Suggested Solutions to Mid-term Test Sample

Answer all the questions and enter your answer on the answer sheet. Answers have to be written in ink.

1. Let y(x) be the solution of the differential equation
$$y'' + 4y - 12x = 0, \qquad y(\pi) = 5 + 3\pi, \quad y'(\pi) = 7.$$
Find the value of $y'(0)$.

2. Let y(x) be the solution of the differential equation
$$y' + y - y^3 = 0, \qquad y(0) = 1$$
Evaluate y(100).

3. A tank initially contains 50 litres of pure water. A solution containing 2g per litre of salt is pumped into the tank at the rate of 3 litres per minute (not 3g per minute). The mixture is stirred
Constantly and flows out at same rate of 3 litres per minute (not 3g per minute). Find the amount of salt in the tank after a long time.

4. An undamped mass spring oscillator of mass 1kg is pulled down from its equilibrium position and released from rest. Given that the period is 2 seconds, find the spring constant.

5. A particle moves along the x-axis with equation of motion
$$\ddot{x} + 6\dot{x} - 16x = 0$$
At t=0 sec the particle is at x=2m and moving to the left with a velocity of 10m/s. When will the particle change direction and go to the right?

1. $$y_h(x) = A\cos 2x + B\sin 2x, \qquad y_p(x) = 3x$$

$$y(\pi) = 5 + 3\pi, \quad y'(\pi) = 7$$
$$\Rightarrow \quad A = 5, \quad B = 2$$

Ans. $y'(0) = 7$

2. Note that (by actual substitution) that y(x)=1 is the solution of the differential equation
$$y' + y - y^3 = 0, \qquad y(0) = 1.$$
Hence y(100)=1.

3. Note that there are two misprints. We should replace 3g per minute at lines 2 & 3 by 3 litres per minute.

The differential equation is $\dfrac{dx}{dt} = 6 - \dfrac{3x}{50}, \qquad x(0) = 0$

Solving we get

$$x(t) = 100 - 100e^{-\frac{3t}{50}}$$

Ans. 100

4. $k = \pi^2$

5. $\dot{x}(t) = \dfrac{6}{5}e^{2t} - \dfrac{56}{5}e^{-8t}$

Particle changes direction when $\dot{x}(t) = 0$, i.e., $t = \dfrac{1}{10}\ln\dfrac{28}{3}$