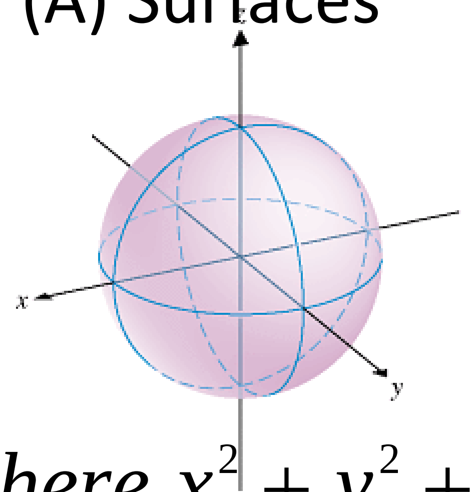
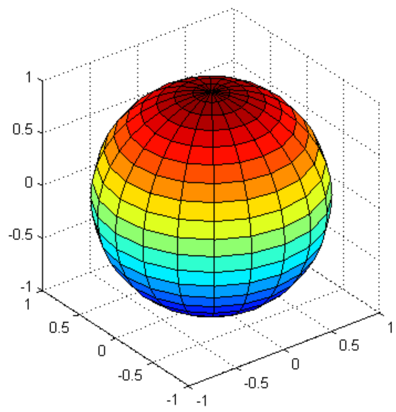
# Surfaces and intersection of two surfaces

