2nd Mid-term Test **@ LT5**

- Monday, March 30th 5:00pm to 5:45pm **@ LT5**
- 10%
- Materials (Lectures 11-17, Tutorials 5-6):
  - Transformers
  - Renewable Energy
  - Per Unit Analysis
- Format:
  - Closed book.
  - 45 minutes.

EE2022 Electrical Energy Systems

Per Unit Analysis- Three Phase

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Department of Electrical and Computer Engineering

Outline

- Single-phase per unit analysis (cont)
- Three-phase transformers
  - Possible connections
  - Phase shift introduced by delta-wye or wye-delta connections
  - Per unit equivalent models
- Three-phase per unit analysis

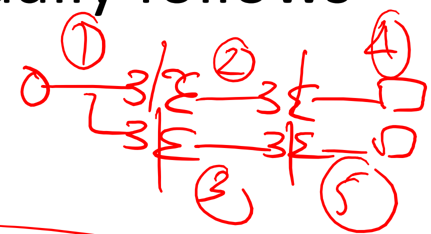
Advantages of Per Unit Analysis

- Simplify calculation by eliminating transformers.
- Helps to spot errors in the data
  - p.u. is more uniform compare to actual impedance value of different sizes of equipments.
- Helps to detect abnormality in the system
  - Operator at control center can spot over/under voltage/current rating easily.

Steps of Per Unit Analysis

1. Choose $S_B^{1\Phi}$ for the system.

2. Select V_B for different zones (usually follows transformer voltage ratings).

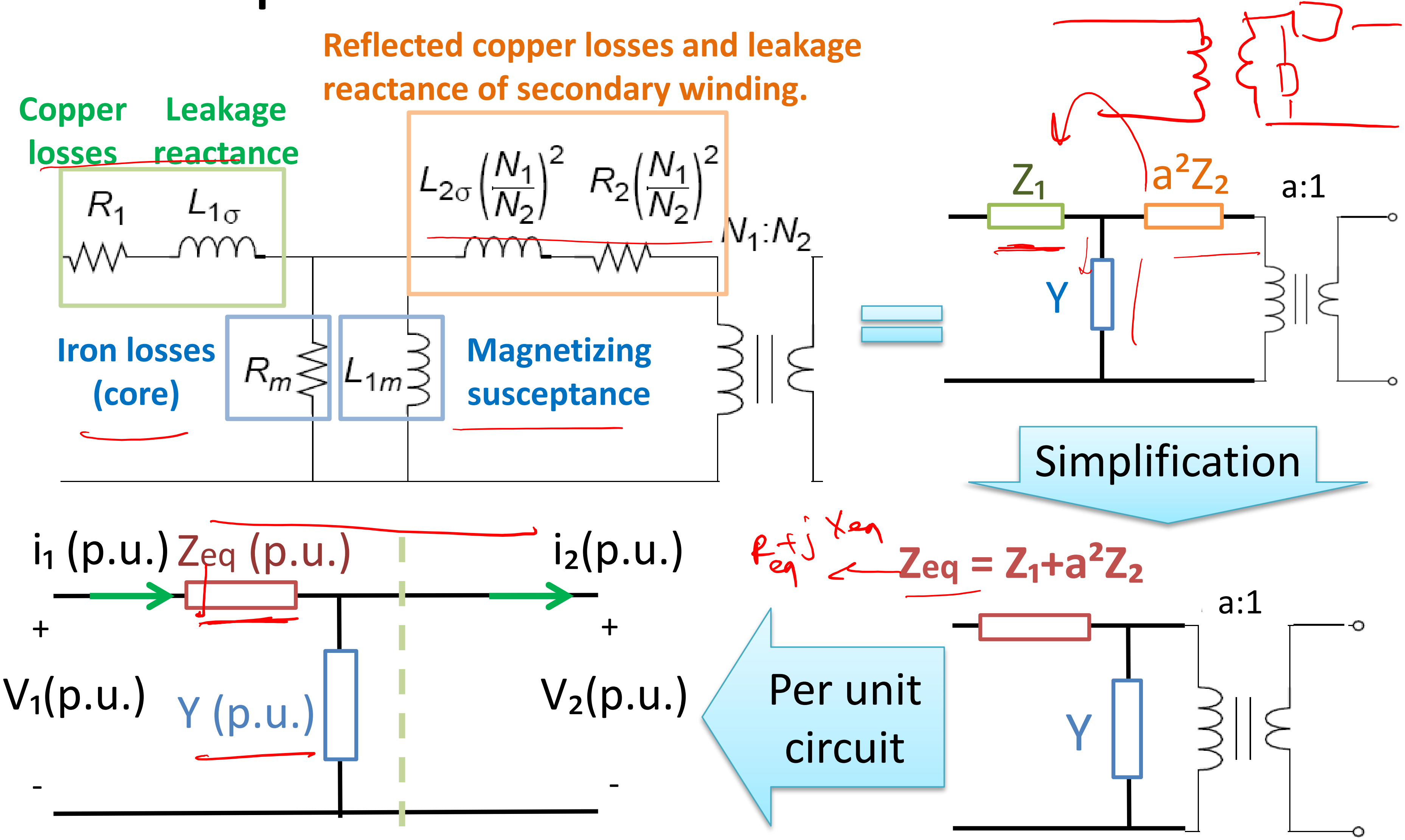


3. Calculate Z_B for different zones.

$Z_B = \frac{V_B^2}{S_B}$

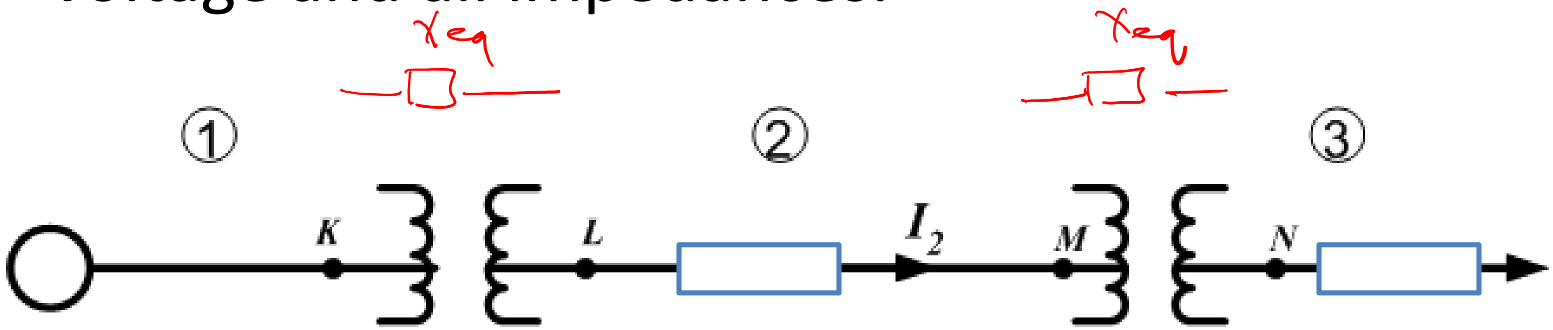
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

An Equivalent Circuit for Transformers



Example : 1Φ, Per Unit Analysis

- Three zones of a single-phase circuit are shown below. Use base value of 30 kVA and 240 V in zone 1, draw per unit circuit and find per unit value of source voltage and all impedances.



