

## MA1506 TUTORIAL 1

1. Solve the following differential equations:

(a)  $x(x+1)y' = 1$       (b)  $(\sec(x))y' = \cos(5x)$

(c)  $y' = e^{(x-3y)}$       (d)  $(1+y)y' + (1-2x)y^2 = 0$

Use [www.graphmatica.com](http://www.graphmatica.com) to sketch the functions you found as solutions of [a]-[d], if  $y = 1/2$  at  $x = 1$ . Graphmatica can *also* directly sketch the graph if you insert the equation itself; for example, in [a] you just enter  $x(x+1)dy = 1 \{1, 1/2\}$ . [curly brackets for the initial conditions, with x followed by y.] Here dy represents  $y'$ . Use this to check that your answers were correct.

2. Experiments show that the rate of change of the temperature of a small iron ball is proportional to the difference between its temperature  $T(t)$  and that of its environment,  $T_{env}$  (which is constant). Write down a differential equation describing this situation. Show that  $T = T_{env}$  is a solution. Does this make sense? The ball is heated to  $300^\circ F$  and then left to cool in a room at  $75^\circ F$ . Its temperature falls to  $200^\circ F$  in half an hour. Show that its temperature will be  $81.6^\circ F$  after 3 hours of cooling.

3. In very dry regions, the phenomenon called **Virga** is very important because it can endanger aeroplanes. [See <http://en.wikipedia.org/wiki/Virga>]. Virga is rain in air that is so dry that the raindrops evaporate before they can reach the ground. Suppose that the volume of a raindrop is proportional to the  $3/2$  power of its surface area. [Why is this reasonable? Note: raindrops are not spherical, but let's assume that they always have the same shape, no matter what their size may be.] Suppose that the rate of reduction of the volume of a raindrop is proportional to its surface area. [Why is this reasonable?] Find a formula for the amount of time it takes for a virga raindrop to evaporate completely, expressed in terms of the constants you introduced and the initial surface area of a raindrop. Check that the units of your formula are correct. Suppose somebody suggests that the rate of reduction of the volume of a raindrop is proportional to the **square** of the surface area. Argue that this cannot be correct.

4. One theory about the behaviour of moths states that they navigate at night by keeping a fixed angle between their velocity vector and the direction of the Moon [or some bright star; see <http://en.wikipedia.org/wiki/Moth>]. A certain moth flies near to a candle and mistakes it for the Moon. What will happen to the moth?

Hints: in polar coordinates  $(r, \theta)$ , the formula for the angle  $\psi$  between the radius vector and the velocity vector is given by  $\tan(\psi) = r \frac{d\theta}{dr}$ . [If you want to derive this formula, remember that the tangential component of a small displacement in polar coordinates  $(r, \theta) \rightarrow (r + dr, \theta + d\theta)$  is  $r d\theta$ , and the radial component is just  $dr$ . Now use simple trigonometry.] Use the formula to solve for  $r$  as a function of  $\theta$ . Note that graphmatica can graph in polar coordinates if you spell out  $\theta$  [as theta!]

5. Solve the following equations:

$$(a) \ y' = \frac{1-2y-4x}{1+y+2x} \quad (b) \ y' = \left( \frac{x+y+1}{x+y+3} \right)^2$$

$$(c) \ x + y + 1 + (-x + y - 3)y' = 0$$