EE2022 ELECTRICAL ENERY SYSTEMS

(Tutorial #1 solutions)

Solution Q1:

(a) $V_1 = \frac{40}{\sqrt{2}}\angle{-90^\circ} + \frac{60}{\sqrt{2}}\angle{-45^\circ} + \frac{30}{\sqrt{2}}\angle{0^\circ} = \frac{109.72}{\sqrt{2}}\angle{-48.69^\circ}$

Hence, $v_1(t) = 109.72\,cos\,(628t - 48.69^\circ)$

(b) $V_2 = \frac{20}{\sqrt{2}}\angle{-90^\circ} + \frac{10}{\sqrt{2}}\angle{60^\circ} + \frac{5}{\sqrt{2}}\angle{70^\circ} = \frac{9.44}{\sqrt{2}}\angle{-44.71^\circ}$

Hence, $v_2(t) = 9.44\,cos\,(314t - 44.71^\circ)$

Solution Q2:

The current through the resistor and capacitor are respectively,

$$\begin{aligned} i_i(t) &= \frac{v(t)}{R} = 7.071\,cos\,(314.16t + 10^\circ) \\ i_2(t) &= C\frac{dv}{dt} = -12.247\,sin\,(314.16t + 10^\circ) \end{aligned}$$

So, the current supplied by the source is

$$\begin{aligned} i(t) &= i_1(t) + i_2(t) \\ &= 7.071\,cos\,(314.16t + 10^\circ) - 12.247\,sin\,(314.16t + 10^\circ) \end{aligned}$$

In phasor form, this current can be written as:

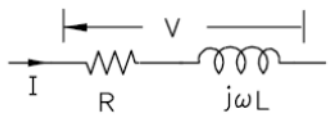
$$I \;=\; 5\angle 10^\circ + 8.66\angle 100^\circ \;=\; 10\angle 70^\circ$$

The time domain expression for the current is

$$i(t) = 14.14\,cos\,(314.16t + 70^\circ)$$

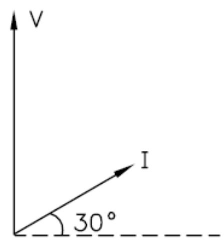
Solution Q3:

Current $i(t) = 2\sqrt{2}\cos\,(5000t + 30^0)mA$



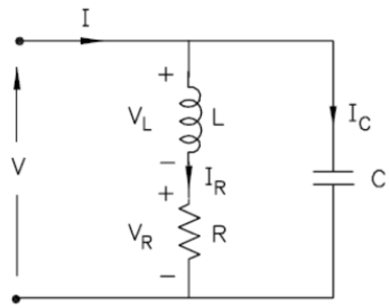
The impedance and voltage phasors can be determined as follows:

$$\begin{aligned} I &= 2 \times 10^{-3} \angle 30^\circ \;\; \mathrm{A} \\ Z &= R + j\omega L = 2309 + j5000 \times 0.8 = 2309 + j4000\Omega \\ V &= Z\,I = (2309 + j4000) \times 2 \times 10^{-3} \angle 30^\circ = 9.24 \angle 90^\circ \;\; \mathrm{V} \\ v(t) &= 9.24\,\sqrt{2}\,cos(5000t + 90^\circ) \;\; \mathrm{V} \end{aligned}$$



Solution Q4:

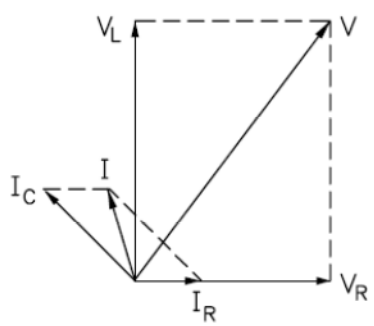
L = 2H, R = 3Ω, C = 0.2μF and $v_R = 6\sqrt{2}cos2t$ volts.



