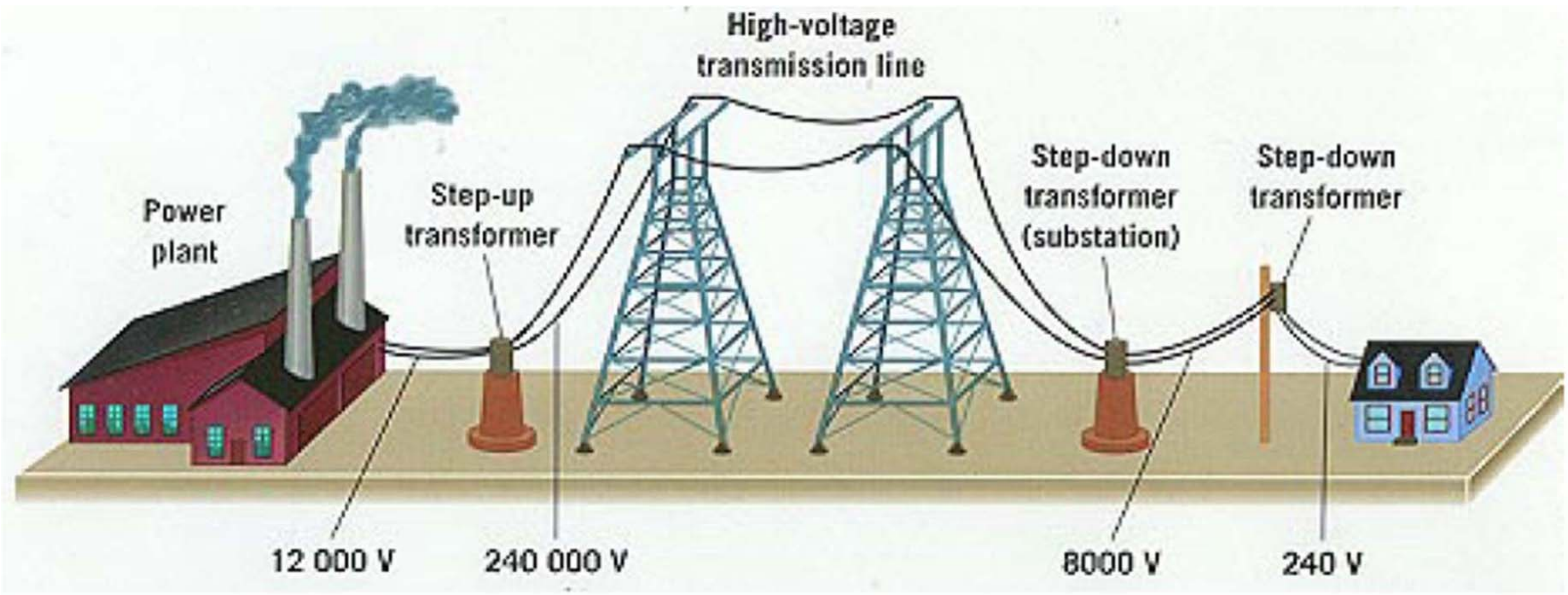
EE2022 Electrical Energy Systems

Transmission Line

Modelling of Power System



Types of Transmission Lines

1. Overhead transmission line
   – Main features of overhead transmission lines
   – Issues with overhead transmission lines
2. Underground cable
   – Types of underground cables
   – Issues with underground cables

➢ Comparison between overhead lines and underground cables.
   – Cost
   – Electrical properties

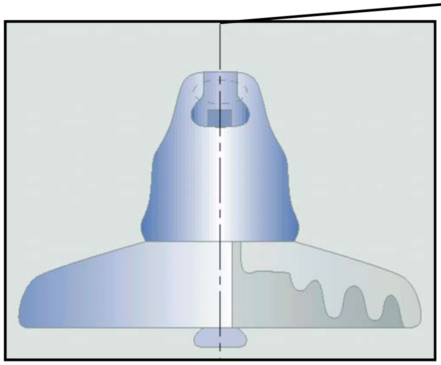
Main features of a transmission line

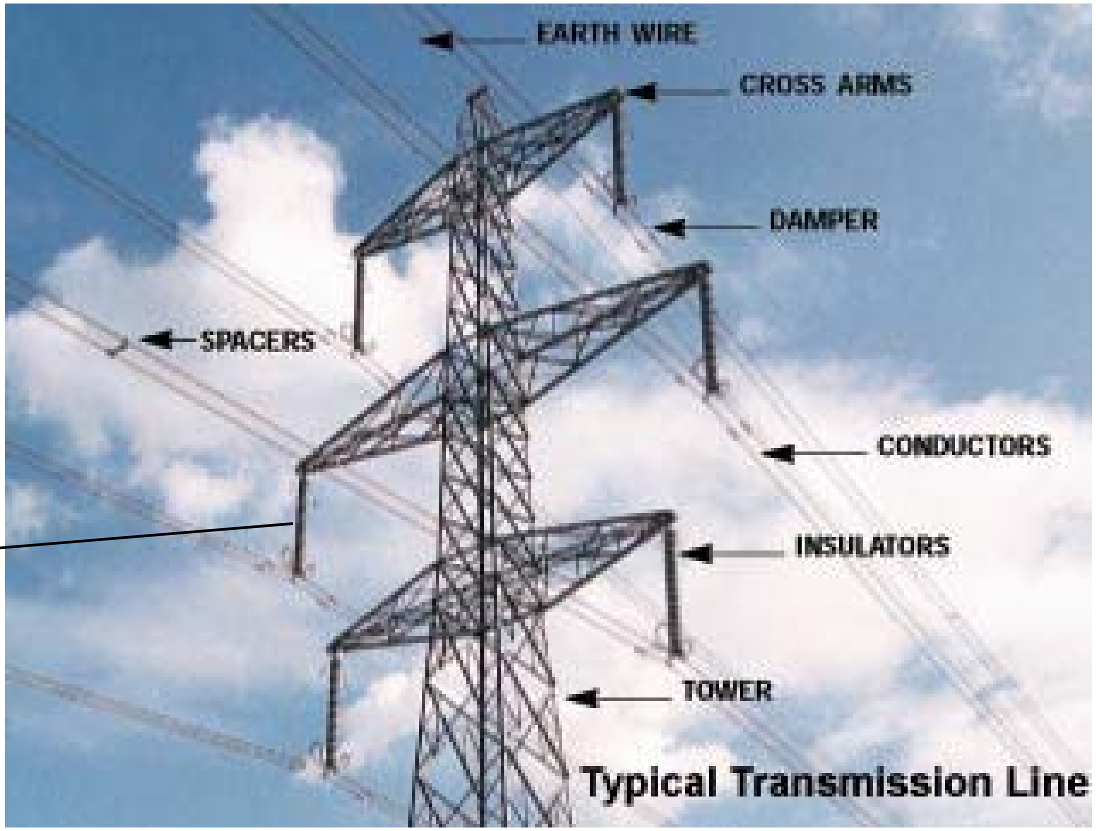
Issues with overhead transmission lines

OVERHEAD TRANSMISSION LINE

Overhead Transmission Line

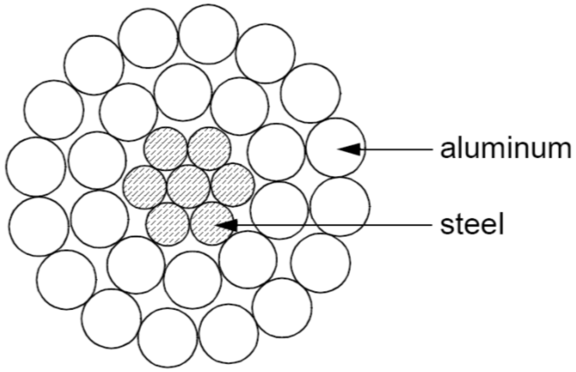
- Support structure
- Conductors
- Insulators
- Shield wires (earth wire)



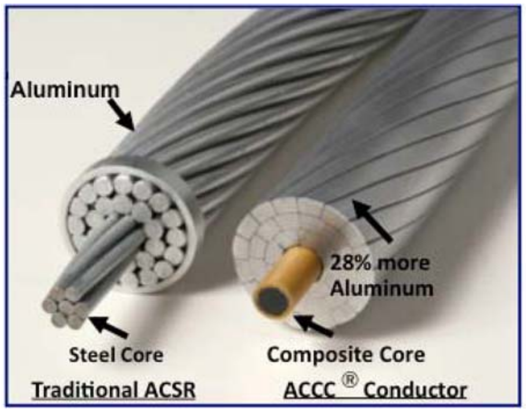


Source: http://www.nationalgrid.com/uk/LandandDevelopment/DDC/devnearohl_final/appendix2/

High Voltage Conductors



Aluminum Conductor Steel Reinforced (ACSR)



