NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2012/2013)

EE3731C – SIGNAL PROCESSING METHODS

November/December 2012 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
2. Answer all the **FOUR (4)** questions.
3. All questions carry equal marks (25 marks).
4. Please write your answers directly in the answer booklets provided.
5. This is a **CLOSED BOOK** examination
6. One A4-size formula sheet is allowed.

Q1 Given a covariance matrix

$$A = \begin{bmatrix} 82 & 34 & 10 \\ 34 & 73 & 19 \\ 10 & 19 & 97 \end{bmatrix}$$

(a) Find its three eigenvectors (hint: the eigenvalues of A are 126, 84, 42).

(15 marks)

(b) Find its first two principal components denoted as P1 and P2.

(5 marks)

(c) Suppose there are three vectors x=(1.73, 1.73, 1.73), y=(0, 0, 0), z=(1.60, 0.80, - 2.4); calculate and draw their projections on P1 and P2.

(5 marks)

Q2 The following two sub-problems are independent.

(a) Given a digital filter with pass-band from 0 Hz to 50 KHz and sampling frequency (Fs) at 1MHz. The input signal S(t)=sin(20,000πt+0.5π) and input noise = white Gaussian noise with variance as 1. Calculate the output signal variance and output noise variance. (hint: white Gaussian noise has a flattened power spectrum from -Fs/2 to Fs/2)

(10 marks)

(b) Design a process to get a high-pass digital filter with cutoff frequency as $\pi/2$, by starting from a low-pass analog filter $H(s)=\frac{1}{1+s}$. For each digital filter, calculate the zero(s), and check whether it is stable. The possibly useful formulas are:

i) Bilinear transformation:

$$s \to k\frac{1-z^{-1}}{1+z^{-1}}, \quad \Omega = k\tan\left(\frac{\omega}{2}\right)$$

ii) Lowpass (θ) → highpass (ϕ) transformation

$$z^{-1} \to -\frac{z^{-1}+\alpha}{1+\alpha z^{-1}}, \quad \alpha = -\frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}$$

(15 marks)

Q3

(a) Consider two multirate systems shown in Fig. Q3a-1.

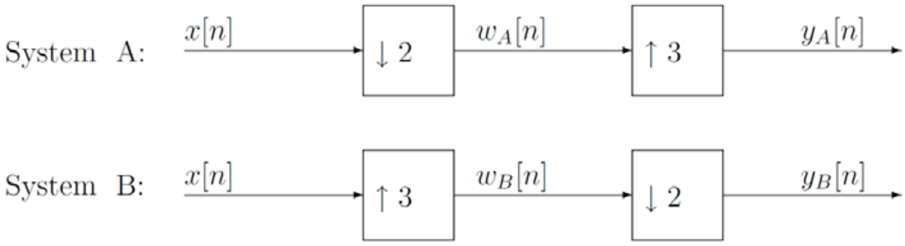

Fig. Q3a-1

i) For an input signal $x[n]$ whose Fourier transform $X(e^{j\omega})$ is shown in Fig. Q3a-2, sketch $W_A(e^{j\omega})$ and $Y_A(e^{j\omega})$, i.e., the Fourier transforms of $w_A[n]$ and $y_A[n]$.

(6 marks)

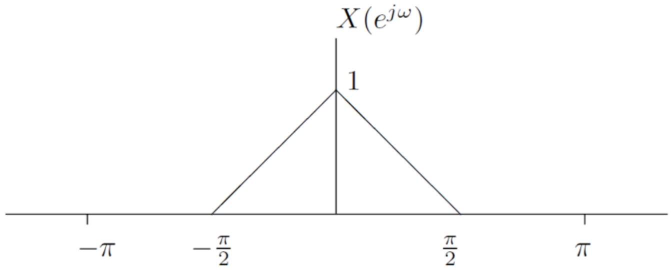

Fig. Q3a-2

ii) Let $X(e^{j\omega})$ denote the Fourier transform of an arbitrary input $x[n]$. Express $Y_B(e^{j\omega})$ in terms of $X(e^{j\omega})$. Your answer should be in the form of an equation, not a sketch.

(6 marks)

iii) For the same input $x[n]$, what is the relationship between the two outputs $y_A[n]$ and $y_B[n]$? Use one sentence to justify you answer.

(3 marks)

(b) Now consider the multirate system shown in Fig. Q3b. Find an expression for $y[n]$ in terms of $x[n]$ by simplifying the system.

(5 marks)

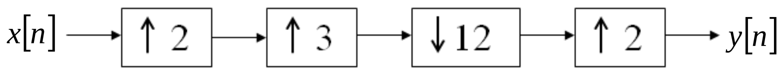

Fig. Q3b

(c) You are asked to convert the sampling rate of an audio signal from 44.1 kHz to 48 kHz using the system shown in Fig.Q3c. Specify your choices of M and L, as well as, the gain and cutoff of the lowpass filter $H(z)$.

(5 marks)

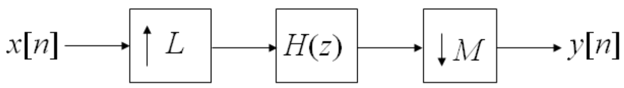

Fig. Q3c

Q4

(a) A zero mean noise process $x[n]$ has the following autocorrelation function

$$R_x(m) = \left(\frac{1}{2}\right)^{|m|}$$

We process $x[n]$ with a linear time-invariant system that satisfies the following difference equation:

$$y[n] = x[n+1] + x[n-1].$$

i) What is the autocorrelation function $R_y[m]$ of the process $y[n]$?

(6 marks)

ii) What is the power spectral density $S_y\left(e^{j\omega}\right)$ of the process $y[n]$?

(Hint: $a^n u[n] \leftrightarrow \frac{1}{1-az^{-1}}$, $|a|<1; -a^n u[-n-1] \leftrightarrow \frac{1}{1-az^{-1}}$, $|a|>1$)

(6 marks)

(b) Consider the power spectral density function given by:

$$S_x\left(e^{j\omega}\right) = 5 + 4\cos(\omega)$$

i) Find the corresponding autocorrelation function $R_x[m]$.

(4 marks)

ii) Find a linear filter whose output process has the same autocorrelation function $R_x[m]$, when excited by white noise of zero mean and unit variance.

(6 marks)

iii) What is the variance of the output process?

(3 marks)

END OF PAPER