NATIONAL UNIVERSITY OF SINGAPORE

PC2232 PHYSICS FOR ELECTRICAL ENGINEERS

(Semester II: AY 2011-12)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **five short** questions in Part I and **three long** questions in Part II. It comprises **twelve** printed pages.
2. Answer **ALL** questions.
3. All answers are to be written on the answer books.
4. This is a **CLOSED BOOK** examination.
5. The total mark for Part I is 40 and that for Part II is 60.
6. A list of constants and formulae can be found on pages 9 – 12 of this question paper. Not all the information will be used in the paper.

PC2232 – PHYSICS FOR ELECTRICAL ENGINEERS

PART I

This part of the examination paper contains **five** short-answer questions from pages 2 – 4. Answer **ALL** questions. The mark for each part is indicated in the square bracket.

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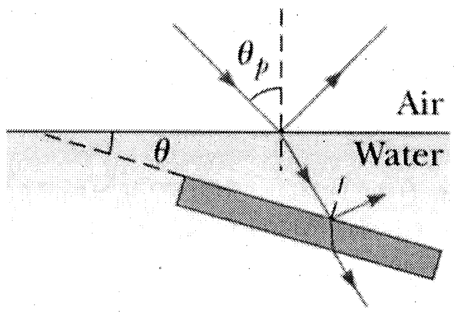
1. A satellite 675 km above the earth's surface transmits sinusoidal electromagnetic waves of frequency 92.4 MHz uniformly in all directions, with a power of 30.0 kW.

   (a) What is the intensity of these waves as they reach a receiver at the surface of the earth directly below the satellite? [2]

   (b) What are the amplitudes of the electric and magnetic fields at the receiver? [3]

   (c) If the receiver has a totally absorbing panel measuring 15.0 cm by 40.0 cm oriented with its plane perpendicular to the direction the waves travel, what average force do these waves exert on the panel? [3]

2. Unpolarized light in air (assume $n_a = 1.00$) strikes the water surface (assume $n_w = 1.33$) at the polarizing angle θ_p. The part of the beam refracted into the water strikes a submerged slab of material with refractive index $n_s = 1.62$ as shown in the figure below. The light reflected from the surface of the slab is completely polarized.



   (a) Copy the figure into the answer booklet. In addition, indicate on each beam, whether it is unpolarized, partially polarized or plane polarized. If it is plane polarized, indicate also its direction of polarization. [2]

   (b) What is the value of angle θ_p and [2]

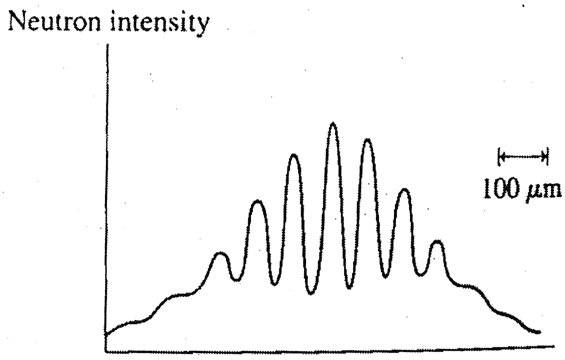
   (c) Calculate the angle θ between the water surface and the surface of the slab. [4]

3. The neutron interference pattern in the figure below was made by shooting neutrons with a speed of 200 m/s through two slits spaced 0.10 mm apart.

   (a) What was the energy, in eV, of the neutrons? [2]

   (b) What was the de Broglie wavelength of the neutrons? [2]

   (c) The pattern was recorded by using a neutron detector to measure the neutron intensity at different positions as indicated in the figure below. Notice the 100 μm scale on the figure. By making appropriate measurements on the figure, determine how far the detector was behind the slits. [4]

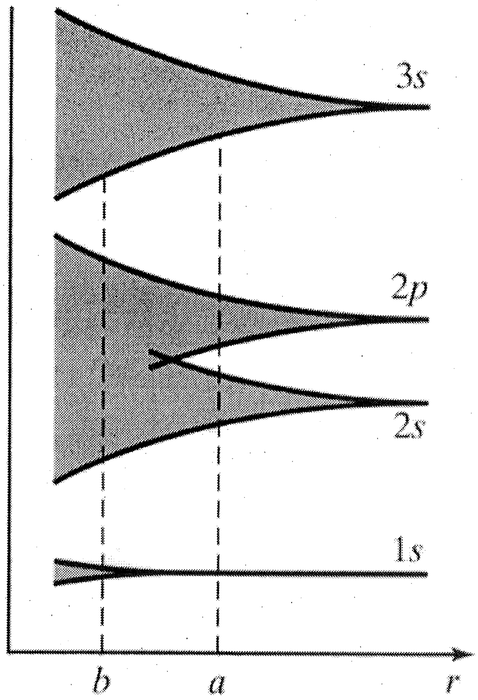


4. A pure, defect-free semiconductor material will absorb the electromagnetic radiation incident on it only if the energy of the individual photons in the incident beam is larger than the band-gap of the semiconductor. The known room-temperature band-gaps for germanium, silicon and gallium arsenide, three widely used semiconductors are 0.66 eV, 1.12 eV and 1.42 eV respectively.

