Trigonometry Final Exam Solutions

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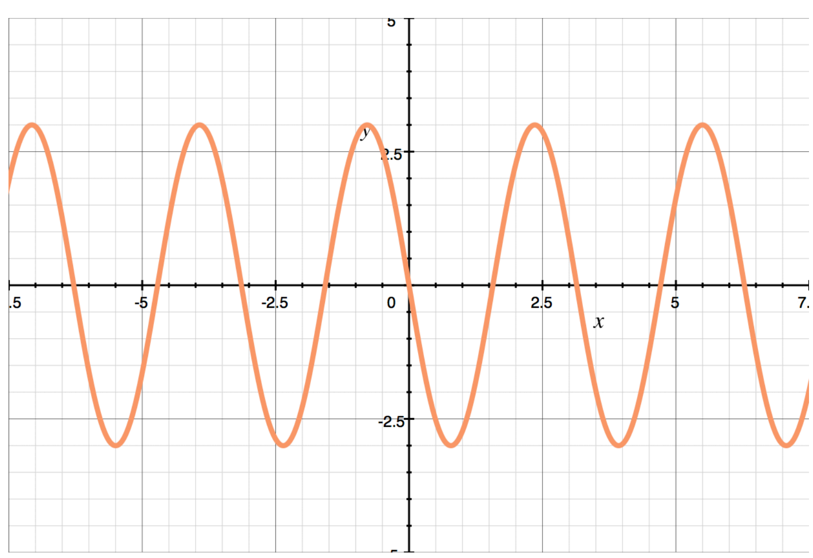
Trigonometry Final Exam Answer Key

1. (5 pts)	A	B	C	D	■
2. (5 pts)	A	B	C	D	■
3. (5 pts)	A	B	C	■	E
4. (5 pts)	A	B	■	D	E
5. (5 pts)	A	■	C	D	E
6. (5 pts)	■	B	C	D	E
7. (5 pts)	■	B	C	D	E
8. (5 pts)	A	B	■	D	E

9. (15 pts) 681 ft

10. (15 pts) $c = 16.95$, $A = 19.79^\circ$, and $B = 52.21^\circ$



11. (15 pts)

12. (15 pts) $\csc^2\theta = \csc^2\theta$

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1. E. Since the central angle θ is in degrees, find the area of the circular sector using the formula.

$$A = \pi r^2 \left(\frac{\theta}{360}\right)$$

Since the diameter is 8 ft, the radius is 4 ft.

$$A = \pi(4)^2 \left(\frac{150}{360}\right)$$

$$A = \pi(16)\left(\frac{5}{12}\right)$$

$$A = \frac{20}{3}\pi$$

2. E. Angular velocity is found using the formula $\omega = v/r$ where ω is angular velocity, v is linear velocity, and r is the radius.

$$\omega = \frac{60 \text{ mi}}{10 \text{ hr-in}}$$

Now we'll convert to revolutions per second using unit conversions.

$$\omega = \frac{60 \text{ mi}}{10 \text{ hr-in}} \cdot \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ rev}}{2\pi}$$

$$\omega = \frac{60 \cdot 5{,}280 \cdot 12 \text{ rev}}{10 \cdot 60 \cdot 60 \cdot 2\pi \text{ sec}}$$

$$\omega = \frac{528 \text{ rev}}{10\pi \text{ sec}}$$

$\omega \approx 16.81$ revolutions per second

3. D. The law of sines is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We know that $A = 126^\circ$, $a = 12$, and $b = x$. Since the sum of the angles of a triangle must equal 180°,

$$B = 180^\circ - 126^\circ - 28^\circ$$

$$B = 26^\circ$$

Substitute the known values into the law of sines.

$$\frac{12}{\sin 126^\circ} = \frac{x}{\sin 26^\circ}$$

Cross multiply and divide both sides by $\sin 126^\circ$.

