PC2232 Physics for Electrical Engineers: Tutorial 2

Question 1: Polarization I

A beam of unpolarized sunlight strikes the vertical plastic wall of a water tank at an unknown angle. Some of the light reflects from the wall and enters the water, as seen in the Fig. 1. The refractive index of the plastic wall is 1.61. If the light that has been reflected from the wall into the water is observed to be completely polarized, what angle does this beam make with the normal inside the water?

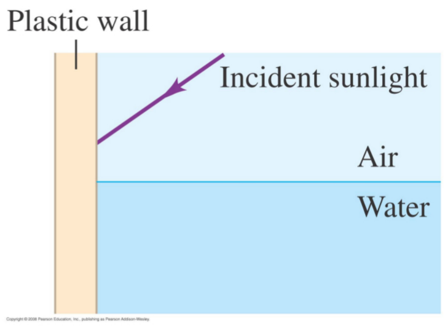

Figure 1:

Question 2: Polarization II

A beam of light traveling horizontally is made of an unpolarized component with intensity I_0 and a polarized component with intensity I_p. The plane of polarization of the polarized component is oriented at an angle of θ with respect to the vertical. The data in the table gives the intensity measured through a polarizer with an orientation of ϕ with respect to the vertical.

ϕ (°)	I_{total} (W/m^2)	ϕ (°)	I_{total} (W/m^2)
0	18.4	100	8.6
10	21.4	110	6.3
20	23.7	120	5.2
30	24.8	130	5.2
40	24.8	140	6.3
50	23.7	150	8.6
60	21.4	160	11.6
70	18.4	170	15.0
80	15.0	170	15.0
90	11.6		

(a) What is the orientation of the polarized component? (That is, what is the angle of θ?)

(b) What are the values of I_0 and I_p?

Question 3: Interference

Consider three very narrow slits with spacings of d and $\frac{3d}{2}$ as shown in the figure. The slits are irradiated from the left with a plane wave of monochromatic light with a wavelength $\lambda = \frac{2d}{5}$. You may assume that the screen is very far away from the slits.

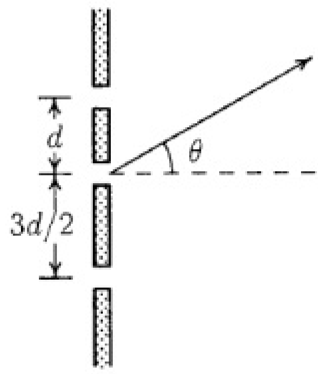

Figure 2:

(a) Derive the condition θ must satisfy such that it is the direction in which waves from *all three slits* interfere constructively, thus giving rise to principal maxima. What are the corresponding values of θ?

(b) Let the bottom slit be covered by a filter which introduces a half-cycle phase change. How many principal maxima will now be observed, and what are their corresponding values of θ?

(c) Now suppose the filter used in part (b) consists of a thin transparent film with an index of refraction $n = 1.33$, and it is now placed directly in front of the bottom slit. Calculate the minimum thickness of this film that would be needed to achieve the half-cycle phase change.

Question 4: Intensity of Interference Pattern

Consider a two-slit interference experiment in which the two slits are of different widths. As measured on a distant screen, the amplitude of the wave from the first slit is E, while amplitude of the wave from the second slit is $2E$.

(a) Show that the intensity at any point of the interference pattern is

$$I = I_0 \left(\frac{5}{9} + \frac{4}{9} \cos \phi \right) \tag{1}$$

where ϕ is the phase difference between the two waves as measured at that particular point on the screen and I_0 is the maximum intensity in the pattern.

(b) Sketch a graph of I versus ϕ. What is the minimum value of the intensity, and for which values of ϕ does it occur?

Question 5: Polarization upon reflection (optional)

In this question, we will derive the expression used in the lecture to determine whether there is a phase change when reflecting normally off a surface, i.e. whether the expression

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_a \tag{2}$$

holds true, where the symbols are as defined in the lecture.

Suppose we choose E_a to be polarized in the x-direction, and B_a to be in the y-direction. Positive-z is then the direction of propagation of the incident plane wave with the boundary at $z = 0$.

(a) Write down the expressions of E_a, B_a, E_r, B_r, E_b, and B_b.

(b) By considering the continuity conditions of both $\vec{E}$ and $\vec{B}$ at the boundary,

$$\vec{E}_a + \vec{E}_r = \vec{E}_b, \quad \text{and} \quad \vec{B}_a + \vec{B}_r = \vec{B}_b, \tag{3}$$

derive Eq. (2) above. Notice that we have assumed non-magnetic materials so that $\mu_a = \mu_b = \mu_0$.

(*The Fresnel equations for incident wave at an oblique angle can also be derived in a similar manner. The mathematics is more tedious due to the extra angular dependence. You may try to derive them on your own or check any EM or Optics book.*)