MA1506 Tutorial 3

Question 1

Solve the following ODE

$$\text{(a)}\ \frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$$

$$\text{(b)}\ y'' - 2y' + y = 0$$

$$\text{(c)}\ y'' + 4y' + 5y = 0$$

$$\text{(d)}\ y'' + y' - 6y = 0,\ y(0) = 0, y'(0) = 5$$

Question 2

Solve the following ODE

$$\text{(a)}\ y'' + 6y' + 9y = 0, \qquad y(0) = 1, \qquad y'(0) = -1$$

$$\text{(b)}\ y'' - 2y' + (1 + 4\pi^2)y = 0, \qquad y(0) = -2, \qquad y'(0) = 2(3\pi - 1)$$

Question 3

Find particular solutions of the following ODE

$$\text{(a)}\ y'' + 2y' + 10y = 25x^2 + 3 \qquad \text{(b)}\ y'' - 6y' + 8y = x^2e^{3x}$$

Question 4

Solve the following ODE by the method of variation of parameters.

$$y'' + 4y = \sin 2t\ \left(\text{i.e., } \frac{d^2y}{dt^2} + 4y = \sin 2t\right)$$

Answers

Q1.

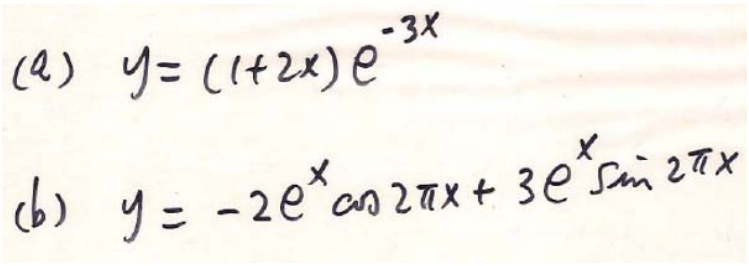
(a) $y = c_1 e^x + c_2 e^{-3x}.$

(b) $y = c_1 e^x + c_2 x e^x.$

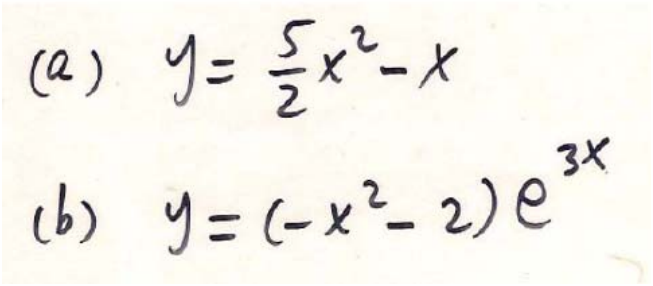
(c) $y = e^{-2x}(c_1 \cos x + c_2 \sin x).$

(d) $y = e^{2x} - e^{-3x}.$

Q2.



Q3.



(c) $y = -\cos x - x \sin x$

Q4.

$$y = y_h + y_p = c_1 \cos 2t + c_2 \sin 2t + \left(-\frac{1}{4}t + \frac{1}{16}\sin 4t\right)\cos 2t + \left(-\frac{1}{16}\cos 4t\right)\sin 2t.$$