# EE3304 (Part II)
# Digital Control Systems

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## *Course Outline*

1. **Background Overview**
2. **Control Performance Specifications**
3. **Time Domain Design – Digital  PID Control**
4. **Frequency Domain Design**
5. **Pole-placement Controller**
6. **Case Study of Real World Application**

## *Primary Textbooks*

- The lecture notes
- GF Franklin, JD Powell and ML Workman, *Digital Control of Dynamic Systems*, 3rd Edition, Addison Wesley, 1998.
  (Chapter 5)

## *References for Background Review*

- EE2010 Lecture Notes
- K Ogata, *Modern Control Engineering*, 7th Edition,

## *Additional note on supporting software*

- Students are expected to be familiar with the computational software tool MATLAB and its package SIMULINK.

If you do not have access to MATLAB, please visit the PC clusters at Engineering:
http://www.eng.nus.edu.sg/eitu/pc.html

## ***What do I expect from you?***

**1.** Be prepared. Roughly go through the lecture notes before the class.

**2.** I am going to spoon-feed you **with lots of questions !**

These questions are designed to arouse your interest and to help you to figure out most of the stuff by **your own thinking**!

You will HAVE FUN by actively participating in thinking and discussing these questions.

It will be a WASTE of TIME if you just want to passively listen to the answers.

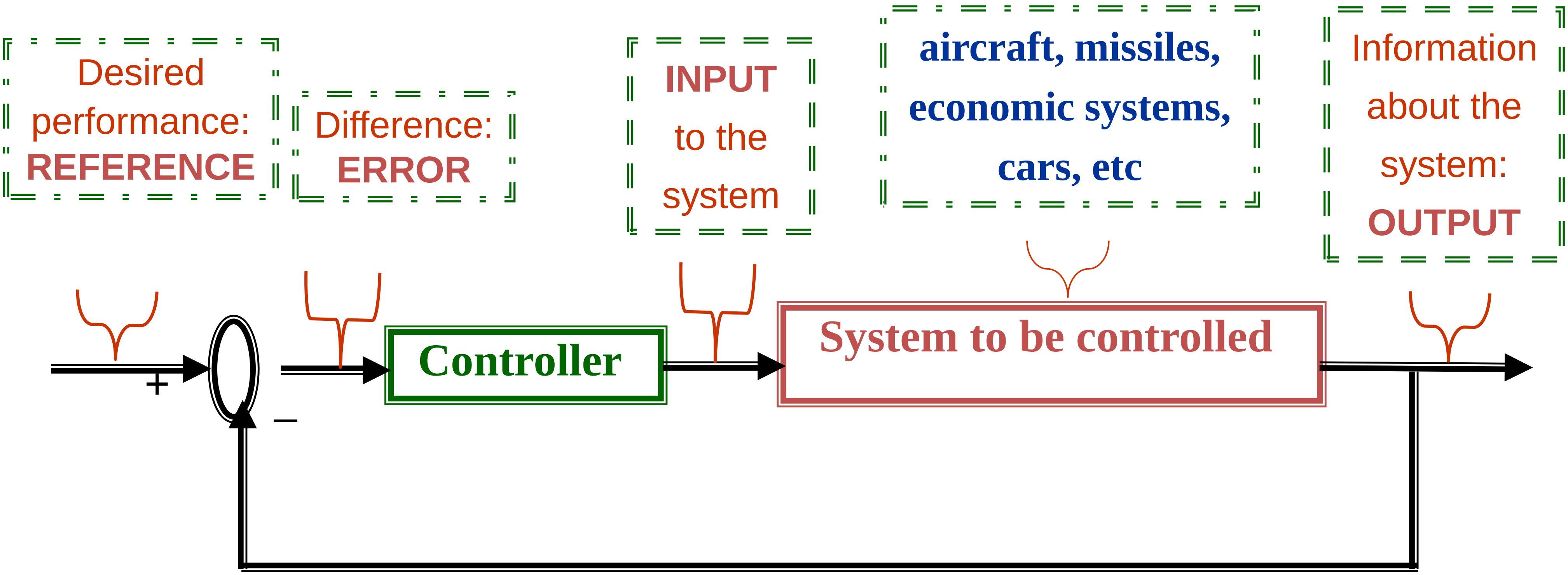
**3.** Do the mini-project by yourself.

You can discuss the questions with your classmates.

But DO NOT copy and paste!

4. Please use Anonymous Feedback in IVLE! Tell me what you want from me!

# What is a feedback control system?




***Objective:*** To make the system OUTPUT and the desired REFERENCE as close as possible, i.e., to make the ERROR as small as possible.

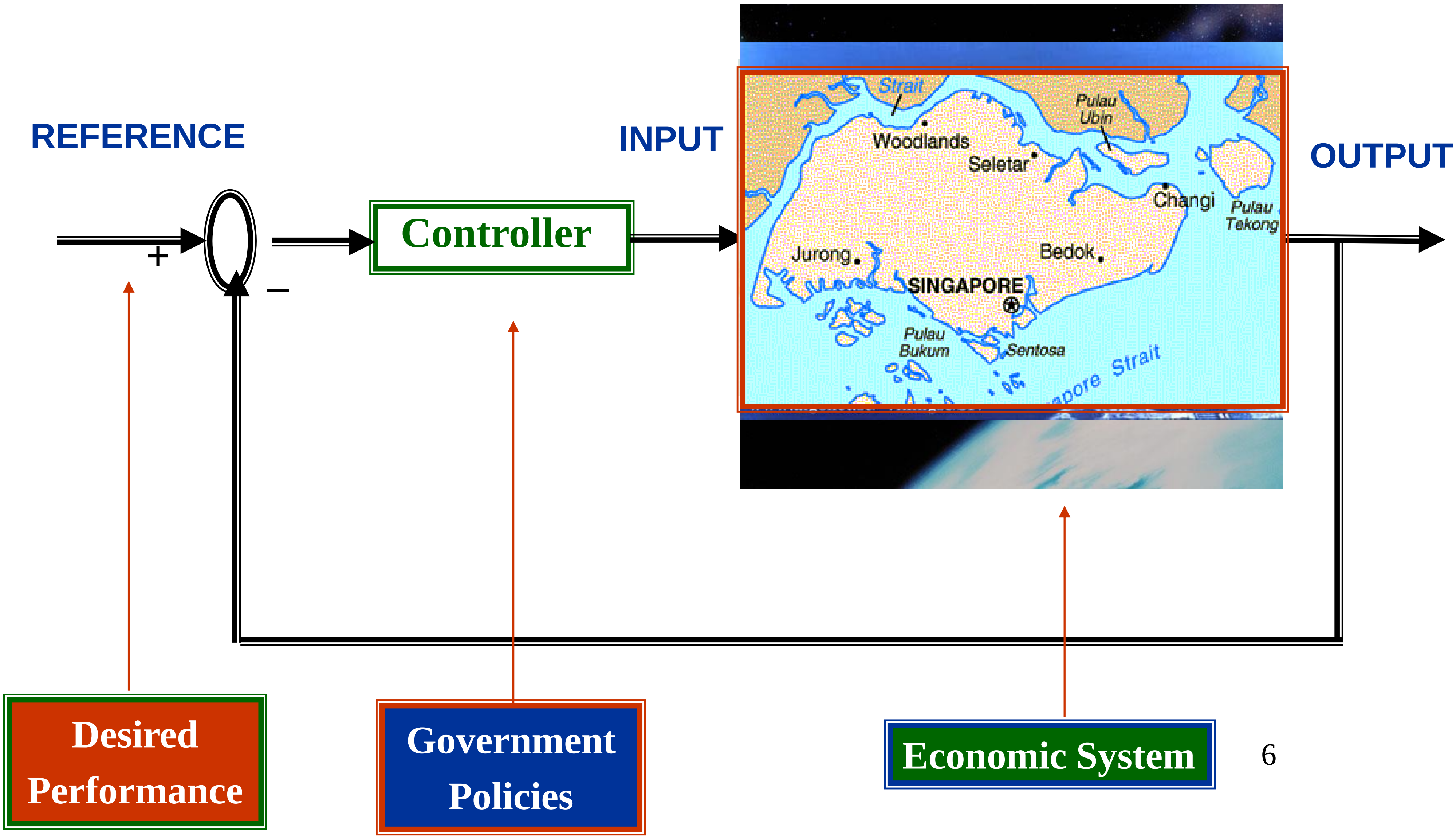
***Key Issues:*** 1) How to describe the system to be controlled? (Modelling)

2) How to design the controller? (Control)



The primary focus of this module is of course: controller design

## *Some Control Systems Examples:*






**Who can give me one example of control system in this classroom?**

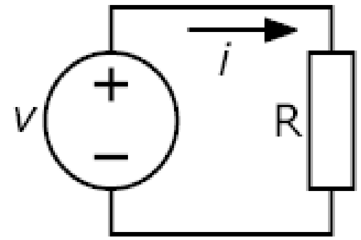
## How to implement the controller in reality?

In the good old days, we used analog circuit design.

## How to implement proportional controller with a simple circuit?

resistor

$$\rightarrow \boxed{K} \rightarrow$$




$$v(t) = Ri(t)$$

## How to implement derivative controller?

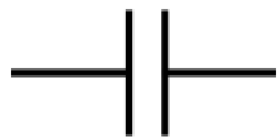
$$\rightarrow \boxed{K_d s} \rightarrow$$


$$v(t) = L\frac{di(t)}{dt}$$

## How to implement integral controller?

$$\rightarrow \boxed{K_i \frac{1}{s}} \rightarrow$$

capacitor



$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$$

## How to implement nonlinear controller?

$$\rightarrow \boxed{f(u)} \rightarrow$$

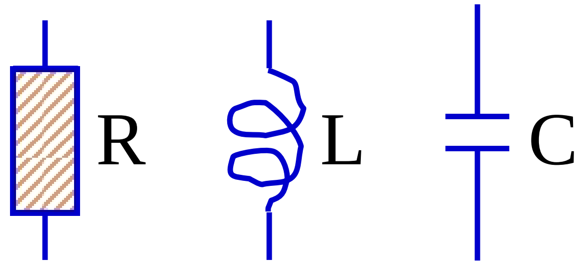
$$y = f(u) = \frac{u}{u^2+1}$$

Not easy!

This is one of the reasons that digital controllers are replacing the analog controllers!

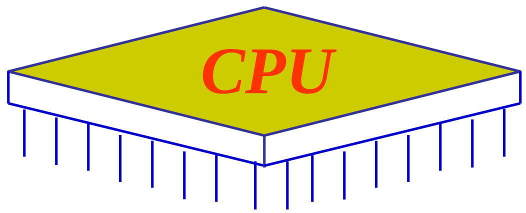
## ***Why digital control?***

The main drive is the cost!




The cost of an analog system increases linearly with the number of control loops;

The initial cost of a digital system might be large, but the cost of adding an additional loop is small.




Flexibility is another advantage

How to change the controller for analog systems?

Rewiring!

How to change the control algorithm for digital system? Reprogramming!

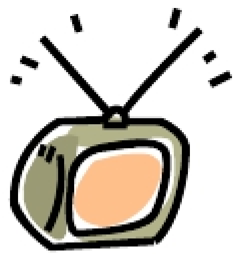
Efficiency is also a factor. For instance, nonlinear control cannot be easily wired, but it can be easily programmed!

Most of control systems nowadays are implemented using either computers such as PCs or Digital Signal Processors (DSP), which are specially designed to carry out computations related to control algorithm realizations.

The advantages of digital controllers using PC or DSP are obvious:

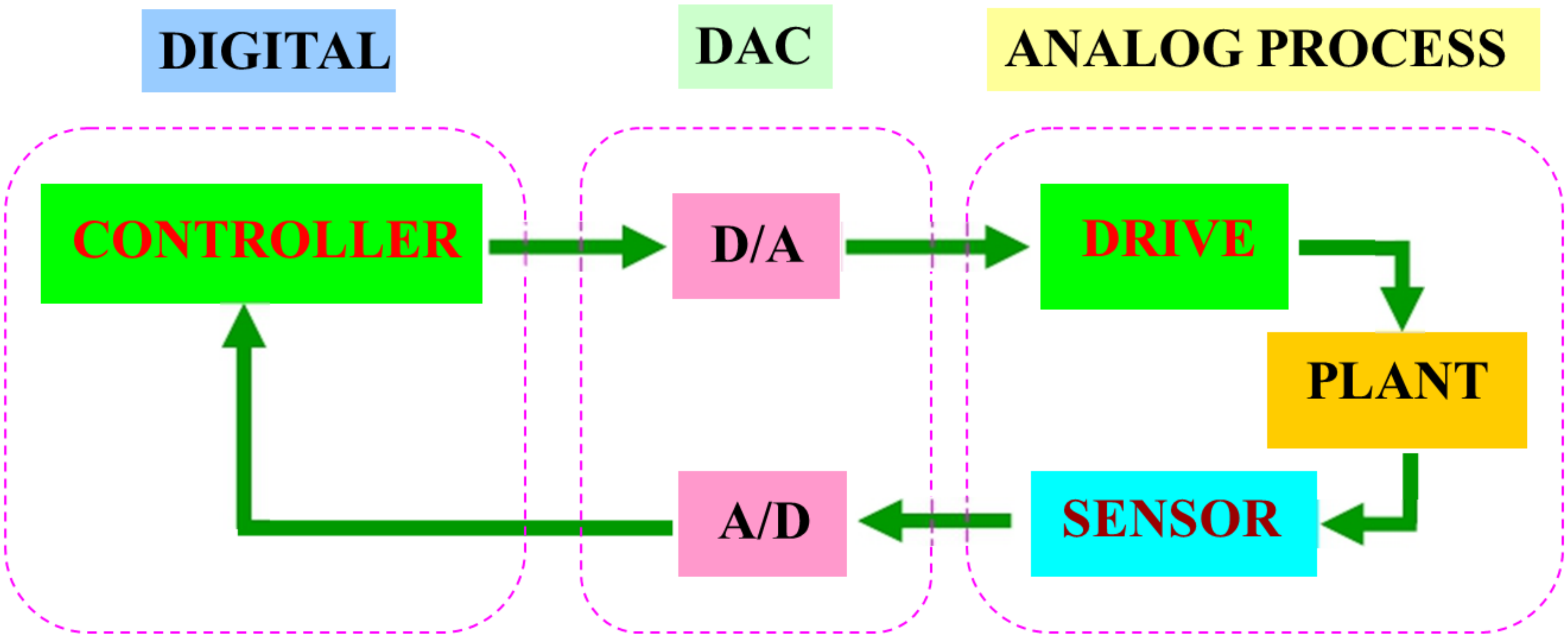
it is fast, reliable, reusable and can be modified through simple coding whenever needed.