   (a) Determine the room-temperature transparency range of these semiconductors. [5]

   (b) Which of these materials could be used for a light detector for the 1550-nm optical communications wavelength? Explain. [3]

5. Consider a hypothetical element that forms a solid with bands as shown in the figure below.



(a) Suppose the isolated atom has configuration $1s^2 2s^2$. If its equilibrium separation is $r_0 = a$, is the solid a conductor or an insulator at 0 K? What if $r_0 = b$? Explain. [4]

(b) Answer both questions for the case that the atomic configuration is

(i) $1s^2 2s^2 2p^1$ and

(ii) $1s^2 2s^2 2p^6$. [4]

PART II

This part of the examination paper contains **THREE** long-answer questions from page 5 to 8. Answer **ALL** questions. The mark for each part is indicated in the square bracket.

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6. (a) Which of the following pairs of energy and magnitude of orbital angular momentum are possible for a hydrogen atom? For those that are possible, to what n and l values do they correspond? For the rest, explain why they are not possible.

   (i) -0.544 eV, 3.655×10^{-34} J · s

   (ii) -1.51 eV, 3.655×10^{-34} J · s

   (iii) -1.51 eV, 2.110×10^{-34} J · s

   (iv) -3.4 eV, 1.492×10^{-34} J · s [6]

   (b) Consider the wave function of one of the $3p$ states of hydrogen:

$$\psi_{310}(r, \theta, \phi) = \frac{2\sqrt{2}}{27\sqrt{\pi}a_0^{3/2}} e^{-r/3a_0} \left(\frac{r}{a_0} - \frac{r^2}{6a_0^2} \right) \cos\theta$$

   where a_0 is the Bohr radius.

   (i) Write down the radial probability density $P(r) = r^2 |R(r)|^2$ for this state. [2]

   (ii) Identify the values of r that represent the minima in $P(r)$ [3]

   (iii) Find the values of r that represent the maxima in $P(r)$. Hence determine the most probable position of the electron in this state. [4]

   (c) Some very short-lived particles known as delta resonances have spin $\frac{3}{2}$. Find

   (i) the magnitude of their spin angular momentum, and [2]

   (ii) the minimum angle between the spin vector and the z axis. [3]

7. (a) Laser light of wavelength 632.8 nm falls normally on a slit that is 0.0250 mm wide. The transmitted light is viewed on a distant screen where the intensity at the centre of the bright fringe is 8.50 W/m^2.

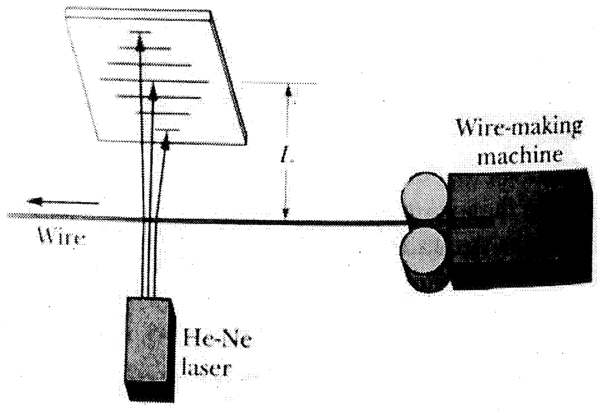
(i) Find the maximum number of totally dark fringes on the screen, assuming the screen to be large enough to show all of them. [4]

(ii) At what angle does the dark fringe that is most distant from the centre occur? [2]

(iii) What is the approximate maximum intensity of the bright fringe that occurs immediately before the dark fringe in part (ii) above? You are to find this value by approximating the angle at which this fringe occurs to be midway between the angles to the dark fringes on either side of it. [6]

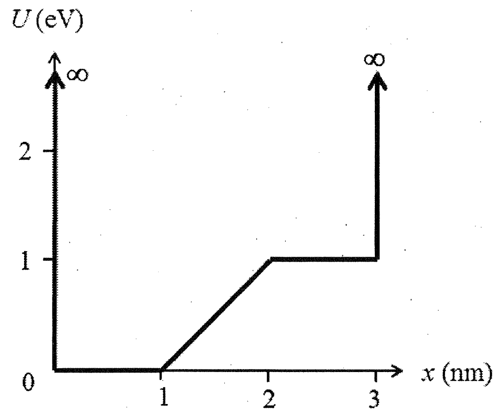
(iv) Sketch the intensity profile for the central 7 bright fringes on the screen along the line perpendicular to the slit. [3]

(b) Wire manufacturers of wire sometimes use a laser to continually monitor the thickness of the product. The wire intercepts the laser beam, producing a diffraction pattern like that of a single slit of the same width as the wire diameter. Suppose light of wavelength 632.8 nm illuminates a wire, and the diffraction pattern appears on the screen at distance $L = 2.60$ m.



