PC2232 Physics for Electrical Engineers: Tutorial 10

Question 1:

Using the functions given in the lectures (up to $n = 3$), verify that for states of the hydrogen atom where $l = n - 1$, the radial probability is of the form

$$P(r) \propto r^{2n} \mathrm{e}^{-2r/na_0}. \tag{1}$$

Show that the most probable radius is given by

$$r = n^2 a_0. \tag{2}$$

Question 2:

Consider an electron in the ground state of a hydrogen atom,

$$\psi_{100}(r, \theta, \phi) = \frac{1}{\sqrt{\pi a_0^3}} \mathrm{e}^{-r/a_0}. \tag{3}$$

(a) Sketch the plots of E and $U(r)$ on the same axes.

(b) Show that, classically, an electron with this energy should not be able to get further than $2a_0$ from the proton.

(c) What is the probability of the electron being found in the classically forbidden region?

Hint: You might find the following integral useful

$$\int \mathrm{d}x\, x^2 \mathrm{e}^{ax} = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) \mathrm{e}^{ax}. \tag{4}$$

Question 3:

A hydrogen atom undergoes a transition from a $4f$ state to the $3d$ ground state. In the absence of a magnetic field, the energy of the photon emitted is 1.88 μm. The atom is then placed in a strong magnetic field in the z-direction. Ignore spin effects and consider only the interaction of the magnetic field with the atom's orbital magnetic moment.

(a) How many different photon wavelengths are observed for the $4f \to 3d$ transition? List the m_l values for the initial and final states for the transition that leads to each photon wavelength?

(b) If the magnetic field is 0.40 T, by how much will these photon wavelengths differ?

Question 4:

(a) If the value of L_z is known, we cannot know either L_x or L_y precisely. But we can know the value of the quantity $\sqrt{L_x^2 + L_y^2}$. Write an expression for this quantity in terms of l, m_l and $\hbar$.

(b) What is the meaning of $\sqrt{L_x^2 + L_y^2}$?

(c) For a state of non-zero orbital angular momentum, find the maximum and minimum values of $\sqrt{L_x^2 + L_y^2}$. Explain your results.

Question 5:

A simplified approach to the question of how l is related to angular momentum can be stated as follows: If L_z can take on only those values $m_l\hbar$, where $m_l = 0, \pm 1, \ldots, \pm l$, then its square is allowed only values $m_l^2\hbar^2$, and the average of L_z^2 should be the sum of its allowed values divided by the number of values $2l + 1$. Because there is no preferred direction in space, the averages of L_x^2 and L_y^2 should be the same, and the sum of all three should give the average of L^2. Given the sum

$$\sum_{n=1}^{N} n^2 = \frac{1}{6} N (N+1)(2N+1), \tag{5}$$

show that by these arguments, the average of L^2 should be $l(l+1)\hbar^2$.

Question 6: (Optional)

The $\psi_{2,1,0}$ state (the $2p$ state in which $m_l = 0$) has most of its probability density along the z-axis, and so is often referred to as the $2p_z$ state. To allow its probability density to stick out in other ways, and thus facilitate various kinds of molecular bonding with other atoms, an atomic electron may assume a wave function that is an algebraic combination of multiple wave functions open to it. One such "hybrid state" is the sum $\psi_{2,1,+1} + \psi_{2,1,-1}$. (*Note*: because the Schrödinger equation is a linear differential equation, a sum of solutions with the same energy is also a solution with that energy. Also, normalization constants may be ignored in the following questions.)

(a) Write this wave function and its probability density in terms of r, θ and ϕ. (Use the Euler formula to simplify your results.)

(b) In which of the following ways does this state differ from its parts (i.e., $\psi_{2,1,+1}$ and $\psi_{2,1,-1}$) and from the $2p_z$ state: Energy? Radial dependence of its probability density? Angular dependence of its probability density?

(c) The state is often referred to as the $2p_y$. Why?

(d) How might we produce a $2p_x$ state?