Trigonometry Workbook

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MATH

NAMING ANGLES

- 1. What do we call an angle of $\theta = 180°$?

- 2. Sketch an acute angle.

- 3. What is the measure of a straight angle?

- 4. Name the angle $\theta = 6°$.

- 5. Sketch a $360°$ angle.

- 6. Give the full set of obtuse angles.

COMPLEMENTARY AND SUPPLEMENTARY ANGLES

1. Find the supplement θ of $7\pi/8$.

2. Angles $\angle 1$ and $\angle 2$ are complementary. Find the supplement of $\angle 2$.

$$m\angle 1 = (2x + 5)^\circ$$

$$m\angle 2 = (x + 4)^\circ$$

3. The complement of θ is $\pi/6$. Find the supplement of θ.

4. Find the complement of 42°.

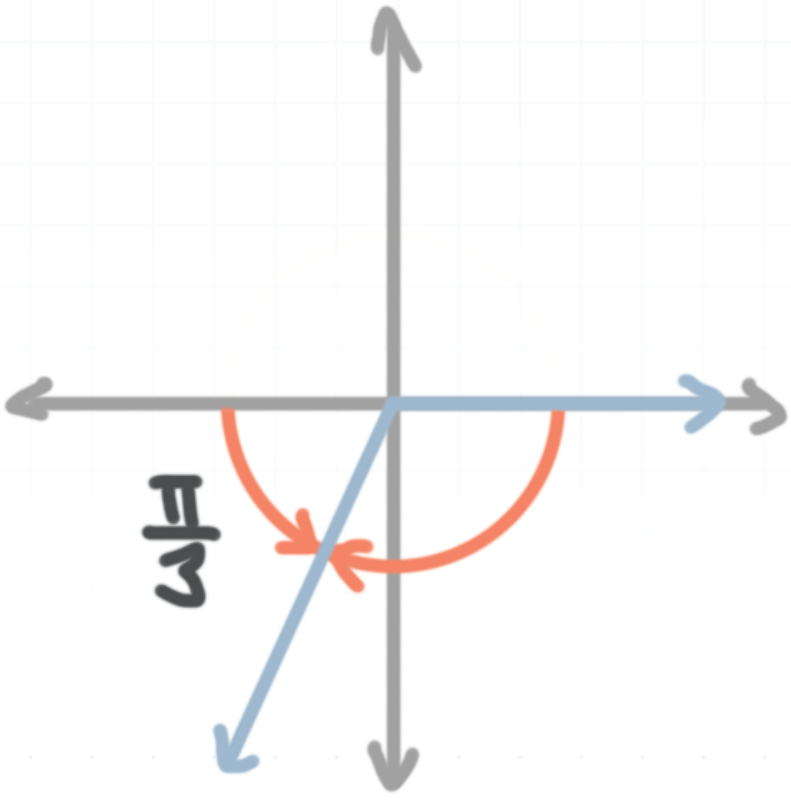
5. Find the angle that's supplementary to $2\pi/3$.

6. True or False? If x and y are complementary angles, then $2(x + y) = 180^\circ$.

POSITIVE AND NEGATIVE ANGLES

1. Sketch $-120°$ in standard position.

2. Find the measure of the unknown negative angle in radians.



3. Sketch $3\pi/2$ in standard position.

4. Sketch $-\pi/2$ in standard position.

5. Sketch $405°$ in standard position.

6. Find an angle between $0°$ and $360°$ that has the same terminal side as a $-675°$ angle.

QUADRANT OF THE ANGLE

1. In which quadrant is the angle $\pi/5$ located?

2. In which quadrant is the angle -820° located?

3. In which quadrant is the angle $-13\pi/4$ located?

4. Of the angles -15°, 240°, -275°, and 550°, assuming all four are sketched in standard position, which one has its terminal side in the fourth quadrant?

5. In which quadrant is the angle $1{,}200^\circ$ located?

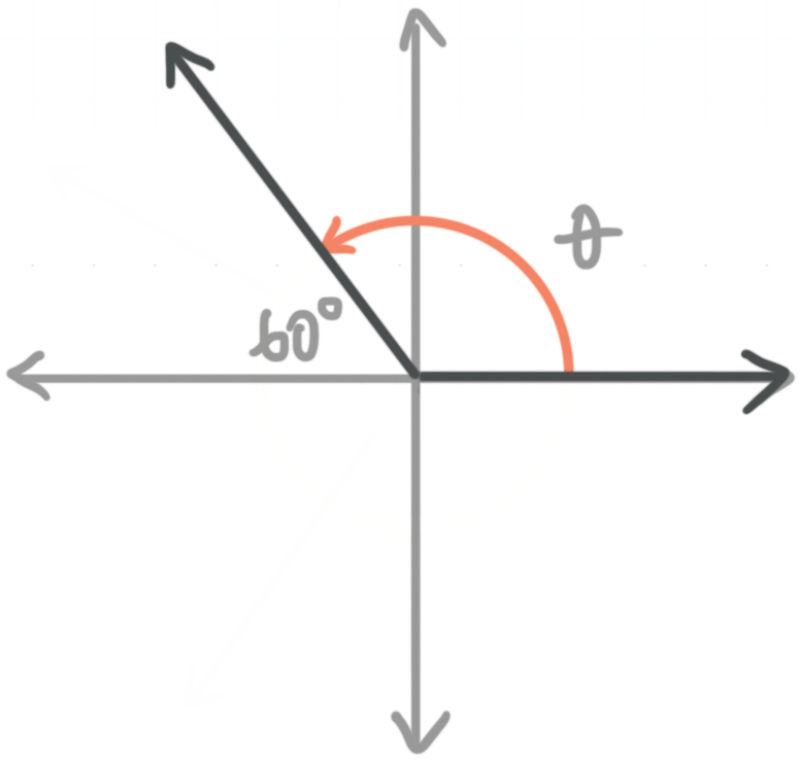
6. On which axis does the angle -7π lie?

DEGREES, RADIANS, AND DMS

■ 1. Convert $65°13'12''$ to a decimal angle.

■ 2. Find the sum $20.25° + 20°2'5''$ in DMS. Note: The sum of two DMS angles is found by adding the two degree parts, adding the two minutes parts, adding the two seconds parts, and then combining the separate sums into one angle.

■ 3. Find the measure of θ in radians.



■ 4. Convert the angle $-9\pi/2$ to degrees.

5. What is the measure of $152.34°$ in DMS?

6. What is the measure of 0.2565π in DMS?

COTERMINAL ANGLES

1. If we start at $-270°$ and move three full rotations clockwise around the origin, at which angle will we arrive?

2. If we start at $5\pi/6$ and move two full rotations counterclockwise around the origin, at which angle will we arrive?