$V_s = 220\angle 0°$ V

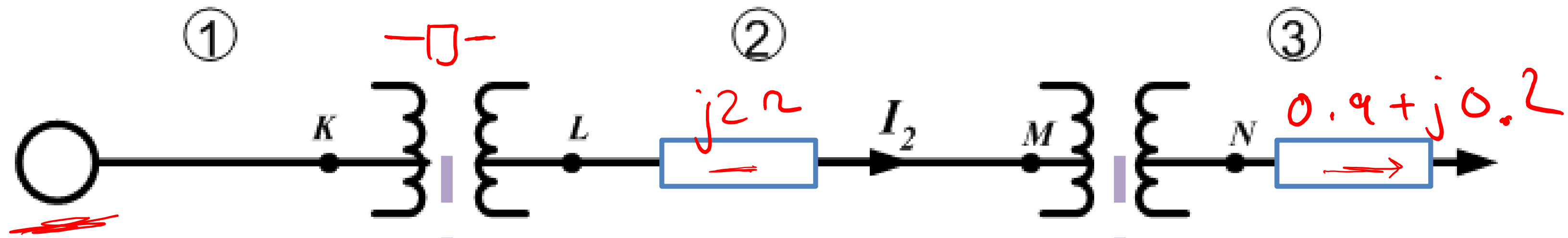
$Z_{line} = j2\,\Omega$

$Z_{load} = 0.9 + j0.2\,\Omega$

T1: 30 kVA
240/480 V
Xeq = 0.1 p.u.

T2: 20 kVA
460/115 V
Xeq = 0.1 p.u.

$S_B = 30\,kVA$



$V_{B1} = 240\,V$

$V_{B2} = \left(\frac{480}{240}\right) \times 240 = 480\,V$

$V_{B3} = \left(\frac{115}{460}\right) \times 480 = 120\,V$

$Z_{B1} = \frac{240^2}{30 \times 10^3} = 1.92\,\Omega$

$Z_{B2} = \frac{480^2}{30 \times 10^3} = 7.68\,\Omega$

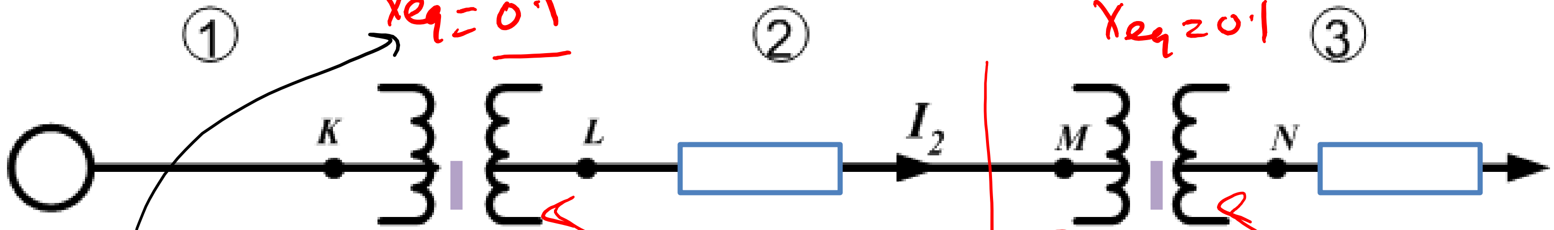
$Z_{B3} = \frac{120^2}{30 \times 10^3} = 0.48\,\Omega$

$V_{source\,pu} = \frac{220\angle 0}{240} = 0.9167\angle 0\ pu$

$Z_{line\,pu} = \frac{j2}{7.68} = j0.2604\ pu$

$Z_{load\,pu} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167\ pu$

$X_{eq} = 0.1$ $\qquad$ $X_{eq} = 0.1$



$$Z_{Bold1} = \frac{240^2}{30\times10^3}$$

$$Z_{Bold} = \frac{480^2}{30\times10^3}$$

$$\frac{460^2}{20\times10^3}$$

$$Z_{Bold} \quad \frac{115^2}{20\times10^3}$$

$$Z_{actual} = \frac{240^2}{30\times10^3}\times 0.1$$

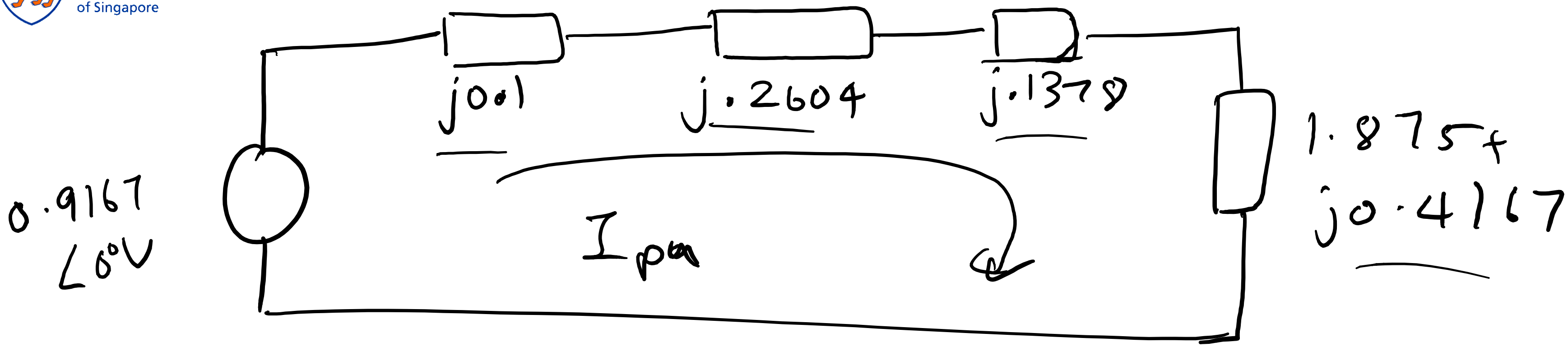
$$Z_{actual} = \frac{480^2}{30\times10^3}\times 0.1$$

$$Z_{pu\,new} = \frac{\frac{240^2}{30\times10^3}\times 0.1}{1.92} = 0.1$$

$$Z_{pu\,new} = \frac{\frac{480^2}{30\times10^3}\times 0.1}{7.68} = 0.1$$

$$Z_{actual} = \frac{460^2}{20\times10^3}\times 0.1$$

$$Z_{pu\,new} = \frac{\frac{460^2}{20\times10^3}\times 0.1}{7.68} = 0.1378\ \text{pu}$$



$$I_{pu} = \frac{V_s}{Z_{eq}} = \frac{0.9167\angle 0}{j.1 + j.2604 + j.1378 + 1.875 + j.4167}$$

