EE3731C: Signal Processing Methods

Tutorial II-1

National University of Singapore

Question #1

Which of the following signals can be down-sampled by a factor of 2 using the system below without any loss of information?

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow y[n] \qquad y[n] = x[nM]$$

a) $x[n] = \delta[n - n_0]$, where n_0 is an unknown integer

b) $x[n] = \cos(\pi n/4)$

c) $x[n] = \cos(\pi n/4) + \cos(3\pi n/4)$

d) $x[n] = \dfrac{\sin(\pi n/3)}{\pi n/3}$

Question #1: Solution

Which of the following signals can be down-sampled by a factor of 2 using the system below without any loss of information?

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow y[n]$$

a) $x[n] = \delta[n - n_0]$, where n_0 is an unknown integer

What is the requirement?

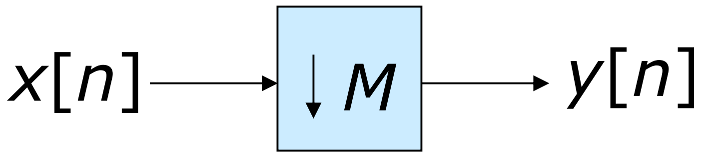
The signal should be bandlimited to $|\omega| < \pi / M$

Is the input signal bandlimited?

No!

Question #1: Solution

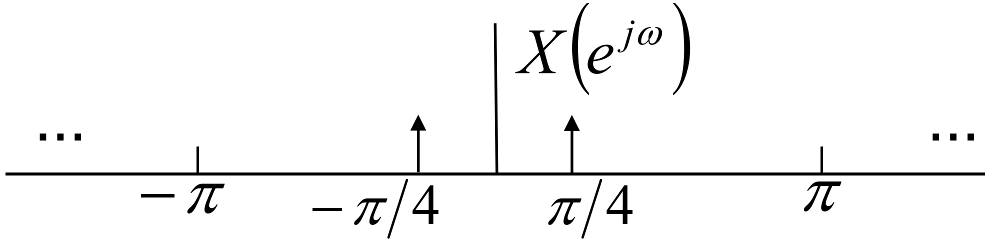
Which of the following signals can be down-sampled by a factor of 2 using the system below without any loss of information?



b) $x[n] = \cos(\pi n/4)$

Is the input signal bandlimited to $|\omega| < \pi/2$?

Yes. $x[n] = \cos(\pi n/4) = \frac{1}{2}\left[\exp(j\pi n/4) + \exp(-j\pi n/4)\right]$



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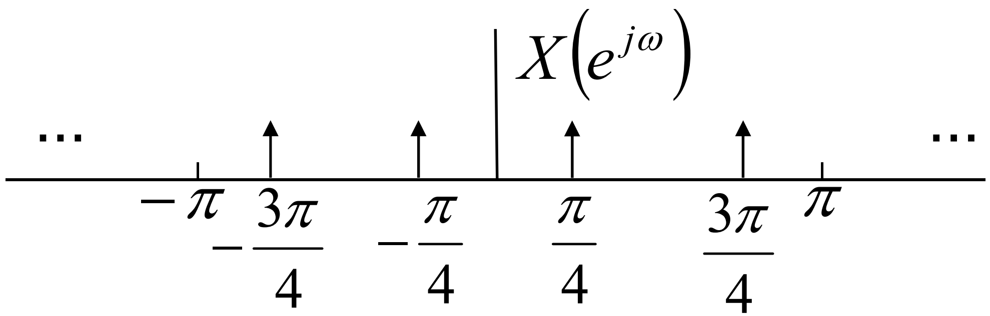

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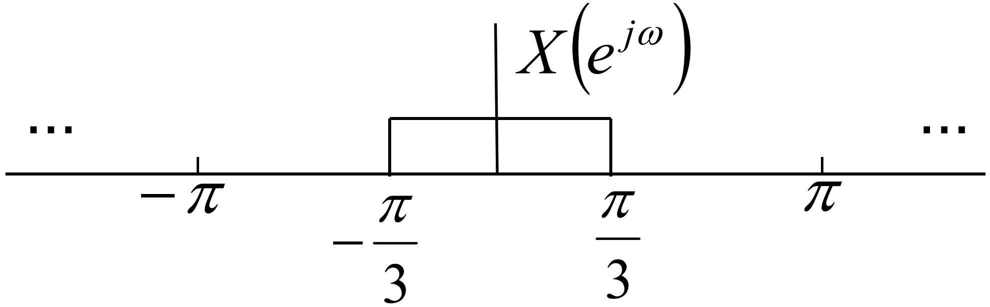


