

EE2022 Electrical Energy Systems

Generators

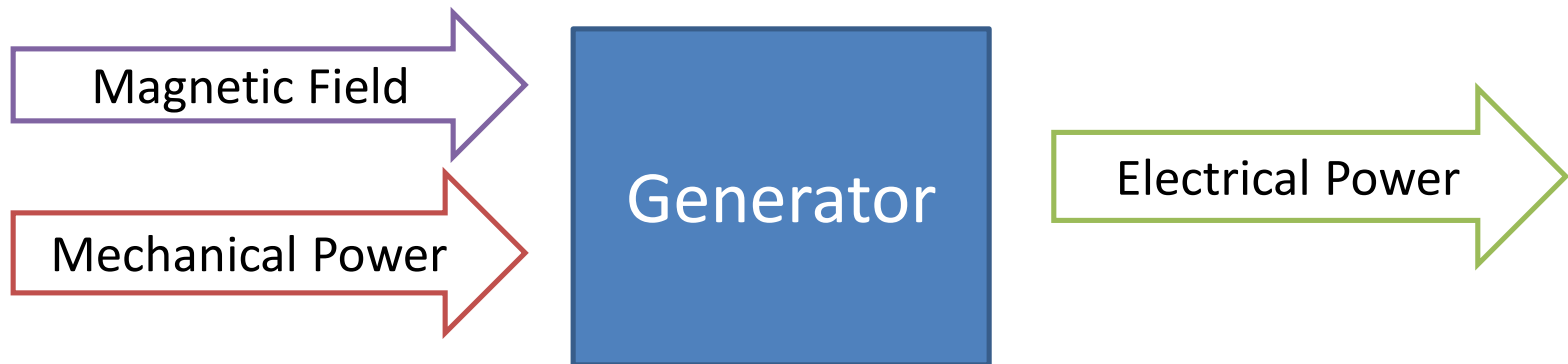
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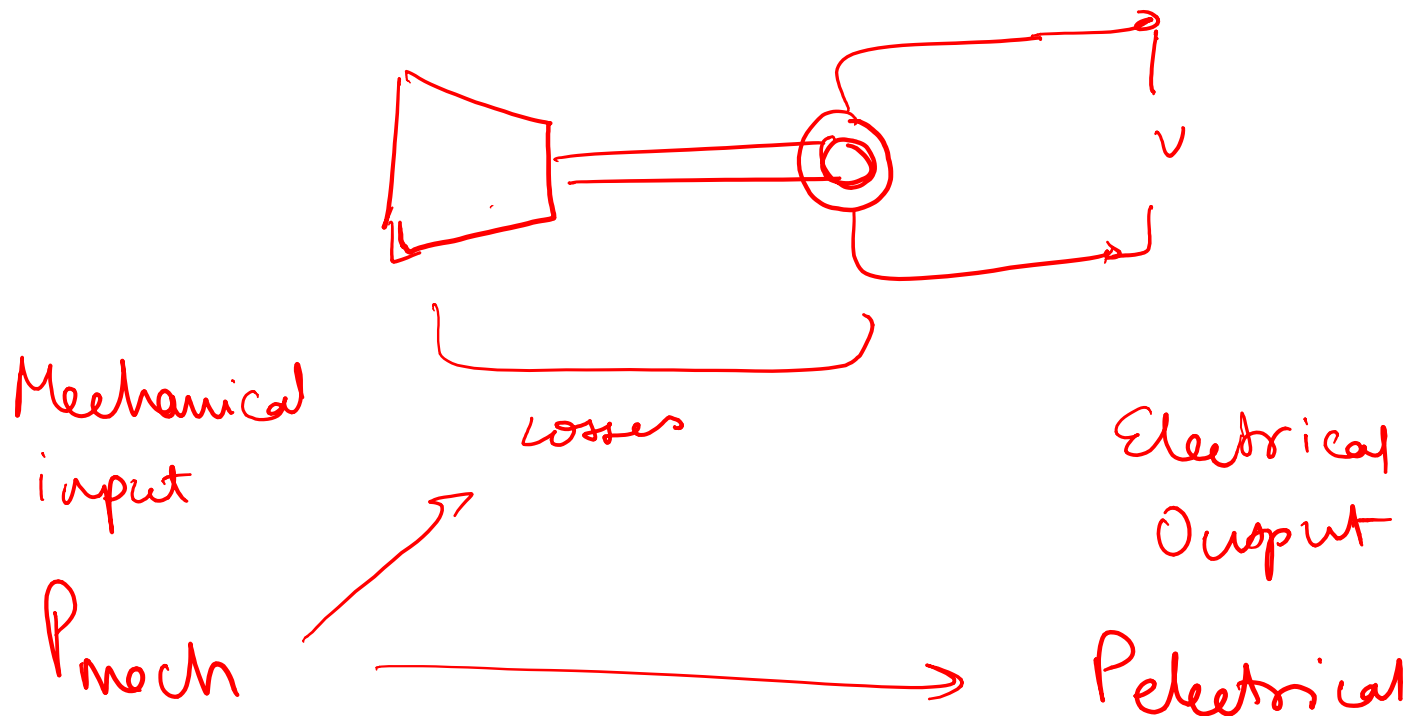
Generator Operation

- In order for a generator to work, we need two inputs:-
 1. Magnetic field at the rotor.
 2. Mechanical power to turn the rotor.



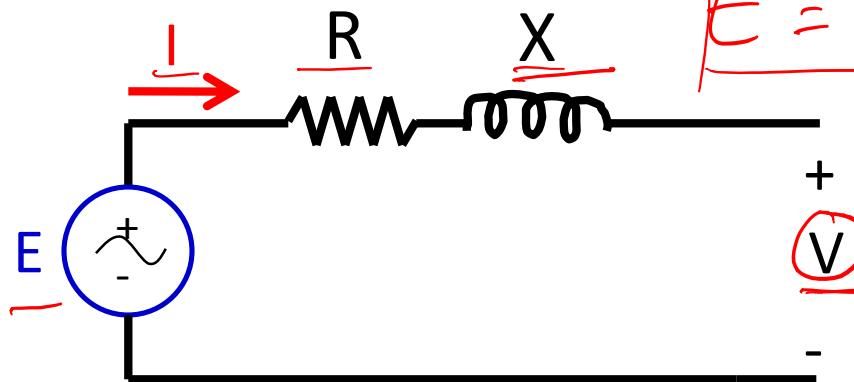
For large generators, we use AC electrical power output and pass it through a rectifier to create DC currents to supply magnetic field circuit.