3. Find the negative and positive coterminal angles that are one full rotation away from $\theta = 200°$.

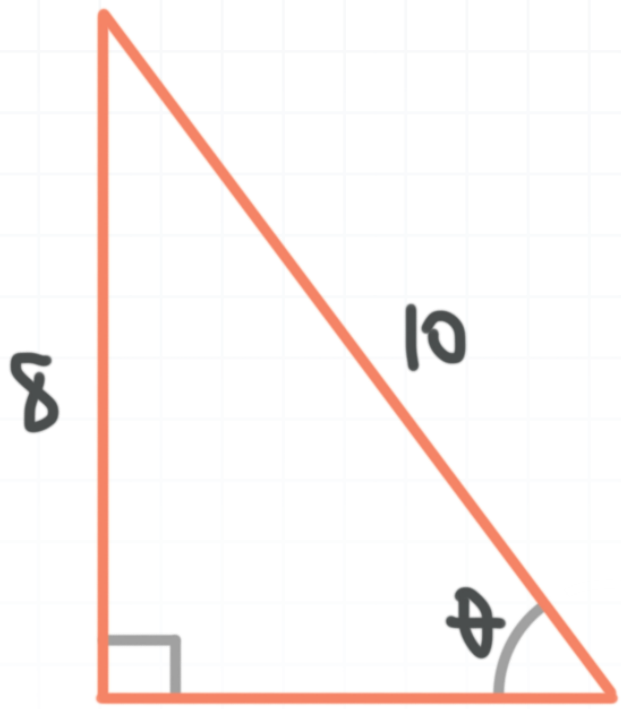
4. Which angle in the interval $[0°,360°)$ is coterminal with $-1{,}624°$?

5. Find the angle in the interval $[0,2\pi)$ that's coterminal with $16\pi/3$.

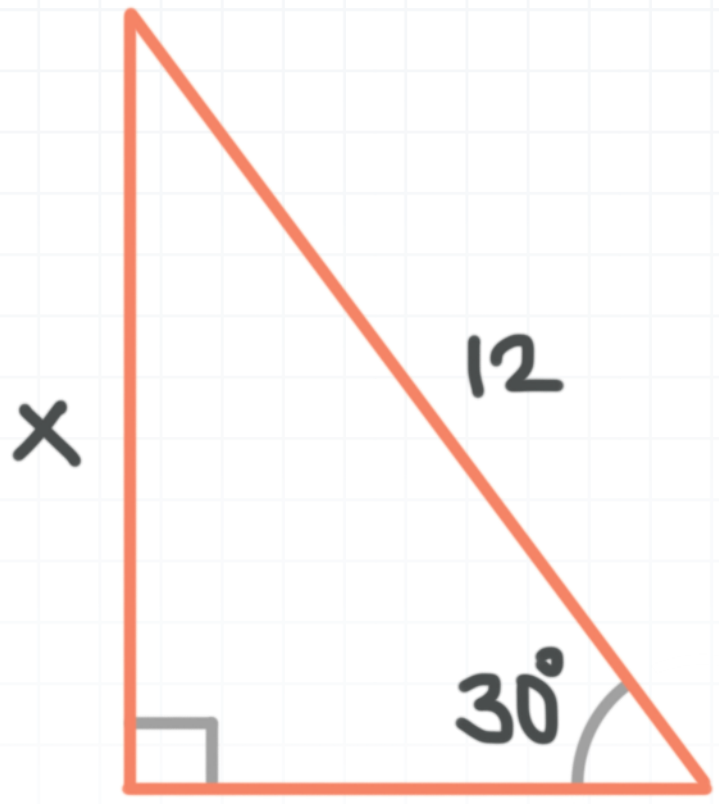
6. Which angle in $[-\pi, \pi)$ is coterminal with $-19\pi/2$?

SINE, COSINE, AND TANGENT

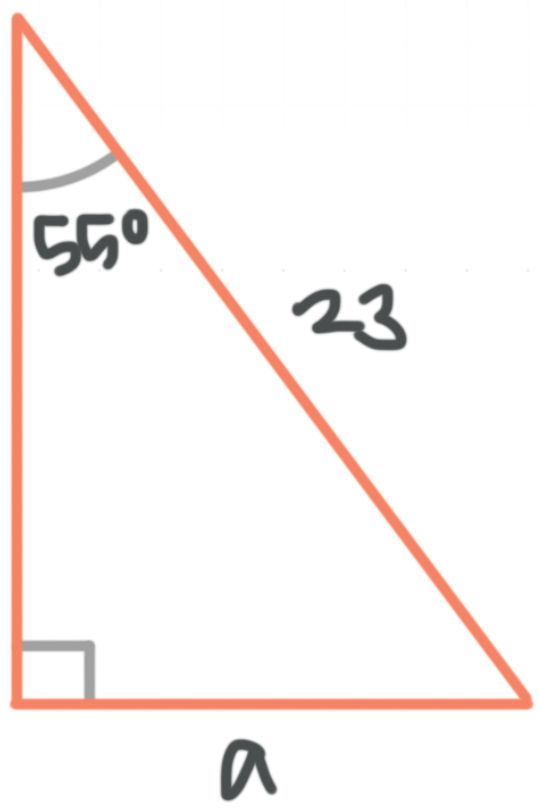
1. Find cosine of the angle θ.



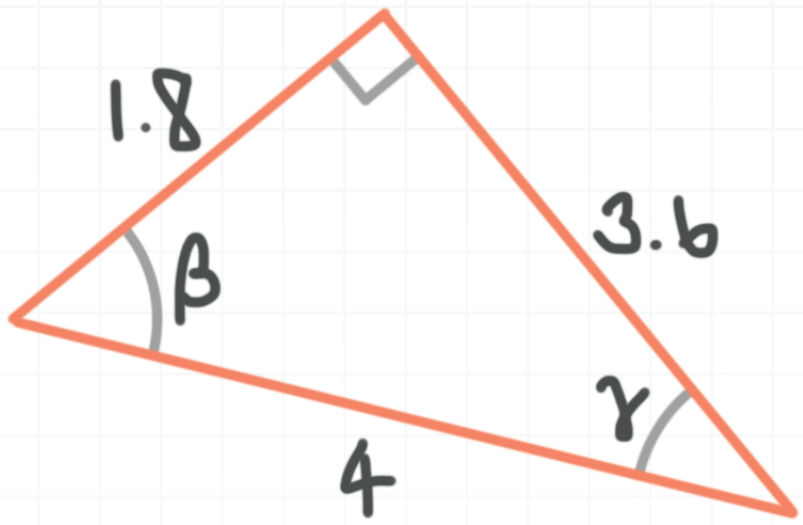
2. Find the measure of the unknown angle of the triangle.



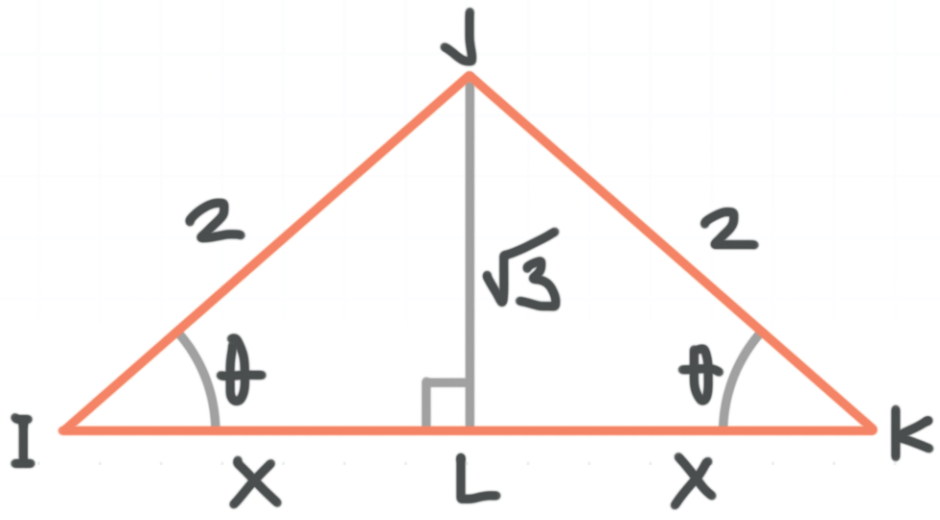
3. Find the equation that would be used to solve for a.



