NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2015/2016)

EE3331C/EE3331E – FEEDBACK CONTROL SYSTEMS

Nov 2015 – Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry equal marks.
4. This is a CLOSED BOOK examination.
5. Some relevant data are provided at the end of this examination paper.
6. Working MUST be provided clearly. Marks will not be awarded if working shown does not match the final answer or when there is no working.

Q.1 A closed-loop system and its root locus plot are shown in Figures Q1-a and Q1-b respectively where $K > 0$.

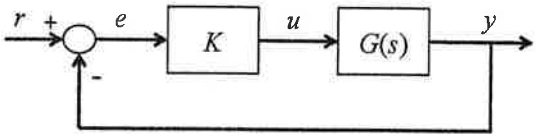

Figure Q1-a: Feedback Control System.

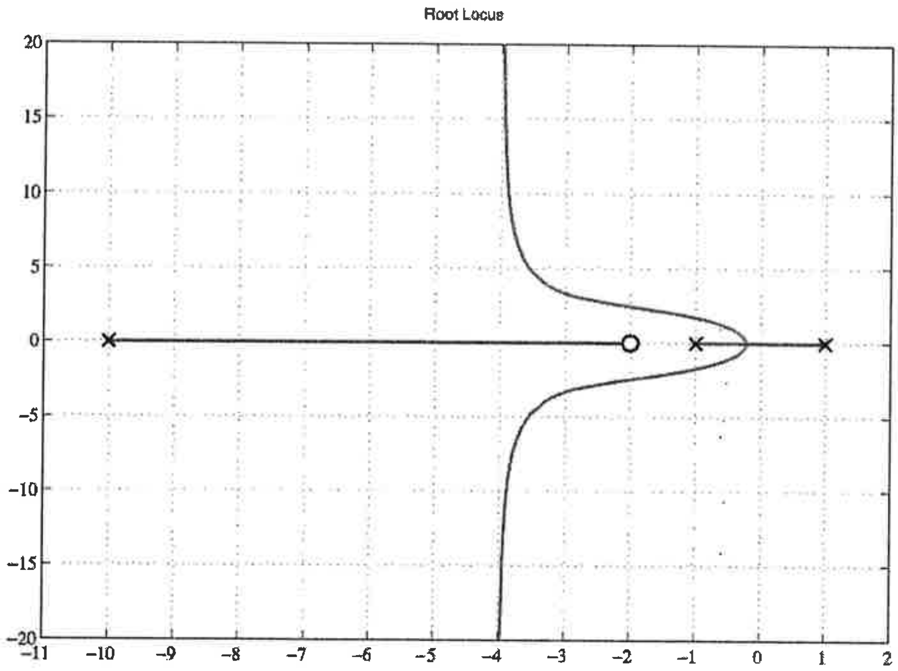

Figure Q1-b: Root Locus plot.

(a) The plant is $G(s) = \dfrac{(s+a)}{(s^3 + bs^2 + cs + d)}$. Find a, b, c and d.

(5 marks)

(b) Find the range of K for the closed-loop system to be stable? Find all the corresponding closed-loop poles. Do not use your calculator to solve polynomial equations when computing the closed-loop poles.

(8 marks)

(c) The poles of the closed-loop system when $K = 21$ are $-8, -1 \pm j\sqrt{3}$. Find a second-order transfer function that may be used to approximate the behaviour of the third-order closed-loop system, $\dfrac{Y(s)}{R(s)}$.

(7 marks)

Q.1 is continued on page 3

(d) Using the transfer function obtained in part 1(c), calculate the time taken for the closed-loop step response to settle within $\pm 5\%$ of the final value (5% settling time). Hint: The unit step responses of second order systems is given by

$$\mathcal{L}^{-1}\left\{\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}\right\} = K\left\{1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left[\left(\omega_n\sqrt{1-\zeta^2}\right)t+\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]\right\}$$

(5 marks)

Q.2 Figure Q2 shows a feedback control system where $G(s) = \dfrac{k_\alpha}{s+\alpha}$.

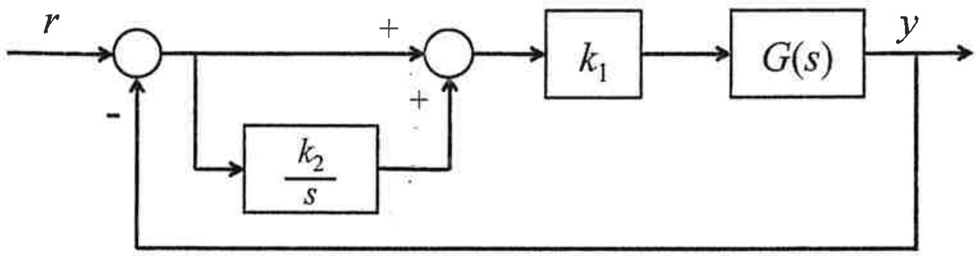

Figure Q2: Feedback Control System.

(a) Find the closed-loop transfer function, $\dfrac{Y(s)}{R(s)}$ in terms of k_1, k_2, k_α and α.

(5 marks)

(b) The feedback control system in Figure Q2 is to be designed to satisfy the following specifications:

- The percentage maximum overshoot to a unit step input is 10%.
- The steady-state error, $(E(s) = R(s) - Y(s))$, for a ramp input to be equal to $\dfrac{a}{8}$, where a is the ramp magnitude.

