EE3304 Mini Project: Control System Design for Balance Systems

Important note: The due date is 25/04/2016. You may choose to hand in your report to the ECE department office, or directly to my office (E4-0807). Late submission is absolutely not allowed. Also only hardcopy will be accepted. Softcopy by email will not be allowed unless with valid reasons. You may work together with your classmates. But do write your report independently. And the results are supposed to be different from each other as the parameters are based upon your matriculation numbers.

In this mini project, you will try to design the digital control system for an inverted pendulum, which is the prototype for widely used balance systems.

1 Introduction of balance systems

In this project, we will consider the control of a balance system. By definition, a balance system is a mechanical system in which the center of mass is balanced above a pivot point [1]. Some real-world examples of balance systems are given in Figure 1.

(a) Segway

(b) A booster rocket at takeoff

(c) WowWee Mip

Figure 1 Balance systems: (a) Segway Personal Transporter (b) a booster rocket and (c) a self-balancing robot called Mip introduced by WowWee Group Limited.

The Segway® Personal Transporter (Figure 1 a) uses a motorized platform to stabilize a person standing on top of it. When the rider leans forward, the transportation device propels itself along the ground but maintains its upright position. Another example is a rocket (Figure 1b), in which a gimbaled nozzle at the bottom of the rocket is used to stabilize the body of the rocket above it. In Figure 1c, the cute little robot can freely self-balance on two dual-directional wheels and can be controlled via simple hand gestures or iOS and Android apps.

2 Cart-pendulum system

The underlying prototype for the balance systems is the famous mechanism: an inverted pendulum on a motorized cart, which is favored by control engineers for verification of many control methods. Its popularity derives in part from the fact that it is unstable without control, that is, the pendulum will simply fall over if the cart isn't moved to balance it. Additionally, the dynamics of the system are nonlinear. In this project, we will consider a two-dimensional inverted pendulum problem where the pendulum is constrained to move in the horizontal plane as shown in Figure 2.

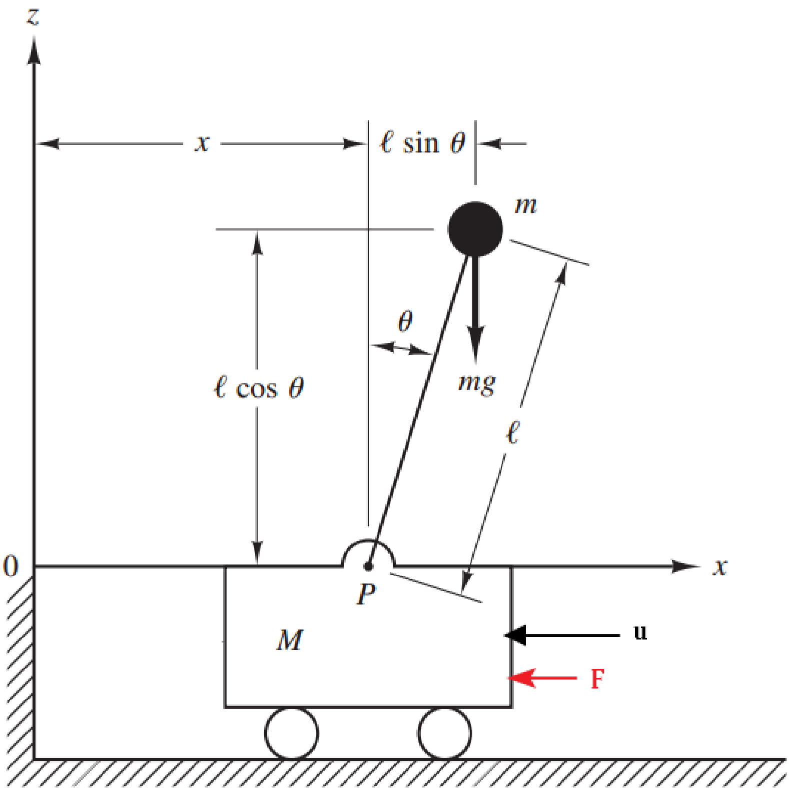

Figure 2 Cart-pendulum system

For this project, let’s assume the following quantities for the cart-pendulum system:

M	Mass of the cart (Kg)	2 + 0.08*(a-b) - 0.03*c
m	Mass of the pendulum (Kg)	0.6 + 0.06*d - 0.03*a - 0.01*b
l	Length to pendulum center of mass (m)	0.5 + 0.015*c - 0.01*(b-d)
F	External disturbance force applied to the cart (N)	Modelled as a step disturbance
x	Cart position coordinate (m)	Free variable
θ	Pendulum angle from vertical (radian)	Control target
u	Force to move the cart (N)	Control input

The numeric values for M, m and l should be determined according to your matriculation number.

