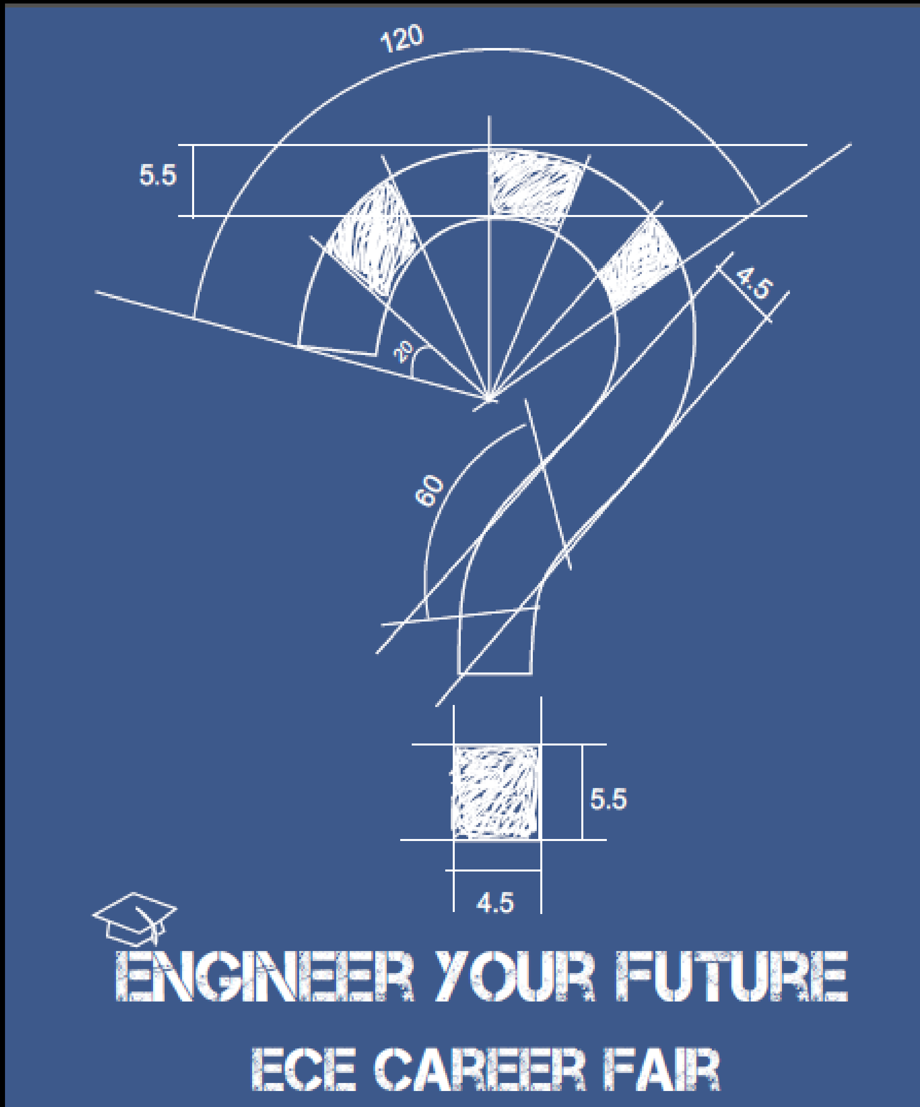

- Network with industry personnel from various companies
- Attend talks by the companies
- Learn more about the available jobs and internships opportunities

Date: 27 March 2015
Time: 12 – 5 PM
Venue: EA Foyer
Attire: Formal

EE2022 Electrical Energy Systems

Per Unit Analysis- Examples and Revision

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Department of Electrical and Computer Engineering

3Φ-Steps of Per Unit Analysis

1. Choose $S_B^{3\Phi}$ for the system.
2. Select $V_B^{\text{line-to-neutral}}$ or $V_B^{\text{line-to-line}}$ for different zones.
3. Calculate Z_{B} for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Note that the **per unit circuit is** the circuit in **per-phase analysis** with normalization of the voltage magnitude at different locations. This means that **the phase of voltage** in per unit analysis **refers to** the **line-to-neutral voltage**.

