Trigonometry Formulas

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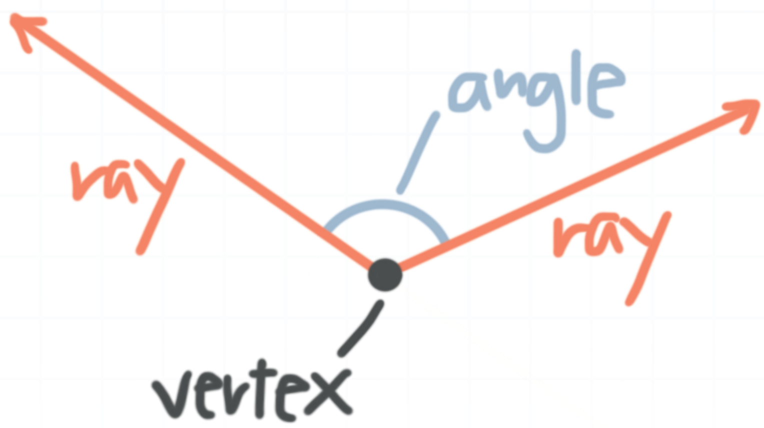
Angles

Naming angles

Ray: a line that's infinitely long in only one direction

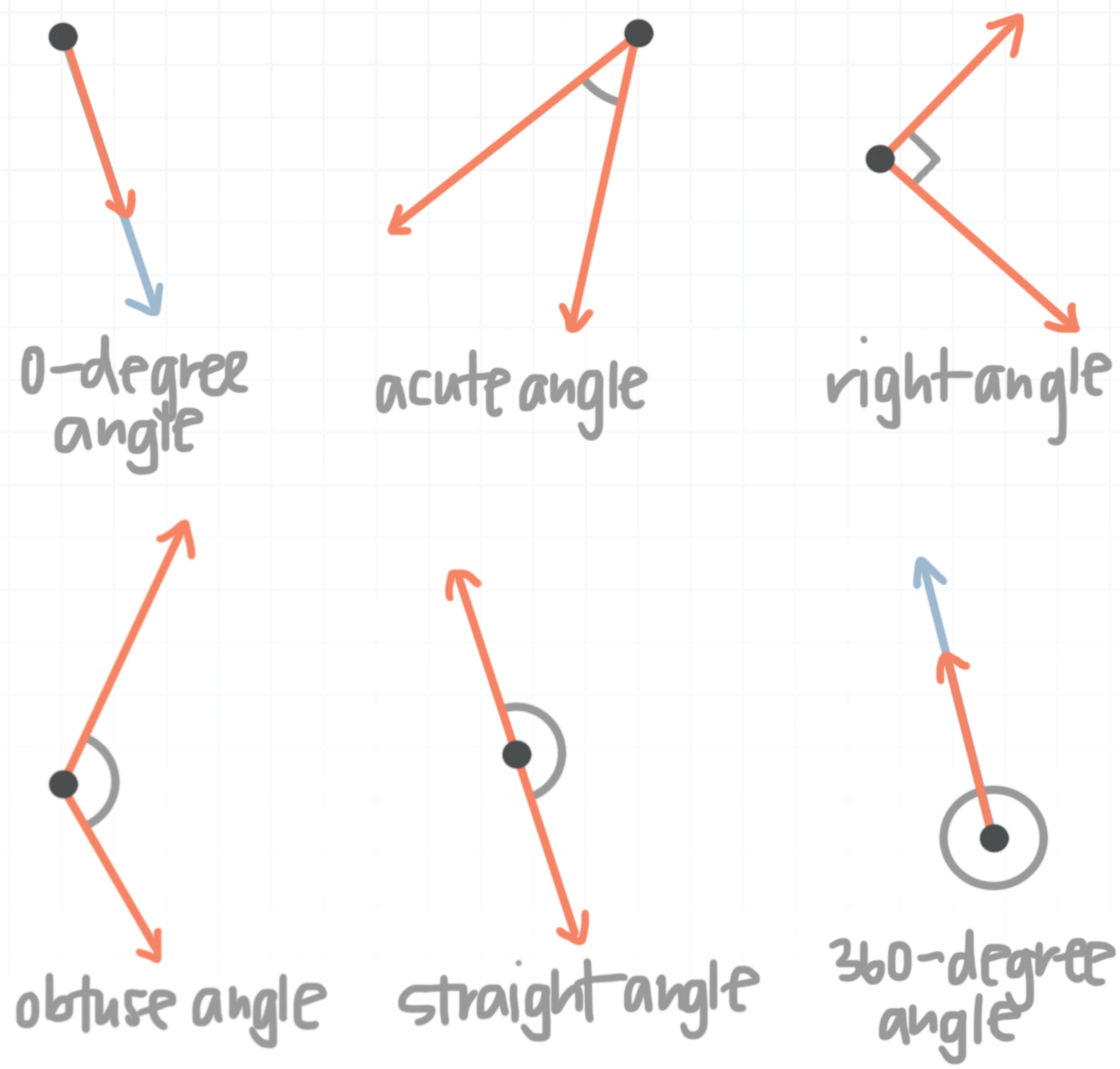
Angle: we get an **angle** when we put two rays together, with their endpoints at the same spot

Vertex: the corner of the angle



Angles:

Angle in degrees	Angle in radians	Angle name
$\theta = 0^\circ$	$\theta = 0$	0° or zero angle
$0^\circ < \theta < 90^\circ$	$0 < \theta < \pi/2$	Acute angle
$\theta = 90^\circ$	$\theta = \pi/2$	Right angle
$90^\circ < \theta < 180^\circ$	$\pi/2 < \theta < \pi$	Obtuse angle
$\theta = 180^\circ$	$\theta = \pi$	Straight angle
$\theta = 360^\circ$	$\theta = 2\pi$	360° or complete angle



Complementary and supplementary angles

Complementary angles: two angles that sum to 90° or $\pi/2$ and form a right angle

Supplementary angles: two angles that sum to 180° or π and form a straight angle

Positive and negative angles in standard position

Standard position: the angle's initial side lies along the positive direction of the x-axis, its vertex is at the origin, and it opens with positive, counterclockwise rotation toward the first quadrant

