NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2014/2015)

EE3331C/EE3331E – FEEDBACK CONTROL SYSTEMS

December 2014 – Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SEVEN** (7) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry equal marks.
4. This is a CLOSED BOOK examination.
5. Some relevant data are provided at the end of this examination paper.
6. Working MUST be provided clearly. Marks will not be awarded if working shown does not match the final answer or when there is no working.

Q.1 Figure Q1 shows a position control system where $G_1(s) = \dfrac{1}{\alpha s}$, $G_2(s) = \dfrac{1}{s}$, and $C(s) = k_3$.

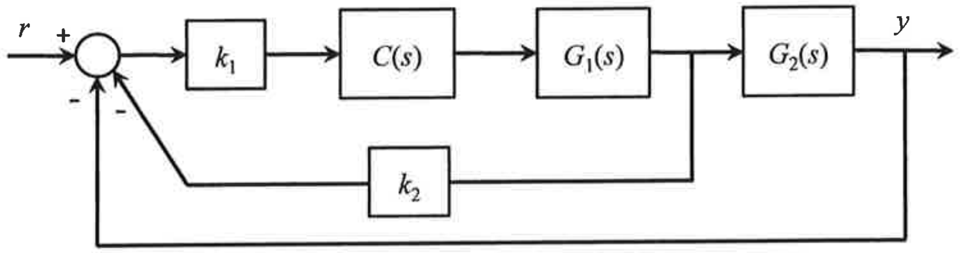

Figure Q1: Position Control System.

(a) Show that the closed-loop transfer function is given by

$$\frac{Y(s)}{R(s)} = \frac{k_1 k_3}{\alpha s^2 + k_1 k_2 k_3 s + k_1 k_3}$$

(6 marks)

(b) Given that $\alpha = 25$, determine the necessary gain, k_2, to maintain a steady-state error $(E(s) = R(s) - Y(s))$ equal to 1 cm when the input is a ramp, $r(t) = t$ (meters).

(6 marks)

(c) Using the value of k_2 from part (b), determine the necessary gain, $k_1 k_3$, such that the maximum overshoot is 10%.

(6 marks)

(d) Sketch the expected output response, $y(t)$, due to an unit step input based on the values in parts (b) and (c). Label the axes and all critical values (e.g. rise-time, 2% settling-time, peak-time, maximum overshoot and steady-state output) on your plot.

(7 marks)

Q.2(a) Write three different transfer functions that will have the asymptotic Bode plot (magnitude) shown in Figure Q2.

(7 marks)

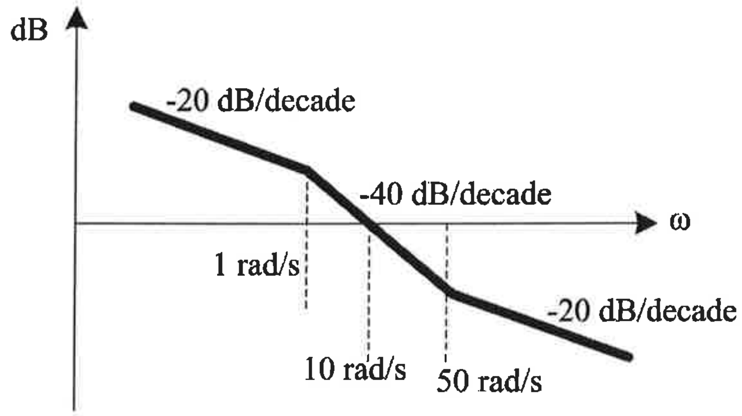

Figure Q2: Asymptotic Bode (magnitude) plot.

(b) For the loop transfer function

$$L(s) = \frac{K}{s(s^2 + 2s + 6)},$$

i. Find the phase-crossover frequency (ω_{cp}).

(4 marks)

ii. Determine K that results in a gain margin of 6 dB.

(4 marks)

(c) Sketch the Nyquist plot of the following transfer function with properly labeled intersections with real axis.

$$C(s)G(s)H(s) = K\frac{(s+1)}{s(2s-1)}$$

Use the Nyquist Stability Criterion to find the range of $K > 0$ such that the the resulting closed loop is stable.

