EE2022 Electrical Energy Systems

Lecture 7:
Three-Phase Circuit Analysis
Summary and Problem Solving

Advantages of Balanced 3-Phase Systems

$|S_{3\phi}| = \sqrt{3}.V.I$ ||| vs. $|S_{1\phi}| = V.I$ ||

- When compared to three single-phase circuits, three-phase circuits have better use of equipment and materials
  - More power can be transmitted per conductor
  - Lesser power losses in the conductors
- This implies reduced capital and operating costs of transmission and distribution.
- We can calculate voltage and current for only one phase and refer to other phases easily.

An Additional Advantage of Balanced 3-Phase Circuit

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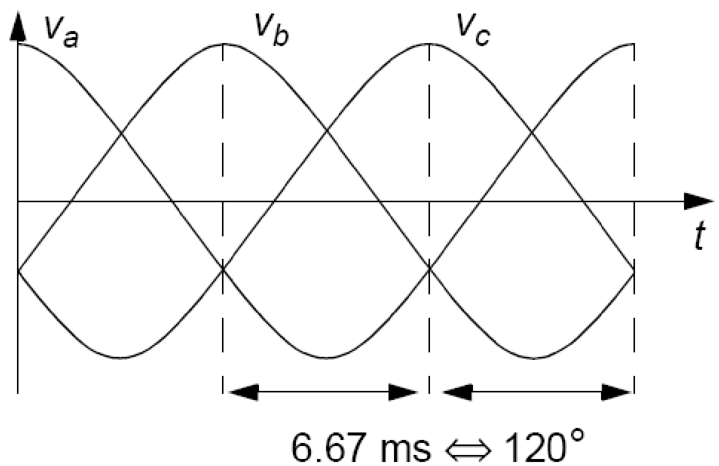
- Constant power transfer to load.
  - This also implies constant mechanical power input for a generator.
  - When mechanical power input is constant, mechanical shaft torque is also constant.
  - This helps to reduce shaft vibration and noise, extending the machine's lifetime.

1-φ. Real power has double frequency
(2ω)

Three-Phase Voltage Sources



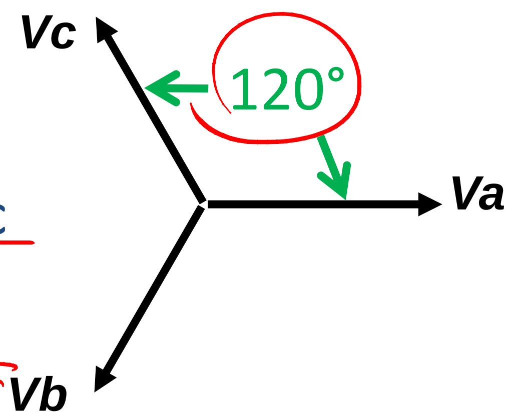
$$v_a = \boxed{\sqrt{2}|V|}\cos(\omega t)$$

$$v_b = \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = \sqrt{2}|V|\cos\left(\omega t - \frac{4\pi}{3}\right)$$

Phase sequence: abc (positive)

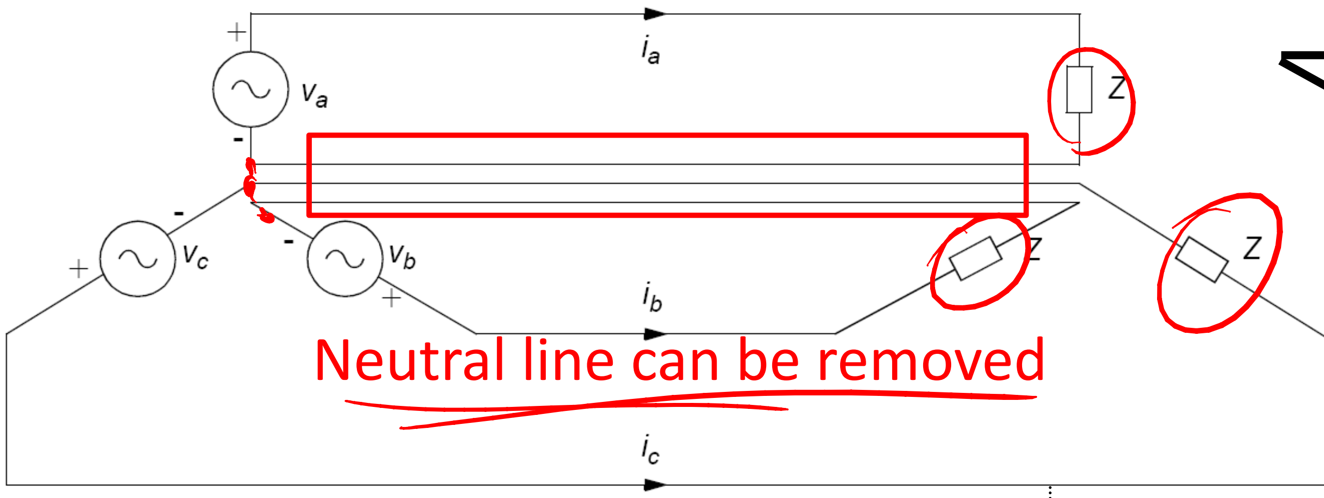
- ve : acb

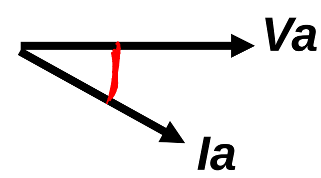


All three voltage sources have the same voltage magnitude,

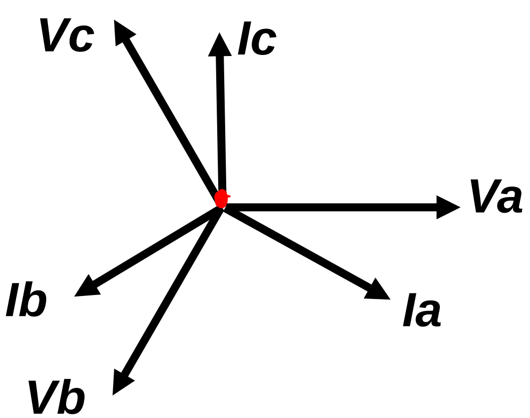
with 120 degrees apart.

