PC2232 Physics for Electrical Engineers

AY2014/15 Semester 2

MIDTERM TEST

6 March, 2015

Time Allowed: ONE hour THIRTY minutes

Instructions:

1. This is a closed-book test.

2. This paper contains 20 multiple-choice questions and 10 printed pages.

3. Each of the questions or incomplete statements is followed by five suggested answers or completions. Select the most appropriate choice in each case and then shade the corresponding bubble on the answer sheet.

4. Only the answer sheet will be collected at the end of the test. Answers written anywhere else will not be marked.

5. Use 2B pencil only. Using any other type of pencil or pen may result in answers unrecognizable by the machine.

6. Answer all questions. Marks will NOT be deducted for wrong answers.

7. Some formulae are given in pp. 9-10. Not all the formulae will be needed in the test.

8. You may not leave the test venue during the first thirty minutes and the last fifteen minutes of the test.

1. Suppose that $(\boldsymbol{E}_1, \boldsymbol{H}_1)$ is a solution of the Maxwell's equations in free space and $(\boldsymbol{E}_2, \boldsymbol{H}_2)$ is another solution. Which of the following is also a solution?

   (A) $(\boldsymbol{E}_1 + j\boldsymbol{E}_2, \boldsymbol{H}_1 + j\boldsymbol{H}_2)$.

   (B) $(\boldsymbol{E}_1 + j\boldsymbol{E}_2, \boldsymbol{H}_1 - j\boldsymbol{H}_2)$.

   (C) $(\boldsymbol{E}_1 + \boldsymbol{E}_2, \boldsymbol{H}_1 - \boldsymbol{H}_2)$.

   (D) $(\boldsymbol{E}_1 + \boldsymbol{E}_2, \boldsymbol{H}_1 + j\boldsymbol{H}_2)$.

   (E) $(\boldsymbol{E}_1 + \boldsymbol{E}_2, \boldsymbol{H}_1 - j\boldsymbol{H}_2)$.

2. Suppose that the time-averaged Poynting vector from the sun is $\bar{\boldsymbol{S}} \approx 1.4\hat{\boldsymbol{z}}$ kW/m^2, which is assumed to be uniform along x and y and pointing in the $\hat{\boldsymbol{z}}$ direction, as shown in the following figure. Assuming that the earth is a perfect sphere with radius $R = 6371$ km and absorbs all the solar power that impinges on its surface, what is the total absorbed solar power? (The circumference of a circle is $2\pi R$, the area of a circle is πR^2, the surface area of a sphere is $4\pi R^2$, and the volume of a sphere is $4\pi R^3/3$.)

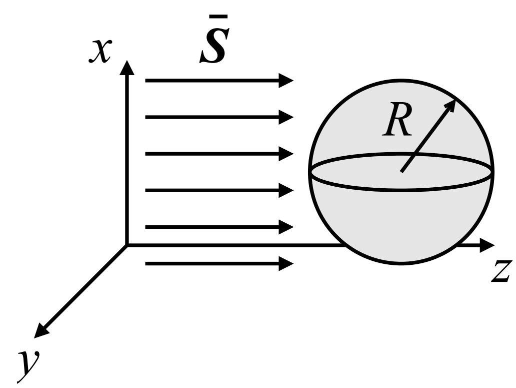
   (A) 5.60×10^7 kW.

   (B) 1.79×10^{14} kW.

   (C) 3.57×10^{14} kW.

   (D) 7.14×10^{14} kW.

   (E) 1.52×10^{21} kW.



3. An electromagnetic plane wave in free space has the following fields:

$$\boldsymbol{E} = \hat{\boldsymbol{x}} E_{\max} \cos(kz - \omega t), \qquad \boldsymbol{H} = \hat{\boldsymbol{y}} \frac{E_{\max}}{Z_0} \cos(kz - \omega t). \tag{1}$$

   What is the time-averaged energy of this wave in a box with volume L^3?

   (A) $\frac{1}{2}\epsilon_0 E_{\max}^2$.

   (B) $\frac{1}{4}\epsilon_0 E_{\max}^2 L^3$.

   (C) $\frac{1}{2}\epsilon_0 E_{\max}^2 L^3$.

