PC2232 (lecture 4) 08/02/2015

(1) Time averaged Poynting vector:

$$\vec{S} = \overline{Re\vec{E} \times Re\vec{H}} = \frac{1}{2}Re\left(\vec{E} \times \vec{H}^*\right)$$

Intensity for a plane wave in medium with refractive index n:

$$I = |\vec{S}| = \frac{n|\tilde{E}|^2}{2Z_0}$$

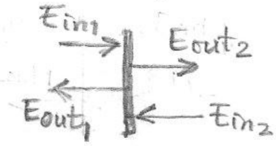
(2) Propagation of time delay:

For a plane wave propagating in x direction in a medium:

$$\tilde{E}_2 = \tilde{E}_1 e^{jKL}$$ where: $\begin{cases} \tilde{E}_1 = \text{Amplitude at } x = x_1 \\ \tilde{E}_2 = \text{Amplitude at } x = x_1 + L \end{cases}$

phase difference only

(3) For a partially reflecting surface:



$$\tilde{E}_{out1} = S_{11}\tilde{E}_{in1} + S_{12}\tilde{E}_{in2}$$

$$\tilde{E}_{out2} = S_{21}\tilde{E}_{in1} + S_{22}\tilde{E}_{in2}$$

$\tilde{E}_{in_1}$ = input at side 1
$\tilde{E}_{in_2}$ = input at side 2
$\tilde{E}_{out_1}$ = output at side 1
$\tilde{E}_{out2}$ = output at side 2

$$\Rightarrow \begin{pmatrix} \tilde{E}_{out_1} \\ \tilde{E}_{out_2} \end{pmatrix} = \underbrace{\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}}_{\text{scattering matrix}} \begin{pmatrix} \tilde{E}_{in_1} \\ \tilde{E}_{in_2} \end{pmatrix}$$

(4) Intensity distribution for loss-less surface:

$$I_{in1} + I_{in2} = I_{out1} + I_{out2}$$

$$\Rightarrow |\tilde{E}_{in_1}|^2 + |\tilde{E}_{in_2}|^2 = |E_{out1}|^2 + |E_{out2}|^2$$

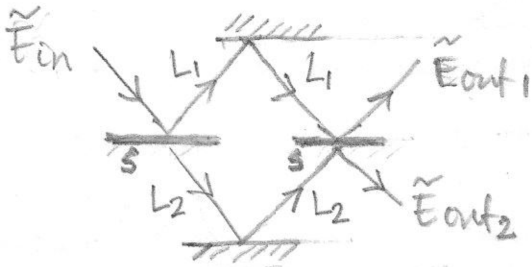
(5) For a 50-50 loss-less beam splitter:

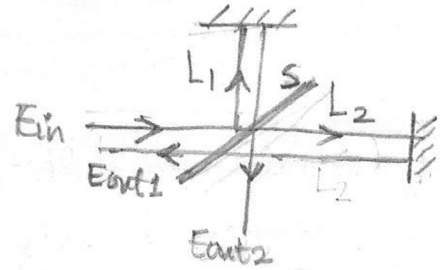
$$S \equiv \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

For loss-less surface in one medium, S is an unitary matrix: $S^H S = I$

$$|\tilde{E}_{out_1}|^2 + |\tilde{E}_{out_2}|^2 = \tfrac{1}{2}|\tilde{E}_{in_1}|^2 + \tfrac{1}{2}|\tilde{E}_{in_2}|^2 + \tfrac{1}{2}|\tilde{E}_{in_1}||\tilde{E}_{in_2}| + \tfrac{1}{2}|\tilde{E}_{in_1}|^2 + \tfrac{1}{2}|\tilde{E}_{in_2}|^2 - \tfrac{1}{2}|\tilde{E}_{in_1}||\tilde{E}_{in_2}|$$
$$= |\tilde{E}_{in_1}|^2 + |\tilde{E}_{in_2}|^2 \longleftarrow \text{Power is conserved for } S$$

(6) Mach-Zehnder & Michelson Interferometers





$$\tilde{E}_{out\,1} = \left(S_{11}^2 e^{2jkL_1} + S_{12}S_{21} e^{2jkL_2}\right)\tilde{E}_{in}$$
$$E_{out\,2} = \left(S_{11}S_{21} e^{2jkL_1} + S_{22}S_{21} e^{2jkL_2}\right)\tilde{E}_{in}$$

