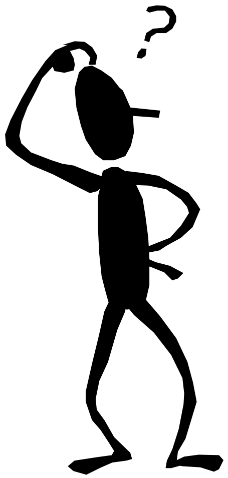


# Lecture 1

# A Motivating Case Study

Consider

- a **practical** temperature control of this LT
- with most simple first-order dynamics

Go through

- continuous-time control
- discrete-time control

To show

- similarity and difference between them
- and elements of the latter

To motivate further study of

- general systems and
- general tools for analysis and design

# 2.1 The Problem Description

The Plant: A Lecture Theater:

***The plant*** is the object to be controlled--- LT.
What to be really controlled is some variable, called
***the output (variable) ---*** the temperature.
To control needs to manipulate some **input**.

## Control Problem

Design a temperature control system

- which consists of the plant and controller
- which has “good” performance

Since the plant is given and usually can not be changed, control engineering is actually to deal with measurement, actuator, controller design and implementation to have “good” performance

# Good Performance

- Stability
  the temperature will not go colder and colder or hotter and hotter or cycles between extremes

- Steady state accuracy
  At the steady state, the temperature should reach the desired temperature

- Transient Response
  the temperature has fast response, little overshoots

# Robust: Performs well under uncertainties

- To disturbances
  On a hot day, is it too warm or on a cold day, is it too cold?

- To errors in modelling
  Suppose our parameters are wrong, does the system still work?

- To partial equipment failure
  If a thermocouple malfunctions, does the whole system collapse?

# Need a solution for controller design!

Relevant Issues:

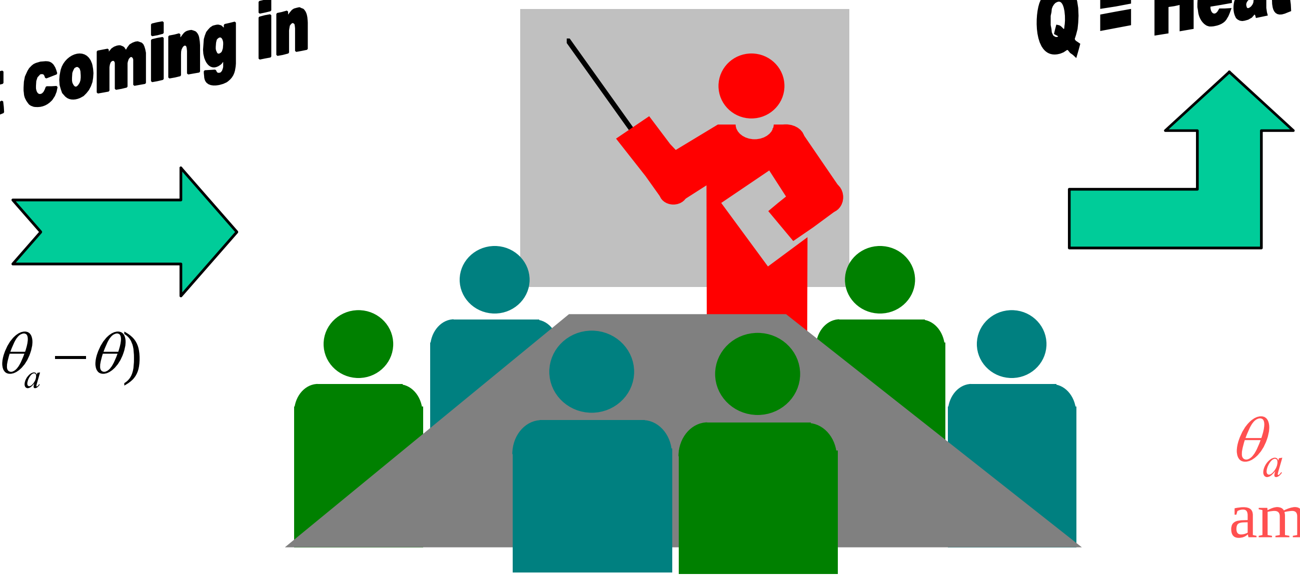
- Performance specifications
  - speed of response, accuracy, robustness
- Model of the lecture theatre
  - Parameters: size of LT, wall insulation properties, ambient temperature
- Digital or analog control
  - digital – choose sampling time
- Controller Structure – PI, PID, adaptive, lead, lag, etc
- Controller parameters
- Simulation and implementation

