PC2232: Tutorial Homework Assignment 1 Solution

Question 1:

Given $a = 6.00 \times 10^{-2}$ m, $f = 7.50 \times 10^9$ Hz, therefore $\lambda = \frac{c}{f} = 0.04$.

(a) Single slit:

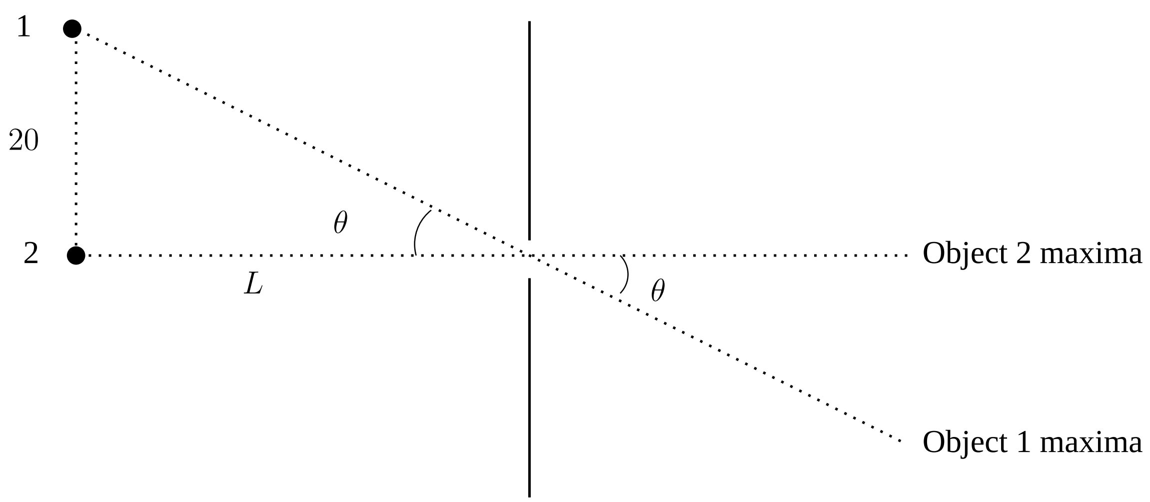
$$\begin{aligned} a\sin\theta &= m\lambda \\ \theta &= \arcsin\frac{m\lambda}{a} = 41.8^\circ. \end{aligned} \tag{1}$$

(b)

$$\begin{aligned} I &= I_{\max}\left(\frac{\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\frac{\pi a\sin\theta}{\lambda}}\right)^2 \\ \frac{I}{I_{\max}} &= \left(\frac{\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\frac{\pi a\sin\theta}{\lambda}}\right)^2 = 0.59. \end{aligned} \tag{2}$$

(c) Resolution criterion $\rightarrow \theta =$ angle at first minima $= 41.8^\circ$. There are two possibilities:

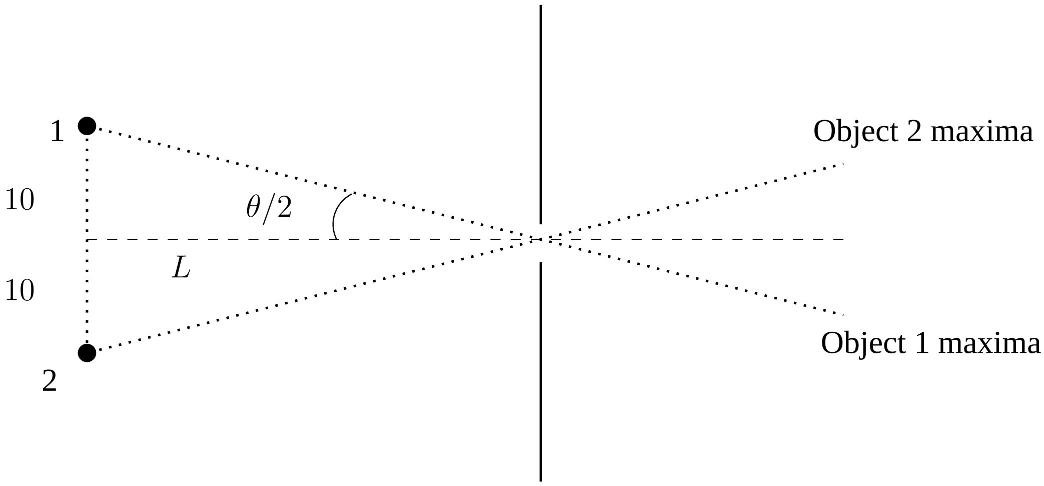
- **First possibility**



In the first possibility shown in the figure above,

$$\tan\theta = \frac{20\text{ cm}}{L}, \quad \rightarrow \quad L = \frac{20\text{ cm}}{\tan 41.8^\circ} = 22.4\text{ cm}. \tag{3}$$

