MA1506 Tutorial 6

Question 1

Suppose we remove 80 bugs per day from the bottle of bugs in Q5 of Tutorial 5,

find the limiting population. [Ans:12]

Question 2

Find the maximum number of bugs per day that can be removed from the bottle in Q5 of Tutorial 5 without causing the population to die out. [Ans: 141]

Question 3

A group of cranes has a logistic equilibrium population of 194,600 with a birth rate of 9.866% per year. Suppose 10,000 cranes are killed per year, find the time when the cranes will extinct.

[Ans: 29.8 years]

Question 4

(a) Suppose you have a sample of 1,000 cells which grow to 1,400 the next day. Given that the birth rate B and the death rate D are constants and that the population is growing at 40% daily, prove that $B\text{-}D = \ln 1.4$

(b) By adjusting the temperature, you are able to double the birth rate while keeping the death rate unchanged. Suppose the cell population started to double everyday henceforth, prove that

B ≈ 35.67%.

Question 5

The differential equation

$$dN/dt = N[0.1\text{-}0.001(N\text{-}100)^2]$$

can be used to model a certain species of whales which has a current population equal to $\hat{N}$. Let A be the largest number such that if $\hat{N} < A$, then the whale population will extinct one day. Prove that A=90. What is the limiting population if

$\hat{N} = 1000$ [Ans: 110]