# 1.2 Continuous time control
## A Plant Model

$\theta$ = room temperature

Heat coming in

Q = Heat removed



$K(\theta_a - \theta)$

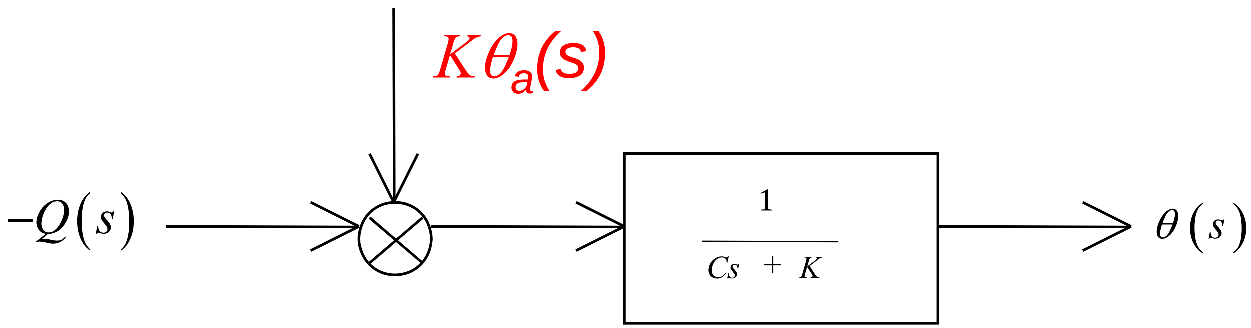
$\theta_a$ = external ambient temperature

$$C\frac{d\theta}{dt} = K(\theta_a - \theta) - Q$$

$$C\frac{d\theta}{dt}=K(\theta_a-\theta)-Q$$

$$Cs\,\theta(s)=K\left[\theta_a(s)-\theta(s)\right]-Q(s)$$

$$\theta(s)=\frac{1}{Cs+K}\left[K\theta_a(s)-Q(s)\right]$$



# Plant Characteristics

Plant Time domain model :

$$C\frac{d\theta}{dt} = K(\theta_a - \theta) - Q$$

Plant s-domain model :

$$\theta(s) = \frac{1}{Cs+K}\left[K\theta_a(s) - Q(s)\right]$$

Plant pole at $s = -K/C$, hence stable if $K>0$, $C>0$.

Plant Time constant = $C/K$

Steady state output when $\frac{d\theta}{dt} \rightarrow 0$, is $\theta_{ss} = \theta_a - \frac{Q}{K}$

