EE2022
Electrical Energy Systems

Lecture 5: AC Power

Problem Solving

Average Power in a Resistor

Instantaneous power:

$$p(t) = V_m\, I_m\, Cos^2\omega t = \frac{1}{2}\left[1 + \cos 2\omega t\right] \times V_m \times I_m$$

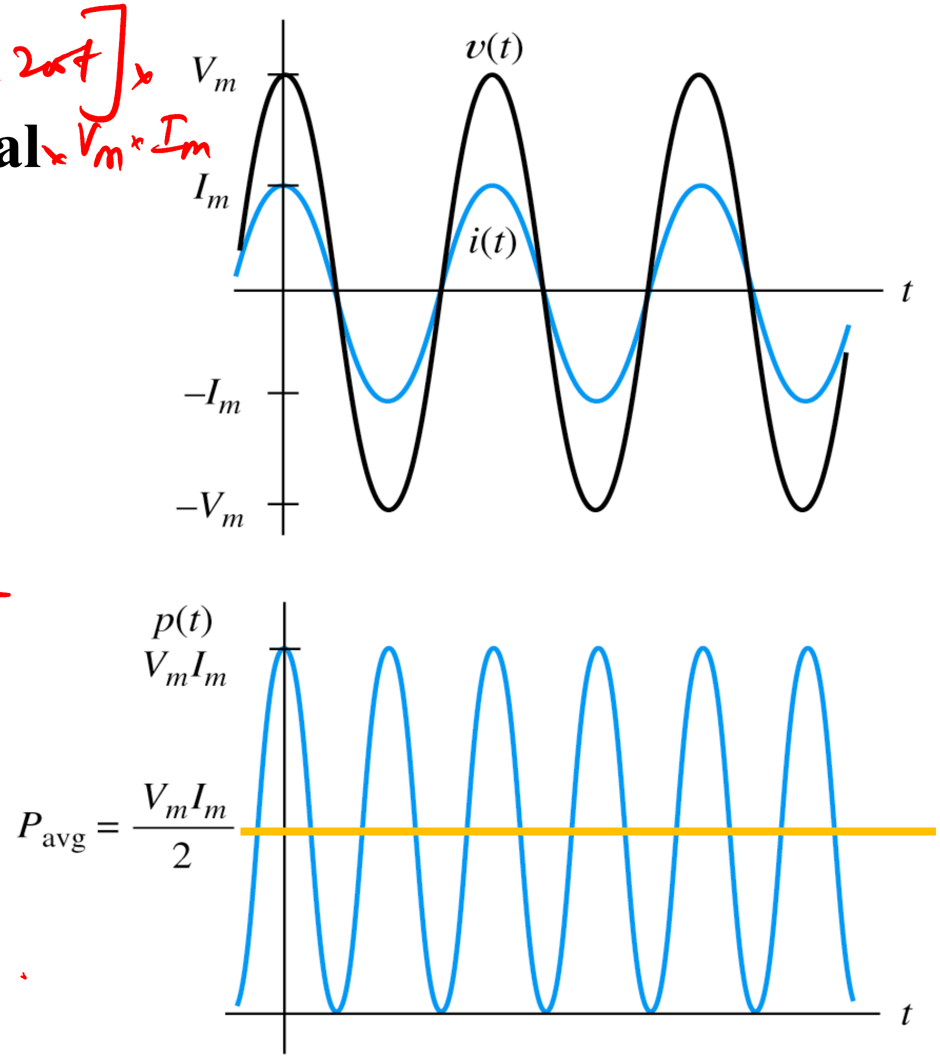
Average of this function is equal to half of the peak amplitude :

$$P_{R,avg} = \frac{V_m I_m}{2}$$

$$P_{R,avg} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

$$V_{rms} = R I_{rms}$$

$$P_{R\,(avg)} = V_{rms} \cdot I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 \cdot R.$$



Current, voltage, and power versus time for a purely resistive l

Average Power in an Inductor

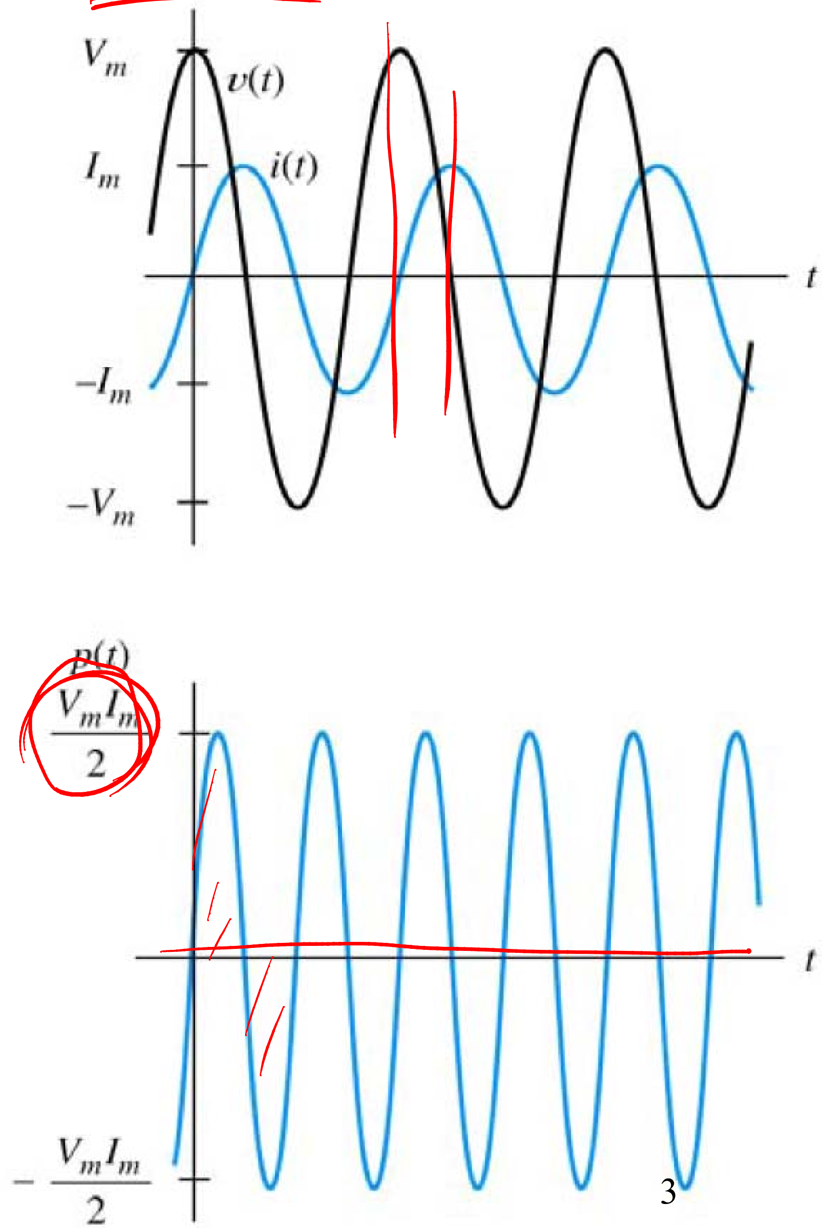
$$p(t) = V_{rms} I_{rms} \sin(2\omega t)$$