As $v_R = 6\sqrt{2}\,cos\,2t$, we have $V_R = 6\angle 0^\circ$. Hence,

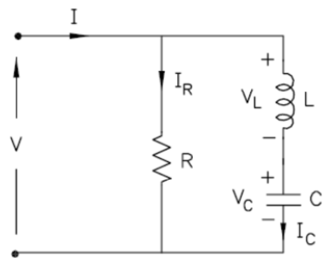
$$
\begin{aligned}
I_R &= \frac{V_R}{R} = \frac{6\angle 0^\circ}{3} = 2\angle 0^\circ \;\; \text{A} \\
V_L &= j\omega L I_R = j2 \times 2 \times 2\angle 0^\circ = 8\angle 90^\circ \;\; \text{V} \\
V &= V_R + V_L = 6\angle 0^\circ + 8\angle 90^\circ = 6 + j8 = 10\angle 53.13^\circ \;\; \text{V} \\
I_C &= j\omega C\, V = j2 \times 0.2 \times (6 + j8) = -3.2 + j2.4 = 4\angle 143.13^\circ \;\; \text{A} \\
I &= I_L + I_C = 2 - 3.2 + j2.4 = -1.2 + j2.4 = 2.68\angle 116.57^\circ \;\; \text{A}
\end{aligned}
$$

The phasor diagram can be drawn as follows:



Solution Q5:

R = 2Ω, L = 3.25mH and C = 100μF, $v_C = 100\sqrt{2}\cos(2000t - 90^0)$ volts



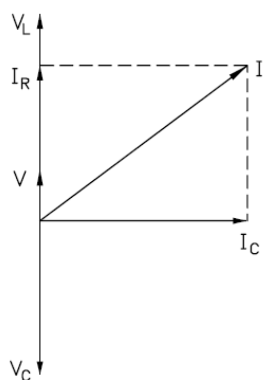
(a) As $v_C = 100\sqrt{2}\,cos(2000t - 90°)$ volts, we get

$$\begin{aligned} V_C &= 100\angle{-90°} \\ I_C &= j\omega C V_C = j2000 \times 10^{-4} \times 100\angle{-90°} = 20\angle 0° \ \ \text{A} \\ V_L &= j\omega L I_C = j2000 \times 3.25 \times 10^{-3} \times 20\angle 0° = 130\angle 90° \ \ \text{A} \end{aligned}$$

(b) So, the voltage V across the circuit and hence currents I_R and I are

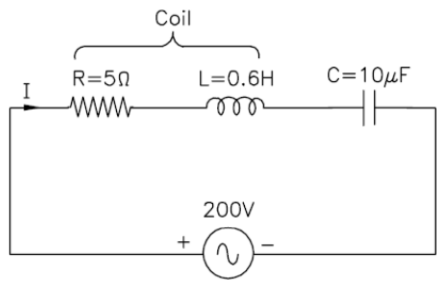
$$\begin{aligned} V &= V_C + V_L = 100\angle{-90°} + 130\angle 90° = -j100 + j130 = j30 \ \ \text{V} \\ I_R &= \frac{V}{R} = \frac{j30}{2} = 15\angle 90° \ \ \text{A} \\ I &= I_C + I_R = 20\angle 0° + 15\angle 90° = 20 + j15 = 25\angle 36.9°\text{A} \end{aligned}$$

(c) Phasor diagram:



(d) $i(t) = 25\sqrt{2}\,cos(2000t + 36.9°)$ A

Solution Q6:



$$
\begin{aligned}
Z &= R + j\left(\omega L - \frac{1}{\omega C}\right) \\
I &= \frac{V}{Z}
\end{aligned}
$$

Current flow in the circuit will be maximum when the impedance is minimum. Hence,

$$
\begin{aligned}
\omega L &= \frac{1}{\omega C} \\
\omega &= \frac{1}{\sqrt{LC}} = 408.248 \ \ \text{rad/s} \\
f &= \frac{\omega}{2\pi} = 64.97 \ \ \text{Hz} \\
I &= \frac{V}{R} = \frac{200}{5} = 40 \ \ \text{A}
\end{aligned}
$$

$$
\begin{aligned}
|V_L| &= \omega L I = 9798 \ \ \text{V} \\
|V_C| &= \frac{I}{\omega C} = 9798 \ \ \text{V} \\
\frac{|V_L|}{|V|} &= \frac{\omega L I}{R I} = \frac{\omega_o L}{R} = \frac{244.95}{5} = 48.99
\end{aligned}
$$

Solution Q7.

