NATIONAL UNIVERSITY OF SINGAPORE

PC2232 Physics for Electrical Engineers

(Semester II: AY 2013-14)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. This assessment paper contains 5 **short** questions in Part I and 2 **long** questions in Part II. It comprises **11** printed pages.
2. Students are required to answer **ALL** questions.
3. All answers are to be written on the answer books.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.
6. The total mark for Part I is 40 and that for Part II is 60.
7. A list of constants and formulae can be found on pages 8-11 of this question paper. Not all the information will be used in the paper.

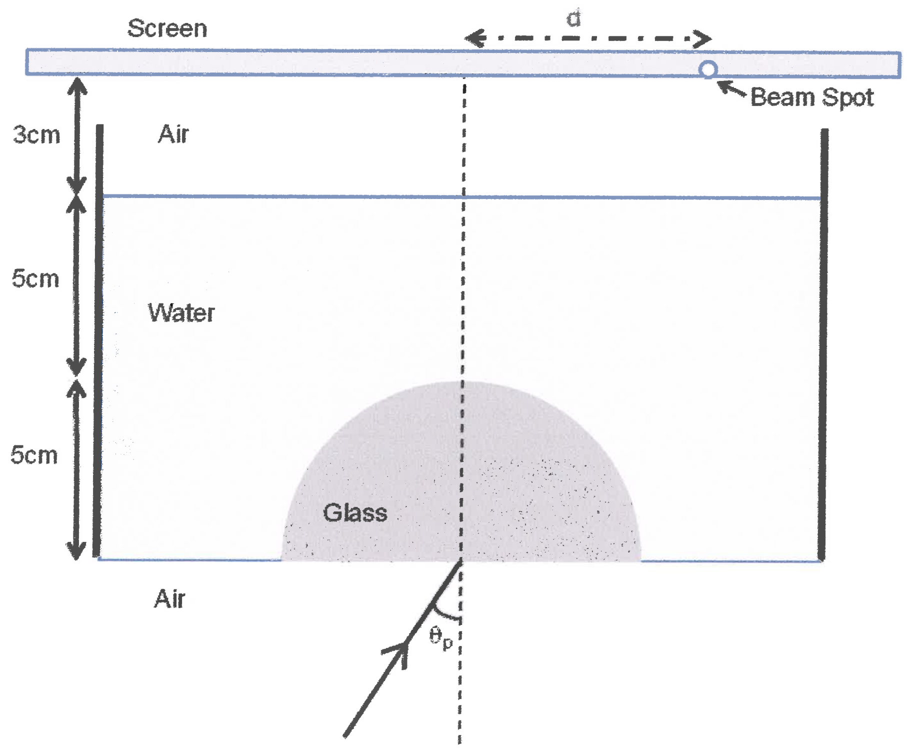
PC2232 – PHYSICS FOR ELECTRICAL ENGINEERS

PART I

This part of the examination paper contains five short-answer questions from pages 2 –4. Answer ALL questions.

---

1. A beam of unpolarized light in air strikes the bottom of a beaker with water and a piece of glass at the polarizing angle θ_p as shown. The shape of the glass is a semicircle and the beam of unpolarized light incident on the glass-air interface at the center of the circle as shown. (assume $n_{air}=1$, n_{water} = 1.33 and n_{glass} = 1.5 and ignore the thickness of the beaker.)



(a) Copy the figure into the answer booklet and add in the reflected beam from the air-glass interface, and refracted beam as the beam travels through the glass, water, eventually out into the air and strike the screen as shown.

(b) What is the value of angle θ_p and what is the unique property of the reflected beam?

(c) The beam eventually hits a screen at a distance d from the center as shown. Calculate the distance d.

2 (a) The Sun is radiating energy at a total power of 3.77 x 10^{26} W. Given that the radius of the Sun is 6.96x10^{8} m and assuming its emissivity to be 0.965. Determine the surface temperature of the Sun.

