EE3731C: Signal Processing Methods

Tutorial II-2

National University of Singapore

Question #1

For each of the following random processes, determine whether X(t) is wide sense stationary and ergodic.

a) $X(t) = \cos(2\pi f t + \theta)\ , \quad \theta \sim U[-\pi, \pi]$

Note that θ remains constant for each single realization.

b) X(*t*) = *A* for all *t*, where *A* is a zero-mean random variable.

Question #1: Solution

a) $X(t) = \cos(2\pi ft + \theta)\ , \quad \theta \sim U[-\pi, \pi]$

$$E\{x[t]\} = E\{\cos(2\pi ft + \theta)\} = \int_{-\pi}^{\pi} \cos(2\pi ft + \theta)\frac{1}{2\pi}d\theta = 0$$

$$R_x[t,\tau] = E\{\cos(2\pi ft + \theta)\cos(2\pi f(t+\tau) + \theta)\}$$

$$= \int_{-\pi}^{\pi} \cos(2\pi ft + \theta)\cos(2\pi f(t+\tau) + \theta)\frac{1}{2\pi}d\theta$$

$$= \int_{-\pi}^{\pi} \cos(2\pi ft + \theta)\cos(2\pi f(t+\tau) + \theta)\frac{1}{2\pi}d\theta$$

$$= \frac{1}{2}\cos(2\pi f\tau)$$

$$\boxed{\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}}$$

Question #1: Solution

For each of the following random processes, determine whether X(t) is wide sense stationary and ergodic.

a) $X(t) = \cos(2\pi f t + \theta)\ , \quad \theta \sim U[-\pi, \pi]$

WSS and ergodic.

b) X(*t*) = *A* for all *t*, where *A* is a zero-mean random variable.

WSS but not ergodic.

Question #2

Find the mean and autocorrelation functions of the following process:

$x[n] = A\cos(\omega n + \phi)$ with random phase ϕ, where A and ω are positive constants and $\phi \sim U(0, 2\pi)$.

Question #2: Solution

$x[n] = A\cos(\omega n + \phi)$ with random phase ϕ, where A and ω are positive constants and $\phi \sim U(0, 2\pi)$.

$$E\{x[n]\} = E\{A\cos(\omega n + \phi)\} = \int_0^{2\pi} A\cos(\omega n + \phi)\frac{1}{2\pi}\,d\phi = 0$$

$$R_x[m] = E\{A\cos(\omega n + \phi)A\cos(\omega(n+m) + \phi)\}$$

$$= \int_0^{2\pi} A^2 \cos(\omega n + \phi)\cos(\omega(n+m) + \phi)\frac{1}{2\pi}\,d\phi$$

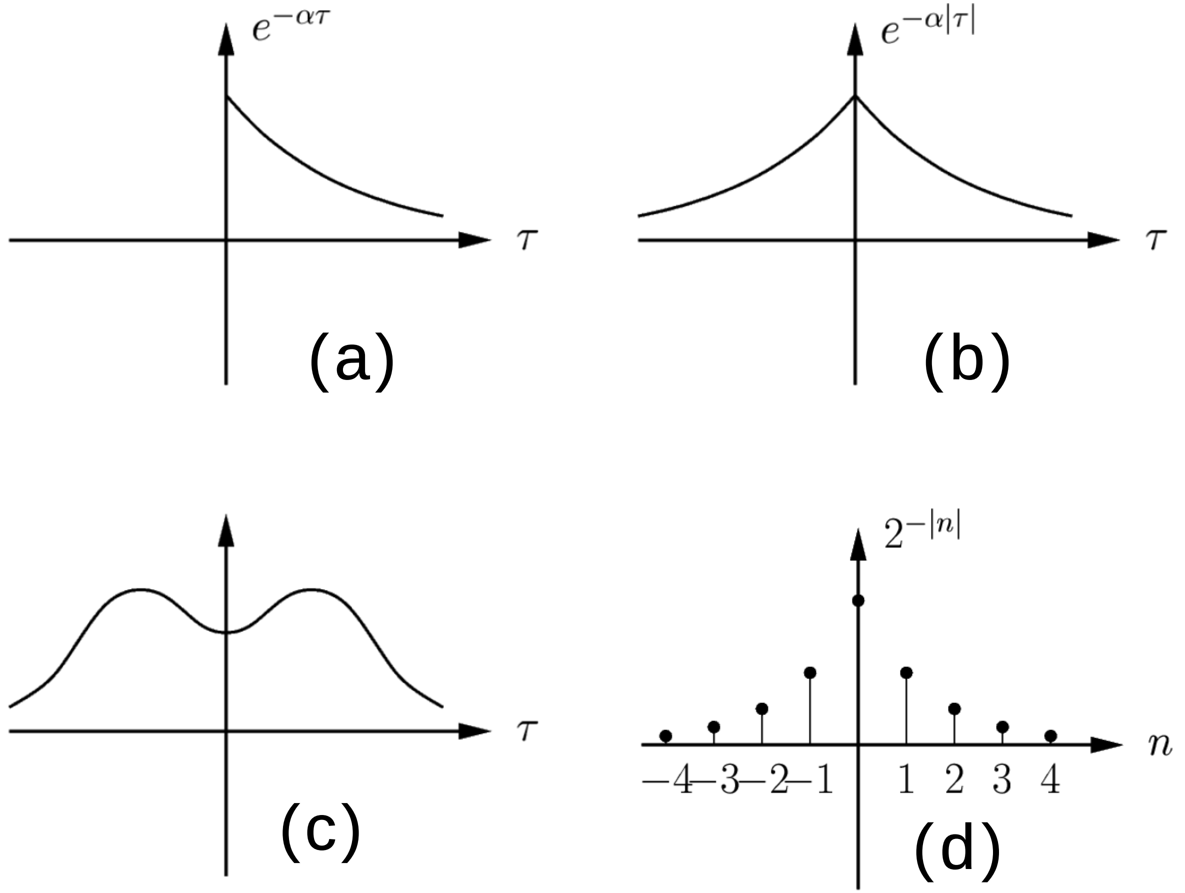
$$= \frac{A^2}{2\pi}\int_0^{2\pi} \cos(\omega n + \phi)\cos(\omega(n+m) + \phi)\,d\phi$$

$$= \frac{A^2}{2}\cos(\omega m)$$

$$\boxed{\cos(\omega n + \phi) = \frac{e^{j(\omega n + \phi)} + e^{-j(\omega n + \phi)}}{2}}$$