   (D) $\epsilon_0 E_{\max}^2 L^3$.

   (E) ∞.

4. Consider a linearly polarized plane wave in free space given by $\boldsymbol{E} = \tilde{E}\left(\cos\theta\hat{\boldsymbol{x}} + \sin\theta\hat{\boldsymbol{y}}\right)\exp(jkz - j\omega t)$. What is its time-averaged Poynting vector?

   (A) $\frac{1}{2Z_0}|\tilde{E}|^2\left(\cos^2\theta\right)\hat{\boldsymbol{z}}$.

   (B) $\frac{1}{2Z_0}|\tilde{E}|^2\left(\sin^2\theta\right)\hat{\boldsymbol{z}}$.

   (C) $\frac{1}{2Z_0}|\tilde{E}|^2\left(\cos\theta + \sin\theta\right)\hat{\boldsymbol{z}}$.

   (D) $\frac{1}{2Z_0}|\tilde{E}|^2(\tan\theta)\hat{\boldsymbol{z}}$.

   (E) $\frac{1}{2Z_0}|\tilde{E}|^2\hat{\boldsymbol{z}}$.

5. Consider a perfectly conducting mirror at $z = 0$, which imposes a boundary condition $\boldsymbol{E}(x, y, z = 0) = 0$. Assuming a solution of

$$\boldsymbol{E} = \hat{\boldsymbol{y}}\left[\tilde{E}_{\text{out}}\exp(jkz - j\omega t) + \tilde{E}_{\text{in}}\exp(-jkz - j\omega t)\right] \tag{2}$$

   for $z \geq 0$ in free space, as shown in the following figure, the reflection coefficient $\tilde{E}_{\text{out}}/\tilde{E}_{\text{in}}$ is

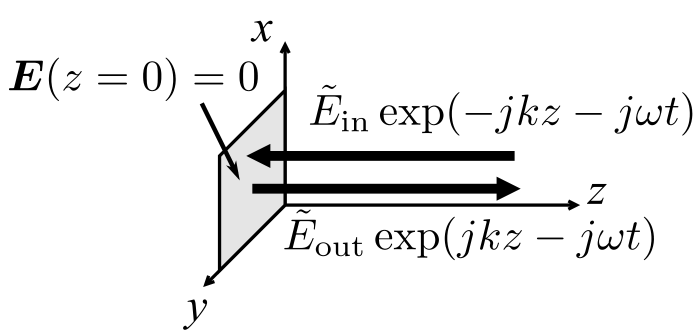
   (A) 1.

   (B) -1.

   (C) j.

   (D) $-j$.

   (E) $1/\sqrt{2}$.



6. Consider an electromagnetic cavity with two perfectly conducting mirrors at $z = 0$ and $z = L$. Assuming the same standing-wave solution in Eq. (2), what is the restriction on the wavenumber k?

   (A) $k = q$, $q = 1, 2, \ldots$

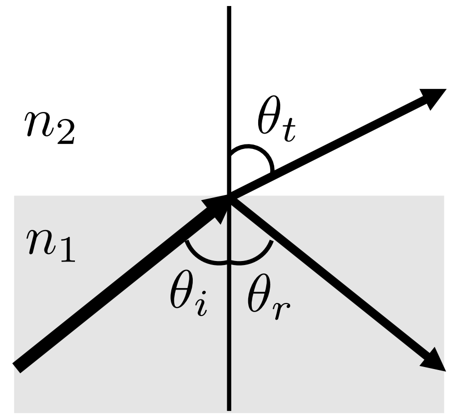
   (B) $k = \pi q$, $q = 1, 2, \ldots$

   (C) $kL = q$, $q = 1, 2, \ldots$

   (D) $kL = \pi q$, $q = 1, 2, \ldots$

   (E) $kL = 2\pi q$, $q = 1, 2, \ldots$

7. Consider the reflection and refraction at a flat interface between two media, as shown in the following figure. The bottom medium has a refractive index of $n_1 = 1.5$ and the top medium is free space. Total internal reflection occurs when



(A) $\theta_i > \sin^{-1}(1/1.5)$.

