EE3331C/EE3331E Feedback Control Systems
Part II, Tutorial 3

Section 1

1. Use the following transfer functions to answer Q1(a)-Q1(d).

$$(i)\ F(s)=\frac{s+10}{s+1},\quad (ii)F(s)=\frac{s-1}{s+1},\quad (ii)F(s)=\frac{s+1}{s-10}$$

a) Map the positive imaginary axis, i.e., $s = j\omega$ where $\omega \in \Re \mid [0,\infty)$, on to the F(s)-plane.

b) Map the top half of the semicircle of the D-contour, i.e., $s = Re^{j\theta}$ where $R \to \infty$ and θ is varied from $90° \to 0°$, on to the F(s)-plane.

c) Combine the sketches from (a) and (b) to get the mapping of the top half of the Nyquist contour. Finally obtain the Nyquist plot by adding the mirror image.

d) Validate the Principle of Argument using these plots.

Solution:

(i)

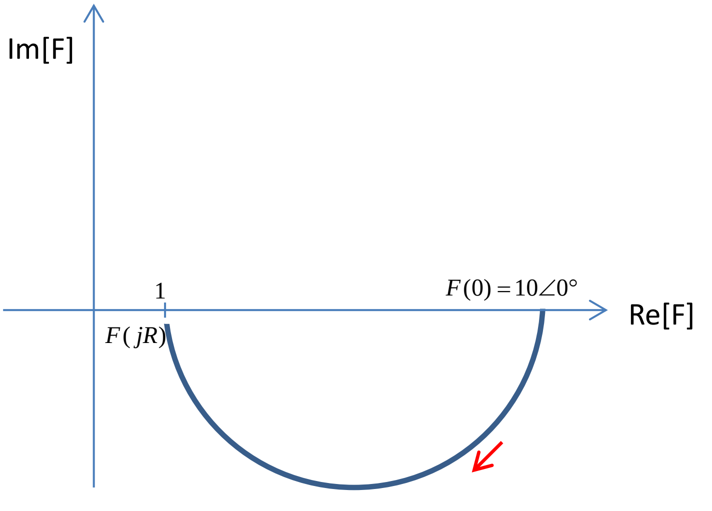
$$(a)\quad F(0)=\frac{0+10}{0+1}=10\angle 0°$$

$$F(jR)=\frac{jR+10}{jR+1}\quad \Rightarrow \quad |F(jR)|=\frac{\sqrt{R^2+100}}{\sqrt{R^2+1}}\cong 1,\quad as\ R\to\infty$$

$$\angle F(jR)=\tan^{-1}\tfrac{R}{10}-\tan^{-1}\tfrac{R}{1}=-\delta,\quad \delta\approx 0$$

$$F(j\omega)=\frac{j\omega+10}{j\omega+1}\quad \Rightarrow \quad |F(j\omega)|=\frac{\sqrt{\omega^2+100}}{\sqrt{\omega^2+1}},\quad \angle F(j\omega)=\tan^{-1}\tfrac{\omega}{10}-\tan^{-1}\tfrac{\omega}{1}$$

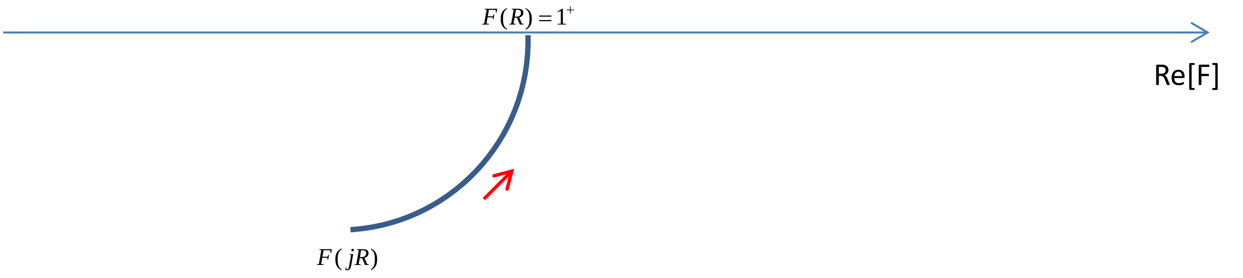
$$\angle F(j\omega)<0\quad for\quad \forall\omega:\quad 0<\omega<\infty$$



