The transfer function model of a piezo-electric micro-actuator is given below. Unit of the transfer function is m/volt

$$G_{ma}(s) = \frac{1000}{s^2 + 1500s + 5625\times 10^6}$$

A sinusoidal voltage as described below is applied to the input of this micro-actuator. Determine the maximum swing of the micro-actuator from its neutral position.

$$v(t) = 1.5\sin(1000t) \quad volt$$

Answer: Frequency response,

$$G_{ma}(j\omega) = \frac{1000}{-\omega^2 + j1500\omega + 5625\times 10^6}$$

$$\begin{aligned} G_{ma}(j1000) &= \frac{1000}{-(1000)^2 + j1500\times 1000 + 5625\times 10^6} \\ &= \frac{1000}{(5625\times 10^6 - 10^6) + j1.5\times 10^6} \\ &= \frac{1000}{5624\times 10^6 + j1.5\times 10^6} \end{aligned}$$

$$\begin{aligned} \left|G_{ma}(j1000)\right| &= \frac{1000}{\sqrt{10^{12}\times 5624^2 + 10^{12}\times 1.5^2}} \\ &= \frac{1000}{10^6\times 5624} \\ &= 1.78\times 10^{-7} \end{aligned}$$

Maximum swing from the neutral position:

$$\begin{aligned} Y_m &= 1.5\times 1.78\times 10^{-7} \\ &= 267\times 10^{-9} \end{aligned}$$

Hand sketch the Bode (magnitude) plots of the following two transfer function

$$G_1(s) = \frac{100(s+10)}{s^2}, \quad G_2(s) = \frac{100(s-10)}{s^2}$$

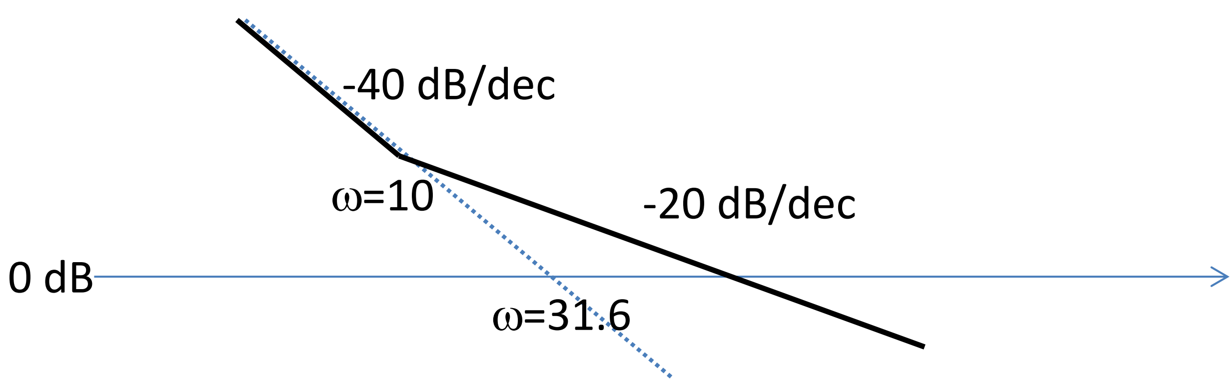
Answer: Two transfer function have identical gain plot

$$G_1(s) = \frac{100 \times 10(0.1s+1)}{s^2}, \quad G_2(s) = \frac{100 \times 10(0.1s-1)}{s^2}$$

Two integrators mean -40 dB/decade gradient in low frequency. The plot intersects the 0-dB line at $\omega 0$ where,

$$\frac{100 \times 10}{\omega_0^2} = 1, \quad \Rightarrow \quad \omega_0 = \sqrt{1000} = 31.6$$

Corner frequency of the zero is 10 rad/s. Gradient of the magnitude plot is increased by +20 dB/decade from frequencies greater than 10 rad/s.



