EE2022 Electrical Energy Systems

Lecture 6:
Three-Phase Circuit Analysis

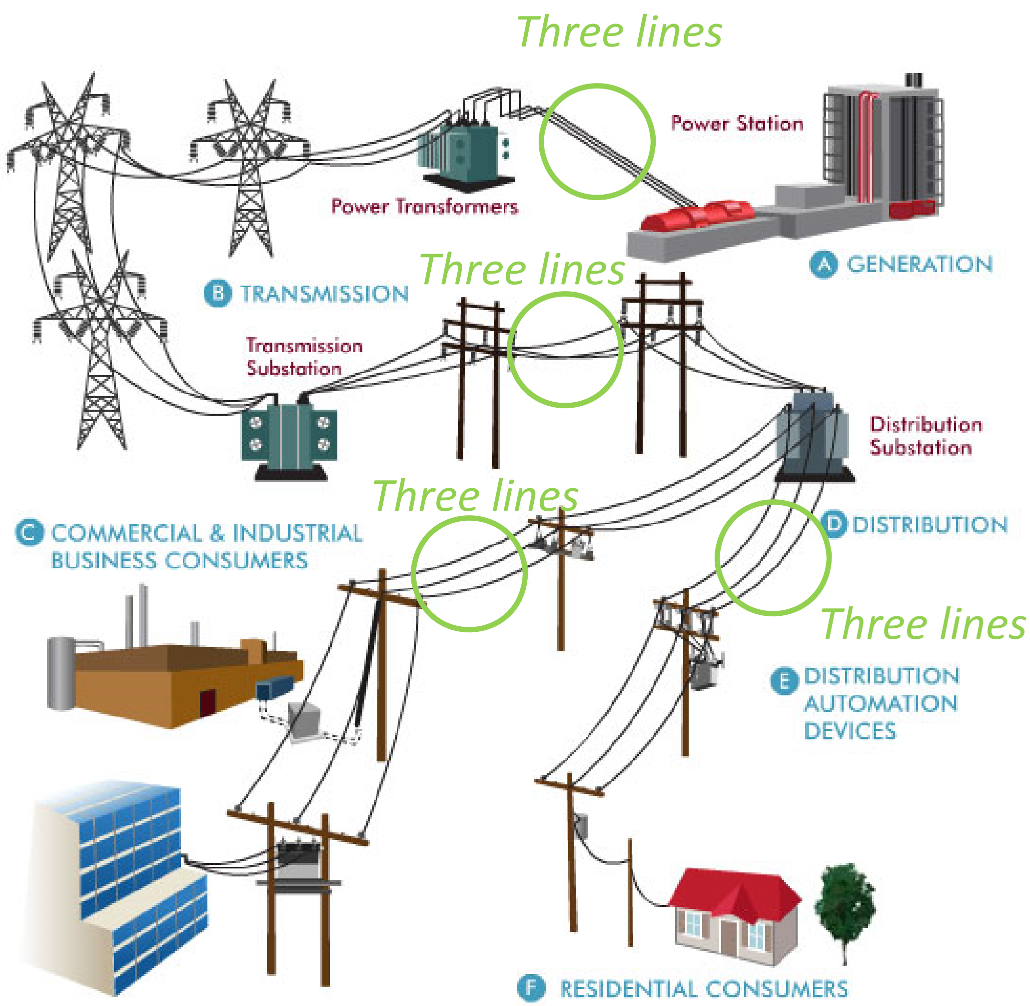
Learning Outcomes

- To calculate the complex power, voltages and currents in single phase and balanced three-phase AC circuits and able to describe their relationships using Phasor diagrams.
  - Be able to solve balanced three-phase circuit problems.
  - Be able to calculate three-phase power.

Outline

- Three-Phase Circuit Analysis
  - Generation, Transmission, and Distribution.
  - Three-phase balanced systems.
  - Advantages of three-phase balanced systems.
- Three-Phase voltage and current
  - Line-to-neutral voltage
  - Line-to-Line voltage
  - Line current.
  - Delta/Wye configuration.

Generation, Transmission and Distribution



Question: How to represent the whole system by an equivalent circuit to find voltage/current at any point?

Answer: Three-phase circuit diagram.

Source: http://venturebeat.com/2010/10/29/super-grid-introduction/

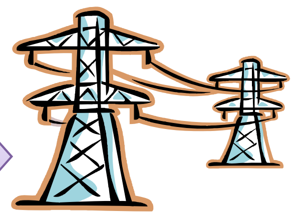
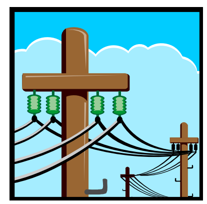
A Three-Phase Circuit System

Generation (11 – 36 KV)



3-Phase Generation system.

Transmission (110 – 765 KV)

3-Phase Transmission system.

Industrial customer
(23 – 138 KV)
Commercial customer
(4.16 – 34.5 KV)
Residential customer
(120 – 240 V)

Generation

Three-phase voltage source

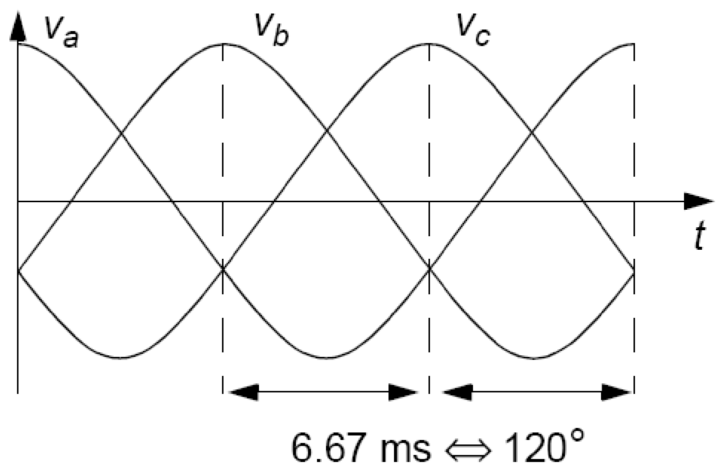
Transmission and Distribution

Three transmission lines

Load

Three-phase load

Three-Phase Voltage Sources

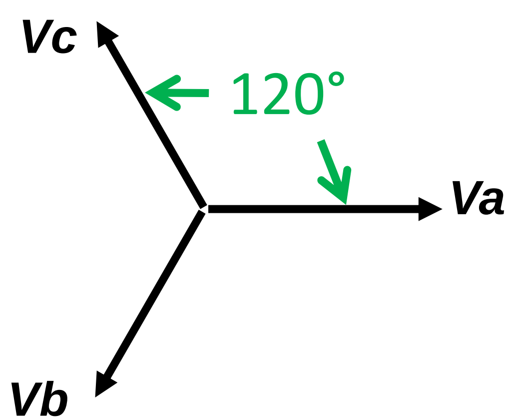


$$v_a = \boxed{\sqrt{2}|V|}\cos(\omega t)$$

$$v_b = \boxed{\sqrt{2}|V|}\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = \boxed{\sqrt{2}|V|}\cos\left(\omega t - \frac{4\pi}{3}\right)$$

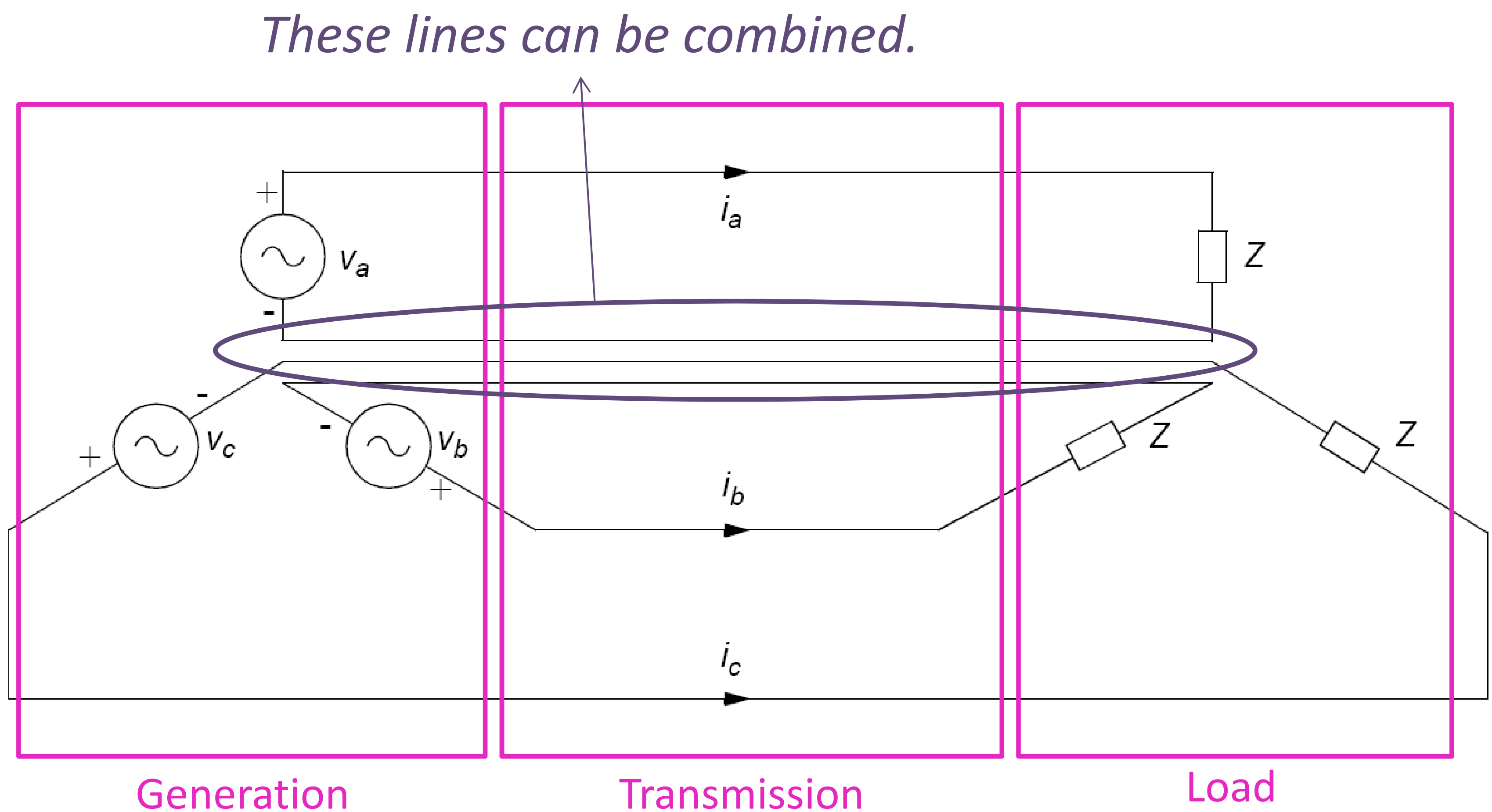
Note that |V| is rms value. – see Lecture#2.



