NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA1506 Laboratory 4 (MATLAB)

For Weeks 10 and 11
This is the last lab session

(1) The motion of a forced undamped oscillator system is governed by

$$\ddot{x} + kx = F\cos\alpha t, \qquad x(0) = 0,\ \dot{x}(0) = 0.$$

(i) For $k = 9$, $F = 80$ and $\alpha = 5$. Plot the solution $x(t)$ from $t = 0$ to eight times the period of the natural frequency.

(ii) For $k = 2500$, $F = 30$ and $\alpha = 45$. Plot the solution $x(t)$ from $t = 0$ to four times the period of the beat frequency.

Remark: You may use the solution for $x(t)$ from your lecture notes. Do not use the numerical solvers in MATLAB.

(2) Let $N(t)$, measured in kilotons be the total mass of a certain species of shrimp in the Pacific Ocean. We use the logistic model to model the change in shrimp population, with $B = 0.71$ per year and the carrying capacity $N_\infty = 71$ kilotons. Using $t = 0$ to represent 2009, the initial mass $N(0) = 0.3N_\infty$.

(i) If harvesting of 12 kilotons per year is allowed. Plot the direction field of

$$\frac{dN}{dt} = -sN^2 + BN - E.$$

(Hint: You can try to use $0 \le t, N \le 80$. Adjust the interval values of t and N until the equilibriums can be seen clearly from your direction field.)

(ii) Identify the equilibriums (if any) and their stability and plot the equilibrium lines into your direction field graph.

(3) A study has determined that the occupation of a young hobbit, as an adult, depends upon the profession of his father and is given by the following stochastic matrix ordered by the professions: 1) chef, 2) gardener, 3) adventurer, 4) farmer and 5) blacksmith.

$$M = \begin{bmatrix} 0.71 & 0.2 & 0.5 & 0.31 & 0.14 \\ 0.12 & 0.2 & 0.1 & 0.15 & 0.05 \\ 0.05 & 0.2 & 0.07 & 0 & 0.41 \\ 0.12 & 0.2 & 0.23 & 0.22 & 0.25 \\ 0 & 0.2 & 0.1 & 0.32 & 0.15 \end{bmatrix}.$$

So the probability of the son of a chef also becoming a chef is 0.71 and a gardener's son has 0.20 probability of taking on any of the five professions.

(i) Write down the matrix M^5, giving your answer accurate up to at least 4 decimal places.

(ii) What is the probability that the fifth generation descendent of a gardener becomes a farmer? (Your answer should be accurate up to 4 decimal places.)