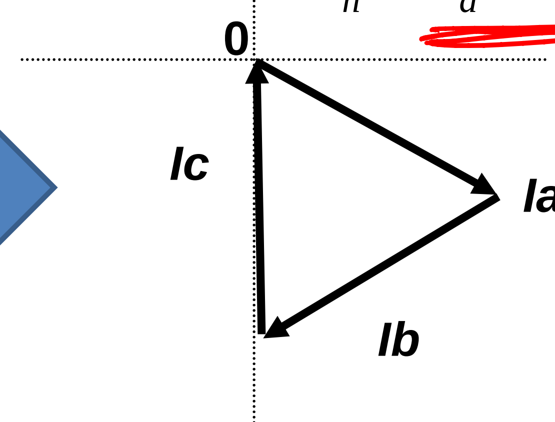
Balanced Three-Phase Circuit





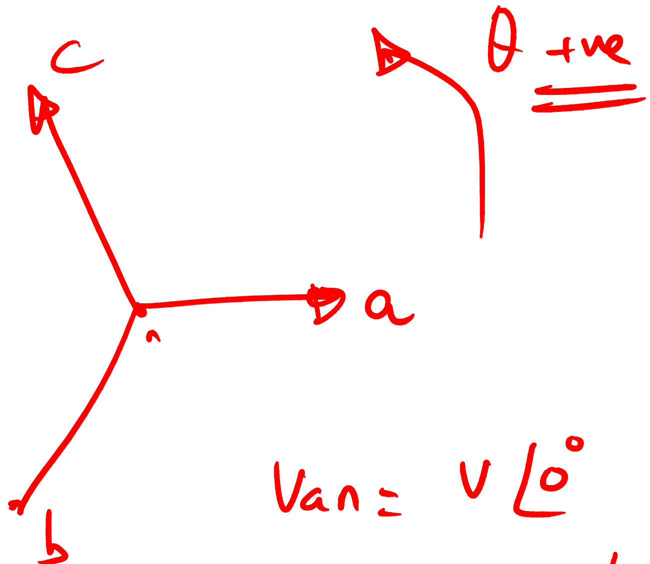
$$I_n = I_a + I_b + I_c = 0$$





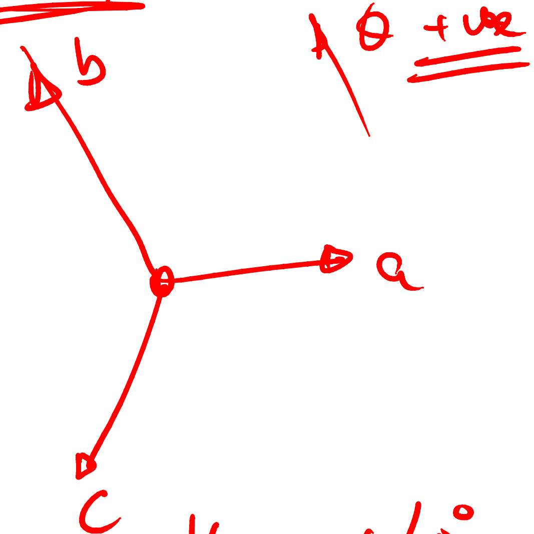
Positive and Negative sequence

abc



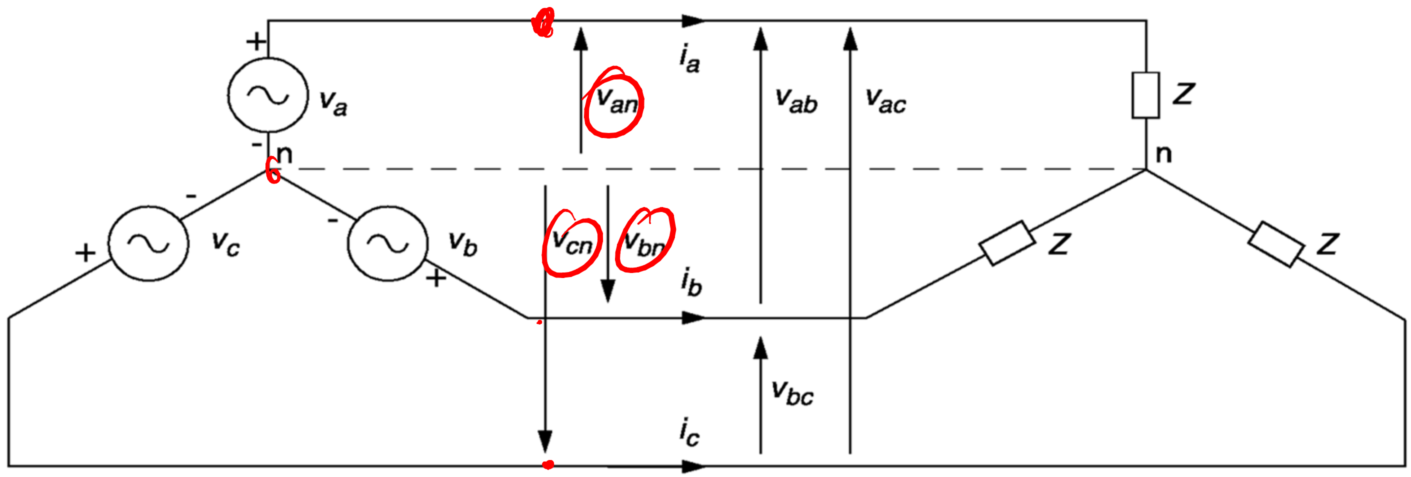
$$V_{an} = V\angle 0^\circ$$
$$V_{bn} = V_{an}\angle -120^\circ$$
$$V_{cn} = V_{bn}\angle -120^\circ$$
$$= V_{an}\angle +120^\circ$$

acb



$$V_{an} = V\angle 0^\circ$$
$$V_{cn} = V_{an}\angle -120^\circ$$
$$V_{bn} = V_{an}\angle +120^\circ$$

Line-To-Neutral (Phase) Voltage

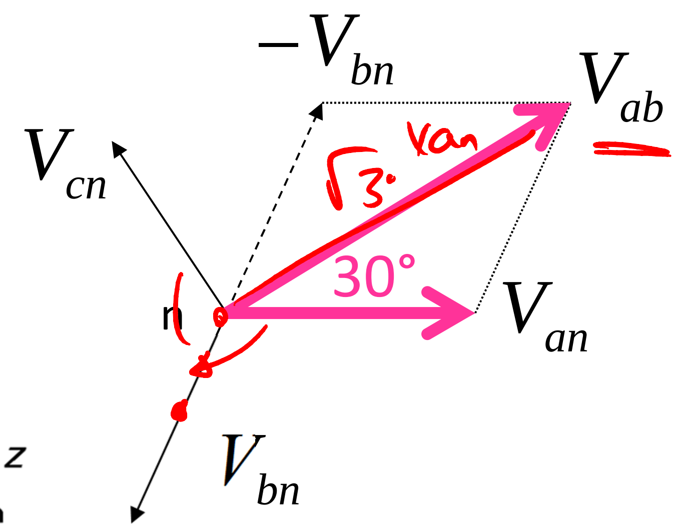


V_{an}, V_{bn}, V_{cn} are called line-to-neutral voltage or phase voltage.

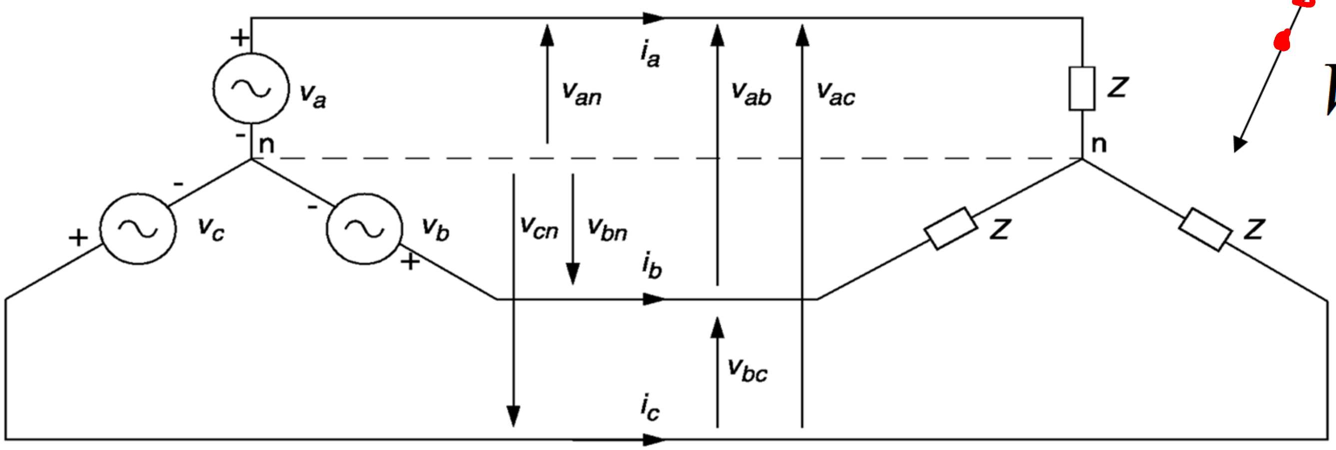
Line-To-Line Voltage

$V_{ab} > V_{an} - V_{bn}$

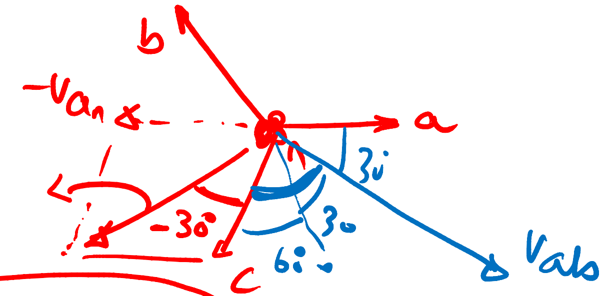
- Voltage is given as line-to-line voltage by convention.
- KVL: $V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} \angle 30°$



$$\boxed{\left|V_{\text{Line-Line}}\right| = \sqrt{3} \left|V_{\text{Line-neutral}}\right|}$$



Example



1. A balanced three-phase Y load has one phase voltage of $V_{cn}=277\angle 45^0$ V. If the phase sequence is negative sequence i.e. acb, find the line voltages V_{ca}, V_{ab}, and V_{bc}.