$$= 0.4395\angle -26.01^\circ \text{ pu}$$

$I_1 = I_{pu} \times I_{B1}$

$I_3 = I_{pu} \times I_{B3}$

$I_2 = I_{pu} \times I_{B2}$

Three-phase transformer connections

Phase shift

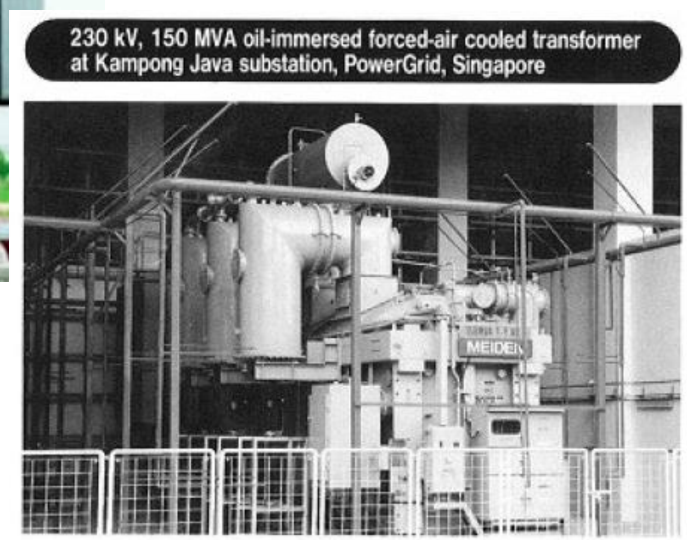
Per-unit equivalent models

THREE PHASE TRANSFORMERS

Three-Phase Transformers

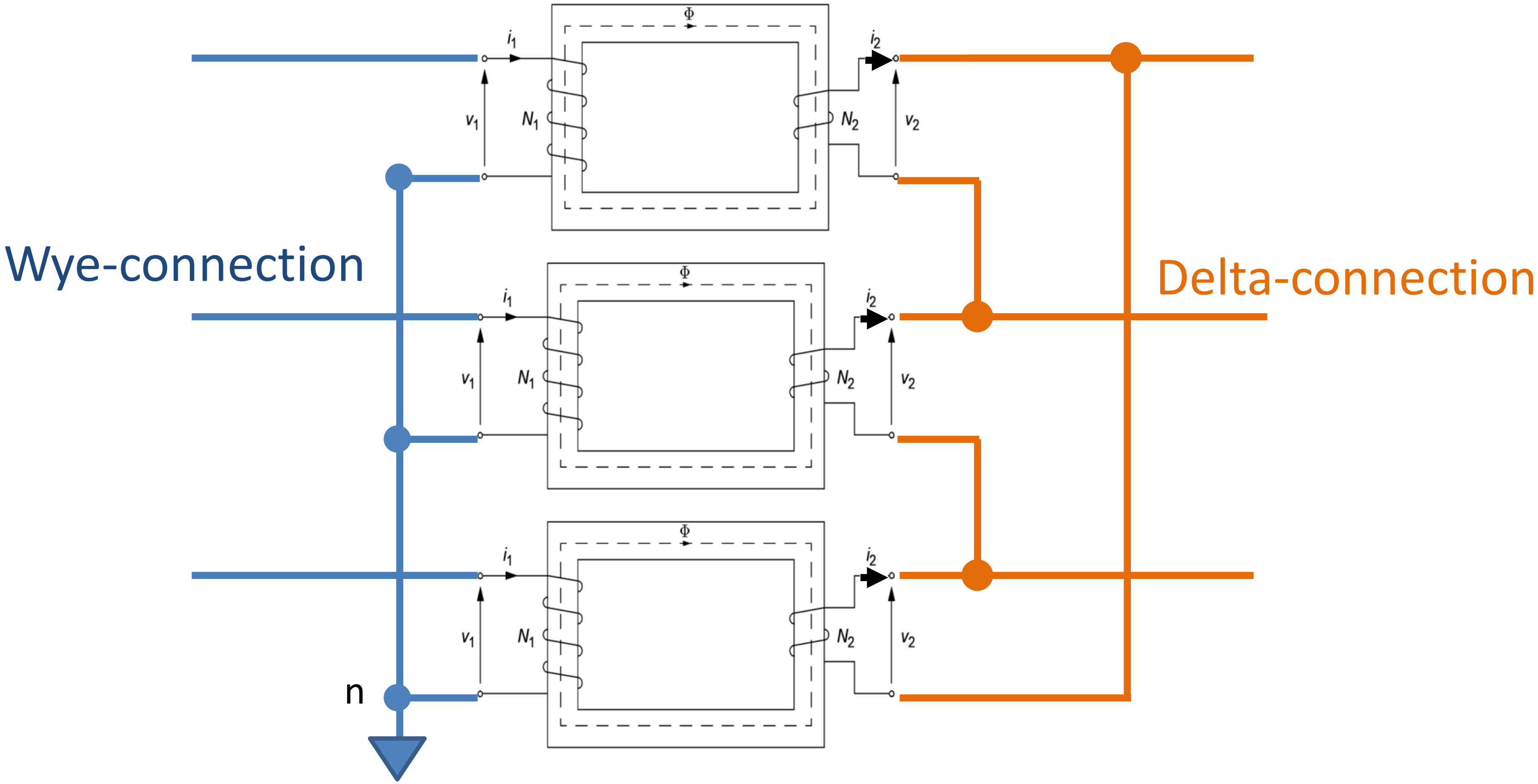
Pole-mounted three single-phase transformers.

