Precalculus
Final Exam Solutions

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MATH

Precalculus Final Exam Answer Key

1. (5 pts) ■ B C D E

2. (5 pts) ■ B C D E

3. (5 pts) A B C D ■

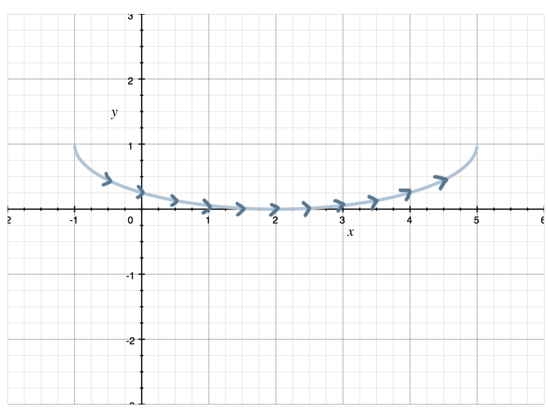
4. (5 pts) A B ■ D E

5. (5 pts) A B ■ D E

6. (5 pts) A B C D ■

7. (5 pts) A B C ■ E

8. (5 pts) A ■ C D E

9. (15 pts) 

10. (15 pts) $(\sqrt{14},0)$ and $(-\sqrt{14},0)$

11. (15 pts) -64

12. (15 pts)

$$z_1 = 2\left(\cos\frac{9\pi}{32} + i\sin\frac{9\pi}{32}\right)$$

$$z_3 = 2\left(\cos\frac{41\pi}{32} + i\sin\frac{41\pi}{32}\right)$$

$$z_2 = 2\left(\cos\frac{25\pi}{32} + i\sin\frac{25\pi}{32}\right)$$

$$z_4 = 2\left(\cos\frac{57\pi}{32} + i\sin\frac{57\pi}{32}\right)$$

Precalculus Final Exam Solutions

1. A. The product A^2 is

$$A^2 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1-1 & -1+1 \\ 1-1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Then the determinant of the zero matrix is $|A^2| = 0$.

2. A. We've been asked to convert the polar coordinates (r, θ) to rectangular coordinates (x, y). Remember that $x = r\cos\theta$ and $y = r\sin\theta$. In this problem, $r = 2$ and $\theta = \pi/3$. So the value of x is

$$x = 2\cos\frac{\pi}{3}$$

$$x = 2\left(\frac{1}{2}\right)$$

$$x = 1$$

and the value of y is

$$y = 2\sin\frac{\pi}{3}$$

$$y = 2\left(\frac{\sqrt{3}}{2}\right)$$

$$y = \sqrt{3}$$

The rectangular coordinates are $(1,\sqrt{3})$.

3. E. Solve the conversion equation $x = r\cos\theta$ for $\cos\theta$.

$$\cos\theta = \frac{x}{r}$$

Now we can substitute into the equation.

$$r = 4\cos\theta$$

$$r = 4\frac{x}{r}$$

$$r^2 = 4x$$

Substitute r^2 using the conversion equation $r^2 = x^2 + y^2$.

$$x^2 + y^2 = 4x$$

This is the equation of a circle. To find the standard form of the equation, we'll need to move all terms to one side and complete the square.

$$x^2 - 4x + y^2 = 0$$

Complete the square by dividing the coefficient of x by 2 and squaring the result.

$$\frac{-4}{2} = -2$$

$$(-2)^2 = 4$$

Add 4 to both sides of the equation.

$$x^2 - 4x + 4 + y^2 = 4$$

Factor $x^2 - 4x + 4$.

$$(x - 2)^2 + y^2 = 4$$

This is the equation of the circle whose center has rectangular coordinates $(2,0)$ and whose radius is 2.

4. C. The polar curve is a rose, and since the petals lie on the x- and y -axes, the equation is in the form $r = a\cos n\theta$. The radius of the petals is 6, so $a = 6$. Since there are 8 petals, $2n = 8$ and $n = 4$. Therefore, the polar equation is $r = 6\cos 4\theta$.