Source: http://www.faqs.org/sec-filings/091214/COMPOSITE-TECHNOLOGY-CORP_10-K/

Insulators

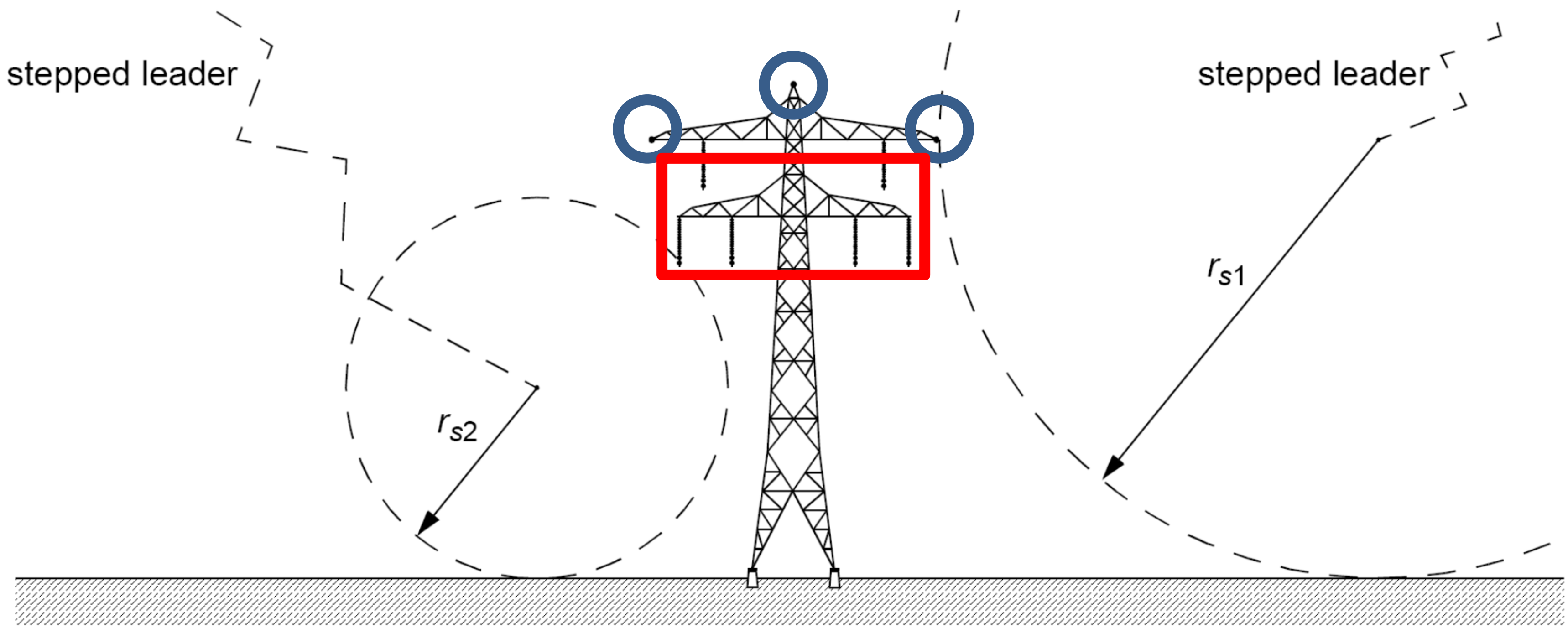
- Insulators are used to isolate the transmission lines from the tower that is connected to ground.
- Traditionally insulators are made from glass or porcelain.
- For new technology, insulators are made from composite materials

Source: Electrical Power System Essentials by Pieter Schavemaker and Lou Van Der Sluis

Shield (Ground) Wires

Small lightening currents can still hit the tower or the three-phase conductors.

Protected against large lightening currents.



Source: Electrical Power System Essentials

Severe Weather Impact on T-Lines

Severe weather swept through west central Minnesota on Aug. 1, 2011. The tower was designed to withstand 120 mph winds. The transmission line operated reliably since it was energized in 1978

2005 Hurricane Rita Damage to Gulf Transmission Lines

Environmental Impact

Ugly power lines

Something you don't see in Singapore.

Underground cables

Types of underground cables

Issues with underground cables

UNDERGROUND CABLE

ADVANTAGES OF UNDERGROUND CABLE

Longer Life Expectancy

Underground systems normally last much longer than overhead wiring.

Reduced Maintenance Costs

Components of underground systems, not being exposed, require less maintenance.

Service Uninterrupted by Storms

Underground systems are protected from ice storms, lightning, and high winds.

Conserve Valuable Land

Land is a valuable resource, particularly in urban areas. Underground systems permit the construction of buildings and other structures over and around them.

Reduced Fire Fighting Hazards

Fire departments are not hampered, or endangered, by poles and overhead wires, when erecting their fire-fighting apparatus.

Prevention of Accidents

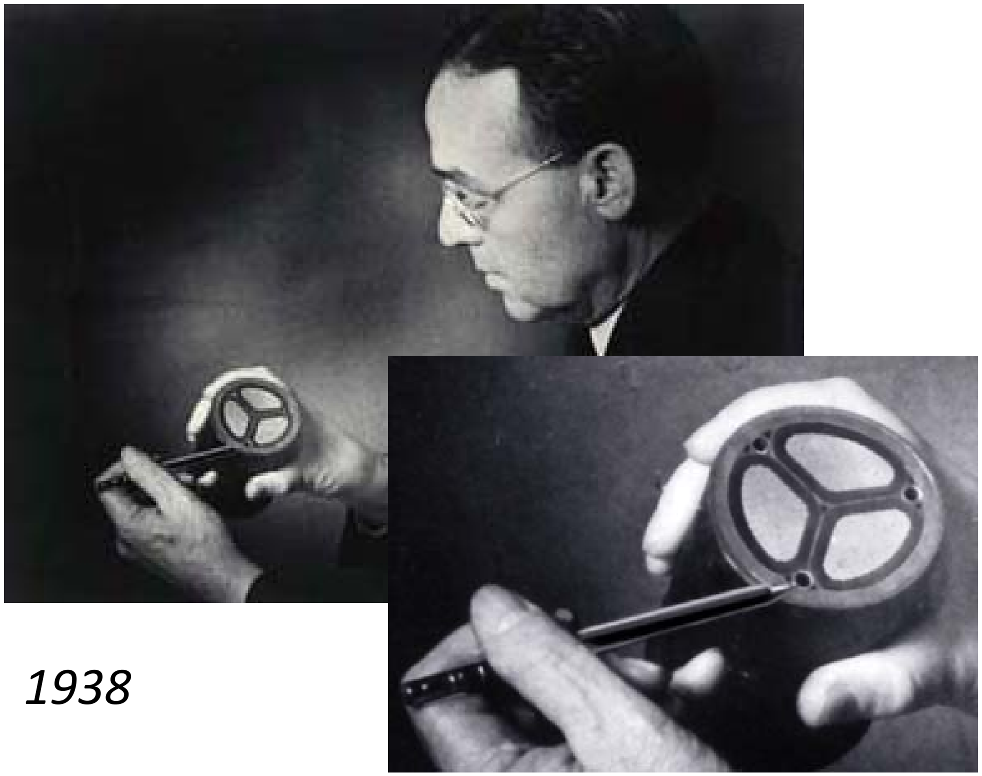
Dangers to persons from the accessibility of high-voltage power, or from poles becoming hazardous to vehicles, and from fallen poles and live wires, are eliminated.

Source: http://coppercanada.ca/publications/pub21e/21e-section3.htm

ADVANTAGES OF UNDERGROUND CABLE

Source: http://coppercanada.ca/publications/pub21e/21e-section3.htm

Underground Cable



1938

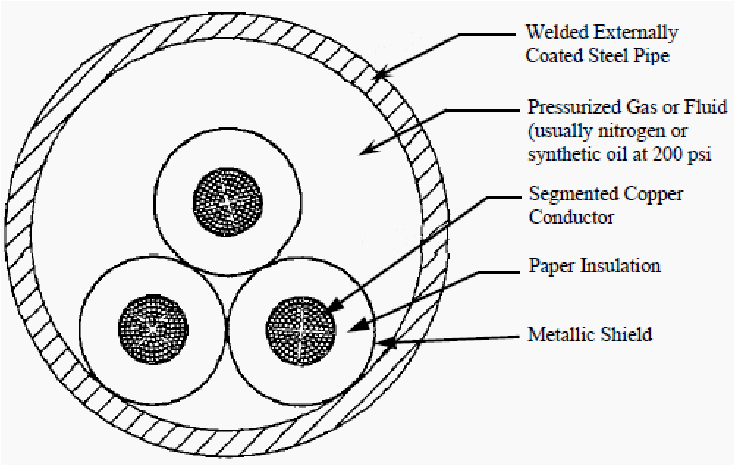
G.B. Shanklin, engineer of the cable section of the General Electric Company, points out one of the three gas channels in a section of gas-filled cable.

- The main difference between underground cables and overhead transmission line is that for underground cable the conductor must be insulated from the ground.

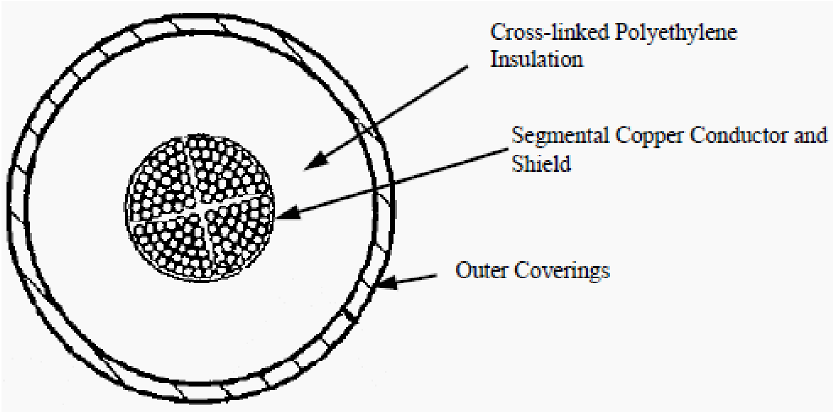
Source: http://scienceservice.si.edu/pages/014005.htm

Types of Underground Cables

- Two types of cables
- Pipe-type
  - All three phase conductors are contained in one pipe.
- Self-contained
  - Individual phase conductor in each cable.



