \cdot I_{R,FL}$

$V_{S,FL} = A \cdot V_{R,NL}$

Rated voltage

$$\left|V_{R,NL}\right| = \left|\frac{V_{S,FL}}{A}\right|$$

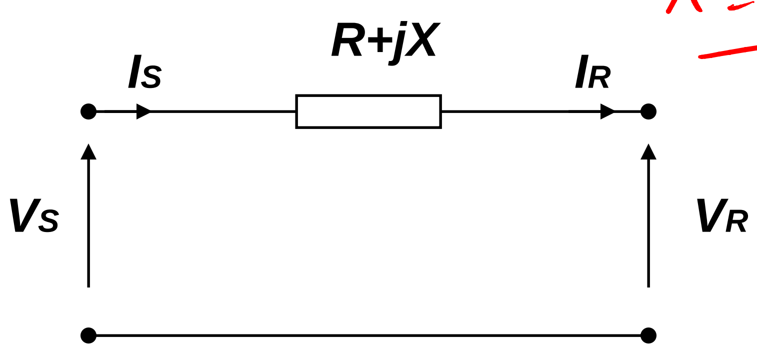
Percent Voltage Regulation

- The variation of line voltage with different loading conditions is called '*voltage regulation*'.
- About 10% voltage change between no load and full load operation is a usual practice for reliable operation.
- Voltage regulation measures the degree of change in voltage when load varies from no-load to full load at a specific power factor.

$$\text{Percent regulation} = \frac{\left(\left|V_{R,NL}\right|\right) - \left|V_{R,FL}\right|}{\left(\left|V_{R,FL}\right|\right)} \times 100$$

Voltage Regulation of a Short Line

- For simplicity, we consider a short transmission line model.



$A = 1 \quad B = Z$

$|V_{R,NL}| = \left|\frac{V_{S,FL}}{A}\right| = |V_{S,FL}|$

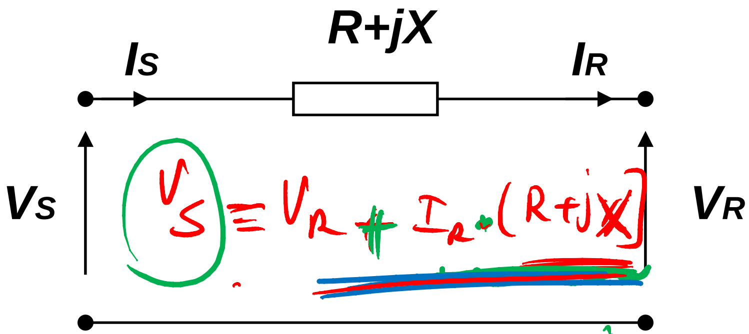
$=V_{S,FL}$

$$\text{Percent regulation} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\%$$

$$= \frac{|V_{S,FL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\%$$

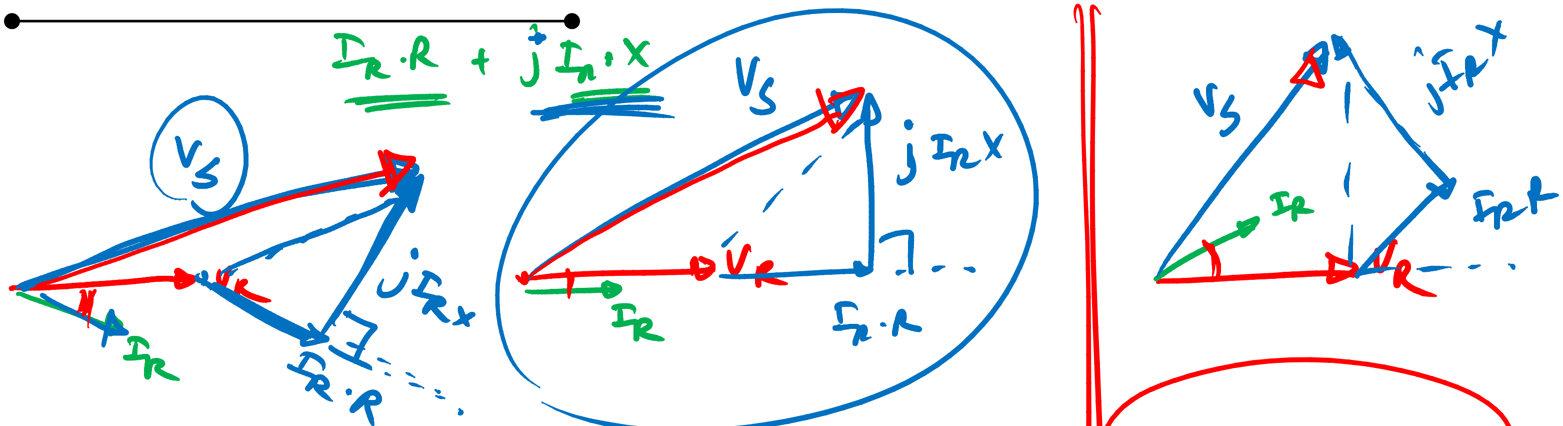
- Note that the receiving end voltage at no load condition is the same as sending end voltage at full load condition.