(Answer: $V_{ca}=480\angle 15^0$ V, $V_{ab}=480\angle 135^0$ V, and $V_{bc}=480\angle -105^0$ V)

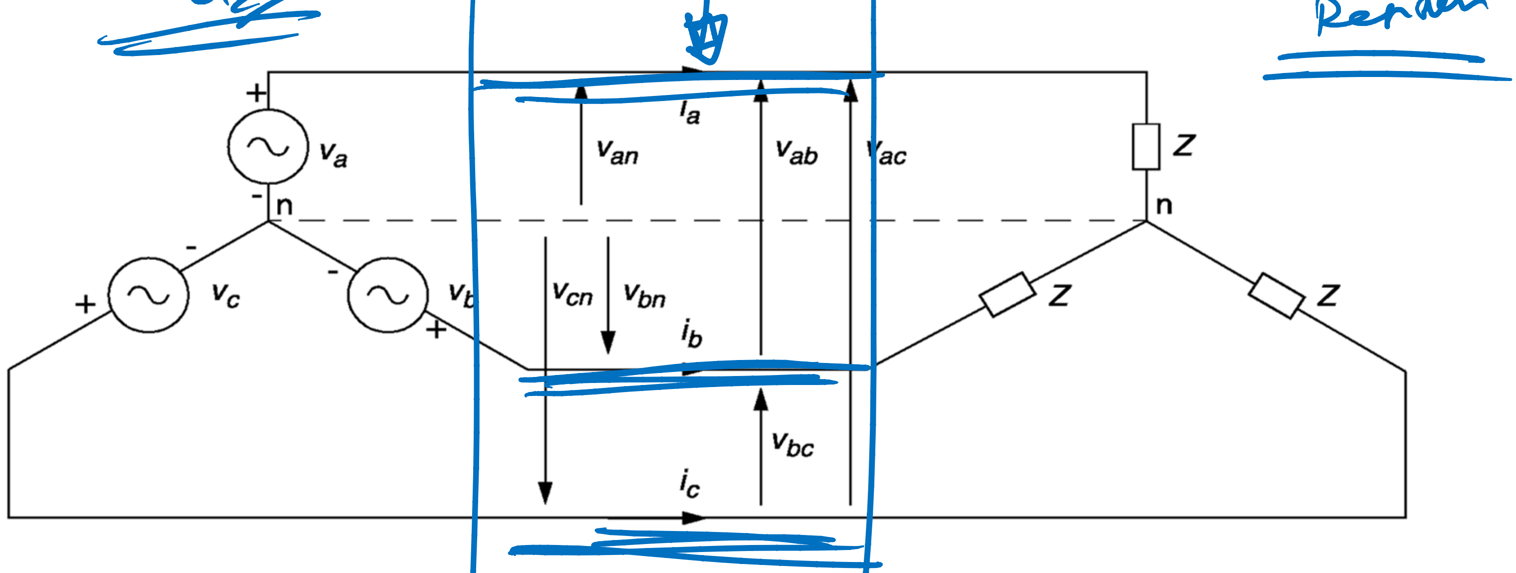
$$V_{ca} = V_{cn} - V_{an}$$
$$= \sqrt{3} . V_{cn} \angle -30^\circ$$
$$= \sqrt{3} \times 277 \angle 45^\circ \angle -30^\circ$$
$$= \sqrt{3} . 277 \angle 15^\circ$$

$$V_{ab} = \sqrt{3} . V_{cn} \angle 90^\circ$$
$$= \sqrt{3} \times 277 \angle 45^\circ \angle 90^\circ$$
$$= \sqrt{3} \times 277 \angle 135^\circ .$$

Line Current

Source Gen

Load Render



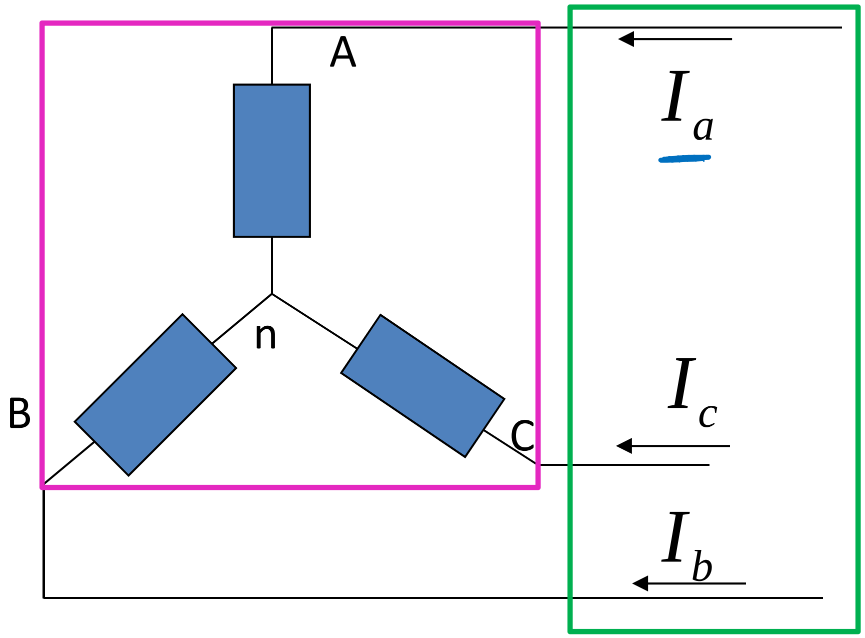
i_a , i_b , i_c are called line currents.

Line Current VS Phase Current

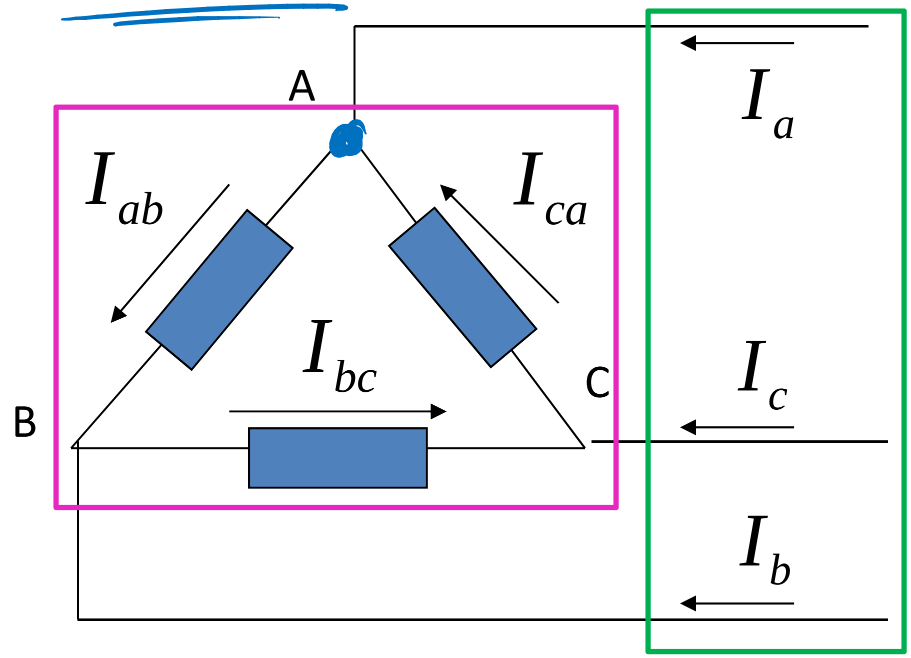
$\sqrt{3}$, $\angle 30^\circ$

$I_a = I_{ab} - I_{ca}$.

Wye Connection



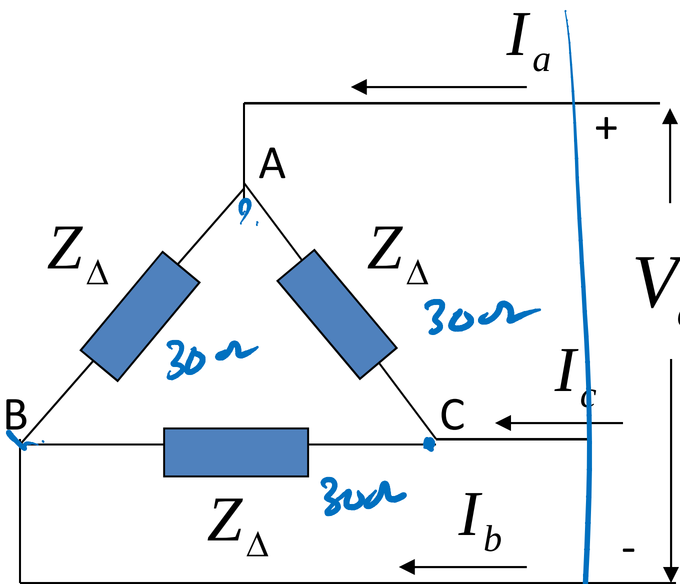
Delta Connection



Currents through the three-phase conductor lines are called 'Line currents'.

Currents carried by the load impedance are called 'Phase currents' or 'Load Current.

