EE2022
Electrical Energy Systems

Lecture 3: Quiz on AC Systems Fundamentals

National University of Singapore

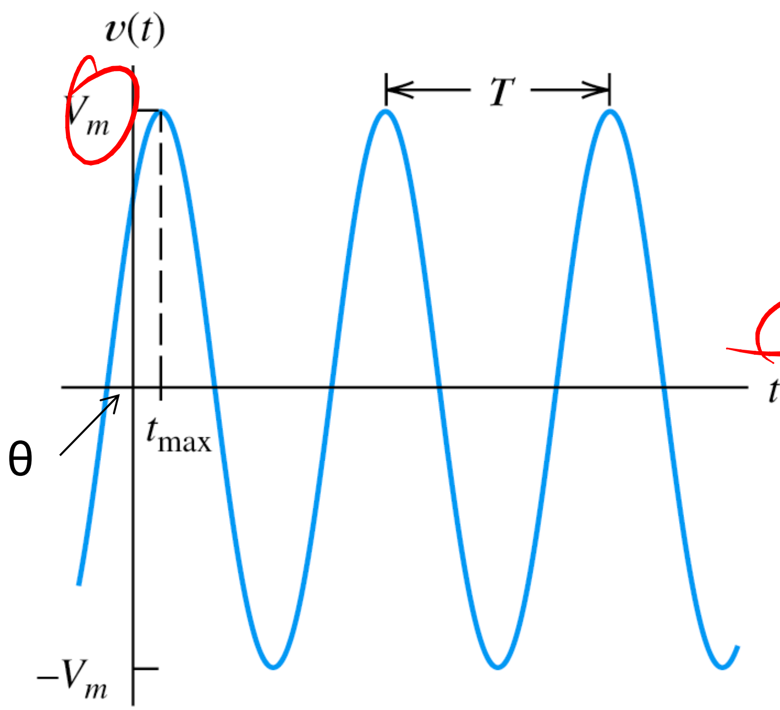
Revision:

1) Why 50/60HZ Sinusoidal AC is used in most power systems?
2) All about sinuosidals: Peak, RMS, Average, frequency, Angular frequency, Phase Angle, Phase difference.
3) Importance of RMS value in Power Calculation.
4) Phasor: What is it? And How does this help?
5) What is impedance? Formulas.
6) How Phasor and Impedance are used in AC analysis.
7) Feeling at home with complex number calculations!

0

230V RMS

Sinusoidal Currents And Voltages



V_m is the peak value

ω is the angular frequency in radians/sec

$2\pi \times f$ $\quad f = \frac{1}{T}$

θ is the phase angle

T is the period

phase angle

A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$.
Note: Assuming that θ is in degrees, we have $t_{\max} = \frac{-\theta}{360} \times T$.
For the waveform shown, θ is -45°.

Root-Mean-Square Values

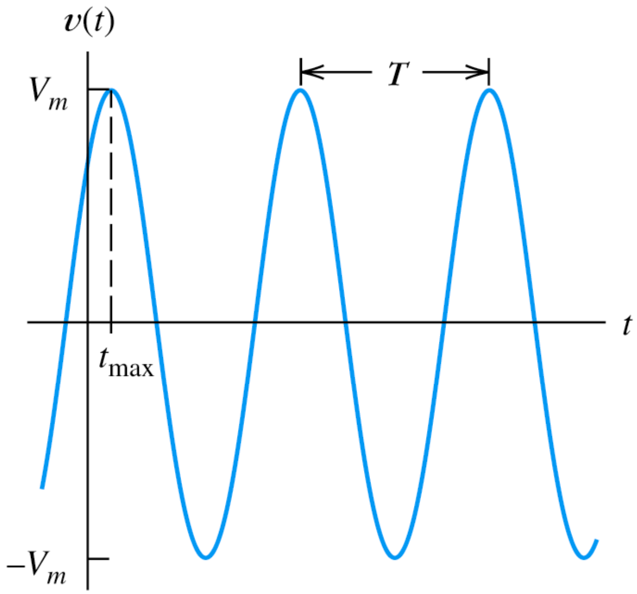


Figure 5.1 A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$.
Note: Assuming that θ is in degrees, we have $t_{\max} = \frac{-\theta}{360} \times T$.
For the waveform shown, θ is -45°.

