NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2012/2013)

EE3331C – FEEDBACK CONTROL SYSTEMS

Nov 2012 **-** Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) printed pages.

2. All questions are compulsory. Answer **ALL** questions.

3. This is a **CLOSED BOOK** examination.

4. On page 6 of this question paper, the following tables are provided:
   a) Useful Laplace Transform Rules
   b) Table of useful Laplace Transforms
   c) Useful design formulae

Q.1 A closed-loop control system and its root locus plot are shown in Figures Q1-a and Q1-b respectively.

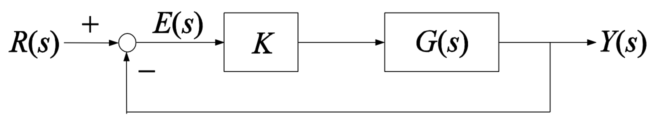

Fig. Q1-a

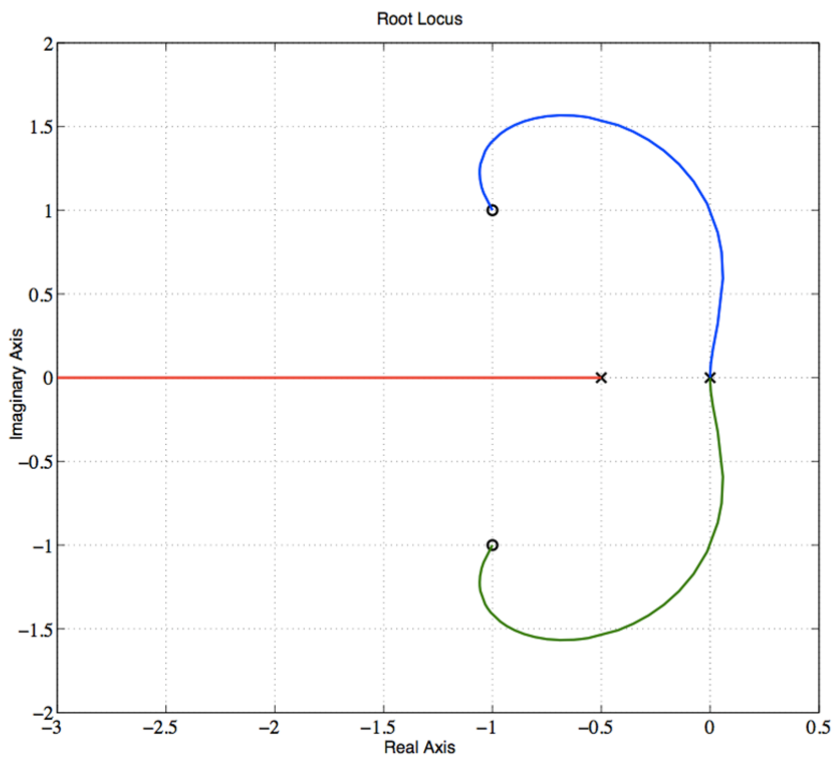

Fig. Q1-b

(a) The transfer function of the plant is $G(s) = \frac{s^2+as+b}{s^3+cs^2+ds+e}$. Find a, b, c, d and e.

[5 marks]

(b) What is the minimum gain, K, for the closed-loop system to be stable?

[7 marks]

(c) Determine the gain, K, for which the complex conjugate poles have a damping ratio of 0.5. Find the locations of the complex poles? [Hint: the third pole is located at s = -2.32]

[7 marks]

(d) Find the value of K such that the steady-state error (E(s) = R(s) – Y(s)) to a parabolic signal, $r(t) = 2t^2$, is 0.5.

[6 marks]

Q.2 A DC motor control system is shown in Fig. Q2-a, where K_1 and $K_2 > 0$.

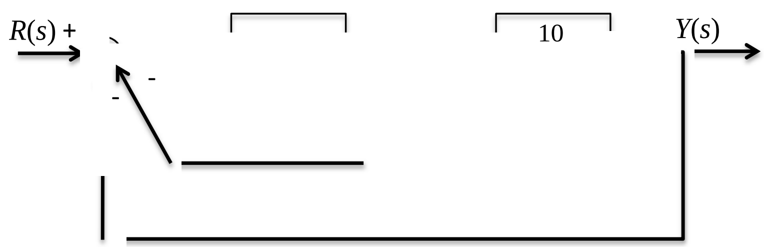

Fig. Q2-a

(a) Sketch the region in the s-plane where the complex poles should be located to meet the following specifications: i) the 2% settling time is less than 0.5 seconds, ii) the maximum overshoot less than 5% for a step input.

