EE2022
Electrical Energy Systems

Lecture 2: AC Systems Fundamentals

National University of Singapore

Why…?

Why AC and not DC ?

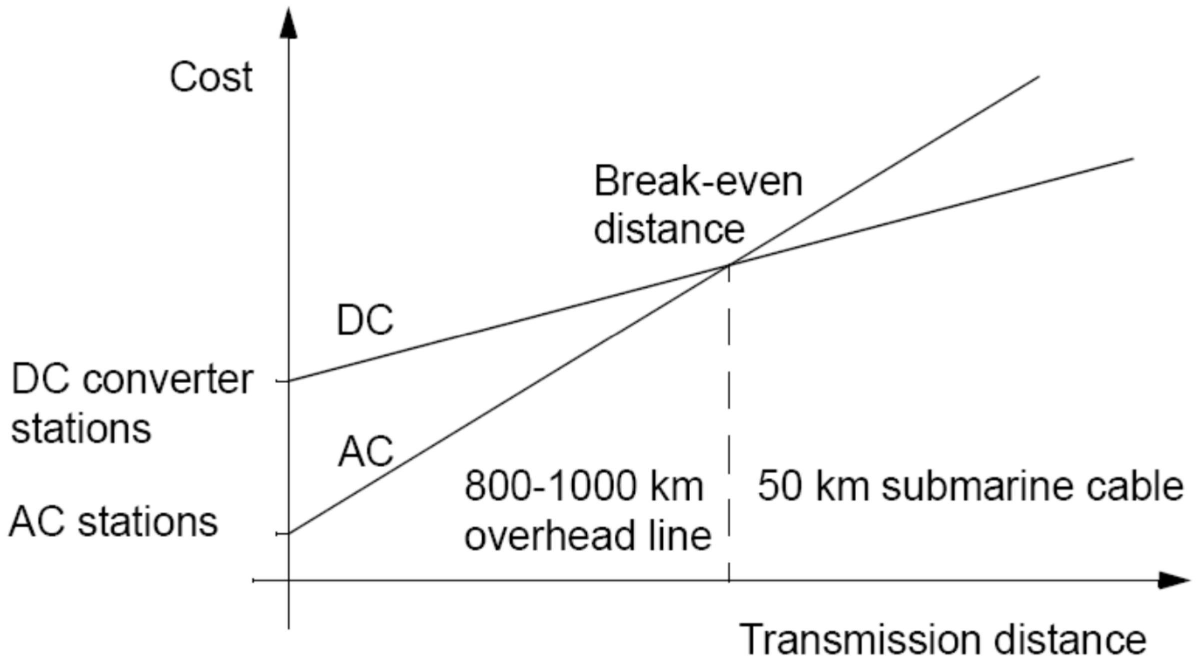
Why a sinusoidal alternating voltage ?

Why 50 Hz (or 60 HZ) ?

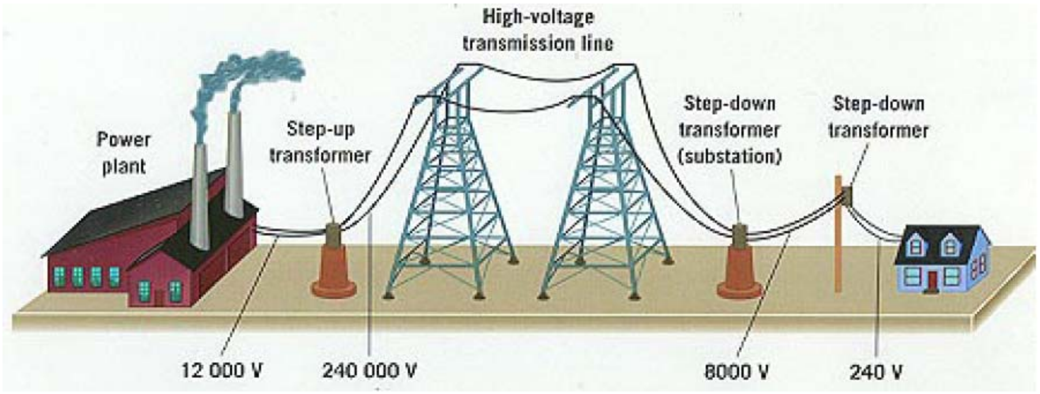
http://en.wikipedia.org/wiki/Utility_frequency

Why AC and not DC ?

Break-even distance for high voltage direct current (HVDC)

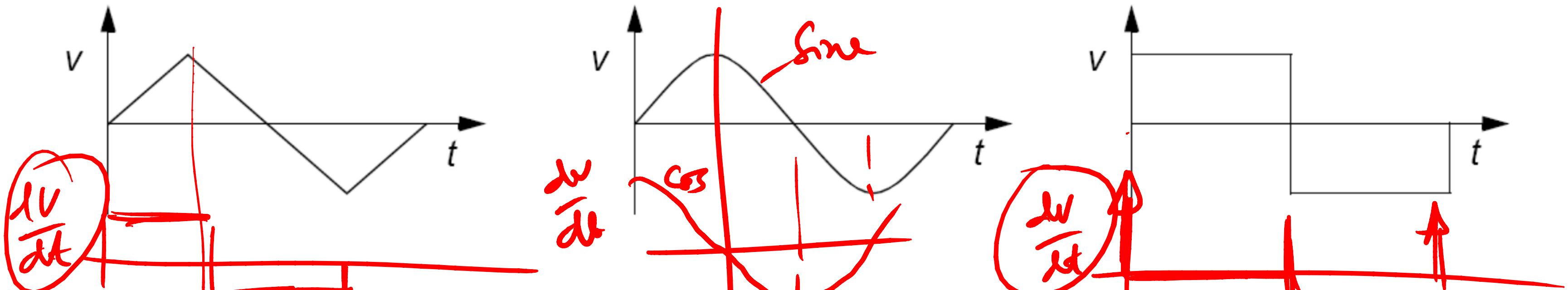


Transformers allow easy transformation of voltage



Why a Sinusoidal Alternating Voltage ?

Triangular, sinusoidal and block signals



- In linear time in-variant dynamic systems, the basic operators are +, -, x, division, differentiation and integration, etc.
- For sinusoids, these operations will result in another sinusoid of same frequency and shape!
- So all components in the system can be designed for this shape.

C: $v - i$: $i_c = C \frac{dV}{dt}$, $v = \frac{1}{C}\int i_c \, dt$

L: $v - i$: $v_L = L \frac{di}{dt}$

The choice of Frequency

50 Hz and 60 Hz

Lamp : light I^2R

- Between 1885 and 1890 in the U.S.A.:
  - 140, 133⅓, 125, 83 ⅓, 66 ⅔, 50, 40, 33 ⅓, 30, 25, 16⅔ Hz
- Nowadays:
  - 60 Hz in North America, Brazil and Japan (has also 50 Hz!)
  - 50 Hz in most other countries

(Singapore)

- Exceptions:
  - 25 Hz Railways (Amtrak)
  - 16⅔ Hz Railways
  - 400 Hz Oil rigs, ships and airplanes

(50/60)

f ↑ → size of the equipment ↓

The choice of Frequency

50 Hz and 60 Hz

- A too low frequency, like 10 or 20 Hz causes flicker
- A too high frequency
  - Increases the hysteresis losses:

$$P_{hys} :: f\psi^{1.5-2.5}$$

  - Increases the eddy current losses:

$$P_{eddy} :: f^2\psi^2$$

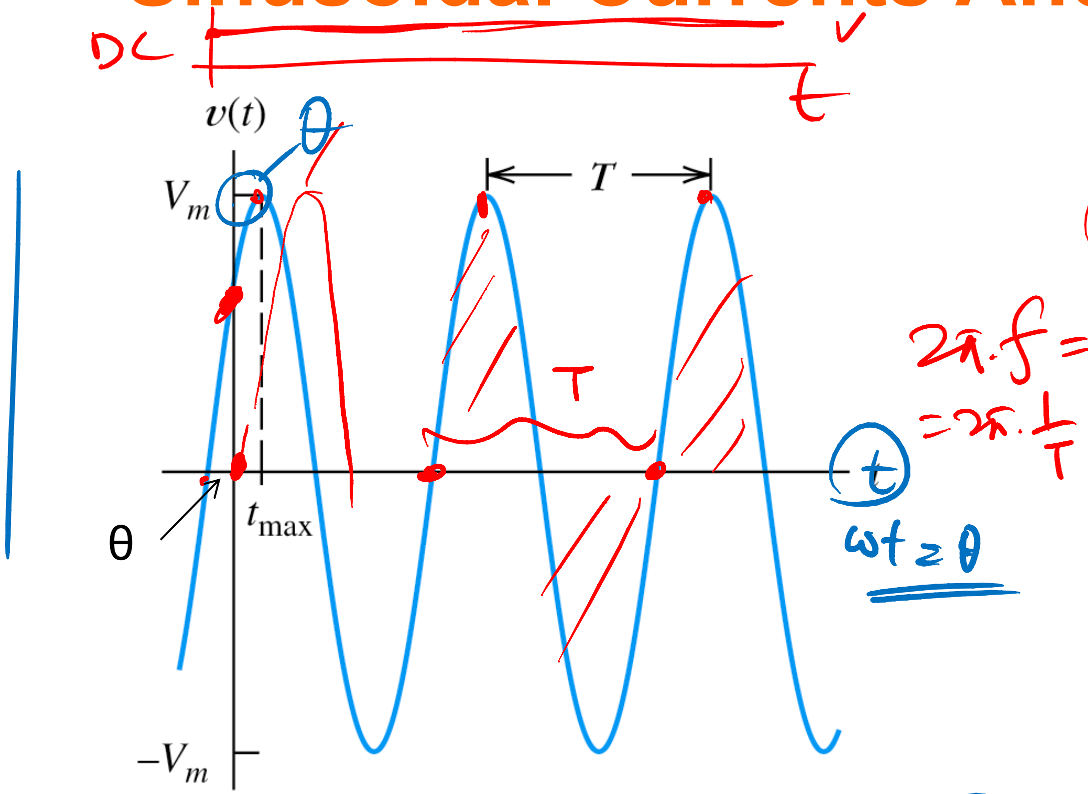
  - Increases the cable and line impedance

Skin effect

upper limit on the freq.

