NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2013/2014)

EE3331C/EE3331E – FEEDBACK CONTROL SYSTEMS

November 2013 – Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry equal marks.
4. This is a CLOSED BOOK examination.
5. Some relevant data are provided at the end of this examination paper.

Q.1 A closed-loop system and its root locus plot are shown in Figures Q1-a and Q1-b respectively.

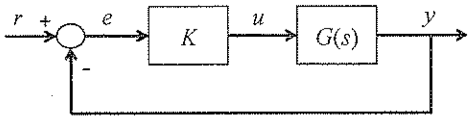

Figure Q1-a: Feedback Control System.

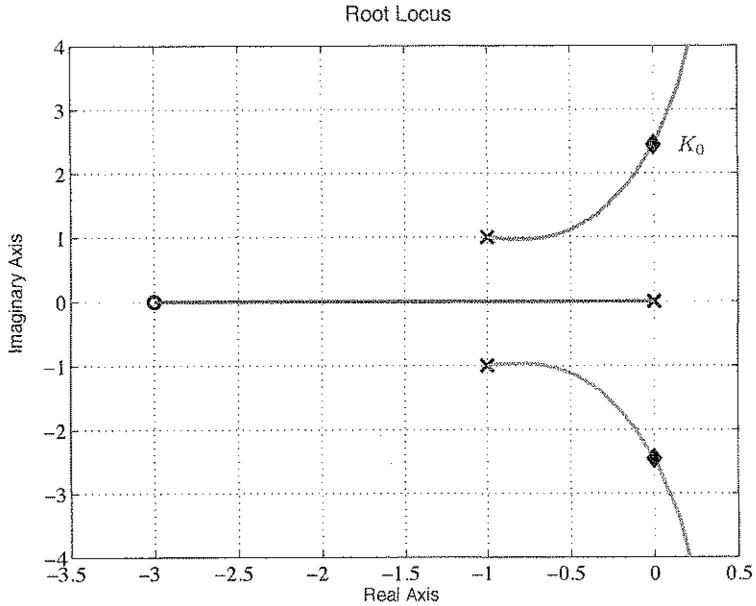

Figure Q1-b: Root Locus plot.

(a) The transfer function is $G(s) = \dfrac{(s+a)}{(s^3 + bs^2 + cs + d)}$. Find a, b, c and d.

(4 marks)

(b) Determine the maximum gain, K_0, before the system becomes unstable. Find the locations of all the closed-loop poles when $K = K_0$.

(10 marks)

(c) When $K = 0.5$, is it reasonable to use the 2% settling-time formula to estimate the settling-time of the closed-loop system? Explain your answer. (Hint: One of the poles is located at $s = -1$ when $K = 0.5$.)

(6 marks)

(d) We are now interested to investigate how the closed-loop poles vary as the parameter, b, in $G(s)$ vary. Explain how the root locus method can be used to study the effect of b on closed-loop stability. What is the new K and $G(s)$ in Figure Q1-a. You may assume that the original K is now equal to 1.

(5 marks)

Q.2 Figure Q2 shows a feedback control system, where k_1,k_2 and $k_3 > 0$.

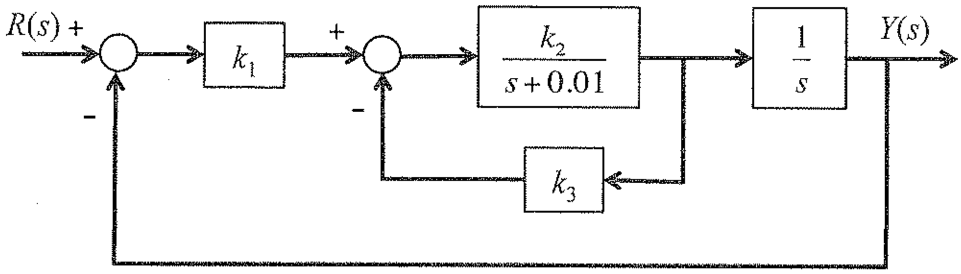

Figure Q2: Feedback Control System.

