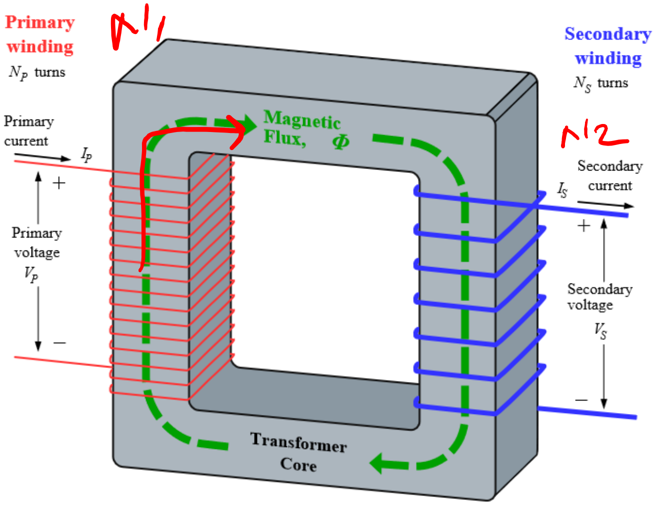
EE2022 Electrical Energy Systems

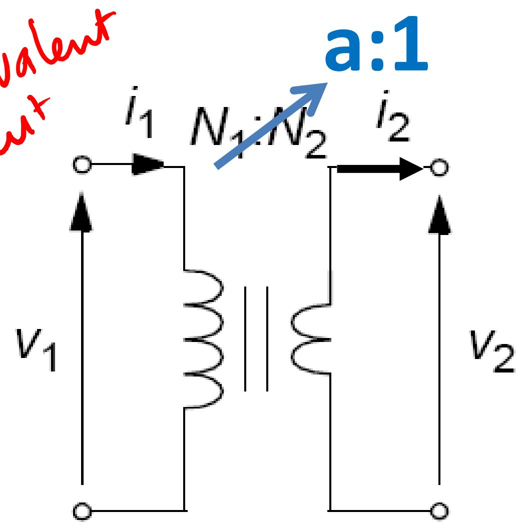
Principle of Transformers

Lecturer : Dr. Sangit Sasidhar (elesang)
Slides prepared by Dr. Panida Jirutitijaroen
Department of Electrical and Computer Engineering

Transformers- Review



Equivalent circuit



- Define turns ratio as:

$$a \equiv \frac{N_1}{N_2}$$

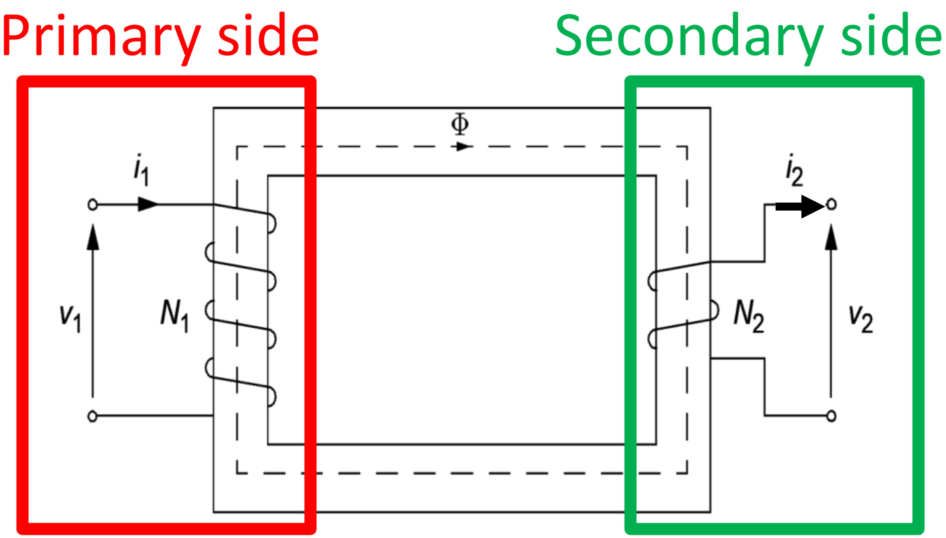
- From Faraday's and Ampere's Law:

$$V_1 = \left(\frac{N_1}{N_2}\right) V_2 = aV_2$$

$$i_2 = \left(\frac{N_1}{N_2}\right) i_1 = ai_1$$

Assumptions

1. No resistance in both windings.
2. No leakage flux around the core.
3. No core resistive loss.
4. Core permeability is infinite.



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$i_1 N_1 = i_2 N_2$$

$\frac{B}{\mu}$ l_{path} =

$\rightarrow \infty$

Outline

- Application of transformers
  – Reflected load
  – Impedance matching for maximum power transfer.
- Practical transformers
  – An equivalent circuit
  – A simplified equivalent circuit
- Transformer parameter tests
  – Short-circuit test
  – Open-circuit test

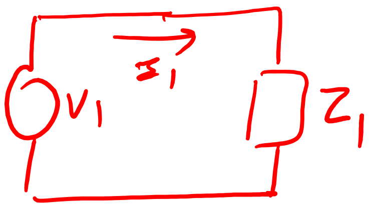
Reflected load

Impedance matching

APPLICATION OF TRANSFORMERS

Reflected Load

- We can reflect a load from one side of a transformer to the other side of the transformer.

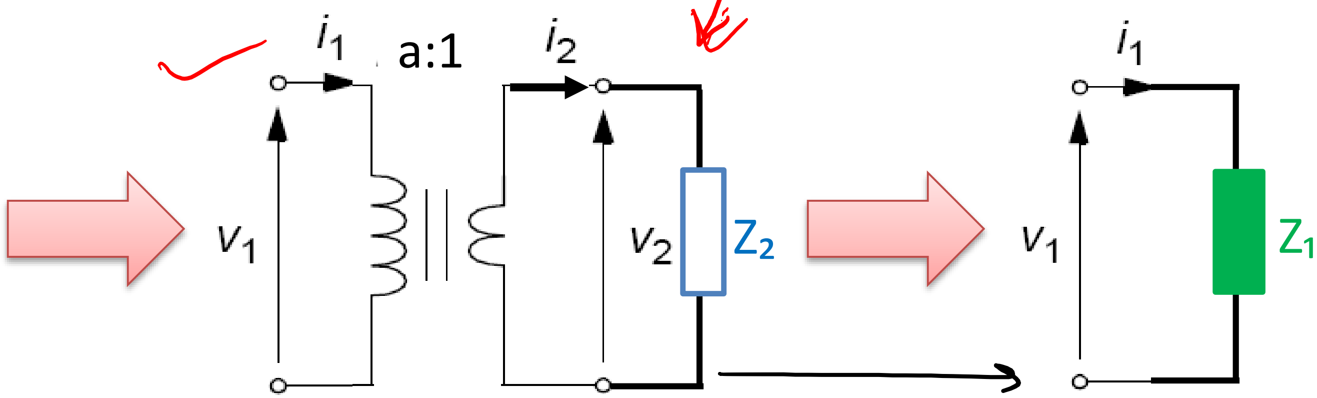


$$V_1 = aV_2$$
$$I_1 = \frac{I_2}{a}$$

$$Z_1 = \frac{V_1}{I_1} = \frac{aV_2}{\frac{I_2}{a}} = \frac{V_2}{I_2}\cdot a^2 \rightarrow a^2 Z_2$$

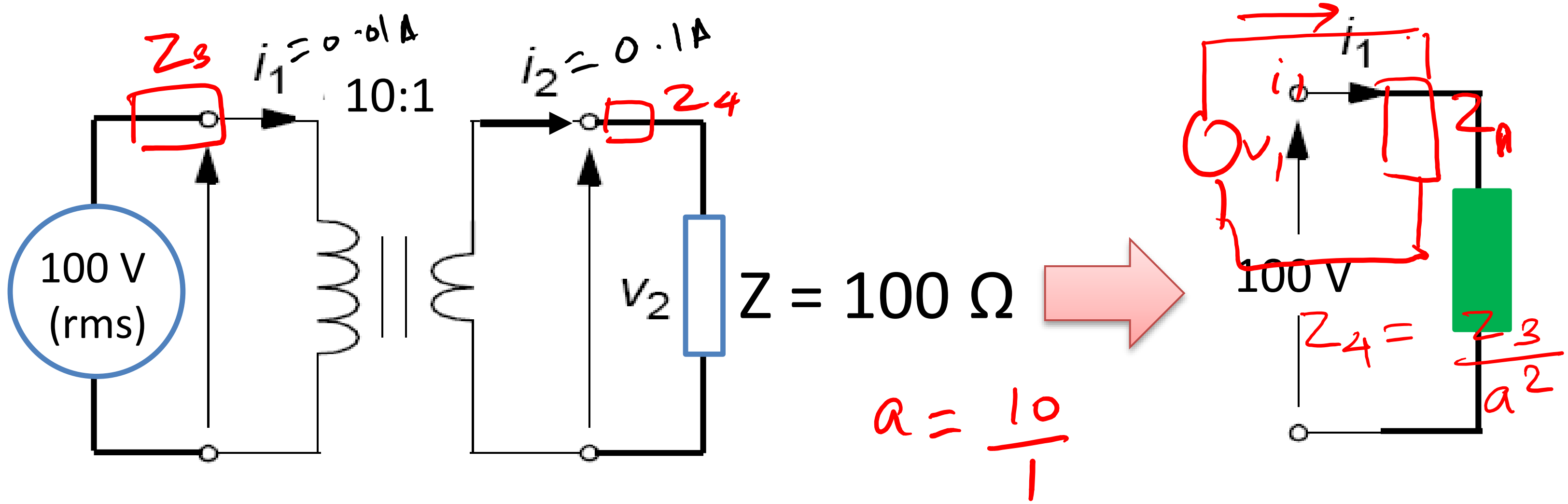
$$\boxed{Z_1 = a^2 Z_2}$$

Interest to find reflected load "Z_1"



Example 1

- Find the reflected load of impedance 100 Ω seen from the primary side of the transformer.



