# EE3731C – Signal Processing Methods

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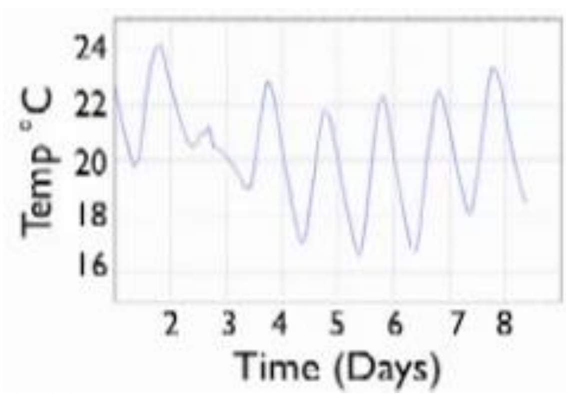
# Recap – What is Signal and Signal Processing

# Signal

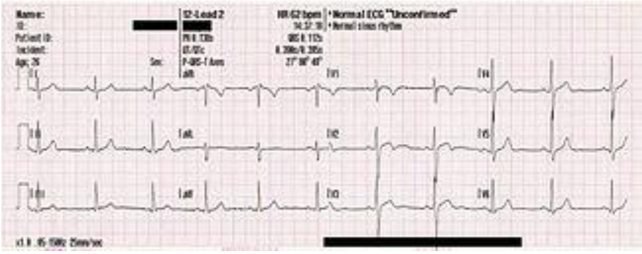
- A function that conveys information about the behavior or attributes of some phenomenon.
  - Can be any quantity exhibiting variation in time or variation in space.

# Signal

- The term “signal” includes, among others, audio, video, speech, image, communication, geophysical, sonar, radar, medical and musical signals.

