NATIONAL UNIVERSITY OF SINGAPORE
Department of Electrical Engineering

EE2022 ELECTRICAL ENERY SYSTEMS
(Solution for Tutorial #6)

5.

Refer to Fig. 1.

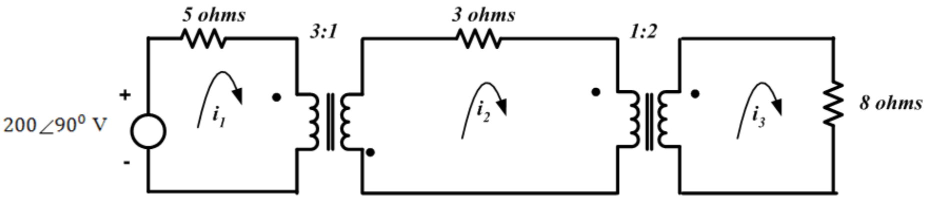
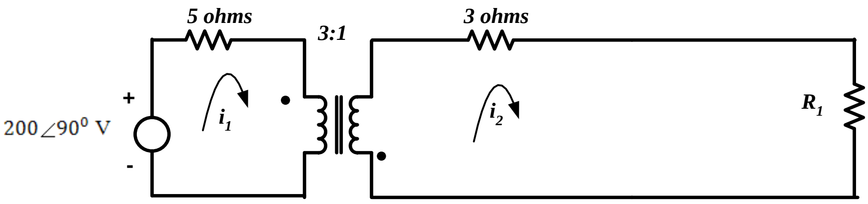
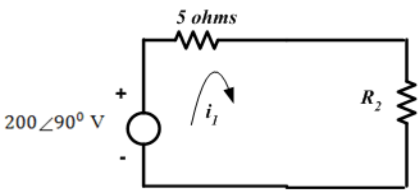


Fig. 1

After reflecting 8Ω to the middle circuit, $R_1 = 8 \times \left(\frac{1}{2}\right)^2 = 2\Omega$.



After reflecting $(3\Omega + R_1)$ which is 5 Ω into the source circuit, $R_2 = 5 \times \left(\frac{3}{1}\right)^2 = 45\Omega$



$$i_1 = \frac{200\angle 90°}{5 + R_2} = \frac{200\angle 90°}{5 + 45} = \frac{200\ \angle 90°}{50} = 4\angle 90°\ A$$

$$|\ i_2| = |\ i_1| \times \left(\frac{3}{1}\right) = 4 \times 3 = 12$$

According to the dot notation, i_1 and i_2 are in the opposite direction. Therefore there is 180^0 phase shift between i_1 and i_2, $\angle\, i_2 = \angle\, i_1 + 180°$

$i_2 = |\ i_2| \angle\, i_2 = 12\angle 270°\ \ A$

According to the dot notation, i_2 and i_3 are in the same direction.

Thus, $i_3 = i_2 \times \left(\frac{1}{2}\right) = 6\angle 270°\ \ A$.

6.

Given the base value of complex power, $S_b = 10\ MVA$. The circuit has three zones separated by two transformers. Given the base voltage at zone B to be 138 kV, we can find the base voltage at zone A and B from transformer voltage ratings.

Base impedance for each zone is found from $Z_b = \frac{V_b^2}{S_b}$, the following table shows the base voltage, base impedance for each zone.

	Zone A	Zone B	Zone C
Base voltage	$\left(\frac{13.8}{138}\right) \times 138 = 13.8\ kV$	$138\ kV$	$\left(\frac{69}{138}\right) \times 138 = 69\ kV$
Base impedance	$\frac{(13.8\ \times 10^3)^2}{10 \times 10^6} = 19.044\ \Omega$	$\frac{(138\ \times 10^3)^2}{10 \times 10^6} = 1904.4\ \Omega$	$\frac{(69 \times 10^3)^2}{10 \times 10^6} = 476.1\ \Omega$

The per unit impedance of the 300 resistive load is found using the base value from

$$Z_{load,p.u.} = \frac{Z_{load}}{Z_{base}}$$

Referred to A,

$$Z_{load,} = 300 \times \left(\frac{138}{69}\right)^2 \times \left(\frac{13.8}{138}\right)^2 = 12 \text{ and } Z_{load,p.u.} = \frac{12}{19.044} = 0.63$$

Referred to B,

$$Z_{load,} = 300 \times \left(\frac{138}{69}\right)^2 = 1200 \text{ and } Z_{load,p.u.} = \frac{1200}{1904.4} = 0.63$$

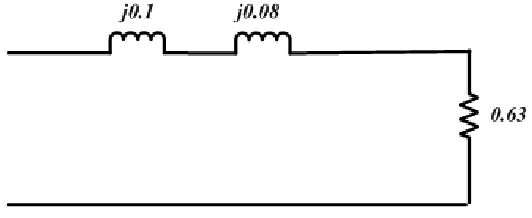
Referred to C,

$$Z_{load,p.u.} = \frac{300}{476.1} = 0.63$$

Note that the per unit reactance of resistive load is the same irrespective of the zone that we refer this resistive load to.

Since the chosen base value for complex power 10000 kVA is the same as the ratings of both transformers and the chosen base voltage values at each zone are the same as voltage ratings of both transformers, we don't need to change the base value of these per unit reactances of both transformers.

Impedance diagram is shown below.