Effect of Different Power Factor



$$\text{Percent regulation} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\%$$

$= V_{S,FL}$



Lagging pf load

The percent voltage regulation is positive (++).

Unity pf load

The percent voltage regulation is positive (+).

Leading pf load

The percent voltage regulation may be negative (-).

In order to minimize voltage regulation, it is ***preferable to have unity power factor load****.*

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Example

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y\left(1+\frac{ZY}{4}\right) & \frac{ZY}{2}+1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

- A three-phase transmission line has a total series impedance of Z = 5+j20 ohm and a total shunt admittance of Y = j133×10^{-6} S.
  - **Find the voltage regulation for the full load.**

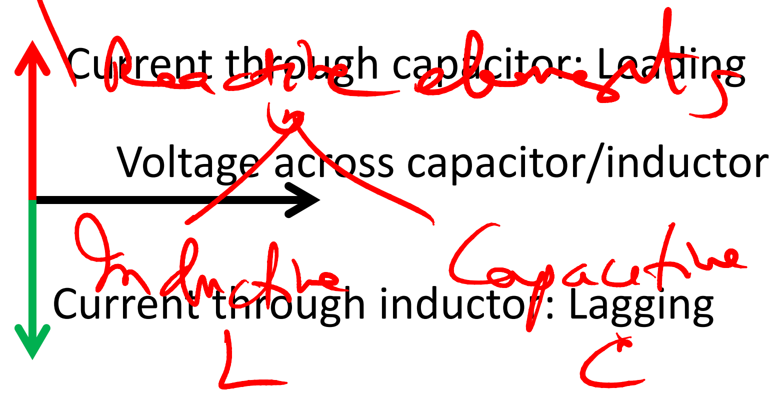
Line Loadability

- Line loadability refers to the maximum amount of MVA to be carried by the transmission line.
- We mainly consider three limits.
  1. Thermal ratings of conductors. I^2R
  2. Voltage-drop limit. $I_R(R+jX)$ voltage drop
  3. Stability limit. – Generator modelling

Line model	Short line	Medium line	Long line
Main concern	Thermal limit	Voltage-drop limit	Voltage-drop limit