Generator Operation



An Equivalent Circuit

- R = resistance in the armature winding.
- X = synchronous reactance, representing flux linkage losses with a leakage reactance in the airgap, X_l and the armature reaction, X_a .



$$E = V + I(R + jX)$$

No load operation

Loaded operation

Phasor diagram

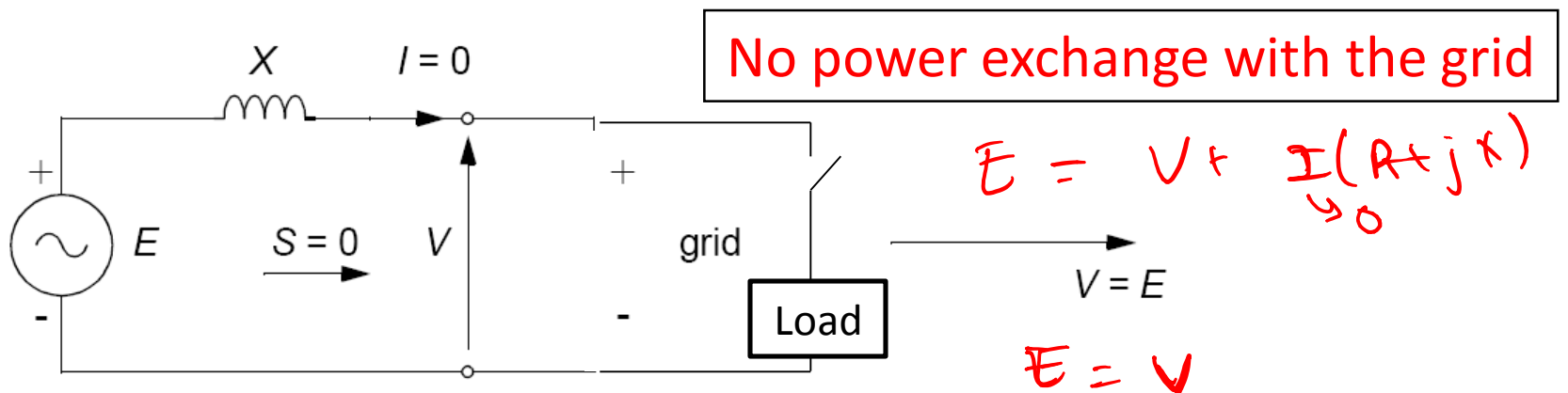
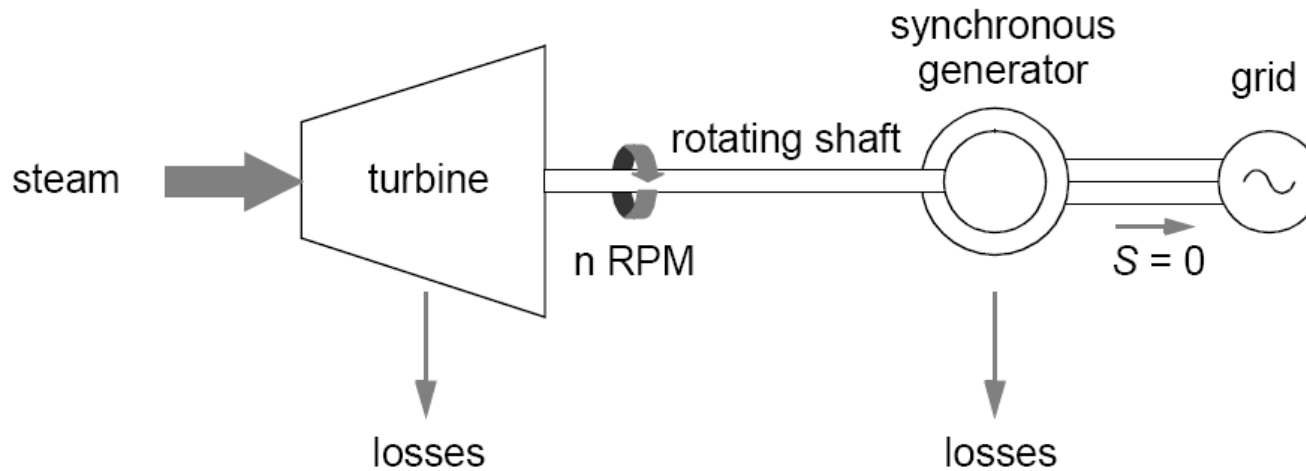
Complex power supplied by a generator

