NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II: 2013/2014)

EE3331C – FEEDBACK CONTROL SYSTEMS

May 2014 – Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SEVEN** (7) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry equal marks.
4. This is a CLOSED BOOK examination.
5. Some relevant data are provided at the end of this examination paper.
6. Working MUST be provided clearly. Marks will not be awarded if working shown does not match the final answer or when there is no working.

Q.1 A closed-loop system and its root locus plot are shown in Figures Q1-a and Q1-b respectively where $K > 0$.

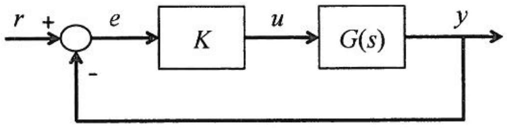

Figure Q1-a: Feedback Control System.

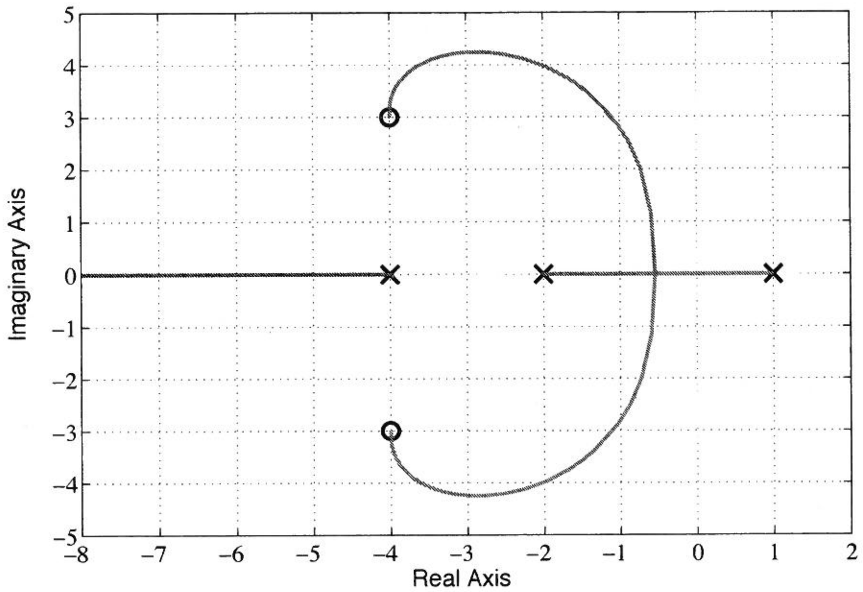

Figure Q1-b: Root Locus plot.

(a) The transfer function is $G(s) = \dfrac{(s^2 + as + b)}{(s^3 + cs^2 + ds + e)}$. Find a, b, c, d and e.

(5 marks)

(b) Determine the DC gain of the closed-loop system. Hence, find the range of K that results in steady-state error of less than 2%.

(5 marks)

(c) Find the value of K that results in a marginally stable closed-loop system.

(5 marks)

(d) Estimate the maximum overshoot due to a unit step input as $K \to \infty$.

(5 marks)

(e) When the pair of complex poles are critically damped, the poles of the closed-loop system are $-4.28, -0.55, -0.55$. Find the range of K that results in step responses with zero overshoot.

(5 marks)

Q.2 Figure Q2 shows a feedback control system.

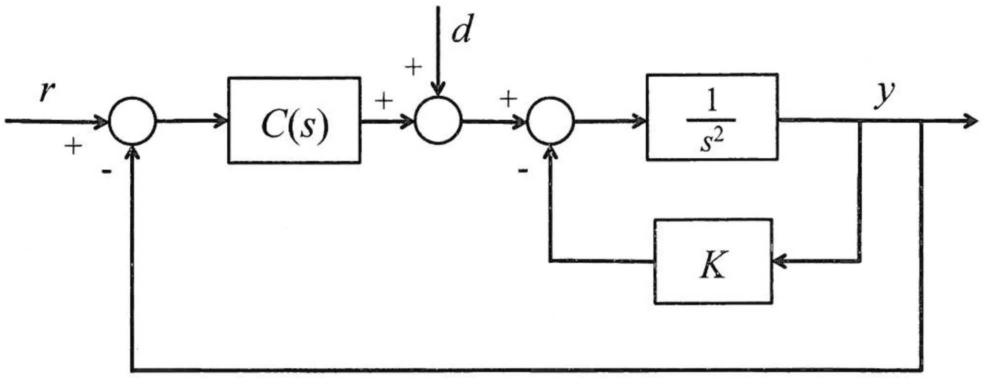

Figure Q2: Feedback Control System.