(a)

$$P = V_{rms} I_{rms} \quad \Rightarrow \quad I_{rms} = \frac{120\ W}{200\ V} = 0.6\ \text{Amp}$$

(b)

$$R_{lamp} = \frac{200\ V}{0.6\ A} = 333.33\ \Omega$$

(c)

With the capacitor connected in series, the total impedance is Z = R_{lamp} – jX_C.

When connected to 240 V source, we still want the voltage across the lamp to the rated value of 200 V, and hence 0.6A current through the lamp.

$$|Z| = \frac{240\ V}{0.6\ A} = 400\ \Omega$$

$$Z = \sqrt{R_{lamp}^2 + X_C^2} \quad \Rightarrow \quad X_C^2 = Z^2 - R_{lamp}^2$$

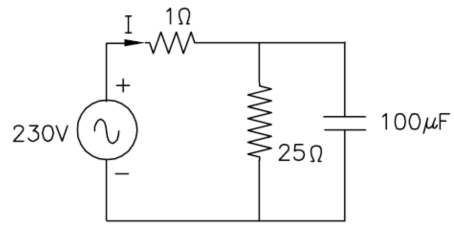
$$X_C = \sqrt{400^2 - (333.33)^2} = 221.11\,\Omega$$

$$X_C = \frac{1}{\omega C} \quad \Rightarrow \quad C = \frac{1}{\omega X_C} = \frac{1}{(2\pi \times 50)(221.11)} = 14.4\ \mu\text{F}$$

EE2022 ELECTRICAL ENERY SYSTEMS

(Tutorial #2 solutions)

Solution Q1:



The load admittance is

$$\begin{aligned} Y_L &= \frac{1}{25} + j377 \times 100 \times 10^{-6} \\ &= 0.04 + j0.0377 = 0.05497\angle 43.3^\circ \end{aligned}$$

So, impedance of the load is

$$Z_L = \frac{1}{Y_L} = 18.193\angle{-43.3^\circ} = 13.239 - j12.478 \ \ \Omega$$

Total impedance of the network is

$$Z = 1 + Z_L = 14.239 - j12.478 = 18.933\angle{-41.23^\circ} \ \ \Omega$$

So, the current supplied by the source is

$$I = \frac{230}{18.933\angle{-41.23^\circ}} = 12.15\angle 41.23^\circ \ \ \mathrm{A}$$

Apparent power delivered to the load is

$$|S_{Load}| = |Z_L||I|^2 = 18.193 \times 12.15^2 = 2686 \ \ \mathrm{VA}$$

Power factor of the load is obtained using the load impedance angle

$$\text{Power Factor} = cos(-43.3^\circ) = 0.728 \ \ \text{leading}$$

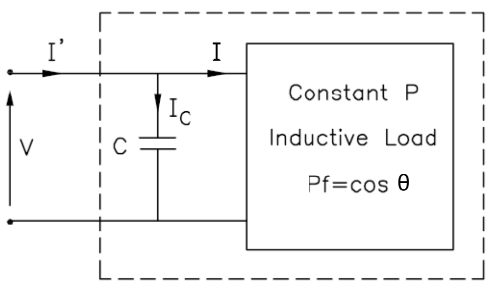
Power factor is leading because of the capacitive load. Apparent power delivered by the source is

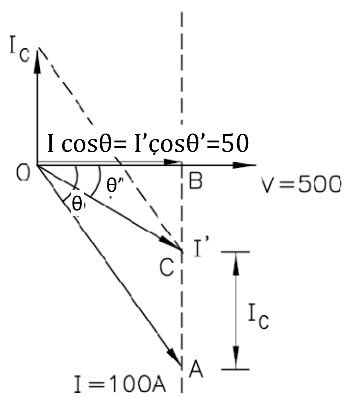
$$|S_{source}| = |V||I| = 230 \times 12.15 = 2794 \ \ \mathrm{VA}$$

Solution Q2:

The power drawn by the load is 25 kW. So,

$$P = |V|\,|I|\,cos\,\theta = 500 \times |I| \times 0.5$$
$$|I| = \frac{25 \times 10^3}{500 \times 0.5} = 100 \;\; \text{A}$$
$$cos\,\theta = 0.5$$
$$\theta = 60^\circ$$
$$I = 100\angle{-60^\circ} \;\; \text{A}$$





