PC2232 Physics for Electrical Engineers
Lecture 2: Light Propagation

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Linearity of Maxwell's Equations

- Derivatives are **linear operators**.
- Suppose we have found a solution $\boldsymbol{E}_1(\boldsymbol{r}, t)$ and $\boldsymbol{H}_1(\boldsymbol{r}, t)$ to the Maxwell's equations (source-free for simplicity):

$$\nabla \cdot \boldsymbol{E}_1 = 0, \qquad \nabla \cdot \boldsymbol{H}_1 = 0, \qquad \nabla \times \boldsymbol{E}_1 = -\mu_0 \frac{\partial \boldsymbol{H}_1}{\partial t}, \qquad \nabla \times \boldsymbol{H}_1 = \epsilon_0 \frac{\partial \boldsymbol{E}_1}{\partial t}. \tag{1}$$

Convince yourself that multiplying the solution by any constant gives another solution, i.e. $(a_1\boldsymbol{E}_1, a_1\boldsymbol{H}_1)$, is also a solution:

$$\nabla \cdot (a_1\boldsymbol{E}_1) = 0, \qquad \nabla \cdot (a_1\boldsymbol{H}_1) = 0, \tag{2}$$

$$\nabla \times (a_1\boldsymbol{E}_1) = -\mu_0 \frac{\partial (a_1\boldsymbol{H}_1)}{\partial t}, \qquad \nabla \times (a_1\boldsymbol{H}_1) = \epsilon_0 \frac{\partial (a_1\boldsymbol{E}_1)}{\partial t}. \tag{3}$$

- The constant a_1 can be complex.
- Suppose that we have found another solution $(\boldsymbol{E}_2, \boldsymbol{H}_2)$. Convince yourself that $(a_1\boldsymbol{E}_1 + a_2\boldsymbol{E}_2, a_1\boldsymbol{H}_1 + a_2\boldsymbol{H}_2)$ is also a solution, where a_2 is another arbitrary constant.

$$\nabla \cdot (a_1\boldsymbol{E}_1 + a_2\boldsymbol{E}_2) = 0, \qquad \nabla \cdot (a_1\boldsymbol{H}_1 + a_2\boldsymbol{H}_2) = 0, \tag{4}$$

$$\nabla \times (a_1\boldsymbol{E}_1 + a_2\boldsymbol{E}_2) = -\mu_0 \frac{\partial (a_1\boldsymbol{H}_1 + a_2\boldsymbol{H}_2)}{\partial t}, \quad \nabla \times (a_1\boldsymbol{H}_1 + a_2\boldsymbol{H}_2) = \epsilon_0 \frac{\partial (a_1\boldsymbol{E}_1 + a_2\boldsymbol{E}_2)}{\partial t}. \tag{5}$$

Superposition of EM Fields

- If $(\boldsymbol{E}_1, \boldsymbol{H}_1), (\boldsymbol{E}_2, \boldsymbol{H}_2), \ldots, (\boldsymbol{E}_N, \boldsymbol{H}_N)$ is a set of solutions, so is

$$\left(\sum_{n=1}^{N} a_n \boldsymbol{E}_n, \sum_{n=1}^{N} a_n \boldsymbol{H}_n \right) \tag{6}$$

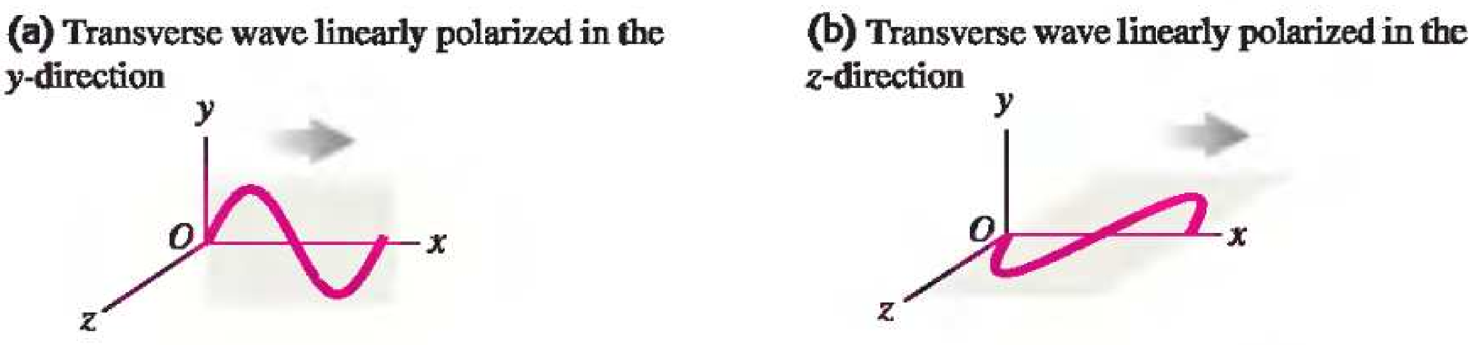
  This is called a **superposition** of EM fields.
- **Be very careful with nonlinear functions of EM fields, e.g., energy density, Poynting vector, however.** For example,

$$\begin{aligned} \text{Energy density of } (a_1 \boldsymbol{E}_1, a_1 \boldsymbol{H}_1) &= a_1^2 \text{Energy density of } (\boldsymbol{E}_1, \boldsymbol{H}_1) \\ &\neq a_1 \text{Energy density of } (\boldsymbol{E}_1, \boldsymbol{H}_1), \end{aligned} \tag{7}$$

$$\begin{aligned} &\text{Energy density of } (\boldsymbol{E}_1, \boldsymbol{H}_1) + \text{Energy density of } (\boldsymbol{E}_2, \boldsymbol{H}_2) \\ &\neq \text{Energy density of } (\boldsymbol{E}_1 + \boldsymbol{E}_2, \boldsymbol{H}_1 + \boldsymbol{H}_2). \end{aligned} \tag{8}$$

Polarizations

- Focus on the electric field. The wave $\hat{\boldsymbol{y}}\tilde{E}\exp(jkx - j\omega t)$ is called **linearly polarized** because the electric field oscillates in one direction only.



