EE3304 DIGITAL CONTROL SYSTEMS PART II
TUTORIAL THREE

Q1. Compare the phase lead and lag compensators, discuss their differences in terms of transfer functions (pole-zero relation), and Bode plots (phase, gain crossover frequency). For a process given by

$$G(s) = k\,\frac{(a - s)^2}{(1 + s)^2}$$

with $a = 1$ and k is a positive constant, which phase compensator is appropriate to apply if the desired gain crossover frequency is 1 *rad/sec*?

Q2. A phase lead compensator is to be designed.

(i) The desired phase margin is 50°, and the settling time is 3 seconds. Find the minimum damping ratio and the approximate bandwidth ($\omega_b \approx \omega_n$). If the damping ratio is chosen to be 0.77, is a sampling interval of 0.1 seconds adequate for the emulation based design?

(ii) Derive the desired characteristic polynomials and desired closed-loop transfer functions (both s domain and z domain).

Q3. A process has the Bode plot shown in Fig.1.

(i) Find the necessary gain k of the lead compensator such that the static error constant is 3.2.

(ii) Find the phase lead needed when the gain crossover frequency is chosen to be 2 rad/sec, where the desired phase margin is 50°.

Q4. A digital controller is given as

$$C(z) = \frac{az - b}{z}, \qquad a > 0$$

where b is a constant. Find the range of b such that $C(z)$ becomes (i) a proportional controller, (ii) a derivative controller, and (iii) a lead compensator. (You may use the w-transform where necessary, and assume that the sampling period is 1).

Q5. A process has the Bode plot shown in Fig.1. Design a phase-lead compensator such that

(i) the steady state error for unit step input is less than 10%..

(ii) the desired phase margin is 40°.

