Answer to Past Exam Papers (Frequency Domain Methods)

Semester 2, AY2014-15

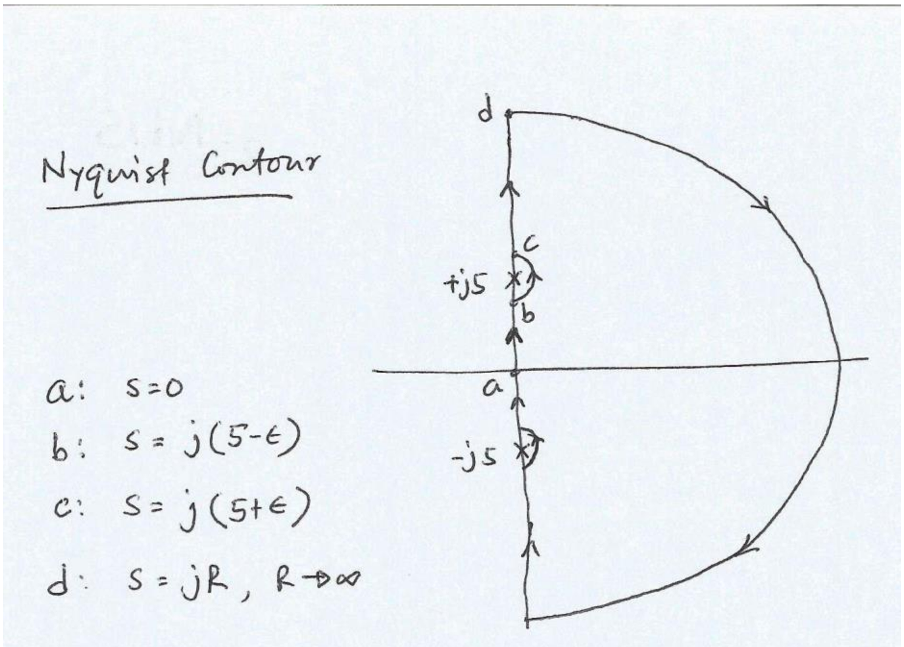
Q.3 (a) $N = 1, \quad K = 10$

Q.3.(b) (i) e_{ss} = 0 for step, e_{ss} = 100% for ramp, and $e_{ss}=\infty$ for parabolic input

(ii) PM of the uncompensated loop is 10.5°

(iii) $C(s) = \dfrac{0.25s+1}{0.3\times 0.25s+1}$

Q.4 (a)



$$L(j\omega) = K\frac{25}{25-\omega^2}$$

For the segment between 'a' and 'b', $(25-\omega^2)>0$ and therefore $\angle L(j\omega) = 0°$