(10 marks)

$$\left[\text{Hint} : \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Q.3 In Figure Q3, the plant is modeled as

$$G_p(s) = \frac{1}{(s+1)}$$

and the reference is

$$r(t) = 2.5 + 2.5\sin(5t).$$

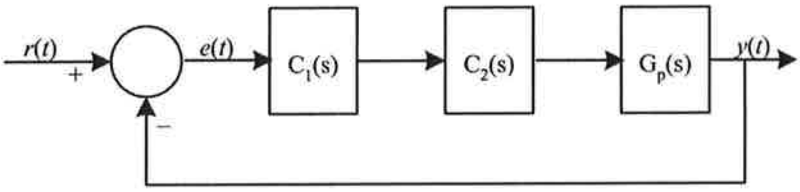

Figure Q3: Closed loop feedback system.

(a) Set $C_1(s) = 1$ and choose $C_2(s)$ such that the steady-state error (e_{ss}) for the given reference signal has an amplitude less than 0.25.

(5 marks)

(b) What is the phase margin with $C_1(s) = 1$ and $C_2(s)$ found in part (a).

(5 marks)

(c) Design the compensator $C_1(s)$ to improve the phase margin to 35°. The gain-crossover frequency of the compensated loop must be greater than the gain-crossover frequency found in part (b).

(12 marks)

$$\left[\text{Hint: for compensator } C(s) = \frac{Ts+1}{\alpha Ts+1}, \quad \omega_m = \frac{1}{T\sqrt{\alpha}}, \quad \sin\phi_m = \frac{1-\alpha}{1+\alpha}\right]$$

(d) Comment on the advantages and disadvantages of the compensator you designed.

(3 marks)

Q.4 A closed-loop system and its root locus plot are shown in Figures Q4-a and Q4-b respectively where $K > 0$.

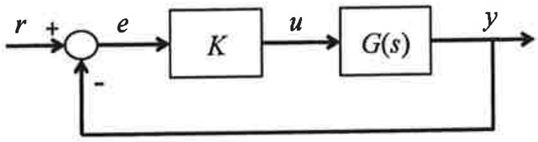

Figure Q4-a: Feedback Control System.

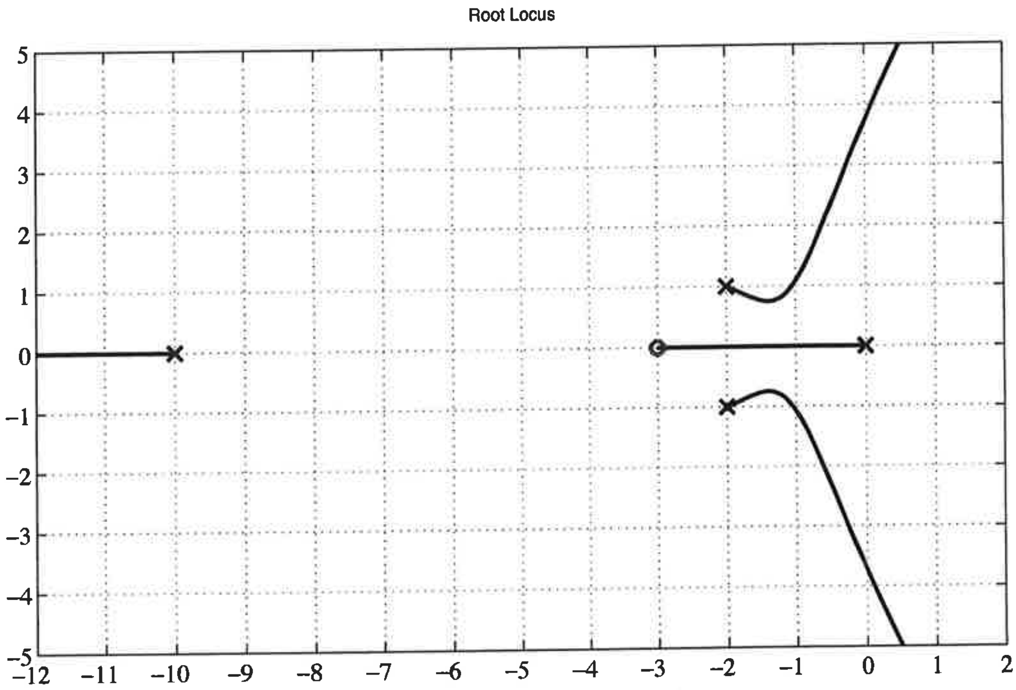

Figure Q4-b: Root Locus plot.

