NATIONAL UNIVERSITY OF SINGAPORE

PC2232 PHYSICS FOR ELECTRICAL ENGINEERS

(Semester II: AY 2012-13)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **five short** questions in Part I and **two long** questions in Part II. It comprises **nine** printed pages.
2. Answer **ALL** questions.
3. All answers are to be written on the answer books.
4. This is a **CLOSED BOOK** examination.
5. The total mark for Part I is 50 and that for Part II is 50.
6. A list of constants and formulae can be found on pages 6 – 9 of this question paper. Not all the information will be used in the paper.

PC2232 – PHYSICS FOR ELECTRICAL ENGINEERS

PART I

This part of the examination paper contains **five** short-answer questions from pages 2 – 3. Answer **ALL** questions. The mark for each question is indicated in the square bracket.

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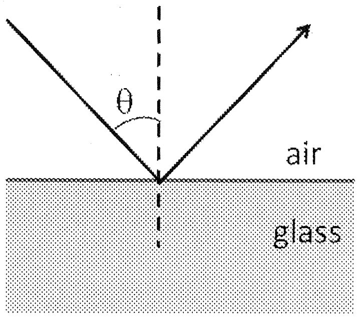
1. Light of intensity I_0 is incident on a air-glass interface at an angle of $\theta = 50°$ as shown in the figure below. The refractive index of glass is 1.53 and that of air is 1.00.



What is the intensity of the reflected light if

(a) the incident light is polarized in the plane of incidence;

(b) the incident light is polarized perpendicular to the plane of incidence and

(c) the incident light is unpolarized? [10]

2. (a) Use the Bohr's model of the hydrogen atom to show that when the electron moves from the state n to the state $n-1$, the frequency of the emitted light is

$$f = \left(\frac{me^4}{8\epsilon_0^2 h^3 n^2}\right)\frac{2n-1}{(n-1)^2}$$

(b) Simplify the above expression as $n \to \infty$.

(c) Hence or otherwise, show that the above equation reduces to the classical frequency one expects the atom to emit. *Hint*: To calculate the classical frequency, note that the frequency of revolution is $v/2\pi r$ where v is the speed of the electron and r is the radius of orbit. [10]

3. Consider a particle of mass m in a two-dimensional rigid rectangular box with sides a (in the x-direction) and b (in the y-direction).

   (a) Extending from the one-dimensional case, write down the expression for the allowed energies and their corresponding wave functions, identified by quantum numbers n_x and n_y, similar to the expressions for infinite square well given on page 9.

   (b) If $b = a/2$, tabulate the lowest six energy levels, indicating also their quantum numbers and degeneracies. [10]

4. (a) List all the possible sets of quantum numbers (n, l, m_l, m_s) of a $2p$ electron.

   (b) Suppose we have an atom such as carbon, which has two $2p$ electrons. Ignoring the Pauli exclusion principle, how many different possible combinations of quantum numbers of the two electrons are there?

   (c) How many different possible combinations are there if Pauli exclusion principle is taken in account?

   (d) Suppose carbon is in an excited state of $2p^1 3p^1$. How many different possible combinations of quantum numbers are there for the two electrons? [10]

5. (a) Given that the energy gap of a certain material is 0.41 eV and that the Fermi energy lies at the middle of the gap, calculate the occupation probability at 295 K for

      (i) a state at the top of the valence band and
      (ii) a state at the bottom of the conduction band.

   (b) How do the occupation probabilities change if the temperature is increased by 100 K? [10]

PC2232 – Physics for Electrical Engineers

PART II

This part of the examination paper contains **TWO** long-answer questions from page 4 to 5. Answer **ALL** questions. The mark for each part is indicated in the square bracket.

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6. (a) A particle is in the nth quantum state of an infinite square well.

   (i) Show that the probability of finding it in the left-hand quarter ($0 \leq x \leq L/4$) of the well is

   $$P = \frac{1}{4} - \frac{\sin(n\pi/2)}{2n\pi} \quad .$$

   (ii) What is the classical probability of finding it the left-hand quarter of the well? Under what circumstances would the probability P approach or be equal to its classical probability? You may want to consider odd and even values of n separately. [8]

   (b) For a finite one-dimensional square well of width L, the well depth U_0 and the particle mass m are related by $\sqrt{mU_0L^2/2\hbar^2} = 4$. A detailed analysis shows that the ground-state energy is given by $E_1 = 0.098U_0$ and the corresponding wave function is approximately

   $$\psi_1(x) = \begin{cases} 17.9\dfrac{1}{\sqrt{L}}\mathrm{e}^{7.60x/L} & : \quad x \leq -\frac{1}{2}L \\ 1.26\dfrac{1}{\sqrt{L}}\cos(2.50x/L) & : \quad -\frac{1}{2}L \leq x \leq \frac{1}{2}L \\ 17.9\dfrac{1}{\sqrt{L}}\mathrm{e}^{-7.60x/L} & : \quad x \geq \frac{1}{2}L \end{cases} \quad .$$

   (i) Show that both $\psi_1(x)$ and its first derivative are continuous at the well edges. *Because the functions given are approximate, your results will not be exactly the same, but should agree to two significant figures.*

   (ii) Plot the function $\sqrt{L}\psi_1(x)$ as a function of x/L, indicating the well edges on your plot.

