MA 1505

(Group C)

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Contents

Reference:
Thomas' Calculus

Webcast of Lectures

CH 1-*Functions* & *Limits*

1.1 *Functions*

Three Examples

- $y = x^2$
- $y = \sqrt{x}$
- $y = \sin x$

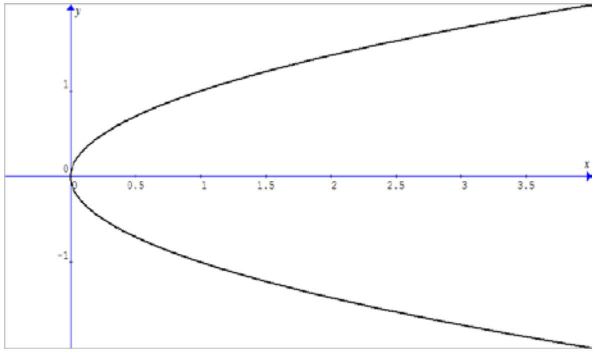
y is a function of x

y is denoted by $f(x)$

A *function* is a rule that assigns to each x a unique value y

x called independent variable

y called dependent variable



$$y^2 = x$$

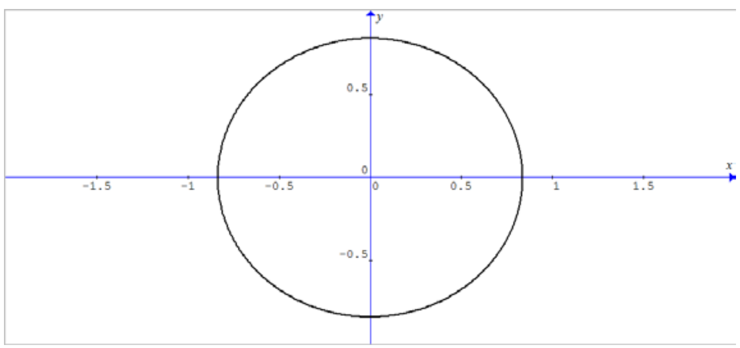
Is y is a function of x?

No

Since for each x,
y has two values

$$y = \pm\sqrt{x}$$

However x is a function of y



$$x^2 + y^2 = 1$$

Is y is a function of x?

No

Since for each x,
y has two values

$$y = \pm\sqrt{1 - x^2}$$

1.2 *Basic Operations on Functions*

Given: functions f & g

(1) ***Sum*** $f+g$: $(f+g)(x) = f(x) + g(x)$

(2) ***Difference*** $f-g$: $(f-g)(x) = f(x) - g(x)$

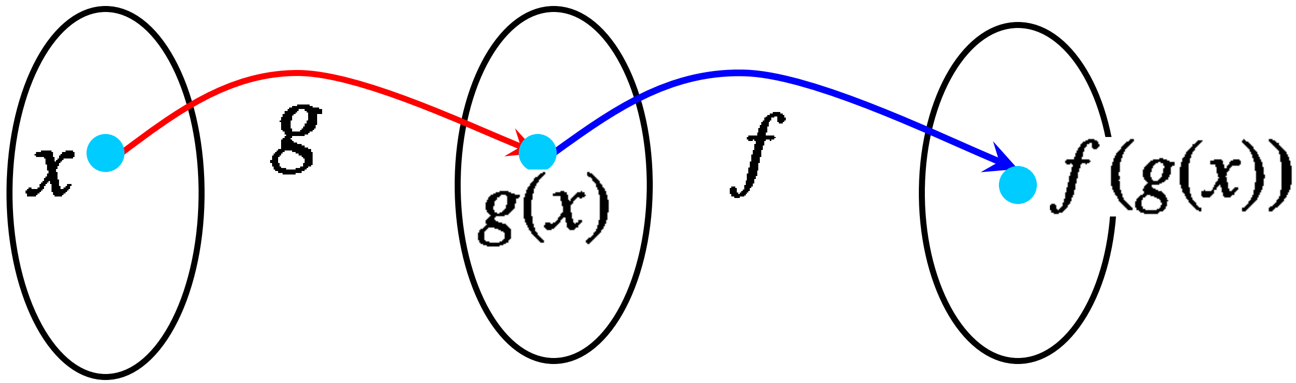
(3) ***Product*** $f \cdot g$: $(f \cdot g)(x) = f(x) \cdot g(x)$