OPERATIONS OF SYNCHRONOUS GENERATOR

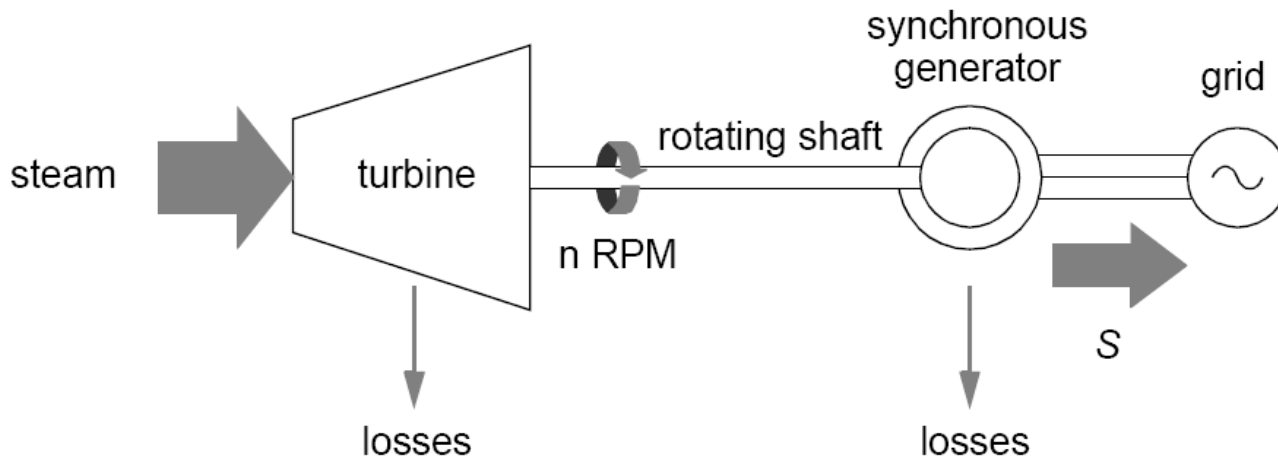
Outline

- Operating consideration of synchronous generators
 - Excitation voltage control
 - Real power control
 - Loading capability

No Load Operation

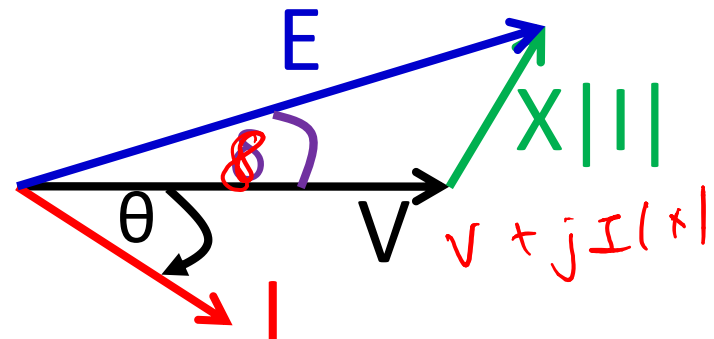
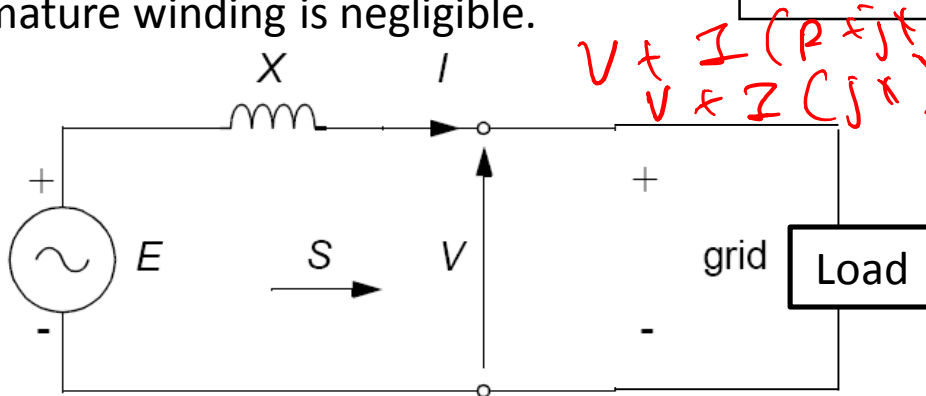


Loaded Operation



Assume that the resistance 'R' in the armature winding is negligible.

Generator injects power into the grid

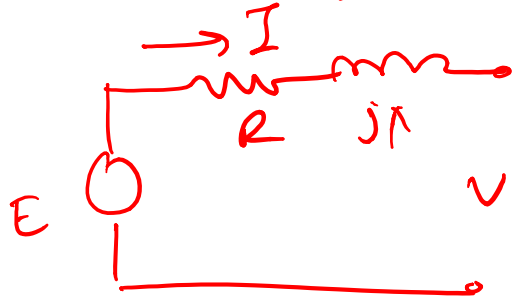


δ is called 'power angle', we'll see why.

Phasor Diagram at Different Operating Conditions

a) lagging load

$$\underline{E} \angle \delta = \underline{V} \angle 0 + \underline{I} |R| \angle -\theta + \underline{I} |X| \angle -\theta + 90^\circ$$

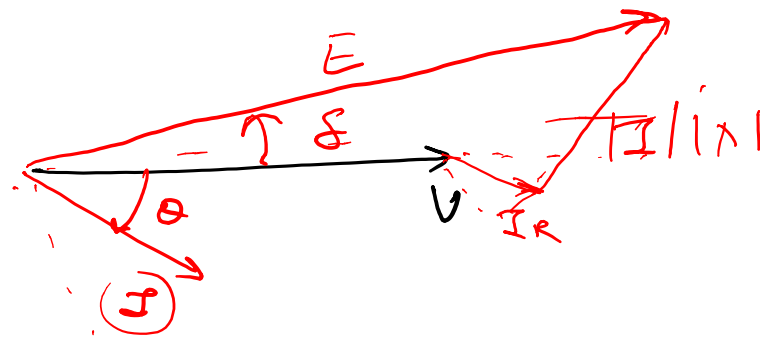


$$I \angle -\theta$$

$$V \angle 0$$

$$R + jX$$

$$\underline{E} \angle \delta = \underline{V} \angle 0 + \underline{I} \angle -\theta (R + jX)$$