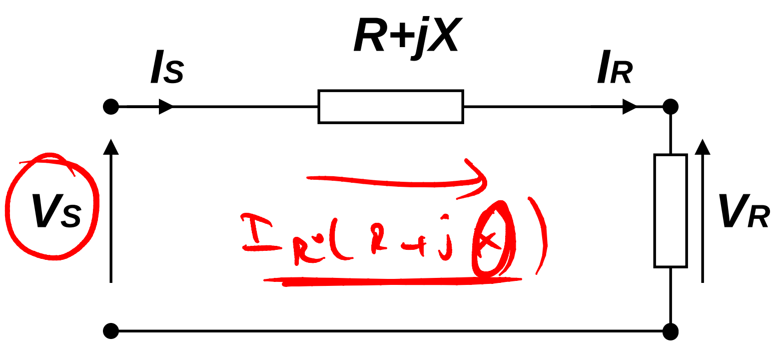
Reactive Compensation Techniques

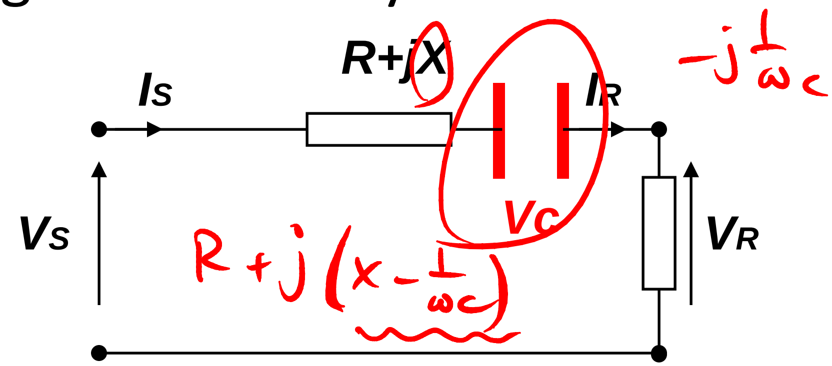
- When the power factor is lagging, the voltage regulation is the highest.
- For medium and long transmission lines, the voltage drop is usually reached before the thermal limit.
- We can use reactive compensation techniques to increase line loadability and maintain voltage at rated value.
- Two main techniques
  - Series compensation (Cap)
  - Shunt compensation (Ind)

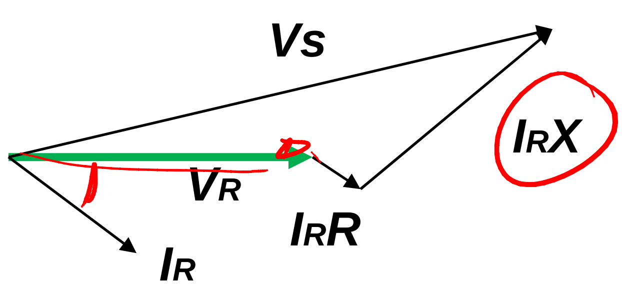


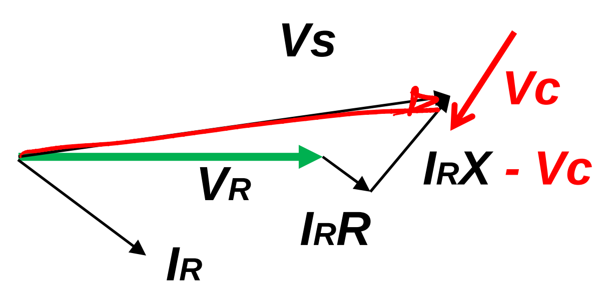
Series Compensation Technique

- **At full load**, the sending end voltage is too high compared to receiving end voltage.
- We can connect the capacitor in series to the load to help reduce the sending end voltage in the heavy load condition.





