NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA1506 Laboratory 2 (MATLAB)
Comments and Suggested Solutions

Exercise 2A

1. This is because the exponential function grows too quickly and the number e^{4t} gets too large.
$$x = \frac{2+2Ae^{4t}}{1-Ae^{4t}} = \frac{\frac{2}{e^{4t}}+2A}{\frac{1}{e^{4t}}-A} \to \frac{2A}{-A} = -2.$$
The lesson to be learnt is that while computer plots are extremely useful, we should not trust them completely. Use your theoretical knowledge to check if you are doing the right thing.

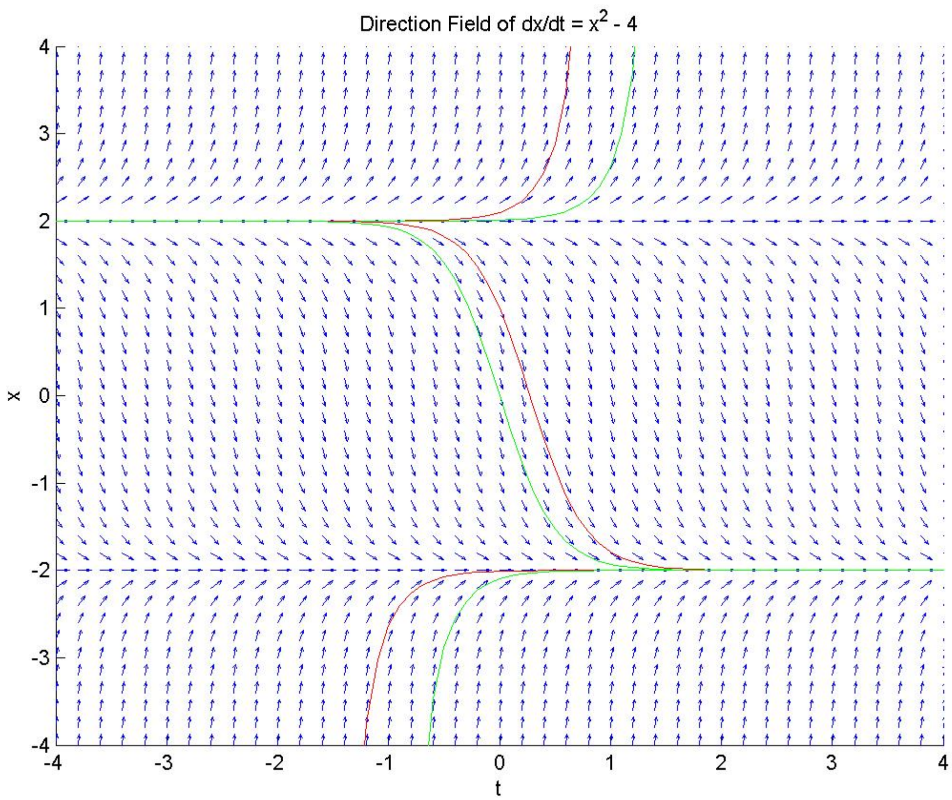
2.

$x(0)$	2.1	2.01	1	0	-2.01	-2.1
$A \approx$	0.0244	0.00244	0.333	1	401	41

Note that for $-2 < x < 2$, the curve is $x = \dfrac{2-2Ae^{4t}}{1+Ae^{4t}}$.

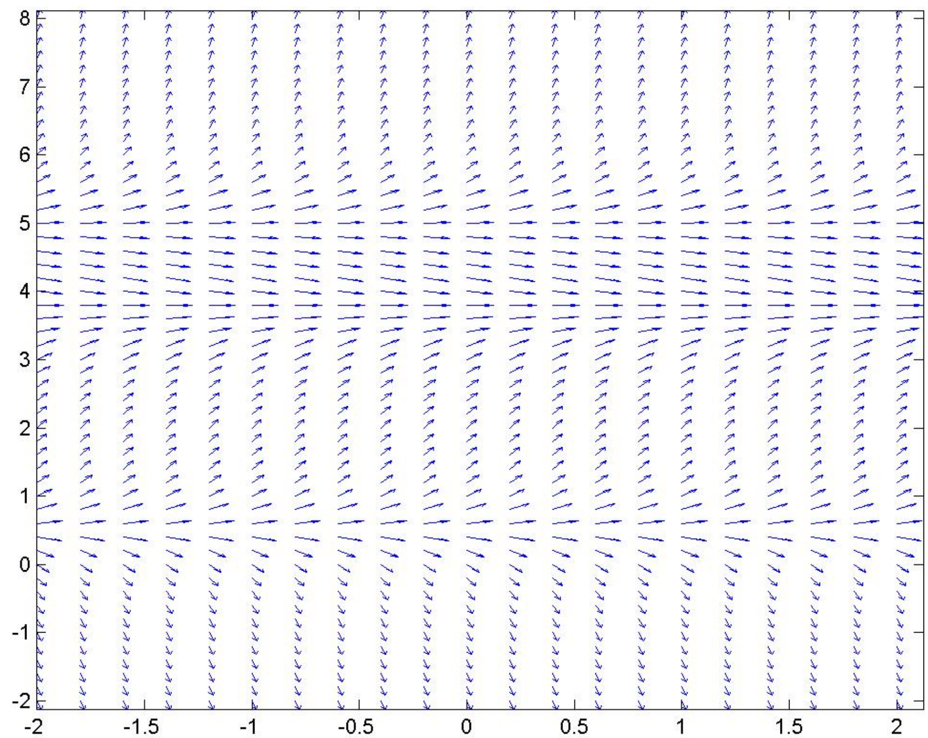
```
>> [T, X]=meshgrid(-4:0.2:4, -4:0.2:4);
>> S = X.^2-4;
>> L = sqrt(1+ S.^2);
>> hold on
>> quiver(T, X, 1./L, S./L, 0.5)
>> axis ([-4 4 -4 4])
>> xlabel('t')
>> ylabel('x')
>> title( 'Direction Field of dx/dt = x^2 - 4')
>> t1=-4:0.1:0.7;
>> x1 = 2*(1+0.0244*exp(4*t1))./(1-0.0244*exp(4*t1));
>> plot(t1,x1,'r')
>> t2=-4:0.1:1.5;
>> x2 = 2*(1+0.00244*exp(4*t2))./(1-0.00244*exp(4*t2));
>> plot(t2,x2,'g')
>> t3=-4:0.1:4;
>> x3 = 2*(1-0.333*exp(4*t3))./(1+0.333*exp(4*t3));
>> plot(t3,x3,'r')
>> t4=-4:0.1:4;
```

```
>> x4 = 2*(1-exp(4*t4))./(1+exp(4*t4));
>> plot(t4,x4,'g')
>> t5=-1.4:0.1:4;
>> x5 = 2*(1+401*exp(4*t5))./(1-401*exp(4*t5));
>> plot(t5,x5,'r')
>> t6=-0.7:0.1:4;
>> x6 = 2*(1+41*exp(4*t6))./(1-41*exp(4*t6));
>> plot(t6,x6,'g')
```