Pipe-Type cable



Self-contained cable

Source: http://electrical-engineering-portal.com/understanding-underground-electric-transmission-cables

NUS National University of Singapore

Types of Insulation

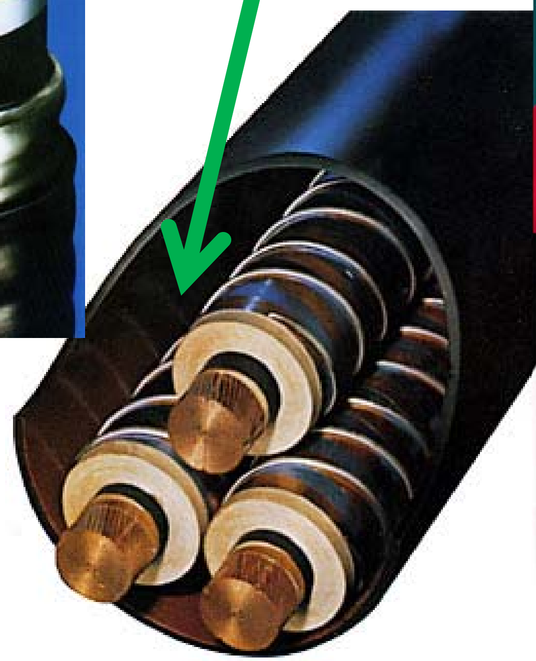
- Three main types of insulation
- Paper-Oil insulation
  - High-pressure, fluid-filled pipe (**HPFF**)
  - Self-contained fluid-filled (**SCFF**)
- Paper-Gas insulation
  - High-pressure, gas-filled pipe (**HPGF**)
- Plastic insulation
  - Solid cable, cross-linked polyethylene (**XLPE**)

High-Voltage Underground Cable

Liquid channel

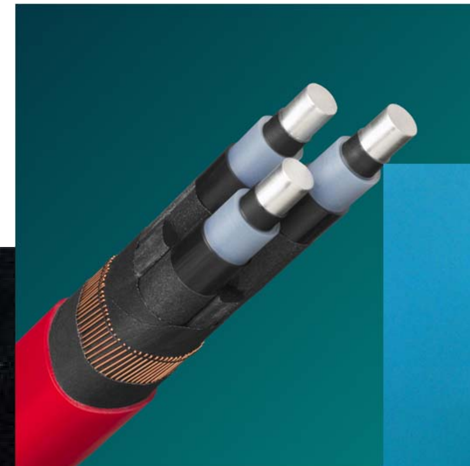
Self Contained Liquid-Filled (SCLF) Cables

Source: http://coppercanada.ca/publications/pub21e/21e-Section6.html



High Pressure Liquid-Filled Pipe-Type Cables (HPLF)

Source: Electrical Power System Essentials by Pieter Schavemaker and Lou Van Der Sluis



6/10 kV cable

Solid cable, cross-linked polyethylene (XLPE)

220/380 kV cable

Singapore Underground Power Cables

Voltage Level	400 KV	230 kV	66 kV
Cable Type	Single-core self-contained	Single-core self-contained	Single-core
Insulator	fluid-filled Polypropylene Laminated Paper (PPLP) insulated with copper conductor and seamless aluminium sheath	Either fluid-filled kraft paper insulated or XLPE insulated with copper conductor and seamless aluminium sheath	XLPE insulated with copper conductor and seamless aluminium sheath
Power rating	1000 MVA	500 MVA	100 MVA

Source: SP powergrid http://www.sppowergrid.com.sg/items2.htm

Overhead/Underground Comparison

Overhead Lines

- Lower construction cost and cable cost.
- Advantage of air for cooling and insulation of the line.
- Vulnerable to strong wind and severe weather.
- Negative visual impact.
- Easier maintenance/repair work

Underground Cables

- Expensive pipe work and cable cost (because of the special insulations)
- Less vulnerable to the severe weather because it is buried underground.
- Environment and aesthetic advantage.
- Tedious and costly maintenance/repair work.

Summary

- Transmission network helps to improve system efficiency, reliability, and reduce frequency deviation.
- Advantages and disadvantages of overhead transmission lines and underground cables

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Transmission Line Parameters

Resistance

Conductance

Inductance

Capacitance

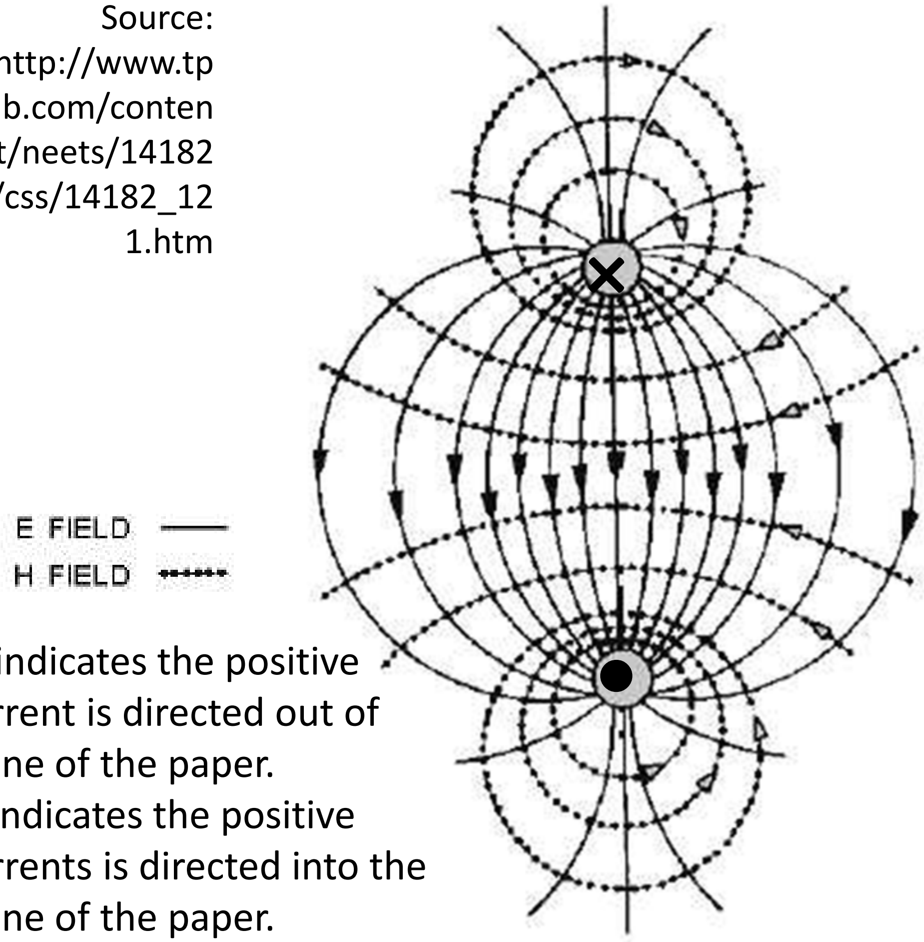
FOUR BASIC PARAMETERS OF TRANSMISSION LINES

Resistance and Conductance

- Resistance
  - Voltage drop along the line from resistive loss
  - This effect can be represented by a resistor along the line.
- Conductance
  - Leakage current through insulators which allows the current to pass the tower to the ground.
  - This effect can be represented by a conductance from a line to the ground.

Inductance and Capacitance

Source: http://www.tpub.com/content/neets/14182/css/14182_121.htm



● indicates the positive current is directed out of plane of the paper.
× indicates the positive currents is directed into the plane of the paper.

- Ampere's Law
  - Current passing through a conductor creates magnetic field around it.
  - This gives inductance property.
- Gauss's Law
  - Electric charge is a source of electric fields.
  - This gives capacitance property.

Corona Discharge

- When the electric field strength is high, air might become electrically ionized and conduct.
- This is called a **‘corona’** effect and can be represented by a conductor from the line to the ground.
- The power loss due to this effect is call corona loss.

How to Reduce Corona Effect?

- Corona effect is caused by electric fields.
- Recall Gauss's Law "Electric charge is a source of electric fields"
- If the current pass through only one conductor per phase, electric charges on each line will be quite high.
- We can **reduce** **electric charges** in each conductor by sending electricity **using bundles of conductors per phase**.
- Not only that we can reduce the electric fields, the *thermal capacity of each line can be increased* because the current per conductor per phase is smaller.

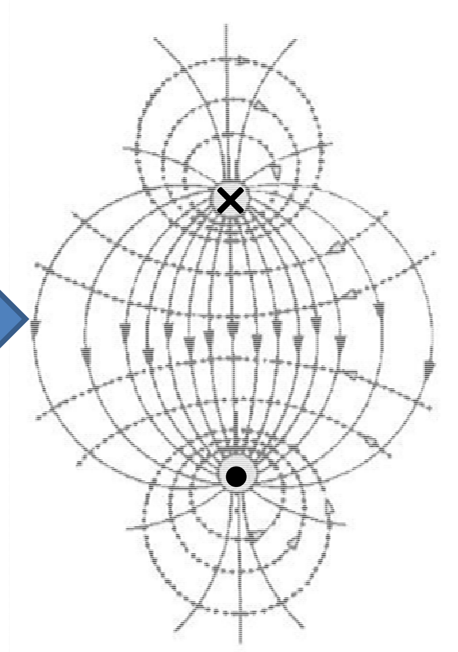
Transmission Line Parameters

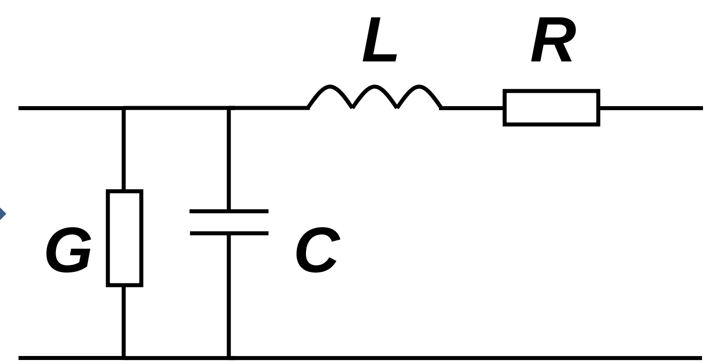
- **R** from Ohmic losses
  - Types, sizes of conductor determine resistance value.
- **G** from insulator leakage current and corona losses
  - Types, number of insulators determine conductance value.
- **L** from magnetic field and **C** from electric field
  - Conductor spacing, bundling, determines magnetic and electric field strength
- All these values can be measured.

