EE3304 (Part II)
Digital Control Systems
Chapter Five
Pole-Placement Controller

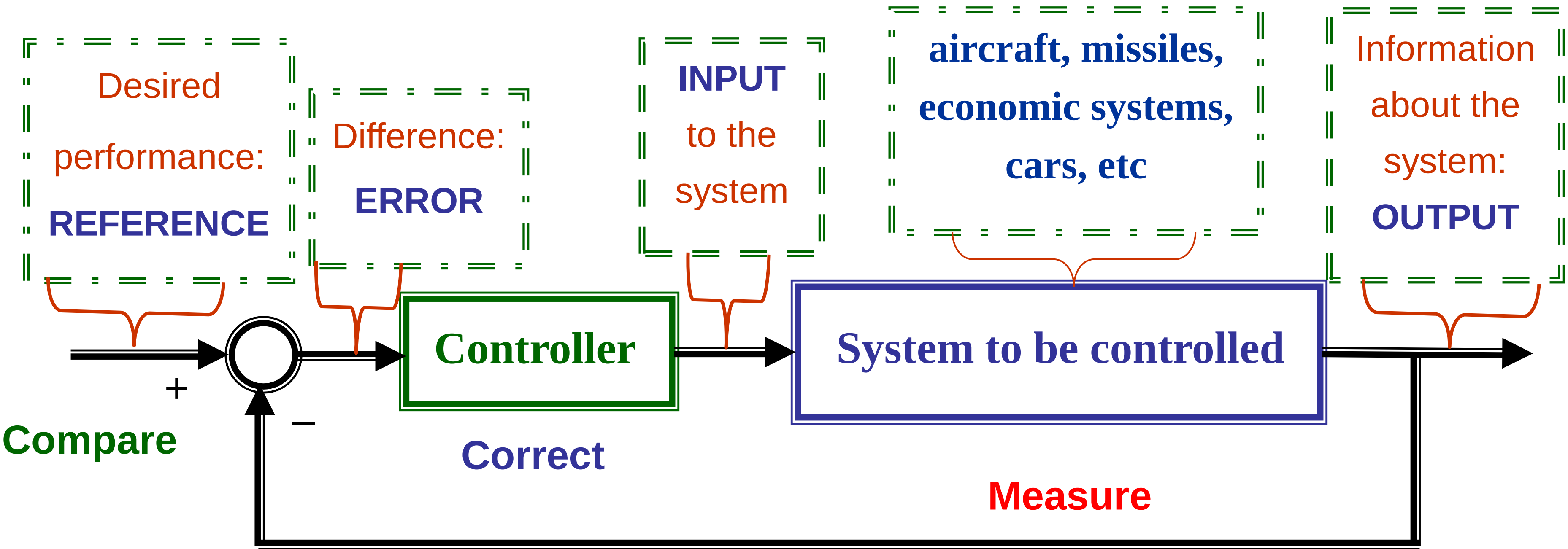
Xiang Cheng

Associate Professor

Department of Electrical & Computer Engineering
The National University of Singapore

Phone: 65166210 Office: Block E4-08-07
Email: elexc@nus.edu.sg

•What is a control system?



•**Feedback: Measure —Compare –-Correct**

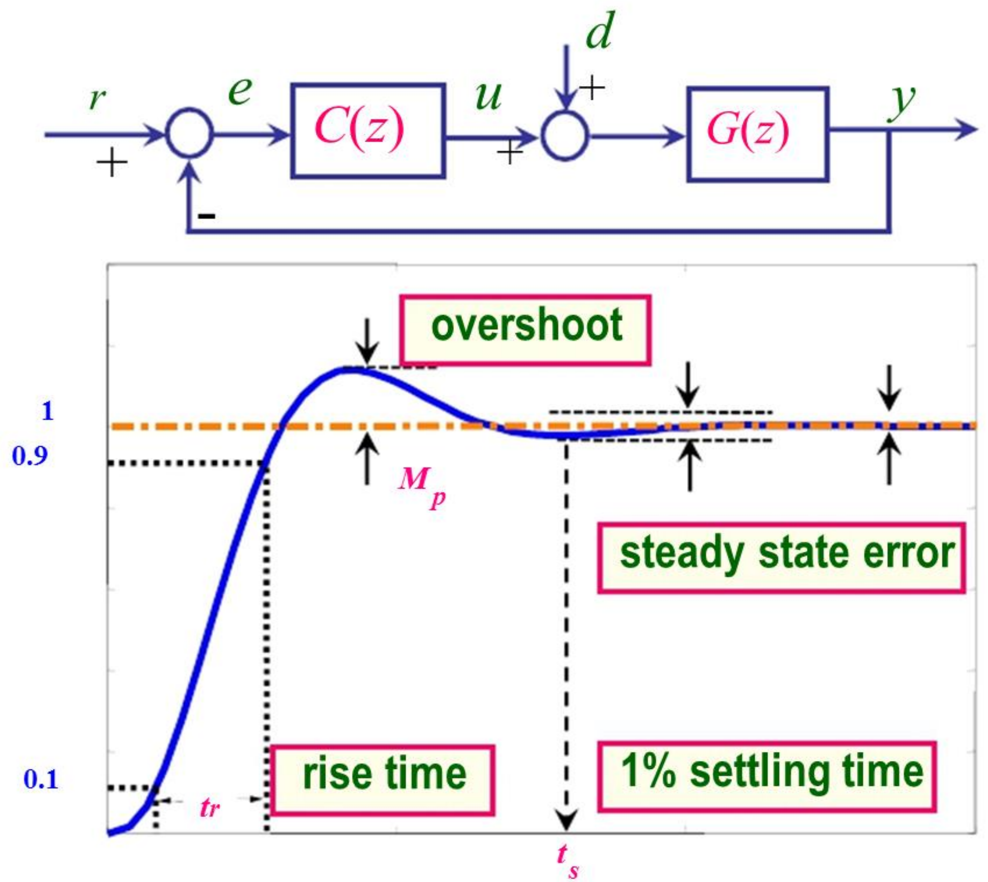
Objective: To make the system OUTPUT and the desired REFERENCE as close as possible, i.e., to make the ERROR as small as possible.

How to specify the reference signal, or the desired output?

Performance Specifications:

Consider a unity feedback system

- stability
- steady state accuracy
- settling time
- overshoot
- rise time
- disturbance rejection
- others



Often the desired CLTF is chosen to be

$$H_{desired}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

A unity desired closed-loop TF can be achieved at steady state.

$(t_s, M_p, t_r) \Leftrightarrow (\zeta, \omega_n) \Leftrightarrow$ Pole location