AC Systems Fundamentals

Sinusoidal Currents And Voltages



- V_m is the peak value
- ω is the angular frequency in radians/sec
- θ is the phase angle
- T is the period

A sinusoidal voltage waveform given by $v(t) = \boxed{V_m \cos(\omega t + \theta)}$.
Note: Assuming that θ is in degrees, we have $t_{max} = \frac{-\theta}{360} \times T$.
For the waveform shown, θ is $-45°$.

Root-Mean-Square Values

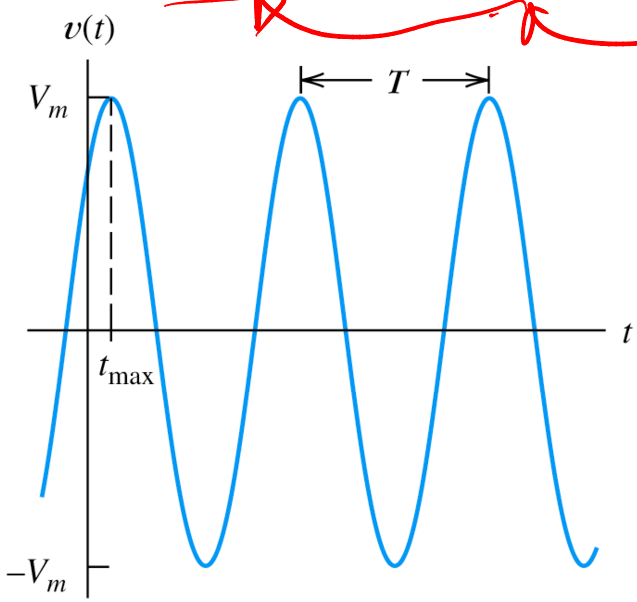


Figure 5.1 A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$. Note: Assuming that θ is in degrees, we have $t_{max} = \frac{-\theta}{360} \times T$. For the waveform shown, θ is -45°.

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$\boxed{= \frac{V_m}{\sqrt{2}}}$$

$$v(t) = V_m \cos(\omega t + \theta)$$

$$v^2(t) = V_m^2 \cdot \cos^2(\omega t + \theta)$$

$$= V_m^2 \cdot \frac{1 + \cos(2\omega t + 2\theta)}{2}$$

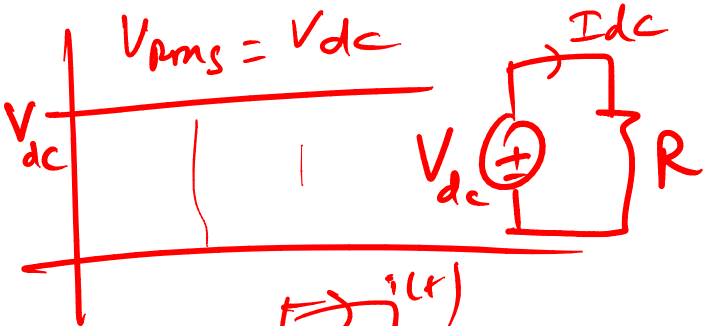
$$= \frac{V_m^2}{2} + \frac{V_m^2}{2} \cdot \cancel{\cos(2\omega t + 2\theta)}$$

$$\int = 0$$

$$\sqrt{2}$$

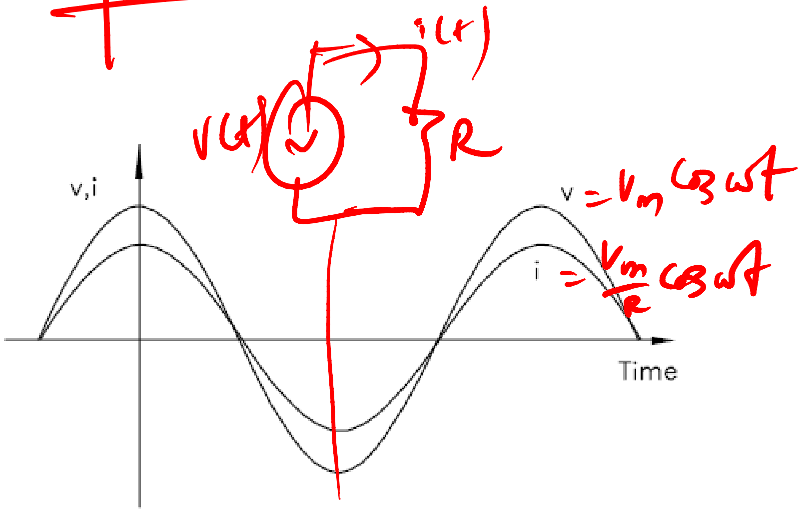
$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

Rationale for using RMS value



The power absorbed by a resistor of R ohms in a d.c. circuit is

$$P = \frac{V_{dc}^2}{R} = I_{dc}^2 R$$



The power absorbed by a resistor of R ohms in an a.c. circuit is

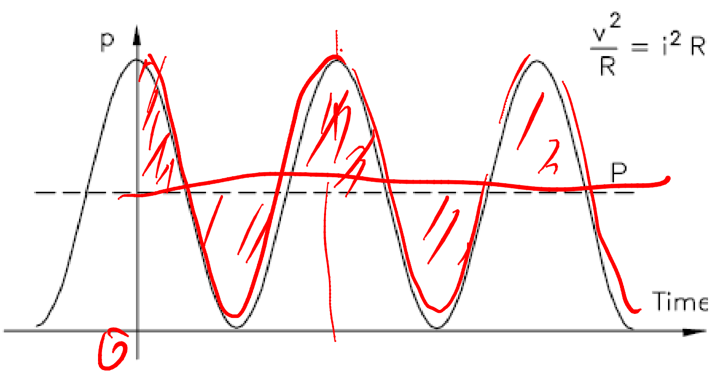
$$P = \frac{I_m^2}{2} R = \frac{V_m^2}{2R}$$

Instantaneous power

$$p(t) = \frac{v^2(t)}{R} = i^2(t).R$$

$$= \frac{V_m^2}{R}.\cos^2 \omega t = \frac{V_m^2}{2R}.\left[1 + \cos 2\omega t\right]$$

$$P_{avg} = \frac{V_m^2}{2R} = \frac{V_x^2}{R} ; \boxed{V_x = V_{rms}}$$



RMS value in Power Calculation

It is desirable to have same form of equation for power in both a.c. and d.c. circuits mainly because of

- Convenience
- Consistence

For d.c. circuits, we have $V_{rms} = V_{dc}$ and $I_{rms} = I_{dc}$. So, for both a.c. and d.c., the power in a resistor is

$$P = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

$$\boxed{P = I^2 R}$$

$I = I_{ms}$.

AC Analysis

Complex number.