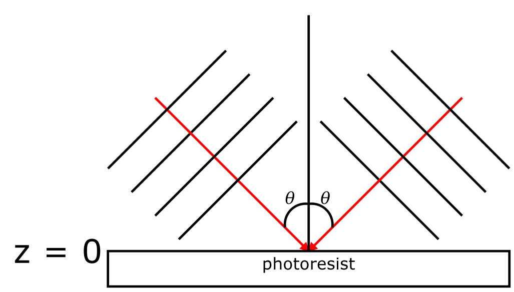
(B) $\theta_i < \sin^{-1}(1/1.5)$.

(C) $\theta_i > \sin^{-1}\left[(1/1.5)^2\right]$.

(D) $\theta_i < \sin^{-1}\left[(1/1.5)^2\right]$.

(E) $\theta_i > \sin^{-1}(\sqrt{1/1.5})$.

8. Consider two optical plane waves impinging on a flat photoresist at $z = 0$ with identical amplitude, as shown in the following figure. Which of the following is a way of reducing the period of the resulting interference pattern?



(A) Decrease the frequency of light.

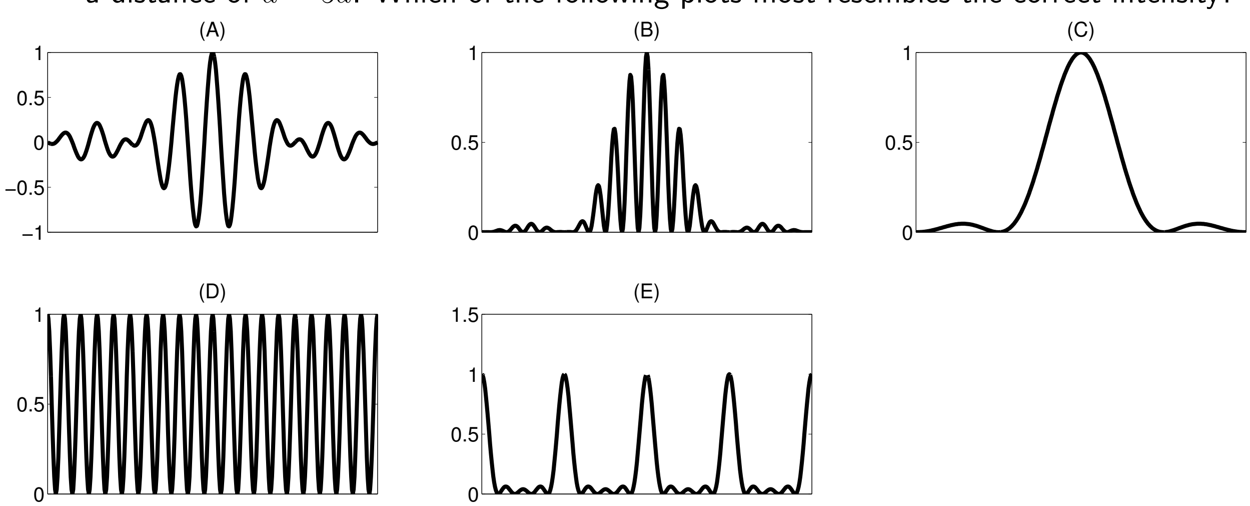
(B) Increase the wavelength of light.

(C) Increase the electric-field amplitude.

(D) Increase the incident angle θ.

(E) Decrease the refractive index of the medium.

9. Consider the Fraunhofer diffraction of two wide slits, each with width a and separated by a distance of $d = 5a$. Which of the following plots most resembles the correct intensity?



10. Consider the focusing of a plane wave onto a spot on the focal plane by a focusing lens with finite aperture. Which of the following is NOT a way of reducing the focus spot size?

    (A) Increase the frequency of light.

    (B) Reduce the wavelength of light.

    (C) Increase the refractive index of the medium between the lens and the focal plane.

    (D) Reduce the focal length while keeping the lens aperture size the same.

    (E) Reduce the lens aperture size while keeping the focal length the same.

11. Which of the following is a valid scattering matrix for a lossless (power-conserving) beam splitter?

    (A) $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$.