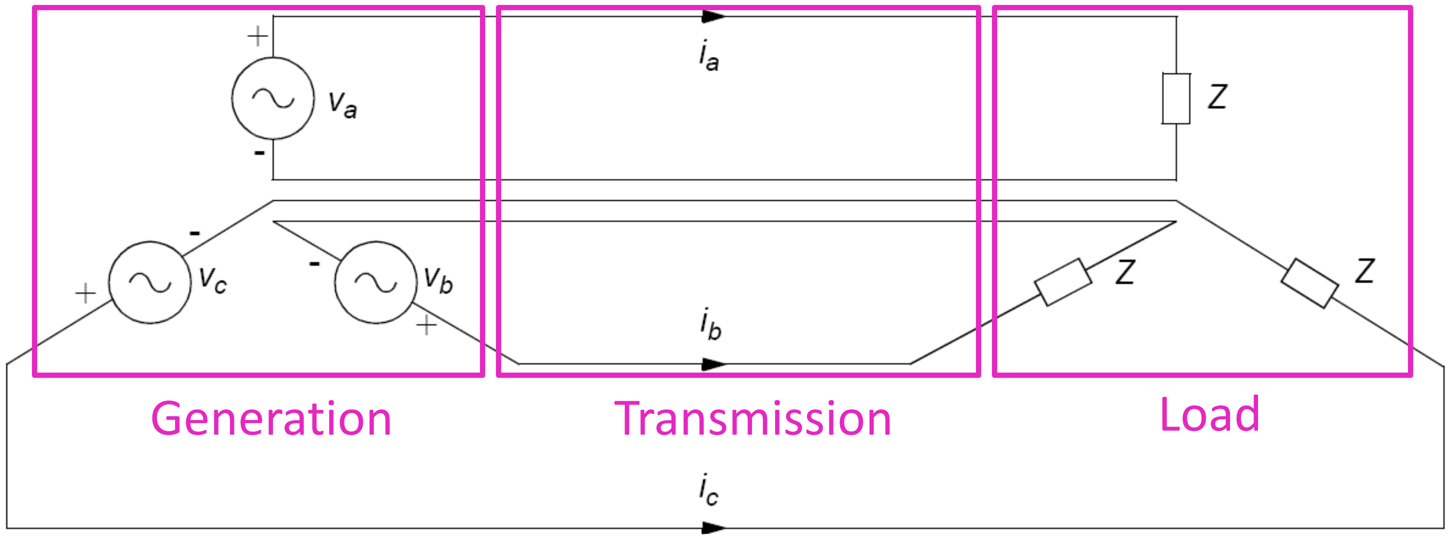
All three voltage sources have the same voltage magnitude,

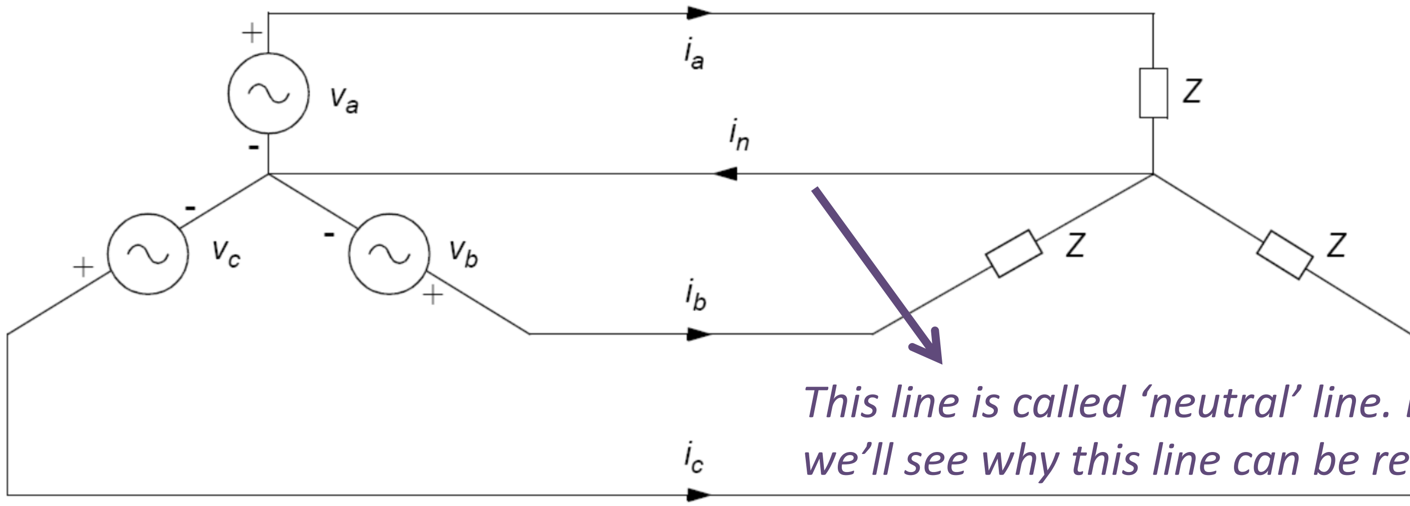
with 120 degrees apart.

Three Single-Phase Circuits



Three Phase Circuit

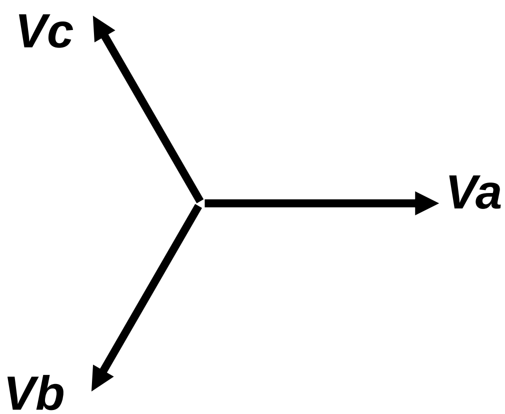




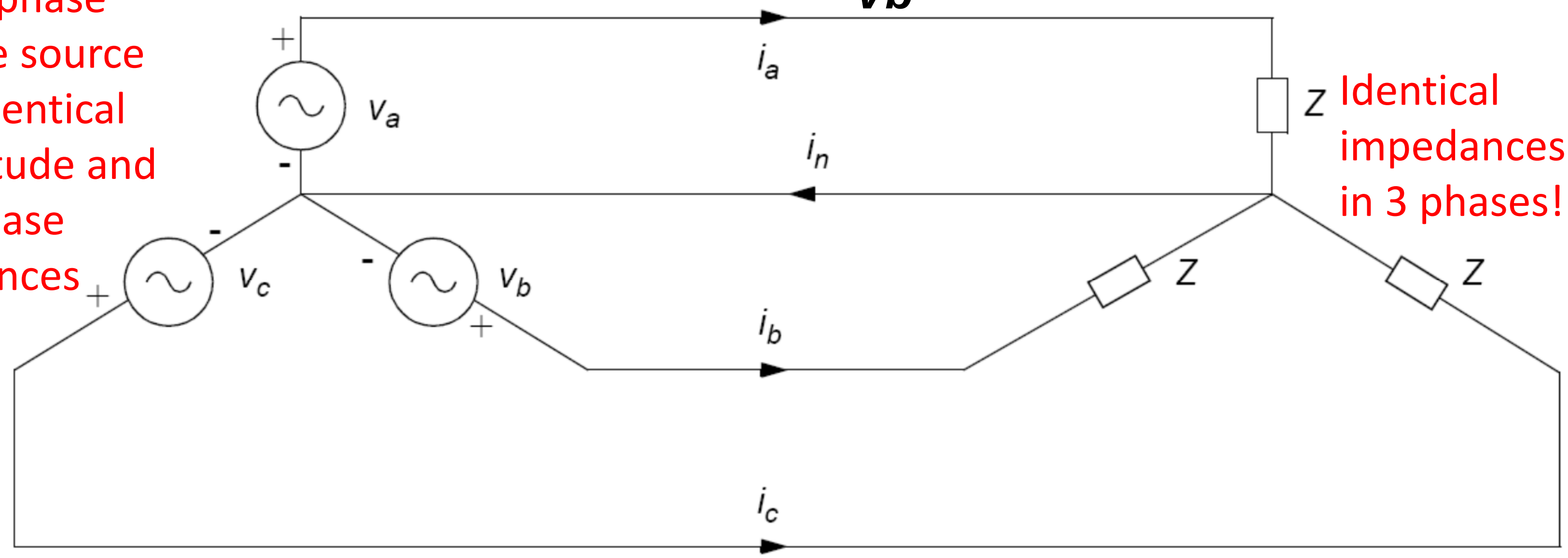
This line is called 'neutral' line. Later on, we'll see why this line can be removed.

Balanced Three-Phase Circuit

Three-phase circuit is said to be balanced when the impedances in the 3 phases are identical.