Single-Phase Single-Line Model

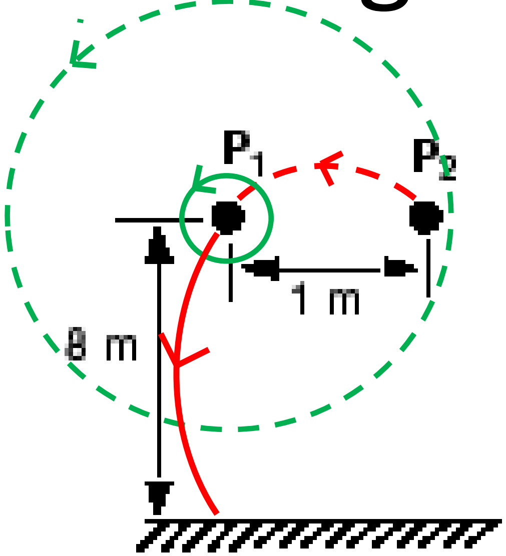
One current-carrying conductor

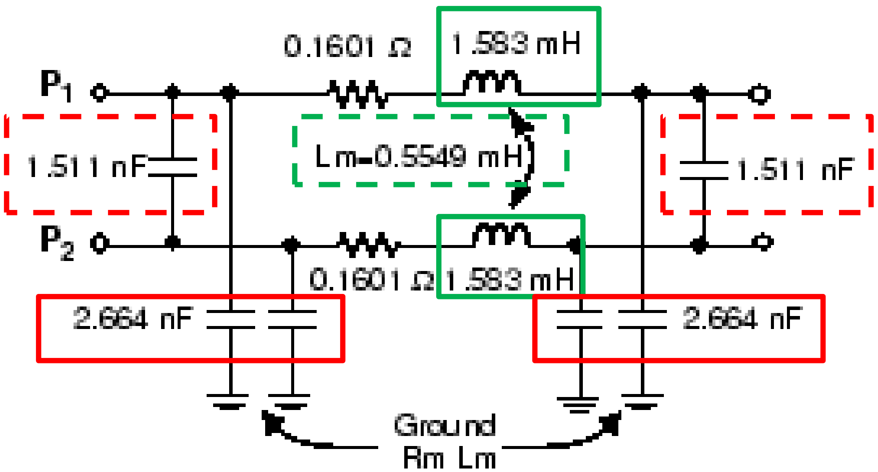
Ground





Single-Phase Two-Line Model

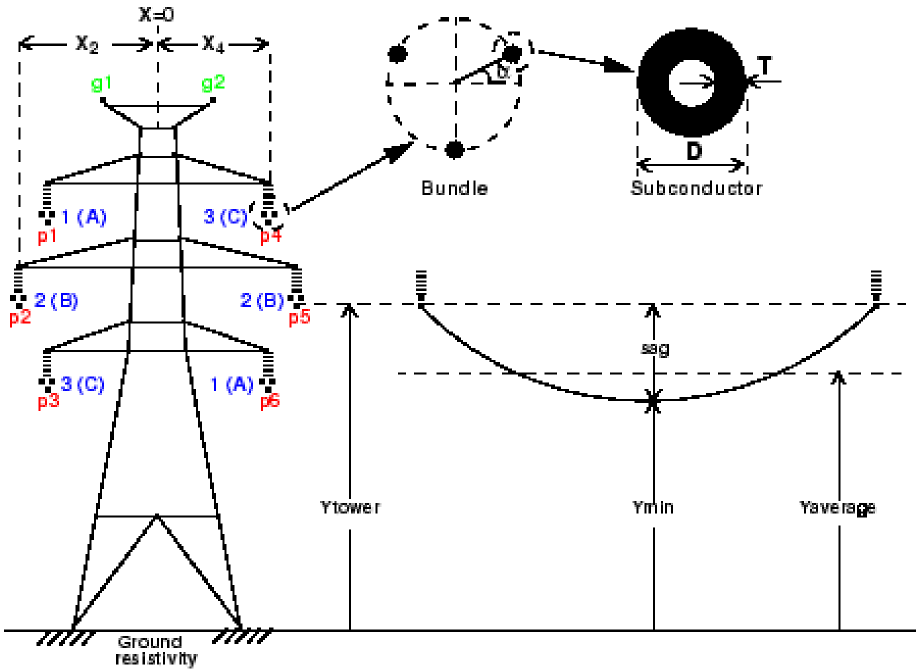




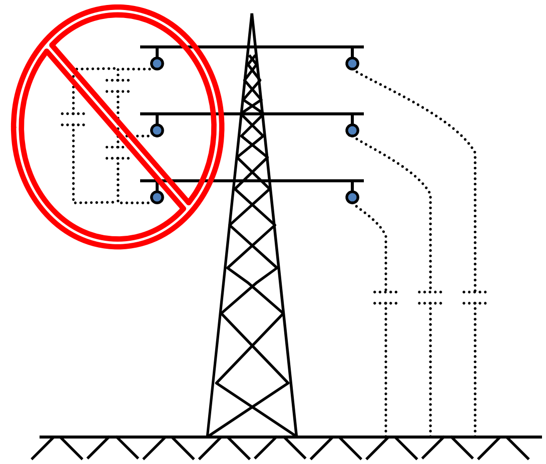
Source:
http://www.mathworks.com/help/toolbox/physmod/powersys/ref/power_lineparam.html#bqs3dxu

- When there is more than one conductor, there will be coupling effect from magnetic field and electric filed.
- This effect is represented by **'mutual' inductance** and **'mutual' capacitance** in the conductor model.
- The magnitude of mutual inductance and capacitance depends on the distance between conductors.

Three-Phase Line, Bundled Conductors



Source: http://www.mathworks.com/help/toolbox/physmod/powersys/ref/power_lineparam.html#bqs3dxu

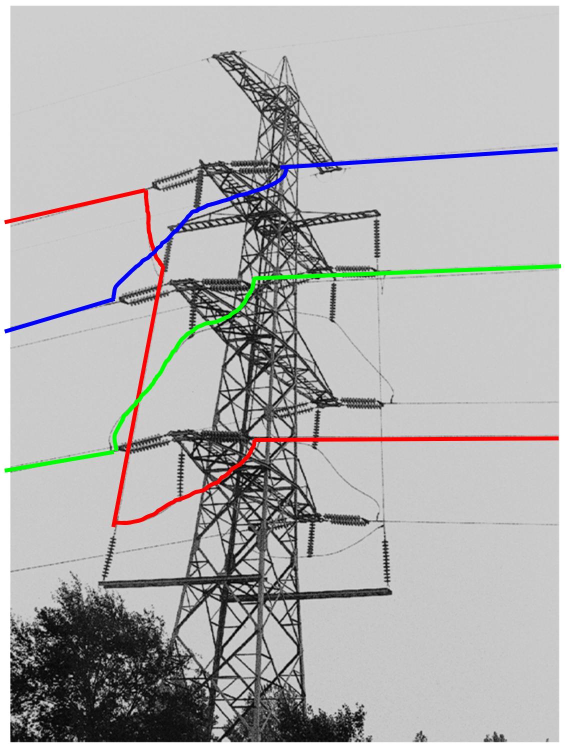


- Unbalanced system

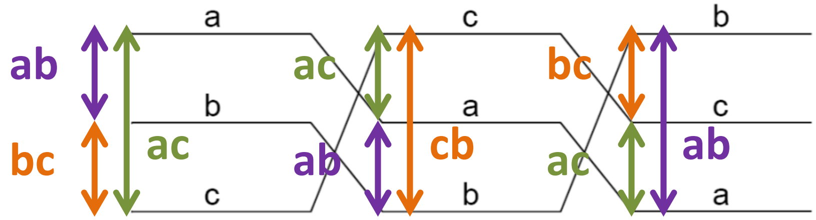
Three-phase line causes **coupling effect of the magnetic field and electric field among three phase conductors**. This is represented as unbalanced capacitance and mutual inductance among the three phase conductors.

Transposition

Source: Electrical Power System Essentials by Pieter Schavemaker and Lou Van Der Sluis



The coupling effect is caused by asymmetric distance between phase conductors.
Simple solution → Make the distance between phase conductor equal.

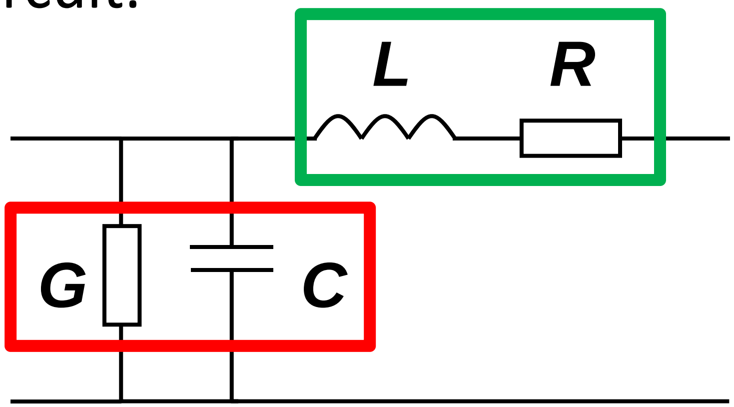


By transposing the line, we can reduce (or eliminate) the coupling effect between phases from both magnetic field and electric field.

This means that we can now use one conductor model to analyze three-phase line.

Per Phase Conductor Model

- We can now use the per phase conductor model to describe the circuit model of each phase in three-phase circuit.



Series Impedance (Ω/m)

$$\boxed{z = r + j\omega l}$$

$$\boxed{y = g + j\omega c}$$

Shunt Admittance (S/m)

- These parameters are given as per unit length of the transmission line.
- We will use this information to derive an equivalent circuit of the transmission line.

