MA1506 Tutorial 2

Question 1
The rate of change of the temperature of a small iron ball is proportional to the difference between its temperature $T(t)$ and that of its surrounding T_{env} (which is constant). Let the ball be heated to 300^0F and $T_{env}=75^0$F. If its temperature falls to 200^0F in half an hour, show that its temperature will be 81.6^0F after 3 hours of cooling.

Question 2
Suppose that the volume of a spherical raindrop is proportional to the 3/2 power of its surface area and the rate of reduction of the volume of the raindrop is proportional to its surface area, find a formula for the amount of time it takes for the raindrop to evaporate completely, giving your answer in terms of the constants you introduced and the initial surface area of the raindrop.

Question 3
Suppose that the population $P(t)$ of a country satisfies the differential equation
$$\frac{dP}{dt} = kP(200 - P)$$
where k is a constant. Given that its population in 1940 was 100 million and was then growing at the rate of 1 million per year, find its population in 2000.

Question 4
As the salt KNO_3 dissolves in methanol, the number $x(t)$ of grams of the salt in a solution after t seconds satisfies the differential equation
$$\frac{dx}{dt} = 0.8x - 0.004x^2$$

(a) What is the maximum amount of salt that will ever dissolve in the methanol?
(b) If x=50 when t=0, how long will it take for an additional 50g of salt to dissolve?

Question 5
One student starts spreading a certain rumor in a class of 1300 students. If the number $R(t)$ of students who have heard the rumor is given by
$$\frac{dR}{dt} = KR(1300 - R)$$
where K is a positive constant. Prove that

$$\frac{1}{R(t)} = \frac{1}{1300} + \frac{1299}{1300} e^{-1300kt}$$

Answers: 2. $(3a/b)(A_0)^{1/2}$ 3. 153.7 million   4. (a) 200g (b) 1.37 sec