PC2232 Physics for Electrical Engineers

AY2014/15 Semester 2

MAKEUP MIDTERM TEST

Time Allowed: ONE hour THIRTY minutes

Instructions:

1. This is a closed-book test.

2. This paper contains 20 multiple-choice questions and 10 printed pages.

3. Each of the questions or incomplete statements is followed by five suggested answers or completions. Select the most appropriate choice in each case and then shade the corresponding bubble on the answer sheet.

4. Only the answer sheet will be collected at the end of the test. Answers written anywhere else will not be marked.

5. Use 2B pencil only. Using any other type of pencil or pen may result in answers unrecognizable by the machine.

6. Answer all questions. Marks will NOT be deducted for wrong answers.

7. Some formulae are given in pp. 9-10. Not all the formulae will be needed in the test.

8. You may not leave the test venue during the first thirty minutes and the last fifteen minutes of the test.

1. Suppose that $(\boldsymbol{E}_1, \boldsymbol{H}_1), (\boldsymbol{E}_2, \boldsymbol{H}_2), \ldots, (\boldsymbol{E}_N, \boldsymbol{H}_N)$ are all solutions of Maxwells equations in free space and they may be complex. Which of the following is also a solution?

   (A) $\left(\sum_{n=1}^{N} \boldsymbol{E}_n \exp(jn\phi), \sum_{n=1}^{N} \boldsymbol{H}_n \exp(-jn\phi)\right)$, where ϕ is an arbitrary constant

   (B) $\left(\sum_{n=1}^{N} \boldsymbol{E}_n \exp(jn\phi), \sum_{n=1}^{N} \boldsymbol{H}_n^* \exp(-jn\phi)\right)$, where ϕ is an arbitrary constant

   (C) $\left(\sum_{n=1}^{N} \boldsymbol{E}_n^* g^n, \sum_{n=1}^{N} \boldsymbol{H}_n^* g^n\right)$, where g is an arbitrary constant

   (D) $\left(\sum_{n=1}^{N} \boldsymbol{E}_n g^n, \sum_{n=1}^{N} \boldsymbol{H}_n^* g^n\right)$, where g is an arbitrary constant

   (E) $\left(\sum_{n=1}^{N} a_n \boldsymbol{E}_n, \sum_{n=1}^{N} a_n \boldsymbol{H}_n^*\right)$, where g is an arbitrary constant

2. Suppose that the time-averaged Poynting vector from the sun is $\bar{\boldsymbol{S}} \approx 1.4\hat{\boldsymbol{z}}$ kW/m^2, which is assumed to be uniform along x and y and pointing in the $\hat{\boldsymbol{z}}$ direction, as shown in the following figure. A solar panel with area A is at an angle with the x-y plane and absorbs all the solar power that impinges on it. Which of the following statements is true?

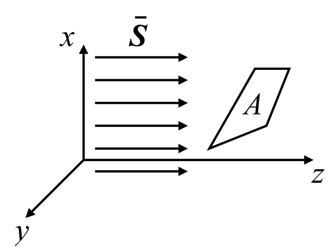
   (A) The absorbed power is 1.4 kW/m^2.

   (B) The absorbed power is equal to 1.4 kW/m^2 times A.

   (C) The absorbed power is more than 1.4 kW/m^2 times A.

   (D) The absorbed power is less than 1.4 kW/m^2 times A.

   (E) None of the above/not enough information given.



3. An electromagnetic plane wave in free space has the following fields:

$$\boldsymbol{E} = (\hat{\boldsymbol{x}}\cos\theta + \hat{\boldsymbol{y}}\sin\theta)E_{\max}\cos(kz - \omega t), \tag{1}$$

$$\boldsymbol{H} = (\hat{\boldsymbol{y}}\cos\theta - \hat{\boldsymbol{x}}\sin\theta)\frac{E_{\max}}{Z_0}\cos(kz - \omega t). \tag{2}$$

   What is the time-averaged energy of this wave in a box with volume L^3?

   (A) $\epsilon_0 E_{\max}^2 L^3$.

   (B) $\frac{1}{2}\epsilon_0 E_{\max}^2 L^3$.

   (C) $\frac{1}{2}(\cos\theta)\epsilon_0 E_{\max}^2 L^3$.

