FOUNDATION OF SENSOR SIGNAL PROCESSING (II)

FEATURE EXTRACTION IN TIME-FREQUENCY DOMAIN

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Module objective

Module: Time-frequency feature extraction for signal processing

Knowledge and understanding

- Understand the fundamentals of time-frequency domain signal representation, transformation, feature extraction, such as Fourier transformation and wavelet transformation

Key skills

- Design, build, implement and evaluate time-frequency feature extraction methods for signal processing

Major reference

- [Introduction] Steven W. Smith, ***The Scientist and Engineer's Guide to Digital Signal Processing***, available at http://www.dspguide.com
- [Practical] J. Unpingco, ***Python for Signal Processing: Featuring IPython Notebooks***, 2014, https://github.com/unpingco/Python-for-Signal-Processing
- [Practical] A. B. Downey, ***Think DSP: Digital Signal Processing in Python***, https://github.com/AllenDowney/ThinkDSP

Topics

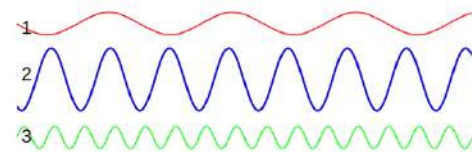
- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

Intelligent sensing and sense making	
Physical Domain (our focus)	Business Domain
• Sensor-driven systems, e.g. robots, drones	• Non-sensor data, e.g. transactions, customer data • Also need sensor data (e.g., location, contact tracing, etc)

Recap: Signal

- A mechanism for conveying information
  - Gestures, traffic lights..

- Electrical engineering: Currents, voltages

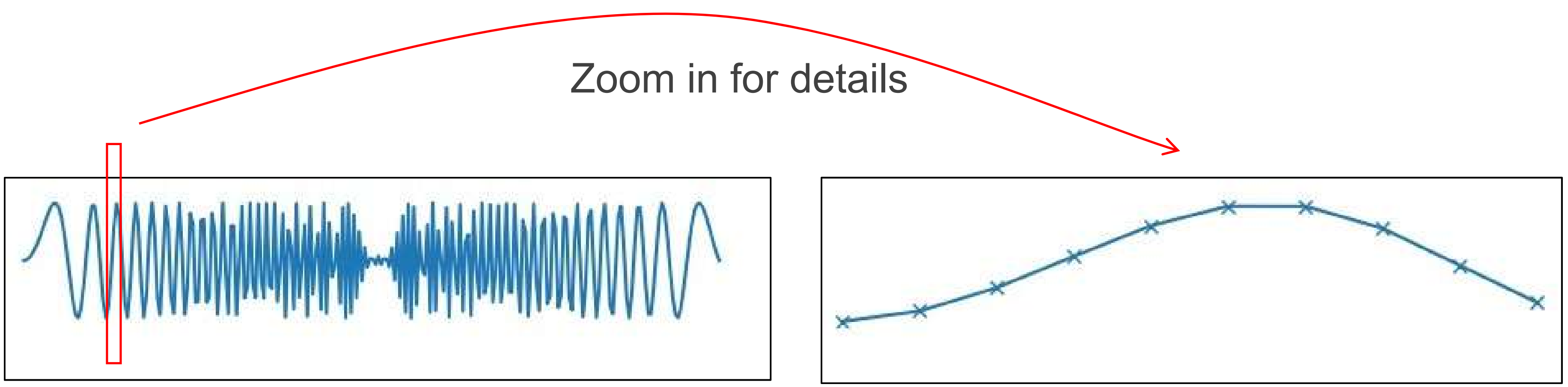


- Digital signals: Ordered collections of numbers that convey information, about a real world phenomenon, such as sounds, images

Source:
1. http://www.publicdomainfiles.com/show_file.php?id=13945761016913
2. https://commons.wikimedia.org/wiki/File:CPT-sound-pitchvolume.svg
3. https://commons.wikimedia.org/wiki/File:A)_Imagen_de_Lenna_en_escala_de_grises;_b)_Imagen_de_Lenna_con_el_filtro_de_Gauss_aplicado..jpg

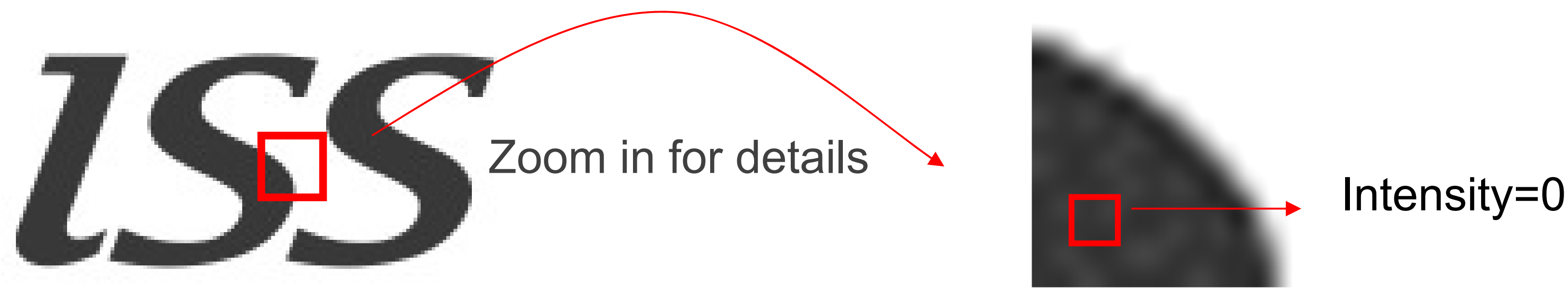
Signal: Audio

- A sequence of numbers
  - The order in which the numbers occur is important
  - Represent a perceivable sound



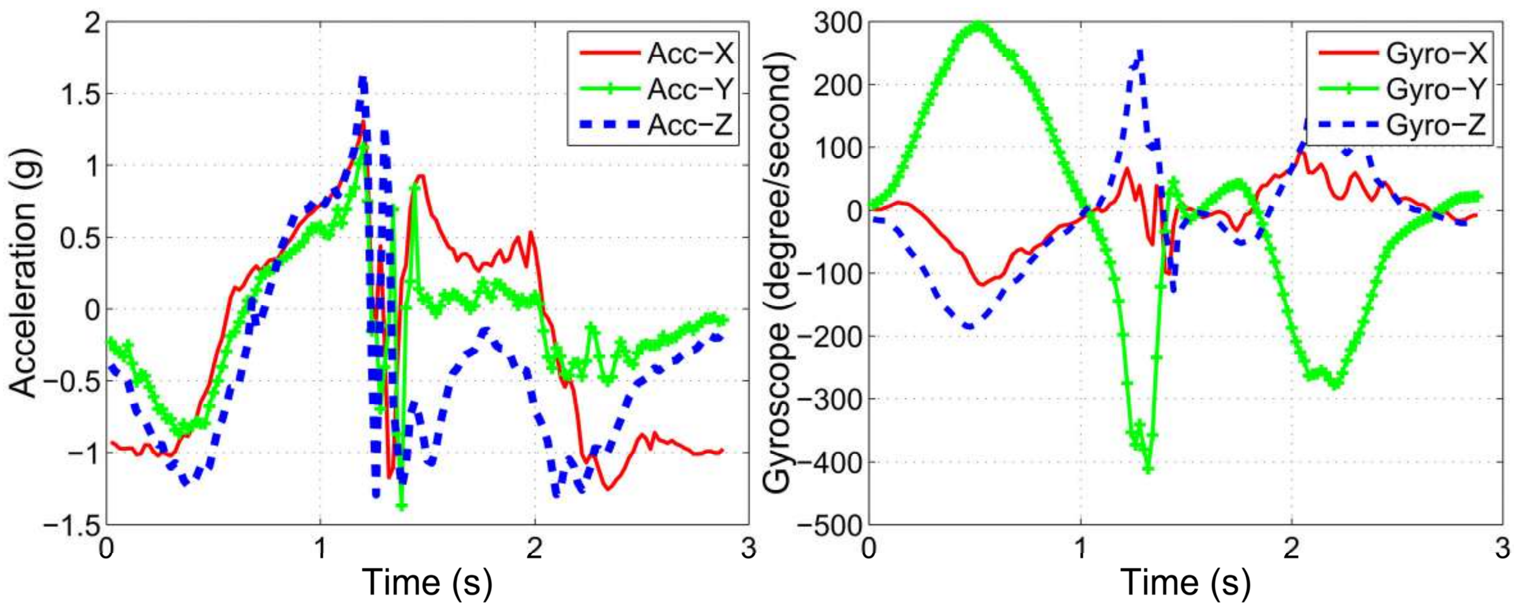
Signal: Image

- A rectangular arrangement (matrix) of numbers
  - sets of numbers (for color images)
- Each pixel represents a visual representation of one of these numbers
  - E.g., 0 is minimum / black, 1 is maximum / white
  - Position / order is important



Signal: Wearable sensor signal

- Human activity classification using wearable sensory data.
- The UTD-MHAD dataset was collected using a wearable inertial sensor in an indoor environment. The dataset contains 27 actions performed by 8 subjects (4 females and 4 males). Each subject repeated each action 4 times. The inertial sensor signals were recorded using the inertial sensor signals (3-axis acceleration and 3-axis rotation signals).



Reference: https://personal.utdallas.edu/~kehtar/UTD-MHAD.html

Signal processing pipeline

Our focus

Sensor	Signal capture	Signal transmission	Feature extraction	Modeling
• Various types of sensors, audio, vision, IoT, etc	• Sensing • Denoising	• Source coding, channel coding, compression	• Deterministic features • Data-driven features • Feature representation learning	• Classification • Regression • Predication

Signal representation/Feature extraction (similar to feature engineering) uses mathematical transformations of raw input data to create new features to be used for further machine learning models.