Three-phase transformers at substations

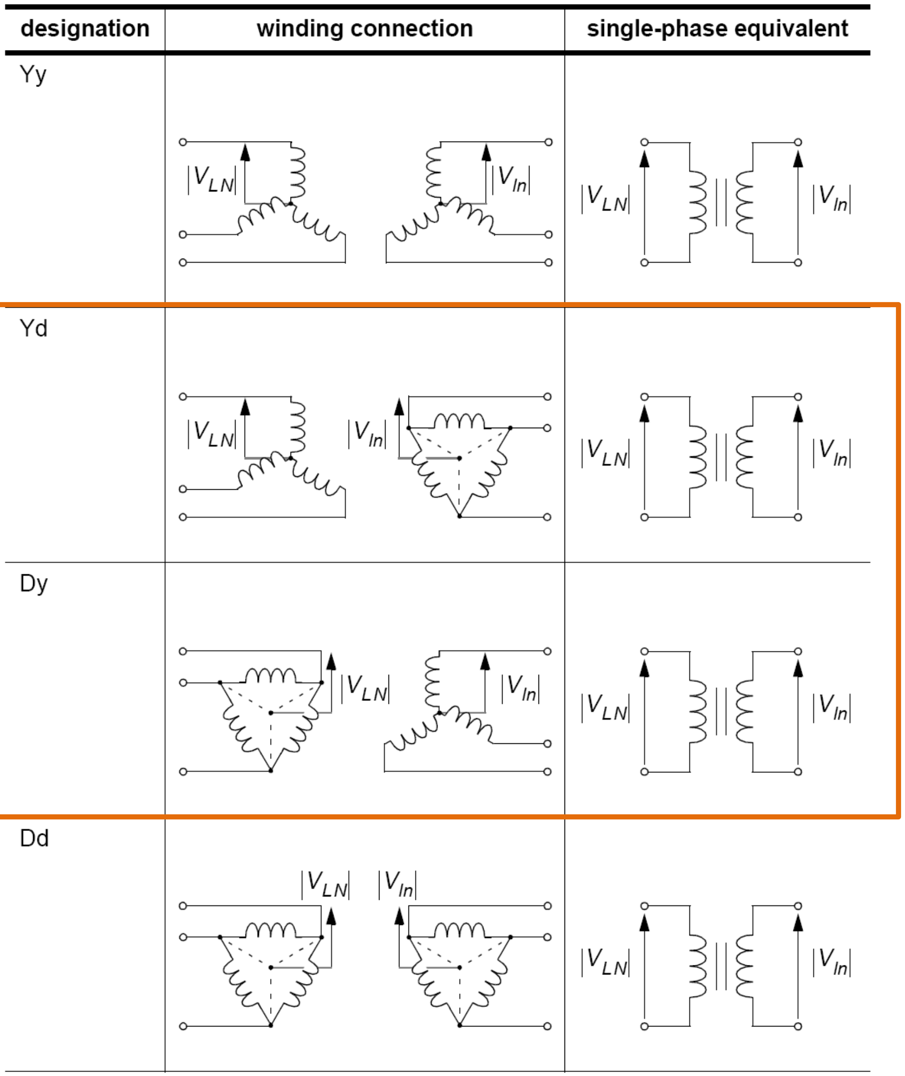


Source: http://www.meidensg.com.sg

Three Single-Phase Transformers

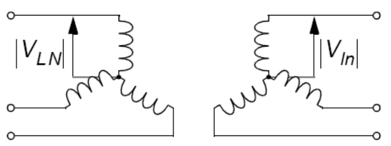
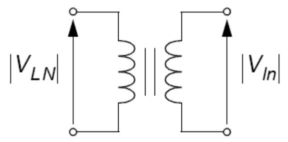
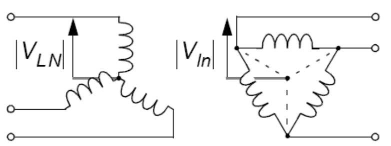
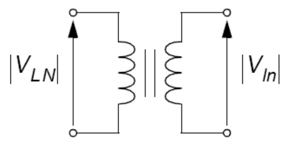
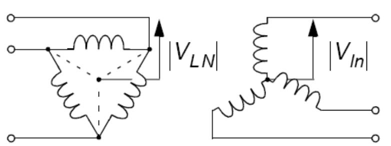
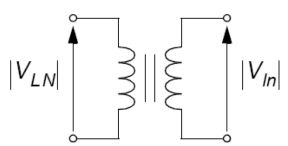
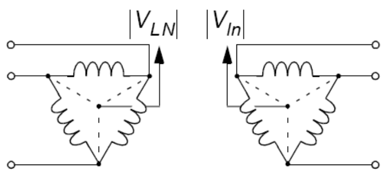
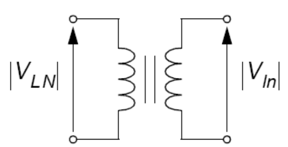


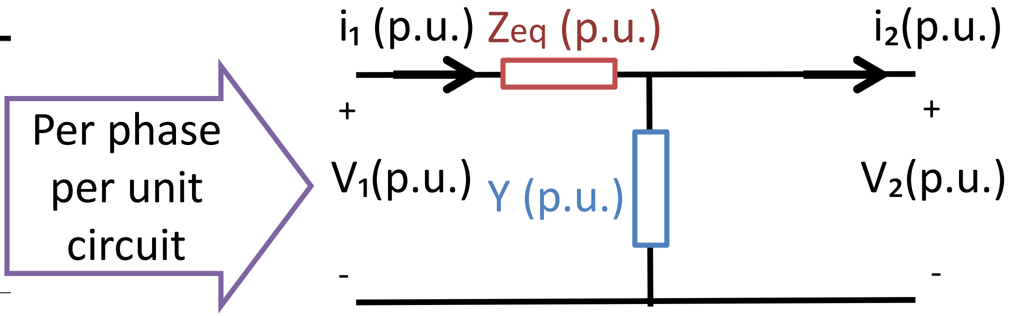
3Φ Transformer Connection



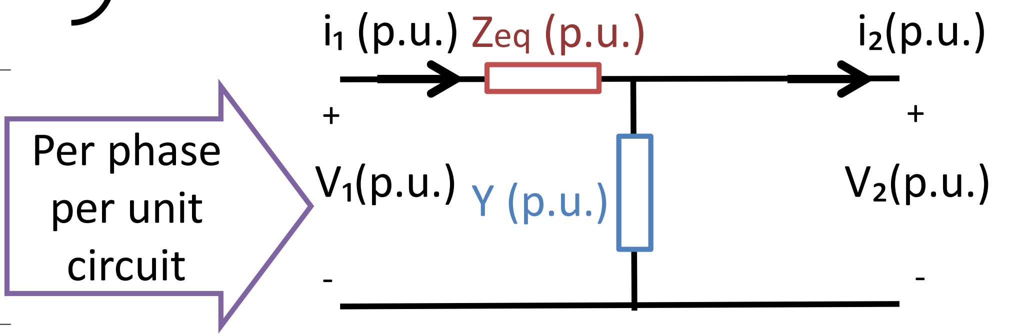
- The voltage rating of a three phase transformer is the ratio between **line-to-line** voltage at the primary side and **line-to-line** voltage at the secondary side.
- The single-phase equivalent shows line-to-neutral voltage.
- For Y-Y and Δ-Δ transformers, voltage and current in both primary and secondary are in phase. The ratio of the voltage and current follows the turn ratio of the transformer.
- The same does not apply to Y-Δ and Δ-Y connections.

3Φ Transformer Connection

designation	winding connection	single-phase equivalent
Yy		
Yd		
Dy		
Dd		

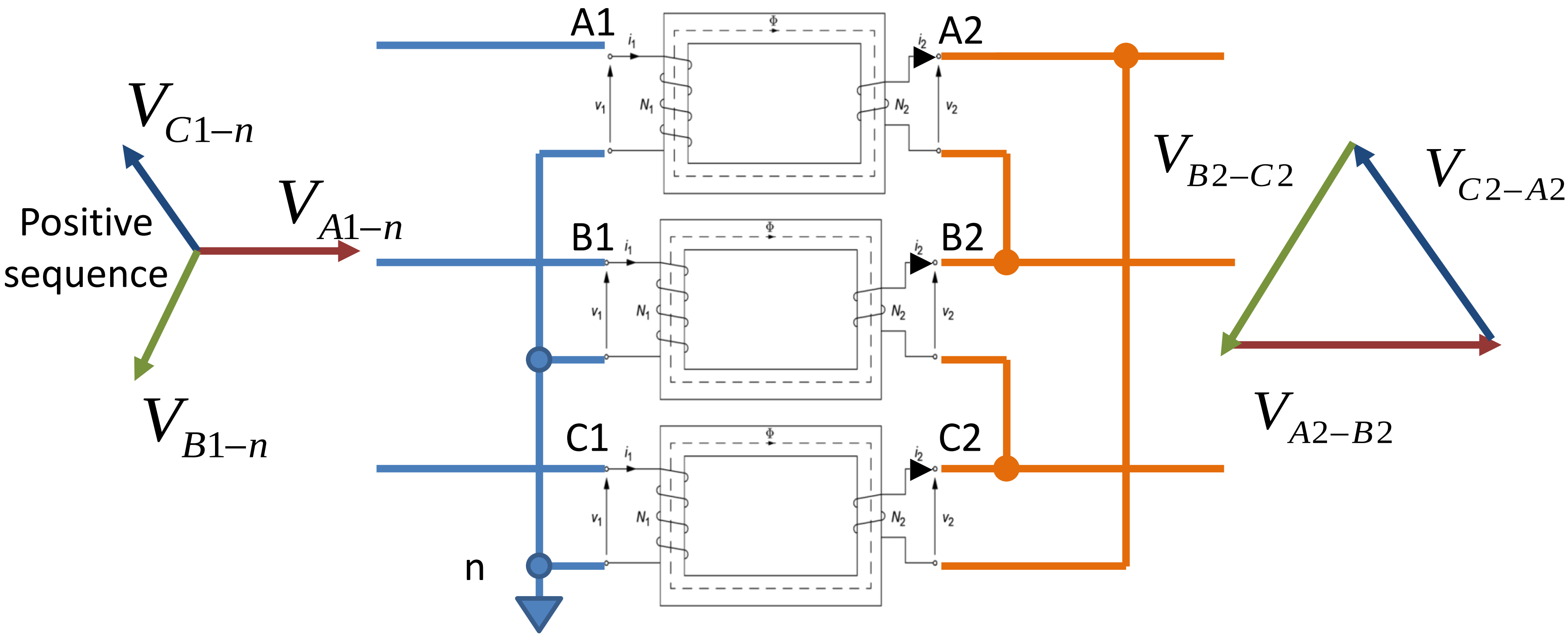


Wye-Delta and Delta-Wye transformer connections introduce some phase shift to the voltage.



