

Q1 (a)  $G(s) = \frac{s^2 + 2s + 2}{s^3 + 0.5s^2}$  NB: There are 2 poles at the origin.

(b) To find the minimum  $K$ , substitute  $s = j\omega$  to the closed-loop pole equation: :-

$$s^3 + (0.5 + K)s^2 + 2Ks + 2K = 0 \quad \text{--- (1)}$$

$$\Rightarrow K = \frac{1}{2}$$

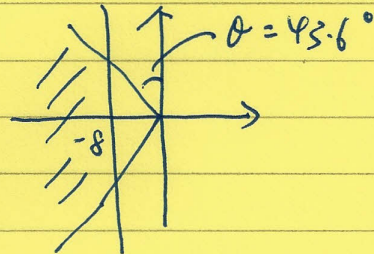
(c) The desired  $\zeta$  poles are:

$$(s + 2.32)(s^2 + \omega_n s + \omega_n^2) = 0 \quad \text{--- (2)} \quad \zeta = 0.5$$

$$\text{① \& ②, } K = 3.58, \quad s = -0.88 \pm j1.52$$

(d)  $K = 2$

Q2 (a)  $M_p < 5\% \Rightarrow \zeta > 0.69$   
 $2\% t_s < 0.5 \Rightarrow \zeta \omega_n > 8$



(b) Find the  $\zeta$  poles from block diagram, compared with 2nd order prototype system.

$\Rightarrow K_1, K_2$  choose  $\zeta$  and  $\omega_n$  at boundary.

(d) Closed-loop poles given by  $s^2 + \underbrace{(10 + 10K_1K_2)}_b s + \underbrace{10K_1}_c = 0$ .

For underdamped system,  $b^2 - 4ac < 0$ .

Q1 (a)  $G(s) = \frac{s^2 + 2s + 2}{s^3}$

(b) Desired poles:  $(s + \alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$  — (1)

Actual poles:  $1 + G(s)K(s) = 0$  — (2)

(1) & (2),  $K = 4$ ,  $s = -2, -1 \pm j1.73$

(c) Poles are:  $s = -1.3, -0.35 \pm j1.72$

(d) The limiting gain can be found by substituting  $s = j\omega$  into (2) which gives  $K = 1$ . The system would be stable for small  $e$ . For large  $e$ , once the gain drops below 1, system is unstable.

Q2 (a)  $G_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{K_1}{s^2 + (2 + K_1 K_2)s + K_1}$

(b)  $2\% t_s \leq 3 \text{ sec.} \Rightarrow \zeta\omega_n \geq \frac{4}{3} \Rightarrow K_1 K_2 \geq \frac{2}{3}$   
 s.s. error for ramp i/p less than 25%.

$\Rightarrow e_{ss} = \lim_{s \rightarrow 0} sE(s) \leq 0.25$

$\Rightarrow \frac{2 + K_1 K_2}{K_1} \leq 0.25$

Choose  $K_1, K_2$  at boundary.