(4) ***Quotient*** f/g : $(f/g)(x) = f(x)/g(x)$

♣ $x^2 + \sin x$, $\ln x - \cos x$,

$x^3 |x|$, $\tan x = \sin x / \cos x$

Composition



$(f \circ g)(x) = f(g(x))$

$f \circ g$: f ***composed with*** g

& read f '***circle***' g

Example

- $f(x) = x - 7$ & $g(x) = x^2$

$(f \circ g)(2) = f(g(2)) =$

$(g \circ f)(2) = g(f(2)) =$

$(f \circ g)(x) = f(g(x)) =$

$(g \circ f)(\mathrm{x}) = g(f(\mathrm{x})) =$

♣ In general, $f \circ g \neq g \circ f$.

1.3 *Limits*

In this section, not interested in the value of f(a).
interested in the values of f(x), when x near a.
interested in the behavior of f near a.

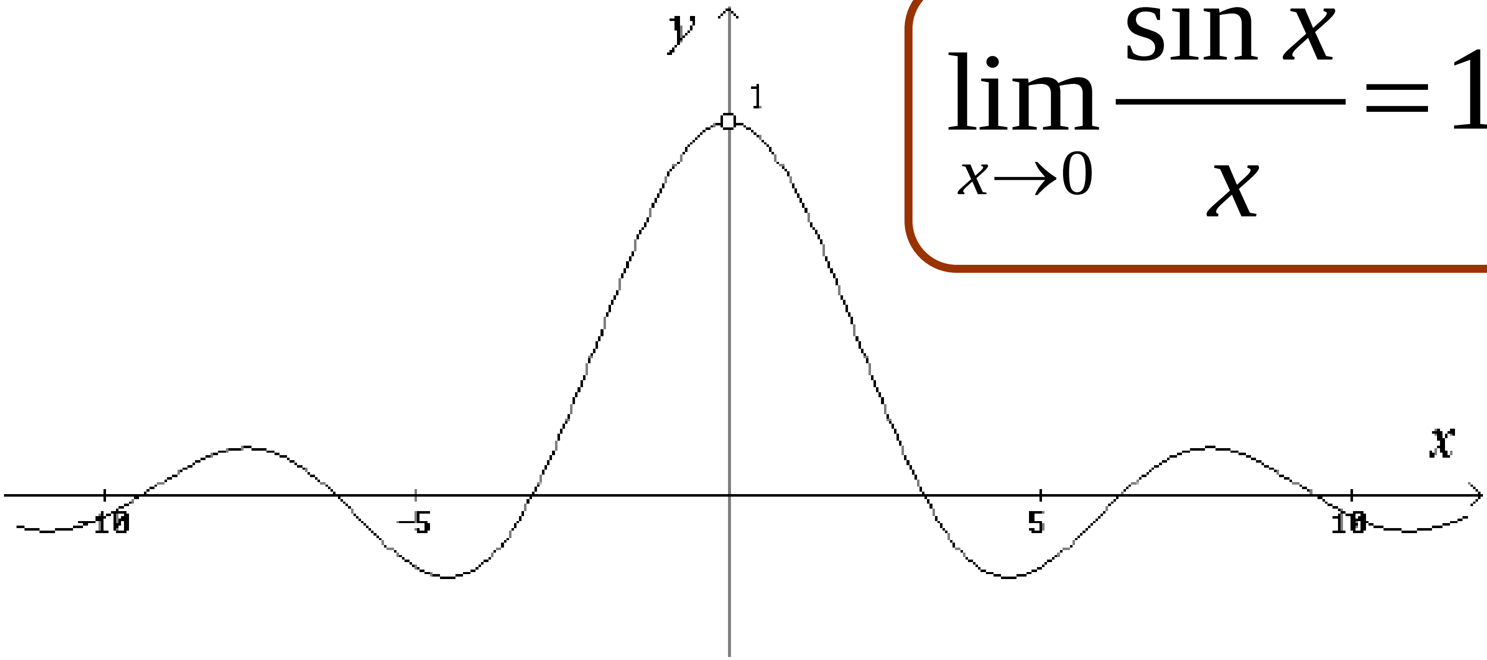
want to know:
when x tends to a, from the left and right,
does f(x) tend to a fixed value?