- **Second possibility**



In the second possibility shown in this figure,

$$\tan\frac{\theta}{2} = \frac{10\text{ cm}}{L}, \quad \rightarrow \quad L = \frac{10\text{ cm}}{\tan\frac{41.8}{2}} = 26.2\text{ cm}. \tag{4}$$

Question 2:

(a) Expressing Planck's formula

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)} \tag{5}$$

in terms of frequency. The formula becomes

$$I(f) = \frac{2\pi hf^5}{c^3\left(e^{hf/kT} - 1\right)} \tag{6}$$

(b) In changing variables, the inegration element also needs to be changed:

$$\lambda = \frac{c}{f}, \quad d\lambda = -\frac{c}{f^2}df. \tag{7}$$

Substituting into the integration,

$$\begin{aligned}\int_0^\infty I(\lambda)\,d\lambda &= \int_\infty^0 \frac{2\pi hf^5}{c^3\left(e^{\frac{hf}{kT}} - 1\right)}\left(-\frac{c}{f^2}\right)df = \frac{2\pi h}{c^2}\int_0^\infty \frac{f^3\,df}{e^{\frac{hf}{kT}} - 1} \\ &= \frac{2\pi h}{c^2}\frac{1}{240}\left(2\pi\frac{kT}{h}\right)^4 = \frac{2\pi^5k^4T^4}{15c^2h^3}.\end{aligned} \tag{8}$$

(c) Total radiated intensity is the integral over all λ,

$$I = \underbrace{\frac{2\pi^5 k^4}{15c^2 h^3}}_{=\sigma} T^4. \tag{9}$$

Calculating σ in terms of the constants,

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3} = 5.67 \times 10^{-8}\ \mathrm{W\,m^2\,K^{-4}}. \tag{10}$$

Question 3:

Absorption $\rightarrow$ atom excited from lower to higher energy level from the state with quantum number n_1 to another state of number n_2, where $n_2 > n_1$. Hence the energy required to effect this transition is

$$E\,(n_1 \rightarrow n_2) = \frac{A}{n_1^2} - \frac{A}{n_2^2}. \tag{11}$$

The energy of the photon is given by

$$E = \frac{hc}{\lambda} \tag{12}$$

where h is Planck's constant and c is the speed of light. So the wavelength of the absorption line is, by comparing the energies in Eq. (11) and (12), is

$$\lambda\,(n_1 \rightarrow n_2) = \frac{\frac{hc}{\lambda}}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}. \tag{13}$$

Adjacent absorption lines must correspond to transitions

$$n_1 \rightarrow n_2, \quad \text{and} \quad n_1 \rightarrow n_2 + 1. \tag{14}$$

The transition from $n_1 \rightarrow n_2 + 1$ requires a larger energy and hence the wavelength will be smaller. The wavelengths are

$$102.8\ \mathrm{nm}\lambda\,(n_1 \rightarrow n_2) = \frac{A}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}, \tag{15}$$

$$97.5\ \mathrm{nm}\lambda\,(n_1 \rightarrow n_2 + 1) = \frac{A}{\left(\frac{1}{n_1^2} - \frac{1}{(n_2+1)^2}\right)}. \tag{16}$$

Since we do not know A, we must first identify n_1 and n_2. Divide Eqs. (15) and (16) to eliminate the dependence on A.

$$\frac{\lambda\left(n_1 \rightarrow n_2\right)}{\lambda\left(n_1 \rightarrow n_2+1\right)}=\frac{\left(\frac{1}{n_1^2}-\frac{1}{(n_2+1)^2}\right)}{\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}=\frac{102.8}{97.5}=1.054. \tag{17}$$

To identify the values of n_1 and n_2, we try the following combinations to check the quantity

$$G=\frac{\left(\frac{1}{n_1^2}-\frac{1}{(n_2+1)^2}\right)}{\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)} \tag{18}$$

to see whether $G=1.054$:

n_2	n_1	G
2	1	1.185185
3	1	1.054688
4	1	1.024
5	1	1.012731
6	1	1.00758

From the table, it is clear that $n_1 = 1$ and $n_2 = 3$ will give the observed ratio of the wavelengths. We can now substitute these values into our expression for $\lambda\left(n_1 \rightarrow n_2\right)$ to obtain the value of A.

$$A=\frac{\frac{hc}{\lambda}}{\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}=\frac{\frac{6.63\times10^{-34}\times3\times10^{8}}{102.8\times10^{-9}}}{\frac{1}{1^2}-\frac{1}{3^2}}=\quad 2.174\times10^{-18}\ \mathrm{J}=13.6\ \mathrm{eV}. \tag{19}$$

Hence the atom is hydrogen.