

Remarks on T1

Q4 Solve $r \frac{d\theta}{dr} = \tan \psi$ ψ is fixed

What will happen to the moth?

To answer this question, discuss
the solution of the above ODE

There are three cases:

$\psi > 90$ $\psi < 90$ $\psi = 90$

We will understand these three cases better
if we sketch the graph of the solution for
each case

Why is this reasonable?

The above question appeared in this tutorial

Use common sense to answer, answer is not unique

Volume of raindrop is proportional to
the $3/2$ power of its surface area

Reasonable since it is true for sphere and cube

Reasonable does not mean that it is true,
you need to further check it up

Q3: “Argue that this cannot be correct”

Two ways to answer

(1) based on a common sense of Physics:

evaporation takes place on the surface of a raindrop, so the rate of reduction of volume of a raindrop is

(2) Assume it is true, then work out the result.

Is the result meaningful?

If not, then the assumption cannot be correct

Initial condition

Very often, initial condition is not given in modelling problems, so we have to set the initial condition. For examples, in Q3, we assume that $\theta = 0, r = R$

in Q4, we assume that $t = 0, V = V_0$

Q5(c)

We shall look at the following three examples before we discuss 5(c).

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{y}{2x} - \frac{x}{2y} \quad \text{Let } v=y/x$$

$$\frac{dy}{dx} = \frac{-x + 2y - 3}{2x - 4y + 5} = \frac{-(x - 2y) - 3}{2(x - 2y) + 5} \quad \text{Let } v=x-2y$$

The above 2nd ODE can be done since two st lines $-x+2y-3$ and $2x-4y+5$ are parallel

$$\frac{dy}{dx} = \frac{x + y}{-x + y} = \frac{1 + \frac{y}{x}}{-1 + \frac{y}{x}} \quad \text{Let } v=y/x$$

Two st lines are NOT parallel, but they intersect at (0,0)

Q5 (c)

Solve
$$\frac{dy}{dx} = \frac{-(x + y + 1)}{-x + y - 3}$$

Two methods just mentioned do not work here

Two st lines are not parallel,

Furthermore, **the intersection is not** at (0,0)

So we may transform these two st lines

(change the coordinate system)

such that they intersect at (0,0).

Then we can solve the problem

Q5 (C)

Need to use the following transformation

$$x = X + \alpha, y = Y + \beta$$

Then choose α, β such that

$$x + y + 1 = X + Y$$

$$-x + y - 3 = -X + Y,$$

Then solve ODE involving X, Y