Average power, P_{avg} = 0

$$P_L(t) = V_{rms} \cdot I_{rms} \cdot \sin 2\omega t$$

$$= \frac{V_{rms}^2}{\omega L} \cdot \sin 2\omega t$$

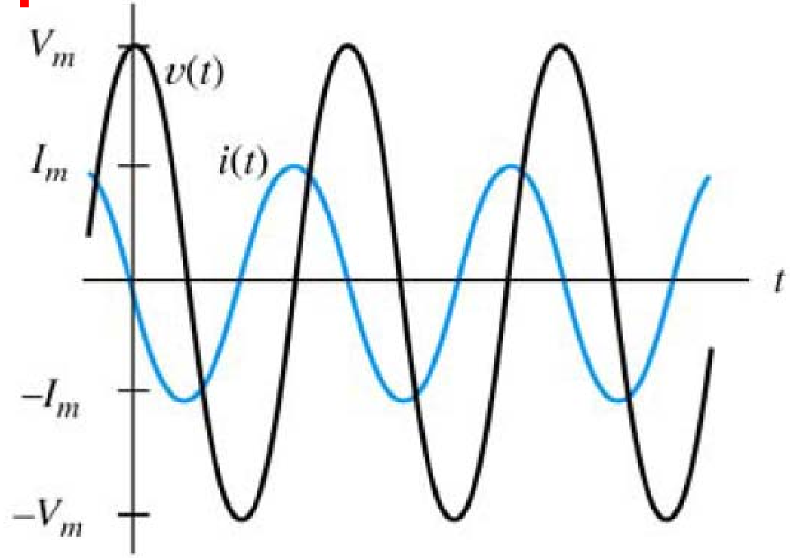
$$= I_{rms}^2 \times \omega L \ \sin 2\omega t .$$



Average Power in a Capacitor

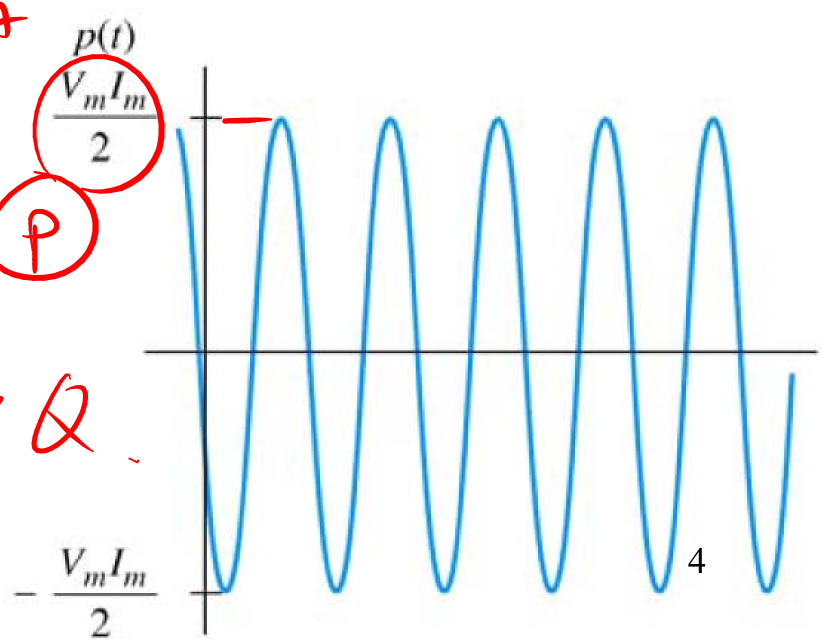
$$p(t) = -V_{rms} I_{rms} \sin(2\omega t)$$

Average power, $P_{avg} = 0$



$$p(t) = -\frac{V_{rms}^2}{1/\omega C} \cdot \sin 2\omega t$$

$$= -I_{rms}^2 \left(\frac{1}{\omega C}\right) \cdot \sin 2\omega t$$



$R \rightarrow$ P_{avg} . $I_{rms}^2 \cdot R \rightarrow P$

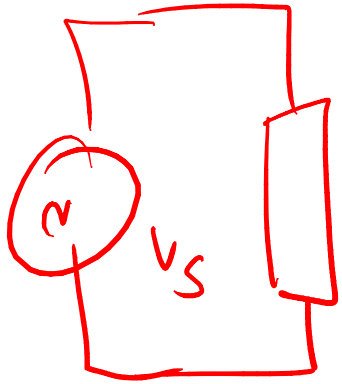
$L \rightarrow$ P_{peak} . $I_{rms}^2 \cdot \omega L$

$C \rightarrow$ P_{peak} . $I_{rms}^2 \cdot \frac{1}{\omega C}$

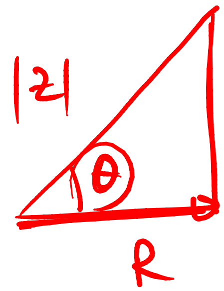
$\Big\} Q$

Real, Reactive and Apparent Power

Generic Load

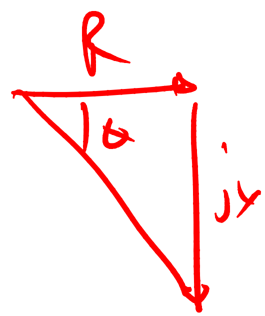


$R + jX = \underline{Z}$



→ Inductive load

$X\ (+ve) = \omega L$



Capacitive

$X\ (-ve) = -\frac{1}{\omega C}$

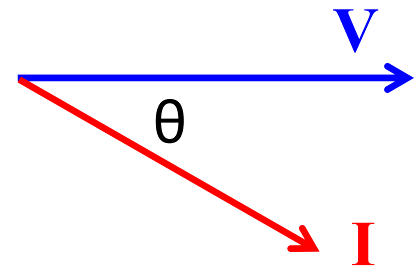
$P = V_{rms} \cdot I_{rms} \cdot \cos\theta$

$= |S| \cdot \cos\theta$

↑ Apparent power.

