EE3331C/EE3331E Feedback Control Systems
Tutorial 3

1. Consider a closed loop system with an open loop transfer function given by

$$KG(s) = \frac{K}{s(s+a)(s+b)}$$

where a and b are unknown constants and $K > 0$. The root locus of the system is given in Figure 1.

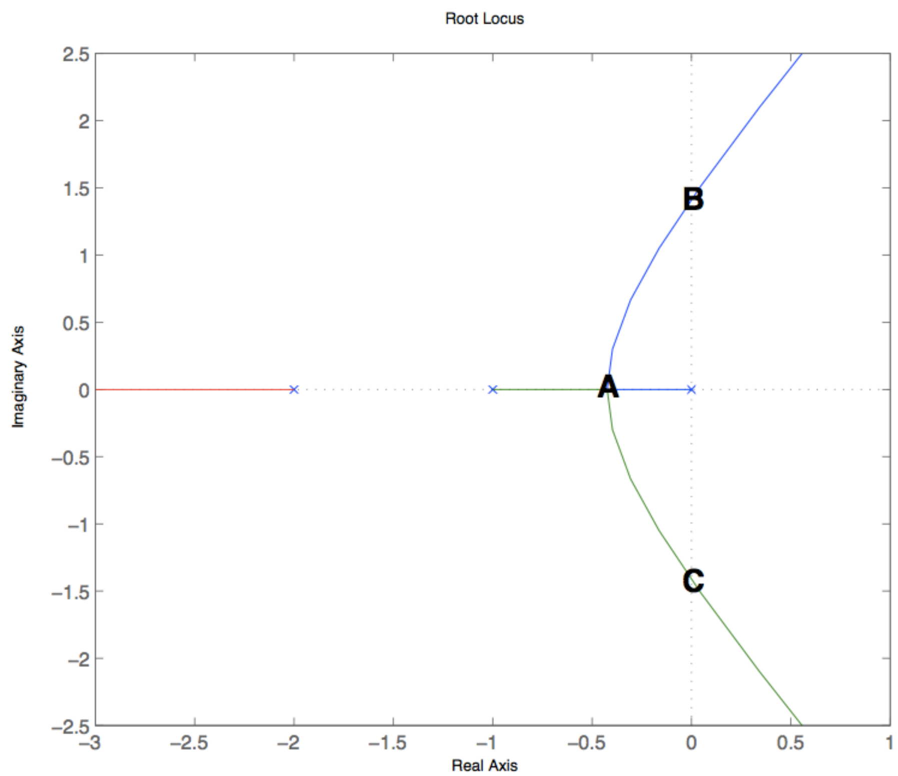

Figure 1: Root Locus

(a) Determine the values of a and b from the root locus.

(b) What is the damping ratio associated with the pole at 'A'? Using the values of a and b from (a) above, determine the closed loop poles at 'A' and the corresponding value of K for this pole. Do not read the value of 'A' from the figure.

(c) What is the damping ratio associated with the poles at 'B' and 'C'? Determine the closed loop poles at 'B' and 'C' and the corresponding value of K.

2. A closed loop system and its root locus (obtained from Matlab) are shown in Figure 2. Recall that such a plot shows the movement of the closed loop poles as K changes. Thus you find that points corresponding to various values of K have been indicated on this root locus.

