

# NATIONAL UNIVERSITY OF SINGAPORE

Department of Electrical Engineering

## EE2022 ELECTRICAL ENERGY SYSTEMS

### (Tutorial #1)

1. Perform the following addition using phasors, and express the result in time domain.

(a)  $v_1(t) = 40\sin 628t + 60\cos(628t - 45^\circ) + 30\cos 628t$

(b)  $v_2(t) = 20\sin 314t + 10\cos(314t + 60^\circ) - 5\sin(314t - 20^\circ)$

Ans: (a)  $109.72\cos(628t - 48.69^\circ)$  volts; (b)  $9.44\cos(314t - 44.71^\circ)$  volts

2. A source delivers a current of  $i(t) = I_m\cos(314.16t + \beta)$  to a circuit consisting of a resistor of  $40\Omega$  and a capacitor of  $137.83\mu\text{F}$  connected in parallel. If the source voltage is  $v(t) = 282.84\cos(314.16t + 10^\circ)$ , determine the currents  $i_1(t)$  and  $i_2(t)$  that flow through the resistor and the capacitor respectively. Express the source current in  $i(t)$  in time domain.

Ans:  $i(t) = 14.14\cos(314.16t + 70^\circ)$

3. A current  $i(t) = 2\sqrt{2}\cos(5000t + 30^\circ)\text{mA}$  flows in a series circuit of  $R=2309\Omega$  and  $L=0.8\text{H}$ .

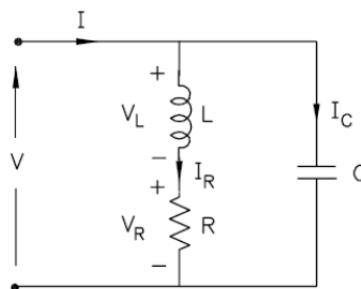
(a) Determine the impedance  $Z$  of the series combination.

(b) Determine the voltage  $v(t)$  across the series combination.

(c) Show the voltage and current phasors on a phasor diagram.

Ans:  $V=9.24\sqrt{2}\cos(5000t + 90^\circ)\text{V}$

4. In the following figure, inductance  $L = 2\text{H}$ , resistance  $R = 3\Omega$ , capacitor  $C = 0.2\text{F}$  and  $v_R = 6\sqrt{2}\cos 2t$  volts. Determine the phasors  $I_R$ ,  $V_L$  and  $I$  show them on a phasor diagram.



Ans:  $V = 10\angle 53.13^\circ\text{V}$ ;  $I = 2.68\angle 116.57^\circ\text{amps}$

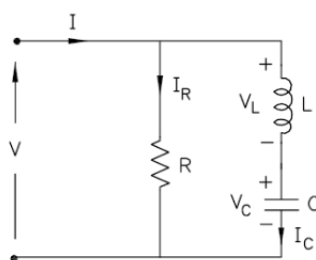
5. In the following circuit,  $R = 2\Omega$ ,  $L = 3.25\text{mH}$  and  $C = 100\mu\text{F}$ . The steady state voltage across  $C$  is  $v_C = 100\sqrt{2}\cos(2000t - 90^\circ)$  volts.

(a) Determine  $V_C$ ,  $I_C$  and  $V_L$

(b) Determine  $I_R$  and  $I$

(c) Draw a phasor diagram showing all the phasors

(d) Determine  $i(t)$ .



Ans:  $V_L = j130$  V,  $I = 25 \angle 36.9^\circ$  amps;

$$(d) i(t) = 25 \sqrt{2} \cos(2000t + 36.9^\circ) \text{ A}$$

6. A coil of resistance  $5\Omega$  and inductance  $0.6\text{H}$  is connected in series with a capacitance of  $10\mu\text{F}$ . If a variable frequency source of  $200\text{V}$  is applied to this circuit, determine the frequency for which the current flow in the circuit will be maximum.

Under this condition, determine the voltage across the inductance and capacitance, and compare them with that of the source.