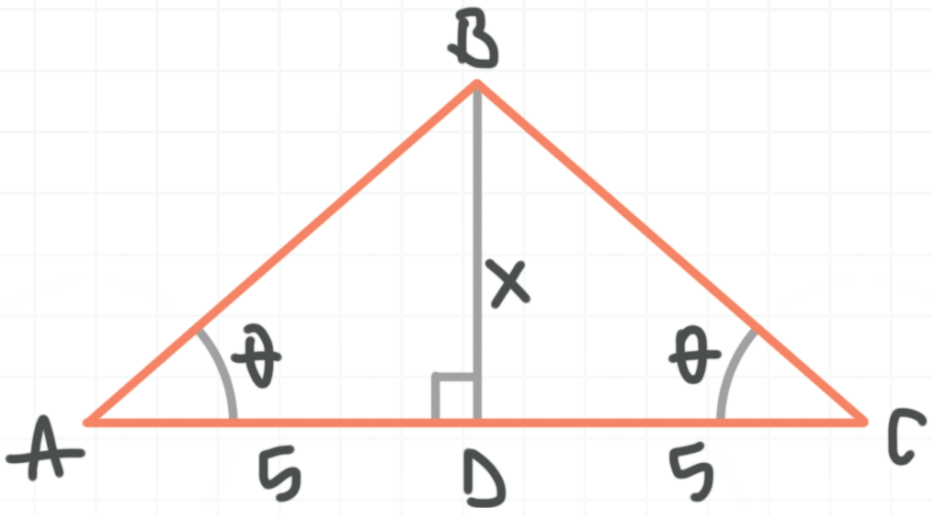
4. Find the sine, cosine, and tangent for β and γ.



■ 5. Find the value of sine of the angle θ, given that the triangle is isosceles (two of the sides have equal length, and the base angles are equal).

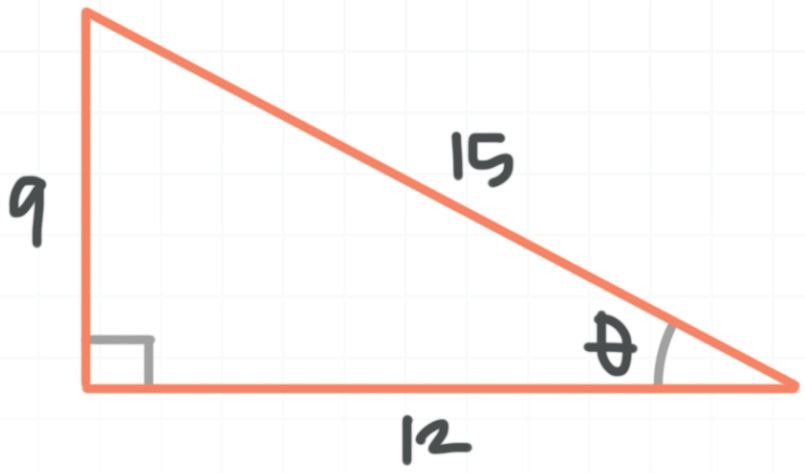


■ 6. Find the equation that would be used to solve for x, given $\overline{AB} = \overline{BC}$ and $\theta = 45°$.

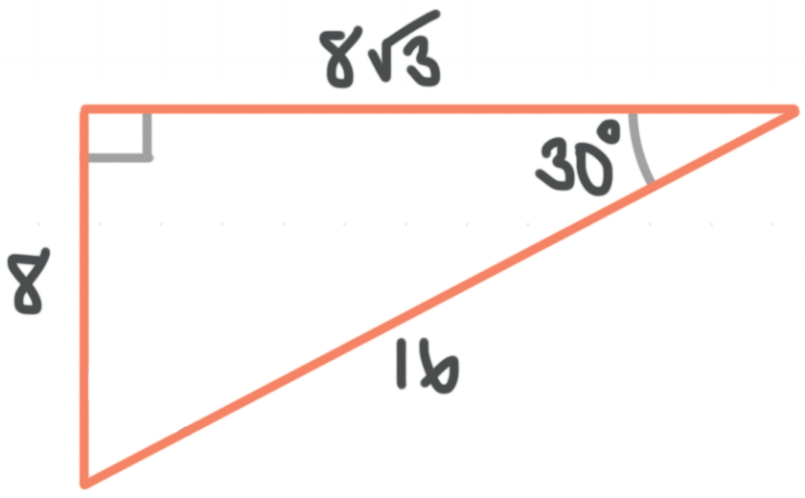


COSECANT, SECANT, COTANGENT, AND THE RECIPROCAL IDENTITIES

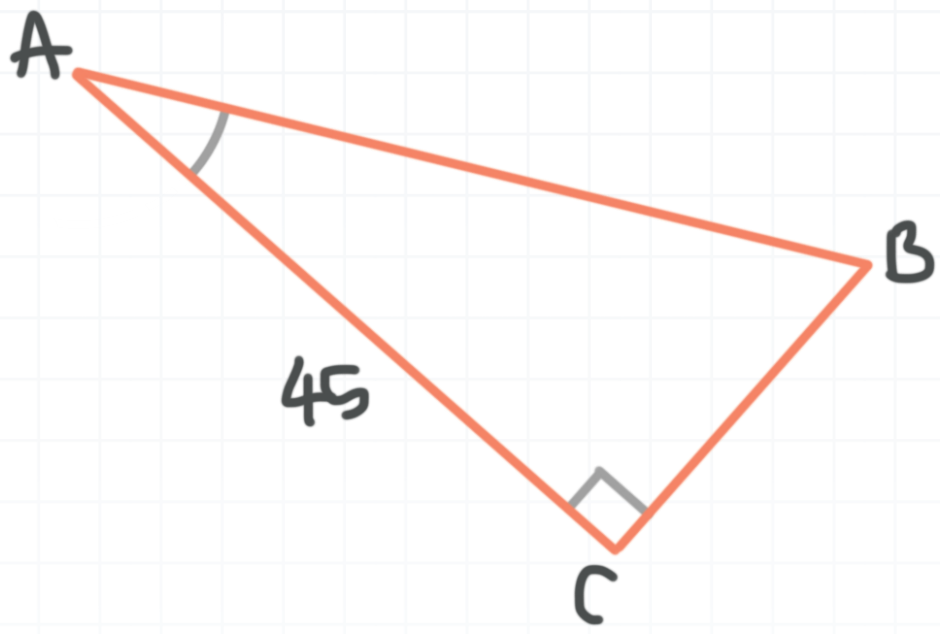
1. Find the value of secant of θ.