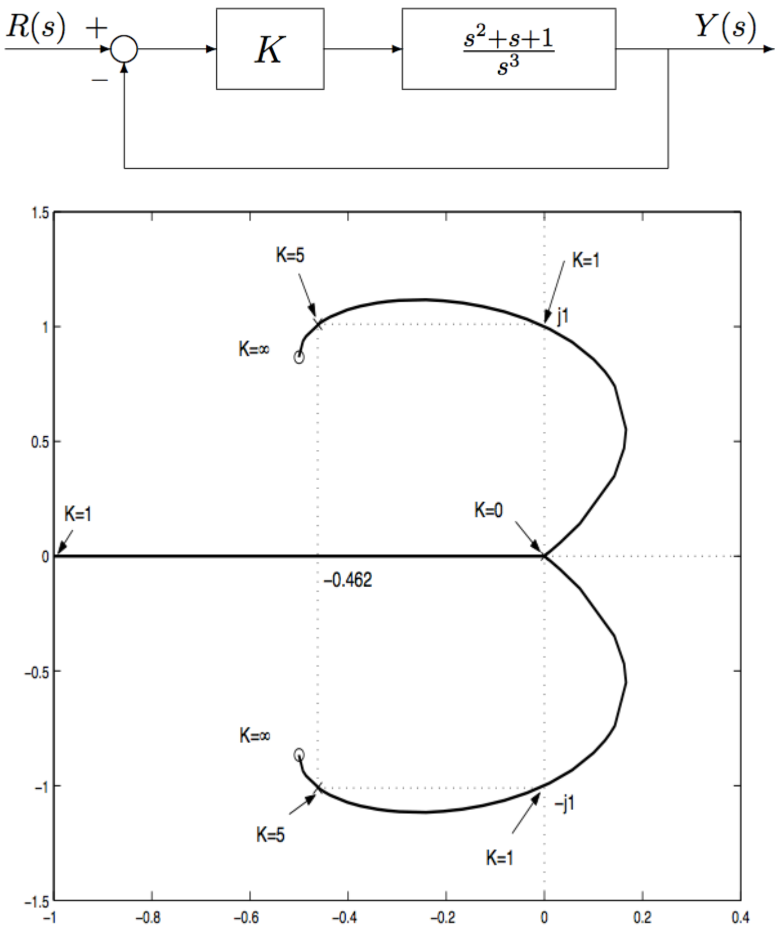

Figure 2: Closed loop system and its root locus

(a) Identify all the closed loop poles when $K = 1$.

(b) Is the closed loop system stable for $K < 1$?

(c) Calculate all the closed loop poles when $K = 5$.

(d) Write down the transfer function of the closed loop system when $K = 5$. By considering only the complex poles, determine the damping ratio of the closed loop system when $K = 5$. Is it reasonable to consider only the pair of complex poles?

(e) Based on the results of part (d) above, estimate the maximum overshoot and the settling time of the closed loop system when the reference input is a unit step function.

3. A position controller for a gasoline engine is shown in Figure 3. The restriction at the carburetor intake and the capacitance of the reduction manifold may be modelled by a first order system whose time constant, τ_t, is 2 seconds. The engine time constant τ_e is equal to 3 seconds while the time constant of the position sensor speed, τ_m, is 1 second.