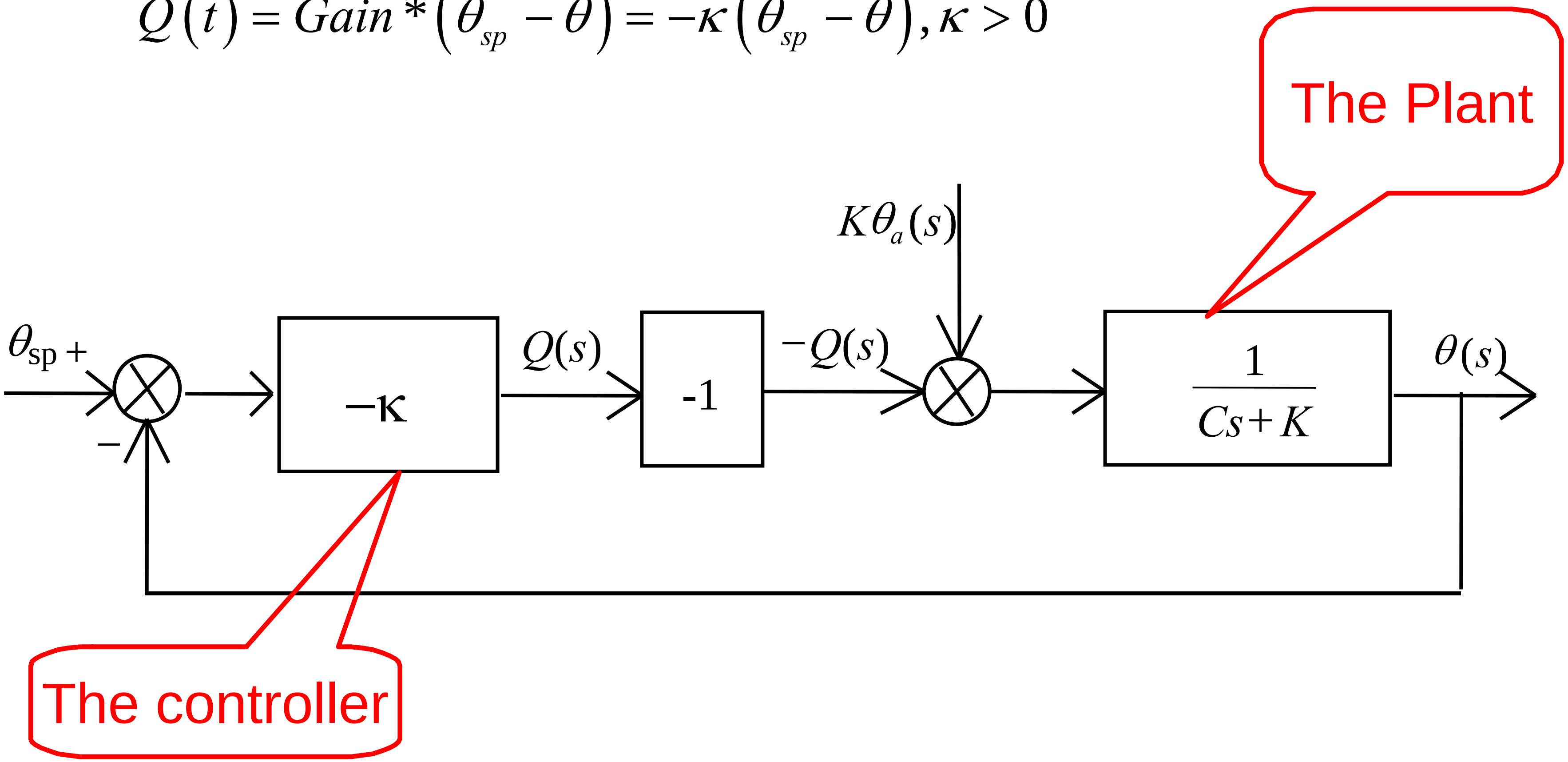
Notice that output depends on $Q$, $K$ and ambient temp, $\theta_a$

$Q$ is the rate of heat energy that the air conditioner is able to remove

Question : What happens when $K$ and $\theta_a$ change?

# Closed-Loop Proportional Control

$$Q(t) = Gain * \left(\theta_{sp} - \theta\right) = -\kappa\left(\theta_{sp} - \theta\right), \kappa > 0$$



# Closed-Loop Control Equation

Plant: $$C\frac{d\theta}{dt} = K\left(\theta_a - \theta\right) - Q$$

Controller: $$Q\left(t\right) = -\kappa\left(\theta_{sp} - \theta\right)$$

CL: $$C\frac{d\theta}{dt} = \kappa\left(\theta_{sp} - \theta\right) + K\left(\theta_a - \theta\right)$$

$$C\frac{d\theta}{dt} + \left(K + \kappa\right)\theta = \kappa\theta_{sp} + K\theta_a$$

Set point Variable

Disturbance Variable

# Steady state analysis

$$C\frac{d\theta}{dt} + (K+\kappa)\theta = \kappa\theta_{sp} + K\theta_a$$

It is always stable. Suppose that $\theta_{sp}$ and $\theta_a$ are constant,

$$\frac{d\theta}{dt} \to 0 \quad \Rightarrow \quad \theta_{ss} = \frac{\kappa}{K+\kappa}\theta_{sp} + \frac{K}{K+\kappa}\theta_a$$

- $\theta_{ss}$ is not equal to $\theta_{sp}$. There is an offset – not a good design
- A change in the ambient temperature would also change the steady state output temperature.
- But if κ is very large, we get better steady state accuracy,

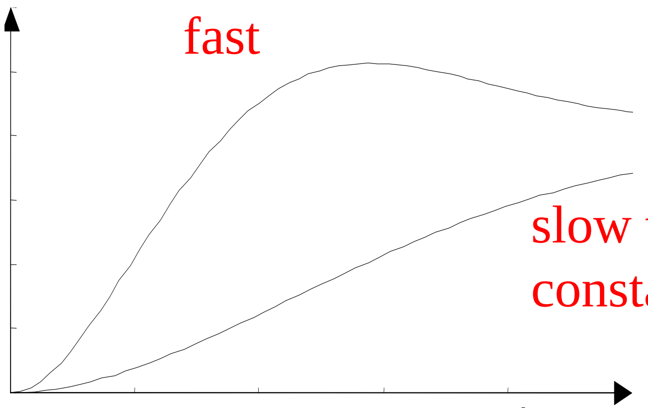
$\theta_{ss} \approx \theta_{sp}$ *for*

$$\frac{\kappa}{K+\kappa} \to 1 \;\; and \;\; \frac{K}{K+\kappa} \to 0$$

# Transient analysis

Suppose $K, \kappa$ , $C$>0.