- Another possibility is $\hat{\boldsymbol{z}}\tilde{E}\exp(jkx - j\omega t)$. This is also linear polarization.
- If we look at the field at $x = 0$, These oscillate like $\hat{\boldsymbol{y}}\tilde{E}\exp(-j\omega t)$ and $\hat{\boldsymbol{z}}\tilde{E}\exp(-j\omega t)$.
- What about

$$\frac{(\hat{\boldsymbol{y}} + \hat{\boldsymbol{z}})}{\sqrt{2}}\tilde{E}\exp(jkx - j\omega t)? \tag{9}$$

- `http://en.wikipedia.org/wiki/Polarization_%28waves%29`

*Circular Polarization

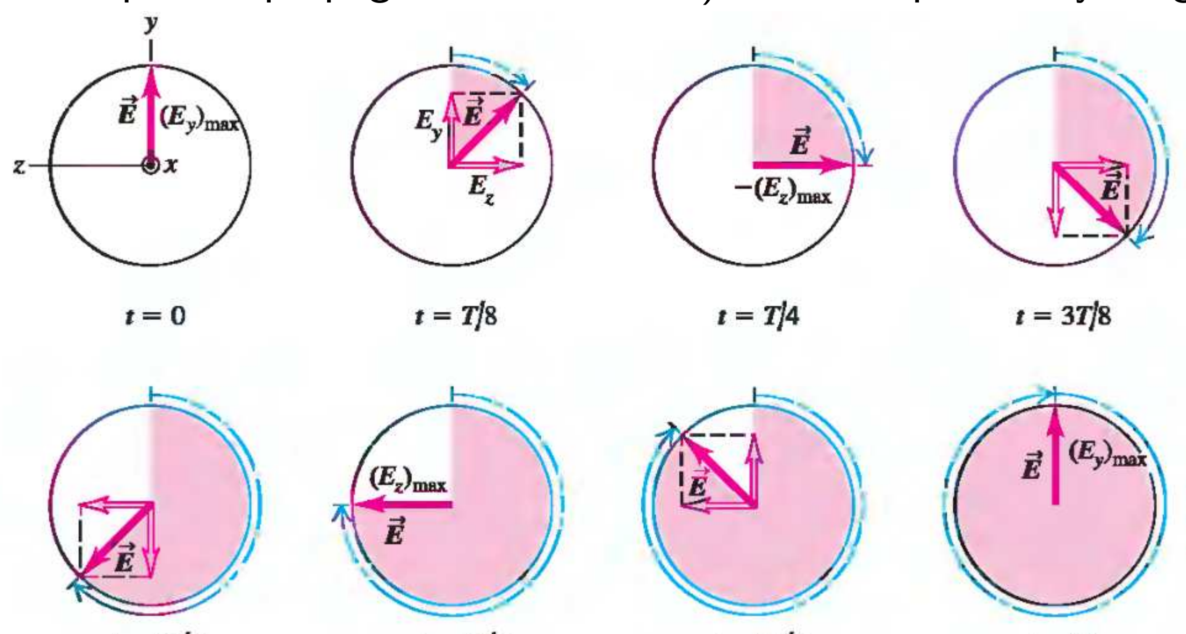
- What about

$$\boldsymbol{E} = \frac{(\hat{\boldsymbol{y}} - j\hat{\boldsymbol{z}})}{\sqrt{2}}\tilde{E}\exp(jkx - j\omega t)? \tag{10}$$

- The factor $-j = \exp\left(-j\frac{\pi}{2}\right)$ means that there is a **phase difference** between the $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{z}}$ components.
- Looking at the field at $x = 0$, the real part of $\hat{\boldsymbol{y}}$ component oscillates like $\cos(\omega t)$ and the real part of $\hat{\boldsymbol{z}}$ component oscillates like $-\sin(\omega t)$.

$$\operatorname{Re}\boldsymbol{E}(x = 0, t) = \cos(\omega t)\hat{\boldsymbol{y}} - \sin(\omega t)\hat{\boldsymbol{z}}. \tag{11}$$

The real electric field is tracing out **a circle**. This is called left **circular polarization** (left-hand rule with respect to propagation direction $\hat{\boldsymbol{x}}$). Another possibility is right circular polarization.



Circular polarization: The $\vec{E}$ vector of the wave has constant magnitude and rotates in a circle.

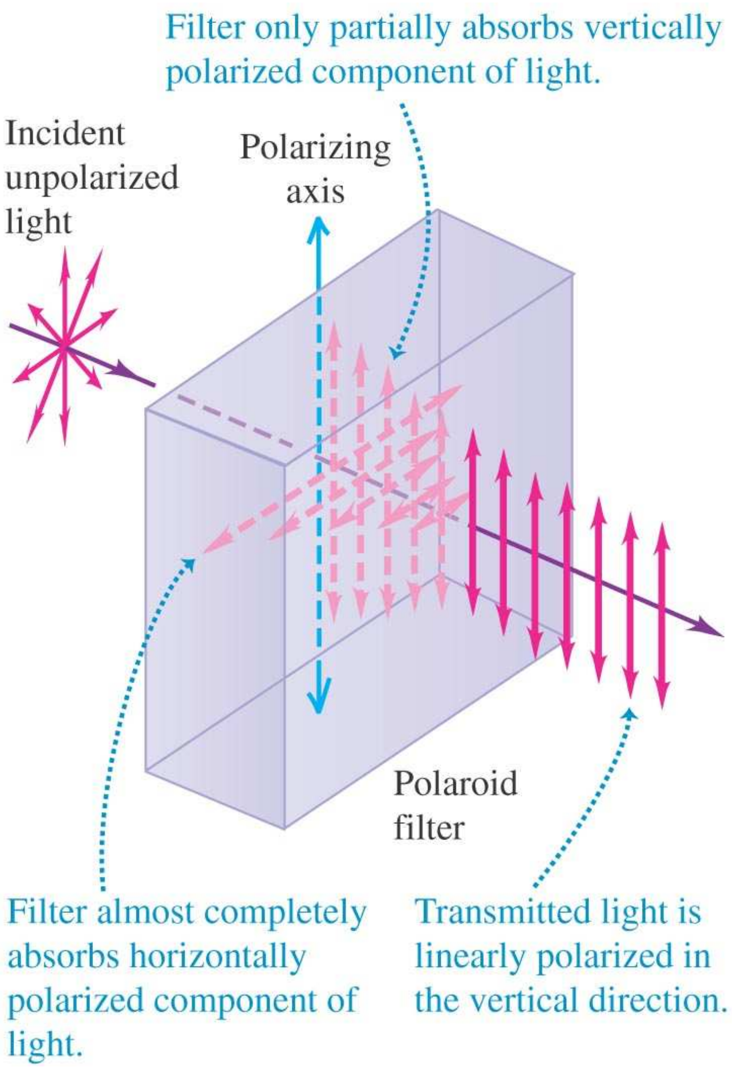
- The polarization of light from the sun and lamps is **unpolarized**, meaning that the polarization is random.
- A polarizing filter, or polarizer, **absorbs** the electric-field component along its polarizing axis, but lets the other component passes through.
- Suppose that the polarizer is at $x = 0$ and the input field is

$$\boldsymbol{E}_{\text{in}} = \left(\tilde{E}_y \hat{\boldsymbol{y}} + \tilde{E}_z \hat{\boldsymbol{z}}\right) \exp(jkx - j\omega t). \quad (12)$$