$Z_2 = 100\,\Omega \Rightarrow Z_1 = a^2 Z_2$

$= 100 \times 100$

$= 10{,}000\,\Omega$

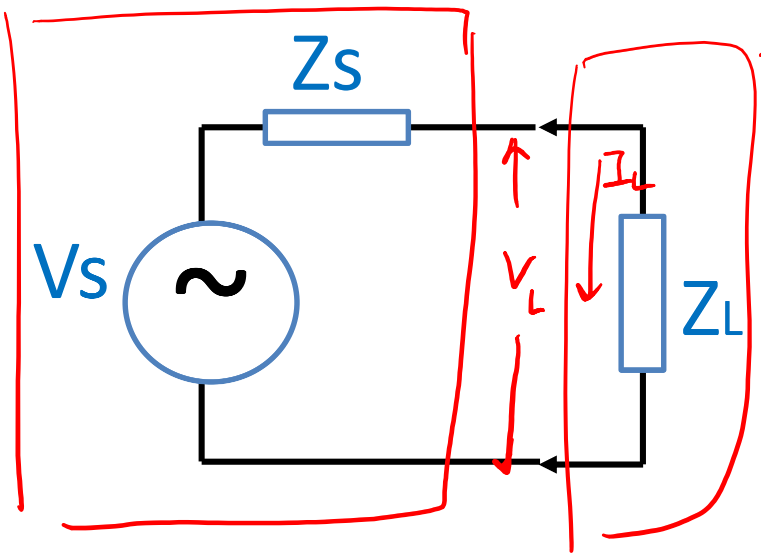
$V_1 = 100\,V$

$Z_1 = 10{,}000\,\Omega \rightarrow I_1 = \frac{100}{10000} = .01\,A$

Impedance Matching

MP3 player

Headphones



- Under what condition that the **maximum power** be *transferred to the load ZL*?

$$V_L = \frac{Z_L}{Z_L+Z_S} \cdot V_S, \quad I_L = \frac{V_S}{Z_L+Z_S}$$

$$S_L = V_L I_L^* = \frac{V_S Z_L}{(Z_L+Z_S)} \cdot \left[\frac{V_S}{(Z_L+Z_S)}\right]^* = \frac{|V_S|^2}{|Z_L+Z_S|^2} \cdot Z_L$$

$$P_L = Re\left(\frac{|V_S|^2}{|Z_L+Z_S|^2} Z_L\right) = \frac{|V_S|^2}{|Z_L+Z_S|^2} \cdot \boxed{R_L}$$

Maximum Power Transfer Theorem

- From real power expression at the load,

$$P_L = \frac{|V_S|^2}{|Z_S + Z_L|^2}\mathrm{Re}\{Z_L\}$$

$Z_L = R_L + jX_L$

$Z_S = R_S + jX_S$

- This means that the maximum power will occur when the denominator is minimum.

$|Z_L + Z_S|^2$ has to be minimum

$\boxed{\mathrm{Im}(Z_L) = -\mathrm{Im}(Z_S)}$

$\longrightarrow |R_L + R_S|^2$ has to be minimum

Maximum Power Transfer Theorem

$$\frac{d}{dR_L}\left(\frac{R_L^2+R_S^2+2R_SR_L}{R_L}\right) \longrightarrow \frac{d}{dR_L}\left(R_L+\frac{R_S^2}{R_L}+2R_S\right)$$

$$\Rightarrow \boxed{1-\frac{R_S^2}{R_L^2}} = 0 \Rightarrow R_L^2 = R_S^2 \quad \text{or} \quad R_L = \pm R_S$$

$$\Rightarrow \boxed{R_L = R_S}$$

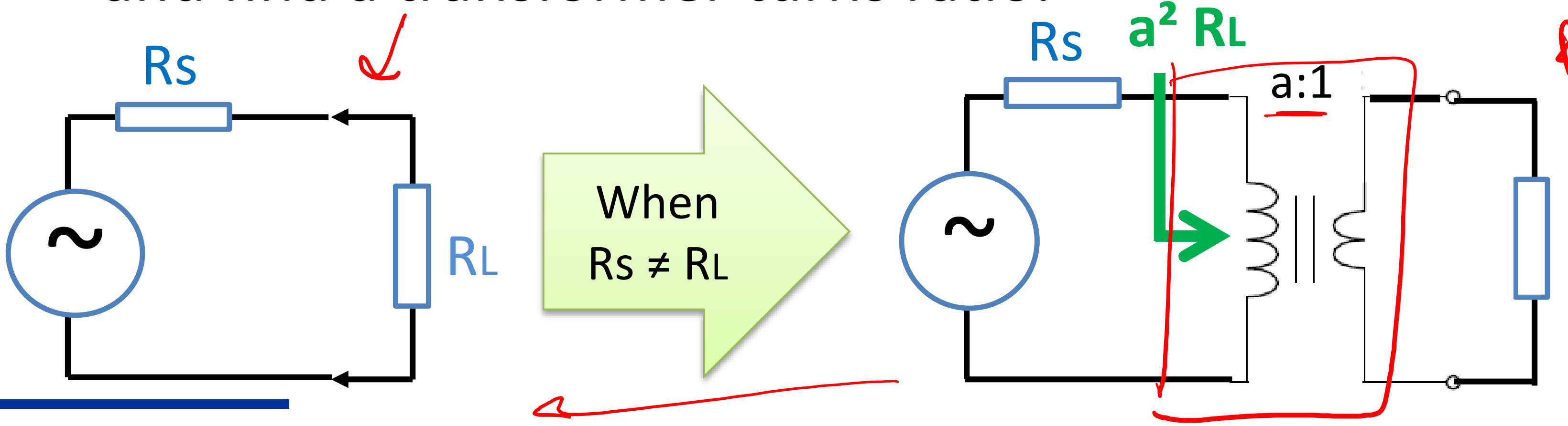
Taking 2nd differential

$$\frac{d}{dR_L}\left(1-\frac{R_S^2}{R_L^2}\right) = +2\frac{R_S^2}{R_L^3}$$

$\Rightarrow$ when $R_L = R_S$ denominator is minimum and maximum power transfer occurs

Impedance Matching for ‘R’

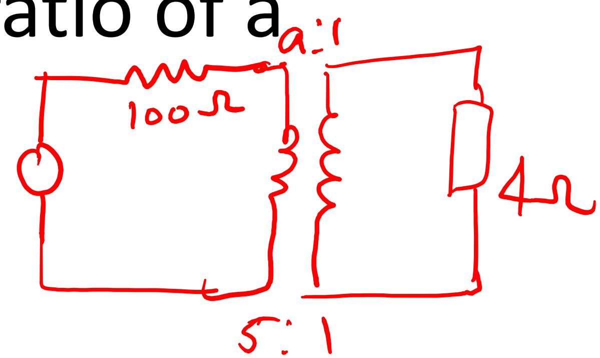
- Maximum power transfer occurs when $R_S = R_L$.
- In the case that we need to connect the voltage source that has internal impedance of R_S to a load R_L that does not satisfy the above condition, we can **design** a **transformer** to *match* impedance for maximum power transfer.
- To find an appropriate transformer, we let $\boxed{R_S = a^2 R_L}$ and find a transformer turns ratio.



$R_L' = R_S$
$= a^2 R_L$

Example 2

- Given the voltage source with an internal resistance of 100 Ω. A transformer is used to connect this voltage source to the load of 4 Ω to achieve maximum power transfer at the load. What should be the turns ratio of a transformer?