   (iii) Calculate the probability of finding the particle in the classically forbidden regions outside the well. [12]

   (c) Sketch the wave functions of the next two higher energy states for the system in (b). For these states, do you expect the probability of finding the particle in the classically forbidden regions to be higher or lower than if it were in the ground state? Give a brief explanation for your answer. [5]

7. (a) A helium-neon laser ($\lambda = 633$ nm) is built with a glass tube of inner diameter 1.0 mm. One mirror is partially transmitting to allow the laser beam out. An electrical discharge in the tube causes it to glow like a neon light. From an optical perspective, the laser beam is a light wave that diffracts out through a 1.0-mm-diameter circular opening.

(i) Can the laser beam exit the mirror without spreading? Give a very brief explanation for your answer, using concepts you have learnt in wave optics and also using the Heisenberg's uncertainty principle.

(ii) The angle θ_1 to the first minimum is called the divergence angle of a laser beam. What is the divergence angle of this laser beam?

(iii) What is the diameter of the laser beam (defined by the first minimum) after it travels A) 3.0 m and B) 1.0 km? [10]

(b) Light from the above laser is expanded by a system of lenses so that the beam is more or less parallel (or collimated) when it passes through two narrow slits separated by 0.20 mm and is observed on a screen 1.0 m from the slits. The location of the central maximum of intensity I_0 is marked on the screen and labelled $y = 0$.

(i) At what distance, on either side of $y = 0$, are the first order bright fringes observed?

(ii) A very thin piece of glass is then placed in one slit so that the wave passing through the glass is delayed by 5.0×10^{-16} s in comparison to the wave going through the other slit. What is the phase difference $\Delta\phi_0$ between the two waves as they leave the slits?

(iii) By how much and in which direction (towards or away from the slit with glass) does the central maximum move due to the introduction of the piece of glass? [9]

(c) The double-slit setup in (b) is replaced by a setup comprising of 5 narrow slits with separation between any two adjacent slits also equals to 0.20 mm and having the same slit width. Draw a graph of intensity vs displacement along a line on the screen perpendicular to the slits. Your graph should have at least 3 principal maxima, including the central maximum. Indicate clearly on the graph the distance between intensity maxima and the intensity of the central maximum in terms of I_0 defined in part (b). [6]

– End of Paper –

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TABLE OF INFORMATION

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\begin{aligned}
\text{Speed of light in vacuum, } c &= 2.998 \times 10^{8} \text{ m/s} \\
\text{Charge of electron, } e &= 1.602 \times 10^{-19} \text{ C} \\
\text{Planck's constant, } h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \\
\hbar &= 1.055 \times 10^{-34} \text{ J} \cdot \text{s} \\
\text{Mass of electron, } m_e &= 9.109 \times 10^{-31} \text{ kg} \\
\text{Mass of proton, } m_p &= 1.673 \times 10^{-27} \text{ kg} \\
\text{Mass of neutron, } m_n &= 1.675 \times 10^{-27} \text{ kg} \\
\text{Boltzmann constant, } k &= 1.381 \times 10^{-23} \text{ J/K} \\
\text{Avogadro's number, } N_A &= 6.022 \times 10^{23} \text{ mol}^{-1} \\
\text{Permeability of free space, } \mu_0 &= 4\pi \times 10^{-7} \text{ Wb/A} \cdot \text{m} \\
\text{Permittivity of free space, } \epsilon_0 &= 8.854 \times 10^{-12} \text{ C}^2\text{/N} \cdot \text{m}^2 \\
\text{Rydberg constant, } R_\infty &= 1.097 \times 10^{7} \text{ m}^{-1} \\
\text{Stefan-Boltzmann constant, } \sigma &= 5.670 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4} \\
\text{Wien's displacement constant, } b &= 2.898 \times 10^{-3} \text{ m K} \\
\text{Bohr radius, } a_0 &= 5.292 \times 10^{-11} \text{ m} \\
\text{Hydrogen ground state energy, } E_1 &= -13.61 \text{ eV}
\end{aligned}
$$

$$\text{Maxwell Equations:} \quad \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$\text{(Integral Form)} \quad \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}}$$

