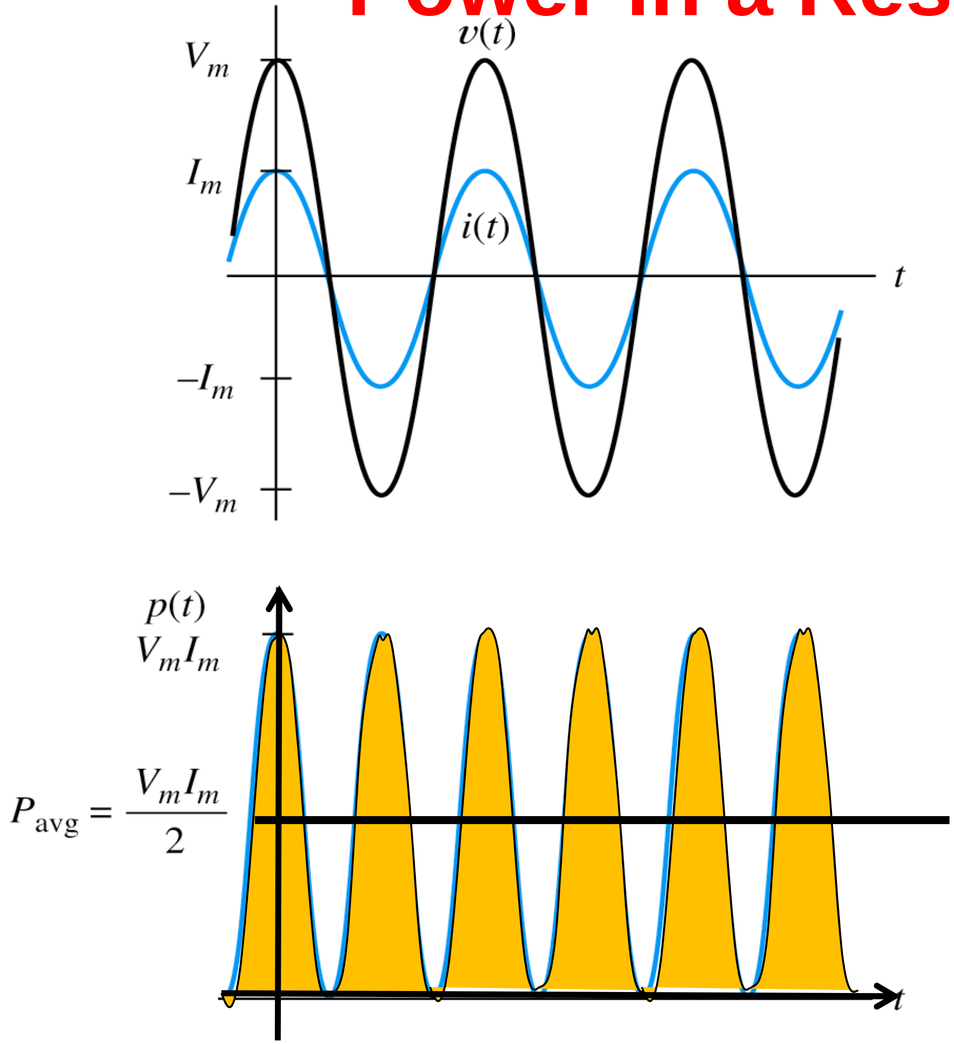
EE2022
Electrical Energy Systems

Lecture 4: AC Power

National University of Singapore

Power in a Resistor



Current, voltage, and power versus time for a purely resistive load.

$v(t) = V_m\ Cos\ \omega t$

$i(t) = I_m\ Cos\ \omega t$

Instantaneous power:
$p(t) = V_m\ I_m\ Cos^2\omega t$

The power wave is square of the cosine function.

Average Power in a Resistor

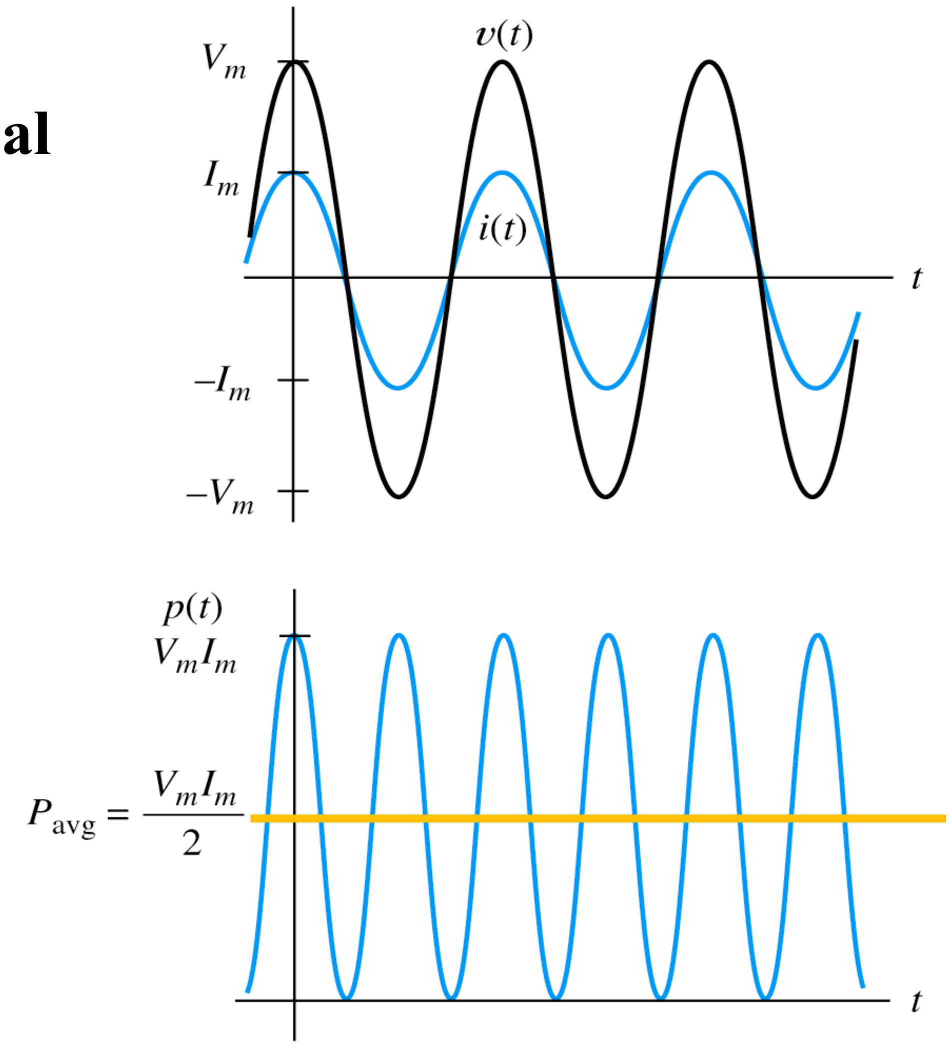
Instantaneous power:
$p(t) = V_m I_m Cos^2\omega t$

Average of this function is equal to half of the peak amplitude :

$$P_{R,avg} = \frac{V_m I_m}{2}$$

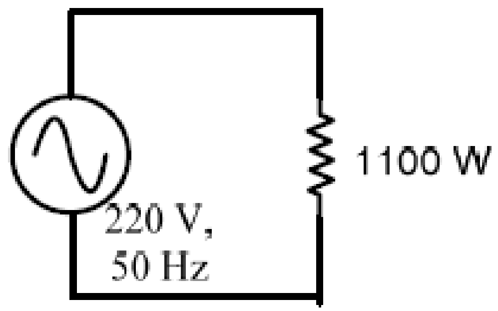
$$P_{R,avg} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

$$V_{rms} = R I_{rms}$$



Current, voltage, and power versus time for a purely resistive l

Example 1: A resistive load is rated as 1100W, 220V, 50 Hz. Calculate the value of load resistance and the amount of current that the load draws from the source.



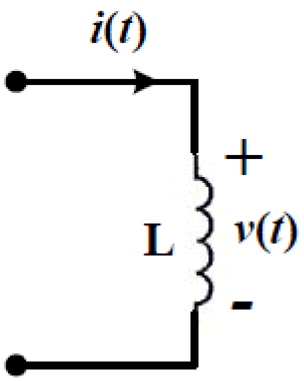
$$P_{R,avg} = \frac{V_{rms}^2}{R}$$

$$1100 = \frac{V_{rms}^2}{R} = \frac{220^2}{R}$$

$$R = \frac{220 \times 220}{1100} = 44\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{44} = 5\ \mathrm{A}$$

Average Power in an Inductor



The voltage and current can be written as:

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - 90°) = I_m \sin(\omega t)$$

The instantaneous power:

$$p(t) = V_m I_m \cos(\omega t)\sin(\omega t)$$

$$= \frac{V_m I_m}{2} \sin(2\omega t)$$

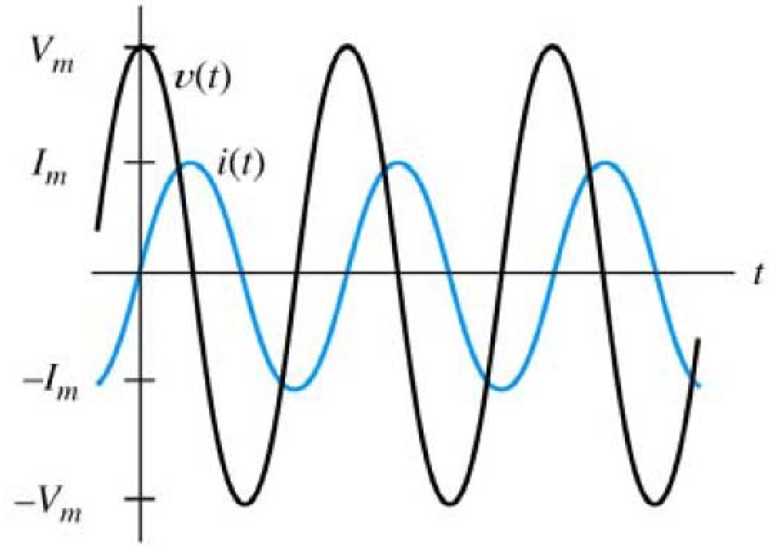
$$= V_{rms} I_{rms} \sin(2\omega t)$$

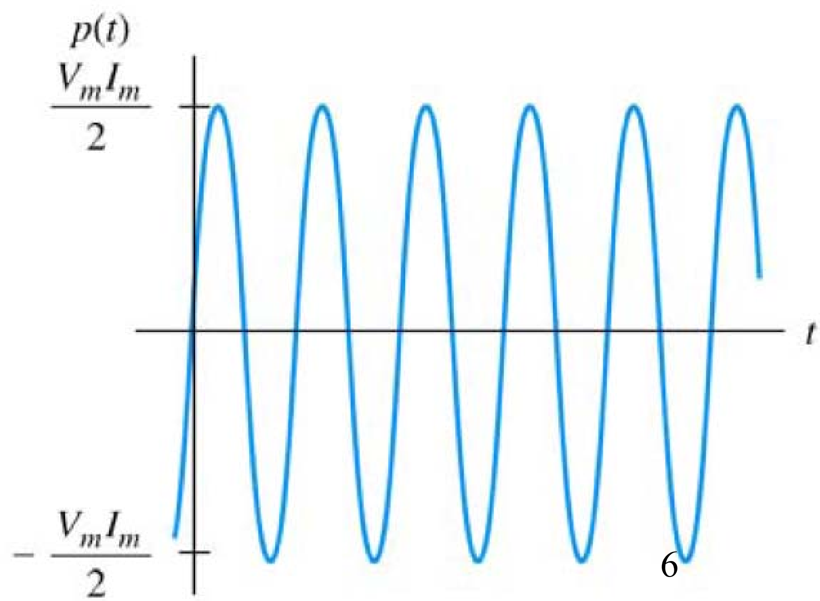
Average Power in an Inductor

$$p(t) = V_{rms} I_{rms} \sin(2\omega t)$$

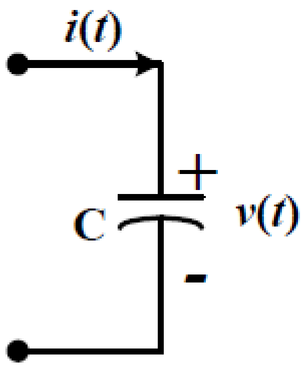
Average power, $P_{avg} = 0$

- Although the time-varying instantaneous power is non-zero, the average power over one cycle is zero
- Instantaneous power is sinusoidal with a frequency twice that of the frequency of voltage and current





Average Power in a Capacitor



The voltage and current can be written as:

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t + 90)$$

$$i(t) = -I_m \sin \omega t$$

The instantaneous power:

$$p(t) = v(t)i(t) = -V_m I_m \cos(\omega t)\sin(\omega t)$$

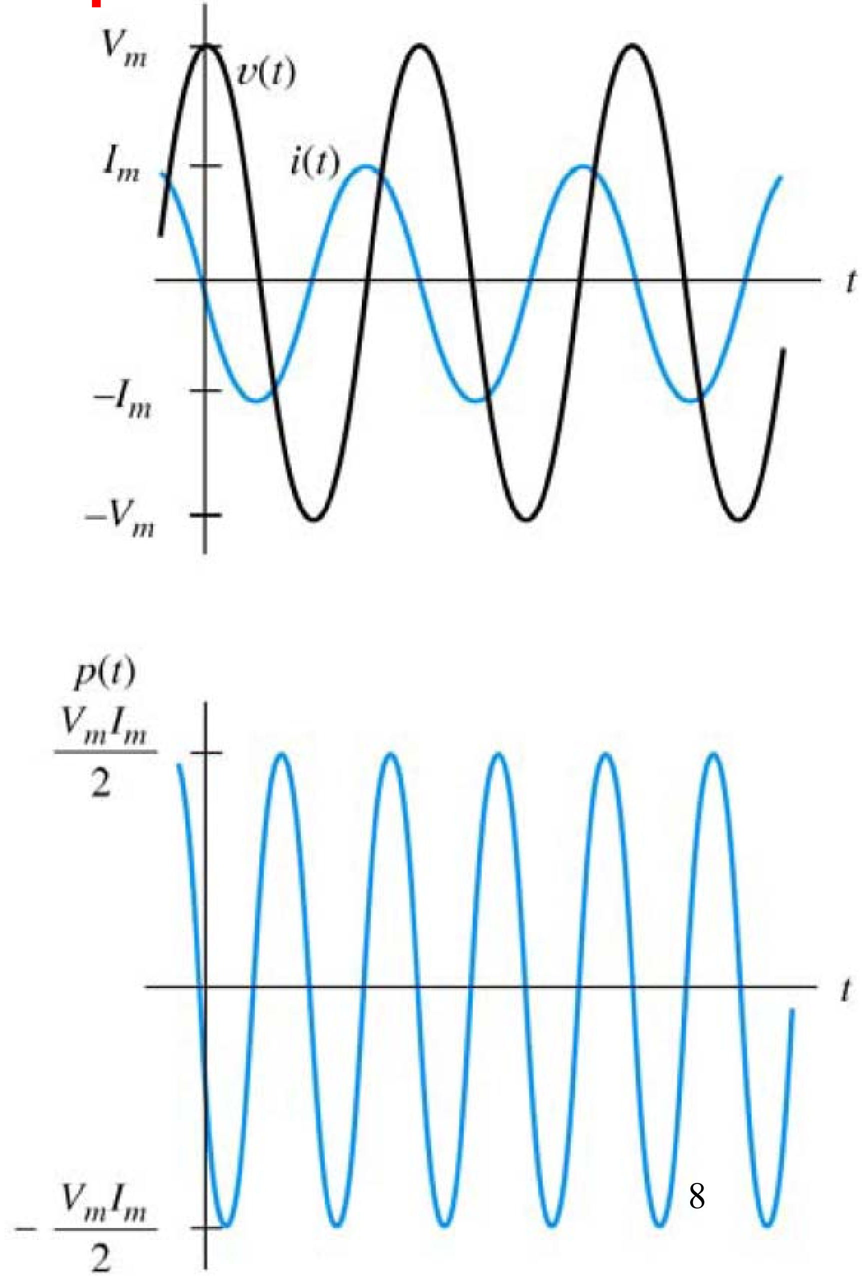
$$p(t) = -\frac{V_m I_m}{2}\sin(2\omega t) = -V_{rms} I_{rms} \sin(2\omega t)$$

Average Power in a Capacitor

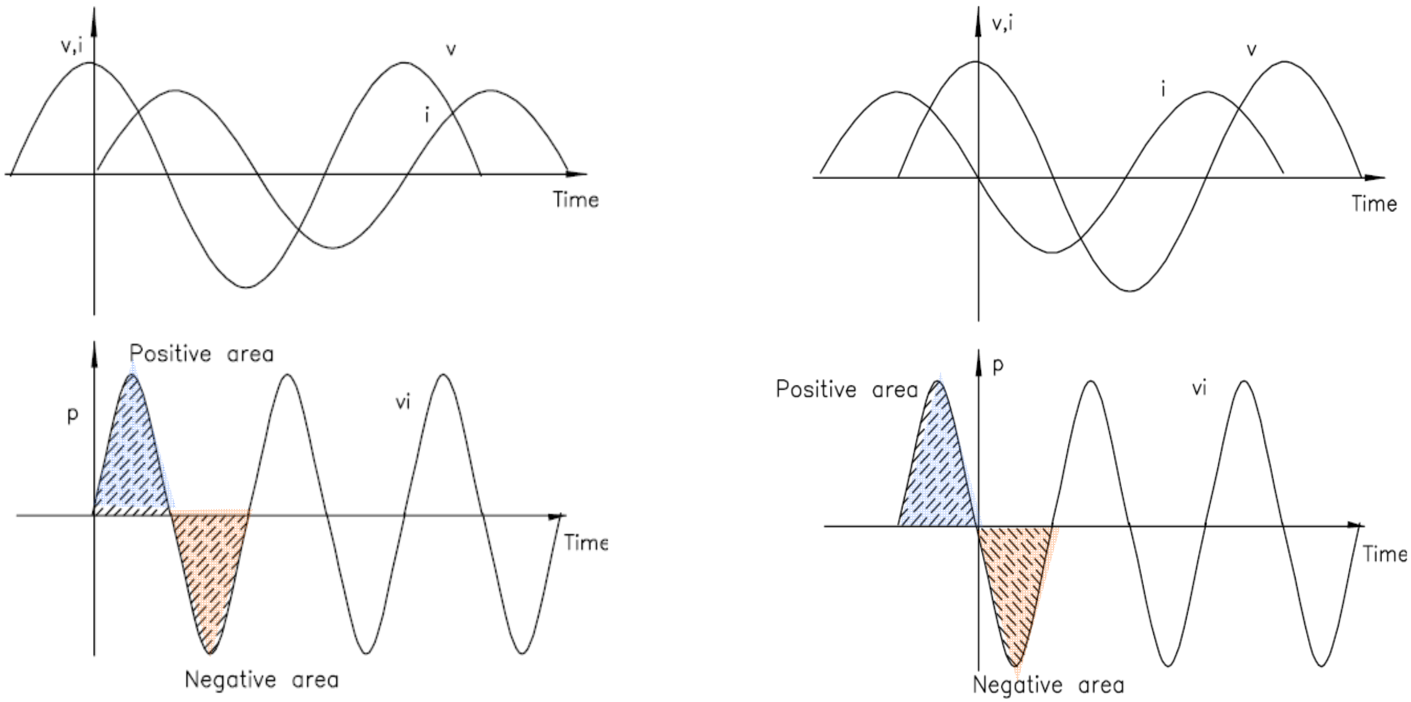
$$p(t) = -V_{rms} I_{rms} \sin(2\omega t)$$

Average power, $P_{avg} = 0$

- The average power over one cycle is zero
- Instantaneous power is sinusoidal with a frequency twice that of the frequency of voltage and current



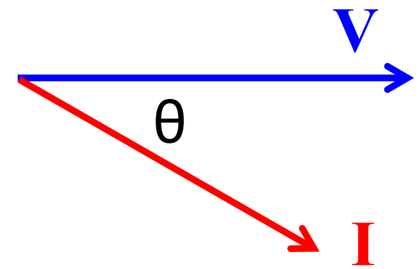
Power in Inductor and Capacitor



Both inductor and capacitor store and release energy during the AC cycle. When both the voltage and current are of the same sign, the element (L or C) draws energy equivalent to the area under the positive half cycle of p(t) from the source and stores it. When they are of opposite signs, it is returning the energy to the source.

Real, Reactive and Apparent Power

Apparent Power



We express power in d.c. and a.c. circuits as follows:

$P_{dc} = V_{dc} I_{dc}$ Watts

$P_{ac} = V_{rms}\, I_{rms} \cos\theta$ Watts

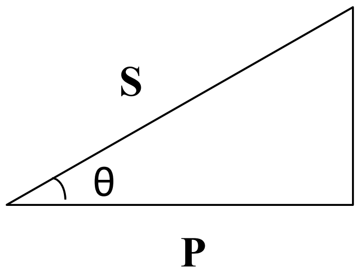
In a.c. circuits an additional term $\cos\theta$ has occurred in the expression for power. The product ($V_{dc}\, I_{dc}$) is expressed in watts. But in a.c. circuits the product ($V_{rms}\, I_{rms} \cos\theta$) is expressed in watts. So let us define ($V_{rms}\, I_{rms}$) as Apparent Power, i.e.

