EE3304 DIGITAL CONTROL SYSTEMS PART II
TUTORIAL ONE

Q1. Consider the analog control system shown in Fig. 1 , where the first-order plant a/(s+a) is controlled by the analog integral controller K/s. Obtain the velocity error constant for the system and the steady-state error in the response to a unit ramp input.

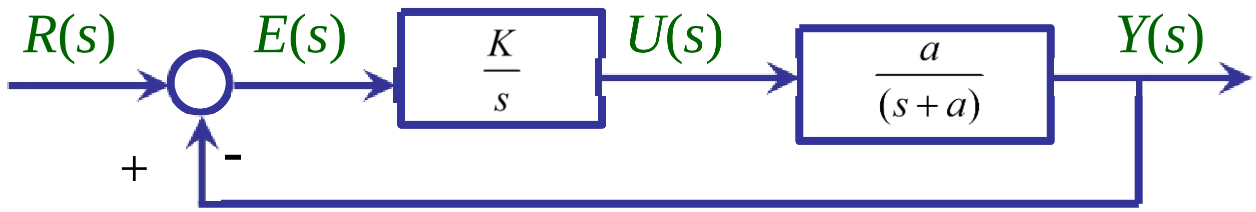


Figure 1 Analog Control System

Next, consider equivalent digital control system. Using equivalent digital controllers obtained by (1) the backward rule and (2) Tustin's rule (i.e., the bilinear rule), show that the velocity error constants for the equivalent digital control systems are the same as that for the analog control system.

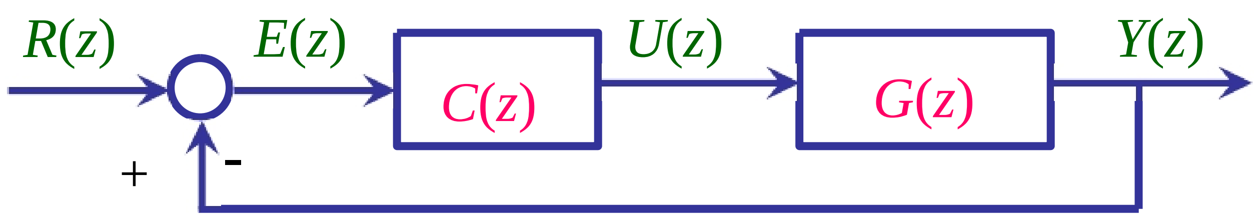


Figure 2 Equivalent Digital Control System

Q2. Consider a unity feedback system with the closed-loop transfer function

$$H(z) = \frac{K(z+c)}{z^2 + az + b}$$

Assume that the open-loop transfer function G(z) involves an integrator, meaning that it has a simple pole at z=1. (The system is of Type 1). Show that the steady-state error in the unit ramp response is given by

$$e_{ss} = \frac{1}{\beta_v} = T\left(\frac{2+a}{1+a+b} - \frac{1}{1+c}\right)$$

where T is the sampling period.

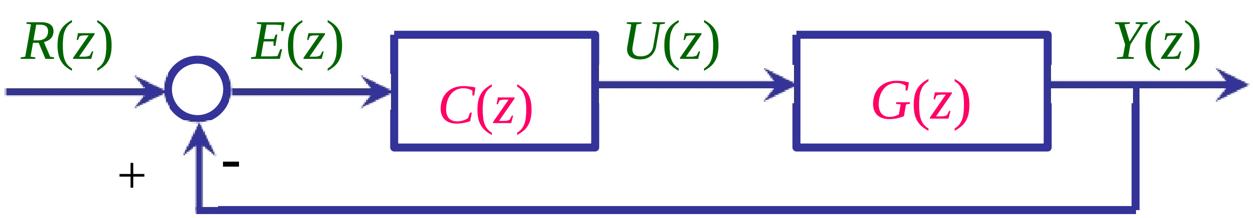
Q3. A process is modeled as $G(z) = \frac{1}{z+1}$. Design an appropriate digital controller C(z) for the unity feedback system such that:

1) place the closed-loop poles within a circle z $\leq$ 0.8;

2) achieve a finite steady state error for ramp input;

3) minimize the steady state error for ramp input.

Show that the steady state error is proportional to the sampling period. What kind of closed-loop response will we have?



Q.4 Derive a desired closed-loop characteristic polynomial for discrete systems such that the resulting closed-loop system response has an overshoot of less than 10% and a settling time of less than 10 seconds for a step reference input, where the sampling period is T=2 seconds.

Q.5 A digital controller $C(z) = \frac{-z+1.5}{z-1}$ is designed for a plant $G(z) = \frac{1}{z+1}$ in a unity feedback system. Select an appropriate sampling period T such that the closed-loop response satisfies

$t_s \leq \frac{0.1 \times 9.2}{\ln 2} = 1.3273$.