$$\text{Maxwell Equations:} \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\text{(Differential Form)} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\text{Speed of light in vacuum, } c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\text{Energy Density of EM wave, } u = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2$$

$$\text{Poynting Vector, } \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\text{Intensity, } I = S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c}$$

$$\text{Fresnel Equations:} \quad \frac{E_{r,\perp}}{E_{a,\perp}} = -\frac{\sin(\theta_a - \theta_b)}{\sin(\theta_a + \theta_b)}$$

$$\frac{E_{b,\perp}}{E_{a,\perp}} = \frac{2 \sin\theta_b \cos\theta_a}{\sin(\theta_a + \theta_b)}$$

$$\frac{E_{r,\parallel}}{E_{a,\parallel}} = \frac{\tan(\theta_a - \theta_b)}{\tan(\theta_a + \theta_b)}$$

$$\frac{E_{b,\parallel}}{E_{a,\parallel}} = \frac{2 \sin\theta_b \cos\theta_a}{\sin(\theta_a + \theta_b) \cos(\theta_a - \theta_b)}$$

$$\text{Malus's Law: } I = I_{\max} \cos^2 \phi$$

$$\text{Brewster's Law: } \tan\theta_{\text{p}} = \frac{n_b}{n_a}$$

$$\text{Reflection at normal incidence: } E_r = \frac{n_a - n_b}{n_a + n_b} E_i$$

$$\text{Single Slit: } I = I_0 \left[\frac{\sin\left[\pi a(\sin\theta)/\lambda\right]}{\pi a(\sin\theta)/\lambda}\right]^2$$

$$\text{Two Slits with finite width: } I = I_0 \cos^2\frac{\phi}{2}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

$$\text{with } \phi = \frac{2\pi d}{\lambda}\sin\theta$$

$$\text{and } \beta = \frac{2\pi a}{\lambda}\sin\theta$$

$$\text{Diffraction Grating: } d\sin\theta = m\lambda$$

$$\text{X-ray Diffraction: } 2d\sin\theta = m\lambda$$

$$\text{Circular Aperture (first dark ring): } \sin\theta_1 = 1.22\frac{\lambda}{D}$$

$$\text{Stefan's Law: } P = \sigma A e T^4$$

$$\text{Wien's Law: } \lambda_{\max}T = b$$

$$\text{Rayleigh-Jeans Formula: } I(\lambda) = \frac{2\pi ckT}{\lambda^4}$$

$$\text{Planck radiation law: } I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$$

$$\text{Photoelectric Effect: } eV_0 = hf - \phi$$

$$\text{Relativistic Total Energy: } E^2 = m^2c^4 + p^2c^2$$

$$\text{Compton's Equation: } \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

$$\text{Rydberg Formula: } \frac{1}{\lambda} = R\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$$

$$\text{Bohr Hydrogen Atom: } E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2}$$

$$\text{and } r_n = \frac{n^2\epsilon_0 h^2}{\pi m e^2}$$

$$\text{de Broglie Wavelength: } \lambda = \frac{h}{mv}$$

$$\text{Heisenberg's Uncertainty Principles: } \Delta p_x \Delta x \geq \hbar;\ \Delta E \Delta t \geq \hbar$$

$$\text{1-D Schrödinger equation:} \quad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

$$\text{1-D Infinite Square Well:} \quad E_n = \frac{n^2h^2}{8mL^2}$$

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad \text{for } 0 \le x \le L$$

$$\text{1-D Harmonic Oscillator:} \quad E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\text{Potential Barrier:} \quad T \approx 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)e^{-2\alpha L}$$

$$\text{where } \alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$$

3-D Schrödinger Equation in rectangular coordinates:

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2m}{\hbar^2}\left[E - U(x,y,z)\right]\psi(x,y,z) = 0$$

3-D Schrödinger Equation in spherical polar coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2m}{\hbar^2}(E - U)\psi = 0$$

$$\text{Angular Momentum:} \quad \vec{L} = \vec{r} \times \vec{p}$$

$$\text{Selection Rules:} \quad \Delta l = \pm 1; \ \Delta m_l = 0, \pm 1$$

$$\text{Magnetic Dipole Moment:} \quad \mu = -\frac{e}{2m}L$$

$$\text{Zeeman Effect:} \quad \Delta E = m_l\frac{e\hbar}{2m}B$$

$$\text{Density of States (Free Electrons):} \quad g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2}$$

$$\text{Fermi-Dirac Distribution:} \quad f(E) = \frac{1}{\mathrm{e}^{(E-E_F)/kT} + 1}$$

$$\text{Trigonometric identity:} \quad \cos 2\theta = 1 - 2\sin^2\theta$$