Apparent Power

We express power in d.c. and a.c. circuits as follows:



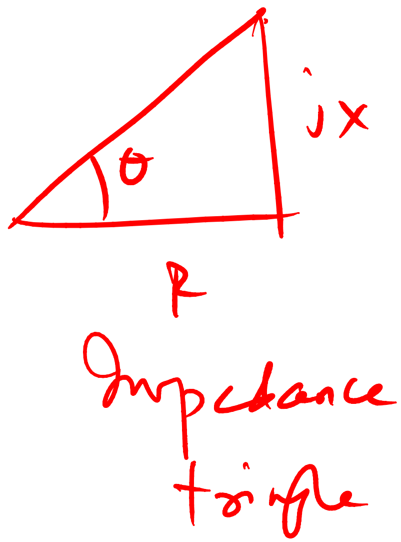
$P_{dc} = V_{dc} I_{dc}$ Watts

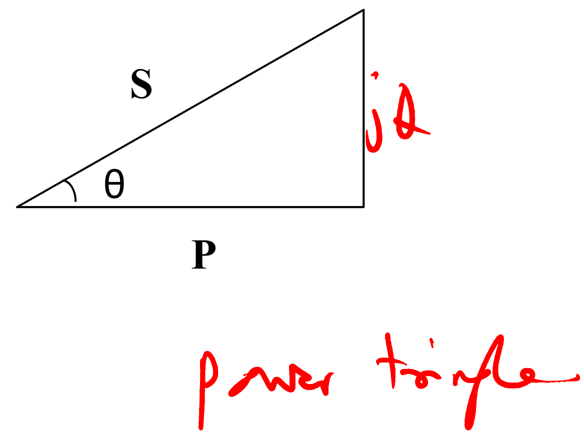
$P_{ac} = V_{rms}\, I_{rms} \cos\theta$ Watts

Apparent Power $|S| = V_{rms}\, I_{rms}$ VA (volt-amperes)

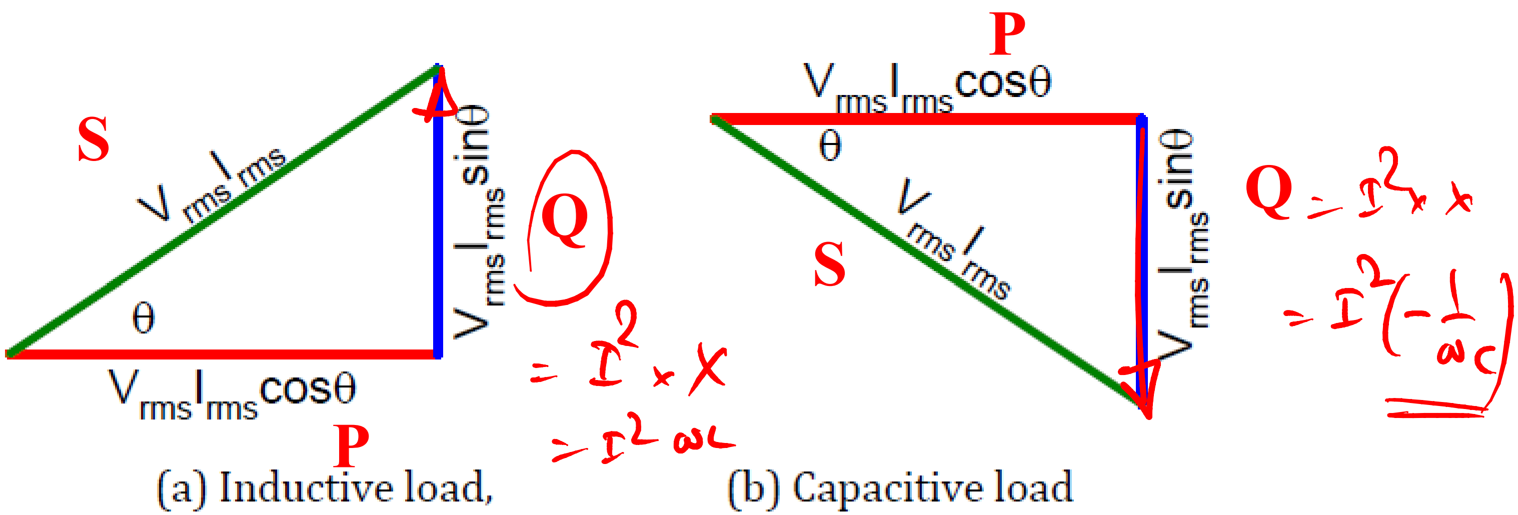
So, the power in a.c. circuit may also be expressed in terms of apparent power as follows:

$P = |S| \cos\theta$





Power Triangle



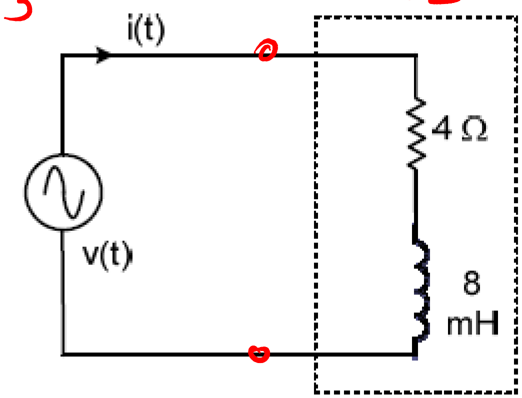
P – Real (or active) power; power consumed in the resistive part of the circuit

Q – Reactive power consumed by the device, due to inductor or capacitor

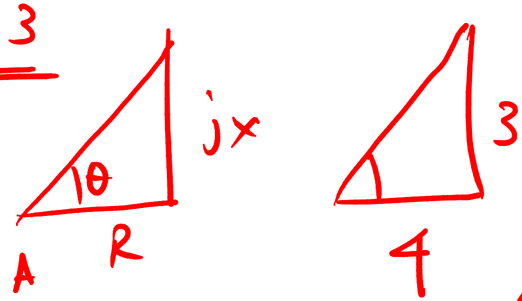
Example 2: **Compute the instantaneous, average, real and reactive powers in the following circuit if $v(t)=14.14\sin(377t)$**

$V = \frac{14.14}{\sqrt{2}}\angle 0^\circ$

$R = 4\,\Omega$, $L = 8\,mH$, $X_L = \omega L = 377 \times 8 \times 10^{-3} = 3\,\Omega$



$Z = R + jX = \underline{\underline{4 + j3}}$



$I = \frac{V}{Z} = 2.0\angle -36.87^\circ\ A$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$

$i(t) = 2.0\sqrt{2}\cdot\sin(377t - 36.87^\circ)\ A.$

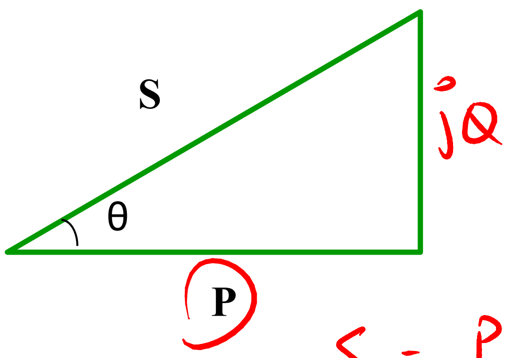
$p(t) = v(t)\cdot i(t) = 14.14 \times 2.0 \times \sqrt{2}\cdot\sin(377t)\cdot\sin(377t - 36.87^\circ)$ W.

