EE3331C/EE3331E Feedback Control Systems
Part II, Tutorial 1 - Solution

Section 2

1. Sketch the Bode plot of

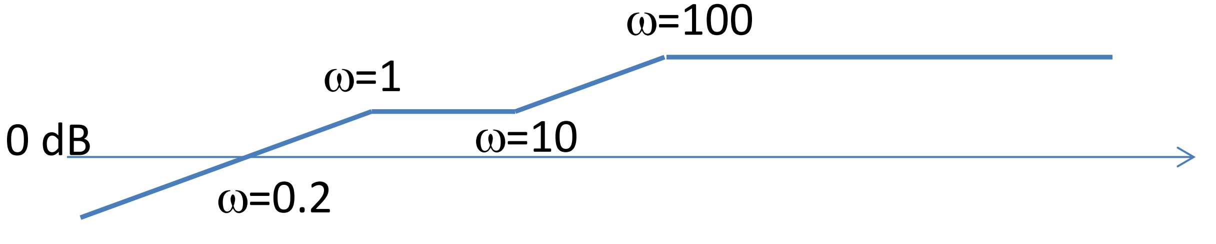
$$(i)\ G(s) = 50\frac{s(s+10)}{(s+1)(s+100)}$$

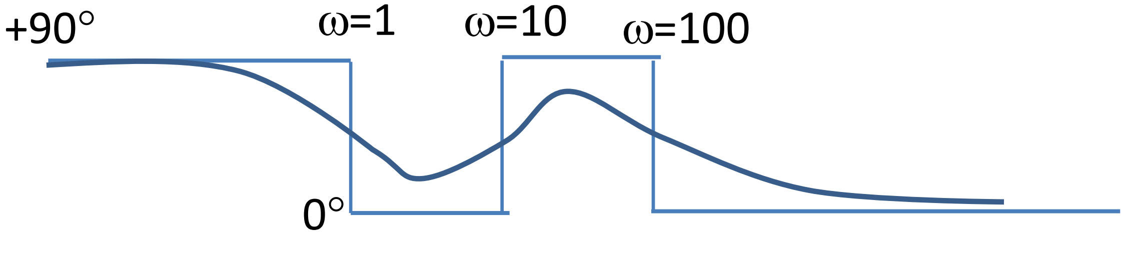
$$(ii)\ \frac{(s-10)}{s(s+1)(s+100)}$$

Solution 1(a)

$$G(s) = 50\frac{s(s+10)}{(s+1)(s+100)} = 5s\frac{(0.1s+1)}{(s+1)(0.01s+1)}$$

The differentiator (5s) has a gradient of +20 dB/dec and intersects 0 dB at ω=0.2 rad/sec.
At ω=1, the gradient is changed by -20 dB/dec
At ω=10, the gradient is changed by +20 dB/dec
At ω=100, the gradient is changed by -20 dB/dec