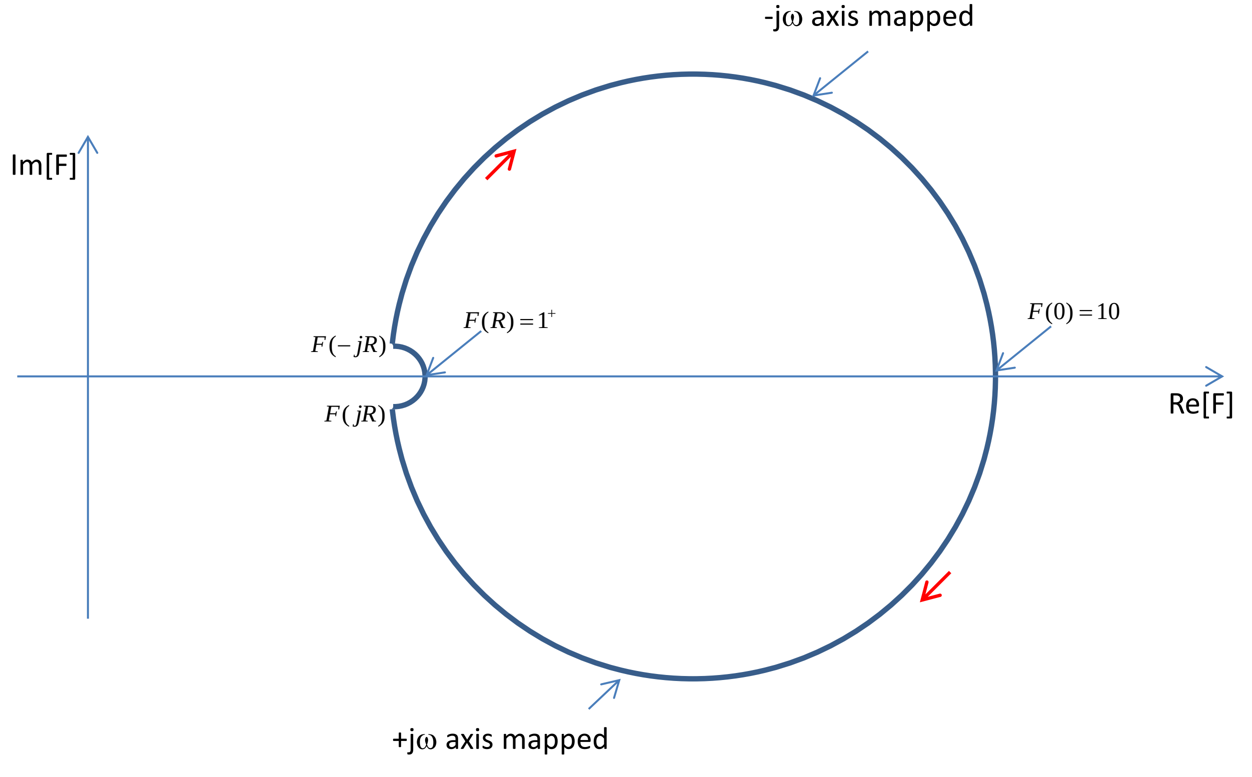
$$(b)\quad F(s)=\frac{Re^{j\theta}+10}{Re^{j\theta}+1},\quad R\to\infty,\theta:90^\circ\to 0^\circ$$

$\theta=90^\circ,\quad F(s)=\dfrac{jR+10}{jR+1}$ [this is same as the end point of Q1(a)]

$\theta=0^\circ,\quad F(R)=\dfrac{R+10}{R+1}\approx 1$ [this is on the real axis; magnitude is slightly more than 1]



(c)



(d)