$|S| = V_{rms}\, I_{rms}$ (product of voltage and current magnitude)

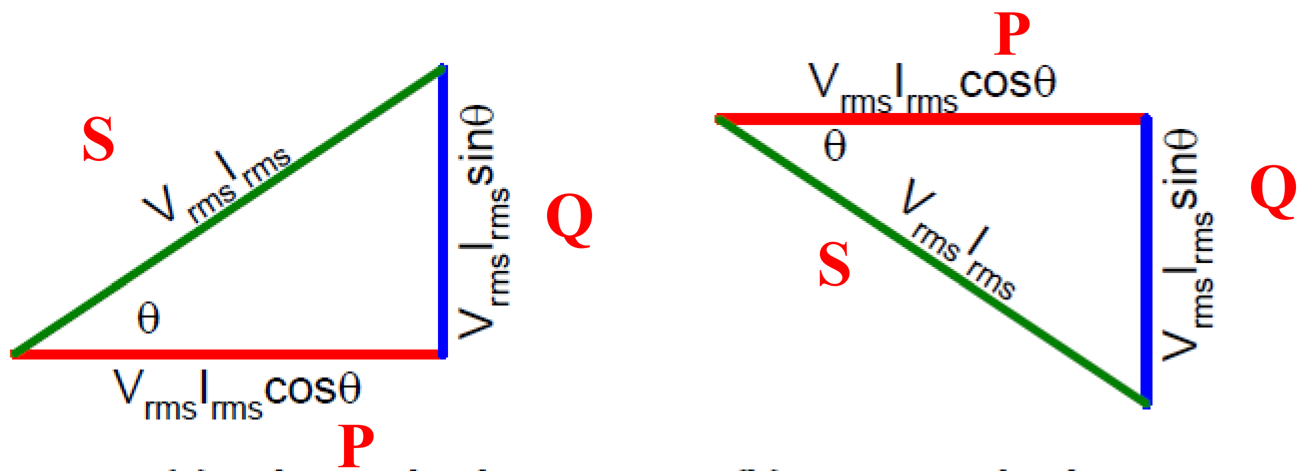
Unit of apparent power = VA (volt-amperes)

So, the power in a.c. circuit may also be expressed in terms of apparent power as follows:

$P = |S| \cos\theta$



Power Triangle

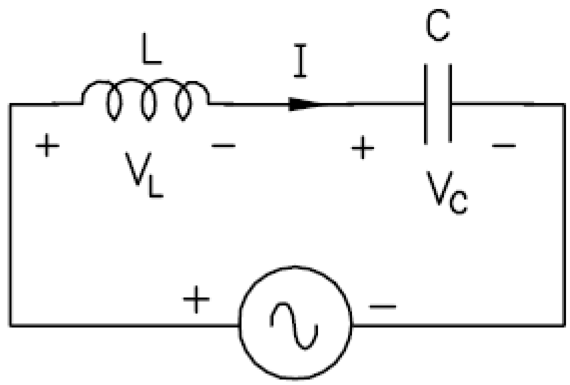


(a) Inductive load, (b) Capacitive load

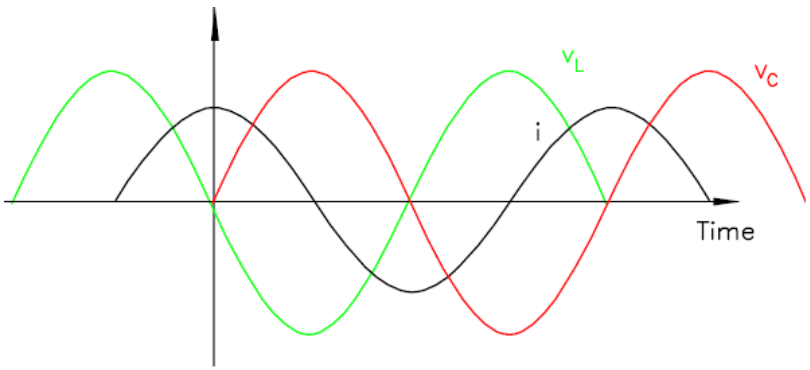
P – Real (or active) power; power consumed in the resistive part of the circuit

Q – Reactive power consumed by the device, due to inductor or capacitor

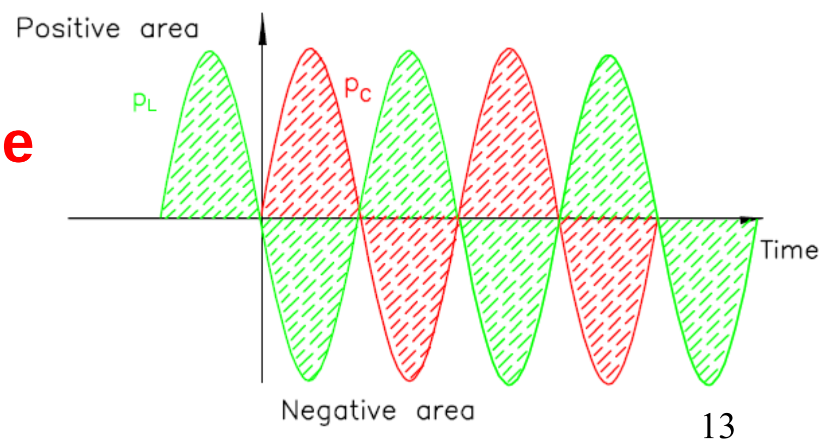
Power In L-C Series Circuit



When $p_L(t)$ is positive, $p_C(t)$ is negative.



Part of the **reactive power** consumption of L is taken care of by the **reactive power** generated by C.

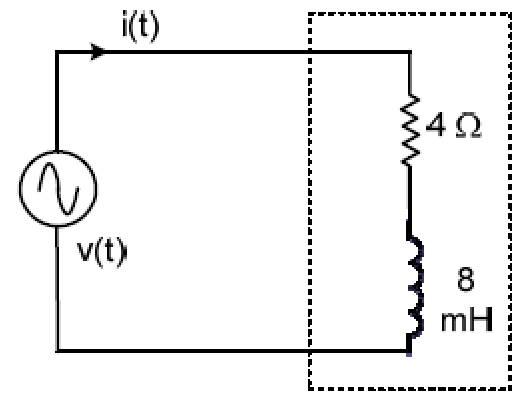


Reactive power is important because transmission lines, transformers, and other elements must be able to withstand the current associated with reactive power.

Example 2: Compute the instantaneous, average, real and reactive powers in the following circuit if *v(t)=14.14 sin (377t)*

$\omega = 377$ rad/s $\qquad Z = R + j\omega L = 4 + j3\ \Omega = 5\angle 36.9°$

$X_L = \omega L = 3\ \Omega$



$$v(t) = 14.14\sin(377t) = 14.14\cos(377t - 90°)$$

$$V = 10\angle -90°$$

$$I = \frac{10\angle -90°}{5\angle 36.9°} = 2\angle -126.9°$$

$$i(t) = 2\sqrt{2}\cos(377t - 126.9°) = 2.28\cos(377t - 126.9°)\ \text{A}$$

Instantaneous power, $p(t) = v(t)i(t) = (14.14\sin(377t)) \times (2.28\cos(377t - 126.9°))$ W

Average power, $P_{avg} = V_{rms} I_{rms} \cos\theta = 10 \times 2 \times \cos(36.9°) = 16$ W

Real power, $P = I_{rms}^2 R = 2^2 \times 4 = 16$ W

Reactive power, $Q = I_{rms}^2 X = 2^2 \times 3 = 12$ VAR

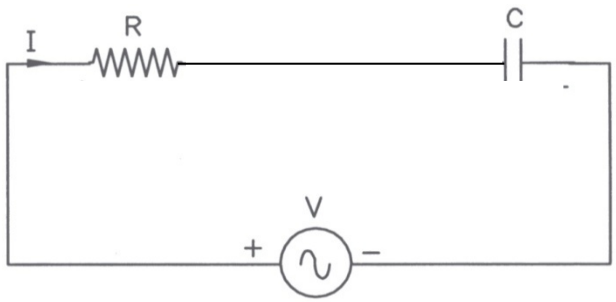
Example 3: A resistive lamp is rated as 200 V, 120 W. (a) Calculate the current drawn by the lamp when connected across 200V supply.

(b) We now want to use this lamp with a 240 V, 50 Hz voltage source. A capacitor bank is connected in series with the lamp so that the voltage across lamp is reduced to the rated 200 V. What is the value of capacitor to be used?

$$P = V_{rms} I_{rms}$$

$$I_{rms} = \frac{120\ W}{200\ V} = 0.6\ \text{A}$$

$$R_{lamp} = \frac{200\ V}{0.6\ A} = 333.33\ \Omega$$



With the capacitor connected in series, the total impedance is $Z = R_{lamp} - jX_C$

$$|Z| = \frac{240\ V}{0.6\ A} = 400\ \Omega$$

$$Z = \sqrt{R_{lamp}^2 + X_C^2} \quad \Rightarrow \quad X_C^2 = Z^2 - R_{lamp}^2$$

$$X_C = \frac{1}{\omega C} \quad \Rightarrow \quad C = \frac{1}{\omega X_C} = \frac{1}{(2\pi \times 50)(221.11)} = 14.4\ \mu\text{F}$$

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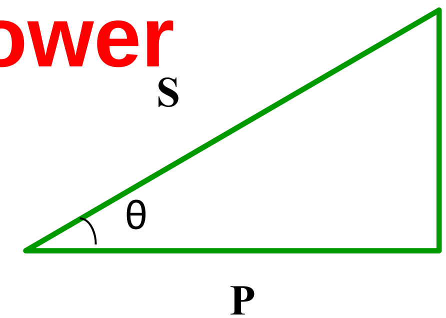
Lecture 4A: Complex Power and Power Factor

Complex Power

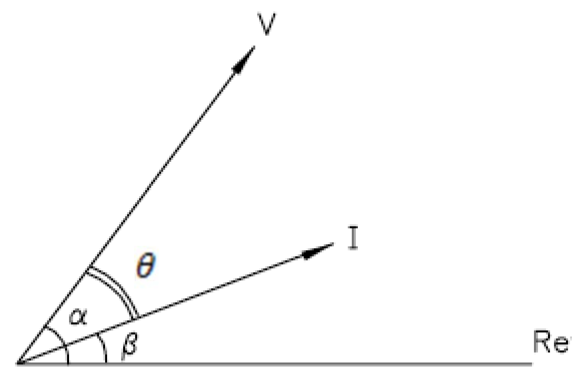
Apparent power, $|S| = V_{rms} I_{rms}$



Complex power can be written in terms of voltage and current phasor:

Complex power, $S = |S|\angle\theta = V_{rms} I_{rms} \angle \theta$

$V = V_{rms} \angle\alpha$ $\quad I = I_{rms} \angle\beta$ $\quad Z = R + jX = |Z| \angle \theta$