## (A) Surfaces

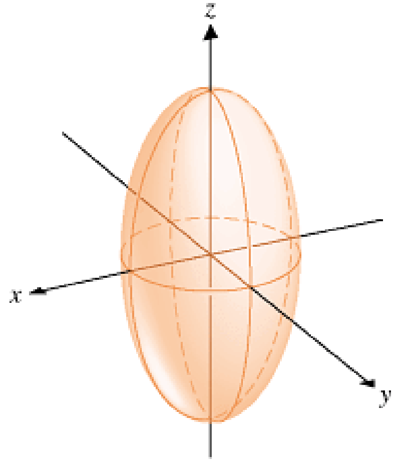


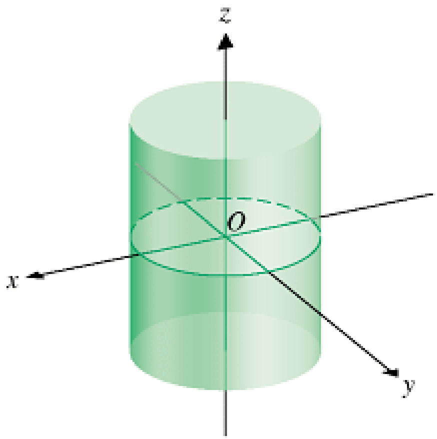

$$sphere\ x^2 + y^2 + z^2 = r^2$$

$$sphere\ (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

$$x^2 + y^2 + z^2 - 2ax - 2by - 2cz + a^2 + b^2 + c^2 = r^2$$

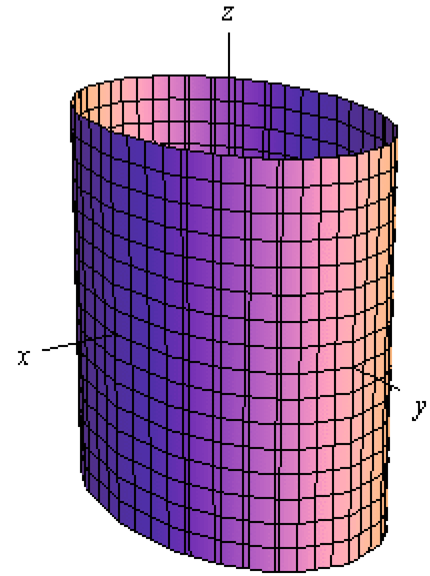



$$ellipsoid\ \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = r^2$$




$$x^2 + y^2 = r^2$$

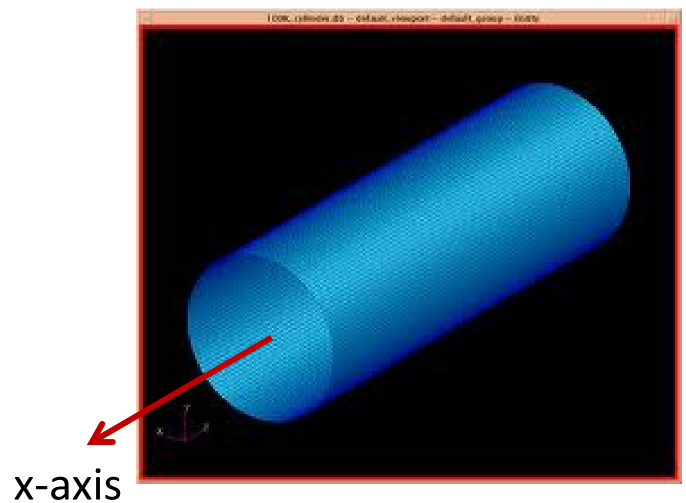
circular cylinder



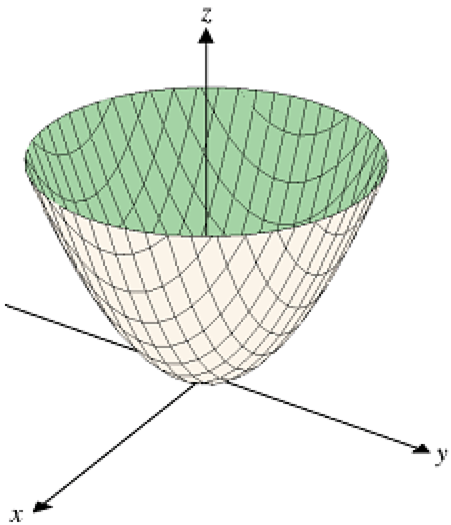

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2$$

elliptic cylinder


$$x^2 + z^2 = R^2$$

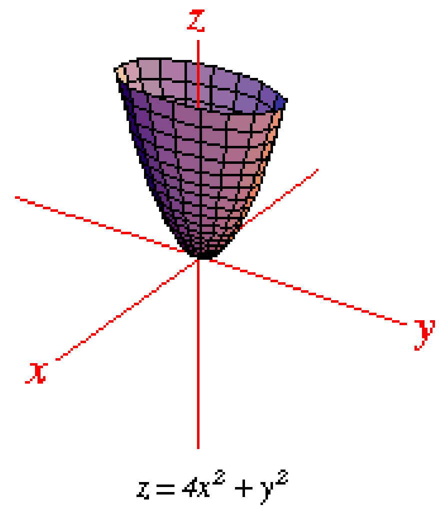



$$y^2 + z^2 = r^2$$



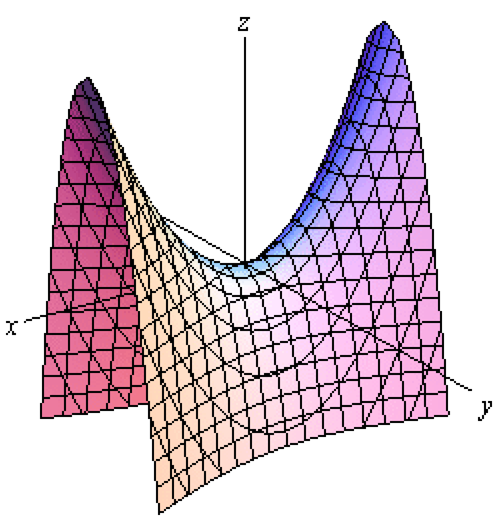

*circular paraboloid*

$$z = x^2 + y^2$$




*elliptic paraboloid*

$$z = 4x^2 + y^2$$




*hyperbolic paraboloid* (saddle)

$$z = x^2 - y^2$$

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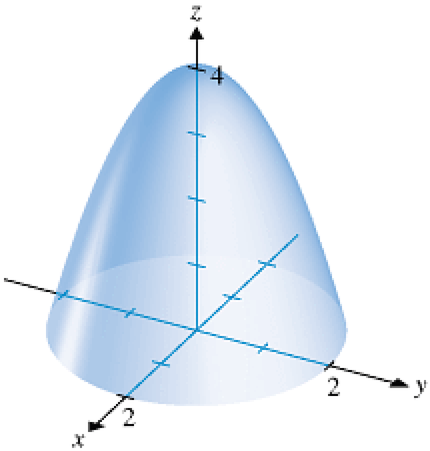
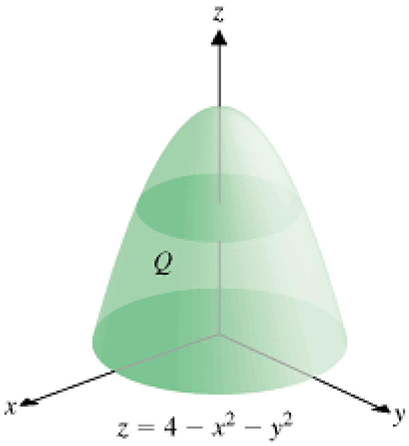
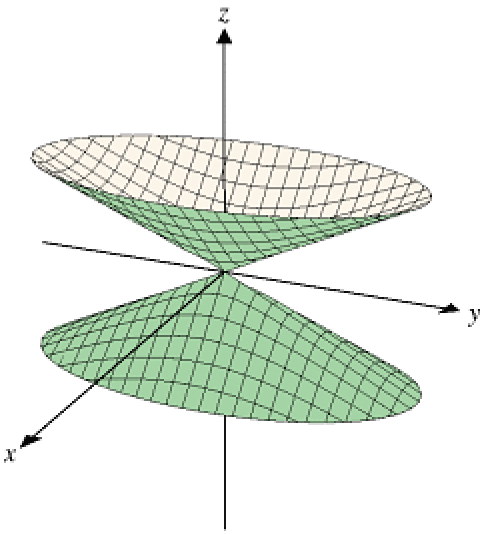