$R_S = a^2 R_L$

$100 = a^2 4$

$a^2 = 25$

$a = 5$

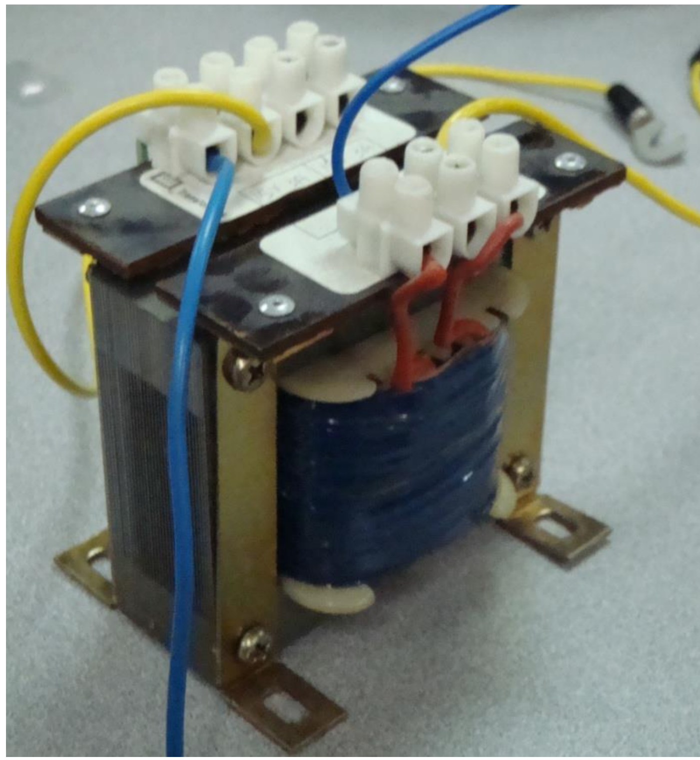
Turns Ratio is $\boxed{5:1}$

Transformer components
Eddy current and hysteresis losses
Magnetizing current losses
A practical transformer model
A simplified model

PRACTICAL TRANSFORMERS

Practical Transformers

Pole-mounted single-phase transformer



Lab-sized single-phase transformers

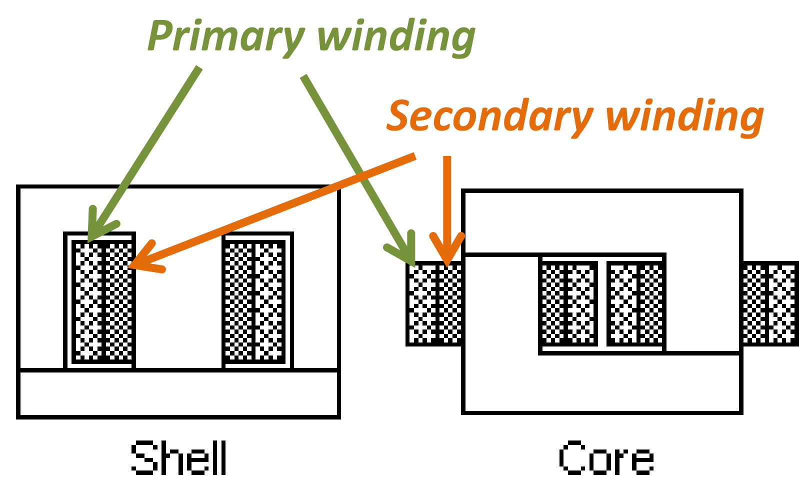
Transformer Components

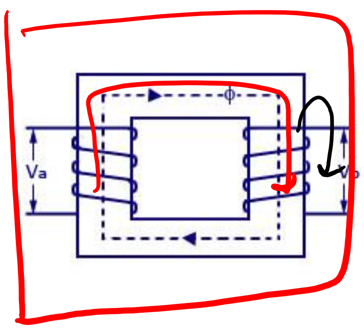
Magnetic core

- Mainly two types of design.
  1. Core type – the magnetic core is surrounded by the windings.
  2. Shell type – the windings are surrounded by the core.

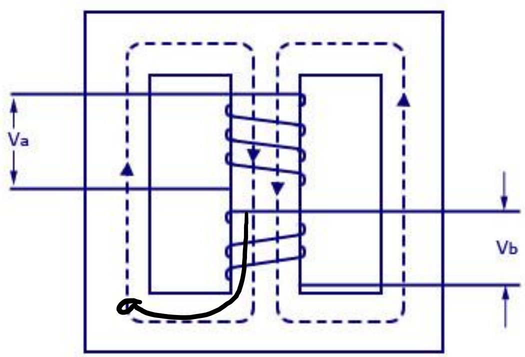
Primary/Secondary winding

- Windings are placed on top of each other to reduce the amount of flux leakage losses.





1. Core type



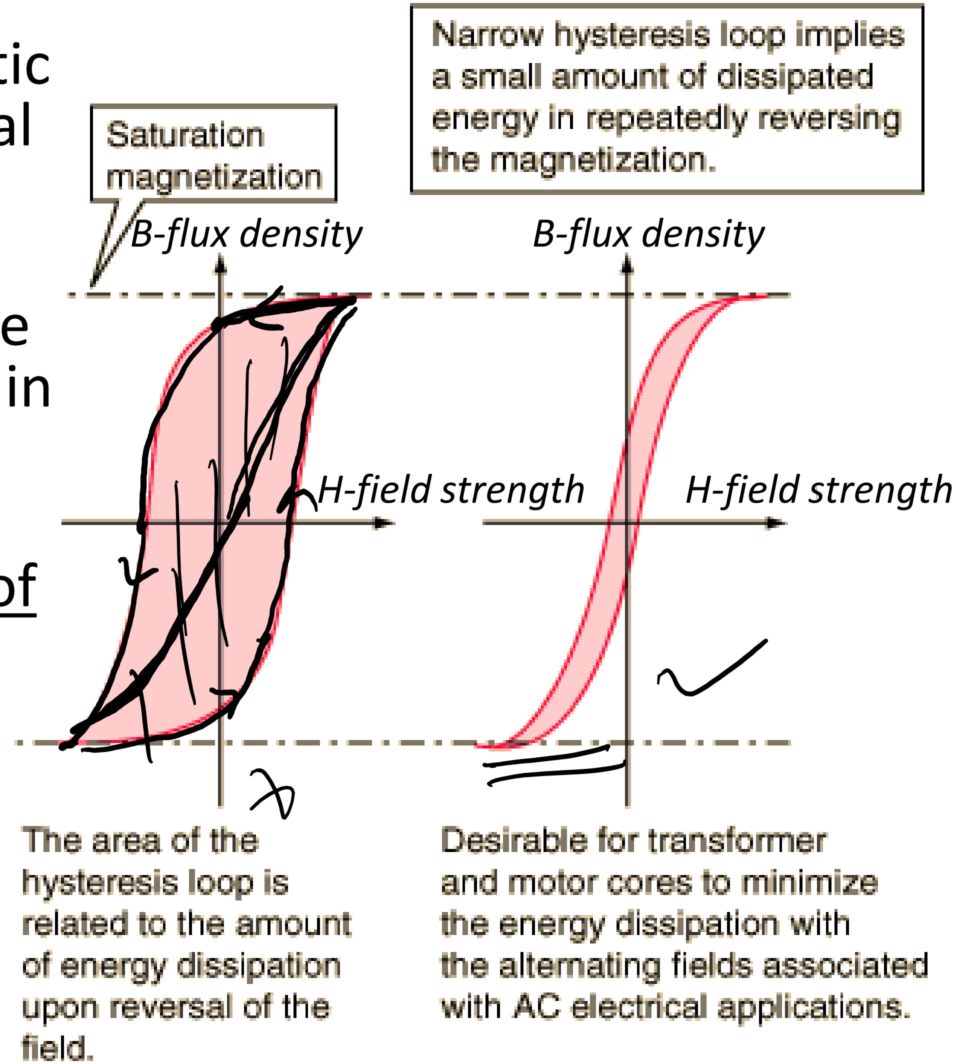
www.CircuitsToday.com

2. Shell type

Hysteresis Losses

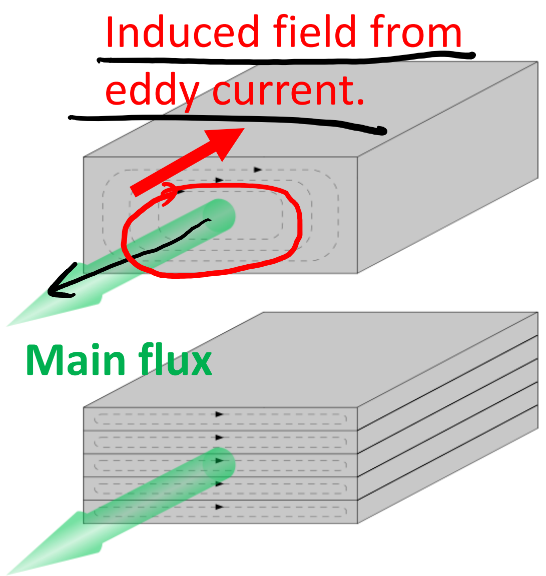
- Hysteresis loop is a characteristic of how a ferromagnetic material is magnetized.
- Each time the direction of magnetic field is reversed, some amount of energy is dissipated in the core.
- This means that <u>this loss is proportional to the frequency of electricity.</u>
- Hysteresis loss produces heat and is represented as a **resistance parallel to** the ideal transformer.

Source: http://hyperphysics.phy-astr.gsu.edu/hbase/solids/hyst.html

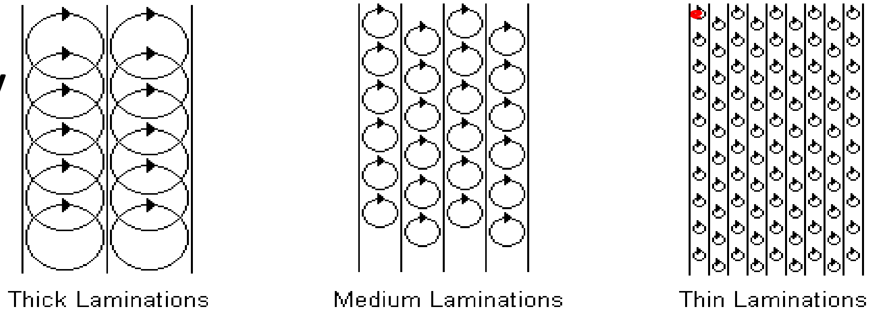


Eddy Current Losses

- Eddy current refers to small circular current in the magnetic core caused by the flux that passes through the core.
- The magnitude of eddy current losses depends on the strength of the main flux, thus the voltage supplied.
- Eddy current loss produces heat and is represented as a **resistance parallel to** the ideal transformer.
- Eddy current loss can be reduced by making the cores from thin sheets of steel i.e. the core is laminated. The thinner the sheets, the smaller are the eddy current losses.