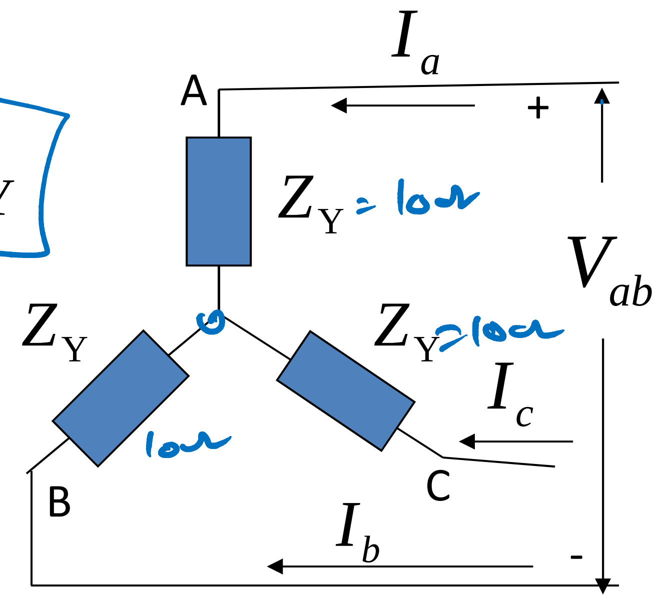
Delta-Wye Load Transformation



$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

$$\boxed{Z_\Delta = 3Z_Y}$$

$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$



$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$Z_\Delta = 12\angle 45^\circ \ \Omega$$

$$Z_Y = \frac{12\angle 45^\circ}{3} = 4\angle 45^\circ \ \Omega$$

Summary

- Three-phase voltage sources
  - Positive and negative sequences
- Balanced three-phase circuit
  - Conditions
  - Advantages
- Balanced three-phase circuit
  - Line-to-neutral (phase) voltage
  - Line-to-line (line) voltage

$$\boxed{\left|V_{\text{Line-Line}}\right| = \sqrt{3}\left|V_{\text{Line-neutral}}\right|}$$

$\sqrt{3} = 1.732$

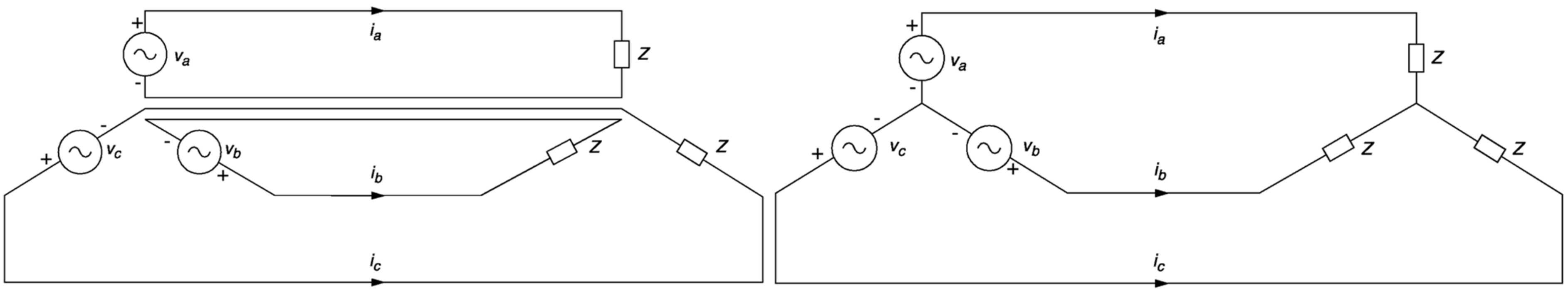
  - Line current
- Wye-Delta connection
- Delta-Wye load transformation

$$\boxed{Z_{\Delta} = 3Z_{Y}}$$

Three Phase Power Calculation

- Three phase power is found from summation of each phase power.

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$



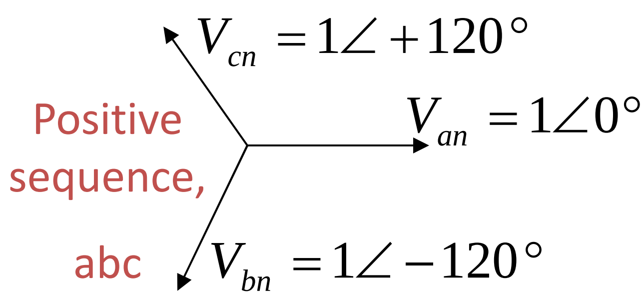
Balanced Three-Phase Power

- From three phase power,

$$S_{3\Phi} = V_{an}I_a^* + V_{bn}I_b^* + V_{cn}I_c^*$$

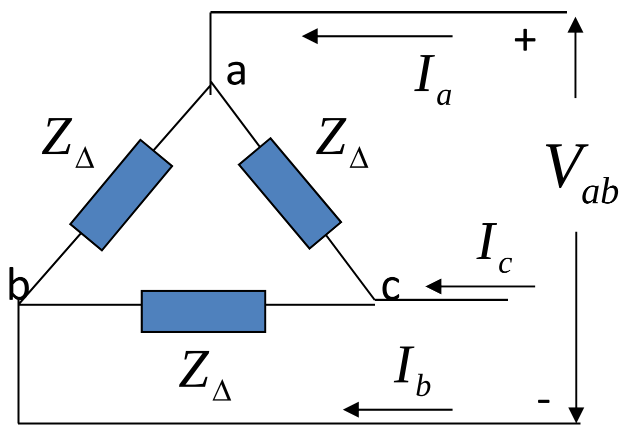
- When the system is balanced, (assume positive sequence) we can write,

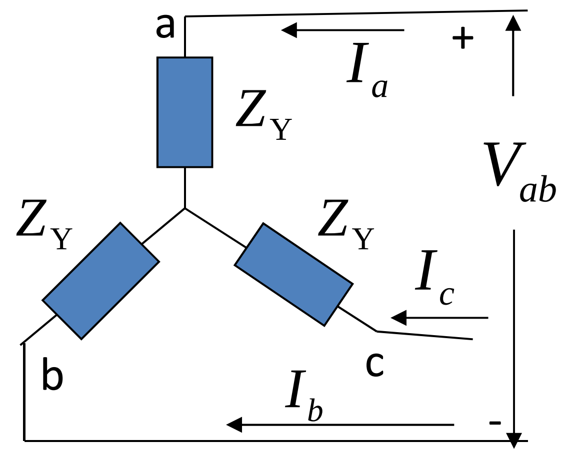
$$S_{3\Phi} = V_{an}I_a^* + V_{an}\angle -120^\circ \left(I_a\angle -120^\circ\right)^* + V_{an}\angle 120^\circ \left(I_a\angle 120^\circ\right)^*$$



$$\boxed{S_{3\Phi} = 3V_{an}I_a^*}$$

Delta/Wye Connected 3-Phase Load





$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

$$|S_{3\Phi}| = 3|V_{ab} I_{ab}^*| = \sqrt{3}|V_{ab}||I_a|$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$|S_{3\Phi}| = 3|V_{an} I_a^*| = \sqrt{3}|V_{ab}||I_a|$$

