EE3304: Digital Control Systems

Case Study: Control Systems Design for Guided Weapons

Prepared by:

Dr. Xiang Cheng

Department of Electrical & Computer Engineering
National University of Singapore

2016

Part I: Missile Control Methods

In the previous classes, you have learned some fundamentals of control engineering. Now it is time to show how classical control theory is applied to real world problems. There are so many control engineering applications. We will only choose the one defense example--we are going to apply our knowledge to the design of autopilot for guided weapons: the missile! Is it exciting? Or is it too complex for you to deal with? Don't worry. After this part is over, you can brag to your friends that you know how to design the control systems for guided missile.

1.1. Historical Background and Overview

Fear of the consequences of a future air war dominates the military thought of all nations. National survival in the event of attack can only be assured if 100 per cent interception can be obtained. Conventional fighter aircraft and anti-aircraft weapons are inadequate for such a task, and the answer to the problem was found to be guided missile.

A guided missile is a military rocket that can be directed in flight to change its flight path. In typical usage the term "missile" refers to guided rockets, and "rockets" to unguided ones. The differences between the two may be fairly minor other than the guidance system.

The first missiles to be used operationally were a series of German missiles of WW2. Most famous of these are the V1 and V2, both of which used a simple mechanical autopilot to keep the missile flying along a pre-chosen route. Less well known were a series of anti-shipping and anti-aircraft missiles, typically based on a simple radio control system directed by the operator.

The V-1 (German: Vergeltungswaffe 1) was the first guided missile used in war and the forerunner of today's cruise missile. The V-1 was developed at Peenemünde by the German Luftwaffe during the Second World War. Between June 1944 and March 29, 1945, it was fired at targets in southeastern England and Belgium, London and Antwerp. V-1s were launched from "ski-jump" launch sites along the French (Pas-de-Calais) and Dutch coasts until the sites were overrun by Allied forces.

Figure 1.1.1 The first missile --- V1

The V-2 Rocket (German: Vergeltungswaffe 2) was the first ballistic missile and first man-made object to achieve suborbital spaceflight, the progenitor of all modern rockets and a direct precursor of the Saturn V moon rocket. The first successful launch of a V-2 was on October 3, 1942 and began operation on September 6, 1944 against Paris, followed by an attack on London two days later. By the end of World War II May 1945 over 3,000 V-2s had been launched. As many as 20,000 slave laborers died constructing V-2s compared to the 7,000 military personnel and civilians that died from the V-2s' use in combat.

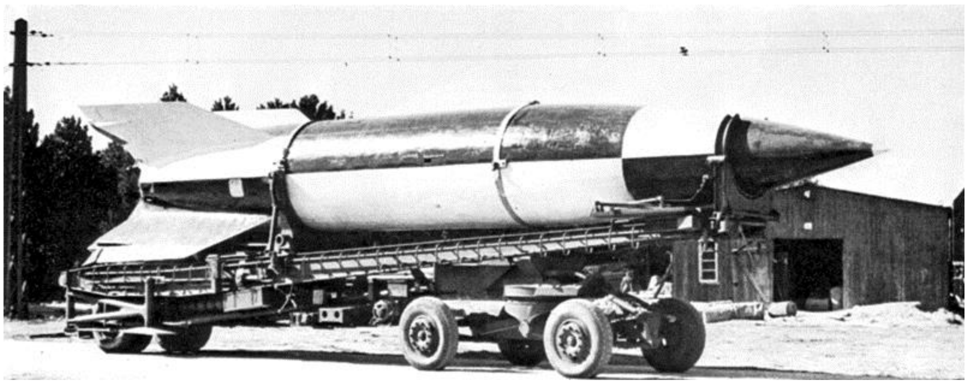

Figure 1.1.2 The first ballistic missile --- V2.

There are many types of missiles that have been developed today such as Ballistic missiles, Cruise missiles, Anti-shipping, Anti-aircraft, Air-to-air, Anti-tank, Anti-ballistic, and Anti-satellite weapon.

Ballistic missiles

Figure 1.1.3a Trident launched from submarine

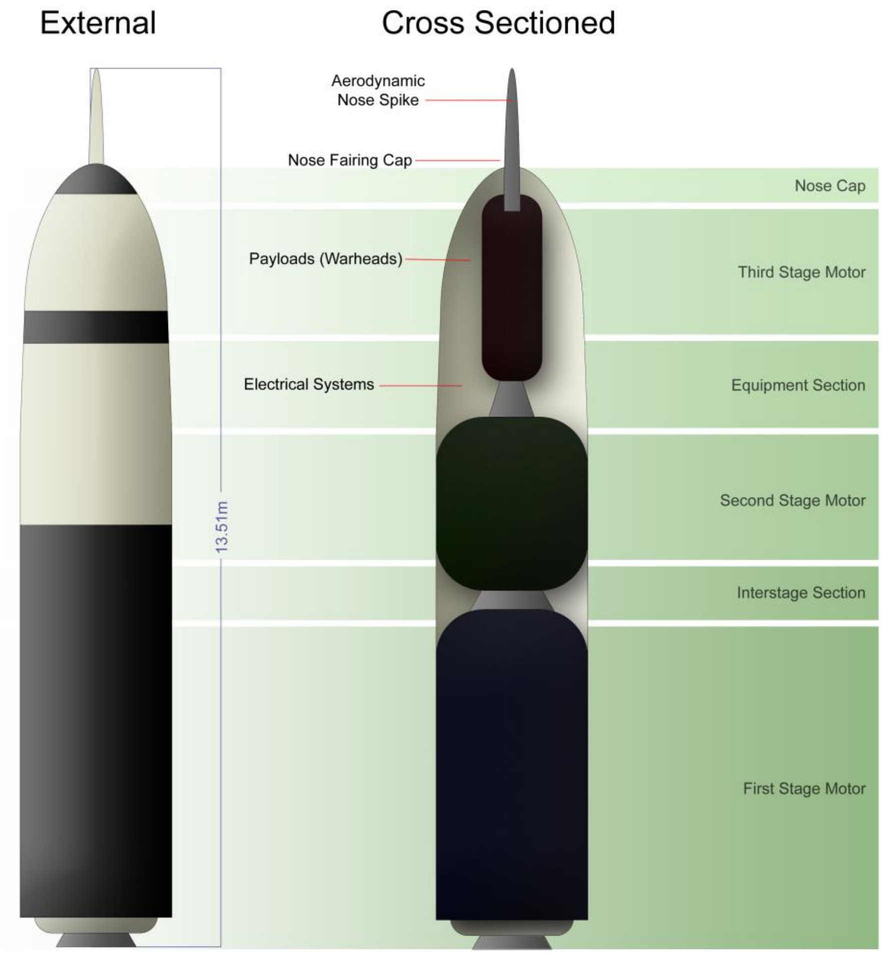

Figure 1.1.3b Diagramatic view of a Trident II D5 missile.

A ballistic missile is a missile that follows a sub-orbital, ballistic flight path with the objective of delivering a warhead to a predetermined target. The missile is only guided during the relatively brief initial powered phase of flight and its course is subsequently governed by the laws of orbital mechanics and ballistics. To date, ballistic missiles have been propelled during powered flight by chemical rocket engines of various types.

A ballistic missile trajectory consists of three parts: the powered flight portion, the free-flight portion which constitutes most of the flight time, and the re-entry phase where the missile re-enters the Earth's atmosphere.

Ballistic missiles can be launched from fixed sites or mobile launchers, including vehicles, aircraft, ships and submarines. The powered flight portion can last from a few tens of seconds to several minutes and can consist of multiple rocket stages.

When in space and no more thrust is provided, the missile enters free-flight. In order to cover large distances, ballistic missiles are usually launched into a high sub-orbital spaceflight; for intercontinental missiles the highest altitude (apogee) reached during free-flight is about 1200 km.

The re-entry stage begins at an altitude where atmospheric drag plays a significant part in missile trajectory, and lasts until missile impact.

The V2 had demonstrated that a ballistic missile could deliver a warhead to a target city with no possibility of interception, and the introduction of nuclear weapons meant it could do useful damage when it arrived. The accuracy of these systems was fairly poor, but post-war development by most military forces improved the basic inertial platform concept to the point where it could be used as the guidance system on ICBMs flying thousands of miles.

<u>Cruise missiles</u>

Figure 1.1.4 Boeing AGM-86 ALCM Tomahawk missile

A cruise missile is a guided missile which uses a lifting wing and most often a jet propulsion system to allow sustained flight. A cruise missile is, in essence, a flying bomb. They are generally designed to carry a large conventional or nuclear warhead many hundreds of miles with excellent accuracy. Modern cruise missiles normally travel at supersonic or at high subsonic speeds, are self-navigating, and fly in a non-ballistic very low altitude trajectory in order to avoid radar detection. In general, cruise missiles are differentiated from unmanned aerial vehicles (UAV) in that the weapon is integrated into the vehicle, and the vehicle is intended to be sacrificed in the mission.

The V1 had been successfully intercepted during the war, but this did not make the cruise missile concept entirely useless. Continued research into much longer ranged and faster versions led to the US's Navaho missile, and its Soviet counterparts, the Burya and Buran cruise missile. However these were rendered largely obsolete by the ICBM, and none was used operationally. Instead shorter-range developments have become widely used as highly accurate attack systems, such as the US Tomahawk missile.

<u>Anti-ship</u>

Another major German missile development project was the anti-shipping class (such as the Fritz X and Henschel Hs 293), intended to stop any attempt at a cross-channel invasion. However the British were able to render their systems useless by jamming their radios. After the war the anti-shipping class slowly developed, and became a major class in the 1960s with the introduction of the low-flying turbojet powered cruise missiles known as "sea-skimmers". These became famous during the Falklands War when an Argentine Exocet (“flying Fish”) missile sank a Royal Navy destroyer.

In 1982, during the Falklands War, Exocets became famous worldwide when Argentine Navy warplanes used them to destroy Royal Navy's destroyer, HMS *Sheffield* on 4 May and sink the support ship *Atlantic Conveyor* on 25 May. As well, an Argentine-converted land-based truck fired an Exocet that badly damaged another destroyer HMS *Glamorgan* on June 12.

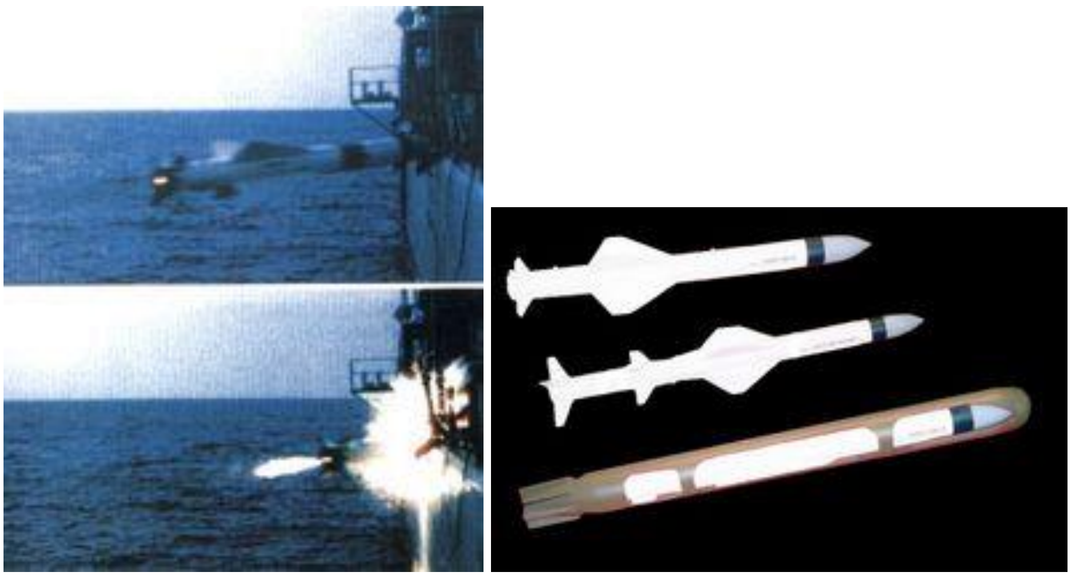

Figure 1.1.5 Anti-shipping missile --- Exocet

Anti-aircraft

By 1944 US and British air-forces were sending huge air-fleets over occupied Europe, increasing the pressure on the Luftwaffe day and night fighter forces. The Germans were keen to get some sort of useful ground-based anti-aircraft system into operation. Several systems were under development, but none had reached operational status before the war's end. The US Navy also started missile research to deal with the Kamikaze threat. By 1950 systems based on this early research started to reach operational service, including the US Army's Nike Ajax, the Navy's "3T's" (Talos, Terrier, Tartar), and soon followed by the Soviet S-25 Berkut and S-75 Dvina and French and British systems.

The very first Stinger fired shot down an Argentine ground attack aircraft during the Falklands War. The CIA supplied nearly 500 Stingers (some sources claim 1500-2000) to the guerrillas fighting Soviet forces in Afghanistan in the 1980s, where they were used quite successfully.

Figure 1.1.6 The Stinger surface-to-air missile. Figure 1.1.7 Launch of a Nike Zeus missile surface-to-air missile

Air-to-air

An air-to-air missile (AAM) is a guided missile fired from an aircraft for the purpose of destroying another aircraft. Air-to-air missiles are broadly grouped into short-range missiles (also called "dogfight" or "within visual range" (WVR) and medium- or long-range missiles (beyond visual range (BVR). Short-range missiles tend to use infrared guidance, while medium- and long-range missiles rely upon some type of radar guidance (and sometimes inertial guidance).

German experience in WWII demonstrated that destroying a large aircraft was quite difficult, and they had invested considerable effort into air-to-air missile systems to do this. This gave birth to the Me262's R4M rockets. It was developed by the Luftwaffe during World War II, and used operationally for a very brief time just prior to the end of the war. In the post-war period the R4M served as the pattern for a number of similar systems, used by almost all interceptor aircraft during the 1940s and '50s. The US Navy and USAF used their superior electronics to deliver a number of such designs in the early 1950s, most famous being the US Navy's AIM-9 Sidewinder and USAF's AIM-4 Falcon. These systems have continued to advance, and modern air warfare consists almost entirely of missile firing.

The Sidewinder was the first truly effective air-to-air missile, named after the Sidewinder snake, which detects its prey via body heat. The first combat use of the Sidewinder was on September 24, 1958. The Sidewinders were used by ROC F-86 Sabres to ambush the PRC MiG-17s as they flew past the Sabres seemingly invulnerable to attack because of their higher altitude ceiling performance.

Figure 1.1.8.a A US Navy VF-103 Jolly Rogers F-14 Tomcat fighter launches an AIM-54 Phoenix long-range air-to-air missile.

Figure 1.1.8.b Sidewinder short-range air-to-air missile.

Anti-tank

By the end of WWII all forces had widely introduced unguided rockets using HEAT warheads as their major anti-tank weapon. However these had a limited useful range of a 100 m or so, and the Germans were looking to extend this with the use of a missile using wire guidance, the X-7. After the war this became a major design class in the later 1950s, and by the 1960s had developed into practically the only non-tank anti-tank system in general use.

A wire-guided missile is a missile guided by signals sent to it via thin wires reeled out during flight. This guidance system is most common for anti-tank missiles, where its ability to be used in areas of limited line-of-sight make it useful, while the range limit imposed by the length of the wire is not a serious concern.

The USSR Sagger missile was used successfully in the 1973 Yom Kippur War by the Syrian and Egyptian armies. Soviet sources claimed that the missile accounted for 800 Israeli tank losses during the war.

Figure 1.1.9 MCLOS wire-guided anti-tank missile.

Anti-ballistic

An anti-ballistic missile (ABM) is a missile designed to counter ballistic missiles. A ballistic missile is used to deliver nuclear, chemical, biological or conventional warheads in a ballistic flight trajectory. The term "anti-ballistic missile" describes any antimissile system designed to counter ballistic missiles. However the term is more commonly used for ABM systems designed to counter long range, nuclear-armed Intercontinental ballistic missiles (ICBMs).

Figure 1.1.10 Four Patriot missiles like the one shown here can be fired from this mobile launcher between loadings.

Throughout the Gulf war in 1991, Patriot missiles attempted engagement of over 40 hostile ballistic missiles. The success of these engagements, and in particular how many of them were real targets is still controversial to this day. The U.S. Army claimed an initial success rate of 80% in Saudi Arabia and 50% in Israel. Those claims were eventually scaled back to 70% and 40%. However, in 1992 Theodore Postol of the Massachusetts Institute of Technology, and Reuven Pedatzur of Tel Aviv University testified before a House Committee stating that, according to their independent analyses, the Patriot system had a success rate of below 10%. Patriot was deployed to Iraq a second time in 2003 with a better performance.

