National University of Singapore
Department of Electrical & Computer Engineering

EE3331C Feedback Control Systems
Mid-Term Quiz

Instructions :

1. There are TWO (2) questions, carrying a total of TWENTY (20) marks. Answer BOTH questions.
2. A formula list is provided on Page 8.
3. Please show all working clearly.
4. Time allowed : 1 hour

Name : ______________________________

Matriculation Number : ____________________

Q1	
Q2	
Total	

1. A closed-loop control system and its root locus plot are shown in Figures 1 and Figure 2 respectively.

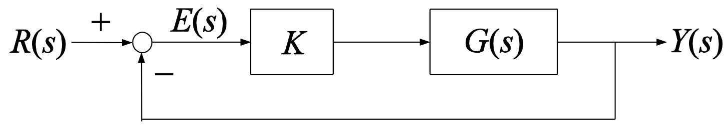

Figure 1: Feedback System

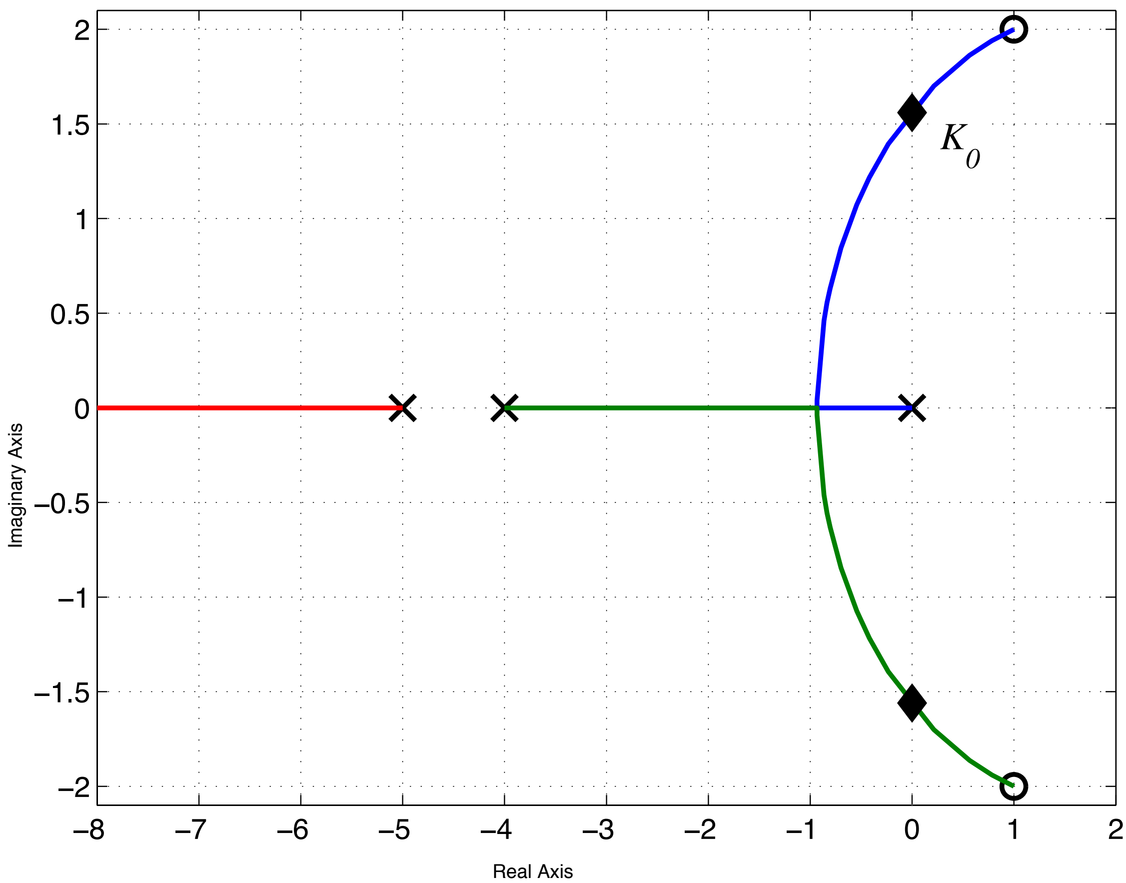

Figure 2: Root Locus Plot

(a) The transfer function of the plant is $G(s) = \dfrac{s^2 + as + b}{s^3 + cs^2 + ds}$. Find a, b, c and d.