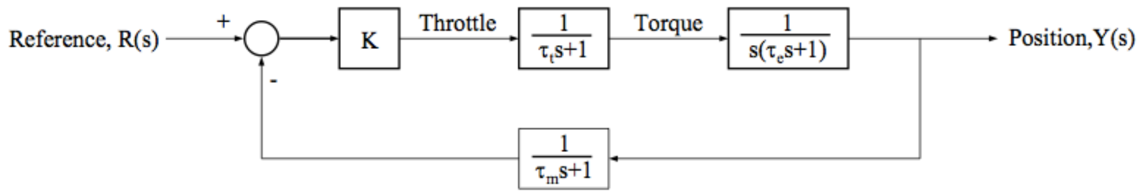

Figure 3: Engine position control system

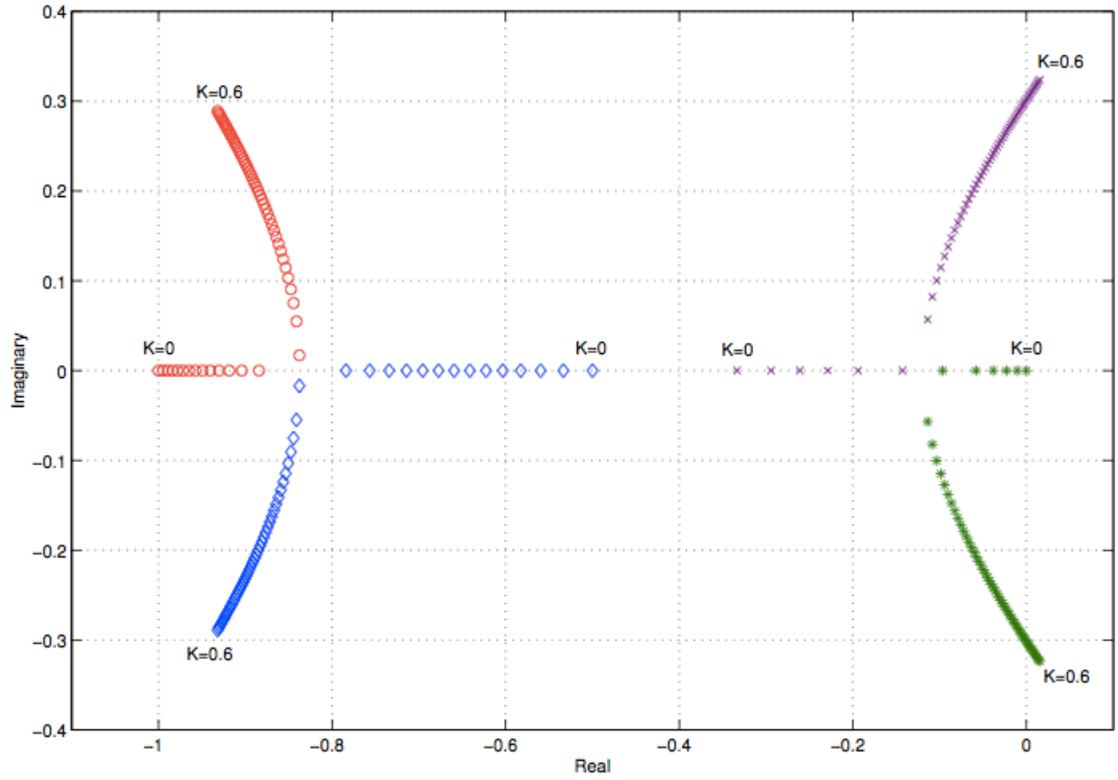

Figure 4: Root locus of Engine position system. Location of system poles when K is varied from 0 to 0.6 in steps of 0.01.

(a) For any particular positive value of the proportional constant, K, how many poles does the closed-loop system have?

(b) Figure 4 shows the location of the poles of the closed-loop system (closed-loop poles) when $K = 0, 0.01, 0.02, \ldots, 0.6$ (The four poles for each value of K are marked by $\times$, $*$, $\circ$ and $\diamond$). The poles of the closed-loop system for selected values of K is also tabulated in Table 1. Using the data, or otherwise,

i. What is the maximum K for which the closed-loop system is stable?
Hint: The closed-loop system become less and less stable as K increases.

ii. Find the system poles when the closed-loop system is marginally stable.

iii. Determine the value of α in Table 1.

iv. Identify the fastest pole and the slowest pole when $K = \alpha$.

K	System poles
0	-1, -0.5, $-\frac{1}{3}$, 0
α	-0.97, -0.62, -0.14, -0.097
0.25	$-0.87 \pm 0.16j$, $-0.047 \pm 0.23j$
0.5	$-0.92 \pm 0.26j$, $0 \pm j\omega$
0.6	$-0.93 \pm 0.29j$, $0.015 \pm 0.32j$

Table 1: System poles for selected values of K.

4. A temperature control system has the block diagram given in Figure 5. The input signal is a voltage and represents the desired temperature, θ_r. Find the steady state error of the system when θ_r is a unit step function and

(a) $D(s) = 1$ (proportional controller, P),

(b) $D(s) = 1 + \frac{0.1}{s}$ (proportional + integral, PI), and

(c) $D(s) = 1 + 0.3s$ (proportional + derivative, PD).

What is the effect of the integral term in the PI controller, and derivative term in the PD controller on the steady state error?

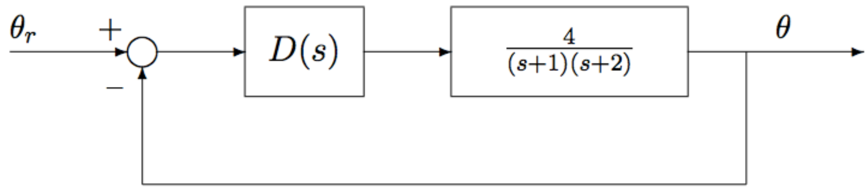

Figure 5: Temperature control system

5. The textbook PID control law is given as

$$G_c(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right)$$

Most industrial PID control law is given as

$$G_c^{'}(s) = K^{'}\left(1 + \frac{1}{sT_i^{'}}\right)\left(1 + sT_d^{'}\right)$$

Obtain expressions for $K^{'}$, $T_i^{'}$ and $T_d^{'}$ in terms of K, T_i and T_d to make the two control law equivalent. Which control law is more general.