7. Consider Fig. 4.

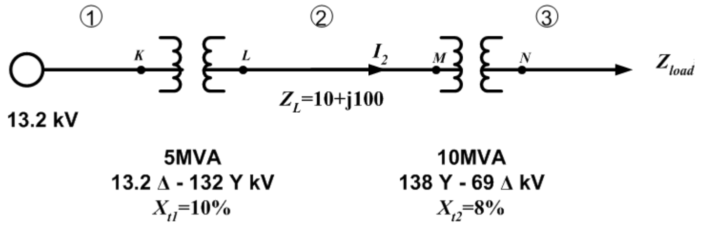


Fig.4

Step 1: Given common 3-phase base VA for entire system, $S_b^{3\Phi} = 10\ MVA$

Step 2: Use a base line-to-line voltage value of 138 kV for the transmission section. Calculate the base voltage in other region of the problem.

Step 3: Find impedance base value for different zone from $Z_b = \frac{V_b^2}{S_b}$, and current base value from $I_b = \frac{S_b^{3\Phi}}{\sqrt{3}V_b^{line-to-line}}$.

Step 4: Calculate the per unit value of impedance from $Z_{p.u.} = \frac{Z}{Z_{base}}$.

The following table shows base values of voltage, current and impedance.

	Zone 1	Zone 2	Zone 3
Base Voltage (line-to-line)	$V_{1,b} = \left(\frac{13.2}{132}\right) \times 138 = 13.8\ kV$	$V_{2,b} = 138\ kV$	$V_{3,b} = \left(\frac{69}{138}\right) \times 138 = 69\ kV$
Base impedance	$Z_{1,b} = \frac{(13.8\ \times 10^3)^2}{10 \times 10^6} = 19.044\ \Omega$	$Z_{2,b} = \frac{(138\ \times 10^3)^2}{10 \times 10^6} = 1904.4\ \Omega$	$Z_{3,b} = \frac{(69 \times 10^3)^2}{10 \times 10^6} = 476.1\ \Omega$
Base current	$I_{1,b} = \frac{10 \times 10^6}{\sqrt{3} \times 13.8 \times 10^3} = 418.4\ \text{A}$	$I_{2,b} = \frac{10 \times 10^6}{\sqrt{3} \times 138 \times 10^3} = 41.84\ \text{A}$	$I_{3,b} = \frac{10 \times 10^6}{\sqrt{3} \times 69 \times 10^3} = 83.67\ \text{A}$

Line impedance per unit value: $Z_{line} = \frac{10+j100}{1904.4} = 5.25 \times 10^{-3} \times (1 + j10) p.u.$

Load impedance per unit value: $Z_{load} = \frac{440}{476.1} = 0.9242\ p.u.$

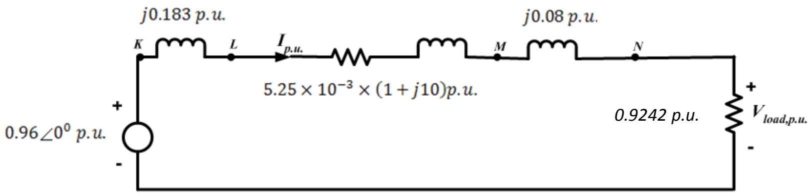
Transformer impedance relative to new base values, $Z_{p.u.,new} = Z_{p.u.,old} \times \left(\frac{V_{b,old}}{V_{b,new}}\right)^2 \times \left(\frac{S_{b,new}}{S_{b,old}}\right)$

$$Z_{tf1} = j0.1 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{10}{5}\right) = j0.183\ p.u.$$

$$Z_{tf2} = j0.08 \times \left(\frac{138}{138}\right)^2 \times \left(\frac{10}{10}\right) = j0.08\ p.u.$$

Source voltage, $|E_{s,p.u.}| = \left(\frac{13.2}{13.8}\right) = 0.9565\ p.u.$ Using E_s as a reference, $E_{s,p.u.} = |E_s|\angle 0° = 0.9565\angle 0°\ p.u.$

Per-unit impedance diagram is given below. Note that we can neglect transformer induced phase shifts. The transmission line current from L to M will have the phase shift by 30 degree but this phase shift will be cancelled at point N.



$$Z_{total,p.u.} = 5.25 \times 10^{-3} \times (1 + j10) + j0.183 + j0.08 + 0.9242 = 0.9815\angle 18.75°\ p.u.$$

$$I_{p.u.} = \frac{E_{s,p.u.}}{Z_{total,p.u.}} = \frac{0.9565\angle 0°}{0.9815\angle 18.75°} = 0.9745\angle -18.75°\ p.u.$$

$$V_{load,p.u.} = I_{p.u.} \times Z_{load,p.u.} = 0.9006\angle -18.75°\ p.u.$$

$$S_{L,p.u.} = V_{load,p.u.} \times I^*_{p.u.} = 0.9006\angle -18.75° \times 0.9745\angle 18.75° = 0.8777\ p.u.$$

$$S_{G,p.u.} = E_{s,p.u.} \times I^*_{p.u.} = 0.9565\angle 0° \times 0.9745\angle 18.75° = 0.9321\ \angle 18.75°\ p.u.$$

The actual power delivered to the load,

$$S_L = S_{L,p.u.} \times S_b = 8.78\ MVA$$

Since the load is resistive, the real power consumed by load is 8.78 MW.

The power supplied by the generator,

$$S_G = S_{G,p.u.} \times S_b = 9.32\angle 18.75°\ MVA$$

$$P_G = 9.32 \times \cos 18.75^0 = 8.83\ MW$$

Efficiency of the overall system,

$$\eta = \frac{P_L}{P_{G_S}} \times 100\% = \frac{8.78}{8.83} \times 100\% = 99.4\%.$$