F(s) has no pole or zero inside the Nyquist contour. The origin is not encircled

$(ii)\ F(s)=\dfrac{s-1}{s+1}$

$(a)\quad F(0)=\dfrac{0-1}{0+1}=1\angle\pm180^\circ$

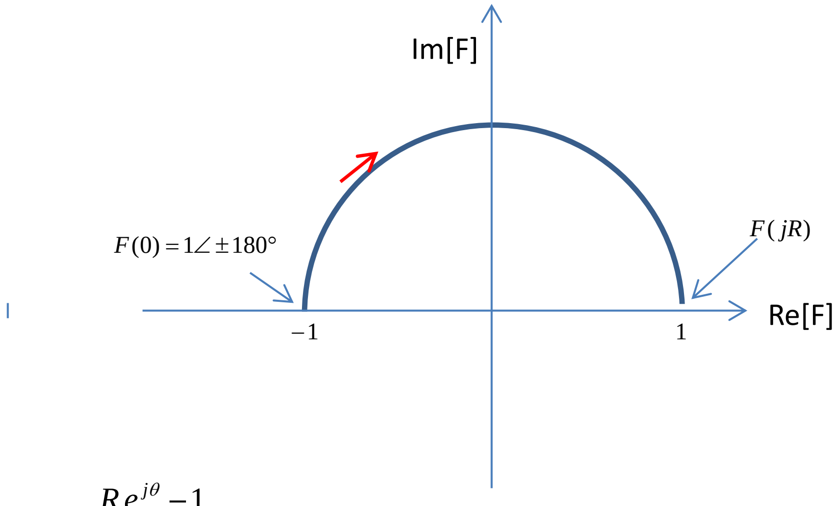
$$F(jR)=\frac{jR-1}{jR+1}\quad\Rightarrow\quad |F(jR)|=\frac{\sqrt{R^2+1}}{\sqrt{R^2+1}}=1 \qquad \angle F(jR)=(180^\circ-\tan^{-1}\tfrac{R}{1})-\tan^{-1}\tfrac{R}{1}$$

$$=180^\circ-2\tan^{-1}\tfrac{R}{1}$$

$$\approx 0^\circ \quad as \quad R\to\infty$$

$$F(j\omega)=\frac{j\omega-1}{j\omega+1}\quad\Rightarrow\quad |F(j\omega)|=\frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+1}}=1,\quad \angle F(j\omega)=180^\circ-\tan^{-1}\tfrac{\omega}{1}-\tan^{-1}\tfrac{\omega}{1}$$

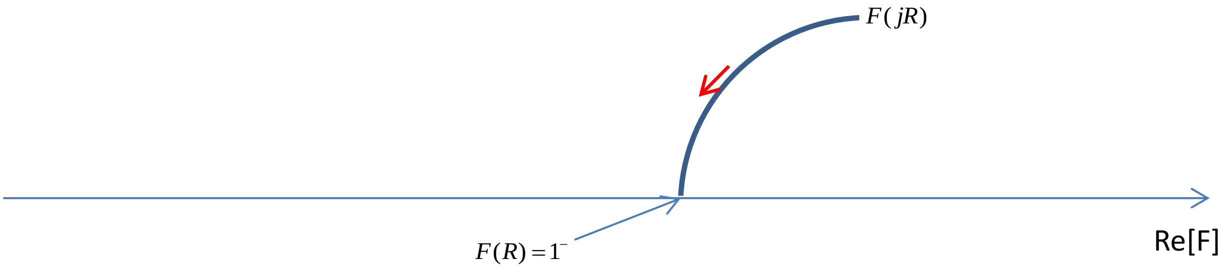
$$180^\circ>\angle F(j\omega)>0 \quad for \quad \forall\omega:\quad 0<\omega<\infty$$



$(b)\quad F(s)=\dfrac{Re^{j\theta}-1}{Re^{j\theta}+1},\quad R\to\infty,\theta:90^\circ\to0^\circ$

$\theta=90^\circ,\quad F(s)=\dfrac{jR-1}{jR+1}$ [this is same as the end point of Q2(a)]

$\theta=0^\circ,\quad F(R)=\dfrac{R-1}{R+1}\approx1$ [this is on the real axis; magnitude is slightly less than 1]



