EE3331C/EE3331E Feedback Control Systems
Tutorial 2

1. Figure 1 shows a heat exchanger (a device for transferring heat from one fluid to another, where the fluids are separated by a solid wall so that they never mix). The temperature of the outgoing fluid, $\theta_2(t)$, needs to be maintained at the desired value, $\theta_r(t)$. Factors which influence the exit temperature are:

   - the value position, $u(t)$, that adjusts the flow of steam into the system,
   - unmeasurable disturbances in the temperature of the incoming fluid stream, $\theta_1(t)$.

   The dynamic behaviour of the heat exchanger may be modelled by the following Laplace Transform relationship:

   $$\theta_2(s) = \frac{2}{(s+1)^2}U(s) + \frac{1}{(s+2)}\theta_1(s)$$

   Suppose the transfer function of the temperature sensor is unity and a negative feedback loop for controlling the outgoing fluid temperature is formed by using a proportional controller, $u(t) = K[\theta_r(t) - \theta_2(t)]$, to compute the valve position. Draw a block diagram of the resulting closed-loop system, clearly labelling each block with its transfer function.

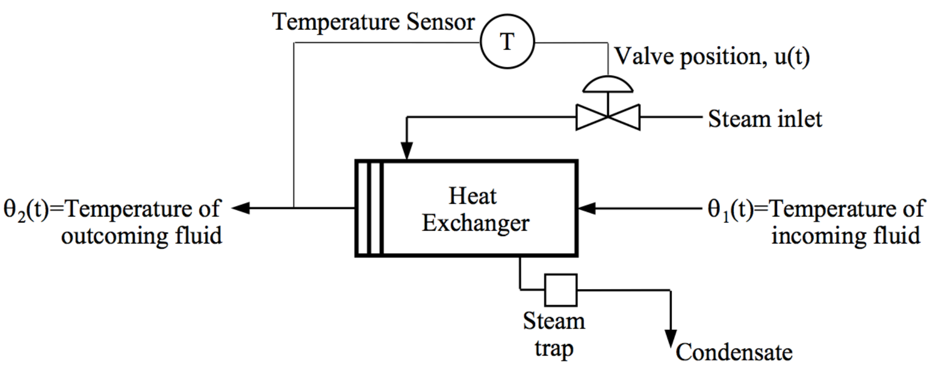

Figure 1: Schematics of a heat exchanger

2. Consider the two control systems shown in Figure 2 where the objective is to track the command signal, $R(s)$. In the open loop control system, the gain K_c is calibrated so that $K_c = 1/10$. In the closed loop control system, K_p of the controller is set to $K_p = 10$. Find the steady state errors ($E(s) = R(s) - Y(s)$) to a unit step input in both cases.

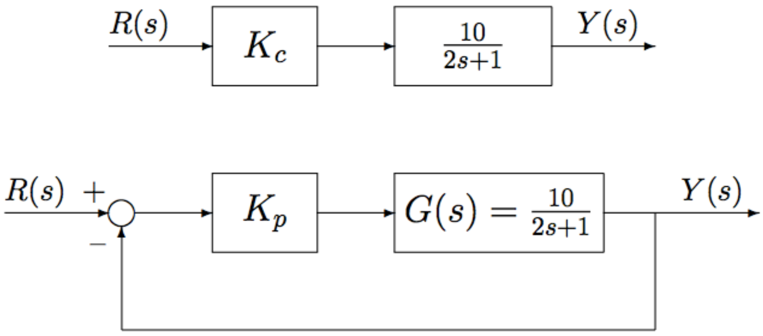

Figure 2: Open and Closed Loop Systems

Suppose now that parameter variations occurring during operating conditions cause $G(s)$ to change to $G(s) = \dfrac{11}{(2.5s+1)}$. What will be the effect on the steady state errors of the open and closed loop systems? Comment on the sensitivity of the two systems to parameter uncertainties.

3. You have been shown how closed loop control has significant benefits in dealing with uncertainties. In this problem, you will find that closed loop control is also able to somewhat linearise the behaviour of a nonlinear plant.

   Consider a unity negative feedback system as shown in Figure 3 where the nonlinear function $f(e) = e^2$. For an input r in the range of 0 to 4, calculate and plot the open loop and closed loop outputs versus the input. Show that the feedback system results in a more linear relationship.

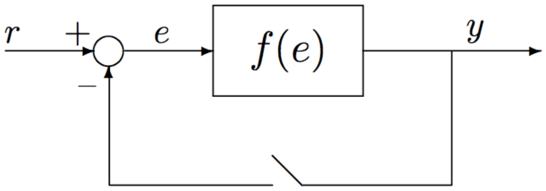

Figure 3: Linearising a nonlinear function

4. An open loop system is modelled by a first order transfer function given by $G(s) = \dfrac{1}{(s+1)}$. The output of $G(s)$ is affected by a constant disturbance given by $d(t) = 0.1$. The open loop system is shown in Figure 4. Calculate the steady state output for a unit step input of $r(t) = 1$. Comment on the effect of the disturbance, $d(t)$, on the steady state output.

