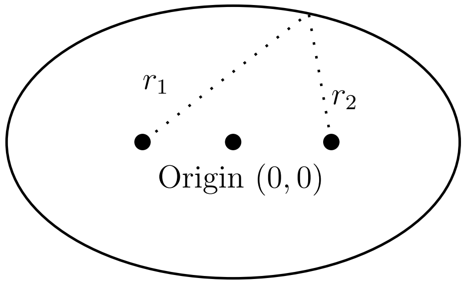
PC2232 Physics for Electrical Engineers: Tutorial 9

Question 1:

Mathematically an ellipse can be expressed as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{1}$$

where a and b are constants. Starting from the statement that the sum of the distance from any point on the ellipse to the two foci remains the same, i.e. the distance $r_1 + r_2 = 2a$ as shown in the following diagram, show that it can lead to Eq. (1). Express the locations of the foci in terms of a and b.



Assuming that you are making a Quantum Mirage with a STM, use a graph paper and draw to scale an ellipse defined by the equation

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1 \tag{2}$$

where all the units are expressed in cm. Mark clearly on your graph where you should place an extra atom inside the quantum corrals and where you expect to find the mirage. One can choose to form the quantum mirage with one ring of atoms, however, what is the advantage of making the mirage with two or even more rings of atoms?

Question 2:

For the cubic 3D infinite well wave function

$$\psi(x, y, z) = A \sin\frac{n_x \pi x}{L} \sin\frac{n_y \pi y}{L} \sin\frac{n_z \pi z}{L}, \tag{3}$$

show that the correct normalization constant is $A = (2/L)^{3/2}$.

Question 3:

In two dimensions, the Schrödinger equation is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi(x,y) = -\frac{2m}{\hbar^2}(E-U)\,\psi(x,y). \tag{4}$$

(a) For an infinite square well,

$$U(x,y) = \begin{cases} 0, & \text{if } 0 < x < L, \quad 0 < y < L \\ \infty, & \text{otherwise} \end{cases} \tag{5}$$

Find the wave functions.

(b) What are the allowed energies E?

(c) List the five lowest energy levels. For each level, write down the number of different states that correspond to that energy.

Question 4: Wavefunction in a 2D potential box

Use any graph plotting software that you can find to draw the wavefunction ψ and $|\psi|^2$ for particle trapped in 2D infinite potential box ($L \times L$). Draw the case for

(a) $n_x = 1$, $n_y = 1$;

(b) $n_x = 1$, $n_y = 2$;

(c) $n_x = 2$, $n_y = 2$;

(d) $n_x = 2$, $n_y = 3$.

(Note: even if you cannot find the software, you can use hand-sketched diagrams).

Question 5:

Consider a cubic 3D infinite well of side length L. There are 15 identical particles of mass m in the well. For some reason, no more than two particles may occupy the same wave function.

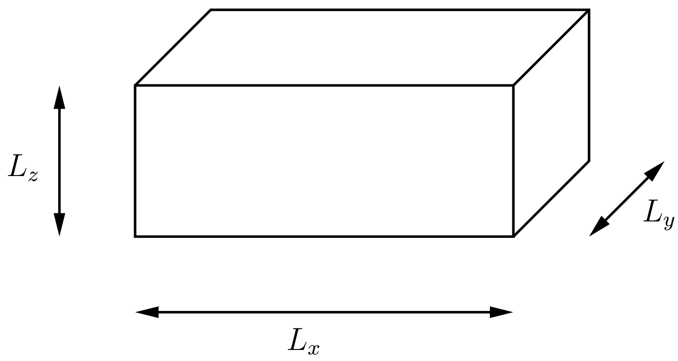
(a) What is the lowest possible *total* energy?

(b) In this minimum total-energy state, at what point(s) would the highest energy particle most likely be found?

(*Note*: Knowing no more than its energy, the highest-energy particle might be in any multiple wavefunction open to it and with equal probability.)

Question 6:

A quantum box as shown in the following figure has the following dimensions: $L_x = 50$ nm, $L_y = 4$ nm, and $L_z = 4$ nm. Can you list the quantum numbers for the first 20 states of such a system? What do you notice? What are the expected lines in the absorption spectrum for an electron (assumed in ground state) confined in this box? (List the first 5 lines with longest wavelength.)



Question 7:

Another quantum box is illustrated in the figure below. Can you create an excel file to compute the energy levels for this system? From this you should be able to identify the first 20 energy levels for this system. Can you point out some of the degenerate states?