- The time constant is $\tau = \dfrac{C}{K+\kappa}$
- Characteristic equation: $\tau$s+1=0
- The pole is at $s = -\dfrac{1}{\tau} < 0$ , stable
- If $\kappa$ is very large, then the time constant is small
- High gain gives us a faster response and better accuracy. So why not use a very large gain?



# Problems with Excessive Gain

- A very small change in $\theta$ results in a large change in $Q$. We may get large changes from noisy measurements.
- Our actuator may not be able to deliver large changes in $Q$.
- System may approach instability. Suppose we have dead time in our plant. We may end up with an unstable system.

$$Q(t) = -\kappa\left[\theta_{sp} - \theta\right]$$

$$\Delta Q(t) = \kappa\left[\Delta\theta\right]$$

Absolute or rate of change

# Simulation: Set-point Response

Suppose $\theta(0)$=23 °C, $\theta_a$=23°C, $\theta_{sp}$=24 °C,
$C$=1, $K$=1, $\kappa$ =1

The CL equation is

$$\frac{d\theta}{dt} + 2\theta = \theta_{sp} + \theta_a$$

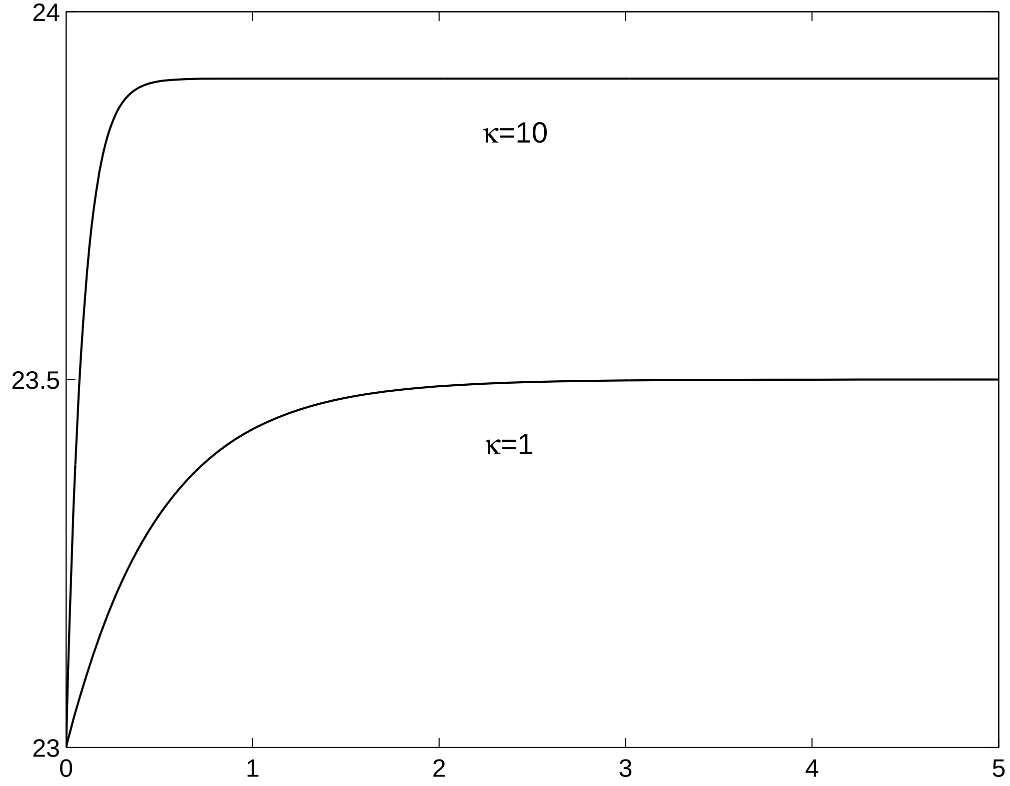
The solution is

$$\theta(t) = 23.5 - 0.5e^{-2t}$$

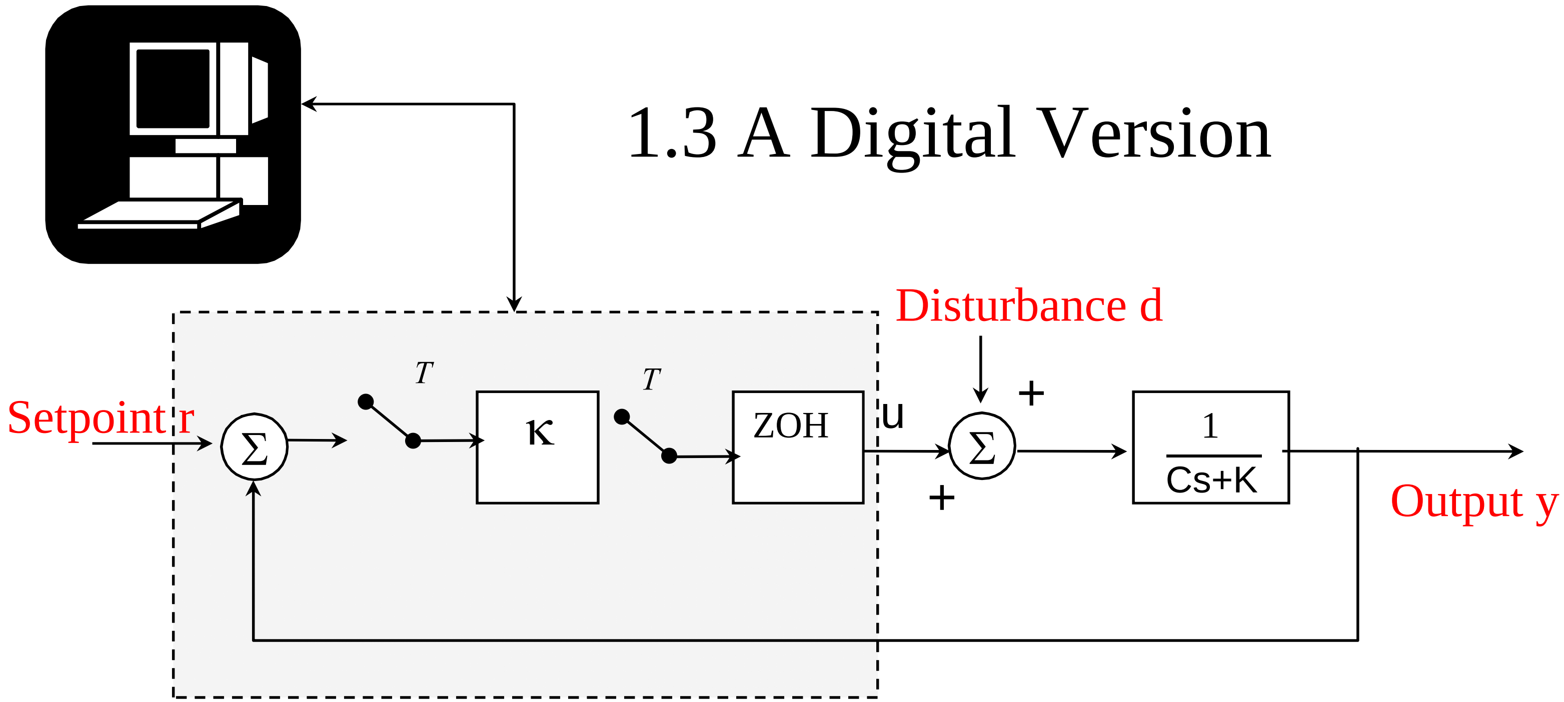
Let now $\kappa$=10, the solution changes to

$$\theta(t) = 23.91 - 0.91e^{-11t}$$

# Simulation results



# 1.3 A Digital Version




Suppose we choose to implement the controller in a computer.
Need to consider sampling of the plant output (temperature).
Then the controller only has information at the sampling instants
Input to the plant is only updated at the sampling instants
What are the consequences? For analysis, we need a digital model!

# Digital model of plant

$$C\frac{d\theta}{dt} = K\left(\theta_a - \theta\right) - Q$$

$$\frac{d\theta}{dt} \approx \frac{\theta\left(t+T\right) - \theta\left(t\right)}{T}$$

A crude approximation

$$C\frac{\theta\left(t+T\right) - \theta\left(t\right)}{T} = K\left(\theta_a - \theta\right) - Q$$

