EE3731C – Signal Processing Methods

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MIT


...because not
everyone
can go to
Caltech

Module Introduction

Purpose

- To provide an introduction to signal processing methods
  - To know general concepts and methods
  - To use general methods for real life applications
- To prepare students for high-level technical electives and graduate modules in signal processing and new media

Prerequisites

- (EE2012 or ST2334) and EE2023
- Math:
  - Calculus
  - Linear algebra
    - Dot and inner product
  - Probability and statistics
    - Noise and uncertainty
    - Confidence level

Grading

- CAs (40%)
  - Each CA 20%
- Final Exam (60%)
  - Close book
  - One A4 size formula sheet is allowed

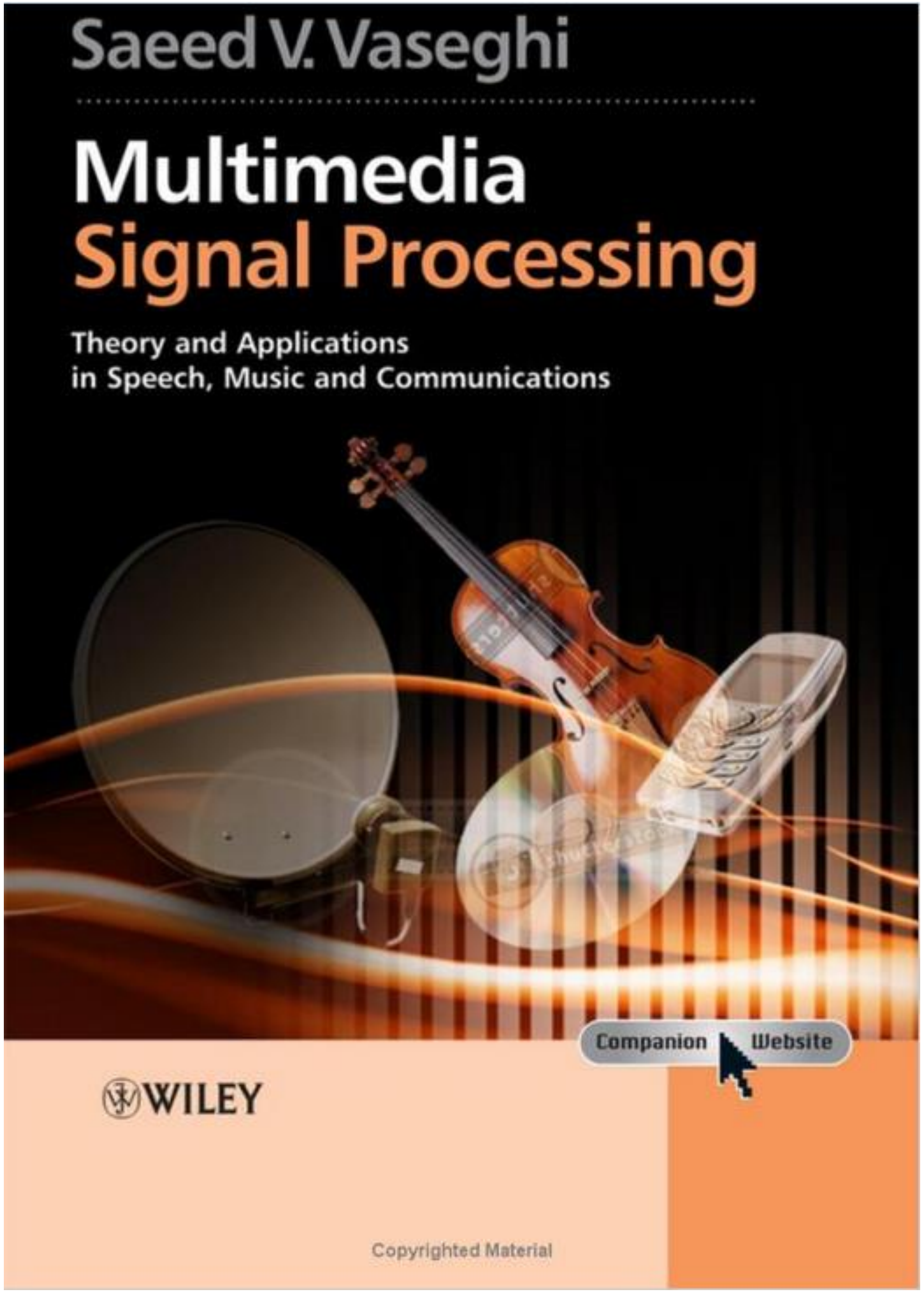
- Lecture: 9am-11pm, Wednesday
- Tutorial: 4pm-5pm, Friday
- Venue: E1-06-01

- Office hours: 5pm-6pm Friday or by appointment
- Office: E4-06-06

- Class Requirements
  - Attend lectures
  - Hand in assignments on time
- IVLE
- TA: Jiang Ming (mjiang@nus.edu.sg)
  - Office hours: 4-5pm, Wednesday
  - Office: E4-06-21

Contents

- Review of Basic Concepts
- PCA and Eigenanalysis
- Digital Filtering
- Multirate Digital Signal Processing
- Probability and Random Signals
- Example Applications



Multimedia Signal Processing: Theory and Applications in Speech, Music and Communications

by Saeed V. Vaseghi

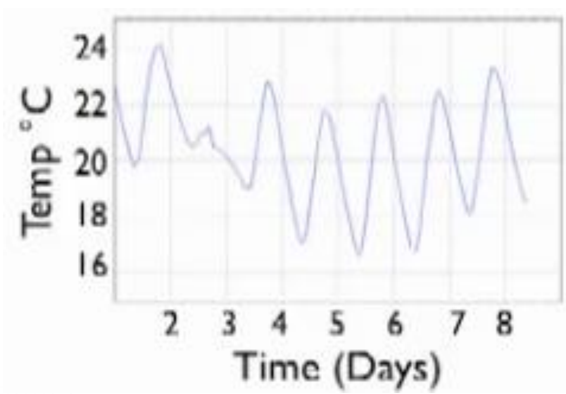
Overview

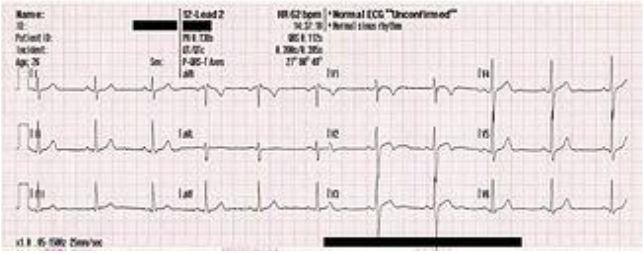
Signal

- A function that conveys information about the behavior or attributes of some phenomenon.
  - Can be any quantity exhibiting variation in time or variation in space.

Signal

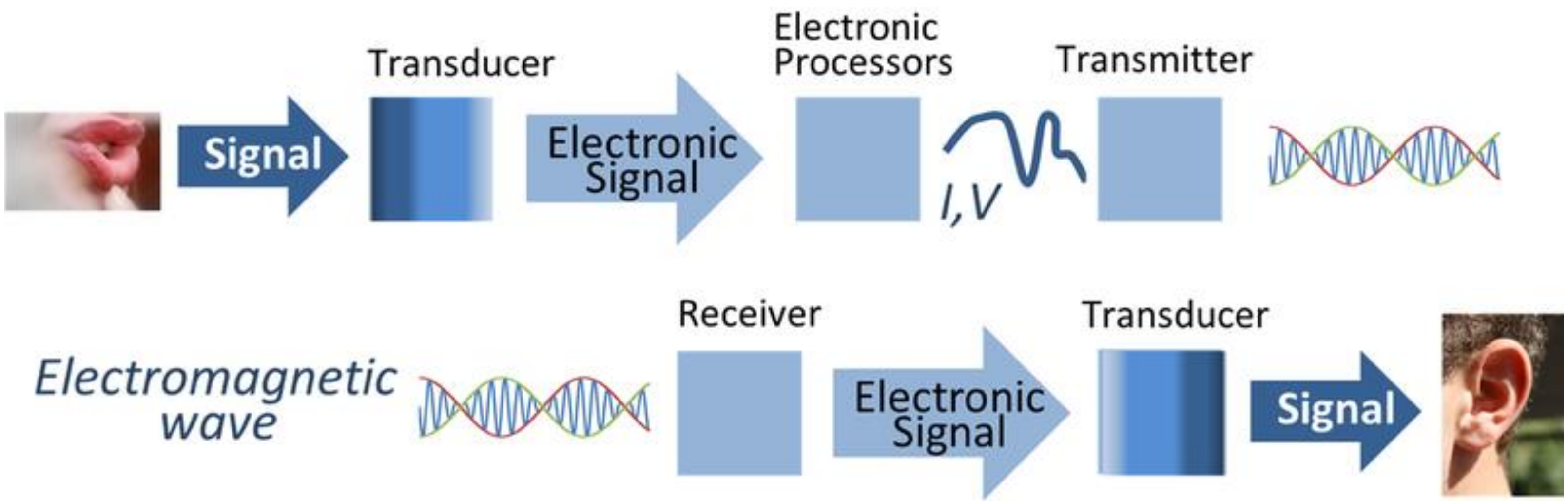
- The term “signal” includes, among others, audio, video, speech, image, communication, geophysical, sonar, radar, medical and musical signals.