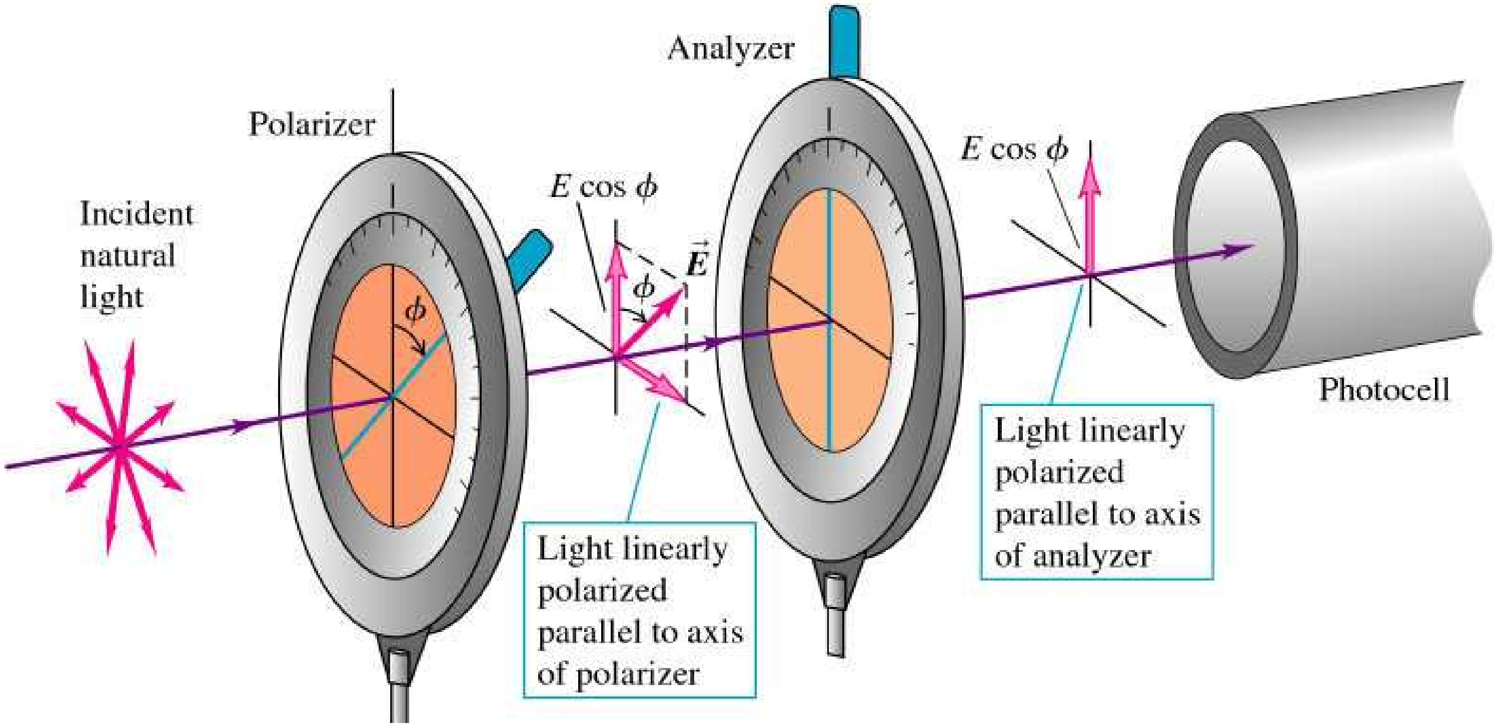
Suppose that the polarizing axis is $\hat{\boldsymbol{y}}$. The ideal behavior of a polarizer is to leave only the $\hat{\boldsymbol{y}}$ component at the output.

$$\boldsymbol{E}_{\text{out}} = \tilde{E}_y \hat{\boldsymbol{y}} \exp(jkx - j\omega t). \quad (13)$$

- Many sunglasses use polarizers to attenuate light.



Analyzer



- Suppose the first polarizing axis is $\cos\phi\hat{\boldsymbol{y}} + \sin\phi\hat{\boldsymbol{z}}$, and the second polarizing axis is $\hat{\boldsymbol{y}}$.
- Malus's law: In terms of intensity (time-averaged Poynting vector), the final output intensity is a factor of $\cos^2\phi$ smaller than the input intensity before the analyzer.
- Application: Liquid crystal display (LCD) `http://www.youtube.com/watch?v=k7xGQKpQAWw`

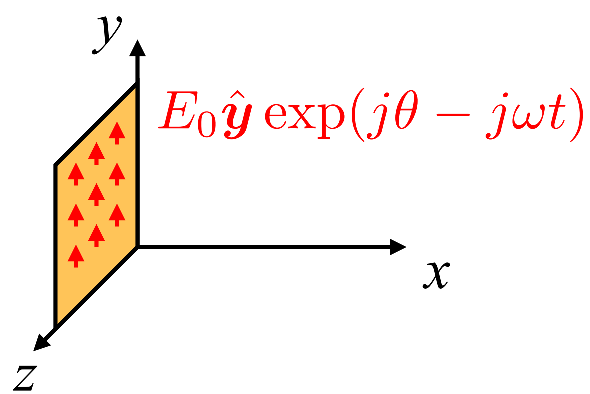
General Strategy of Solving Maxwell's Equations

1. Find as many solutions as you can to the equations.
2. Find a **superposition** of the solutions that satisfy the **boundary conditions** on $\boldsymbol{E}$ and $\boldsymbol{H}$.

- Simple example:

  Suppose that, at the 2D plane $x = 0$, the electric field is given by

$$\boldsymbol{E}(x=0,y,z,t) = \hat{\boldsymbol{y}} E_0 \exp(j\theta - j\omega t), \tag{14}$$



  which is assumed to be independent of y and z. What are the EM fields for $x > 0$?