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \frac{V_m}{\sqrt{2}}$$

RMS value in Power Calculation

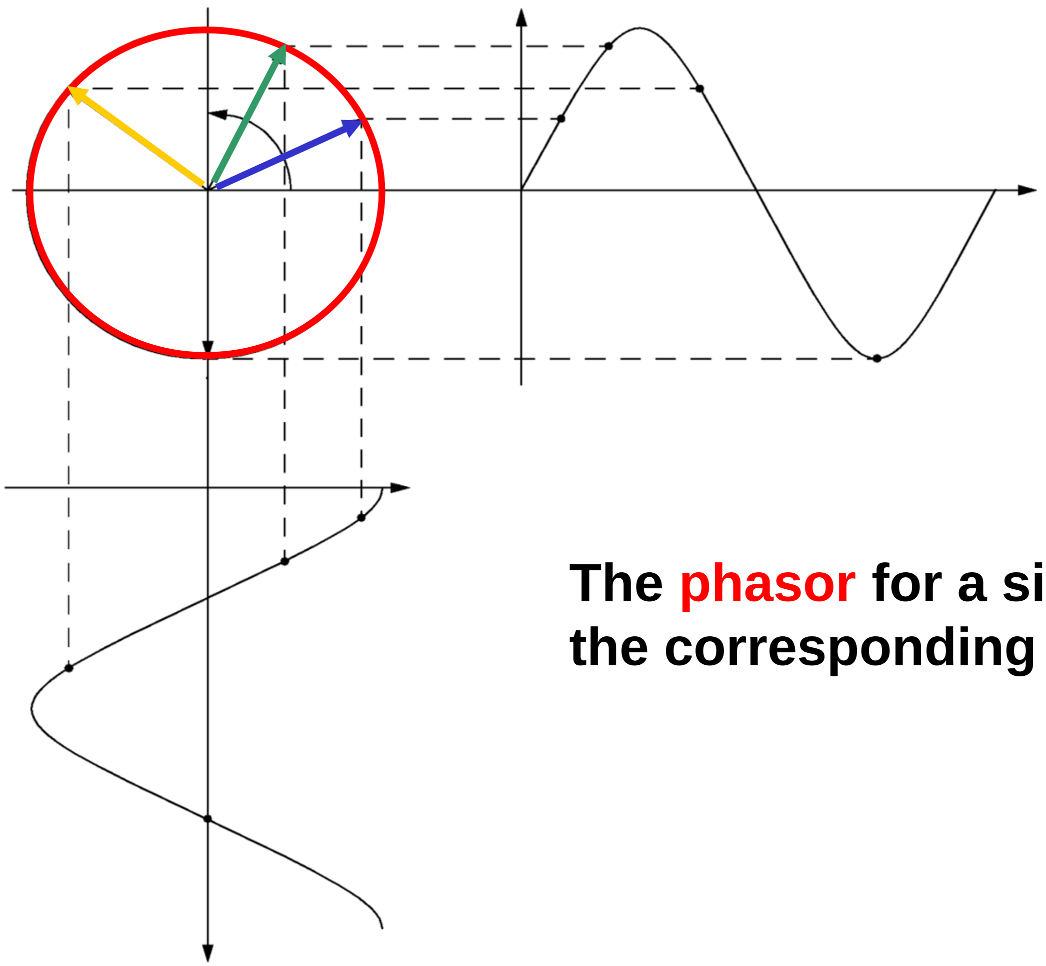
It is desirable to have same form of equation for power in both a.c. and d.c. circuits mainly because of

- Convenience
- Consistence

For d.c. circuits, we have $V_{rms} = V_{dc}$ and $I_{rms} = I_{dc}$. So, for both a.c. and d.c., the power in a resistor is

$$P = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

What is Phasor?



The **phasor** for a sinusoid is a snapshot of the corresponding rotating vector at $t = 0$.