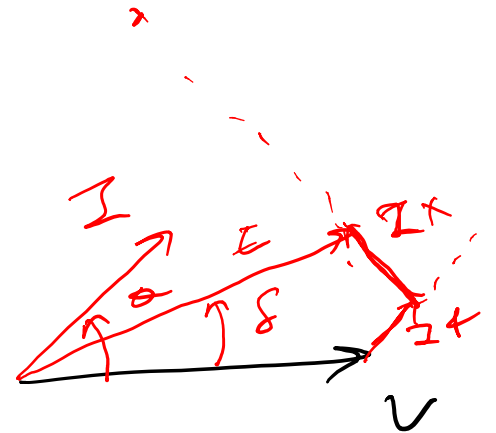
Phasor Diagram at Different Operating Conditions

c) Leading pf

$$I \angle \theta$$

$$E \angle \delta = V \angle 0 + \underline{I \angle \theta} (R + jX)$$

$$= V \angle 0 + |I||R| \angle \theta + |I||X| \angle \theta + 90$$



Phasor Diagram at Different Operating Conditions

b) Unity power factor

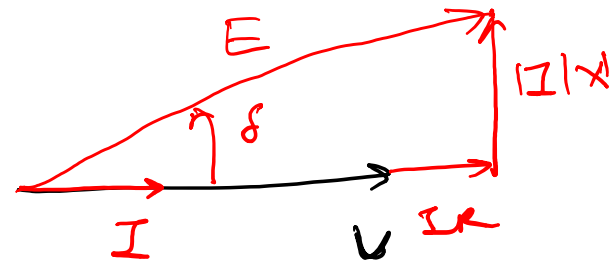
$$I \angle 0^\circ$$

$$V \angle 0$$

$$R + jX$$

$$E \angle \delta = V \angle 0 + I \angle 0 (R + jX)$$

$$= V \angle 0 + |I||R| \angle 0 + |I||X| \angle 90$$



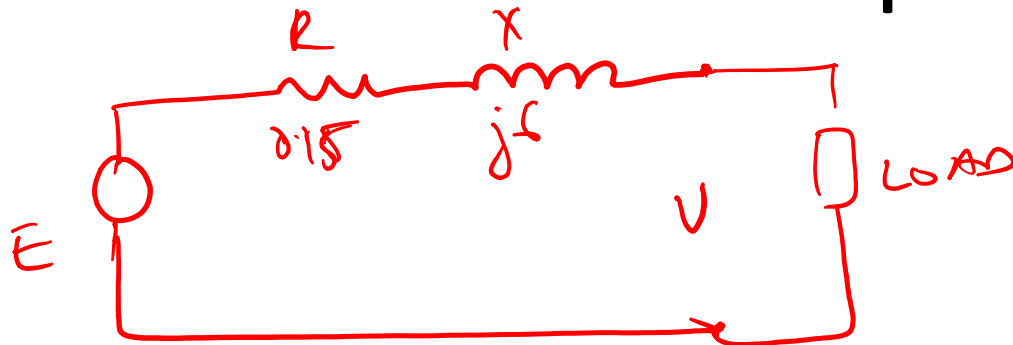
if we have load
 $\delta \rightarrow$ positive
 $\delta = 0 \Rightarrow$ no power

Example 1

- Given an equivalent circuit of three-phase wye-connected synchronous generator with a terminal voltage of 600 V per phase, stator reactance of 6Ω per phase and an armature resistance of 0.15Ω per phase. If the machine is connected to a resistive load that draws 20 A, find the internal EMF and draw a Phasor diagram.

$E ?$

Example 1



$$I = 20 \angle 0^\circ \text{ A} \\ (\text{Resistive})$$

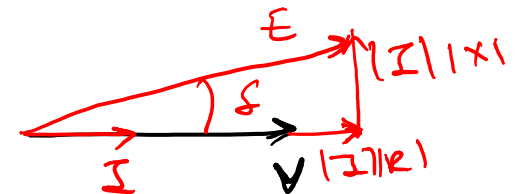
$$V = 600 \angle 0^\circ \text{ V}$$

$$E = V + I(R + jX)$$

$$= 600 \angle 0^\circ + 20 \angle 0^\circ (0.15 + j6)$$

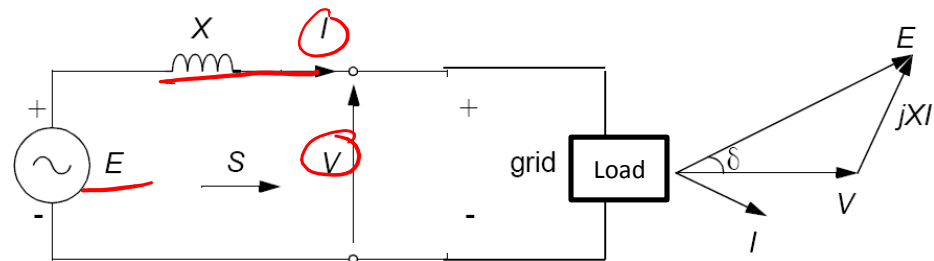
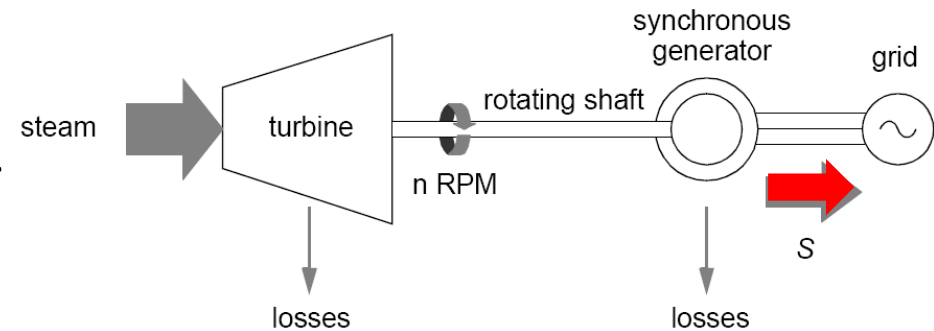
$$E = 614.82 \angle 11.26^\circ \text{ V}$$