(b) Find the value of K_0 in Figure 2.

(c) What is the steady-state error ($E(s) = R(s) - Y(s)$), when the reference signal, $r(t)$ is a unit ramp function, t. You may assume that $K < K_0$, i.e. the closed-loop system is stable.

(10 Marks)

2. Figures 3 and 4 shows a series RLC circuit and its output voltage, $v_o(t)$.

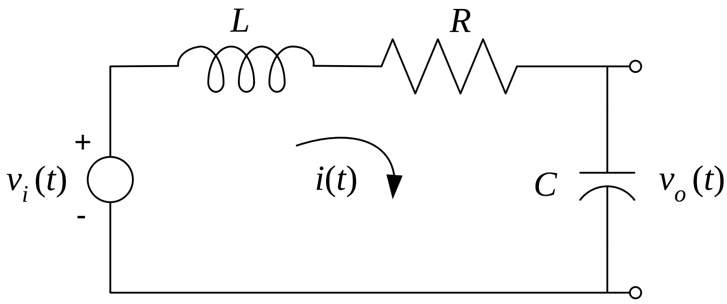

Figure 3: Series RLC circuit

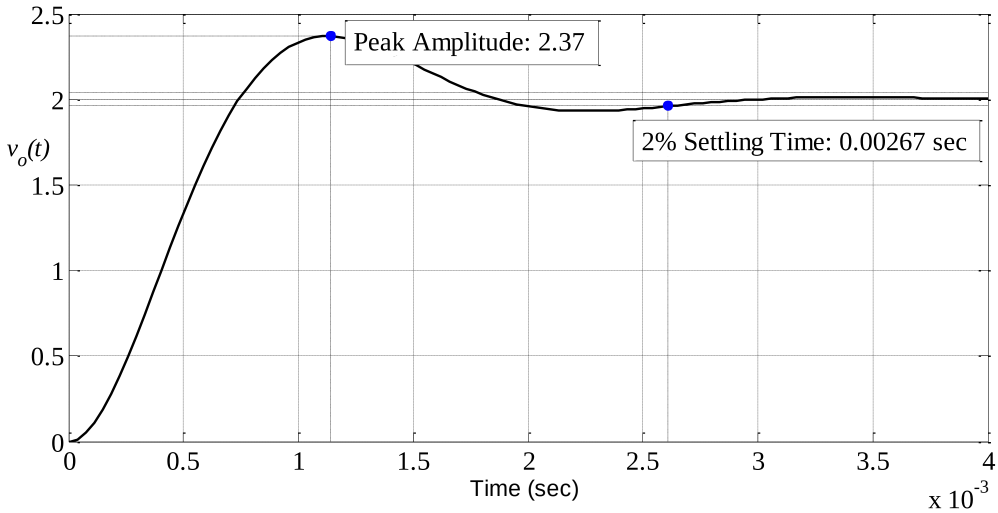

Figure 4: Output voltage of series RLC circuit

(a) Derive the differential equation relating the input voltage, $v_i(t)$, and output voltage, $v_o(t)$. Assuming zero initial conditions, show that

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

(b) Figure 4 shows the output voltage, $v_o(t)$, of the series RLC circuit in part (a) when $v_i(t)$ is a step voltage of 2V. Estimate L and C given that $R = 3\Omega$.

(10 Marks)

SOME USEFUL LAPLACE TRANSFORM RULES

Transform of derivatives, $\mathcal{L}\left\{\frac{dy(t)}{dt}\right\}$	$sY(s) - y(0)$
Transform of integral, $\mathcal{L}\left\{\int_0^t y(\tau)d\tau\right\}$	$\dfrac{Y(s)}{s}$
Shift in time domain, $\mathcal{L}\{y(t-L)u(t-L)\}$	$Y(s)e^{-sL}$
Shift in s-domain, $\mathcal{L}\left\{y(t)e^{-at}\right\}$	$Y(s+a)$
Final Value Theorem	$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