5. C. First, find the equation of the parabola in the standard form $x - h = a(y - k)^2$.

$$y^2 + 4y - x + 1 = 0$$

$$x - 1 = y^2 + 4y$$

$$x - 1 + 4 = y^2 + 4y + 4$$

$$x + 3 = (y + 2)^2$$

The focus is at $(h + (1/4)a, k)$.

$$\left(-3+\left(\frac{1}{4}\right)(1), -2\right)$$

$$\left(\frac{-12}{4}+\frac{1}{4}, -2\right)$$

$$\left(\frac{-11}{4}, -2\right)$$

6. E. Start by finding the standard form of the ellipse.

$$x^2-2x+2y^2+12y-17=0$$

$$x^2-2x+2(y^2+6y)=17$$

$$x^2-2x+1+2(y^2+6y+9)=17+1+18$$

$$(x-1)^2+2(y+3)^2=36$$

$$\frac{(x-1)^2}{36}+\frac{(y+3)^2}{18}=1$$

Now we see that $a=\sqrt{36}=6$ and $b=\sqrt{18}$. To find eccentricity, we need to find c/a. Use the formula $b^2+c^2=a^2$ to find c.

$$(\sqrt{18})^2+c^2=6^2$$

$$18+c^2=36$$

$$c^2=18$$

$$c = \sqrt{18}$$

Therefore,

$$\frac{c}{a} = \frac{\sqrt{18}}{6} \approx 0.7071$$

7. D. Rewrite the equation so that all the i terms are written as multiples of i^2 and i.

$$5i^2 - 2i^3 + i^4 + 4i$$

$$5i^2 - 2i^2i + i^2i^2 + 4i$$

Substitute -1 for i^2.

$$5(-1) - 2(-1)i + (-1)(-1) + 4i$$

$$-5 + 2i + 1 + 4i$$

$$-4 + 6i$$

8. B. To multiply complex numbers, multiply the outside numbers and add the angles.

$$\left[2\sqrt{3}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]\left[\frac{4}{\sqrt{3}}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]$$

$$2\sqrt{3} \cdot \frac{4}{\sqrt{3}} \left[\cos \left(\frac{3\pi}{4} + \frac{\pi}{3} \right) + i \sin \left(\frac{3\pi}{4} + \frac{\pi}{3} \right) \right]$$

$$8 \left[\cos \left(\frac{9\pi}{12} + \frac{4\pi}{12} \right) + i \sin \left(\frac{9\pi}{12} + \frac{4\pi}{12} \right) \right]$$

$$8 \left(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12} \right)$$

9. Start by solving the equations for $\sin t$ and $\cos t$. We get

$$x = 2 + 3 \sin t$$

$$\sin t = \frac{x-2}{3}$$

and

$$y = 1 - \cos t$$

$$\cos t = 1 - y$$

Use the basic Pythagorean identity $\sin^2 t + \cos^2 t = 1$ and substitute the results above.