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		$b^k \quad \left(b = e^{-aT}\right)$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$	$\frac{z\left(1-e^{-aT}\right)}{a(z-1)\left(z-e^{-aT}\right)}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z\left(e^{-aT}-e^{-bT}\right)}{\left(z-e^{-aT}\right)\left(z-e^{-bT}\right)}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}\left(1-e^{-at}-ate^{-at}\right)$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$

$$H(z)=\frac{b_1 z+b_2}{z^2+a_1 z+a_2}$$

$G(s)$	$H(q)$ or the coefficients in $H(q)$
$\frac{1}{s}$	$\frac{h}{q-1}$
$\frac{1}{s^2}$	$\frac{h^2(q+1)}{2(q-1)^2}$
$\frac{1}{s^m}$	$\frac{q-1}{q}\lim_{a\to 0}\frac{(-1)^m}{m!}\frac{\partial^m}{\partial a^m}\left(\frac{q}{q-e^{-ah}}\right)$
e^{-sh}	q^{-1}
$\frac{a}{s+a}$	$\frac{1-\exp(-ah)}{q-\exp(-ah)}$
$\frac{a}{s(s+a)}$	$b_1=\frac{1}{a}(ah-1+e^{-ah})$ $\quad b_2=\frac{1}{a}(1-e^{-ah}-ahe^{-ah})$ $a_1=-(1+e^{-ah})$ $\quad a_2=e^{-ah}$
$\frac{a^2}{(s+a)^2}$	$b_1=1-e^{-ah}(1+ah)$ $\quad b_2=e^{-ah}(e^{-ah}+ah-1)$ $a_1=-2e^{-ah}$ $\quad a_2=e^{-2ah}$
$\frac{s}{(s+a)^2}$	$\frac{(q-1)he^{-ah}}{(q-e^{-ah})^2}$
$\frac{ab}{(s+a)(s+b)}$ $a\neq b$	$b_1=\frac{b(1-e^{-ah})-a(1-e^{-bh})}{b-a}$ $b_2=\frac{a(1-e^{-bh})e^{-ah}-b(1-e^{-ah})e^{-bh}}{b-a}$ $a_1=-(e^{-ah}+e^{-bh})$ $a_2=e^{-(a+b)h}$

return1

return2

$$H(z)=\frac{b_1 z+b_2}{z^2+a_1 z+a_2}$$

return

$G(s)$	$H(q)$ or the coefficients in $H(q)$
$\frac{(s+c)}{(s+a)(s+b)}$ $a\neq b$	$b_1=\frac{e^{-bh}-e^{-ah}+(1-e^{-bh})c/b-(1-e^{-ah})c/a}{a-b}$ $b_2=\frac{c}{ab}e^{-(a+b)h}+\frac{b-c}{b(a-b)}e^{-ah}+\frac{c-a}{a(a-b)}e^{-bh}$ $a_1=-e^{-ah}-e^{-bh}$ $\quad a_2=e^{-(a+b)h}$
$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$b_1=1-\alpha\left(\beta+\frac{\zeta\omega_0}{\omega}\gamma\right)$ $\quad \omega=\omega_0\sqrt{1-\zeta^2}$ $\quad \zeta<1$ $b_2=\alpha^2+\alpha\left(\frac{\zeta\omega_0}{\omega}\gamma-\beta\right)$ $\quad \alpha=e^{-\zeta\omega_0 h}$ $a_1=-2\alpha\beta$ $\quad \beta=\cos(\omega h)$ $a_2=\alpha^2$ $\quad \gamma=\sin(\omega h)$
$\frac{s}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$b_1=\frac{1}{\omega}e^{-\zeta\omega_0 h}\sin(\omega h)$ $\quad b_2=-b_1$ $a_1=-2e^{-\zeta\omega_0 h}\cos(\omega h)$ $\quad a_2=e^{-2\zeta\omega_0 h}$ $\omega=\omega_0\sqrt{1-\zeta^2}$
$\frac{a^2}{s^2+a^2}$	$b_1=1-\cos ah$ $\quad b_2=1-\cos ah$ $a_1=-2\cos ah$ $\quad a_2=1$
$\frac{s}{s^2+a^2}$	$b_1=\frac{1}{a}\sin ah$ $\quad b_2=-\frac{1}{a}\sin ah$ $a_1=-2\cos ah$ $\quad a_2=1$
$\frac{a}{s^2(s+a)}$	$b_1=\frac{1-\alpha}{a^2}+h\left(\frac{h}{2}-\frac{1}{a}\right)$ $\quad \alpha=e^{-ah}$ $b_2=(1-\alpha)\left(\frac{h^2}{2}-\frac{2}{a^2}\right)+\frac{h}{a}(1+\alpha)$ $b_3=-\left[\frac{1}{a^2}(\alpha-1)+\alpha h\left(\frac{h}{2}+\frac{1}{a}\right)\right]$ $a_1=-(\alpha+2)$ $\quad a_2=2\alpha+1$ $\quad a_3=-\alpha$