Ans: 48.99

7. A purely resistive lamp is rated at  $200\text{V}$  (rms),  $120\text{W}$  (i.e., it dissipates  $120\text{W}$  power when connected to a  $200\text{V}$  (rms) AC source).
- What is the RMS value of current drawn from the power supply?
  - Calculate the resistance of the lamp.
  - We want to connect this lamp across a supply voltage of  $240\text{V}$  (rms),  $50\text{Hz}$ . In order to keep the voltage across the lamp to the rated value of  $200\text{V}$ , a capacitor is connected in series with the lamp. What is the value of the capacitor to be used?

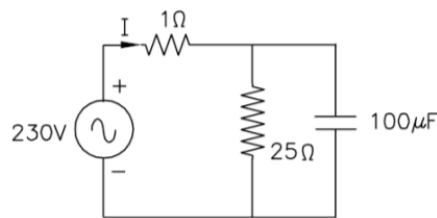
Ans:  $0.6\text{A}$ ;  $333.33\text{ ohms}$ ;  $14.4\mu\text{F}$

# NATIONAL UNIVERSITY OF SINGAPORE

## Department of Electrical Engineering

### EE2022 ELECTRICAL ENERGY SYSTEMS (Tutorial #2)

1. A load consists of a  $25\ \Omega$  resistor connected in parallel with a  $100\ \mu\text{F}$  capacitor. Power is delivered to the load over a short transmission line of resistance  $1\ \Omega$  from a 230 volts source as shown below. Find the apparent power delivered to the load and power factor of the load. Also determine the apparent power supplied by the source. Assume  $\omega = 377\ \text{rad/s}$ .

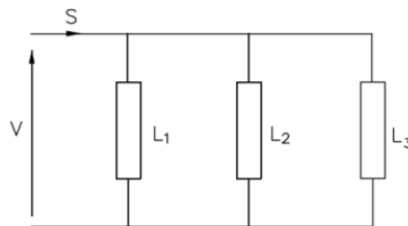


Ans:  $|S_{\text{load}}| = 2686\ \text{VA}$ ;  $|S_{\text{source}}| = 2794\ \text{VA}$

2. A load connected across a 500 V, 50Hz line draws 25 kW at 0.5 power factor lagging. Determine the current drawn by the load. A capacitor C is now connected in parallel with the load to improve the power factor. What must be the value of C to make the overall power factor 0.95 lagging?

Ans:  $448.2\ \mu\text{F}$

3. The power consumption of loads  $L_1$ ,  $L_2$  and  $L_3$  connected in parallel as shown below, is respectively 5 kW at 0.8 p.f. lagging, 10 kW at 0.6 p.f. lagging and 15 kW at 0.8 p.f. leading. Determine the total complex power delivered by the source. What is the power factor of the circuit as seen by the source?



Ans: 0.982 lagging

4. A load connected across a 200 V, 50Hz line draws 10 kW at 0.5 power factor lagging. Determine the current delivered by the source. If a capacitor of  $1000\ \mu\text{F}$  is connected in parallel with the load, what will be the current drawn from the source? Also, determine the overall power factor of the system (load and the capacitor connected in parallel) as seen by the source.

Ans: 0.902 lagging

5. An industry draws 1500 kW of power at 500V from a substation over a transmission line having a resistance of  $0.005\Omega$  and a reactance of  $0.01\Omega$ . Determine the current flow from the substation if the power factor of the industrial load is 0.6 lagging. What would be the current flow if the power factor could be improved to unity? Compare the transmission line losses in both the cases. Ignore the voltage drop across the transmission line.

Ans:  $|I_1| = 5000 \text{ A}$ ;  $|I_2| = 3000 \text{ A}$

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**EE2022 ELECTRICAL ENERGY SYSTEMS**  
**(Tutorial #3 Three-Phase Circuit Analysis)**

Tutors will discuss problem 6 to 9.

Problems 1 to 5 are meant for you to practice on your own. Solving these problems will help you to check your understanding of the basic concepts. You are advised to work on all problems before the tutorial class.