e.g. $z_1 = 1 + j2$

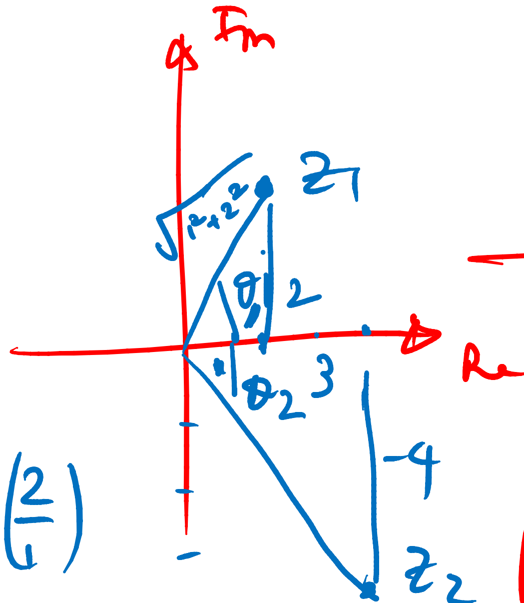
$z_2 = 3 - j4$

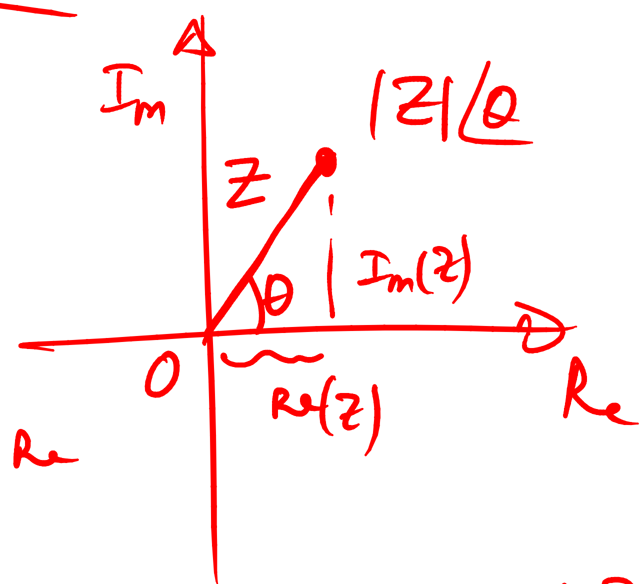
Rectangular form

$z_1 = \sqrt{5}\,\angle \tan^{-1}\left(\frac{2}{1}\right)$

$z_2 = \sqrt{3^2 + (-4)^2}\,\angle \tan^{-1}\left(\frac{-4}{3}\right)$

Polar form





$|Z|\angle\theta = Re(z) + j\,Im(z)$

$j = \sqrt{-1}$, $\boxed{j = i}$

Exponential form: $|Z| \cdot e^{j\theta} = |Z|(\cos\theta + j\sin\theta)$

Complex algebra

$z_1 + z_2$

$z_1 - z_2$

$z_1 \cdot z_2$

$\dfrac{z_1}{z_2}$

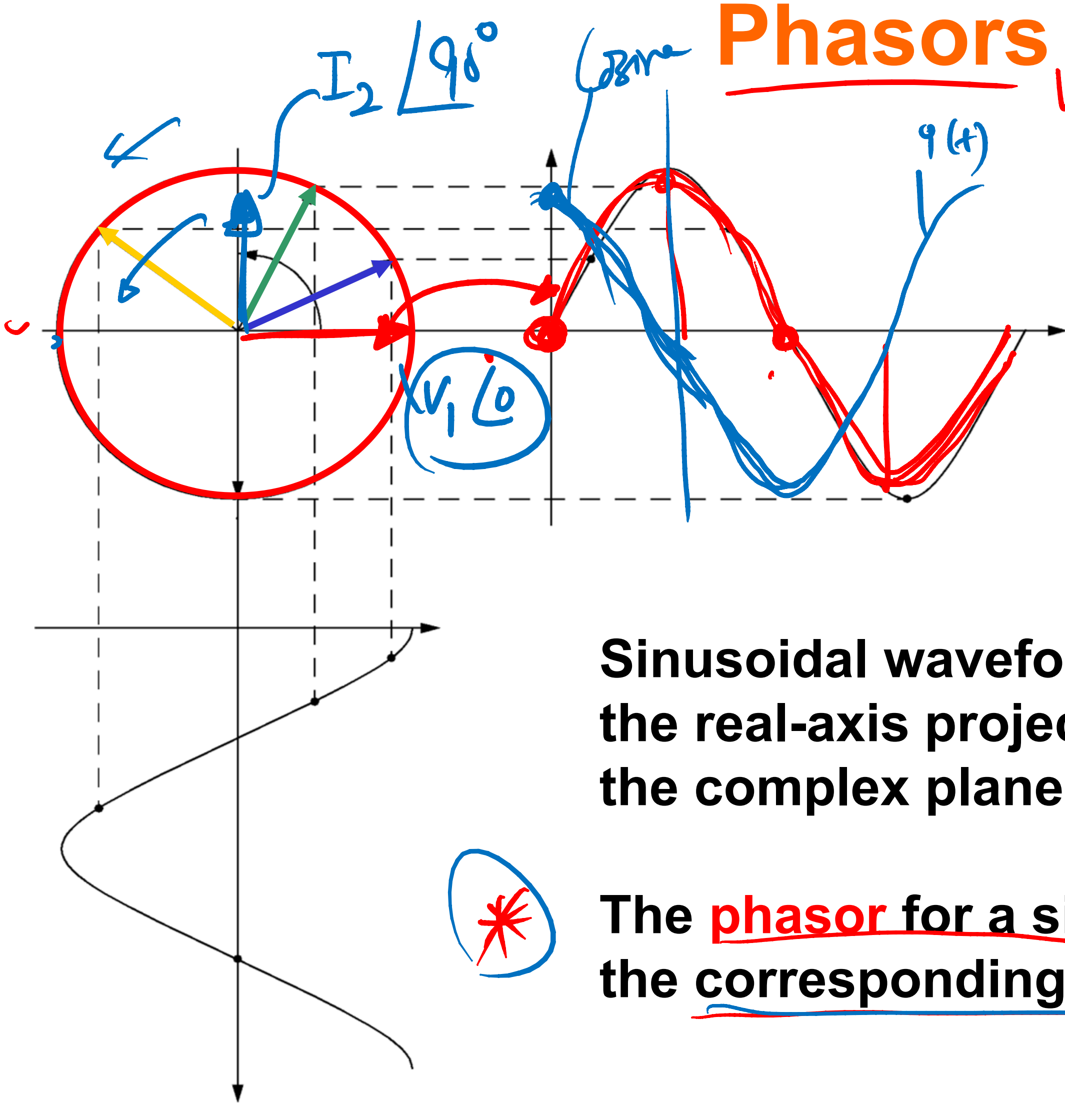
AC Analysis.

① Phasor

② Impedance

Complex number.

Phasors



$V_1 \cdot \sin(\omega t) = v_1(t)$

$i(t) = \dot{I}_2 \cdot \cos(\omega t)$

(why) later

Sinusoidal waveforms can be visualized as the real-axis projection of vectors rotating in the complex plane.

The phasor for a sinusoid is a snapshot of the corresponding rotating vector at $t = 0$.

Phasor Representation of a Sinusoidal waveform

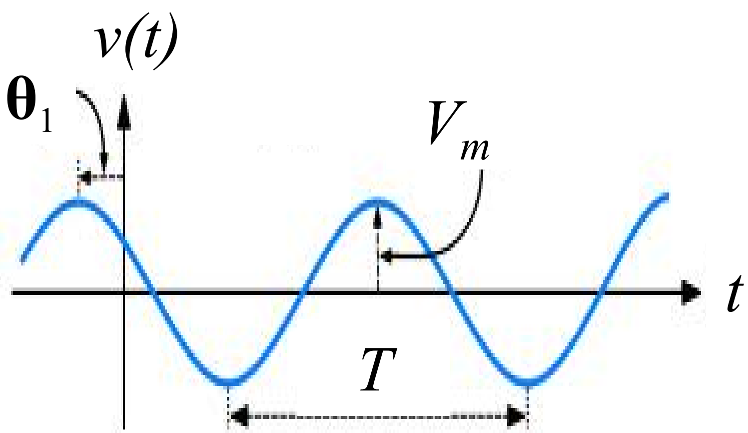
$$\text{Time function}: \quad v_1(t) = V_m \cos(\omega t + \theta_1)$$

$$\text{Phasor}: \quad \mathbf{V}_1 = V_1 \angle \theta_1 \quad \leftarrow\rightarrow \; V_m \angle \theta_1$$

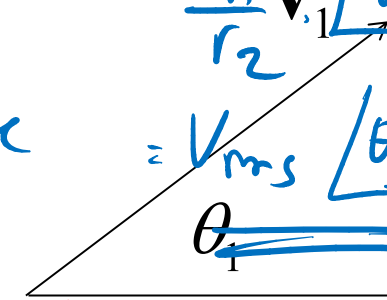
$$\downarrow$$

$$\frac{V_m}{\sqrt{2}} \angle \theta_1 = V_{rms} \angle \theta_1$$

Where V_1 is the RMS value of voltage



Phase

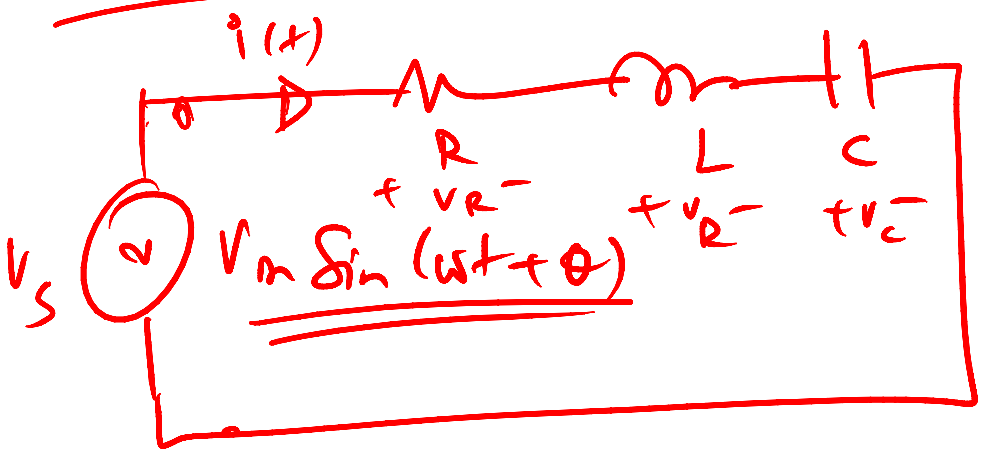


$$I_m \sin(\omega t + \theta)$$

$$\downarrow$$

$$\frac{I_m}{\sqrt{2}} \angle \theta$$

AC analysis



KVL, KCL etc.

$$v_s = v_R + v_L + v_c$$

$$V_m \sin(\omega t + \theta) = iR + L\frac{di}{dt} + \frac{\int i\,dt}{C}$$

$$V_m \sin(\omega t + \theta) = iR + L \cdot \frac{di}{dt} + \frac{1}{C}\int i \cdot dt$$

$$\frac{d}{dt} \div \quad V_m \cdot \omega \cos(\omega t + \theta) = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} \cdot i$$

Solve the diff. eqn $\longrightarrow i(t)$

Phasor :

impedance :

Sinusoidal $\longrightarrow$ Complex number

R, L, C $\longrightarrow$

$R \to R$

$L \to j\omega L$

$C \to \dfrac{1}{j\omega C}$

Complex number

Convention used for Phasors

So far, we have used peak amplitude of cosine wave as the length of phasor.

In AC systems, the **RMS** value of a sine wave or cosine wave is typically used.

$$V_{rms} = \frac{V_m}{\sqrt{2}}, \; I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$V_m = \sqrt{2}V_{rms}, \; I_m = \sqrt{2}I_{rms}$$

Using RMS value doesn't alter the relationship between phasors.