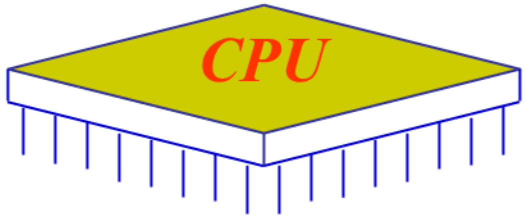
We are moving from analog world to digital world: digital cameras, digital TV, digital communication (your hand phone), digital computer, DVD player, MP3, etc. 21th century is a digital century.



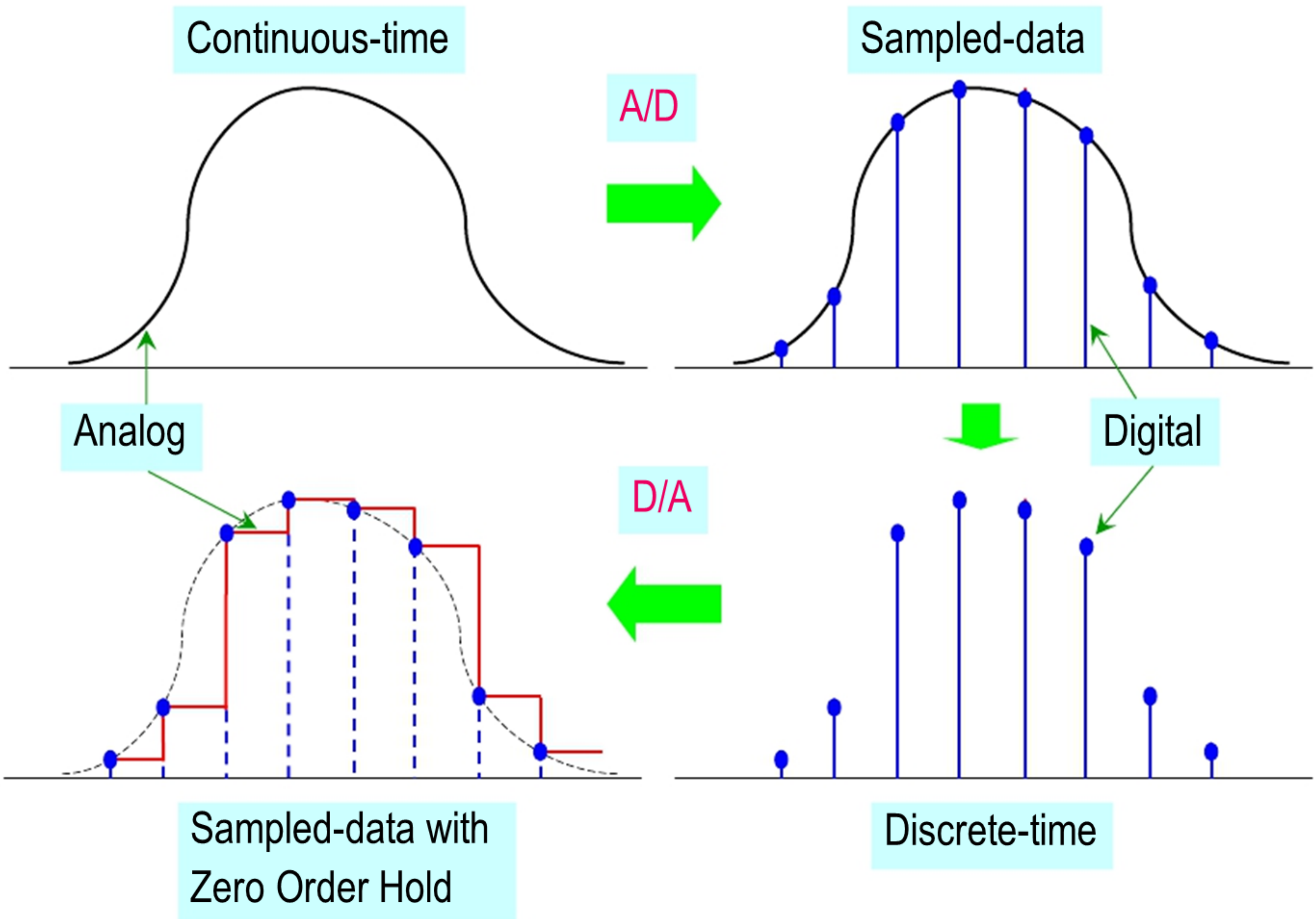
**<u>What is the nature of digital signals compared to analog signals?</u>**

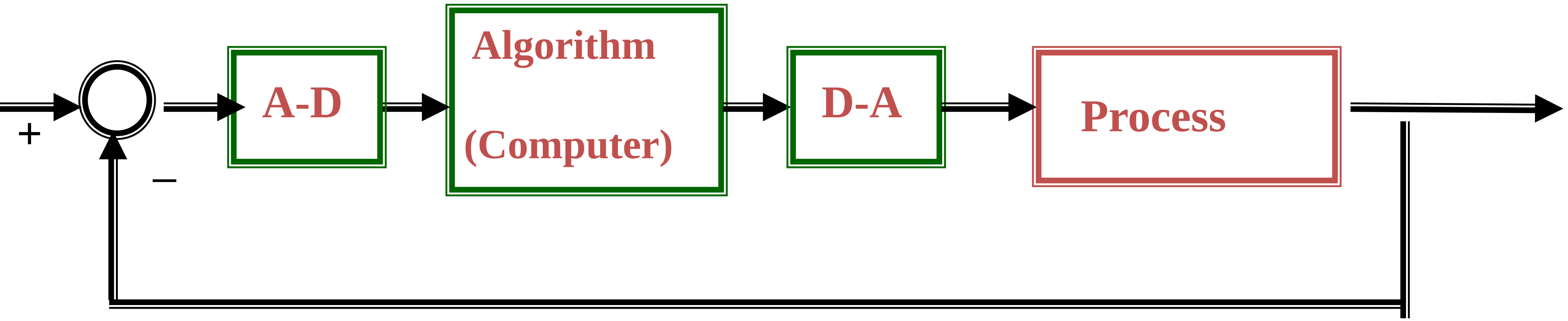
Digital signals are only defined on a sequence of discrete-time instances. It is discrete-time instead of continuous time.



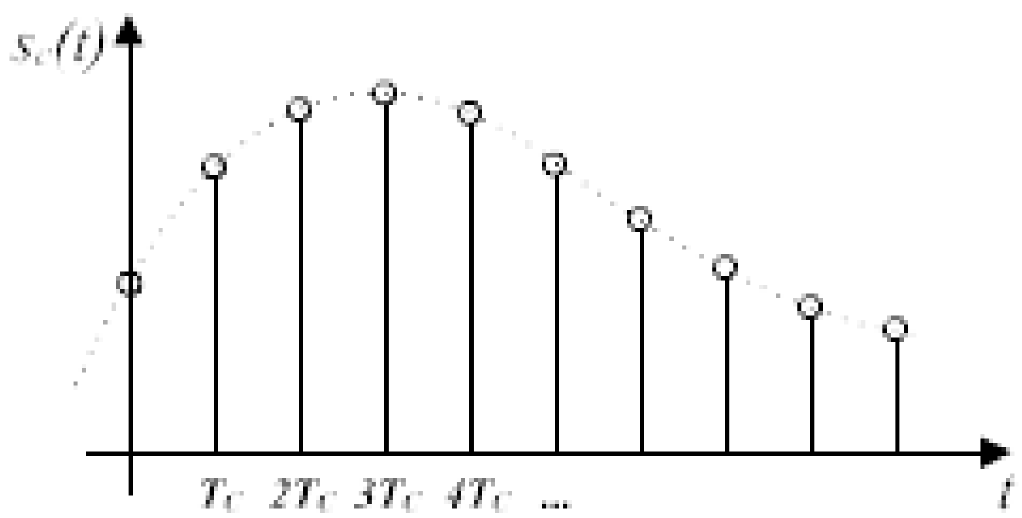

| | |
|---|---|
| D/A | Zero Order Hold |
| A/D | Sampler |


Continuous-time
A/D
Sampled-data
Analog
Digital
D/A
Sampled-data with
Zero Order Hold
Discrete-time

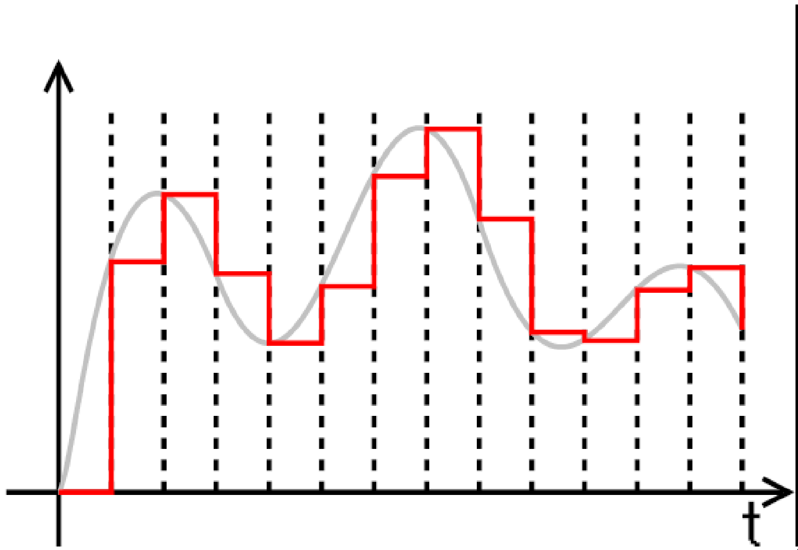



**How to do the A-D?**

Sampling!



**How to do the D-A? What's the natural way of converting the discrete-time signal into continuous-time signal?**

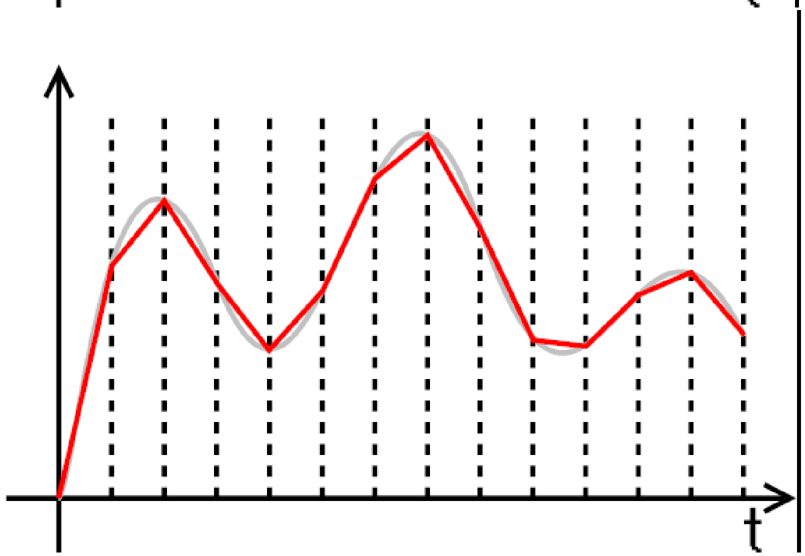



Zero-order Hold

**Is the resulting analog signal continuous or dis-continuous?**

It is dis-continuous.

**Can we make it piece-wise continuous?**




First-order Hold

Which one looks better?

Why don't we use First-order Hold to convert the control signal from discrete-time to continuous time?

For feedback control system, the control signal depends upon the measured output.