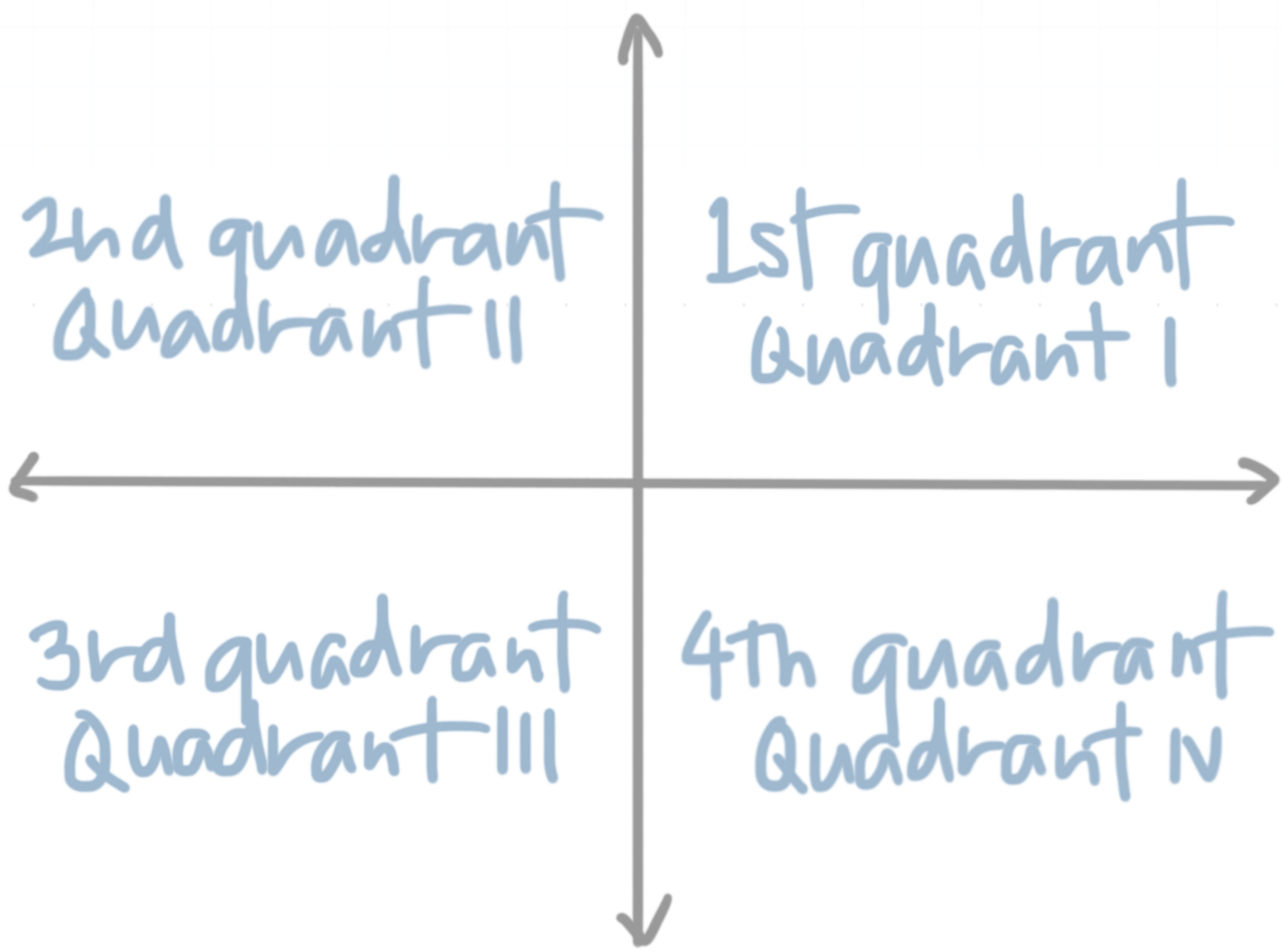
Initial side: the side where the angle begins

Terminal side: the side where the angle ends

Positive angle: rotation from the initial side to the terminal side is counterclockwise

Negative angle: rotation from the initial side to the terminal side is clockwise

Quadrants of the Cartesian (rectangular) coordinate system:

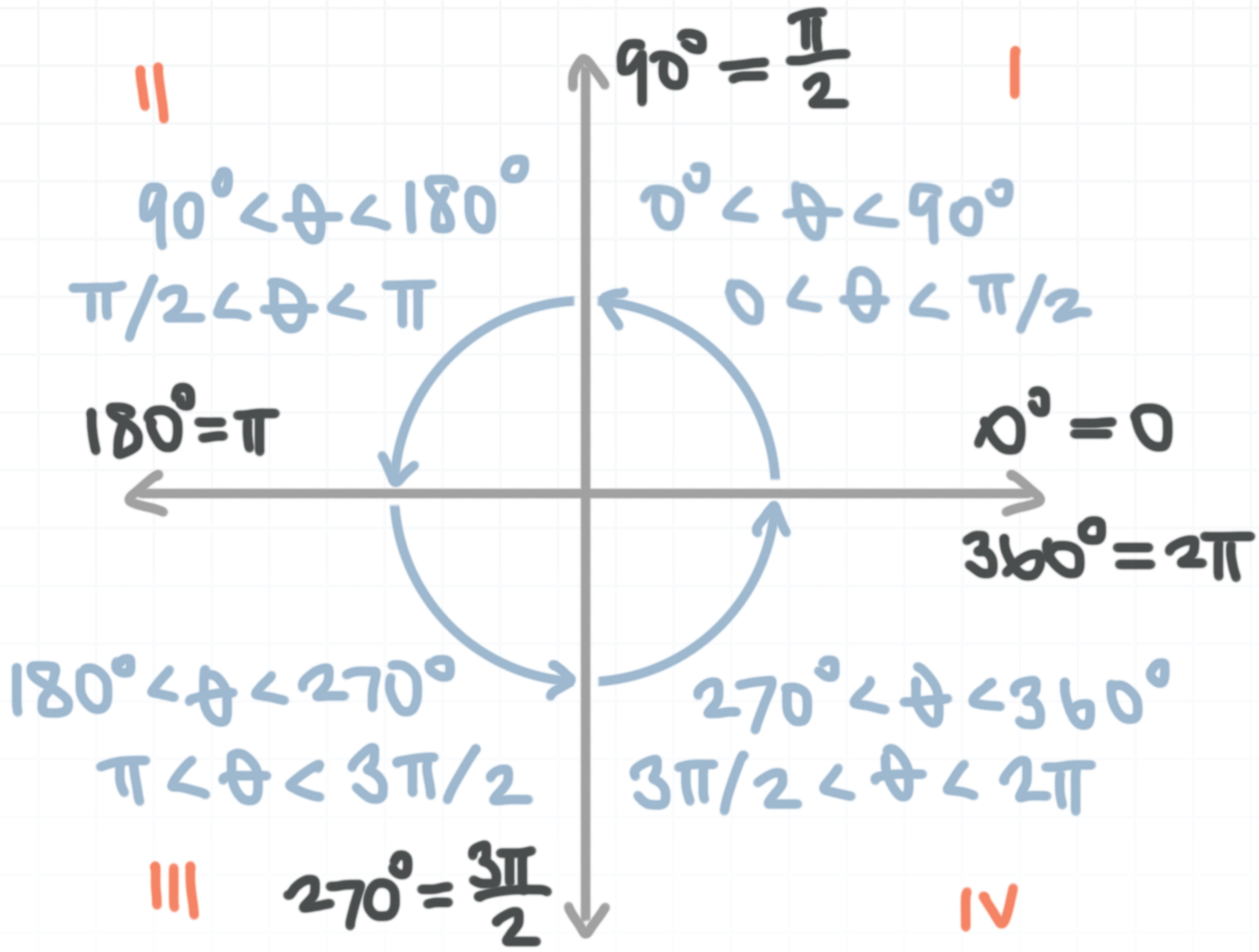


Quadrantal angles: angles in which the terminal side falls exactly on one of the axes

Axis	Degrees	Radians
Positive x-axis	0° or -360°	0 or -2π
Positive y-axis	90° or -270°	$\pi/2$ or $-3\pi/2$
Negative x-axis	180° or -180°	π or $-\pi$
Negative y-axis	270° or -90°	$3\pi/2$ or $-\pi/2$
Positive x-axis	360°	2π

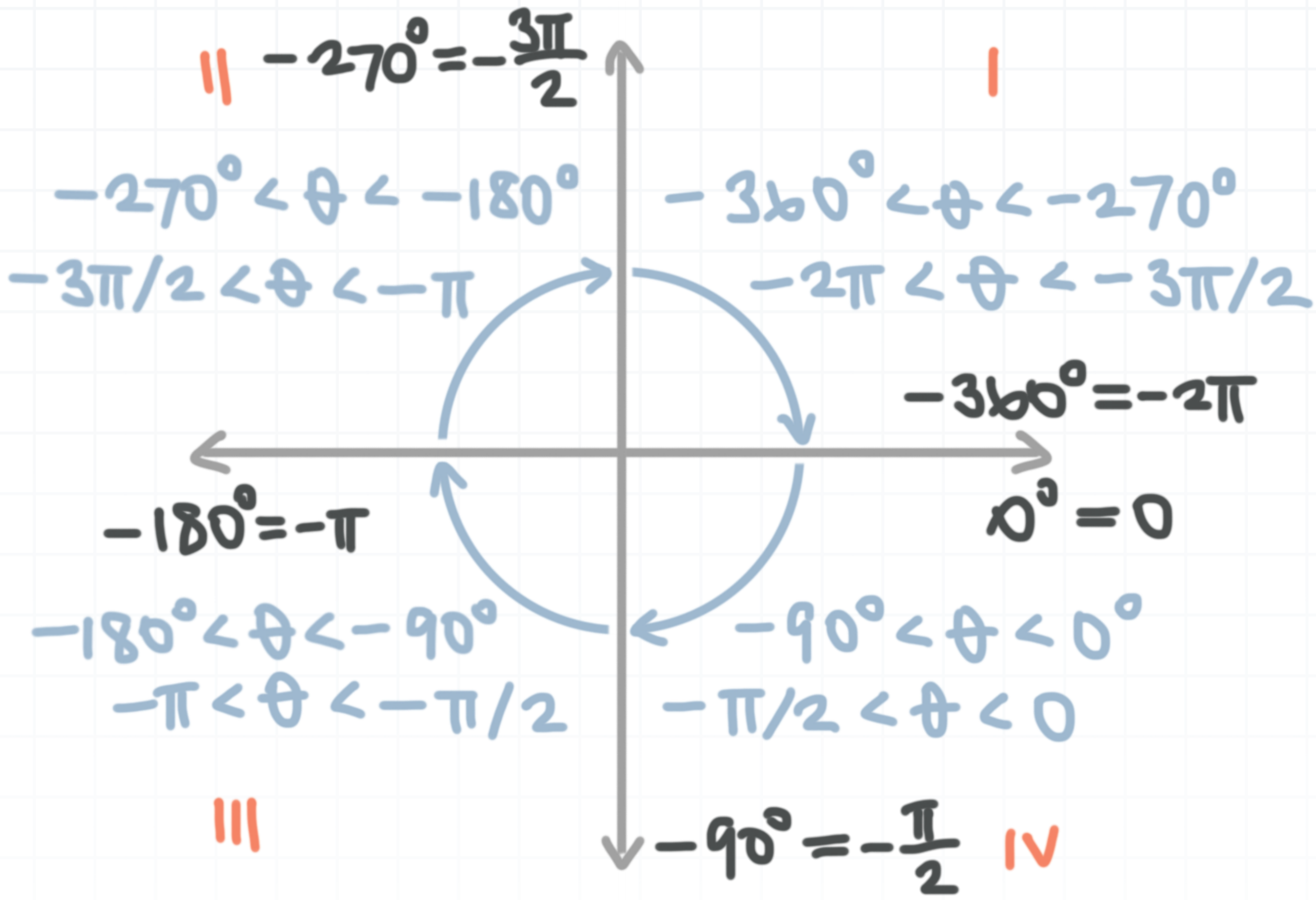
Positive angle definitions of each quadrant:

Quadrant	Degrees	Radians
First	$0^\circ < \theta < 90^\circ$	$0 < \theta < \pi/2$
Second	$90^\circ < \theta < 180^\circ$	$\pi/2 < \theta < \pi$
Third	$180^\circ < \theta < 270^\circ$	$\pi < \theta < 3\pi/2$
Fourth	$270^\circ < \theta < 360^\circ$	$3\pi/2 < \theta < 2\pi$



Negative angle definitions of each quadrant:

Quadrant	Degrees	Radians
Fourth	$-90^\circ < \theta < 0^\circ$	$-\pi/2 < \theta < 0$
Third	$-180^\circ < \theta < -90^\circ$	$-\pi < \theta < -\pi/2$
Second	$-270^\circ < \theta < -180^\circ$	$-3\pi/2 < \theta < -\pi$
First	$-360^\circ < \theta < -270^\circ$	$-2\pi < \theta < -3\pi/2$



Degrees, radians, and DMS

DMS: A third angle-measurement system defined by degrees, minutes, and seconds

Degree: 1°

Minute: 1/60th of a degree

Second: 1/60th of a minute, or 1/3,600th of a degree

Converting between coordinate systems:

	To radians	To degrees	To DMS
From radians	-	Multiply by 180°/π	Convert through degrees first
From degrees	Multiply by π/180°	-	Convert fraction of a degree to minutes, then fraction of a minute to a second
From DMS	Convert through degrees first	Convert seconds to minutes, then minutes to degrees	-