1. A balanced three-phase Y load has one phase voltage of  $V_{cn}=277 \angle 45^\circ$  V. If the phase sequence is negative sequence i.e. acb, find the line voltages  $V_{ca}$ ,  $V_{ab}$ , and  $V_{bc}$ .

(Answer:  $V_{ca}=480 \angle 15^\circ$  V,  $V_{ab}=480 \angle 135^\circ$  V, and  $V_{bc}=480 \angle -105^\circ$  V)

2. What are the phase voltages for a balanced three-phase Y load, if  $V_{ba}=12470 \angle -35^\circ$  V? The phase sequence is positive sequence i.e. abc.

(Answer:  $V_{bn}=7200 \angle -5^\circ$  V,  $V_{an}=7200 \angle 115^\circ$  V, and  $V_{cn}=7200 \angle -125^\circ$  V)

3. A balanced Y load of  $40 \Omega$  resistors is connected to a 480 V, three-phase, three-wire source. Find the rms line current.

(Answer: 6.93 A)

4. In a three-phase, three-wire circuit, find the phasor line currents to a balanced Y load for which  $Z_Y=60 \angle -30^\circ \Omega$  and  $V_{cb}=480 \angle 65^\circ$  V. The phase sequence is abc.

(Answer:  $I_a=4.62 \angle 5^\circ$  A,  $I_b=4.62 \angle -115^\circ$  A, and  $I_c=4.62 \angle 125^\circ$  A)

5. Calculate the total average power delivered by a three-phase source with the line to line voltage of 500 V to each of the following balanced Y connected loads with  $Z_L$  equal to:

- a)  $(30+j0) \Omega$ ;
- b)  $(30+j72) \Omega$ ;
- c)  $(30-j12.5) \Omega$ .

(Answer: (a) 8333.33 W, (b) 1232.75 W, (c) 7100.58 W)

6. Find the rms line voltage ( $V_{Line-Line}$ ) at the source of the circuit in Fig. 1. As shown, rms phase voltage is 100V and each line impedance is  $2+j3\ \Omega$ .

(Answer: 173 V)

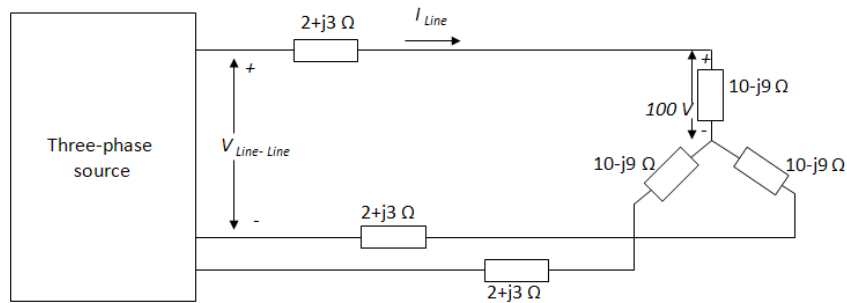


Fig. 1

7. A 208-V three-phase circuit has two balanced loads, one a  $\Delta$ -connected load of  $21\angle 30^\circ\ \Omega$  impedances and the other a Y-connected load of  $9\angle -60^\circ\ \Omega$  impedances. Find the total rms line current and also the total average power absorbed by the two loads.

(Answer: 21.7 A, 7744.15 W)

8. In a 208-V three-phase circuit a balanced  $\Delta$  load absorbs 2 kW at a 0.8 leading power factor. Find  $Z_\Delta$ .

(Answer:  $51.9\angle -36.87^\circ\ \Omega$ )

9. Two balanced three-phase motor loads comprising of an induction motor and a synchronous motor are connected in parallel. An induction motor draws 400 kW at 0.8 power factor lagging and a synchronous motor draws 150 kVA at 0.9 power factor leading. Both motor loads are supplied by a balanced three-phase 4160 V source. If the cable impedance between the source and load is neglected,

- Draw the power triangle for each motor and for the combined-motor load.
- Determine the power factor of the combined-motor load.
- Determine the magnitude of the line current delivered by the source.
- A delta connected capacitor bank is now installed in parallel with the combined-motor load. What value of capacitive reactance is required in each phase of the capacitor bank to make the source power factor unity?
- Determine the magnitude of the line current delivered by the source with the capacitor bank installed.