Example 1

- The one-line diagram of a three-phase power system is shown below. Select a **common base of 100 MVA** and 22kV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in per-unit. The manufacturer's data for each device is given as follow. 100MVA

1 – G1: 90 MVA 22kV X = 18% = 0.18

1-2 – T1: 50 MVA 22/220kV X = 10% = 0.1

2-4 – T2: 40 MVA 220/11kV X = 6% = 0.06

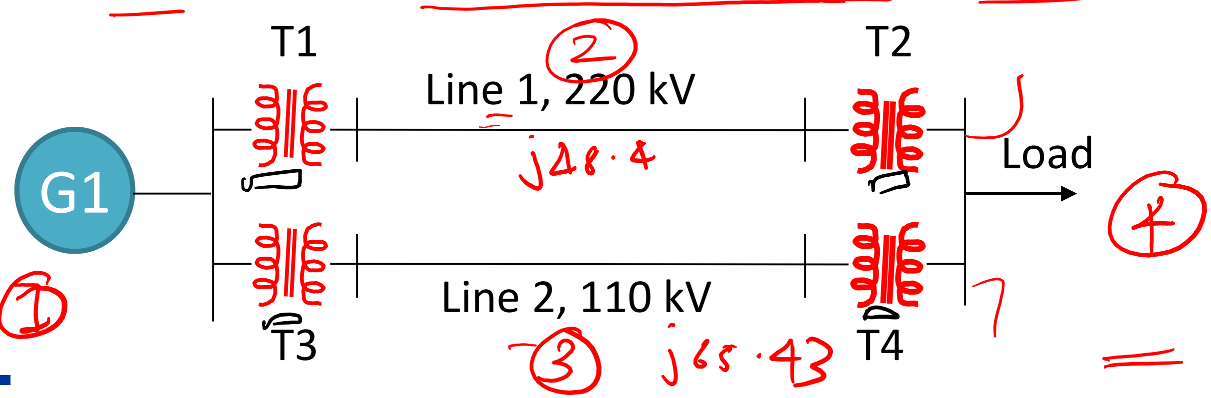
1-3 – T3: 40 MVA 22/110kV X = 6.4% = 0.064

3-4 – T4: 40 MVA 110/11kV X = 8% = 0.08

– Three-phase Y-connected load absorbs 57 MVA 0.6 pf lagging at 10.45kV.

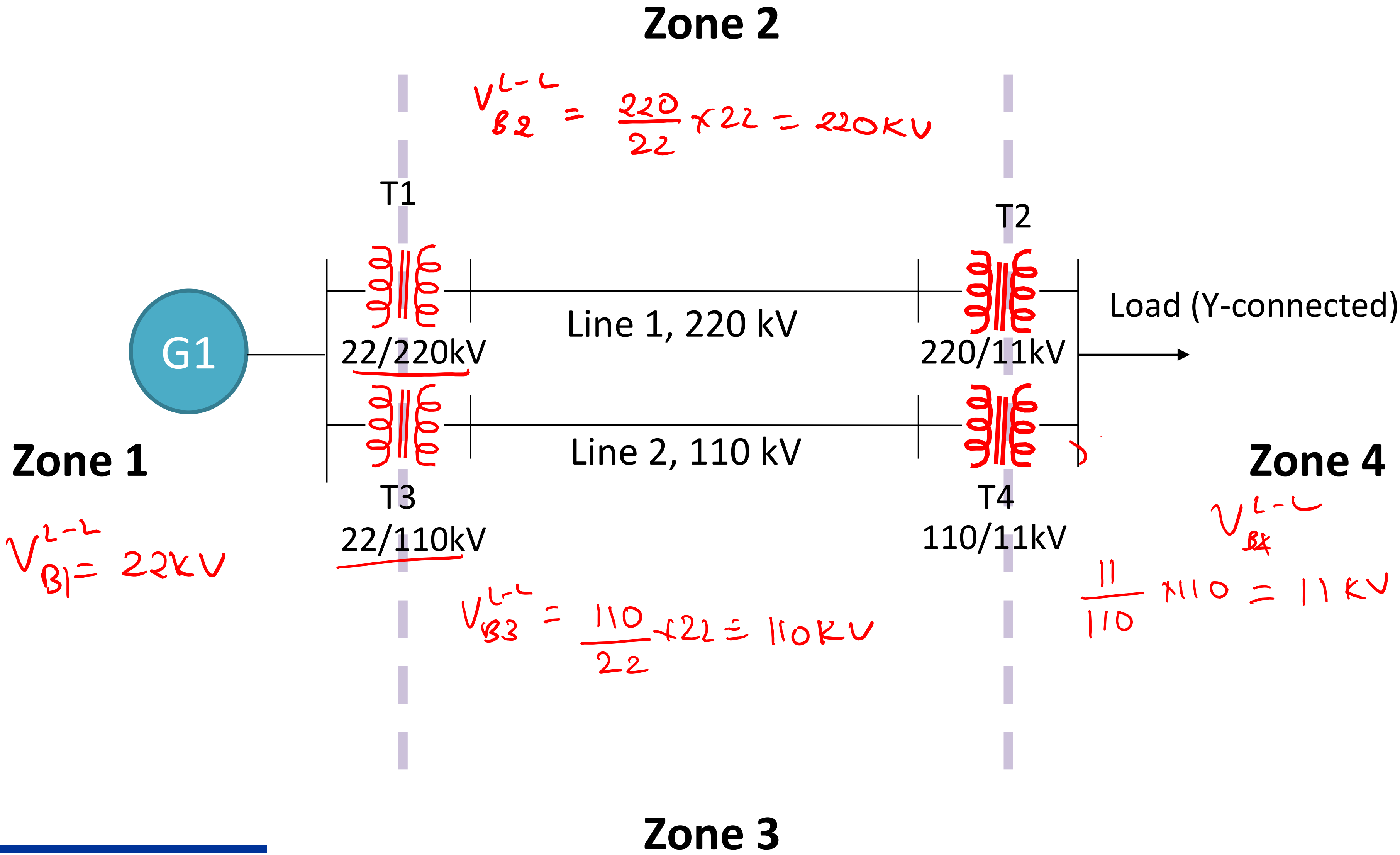
– Line 1 and line 2 have per phase reactances of 48.4 and 65.43 Ω respectively.

