MA1506 Tutorial 4

Question 1

An undamped mass spring oscillator of mass 1kg is pulled down from its equilibrium position and released from rest. Given that the period is 2 seconds, find the spring constant.

Question 2

A particle moves along the x-axis with equation of motion $\ddot{x} + 6\dot{x} - 16x = 0$
At t=0 sec the particle is at x=2m and moving to the left with a velocity of 10m/s.
When will the particle change direction and go to the right?

Question 3

A simple pendulum of length 0.5m is displaced an angle 0.1 radian from the equilibrium position and released from rest. Determine the resulting motion.

Question 4

Find the equilibrium points of $\ddot{X} = \cos X$ and determine their nature.

Question 5

A certain electrical circuit has a resistor 5 ohms, an inductor of inductance 0.05 henries, and a capacitor $4\text{x}10^{-4}$ farads. A varying voltage given by V=200cos(100t) is applied. Find the current in this curcuit if the initial current and intial voltage on the capactor are both zero.

Question 6

Suppose that

$$x(t) = 2te^{-2t} + \sqrt{3}\,e^{-2t} + 2\cos(\sqrt{6}\,t - \delta)$$

is the solution of the mass-spring system given by

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos \alpha t$$
$$x(0) = x_0, \qquad \dot{x}(0) = v_0$$

Assume that the homogeneous solution is not identically zero.

(i) Find the transient solution.

(ii) Classify the system as under damped, critically damped or over damped.

(iii) If the mass is 1kg, find the value of b.

(iv) Find the angular frequency α of the forcing function.

(v) Find the forcing amplitude F_0 .

Answers:

1. π^2

2. 0.223 sec

3 $\frac{1}{10}\cos\sqrt{2g}t$

4. Stable $\frac{\pi}{2}$, unstable $\frac{3\pi}{2}$

5. $22.13e^{-50t}\sin(t50\sqrt{19}) - 2.35e^{-50t}\cos(t50\sqrt{19}) + 2.35\cos(100t) - 9.41\sin(100t)$

6. (i) $2te^{-2t} + \sqrt{3}e^{-2t}$

(ii) critically damped

(iii) 4

(iv) $\sqrt{6}$

(v) 20