Remarks of Tutorial 4

Q1

(a) Recall : A solution of a given ODE is said to be an equilibrium solution (point) if it is a CONSTANT solution.

If x_E is an equilibrium pt of $\ddot{x} = f(x)$

then $\ddot{x}_E = f(x_E)$

Hence $f(x_E) = 0$

So finding equilibrium pts is finding solutions of f(x)=0

(b) Stability of an equilibrium pt

To discuss stability of an equilibrium pt , we just need to look at those x near the equilibrium pt.

Hence we want to find the approximate values of f(x) where x near the equilibrium pt

We can use the tangent line (or Taylor series) at equilibrium pt to approximate f(x)

$$f'(x_E) \approx \frac{f(x) - f(x_E)}{x - x_E}$$

$$f(x) \approx f(x_E) + f'(x_E)(x - x_E)$$

$$f(x) \approx f'(x_E)x - f'(x_E)x_E + f(x_E)$$

So near the equilibrium pt,
the given ODE can be approximated by the following 2nd order linear nonhomogeneous ODE

$$\ddot{x} - f'(x_E)x = -f'(x_E)x_E + f(x_E)$$

The general solution is $x(t) = x_h(t) + x_p(t)$

where $x_h(t)$ is the general solution of

$\ddot{x} - f'(x_E)x = 0$ i.e. $\ddot{x} = f'(x_E)x$

where $x_p(t)$ is a particular solution of nonhomo. ODE

Note that $x_p(t)$ is a constant function

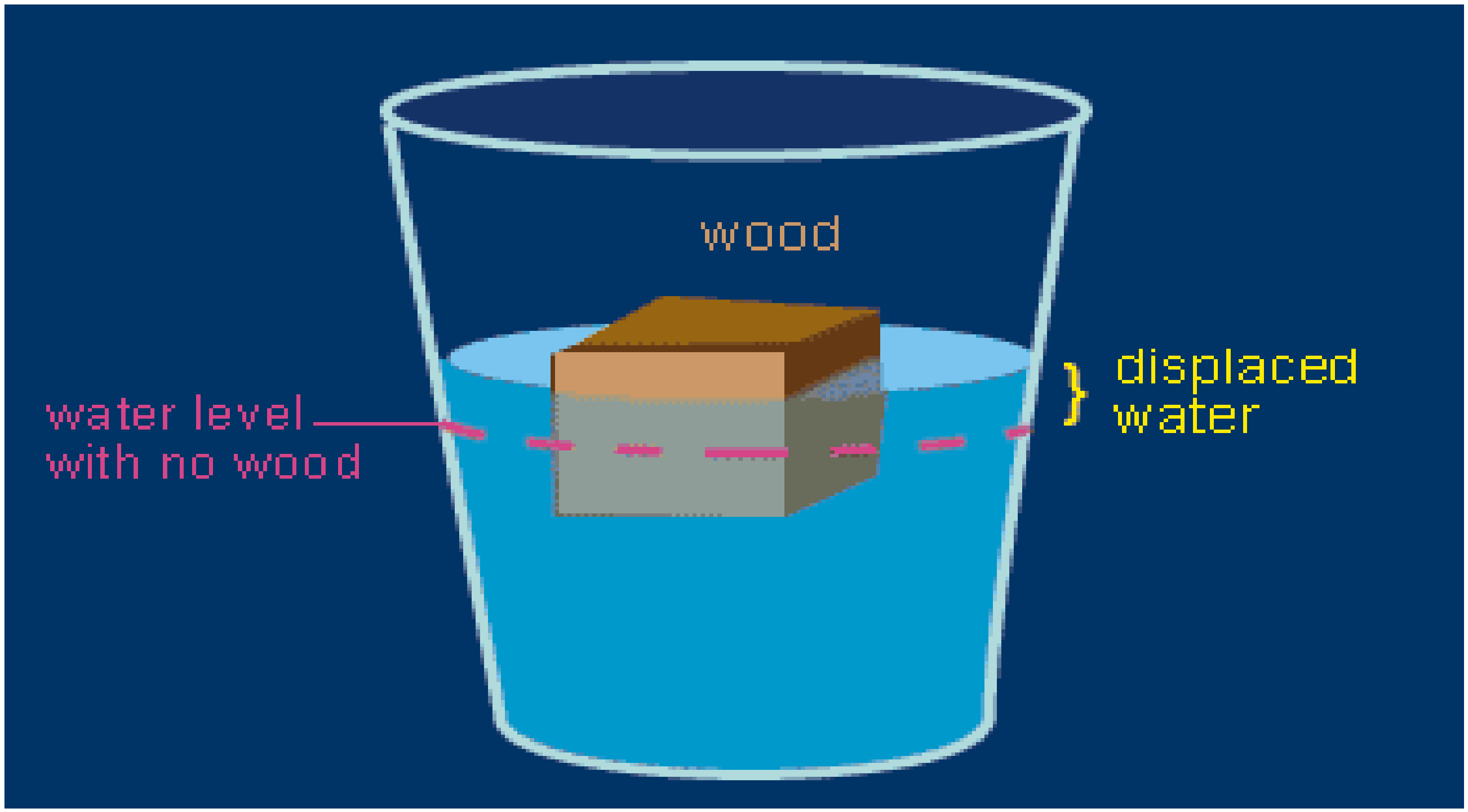
Hence discussing the stability of

$$\ddot{x} - f'(x_E)x = -f'(x_E)x_E + f(x_E)$$

is equivalent to discussing the stability of

$$\ddot{x} = f'(x_E)x$$

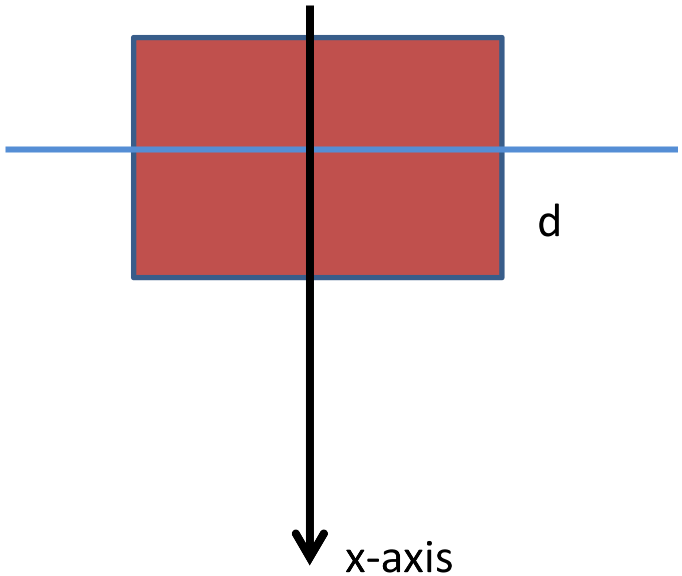
Q4 Buoyancy force

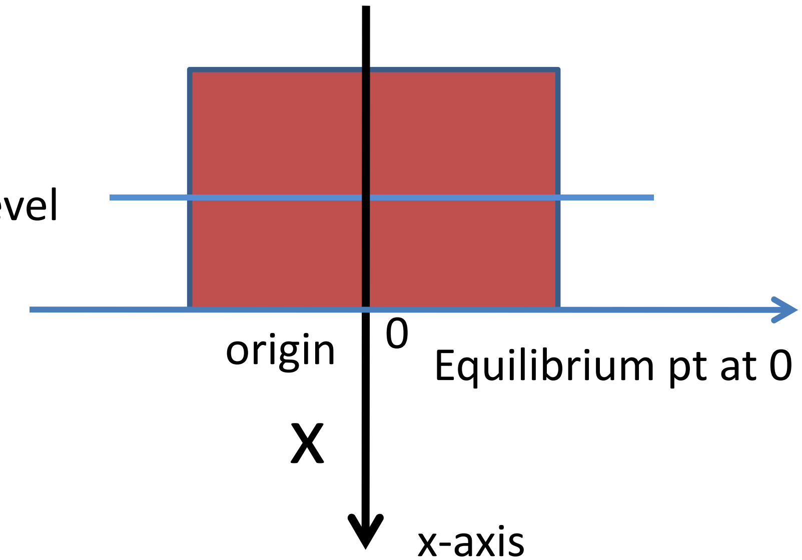