Question #2

In the multirate system shown below, $H(z)$ represents a lowpass filter with Gain = L and cutoff $\omega_c = \min(\pi/L, \pi/M)$

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow \boxed{H(z)} \longrightarrow \boxed{\downarrow M} \longrightarrow y[n]$$

Determine the corresponding output $y[n]$ for the following input signal $x[n]$ and the up-sampling and down-sampling rate of L and M.

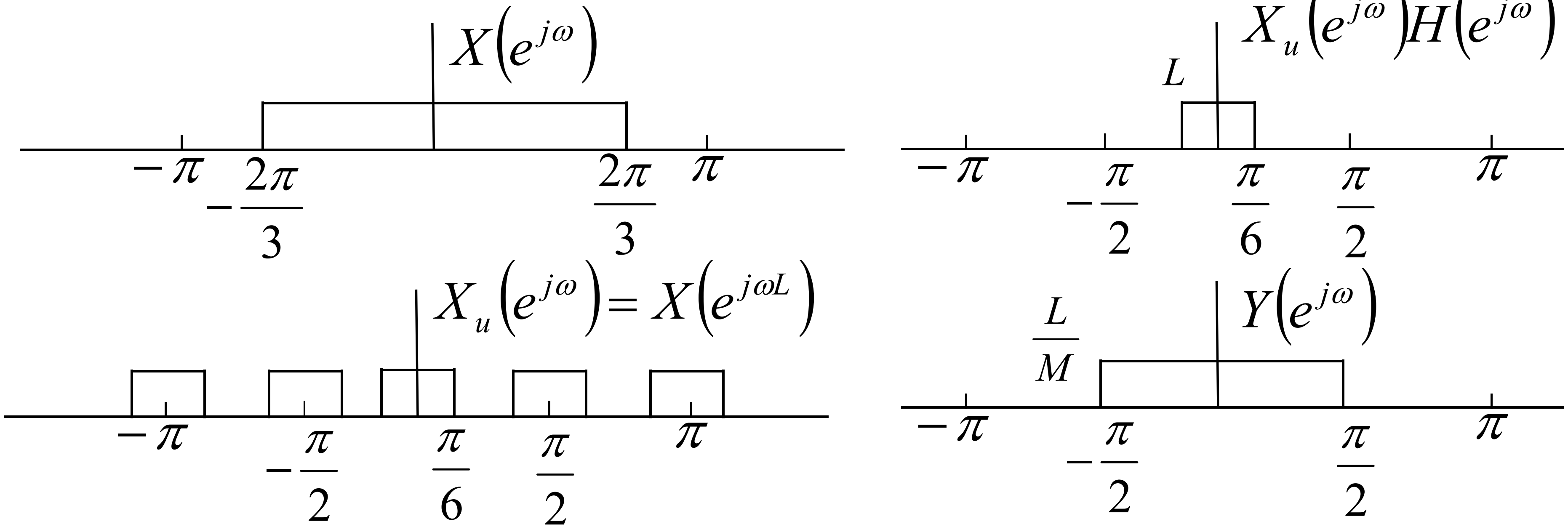
$$x[n] = \frac{\sin(2\pi n/3)}{\pi n}, \quad L = 4, \quad M = 3$$