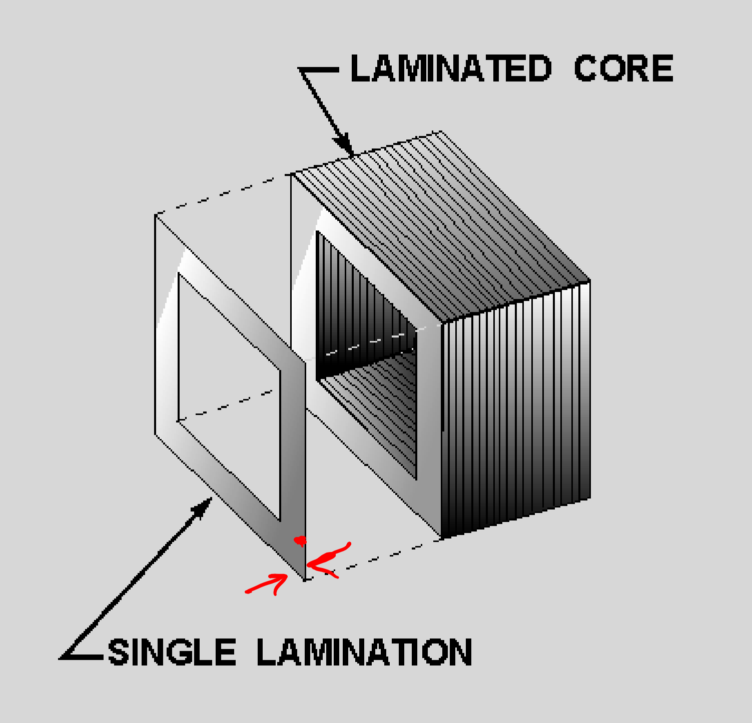


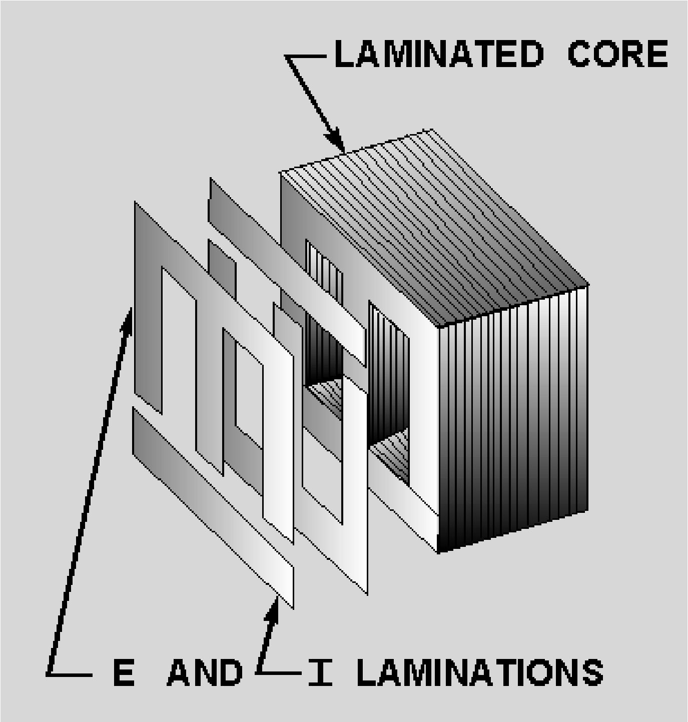
Eddy currents will induce a magnetic field that opposes to the direction of the main flux.



Source: http://sound.westhost.com/xfmr2.htm

Core Construction of Transformers



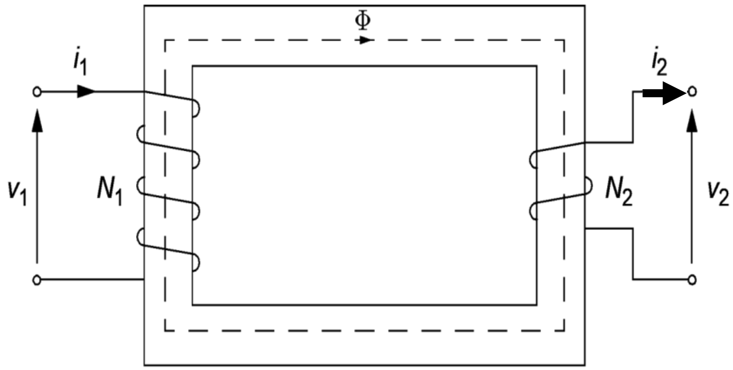


Source: tpub.com

Ideal VS Practical Transformer

Ideal transformer

1. Zero resistance in the both windings.
2. No leakage flux around the core.
3. No core resistive loss.
4. Core permeability is infinite



Practical transformer

1. Winding losses (copper losses) represented as ***resistance in both windings***.
2. Leakage flux around the core represented as ***inductance in both windings***.
3. Core resistive losses (hysteresis loss + eddy current loss) represented as ***resistance in parallel to the core***.
4. Magnetic core permeability is finite.

How to represent this effect?

Finite Core Permeability

- From Ampere's law applied to transformers,

$$\boxed{\frac{Bl_{path}}{\mu} = i_1N_1 - i_2N_2}$$

Recall that in ideal transformer, μ is infinite so $i_1N_1 = i_2N_2$

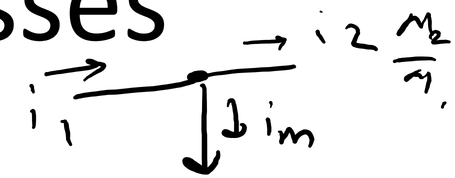
- When the core material has finite core permeability ($\mu \neq \infty$), and $V_1 = N_1(j\omega)BA$ (Faraday's law), **assuming a** **constant** ***μ***,

$$B = \frac{V_1}{N_1 j\omega A}$$

$$\frac{B\, l_{path}}{\mu N_1} = i_1 - i_2\frac{N_2}{N_1}$$

$$\Rightarrow \underbrace{\boxed{i_1 - i_2\frac{N_2}{N_1}}}_{i_m} = \frac{V_1}{\mu N_1^2 (j\omega) A}\cdot l_{path} = -j\left(\frac{l_{path}}{\mu N_1^2 \cdot \omega \cdot A}\right)V_1$$