The power factor is improved to 0.95 lagging by connecting a capacitor across the load. Hence,

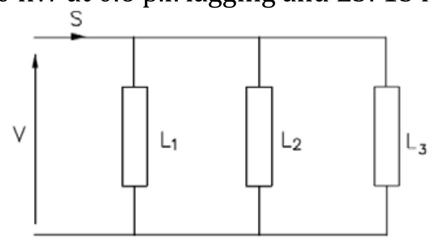
cosθ'= 0.95

θ'= 18.19 deg

$$\tan\theta' = \frac{CB}{OB} = \frac{AB - |I_C|}{50} = \frac{100\,sin\,\phi - |I_C|}{50} = \frac{86.8 - |I_C|}{50}$$
$$|I_C| = 70.37 \;\; \text{A} = \omega C|V| = 2\pi \times 50 \times C \times 500$$
$$C = 448.2 \;\; \mu\text{F}$$

Solution Q3:

L1: 5 kW at 0.8 p.f. lagging, L2: 10 kW at 0.6 p.f. lagging and L3: 15 kW at 0.8 p.f. leading



The complex power consumption of the loads are computed as follows:

$$
\begin{aligned}
P_1 &= |S_1| \times 0.8 \\
|S_1| &= 6250 \ \text{VA} \\
S_1 &= 6250\angle 36.87^\circ = 5000 + j3750 \\
P_2 &= |S_2| \times 0.6 \\
|S_2| &= 16666.67 \ \text{VA} \\
S_2 &= 16666.67\angle 53.13^\circ = 10000 + j13333.33 \\
P_3 &= |S_3| \times 0.8 \\
|S_3| &= 18750 \ \text{VA} \\
S_3 &= 18750\angle -36.87^\circ = 15000 - j11250
\end{aligned}
$$

Hence, overall power factor as seen by the source can be calculated as:

$$
\begin{aligned}
S &= S_1 + S_2 + S_3 \\
&= 30000 + j5833.33 \\
&= 30561.87/11^\circ \ \text{VA} \\
\text{p.f.} &= cos\, 11^\circ = 0.982 \ \text{lagging}
\end{aligned}
$$

Solution Q4:

As the power delivered is 10kW at 0.5 p.f. lagging,

$$\begin{aligned} |I| &= \frac{P}{V\,cos\,\phi} = \frac{10000}{200 \times 0.5} = 100 \text{ A} \\ I &= 100\angle 60^\circ = 50 - j86.6 \text{ A} \end{aligned}$$

When a capacitor of $1000\mu F$ is connected across the supply, the capacitor current is

$$I_C = j\omega CV = j314 \times 1000 \times 10^{-6} \times 200 = j62.8 \text{ A}$$

So the total current drawn from the supply is

$$I_T = I + I_C = 50 - j23.8 = 55.38\angle{-25.45^\circ}$$

The power factor as seen by the source is

$$\text{p.f.} - cos\,(-25.45^\circ) - 0.902 \text{ lagging}$$

Solution Q5:

Current drawn from the substation is

$$|I_1| = \frac{P}{|V|\,cos\,\theta} = \frac{1500 \times 10^3}{500 \times 0.6} = 5000 \text{ A}$$

If the power factor could be improved to unity, then current drawn would be

$$|I_2| = \frac{P}{|V|\,cos\,\theta} = \frac{1500 \times 10^3}{500 \times 1.0} = 3000 \text{ A}$$

The transmission line losses in both the cases are

$$P_{loss1} = R\,|I_1|^2 = 0.005 \times 5000^2 = 125 \text{ kW}$$

$$P_{loss2} = R\,|I_2|^2 = 0.005 \times 3000^2 = 45 \text{ kW}$$

So, cost of energy wasted will be high in the first case.

This example illustrates that for the same real power demand, if the power factor of the load decreases, then |I| increases, resulting in heavy transmission line losses.

A poor power factor results in higher current and hence higher power loss.