Question #2: Solution

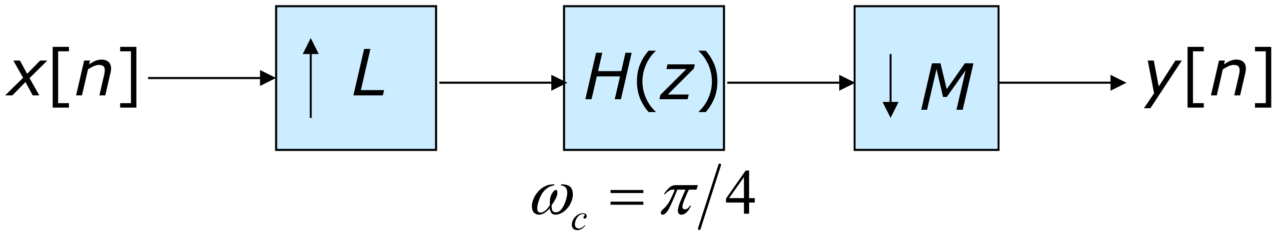
$$x[n] \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow M} \rightarrow y[n]$$

$$\omega_c = \pi/4$$

$$x[n] = \frac{\sin(2\pi n/3)}{\pi n}, \quad L = 4, \quad M = 3$$



Question #2: Solution



$$x[n] = \frac{\sin(2\pi n/3)}{\pi n}, \quad L = 4, \quad M = 3$$

Since *L*>*M*, the system will not introduce aliasing.

The sampling rate conversion factor is *L*/*M* = 4/3

$$y[n] = \frac{4}{3}\frac{\sin(\pi n/2)}{\pi n} = \frac{4\sin(\pi n/2)}{3\pi n}$$

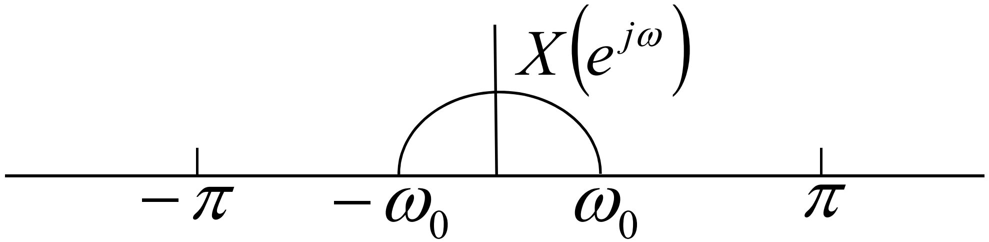
Question #3

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow \boxed{H(z)} \longrightarrow \boxed{\downarrow M} \longrightarrow y[n]$$

$H(z)$: a lowpass filter with Gain = L and cutoff

$$\omega_c = \min(\pi/L, \pi/M)$$

The Fourier transform of the input signal is given by



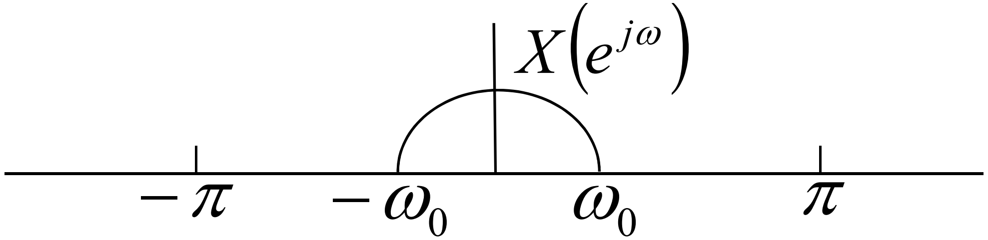
For each of the following choices of L and M, specify the maximum possible value of ω_0 such that $Y(e^{j\omega}) = aX(e^{j\omega L/M})$ for some constant a.

a) $L = 2, \quad M = 3$

b) $L = 3, \quad M = 2$

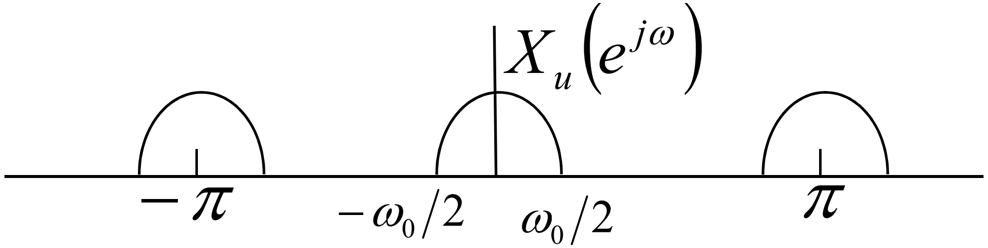
Question #3: Solution

$x[n] \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow M} \rightarrow y[n]$