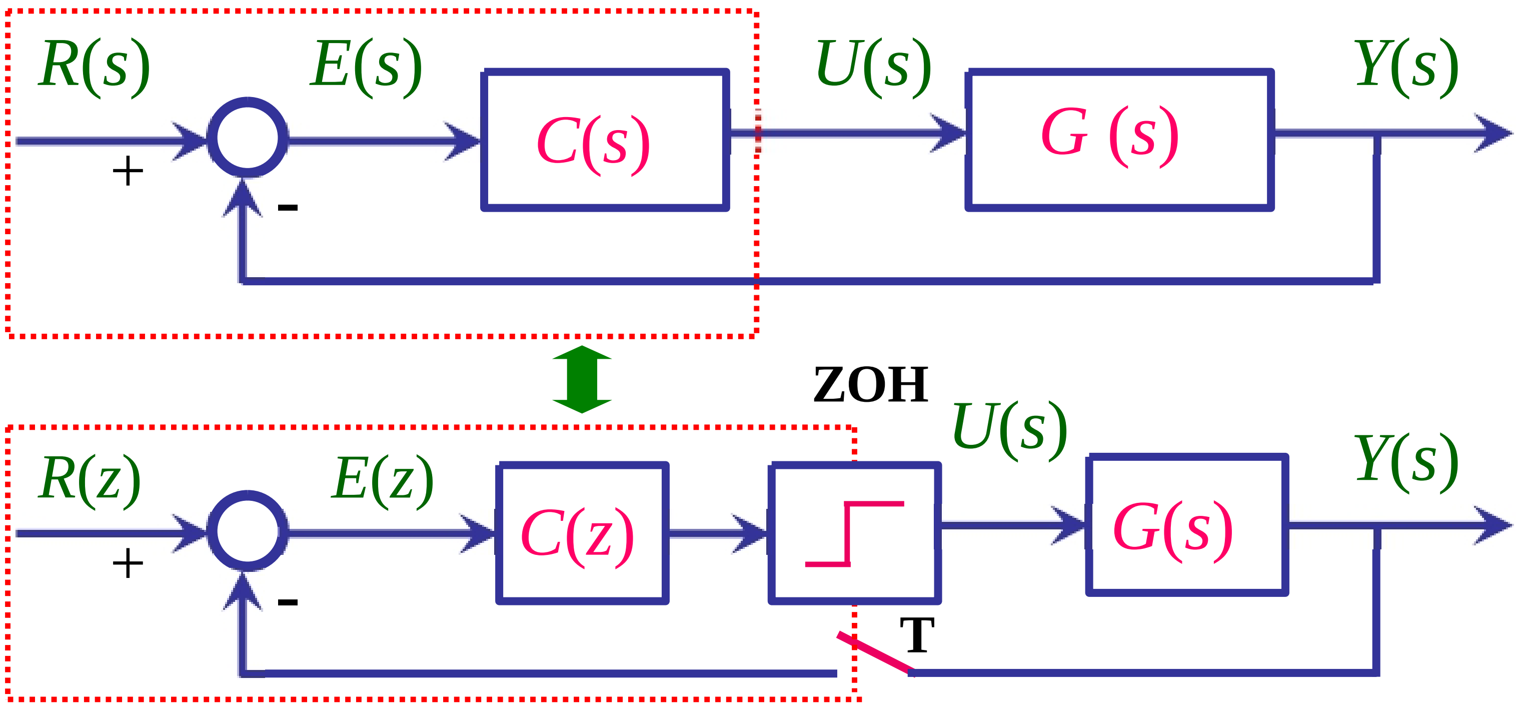
**Can we measure the future output now?**

No.

**Imagine that you are the engineer, and you know how to design the analog controller. What is the easiest way to convert that into a digital controller?**

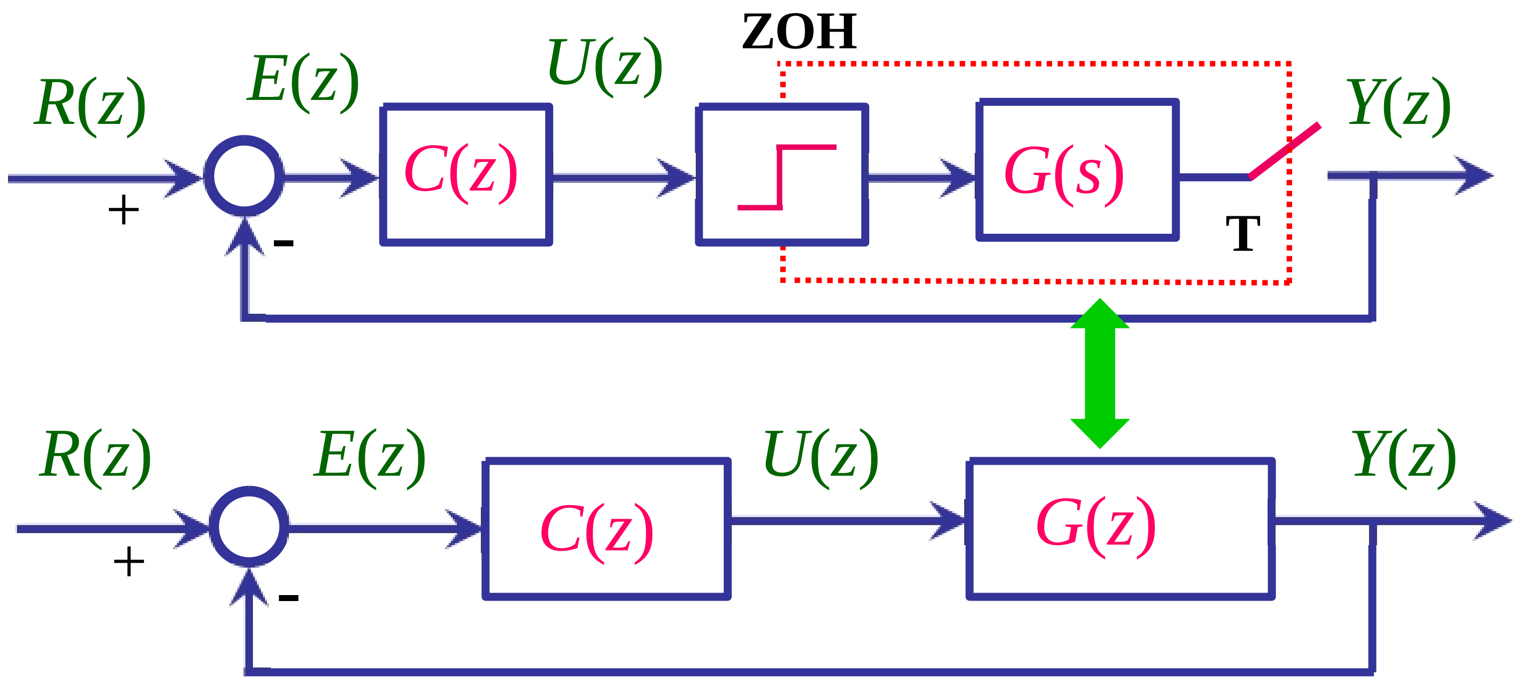
There are two ways to design a digital controller or a discrete-time control system:

- Follow whatever we have learnt in EE2010 to design a continuous-time controller and then discretize it using bilinear transformation or any other discretization technique to obtain an equivalent digital controller (**Emulation**).




The above design works very well if sampling period T is sufficiently small.

• Alternatively, one could discretize the plant by zero-order-hold first to obtain a sampled-data system or discrete-time system and then apply digital control system design techniques to design a digital controller (**Discrete-Time System Design**)



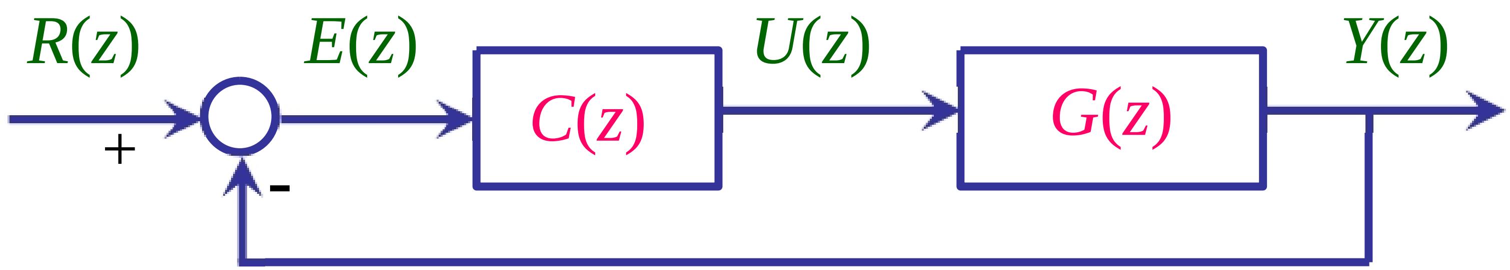

It has been shown in Part I that

$$G(z) = (1 - z^{-1}) Z\{\frac{G(s)}{s}\}$$

You can also use Table 2.1 or command "c2d" in MATLAB

***Feedback control in discrete setting***

Let us examine the following block diagram of control system:




**How to write down transfer functions for basic block diagrams?**

What is the feed-forward T.F. from R to Y?

$C(z)\, G\, (z)$

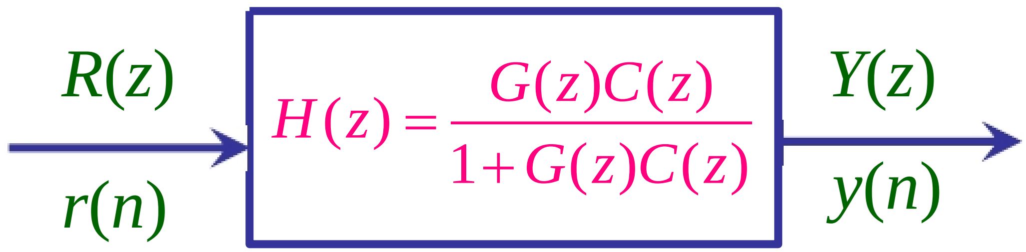
What is the open loop T.F.?

$C(z)\, G\, (z)$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{R(z)} = \frac{C(z)G(z)}{1+C(z)G(z)}$$

Closed-loop transfer function from $R$ to $Y$

Thus, the block diagram of the control system can be simplified as,




The problem becomes how to choose an appropriate $C(z)$ such that the closed loop $H(z)$ will have desired properties.

**<u>What are the desired properties that we want the closed-loop system to have?</u>**

Before we design any control system, we need to know the goal , or the performance specifications!

# ***Break***

***State-of-the-art control systems***

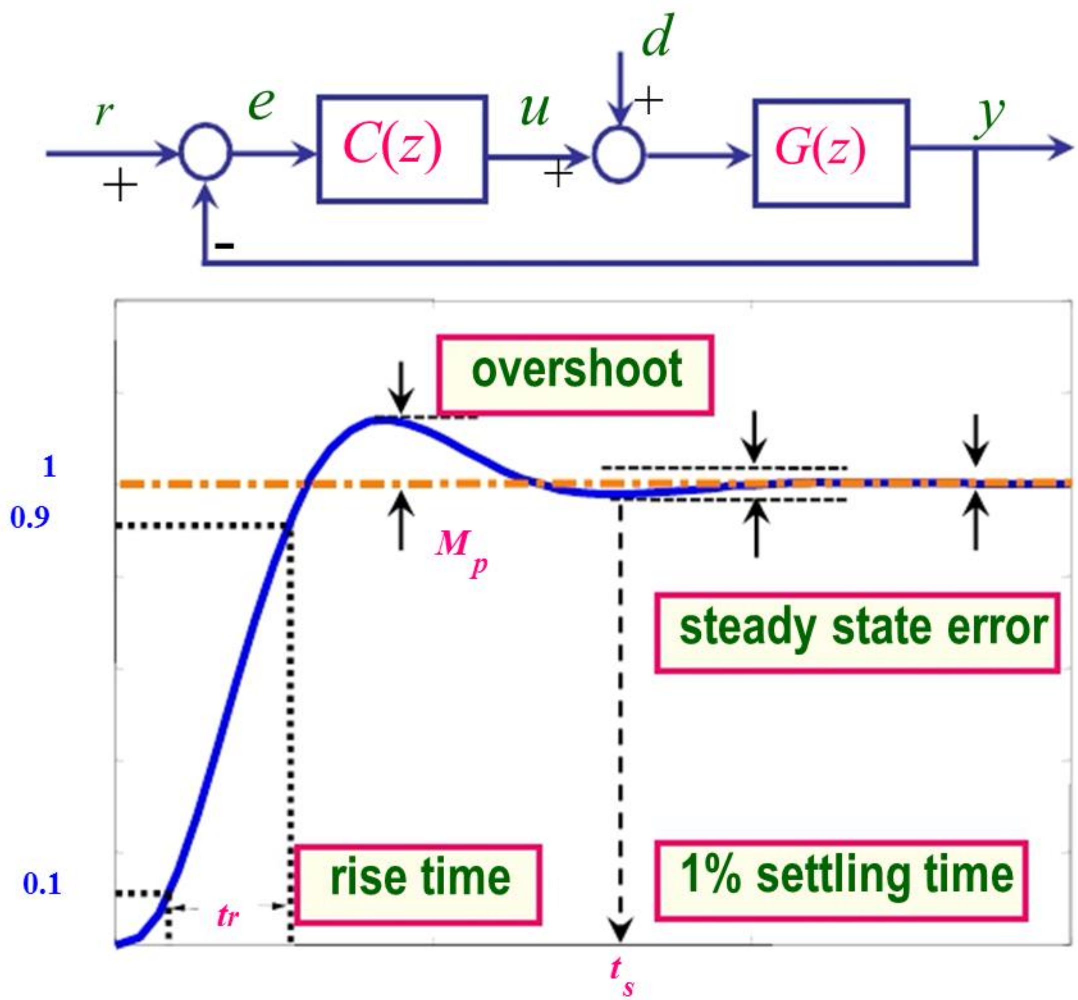
***Big Dog***

# *Control Performance Specifications*

*Time domain design specifications*:

- stability
- steady state accuracy
- settling time
- overshoot
- rise time
- disturbance rejection
- others

Consider a unity feedback system

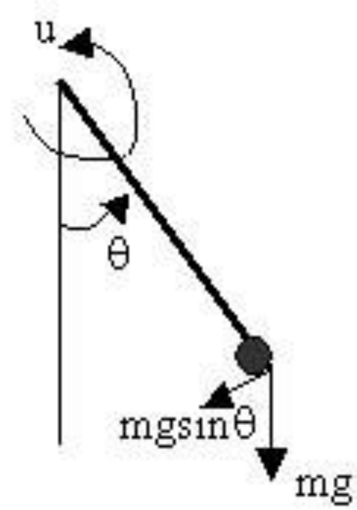


**Among all these requirements, which one if the No. 1 requirement?**

First Requirement: The whole system has to be stable!