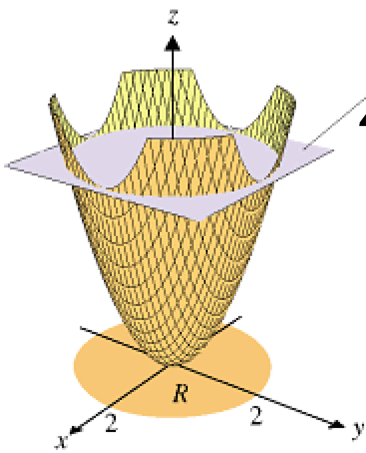

$$z = 4 - x^2 - y^2$$

Inverted paraboloid



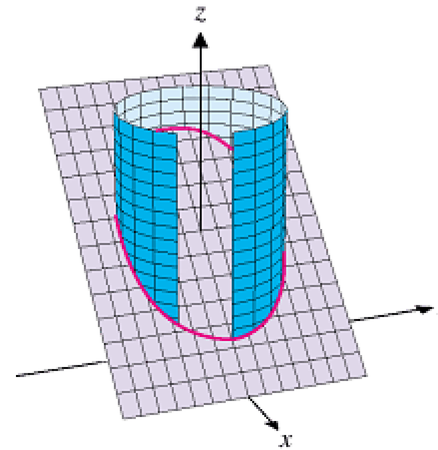

$$elliptic\ cone\ x^2 + \frac{y^2}{4} = z^2$$

# (B) Intersection of two surfaces

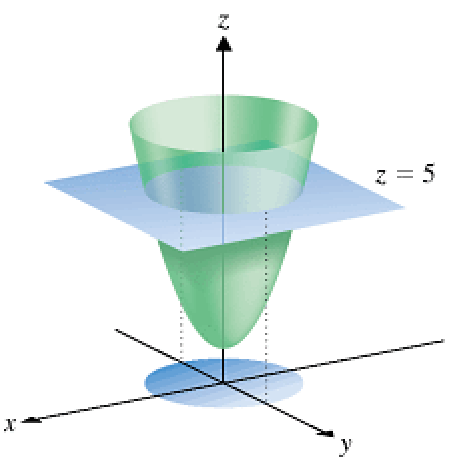


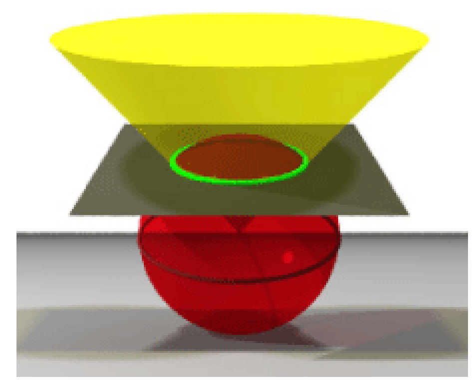

$Z = 4$

$z = x^2 + y^2$

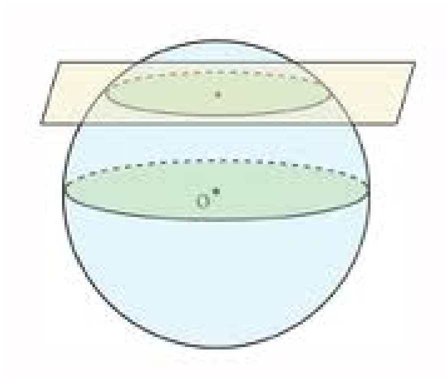


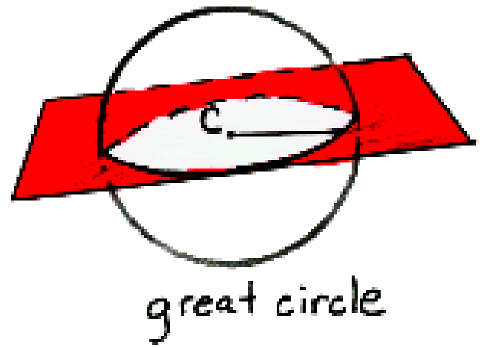

C is the intersection of the circular cylinder $x^2+y^2 = 4$ and the plane $x+z = 3$

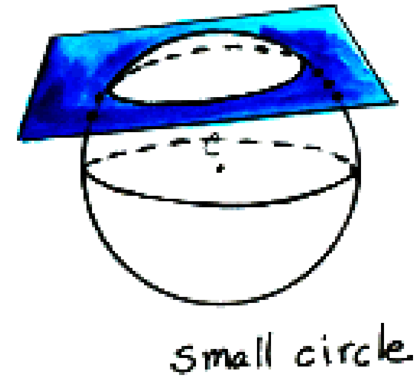


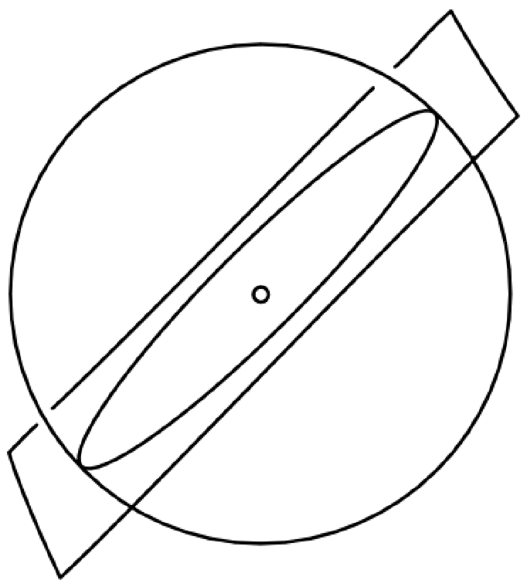



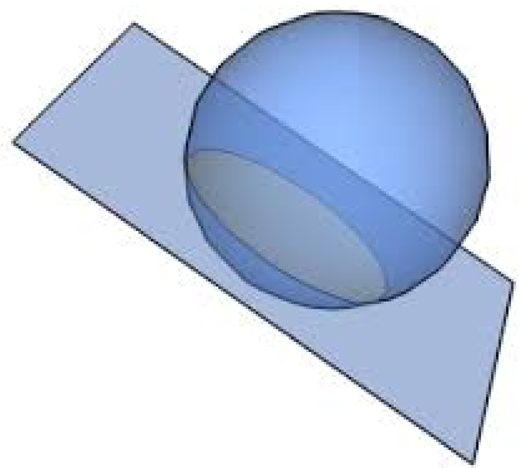
# Intersection of plane and sphere is always circle



