PC2232 (lecture 2) 01/02/2015

Wave equation (Solution of Maxwell's equations)

General Form:

$$\vec{E}(x,y,z,t) = \tilde{E}_m \, e^{-j(\omega t - \vec{k}\cdot\vec{r} - \phi)}$$ (exponential form)

$\tilde{E}_m$ = Amplitude vector = $E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$

E_x, E_y, E_z can be complex numbers showing phase difference between x, y, z components

$\vec{k}$ = Propagation vector = $k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$

$\vec{r}$ = Position vector of any point on the wave front
$= x\hat{x} + y\hat{y} + z\hat{z}$

ϕ = Fixed phase.

ω = Angular frequency.

Wave parameter definitions:

(1) $k = |\vec{k}|$ = wave number = propagation constant
= How many wavelengths make 2π metres. (spacial)

(2) λ = wave length = How many meters travelled in one cycle (spacial)

(3) f = frequency = How many cycles per second (temporal)

(4) T = time period = time taken for one cycle (temporal)

(5) v = Phase velocity = distance travelled by a fixed point in one second. (spacial)

(6) ω = How many time periods make 2π seconds (temporal)

Basic Relationships between wave parameters

(1) $\omega = 2\pi f = \frac{2\pi}{T}$; $f = 1/T$

(2) $v = f\lambda$

(3) $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$

Wave parameters in non-magnetic mediums (ω, f remain unchanged)

(1) ϵ = permittivity $= \epsilon_0 \epsilon_r$, $\epsilon_r > 1$

(2) μ = permeability $= \mu_0 \mu_r = \mu_0$ $(\mu_r = 1)$

(3) c = speed of light in free space $= \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

v = speed of light in a medium $= \frac{1}{\sqrt{\mu_0 \epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$

(4) λ_0 = Wave length in free space $= \frac{c}{f}$

λ = wavelength in a medium $= \frac{v}{f} = \frac{\lambda_0}{\sqrt{\epsilon_r}}$

(5) k_0 = k in free space $= \frac{\omega}{c}$

k = in any medium $= \frac{\omega}{v} = \sqrt{\epsilon_r}\, k_0$

(6) n = Refractive index $= \frac{c}{v} = \sqrt{\epsilon_r} \geq 1$

(7) $v = \frac{c}{n}$

$\lambda = \frac{\lambda_0}{n}$

$k = n k_0$

(8) Z_0 = Intrinsic impedance of free space

$= \sqrt{\frac{\mu_0}{\epsilon_0}}$

Z = Intrinsic impedance of a medium

$= \sqrt{\frac{\mu_0}{\epsilon}} = Z_0/\sqrt{\epsilon_r} = Z_0/n$

Boundary Conditions:

(1) $E = 0$ on the surface and inside perfect conductors.

(2) Tangential component of $\vec{E} = \hat{n} \times \vec{E}$ and tangential component of $\vec{H} = \hat{n} \times \vec{H}$ are continuous at a boundary.

(3) Normal component of $\vec{B} = (\vec{B} \cdot \hat{n})\hat{n}$ and normal component of $\vec{D} = (\vec{D} \cdot \hat{n})\hat{n}$ are continuous at a boundary.

Law of Reflection:

(1) Incident Angle θ_i = Reflected angle θ_r

Snell's Law of Refraction:

$$n_1 \sin\theta_i = n_2 \sin\theta_t \quad (\theta_t = \text{refracted angle})$$

While passing from refractive indices n_1 to n_2

Amplitudes of reflected and refracted wave

$\tilde{E}_r = S_r \tilde{E}_i$ & $\tilde{E}_t = S_t \tilde{E}_i$ (S_r & S_t are reflection & trasmission coefficients)

TE polarization $\Big\}$ $$S_r = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \quad \& \quad S_t = \frac{2 n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

TM polarization $\Big\}$ $$S_r = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} \quad \& \quad S_t = \frac{2 n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$