MA1506 Tutorial 5

Question 1

Consider the spring mass system whose motion is governed by

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 17 \sin 2t$$

With initial conditions y(0)=−2 and y ′(0)=0.

(a) Determine whether the motion is overdamped , crtitically damped or underdamped.
(b) Find the steady state and transient parts of the solution.

[Ans: (a) under damped (b) $2e^{-t}\cos t$; −4cos2t+sin2t]

Question 2

Find the total energy of the system

$$3\ddot{x} + x = 0, \quad x(0) = 1, \dot{x}(0) = -2$$

[Ans: 13/2 units]

Question 3

Consider the spring mass system whose motion is governed by

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-t}\cos 2t$$

(a) Describe the variation with time of the applied external force.
(b) Determine the motion of the mass. What happens as t tends to infinity?

[Ans: (a) External force tends to zero as t tends to infinity

(b) y(t) tends to zero as t tends to infinity]

Question 4

The bacteria in a certain culture number 10,000 initially. Two and a half hours later, there are 11,000 of them. Assuming a Malthus model, find the number of bacteria 10 hours after the start of the experiment. How long will it take for the number to reach 20,000?

[Ans: 14,600, 18.18 hrs]

Question 5

You have 200 bugs in a bottle and you supply them with food every day. After two days, there are 360 bugs. Given that the birth rate of bug is 150% per day and assumes that the population obeys the logistic model, find the number of bugs 3 days later. Predict how many bugs will you have eventually.

[Ans: 371, 376]