MA1506 Tutorial 5 Solution

1.

(a) The motion is under damped.

(b) $y(t)=2e^{-t}\cos 2t - 4\cos 2t + \sin 2t$

Transient part: $2e^{-t}\cos 2t$; Steady part: $-4\cos 2t + \sin 2t$

2.

$$3\ddot{x} + x = 0$$

$$\therefore \frac{3}{2}\dot{x}^2 + \frac{1}{2}x^2 = \text{constant} = \text{total energy}$$

at $t=0 \Rightarrow x(0)=1,\ \dot{x}(0)=-2$

$$\therefore \frac{3}{2}(-2)^2 + \frac{1}{2}(1)^2 = \frac{13}{2}$$

$$\therefore \text{total energy} = \frac{13}{2} \text{ units}$$

3.

(a) As t tends to infinity, F(t) exhibits oscillatory behaviour with decreasing amplitude. In fact, $\lim_{t\to\infty} F(t) = 0$

(b) $y_h(t) = e^{-t}(A\cos 2t + B\sin 2t)$

$y_p(t) = te^{-t}\sin 2t$

$y(t) = y_h(t) + y_p(t) \to 0 \qquad \text{as} \qquad t \to \infty$