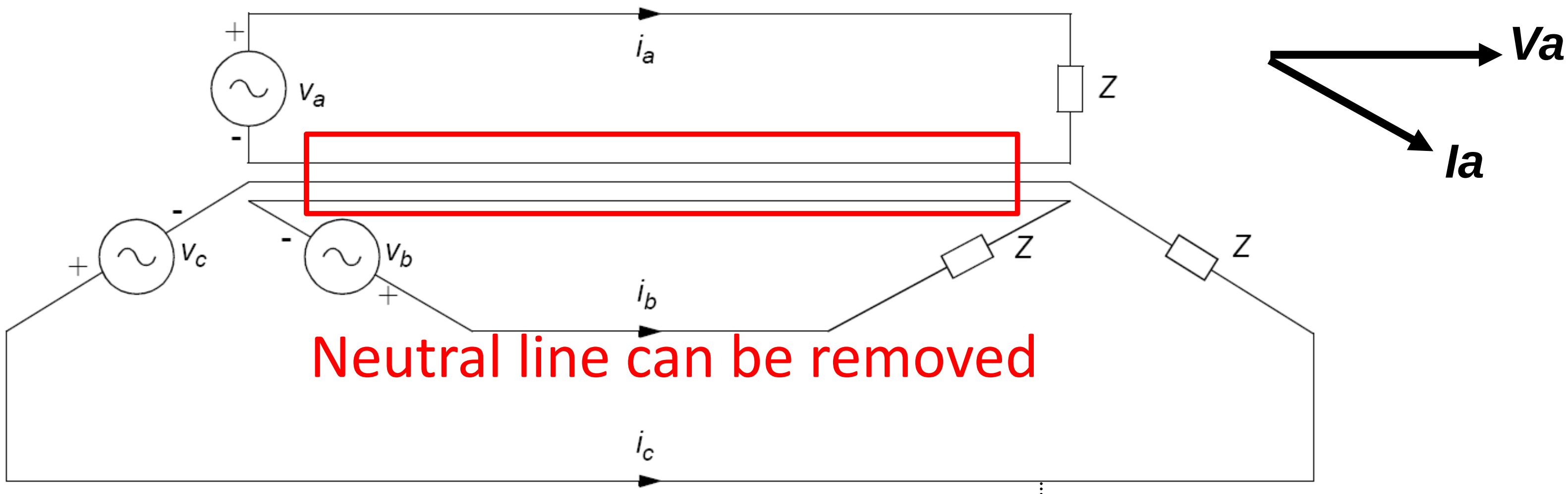


Three-phase voltage source with identical magnitude and 120 phase differences

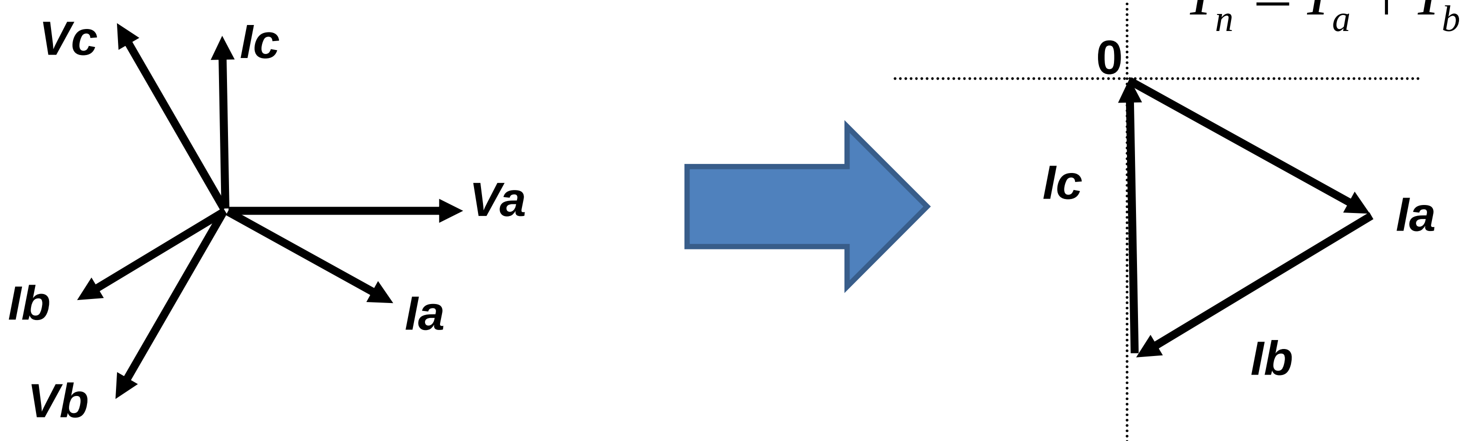


Identical impedances in 3 phases!

Balanced Three-Phase Circuit

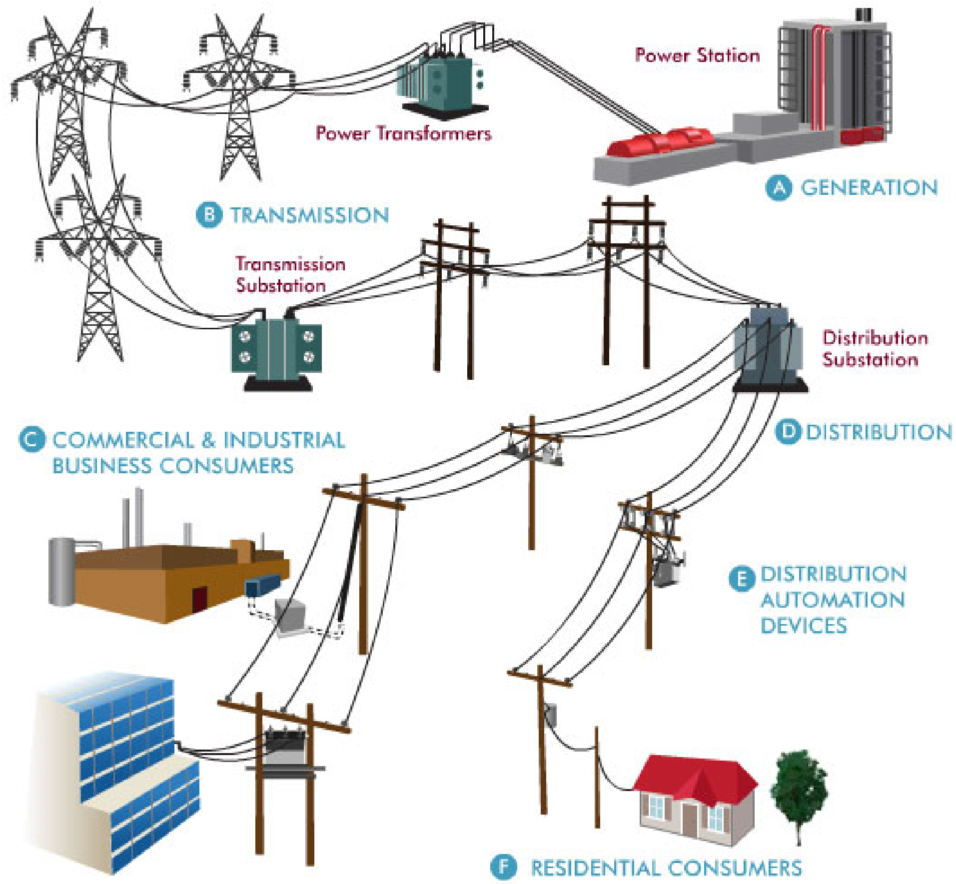


$$I_n = I_a + I_b + I_c = 0$$



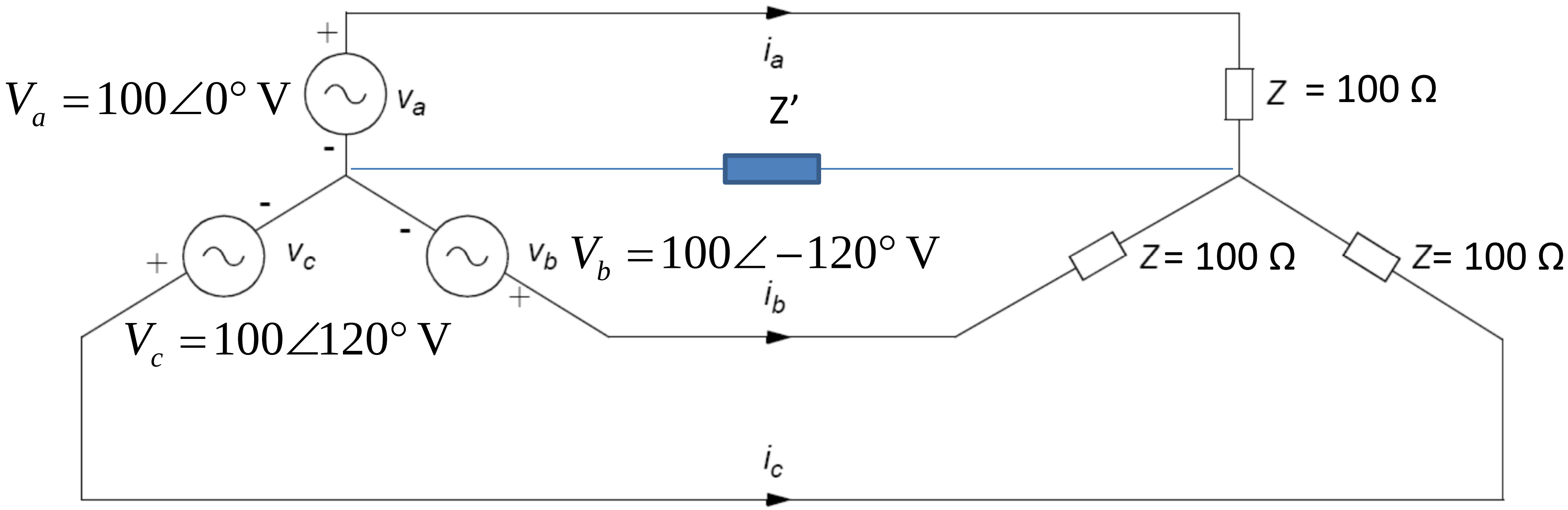
Advantages of Balanced 3-Phase Systems

- When compared to three single-phase circuits, three-phase circuits have better use of equipment and materials
  - More power can be transmitted per conductor
  - Lesser power losses in the conductors
- This implies reduced capital and operating costs of transmission and distribution.
- We can calculate voltage and current for only one phase and refer to other phases easily.



Example 1

- Consider the following three-phase system shown below. Find the current i_a when z' = 10 Ω.



