Tutorial 2 Q1 (g) and Q5(f)

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} \qquad 1^{\infty}\ \mathit{form}$$

$$=\lim_{x\to 0} e^{\ln\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}} = \lim_{x\to 0} e^{\frac{1}{x^2}[\ln\sin x - \ln x]}$$

$$= e^{\lim\limits_{x\to 0}\frac{1}{x^2}[\ln\sin x - \ln x]}$$

$$\lim_{x\to 0}\frac{1}{x^2}[\ln\sin x - \ln x] =$$

$$\lim_{x\to 0}\frac{x\cos x - \sin x}{2x^2\sin x}$$

$$=\lim_{x\to 0}\frac{x(1-x^2/2!+x^4/4!+\ldots)-(x-x^3/3!+x^5/5!+\ldots)}{2x^2(x-x^3/3!+..)}$$

$$=\lim_{x\to 0}\frac{-x^3/2!+x^5/4!+x^3/3!-x^5/5!+\ldots}{2x^3(1-x^2/3!)}$$

$$=\frac{-\frac{1}{2!}+\frac{1}{3!}}{2}$$

Why

$$\boxed{=\lim_{x\to 0}\frac{-1/2!+x^5/x^3 4!+1/3!-x^5/x^3 5!+\ldots}{2(1-x^2/3!)}}$$

Q5 (f) You are asked to use the method

of integration by parts to find the following integral:

$$\int\{\sin e^{-x} + e^{x}\cos e^{-x}\}dx$$

Recall integration by parts

$$\int udv = uv - \int vdu$$

There are two integrals in this Q

$$\int\{\sin e^{-x} + e^{x}\cos e^{-x}\}dx$$

So we may guess that we should apply IP to each integral. The 2nd one is easier.

$$\int e^{x}\cos e^{-x}dx = \int \cos e^{-x}de^{x} \ .$$

Let $v = e^{x}, u = \cos e^{-x}$.

Then we will realize that in fact the given integral= $\int udv + \int vdu = uv = e^{x}\cos e^{-x}$