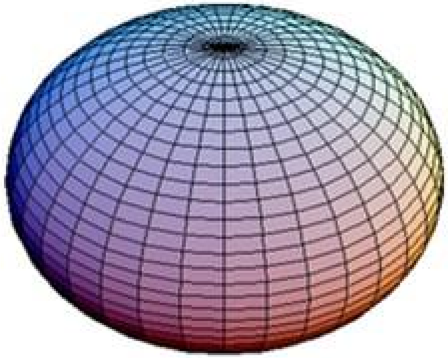


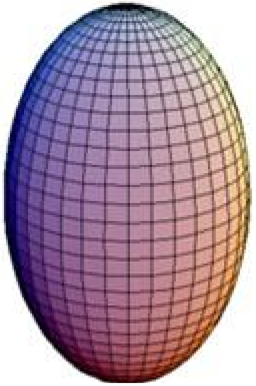



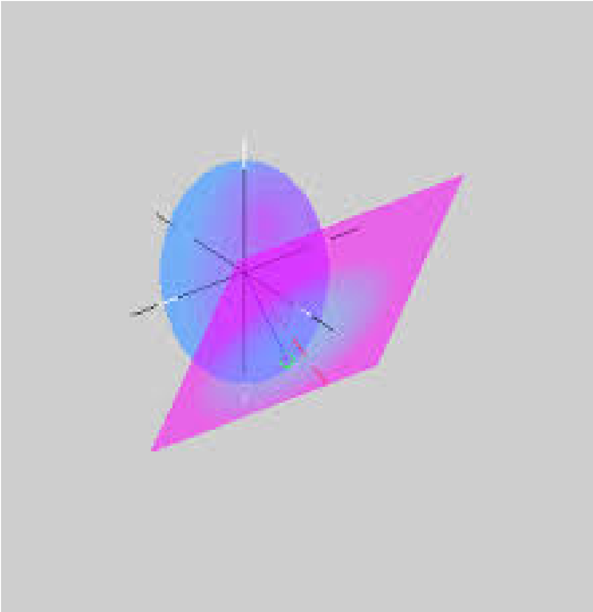



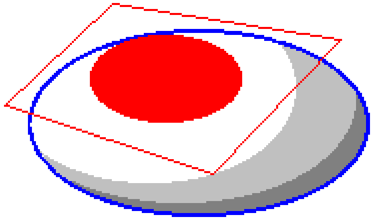


# Intersection of  plane and ellipsoid is always ellipse

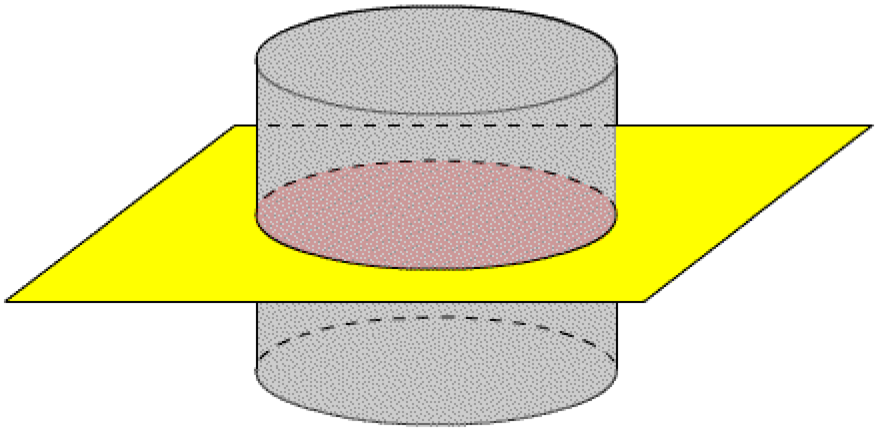


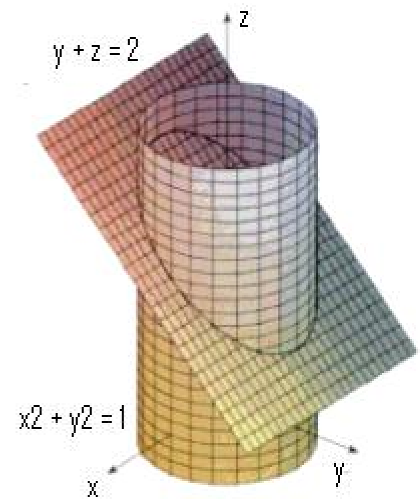


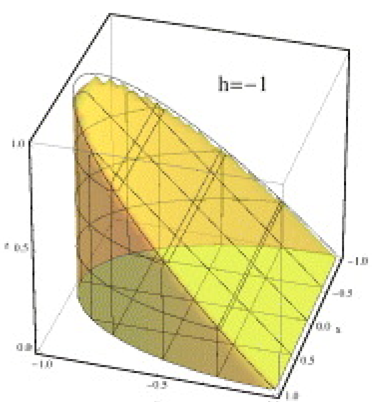




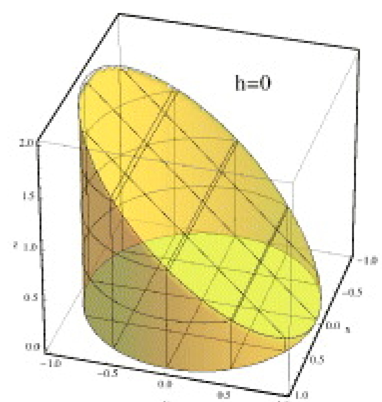
# Intersection of plane and cylinder



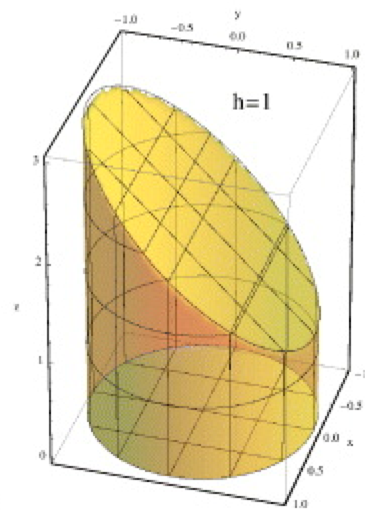


(a)

