PC2232: Physics for Electrical Engineers

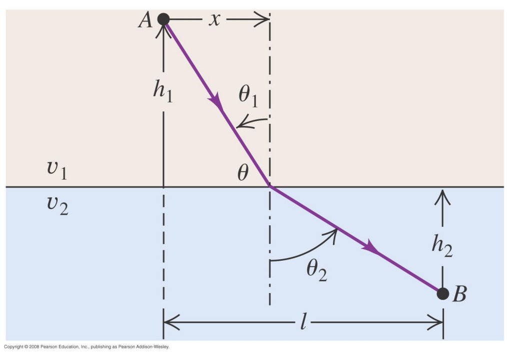
Tutorial 1

1. **EM waves**
   A 550-nm harmonic EM-wave whose electric field is in the z-direction is traveling in the y-direction in vacuum.
   (a) What is the frequency of the wave?
   (b) Determine both ω and k for this wave.
   (c) If the electric field amplitude is 600 V/m, what is the amplitude of the magnetic field?
   (d) Write an expression for both $\vec{E}(t)$ and $\vec{B}(t)$ given that each is zero at $x = 0$ and $t = 0$. Put in all the appropriate units.

2. **EM waves in material**
   An electromagnetic wave with frequency 5.70×10^{14} Hz propagates with a speed of 2.17×10^{8} m/s in a certain piece of glass. Find
   (a) The wavelength of the wave in the glass
   (b) The wavelength of a wave of the same frequency propagating in air
   (c) The index of refraction n of the glass for an electromagnetic wave with this frequency
   (d) The dielectric constant for glass at this frequency, assuming that the relative permeability is unity.

3. **Energy and Momentum in EM waves**
   Public television station KQED in San Francisco broadcasts a sinusoidal radio signal at a power of 316 kW. Assume that the wave spreads out uniformly into a hemisphere above the ground. Consider a home 5.00 km away from the antenna.
   (a) What average pressure does this wave exert on a totally reflecting surface?
   (b) What are the amplitudes of the electric and magnetic fields of the wave?
   (c) What is the average density of the energy this wave carries?
   (d) From the energy density in part (c), what percentage is due to the electric field and what percentage is due to the magnetic field?

4. **Fermat's Principle of Least Time**
   A ray of light goes from point A in a medium in which the speed of light is v_1 to point B in a medium in which the speed is v_2 as shown in the figure below. The ray strikes the interface a horizontal distance x to the right of point A.



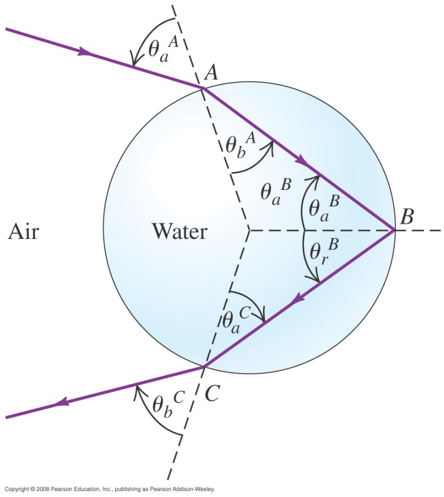
(a) Show that the time t required for the light to travel from A to B is

$$t = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l-x)^2}}{c}$$

(b) Take the derivative of t with respect to x. Set the derivative equal to zero to show that this time reaches its *minimum* value when $n_1 \sin\theta_1 = n_2 \sin\theta_2$. This is Snell's law, and corresponds to the actual path taken by the light.

This is an example of Fermat's *principle of least time*, which states that among all possible paths between two points, the one actually taken by a ray of light is that for which the time of travel is a *minimum*. (In fact, there are some cases in which the time is a maximum rather than a minimum.)

5. **Rainbow**
A rainbow is produced by the reflection of sunlight by spherical drops of water in the air. The figure below shows a ray that refracts into a drop at point A, is reflected from the back surface of the drop at point B, and refracts back into the air at point C. The angles of incidence and refraction, θ_a and θ_b, are shown at points A and C, and the angles of incidence and reflection, θ_a and θ_r, are shown at point B.



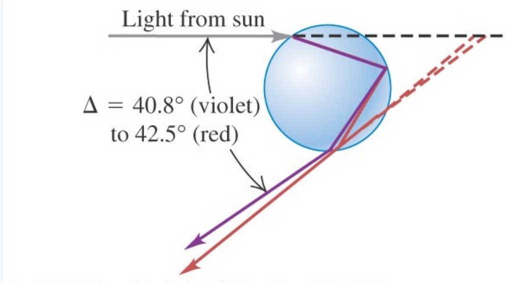
(a) Show that $\theta_a^B = \theta_b^A$, $\theta_a^C = \theta_b^A$ and $\theta_b^C = \theta_a^A$

(b) Show that the angle in radians between the ray before it enters the drop at A and after it exits at C (the total angular deflection of the ray) is $\Delta = 2\theta_a^A - 4\theta_b^A + \pi$. (*Hint*: Find the angular deflections that occur at A, B and C, and add them to get Δ.)

(c) Use Snell's law to write Δ in terms of θ_a^A and n, the refractive index of the water in the drop.

(d) A rainbow will form when the angular deflection Δ is *stationary* in the incident angle θ_a^A – that is, when $\dfrac{\mathrm{d}\Delta}{\mathrm{d}\theta_a^A} = 0$. If this condition is satisfied, all the rays with the incident angles close to θ_a^A will be sent back in the same direction, producing a bright zone in the sky. Let θ_1 be the value of θ_a^A for which this occurs. Show that $\cos^2\theta_1 = \frac{1}{3}(n^2 - 1)$.
(*Hint*: You may find the derivative formula $\dfrac{\mathrm{d}\arcsin u(x)}{\mathrm{d}x} = \left(1 - u^2\right)^{-\frac{1}{2}} \left(\dfrac{\mathrm{d}u}{\mathrm{d}x}\right)$ helpful.)

(e) The index of refraction in water is 1.342 for violet light and 1.330 for red light. Use the results of parts (c) and (d) to find θ_1 and Δ for violet and red light. Do your results agree with the angles shown in the figure below?



When you view the rainbow, which colour, red or violet, is higher above the horizon?

6. **EM waves (Optional)**

(a) An electromagnetic wave is specified (in SI units) by the following function:

$$\vec{E} = \left(-6\hat{i} + 3\sqrt{5}\hat{j}\right)\left(10^4 V m^{-1}\right) e^{i\left(\left(\sqrt{5}x+2y\right)\left(\frac{\pi}{3}\right)\left(10^7\right)-9.42\times10^{15}t\right)}$$

Find

i. the direction along which the electric field oscillates,
ii. the scalar value of amplitude of the electric field,
iii. the direction of propagation of the waves,
iv. the propagation number and wavelength,
v. the frequency and angular frequency,
vi. the speed.