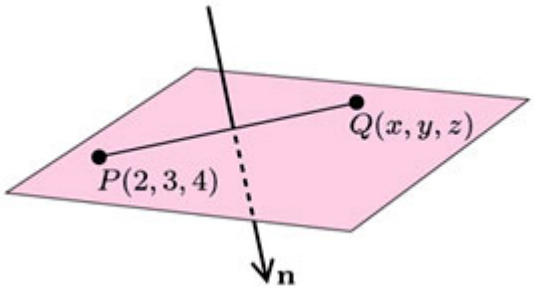
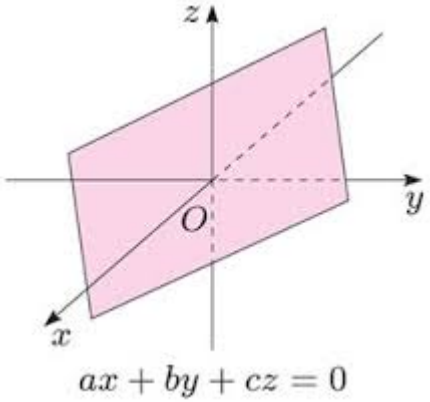
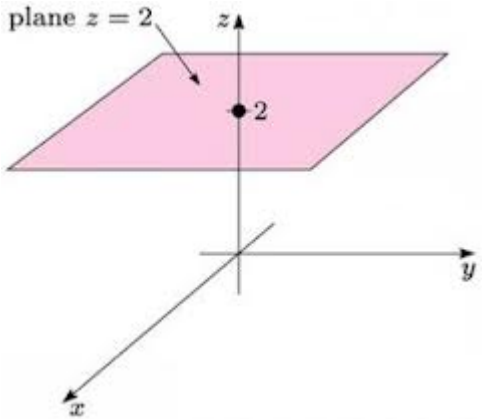
Vector equations(parametric equations) of surfaces

For the domain of $\mathrm{r}(u,v)$ **see past exam papers**

(A) Planes







(1) $2x-5y+3z=4$

$$Let\ x=u,\ y=v.\ Then\ z=\frac{1}{3}(4-2u+5v)$$

$$\mathrm{r}(u,v)=u\mathrm{i}+v\mathrm{j}+(1/3)(4-2u+5v)\mathrm{k}$$

(2) $3x - y = 5$

$let\ z = u,\ x = v.$ Then $y = 3v - 5$

$\mathrm{r}(u,v) = v\mathrm{i} + (3v - 5)\,\mathrm{j} + u\mathrm{k}$

(3) $z = 5$

$let\ x = u, y = v.\ Then$

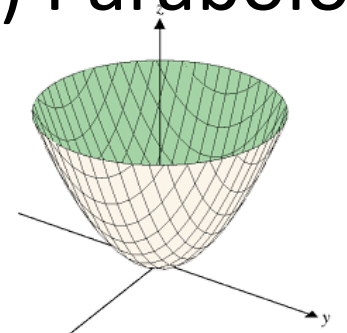
$\mathrm{r}(u,v) = u\mathrm{i} + v\mathrm{j} + 5\mathrm{k}$

(4) $x = 3$

$let\ y = u, z = v.\ Then$

$\mathrm{r}(u,v) = 3\mathrm{i} + u\mathrm{j} + v\mathrm{k}$

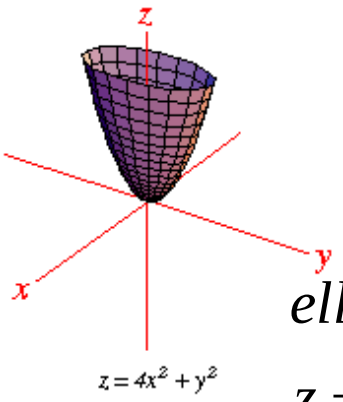
(B) Paraboloid



circular paraboloid

$$z = x^2 + y^2$$

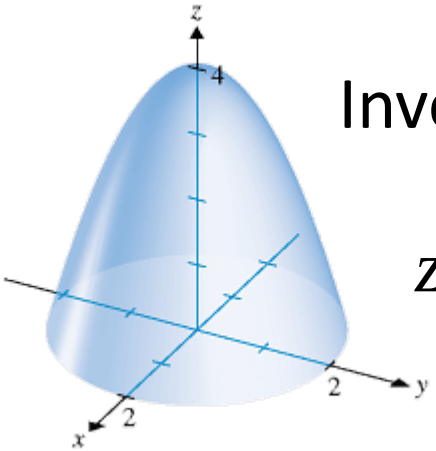
$$\mathrm{r}(u,v) = u\mathrm{i} + v\mathrm{j} + (u^2 + v^2)\mathrm{k}$$