- Use unstructured data sources
- Create features that are more easily interpreted
- Enhance creativity by using large sets of features

Reference: Chapter 5 Feature engineering, H. Brink, J. W. Richards, M. Fetherolf, Real-world machine learning, Manning, 2017.

Topics

- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

Time-frequency analysis of signal

One of top 10 algorithms in 20th century!

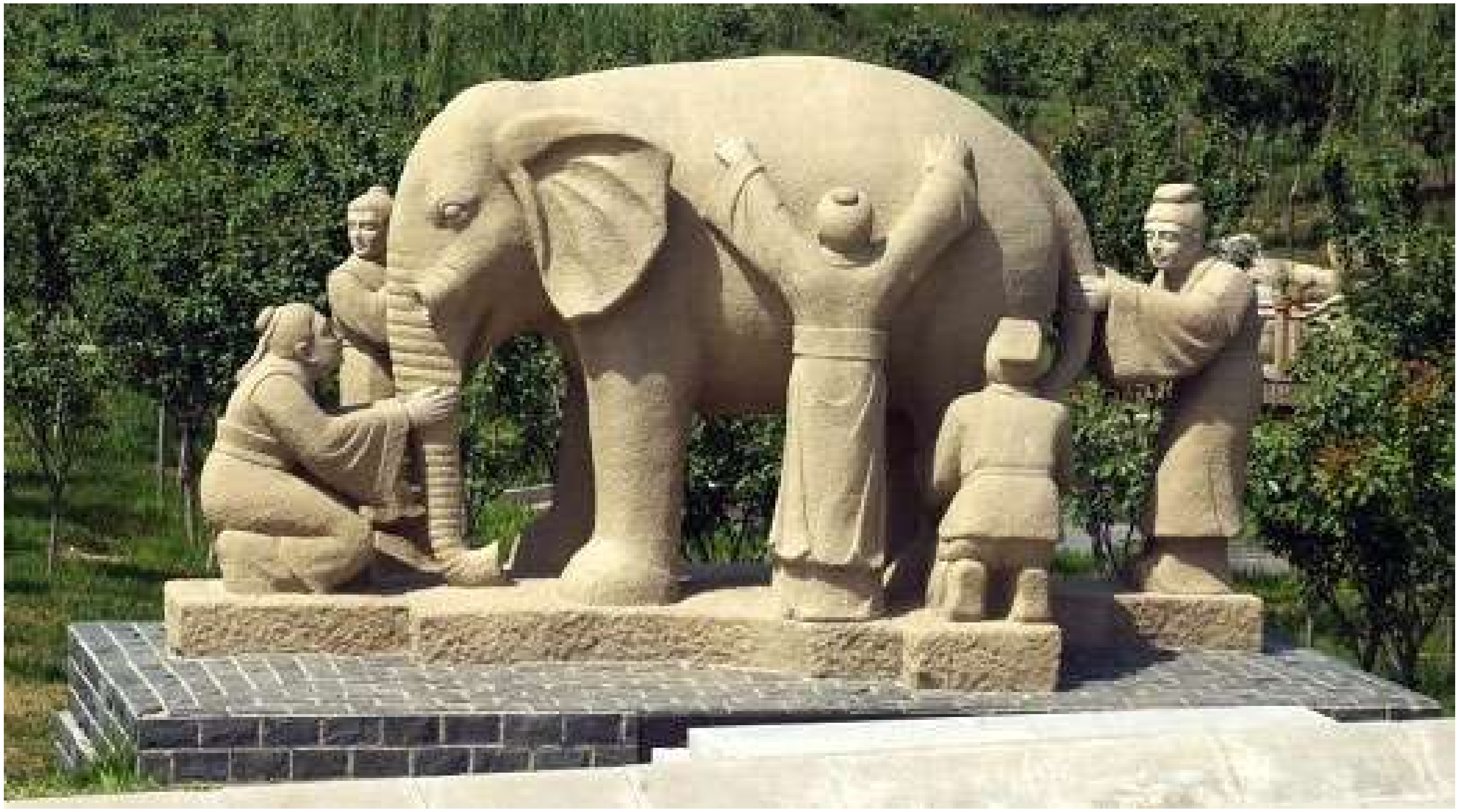
1. Metropolis algorithm for Monte Carlo
2. Simplex method for linear programming
3. Krylov subspace iteration
4. Decomposition approach to matrix computation (Singular value)
5. The Fortran compiler
6. QR algorithm for eigenvalues
7. Quick sort
8. Fast Fourier transform
9. Integer relation detection
10. Fast multipole

Source: https://www.computer.org/csdl/mags/cs/2000/01/c1022.html

Signal decomposition

Intuition: Describe this image so that a listener can visualize what you are describing.

- Pixel-based descriptions are uninformative
- Content-based descriptions are infeasible in the general case



Source: https://cellcode.us/quotes/and-parable-blind-men-elephant.html

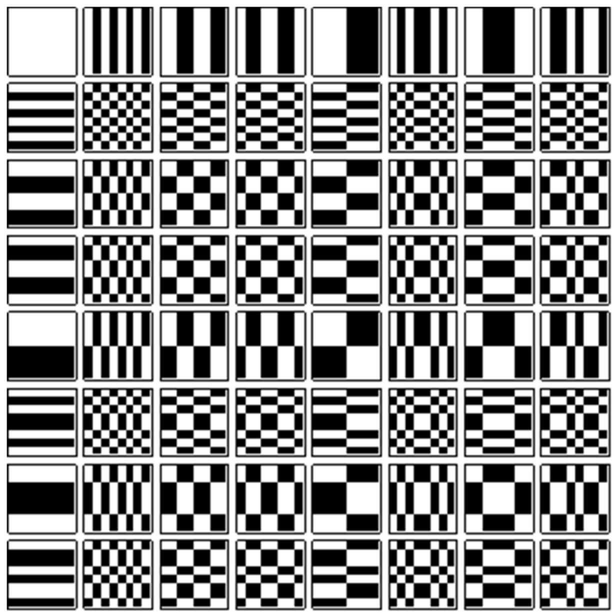
Signal decomposition

Objective: Decompose signal into ‘basis’, which is determined by certain frequency.

- Image signal decomposition: Checkerboard basis
- Images have some fast varying regions. For example, a first checkerboard picture with constant color. A second checkerboard picture that has fast changes
- How about more checkerboard?

Input image

A set of basis (checkerboard)



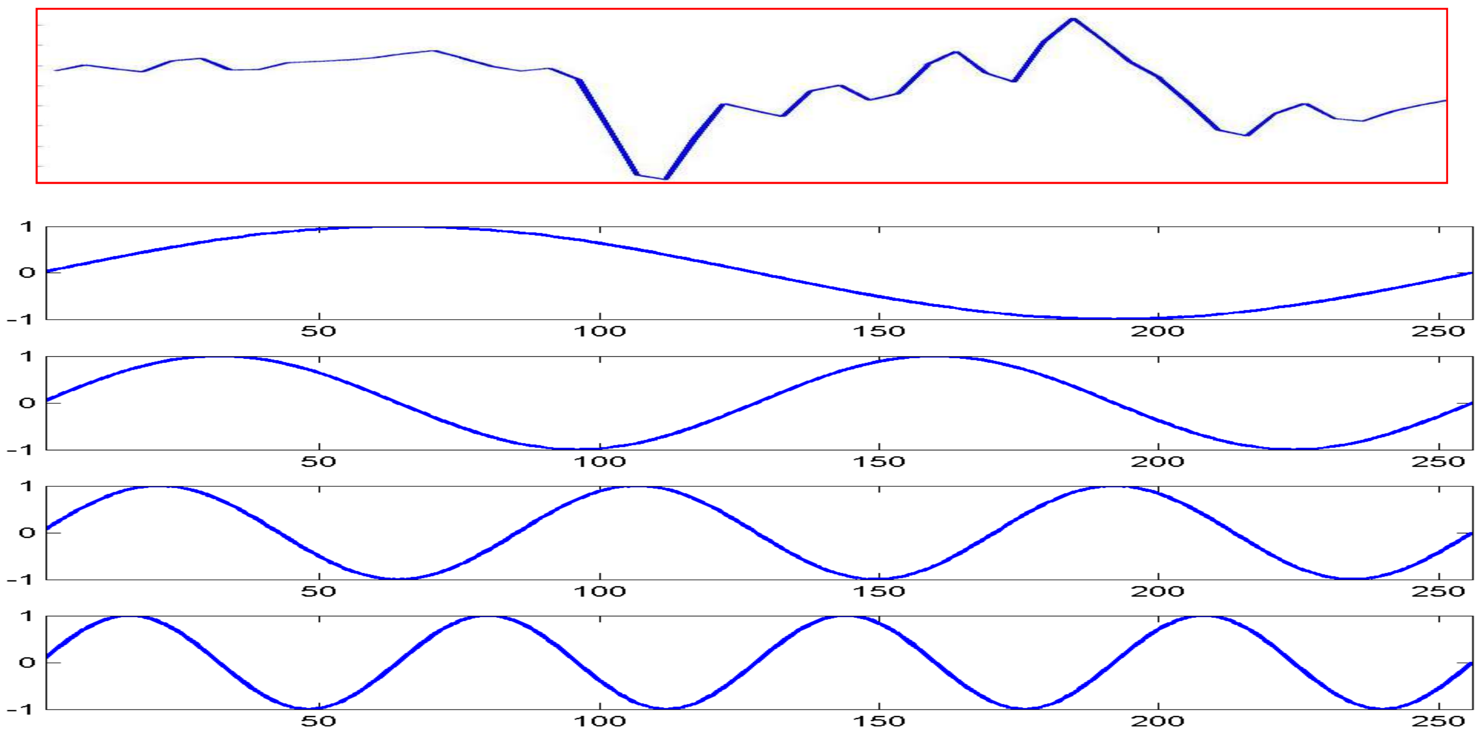
Note: The images are NOT visualized in the actual scale.

