Precalculus Workbook

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MATH

POLAR COORDINATES

■ 1. Find the rectangular point that's equivalent to the polar point.

$$(r,\theta) = \left(-14, \frac{5\pi}{6}\right)$$

■ 2. Convert the rectangular point $(x, y) = (5\sqrt{2}, 5\sqrt{2})$ to polar coordinates.

■ 3. Convert the rectangular point to polar coordinates.

$$(x, y) = \left(\cos\left(\frac{2\pi}{7}\right), \sin\left(\frac{2\pi}{7}\right)\right)$$

■ 4. Convert the polar point into rectangular coordinates.

$$(r,\theta) = \left(6, \frac{\pi}{6}\right)$$

■ 5. Convert the rectangular point $(x, y) = (2\sqrt{3}, -6)$ to polar coordinates.

■ 6. Convert the polar point $(r,\theta) = (8, 2\pi)$ into rectangular coordinates.

MULTIPLE WAYS TO EXPRESS POLAR POINTS

■ 1. Find five polar points that are equivalent to $(6, -\pi/4)$, where the angle θ lies within $(-3\pi, 3\pi)$.

■ 2. Find a point that's equivalent to $(3, 101\pi/2)$, with a positive radius r and a polar angle θ that lies within $(0, 2\pi]$.

■ 3. How many points are equivalent to the point $(1, \pi/7)$ and have a polar angle in the interval $0 < \theta < 5\pi$?

■ 4. Find four polar points that are equivalent to the point $(-10, 16\pi/7)$ and have a polar angle in the interval $(-2\pi, 2\pi)$.

■ 5. How many points in $0 < \theta < 3\pi/2$ are equivalent to the point $(2, -2\pi/3)$?

■ 6. Find a point equivalent to $(-1, 53\pi/6)$ with positive radius r and a polar angle θ within $(0, 2\pi]$.

CONVERTING EQUATIONS

■ 1. Convert the rectangular equation into polar coordinates.

$$(x+9)^2+(y-13)^2=64$$

■ 2. Convert the rectangular equation into polar coordinates.

$$\frac{(x+5)^2}{9}+\frac{(y-7)^2}{4}=1$$

■ 3. Convert the rectangular equation into polar coordinates.

$$x=\frac{1}{y+3}$$

■ 4. Convert the polar equation to rectangular coordinates.

$$r^2=4\sin^2\theta$$

■ 5. Convert the polar equation to rectangular coordinates.

$$\theta=r^2+1$$

6. Convert the polar equation to rectangular coordinates.

$$r = e^{\theta}$$

GRAPHING POLAR CURVES IN A RECTANGULAR SYSTEM

■ 1. Sketch the graph of $r = 3\sin\theta - 3$ in a rectangular coordinate system.

■ 2. Convert $r = 4\cos\theta$ to rectangular coordinates and then sketch the graph of the resulting equation.

■ 3. Convert $r = 6\sin\theta - 2\cos\theta$ into rectangular coordinates and then sketch the graph of the resulting equation.

■ 4. Convert $r^2\sin(2\theta) = 1$ into rectangular coordinates and then sketch the graph of the resulting equation. Hint: Apply the trigonometric identity $\sin(2\theta) = 2\sin\theta\cos\theta$.

■ 5. Convert $r\cos\theta - \tan\theta + 2 = 0$ into rectangular coordinates and then sketch the graph of the resulting equation.

■ 6. Convert the polar equation into rectangular coordinates and the sketch the graph of the resulting equation.

$$r^2 = \frac{\sin\theta}{\cos^3\theta}$$

GRAPHING CIRCLES

■ 1. Sketch the graph of $r = -1$.

■ 2. Sketch the graph of $r = 8\sin\theta$.

■ 3. Sketch the graph of $r = 5\cos\theta$.

■ 4. Sketch the graph of $r = -3\cos\theta$.

■ 5. Sketch the graph of $r = -\sin\theta$.

■ 6. Sketch the graph of $r^2 = 4\sin^2\theta$.

GRAPHING ROSES

■ 1. Sketch the graph of the rose $r = 2\sin(5\theta)$.

■ 2. Sketch the graph of the rose $r = -4\cos(3\theta)$.

■ 3. Sketch the graph of the rose $r = -3\sin(2\theta)$.

■ 4. Sketch the graph of the rose $r = 7\cos(4\theta)$.

■ 5. How many petals will the rose have?

$$r = 5\cos(12\theta)$$

■ 6. Write an equation of a rose that has 9 petals and passes through the point $(6,3\pi/2)$.

GRAPHING CARDIOIDS

■ 1. Create a table of values for $r = 5 - 5\cos\theta$ over the interval $0 \le \theta \le 2\pi$, then use it to sketch the graph.

■ 2. Create a table of values for $r = 3 - 3\sin\theta$ over the interval $0 \le \theta \le 2\pi$, then use it to sketch the graph.

■ 3. Create a table of values for $r = -2 - 2\sin\theta$ over the interval $0 \le \theta \le 2\pi$, then use it to sketch the graph.

■ 4. Create a table of values for $r = -3 - 3\cos\theta$ over the interval $0 \le \theta \le 2\pi$, then use it to sketch the graph.

■ 5. Sketch the graph of the cardioid $r = 2 + 2\cos\theta$.

■ 6. Find the equation of the cardioid that extends out a distance of 2 from the pole, is symmetric across the vertical axis, and mostly sits above of the horizontal axis. Then sketch its graph.

GRAPHING LIMACONS

■ 1. Sketch the graph of $r = 2 + 5\cos\theta$.

■ 2. Sketch the graph of $r = 4 - \sin\theta$.

■ 3. Sketch the graph of $r = 7 - 6\cos\theta$.

■ 4. Sketch the graph of $r = 3 + 2\sin\theta$.

■ 5. Sketch the graph of $r = 1 - 20\sin\theta$.

■ 6. Sketch the graph of $r = 9 + 4\cos\theta$.

GRAPHING LEMNISCATES

■ 1. Sketch the graph of $r^2 = -4\cos(2\theta)$.

■ 2. Sketch the graph of $r^2 = 7\sin(2\theta)$.

■ 3. Sketch the graph of $r^2 = 4\cos(2\theta)$.

■ 4. Sketch the graph of $r^2 = -16\sin(2\theta)$.

■ 5. Find the equation of the lemniscate that has loops that sit along the horizontal axis and extend out to a distance of 5, then sketch its graph.

■ 6. Find the equation of the lemniscate that has loops that sit in the first and third quadrants and extend out to a distance of $\sqrt{5}$, then sketch its graph.

INTERSECTION OF POLAR CURVES

■ 1. Find the points of intersection of $r = 4 - 4\cos\theta$ and $r = 4 + 4\sin\theta$.

■ 2. Find the points of intersection of $r = 3 - 4\sin\theta$ and $r = -\sin\theta$.

■ 3. Find the points of intersection of $r = -3 - 3\sin\theta$ and $r = 3 - \sin\theta$.

■ 4. Find the points of intersection of $r = 1 + 2\sin\theta$ and $r = 1 + 2\cos\theta$.