Example 1: Converting sinusoidal time-domain expression to phasor form

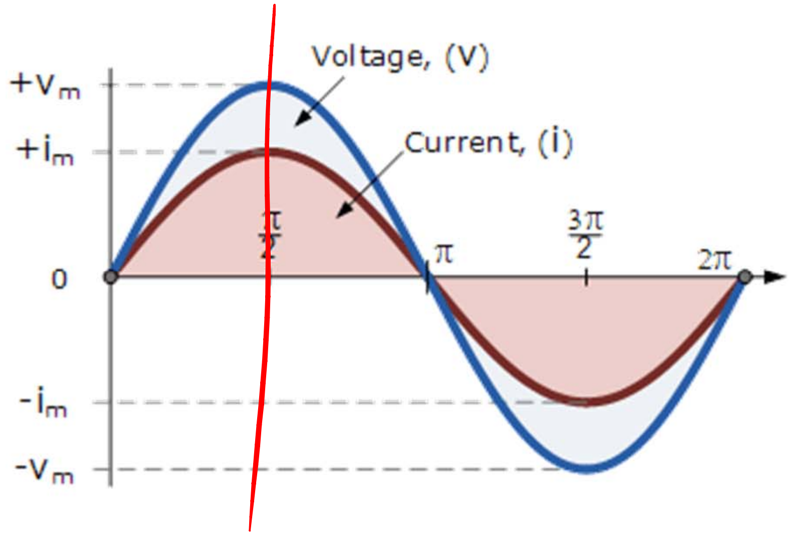
$$v(t) = \sqrt{2}.20\cos(\omega t - 45)$$

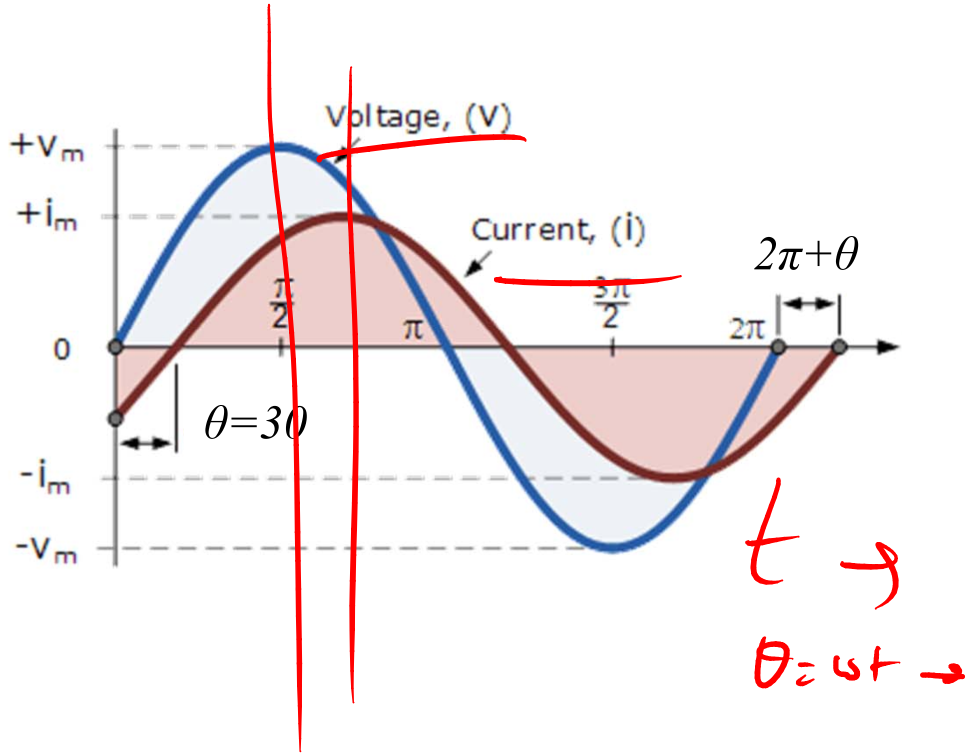
$$Vrms = Vm/\sqrt{2} = 20$$

$$\mathbf{V}_1 = 20\angle -45^\circ$$

Phase Relationships

Sometimes when we are analysing alternating waveforms we may need to know the position of the phasor, representing the alternating quantity at some particular instant in time especially when we want to compare two different waveforms on the same axis