**Why is stability so critical for the control system design?**

- **The fundamental question: what would happen if there is a small disturbance striking the system?**




•The pendulum example

**Stable**: small deviation from the equilibrium position will remain small.

**Asymptotic Stable**: small deviation from the equilibrium position will not only remain small, but also decrease to zero.

**Unstable**: small deviation from the equilibrium position will result in huge deviation in the long run.

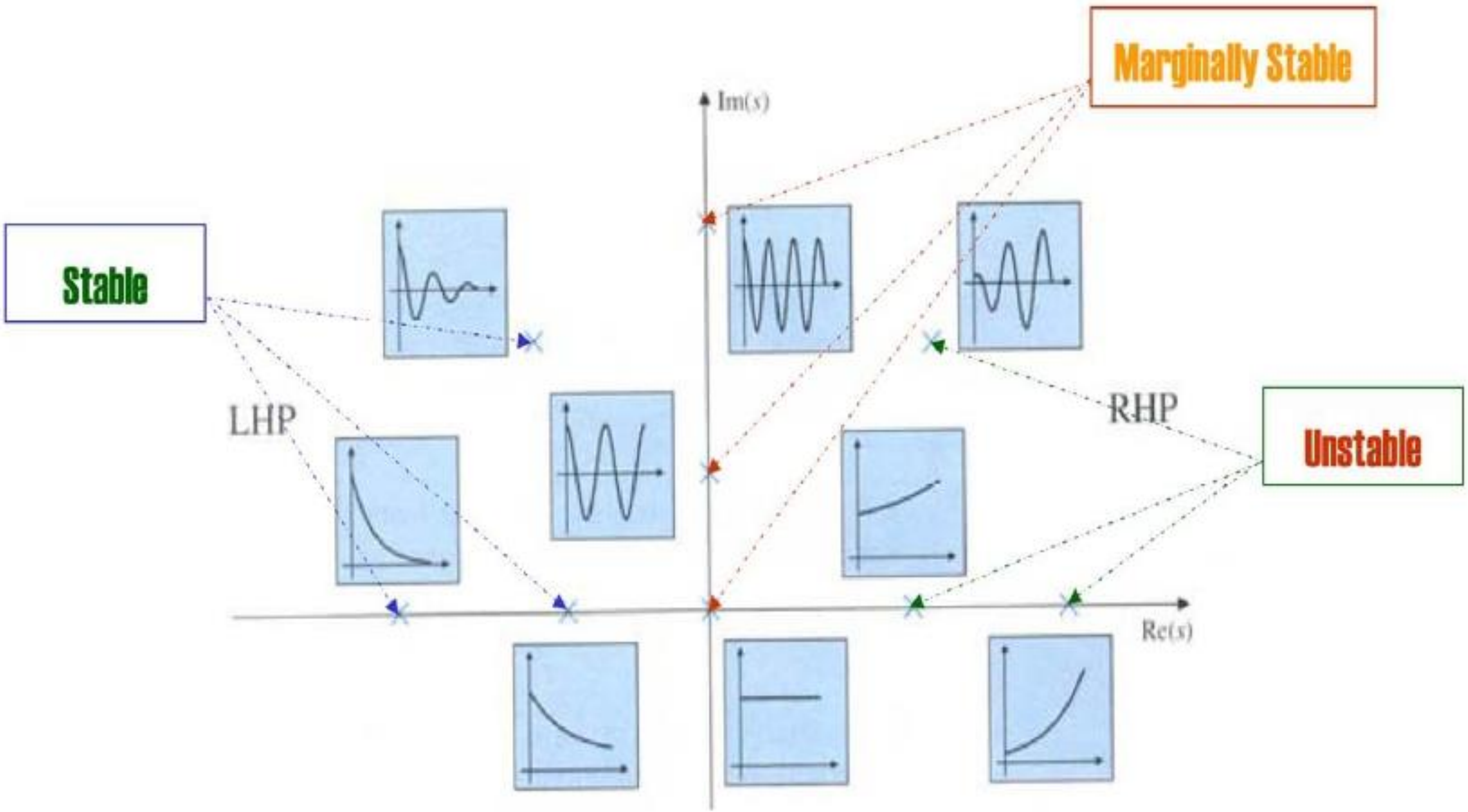
**Stabilization:** In most of the control problem in industry, the objective is to make the system stable by proper design of the controller, such that any small deviation from the desired point will be corrected.

•**A Stabilization Example**

*Stability criterion — Continuous-time systems*

**<u>Where are the poles for stable system in continuous-time system?</u>**

A continuous-time system is said to be stable if its poles lie in the left half of the s-plane. It is unstable if it has poles in the right half of the s-plane.

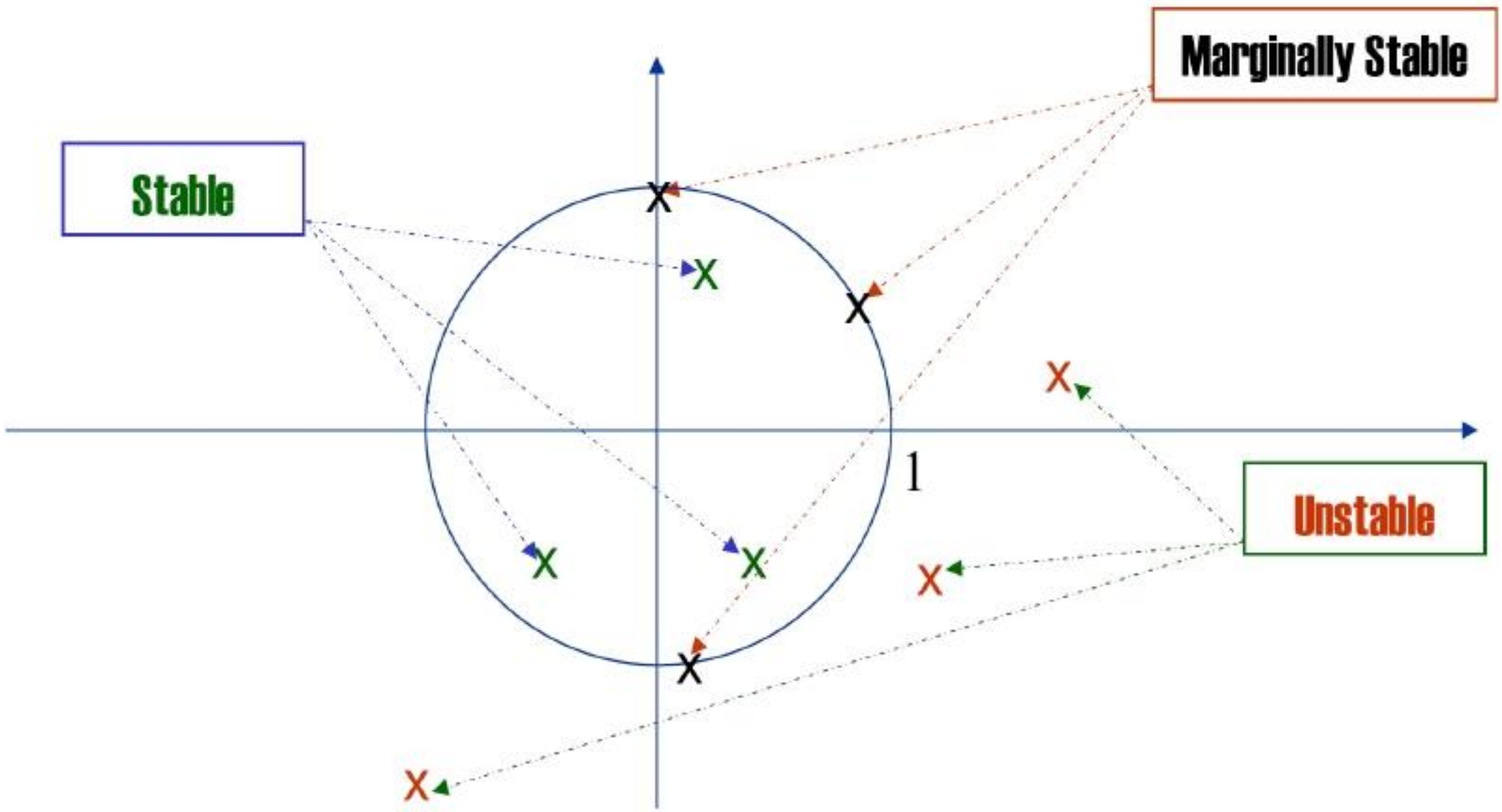





Things are little bit tricky when the poles lie on the imaginary axis.

## Where are the poles for stable system in discrete-time system?

A discrete-time system is said to be stable if its poles lie inside the unit circle. It is unstable if it has poles outside the unit circle.






Things are little bit tricky when the poles lie exactly on unit circle.

The relationship between poles and stability:

Partial Fraction expansion of Transfer Function (Impulse Response)

$$H(s)=\frac{1}{s^2-1}=\frac{1}{2}\left(\frac{1}{s-1}-\frac{1}{s+1}\right)$$

$$H(z)=\frac{z+1}{(z-0.5)(z-2)}$$

If λ is the pole, then

$\frac{1}{s-\lambda}$ must be one component in H(s)

$\frac{1}{z-\lambda}$ must be one component in H(z)/z

$e^{\lambda t}$ is a component of the impulse response

$\lambda^k$ is a component of the impulse response

$\lambda=\sigma+j\omega$ $e^{\sigma t}$ --stability

$e^{\lambda t}=e^{\sigma t}e^{j\omega t}$ $e^{j\omega t}$ ---oscillation

$\sigma<0$ $e^{\sigma t}\rightarrow 0$ Stable

$\sigma>0$ $e^{\sigma t}\rightarrow\infty$ Unstable

$\sigma=0$ $e^{\sigma t}=1$ Marginal Stable

$\lambda=\rho e^{j\theta}$ $\rho^k$ ---stability

$\lambda^k=\rho^k e^{j\theta k}$ $e^{j\theta k}$ ---oscillation

$\rho<1$ $\rho^k\rightarrow 0$ Stable

$\rho>1$ $\rho^k\rightarrow\infty$ Unstable

$\rho=1$ $\rho^k=1$ Marginal Stable

## Multiplicity of the poles

Partial Fraction expansion of Transfer function (Impulse Response)

$$H(s)=\frac{3s+1}{(s-1)(s+1)^2}=\frac{1}{s-1}-\frac{1}{s+1}+\frac{1}{(s+1)^2}$$

If λ is the multiple pole, then

$\frac{1}{(s-\lambda)^2}$ must be one component in H(s)

$te^{\lambda t}$ is component of the impulse response

$\lambda=\sigma+j\omega$ $\quad te^{\sigma t}$ ---stability

$te^{\lambda t}=te^{\sigma t}e^{j\omega t}$ $\quad e^{j\omega t}$ ---oscillation

$\sigma<0 \quad te^{\sigma t}\to 0$ Stable

$\sigma>0 \quad te^{\sigma t}\to\infty$ Unstable

$\sigma=0 \quad te^{\sigma t}=t\to\infty$ Unstable

$\frac{1}{(z-\lambda)^2}$ must be one component in H(z)/z

$k\lambda^k$ is component of the impulse response

$\lambda=\rho e^{j\theta}$ $\quad k\rho^k$ ---stability

$k\lambda^k=k\rho^k e^{j\theta k}$ $\quad e^{j\theta k}$ ---oscillation

$\rho<1 \quad k\rho^k\to 0$ Stable

$\rho>1 \quad k\rho^k\to\infty$ Unstable

$\rho=1 \quad k\rho^k=k\to\infty$ Unstable

Stability criterion

Continuous-time

$$\lambda = \sigma + j\omega$$

$\sigma < 0$ Stable

$\sigma > 0$ Unstable

$\sigma = 0$

Single

Multiple

Marginal Stable

Unstable

Discrete-time

$$\lambda = \rho e^{j\theta}$$

$\rho < 1$ Stable

$\rho > 1$ Unstable

$\rho = 1$

Single

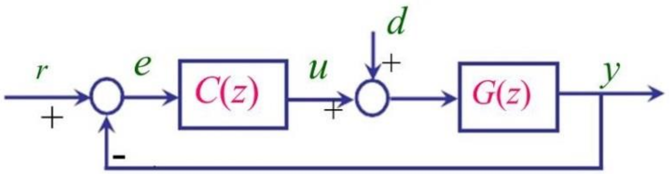
Multiple

Marginal Stable

Unstable

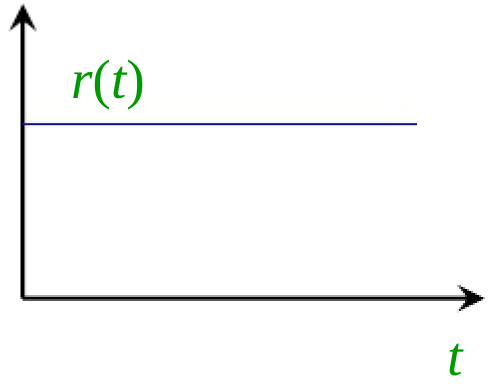
After the stability is guaranteed, we may consider other requirements:

***Steady state accuracy — Continuous time systems***




Case 1:

$r(t) = 1$ is a unit step reference, $R(s) = \dfrac{1}{s}$




$$E(s) = \frac{R(s)}{1+G(s)C(s)} = \frac{1}{1+G(s)C(s)}\frac{1}{s}$$

**How to compute the steady state value of the error ?**

Final Value Theorem: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+G(s)C(s)}\frac{1}{s}$$

$$= \frac{1}{1+G(0)C(0)} = \frac{1}{1+\beta_p}$$

**position error constant**

$$\beta_p = \lim_{s\to 0} G(s)C(s)$$

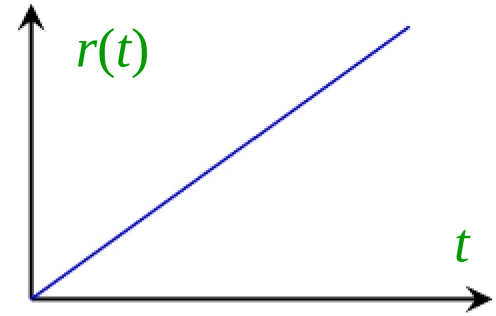
**Is it possible to make the steady state error go to zero?**

Make the position error constant $\beta_p$ infinity!