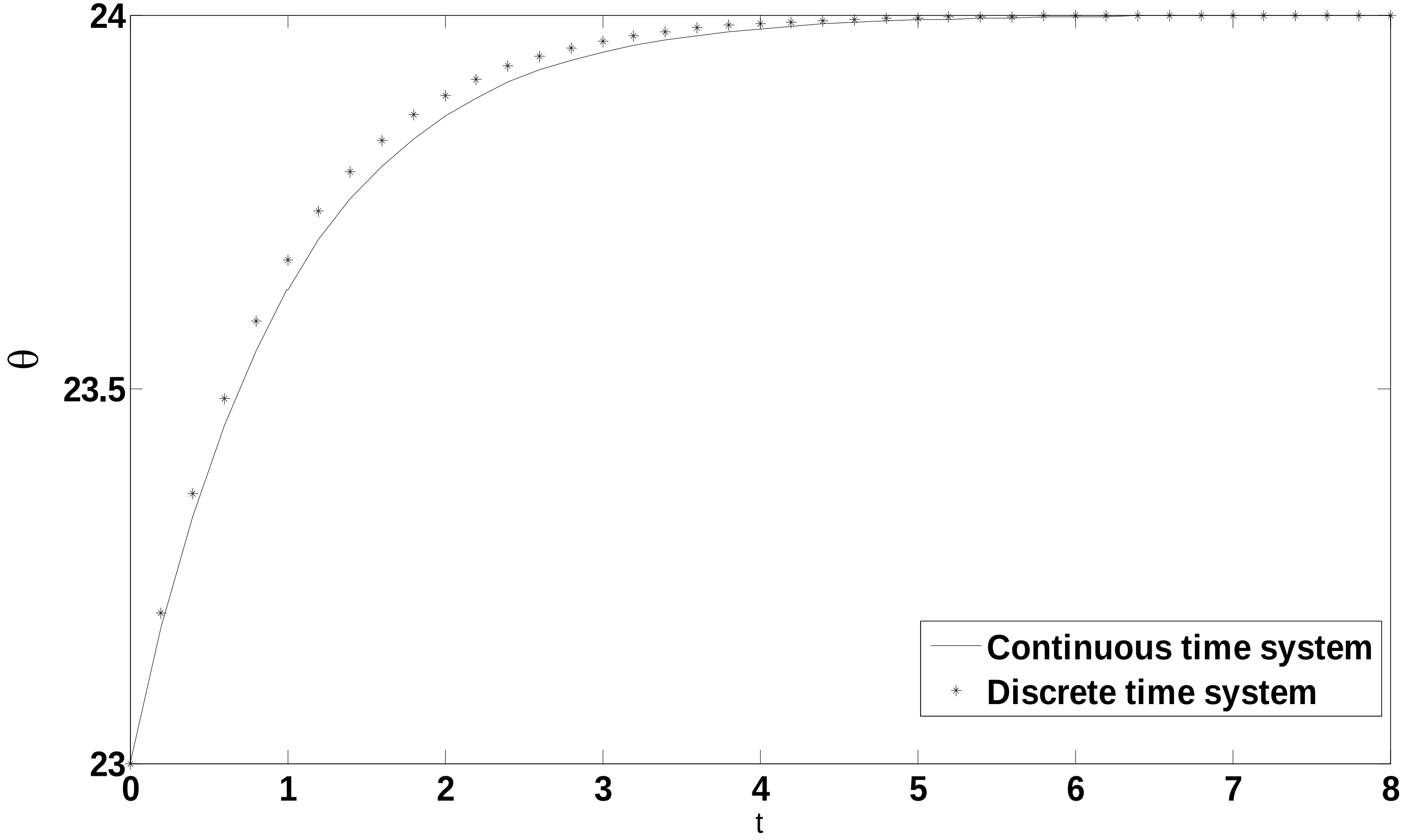
$$\theta\left(t+T\right) + \left[\frac{KT}{C} - 1\right]\theta\left(t\right) = \frac{KT}{C}\theta_a\left(t\right) - \frac{T}{C}Q\left(t\right)$$

Let $t=kT$, or index t by $k$

We have a difference equation

# Simulation comparison: continuous and discrete time system

- Let $Q$=0, $C$=1, $K$=1, $T$=0.2, $\theta_a$=24°C,$\theta$(0)=23°C,



# The Closed-Loop Control

$$\theta(k+1)+\left[\frac{KT}{C}-1\right]\theta(k)=\frac{KT}{C}\theta_a(k)-\frac{T}{C}Q(k)$$

$$Q(k)=-\kappa\left[\theta_{sp}(k)-\theta(k)\right],\kappa>0$$

$$\theta(k+1)+\left[\frac{KT}{C}-1\right]\theta(k)=\frac{T}{C}\kappa\left[\theta_{sp}(k)-\theta(k)\right]+\frac{KT}{C}\theta_a(k)$$

$$\theta(k+1)+\left[\frac{KT}{C}+\frac{\kappa T}{C}-1\right]\theta(k)=\frac{\kappa T}{C}\theta_{sp}(k)+\frac{KT}{C}\theta_a(k)$$

Notice how T affects the closed loop equations

The closed loop difference equation

## Closed-Loop steady state

$$\theta(k+1)+\left[\frac{KT}{C}+\frac{\kappa T}{C}-1\right]\theta(k)=\frac{\kappa T}{C}\theta_{sp}(k)+\frac{KT}{C}\theta_a(k)$$

If the inputs are constant and the system is stable, the output will reach some constant eventually, which does not depend on *T*.