Coterminal angles

Coterminal angles: angles whose terminal sides lie on top of each other

The six trig functions

Sine, cosine, and tangent

Right triangle: a triangle that includes exactly one 90° interior angle

Hypotenuse: the side opposite the right angle, which will always be the longest side

Legs: the other two sides of the right triangle that aren't the hypotenuse, and the sides that border the right angle

Trigonometry: the study of triangles

Trigonometric function: a function that gives the relationship between different parts of a triangle

SOH-CAH-TOA:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

SOH: sine, opposite, hypotenuse

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

CAH: cosine, adjacent, hypotenuse

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

TOA: tangent, opposite, adjacent

Sine, cosine, and tangent as circular functions:

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

Cosecant, secant, cotangent, and the reciprocal identities

The six trig functions:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

The six circular functions:

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

The reciprocal identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

The quotient identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

The Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1$$

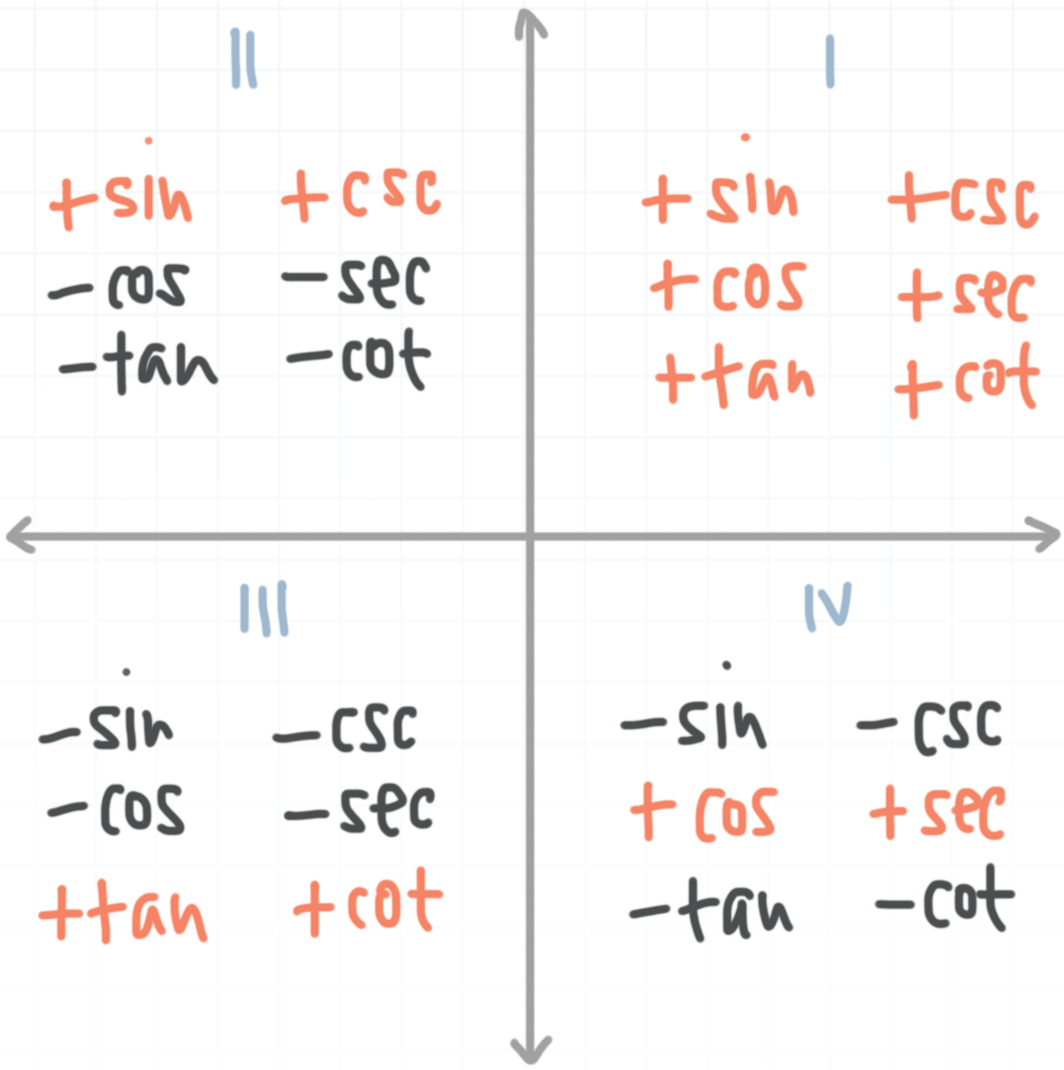
$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

Signs by quadrant

Quadrant	Sign on x	Sign on y	Sign on r
I	+	+	+
II	-	+	+
III	-	-	+
IV	+	-	+

	I	II	III	IV
sin	+	+	-	-
csc	+	+	-	-
cos	+	-	-	+
sec	+	-	-	+
tan	+	-	+	-
cot	+	-	+	-