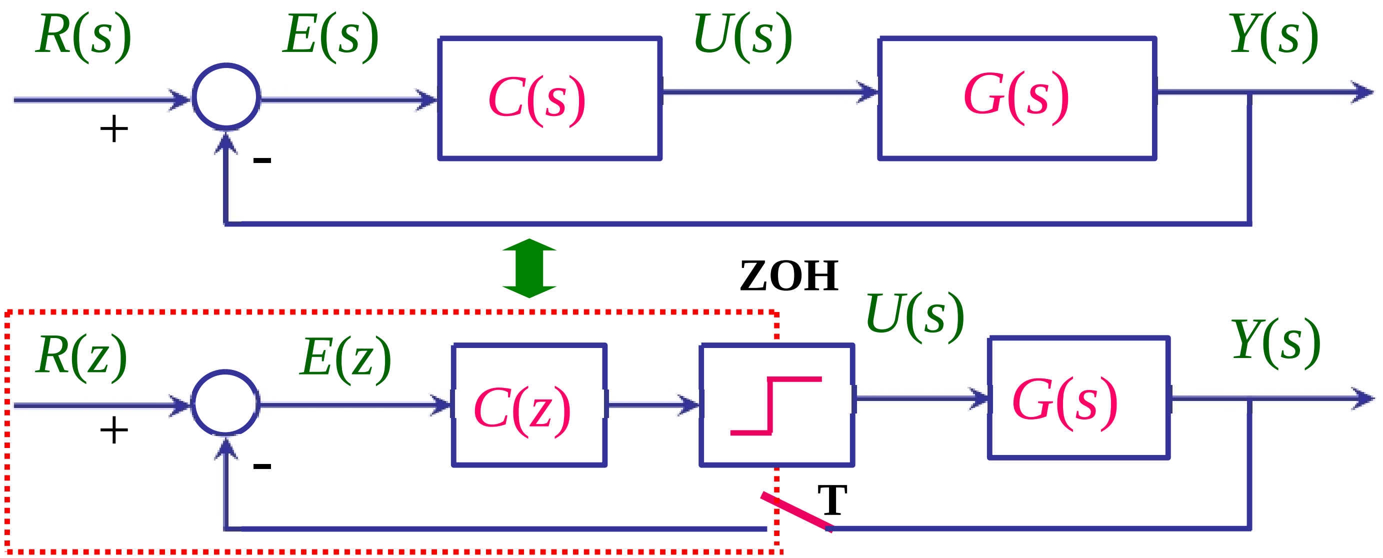
desired pole location in s-plane

$z = e^{Ts}$

desired pole location in z-plane

Digital control system design using emulation

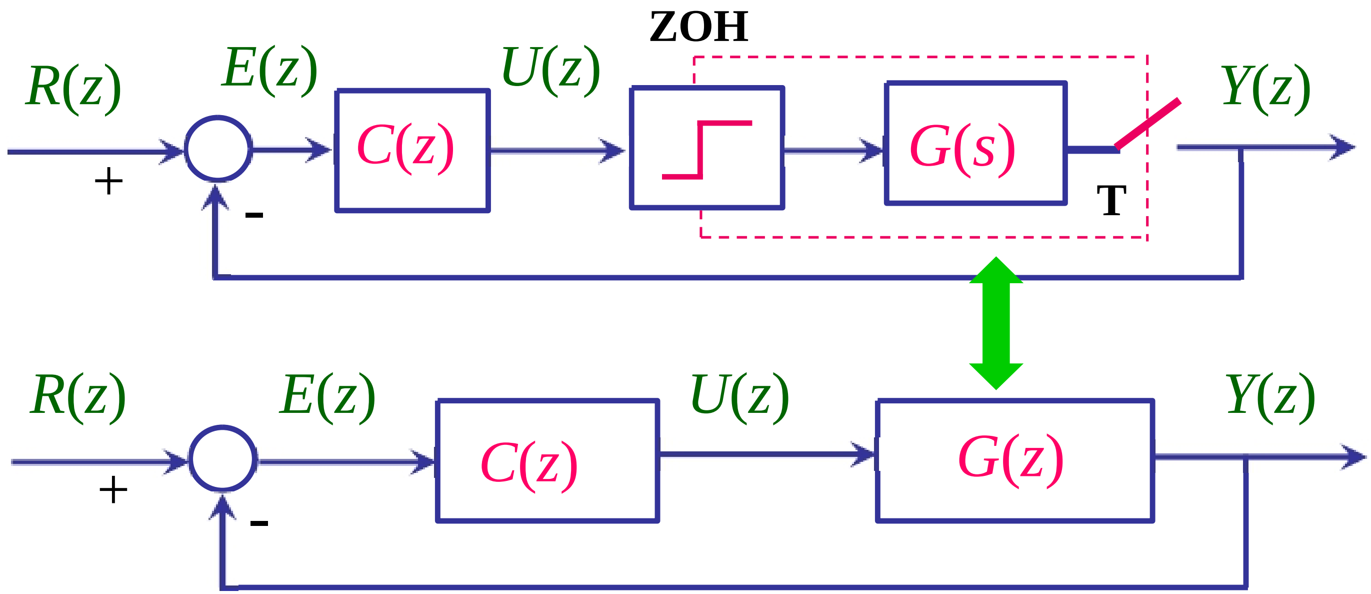
In this approach, we design a continuous-time controller that meets all design specifications and then discretize it using a bilinear transformation or other discretization technique to obtain an equivalent digital controller.



This method works if the sampling rate is 30 times faster than the system bandwidth. Further refinement is necessary for the case where the sampling rate is 6 times the bandwidth.

Digital controller design based on discrete-time system

We can discretize the continuous-time plant first or directly work on a discrete-time plant to design a digital controller



where $G(z) = (1 - z^{-1})Z[\frac{G(s)}{s}]$ and $C(z)$ is a digital controller, e.g. P, PI, PD, PID, etc.

Digital PI Control

$$\boxed{C(z) = K_p + K_i \frac{Tz}{z-1}}$$

Digital PD Control

$$\boxed{C(z) = K_p + K_d \frac{z-1}{Tz}}$$

Digital PID Control

$$\boxed{C(z) = K_p + K_i \frac{Tz}{z-1} + K_d \frac{z-1}{Tz}}$$

How to design the control parameters?

If the TF is not known, then use auto-tuning methods

If the TF is known, then choose the parameters to match the reference model.

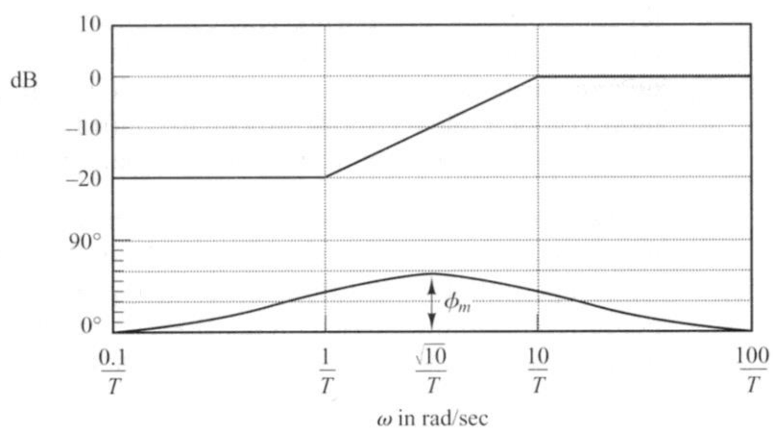
Frequency domain design methods

A common approach to the frequency domain design is that we first adjust the open-loop gain so that the requirement on the steady-state accuracy is met.

Then the magnitude and phase curves of the uncompensated open loop (with the open-loop gain just adjusted) is plotted. If the specifications on the phase margin and gain margin are not satisfied, then a suitable compensator that will re-shape the open-loop transfer function is determined.

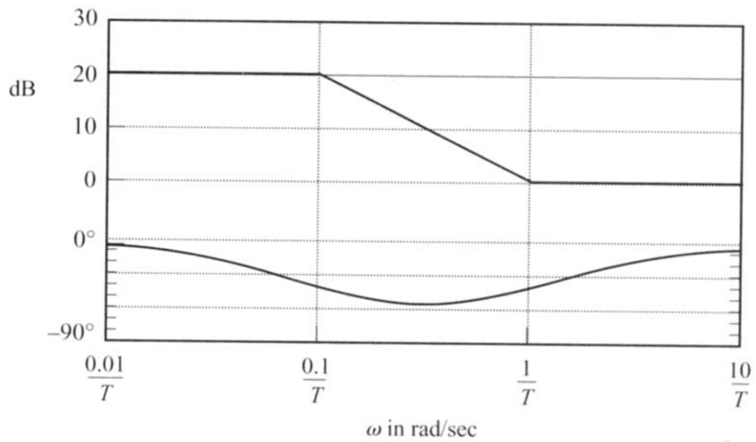
Lead Compensator

$$C(s) = k\frac{\tau s + 1}{\alpha\tau s + 1}, \alpha < 1$$



Lag Compensator

$$C(s) = k\frac{\tau s + 1}{\alpha\tau s + 1}, \alpha > 1$$

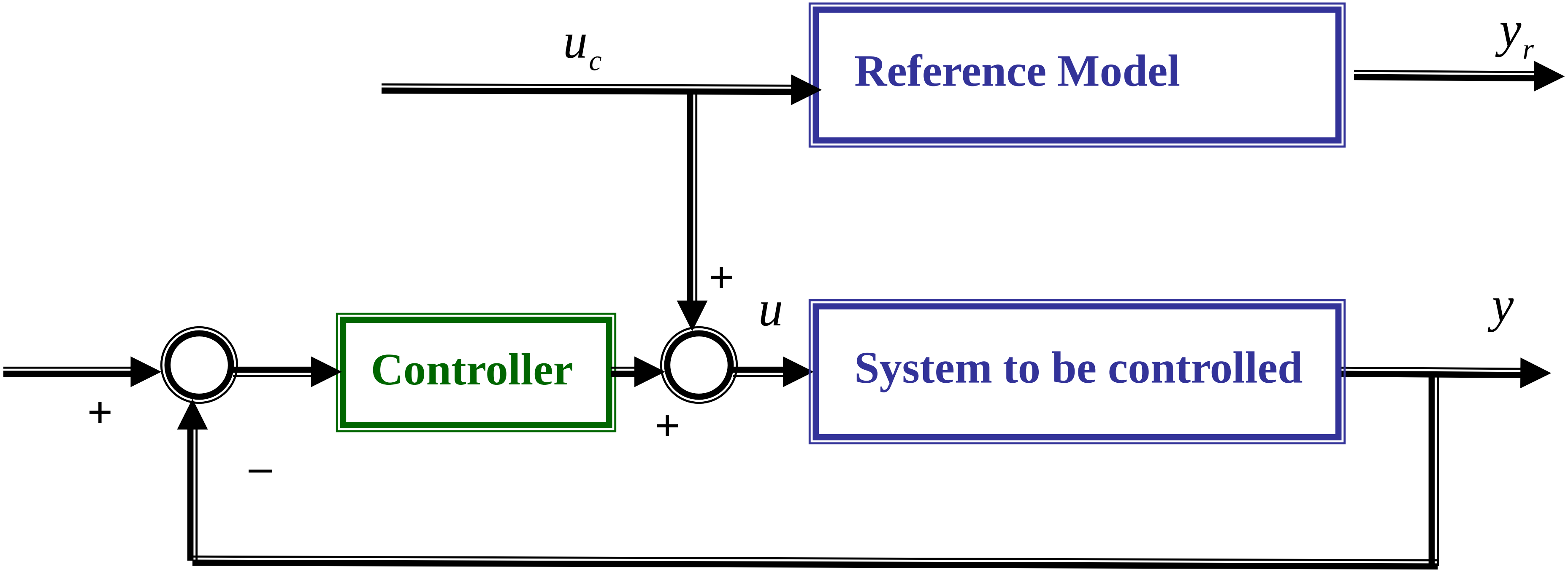


If the transfer function G(s) is not available, can we use PID or lead-compensator?

Yes. Both methods can be model-free in some sense.