If G(s)C(s) has at least ONE integrator 1/s!

## Case 2:

$r(t) = t$ is a unit ramp reference, $R(s) = \frac{1}{s^2}$




Use final value theorem

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{1}{1 + G(s)C(s)} \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{sG(s)C(s)} = \frac{1}{\lim_{s \to 0} sG(s)C(s)} = \frac{1}{\beta_v}$$

**velocity error constant**

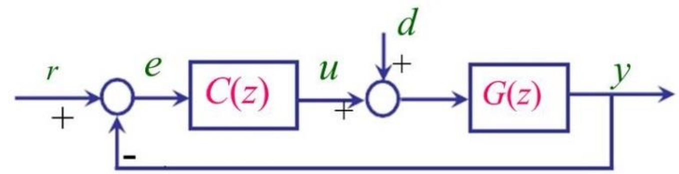
$$\beta_v = \lim_{s \to 0} sG(s)C(s)$$

**Is it possible to make error go to zero?**

Make the velocity error constant $\beta_v$ infinity!

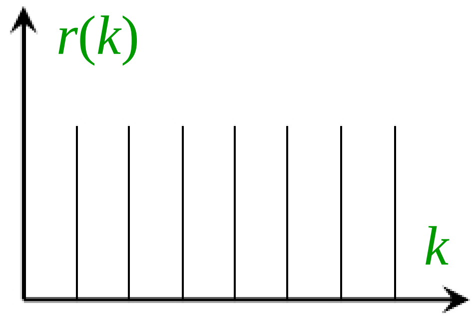
If G(s)C(s) has at least TWO integrator 1/s!

***Steady state accuracy — Discrete time systems***




Case 1:

$r(k) = 1$ is a unit step reference, $R(z) = \dfrac{z}{z-1}$




$$E(z) = \frac{R(z)}{1+G(z)C(z)} = \frac{1}{1+G(z)C(z)}\frac{z}{z-1}$$

Use final value theorem $\lim\limits_{t\to\infty} f(k) = \lim\limits_{z\to 1}(z-1)F(z)$

$$e(\infty) = \lim_{z\to 1}(z-1)E(z) = \lim_{z\to 1}(z-1)\frac{1}{1+G(z)C(z)}\frac{z}{z-1}$$

$$= \lim_{z\to 1}\frac{1}{1+G(z)C(z)} = \frac{1}{1+\beta_p}$$

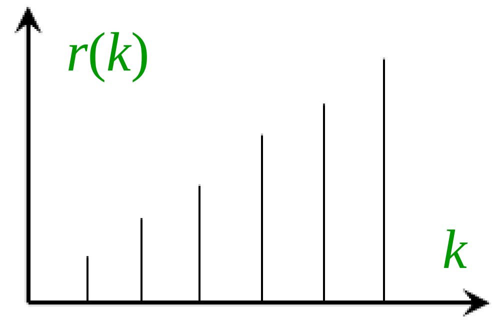
**position error constant**

Similarly, as in the continuous time case, we need the position error constant to be infinity, which implies $G(z)C(z)$ has at least a factor $1/(z-1)$ (TYPE 1 SYSTEM) in order to make $e(\infty) = 0$. If $\beta_p$ is a finite scalar, the open loop system $G(z)C(z)$ is said to be TYPE 0 SYSTEM.

$$\beta_p = \lim_{z\to 1} G(z)C(z)$$

## Case 2:

$r(k) = k$ is a unit ramp reference, $R(z) = \dfrac{Tz}{(z-1)^2}$




Use final value theorem

$$e(\infty) = \lim_{z\to 1}(z-1)E(z) = \lim_{z\to 1}(z-1)\frac{1}{1+G(z)C(z)}\frac{Tz}{(z-1)^2}$$

$$= \lim_{z\to 1}\frac{1}{(z-1)G(z)C(z)/T} = \frac{1}{\lim_{z\to 1}(z-1)\dfrac{G(z)C(z)}{T}} = \frac{1}{\beta_v}$$

**velocity error constant**

Similarly, we need the velocity error constant to be infinity, which implies $G(z)C(z)$ has at least a factor $1/(z-1)^2$ (TYPE 2 SYSTEM) in order to make $e(\infty) = 0$.

$$\beta_v = \lim_{z\to 1}\frac{z-1}{T}G(z)C(z)$$

**Notes**:

- The similarity between continuous-time and discrete-time steady state properties.

**e.g.** • number of integrators needed for a reference signal

• definitions of error constants

**<u>How many integrators are needed for perfectly tracking a constant reference?</u>**

One integrator is needed in the open TF $G(z)C(z)$.

**<u>How many integrators are needed for perfectly tracking a ramp reference output?</u>**

The control system for perfectly tracking a ramp signal will be more complex as 2 integrators are needed in $G(z)C(z)$.

More integrators will make it more difficult to stabilize the whole closed loop.

# ***Break***

***State-of-the-art control systems***

***Big Dog***

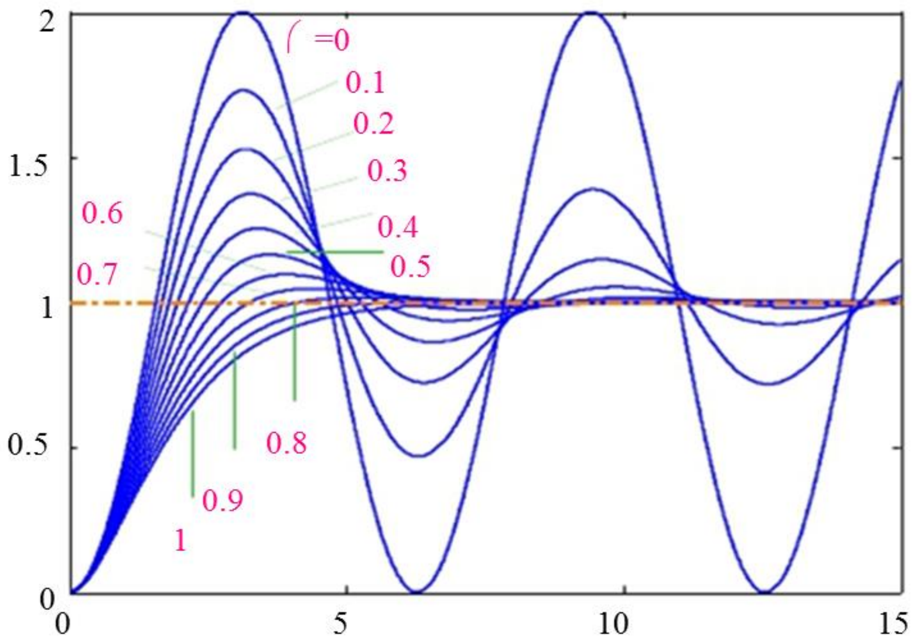
Once ***steady state accuracy*** is assured, we need to consider how fast the system can approach the steady state.

## ***Transient behavior of 2nd order systems***

Consider the following block diagram with a standard 2nd order system under unit step

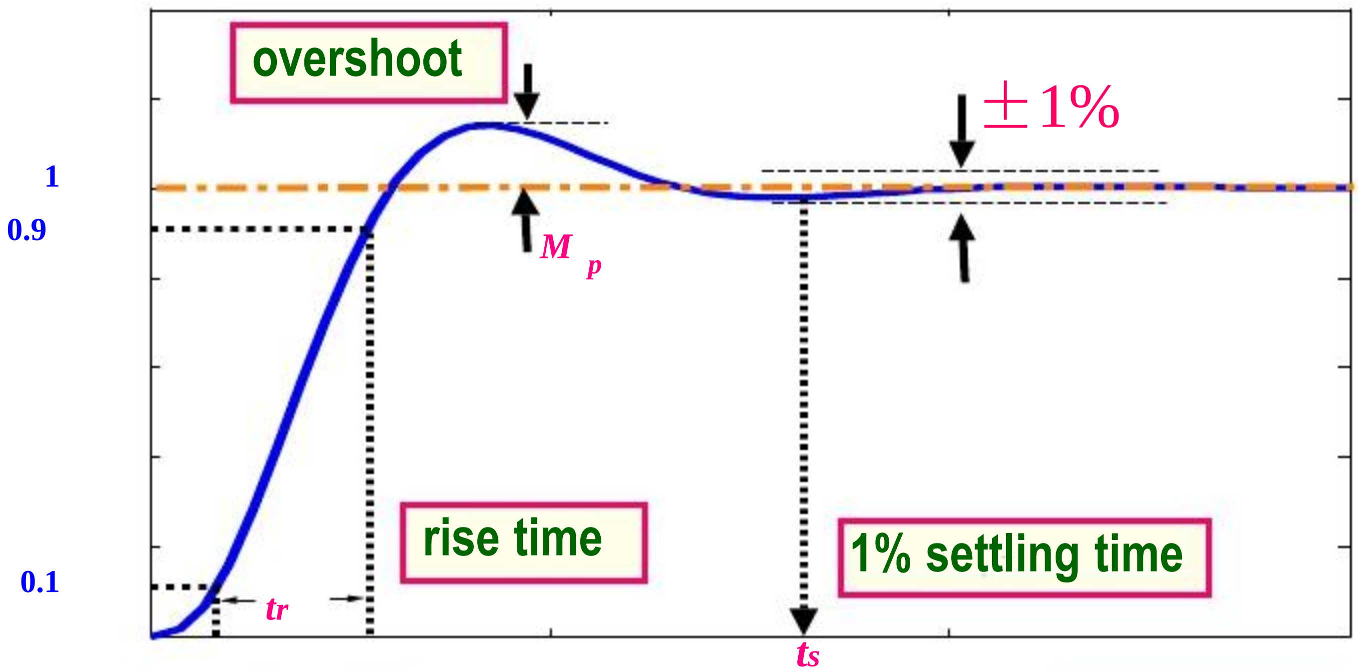
$$R(s) = \frac{1}{s} \qquad r(t) = 1 \longrightarrow \boxed{H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \longrightarrow Y(s)$$

The behavior of the system is as follows:




The system behavior is fully characterized by 2 factors: the damping ratio $\zeta$, and the natural frequency $\omega_n$.

## *Settling time, overshoot and rise time — Continuous time*




$$t_r \cong \frac{1.8}{\omega_n}$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$t_s \cong \frac{4.6}{\zeta\omega_n}$$

$$(t_s, M_p, t_r) \Leftrightarrow (\zeta, \omega_n) \Leftrightarrow \text{Pole location}$$



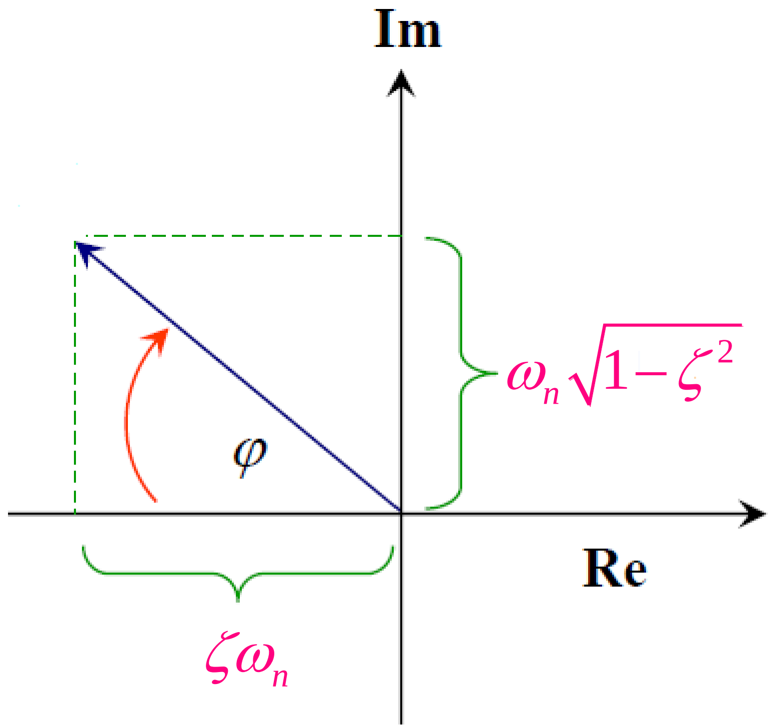
Are the locations of the poles also related to the step response?

Yes. The poles play the major role to affect the system response!

