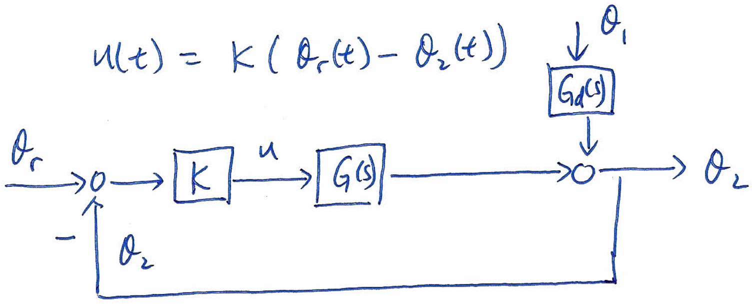
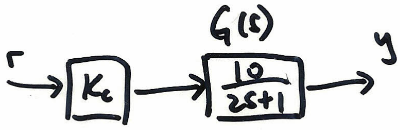
Q1.

$$\theta_2(s) = \underbrace{\frac{2}{(s+1)^2}}_{G(s)} U(s) + \underbrace{\frac{1}{(s+2)}}_{G_d(s)} \theta_1(s)$$

$$u(t) = K(\theta_r(t) - \theta_2(t))$$



Q2. OL :



$r(t) = 1$

$R(s) = \frac{1}{s}$

steady-state error, $e = r - y$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s[R(s) - Y(s)]$$

$$= \lim_{s\to 0} s[R(s) - G(s)K_c R(s)] \quad — (1)$$

$$= \lim_{s\to 0} s\left[\frac{1}{s} - \frac{10K_c}{(2s+1)s}\right]$$

$$= 1 - 10K_c \quad , K_c = 1/10$$

$$= 0 \; *$$

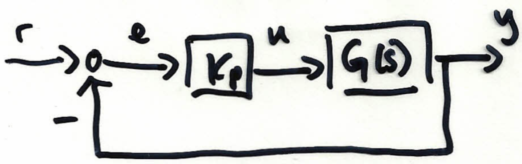
$$G'(s) = \frac{11}{(2.5s+1)}$$

From (1), $e_{ss} = \lim_{s\to 0} s[R(s) - G'(s)K_c R(s)]$

$$= \lim_{s\to 0} s\left[\frac{1}{s} - \frac{11K_c}{(2.5s+1)s}\right]$$

$$= 1 - 11K_c \quad , K_c = 1/10$$

$$= -0.1 \; *$$

Q2. 

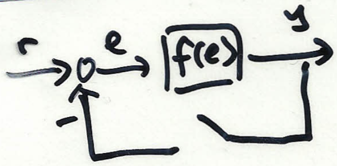
$$G(s) = \frac{10}{2s+1}$$

$$e = r - y$$
$$= r - Gu = r - GK_p e$$
$$\frac{e}{r} = \frac{1}{1+GK_p}$$

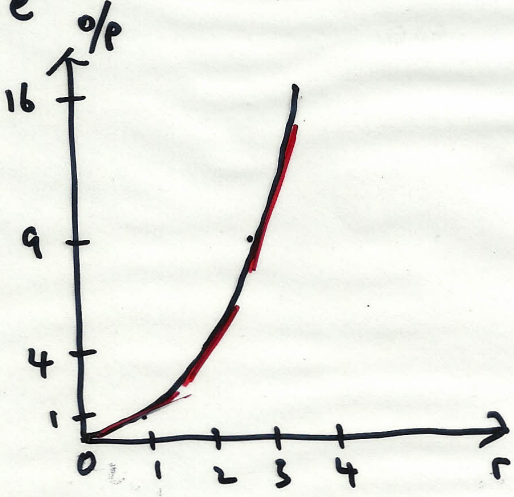
$$e_{ss} = \lim_{s\to 0} sE(s)$$
$$= \lim_{s\to 0} s\frac{1}{1+G(s)K_p}R(s) \quad \text{— (2)}$$
$$= \lim_{s\to 0} s\frac{1}{1+\frac{10K_p}{2s+1}}\cdot\frac{1}{s}$$
$$= \frac{1}{1+10K_p}, \quad K_p = 10$$
$$= \frac{1}{101} = 0.0099$$