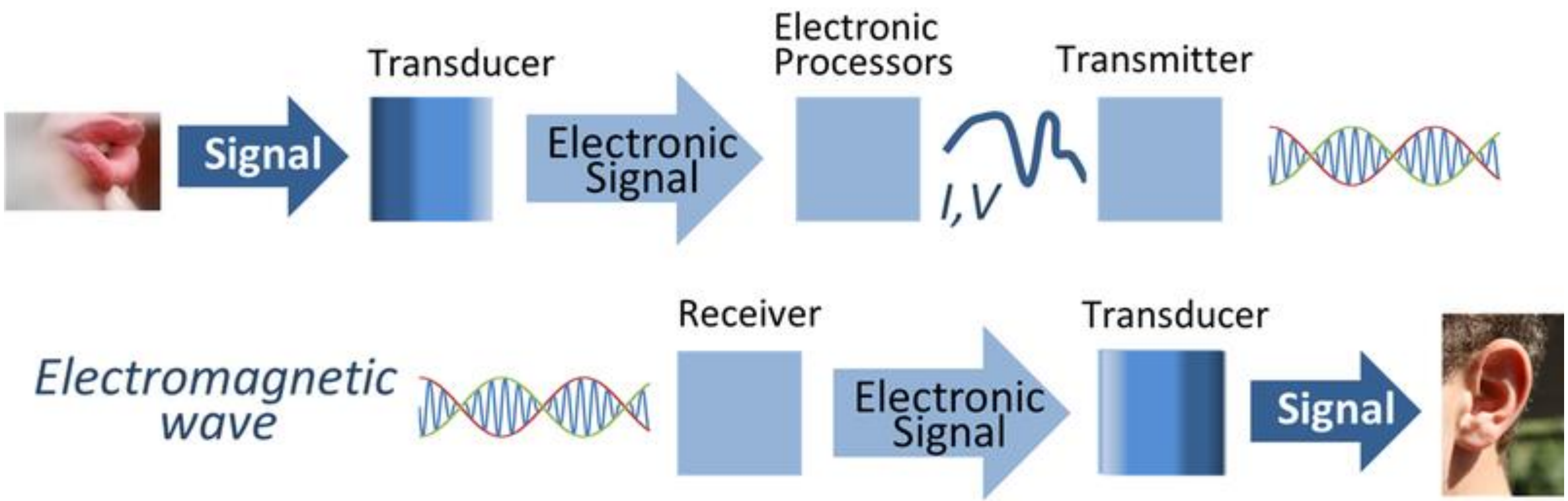






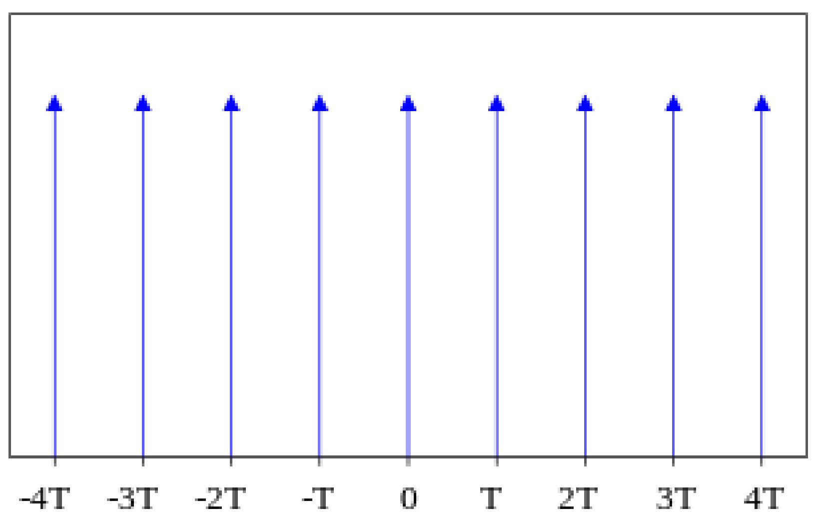
# Signal Processing

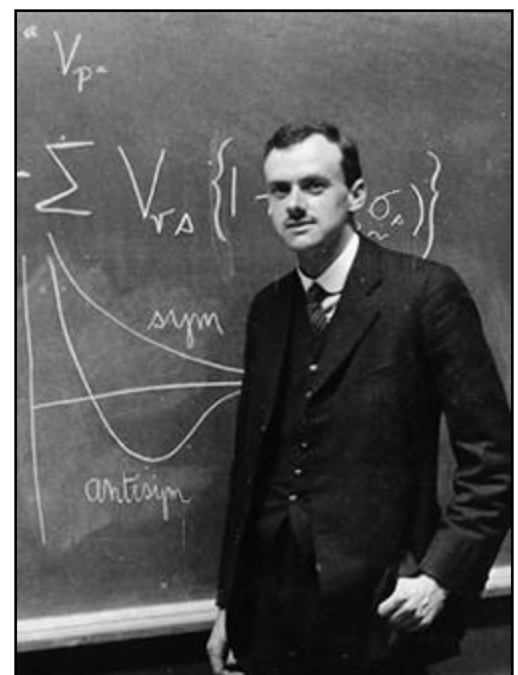
- Deals with operations on or analysis of signals, or measurements of time-varying or spatially varying physical quantities.



# Digital Filtering

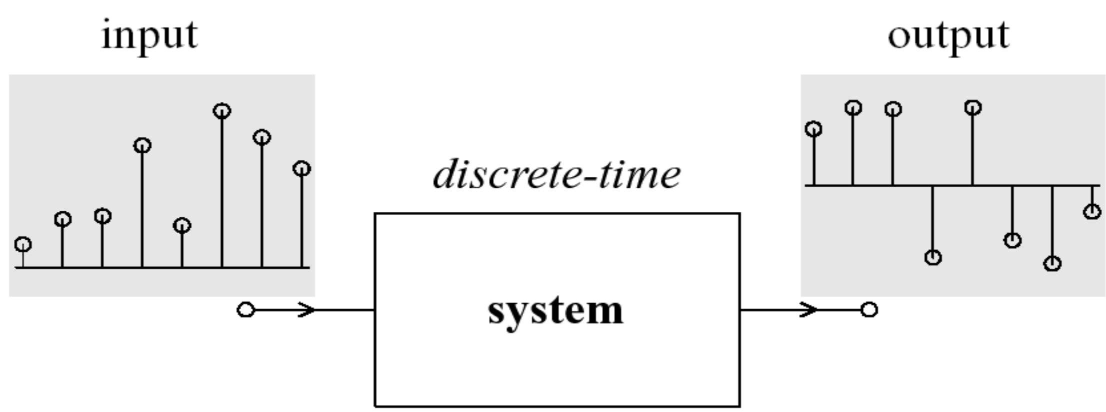
- Digital filtering
  - Discrete sampling of continuous signals
- Mathematically: sampling = multiplying by the Dirac Comb (impulse train, or sampling function)



# Discrete Time System

- Discrete time system
  - Given a discrete input series, what we need to build is a system which takes the input signal and alters it such as the output has some more desired properties.




- The most useful class of systems to do that are digital filters.

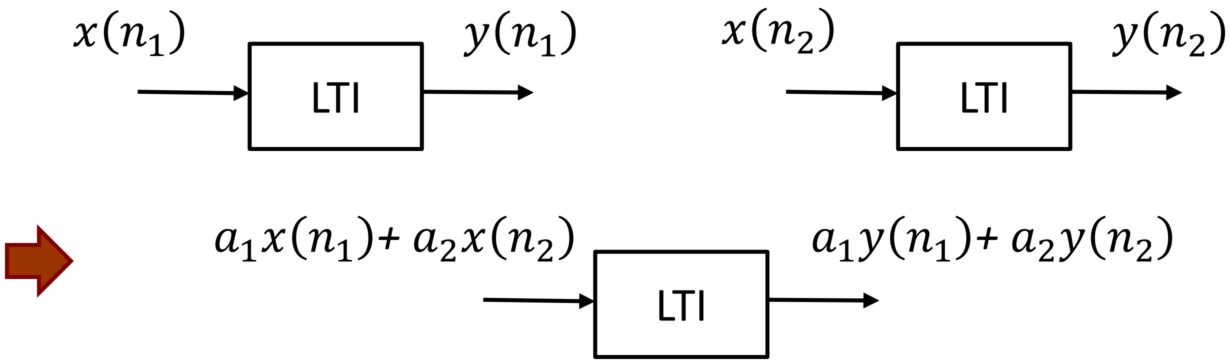
# Filter Operations

*The name "filter" is used because these signal-processing elements typically "pass" or amplify certain frequency components of the signal, while they "stop" or attenuate others.*