  - We know that a solution is

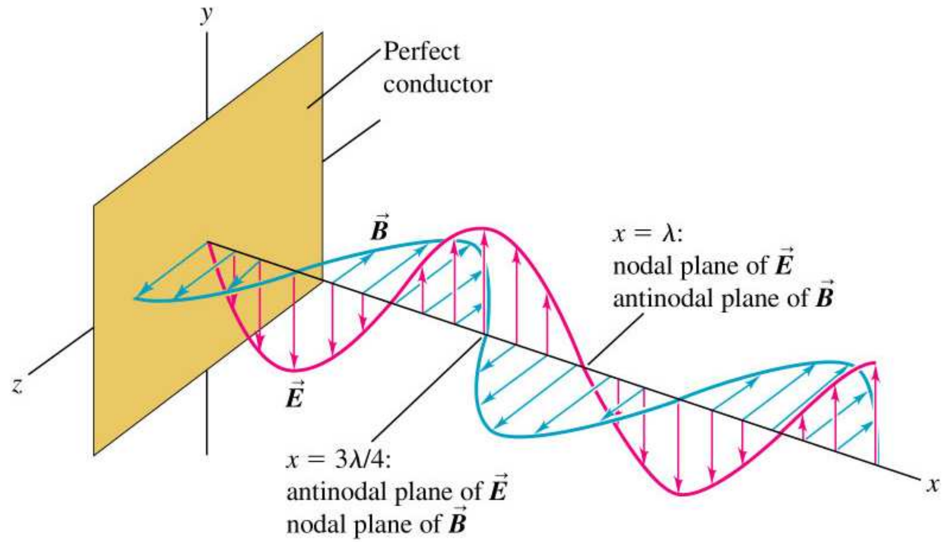
$$\boldsymbol{E}(\boldsymbol{r},t) = \hat{\boldsymbol{y}} \tilde{E} \exp(jkx - j\omega t), \qquad \boldsymbol{H}(\boldsymbol{r},t) = \hat{\boldsymbol{z}} \frac{\tilde{E}}{Z_0} \exp(jkx - j\omega t). \tag{15}$$

  - Plug in $x = 0$ for $\boldsymbol{E}(\boldsymbol{r},t)$,

$$\boldsymbol{E}(\boldsymbol{r},t)\Big|_{x=0} = \hat{\boldsymbol{y}} \tilde{E} \exp(-j\omega t) = \hat{\boldsymbol{y}} E_0 \exp(j\theta - j\omega t), \qquad \tilde{E} = E_0 e^{j\theta}. \tag{16}$$

  - Here, we see that ω and $\tilde{E}$ are determined by the boundary conditions. k is then determined by ω.
  - *Is this the only possible solution?

Reflection



- Consider a perfect conductor surface at $x = 0$. Boundary condition:

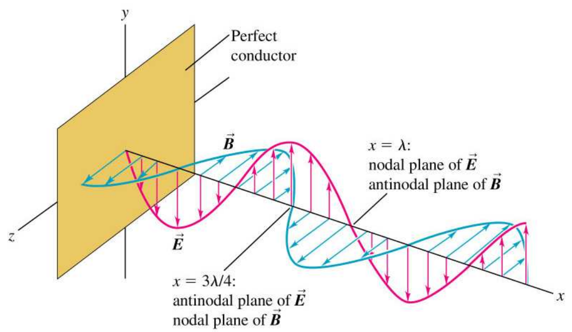
$$\boldsymbol{E}(x = 0, t) = 0. \tag{17}$$

- Consider an incoming wave propagating in $-\hat{\boldsymbol{x}}$ direction:

$$\boldsymbol{E}_{\text{in}}(x,t) = -\hat{\boldsymbol{y}}\tilde{E}\exp(-jkx - j\omega t), \qquad \boldsymbol{H}_{\text{in}}(x,t) = \hat{\boldsymbol{z}}\frac{\tilde{E}}{Z_0}\exp(-jkx - j\omega t). \tag{18}$$

This by itself doesn't satisfy the boundary condition.

Standing Wave



- To satisfy the boundary condition, let us exploit the **linearity** of the Maxwell's equations and find a new solution by adding different solutions together. The correct way is to add a **reflected** wave:

$$\boldsymbol{E} = \boldsymbol{E}_{\text{in}} + \boldsymbol{E}_{\text{out}}, \qquad \boldsymbol{E}_{\text{out}}(x,t) = \hat{\boldsymbol{y}}\tilde{E}\exp(jkx - j\omega t), \tag{19}$$

$$\boldsymbol{H} = \boldsymbol{H}_{\text{in}} + \boldsymbol{H}_{\text{out}}, \qquad \boldsymbol{H}_{\text{out}}(x,t) = \hat{\boldsymbol{z}}\frac{\tilde{E}}{Z_0}\exp(jkx - j\omega t). \tag{20}$$

  This ensures that $\boldsymbol{E}$ satisfies the wave equation for $x > 0$ and also the boundary condition at $x = 0$.
- Take $\tilde{E}$ to be real and positive for simplicity. The total fields are

$$\boldsymbol{E} = \hat{\boldsymbol{y}}2\tilde{E}\sin(kx)je^{-j\omega t}, \qquad \boldsymbol{H} = \hat{\boldsymbol{z}}2\frac{\tilde{E}}{Z_0}\cos(kx)e^{-j\omega t}. \tag{21}$$

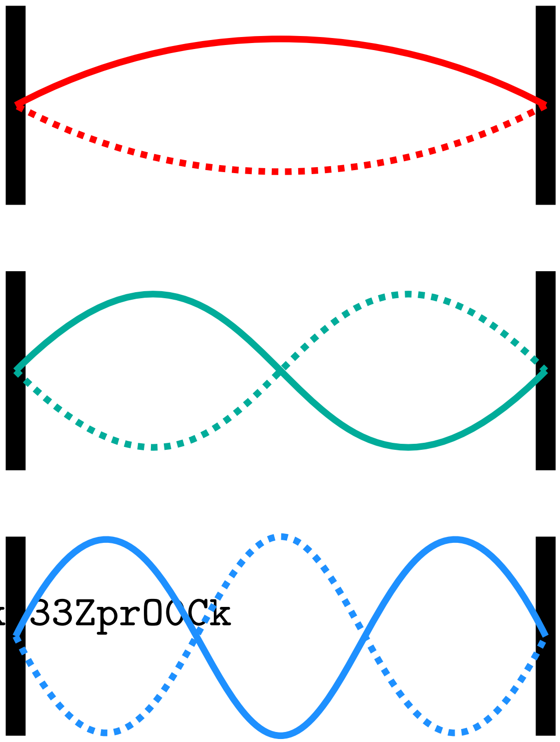
  This is a **standing wave**. The electric field is **always** zero at $x = m\lambda/2, m = 0, \pm1, \pm2, \dots$ (nodes). The magnetic field is **always** zero at $x = m\lambda/2 + \lambda/4$.
- http://en.wikipedia.org/wiki/Standing_wave

- Consider two parallel conductor surfaces separated by L. The standing-wave solution still works.
- There is an additional restriction however. For $\boldsymbol{E}=0$ at $x=0$ and $x=L$, the surfaces must be at the nodes, and

$$\frac{q\lambda}{2}=L, \qquad \lambda=\frac{2L}{q}, \qquad q=1,2,\ldots \tag{22}$$

  This means that only discrete wavelengths and frequencies can exist in a cavity:

$$\nu=\frac{c}{\lambda}=\frac{qc}{2L}. \tag{23}$$



- Example: microwave oven: `https://www.youtube.com/watch?v=k[illegible]33Zpr00Ck`
- An optical cavity is an important component of lasers (more on this later).