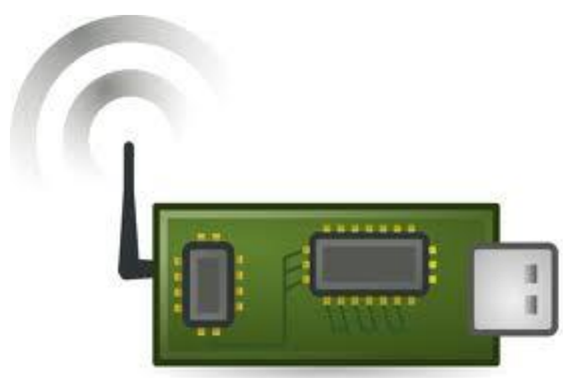
Signal Processing

- Deals with operations on or analysis of signals, or measurements of time-varying or spatially varying physical quantities.



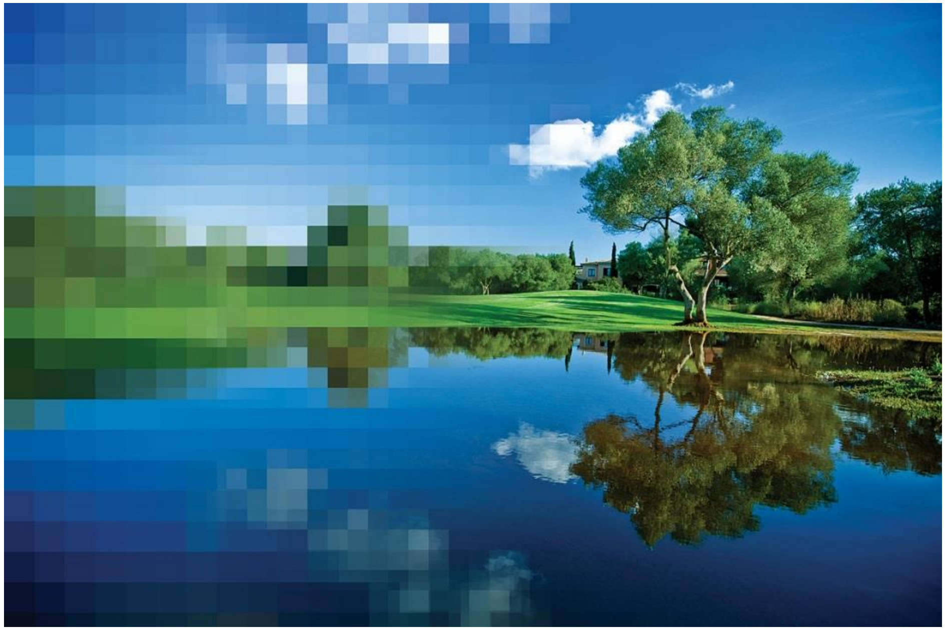
Typical Operations

- Signal acquisition and reconstruction
  – Involves measuring a physical signal, storing it, and possibly later rebuilding the original signal.



- Quality improvement
  - e.g., noise reduction, image enhancement, and echo cancellation.

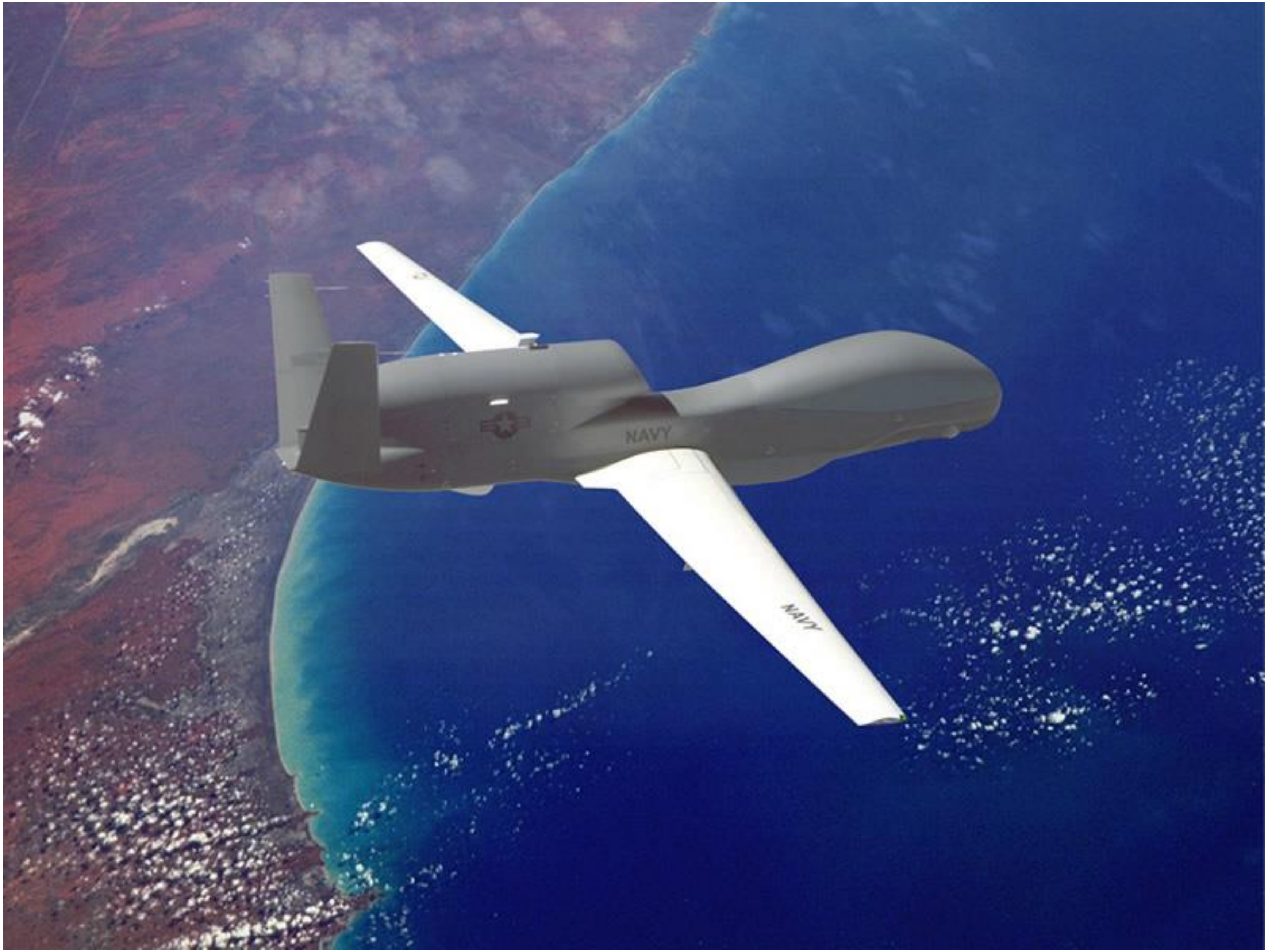
- Signal compression
  - e.g., audio compression, image compression, and video compression.



- Feature extraction,
  - e.g., in image understanding and speech recognition.

