EE3304 DIGITAL CONTROL SYSTEMS PART II
TUTORIAL FOUR

Q.1. Consider the system given by the transfer function

$$G(z) = \frac{z+0.9}{z^2 - 2.5z + 1}$$

Design a pole placement controller in the form of

$$U(z) = \frac{T(z)}{R(z)} U_c(z) - \frac{S(z)}{R(z)} Y(z)$$

such that the closed-loop system has the desired characteristic polynomial

$$A_m(z) = z^2 - 1.8z + 0.9$$

Let the polynomial Ao(z) have as low order as possible and place all of its poles in the origin. Design the controller such that the steady-state gain from the command signal $u_c(k)$ to the output y(k) is one. Consider the following two cases:

a) The process zero is canceled.

b) The process zero is not canceled.

Simulate the step responses of the two cases (letting uc(k) = 1), and plot out the corresponding output and input signals. Discuss the differences between the two controllers. Which one should be preferred?

Solution:

a) The process zero is canceled.

The closed-loop transfer function has the form

$$H_{cl}(z) = \frac{B_m}{A_m} = \frac{BT}{AR+BS}$$

To cancel the process zero, let $R = B = z + 0.9$

$$\text{Let} \quad S = s_0 z + s_1 \quad \Rightarrow$$

$$(z+0.9)(z^2 - 2.5z + 1) + (z+0.9)(s_0 z + s_1) = \underbrace{(z^2 - 1.8z + 0.9)}_{A_m}(z+0.9)$$

Therefore we have $R = z + 0.9,\ S = 0.7z - 0.1.$

Choose $T(z) = t_0$ such that

$$H_{cl}(z)\big|_{z=1} = 1 \quad \Rightarrow \quad \frac{t_0}{z^2 - 1.8z + 0.9\big|_{z=1}} = \frac{t_0}{0.1} = 1 \Rightarrow \quad T(z) = 0.1$$

The controller is:

$$u(k) = -0.9u(k-1) + 0.1u_c(k-1) - 0.7y(k) + 0.1y(k-1),\ k \geq 1$$

b) The process zero is not canceled.

$$\begin{aligned}
&\text{Let } R = z + r_1,\ S = s_0 z + s_1 \\
&AR + BS = (z^2 - 2.5z + 1)(z + r_1) + (z + 0.9)(s_0 z + s_1) \\
&= z^3 + (r_1 + s_0 - 2.5)z^2 + (1 - 2.5r_1 + s_1 + 0.9s_0)z + 0.9s_1 + r_1 = z\underbrace{(z^2 - 1.8z + 0.9)}_{A_m}
\end{aligned}$$

Therefore we have

$$\begin{aligned}
&r_1 + s_0 - 2.5 = -1.8 \\
&1 - 2.5r_1 + s_1 + 0.9s_0 = 0.9 \\
&0.9s_1 + r_1 = 0
\end{aligned}$$

So we get $R = z + 0.1618\ ,\quad S = 0.5382z - 0.1798$

Let $T(z) = t_0 z$

$$H_{cl}(z)\big|_{z=1} = \frac{t_0 z(z + 0.9)}{z(z^2 - 1.8z + 0.9)}\bigg|_{z=1} = 1 \quad \Rightarrow \quad t_0 = 0.0526$$

The controller is:

$$u(k) = -0.1618u(k-1) + 0.0526u_c(k) - 0.5382y(k) + 0.1798y(k-1)$$

Simulation of the two cases can be obtained easily by building the corresponding block diagram in Simulink.