Ans: $i_a = 1\angle 0°$ A, $i_b = 1\angle -120°$ A, $i_c = 1\angle 120°$ A

Line-to-Neutral Voltage

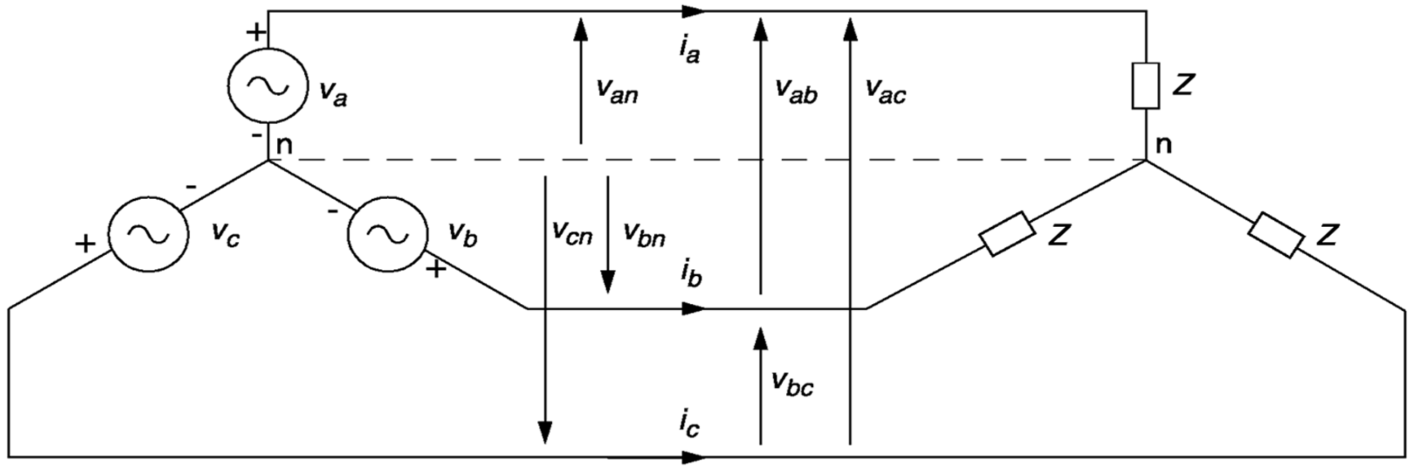
Line-to-line voltage

Line current

Wye-Delta connection

THREE-PHASE CURRENT AND VOLTAGE

Line-To-Neutral (Phase) Voltage

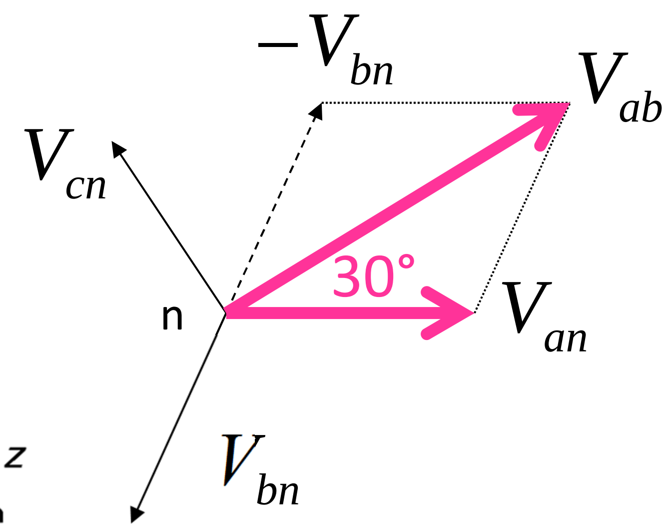


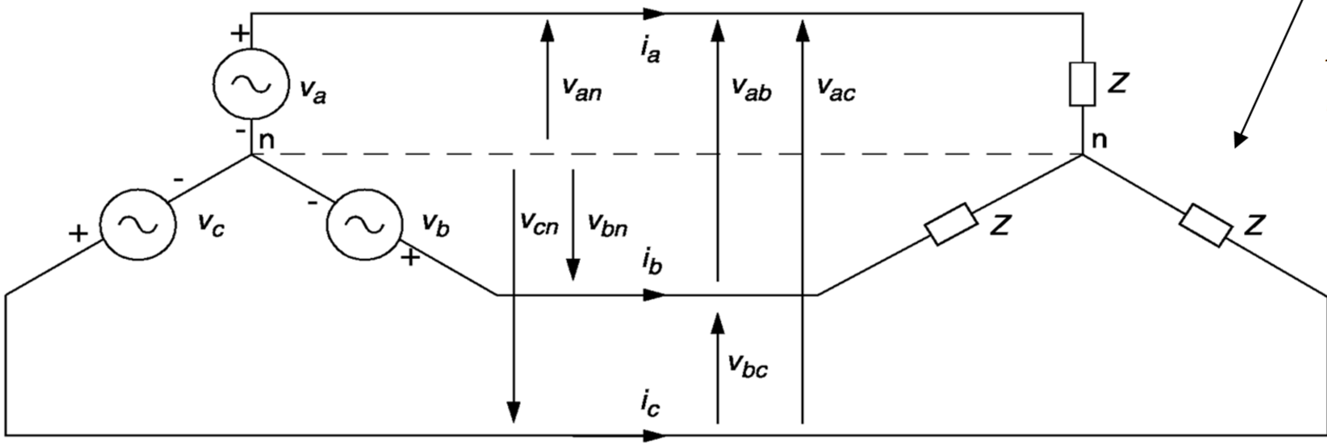
V_{an}, V_{bn}, V_{cn} are called line-to-neutral voltage or phase voltage [R3].

Line-To-Line Voltage

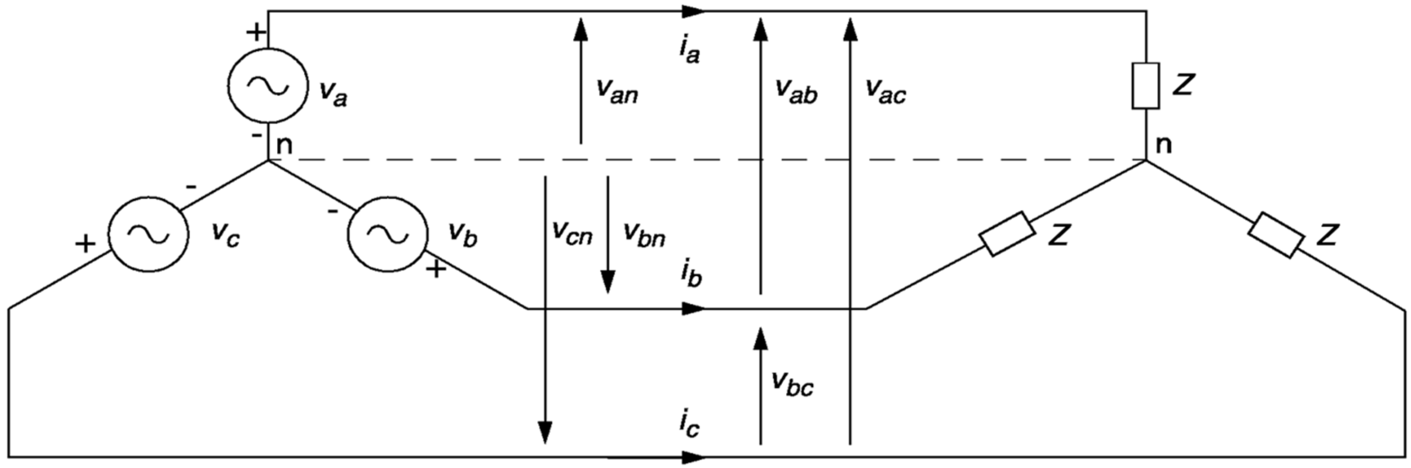
- Voltage is given as line-to-line voltage by convention.
- KVL: $V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} \angle 30°$

$$\boxed{\left| V_{\text{Line-Line}} \right| = \sqrt{3} \left| V_{\text{Line-neutral}} \right|}$$





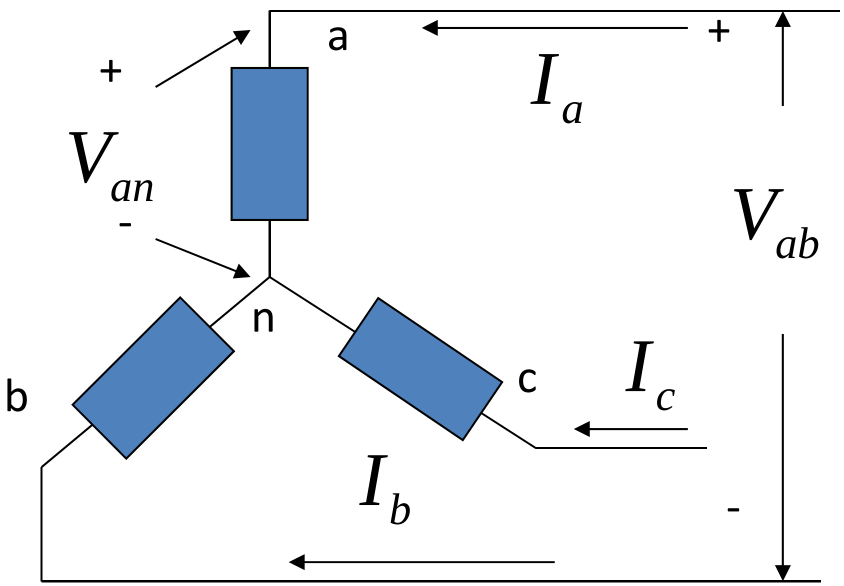
Line Current



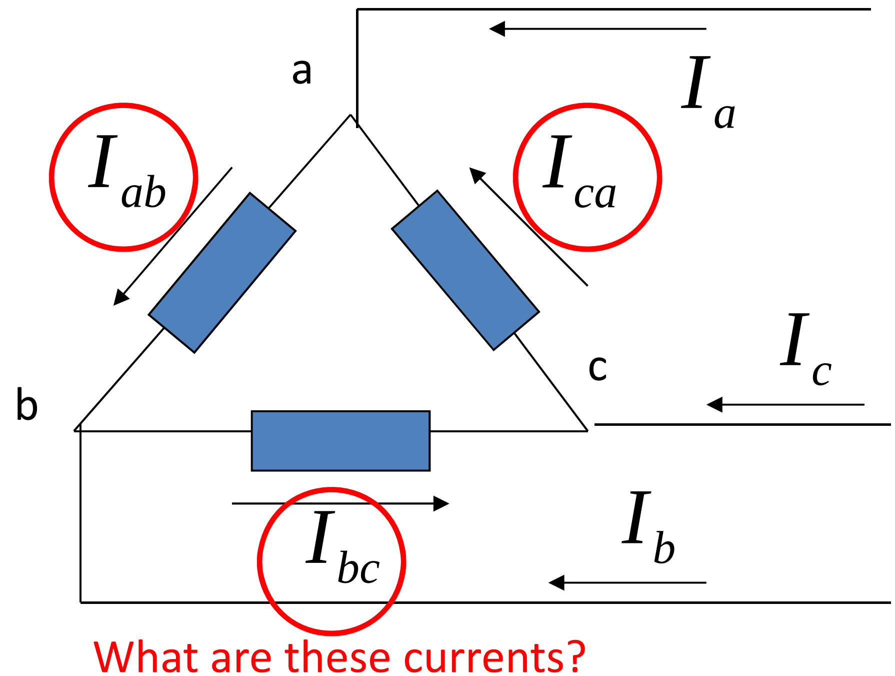
i_a , i_b , i_c are called line currents.

3-Phase Circuit Connection

Wye Connection

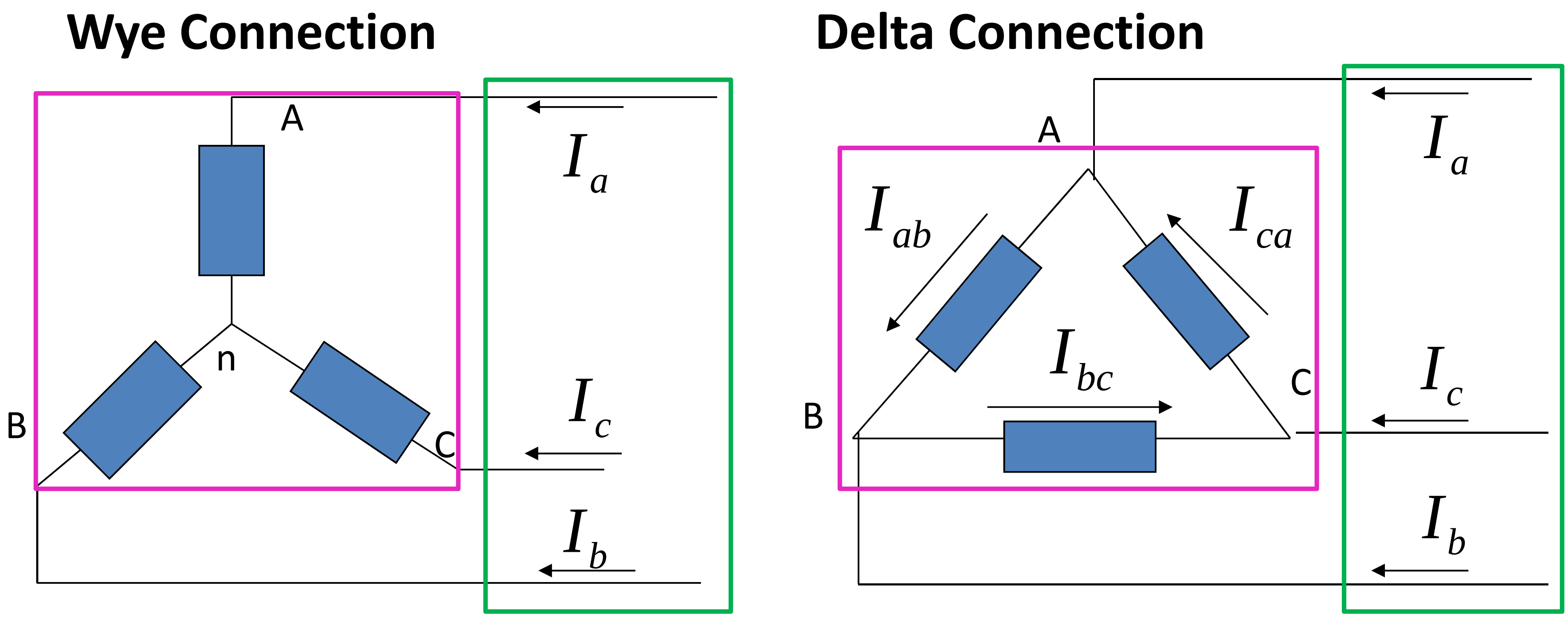


Delta Connection



These two types of connections apply to both three-phase voltage sources and three-phase loads.