## Poles of the second order system:

$$H(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$s=-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$




Magnitude:

$$\sqrt{(\zeta\omega_n)^2+(\omega_n\sqrt{1-\zeta^2})^2}=\omega_n$$

Phase:

$$\cos\varphi=\frac{\zeta\omega_n}{\omega_n}=\zeta$$

Example 1: How to specify the poles using the settling time, rise time and overshoot?

$$(ts, Mp, tr) \le (3.45, 0.1, 0.9)$$

$$t_r : \frac{1.8}{\omega_n} \le 0.9$$

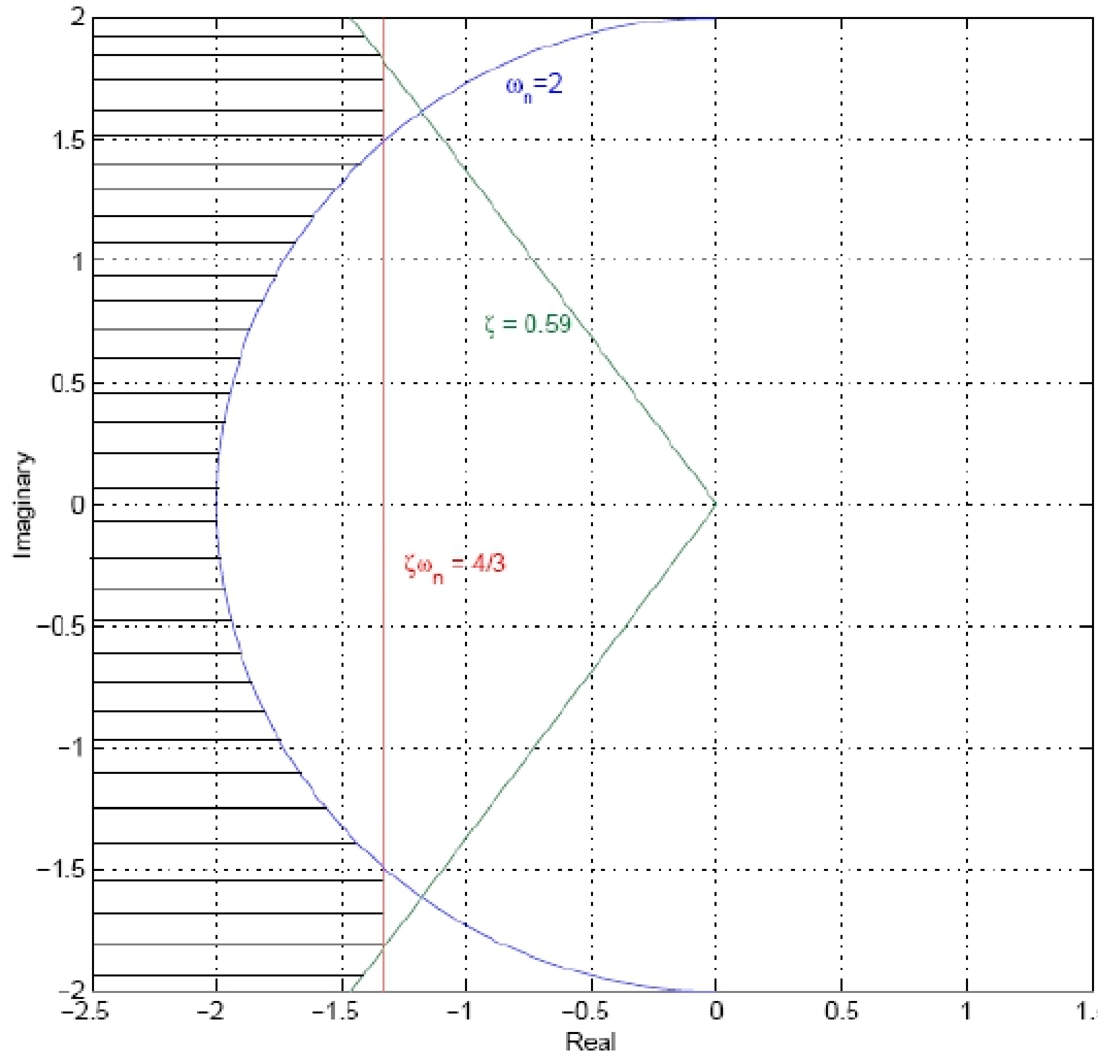
$$\Rightarrow \omega_n \ge 2$$

$$M_p : e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \le 0.1$$

$$\Rightarrow \zeta \ge 0.59$$

$$t_s : \frac{4.6}{\zeta\omega_n} \le 3.45$$

$$\Rightarrow \zeta\omega_n \ge \frac{4}{3}$$




All the poles have to be in the shaded area!

## How to handle discrete-time process?

**<u>If you know the poles of the continuous-time system, can you compute the poles for the corresponding discrete-time system?</u>**

$$s \Rightarrow z = e^{Ts}$$

$$s = \sigma + j\omega \Rightarrow z = e^{Ts} = e^{T(\sigma + j\omega)} = e^{\sigma T} e^{j\omega T}$$

$$(t_s, M_p, t_r) \Leftrightarrow (\zeta, \omega_n) \Leftrightarrow$$ Pole location

desired pole location in $s$-plane

$$z = e^{Ts}$$

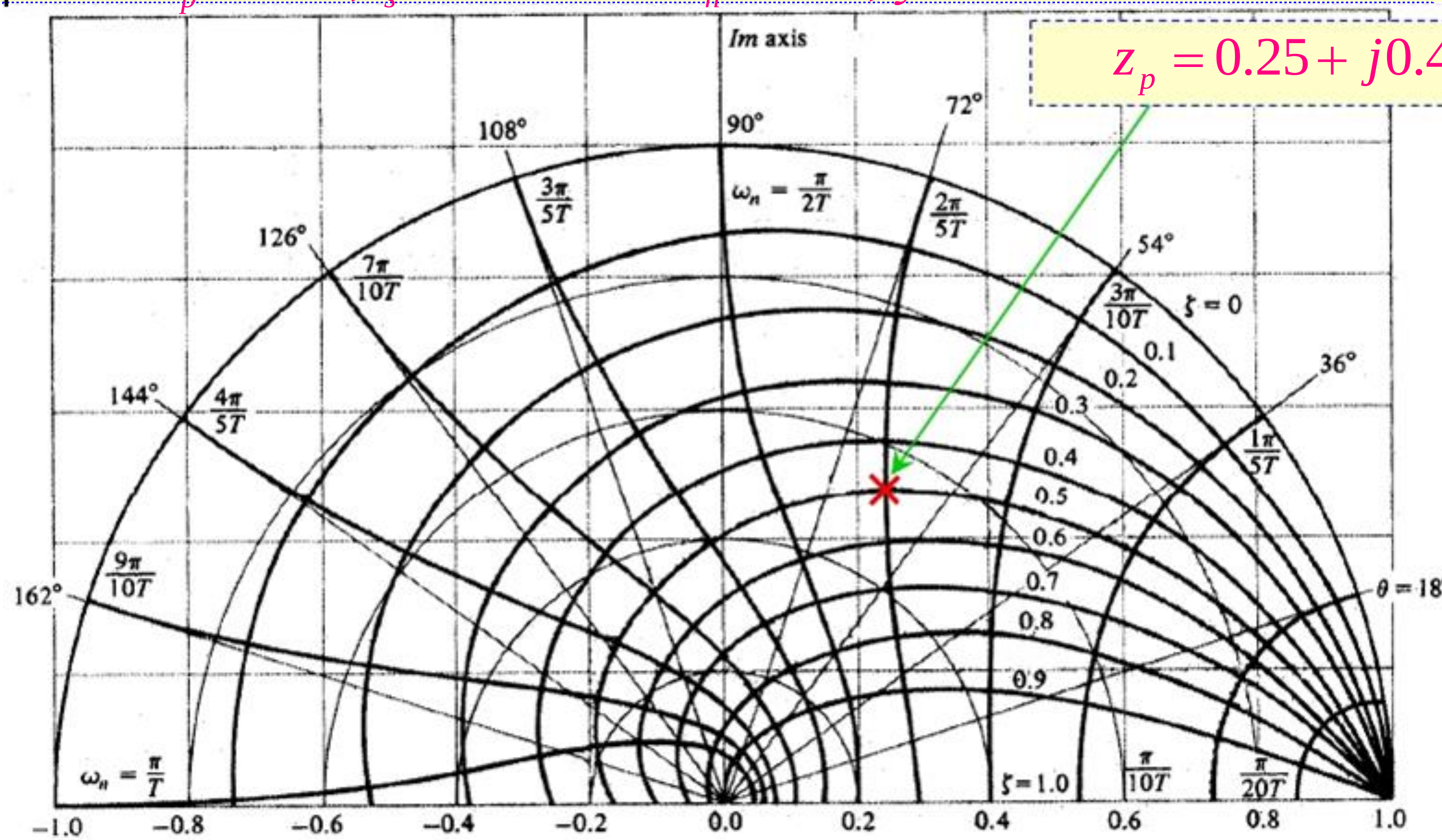
desired pole location in z-plane

Example 2: How to specify the poles in the discrete-time system?

***Settling time, overshoot and rise time — Discrete time***

Example : $M_p = 18\%, t_s = 7.3\text{sec} \Rightarrow \omega_n = 1.26, \zeta = 0.5$ with $T = 1$

$z_p = 0.25 + j0.47$




$z$ = plane loci of roots of constant $\xi$ and $\omega_n$

$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

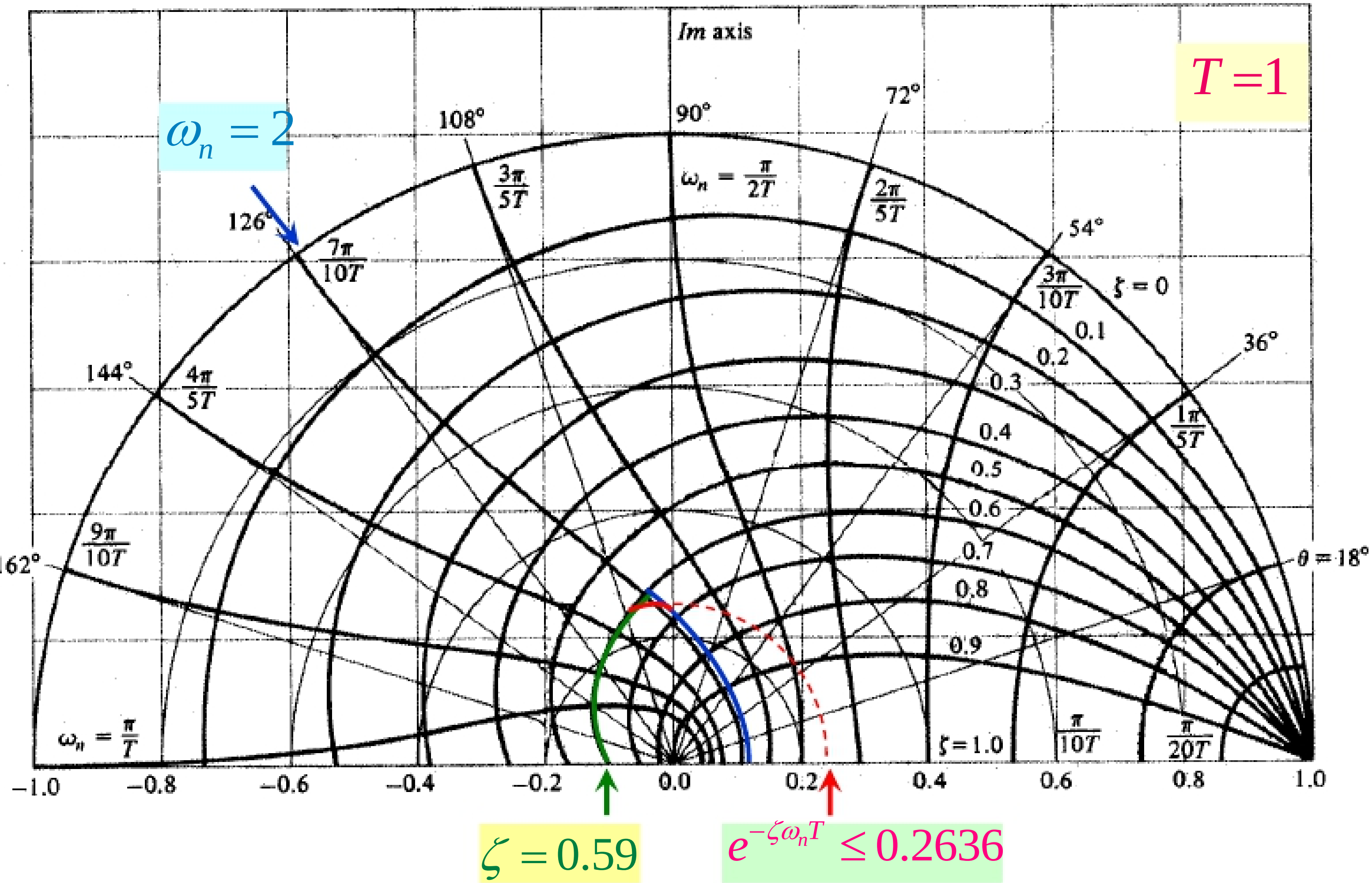
$z = e^{Ts}$ T = sampling period

$$z_p = e^{1\bullet(-0.5\times1.26 + j1.26\sqrt{1-0.5^2})} = 0.25 + j0.47$$

Example 3: Find out the poles in discrete-time system for example 1:

$$\omega_n \geq 2 \qquad \zeta \geq 0.59 \qquad \zeta\omega_n \geq \frac{4}{3}$$

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \Longrightarrow z = e^{Ts} = e^{-T\zeta\omega_n \pm jT\omega_n\sqrt{1-\zeta^2}} = e^{-T\zeta\omega_n}e^{\pm jT\omega_n\sqrt{1-\zeta^2}}$$



Now we are pretty clear about how to find out the poles to satisfy transient response requirements for second order system.