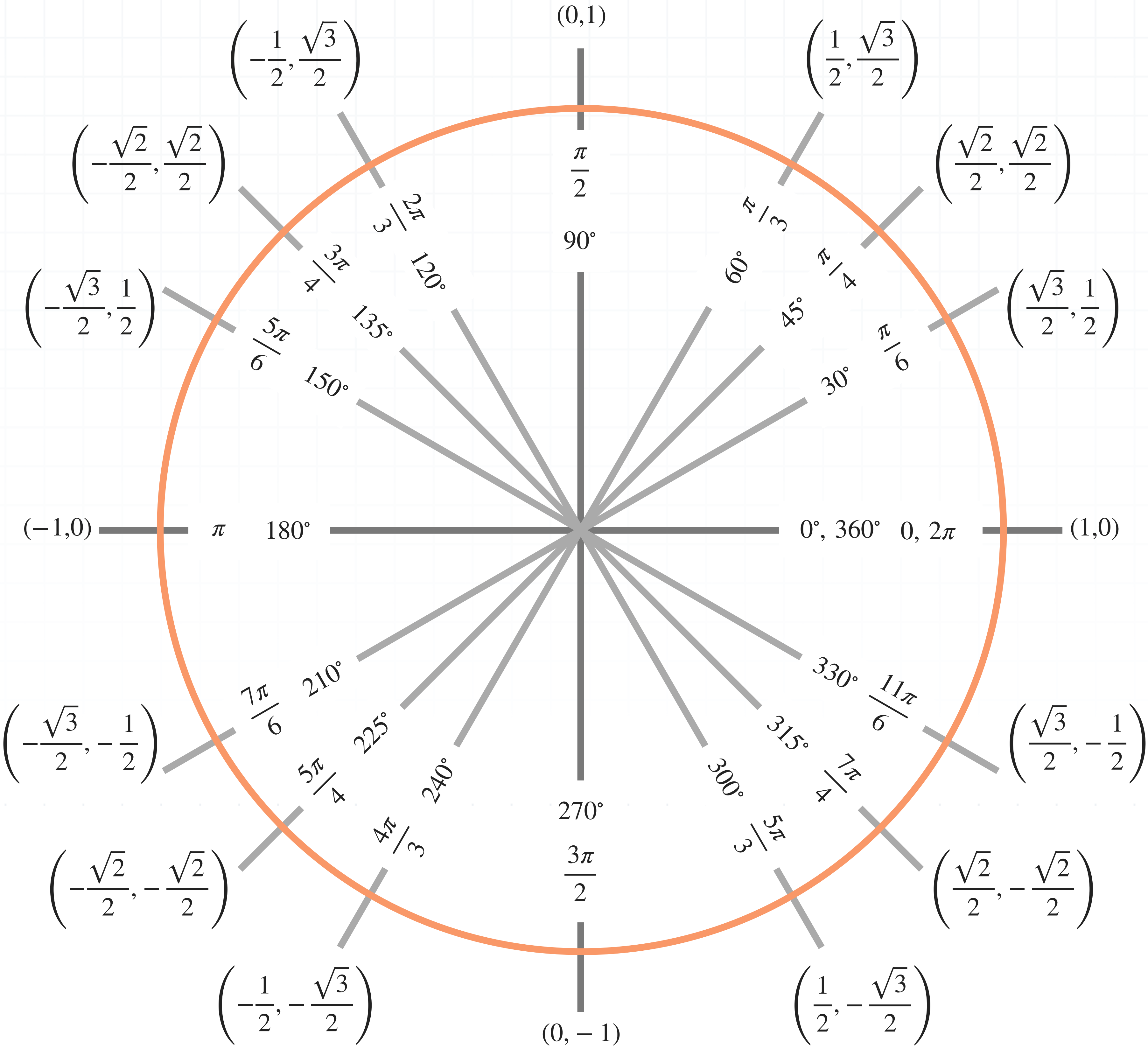


When the trig functions are undefined

	sin	csc	cos	sec	tan	cot
0°=0	0	Undefined	1	1	0	Undefined
90°=π/2	1	1	0	Undefined	Undefined	0
180°=π	0	Undefined	-1	-1	0	Undefined
270°=3π/2	-1	-1	0	Undefined	Undefined	0
360°=2π	0	Undefined	1	1	0	Undefined

The unit circle

The unit circle



Reference angles

Reference angle: for an angle θ in standard position, the reference angle is the positive acute angle formed by the x-axis and the terminal side of θ

θ's quadrant	β in radians	β in degrees
I	$\beta = \theta$	$\beta = \theta$
II	$\beta = \pi - \theta$	$\beta = 180^\circ - \theta$
III	$\beta = \theta - \pi$	$\beta = \theta - 180^\circ$
IV	$\beta = 2\pi - \theta$	$\beta = 360^\circ - \theta$

Even-odd identities

Even functions: $f(-x) = f(x)$

Odd functions: $f(-x) = -f(x)$

Even-odd identities: Cosine and secant are even functions, while sine, cosecant, tangent, and cotangent are odd functions

$\sin(-\theta) = -\sin\theta$ $\quad$ $\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\quad$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

The set of all possible angles

Complete set of coterminal angles: $\alpha = \theta + n(360^\circ)$ in degrees and $\alpha = \theta + n(2\pi)$ in radians

Solving right triangles

Sum of the interior angles of a triangle: $m\angle A + m\angle B + m\angle C = 180^\circ$

Pythagorean theorem: $a^2 + b^2 = c^2$

Angles of elevation and depression

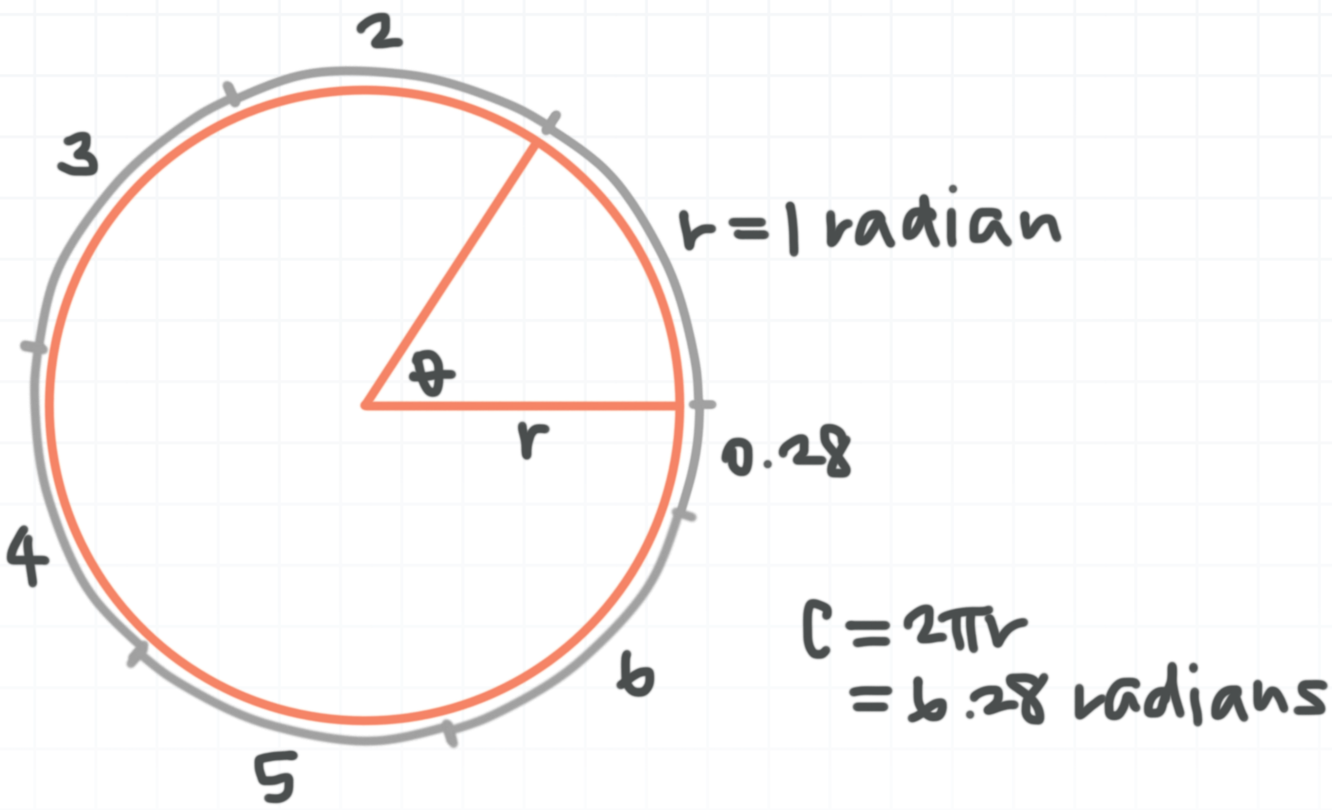
Angle of elevation: a positive angle in standard position, measured from the horizontal side of the angle along the positive side of the x-axis, *up to* the terminal side of the angle