Note

$$As \quad k\rightarrow\infty, \quad \theta(k+1)=\theta(k)=\theta_{ss}$$

$$\theta_{ss}+\left[\frac{KT}{C}+\frac{\kappa T}{C}-1\right]\theta_{ss}=\frac{\kappa T}{C}\theta_{sp}+\frac{KT}{C}\theta_a$$

What happened to *T*?
Is this reasonable?

$$\theta_{ss}=\frac{\kappa}{K+\kappa}\theta_{sp}+\frac{K}{K+\kappa}\theta_a$$

looks familiar?

# Closed-Loop transient

• CL characteristic equation: $$z+\left[\frac{KT}{C}+\frac{\kappa T}{C}-1\right]=0$$

• CL pole at $$z=1-\left[\frac{KT}{C}+\frac{\kappa T}{C}\right]$$

• Suppose K=1, C=1, κ=1: z=1-2T. When $0<T<1$, the magnitude of pole, |z| < 1, hence closed-loop is stable.

•Transient behaviour depends on pole position ( $\kappa$ , $T$ et. al)

• Note the special behaviour when the pole is at z=0.

# Response with respect to κ

- Suppose $C$=1, $K$=1, $\kappa$=1
  $\theta$(0)=23 °C, $\theta_a$=23°C, $\theta_{sp}$=24 °C
- The CL equation is

$$\theta(k+1)+(2T-1)\theta(k)=T\theta_{sp}(k)+T\theta_a(k)$$

$$\theta(k+1)=-(2T-1)\theta(k)+T\theta_{sp}(k)+T\theta_a(k)$$

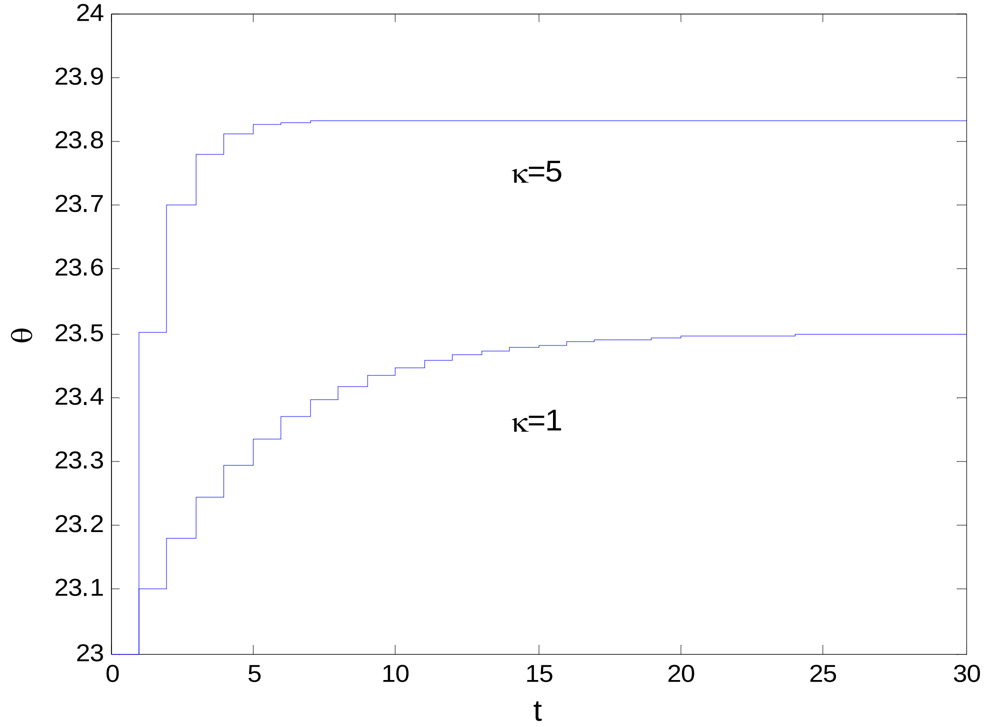
- It can be solved recursively as follows, (for $T$=0.1)