Line Current VS Phase Current



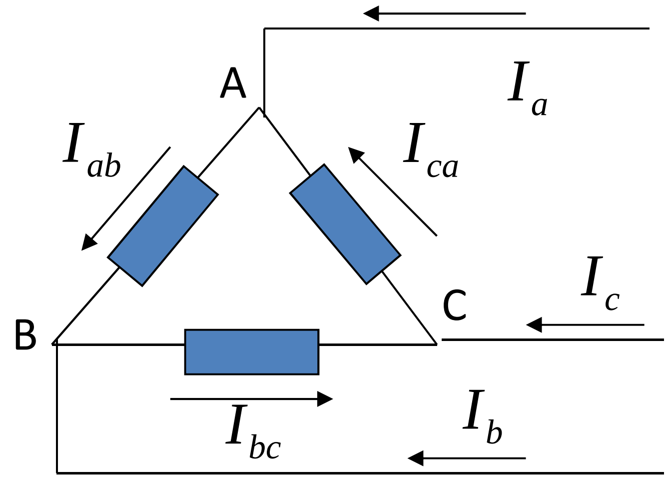
Currents through the three-phase conductor lines are called 'Line currents'.
Currents carried by the load impedance are called 'Phase currents' or 'Load Current.

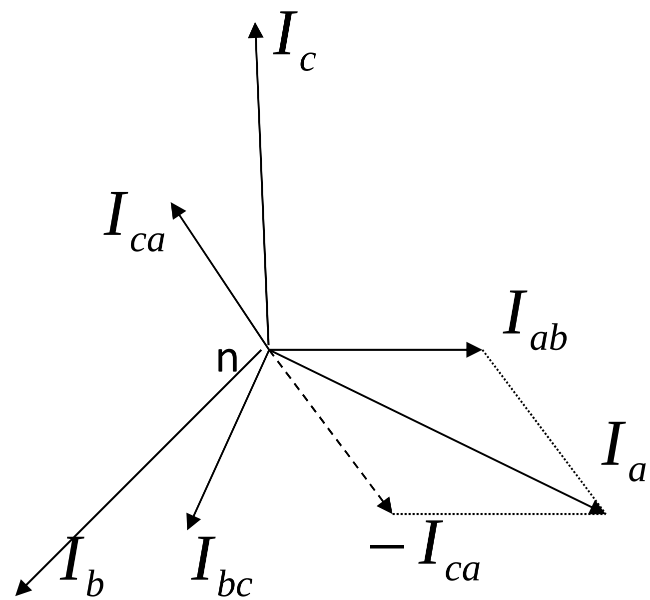
Delta-Connected Load

- I_{ab} , I_{bc} , I_{ca} are called Phase currents.
- We can find relationship between line currents and phase currents using KCL,

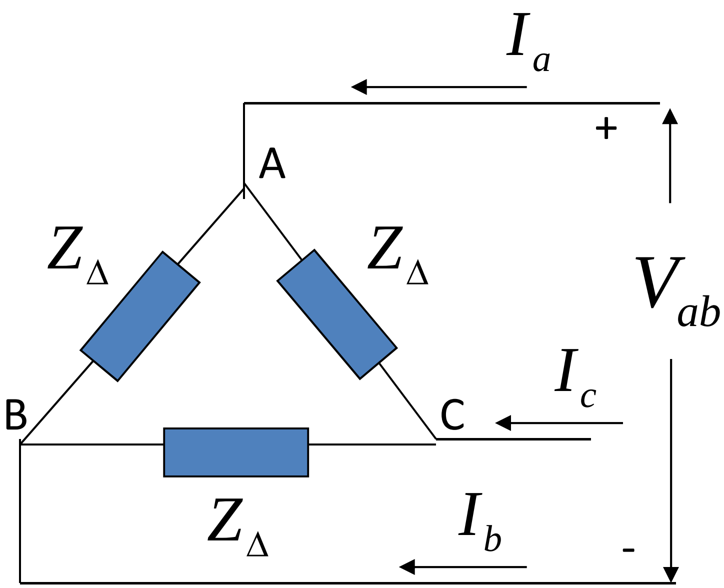
$$I_a = I_{ab} - I_{ca} = \sqrt{3} I_{ab} \angle -30°$$

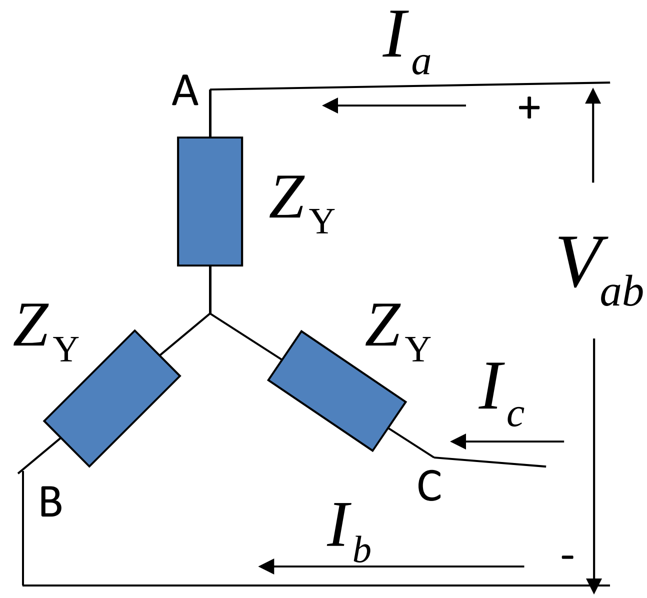
$$\boxed{\left| I_{\text{Line}} \right| = \sqrt{3} \left| I_{\text{Phase}} \right|}$$





Delta-Wye Load Transformation





$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$Z_\Delta = \frac{V_{ab}}{I_{ab}} = \frac{\sqrt{3} V_{ab}}{I_a \angle 30^\circ}$$

$$\boxed{Z_\Delta = 3 Z_Y}$$

$$Z_Y = \frac{V_{an}}{I_a} = \frac{V_{ab}}{\sqrt{3} I_a \angle 30^\circ}$$

Example 3

- For a balanced Y-connected three phase voltage source and Y-connected load system with a line voltage of 440 V and three equal resistive loads of 100 Ω per phase, assume positive sequence, what will be the magnitudes of
  - (a) the line-to-neutral voltage,
  - (b) the phase current,
  - (c) the line current?

Example 4

- For a balanced Y-connected three phase generator with the line-to-neutral voltage of 80 V, Δ-connected load of 120 Ω, assume positive sequence, find
  - (a) the line-to-line voltage,
  - (b) the voltage across a resistor,
  - (c) the current through a resistor?

Summary

- Three-phase voltage sources
  - Positive and negative sequences
- Balanced three-phase circuit
  - Conditions
  - Advantages
- Balanced three-phase circuit
  - Line-to-neutral (phase) voltage
  - Line-to-line (line) voltage
  - Line current

$$\boxed{\left|V_{\text{Line-Line}}\right| = \sqrt{3}\left|V_{\text{Line-neutral}}\right|}$$

- Wye-Delta connection
- Delta-Wye load transformation

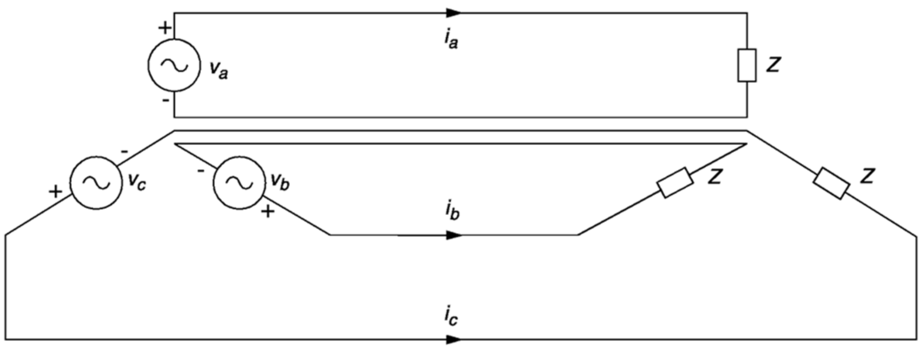
$$\boxed{Z_{\Delta} = 3Z_{\text{Y}}}$$

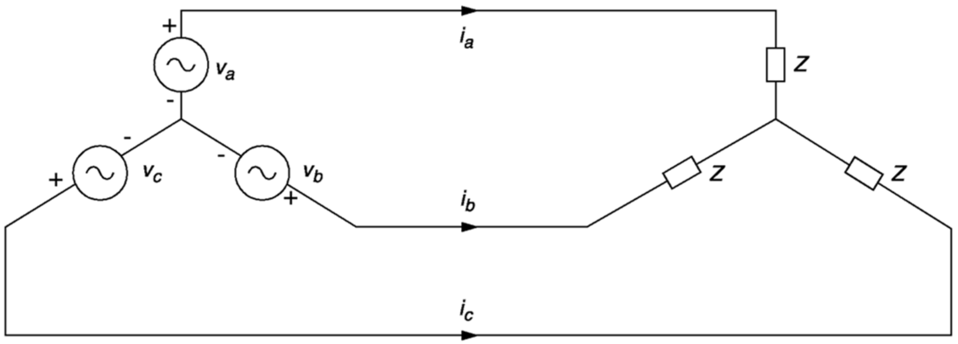
- Three-Phase Circuit Analysis
  - Three-phase complex power
  - Per Phase analysis

Three Phase Power Calculation

- Three phase power is found from summation of each phase power.