<u>Anti-satellite weapon (ASAT)</u>

Anti-satellite weapons (ASATs) are space weapons designed to destroy satellites for strategic military purposes. Currently, only the USA, the former USSR and the People's Republic of China are known to have developed these weapons, with India claiming the technical capability to develop such weapons. On January 11, 2007, China destroyed an old orbiting weather satellite, the world's first test since the 1980s.

At 5:28 p.m. EST January 11, 2007, the People's Republic of China successfully destroyed a defunct weather satellite, FY-1C. The destruction was carried out by a modified medium-range ballistic missile DF-21 with kinetic ASAT warhead. FY-1C was a weather satellite orbiting Earth in polar orbit at an altitude of about 537 miles (865km), with a mass of about 750 kg. Launched in 1999, it was the fourth satellite in the Feng Yun series. The missile was launched from a mobile Transporter-Erector-Launcher (TEL) vehicle from land and the warhead destroyed the satellite in a head-head-collision manner at an extremely high relative velocity.

Figure 1.1.11 U.S. Vought ASM-135 ASAT missile launch on Sep. 13, 1985

1.2. Definitions and Notations

As described above, a guided missile is one which is usually fired in a direction approximately towards the target and subsequently receives steering commands from the guidance system to improve its accuracy. Inertial guidance is often used in medium and long range missiles (over 40 km say) when the intention is to hit a given map reference. The techniques used in such systems are quite different form those used in most short and medium range systems. The guidance-control systems covered in this module are command systems and homing systems. There is much in common between these two systems; for instance one has to track the target in both systems. In command systems the tracker is usually stationary or moving slowly (e.g. the target tracker could be on a ship). In homing systems the target tracker is in the missile and in such a case it is the relative movement of target and missile which is relevant.

Before going into mathematical detail concerning the motion of a missile in space as a result of guidance commands, some definitions and discussion are desirable. For instance, are we going to let the missile roll freely or are we going to control its orientation in roll? Are we going to maneuver the missile by moving control surfaces or are we going to alter the direction of thrust? This section is therefore mainly descriptive and discusses ways and means.

It is convenient to start with a definition of the task of a missile control system. It is one of the tasks of the guidance system to detect whether the missile is flying too high or too low, or too much to the left or right. It measures these deviations or errors and sends signals to the control system to reduce these errors to zero. The task of the control system therefore is to maneuver the missile quickly and efficiently as a result of these signals.

Suppose the guidance "sees" the missile at m relative to its own bore-sight as shown in Fig. 1.2.1, and that we interpret this to mean that the missile is too far to the right and too low. In a Cartesian system the guidance angular error detector produces two signals, a left-right signal and an up-down signal which are transmitted to the missile by a wire or radio link to two separate servos, say rudder servos and elevator servos. Fig 1.2.1 shows that this same information could be expressed in polar co-ordinates i.e. R and Φ. If the same information is expressed in another way then the control system must be mechanized differently. The usual method is to regard the Φ signal as a command to roll through an angle Φ measured from the vertical and then to maneuver outwards by means of the missile's elevators. The method of control compatible with polar commands is called polar control or twist and steer.

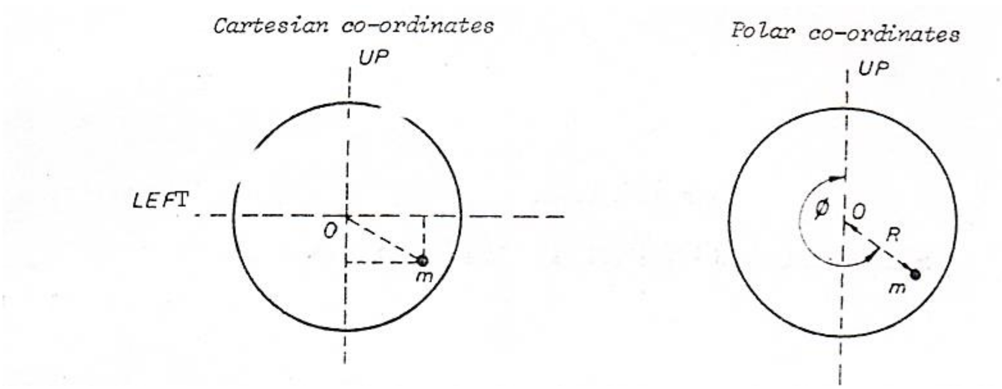

Figure 1.2.1 Cartesian and Polar coordinates.

And finally, by way of introduction, we should perhaps be rather more specific concerning the words "elevators" and "rudders"; and "ailerons" should also be defined. Guided missile usually has one or two axes of symmetry. If a missile has four control surfaces as shown in Fig 1.2.2 one regards surface 1 and 3 as elevators and 2 and 4 as rudders even if the missile should roll subsequently.

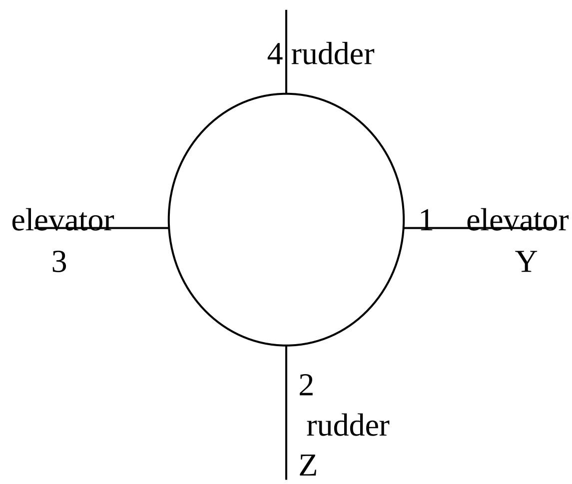

Figure 1.2.2 Rudders and Elevators.

If 1 and 3 are mechanically linked together such that a servo must impart the same rotation to both then these surfaces are elevators pure and simple. The same argument applies to the rudders. When surfaces 1 and 3 each have their own servo it is possible for them to act as ailerons. If looking in the direction y one surface is rotated ξ° c1ockwise and the other surface ξ° anti-c1ockwise then a pure couple is imparted to the missile about the fore and aft axis and this will tend to make the missile roll. Such control surfaces are now called ailerons. We can double the power of the ailerons by doing the same thing to control surfaces 2 and 4. If now the aerodynamics is linear i.e. the normal forces are proportional to incidence then the principle of superposition applies. Commands for elevator, rudder and aileron movements can be added electrically resulting in unequal movements to opposite control surfaces. In this way we have the means to control roll motion as well as the up-down (i.e. pitch) motion and left-right (i.e. yaw) motion.

1.3. Why Not Manoeuvre By Banking?

The conventional method of altering course to the left or right in a glider or airplane is to use the ailerons to bank i.e. roll by an angle Φ, see Fig 1.3.1.

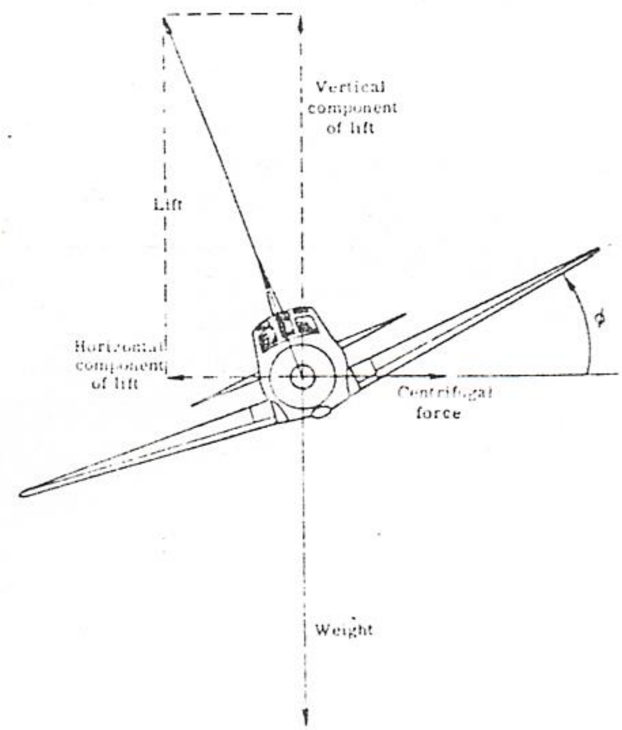

Figure 1.3.1 Forces acting on a banking aircraft.

If the lift force is increased slightly by the use of the elevators so that the vertica1 component of lift equals the weight then there is a horizontal component lift equal to the total lift times sinΦ. It is this component of lift which causes the flight path to change. This is the preferred method of maneuvering since lifting forces are most efficiently generated perpendicular to the wings; the lift-to-drag ratio is a maximum in this condition. Moreover, from a passenger's point of view this method of maneuver is best for his comfort as the total force he experiences is always symmetrically through the seat of his pants. Nevertheless the great majority of missiles have a symmetrical cruciform configuration i.e. two sets of wings and two sets of control surfaces. Why is this? There are two main objections to maneuvering by ailerons and elevators. Firstly, it is a slow process as the full maneuver cannot start to take place until the full bank angle is achieved. This might not be acceptable with some systems designed to hit fast moving targets. Moreover, it is not a very precise method of maneuvering. If the elevators are moved at the same time as the ailerons there will be some movement in the plane perpendicular to the desired one. If one waits until banking is complete there is an additional delay.

1.4 Roll Control

With Cartesian control ideally one would like the missile to remain in the same roll orientation as at launch during the whole flight. Up-down guidance signals if sent to the elevator servos should then result in a vertical maneuver of the missile; and left-right signals if sent to the rudder servos should result in a horizontal maneuver. However a missile is not designed like an airplane and there is no tendency to remain in the same roll orientation. In fact it will tend to roll due to any of the following causes:

(a) Accidental rigging errors which cannot be eliminated entirely
(b) Asymmetrical loading of the lifting and control surfaces in supersonic flight which occur when pitch and yaw incidences occur simultaneously and are not equal. This effect can be considerable but can be minimized by good design and tends to be small for long thin missiles.
(c) Atmospheric disturbances especially if the missile is flying close to the ground.

If the guidance error detector is on the ground, up-down and left-right signals can be implemented correctly if a roll gyro and resolver exist in the missile to ensure that the commands are mixed in the correct proportions to the elevators and rudders. However, it will be demonstrated in a later section that high roll rates cause cross-coupling between the two channels and tend to unstabilize the system. Conversely, if the guidance error detector is in the missile (e.g. all homing systems), then the guidance system and the control system share the same reference axes; if the guidance error detector rotates so does the control system and the necessity of resolving guidance signals due to missile rotation does not arise.

1.5. Aerodynamic Lateral Control

With a Cartesian control system the pitch control system is made identical to the yaw control system so we need discuss one channel only; in this respect the nomenclature differs from that used in aircraft. With missiles, lateral movement usually means up-down or left-right. With polar control one rolls and elevates. The following remarks apply to the elevation channel in twist and steer missiles also.

Rear control surfaces

The majority of tactical missiles have fixed main lifting surfaces (often called wings) with their centre of pressure somewhere near the missile centre of gravity, and rear control surfaces. Rear control surfaces often make for a convenient arrangement of components. Usually it is desirable to have the propulsion system placed centrally in the missile so that the centre of gravity movement due to propellant usage is minimized. It is convenient and sometimes essential to have the warhead and fuse at the front together with any associated electronics including the guidance receiver. This leaves the control system to occupy the rear end with the propulsion blast pipe passing through its centre. If there are four servos it is not impossible to design a neat servo package round this pipe.

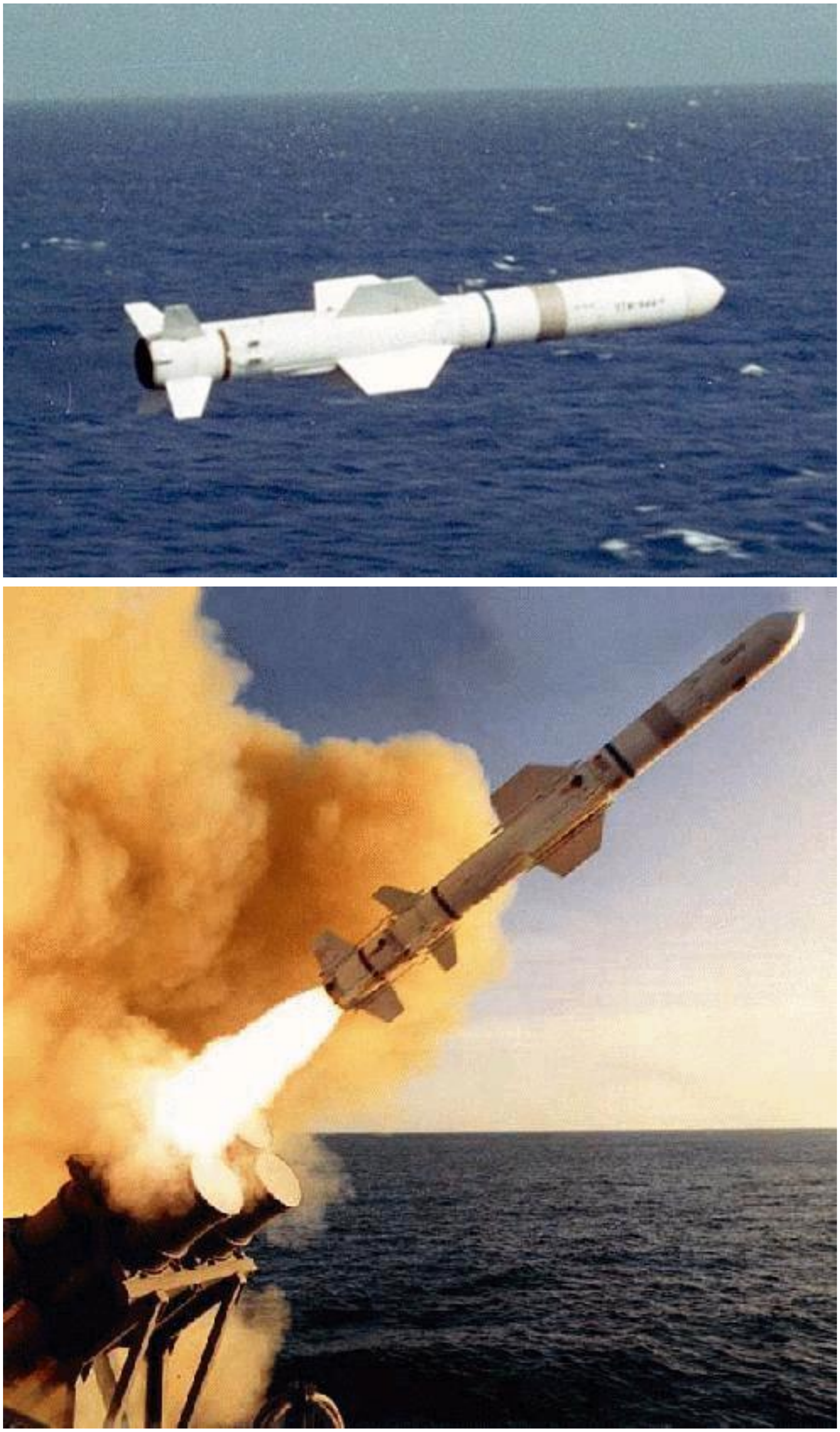

Figure 1.5.1 Harpoon—Anti –ship missile, standard cruciform real control surfaces

When considering the lateral forces and moments on missiles it is convenient first of all to consider the combined normal forces due to incidence on the body, wings and control surfaces as acting through a point on the body called the centre of pressure (c.p.) and to regard the control surfaces as permanently locked in the central position. If the c.p. is ahead of the centre of gravity (c.g.) then the missile is said to be statically unstable. If it coincides with the c.g. then it is said to be neutrally stable and if it is behind the c.g. it is said to be statically stable. This of course is the reason why feathers are placed at the rear end of an arrow to move the c.p. aft. These three possible conditions are shown in Fig 1.5.2 to 1.5.4.

