PC2232 Physics for Electrical Engineers: Tutorial 8

Question 1: Stirling's approximation

[Griffiths 5.27] Stirling's approximation, given in pg 7 of Lecture 11, is

$$\ln z! \simeq z \ln z - z, \quad \text{for } z \gg 1. \tag{1}$$

This relation, being an approximation, is of course not expected to be 100% accurate.

(a) Find the percentage error of in Stirling approximation for $z = 10$.

(b) What is the smallest integer z such that the error is less than 5%?

Question 2: Two-dimensional density of states

Consider spin-1/2 electrons in a two-dimensional box with the length of sides L_x and L_y. Calculate the density of states, $\rho(E)$ for this case.

Question 3: Bose-Einstein distribution

The Fermi-Dirac distribution was obtained for *fermions*. Here, calculate the corresponding distribution for *bosons*, where the multiplicity W for bosons is given by

$$W(N_1, \ldots, N_N) = \prod_{n=1}^{\infty} \frac{(N_n + d_n - 1)!}{N_n!\,(d_n - 1)!}, \tag{2}$$

under the constraints of number and energy conservation. *Hint*: use the method of Lagrange multipliers similar to the fermion case:

$$G = \ln W + \alpha \left(N - \sum_n N_n \right) + \beta \left(E - \sum_n N_n E_n \right). \tag{3}$$

Question 4: Blackbody radiation

According to Plank's hypothesis, each photon of light carries an energy given by $E = \hbar\omega$. You are given that for photons, the number of states per k-space volume is the same as for electrons,

$$C(k_x, k_y, k_z) = \frac{2V}{\pi^3}, \tag{4}$$

where V is the real-space volume of the box. Show that, for an infinitesimal interval of energy between E and $E + \mathrm{d}E$, there are d_k states that can carry this energy, where d_k is given by

$$d_k = \frac{V k^2 \mathrm{d}k}{\pi^2}. \tag{5}$$

Show that the energy density for photons in the box, defined by

$$\frac{N\hbar\omega}{V} = \rho(\omega)\,\mathrm{d}\omega = \frac{\hbar\omega^3\,\mathrm{d}\omega}{\pi^2 c^3\left(\mathrm{e}^{\hbar\omega/k_\mathrm{B}T} - 1\right)}. \tag{6}$$

This is the well-known spectrum describing blackbody radiation.

Question 5: Wien's Law

(a) Use Eq. (6) to determine the energy density in the *wavelength* range $\mathrm{d}\lambda$. *Hint*: Set $\rho(\omega)\mathrm{d}\omega = \bar{\rho}(\lambda)\mathrm{d}\lambda$.

(b) Find the *peak intensity wavelength*, $\lambda = \lambda_\mathrm{max}$ for $\bar{\rho}(\lambda)$. Express λ_max in terms of temperature and the other physical constants. *Hint*: You may need the numerical solution to the following trancedental equation:

$$(5 - x)\,\mathrm{e}^x = 5, \quad \rightarrow \quad x \simeq 4.965\ldots \tag{7}$$