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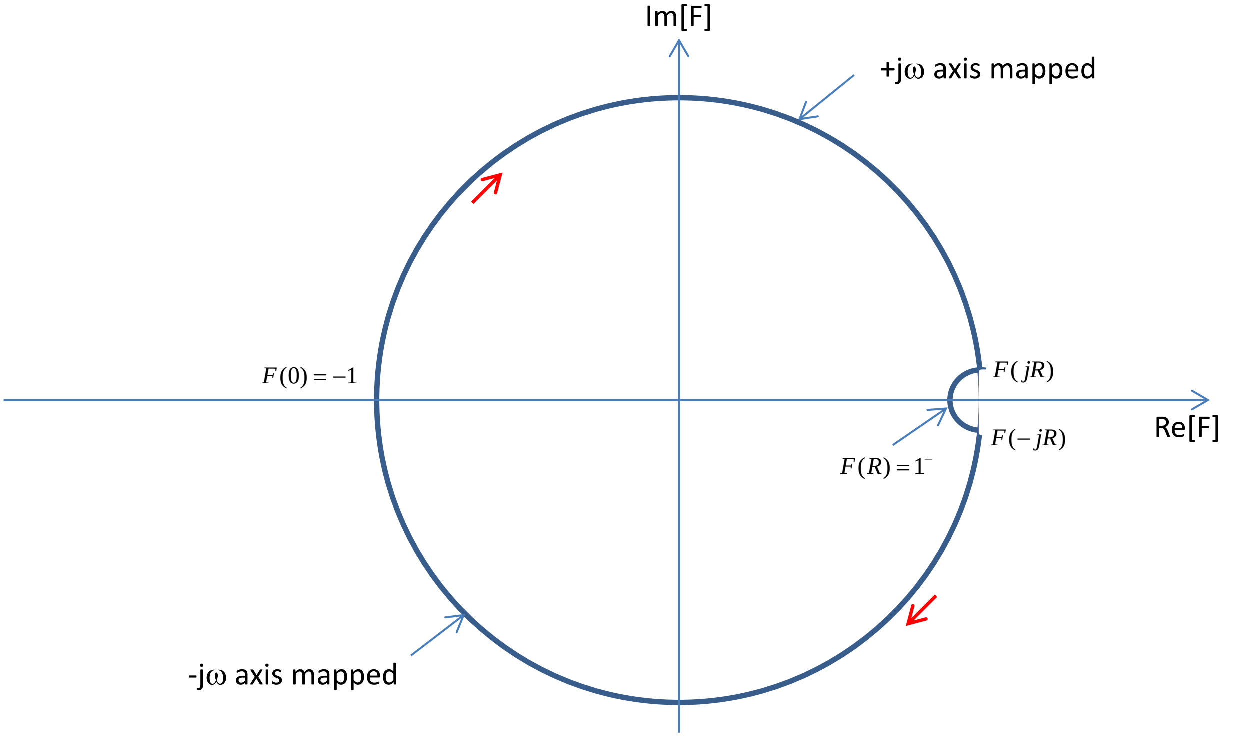


(d)

F(s) has one zero inside the Nyquist contour. The origin is encircled once clockwise.

$$(iii)\ F(s)=\frac{s+1}{s-10}$$

$$(a)\quad F(0)=\frac{0+1}{0-10}=0.1\angle\pm180^\circ$$

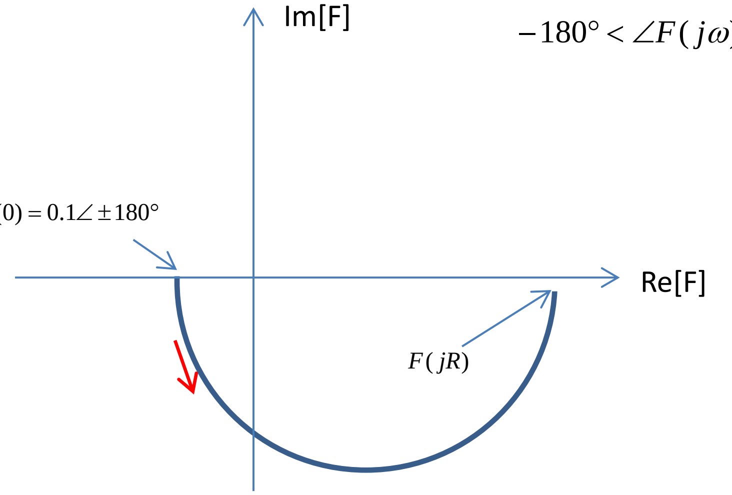
$$F(jR)=\frac{jR+1}{jR-10}\quad\Rightarrow\quad |F(jR)|=\frac{\sqrt{R^2+1}}{\sqrt{R^2+100}}\approx 1\ as\ R\to\infty$$

$$\angle F(jR)=\tan^{-1}\tfrac{R}{1}-(180^\circ-\tan^{-1}\tfrac{R}{10})$$
$$=-180^\circ+\tan^{-1}\tfrac{R}{1}+\tan^{-1}\tfrac{R}{10}$$
$$\approx 0^\circ\quad as\quad R\to\infty$$

$$F(j\omega)=\frac{j\omega+1}{j\omega-10}\quad\Rightarrow\quad |F(j\omega)|=\frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+100}},$$

$$\angle F(j\omega)=\tan^{-1}\tfrac{\omega}{1}-180^\circ+\tan^{-1}\tfrac{\omega}{10}$$