3. Some trial and error should show that the window $-2 \leq x \leq 2, -2 \leq y \leq 8$ is suitable.

```
>> [X, Y]=meshgrid(-2:0.2:2, -2:0.2:8);
>> S = 3*sin(Y)+Y-2;
>> L = sqrt(1+ S.^2);
>> quiver(X, Y, 1./L, S./L, 0.5)
>> axis tight
```



There are 3 equilibriums at $y \approx 5, 3.9$ and 0.5, with the equilibrium at $y \approx 3.9$ the only stable one. It is not difficult to use MATLAB to find the roots of $3 \sin y + y - 2$ more precisely, giving us 0.5170, 3.7745 and 4.9295.

```
>> f = inline('3*sin(y)+y-2','y')
>> fzero(f,5)
>> fzero(f,3.9)
>> fzero(f,0.5)
```

The first command creates a function defined within the single quotation marks. The second argument indicates that the function is a function of y. The fzero command will find the roots of a function, but we need to give it a starting point to search. Hence for the three roots we need to indicate the three approximate starting points.

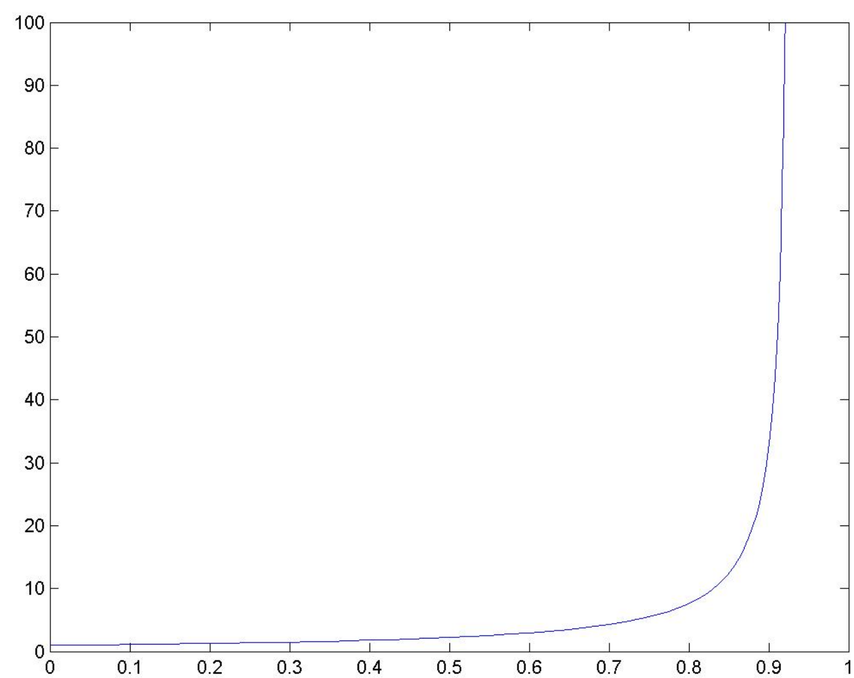
Exercise 2B

1.
```
>> f = inline ('(x+1)/sqrt(x)', 'x', 'y');
>> [x, fa] = ode45(f,[1  4],2);
>> fa(end)

ans =

    8.6667
```

The last command returns the last value in the array fa, which is $fa(4)$, since we only approximated from $1 \leq x \leq 4$. Note that ode45 requires your input function to have two variables although this d.e. only has a single variable x.

2.
```
>> f = inline('t+x.^2','t','x');
>> [ta, xa]=ode45(f,[0 1],1);
>> plot (ta,xa)
>> axis( [ 0 1 0 100])
```

The error message probably arose due to large numerical values. Note that **xa(end)** $> 2.4 \times 10^{14}$.



3. Replace the last line of myfunction.m with

```
xdot(2) = -2*x(2)-x(1) + 2 +exp(2*t);
```

```
>> [ta,xa]=ode45('myfunction',[0 1],[37/9 -7/9]);
>> xa(end,1)

ans =

    3.9246
```

—The End—