The step responses of the two systems are shown in Figure 1.1

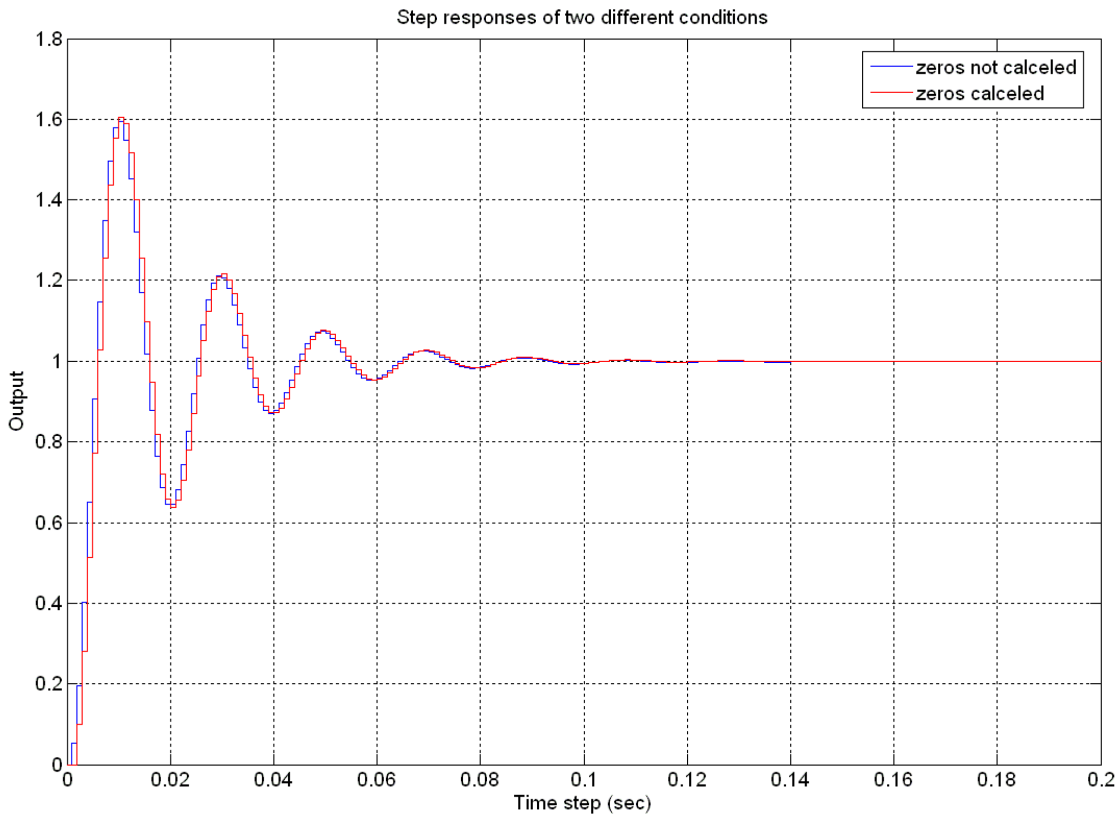

Figure 1.1 The step responses of the system.

You will find that the system performances (outputs of the system) under the 2 cases are almost the same.

The difference lies in the control signal as shown in Figure 1.2. The control effort based on the design without zero cancellation is easier for implementation as it is smaller in magnitude and smoother.

The control input for the case of zero cancellation is oscillating (chattering) due to the fact that the process zero z = - 0.9 (negative) is now the pole of the transfer function from the command signal u_c to the input u,

$$\frac{U}{U_C} = \frac{TA}{AR+BS} = \frac{0.1\left(z^2 - 2.5z + 1\right)}{\left(z+0.9\right)\left(z^2 - 1.8z + 0.9\right)}$$

The negative pole would result in oscillation. Overall, we can conclude that the design of case (b) is preferred.