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EE2022 ELECTRICAL ENERY SYSTEMS
(Solution for Tutorial #3)

6. Line current, $I_{line} = \frac{100}{|10-j9|} = 7.43$ A

   Line-to-neutral voltage at the source,

$$|V_{Line-neutral}| = |\mathrm{I_{line}}| \times |\mathrm{Z_{total}}| = 7.43|(2+j3)+(10-j9)| = 99.7 \text{ V}.$$

   Line voltage at the source,

$$V_{Line-Line} = \sqrt{3} \times |V_{Line-neutral}| = \sqrt{3} \times 99.7 = 173 \text{ V}.$$

7. We first need to combine the two loads, Δ is transformed to Y,

$$\mathrm{Z_Y} = \frac{\mathrm{Z_\Delta}}{3} = \frac{21\angle 30°}{3} = 7\angle 30° \ \Omega.$$

   Now we have two balanced Y connected loads of $9\angle{-60^0}\ \Omega$ and $7\angle 30^0\ \Omega$ in parallel.

   The total load impedance per phase, $Z_{Y,total} = \dfrac{(7\ \angle 30^0)\times(9\ \angle -60^0)}{(7\ \angle 30^0 + 9\ \angle -60^0)} = 5.53\ \angle -7.87^0\ \ \Omega$

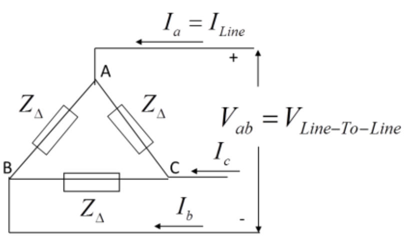
   The rms line current is found from,

$$I_{line} = \frac{V_{line-to-neutral}}{|\mathrm{Z_Y}|} = \frac{V_{line-to-line}}{\sqrt{3}|\mathrm{Z_Y}|} = \frac{208}{\sqrt{3}\times 5.53} = 21.7 \text{ A}$$

   The total power absorbed by the two loads is ,

$$|P_{3\Phi}| = \sqrt{3}|V_{line-to-line}||I_{line}| \times \text{p.f.} = \sqrt{3} \times 208 \times 21.7 \times \cos(7.87) = 7744.15 \text{ W}$$

8.



   For a balanced three-phase load, $|P_{3\Phi}| = \sqrt{3}|V_{line-to-line}||I_{line}| \times \text{p.f.}$ By substituting $|V_{line-to-line}| = 208$ V, $P_{3\Phi} = 2000\ W$, p.f. $= 0.8$ in the above equation, we have $|I_{line}| = 6.94$ A.

   The phase current that pass through an impedance z can be found from

$$|I_{Phase}| = \frac{|I_{line}|}{\sqrt{3}} = 4.01 \text{ A}.$$

   Let $V_{line-to-line} = 208\angle 0°$, the angle of the phase current can be found from power factor leading.

$$\angle I_{Phase} = +\cos^{-1} 0.8 = 36.87°$$

Note that the phase angle is positive because the power factor is leading.

We can find the impedance of Δ-connected load as follows.

$$Z_\Delta = \frac{V_{ab}}{I_{ab}} = \frac{V_{line-to-line}}{I_{phase}} = \frac{208\angle 0°}{4.01\angle 36.87°} = 51.9\angle -36.87 = 41.44 - \mathrm{j}31.08\ \Omega$$

9.

a) <u>Power triangle for the induction motor,</u>

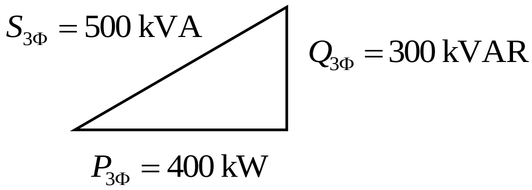
From $P_{3\Phi} = |S_{3\Phi}| \times p.f$, given that the real power, $P_{3\Phi} = 400\ kW$, we can find,

$$|S_{3\Phi}| = \frac{P_{3\Phi}}{p.f} = \frac{400}{0.8} = 500\ kVA.$$

Then, the reactive power can be found from,

$$Q_{3\Phi} = |S_{3\Phi}| \times sin(\cos^{-1}(p.f.)) = 500 \times \sin(\cos^{-1} 0.8) = 300\ \mathrm{kVAR}.$$

Since the power factor is *lagging*, this reactive power is *absorbed* by the induction motor. The power triangle is given below.