SOME USEFUL LAPLACE TRANSFORMS

Function, $f(t)$	Laplace Transform, $F(s)$	Function, $f(t)$	Laplace Transform, $F(s)$
delta function, $\delta(t)$	1	$\sin\omega t$	$\dfrac{\omega}{s^2+\omega^2}$
unit step function, $u(t)$	$\dfrac{1}{s}$	$\cos\omega t$	$\dfrac{s}{s^2+\omega^2}$
t	$\dfrac{1}{s^2}$	$1-e^{-at}$	$\dfrac{a}{s(s+a)}$
ke^{-at}	$\dfrac{k}{s+a}$	$t-\dfrac{1}{a}\left(1-e^{-at}\right)$	$\dfrac{a}{s^2(s+a)}$

SOME DESIGN FORMULAE FOR UNDERDAMPED 2^{nd} ORDER SYSTEM

Standard 2^{nd} order system : $\dfrac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Maximum overshoot, M_p	$M_p = Ke^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$
Settling time (2%), t_s	$t_s = \dfrac{4}{\zeta\omega_n}$
Rise time, t_r	$t_r = \dfrac{1.8}{\omega_n}$
Peak time, t_p	$t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Q1.

(a). zeros: $s = 1 \pm j2$

$(s-1+j2)(s-1-j2) = s^2 - 2s + 5$

$\Rightarrow a = -2,\ b = 5$

poles :- $s = 0, -4, -5$

$\Rightarrow c = 9,\ d = 20$

(b) $K = K_0$,

& poles :- $1 + GK = 0$

$$\Rightarrow 1 + \frac{K(s^2-2s+5)}{s(s^2+9s+20)} = 0$$

$$\Rightarrow s^3 + 9s^2 + 20s + Ks^2 - 2Ks + 5K = 0$$

$s = j\omega$, $-j\omega^3 - (9\omega^2 + K\omega^2) + j(20-2K)\omega + 5K = 0$

real: $-(9+K)\omega^2 + 5K = 0$

imag: $-\omega^3 + (20-2K)\omega = 0$

$\Rightarrow \omega^2 = 20 - 2K.$

$-(9+K)(20-2K) + 5K = 0$

$-(180 + 20K - 18K - 2K^2) + 5K = 0$

$\Rightarrow 2K^2 \cancel{+} 2K - 180 + 5K = 0$

$2K^2 + 3K - 180 = 0$

$$K = \frac{-3 \pm \sqrt{9 + 4(2)(180)}}{4}$$

$\Rightarrow K = 8.77,\ -10.27$

(c). $e_{ss} = \lim_{s \to 0} sE(s)$

$$= \lim_{s\to 0} s \frac{1}{1+GK} R(s)$$

$$= \lim_{s\to 0} s \frac{1}{1 + \frac{K_1(s^2-2s+5)}{s(s^2+9s+20)}} \cdot \frac{1}{s^2}$$

$$= \lim_{s\to 0} s \frac{s}{s + \frac{K_1(5)}{20}} \cdot \frac{1}{s^2}$$

$$= \frac{20}{5K_1} = \frac{4}{K_1}$$ #

Q2. (a). KVL,

$$v_o(t) + L\frac{di(t)}{dt} + Ri(t) = v_i(t) \quad , \quad i(t) = C\frac{dv(t)}{dt}$$

$$\Rightarrow LC\frac{d^2v_o(t)}{dt^2} + RC\frac{dv_o(t)}{dt} + v_o(t) = v_i(t).$$

Take LT,

$$(LCs^2 + RCs + 1)V_o(s) = V_i(s)$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2+RCs+1}$$

$$= \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

(b). c.f. $s^2 + 2\zeta\omega_n s + \omega_n^2$

$$\Rightarrow \omega_n^2 = \frac{1}{LC} \quad , \quad 2\zeta\omega_n = \frac{R}{L} \quad , \quad R = 3\Omega$$

From the graph,

$$\% M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = \frac{0.37}{2}$$

$$\Rightarrow \zeta = \sqrt{\frac{(\ln(\frac{0.37}{2}))^2}{\pi^2 + (\ln(\frac{0.37}{2}))^2}} = 0.47$$

$$2\% \ t_s = 4/\zeta\omega_n = 0.00267 \quad \Rightarrow \omega_n = \frac{4}{(0.47)(0.00267)} = 3187.5$$

$$\Rightarrow \zeta\omega_n = \frac{4}{0.00267}$$

$$L = \frac{R}{2}\,\frac{0.00267}{4} = 1\,mH. \ *$$

$$C = \frac{1}{L\omega_n^2} = 100\,\mu F \ *$$