150 kV Line/Cable Parameters

Overhead Transmission Line

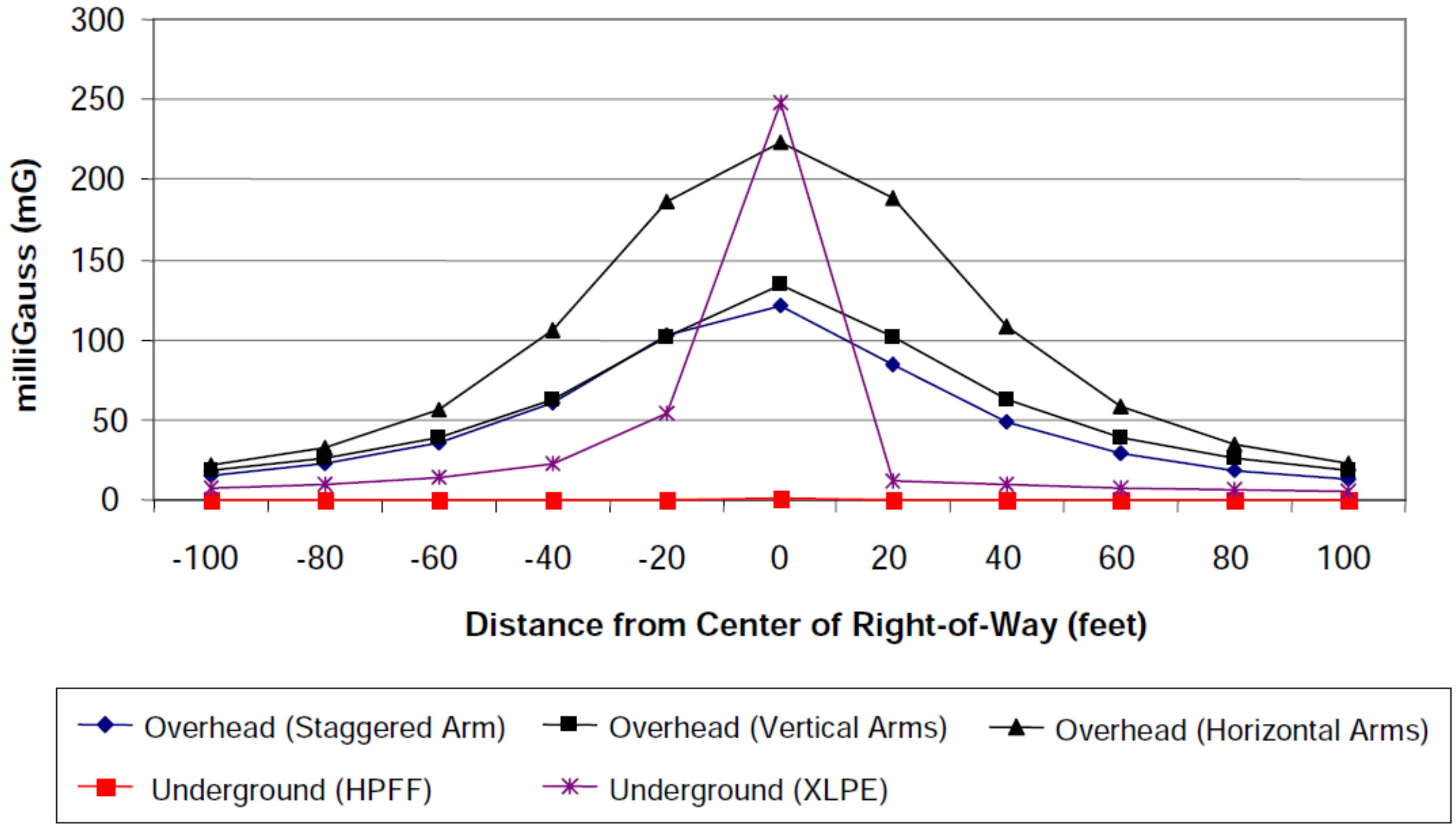
- R = 0.125 Ω/km
- XL = 0.425 Ω /km
- C = 7.7 nF/km
- 130 MVA rating.

Underground Cable

- R = 0.12 Ω/km
- XL = 0.166 Ω/km
- C = 210 nF/km
- 135 MVA rating.

The main difference in electrical properties between overhead transmission lines and underground cables.

Line/Cable Magnetic Field



Underground cable seems to filter the magnetic field better than overhead line in a longer distance from center of Right-of-Way.

Source: http://*jlarc.state.**va**.us/reports/Rpt343.pdf*

Summary

- Basic parameters of conductor model
  - Resistance to represent resistive loss along the line.
  - Conductance to represent leaking currents to the insulator and corona effect.
  - Inductance to represent magnetic field.
  - Capacitance to represent electric field.
- For three-phase lines, there will be some coupling effect of inductance and capacitance that depends on the distance between conductors. This coupling effect is eliminated by ‘transposing’ the three lines.

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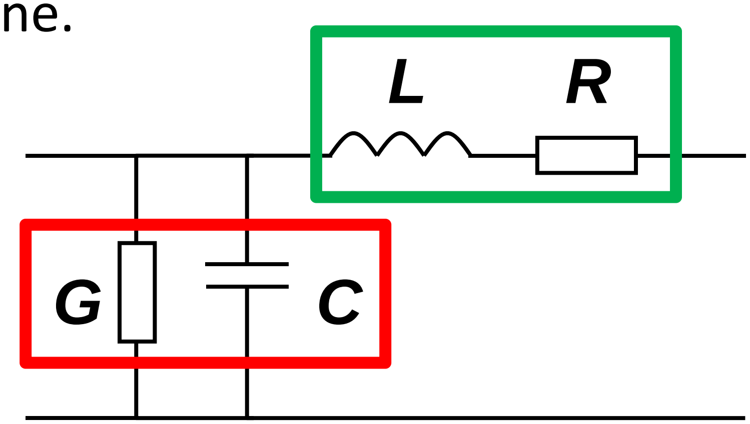
Transmission Line Modeling

A Per Phase Conductor Model

- R for resistive loss in the conductor.
- G for leakage current through insulators and corona losses.
- L to represent magnetic field.
- C to represent electric field.
- Although there are some coupling effects of magnetic field and electric field between phase conductors, they are eliminated by transposing the lines.
- These parameters are given as per unit length of the transmission line.



Series Impedance (Ω/m)

$$\boxed{z = r + j\omega l}$$

$$\boxed{y = g + j\omega c}$$

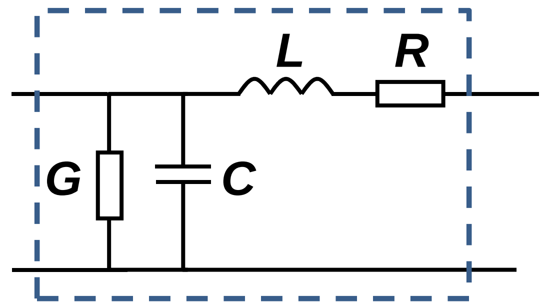
Shunt Admittance (S/m)

Outline

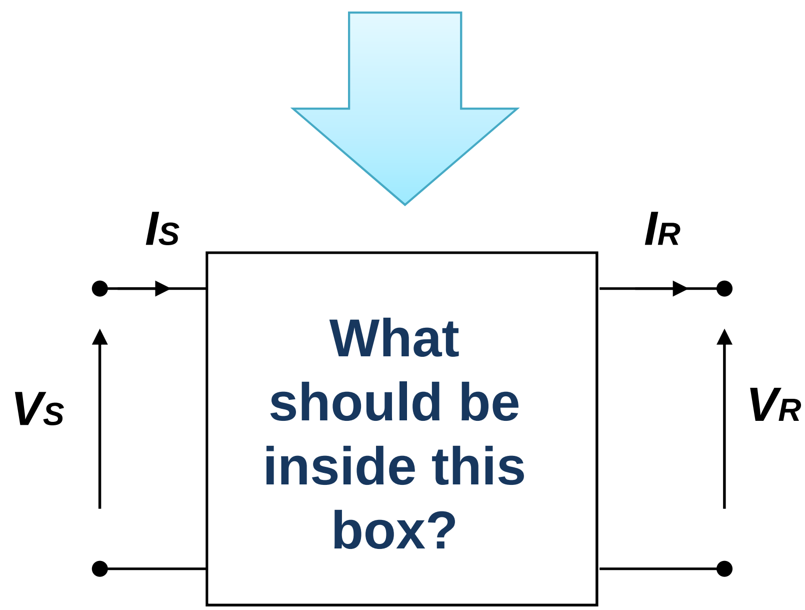
- Transmission line modeling
  – Short length model
  – Medium length model
  – Long length model
- Transmission line operation
  – Voltage regulation
  – Line loadability
  – Transmission line efficiency

Equivalent Circuit of A Transmission Line

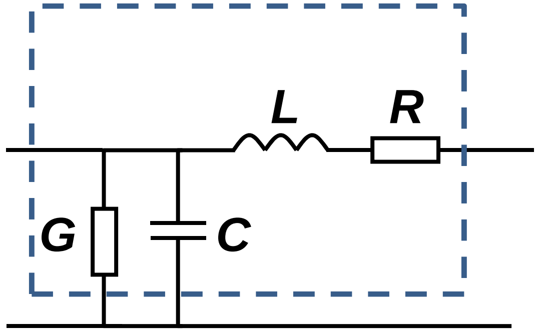
- An equivalent circuit of a transmission line is given in *per-phase* representation.



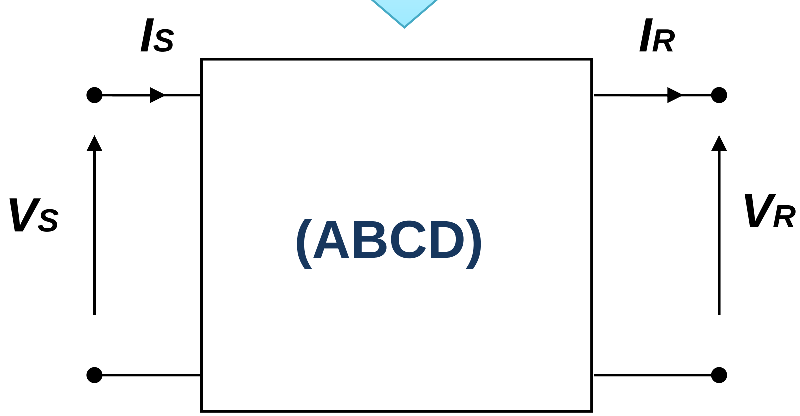
Conductor per phase model



Equivalent Circuit of A Transmission Line : Matrix Representation



Conductor per phase model



$$V_S = AV_R + BI_R$$

$$I_S = CV_R + DI_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

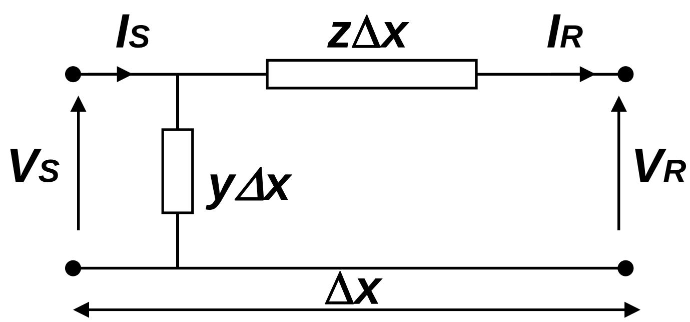
A, B, C, and D are parameters to be found.

Purposes of Equivalent Circuit

- To calculate the voltage at the receiving end when the sending end voltage is known or vice versa.
  - This is used to find the voltage difference between sending and receiving end.
- To find the amount of real and reactive power transfer in the line.
  - To make sure that the power does not exceed the heating limit by the lines.
  - For transmission line efficiency calculation.