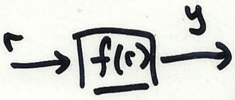
$$G'(s) = \frac{11}{2.5s+1}$$

From (2), $e_{ss} = \lim_{s\to 0} s\frac{1}{1+\frac{11K_p}{2.5s+1}}\cdot\frac{1}{s}$
$$= \frac{1}{1+11K_p}$$
$$= \frac{1}{111} = 0.009$$

Q3. 

$f(e) = e^2$

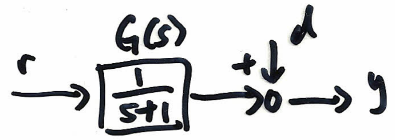


w/o fb: 

$y = e^2 = r^2, \quad \frac{dy}{dr} = 2r$

with fb: $y = e^2 = (r-y)^2$
$$= r^2 - 2ry + y^2$$
$$y^2 - (2r+1)y + r^2 = 0$$
$$y = \frac{1}{2} + r \pm\sqrt{r+\frac{1}{4}}$$
$$\frac{dy}{dr} = 1 \pm \frac{1}{\sqrt{4r+1}}$$

Q4.

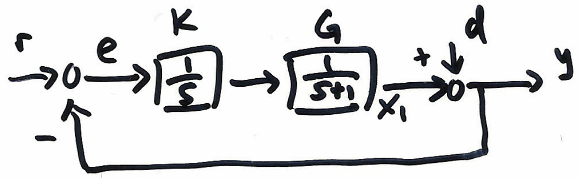


$$y_{ss} = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left[G(s)R(s) + D(s)\right]$$

$$= \lim_{s\to 0} s\left[\frac{1}{s+1}\cdot\frac{1}{s} + \frac{0.1}{s}\right]$$

$$= 1 + 0.1 = 1.1$$

CL:



$$y_{ss} = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left[Y_r(s) + Y_d(s)\right]$$

$$= \lim_{s\to 0} s\left[\frac{GK}{1+GK}R(s) + \frac{1}{1+GK}D(s)\right]$$

$$= \lim_{s\to 0} s\left[\frac{\frac{1}{s(s+1)}}{1+\frac{1}{s(s+1)}}\cdot\frac{1}{s} + \frac{1}{1+\frac{1}{s(s+1)}}\cdot\frac{0.1}{s}\right]$$

$$= \lim_{s\to 0} s\left[\frac{1}{s(s+1)+1}\cdot\frac{1}{s} + \frac{s(s+1)}{s(s+1)+1}\cdot\frac{0.1}{s}\right]$$

$$= 1 + 0$$

$$= 1 \ \#$$

$d = 0$

$$y = Gu = GKe$$

$$= GK(r - y)$$

$$\frac{y}{r} = \frac{GK}{1+GK}$$

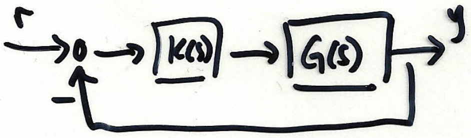
$r = 0$

$$y = x_1 + d$$

$$= GKe + d$$

$$= GK(-y) + d$$

$$\frac{y}{d} = \frac{1}{1+GK}$$

Q5. 