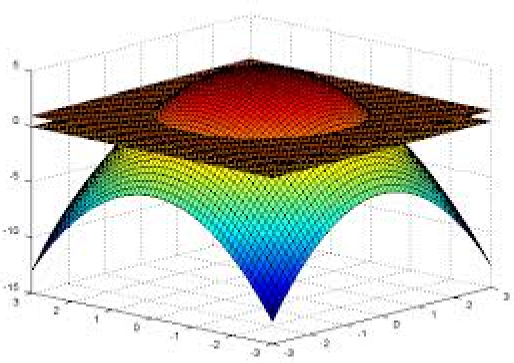


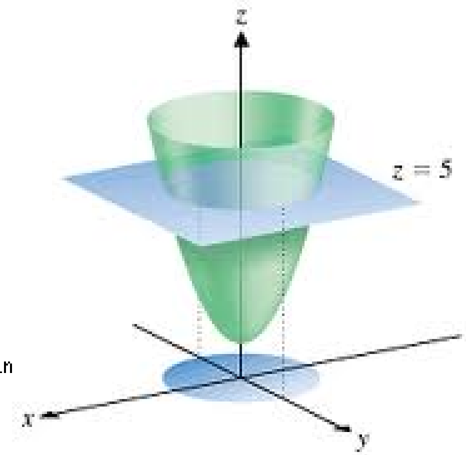
(b)



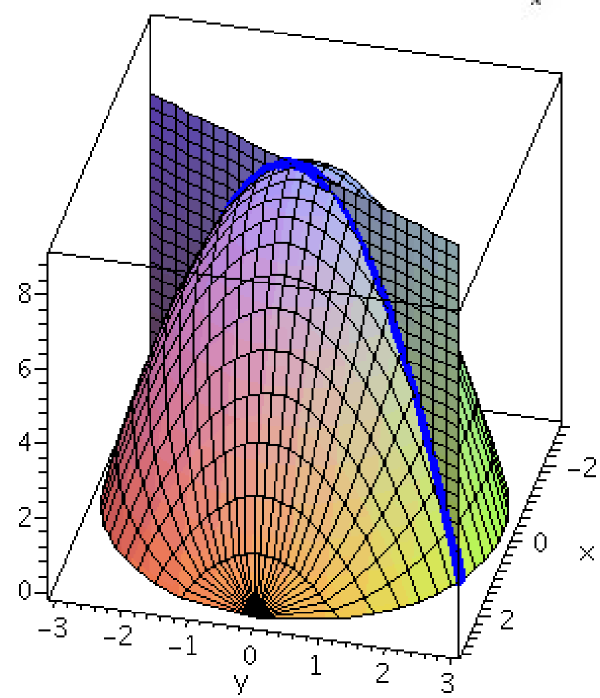
(c)