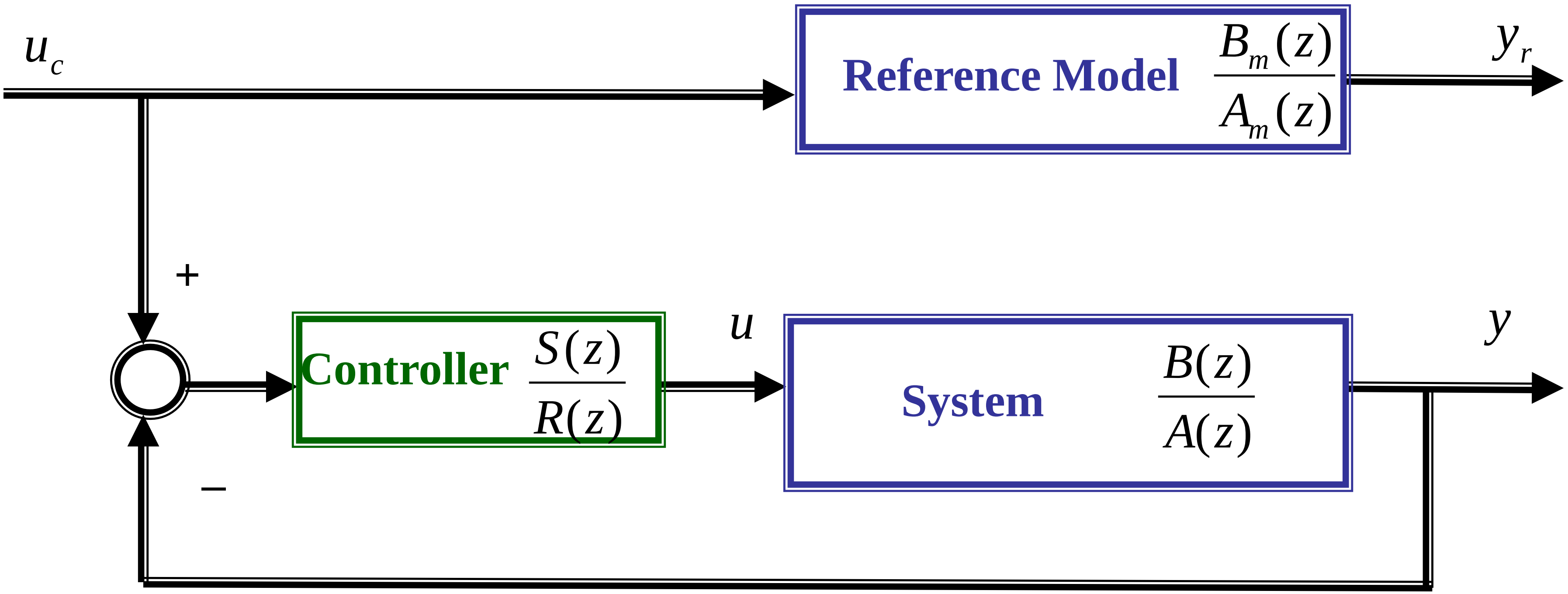
If the model is available, we have plenty of other model-based approaches.

Model-reference control:



Objective: To make the closed loop model follow the reference model as close as possible.

Error Feedback control (unity feedback control system):



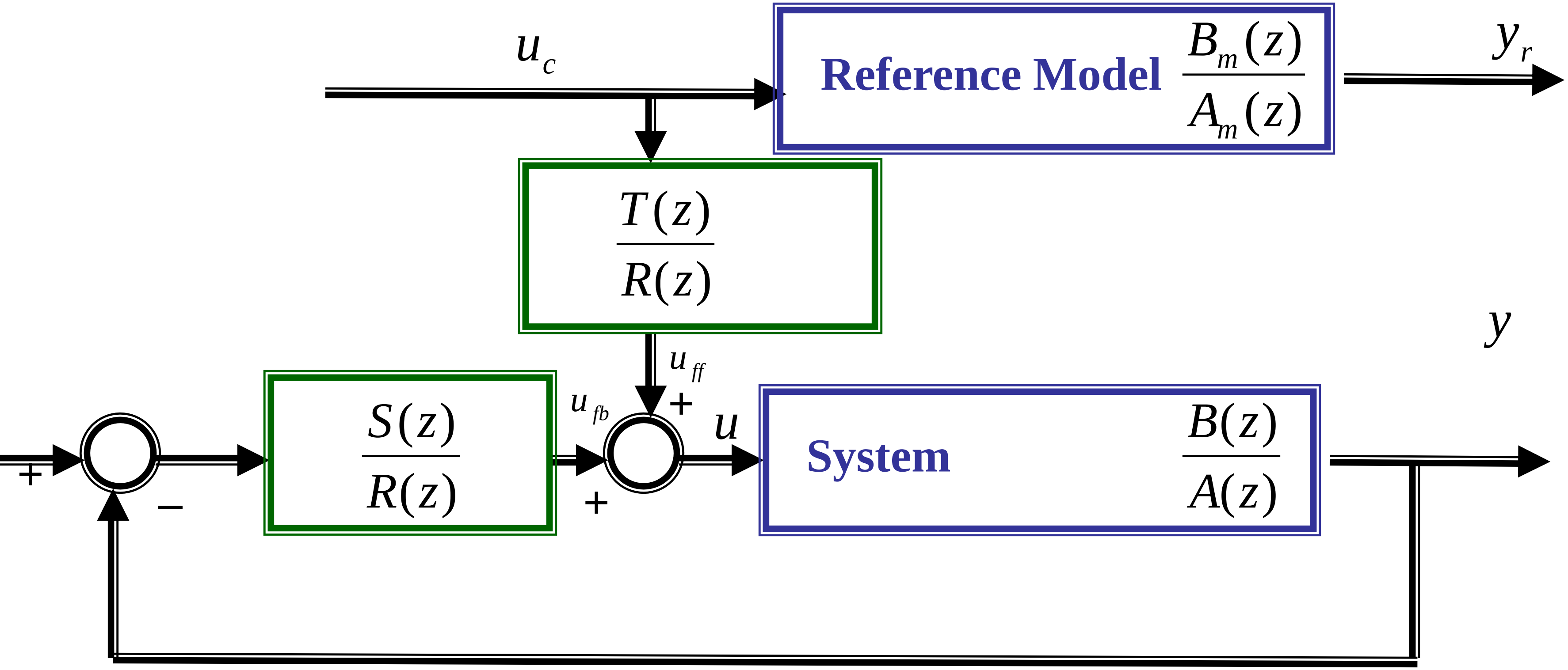
What's the closed loop transfer function? How to write down TF?

$$\frac{Y(z)}{U_c(z)} = \frac{\frac{S(z)}{R(z)}\frac{B(z)}{A(z)}}{1+\frac{S(z)}{R(z)}\frac{B(z)}{A(z)}} = \frac{S(z)B(z)}{R(z)A(z)+S(z)B(z)}$$

Can we match the reference model?

Design R(z) and S(z) so that $R(z)A(z)+S(z)B(z)=A_m(z)$

But how to match $B_m(z)$? Impossible! We need more freedom!



What's the closed loop transfer function from command signal Uc to y?

Let's first get the T.F. from u_{ff} to y:

What is the feed-forward T.F.?

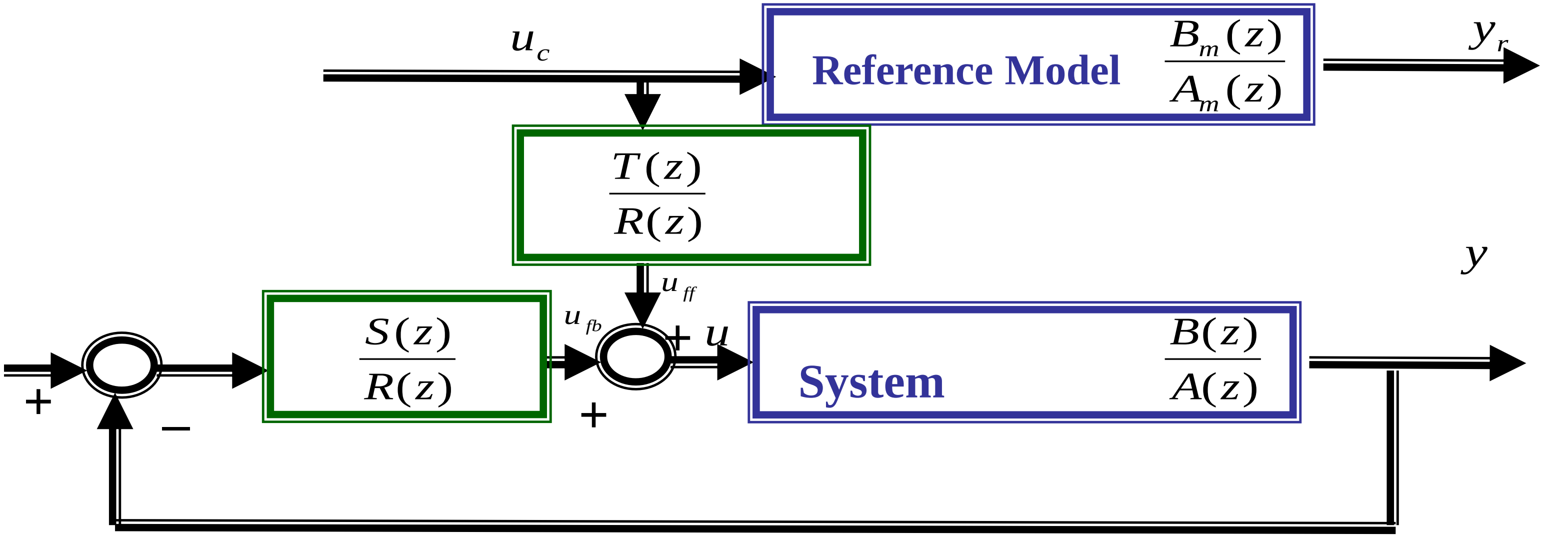
$$\frac{B(z)}{A(z)}$$

What is the open-loop T.F.?

$$\frac{B(z)}{A(z)}(-1)\frac{S(z)}{R(z)}(-1) = \frac{B(z)}{A(z)}\frac{S(z)}{R(z)}$$

$$\frac{Y(z)}{U_{ff}(z)} = \frac{\frac{B(z)}{A(z)}}{1+\frac{B(z)}{A(z)}\frac{S(z)}{R(z)}} \Longrightarrow \frac{Y(z)}{U_{ff}(z)} = \frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)}$$

Two-Degree-of-Freedom Controller



$$\frac{Y(z)}{U_{ff}(z)} = \frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)} \qquad \frac{U_{ff}(z)}{U_c(z)} = \frac{T(z)}{R(z)}$$

$$\frac{Y(z)}{U_c(z)} = \frac{Y(z)}{U_{ff}(z)}\frac{U_{ff}(z)}{U_c(z)} = \frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)}\frac{T(z)}{R(z)} = \frac{B(z)T(z)}{A(z)R(z)+B(z)S(z)}$$

How to design the feedback controller $\frac{S(z)}{R(z)}$? Match the poles or zeros?