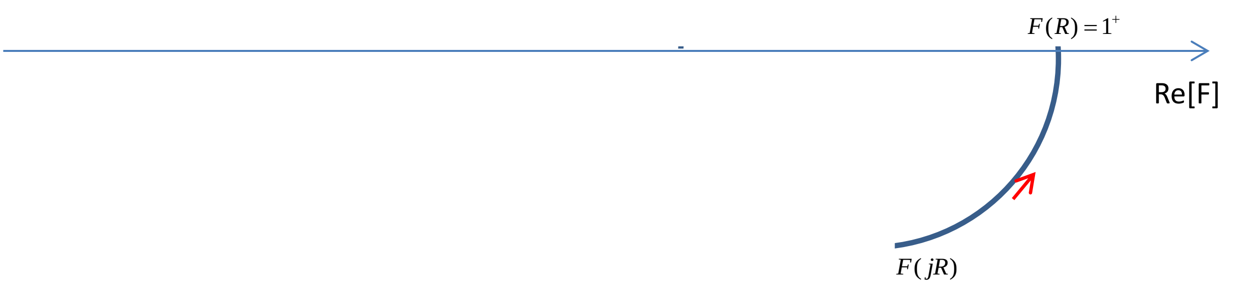
$$-180^\circ<\angle F(j\omega)<0\quad for\quad \forall\omega:\quad 0<\omega<\infty$$



$$(b)\quad F(s)=\frac{Re^{j\theta}-1}{Re^{j\theta}+10},\quad R\to\infty,\ \theta:90^\circ\to0^\circ$$

$\theta=90^\circ,\quad F(s)=\dfrac{jR-1}{jR+10}$ [this is same as the end point of Q2(a)]

$\theta=0^\circ,\quad F(R)=\dfrac{R-1}{R+10}\approx1$ [this is on the real axis; magnitude is slightly **more** than 1]



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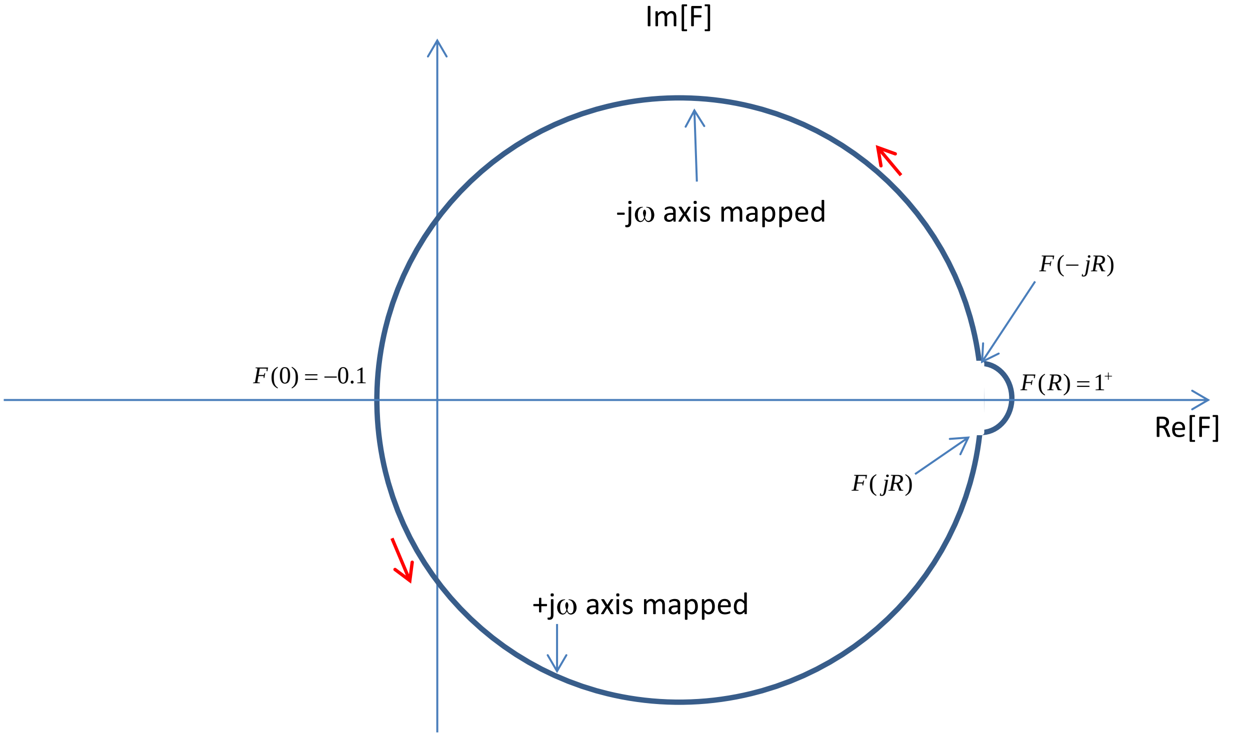