    (B) $\begin{pmatrix} j/\sqrt{2} & j/\sqrt{2} \\ j/\sqrt{2} & j/\sqrt{2} \end{pmatrix}$.

    (C) $\begin{pmatrix} 1/\sqrt{2} & j/\sqrt{2} \\ j/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$.

    (D) $\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$.

    (E) $\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix}$.

12. Consider the Mach-Zehnder interferometer in the following figure. Suppose that the first input amplitude $\tilde{E}_1 = 0$, the second input amplitude is $\tilde{E}_2$, the beam splitters are identical with the following scattering matrix:

$$\begin{pmatrix} \tilde{E}_1' \\ \tilde{E}_2' \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} \tilde{E}_1 \\ \tilde{E}_2 \end{pmatrix}, \qquad \begin{pmatrix} \tilde{E}_{\mathrm{out1}} \\ \tilde{E}_{\mathrm{out2}} \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} \tilde{E}_1'' \\ \tilde{E}_2'' \end{pmatrix}, \tag{3}$$

$$\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}, \tag{4}$$

and the two arms have the same length, identical perfect mirrors, and the same refractive index. What are $|\tilde{E}_{\text{out1}}|^2$ and $|\tilde{E}_{\text{out2}}|^2$?

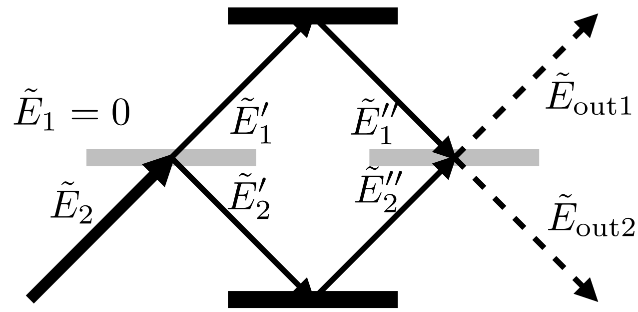
(A) $|\tilde{E}_{\text{out1}}|^2 = 0, |\tilde{E}_{\text{out2}}|^2 = |\tilde{E}_2|^2$.

(B) $|\tilde{E}_{\text{out1}}|^2 = |\tilde{E}_{\text{out2}}|^2 = |\tilde{E}_2|^2$.

(C) $|\tilde{E}_{\text{out1}}|^2 = |\tilde{E}_{\text{out2}}|^2 = |\tilde{E}_2|^2/2$.

(D) $|\tilde{E}_{\text{out1}}|^2 = |\tilde{E}_{\text{out2}}|^2 = |\tilde{E}_2|^2/\sqrt{2}$.

(E) $|\tilde{E}_{\text{out1}}|^2 = |\tilde{E}_{\text{out2}}|^2 = 0$.



13. Consider a Fabry-Perot interferometer consisting of a lossless dielectric slab with refractive index n in free space and normal incidence ($\theta_i = 0$). The power transmission coefficient is

$$\frac{|\tilde{E}_{\text{out2}}|^2}{|\tilde{E}_1|^2} = \frac{|s_{12}s_{21}|^2}{|1 - s_{22}^2 e^{2jkL}|^2}, \qquad k = \frac{\omega n}{c}. \tag{5}$$

For a given free-space wavelength λ_0, what is the thickness L such that the power transmission is maximum?

(A) $L = q\lambda_0, q = 0, 1, 2, \ldots$

(B) $L = q\lambda_0/2, q = 0, 1, 2, \ldots$

(C) $L = q\lambda_0/n, q = 0, 1, 2, \ldots$

(D) $L = q\lambda_0/(2n), q = 0, 1, 2, \ldots$

(E) $L = \pi q\lambda_0/n, q = 0, 1, 2, \ldots$

14. If the optical loss coefficient of a lossy medium measured in decibel is 0.2 dB/km and $S(z)$ is the optical intensity of a plane wave at propagation distance z, what is $S(z)/S(0)$ after 50 km?

(A) 10^{-10}.

(B) 2.06×10^{-9}.

(C) 10^{-1}.

(D) 0.135.

(E) $1/\sqrt{10}$.

15. What is the group velocity of an electromagnetic wave in a dielectric with real and constant (dispersionless) refractive index n?