Default conventions used for phasor:

- time domain signal is expressed as cosine function
- length of the phasor is RMS value of the waveform

Example 1: Converting sinusoidal time-domain expression to phasor form

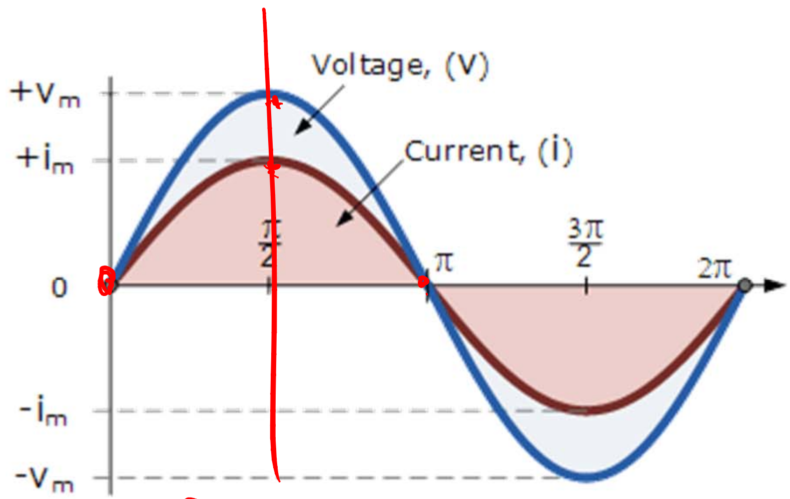
$$v(t) = \sqrt{2}.20\cos(\omega t - 45)$$

$$Vrms = Vm / \sqrt{2} = 20$$

$$\boxed{\mathbf{V}_1 = 20\angle -45^\circ}$$

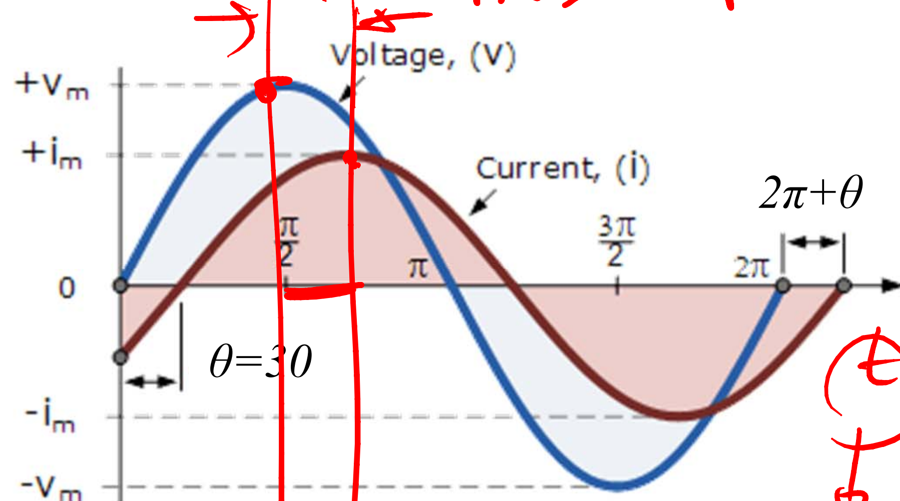
Phase Relationships

Sometimes when we are analysing alternating waveforms we may need to know the position of the phasor, representing the alternating quantity at some particular instant in time especially when we want to compare two different waveforms on the same axis



in-phase

Phase difference



$\theta = \omega \cdot t$

$= 2\pi f \cdot t$

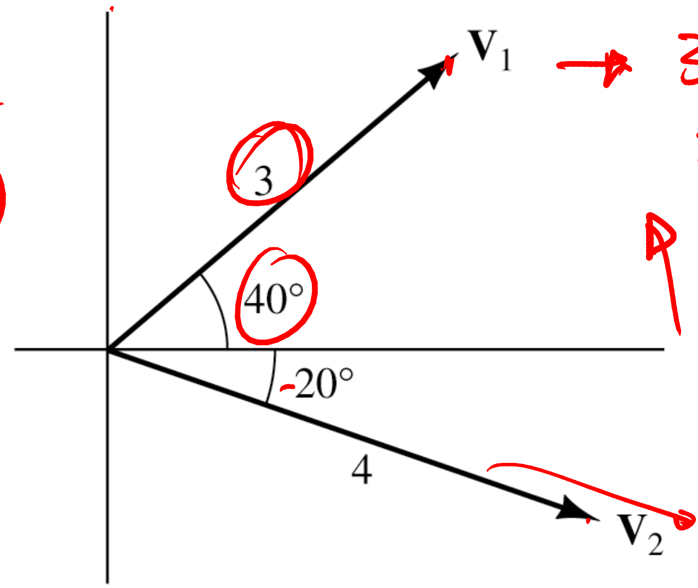
$= \frac{2\pi}{T} \cdot t$

Phase Relationships

To determine phase relationships from a phasor diagram, consider the phasors to rotate counterclockwise.
Then when standing at a fixed point, if V_1 arrives first followed by V_2 after a rotation of θ, we say that V_1 leads V_2 by θ.
Alternatively, we could say that V_2 lags V_1 by θ.

phase diff $= \theta_1 - \theta_2$
$= 40 - (-20^\circ)$
$= 60^\circ$

V_1 leads V_2 by 60°



$\rightarrow 3\angle 40^\circ \rightarrow \theta_1 = 40^\circ$

$3\sqrt{2}(\cos \omega t + 40^\circ)$

$\theta_1 = 40^\circ$

+ve θ

if −ve θ

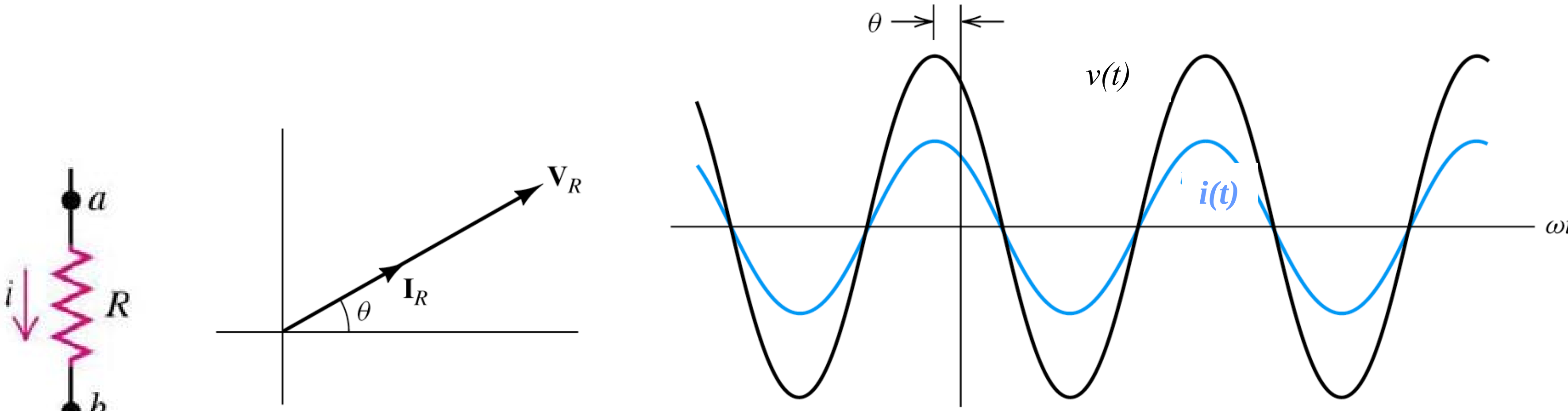
$4\sqrt{2}(\cos \omega t - 20^\circ)$

$\theta_2 = -20^\circ$

Because the vectors rotate counterclockwise, v_1 leads v_2 by 60° (or, equivalently, v_2 lags v_1 by 60°.)

Impedance for Circuit Components R, L, C

Voltage and current in a Resistance



(a) Phasor diagram

(b) Current and voltage versus time

For a pure resistance, current and voltage are in phase.

$V = iR$

$i(t) = I_m \cos(\omega t + \theta)$

$V = R \cdot i = R \cdot I_m \cos(\omega t + \theta)$

$I_{phasor} = \frac{I_m}{\sqrt{2}} \angle \theta$

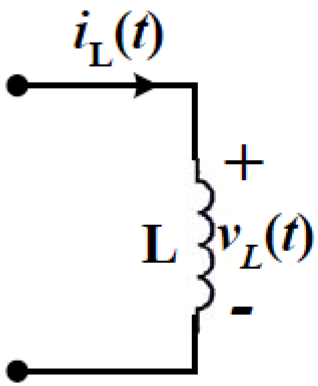
$V = Z \cdot I$

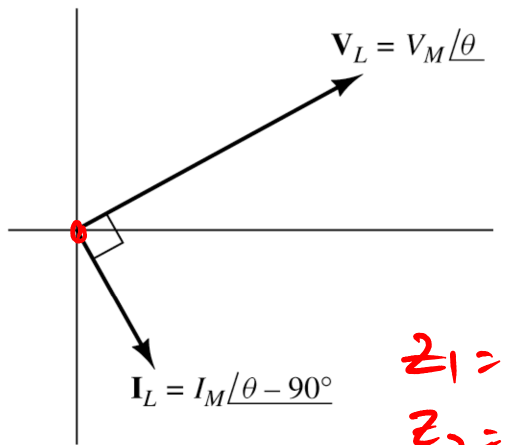
Impedance

$V_{phasor} = \frac{R \cdot I_m}{\sqrt{2}} \angle \theta$

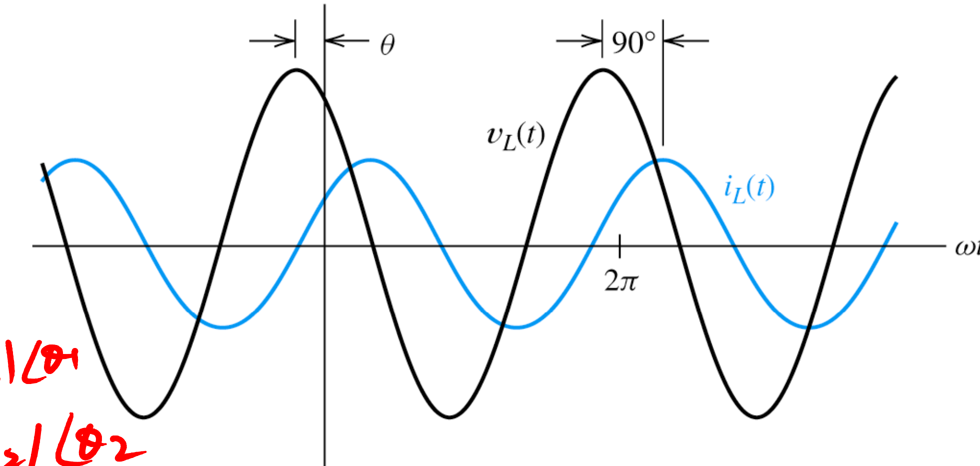
$\boxed{Z_R = R}$

Voltage and current in an Inductor





(a) Phasor diagram



(b) Current and voltage versus time

$Z_1 = |Z_1|\angle\theta_1$

$Z_2 = |Z_2|\angle\theta_2$

$\frac{Z_1}{Z_2} = \frac{|Z_1|}{|Z_2|}\angle\theta_1-\theta_2$

Current lags voltage by 90° in a pure inductance.