   (D) $\frac{1}{2}(\cos^2\theta)\epsilon_0 E_{\max}^2 L^3$.

   (E) ∞.

4. Consider an elliptically polarized plane wave in free space given by $\boldsymbol{E} = \tilde{E}\left(\cos\theta\hat{\boldsymbol{x}} + e^{i\phi}\sin\theta\hat{\boldsymbol{y}}\right)\exp(jkz - j\omega t)$, where ϕ and θ are arbitrary constants. What is its time-averaged Poynting vector?

   (A) $\frac{1}{2Z_0}|\tilde{E}|^2\hat{\boldsymbol{z}}$.

   (B) $\frac{1}{2Z_0}|\tilde{E}|^2\left(\sin^2\theta\right)\hat{\boldsymbol{z}}$.

   (C) $\frac{1}{2Z_0}|\tilde{E}|^2\left(\cos^2\theta\right)\hat{\boldsymbol{z}}$.

   (D) $\frac{1}{2Z_0}|\tilde{E}|^2\left(\cos\theta + \sin\theta\right)\hat{\boldsymbol{z}}$.

   (E) $\frac{1}{2Z_0}|\tilde{E}|^2(\tan\theta)\hat{\boldsymbol{z}}$.

5. Consider a perfectly conducting mirror in the x-y plane at $z = 0$, which imposes a boundary condition $\boldsymbol{E}(x, y, z = 0) = 0$. Assuming a solution of

$$\boldsymbol{E} = \hat{\boldsymbol{y}}\left[\tilde{E}_{\text{out}}\exp(jk_x x + jk_z z - j\omega t) + \tilde{E}_{\text{in}}\exp(jk_x x - jk_z z - j\omega t)\right] \qquad (3)$$

   for $z \geq 0$ in free space. The reflection coefficient $\tilde{E}_{\text{out}}/\tilde{E}_{\text{in}}$ is

   (A) 1.

   (B) -1.

   (C) $\tan^{-1}(k_x/k_z)$.

   (D) $-\tan^{-1}(k_x/k_z)$.

   (E) $1/\sqrt{2}$.

6. Consider an electromagnetic cavity with two perfectly conducting mirrors at $z = 0$ and $z = L$. Assuming the same standing-wave solution in Eq. (3), what is the restriction on the wavenumber k_z?

   (A) $k_z = q$, $q = 1, 2, \ldots$

   (B) $k_z = \pi q$, $q = 1, 2, \ldots$

   (C) $k_z L = q$, $q = 1, 2, \ldots$

   (D) $k_z L = \pi q$, $q = 1, 2, \ldots$

   (E) $k_z L = 2\pi q$, $q = 1, 2, \ldots$

7. Consider the reflection and refraction at a flat interface between two media, as shown in the following figure. The bottom medium has a permittivity constant of $\epsilon_1 = 1.5\epsilon_0$ and the top medium is free space. Total internal reflection occurs when

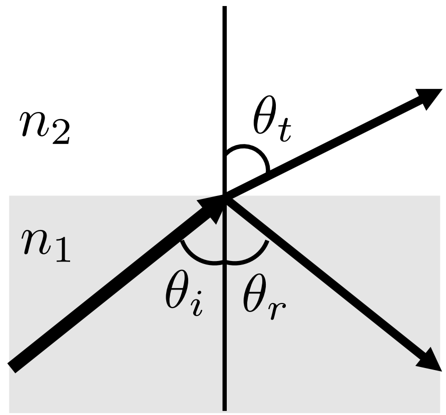
   (A) $\theta_i > \sin^{-1}(1/1.5)$.

   (B) $\theta_i < \sin^{-1}(1/1.5)$.

   (C) $\theta_i > \sin^{-1}\left[(1/1.5)^2\right]$.

   (D) $\theta_i < \sin^{-1}\left[(1/1.5)^2\right]$.

   (E) $\theta_i > \sin^{-1}(\sqrt{1/1.5})$.



8. Consider two optical plane waves impinging on a flat photoresist at $z = 0$ with identical amplitude, as shown in the following figure. Which of the following is a way of reducing the period of the resulting interference pattern?