$P_{avg} = P = V_{rms}\cdot I_{rms}\cdot\cos\theta = \frac{14.14}{\sqrt{2}} \times 2.0 \times \cos(36.87^\circ)$ W

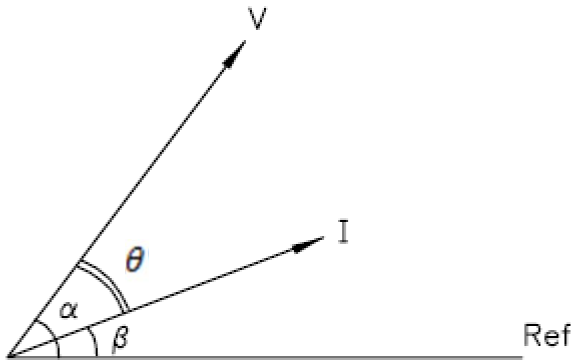
$Q = V_{rms}\cdot I_{rms}\cdot\sin\theta = \frac{14.14}{\sqrt{2}} \times 2.0\cdot\sin(36.87^\circ)$ VAR

Complex Power

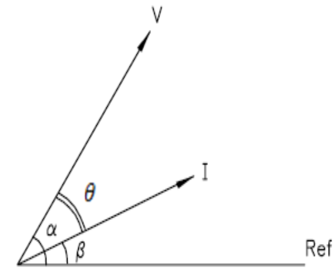
Apparent power, $|S| = V_{rms}\ I_{rms}$



$S = P + \hat{j}Q$



Complex Power

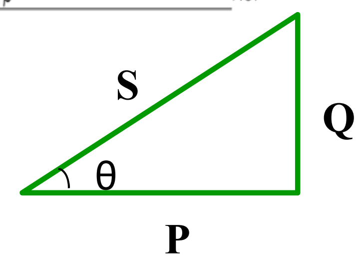


1 **Complex power S = V.I***

Complex power, $S = V_{rms}\, I_{rms} \cos\theta + j\, V_{rms}\, I_{rms} \sin\theta$

$S = |S| \cos\theta + j\, |S| \sin\theta$

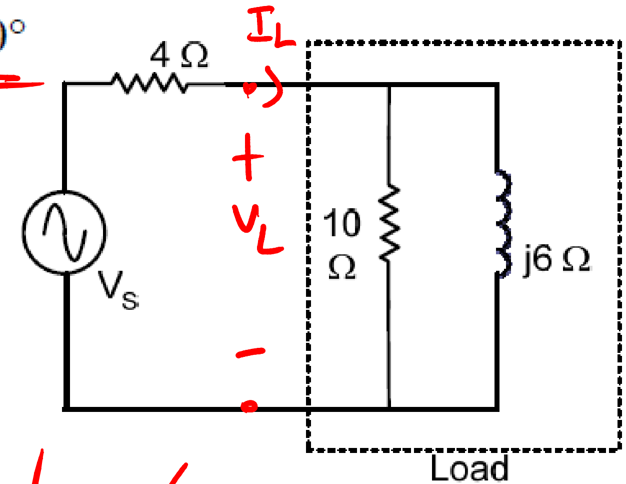
2 $= P + jQ$



S	**Complex power**	VA	$S = \lvert S\rvert \angle\theta = P + jQ = VI^* = \\ = \lvert V\rvert \lvert I\rvert \angle\theta = \lvert V\rvert \lvert I\rvert (\cos\theta + j\sin\theta)$
|*S*|	**Apparent power**	VA	$\lvert S\rvert = \lvert V\rvert \lvert I\rvert = \sqrt{P^2 + Q^2}$
P	**Active power** Average power, Real power	W	$P = \mathrm{Re}(S) = \lvert S\rvert \cos(\theta) = \lvert V\rvert \lvert I\rvert \cos(\theta)$
Q	**Reactive power**	var	$Q = \mathrm{Im}(S) = \lvert S\rvert \sin(\theta) = \lvert V\rvert \lvert I\rvert \sin(\theta)$

Example 1: A voltage source with series resistor is connected to a parallel combination of inductor and resistor. Find the complex power, and hence real power and reactive power delivered to the load.

$V_S = 110\angle 0°$



$S = V_L \cdot I_L^* = P + jQ.$

$V_L = V_s \times \dfrac{Z_L}{4 + Z_L} = 110\angle 0° \times \dfrac{5.14\angle 59°}{4 + 5.14\angle 59°}$

$= 70.9\angle 25.46°.$

$I_L = \dfrac{V_L}{Z_L} = \dfrac{70.9\angle 25.46°}{5.14\angle 59°} = 13.79\angle -33.54°$

$Z_L = \dfrac{10 \times (j6)}{10 + j6} = 5.14\angle 59°$

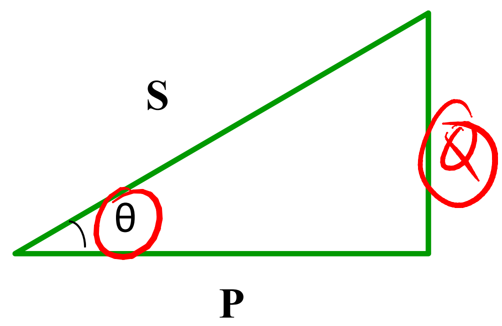
$S = V_L \cdot I_L^* = 70.9\angle 25.46° \times 13.79\angle 33.54°$

$= \underbrace{977.71\angle 59°}_{S} = \underbrace{503.56}_{P} + j\underbrace{838.}_{Q}$