2. Find the exact value of the six trigonometric functions for $\theta = 30°$.



3. Given right triangle ABC, $\sin A = 28/53$. Find the exact value of secant, cosecant, and cotangent for the angle A.



4. Find $\csc\theta$, if $\sin\theta = 8/17$.

5. If $\sec\theta = 61/60$ and $\tan\theta = 11/60$, determine the values of the other four trigonometric functions.

6. Given the value of $\cot\theta$, find the value of $\tan\theta$.

$$\cot\theta = \frac{63}{16}$$

THE QUOTIENT IDENTITIES

1. If $\sin\theta = 16/65$ and $\cos\theta = 63/65$, find $\cot\theta$.

2. If $\tan\theta = 4/3$ and $\cos\theta = 3/5$, find $\sin\theta$.

3. If $\cot\theta = \sqrt{13}/6$ and $\csc\theta = -7/6$, find $\cos\theta$.

4. If $\cot\theta = -12/5$ and $\cos\theta = 12/13$, find $\sin\theta$.

5. If $\sin\theta = 39/89$ and $\tan\theta = -39/80$, find $\cos\theta$.

6. If $\tan\theta = 8/15$ and $\sec\theta = 17/15$, find $\sin\theta$.

THE PYTHAGOREAN IDENTITIES

■ 1. Find the positive value of $\cos(49.3°)$ if $\sin(49.3°) = 0.758$.

■ 2. In a right triangle, sine of the acute angle is $1/5$. What are the positive values of the cosine and cotangent of this angle?

■ 3. If $\sin\theta = 12/13$, what is the negative value of $\cot\theta$?

■ 4. If $\theta = 6\pi/5$ and $\sin\theta = -0.588$, what is the negative value of $\cos\theta$?

■ 5. If θ is an angle in the second quadrant such that $\cos\theta = -0.412$, what is the negative value of $\tan\theta$?

■ 6. Evaluate the expression if $\cos\theta = 1/\sqrt{3}$.

$$\tan^2\theta + \sin^2\theta + \sec^2\theta$$

SIGNS BY QUADRANT

- 1. Find $\sin\theta$ if the angle θ lies in the interval $[0°,180°)$ and $\cos^2\theta - 0.36 = 0$.

- 2. Find $\cot\theta$ if $\cos\theta = 0.6$ and the angle θ is in the interval $[5\pi,6\pi)$.

- 3. Find $\sin\theta$ if $\sec\theta = 3$ and $\cot\theta < 0$.

- 4. At the angle $-340°$, what are the signs of sine and cosine.

- 5. In which quadrant does the angle θ lie, if $\tan\theta$ is positive and $\sec\theta$ is negative?

- 6. Find the largest among the values of the six trig functions of θ if $\cos\theta = -0.1$ and θ lies in the third quadrant.

WHEN THE TRIG FUNCTIONS ARE UNDEFINED

1. For what angle is $\cot\theta$ undefined in the interval $(0,2\pi]$?

2. Determine whether or not $\cot(-43\pi/4)$ is defined.

3. Which trigonometric functions are undefined for $\theta = \pi/2$?

4. Which of the six trigonometric functions are undefined along the y-axis (when $x = 0$)?

5. Find the angle where $\tan\theta$ is undefined in the given interval.

$$\left(\frac{7\pi}{3}, \frac{25\pi}{6}\right)$$

6. Find the values of all six trig functions at $\theta = \pi$, and say whether or not any of them are undefined at this angle.

THE UNIT CIRCLE

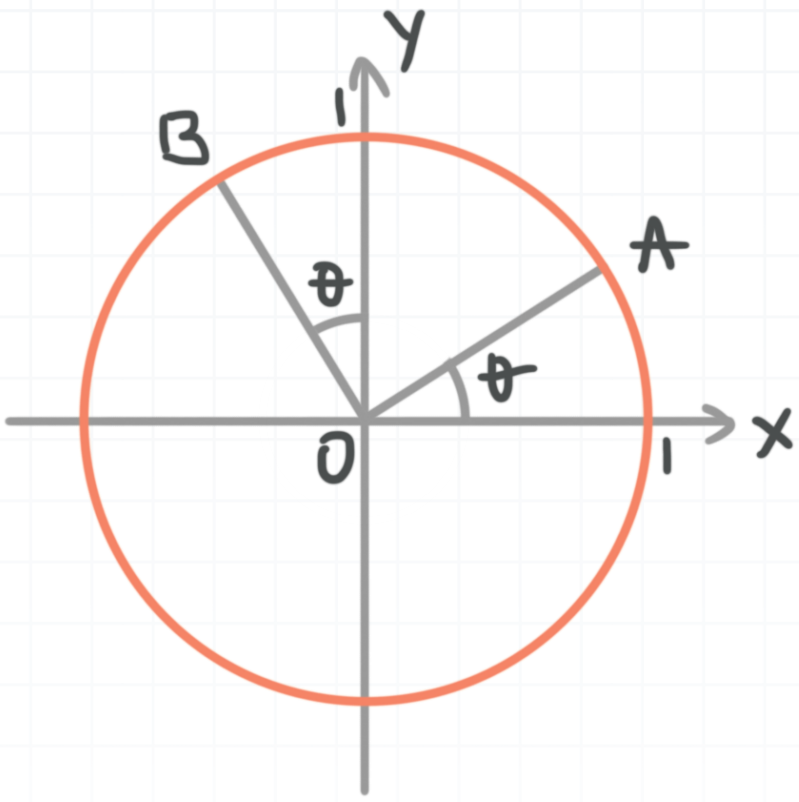
■ 1. What is the coordinate point associated with $\theta = 2\pi/3$ along the unit circle?

■ 2. The terminal side of the angle θ in $[0,2\pi)$ intersects the unit circle at the given point. Find the measure of θ in degrees.

$$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

■ 3. Find $\sin\theta$ if $\theta \in [0,2\pi)$ and $\cos\theta = \sin\theta$.

■ 4. The points A and B lie on the unit circle in quadrants I and II respectively. The angle between OA and the positive x-axis is θ. The angle between OB and the positive y-axis is θ. Find the sine of $\angle AOB$.



5. Evaluate the expression.

$$2\csc\left(\frac{49\pi}{6}\right) - 3\cos\left(\frac{13\pi}{3}\right) + \tan\left(\frac{25\pi}{4}\right)$$

6. Find the angle θ in the interval $[0,2\pi)$.

$$\sin\theta = \frac{1}{2} \text{ and } \cos\theta = -\frac{\sqrt{3}}{2}$$

NEGATIVE ANGLES AND ANGLES MORE THAN ONE ROTATION

1. θ has a measure of $42°$. If it's rotated 6 full rotations in the negative direction, find its measure after the rotations.

2. Find the values of the six trig functions at $\theta = -11\pi/3$.

3. Find the angle in the interval $[0°,360°)$ that's coterminal with $-427°$.

4. Find the angle in the interval $[0,2\pi)$ that's coterminal with $\theta = -65\pi/6$.

5. θ has a measure of $5\pi/3$. If it's rotated 8 full rotations in the negative direction, find its measure after the rotations.

6. Find the value of all six trig functions at $\theta = -23\pi/6$.

COTERMINAL ANGLES IN A PARTICULAR INTERVAL

- 1. Find the angle in the interval $[-90°,90°]$ that's coterminal with $-748°$.

- 2. Find the angle in $[-4\pi, -2\pi]$ that's coterminal with $-21\pi/4$.

- 3. Find the values of the six trig functions at the angle in the interval $[-\pi/6,11\pi/6]$ that's coterminal with $43\pi/2$. $\cot\left(\frac{3\pi}{2}\right) = \frac{0}{1} = 0$

- 4. Find the angle in the interval $[-550°, -190°]$ that's coterminal with $367°$.