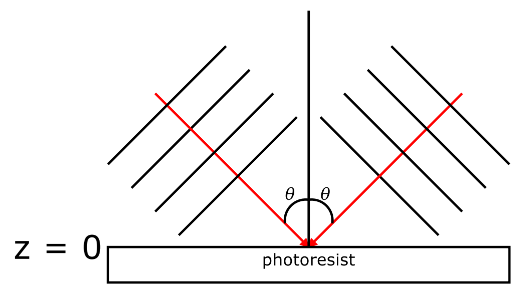
   (A) Decrease the frequency of light.

   (B) Increase the wavelength of light.

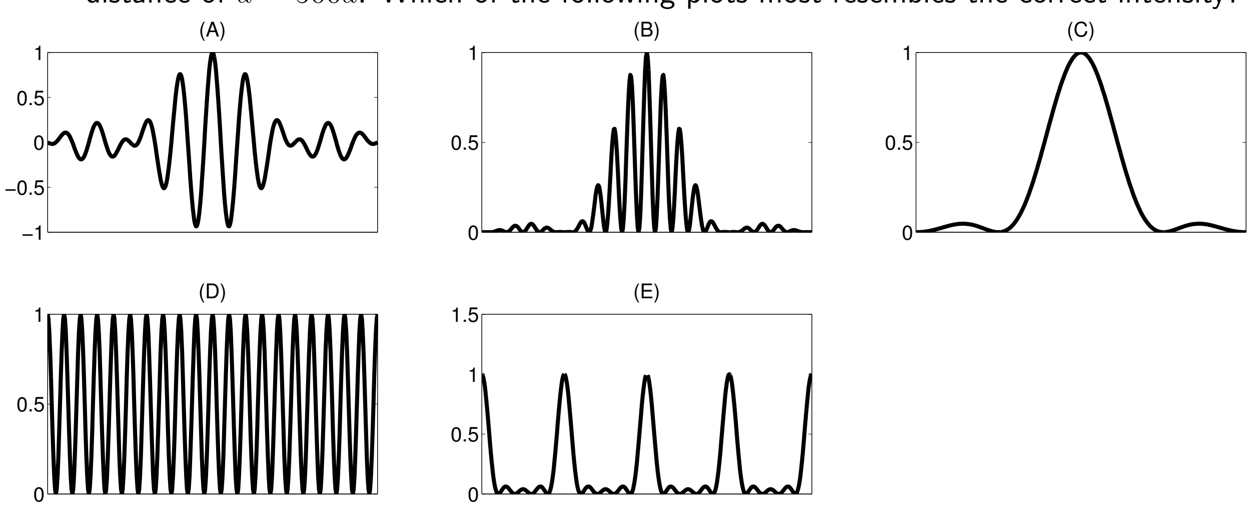
   (C) Increase the electric-field amplitude.

   (D) Decrease the incident angle θ.

   (E) Increase the refractive index of the medium.



9. Consider the Fraunhofer diffraction of two slits, each with width a and separated by a distance of $d = 500a$. Which of the following plots most resembles the correct intensity?



10. Consider the focusing of a plane wave onto a spot on the focal plane by a focusing lens with finite aperture. Which of the following is NOT a way of reducing the focus spot size?

   (A) Increase the lens aperture size while keeping the focal length the same.

   (B) Increase the focal length while keeping the lens aperture size the same.

   (C) Increase the frequency of light.

   (D) Reduce the wavelength of light.

   (E) Increase the refractive index of the medium between the lens and the focal plane.

11. Which of the following is a valid scattering matrix for a lossless (power-conserving) beam splitter?

   (A) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, where θ is an arbitrary real constant.

   (B) $\begin{pmatrix} \cos\theta & e^{i\phi}\sin\theta \\ e^{i\phi}\sin\theta & \cos\theta \end{pmatrix}$, where θ is an arbitrary real constant.

   (C) $\begin{pmatrix} \cos\theta & e^{i\phi}\sin\theta \\ -e^{-i\phi}\sin\theta & \cos\theta \end{pmatrix}$, where θ and ϕ are arbitrary real constants.

   (D) $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$.

   (E) $\begin{pmatrix} 1/\sqrt{2} & j/\sqrt{2} \\ -j/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$.