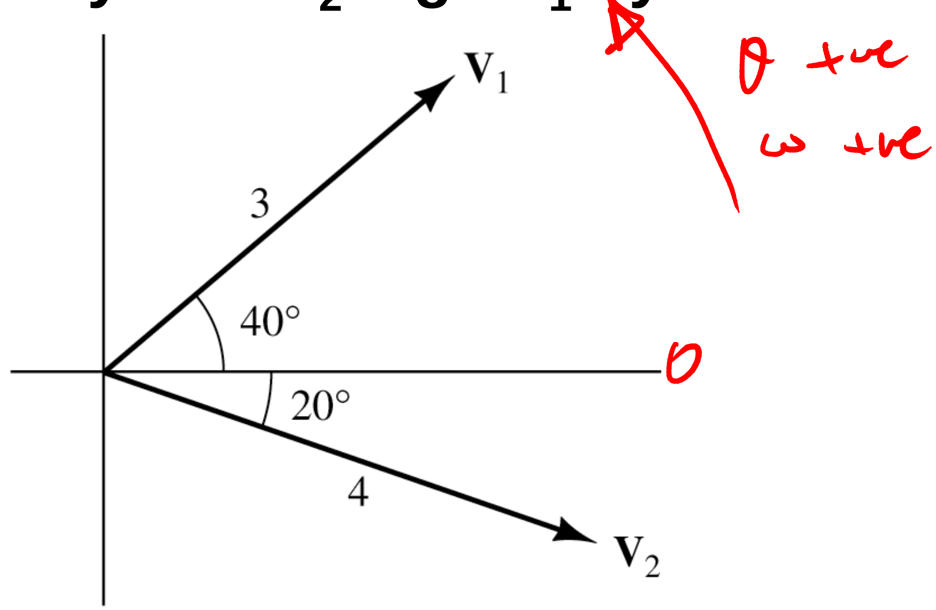


Phase Relationships

To determine phase relationships from a phasor diagram, consider the phasors to rotate counterclockwise.

Then when standing at a fixed point, if V_1 arrives first followed by V_2 after a rotation of θ , we say that V_1 leads V_2 by θ .

Alternatively, we could say that V_2 lags V_1 by θ .



Because the vectors rotate counterclockwise, $\mathbf{v}_1$ leads $\mathbf{v}_2$ by 60° (or, equivalently, $\mathbf{v}_2$ lags $\mathbf{v}_1$ by 60°.)

Impedance for Circuit Components R, L, C

$R: \quad v = R \cdot i \rightarrow V = R \cdot I$

$$\boxed{Z = \frac{V}{I}}$$

$L: \quad v = \underline{L} \cdot \left(\frac{di}{dt}\right) \rightarrow V = j\omega L \cdot I$

