EE2022 Electrical Energy Systems

Lecture 6:
Three-Phase Circuit Analysis

Learning Outcomes

- To calculate the complex power, voltages and currents in single phase and balanced three-phase AC circuits and able to describe their relationships using Phasor diagrams.
  - Be able to solve balanced three-phase circuit problems.
  - Be able to calculate three-phase power.

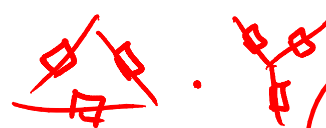
Outline

$\sqrt{2} \rightarrow X_{rms} = \frac{X_m}{\sqrt{2}}$

- Three-Phase Circuit Analysis
  - Generation, Transmission, and Distribution.
  - Three-phase balanced systems.
  - Advantages of three-phase balanced systems.
- Three-Phase voltage and current
  - Line-to-neutral voltage
  - Line-to-Line voltage
  - Line current.
  - Delta/Wye configuration.

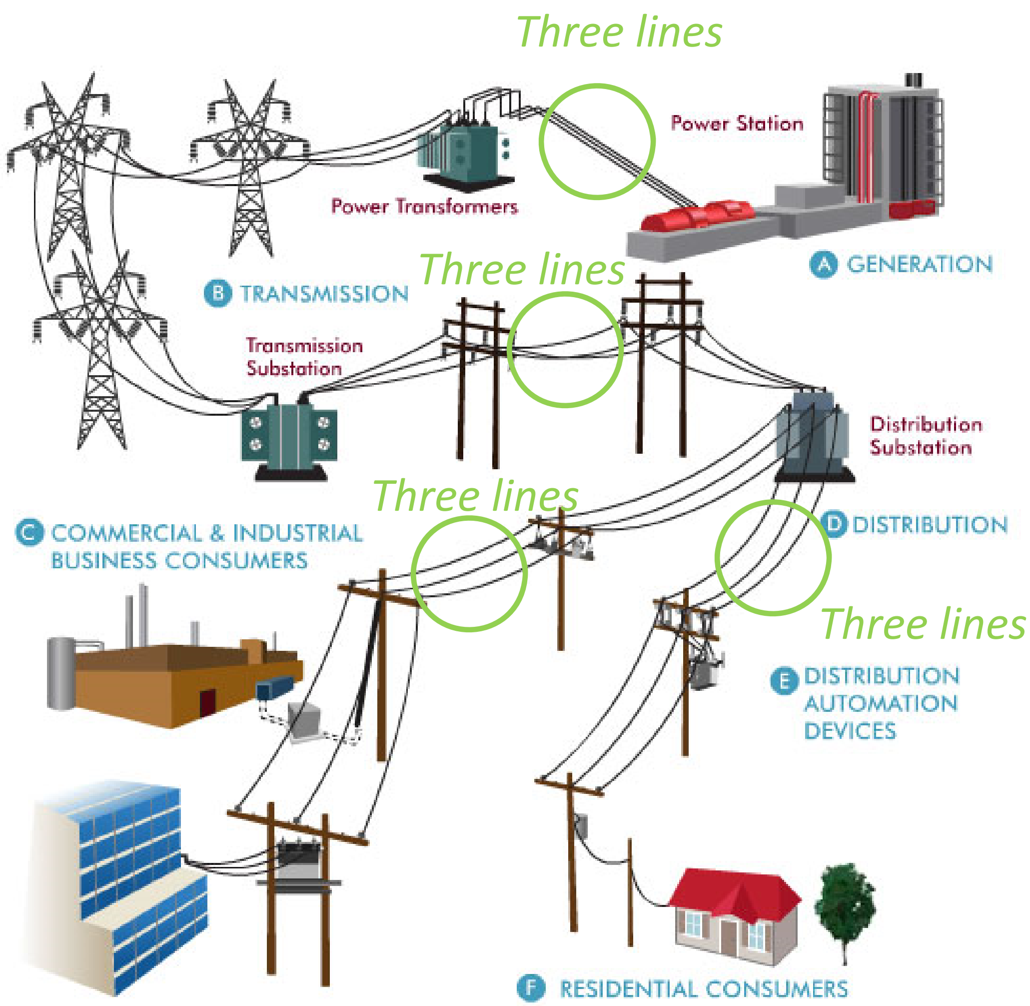
Line voltage/current

phase -11-



line . $\sqrt{3}$. phase

Generation, Transmission and Distribution



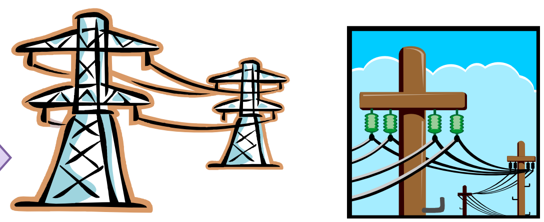
Source: http://venturebeat.com/2010/10/29/super-grid-introduction/

A Three-Phase Circuit System

Generation (11 – 36 KV)

3-Phase Generation system.

Transmission (110 – 765 KV)



3-Phase Transmission system.

Industrial customer (23 – 138 KV)
Commercial customer (4.16 – 34.5 KV)
Residential customer (120 – 240 V)

Generation

Three-phase voltage source

Transmission and Distribution

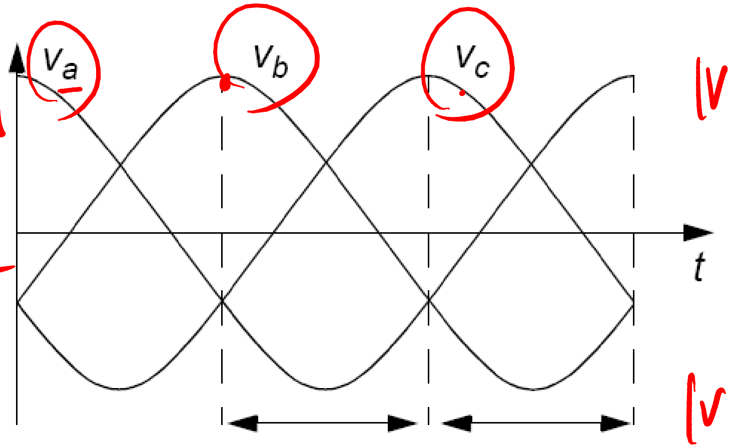
Three transmission lines

Load

Three-phase load

Three-Phase Voltage Sources

abc
bca
cab
acb
cba
bac



$$|V|\angle 0 \leftarrow v_a = \boxed{\sqrt{2}|V|}\cos(\omega t) \quad \text{phase } \theta = 0$$

$$|V|\angle -120^\circ \quad v_b = \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right) \quad \theta = -120^\circ$$

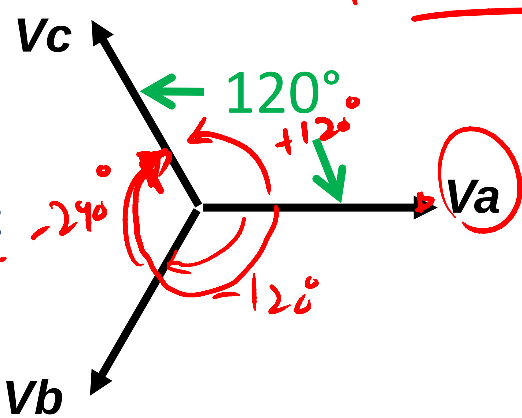
$$|V|\angle 240^\circ \quad v_c = \sqrt{2}|V|\cos\left(\omega t - \frac{4\pi}{3}\right) \quad \theta = -240^\circ$$

$V_b \quad V_a \quad V_c$

Phasor diagram

$\frac{2\pi}{3}$ radian = 120°

Phase sequence: abc (positive)

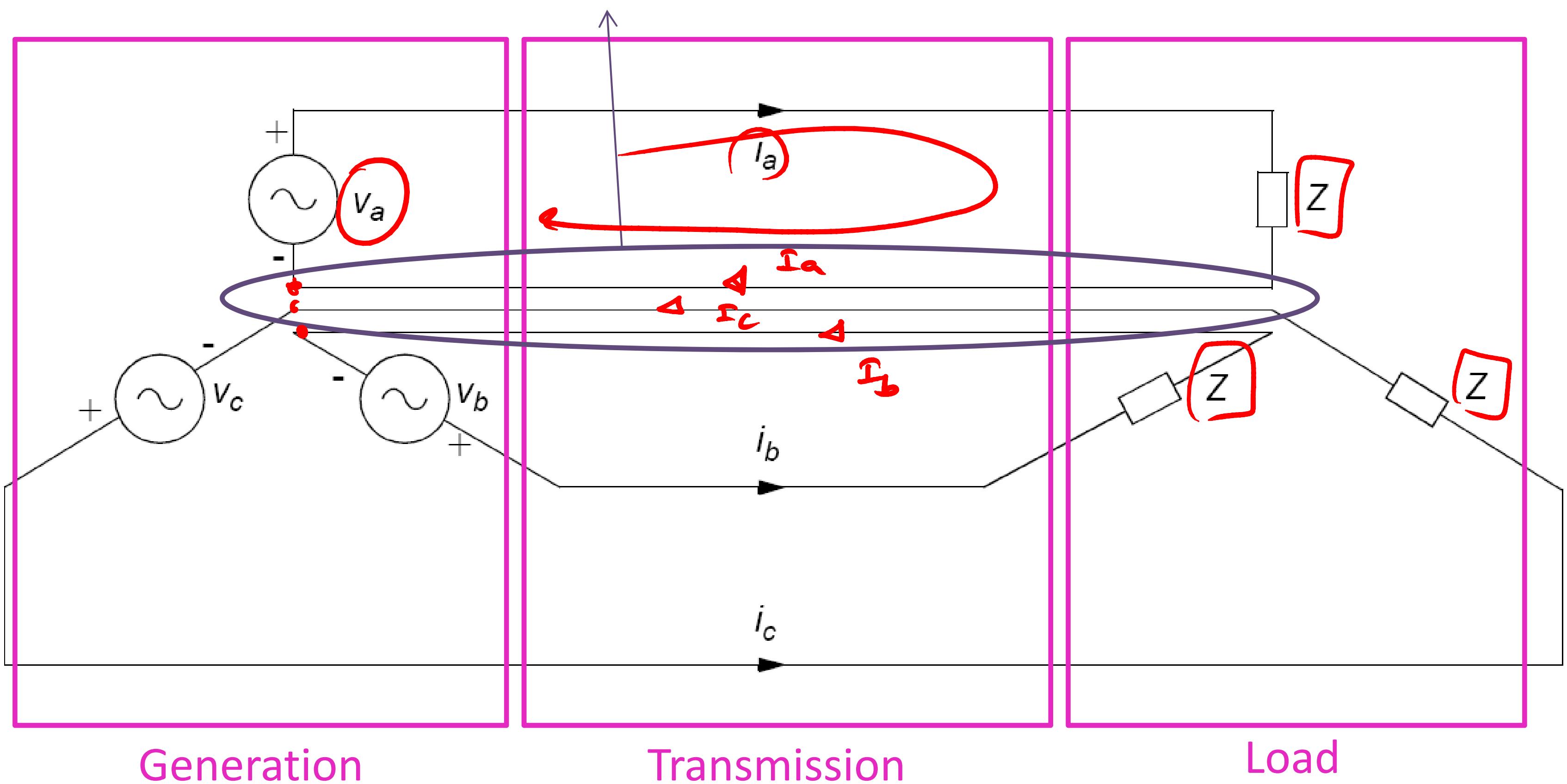


All three voltage sources have the same voltage magnitude,

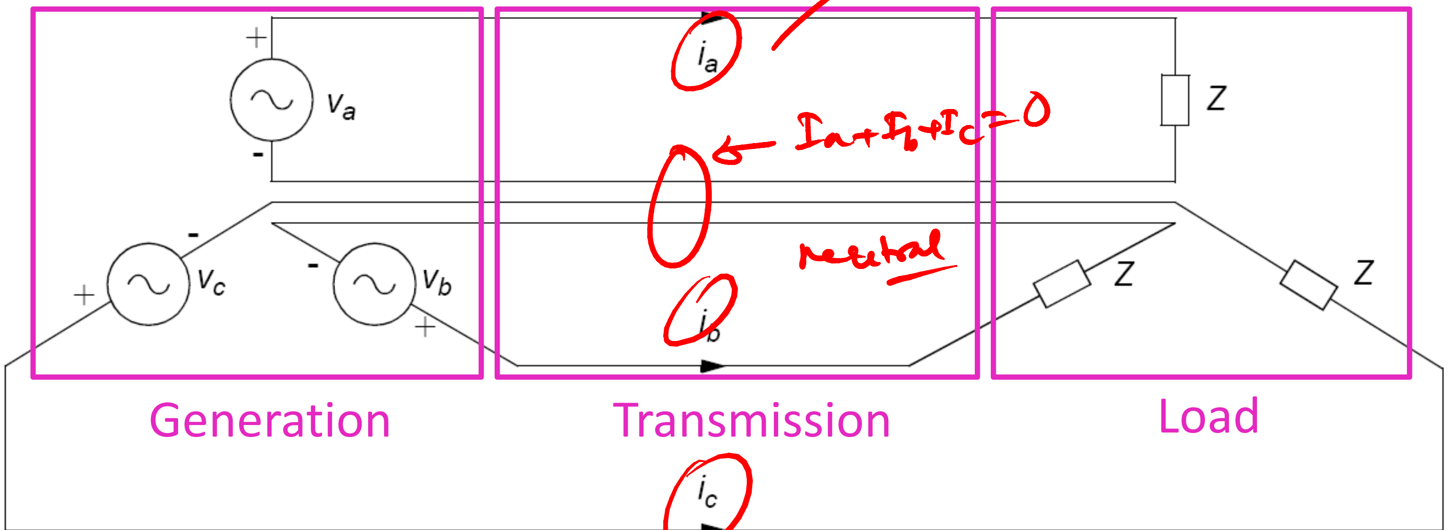
with 120 degrees apart.