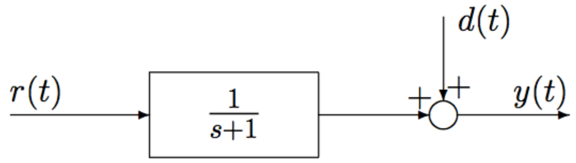

Figure 4: Open loop system

G(s) is now controlled in a closed loop unity feedback configuration as shown in Figure 5. Calculate the steady state output $y(t)$ when the reference signal is $r(t) = 1$ and $d(t) = 0.1$. Comment on the role of the feedback system in disturbance rejection.

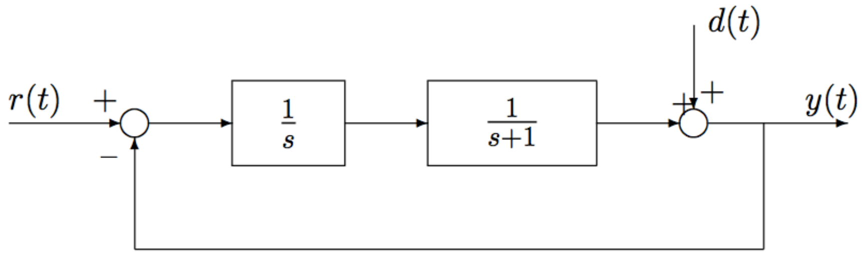

Figure 5: Closed loop control system

5. Consider the control system shown in Figure 6.

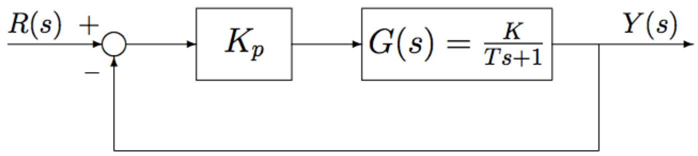

Figure 6: Closed loop control system

(a) Show that the closed loop system will always be stable for all $K_p > 0$, $K > 0$ and $T > 0$.

(b) Suppose the proportional controller, K_p is now replaced by a proportional plus integral (PI) controller given by

$$K(s) = Kp\left(1 + \frac{1}{T_i s}\right)$$

Is the closed loop system still stable for all values of $K_p > 0$ and $T_i > 0$?

(c) Suppose $G(s)$ in Figure 6 is now replaced by a double integrator given by $G(s) = \frac{1}{s^2}$. Can the closed loop system be stabilized by any value of K_p?

Describe the expected performance of this closed loop system.

(d) If K_p is now replaced by a proportional + integral + derivative (PID) controller of the form

$$K(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right),$$

determine the value of K_p for which the closed loop system is marginally stable when $T_i = 0.2$ and $T_d = 0.1$. Find all the closed loop poles when $K_p = 100$.

6. Figure 7 shows a position control system. Compute the values of K_c and T_d so that the characteristic equation of the closed loop system will have roots at $-1 \pm j\sqrt{3}$. Determine its damping ratio and natural frequency. Sketch the expected closed loop step response.

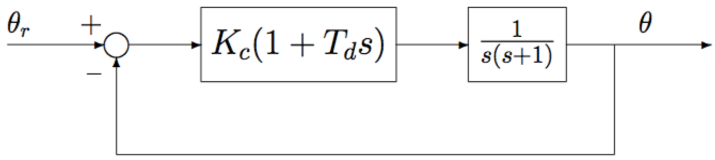

Figure 7: Position control system

7. Consider the control of a satellite-tracking antenna system. The antenna and drive parts have a moment of inertia J and a damping B; these arise to some extent from bearing and aerodynamic friction, but mostly from the back emf of the DC drive motor. The equations of motion are

$$J\ddot{\theta} + B\dot{\theta} = T_c$$

where T_c is the torque from the drive motor. Assume that $J = 600,000$ kg·m^2 and $B = 20,000$ N·m·sec.

(a) Find the transfer function between the applied torque T_c and the antenna angle θ.

(b) Suppose the applied torque is computed so that θ tracks a reference command θ_r according to the feedback law, $T_c = K(\theta_r - \theta)$, where K is the feedback gain. Find the transfer function between θ_r and θ.

(c) What is the maximum value of K that can be used if you wish to have an overshoot $M_p < 10\%$?

(d) What values of K will provide a rise time of less than 80 sec? (Ignore the M_p constraints.)