Repeated Real Roots

$$F(s) = \frac{s+3}{(s+1)(s+2)^2}$$

$$= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$\Rightarrow \quad A = (s+1)F(s)\Big|_{s=-1} = \frac{s+3}{(s+2)^2}\Big|_{s=-1} = 2$$

$$B = \frac{d}{ds}\left[(s+2)^2 F(s)\right]\Big|_{s=-2} = \frac{d}{ds}\left[\frac{s+3}{s+1}\right]\Big|_{s=-2} = -2$$

$$C = (s+2)^2 F(s)\Big|_{s=-2} = \frac{s+3}{s+1}\Big|_{s=-2} = -1$$

$$\Rightarrow F(s) = \frac{2}{(s+1)} - \frac{2}{(s+2)} - \frac{1}{(s+2)^2}$$

$$\Rightarrow f(t) = 2e^{-t} - 2e^{-2t} - te^{-2t}$$

Distinct Complex Roots

$$F(s) = \frac{1}{s(s^2+s+1)} \quad — ①$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+s+1} \quad — ②$$

$$A = sF(s)\Big|_{s=0} = 1$$

To compute B & C, equate ① & ②

$$\Rightarrow 1 = A(s^2+s+1) + (Bs+C)s$$

$$\Rightarrow B = -1, \; C = -1$$

$$\Rightarrow F(s) = \frac{1}{s} - \frac{s+1}{s^2+s+1}$$

$$= \frac{1}{s} - \frac{s+1}{(s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$= \frac{1}{s} - \frac{(s+\frac{1}{2}) + \frac{1}{\sqrt{3}}(\frac{\sqrt{3}}{2})}{(s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$f(t) = 1 - e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)$$