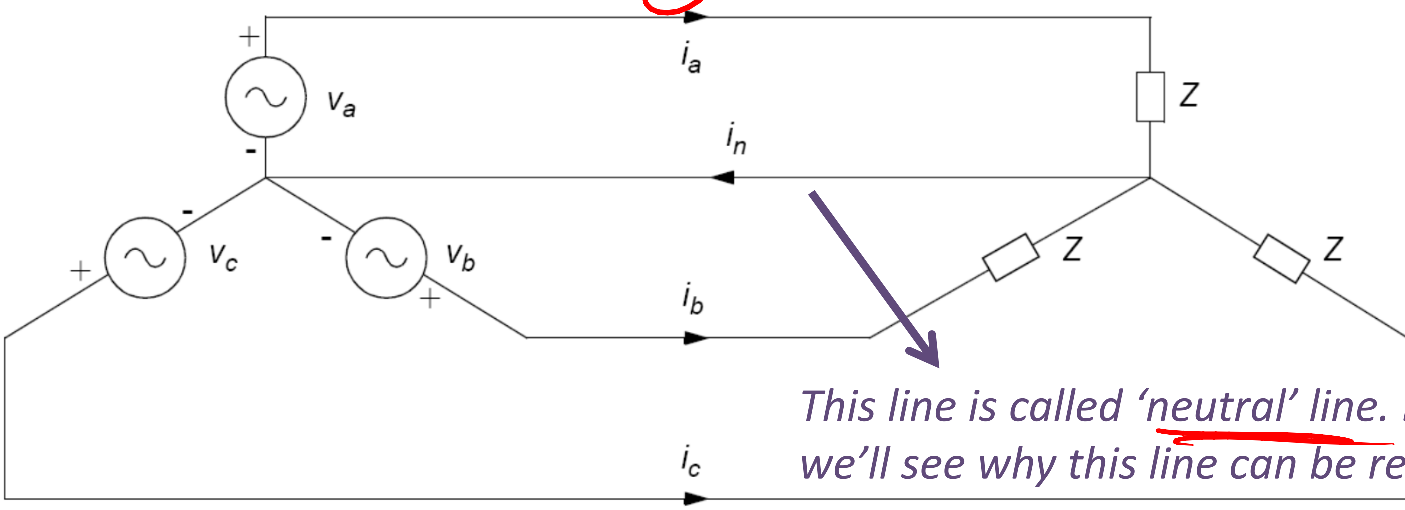
Three Single-Phase Circuits

These lines can be combined.



Three Phase Circuit

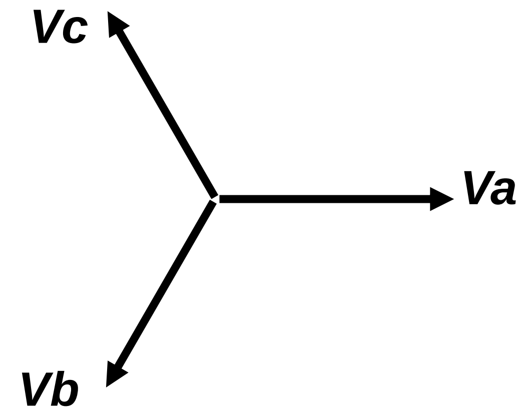




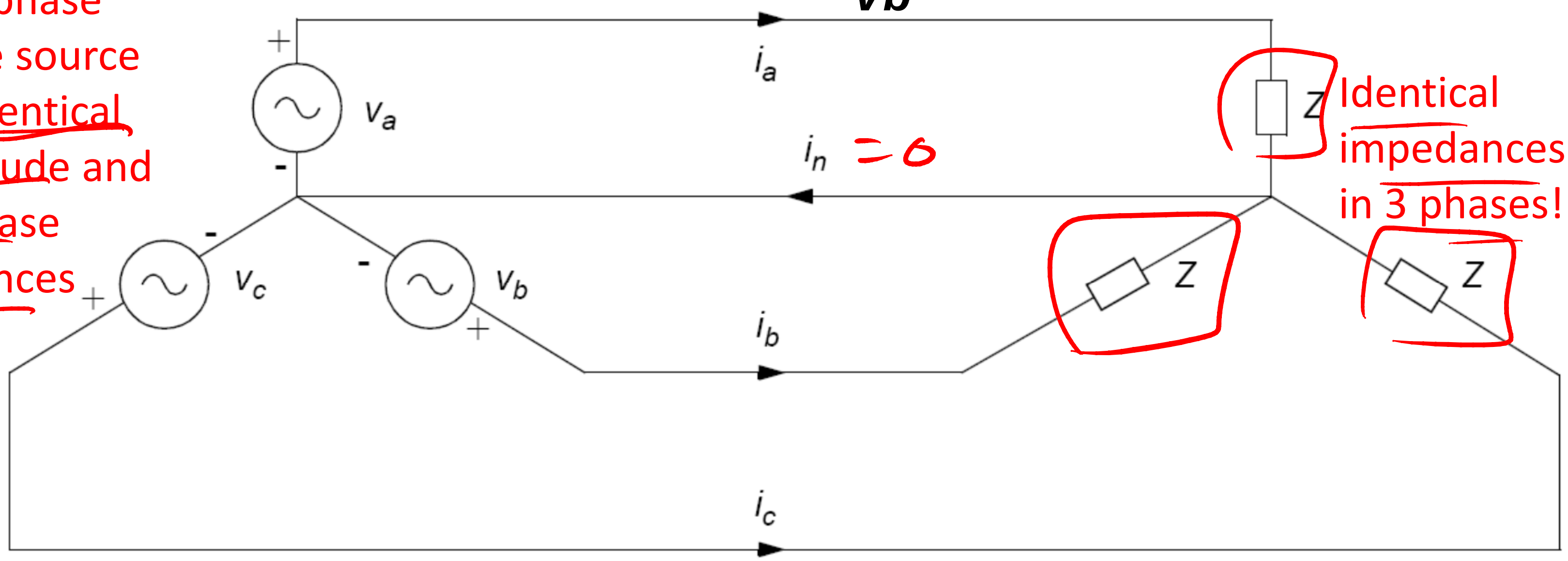
This line is called 'neutral' line. Later on, we'll see why this line can be removed.

Balanced Three-Phase Circuit

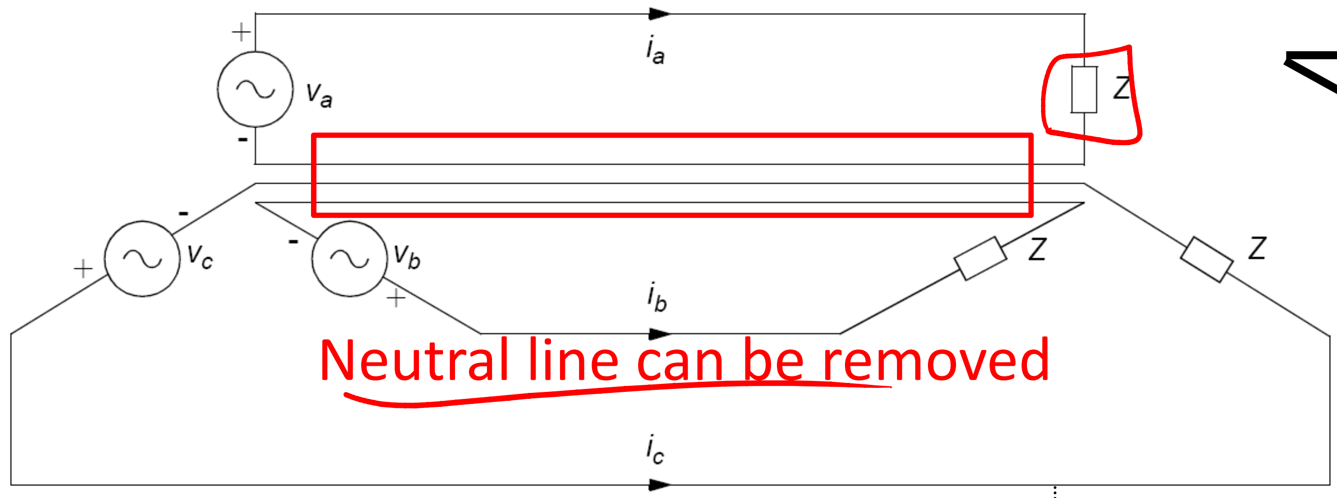
Three-phase circuit is said to be balanced when the impedances in the 3 phases are identical.

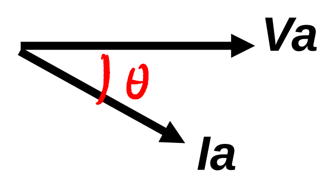


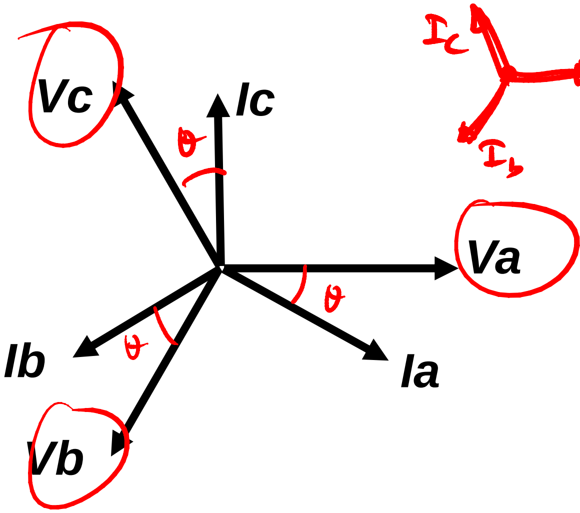
Three-phase voltage source with identical magnitude and 120 phase differences

