Examination for Semester II, 2012/13

Q.4 (a)

The specifications for this problem have no requirement involving system type or error constant. ~~Our~~ Concern for changing system type and error constant by introducing compensator is not valid here. We can choose K as the last parameter.

$$G(s) = \frac{1}{s(s+2)}$$

$$G(j\omega) = \frac{1}{j\omega(2+j\omega)}$$

at $\omega = 6.6$ rad/s, $\quad G(j6.6) = \dfrac{1}{(j6.6)(2+j6.6)}$

$$\phi(6.6) = -90^\circ - \tan^{-1}\frac{6.6}{2}$$
$$= -163^\circ$$

$$PM_u = 17^\circ$$

Target PM : 50°

Required phase lead $\Rightarrow$ $50^\circ - 17^\circ = 33^\circ$

Let $\phi_m = 33^\circ$ [No allowance is required as we can ensure 6.6 to be ω_{cg} by adjusting K in the end].

Then $\alpha = \dfrac{1-\sin 33^\circ}{1+\sin 33^\circ} = 0.29 \approx 0.3$

$$\omega_m = 6.6 \text{ rad/s}$$

$$T = \frac{1}{6.6\sqrt{0.3}} = 0.28$$

$$C(s) = \frac{0.28s+1}{0.3 \times 0.28 s + 1}$$

$$|C(j6.6)| = \frac{1}{\sqrt{0.3}} = 1.8$$

$$|G(j6.6)| = \frac{1}{6.6\sqrt{6.6^2+4}} = 0.02$$

$$K|C(j6.6)||G(j6.6)| = 1 \Rightarrow K = 27.8$$

$$G_c(s) = 27.8 \frac{(0.28s+1)}{(0.3 \times 0.28s+1)}$$

Verify

$$\left| K\,C(j\omega)G(j\omega) \right|_{\omega=6.6} = 1.$$

$$\angle K C(j\omega) G(j\omega)_{\omega=6.6} = -90^\circ - \tan^{-1}\frac{6.6}{2} + \tan^{-1}(0.28 \times 6.6) - \tan^{-1}(0.3 \times 0.28 \times 6.6)$$

$$= -90^\circ - 73^\circ + 61.6^\circ - 29^\circ$$

$$= -130.4^\circ$$

$PM \approx 50^\circ$