PC2232 Physics for Electrical Engineers: Tutorial 1

Question 1: Scalar derivatives

Show that

$$\vec{\nabla}\left(\vec{k}\cdot\vec{r}\right)=\vec{k}, \tag{1}$$

where $\vec{k}$ is a constant vector and $\vec{r}=x\hat{x}+y\hat{y}+z\hat{z}$. Using the above results, verify Eq. (45) of the Lecture 1 notes. In other words, show that

$$\vec{\nabla}\cdot\left(\vec{E}_0\mathrm{e}^{j\vec{k}\cdot\vec{r}}\right)=j\vec{k}\cdot\vec{E}_0\mathrm{e}^{j\vec{k}\cdot\vec{r}}, \tag{2}$$

where $\vec{E}_0$ is a constant.

Question 2: Linearity

Suppose that $(\vec{E}_1,\vec{H}_1)$ and $(\vec{E}_2,\vec{H}_2)$ are solutions of Maxwell's equations.

(a) Show that $(a_1\vec{E}_1+a_2\vec{E}_2,\ a_1\vec{H}_1+a_2\vec{H}_2)$ is also a solution for any complex constants a_1 and a_2.

(b) If $\left(\vec{E}_i,\vec{H}_i\right)$, $i=1,\ldots,N$ are N solutions of Maxwell's equations, show that

$$\left(\sum_{i=1}^{N}a_i\vec{E}_i,\ \sum_{i=1}^{N}a_i\vec{H}_i\right) \tag{3}$$

is a solution for any complex constants a_i for $i=1,\ldots,N$.

Question 3: Wave equation

Verify that any function of the form $f(x-vt)$ is a solution to the wave equation,

$$\frac{\partial^2 f}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}, \tag{4}$$

provided that $v^2=c^2$.

Question 4: Time average

Verify that the time average of the function $\cos^2(kx-\omega t)$ is equal to $1/2$.

Question 5: Power flow

A solar panel exposed to an electromagnetic radiation with a Poynting vector given by (in Cartesian coordinates)

$$\vec{S} = 10kx\,\hat{z}\ \mathrm{Wm}^{-2}, \tag{5}$$

where k is a constant. The solar panel has length a and width b and is placed on a plane perpendicular to the z-axis, as shown in Fig. 1. Calculate the power flow received by the solar panel. [Ans: $P = 5ka^2b$ W]

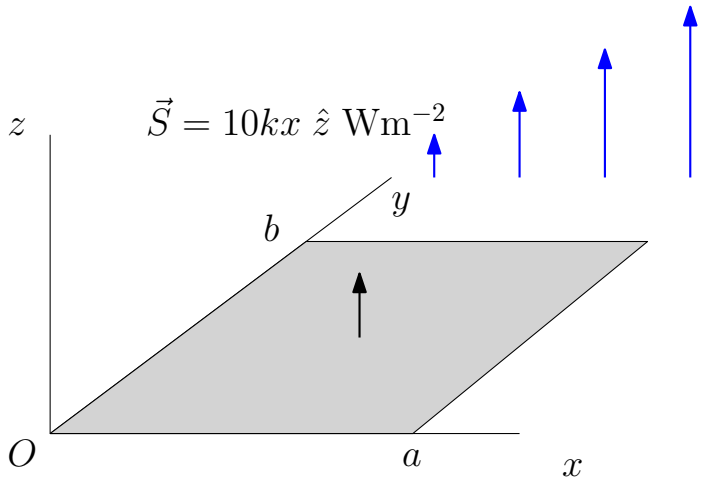

Figure 1: Solar panel in an electromagnetic wave.

Question 6: Energy and Momentum in EM waves (Optional)

Public television station KQED in San Fransisco broadcasts a sinusoidal radio signal at a power of 316 kW. Assume that the wave spreads out uniformly into a hemisphere above the ground. Consider a home 5.00 km away from the antenna.

(a) What average pressure does this wave exert on a totally reflecting surface?

(b) What are the amplitudes of the electric and magnetic fields of the wave?

(c) What is the average density of the energy carried by this wave?

(d) From the energy density in part (c), what percentage is due to the electric field and what percentage is due to the magnetic field?

Question 7: EM Waves (Optional)

An electromagnetic wave is specified (in SI units) by the following function:

$$\vec{E}(t) = \left(-6\ \hat{x} + 3\sqrt{5}\ \hat{y}\right)\left(10^4\ \mathrm{Vm}^{-1}\right) \mathrm{e}^{j\left[\left(\sqrt{5}x+2y\right)\left(\frac{\pi}{3}\right)\left(10^7\right)-\left(9.42\times10^{15}\ t\right)\right]}. \tag{6}$$

Find

(a) the direction along which the electric field oscillates,

(b) the scalar value of the amplitude of the electric field,

(c) the direction of propagation of the waves,

(d) the propagation number and wavelength,

(e) the frequency and angular frequency,

(f) the speed.