Equations of planes and tangent planes

First recall Chapter 5, Section 5

Equations of Planes

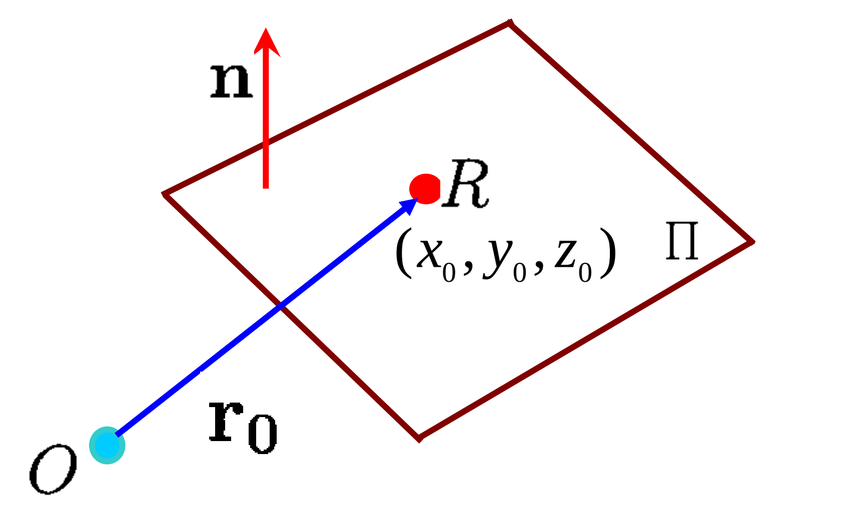
5.5 ***Planes*** in **Space**

♣ A ***plane*** **Π** in space is determined by

(i) a ***point*** on the plane &

(ii) its ***orientation*** (indicated by a ***normal*** to **Π**)

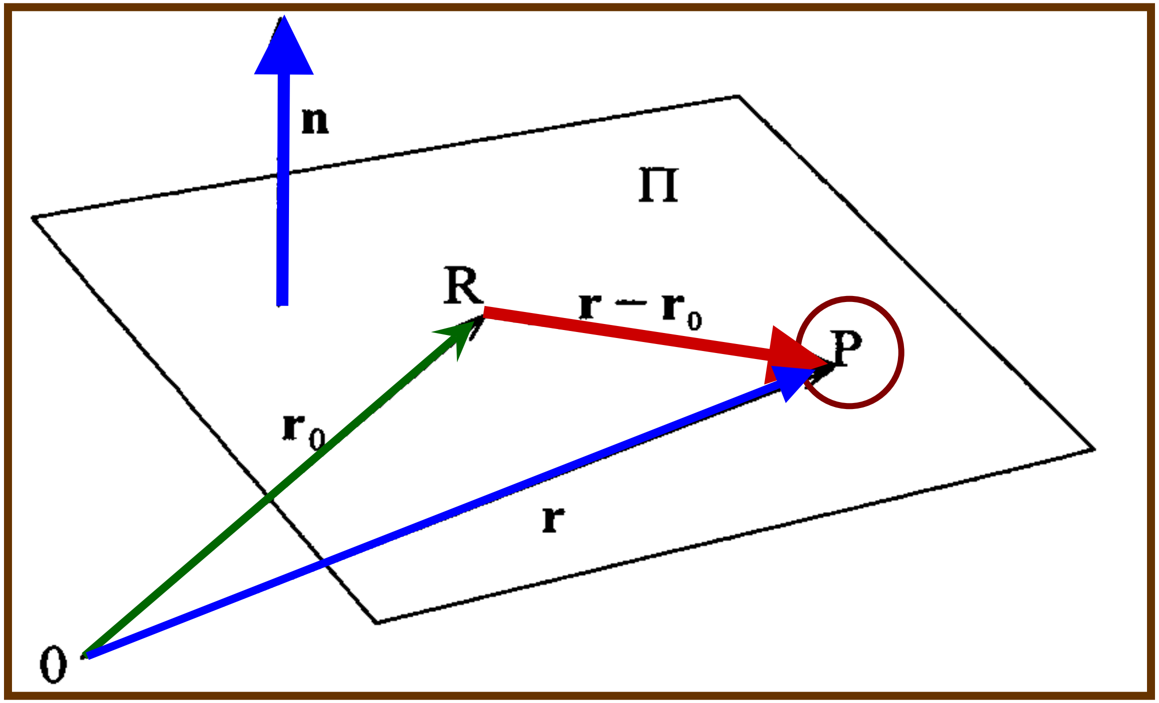
Problem Given point ***R*** in plane **Π** with position vector $\mathbf{r}_0 = x_0\mathbf{i} + y_0\mathbf{j} + z_0\mathbf{k}$ & normal $\mathbf{n} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ to **Π** :