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$





Balanced Three-Phase Power

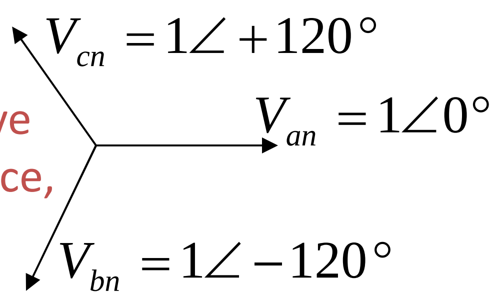
- From three phase power,

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

- When the system is balanced, (assume positive sequence) we can write,

$$S_{3\Phi} = V_{an} I_a^* + V_{an}\angle -120^\circ \left(I_a \angle -120^\circ\right)^* + V_{an}\angle 120^\circ \left(I_a \angle 120^\circ\right)^*$$

Positive sequence, abc



$$\boxed{S_{3\Phi} = 3 V_{an} I_a^*}$$

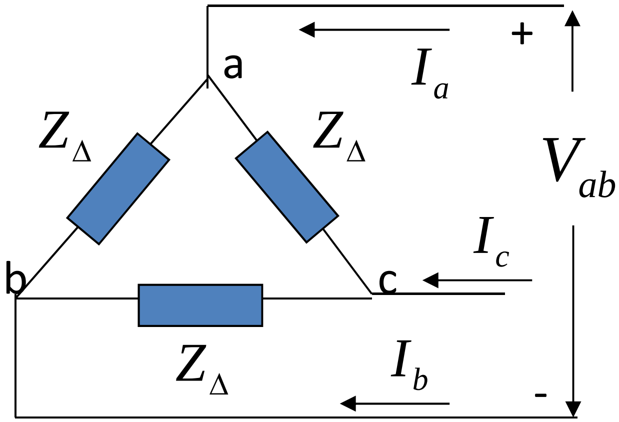
Balanced Three-Phase Load

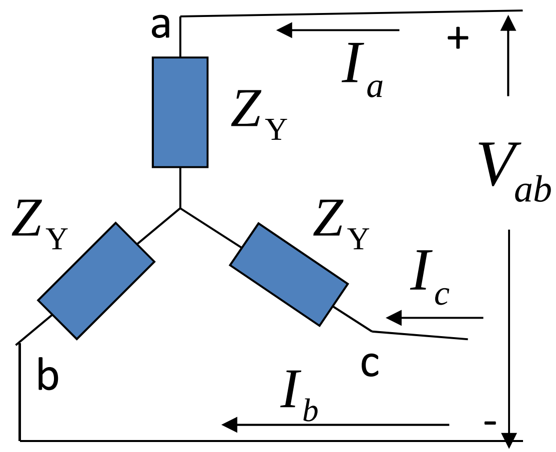
- Three-phase load can be connected in either Wye or Delta connection.
- 3-phase load parameter is given as total apparent power ($|S_{3\Phi}|$) with power factor.
- The voltage given is **Line-to-line voltage**.
- We can find three-phase real and reactive power as follows.

$$P_{3\Phi} = 3P_{1\Phi} = |S_{3\Phi}| \times \text{p.f.}$$

$$Q_{3\Phi} = 3Q_{1\Phi} = |S_{3\Phi}| \times \sin\left(\cos^{-1}(\text{p.f.})\right) = |S_{3\Phi}| \times \sin\phi$$

Delta/Wye Connected 3-Phase Load





$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

$$|S_{3\Phi}| = 3 |V_{ab} I_{ab}^*| = \sqrt{3} |V_{ab}| |I_a|$$

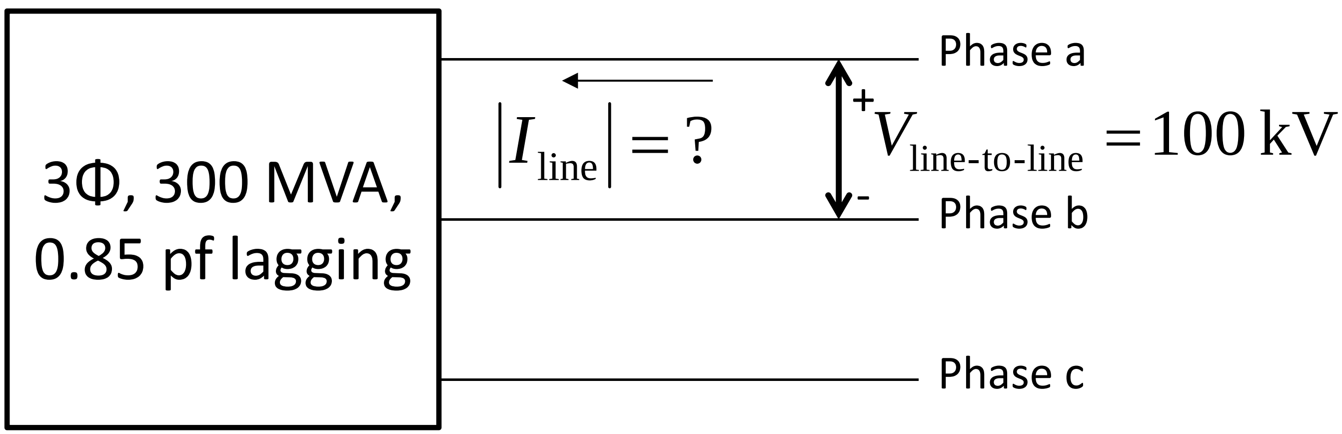
$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$|S_{3\Phi}| = 3 |V_{an} I_a^*| = \sqrt{3} |V_{ab}| |I_a|$$

$$\boxed{|S_{3\Phi}| = \sqrt{3} |V_{\text{Line-To-Line}}| |I_{\text{Line}}|}$$

Example 1

- A 3Φ load of 300 MVA, 100 kV at 0.85 p.f. lagging, find
  - The magnitude of line current $\left|I_{\text{Line}}\right|$
  - Three-phase (real) power P_{Load}



Ans: 1732 A, 255 MW.

Instantaneous Three-Phase Power

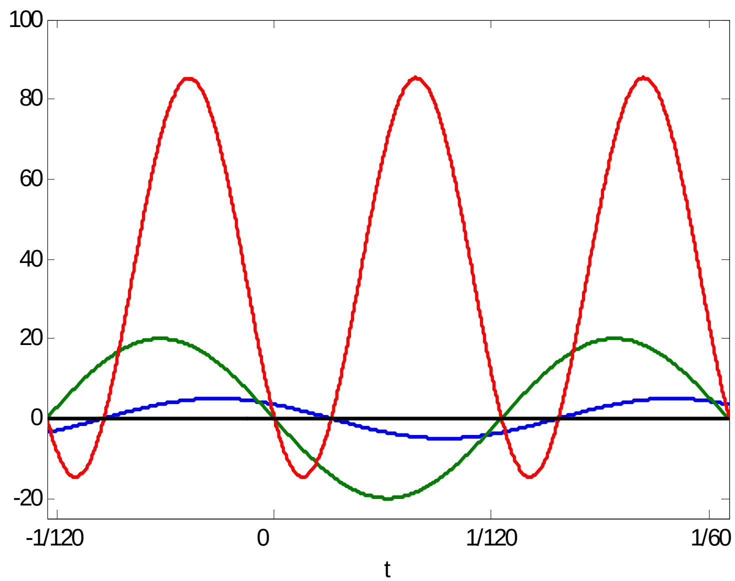
- Given by,

$$p_{3\Phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t)$$

- Recall that single phase instantaneous power,

$$v(t) = |V|\cos(\omega t + \theta_v)$$
$$i(t) = |I|\cos(\omega t + \theta_i)$$

$$p(t) = v(t) \times i(t)$$
$$= |V||I|\cos(\theta_v - \theta_i)$$
$$+ |V||I|\cos(2\omega t + \theta_v + \theta_i)$$



Instantaneous Three-Phase Power

- For a balanced three-phase system,

=0

$$\begin{aligned} p_{3\Phi}(t) = {} & |V_a||I_a|\cos(\theta_v - \theta_i) + |V_a||I_a|\cos(2\omega t + \theta_v + \theta_i) \\ & + |V_b||I_b|\cos(\theta_v - \theta_i) + |V_b||I_b|\cos(2\omega t + \theta_v + \theta_i - 240^\circ) \\ & + |V_c||I_c|\cos(\theta_v - \theta_i) + |V_c||I_c|\cos(2\omega t + \theta_v + \theta_i - 480^\circ) \end{aligned}$$

- We can find three phase instantaneous power as,

$$p_{3\Phi}(t) = 3|V_a||I_a|\cos\phi = 3P$$

- **Constant** power transfer to load.

An Additional Advantage of Balanced 3-Phase Circuit

- Constant power transfer to load.
  - This also implies constant mechanical power input for a generator.
  - When mechanical power input is constant, mechanical shaft torque is also constant.
  - This helps to reduce shaft vibration and noise, extending the machine's lifetime.

Assumption

Single-line diagram

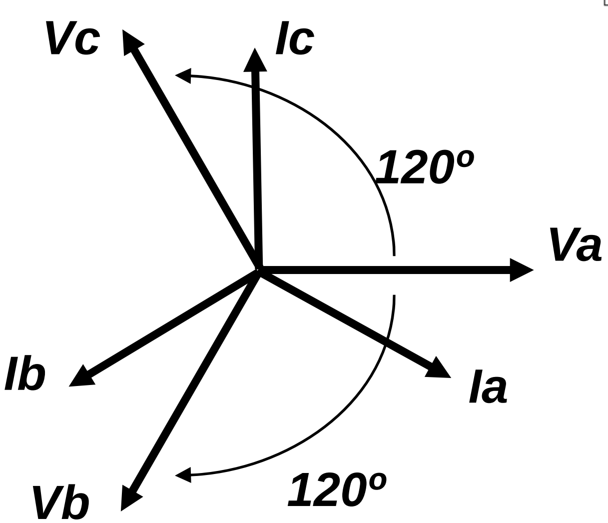
Example example example…

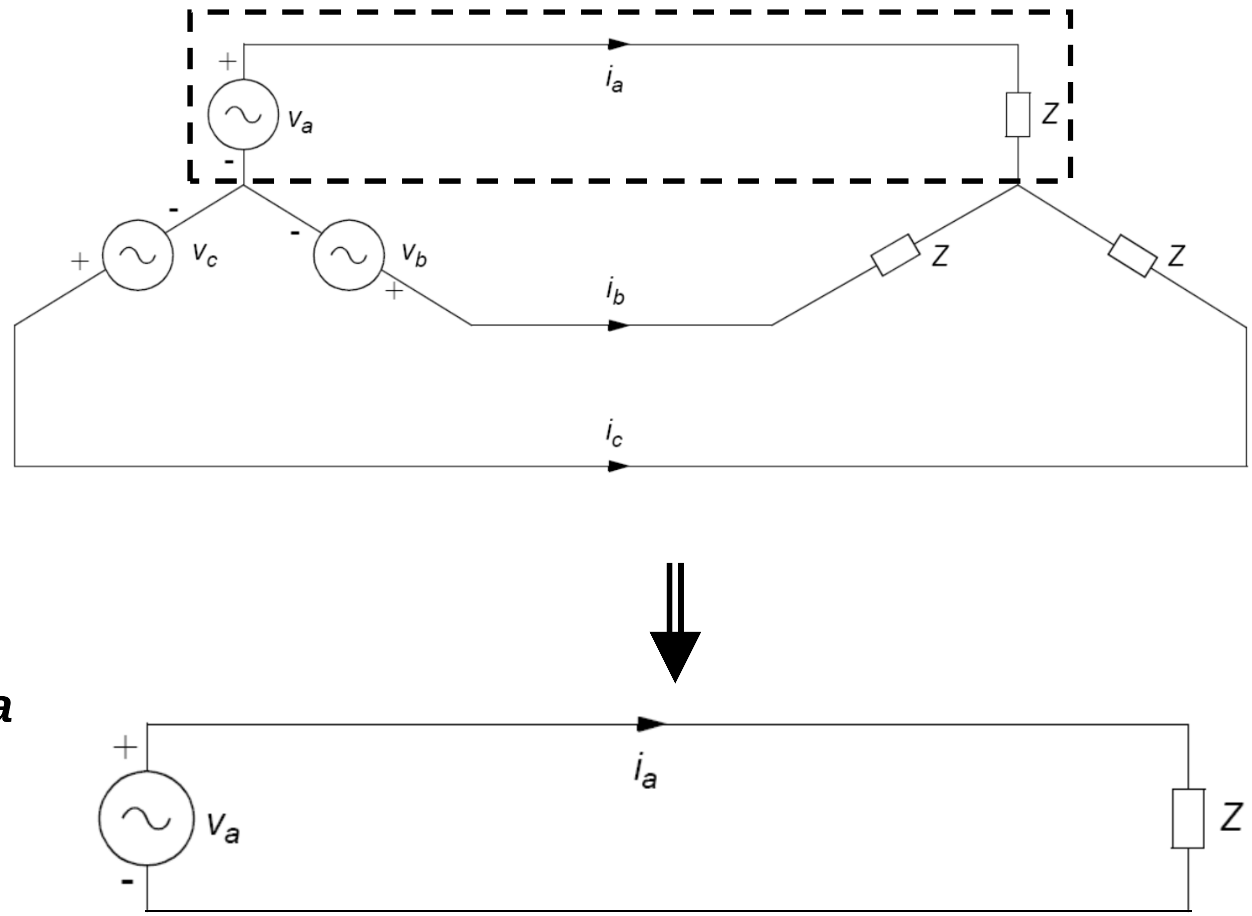
PER PHASE ANALYSIS

Per Phase Analysis: Assumption

It must be balanced three-phase circuit.

