Trigonometry Final Exam Solutions

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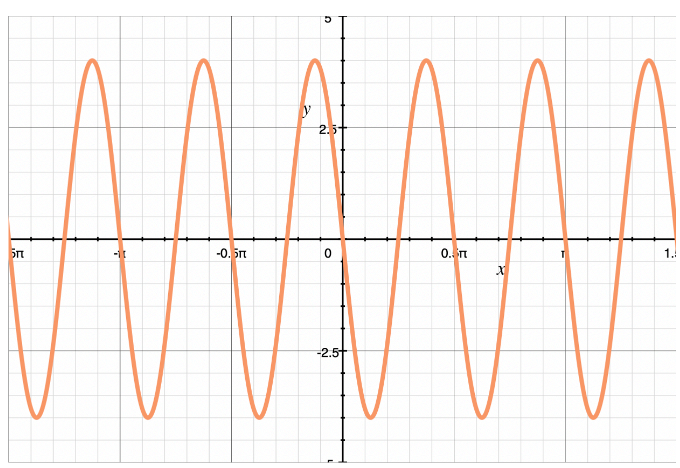
Trigonometry Final Exam Answer Key

1. (5 pts)	A	■	C	D	E
2. (5 pts)	A	B	C	■	E
3. (5 pts)	A	B	■	D	E
4. (5 pts)	■	B	C	D	E
5. (5 pts)	A	■	C	D	E
6. (5 pts)	A	B	C	■	E
7. (5 pts)	A	B	C	D	■
8. (5 pts)	■	B	C	D	E

9. (15 pts) 374 ft

10. (15 pts) 45.538°

11. (15 pts)



12. (15 pts) $c = 18.33$, $A = 10.89^\circ$, and $B = 49.11^\circ$

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1. B. Angular velocity is given by $\omega = v/r$, where ω is angular velocity, v is linear velocity, and r is the radius.

$$\omega = \frac{72 \text{ mi}}{9 \text{ hr-in}}$$

Now we'll convert to revolutions per second using unit conversions.

$$\omega = \frac{72 \text{ mi}}{9 \text{ hr-in}} \cdot \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ rev}}{2\pi}$$

$$\omega = \frac{72 \cdot 5{,}280 \cdot 12 \text{ rev}}{9 \cdot 60 \cdot 60 \cdot 2\pi \text{ sec}}$$

$$\omega = \frac{352 \text{ rev}}{5\pi \text{ sec}}$$

$$\omega \approx 22.4 \text{ revolutions per second}$$

2. D. Since the central angle θ is in degrees, find the area of the circular sector using

$$A = \pi r^2 \left(\frac{\theta}{360}\right)$$

Since the diameter is 20 cm, the radius is 10 cm.

$$A = \pi(10)^2\left(\frac{120}{360}\right)$$

$$A = \pi(100)\left(\frac{1}{3}\right)$$

$$A = \frac{100}{3}\pi$$

3. C. The law of sines is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We know that $B = 35^\circ$, $b = 7$, $C = 105^\circ$, and $c = x$. Since the sum of the interior angles of a triangle is 180°,

$$A = 180^\circ - 35^\circ - 105^\circ$$

$$A = 40^\circ$$

Use the second and third parts of the law of sines equation.

$$\frac{7}{\sin 35^\circ} = \frac{x}{\sin 105^\circ}$$

$$x \sin 35^\circ = 7 \sin 105^\circ$$

$$x = \frac{7 \sin 105^\circ}{\sin 35^\circ}$$

$$x \approx 11.8$$

4. A. The period of a tangent function is $\pi/|b|$, where b is the coefficient on θ.

$$\frac{\pi}{\frac{1}{6}}$$

$$\pi\left(\frac{6}{1}\right)$$

$$6\pi$$

5. B. Use Heron's formula to find the area of a triangle. If a, b, and c are the lengths of the sides of the triangle, then half the perimeter of the triangle is

$$s=\frac{1}{2}(a+b+c)$$

$$s=\frac{1}{2}(15+17+26)$$

$$s=\frac{1}{2}(58)$$

$$s=29$$

Plugging this value into Heron's formula, we get

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{29(29-15)(29-17)(29-26)}$$

$$A = \sqrt{29(14)(12)(3)}$$

$$A = \sqrt{14{,}616}$$

$$A \approx 121$$

6. D. From just the unit circle, we wouldn't know the value of tangent at $7\pi/6$, but we can rewrite $7\pi/6$ as

$$\frac{7\pi}{6} = \frac{6\pi+\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \pi + \frac{\pi}{6}$$