Wye-Delta 3Φ Transformers

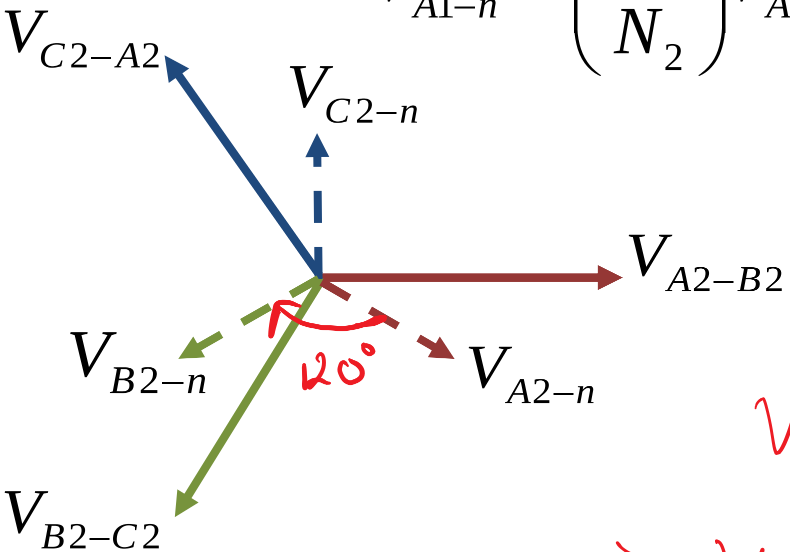
Wye-connected Delta-connected



Line-to-neutral voltage $V_{A1-n} = \left(\frac{N_1}{N_2}\right) V_{A2-B2}$ ***Line-to-line*** *voltage!!*

Wye-Delta 3Φ Transformer

$$V_{A1-n} = \left(\frac{N_1}{N_2}\right) V_{A2-B2}$$



For a positive sequence,

$$V_{A2-B2} = V_{A2N} - V_{B2N}$$

$$= V_{A2N} - V_{A2N}\angle -120^\circ$$

$$V_{A2B2} = \sqrt{3}\, V_{A2N} \angle 30^\circ$$

$$\Rightarrow V_{A1N} = \boxed{\left(\frac{N_1}{N_2}\right)\sqrt{3}}\, V_{A2N} \angle 30^\circ$$

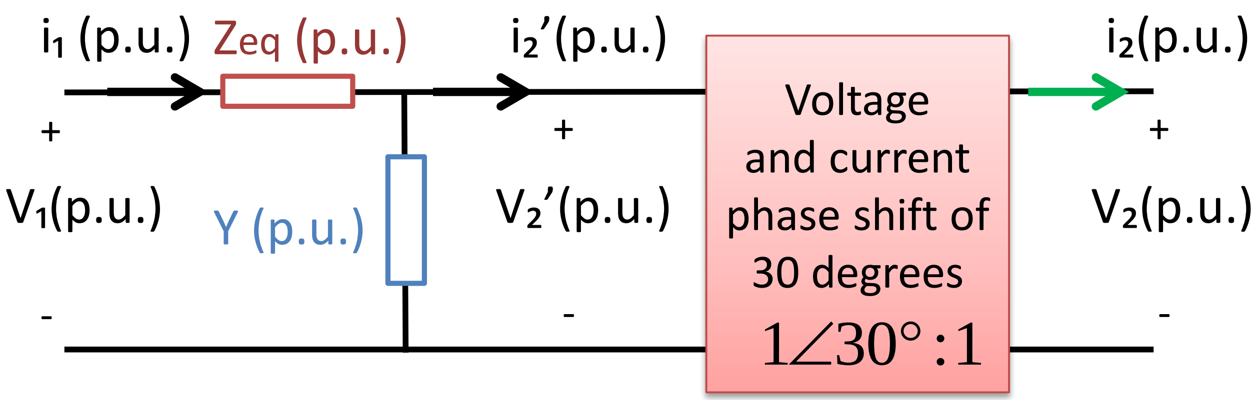
The line-to-neutral voltage at the primary side leads the line-to neutral voltage at the secondary side by 30 degrees. The same is true for the current.

Y-Δ 3Φ Transformer: Per Phase Model

For a positive sequence voltage source,

$$\frac{V_{A1N}}{V_{A2N}} = \frac{1\angle 30^\circ}{1}$$

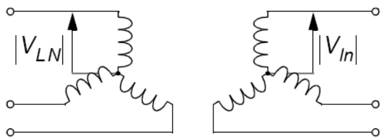
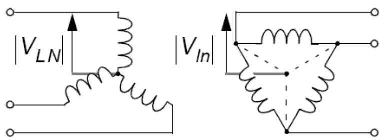
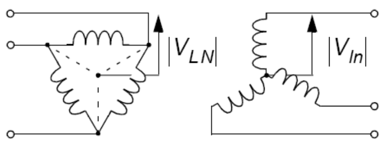
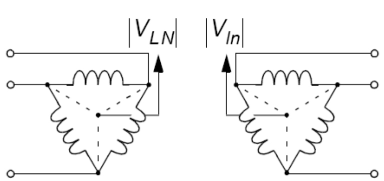
$$V_{A1-n} = \left(\frac{N_1}{N_2}\right)\sqrt{3}V_{A2-n}\angle 30^\circ \Rightarrow \angle V_{A1-n} : \angle V_{A2-n} = 1\angle 30^\circ : 1$$