$$\delta = \text{power angle} = 11.26^\circ$$



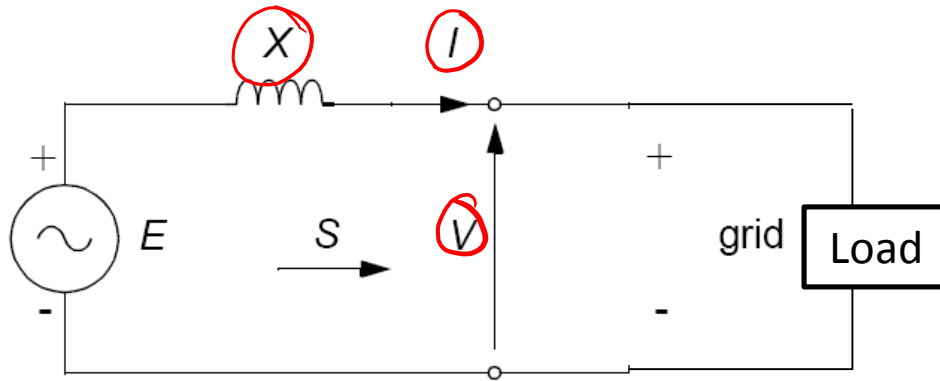
Complex Power Supplied

- Complex power 'S' supplied by a generator can be calculated.
- Using the equivalent circuit to find complex power in terms of excitation voltage 'E', terminal (grid) voltage 'V', and synchronous reactance 'X'.
- Typically, the resistance 'R' in the armature winding is negligible when compared to a synchronous reactance. **In this analysis, we omit the resistance.**



$$S = \sqrt{3} V_L I_L^{\circ}$$

Complex Power Supplied Per Phase



$$S = \underline{V} \underline{I}^*$$

$$\underline{E} = \underline{V} + \underline{I}(R + jX)$$

$$= \underline{V} + j\underline{I}X$$

$$\underline{I} = \frac{\underline{E} \angle \delta - \underline{V} \angle 0}{jX}$$

$$\underline{I}^* = \frac{\underline{E} \angle -\delta - \underline{V} \angle 0}{-jX}$$

Complex Power Supplied Per Phase

$$S = V I^*$$

$$= V I_0 \cdot \left(\frac{E \angle -\delta - V \angle 0}{-jX} \right)$$

$$= \frac{|V||E| \angle -\delta}{-jX} + \frac{|V|^2}{jX}$$

$$= \frac{|V||E| \cos(-\delta) + j|V||E| \sin(-\delta)}{-jX} + \frac{|V|^2}{jX}$$

$$= \frac{|V||E| \cos \delta}{-jX} + \frac{|V||E| \sin \delta}{|X|} + \frac{|V|^2}{jX}$$

$$S = \frac{|V||E| \sin \delta}{X}$$

$$+ j \left(\frac{|V||E| \cos \delta}{X} - \frac{|V|^2}{X} \right)$$

Three-Phase Complex Power Supplied

- We have, $S_{3\phi} = 3 S_{1\phi}$

$$= 3 \frac{|V||E|}{|X|} \sin \delta + j \left(3 \frac{|V||E|}{|X|} \cos \delta - \frac{3|V|^2}{X} \right)$$

$$P_{3\phi} = \frac{3|V||E|^2}{|X|} \sin(\delta)$$

$$Q_{3\phi} = \frac{3|V||E|}{|X|} \cos \delta - \frac{3|V|^2}{|X|}$$

$$P_{3\phi} = \frac{3|V||E|}{X} \sin \delta$$

$$Q_{3\phi} = \frac{3|V||E|}{X} \cos \delta - \frac{|V|^2}{X}$$

Steady state operation of a generator

Real power output

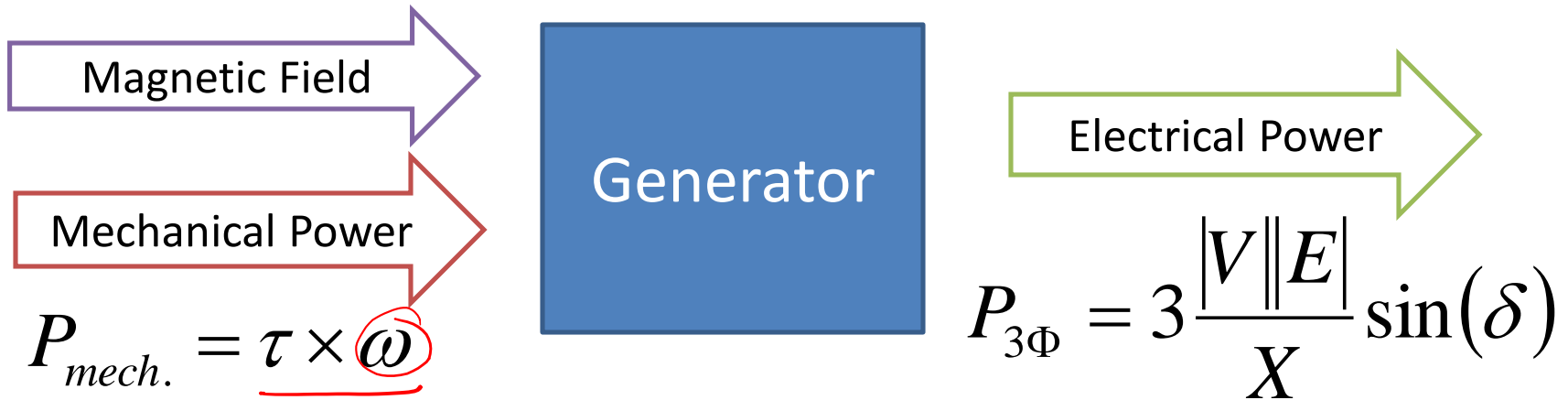
Power angle

Maximum power transfer

Reactive power output

REAL AND REACTIVE POWER OUTPUT

Steady State Operation



- When electrical load is increased, we need to increase mechanical power input.
- The speed of the rotor (ω) needs to be constant because **rotor speed determines the voltage frequency and the frequency needs to be kept constant.**
- We can only increase the **mechanical ‘Torque’** to supply additional electrical load while maintaining the same speed.

Real Power Output

- From
$$P_{3\Phi} = 3 \frac{|V||E|}{X} \sin(\delta)$$
- $|V|$ and X are constant values.
- $|E|$ depends on the magnitude of magnetic field at the rotor.
- When the magnetic field is kept constant and mechanical power input is increased, the electrical power output will be increased.
- Since $|V|$, $|E|$, and X are kept constant, **power angle** will be increased.