Reference: Hamamard basis image, https://en.wikipedia.org/wiki/Hadamard_transform

Signal decomposition

Objective: Decompose signal into ‘basis’, which is determined by certain frequency.

- Sound: Sinusoids basis
- They are orthogonal
- They can represent rounded shapes nicely.



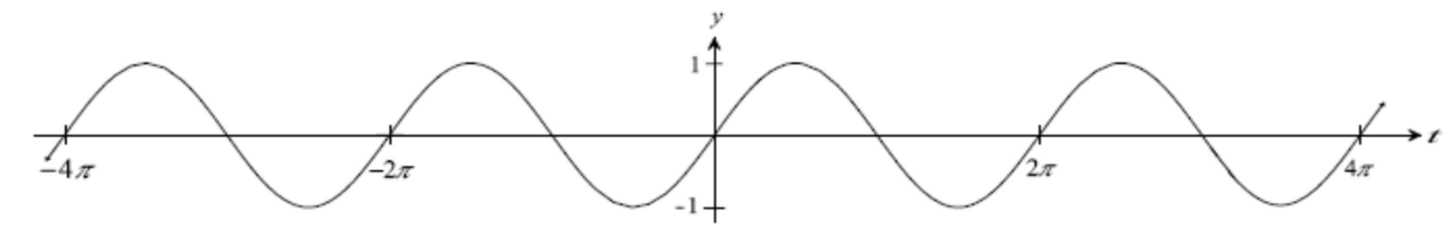
Sine and Cosine functions

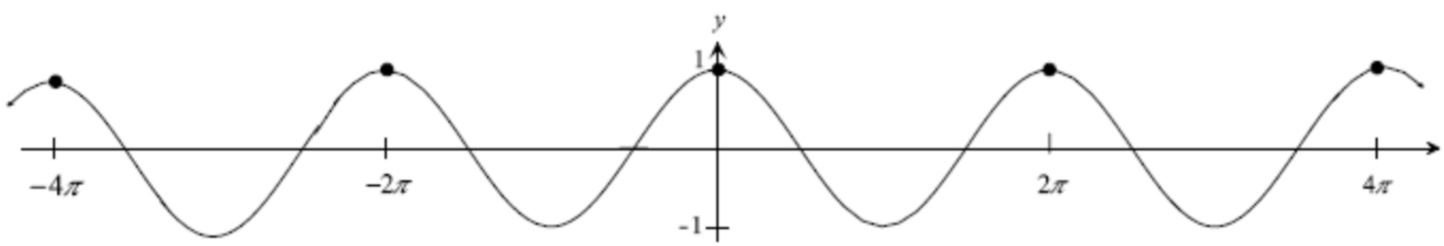
- Periodic functions
- General form of sine and cosine functions:

$\lvert A \rvert$	amplitude
$\frac{2\pi}{\lvert\alpha\rvert}$	period
b	phase shift

$$y(t) = A\sin(\alpha t + b) \qquad y(t) = A\cos(\alpha t + b)$$

Example: $A = 1, b = 0, \alpha = 1$ $\quad$ $\text{period} = 2\pi$





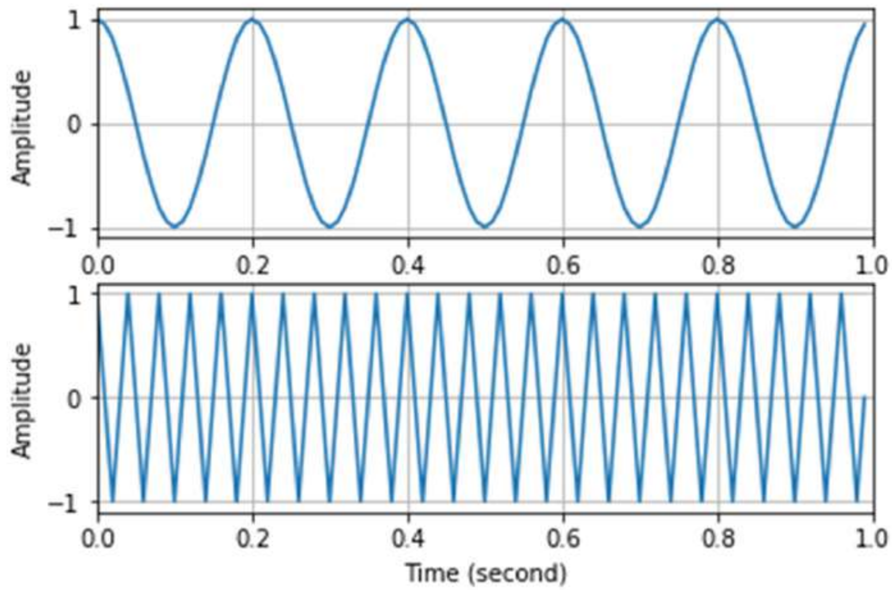
Fourier analysis

Example: FFT function in SciPy, https://docs.scipy.org/doc/scipy/reference/tutorial/fft.html