■ 5. How many points of intersection exist for the graphs of the polar equations? Hint: Think just about the shape of each curve, without trying to solve algebraically for the points of intersection.

$$r = 5\sin(3\theta)$$

$$r = 4$$

■ 6. How many points of intersection exist for the graphs of the polar equations? Hint: Think just about the shape of each curve, without trying to solve algebraically for the points of intersection.

$$r = 2\sin(2\theta)$$

$$r^2 = 9\sin(2\theta)$$

COMPLEX NUMBERS

■ 1. Simplify the imaginary number.

$$i^{437}$$

■ 2. Simplify the imaginary number.

$$i^{2,314}$$

■ 3. Name the real and imaginary parts of the complex number.

$$z = -5 + 17i$$

■ 4. Name the real and imaginary parts of the complex number.

$$z = \sqrt{7} - 4\pi i$$

■ 5. How can the numbers be classified?

$$z = -3 + 9i$$

$$z = 0 - 15i$$

$$z = 6 + 0i$$

6. How can the numbers be classified?

$$z = 0 - \pi i$$

$$z = -\sqrt{5} + 0i$$

$$z = -11 + \frac{2}{3}i$$

COMPLEX NUMBER OPERATIONS

■ 1. Find the sum and difference of the complex numbers.

$$\frac{7}{5} - \frac{2}{3}i$$

$$\frac{7}{2} - \frac{8}{3}i$$

■ 2. Find the product of the complex numbers.

$$-7i$$

$$-5 + 9i$$

■ 3. Find the product of the complex numbers.

$$5 - 2i$$

$$6 - 11i$$

■ 4. Divide the complex number $-4 + 15i$ by the imaginary number $5i$.

■ 5. Find the complex conjugate of each complex number.

$9 - 9i$

$-3 + 13i$

$11 - 22i$

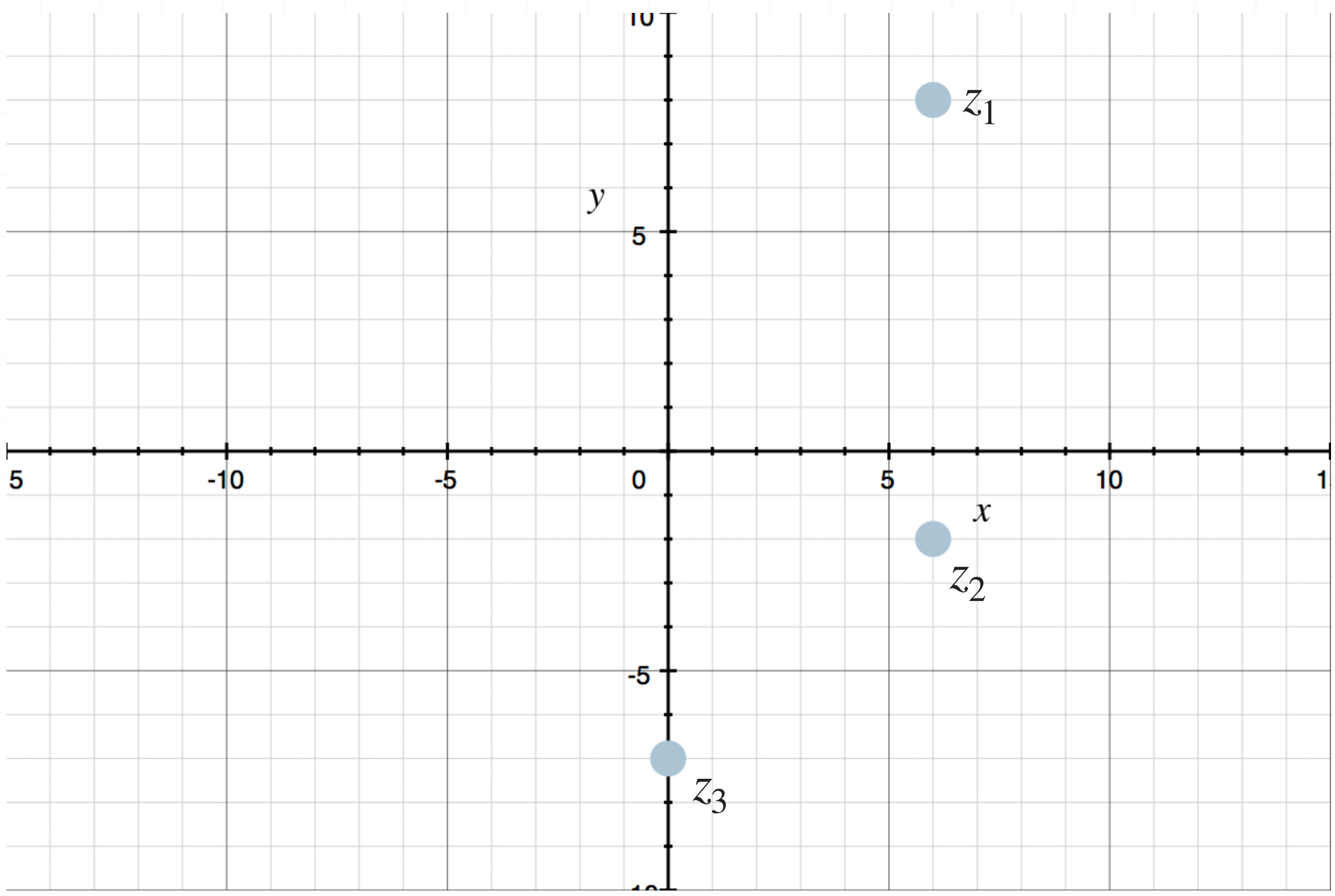
■ 6. Express the fraction in the form $a + bi$ where a and b are real numbers.

$$\frac{-3 + 7i}{4 - 5i}$$

GRAPHING COMPLEX NUMBERS

■ 1. Graph $-3+5i$, $2-4i$, and 5 in the complex plane.

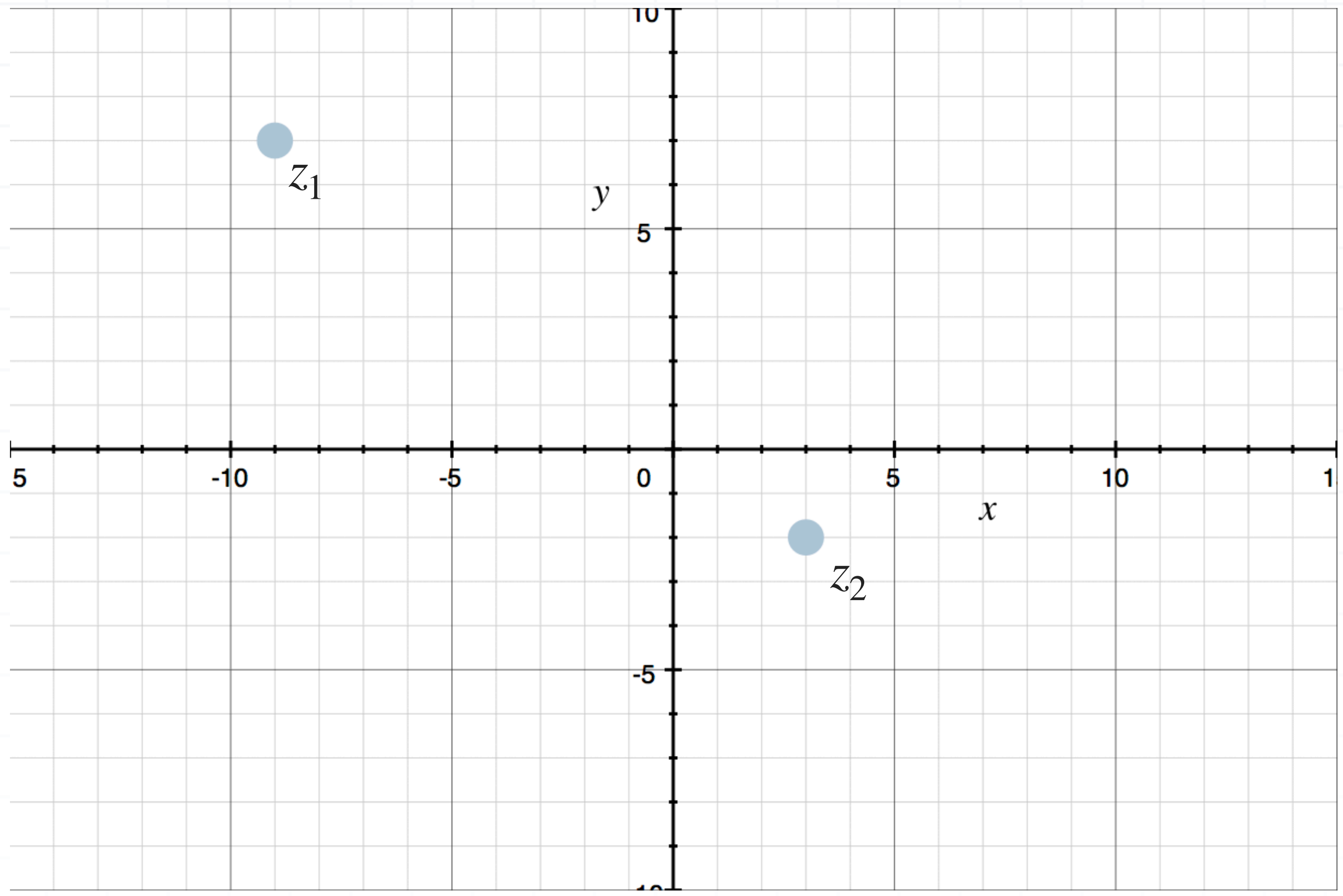
■ 2. Which three complex numbers are represented in the graph?