So the original expression can be rewritten as

$$\tan\left(\frac{7\pi}{6}\right) = \tan\left(\pi + \frac{\pi}{6}\right)$$

and we can plug this new right side into the sum identity for the tangent function.

$$\tan(\theta+\alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$$

$$\tan\left(\pi + \frac{\pi}{6}\right) = \frac{\tan\pi + \tan\left(\frac{\pi}{6}\right)}{1 - \tan\pi\tan\left(\frac{\pi}{6}\right)}$$

$$\tan\left(\pi+\frac{\pi}{6}\right)=\frac{0+\frac{\sqrt{3}}{3}}{1-(0)\left(\frac{\sqrt{3}}{3}\right)}$$

$$\tan\left(\pi+\frac{\pi}{6}\right)=\frac{\frac{\sqrt{3}}{3}}{1}$$

$$\tan\left(\pi+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$$

7. E. Use the sum of cosines to find the exact value, since $60^\circ+45^\circ=105^\circ$.

$$\cos(\theta+\alpha)=\cos\theta\cos\alpha-\sin\theta\sin\alpha$$

$$\cos(105^\circ)=\cos 60^\circ\cos 45^\circ-\sin 60^\circ\sin 45^\circ$$

$$\cos(105^\circ)=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$\cos(105^\circ)=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$$

$$\cos(105^\circ)=\frac{\sqrt{2}-\sqrt{6}}{4}$$

8. A. Remember that a sine function written in the form $a\sin(b(\theta+c))+d$ has a vertical stretch/compression of a, horizontal stretch/compression of b, horizontal shift of c, and a vertical shift of d.