$$x \sin 126^\circ = 12 \sin 26^\circ$$

$$x = \frac{12 \sin 26^\circ}{\sin 126^\circ}$$

$$x = 6.5$$

4. C. Use Heron's formula to find the area of a triangle. If a, b, and c are the lengths of the sides of the triangle, then part of Heron's formula says

$$s = \frac{1}{2}(a + b + c)$$

$$s = \frac{1}{2}(14 + 18 + 22)$$

$$s = \frac{1}{2}(54)$$

$$s = 27$$

Plugging this value into Heron's formula, we get

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

$$A = \sqrt{27(27 - 14)(27 - 18)(27 - 22)}$$

$$A = \sqrt{27(13)(9)(5)}$$

$$A = \sqrt{15{,}795}$$

$$A \approx 126$$

5. B. The period of a cosine function is $2\pi/b$ where b is the coefficient on θ.

$$\frac{2\pi}{\frac{1}{4}}$$

$$2\pi\left(\frac{4}{1}\right)$$

$$8\pi$$

6. A. Remember that a sine function written in the form $a\sin(b(\theta+c))+d$ has a vertical stretch of a, horizontal compression of b, horizontal shift to the left of c, and a vertical shift up of d.

The equation

$$y=4\sin\left(3\theta-\frac{\pi}{4}\right)+5$$

$$y=4\sin\left(3\left(\theta-\frac{\pi}{12}\right)\right)+5$$

has a vertical stretch of 4, a horizontal compression of 3, a horizontal shift to the right of $\pi/12$, and a vertical shift up of 5.

7. A. Use the sum of cosines to find the exact value, since $30^\circ+45^\circ=75^\circ$.

$$\cos(\theta+\alpha)=\cos\theta\cos\alpha-\sin\theta\sin\alpha$$

$$\cos 75^\circ=\cos(30^\circ+45^\circ)$$

$$\cos(75^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$

$$\cos 75^\circ = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\cos 75^\circ = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos 75^\circ = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos 75^\circ = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\cos 75^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$$

8. C. Use the product identity for $\cos\theta \sin\alpha$.

$$\cos\theta \sin\alpha = \frac{1}{2}[\sin(\theta + \alpha) - \sin(\theta - \alpha)]$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2}[\sin(75^\circ + 15^\circ) - \sin(75^\circ - 15^\circ)]$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2}(\sin 90^\circ - \sin 60^\circ)$$

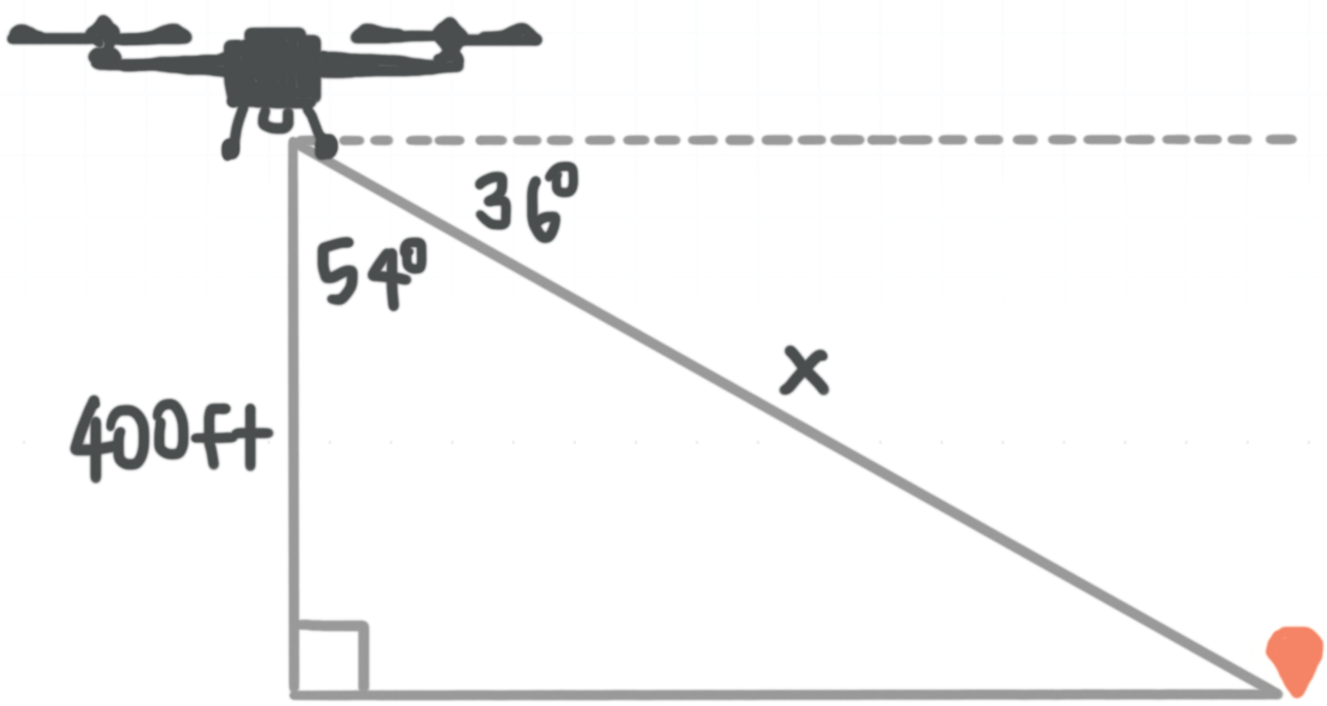
$$\cos 75^\circ \sin 15^\circ = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2}\left(\frac{2}{2} - \frac{\sqrt{3}}{2}\right)$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2}\left(\frac{2 - \sqrt{3}}{2}\right)$$