Power Angle

- From
$$P_{3\Phi} = 3 \frac{|V||E|}{X} \sin(\delta)$$
- Consider three cases:

Power angle	Real power output	Operation mode
$\delta > 0$	$P > 0$	Supply power as generator
$\delta = 0$	$P = 0$	No power exchange
$\delta < 0$	$P < 0$	Absorb power as a motor

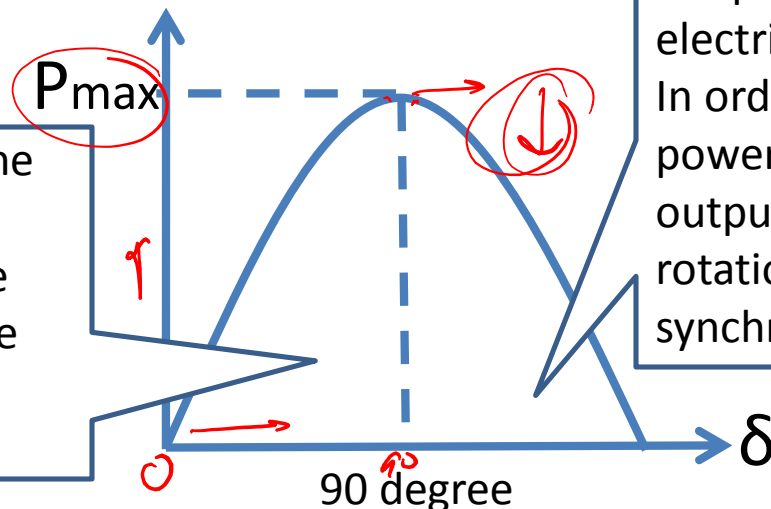
This is why δ is called 'power angle'!

Maximum Power Transfer

- In theory, the power angle $\delta \leq 90$ degree.
- This limitation is called “Steady-state stability limit”.
- Above 90 degree, generator will lose synchronism.
- The maximum power transfer is the real power output when the power angle is 90 degrees.

$$P_{3\phi} = 3 \frac{|V||E|}{X} \sin(\delta) = A \sin \delta$$

Under this region, when the mechanical power input increases, the power angle increase and as a result the electrical power output increases



Under this region, when the mechanical power input increases, the power angle increase BUT the electrical power output decreases. In order to balance mechanical power input to electrical power output, the machine will adjust its rotational speed, hence out of synchronism.

$$P_{max} = \frac{3}{2} \omega = \frac{3}{2} V I$$

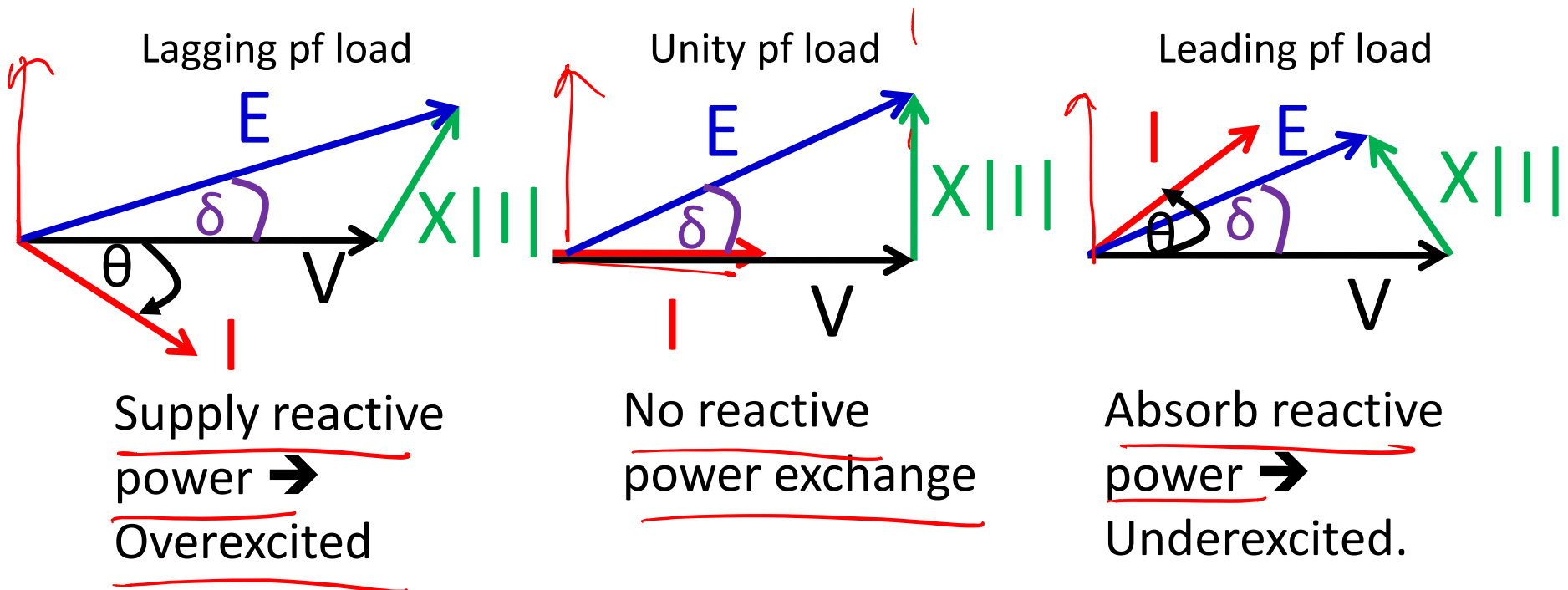
Reactive Power Output

- From $Q_{3\Phi} = 3 \frac{|V||E|}{X} \cos(\delta) - 3 \frac{|V|^2}{X} = 3 \frac{|V|}{X} \{ |E| \cos(\delta) - |V| \}$
- Reactive power control is done by **adjusting $|E|$** . (Although the power angle also affects the reactive power output, internal voltage magnitude dominates the final output.)
- Consider three cases,