- 5. Find the angle in $[7\pi,9\pi]$ that's coterminal with $7\pi/6$.

- 6. Find the values of the six trig functions at the angle in the interval $[-\pi, \pi]$ that's coterminal with $-22\pi/3$.

REFERENCE ANGLES

1. Find the reference angle for $\theta = -16\pi/3$.

2. Which of the angles $19°$, $119°$, $1{,}019°$, and $2{,}019°$ has the smallest reference angle?

3. Sketch the angle $4\pi/3$ in standard position and find its reference angle.

4. Find the reference angle for $\theta = 438°$.

5. Which of the angles $68°$, $168°$, $1{,}068°$, or $2{,}068°$ has the largest reference angle?

6. Find the values of the six trig functions at the reference angle for $\theta = -27\pi/4$.

SYMMETRY ACROSS AXES

■ 1. Use the unit circle to find the angle that has the same cosine as the angle $\theta = 11\pi/6$.

■ 2. If θ is an angle such that $\cos\theta = -0.567$, what is a possible value of $\cos(\theta + 900^\circ)$?

■ 3. If θ is an angle in the fourth quadrant such that $\cos\theta = 2/5$, find $\cos(\theta - 5\pi)$ and $\sin(\theta - 5\pi)$.

■ 4. Use the unit circle to find an angle that has the same sine as the angle $\theta = 225^\circ$.

■ 5. If θ is an angle such that $\cos\theta = 3/5$, what are two possible values of $\sin(\theta + 9\pi)$?

■ 6. If θ is an angle in the third quadrant such that $\sin\theta = -0.255$, what is the value of $\cos(\theta - 11\pi)$?

EVEN-ODD IDENTITIES

1. Which of the six trig functions are even?

2. Evaluate the expression.

$$\sec(-45^\circ) \cdot \csc(-45^\circ) \cdot \tan(-45^\circ) \cdot \cot(-45^\circ)$$

3. List the values in order from smallest to largest.

$$\sin\left(-\frac{\pi}{6}\right),\ \cos\left(-\frac{\pi}{6}\right),\ \tan\left(-\frac{\pi}{6}\right)$$

4. Evaluate the expression.

$$\cos(-30^\circ) + \cos(-45^\circ) + \sin(-45^\circ) + \sin(-60^\circ)$$

5. Find the values of cotangent and cosecant at $\theta = -29\pi/2$.

6. Which of the six trig functions has the largest value at $\theta = -\pi/3$?

THE SET OF ALL POSSIBLE ANGLES

■ 1. A trigonometric equation has two solutions in $[0,2\pi)$, which are $\pi/4$ and $5\pi/4$. Give the complete solution set.

■ 2. Solve the trigonometric equation.

$$\cos(3x + 5\pi) = 0$$

■ 3. Find the set of coordinate points in the xy-plane that satisfy the equation.

$$\sin x \cos y = 0$$

■ 4. Solve the equation $\cos\theta + \sqrt{2} = -\cos\theta$ for all possible values of θ.

■ 5. Solve the trigonometric equation.

$$\sin\left(\frac{x}{2}\right) = -\frac{1}{2}$$

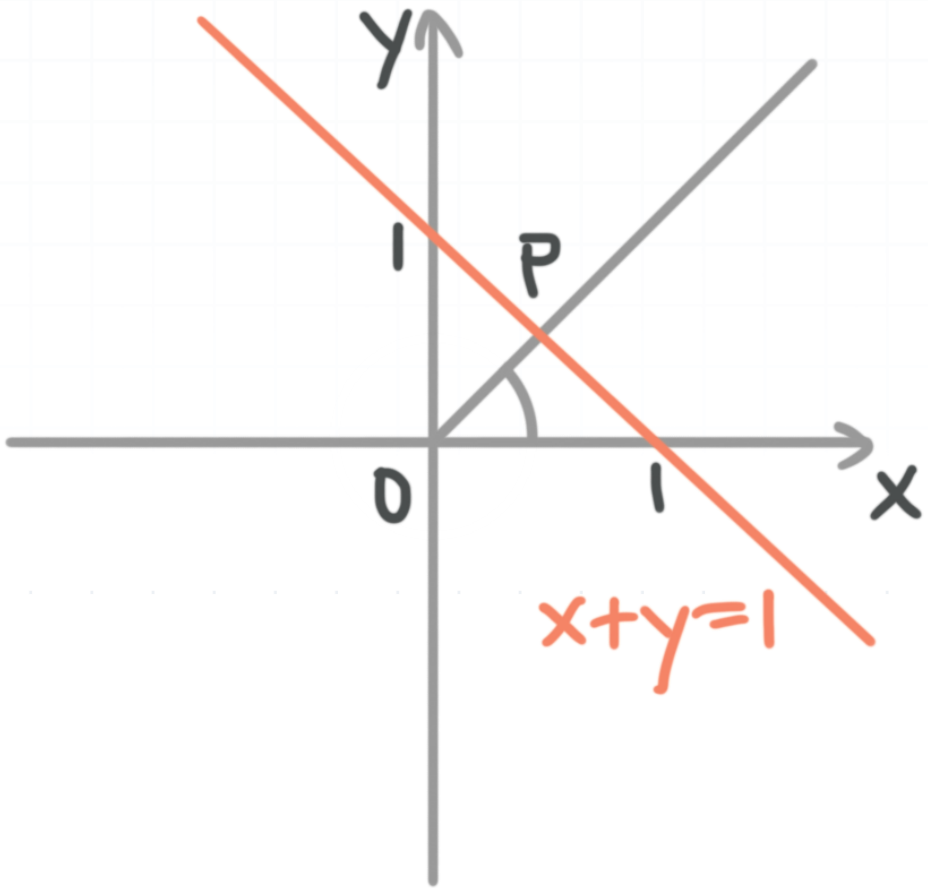
■ 6. Solve the trigonometric equation.

$4\cos^2\theta = 1$

POINTS NOT ON THE UNIT CIRCLE

■ 1. On the circle with center at the origin and radius 5, find the point(s) where $\cos\theta = -0.6$.