EM Waves in a Medium

- So far we have assumed vacuum (free space). In transparent media such as air, glasses, crystals, the Maxwell's equations must be modified.
- For our purpose, we consider mainly **dielectric media** where the only change of the equations is to replace ϵ_0 with the permittivity of the medium ϵ.
- If ϵ is constant, all the previous analyses are still valid, except that we need to replace ϵ_0 with ϵ. This means that the speed of light in a medium is now

$$v = \frac{1}{\sqrt{\epsilon\mu_0}}, \tag{24}$$

  which is different from c, the speed of light in vacuum.
- The **refractive index** n is defined as

$$n \equiv \frac{c}{v}. \tag{25}$$

  For dielectric media, $n = \sqrt{\epsilon/\epsilon_0}$. It is a measure of how much light is slowed down in a medium relative to vacuum. Usually $n \geq 1$.
- In most EM problems involving many media, the frequency ω is given and determined by the frequency of the external source or boundary condition. Because of the linear-time-invariant property of the Maxwell's equations, **the frequency ω is the same everywhere**, and only the wavelengths and the wavenumbers differ depending on the refractive indices:

$$k = \frac{\omega}{v} = \frac{\omega n}{c} = k_0 n, \qquad \lambda = \frac{2\pi}{k} = \frac{\lambda_0}{n}. \tag{26}$$

  k_0 and λ_0 denote free-space values. If $n \geq 1$, the wavelength λ is smaller in a medium.

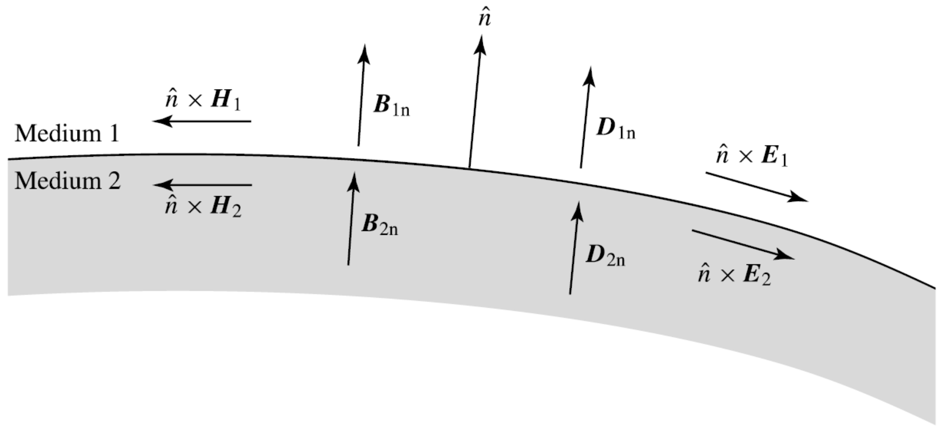
Refractive Indices of Some Materials

Substance	Index of Refraction, n
Solids	
Ice (H_2O)	1.309
Fluorite (CaF_2)	1.434
Polystyrene	1.49
Rock salt (NaC1)	1.544
Quartz (SiO_2)	1.544
Zircon ($ZrO_2 \cdot SiO_2$)	1.923
Diamond (C)	2.417
Fabulite ($SrTiO_3$)	2.409
Rutile (TiO_2)	2.62
Glasses (typical values)	
Crown	1.52
Light flint	1.58
Medium flint	1.62
Dense flint	1.66
Lanthanum flint	1.80

- This is a very simplistic treatment of EM in medium. In reality,
  - ϵ and n usually depend on frequency and are complex (lead to dispersion and loss, very important for optical communications),
  - ϵ may even be a matrix (tensor) instead of a scalar (speed depends on electric-field direction, polarization optics),
  - the permeability may also change from μ_0 (not usual in optics).

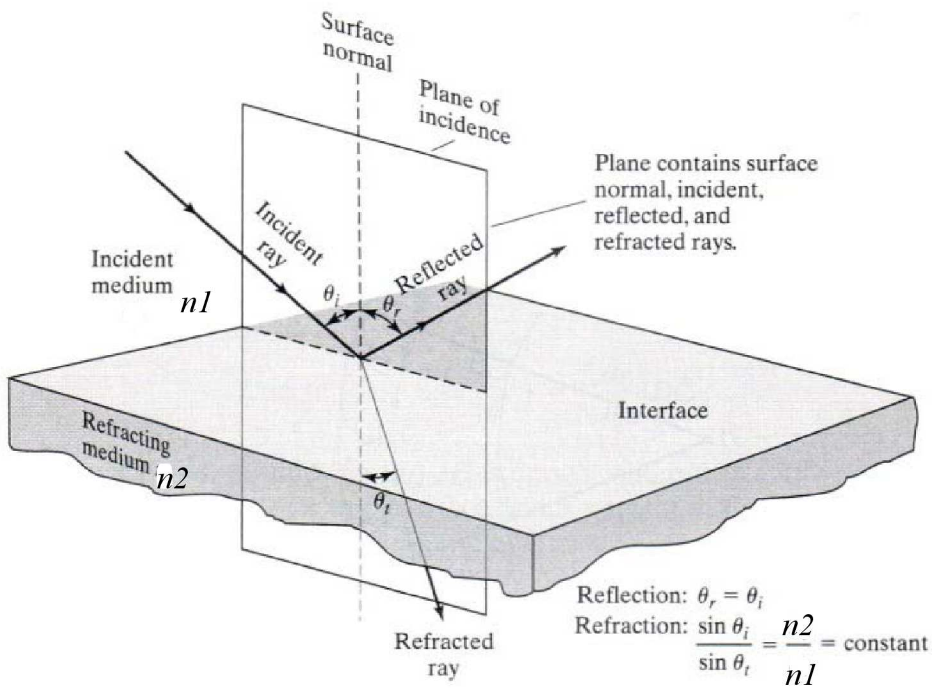
Boundary Conditions Between Two Media

- Suppose there are two media with different refractive indices and there is no free charge or free current along the interface. From Maxwell equations (integral form, see EE2011), we obtain the following boundary conditions at the interface:



$$\hat{\boldsymbol{n}} \times \boldsymbol{E}_1 = \hat{\boldsymbol{n}} \times \boldsymbol{E}_2, \qquad \hat{\boldsymbol{n}} \times \boldsymbol{H}_1 = \hat{\boldsymbol{n}} \times \boldsymbol{H}_2, \qquad \boldsymbol{B}_{1\mathrm{n}} = \boldsymbol{B}_{2\mathrm{n}}, \qquad \boldsymbol{D}_{1\mathrm{n}} = \boldsymbol{D}_{2\mathrm{n}}. \tag{27}$$

- $\hat{\boldsymbol{n}}$ is the unit vector perpendicular to the surface.
- Do not confuse $\hat{\boldsymbol{n}}$ here with refractive index. You should get used to recycling symbols and they may be used to mean different things in different contexts.
- The subscript n denotes that it is the component of the vector parallel to $\hat{\boldsymbol{n}}$.
- We will just assume $\boldsymbol{B} = \mu_0 \boldsymbol{H}$ and $\boldsymbol{D} = \epsilon \boldsymbol{E}$.
- **Linearity** still works when the problem involves many different media.