find an ***equation*** for **Π**

Vector equation for plane $\mathbf{\Pi}$

- Let ***P*** be a point in plane $\mathbf{\Pi}$ with position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Then



$$(\mathbf{r} - \mathbf{r}_0) \cdot \mathbf{n} = 0.$$

5.5.1 *Cartesian* equation for Π

$$\boxed{(\mathbf{r}-\mathbf{r}_0)\cdot\mathbf{n}=0}$$

$$\mathbf{r}-\mathbf{r_0}=(x-x_0)\mathbf{i}+(y-y_0)\mathbf{j}+(z-z_0)\mathbf{k}$$

$$\mathbf{n}=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$$

$$(\mathbf{r}-\mathbf{r_0})\cdot\mathbf{n}=a(x-x_0)+b(y-y_0)+c(z-z_0)$$

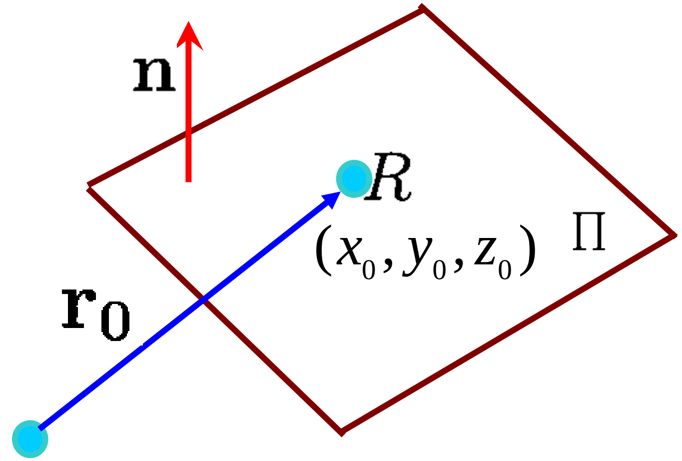
$$\boxed{a(x-x_0)+b(y-y_0)+c(z-z_0)=0}$$

or

$$\boxed{\begin{gathered}ax+by+cz=d,\\ \text{where } d=ax_0+by_0+cz_0\end{gathered}}$$

