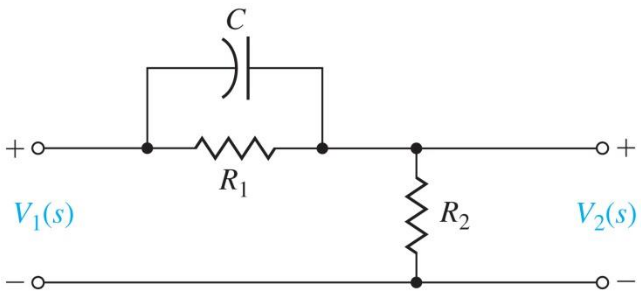
Circuit Implementation of Lead Compensator and Lag Compensator



For an RC circuit, current waveform leads in phase with reference to the waveform of the applied voltage.

Current $i(t)$ leads input voltage $v_1(t)$. The output voltage $v_2(t)$ is in-phase with the current. Therefore, voltage waveform at the output leads in phase the voltage waveform at the input.

In transfer function form: $$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \left(R_1 \,||\, \frac{1}{Cs}\right)}$$

Substituting $$\left(R_1 \,||\, \frac{1}{Cs}\right) = \frac{R_1 \times \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{R_1}{R_1 Cs + 1}$$

$$G(s) = \frac{R_2}{R_2 + \frac{R_1}{R_1 Cs + 1}}$$

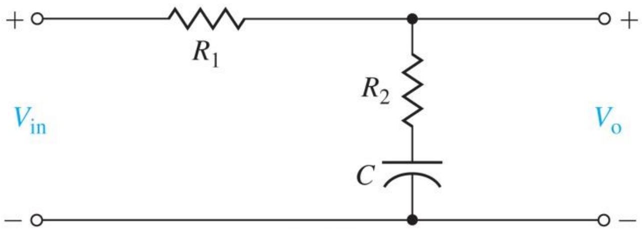
$$= \frac{R_2(R_1 Cs + 1)}{R_2 R_1 Cs + R_2 + R_1}$$

$$= \left(\frac{R_2}{R_1 + R_2}\right) \frac{(R_1 Cs + 1)}{\left(\frac{R_2}{R_1 + R_2} R_1 Cs + 1\right)}$$

Let, $R_1 C = T$ & $\dfrac{R_2}{R_1 + R_2} = \alpha$

$$G(s) = \alpha \frac{(Ts + 1)}{(\alpha Ts + 1)}$$ [It is obvious that 0<α<1, as in lead compensator.]

This circuit's dc-gain is α. Putting an amplifier with gain ($1/\alpha$) in cascade can bring the dc-gain to unity.



Current $i(t)$ leads input voltage $v_{\text{in}}(t)$ by $\theta_1 = \tan^{-1}\dfrac{1}{\omega\tau_{12}}$

τ_{12} is time constant of the whole circuit, i.e., $\tau_{12} = (R_1 + R_2)C$

Current $i(t)$ lead output voltage $v_{\text{o}}(t)$ by $\theta_2 = \tan^{-1}\dfrac{1}{\omega\tau_2}$

τ_2 is time constant of the vertical branch, i.e., $\tau_2 = R_2 C$

Considering waveform of current $i(t)$ as reference, both $v_{\text{in}}(t)$ and $v_{\text{o}}(t)$ lags the current waveform. Input voltage lags current by θ_1 and output voltage by θ_2.

As $\tau_{12} > \tau_2$, $\tan^{-1}\dfrac{1}{\omega\tau_{12}} < \tan^{-1}\dfrac{1}{\omega\tau_2} \Rightarrow \theta_1 < \theta_2$

Waveform of $v_{\text{o}}(t)$ is more lagging than the waveform of $v_{\text{in}}(t)$ with reference to the current waveform. The output voltage lags the input voltage.

In transfer function form:

$$G(s) = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{R_2Cs + 1}{(R_1 + R_2)Cs + 1} = \frac{(R_2Cs + 1)}{\left(\frac{R_1+R_2}{R_2} R_2Cs + 1\right)}$$

Let, $R_2C = T$ & $\dfrac{R_1 + R_2}{R_2} = \alpha$

$$G(s) = \frac{(Ts + 1)}{(\alpha Ts + 1)}$$

[It is obvious that α>1, as in lag compensator.]

This circuit's dc-gain is 1.