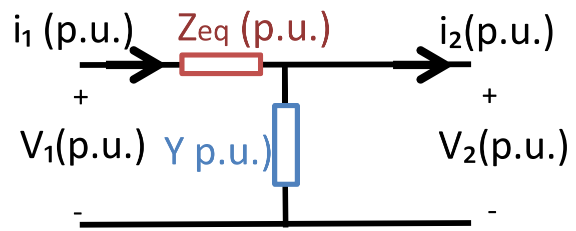


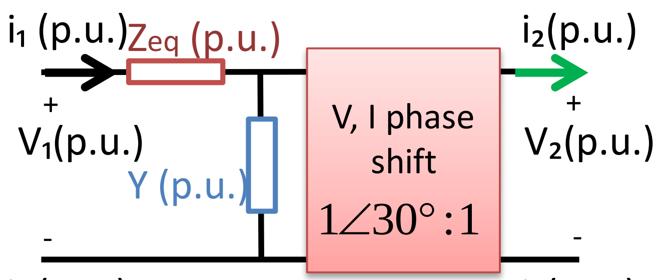
$$i_2 = i_2' \angle -30^\circ$$

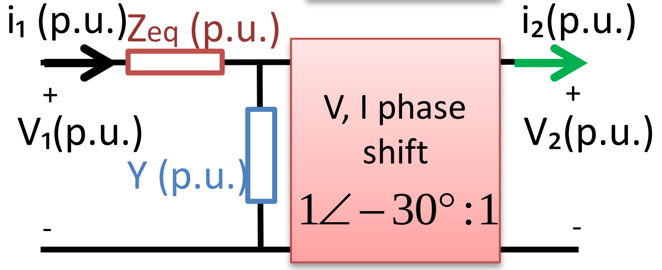
$$V_2 = V_2' \angle -30^\circ$$

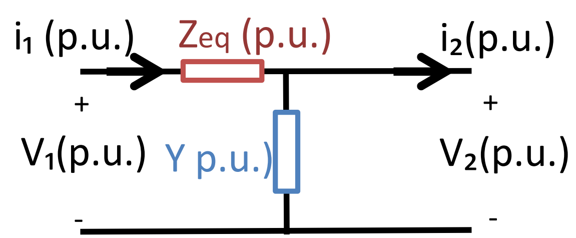
3Φ Transformer Per Unit Model

designation	winding connection
Yy	
Yd	
Dy	
Dd	









Three-Phase Transformer Nameplate

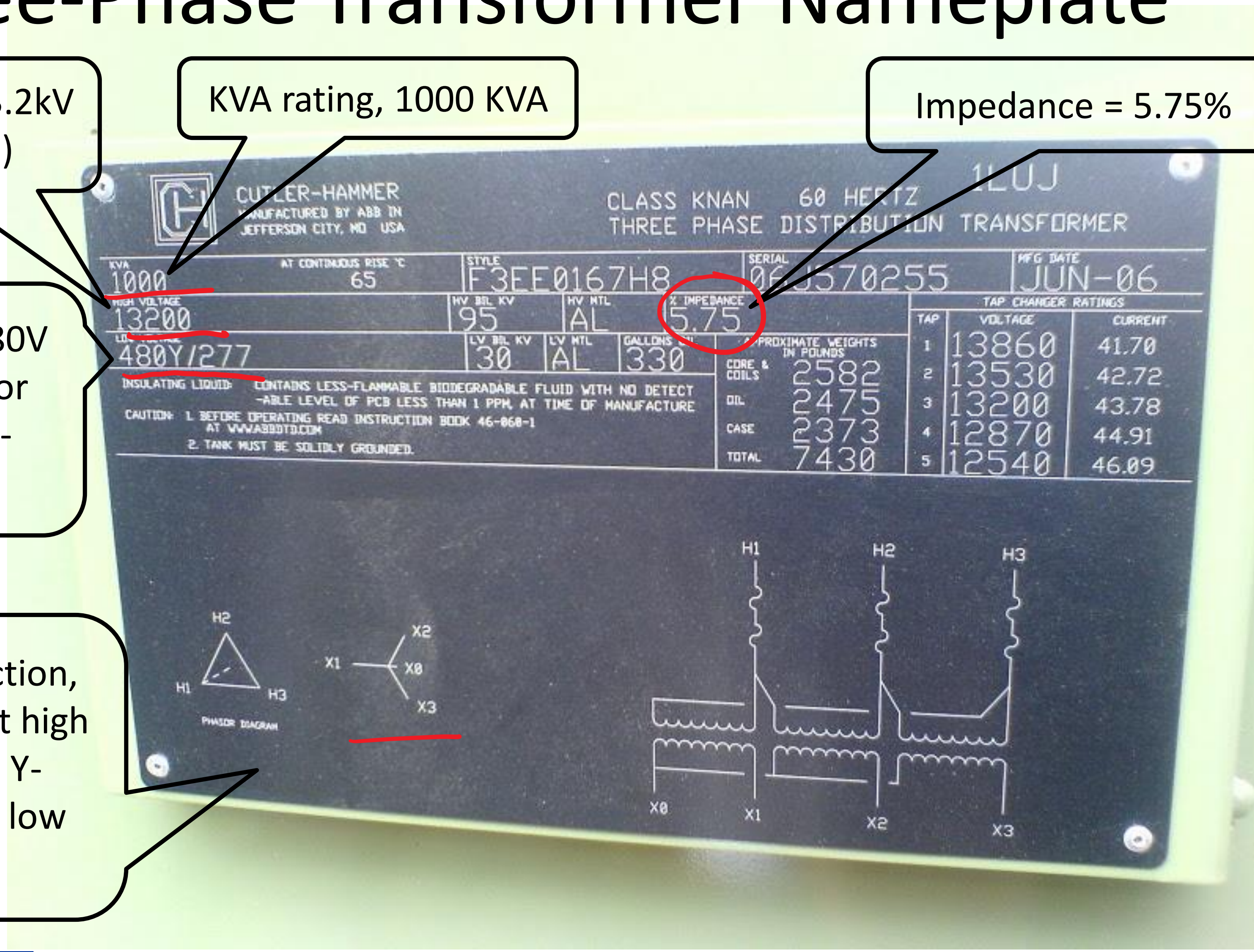
High voltage 13.2kV (line-to-line)

KVA rating, 1000 KVA

Impedance = 5.75%

Low voltage 480V (line-to-line) or 277V(line-to-neutral)

Circuit connection, Δ-connected at high voltage and Y-connected at low voltage



Per Unit Quantity: 3Φ

Base Value: 3Φ

Steps of Calculation: 3Φ

THREE PHASE PER UNIT ANALYSIS

Recall: Per Unit Quantity

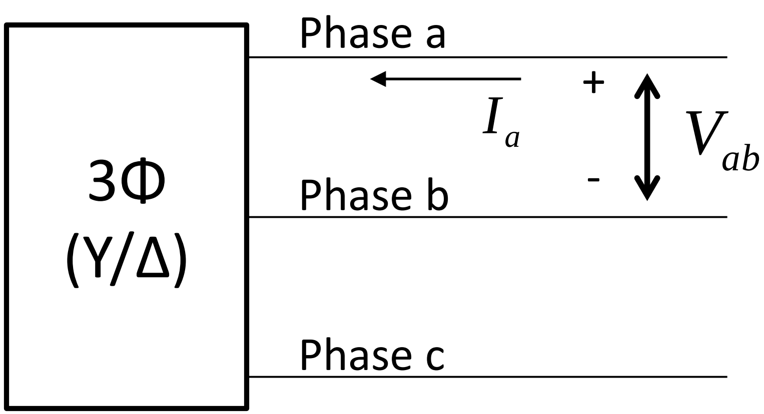
- The per unit quantity of voltage, current, power and impedance is found from dividing the actual quantity by a base value of that quantity.

$$\text{per} - \text{unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

- Per unit value is denoted by ‘p.u.’.
- The base value quantity typically follows transformer rating.
- The per unit values are the same irrespective of the sides of the transformer.

Per Unit Analysis in 3Φ Circuit

- Recall that in three phase circuit:
  - Voltage is given as line-to-line voltage.
  - Current is given as line current.
  - Apparent power is given as three-phase power.



$$\boxed{\left|S_{3\Phi}\right| = \sqrt{3}\left|V_{\text{Line-To-Line}}\right|\left|I_{\text{Line}}\right|}$$

Base Value for Voltage: 3Φ Case

- Base values are **real numbers**, denote by subscript '*B*'.
- Voltage base values follow transformer voltage ratings.

$$V_B^{line-to-neutral} = \frac{V_B^{line-to-line}}{\sqrt{3}}$$

- Note that the voltage ratios are given as line-to-line voltage at both ends. This means that it already incorporates the factor of $\sqrt{3}$ term in Y or Δ configurations. (see slide 17 again)

Base Value for Power: 3Φ Case

- Only **single** base complex power $S_B^{3\Phi}$ in the system.
- The base value of power is used to normalize the quantity. Thus, the base values of real power, reactive power, and complex power are all the same real number.

$S_B^{3\phi} = P_B^{3\phi} = Q_B^{3\phi}$

Base Value for Current and Impedance: 3Φ Case

- Current base values are calculated from the base power and base voltage.

$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\, V_B^{L-L}}$$

- Impedance base values (same value for impedance, resistance, or reactance) are calculated from voltage and current.

$$Z_B = \frac{V_B^{L-N}}{I_B^{L-N}} = \frac{\left(V_B^{L-L}\right)^2}{S_B^{3\phi}}$$

Change of Base Value

- Manufacturers usually specify equipment impedances in per unit values together with voltage ratings (V) and apparent power rating (VA).
- The impedance base values can be found from the ratings of the equipment.
- Different equipment has different ratings.
- We may need to calculate per unit values on the new basis.
- Note that this process is exactly the same as in single-phase case.