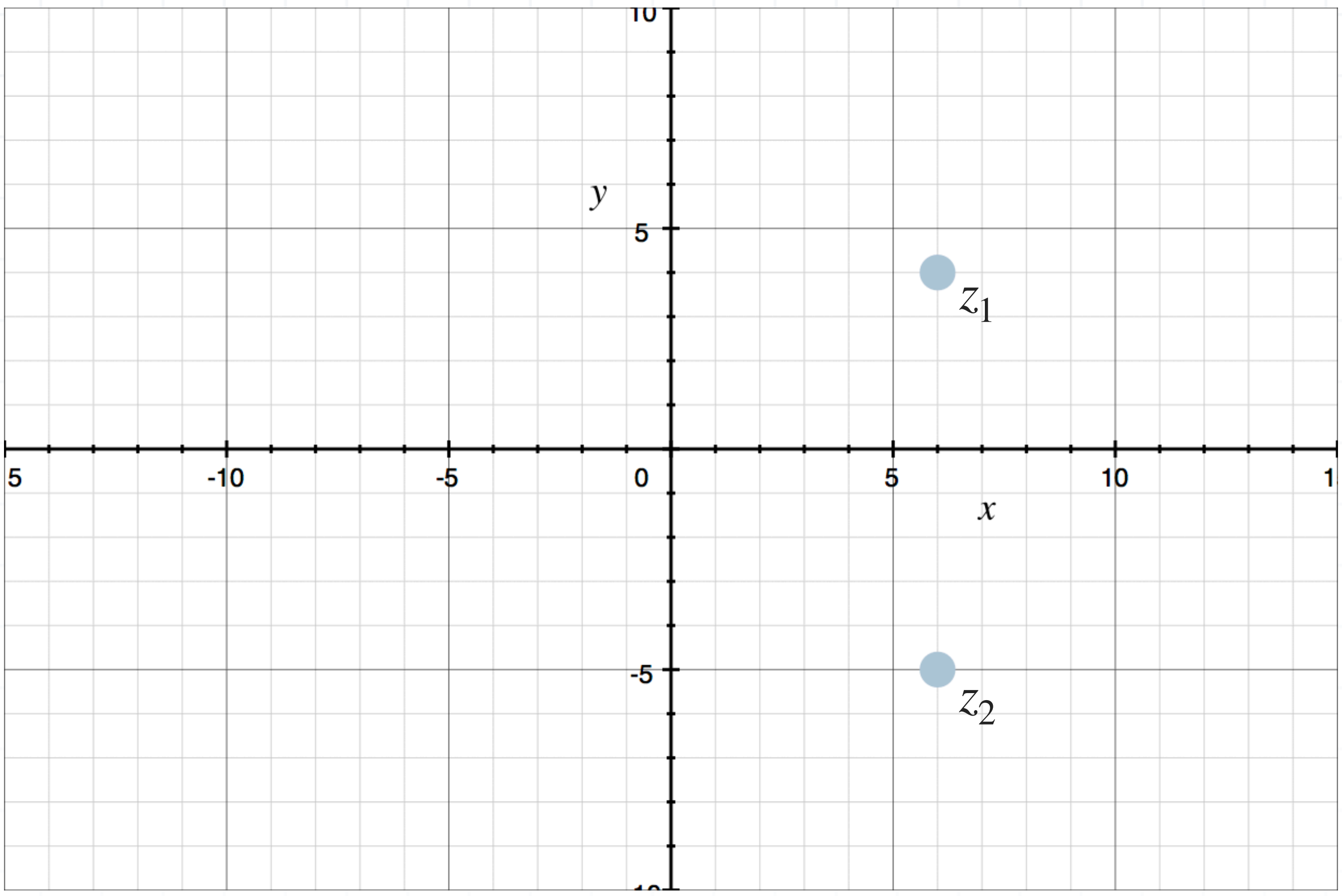
■ 3. Graph the sum of the complex numbers $5-4i$ and $-1+10i$.

■ 4. Graph the difference of the complex numbers $8-7i$ and $13-4i$.

■ 5. Graph the sum of the complex numbers z_1 and z_2.



■ 6. Graph the difference of the complex numbers z_1 and z_2.



y
x
5
-5
10
0
-10
z_1
z_2

DISTANCES AND MIDPOINTS

■ 1. Find the distance between $s = 5 + 3i$ and $t = 1 - i$.

■ 2. Find the distance between $u = -5 - 3i$ and $v = 4 + 2i$.

■ 3. Find the distance between $w = 2 + 6i$ and $z = -2 - 6i$.

■ 4. Find the midpoint between $s = 5 + 3i$ and $t = 1 - i$.

■ 5. Find the midpoint between $u = -7 - 5i$ and $z = 2 + 2i$.

■ 6. Graph the midpoint between $w = 6 + 8i$ and $z = 2 + 4i$.

COMPLEX NUMBERS IN POLAR FORM

■ 1. If the complex number $6 - 2i$ is expressed in polar form, which quadrant contains the angle θ?

■ 2. Find r for the complex number.

$$-9 - 3i$$

■ 3. What is the polar form of the complex number?

$$5 + 12i$$

■ 4. Write the complex number in polar form.

$$11i$$

■ 5. What is the polar form of the complex number?

$$z = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

6. Write the complex number in polar form.

$$-5$$

MULTIPLYING AND DIVIDING POLAR FORMS

■ 1. What is the product z_1z_2 of the complex numbers in polar form?

$$z_1 = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$z_2 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

■ 2. What is the product z_1z_2 of the complex numbers in polar form?

$$z_1 = \sqrt{3}\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right)$$

$$z_2 = \frac{\sqrt{5}}{3}\left(\cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}\right)$$

■ 3. What is the quotient z_1/z_2 of the complex numbers in polar form?

$$z_1 = 12\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$z_2 = 15\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

■ 4. What is the quotient z_1/z_2 of the complex numbers in polar form?

$$z_1 = \sqrt{7}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

$$z_2 = \frac{1}{\sqrt{2}}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

■ 5. What is the product z_1z_2 of the complex numbers in polar form?

$$z_1 = \frac{\sqrt{15}}{4}\left(\cos\frac{7\pi}{2} + i\sin\frac{7\pi}{2}\right)$$

$$z_2 = \frac{1}{\sqrt{5}}\left(\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}\right)$$