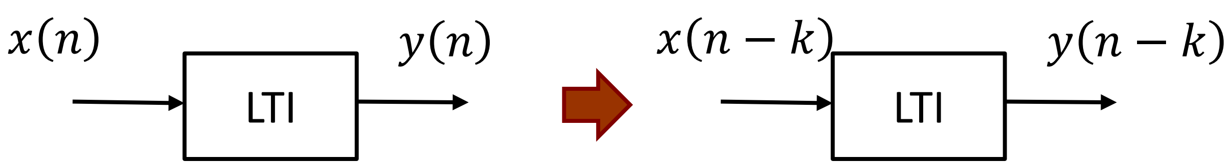
# Linear Time-Invariant (LTI) Systems

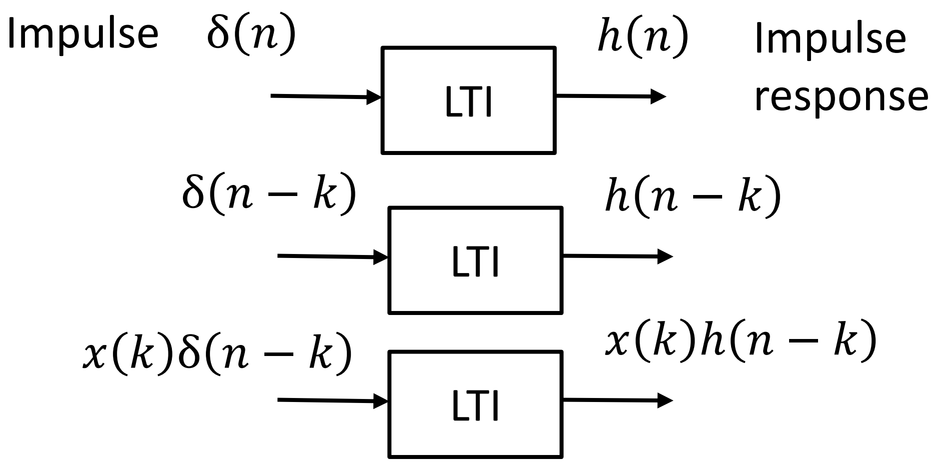
An important category of discrete-time systems are those which have the properties of

- **Linearity** (principle of superposition and scaling)