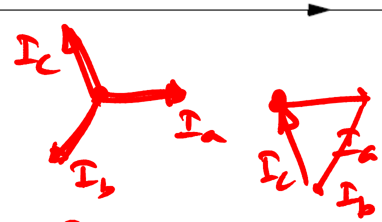


Balanced Three-Phase Circuit

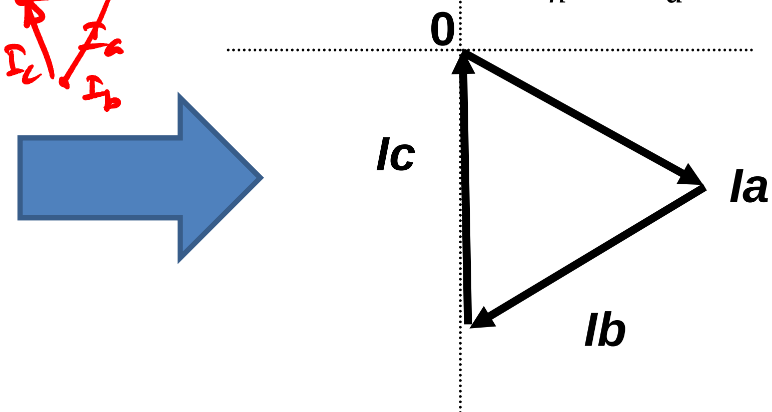






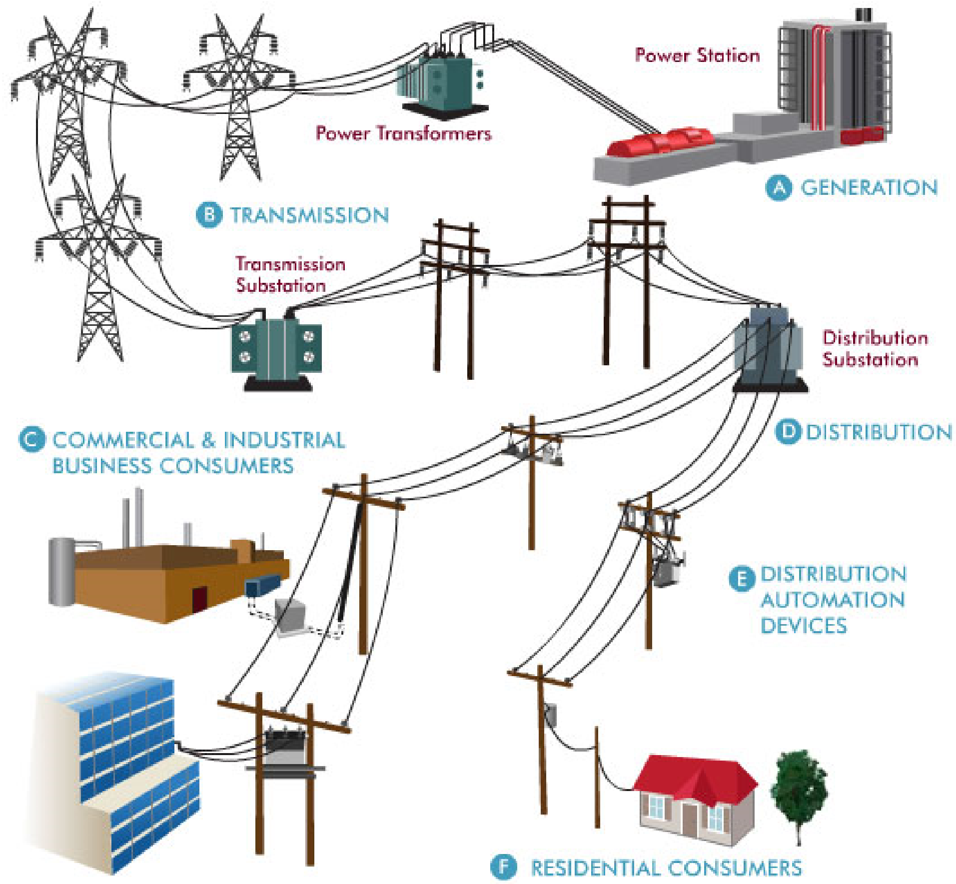


$$I_n = I_a + I_b + I_c = 0$$



Advantages of Balanced 3-Phase Systems

- When compared to three single-phase circuits, three-phase circuits have better use of equipment and materials
  - More power can be transmitted per conductor
  - Lesser power losses in the conductors
- This implies reduced capital and operating costs of transmission and distribution.
- We can calculate voltage and current for only one phase and refer to other phases easily.

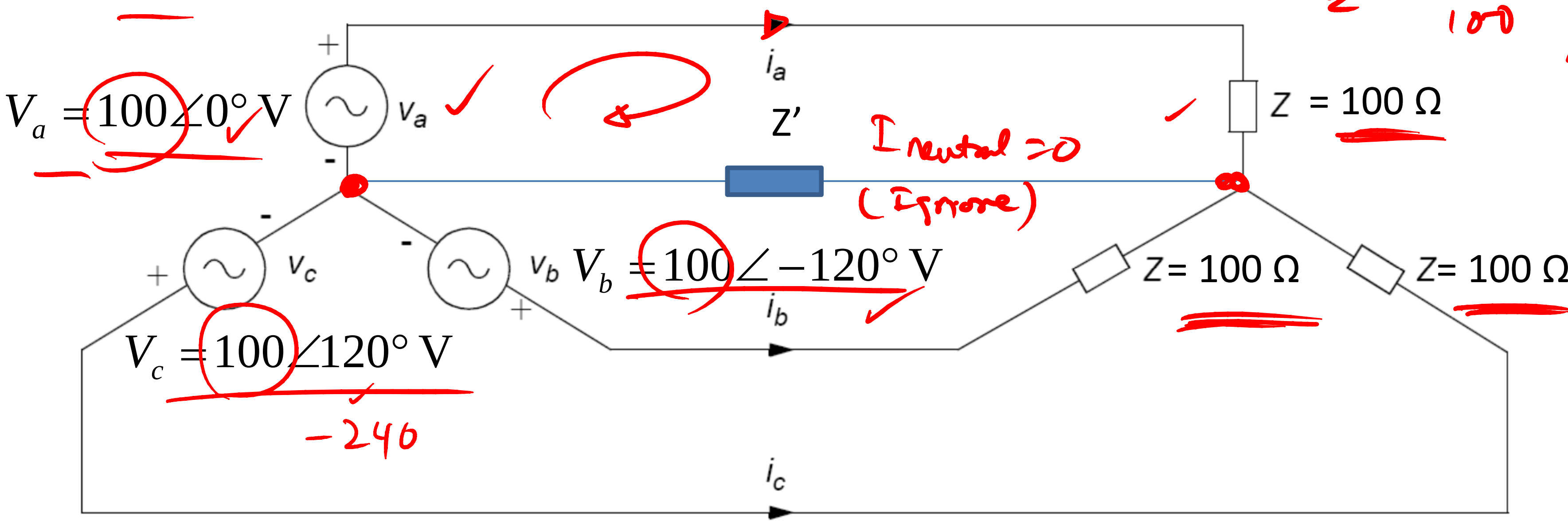


Example 1

3 phase → Single phase

- Consider the following three-phase system shown below. Find the current i_a when z' = 10 Ω.

$-V_a + I_a \cdot Z = 0 \Rightarrow I_a = \frac{V_a}{Z} = \frac{100\angle 0}{100} = 1\angle 0$ A



Ans: $i_a = 1\angle 0° \text{ A}, i_b = 1\angle -120° \text{ A}, i_c = 1\angle 120° \text{ A}$

Line-to-Neutral Voltage

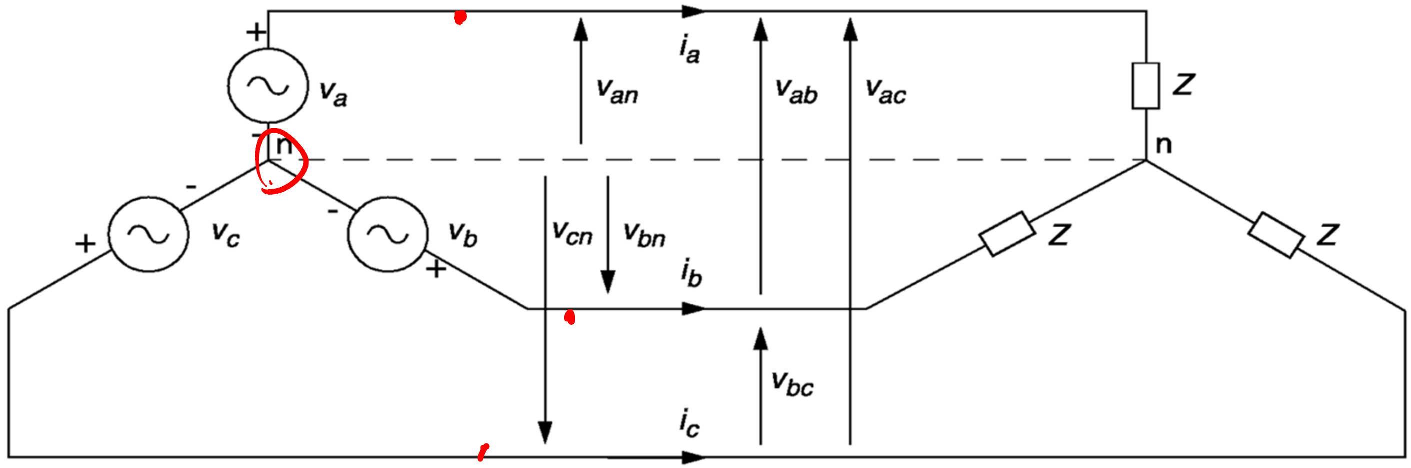
Line-to-line voltage

Line current

Wye-Delta connection

THREE-PHASE CURRENT AND VOLTAGE

Line-To-Neutral (Phase) Voltage

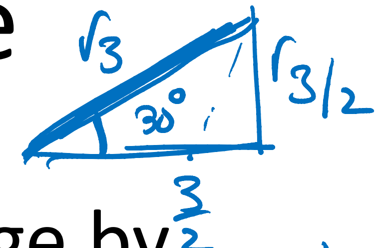


V_{an}, V_{bn}, V_{cn} are called line-to-neutral voltage or phase voltage.

Line-To-Line Voltage

V_{ab}, V_{bc}, V_{ca}

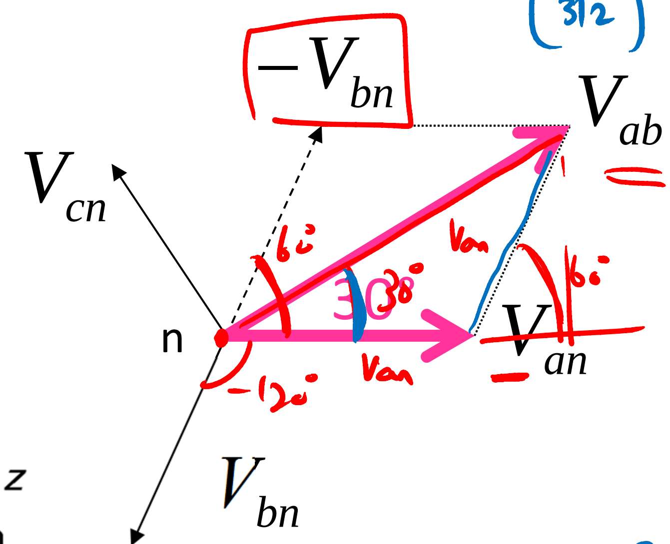
$\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{3}$

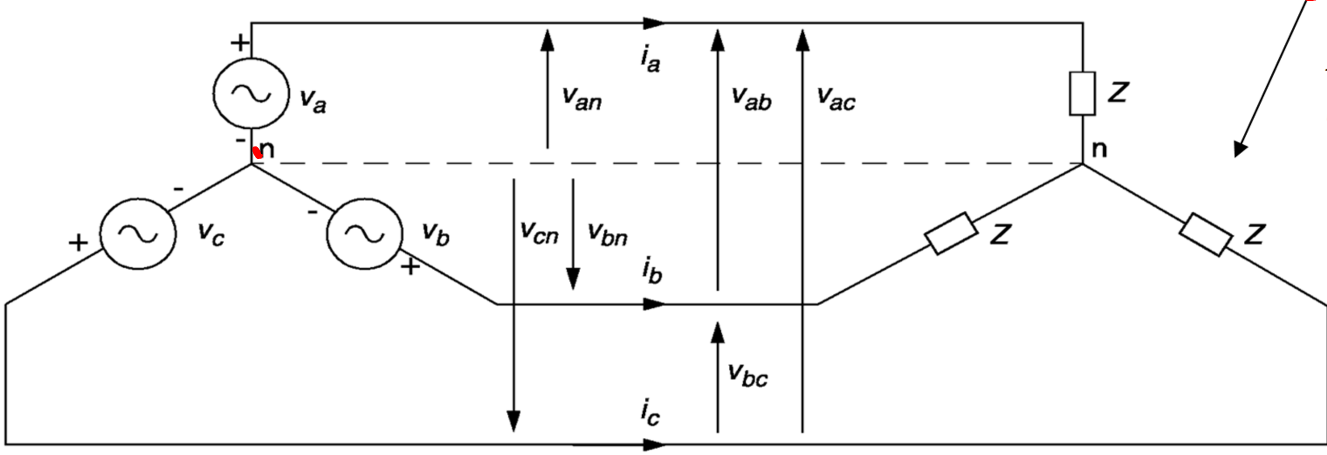


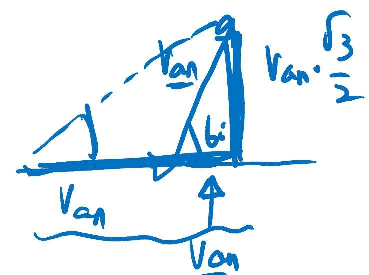
$\tan^{-1}\left(\frac{\sqrt{3}/2}{3/2}\right)=30^\circ$

- Voltage is given as line-to-line voltage by convention.
- KVL: $V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_{an}\angle 30^\circ$

$$\boxed{\left|V_{\text{Line-Line}}\right| = \sqrt{3}\left|V_{\text{Line-neutral}}\right|}$$