■ 6. Suppose that a complex number z is the product $z_1 \cdot z_2$ of the given complex numbers. If z is expressed in polar form, $r(\cos\theta + i\sin\theta)$, where is θ located?

$$z_1 = 3\sqrt{5}\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$$

$$z_2 = 6\left(\cos\frac{7\pi}{10} + i\sin\frac{7\pi}{10}\right)$$

POWERS OF COMPLEX NUMBERS AND DE MOIVRE'S THEOREM

■ 1. Find z^5 in polar form.

$$z = 2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

■ 2. Find z^7 in polar form.

$$z = \sqrt{5}\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$$

■ 3. Find z^6 in rectangular form $a + bi$.

$$z = \frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$$

■ 4. Find z^3 in rectangular form $a + bi$.

$$z = 2\sqrt{6}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$

■ 5. Find z^5 in polar form.

$$z = -4 - 4i$$

6. Find z^4 in rectangular form $a + bi$.

$$z = \sqrt{6} - \sqrt{2}i$$

COMPLEX NUMBER EQUATIONS

■ 1. Find the solutions of the complex equation.

$$z^2 = 49$$

■ 2. Find the solution of the complex equation that lies in the third quadrant.

$$z^3 = 216$$

■ 3. Find the solutions of the complex equation.

$$z^4 = 256$$

■ 4. Find the solutions of the complex equation.

$$z^6 = 729$$

■ 5. Find the solutions of the complex equation.

$$z^5 = 32$$

6. How many solutions of the complex equation lie in the second quadrant?

$$z^8 = 256$$

ROOTS OF COMPLEX NUMBERS

■ 1. Find the cube roots of the complex number.

$$z = 27\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

■ 2. Find the 4th roots of the complex number.

$$z = 256(\cos 60^\circ + i\sin 60^\circ)$$

■ 3. Find the 5th roots of the complex number that lies in the first quadrant of the complex plane.

$$z = 25(\cos 80^\circ + i\sin 80^\circ)$$

■ 4. Find the 4th roots of the complex number.

$$z = 34\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$$

■ 5. Find the 6th roots of the complex number that lies in the second quadrant of the complex plane.

$$z = 11\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

6. Find the 7th roots of the complex number.

$$z = 20(\cos 120^\circ + i\sin 120^\circ)$$

MATRIX DIMENSIONS AND ENTRIES

■ 1. Give the dimensions of the matrix.

$$D = \begin{bmatrix} 11 & 9 \\ -4 & 8 \end{bmatrix}$$

■ 2. Give the dimensions of the matrix.

$$A = \begin{bmatrix} 3 & 5 & -2 & 1 & 8 \end{bmatrix}$$

■ 3. Given matrix J, find $J_{4,1}$.

$$J = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 1 \end{bmatrix}$$

■ 4. Given matrix C, find $C_{1,2}$.

$$C = \begin{bmatrix} 3 & 12 \\ 1 & 4 \\ 9 & 5 \\ -3 & 2 \end{bmatrix}$$

■ 5. Given matrix N, state the dimensions and find $N_{1,3}$.

$$N = \begin{bmatrix} 1 & 5 & 9 \\ 14 & -8 & 6 \end{bmatrix}$$

■ 6. Given matrix S, state the dimensions and find $S_{3,4}$.

$$S = \begin{bmatrix} 3 & 6 & -7 & 1 & 0 \\ 0 & 9 & 15 & 3 & 4 \\ 4 & 0 & 2 & 11 & 8 \\ -5 & 8 & 7 & 9 & 2 \end{bmatrix}$$

REPRESENTING SYSTEMS WITH MATRICES

■ 1. Represent the system with an augmented matrix called A.

$$-2x + 5y = 12$$

$$6x - 2y = 4$$

■ 2. Represent the system with an augmented matrix called D.

$$9y - 3x + 12 = 0$$

$$8 - 4x = 11y$$

■ 3. Represent the system with an augmented matrix called H.

$$4a + 7b - 5c + 13d = 6$$

$$3a - 8b = -2c + 1$$

■ 4. Represent the system with an augmented matrix called M.

$$-2x + 4y = 9 - 6z$$

$$7y + 2z - 3 = -3t - 9x$$

■ 5. Represent the system with an augmented matrix called A.

$$3x - 8y + z = 7$$

$$2z = 3y - 2x + 4$$

$$5y = 12 - 9x$$

■ 6. Represent the system with an augmented matrix called K.

$$-4b + 2c = 3 - 7a$$

$$9c = 4 - 2b$$

$$8a - 2c = 5b$$

SIMPLE ROW OPERATIONS

■ 1. Write the new matrix after $R_1 \leftrightarrow R_2$.

$$\begin{bmatrix} 2 & 6 & -4 & 1 \\ 8 & 2 & 1 & -5 \end{bmatrix}$$

■ 2. Write the new matrix after $R_2 \leftrightarrow R_4$.

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 6 & 1 & 5 & -4 \\ -7 & 7 & 0 & 3 \\ 9 & 2 & 8 & 3 \end{bmatrix}$$

■ 3. Write the new matrix after $R_1 \leftrightarrow 3R_2$.

$$\begin{bmatrix} 9 & 2 & -7 \\ 1 & 6 & 4 \end{bmatrix}$$

■ 4. Write the new matrix after $3R_2 \leftrightarrow 3R_4$.

$$\begin{bmatrix} 0 & 11 & 6 \\ 7 & -3 & 9 \\ 8 & 8 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

5. Write the new matrix after $R_1 + 2R_2 \to R_1$.

$$\begin{bmatrix} 6 & 2 & 7 \\ 1 & -5 & 15 \end{bmatrix}$$

6. Write the new matrix after $4R_2 + R_3 \to R_3$.

$$\begin{bmatrix} 13 & 5 & -2 & 9 \\ 8 & 2 & 0 & 6 \\ 4 & 1 & 7 & -3 \end{bmatrix}$$

GAUSS-JORDAN ELIMINATION AND REDUCED ROW-ECHELON FORM

■ 1. Use Gauss-Jordan elimination to solve the system.

$$x + 2y = -2$$

$$3x + 2y = 6$$

■ 2. Use Gauss-Jordan elimination to solve the system.

$$2x + 4y = 22$$

$$3x + 3y = 15$$

■ 3. Use Gauss-Jordan elimination to solve the system.

$$x - 3y - 6z = 4$$

$$y + 2z = -2$$

$$-4x + 12y + 21z = -4$$

■ 4. Use Gauss-Jordan elimination to solve the system.

$$2y + 4z = 4$$

$$x + 3y + 3z = 5$$

$$2x + 7y + 6z = 10$$

■ 5. Use Gauss-Jordan elimination to solve the system.

$$3x + 12y + 42z = -27$$

$$x + 2y + 8z = -5$$

$$2x + 5y + 16z = -6$$

■ 6. Use Gauss-Jordan elimination to solve the system.

$$4x + 8y + 4z = 20$$

$$4x + 6y = 4$$

$$3x + 3y - z = 1$$

MATRIX ADDITION AND SUBTRACTION

■ 1. Add the matrices.

$$\begin{vmatrix} 7 & 6 \\ 17 & 9 \end{vmatrix} + \begin{vmatrix} 0 & 8 \\ -2 & 5 \end{vmatrix}$$

■ 2. Add the matrices.

$$\begin{vmatrix} 8 & 3 \\ -4 & 7 \\ 6 & 0 \\ 1 & 13 \end{vmatrix} + \begin{vmatrix} 6 & 7 \\ 2 & -3 \\ 9 & 11 \\ 7 & -2 \end{vmatrix}$$

