PC2232: Tutorial Homework Assignment 1

Due date: Thursday, 5 March 2015, before 6pm

Question 1: EM Wave in Media

(a) Given a material with real permittivity $\boldsymbol{\epsilon}(\omega)$.

i. Express $n(\omega)$ in terms of $\boldsymbol{\epsilon}(\omega)$ and any other constants

ii. Explain why ω is constant when light passes throught from vacuum into a material with refractive index $n(\omega)$*

iii. Consider the plane wave solution to our Maxwell's equation in vacuum†.:

$$|E_0| = \tilde{E}\mathrm{e}^{j\vec{k_0}\cdot\vec{r}}\mathrm{e}^{-j\omega t}$$

For a plane wave solution in a material of refractive index $n(\omega)$, express the following quantities in terms of $n(\omega)$‡:

- $|E|$
- $|H|$
- $|\bar{S}|$

(b) Given a material with complex permittivity. Therefore, k and n are also complex:

$$k = k' + jk'' \qquad\qquad n = n' + jn''$$

Show that this leads to power loss along the direction of propagation.
For simplicity, assume your wave propagates in the z direction.

(c) Given that the loss coefficient is defined as:

$$\alpha = -\frac{1}{z}\left(10\log_{10}\left[\frac{\bar{S}(z)}{\bar{S}(0)}\right]\right)$$

Show that for the complex k above, we will obtain the following equation

$$\alpha = 20k''\log_{10}\mathrm{e}$$

(d) Given: $k'' = 0.04\text{km}^{-1}$
Determine $\dfrac{S(z)}{S(0)}$ after $z = 5\text{km}$

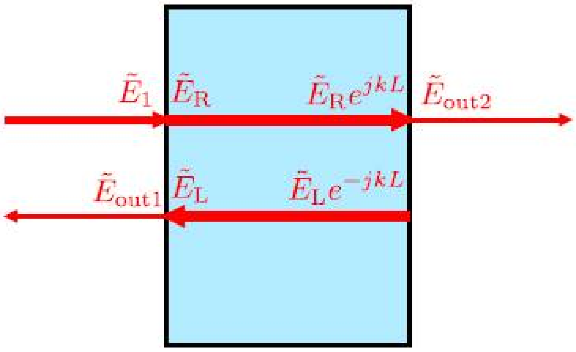
*Understand that when we write $n(\omega)$, we mean that the refractive index changes according to ω, not that ω changes according to n

†k_0 is the value of k in vacuum

‡And any other quantity already given in the plane wave solution above

Question 2: Fabry-Perot interferometer

A Fabry-Perot interferometer basically consists of a dielectric slab (refractive index of n) in free space. It works on the basis of multiple reflections[§].



(a) Work out the scattering matrix for the first interface if we have a TM input with normal incident. You are allowed to use equations from the lecture notes. State clearly where the equations you use come from.

(b) Given:

$$E_{\mathrm{out}_2} = s_{12}s_{21}\mathrm{e}^{jkL}\sum_{m}^{\infty}\left(s_{22}\mathrm{e}^{2jkL}\right)^m \tilde{E}_{\mathrm{in}}$$

Using the identity for the infinite geometric series (given in lecture notes pg 20, lecture 4), obtain the final expression for E_{out}

(c) For $n = 4$, obtain the conditions for L for minimum transmission. Give your answer in terms of wavelength of light in free space (λ_0)

(d) Given that the refractive index of the medium inside a Fabry-Perot interferometer is complex such that $k = k' + jk''$. Show that it is now possible to end up with a non-converging sum for E_{out_2}.

[§]Refer to pg 15 of lecture 4