(a) Show that the closed-loop transfer function is given by

$$\frac{Y(s)}{R(s)} = \frac{C(s)}{s^2 + K + C(s)}$$

(3 marks)

(b) Assume that $C(s) = k_p$, sketch the expected output response, $y(t)$, for $t > 0$, due to a unit step input. Label the axes and all critical values on your plot.

(5 marks)

(c) What condition must $C(s)$ satisfy so that the system can track a ramp input with constant steady-state error?

(7 marks)

(d) For the transfer function, $C(s)$, that stabilizes the system and satisfies the condition in Q.2(c), find the type of disturbances, $d(t)$ (examples, step, ramp or parabolic), that the system can reject with zero steady-state error.

(5 marks)

(e) Assume that the controller, $C(s)$, is a PID controller given by

$$C(s) = k_p + \frac{k_i}{s} + k_d s$$

What constraints must k_p, k_i and k_d satisfy for the closed-loop system to be stable.
[Hint: A third order polynomial, $s^3 + a_2 s^2 + a_1 s + a_0$, will have all its roots with negative real-parts if $a_0 > 0, a_1 > 0, a_2 > 0$ and $a_2 a_1 > a_0$.]

(5 marks)

Q.3(a) Transfer function model of a plant is,

$$G(s) = \frac{1}{s(s+1)}$$

The controller is $G_c(s) = K$, where K is a constant. Find the value of K such that steady-state error for unit ramp input is 0.1. Find the gain margin and the phase margin.

(6 marks)

(b) What are the effects on gain margin, phase margin and steady-state error, if the gain K is increased?

(3 marks)

(c) Can the closed loop be unstable if the controller of Q.3(a) is implemented digitally? If it can, determine the condition for which the closed loop is marginally stable.

(4 marks)

(d) Sketch the Nyquist plot of the loop transfer function

$$L(s) = \frac{A}{s^2(\tau s+1)}$$

Both A and τ are positive real numbers. Show using Nyquist stability criterion that the resulting closed loop is unstable for all values of A.

(7 marks)

(e) Explain how the system of Q.3(d) can be stabilized by cascading another transfer function to $L(s)$.

(5 marks)

Q.4 Figure Q4-a shows a feedback control system.

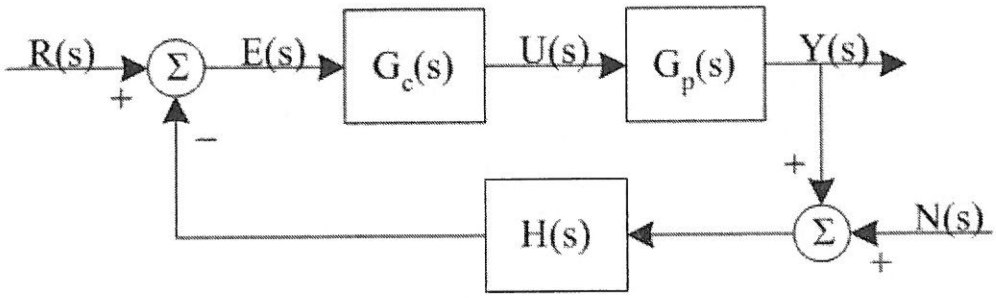

Figure Q4-a: Feedback Control System.

(a) For the feedback system shown above, the reference input is a sinusoidal signal of frequency $\omega < 1$ rad/s. It is desired to have steady-state error less than 2% for these reference signals. The spectrum of the sensor noise lies in frequencies $\omega > 100$ rad/s. It is desired to have the effect of noise on output less than 5% of the noise level. Show that a sufficient condition for meeting the specification is that the magnitude plot of the loop transfer function $L(s) = G_c(s)G_p(s)H(s)$ lies outside the shaded regions in Figure Q4-b.

