

Consider the following 2nd order D.E.

$$y'' + p(x)y' + q(x)y = r(x) \quad \text{--- (1)}$$

General soln :-

$$y = y_h + y_p$$

① For the homogeneous soln :-

Solve the characteristic equation of (1)

$$\Rightarrow \lambda^2 + a\lambda + b = 0 \quad \text{if } p(x) = a, q(x) = b$$

Case	Type of Roots	General Soln
I	Distinct real λ_1, λ_2	$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$
II	Double $-\frac{1}{2}a$	$y = (C_1 + C_2 x) e^{-ax/2}$
III	Complex $-\frac{1}{2}a \pm i\omega$	$y = e^{-ax/2} (A \cos \omega x + B \sin \omega x)$

② For the particular soln :-

Term in $r(x)$	Choice of y_p
$k e^{\gamma x}$	$C e^{\gamma x}$
$k x^n (n=0, 1, \dots)$	$K_n x^n + K_{n-1} x^{n-1} + \dots + K_1 x + K_0$
$k \cos \omega x$ $k \sin \omega x$	$\left. \begin{array}{l} \\ \end{array} \right\} K \cos \omega x + M \sin \omega x$
$k e^{ax} \cos \omega x$ $k e^{ax} \sin \omega x$	$\left. \begin{array}{l} \\ \end{array} \right\} e^{ax} (K \cos \omega x + M \sin \omega x)$