elliptic paraboloid

$$z = 4x^2 + y^2$$

$$\mathrm{r}(u,v) = u\mathrm{i} + v\mathrm{j} + (4u^2 + v^2)\mathrm{k}$$



Inverted paraboloid

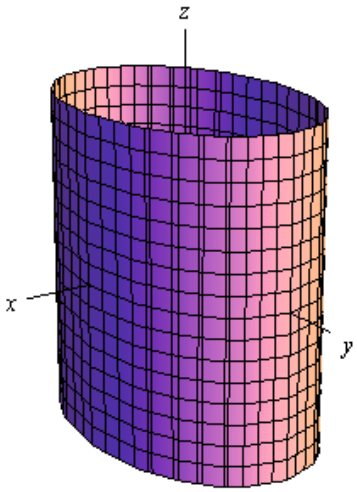
$$z = 4 - x^2 - y^2$$

$$\mathrm{r}(u,v) = u\mathrm{i} + v\mathrm{j} + (4 - u^2 - v^2)\mathrm{k}$$

(C) Cone and elliptic cylinder

$$z = \sqrt{x^2 + y^2}$$

$$\mathrm{r}(u,v) = u\mathrm{i} + v\mathrm{j} + (\sqrt{u^2 + v^2})\mathrm{k}$$

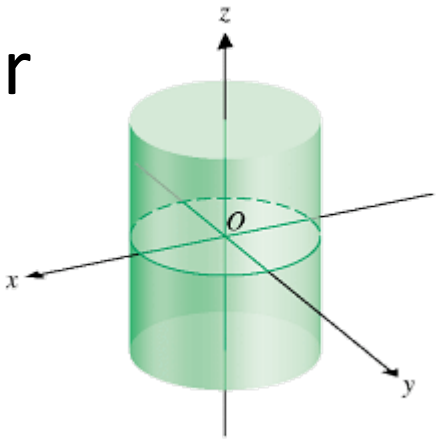


elliptic cylinder

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = c^2$$

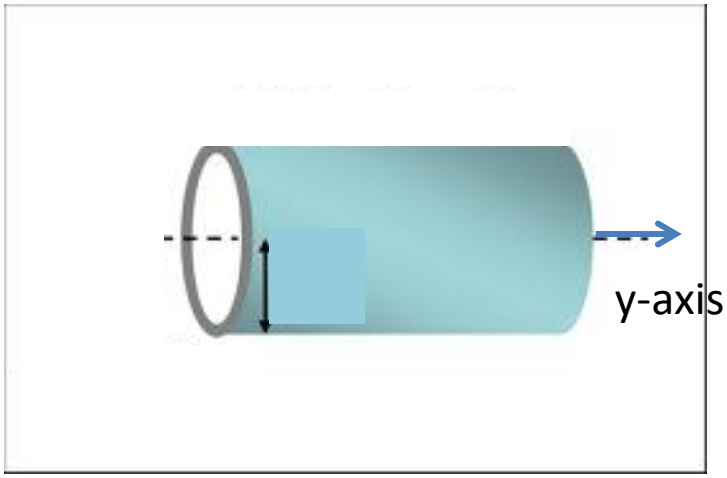
$$\mathrm{r}(u,v) = (\mathrm{c})(a)(\cos u)\mathrm{i} + (\mathrm{c})(b)(\sin u)\ \mathrm{j} + v\mathrm{k}$$