$S = V_{rms} \angle\alpha \; I_{rms} \angle\beta$

$$I = \frac{V}{Z} = \frac{V_{rms}\angle\alpha}{|Z|\angle\theta}$$

$I = I_{rms} \angle\alpha-\theta$

Defining I* as the complex conjugate of I, i.e., If $I = I_{rms} \angle\beta$, then $I^* = I_{rms} \angle{-\beta}$

$I^* = I_{rms} \angle{-\alpha+\theta}$

Complex power $S = (V_{rms} \angle\alpha)(I_{rms}\angle\theta-\alpha)$

Or, Complex power $S = V\,I^*$

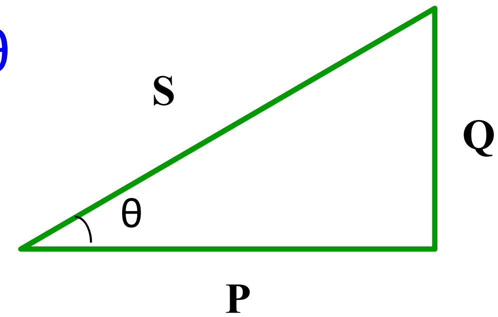
Complex Power

We can show that,

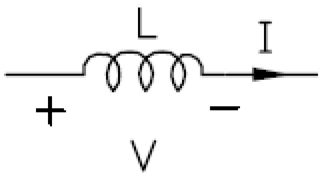
Complex power, $S = V_{rms} I_{rms} \cos\theta + j\, V_{rms} I_{rms} \sin\theta$

Or, $S = |S| \cos\theta + j\, |S| \sin\theta$

$= P + jQ$



S	**Complex power**	VA	$S = \lvert S\rvert\angle\theta = P + jQ = VI^{*} = \; = \lvert V\rvert\lvert I\rvert\angle\theta = \lvert V\rvert\lvert I\rvert(\cos\theta + j\sin\theta)$
$\lvert S\rvert$	**Apparent power**	VA	$\lvert S\rvert = \lvert V\rvert\lvert I\rvert = \sqrt{P^2 + Q^2}$
P	**Active power** Average power, Real power	W	$P = \mathrm{Re}(S) = \lvert S\rvert\cos(\theta) = \lvert V\rvert\lvert I\rvert\cos(\theta)$
Q	**Reactive power**	var	$Q = \mathrm{Im}(S) = \lvert S\rvert\sin(\theta) = \lvert V\rvert\lvert I\rvert\sin(\theta)$



Power in an Inductor

The impedance of a pure inductor is $Z_L = j\omega L = \omega L\angle 90^\circ$

Taking the voltage V as reference phasor,

$$V = |V|\angle 0^\circ$$

$$I = \frac{V}{Z_L} = \frac{|V|\angle 0^\circ}{j\omega L} = \frac{|V|}{\omega L}\angle -90^\circ = |I|\angle -90^\circ$$

So, the complex power in an inductor is $S_L = P_L + jQ_L = V\,I^*$

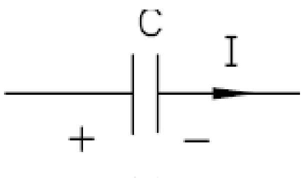
$$= Z_L I \times I^* = j\omega L|I|^2$$

real and reactive power consumed by the inductor

$$P_L = |V||I|\,cos\,90^\circ = 0$$

$$Q_L = |V||I|\,sin\,90^\circ = |V||I| = \omega L|I|^2$$

Note: The magnitude of phasor |V| or |I| is the same as its rms value, and is used interchangeably



Power in a Capacitor

The impedance of a capacitor

$$Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

Taking the voltage V as reference phasor, $V = |V|\angle 0^\circ$

$$I = \frac{V}{Z_C} = \omega C|V|\angle 90^\circ = |I|\angle 90^\circ$$

complex power in a capacitor $S_C = P_C + jQ_C = V\,I^*$

$$= Z_C I \times I^* = -j\frac{|I|^2}{\omega C}$$

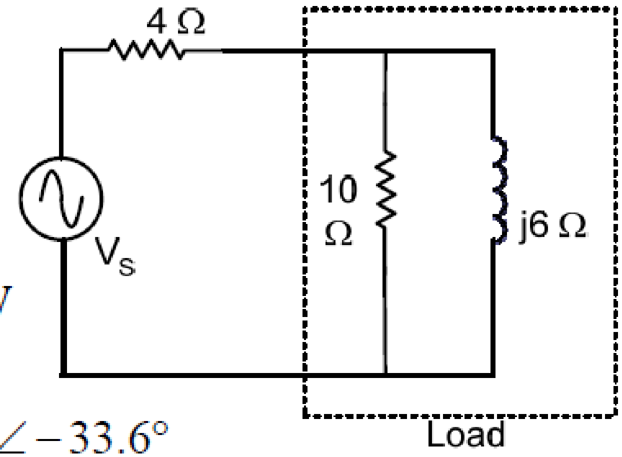
real and reactive power consumed by the capacitor

$$P_C = |V||I|\,cos\,(-90^\circ) = 0$$

$$Q_C = |V||I|\,sin\,(-90^\circ) = -|V||I| = -\frac{|I|^2}{\omega C} = -\omega C|V|^2$$

Reactive power drawn by a capacitor is negative. So, as a convention, it is assumed that a capacitor generates reactive power while an inductor consumes reactive power.

Example 1: A voltage source with series resistor is connected to a parallel combination of inductor and resistor. Find the load voltage and load current. Use these values to find complex power and hence real power and reactive power. Also find power delivered to the load. $V_S = 110\angle 0^\circ$



$$Z_L = (10\Omega) \,||\, (j6\Omega) = \frac{10 \times j6}{10 + j6} = 5.145\angle 59^\circ \Omega$$

Voltage across the load,

$$V_L = \frac{Z_L}{4 + Z_L}(110\angle 0^\circ) = \frac{5.145\angle 59^\circ}{4 + 5.145\angle 59^\circ}(110\angle 0^\circ) = 70.9\angle 25.44^\circ \text{ V}$$

Current flowing through the load, $I_L = \frac{70.9\angle 25.44^\circ}{5.145\angle 59^\circ} = 13.8\angle -33.6^\circ$

Complex power S = $V_L I^*_L = (70.9\angle 25.44^\circ)(13.8\angle 33.6^\circ) = 978\angle 59^\circ$

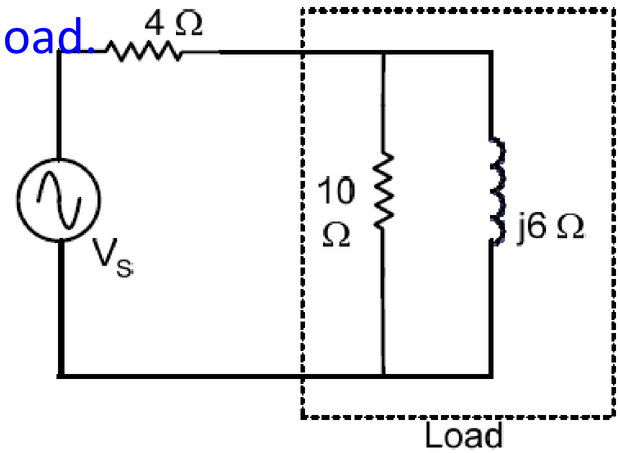
Complex power S = 503 + j839 W

Real power, P = 503 W, Reactive power Q = 839 VAr

Complex power delivered by the source = $V_s I^*_s = (110\angle 0^\circ)(13.8\angle 33.6^\circ) = 1518\angle 33.6^\circ$

$$= 1264 + j838$$

Example 2: Calculate the real and reactive power in the circuit of example 1, if the inductor is removed. Also find power delivered to the load.



$Z_L = 10\ \Omega \qquad V_S = 110\angle 0°$

Voltage across the load,

$$V_L = \frac{Z_L}{4+Z_L}(110\angle 0°) = \frac{10}{4+10}(110\angle 0°) = 78.6\angle 0°$$

Current flowing through the load, $I_L = \dfrac{78.6\angle 0°}{10} = 7.86\angle 0°$

Complex power $S = V_L I^*_L = (78.6\angle 0°)(7.86\angle 0°) = 617\angle 0° = 617 + j0\,\text{W}$

Real power, P = 617 W, Reactive power Q = 0 VAr

Complex power delivered by the source = $V_s I^*_s = (110\angle 0°)(7.86\angle 0°) = 864.6\angle 0°$

Comparing the percentage of real power transferred by the source to the load in examples 1 and 2,

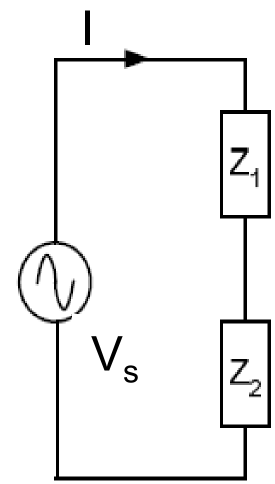
Example 1: $100 \times \dfrac{503}{1264} = 39.8\%$

Example 2 : (without inductor) $100 \times \dfrac{617}{864.6} = 71.4\%$

The reactive element in load makes the transfer of real power from the source to load less efficient!

Complex Power of Series and Parallel-connected loads

Series connected load: If two loads Z_1 and Z_2 are connected in **series** across a voltage source,



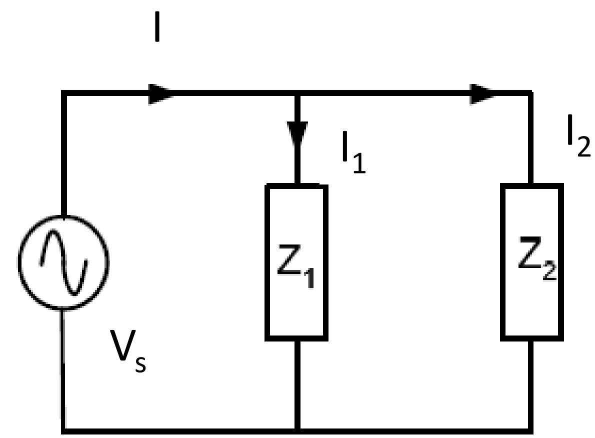
For load 1: $S_1 = V_1 I^* = P_1 + jQ_1$

For load 2: $S_2 = V_2 I^* = P_2 + jQ_2$

Total complex power $S = V_s I^* = (V_1 + V_2) I^* = V_1 I^* + V_2 I^*$

Or, $S = P_1 + jQ_1 + P_2 + jQ_2 = (P_1 + P_2) + j(Q_1 + Q_2)$

Parallel connected load: If two loads Z_1 and Z_2 are connected in **parallel**,



For load 1: $S_1 = V_s I_1^* = P_1 + jQ_1$

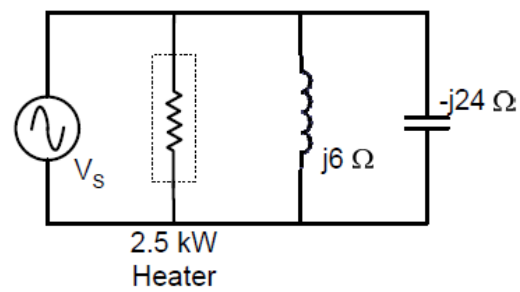
For load 2: $S_2 = V_s I_2^* = P_2 + jQ_2$

Total complex power $S = V_s I^* = V_s (I_1^* + I_2^*)$

Or, $S = P_1 + jQ_1 + P_2 + jQ_2 = (P_1 + P_2) + j(Q_1 + Q_2)$

- **Complex power of a network of interconnected loads is equal to the sum of complex powers of individual loads.**
- **Real (or Reactive) power of a network of interconnected loads is equal to the sum of real (or reactive) powers of individual loads.**

Example 3: A voltage source supplies power to an electric heater (with resistance R), an inductive element and a capacitor as shown below. The supply voltage is 120 V , 50 Hz and the power consumed in the heater is 2.5 kW. Find the reactive power and complex power provided by the source, and the current *i(t)* drawn from the supply.