**How to deal with higher order system?**

Example 4:
$$H(s)=\frac{2}{(s^2+s+1)(s+2)}$$

Step one: Compute the poles

$$s=-2 \quad s=-0.5\pm j\frac{\sqrt{3}}{2}$$

Step two: identify the fast and slow modes by checking the real parts of the poles:

$$s=-2 \Longrightarrow e^{-2t}$$

$$s=-0.5\pm j\frac{\sqrt{3}}{2} \Longrightarrow e^{-0.5t}e^{j\frac{\sqrt{3}}{2}t}$$

**Which one decreases to zero faster ?** $e^{-2t}$

The bigger the absolute value of the real part, the faster it goes to zero!

**To approximate the system, shall we ignore the fast mode or the slow mode?**

The dynamics of the system is dominated by the slow mode, and the fast model can be ignored.

$$H(s)=\frac{2}{(s^2+s+1)(s+2)}\approx\frac{1}{(s^2+s+1)}$$

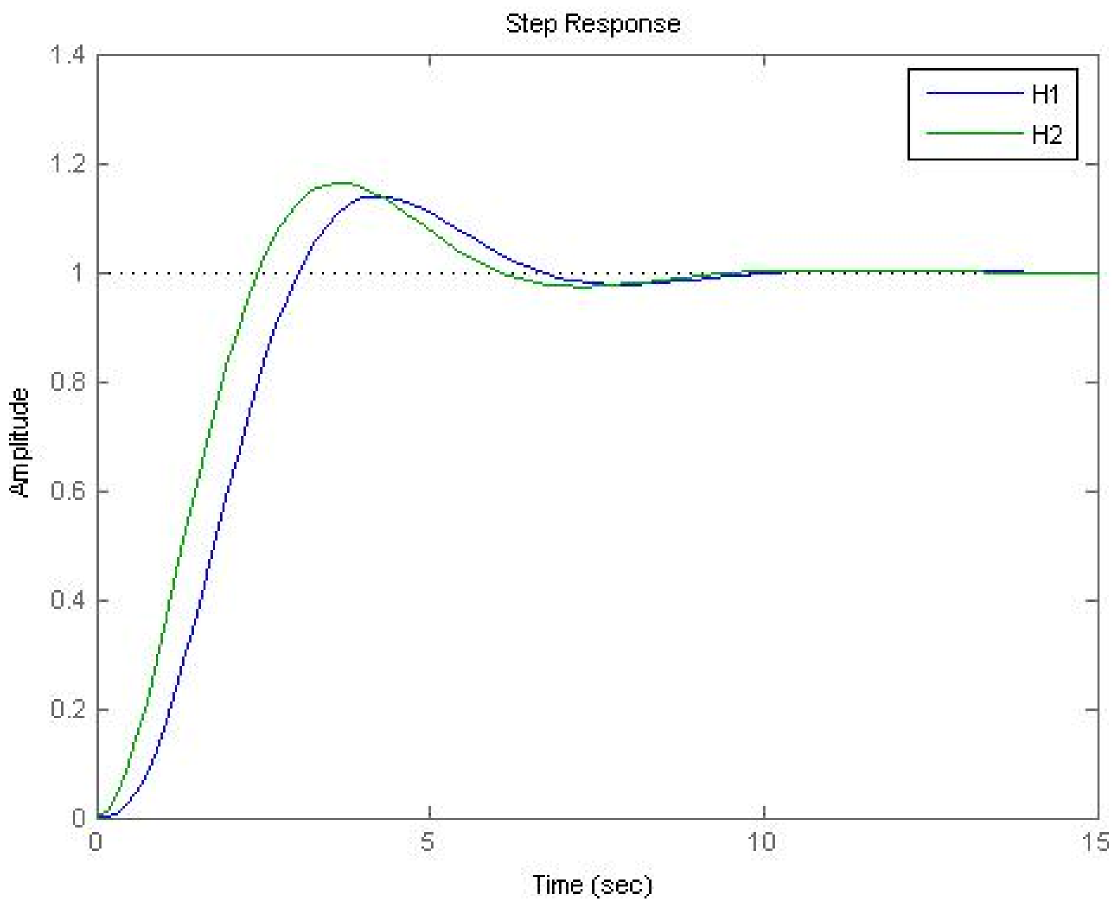


The higher order system can be approximated by second order or even first order system!

How good is the approximation?

The step responses of the two systems:

$$H_1(s) = \frac{2}{(s^2+s+1)(s+2)} \qquad H_2(s) = \frac{1}{(s^2+s+1)}$$




There are minor differences in the transient part.

The steady state responses are almost the same!

The higher order system can be approximated reasonably well by second order or even first order system!

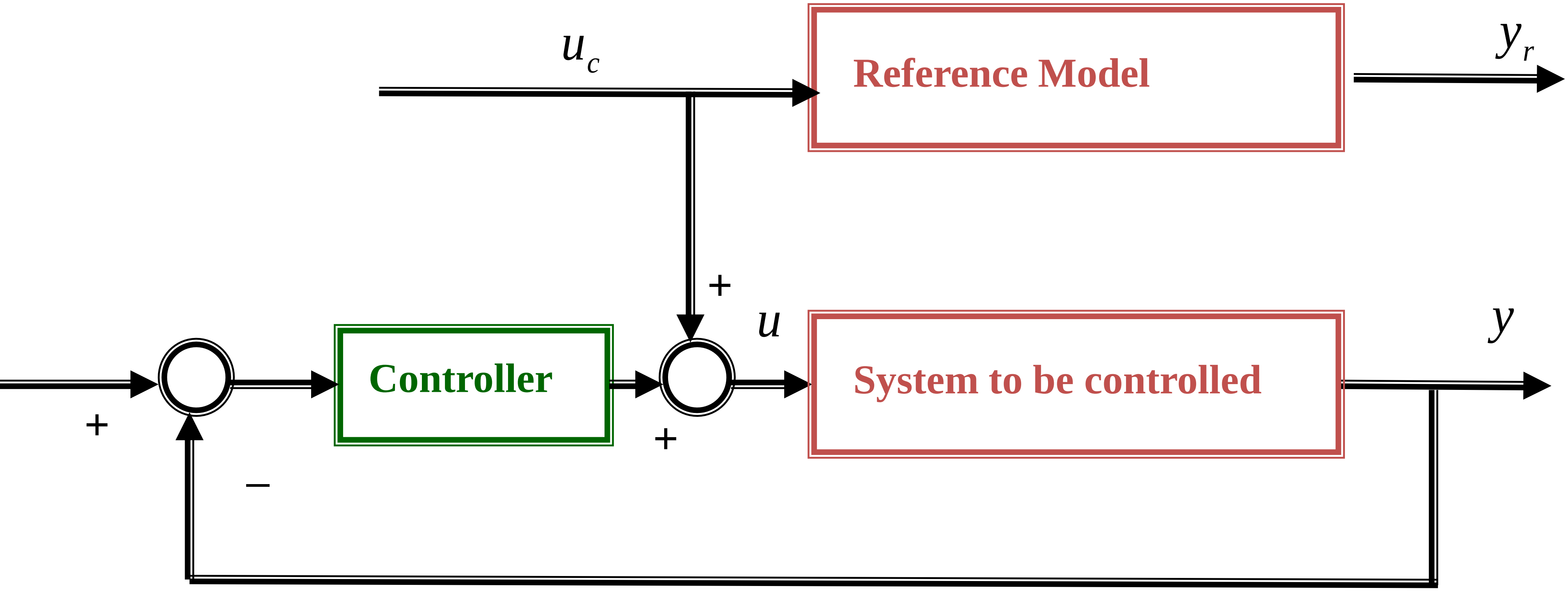
**Notes**:

1. Transient behavior of engineering systems above 2nd order cannot be simply modeled, and the common way is to approximate it by a 2nd order by identifying the fast and slow modes of the system.

2. We can design a desired reference model $H_{desired}(s/z)$, which satisfies all time domain specifications. Then the controller design task renders to the selection of an appropriate $C(s)$ so that $H(s/z) \rightarrow H_{desired}(s/z)$.

In our daily life, we learn from a good example, as a reference model, and adjust our behavior or manner accordingly (control actions).

## Model-reference control:




***Objective:*** To make the closed loop model follow the reference model as close as possible.

How to specify the appropriate reference model in practice?

It depends upon the performance specifications.

In many cases, the reference model can be specified by a second order system which satisfies all the performance requirements.

Often the desired CLTF is chosen to be

$$\boxed{H_{desired}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

Note $H_{desired}$ $(s)$ can be expressed by its poles.

Denote the location of desired poles

$$s_p = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} \qquad \bar{s}_p = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$$

The desired CLTF can be expressed as

$$H_{desired}(s) = \frac{\omega_n^{\ 2}}{(s - s_p)(s - \bar{s}_p)}$$

### ***Desired CLTF — discrete time system***

From time specifications the desired poles are determined

$$z = z_p \qquad z = \bar{z}_p$$

$$r(k) = 1 \qquad H_{desired}(z) = \frac{?}{(z - z_p)(z - \bar{z}_p)} \qquad y(k)$$

There is no simple formula to get the zeros of the discrete reference model. We need to use the following formula to convert the continuous-time TF to discrete-time TF using ZOH

$$H_{desired}(z) = (1 - z^{-1})Z[\frac{H_{desired}(s)}{s}]$$

You can also use the table 2.1, or use the command “c2d” in MATLAB.

Disturbance Rejection

**<u>Disturbance may occur at any moment and everywhere!</u>**

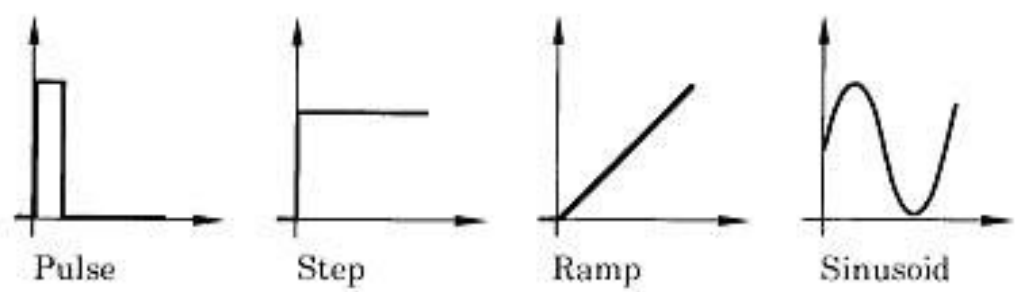
Load Disturbances:
Most often encountered

**Wind gusts on an antenna**
**Waves on a ship**
**Load on a motor**
**Variations in temperature**

Measurement errors

Parameter Variations

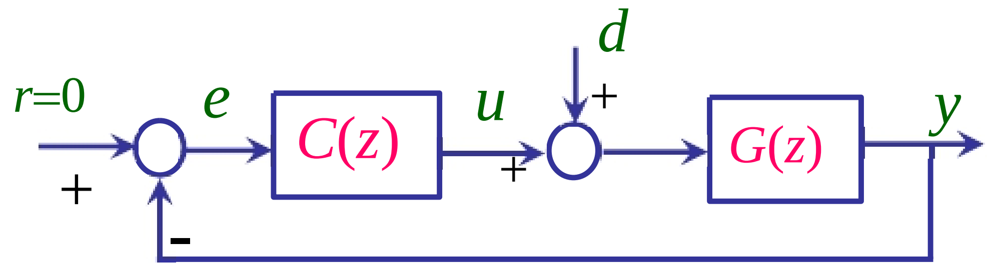
Simple models:




Real world examples:
[Ship hit by bomb.]
[Ship hit by wave.]

## *Disturbance rejection*

Recall the block diagram below. We set the reference $r = 0$ for simplicity and for the study of the effects of the disturbance $d$ (unwanted signal) on the system output.




Note that

$$Y(z) = G(z)[D(z) - C(z)Y(z)]$$

$$Y(z) = \frac{G(z)}{1 + G(z)C(z)} D(z)$$

We can also obtain the TF without using the algebraic equations.