12. Consider the Mach-Zehnder interferometer in the following figure. Suppose that the first input amplitude is $\tilde{E}_1$, the second input amplitude is $\tilde{E}_2$, the beam splitters are identical with the following scattering matrix:

$$\begin{pmatrix} \tilde{E}_1' \\ \tilde{E}_2' \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} \tilde{E}_1 \\ \tilde{E}_2 \end{pmatrix}, \qquad \begin{pmatrix} \tilde{E}_{\text{out}1} \\ \tilde{E}_{\text{out}2} \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} \tilde{E}_1'' \\ \tilde{E}_2'' \end{pmatrix}, \tag{4}$$

$$\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}, \tag{5}$$

and the two arms have the same length, identical perfect mirrors, and the same refractive index. What are $|\tilde{E}_{\text{out}1}|^2$ and $|\tilde{E}_{\text{out}2}|^2$?

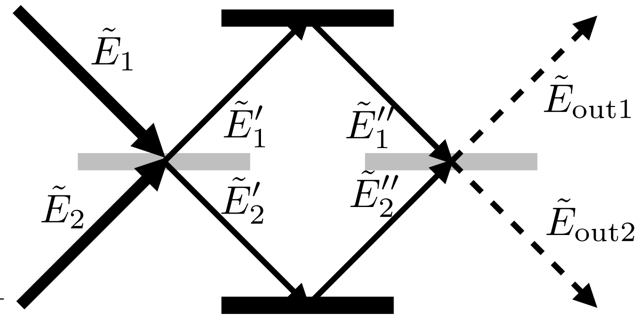
(A) $|\tilde{E}_{\text{out}1}|^2 = |\tilde{E}_1|^2, |\tilde{E}_{\text{out}2}|^2 = |\tilde{E}_2|^2$.

(B) $|\tilde{E}_{\text{out}1}|^2 = |\tilde{E}_2|^2, |\tilde{E}_{\text{out}2}|^2 = |\tilde{E}_1|^2$.

(C) $|\tilde{E}_{\text{out}1}|^2 = 0$, $|\tilde{E}_{\text{out}2}|^2 = |\tilde{E}_2|^2$.

(D) $|\tilde{E}_{\text{out}1}|^2 = |\tilde{E}_1|^2$, $|\tilde{E}_{\text{out}2}|^2 = 0$.

(E) $|\tilde{E}_{\text{out}1}|^2 = |\tilde{E}_{\text{out}2}|^2 = (|\tilde{E}_1|^2 + |\tilde{E}_2|^2)/2$.



13. Consider a Fabry-Perot interferometer consisting of a lossless dielectric slab with refractive index n in free space and normal incidence ($\theta_i = 0$). The power transmission coefficient is

$$\frac{|\tilde{E}_{\text{out}2}|^2}{|\tilde{E}_1|^2} = \frac{|s_{12}s_{21}|^2}{|1 - s_{22}^2 e^{2jkL}|^2}, \qquad k = \frac{\omega n}{c}. \tag{6}$$

For a given free-space wavelength λ_0, what is the thickness L such that the power reflection is maximum?

(A) $L = q\lambda_0/2, q = 0, 1, 2, \ldots$

(B) $L = \lambda_0/4 + q\lambda_0/2, q = 0, 1, 2, \ldots$

(C) $L = q\lambda_0/(2n), q = 0, 1, 2, \ldots$

(D) $L = \lambda_0/(4n) + q\lambda_0/(2n), q = 0, 1, 2, \ldots$

(E) $L = \pi\lambda_0/(4n) + \pi q\lambda_0/(2n), q = 0, 1, 2, \ldots$

14. If the optical loss coefficient of a lossy medium measured in decibel is 0.2 dB/km and $S(z)$ is the optical intensity of a plane wave at propagation distance z, what is $S(z)$ after 50 km?

(A) 10^{-10}.

(B) 2.06×10^{-9}.

(C) 10^{-1}.

(D) 0.135.

(E) Not enough information given.

15. What is the group velocity of a nonrelativistic free quantum particle with mass m and wavenumber k?

   (A) $\hbar k/m$.

   (B) $\hbar k/(2m)$.

   (C) c.

   (D) $\hbar^2 k^2/(2m)$.

   (E) c/m.