■ 3. Subtract the matrices.

$$\begin{vmatrix} 7 & 9 \\ 4 & -1 \end{vmatrix} - \begin{vmatrix} 3 & 8 \\ 12 & -3 \end{vmatrix}$$

■ 4. Subtract the matrices.

$$\begin{vmatrix} 8 & 11 & 2 & 9 \\ 6 & 3 & 16 & 8 \end{vmatrix} - \begin{vmatrix} 6 & 11 & 7 & -4 \\ 5 & 8 & 1 & 15 \end{vmatrix}$$

■ 5. Solve for m.

$$\begin{vmatrix} 6 & 5 \\ 9 & -9 \end{vmatrix} + \begin{vmatrix} 3 & 7 \\ 1 & 6 \end{vmatrix} = m + \begin{vmatrix} 7 & 12 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 8 \\ 4 & -7 \end{vmatrix}$$

■ 6. Solve for n.

$$\begin{vmatrix} 4 & 12 \\ 9 & 8 \end{vmatrix} - \begin{vmatrix} 0 & 3 \\ 9 & 9 \end{vmatrix} = n - \begin{vmatrix} 6 & 3 \\ 5 & 11 \end{vmatrix} + \begin{vmatrix} 7 & -4 \\ -18 & 1 \end{vmatrix}$$

SCALAR MULTIPLICATION AND ZERO MATRICES

■ 1. Use scalar multiplication to simplify the expression.

$$\frac{1}{4}\begin{bmatrix} 12 & 8 & 3 \\ 2 & -16 & 0 \\ 1 & 5 & 7 \end{bmatrix}$$

■ 2. Solve for y.

$$4\begin{bmatrix} 2 & 9 \\ -5 & 0 \end{bmatrix} + y = 5\begin{bmatrix} 1 & -3 \\ 6 & 8 \end{bmatrix}$$

■ 3. Solve for n.

$$-2\begin{bmatrix} 6 & 5 \\ 0 & 11 \end{bmatrix} = n - 4\begin{bmatrix} 2 & 4 \\ -1 & 9 \end{bmatrix}$$

■ 4. Add the zero matrix to the given matrix.

$$\begin{bmatrix} 8 & 17 \\ -6 & 0 \end{bmatrix}$$

■ 5. Find the opposite matrix.

$$\begin{vmatrix} 6 & 8 & 0 \\ 2 & -3 & 11 \\ 4 & 12 & 9 \end{vmatrix}$$

■ 6. Multiply the matrix by a scalar of 0.

$$\begin{vmatrix} 14 & -1 & 7 & 5 \\ 3 & 7 & 18 & -4 \end{vmatrix}$$

MATRIX MULTIPLICATION

■ 1. If matrix A is 3×3 and matrix B is 3×4, say whether AB or BA is defined, and give the dimensions of any product that's defined.

■ 2. Find the product of matrices A and B.

$$A = \begin{bmatrix} 2 & 6 \\ -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 0 \\ 5 & -4 \end{bmatrix}$$

■ 3. Find the product of matrices A and B.

$$A = \begin{bmatrix} 5 & -1 \\ 0 & 11 \\ 7 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 1 & 8 \\ -3 & 0 & 4 \end{bmatrix}$$

■ 4. Find the product of matrices A and B.

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 8 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 \\ 4 & 8 \end{bmatrix}$$

■ 5. Use the distributive property to find $A(B + C)$.

$$A = \begin{bmatrix} 2 & 0 \\ 4 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 \\ 5 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & 1 \\ 3 & -1 \end{bmatrix}$$

■ 6. Find the product of matrices A and B.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -2 & 8 & 1 \\ 7 & 3 & 5 & 2 \end{bmatrix}$$

IDENTITY MATRICES

■ 1. Write the identity matrix I_4.

■ 2. If we want to find the product IA, where I is the identity matrix and A is a 4×2, then what are the dimensions of I?

■ 3. If we want to find the product IA, where I is the identity matrix and A is a 3×4, then what are the dimensions of I?

■ 4. If we want to find the product IA, where I is the identity matrix and A is given, then what are the dimensions of I? What is the product IA?

$$A = \begin{bmatrix} 2 & 8 \\ -2 & 7 \\ 3 & 5 \end{bmatrix}$$

■ 5. If we want to find the product IA, where I is the identity matrix and A is given, then what are the dimensions of I? What is the product IA?

$$A = \begin{bmatrix} 7 & 1 & 3 & -2 \\ 5 & 5 & 2 & 9 \end{bmatrix}$$

6. If A is a 2×4 matrix what are the dimensions of the identity matrix that make the equation true?

$$A \cdot I = A$$

TRANSFORMATIONS

■ 1. Find the resulting vector $\overrightarrow{b}$ after $\overrightarrow{a} = (1,6)$ undergoes a transformation by matrix M.

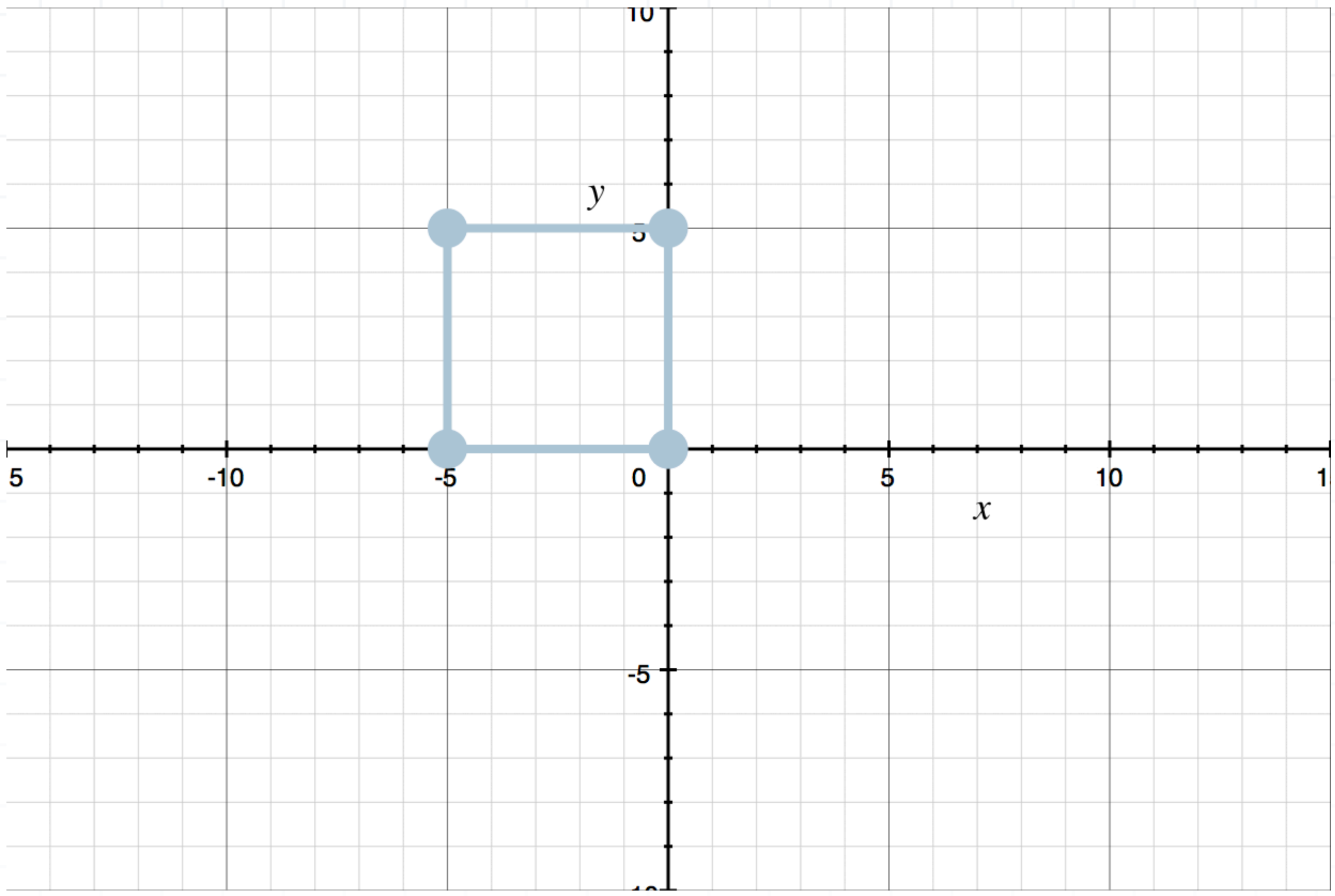
$$M = \begin{bmatrix} -7 & 1 \\ 0 & -2 \end{bmatrix}$$