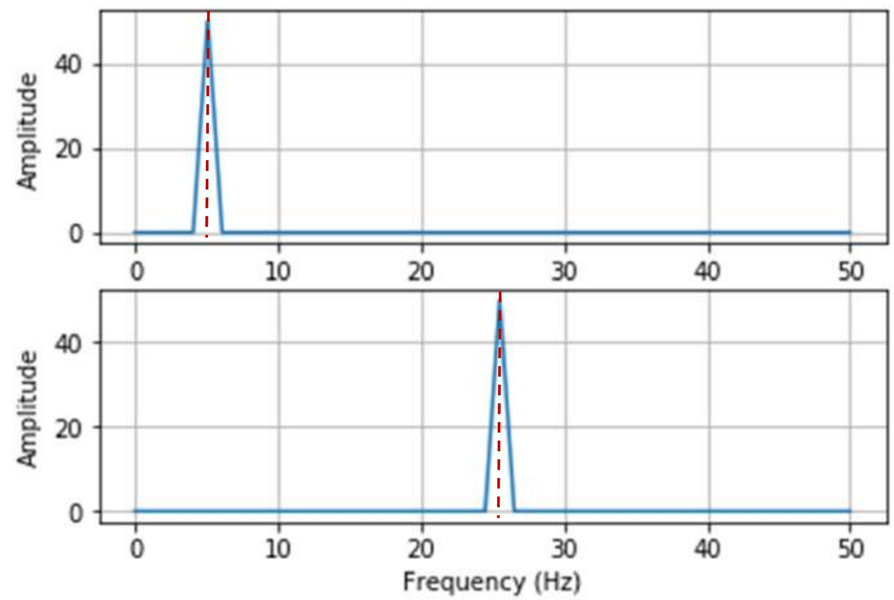
$$f(t) = \cos(2\pi \cdot 5 \cdot t)$$

The 5th basis function has the peak

Original signal in time domain



Fourier domain



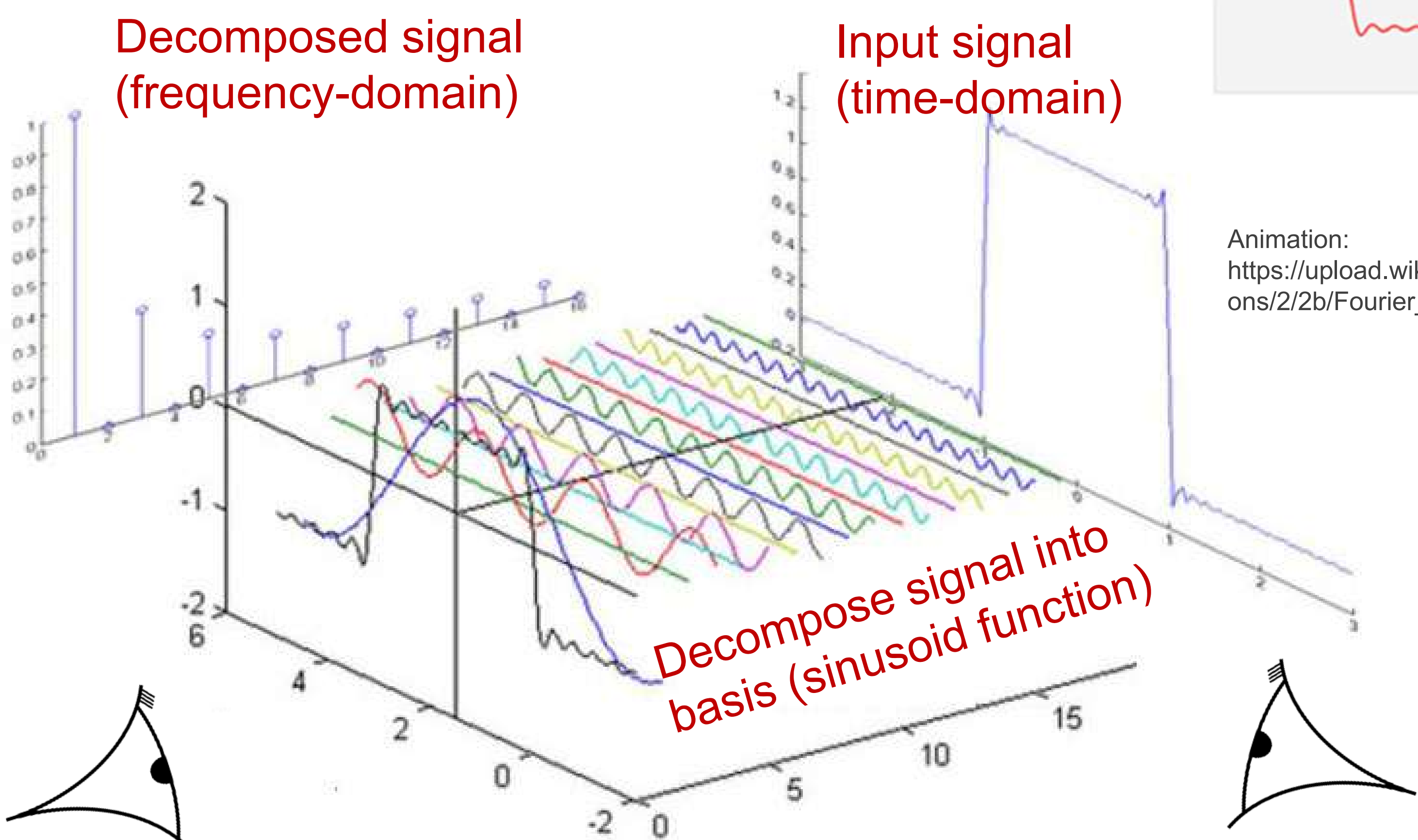
$$f(t) = \cos(2\pi \cdot 25 \cdot t)$$

The 25th basis function has the peak

Signal decomposition

Idea: Decompose the input signal from the time domain into a set of basis (that are sinusoid functions) to obtain the decomposed signal in the frequency domain.

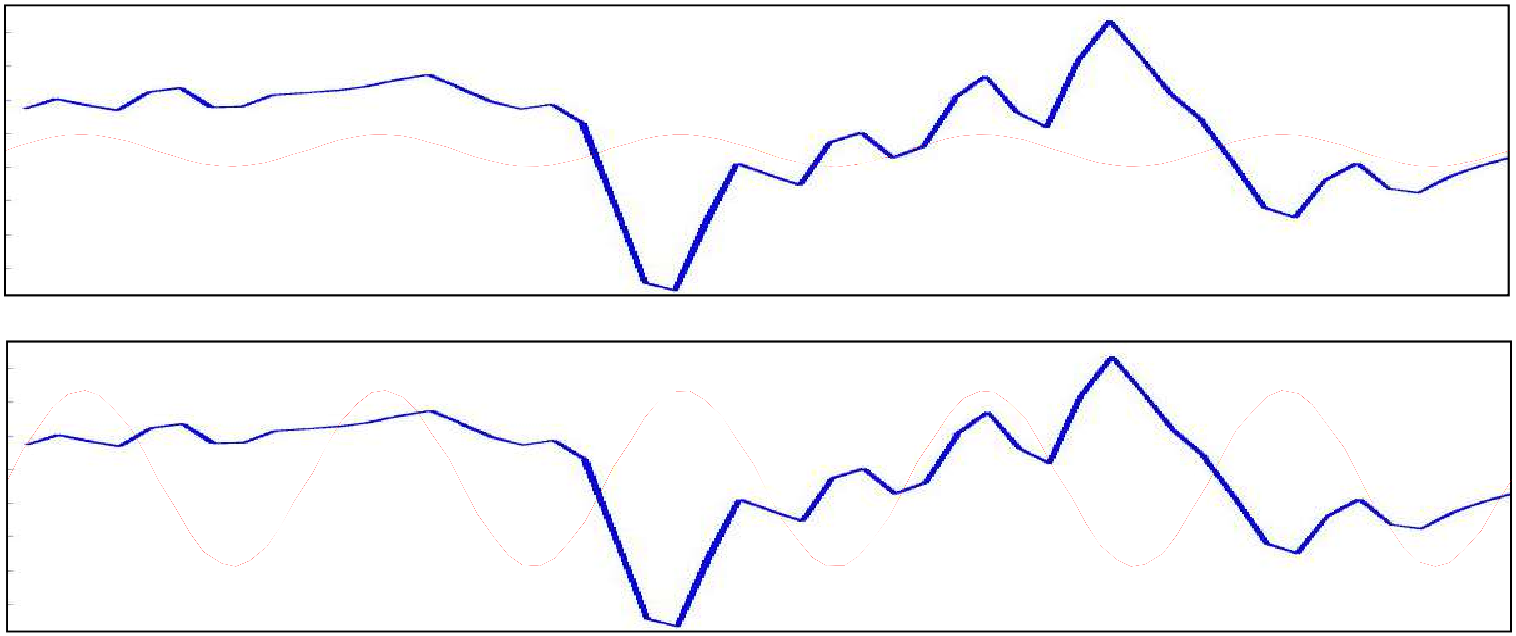
Animation:
https://upload.wikimedia.org/wikipedia/commons/2/2b/Fourier_series_and_transform.gif



View for time-domain

View for frequency-domain

Signal decomposition as optimization



Decompose signal (blue plot) into a set of sinusoid functions (red plot)

- Idea 1: Each sinusoid's amplitude is <u>adjusted</u> until it gives the smallest error. The amplitude is the weight of the sinusoid.
- Idea 2: Move the sinusoid left/right, and at each shift, try all amplitudes. Find the combination of amplitude and phase that results in the smallest error.

Fourier analysis

Signal (Fourier domain)	Apply all signal values	Signal (time domain)	Basis (sinusoid functions)

$$\text{Forward: } F(u) = \sum_{x=0}^{N-1} f(x) e^{\frac{-i2\pi ux}{N}}, \text{where } u = 0,1,\cdots,N-1$$

$$\text{Inverse: } f(x) = \frac{1}{N} \sum_{u=0}^{N-1} F(u) e^{\frac{i2\pi ux}{N}}, \text{where } x = 0,1,\cdots,N-1$$

Note: $e^{ix} = \cos x + i \sin x$; $e^{i\pi} = \cos \pi + i \sin \pi = -1$, Reference: https://en.wikipedia.org/wiki/Euler%27s_identity

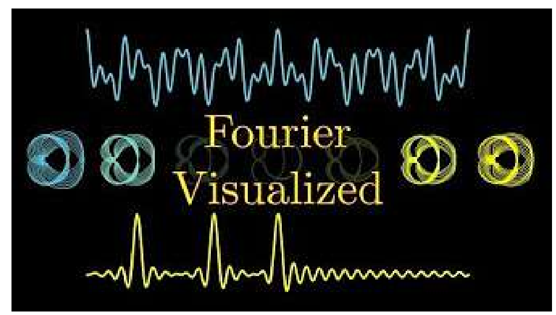
Example
- Signal: $f(x) = [2, 3, 4, 4]$
- Fourier coefficients: $\mathrm{F}(u) = [13, (-2+i), -1, (-2-i)]$, where i is the imaginary unit

$$F(0) = \sum_{x=0}^{3} f(x) e^{\frac{-i2\pi 0x}{4}} = 2 + 3 + 4 + 4 = 13$$

$$F(1) = \sum_{x=0}^{3} f(x) e^{\frac{-i2\pi x}{4}} = 2e^{0} + 3e^{-i\pi/2} + 4e^{-i\pi} + 4e^{-i3\pi/2} = -2 + i$$

$$F(2) = \sum_{x=0}^{3} f(x) e^{\frac{-i4\pi x}{4}} = 2e^{0} + 3e^{-i\pi} + 4e^{-i2\pi} + 4e^{-i3\pi} = -1$$

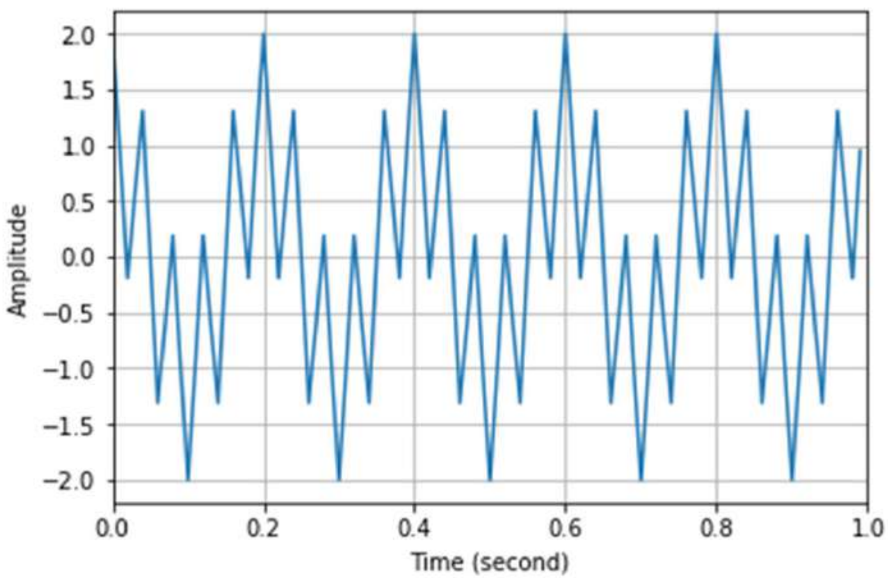
$$F(3) = \sum_{x=0}^{3} f(x) e^{\frac{-i6\pi x}{4}} = 2e^{0} + 3e^{-i3\pi/2} + 4e^{-i3\pi} + 4e^{-i9\pi/2} = -2 - i$$