Applications

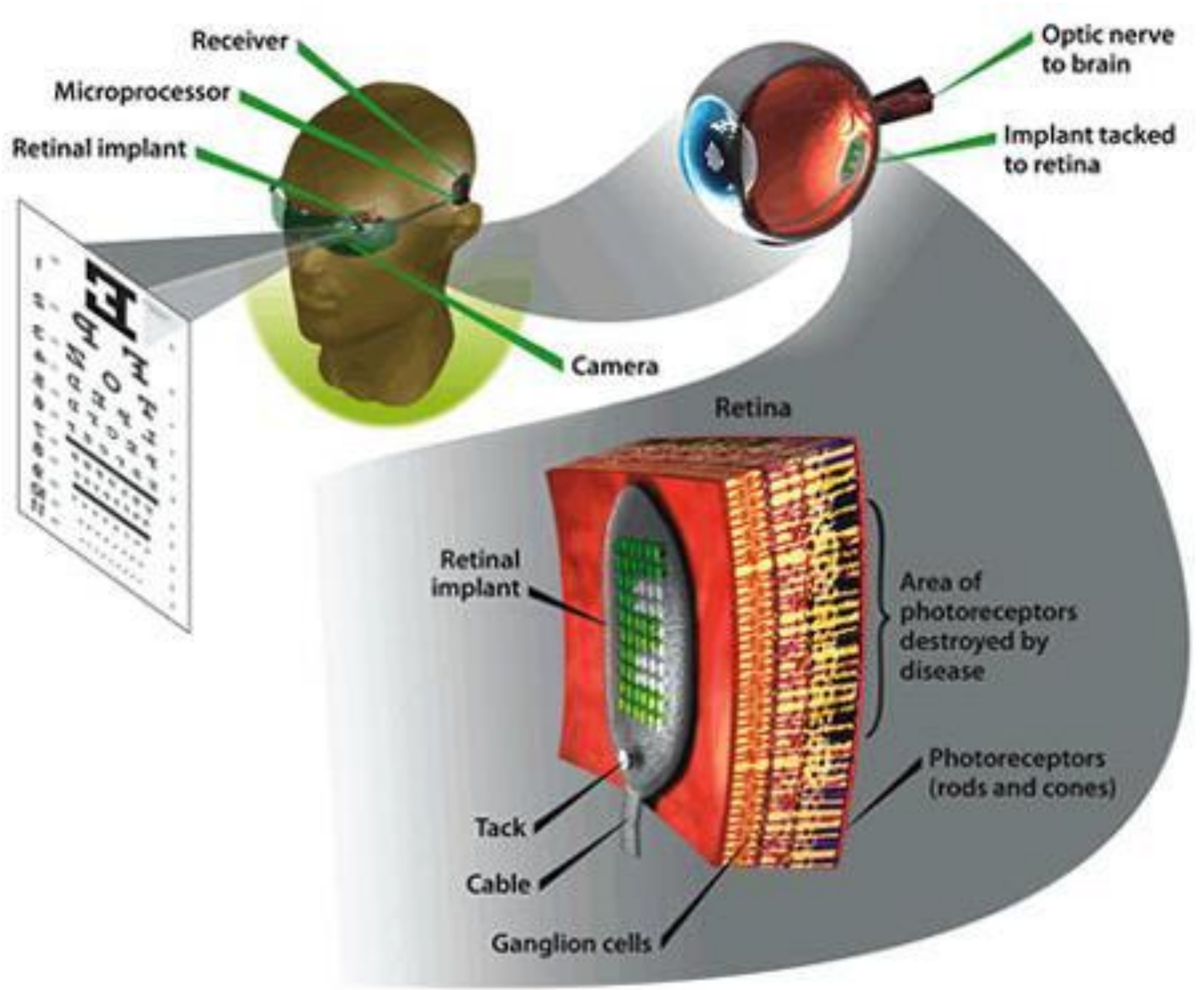
- Consumer electronics
  - Camera, speaker, mobile devices
- Medical
  - Body signal monitoring, medical imaging
- Military
  - Target detection/tracking
- Remote sensing
  - Weather monitoring



Military application

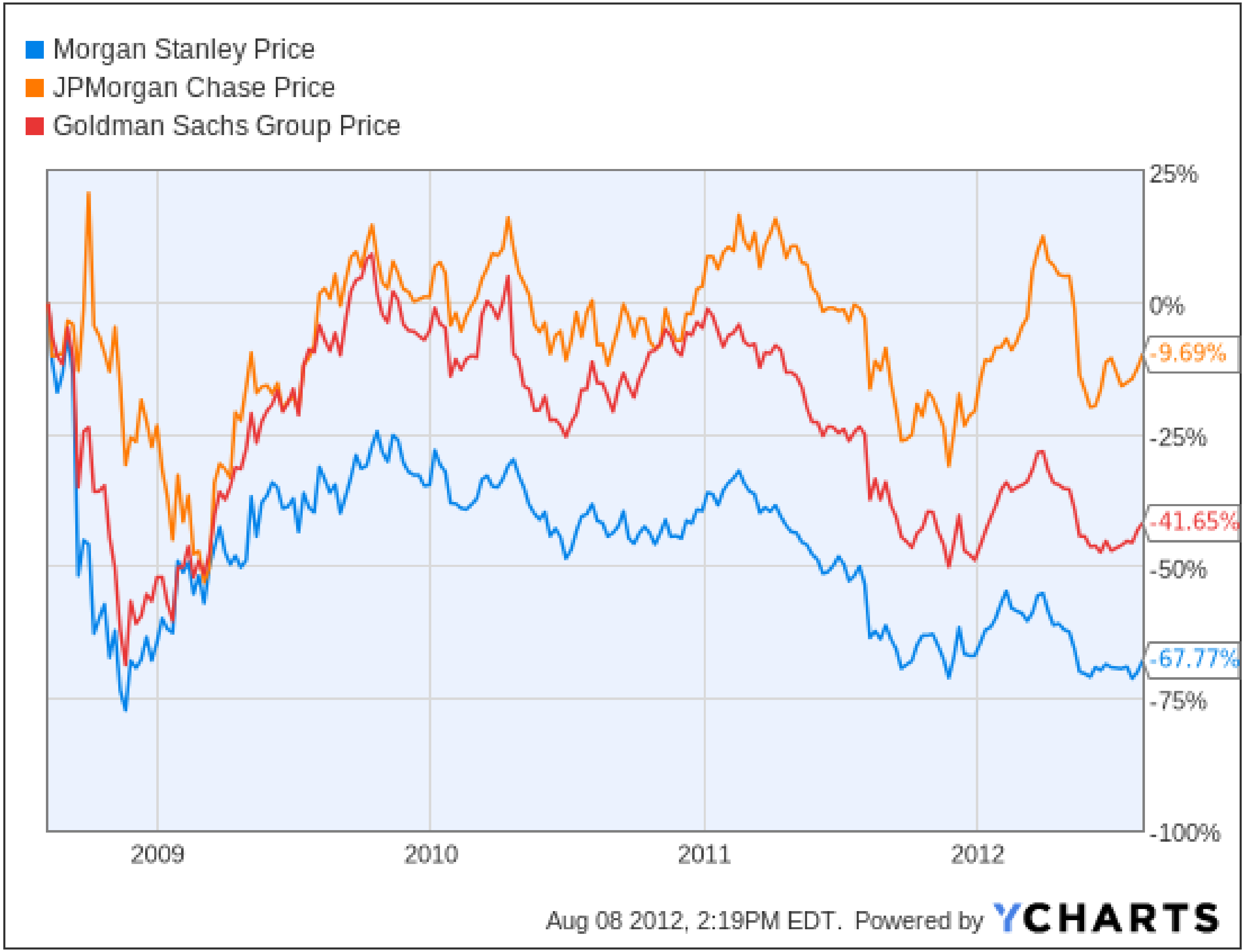
Global Hawk – 20km high, 100km radius, unmanned