The four symbols, a, b, c, d, are the last four digits of your matriculation number. For example, if your matric number is A0124789X, then the last four digits are 4789. Therefore a=4, b=7, c=8 and d=9. And the corresponding parameters are computed as

M = 2 + 0.05*(a-b) - 0.03*c = 2 + 0.05*(4-7) - 0.03*8 = 1.61

m = 0.6 + 0.06*d - 0.03*a - 0.01*b = 0.6 + 0.06*9 - 0.03*4 - 0.01*7 = 0.95

l = 0.5 + 0.015*c - 0.01*(b-d) = 0.5 + 0.015*8 - 0.01*(7-9) = 0.64

3 Dynamic model of a cart-pendulum system

For model-based control system design, the first step is to derive the dynamic model, which is usually a differential equation in continuous time. For the cart-pendulum system given in Figure 2, we assume that the mass is concentrated at the top of the rod; and in this case, the moment of inertia of the pendulum about its center of gravity is very small and we assume it to be 0. Besides, since we must keep the inverted pendulum vertical, we can assume that the angle $\theta(t)$ and angular velocity $\dot{\theta}(t)$ are small quantities such that $\sin\theta \simeq \theta$ and $\theta\dot{\theta}^2 = 0$. By this approximation, we can get a linearized model for this cart-pendulum system, which is governed by two equations as follows

$$(M+m)\ddot{x} + ml\ddot{\theta} = -u \tag{1}$$

$$ml^2\ddot{\theta} + ml\ddot{x} = mgl\theta \tag{2}$$

For detailed derivations of the above model, please refer to [2].

4 Control system design

4.1 Control objective

Try to control the pendulum's angle θ **without** regard to the cart's position *x*. To be specific, we want to keep the pendulum angle θ to a reference value, even when an external disturbance force *F* applies to the cart.

4.2 Controller design

Since we attempt to maintain θ to be the given reference *r*, the block diagram for this closed-loop control system is shown in Figure 3. (Note: this figure is just for illustration purpose and the structure of the controller may vary according to different control strategies.) *F* is the external disturbance force, which is an unknown constant.

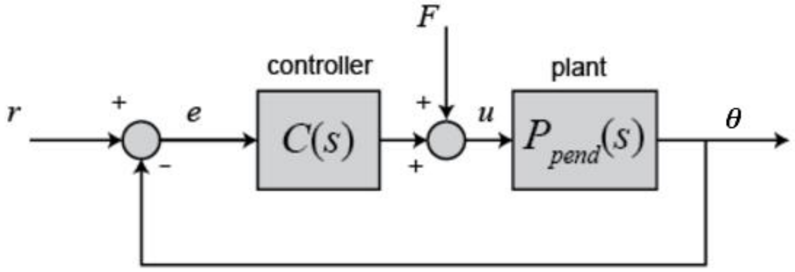

Figure 3 Block diagram of the closed-loop system

Please finish the following two tasks.

Task 1: get the transfer function

Find the transfer function $P_{pend}(s) = \dfrac{\Theta(s)}{U(s)}$ from the dynamic model given in equations (1) and (2). Is this open-loop system stable? Justify your answers.

Task 2: design digital controllers

The **design specifications** for the **unit step response** are given as follows:

1) The settling time is less than 4 seconds.
2) The overshoot is less than 10%.
3) The steady state error is less than 5%.
4) The system is able to reject step disturbance *F*.

Design the **digital** controller using the following two approaches:

1) The PID controller. It could be PI, or PD, or PID.
2) The pole-placement controller (Two-Degree-of-Freedom Controller).

For each approach, choose an appropriate sampling period. Please provide all the details including the MATLAB codes, SIMULINK Block Diagrams and all the plots of the inputs and outputs. Compare the two approaches and make your recommendation.

Note that there are no unique answers to the above design questions. You need to make your own judgement assuming you are the engineer responsible for the control system design in the real world. There are three major factors you should consider when you design and justify your controller:

1. Speed --- Transient response
2. Accuracy --- Steady state error
3. Cost ---- Size of the control signals

5 Reference

[1] Aström, K.J. and Murray, R.M., 2010. Feedback systems: an introduction for scientists and engineers. Princeton university press.

[2] Ogata, K., 2010. Modern control engineering, 5th edition. Prentice Hall PTR.