$$Y(e^{j\omega}) = aX(e^{j\omega L/M})$$

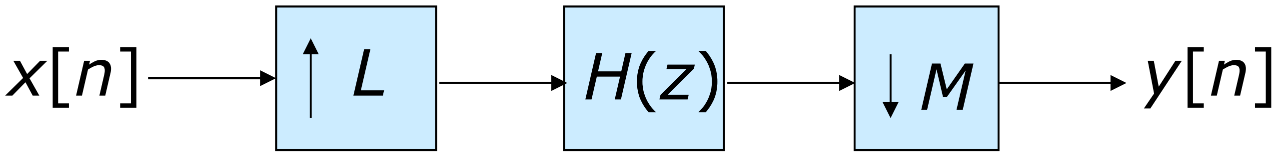
a) $L = 2, \quad M = 3$ $\qquad \omega_c = \min(\pi/M, \pi/L) = \pi/3$

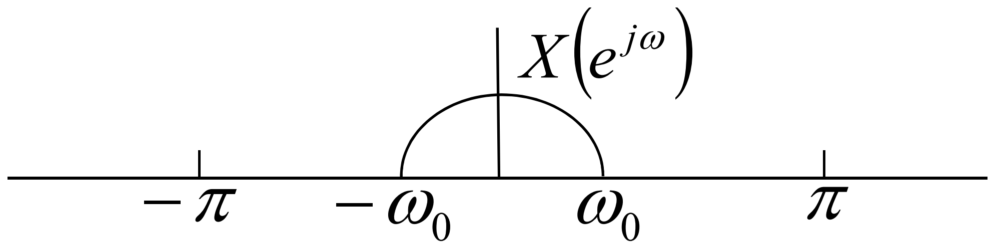


$$\frac{\omega_0}{2} \le \pi/3$$

$$\omega_0 \le 2\pi/3$$

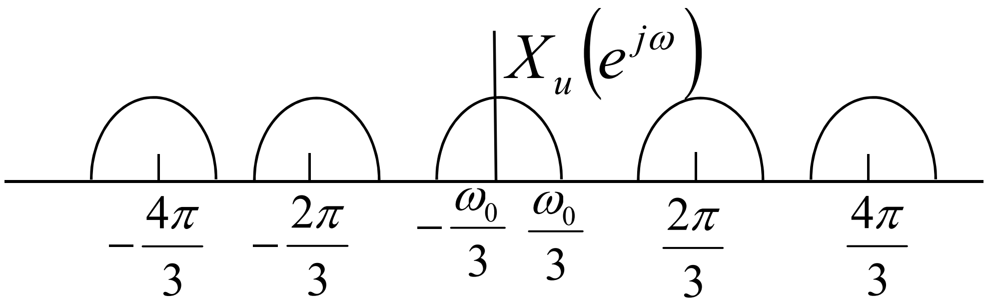
Question #3: Solution





$$Y\left(e^{j\omega}\right) = aX\left(e^{j\omega L/M}\right)$$

b) $L = 3, \quad M = 2$ $\qquad \omega_c = \min(\pi / M, \pi / L) = \pi / 3$

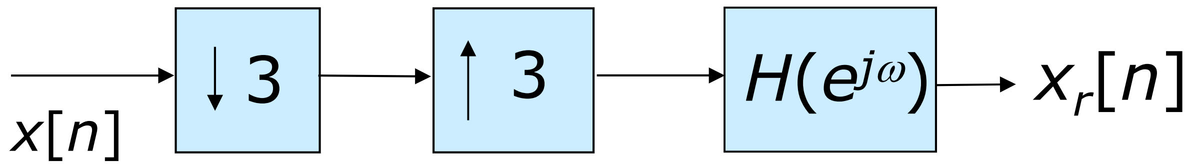


$$\frac{\omega_0}{3} \leq \pi / 3$$

$$\omega_0 \leq \pi$$

Question #4

In the system shown below,



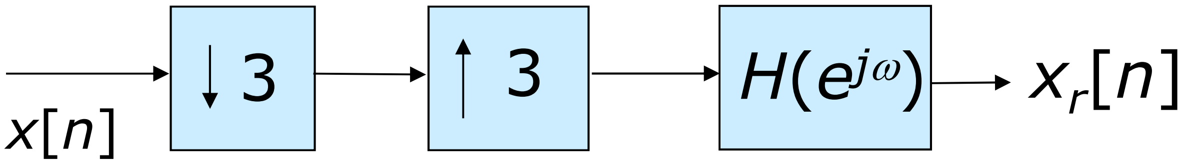
we have