Google earth

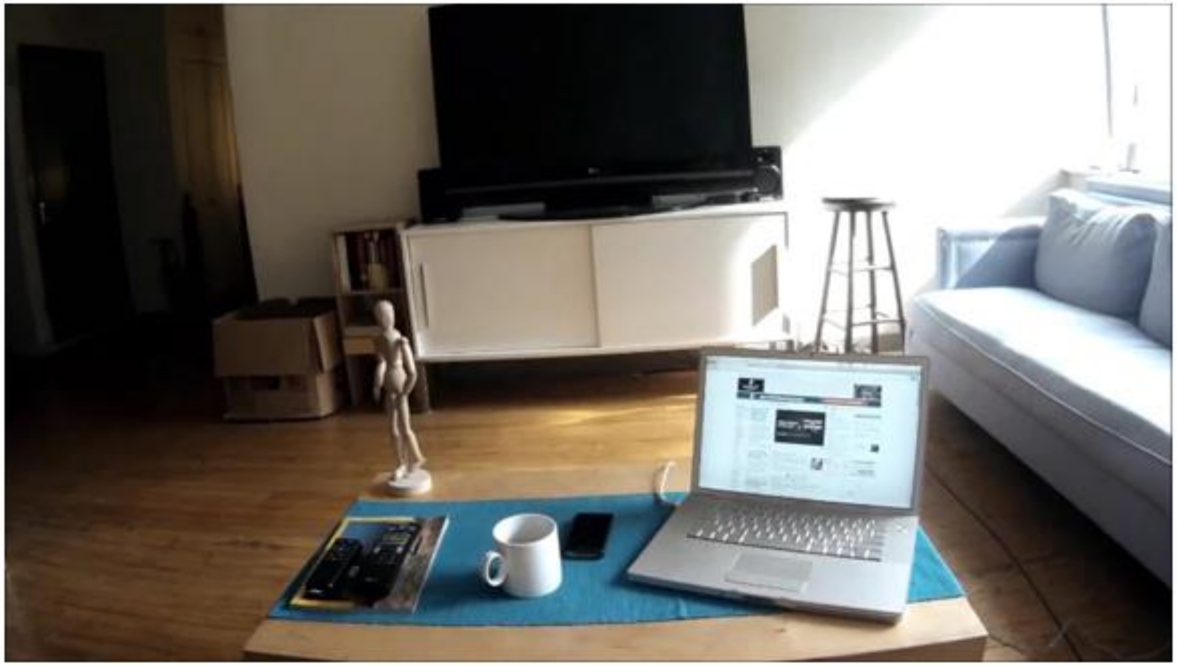


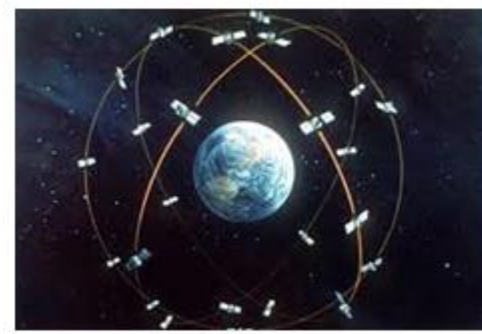
Adapted, with permission, from *IEEE Engineering in Medicine and Biology* **24**:15 (2005).

Medical applications



Where are M.I.T Grads Going?   Wall Street …

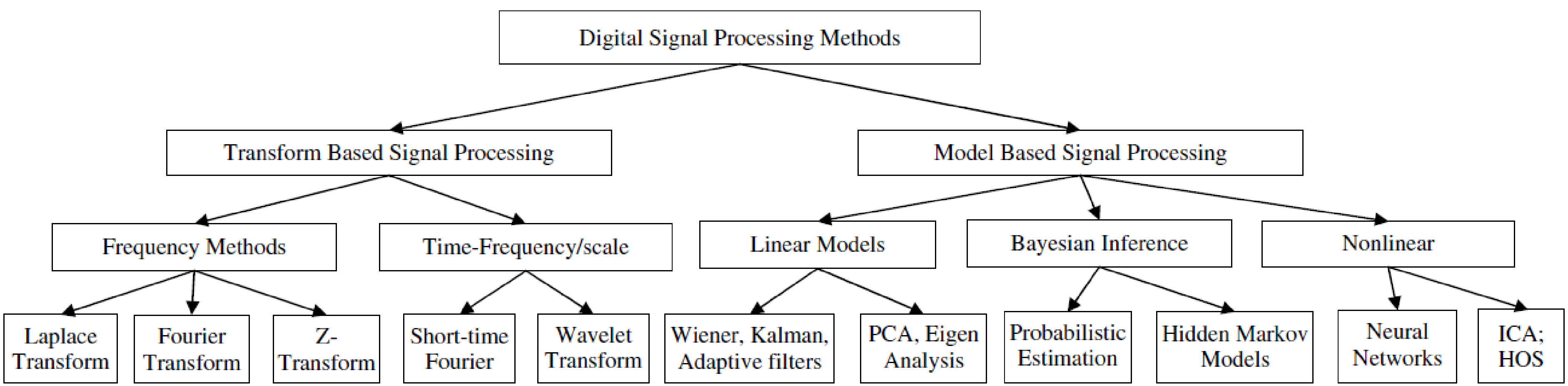




Consumer electronics

Categories

- Analog signal processing:
  - for signals that have not been digitized.
- Digital Signal Processing:
  - the processing of digitized discrete-time sampled signals.



Models

- Models to differentiate “signals” from “noise”

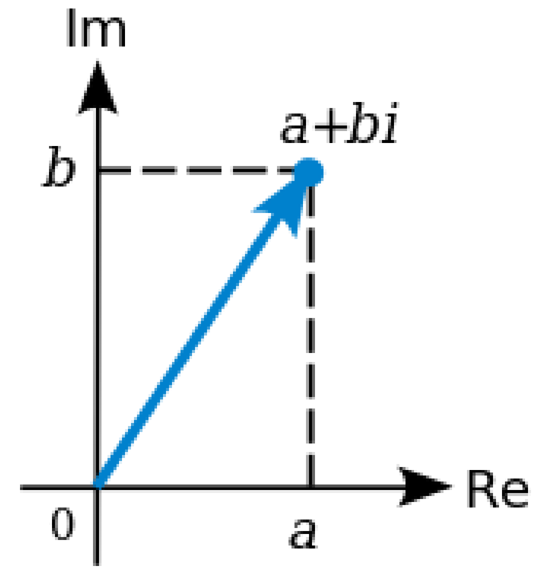
  - Models of image
  - Models of video
  - Models of audio
  - Models of biological signals
  - Models of cellphone signals

- Models are derived from prior information
- Poor models may lead to poor performance

Basic Concepts and Fourier Transforms

Complex Numbers

- z = a + bi
  - z: complex number
  - a: real part
  - b: imaginary part
  - i: imaginary unit ($i^2 = -1$)



- Extends a 1d number line to a 2d complex plane
  - (a, b) in the complex plane
  - Real part is zero: purely imaginary
  - Imaginary part is zero: real number

Operations

- Addition

  a + bi + c + di = (a+c)+(b+d)i

- Multiplication

  (a + bi)(c + di) = (ac-bd) + (ad+bc)i

- Conjugate $\bar{z}$ = a – bi
  - Geometrically, it is the reflection of z about the real axis.
  - $\bar{\bar{z}} = z$

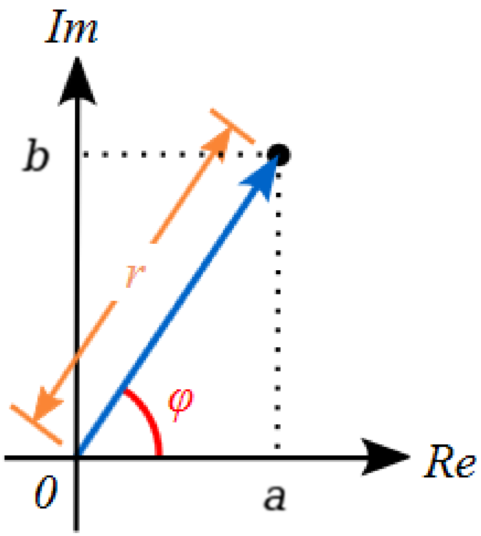
Polar Form

- Magnitude (modulus):

$$r = |z| = \sqrt{a^2 + b^2}$$

- Phase (argument):