(d)

F(s) has one pole inside the Nyquist contour. The origin is encircled once counterclockwise.

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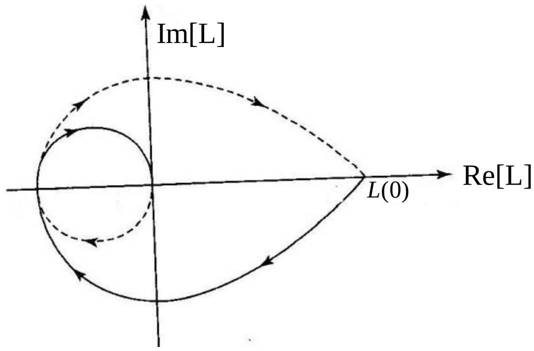
Section 2

1. Sketch the Nyquist plot and check closed loop stability for the following loop transfer functions.

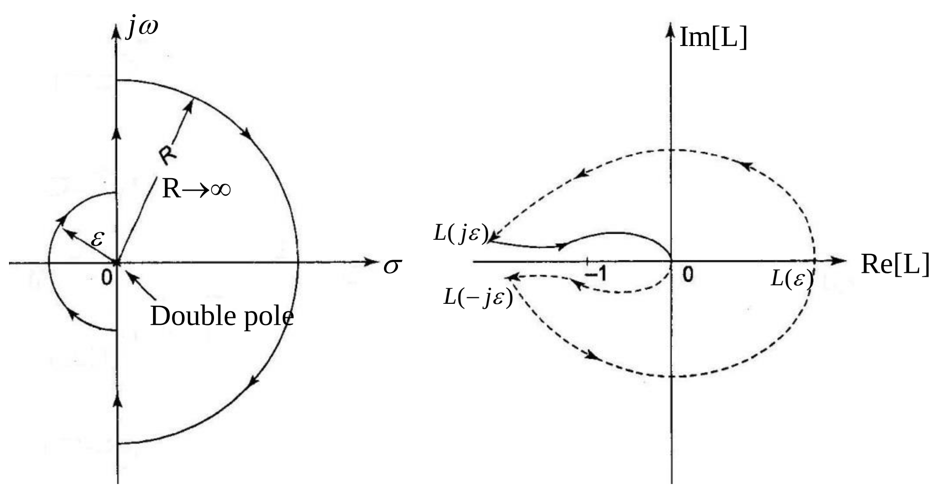
$$(i)\ L(s)=\frac{50}{(s+1)(s+2)(s+3)},\quad (ii)\ L(s)=\frac{(s+2)}{(s+1)(s-1)},\quad (iii)\ L(s)=\frac{(s+5)^2}{s^2(s+1)}$$

2. Nyquist plots are shown for the following loop transfer functions. Determine the range of K that makes the closed loop stable. {For the function (ii), the Nyquist contour mapped is also shown. This is not required for the other two functions as they have no integrator.} If the CL is unstable, determine the number of unstable poles.

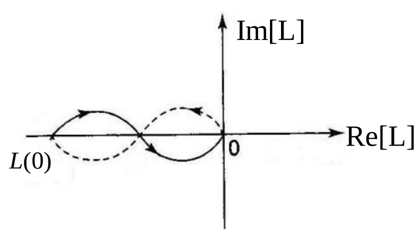
$$(i)\ L(s)=\frac{K}{(s+2)(s^2+2s+5)}$$



$$(ii)\ L(s)=\frac{K}{s^2(s+1)}$$



$$(iii)\ L(s)=K\frac{(s+2)}{(s+1)(s-3)}$$



3. Using Nyquist stability criterion, find the range of K for which the closed loop is stable if the loop transfer function is

$$L(s)=K\frac{(s+1)^2}{s^3}.$$