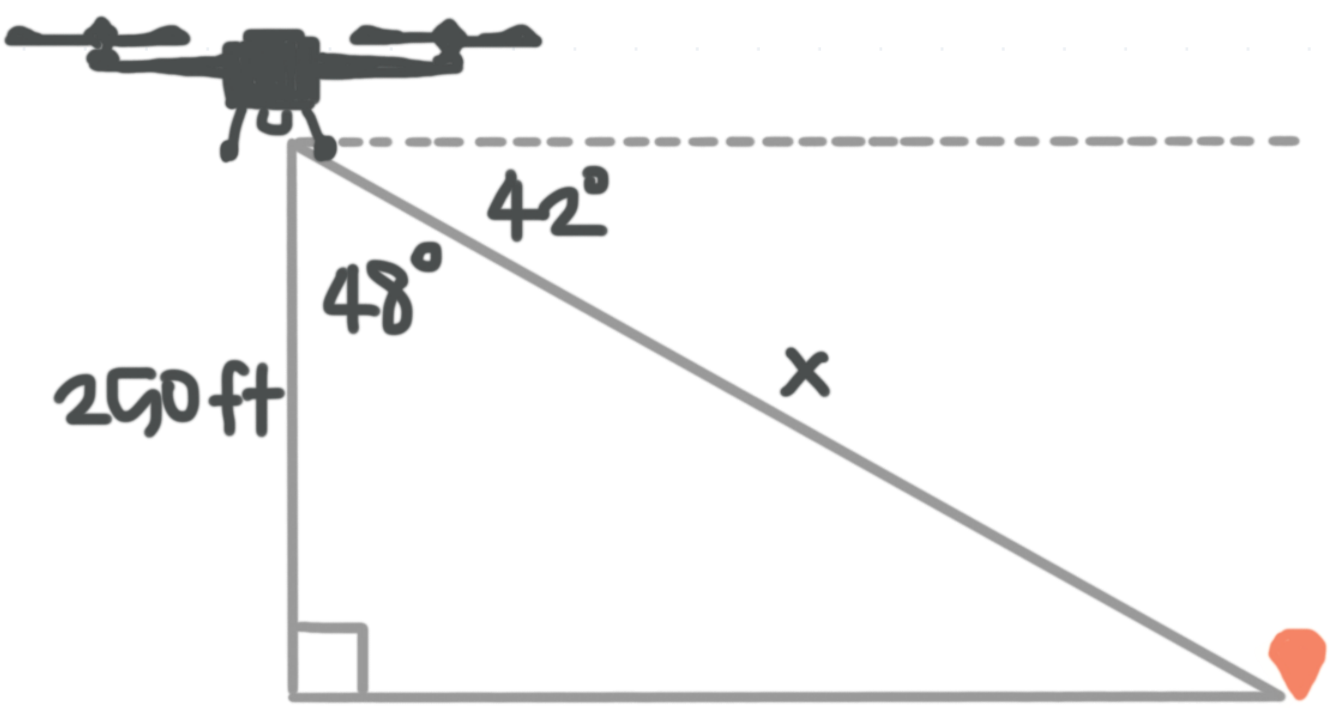
The equation

$$y = 2\sin\left(5\theta + \frac{\pi}{4}\right) - 3$$

$$y = 2\sin\left(5\left(\theta + \frac{\pi}{20}\right)\right) - 3$$

has a vertical stretch of 2, horizontal compression of 5, horizontal shift to the left of $\pi/20$, and vertical shift down of 3.

9. Draw a diagram.



Since the angle of depression is 42°, the angle inside the triangle is $90^\circ - 42^\circ = 48^\circ$. To find x, the slant distance from the drone to the object on the ground, use cosine because

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

The adjacent side is 250 ft and the hypotenuse is x.

$$\cos 48^\circ = \frac{250}{x}$$

$$x\cos 48^\circ = 250$$

$$x = \frac{250}{\cos 48^\circ}$$

$$x \approx 374 \text{ ft}$$

10. The 45° part of $45^\circ 32' 16''$ is already in degrees, so we only need to convert the minutes and seconds portions of the DMS angle.

We need to convert $16''$ from seconds to minutes. We know that $1' = 60''$, so we'll multiply $16''$ by $1'/60''$ in order to cancel the seconds and be left with just minutes.

$$16''\left(\frac{1'}{60''}\right)$$

$$\left(\frac{16}{60}\right)'$$

$$\approx 0.266'$$

Then the total minutes in $45^\circ 32' 16''$ is

$$(32 + 0.266)'$$

$$32.266'$$

To convert this value for minutes into degrees, we'll multiply by $1^\circ/60'$ in order to cancel the minutes and be left with an approximate value for degrees.