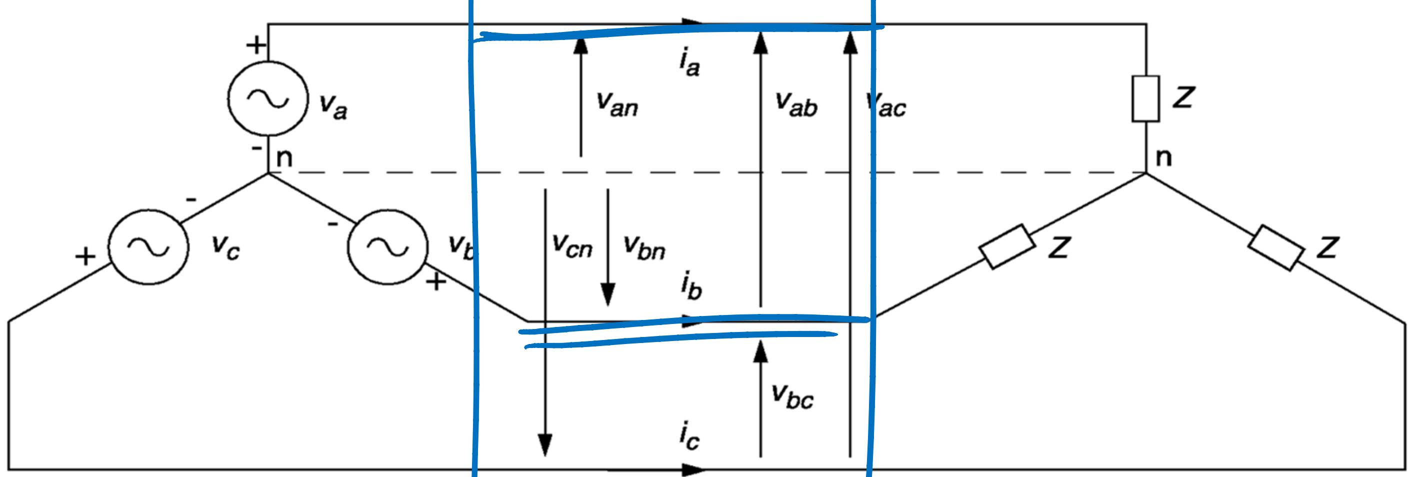


$\frac{3}{2}\cdot V_{an} + j\frac{\sqrt{3}}{2}\cdot V_{an}$

Line Current

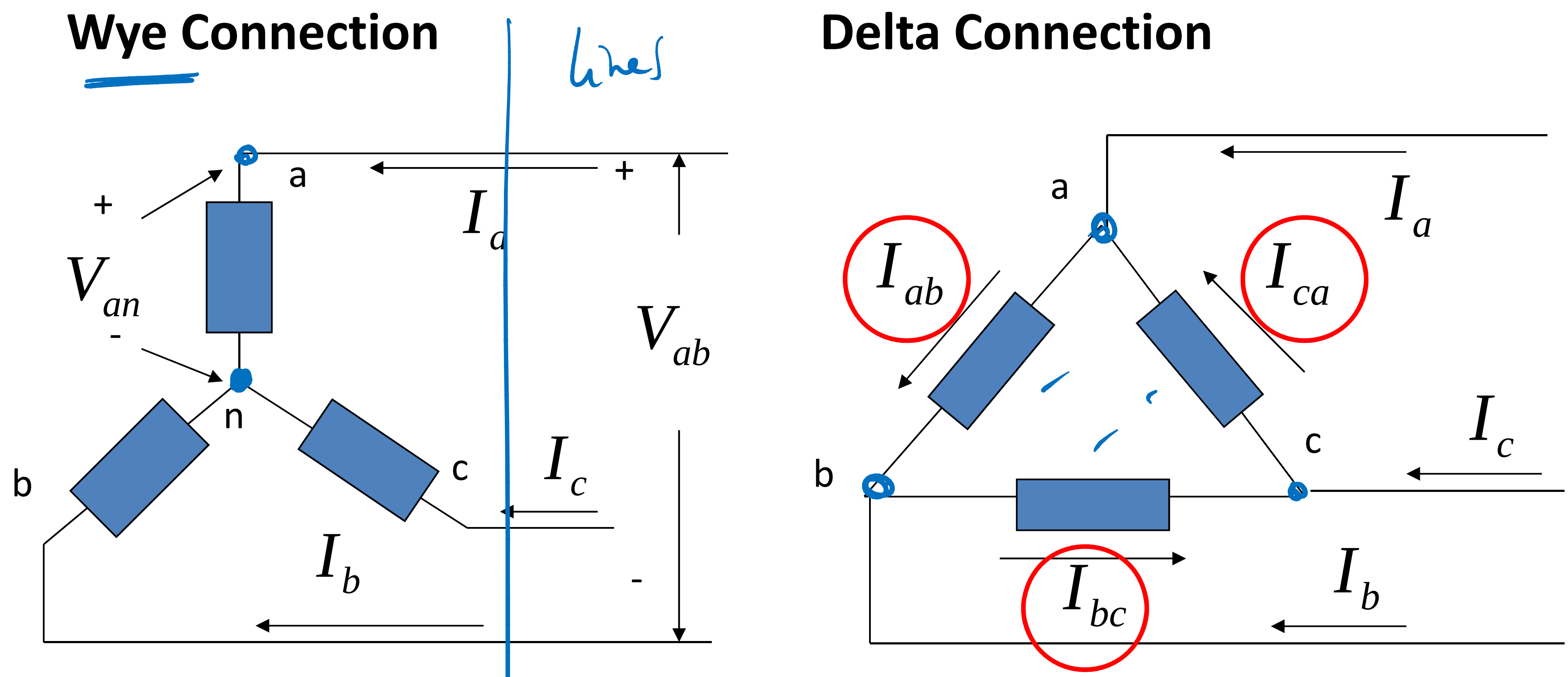
Source
Gen

Load
Renter



i_a , i_b , i_c are called line currents.

3-Phase Circuit Connection



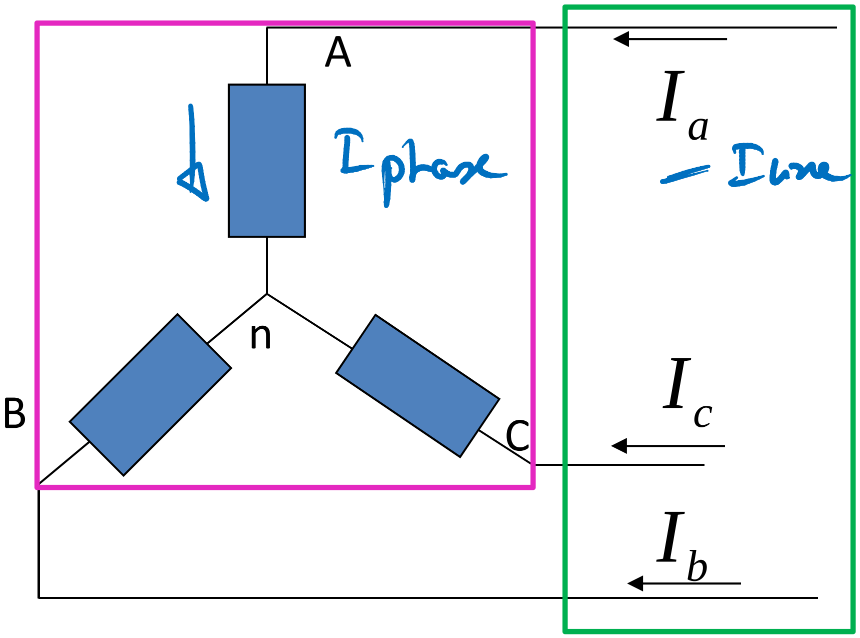
These two types of connections apply to both three-phase voltage sources and three-phase loads.

Line Current VS Phase Current

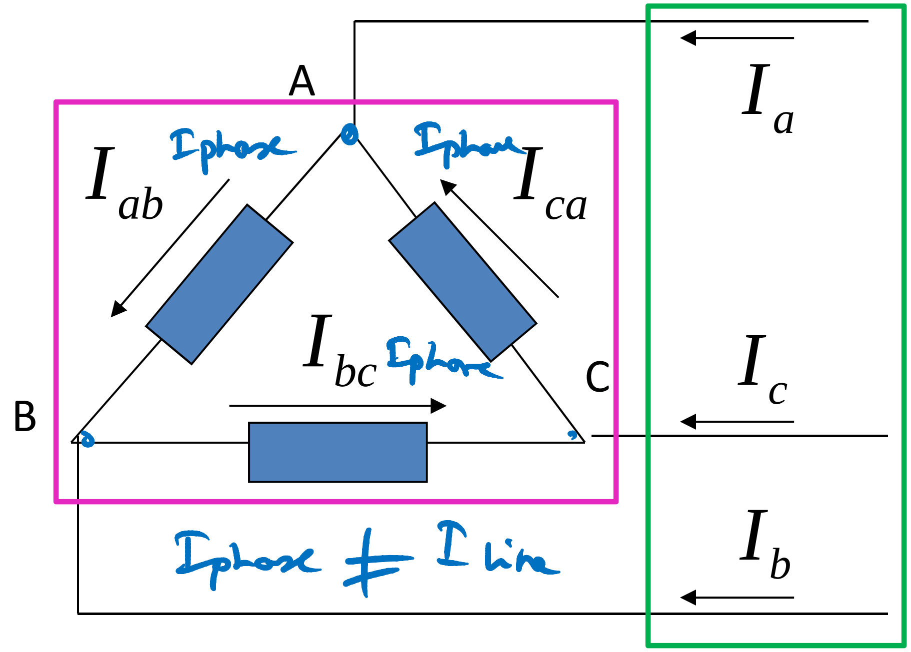
$I_{phase} = I_{line}$

current in each load.

Wye Connection



Delta Connection



Currents through the three-phase conductor lines are called 'Line currents'.

Currents carried by the load impedance are called 'Phase currents' or 'Load Current.

Delta-Connected Load

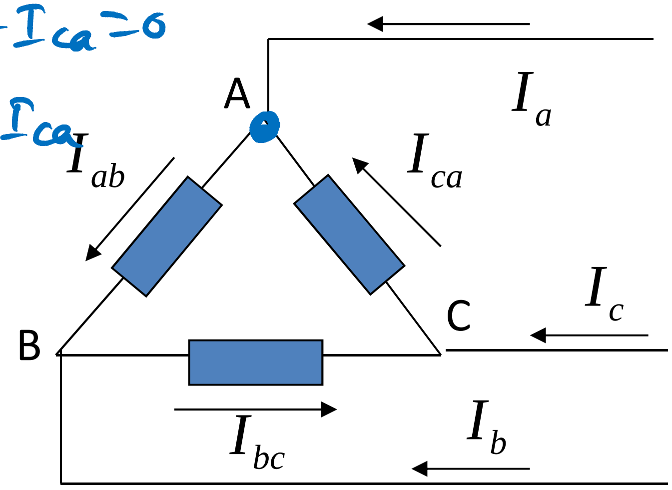
$-I_a + I_{ab} - I_{ca} = 0$

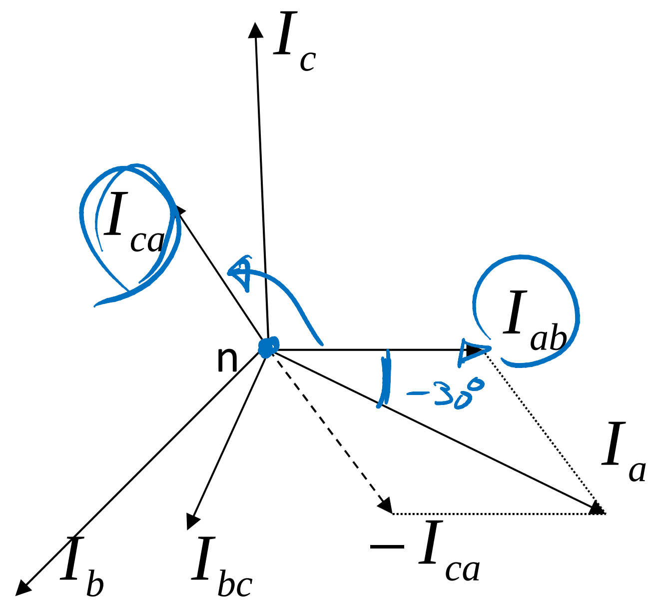
$I_a = I_{ab} - I_{ca}$

- I_{ab} , I_{bc} , I_{ca} are called Phase currents.
- We can find relationship between line currents and phase currents using KCL,

$$I_a = I_{ab} - I_{ca} = \sqrt{3} I_{ab} \angle -30°$$

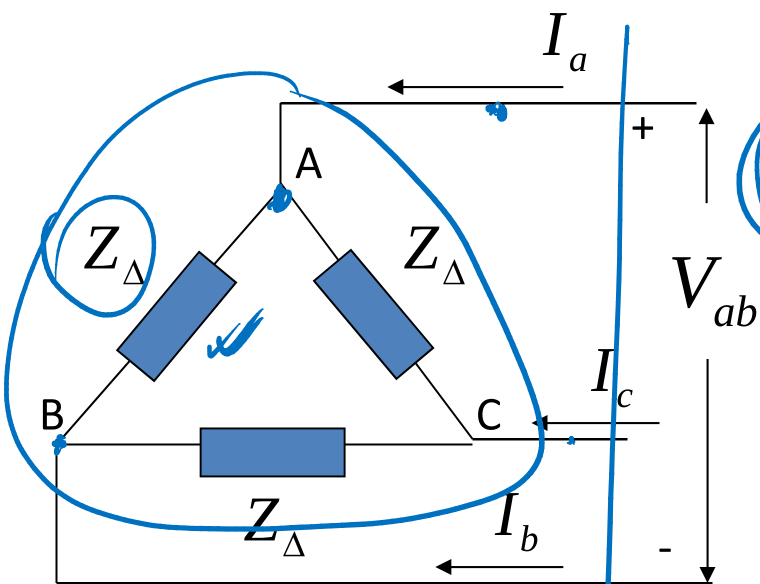
$$\boxed{|I_{\text{Line}}| = \sqrt{3} |I_{\text{Phase}}|}$$





