EE2022
Electrical Energy Systems
Lecture 10: Summary

National University of Singapore

EE2022 Syllabus – Part 1

1. Introduction

Energy and its various forms; Importance of Electrical Energy in secondary form

2. Introduction to Electrical Power

Active, Reactive and Apparent Power; Concept of Power factor, lagging, leading and unity power factor operation

3. Three phase systems

Single- and three-phase power system; Star and Delta connection; Relationship between phase and line quantities; per-unit representation.

4. Generation, Transmission and Distribution Network

Transmission line modelling; Calculation of transmission line parameters;

Why…?

Why AC and not DC ?

Why a sinusoidal alternating voltage ?

Why 50 Hz (or 60 HZ) ?

http://en.wikipedia.org/wiki/Utility_frequency

The choice of Frequency

50 Hz and 60 Hz

- Between 1885 and 1890 in the U.S.A.:
  - 140, 133⅓, 125, 83 ⅓, 66 ⅔, 50, 40, 33 ⅓, 30, 25, 16⅔ Hz

- Nowadays:
  - 60 Hz in North America, Brazil and Japan (has also 50 Hz!)
  - 50 Hz in most other countries

- Exceptions:
  - 25 Hz Railways (Amtrak)
  - 16⅔ Hz Railways
  - 400 Hz Oil rigs, ships and airplanes

The choice of Frequency

50 Hz and 60 Hz

- A too low frequency, like 10 or 20 Hz causes flicker
- A too high frequency
  - Increases the hysteresis losses:

$$P_{hys} :: f\psi^{1.5-2.5}$$

  - Increases the eddy current losses:

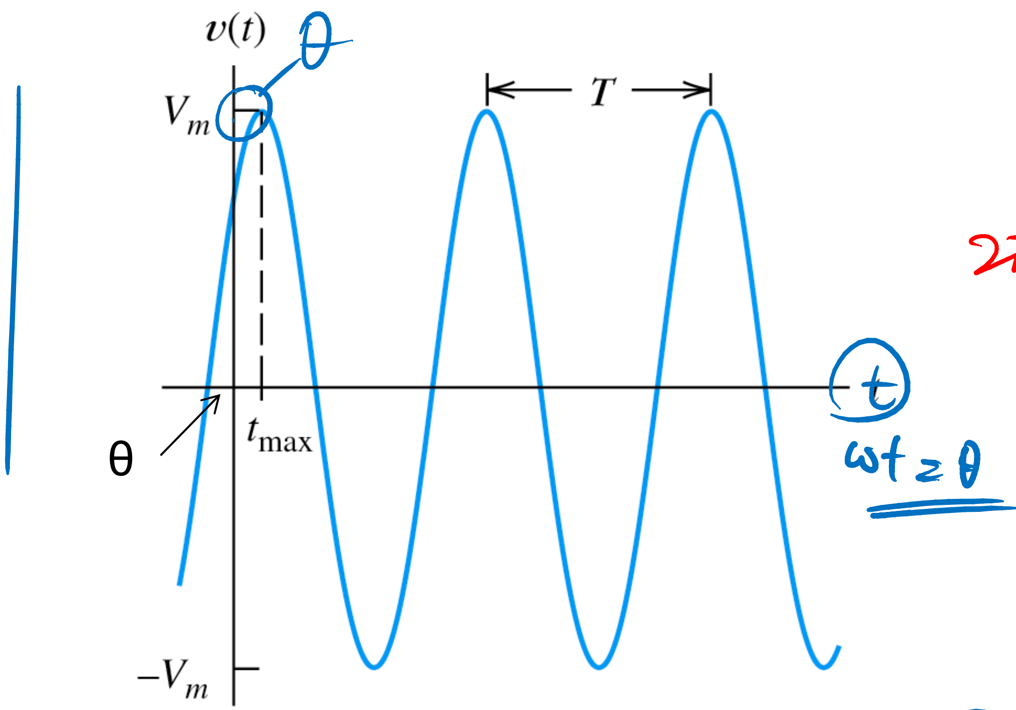
$$P_{eddy} :: f^2\psi^2$$

  - Increases the cable and line impedance

$j\omega L$

$j2\pi f.L$

Sinusoidal Currents And Voltages



V_m is the peak value

$2\pi f$ ω is the angular frequency in radians/sec

θ is the phase angle

$\frac{1}{f}$ = T is the period

A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$.

Note: Assuming that θ is in degrees, we have $t_{\text{max}} = \frac{-\theta}{360} \times T$.

For the waveform shown, θ is -45°.

Root-Mean-Square Values

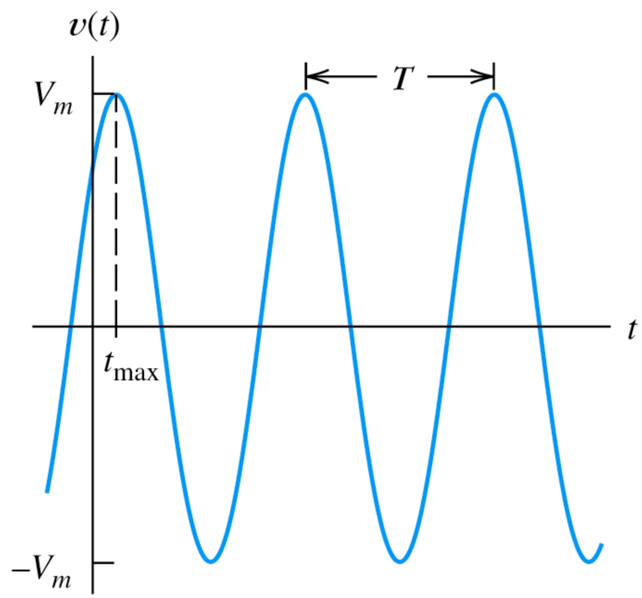
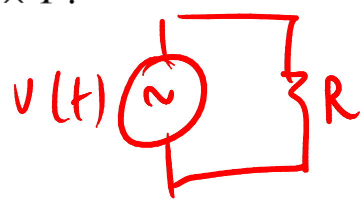


Figure 5.1 A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$. Note: Assuming that θ is in degrees, we have $t_{max} = \frac{-\theta}{360} \times T$. For the waveform shown, θ is -45°.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$\boxed{= \frac{V_m}{\sqrt{2}}}$$

power calculations.



$$\boxed{V_{rms}}$$

Avg power $= \dfrac{(V_{rms})^2}{R}$

$= \dfrac{V_{dc}^2}{R}$

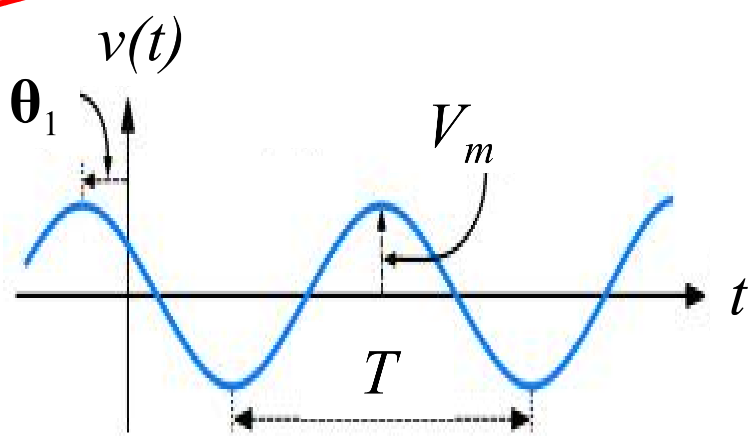
Phasor Representation of a Sinusoidal waveform