$$H\left(e^{j\omega}\right)=\begin{cases}3, & |\omega|<\pi/3,\\ 0, & \pi/3\le|\omega|\le\pi.\end{cases}$$

For each of the following input signals $x[n]$, indicate whether the output $x_r[n]=x[n]$.

a) $x[n]=\cos(\pi n/4)$

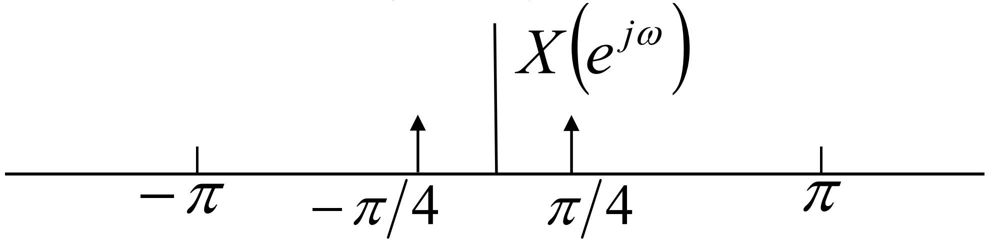
b) $x[n]=\cos(\pi n/2)$

Question #4: Solution



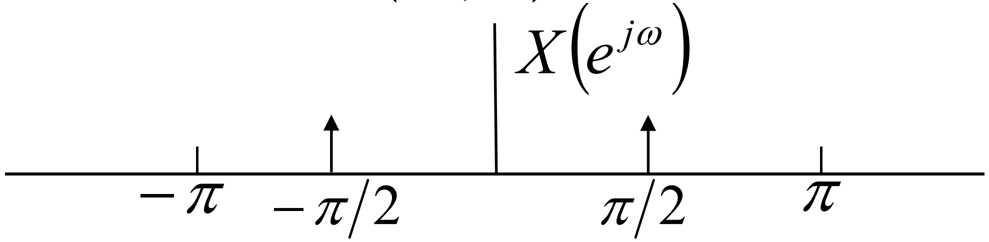
$$H\left(e^{j\omega}\right)=\begin{cases}3, & |\omega|<\pi/3,\\ 0, & \pi/3\le|\omega|\le\pi.\end{cases}$$

a) $x[n]=\cos(\pi n/4)$



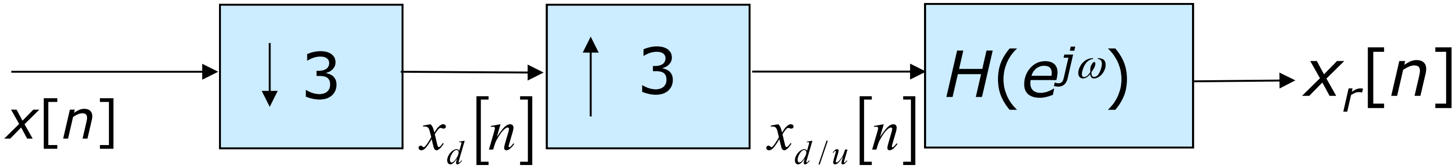
Yes. There is no aliasing because the signal is bandlimited to $|\omega|<\pi/3$

b) $x[n]=\cos(\pi n/2)$



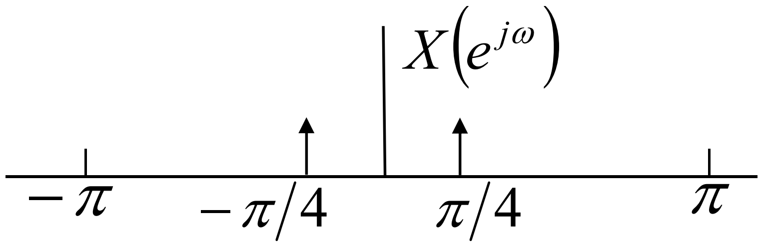
No. Aliasing occurs because the signal is not bandlimited to $|\omega|<\pi/3$