Suppose f(x) tends to a fixed value L. Then we say :The limit of f(x) is L as x tends to a

Write: $$\boxed{\lim_{x \to a} f(x) = L}$$

1.3.1 *Examples*

1. $f(x) = \sin x / x \quad (x \neq 0)$



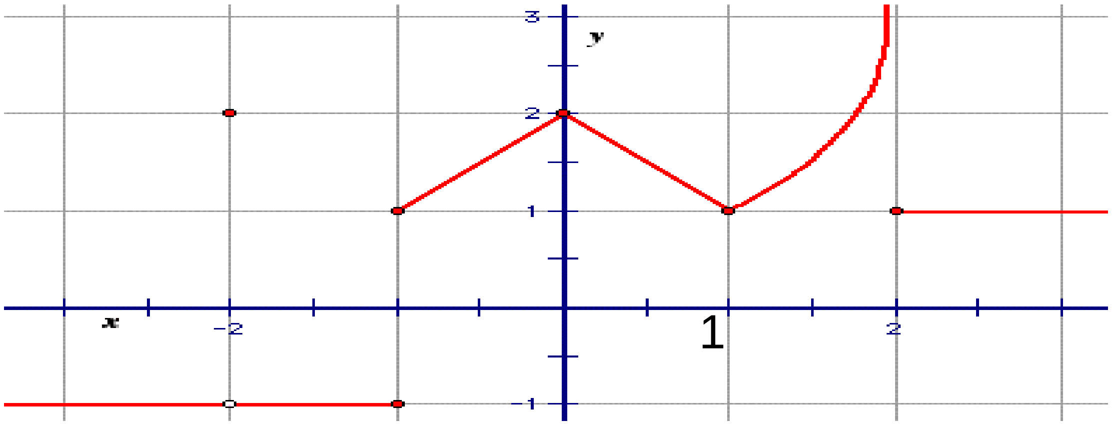
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

How to draw graph

http://www.graphmatica.com/

We can find limit if the graph is given
We shall give more examples

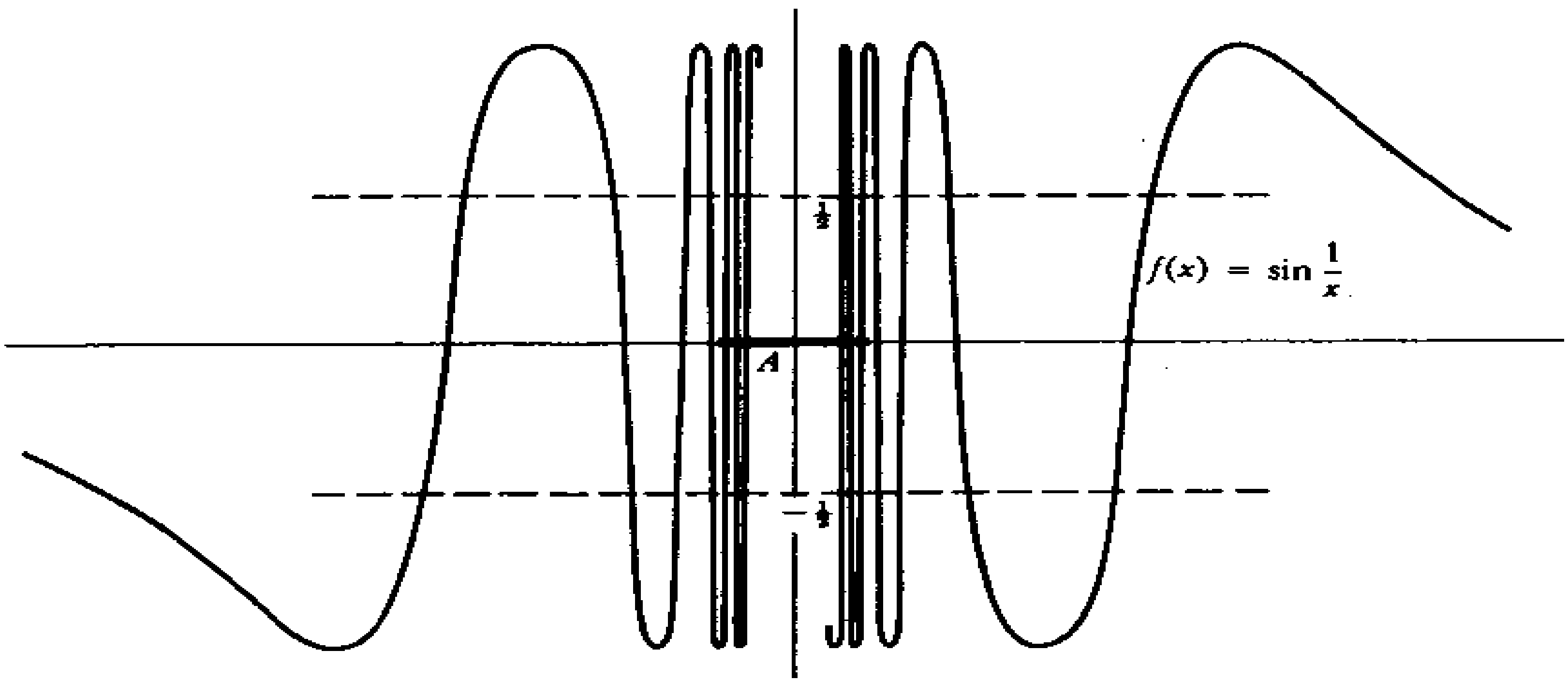
2.