$(\sqrt{3} \cdot V_{phase})$

$$\boxed{|S_{3\Phi}| = \sqrt{3}|V_{\text{Line-To-Line}}||I_{\text{Line}}|}$$

$\sqrt{3} \cdot V \cdot I$

$S_{3\phi} \neq 3 \cdot V \cdot I$

Assumption

Single-line diagram

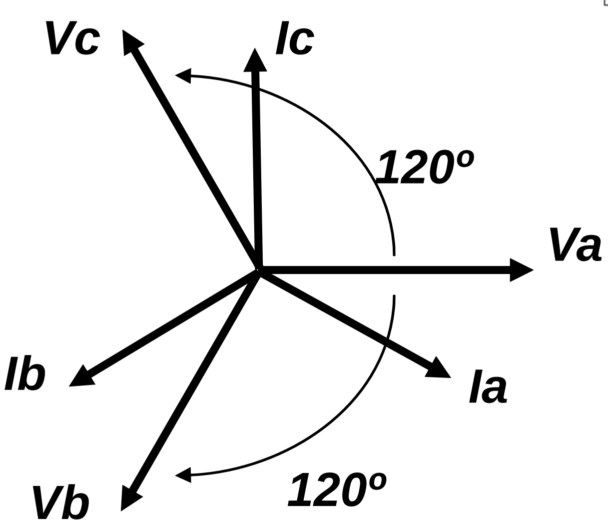
Example example example…

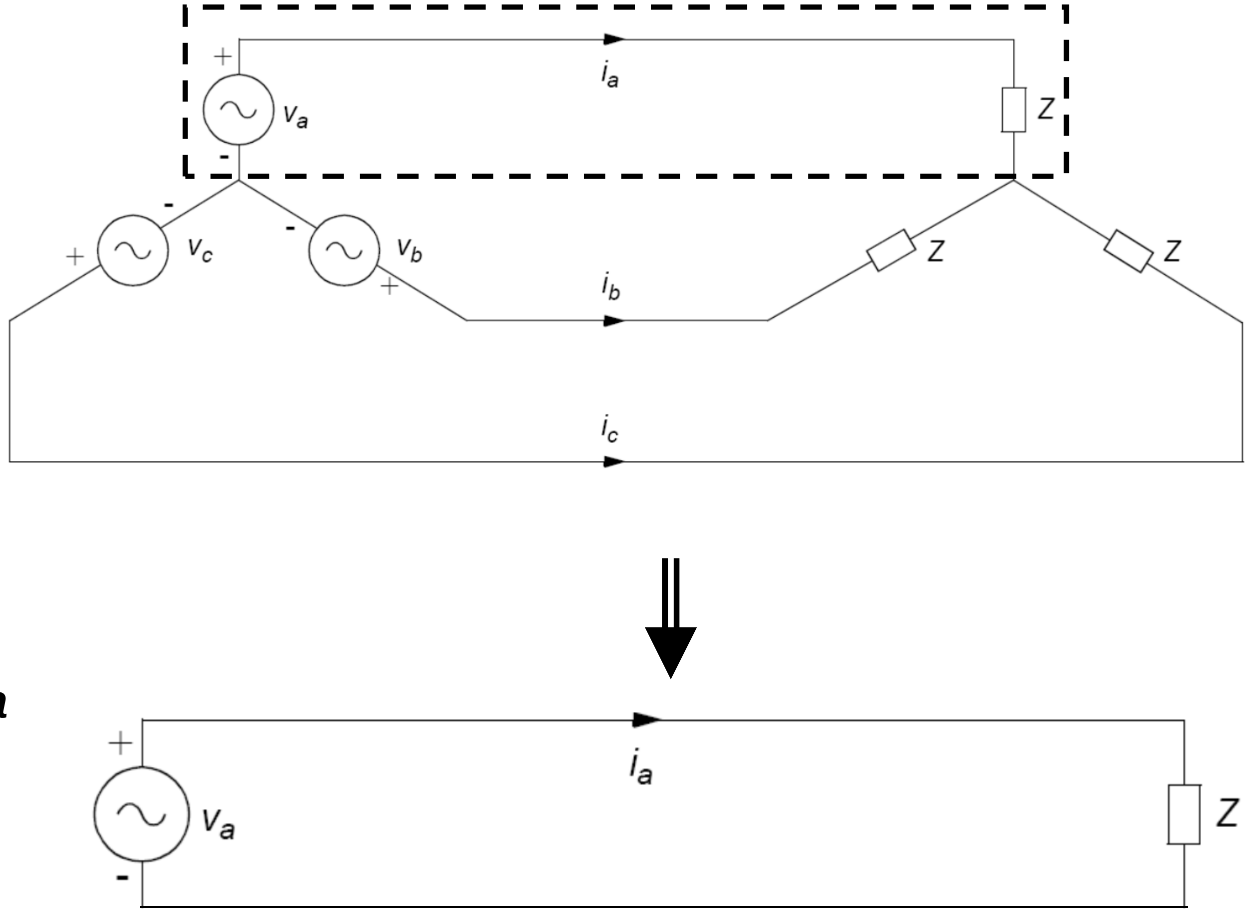
PER PHASE ANALYSIS

Per Phase Analysis: Assumption

It must be balanced three-phase circuit.

$$I_n = I_a + I_b + I_c = 0$$





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Steps of Per Phase Analysis

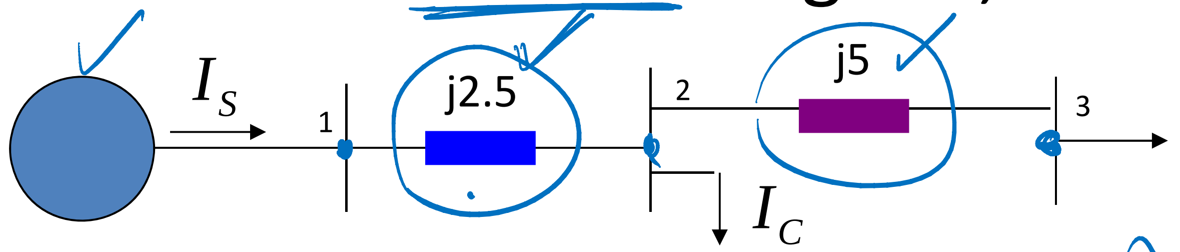
- Make sure that the three-phase system is balanced.
  - The three-phase sources need to have the same magnitude with 120 degree phase difference.
  - The three-phase impedances must be of the same value (both phase and magnitude).
- **Convert** all Delta-connected sources/loads to Wye-connected sources/loads.
- Per phase analysis reduce three-phase circuit to single-phase circuit. We can apply the same concept used in single-phase.