If the desired wire diameter is 1.37 mm, what is the observed distance between the two tenth-order minima (one on each side of the central maximum)? [5]

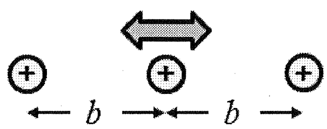
8. (a) The graph below shows the potential energy function $U(x)$ of a particle. Solution of the Schrödinger equation finds that the $n = 3$ level has $E_3 = 0.50$ eV and that the $n = 6$ level has $E_6 = 2.00$ eV.



(i) Redraw this figure in your answer booklet and add to it the energy lines for the $n = 3$ and $n = 6$ states. [1]

(ii) Sketch the $n = 3$ and $n = 6$ wave functions. Show them as oscillating about the appropriate energy lines. The sketches should be reasonably accurate with important features carefully illustrated. [8]

(b) In most metals, the atomic ions form a regular arrangement called a crystal lattice. The conduction electrons in the sea of electrons move through the lattice. The figure below is a one-dimensional model of a crystal lattice. The ions have a mass m, charge e, and an equilibrium separation b.



(i) Suppose the middle charge is displaced a very small distance x (where $x \ll b$) from its equilibrium position while the outer charges remain fixed. Show that the net electric force on the middle charge is given approximately by

$$F = -\frac{e^2}{\pi \epsilon_0 b^3} x$$

In other words, the charge experiences a linear restoring force. [4]

Question continues on the next page.

(ii) Suppose this crystal consists of aluminum ions of mass 4.48×10^{-27} kg with an equilibrium spacing of 0.30 nm. What are the energies of the four lowest states of these ions due to the vibrational motion? [5]

(iii) What is the wavelength of photons emitted during the quantum jump between adjacent energy levels? Is this wavelength in the infrared, visible or ultraviolet portion of the spectrum [2]

– End of Paper –

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TABLE OF INFORMATION

Speed of light in vacuum, c	$=$	2.998×10^{8} m/s
Charge of electron, e	$=$	1.602×10^{-19} C
Planck's constant, h	$=$	6.626×10^{-34} J $\cdot$ s
$\hbar$	$=$	1.055×10^{-34} J $\cdot$ s
Mass of electron, m_e	$=$	9.109×10^{-31} kg
Mass of proton, m_p	$=$	1.673×10^{-27} kg
Mass of neutron, m_n	$=$	1.675×10^{-27} kg
Boltzmann constant, k	$=$	1.381×10^{-23} J/K
Avogadro's number, N_A	$=$	6.022×10^{23} mol^{-1}
Permeability of free space, μ_0	$=$	$4\pi \times 10^{-7}$ Wb/A $\cdot$ m
Permittivity of free space, ϵ_0	$=$	8.854×10^{-12} C^2/N $\cdot$ m^2
Rydberg constant, R_∞	$=$	1.097×10^{7} m^{-1}
Stefan-Boltzmann constant, σ	$=$	5.670×10^{-8} Wm^{-2}K^{-4}
Wien's displacement constant, b	$=$	2.898×10^{-3} m K
Bohr radius, a_0	$=$	5.292×10^{-11} m
Hydrogen ground state energy, E_1	$=$	-13.61 eV