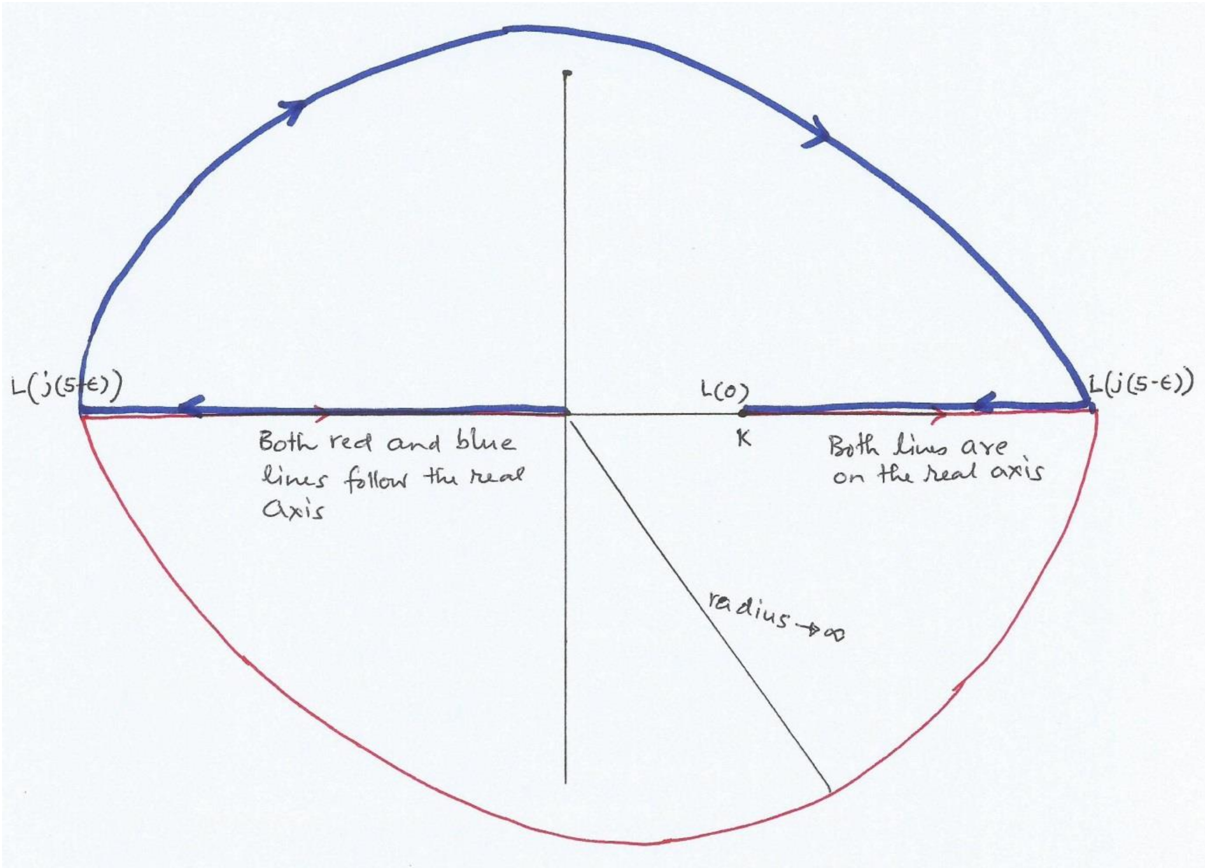
$|L(j\omega)|$ increases with increasing value of ω in this range and approaches ∞ as ω approaches $(5-\varepsilon)$.

For the segment between 'a' and 'b', $(25-\omega^2)<0$ and therefore $\angle L(j\omega) = -180°$

$|L(j\omega)|$ decreases with increasing value of ω in this range and approaches 0 as ω approaches ∞.

For the segment between 'b' and 'c', In the s-plane, the pole is bypassed via an arc rotating by 180° in counter-clockwise direction. In the L(s)-plane, the corresponding L(s) plot is rotated by 180° in clockwise direction at an infinite radius.

The complete Nyquist plot is shown below:



The Nyquist plot passes through the point (-1,0). Hence the closed loop is marginally stable.

(ii) With PD controller, we add a zero to the loop transfer function.

$$G_c(s) = K_P + K_D s = K_P(\tau s + 1)$$

So the modified loop transfer function is: $L(s) = K_{new}\dfrac{(\tau s+1)}{s^2+25}$

$$L(j\omega) = K_{new}\frac{(j\omega\tau+1)}{25-\omega^2}$$

For the segment between 'a' and 'b', $(25-\omega^2)>0$ and therefore $\angle L(j\omega) = \tan^{-1}(\omega\tau)$

$$|L(j\omega)| = K_{new}\frac{\sqrt{\omega^2\tau^2+1}}{25-\omega^2}$$

$|L(j\omega)|$ increases with increasing value of ω in this range and approaches ∞ as ω approaches $(5-\varepsilon)$.

$$\angle L(j\omega) = \tan^{-1}(\omega\tau)$$

$\angle L$ increases with increasing value of ω and approaches $\tan^{-1}(5-\varepsilon)\tau \approx \tan^{-1}(5\tau)$ as ω approaches $(5-\varepsilon)$.