- Suppose that the first medium has refractive index n_1 and the second medium has refractive index n_2 with a flat interface. With an **incident** sinusoidal plane wave from medium 1:

$$\boldsymbol{E}_i = \tilde{\boldsymbol{E}}_i \exp(j\boldsymbol{k}_i \cdot \boldsymbol{r} - j\omega t), \tag{28}$$

  the complete solution (finding the right superposition of waves and matching of boundary conditions) involves a **reflected** plane wave and a **refracted** (also called transmitted) plane wave:
- Instead of going through the math in detail, I will just tell you the solutions.
- `PhET/en/simulation/bending-light.html`

Preliminary Calculations

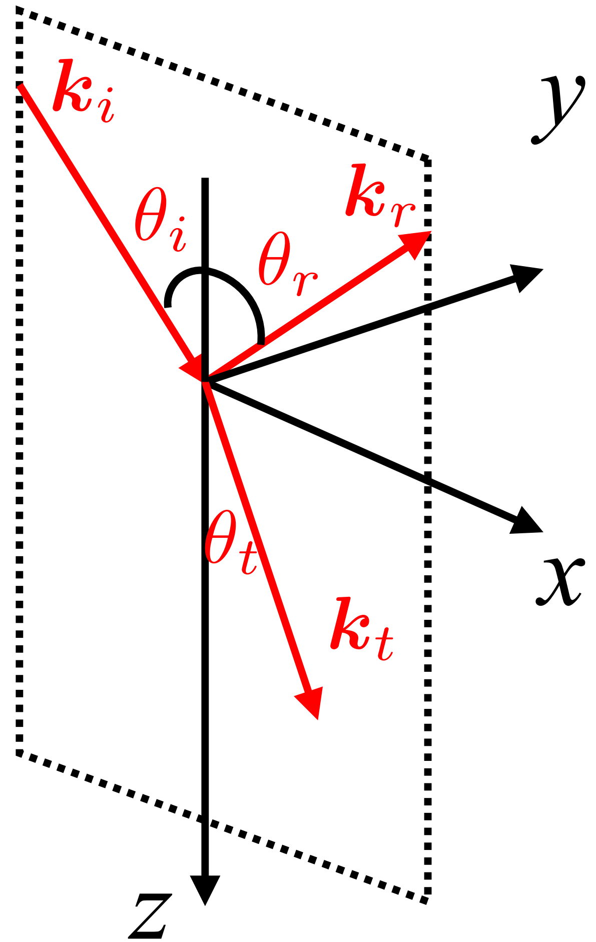
- Fact: all three waves oscillate at the same frequency ω (assuming field-independent refractive indices, Maxwell's equations are *linear-time-invariant*; see Signals and Systems module).
- Without loss of generality, suppose that the interface is at $z = 0$ (flat along x and y), and the incident wavevector $\boldsymbol{k}_i$ is in the x-z plane.
- Assume plane wave solutions:

$$z < 0 : \boldsymbol{E} = \boldsymbol{E}_i + \boldsymbol{E}_r, \tag{29}$$

$$\boldsymbol{E}_i = \tilde{\boldsymbol{E}}_i \exp(j\boldsymbol{k}_i \cdot \boldsymbol{r} - j\omega t), \tag{30}$$

$$\boldsymbol{E}_r = \tilde{\boldsymbol{E}}_r \exp(j\boldsymbol{k}_r \cdot \boldsymbol{r} - j\omega t), \tag{31}$$

$$z > 0 : \boldsymbol{E} = \boldsymbol{E}_t = \tilde{\boldsymbol{E}}_t \exp(j\boldsymbol{k}_t \cdot \boldsymbol{r} - j\omega t). \tag{32}$$



- Given ω, we require

$$|\boldsymbol{k}_i| = |\boldsymbol{k}_r| = \frac{\omega n_1}{c} \equiv k_1, \qquad |\boldsymbol{k}_t| = \frac{\omega n_2}{c} \equiv k_2. \tag{33}$$

- The incident angle is given by θ_i. This means

$$\boldsymbol{k}_i = k_1 \left(\sin\theta_i \hat{\boldsymbol{x}} + \cos\theta_i \hat{\boldsymbol{z}}\right). \tag{34}$$

- Fact: the wavevector components parallel to the interface are all equal:

$$\boldsymbol{k}_{i\parallel} = \boldsymbol{k}_{r\parallel} = \boldsymbol{k}_{t\parallel}. \tag{35}$$

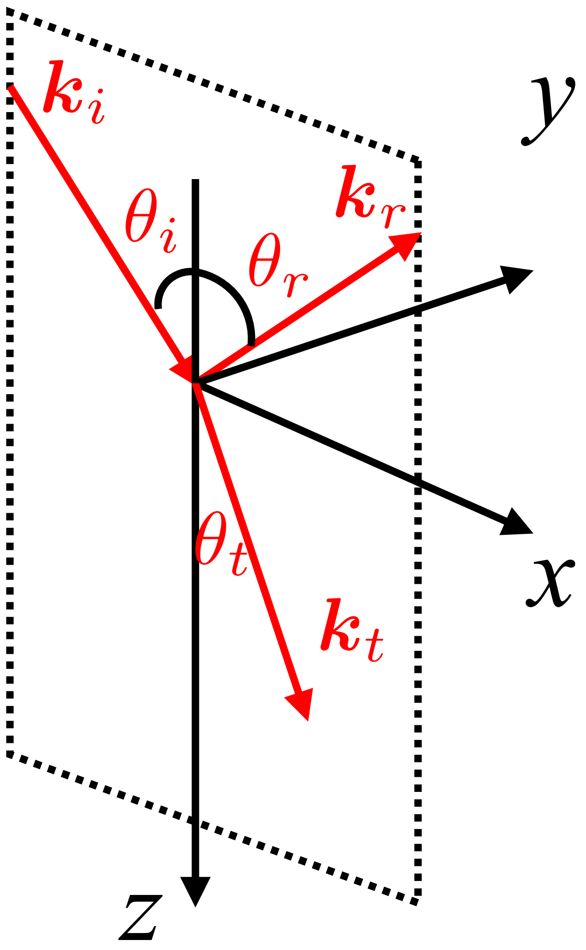
In our geometry, this means

$$k_1 \sin\theta_i \hat{\boldsymbol{x}} = k_1 \sin\theta_r \hat{\boldsymbol{x}} = k_2 \sin\theta_t \hat{\boldsymbol{x}}, \qquad k_2 \equiv \frac{\omega n_2}{c}, \tag{36}$$

$$\boxed{\theta_i = \theta_r,} \qquad \boxed{n_1 \sin\theta_i = n_2 \sin\theta_t.} \tag{37}$$