SUMMARY

Equations for Π



$$\mathbf{r_0} = x_0\mathbf{i} + y_0\mathbf{j} + z_0\mathbf{k}$$

$$\mathbf{n} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

♣ **Vector equation :**

$$(\mathbf{r} - \mathbf{r}_0) \cdot \mathbf{n} = 0$$

♣ **Cartesian equation :**

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

♣ **Cartesian equation simplified :**

$$ax + by + cz = d,$$

where $d = ax_0 + by_0 + cz_0$

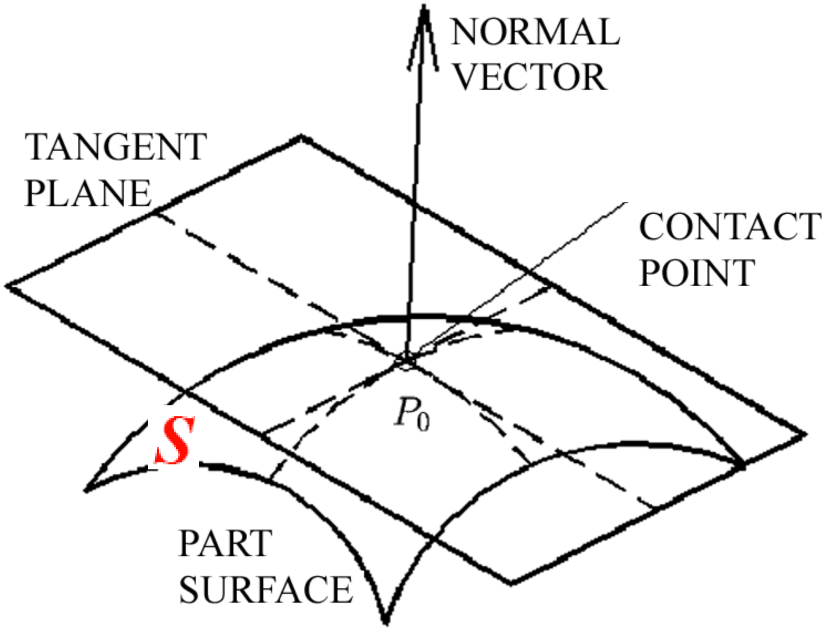
- # Tangent Planes

Given : surface ***S***

$$\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$$

& a point P_0 with position vector $\mathbf{r}_0 = \mathbf{r}(u_0, v_0)$

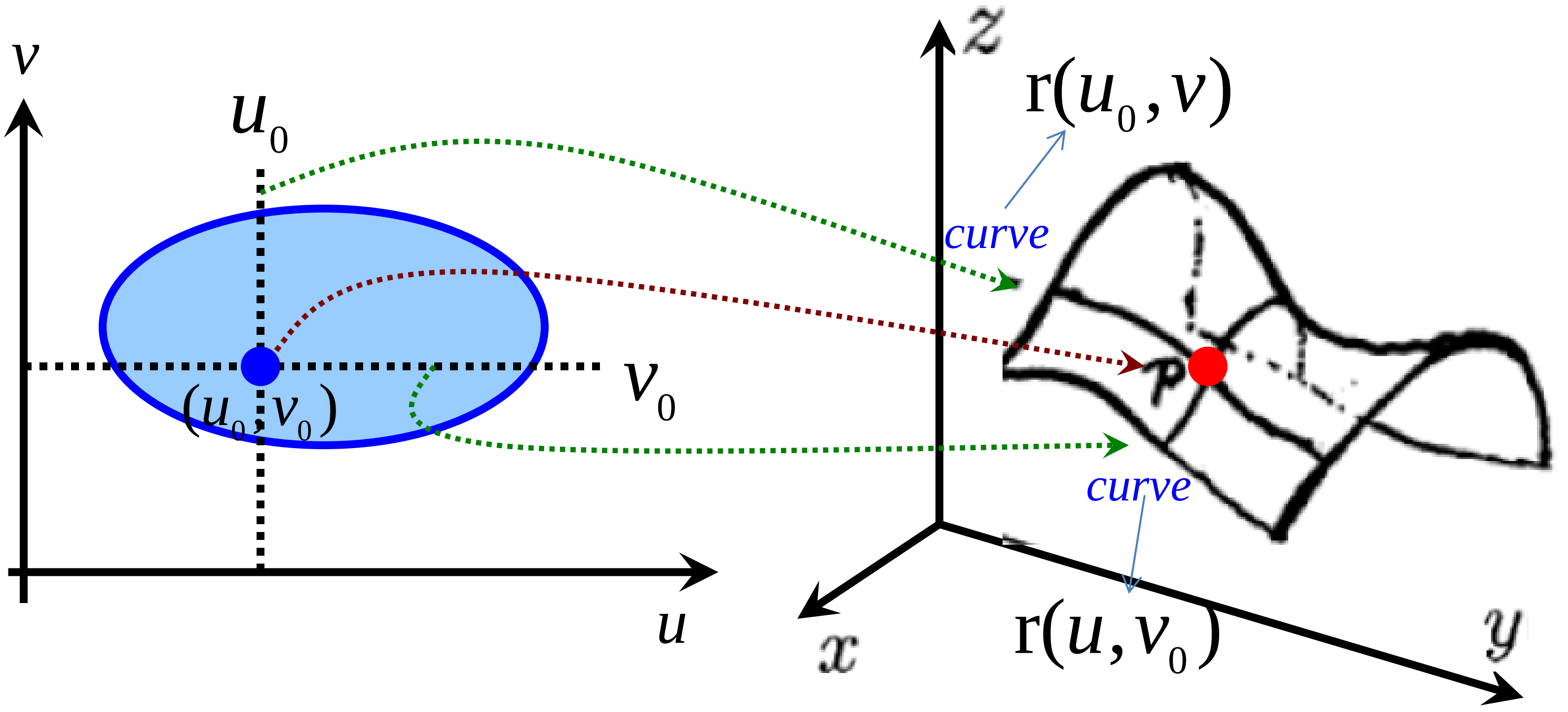
Find : the equation of the **tangent plane** to ***S*** at P_0