$\underline{Z_L} = j\omega L$

$C: \quad v = \frac{1}{C} \int i \, dt$

$V = \frac{1}{j\omega C} \cdot I$

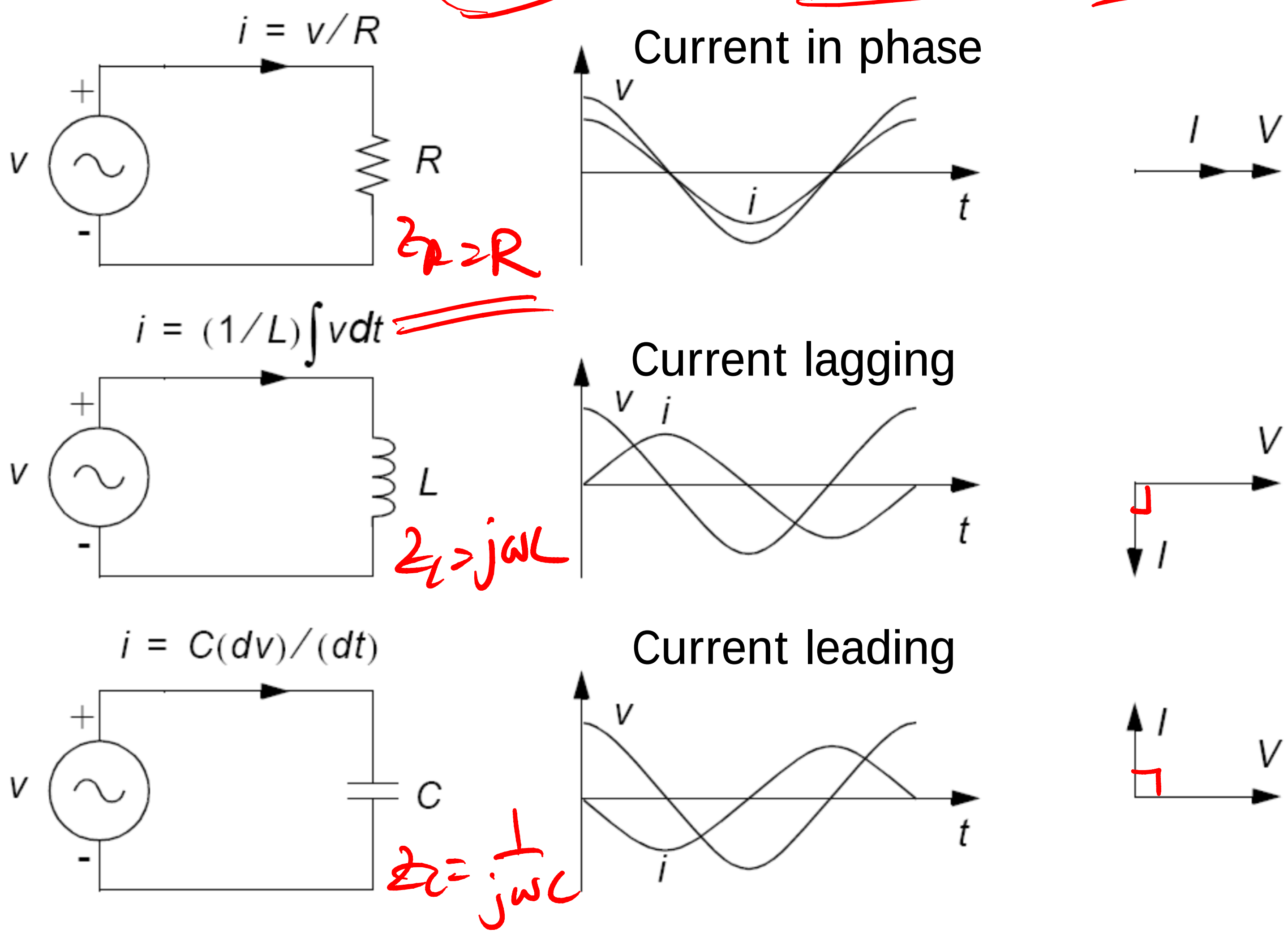
$Z_C = \frac{1}{j\omega C}$

$$\frac{d}{dt}(\sin \omega t) = \underline{(\cos \omega t)} \times \omega$$

$$= \sin(\omega t + 90^\circ) \times \underline{\omega}, \quad j\underline{\omega}$$

$$\int \sin \omega t \cdot dt = \frac{-\cos \omega t}{\omega} = \frac{\sin(\omega t - 90^\circ)}{\omega} : -j\frac{1}{\omega} = \frac{1}{j\omega}$$

Summary: Time ⇔ Phasor domains



Example 3: Using Phasors to Add Sinusoids

$$v_1(t) = \sqrt{2}.20\cos(\omega t - 45) \qquad v_2(t) = \sqrt{2}.10\cos(\omega t - 30)$$

$v_1(t) + v_2(t) = \sqrt{2} \times 20 \cos(\omega t - 45^\circ) + \sqrt{2} \times 10 \cos(\omega t - 30^\circ)$