(a) $G(s) = \frac{K}{Ts+1}$, $K(s) = K_p$

¢ poles :- $1 + G(s)K(s) = 0$

$$1 + \frac{KK_p}{Ts+1} = 0$$

$$\Rightarrow s = -\frac{1}{T}(1 + KK_p)$$

$$< 0 \quad \text{for } T, K, K_p > 0$$

(b) $K(s) = K_p\left(1 + \frac{1}{T_i s}\right)$

¢ poles :- $1 + G(s)K(s) = 0$

$$1 + \frac{KK_p}{Ts+1}\left(\frac{T_i s + 1}{T_i s}\right) = 0$$

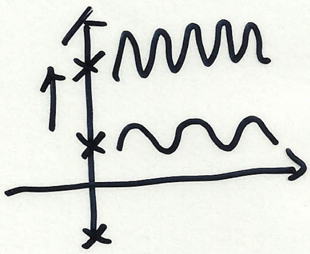
$$s^2 + \underbrace{\left(\frac{1+KK_p}{T}\right)}_{>0} s + \underbrace{\frac{KK_p}{TT_i}}_{>0} = 0 \qquad K_p, T_i > 0$$

(c) $G(s) = \frac{1}{s^2}$, $K(s) = K_p$

¢ poles :- $1 + GK = 0$

$$1 + \frac{K_p}{s^2} = 0$$

$$\Rightarrow s = \pm j\sqrt{K_p}$$



¢ t.f. $H(s) = \frac{GK}{1+GK} = \frac{K_p}{s^2 + K_p} = \frac{Y(s)}{R(s)}$

$$Y(s) = \frac{K_p}{s^2 + K_p} \cdot R(s)$$

$$= \frac{K_p}{s^2 + K_p} \cdot \frac{1}{s}$$

$$y(t) = 1 - \cos\sqrt{K_p}\,t$$

Q5(d). $K(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$, $G(s) = \frac{1}{s^2}$

Find K_p , $T_i = 0.2$, $T_d = 0.1$ such that c/l is marginally stable

c/l poles:- $1 + GK = 0$

$$1 + \frac{1}{s^2} K_p \left(\frac{T_i T_d s^2 + T_i s + 1}{T_i s}\right) = 0$$

$$\Rightarrow s^3 + 0.1 K_p s^2 + K_p s + 5 K_p = 0 \quad \text{— ①}$$

let $s = j\omega$

$$-j\omega^3 - 0.1 K_p \omega^2 + j K_p \omega + 5 K_p = 0$$

Re : $-0.1 K_p \omega^2 + 5 K_p = 0$

$$\omega^2 = 50$$

$$\omega = \sqrt{50}$$

Im: $-\omega^3 + K_p \omega = 0$

$$K_p = \omega^2 = 50 \ *$$

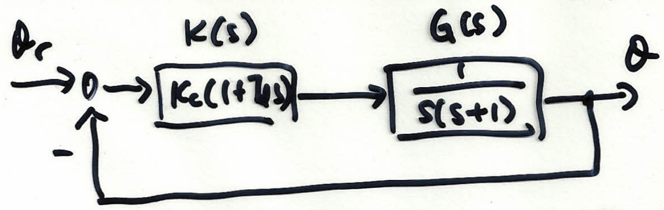
When $K_p = 100$, find c/l poles:-

Sub $K_p = 100$ into ①

$$s^3 + 10 s^2 + 100 s + 500 = 0$$

$$s = -6.5 \ , \ -1.79 \pm j8.61$$

Q6.



- Desired ¢ poles:- $s = -1 \pm j\sqrt{3}$

$$(s+1+j\sqrt{3})(s+1-j\sqrt{3}) = s^2+2s+4 = 0 \quad — ①$$

- Actual ¢ poles:-

$$1 + G(s)K(s) = 0$$

$$1 + \frac{1}{s(s+1)} K_c(1+T_d s) = 0$$

$$s^2 + (1+K_cT_d)s + K_c = 0 \quad — ②$$

① & ②
s^0: $K_c = 4$
s^1: $1 + K_cT_d = 2 \Rightarrow T_d = \frac{1}{4}$ *

- 2^{nd} order system, c.e.