$$32.266' \left(\frac{1^\circ}{60'}\right)$$

$$\left(\frac{32.266}{60}\right)^\circ$$

$$\approx 0.538^\circ$$

Putting this together with the 55° from the original angle, we get approximately

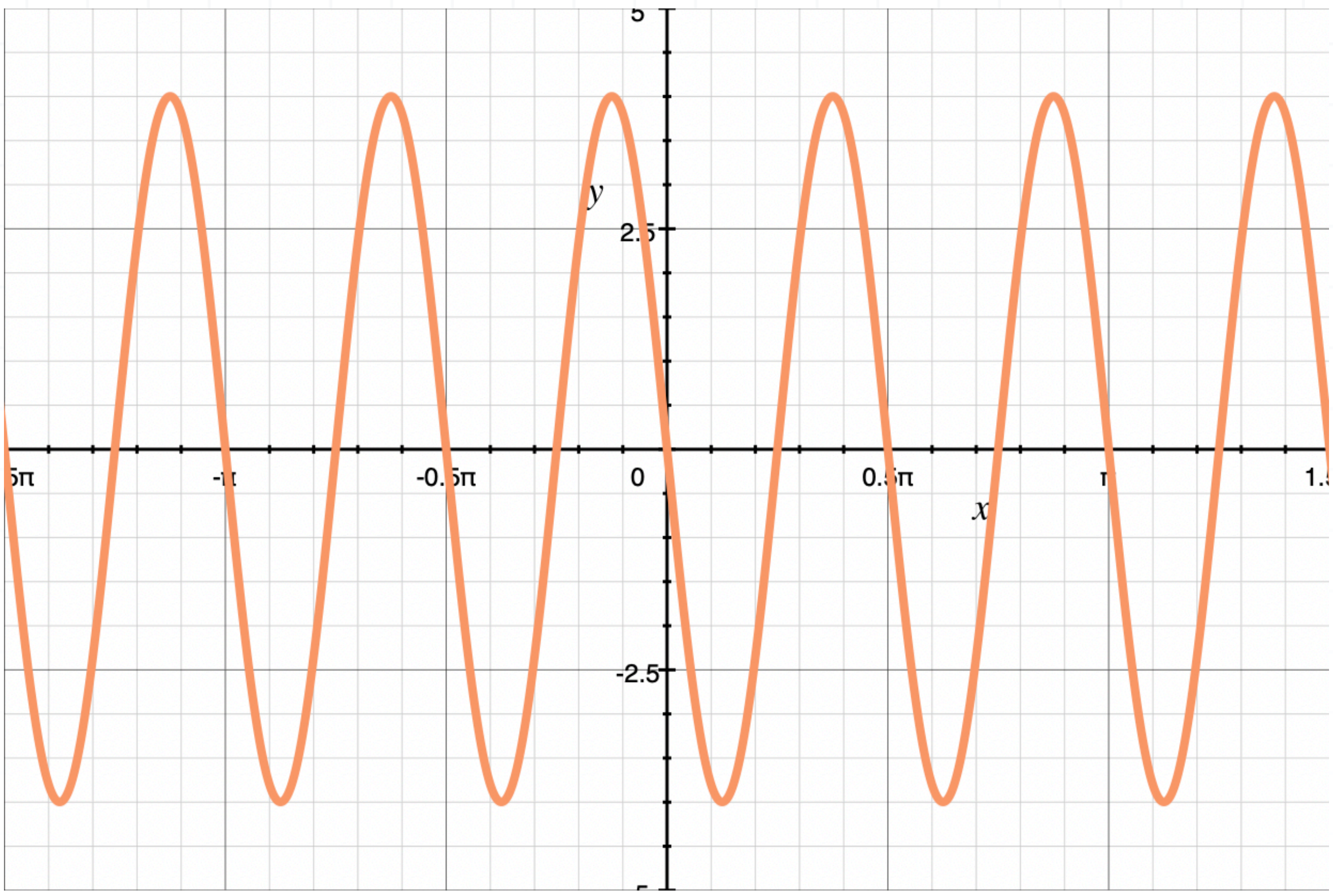
$$(45 + 0.538)^\circ$$

$$45.538^\circ$$

11. Since the function is a sine function, its graph will pass through the origin. From the origin, it will decrease to -4, since the amplitude is 4 and the function is negative. The period is

$$\frac{2\pi}{|b|} = \frac{2\pi}{4} = \frac{\pi}{2}$$

With this period, then two periods can be graphed between 0 and π. The zeros on that interval are at 0, $\pi/4$, $\pi/2$, $3\pi/4$, and π.



12. Use the law of cosines to find side c.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 4^2 + 16^2 - 2(4)(16)\cos 120^\circ$$

$$c^2 = 16 + 256 - 128\cos 120^\circ$$

$$c = \sqrt{16 + 256 - 128\cos 120^\circ}$$

$$c \approx 18.33$$

Use the law of sines to find angle A.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{4}{\sin A} = \frac{18.33}{\sin 120^\circ}$$

$$18.33 \sin A = 4 \sin 120^\circ$$

$$\sin A = \frac{4 \sin 120^\circ}{18.33}$$

Apply the inverse sine function to both sides.

$$A = \sin^{-1}\left(\frac{4 \sin 120^\circ}{18.33}\right)$$

$$A \approx 10.89^\circ$$

Since the sum of the interior angles of a triangle is 180°, subtract the measures of angles A and C from 180° to find angle B.

$$B = 180^\circ - A - C$$

$$B = 180^\circ - 10.89^\circ - 120^\circ$$

$$B = 49.11^\circ$$