[5 marks]

(b) Select suitable values of K_1 and K_2 to meet the specifications in Q2(a).

[8 marks]

(c) Sketch the expected output response, $y(t)$, for $t > 0$, due to step input of magnitude 2. Label the axes and all critical values on your plot.

[5 marks]

(d) Using the values of K_1 from part Q2(b), find the range of values of K_2 such that the closed-loop system has an underdamped response to a step input. Suppose K_2 undergoes a small perturbation: $K_2 \rightarrow K_2 + \delta K_2$, what effect does this have on the system response?

[7 marks]

Q.3

(a) Sketch the polar plot for the transfer function $\frac{K(s-1)}{s(s+2)}$.

[4 marks]

(b) Draw the Nyquist plot for the system in Fig. Q3-b. Using the Nyquist stability criterion, determine the range of K for which the system is stable. Consider both positive and negative values of K.

[8 marks]

$$G(s) = \frac{1}{s^2 + 2s + 2}, \qquad H(s) = \frac{1}{s + 1}$$

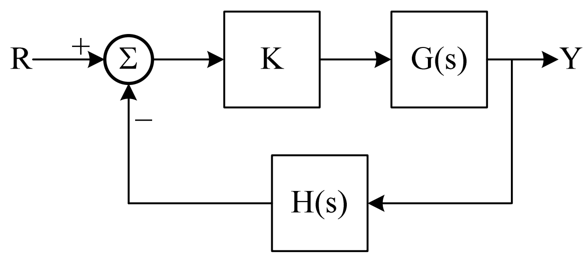

Fig. Q3-b

(c) A magnetic tape-drive speed control system is shown in Fig. Q3-c. Parameters of different components are, J = 2 N.m.sec^2/rad, b = 1 N.m.sec, τ_m = 1 sec and K = 0.7.

i. What is the gain margin of this system?

Hint: $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

[4 marks]

ii. Estimate the gain crossover frequency from the asymptotic Bode magnitude plot and use it to determine the phase margin. Is this a good system design?

[9 marks]

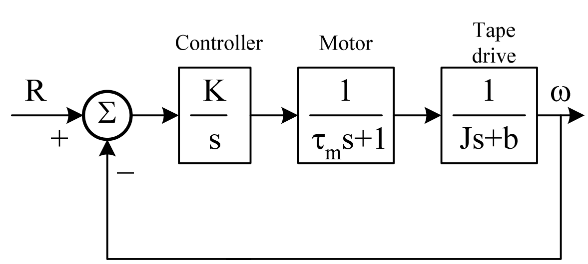

Fig. Q3-c

Q.4

Consider the open loop transfer function

$$G(s) = \frac{K}{s(s+8)}$$

where, K is the gain of the amplifier.

(a) Select the gain K such that the resulting velocity error constant $K_v \geq 100 \text{ sec}^{-1}$. What is the corresponding phase margin?

[6 marks]

(b) Design a compensator such that the phase margin of the compensated system is increased to 50° without compromising the low frequency characteristics.

[16 marks]

(c) What is the gain margin of the compensated system?

[3 marks]

END OF PAPER

Some Useful Laplace Transform Rules

Transform of derivative, $L\left\{\frac{dy}{dt}\right\}$	$sY(s) - y(0)$
$L\left\{\frac{d^2y}{dt^2}\right\}$	$s^2Y(s) - sy(0) - y'(0)$
Transform of integral, $L\left\{\int_0^t y(\tau)d\tau\right\}$	$\frac{Y(s)}{s}$
Shift in time-domain, $L\{y(t-t_d)u(t-t_d)\}$	$Y(s)e^{-st_d}$
Shift in s-domain, $L\{y(t)e^{-at}\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

Laplace Transform of Commonly Used Functions

Function f(t)	Laplace Transform F(s)	Function f(t)	Laplace Transform F(s)
Unit impulse, δ(t)	1	Unit step, u(t)	$\frac{1}{s}$
Ramp function, t	$\frac{1}{s^2}$	Exponential function, ke^{-at}	$\frac{k}{s+a}$
Sine function, sin(ωt)	$\frac{\omega}{s^2+\omega^2}$	Cosine function, cos(ωt)	$\frac{s}{s^2+\omega^2}$
Parabolic function, t^2	$\frac{2}{s^3}$		

Step Response of Underdamped Second Order System

Model of the prototype 2nd order underdamped system $G(s) = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Maximum overshoot, M_p	$M_p = Ke^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}$
Settling time, t_s	$t_s = \frac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \frac{1.8}{\omega_n}$
Time of the 1st peak, t_p	$t_p = \frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$