16. Consider a quarter-wave plate that introduces a net phase difference $\theta = \pi/2$ (in radian) between a wave polarized in $\hat{\boldsymbol{x}}$ and a wave polarized in $\hat{\boldsymbol{y}}$. If the input is a linearly polarized wave with polarization in the direction $\hat{\boldsymbol{x}}$, what is the output polarization vector?

   (A) $\hat{\boldsymbol{x}}$.

   (B) $\hat{\boldsymbol{y}}$.

   (C) $(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})/\sqrt{2}$.

   (D) $(\hat{\boldsymbol{x}} - j\hat{\boldsymbol{y}})/\sqrt{2}$.

   (E) $(\hat{\boldsymbol{x}} + j\hat{\boldsymbol{y}})/\sqrt{2}$.

17. An electron can have two spin values, $s_z = 1/2$ or $s_z = -1/2$. The wavefunction for the x spin component in terms of that for the z component is

$$\begin{aligned} \psi_x(s_x = +1/2) &= \left[\psi_z(s_z = +1/2) + \psi_z(s_z = -1/2)\right]/\sqrt{2}, \\ \psi_x(s_x = -1/2) &= \left[\psi_z(s_z = +1/2) - \psi_z(s_z = -1/2)\right]/\sqrt{2}. \end{aligned} \tag{7}$$

   Suppose that s_z has been measured and the outcome is $s_z = -1/2$. If one then measures s_z again, what is the probability distribution of the second outcome with respect to s_z?

   (A) $P_z(s_z = 1/2) = 1/\sqrt{2}$, $P_z(s_z = -1/2) = 1/\sqrt{2}$.

   (B) $P_z(s_z = 1/2) = 1/\sqrt{2}$, $P_z(s_z = -1/2) = -1/\sqrt{2}$.

   (C) $P_z(s_z = 1/2) = 0$, $P_z(s_z = -1/2) = 1$.

   (D) $P_z(s_z = 1/2) = 1$, $P_z(s_z = -1/2) = 0$.

   (E) $P_z(s_z = 1/2) = 1/2$, $P_z(s_z = -1/2) = 1/2$.

18. Given Eqs. (7) in the previous question, what is $\psi_z(s_z)$ if $\psi_x(s_x = 1/2) = 1/\sqrt{2}$ and $\psi_x(s_x = -1/2) = 1/\sqrt{2}$?

(A) $\psi_z(s_z = 1/2) = 0$, $\psi_z(s_z = -1/2) = 1$.

(B) $\psi_z(s_z = 1/2) = 1$, $\psi_z(s_z = -1/2) = 0$.

(C) $\psi_z(s_z = 1/2) = 1/\sqrt{2}$, $\psi_z(s_z = -1/2) = 1/\sqrt{2}$.

(D) $\psi_z(s_z = 1/2) = 1/\sqrt{2}$, $\psi_z(s_z = -1/2) = -1/\sqrt{2}$.

(E) Not enough information given.

19. For an electron in free space with a flat potential $U(\boldsymbol{r}) = U_0$, where U_0 is a constant, and total energy $E = 2U_0$, how is its de Broglie wavelength changed if the total energy E is increased to $4U_0$?

(A) the wavelength is decreased by a factor of 2.

(B) the wavelength is decreased by a factor of 3.

(C) the wavelength is decreased by a factor of $\sqrt{2}$.

(D) the wavelength is decreased by a factor of $\sqrt{3}$.

(E) the wavelength is decreased a factor of $\sqrt{4}$.

20. Consider a potential $U(x) = U_0$ for $x < 0$ and $U(x) = 0$ for $x \geq 0$ for a one-dimensional quantum nonrelativistic particle with energy E, as shown in the following figure. If $E < U_0$ and $\tilde{\psi}(x) = A\exp(ikx) + B\exp(-ikx)$ for $x \geq 0$, what is A/B in the limit of $U_0 \to \infty$?