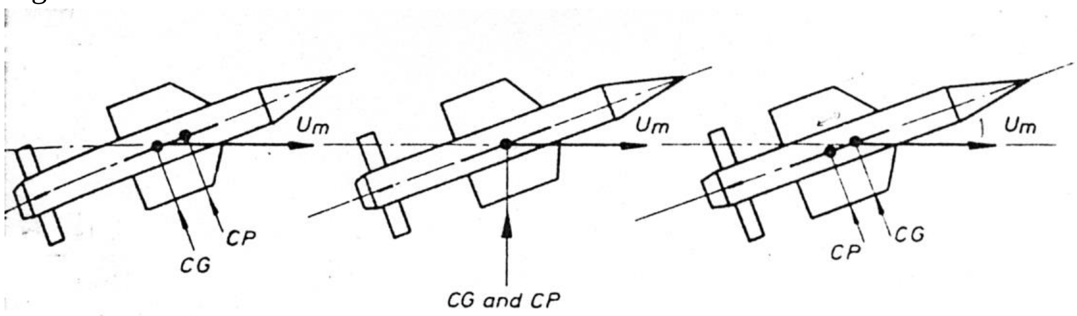

Figure 1.5.2 Unstable Figure 1.5.3 Neutrally Stable Figure 1.5.4 Stable

The missiles are shown with a small incidence i.e. the body is not pointing in the same direction as the missile velocity vector U. In the unstable condition any perturbation of the body away from the direction of the velocity vector results in a moment about the c.g. which tends to increase this perturbation. Conversely in the stable condition any perturbation of the body direction results in a moment which tends to oppose or decrease this perturbation. The distance of the c.p. to the c.g. is called the static margin. Since lateral force and hence lateral maneuver by aerodynamic means is obtained by exerting a moment on the body such that some incidence results it follows that if the static margin is excessive, the missile is unnecessarily stable and control moments will be relatively ineffective in producing a sizeable maneuver. There has to be a compromise here between stability and maneuverability.

Now consider a missile whose forward speed is constant, with a steady body and wing incidence of β and a control surface movement from the central position of ζ. We will consider motion in the horizontal plane only and assume the missile is not rolling; the effects of gravity are zero in this plane. Fig 3.4-4 shows the normal force N due to the body, wings and rear control surfaces assumed to be in the central position; this force N acts through the c.p. But there will be an additional force N_c due to the control surfaces being deflected by an amount ζ. Let this force act at a distance l_c from the c.g. Neglecting the small damping moment due to the fact that the missile is executing a steady turn, this picture can represent dynamic equilibrium if the rudder moment $N_c\, l_c$ is numerically equal to Nx* where x* is the static margin. If l_c /x* = 10 say then N = $10\,N_c$, and the total lateral force = $9\,N_c$. Note that this force is in the opposite sense to N_c. Since x* is typically 5% or1ess of the body length it is easily seen that a small change in the static margin can significantly affect the maneuverability of the missile. Thus the standard method of obtaining a large lateral force on a missile is to have a large moment arm by placing the control surfaces as far from the c.g. as possible, and by designing a small static margin. If a missile has no autopilot (i.e. no instrument feedback) a sizeable static margin has to be allowed to ensure stability in flight, say 5% or more of the overall length. With instrument feedback zero or even negative static margins can be used, thus assisting maneuverability. Before leaving rear controls it should be noted that the overall c.p. can never be regarded as in a fixed position. The c.p. of the body in particular will vary with incidence and with Mach number.

Forward control surfaces

Since the main objective of sitting a control surface is to place it as far from the c.g. as possible, a position as far forward as is practicable appears a logical choice. Forward control surfaces are often called "canards" named after ducks who apparently steer themselves by moving their head. Fig 3.4-6 shows another possible case of dynamic equilibrium. In this case it is seen that the lateral force due to the missile as a whole now adds to the force due to the deflection of the control surface and therefore if l_c/x*=10 as before, then the total normal force is 11 N_c compared with 9 N_c with rear controls. Also, the final sense of the total normal force is in the same sense as the control force. Canards therefore are slightly more efficient in the use of lateral control forces. If the reader thinks that canards will automatically render the missile unstable he will note that the canard controlled missile has been drawn with the main lifting surfaces rather further aft to make the overall c.p. aft of the c.g. It is the position of the overall c.p. relative to the c.g. with the control surfaces centralized which is the stability criterion.

Since canards appear to be superior to rear controls why do we find so many missiles with rear controls? Firstly, we shall see that with feedback instruments in a control system the static margin can be made zero or even negative while maintaining overall stability; so the difference in total normal force available can often be negligible. Secondly, there is the question of convenience in packaging as already mentioned which usually favors rear controls.

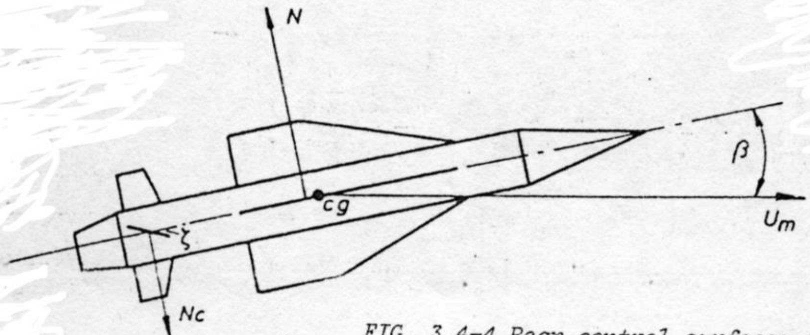

FIG. 3.4-4 Rear control surfaces supersonic

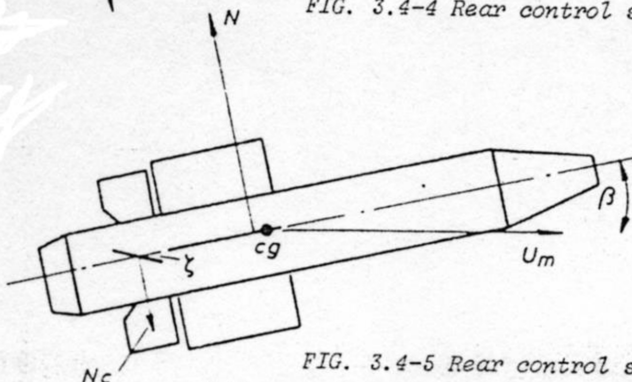

FIG. 3.4-5 Rear control surfaces subsonic

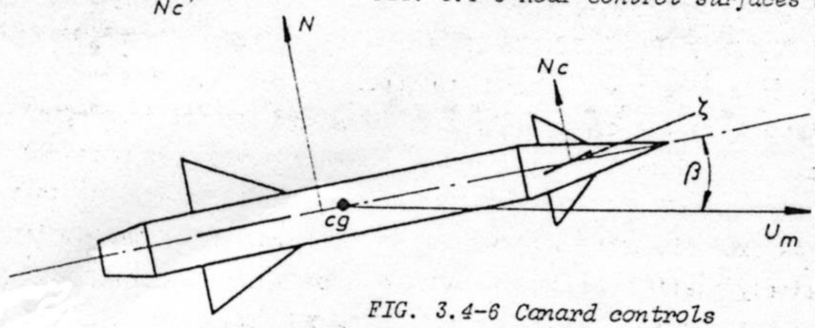

FIG. 3.4-6 Canard controls

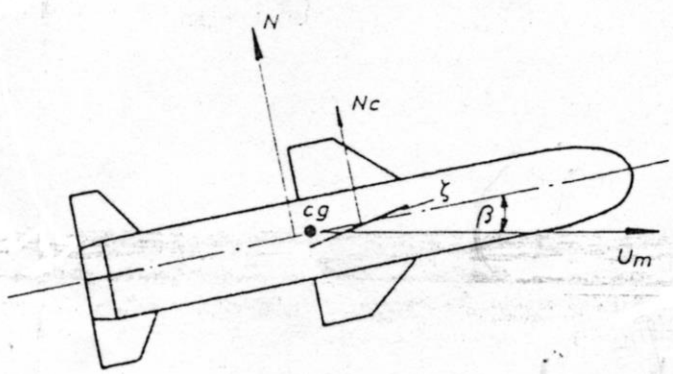

FIG. 3.4-7 Moving wings

Figure 1.5.5 Penguin--- air-to-surface Anti-Ship missile--- Canards

Moving wings

To use servos to move the main lifting surfaces and employ small fixed rear stabilizing surfaces is unusual. There could be the rare occasion when the servos are most conveniently placed near the centre of the missile. For instance, if a medium range missile has two separate motors, a boost motor and a sustain motor, the former may occupy the whole of the rear end of the missile and the sustainer motor may occupy most of the remaining rear half of the body, discharging to atmosphere through two inclined nozzles. In such a case there is just no room to install servos at the rear. However there are some distinct penalties involved in the use of moving wings. Clearly the servos will be appreciably larger to cope with the increased inertia of the load and the larger aerodynamic hinge moments. Also moving wings are an inefficient way of producing a large normal force due to the small moment arm available. Owing to the fact that the whole bending moment at the wing root has to be taken by a shaft, the wing will have to be designed much thicker around the mid chord. This not only increases the structure weight but at supersonic speeds it will increase the drag, the pressure drag varying with the thickness-to-chord ratio squared. It is desirable to make the centre section of the missile body square in cross section to eliminate a 1arge wing-body gap when the wing is deflected; such a gap considerably reduces the generated normal force. And finally since the moment arm is small, the position of the c.g. is critical as a small shift will make an appreciable change in the control moment arm. Nevertheless, if the maximum g requirements are low and the speed is subsonic, such as for an anti-ship missile, the overall weight penalty may not be excessive if small moving wings are used. The Akash (Sky) is a medium-range, theatre defense, surface-to-air missile with moving wings.

Figure 1.5.6 Akash (Sky), surface-to-air missile with moving wings.

1.6. Polar Control V.S. Cartesian Control

The great majority of tactical missiles are required to have the same degree of maneuverability vertically and horizontally; this is why the conventional configuration is a cruciform one with two equal pairs of lifting surfaces and two pairs of control surfaces.

If no form of roll control is required only two servos are necessary provided opposite control surfaces are linked together. If some form of roll control is required at least three servos are necessary and in practice four identical servos are usual. The advantage of adopting a twist and steer method of control is to be able to use only one pair of lifting surfaces and one pair of control surfaces. This cuts down weight and drag and such a configuration will certainly be advantageous for horizontal storage between decks of a ship or for launching under the wings of an aircraft. As an example of the drag bonus, about a half of the total drag of a conventional supersonic missile comes from the four wings and four control surfaces. Clearly there are advantages. The method of maneuvering the missile is this. The command goes as a positive command to one control surface and a negative command to the other. This causes the missile to roll. Nevertheless, maneuver cannot be so efficient and fast as with Cartesian control.

There are also some difficulties in ensuring a stable system with polar control used in a homing system, and in assessing the system performance. With Cartesian control it is possible to resolve target and missile motion into two planes and to consider the pitch and yaw channels as independent two-dimensional problems. This simplification is not possible in the case of polar control; indeed the equations of motion which result are, in general, not susceptible to analysis. Detailed three-dimensional simulation has to be employed. Bearing in mind the fact that Cartesian control must be a quicker method of moving laterally in any one direction and that analysis of the performance of Cartesian system is simpler it is understandable why the great majority of missile system use this method. Clearly therefore, polar control is not feasible if roll position stabilization is required; nor in the case of homers, if a high roll rate might disturb the homing head unduly. Since so few missiles use polar control it is clear that most designers do not consider the advantage in saving weight drag and space do not compensate for the loss of accuracy and speed of response.

Bloodhound, the British surface-to-air missile, is the well known example of polar control and the argument for its adoption stems primarily from the choice of a ram jet for the propulsion motor. The forward wings can control the pitch and roll, and the rear wings serve as stabilizer.

Figure 1.6.1 Bloodhound ---- Example of Polar Control

1.7 Thrust Vector Control

A completely different method of steering a missile is to alter the direction of the efflux from the propulsion motor and such a method is known as thrust vector control (TVC). This method of control is clearly not primarily dependant on the dynamic pressure of the atmosphere, but on the other hand it is inoperative after motor burn-out. TVC is therefore likely to have a limited application. The following situations make TVC essential or desirable:

(a) It is essential to use TVC in the vertical launch phase of an inter-continental ballistic missiles (ICBM) as these missiles whose total weight is well over 90% fuel have to be launched extremely gradually to avoid dynamic loading. Aerodynamic control would be completely ineffective for some time and the missile would topple over due to a small inevitable thrust misalignment unless an attitude sensor and TVC were used.

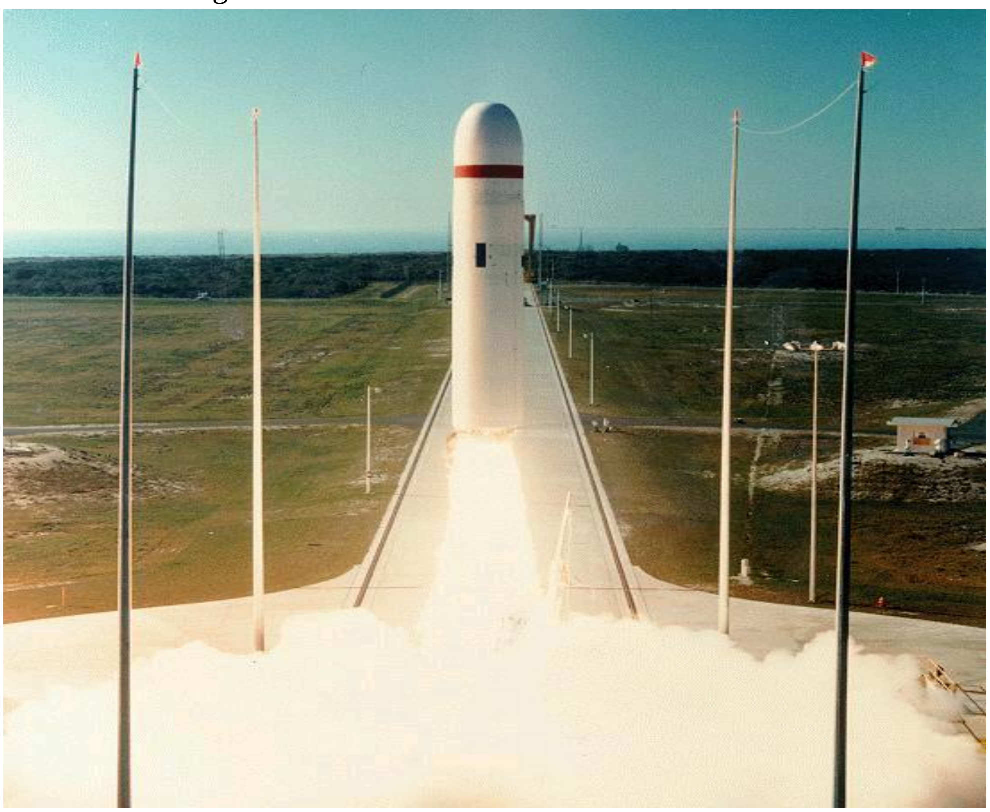

Figure 1.7.1 Trident---Inter-Continental-Ballistic-Missile (ICBM)----no control surfaces!

(b) If a missile is separated some distance from its controller such as in the anti-tank system Swing-fire and rapid gathering is required to achieve a short minimum range then it must be possible to maneuver the missile almost immediately after launch. Note that the control surface method works only when there is relative high speed wind (Air and speed), and TVC can overcome this limitation.

(c) In a short range air-to-air missile, one may be trying to hit a fast crossing target with no aim-off and with a flight time of a few seconds. The exceptional maneuverability one can obtain with TVC would give the system a better coverage.

Figure 1.7.2 Javelin—Anti-tank missile

(d) It can be argued that some systems would be cheaper and simpler if one launched vertically and then turned over rapidly, thus eliminating an expensive and heavy launcher. Vertical launch followed by a rapid turnover is an attractive concept for missiles carried and launched from a vehicle; 360° arc of fire is obtainable, and storage and reloading is almost certainly facilitated. This is especially so for missiles launched from a ship. With a conventional trainable launcher there may be considerable blind arcs due to other equipment occupying part of the deck space.
(f) Submarine launched missiles surfacing in different sea conditions may well need very early course correction.

Figure 1.7.3 Tomahawk is an all-weather submarine or ship-launched land-attack cruise missile. Both TVC and Aerodynamic control methods are involved.

Part II Aerodynamic Equations, Derivatives and Transfer Functions

2.1. Notations and Conventions

To obtain the transfer function of the missile it is first necessary to obtain the equations of motion for the missile. The equations of motion are derived by applying Newton's laws of motion, which relate the summation of the external forces and moments to the linear and angular accelerations of the system or body. To make this application, certain assumptions must be made and an axis system defined. The center of the axis system is, by definition, located at the center of gravity of the missile. In general, the axis system is fixed to the missile and rotates with it. Such a set of axes is referred to as "body axes."