$$\lim_{x \to 1} f(x) = 1$$

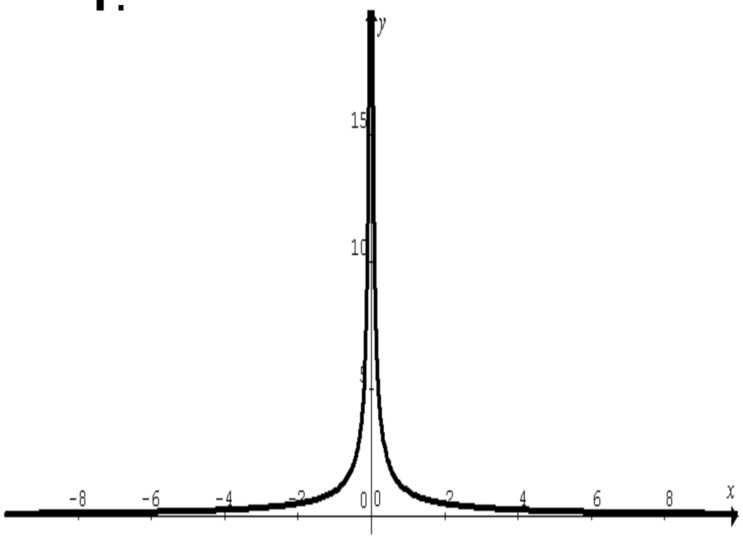
$$\lim_{x \to -2} f(x) = -1$$

3. $$f(x) = \sin(\frac{1}{x}) \quad (x \neq 0)$$

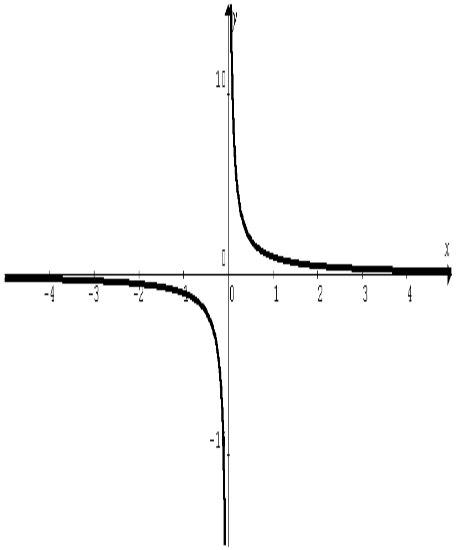


$$\lim_{x \to 0} f(x)$$ ***doesn't exist***

4.