Find power in each load –

Heater: $P_H = 2.5$ kW $\quad Q_H = 0$ VAR

Inductor: $P_L = 0$ W $\quad Q_L = \frac{120^2}{6} = 2.4 \text{ kVAR}$

Capacitor: $PC = 0$ W $\quad Q_C = -\frac{120^2}{24} = -600 \text{ VAR}$

Total real power: $P = P_H + P_L + P_C = 2.5$ kW

Total reactive power: $Q = Q_H + Q_L + Q_C = 0 + 2400 - 600 = 1.8$ kVAR

Total complex power: $S = P + jQ = 2500 + j1800$

$$|S| = \sqrt{P^2 + Q^2} = 3081 \text{ VA}$$

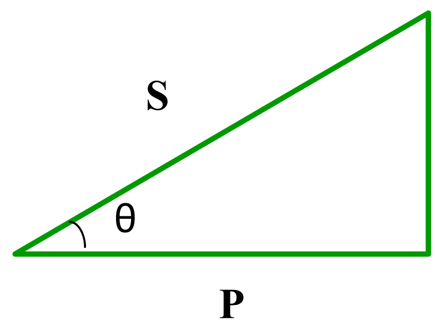
$$I_{rms} = \frac{|S|}{V_{rms}} = \frac{3081}{120} = 25.7 \text{ A}$$

$$i(t) = 25.7\sqrt{2}\cos(314t - 35.75°) \text{ A}$$

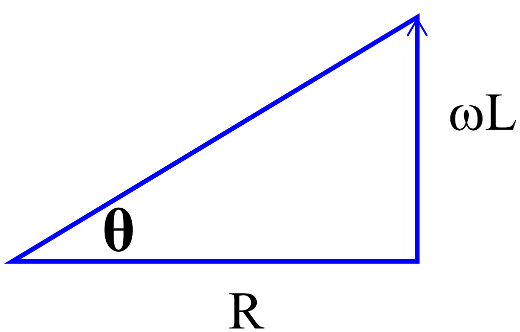
Power Factor of an a.c. circuit

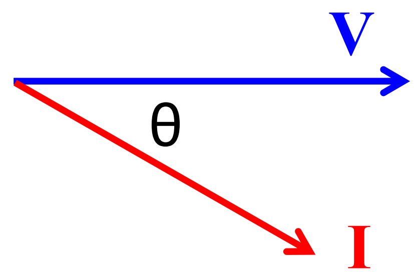
Now, we will define Power Factor of an a.c. circuit as the ratio of real power to the apparent power.

$$\text{Power Factor} = \frac{P}{|S|} = \frac{P}{|V|\,|I|} = cos\ \theta$$



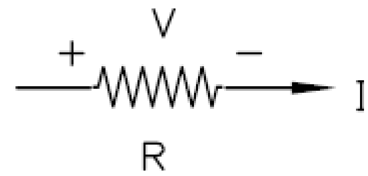
Power factor angle (θ) is the same in power triangle, impedance triangle, and the angle between voltage and current.





Unity Power Factor

Consider a device that is purely resistive, i.e. Z = R.

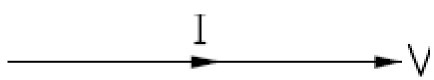


Taking voltage V as reference phasor, we have
$V = |V| \angle 0^o$

Both the voltage and current are in phase. So, the power in a resistor is

$$P = |V|\,|I|\,cos\,0^\circ = |V|\,|I|$$

The power factor of a purely resistive network is unity.



Lagging Power Factor

Consider an inductive device or load, the impedance of which is given by

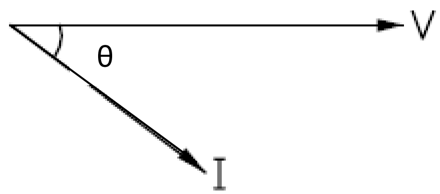
$$Z = R + j\omega L = R + jX_L = |Z| \angle \theta$$

Taking voltage V as reference phasor, we have

$$V = |V| \angle 0^\circ$$

$$I = \frac{|V| \angle 0^\circ}{|Z| \angle \theta} = \frac{|V|}{|Z|} \angle -\theta = |I| \angle -\theta$$

As the impedance angle is positive, the current is lagging behind the voltage as shown in the phasor diagram below:



The real power in the device is

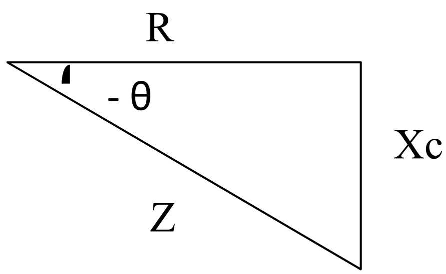
$$P = |V|.|I| \cos\theta$$

Power factor is said to be lagging, if the current lags behind the voltage. In other words, p.f. is lagging when the impedance angle θ is positive, i.e. When the load or device is inductive in nature.

Leading Power Factor

Consider a capacitive device or load, the impedance of which is given by

$$Z = R - j\frac{1}{\omega C} = R - jX_C = |Z| \angle -\theta$$



Taking the voltage V as reference phasor,

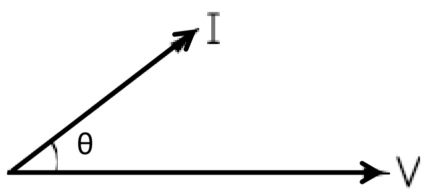
$$V = |V| \angle 0°$$

$$I = \frac{|V| \angle 0°}{|Z| \angle -\theta} = \frac{|V|}{|Z|} \angle \theta = |I| \angle \theta$$

As the impedance angle is negative, the current is leading the voltage as shown in the phasor diagram below:

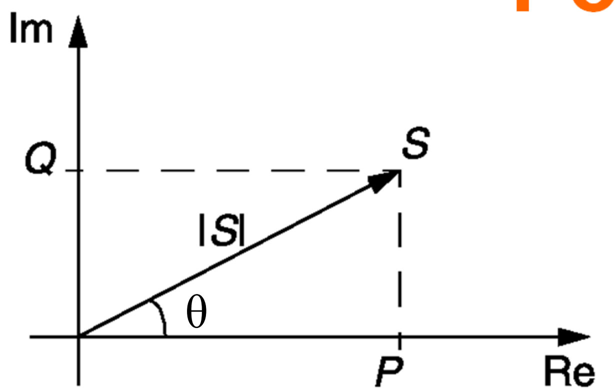
The real power in the device is

$$P = |V|.|I|cos\theta$$



Power factor is said to be leading, if the current leads the voltage. In other words, p.f. is leading when the impedance angle θ is negative, i.e. when the load or device is capacitive in nature.

Power Factor

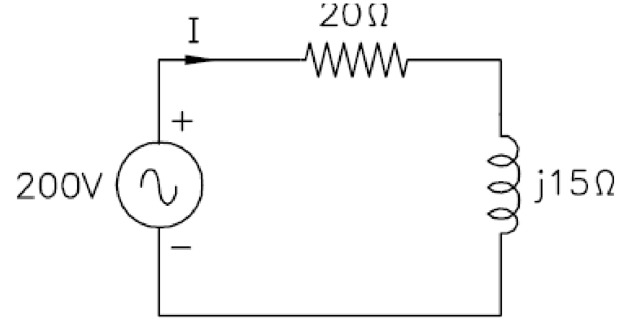


$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$$\cos(\theta) = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}}$$

$$\cos(\theta) = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$$

Example 4: Consider the simple AC circuits shown below. Determine the apparent, real and reactive power delivered by the source. What is the power factor of the circuit?



Taking voltage as the reference phasor,

$$V = 200\angle 0^\circ$$

$$Z = 20 + j15 = 25\angle 36.87^\circ$$

$$I = \frac{V}{Z} = \frac{200}{25\angle 36.87^\circ} = 8.0\angle -36.87^\circ \text{ A}$$

$$S = VI^* = 200 \times 8.0\angle 36.87^\circ$$

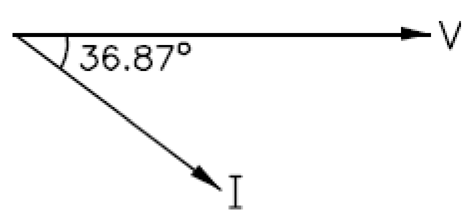
$$= 1600\angle 36.87^\circ = 1280 + j960$$

$$\text{Apparent Power} = 1600 \text{ VA}$$

$$\text{Real Power} = 1280 \text{ W}$$

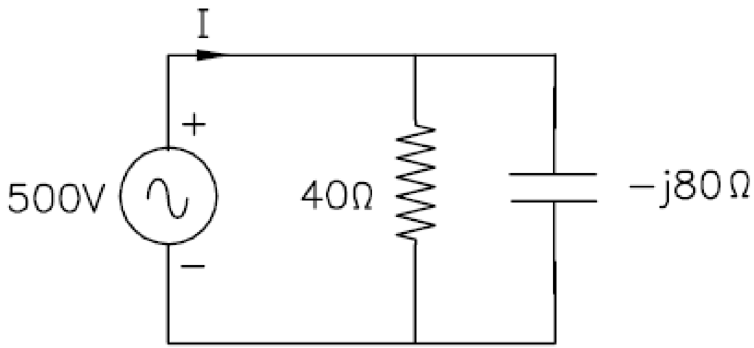
$$\text{Reactive Power} = 960 \text{ VAr}$$

$$\text{Power factor} = cos\,36.87^\circ = 0.8 \text{ lagging}$$



Example 5: In the parallel R-C circuit shown below, determine the apparent power, real power and reactive power delivered by the source.