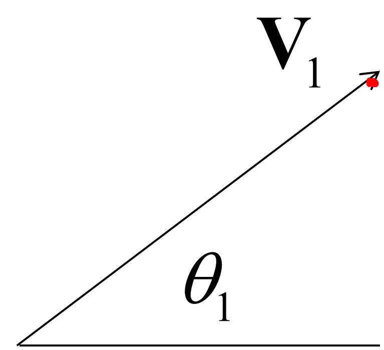
$$\text{Time function}: \quad v_1(t) = V_m \cos(\omega t + \theta_1)$$

$\frac{d}{dt}$, $\int dt$, Phasor : $\quad \mathbf{V}_1 = V_1 \angle \theta_1$

Where ~~V_1~~ is ~~the~~ RMS value of voltage

① Phasor ② Impedance





Example - Phasors

$$10 \sin(200t + 45^\circ) + 20 \cos(200t - 60^\circ) + 15 \sin(200t + 90^\circ)$$

$$'' \quad + 20 \sin(200t - 60^\circ + 90^\circ) + \quad ''$$

Phasors - (same frequency)

$$\frac{10}{\sqrt{2}} \angle 45^\circ + \frac{20}{\sqrt{2}} \angle 30^\circ + \frac{15}{\sqrt{2}} \angle 90^\circ =$$

$$= \frac{1}{\sqrt{2}} \left[10 \angle 45^\circ + 20 \angle 30^\circ + 15 \angle 90^\circ \right] = \left(\frac{40.29}{\sqrt{2}} \right) \angle 52.75^\circ .$$

Result : $40.29 \cdot \sin(200t + 52.75^\circ)$.

Example

R L C

Impedance = Resistance + j Reactance

$$Z = R + jX$$

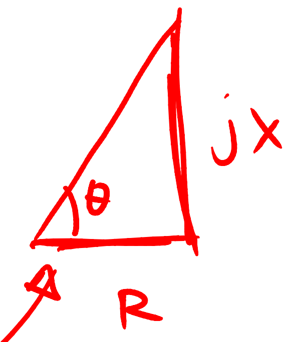
R  R

L  $j\omega L$

C  $-j\dfrac{1}{\omega C}$

$X = \omega L$

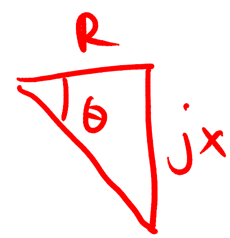
$C = -\dfrac{1}{\omega C}$



$$\angle I = \angle V - \angle Z$$

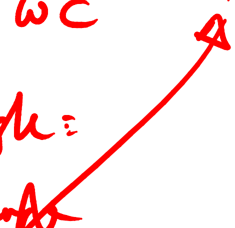
$$I = \frac{V}{Z} =$$

Impedance angle = power factor angle



Current will lag voltage by θ (if θ +ve)

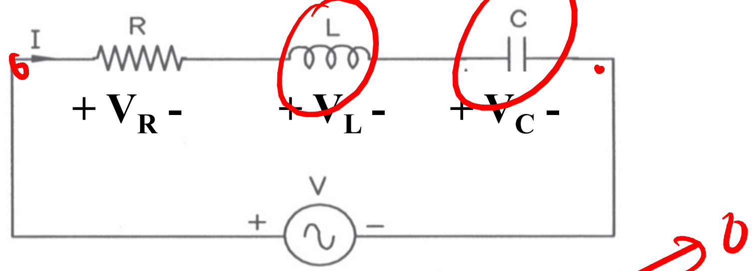
" " lead " by θ (if θ −ve)



Series Resonance

Series circuit comprising inductor, capacitor and resistance has some interesting behavior.

This can be analyzed when we connect the circuit across variable frequency source.

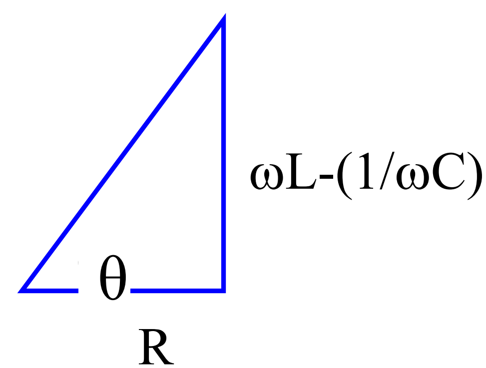


Impedance: $$Z = R + j\omega L - \frac{j}{\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= |Z| \angle \theta$$

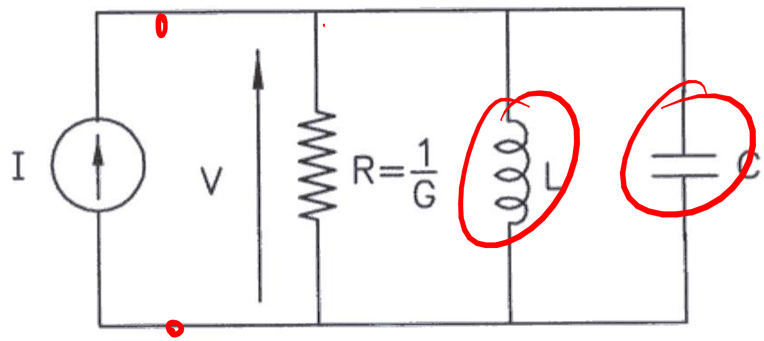
Magnitude and phase angle: $$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan \theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$



Current flow in the circuit $$I = \frac{V}{R + j\left(\omega L - \frac{j}{\omega C}\right)}$$

Parallel RLC Circuits



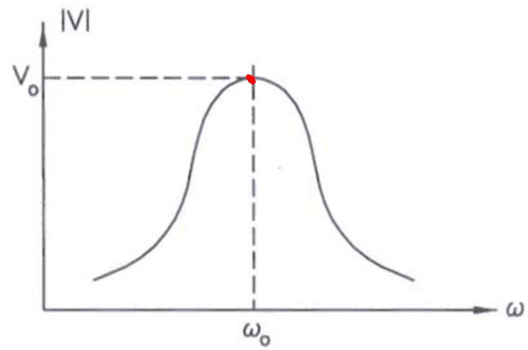
Similar behavior is observed when R, L and C are connected in parallel.