Choose R(z) and S(z) such that $A_{cl}(z) = A(z)R(z)+B(z)S(z) = A_m(z)A_o(z)$

Why not directly match $A_m(z)$?

The order of $A_m(z)$ (normally 2nd order) may be lower than the closed loop!

$$\frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A(z)R(z)+B(z)S(z)} = \frac{T(z)B(z)}{A_m(z)A_o(z)} \Longrightarrow \frac{B_m(z)}{A_m(z)}$$

How to design the feedforward controller $\frac{T(z)}{R(z)}$? R(z) is already fixed!

- <u>How to choose T(z)?</u>

Compare it to the reference model, one simple way to choose T(z) is $T(z) = t_o A_o(z)$

So the closed loop system becomes: $\frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A_m(z)A_o(z)} = \frac{t_o B(z)}{A_m(z)}$

At least the poles match those of the reference model!

<u>What is the steady state gain (DC gain) of</u> $\frac{t_o B(z)}{A_m(z)}$? $\frac{t_o B(1)}{A_m(1)}$

<u>How to choose</u> t_o <u>to make the static gain unity?</u> $t_o = \frac{A_m(1)}{B(1)}$

- <u>Is perfect tracking attainable? Can we match the zeros?</u>

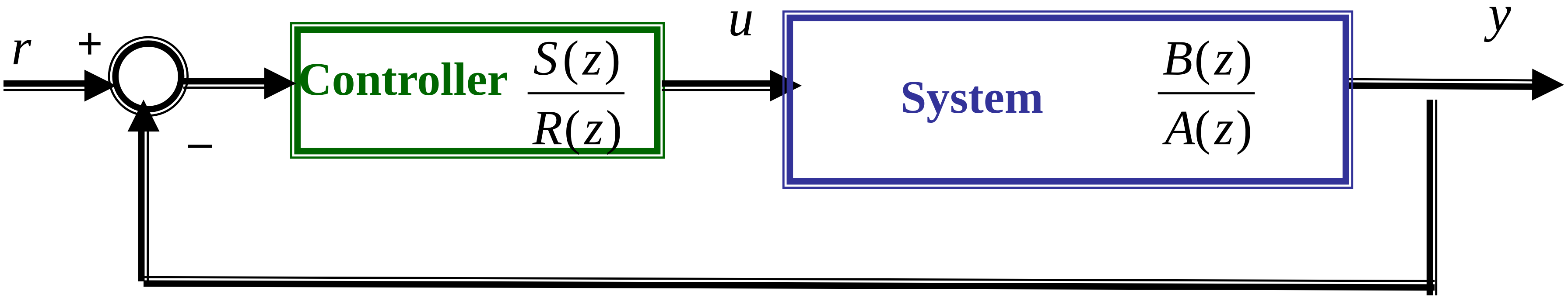
$$T(z) = A_o(z)B_m(z) \Longrightarrow \frac{Y(z)}{U_c(z)} = \frac{B_m(z)B(z)}{A_m(z)}$$

- <u>Is it possible to get rid of</u> $B(z)$? Yes. Under certain conditions!

•This is the overall picture of the design of the two degree of freedom controller.

But how to solve the equation: $A_{cl}(z) = A(z)R(z) + B(z)S(z)$?

Let's try to design the feedback controller first.



$$\frac{Y(z)}{R(z)} = \frac{\frac{S(z)}{R(z)}\frac{B(z)}{A(z)}}{1+\frac{S(z)}{R(z)}\frac{B(z)}{A(z)}} = \frac{S(z)B(z)}{R(z)A(z)+S(z)B(z)}$$

$$R(z)A(z)+S(z)B(z) \rightarrow A_{cl}(z) = A_m(z)A_o(z)$$

Design example: the double integrator

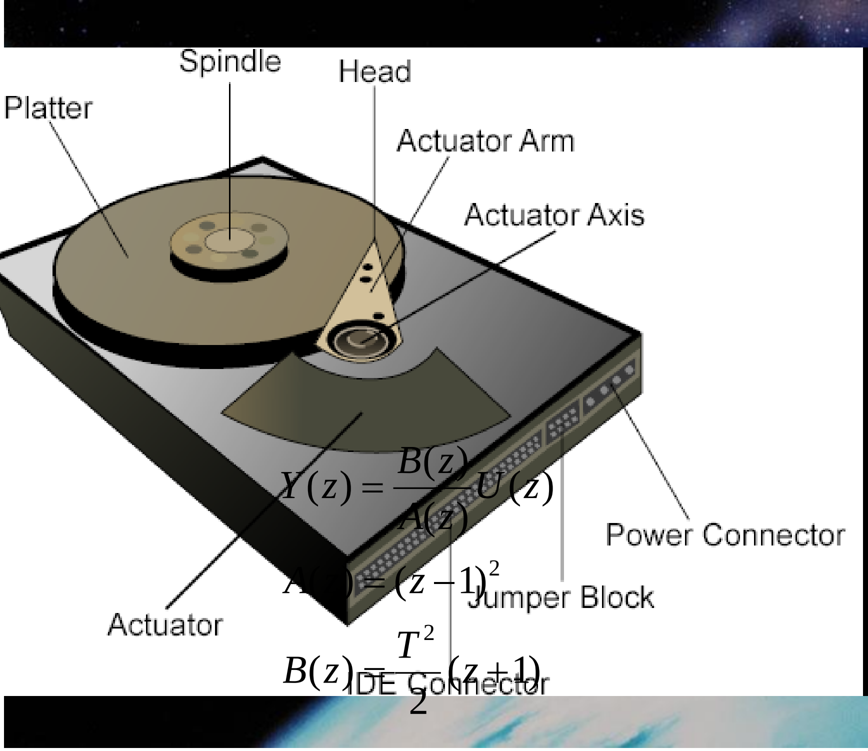
$$\ddot{y}(t) = u(t)$$

$$Y(s) = \frac{1}{s^2} U(s)$$

Real World Examples

Zero-order hold

$$Y(z) = \frac{\frac{T^2}{2}(z+1)}{(z-1)^2} U(z)$$



$$Y(z) = \frac{B(z)}{A(z)} U(z)$$

$$A(z) = (z-1)^2$$

$$B(z) = \frac{T^2}{2}(z+1)$$

Let's try the simplest controller, the proportional controller, $R = 1, S = s_0$

$$RA + SB = (z-1)^2 + \frac{T^2}{2} s_0 (z+1) = z^2 + (\frac{T^2}{2} s_0 - 2) z + \frac{T^2}{2} s_0 + 1$$

$$\Longrightarrow \quad A_{cl} = z^2 + p_1 z + p_2$$

Can we match them?

Two equations with one parameter s_0----Mission Impossible!

So we have to give more degrees of freedom to the controller!

Then try the first-order controller, $R = z + r_1$

$S = s_0 z + s_1$

Compare it to the P controller, how many extra control parameters? Two more!

Increase the order of the controller by ONE $\Longrightarrow$ Degree of freedom jumps by TWO

$$AR + BS = (z-1)^2(z + r_1) + \frac{T^2}{2}(z+1)(s_0 z + s_1)$$

$$= z^3 + (r_1 + \frac{T^2}{2}s_0 - 2)z^2 + (1 - 2r_1 + \frac{T^2}{2}(s_0 + s_1))z + r_1 + s_1\frac{T^2}{2}$$

•If the desired C.P. is $A_{cl} = z^3 + p_1 z^2 + p_2 z + p_3$

Match the coefficients: •How many equations? •How many design parameters?

So we can easily compute the design parameters by solving the three equations.

•What if the desired C.P. of the reference model is $A_m = z^2 + p_1 z + p_2$?

•Can we match A_m direcly? No!

Let's introduce A_o such that $A_{cl} = A_m A_o$ $\quad A_o = z$ $\Longrightarrow$ $A_{cl} = z^3 + p_1 z^2 + p_2 z$

Can we match A_{cl} now? Yes!

What is the lesson? The controller must have sufficient degree of freedom!

Now let's consider the following question:

Given polynomials A(z) and B(z), and a third polynomial , $A_{cl}(z)$

can we find R(z) and S(z) such that

$$A(z)R(z) + B(z)S(z) = A_{cl}(z)$$

- Let's consider second order system

$$A(z) = a_0 z^2 + a_1 z + a_2$$
$$B(z) = b_0 z^2 + b_1 z + b_2$$
$$A_{cl}(z) = p_0 z^3 + p_1 z^2 + p_2 z + p_3$$

- Design a first order controller

$$R(z) = r_0 z + r_1$$
$$S(z) = s_0 z + s_1$$

- From $A(z)R(z) + B(z)S(z) = A_{cl}(z)$

compare the coefficients

$$\begin{array}{lll} z^3: & p_0 & a_0 r_0 + b_0 s_0 \\ z^2: & p_1 & a_1 r_0 + a_0 r_1 + b_1 s_0 + b_0 s_1 \\ z: & p_2 & a_2 r_0 + a_1 r_1 + b_2 s_0 + b_1 s_1 \\ 1: & p_3 & a_2 r_1 + b_2 s_1 \end{array}$$

- How many equations?

Four

- How many design parameters?

Four

Rewrite it in a compact matrix form,

$$\begin{bmatrix} a_0 & 0 & b_0 & 0 \\ a_1 & a_0 & b_1 & b_0 \\ a_2 & a_1 & b_2 & b_1 \\ 0 & a_2 & 0 & b_2 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

•Sylvester matrix

•What is the condition on the Sylvester matrix for getting the solution?

The solution exists as long as the Sylvester matrix is non-singular!

•What is the condition for non-singularity of Sylvester matrix?