$V_B^{L-L} = 22kV$



② Base Value for V

① $S_B^{3\phi} = 100MVA$



$V_{B2}^{L-L} = \frac{220}{22} \times 22 = 220kV$

$V_{B1}^{L-L} = 22kV$

$V_{B3}^{L-L} = \frac{110}{22} \times 22 = 110kV$

V_{B4}^{L-L}

$\frac{11}{110} \times 110 = 11kV$

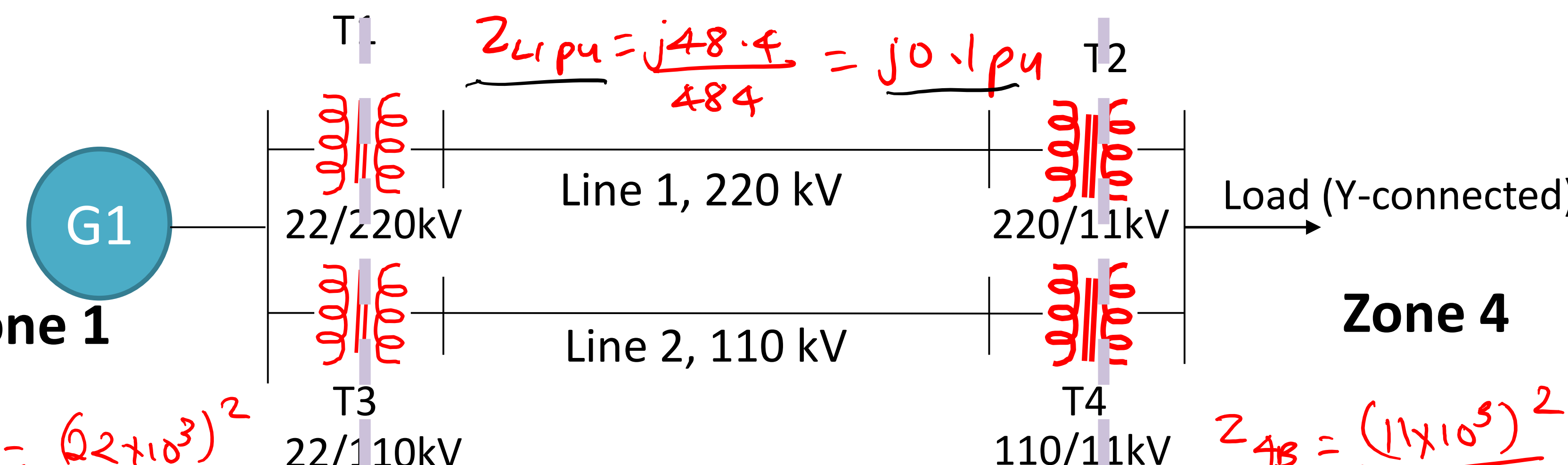
③ Base Impedances

$$Z_B = \frac{(V_B^{L-L})^2}{S_B^{3\phi}}$$

Zone 2

$$Z_{2B} = \frac{(220 \times 10^3)^2}{100 \times 10^6} = 484\,\Omega$$

$$Z_{L1pu} = \frac{j48.4}{484} = j0.1\,pu$$



$$Z_{1B} = \frac{(22 \times 10^3)^2}{100 \times 10^6} = 4.84\,\Omega$$

$$Z_{3B} = \frac{(110 \times 10^3)^2}{100 \times 10^6} = 121\,\Omega$$

$$Z_{4B} = \frac{(11 \times 10^3)^2}{100 \times 10^6} = 1.21\,\Omega$$

$$Z_{L2pu} = \frac{j65.43}{121} = j0.54\,pu$$

Zone 3

Load Impedance

Zone ④

$pf = 0.6$ lag

$\angle = 53.13$

$V_{LOAD}^{L-L} = 10.45\,kV$ $S_{LOAD}^{3\phi} = 57\,MVA$ I_{LOAD} ?

Z_Y ?