(D) Circular cylinder



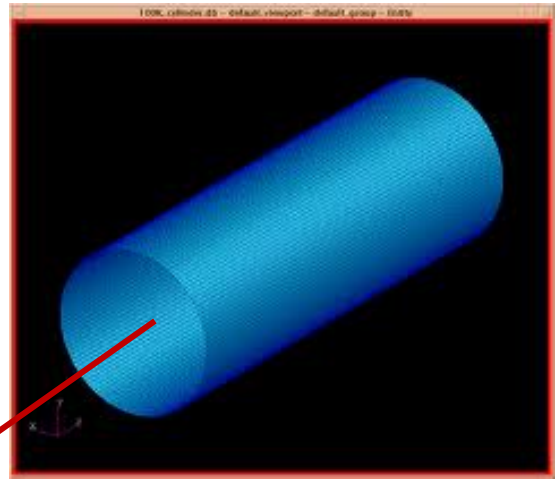
$$x^2 + y^2 = a^2$$

$$\mathrm{r}(u,v) = (a\cos u)\mathrm{i} + (a\sin u)\,\mathrm{j} + v\mathrm{k}$$



$$x^2 + z^2 = a^2$$

$$\mathrm{r}(u,v) = (a\cos u)\mathrm{i} + v\mathrm{j} + (a\sin u)\mathrm{k}$$



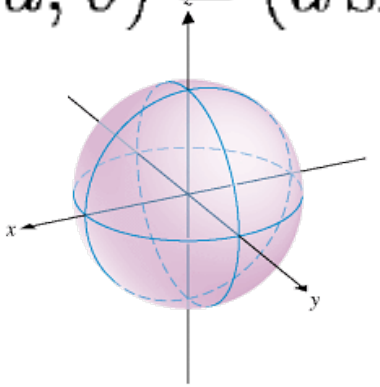
x-axis

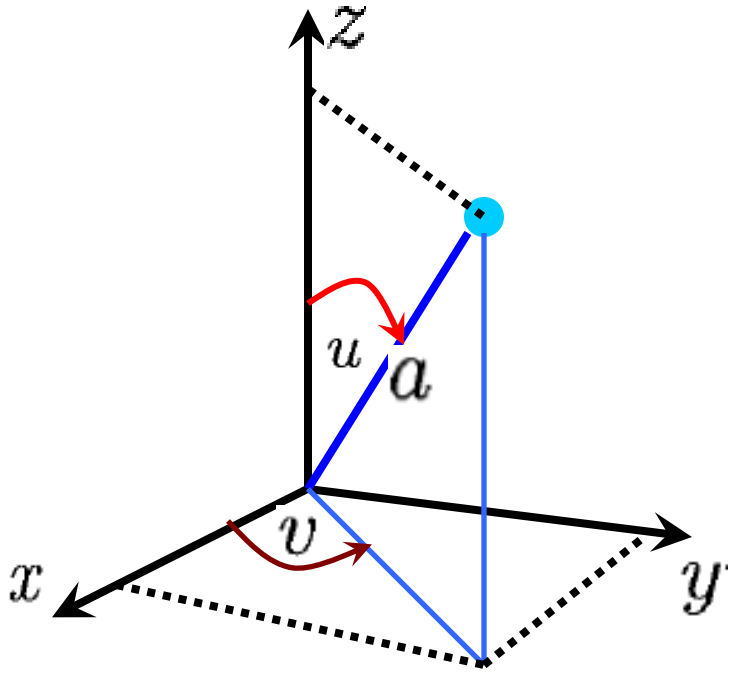
$$y^2 + z^2 = a^2$$

$$\mathrm{r}(u,v) = v\mathrm{i} + (a\cos u)\,\mathrm{j} + (a\sin u)\mathrm{k}$$

(E) **Spheres** ($x^2 + y^2 + z^2 = a^2$ with **radius** a)

$$\mathbf{r}(u, v) = (a \sin u \cos v)\mathbf{i} + (a \sin u \sin v)\mathbf{j} + (a \cos u)\mathbf{k}.$$





- **Full sphere** :

$$0 \le u \le \pi, \quad 0 \le v \le 2\pi$$

- **Upper hemisphere** :

$$0 \le u \le \pi/2, \quad 0 \le v \le 2\pi$$