Magnetizing Current Losses



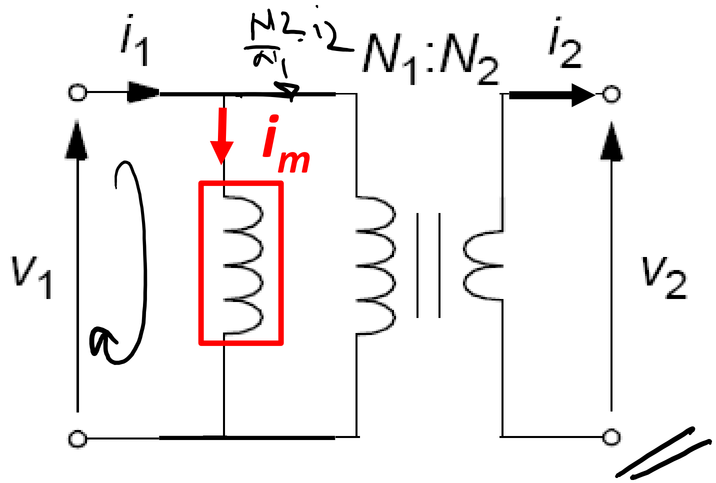
- From the magnetizing current,

$$i_m = -\underline{j}\left(\frac{l_{path}}{\omega N_1^2 A\mu}\right)V_1$$

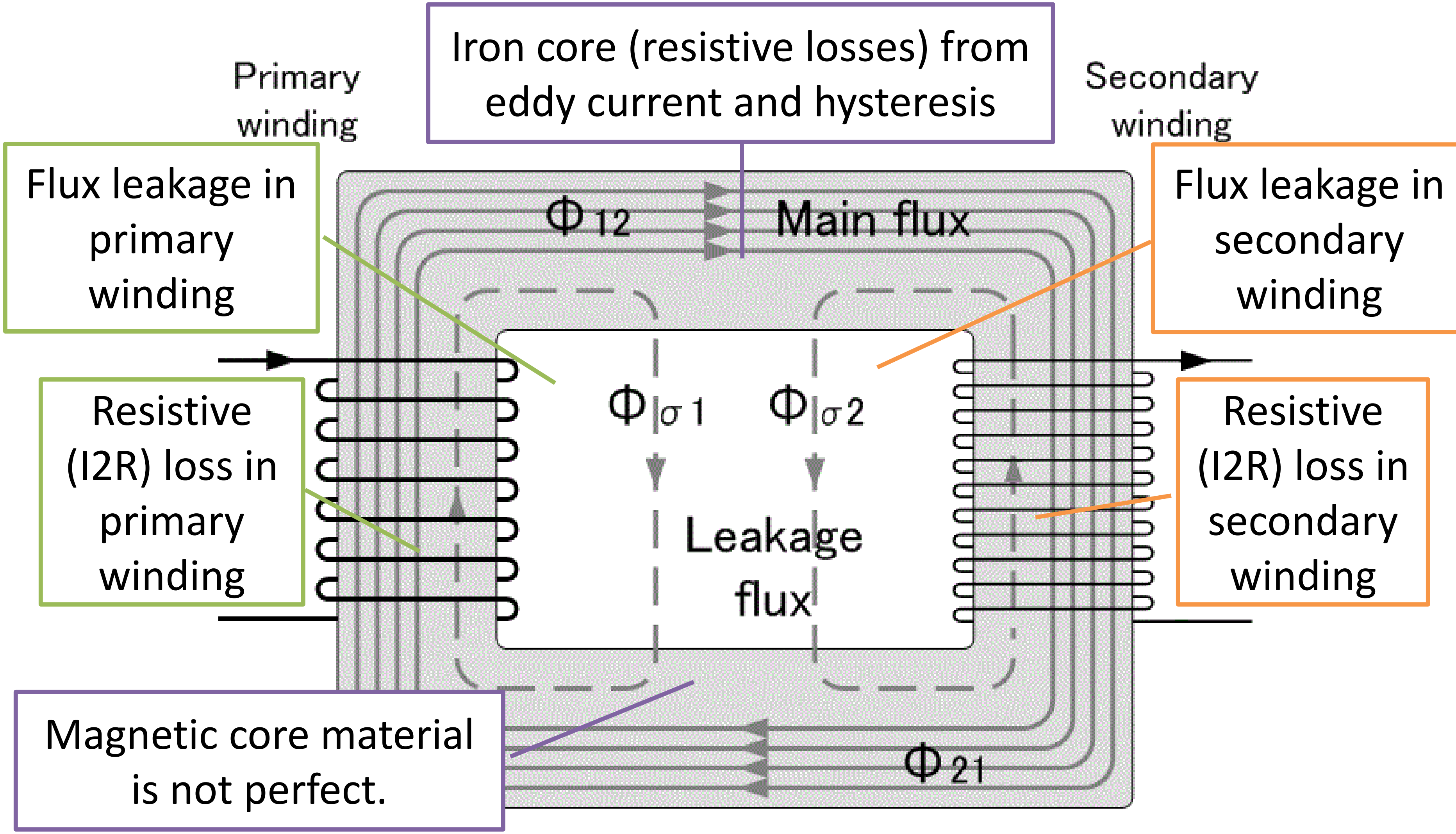
$(-j = \angle -90^\circ)$

- We find that the current lags the voltage V_1 by 90°(-j).
- As such, we can use an **inductor** to represent the effect of finite magnetic core permeability in the equivalent circuit.

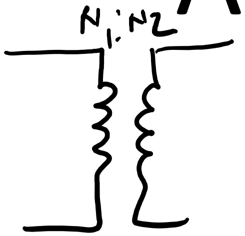
$$i_1 - \left(\frac{N_2}{N_1}\right)i_2 = i_m$$