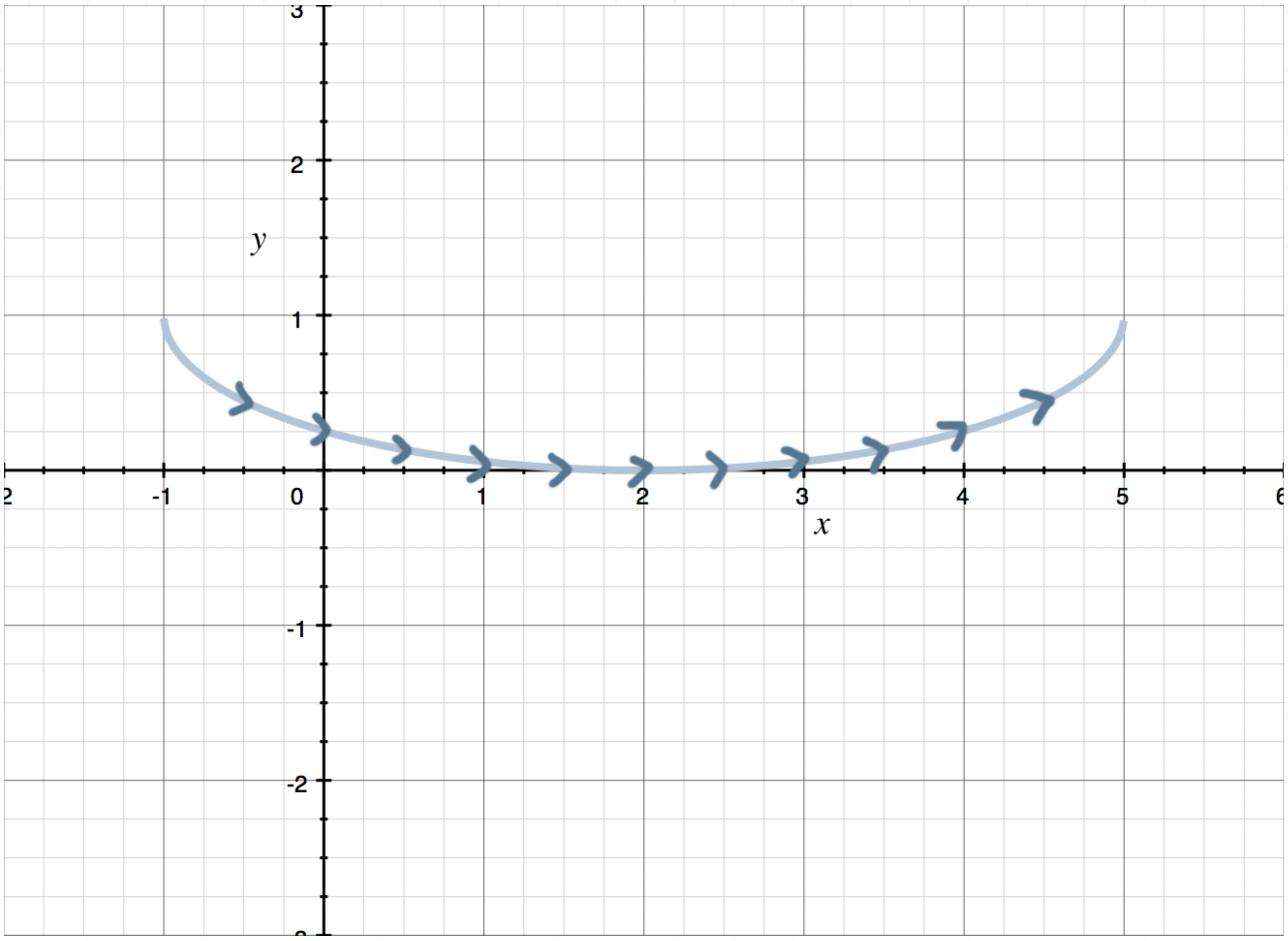
$$\left(\frac{x-2}{3} \right)^2 + (1-y)^2 = 1$$

$$\frac{(x-2)^2}{9} + (1-y)^2 = 1$$

This is the equation of an ellipse, and we now know the shape of the graph. Let's make a table of values to help us sketch the graph.

t	$x = 2 + 3\sin t$	$y = 1 - \cos t$
$-\frac{\pi}{2}$	$2 + 3\sin\left(-\frac{\pi}{2}\right) = 2 + 3(-1) = -1$	$1 - \cos\left(-\frac{\pi}{2}\right) = 1 - 0 = 1$
$-\frac{\pi}{4}$	$2 + 3\sin\left(-\frac{\pi}{4}\right) = 2 - \frac{3\sqrt{2}}{2} \approx -0.21$	$1 - \cos\left(-\frac{\pi}{4}\right) = 1 - \frac{\sqrt{2}}{2} \approx 0.29$
0	$2 + 3\sin 0 = 2 + 3(0) = 2$	$1 - \cos 0 = 1 - 1 = 0$
$\frac{\pi}{4}$	$2 + 3\sin\frac{\pi}{4} = 2 + \frac{3\sqrt{2}}{2} \approx 4.21$	$1 - \cos\frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2} \approx 0.29$
$\frac{\pi}{2}$	$2 + 3\sin\frac{\pi}{2} = 2 + 3(1) \approx 5$	$1 - \cos\frac{\pi}{2} = 1 - 0 = 1$