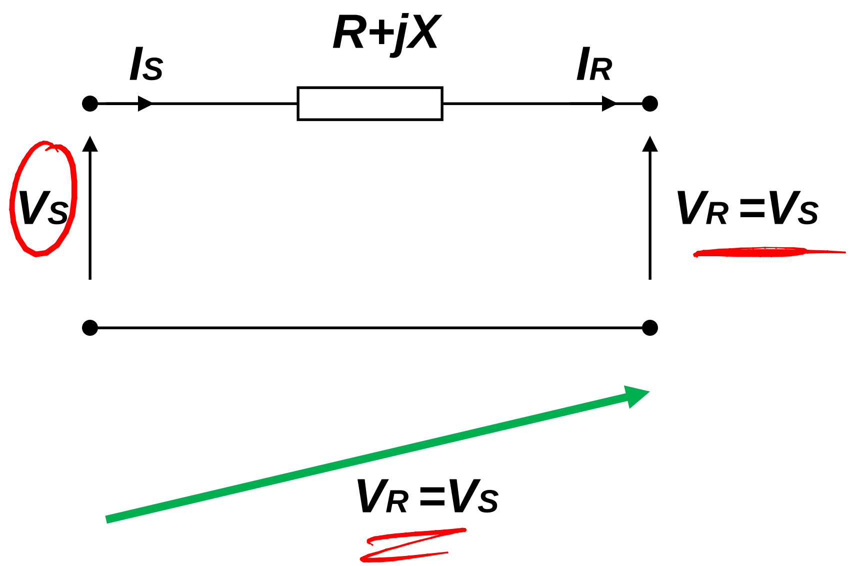


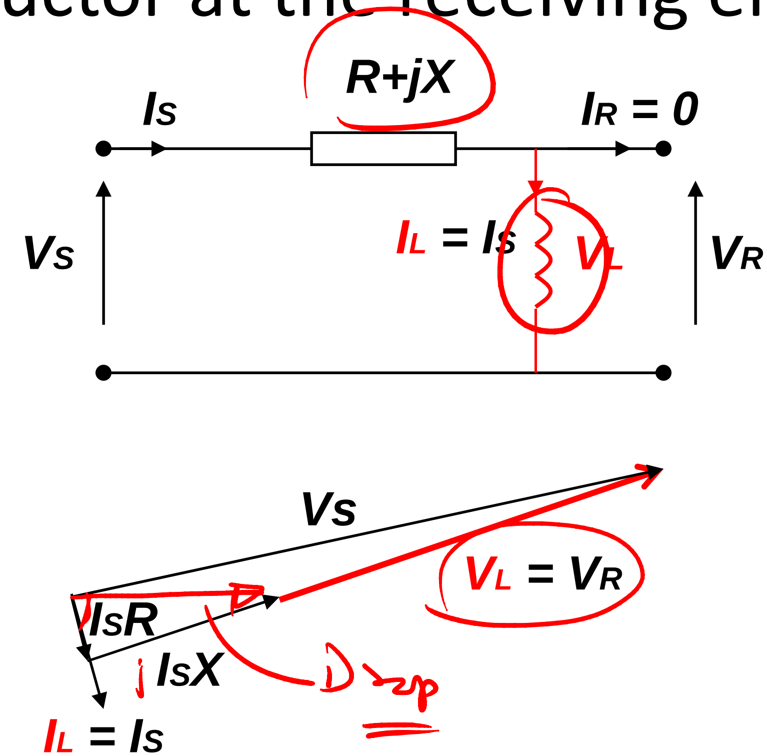


Shunt Compensation Technique

→ no voltage drop

- **At no load**, the receiving end voltage is higher than the usual full load case
- We can connect the inductor at the receiving end





Compensation Techniques

Series compensation

- Series capacitor
- Use during heavy load condition to boost up the voltage magnitude.

Source: Siemens

Shunt compensation

- Shunt reactor
- Use during light load condition to dampen the voltage magnitude.

Source: ABB

Transmission Line Efficiency

- We can compute the transmission line efficiency (%) from the ratio of the real power at the receiving end to real power at the sending end.

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100$$

$$S_{R,3\Phi} = 3V_{R,\text{line-to-neutral}} I_R^* = P_{R,3\Phi} + jQ_{R,3\Phi}$$

$$S_{S,3\Phi} = 3V_{S,\text{line-to-neutral}} I_S^* = P_{S,3\Phi} + jQ_{S,3\Phi}$$

Example

- At 'Full Load', **find the efficiency of the transmission line.**

Quiz

$V_R = \frac{11\times10^3}{\sqrt{3}} \angle 0$

- A 3-phase load draws 1MW power at 11kV.
  - Find the current drawn at power factor of
    - 0.9 lagging
    - Unity
    - 0.9 leading

$\underset{(F.L)}{I_R} = \frac{P_{3\phi}}{p.f.} \times \frac{1}{\sqrt{3}\times V_{LL}} \angle \pm\cos^{-1}(p.f.)$

- The three-phase transmission line has a total series impedance of Z = 5+j20 ohm and a total shunt admittance of Y = j133×10^{-6} S.