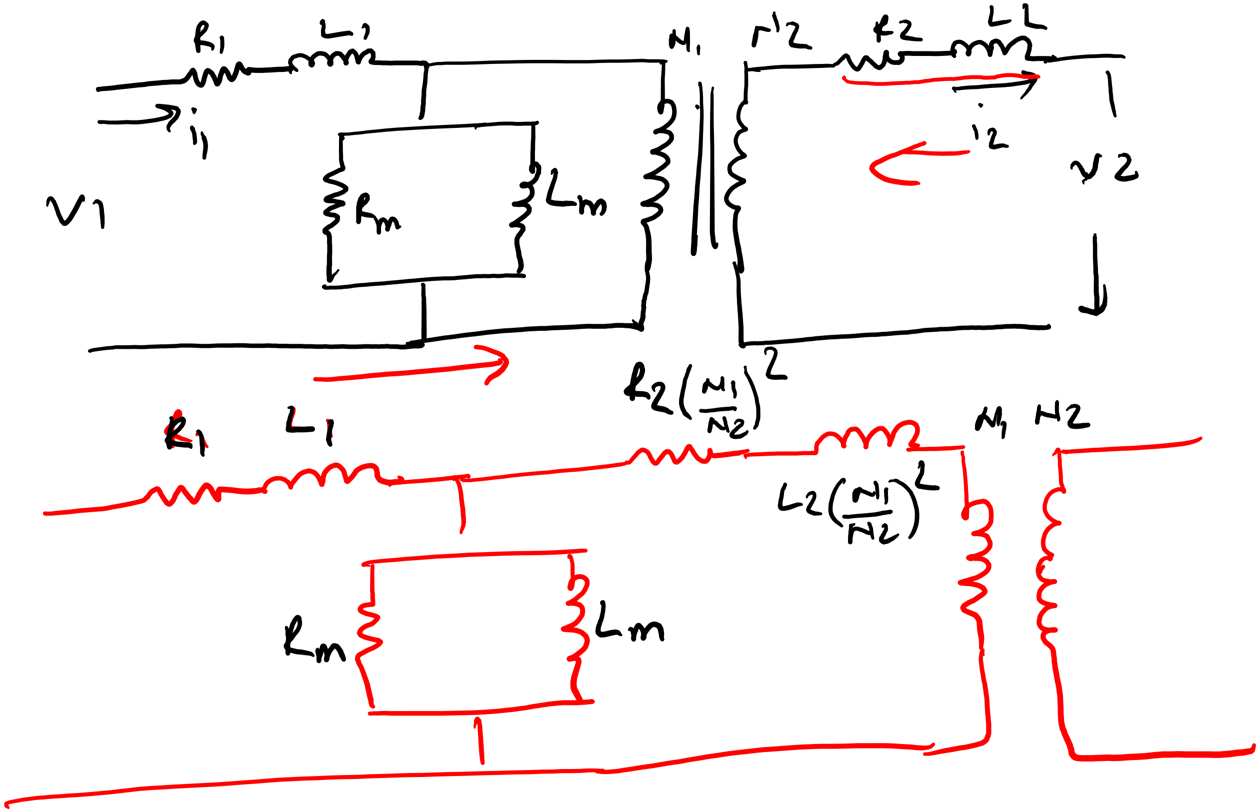


Practical Transformers



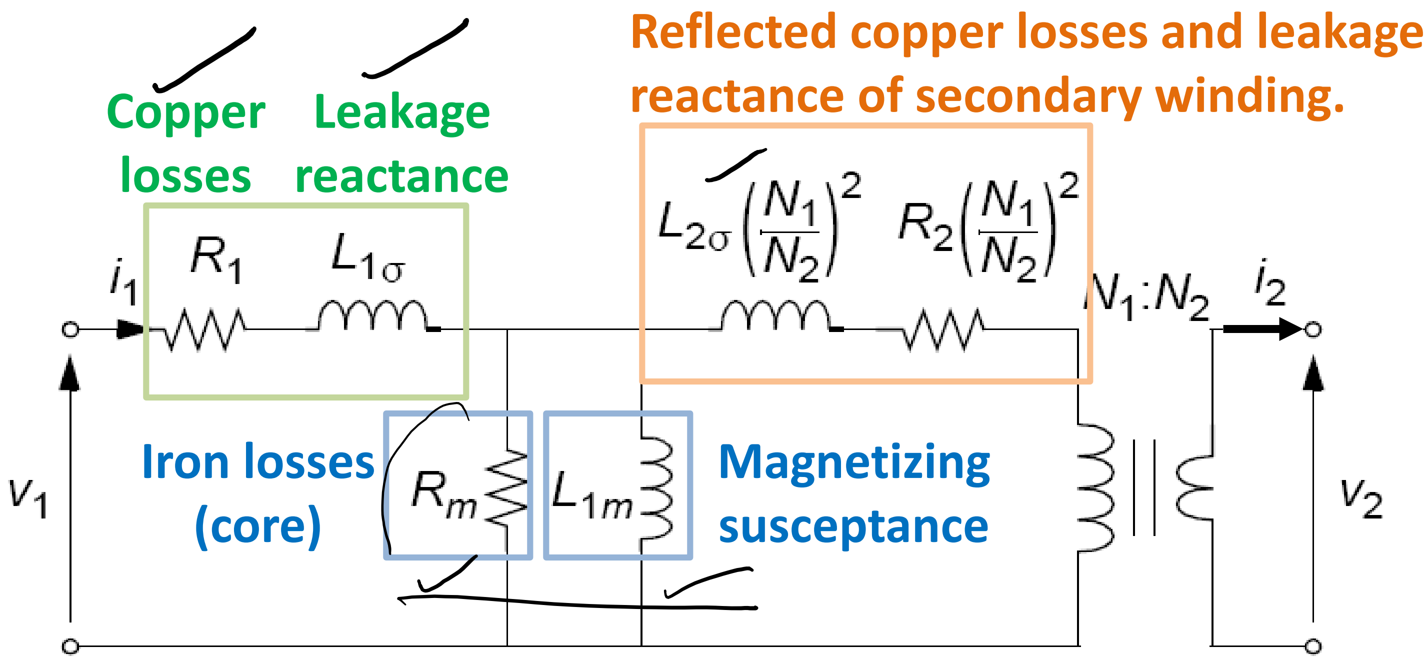
A Practical Transformer Model





A Practical Transformer Model

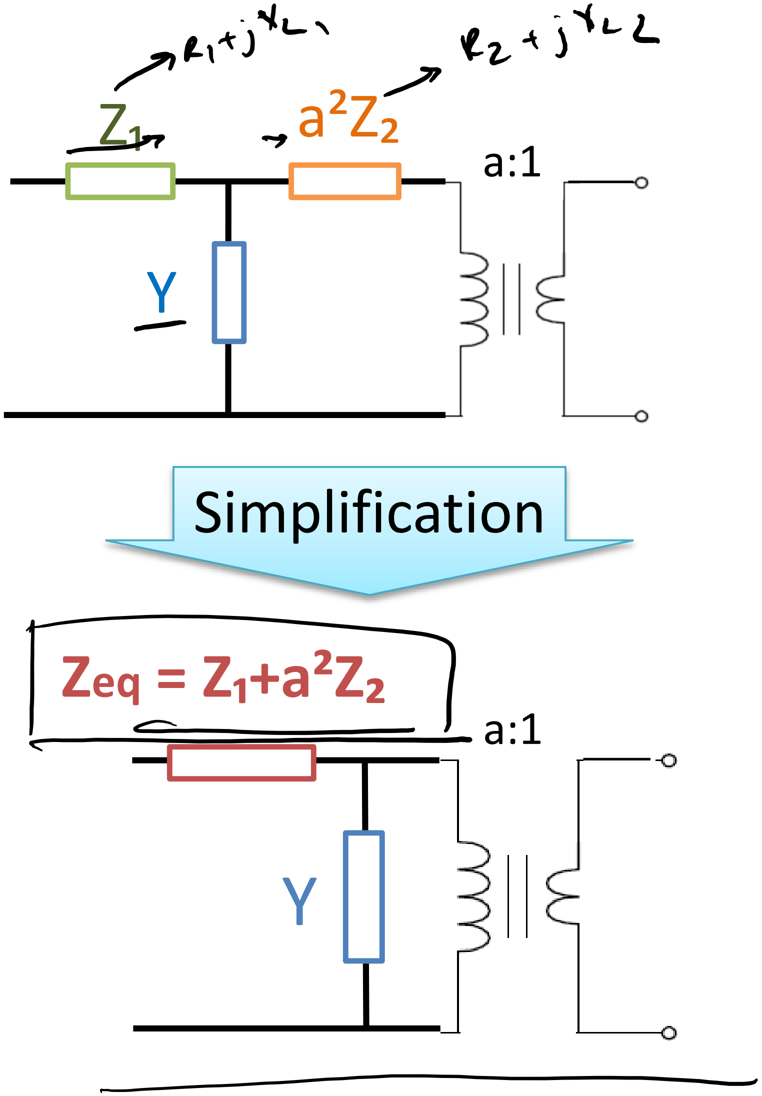
By convention, the **primary side** of transformer is the side with a **higher number** of turns.



Note that in Chapter 3 [Glover, Sarma, and Overbye, "Power System Analysis and Design"], the core losses are represented as 'shunt admittance', Y = G –jB where G and B are positive. The imaginary part is negative to represent inductive property.

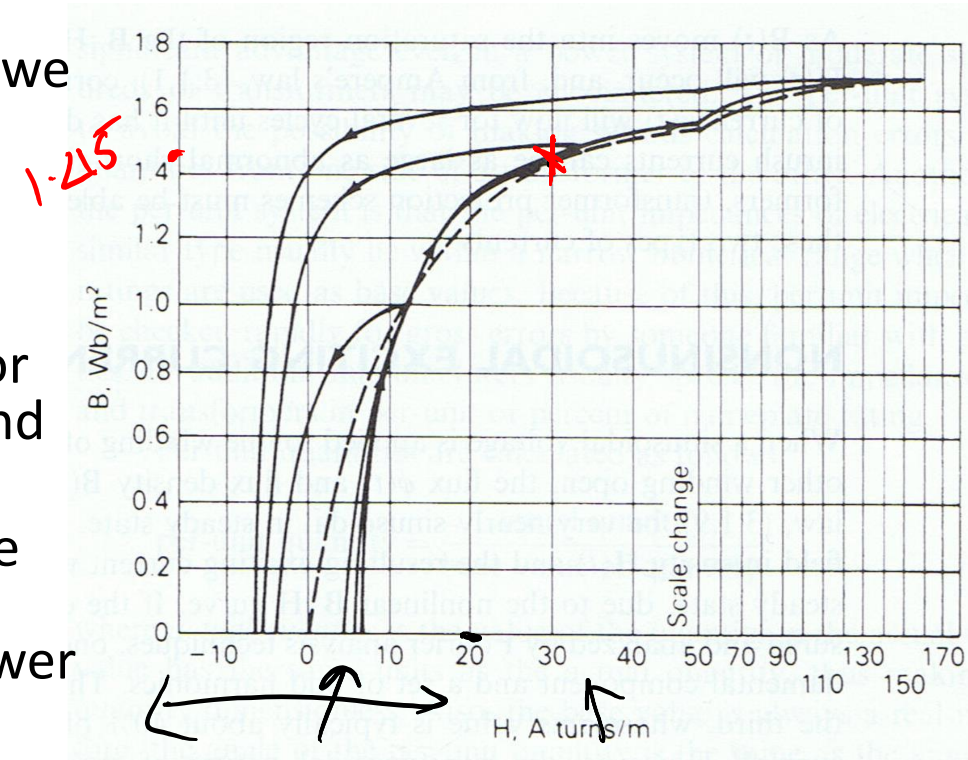
A Simplified Model

- Typically the admittance, Y, is very small i.e. resistance is very large.
- This means that the currents flowing through Z_1 and a^2Z_2 are almost the same.
- We can simply combine Z_1 and a^2Z_2 to "Z_{eq}", the equivalent series impedance.



Saturation

- In practical transformer model, we assume **a constant core permeability** thus linear relationship between B and H.
- In fact, the B-H curve for ferromagnetic materials used for transformer core is nonlinear and has multiple values.
- As H increases, the core become **saturated** i.e. the magnetic flux density B increase at a much lower rate.
- This means that there will be high magnetizing current flow making the transformer to heat up.
- This effect is **NOT included** in the equivalent circuit.



B-H curve is approximated by a dashed line.

$B = \mu H$

B = Magnetic flux density (Weber/m² or Tesla)

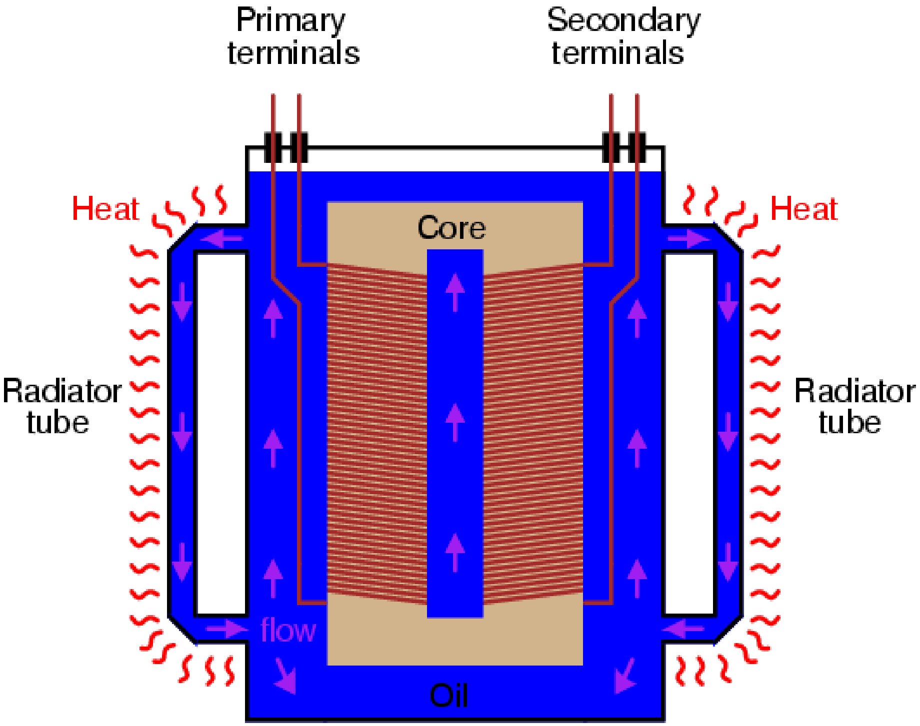
H = Magnetic field intensity (A/m)

μ = Magnetic core permeability (H/m)

Transformer Heating

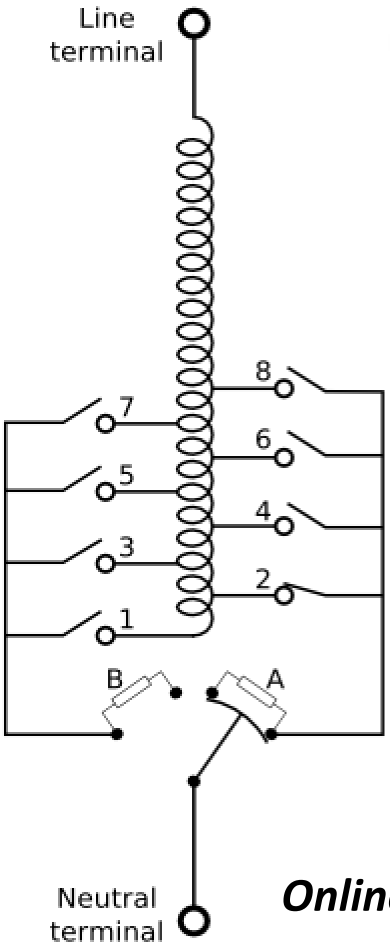
- Heating is caused by high loading of the transformer, eddy current losses, saturation.
- Heating can lead to winding insulation damage, short circuit, and even explosion.
- In order to prevent overheating, transformers are usually cooled by a fan and a convection oil to reduce heat inside the iron core.