The admittance of the circuit is $$Y = G + j\omega C - \frac{j}{\omega L}$$

$$\omega_o = \frac{1}{\sqrt{L \cdot C}}$$

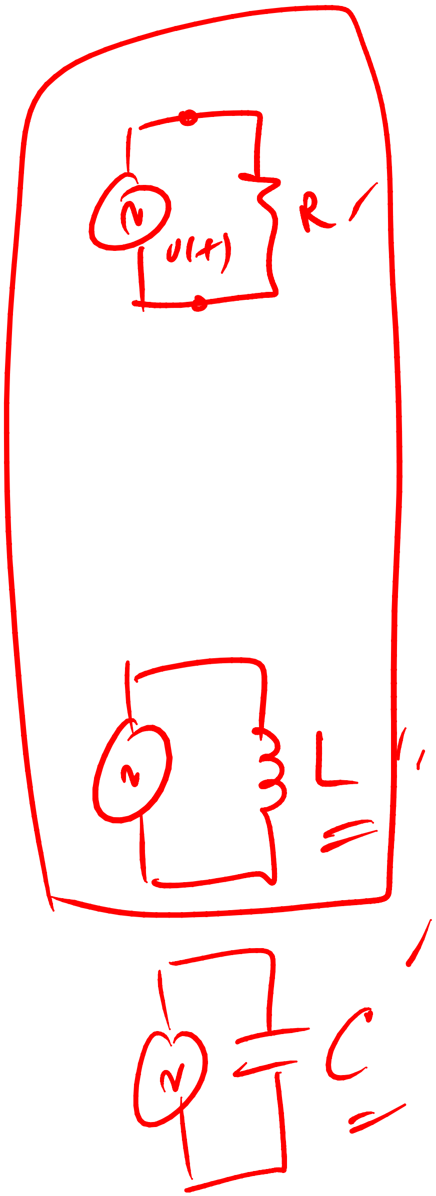
$$|V| = \frac{|I|}{|Y|} = \frac{|I|}{\sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

At resonance: $$\omega_o L = \frac{1}{\omega_o C}$$



- **What is power in AC circuits for R, L, C?**
- **Power in Generic AC Load** $\boxed{R+jX}$
- **Real, Reactive and Apparent Power**
- **Power Triangle (Complex Power)**
- **Power Factor**
- **Power Factor Compensation**

Power in R, L, C



$$v(t) = V_m \cdot \sin(\omega t)$$

$$i(t) = \frac{V_m}{R} \cdot \sin \omega t$$

$$p(t) = \frac{V_m^2}{R} \cdot \sin^2 \omega t = \frac{V_m^2}{R} \cdot \left[1 - \cos 2\omega t\right]$$

↑ Instantaneous power.

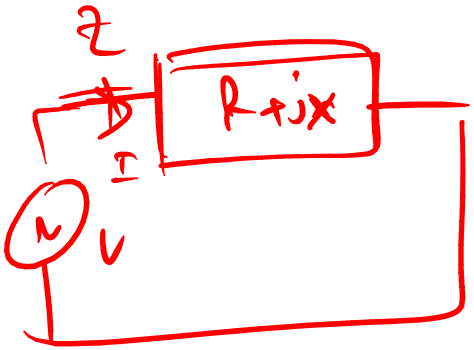
→ twice

Frequency of oscillation of power

$$p(t) = \frac{V_m^2}{\omega L} \cdot \sin \omega t \cdot \cos \omega t = \frac{1}{2} \cdot \frac{V_m^2}{\omega L} \cdot \sin(2\omega t)$$

$$p(t) = -V_m^2 \cdot \omega C \cdot \sin \omega t \cdot \cos \omega t = -\frac{V_m^2}{2} \cdot \omega C \cdot \sin(2\omega t)$$

Power in Generic AC Load

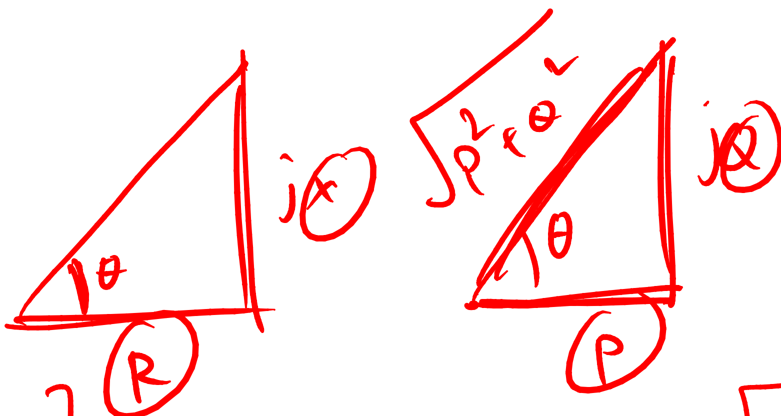


$$I = \frac{V}{Z}$$

$$\text{power} = I^2 \cdot Z = I^2 \cdot (R + jX)$$

$$= \underbrace{I^2 R}_{P} + j\,\underbrace{I^2 X}_{Q}$$

Real power (Average power) — Reactive power.



$\cos\left[\tan^{-1}\frac{X}{R}\right]$ Apparent power $|S| = \sqrt{P^2 + Q^2}$

$P = |S| \cdot \cos\theta$

$Q = |S| \cdot \sin\theta$

$\cos\left(\tan^{-1}\frac{Q}{P}\right)$ $|S| = V_{rms} I_{rms}$

$\boxed{\cos\theta = \text{power factor}}$

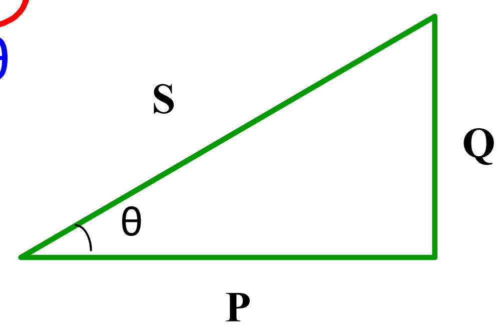
Complex Power

We can show that,

Complex power S = V. I*

Complex power, $S = V_{rms}\, I_{rms} \cos\theta + j\, V_{rms}\, I_{rms} \sin\theta$

Or, $S = |S| \cos\theta + j\, |S| \sin\theta = P + jQ$

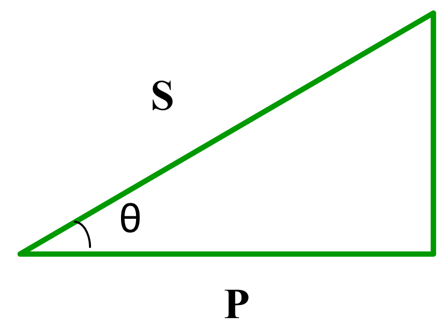


S	**Complex power**	VA	$S = \lvert S\rvert \angle\theta = P + jQ = VI^* = \\ = \lvert V\rvert \lvert I\rvert \angle\theta = \lvert V\rvert \lvert I\rvert (\cos\theta + j\sin\theta)$
$\lvert S\rvert$	**Apparent power**	VA	$\lvert S\rvert = \lvert V\rvert \lvert I\rvert = \sqrt{P^2 + Q^2}$
P	**Active power** Average power, Real power	W	$P = \mathrm{Re}(S) = \lvert S\rvert \cos(\theta) = \lvert V\rvert \lvert I\rvert \cos(\theta)$
Q	**Reactive power**	var	$Q = \mathrm{Im}(S) = \lvert S\rvert \sin(\theta) = \lvert V\rvert \lvert I\rvert \sin(\theta)$

