NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II: 2011/2012)

EE3331C – FEEDBACK CONTROL SYSTEMS

May 2012 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) printed pages.

2. All questions are compulsory. Answer **ALL** questions.

3. This is a **CLOSED BOOK** examination.

4. On page 6 of this question paper, the following tables are provided:
   a) Useful Laplace Transform Rules
   b) Table of useful Laplace Transforms
   c) Useful design formulae

Q.1

(a) The open-loop transfer function of a unity feedback system is $G(s) = \frac{K}{(s+2)(s^2+4s+3)}$.

Find the value of K such that the steady-state error to a unit step input is $\frac{3}{8}$.

[4 marks]

(b) Using the information available from the root locus of KG(s) shown in Fig. Q1-b, determine the transfer function G(s). Assume the gain cross-over frequency of G(s) to be 1 rad/s.

[6 marks]

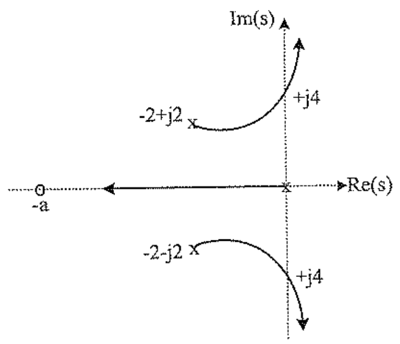

Fig. Q1-b

(c) Fig. Q1-c shows the Nyquist plot of an open loop transfer function that has one pole in the RHP. Determine the range of the value of K for which the closed loop is stable.

[6 marks]

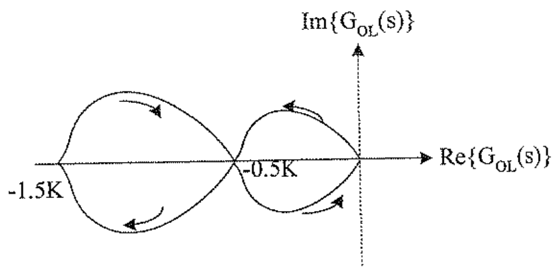

Fig. Q1-c

(d) Between a PI controller and a phase-lag compensator, which one will give more improvement in the steady state error? Why?

[4 marks]

(e) The open loop transfer function of a unity feedback system is $G_{ol}(s) = \frac{K}{s(s+10)}$. Find the range of K such that the peak overshoot to unit step input is less than 12%.

[5 marks]

Q.2 Suppose you are to design a unity feedback PI controller $G_c(s) = K\left(1 + \frac{1}{T_i s}\right)$ for a first-order plant described by the transfer function model $G_p(s) = \frac{A}{s+p}$.

(a) Determine the permissible ranges of natural frequency (ω_n) and damping factor (ζ) of the closed loop system such that the closed loop poles lie within the shaded region shown in Fig. Q2-a. In the figure, lengths *ab* and *ac* are 1 unit each.

[6 marks]

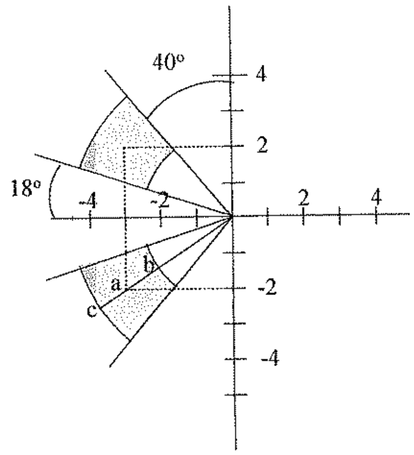

Fig. Q2-a

(b) Choose the values of K and T_i so that the closed loop poles are at the center of the shaded regions. Assume $A = 2, p = 2$ for this part of the question.

[7 marks]

(c) Prove that no matter what the values of A and p are, the PI controller provides enough flexibility to place the poles anywhere in the complex (left-half) plane.

[6 marks]

(d) With the controller designed in part (b), find the steady state error for reference input,

$$r(t) = \begin{cases} 0, & t < 0 \\ 2 + t, & t \geq 0 \end{cases}$$

[6 marks]

Q.3

(a) Sketch the polar plot of $G(s) = \frac{5(s+2)}{s(s-2)}$.

[5 marks]

(b) For the following unity gain negative feedback system, sketch the Nyquist contour and find its mapping on the G(s) plane. Use the Nyquist plot to determine the range of K for which the closed loop system is stable.

