PC2232: Tutorial Homework Assignment 2

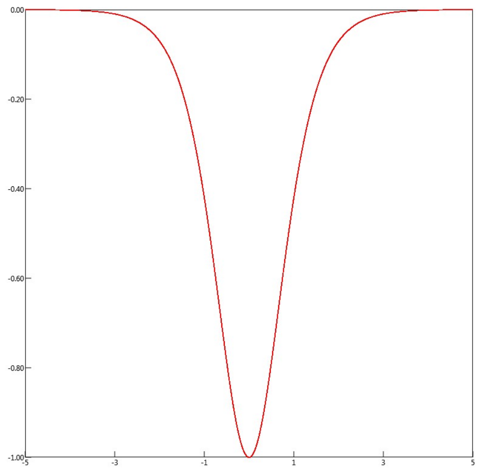
Due date: Tuesday, 14 April 2015, before 6pm

Question 1: Schrödinger Equation

Hint: For this question, understand that bound state is a state with energy level lower than the potential of both the region on the left and the region on the right of the 'well'.

(a) Given the following potential

$$U(x) = -\frac{\hbar^2 a^2}{m}\mathrm{sech}^2(ax)$$



i. Show that $\psi = A\,\mathrm{sech}\,(ax)$ is a bound state solution to the Schrödinger equation. The following definitions might help:

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \cosh^2 x + \sinh^2 x = 1$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) = \cosh x \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = \sinh x$$

ii. Show that $\psi_k(x) = B\left(\frac{ik - a\tanh[ax]}{ik + a}\right)\mathrm{e}^{ikx}$ is a free particle solution to the Schrödinger equation*. Notice that both these solutions are for all of x†.

iii. Recall that $\tanh x = \dfrac{1+\mathrm{e}^{-2x}}{1-\mathrm{e}^{-2x}}$.

Therefore, show that for the free particle solution incident on this potential, there is no reflected wave. Hint: Consider asymptotic behaviour of the wave.

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*k here is the regular k that we see: $k = \dfrac{2mE}{\hbar}$

†That is, there are no different regions here like the ones in the infinite square well. This one solution works for all of x

(b) Given the following potential:

$$U(x) = \begin{cases} \infty & (x < 0) \\ -\dfrac{32\pi^2\hbar^2}{mL^2} & (0 \le x \le L) \\ 0 & (x > L) \end{cases}$$

i. How many bound energy states are there?

ii. Solve the Schrödinger equation for this potential[‡]
For the free particle solution, note that you have to keep both terms in region III. Explain why this is so.

iii. Briefly discuss how you would find energy values of the bound states[§]

Question 2: Fermi-Dirac distribution

(a) Recalling that the Fermi-Dirac distribution is the following equation:

$$f(E_n) = \frac{N_n}{d_n} = \frac{1}{e^{\frac{E_n - E_F}{k_B T}} + 1}$$

i. Set T to 0. Determine $f(E)$ for the two following cases

- $E_n < E_F$
- $E_n > E_F$

ii. Keeping the above values in mind, in your own words, explain the two following concept:

- Fermi Energy
- Fermi-Dirac distribution[¶]

(b) Pure germanium has a band gap of 0.67 eV. The Fermi energy is in the middle of the gap.

i. For temperatures of 250 K, 300 K, and 350 K, calculate the probability that a state at the bottom of the conduction band is occupied[‖].

ii. For each temperature in part (a), calculate the probability that a state at the top of the valence band is empty.

[‡]Hint I: Note that there are two cases to consider. Hint II: We do not need the numerical values of the normalization and quantization condition. (That is, we do not need the numerical values of k, α and A, $B \ldots$)

[§]No mathematical calculation is necessary, and we are looking for less than 5 sentences

[¶]That is, what exactly is $f(E)$, what does it mean? Hint I: Your answer should be for a particular energy level and temperature. Hint II: Recall value of $f(E)$ is when T is set to 0 as calculated earlier. Understanding the meaning of the $f(E)$ values obtained should help you solve this.

[‖]Understanding part (a) of this question might help you solve this