So far, we have learnt that stability of the closed loop can be checked by finding the gain-crossover frequency (ω_{cg}) and phase-crossover frequency (ω_{cp}) only. Using these two parameters only, check closed loop stability for the following three loop transfer functions.

$$L_1(s) = \frac{100}{s(s+2)}, \quad L_2(s) = \frac{100}{s(s^2+6s+25)}, \quad L_3(s) = \frac{200}{s(s^2+6s+25)}$$

Answer:

$$(a)\ L_1(s) = \frac{100}{s(s+2)} \qquad L_1(j\omega) = \frac{100}{j\omega(j\omega+2)}$$

$$|L| = \frac{100}{\omega\sqrt{\omega^2+4}}$$

DC-gain is infinite and gain is monotonically decreasing function of ω. So, there exists a frequency $\omega_{cg} < \infty$ such that

$$|L(j\omega_{cg})| = \frac{100}{\omega_{cg}\sqrt{\omega_{cg}^2+4}} = 1$$

$$\angle L = -90° - \tan^{-1}\frac{\omega}{2} \qquad -90° - \tan^{-1}\frac{\omega_{cp}}{2} = -180°$$

$$\omega_{cp} = \infty$$

The resulting closed loop is stable.

$$(b)\ L_2(s) = \frac{100}{s(s^2+6s+25)} \qquad L_2(j\omega) = \frac{100}{j\omega(-\omega^2 + j6\omega + 25)}$$

Find phase crossover frequency by solving $\angle L_2(j\omega_{cg}) = -180°$

$$\angle L_2 = -90° - \tan^{-1}\frac{6\omega}{25-\omega^2}$$

$$\angle L_2 = -180 \quad \Rightarrow \quad \tan^{-1}\frac{6\omega_{cp}}{25-\omega_{cp}^2} = -90°$$

$$|L_2| = \frac{100}{\omega\sqrt{(25-\omega^2)^2+36\omega^2}}$$

$$\frac{6\omega_{cp}}{25-\omega_{cp}^2} = \tan 90° = \infty$$

$$\omega_{cp}^2 = 25 \quad \Rightarrow \quad \omega_{cp} = 5$$

$$|L_2(j\omega_{cp})| = \frac{100}{5\sqrt{(25-5^2)+36\times 5^2}} = \frac{100}{5\times 6\times 5} = 0.66$$

DC-gain is infinite. And the gain $\to 0$ as $\omega \to \infty$. So, there exists a frequency $\omega_{cg} < \infty$ at which gain is 1. For $\omega > 5$, the gain decreases. So $\omega_{cg} < 5$.

The resulting closed loop is stable.

$$(b)\ L_2(s) = \frac{100}{s(s^2+6s+25)} \qquad L_2(j\omega) = \frac{100}{j\omega(-\omega^2 + j6\omega + 25)}$$

Alternative method to find ω_{cp}. $\qquad \angle L_2(j\omega_{cg}) = -180°$

At phase-crossover frequency (if it exists), the phase is -180°, i.e., $L(j\omega_{cp})$ is a negative real number.

$$\mathrm{Im}\{L(j\omega_{cp})\} = 0$$

$$L_2(j\omega) = \frac{100}{j\omega(25-\omega^2) - 6\omega^2}$$

$L_2(j\omega_{cp})$ is negative real if, $\qquad (25-\omega_{cp}^2) = 0 \quad \Rightarrow \quad \omega_{cp} = 5$

$$(c)\ L_3(s) = \frac{200}{s(s^2+6s+25)} \qquad L_3(j\omega) = \frac{200}{j\omega(-\omega^2 + j6\omega + 25)}$$

Phase crossover frequency is same as in problem (b)

$$\omega_{cp} = 5$$

$$|L_3| = \frac{200}{\omega\sqrt{(25-\omega^2)^2 + 36\omega^2}} \qquad |L_3(j\omega_{cp})| = \frac{200}{5\sqrt{(25-5^2) + 36\times 5^2}}$$

$$= \frac{200}{5\times 6\times 5} = 1.33$$

DC-gain is infinite. And the gain $\rightarrow 0$ as $\omega \rightarrow \infty$. So, there exists a frequency $\omega_{cg} < \infty$ at which gain is 1.

Gain is 1.33 at ω_{cp}= 5. So the gain-crossover frequency is greater than 5 rad/s.

$$\omega_{cg} > \omega_{cp}$$

The resulting closed loop is unstable.