$$k=0:\theta(1)=-(2\times0.1-1)\times23+0.1\times24+0.1\times23=23.1$$

$$k=1:\theta(2)=-(2\times0.1-1)\times23.1+0.1\times24+0.1\times23=23.18$$

- Repeat the above procedure for κ=5

# Simulation results



# Responses with Respect to T

C=1,

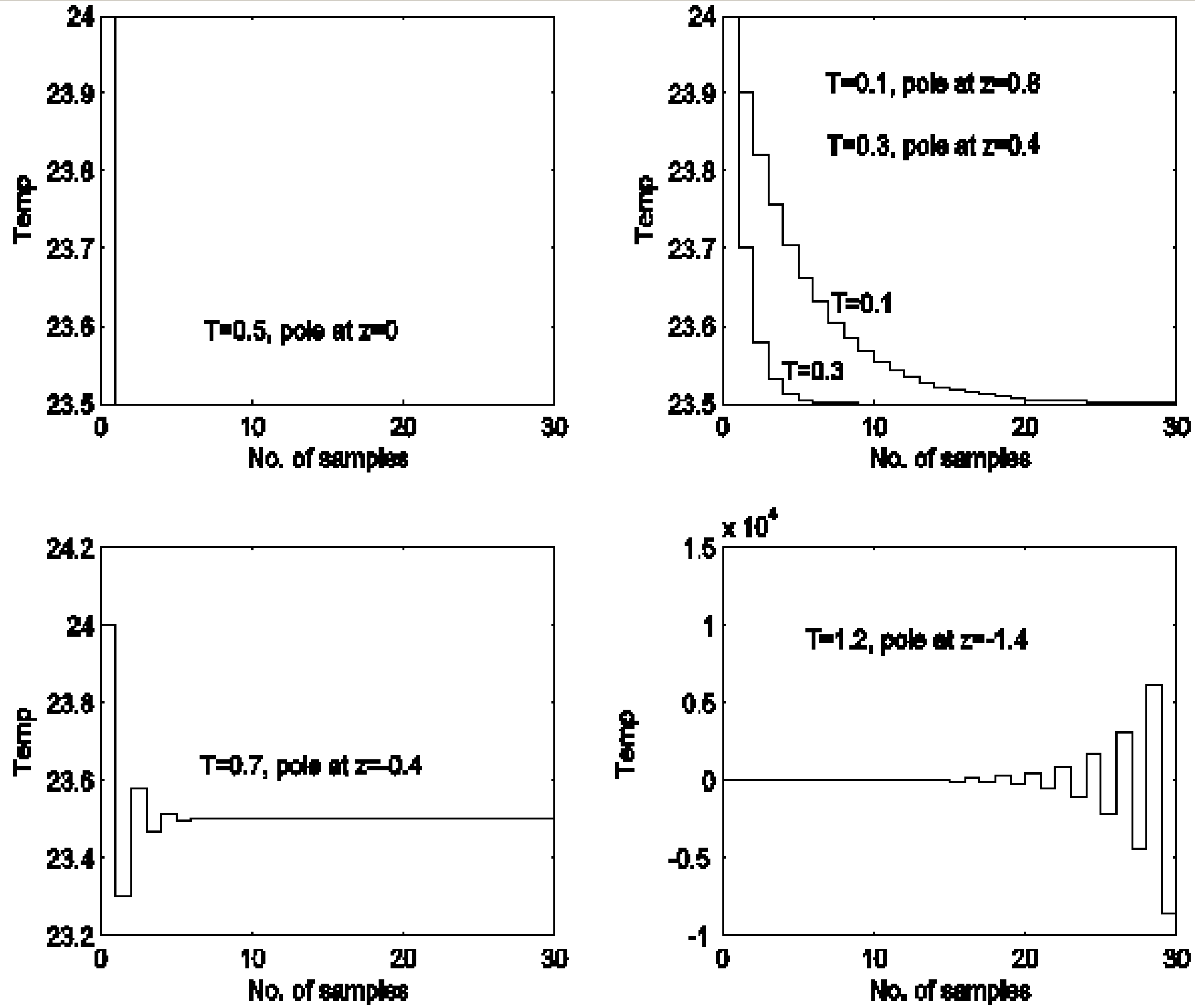
K=1,

κ=1,

$\theta_a$=23°C,

$\theta_{sp}$=23 °C,

$\theta(0)$=24 °C,




Note the different behaviours due to the sampling interval.