$$I_n = I_a + I_b + I_c = 0$$

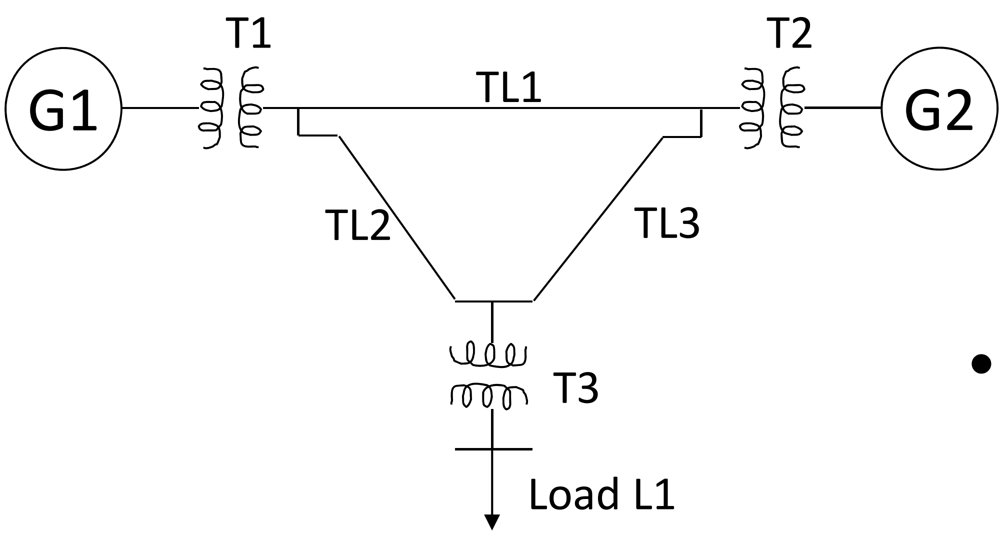




Steps of Per Phase Analysis

- Make sure that the three-phase system is balanced.
  - The three-phase sources need to have the same magnitude with 120 degree phase difference.
  - The three-phase impedances must be of the same value (both phase and magnitude).
- **Convert** all Delta-connected sources/loads to Wye-connected sources/loads.
- Per phase analysis reduce three-phase circuit to single-phase circuit. We can apply the same concept used in single-phase.

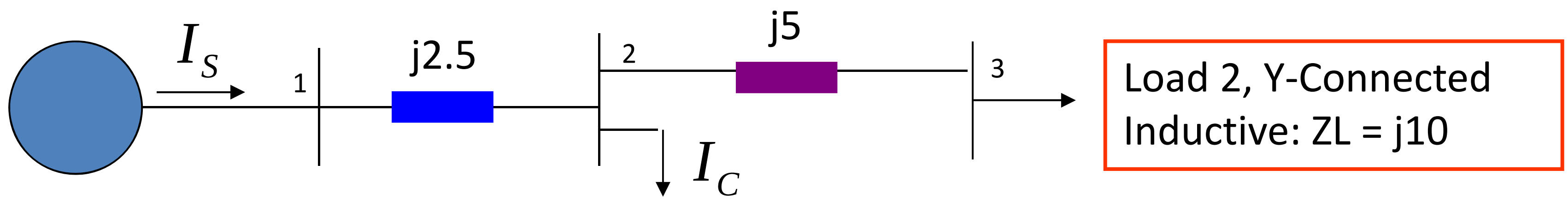
Single-Line Diagram



- Show the interconnections of a transmission system
  - Generator
  - Load
  - Transmission line
  - Transformer
- This is a representation of a 3Φ circuit. Each line represents three conductors in three-phase system.

Example 2

- Given a one-line diagram,



Source,Δ-Connected

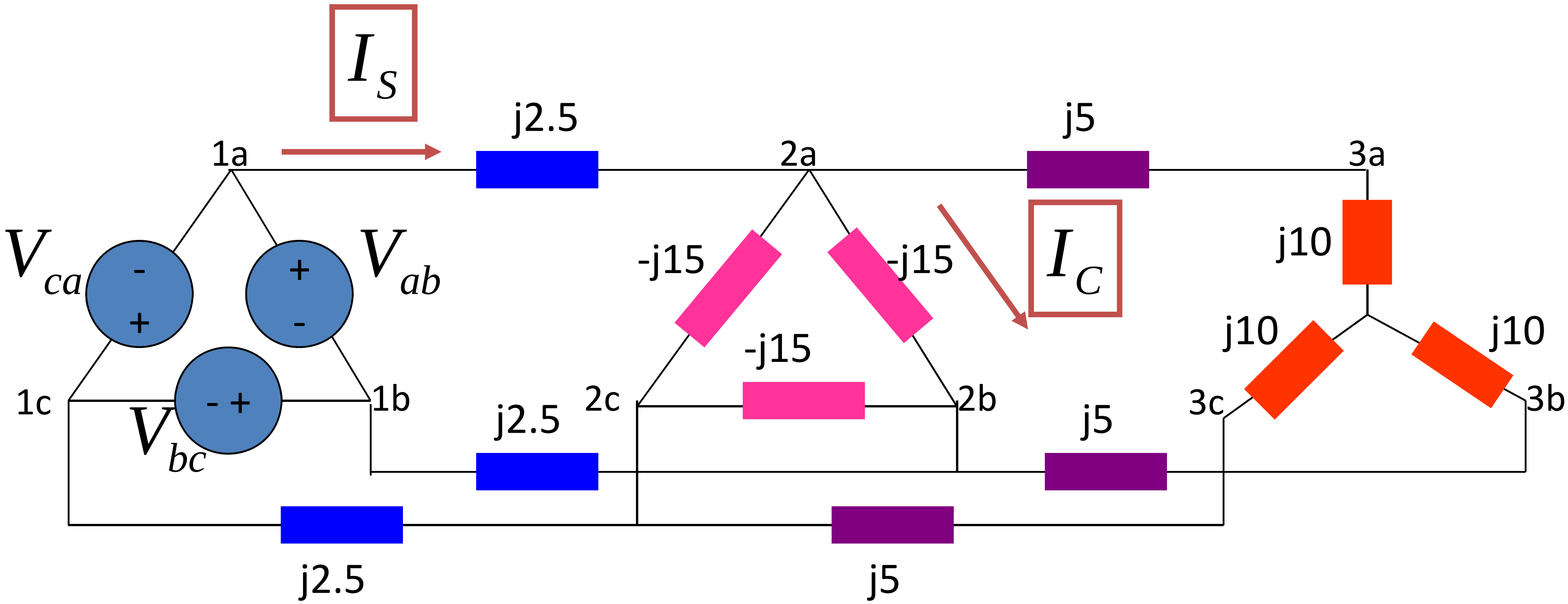
Positive sequence.

Load 1, Δ-Connected
Capacitor: ZC = -j15

If the voltage source is $\left|V_{\text{Line-Line}}\right| = \sqrt{3}\ \text{V}$. Find,