(Answer: (b) 0.916 lagging, (c) 81.1 A, (d)  $-j221.3\ \Omega$ , (e) 74.3 A)

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**EE2022 ELECTRICAL ENERGY SYSTEMS**  
**(Tutorial #4 Electric Power Transmission)**

Tutors will discuss problem 4 to 6.

Problems 1 to 4 are meant for you to practice on your own. Solving these problems will help you to check your understanding of the basic concepts. You are advised to work on all problems before the tutorial class.

1. What are the advantages and disadvantages of both overhead transmission lines and underground cables?
2. Consider a three-phase transmission line operating at the sending end voltage of 500kV. The series impedance is  $z = 0.045 + j0.4 \Omega$  per phase per km and the shunt admittance is  $y = j4 \times 10^{-6}$  Siemens per phase per km. For the following cases, evaluate transmission matrix for short length, medium length and long length models, and find the receiving end voltage (per phase) at no load using all three models. What can you conclude from this exercise?
  - (a) When the length of the transmission line is 50 km
  - (b) When the length of the transmission line is 100 km
  - (c) When the length of the transmission line is 250 km

Answer: Per phase receiving end voltage at no load (kV)

	short length model	medium length model	long length model
$l=50$ km	$288.7 \angle 0^\circ$	$289.28 \angle -0.01^\circ$	$289.3 \angle -0.01^\circ$
$l=100$ km	$288.7 \angle 0^\circ$	$291 \angle -0.052^\circ$	$291.03 \angle -0.05^\circ$
$l=250$ km	$288.7 \angle 0^\circ$	$303.9 \angle -0.34^\circ$	$303.76 \angle -0.33^\circ$

3. A three phase 765 kV, 60 Hz, 300 km line has the following positive sequence series impedance and shunt admittance;  $z = 0.0165 + j0.3306 \Omega/\text{km}$  and  $y = j4.674 \times 10^{-6} \text{ S/km}$ . Calculate ABCD parameters in a nominal  $\Pi$  circuit.

(Answer:  $A = D = 0.9305 + j0.0035$ ,  $B = 4.95 + j99.18\Omega$ ,  $C = (-2.4 \times 10^{-6} + j0.0014) \text{ S}$ )

4. A 69 kV three-phase short transmission line is 16 km long. The line has a per phase series impedance of  $0.125 + j0.4375 \Omega/\text{km}$ . Determine the sending end voltage per phase, voltage regulation, and the transmission efficiency when the line delivers 70 MVA, 0.8 lagging power factor at 64 kV.

(Answer: 40.71 kV, 10.17%, and 95.91%)

5. A 200 km, 230 kV, 60 Hz three phase line has a per phase series impedance,  $z = 0.08 + j0.48 \Omega/\text{km}$  and a per phase shunt admittance  $y = j3.33 \times 10^{-6} \text{ S/km}$ . At full load, the line delivers 250 MW at 0.99 p.f. lagging and at 220 kV. Using the nominal  $\Pi$  circuit, find sending end voltage and current per phase.

(Answer:  $155.40 \angle 23.58^\circ \text{ kV}$ ,  $635.38 \angle -0.34^\circ \text{ A}$ )

6. A 345 kV, 50 Hz, three-phase transmission line is 130 km long. The resistance per phase is  $0.036 \Omega/\text{km}$  and the inductance per phase is  $0.8 \text{ mH/km}$ . The shunt capacitance is  $0.0112 \mu\text{F/km}$  where shunt conductance is negligible. The receiving end load is 270 MVA with 0.8 lagging power factor at 325 kV. Use nominal  $\Pi$  model, determine the voltage regulation and transmission line efficiency. If the power factor is corrected to 0.95 lagging, keeping the receiving end MVA constant, what will be the new voltage regulation and transmission line efficiency? What can you conclude from this problem?

(Answer: 6.19%, 98.7%, 4.06%, 98.8%)