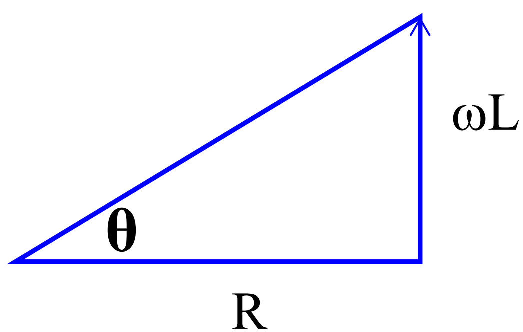
Power Factor of an a.c. circuit

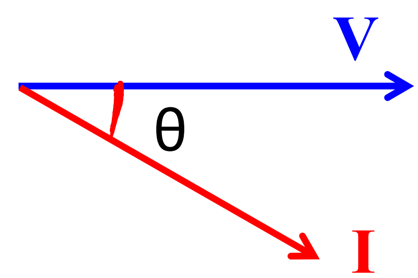
Now, we will define Power Factor of an a.c. circuit as the ratio of real power to the apparent power.

$$\text{Power Factor} = \frac{P}{|S|} = \frac{P}{|V|\,|I|} = \cos\theta$$

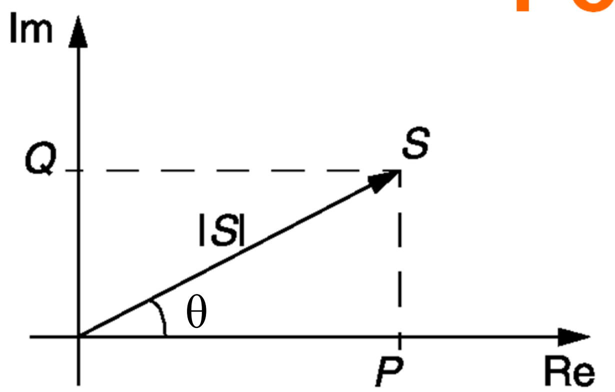


Power factor angle (θ) is the same in power triangle, impedance triangle, and the angle between voltage and current.





Power Factor



$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$$\cos(\theta) = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}}$$

$$\cos(\theta) = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$$

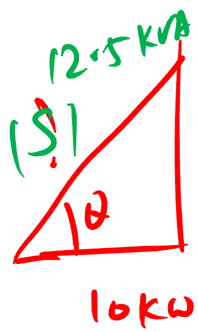
Example – Different types of power

① Load draws 10 kW at 230 V p.f. of 0.8 lag

② 〃 15 kVA at 230 V p.f. of 0.9 lead.

R + jX

①



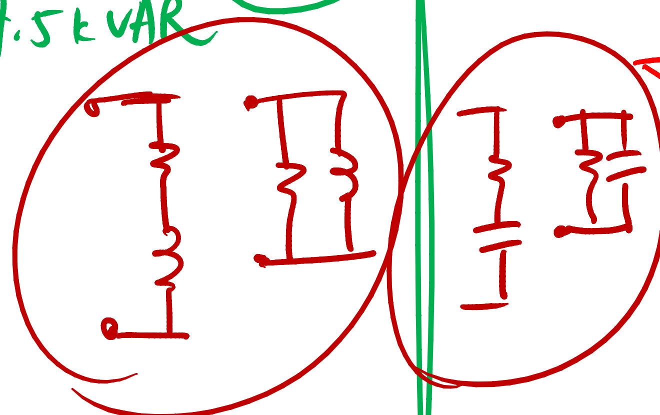
? Q = 7.5 kVAR

$\theta = \cos^{-1}(0.8)$

$= 36.87^\circ$

$|S| = \frac{P}{p.f.} = 12.5$

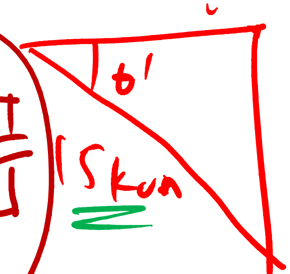
$Q = |S| \cdot \sin(36.87^\circ) = 7.5$ kVAR



②

R + jX

P ? = 13.5 kW

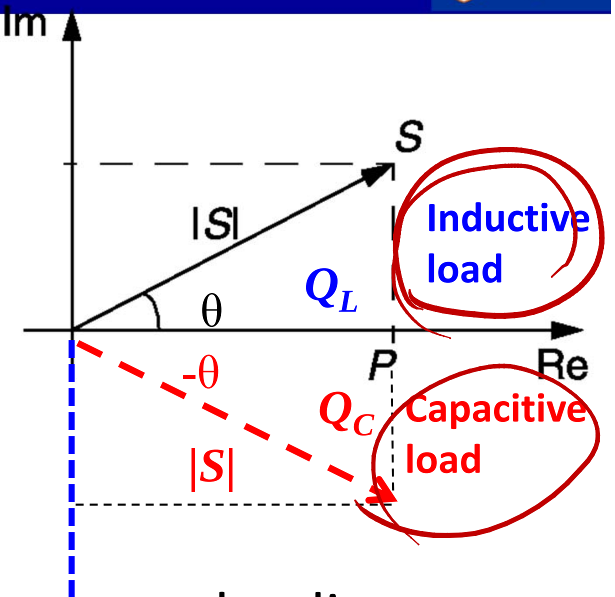


Q ? − 6.54 kVAR

$\theta' = -\cos^{-1}(0.9)$

$= -25.84^\circ$

Power Factor



$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$0 \leq \theta \leq 90^o$

Hence, power factor lies between: $0 \leq$ p.f. ≤ 1

Since cos (+θ) = cos (-θ), we identify the power factor as leading or lagging.

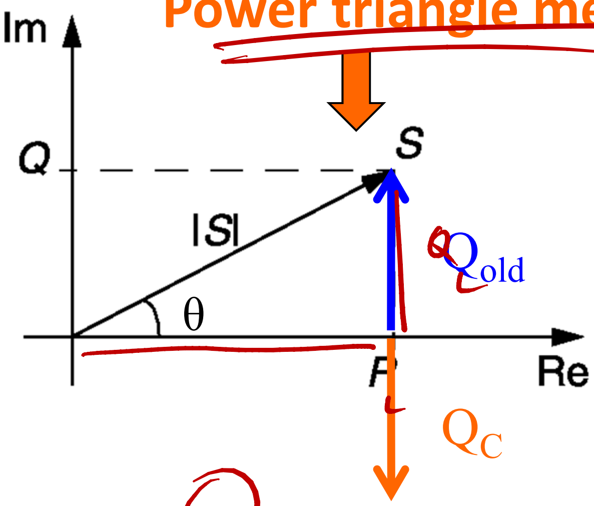
For inductive load: Current lags behind voltage (Reactive power +ve)
Hence, power factor is lagging p.f.

For capacitive load: Current leads the voltage (Reactive power is -ve)
Hence, power factor is leading p.f.