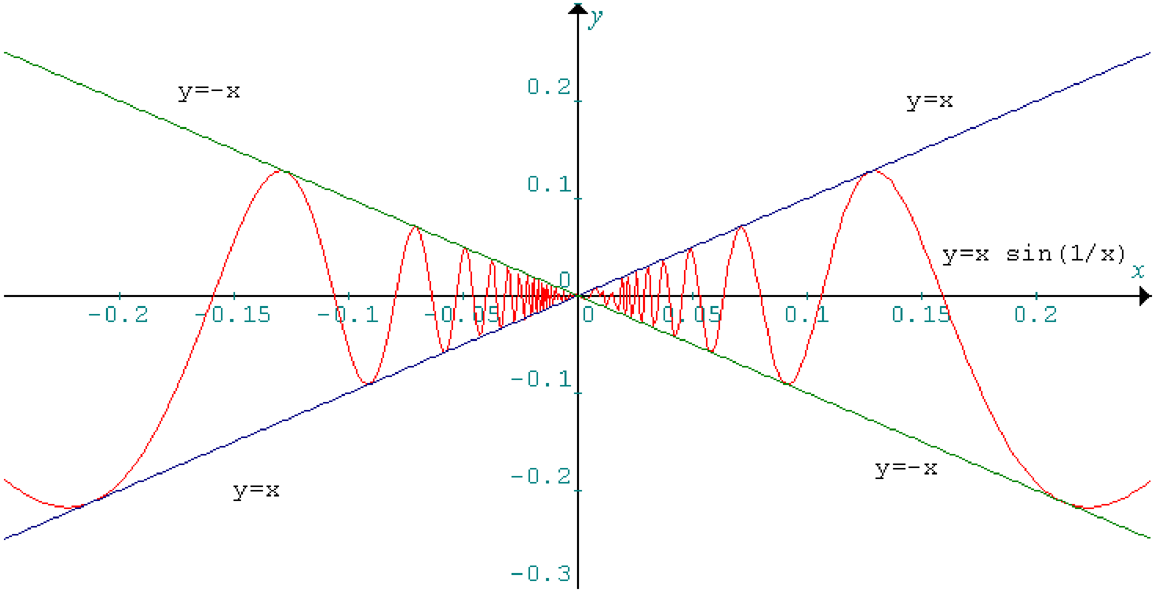


$$\lim_{x\to 0} f(x) = \infty$$



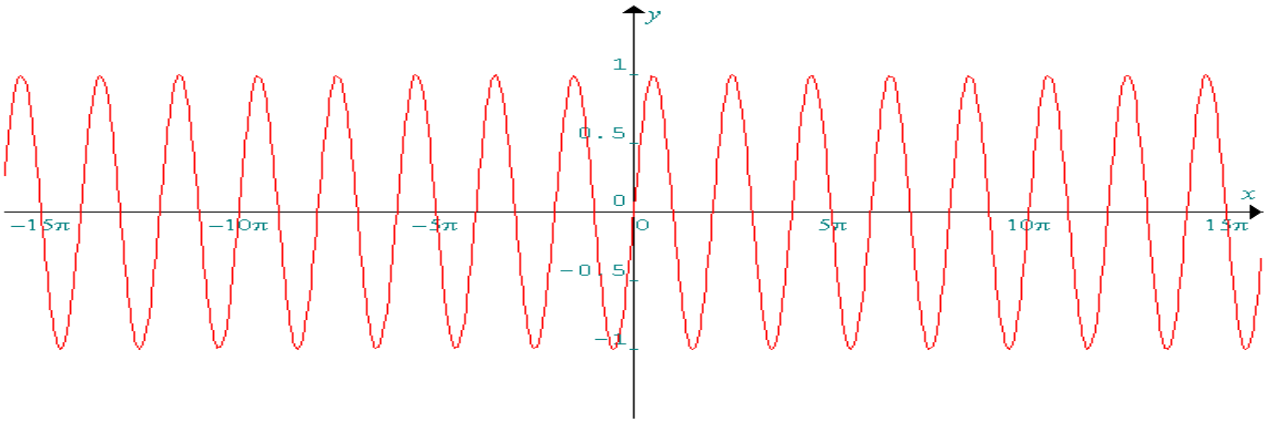
$$\lim_{x\to 0} f(x)$$ does not exist

5.



$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0$$

6.



The above graph of sinx oscillates between 1 and -1

$\lim_{x\to\infty} \sin x$ does not exist

$\lim_{x\to-\infty} \sin x$ does not exist

1.3.2 *Informal Definition*

If $f(x)$ tends to a fixed value L when x tends to a,

We say: the ***limit*** of $f(x)$ is L as x tends to a.

Write:

$$\boxed{\lim_{x \to a} f(x) = L}$$

1.3.3 *Rules* of *Limits*

Suppose $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = L'$, then

(i) $\lim_{x\to a}(f \pm g)(x) = L \pm L'$;

(ii) $\lim_{x\to a}(fg)(x) = LL'$;

(iii) $\lim_{x\to a} \dfrac{f}{g}(x) = \dfrac{L}{L'}$ provided $L' \neq 0$;

(iv) $\lim_{x\to a} kf(x) = kL$ for any real number k.

