

**NATIONAL UNIVERSITY OF SINGAPORE**

**EXAMINATION FOR**

(Semester II : 2011/2012)

**EE3304 - DIGITAL CONTROL SYSTEMS**

April/May 2012 - Time Allowed: 2 Hours

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INSTRUCTIONS TO CANDIDATES

- (a) This examination paper contains **FOUR (4)** questions, and comprises **FIVE (5)** pages.
- (b) All questions are compulsory. Answer **ALL** questions.
- (c) This is a **CLOSED** book examination. Each student is allowed to bring **ONE (1)** sheet of A4 size paper.
- (d) Programmable calculators are not allowed.

**Q.1 (a)** A continuous signal ( $t$  is in the unit of second) to be sampled is

$$f(t) = \cos 4\pi t + \cos 6\pi t + \cos 8\pi t.$$

You are given two possible sampling frequencies:  $5\text{Hz}$  and  $10\text{Hz}$ . Which one do you prefer? And explain your choice.

(6 marks)

(b) Consider the function shown in Figure 1, where  $x(0)=0$ ,  $x(1)=1$ ,  $x(2)=2$ ,  $x(k)=1$ ,  $k \geq 3$ . Obtain its z-transform, assuming that the sampling period  $T$  is 1 second.

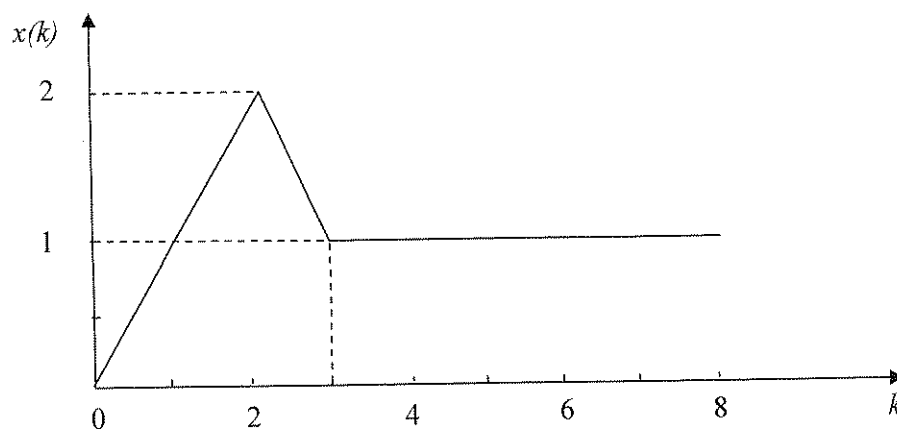


Figure 1

(9 marks)

(c) The discrete convolution sum is given by

$$x(k) = \sum_{j=0}^k h(k-j)u(j)$$

if  $u(k) = h(k) = 0$ ,  $k < 0$ . Show

$$X(z) = H(z)U(z)$$

(10 marks)

**Q.2** Consider a digital control system shown in Figure 2, where the sampling period  $T = 1s$ ,  $D(z) = k(z-1)$ , and  $n \geq 1$  is a fixed integer.

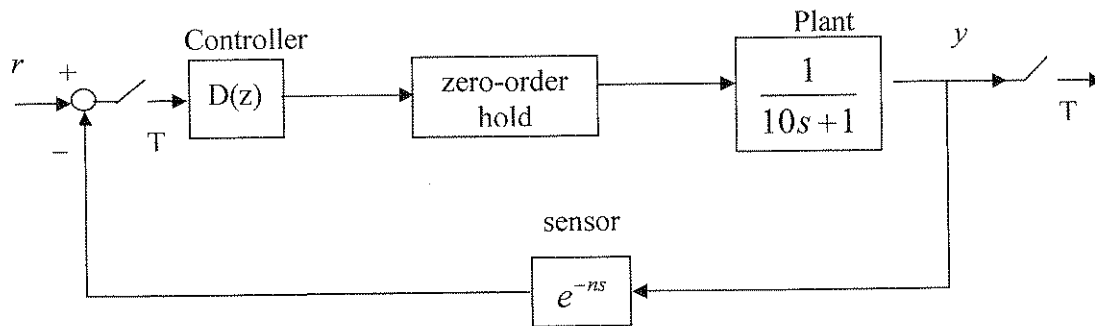


Figure 2

- (a) Determine the discrete-time transfer function of the closed-loop system,  $\frac{Y(z)}{R(z)}$ .  
(13 marks)
- (b) Let  $n=1$  and  $k=1$ . Find the output time response when the input is the unit pulse.  
(9 marks)
- (c) Is the closed-loop system stable and why?  
(3 marks)

**Q.3** A process model is given by

$$\frac{1}{z-2}.$$

The digital controller is designed in a unit feedback configuration to improve the performance of the system.

- (a) Assume that the model is not known and the PID controller is used, is it possible to use the Ziegler-Nicholes (ZN) auto-tuning methods to design the control parameters for the PID controller? If yes, then which ZN method can be applied? Justify your answers.

(3 marks)

- (b) Is it possible to apply the frequency domain design methods such as phase lead or lag compensators to improve the performance of the system? If yes, then which type of compensator is preferred? Justify your answers.

(3 marks)

- (c) Assume that the process model is known, design a first order digital controller to meet following requirements:

- 1) place the closed-loop poles within a circle  $z \leq 0.9$ ;
- 2) achieve a finite steady state error for a ramp input;
- 3) minimize the steady state error for a ramp input.

Show that the steady state error for a ramp input is proportional to the sampling period.

(19 marks)

**Q.4 (a)** A process is described by the transfer function

$$H(z) = \frac{\alpha z + 1}{z^2 + z}.$$

Design a pole placement controller in the form of

$$U(z) = \frac{T(z)}{R(z)} U_c(z) - \frac{S(z)}{R(z)} Y(z)$$

such that the closed loop transfer function from the command signal,  $u_c(k)$ , to the system output,  $y(k)$ , follows the reference model,  $\frac{1}{z^2}$ . Discuss the condition on the parameter  $\alpha$  such that perfect tracking is attainable.

(12 marks)

(b) Assuming the parameter  $\alpha = 0$ , design a one-step-ahead controller to make the output of the system,  $y(k)$ , follow any arbitrary desired output,  $y^*(k)$ .

(8 marks)

(c) Is it possible to simply regard the one-step-ahead controller designed in (b) as a special type of pole-placement controller? Justify your answer.

(5 marks)

**END OF PAPER**