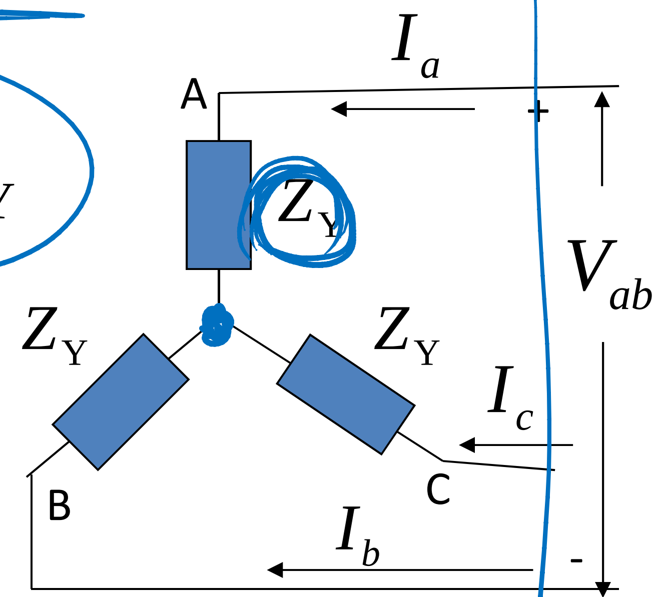
Delta-Wye Load Transformation

Equivalence



$$Z_\Delta = 3Z_Y$$

$$Z_y = \frac{Z_\Delta}{3}$$



$$I_a = \sqrt{3} I_{ab} \angle -30°$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30°$$

3x 30Ω connected in Δ. → $Z_y = \frac{30}{3} = 10\Omega$

$$Z_\Delta = \frac{V_{LL}}{I_{ph}} = \frac{V_{ab}}{I_{ab}}$$

$$Z_y = \frac{V_{phase}}{I_{phase}} = \frac{V_{an}}{I_a}$$

$$\frac{Z_\Delta}{Z_y} = \frac{V_{ab}}{I_{ab}} \div \frac{V_{an}}{I_a} = \frac{V_{ab}}{V_{an}} \cdot \frac{I_a}{I_{ab}} = \sqrt{3} \cdot \sqrt{3} = 3$$

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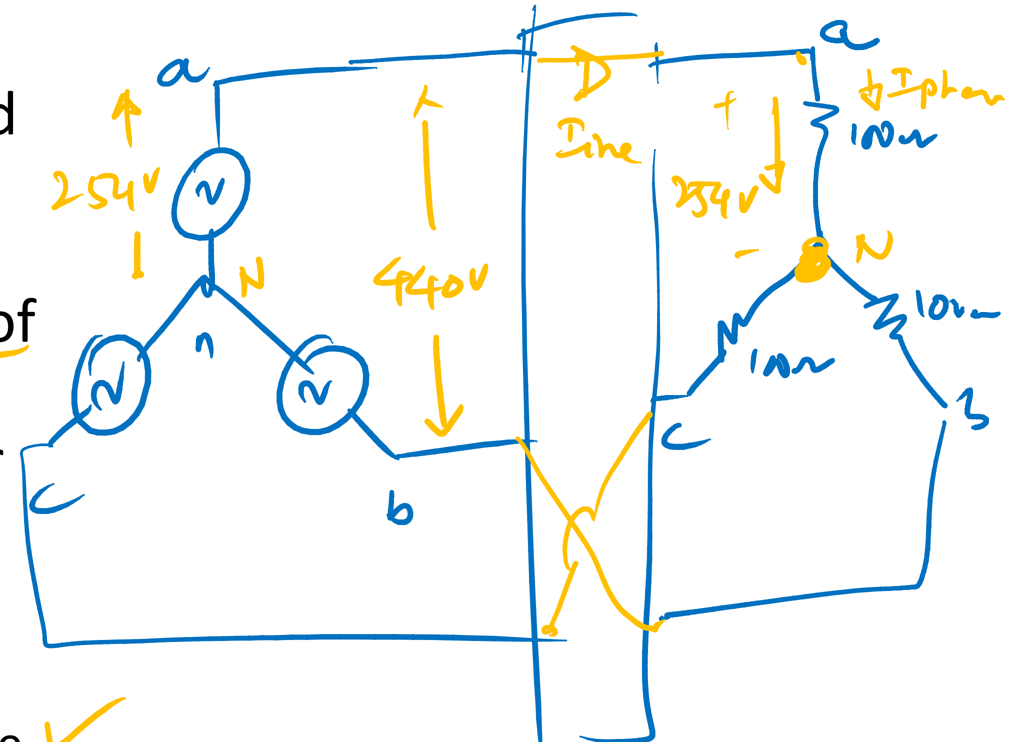
a b c (+ve sequ)

Example 3

−ve seq: acb

- For a balanced Y-connected three phase voltage source and Y-connected load system with a line voltage of 440 V and three equal resistive loads of 100 Ω per phase, assume positive sequence, what will be the magnitudes of
  - (a) the line-to-neutral voltage,
  - (b) the phase current,
  - (c) the line current?



$V_{LL} = 440\text{ V}$

(a) $V_{LN} = \dfrac{440}{\sqrt{3}} = 254\text{ V}$

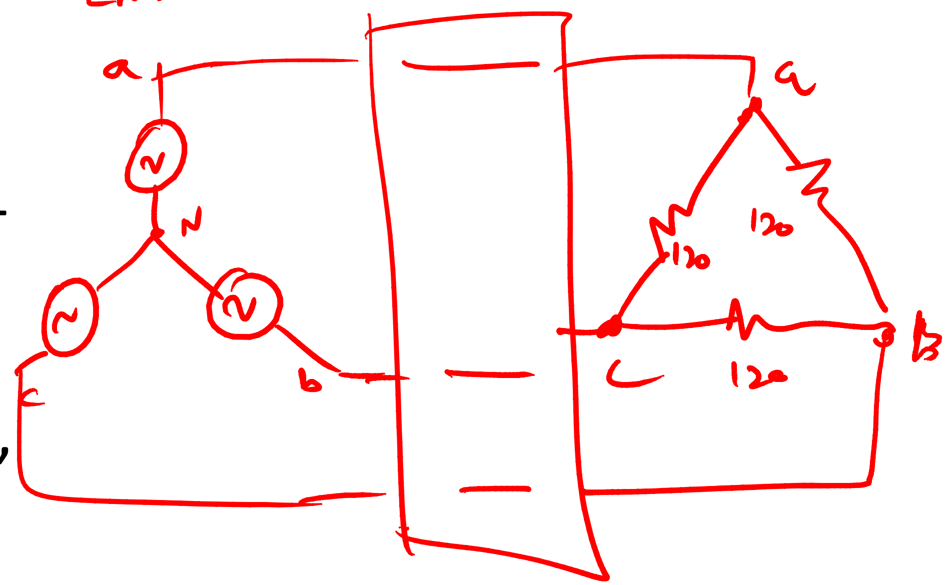
(b) $\dfrac{254}{100} = 2.54\text{ A}$

(c) $I_{line} = 2.54\text{ A}$

Example 4

$V_{LN} = 80V.$

- For a balanced Y-connected three phase generator with the line-to-neutral voltage of 80 V, Δ-connected load of 120 Ω, assume positive sequence, find
  - (a) the line-to-line voltage,
  - (b) the voltage across a resistor,
  - (c) the current through a resistor?



(a) $V_{LL} = \sqrt{3} \times 80 = 138.56 \text{ V}.$

(b) Voltage across each resistor =

$\Rightarrow V_{LL} = 138.5$

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Summary

- Three-phase voltage sources
  - Positive and negative sequences
- Balanced three-phase circuit
  - Conditions
  - Advantages
- Balanced three-phase circuit
  - Line-to-neutral (phase) voltage
  - Line-to-line (line) voltage
  - Line current

$$\boxed{\left|V_{\text{Line-Line}}\right| = \sqrt{3}\left|V_{\text{Line-neutral}}\right|}$$

$\angle 30^{\circ}$

- Wye-Delta connection
- Delta-Wye load transformation

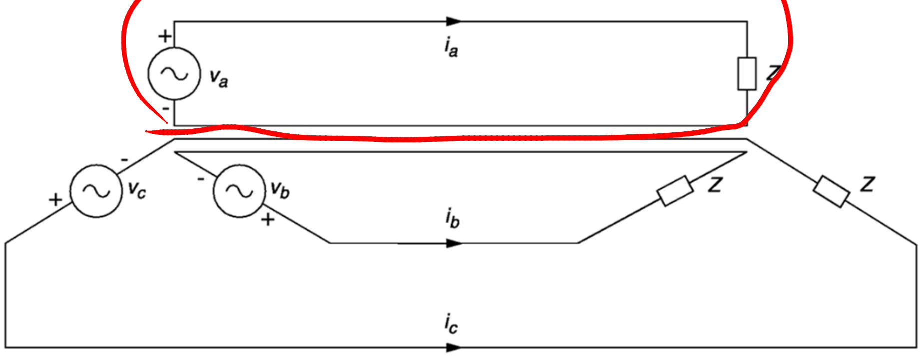
$$\boxed{Z_{\Delta} = 3Z_{Y}}$$

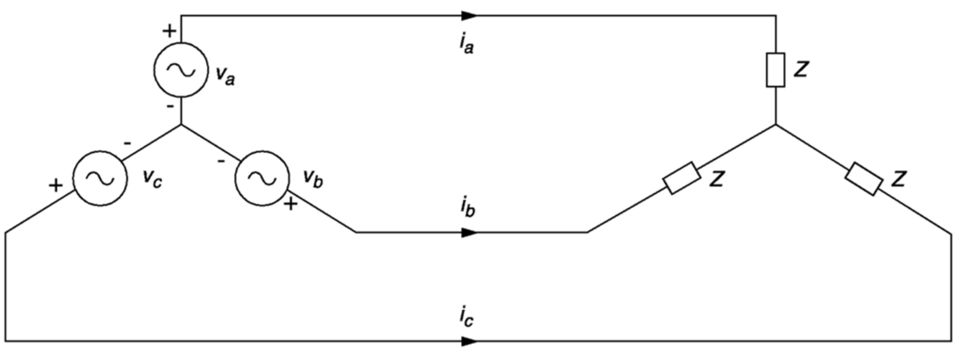
- Three-Phase Circuit Analysis
  - Three-phase complex power
  - Per Phase analysis → balanced 3ph. system.

Three Phase Power Calculation

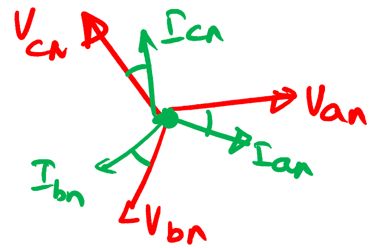
- Three phase power is found from summation of each phase power.