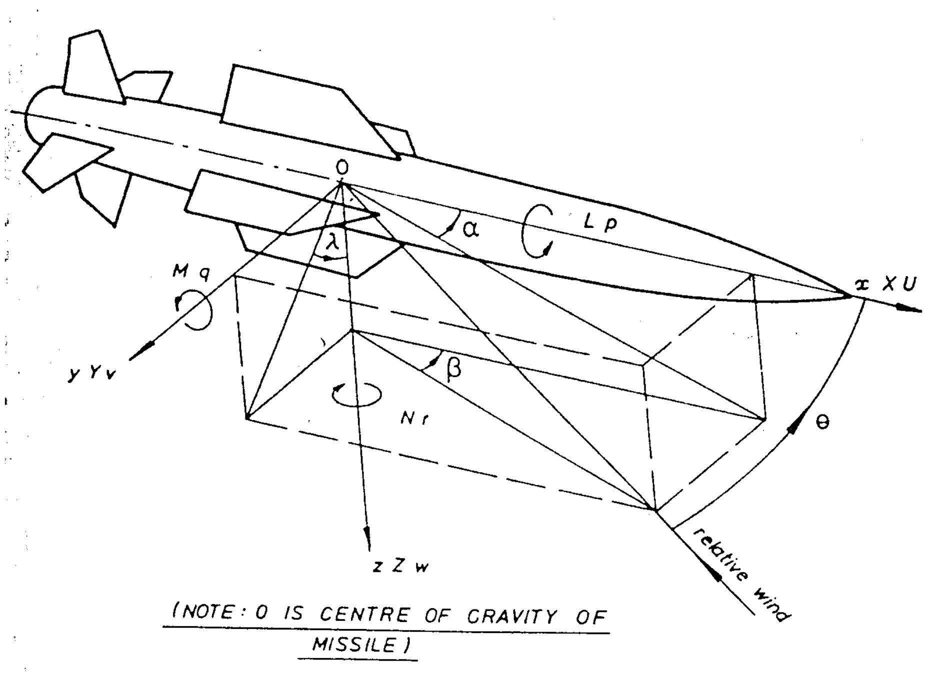

Figure 2.1 Reference axis system of guided weapons

The reference axis system standardized in the guided weapons industry is centered on the center of gravity and fixed in the body as follows:

x axis, called the roll axis, forwards along the axis of symmetry if one exists, but in any case in the plane of symmetry.

y axis, called the pitch axis, outwards and to the right, if viewing the missile from behind.

z axis, called the yaw axis, downwards in the plane of symmetry to form a right handed orthogonal system with the other two.

Table 2.1 defines the forces and moments acting on the missile, the linear and angular velocities, and the moments of inertia. The moments of inertia about O are defined as:

$$
\begin{aligned}
I_{xx} &= \sum_i m_i (y_i^2 + z_i^2) \\
I_{yy} &= \sum_i m_i (x_i^2 + z_i^2) \\
I_{zz} &= \sum_i m_i (y_i^2 + x_i^2)
\end{aligned}
\tag{2.1}
$$

The products of inertia are defined as:

$$
\begin{aligned}
I_{xy} &= -\sum_i m_i x_i y_i \\
I_{xz} &= -\sum_i m_i x_i z_i \\
I_{yz} &= -\sum_i m_i y_i z_i
\end{aligned}
\tag{2.2}
$$

The yaw plane is the Oxy plane and the pitch plane is the Oxz plane. The following angles are defined:

α: incidence in the pitch plane. β: incidence in the yaw plane.

λ: incidence plane angle. θ: total incidence.

Table 2.1. Forces, moments, velocities and moments of inertia

	Roll axis x	Pitch axis y	Yaw axis z
Unit vectors along each axis	$\vec{i}$	$\vec{j}$	$\vec{k}$
Component of Angular velocity along each axis	p	q	r
Component of Linear velocity along each axis	U	v	w
Component of force acting on missile along each axis	X	Y	Z
Moments acting on missile about each axis	L	M	N
Moments of inertial about each axis	I_{xx}	I_{yy}	I_{zz}
Products of inertia	I_{yz}	I_{xz}	I_{xy}

The reason why U, the missile velocity along the x axis is denoted by a capital letter is to emphasize that it is a large positive quantity changing at most only a few percent per second. The angular rates and components of velocity along the pitch and yaw axes however tend to be much smaller quantities which can be positive or negative and can have much larger rates of change.

Very often a student has difficulty understanding what is meant by the velocity of a body with respect to an axis system that is moving with the body. How can there be any relative velocity in this situation? Statements about the velocity along the *OX* axis refer to the component of velocity with respect to inertial space taken along the instantaneous direction of the *OX* axis at any instant. The missile has some resultant velocity vector with respect to inertial space. This vector is resolved into the instantaneous missile axes to obtain the velocity components *U*, *v*, and *w*. This resolution also applies to the angular velocity. Resolve the instantaneous angular velocity vector with respect to inertial space, into the instantaneous direction of the *OX OY* and *OZ* axes to obtain *p*, *q*, and *r*, respectively. It should be recalled that *p*, *q*, and *r* are the components of the total angular velocity of the body (missile) with respect to inertial space. Thus, they are the angular velocities that would be measured by rate gyros fixed to these axes. It should also be recalled that inertial space is that space where Newton's laws apply. In general, a set of axes with their origin at the center of the earth but not rotating with the earth may be considered as an inertial coordinate system. Thus, the earth rotates once a day with respect to such an axis system.

2.2. Development of the Equations of Motion

The equations of motion for the guided missile can be derived from Newton's second law of motion, which states that the summation of all external forces acting on a body must be equal to the time rate of change of its momentum, and the summation of the external moments acting on a body must be equal to the time rate of change of its angular momentum. The time rates of change are all taken with respect to inertial space. The laws can be expressed by two vector equations,

$$\sum \vec{F} = \frac{d}{dt}(m\vec{V}) \tag{2.3}$$

and

$$\sum \vec{M} = \frac{d}{dt}(\vec{H}) \tag{2.4}$$

where

$$\sum \vec{F} = X\vec{i} + Y\vec{j} + Z\vec{k}$$

$$\sum \vec{M} = L\vec{i} + M\vec{j} + N\vec{k} \tag{2.5}$$

$$\vec{V} = U\vec{i} + v\vec{j} + w\vec{k} \tag{2.6}$$

Let the angular velocity be denoted as $\vec{\omega}$, then

$$\vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k} \qquad (2.7)$$

The angular momentum can be expressed as

$$\vec{H} = \vec{I} \cdot \vec{\omega} \qquad (2.8)$$

where

$$\vec{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \qquad (2.9)$$

Before proceeding with the derivation, it is necessary to make some assumptions:

1. The mass of the guided missile remains constant during any particular dynamic analysis. Actually, there is considerable difference in the mass of a missile with and without fuel, but the amount of fuel consumed during the period of the dynamic analysis may be safely neglected.
2. The missile is a rigid body. Thus, any two points on or within the missile remain fixed with respect to each other. This assumption greatly simplifies the equations and is quite valid for guided weapon.
3. There are two planes of the symmetry, OXZ plane and OXY plane. For most of the guided missiles, they are always symmetrical to the OXZ plane, and reasonably symmetrical to the OXY plane. This assumption will significantly simplify the dynamical equations of the angular momentums. Note this is the main difference between airplane and missile. For airplane, there is only one symmetrical plane.
4. The earth is an inertial reference, and unless otherwise stated, the atmosphere is fixed with respect to the earth. Although this assumption is invalid for the analysis of inertial guidance systems, it is valid for analyzing automatic control systems for both aircraft and missiles, and it greatly simplifies the final equations. The validity of this assumption is based upon the fact that normally the gyros and accelerometers used for control systems are incapable of sensing the angular velocity of the earth or accelerations resulting from this angular velocity such as the Coriolis accelerations.

It is now time to consider the motion of a missile with respect to the earth. Equation (2.4) can be expanded to obtain

$$\sum \vec{F} = m\frac{d\vec{V}}{dt} \qquad (2.10)$$

It is necessary to obtain an expression for the time rate of change of the linear velocity vector with respect to the inertial space (the earth here). This process is complicated by the fact that the velocity may be rotating (the body axis is rotating) while it is changing in magnitude. This fact leads to the expression for the total derivative of a vector given below.

$$\frac{d\vec{V}}{dt} = \frac{dU}{dt}\vec{i} + \frac{dv}{dt}\vec{j} + \frac{dw}{dt}\vec{k} + U\frac{d\vec{i}}{dt} + v\frac{d\vec{j}}{dt} + w\frac{d\vec{k}}{dt} \qquad (2.11)$$

where

$$\frac{d\vec{i}}{dt} = \vec{\omega} \times \vec{i}$$
$$\frac{d\vec{j}}{dt} = \vec{\omega} \times \vec{j} \qquad (2.12)$$
$$\frac{d\vec{k}}{dt} = \vec{\omega} \times \vec{k}$$

Therefore,

$$\frac{d\vec{V}}{dt} = \frac{dU}{dt}\vec{i} + \frac{dv}{dt}\vec{j} + \frac{dw}{dt}\vec{k} + \vec{\omega} \times \vec{V} \qquad (2.13)$$

where

$$\vec{\omega} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ U & v & w \end{vmatrix} = (qw - rv)\vec{i} + (rU - pw)\vec{j} + (pv - qU)\vec{k} \qquad (2.14)$$

The three equations for linear motion are obtained as follows:

$$X = m(\dot{U} + wq - vr)$$
$$Y = m(\dot{v} + rU - pw) \qquad (2.15)$$
$$Z = m(\dot{w} + pv - qU)$$

Cross-Coupling

The term $-mpw$ implies that there is a force in the y direction due to incidence in the pitch (w) and roll motion (p). In other words the pitching motion of the missile is coupled to the yawing motion on account of roll rate. The term mpv is also saying that the yawing motion induces forces in the pitch plane if rolling motion is present. This is most undesirable as we require these two “channels” to be completely uncoupled. Ideally rudder movements should produce forces and moments in the yaw plane and result in yawing motion only. Elevators should result in a maneuver in the pitch plane. Cross-coupling between the planes must contribute to system inaccuracy. To reduce these undesirable effects the designer tries to keep roll rates as low as possible, and in a simplified analysis one usually neglects the term pv and pw if roll rates are expected to be small and incidence (v and w are proportional to incidence) is not large.

To obtain the equations of angular motion, we will use the assumption of two planes of symmetry to simplifies the expression for the angular momentum as

$$\vec{H} = I_{xx}p\vec{i} + I_{yy}q\vec{j} + I_{zz}r\vec{k} \qquad (2.16)$$

since all the products of inertia are zero, and in addition, $I_{yy} = I_{zz}$. Similarly, we will have

$$\frac{d\vec{H}}{dt} = I_{xx}\dot{p}\vec{i} + I_{yy}\dot{q}\vec{j} + I_{zz}\dot{r}\vec{k} + \vec{\omega}\times\vec{H} \qquad (2.17)$$

where

$$\vec{\omega}\times\vec{H} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ I_{xx}p & I_{yy}q & I_{zz}r \end{vmatrix} = qr(I_{zz} - I_{yy})\vec{i} + pr(I_{xx} - I_{zz})\vec{j} + pq(I_{yy} - I_{xx})\vec{k} \qquad (2.18)$$

Note that $I_{yy} = I_{zz}$, and we have following three equations for angular motion:

$$\begin{aligned} L &= I_{xx}\dot{p} \\ M &= I_{yy}\dot{q} + pr(I_{xx} - I_{zz}) \\ N &= I_{zz}\dot{r} + pq(I_{yy} - I_{xx}) \end{aligned} \qquad (2.19)$$

If the roll rate p is very small and hence neglected, then we will have following six equations

$$\begin{aligned} X &= m(\dot{U} + wq - vr) \\ Y &= m(\dot{v} + rU) \\ Z &= m(\dot{w} - qU) \\ L &= I_{xx}\dot{p} \\ M &= I_{yy}\dot{q} \\ N &= I_{zz}\dot{r} \end{aligned} \qquad (2.20)$$

These are nonlinear equations since all the external forces and moments are nonlinear functions of the linear and angular velocities as well as the control variables such as the aileron deflection ξ, the elevator deflection η and the rudder deflection ζ. In order to obtain the transfer functions of the system, we have to linearize the above equations.

2.3 Linearization and Aerodynamic derivatives.

Unfortunately there is no general solution to any non-linear differential equation. Yet we can make a fairly accurate estimate of performance, certainly from the stability aspect by linearizing the equations and taking small perturbations about a given operating point to obtain the slopes of the curves.

We can linearize the functions of the external forces and moments as following

$$\begin{aligned} Y &= Y_v v + Y_r r + Y_\zeta \varsigma \\ Z &= Z_w w + Z_q q + Z_\eta \eta \\ L &= L_p p + L_\xi \xi \\ M &= M_w w + M_q q + M_\eta \eta \\ N &= N_v v + N_r r + N_\zeta \varsigma \end{aligned} \qquad (2.21)$$

All the linear coefficients appearing on the right side of equations (2.21) such as Y_ζ and N_v are called aerodynamic derivatives. Aerodynamic derivatives are devices enabling control engineers to obtain transfer functions defining the response of a missile to aileron, elevator or rudder inputs. They can be determined by experiments.

For example, consider now the graphs shown in Fig. 2.3.1, which show rolling moment $L(\xi)$ as a function of aileron angle ξ for a particular cruciform missile with rear controls for sea level at M = 1.9.

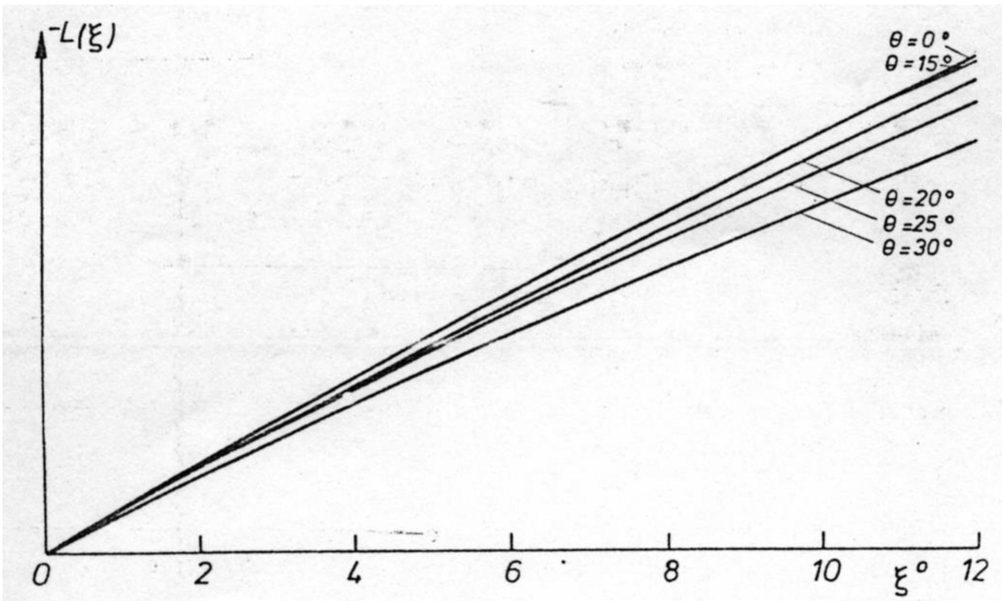

Figure 2.3.1 Aileron effectiveness

$L(\xi)$ is not a linear function of ξ *for* two reasons:
(a) aileron effectiveness decreases with total incidence θ
(b) for a given θ, $L(\xi)$ is not a linear function of ξ**,** although the graph does at least pass through the origin.

The corresponding aerodynamic derivative L_ξ is defined as:

$$L_\xi = \frac{\partial L(\xi)}{\partial \xi}$$

The incremental moment L due to a small increment ξ is therefore given by

$$L = L_\xi \xi$$

where the value of L_ξ depends on the operating point. Bearing in mind that in most applications ξ is unlikely to exceed a few degrees we might reasonably regard L_ξ as a constant.

2.4 Aerodynamic Transfer functions

We will now write down the linearized control equations for a missile, omitting the force equation along the x axis as this neither affects the roll, pitch or yaw motions. We will also omit the gravity term and lift term since they usually cancel each other out. Then the equations for controller design can be written

$$mf_y = m(\dot{v} + rU) = Y = Y_v v + Y_r r + Y_\varsigma \varsigma \quad (2.22)$$

i.e. $$f_y = \dot{v} + rU = y_v v + y_r r + y_\varsigma \varsigma \quad (2.23)$$

$$mf_z = m(\dot{w} - qU) = Z = Z_w v + Z_q q + Z_\eta \eta \quad (2.24)$$

i.e. $$f_z = \dot{w} - qU = z_w v + z_q q + z_\eta \eta \quad (2.25)$$

$$I_{xx}\dot{p} = L = L_p p + L_\xi \xi \quad (2.26)$$

i.e. $$\dot{p} = l_p p + l_\xi \xi \quad (2.27)$$

$$I_{yy}\dot{q} = M = M_w w + M_q q + M_\eta \eta \quad (2.28)$$

i.e. $$\dot{q} = m_w w + m_q q + m_\eta \eta \quad (2.29)$$

$$I_{zz}\dot{r} = N = N_v v + N_r r + N_\varsigma \varsigma \quad (2.30)$$

i.e. $$\dot{r} = n_v v + n_r r + n_\varsigma \varsigma \quad (2.31)$$

Important transfer functions can then be obtained from above equations as follows.