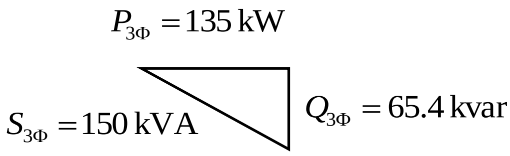
<u>Power triangle for the synchronous motor,</u>

$$S_{3\Phi} = 150\ kVA$$

$$P_{3\Phi} = |S_{3\Phi}| \times p.f = 150 \times 0.9 = 135\ kW.$$

Since the power factor is *leading*, this reactive power is *injected* by the synchronous motor.

$$Q_{3\Phi} = |S_{3\Phi}| \times sin(\cos^{-1}(p.f.)) = 150 \times \sin(-\cos^{-1} 0.9) = -65.4\ \mathrm{kVAR}.$$



<u>Power triangle for the combined-motor load,</u>

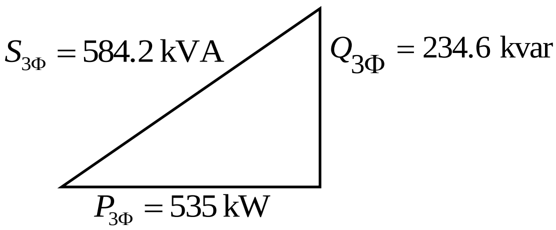
$$P_{3\Phi} = 400 + 135 = 535\ kW.$$

$$Q_{3\Phi} = 300 - 65.4 = 234.6\ kVAR$$

Since the reactive power is positive, this reactive power is *absorbed* by the combined-motor load.

The magnitude of apparent power is found below.

$$S_{3\Phi} = \sqrt{|P_{3\Phi}|^2 + |Q_{3\Phi}|^2} = 584.2\ kVA.$$



b) Power factor of the combined-motor load,

$$p.f. = \frac{P_{3\Phi}}{|S_{3\Phi}|} = 0.916$$

Since the load absorbs reactive power, the power factor is 0.916 lagging.

c) From $|S_{3\Phi}| = \sqrt{3}|V_{line-to-line}||I_{line}|$, we can find the line current below.

$$|I_{line}| = \frac{|S_{3\Phi}|}{\sqrt{3}|V_{line-to-line}|} = \frac{584.2 \times 10^3}{\sqrt{3} \times 4160} = 81.1\ A$$

d) To make the source power factor unity, the reactive power supplied by the capacitor bank, $Q_{c,3\Phi} = -234.6$ kVAR.

For a delta connected capacitor bank, the voltage applied to the capacitor at each phase is the line-to-line voltage.

$$Q_{c,1\Phi} = \frac{Q_{c,3\Phi}}{3} = -78.2\ kVAR$$

The capacitive reactance at each phase, $X_{c,1\Phi}$, can be found from $Q_{c,1\Phi} = \frac{|V_{line-to-line}|^2}{X_{c,1\Phi}}$.

We have,

$$X_{c,1\Phi} = \frac{|V_{line-to-line}|^2}{Q_{c,1\Phi}} = \frac{4160^2}{-78.2 \times 10^3} = -221.3\ \Omega.$$

The capacitive reactance is then -j221.3 Ω.

e) With the capacitor bank installed, power factor=1.

Active power delivered by the source, $P_{3\Phi} = \sqrt{3}|V_{line-to-line}||I_{line}| \times \mathrm{p.f.} = 535\ \mathrm{kW}$.

With the capacitor bank installed, power factor=1. The line current magnitude is found below.

$$|I_{line}| = \frac{|P_{3\Phi}|}{\sqrt{3}|V_{line-to-line}| \times \mathrm{p.f.}} = \frac{535 \times 10^3}{\sqrt{3} \times 4160 \times 1} = 74.3\ A$$

Note that once the power factor is adjusted to 1, the line current magnitude is reduced from 81.1 A to 74.3 A. This helps to reduce the power losses in the transmission lines.

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EE2022 ELECTRICAL ENERY SYSTEMS
(Solution for Tutorial #4)

4.

Using the short length model, $l = 16\ km$, $Z = 16 \times (0.125 + j0.4375) = 2 + j7\ \Omega$. Per phase equivalent model is shown in Fig. 1.