$\cdot\ V_{LL} \rightarrow V_{LN}$

$\cdot\ \bar{Z}_{\Delta} \rightarrow \bar{Z}_{Y}$

Example

- Given a one-line diagram,

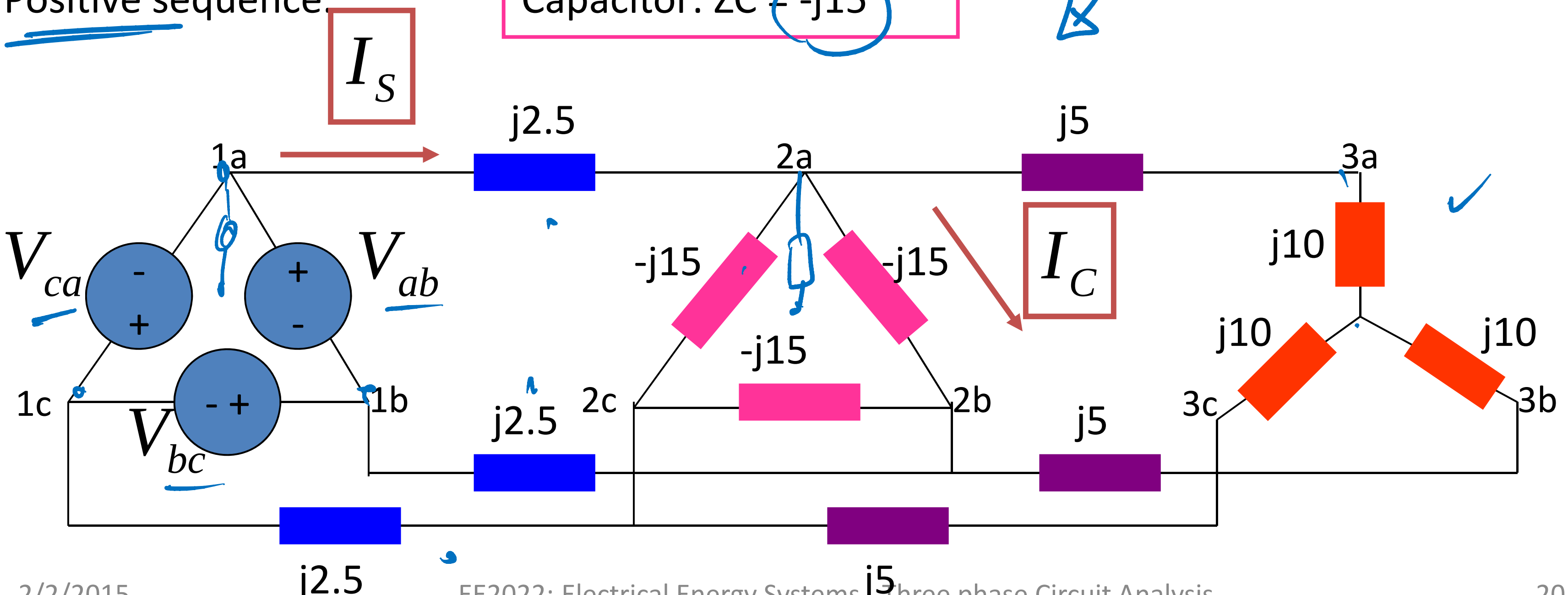


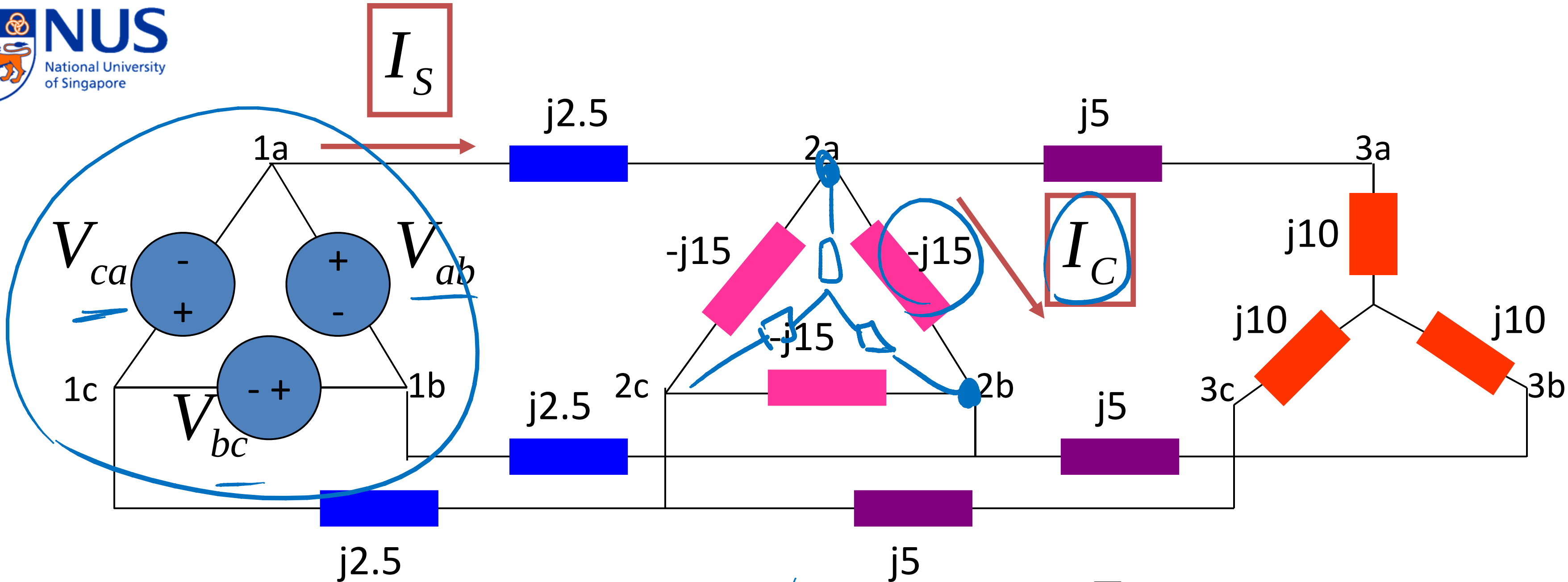
Load 2, Y-Connected
Inductive: ZL = j10

Source,Δ-Connected

Positive sequence.

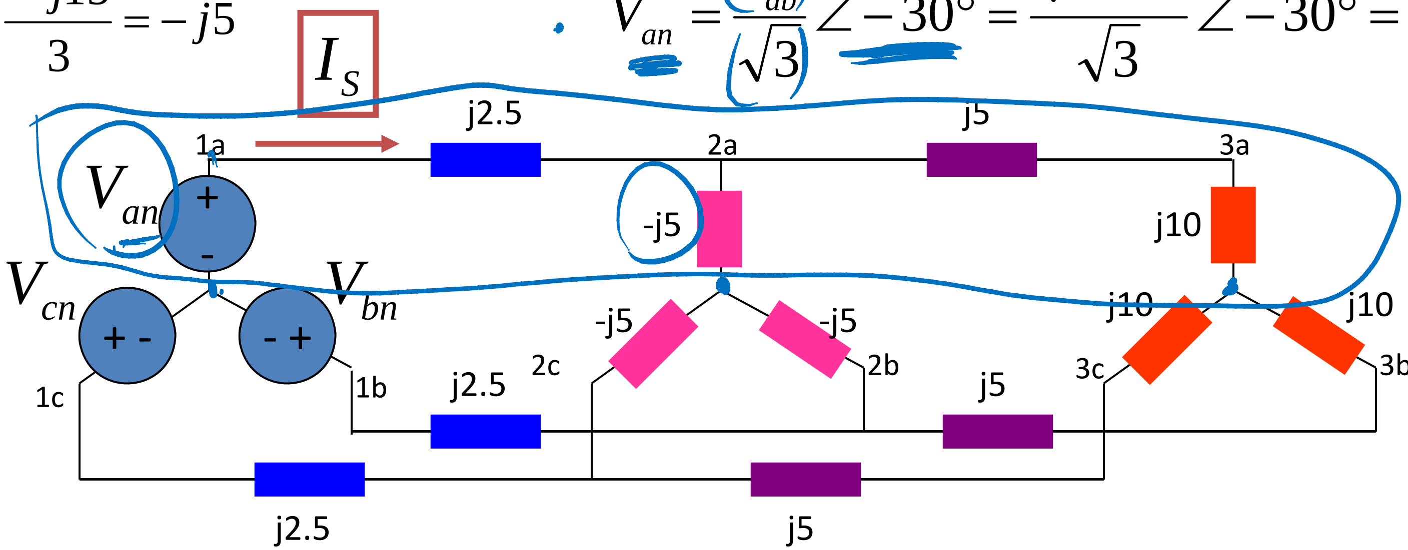
Load 1, Δ-Connected
Capacitor: ZC = -j15



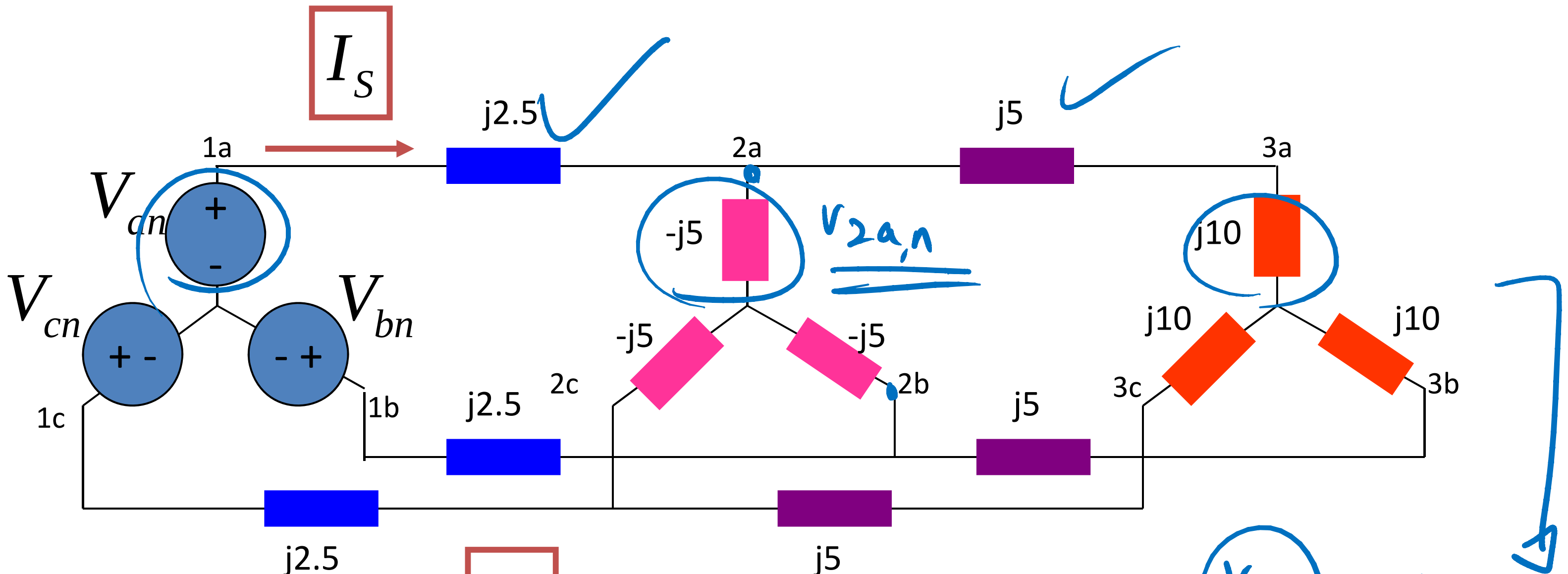


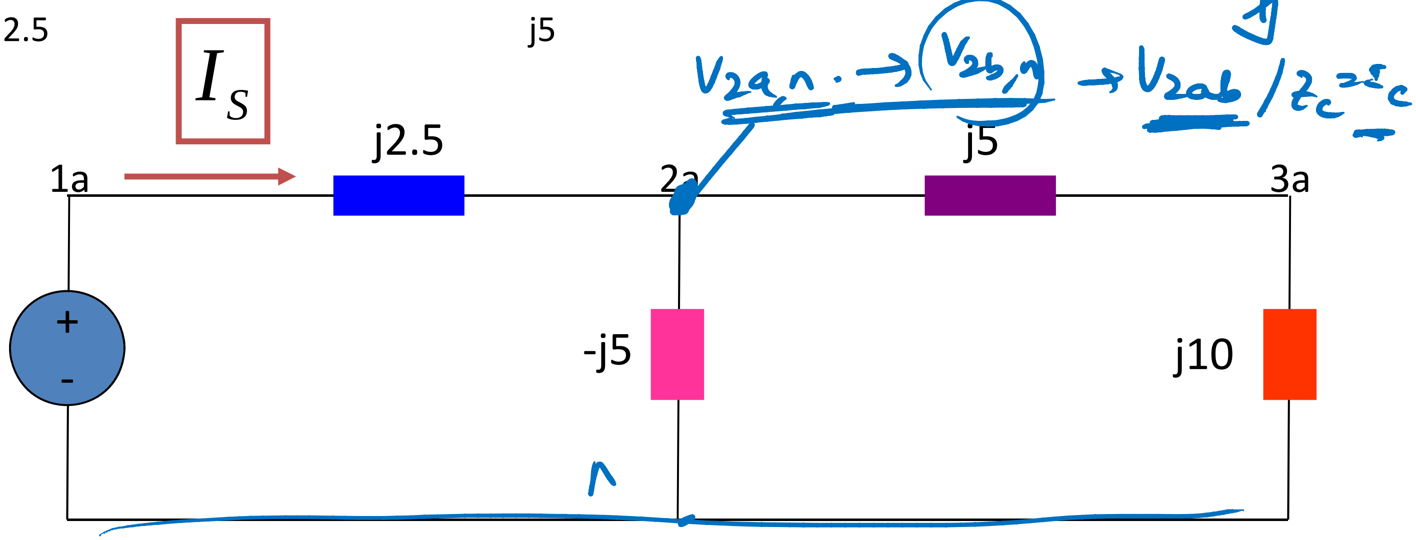
$$Z_Y = \frac{Z_\Delta}{3} = \frac{-j15}{3} = -j5$$