$$S_{LOAD}^{3\phi} = \frac{3V_{l-n}^2}{Z_Y^{\phi}} = \frac{V_{L-L}^2}{Z_Y^{\phi}}$$

$$\Rightarrow Z_y^{\phi} = \frac{V_{LL}^2}{S_{LOAD}^{3\phi}} = \frac{(10.45\times10^3)^2}{57\times10^6\angle 53.13} = 1.1495 - j1.5327\,\Omega$$

$$Z_y = 1.1495 + j1.5327\,\Omega$$

$$Z_{4B} = 1.21\,\Omega \Rightarrow Z_{y\,pu} = \frac{1.1495 + j1.5327}{1.21}$$

$$\boxed{Z_{y\,pu} = 0.95 + j1.2667\ p.u.}$$

④ P.U. Impedance with Change of Base

$$Z_{pu}^{new} = Z_{pu}^{old} \times \left(\frac{V_{B\,OLD}^{L-L}}{V_{B\,NEW}^{L-L}}\right)^2 \times \left(\frac{S_{NEW}^{3\phi}}{S_{OLD}^{3\phi}}\right) = Z_{pu}^{old} \times \frac{S_{NEW}^{3\phi}}{S_{OLD}^{3\phi}}$$

G1 90 MVA 22 kV $X = 18\%$ $\Rightarrow$ $X_G = j0.18 \times \frac{100}{90} = j0.2\ pu$

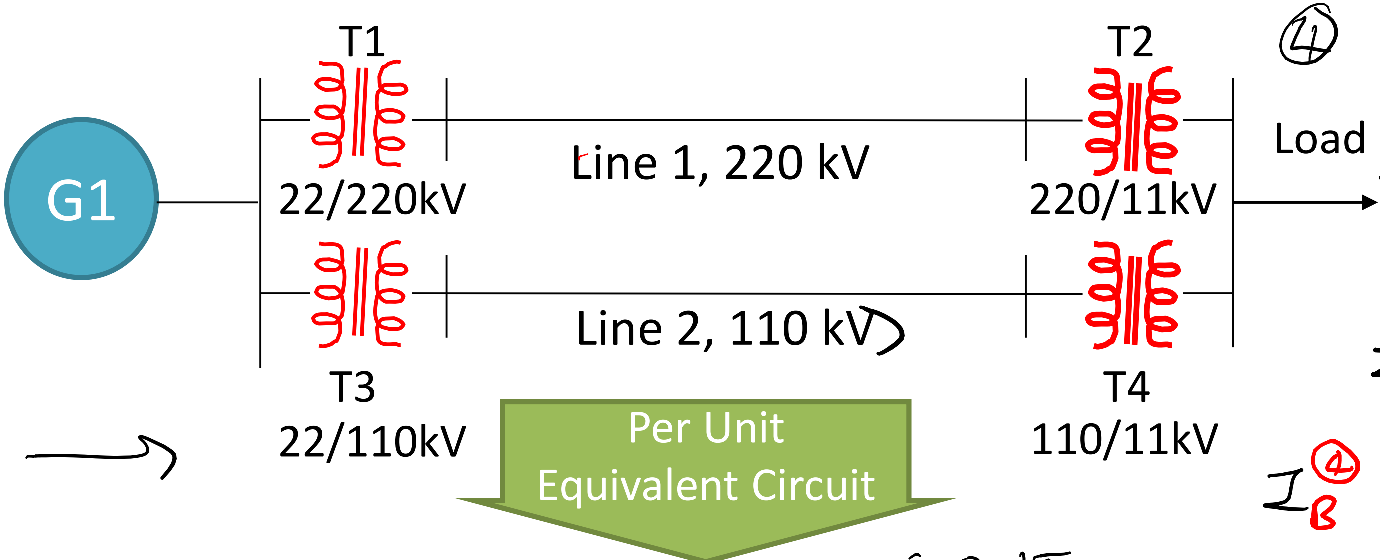
T1 80 MVA 22/220 kV $X = 10\%$ $\Rightarrow$ $X_{T_1} = j0.1 \times \frac{100}{50} = j0.2\ pu$