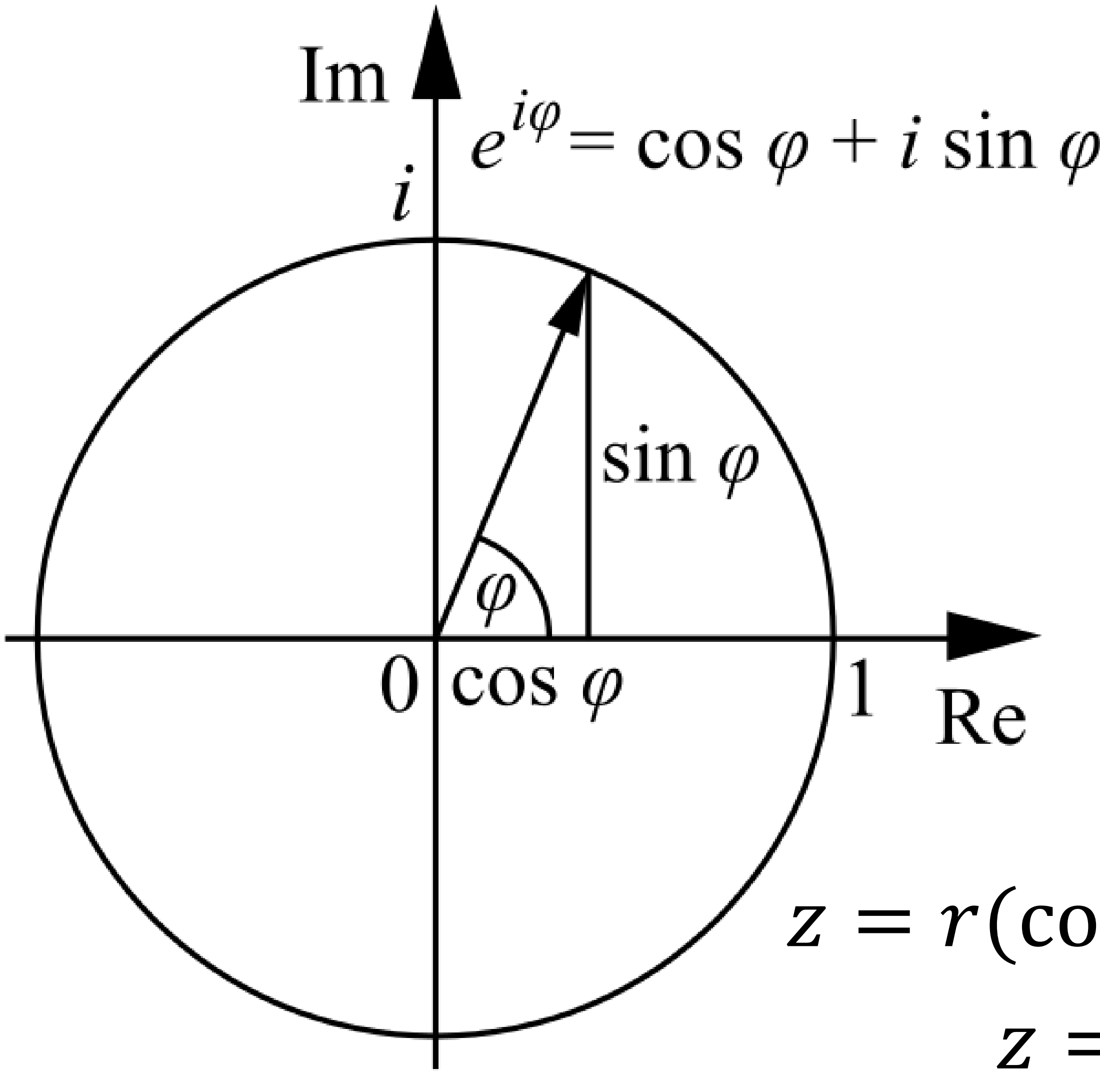
$$\varphi(z) = tan^{-1}(b/a)$$



- r and φ locate a point on an Argand diagram.
- Recovering the rectangular coordinates from the polar form with trigonometric from

$$z = r(\cos\varphi + i\sin\varphi)$$

Euler's Formula



$$z = r(\cos\varphi + i\sin\varphi)$$
$$z = re^{i\varphi}$$

Operations and Properties

- $e^a\, e^b = e^{a+b}$
  - valid for any complex numbers *a* and *b:*

    $xy = r(x)e^{i\varphi(x)}\, r(y)\, e^{i\varphi(y)} = r(x)\, r(y)\, e^{i(\varphi(x)+\varphi(y))}$

- $|e^{i\varphi}| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1$
- $\sin\varphi = \dfrac{e^{i\varphi} - e^{-i\varphi}}{2i}$
- $\cos\varphi = \dfrac{e^{i\varphi} + e^{-i\varphi}}{2}$

Signal in the Time Domain

- Time domain analysis is the analysis of signals with respect to time.
- In the time domain, the signal's value is known for
  - all real numbers, for the case of continuous time; or
  - at various separate instants in the case of discrete time.
- A time-domain graph shows how a signal changes with time.

Signal in the Frequency Domain

- Frequency = Rate of Change
- Frequency domain analysis is the analysis of signals with respect to frequency, rather than time.
- A frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies.

Why Fourier Transform?

- Connect the time domain and the frequency domain.

- Why frequency domain?
  - Perform useful analysis in the frequency domain that is too hard to do in the time domain.

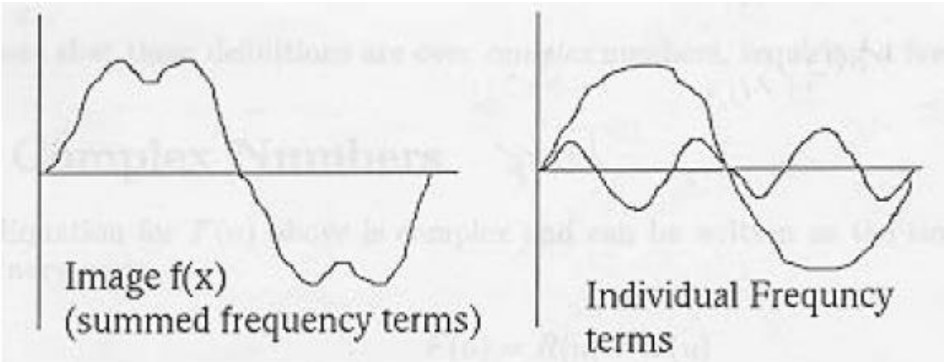
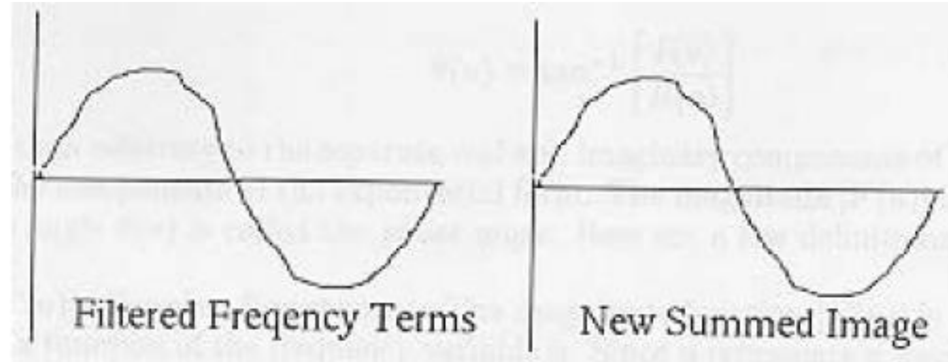
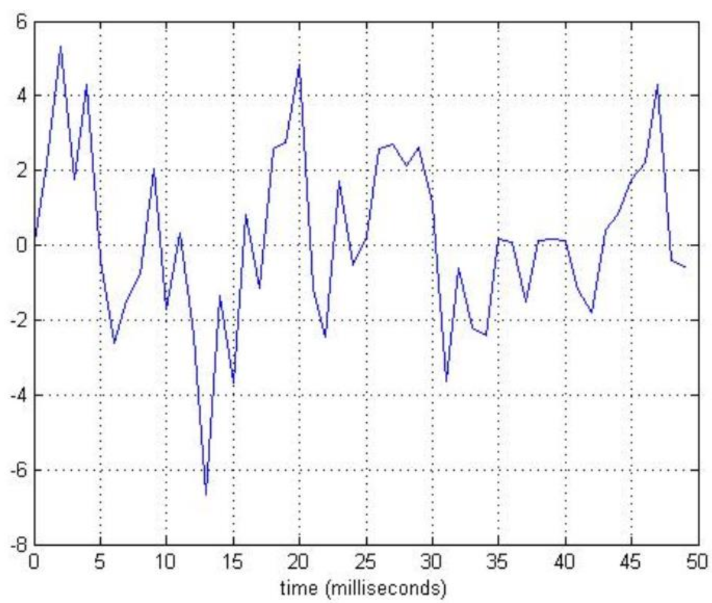
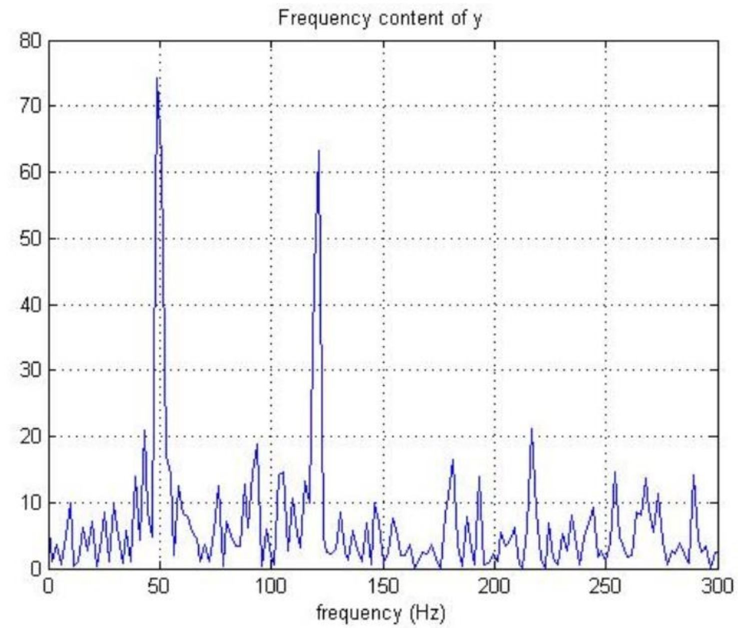