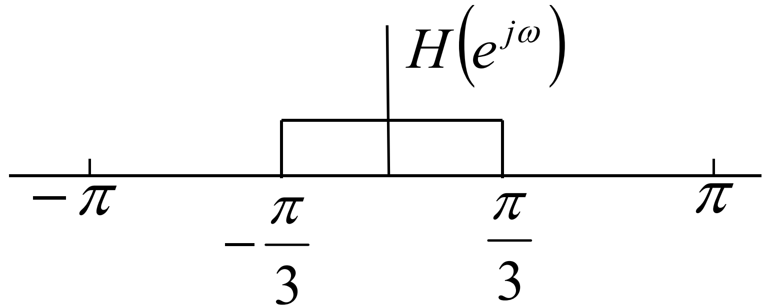
Question #4: Solution

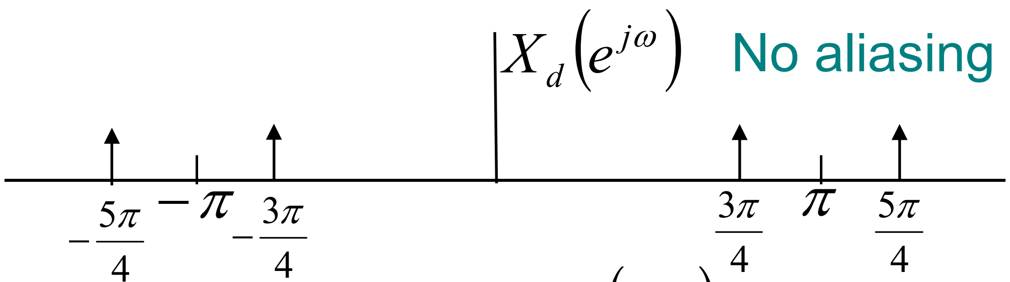


a) $x[n] = \cos(\pi n/4)$

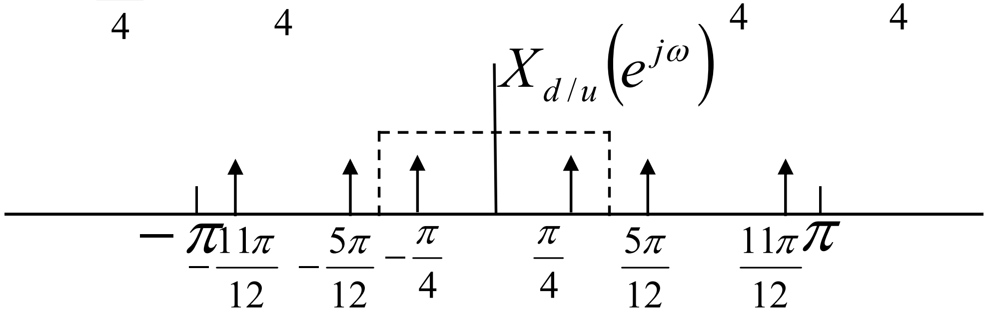
$$H(e^{j\omega}) = \begin{cases} 3, & |\omega| < \pi/3, \\ 0, & \pi/3 \le |\omega| \le \pi. \end{cases}$$