    (A) c.

    (B) c/n.

    (C) c/n^2.

    (D) nc.

    (E) n/c.

16. Consider a half-wave plate that introduces a net phase difference $\theta = \pi$ (in radian) between a wave polarized in $\hat{\boldsymbol{x}}$ and a wave polarized in $\hat{\boldsymbol{y}}$. If the input is a linearly polarized wave with polarization in the direction $(\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}})/\sqrt{2}$, what is the output polarization vector?

    (A) $\hat{\boldsymbol{x}}$.

    (B) $\hat{\boldsymbol{y}}$.

    (C) $(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})/\sqrt{2}$.

    (D) $(\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}})/\sqrt{2}$.

    (E) $(\hat{\boldsymbol{x}} + j\hat{\boldsymbol{y}})/\sqrt{2}$.

17. An electron can have two spin values, $s_z = 1/2$ or $s_z = -1/2$. The wavefunction for the x spin component in terms of that for the z component is

$$\begin{aligned}\psi_x\left(s_x = +1/2\right) &= \left[\psi_z\left(s_z = +1/2\right) + \psi_z\left(s_z = -1/2\right)\right]/\sqrt{2}, \\ \psi_x\left(s_x = -1/2\right) &= \left[\psi_z\left(s_z = +1/2\right) - \psi_z\left(s_z = -1/2\right)\right]/\sqrt{2}.\end{aligned} \tag{6}$$

    Suppose that s_z has been measured and the outcome is $s_z = -1/2$. If one then measures s_x, what is the probability distribution of s_x?

    (A) $P_x(s_x = 1/2) = 1/\sqrt{2}$, $P_x(s_x = -1/2) = 1/\sqrt{2}$.

    (B) $P_x(s_x = 1/2) = 1/\sqrt{2}$, $P_x(s_x = -1/2) = -1/\sqrt{2}$.

    (C) $P_x(s_x = 1/2) = 0$, $P_x(s_x = -1/2) = 1$.

    (D) $P_x(s_x = 1/2) = 1$, $P_x(s_x = -1/2) = 0$.

    (E) $P_x(s_x = 1/2) = 1/2$, $P_x(s_x = -1/2) = 1/2$.

18. Given Eqs. (6) in the previous question, what is $\psi_z(s_z)$ if $\psi_x(s_x = 1/2) = 1$ and $\psi_x(s_x = -1/2) = 0$?

(A) $\psi_z(s_z = 1/2) = 1/\sqrt{2}$, $\psi_z(s_z = -1/2) = 1/\sqrt{2}$.

(B) $\psi_z(s_z = 1/2) = 1/\sqrt{2}$, $\psi_z(s_z = -1/2) = -1/\sqrt{2}$.

(C) $\psi_z(s_z = 1/2) = 0$, $\psi_z(s_z = -1/2) = 1$.

(D) $\psi_z(s_z = 1/2) = 1$, $\psi_z(s_z = -1/2) = 0$.

(E) $\psi_z(s_z = 1/2) = 1/2$, $\psi_z(s_z = -1/2) = 1/2$.

19. For an electron in free space, how is its de Broglie wavelength changed if its energy is doubled?

(A) the wavelength is increased by a factor of 2.

(B) the wavelength is decreased by a factor of 2.

(C) the wavelength is decreased by a factor of 4.

(D) the wavelength is increased by a factor of $\sqrt{2}$.

(E) the wavelength is decreased by a factor of $\sqrt{2}$.

20. Consider a potential $U(x) = U_0$ for $x < 0$ and $U(x) = 0$ for $x \geq 0$ for a one-dimensional quantum nonrelativistic particle with energy E, as shown in the following figure. If $E < U_0$ and $\tilde{\psi}(x) = A\exp(ikx) + B\exp(-ikx)$ for $x \geq 0$, what is $|A|^2/|B|^2$?