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30^\circ = \frac{\sqrt{3}\angle 0^\circ}{\sqrt{3}} \angle -30^\circ = 1\angle -30^\circ$$



Example 2: 1-Phase diagram

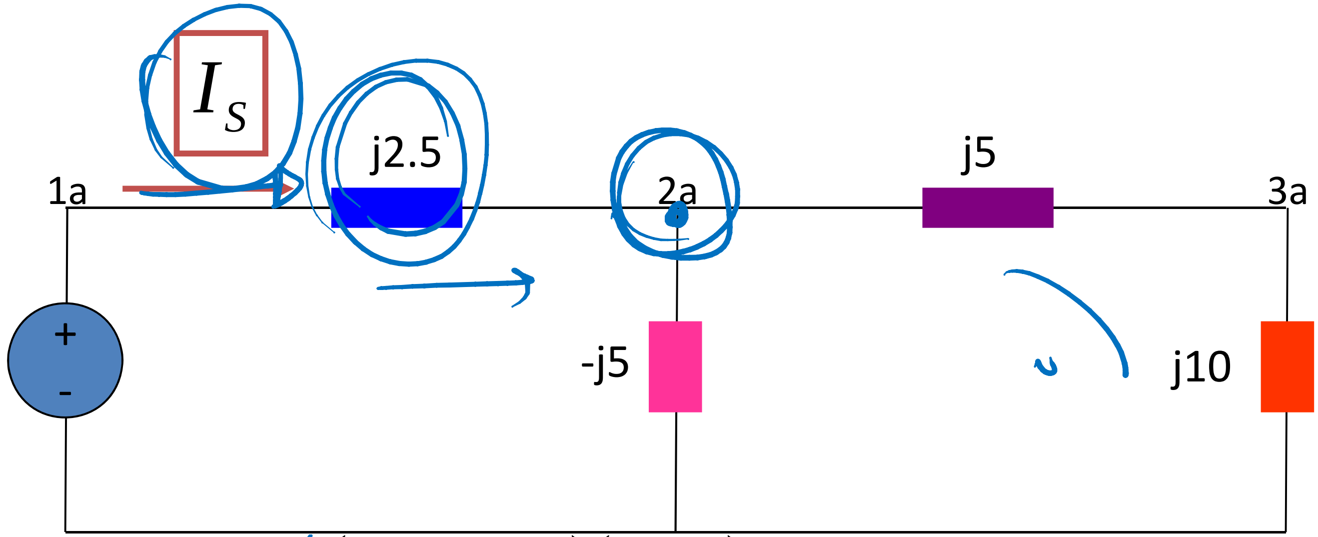


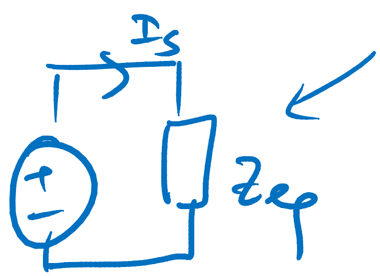


$$V_{an} = 1\angle -30^\circ$$

Example 2: 1-Phase diagram

$$V_{an} = 1\angle -30^\circ$$



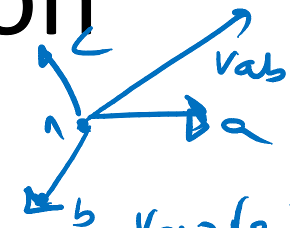


$$Z_{eq} = j2.5 + \frac{(j10+j5)(-j5)}{(j10+j5)+(-j5)} = -j5$$

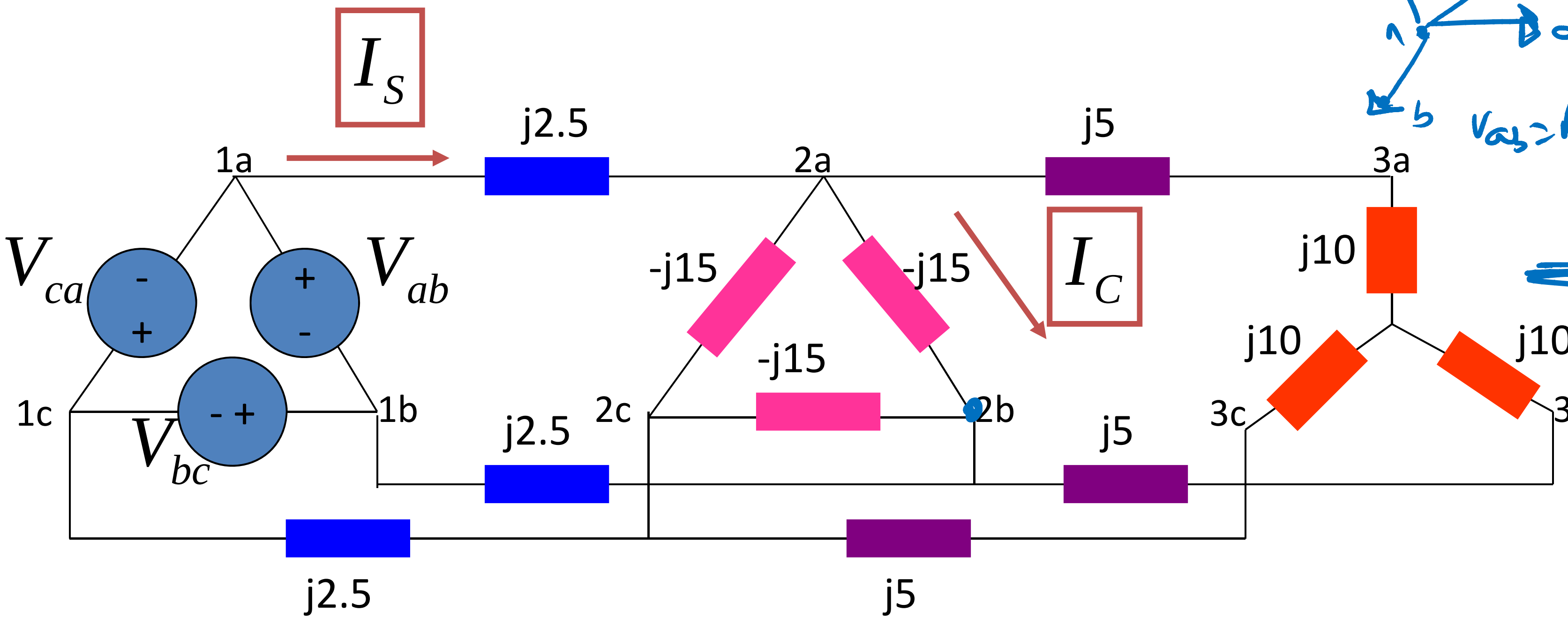
$$I_S = \frac{V_{an}}{Z_{eq}} = \frac{1\angle -30^\circ}{-j5} = \frac{1\angle -30^\circ}{5\angle -90^\circ} = 0.2\angle 60^\circ \text{ A}$$

$V_{2a} = V_{an} - j2.5\times I_S = 1.5\angle -30^\circ$ We will use this to find $\boxed{I_C}$