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$





Balanced Three-Phase Power



V_{an}

$V_{bn} = V_{an}\angle -120^\circ$

$V_{cn} = V_{an}\angle +120^\circ$

I_a

$I_b = I_a\angle -120^\circ$

$I_c = I_a\angle +120^\circ$

- From three phase power,

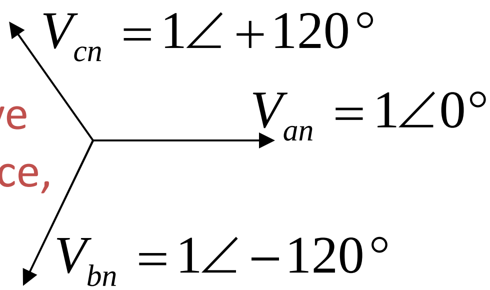
$$S_{3\Phi} = V_{an}I_a^* + V_{bn}I_b^* + V_{cn}I_c^*$$

- When the system is balanced, (assume positive sequence) we can write,

$$S_{3\Phi} = V_{an}I_a^* + V_{an}\angle -120^\circ\left(I_a\angle -120^\circ\right)^* + V_{an}\angle 120^\circ\left(I_a\angle 120^\circ\right)^*$$

$$= V_{an}\cdot I_a^* + V_{an}\cdot I_a^* + V_{an}\cdot I_a^*$$

Positive sequence, abc



$$\boxed{S_{3\Phi} = 3V_{an}I_a^*}$$

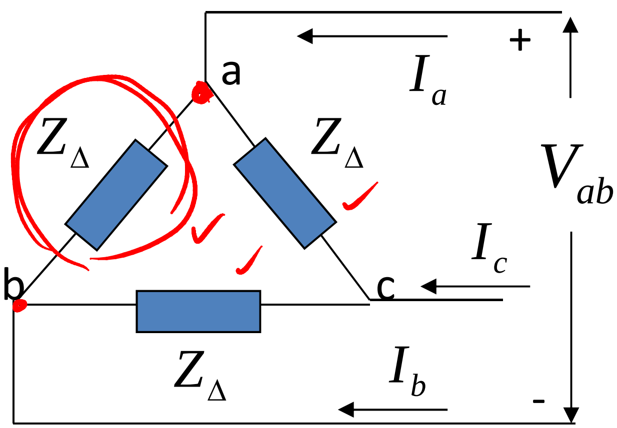
Balanced Three-Phase Load

- Three-phase load can be connected in either Wye or Delta connection.
- 3-phase load parameter is given as total apparent power ($|S_{3\Phi}|$) with power factor.
- The voltage given is **Line-to-line voltage**.
- We can find three-phase real and reactive power as follows.

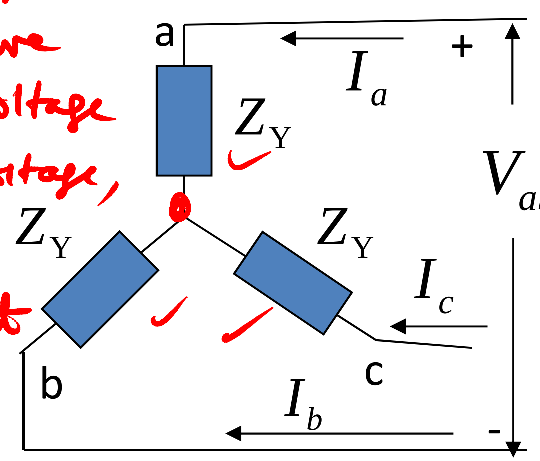
$$P_{3\Phi} = 3P_{1\Phi} = |S_{3\Phi}| \times \text{p.f.}$$

$$Q_{3\Phi} = 3Q_{1\Phi} = |S_{3\Phi}| \times \sin\left(\cos^{-1}(\text{p.f.})\right) = |S_{3\Phi}| \times \sin\phi$$

Delta/Wye Connected 3-Phase Load



In 3ph system whenever we mention voltage it's line voltage, current → line current



$V = V_{an}$
$I = I_a$

$$I_a = \sqrt{3} I_{ab} \angle -30°$$, $V = V_{ab}$; $I = I_{ab}$

$$V_{ab} = \sqrt{3} V_{an} \angle 30°$$

$$|S_{3\Phi}| = 3 |V_{ab} I_{ab}^*| = \sqrt{3} |V_{ab}| |I_a|$$

$$|S_{3\Phi}| = 3 |V_{an} I_a^*| = \sqrt{3} |V_{ab}| |I_a|$$

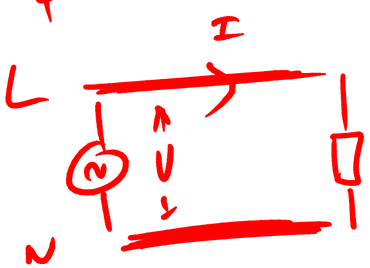
$$\boxed{|S_{3\Phi}| = \sqrt{3} |V_{\text{Line–To–Line}}| |I_{\text{Line}}|}$$

$|S_{3\phi}| = \sqrt{3} \cdot V \cdot I$

Conductor utilization in 1ϕ vs 3ϕ

1-ϕ

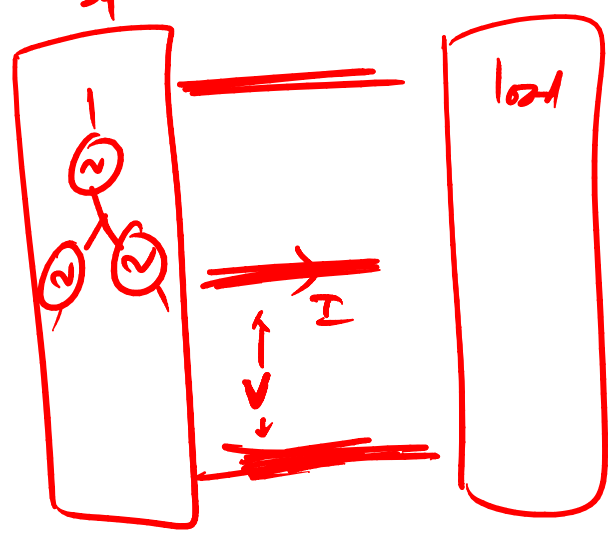
$$|S|_{1\phi} = V.I$$



2 conductors

3-ϕ

$$|S|_{3\phi} = \sqrt{3}.\ V.\ I$$



3 conductors

$\frac{1}{2}.X$ $\quad \frac{V.I}{2}$ $\leftarrow$ power per conductor $\rightarrow$ $\frac{\sqrt{3}.V.I}{3}$ $\quad \frac{1}{\sqrt{3}}.X$

Example 1

$P = |S| . p.f.$

apparent power

$P_{3\phi} = |S_{3\phi}| \times p.f.$

- A 3Φ load of 300 MVA, 100 kV at 0.85 p.f. lagging, find

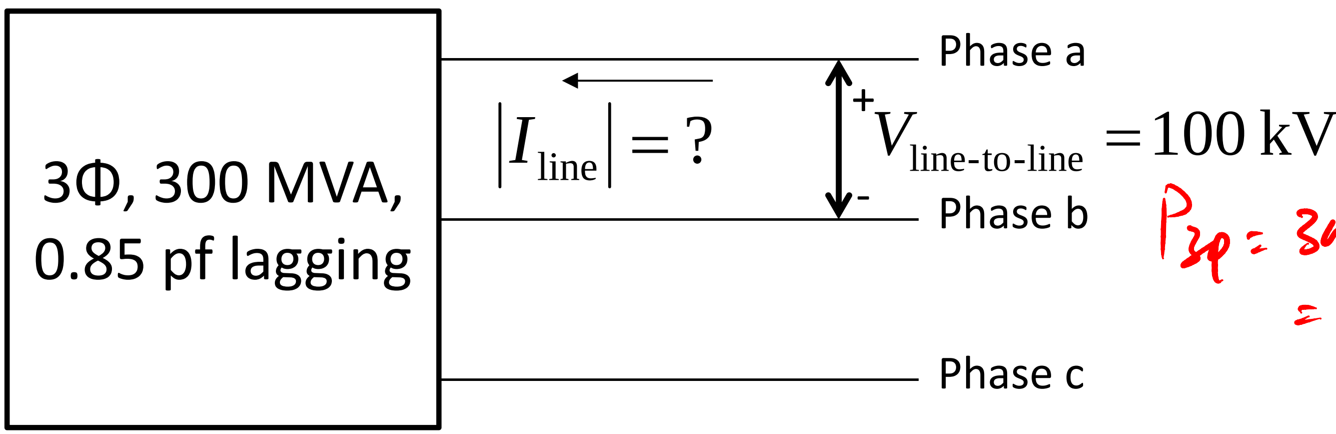
  $|S_{3\phi}|$ V_{LL}

  - The magnitude of line current $|I_{\text{Line}}|$
  - Three-phase (real) power P_{Load}

$|S_{3\phi}| = \sqrt{3} . V . I$

$300 \times 10^6 = \sqrt{3} . 100 \times 10^3 \times I$

$I = 1732 \text{ A}.$



$P_{3\phi} = 300 \times 10^6 \times 0.85$

$= 255 \text{ MW}.$

Ans: 1732 A, 255 MW.

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Instantaneous Three-Phase Power

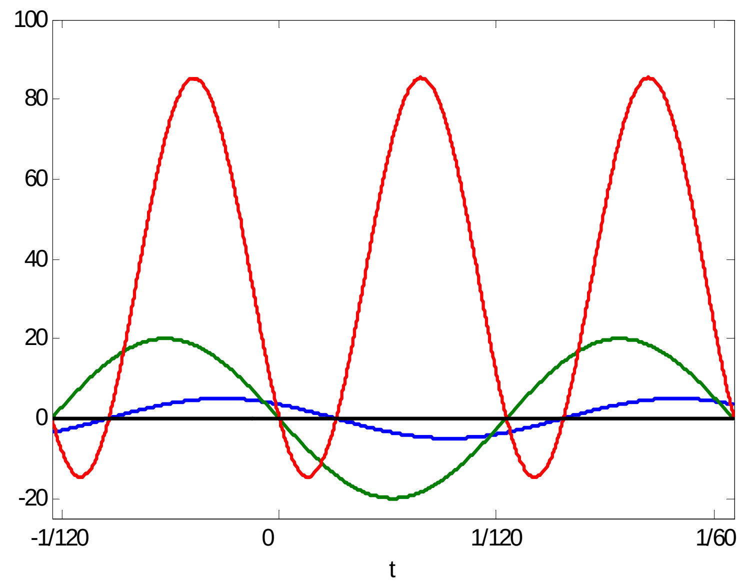
- Given by,

$$p_{3\Phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t)$$

- Recall that single phase instantaneous power,

$$v(t) = |V|\cos(\omega t + \theta_v)$$
$$i(t) = |I|\cos(\omega t + \theta_i)$$

$$p(t) = v(t) \times i(t)$$
$$= |V||I|\cos(\theta_v - \theta_i)$$
$$+ |V||I|\cos(2\omega t + \theta_v + \theta_i)$$



Instantaneous Three-Phase Power

- For a balanced three-phase system,

=0

$$\begin{aligned} p_{3\Phi}(t) = {} & |V_a||I_a|\cos(\theta_v - \theta_i) + |V_a||I_a|\cos(2\omega t + \theta_v + \theta_i) \\ & + |V_b||I_b|\cos(\theta_v - \theta_i) + |V_b||I_b|\cos(2\omega t + \theta_v + \theta_i - 240^\circ) \\ & + |V_c||I_c|\cos(\theta_v - \theta_i) + |V_c||I_c|\cos(2\omega t + \theta_v + \theta_i - 480^\circ) \end{aligned}$$

- We can find three phase instantaneous power as,

$$p_{3\Phi}(t) = 3|V_a||I_a|\cos\phi = 3P$$

- **Constant** power transfer to load.

An Additional Advantage of Balanced 3-Phase Circuit

2

- Constant power transfer to load.
  - This also implies constant mechanical power input for a generator.
  - When mechanical power input is constant, mechanical shaft torque is also constant.
  - This helps to reduce shaft vibration and noise, extending the machine's lifetime.