# Intersection of  plane and paraboloid

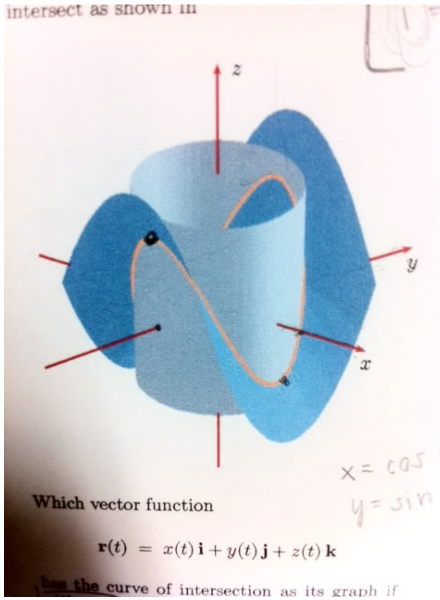


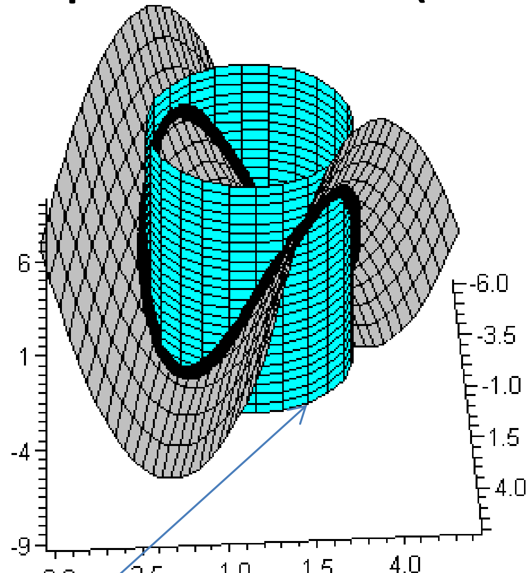


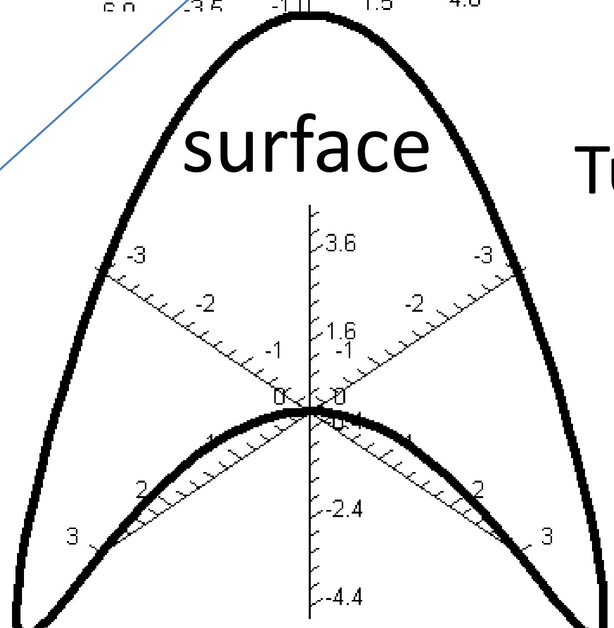

The curve of intersection of the paraboloid z=9-x^2-y^2 an



# Intersection of hyperbolic paraboloid (saddle) and cylinder

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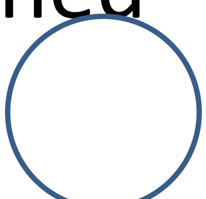
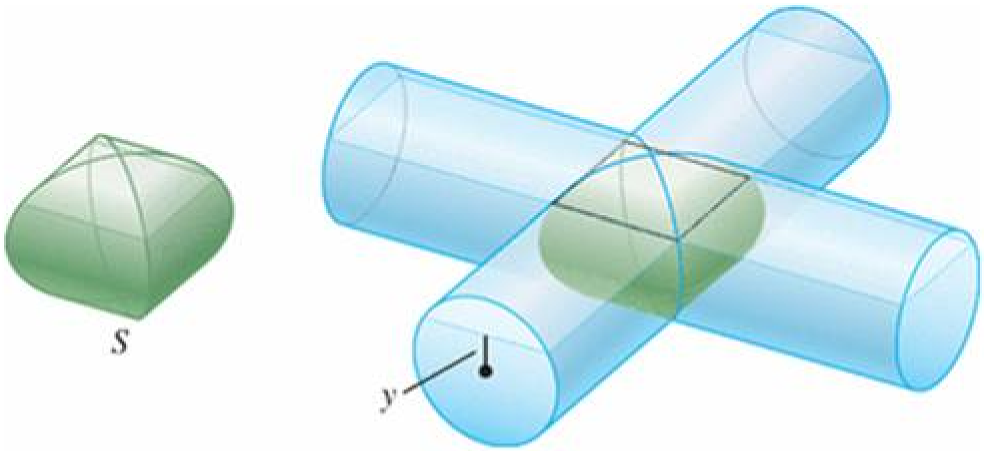
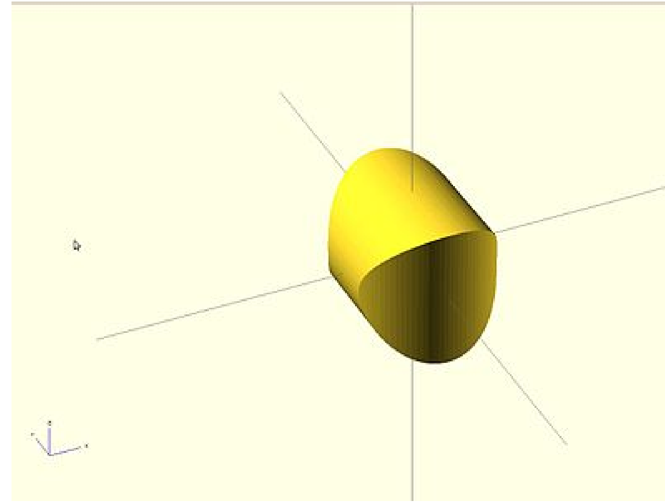
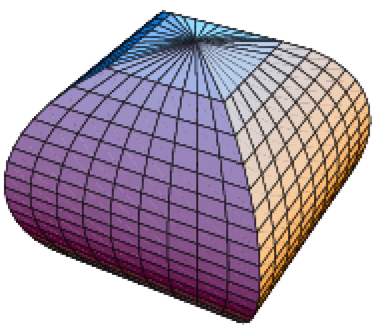
Surface defined on the base of cylinder



# Intersection of two cylinders

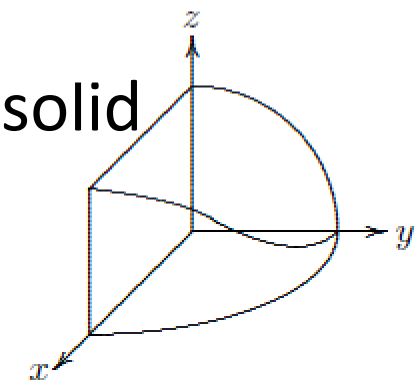




Volume of the solid

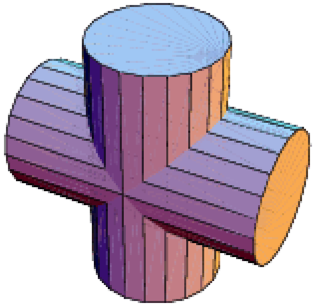
= 8 times of the volume of this solid

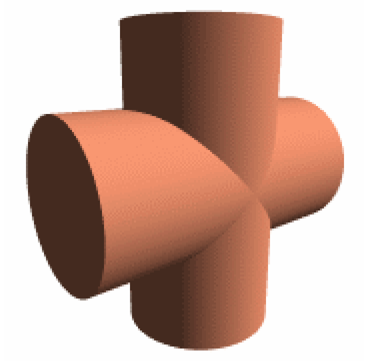



Tutorial 8 Q2

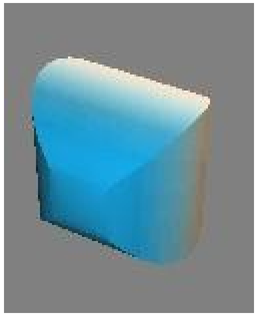
http://www.math.tamu.edu/~Tom.Kiffe/calc3/newcylinder/2cylinder.html