(6 marks)

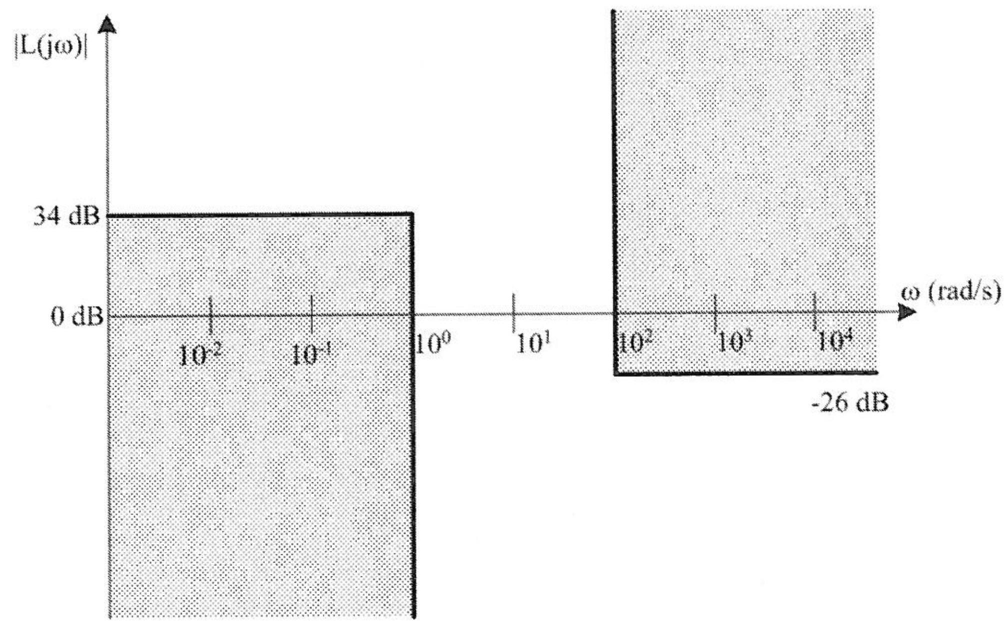

Figure Q4-b: Specifications for $L(j\omega)$.

Q.4 is continued on page 6

(b) Assume $G_c(s) = 1$, $H(s) = 1$ and

$$G_p(s) = \frac{K}{s(0.1s+1)}$$

Determine the value of K such that the loop transfer function meets the specifications for $\omega < 1$ rad/s. What is the phase margin with this value of K?

(7 marks)

(c) Design a first-order compensator such that the phase margin is increased to 45^o. Useful formulae for compensator design are,

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

and

$$\sin(\phi_m) = \frac{1-\alpha}{1+\alpha}$$

(8 marks)

(d) Does the compensator of Q4(c) meet the high frequency specification? If not, explain why it cannot be met using a first-order compensator alone.

(4 marks)

END OF PAPER

DATA SHEET :

SOME USEFUL LAPLACE TRANSFORM RULES

Transform of derivatives, $\mathcal{L}\left\{\frac{dy(t)}{dt}\right\}$	$sY(s) - y(0)$
Transform of integral, $\mathcal{L}\left\{\int_0^t y(\tau)d\tau\right\}$	$\dfrac{Y(s)}{s}$
Shift in time domain, $\mathcal{L}\{y(t-L)u(t-L)\}$	$Y(s)e^{-sL}$
Shift in s-domain, $\mathcal{L}\{y(t)e^{-at}\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

SOME USEFUL LAPLACE TRANSFORMS

Function, $f(t)$	Laplace Transform, $F(s)$	Function, $f(t)$	Laplace Transform, $F(s)$
delta function, $\delta(t)$	1	$\sin\omega t$	$\dfrac{\omega}{s^2+\omega^2}$
unit step, $u(t)$	$\dfrac{1}{s}$	$\cos\omega t$	$\dfrac{s}{s^2+\omega^2}$
t	$\dfrac{1}{s^2}$	$1-e^{-at}$	$\dfrac{a}{s(s+a)}$
ke^{-at}	$\dfrac{k}{s+a}$	$t - \dfrac{1}{a}\left(1-e^{-at}\right)$	$\dfrac{a}{s^2(s+a)}$

SOME DESIGN FORMULAE FOR UNDERDAMPED 2^{nd} ORDER SYSTEM

Standard 2^{nd} order system : $\dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Percentage overshoot, $\%M_p$	$\%M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$
Settling time (2%), t_s	$t_s = \dfrac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \dfrac{1.8}{\omega_n}$
Peak time, t_p	$t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$