- **Time-invariance**






By superposition

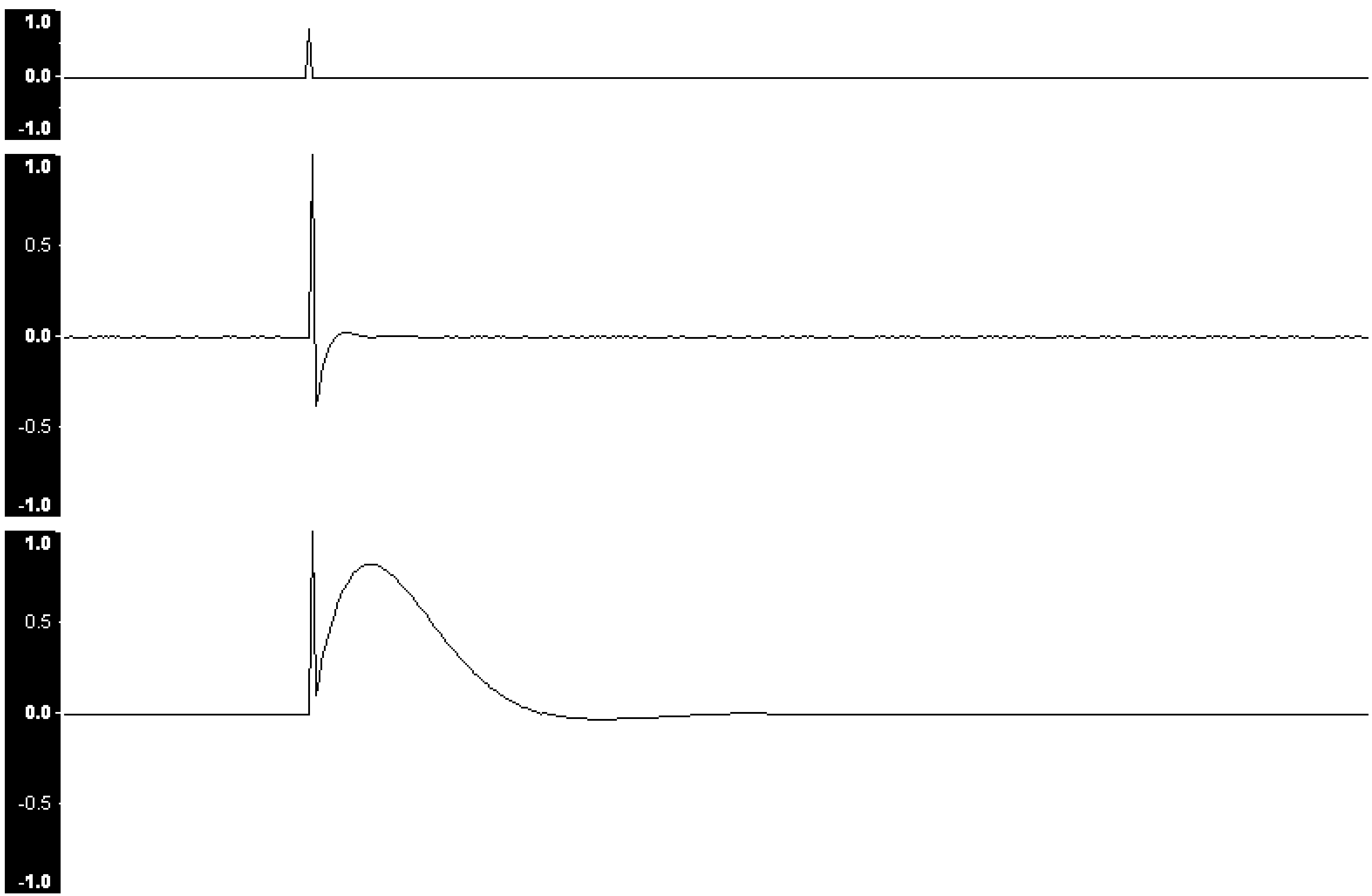
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) * h(n)$$

**Discrete Convolution**

The **impulse** can be modeled as

- a Dirac delta function for continuous-time systems, or
- the Kronecker delta for discrete-time systems.

The **impulse response** of a dynamic system is its output when presented with an impulse.


1.0
0.0
-1.0
1.0
0.5
0.0
-0.5
-1.0
1.0
0.5
0.0
-0.5
-1.0

# Finite Impulse Response (FIR)

- A filter whose impulse response is of *finite* duration, because it settles to zero in finite time.
  - The impulse response of an Nth-order discrete-time FIR filter (i.e., with a Kronecker delta impulse input) lasts for $N + 1$ samples, and then settles to zero.
  - FIR filters can be discrete-time or continuous-time, and digital or analog.

- The output *y* of a LTI system is determined by convolving its input signal *x* with its impulse response *h*.

- Formally, for a discrete-time FIR filter,

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{N} x(k)h(n-k) \\
&= x(0)h(n) + x(1)h(n-1) + \cdots + x(N)h(n-N)
\end{aligned}
$$

  - $x(n)$: input signal
  - $y(n)$: output signal
  - $h(k)$: filter coefficients
  - $N$: filter order

# Infinite Impulse Response

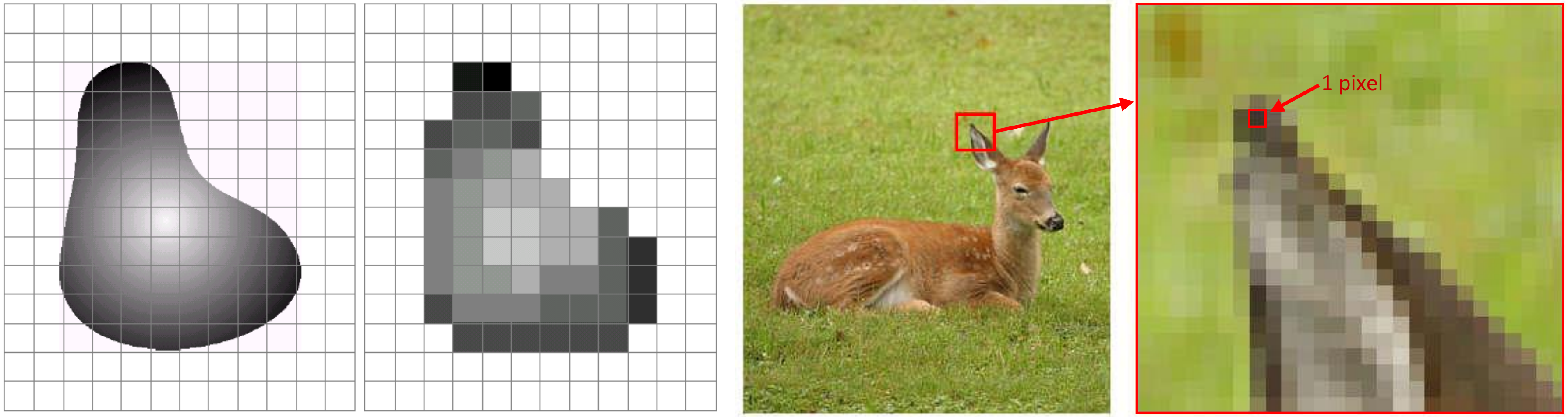
- Having an impulse response which does not become exactly zero past a certain point, but continues indefinitely.