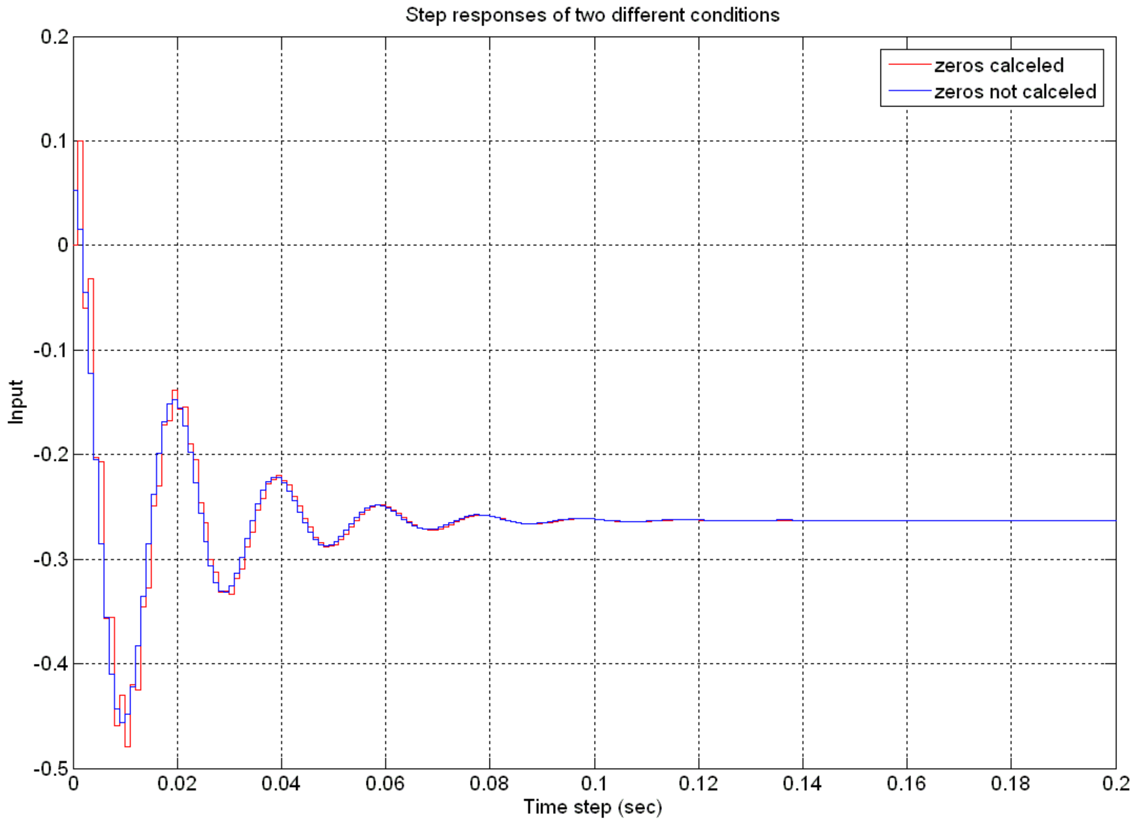

Figure 1.2 The control input for the two cases.

Q.2. Assume that the process is described by the transfer function

$$G(z) = \frac{0.4z + 0.3}{z^2 - 1.6z + 0.65}$$

Design a pole placement controller in the form of

$$U(z) = \frac{T(z)}{R(z)} U_c(z) - \frac{S(z)}{R(z)} Y(z)$$

to satisfy the following specifications:

- Static gain = 1
- Cancellation of process zero
- Disturbance rejection for constant disturbance
- Desired characteristic polynomial

$$A_m(z) = z^2 - 0.7z + 0.25$$

The closed loop transfer function from the Uc to Y is

$$\frac{Y(z)}{U_c(z)} = \frac{BT}{AR + BS}$$

Since $B(z) = 0.4(z+0.75)$ is stable, we can choose $R(z) = (z+0.75)R'$ to cancel the stable zero. Since we also want to reject constant disturbance, R(z) should contain the factor (z-1). Overall, we may choose

$$R(z) = (z+0.75)(z-1) = z^2 - 0.25z - 0.75$$

and let

$$S(z) = s_0 z^2 + s_1 z + s_2$$

The Diophantine equation is now

$$AR + BS = A_{cl}$$

$$(z^2 - 1.6z + 0.65)(z-1)(z+0.75) + 0.4(z+0.75)(s_0 z^2 + s_1 z + s_2) = (z+0.75)(z^2 - 0.7z + 0.25)A_o(z)$$

For simplicity, let $A_o(z) = z$. So we have

$$z^3 + (0.4s_0 - 2.6)z^2 + (2.25 + 0.4s_1)z + 0.4s_2 - 0.65 = z^3 - 0.7z^2 + 0.25z$$

which gives

$$0.4s_0 - 2.6 = -0.7$$
$$2.25 + 0.4s_1 = 0.25$$
$$0.4s_2 - 0.65 = 0$$

And we get

$$S(z) = 2.5(1.9z^2 - 2z + 0.65) = 4.75z^2 - 5z + 1.625$$

Let $T(z) = t_0 z$

The closed-loop TF is

$$H_{cl}(z) = \frac{Y(z)}{U_c(z)} = \frac{BT}{AR+BS} = \frac{0.4(z+0.75)t_0 z}{(z+0.75)(z^2 - 0.7z + 0.25)z} = \frac{0.4t_0}{(z^2 - 0.7z + 0.25)}$$

The steady state gain is

$$H_{cl}(z)|_{z=1} = \frac{0.4t_0}{(z^2 - 0.7z + 0.25)}|_{z=1} = \frac{t_0}{1.375} = 1$$

So $t_0 = 1.375$

Q.3. Assume that the process is described by the transfer function

$$G(z) = \frac{z-0.5}{z^2-4z+3}$$

Design a pole placement controller in the form of

$$U(z) = \frac{T(z)}{R(z)}U_c(z) - \frac{S(z)}{R(z)}Y(z)$$

such that the closed loop transfer function from the command signal, $u_c(k)$, to the system output, $y(k)$, follows the reference model, $\frac{1}{z^2}$.