$\boxed{-\sin\theta = \cos(\theta+90^\circ)}$

$V = L\frac{di}{dt}$

$i = I_m\cos(\omega t+\theta)$

$V = L\frac{di}{dt} = L\cdot I_m\cdot(-\omega)\cdot\sin(\omega t+\theta)$

$= L\cdot I_m\cdot\omega(\cos(\omega t+\theta+90^\circ))$

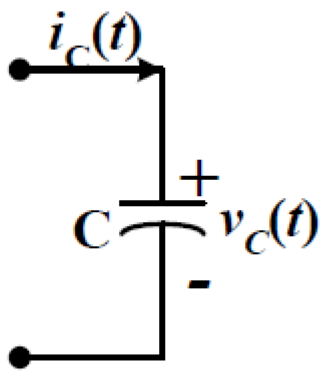
$I_{phasor} = \frac{I_m}{\sqrt{2}}\angle\theta$

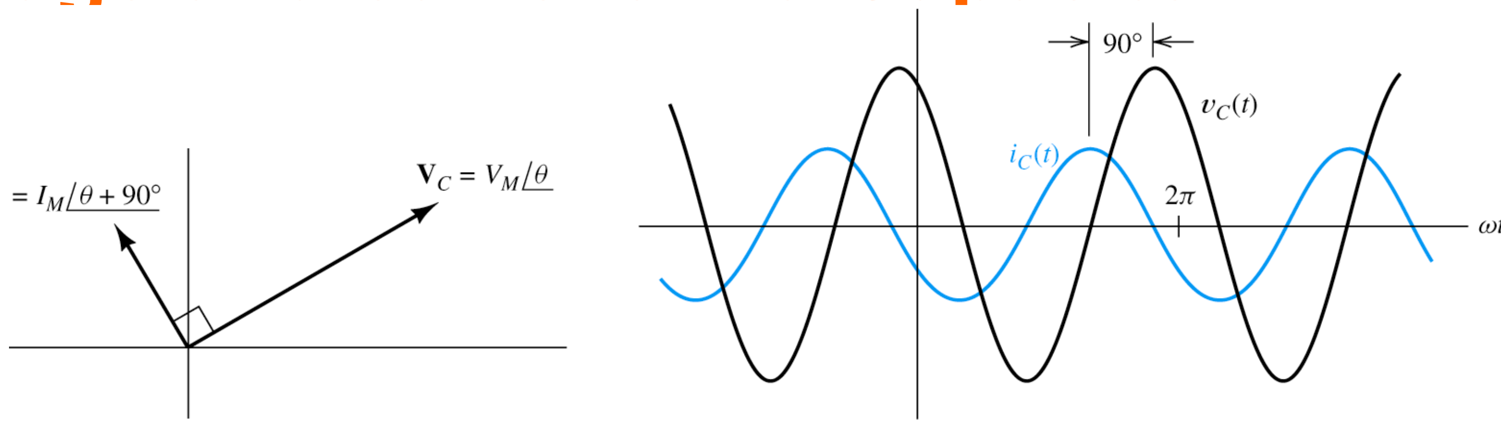
$V_{phasor} = L\cdot I_m\cdot\omega\angle\theta+90^\circ$

$\rightarrow Z_L = \frac{V_{phasor}}{I_{phasor}} = L\cdot\omega\cdot\angle 90^\circ$

$= j\omega L$

Voltage and current in a Capacitor





(a) Phasor diagram

(b) Current and voltage versus time

Current leads voltage by 90° in a pure capacitance.

$$i = C \cdot \frac{dV}{dt}$$

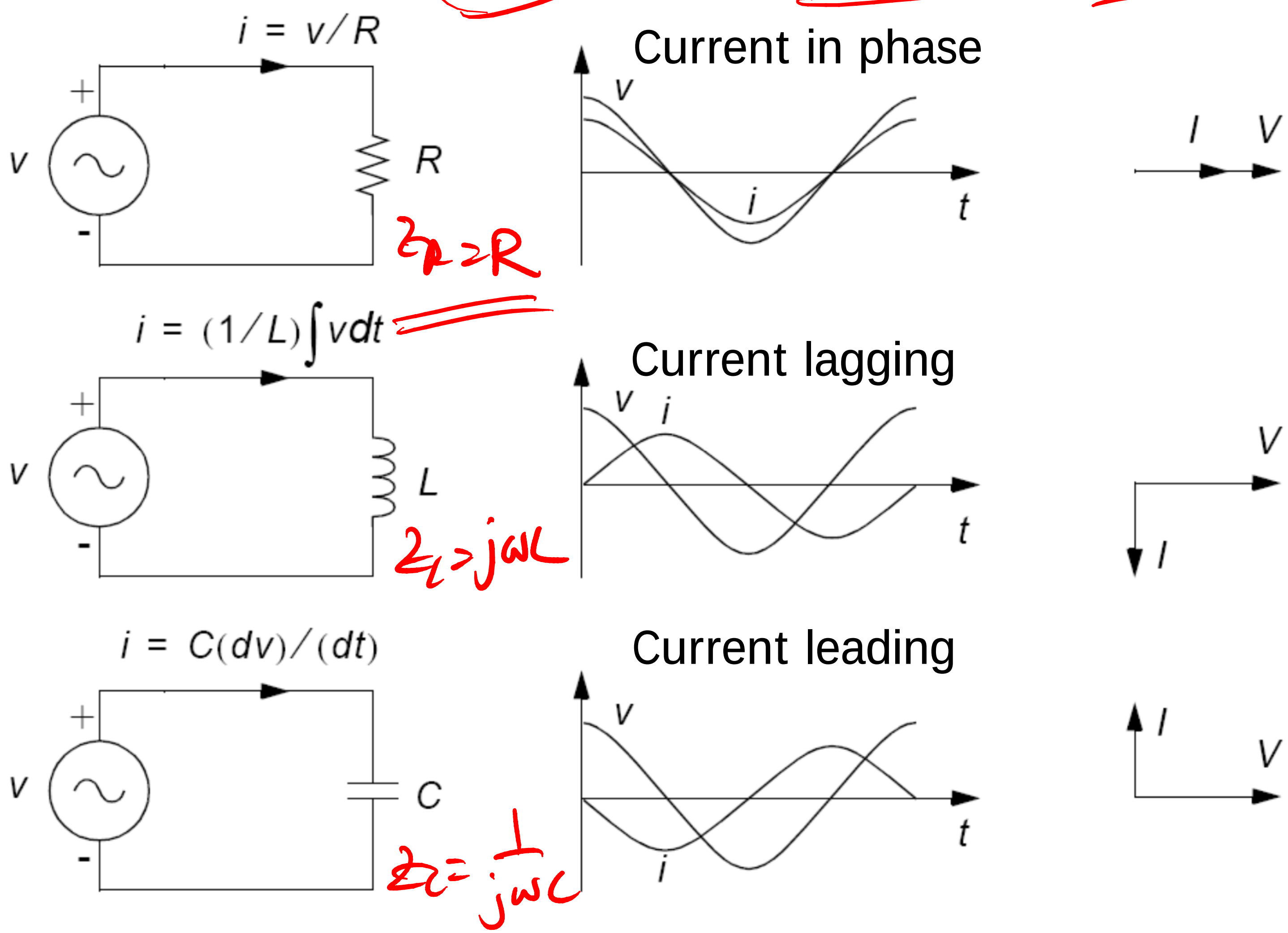
$$V = V_m \cos(\omega t + \theta) \qquad V_{phasor} = \frac{V_m}{\sqrt{2}} \angle\theta$$

$$i = V_m C \cdot (-\omega) \sin(\omega t + \theta) = C \cdot \omega V_m \cos(\omega t + \theta + 90°)$$

$$I_{phasor} = \frac{C\omega \cdot V_m}{\sqrt{2}} \angle\theta + 90°$$

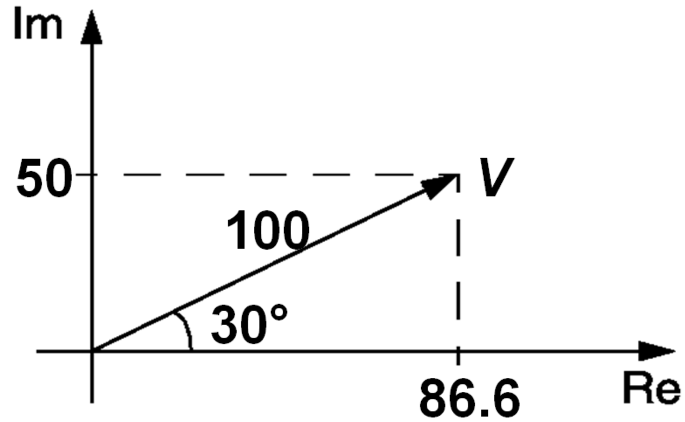
$$Z_C = \frac{V_{phasor}}{I_{phasor}} = \frac{V_m/\sqrt{2} \angle\theta}{C \cdot \omega \cdot V_m/\sqrt{2} \angle\theta + 90°} = \frac{1}{C\omega} \angle -90° = \frac{1}{jC\omega}$$

Summary: Time ⇔ Phasor domains



A sinusoidal waveform can be represented as follows:

$$v(t) = V_m \cos(\omega t + 30) = 141.42\cos(\omega t + 30)$$



Phasor representation:

Amp

phase angle

$$V = 100\angle 30$$

$$V = 86.6 + j50$$

Example 3: Using Phasors to Add Sinusoids

$$v_1(t) = \sqrt{2}.20\cos(\omega t - 45) \qquad v_2(t) = \sqrt{2}.10\cos(\omega t - 30)$$

$v_1(t) + v_2(t) = \sqrt{2} \times 20 \cos(\omega t - 45^\circ) + \sqrt{2} \times 10 \cos(\omega t - 30^\circ)$

