Another proof of Q 7 Tutorial 1

Q7 is an exercise of Calculus. Hence we shall use differentiation or integration to handle this Q. The solution provided is by differentiation. Now we shall use the inverse operation, integration, to prove the equality.

$$\int \tan x dx = \ln \sec x$$

$$\int \cot x dx = \ln \sin x$$

$$\frac{1}{2}\tan\frac{x}{2}+\frac{1}{2^2}\tan\frac{x}{2^2}+\ldots+\frac{1}{2^n}\tan\frac{x}{2^n}=\frac{1}{2^n}\cot\frac{x}{2^n}-\cot x$$

$$\Leftrightarrow \int\frac{1}{2}\tan\frac{x}{2}+\int\frac{1}{2^2}\tan\frac{x}{2^2}+\ldots+\int\frac{1}{2^n}\tan\frac{x}{2^n}=\int\frac{1}{2^n}\cot\frac{x}{2^n}-\int\cot x$$

$$\Leftrightarrow \ln\sec\frac{x}{2}+\ln\sec\frac{x}{2^2}+\ldots+\ln\sec\frac{x}{2^n}=\ln\sin\frac{x}{2^n}-\ln\sin x+c$$

$$\ln\left[\sec\frac{x}{2}\sec\frac{x}{2^2}\ldots\sec\frac{x}{2^n}\right]=\ln\frac{\sin\frac{x}{2^n}}{\sin x}+c$$

$$\Leftrightarrow \frac{1}{\cos\frac{x}{2}\cos\frac{x}{2^2}\ldots\cos\frac{x}{2^n}} = e^c\,\frac{\sin\frac{x}{2^n}}{\sin x}$$

$$\Leftrightarrow \sin x = e^c \cos\frac{x}{2}\cos\frac{x}{2^2}\ldots\cos\frac{x}{2^n}\sin\frac{x}{2^n}$$

This equality holds, see the solution of Q7