(b) Given that the intensity of the solar radiation reaching Mars is 5.77 x 10^{2} W/m^{2}, determine the distance of Mars from the Sun assuming that orbit of Mars is a uniform circle with Sun at the center.

(c) What would be the equilibrium temperature of Mars so that at steady state, it radiates as much energy as it absorbs? Note the radius of Mars is 3,390,000 m, and assume its emissivity to be 0.9 and it has no atmosphere.

3 (a)The energy, magnitude of orbital angular momentum and magnitude of component of the angular momentum in z-direction for a hydrogen atom is given by

$$E = -1.51 eV$$
$$L = 2.583 \times 10^{-34} J.s$$
$$L_z = -1.055 \times 10^{-34} J.s$$

Identify the principal quantum number n, the orbital quantum number l and magnetic quantum number m_l.

(b) Write down the corresponding wavefunction.

(c) Write down the radial probability density $P(r) = r^2|R(r)|^2$ for this state and find the value of r that represents the maximum in $P(r)$.

4 (a) Show by direct substitution in the Schrodinger equation for the one-dimensional harmonic oscillator that the wavefunction

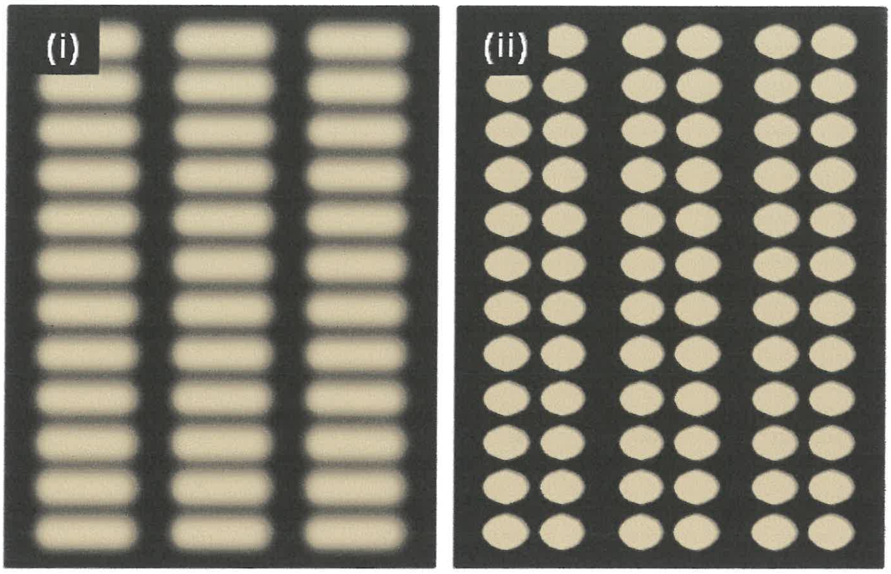
$$\psi(x) = \left(\frac{\beta}{8\sqrt{\pi}}\right)^{1/2}\left(4\beta^2x^2 - 2\right)e^{-\frac{1}{2}\beta^2x^2} \quad \text{where } \beta = \left(\frac{mk'}{\hbar^2}\right)^{1/4}$$

is a solution with the energy corresponding to $E = \frac{5}{2}\hbar\omega$.

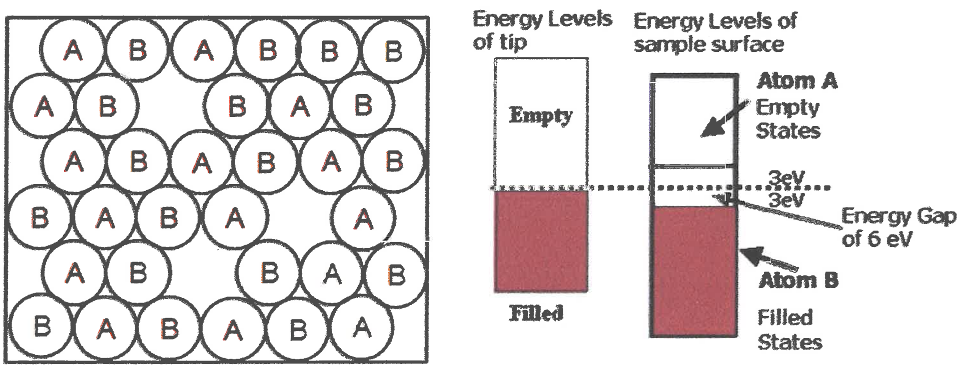
(b) Sketch this wavefunction and identify the quantum number for this state.

