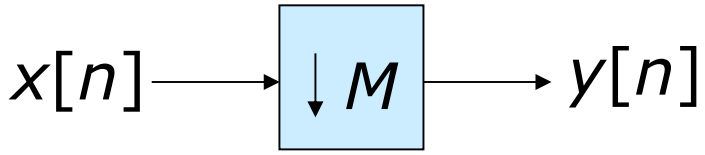
EE3731C: Signal Processing Methods

Tutorial II-1

Question #1

Which of the following signals can be down-sampled by a factor of 2 using the system below without any loss of information?



a) $x[n] = \delta[n - n_0]$, where n_0 is an unknown integer

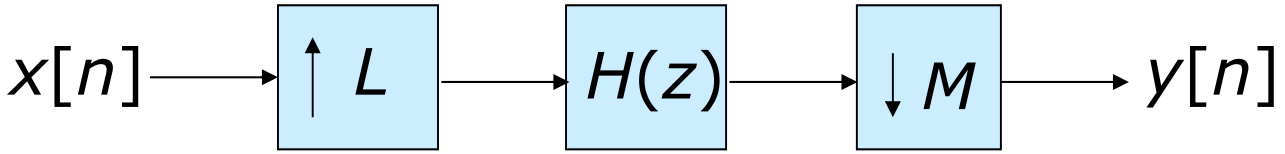
b) $x[n] = \cos(\pi n/4)$

c) $x[n] = \cos(\pi n/4) + \cos(3\pi n/4)$

d) $x[n] = \dfrac{\sin(\pi n/3)}{\pi n/3}$

Question #2

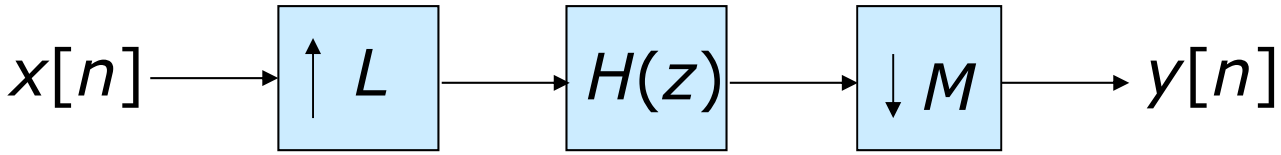
In the multirate system shown below, $H(z)$ represents a lowpass filter with Gain = L and cutoff $\omega_c = \min(\pi/L, \pi/M)$



Determine the corresponding output $y[n]$ for the following input signal $x[n]$ and the up-sampling and down-sampling rate of L and M.

$$x[n] = \frac{\sin(2\pi n/3)}{\pi n}, \quad L = 4, \quad M = 3$$

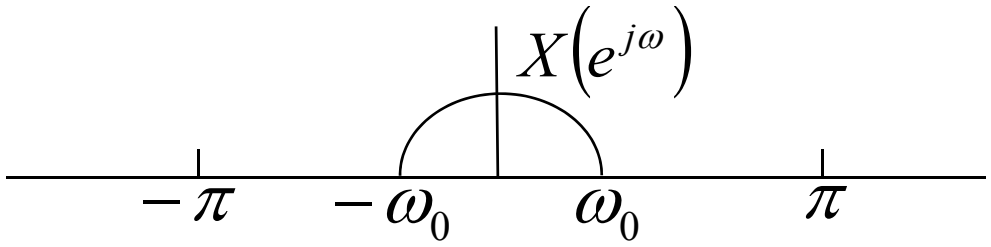
Question #3



$H(z)$: a lowpass filter with Gain = L and cutoff

$$\omega_c = \min(\pi/L, \pi/M)$$

The Fourier transform of the input signal is given by



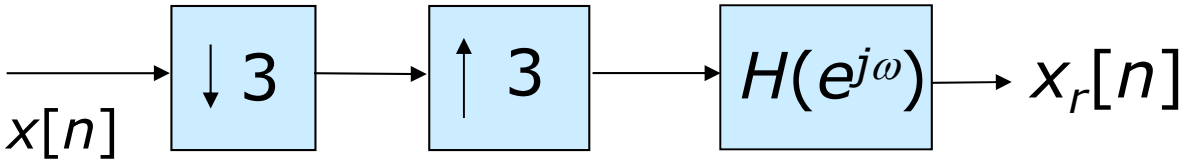
For each of the following choices of L and M, specify the maximum possible value of ω_0 such that $Y(e^{j\omega}) = aX(e^{j\omega L/M})$ for some constant a.

a) $L = 2, \quad M = 3$

b) $L = 3, \quad M = 2$

Question #4

In the system shown below,



we have $$H\left(e^{j\omega}\right)=\begin{cases}3, & |\omega|<\pi/3,\\ 0, & \pi/3\le|\omega|\le\pi.\end{cases}$$

For each of the following input signals $x[n]$, indicate whether the output $x_r[n]=x[n]$.

a) $x[n]=\cos\left(\pi n/4\right)$

b) $x[n]=\cos\left(\pi n/2\right)$

Question #5

Consider the multirate system shown below. Find an expression for $y[n]$ in terms of $x[n]$ by simplifying the system.