$$\cos 75^\circ \sin 15^\circ = \left(\frac{2 - \sqrt{3}}{4}\right)$$

9. Draw a diagram.



Since the angle of depression is 36°, the angle inside the triangle is $90^\circ - 36^\circ = 54^\circ$. To find x, the slant distance from the drone to the object on the ground, use cosine because

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

The adjacent is 400 ft and the hypotenuse is x.

$$\cos 54^\circ = \frac{400}{x}$$

Cross multiply and divide both sides by $\cos 54^\circ$.

$$x\cos 54^\circ = 400$$

$$x = \frac{400}{\cos 54^\circ}$$

$$x \approx 680.52 \text{ ft}$$

$$x \approx 681 \text{ ft}$$

10. Use the law of cosines to find side c.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 6^2 + 14^2 - 2(6)(14)\cos 108^\circ$$

$$c^2 = 36 + 196 - 168\cos 108^\circ$$

$$c = \sqrt{36 + 196 - 168\cos 108^\circ}$$

$$c \approx 16.85$$

Use the law of sines to find angle A.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{6}{\sin A} = \frac{16.85}{\sin 108^\circ}$$

Cross multiply and divide both sides by 16.85.

$$16.85 \sin A = 6 \sin 108^\circ$$

$$\sin A = \frac{6 \sin 108^\circ}{16.85}$$

Take the $\sin^{-1}$ of both sides.

$$A = \sin^{-1}\left(\frac{6 \sin 108^\circ}{16.85}\right)$$

$$A \approx 19.79^\circ$$

Since the sum of all the angles in a triangle is 180°, subtract the measures of angles A and C from 180° to find angle B.