# 2D Filtering and Applications to Image Processing

***"One picture is worth more than ten thousand words"***

# What is a Digital Image?

- A **digital image** is a representation of a two-dimensional image as a finite set of digital values, called picture elements or pixels.



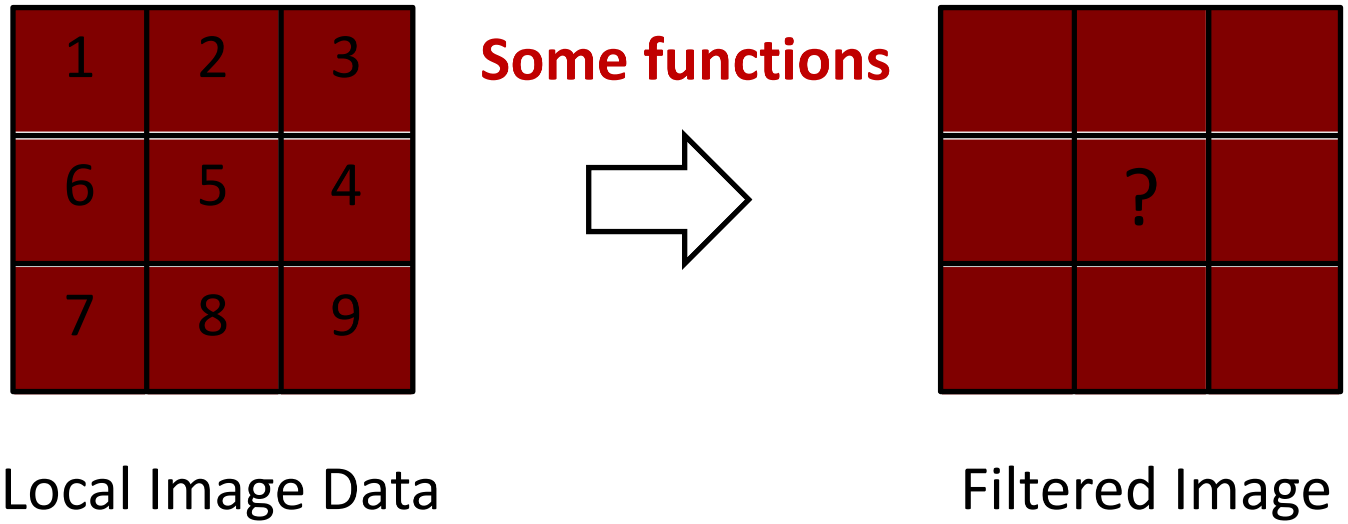

*Digitization* implies that a digital image is an *approximation* of a real scene.

# Image Formats

- Common image formats include:
  - 1 sample per pixel (B&W or Grayscale)
  - 3 samples per pixel (Red, Green, and Blue)

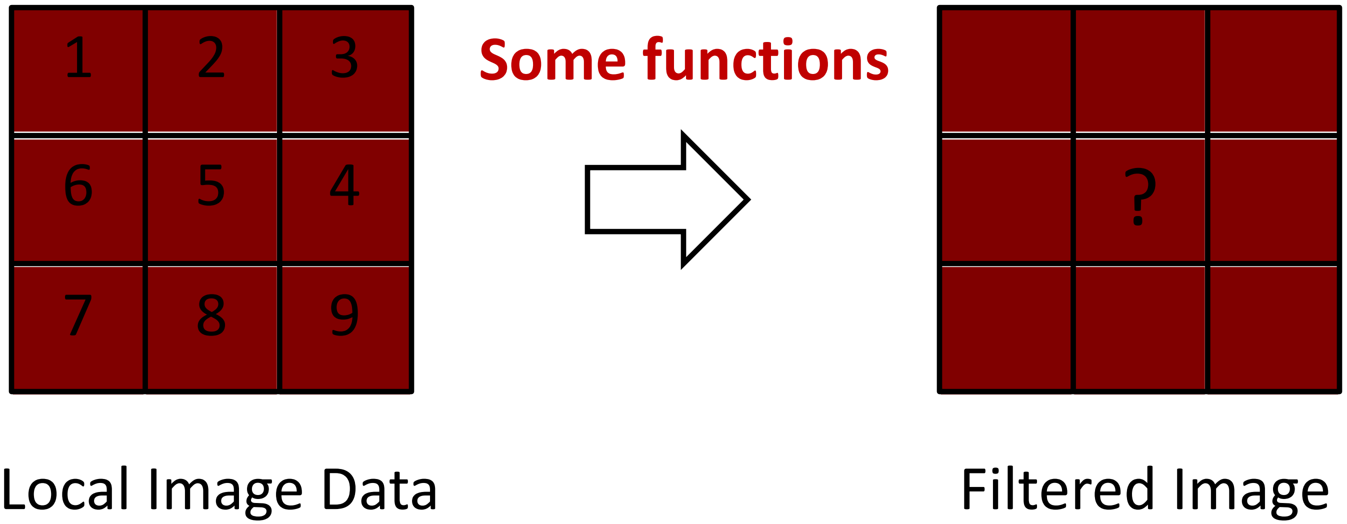
# What is Image Filtering?