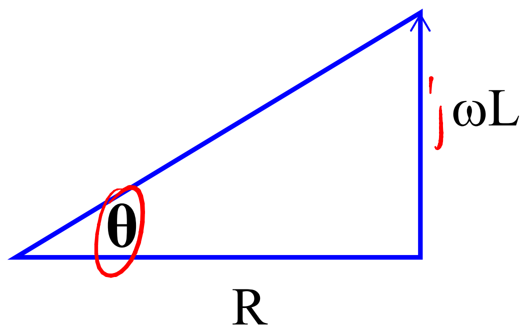
Power Factor of an a.c. circuit

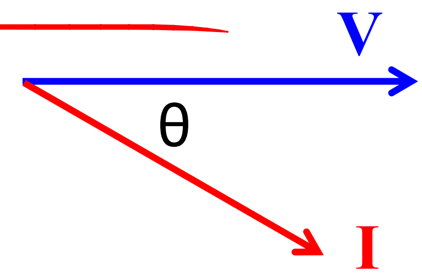
Now, we will define Power Factor of an a.c. circuit as the ratio of real power to the apparent power.

$$\text{Power Factor} = \frac{P}{|S|} = \frac{P}{|V|\,|I|} = \cos\theta$$



Power factor angle (θ) is the same in power triangle, impedance triangle, and the angle between voltage and current.



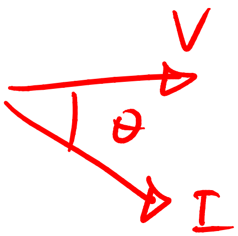


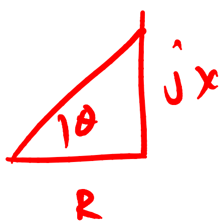
Leading \ Lagging Power Factor

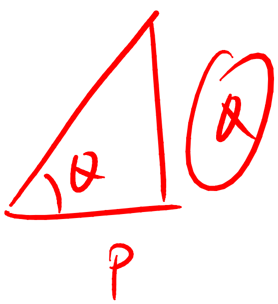
Lagging

↓

Inductive





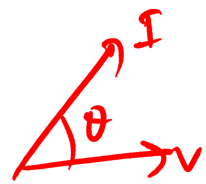


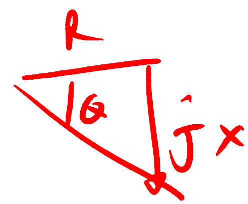
+ve

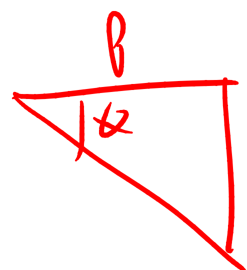
(Leading)

↓

capacitive.

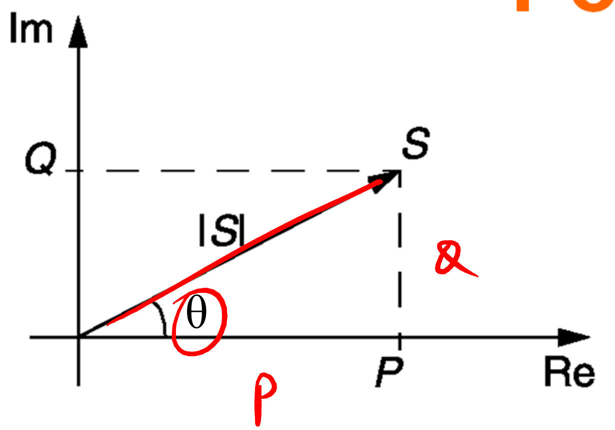






Q -ve

Power Factor



$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$$|S| = \sqrt{P^2+Q^2}$$

$$\text{P. f.} = \cos(\theta) = \frac{P}{|S|} = \frac{P}{\sqrt{P^2+Q^2}} \quad (1)$$

$$\cos(\theta) = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right) \quad (2)$$

Power Factor Correction

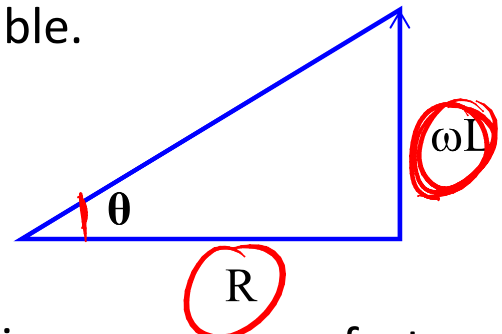
Loads are usually connected to a fixed voltage supply, e.g. 220V, 50 Hz in Singapore, hence V_{rms} is given.

utility

$$p.f. = \frac{P}{V_{rms} I_{rms}} \qquad Or \quad \boxed{I_{rms} = \frac{P}{V_{rms} \times \cos\theta}}$$

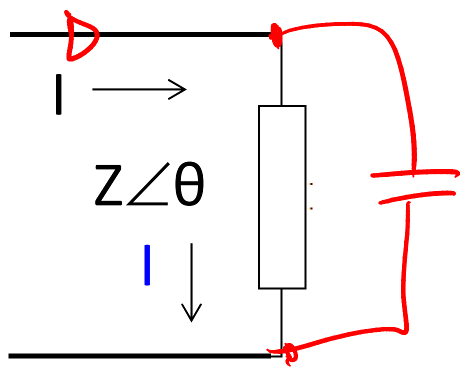
To deliver a certain amount of power to the load, current will be larger if power factor is smaller.

Hence it is desirable that power factor be as close to 1 as possible.
i.e. θ should be as small as possible.



Once the load is connected, its θ cannot be changed.

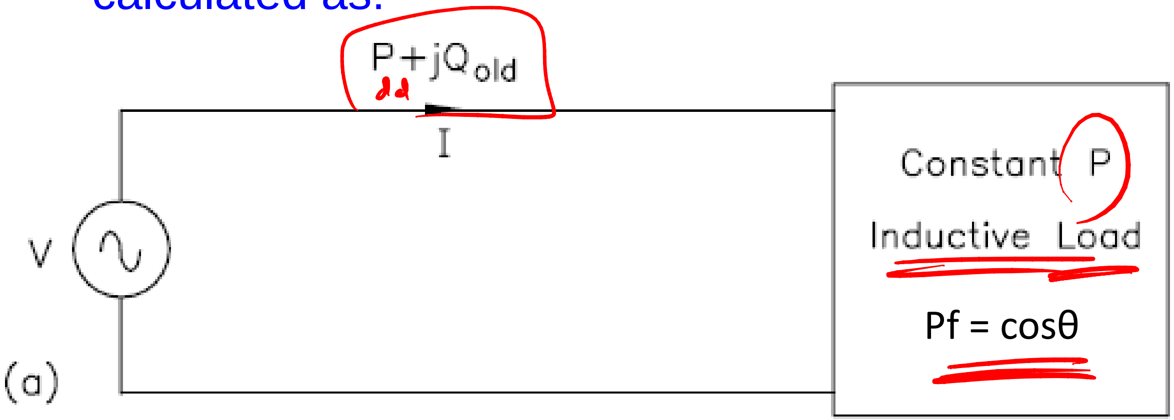
Another reactive element can be added parallel to the load to improve power factor.