Find k_1 and k_2 to meet the specifications. You may express your answer in terms of k_α, α and a.

(15 marks)

(c) Prove that no matter what the values of K_α and α are, the controller provides enough flexibility to place the poles anywhere in the complex (left-half) plane.

(5 marks)

Q.3 (a) Find the phase-crossover frequency, ω_{cp}, of the transfer function

$$\frac{10}{(s+2)(s^2+2s+5)}.$$

(4 marks)

(b) Sketch the Bode magnitude and phase plots of

$$G(s) = \frac{(s+2)(s-5)}{(s+1)(s+10)}.$$

(6 marks)

(c) The Nyquist contour and the Nyquist plot of the loop transfer function $L(s) = \dfrac{K(s-1)}{s(s+2)}$ are shown in Figure Q3. Find the range of K for which the resulting closed loop is stable.

(5 marks)

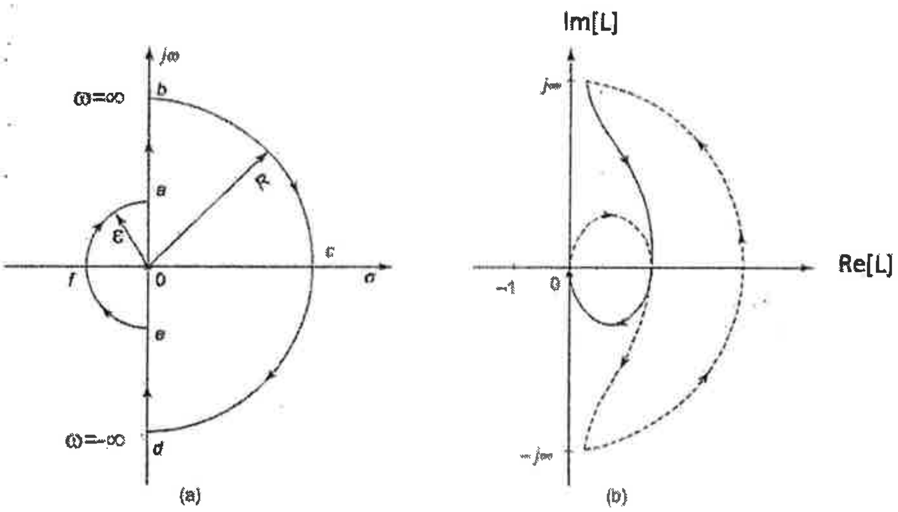

Figure Q3: The Nyquist contour and the Nyquist plot.

(d) Sketch the Nyquist plot for the loop transfer function

$$L(s) = \frac{K(1+4s)}{s^2(1+s)(1+2s)}.$$

(7 marks)

(e) How can you find the velocity error constant, k_v, of a type 1 system from its Bode plot?

(3 marks)

Q.4(a) Find the value of K such that gain margin is 10 dB for the loop transfer function

$$L(s) = \frac{K}{(s+10)(s+1)^2}.$$

(5 marks)

(b) What are the principal effects of lead compensation on system performance?

(4 marks)

(c) The uncompensated loop transfer function of a unity feedback system is

$$L_u(s) = \frac{40}{s(s+2)}.$$

i. Show that the steady-state error is 0.05 with unit ramp input.

(2 marks)

ii. What is the error if the reference is sinusoid with frequency 1 rad/sec and amplitude of 1 unit?

(4 marks)

iii. Design a compensator to improve the phase margin to 50° without compromising the performance for sinusoidal reference input.

(10 marks)

$$\left[\text{Hint: for compensator } C(s) = \frac{Ts+1}{\alpha Ts+1}, \quad \omega_m = \frac{1}{T\sqrt{\alpha}}, \quad \sin\phi_m = \frac{1-\alpha}{1+\alpha}\right]$$

END OF PAPER

DATA SHEET :

SOME USEFUL LAPLACE TRANSFORM RULES

Transform of derivatives, $\mathcal{L}\left\{\frac{dy(t)}{dt}\right\}$	$sY(s) - y(0)$
Transform of integral, $\mathcal{L}\left\{\int_0^t y(\tau)d\tau\right\}$	$\frac{Y(s)}{s}$
Shift in time domain, $\mathcal{L}\{y(t-L)u(t-L)\}$	$Y(s)e^{-sL}$
Shift in s-domain, $\mathcal{L}\{y(t)e^{-at}\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

SOME USEFUL LAPLACE TRANSFORMS

Function, $f(t)$	Laplace Transform, $F(s)$	Function, $f(t)$	Laplace Transform, $F(s)$
delta function, $\delta(t)$	1	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
unit step, $u(t)$	$\frac{1}{s}$	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
t	$\frac{1}{s^2}$	$1-e^{-at}$	$\frac{a}{s(s+a)}$
ke^{-at}	$\frac{k}{s+a}$	$t-\frac{1}{a}\left(1-e^{-at}\right)$	$\frac{a}{s^2(s+a)}$

SOME DESIGN FORMULAE FOR UNDERDAMPED 2^{nd} ORDER SYSTEM

Standard 2^{nd} order system : $\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Percentage overshoot, $\%M_p$	$\%M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$
Settling time (2%), t_s	$t_s = \frac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \frac{1.8}{\omega_n}$
Peak time, t_p	$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$