EE3331C Feedback Control Systems Tutorial 1

* Solve the following problems using Laplace Transforms:

(a) $F(s) = \dfrac{10}{s(s+1)(s+10)}$ $\qquad f(t) = 1(t) - \frac{10}{9}e^{-t} + \frac{1}{9}e^{-10t}$

(b) $F(s) = \dfrac{3s+2}{s^2+4s+20}$ $\qquad f(t) = 3e^{-2t}\cos(4t) - e^{-2t}\sin(4t)$

(c) $F(s) = \dfrac{1}{s(s+2)^2}$ $\qquad f(t) = \frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}te^{-2t}$

(d) $F(s) = \dfrac{e^{-s}}{s^2}$ $\qquad f(t) = (t-1)1(t)$

(e) $\ddot{y}(t) - 2\dot{y}(t) + 4y(t) = 0$, $y(0) = 1$ and $\dot{y}(0) = 2$
. Ans: $y(t) = e^t \cos\sqrt{3}t + \frac{1}{\sqrt{3}}e^t \sin\sqrt{3}t$

(f) $\ddot{y}(t) + y(t) = t$, $y(0) = 1$ and $\dot{y}(0) = -1$ $\qquad$ Ans: $y(t) = t + \cos t - 2\sin t$

(g) Find $F(s)$ for

- $f(t) = t\sin 3t - 2t\cos t$,
- $f(t) = \int_0^t \cos(t-\tau)\sin\tau d\tau$

(h) For each of the transfer function, write, by inspection, the general form of the step response:

- $G(s) = \dfrac{400}{s^2+12s+400}$
- $G(s) = \dfrac{225}{s^2+30s+225}$
- $G(s) = \dfrac{625}{s^2+625}$

1. An input $u(t) = t$, $t \geq 0$, is applied to a black box with a transfer function $G(s)$. The resulting output response, when the initial conditions are zero is

$$y(t) = e^{-t} - \frac{1}{4}e^{-2t} - \frac{3}{4} + \frac{1}{2}t, \qquad t \geq 0.$$

Determine $G(s)$ for this system.

2. Consider the following electrical circuit:

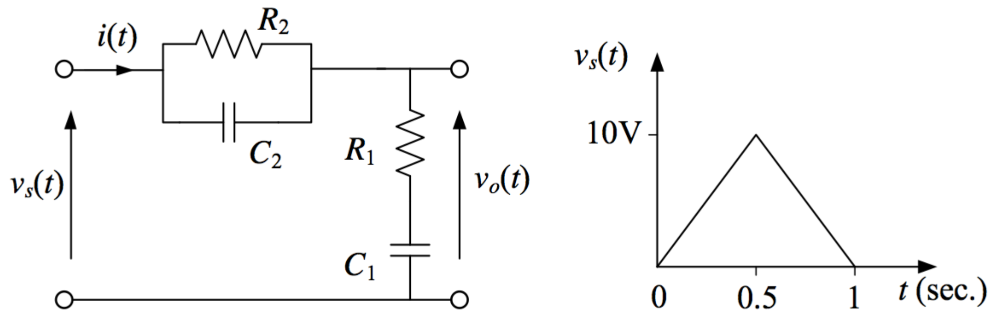

Figure 1: Electrical circuit

(a) Find the transfer function, $G(s) = \dfrac{V_o(s)}{V_s(s)}$.

(b) Find the Laplace Transform for the input signal, $v_s(t)$.

(c) Given that $R_1 = R_2 = 1\text{M}\Omega$, $C_1 = 1\mu\text{F}$, and $C_2 = 0$, find an expression for the output, $v_o(t)$, for all $t \geq 0$. What is the output voltage, $v_o(t)$ at $t = 1$ second?

3. For the second-order system with transfer function $G(s) = \dfrac{5}{s^2 + 9s - 10}$. Determine (a) DC gain; and (b) the final value to a step input.

4. An armature-controlled DC motor is driving a load, it has the following transfer function

$$\frac{\omega(s)}{V(s)} = \frac{K}{\tau s + 1}$$

The input voltage is 5V. The speed at $t = 2$ seconds is 30 rad/s, and the steady-state speed is 70 rad/s when $t \to \infty$. Determine K and τ.

5. The step/impulse response of four processes are shown in Figure 2. Assume that the step/impulse signal is introduced at $t = 0$ and the transfer function of all the systems assume the following form

$$G_i(s) = \frac{K}{as^2 + bs + c} e^{-sL}$$

Determine the parameters K, a, b, c and L for all four systems.