Plot the points in order, drawing arrows in the same direction as you plot the points (counter-clockwise).



10. Square each equation and simplify. For x we get

$$x = 7t + \frac{1}{2t}$$

$$x^2 = \left(7t + \frac{1}{2t}\right)^2$$

$$x^2 = 49t^2 + \frac{1}{4t^2} + 7$$

and for y we get

$$y = 7t - \frac{1}{2t}$$

$$y^2 = \left(7t - \frac{1}{2t}\right)^2$$

$$y^2 = 49t^2 + \frac{1}{4t^2} - 7$$

To find the vertices, subtract y^2 from x^2.

$$x^2 - y^2 = 49t^2 + \frac{1}{4t^2} + 7 - \left(49t^2 + \frac{1}{4t^2} - 7\right)$$

$$x^2 - y^2 = 49t^2 + \frac{1}{4t^2} + 7 - 49t^2 - \frac{1}{4t^2} + 7$$

$$x^2 - y^2 = 14$$

Therefore, the vertices of the hyperbola are at

$(\sqrt{14},0)$ and $(-\sqrt{14},0)$

11. To find the complex number raised to the power of 4, raise the outside number to the power of 4 and multiply the angle by 4.

$$(2\sqrt{2})^4 \left[\cos\left(4 \cdot \frac{5\pi}{4}\right) + i \sin\left(4 \cdot \frac{5\pi}{4}\right)\right]$$

$$16 \cdot 4(\cos 5\pi + i \sin 5\pi)$$

$$64 \cos 5\pi + 64i \sin 5\pi$$

$$64(-1) + 64(0)$$

-64

12. There will be 4 roots since we're taking the fourth root of a complex number. Find the first root by taking the fourth root of the outside number and dividing the angle by 4.

$$\sqrt[4]{16}\left[\cos\left(\frac{1}{4}\cdot\frac{9\pi}{8}\right)+i\sin\left(\frac{1}{4}\cdot\frac{9\pi}{8}\right)\right]$$

$$z_1=2\left(\cos\frac{9\pi}{32}+i\sin\frac{9\pi}{32}\right)$$

To find the other three roots, divide 2π by 4 (since we're taking the fourth root) and add the result to the previous angle.

$$\frac{2\pi}{4}=\frac{\pi}{2}=\frac{16\pi}{32}$$

Then the second root is

$$z_2=2\left[\cos\left(\frac{9\pi}{32}+\frac{16\pi}{32}\right)+i\sin\left(\frac{9\pi}{32}+\frac{16\pi}{32}\right)\right]$$

$$z_2=2\left(\cos\frac{25\pi}{32}+i\sin\frac{25\pi}{32}\right)$$

the third root is

$$z_3=2\left[\cos\left(\frac{25\pi}{32}+\frac{16\pi}{32}\right)+i\sin\left(\frac{25\pi}{32}+\frac{16\pi}{32}\right)\right]$$

$$z_3 = 2\left(\cos\frac{41\pi}{32} + i\sin\frac{41\pi}{32}\right)$$

and the fourth root is

$$z_4 = 2\left[\cos\left(\frac{41\pi}{32} + \frac{16\pi}{32}\right) + i\sin\left(\frac{41\pi}{32} + \frac{16\pi}{32}\right)\right]$$

$$z_4 = 2\left(\cos\frac{57\pi}{32} + i\sin\frac{57\pi}{32}\right)$$

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