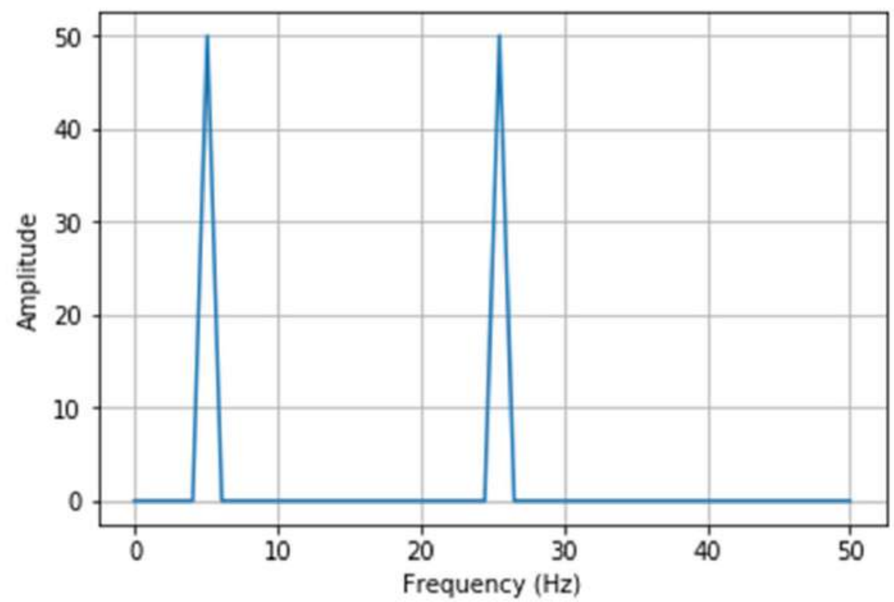


What is the Fourier Transform? A visual introduction
A 20-minute video tutorial on
https://www.youtube.com/watch?v=spUNpyF58BY
4.4 millions views since Janurary 2018

Fourier analysis

Challenge: Provides good localization in the frequency domain but poor localization in the time domain. Has knowledge of what frequencies exist, but no information about where these frequencies are located in time.

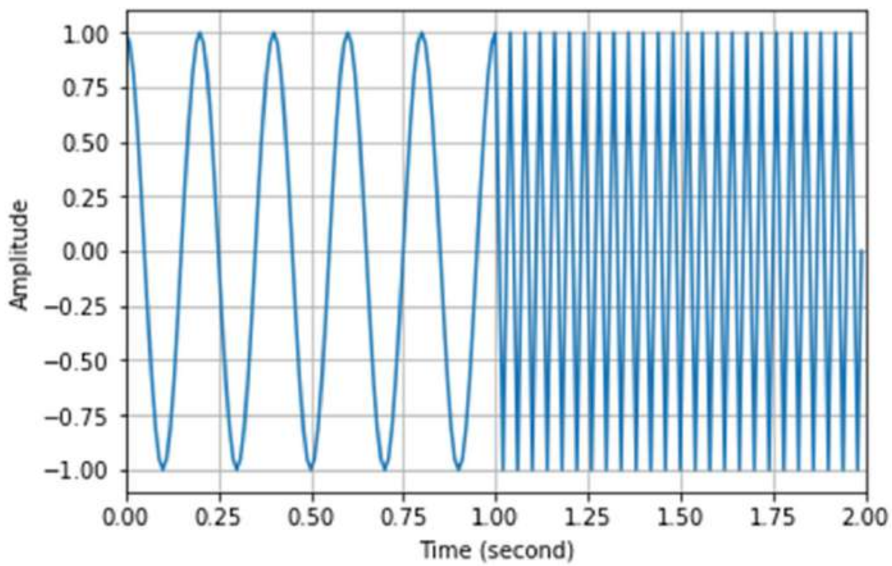


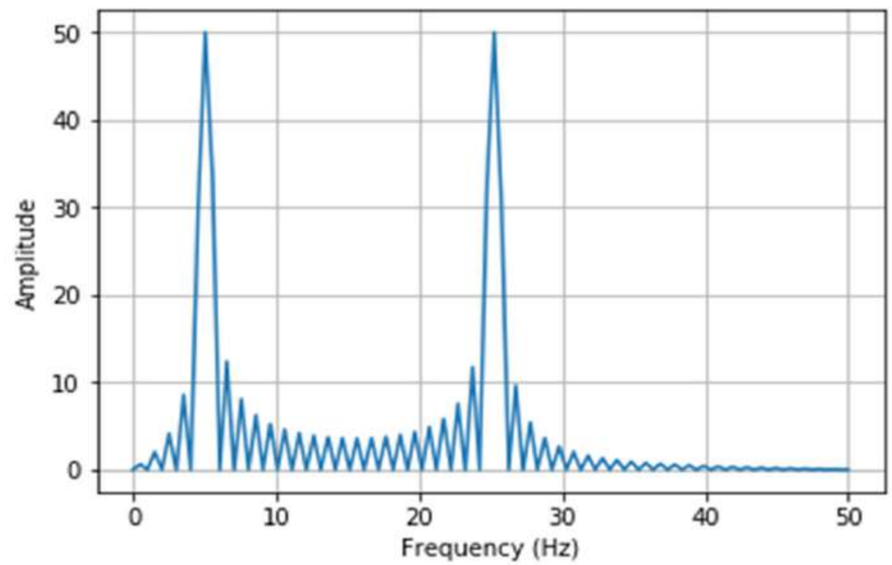


$$f(t) = \cos(2\pi \cdot 5 \cdot t) + \cos(2\pi \cdot 25 \cdot t)$$

Fourier analysis

Challenge: Provides good localization in the frequency domain but poor localization in the time domain. Has knowledge of what frequencies exist, but no information about where these frequencies are located in time.

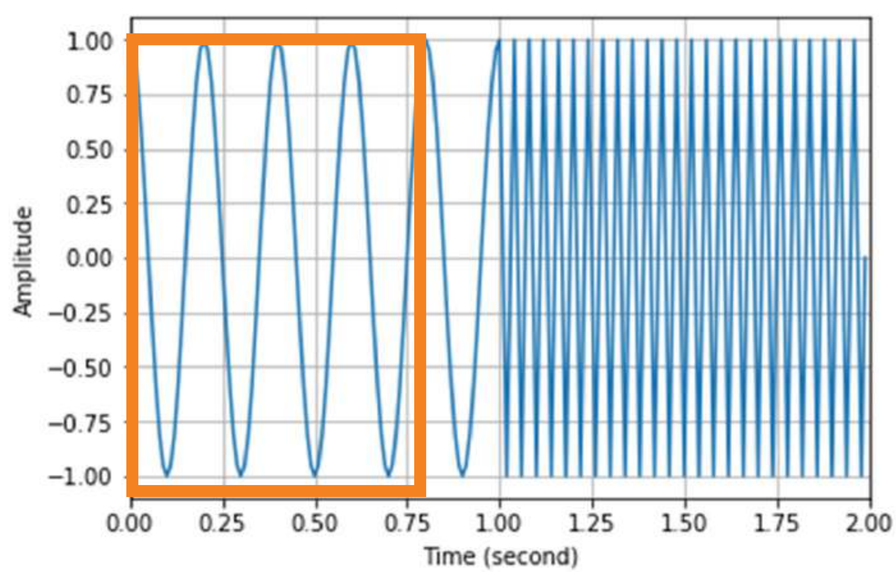




$$f(t) = \begin{cases} \cos(2\pi \cdot 5 \cdot t) & 0 \leq t < 1 \\ \cos(2\pi \cdot 25 \cdot t) & 1 \leq t < 2 \end{cases}$$

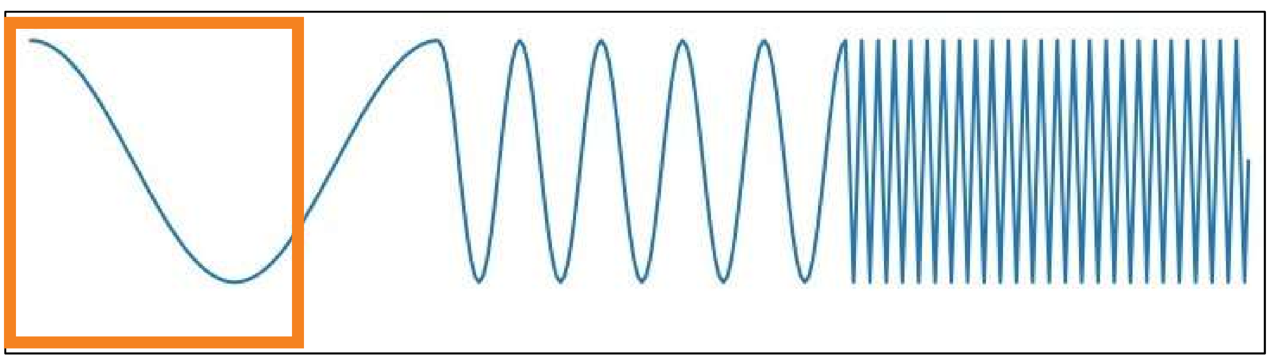
Short time Fourier transform

- Segment the signal into narrow time intervals (i.e., narrow enough to be considered stationary) and take the Fourier transform of each segment.
- Each Fourier transform provides the spectral information of a separate time-slice of the signal, providing simultaneous time and frequency information.