■ 2. Sketch triangle $\triangle ABC$ with vertices $(2,3)$, $(-3,-1)$, and $(1,-4)$, and the transformation of $\triangle ABC$ after it's transformed by matrix L.

$$L = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

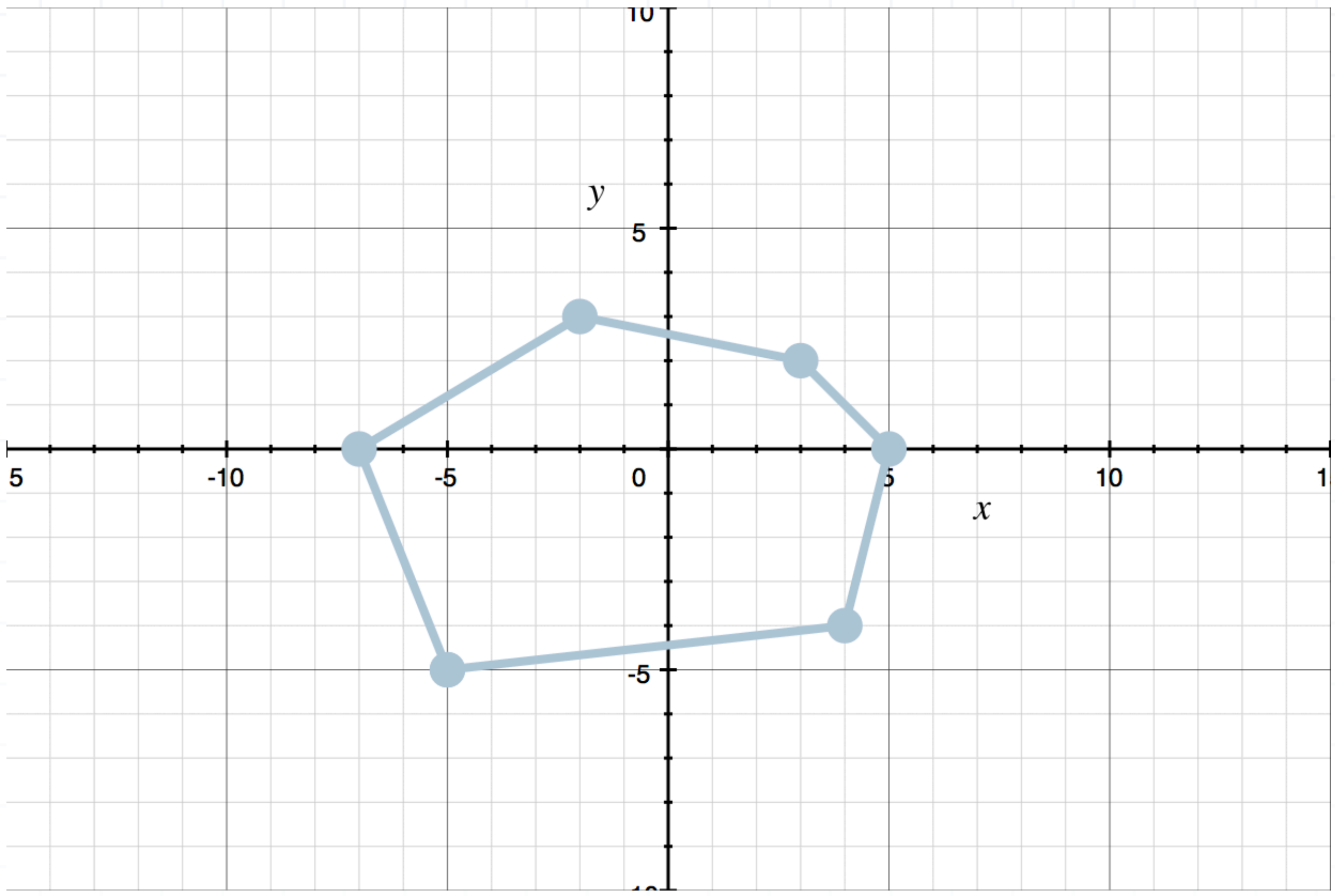
■ 3. Sketch the transformation of the square in the graph after it's transformed by matrix Z.

$$Z = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$



■ 4. Sketch the transformation of the hexagon after it's transformed by matrix Y.

$$Y = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$$



■ 5. What happens to the unit vector $\vec{a}=(1,0)$ after the transformation given by matrix K.

$$K=\begin{bmatrix}3 & -5\\ -1 & 0\end{bmatrix}$$

■ 6. What happens to the unit vector $\vec{b}=(0,1)$ after the transformation given by matrix K.

$$K=\begin{bmatrix}3 & -5\\ -1 & 0\end{bmatrix}$$

MATRIX INVERSES, AND INVERTIBLE AND SINGULAR MATRICES

■ 1. Find the determinant of the matrix.

$$B = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

■ 2. Find the determinant of the matrix.

$$B = \begin{bmatrix} 1 & -6 \\ 5 & 5 \end{bmatrix}$$

■ 3. Find the inverse of matrix G.

$$G = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

■ 4. Find the inverse of matrix N.

$$N = \begin{bmatrix} 11 & -4 \\ 5 & -3 \end{bmatrix}$$

■ 5. Is the matrix invertible or singular?

$$Z = \begin{bmatrix} 4 & 2 \\ -2 & -1 \end{bmatrix}$$

■ 6. Is the matrix invertible or singular?

$$Y = \begin{bmatrix} 0 & 6 \\ 2 & -1 \end{bmatrix}$$

SOLVING SYSTEMS WITH INVERSE MATRICES

■ 1. Use an inverse matrix to solve the system.

$$-4x + 3y = -14$$

$$7x - 4y = 32$$

■ 2. Use an inverse matrix to solve the system.

$$6x - 11y = 2$$

$$-10x + 7y = -26$$

■ 3. Use an inverse matrix to solve the system.

$$13y - 6x = -81$$

$$7x + 17 = -22y$$

■ 4. Sketch a graph of vectors to visually find the solution to the system.

$$3x = 3$$

$$x - y = -2$$

5. Sketch a graph of vectors to visually find the solution to the system.

$$-y = -4$$

$$2x - y = -2$$

6. Sketch a graph of vectors to visually find the solution to the system.

$$x - y = 0$$

$$x + y = 2$$

SOLVING SYSTEMS WITH CRAMER'S RULE

■ 1. Use Cramer's Rule to find the expression that would give the solution for x. You do not need to solve the system.

$$2x - y = 5$$

$$x + 3y = 15$$

■ 2. Use Cramer's Rule to find the expression that would give the solution for x. You do not need to solve the system.

