P99

$$y'' - 3y' - 4y = 2\sin x$$

Let $y_p = A\cos x + B\sin x$

$$y_p' = -A\sin x + B\cos x$$

$$y_p'' = -A\cos x - B\sin x$$

Subst into $y'' - 3y' - 4y = 2\sin x$

$$(-A\cos x - B\sin x) - 3(-A\sin x + B\cos x)$$
$$- 4(A\cos x + B\sin x) = 2\sin x$$

$$\cos x[-A - 3B - 4A] + \sin x[-B + 3A - 4B] = 2\sin x$$

$$\Rightarrow \begin{cases} -5A - 3B = 0 \\ -5B + 3A = 2 \end{cases}$$

$$A = 3/17 \qquad B = -5/17$$

P107 Solve $y'' - 3y' - 4y = e^{2x}$

We first find y_h by solving

$$y'' - 3y' - 4y = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\therefore \; y_h = C_1 e^{4x} + C_2 e^{-x}$$

Now the RHS is e^{2x} which does not appear in y_h.

So let $y_p = A e^{2x}$

$$y_p' = 2A e^{2x}$$

$$y_p'' = 4A e^{2x}$$

Subst. into $y'' - 3y' - 4y = e^{2x}$

$$4Ae^{2x} - 6Ae^{2x} - 4Ae^{2x} = e^{2x}$$

$$\Rightarrow \; (4A - 6A - 4A) = 1 \;\Rightarrow A = -\tfrac{1}{6}$$

Solve $y'' - 3y' - 4y = e^{4x}$

$$\begin{cases} y'' - 3y' - 4y = 0 \\ \Rightarrow y_h = C_1 e^{4x} + C_2 e^{-x} \end{cases}$$

Now RHS is e^{4x} which already appears in y_h. So must try $y_p = A x e^{4x}$ & not $y_p = A e^{4x}$

$$y_p = A x e^{4x}$$

$$y_p' = A e^{4x} + 4A x e^{4x}$$

$$y_p'' = 4A e^{4x} + [4A e^{4x} + 16 A x e^{4x}]$$

Subt. into $y'' - 3y' - 4y = e^{4x}$

$$(8+16x)Ae^{4x} - (3+12x)Ae^{4x} - 4Axe^{4x} = e^{4x}$$

$$\Rightarrow 5Ae^{4x} = e^{4x} \Rightarrow A = 1/5.$$

P108

Solve $\quad y''+2y'+y=e^{-x}$

$$\begin{cases} y''+2y'+1=0 \\ \Rightarrow \quad y_h = c_1 e^{-x} + c_2 x e^{-x} \end{cases}$$

Now the RHS is e^{-x} which appears in y_h as e^{-x} & xe^{-x}, so we must try

$$y_p = x^2 A e^{-x}$$

$$y_p' = 2Axe^{-x} - Ax^2e^{-x}$$

$$y_p'' = (2Ae^{-x} - 2Axe^{-x}) - (2Axe^{-x} - Ax^2e^{-x})$$

$$= 2Ae^{-x} - 4Axe^{-x} + Ax^2e^{-x}$$

Subst into

$$y''+2y'+y=e^{-x}$$

P108 (cont)

$$(2-4x+x^2)Ae^{-x}+(4x-2x^2)Ae^{-x}$$
$$+\,x^2Ae^{-x}=e^{-x}$$

$$\therefore\quad 2A=1$$
$$A=1/2$$

$$\therefore\ y_p=\tfrac{1}{2}x^2e^{-x}$$

general solution

$$y=y_h+y_p$$
$$=\left(C_1e^{-x}+C_2xe^{-x}\right)+\tfrac{1}{2}x^2e^{-x}$$