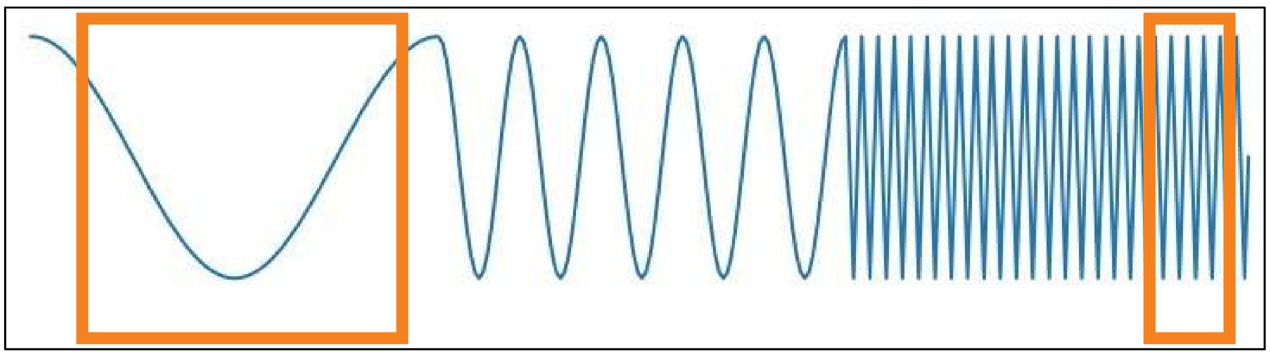
Short time Fourier transform

- Choose a window function of finite length
- Place the window on start of the signal
- Truncate the signal using this window
- Compute Fourier transform of the truncated signal
- Incrementally slide the window to the right
- Repeat until window reaches the end of the signal



Short time Fourier transform

- A compromise between time-based and frequency-based views of a signal. The precision is determined by the size of the window.
  - Wide window: good frequency resolution, poor time resolution. Wider windows are more appropriate at low frequencies
  - Narrow window: good time resolution, poor frequency resolution. Narrower windows are more appropriate at high frequencies.

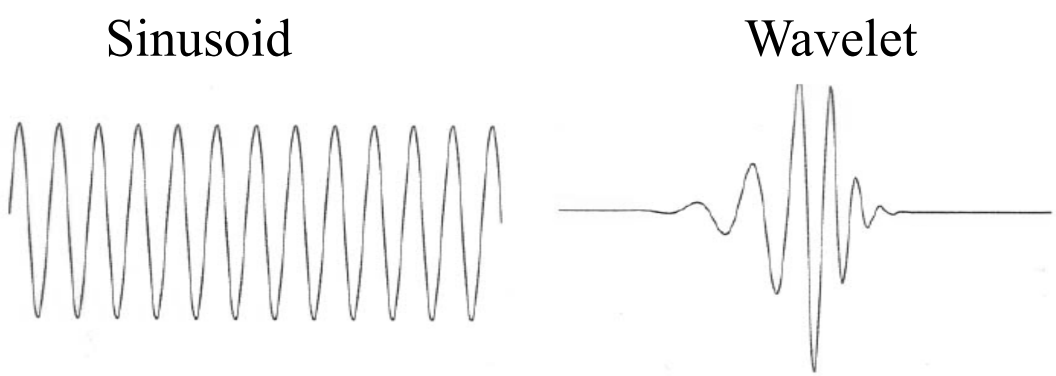


Topics

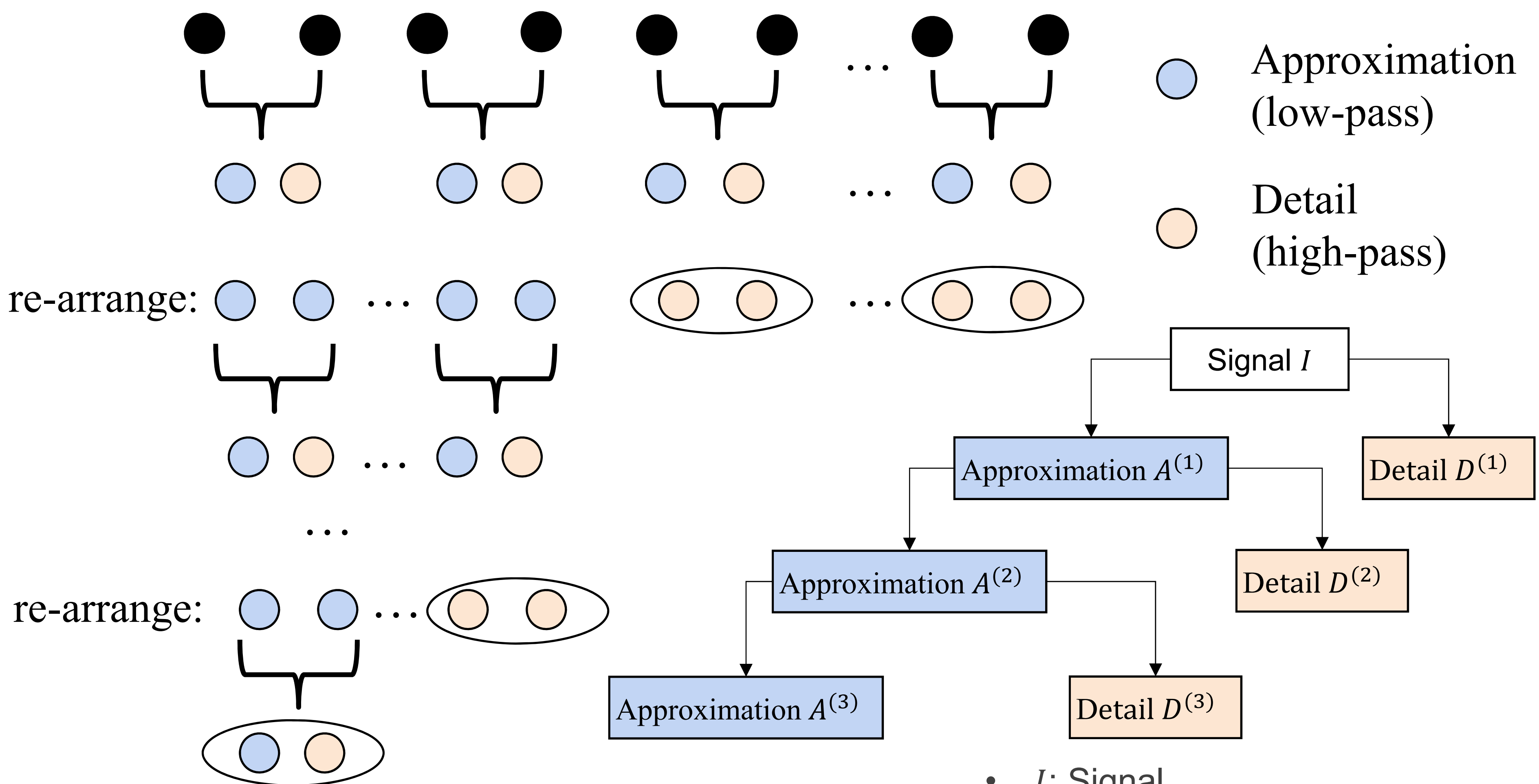
- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

What is a wavelet?

- A function that "waves" above and below the x-axis with the following properties
  - Varying frequency
  - Limited duration
- This is in contrast to sinusoids, used by Fourier transform, which have infinite duration and constant frequency.



Overview: Signal decomposition



- I: Signal
- A: Approximation coefficient
- D: Detail coefficient

Example 1: Decomposition

- Decompose signal by averaging and subsampling the signal together to get new approximation coefficients and detail coefficients.
- Note: Haar wavelet is used in this example. There are many other types of wavelet (choices of filters, see slide 32) in the literature.

input: $[9 \quad 7 \quad 3 \quad 5]$

Average filter [½, ½] with duplication padding

$(9+7)/2=8$

$[8 \quad 5 \quad 4 \quad 5]$

Difference filter [½, -½] with duplication padding

$(9-7)/2=1$

$[1 \quad 2 \quad -1 \quad 0]$

Down-sampling ratio 2

$[8 \quad 4]$

Down-sampling ratio 2

$[1 \quad -1]$

Average filter [½, ½] with duplication padding

$[6 \quad 4]$

Difference filter [½, -½] with duplication padding

$[2 \quad 0]$

Down-sampling ratio 2

$[6]$

Down-sampling ratio 2

$[2]$

Result: Wavelet coefficients
$[6 \quad 2 \quad 1 \quad -1]$

Example 1: Reconstruction

- The original signal can be reconstructed by adding or subtracting the detail coefficients from the approximation coefficients.