1. Current magnitude supplied by source, $|I_S|$, and,
2. Current magnitude through a capacitor, $|I_C|$.

Example 2: 3Φ Circuit Diagram



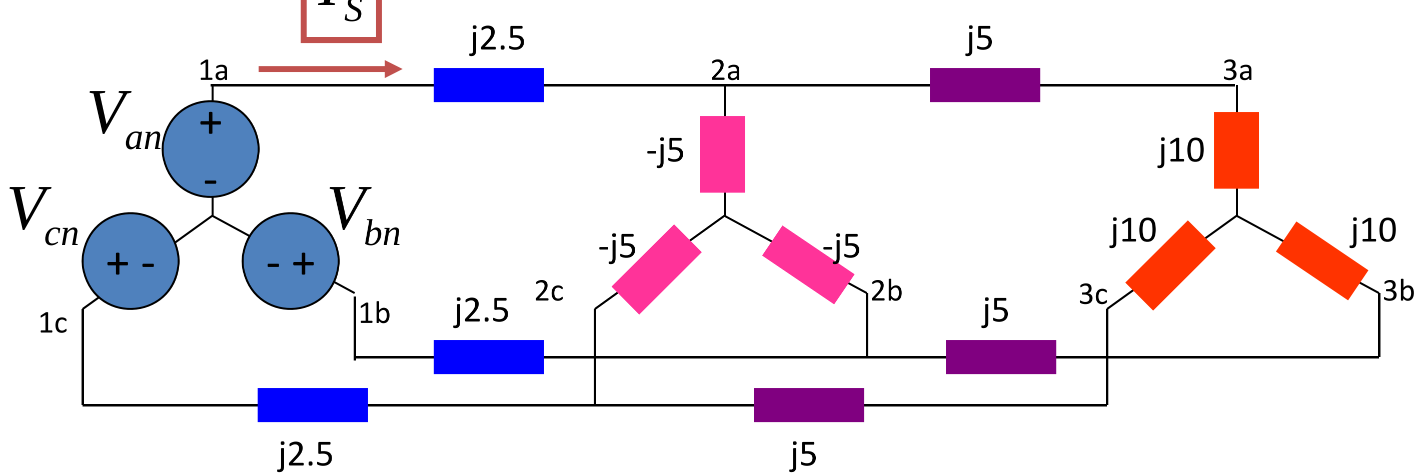
Example 2: Convert from Δ → Y

$$Z_Y = \frac{Z_\Delta}{3} = \frac{-j15}{3} = -j5$$

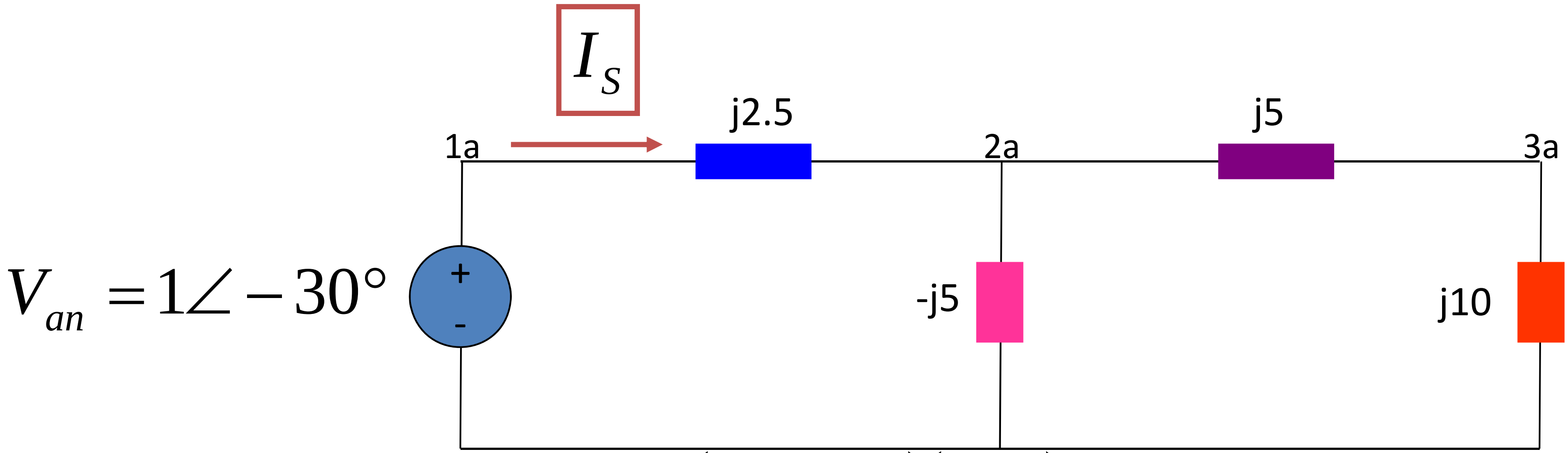
Use voltage source as angle reference

$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30° = \frac{\sqrt{3}\angle 0°}{\sqrt{3}}\angle -30° = 1\angle -30°$$

I_S



Example 2: 1-Phase diagram

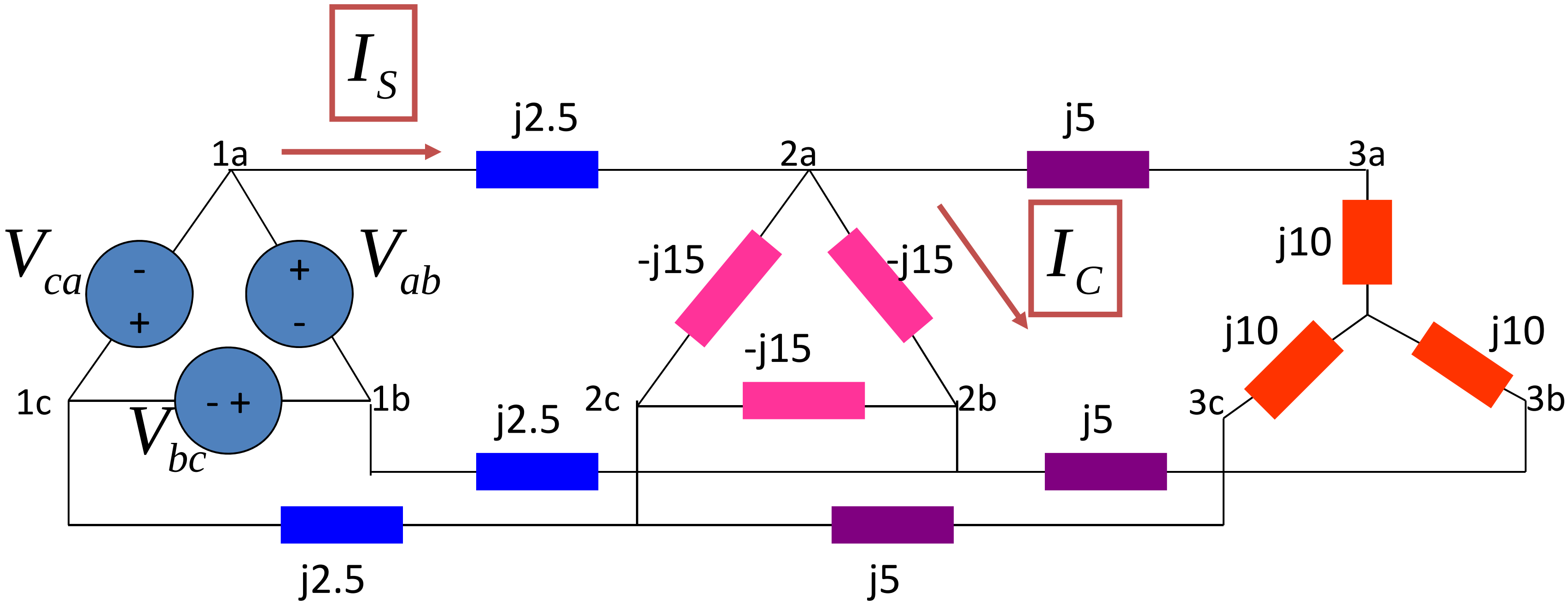


$$Z_{eq} = j2.5 + \frac{(j10 + j5)(-j5)}{(j10 + j5) + (-j5)} = -j5$$

$$I_S = \frac{V_{an}}{Z_{eq}} = \frac{1\angle -30°}{-j5} = \frac{1\angle -30°}{5\angle -90°} = 0.2\angle 60° \text{ A}$$

$V_{2a} = V_{an} - j2.5 \times I_S = 1.5\angle -30°$ We will use this to find $\boxed{I_C}$

Example 2: Final Calculation



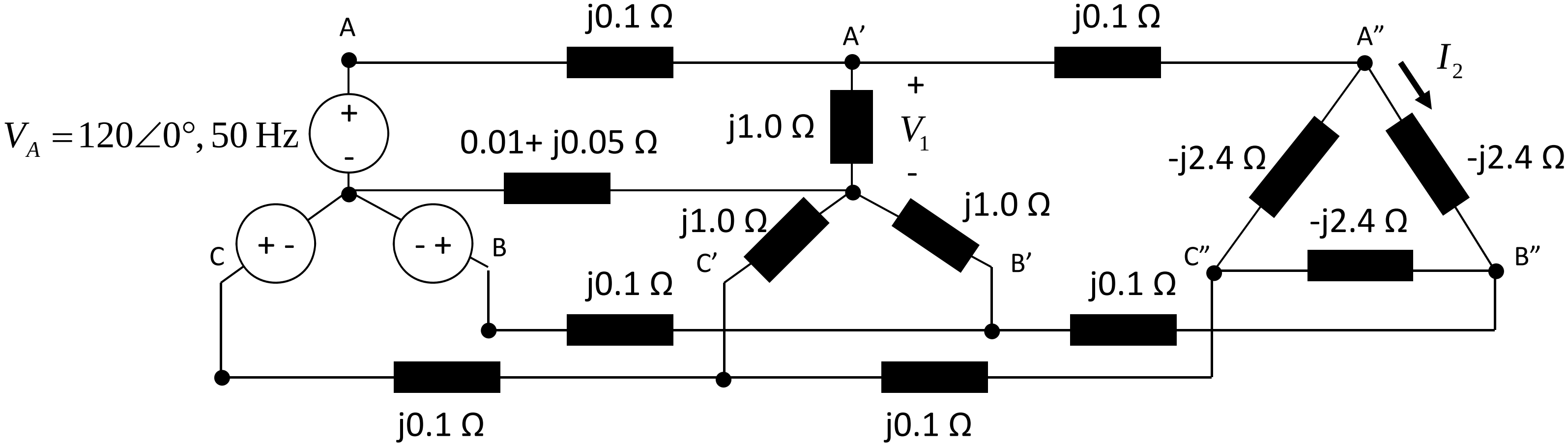
$$V_{2b} = V_{2a}\angle -120°$$

$$I_C = \frac{V_{2a} - V_{2b}}{-j15} = \frac{1.5\angle -30° - 1.5\angle(-30° - 120°)}{15\angle -90°} = \frac{\sqrt{3}}{10}\angle 90°\ \text{A}$$

Ans: $I_S = 0.2\angle 60°, I_C = 0.1732\angle 90°$

Practice Problem 1

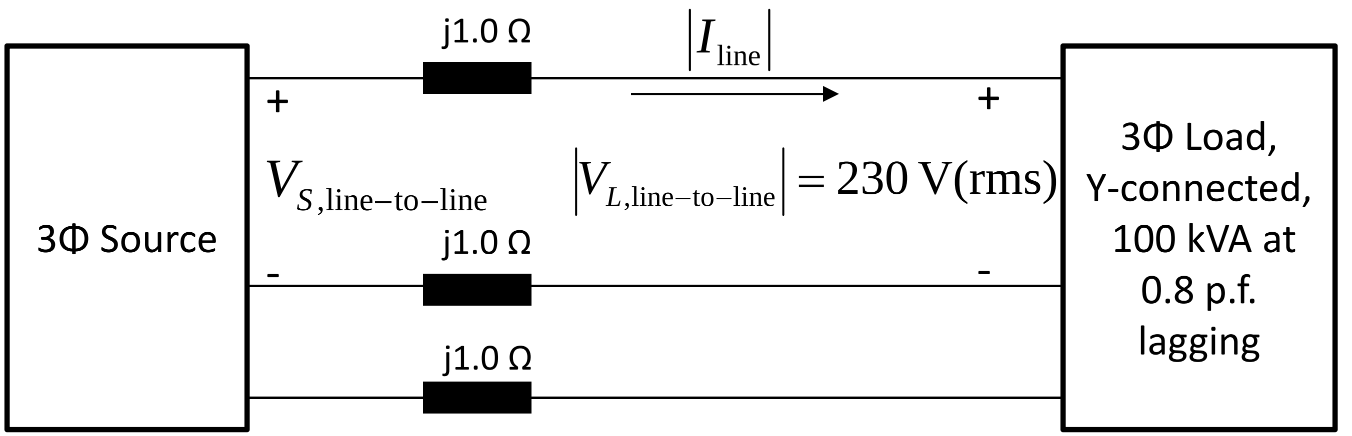
- (Final EE2022 AY2011/12 semester 1) Find the voltage V1 and current I2.
  - *Hint: remove the impedances that are in neutral line (why can we do this?) and transform delta to wye connection.*



Ans: $V_1 = 125.37\angle 0°, I_2 = 103.41\angle 120°$

Practice Problem 2

- (Final EE2022 AY2011/12 semester 1) A balanced three-phase voltage source feeds a balanced three-phase load shown below, find $\left|V_{S,\text{line-to-line}}\right|$.
  - *Hint: Assume that line-to-line voltage at the load has reference angle of 0 degree. Then, find line current magnitude and angle at the load. Calculate line-to-neutral voltage from source using V_source(line-to-neutral) = I_line×j1.0 + V_load(line-to-neutral). Then, use relationship between line-to-line voltage and line-to-neutral voltage to find V_source(line-to-line).*



Ans: 601.61 V

Summary

- Three-phase complex power,

$$\boxed{\left|S_{3\Phi}\right| = \sqrt{3}\left|V_{\text{Line-To-Line}}\right|\left|I_{\text{Line}}\right|}$$

- Per phase analysis
  - Only applied to balanced three-phase circuit.
  - Need to **convert** all Delta-connected load/sources to Wye-connected load/sources.
  - Same analysis as single-phase circuit.