For example: If the load is $10\angle 25^o$ the power factor is 0.906 lagging
If the load is $20\angle -30^o$ the power factor is 0.87 leading

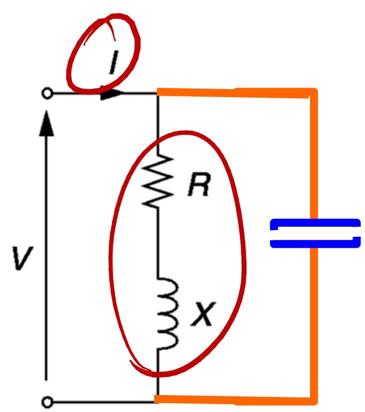
Power Factor Correction

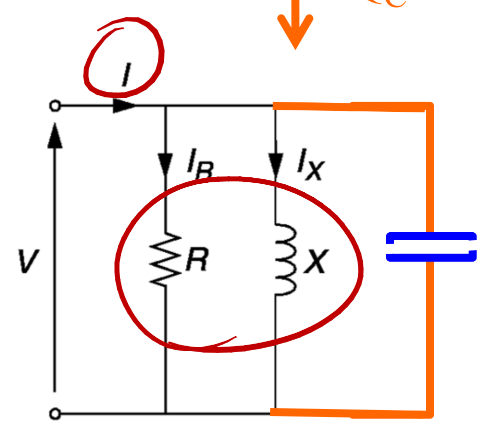
Power triangle method

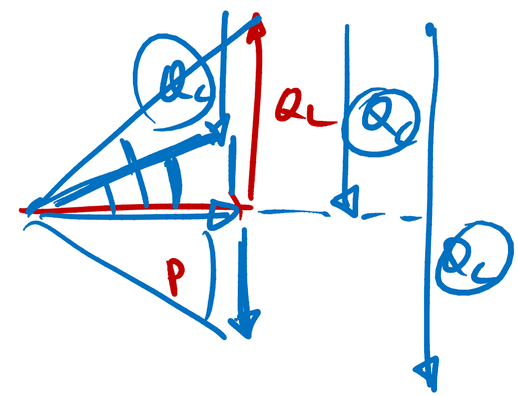


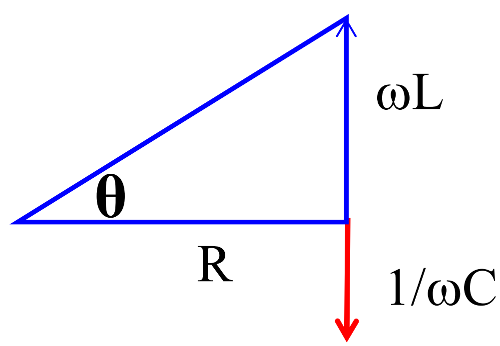
$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$$|I| = P/|V|\cos(\theta)$$







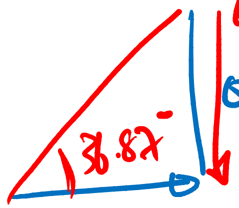


Example

Load is $20 + j10\ \Omega$

0.8 to 0.9

Load draws 10 kW, at 0.8 lagging p.f. from a 230V/50 Hz source.



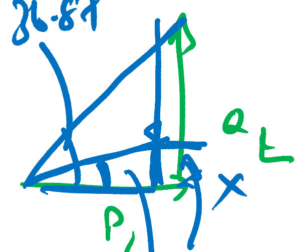
$P_L + jQ_L$

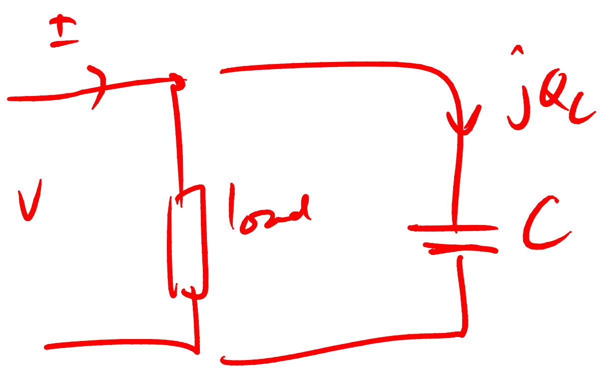
$P_L + jQ_L + jQ_c$

① Unity p.f.

$Q_L + Q_c = 0$

$Q_c = -Q_L = -7.5$ kVAR.





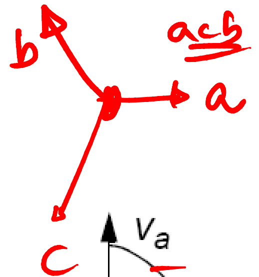
$Q_c = -V^2 \omega C$

$$C = \frac{-Q_c}{V^2 \cdot 2\pi \cdot 50} = \frac{+7.5 \times 10^3}{230^2 \times 2\pi \times 50}\ \text{F}$$

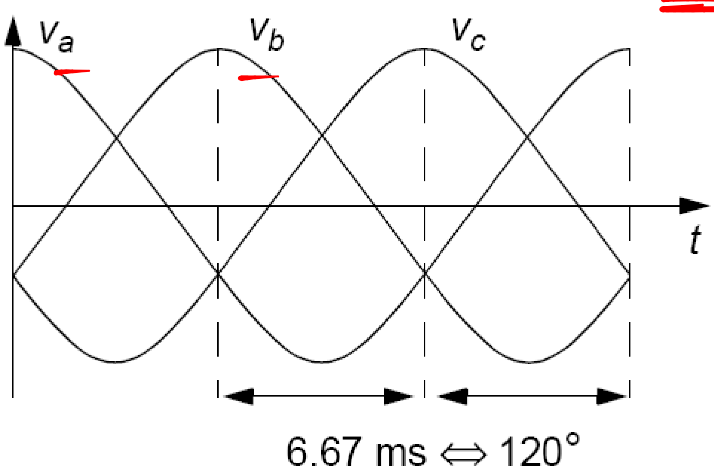
$Q_c + Q_L = 4.84$

$Q_c = 4.84 - 7.5 = -2.66$ kVAR

$X = P_L \cdot \tan 25.84° = 4.84$ kVAR



Three-Phase Voltage Sources

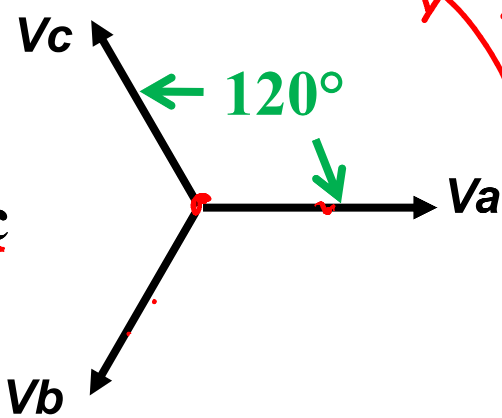


$$v_a = \sqrt{2}|V|\cos(\omega t)$$

$$v_b = \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = \sqrt{2}|V|\cos\left(\omega t - \frac{4\pi}{3}\right)$$

Phase sequence: abc (positive)



All three voltage sources have the same voltage magnitude,

with 120 degrees apart.