For the segment between 'c' and 'd', $(25-\omega^2)<0$ and therefore $\angle L(j\omega) = -180° + \tan^{-1}(\omega\tau)$

$$|L(j\omega)| = K_{new}\frac{\sqrt{\omega^2\tau^2+1}}{25-\omega^2}$$

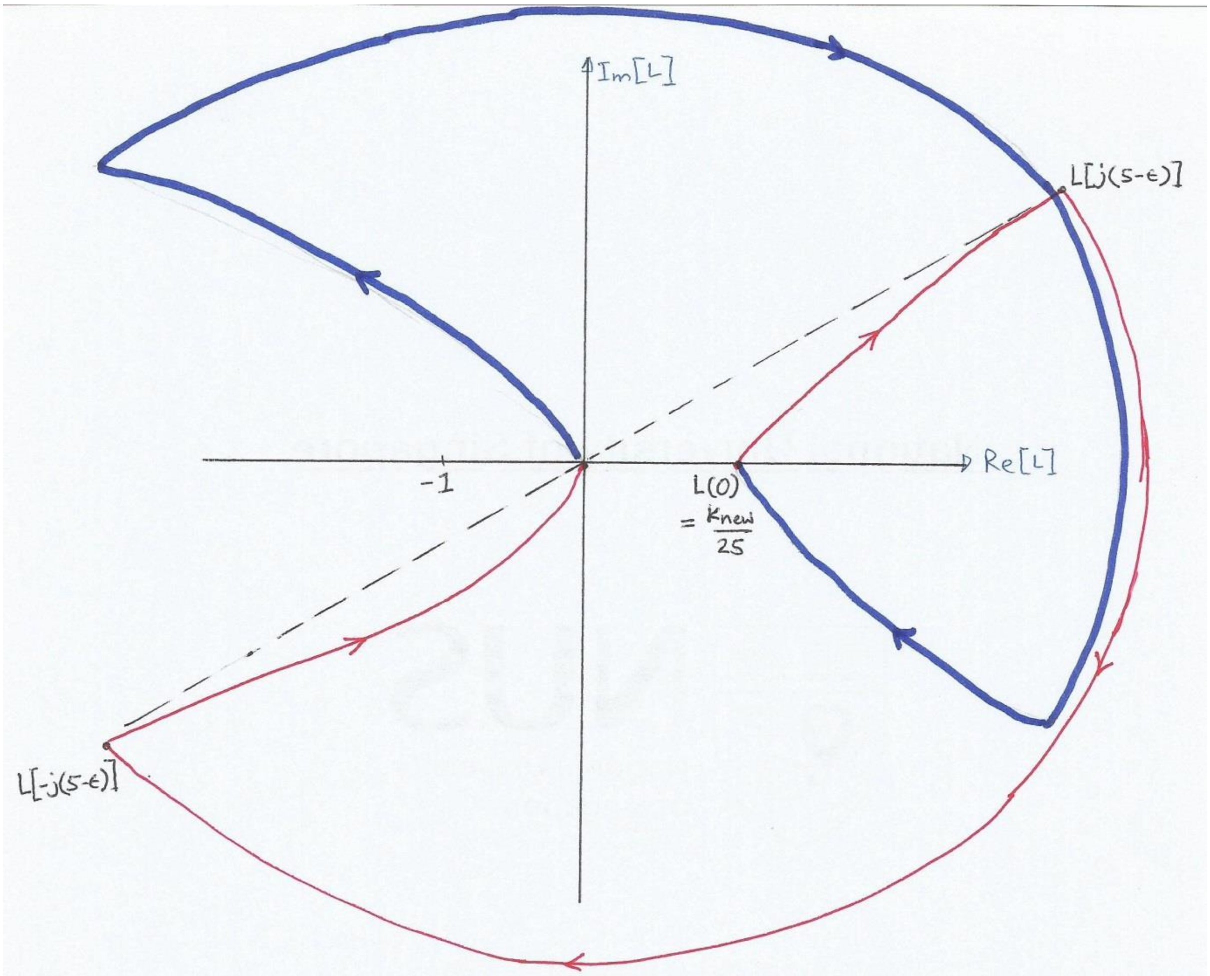
$|L(j\omega)|$ decreases with increasing value of ω in this range and approaches 0 as ω approaches ∞.

$$\angle L(j\omega) = -180° + \tan^{-1}(\omega\tau)$$

$\angle L$ approaches -90° as ω approaches ∞.

