PC2232 Physics for Electrical Engineers: Tutorial 3

Question 1: Tsunami!

On December 26, 2004, a violent magnitude 9.1 earthquake occurred off the coast of Sumatra. This quake triggered a huge tsunami (similar to a tidal wave) that killed more than 150,000 people. Scientists observing the wave on the open ocean measured the time between crests to be 1.0 hour and the speed of the wave to be 800 km/h. Computer models of the evolution of this enormous wave showed that it bent around the continents and spread to all the oceans of the earth. When the wave reached the gaps between continents, it diffracted between them as through a slit.

(a) What is the wavelength of this tsunami?

(b) The distance between the southern top of African and northern Antartica is about 4500 km, while the distance between the southern end of Australia and Antartica is about 3700 km. As an approximation, we can model this wave's behaviour by using Fraunhofer diffraction. Find the smallest angle away from the central maximum for which the waves would cancel after going through each of these gaps.

Question 2: Multiple slits

Laser light of wavelength 500.0 nm illuminates two identical slits, producing an interference pattern on a screen 90.0 cm from the slits. The bright bands are 1.00 cm apart, and the third bright bands on either side of the central maximum are missing in the pattern. Find the width and the separation of the two slits.

Question 3: Diffraction Grating

The wavelength range of the visible spectrum is approximately 400-700 nm. White light falls at normal incidence on a diffraction grating that has 350 slits/mm. Find the angular width of the visible spectrum in

(a) The first order and

(b) The third order.

Question 4: Circular Aperture

When Mars is nearest the Earth, the distance separating the two planets is 88.6×10^6 km. Mars is viewed through a telescope whose mirror has a diameter of 30.0 cm.

(a) If the wavelength of the light is 590 nm, what is the angular resolution of the telescope?

(b) What is the smallest distance that can be resolved between two points on Mars?

Question 5: Blackbody Radiation—Sirius B

The brightest star in the sky is Sirius, the Dog Star. It is actually a binary system of two stars, the smaller one, (Sirius B) being a white dwarf. Spectral analysis of Sirius B indicates that its surface temperature is $24,000$ K and that it radiates energy at a total rate of 1.0×10^{25} W. Assume that it behaves like an ideal blackbody.

(a) What is the total radiated intensity of Sirius B?

(b) What is the peak intensity wavelength? Is this wavelength visible to humans?

(c) What is the radius of Sirius B? Express your answer in kilometers and as a fraction of our sun's radius.

(d) Which star radiates more *total* energy per second, the hot Sirius B or the (relatively) cool sun with a surface temperature of 5800 K? To find out, calculate the ratio of the total power radiated by our sun to the power radiated by Sirius B.

Question 6: Planck's Theory (Optional)

(a) Write the Planck distribution law in terms of the frequency f rather than wavelength λ, to obtain $I(f)$

(b) Show that

$$\int_0^\infty I(\lambda)\,\mathrm{d}\lambda = \frac{2\pi^5 k^4}{15c^2h^3}T^4, \tag{1}$$

where $I(\lambda)$ is the Planck distribution formula given by Planck's radiation law:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(\mathrm{e}^{\frac{hc}{\lambda kT}} - 1\right)}. \tag{2}$$

(*Hint*: Change the integration variable from λ to f.)

You will need to use the following tabulated integral:

$$\int_0^\infty \frac{x^3\,\mathrm{d}x}{\mathrm{e}^{\alpha x} - 1} = \frac{1}{240}\left(\frac{2\pi}{\alpha}\right)^4. \tag{3}$$

(c) The result of (b) is I and has the form of the Stefan-Boltzmann law, $I = \sigma T^4$. Evaluate the constants in (b) to show that σ has the value $\sigma = 5.670400(40) \times 10^{-8}$ $\mathrm{Wm^2K^{-4}}$

Question 7: Single Slit Diffraction (Optional)

Suppose that the single slit shown in the figure is 6.00 cm wide and in front of a microwave source operating at 7.50 GHz.

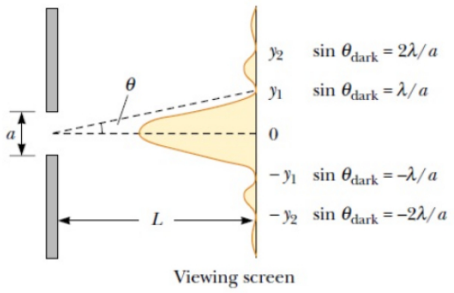

Figure 1:

(a) Calculate the angle subtended by the first minimum in the diffraction pattern.

(b) What is the relative intensity $I/I_{\max}$ at $\theta = 15.0°$?

(c) Assume that two such sources, separated laterally by 20.0 cm are behind the slit. What must be the maximum distance between the plane of the sources and the slit be if the diffraction patterns are to be resolved? (In this case, the approximation $\sin\theta \simeq \tan\theta$ is not valid because of the relatively small value of $\frac{a}{\lambda}$.)