Image f(x)
(summed frequency terms)
Individual Frequncy terms

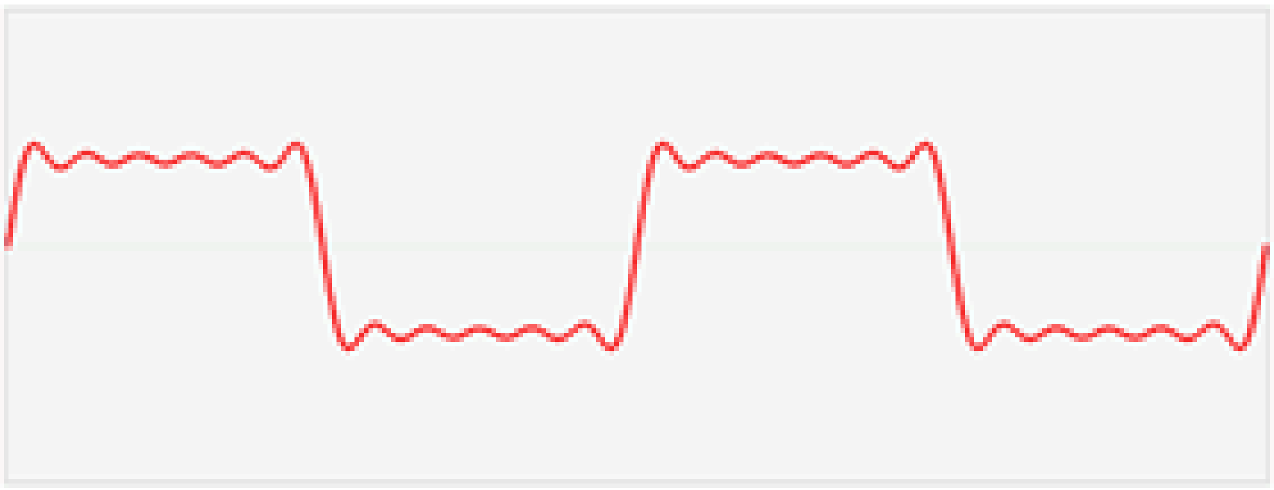

Filtered Freqency Terms
New Summed Image


time (milliseconds)

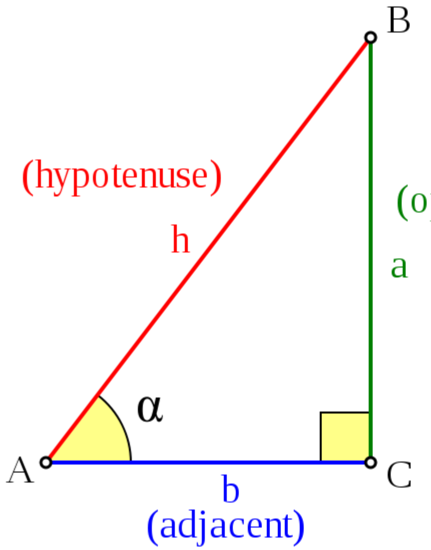

Frequency content of y
frequency (Hz)

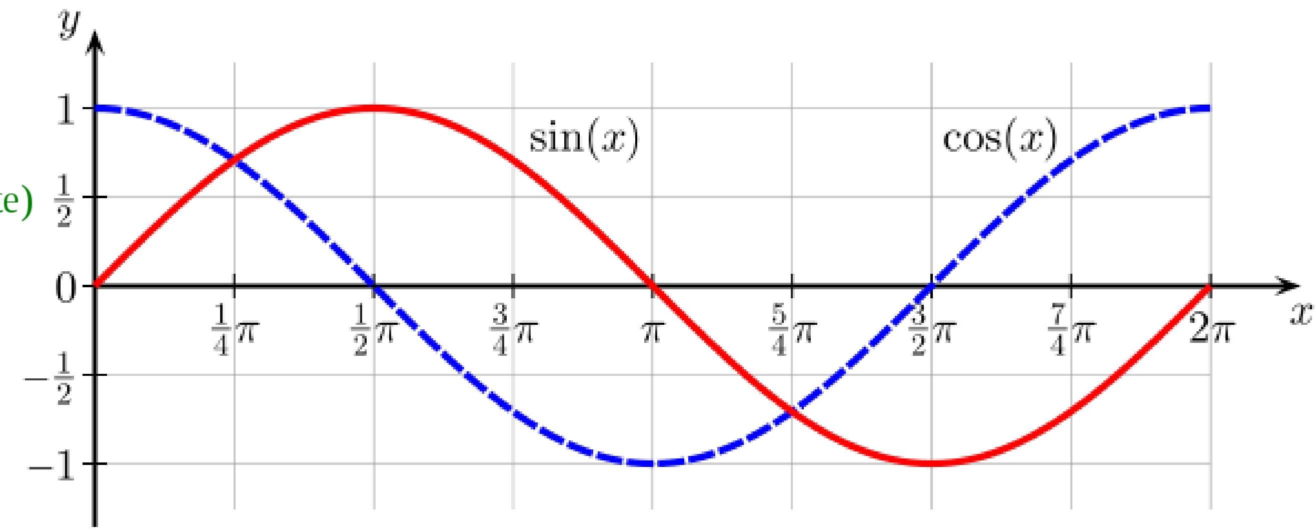
- The Fourier Transform changes signals from the time domain to the frequency domain and back again.

- Any time domain signal can be represented as a sum of sine waves.