Angle of depression: a negative angle in standard position, measured from the horizontal side of the angle along the positive side of the x-axis, *down to* the terminal side of the angle

Angles in circles

Radians and arc length

Radian: an angle that has its vertex at the center of a circle and that intercepts an arc of the circle equal in length to the circle's radius has an angle measure of one radian



Circumference of a circle: $C = 2\pi r$

Circular arc: an arc follows the perimeter of a circle

Arc length: $s = r\theta$, where s is the length of the arc, r is the radius of the circle, and θ is the central angle that carves out that particular arc. Note: θ has to be in radians

Area of a circular sector

Circular sector: a wedge in a circle, like a piece in a pie

Semicircle: exactly half of the circle

Minor arc: an arc shorter than half of the circle

Major arc: an arc longer than half of the circle

Central angle: the angle at the center of the circle that defines the circular sector

Area of a circular sector:

$$A = \frac{1}{2}r^2\theta \text{ where } \theta \text{ is defined in radians}$$

$$A = \left(\frac{\pi}{360}\right)r^2\theta \text{ where } \theta \text{ is defined in degrees}$$

Equation of the circle: $(x-h)^2 + (y-k)^2 = r^2$

Trig functions of real numbers

The six circular functions: the sine, cosine, tangent, cosecant, secant, and cotangent functions

Linear and angular velocity

Velocity: defined by magnitude and direction

Speed: only the magnitude portion of velocity

Linear velocity: how fast the length of an arc is changing, $v = s/t$, where v is linear velocity, s is arc length, and t is time

Angular velocity: the rate of change of the interior angle (the rate at which the central angle is swept out as we move around the circle), $\omega = \theta/t$, where ω (omega) is angular velocity, and θ is the radian measure of the interior angle at time t

Relating linear and angular velocity

Linear velocity: $v = r\omega$

Graphing trig functions

Sketching sine and cosine

$$y = a\sin(b(x + c)) + d$$

$$y = a\cos(b(x + c)) + d$$

Sketching cosecant and secant

1. Sketch the corresponding reciprocal function. For cosecant this will be sine, for secant it'll be cosine.

2. Sketch in vertical asymptotes where the sine or cosine curve crosses its own midline, since these are the points at which the secant or cosecant functions are undefined.

3. Sketch the U-shapes of the cosecant or secant curve at the maximum and minimum points of the sine or cosine graph, and in between the vertical asymptotes.

Period and amplitude

Amplitude: the distance from the “midline” of the function’s curve to the very top of the curve or the very bottom of the curve (which isn’t defined for tangent, cosecant, secant, or cotangent),

$$|a| = \frac{\text{max} - \text{min}}{2}$$

Period: one full rotation of the function, or how long it takes for the function to get back to its “starting point”:

$2\pi/|b|$ for sine, cosine, secant, and cosecant

$\pi/|b|$ for tangent and cotangent

Sketching tangent and cotangent

1. For $y = a\tan(bx)$, solve the equations $bx = -\pi/2$ and $bx = \pi/2$ to get adjacent vertical asymptotes, and for $y = a\cot(bx)$, solve $bx = 0$ and $bx = \pi$ to get adjacent vertical asymptotes.
2. Sketch this pair of vertical asymptotes, then divide the interval in between the pair into four equal parts.
3. Evaluate the function at each quarter line and the midline of the interval.

4. Join those points with a smooth curve, letting the curve approach the vertical asymptotes, then once the function is sketched in one interval, duplicate that curve in other intervals to the left and right.

Where tangent is defined:

tangent is undefined when $\cos x = 0$, so the graph of tangent will have vertical asymptotes where the cosine function crosses the x-axis

tangent is 0 when $\sin x = 0$, so the graph of tangent will cross the x-axis where the sine function crosses the x-axis

Where cotangent is defined:

cotangent is undefined when $\sin x = 0$, so the graph of cotangent will haver vertical asymptotes where the sine function crosses the x-axis.

cotangent is 0 when $\cos x = 0$, so the graph of cotangent will cross the x-axis where the cosine function crosses the x-axis

Horizontal and vertical shifts

Phase shift: a horizontal shift, when the graph is shifted horizontally to the left or right. The shift is determined by c. If c is positive, the graph is shifted c units to the left, but if c is negative, the graph is shifted c units to the right.

Vertical shift: a vertical shift, when the graph is shifted vertically up or down. The shift is determined by d. If d is positive, the graph is shifted d units up, but if d is negative, the graph is shifted d units down.

Graphing transformations

Transformations: horizontal and vertical stretches, compressions, shifts, and reflections

Graphing transformations:

1. **horizontally stretch** or compress the function by a factor of b
2. **horizontally shift** to the left or right by c
3. **reflect** over the y-axis if b is negative
4. **vertically stretch** or compress the function by a factor of a
5. **reflect** over the x-axis if a is negative
6. **vertically shift** the function upward or downward by d

Graphing combinations

Combination: a combination of two functions is the result of adding, subtracting, multiplying, or dividing those functions

Inverse trig functions

Inverse trig relations

Function: a function will always pass the Vertical Line Test, which means that no perfectly vertical line will intersect the curve at more than one point

Inverse trig relation: the inverse of a trig function, which itself is not a function because it does not pass the Vertical Line Test

The inverse of	is written as	and is equivalent to
$y = \sin x$	$y = \sin^{-1} x$ or $y = \arcsin x$	$x = \sin y$
$y = \cos x$	$y = \cos^{-1} x$ or $y = \arccos x$	$x = \cos y$
$y = \tan x$	$y = \tan^{-1} x$ or $y = \arctan x$	$x = \tan y$
$y = \csc x$	$y = \csc^{-1} x$ or $y = \text{arccsc} x$	$x = \csc y$
$y = \sec x$	$y = \sec^{-1} x$ or $y = \text{arcsec} x$	$x = \sec y$
$y = \cot x$	$y = \cot^{-1} x$ or $y = \text{arccot} x$	$x = \cot y$