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad — ③$$

① & ③
s^0: $\omega_n^2 = 4 \Rightarrow \omega_n = 2$
s^1: $2\zeta\omega_n = 2 \Rightarrow \zeta = 0.5$ *

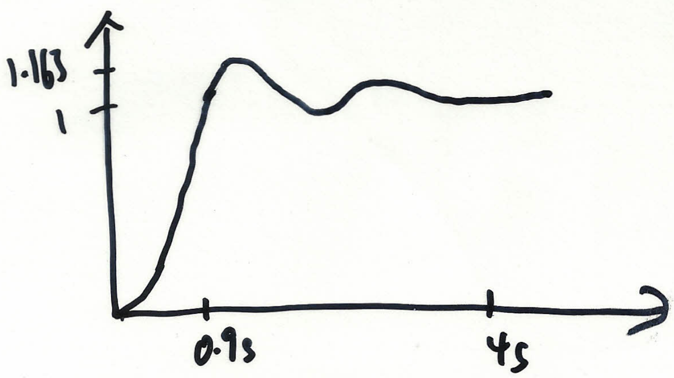
$$H_2(s) = \frac{GK}{1+GK} = \frac{K_c(1+T_d s)}{s(s+1)+K_c(1+T_d s)} = \frac{K_c(1+T_d s)}{s^2+(1+K_cT_d)s+K_c}$$

///

¢ step response

- $t_r = \frac{1.8}{\omega_n} = 0.9s$
- $t_s = 4/\zeta\omega_n = 4s$
- $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.163$