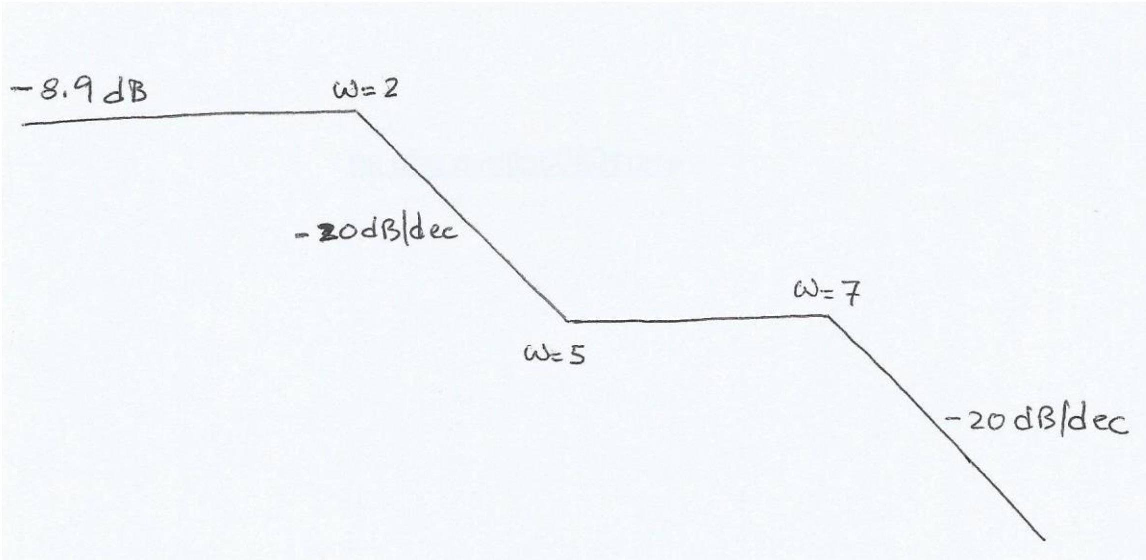
For the segment between 'b' and 'c', In the s-plane, the pole is bypassed via an arc rotating by 180° in counter-clockwise direction. In the L(s)-plane, the corresponding L(s) plot is rotated by 180° in clockwise direction at an infinite radius.

The complete Nyquist plot is shown below:



The Nyquist plot doesn't encircle the point (-1,0). Hence the closed loop is stable.

Q.4 (b) (i)



(ii) $\dfrac{s-5}{s^2+9s+14}$

Q.4 (c) ω_m = 20 rad/s, ϕ_m = 36.9°

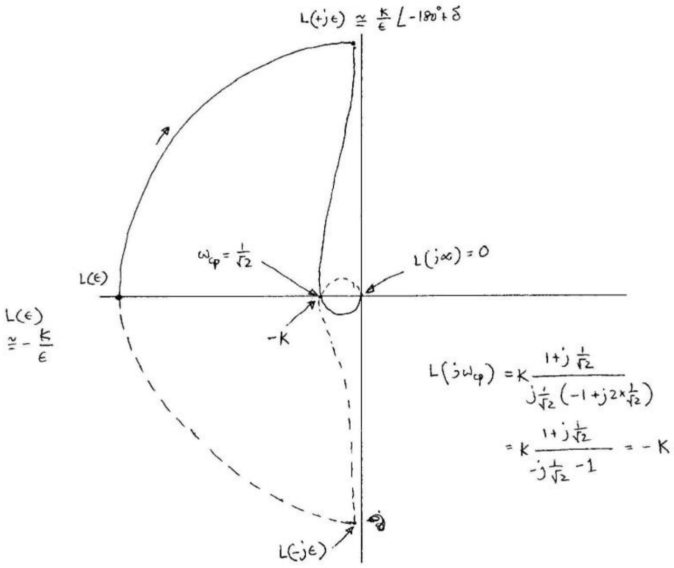
Semester 1, AY2014-15

Q.2 (a) $\dfrac{1.97(s+50)}{s(s+1)}$

Q.2 (b) $\omega_{cp} = \sqrt{6}\, rad/s, \quad K = 6$

Q.2 (c) The resulting closed loop is stable for K>1

The Nyquist plot is shown below.



Q.3 (a) $C_2(s) = \dfrac{K}{s}, \quad K > 279.9$ Let K = 280.

Q.3 (b) $\omega_{cg} = 16.7\, rad/s, \quad PM \cong 3.5°$

Q.3 (c) lead compensator is required as gain-crossover is to be higher than that without compensation

Following parameters of the compensator are obtained using $\phi_m \approx 35°$

$$\alpha = 0.27, \quad T \cong 0.08, \quad C(s) = \frac{0.08s+1}{0.27 \times 0.09s+1}$$

Semester 2, AY2013-14

Q.3 (a) K=10, GM infinity, PM=18°

Q.3 (b) with increasing values of K, phase margin decreases, gain margin remains infinity.