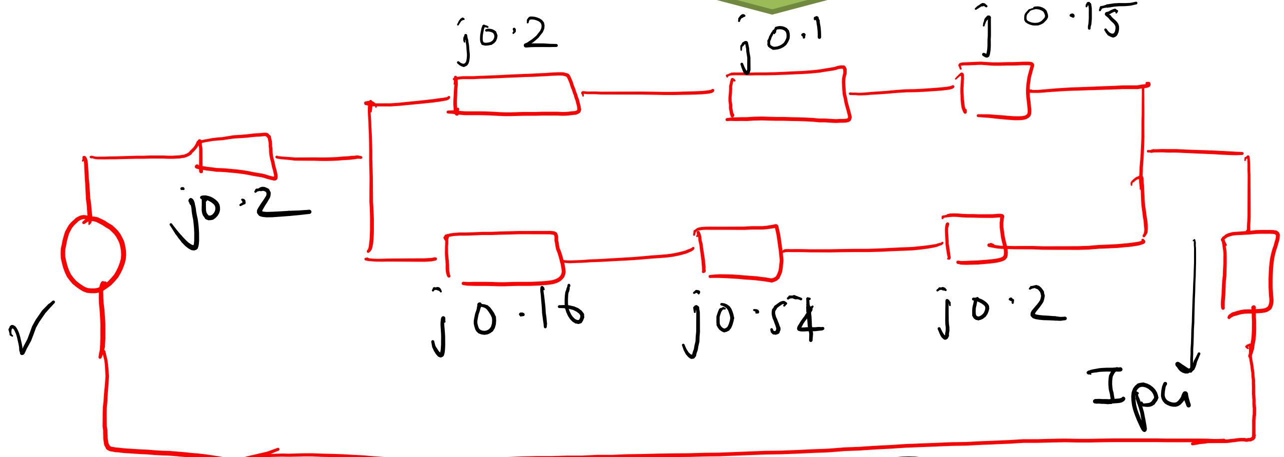
T2 40 MVA 220/11 kV $X = 6\%$ $\Rightarrow$ $X_{T2} = j0.06 \times \frac{100}{40} = j0.15\ pu$

T3 40 MVA 22/110 kV $X = 6.4\%$ $\Rightarrow$ $X_{T3} = j0.064 \times \frac{100}{40} = j0.16\ pu$

T4 40 MVA 110/11 kV $X = 8\%$ $\Rightarrow$ $X_{T4} = j0.08 \times \frac{100}{40} = j0.2\ pu$