# Intersection of two cylinders

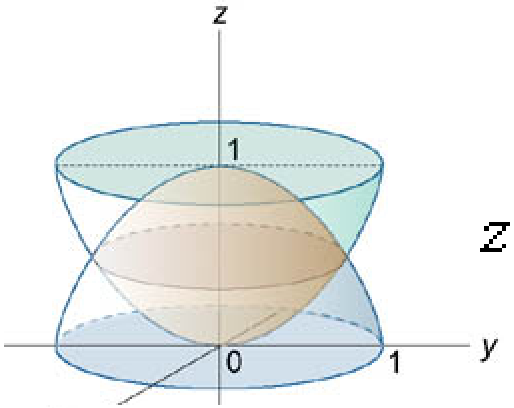




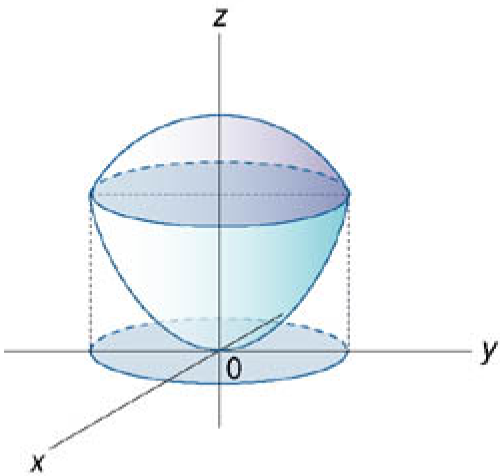
animation

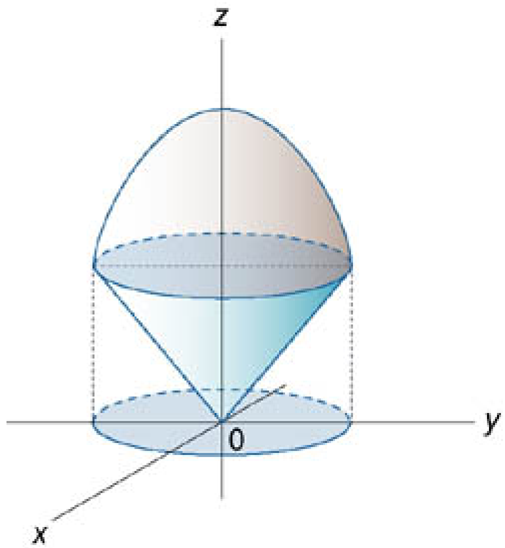


# Intersection of two surfaces



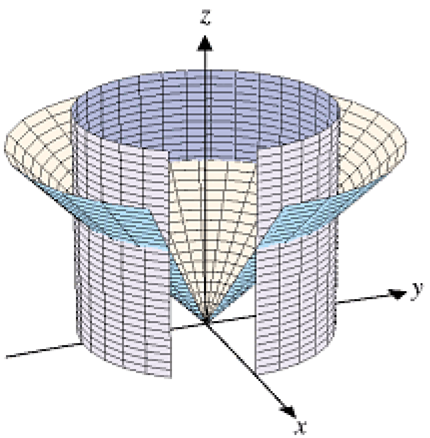

$$z_1 = x^2 + y^2 \quad \text{and} \quad z_2 = 1 - x^2 - y^2.$$



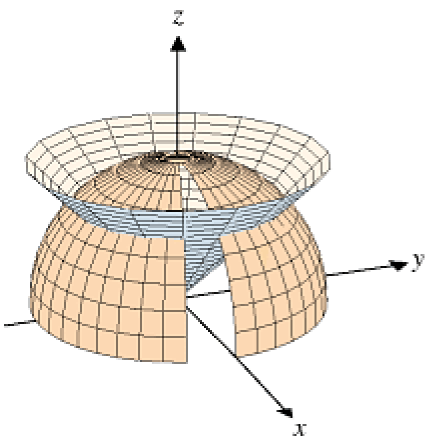




The sphere $x^2 + y^2 + z^2 = 6$ and the paraboloid $x^2 + y^2 = z$.

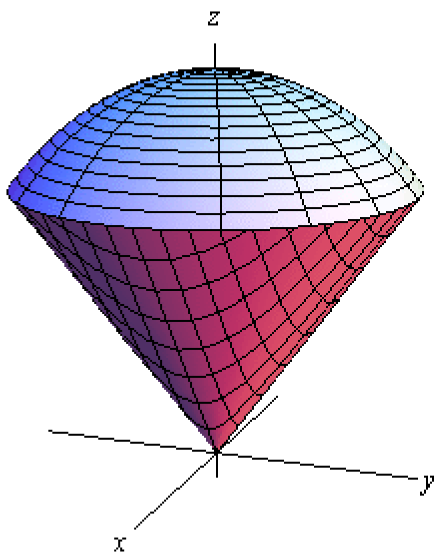
The paraboloid z = 2 – x^2 – y^2 and the conic surface