Given the wavelet coefficients (obtained in previous slide) $[6 \quad 2 \quad 1 \quad -1]$

$$[6] \xrightarrow{2} [8 \quad 4] \xrightarrow{[1\ -1]} [9 \quad 7 \quad 3 \quad 5]$$

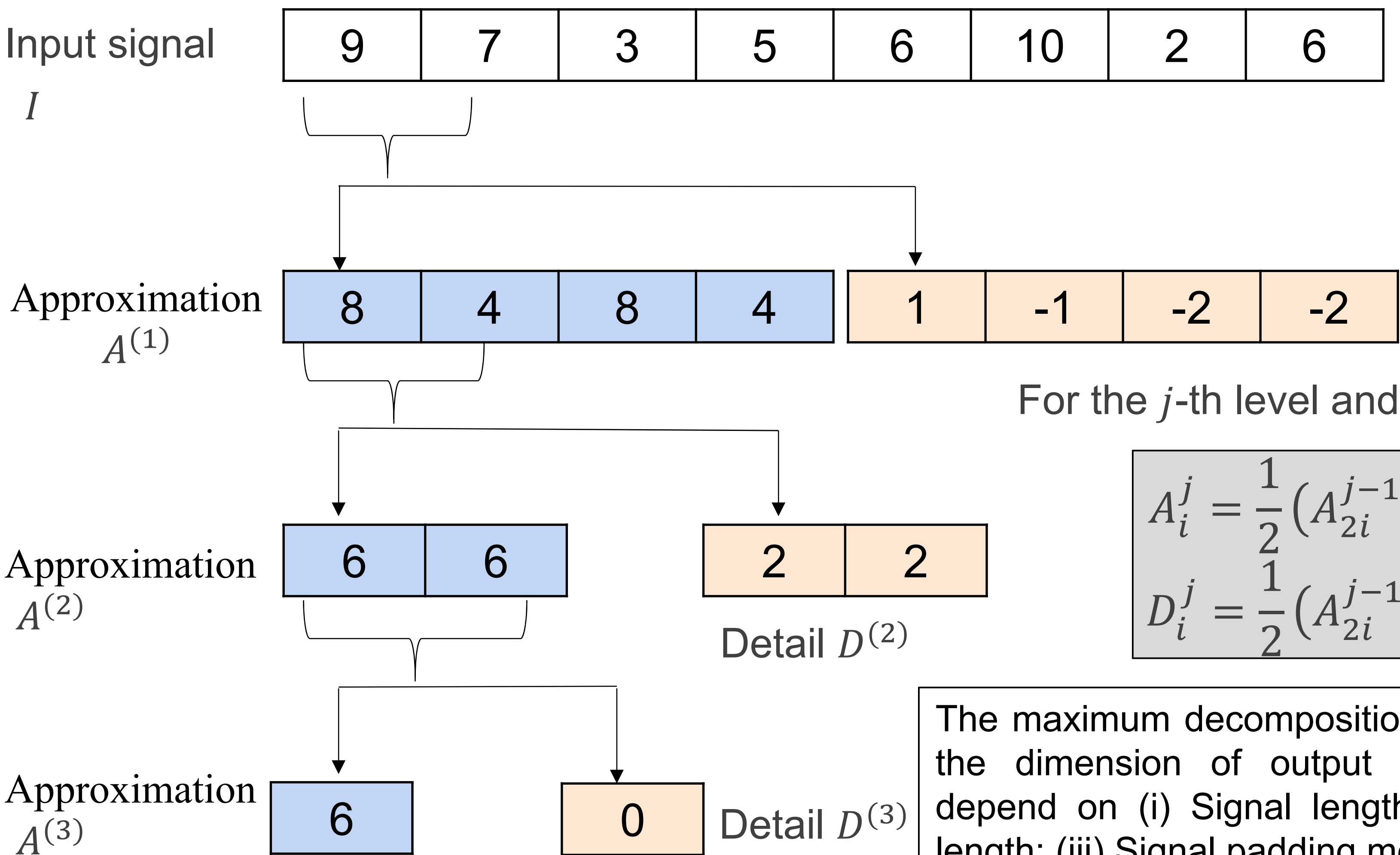
$8 = (6+2)$, $4 = (6-2)$

$9 = (8+1)$, $7 = (8-1)$, $3 = (4+(-1))$, $5 = (4-(-1))$

Note: This process is only applicable for the example in the previous slide. Other types of wavelet filters will have different calculation processes.

Example 2: Signal decomposition

- The detail layer is produced by convolving with [½, -½] (then subsampling)
- The approximate layer is produced by convolving with [½ ½] (then subsampling)



For the j-th level and i-th index

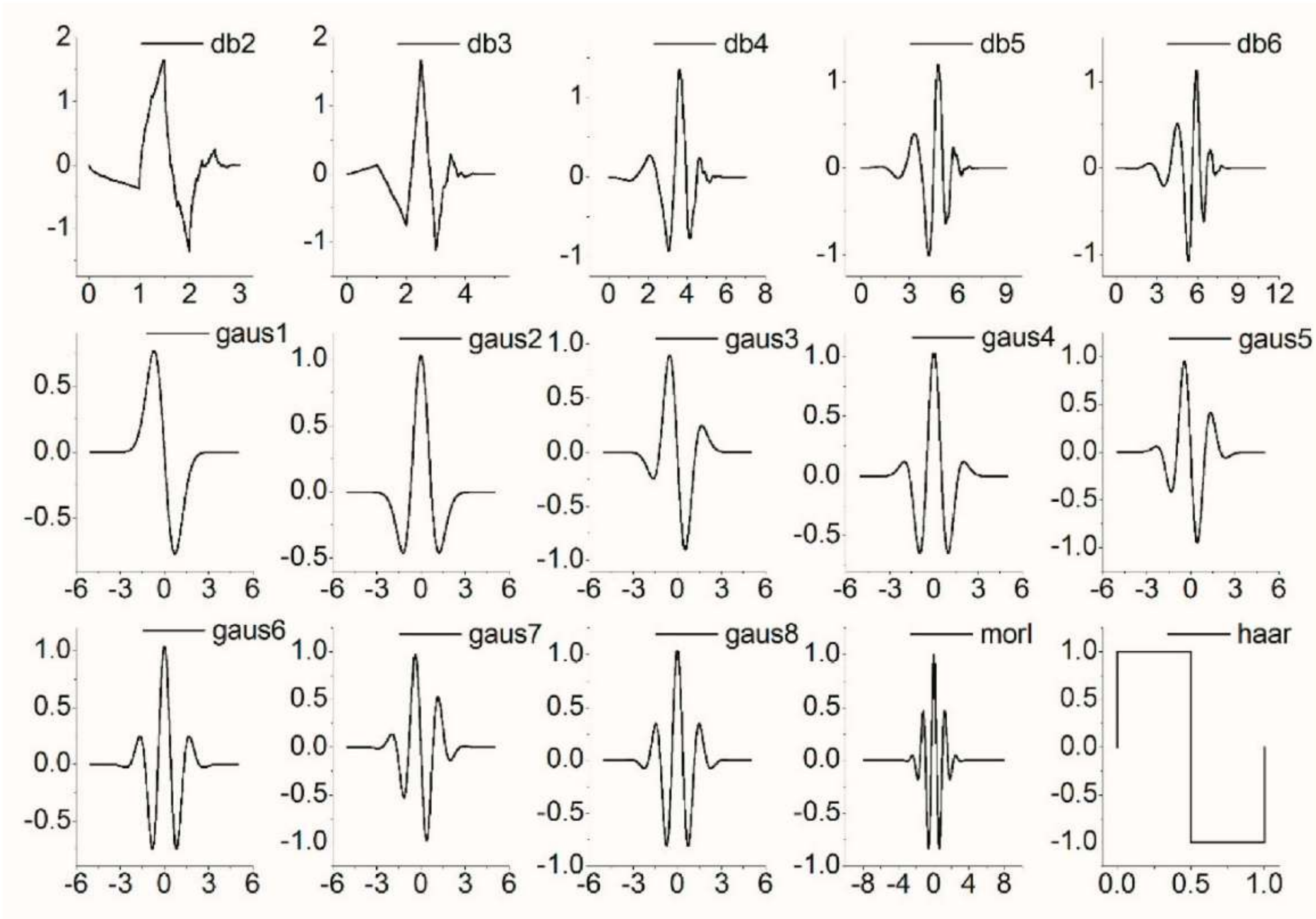
$$A_i^j = \frac{1}{2}\left(A_{2i}^{j-1} + A_{2i+1}^{j-1}\right)$$

$$D_i^j = \frac{1}{2}\left(A_{2i}^{j-1} - A_{2i+1}^{j-1}\right)$$

The maximum decomposition level and the dimension of output coefficients depend on (i) Signal length; (ii) Filter length; (iii) Signal padding method.

Reference: https://pywavelets.readthedocs.io/en/latest/ref/dwt-discrete-wavelet-transform.html#maximum-decomposition-level-dwt-max-level-dwtn-max-level

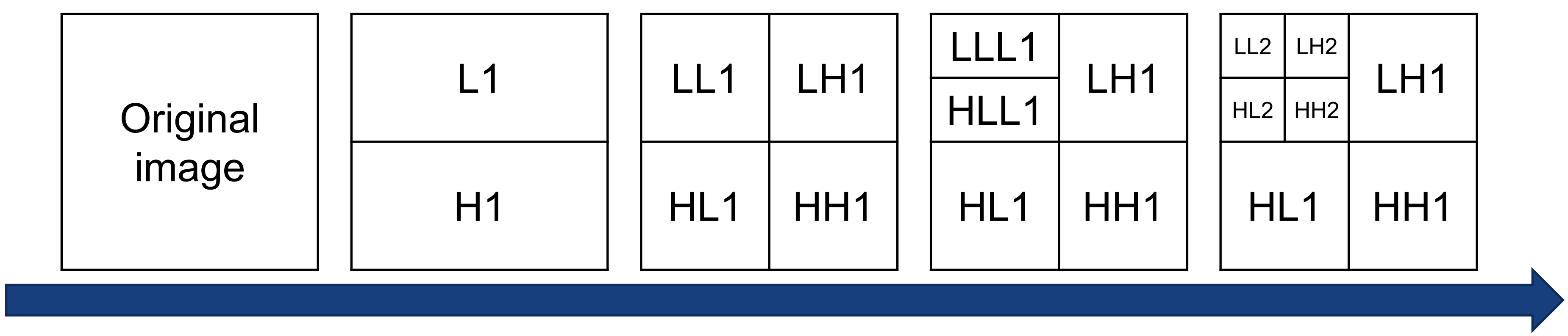
Wavelet: Other choice of filters



Source: C. Xia and C. Liu, "Identification and Representation of Multi-Pulse Near-Fault Strong Ground Motion Using Adaptive Wavelet Transform," Applied Sciences, Vol. 9, No. 2, pp. 259, 2019 , https://www.mdpi.com/2076-3417/9/2/259