(a) Show that the closed-loop transfer function is given by

$$\frac{Y(s)}{R(s)} = \frac{k_1 k_2}{s^2 + (0.01 + k_2 k_3)s + k_1 k_2}$$

(7 marks)

(b) Determine the steady-state error ($E(s) = R(s) - Y(s)$) to a ramp input $r(t) = t$, $t \geq 0$ in terms of k_1, k_2 and k_3. Let $k_2 = 10$ and $k_3 = 0.05$, and find k_1 such that the steady-state error to the unit ramp input is equal to 1.

(7 marks)

(c) Using the values of k_1, k_2 and k_3 from part 2(b), sketch the output response, $y(t)$ when the input signal, $r(t)$, is a step of magnitude of 2. Label your axes and indicate clearly the rise-time, 2% settling time and maximum overshoot.

(7 marks)

(d) Discuss the effect of varying k_1 and k_3 on the closed-loop step response.

(4 marks)

Q.3(a) Sketch the Bode (magnitude) plot for the transfer function,

$$G(s) = \frac{4(s+2)}{s(s+50)}$$

Indicate the slopes of different segments of the plot and the gain-crossover frequency. Write another stable transfer function (without any transportation delay) that has magnitude plot identical to that of the above transfer function but different phase plot.

(7 marks)

(b) For a unity negative feedback system, the characteristic equation is,

$$(s+2)(s^2+2s+5)+K=0, \qquad K>0$$

Use Nyquist stability criterion to find the range of K for which the closed loop is stable.

$$\left[\text{Hint} : \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

(13 marks)

(c) Find the phase margin of the following transfer function

$$G(s) = \frac{3}{s(s+3)}e^{-2s}$$

(5 marks)

Q.4 (a) Consider the following compensator transfer function

$$G_c(s) = \frac{Ts+1}{\alpha Ts+1}, \qquad 0 < \alpha < 1$$

Show that maximum phase contribution of this transfer function occurs at the frequency (ω_m) where,

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

$$\left[\text{Hint : } \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}\right]$$

(5 marks)

(b) Find an expression for the maximum phase contribution (ϕ_m) of the compensator in Q.4(a) in terms of the compensator parameters and show that $\phi_m > 0$.

(5 marks)

(c) The open loop transfer function of a unity feedback system is

$$G_o(s) = \frac{K}{s(s+2)}, \qquad K > 0$$

Design a lead compensator to achieve velocity error constant of 20 $\sec^{-1}$ and a phase margin of at least 50^o.

(15 marks)

END OF PAPER

DATA SHEET :

SOME USEFUL LAPLACE TRANSFORM RULES

Transform of derivatives, $\mathcal{L}\left\{\frac{dy(t)}{dt}\right\}$	$sY(s) - y(0)$
Transform of integral, $\mathcal{L}\left\{\int_0^t y(\tau)d\tau\right\}$	$\frac{Y(s)}{s}$
Shift in time domain, $\mathcal{L}\{y(t-L)u(t-L)\}$	$Y(s)e^{-sL}$
Shift in s-domain, $\mathcal{L}\{y(t)e^{-at}\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

SOME USEFUL LAPLACE TRANSFORMS

Function, $f(t)$	Laplace Transform, $F(s)$	Function, $f(t)$	Laplace Transform, $F(s)$
delta function, $\delta(t)$	1	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
unit step, $u(t)$	$\frac{1}{s}$	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
t	$\frac{1}{s^2}$	$1-e^{-at}$	$\frac{a}{s(s+a)}$
ke^{-at}	$\frac{k}{s+a}$	$t-\frac{1}{a}\left(1-e^{-at}\right)$	$\frac{a}{s^2(s+a)}$

SOME DESIGN FORMULAE FOR UNDERDAMPED 2^{nd} ORDER SYSTEM

Standard 2^{nd} order system : $\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Percentage overshoot, $\%M_p$	$\%M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$
Settling time (2%), t_s	$t_s = \frac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \frac{1.8}{\omega_n}$
Peak time, t_p	$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$