Per Unit Equivalent Circuit





0.95 + j 1.267 pu

(4) $S^{3\phi} = \sqrt{3}\, V_{LL} I_L$

$$I_L = \frac{57\times 10^6}{\sqrt{3}\times 10.45\times 10^3}$$

$$= 3149.18\angle -53.13^\circ$$

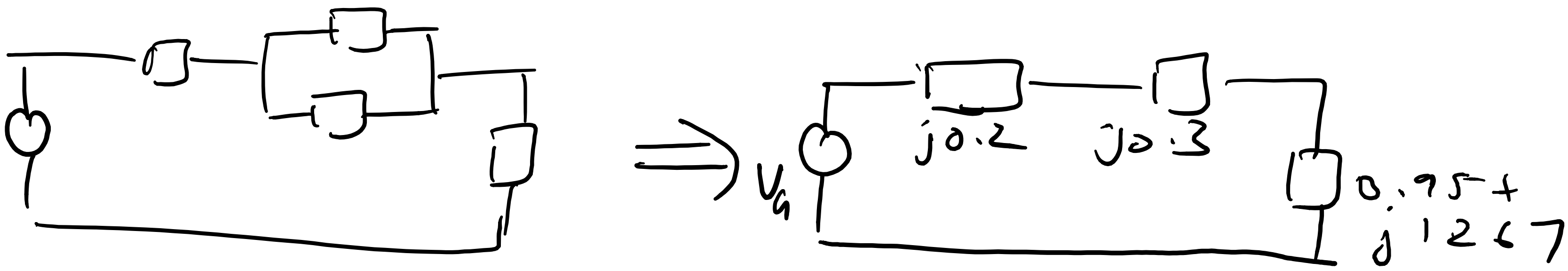
$$I_B^{(4)} = \frac{S^{3\phi}}{\sqrt{3}\,V_{L-B}} = \frac{100\times 10^6}{\sqrt{3}\times 11\times 10^3}$$

$$= 5248.6\ \text{A}$$

$$I_{pu} = \frac{3149.18\angle -53.13}{5248.6}$$

$$\boxed{I_{pu} = 0.6\angle -53.13}$$

Solve and Convert to Actual Quantities



$$I_{pu} = \frac{V_{G\,pu}}{Z_{pu}} \Rightarrow V_{G\,pu} = I_{pu} \times Z_{pu}$$

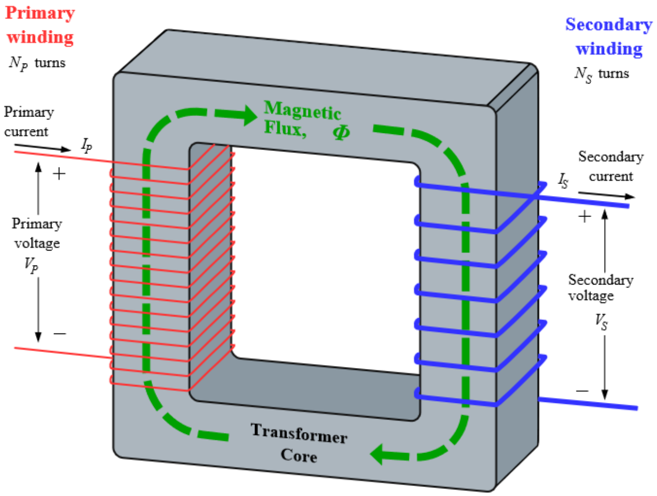
$$= (0.6 \angle -53.13) \times (j0.2 + j0.3 + 0.95 + j1.267)$$

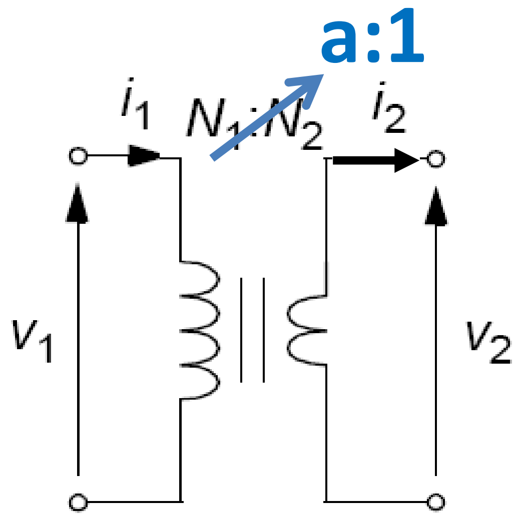
$$V_{G\,pu} = 1.204 \angle 8.606 \text{ pu}$$

$$V_G = V_{G\,pu} \times V_{B1} = 1.204 \angle 8.606 \times 22 \times 10^3$$

$$= 26.481 \angle 8.606 \text{ kV}$$

Transformers- Review





- Define turns ratio as:

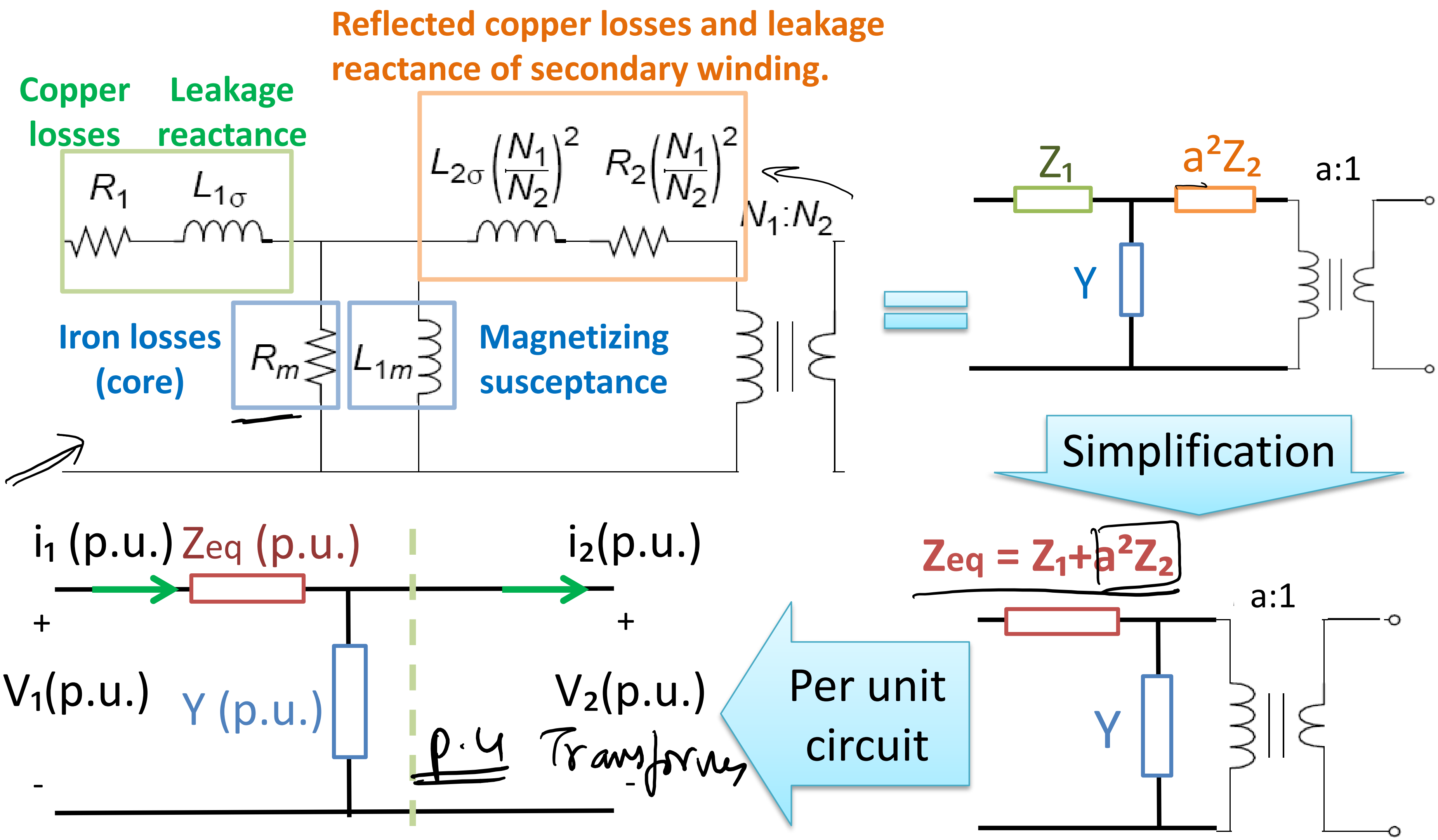
$$a \equiv \frac{N_1}{N_2}$$

- From Faraday's and Ampere's Law:

$$V_1 = \left(\frac{N_1}{N_2}\right) V_2 = aV_2$$

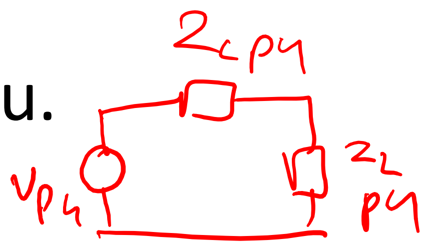
$$i_2 = \left(\frac{N_1}{N_2}\right) i_1 = ai_1$$