2D wavelet transformation

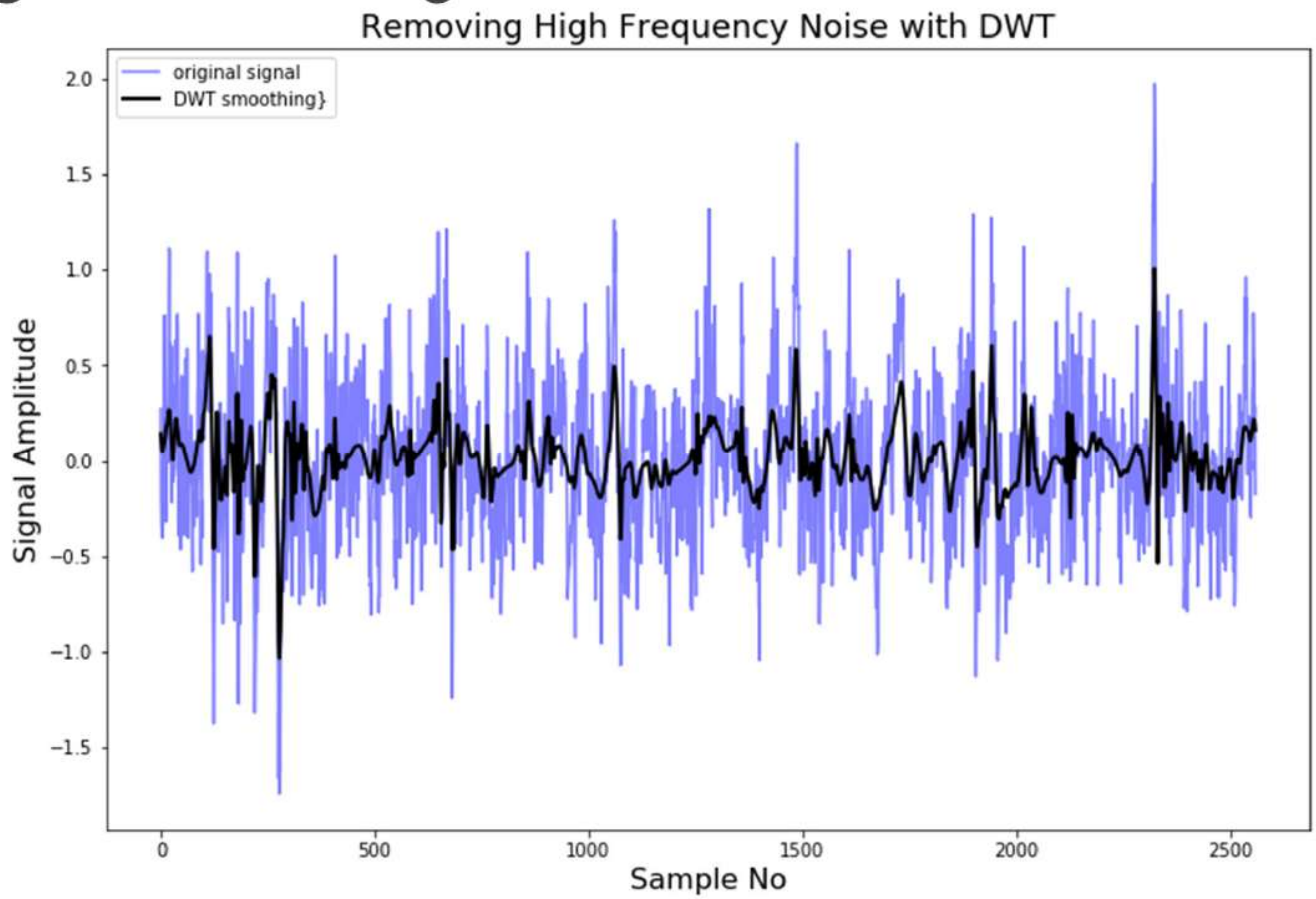
- LL: The upper left quadrant is filtered by the analysis low-pass filter along the rows and then filtered along the corresponding columns with the analysis low-pass filter. It represents the approximated version of the original at half the resolution.
- HL/LH: The lower left and the upper right blocks are filtered along the rows and columns with low-pass filter and high-pass filter, alternatively. The LH block contains vertical edges. In contrast, the HL blocks shows horizontal edges.
- HH: The lower right quadrant is derived analogously to the upper left quadrant but with the use of the analysis high pass filter, where we find edges of the original image in diagonal direction.



L: Low-pass filter, H: High-pass filter

Wavelet application: Denoising

- Signal denoising



Reference: http://ataspinar.com/2018/12/21/a-guide-for-using-the-wavelet-transform-in-machine-learning/

Wavelet application: Feature extraction

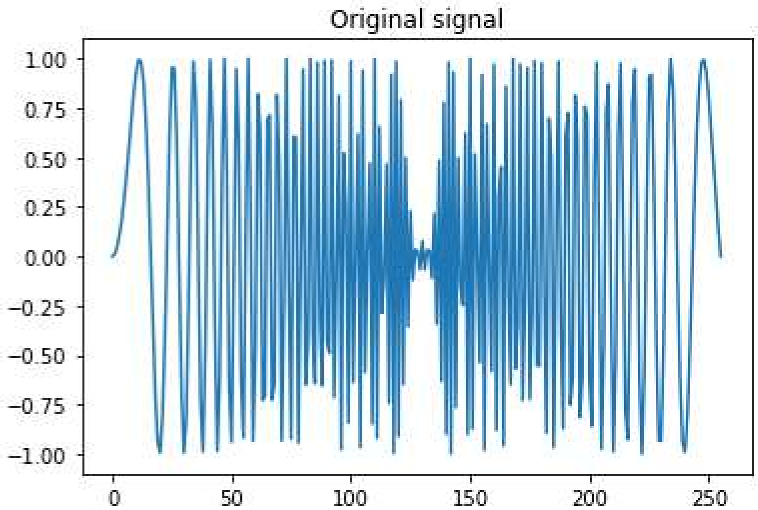
- Perform wavelet transformation on signal
- Extract statistical features
  - Auto-regressive model coefficient values
  - Statistical features like variance, mean, median, etc.
- Perform signal classification or other machine learning tasks

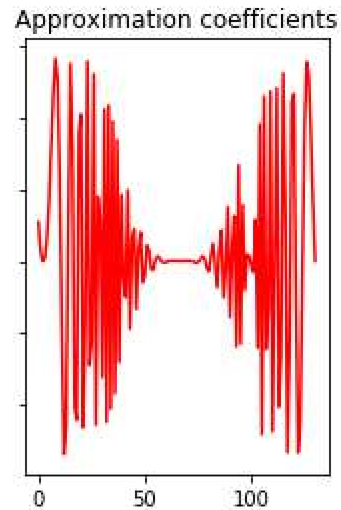
Workshop

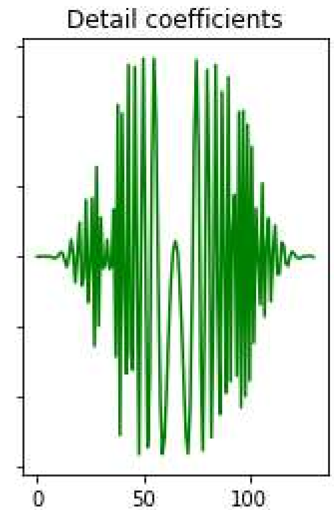
Objective

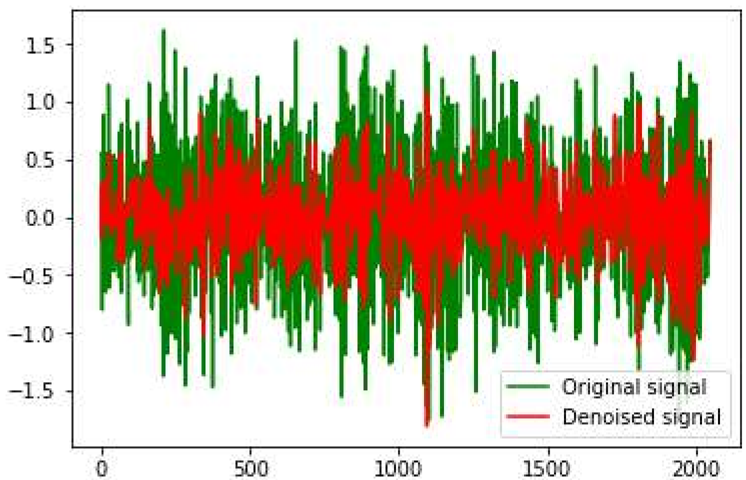
- Perform wavelet decomposition on signal
- Perform wavelet-based signal denoising
- Extract statistical features from wavelet coefficients from wearable sensor data, and then perform classification for human activity classification

Workshop









Human Activity Recognition Using Smartphones Data Set

- Each person performed six activities (WALKING, WALKING_UPSTAIRS, WALKING_DOWNSTAIRS, SITTING, STANDING, LAYING) wearing a smartphone (Samsung Galaxy S II) on the waist. Using its embedded accelerometer and gyroscope, we captured 3-axial linear acceleration and 3-axial angular velocity at a constant rate of 50Hz.
- Reference: https://archive.ics.uci.edu/ml/datasets/Human+Activity+Recognition+Using+Smartphones

Summary

- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

Thank you!

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