Sine and Cosine Functions

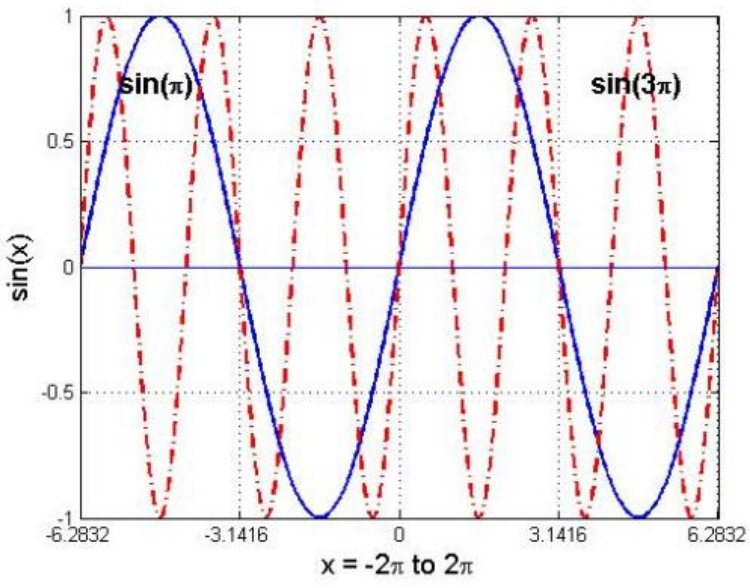


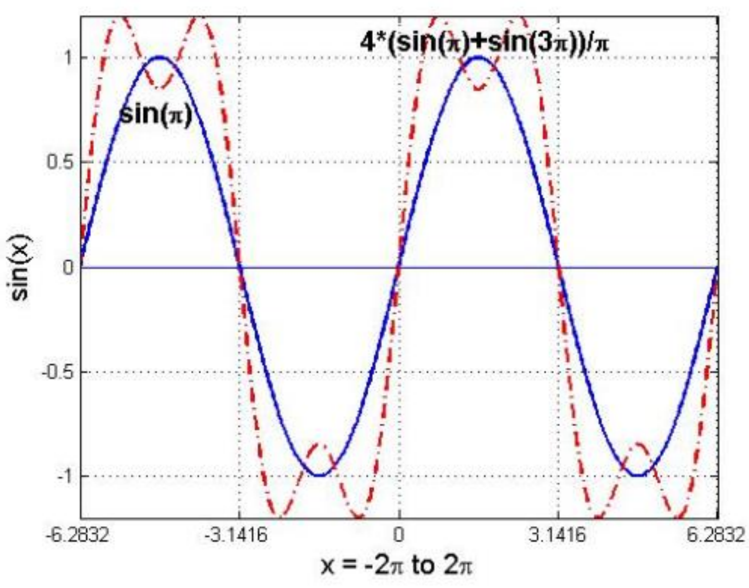


- The sine and cosine functions are related:
  - Out of phase by 90°: $\sin \varphi = \cos\left(\frac{\pi}{2} - \varphi\right)$
  - For a given angel, give the respective x, y coordinates on a unit circle: $\cos^2\varphi + \sin^2\varphi = 1$

Fourier’s Basic Idea

- Sine and cosine functions have period 2π.
- The linear combination of them still have a period 2π.

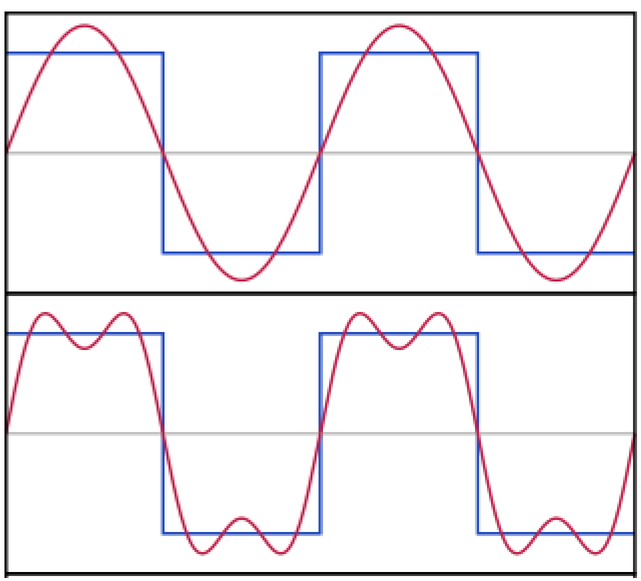


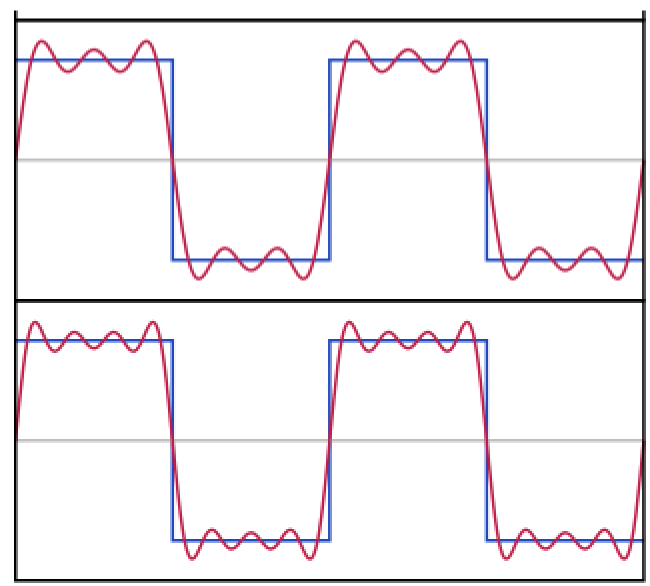


Fourier Series

- For any periodic signals, we can decompose them into the sum of a (possibly infinite) set of simple oscillating functions, namely sines and cosines (or complex exponentials)

$$f(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx)$$





Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx)$$

- To find the coefficients, multiply both sides by *cosmx*, or *sinmx*, and integrating it over [-π, π]

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos mx dx$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

Hint: Orthogonality relations of sine and cosine functions

Fourier Transform

- For a continuous function $f(x)$ of a single variable x representing time (or distance), its Fourier Transform is
  - $F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx$
    - u: temporal (spatial) frequency

- In general, $F(u)$ is a complex number even if the original data is real.

Inverse Fourier Transform

- $f(x)=\int_{-\infty}^{\infty} F(u)e^{i2\pi ux}du$

- Exponential term has the opposite sign as the forward Fourier Transform.

Properties of Fourier Transform

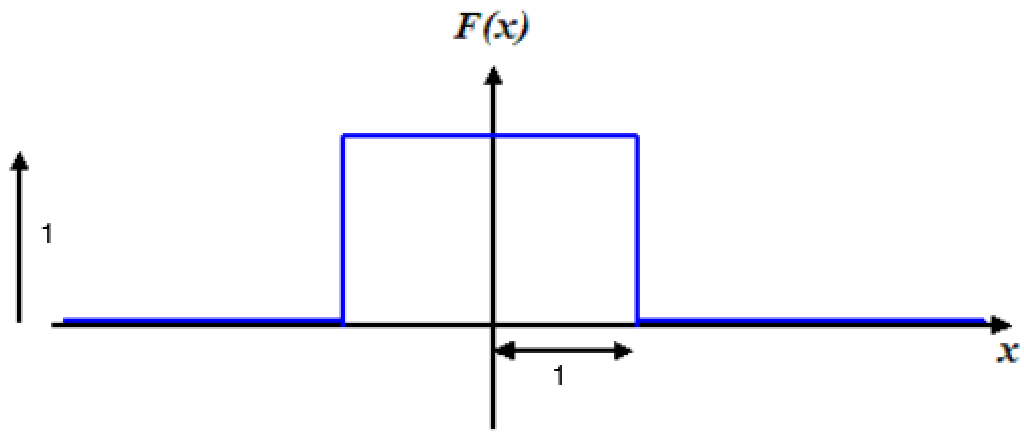
Assume $FT\big(f(x)\big) = F(u), FT\big(g(x)\big) = G(u)$

- Linearity property
  - $FT(af(x) + bg(x)) = aF(u) + bG(u)$
- Shift property
  - $FT\big(f(x-a)\big) = e^{-i2\pi ua}F(u)$
- Scaling property
  - $FT\big(f(cx)\big) = \frac{F(\frac{u}{c})}{|c|}$
- Convolution property
  - $FT(f(x) * g(x)) = F(u)G(u)$

A Classic Example

- Compute a Fourier Transform of the following:

$$f(x) = \begin{cases} 1, |x| \leq 1 \\ 0, x > 1 \end{cases}$$



Sinc Function

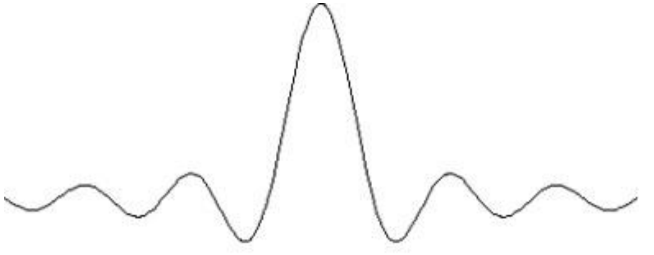
- $F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx$

$= \int_{-1}^{1} 1 e^{-i2\pi ux} dx = \frac{1}{-i2\pi ux}(e^{i2\pi u} - e^{-i2\pi u})$

Since $sin\varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}$

$$F(u) = \frac{sin2\pi u}{\pi u}$$

- $F(u)$ is referred to as the Sinc Function.



