EE3731C:
Signal Processing Methods

Tutorial II-3

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Question #1

A one dimensional random walk starts at $S_0 = 0$ and at each step moves by ± 1 with equal probability. To define this walk formally, take independent random variables $Z_1, Z_2, \ldots,$ each of which is 1 with probability 1/2 and −1 with probability 1/2, and set

$$S_n = Z_1 + Z_2 + \ldots + Z_n$$

This sequence $\{S_n\}$ is called the simple random walk on the integers.

(i) Find $E[S_n]$ and $E[S_n^2]$.

(ii) What is the root mean square displacement of the walk after n steps?

Question #2

Let $w[n]$ denote a Gaussian white noise sequence with mean zero and variance 1. Determine the mean and autocorrelation functions of $x[n]$ in the following cases.

(i) $x[n] = 0.8\, x[n-1] + 0.5\, w[n]$

(ii) $x[n] = 0.4w[n] - 0.1w[n-1]$

Hints: Since $x[n]$ is generated by the same rule at all times, $x[n]$ will be stationary. When $x[n-1]$ is a function past inputs and past noise values, it is independent of $w[n]$ because the noise is independent.

Question #3

Let $w[n]$ denote a Gaussian white noise sequence with mean zero and variance 1. Find the power spectrum of the random process $x[n]$ in the following cases.

(i) $x[n] = 0.8\,x[n-1] + 0.5\,w[n]$

(ii) $x[n] = 0.4w[n] - 0.1w[n-1]$

Question #4

Let a WSS random process $x[n]$ be the input to an LTI system with known impulse response $h[n]$. Suppose that the input $x[n]$ is not observable, but we can measure the output $y[n]$.

i. Propose an estimate of the power spectrum of $y[n]$ (i.e., describe the processing you would apply to the observed output y[n] to estimate the power spectrum $S_y(e^{j\omega})$). Hint: Consider using the DFT to estimate the "power" of the output at each frequency.

ii. How would you use the estimate of $S_y(e^{j\omega})$ to estimate the power spectrum of the unobservable input $x[n]$?

Question #5

A zero mean noise process $x[n]$ has the following autocorrelation function

$$R_x(m) = \left(\frac{1}{2}\right)^{|m|}$$

We process $x[n]$ with a linear time-invariant system that satisfies the following difference equation:

$$y[n] = x[n+1] + x[n-1]$$

What is the autocorrelation function of $y[n]$?