- Modify the pixels in an image based on some function of a local neighborhood of the pixels.



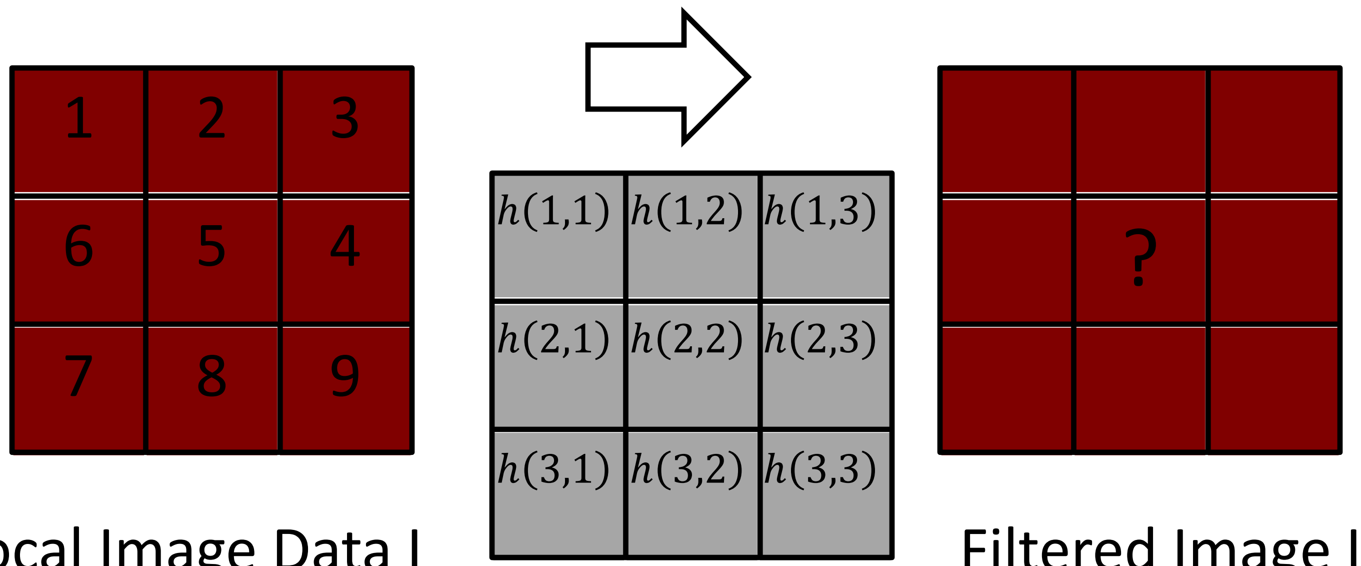
# Linear Functions

- Linear filtering is the simplest one
  - Replacing each pixel by a linear combination of its neighbors



# Convolution

- The prescription for the linear combination is called the "convolution kernel".