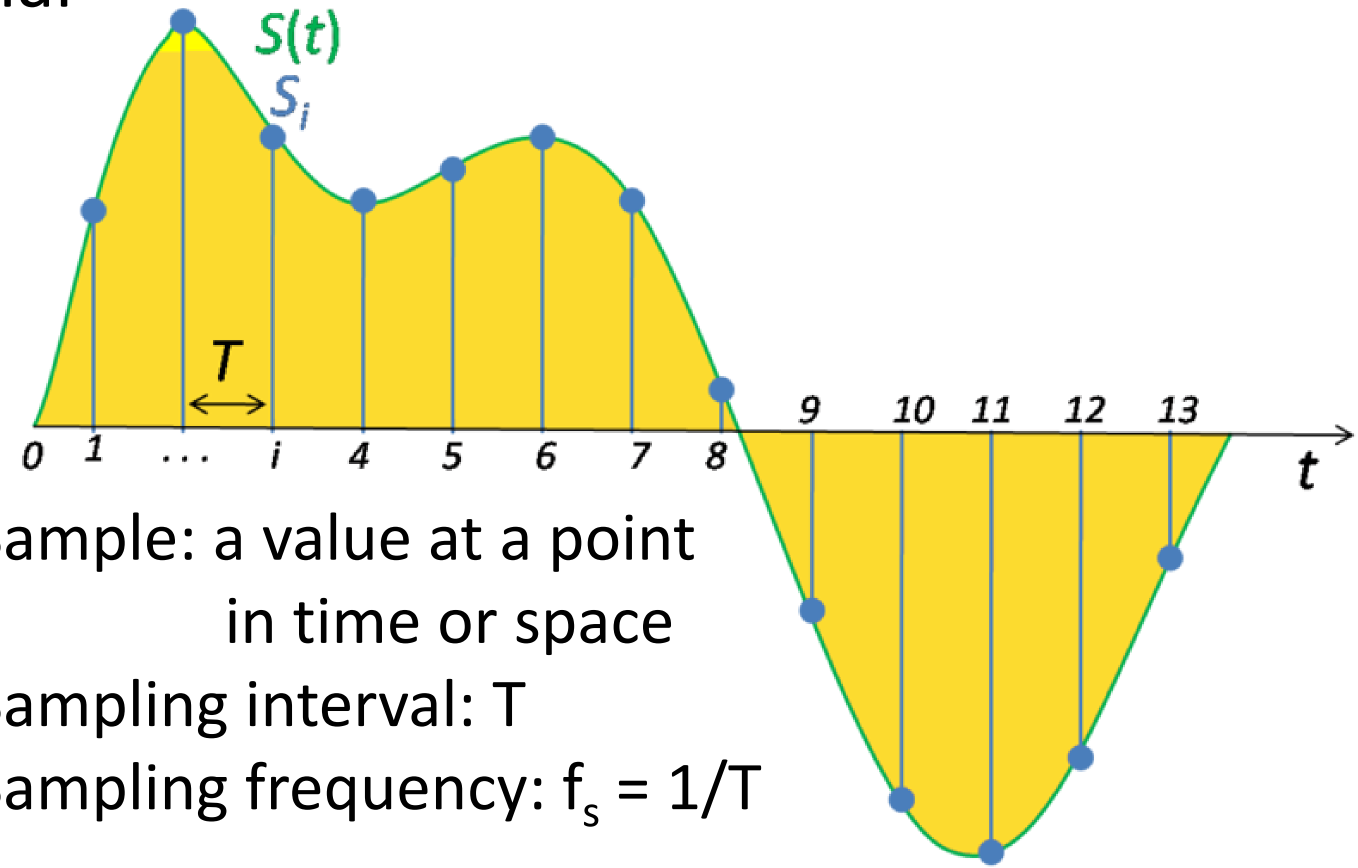
2D Fourier Transform

- For a continuous function $f(x, y)$ of variables x and y, its Fourier Transform is
  - $F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i2\pi(ux+vy)}\, dxdy$
    - u, v: temporal (spatial) frequency

- Inverse Transform
  - $f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{i2\pi(ux+vy)}\, dud$v

Sampling

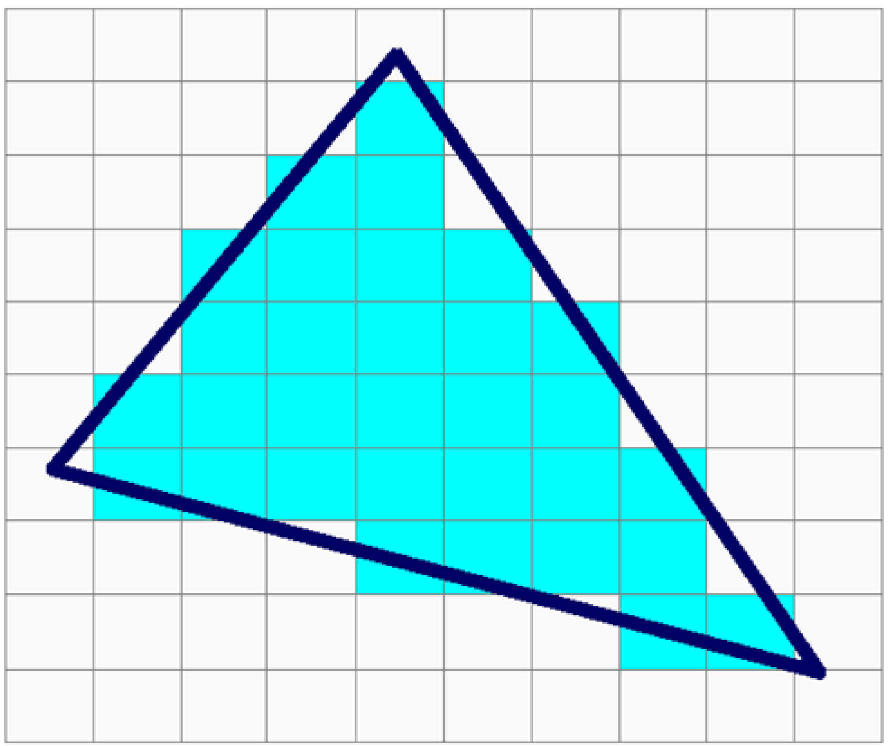
- The reduction of a continuous signal to a discrete signal



  - Sample: a value at a point in time or space
  - Sampling interval: T
  - Sampling frequency: $f_s = 1/T$

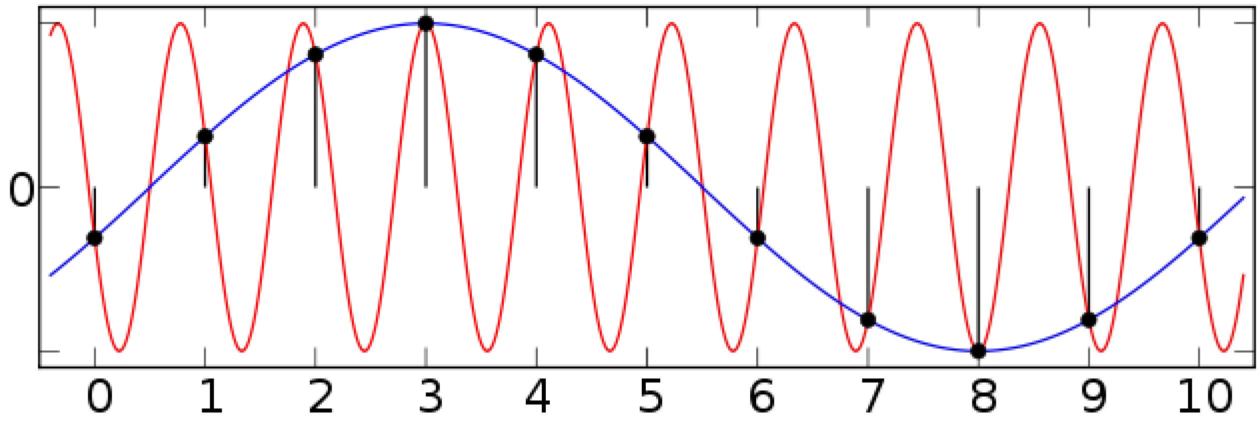
Aliasing

- Artifacts due to undersampling or poor reconstruction.



Aliasing

- Formally, aliasing is when a high frequency signal masquerades as a lower frequency signal.



Two different sinusoids that fit the same set of samples.

Discrete Fourier Transform

- The DFT requires an input function that is discrete.
- Such inputs are often created by digitally sampling a continuous function.
- Forward DFT (assume N samples from 0 to N-1):
  - $F(u)=\frac{1}{N}\sum_{x=0}^{N-1} f(x)e^{-i2\pi ux/N}$
- Inverse DFT:
  - $f(x)=\sum_{x=0}^{N-1} F(u)e^{i2\pi ux/N}$
- 2D DFT (for a NxM grid in x and y),
  - $F(u,v)=\frac{1}{NM}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y)e^{-i2\pi(\frac{ux}{N}+\frac{vy}{M})}$