Let's take a look at a simple example:

$$\frac{Y(z)}{U(z)} = \frac{(z+1)}{(z+2)(z+1)}$$

$$A(z) = (z+2)(z+1)$$

$$B(z) = z+1$$

$$\begin{aligned} A(z)R(z) + B(z)S(z) &= (z+2)(z+1)R(z) + (z+1)S(z) \\ &= (z+1)[(z+2)R(z) + S(z)] \end{aligned}$$

Is it possible to choose R and S such that AR+BS matches any A_{cl}?

For example, can we make $(z+1)[(z+2)R(z)+S(z)] = z^3$?

It is impossible! Unless A_{cl} contains the same factor (z+1).

So one condition to assure Sylvester matrix is nonsingular:
A(z) and B(z) should not have any common factors.

•Sylvester Theorem:

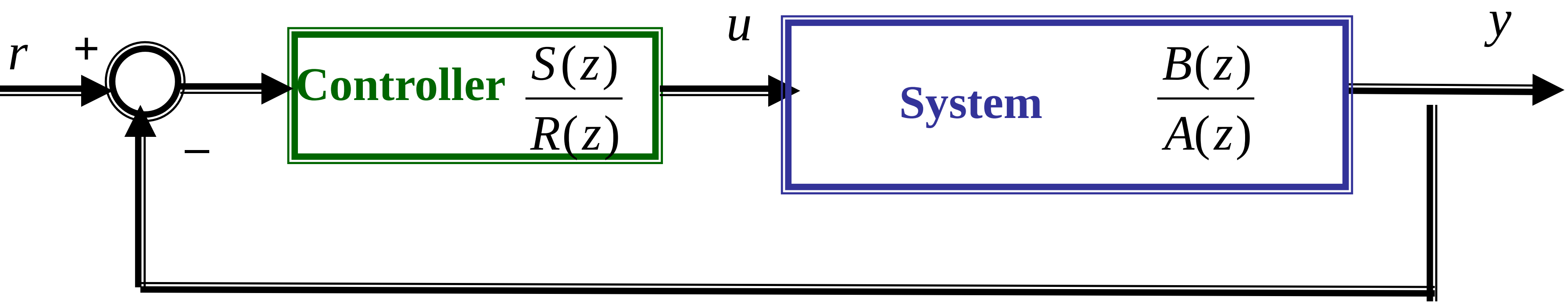
The Sylvester matrix is nonsingular if and only if the two polynomials A(z) and B(z) have no common factors.

$$A(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n$$
$$B(z) = b_0 z^n + b_1 z^{n-1} + \cdots + b_n$$

Sylvester matrix is 2nx2n matrix:

$$M_s = \underbrace{\begin{bmatrix} a_0 & 0 & \cdots & 0 & b_0 & 0 & \cdots & 0 \\ a_1 & a_0 & \ddots & \vdots & b_1 & b_0 & \ddots & \vdots \\ \vdots & a_1 & \ddots & 0 & \vdots & b_1 & \ddots & 0 \\ \vdots & \vdots & \ddots & a_0 & \vdots & \vdots & \ddots & b_0 \\ a_n & \vdots & \vdots & a_1 & b_n & \vdots & \vdots & b_1 \\ 0 & \ddots & \vdots & \vdots & 0 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_n & 0 & \cdots & 0 & b_n \end{bmatrix}}_{<------n------><------n------>}$$

The basic solution for pole placement:



Assumptions: the order of the system is n: Deg A=n, Deg B <= n

The order of controller is n-1

$$A(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n \qquad R(z) = r_0 z^{n-1} + r_1 z^{n-2} + \cdots + r_{n-1}$$

$$B(z) = b_0 z^n + b_1 z^{n-1} + \cdots + b_n \qquad S(z) = s_0 z^{n-1} + s_1 z^{n-2} + \cdots + s_{n-1}$$

How many design parameters? 2n

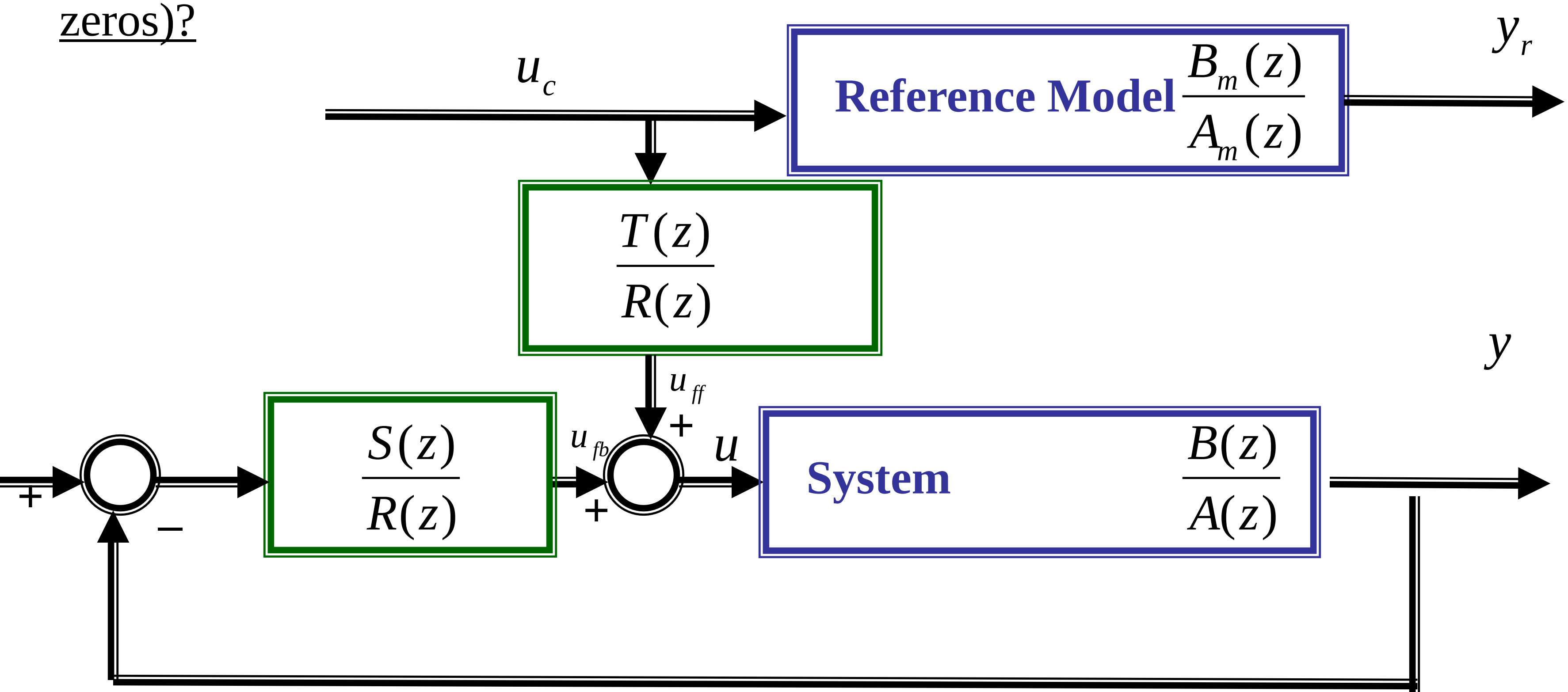
The order of the closed loop: 2n-1 $\qquad A_{cl}(z) = p_0 z^{2n-1} + p_1 z^{2n-2} + \cdots + p_{2n-1}$

Then if A and B have no common factors, R and S can be determined from

$$A(z)R(z) + B(z)S(z) = A_{cl}(z) \quad \Longrightarrow \quad \begin{bmatrix} r_0 \\ \vdots \\ r_{n-1} \\ s_0 \\ \vdots \\ s_{n-1} \end{bmatrix} = M_s^{-1} \begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ \vdots \\ \vdots \\ p_{2n-1} \end{bmatrix}$$

return

Tracking Problem: How to match the reference model completely (both poles and zeros)?



What's the closed loop transfer function from command signal Uc to y?

$$\frac{Y(z)}{U_{ff}(z)} = \frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)} \quad \xrightarrow{\frac{U_{ff}(z)}{U_c(z)} = \frac{T(z)}{R(z)}} \quad \frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A(z)R(z)+B(z)S(z)}$$

If we let

$$A(z)R(z)+B(z)S(z) = A_{cl}(z) = A_m(z)A_o(z)$$

Then we have $$\frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A_m(z)A_o(z)}$$

$$\frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A(z)R(z)+B(z)S(z)} = \frac{T(z)B(z)}{A_{cl}(z)}$$

$$A_{cl}(z) = A_m(z)A_o(z) \Longrightarrow \frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A_m(z)A_o(z)}$$

How to choose T(z)?

$$T(z) = B_m(z)A_o(z)$$

$$\frac{Y(z)}{U_c(z)} = \frac{B_m(z)A_o(z)B(z)}{A_m(z)A_o(z)} = \frac{B_m(z)B(z)}{A_m(z)}$$

How to get rid of B(z)?

$$\frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A(z)R(z)+B(z)S(z)} = \frac{T(z)B(z)}{A_{cl}(z)}$$

How to get rid of B(z)?

Remember that we are free to choose $A_{cl}(z)$ in anyway we like.

Can we choose $A_{cl}(z)$ properly such that B(z) can be canceled out?

Yes if B(z) is stable!

$$A_{cl}(z) = A_m(z)A_o(z)B(z)$$

Choose R(z) and S(z) such that

$$A_{cl}(z) = A(z)R(z)+B(z)S(z) = A_m(z)A_o(z)B(z)$$

$$A(z)R(z) = A_m(z)A_o(z)B(z) - B(z)S(z) = B(z)(A_m(z)A_o(z) - S(z))$$

What is the design requirement on R(z)?

We just let R(z) contain B(z) as a factor.

<u>In total, we have</u>

$$\frac{Y(z)}{U_c(z)} = \frac{T(z)B(z)}{A(z)R(z)+B(z)S(z)} = \frac{B_m(z)A_o(z)B(z)}{A_m(z)A_o(z)B(z)} = \frac{B_m(z)}{A_m(z)}$$

Example: $H(z)=\dfrac{(z-b)}{(z-1)(z-a)}$ |b|<1 and a>1 Reference model: $H_m(z)=\dfrac{z(1+p_1+p_2)}{z^2+p_1z+p_2}$

Z=b is a stable zero, which may be cancelled out.