An Equivalent Circuit for Transformers



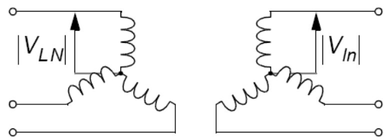
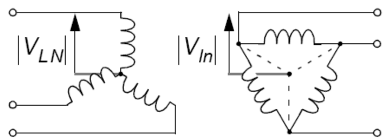
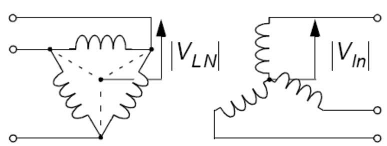
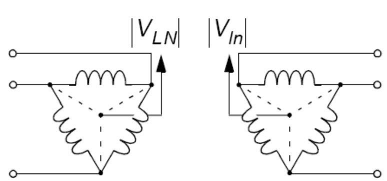
1Φ-Steps of Per Unit Analysis

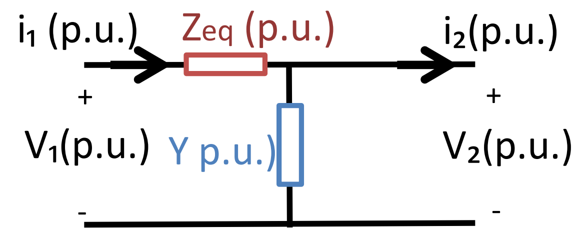
1. Choose $S_B^{1\Phi}$ for the system.
2. Select V_B for different zones (usually follows transformer voltage ratings).
3. Calculate Z_B for different zones. $= \frac{V_B^2}{S_B}$
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.

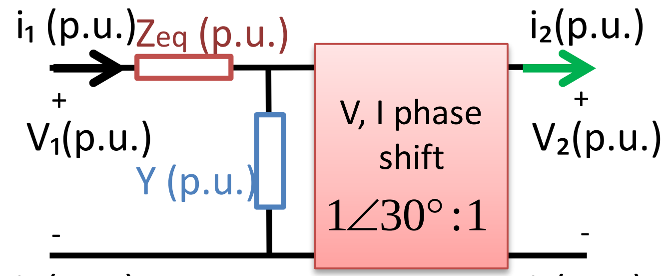


6. Convert back to actual quantities if needed.

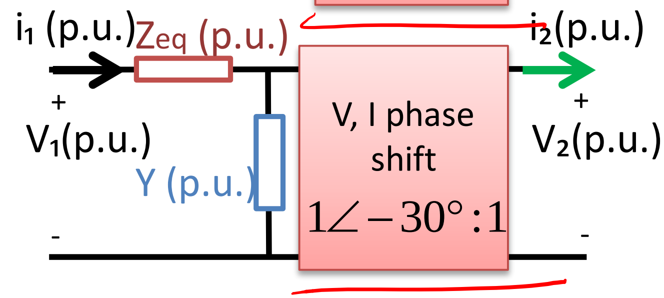
3Φ Transformer Per Unit Model

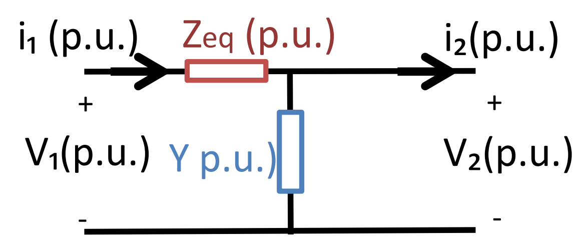
designation	winding connection
Yy	
Yd	
Dy	
Dd	





$1\,pu\angle 0$





1 Φ vs 3 Φ Per Unit Analysis

$$\eta = \frac{P_L \, pu}{P_S \, pu} = \frac{P_L / S_B}{P_S / S_B} = \frac{P_L}{P_S}$$

	1 Φ	3 Φ
①	S_B	$S_B^{3\phi}$
②	V_B each zone	V_B^{L-L} each zone
③	$Z_B = \frac{V_B^2}{S_B}$ each zone	$Z_B = \frac{(V_B^{L-L})^2}{S_B^{3\phi}}$ each zone
④	$I_B = \frac{S_B}{V_B}$ each zone	$I_B = \frac{S_B^{3\phi}}{\sqrt{3}\, V_{L-L}}$ each zone

$$Z_{pu\,new} = Z_{pu\,old} \cdot \frac{Z_{B\,OLD}}{Z_{B\,NEW}} = Z_{pu\,old} \left(\frac{V_{B\,OLD}}{V_{B\,NEW}}\right)^2 \left(\frac{S_{NEW\,B}}{S_{B\,OLD}}\right)$$