Line-To-Line Voltage

a b c   ab, bc, ca // ba, cb, ac

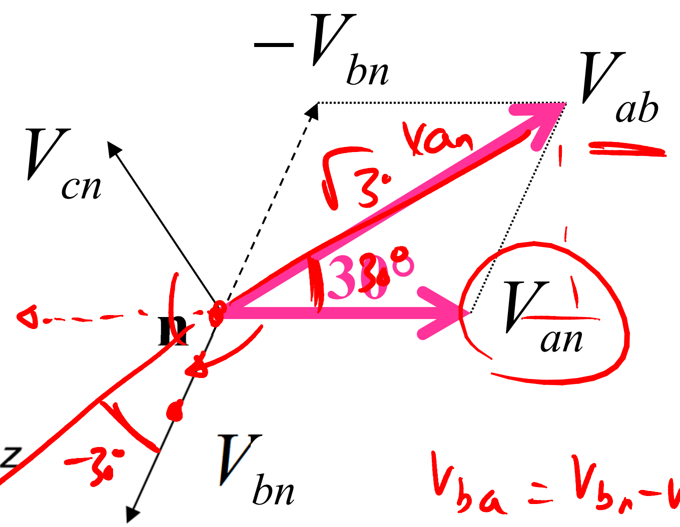
$V_{ab} = V_{an} - V_{bn}$

Voltage is given as line-to-line voltage by convention.

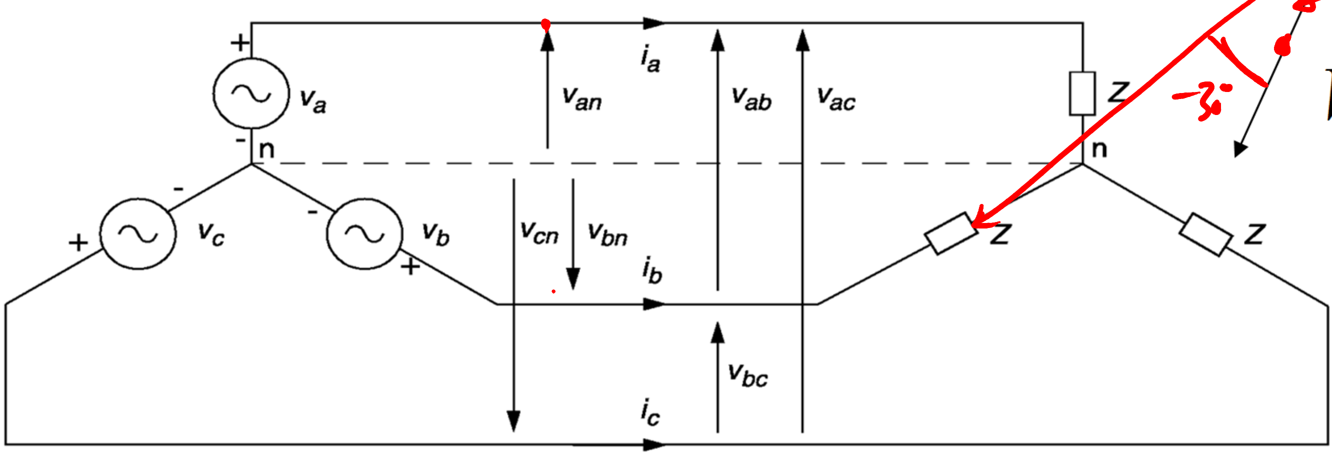
KVL:

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_{an}\angle 30°$$

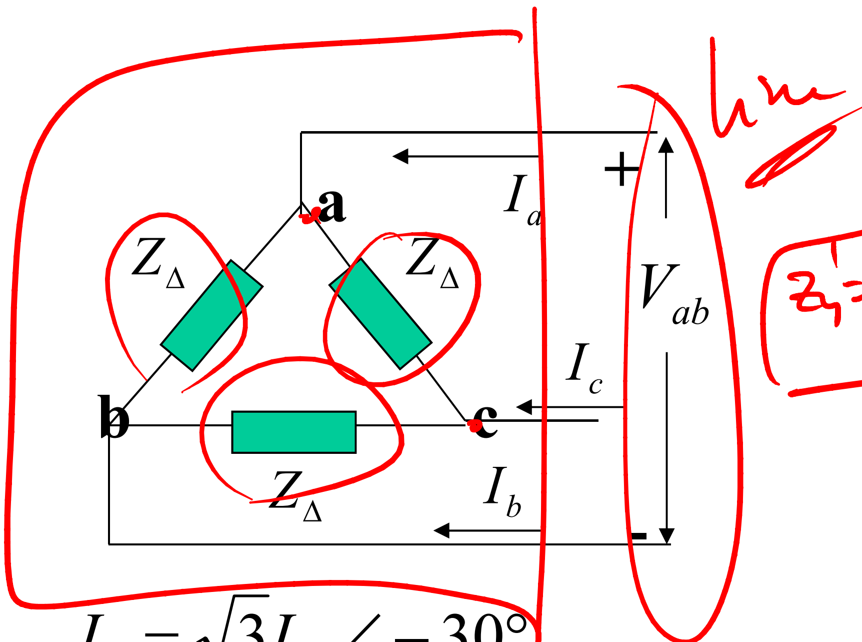
$$\boxed{\left|V_{\text{Line-Line}}\right| = \sqrt{3}\left|V_{\text{Line-neutral}}\right|}$$

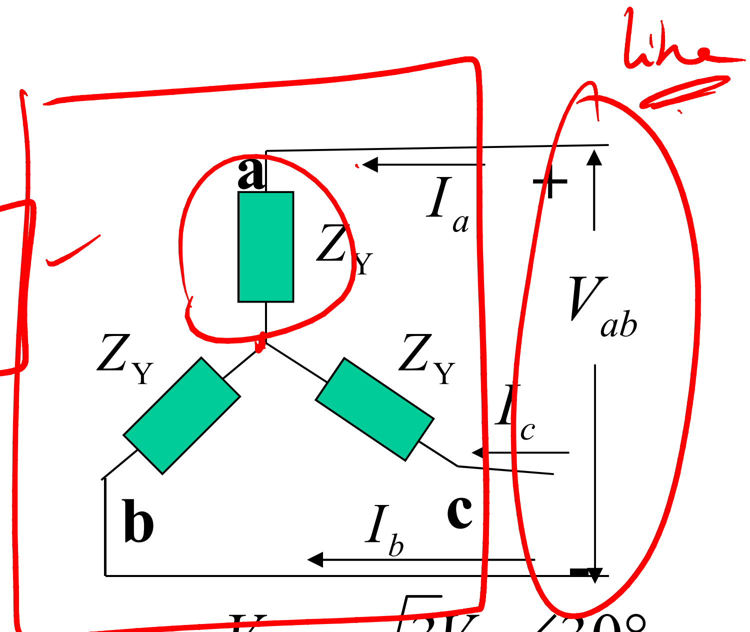


$V_{ba} = V_{bn} - V_{an}$



Delta/Wye Connected 3-Phase Load





$$I_a = \sqrt{3} I_{ab} \angle -30°$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30°$$