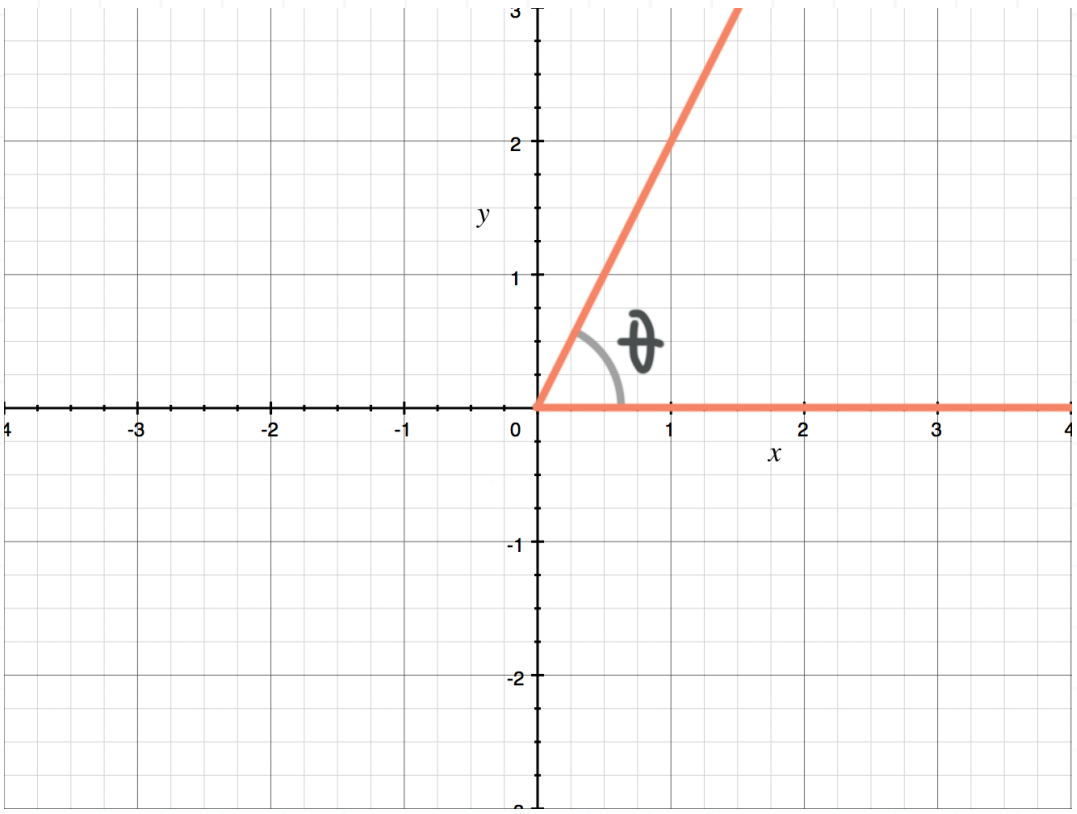
■ 2. Find the point P on the line $x + y = 1$ where sine of the angle between P and the positive direction of the x-axis is $\sqrt{2}/2$.



■ 3. An airplane flew 3 hours North-North-East (NNE) with a constant velocity of 120 km/h. How far north is the plane from the airport and how far east is the plane from the airport?

■ 4. An airplane flew due East with a constant velocity and reached its destination in 12 minutes. Then the plane turned North and flew with the same velocity an additional 5 minutes. Find the cosine of the angle between the line passing through the initial and current positions of the plane, and the direction of due East.

■ 5. Find the sine of the acute angle between the line $y = 2x$ and the positive direction of the x-axis.



6. Find the set of all possible angles θ that pass through the point $P(-3, -3\sqrt{3})$.

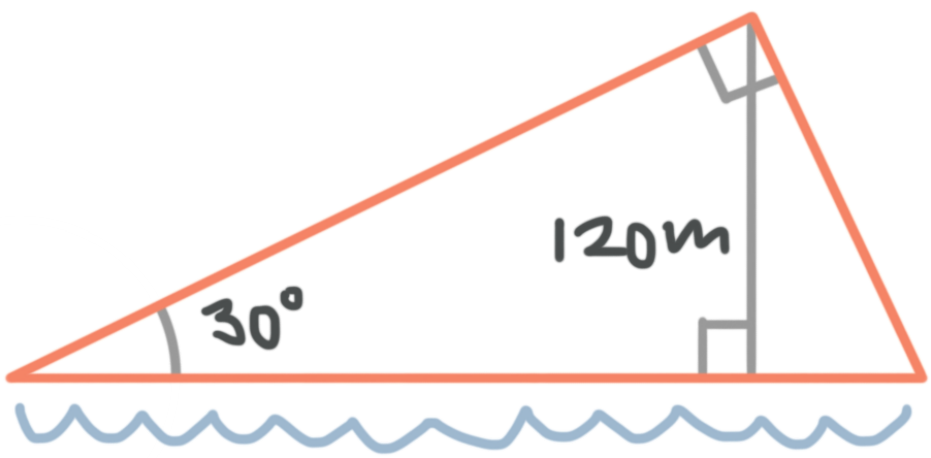
SOLVING RIGHT TRIANGLES

■ 1. A right triangle has a hypotenuse with length 5 and one $35°$ interior angle. Solve the triangle.

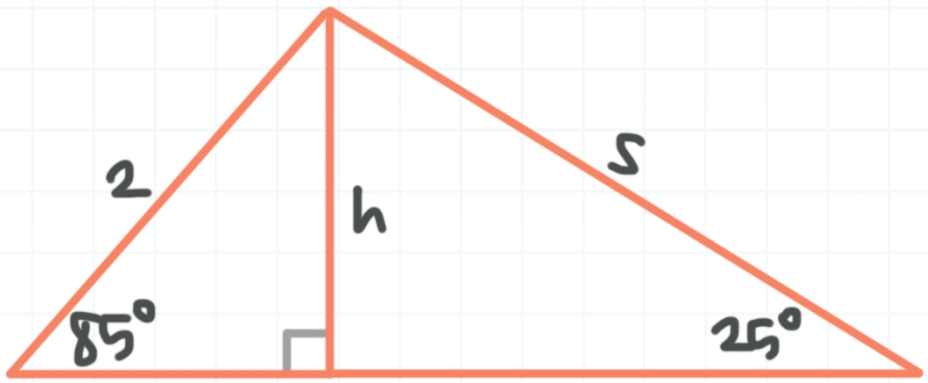
■ 2. A right triangle has a leg with length 7.5 which is opposite of an angle of $76°$. Solve the triangle.

■ 3. A plot of land shaped like a right triangle has its hypotenuse along a river. The angle between one of the legs and the river is $30°$, and the distance from the river to the furthest point of the plot is 120 m.

The land owner decides to build a fence around the plot and needs to find the total length of the fence. Since the fence is sold by sections of 10 meters, round up to the nearest 10 meters.



■ 4. Find the values of h and s, rounded to the nearest whole number.