$$
\begin{aligned}
\text{Maxwell Equations:} \quad \oint \vec{E}\cdot d\vec{A} &= \frac{Q_{\text{encl}}}{\epsilon_0} \\
\text{(Integral Form)} \quad \oint \vec{E}\cdot d\vec{l} &= -\frac{d\Phi_B}{dt} \\
\oint \vec{B}\cdot d\vec{A} &= 0 \\
\oint \vec{B}\cdot d\vec{l} &= \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \\
\text{Maxwell Equations:} \quad \nabla\cdot\vec{E} &= \frac{\rho}{\epsilon_0} \\
\text{(Differential Form)} \quad \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\
\nabla\cdot\vec{B} &= 0 \\
\nabla\times\vec{B} &= \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \\
\text{Speed of light in vacuum, } c &= \frac{1}{\sqrt{\epsilon_0\mu_0}} \\
\text{Energy Density of EM wave, } u &= \epsilon_0 E^2 = \frac{1}{\mu_0}B^2 \\
\text{Poynting Vector, } \vec{S} &= \frac{1}{\mu_0}\vec{E}\times\vec{B} \\
\text{Intensity, } I &= S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} \\
\text{Fresnel Equations:} \quad \frac{E_{r,\perp}}{E_{a,\perp}} &= -\frac{\sin(\theta_a-\theta_b)}{\sin(\theta_a+\theta_b)} \\
\frac{E_{b,\perp}}{E_{a,\perp}} &= \frac{2\sin\theta_b\cos\theta_a}{\sin(\theta_a+\theta_b)} \\
\frac{E_{r,\parallel}}{E_{a,\parallel}} &= \frac{\tan(\theta_a-\theta_b)}{\tan(\theta_a+\theta_b)} \\
\frac{E_{b,\parallel}}{E_{a,\parallel}} &= \frac{2\sin\theta_b\cos\theta_a}{\sin(\theta_a+\theta_b)\cos(\theta_a-\theta_b)} \\
\text{Malus's Law: } I &= I_{\max}\cos^2\phi \\
\text{Brewster's Law: } \tan\theta_{\text{p}} &= \frac{n_b}{n_a} \\
\text{Reflection at normal incidence: } E_r &= \frac{n_a-n_b}{n_a+n_b}E_i
\end{aligned}
$$

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\begin{aligned}
\text{Single Slit: } I &= I_0 \left[\frac{\sin\left[\pi a(\sin\theta)/\lambda\right]}{\pi a(\sin\theta)/\lambda} \right]^2 \\
\text{Two Slits with finite width: } I &= I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 \\
\text{with } \quad \phi &= \frac{2\pi d}{\lambda} \sin\theta \\
\text{and } \quad \beta &= \frac{2\pi a}{\lambda} \sin\theta \\
\text{Diffraction Grating: } d\sin\theta &= m\lambda \\
\text{X-ray Diffraction: } 2d\sin\theta &= m\lambda \\
\text{Circular Aperture (first dark ring): } \sin\theta_1 &= 1.22\frac{\lambda}{D} \\
\text{Stefan's Law: } P &= \sigma A e T^4 \\
\text{Wien's Law: } \lambda_{\max} T &= b \\
\text{Rayleigh-Jeans Formula: } I(\lambda) &= \frac{2\pi ckT}{\lambda^4} \\
\text{Planck radiation law: } I(\lambda) &= \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)} \\
\text{Photoelectric Effect: } eV_0 &= hf - \phi \\
\text{Relativistic Total Energy: } E^2 &= m^2c^4 + p^2c^2 \\
\text{Compton's Equation: } \lambda' - \lambda &= \frac{h}{mc}(1 - \cos\theta) \\
\text{Rydberg Formula: } \frac{1}{\lambda} &= R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \\
\text{Bohr Hydrogen Atom: } E_n &= -\frac{me^4}{8\epsilon_0^2 n^2 h^2} \\
\text{and } \quad r_n &= \frac{n^2 \epsilon_0 h^2}{\pi m e^2} \\
\text{de Broglie Wavelength: } \lambda &= \frac{h}{mv} \\
\text{Heisenberg's Uncertainty Principles: } \Delta p_x \Delta x &\geq \hbar; \ \Delta E \Delta t \ \geq \ \hbar
\end{aligned}
$$

$$\text{1-D Schrödinger equation:} \quad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

$$\text{1-D Infinite Square Well: } E_n = \frac{n^2h^2}{8mL^2}$$

$$\text{1-D Harmonic Oscillator: } E_n = \left(n+\frac{1}{2}\right)\hbar\omega$$

$$\text{Potential Barrier: } T \approx 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2\alpha L}$$

$$\text{where } \alpha = \sqrt{\frac{2m(U_0-E)}{\hbar^2}}$$

3-D Schrödinger Equation in rectangular coordinates:

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2m}{\hbar^2}\left[E-U(x,y,z)\right]\psi(x,y,z) = 0$$

3-D Schrödinger Equation in spherical polar coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

$$\text{Angular Momentum: } \vec{L} = \vec{r}\times\vec{p}$$

$$\text{Selection Rules: } \Delta l = \pm 1;\ \Delta m_l = 0, \pm 1$$

$$\text{Magnetic Dipole Moment: } \mu = -\frac{e}{2m}L$$

$$\text{Zeeman Effect: } \Delta E = m_l\frac{e\hbar}{2m}B$$

$$\text{Density of States (Free Electrons): } g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2}$$

$$\text{Fermi-Dirac Distribution: } f(E) = \frac{1}{\mathrm{e}^{(E-E_F)/kT}+1}$$