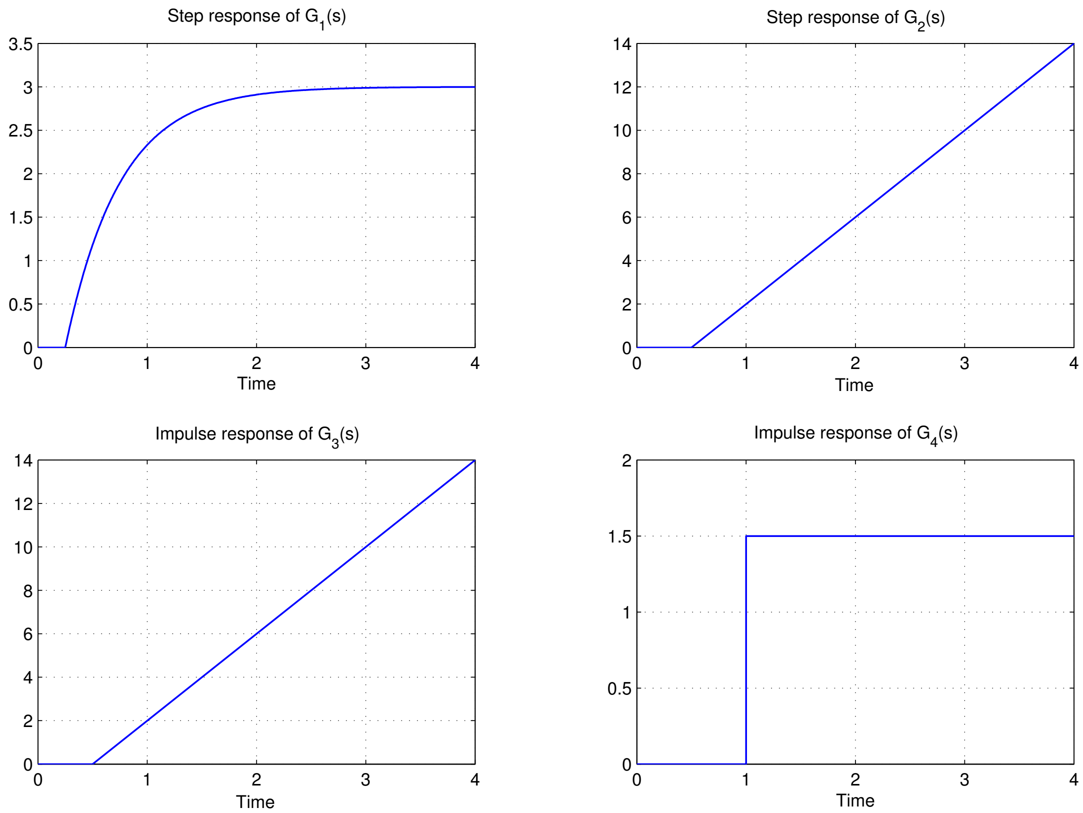

Figure 2: Step and impulse responses

6. Suppose a second order system has a transfer function given by $G(s) = \dfrac{K}{s^2 + bs + c}$. The step response of $G(s)$ to a step input of 0.5 units gave rise to the following output trajectory. Find the parameters K, b and c in $G(s)$.

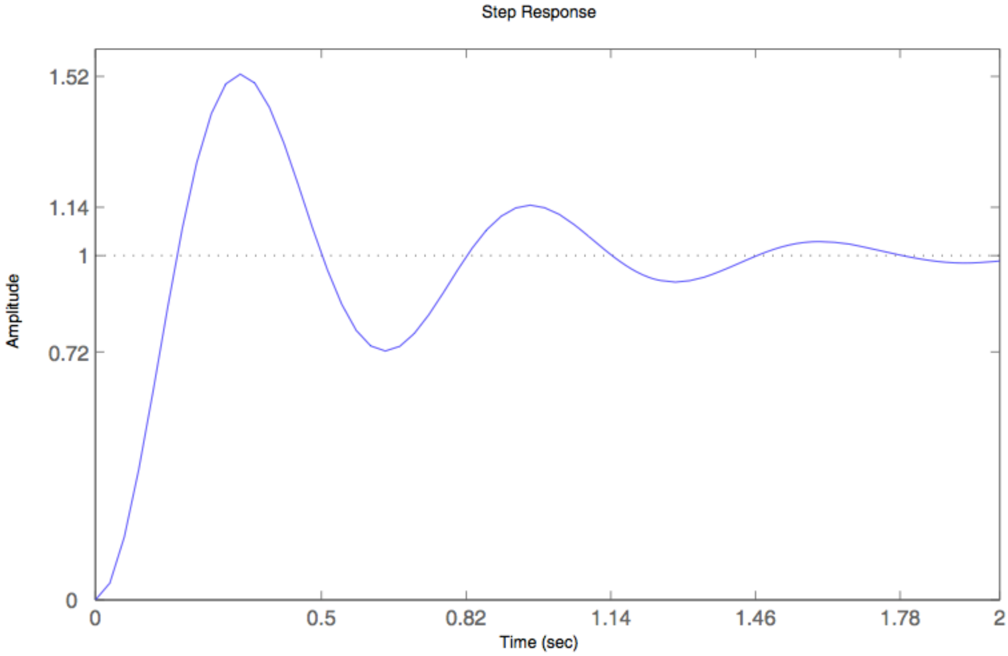

Figure 3: Second-order step response

7. *For each pair of second-order system specifications below, find the location of the second-order pair of poles:

   - $\%M_p = 10\%$, $t_s = 0.5$ sec
   - $\%M_p = 15\%$, $t_p = 0.25$ sec
   - $t_r = 1$ sec, $t_s = 5$ sec