This means that the wavevectors are all **in the same plane** (no $\hat{\boldsymbol{y}}$ component), **incident angle is equal to the reflected angle**, and **Snell's law** $n_1 \sin\theta_i = n_2 \sin\theta_t$ holds.

- `PhET/sims/bending-light`

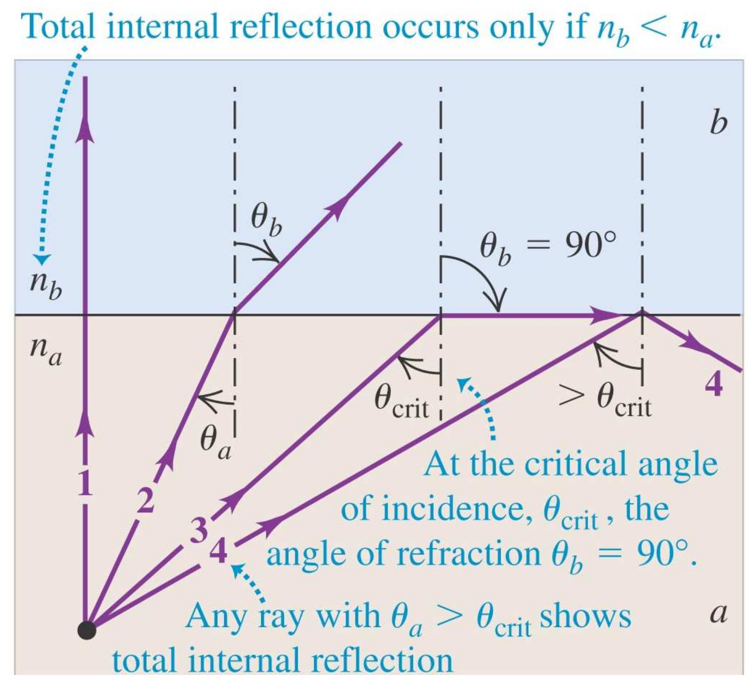


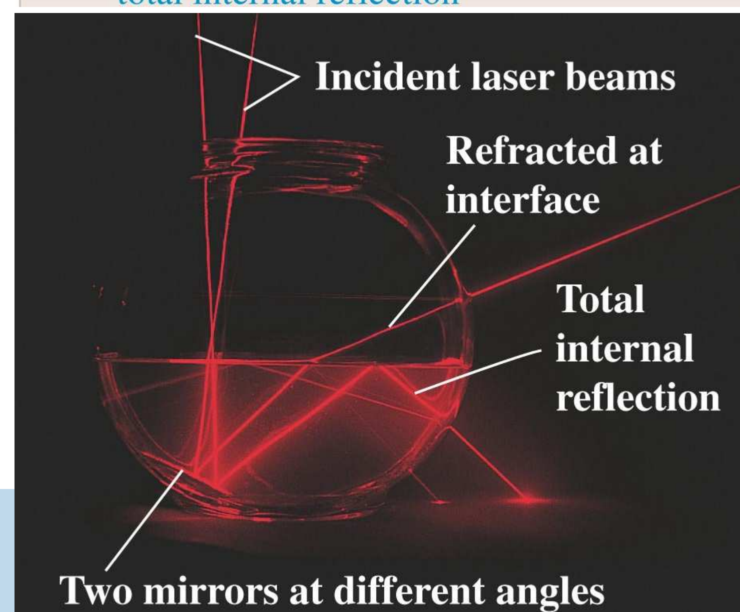
- Snell's law: $k_{ix} = n_1 \sin\theta_i = n_2 \sin\theta_t = k_{tx}$.
- If $n_1 > n_2$, we can have a situation where

$$n_1 \sin\theta_i > n_2, \qquad \sin\theta_t = \frac{n_1}{n_2}\sin\theta_i > 1. \tag{38}$$

  When this happens, the wavevector component along the interface is larger than the wavenumber $k_2 \equiv \omega n_2/c$ in the second medium!
- **Critical angle** θ_c is defined by

$$\boxed{n_1 \sin\theta_c = n_2.} \tag{39}$$





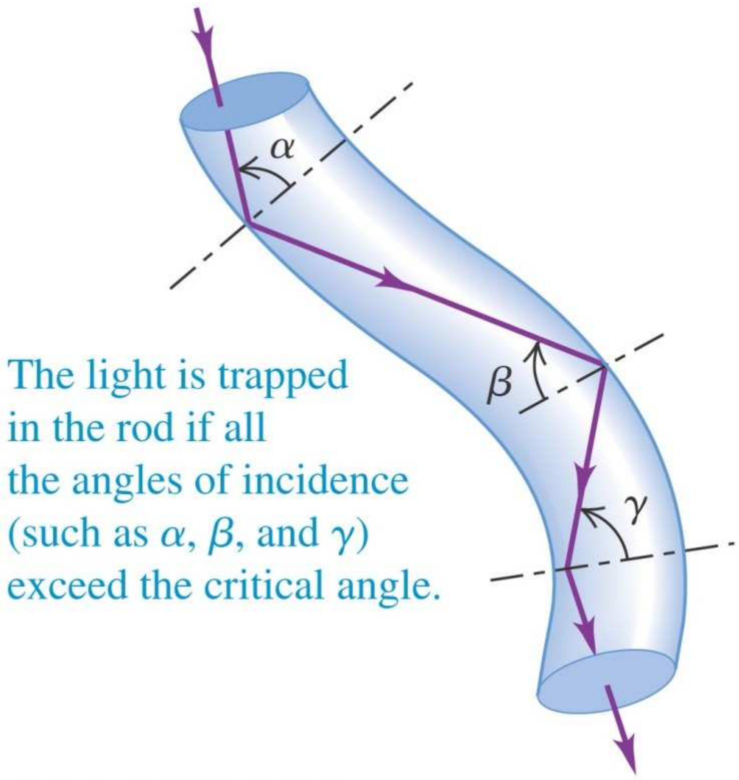
- The wave is completely reflected (**total internal reflection**) for any incident angle larger than the critical angle.
- *Recall $\sqrt{k_{tx}^2 + k_{tz}^2} = k_2$. What happens when k_{tx} is larger than k_2?

$$k_{tz} = \pm\sqrt{k_2^2 - k_{tx}^2} = \text{imaginary for } |k_{tx}| > k_2. \tag{40}$$

- *The refracted wave decays exponentially from surface (**evanescent wave**).