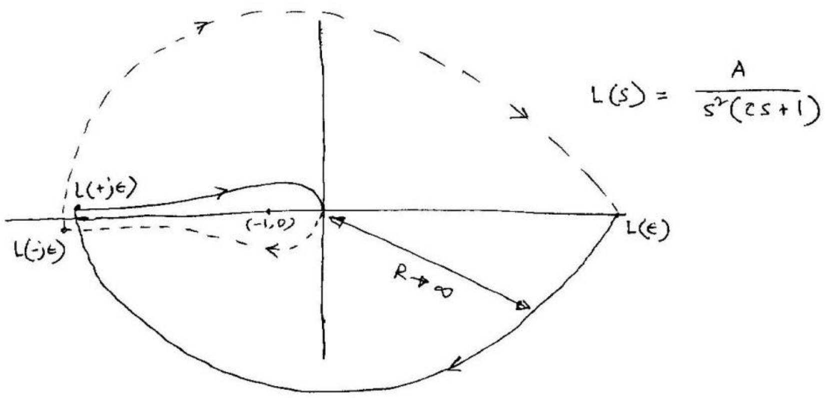
e_{ss} for step input is zero, e_{ss} for ramp input decreases with increasing K

Q.3 (c) digital implementation introduces delay which may make the closed loop unstable

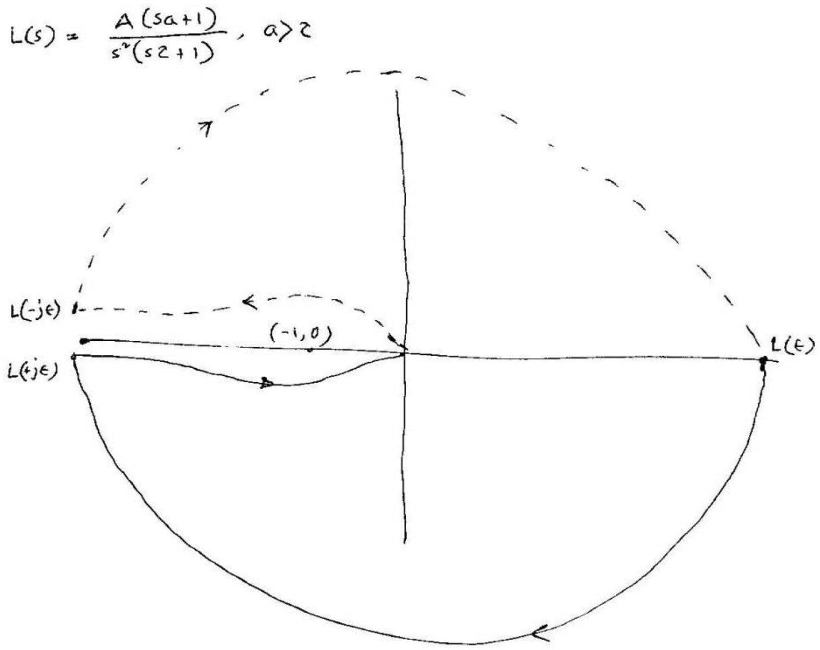
If computational delay is t_d second, the system is marginally stable if there exists a frequency ω_x such that,

$$K\left|G_p(j\omega_x)\right|=1, \quad \angle G_p(j\omega_x)-(\omega_x t_d)\times\frac{180^\circ}{\pi}=-180^\circ$$

Q.3 (d) Sketch the Nyquist plot and show that (-1,0) is encircled twice regardless the value of A



Q.3 (e) Redraw the Nyquist plot after adding either one differentiator or one real-axis zero (as+1) with a>τ. Show that the new Nyquist plot doesn't encircle (-1,0). It can also be done by adding two zeros.