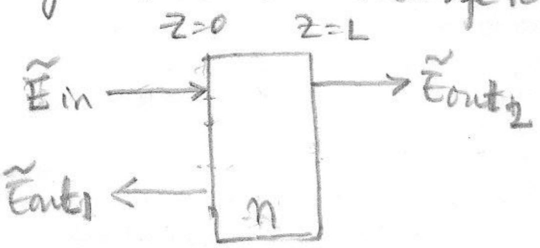
For 50-50 beam splitter:

$$\tilde{E}_{out_1} = \tfrac{1}{2}\left(e^{2jkL_1} + e^{2jkL_2}\right)\tilde{E}_{in}$$
$$\tilde{E}_{out_2} = \tfrac{1}{2}\left(e^{2jkL_1} - e^{2jkL_2}\right)E_{in}$$

Power distribution:

$$|\tilde{E}_{out_1}|^2 = \cos^2[k(L_1 - L_2)]\,|\tilde{E}_{in}|^2$$
$$|\tilde{E}_{out_2}|^2 = \sin^2[k(L_1 - L_2)]\,|\tilde{E}_{in}|^2$$

(7) Fabry-Perot Interferometer:



TE polarization, normal incidence

Interface at $z=0$:

$$S_0 = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} \frac{1-n}{1+n} & \frac{2n}{1+n} \\ \frac{2}{1+n} & \frac{n-1}{1+n} \end{pmatrix}$$

Interface at $z=L$:

$$S_L = \begin{pmatrix} S_{22} & S_{21} \\ S_{12} & S_{11} \end{pmatrix} = \begin{pmatrix} \frac{n-1}{1+n} & \frac{2}{1+n} \\ \frac{2n}{1+n} & \frac{1-n}{1+n} \end{pmatrix}$$

not a unitary matrix

$$\tilde{E}_{out_1} = \left(S_{11} + \frac{S_{12} S_{22} S_{21} e^{2jkL}}{1 - S_{22}^2 e^{2jkL}} \right) \tilde{E}_1$$

$$\tilde{E}_{out_2} = \frac{S_{12} S_{21} e^{jkL}}{1 - S_{22}^2 e^{2jkL}} \tilde{E}_1$$

Maxima for transmitted intensity ratio:

$T = \frac{|\tilde{E}_{out_2}|^2}{|\tilde{E}_1|^2}$ happens for $f = \frac{qc}{2nL}$, $q = 0, 1, 2, \ldots$

The minima for reflected intensity ratio:

$R = \frac{|\tilde{E}_{out_1}|^2}{|\tilde{E}_1|^2}$ happens for $f = \frac{qc}{2nL}$, $q = 0, 1, 2, \ldots$

Lecture - 2

Standing Waves

$\vec{E}_{in}(x,t) = -\hat{y} E_0 e^{-jkx - j\omega t}$, $\vec{H}_{in}(x,t) = \hat{z} \frac{E_0}{Z_0} e^{-jkx - j\omega t}$

$\vec{E}_{out}(x,t) = \hat{y} E_0 e^{jkx - j\omega t}$, $\vec{H}_{out}(x,t) = \hat{z} \frac{E_0}{Z_0} e^{jkx - j\omega t}$

in = frorward travelling, out = backward travelling

$$\left.\begin{aligned} \vec{E} &= \vec{E}_{in} + \vec{E}_{out} = 2\hat{y} E_0 \sin(kx) e^{-j\omega t} \\ \vec{H} &= \vec{H}_{in} + \vec{H}_{out} = 2\hat{z} \frac{E_0}{Z_0} \cos(kx) e^{-j\omega t} \end{aligned}\right\} \text{standing waves.}$$

$$\left.\begin{aligned} &\vec{E} \text{ is } 0 \text{ for } x = m\lambda/2 \\ &\vec{H} \text{ is } 0 \text{ for } x = m\lambda/2 + \lambda/4 \end{aligned}\right\} m = 0, \pm 1, \pm 2 \ldots$$

Optical cavity

For a length L of the cavity, wavelengths that can exist are:

$$\left.\begin{aligned} \lambda &= \frac{2L}{q} \\ f &= \frac{qc}{2L} \end{aligned}\right\} q = 1, 2, \ldots$$