$V_1 = 20\angle{-45^\circ}$ $\qquad V_2 = 10\angle{-30^\circ}$

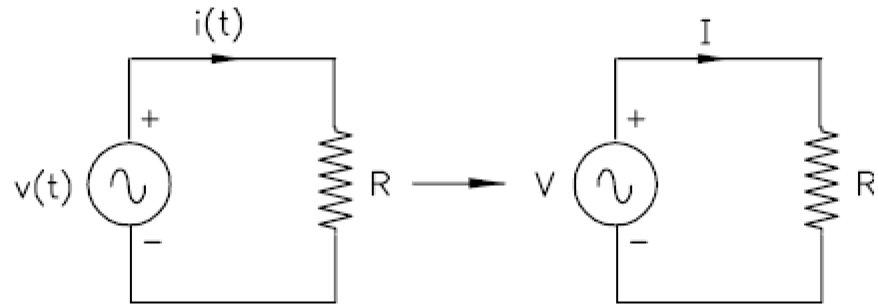
$v_1(t) + v_2(t) =$ $\qquad V_1 + V_2 = 20\angle{-45^\circ} + 10\angle{-30^\circ}$

$= 29.77\angle{-40^\circ}$

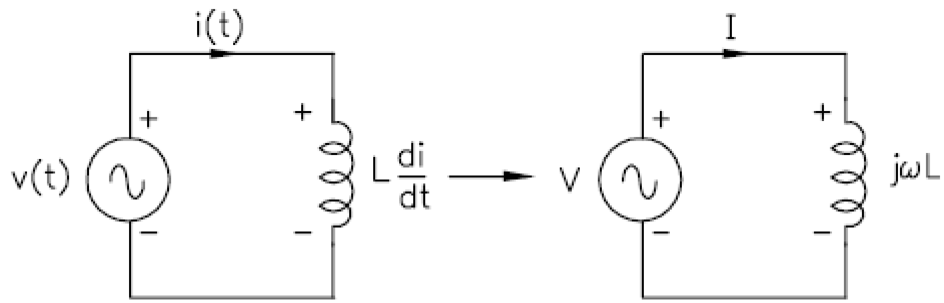
$= 29.77\sqrt{2}\cos(\omega t - 40^\circ)$