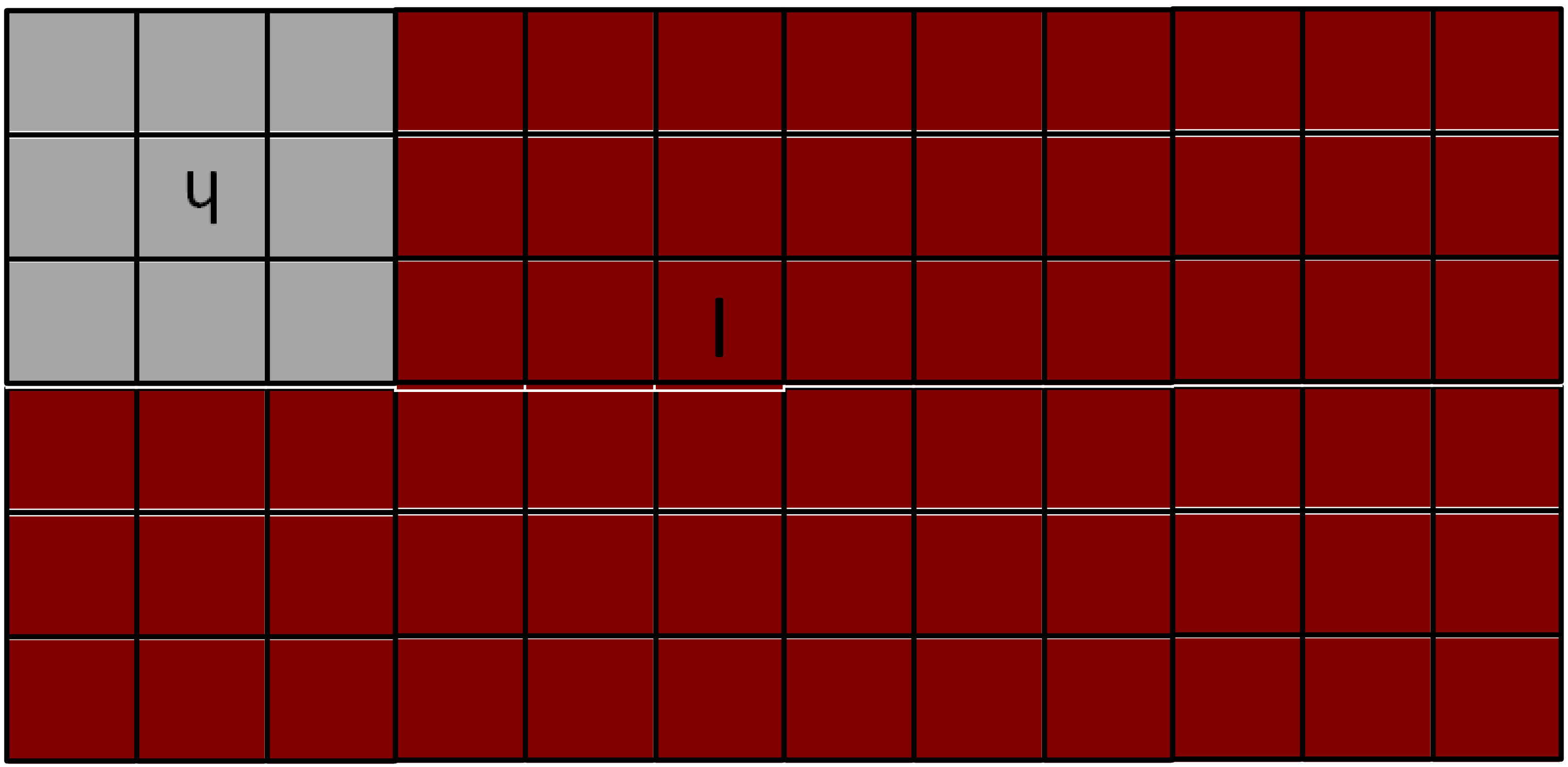



Local Image Data I

Convolution Kernel

Filtered Image I'

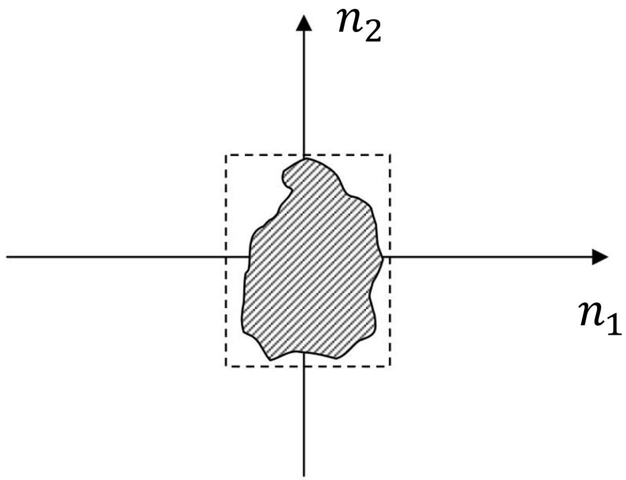
$$
\begin{aligned}
I'(i,j) = {} & h(1,1)I(i-1,j-1) + h(1,2)I(i-1,j) + h(1,3)I(i-1,j+1) + \\
& h(2,1)I(i,j-1) + h(2,2)I(i,j) + h(2,3)I(i,j+1) + \\
& h(3,1)I(i+1,j-1) + h(3,2)I(i+1,j) + h(3,3)I(i+1,j+1)
\end{aligned}
$$