Inverse trig functions

Inverse trig function: when we limit the range of an inverse trig function, such that it will pass the Vertical Line Test, it becomes an inverse trig function

Inverse function	Domain	Range

$y = \sin^{-1} x$	$x = [-1,1]$	$y = \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
$y = \cos^{-1} x$	$x = [-1,1]$	$y = [0,\pi]$
$y = \tan^{-1} x$	$x = (-\infty, \infty)$	$y = \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
$y = \csc^{-1} x$	$x = (-\infty, -1] \cup [1,\infty)$	$y = \left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$
$y = \sec^{-1} x$	$x = (-\infty, -1] \cup [1,\infty)$	$y = \left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$
$y = \cot^{-1} x$	$x = (-\infty, \infty)$	$y = (0,\pi)$

Trig functions of inverse trig functions

Inverse operations with trig functions: Taking the trig function of an inverse trig function, or vice versa, are opposite operations that undo each other, within particular intervals

$\sin^{-1}(\sin x) = x$	for $x = [-\pi/2, \pi/2]$
$\sin(\sin^{-1} x) = x$	for $x = [-1,1]$
$\cos^{-1}(\cos x) = x$	for $x = [0,\pi]$
$\cos(\cos^{-1} x) = x$	for $x = [-1,1]$
$\tan^{-1}(\tan x) = x$	for $x = (-\pi/2, \pi/2)$

$\tan(\tan^{-1} x) = x$	for $x = (-\infty, \infty)$
$\csc^{-1}(\csc x) = x$	for $x = [-\pi/2,0)$ or $x = (0,\pi/2]$
$\csc(\csc^{-1} x) = x$	for $x = (-\infty, -1]$ or $x = [1,\infty)$
$\sec^{-1}(\sec x) = x$	for $x = [0,\pi/2)$ or $x = (\pi/2,\pi]$
$\sec(\sec^{-1} x) = x$	for $x = (-\infty, -1]$ or $x = [1,\infty)$
$\cot^{-1}(\cot x) = x$	for $x = (0,\pi)$
$\cot(\cot^{-1} x) = x$	for $x = (-\infty, \infty)$

Trig functions of inverse trig functions:

	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$	$\csc^{-1} x$	$\sec^{-1} x$	$\cot^{-1} x$
sin of	x	$\sqrt{1-x^2}$	$\frac{x}{\sqrt{x^2+1}}$	$\frac{1}{x}$	$\sqrt{1-\frac{1}{x^2}}$	$\frac{1}{x\sqrt{\frac{1}{x^2}+1}}$
cos of	$\sqrt{1-x^2}$	x	$\frac{1}{\sqrt{x^2+1}}$	$\sqrt{1-\frac{1}{x^2}}$	$\frac{1}{x}$	$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$
tan of	$\frac{x}{\sqrt{1-x^2}}$	$\frac{\sqrt{1-x^2}}{x}$	x	$\frac{1}{x\sqrt{1-\frac{1}{x^2}}}$	$x\sqrt{1-\frac{1}{x^2}}$	$\frac{1}{x}$
csc of	$\frac{1}{x}$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{\sqrt{x^2+1}}{x}$	x	$\frac{1}{\sqrt{1-\frac{1}{x^2}}}$	$x\sqrt{\frac{1}{x^2}+1}$
sec of	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{x}$	$\sqrt{x^2+1}$	$\frac{1}{\sqrt{1-\frac{1}{x^2}}}$	x	$\sqrt{\frac{1}{x^2}+1}$
cot of	$\frac{\sqrt{1-x^2}}{x}$	$\frac{x}{\sqrt{1-x^2}}$	$\frac{1}{x}$	$x\sqrt{1-\frac{1}{x^2}}$	$\frac{1}{x\sqrt{1-\frac{1}{x^2}}}$	x

Trig identities

Sum-difference identities for sine and cosine

The fundamental identities: the reciprocal identities, quotient identities, Pythagorean identities, and the even-odd identities

Sum-difference identities for sine and cosine:

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

$$\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$$

$$\cos(\theta + \alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$$

$$\cos(\theta - \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha$$

Cofunction identities

In degrees:

$$\sin\theta = \cos(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

$$\cos\theta = \sin(90^\circ - \theta) \qquad \sec\theta = \csc(90^\circ - \theta)$$

$$\tan\theta = \cot(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta)$$

In radians:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) \qquad \sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

Sum-difference identities for tangent

Sum-difference identities for tangent:

$$\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$$

$$\tan(\theta-\alpha)=\frac{\tan\theta-\tan\alpha}{1+\tan\theta\tan\alpha}$$

Double-angle identities

$$\sin 2\theta=2\sin\theta\cos\theta$$

$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$$

$$\cos 2\theta=\cos^2\theta-\sin^2\theta$$

$$\cos 2\theta=1-2\sin^2\theta$$

$$\cos 2\theta=2\cos^2\theta-1$$

Half-angle identities

$$\cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\tan\frac{\theta}{2}=\pm\frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}}$$

$$\tan\frac{\theta}{2}=\frac{\sin\theta}{1+\cos\theta}$$

$$\tan\frac{\theta}{2}=\frac{1-\cos\theta}{\sin\theta}$$

Product-to-sum identities

$$\sin\theta\cos\alpha=\frac{1}{2}\left[\sin(\theta+\alpha)+\sin(\theta-\alpha)\right]$$

$$\cos\theta\sin\alpha=\frac{1}{2}\left[\sin(\theta+\alpha)-\sin(\theta-\alpha)\right]$$

$$\cos\theta\cos\alpha=\frac{1}{2}\left[\cos(\theta+\alpha)+\cos(\theta-\alpha)\right]$$

$$\sin\theta\sin\alpha=\frac{1}{2}\left[\cos(\theta-\alpha)-\cos(\theta+\alpha)\right]$$

Sum-to-product identities

$$\sin\theta+\sin\alpha=2\sin\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right)$$

$$\sin\theta-\sin\alpha=2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right)$$

$$\cos\theta+\cos\alpha=2\cos\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right)$$

$$\cos\theta-\cos\alpha=-2\sin\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right)$$

Proving the trig equation

1. Try to express every trig function in the equation in terms of sine and cosine. For cosecant and secant, we'll do this with the reciprocal identities, and for tangent and cotangent we'll do this with the quotient identities.