If the load is originally inductive, choose a capacitor
If the load is originally capacitive, choose an inductor as correcting device

Power Factor Correction

If P is the active power consumption of the load, the current drawn by the load can be calculated as:



$\checkmark\ P = 1000\,W = |S| \cos\theta$.

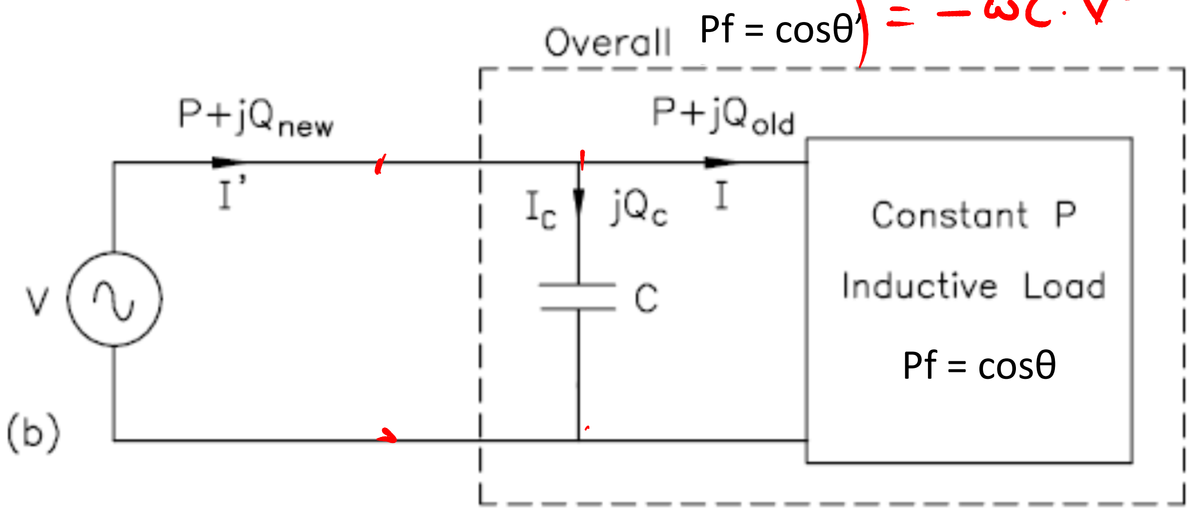
$\checkmark\ Pf = 0.8$ lagging $= \cos\theta$.

$\rightarrow Q = |S| \sin\theta$

$= \frac{P}{p.f.} \times \sin(\cos^{-1}(p.f.))$

When a capacitor is connected to improve the power factor, the current drawn by this capacitor: $Q_c = Q_{new} - Q_{old}$

$= -\omega C \cdot V^2$



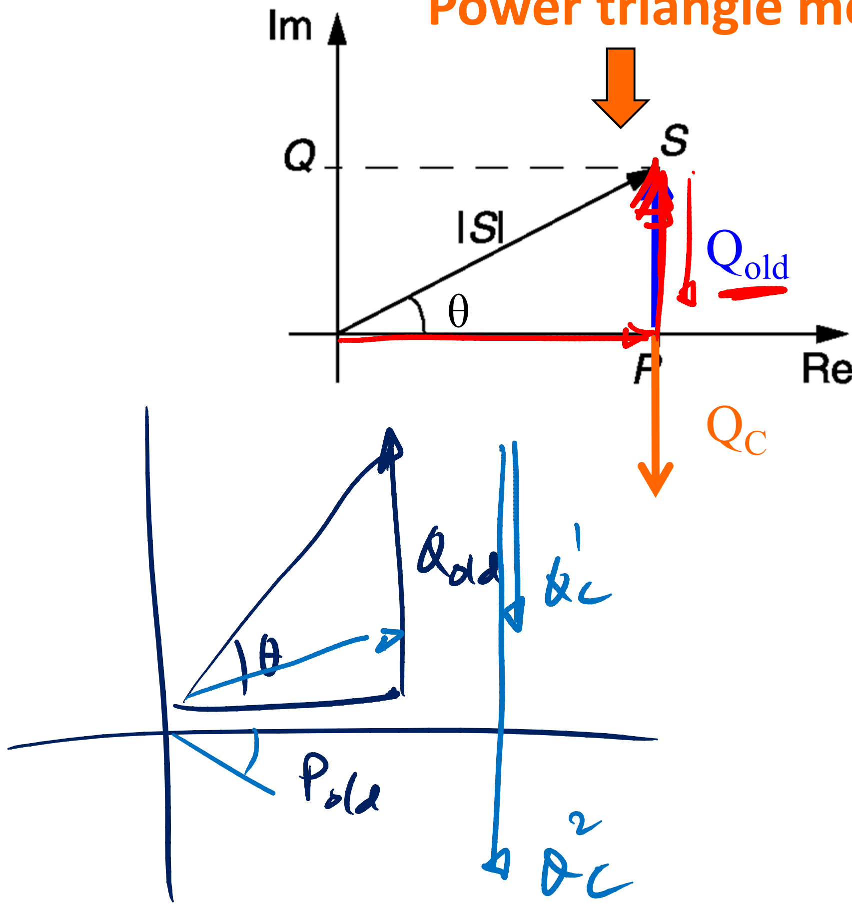
$P_{new} + jQ_{new} = P_{old} + j(Q_{old} + Q_c)$

$P.f.\ new = \cos\theta'$

$Q_{new} = \frac{P_{old}}{\cos\theta'} \cdot \sin\theta'$.

Power Factor Correction

Power triangle method



$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

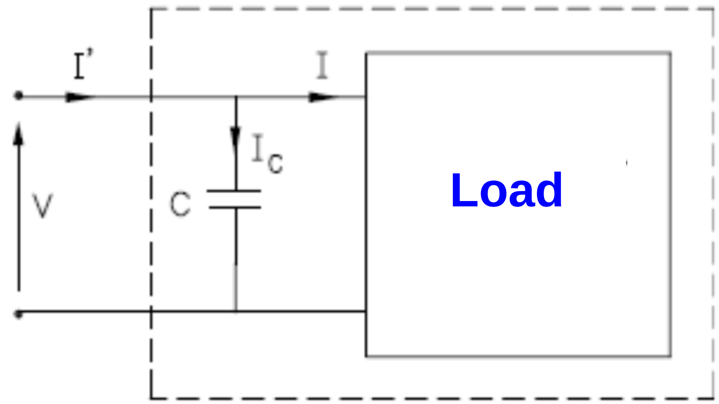
$$|I| = P / |V|\cos(\theta)$$

Example 2 - Power Factor Correction

A load connected across a 200 V, 50Hz line draws 10 kW at 0.5 power factor lagging. A capacitor C is now connected in parallel with the load to improve the power factor. What must be the value of C to make the overall power factor

(i) 0.9 lagging, (ii) unity and (iii) 0.8 leading?