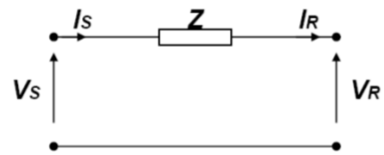

Fig.1

The voltage and current at sending and receiving end can be found from the following.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & 2+j7 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The receiving end voltage is given as 64 kV line-to-line. Let the voltage at the load be reference angle, we can find the voltage per phase as follows.

$$V_R = \frac{64 \times 10^3}{\sqrt{3}} \angle 0° = 36.95 \angle 0°\ kV.$$

The receiving end current, I_R, can be found from the complex power consumed by the load at the receiving end, denoted by $S_{3\Phi,\mathrm{R}}$, of 70 MVA, 0.8 lagging.

$$|I_R| = \frac{|S_{1\Phi,\mathrm{R}}|}{|V_R|} = \frac{|S_{3\Phi,\mathrm{R}}|}{3|V_R|} = \frac{70 \times 10^6}{3 \times 36.95 \times 10^3} = 631.48\ A$$

The angle of receiving end current is found from power factor. The angle is negative because power factor is lagging.

$$\angle I_R = -\cos^{-1}(0.8) = -36.87°$$

Sending end voltage is found below.

$V_S = V_R + (2 + j7) \times\ I_R = 36950.42 \angle\ 0° + (2 + j7) \times (631.48 \angle\ -36.87°) = 40.71\ \angle\ 3.91°\ kV$.
Note that the no load voltage for short transmission line model is the same as sending end voltage at full load.

$$Voltage\ regulation = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\% = \frac{40.71 - 36.95}{36.95} \times 100\% = 10.17\%$$

The complex power delivered at the load is given as 70 MVA, 0.8 lagging. Thus, real power delivered at the load is

$$P_{3\Phi,\mathrm{R}} = |S_{3\Phi}| \times p.f. = 70 \times 0.8 = 56\ MW.$$

We then find real power at the sending end. For a short transmission line, sending end current is the same as receiving end current, $I_S = I_R = 631.48 \angle -36.87°$.
The sending end three-phase complex power is found from $S_{3\Phi,S} = 3V_S {I_S}^*$,

$$S_{3\Phi,S} = 3 \times 40707.94 \angle\ 3.91° \times 631.48 \angle\ 36.87° = 58.39 + j50.37\ MVA$$

Three-phase real power at sending end $P_{3\Phi,S} = 58.39\ MW$. The transmission efficiency is found below.

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100\% = \frac{56}{58.39} \times 100\% = 95.91\ \%$$

5.

The nominal pi-circuit is a medium-length line model, which is shown in Fig. 2.

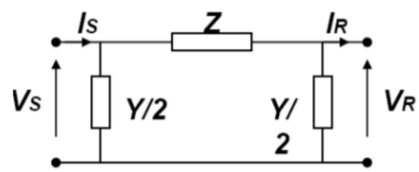

Fig.2

Given per phase series impedance per km and shunt admittance per km, the total impedance and admittances are found.

$$Z = 200 \times (0.08 + \mathrm{j}0.48\ ) = 16 + j96\ \Omega$$
$$Y = 200 \times (\mathrm{j}3.33 \times 10^{-6}) = (\mathrm{j}6.66 \times 10^{-4})\mathrm{siemens}$$

The ABCD parameters in the medium-length model can be found from below.

$A = D = \frac{ZY}{2} + 1 = 0.9680 + j0.0053$
$B = Z = 16 + j96\ \ \Omega$
$C = Y\left(1 + \frac{ZY}{4}\right) = (-1.7742 \times 10^{-6} + j6.5535 \times 10^{-4}\ )S$

At full load, the line delivers 250 MW at 0.99 p.f. lagging at 220 kV (line-to-line). We first find the receiving end voltage per phase.

$$V_R = \frac{220 \times 10^3}{\sqrt{3}} \angle 0° = 127.02\angle 0°\ kV.$$

The receiving end current, I_R, can be found from the complex power consumed by the load at the receiving end, denoted by $P_{3\Phi,\mathrm{R}}$, of 250 MW, 0.99 lagging.

$$|I_R| = \frac{|P_{3\Phi,\mathrm{R}}|}{3|V_R| \times \mathrm{p.f.}} = \frac{250 \times 10^6}{3 \times 127.02 \times 10^3 \times 0.99} = 662.69\ A.$$

The angle of receiving end current is found from power factor. The angle is negative because power factor is lagging.

$$\angle I_R = -\cos^{-1}(0.99) = -8.11°$$

Using ABCD parameters found earlier, together with receiving end voltage and current, the sending end voltage and current is found.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 155.40\angle 23.58°\ kV \\ 635.38\angle -0.34°\ A \end{bmatrix}$$

6.