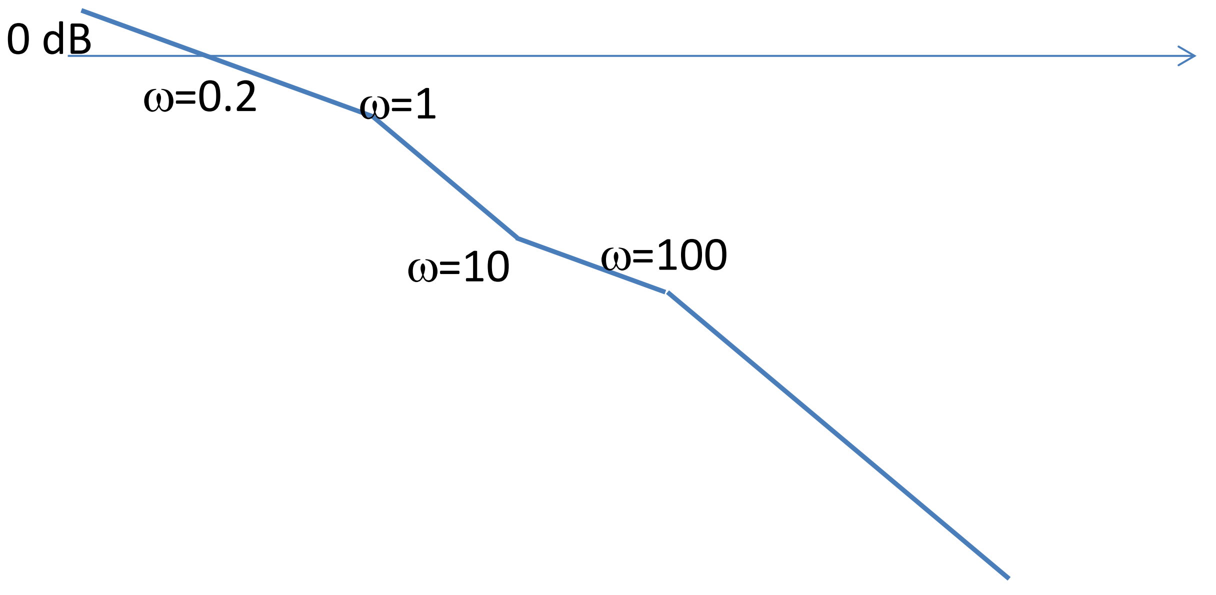




Solution 1(b)

$$G(s)=\frac{(s-10)}{s(s+1)(s+100)}=\frac{0.1}{s}\frac{(0.1s-1)}{(s+1)(0.01s+1)}$$

The integrator (0.1/s) has a gradient of -20 dB/dec and intersects 0 dB at ω=0.1 rad/sec.
At ω=1, the gradient is changed by -20 dB/dec
At ω=10, the gradient is changed by +20 dB/dec
At ω=100, the gradient is changed by -20 dB/dec

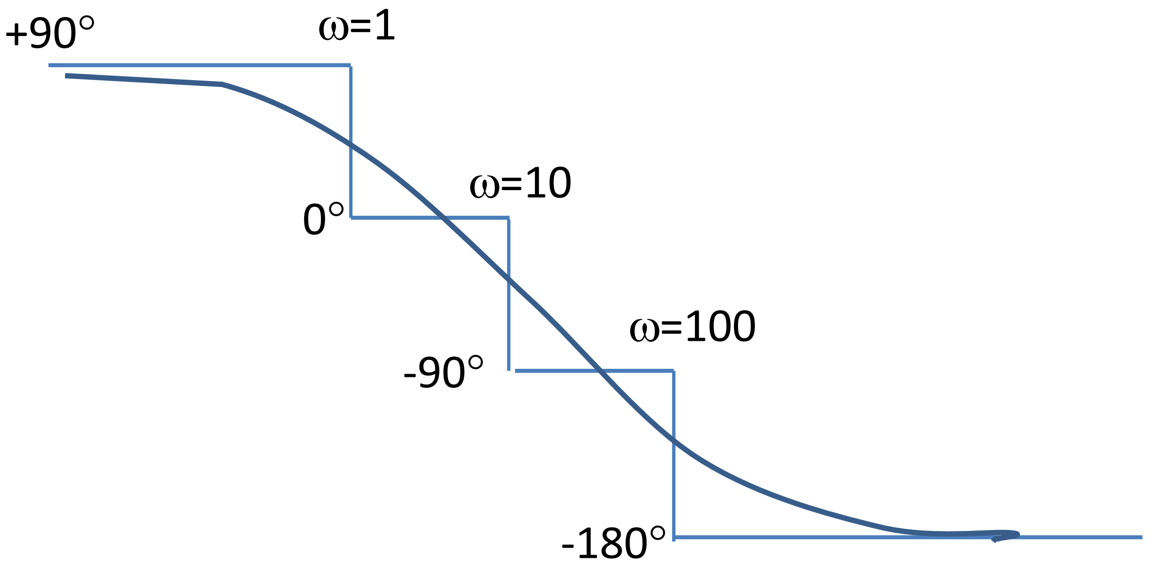


The integrator (0.1/s) gives -90° phase for all ω.
Let the phase of the right hand side zero be ϕ_1. Then,