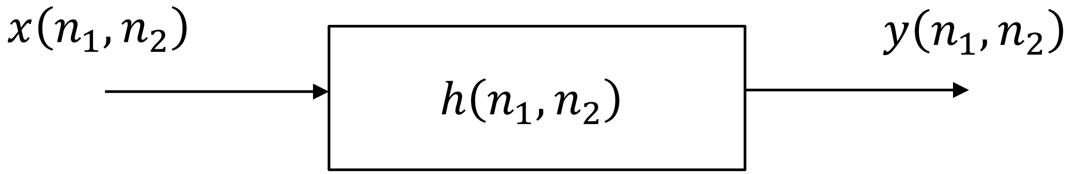

$$I'(i,j) = I * h = \sum_{k,l} I(i-k, j-l)h(k,l)$$

# Impulse Response in 2D

- An FIR filter has an impulse response $h(n_1, n_2)$ with finite support

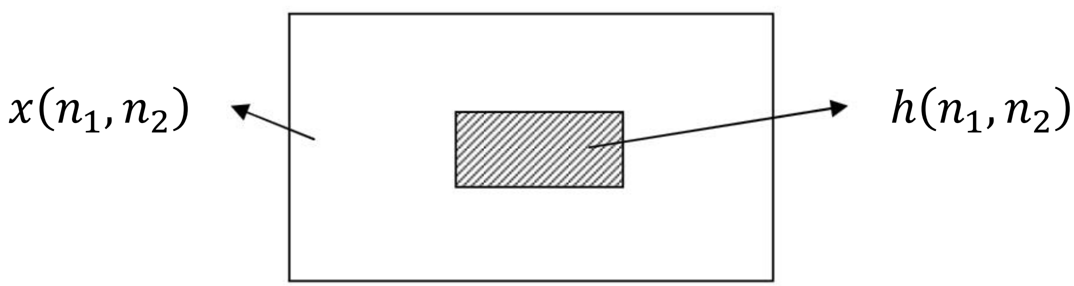



$h(n_1, n_2)$ has only nonzero samples in region of support and it is zero outside




2D: $y(n_1, n_2) = \sum_{k_1=0}^{N_1} \sum_{k_2=0}^{N_2} x(k_1, k_2) h(n_1 - k_1, n_2 - k_2)$

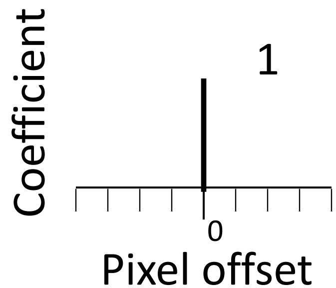
(1D: $y(n) = \sum_{k=0}^{N} x(k) h(n-k)$)




2D convolution provides a way to implement a 2D FIR filter (direct implementation).

# Filtering Examples - Identical

Original

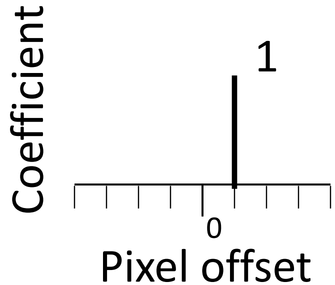



| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

Filtered

# Filtering Examples - Shifting

Original

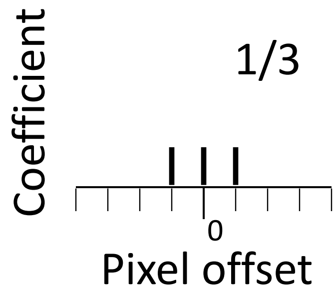



| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Filtered

# Filtering Examples - Blurring

Original

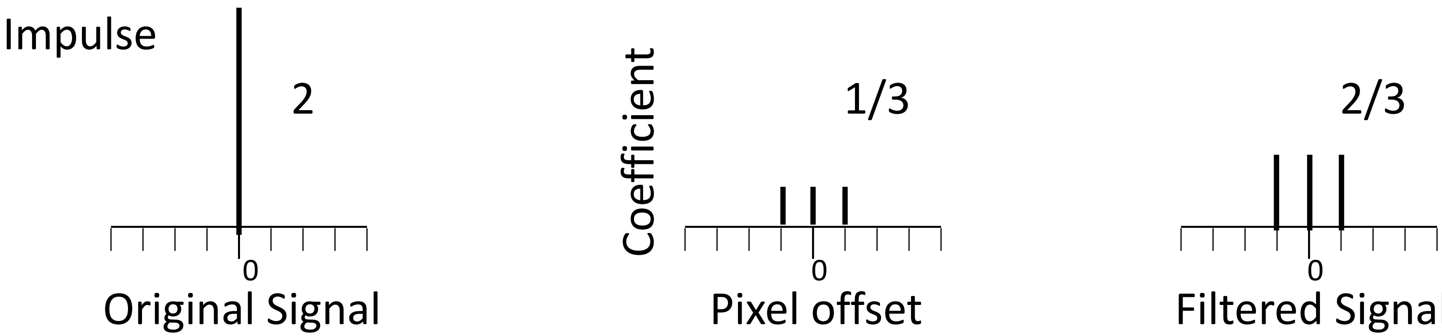



(1/9)

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Filtered

# Filtering Examples - Blurring



# More Filters and Operations…

- Low-pass vs. High-pass
- Mean, Median vs. Gaussian
- ……
- Smoothing
- Sharpening
- Denoising
- Edge detection
- ……

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Some of the materials are from the Internet and the textbooks