Roll rate/aileron $\frac{p}{\xi}$

This is the simplest aerodynamic transfer function. From Equation (2.27), we obtain

$$\frac{p}{\xi} = \frac{l_\xi}{s - l_p} = \frac{-l_\xi / l_p}{T_a s + 1} \quad (2.32)$$

where $-l_\xi / l_p$ can be regarded as a steady state gain and $T_a = \frac{1}{-l_p}$ can be regarded as an aerodynamic time constant.

Lateral acceleration /rudder $\frac{f_y}{\varsigma}$

Equations (2.23) and (2.31) can be used to eliminate r and v to yield

$$\frac{f_y}{\varsigma} = \frac{y_\varsigma s^2 - y_\varsigma n_r s - U(n_\varsigma y_v - n_v y_\varsigma)}{s^2 - (y_v + n_r)s + y_v n_r + U n_v} \quad (2.33)$$

The reader will notice that y_r does not appear in this transfer function since it is a very small quantity and in this context it is usually omitted. Some consideration of the individual terms is worthwhile. Clearly the undamped natural frequency ω_n is given as

$$\omega_n^2 = y_v n_r + U n_v$$

and is usually referred to as the weather cock frequency; it reflects the tendency for a stable missile to return to the unperturbed zero incidence position.

Consider now a typical surface-to-air missile with rear controls whose main yaw derivatives for M=1.4 and height 1500 m (U=467m/s) are:

y_v	n_v	y_ς	n_ς	n_r
-2.74	0.309	197	-534	-2.89

The missile is 2m long and $I_{zz} = 13.8 kgm^2$ and m=53 kg. Inserting the values for the derivatives we obtain

$$\frac{f_y}{\varsigma} = \frac{197s^2 + 570s - 467(1460 - 60.8)}{s^2 + (2.74 + 2.89)s + (7.9 + 144)}$$

Thus $\omega_n^2 = 7.9 + 144$ and $\omega_n = 12.4$ rad/s . The damping ratio ζ is given by $2\zeta\omega_n = 2.74 + 2.89$ and hence $\zeta = 0.23$. The damping terms vary with air density, and so do the force and moment terms, but the inertia remains the same. The steady state gain is -467(1460-60.8)/(7.9+144) = -4300. This means that if the aerodynamics were linear 0.1rad rudder deflection wou1d produce a lateral acceleration in the -y direction of 430m/s^2 i.e. about 43g. This appears to be a very high aerodynamic gain, but there is often a good reason for this as will be discussed under autopilot design.

Now a second order response (surely the airframe is yet another example of a damped spring-mass system) is completely defined by the steady state gain, the undamped natural frequency and the damping ratio.

What happens if we take the controls to the front of the missile and move the wings back so that the position of the c.p. remains unchanged? Assume all aerodynamic derivatives are unchanged numerically; there will be a change in the algebraic sign of n_ς i.e. it is now positive. This will have the effect of changing one term in the numerator from -467 (1460 - 60.8) to -467 (-1460 - 60.8) = 710,000. The steady state gain is now 710, 000/152 = 4670 an increase of about *9%*. The reason for the change in algebraic sign of the steady state gain is due to the fact that +ve canard rudder deflection produces +ve incidence.

Figures 2.5.1 (a) and (b) show the time response for f_y, for a step rudder input for the tail controlled missile compared with a similar missile with canards. For convenience of comparison both responses are regarded as +ve.

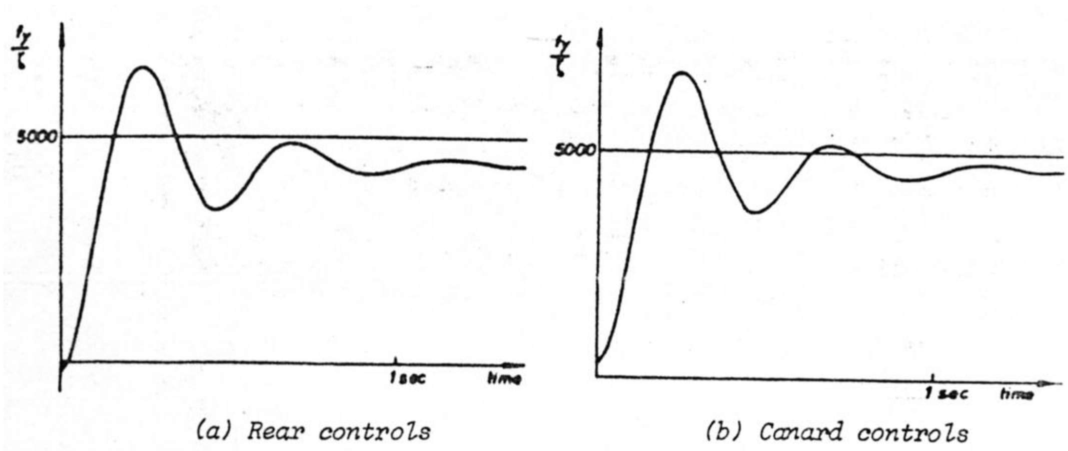

Figure 2.5.1 Lateral Acceleration for a Step Rudder Input

Finally, consider the original rear controlled missile whose c.g. has moved forward slightly to increase the small static margin by a factor of four. The steady state gain is decreased approximately by a factor of four, the weathercock frequency is doubled and the damping ratio is halved. The significance of the static margin should now be apparent. A large static margin results in:

(a) a small steady state gain
(b) a high weathercock frequency
(c) a very 1ow damping ratio

A small static margin results in:

(a) a large steady state gain
(b) a 1ow weathercock frequency
(c) a 1ow or moderate damping ratio.

It follows therefore that the position of the c.p. and the c.g. are of vital interest to the missile control system designer.

For a symmetrical missile the transfer function f_z / η must be essentially the same as for f_y / ς. There are some changes in sign however. For instance +ve rear elevator deflection eventually produces acceleration in the +ve z direction.

Body rate/rudder $\frac{r}{\varsigma}$,

To obtain this transfer function v must be eliminated from equations (2.23) and (2.31) to yield:

$$\frac{r}{\varsigma} = \frac{n_\varsigma s - n_\varsigma y_v + n_v y_\varsigma}{s^2 - (y_v + n_r)s + y_v n_r + U n_v} \qquad (2.34)$$

If one compares this with equation (2.33) one sees that the undamped natural frequency and the damping ratio are the same; indeed, lateral acceleration, body rate and incidence are all oscillatory modes of the missile and these modes are identical in frequency and damping. Only the initial values and slopes and the steady state gains are different.

Part III Autopilot Design for Guided Missiles

3.1 Introduction

The name "guided missile" implies that the missile is controlled either by an internal guidance system or by commands transmitted to the missile by radio from the ground or launching vehicle.

Before discussing some typical guidance systems, the terms "navigation system," "guidance system," and "control system" are defined. A navigation system is one that automatically determines the position of the vehicle with respect to some reference frame, for example, the earth, and displays this to an operator. If the vehicle is off course, it is up to the operator to make the necessary correction. A guidance system, on the other hand, automatically makes the necessary correction to keep the vehicle on course by sending the proper signal to the control system or autopilot. The guidance system then performs all the functions of a navigation system plus generating the required correction signal to be sent to the control system. The control system controls the direction of the motion of the vehicle or simply the orientation of the velocity vector.

The type of guidance system used depends upon the type and mission of the missile being controlled, and they can vary in complexity from an inertial guidance system for long-range surface-to-surface or air-to-surface winged missiles to a simple system where the operator visually observes the missile and sends guidance commands via a radio link. In any case, the guidance command serves as the input to the missile control system. The command may be in the form of a heading or attitude command, a pitching or turning rate command, or a pitch or yaw acceleration command, depending upon the type of guidance scheme used. We will deal with the control system in the missile that receives the signal from the guidance system and not with the guidance system itself. Such kind of system is sometimes called autopilot.

An autopilot is a closed loop system and it is a minor loop inside the main guidance loop; not all missile systems require an autopilot. For example, the anti-tank missile. A missile will maneuver up-down or left-right in an apparently satisfactory manner if a control surface is moved or the direction of thrust altered. If the missile carries accelerometers and/or gyros to provide additional feedback into the missile servos to modify the missile motion then the missile control system consisting of servos, control surfaces or thrust vector elements, the airframe, and feedback instruments plus control electronics is usually called an autopilot, but this definition is not universally accepted. Broadly speaking autopilots either control the motion in the pitch and yaw planes, in which case they are called lateral autopilots, or they control the motion about the fore and aft axis in which case they are called roll autopilots. This contrasts with the usual definition of aircraft autopilots; those designed to control the motion in the pitch plane are called longitudinal autopilots and only those to control the motion in yaw are called lateral autopilots. For instance, an aircraft autopilot designed to keep the heading constant would be called a lateral autopilot. For a symmetrical cruciform missile however pitch and yaw autopilots are often identical one injects a bias in the vertical plane to offset the effect of gravity but this does not affect the design of the autopilot.

Of the various types of guided missiles, those that are flown in the same manner as manned aircraft (that is, missiles that are banked to turn, such as cruise missiles and remotely piloted vehicles) will not be discussed. For example the Tomahawk.

Figure 3.1.1 Tomahawk cruise missile.

Of interest in this module are other "aerodynamic missiles" (which use aerodynamic lift to control the direction of flight), such as the Sidewinder, Patriot, etc. One feature of these missiles is that they are roll stabilized; thus there is no coupling between the longitudinal and the lateral modes, which simplifies the analysis. Since the missiles to be studied in this section are roll stabilized, a system for accomplishing this is discussed first.

3.2 Roll Stabilization

Roll stabilization can be accomplished by different means, depending on the type of missile. For aerodynamic missiles, the required rolling moment is achieved by differential movement of the control surfaces. For ballistic missiles, the rolling moment can be obtained by differential swiveling of small rockets mounted on the side of the missile, as is done on the Atlas, or by differential swiveling of the two main rocket engines if more than one engine is used.

The Atlas has vernier engines, located on opposite sides of the missile above the booster engine fairing. They controlled the roll of the missile and trim its final flight velocity.

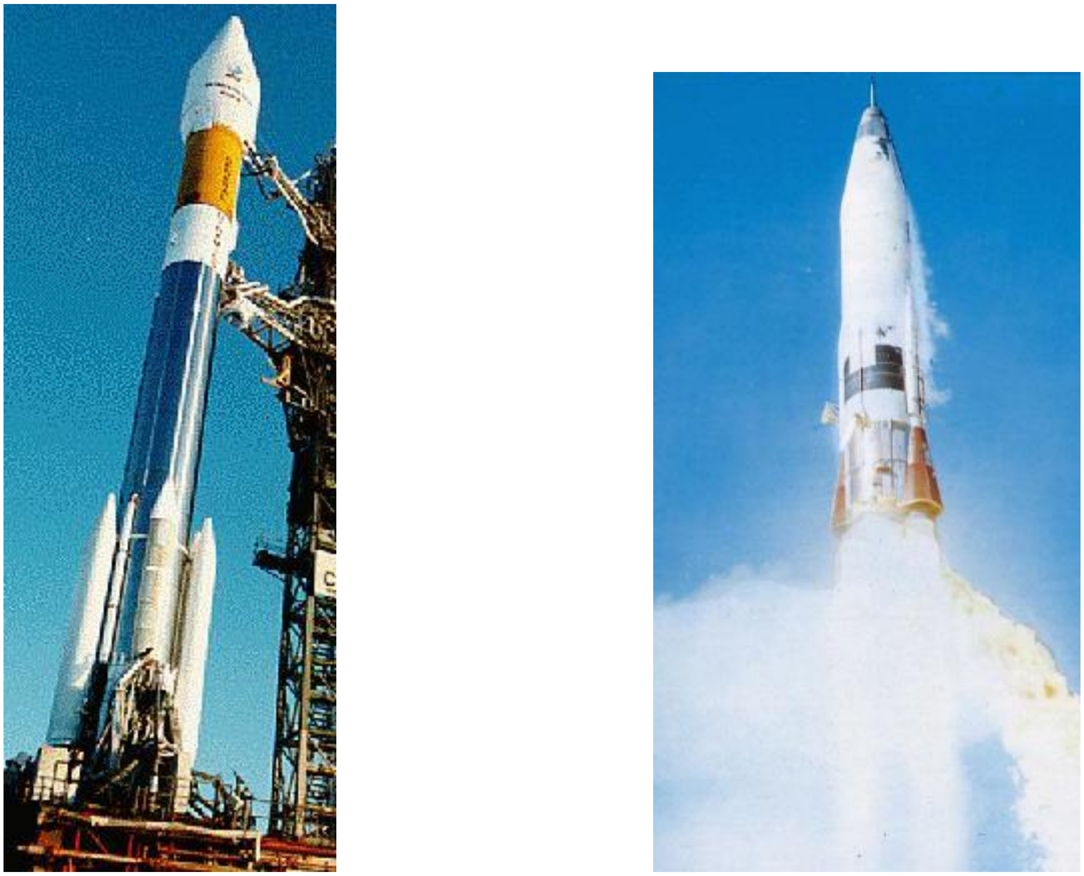

Figure 3.2.1 Atlas ICBM

The next problem is the detection of the rolling motion so that it can be controlled, and the reduction of roll rate to zero or maintenance of the roll angle equal to some specified reference.

Consider an air-to-air homing missile which is roll position stabilized and due largely to the variability in the launch speed can have a velocity in the range M=1.4 to M=2.8.

Table 3.2.1 Roll Aerodynamic Derivatives, Gains and Time Constants

	M = 1.4	M = 1.6	M = 1.8	M = 2.0	M = 2.4	M = 2.8
$-L_\xi$	7050	8140	9100	10,200	11,700	13,500
$-L_p$	22.3	24.9	27.5	30.3	34.5	37.3
$T_a = \frac{-1}{\ell_p} = \frac{-A}{L_p}$	0.043	0.0385	0.0349	0.0316	0.0278	0.0257
$\frac{\ell_\xi}{\ell_p} = \frac{L_\xi}{L_p}$	316	327	331	336	340	362

Table 3.2.1 shows the variability of the roll derivatives and time constant over this range for a missile at the height of 1500m.

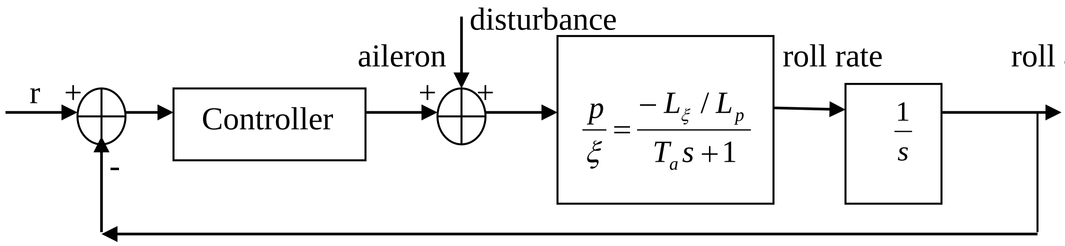

Figure 3.2.2 Roll position control autopilot

Fig 3.2.2 shows the roll position control loop with the demand roll position equal to zero. To simplify the discussion, we have ignored the models of the actuators and sensors in the block diagram, such that we can just focus upon the controller design.

The open loop transfer function without any controller is

$$H(s) = \frac{-\frac{L_\xi}{L_p}}{s(T_a s + 1)} \tag{3.1}$$

and becomes the following transfer function by inserting all these parameter values for the case of M=2.8.

$$H(s) = \frac{-362}{s(0.0257s + 1)} \tag{3.2}$$

The system is marginally stable, and the step response of this system (obtained by MATLAB command step(sys)) is shown in Fig. 3.2.3. Obviously if there is a constant disturbance exerting on the missile, then it will keep on rolling, which is undesirable at all.

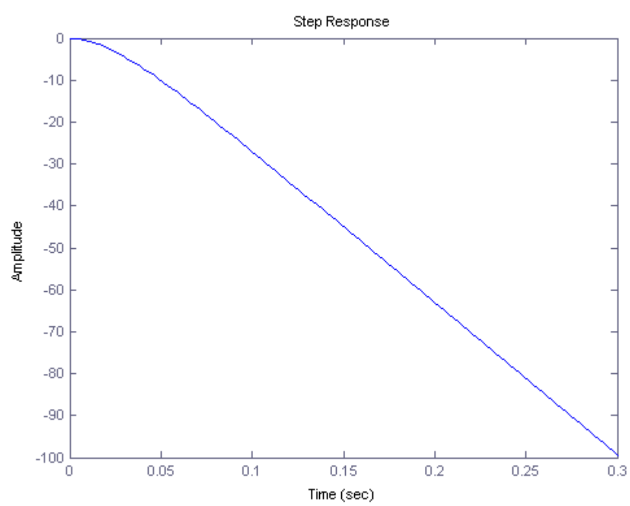

Figure 3.2.3 Step response of the open-loop system.