Source: abb.com

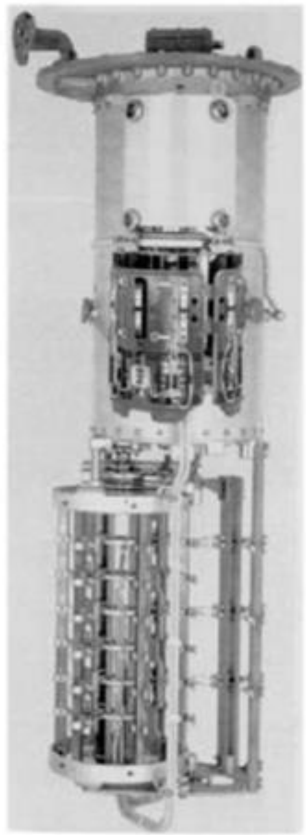


Source: http://openbookproject.net/electricCircuits/AC/AC_9.html

Tap Changing Transformer

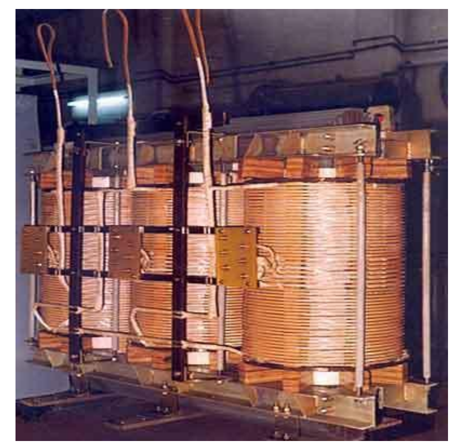


Online



Design

- Transformers with tap changer are used to adjust the voltage by changing the turns ratio.
- Tap-changing transformers are used to regulate the voltage at the end users to be at the desired value.
- This can be done both off-line and on-line.



Off-line

Source: http://en.wikipedia.org/wiki/Tap_%28transformer%29
http://www.powertransformerdesign.net

Short-circuit test

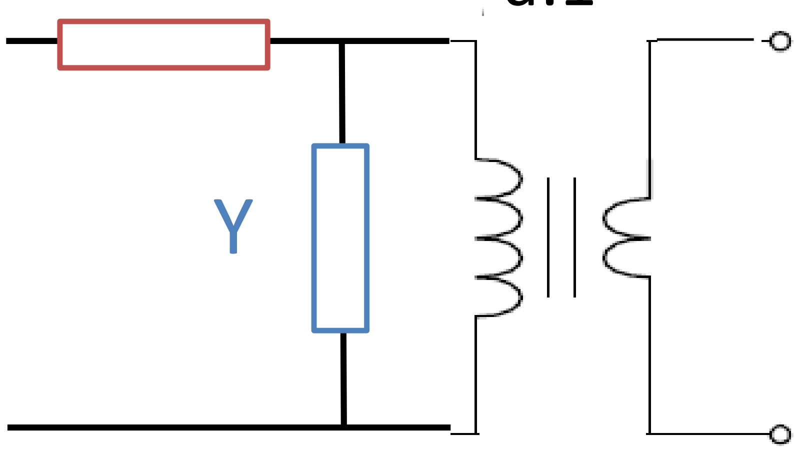
Open-circuit test

TRANSFORMER PARAMETER TESTS

Transformer Parameters

- Series impedance
  - Z_1 and Z_2 are series impedances representing the resistive loss and flux linkage loss in the two windings.
- Shunt admittance
  - Y is a shut admittance representing iron core loss and magnetizing susceptance.

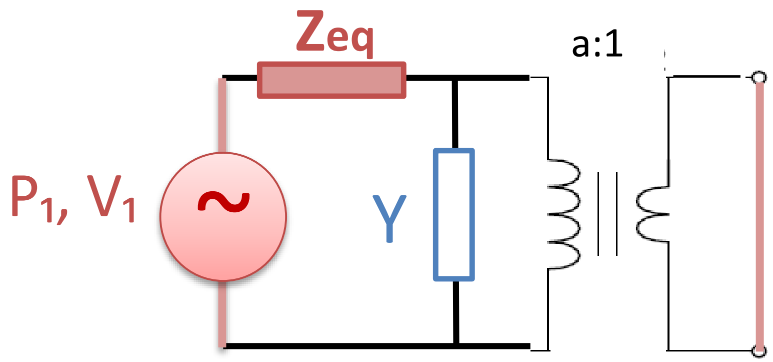
$Z_{eq} = Z_1+a^2Z_2$



Note that by convention, the primary side of a transformer is the side with a higher number of turns. This means that a > 1.

Short-Circuit Test

- To find equivalent series impedance.
- Short circuit the *secondary* side.
- Apply **rated current** (implies small voltage) at the primary side.
- Measure real power and voltage at the primary side.



Note that we want small voltage applied to the primary side so that there will be large amount of current passing through the impedance. This will allow more accurate calculation of the series impedance

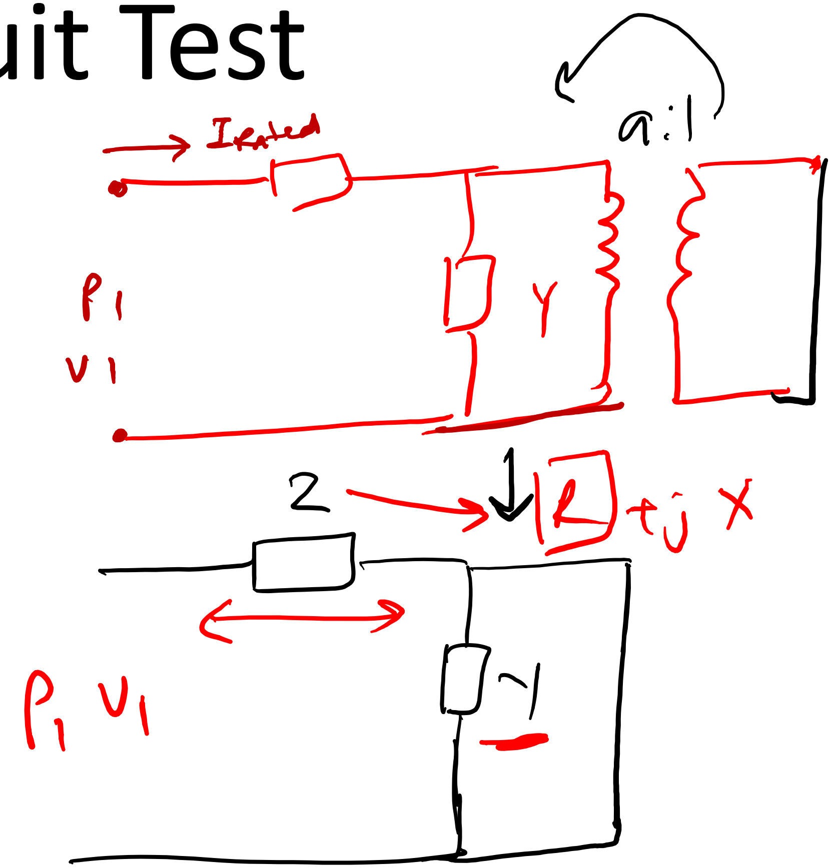
Short-Circuit Test

Steps

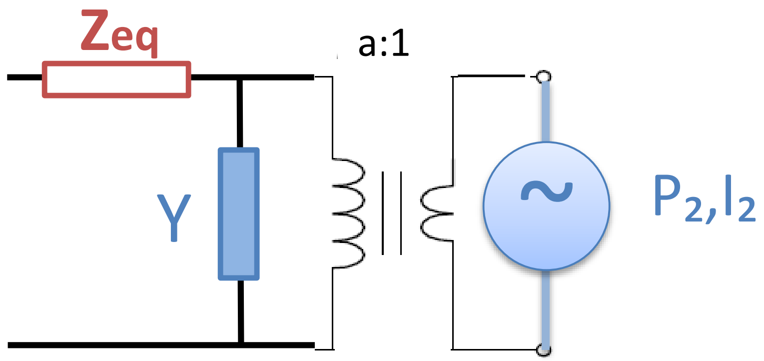
1. Find $Z_{eq} = \dfrac{V_1}{I_{1\,rated}}$

