PC2232 Physics for Electrical Engineers: Tutorial 2

Question 1: Polarization I

A beam of unpolarized sunlight strikes the vertical plastic wall of a water tank at an unknown angle. Some of the light reflects from the wall and enters the water, as seen in the Fig. 1. The refractive index of the plastic wall is 1.61. If the light that has been reflected from the wall into the water is observed to be completely polarized, what angle does this beam make with the normal inside the water?

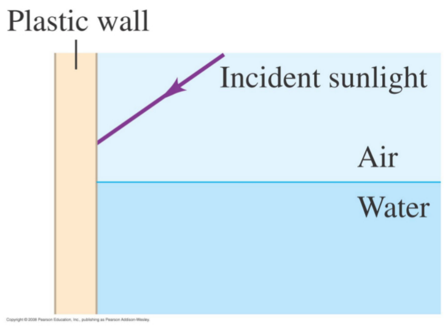

Figure 1:

Question 2: Light intensity

A beam of light traveling horizontally is made of an unpolarized component with intensity I_0 and a polarized component with intensity I_p. The plane of polarization of the polarized component is oriented at an angle of θ with respect to the vertical. The data in the table gives the intensity measured through a polarizer with an orientation of ϕ with respect to the vertical.

ϕ (°)	I_{total} (W/m^2)	ϕ (°)	I_{total} (W/m^2)
0	18.4	100	8.6
10	21.4	110	6.3
20	23.7	120	5.2
30	24.8	130	5.2
40	24.8	140	6.3
50	23.7	150	8.6
60	21.4	160	11.6
70	18.4	170	15.0
80	15.0	170	15.0
90	11.6		

(a) What is the orientation of the polarized component? (That is, what is the angle of θ?)

(b) What are the values of I_0 and I_p?

Question 3: Polarization II

Consider an electromagnetic wave propagating in the z direction. The electric field can be decomposed into the x and y axis as follows:

$$\vec{E} = E_x \hat{x} + E_y \hat{y}. \tag{1}$$

The propagating wave is sinusoidal where the components are given by

$$E_x = E_0 \sin(kz - \omega t), \quad E_y = E_0 \sin(kz - \omega t + \phi). \tag{2}$$

Show that

$$E_y = E_x \cos\phi + \sqrt{E_0^2 - E_x^2} \sin\phi. \tag{3}$$

What is the shape traced out by the electric field vector at $z = 0$ over one period if

(a) $\phi = 0$? (Linear polarization)

(b) $\phi = \pi/2$? (Circular polarization)

(c) $\phi = \pi/4$? (Elliptical polarization)

[*Hint*: Find the locus of the points (E_x, E_y) on the (x, y) axis. The equations of a line, circle and ellipse are respectively $x = y$, $x^2 + y^2 = R^2$ and $x^2/a^2 + y^2/b^2 = 1$ for constants a, b and R.]

Finally, write down the Poynting vector of an elliptically polarized light.

Question 4: Electromagnetic energy in a cavity

Consider the standing wave solution in a cavity [Eq. (22) in Lecture 2 notes], where

$$\vec{E} = 2\tilde{E}\mathrm{e}^{-j\omega t}\sin(kx)\,\hat{y}, \quad \vec{H} = \frac{2\tilde{E}}{Z_0}\mathrm{e}^{-j\omega t}\cos(kx)\,\hat{z}. \tag{4}$$

Write down the Poynting vector for the standing wave.

(a) Does energy flow out of the sides of the cavity?

(b) What is the Poynting vector averaged over time?

Question 5: Microwave

In a microwave oven with the dimensions * $L_x = 315\,\mathrm{mm}$, $L_y = 348\,\mathrm{mm}$, $L_z = 227\,\mathrm{mm}$, we assume that the standing electromagnetic wave propagates only in the x and y direction. Write down the standing wave solution that can exist within the oven, assuming all sides are perfectly reflecting. What are the possible modes in the x and y direction (q_x and q_y respectively) for the wave?

*Panasonic Microwave Oven (25 L)NNST342MYPQ

Question 6: Polarization upon reflection (optional)

In this question, we will derive the expression used in the lecture to determine whether there is a phase change when reflecting normally off a surface, i.e. whether the expression

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_a \tag{5}$$

holds true, where the symbols are as defined in the lecture.

Suppose we choose E_a to be polarized in the x-direction, and B_a to be in the y-direction. Positive-z is then the direction of propagation of the incident plane wave with the boundary at $z = 0$.

(a) Write down the expressions of E_a, B_a, E_r, B_r, E_b, and B_b.

(b) By considering the continuity conditions of both $\vec{E}$ and $\vec{B}$ at the boundary,

$$\vec{E}_a + \vec{E}_r = \vec{E}_b, \quad \text{and} \quad \vec{B}_a + \vec{B}_r = \vec{B}_b, \tag{6}$$

derive Eq. (5) above. Notice that we have assumed non-magnetic materials so that $\mu_a = \mu_b = \mu_0$.

(*The Fresnel equations for incident wave at an oblique angle can also be derived in a similar manner. The mathematics is more tedious due to the extra angular dependence. You may try to derive them on your own or check any EM or Optics book.*)