(Use the Short-line model)

- At 'Full Load', for all three power factors
  - **Find the voltage at the sending end.**
  - **Find the current at the sending end.**
  - **Find the voltage regulation.**
  - **Find the efficiency of the transmission line.**

$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

$\frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\%$

$\underset{(FL)}{V_S} = V_R + Z \cdot I_{R,FL}$

$\underset{(FL)}{I_S} = \underset{(FL)}{I_R}$

$|V_{S,FL}| = |V_{R,NL}|$

$\eta = \frac{P_{R,3\phi}}{P_{S,3\phi}} = \frac{1\times10^6}{3\times Re(V_S \cdot I_S^*)}$

Problem with sqrt(complex number)

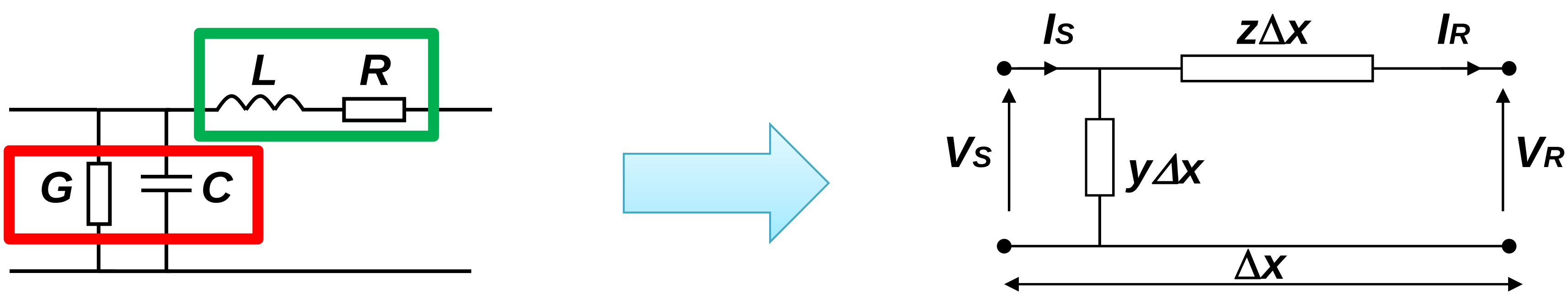
Problem with Hyperbolic function of a complex number

Comparison of long-line differential equations in EE2022 and EE2011

APPENDIX

Long Line: A Distributed Model

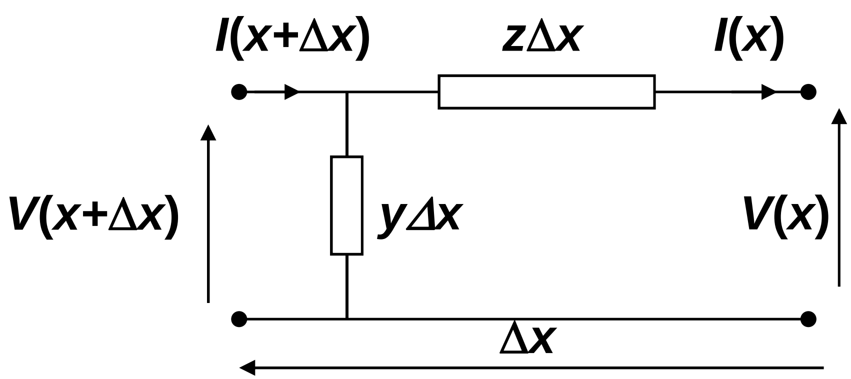
- Parameter are given as per-length and are uniformly distributed along the transmission lines.
- A distributed model accounts for this distributed nature of transmission-line parameters.
- This model provides exact transmission line equations and is suitable for long-length transmission lines.



$$z = r + j\omega l$$ **Series Impedance (Ω/m)**

$$y = g + j\omega c$$ **Shunt Admittance (S/m)**

Long-Line: A Distributed Model



Long-Line: Differential Equations

$$\left.\begin{aligned}\frac{dI}{dx} &= yV \\ \frac{dV}{dx} &= zI\end{aligned}\right\} \frac{d^2V}{dx^2} = z\frac{dI}{dx} = zyV$$