Solution: Take source voltage as reference. So, current drawn at 0.5 p.f. lagging is

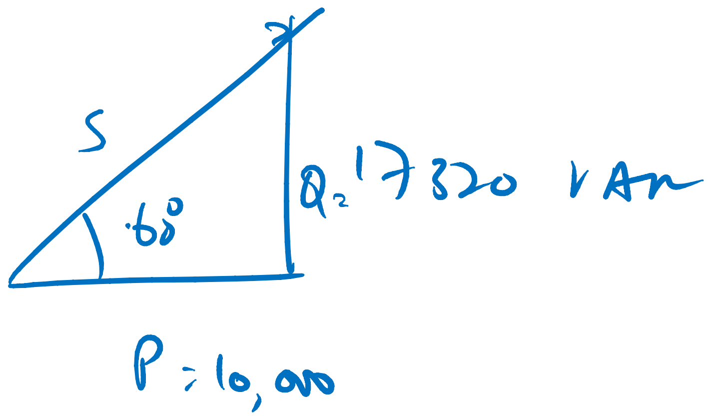


$P = 10\text{ kW}$

$p.f. = 0.5$ lag

$$Q_{old} = \frac{P}{p.f.} \times \sin(\cos^{-1} p.f.) = \frac{10000}{0.5} \times \sin(\cos^{-1} 0.5)$$

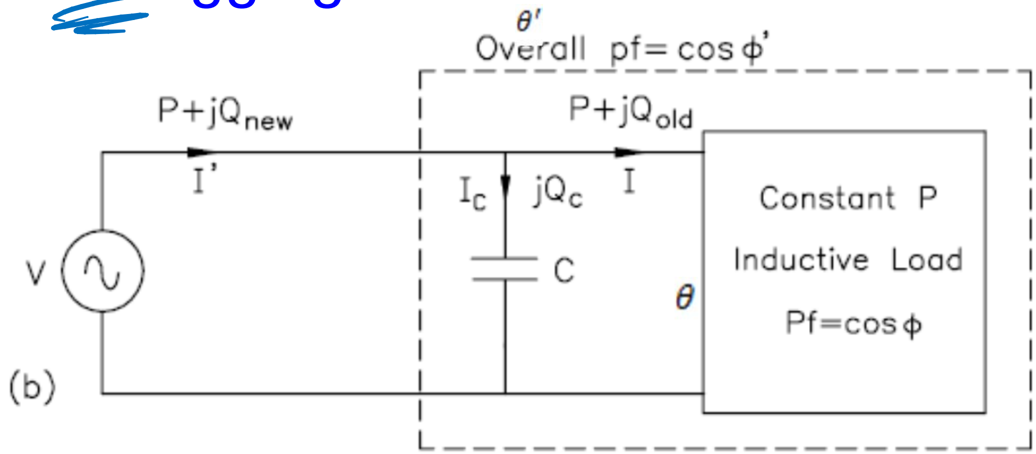
$$= 17320 \text{ VAR}.$$



Old.

(load)

Case 1: 0.9 lagging



$$P_{new} + j\,Q_{new} = P_{old} + j\,Q_{new}$$

$$= 10000 + j\,Q_{new}$$

$$Q_{new} = \frac{P}{p.f.} \times \sin\left(\cos^{-1}(p.f.\ new)\right)$$

$$= \frac{10000}{0.9} \cdot \sin\left(\cos^{-1}(0.9)\right) = 4843 \text{ VAR}$$

$$Q_c = Q_{new} - Q_{old}$$

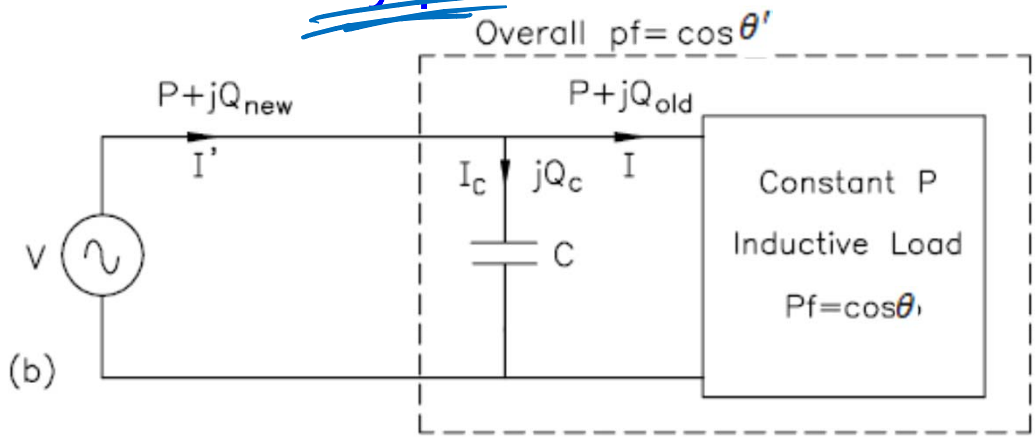
$$= 4843 - 17320$$

$$= -12477$$

$$= -\omega \cdot C \cdot V^2$$

$$\Rightarrow C = \frac{12477}{2\pi 50} \times \frac{1}{200} \text{ F}.$$

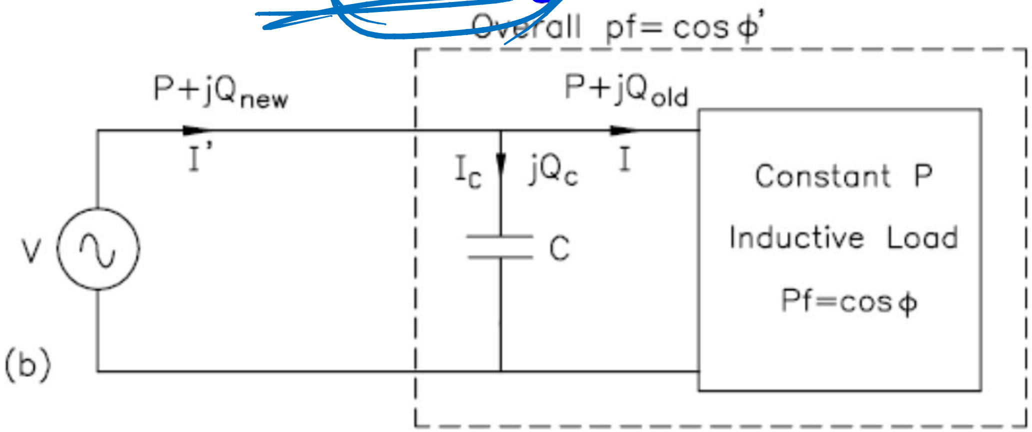
Case 2: Unity p.f.