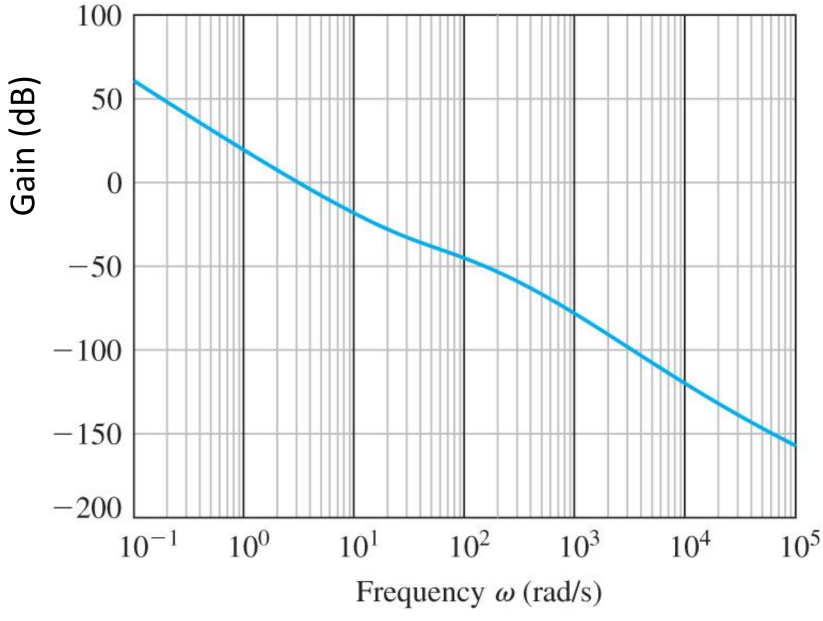
$$\begin{aligned}\phi_1 &= \angle(0.1j\omega-1)\\ &= 180° - \tan^{-1}(0.1\omega)\end{aligned}$$

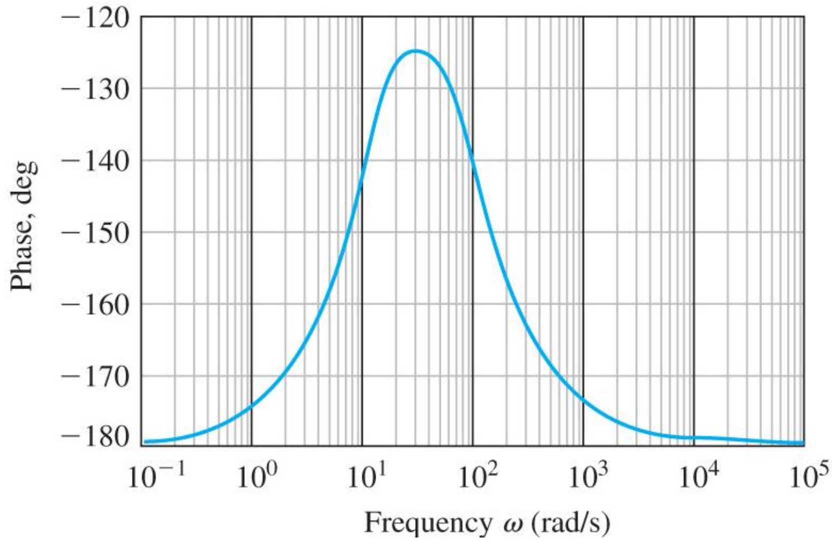
The RHP zero contributes +180° phase for all ω and the phase variation is like a pole with the corner frequency of the zero.

Low frequency phase is -90°+180°= 90°.



2. Estimate the transfer function from the Bode plot given





Low frequency gradient in magnitude plot is -40 dB/decade. There are two integrators. $\frac{K}{s^2}$

It intersects the 0dB line at ω = 3 rad/sec. $\frac{K}{3^2} = 1 \quad \Rightarrow \quad K = 9$

High frequency gradient is -40 dB/decade. Therefore, (n-m) = 2.

Upward bend is at approximately ω=10 rad/sec and downward bend at approximately ω=100 rad/s.

$$G_{est}(s) = \frac{9}{s^2}\frac{(0.1s+1)}{(0.01s+1)}$$

Given phase plot conforms with the phase of this transfer function.