PC2232: Tutorial Homework Assignment 1

Due date: Friday, 21 February 2014

Question 1:

Suppose that the single slit shown in the figure is 6.00 cm wide and in front of a microwave source operating at 7.50 GHz.

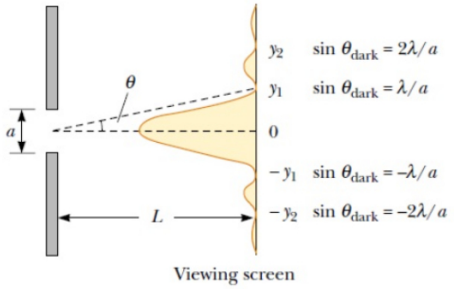

Figure 1:

(a) Calculate the angle subtended by the first minimum in the diffraction pattern.

(b) What is the relative intensity $I/I_{\max}$ at $\theta = 15.0°$?

(c) Assume that two such sources, separated laterally by 20.0 cm are behind the slit. What must be the maximum distance between the plane of the sources and the slit be if the diffraction patterns are to be resolved? (In this case, the approximation $\sin\theta \simeq \tan\theta$ is not valid because of the relatively small value of $\frac{a}{\lambda}$.)

Question 2:

(a) Write the Planck distribution law in terms of the frequency f rather than wavelength λ, to obtain $I(f)$

(b) Show that

$$\int_0^\infty I(\lambda)\,\mathrm{d}\lambda = \frac{2\pi^5 k^4}{15c^2h^3}T^4, \tag{1}$$

where $I(\lambda)$ is the Planck distribution formula given by Planck's radiation law:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(\mathrm{e}^{\frac{hc}{\lambda kT}} - 1\right)}. \tag{2}$$

(*Hint*: Change the integration variable from λ to f.)

You will need to use the following tabulated integral:

$$\int_0^\infty \frac{x^3\,\mathrm{d}x}{\mathrm{e}^{\alpha x} - 1} = \frac{1}{240}\left(\frac{2\pi}{\alpha}\right)^4. \tag{3}$$

(c) The result of (b) is I and has the form of the Stefan-Boltzmann law, $I = \sigma T^4$. Evaluate the constants in (b) to show that σ has the value $\sigma = 5.670400(40) \times 10^{-8}$ $\mathrm{Wm^2K^{-4}}$

Question 3:

An atom has energy level $E_n = -A/n^2$ where n is an integer and A is a constant. Among the spectral lines that the atom can absorb at room temperature are two adjacent lines with wavelength 97.5 nm and 102.8 nm. Find the value of the constant A in electron volts. Assume the atom is in the ground state.