$V_1 = 20\angle -45^\circ$ $\qquad$ $V_2 = 10\angle -30^\circ$

$v_1(t) + v_2(t) =$ $\qquad$ $V_1 + V_2 = 20\angle -45^\circ + 10\angle -30^\circ$

$= 29.77\angle -40^\circ$

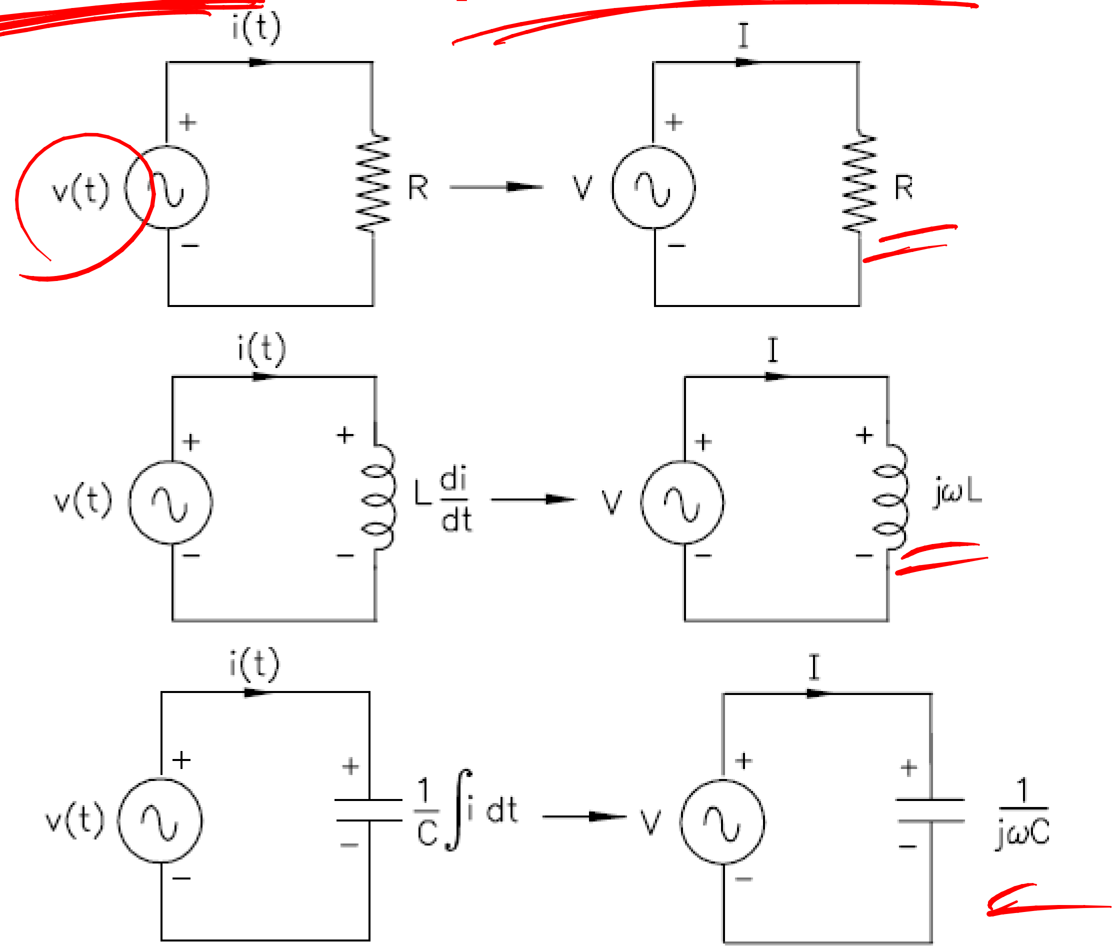
$= 29.77\sqrt{2}\cos(\omega t - 40^\circ)$

Equivalent time domain and phasor representations

A.C. circuit with a resistance

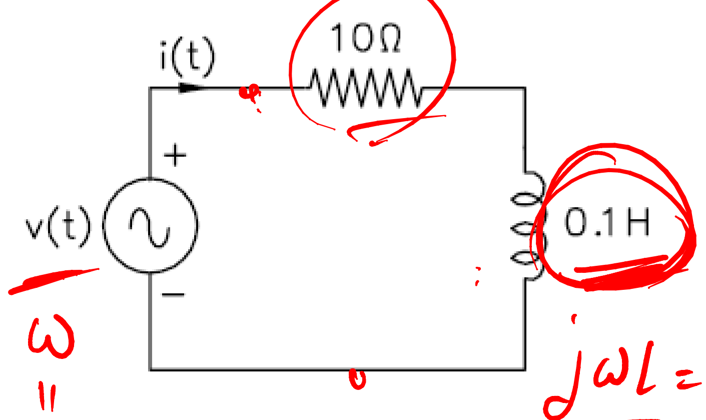
A.C. circuit with an inductance

A.C. circuit with a capacitance

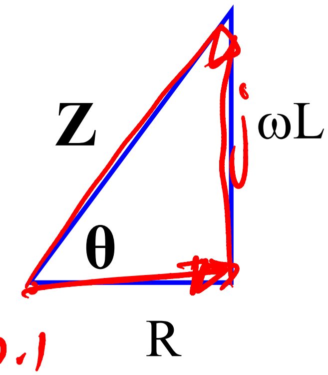


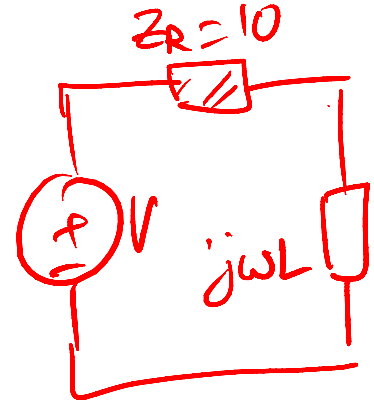
Element	Voltage	Current	Impedance
R	$\lvert V\rvert\angle 0^\circ$	$\frac{\lvert V\rvert\angle 0^\circ}{R}$	R
L	$\omega L\lvert I\rvert\angle 90^\circ$	$\lvert I\rvert\angle 0^\circ$	$j\omega L$
C	$\lvert V\rvert\angle 0^\circ$	$\omega C\lvert V\rvert\angle 90^\circ$	$\frac{1}{j\omega C}$

Complex Impedance



$\omega = 100$

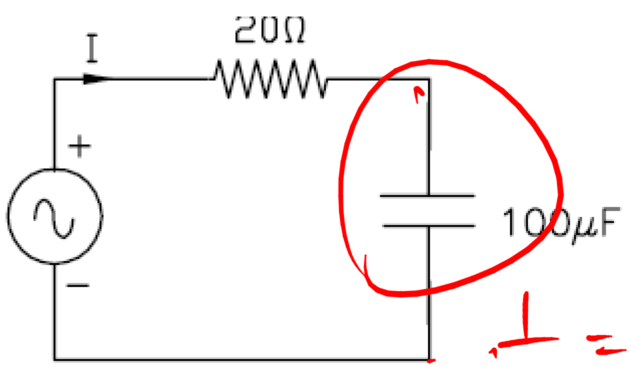


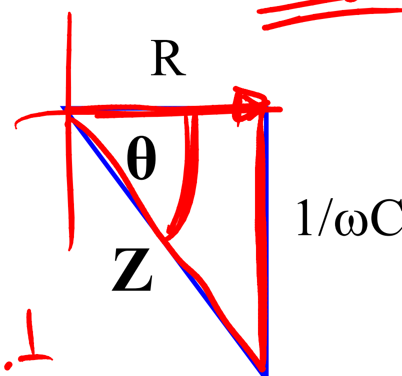


$j\omega L = j\,100 \times 0.1$

$= j\,10$

$Z = 10 + j10 = 10\sqrt{2}\angle 45°$





$\frac{1}{j\omega C} = -j \cdot \frac{1}{\omega C}$

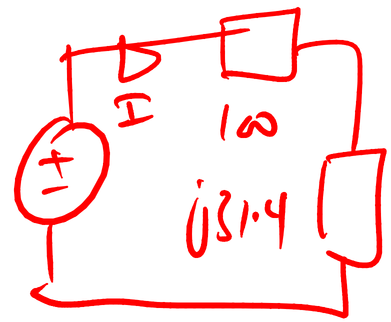
$\boxed{\frac{1}{j} = -j}$

$j^2 = -1$

Example 4: A time-varying voltage *v(t)* is applied across a series combination of 100Ω resistor and 0.1 H inductor. Find the peak amplitude of the resulting current waveform.

$$v(t) = 100\sin(314t)$$

$\frac{100}{\sqrt{2}}\angle 0^\circ$



$R = 100\,\Omega \rightarrow Z_R = 100$

$L = 0.1\,H \rightarrow Z_L = j\omega L = j\,314 \times 0.1 = j\,31.4$

$$I = \frac{V}{Z} = \frac{100/\sqrt{2}\angle 0^\circ}{100 + j31.4} = \underbrace{0.674}_{I_{rms}} \angle -17.43^\circ$$

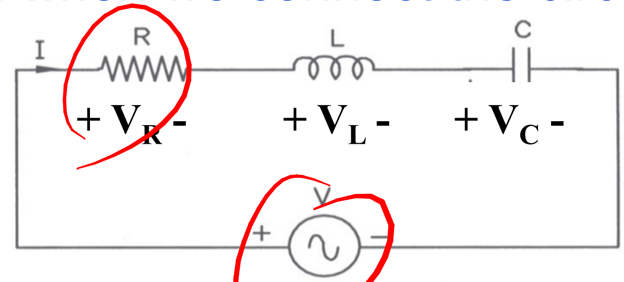
$I_m = \sqrt{2} \times I_{rms} = \sqrt{2} \times 0.674 = 0.953$ A.

$i(t) = I_m \sin(\omega t + \theta) = 0.953 \sin(314t - 17.43^\circ)$ A.

Series Resonance

Series circuit comprising inductor, capacitor and resistance has some interesting behavior.

This can be analyzed when we connect the circuit across variable frequency source.