*** The additional notes below are not found in the lecture notes**

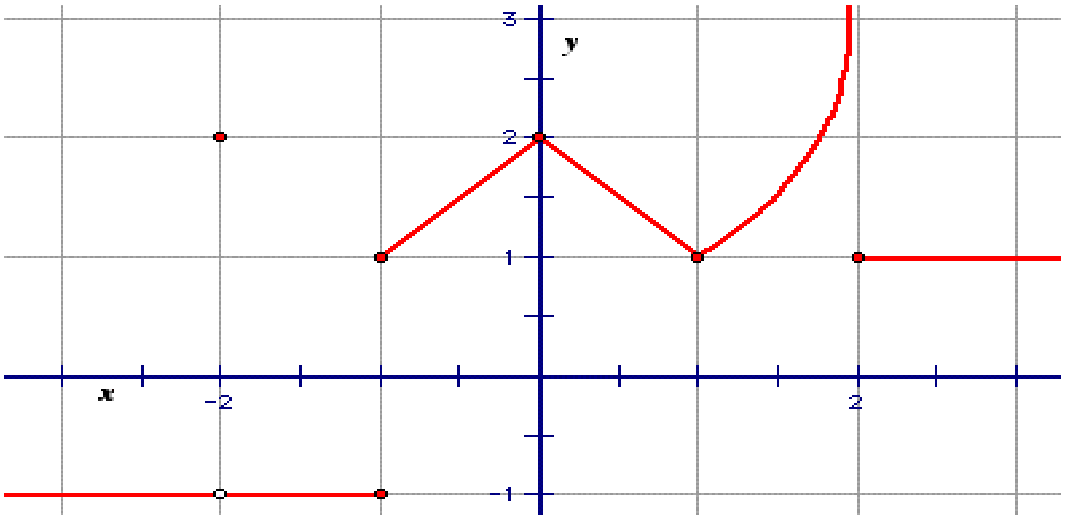
1.3.4 Continuity

Intuitively, a function f is continuous at a ,
if f(x) tends to f(a) whenever x tends to a.
In other words

$$\lim_{x \to a} f(x) = f(a)$$

Hence limit at a can be computed by substitution if f is continuous at a.

A function f is discontinuous at a
if f has a jump at a.



The above function is continuous everywhere except at x=

What functions are continuous?

A Polynomial function is continuous everywhere

$$f(x) = x^3 + 4x^2 + x + 10$$

$$\lim_{x \to 3} f(x) = f(3) = 3^3 + (4)(3^2) + 3 + 10$$

A rational function is continuous everywhere except at those points where its denominator is zero.

Rational function=poly/poly

$$f(x) = \frac{x^2 + 2x + 4}{x^2 - 5x + 6} = \frac{x^2 + 2x + 4}{(x-3)(x-2)}$$

f is continuous everywhere except at 3, 2

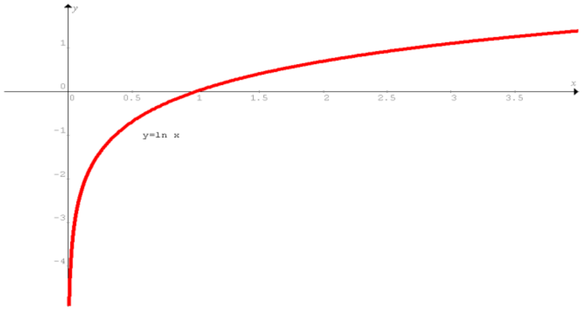
Functions $\sin x$ $\cos x$ $|x|$ a^x

are continuous everywhere

$$\lim_{x\to 2}\sin x = \sin 2 \qquad \lim_{x\to 4}\cos x = \cos 4$$

$$\lim_{x\to -6}|x| = |-6| = 6 \qquad \lim_{x\to 8} a^x = a^8$$

ln x is continuous on $(0,\infty)$ denotes interval from 0 to infinity, 0 not included



$$\cos\left(\frac{\left|x^4 - 3x + x^{\frac{1}{3}} + e^x\right|}{x^2 - x - 6}\right)$$

is continuous everywhere except at 3 and -2.

$$\lim_{x\to 1}\cos\left(\frac{\left|x^4 - 3x + x^{\frac{1}{3}} + e^x\right|}{x^2 - x - 6}\right) = \cos\left(\frac{\left|1^4 - (3)(1) + 1^{\frac{1}{3}} + e^1\right|}{1^2 - 1 - 6}\right)$$

We have learnt two methods to find limits

namely by graph and substitution

We will learn another method
L' Hospital's rule
in Chapter 2

End