Per Unit Value for Impedance

$$Z^{3\phi}_{ACTUAL} = Z^{3\phi pu}_{OLD} \, Z^{3\phi}_{BOLD} = Z^{3\phi pu}_{NEW} \, Z^{3\phi}_{BNEW}$$

$$\Rightarrow Z^{3\phi\,NEW}_{pu} = \frac{Z^{3\phi}_{BOLD}}{Z^{3\phi}_{BNEW}} \cdot Z^{3\phi\,OLD}_{pu}$$

$$= \frac{\left(V^{L-L}_{BOLD}\right)^2}{S^{3\phi}_{BOLD}} \times \frac{S^{3\phi}_{BNEW}}{\left(V^{L-L}_{BNEW}\right)^2} \cdot Z^{3\phi\,OLD}_{pu}$$

$$\boxed{Z^{3\phi\,NEW}_{pu} = \left(\frac{V^{L-L}_{BOLD}}{V^{L-L}_{BNEW}}\right)^2 \times \left(\frac{S^{3\phi}_{BNEW}}{S^{3\phi}_{BOLD}}\right) \times Z^{3\phi\,OLD}_{pu}}$$

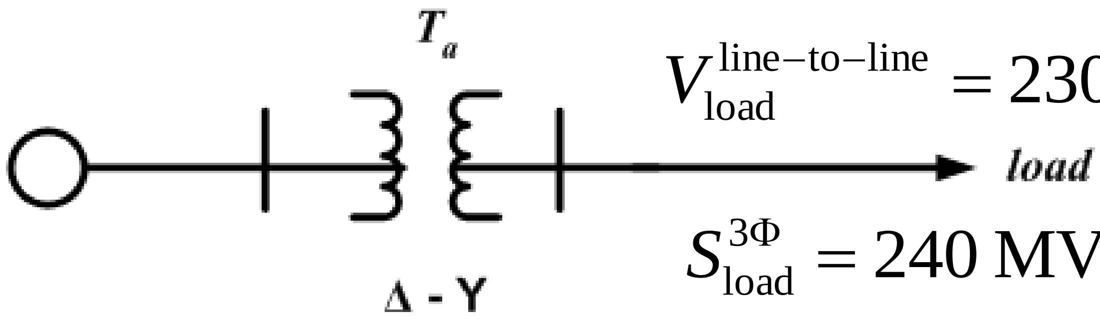
Steps of Calculation: 3Φ Case

1. Choose $S_B^{3\Phi}$ for the system.
2. Select $V_B^{\text{line-to-neutral}}$ or $V_B^{\text{line-to-line}}$ for different zones.
3. Calculate Z_{B} for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Note that the **per unit circuit is** the circuit in **per-phase analysis** with normalization of the voltage magnitude at different locations. This means that **the phase of voltage** in per unit analysis **refers to** the **line-to-neutral voltage**.

Example : Δ-Y 3Φ Transformer

- Three-phase generator rated 300 MVA,23 kV supplying a system Y-connected load of 240 MVA, 0.9 power factor lagging at 230 kV through a 330 MVA 23Δ/ 230Y kV **step-up** transformer of leakage reactance 11%.
- Use **base values at the load of 100 MVA and 230 kV**, find the current supplied by the generator. Use *load voltage for an angle reference*.



$$V_{\text{load}}^{\text{line-to-line}} = 230\,\text{kV}$$

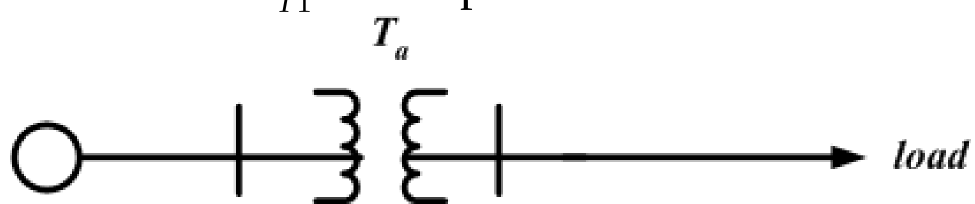
$$S_{\text{load}}^{3\Phi} = 240\,\text{MVA}, 0.9 \text{ pf lagging}$$

Example : Base Values

330 MVA

23Δ/ 230Y kV

$X_{T1} = 0.11\,\text{p.u.}$



$S_B^{3\phi} = 100\,\text{MVA}$

Δ - Y

$V_{B1}^{L-L} = \frac{23}{\cancel{230}} \times \cancel{230} = 23\,\text{kV}$

$V_{B2}^{L-L} = 230\,\text{kV}$

$Z_{B1} = \frac{\left(V_{B1}^{L-L}\right)^2}{S_B^{3\phi}} = \frac{(23\times10^3)^2}{100\times10^6} = 5.29\,\Omega$

$Z_{B2} = \frac{\left(V_{B2}^{L-L}\right)^2}{S_B^{3\phi}} = \frac{(230\times10^3)^2}{100\times10^6} = 529\,\Omega$

$I_{B1} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B1}^{L-L}} = \frac{100\times10^6}{\sqrt{3}\times23\times10^3}$

$= 2510.2\,\text{A}$

$I_{B2} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B2}^{L-L}} = \frac{100\times10^6}{\sqrt{3}\times230\times10^3}$

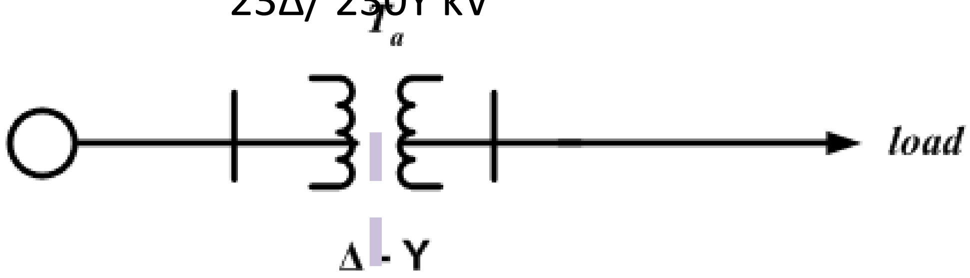
$= 251.02\,\text{A}$

Example : Transformer Reactance

330 MVA $X_{T1} = 0.11\,\text{p.u.}$

23Δ/ 230Y kV

$$S_B^{3\Phi} = 100\ \text{MVA}$$



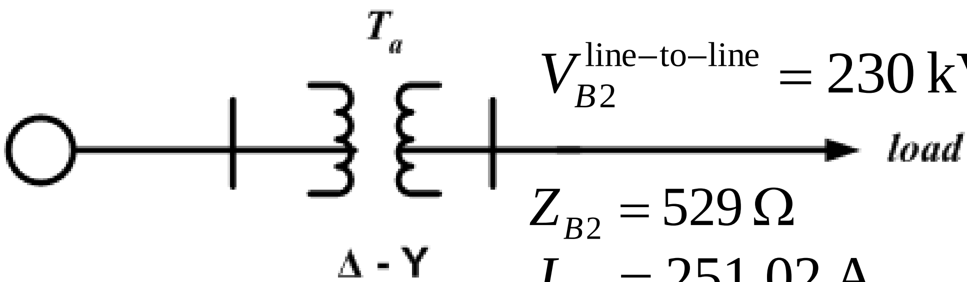
$$X_{pu}^{NEW} = \left(\frac{V_{BOLD}^{L-L}}{V_{BNEW}^{L-L}}\right)^2 \times \left(\frac{S_{BNEW}^{3\phi}}{S_{BOLD}^{3\phi}}\right) \times X_{pu}^{OLD} = \left(\frac{\cancel{23\times10^3}}{\cancel{23\times10^3}}\right)^2 \times \left(\frac{100\times10^6}{330\times10^6}\right) \times 0.11$$

$$= 0.0333\ pu$$

$$X_{pu}^{NEW} = \left(\frac{\cancel{230\times10^3}}{\cancel{230\times10^3}}\right)^2 \times \left(\frac{100\times10^6}{330\times10^6}\right) \times 0.11 = 0.0333\ pu.$$