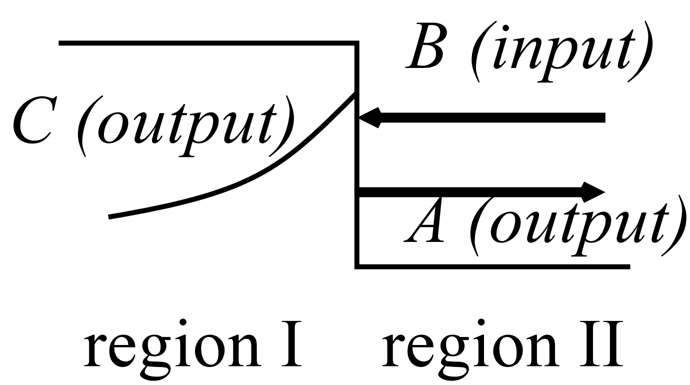
(A) 0.

(B) $1/\sqrt{2}$.

(C) $1/2$.

(D) 1.

(E) 2.



END OF PAPER

Appendix A: Table of Information

1. Maxwell's equations in free space

$$\nabla \cdot \boldsymbol{E} = 0, \qquad \nabla \cdot \boldsymbol{H} = 0, \qquad \nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}, \qquad \nabla \times \boldsymbol{H} = \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}. \tag{A1}$$

2. Speed of light in free space

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}. \tag{A2}$$

3. Free-space impedance

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{A3}$$

4. Dispersion relation in free space

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi\nu}{c}. \tag{A4}$$

5. Poynting vector

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}. \tag{A5}$$

6. Electromagnetic energy density

$$u = \frac{1}{2}\epsilon_0 \boldsymbol{E} \cdot \boldsymbol{E} + \frac{1}{2}\mu_0 \boldsymbol{H} \cdot \boldsymbol{H}. \tag{A6}$$

7. Snell's law

$$n_1 \sin\theta_i = n_2 \sin\theta_t, \tag{A7}$$

   where θ_i is the incident angle, θ_t is the transmitted angle, n_1 is the refractive index of the medium with the incident light, and n_2 is that of the medium with the transmitted light.

8. Fraunhofer diffraction

$$\mathcal{E}(x, z) \propto \int_{-\infty}^{\infty} dx' \mathcal{E}(x', 0) \exp\left(-j \frac{2\pi x x'}{\lambda z}\right). \tag{A8}$$

9. Unitary matrix

$$\boldsymbol{s}\boldsymbol{s}^\dagger = \boldsymbol{I}, \qquad \left(\boldsymbol{s}^\dagger\right)_{nm} = \boldsymbol{s}^*_{mn}. \tag{A9}$$

10. Fresnel reflection and transmission coefficients

    - TE polarization (s-polarization):

$$r_s \equiv \frac{\tilde{E}_r}{\tilde{E}_i} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}, \qquad t_s \equiv \frac{\tilde{E}_t}{\tilde{E}_i} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}, \tag{A10}$$

    - TM polarization (p-polarization):

$$r_p \equiv \frac{\tilde{E}_r}{\tilde{E}_i} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}, \qquad t_p \equiv \frac{\tilde{E}_t}{\tilde{E}_i} = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}. \tag{A11}$$

11. Phase and group velocities:

$$v_p = \frac{\omega}{k}, \qquad v_g = \frac{d\omega}{dk}. \tag{A12}$$

12. Energy and momentum of a quantum plane wave

$$\psi(\boldsymbol{r}, t) \propto \exp(i\boldsymbol{k} \cdot \boldsymbol{r} - i\omega t), \qquad E = \hbar\omega, \qquad \boldsymbol{p} = \hbar\boldsymbol{k}. \tag{A13}$$

13. Born's rule

$$P(x, t) = |\psi(x, t)|^2. \tag{A14}$$

14. Kinetic energy

$$E = \frac{p^2}{2m}. \tag{A15}$$

15. Time-dependent Schrödinger equation

$$i\hbar \frac{\partial \psi(\boldsymbol{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\boldsymbol{r})\right] \psi(\boldsymbol{r}, t). \tag{A16}$$

16. Time-independent Schrödinger equation

$$E\tilde{\psi}(\boldsymbol{r}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\boldsymbol{r})\right] \tilde{\psi}(\boldsymbol{r}). \tag{A17}$$

17. One-dimensional time-independent Schrödinger equation

$$E\tilde{\psi}(x) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right] \tilde{\psi}(x). \tag{A18}$$