What is the power factor of the circuit?



$$V = 500\angle 0^\circ$$

$$Y = Y_1 + Y_2 = \frac{1}{40} + \frac{1}{-j80}$$

$$= 0.025 + j0.0125 = 0.02795\angle 26.57^\circ$$

$$I = YV = 13.975\angle 26.57^\circ \text{ A}$$

$$S = VI^* = 500 \times 13.975\angle -26.57^\circ$$

$$= 6987.5\angle -26.57^\circ \quad = 6250 - j3125$$

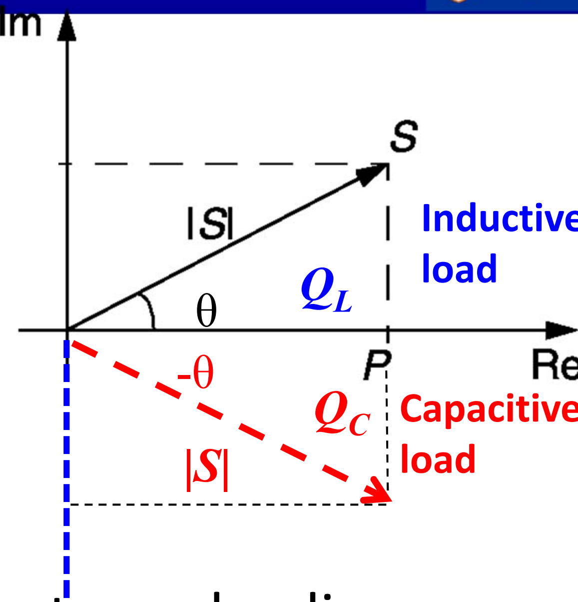
$$\text{Apparent Power} = 6987.5 \text{ VA}$$

$$\text{Real Power} = 6250 \text{ W}$$

$$\text{Reactive Power} = -3125 \text{ VAr}$$

$$\text{Power factor} = cos\left(-26.57^\circ\right) \quad = 0.894 \text{ leading}$$

Power Factor



$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$0 \leq \theta \leq 90^o$

Hence, power factor lies between: $0 \leq$ p.f. ≤ 1

Since cos (+θ) = cos (-θ), we identify the power factor as leading or lagging.

For inductive load: Current lags behind voltage
Hence, power factor is lagging p.f.
For capacitive load: Current leads the voltage
Hence, power factor is leading p.f.

For example: If the load is $10\angle 25^o$ the power factor is 0.906 lagging
If the load is $20\angle -30^o$ the power factor is 0.87 leading

Power Factor Correction

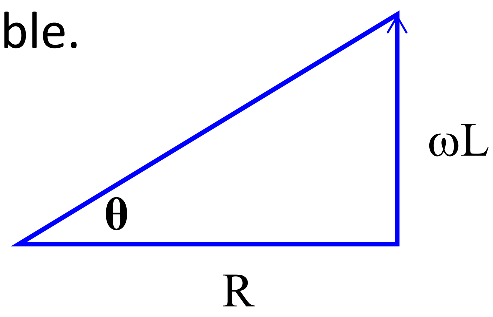
Loads are usually connected to a fixed voltage supply, e.g. 220V, 50 Hz in Singapore, hence V_{rms} is given.

$$p.f. = \frac{P}{V_{rms} I_{rms}} \qquad Or, I_{rms} = \frac{P}{V_{rms} \times \cos\theta}$$

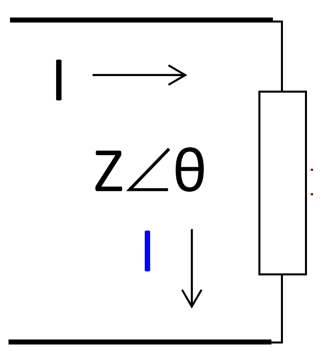
To deliver a certain amount of power to the load, current will be larger if power factor is smaller.

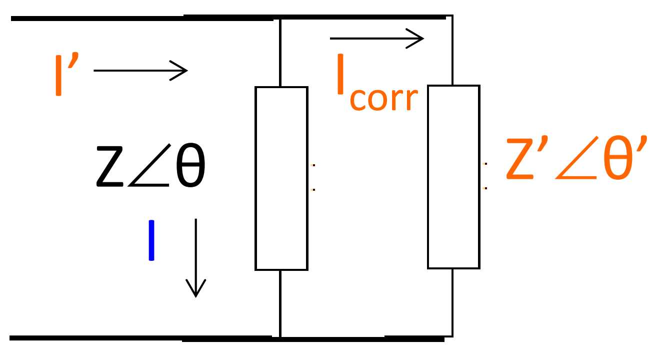
Hence it is desirable that power factor be as close to 1 as possible.
i.e. θ should be as small as possible.

Once the load is connected, its θ cannot be changed.



Another reactive element can be added parallel to the load to improve power factor.





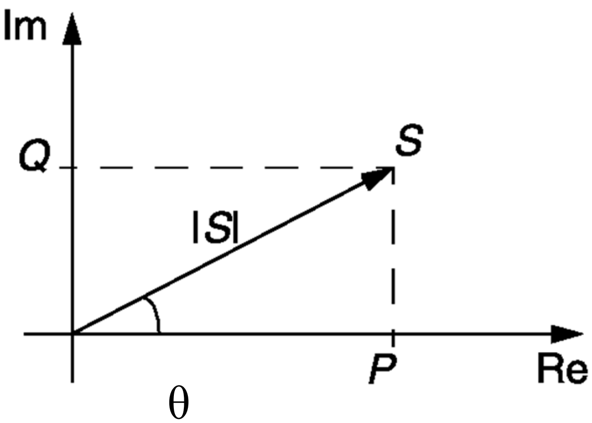
If the load is originally inductive, choose a capacitor
If the load is originally capacitive, choose an inductor as correcting device

Power Factor Correction

If the power factor is poor (i.e., very low), current drawn by the load will be high.

This will result in

- Poor voltage regulation at the load
- Heavy transmission lines losses
- High operating cost



$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$$|I| = P/|V|\cos(\theta)$$

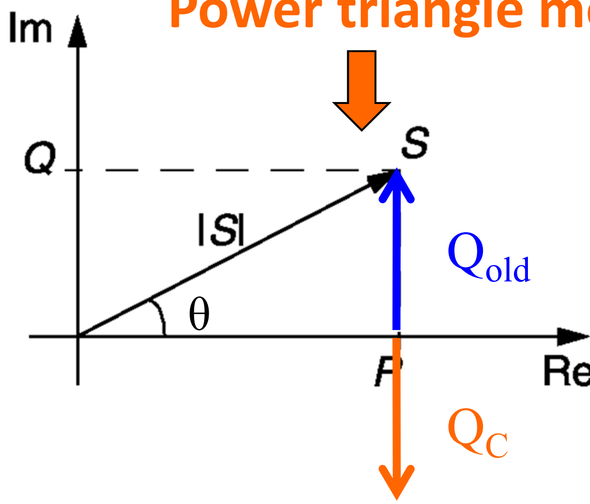
A capacitor can be added in parallel to the load for power factor correction.

There are various ways to find the value of capacitor:

- **Impedance triangle method**
- **Power triangle method**
- **Power equations**

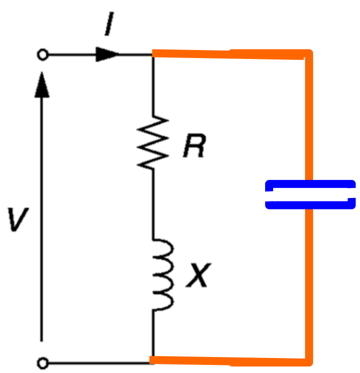
Power Factor Correction

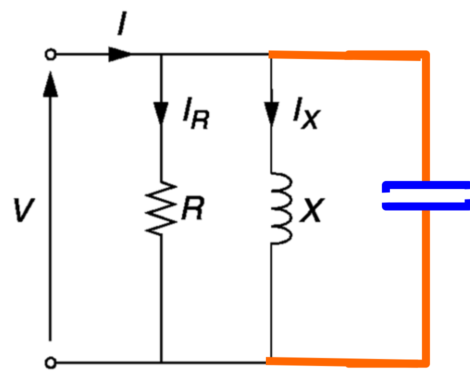
Power triangle method



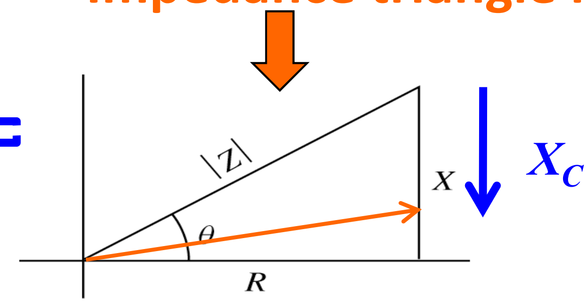
$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$$|I| = P/|V|\cos(\theta)$$

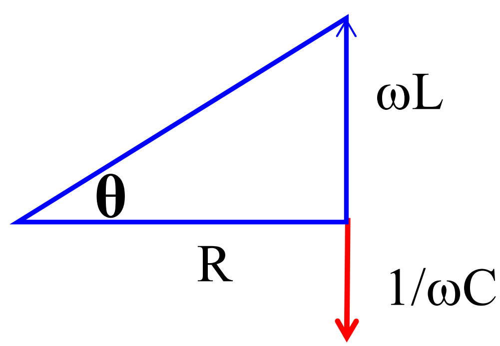




Impedance triangle method

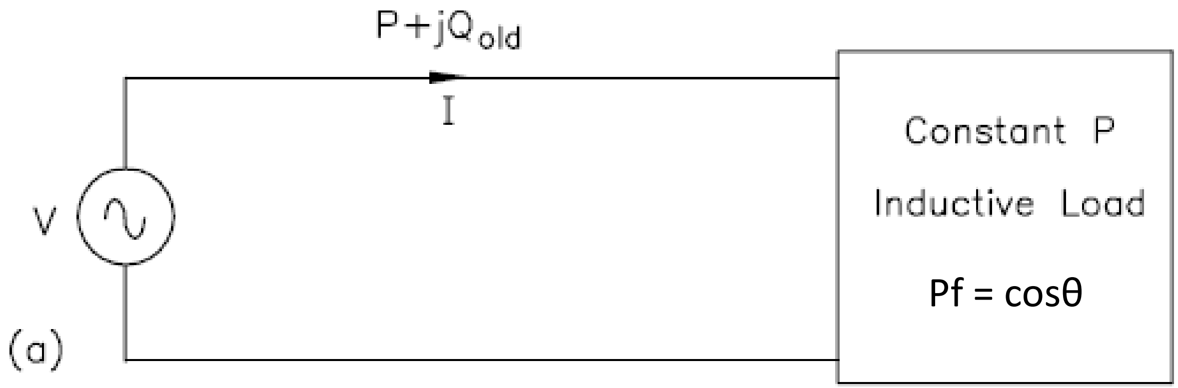


The load impedance in the complex plane.