Example : Load Voltage and Current

$$S_B^{3\Phi} = 100\ \text{MVA}$$



$$V_{B2}^{\text{line-to-line}} = 230\ \text{kV}$$

$$Z_{B2} = 529\ \Omega$$

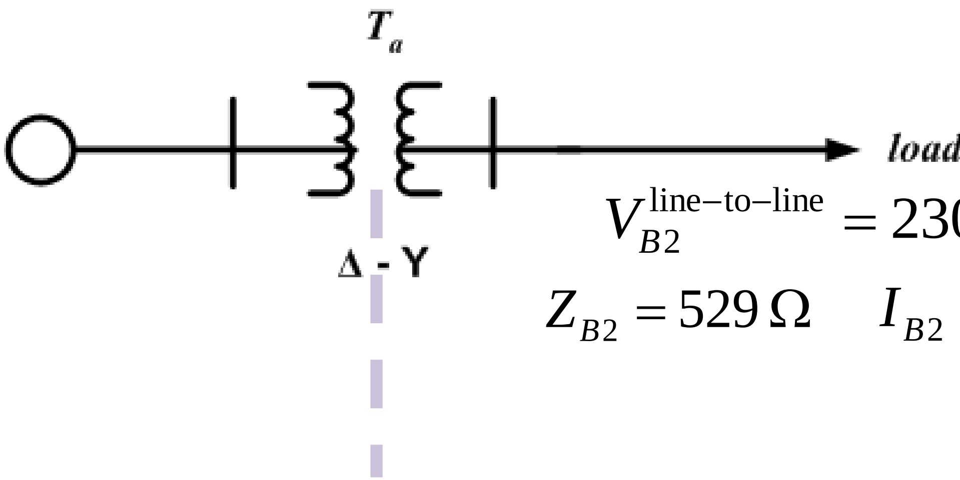
$$I_{B2} = 251.02\ \text{A}$$

$$S_L = \sqrt{3}\, V_{LL} I_L \Rightarrow I_L = \frac{S_L}{\sqrt{3}\, V_{L-L}}$$

$$\Rightarrow I_L = \frac{240\times10^6}{\sqrt{3}\times 230\times10^3} = 602.45\ \text{A}$$

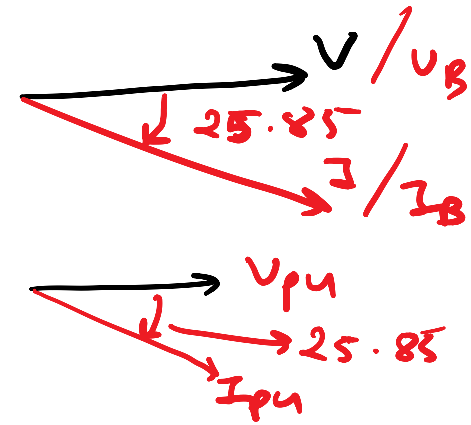
$$\angle I_L = \cos^{-1} 0.9 = -25.85^\circ \Rightarrow \boxed{I_L = 602.45\angle -25.85}$$

Example : Per Unit Values



$$V_{B2}^{\text{line-to-line}} = 230\,\text{kV}$$

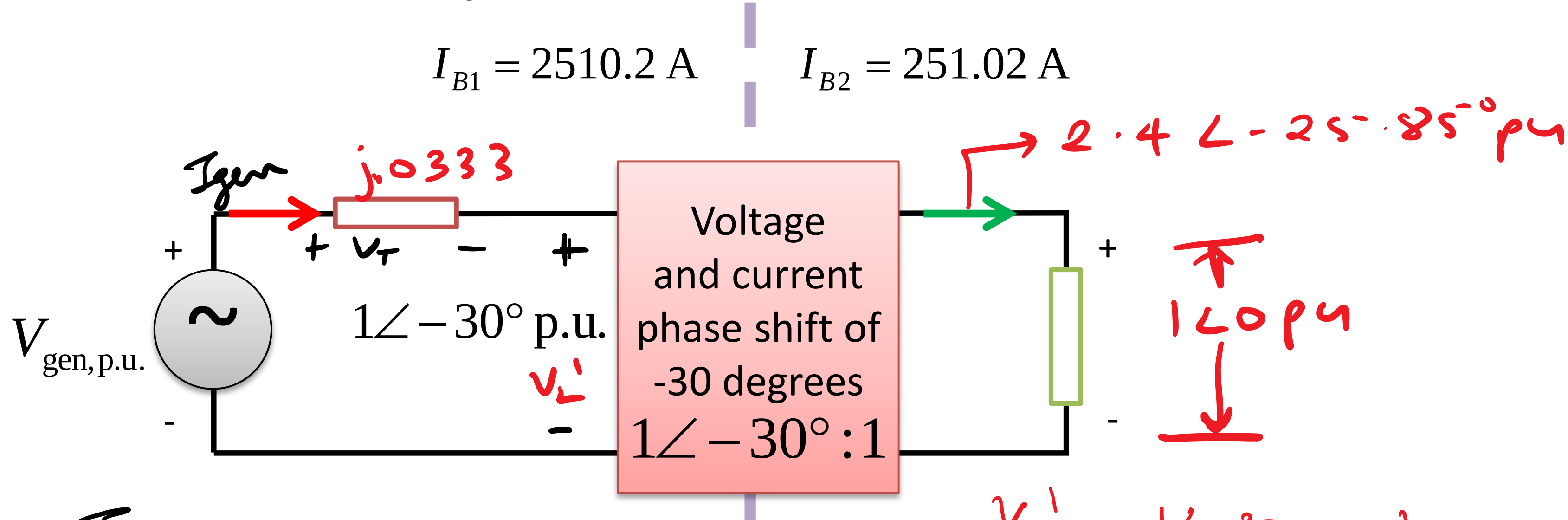
$$Z_{B2} = 529\,\Omega \qquad I_{B2} = 251.02\ \text{A}$$



$$V_{LOAD\,pu} = \frac{V_{LOAD}^{L-L}}{V_B^{L-L}} \angle 0 = \frac{230\times10^3}{230\times10^3}\angle 0 = 1\angle 0\ pu$$

$$I_{LOAD,pu} = \frac{I_{LOAD}}{I_B} = \frac{602.45\angle -25.85}{251.02} = 2.4\angle -25.85\,pu$$

Example : Per Unit Circuit



$$\frac{I_{gen\,pu}}{I_{L\,pu}} = \frac{1\angle -30}{1}$$

$$\Rightarrow I_{gen\,pu} = 2.4\angle -25.85 \times 1\angle -30$$

$$= 2.4\angle -55.85^\circ \text{ pu}$$

$$I_{gen} = (2.4\angle -55.85) \times 2510.2 \Rightarrow \boxed{I_{gen} = 6024.5\angle -55.85 \text{ A}}$$

$$\frac{V_L'}{V_L} = \frac{1\angle -30}{1} \Rightarrow V_L' = 1\angle -30$$

$$\boxed{V_{gen\,pu} = V_T + V_L' = (I_{gen} \times j.033) + 1\angle -30}$$

Summary

→ Revision of 1-ϕ per unit analysis

→ 3ϕ transformers $\begin{cases} \Delta Y \\ Y\Delta \end{cases}$ } Phase Shift

→ 3ϕ - per unit analysis

	1 ϕ	3ϕ
1)	$S_B^{1\phi}$	$S_B^{3\phi}$
2)	V_{B1}, V_{B2}, ... each zone	V_{B1}^{L-L}, V_{B2}^{L-L}, ... each zone
3)	$Z_{B1} = \dfrac{V_{B1}^2}{S_B^{1\phi}}$ .... each zone	$Z_B^{3\phi} = \dfrac{(V_{B1}^{L-L})^2}{S_B^{3\phi}}$ each zone
4)	$I_{B1} = \dfrac{S_B^{1\phi}}{V_{B1}}$, each zone	$I_{B1}^{3\phi} = \dfrac{S_B^{3\phi}}{\sqrt{3}\, V_{B1}^{L-L}}$ each zone