NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II: 2012/2013)

EE3331C – FEEDBACK CONTROL SYSTEMS

May 2013 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) printed pages.

2. All questions are compulsory. Answer **ALL** questions.

3. This is a **CLOSED BOOK** examination.

4. Show all your working clearly.

5. On page 5 of this question paper, the following tables are provided:
   a) Useful Laplace Transform Rules
   b) Table of useful Laplace Transforms
   c) Useful design formulae

Q.1 A closed-loop system and its root locus plot are shown in Figures Q1-a and Q1-b respectively.

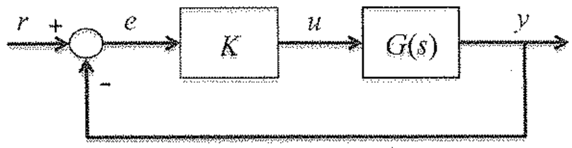

Fig. Q1-a

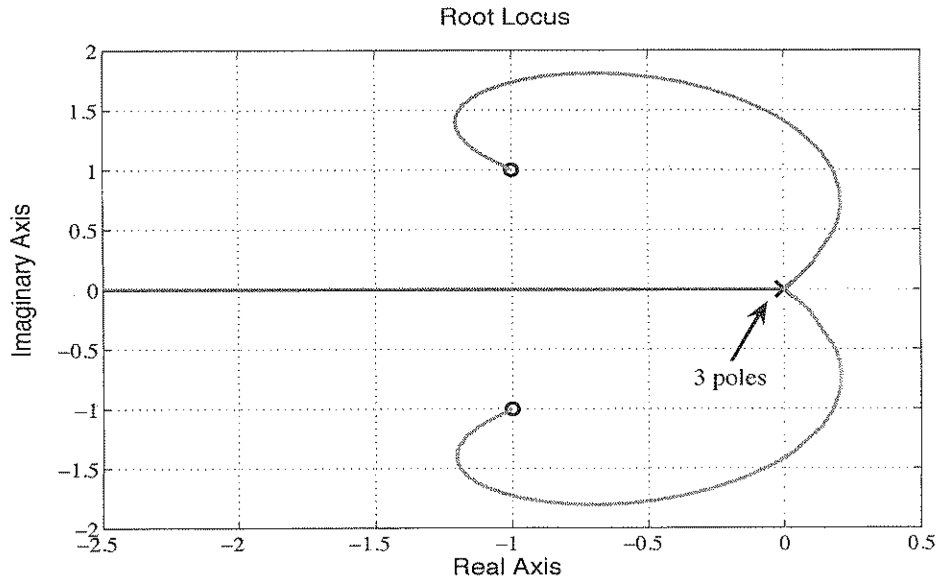

Fig. Q1-b

(a) The transfer function of the plant is $G(s) = \frac{s^2+as+b}{s^3+cs^2+ds+f}$. Find a, b, c, d and f.

[5 marks]

(b) Determine the gain, K, for which the complex conjugate poles have a damping ratio of 0.5. Find the locations of all the closed-loop poles.

[10 marks]

(c) Find all the closed-loop poles when the unit impulse response of the closed-loop system is given by $0.56e^{-1.3t} + 1.44e^{-0.35t}\cos(1.72t) + 0.72e^{-0.35t}\sin(1.72t)$.

[4 marks]

(d) Assume that, due to some unknown mechanism, the amplifier output is given by the following saturation non-linearity (instead of the proportional gain, K):

$$u = \begin{cases} 2e, & |e| \leq 1; \\ 2, & e > 1; \\ -2, & e < -1. \end{cases}$$

Describe qualitatively how you would expect the system to respond to a unit step input.

[6 marks]

Q.2 A wielding head for an auto body requires an accurate control system for positioning the wielding head. The feedback control system is shown in Figure Q2-a, where K_1 and $K_2 > 0$.

