MA1506 Tutorial 1

Question 1

Solve the following 1st order ODEs.

(a) $\frac{dy}{dx} = \frac{-x\sqrt{1-y^2}}{y\sqrt{1-x^2}}$

(b) $y' = \frac{1+y^2}{xy+x^3y}$

(c) $\frac{dy}{dx} - (\tan x)y = \cos x$

(d) $\frac{dy}{dx} + 2ty = t,$
$y(1) = 2$

(e) $x\frac{dy}{dx} + y - e^x = 0,$
$y(1) = e$

Question 2

Solve the following ODEs

(a) $x(1+x)y' = 1$

(b) $y' \sec x = \cos 5x$

(c) $y' = e^{(x-3y)}$

(d) $(1+y)y' + (1-2x)y^2 = 0$

Question 3

Solve the following differential equations

(a) $xy' + (1+x)y = e^{-x}, \quad x > 0$

(b) $y' - (1+\frac{3}{x})y = x+2, \quad y(1) = e-1, \quad x > 0$

(c) $y' + y + \frac{x}{y} = 0$

(d) $2xyy' + (x-1)y^2 = x^2e^x, \quad x > 0$

Question 4

Solve the following differential equations

(a) $y' = \dfrac{1-2y-4x}{1+y+2x}$

(b) $y' = \left(\dfrac{x+y+1}{x+y+3}\right)^2$

Answers

Question 1 $(1-y^2)^{1/2} + (1-x^2)^{1/2} = C$

$(1+y^2)(1+x^2) = cx^2$

$y = \dfrac{1}{\cos x}\left(\dfrac{x}{2} + \dfrac{\sin 2x}{4} + C\right)$

$y = \dfrac{1}{2} + \dfrac{3}{2}e^{1-t^2}$

$xy = e^x$

Question 2

$y = \ln\left|\dfrac{x}{1+x}\right| + c$

$y = \dfrac{1}{2}\left[\dfrac{1}{6}\sin 6x + \dfrac{1}{4}\sin 4x\right] + c$

$\dfrac{1}{3}e^{3y} = e^x + c$

$\ln|y| - \dfrac{1}{y} = x^2 - x + c$

Question 3

$$y = e^{-x} + cx^{-1}e^{-x}$$

$$y = -x + x^3 e^x$$

$$y^2 = \frac{1}{2} - x + ce^{-2x}$$

$$y^2 = \frac{1}{2}xe^x + cxe^{-x}$$

Question 4

$$(2x + y) + \frac{1}{2}(2x + y)^2 = 3x + c$$

$$x + y + \ln|(x + y)^2 + 4x + 4y + 5| = 2x + c$$