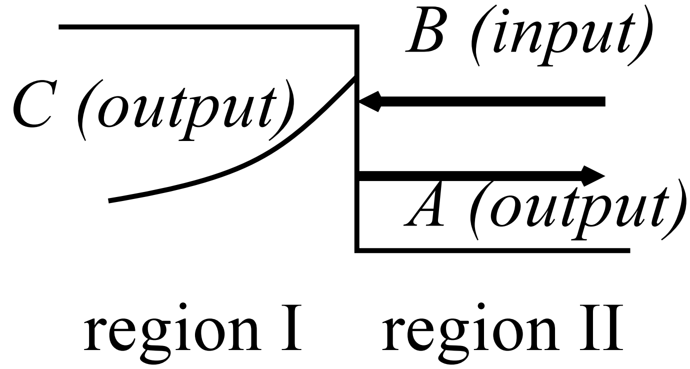
(A) 0.

(B) 1.

(C) -1.

(D) $1/\sqrt{2}$.

(E) $-1/\sqrt{2}$.



END OF PAPER

Appendix A: Table of Information

1. Maxwell's equations in free space

$$\nabla \cdot \boldsymbol{E} = 0, \qquad \nabla \cdot \boldsymbol{H} = 0, \qquad \nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}, \qquad \nabla \times \boldsymbol{H} = \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}. \tag{A1}$$

2. Speed of light in free space

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}. \tag{A2}$$

3. Free-space impedance

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{A3}$$

4. Dispersion relation in free space

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi\nu}{c}. \tag{A4}$$

5. Poynting vector

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}. \tag{A5}$$

6. Electromagnetic energy density

$$u = \frac{1}{2}\epsilon_0 \boldsymbol{E} \cdot \boldsymbol{E} + \frac{1}{2}\mu_0 \boldsymbol{H} \cdot \boldsymbol{H}. \tag{A6}$$

7. Snell's law

$$n_1 \sin\theta_i = n_2 \sin\theta_t, \tag{A7}$$

   where θ_i is the incident angle, θ_t is the transmitted angle, $n_1 = \sqrt{\epsilon_1/\epsilon_0}$ is the refractive index of the medium with the incident light, and $n_2 = \sqrt{\epsilon_2/\epsilon_0}$ is that of the medium with the transmitted light.

8. Fraunhofer diffraction

$$\mathcal{E}(x, z) \propto \int_{-\infty}^{\infty} dx' \mathcal{E}(x', 0) \exp\left(-j\frac{2\pi x x'}{\lambda z}\right). \tag{A8}$$

9. Unitary matrix

$$\boldsymbol{s}\boldsymbol{s}^\dagger = \boldsymbol{I}, \qquad \left(\boldsymbol{s}^\dagger\right)_{nm} = \boldsymbol{s}^*_{mn}. \tag{A9}$$

10. Fresnel reflection and transmission coefficients

- TE polarization (s-polarization):

$$r_s \equiv \frac{\tilde{E}_r}{\tilde{E}_i} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}, \qquad t_s \equiv \frac{\tilde{E}_t}{\tilde{E}_i} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t}, \tag{A10}$$

- TM polarization (p-polarization):

$$r_p \equiv \frac{\tilde{E}_r}{\tilde{E}_i} = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t}, \qquad t_p \equiv \frac{\tilde{E}_t}{\tilde{E}_i} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}. \tag{A11}$$

11. Phase and group velocities:

$$v_p = \frac{\omega}{k}, \qquad v_g = \frac{d\omega}{dk}. \tag{A12}$$

12. Energy and momentum of a quantum plane wave

$$\psi(\boldsymbol{r},t) \propto \exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t), \qquad E = \hbar\omega, \qquad \boldsymbol{p} = \hbar\boldsymbol{k}. \tag{A13}$$

13. Born's rule

$$P(x,t) = |\psi(x,t)|^2. \tag{A14}$$

14. Kinetic energy

$$E = \frac{p^2}{2m}. \tag{A15}$$

15. Time-dependent Schrödinger equation

$$i\hbar\frac{\partial\psi(\boldsymbol{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\boldsymbol{r})\right]\psi(\boldsymbol{r},t). \tag{A16}$$

16. Time-independent Schrödinger equation

$$E\tilde{\psi}(\boldsymbol{r}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\boldsymbol{r})\right]\tilde{\psi}(\boldsymbol{r}). \tag{A17}$$

17. One-dimensional time-independent Schrödinger equation

$$E\tilde{\psi}(x) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right]\tilde{\psi}(x). \tag{A18}$$