MA1506 TUTORIAL 5

Question 1

The bacteria in a certain culture number 10000 initially. Two and a half hours later there are 11000 of them. Assuming a Malthus model, how many bacteria will there be 10 hours after the start of the experiment? How long will it take for the number to reach 20000? [Answers: About 14600; about 18.18 hours.]

Question 2

Read the wikipedia article on Overpopulation. Perhaps Malthus was not so wrong after all? It has been suggested that the Earth's population explosion problem can be solved by sending excess population to colonise other planets. Assuming that a fixed number of colonists are sent out each year, and that the Malthus model would hold if there were no emigration, set up an ODE to describe this plan. Solve it. Next, modify the model by assuming that the rate of emigration is proportional to time [that is, we send more and more people out each year]. What happens now?

Question 3

On the island of Orpsengia, the human birth and death rates per capita are constant; and the population of the island has been doubling every 20 years. However, one day several pirate ships arrive. All of the island women under the age of 50, tired of being ordered about by their mothers-in-law and ignored by their husbands, decide to elope with the glamorous pirates, taking their children with them. After that, the remaining population of Orpsengia declines by half over the next ten years. What was the original birth rate per capita on Orpsengia? You will have to make several simplifying assumptions to solve this problem; that is ok as long as you list your assumptions carefully! [Answer: about 10.4%.]

Question 4

One of the reasons that it is so difficult to predict the future population of a country is an effect called *population momentum* [see wikipedia]. This is simply the fact that there is a *delay* between changes in birth rates per capita and the resulting change in the population itself. Suppose that the birth rate per capita of a certain country is not constant but rather given by the expression [where t represents time]

$$B(t) \;=\; 3 \;-\; 3\tanh(3t).$$

Graph this function and show that it represents a birth rate per capita of nearly 6 [in some units] in the past [$t \to -\infty$] and almost zero in the future [$t \to \infty$]. Notice that the decline is centred on t = 0 [so that B = 3 at t = 0]. Assuming that the population N(t) satisfies N(0) = 1, and that the death rate per capita is [as in the logistic model] given by $0.2\,N$, show that the population reaches its maximum, and *then* begins to decline, some time *after* t = 0 [at about t = 0.47], so there is indeed a delay. [You can do this either by solving the differential equation [hint: Bernoulli] and graphing the solution, or by directly using a computer to graph the solution of the equation.]

Question 5

You have 200 bugs in a bottle. Every day you supply them with food and count them. After two days you have 360 bugs. It is known that the birth rate for this kind of bug is 150% per day. [Is this a sensible way of stating a birth rate per capita? Why?] Assuming that the population is given by a logistic model, find the number of bugs after 3 days. Predict how many bugs you will have eventually. [Answers: about 372; about 376.]

Question 6

In the logistic model, we assume that the death rate per capita is given by $\mathrm{D} = \mathrm{sN}$. But this does not really make sense for low populations, because there will always be a minimum non-zero death rate per capita due to death from old age. So actually an equation like $\mathrm{D} = \mathrm{D}_0 + \mathrm{sN}$, where D_0 is a positive constant, would make more sense, since now D can never fall below D_0. What differential equation do you get now? Solve and discuss.