$\gamma = \sqrt{zy}$ *γ is called **propagation constant** (1/m).*

$$\boxed{V = k_1 e^{\gamma x} + k_2 e^{-\gamma x}}$$

$$\boxed{I = \frac{1}{z}\frac{dV}{dx} = \frac{k_1\gamma}{z}e^{\gamma x} - \frac{k_2\gamma}{z}e^{-\gamma x}}$$

Long-Line: Differential Equations

We can find the constants k_1 and k_2 from the fact that at the receiving end of the line (x=0), $V = V_R$ and $I = I_R$.

$$\left.\begin{aligned} V_R &= k_1 + k_2 \\ I_R &= \frac{k_1\gamma}{z} - \frac{k_2\gamma}{z} \end{aligned}\right\} \quad \begin{aligned} k_1 &= \frac{V_R + I_R(z/\gamma)}{2} \\ k_2 &= \frac{V_R - I_R(z/\gamma)}{2} \end{aligned}$$

Define ***Z_c*** **as a** **characteristic impedance (Ω).**

$$\frac{z}{\gamma} = \frac{z}{\sqrt{zy}} = \sqrt{\frac{z}{y}} \equiv Z_c \quad \Longrightarrow \quad \begin{aligned} k_1 &= \frac{V_R + Z_c I_R}{2} \\ k_2 &= \frac{V_R - Z_c I_R}{2} \end{aligned}$$

Long Line: V&I Equations

- Substitute k_1 and k_2, we have the voltage and current equations at any point *x* from the load (receiving end) as follows.

$$V = V_R\left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right)+Z_c I_R\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right) \qquad I = I_R\left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right)+\frac{V_R}{Z_c}\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)$$

$$\sinh(\gamma x)=\frac{e^{\gamma x}-e^{-\gamma x}}{2} \qquad \Downarrow \qquad \cosh(\gamma x)=\frac{e^{\gamma x}+e^{-\gamma x}}{2}$$

$$\boxed{\begin{aligned} V &= V_R\cosh(\gamma x)+Z_c I_R\sinh(\gamma x) \\ I &= I_R\cosh(\gamma x)+\frac{V_R}{Z_c}\sinh(\gamma x)\end{aligned}}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c\sinh(\gamma l) \\ \dfrac{\sinh(\gamma l)}{Z_c} & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Problem with Sqrt(zy) or Sqrt(z/y)?

- zy and z/y are complex numbers, how to find sqrt of complex numbers?
- You can use:

$$\sqrt{r\{\cos(\beta)+j\sin(\beta)\}}=\sqrt{r}\left[\cos\frac{\beta}{2}+j\sin\frac{\beta}{2}\right]$$

- For example, let

$$zy=r\{\cos(\beta)+j\sin(\beta)\}$$

- This means that:

$$r=|zy| \qquad \beta=\angle zy$$

Magnitude of zy phasor — Phase of zy phasor

Problem with Hyperbolic Function?

- How to find sinh(x+jy) or cosh(x+jy)?
- Try this:
  - sinh(x + jy) = sinh(x)cos(y) + j cosh(x)sin(y)
  - cosh(x + jy) = cosh(x)cos(y) + j sinh(x)sin(y)
- **Warning:** Familiarize with your calculator. You need to check whether the setting of cos() and sin() is in 'radian' or in 'degree'. In this formula, y is in radian.

- For example,

  cosh(0.0064+0.052i)

  = cosh(0.0064)cos(**0.052 radian**) + j sinh(0.0064)sin(**0.052 radian**)

  = 0.998668757010298 + 0.000332652309305i

Alternatively,…

- From,

$$\sinh(\gamma x) = \frac{e^{\gamma x} - e^{-\gamma x}}{2} = \frac{e^{x\sqrt{zy}} - e^{-x\sqrt{zy}}}{2}$$

$$\cosh(\gamma x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} = \frac{e^{x\sqrt{zy}} + e^{-x\sqrt{zy}}}{2}$$

  – x is the length of a transmission line.

- Use this:

$$e^{a+jb} = e^{a}\left(\cos(b) + j\sin(b)\right)$$