Assumption

Single-line diagram

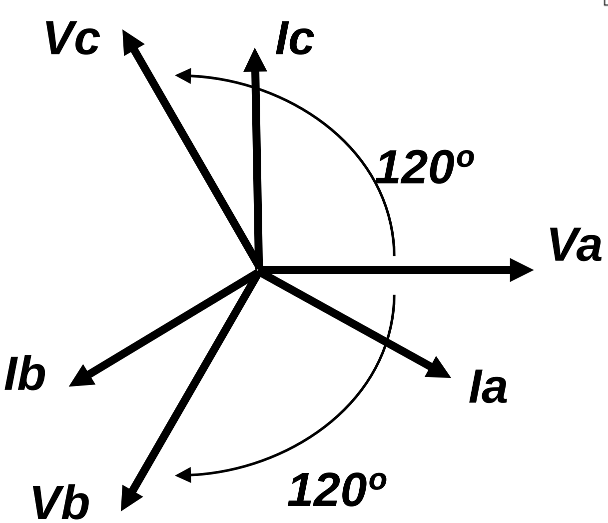
Example example example…

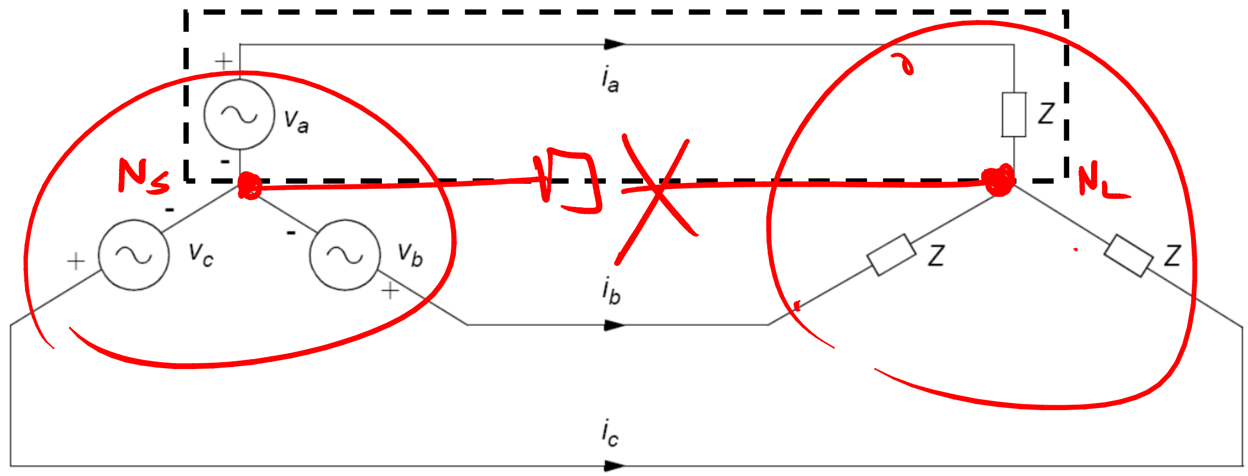
PER PHASE ANALYSIS

Per Phase Analysis: Assumption

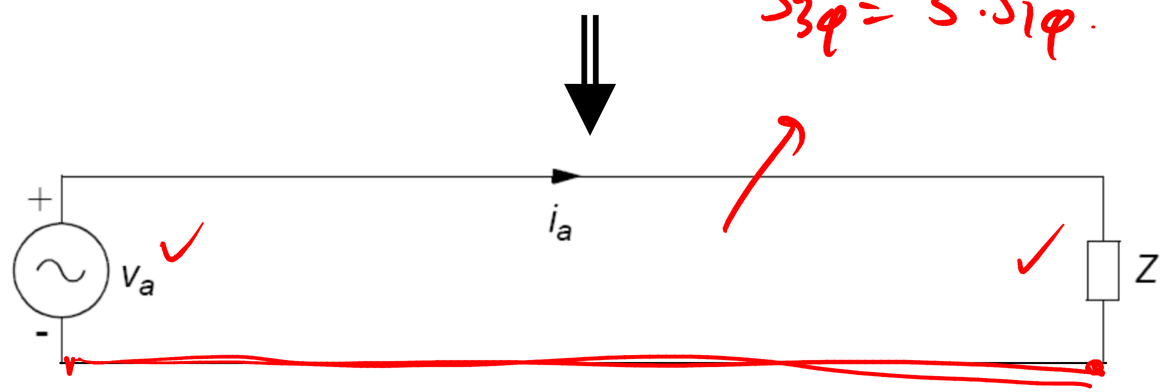
It must be balanced three-phase circuit.

$$I_n = I_a + I_b + I_c = 0$$





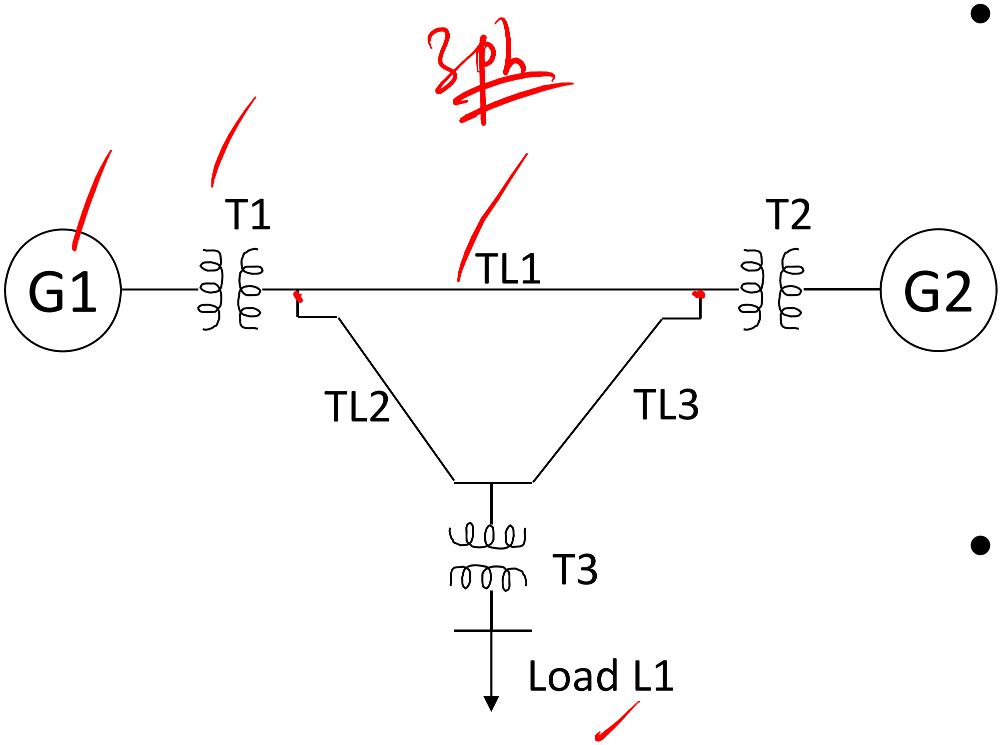
$S_{3\varphi} = 3 \cdot S_{1\varphi}$



Steps of Per Phase Analysis

- Make sure that the three-phase system is balanced.
  - The three-phase sources need to have the same magnitude with 120 degree phase difference.
  - The three-phase impedances must be of the same value (both phase and magnitude).
- **Convert** all Delta-connected sources/loads to Wye-connected sources/loads.
- Per phase analysis reduce three-phase circuit to single-phase circuit. We can apply the same concept used in single-phase.

Single-Line Diagram

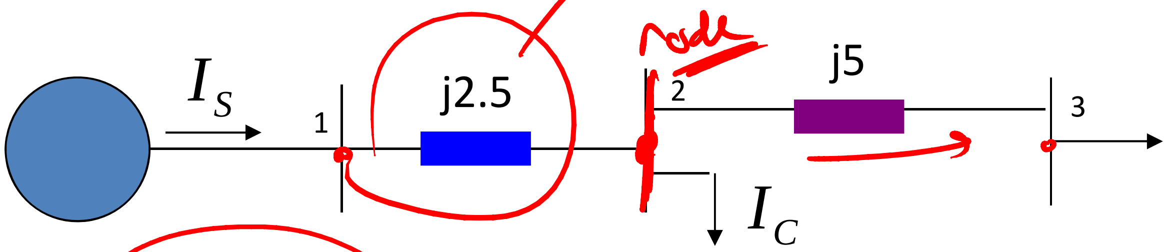


- Show the interconnections of a transmission system
  - Generator
  - Load
  - Transmission line
  - Transformer
- This is a representation of a 3Φ circuit. Each line represents three conductors in three-phase system.

Example 2

reactance, for line inductance

- Given a one-line diagram,



Load 2, Y-Connected
Inductive: ZL = j10

Source, Δ-Connected

Positive sequence.

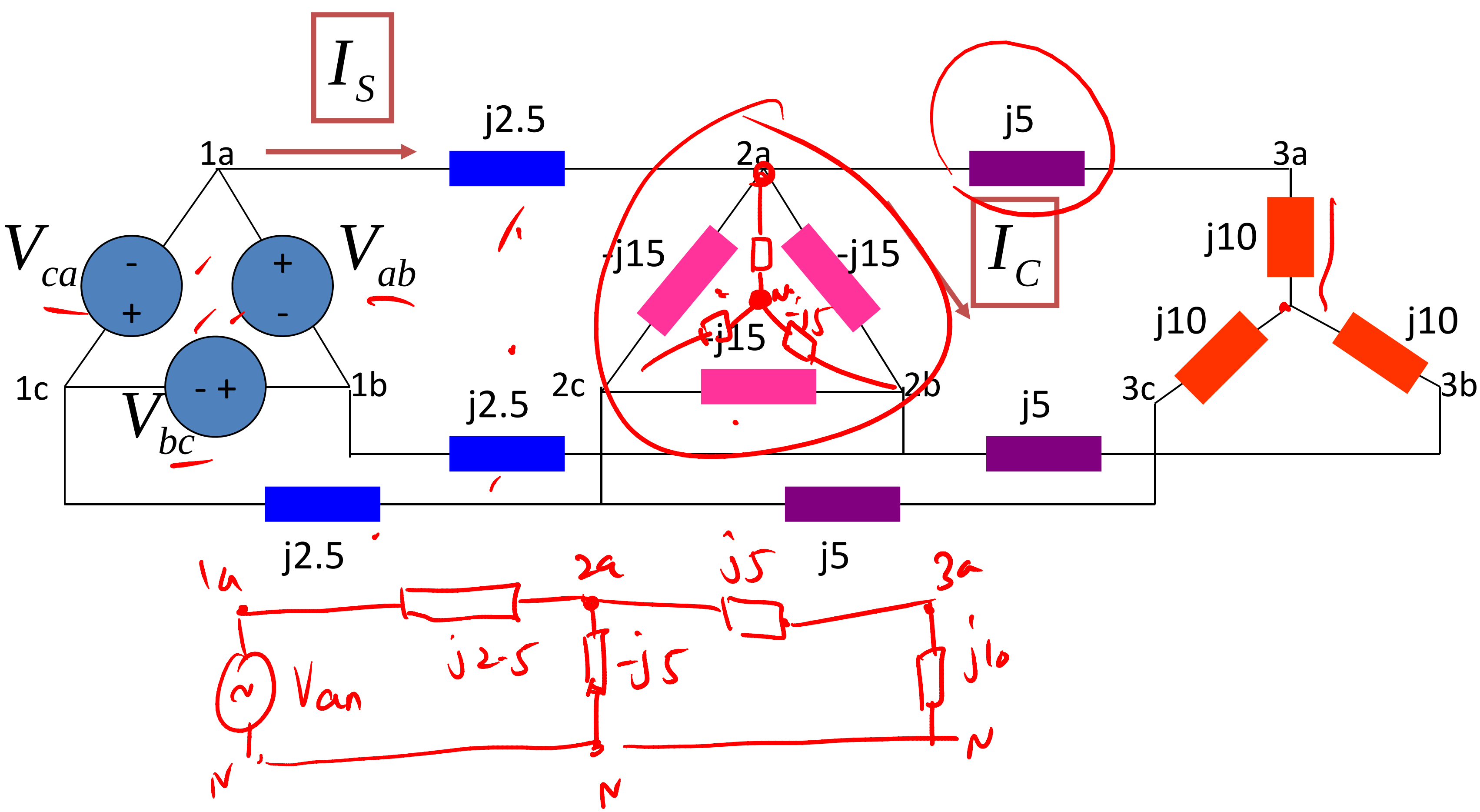
Load 1, Δ-Connected
Capacitor: ZC = -j15

$\rightarrow Z_Y = \frac{Z_\Delta}{3} = \frac{-j15}{3} = -j5$

If the voltage source is $\left|V_{\text{Line-Line}}\right| = \sqrt{3}$ V. Find,

1. Current magnitude supplied by source, $\left|I_S\right|$, and,
2. Current magnitude through a capacitor, $\left|I_C\right|$.

