MA1506 Tutorial 7

Question 1

In the fishing model

$$\frac{dN}{dt} = -sN^2 + BN - E$$

assume that the fishing rate is less than $B^2/4s$ so that we have two equilibrium points $\beta_2 > \beta_1$. Find the harvesting rate E (in terms of B and s) such that $\beta_1 = 0.9\beta_2$.

[Ans: 0.9972 $B^2/4s$]

Question 2

$$\frac{dN}{dt} = N[B - s(N-u)^2]$$

is another fishing model in which N(t) is the population with B, s and u being constants such that $u>(B/2s)^{1/2}$, prove that extinction will result if $N<u-(B/2s)^{1/2}$. Find the limiting population if $N>u-(B/2s)^{1/2}$. [$u + (B/2s)^{1/2}$]

Question 3

A cantilevered beam of length L is fixed at one end with the other end free. If the moment function M(x) of the beam about the point distance x from the fixed end is given by

$$M(x)=\alpha(-x^2+x^3/3L+Lx-L^2/3),$$

find the deflection y(x) of the beam. (You may assume that $y(0)=y'(0)=0$)

[Ans: $(-\alpha x^2/60EIL)(5Lx^2-x^3-10L^2x+10L^3)$]

Question 4

In the chemical decomposition A→B→C, the concentration [B(t)] of B follows the rate given by

$$\frac{d[B(t)]}{dt} = k_1[A(0)]e^{-k_1 t} - k_2[B(t)]$$

where k_1, k_2 are the rate reactions. If [B(0)]=0, find [B(1)].

[Ans: $\frac{k_1[A(0)]}{k_2 - k_1}(e^{-k_1} - e^{-k_2})$]

Question 5

Solve the following systems of equations

(a) $x+2y=1,\ \ 2x-y=0;$
(b) $x+2y=1,\ \ 3x+6y=3;$
(c) $x+2y=1,\ \ 3x+6y=5.$

[Ans: (a) $x=1/5$, $y=2/5$; (b) all points on $x+2y=1$; (c) no solution]