**<u>What is the feedforward TF from disturbance to output?</u>**

G(z)

**<u>What is the open loop TF ?</u>**

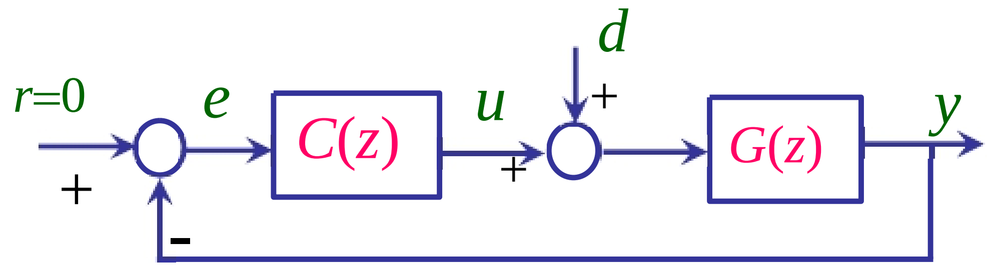
G(z)C(z)

Therefore, we can get the closed loop TF

$$\frac{Y(z)}{D(z)} = \frac{G(z)}{1 + G(z)C(z)}$$

## *Disturbance rejection*

Recall the block diagram below. We set the reference $r = 0$ for simplicity and for the study of the effects of the disturbance $d$ (unwanted signal) on the system output




$$Y(z) = \frac{G(z)}{1+G(z)C(z)} D(z)$$

Consider the case when $d$ is a unit step signal, we use the final value theorem

$$y(\infty) = \lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)\frac{G(z)}{1+G(z)C(z)}\frac{z}{z-1} = \frac{G(1)}{1+G(1)C(1)}$$

**Is it possible to reject the effect of the disturbance, i.e. make y go to zero?**

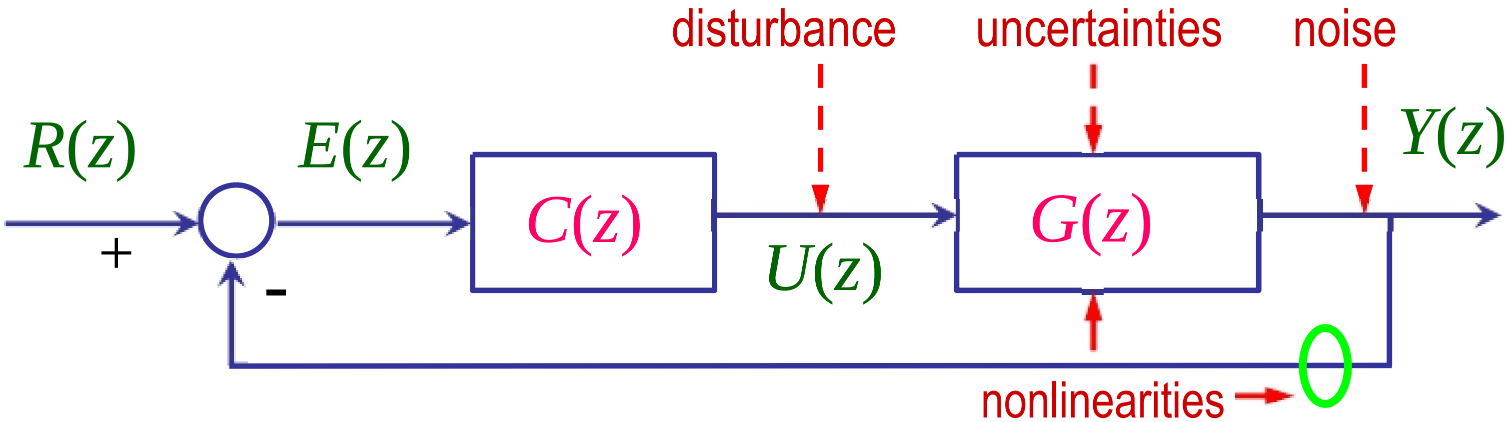
Make C(1) as infinity -→ put an integrator 1/(z-1) in the controller!

So it is very easy to cope with disturbance by adding an integrator in the controller!

How about making G(1)=0?

We can design the controller, C(z), in anyway we like.
But we cannot change the system, G(z), directly!

***Other considerations***

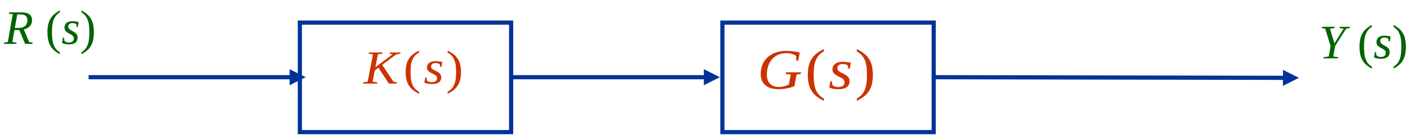



- Plant nonlinearities $\longrightarrow$ nonlinear control
- Plant parameter uncertainties $\longrightarrow$ robust control, adaptive control
- Control input nonlinearities $\longrightarrow$ nonlinear control
- Sensor nonlinearities $\longrightarrow$ nonlinear control
- Disturbance rejection $\longrightarrow$ robust control, optimal control

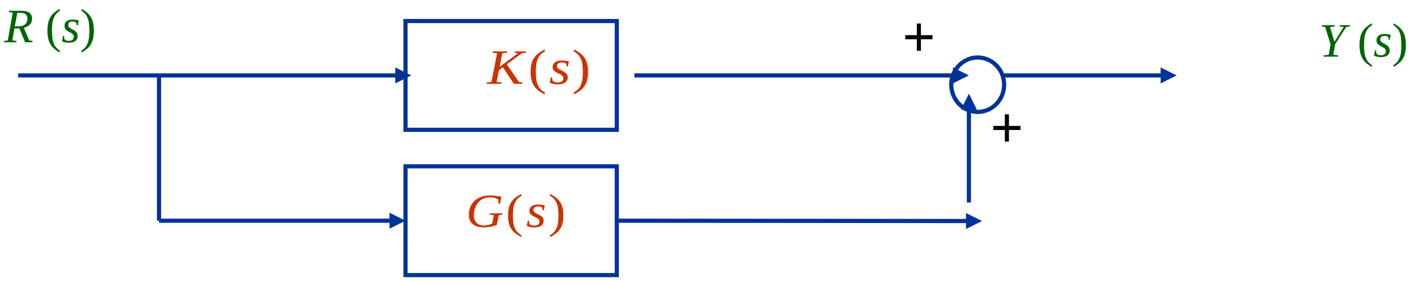
Q & A…

THANK YOU !

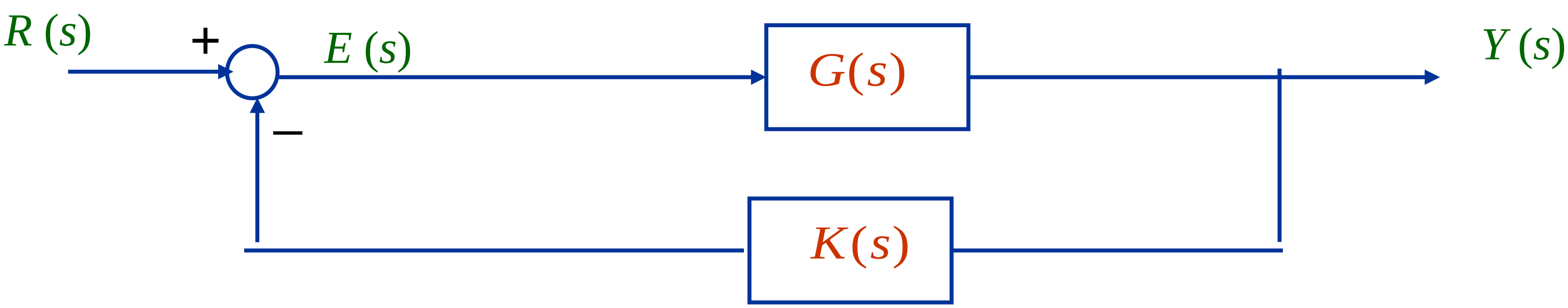
## *Transfer functions Y/R for basic block diagrams*




$K(s)G(s)$




$K(s)+G(s)$




$$Y(s)=G(s)E(s)=G(s)(R(s)-K(s)Y(s))$$

$$\Rightarrow \boxed{H(s)=\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G(s)K(s)}}$$

Closed-loop transfer function from $R$ to $Y$.

$G(s)$--- *Feedforward TF*

$G(s)K(s)$----*open loop TF*

**return**

| Entry # | Laplace Domain | Time Domain | Z Domain (t=kT) |
|---|---|---|---|
| 1 | $1$ | $\delta(t)$ unit impulse | $1$ |
| 2 | $\frac{1}{s}$ | $u(t)$ unit step | $\frac{z}{z-1}$ |
| 3 | $\frac{1}{s^2}$ | $t$ | $\frac{Tz}{(z-1)^2}$ |
| 4 | $\frac{1}{s+a}$ | $e^{-at}$ | $\frac{z}{z-e^{-aT}}$ |
| 5 | | $b^k \quad (b = e^{-aT})$ | $\frac{z}{z-b}$ |
| 6 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ | $\frac{Tze^{-aT}}{(z-e^{-aT})^2}$ |
| 7 | $\frac{1}{s(s+a)}$ | $\frac{1}{a}(1-e^{-at})$ | $\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$ |
| 8 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ | $\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$ |
| 9 | $\frac{1}{s(s+a)(s+b)}$ | $\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$ | |
| 10 | $\frac{1}{s(s+a)^2}$ | $\frac{1}{a^2}(1-e^{-at}-ate^{-at})$ | |
| 11 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ | |
| 12 | $\frac{b}{s^2+b^2}$ | $\sin(bt)$ | $\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$ |
| 13 | $\frac{s}{s^2+b^2}$ | $\cos(bt)$ | $\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$ |
| 14 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin(bt)$ | $\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$ |
| 15 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos(bt)$ | $\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$ |

return

$$H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}$$

| $G(s)$ | $H(q)$ or the coefficients in $H(q)$ |
|---|---|
| $\frac{1}{s}$ | $\frac{h}{q-1}$ |
| $\frac{1}{s^2}$ | $\frac{h^2(q+1)}{2(q-1)^2}$ |
| $\frac{1}{s^m}$ | $\frac{q-1}{q} \lim_{a \to 0} \frac{(-1)^m}{m!} \frac{\partial^m}{\partial a^m} \left( \frac{q}{q - e^{-ah}} \right)$ |
| $e^{-sh}$ | $q^{-1}$ |
| $\frac{a}{s+a}$ | $\frac{1 - \exp(-ah)}{q - \exp(-ah)}$ |
| $\frac{a}{s(s+a)}$ | $b_1 = \frac{1}{a}(ah - 1 + e^{-ah})$ $\quad b_2 = \frac{1}{a}(1 - e^{-ah} - ahe^{-ah})$ <br> $a_1 = -(1 + e^{-ah})$ $\quad a_2 = e^{-ah}$ |
| $\frac{a^2}{(s+a)^2}$ | $b_1 = 1 - e^{-ah}(1 + ah)$ $\quad b_2 = e^{-ah}(e^{-ah} + ah - 1)$ <br> $a_1 = -2e^{-ah}$ $\quad a_2 = e^{-2ah}$ |
| $\frac{s}{(s+a)^2}$ | $\frac{(q-1)he^{-ah}}{(q - e^{-ah})^2}$ |
| $\frac{ab}{(s+a)(s+b)}$ <br> $a \neq b$ | $b_1 = \frac{b(1 - e^{-ah}) - a(1 - e^{-bh})}{b - a}$ <br> $b_2 = \frac{a(1 - e^{-bh})e^{-ah} - b(1 - e^{-ah})e^{-bh}}{b - a}$ <br> $a_1 = -(e^{-ah} + e^{-bh})$ <br> $a_2 = e^{-(a+b)h}$ |

return1

return2

$$H(z)=\frac{b_1 z+b_2}{z^2+a_1 z+a_2}$$

return

| $G(s)$ | $H(q)$ or the coefficients in $H(q)$ |
| --- | --- |
| $\dfrac{(s+c)}{(s+a)(s+b)}$ <br> $a \neq b$ | $b_1=\dfrac{e^{-bh}-e^{-ah}+(1-e^{-bh})c/b-(1-e^{-ah})c/a}{a-b}$ <br> $b_2=\dfrac{c}{ab}e^{-(a+b)h}+\dfrac{b-c}{b(a-b)}e^{-ah}+\dfrac{c-a}{a(a-b)}e^{-bh}$ <br> $a_1=-e^{-ah}-e^{-bh} \qquad a_2=e^{-(a+b)h}$ |
| $\dfrac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$ | $b_1=1-\alpha\left(\beta+\dfrac{\zeta\omega_0}{\omega}\gamma\right) \qquad \omega=\omega_0\sqrt{1-\zeta^2} \qquad \zeta<1$ <br> $b_2=\alpha^2+\alpha\left(\dfrac{\zeta\omega_0}{\omega}\gamma-\beta\right) \qquad \alpha=e^{-\zeta\omega_0 h}$ <br> $a_1=-2\alpha\beta \qquad \beta=\cos(\omega h)$ <br> $a_2=\alpha^2 \qquad \gamma=\sin(\omega h)$ |
| $\dfrac{s}{s^2+2\zeta\omega_0 s+\omega_0^2}$ | $b_1=\dfrac{1}{\omega}e^{-\zeta\omega_0 h}\sin(\omega h) \qquad b_2=-b_1$ <br> $a_1=-2e^{-\zeta\omega_0 h}\cos(\omega h) \qquad a_2=e^{-2\zeta\omega_0 h}$ <br> $\omega=\omega_0\sqrt{1-\zeta^2}$ |
| $\dfrac{a^2}{s^2+a^2}$ | $b_1=1-\cos ah \qquad b_2=1-\cos ah$ <br> $a_1=-2\cos ah \qquad a_2=1$ |
| $\dfrac{s}{s^2+a^2}$ | $b_1=\dfrac{1}{a}\sin ah \qquad b_2=-\dfrac{1}{a}\sin ah$ <br> $a_1=-2\cos ah \qquad a_2=1$ |
| $\dfrac{a}{s^2(s+a)}$ | $b_1=\dfrac{1-\alpha}{a^2}+h\left(\dfrac{h}{2}-\dfrac{1}{a}\right) \qquad \alpha=e^{-ah}$ <br> $b_2=(1-\alpha)\left(\dfrac{h^2}{2}-\dfrac{2}{a^2}\right)+\dfrac{h}{a}(1+\alpha)$ <br> $b_3=-\left[\dfrac{1}{a^2}(\alpha-1)+\alpha h\left(\dfrac{h}{2}+\dfrac{1}{a}\right)\right]$ <br> $a_1=-(\alpha+2) \qquad a_2=2\alpha+1 \qquad a_3=-\alpha$ |