8. *Consider the following second-order system with an extra pole:

$$G(s) = \frac{\omega_n^2 p}{(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2}$$

   Show that the unit step response is

$$y(t) = 1 + Ae^{-pt} + Be^{-\sigma t}\sin(\omega_d t - \theta)$$

9. Identify the transfer functions corresponding to the step responses shown in the Figure 4. The transfer functions are

$$G_a(s) = \frac{2}{s^2 + 0.8s + 4}, \qquad G_b(s) = \frac{1}{s^2 + 0.4s + 1}, \qquad G_c(s) = \frac{4}{s^2 + 0.8s + 4},$$
$$G_d(s) = \frac{0.5}{s^2 + 0.4s + 1}, \qquad G_e(s) = \frac{0.5}{s^2 + 1.4s + 1}.$$

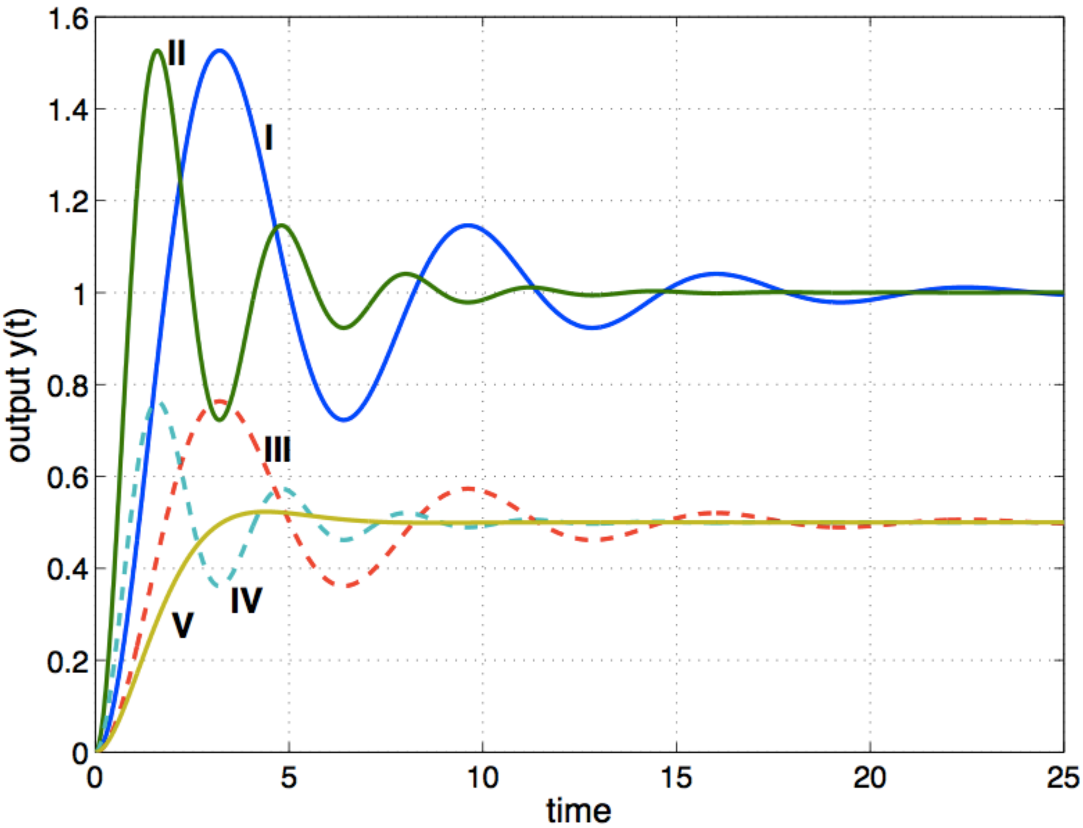

Figure 4: Step responses

10. A second-order system has the transfer function, $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$. The system specification for a step input are as follows:

- Percentage overshoot, $5\% <$ P.O. $< 10\%$
- Settling time, $t_s < 4$ seconds
- Peak time, $t_p < 1$ second

Show the permissible area for the poles of $G(s)$ in order to achieve the aboves specifications.

11. **Consider the series RL circuit in Figure 5 which contains a nonlinear resistor whose voltage current relationship is defined as $v_r(t) = 10\ln\left(\frac{1}{2}i(t)\right)$

    (a) Linearize the nonlinear differential equation at the operating point, $v_0 = 20$V.

    (b) Find the transfer function, $\frac{I(s)}{V(s)}$.

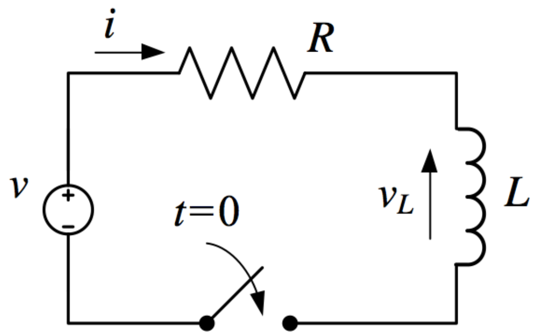

Figure 5: Nonlinear RL circuit