This is a medium length line as shown in Fig. 2.
Given per phase series impedance per km and shunt admittance per km, the total impedance and admittances are found.

$$Z = 130 \times (0.036 + \mathrm{j} \times 100\,\Pi \ \times 0.8 \times 10^{-3}\ ) = 4.68 + j32.67\ \Omega$$

$$Y = 130 \times (\mathrm{j} \times 100\,\Pi \ \times 0.0112\ \times 10^{-6}) = (\mathrm{j}4.57 \times 10^{-4})\ \text{Siemens}$$

The ABCD parameters in the medium-length model are found below.

$A = D = \frac{ZY}{2} + 1 = 0.9925 + j0.0011$
$B = Z = 4.68 + j32.67$
$C = Y\left(1 + \frac{ZY}{4}\right) = (-0.2448 \times 10^{-8} + j0.4557 \times 10^{-5}\ )$

The receiving end load is 270 MVA with 0.8 lagging power factor at 325 kV. $V_R = \frac{325}{\sqrt{3}}\angle 0°$ kV,
Find the receiving end current, I_R,

$$|I_R| = \frac{|S_{3\Phi,\mathrm{R}}|}{3|V_R|} = \frac{270 \times 10^6}{3 \times 187.64 \times 10^3} = 479.65\ A.$$

<u>When power factor is 0.8, lagging.</u>
$I_R = |I_R|\angle - cos^{-1}0.8 = 479.64\angle - 36.87°A$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} 187.64 \times 10^3\angle 0° \\ 479.64\angle - 36.87° \end{bmatrix} = \begin{bmatrix} 197764\angle 3.30°\ V \\ 430.27\angle - 27.66°\ A \end{bmatrix}$$

At full load, $|V_{R,FL}| = 187.64\ kV$.
At no load, $I_R = 0, |V_{R,NL}| = \left|\frac{V_S}{A}\right| = \left|\frac{197764\angle 3.30°}{(0.9925+j0.0011)}\right| = 199.25\ kV$

$$\%\ Regulation = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\% = \frac{199.25 - 187.64}{187.64} \times 100\% = 6.19\%.$$

Transmission line efficiency:

$$S_{3\Phi,S} = 3V_sI_s^* = 3 \times 197764\angle 3.30° \times 430.26\angle - 27.66° = 218.9 + j131.3\ MVA.$$

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100\% = \frac{270 \times 0.8}{218.9} \times 100\% = 98.7\%$$

<u>When power factor is 0.95, lagging.</u>
$I_R = |I_R|\angle - cos^{-1}p.f. = 479.64\angle - 18.19°A$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} 187.64 \times 10^3\angle 0° \\ 479.64\angle - 18.19° \end{bmatrix} = \begin{bmatrix} 193796\angle 4.26°\ V \\ 456.70\angle - 7.88°\ A \end{bmatrix}$$

At full load, $|V_{R,FL}| = 187.64\ kV$.
At no load, $I_R = 0, |V_{R,NL}| = \left|\frac{V_S}{A}\right| = \left|\frac{193796\angle 4.26°}{(0.9925+j0.0011)}\right| = 195.26\ kV$

$$\%\ Regulation = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\% = \frac{195.26 - 187.64}{187.64} \times 100\% = 4.06\%.$$

Transmission line efficiency:

$$S_{3\Phi,S} = 3V_sI_s^* = 3 \times 193796\angle 4.26° \times 456.70\angle 7.88° = 259.58 + j55.84\ MVA.$$

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100\% = \frac{270 \times 0.95}{259.58} \times 100\% = 98.8\%$$

When the power factor is close to one, the transmission line efficiency and voltage regulation is improved.