Remarks on tutorial 3

Q4 (A) Chain Rule $\frac{df(x)}{dy} = \frac{df(x)}{dx}\frac{dx}{dy}$

Apply Chain Rule to the right-hand side of the following equation

$$\frac{d^2y}{dx^2} = \frac{d}{dy}\left[\frac{1}{2}(y')^2(x)\right]$$

(B) Suppose that the earth were to stop moving. Then the earth would fall towards the sun according to

$$\ddot{r} = -\frac{GM}{r^2}$$

Use the first part result of Q4, we get

$$\frac{d}{dr}\left[(r')^2\right] = -\frac{GM}{r^2}$$

$$d\left[(r')^2\right] = -\frac{GM}{r^2}dr$$

Now Integrate both sides

Remember $\int df(x) = \int f'(x)dx = f(x)$

(C) From (B) , we have

$$(\dot{r})^2 = 2GM\left(\frac{1}{r} - \frac{1}{R}\right) \qquad \dot{r} = -\sqrt{2GM\left(\frac{1}{r} - \frac{1}{R}\right)}$$

(D) Initial condition

$$t = 0, r = R, \dot{r} = 0$$

(E) Now find t when $r = \frac{2}{3}R$

i.e., reach the orbit of Venus

(F) We need to find the integral

$$\int \frac{1}{\sqrt{\frac{1}{r} - \frac{1}{R}}} dr$$

Let x=r/R Then we need to find the integral

$$\int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$