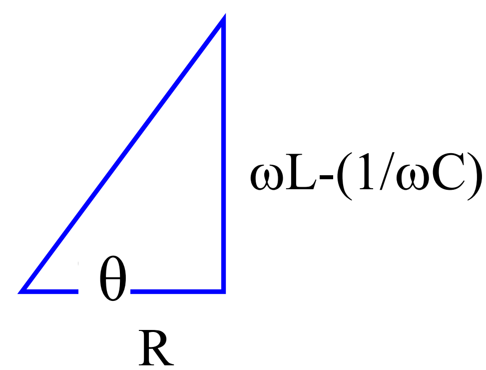
Impedance: $$Z = R + j\omega L - \frac{j}{\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) \rightarrow 0$$

$$\boxed{\omega L = \frac{1}{\omega C}}$$

$$= |Z| \angle \theta$$

Magnitude and phase angle: $$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

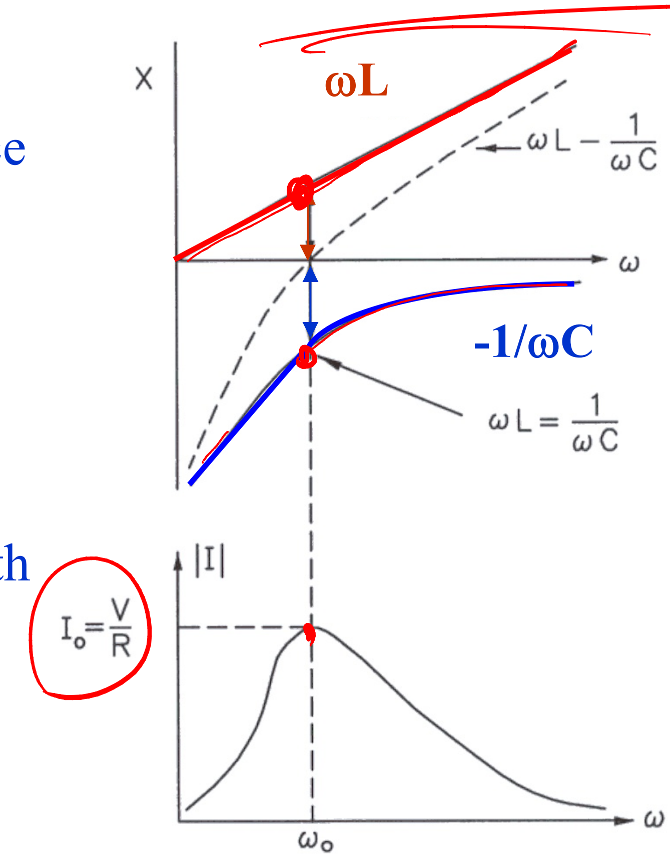


Current flow in the circuit $$I = \frac{V}{R + j\omega L - \frac{j}{\omega C}}$$

Frequency response of Series resonant circuit

Variation of inductive and capacitive reactance with respect to frequency

Variation of current with respect to frequency



$$I = \frac{V}{R + j\omega L - \dfrac{j}{\omega C}}$$

The resonant frequency (ω_o) is defined to be the frequency at which the circuit impedance is purely resistive (i.e. total reactance is zero).

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

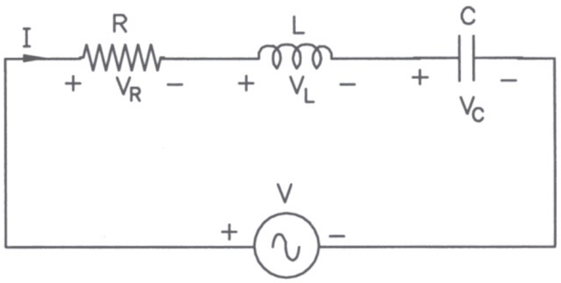
At resonant frequency:

Impedance of the inductance = Impedance of the capacitance

$$\boxed{\omega_o L = \frac{1}{\omega_o C}}$$

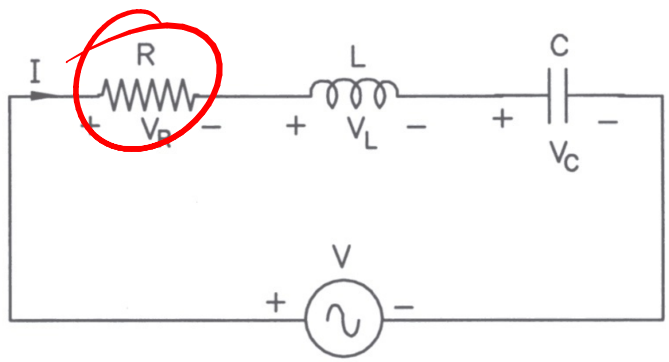
$$\omega_o^2 = \frac{1}{LC}$$

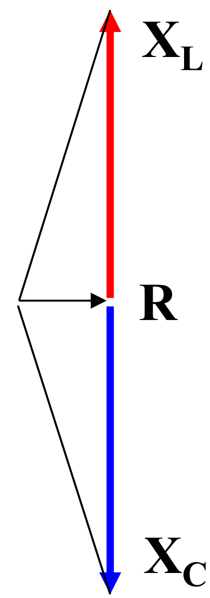
$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$



Note that at resonance:

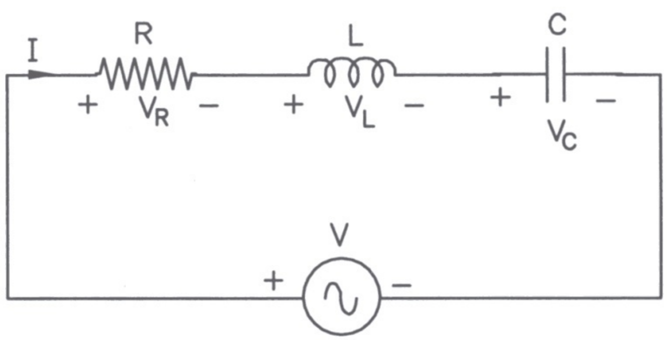
The admittance or impedance is purely resistive. In other words, the L-C series combination acts as a short circuit.





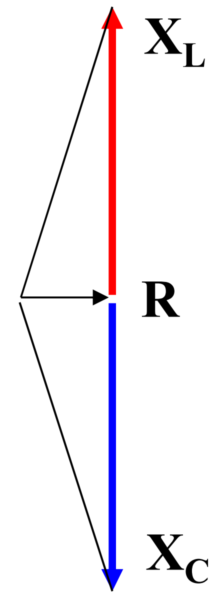
Note that at resonance:

The magnitude of I is maximum



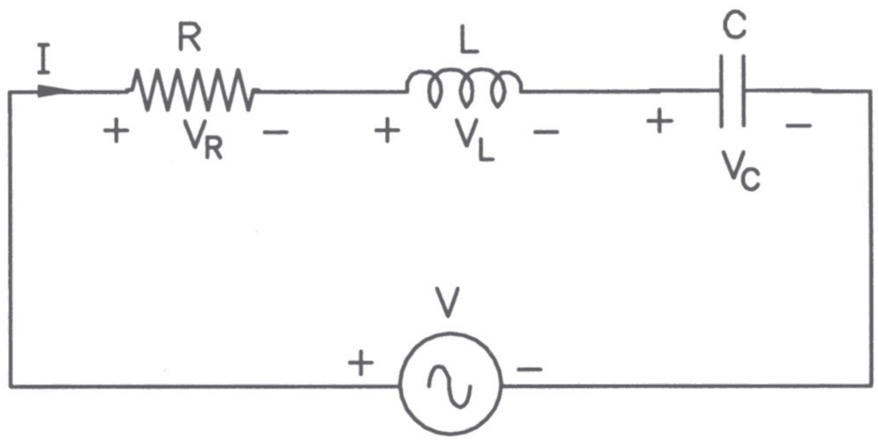
$$I = \frac{V}{R + j\omega L - \frac{j}{\omega C}}$$

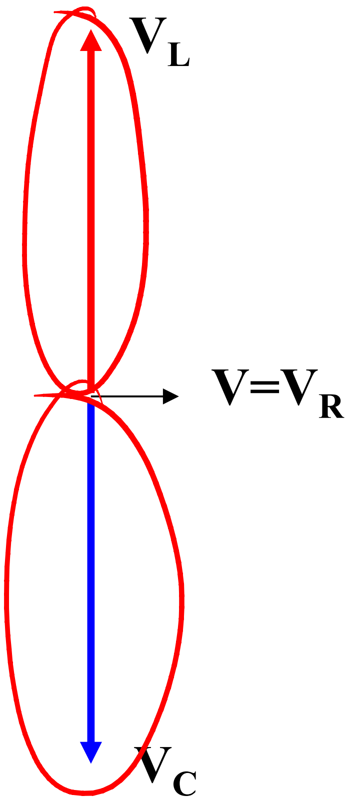
$$I_o = \frac{V}{R}$$



Note that at resonance:

The voltage across the inductor or capacitor can be much higher than the source voltage.





Frequency Response of Series RLC Circuits

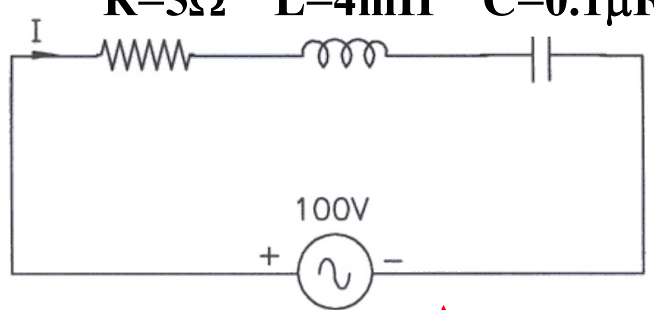
Current $$I = \frac{V}{R + j\omega L - \dfrac{j}{\omega C}}$$

The power dissipated in the circuit is maximum at resonance $$P(\omega_o) = \left|I_o\right|^2 R = \frac{\left|V\right|^2}{R}$$

Example 5

For the circuit shown below, determine the frequency at which resonance occurs. What is the current flow in the circuit at that frequency?