(a) The transfer function of the plant is $G(s) = \dfrac{(s+a)}{(s^4 + bs^3 + cs^2 + ds + e)}$. Find a, b, c, d and e.

(6 marks)

(b) Find the maximum gain, K, before the system becomes unstable?

(7 marks)

(c) The impulse response of the closed-loop system has the form: $Ae^{-10.5t} + Be^{-2.5t} + Ce^{-0.5t}\cos(2.3t + \phi)$, where A, B, C and ϕ are constants. Find the closed-loop pole locations and the corresponding gain K.

(6 marks)

Q.4 is continued on page 6

(d) We are now interested to investigate how the closed-loop poles vary as the parameter, d, in $G(s)$ vary. Figure Q4-d shows 2 root locus plots (A and B), which plot (A or B) shows how the closed-loop poles vary as d vary? Explain your answer.

(6 marks)

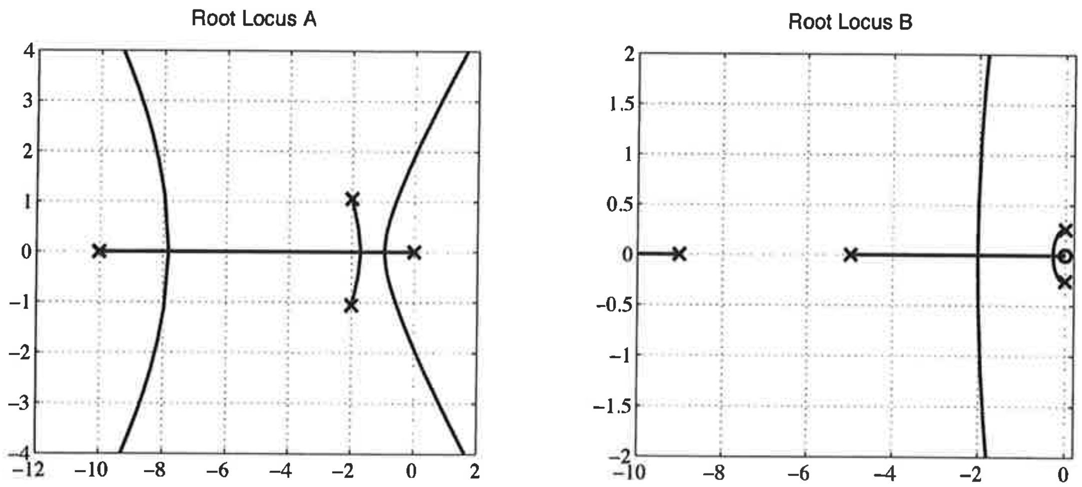

Figure Q4-d: Root Locus plots.

END OF PAPER

DATA SHEET :

SOME USEFUL LAPLACE TRANSFORM RULES

Transform of derivatives, $\mathcal{L}\left\{\frac{dy(t)}{dt}\right\}$	$sY(s) - y(0)$
Transform of integral, $\mathcal{L}\left\{\int_0^t y(\tau)d\tau\right\}$	$\frac{Y(s)}{s}$
Shift in time domain, $\mathcal{L}\{y(t-L)u(t-L)\}$	$Y(s)e^{-sL}$
Shift in s-domain, $\mathcal{L}\{y(t)e^{-at}\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

SOME USEFUL LAPLACE TRANSFORMS

Function, $f(t)$	Laplace Transform, $F(s)$	Function, $f(t)$	Laplace Transform, $F(s)$
delta function, $\delta(t)$	1	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
unit step, $u(t)$	$\frac{1}{s}$	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
t	$\frac{1}{s^2}$	$1-e^{-at}$	$\frac{a}{s(s+a)}$
ke^{-at}	$\frac{k}{s+a}$	$t - \frac{1}{a}\left(1-e^{-at}\right)$	$\frac{a}{s^2(s+a)}$

SOME DESIGN FORMULAE FOR UNDERDAMPED 2^{nd} ORDER SYSTEM

Standard 2^{nd} order system : $\dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Percentage overshoot, $\%M_p$	$\%M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$
Settling time (2%), t_s	$t_s = \frac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \frac{1.8}{\omega_n}$
Peak time, t_p	$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$