Let's try the first order controller first. $R(z)=(z-b), S(z)=s_0z+s_1$

The closed loop:

$$R(z)A(z)+S(z)B(z)=(z-b)(z-1)(z-a)+(s_0z+s_1)(z-b)=(z-b)(z^2+(s_0-a-1)z+a+s_1)$$

The desired closed loop: $A_{cl}(z)=(z-b)(z^2+p_1z+p_2)$

Match the polynomials:

$$(s_0-a-1)=p_1 \qquad s_0=p_1+a+1$$
$$a+s_1=p_2 \quad \Longrightarrow \quad s_1=p_2-a$$

The closed loop T.F. from the command signal to the output:

$$\frac{Y(z)}{U_c(z)}=\frac{(z-b)T(z)}{(z-b)(z^2+p_1z+p_2)}\approx\frac{T(z)}{(z^2+p_1z+p_2)}$$

How to choose T(z) to match the reference model? $T(z)=z(1+p_1+p_2)$

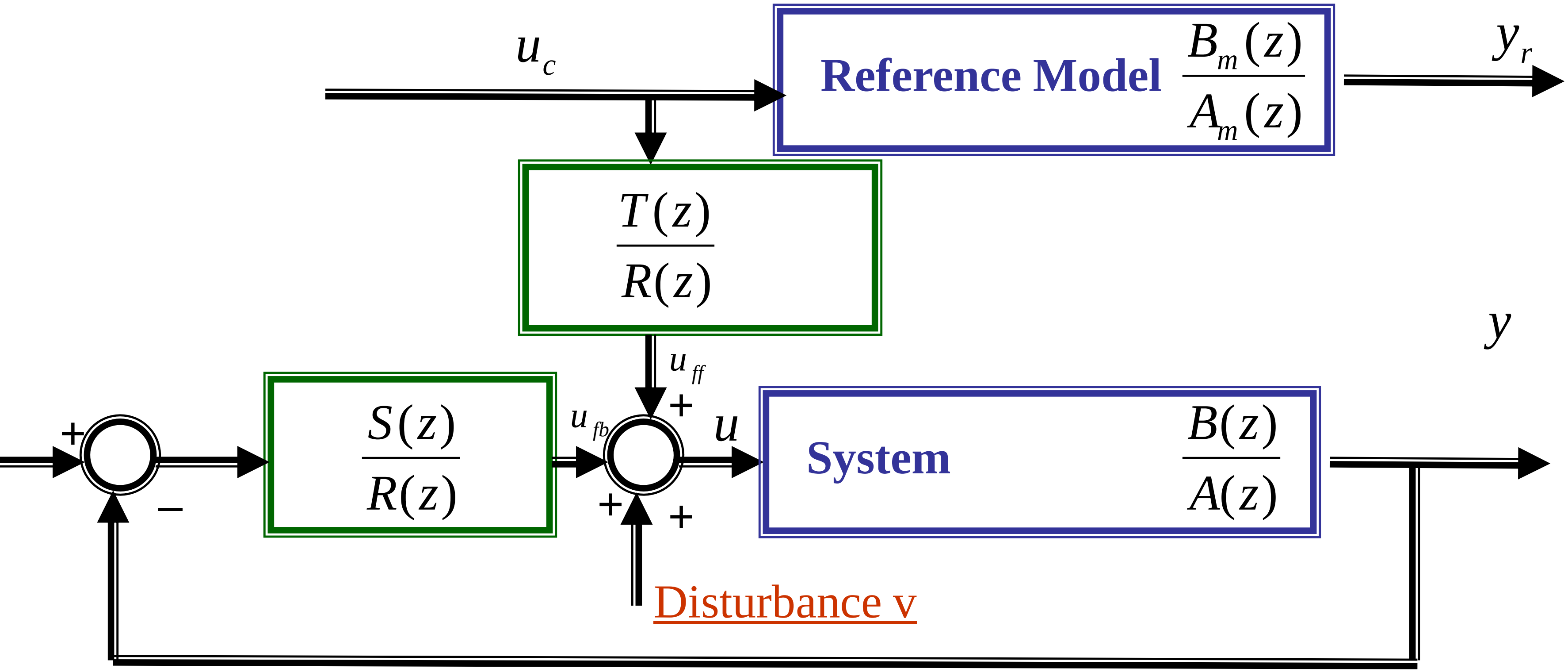
And the resulting controller is

$$U(z)=\frac{T(z)}{R(z)}U_c(z)-\frac{S(z)}{R(z)}Y(z)$$
$$(z-b)U(z)=z(1+p_1+p_2)U_c(z)-(s_0z+s_1)Y(z)$$

How to implement it by computer?

$$u(k)=bu(k-1)+(1+p_1+p_2)u_c(k)-s_0y(k)-s_1y(k-1)$$

Disturbance Rejection Problem



The simplest case: $A(z)Y(z) = B(z)(U(z)+V(z))$

Let's find the transfer function from disturbance v to output y:

What's the feed-forward transfer function from v to y? $\frac{B(z)}{A(z)}$

What's the open-loop transfer function from v to y? $\frac{B(z)}{A(z)}(-1)\frac{S(z)}{R(z)}(-1) = \frac{B(z)}{A(z)}\frac{S(z)}{R(z)}$

$$\frac{Y(z)}{V(z)} = \frac{\frac{B(z)}{A(z)}}{1+\frac{B(z)}{A(z)}\frac{S(z)}{R(z)}} = \frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)} = \frac{B(z)R(z)}{A_{cl}(z)}$$

So overall, the output is affected by both the disturbance and the command signal

$$Y = \frac{BRV}{(RA + BS)} + \frac{BTU_c}{(RA + BS)}$$

Let's consider the simple case that the disturbance is an unknown constant. Then the corresponding steady state output subjected to this constant input is simplly the disturbance constant multiplied by the DC gain.

<u>How do we compute the DC gain of a continuous-time transfer function G(s) ?</u>

$$G(s)|_{s=0} = G(0)$$

<u>How do we compute the DC gain of a discrete-time transfer function G(z) ?</u>

$$G(z)|_{z=1} = G(1)$$

<u>What is the DC gain from the disturbance to the output?</u>

$$\left.\frac{B(z)R(z)}{A_{cl}(z)}\right|_{z=1} = \frac{B(1)R(1)}{A_{cl}(1)}$$

<u>How to design R(z) such that the DC gain is 0, i.e. R(1)=0?</u>

$$R = (z - 1)R'(z)$$

<u>This is the same strategy discussed in chapter two: adding an integrator!</u>

Example: $y(k+1)=3y(k)+u(k)+v(k)$ $\Longrightarrow$ A(z)Y(z)=B(z)(U(z)+V(z))

where v(k) is a constant disturbance:

How to design R(z)? $R=(z-1)R'(z)$

What is A(z) and B(z)? A(z)=z-3 B(z)=1

The simplest controller is the first order controller

$$R(z)=(z-1)$$

$$S(z)=s_0 z+s_1$$

The closed loop: $AR+BS=(z-3)(z-1)+s_0 z+s_1=z^2+(s_0-4)z+s_1+3$

If the desired C.P. is $A_{cl}=z^2+p_1 z+p_2$

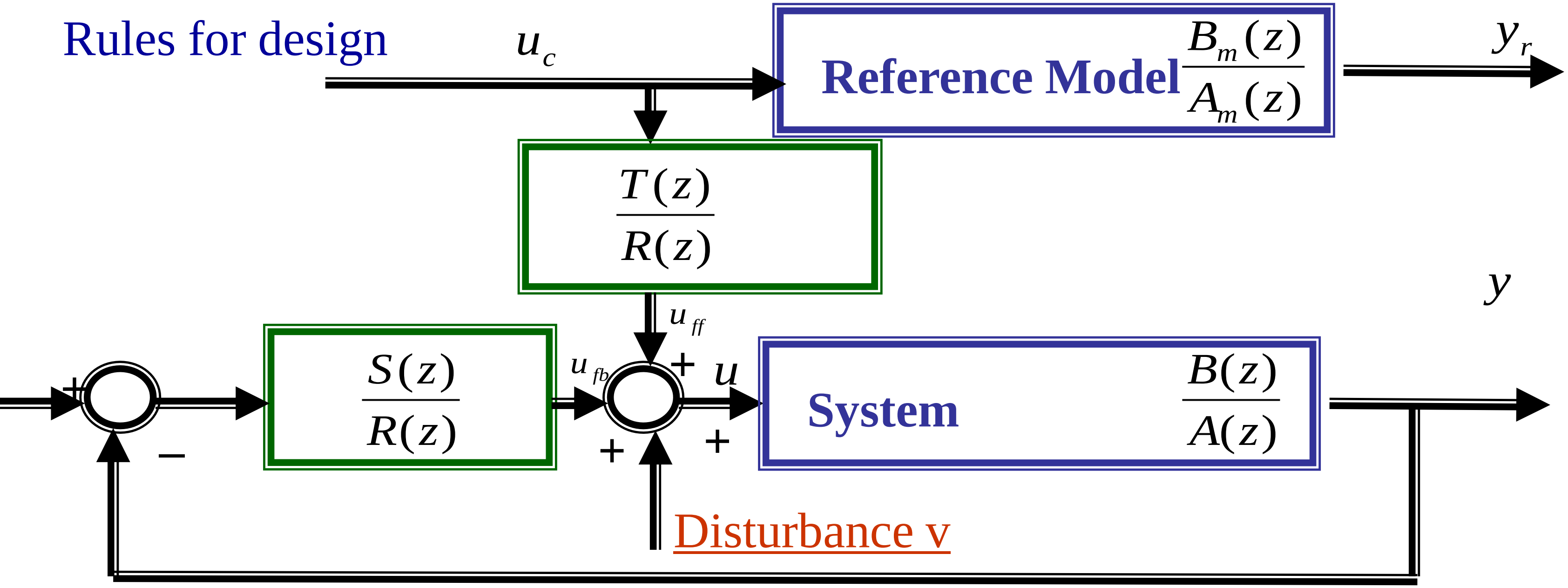
Can you match them? Of course!

$$y=\frac{BRv}{(RA+BS)}+\frac{BTu_c}{(RA+BS)}=\frac{Tu_c}{z^2+p_1 z+p_2}$$

How to choose T?