6. Discuss why in practice, the derivative controller

$$U(s) = k_d s E(s)$$

is replaced by

$$U(s) = k_d \frac{s}{1 + k_d \frac{s}{N}} E(s)$$

where N is a positive constant usually in the range of 3 to 20.

7. The engine, body and tires of a racing vehicle affect the acceleration and speed attainable. The speed control of a car is represented in Figure 6, where $Y(s)$ is the speed. The driver wants to drive the car at 100 km/hr. What speed can he attain

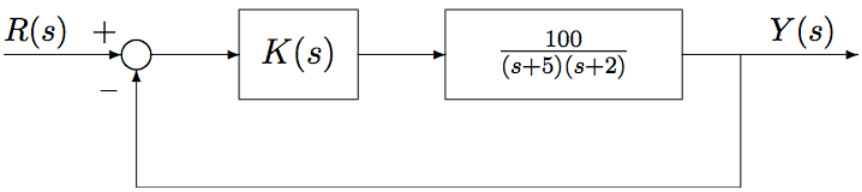

Figure 6: Racing vehicle control system

if he tries to drive the car at 100 km/hr, assuming $K(s) = 1$? Suggest a controller that will help him attain the speed that he wants for his racing car.

8. The control of one axis of a robot can be represented by the block diagram in Figure 7. Determine the value of K so that the robot reaches steady state in this axis in minimum time with no overshoot.

   For the value of K obtained above, determine the steady state error, $e = \theta_r - \theta$, for an input, $\theta_r(t) = 2 + 0.1t$.

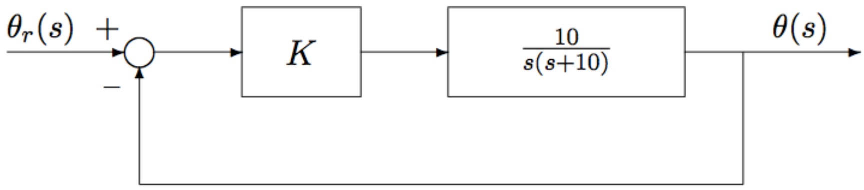

Figure 7: Robot control system

9. Consider the model of a telescope-pointing system shown in Figure 8. The goal is

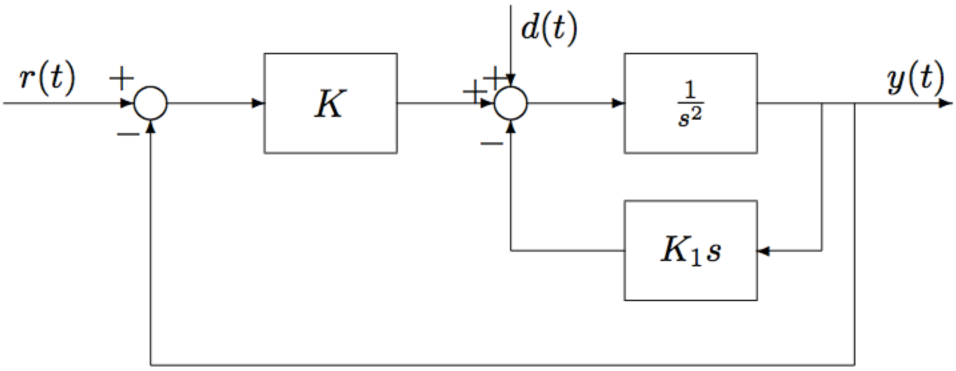

Figure 8: Telescrope-Pointing system

   to design K_1 and K so that

   - the percentage overshoot of the output to a step command, $r(t)$, is less than 10%,
   - the steady state error to a ramp command is minimized, and
   - the effect of a step disturbance, $d(t)$, is reduced.

   Find suitable values of K_1 and K so that the closed loop system will satisfy the design specifications above.