(c) Identify the minima of the probability density for this state.

5. (a) The following images show STM images obtained from the surface of Si(100) (2x1). Explain why the image (i) looks different from image (ii) despite the fact that both images were obtained from identical region on the sample.



(b) The following figure shows the arrangement of atoms A and B on the surface of a sample as well as the electron energy levels of the STM tip and sample with no applied voltage bias. If you are interested in determining the ratio of the number of atoms A to the number of atoms B on this surface, what are the steps you will carry out to obtain this value? Note that the surface is not perfect and voids are present.



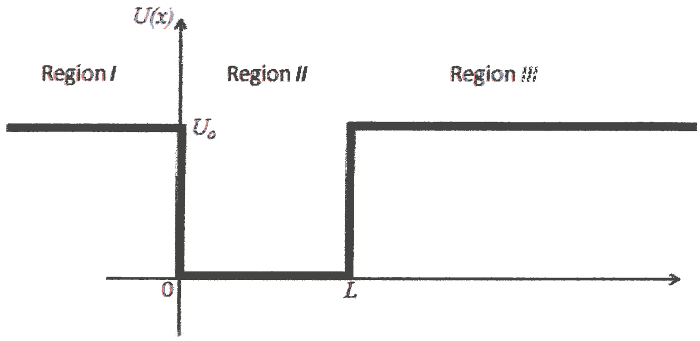
PC2232 – Physics for Electrical Engineers

PART II

This part of the examination paper contains TWO long-answer questions from page 5 to 7. Answer ALL questions.

---

6. (a) The following figure shows the potential energy profile of a 1D finite potential well. Starting with the Schrodinger Equation, derive the expressions for the wavefunction for region I, II and III as identified in the figure. Note that you do not need to normalize the wavefunction.



(b) Using the results you have obtained in (a) and make use of appropriate boundary conditions, show that the energy E of the particle in such a potential and is given by