In order to stabilize the system, the simplest controller is the proportional controller as shown in Fig. 3.2.4. It is important to note that the gain is negative signed such that the whole closed-loop is stable.

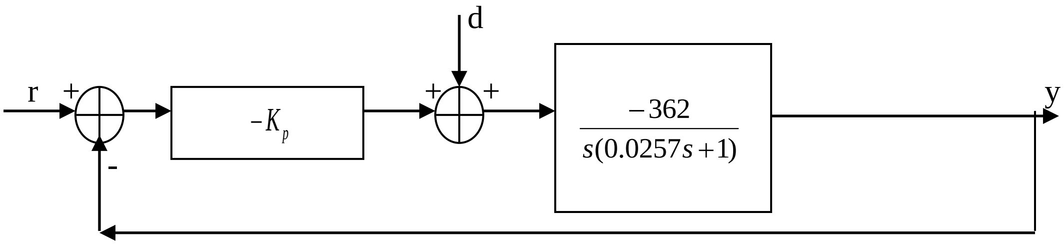

Figure 3.2.4 Proportional control of roll motion.

The closed-loop transfer function from the reference signal r to the roll angle y is then

$$\frac{Y(s)}{R(s)} = \frac{\dfrac{K_p 362}{s(0.0257s+1)}}{1+\dfrac{K_p 362}{s(0.0257s+1)}} = \frac{K_p 362}{s(0.0257s+1)+K_p 362} \qquad (3.3)$$

The closed-loop system is stable now. The transfer function from the disturbance d to the roll angle y is

$$\frac{Y(s)}{D(s)} = \frac{\dfrac{-362}{s(0.0257s+1)}}{1+\dfrac{K_p 362}{s(0.0257s+1)}} = \frac{-362}{s(0.0257s+1)+K_p 362} \qquad (3.4)$$

And the steady-state gain of above transfer function is

$$\left.\frac{Y(s)}{D(s)}\right|_{s=0} = \frac{-1}{K_p}$$

The response of the missile when the disturbance is a step-input with the proportional gain $K_p = 10$ is shown in Fig. 3.2.5.

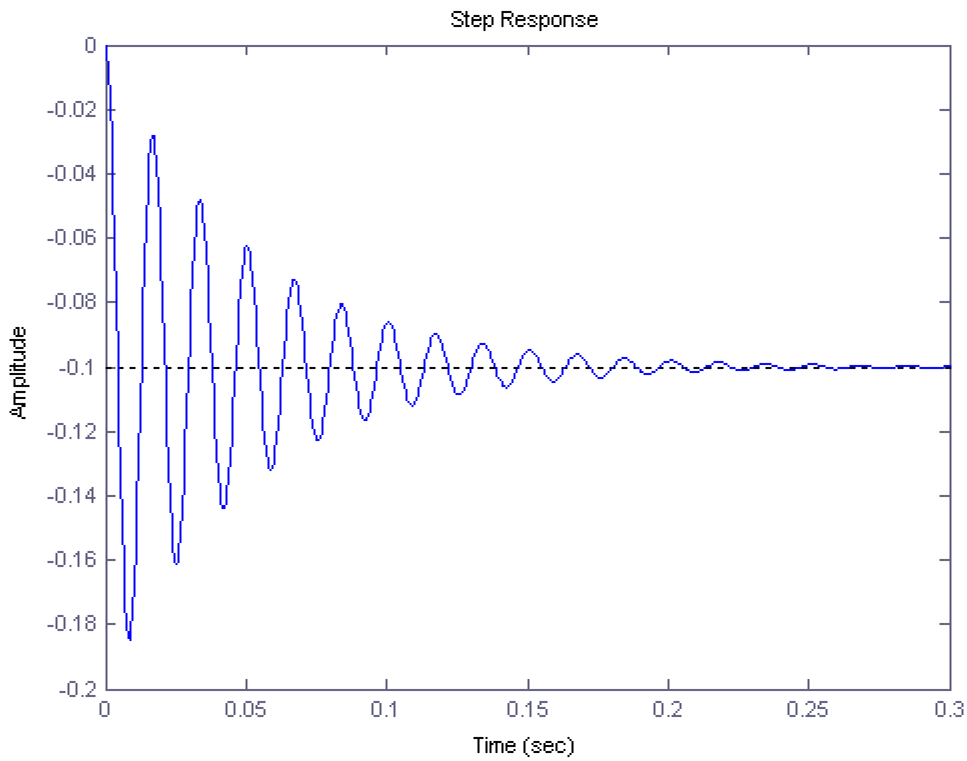

Figure 3.2.5 Step response of closed-loop system when proportional controller is used.

It can be seen that the roll angle converges to a small constant value of -0.1 within 0.3 seconds, and hence the roll rate would be zero, and the objective of roll stabilization is achieved. However, it is evident that the movement is highly oscillatory due to the small damping ratio of the closed loop system as shown below. If we rewrite the transfer function in the form of

$$\frac{Y(s)}{R(s)} = \frac{K_p 362}{s(0.0257s+1)+K_p 362} = \frac{K_p 362/0.0257}{s^2 + s/0.0257 + K_p 362/0.0257} \qquad (3.5)$$

And compare it to the prototype of second-order system

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

we have

$$\omega_n^2 = K_p 362/0.0257$$

$$2\zeta\omega_n = 1/0.0257$$

and

$$\omega_n = \sqrt{K_p 362 / 0.0257}$$
$$\zeta = \frac{1}{2\sqrt{K_p 362 \times 0.0257}} .$$

When $K_p = 10$, it results in

$$\omega_n = 375.3$$
$$\zeta = 0.05$$

The damping ratio is only 0.05, which can well explain the oscillation with high frequency. In order to prevent this from happening, the damping ratio has to be increased to at least 0.7. Although the damping ratio can be simply increased by decreasing the proportional gain K_p, it is not a good solution since we also want to keep a low value of steady state gain ($1/K_p$) such that the steady-state roll angle is small. How do we increase the damping ratio then?

Let's try to use frequency domain design to solve this problem. The open loop transfer function with the proportional gain $K_p = 10$, but without any phase lead compensators is

$$G(s) = \frac{3620}{s(0.0257s+1)} \qquad (3.6)$$

The Bode diagram is then plotted in Fig. 3.2.6. The phase margin can be obtained using Matlab command "margin(sys)" as $6°$, which obviously is too small.

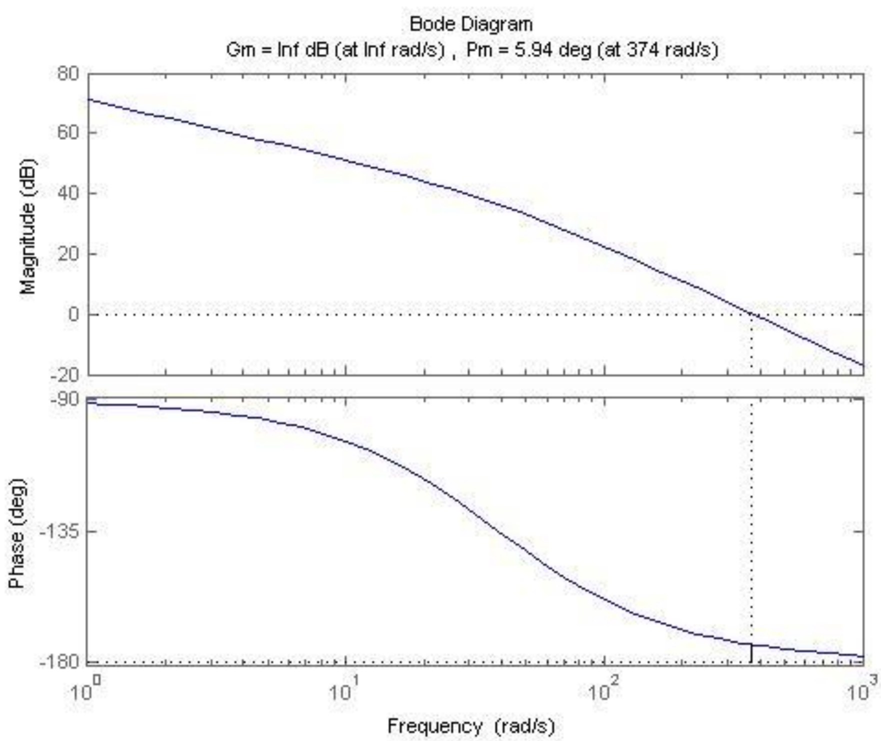

Figure 3.2.6 Bode plot of system without lead compensation

We need to increase the phase margin to around $\phi_{desire} = 70^\circ$. To this end, a lead compensator is chosen to be

$$C(s) = \frac{\tau s + 1}{\alpha \tau s + 1}. \tag{3.7}$$

Following the **Lead Compensation Design Procedure** (Chapter Four, section 4.9.3) provided in the lecture notes, find the gain crossover frequency ω_g by checking the Bode plot.

$$\omega_g = 374 rad/s$$

Then decide the compensator parameters by using:

$$\phi_m = \phi_{desire} - \phi + (5^\circ \to 12^\circ) \tag{3.8}$$

Let's try

$$\phi_m = \phi_{desire} - \phi + 6^\circ = 70^\circ - 6^\circ + 6^\circ = 70^\circ$$

Then

$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m} = 0.031 \tag{3.9}$$

Find out the frequency, ω_m, at which the magnitude of the uncompensated system, $|G(j\omega)|$, is equal to $-20\log_{10}(1/\sqrt{\alpha}) = -15$ dB (you can do this by zoom in the Bode diagram of G(s)),

$$\omega_m = 890$$

and then compute

$$\tau = \frac{1}{\omega_m \sqrt{\alpha}} = 0.0064\,. \tag{3.10}$$

Therefore, the lead compensator is

$$C(s) = \frac{\tau s + 1}{\alpha \tau s + 1} = \frac{0.0064 s + 1}{0.0002 s + 1} \tag{3.11}$$

Evaluate the compensated system via the Bode plots (use command "margin(C(*s*)*G(*s*))").

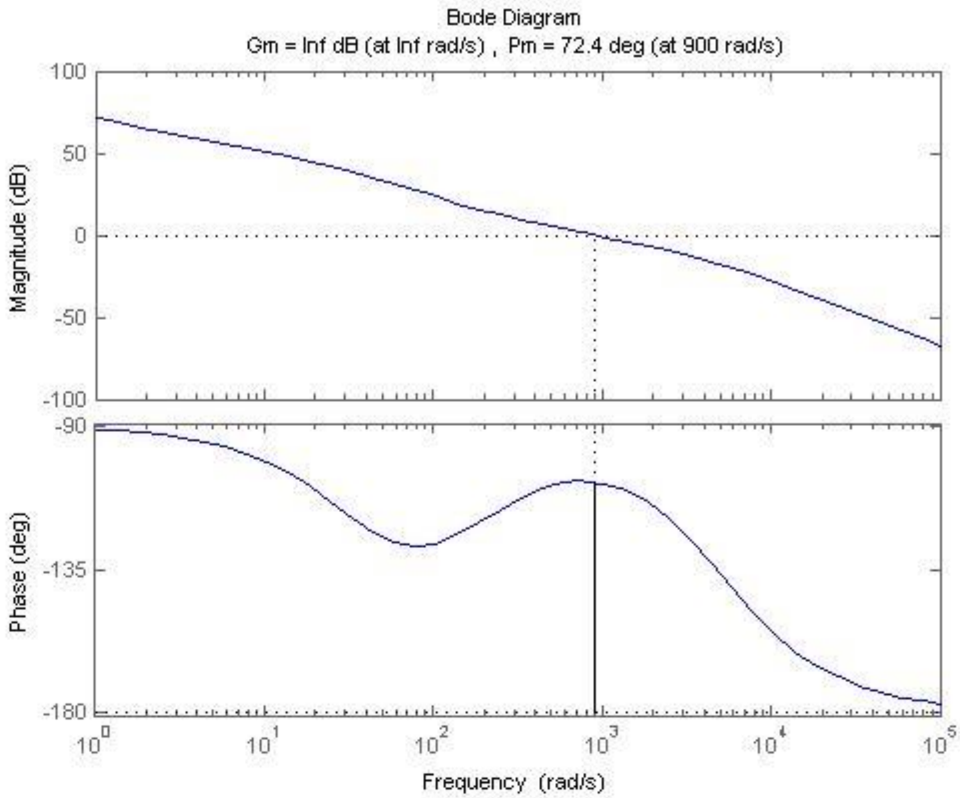

Figure 3.2.7 Bode plot of system with lead compensation

The phase margin now is $72.4°$, which is close to the desired value of $70°$.

The overall feedback control system with the phase lead compensator is shown below.

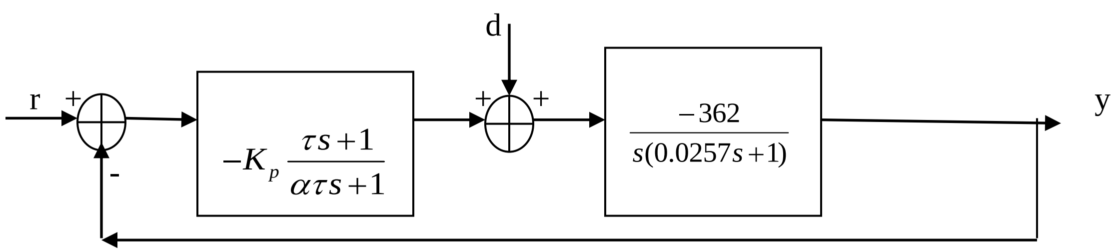

Figure 3.2.8 roll stabilization with lead compensator.

The closed-loop transfer function from the reference signal r to the roll angle y is then

$$\frac{Y(s)}{R(s)} = \frac{\frac{K_p(\tau s+1)}{(\alpha\tau s+1)}\frac{362}{s(0.0257s+1)}}{1+\frac{K_p(\tau s+1)}{(\alpha\tau s+1)}\frac{362}{s(0.0257s+1)}} = \frac{3620(\tau s+1)}{s(0.0257s+1)(\alpha\tau s+1)+3620(\tau s+1)}$$

$$= \frac{23.17s+3620}{5.14\times10^{-6}s^3+0.0259s^2+24.17s+3620} \tag{3.12}$$

The transfer function from the disturbance d to the roll angle y is used, is then

$$\frac{Y(s)}{D(s)} = \frac{\frac{-362}{s(0.0257s+1)}}{1+\frac{K_p(\tau s+1)}{(\alpha\tau s+1)}\frac{362}{s(0.0257s+1)}} = \frac{-362(\alpha\tau s+1)}{s(0.0257s+1)(\alpha\tau s+1)+3620(\tau s+1)}$$

$$= \frac{-0.0724s-362}{5.14\times10^{-6}s^3+0.0259s^2+24.17s+3620} \tag{3.13}$$

The response of the missile for the step disturbance is then shown in Fig. 3.2.9. Compared to the results obtained by P control as shown in Fig 3.2.5, not only the high frequent oscillation disappears, but also the settling time is decreased dramatically to around 0.03 seconds from 0.3 seconds.

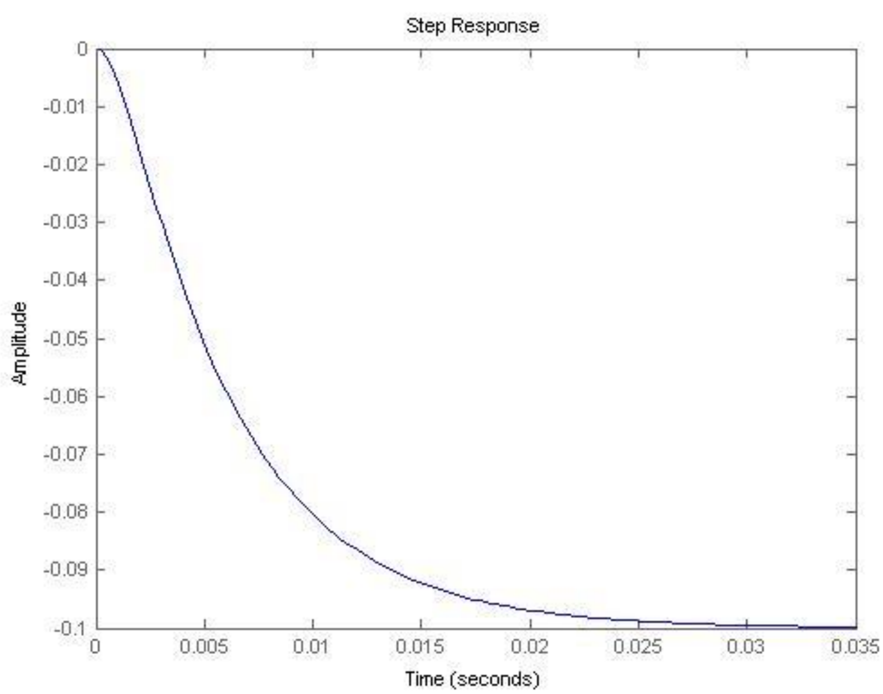

Figure 3.2.9 Step disturbance rejection by analog lead compensator.