$$ax + by = e$$

$$cx + dy = f$$

■ 3. Use Cramer's Rule to find the expression that would give the solution for y. You do not need to solve the system.

$$3x + 4y = 11$$

$$2x - 3y = -4$$

■ 4. Use Cramer's Rule to find the expression that would give the solution for y. You do not need to solve the system.

$$ax + by = e$$

$$cx + dy = f$$

■ 5. Use Cramer's Rule to solve for x.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

■ 6. Use Cramer's Rule to solve for y.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

FRACTION DECOMPOSITION

■ 1. Find the form of partial fractions decomposition of the rational function.

$$f(x) = \frac{6x + 16}{x^2 + 10x + 21}$$

■ 2. Identify the repeated factors in the denominator of rational function.

$$f(x) = \frac{3x + 7}{(x - 1)(x^2 - 1)(x^2 + 1)^3(x^2 - 2x - 3)}$$

■ 3. How many fractions will exist in the partial fractions decomposition of the function?

$$f(x) = \frac{1}{(x^2 + 1)(x^4 + 5x^2 + 6)}$$

■ 4. Find the form of the partial fractions decomposition of the function, without solving for the constants.

$$f(x) = \frac{1}{(x^2 + 1)(x^4 - 1)}$$

5. Find the form of the partial fractions decomposition of the function, without solving for the constants.

$$f(x) = \frac{x^2}{(x^2 + 2x)(x^2 + 2x + 2)}$$

6. Find the form of the partial fractions decomposition of the function, without solving for the constants.

$$f(x) = \frac{x^2 + 2}{(1 - x)(1 - 2x)(1 - 3x)}$$

DISTINCT LINEAR FACTORS

■ 1. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{4}{(3x-1)(x+1)}$$

■ 2. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{24}{x(x+4)(x-2)}$$

■ 3. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{3x}{(x+2)(x+5)}$$

■ 4. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{6x}{(x^2-1)(x-2)}$$

■ 5. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x+1}{9x^3 - x}$$

■ 6. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{6x - 24}{(x^2 - 1)(x^2 - 4)}$$

REPEATED LINEAR FACTORS

■ 1. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^3 - 2x^2 + 2x - 3}{(x-2)^4}$$

■ 2. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^3 + 3x^2 + 3x + 3}{(x+1)^4}$$

■ 3. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x}{(x+2)^3}$$

■ 4. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{2x + 5}{x^2 - 2x + 1}$$

■ 5. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^2 - 21x + 100}{(x - 10)^3}$$

6. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^2 + 8x}{(x + 8)^3}$$

DISTINCT QUADRATIC FACTORS

■ 1. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{3x^3 + 3}{(x^2 + 2)(x^2 + 5)}$$

■ 2. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{4x^2 + 4x}{(2x^2 + 2)(3x^2 + 1)}$$

■ 3. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{6x^3 - 13x^2 + 4x - 3}{(4x^2 + 1)(x^2 + x + 1)}$$

■ 4. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^3 + 1}{(x^2 + 1)(x^2 + x + 2)}$$

■ 5. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x+3}{(x^2+x+1)(x^2+2x+2)}$$

■ 6. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{3x^4+3x^3+3x^2}{(x^2+1)(2x^2+5)(x^2+x+1)}$$

REPEATED QUADRATIC FACTORS

■ 1. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{10x^3 - 7}{(5x^2 + 3)^2}$$

■ 2. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{3x^3 + 2x^2 + 30x + 16}{(x^2 + 9)^2}$$

■ 3. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^3 - 3x^2 - 11}{(x^2 + x + 2)^2}$$

■ 4. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{9x^4 + 6x^2 + x + 3}{(3x^2 + 1)^3}$$

■ 5. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^3 + 5x^2 + 13x + 2}{(x^2 + 4x + 6)^2}$$

■ 6. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x^7}{(x^2 + 1)^4}$$

MIXED FACTORS

■ 1. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{2x^4 + 16}{x(x^2 + 2)^2}$$

■ 2. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{4x + 8}{(x^2 - 1)(2x + 2)(2x + 1)}$$

■ 3. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{2x^3 + 7x^2 - 2x + 5}{x^4 - 1}$$

■ 4. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{36}{(x + 2)(x^2 - 1)^2}$$

■ 5. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{4x^4 + 7x^3 + 4x^2 + 3x - 2}{x^3(x^2 + x + 1)}$$

■ 6. Rewrite the function as its partial fractions decomposition.

$$f(x) = \frac{x + 1}{(x^2 + 1)(x^2 + x + 1)}$$

IDENTIFYING CONIC SECTIONS

■ 1. Identify the equation as a circle, ellipse, parabola, or hyperbola.

$$5y^2 - 2 = x + 3y + 6$$

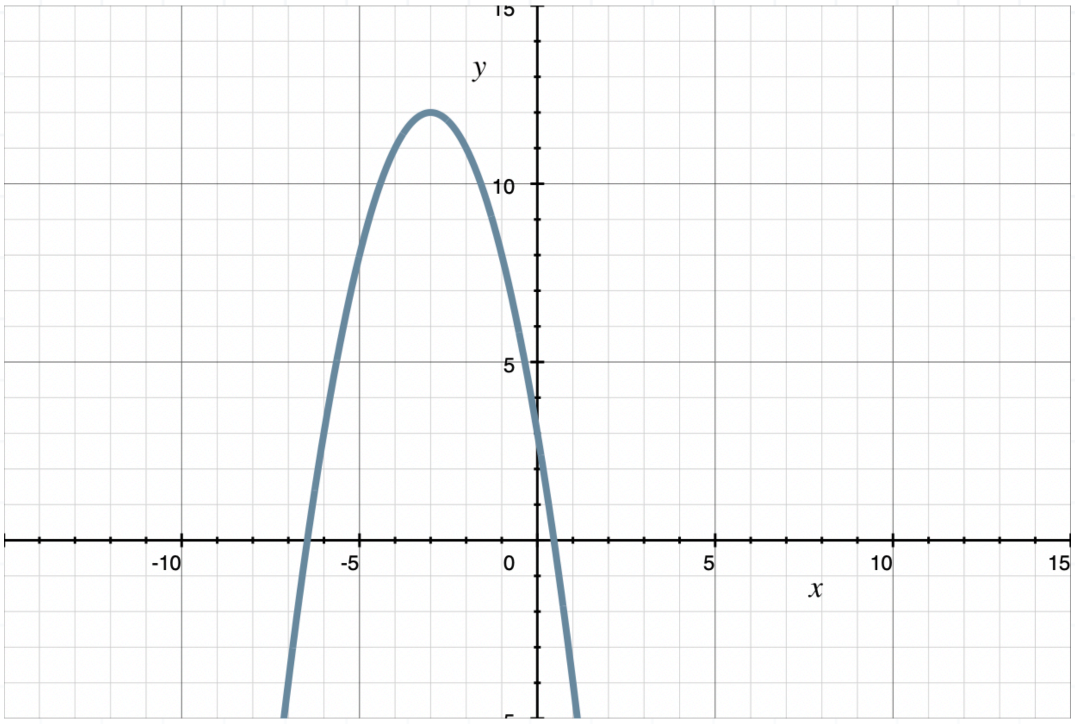
■ 2. Identify the equation as a circle, ellipse, parabola, or hyperbola.

$$x^2 - 5x + 2y = 1 - y^2$$

■ 3. Identify the equation as a circle, ellipse, parabola, or hyperbola.

$$8y^2 - 9x + 2y = -2x^2 + 6$$

■ 4. Identify the graph as a circle, ellipse, parabola, or hyperbola.