$$Q_{new} = 0 \qquad Q_c = Q_{new} - Q_{old} = -17320.$$

$$C = \frac{Q_c}{\omega V^2} = \frac{17320}{2\pi \times 50 \times 200} \text{ F.}$$

Case 3: 0.8 leading



$$Q_{new} = \frac{P}{P.f.} \cdot \sin(-\cos^{-1} 0.8)$$

$$= \frac{10000}{0.8} \cdot \sin(-36.87^\circ) = -7500 \text{ VAR}.$$

$$Q_c = Q_{new} - Q_{old} = -7500 - 17320 \text{ VAR}$$

$$\Rightarrow C.$$

Transmission Line Loss

Industrial load connected to a substation

V_S = Substation or sending end voltage.

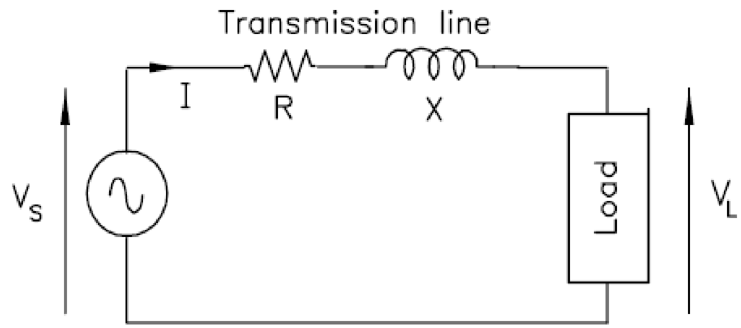
V_L = Voltage at the load.

I = Current drawn by the load.

R = Resistance of the transmission line.

X = Reactance of the transmission line.

The transmission line loss is given by: $P_{loss} = |I|^2 R$



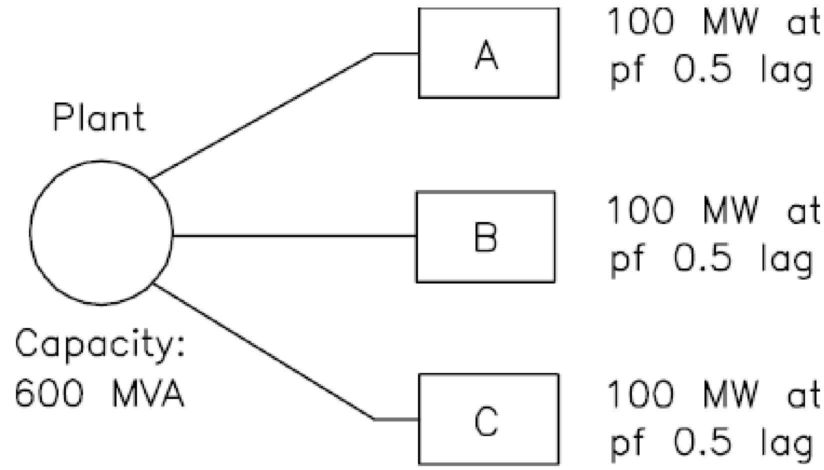
For the same real power demand, if the power factor of the load decreases, then |I| increases as shown by the following equation. This results in heavy line losses.

$$|I| = \frac{P}{|V| cos\theta}$$

A poor power factor results in higher current and hence higher power loss.

Capital Cost of Power Plants

Three industrial customers A, B and C are drawing power from the Power Plant. Let the load at each of the three industries be 100 MW at a p.f. of 0.5 lagging. The maximum demand of each consumer is



$$\text{Max. Demand} = |V||I| = \frac{P}{\cos\theta} = \frac{100}{0.5} = 200\ \text{MVA}$$

So the installed capacity of the plant should be

$$\sum \text{Max. Demand} = 600\ \text{MVA}$$

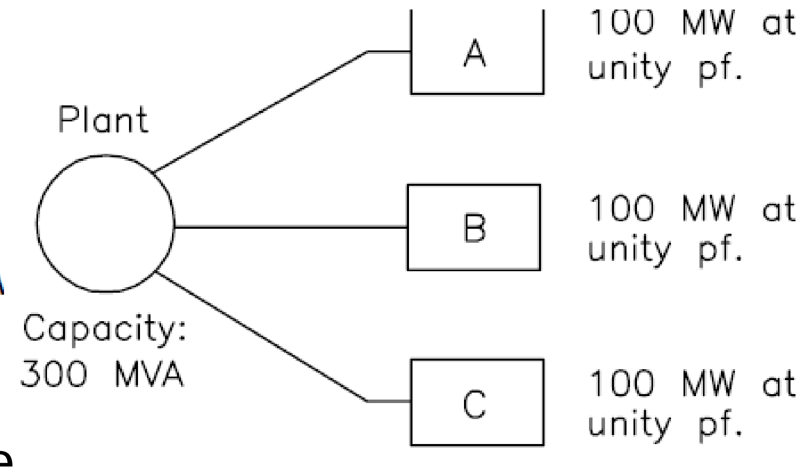
If \$X is the capital cost per annum per MVA of the plant, the total annual capital cost is

Capital cost = \$600X

The three consumers will share the capital cost equally in this case and each will pay \$200X.

If the load at each of the three industries is 100 MW at unity p.f., the maximum demand of each consumer is

$$\text{Max. Demand} = |V||I| = \frac{P}{cos\ \theta} = \frac{100}{1.0} = 100\ \text{MVA}$$



So the installed capacity of the plant should be

$$\sum \text{Max. Demand} = 300\ \text{MVA}$$

As the plant capacity is only 300 MVA, the total annual capital cost in this case is

Capital cost = $300X

The three consumers will share the capital cost equally in this case also and each will pay $100X.

Two points become evident from this example:

- Plant capacity gets affected by the maximum demand of each consumer.
- Annual capital cost gets affected by the power factor of the load.

A poor power factor results in higher capital cost.