2. Make sure all the angles are the same. For example, when we have $\sin(2\alpha)$ on one side of the equation, and $\sin\alpha$ on the other side, it's difficult to prove the equation. The same applies for addition and subtraction: don't try working with $\sin(\alpha+\beta)$ and $\sin\alpha$.

3. Try rewriting the more complicated side of the equation in order to match the simpler side.

4. If we need to add more powers or remove them, we can use the Pythagorean identities like $\cos^2 x + \sin^2 x = 1$. We can always multiply by 1 without changing the meaning, so we can always multiply by $\cos^2 x + \sin^2 x$.

5. Look for fractions that can be combined or pulled apart, and consider whether or not the equation might be factorable.

6. Look for a trig function that links the trig functions in the equation. For instance, if we have sine and cotangent in an equation, we know that tangent is a linking function, because tangent=sine/cosine, and tangent=1/cotangent.

7. If we ever have a value which is the sum or difference of a constant and a trig function, like $1+\cos\theta$, consider multiplying by the conjugate. The conjugate is the same two terms, but with the opposite sign between them. So the conjugate of $1+\cos\theta$ is $1-\cos\theta$.

The law of sines and law of cosines

Law of sines

Oblique triangles: triangles that aren't right (they don't include a right angle)

Similar triangles: triangles with the same shape but different size

Solving oblique triangles:

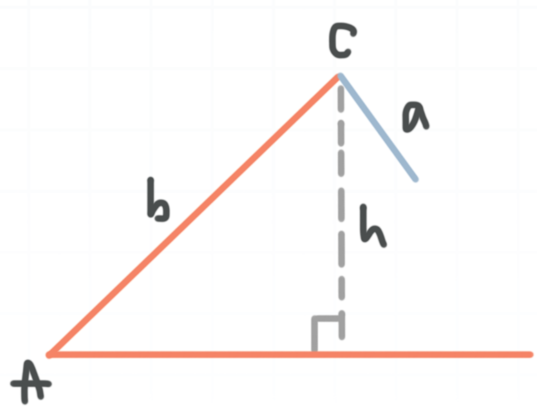
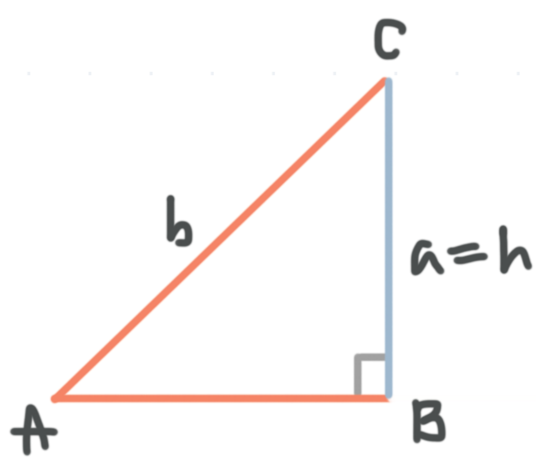
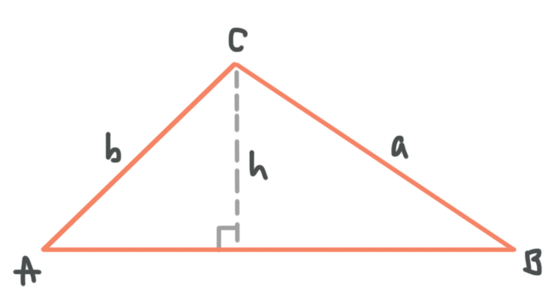
Known information	How to solve
SAA or ASA One side and two angles	1. Use A+B+C=180° to find the remaining angle 2. Use law of sines to find the remaining sides
SAS Two sides and the included angle	1. Use law of cosines to find the third side 2. Use law of sines to find another angle 3. Use A+B+C=180° to find the remaining angle
SSS Three sides	1. Use law of cosines to find the largest angle 2. Use law of sines to find either remaining angle 3. Use A+B+C=180° to find the remaining angle
SSA Two sides and a non-included angle	**The ambiguous case**. If two triangles exist, use this same set of steps to find both triangles. 1. Use law of sines to find an angle 2. Use A+B+C=180° to find the remaining angle 3. Use law of sines to find the remaining side

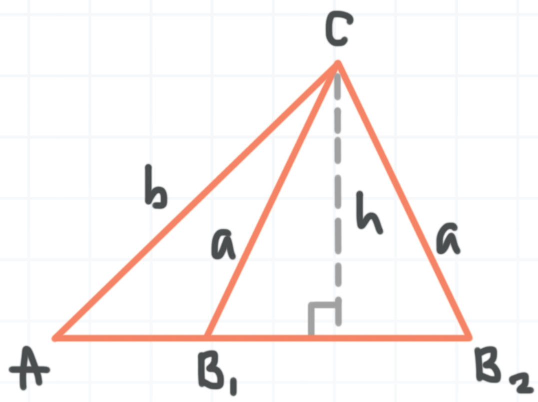
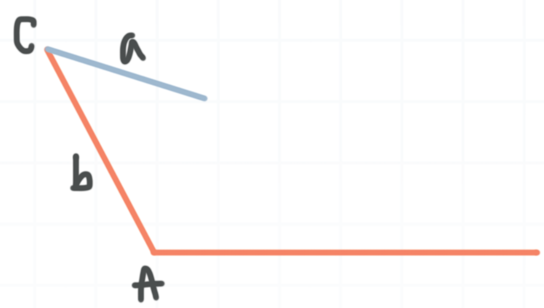
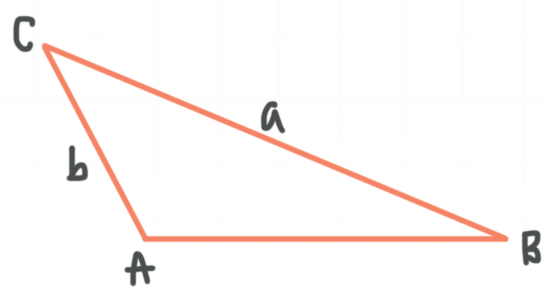
Law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The ambiguous case of the law of sines

Case	# of triangles	Sketch	Conditions
A is acute			
I	0		$a < h$, $h = b \sin A$
II	1		$a = h$
III	1		$a > b$

IV	2		$b > a > h$
A is obtuse			
V	0		$a \leq b$
VI	1		$a > b$

Area from the law of sines

Law of sines for the area of a triangle:

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$\text{Area} = \frac{1}{2}ac\sin B$$

$$\text{Area} = \frac{1}{2}bc\sin A$$

Law of cosines

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Heron's formula

Heron's formula for area of a triangle:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

W W W . K R I S T A K I N G M A T H . C O M