Equivalent time domain and phasor representations

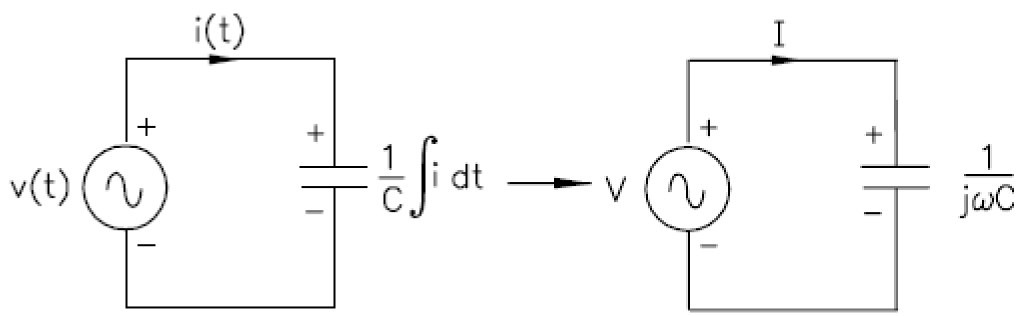
A.C. circuit with a resistance



A.C. circuit with an inductance

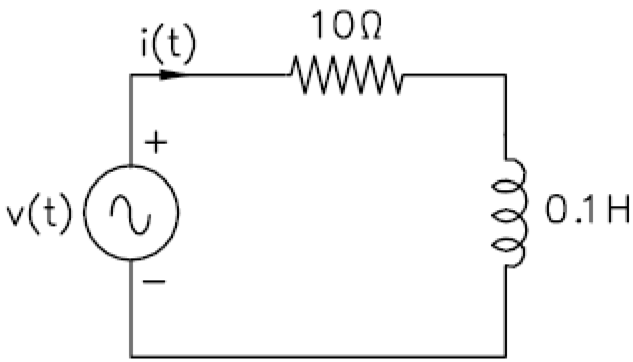


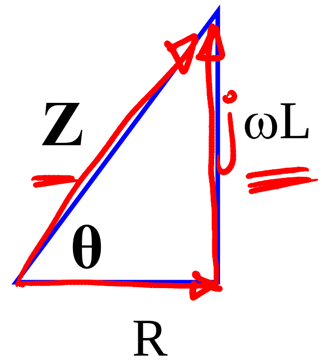
A.C. circuit with a capacitance

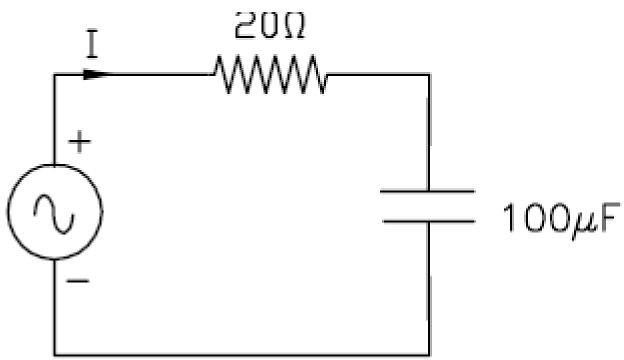


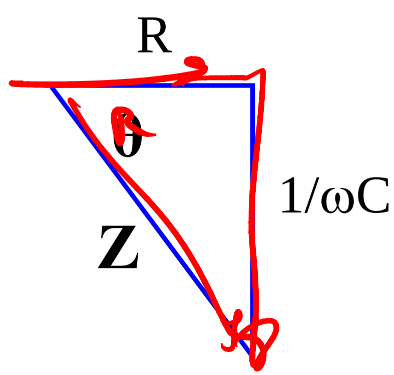
Element	Voltage	Current	Impedance
R	$\|V\|\angle 0^\circ$	$\frac{\|V\|\angle 0^\circ}{R}$	R
L	$\omega L\|I\|\angle 90^\circ$	$\|I\|\angle 0^\circ$	$j\omega L$
C	$\|V\|\angle 0^\circ$	$\omega C\|V\|\angle 90^\circ$	$\frac{1}{j\omega C}$

Complex Impedance









$= -j\frac{1}{\omega C}$

Class Quiz

* write Matric No. and Name

* Use 1 page per question.

$$\text{Q1: } v_1(t) = 5\sqrt{2}\sin(314t + 20^0),\ v_2(t) = 10\sqrt{2}\cos(314t + 40^0)$$

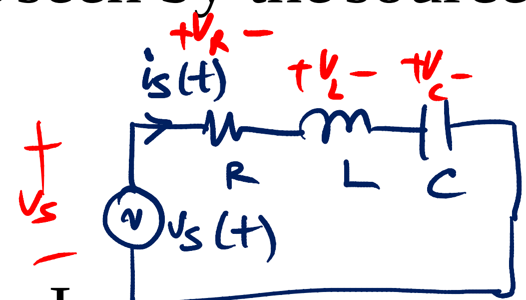
$$v_3(t) = 15\sqrt{2}\cos(314t - 20^0),\ v_4(t) = 5\sqrt{2}\sin(314t + 240^0)$$

Find : (a)Phasors $\mathrm{V}_1, \mathrm{V}_2, \mathrm{V}_3, \mathrm{V}_4$ $(b)\mathrm{V} = \mathrm{V}_1 - \mathrm{V}_2 + \mathrm{V}_3 - \mathrm{V}_4$

$(c)v(t) = v_1(t) - v_2(t) + v_3(t) - v_4(t)$

Q2 : Find : (a) $\mathrm{Z}_\mathrm{R}, Z_L, Z_C$ (b)Overall impedance seen by the source

(c)Current (Phasor) drawn from the source, I_S



(d)Voltage phasors $\mathrm{V}_\mathrm{R}, \mathrm{V}_\mathrm{L}, \mathrm{V}_\mathrm{C}$.

(e)Draw phasor diagram showing $\mathrm{V}_\mathrm{S}, \mathrm{V}_\mathrm{R}, \mathrm{V}_\mathrm{L}, \mathrm{V}_\mathrm{C}, \mathrm{I}_\mathrm{S}$.

V_s = 230V rms, 50 Hz.

$R = 10\Omega, L = 100mH, C = 47\mu F$