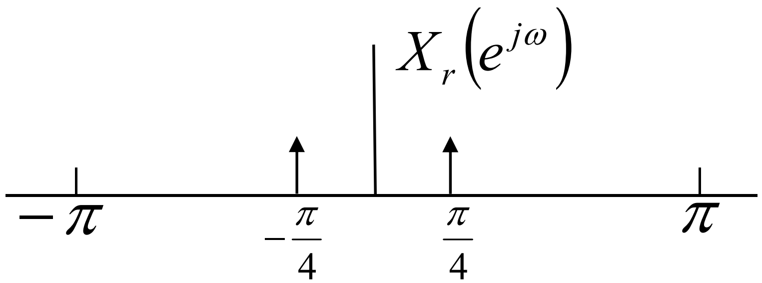






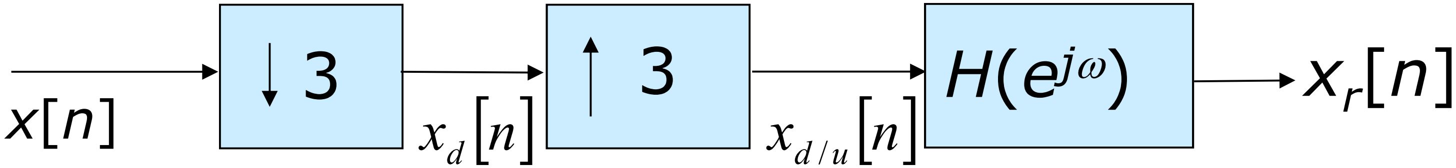
No aliasing



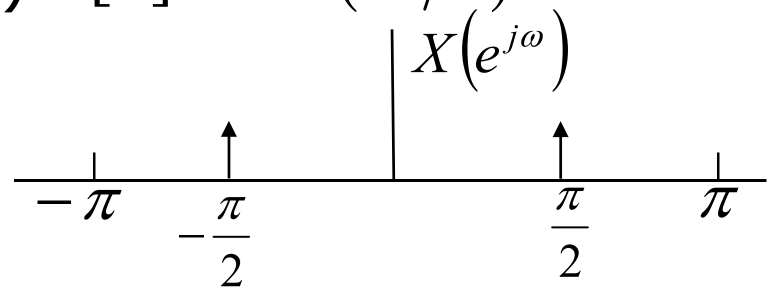


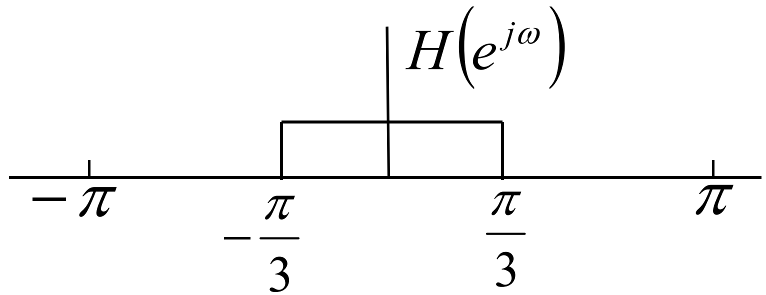
Same as input

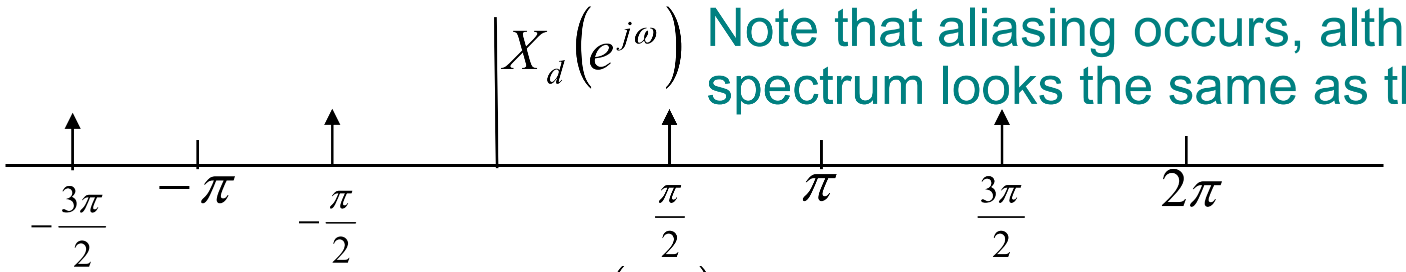
Question #4: Solution



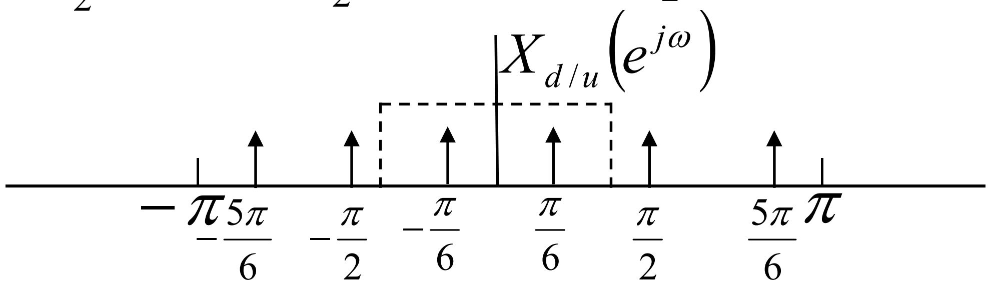
b) $x[n] = \cos(\pi n/2)$

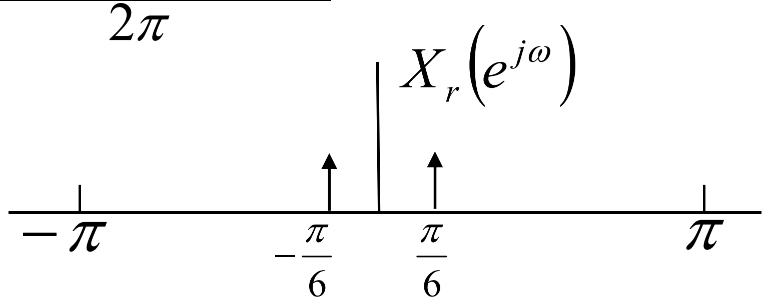






Note that aliasing occurs, although the spectrum looks the same as that of the input.

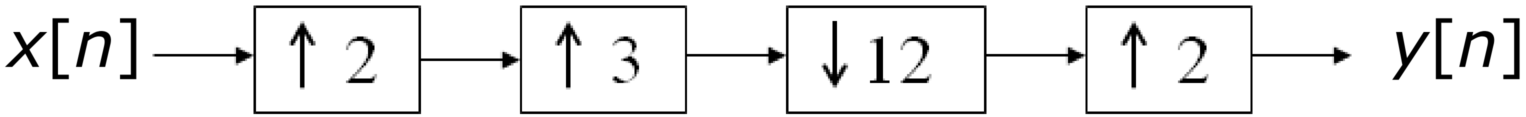