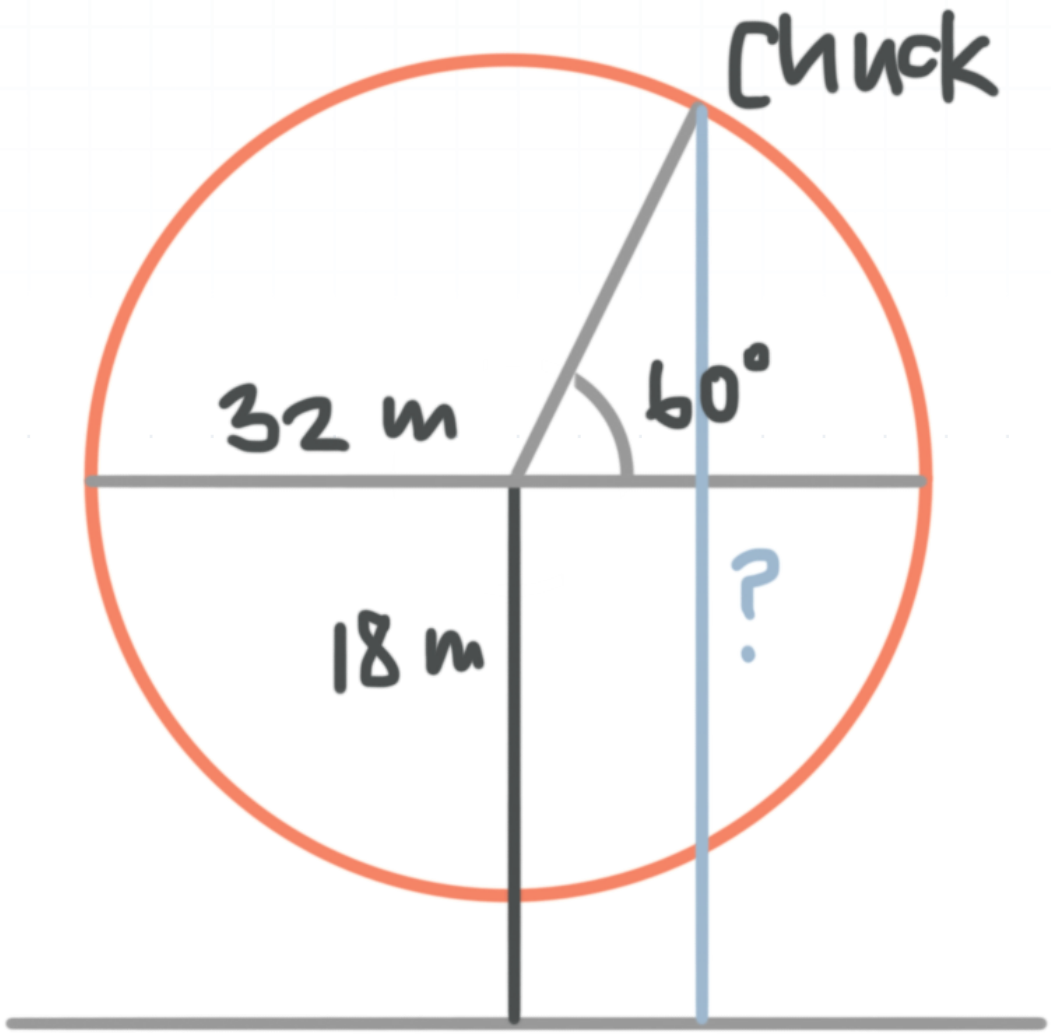
■ 5. The length of one leg of a right triangle is 20, and the angle opposite the other leg has a measure of $40°$. What is the length of the other leg?

■ 6. A right triangle has a leg with length 6. The angle opposite that leg is $24°$. Find the measures of all three interior angles and the lengths of all three sides.

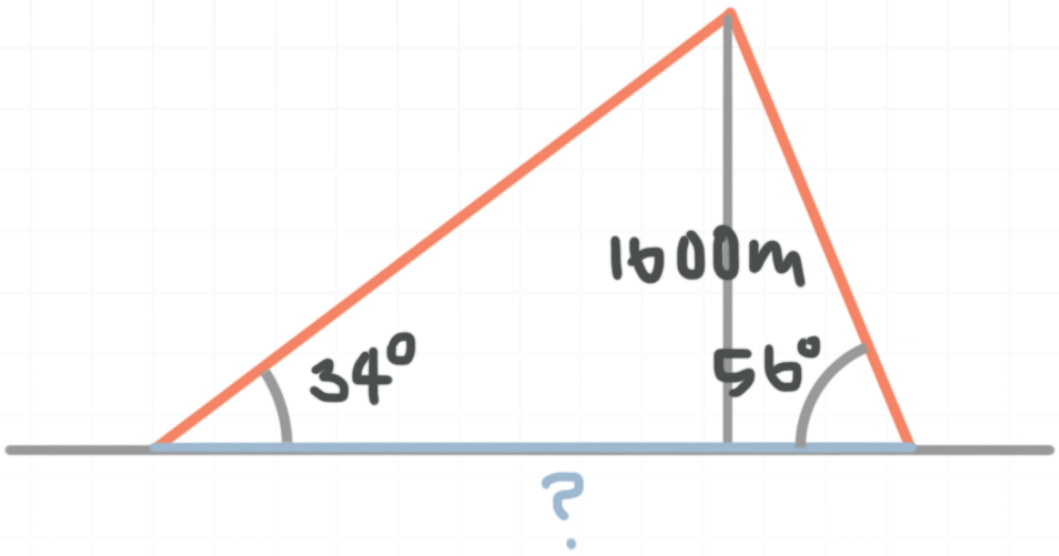
ANGLES OF ELEVATION AND DEPRESSION

■ 1. Find $\cos\theta$, where θ is the angle of elevation of the point $(2,3)$ with respect to the point $(-1,-1)$.

■ 2. Chuck rides a ferris wheel at a carnival. Find the distance from Chuck to the ground (rounded to the nearest meter), if the center of the wheel is 18 m from the ground, the diameter of the wheel is 32 m, and the angle of elevation of Chuck with respect to the center of the wheel is $60°$.



■ 3. A company considers building a tunnel through a mountain. The height of the mountain is $1{,}600$ m above the ground, and the angles of elevation of the peak with respect to the left and right endpoints of the tunnel are $34°$ and $56°$ respectively. Find the length of the tunnel to the nearest meter.

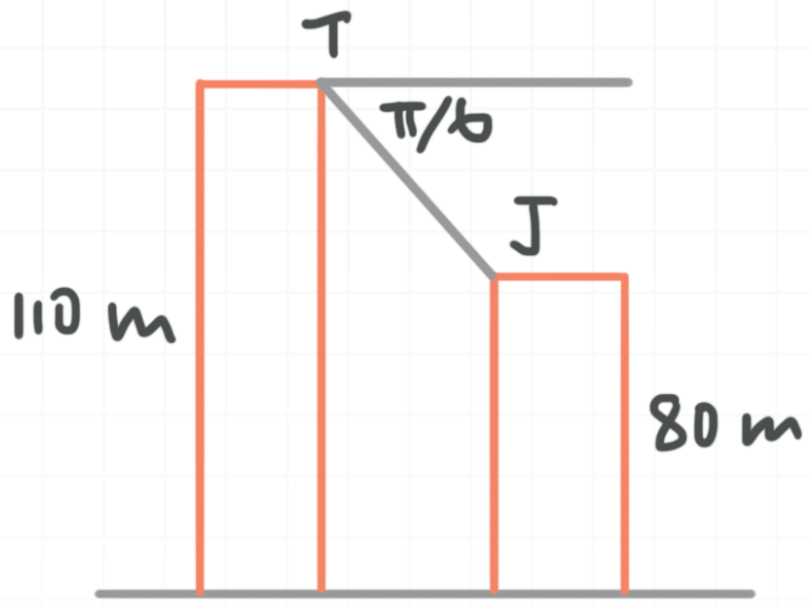


■ 4. Suppose you measure the horizontal distance of a cell-phone tower from a nearby point on the ground A to be 24 feet. If the angle of elevation of the top of the tower with respect to A is $78°$, how far is the top of the tower from A, and what is the height of the tower?

■ 5. Find $\sin\theta$, where θ is the angle of depression of the point $(3, -4)$ with respect to origin.

