Chapter 1

Functions: limits and continuity

Key Results

- General notion of functions
- Informal definition of the limit of functions
- Some rules of limits

Terminology (review)

- The symbol $y = f(x)$ denotes the statement "y is a function of x".
- A function represents a rule that assigns a unique value y to each value x.
- Input x and output y. Input-output process.
- x is the independent variable.
- y is the dependent variable.

Domain, Codomain, Range

- Symbolically,

$$
\begin{aligned}
f\,:\; D &\longrightarrow \mathbb{R} \\
x &\longmapsto y = f(x)
\end{aligned}
$$

- *D* is domain – set of inputs.
- **R** is codomain – indicates that output values of *f* are real numbers.
- actual collection of *y* values is the range.

Composition

Consider two functions

$$f(x) = \boxed{x - 7} \quad \text{and} \quad g(x) \boxed{= x^2}$$

$$(f \circ g)(2) = f(\boxed{g(2)}) = f(4) = -3$$

$$(g \circ f)(2) = g(\boxed{f(2)}) = g(-5) = 25$$

For most functions f and g,

$$\boxed{f \circ g \neq g \circ f}$$

Limits

Consider the function

$$f(x) = \frac{\sin(x)}{x}$$

x cannot be zero

- Describe the behaviour of *f* as *x* tends to 0.

- Note that *f* is defined for every nonzero value of *x*.

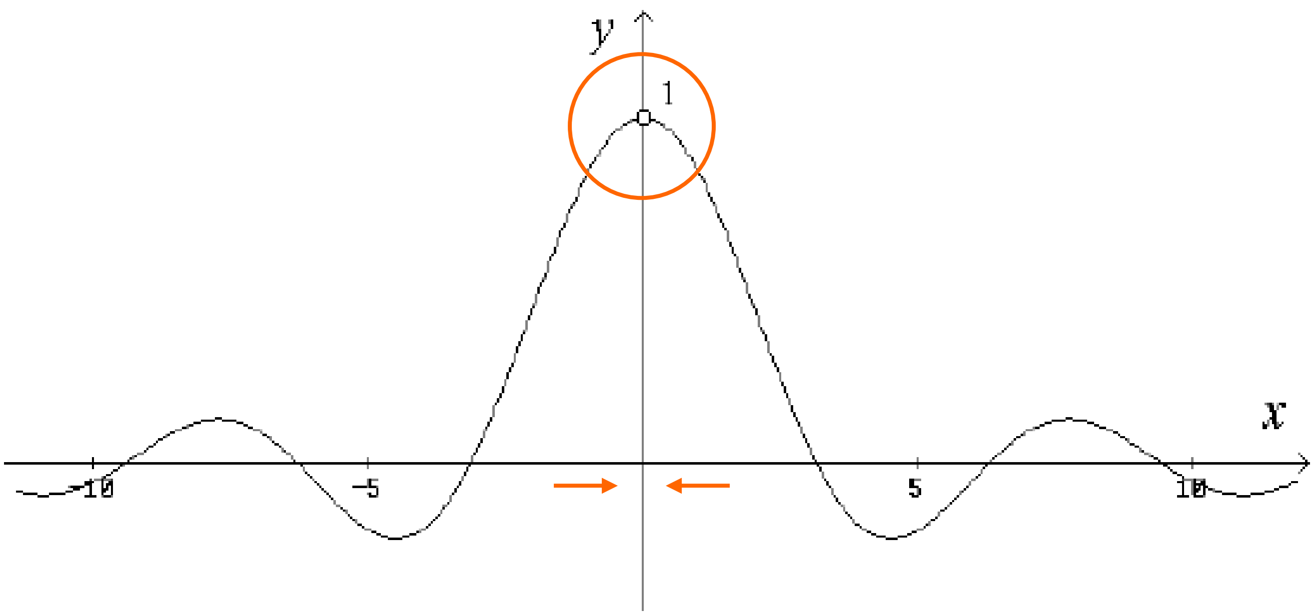
$$f'(a) = \boxed{\lim_{x \to a}} \frac{f(x) - f(a)}{x - a}$$

Note that

$$\frac{\sin(0)}{0} = \frac{0}{0}$$

does not make sense. It is undefined.

How to study the values of f for values of x close to 0?



plot the graph of $\frac{\sin(x)}{x}$

When x gets closer and closer to 0 from either side,

$$\frac{\sin(x)}{x} \text{ approaches } 1$$

Write

$$\boxed{\lim_{x \to 0} \frac{\sin(x)}{x} = 1}$$

Read as

"the limit of $\frac{\sin(x)}{x}$ as x tends to 0 is equal to 1"

Informal Definition

- If $f(x)$ gets arbitrarily close to L when x is sufficiently close to x_0, then L is the limit of $f(x)$ as x tends to x_0.

- Write

$$\lim_{x \to x_0} f(x) = L$$

Rules of Limits

Suppose

$$\lim_{x \longrightarrow a} f(x) = L \text{ and } \lim_{x \longrightarrow a} g(x) = L'$$

Then

$$\text{(i)} \lim_{x \longrightarrow a} (f \pm g)(x) = L \pm L';$$

$$\text{(ii)} \lim_{x \longrightarrow a} (fg)(x) = LL'$$

$$\text{(iii)} \lim_{x \longrightarrow a} \frac{f}{g}(x) = \frac{L}{L'} \text{ provided } L' \neq 0;$$

$$\text{(iv)} \lim_{x \longrightarrow a} kf(x) = kL \text{ for any real number } k.$$