In the following, we just discretize the lead compensator C(s) to be C(z) by using bilinear transform ($s = \frac{2}{T} \cdot \frac{z-1}{z+1}$). The bandwidth of the closed-loop system is found to be 1216.7 rad/s=193.6Hz (using MATLAB command bandwidth(sys)).

Choose a sampling rate at least 5 times the bandwidth, T=0.001s, the discretized controller is

$$
\begin{aligned}
C(z) &= -K_p\, C(s)\big|_{s=\frac{2}{T}\frac{z-1}{z+1}} = -10\,\frac{0.0064s+1}{0.0002s+1}\bigg|_{s=\frac{2}{T}\frac{z-1}{z+1}} \\
&= -10\,\frac{0.0064\frac{2}{T}\frac{z-1}{z+1}+1}{0.0002\frac{2}{T}\frac{z-1}{z+1}+1} \\
&= \frac{\text{-98.57 z +84.29}}{\text{z + 0.4286}}
\end{aligned}
\tag{3.14}
$$

You can also obtain the above result using MATLAB command: c2d(C(s),T,'tustin').

Simulate the unit step response of the closed-loop system with the digital lead compensator with T=0.001s. The block diagram used in MATLAB SIMULINK is shown below in Fig. 3.2.10

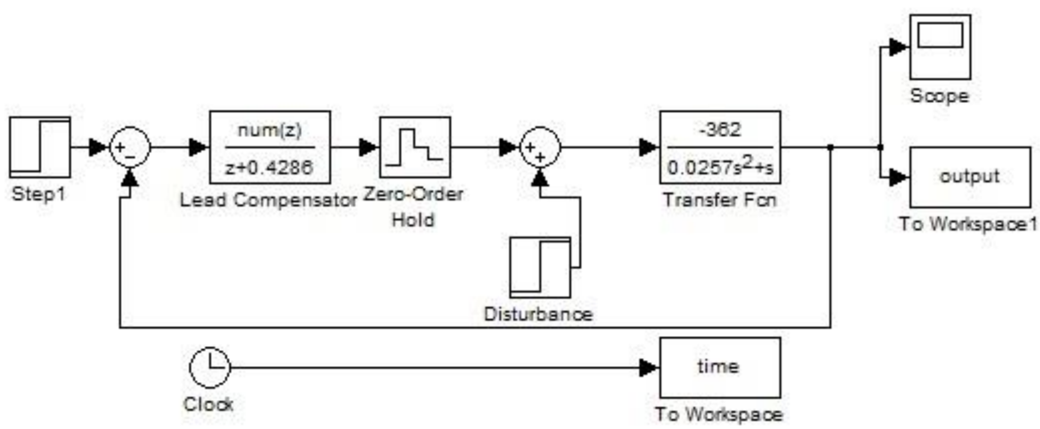

Figure 3.2.10 Simulink Block Diagram for roll stabilization with lead compensator.

You may save the block diagram as image using "PrtScn" in the keyboard combined with "paint". It is important to set the sampling period correctly for both the blocks of transfer function of the digital controller and the zero-order hold. Otherwise, you may get something unexpected.

The response of the missile for the step disturbance is also shown in Fig. 3.2.11. It is obvious that the performance is about the same as that from the analog controller. There is no oscillation and the settling time is around 0.03 seconds. Therefore, it may be concluded that the roll stabilization can be well achieved with digital lead compensator.

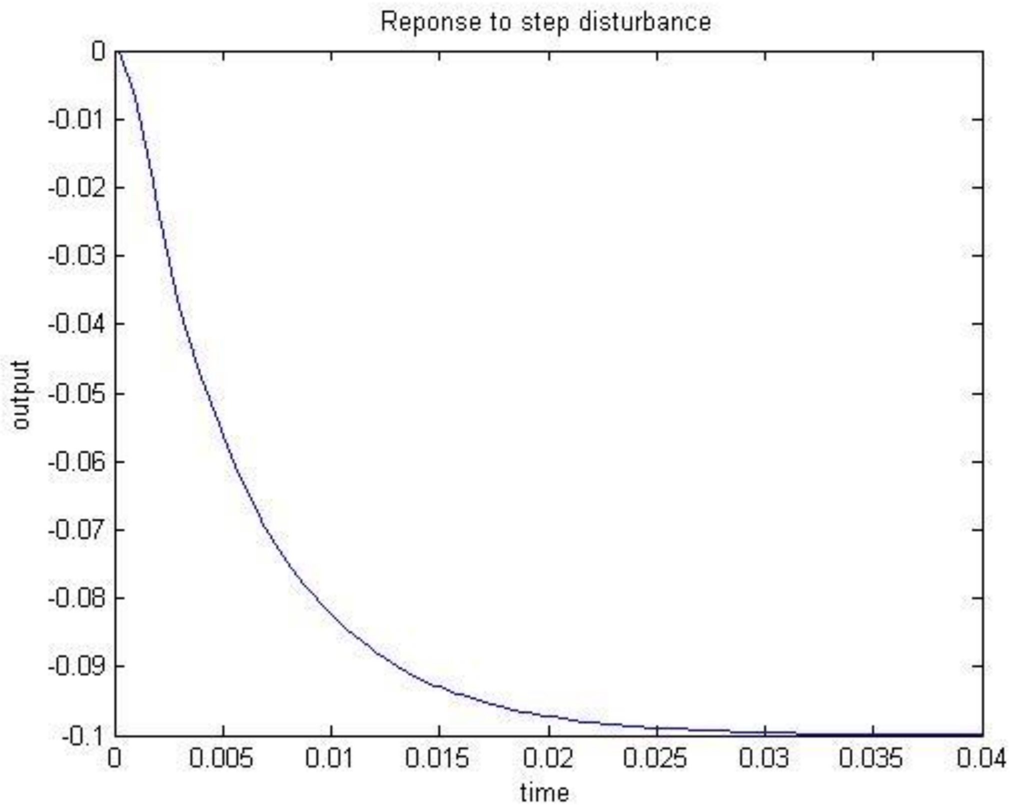

Figure 3.2.11 Step disturbance rejection by digital lead compensator.

3.3. Lateral autopilot design

Identical autopilots are used to control the pitch and yaw motions if the missile has two planes of symmetry, so we need consider one channel only, the yaw autopilot say. The transfer function from rudder angle to body rate is,

$$\frac{r}{\varsigma} = G_p(s) = \frac{n_\varsigma s - n_\varsigma y_v + n_v y_\varsigma}{s^2 - (y_v + n_r)s + y_v n_r + U n_v} \qquad (3.15)$$

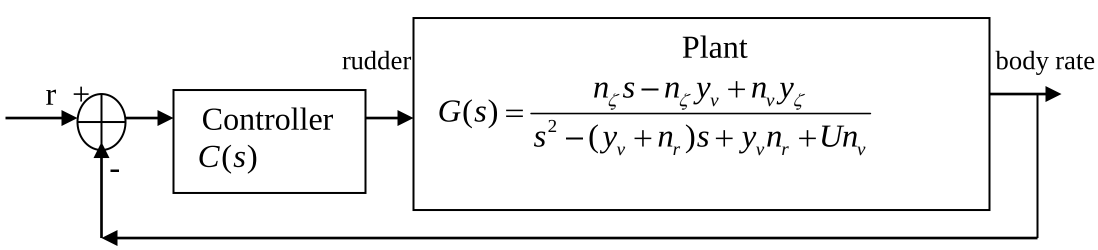

Figure 3.3.1 Lateral Autopilot Design.

Figure 3.3.1 shows the arrangement for a missile with rear controls. The objective of the lateral autopilot is for command signal following. The command signal is given by the guidance system, for instance, a desired body rate r. Then the task of the autopilot is to assure that the body rate of the missile, i.e., the output y, follows r as fast as possible.

In order to design the autopilot for a rear controlled missile we must make a first estimate of the required aerodynamic derivatives. We estimate that the missile may have a forward velocity of 500 m/s, i.e. 1.5 M (500/343). The missile's maximum velocity will be $\sqrt{2}\times 500$ m/s and the minimum velocity will be $500/\sqrt{2}$ m/s i.e. a speed variation of 2 to 1 overall. The aerodynamic derivatives are estimated and tabulated in Table 3.3.1.

Table 3.3.1 Aerodynamic derivatives under different flight conditions

U(m/s)	y_v	n_v	y_ς	n_ς	n_r
500	-3	+1	+180	-500	-3
$500/\sqrt{2}$	-3.0/$\sqrt{2}$	1/$\sqrt{2}$	180/2	-500/2	-3/2
$\sqrt{2}\times 500$	$\sqrt{2}\times$(-3)	$\sqrt{2}$	$2\times$ (180)	$2\times$ (-500)	$2\times$(-3)

Let's first design the autopilot for the nominal speed of 500 m/s. The transfer function from the rudder angle to the yaw body rate is

$$G(s)=\frac{n_\zeta s-n_\zeta y_v+n_v y_\zeta}{s^2-(y_v+n_r)s+y_v n_r+Un_v}=\frac{-500s-1320}{s^2+6s+509} \quad (3.16)$$

The open loop weather cock mode is defined by

$\omega_n{}^2=Un_v+y_v n_r=509$ and $2\zeta\omega_n=-(y_v+n_r)=6$. Hence $\omega_n=22.56$ and $\zeta=0.13$. The step response of the open loop system is shown in Figure 3.3.2.

Although the open-loop system is stable, the performance is not satisfactory due to a number of factors.

- The response is highly oscillatory.
- The response is slow.
- The steady-state gain is not unity.

Thus the design objective of the controller is to fulfill following requirements:

- The maximum overshoot is less than 20%.
- The settling time is less than 0.1 seconds.
- Steady state gain is one such that the steady state error is zero
- Disturbance rejection is required for load disturbances.

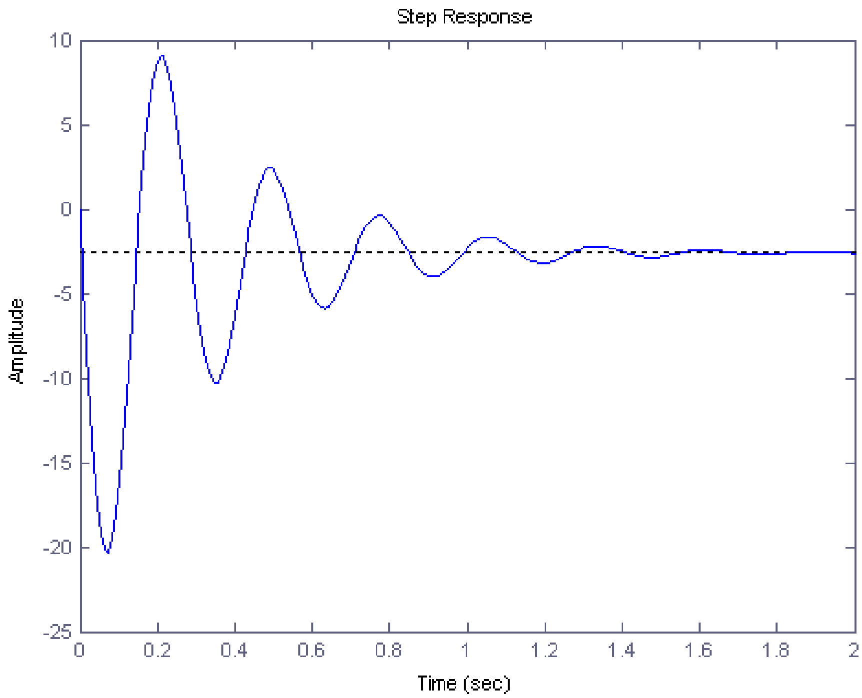

Figure 3.3.2 The step response of the open-loop system.

The characteristic polynomial in continuous time domain has the form of:

$$s^2 + 2\varsigma\omega_n s + \omega_n^2$$

Overshoot less than 20%: $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} < 20\% \Rightarrow \zeta > 0.456$. Let $\zeta = 0.5$

Settling time less than 0.1 seconds: $t_s = \dfrac{4.6}{\zeta\omega_n} < 0.1 \Rightarrow \omega_n = \dfrac{4.6}{0.1\zeta} = 92$.

The reference model is then

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{8464}{s^2 + 92s + 8464} \tag{3.17}$$

Let's first design the analog controller and use the emulation design to get the digital controller.

The closed-loop transfer function from the command signal r to output y,

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} \tag{3.18}$$

If the simplest type of feedback controller, P control, is used,

$$C(s) = -K_p\,, \qquad (3.19)$$

Again, the sign of the gain is negative is to make sure that the open-loop gain is positive such that the closed-loop is stable. The transfer function is then,

$$\frac{Y(s)}{R(s)} = \frac{-K_p G(s)}{1 - K_p G(s)} = \frac{-K_p(-500s - 1320)}{s^2 + (6 + 500K_p)s + (509 + 1320K_p)} \qquad (3.20)$$

Therefore, the resulting natural frequency ω_n and μ can be obtained by

$$\begin{aligned} \omega_n^{\,2} &= 509 + 1320K_p \\ 2\zeta\omega_n &= 6 + 500K_p \end{aligned}\,. \qquad (3.21)$$

Obviously the P controller does not have the freedom to satisfy both requirements on damping ratio and natural frequency at the same time. For instance, if we first try to meet the frequency requirement by setting $8464 = 509 + 1320K_p$, which results in $K_p = 6$. But then the corresponding damping ratio would be $\zeta = (6 + 500K_p)/2\omega_n = 16$, which is over-damped. Another problem with the P control is that the steady state gain is 0.94, which implies that the body rate of the missile, i.e. the output y, cannot follow the command signal exactly.

Therefore, a simple P controller cannot meet all the design specifications. Hence we have to consider more sophisticated controller, like the PD, or PI controller. Remember that the purpose of D control is usually for increasing the damping ratio. But the damping ratio for this lateral autopilot is already too high. Therefore, PI controller should be considered instead of PD controller.

Normally, the main purpose of Integral control is to improve the steady state response and set the steady state gain to be unity, which perfectly fits this example. Although the Integral control maydecrease the damping ratio, this effect is actually needed for this particular example of autopilot design. Hence let's use the PI controller in the form of,

$$C(s) = -(K_p + K_i \frac{1}{s})\,. \qquad (3.22)$$

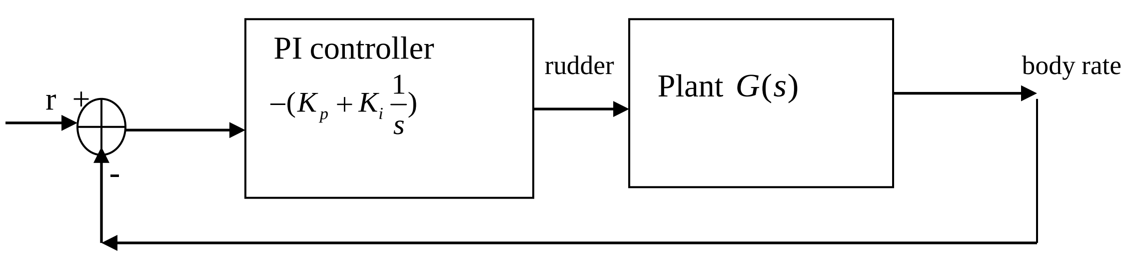

Figure 3.3.3 Lateral Autopilot Design with PI controller.