Q.4 (a)

$$\varepsilon_s < 0.02 \quad \Rightarrow \quad |L(j\omega)| > \frac{1+\varepsilon_s}{\varepsilon_s} = 51, \quad 20\log(51) = 34$$

$$\varepsilon_T < 0.05 \quad \Rightarrow \quad |L(j\omega)| < \frac{\varepsilon_T}{1+\varepsilon_T} = 0.05, \quad 20\log(0.05) = -26$$

Q.4 (b) K>50

With K = 50, phase margin is 25°

Q.4 (c) Lead compensator designed with buffer phase of 5°

$$C(s) = \frac{0.06s+1}{0.4\times 0.06s+1}$$

Compensator is designed when K = 50 chosen in Q.4(b).

$\phi_m = 25°$, $\alpha = 0.4$, $\omega_m = 27.4$

Q.4 (d) For the compensated loop transfer function,

$$\begin{aligned}|L(j100)| &= |L_u(j100)| \times |C(j100)| \\ &= |L_u(j100)| \times 2.34\end{aligned}$$

Gain at ω=100 of the compensated system is increased by 20log(2.34) = 7.4 dB

Semester 1, AY2013-14

Q.3 (b) Stable for K<26

Q.3 (c) PM = -36.5°

Q.4 (c) K=40 to meet the requirement of velocity error constant

$\alpha = 0.25, \quad \omega_m = 8.83\, rad/s, \quad T = 0.23$

$$C(s) = \frac{0.23s+1}{0.25 \times 0.23s+1}$$

This result is obtained with phase margin allowance, δ_ϕ=5°. Compensated phase margin is 49.5°, ω_{cg} = 8.89 rad/s

Semester 2, AY2012-13

Q.3 (b) Stable for K>1.5

Q.3 (c) GM = 6 dB

Q.4 (a) Q(3) of Tutorial 2/ solution provided in Tutorial

Q.4 (b) $u(t) = 3.7\sin(5t + 137.7°)$

Semester 1, AY2012-13

Q.3 (b) For positive K, stable if 0< K <10

For negative K, stable if -2< K < 0

Q.3(c) (i) GM 6.6 dB

(ii) $\omega_{cg} \approx$ 0.6 rad/s, PM $\approx$ 9°.

It is not good design as the resulting damping is very low.

Q.4 (a)(b) Q(6) of Tutorial 3/ solution provided

Q.4 (c) infinite GM

Semester 2, AY2011-12

Q.3 (b) stable if K > (1/2)

Q.3 (c)

PM is $\phi = 40°$.

There are several GMs. If the system gain is increased by $1/\alpha$ (multiplied by $1/\alpha$) or decreased by the factor β (divided by β), the system becomes unstable.

It is obvious from the plot that the transfer function contains one integrator (low frequency point starts along the negative imaginary axis). We can complete the Nyquist plot by adding an arc of infinite radius and taking mirror image.

If (-1,0) is to the left of ω_0, there will be no encirclement. Closed loop is stable.

If the gain is increased so that (-1,0) lies between ω_0 and ω_L, there will be two clockwise encirclement. CL is unstable with two RHP poles.

If the gain is further increased so that the point (-1,0) lies between ω_L and ω_H, resulting in one CW and one CCW encirclement. CL is stable.

If gain is further increased so that the point (-1,0) lies between ω_H and the origin, there will be two CW encirclement. CL is unstable with two RHP poles.

Q.3(d) $\omega_{cp} \approx 2.45$ rad/s, GM ≈ 10.5 dB.

Q.4(a) this problem is better solved using root locus method (it is not taught anymore)

We can approximate the specifications in frequency domain and then solve using frequency domain method of compensator design. Following answer is obtained using that approach.

Maximum overshoot 16% $\Rightarrow$ $\zeta \geq 0.5$ frequency-domain approximation: PM$\geq 50°$

Settling time 0.5s and $\zeta=0.5 \Rightarrow$ $\omega_n = 16$ rad/s frequency-domain approx.: $\omega_{cg} = 12.6$ rad/s

$$\omega_{cg} = \omega_n \sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}$$

Controller parameters required to meet these new specifications are,

$$K = 133, \quad \alpha = 0.5, \quad \omega_m = 12.6\ rad/s, \quad T = 0.11,$$

$$L(s) = \frac{0.11s + 1}{0.5 \times 0.11s + 1}\left(\frac{133}{s(s+8)}\right)$$

Q.4 (b) $y_{ss}(t) = 5.77 \sin\left(\sqrt{3}\,t - 90°\right)$