Transmission Line Models

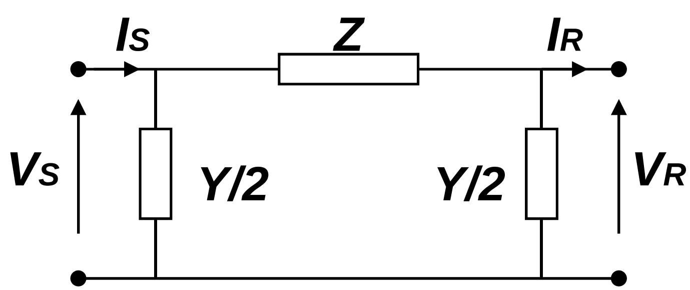
Long-Line Model :Distributed Model



z: Series Impedance (Ω/m)
y: Shunt Admittance (S/m)
Δx: distance (m)

Simplification

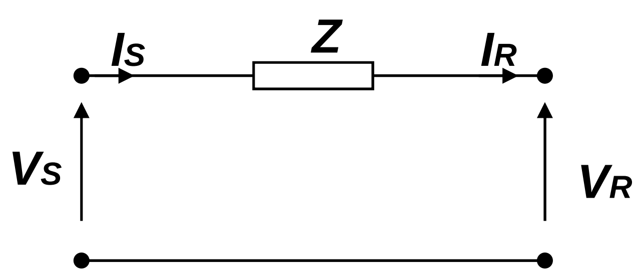
Medium-Line Model: Lumped Model



Z: Series Impedance (Ω) = zΔx
Y: Shunt Admittance (S) = yΔx

Simplification

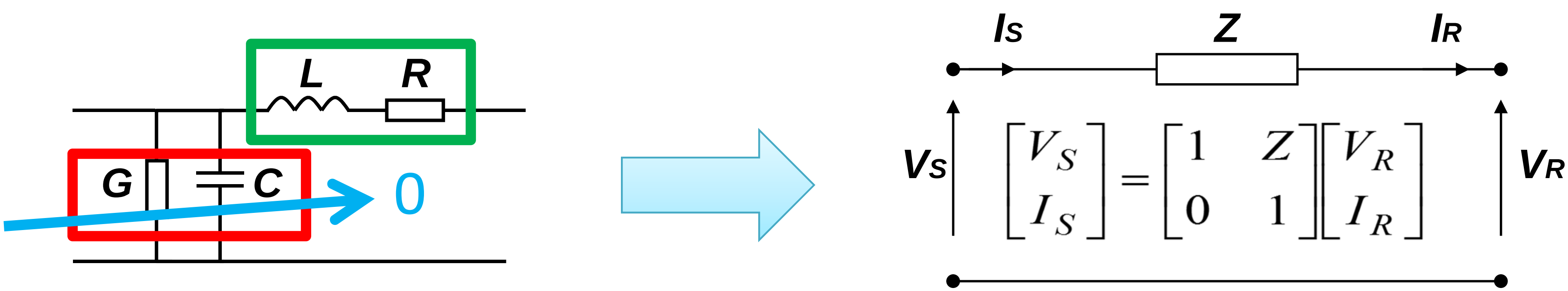
Short-Line Model



Z: Series Impedance (Ω) = zΔx
Y ≈ 0

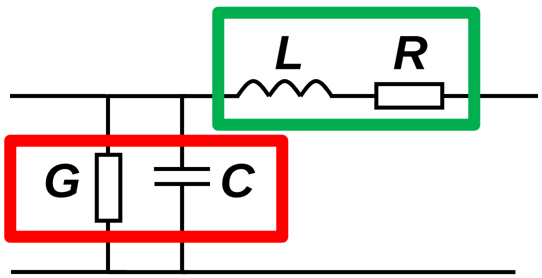
Short Line: A Simplified Model

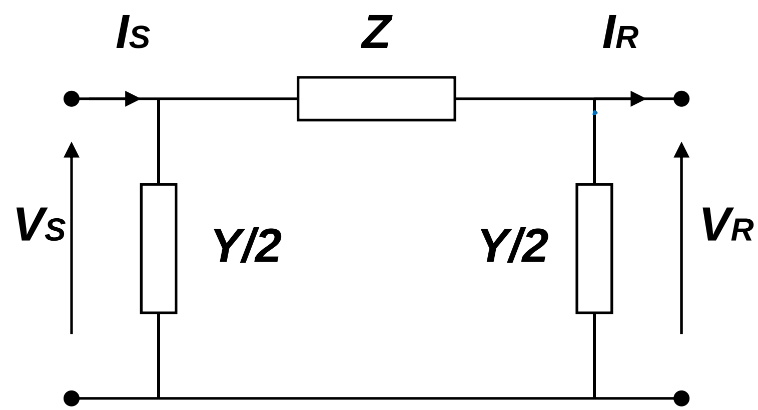
- In this model, we ignore the shunt admittance and only consider series impedance.



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Medium Line: A Lumped Model 'nominal π circuit'.





$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y\left(1+\frac{ZY}{4}\right) & \frac{ZY}{2}+1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Equivalent Models: Summary

Short (<80 km)

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Medium (80 km..240 km)

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y\left(1+\frac{ZY}{4}\right) & \frac{ZY}{2}+1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Long (>240 km), 'l' is the length of the transmission line.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{\sinh(\gamma l)}{Z_c} & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The ranges of transmission length for each model are only suggestion! The selection of an appropriate model mainly depends on the conductor parameter and the application.

Example 1

- A three-phase transmission line is 40 km long. It has a total series impedance of $Z = 5+j20$ ohm and a total shunt admittance of $Y = j133\times10^{-6}$ S. 'Full Load' at the receiving end of the line is a Y-connected load of 40 MW at a voltage of 220 kV with a power factor 0.9 lagging.
  - **Find the voltage at the sending end using three line models.**

Example 1: Model Comparison

- Consider the full load case at the receiving end, we vary the length of the transmission line and find the sending end voltage.
- Note that the receiving end voltage magnitude is 127.017 kV.

Line Model	Sending End Voltage Magnitude in kV when line length is **40 km**	Sending End Voltage Magnitude in KV when line length is **150 km**	Sending End Voltage Magnitude in kV when line length is **400 km**
Short	128.55	132.96	143.59
Medium	128.38	130.62	127.54
Long	128.38	130.57	126.88

What can we say about the choice of model?

Full load VS No load condition

Voltage regulation

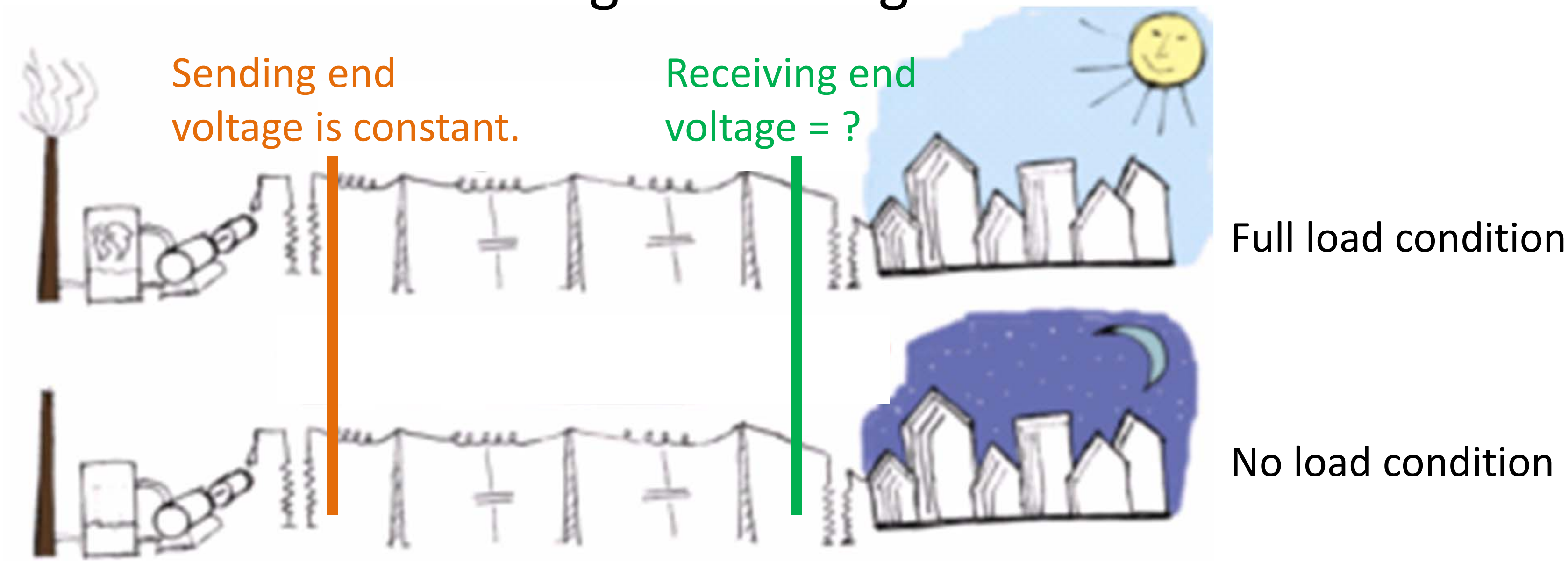
Line loadability

Transmission line efficiency

TRANSMISSION LINE OPERATION

Full Load VS No Load Conditions

- If the *sending end voltage is kept constant*, receiving end voltage will vary between full load and no load conditions.
- We are interested to know the magnitude of this variation in the receiving end voltage.



Example 2

- From Example 1, we now disconnect the three-phase load at the receiving end (no load condition). Let us fix the sending end voltage to be the same value computed from the previous ‘Full Load’ case, find the receiving end voltage using three line models.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \xrightarrow{\text{No load}} \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ 0 \end{bmatrix}$$

$$\left| V_{R,NL} \right| = \left| \frac{V_{S,FL}}{A} \right|$$

Example 2: Voltage Differences FL/NL

Receiving end voltage (kV)	40 km	150 km	400 km
Full load	127.02	127.02	127.02
No load	128.55	150.55	145.78

We can see that the **longer** the transmission line, the **higher** the magnitude of **voltage variation**.