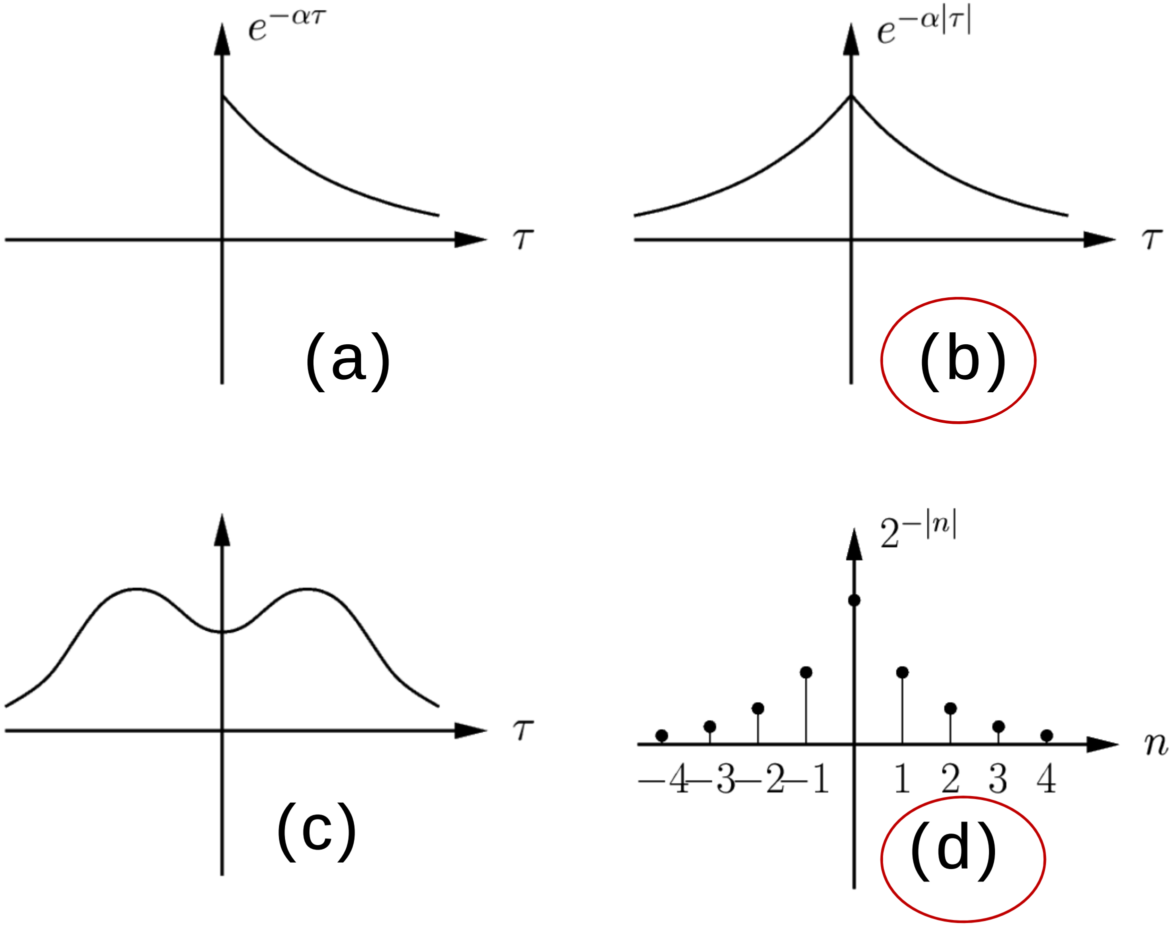
Question #3

Which of the followings are autocorrelation functions of WSS processes?



Question #3: Solution

Based on the properties of the autocorrelation function: 1) even; and 2) peaks at zero.



Question #4

$x[n]$ is Gaussian i.i.d., that is, $x[n]$ are sampled independently and identically from the same Gaussian distribution with zero mean and variance σ^2.

Let $y[n] = x[n] + x[n-1]$. Find the cross-correlation function between $x[n]$ and $y[n]$, as well as, the autocorrelation function of $y[n]$.

Question #4: Solution

The autocorrelation function of $x[n]$ is:

$$R_x[m] = E\{x[n]x[n+m]\} = \begin{cases} \sigma^2 & m = 0 \\ 0 & m \neq 0 \end{cases} \qquad R_x[m] = \sigma^2 \delta[m]$$

The cross-correlation function between $x[n]$ and $y[n]$

$$R_{xy}[m] = E\{x[n]y[n+m]\} = E\{x[n](x[n+m] + x[n+m-1])\}$$

$$R_{xy}[m] = \sigma^2(\delta[m] + \delta[m-1])$$

The autocorrelation function of $y[n]$

$$\begin{aligned} R_y[m] &= E\{y[n]y[n+m]\} \\ &= E\{(x[n] + x[n-1])(x[n+m] + x[n+m-1])\} \\ &= \sigma^2(\delta[m-1] + 2\delta[m] + \delta[m+1]) \end{aligned}$$