[10 marks]

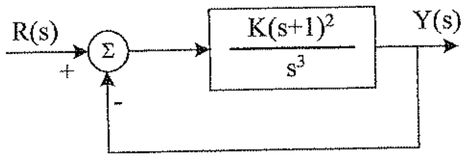

Fig. Q3-b

(c) The Nyquist plot for some practical control system is shown below where $\alpha = 0.4$, $\beta = 1.3$ and $\phi = 40°$. What are the gain and phase margins of the system? Assuming that the open loop system has no pole in the right-half plane, describe what happens to the stability of the system as the gain goes from 0 to a very large value.

[5 marks]

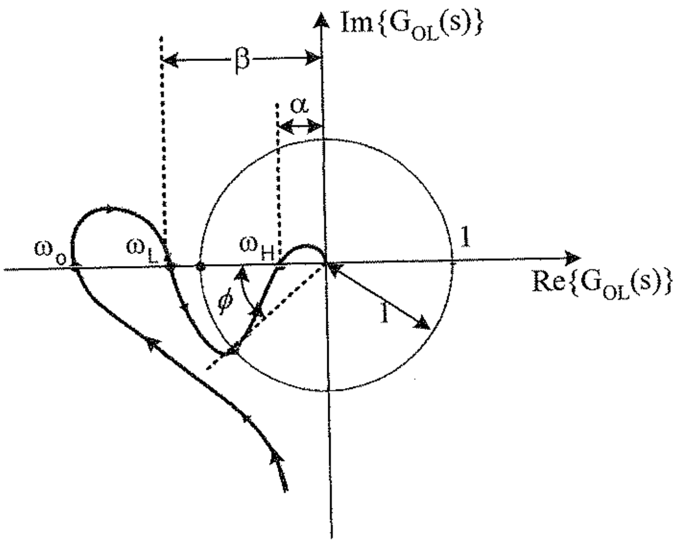

Fig. Q3-c

(d) Find the phase crossover frequency and gain margin for the open loop transfer function $G_{ol}(s) = \frac{6}{(s^2+2s+2)(s+2)}$.

[5 marks]

Q.4

(a) You are to design a controller for a plant with transfer function $G_p(s) = \frac{1}{s(s+8)}$. Design specifications are,

Maximum overshoot:	16%
Settling time:	0.5 s

(i) Choose a proportional gain K such that the closed loop system satisfies the overshoot specification. What is the settling time for this design?

[5 marks]

(ii) Design a lead compensator such that the settling time specification is met without affecting the overshoot obtained in part (i).

[15 marks]

(b) If a sinusoidal input $u(t) = 10\sin(\sqrt{3}t)$ is applied to a 2nd order dynamic system $G(s) = \frac{3}{s^2+3s+3}$, find the steady-state sinusoidal output of the system.

[5 marks]

END OF PAPER

Some Useful Laplace Transform Rules

Transform of derivative, $L\left\{\frac{dy}{dt}\right\}$	$sY(s) - y(0)$
$L\left\{\frac{d^2y}{dt^2}\right\}$	$s^2Y(s) - sy(0) - y'(0)$
Transform of integral, $L\left\{\int_0^t y(\tau)d\tau\right\}$	$\frac{Y(s)}{s}$
Shift in time-domain, $L\{y(t-t_d)u(t-t_d)\}$	$Y(s)e^{-st_d}$
Shift in s-domain, $L\{y(t)e^{-at}\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

Laplace Transform of Commonly Used Functions

Function f(t)	Laplace Transform F(s)	Function f(t)	Laplace Transform F(s)
Unit impulse, δ(t)	1	Unit step, u(t)	$\frac{1}{s}$
Ramp function, t	$\frac{1}{s^2}$	Exponential function, ke^{-at}	$\frac{k}{s+a}$
Sine function, sin(ωt)	$\frac{\omega}{s^2+\omega^2}$	Cosine function, cos(ωt)	$\frac{s}{s^2+\omega^2}$

Step Response of Underdamped Second Order System

Model of the prototype 2nd order underdamped system $G(s) = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Maximum overshoot, M_p	$M_p = Ke^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}$
Settling time, t_s	$t_s = \frac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \frac{1.8}{\omega_n}$
Time of the 1st peak, t_p	$t_p = \frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$