Voltage Regulation

- The variation of line voltage with different loading conditions is called '*voltage regulation*'.
- About 10% voltage change between no load and full load operation is a usual practice for reliable operation.
- Voltage regulation measures the degree of change in voltage when load varies from no-load to full load at a specific power factor.

$$\text{Percent regulation} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

Example 2

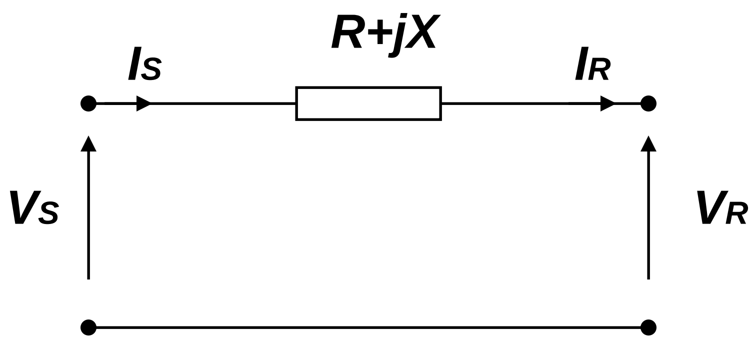
- Calculate percent voltage regulation from Example 2.

Receiving end voltage (kV)	40 km	150 km	400 km
Full load	127.02	127.02	127.02
No load	128.55	150.55	145.78
% VR	1.2	18.5	14.7

Recall that in this example, the three-phase load has a lagging power factor, we will now consider different cases of power factor.

Voltage Regulation of a Short Line

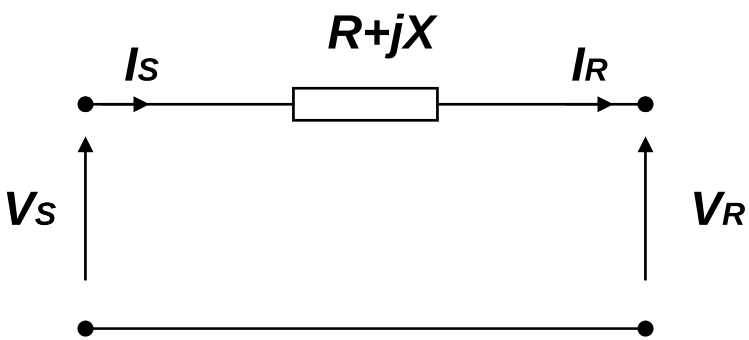
- For simplicity, we consider a short transmission line model.



$$\text{Percent regulation} = \frac{\overset{=V_{S,FL}}{\left|V_{R,NL}\right|} - \left|V_{R,FL}\right|}{\left|V_{R,FL}\right|} \times 100\%$$

- Note that the receiving end voltage at no load condition is the same as sending end voltage at full load condition.

Effect of Different Power Factor



$$\text{Percent regulation} = \frac{\cancelto{=V_{S,FL}}{\left|V_{R,NL}\right|} - \left|V_{R,FL}\right|}{\left|V_{R,FL}\right|} \times 100\%$$

Lagging pf load

The percent voltage regulation is positive (++).

Unity pf load

The percent voltage regulation is positive (+).

Leading pf load

The percent voltage regulation may be negative (-).

In order to minimize voltage regulation, it is ***preferable to have unity power factor load****.*

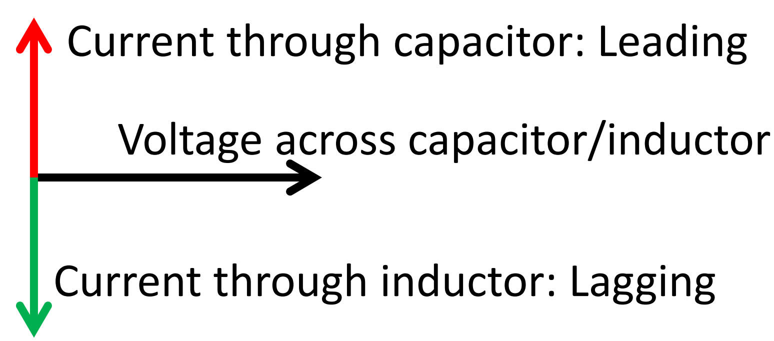
Line Loadability

- Line loadability refers to the maximum amount of MVA to be carried by the transmission line.
- We mainly consider three limits.
  1. Thermal ratings of conductors.
  2. Voltage-drop limit.
  3. Stability limit.

Line model	Short line	Medium line	Long line
Main concern	Thermal limit	Voltage-drop limit	Voltage-drop limit

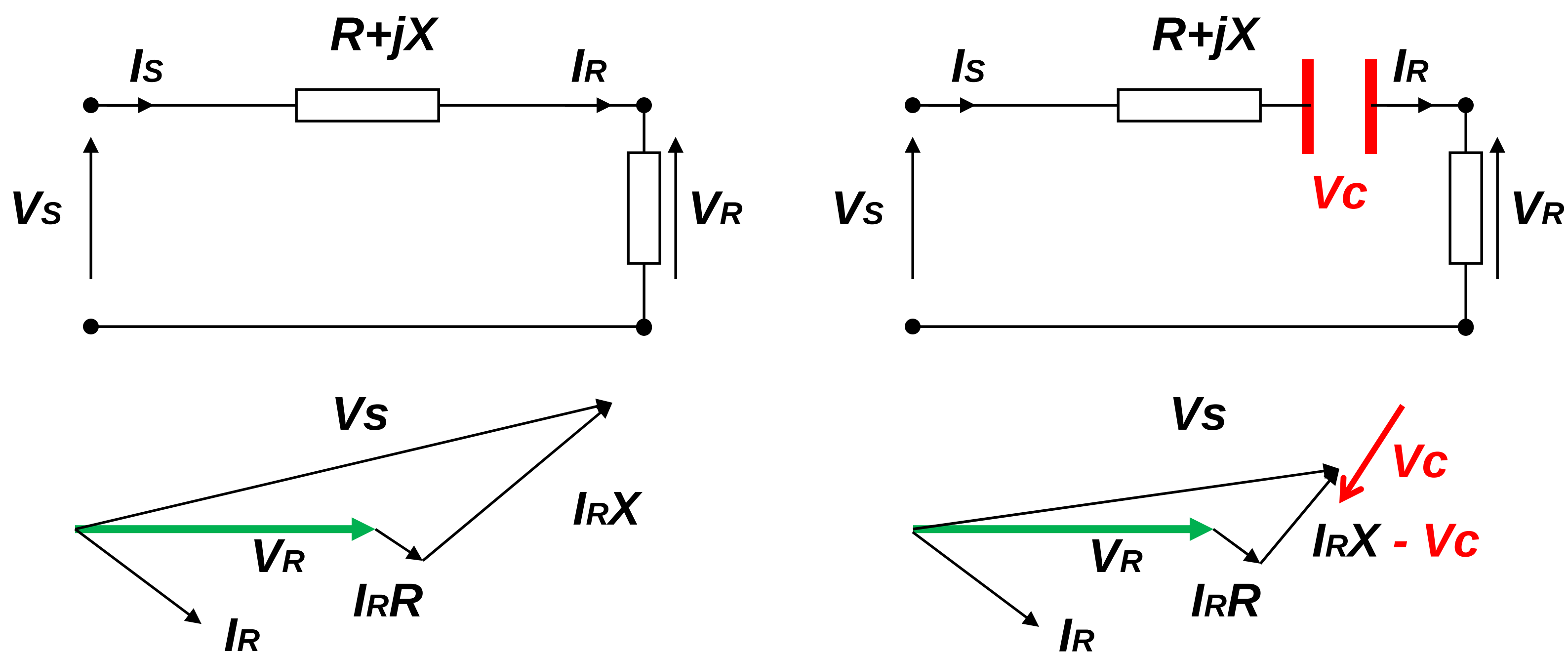
Reactive Compensation Techniques

- when the power factor is lagging, the voltage regulation is the highest.
- For medium and long transmission lines, the voltage drop is usually reached before the thermal limit.
- We can use reactive compensation techniques to increase line loadability and maintain voltage at rated value.
- Two main techniques
  - Series compensation.
  - Shunt compensation.



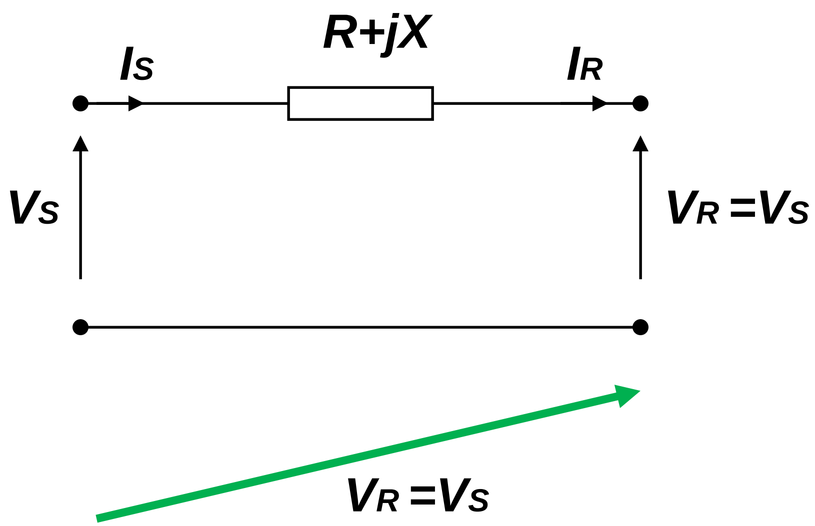
Series Compensation Technique

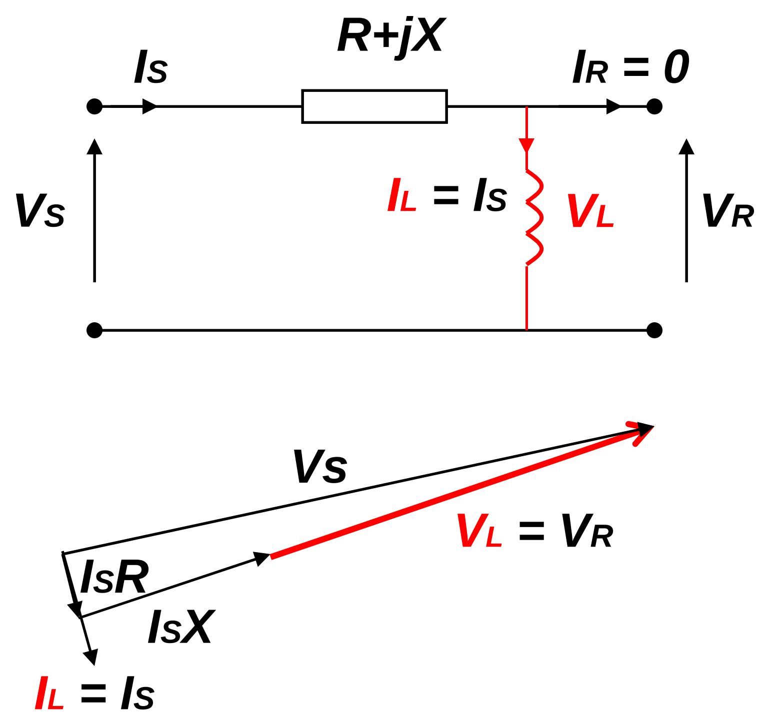
- **At full load**, the sending end voltage is too high compared to receiving end voltage.
- We can connect the capacitor in series to the load to help reduce the sending end voltage in the heavy load condition.