Power Factor Correction

If P is the active power consumption of the load, the current drawn by the load can be calculated as:



$$P=|V|.|I|\cos\theta$$

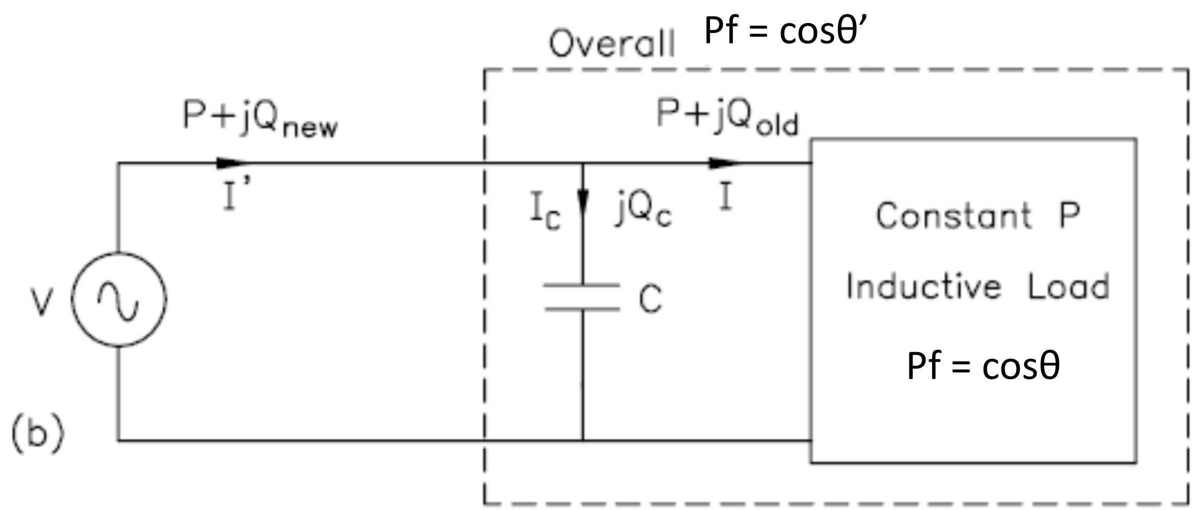
$$|I| = \frac{P}{|V|\cos\theta}$$

$$I = |I|\angle -\theta$$

The apparent power:

$$S_{old} = VI^* = P+jQ_{old}$$

When a capacitor is connected to improve the power factor, the current drawn by this capacitor: $I_c = j\omega CV = \omega CV \angle 90^o$



Applying KCL, the total current drawn by the load is:

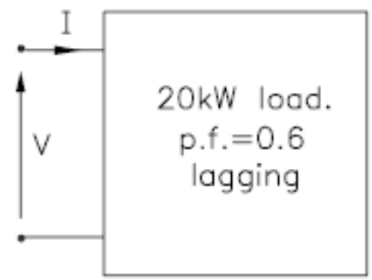
$$I' = I + I_c$$

$$P=|V|.|I'|\cos\theta'$$

$$S_{new} = VI'^* = P+jQ_{new}$$

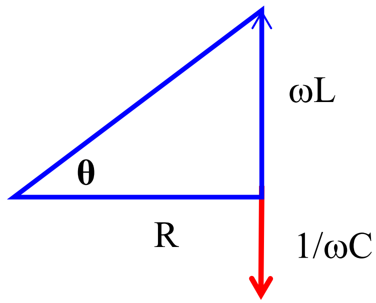
Example 6 - Power Factor Correction

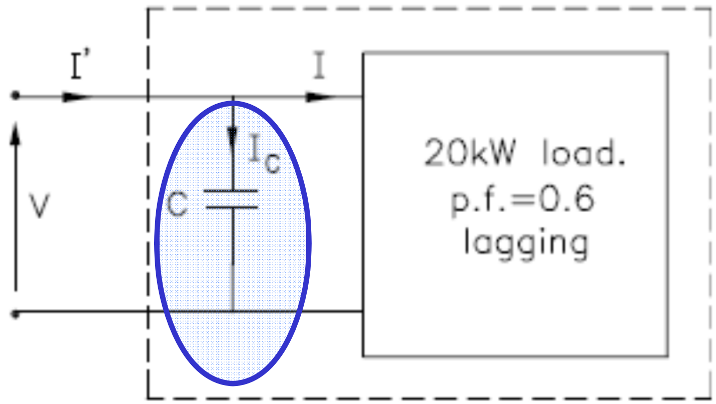
A load connected across a 230V, 50Hz line draws 20kW at 0.6 p.f. lagging. Determine the current drawn by the load. If a capacitor of 800 μF is connected in parallel with the load, what will be the current drawn from the source? Also determine the overall power factor of the system (load and capacitor connected in parallel) as seen by the source.



$$V = 230\angle 0^\circ$$

$$|I| = \frac{20000}{230 \times 0.6} = 144.93 \text{ A}$$





As cos θ = 0.6 lagging, the impedance angle θ is

$\theta = \cos^{-1} 0.6 = 53.13^\circ$

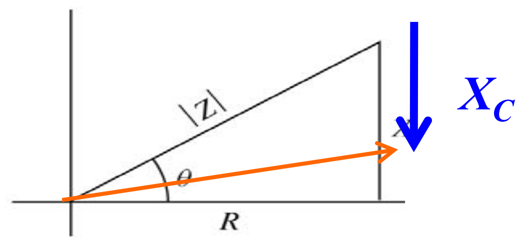
$I = 144.93 \angle -53.13 = 86.96 - j\,115.94$ A

$I_C = j\omega CV = j\,314 \times 800 \times 10^{-6} \times 230 = j57.78$

$I' = I + I_C = 86.96 - j\,115.94 + j57.78$

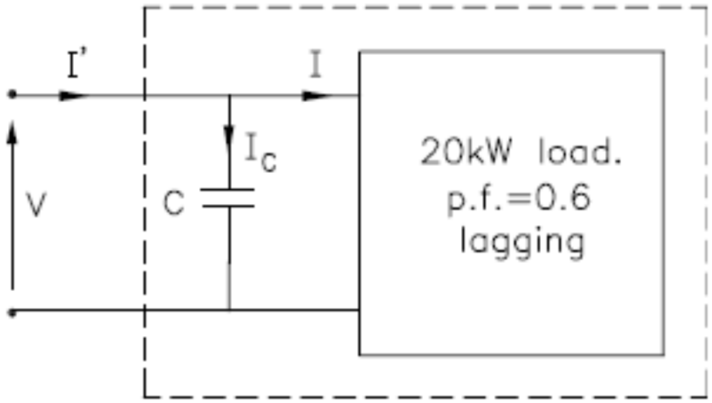
$= 104.62 \angle -33.78^\circ$

Power factor = cos 33.78° = 0.83 lagging



The load impedance in the complex plane.

How to Determine C, given cosθ' ?



$S_{new} = P+jQ_{new} = P+ jQ_{old} +jQ_c$

$Q_{new} = Q_{old} + Q_c$

Or, $Q_c = Q_{new} - Q_{old}$

So, the capacitance C needed to improve the power factor of the load can be calculated as

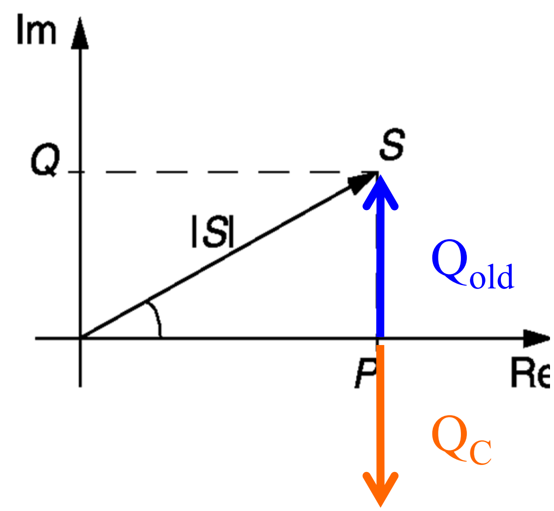
$$\mathbf{Q_c = -\omega C |V|^2}$$

If the overall power factor is improved to unity,

$$\text{Cos } \theta' = 1$$

$$S_{new} = |V|.|I'|\cos\theta' + j\,|V|.|I'|\sin\theta' = P+jQ_{new}$$

$$Q_{new} = |V|.|I'|\sin\theta' = 0$$



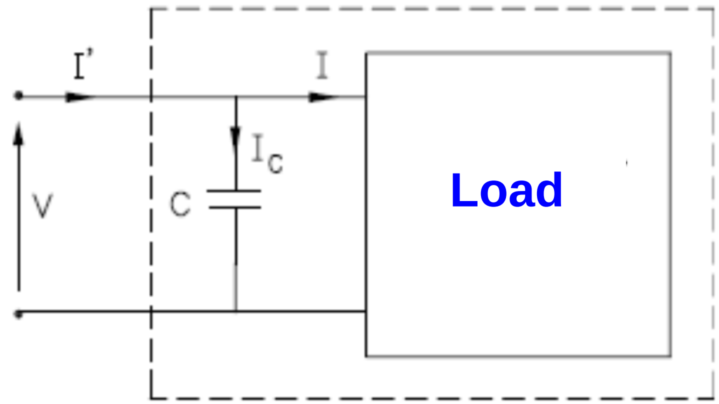
Then capacitor supplies 100% of the reactive power required by the load.

Example 2 - Power Factor Correction

A load connected across a 200 V, 50Hz line draws 10 kW at 0.5 power factor lagging. Determine the current drawn by the load. A capacitor C is now connected in parallel with the load to improve the power factor. What must be the value of C to make the overall power factor

(i) 0.9 lagging, (ii) unity and (iii) 0.8 leading?