Solution: The closed loop transfer function from the Uc to Y is

$$\frac{Y(z)}{U_c(z)} = \frac{BT}{AR+BS}$$

Since $B(z) = z - 0.5$ is stable, we can choose $R(z) = (z - 0.5)$ to cancel the stable zero to match H_m.

$$\frac{Y(z)}{U_c(z)} = \frac{BT}{AR+BS} = \frac{T}{A+S} = \frac{1}{z^2} \quad \Rightarrow \quad S = 4z-3, T = 1$$

Therefore, the controller is

$$u(k) = 0.5u(k-1) + u_c(k-1) - 4y(k) + 3y(k-1), \quad k \geq 1$$

Q.4. In some systems the process output y(k) and the command signals $u_c(k)$ are not available because only the error $e = u_c - y$ is measured. This case is called error feedback. A typical case is a CD player in which only the deviation from the track can be measured. This means that a two-degree-of-freedom controller cannot be used. Mathematically it means that the polynomials S and T are identical and the control law becomes

$$U(z) = \frac{S(z)}{R(z)}(U_c(z) - Y(z))$$

Assume that the process is described by the same transfer function

$$G(z) = \frac{z-0.5}{z^2-4z+3}$$

as that for Q.3. Design an error feedback controller such that the closed loop transfer function from the command signal, $u_c(k)$, to the system output, $y(k)$, follows the reference model, $\frac{1}{z^2}$.

Solution: For error feedback control system, it can be easily obtained that the closed-loop transfer function from the command signal $u_c(k)$ to the output y(k) is

$$\frac{Y(z)}{U_c(z)} = \frac{\frac{S}{R}\frac{B}{A}}{1+\frac{S}{R}\frac{B}{A}} = \frac{C(z)G(z)}{1+C(z)G(z)}$$

Where $C(z) = \frac{S(z)}{R(z)}, G(z) = \frac{B(z)}{A(z)}$

And let

$$\frac{Y(z)}{U_c(z)} = \frac{C(z)G(z)}{1+C(z)G(z)} = H_m(z)$$

C(z) can be solved from above equation as

$$C(z) = \frac{H_m(z)}{G(z)(1-H_m(z))} = \frac{\frac{B_m}{A_m}}{\frac{B}{A}(1-\frac{B_m}{A_m})} = \frac{AB_m}{B(A_m - B_m)}$$

So

$$R(z) = B(A_m - B_m)$$
$$S(z) = AB_m$$

For this case,

$G(z) = \frac{z-0.5}{z^2-4z+3}$ and $H_m(z) = \frac{1}{z^2}$, and we have the solution

$$C(z) = \frac{z^2-4z+3}{(z-0.5)(z^2-1)} = \frac{z^2-4z+3}{z^3-0.5z^2-z+0.5}$$

It seems that the pure error feedback controller can also solve the problem. But let's check out the closed-loop transfer function in greater details. In this case, we have

$$\frac{Y(z)}{U_c(z)} = \frac{\frac{S}{R}\frac{B}{A}}{1+\frac{S}{R}\frac{B}{A}} = \frac{BS}{AR+BS}$$
$$= \frac{BAB_m}{AB(A_m - B_m) + BAB_m} = \frac{ABB_m}{ABA_m}$$

We can see that there are common factors AB in the closed loop transfer function. Only if both A(z) and B(z) are stable, we can match the reference model completely. Otherwise, the closed loop is unstable. For this example, A(z) is unstable, so the closed loop will be unstable and the error feedback controller cannot make the overall system match the reference model.

However, if only pole placement is required, then we can easily design R and S to solve the Diophantine equation

$$AR + BS = A_m A_o = z^3 .$$