Shunt Compensation Technique

- **At** **no load**, the receiving end voltage is higher than the usual full load case
- We can connect the inductor at the receiving end





Compensation Techniques

Series compensation

- Series capacitor
- Use during heavy load condition to boost up the voltage magnitude.

Source: Siemens

Shunt compensation

- Shunt reactor
- Use during light load condition to dampen the voltage magnitude.

Source: ABB

Transmission Line Efficiency

- We can compute the transmission line efficiency (%) from the ratio of the real power at the receiving end to real power at the sending end.

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100$$

$$S_{R,3\Phi} = 3V_{R,\text{line-to-neutral}} I_R^* = P_{R,3\Phi} + jQ_{R,3\Phi}$$

$$S_{S,3\Phi} = 3V_{S,\text{line-to-neutral}} I_S^* = P_{S,3\Phi} + jQ_{S,3\Phi}$$

Summary: T-Line Models

- Three models
  - Short line model
  - Medium line model
  - Long line model
- The choice of model depends on transmission line parameters and its length.

Summary: T-Line Operations

- Voltage regulation refers to variations of voltage at the receiving end **between full load and no load conditions**.
- It is desirable to have power factor at load close to one in order to keep the voltage regulation small.
- In order to reduce the voltage regulation we can compensate the receiving end voltage by
  - Connecting a capacitor in series to the load to increase the receiving end voltage during heavy load condition.
  - Connecting an inductor in parallel to the load to reduce the receiving end voltage during light load condition.
- The transmission line efficiency is the ratio of real power at receiving end to real power at sending end.

Problem with sqrt(complex number)

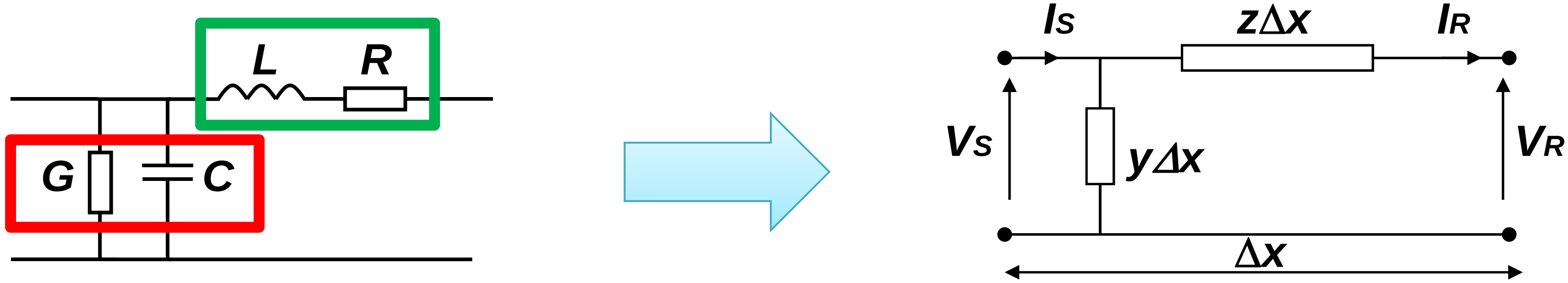
Problem with Hyperbolic function of a complex number

Comparison of long-line differential equations in EE2022 and EE2011

APPENDIX

Long Line: A Distributed Model

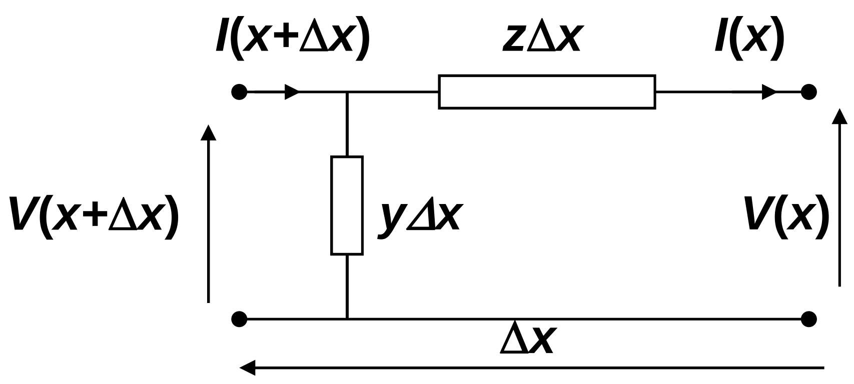
- Parameter are given as per-length and are uniformly distributed along the transmission lines.
- A distributed model accounts for this distributed nature of transmission-line parameters.
- This model provides exact transmission line equations and is suitable for long-length transmission lines.



$$z = r + j\omega l$$ **Series Impedance (Ω/m)**

$$y = g + j\omega c$$ **Shunt Admittance (S/m)**

Long-Line: A Distributed Model



Long-Line: Differential Equations

$$\left.\begin{aligned}\frac{dI}{dx} &= yV \\ \frac{dV}{dx} &= zI\end{aligned}\right\} \frac{d^2V}{dx^2} = z\frac{dI}{dx} = zyV$$

$$\gamma = \sqrt{zy}$$

γ *is called* ***propagation constant*** *(1/m).*

$$\boxed{V = k_1 e^{\gamma x} + k_2 e^{-\gamma x}}$$

$$\boxed{I = \frac{1}{z}\frac{dV}{dx} = \frac{k_1\gamma}{z}e^{\gamma x} - \frac{k_2\gamma}{z}e^{-\gamma x}}$$

Long-Line: Differential Equations

We can find the constants k_1 and k_2 from the fact that at the receiving end of the line (x=0), V = V_R and I = I_R.

$$\left.\begin{aligned} V_R &= k_1 + k_2 \\ I_R &= \frac{k_1\gamma}{z} - \frac{k_2\gamma}{z} \end{aligned}\right\} \quad \begin{aligned} k_1 &= \frac{V_R + I_R(z/\gamma)}{2} \\ k_2 &= \frac{V_R - I_R(z/\gamma)}{2} \end{aligned}$$

Define Zc **as a characteristic impedance (Ω).**

$$\frac{z}{\gamma} = \frac{z}{\sqrt{zy}} = \sqrt{\frac{z}{y}} \equiv Z_c \quad \Longrightarrow \quad \begin{aligned} k_1 &= \frac{V_R + Z_c I_R}{2} \\ k_2 &= \frac{V_R - Z_c I_R}{2} \end{aligned}$$

Long Line: V&I Equations

- Substitute k_1 and k_2, we have the voltage and current equations at any point *x* from the load (receiving end) as follows.

$$V = V_R\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + Z_c I_R\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) \qquad I = I_R\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + \frac{V_R}{Z_c}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

$$\sinh(\gamma x) = \frac{e^{\gamma x} - e^{-\gamma x}}{2} \qquad \Downarrow \qquad \cosh(\gamma x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2}$$

$$\boxed{\begin{aligned} V &= V_R \cosh(\gamma x) + Z_c I_R \sinh(\gamma x) \\ I &= I_R \cosh(\gamma x) + \frac{V_R}{Z_c}\sinh(\gamma x) \end{aligned}}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \dfrac{\sinh(\gamma l)}{Z_c} & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Problem with Sqrt(zy) or Sqrt(z/y)?

- zy and z/y are complex numbers, how to find sqrt of complex numbers?
- You can use:

$$\sqrt{r\{\cos(\beta)+j\sin(\beta)\}}=\sqrt{r}\left[\cos\frac{\beta}{2}+j\sin\frac{\beta}{2}\right]$$

- For example, let

$$zy=r\{\cos(\beta)+j\sin(\beta)\}$$

- This means that:

$$r=|zy|$$

Magnitude of zy phasor

$$\beta=\angle zy$$

Phase of zy phasor

Problem with Hyperbolic Function?

- How to find sinh(x+jy) or cosh(x+jy)?
- Try this:
  - sinh(x + jy) = sinh(x)cos(y) + j cosh(x)sin(y)
  - cosh(x + jy) = cosh(x)cos(y) + j sinh(x)sin(y)
- **Warning:** Familiarize with your calculator. You need to check whether the setting of cos() and sin() is in 'radian' or in 'degree'. In this formula, y is in radian.

- For example,
  cosh(0.0064+0.052i)
  = cosh(0.0064)cos(**0.052 radian**) + j sinh(0.0064)sin(**0.052 radian**)
  = 0.998668757010298 + 0.000332652309305i

Alternatively,…

- From,

$$\sinh(\gamma x) = \frac{e^{\gamma x} - e^{-\gamma x}}{2} = \frac{e^{x\sqrt{zy}} - e^{-x\sqrt{zy}}}{2}$$

$$\cosh(\gamma x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} = \frac{e^{x\sqrt{zy}} + e^{-x\sqrt{zy}}}{2}$$

  – x is the length of a transmission line.

- Use this:

$$e^{a+jb} = e^{a}\left(\cos(b) + j\sin(b)\right)$$