■ 5. Identify the equation as a circle, ellipse, parabola, or hyperbola.

$$11x + 12y^2 - 2 = 9y - 12x^2 + 15$$

■ 6. Identify the equation as a circle, ellipse, parabola, or hyperbola.

$$-5x + 14y - 4x^2 = 25 - 2y^2$$

CIRCLES

■ 1. If the center of a circle is $(-4,1)$ and a point on the circle is $(0,-2)$, find the equation of the circle.

■ 2. If the center of a circle is $(7,-2)$ and a point on the circle is $(10,-4)$, find the equation of the circle.

■ 3. Graph the circle.

$$(x-1)^2+(y+9)^2=49$$

■ 4. Find the center and radius of the circle.

$$(x-7)^2+(y+11)^2=18$$

■ 5. Find the center and radius of the circle.

$$x^2+y^2-4x-2y+1=0$$

■ 6. Find the center and radius of the circle.

$$x^2 + y^2 + 12x - 26y + 173 = 0$$

ELLIPSES

■ 1. Sketch the graph of the ellipse by finding its center and major and minor radii.

$$\frac{(x-4)^2}{9}+\frac{(y-3)^2}{25}=1$$

■ 2. Sketch the graph of the ellipse by finding its center and major and minor radii.

$$\frac{(x-6)^2}{9}+\frac{(y+4)^2}{4}=1$$

■ 3. Find the coordinates of the foci of the ellipse.

$$\frac{(x+7)^2}{4}+\frac{(y+6)^2}{20}=1$$

■ 4. Find the coordinates of the foci of the ellipse.

$$\frac{(x-3)^2}{8}+\frac{(y-6)^2}{5}=1$$

■ 5. Sketch the graph of the ellipse.

$$x^2 - 12y + 37 = 6 - 3y^2 - 10x$$

■ 6. Sketch the graph of the ellipse.

$$14y - 24x + 85 = 16 - 4x^2 - y^2$$

PARABOLAS

■ 1. Find the equation of the parabola with a focus at $(-1,9)$ and a directrix at $y=7$.

■ 2. Find the equation of the parabola with a focus at $(3,-7)$ and a directrix at $y=-3$.

■ 3. Find the focus and directrix of the parabola.

$$y=x^2-3$$

■ 4. Find the focus and directrix of the parabola.

$$y=-\frac{1}{3}(x-1)^2+2$$

■ 5. Find each piece of the parabola from its equation.

$$y=\frac{1}{2}x^2+4$$

6. Find each piece of the parabola from its equation.

$$x = -\frac{2}{3}(y+2)^2 + 1$$

HYPERBOLAS

■ 1. Find the asymptotes, foci, vertices, and directrices of the hyperbola.

$$\frac{y^2}{4} - \frac{x^2}{25} = 1$$

■ 2. Find the asymptotes, foci, vertices, and directrices of the hyperbola.

$$\frac{x^2}{4} - \frac{y^2}{81} = 1$$

■ 3. Find the asymptotes, foci, vertices, and directrices of the hyperbola.

$$\frac{(y-3)^2}{36} - \frac{(x+2)^2}{9} = 1$$

■ 4. Find the asymptotes, foci, vertices, and directrices of the hyperbola.

$$\frac{(x+1)^2}{25} - \frac{(y+4)^2}{144} = 1$$

■ 5. Sketch the graph of the hyperbola.

$$\frac{y^2}{16} - \frac{x^2}{4} = 1$$

■ 6. Sketch the graph of the hyperbola.

$$\frac{(x+1)^2}{2} - \frac{(y-1)^2}{12} = 1$$

ROTATING AXES

■ 1. Find the angle of rotation of the conic.

$$3x^2 + 2xy + y^2 - y - 12 = 0$$

■ 2. Find the vertex of the parabola.

$$x^2 + 2xy + y^2 = 2x - 2y + 4$$

■ 3. Sketch the graph of $x^2 + \sqrt{3}xy = 1$.

■ 4. Find foci of the conic.

$$2x^2 - 4xy + 5y^2 - 4x - 8y + 8 = 0$$

■ 5. Use the discriminant to determine the shape of the conic.

$$-2x^2 - xy - y^2 + 4x + y + 3 = 0$$

■ 6. Use the discriminant to determine the shape of the conic.

$$25x^2 + 30xy + 9y^2 - 12x - 8 = 0$$

POLAR EQUATIONS OF CONICS

■ 1. A hyperbola has vertices at $(2,-1)$ and $(2,3)$, directrices $y=0$ and $y=2$, and foci at $(2,5)$ and $(2,-3)$. Find the eccentricity of hyperbola.

■ 2. Find the eccentricity of the conic.

$$\frac{(x+1)^2}{4}-\frac{(y-1)^2}{32}=1$$

■ 3. Find the foci of the ellipse.

$$r=\frac{5}{3-2\cos\theta}$$

■ 4. A conic has a focus at $(0,0)$ with a corresponding directrix of $y=-5$ that passes thought the point $(5,0)$. Write the conic equation in polar coordinates.

■ 5. Determine the shape of the conic section.

$$r=\frac{10}{6+4\cos\theta}$$

■ 6. Find the equation of the conic section that has eccentricity $e = 5/4$, directrix $x = -2$, and is rotated by $\alpha = \pi/3$.

PARAMETRIC CURVES AND ELIMINATING THE PARAMETER

■ 1. Sketch the curve defined by the parametric equations.

$$x = \arcsin t$$

$$y = \arccos t$$

■ 2. Sketch the curve defined by the parametric equations.

$$x = 1 + 2\cos t$$

$$y = 3 + \sin t$$

■ 3. Eliminate the parameter.

$$x = t^2 + 3t - 4$$

$$y = \sqrt[3]{t}$$

■ 4. Eliminate the parameter.

$$x = t + \frac{1}{t}$$

$$y = t^2 + \frac{1}{t^2}$$

■ 5. Eliminate the parameter.

$$x = 2 + 3\cos t$$

$$y = 4 - \cos t$$

■ 6. Eliminate the parameter.

$$x = 1 + 2\cos t$$

$$y = 5 + 3\sin t$$

DIRECTION OF THE PARAMETER

■ 1. Sketch the graph of the parametric curve and indicate the direction of increasing t for $t > 0$.

$$x = t^2$$

$$y = t - 2$$

■ 2. Sketch the graph of the parametric curve and indicate the direction of increasing t.

$$x = 1 + \cos t$$

$$y = 2 + \sin t$$

■ 3. Sketch the graph of the parametric curve and indicate the direction of increasing t.

$$x = t^2$$

$$y = t^3$$

■ 4. Sketch the graph of the parametric curve and indicate the direction of increasing t on the interval $\pi/6 < t < \pi/3$.

$$x = \tan t$$

$$y = \cot t$$

■ 5. Sketch the graph of the parametric curve and indicate the direction of increasing t.

$$x = \sin t + \cos t$$

$$y = \sin t - \cos t$$

■ 6. Sketch the graph of the parametric curve and indicate the direction of increasing t.

$$x = t^2 + t$$

$$y = 1 - t$$

FINDING THE PARAMETRIC REPRESENTATION

■ 1. Express the rectangular equation in parametric form.

$$(x-3)^2-(y-4)^2=4$$

■ 2. Express the rectangular equation in parametric form.

$$x^2+2xy+x+4y-5=0$$

■ 3. Express the rectangular equation in parametric form.

$$x^2+4y^2-6x+8y=0$$

■ 4. Express the rectangular equation in parametric form.

$$x^2+4xy+4y^2=0$$

■ 5. Express the rectangular equation in parametric form.

$$x^2y^2-y^2+1=0$$

■ 6. Express the rectangular equation in parametric form.

$$xy + \sin y - 1 = 0$$

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