$$B = 180^\circ - A - C$$

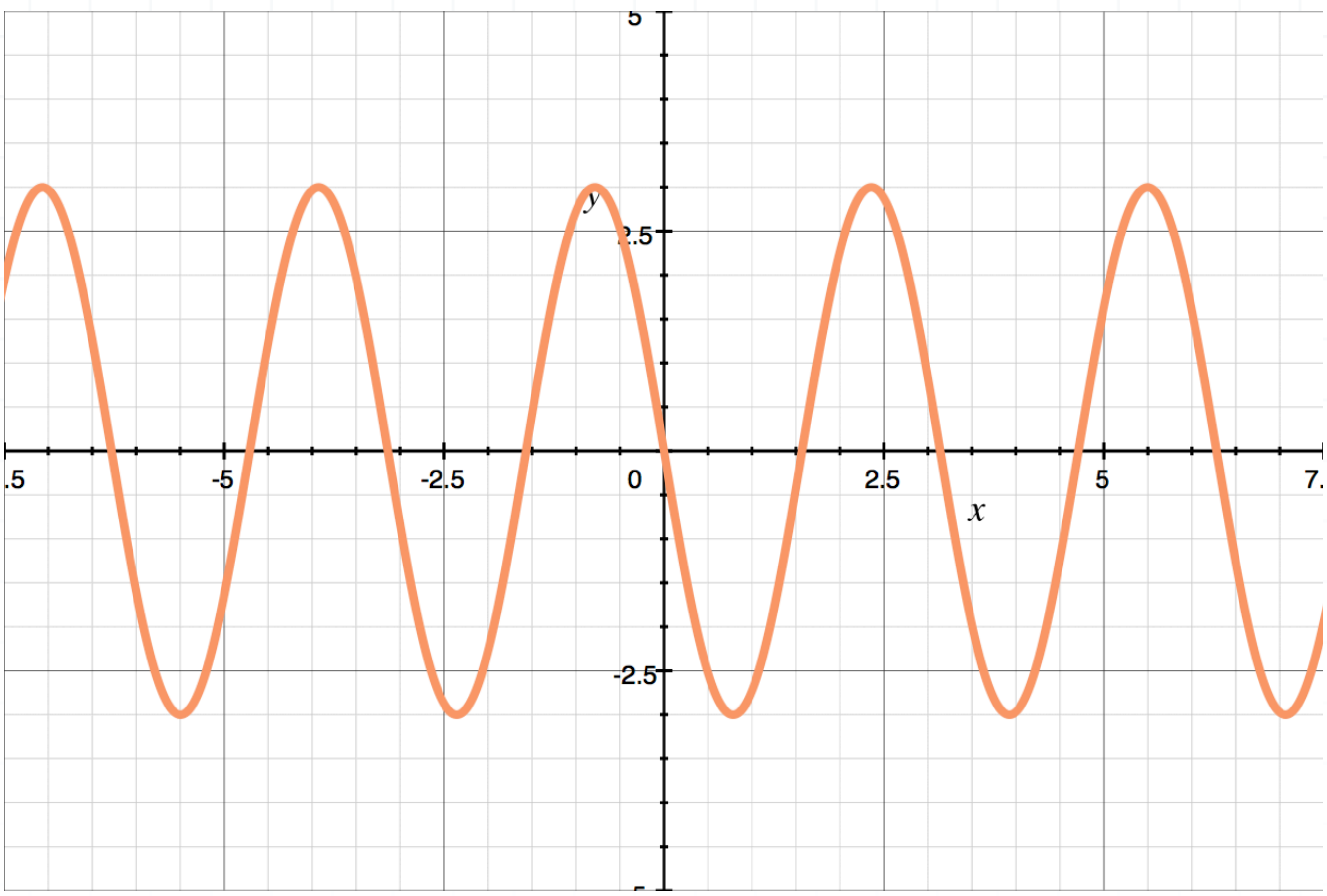
$$B = 180^\circ - 19.79^\circ - 108^\circ$$

$$B = 52.21^\circ$$

11. Since the graph is the sine function, it will go through the origin. From the origin it will go down to -3 since the amplitude is 3 and the function is negative. The period is

$$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$

so two periods can be graphed from $-\pi$ to π. There are minimums at $-3\pi/4$ and $\pi/4$ and maximums at $-\pi/4$ and $3\pi/4$. The zeros are at $-\pi$, $-\pi/2$, 0, $\pi/2$, and π.



12. To show a trig identity is true, work with one side of the equation until it equals the other side.

$$\frac{\sec^2\theta}{\sec^2\theta - 1} = \csc^2\theta$$

Substitute $\tan^2\theta$ in for $\sec^2\theta - 1$ using the Pythagorean identity $\tan^2\theta = \sec^2\theta - 1$.

$$\frac{\sec^2\theta}{\tan^2\theta} = \csc^2\theta$$

It's often helpful to change everything into terms of sine and cosine when proving trig identities. Remember that $\sec\theta = 1/\cos\theta$ and $\tan\theta = \sin\theta/\cos\theta$.

$$\frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \csc^2\theta$$

Simplify the fraction by multiplying by a well-chosen 1.

$$\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} \cdot \frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \csc^2\theta$$

$$\frac{1}{\sin^2\theta} = \csc^2\theta$$

The reciprocal identity states that $\csc\theta = 1/\sin\theta$.

$$\csc^2\theta = \csc^2\theta$$

We've now shown that the equation is a true identity.

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