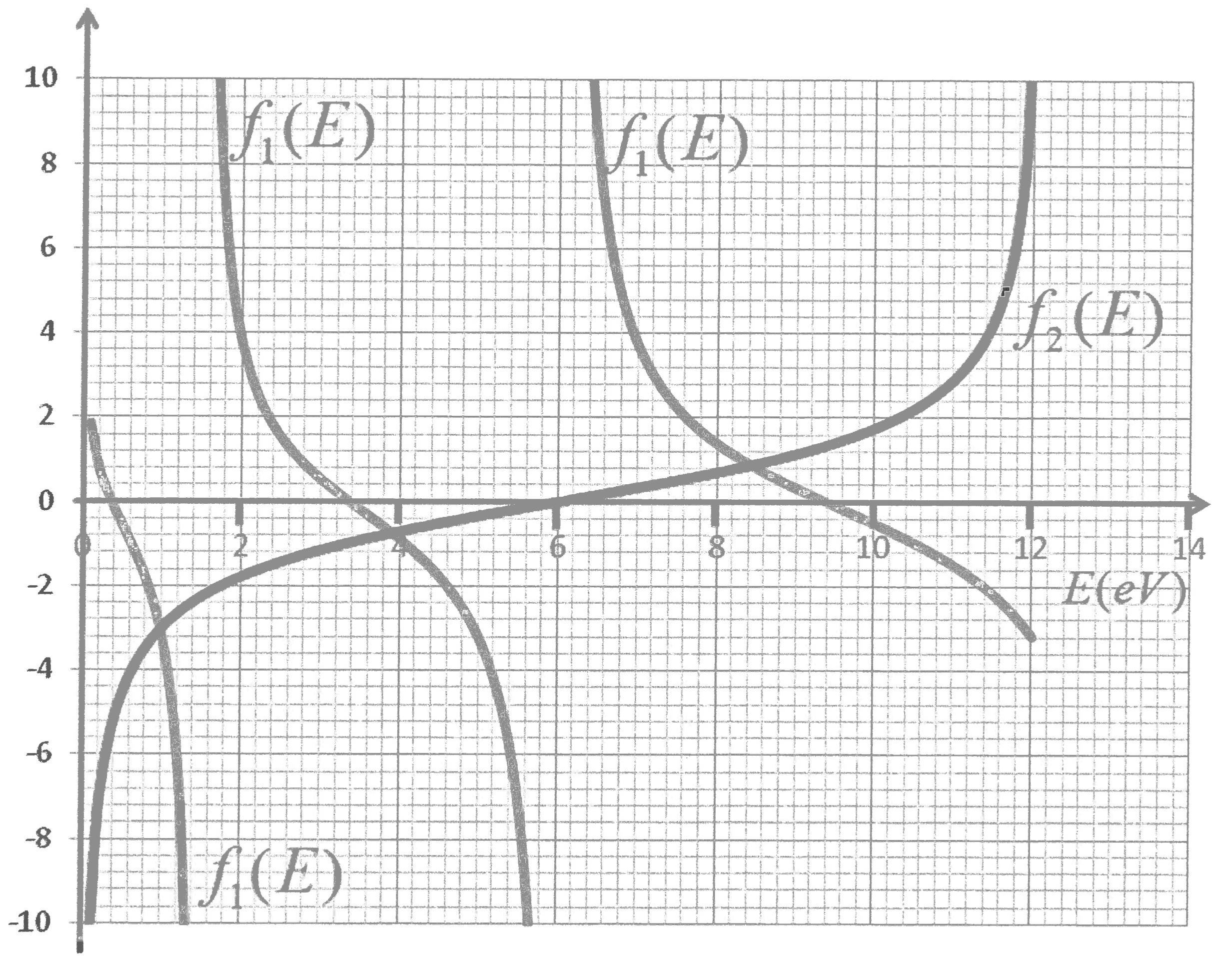
$$2\cot\left(\sqrt{\frac{2mE}{\hbar^2}}L\right) = \sqrt{\frac{E}{U_o - E}} - \sqrt{\frac{U_o - E}{E}}$$

(c) Discuss what happen to the energy of the particle in this case if $U_o \rightarrow$ infinity.

(d) Consider an electron trapped in a finite potential well with a width of L=0.5 nm, and $U_o = \frac{\hbar^2}{mL^2}$. The following plot shows graphs of the two functions

$$f_1(E) = 2\cot\left(\sqrt{\frac{2mE}{\hbar^2}}L\right) \quad \text{and} \quad f_2(E) = \sqrt{\frac{E}{U_o - E}} - \sqrt{\frac{U_o - E}{E}}$$

as a function of the energy E.



From the graph, identify the number of bound state in this case, sketch the wavefunction and determine the corresponding energy of the bound states from the graph.

(e) If the electron makes a transition from the first excited state to the ground state by emitting a photon, find the wavelength of the photon.

7(a) What is the meaning of Density of State?

(b) In the case of Free-Electron Models of Metals, show that the density of state $g(E)$ is given by

$$g(E) = \frac{(2m)^{3/2} L^3}{2\pi^2 \hbar^3} E^{1/2}$$

(c) The Fermi-Dirac Distribution $f(E)$ is given by

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

Draw sketches of the Fermi Dirac Distribution (i.e. $f(E)$ versus E) for the case of $kT = 0$ and $kT = E_F/10$. Explain the significance of the difference in the sketches you have drawn.

(d) Show that when $T = 0$ K, the Fermi Energy of the 3D free electron system, E_{F0}, is related to density of free electron, n_e by the following equation

$$E_{F0} = \frac{3^{2/3} \pi^{4/3} \hbar^2}{2m} n_e^{2/3}$$

(e) The Fermi energy of aluminum (Al) is 11.63 eV. (i) Assuming that the free electron model applies to aluminum, calculate the number of free electrons per unit volume at low temperatures. (ii) From the results you have obtained from (i), determine how many free electrons are contributed by each Al atom.