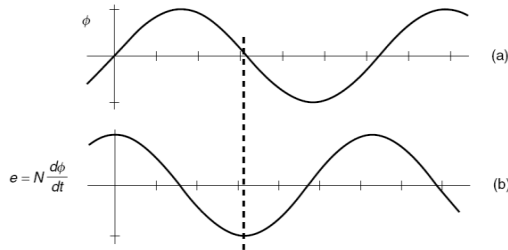
Cases	Reactive power output	Operation mode
$ E \cos \delta > V $	$Q > 0$	Supply reactive power. This mode is called 'Overexcited'.
$ E \cos \delta = V $	$Q = 0$	No reactive power exchange
$ E \cos \delta < V $	$Q < 0$	Absorb reactive power. This mode is called 'Underexcited'.

Reactive Power Exchange

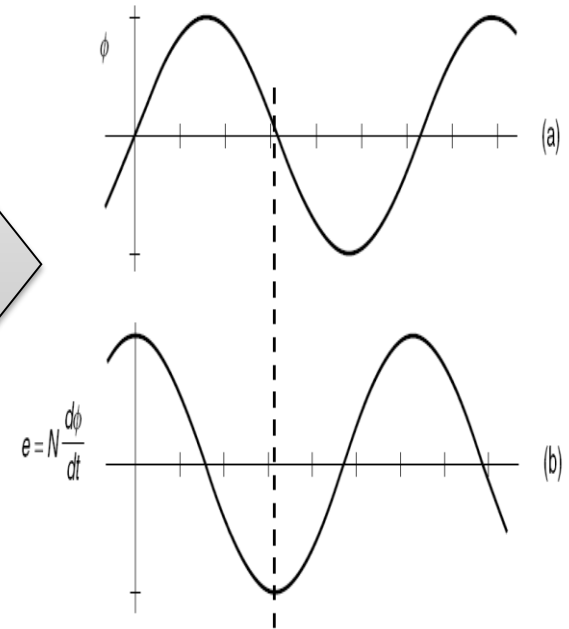
- We can vary the magnitude of excitation voltage to either supply or absorb variable amount of reactive power.



Adjusting Excitation Voltage



Magnetic field will still rotate at the same frequency with higher magnitude.



The magnetic field can be intensified with higher field current magnitude. As a result, excitation voltage of a generator will be increased when we increase the magnitude of magnetic field.

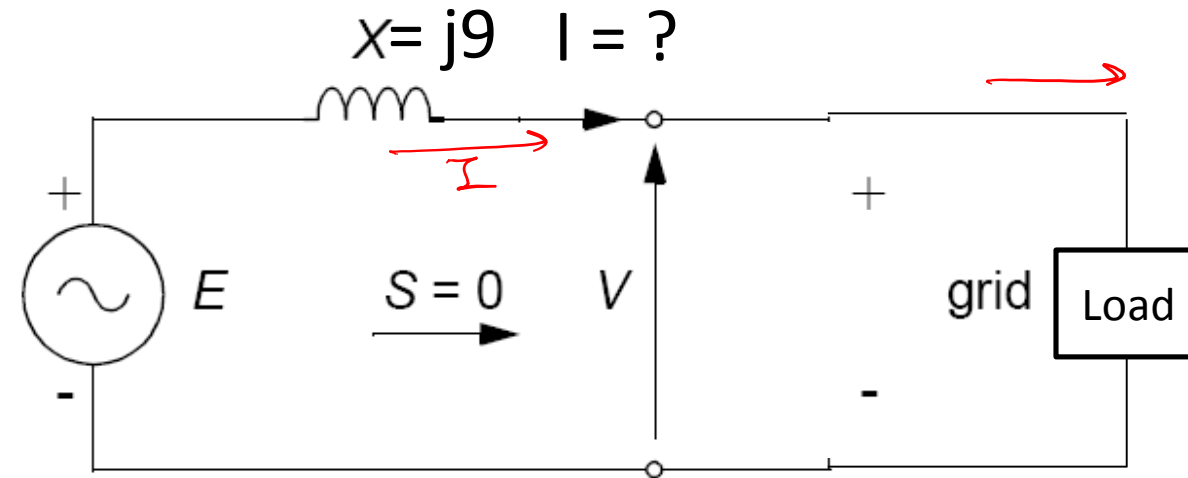
Example 2

This voltage is line-to-line voltage.

- A 50-MVA, 30kV, three-phase, wye-connected 60 Hz synchronous generator has a synchronous reactance of $9\ \Omega$ per phase and a negligible resistance. The generator is supplying to the system a rated power at 0.8 p.f. lagging at the rated terminal voltage.
 - (a) Determine the excitation voltage per phase (E) and the power angle (δ).
 - (b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering 25 MW. Determine the armature current and the new power factor.
 - (c) what is the maximum power transfer by this generator at the current excitation?

|E| same

Example 2(a) Excitation Voltage



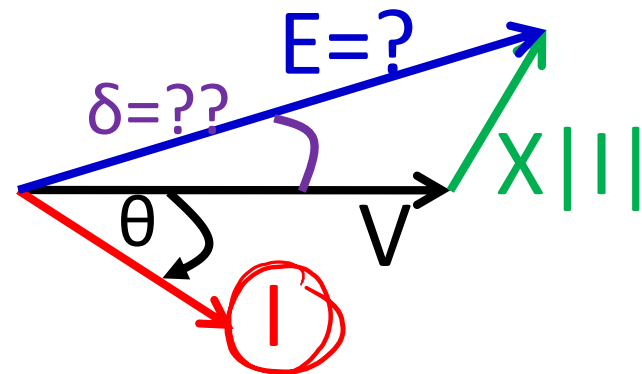
Line-to-line voltage

$$V = \frac{30 \times 10^3}{\sqrt{3}} \angle 0^\circ \text{ V}$$

$$E = I(jX) + V$$

➔ Need to find the current !!!

