Remarks on Chapter 4

(1) $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$

when n=0, we get

$$\frac{f^{(0)}(a)}{0!}=f(a)$$

Here we define

$f^{(0)}(a)=f(a)$

$0!=1$

(2)The power series about the point (called centre) a is of the form

$$\sum_{n=0}^{\infty}c_n(x-a)^n$$

Is $\sum_{n=0}^{\infty}\frac{1}{n}(5x+7)^n$ a power series?

ANS:YES, since

$$\sum_{n=0}^{\infty}\frac{1}{n}(5x+7)^n=\sum_{n=0}^{\infty}\frac{5^n}{n}(x-(7/5))^n$$

So the centre of the power series is -7/5. How to find the radius of convrergence? Two ways:

(a) Start from the standard form,

$$\sum_{n=0}^{\infty} \frac{5^n}{n}(x-(7/5))^n$$

use ratio test , get

$5\left|x-(-7/5)\right|<1$ Hence

$\left|x-(-7/5)\right|<1/5$

(b) Or start from the given form $\sum_{n=0}^{\infty} \frac{1}{n}(5x+7)^n$

Use ratio test , get

$\left|5x+7\right|<1$ Hence $5\left|x-(-7/5)\right|<1$

So $\left|x-(-7/5)\right|<1/5$