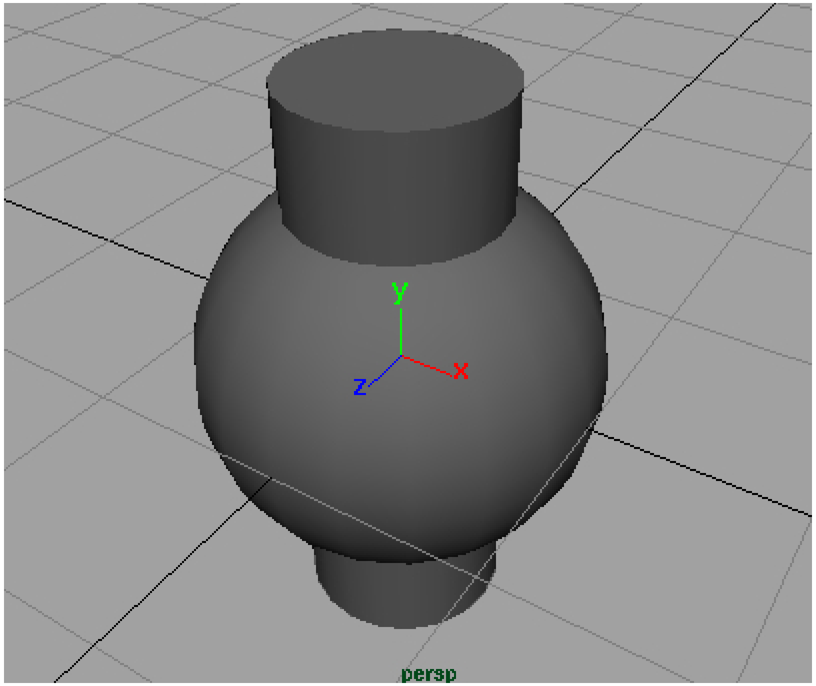


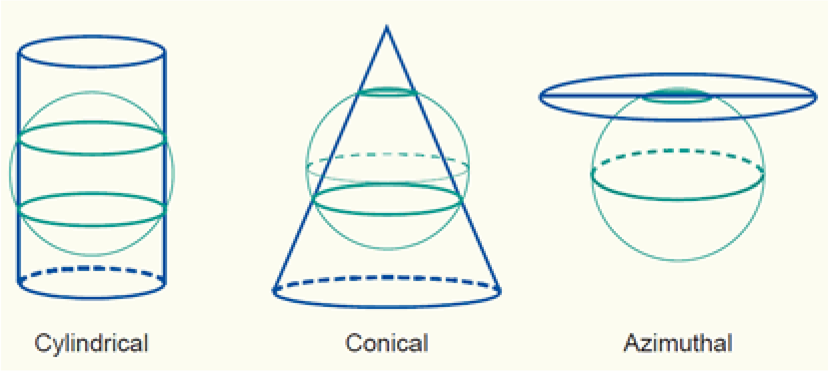

cone $z = \sqrt{x^2 + y^2}$ and cylinder $x^2 + y^2 = 4$

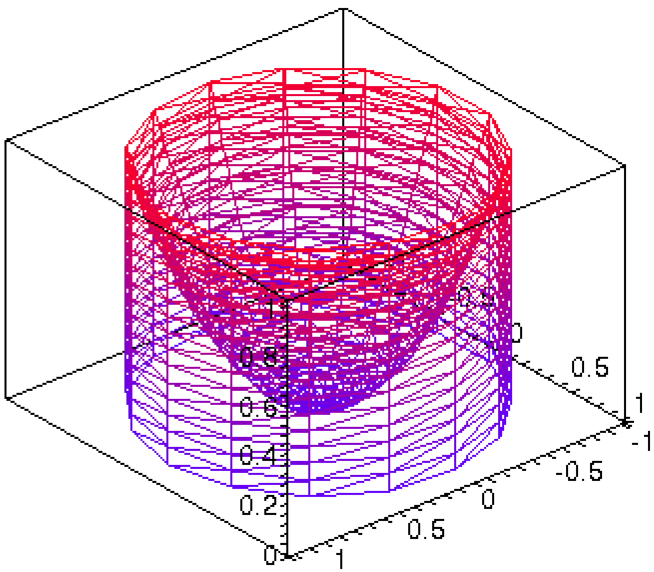


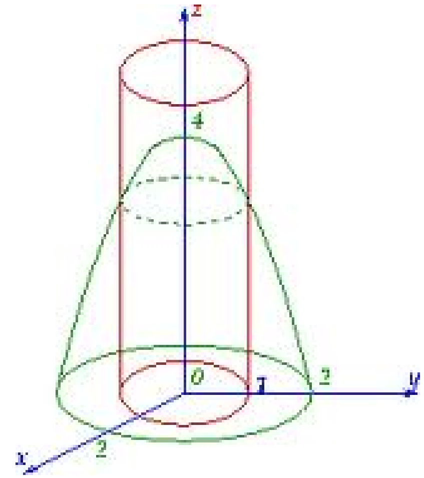

the sphere
and  the cone.


z
y
x


Y
Z
X
persp


Cylindrical
Conical
Azimuthal


1
0.8
0.6
0.4
0.2
0
1
0.5
0
-0.5
-1
1
0.5
0
-0.5


z
4
0
1
2
y
x
2

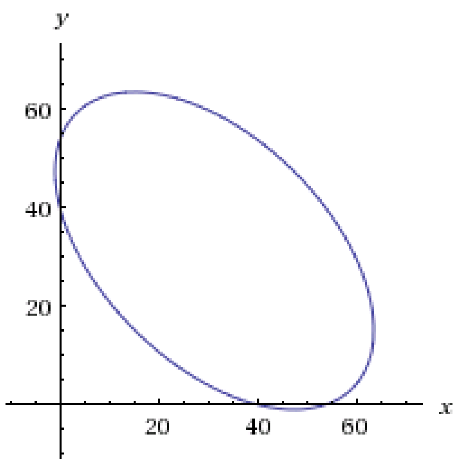
# Example 1

Determine projection into x-y plane of the curve of intersection of a plane and a sphere

$$x + y + z = 94 \qquad\qquad x^2 + y^2 + z^2 = 4506$$

Solve the above, get rid of z, we get

$$x^2 + y^2 + (94 - x - y)^2 = 4506$$ projection into x-y plane




which is an ellipse

# Example 2

Determine projection into x-y plane of the curve of intersection of the following surfaces

$$z = 1 - y^2$$

$$z = x^2 + y^2$$

Solve the above, get rid of z, we get

$$1 - y^2 = x^2 + y^2$$ projection into x-y plane

so

$$x^2 + 2y^2 = 1, \text{ which is an ellipse}$$

# Example 3

Determine projection into x-y plane of the curve of intersection of the following surfaces

$$z = 2x^2 + 3y^2$$

$$z = 5 - 3x^2 - 2y^2$$

Solve the above, get rid of z, we get

$$5x^2 + 5y^2 = 5$$ projection into x-y plane

$$x^2 + y^2 = 1, \text{ which is a circle}$$

# Appendix

| | |
|---|---|
| http://www.mhhe.com/math/calc/smithminton2e/cd/folder_structure/text/chap14/section04.htm | Green's theorem |
| http://www.mhhe.com/math/calc/smithminton2e/cd/folder_structure/text/chap10/section06.htm | Surfaces in space |
| http://www.mhhe.com/math/calc/smithminton2e/cd/folder_structure/text/chap14/section03.htm | Indep of path |
| http://www.mhhe.com/math/calc/smithminton2e/cd/folder_structure/text/chap14/section08.htm | Stokes' Theorem |
| http://www.mhhe.com/math/calc/smithminton2e/cd/folder_structure/text/chap14/section06.htm | Surface integral |