Example 2: 3Φ Circuit Diagram

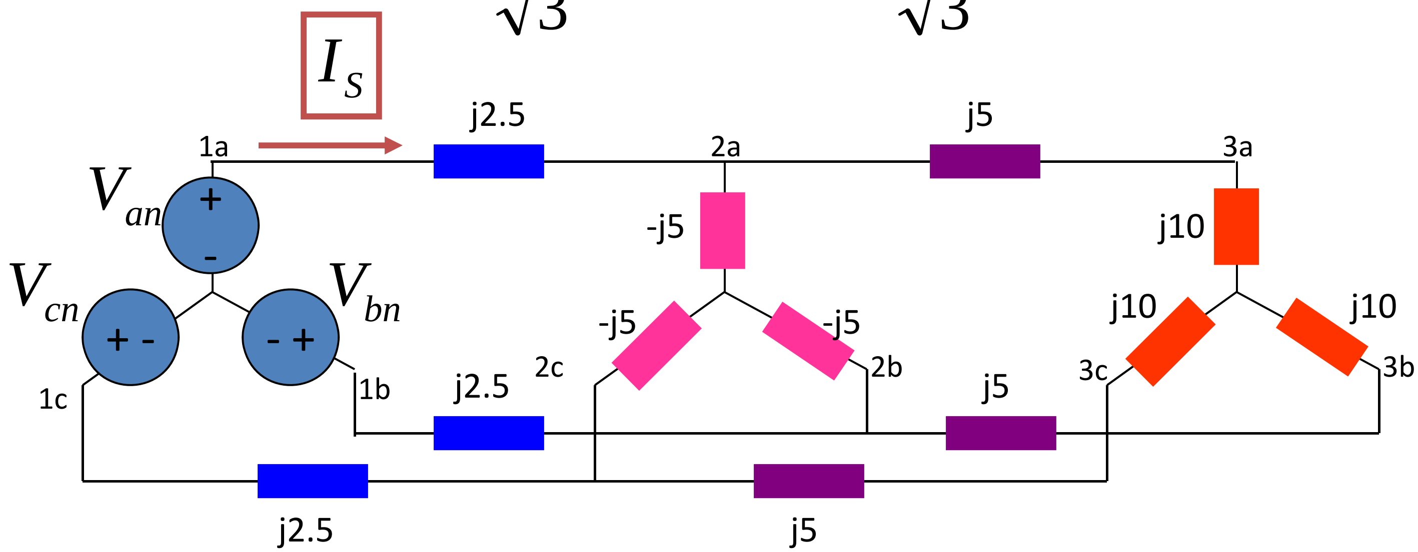


Example 2: Convert from Δ → Y

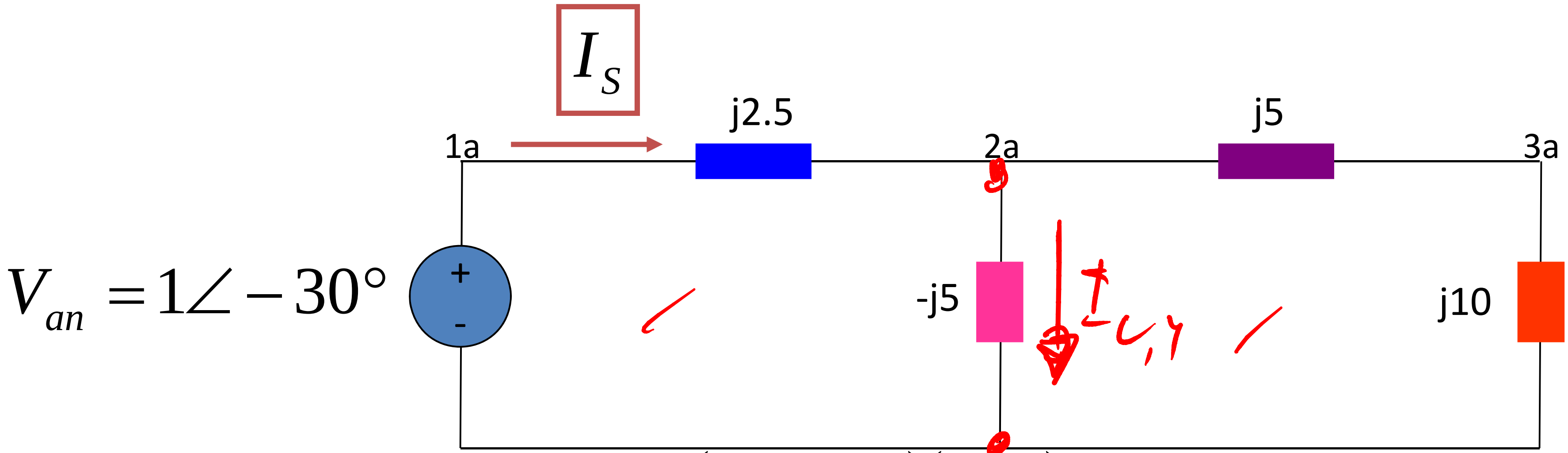
$$Z_Y = \frac{Z_\Delta}{3} = \frac{-j15}{3} = -j5$$

Use voltage source as angle reference

$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30^\circ = \frac{\sqrt{3}\angle 0^\circ}{\sqrt{3}}\angle -30^\circ = 1\angle -30^\circ$$



Example 2: 1-Phase diagram

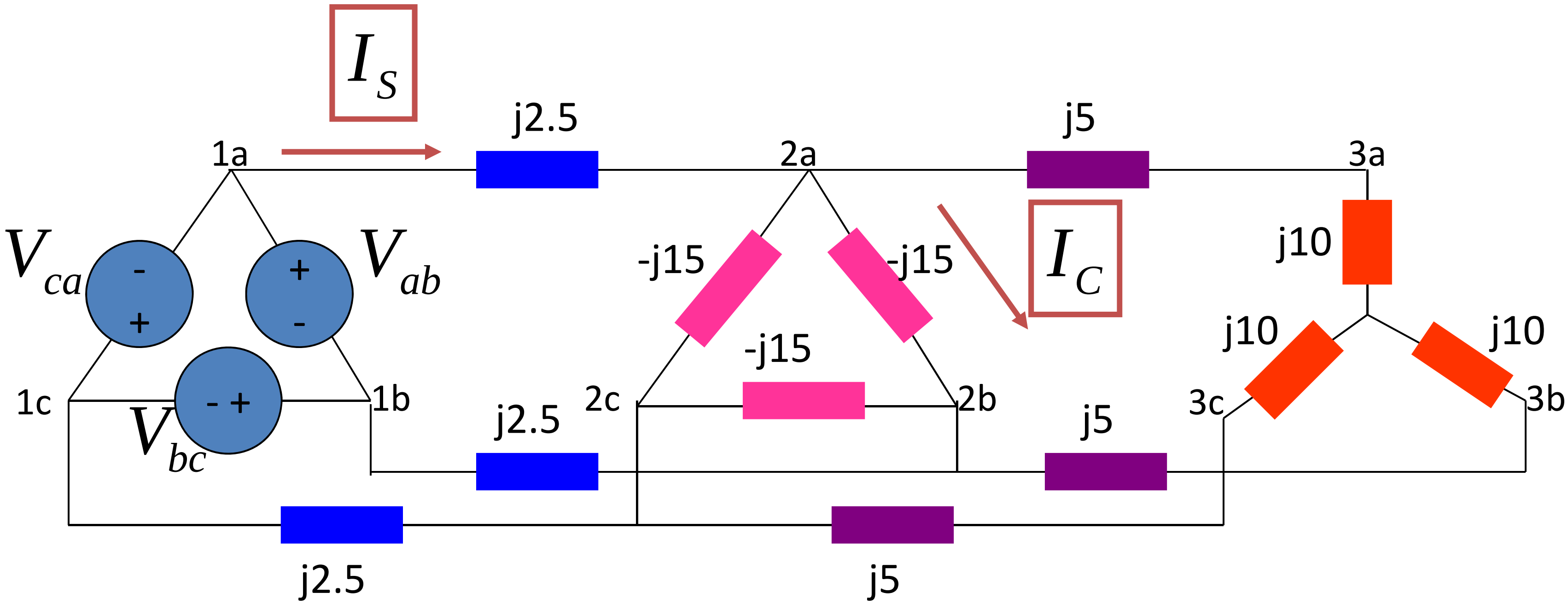


$$Z_{eq} = j2.5 + \frac{(j10 + j5)(-j5)}{(j10 + j5) + (-j5)} = -j5$$

$$I_S = \frac{V_{an}}{Z_{eq}} = \frac{1\angle -30°}{-j5} = \frac{1\angle -30°}{5\angle -90°} = 0.2\angle 60° \text{ A}$$

$V_{2a} = V_{an} - j2.5 \times I_S = 1.5\angle -30°$ We will use this to find $\boxed{I_C}$

Example 2: Final Calculation



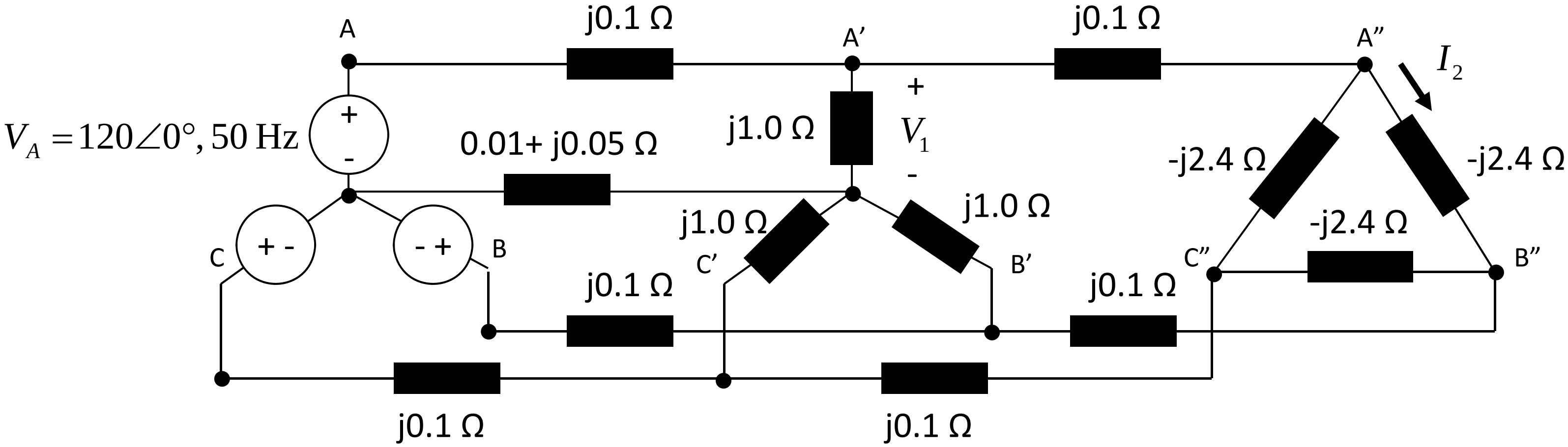
$$V_{2b} = V_{2a}\angle -120^\circ$$

$$I_C = \frac{V_{2a} - V_{2b}}{-j15} = \frac{1.5\angle -30^\circ - 1.5\angle\left(-30^\circ - 120^\circ\right)}{15\angle -90^\circ} = \frac{\sqrt{3}}{10}\angle 90^\circ \text{ A}$$

Ans: $I_S = 0.2\angle 60^\circ, I_C = 0.1732\angle 90^\circ$

Practice Problem 1

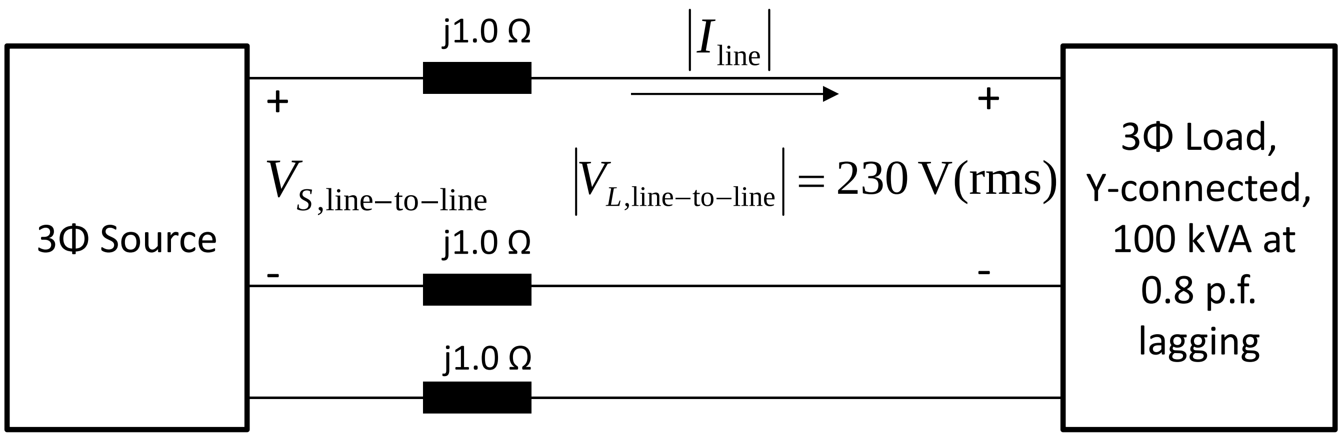
- (Final EE2022 AY2011/12 semester 1) Find the voltage V1 and current I2.
  - *Hint: remove the impedances that are in neutral line (why can we do this?) and transform delta to wye connection.*



Ans: $V_1 = 125.37\angle 0°, I_2 = 103.41\angle 120°$

Practice Problem 2

- (Final EE2022 AY2011/12 semester 1) A balanced three-phase voltage source feeds a balanced three-phase load shown below, find $\left|V_{S,\text{line-to-line}}\right|$.
  - *Hint: Assume that line-to-line voltage at the load has reference angle of 0 degree. Then, find line current magnitude and angle at the load. Calculate line-to-neutral voltage from source using V_source(line-to-neutral) = I_line×j1.0 + V_load(line-to-neutral). Then, use relationship between line-to-line voltage and line-to-neutral voltage to find V_source(line-to-line).*



Ans: 601.61 V

Summary

- Three-phase complex power,

$$\boxed{\left|S_{3\Phi}\right| = \sqrt{3}\left|V_{\text{Line–To–Line}}\right|\left|I_{\text{Line}}\right|}$$

- Per phase analysis
  - Only applied to balanced three-phase circuit.
  - Need to **convert** all Delta-connected load/sources to Wye-connected load/sources.
  - Same analysis as single-phase circuit.