- ***Parametric surfaces*** in space :

$$\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k} \qquad (1)$$

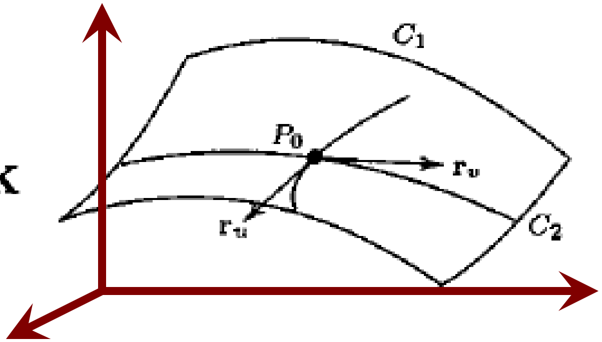
where u and v are two independent parameters.



- **Fix** $v = v_0$. **Curve** C_1

$$\mathbf{r}(u, v_0) = x(u, v_0)\mathbf{i} + y(u, v_0)\mathbf{j} + z(u, v_0)\mathbf{k}$$

Tangent vector to C_1 at P_0



$$\mathbf{r}_u = \frac{\partial x}{\partial u}(u_0, v_0)\mathbf{i} + \frac{\partial y}{\partial u}(u_0, v_0)\mathbf{j} + \frac{\partial z}{\partial u}(u_0, v_0)\mathbf{k}$$

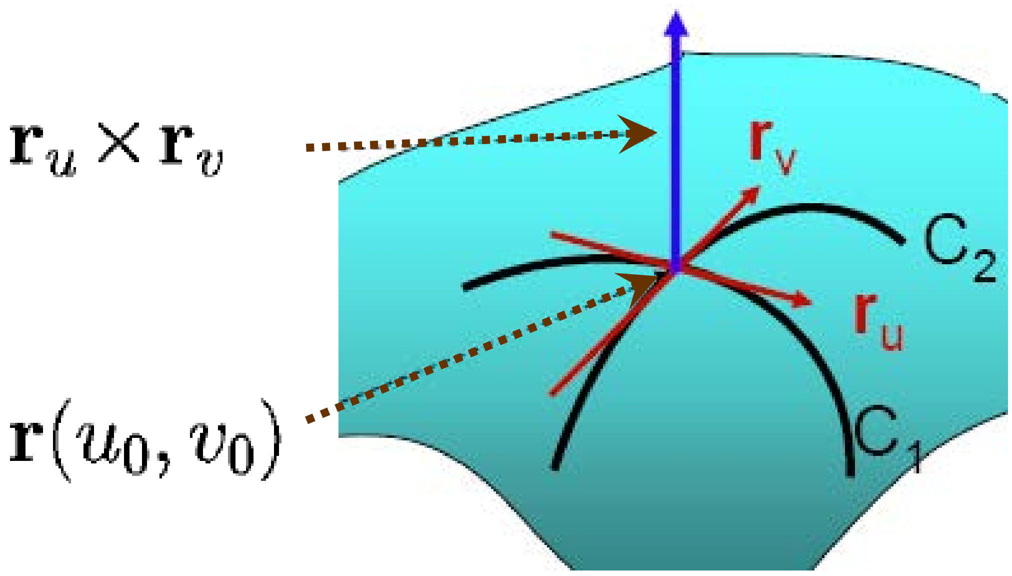
- **Fix** $u = u_0$. **Curve** C_2

$$\mathbf{r}(u_0, v) = x(u_0, v)\mathbf{i} + y(u_0, v)\mathbf{j} + z(u_0, v)\mathbf{k}$$

Tangent vector to C_2 at P_0

$$\mathbf{r}_v = \frac{\partial x}{\partial v}(u_0, v_0)\mathbf{i} + \frac{\partial y}{\partial v}(u_0, v_0)\mathbf{j} + \frac{\partial z}{\partial v}(u_0, v_0)\mathbf{k}$$

- As $\mathbf{r}_u$ and $\mathbf{r}_u$ lie in the tangent plane to S at P_0, the cross product $\mathbf{r}_u \times \mathbf{r}_v$ provides a **normal** vector to the tangent plane. Thus the **equation** of the **tangent plane** is : $\boxed{(\mathbf{r} - \mathbf{r}_0) \cdot (\mathbf{r}_u \times \mathbf{r}_v) = 0}$



See CH5, 5.5 planes in Space

$(r-r_0)\bullet n = 0$