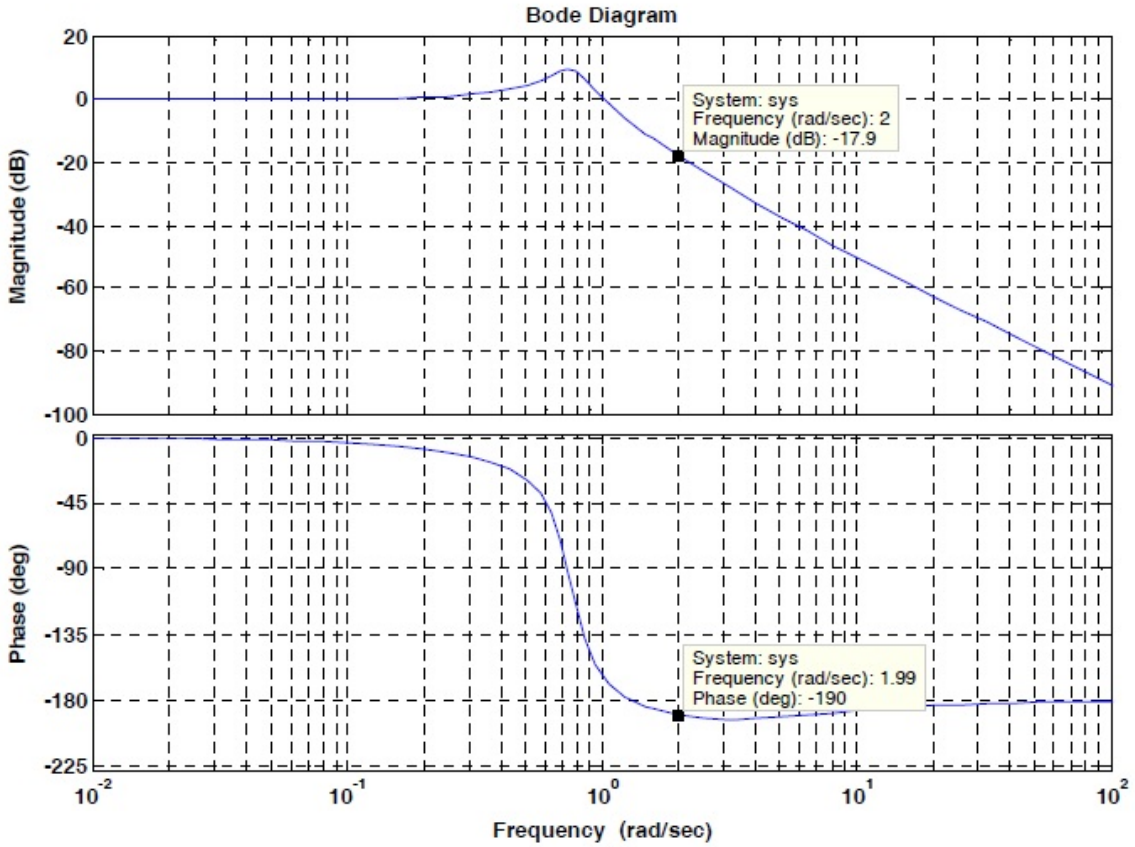

Figure 1 Bode plot for Q3 and Q5

Q6. The Bode plots of a plant are shown in Fig.2. Find the phase margin from the Bode plot. In order to achieve a phase margin of 50 and meanwhile keep the bandwidth above 2 rad/sec, should we use a lead compensator or a lag compensator? Briefly explain the reason for your selection.

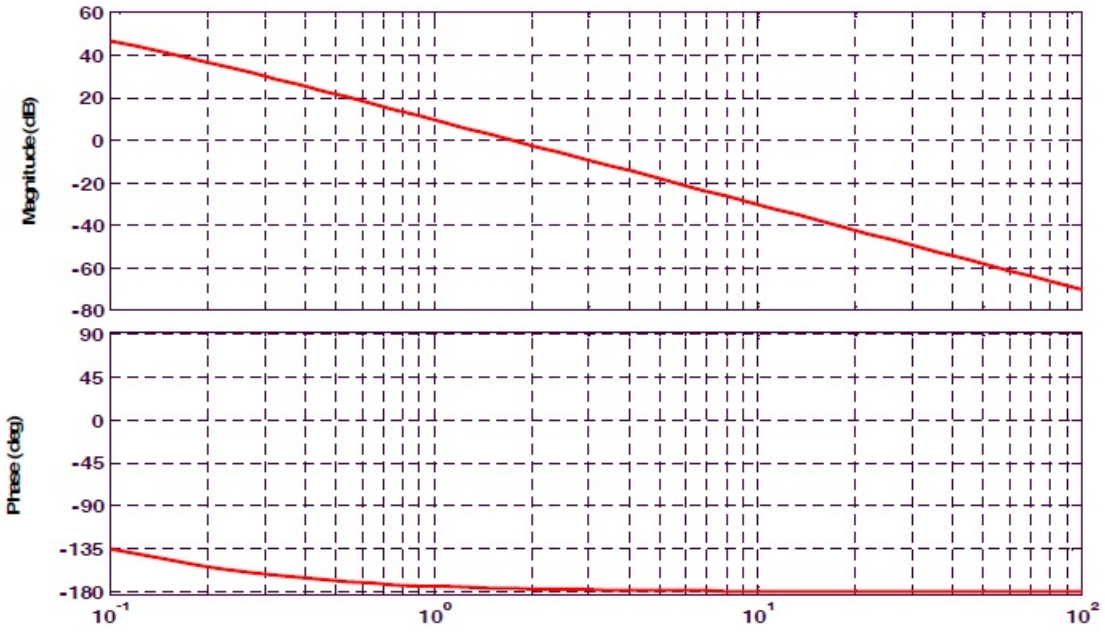

Figure 2 Bode plot for Q6

Q7. Design a digital lead compensator for the system shown in Fig. 3.

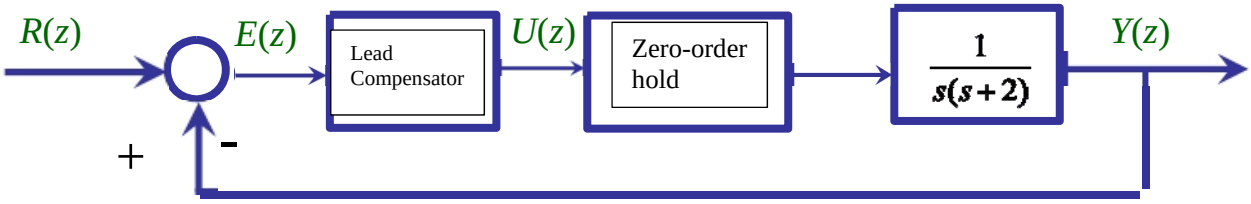

Figure 3 Digital control system for Q7

Use the frequency domain design method in the w-domain. The design specifications are that the phase margin be 50^{o}, the gain margin be at least 10dB, and the velocity error constant be 5. The sampling period is T=0.1.