2. Find $R_{eq} = \dfrac{P_1}{I^2_{1\,rated}}$

3. Find $X_{eq} = \sqrt{Z^2_{eq} - R^2_{eq}}$



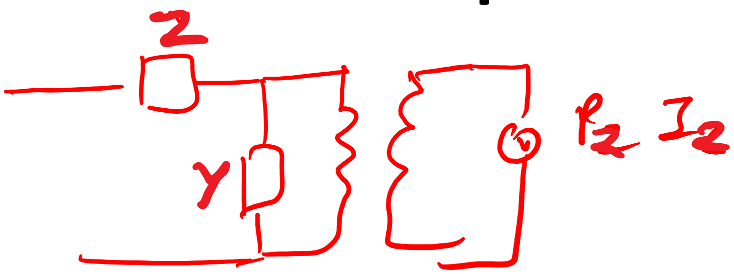
Open-Circuit Test

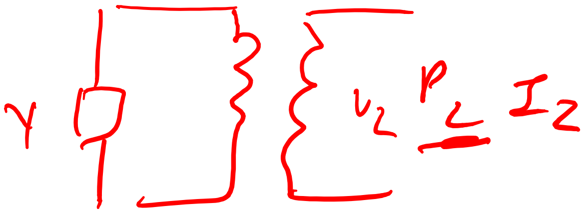


Note that we want rated voltage at the secondary side so that there will be large amount of current passing through the admittance Y (I=YV). This will allow more accurate calculation of the shunt admittance.

- To find equivalent shunt admittance.
- Open circuit the *primary* side.
- Apply **rated voltage** at the secondary side.
- Measure real power and current at the secondary side.

Open-Circuit Test





$\underline{Y} = \underline{G} - j\underline{B}$

Steps

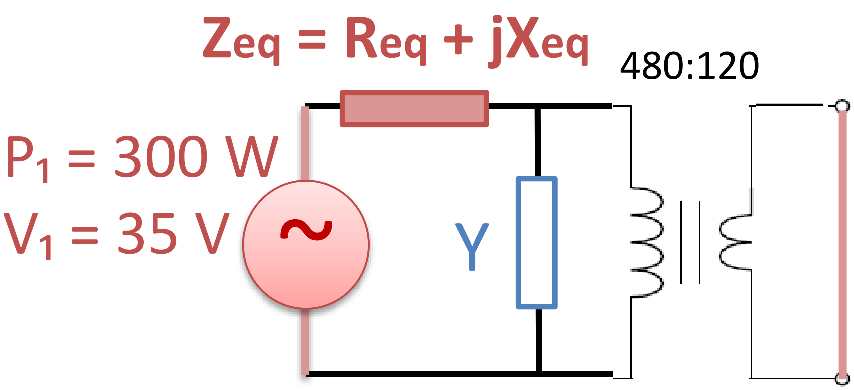
1. Find $G_{eq} = \dfrac{P_1}{V_{1\,rated}^2}$

2. Find $Y_{eq} = \dfrac{I_1}{V_{1rated}} = \dfrac{I_2 / a}{V_{1rated}}$

3. Find $B_{eq} = \sqrt{Y_{eq}^2 - G_{eq}^2}$

Example 3: Short Circuit Test

- Consider a single-phase 20kVA, 480/120 V 60 Hz transformer. During short circuit test, rated current is applied to the primary side. The voltage of **35 V** and real power of **300 W** are measured. Find equivalent series impedance of this transformer.



$$X = \sqrt{Z^2 - R^2}$$

$$|I_{1,rated}| = \frac{|S_{rated}|}{|V_{1,rated}|} = \frac{20\times 10^3}{480} = 41.667\text{ A}$$

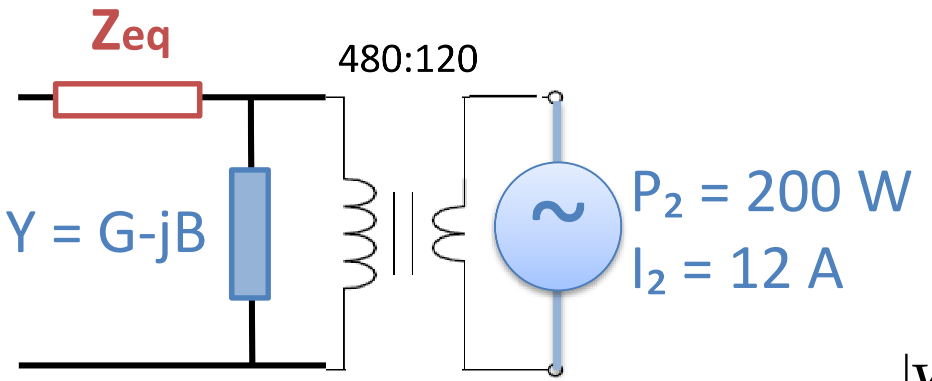
$$R_{eq} = \frac{P_1}{|I_{1,rated}|^2} = \frac{300}{41.667^2} = 0.1728\,\Omega$$

$$|Z_{eq}| = \frac{|V_1|}{|I_{1,rated}|} = \frac{35}{41.667} = 0.84\,\Omega$$

$$X_{eq} = \sqrt{0.84^2 - 0.1728^2} = 0.822\,\Omega$$

Example 4: Open Circuit Test

- Consider the same transformer as Example 2. During open circuit test: rated voltage applied to secondary side, then **I_2 = 12 A** and **P_2 = 200 W**. Find equivalent shunt admittance Y of this transformer.



$$|V_{2,rated}| = 120\text{ V}, |V_{1,rated}| = 480\text{ V}$$

$$G_{eq} = \frac{P_1}{|V_{1,rated}|^2} = \frac{200}{480^2} = 0.000868\, S$$

$$|Y| = \frac{|I_1|}{|V_{1,rated}|} = \frac{|I_2|/a}{|V_{1,rated}|} = \frac{12/4}{480} = 0.00625\, S$$

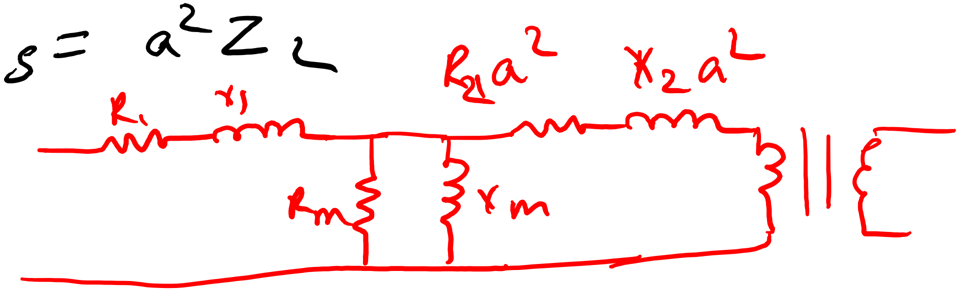
$$B = \sqrt{0.00625^2 - 0.000868^2} = 0.00619\, S$$

Summary

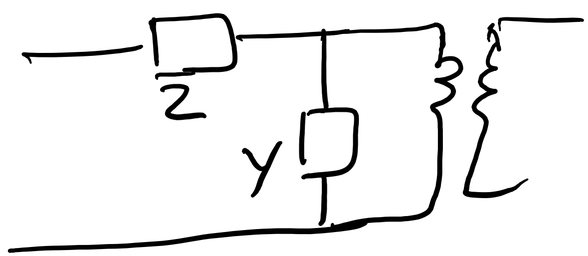
→ Reflected load $Z_1 = a^2 Z_2$

→ Impedence matching $Z_S = a^2 Z_L$

→ Practical transformers

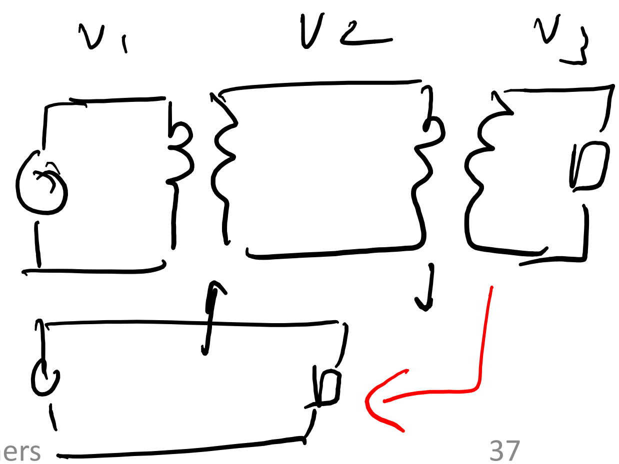


→ Simplified Model



→ Short Circuit Test → Z_{eq}

→ Open Circuit Test → Y



e-Learning Week

- Submit a 1 page write up debating on the following topic:
  - Nuclear Energy: Boon or Bane
- Critique on a fellow classmate's write-up
- Submit the write up by March 12, 2015
- Submit the critique by March 15, 2015
- 2% of your overall grade