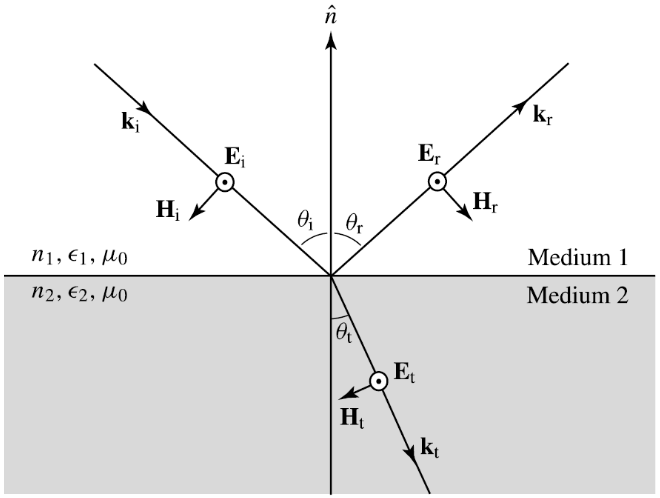
Application: Optical Fibers

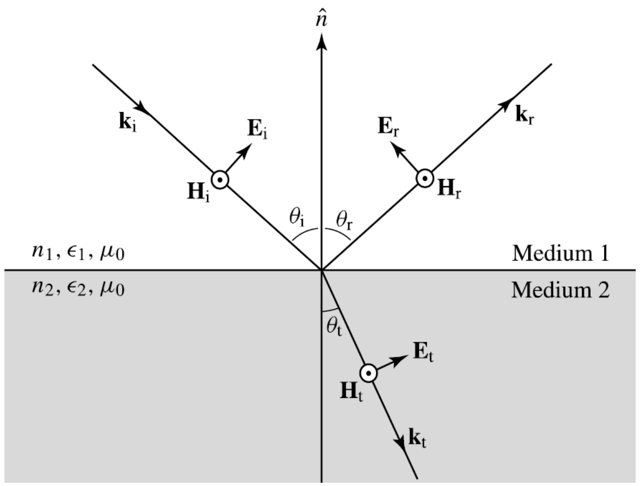
- An **optical fiber** consists of a tube of dielectric **core** surrounded by a cladding with lower refractive index.
- Light is trapped inside the core due to total internal reflection.
- Note that the interface is curved, plane wave solutions are not valid. The concept of TIR still applies to **cylindrical waves** however.
- **Optical communications**: Silica optical fibers have very low loss (~ 0.2 dB/km) across a very large bandwidth. Optical signals can be carried over very long distance without suffering from much loss.



TE and TM Polarizations

- Given an incident wavevector $\boldsymbol{k}_i$, two situations occur depending on the vectoral direction of the incident field $\tilde{\boldsymbol{E}}_i$.





- In the left figure, where $\tilde{\boldsymbol{E}}_i$ is parallel to the interface, the incident wave is called **TE (transverse-electric) polarized**, or s polarized.
- The electric fields of the incident and refracted waves are all parallel to the interface due to boundary conditions.
- In the right figure, where the magnetic field of the incident wave is parallel to the interface, the incident wave is called **TM (transverse-magnetic) polarized**, or p polarized. The magnetic fields of the incident and refracted waves are all parallel to the interface due to boundary conditions.
- Any other polarization for the same $\boldsymbol{k}_i$ can be written as a superposition of the TE and TM polarizations.

Reflection and Transmission Coefficients

- The reflected wave amplitude and the refracted wave amplitude can be computed by matching the boundary conditions.
- Let $\tilde{\boldsymbol{E}}_i = \hat{\boldsymbol{e}}_i \tilde{E}_i$, $\tilde{\boldsymbol{E}}_r = \hat{\boldsymbol{e}}_r \tilde{E}_r$, $\tilde{\boldsymbol{E}}_t = \hat{\boldsymbol{e}}_t \tilde{E}_t$. $\hat{\boldsymbol{e}}_i$, $\hat{\boldsymbol{e}}_r$, $\hat{\boldsymbol{e}}_t$ are unit vectors indicated in the previous figures.
- The important point here is that $\tilde{E}_r$ and $\tilde{E}_t$ are proportional to $\tilde{E}_i$. The proportionality coefficients depend on the polarization, the refractive indices, and the angles. Let

$$\boxed{\tilde{E}_r = r_s \tilde{E}_i,} \qquad \boxed{\tilde{E}_t = t_s \tilde{E}_i} \tag{41}$$

for TE polarization (s polarization) and

$$\boxed{\tilde{E}_r = r_p \tilde{E}_i,} \qquad \boxed{\tilde{E}_t = t_p \tilde{E}_i} \tag{42}$$

for TM polarization (p polarization).
- After some algebra, the reflection and transmission coefficients are

$$r_s \equiv \frac{\tilde{E}_r}{\tilde{E}_i} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}, \qquad t_s \equiv \frac{\tilde{E}_t}{\tilde{E}_i} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}, \tag{43}$$

for TE polarization.
- For TM, the reflection and transmission coefficients are

$$r_p \equiv \frac{\tilde{E}_r}{\tilde{E}_i} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}, \qquad t_p \equiv \frac{\tilde{E}_t}{\tilde{E}_i} = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}. \tag{44}$$

Suggested Problems

- What is the Poynting vector of a circularly/elliptically polarized wave? Intensity?
- Malus's law.
- What is the Poynting vector of a standing wave? Averaged over time? What is the energy absorbed by the mirror for perfect reflection?
- Microwave oven cavity: what is the spacing of the resonant frequencies for typical microwave oven size? What is q for microwave (roughly)? What if the cavity is filled with a dielectric with a certain refractive index?
- Wavelengths: for a given frequency, e.g., green, what is the wavelength in free space? in air? in water? in glass? in diamond?
- Refractive index: what is the speed of light and the refractive index if μ_0 is replaced by a different constant μ?
- Reflection and refraction: What is the Poynting vector in the two media?
- Total internal reflection: critical angle for air, water, glass, etc.?
- Brewster angle.
- Linearity: if two plane waves with different incident angles are incident on a flat surface, what is the resulting solution?