$$|S_{3\Phi}| = 3|V_{ab} I_{ab}^*| = \sqrt{3}|V_{ab}||I_a|$$

$$|S_{3\Phi}| = 3|V_{an} I_a^*| = \sqrt{3}|V_{ab}||I_a|$$

$(\sqrt{3} \cdot V_{phase})$

$$\boxed{|S_{3\Phi}| = \sqrt{3}|V_{\text{Line-To-Line}}||I_{\text{Line}}|}$$

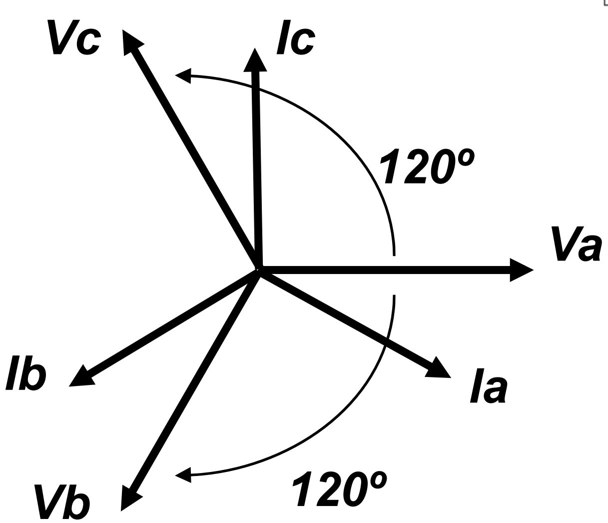
$\sqrt{3} \cdot V \cdot I$

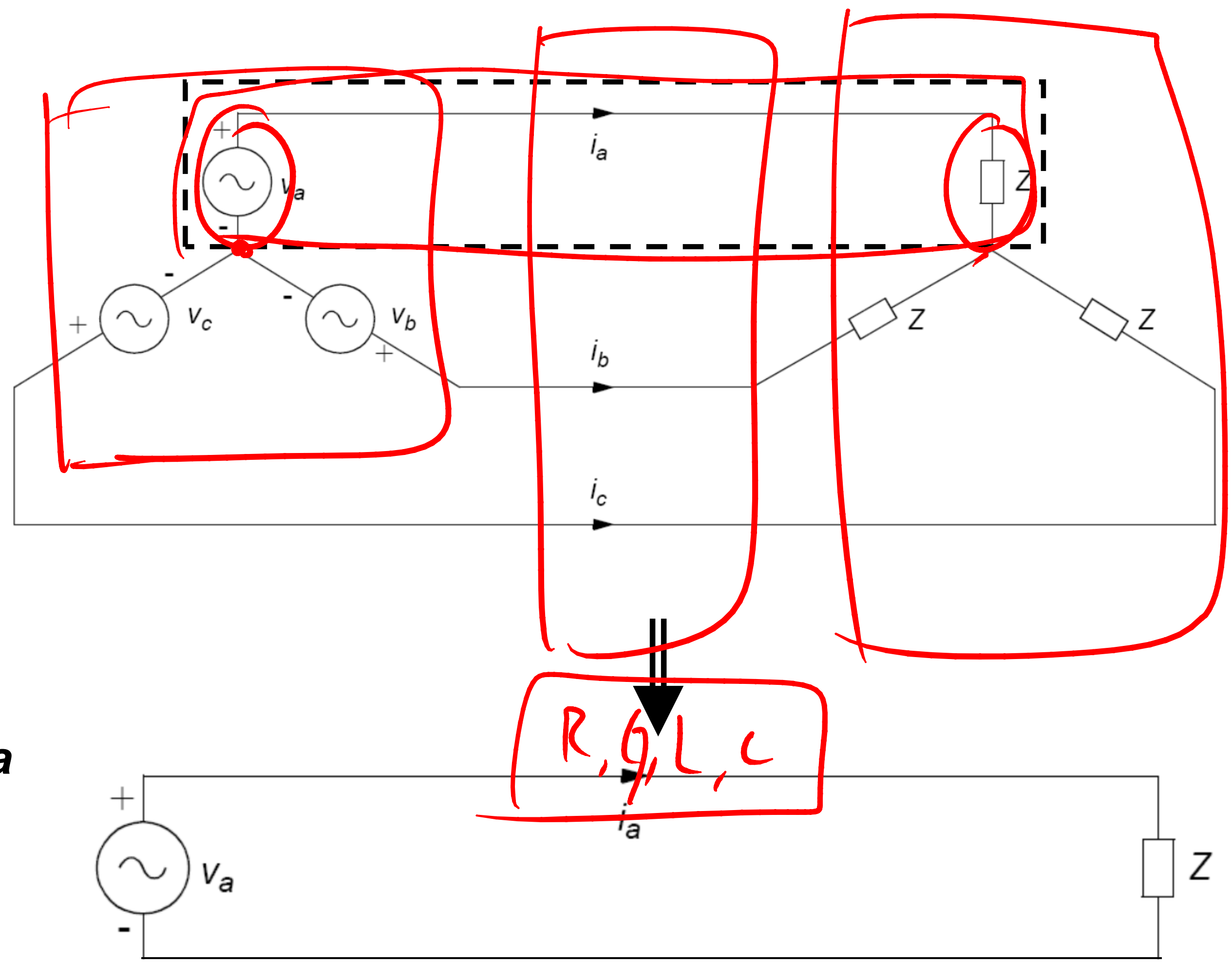
$S_{3\phi} \neq 3 \cdot V \cdot I$

Per-Phase Analysis: Assumption

It must be balanced three-phase circuit.

$$I_n = I_a + I_b + I_c = 0$$





Steps of Per Phase Analysis

Make sure that the three-phase system is balanced.

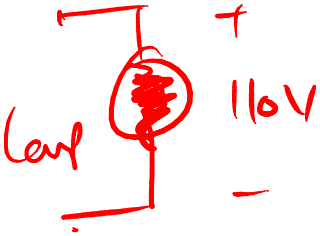
- The three-phase sources need to have the same magnitude with 120 degree phase difference.
- The three-phase impedances must be of the same value (both phase and magnitude).

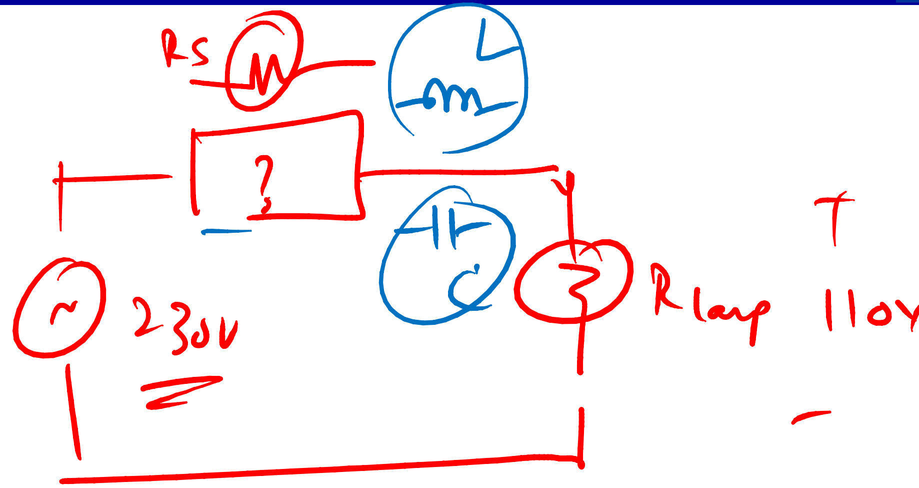
$V_{LL} \rightarrow \quad V_{LN} = \frac{V_{LL}}{\sqrt{3}}$

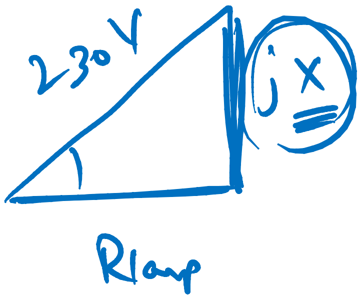
Convert all Delta-connected sources/loads to Wye-connected sources/loads.