Also, determine the ratio of voltage across the inductor to that across the resistor.



$$\omega_o = \frac{1}{\sqrt{LC}} = 5\times10^4 \, rad/s$$

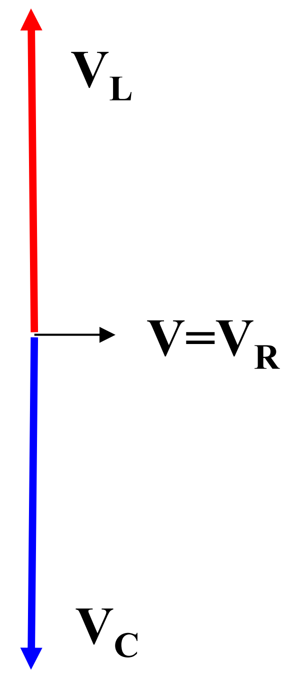
$$f_o = \frac{\omega_o}{2\pi} = 7.958kHz$$

$$I_o = \frac{V}{R} = \frac{100}{5} = 20A$$

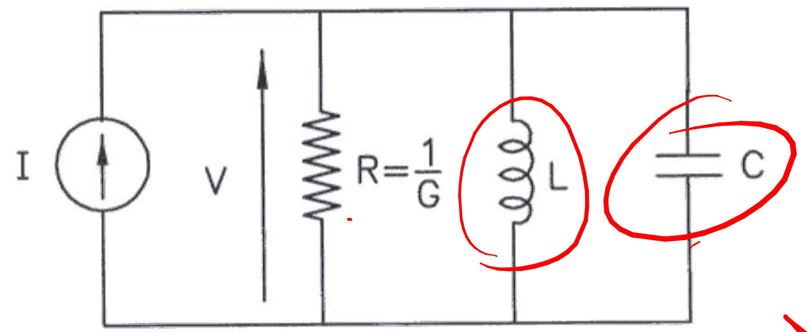
$$|V_L| = \omega_o L I_o = 4000\ V$$

$$|V_C| = \frac{I_o}{\omega_o C} = 4000\ V$$

$$\frac{|V_L|}{|V_R|} = \frac{\omega_o L I_o}{R I_o} = \frac{\omega_o L}{R} = \frac{200}{5} = 40$$



Parallel RLC Circuits



Similar behavior is observed when R, L and C are connected in parallel.

$\frac{1}{\text{Impedance}}$

$$Y = \frac{1}{Z}$$

The admittance of the circuit is $$Y = G + j\omega C - \frac{j}{\omega L}$$

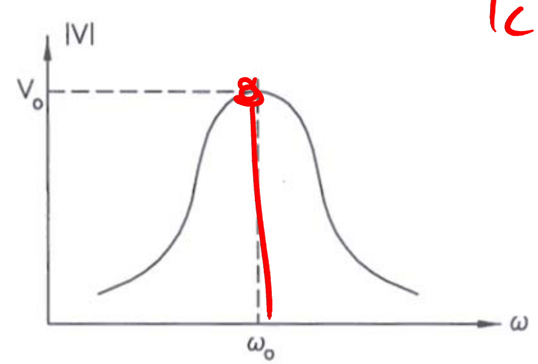
$G = \frac{1}{R}$

$Y_L = \frac{1}{j\omega L}$

$Y_C = j\omega C$

$$|V| = \frac{|I|}{|Y|} = \frac{|I|}{\sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

At resonance: $$\omega_o L = \frac{1}{\omega_o C}$$



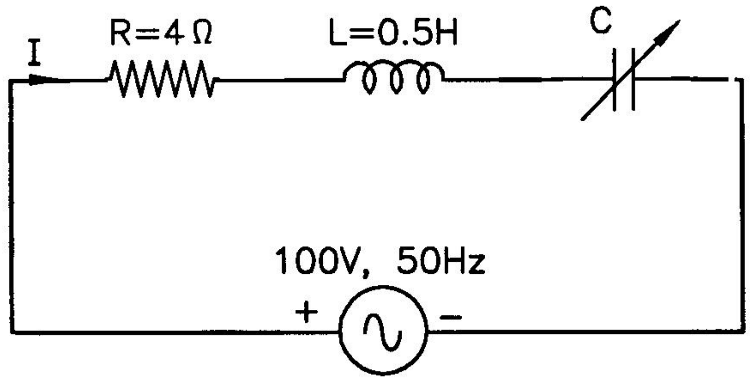
Example 6

A circuit, having a resisance of 4Ω, an inductance of 0.5H and a variable capacitor in series, is connected across a 100V, 50Hz supply. Calculate

(a) capacitance to give resonance,

(b) the voltage across the inductance and capacitance and

(c) the ratio of voltage across the inductor to that across the resistor.



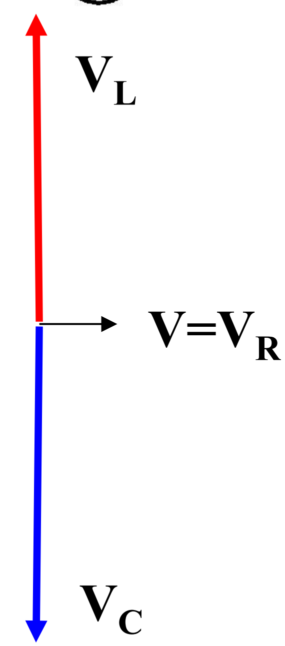
(a)For resonance
$$2\pi f L = \frac{1}{2\pi f C}$$

$$C = \frac{1}{(2\pi \times 50)^2 \times 0.5} = 20.3\mu F$$

(b) At resonance
$$I = \frac{V}{R} = \frac{100}{4} = 25A$$

$$|V_L| = 2\pi f L I = 2 \times 3.14 \times 50 \times 0.5 \times 25 = 3925V$$

$$|V_C| = |V_L| = 3925V$$



(c)Ratio $|V_L|$ to $|V_R|$
$$\frac{|V_L|}{|V_R|} = \frac{2\pi \times 50 \times 0.5}{4} = 39.25$$

AC Analysis.

? Sinusoidals → any operation $\frac{d}{dt}$ $\int dt$ → Sin

AC analysis. Phasor, Impedance → Complex numbers.

Find phasor

$v_1(t) = 100 \sin(314t + 45^\circ) \rightarrow \frac{100}{\sqrt{2}} \angle 45^\circ$

$v_2(t) = 50 \cos(314t + 30^\circ) \rightarrow \frac{50}{\sqrt{2}} \angle 120^\circ$

$= 50 \sin(314t + 30 + 90^\circ)$

$\frac{100}{\sqrt{2}} \angle 45^\circ$

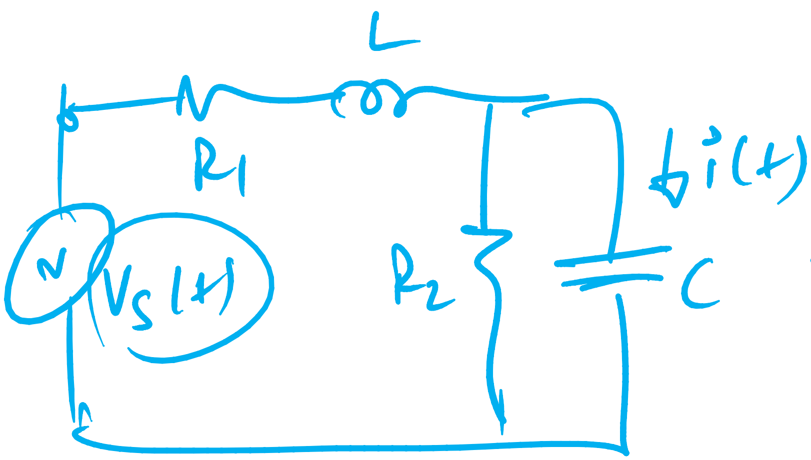
$\frac{50}{\sqrt{2}} \angle 30^\circ$

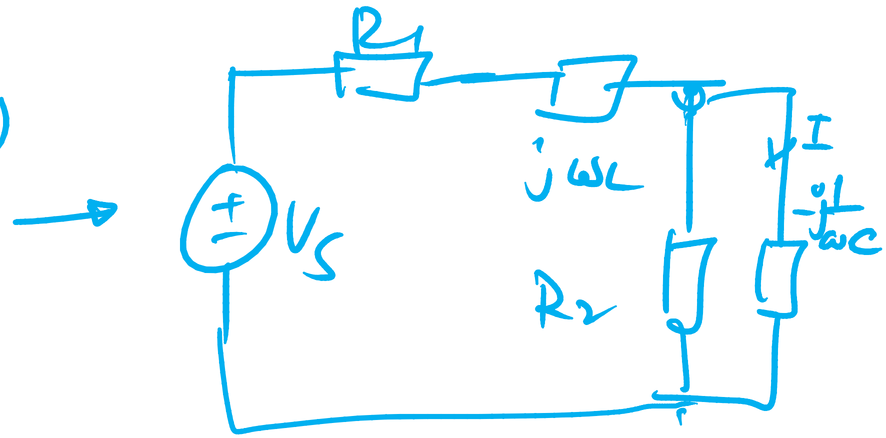
Impedance

R $\quad v \rightarrow i \quad Z_R = R$,

L $\quad v \rightarrow i \quad Z_L = j\omega L$,

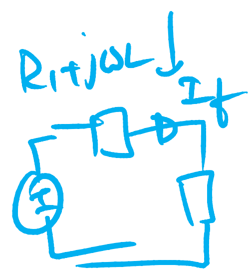
C $\quad v \rightarrow i \quad Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$





$I = I_t \times \frac{R_2}{R_2 - j\frac{1}{\omega C}}$

$\times\, i(t)$



$\frac{R_2 \times (-j/\omega C)}{R_2 + (-j/\omega C)}$

Resonance

Series $L-C$ $\quad j\omega L - j\frac{1}{\omega C} = 0$

Parall $\rightarrow L-C$ $\quad \frac{1}{j\omega C} + j\omega C = 0$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$ resonance frequenc