Different from input

Question #5

Consider the multirate system shown below. Find an expression for $y[n]$ in terms of $x[n]$ by simplifying the system.



Question #5: Solution

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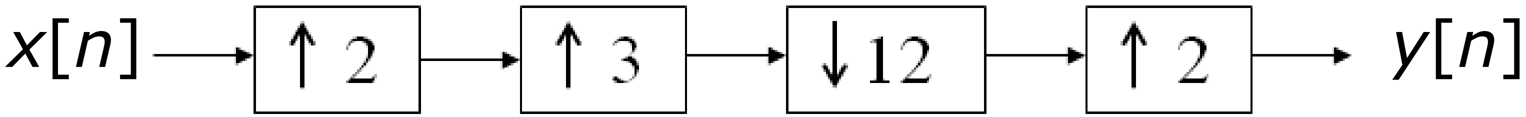
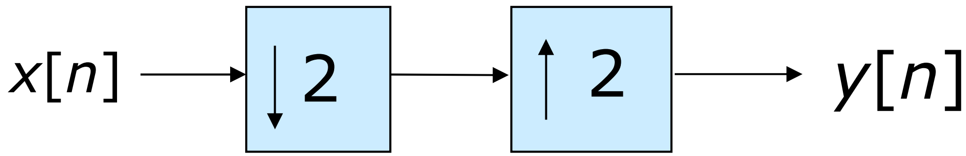


The system can be simplified to



Hence, $y[n] = x[n]$, if n is even; $y[n] = 0$, if n is odd.

Alternatively, it can be written as: $y[n] = \dfrac{1+(-1)^n}{2}x[n]$