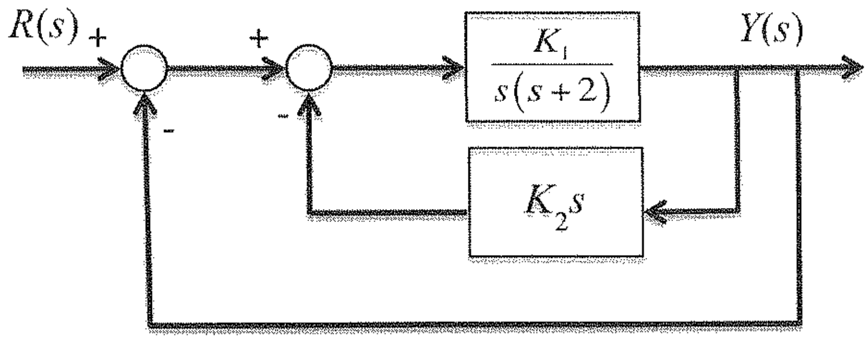

Fig. Q2-a

(a) Find the closed-loop transfer function, $\frac{Y(s)}{R(s)}$.

[5 marks]

(b) The feedback control system in Figure Q2-a is to be designed to satisfy the following specifications:

1. The settling time to within 2% of the final value ≤ 3 sec.
2. The steady-state error, ($E(s) = R(s) - Y(s)$), for a ramp input $\leq 25\%$ of input slope.

Select suitable values of K_1 and K_2 to meet the specifications.

[15 marks]

(c) Sketch the expected output response, $y(t)$, for $t > 0$, due to step input of magnitude 2. Label the axes and all critical values on your plot.

[5 marks]

Q.3

(a) Sketch the polar plot for the transfer function,

$$G(s) = \frac{4}{s^2 + 4}.$$

[6 marks]

(b) A unity negative feedback system has open loop transfer function,

$$L(s) = \frac{K(s+5)^2}{s^2(s+1)}, \quad K > 0.$$

Use Nyquist stability criterion to find the range of K for which the closed loop is stable.

[13 marks]

(c) Determine the gain margin of the open loop transfer function

$$G(s) = \frac{3(2-s)}{s^2 + 6s + 5}.$$

[6 marks]

[Hint: $\tan(A \pm B) = \frac{\tan A \pm \tan A}{1 \mp \tan A \tan B}$]

Q.4

(a) A plant is modeled using the transfer function

$$G(s) = \frac{1}{s(s+2)}.$$

Design a feedback controller to achieve the following design targets.

Gain crossover frequency:	6.6 rad/s
Phase margin:	$\geq 50°$

[16 marks]

(b) Transfer function of a dynamic system is $\frac{16}{(s+1)(s+3)}$.

What is the input applied to this system if the steady state signal observed at the output is $y(t) = 2\sin(5t)$?

[4 marks]

(c) Explain, using suitable sketch, how do you determine the velocity error constant k_v of a Type 1 system from its Bode plot.

[5 marks]

Some Useful Laplace Transform Rules

Transform of derivative, $L\left\{\frac{dy}{dt}\right\}$	$sY(s) - y(0)$
$L\left\{\dfrac{d^2y}{dt^2}\right\}$	$s^2Y(s) - sy(0) - y'(0)$
Transform of integral, $L\left\{\int_0^t y(\tau)d\tau\right\}$	$\dfrac{Y(s)}{s}$
Shift in time-domain, $L\{y(t-t_d)u(t-t_d)\}$	$Y(s)e^{-st_d}$
Shift in s-domain, $L\{y(t)e^{-at}\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

Laplace Transform of Commonly Used Functions

Function $f(t)$	Laplace Transform $F(s)$	Function $f(t)$	Laplace Transform $F(s)$
Unit impulse, $\delta(t)$	1	Unit step, $u(t)$	$\dfrac{1}{s}$
Ramp function, t	$\dfrac{1}{s^2}$	Exponential function, ke^{-at}	$\dfrac{k}{s+a}$
Sine function, $\sin(\omega t)$	$\dfrac{\omega}{s^2+\omega^2}$	Cosine function, $\cos(\omega t)$	$\dfrac{s}{s^2+\omega^2}$
Parabolic function, t^2	$\dfrac{2}{s^3}$		

Step Response of Underdamped Second Order System

Model of the prototype 2nd order underdamped system $G(s) = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Percentage Maximum overshoot, $\%M_p$	$\%M_p = e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}$
2% Settling time, t_s	$t_s = \dfrac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \dfrac{1.8}{\omega_n}$
Time of the 1st peak, t_p	$t_p = \dfrac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$