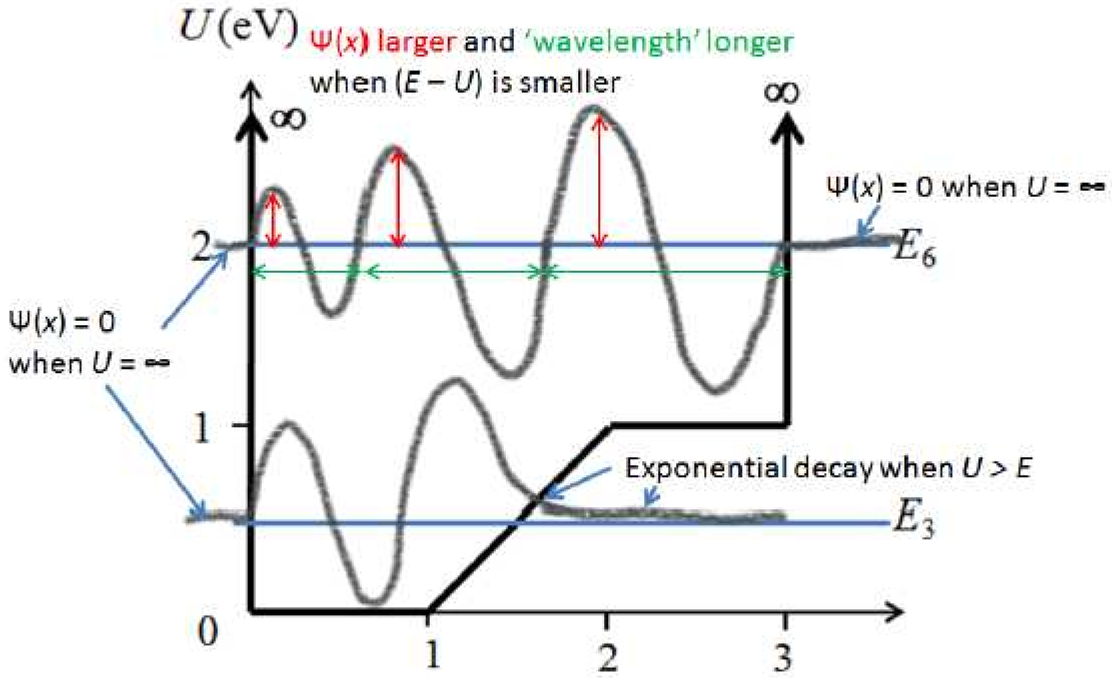
PC2232: Tutorial Homework Assignment 2

Due date: Wednesday, 16 April 2014

Question 1:

The energies and wave functions of 3rd and 6th levels are shown below. Note that there are 3 turning points in $\psi_3(x)$ and six turning points in $\psi_6(x)$.



Question 2:

For the hydrogen atom, the energies must be given by

$$E_n = \frac{13.6 \text{ eV}}{n^2}, \quad \text{and} \quad \left|\vec{L}\right| = \hbar\sqrt{l(l+1)}, \quad \text{where } 0 \leq l < n. \tag{1}$$

On checking each pair of values, we have:

(a) $n = 5$, $l = 3$. So this is a possible state.

(b) $n = 3$, $l = 3$. Not possible as $l = n$, instead of $l < n$.

(c) $n = 3$, but $L = 2\hbar$. Not possible since $L \neq \hbar\sqrt{l(l+1)}$.

(d) $n = 2$, $l = 1$. This is a possible state.

So only (a) and (d) are possible states.

Question 3:

(a)

$$\psi_{310}(r,\theta,\phi) = \frac{2\sqrt{2}}{27\sqrt{\pi}a_0^{3/2}} \mathrm{e}^{-r/3a_0} \left(\frac{r}{a_0} - \frac{r^2}{6a_0^2}\right) \cos\theta. \tag{2}$$

Consider only the radial part and ignoring the normalization constants,

$$\begin{aligned} P(r) =& r^2 \left|R(r)\right|^2 = r^2 \left[\mathrm{e}^{-r/3a_0} \left(\frac{r}{a_0} - \frac{r^2}{6a_0^2}\right)\right]^2 \\ =& \mathrm{e}^{-2r/3a_0} \left(\frac{r^4}{a_0^4} - \frac{r^5}{3a_0^3} + \frac{r^6}{36a_0^4}\right). \end{aligned} \tag{3}$$

(b) The mimimum value of $P(r)$ is zero, so

$$\begin{aligned} P(r) =& r^2 \mathrm{e}^{-2r/3a_0} \left(\frac{r}{a_0}\right)^2 \left(1 - \frac{r}{6a_0}\right)^2 = 0 \\ & \rightarrow r = 0, \quad \text{or} \quad r = 6a_0. \end{aligned} \tag{4}$$

(c) The maxima occurs when $P(r) = r^2 \left|R(r)\right|^2$ is maximum. We can simplify the calculations by noticing that this condition is the same as when $rR(r)$ is maximum. This happens when

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}r}\left(rR(r)\right) =& 0, \\ \frac{\mathrm{d}}{\mathrm{d}r}\left[\mathrm{e}^{-r/3a_0} \left(\frac{r^2}{a_0} - \frac{r^3}{6a_0^2}\right)\right] =& 0 \\ -\frac{1}{3a_0}\mathrm{e}^{-r/3a_0} \left(\frac{r^2}{a_0} - \frac{r^3}{6a_0^2}\right) + \mathrm{e}^{-r/3a_0} \left(\frac{2r}{a_0} - \frac{3r^2}{6a_0^2}\right) =& 0 \\ \mathrm{e}^{-r/3a_0} \left(-\frac{r^2}{3a_0^2} + \frac{r^3}{18a_0^3} + \frac{2r}{a_0} - \frac{3r^2}{6a_0^2}\right) =& 0 \\ \mathrm{e}^{-r/3a_0} \frac{r}{18a_0^3} \left(r - 3a_0\right)\left(r - 12a_0\right) =& 0. \end{aligned} \tag{5}$$

We have shown that $r = 0$ gives minimum value for $P(r)$. Thus the maxima of $P(r)$ occurs when $r = 3a_0$ and $r = 12a_0$. $P(r = 3a_0) = 2.74a_0^2$ and $P(r = 12a_0) = 6.96a_0^2$. Therefore the most probable position is $r = 12a_0$.