To satisfy other requirements like zero placement.

Rules for design



Occam's razor -- the simpler, the better.

Separation Property: Design feedback controller first, then build feedforward controller.

Step One: Figure out the design requirements on R(z).

•If need to cancel stable zeros for perfect match: R(z) must contain B(z)

•Constant disturbance rejection: R(z) must contain (z-1).

•Causality conditions: $Deg(R(z)) \geq Deg(S(z))$

Step Two: Design R and S by solving the equation $AR + BS = A_{cl}$

Step Three: Choose T at the final stage, to satisfy other design requirements.

The order of controller can be increased in order to meet other design requirements.

Stable zeros condition is required for perfect tracking

(perfect match of the reference model)

Design Example*: *speed control system

Consider the vehicle, which has a weight m = 1000 kg. Assuming the average friction coefficient b= 100, design a speed control system such that the vehicle can reach 100 km/h from 0 km/h in 8 s with an overshoot less than 5%.

Assuming the sampling period $T = 0.6$ seconds, design a digital pole placement controller that achieves the above specifications.

This is the design example where the PID controller did not work out very well. In particular, the overshoot was over 20%. Let's see if the pole-placement controller can do a better job.

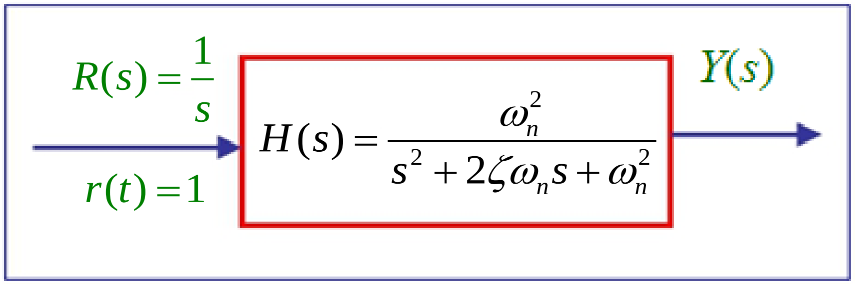
We have already obtained the discrete-time T.F. from the continuous time T.F.

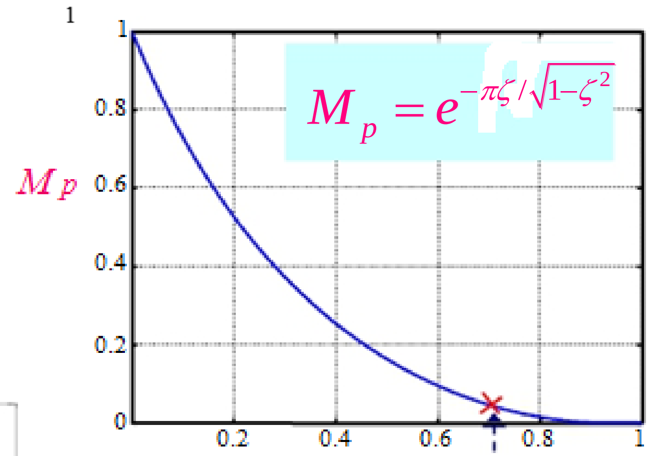
$$G(s) = \frac{1}{1000s + 100}$$

$$G(z) = (1 - z^{-1})Z[\frac{G(s)}{s}] = \frac{0.00058}{z - 0.942}$$

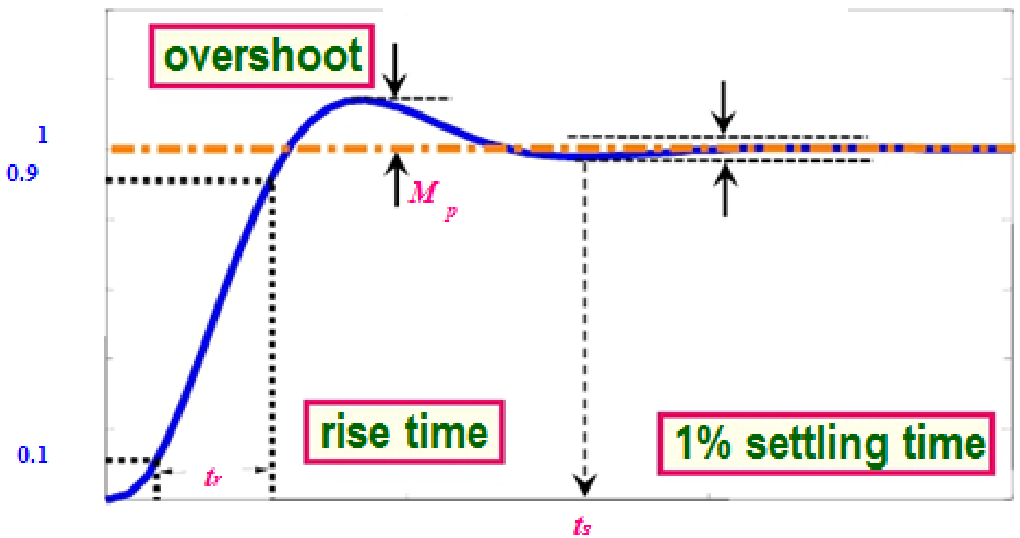
Get the continuous-time reference model first:

First derive ξ and ω_n from the design specifications:





$$\zeta \geq 0.6901 \quad \Rightarrow \quad \zeta = 0.7$$



$$t_s \cong \frac{4.6}{\zeta\omega_n} \Rightarrow \omega_n \cong \frac{4.6}{t_s\zeta}$$

$$\Rightarrow \omega_n = \frac{4.6}{8 \times 0.7} = 0.82$$

Desired CLTF $\Rightarrow$ $H_d(s) = \dfrac{0.67}{s^2 + 1.15s + 0.67}$

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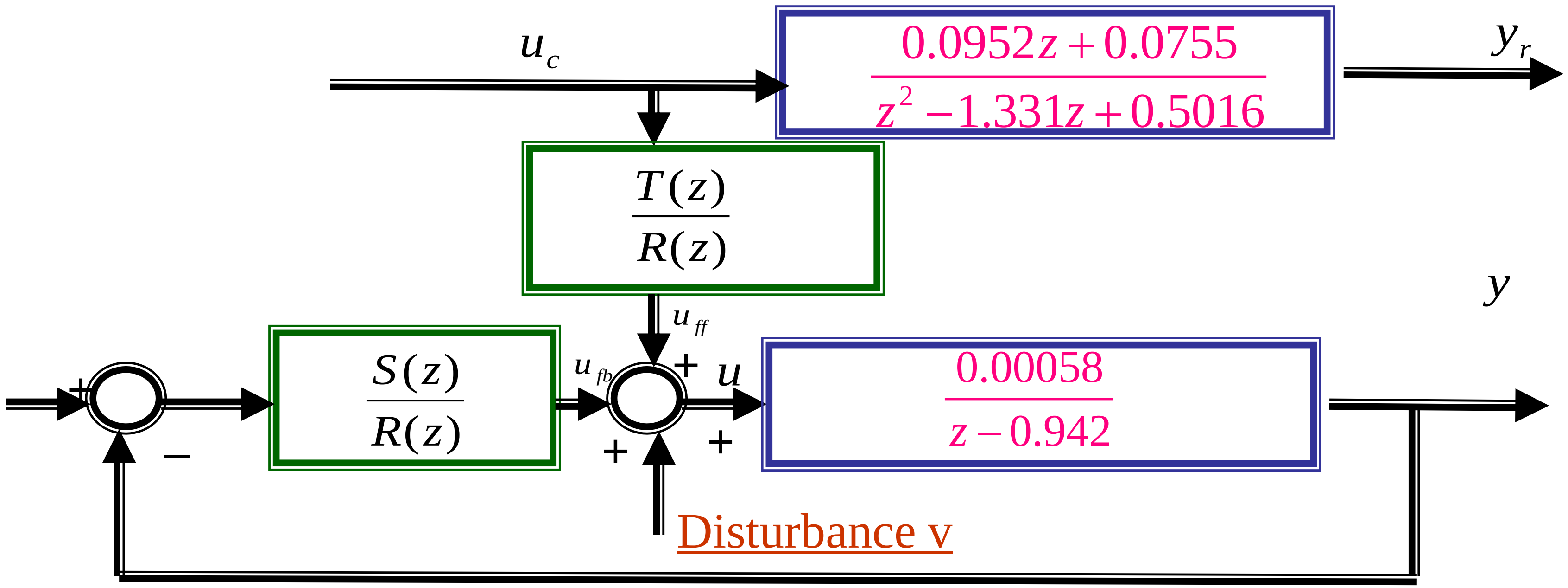
Convert the desired reference model from continuous time to discrete-time

$$H_d(z) = (1 - z^{-1}) Z[\frac{H_d(s)}{s}]$$

You can use the conversion table or the MATLAB command c2d.

$$H_d(z) = \frac{0.0952z + 0.0755}{z^2 - 1.331z + 0.5016}$$



We want to reject the constant disturbance, what should we include in R(z)?