$Z_{\Delta} \rightarrow \quad Z_{Y} = \frac{Z_{\Delta}}{3}$

Per phase analysis reduce three-phase circuit to single-phase circuit. We can apply the same concept used in single-phase.







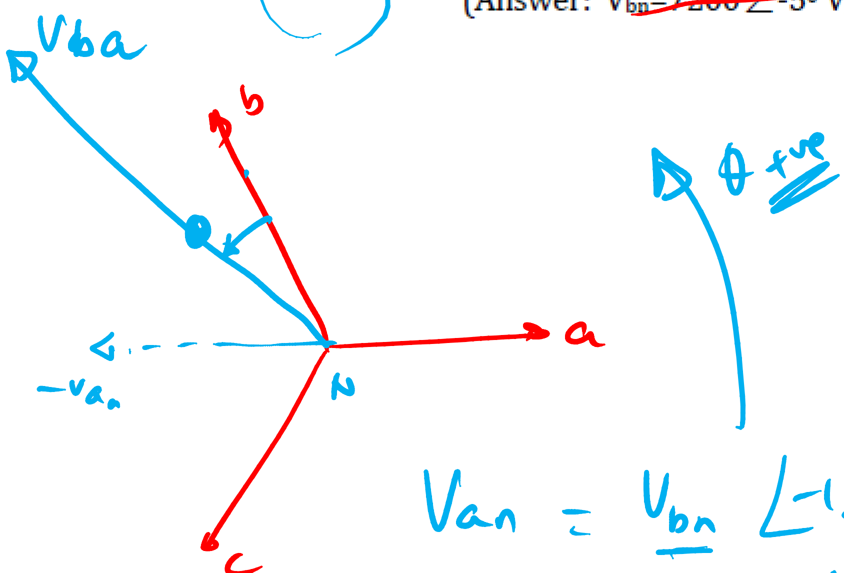
$$\sqrt{230^2 - 110^2}$$

$$230 \times \left(\frac{R_{lamp}}{R_S + R_{lamp}} \right) = 110\,V \Rightarrow R_S$$

X → L, C

2. What are the phase voltages for a balanced three-phase Y load, if $V_{ba}=12470\angle -35^{0}$ V? The phase sequence is ~~positive~~ −ve sequence i.e. ~~abc~~. acb

(Answer: ~~$V_{bn}=7200\angle -5^{0}$ V, $V_{an}=7200\angle 115^{0}$ V, and $V_{cn}=7200\angle -125^{0}$ V~~)



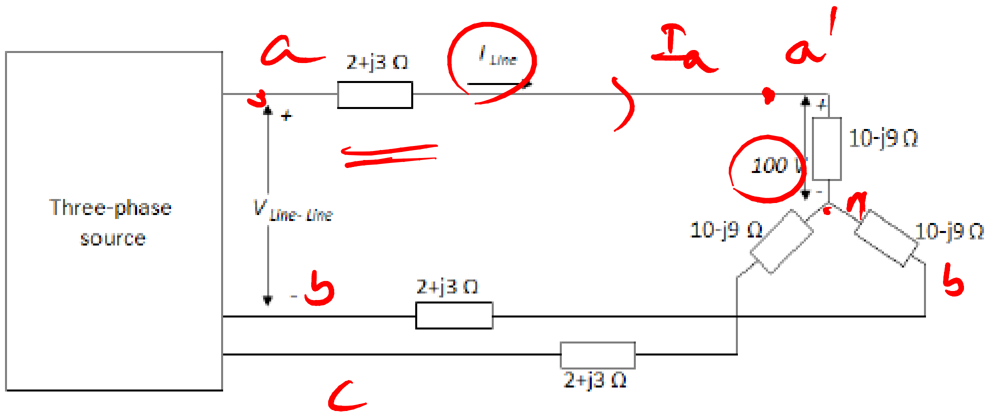
$$V_{ba} = V_{bn} - V_{an}$$
$$= \sqrt{3}\, V_{bn} \angle +30^\circ$$
$$V_{bn} = \frac{V_{ba}}{\sqrt{3}} \angle -30^\circ$$
$$= \frac{12470}{\sqrt{3}} \angle -35-30^\circ$$
$$= \frac{12470}{\sqrt{3}} \angle -65^\circ$$

$$V_{an} = V_{bn} \angle -120^\circ$$
$$V_{cn} = V_{bn} \angle +120^\circ$$

$V_{a'n}$ ✓

$V_{a'n} = 100\angle 0^\circ$

V_{ab} ?



$$I_a = \frac{100\angle 0^\circ}{10-j9} = 7.43\angle 41.98^\circ \text{ A.}$$

$$V_{an} = V_{a'n} + I_a \times (2+j3) = 99.72\angle 15.41^\circ$$

$$V_{ab} = \sqrt{3}\, V_{an} \angle 30^\circ$$

$$V_{LL} = \sqrt{3}\, V_{an}.$$

8. In a 208-V three-phase circuit a balanced Δ load absorbs 2 kW at a 0.8 leading power factor. Find Z_Δ.

Y

Z_Y.

(Answer: 51.9∠-36.87° Ω)

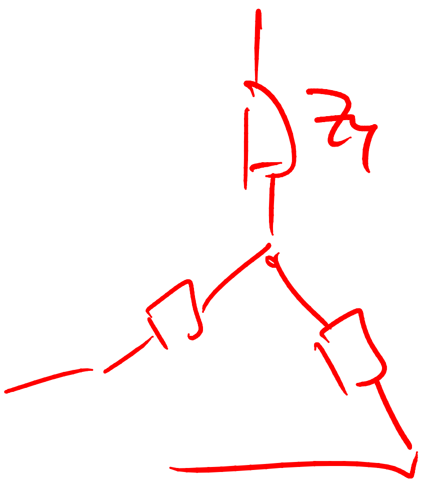
$V = 208\ \text{V}$

$P_{1\phi} = \frac{2000}{3}$. $p.f. = 0.8$ leading

$P = \frac{V^2}{|Z_\Delta|} \times (p.f.)$

$Z_\Delta = \frac{V^2}{P} \cdot p.f.$

$= \frac{208^2}{2000/3} \times 0.8$



$V_{LN} = \frac{208}{\sqrt{3}}$.

$Z_Y = \frac{V^2}{P} \cdot p.f.$

$= \frac{(208/\sqrt{3})^2}{2000/3} \times p.f.$