(f) Derive an expression for the average free-electron energy in a metal for the case when $T = 0$ K. Express your answer in terms of E_{F0}.

END of Paper (SCH)

TABLE OF INFORMATION

Speed of light in vacuum, $c = 2.998 \times 10^{8}$ m/s

Charge of electron, $e = 1.602 \times 10^{-19}$ C

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Mass of electron, $m_e = 9.109 \times 10^{-31}$ kg

Mass of proton, $m_p = 1.673 \times 10^{-27}$ kg

Boltzmann constant, $k = 1.381 \times 10^{-23}$ J/K

Avogadro's number, $N_A = 6.022 \times 10^{23}$ mol^{-1}

Permeability of free space, $\mu_o = 4\pi \times 10^{-7}$ Wb/Am

Permittivity of free space, $\varepsilon_o = 8.854 \times 10^{-12}$ C^2/Nm^2

Rydberg constant, $R = 1.097 \times 10^{7}$ m^{-1}

Stefan-Boltzmann constant, $\sigma = 5.670 \times 10^{-8}$ Wm^{-2}K^{-4}

Wien's displacement constant, $b = 2.898 \times 10^{-3}$ mK

Bohr radius, $a_o = 5.29 \times 10^{-11}$ m

Density of Al = 2.70 g/cm^3

Molar mass of Al = 26.98 g/mol

List of Formulae

Speed of light in vacuum $c = \dfrac{1}{\sqrt{\varepsilon_o \mu_o}}$

Speed of light in matter $v = \dfrac{1}{\sqrt{\varepsilon\mu}} = \dfrac{1}{\sqrt{KK_m}} \dfrac{1}{\sqrt{\varepsilon_o \mu_o}} = \dfrac{c}{\sqrt{KK_m}}$

Index of refraction $n = \dfrac{c}{v}$

Energy Density of EM Wave $u = \varepsilon_o E^2 = \dfrac{1}{\mu_o} B^2$

Poynting Vector $\vec{S} = \dfrac{1}{\mu_o} \vec{E} \times \vec{B}$

Intensity $I = S_{av} = \dfrac{E_{\max} B_{\max}}{2\mu_o} = \dfrac{E_{\max}^2}{2\mu_o c}$

Laws of Refraction $n_a \sin(\theta_a) = n_b \sin(\theta_b)$

Brewster's Law $\tan(\theta_p) = \dfrac{n_b}{n_a}$

Fresnel's Equations

$$\frac{E_{r,\perp}}{E_{a,\perp}} = -\frac{\sin(\theta_a - \theta_b)}{\sin(\theta_a + \theta_b)}$$

$$\frac{E_{b,\perp}}{E_{a,\perp}} = \frac{2\sin(\theta_b)\cos(\theta_a)}{\sin(\theta_a + \theta_b)}$$

$$\frac{E_{r,\parallel}}{E_{a,\parallel}} = \frac{\tan(\theta_a - \theta_b)}{\tan(\theta_a + \theta_b)}$$

$$\frac{E_{b,\parallel}}{E_{a,\parallel}} = \frac{2\sin(\theta_b)\cos(\theta_a)}{[\sin(\theta_a + \theta_b)\cos(\theta_a - \theta_b)]}$$

For normal incidence $E_r = \dfrac{n_a - n_b}{n_a + n_b} E_i$

Stefan's Law $P = \sigma A e T^4$

Rayleigh-Jeans Formula $I(\lambda) = \dfrac{2\pi c k T}{\lambda^4}$

Planck's Radiation Law $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$

Photoelectric Effect $eV_o = hf - \phi$

Compton Formula $\lambda' - \lambda = \dfrac{h}{mc}(1 - \cos\theta)$

Rydberg Formula $\dfrac{1}{\lambda} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$ Energy of Hydrogen Atom $E_n = -\dfrac{hcR}{n^2}$