(z -1)

Try $R = z - 1$ and $S = s_0 z + s_1$, it follows that

$$AR + BS = (z - 0.942)(z - 1) + 0.00058(s_0 z + s_1)$$
$$= z^2 + (0.00058 s_0 - 1.942) z + 0.00058 s_1 + 0.942$$

Compare with the $A_m(z)$

$$z^2 - 1.331z + 0.5016$$

Match the coefficients, we have

$$(0.00058 s_0 - 1.942) = -1.331 \qquad 0.00058 s_1 + 0.942 = 0.5016$$

$$\Rightarrow \quad s_0 = 1053.4 \qquad s_1 = -759.3$$

We have

$$R = z - 1, \quad S = 1053.4z - 759.3$$

$$\frac{Y(z)}{U_c(z)} = \frac{T(z)}{R(z)} \frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)} = \frac{B(z)T(z)}{A(z)R(z)+B(z)S(z)}$$

$$\frac{Y(z)}{U_c(z)} = \frac{B(z)T(z)}{A_m(z)}$$

Design $T(z)$ to match $\frac{B_m(z)}{A_m(z)}$

$$\frac{B(z)T(z)}{A_m(z)} = \frac{B_m(z)}{A_m(z)} \Longrightarrow$$

$$T(z) = \frac{B_m}{B} = \frac{0.0952z + 0.0755}{0.00058} = 164.14z + 130.17$$

For feed-forward controller, we have

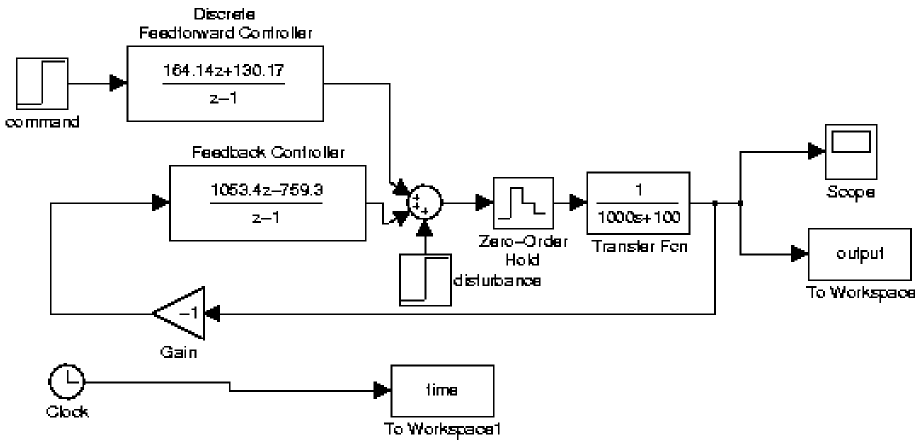
$$H_{ff}(z) = \frac{T}{R} = \frac{164.14z + 130.17}{(z-1)}$$

Therefore, the two-degree of freedom controller has the form of,

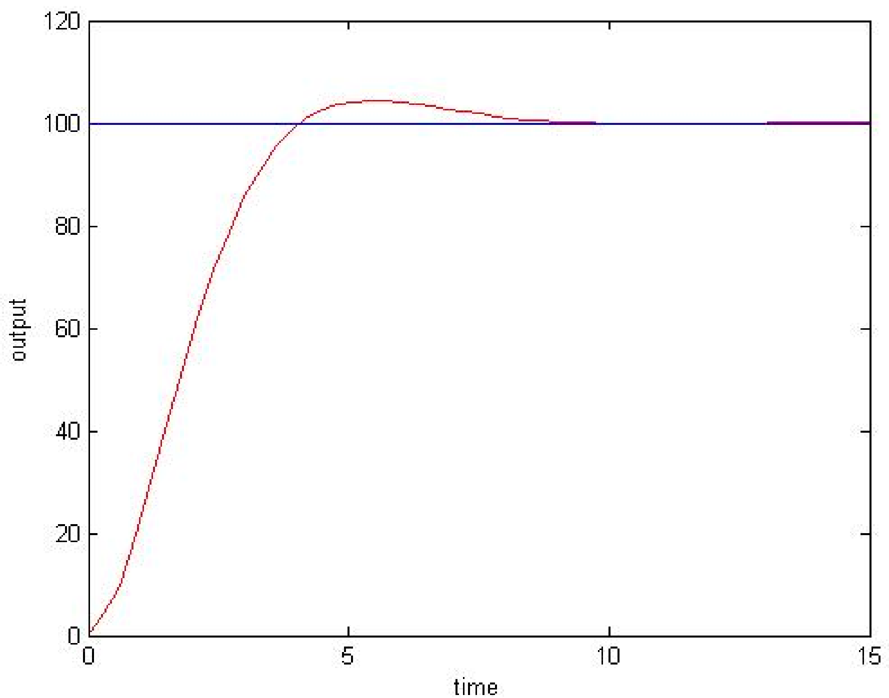
$$U(z) = -\frac{1053.4z - 759.3}{z-1} Y(z) + \frac{164.14z + 130.17}{z-1} U_c(z)$$

Verification through SIMULINK

Now simulate the digital two-degree-of-freedom control system response with actual plant



Does it meet all the performance requirements?



Yes.
The overshoot is less than 5%.
The settling time is around 8s.

The two-degree-of-freedom controller can perform better than PID controller!

Q & A…

THANK YOU !

$$H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}$$

return

$G(s)$	$H(q)$ or the coefficients in $H(q)$
$\frac{1}{s}$	$\frac{h}{q-1}$
$\frac{1}{s^2}$	$\frac{h^2(q+1)}{2(q-1)^2}$
$\frac{1}{s^m}$	$\frac{q-1}{q} \lim_{a \to 0} \frac{(-1)^m}{m!} \frac{\partial^m}{\partial a^m} \left(\frac{q}{q - e^{-ah}} \right)$
e^{-sh}	q^{-1}
$\frac{a}{s+a}$	$\frac{1 - \exp(-ah)}{q - \exp(-ah)}$
$\frac{a}{s(s+a)}$	$b_1 = \frac{1}{a}(ah - 1 + e^{-ah})$ $\quad b_2 = \frac{1}{a}(1 - e^{-ah} - ahe^{-ah})$ $a_1 = -(1 + e^{-ah})$ $\quad a_2 = e^{-ah}$
$\frac{a^2}{(s+a)^2}$	$b_1 = 1 - e^{-ah}(1 + ah)$ $\quad b_2 = e^{-ah}(e^{-ah} + ah - 1)$ $a_1 = -2e^{-ah}$ $\quad a_2 = e^{-2ah}$
$\frac{s}{(s+a)^2}$	$\frac{(q-1)he^{-ah}}{(q - e^{-ah})^2}$
$\frac{ab}{(s+a)(s+b)}$ $a \neq b$	$b_1 = \frac{b(1 - e^{-ah}) - a(1 - e^{-bh})}{b - a}$ $b_2 = \frac{a(1 - e^{-bh})e^{-ah} - b(1 - e^{-ah})e^{-bh}}{b - a}$ $a_1 = -(e^{-ah} + e^{-bh})$ $a_2 = e^{-(a+b)h}$

$$H(z)=\frac{b_1 z+b_2}{z^2+a_1 z+a_2}$$

return

$G(s)$	$H(q)$ or the coefficients in $H(q)$
$\dfrac{(s+c)}{(s+a)(s+b)}$ $a \neq b$	$b_1=\dfrac{e^{-bh}-e^{-ah}+(1-e^{-bh})c/b-(1-e^{-ah})c/a}{a-b}$ $b_2=\dfrac{c}{ab}e^{-(a+b)h}+\dfrac{b-c}{b(a-b)}e^{-ah}+\dfrac{c-a}{a(a-b)}e^{-bh}$ $a_1=-e^{-ah}-e^{-bh}$ $\quad a_2=e^{-(a+b)h}$
$\dfrac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$b_1=1-\alpha\left(\beta+\dfrac{\zeta\omega_0}{\omega}\gamma\right)$ $\quad \omega=\omega_0\sqrt{1-\zeta^2}$ $\quad \zeta<1$ $b_2=\alpha^2+\alpha\left(\dfrac{\zeta\omega_0}{\omega}\gamma-\beta\right)$ $\quad \alpha=e^{-\zeta\omega_0 h}$ $a_1=-2\alpha\beta$ $\quad \beta=\cos(\omega h)$ $a_2=\alpha^2$ $\quad \gamma=\sin(\omega h)$
$\dfrac{s}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$b_1=\dfrac{1}{\omega}e^{-\zeta\omega_0 h}\sin(\omega h)$ $\quad b_2=-b_1$ $a_1=-2e^{-\zeta\omega_0 h}\cos(\omega h)$ $\quad a_2=e^{-2\zeta\omega_0 h}$ $\omega=\omega_0\sqrt{1-\zeta^2}$
$\dfrac{a^2}{s^2+a^2}$	$b_1=1-\cos ah$ $\quad b_2=1-\cos ah$ $a_1=-2\cos ah$ $\quad a_2=1$
$\dfrac{s}{s^2+a^2}$	$b_1=\dfrac{1}{a}\sin ah$ $\quad b_2=-\dfrac{1}{a}\sin ah$ $a_1=-2\cos ah$ $\quad a_2=1$
$\dfrac{a}{s^2(s+a)}$	$b_1=\dfrac{1-\alpha}{a^2}+h\left(\dfrac{h}{2}-\dfrac{1}{a}\right)$ $\quad \alpha=e^{-ah}$ $b_2=(1-\alpha)\left(\dfrac{h^2}{2}-\dfrac{2}{a^2}\right)+\dfrac{h}{a}(1+\alpha)$ $b_3=-\left[\dfrac{1}{a^2}(\alpha-1)+\alpha h\left(\dfrac{h}{2}+\dfrac{1}{a}\right)\right]$ $a_1=-(\alpha+2)$ $\quad a_2=2\alpha+1$ $\quad a_3=-\alpha$

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		$b^k \quad (b = e^{-aT})$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$	$\frac{z\left(1-e^{-aT}\right)}{a(z-1)\left(z-e^{-aT}\right)}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z\left(e^{-aT}-e^{-bT}\right)}{\left(z-e^{-aT}\right)\left(z-e^{-bT}\right)}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}\left(1-e^{-at}-ate^{-at}\right)$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$

return