When you place a block of wood in a pail of water, the block displaces some of the water, and the water level goes up.

weight of the wood=weight of displaced water

Q4





$$m\ddot{x} = -\rho A x g = -(\rho A g)x$$

$$mg = \rho A d g = (\rho A g)d$$

Spring (restoring) constant

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$$m\ddot{x} = -kx$$ K=spring (restoring) constant

$$k = \rho A g$$

This formula holds for any right prism with cross-section area A, e.g., cylinder

$$\ddot{x} = -\frac{k}{m}x$$

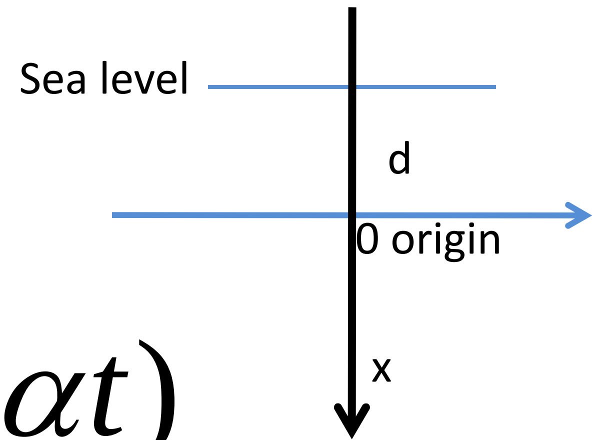
$\frac{k}{m}$ can also be in terms of d since

$$mg = \rho A d g = (\rho A g)d$$

Hence $\ddot{x} = -\frac{g}{d}x$

look at mass-spring system, it has similar results

$$m\ddot{x} + kx = 0$$



$$m\ddot{x} + b\dot{x} + kx = F_0 \cos(\alpha t)$$

where $k = \rho A g$ or $k = \dfrac{mg}{d}$

We can use solutions (given in L N) of the above ODEs without proof to discuss Q4

Q3 The amplitude response function is given by

$$A(\alpha) = \frac{F_0 / m}{\sqrt{(\omega^2 - \alpha^2)^2 + \dfrac{b^2}{m^2}\alpha^2}}$$

Let $$f(\alpha) = (\omega^2 - \alpha^2)^2 + \frac{b^2}{m^2}\alpha^2$$ where α≥0

Hence A(α) is increasing (decreasing)
iff f(α) is increasing (decreasing)

Find $f'(\alpha)$

Use $f'(\alpha)$ to find intervals on which f is increasing and decreasing

Hence we can find the minimum of f and consequently the maximum of A

Note that there are two cases

$$\omega^2 \geq \frac{b^2}{2m^2} \qquad \omega^2 < \frac{b^2}{2m^2}$$

$$\sqrt{2}\omega m \geq b \qquad \sqrt{2}\omega m < b$$