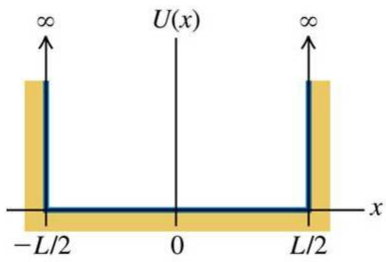
PC2232 Physics for Electrical Engineers: Tutorial 6& 7

Question 1:

In the lecture, we considered a box with walls at $x = 0$ and $x = L$. Now consider a box with a width L but centered at $x = 0$ so that it extends from $x = -L/2$ to $x = L/2$. Note that the box is symmetric about $x = 0$.



(a) Consider possible wavefunctions of the form $\psi(x) = A \sin kx$. Apply the boundary conditions at the wall to obtain the allowed energy levels.

(b) Another set of possible wavefunctions are functions of the form $\psi(x) = B \cos kx$. Apply the boundary conditions at the wall to obtain the allowed energy levels.

(c) Compare the energies obtained in parts (a) and (b) to the set obtained in the lecture.

(d) An odd function f satisfies the condition $f(x) = -f(-x)$, and an even function g satisfies $g(x) = g(-x)$. Of the wavefunctions from parts (a) and (b), which are odd and which are even?

Question 2:

With the help of the applet on bound states used in the lecture (which you may download from `http://phet.colorado.edu./simulations`) or otherwise, answer the following questions.

(a) Consider a finite potential square well of width 1.0 nm and a potential height U_0 of 10.0 eV, how many bound states are allowed? List their energy levels.

(b) Sketch wavefunctions and probability density for the lowest and highest energy levels in this system. What can be said about the probability of finding the atoms beyond the classically allowed region?

(c) With the same width of 1.0 nm, what are the corresponding energy levels for an infinite potential well? (You have to calculate this from the formula given in the lecture.) Compare the values with those of the finite square well.

(d) When the width is reduced while the potential height is kept at 10.0 eV, describe and explain what happens to the number and values of allowed energy levels. What is the approximate maximum width that would allow only one bound state in the well?

(e) If the energy of the particle E is above the potential height U_0, the particle will behave as a free particle. Sketch the wavefunction for the case where $E = 1.0$ eV, $U_0 = 0.9$ eV and $L = 4.0$ nm. (You should sketch the wavefunction based on you understanding of the lecture and then check it with the applet on tunneling shown during the lecture. You may download this applet from the same website listed above.)

Question 3:

(a) Show by direct substitution in the Schrödinger equation for the one-dimensional harmonic oscillator that the wavefunction

$$\psi(x) = A_1 x \mathrm{e}^{-\alpha^2 x^2/2}, \quad \text{where} \quad \alpha^2 = \frac{m\omega}{\hbar} \tag{1}$$

is a solution with energy corresponding to $E_1 = \frac{3}{2}\hbar\omega$.

(b) Find the normalization constant A_1.

(c) Show that the probability density has a minimum at $x = 0$ and maxima at $x = \pm 1/\alpha$, corresponding to the classical turning points for the ground state $n = 0$.

Hint: The following definite integral may be useful

$$\int_{-\infty}^{\infty} x^2 \mathrm{e}^{-ax^2}\, \mathrm{d}x = \frac{\sqrt{\pi}}{2} a^{-3/2}. \tag{2}$$

Question 4:

A quantum particle of mass m_1 is in a square well with infinitely high walls and length 3 nm. Rank the situation (a) through (f) according to the energy from highest to lowest, noting any cases of equality.

(a) The particle of mass m_1 is in the ground state ($n = 1$) of the well.

(b) The same particle is in the $n = 2$ excited state of the same well.

(c) A particle of mass $2m_1$ is in the ground state of a well of length 1.5 nm.

(d) A particle of mass m_1 is in the ground state of a well of length 3 nm but with finite potential wells.

(e) A particle of mass m_1 is in the $n = 2$ state of a well of length 3 nm but with finite potential wells.

(f) A particle of mass m_1 is in the ground state of a well of length 3 nm, and the uncertainty principle has become inoperative, that is, Planck's constant has been reduced to zero.

Question 5:

A quantum particle of mass m is confined in a one-dimensional potential well of length $2L$. Its potential energy is infinite for $x < -L$ and for $x > +L$. Inside the region $-L < x < L$, its potential energy is given by

$$U(x) = -\frac{\hbar^2 x^2}{mL^2\left(L^2 - x^2\right)}. \tag{3}$$

(a) Verify that

$$\psi(x) = A\left(1 - \frac{x^2}{L^2}\right) \tag{4}$$

is a possible solution for the region $-L < x < L$. What is the wave function of the particle for $x < -L$ and $x > +L$?

(b) Write down the energy of the particle in this state in terms of $\hbar$, m and L.

(c) Sketch the wave function and the probability density function of this state.

(d) Find an expression for A that normalizes the wave function.

(e) Determine the probability that the particle is located between $x = -L/3$ and $x = L/3$.

Question 6:

Particles of energy E are incident from the left, where $U(x) = 0$, and at the origin encounter an abrupt drop in potential energy, whose depth is $-3E$.

(a) Classically what would the particles do, and what would happen to their kinetic energy?

(b) Applying quantum mechanics: Assume an incident wave of the form $\psi_{\text{inc}}(x) = \mathrm{e}^{\mathrm{i}kx}$, where the normalization constant has been given the simple value of 1. Determine completely the wave function everywhere, including numeric values for the multiplicative constants.

(c) What is the probability that incident particles will be reflected?

Question 7:

In the $E > U_0$ potential barrier, there should be no reflected wave when the incident wave is at one of the transmission resonances. Prove this by assuming that a beam of particles is incident at the first transmission resonance, $E = U_0 + \frac{\pi^2\hbar^2}{2mL^2}$ prove that this is true by utilizing the continuity conditions. Hint: Your aim is to show that $B = 0$. (Note: k' is particularly simple in this special case.)