Example 2: Final Calculation



$V_{ab} = \sqrt{3} \cdot V_a \angle 30°$



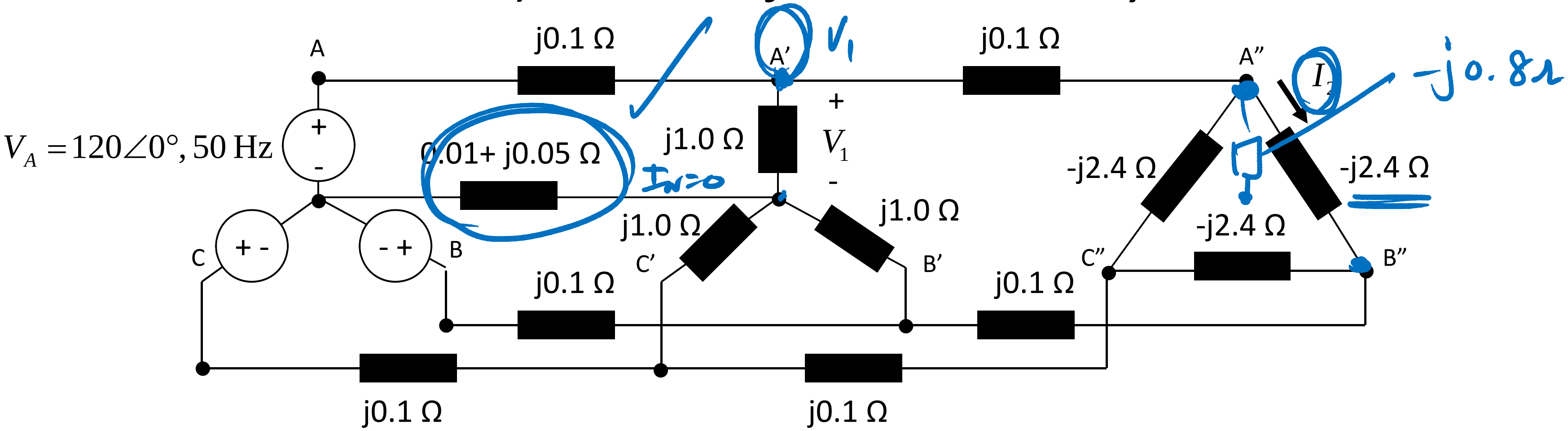
$$V_{2b} = V_{2a}\angle -120°$$

$$I_C = \frac{V_{2a} - V_{2b}}{-j15} = \frac{1.5\angle -30° - 1.5\angle(-30° - 120°)}{15\angle -90°} = \frac{\sqrt{3}}{10}\angle 90°\ \text{A}$$

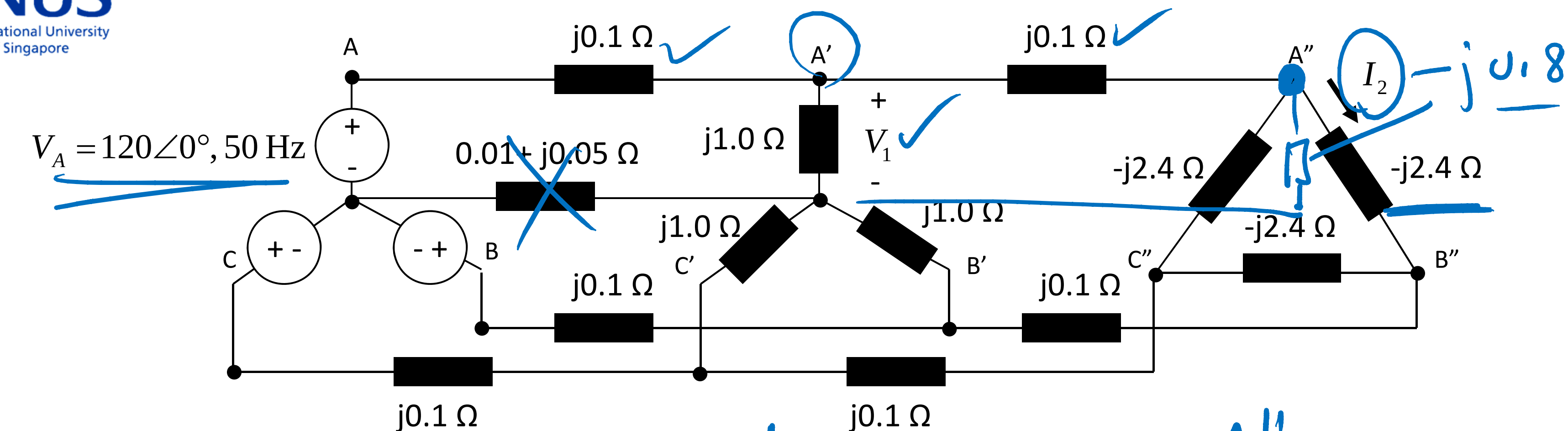
Ans: $I_S = 0.2\angle 60°, I_C = 0.1732\angle 90°$

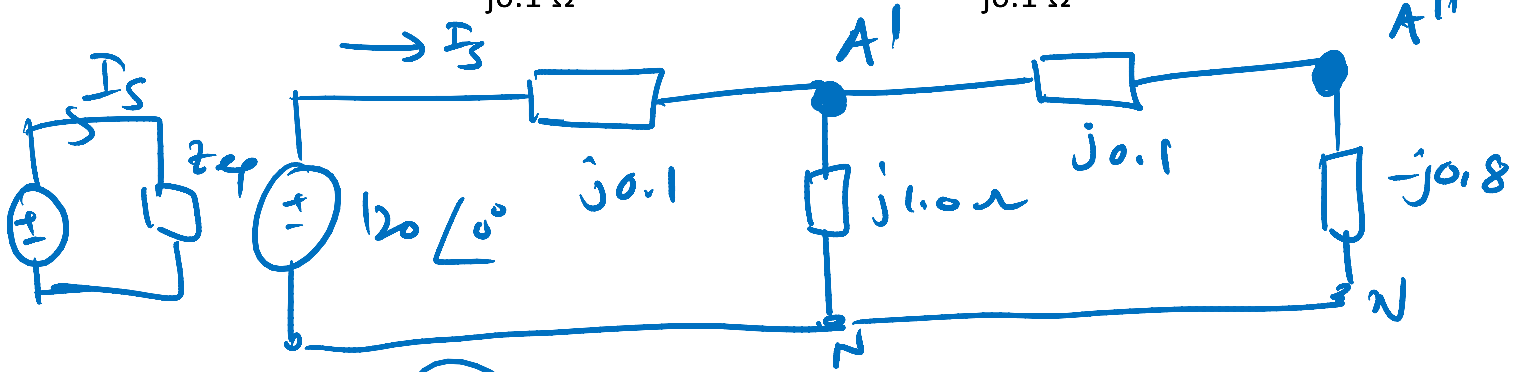
Practice Problem 1

- (Final EE2022 AY2011/12 semester 1) Find the voltage V1 and current I2.
  - *Hint: remove the impedances that are in neutral line (why can we do this?) and transform delta to wye connection.*



Ans: $V_1 = 125.37\angle 0°, I_2 = 103.41\angle 120°$





① Find I_s,

② $V_{A'N} = 120\angle 0^\circ - I_s \times j0.1$

③ $V_{A''N} = V_{A'N} \times \dfrac{-j0.8}{j0.1 - j0.8}$

④ $V_{A''B''} = \sqrt{3}\cdot V_{A''N}\angle 30^\circ$ —

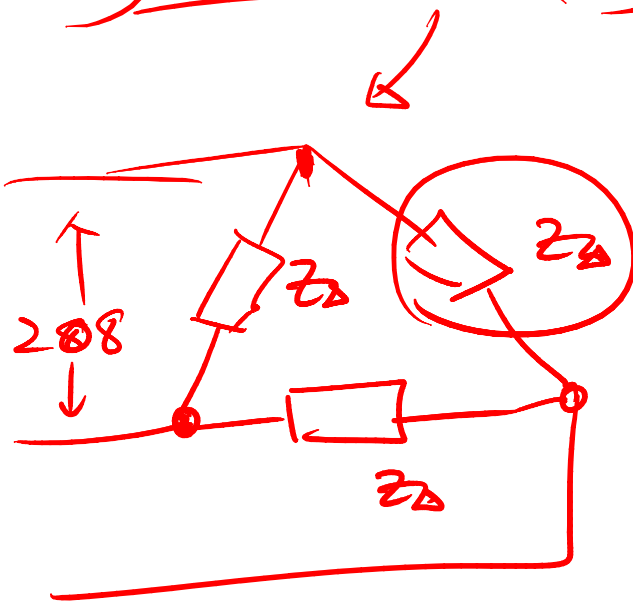
⑤ $I_2 = \dfrac{V_{A''B''}}{-j2.4}$

8. In a 208-V three-phase circuit a balanced Δ load absorbs 2 kW at a 0.8 leading power factor. Find Z Δ.

(Answer: 51.9∠-36.87° Ω)

$V = 208$, Z_Δ? p.f. $= 0.8$, $Z_\Delta = \frac{|V|^2 \times p.f.}{P} = \frac{208^2 \times 0.8}{2000/3} = 51.91$

$\theta = -\cos^{-1}(0.8)$



$V_{LL} = 208$ V.

$P_{3\phi} = 2$ kW.

p.f. $= 0.8$ leading.

Z_Δ?

$Z_\Delta = 51.9\angle -36.87^\circ$

Power consumed by each element is $P_{1\phi} = \frac{2000}{3}$.

Voltage across each element $= 208V$

$P = |S| \times p.f. = V.(I)(p.f.) = V.\frac{V}{Z}.p.f. = \frac{|V|^2}{Z_\Delta}.p.f.$