Solution: Take source voltage as reference. So, current drawn at 0.5 p.f. lagging is



$$V = 200\angle 0^\circ$$

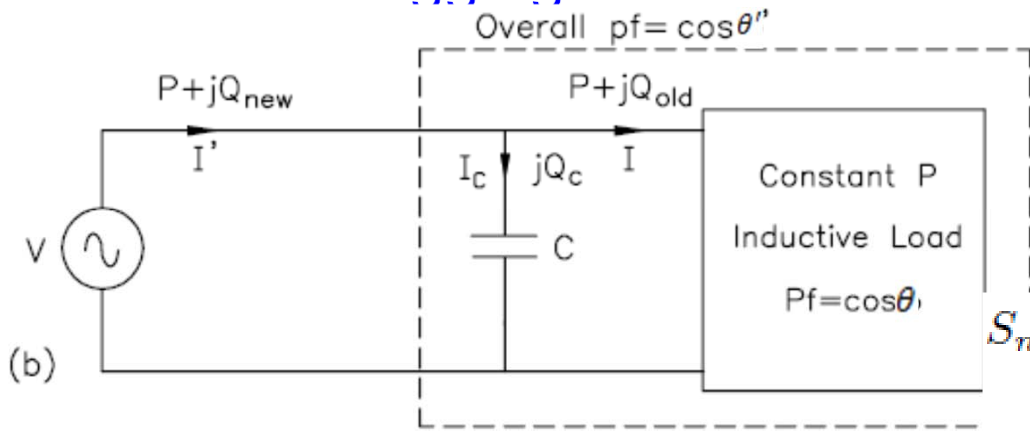
$$|I| = \frac{P}{|V|cos\theta} = \frac{10000}{200 \times 0.5} = 100 \text{ A}$$

$$\theta = cos^{-1}0.5 = 60^\circ$$

$$I = 100\angle -60^\circ$$

$$S_{old} = P + jQ_{old} = V\,I^* = 200 \times 100\angle 60^\circ = 10000 + j17320.5$$

Case 1: 0.9 lagging



$$|I'| = \frac{P}{|V|\,cos\,\theta'} = \frac{10000}{200 \times 0.9} = 55.56 \text{ A}$$

$$\theta' = cos^{-1}0.9 = 25.84^\circ$$

$$I' = 55.56\angle{-25.84^\circ}$$

$$S_{new} = P + jQ_{new} = V\,I'^*$$

$$= 200 \times 55.56\angle 25.84 = 10000 + j4843.3$$

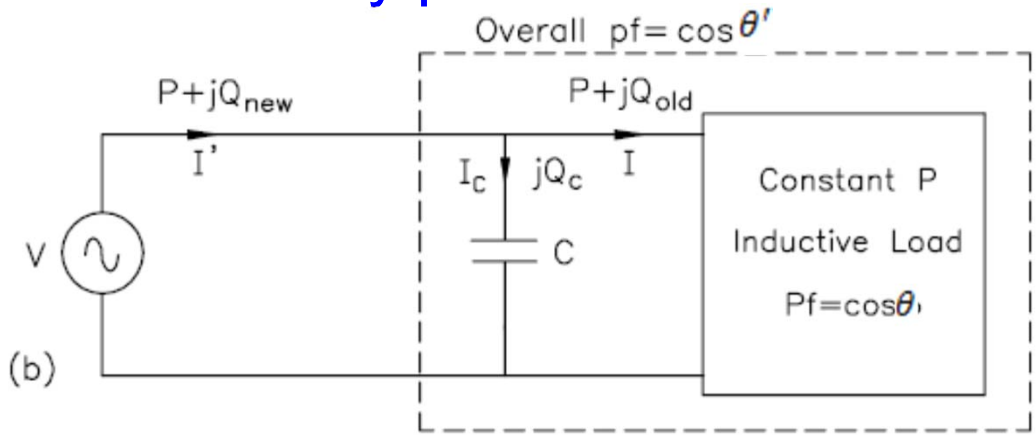
Hence, Q_C and C may be obtained from

$$Q_C = Q_{new} - Q_{old}$$

$$Q_C = -\omega C|V|^2 = -12476.7 \text{ VAr}$$

$$C = \frac{12476.7}{314 \times 200^2} = 993.4 \;\; \mu F$$

Case 2: Unity p.f.



$$S_{new} = P + jQ_{new} = 10000 + j0$$

$$Q_{new} = 0$$

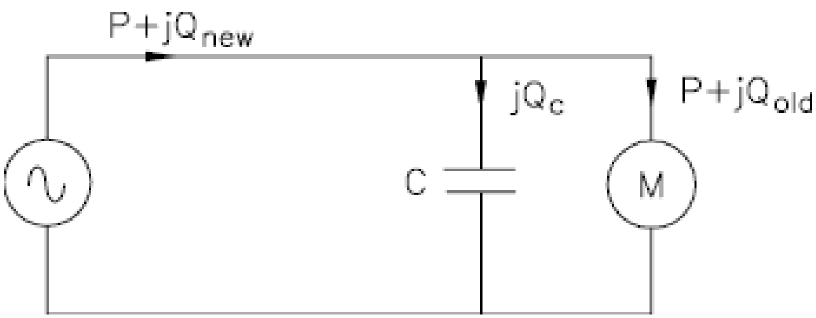
$$Q_C = Q_{new} - Q_{old} = -Q_{old}$$

$$= -\omega C|V|^2$$

$$C = \frac{17320.5}{314 \times 200^2} = 1378.3 \ \ \mu F$$

Example 3

A 5000 W electric motor is connected to a source of 230 V, 50 Hz and the result is a lagging power factor of 0.8. To correct the power factor to 0.95 lagging, a capacitor is placed in parallel with the motor. Calculate the current drawn from the source with and without capacitor. Determine the value of the capacitor required to make the correction.



$$|I| = \frac{P}{|V|\cos\theta} = \frac{5000}{230 \times 0.8} = 27.174 \text{ A}$$

$$\theta = cos^{-1} 0.8 = 36.87^\circ$$

$$I = 27.174\angle{-36.87^\circ}$$

$$S_{old} = V\,I^* = P + jQ_{old}$$

$$= 230 \times 27.174\angle 36.87^\circ = 5000 + j3750$$

$$|I'| = \frac{P}{|V|\cos\theta'} = \frac{5000}{230 \times 0.95} = 22.883 \text{ A}$$

$$\theta'' = cos^{-1} 0.95 = 18.19^\circ$$

$$I' = 22.883\angle{-18.19^\circ}$$

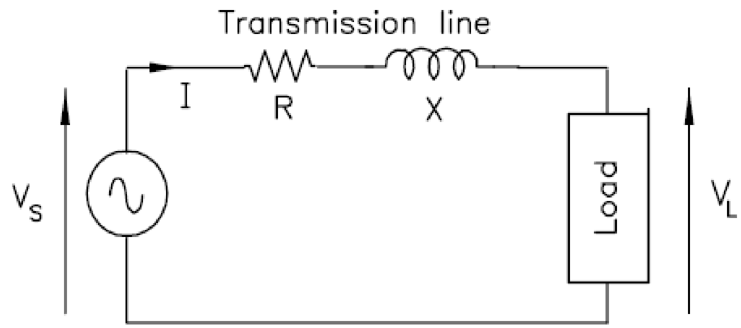
$$S_{new} = V\,I'^* = P + jQ_{new} = 230 \times 22.883\angle 18.19^\circ = 5000 + j1643$$

$$Q_C = Q_{new} - Q_{old}$$

$$Q_C = -\omega C|V|^2 = -2107 \text{ VAr}$$

$$C = \frac{2107}{100\pi \times 230^2} = 126.85 \ \mu F$$

Transmission Line Loss



Industrial load connected to a substation

V_S = Substation or sending end voltage.

V_L = Voltage at the load.

I = Current drawn by the load.

R = Resistance of the transmission line.

X = Reactance of the transmission line.

The transmission line loss is given by: $P_{loss} = |I|^2 R$

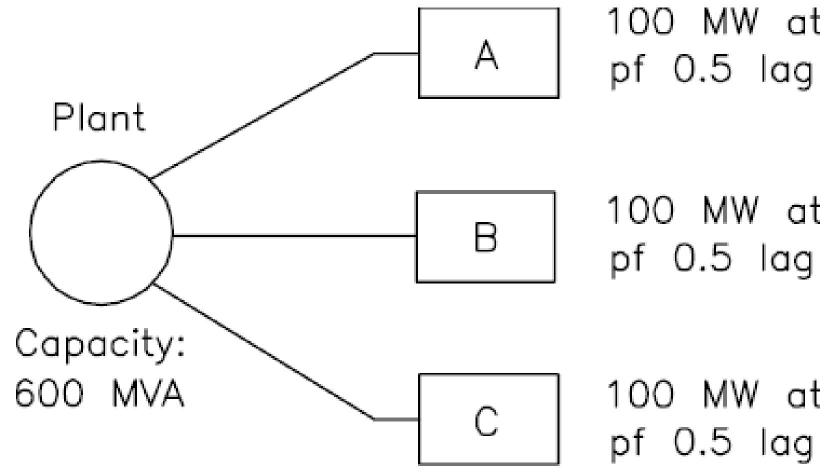
For the same real power demand, if the power factor of the load decreases, then |I| increases as shown by the following equation. This results in heavy line losses.

$$|I| = \frac{P}{|V|cos\theta}$$

A poor power factor results in higher current and hence higher power loss.

Capital Cost of Power Plants

Three industrial customers A, B and C are drawing power from the Power Plant. Let the load at each of the three industries be 100 MW at a p.f. of 0.5 lagging. The maximum demand of each consumer is



$$\text{Max. Demand} = |V||I| = \frac{P}{cos\ \theta} = \frac{100}{0.5} = 200\ \text{MVA}$$

So the installed capacity of the plant should be

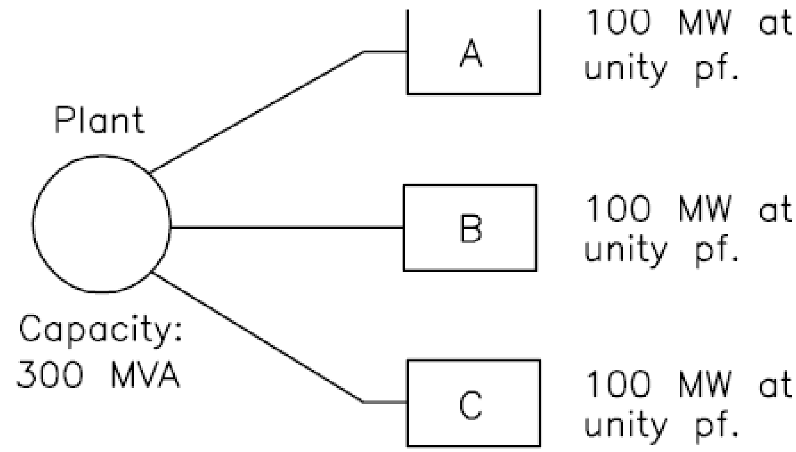
$$\sum \text{Max. Demand} = 600\ \text{MVA}$$

If \$X is the capital cost per annum per MVA of the plant, the total annual capital cost is

Capital cost = \$600X

The three consumers will share the capital cost equally in this case and each will pay \$200X.

If the load at each of the three industries is 100 MW at unity p.f., the maximum demand of each consumer is



$$\text{Max. Demand} = |V||I| = \frac{P}{cos\ \theta} = \frac{100}{1.0} = 100 \text{ MVA}$$

So the installed capacity of the plant should be

$$\sum \text{Max. Demand} = 300 \text{ MVA}$$

As the plant capacity is only 300 MVA, the total annual capital cost in this case is

Capital cost = $300X

The three consumers will share the capital cost equally in this case also and each will pay $100X.

Two points become evident from this example:

- Plant capacity gets affected by the maximum demand of each consumer.
- Annual capital cost gets affected by the power factor of the load.

A poor power factor results in higher capital cost.