Bohr Hydrogen Atom Energy $E_n = -\dfrac{me^4}{8\varepsilon_o^2 n^2 h^2}$ Radius $r = \varepsilon_o \dfrac{n^2 h^2}{\pi me^2}$

Bohr Hydrogen-like System $E_n = -\dfrac{mZ^2 e^4}{8\varepsilon_o^2 n^2 h^2}$ $r = \varepsilon_o \dfrac{n^2 h^2}{\pi mZe^2}$

$E_n = -\dfrac{RhcZ^2}{n^2}$ $E_n = -\dfrac{2.18 \times 10^{-18} Z^2}{n^2}$ *Joule* $E_n = -\dfrac{13.607 Z^2}{n^2}$ *eV*

De Broglie's Wavelength $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$

Heisenberg Uncertainty Principles $\Delta x \Delta p \geq \dfrac{h}{2\pi}$ $\Delta E \Delta t \geq \dfrac{h}{2\pi}$

Schrodinger Equation (1D)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

For Harmonic Oscillator $U(x) = \dfrac{1}{2}k'x^2$

Hydrogen Atom

Spherical Harmonics

$$Y_{0,0}(\theta,\phi)=\frac{1}{\sqrt{4\pi}}$$

$$Y_{1,-1}(\theta,\phi)=\sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi}, \qquad Y_{1,0}(\theta,\phi)=-\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{1,1}(\theta,\phi)=-\sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi}$$

$$Y_{2,2}(\theta,\phi)=\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\phi}, \quad Y_{2,1}(\theta,\phi)=-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\phi},$$

$$Y_{2,0}(\theta,\phi)=-\sqrt{\frac{5}{16\pi}}\left(3\cos_2\theta-1\right), \quad Y_{2,-1}(\theta,\phi)=\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{-i\phi},$$

$$Y_{2,-2}(\theta,\phi)=\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{-2i\phi}$$

Radial Solutions

$$R_{1,0}(r)=\frac{1}{a_o^{3/2}}2e^{-r/a_o}$$

$$R_{2,0}(r)=\frac{1}{(2a_o)^{3/2}}2\left(1-\frac{r}{2a_o}\right)e^{-r/2a_o}$$

$$R_{2,1}(r)=\frac{1}{(2a_o)^{3/2}}\frac{r}{\sqrt{3}a_o}e^{-r/2a_o}$$

$$R_{3,0}(r)=\frac{1}{(3a_o)^{3/2}}\left(2-\frac{4r}{3a_o}+\frac{4r^2}{27a_o^2}\right)e^{-r/3a_o}$$

$$R_{3,1}(r)=\frac{1}{(3a_o)^{3/2}}\frac{4\sqrt{2}r}{9a_o}\left(1-\frac{r}{6a_o}\right)e^{-r/3a_o}$$

$$R_{3,2}(r)=\frac{1}{(3a_o)^{3/2}}\frac{2\sqrt{2}r^2}{27\sqrt{5}a_o^2}e^{-r/3a_o}$$

1D Infinite Potential Well

$$\psi(x) = \begin{cases} 0 & : \quad x < 0 \\ A \sin \dfrac{n\pi x}{L} & : \quad 0 \le x \le L \\ 0 & : \quad x > L \end{cases} \qquad A = \sqrt{\frac{2}{L}}$$

$$E_n = \frac{n^2 h^2}{8mL^2}$$

3D Infinite Potential Well

$$E_{n_x, n_y, n_z} = \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right) \frac{\pi^2 \hbar^2}{2m}$$

$$\psi_{n_x, n_y, n_z}(x, y, z) = A \sin \frac{n_x \pi}{L_x} \sin \frac{n_y \pi y}{L_y} \sin \frac{n_z \pi z}{L_z}$$

Hydrogen Atom

$$\psi_{nlm_l}(r, \theta, \phi) = R_{nl}(r) Y_{lm_l}(\theta, \phi)$$