NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II : 2009/2010)

EE3304 - DIGITAL CONTROL SYSTEMS

April/May 2010 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions, and comprises **FOUR (4)** pages.
2. All questions are compulsory. Answer **ALL** questions.
3. This is a CLOSED book examination. Each student is allowed to bring **ONE** (1) relevant data sheet of A4 size.
4. No programmable calculator is allowed.

Q1. (a) Find the unit step response for the system:

$$y(k) = 1.3y(k-1) - 0.4y(k-2) + 3u(k).$$

(8 marks)

(b) A continuous signal given by

$$f(t) = 6\sin 7t - 8\sin 9t$$

is sampled at ω_s. Give the minimum value of ω_s which can avoid the aliasing problem, and show all the frequency components in the resulting sampled signal.

(6 marks)

(c) State and prove the initial value theorem of z-transform. Use it to find the initial value for

$$X(z) = \frac{2z^2 + 3z + 1}{z^2 - 0.5z + 1}.$$

(11 marks)

Q2. Consider the computer control system shown in Figure 1, where the sampling period $T = 1\text{sec.}$, $G(s) = \dfrac{3}{s+2}$, $D(z)=K$, and $W(s)= e^{-s}$.

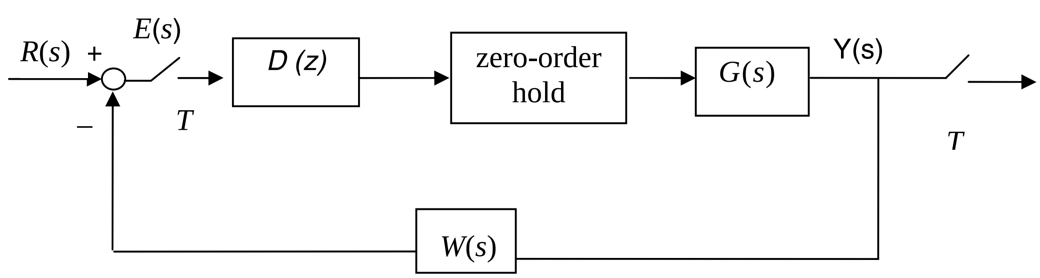

Figure 1

(a) Find the discrete-time closed-loop transfer function of the system, $\dfrac{Y(z)}{R(z)}$.

(15 marks)

(b) Check the closed-loop stability for K=0.5.

(4 marks)

(c) Find the range for positive K such that the closed-loop system is stable.

(6 marks)

Q3. An open-loop transfer function with a proportional controller is

$$\frac{l_p(z-b)}{z(z-a)},$$

where a and b are real numbers, $a \neq b$, and l_p is the proportional control gain.

(a) Assuming that the closed-loop system is stable, calculate the steady state error with respect to a unit step reference. Can the steady state error be eliminated?

(7 marks)

(b) Assume that both desired closed-loop poles locate at z_p. Find the relationship between parameters a and b, such that the pole placement can be achieved by the proportional controller.

(8 marks)

(c) Assume that the closed-loop poles achieved by the proportional controller are $z_p = \alpha + j\beta$ and $\bar{z}_p = \alpha - j\beta$, where α and β are real numbers. Show that the settling time is proportional to the sampling period T but reciprocal to the quantity $\ln(\alpha^2 + \beta^2)^{-1}$. How does the closed-loop response vary when $\alpha^2 + \beta^2$ varies from 0 to 1?

(10 marks)

Q4 (a) Write down the digital D controller with w transform, sketch the Bode plot, and indicate the roll-off rate and corner frequency.

(8 marks)

(b) Discuss the applicability of two Ziegler-Nichols auto-tuning methods to the following plant,

$$\frac{s+b}{s(s+1)(s+10)},$$

when the parameter b takes different values from $[0,\infty)$.

(7 marks)

(c) A process model is

$$Y(z)=\frac{1}{z-1}U(z),$$

where $U(z)$ and $Y(z)$ are Z-transforms of the control input, $u(k)$, and plant output, $y(k)$, respectively. Design a model predictive control law through minimizing the following objective function,

$$J(u)=[\hat{y}(k+1\,|\,k)-r(k+1)]^2+\lambda u^2(k),$$

where $r(k)$ is a unit step reference signal. Show that the steady state error is zero.

(10 marks)

END OF PAPER