Hint: The generator is delivering rated power to the system at a 0.8 power factor lagging.



Example 2(a) Armature Current

$$S = 50 \text{ MVA} = 3 V_{ph} I_{ph} = \sqrt{3} V_{LL} I_L$$

$$V_{ph} = \frac{30 \text{ kV}}{\sqrt{3}} \Rightarrow 3 \times 17.32 \times 10^3 \times I_{ph} = 50 \times 10^6$$

$$= 17.32 \text{ kV}$$

$$\Rightarrow I = \frac{50 \times 10^6}{3 \times 17.32 \times 10^3} = 962.25 \text{ A}$$

$$\angle I = \cos^{-1}(0.8)$$

$$= 36.87$$

$$\Rightarrow \boxed{I = 962.25 \angle -36.87^\circ \text{ A}}$$

Example 2(a) Excitation Voltage

$$E \angle \delta = V + I (jX)$$

$$= 17.32 \times 10^3 \angle 0^\circ + 962.25 \angle -36.87^\circ j 9$$

$$= 23558.43 \angle 17.1^\circ$$

$$E = 23558.43 \text{ V}$$

$$\delta = \text{power angle} = 17.1^\circ$$

Example 2(b) New Power Angle

$$|E| = 23558.43 \text{ V}$$

$$|V| = 17.32 \times 10^3 \text{ V}$$

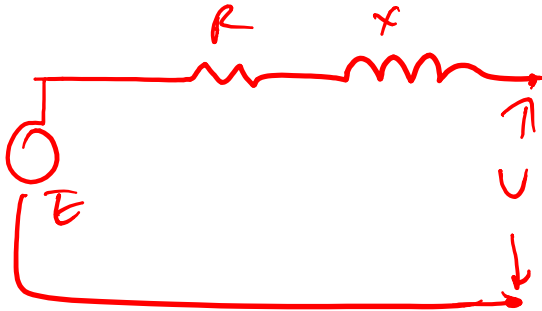
$$X = j9$$

$$P = 25 \text{ MW} = \frac{3|V||E|}{X} \sin \delta'$$

$$\sin \delta = \frac{P \cdot X}{3|V||E|} \Rightarrow \delta = \sin^{-1} \left(\frac{P \cdot X}{3|V||E|} \right)$$

$$= \sin^{-1} \left(\frac{25 \times 10^6 \times 9}{3 \times 17.32 \times 10^3 \times 23558.43} \right) = 10.6^\circ$$

Example 2(b) Armature Current



$$I = \frac{E - V}{jX}$$

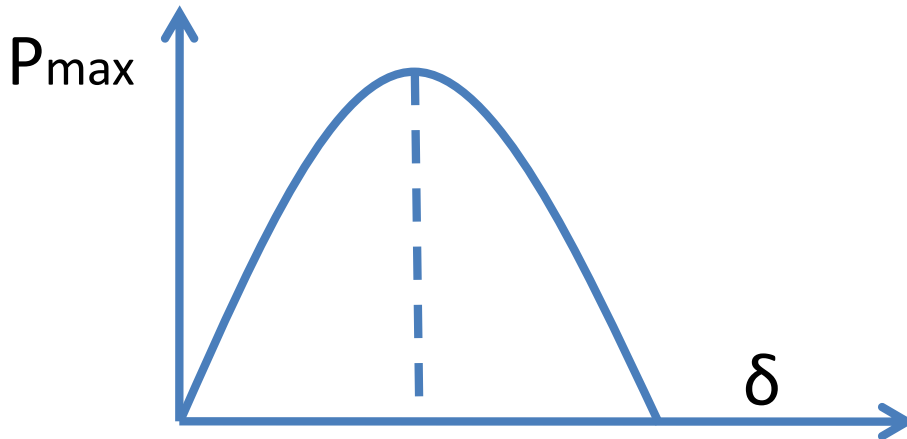
$$= \frac{E \angle \delta' - V \angle 0}{jX}$$

$$= \frac{23558.43 \angle 10.6 - 17.32 \times 10^3}{92 \angle 90^\circ}$$

$$= \underline{207 \angle -53.4}$$

$$\text{Power factor} = \cos(53.4) = 0.596 \text{ lagging.}$$

Example 2(c) Maximum Power Transfer



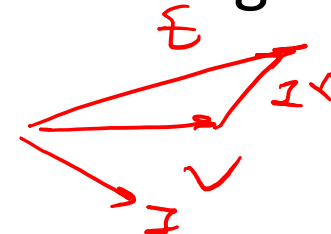
$$\begin{aligned}
 P_{3\phi \text{ max}} &= \frac{31011 \text{ E} (1)}{(\times 1)} \\
 &= \frac{3 \times 17.32 \times 10^3 \times 23558.43}{9} \\
 &= \underline{136 \text{ MW}}
 \end{aligned}$$

Example 2: Points to Note

- From (b), when we adjust *only* mechanical power δ input, we see that the current magnitude and power factor has changed.

$$\underline{I = 962.25 \angle -36.87^\circ} \Rightarrow \underline{I = 807 \angle -53.4^\circ}$$

- If we want to keep the power factor constant, we need to adjust the excitation voltage magnitude too!
- We will illustrate this effect using Phasor diagram in the next section.



Summary

→ $E \angle \delta = V \angle 0 + \underline{I (R + jX)}$

↗ Lagging
↘ Unity
 Leading.

→ Complex power $P_{3\phi} = \frac{3|V||E|}{|X|} \sin \delta$

$$Q_{3\phi} = \frac{3|V||E|}{|X|} \cos \delta - \frac{3|V|^2}{X}$$

→ $\delta \rightarrow$ Power angle

→ Real power & reactive power

↗ over excited
↘ under excited.

$|E|$ or δ

$$\frac{f(E)(V)X}{\cos \theta}$$

→ $I = \frac{E \angle \delta - V \angle 0}{jX}$