$$H_1(s) = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

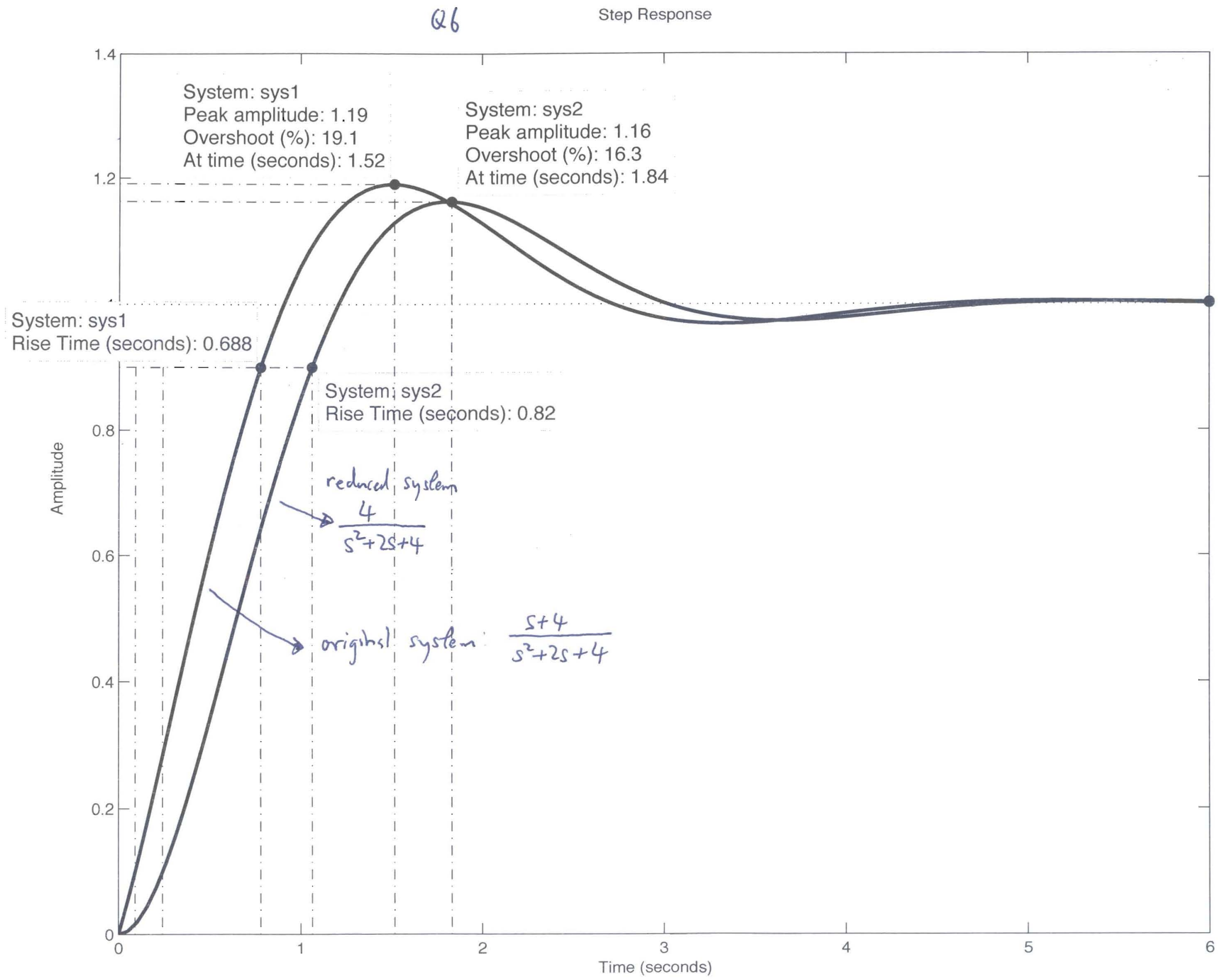


Match DC gain

$$H_1(s)\big|_{s=0} = H_2(s)\big|_{s=0}$$

$$K = \frac{K_c}{K_c} = 1$$

Q6


Step Response
System: sys1
Peak amplitude: 1.19
Overshoot (%): 19.1
At time (seconds): 1.52
System: sys2
Peak amplitude: 1.16
Overshoot (%): 16.3
At time (seconds): 1.84
System: sys1
Rise Time (seconds): 0.688
System: sys2
Rise Time (seconds): 0.82
reduced system
$\frac{4}{s^2+2s+4}$
original system: $\frac{s+4}{s^2+2s+4}$
Amplitude
Time (seconds)
1.4
1.2
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6

Q7. $J\ddot{\theta} + B\dot{\theta} = T_c$

(a). Take LT, assume zero i.c.

$$s^2 J\theta(s) + sB\theta(s) = T_c(s)$$

$$\frac{\theta(s)}{T_c(s)} = \frac{1}{s(Js+B)} = G(s) = \frac{1.67\times10^{-6}}{s(s+\frac{1}{30})}$$

(b). $\theta(s) = G(s)T_c(s)$

$$= G(s)K(\theta_r(s) - \theta(s))$$

$$\frac{\theta(s)}{\theta_r(s)} = \frac{G(s)K}{1+G(s)K}$$

$$= \frac{1.67K\times10^{-6}}{s^2+\frac{1}{30}s+1.67K\times10^{-6}} \qquad \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

(c) $M_p < 10\%$

$$\Rightarrow e^{-\pi\zeta/\sqrt{1-\zeta^2}} < 0.1$$

$$\Rightarrow \zeta > 0.591$$

s^1: $2\zeta\omega_n = \frac{1}{30}$, $\zeta = \frac{1}{60\omega_n} > 0.591$

$$\Rightarrow \omega_n < 0.0282$$

s^0: $\omega_n^2 = 1.67K\times10^{-6} < 0.0282^2$

$$\Rightarrow K < 477 \;\#$$

(d) $t_r = \frac{1.8}{\omega_n} < 80$

$$\Rightarrow \omega_n > \frac{1.8}{80}$$

$$1.67K\times10^{-6} > \left(\frac{1.8}{80}\right)^2$$

$$\Rightarrow K > 304 \;\#$$