The closed loop transfer function from the reference signal r to the output y is

$$
\begin{aligned}
\frac{Y(s)}{R(s)} &= \frac{C(s)G(s)}{1+C(s)G(s)} \\
&= \frac{-(K_p + K_i \frac{1}{s})(\frac{-500s-1320}{s^2+6s+509})}{1-(K_p + K_i \frac{1}{s})(\frac{-500s-1320}{s^2+6s+509})} \\
&= \frac{(K_p s + K_i)(500s+1320)}{s(s^2+6s+509)+(K_p s+K_i)(500s+1320)} \\
&= \frac{(K_p s + K_i)(500s+1320)}{s^3+(6+500K_p)s^2+(509+1320K_p+500K_i)s+1320K_i}
\end{aligned}
$$

Compared to the reference model, it is a third order system. We need to design another pole in the reference model, let

$$
\begin{aligned}
&(s+a)(s^2+2\zeta\omega_n s+\omega_n^2) = s^3+(a+2\zeta\omega_n)s^2+(2\zeta\omega_n a+\omega_n^2)s+\omega_n^2 a \\
&= s^3+(a+92)s^2+92(a+92)s+92^2 a
\end{aligned}
$$

Match the coefficients of the denominators, we have

$$
\begin{aligned}
&6+500K_p = a+92 \\
&509+1320K_p+500K_i = 92(a+92) \\
&1320K_i = 92^2 a
\end{aligned}
$$

We get

$$
\begin{aligned}
K_p &= 0.1770 \\
K_i &= 15.8991 \\
a &= 2.4795
\end{aligned}
$$

The step response of the closed loop system is plotted in Fig. 3.3.4 with

$$
\begin{aligned}
K_p &= 0.1770 \\
K_i &= 15.8991
\end{aligned}
$$

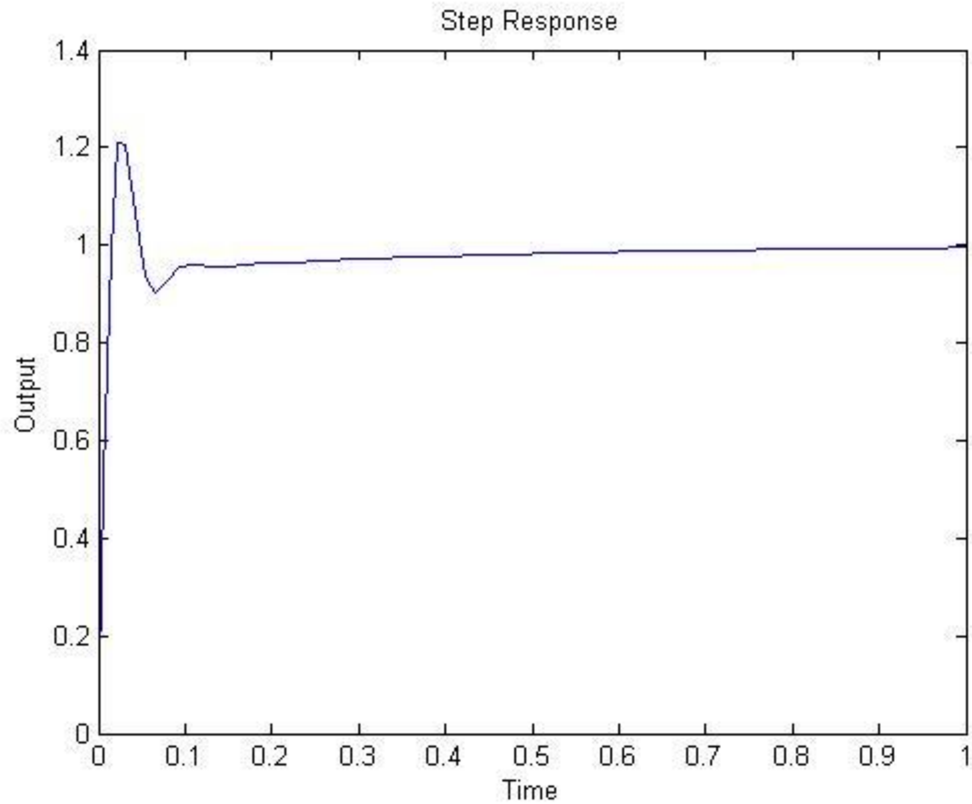

Figure 3.3.4 Step response with PI controller.

Obviously, although the maximum overshoot meets the requirement, the settling time is around 0.5 seconds, which is too slow. This is because that the dominant slow mode is –a=-2.5, instead of the desired mode

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -46 \pm j79.674$$

Therefore we need to add more freedom to the controller. Let's try the two-degree-of-freedom controller.

First let's choose the sampling period. Since the desired natural frequency is $\omega_n = 92rad/\sec = 14.6Hz$, the bandwidth of the desired closed loop is around 15Hz. 100Hz might be good enough. Then the sampling period is T=0.01.

From table 2.1, or using "c2d" command in MATLAB, we can obtain the reference model in the discrete-time as

$$\frac{8464}{s^2+92s+8464} \quad \rightarrow \quad H_d(z) = \frac{B_m(z)}{A_m(z)} = \frac{0.2981\text{ z} + 0.2179}{z^2 - 0.8826z + 0.3985} \qquad (3.23)$$

and the transfer function of the sampled system as

$$\frac{-500s-1320}{s^2+6s+509} \quad \rightarrow \quad G(z)=\frac{B(z)}{A(z)}=\frac{-4.876\,z+4.749}{z^2-1.893z+0.9418} \qquad (3.24)$$

The two-degree-of-freedom controller will be used as shown in Fig. 3.3.5. We need to design the feedback controller and feed-forward controller.

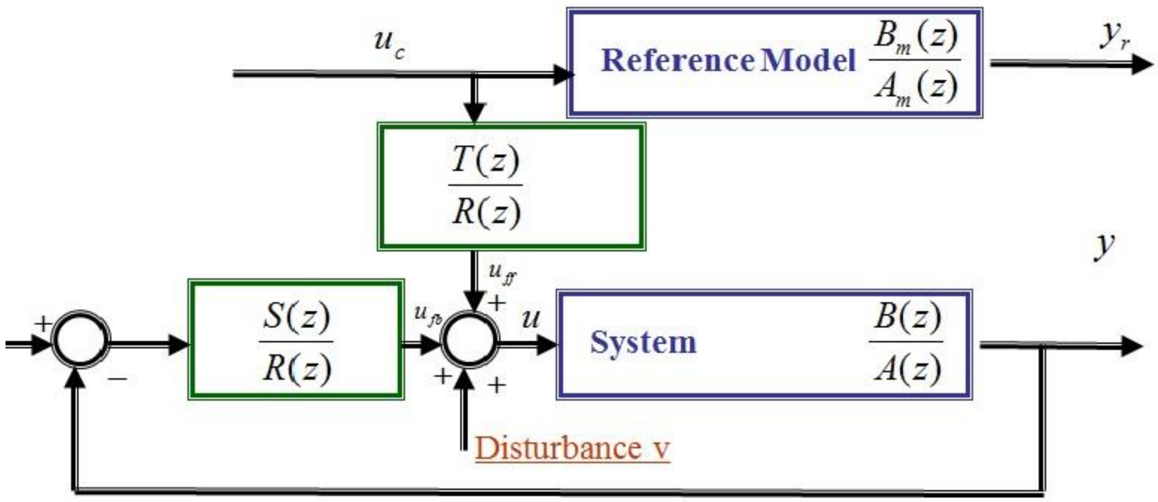

Figure 3.3.5 Two-Degree-of-Freedom Controller.

Since we want to reject load disturbance, (z-1) has to be a factor of R(z). Notice that B(z) is stable, so we Let

$$R(z)=(z-1)B(z)$$
$$S(z)=s_0z^2+s_1z+s_2$$

It follows that

$$AR+BS=(z^2-1.893z+0.9418)(z-1)B+B(s_0z^2+s_1z+s_2)$$
$$=B(z^3+(s_0-2.893)z^2+(s_1+2.8348)z+s_2-0.9418)$$

Compared with the desired polynomial Am(z), we need to add Ao(z) such that

$$A_{cl}(z)=B(z)A_m(z)A_o(z)=B(z)(z^2-0.8826z+0.3985)z$$
$$=B(z)(z^3-0.8826z^2+0.3985z)$$

where we design Ao(z)=z for simplicity. Comparing the coefficients, we have the equations:

$$s_0-2.893=-0.8826$$
$$s_1+2.8348=0.3985$$
$$s_2-0.9418=0$$

from which the control parameters are obtained as

$$s_0 = 2.0104$$
$$s_1 = -2.4363$$
$$s_2 = 0.9418$$

Therefore the feedback controller is

$$R(z) = (z-1)(-4.876\text{ z} + 4.749) = -4.876z^2 + 9.625z-4.749$$
$$S(z) = 2.0104z^2 - 2.4363z+0.9418$$

The closed loop transfer function from command signal to the output is

$$\frac{Y(z)}{U_c(z)} = \frac{T(z)}{R(z)}\frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)} = \frac{B(z)T(z)}{B(z)A_m(z)A_o(z)} = \frac{T(z)}{A_m(z)A_o(z)}$$

Design T(z) to match $\dfrac{B_m(z)}{A_m(z)}$

$$\frac{T(z)}{A_m(z)A_o(z)} = \frac{B_m(z)}{A_m(z)}$$

$$T(z) = B_m(z)A_o(z) = (0.2981\text{ z} + 0.2179)\text{z}=0.2981\text{ z}^2 + 0.2179\text{z}$$

Overall, the two-degree of freedom controller has the form of,

$$U(z) = -\frac{2.0104z^2 - 2.4363z+0.9418}{-4.876z^2 + 9.625z-4.749}Y(z) + \frac{0.2981\text{ z}^2 + 0.2179\text{z}}{-4.876z^2 + 9.625z-4.749}U_c(z) \quad (3.25)$$

Now simulate the digital two-degree-of-freedom control system response with actual plant. The SIMULINK block diagram is shown in Fig. 3.3.6. The step response is shown in Fig. 3.3.7. It is obvious that both requirements on overshoot and settling time are met.

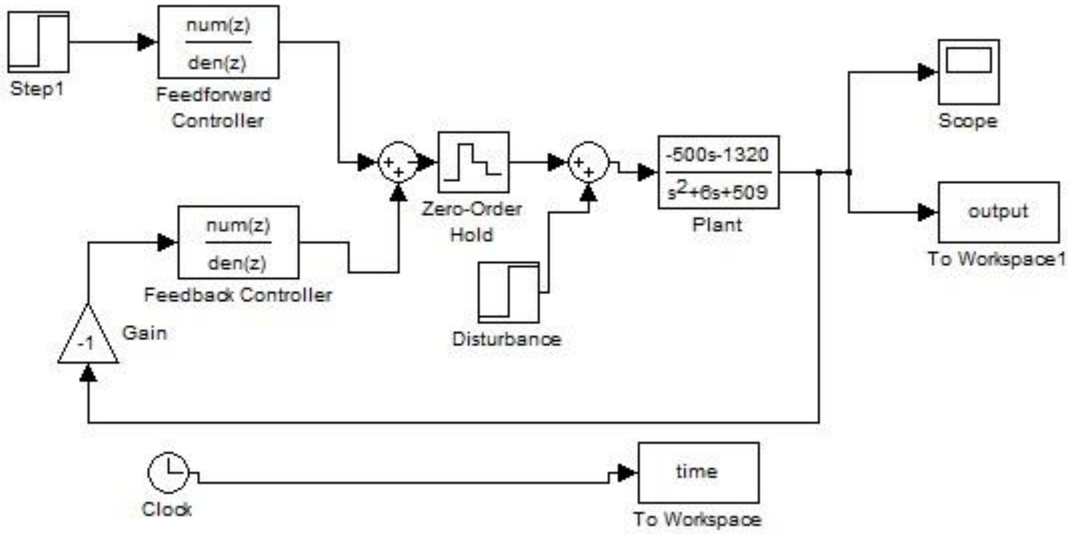

Figure 3.3.6 Simulink Block Diagram for Two-Degree-of-Freedom Controller.

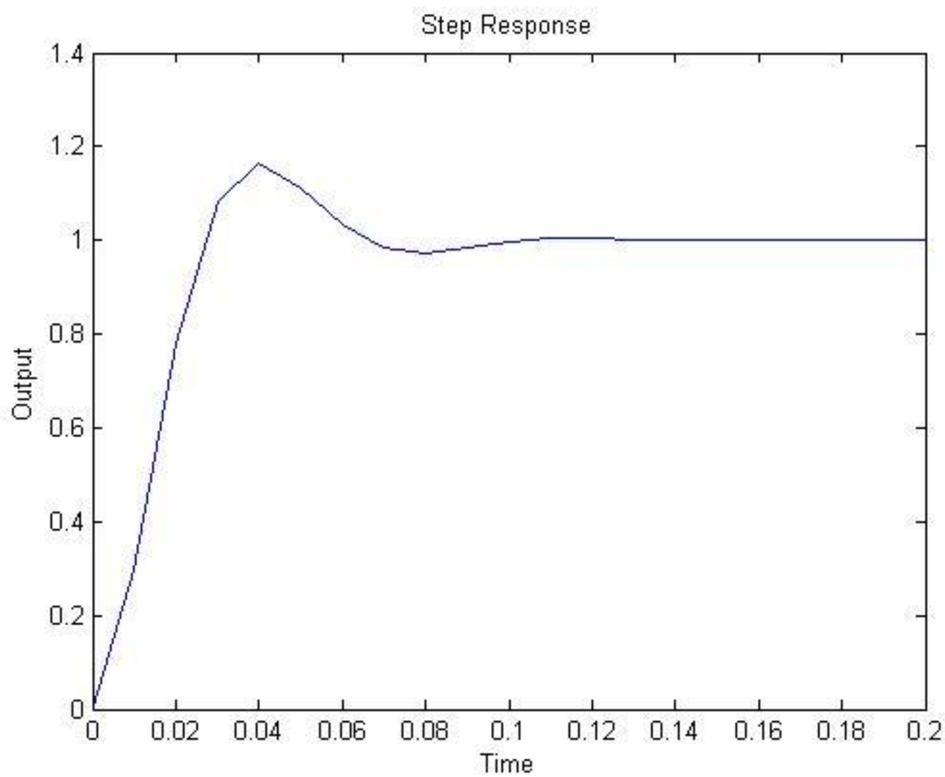

Figure 3.3.7 Step response of the system with Two-Degree-of-Freedom Controller.

In the next simulation, we want to see whether the effect of the disturbance can be eliminated. We simulate the response of the missile when the command signal is 10m/s while there is a step disturbance applied to the system at t=0.2s. It can be seen from Fig.

3.3.8 that the missile is affected by the disturbance immediately after t=0.2s, and then the effect of disturbance disappears around t=0.3s, and the missile can follow the command very well.

Therefore, it may be concluded that the lateral autopilot can be designed with the two-degree-of-freedom digital controller.

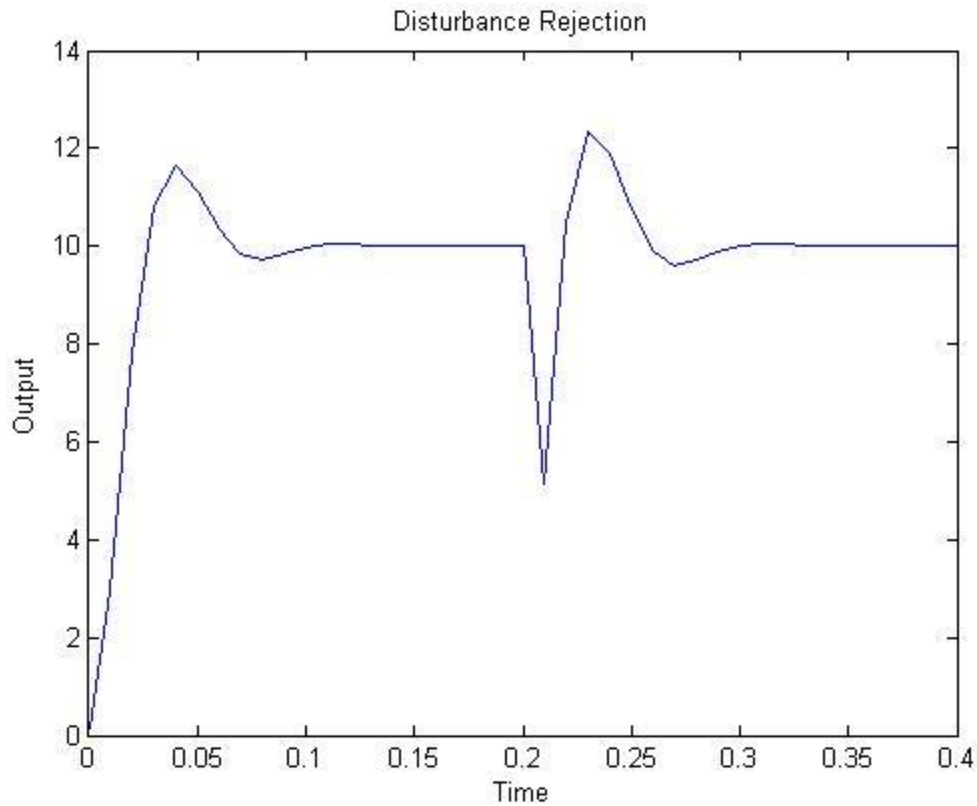

Figure 3.3.8 Command following with disturbance rejection by Two-Degree-of-Freedom Controller.

Most of the material covered in the notes is extracted from following sources.

References:

1. P. Garnell, Guided weapon control systems, 2^{nd} edition, Pergamon Press, 1980.
2. J. H. Blakelock, Automatic Control of Aircraft and Missiles, 2^{nd} edition, John Wiley & Sons, 1991
3. K. Ogata, Modern Control Engineering, 4^{th} edition, Prentice-Hall, 2002
4. http://en.wikipedia.org/wiki/Guided_missile