■ 6. Tom stands on the roof of a 110 m high building, and he looks down at Jerry, who's standing on the roof of a neighboring 80 m high building. The

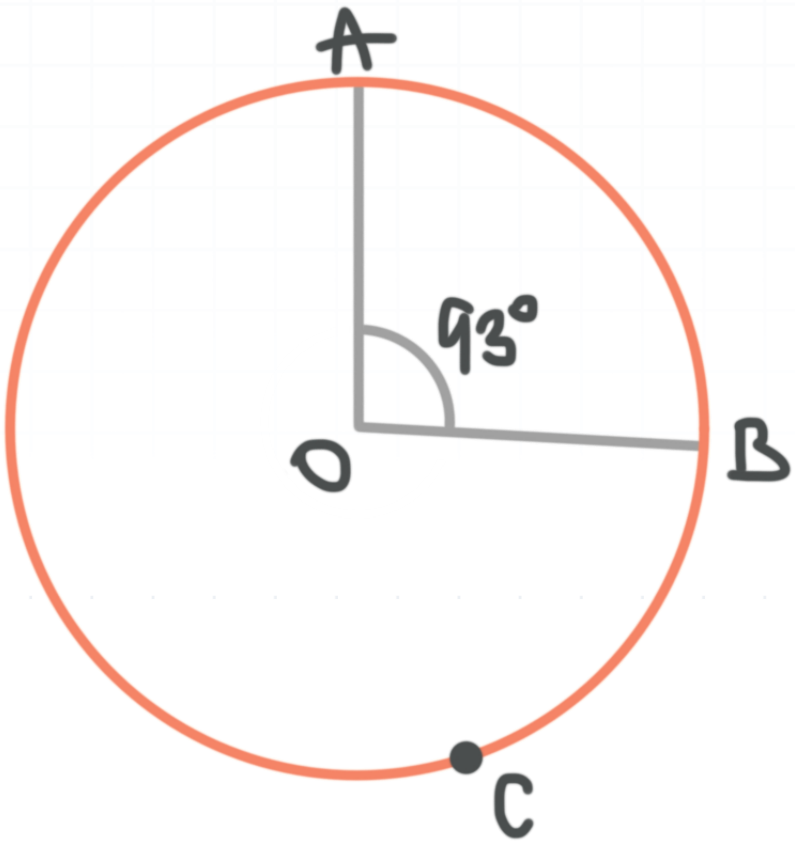
angle of depression of Jerry with respect to Tom is $\pi/6$. Find the distance between the buildings to the nearest meter.



RADIANS AND ARC LENGTH

■ 1. Find the degree measure of the central angle if the length of an arc carved out by this central angle is 9.42 and the radius of the circle is $r = 6$.

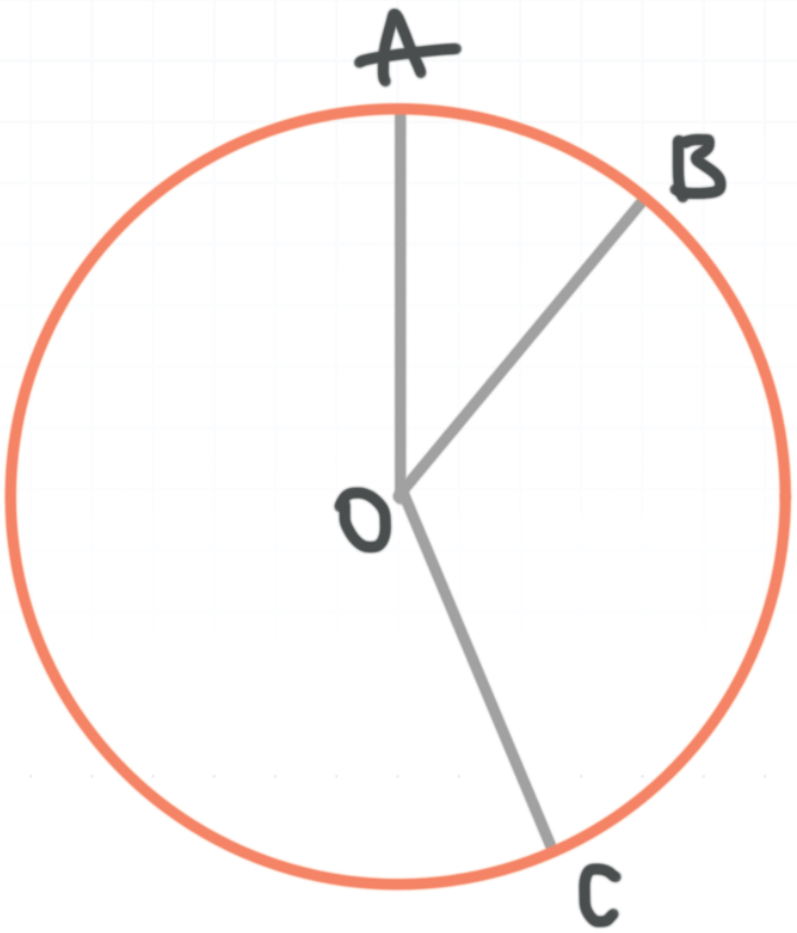
■ 2. In circle O, the diameter is 30 cm, and the measure of arc AB is $93°$. Find the length of arc ACB.



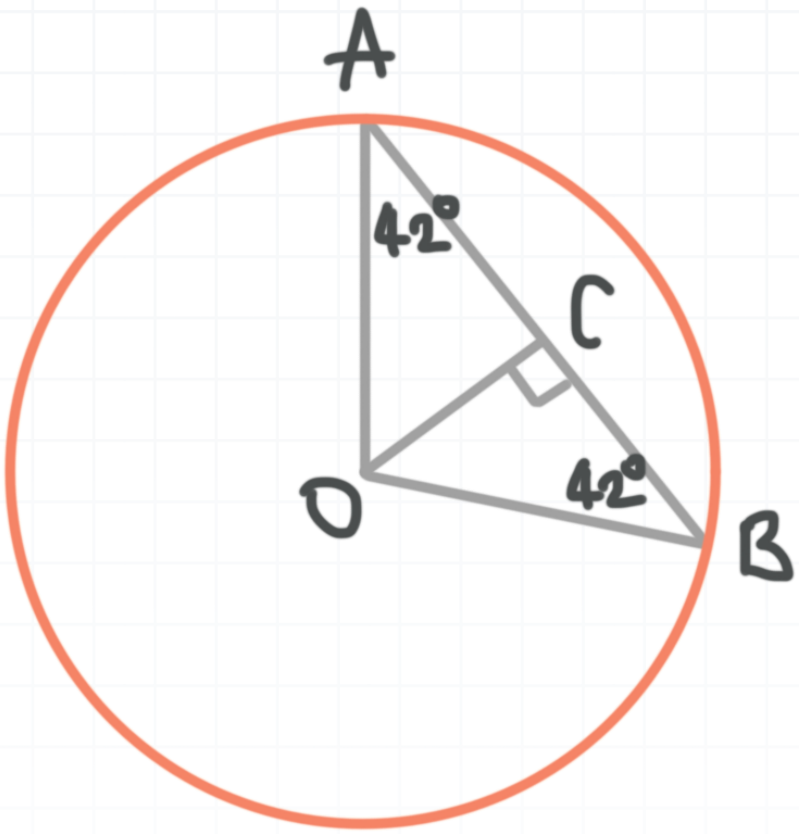
■ 3. A circle has a central angle of $35°23'6''$ which subtends an arc of length 3π cm. Find the diameter of the circle to the nearest centimeter.

■ 4. A circle has a radius of 19 cm. Find the central angle that subtends an arc of length 47.5 cm, rounding the answer to the nearest second.

■ 5. If AOB is a central angle of $53°$, the angle $BOC = 122°$, and the radius is 9 cm, then find the length of the arc ABC. Use $\pi = 3.14$ and round the answer to one decimal place.

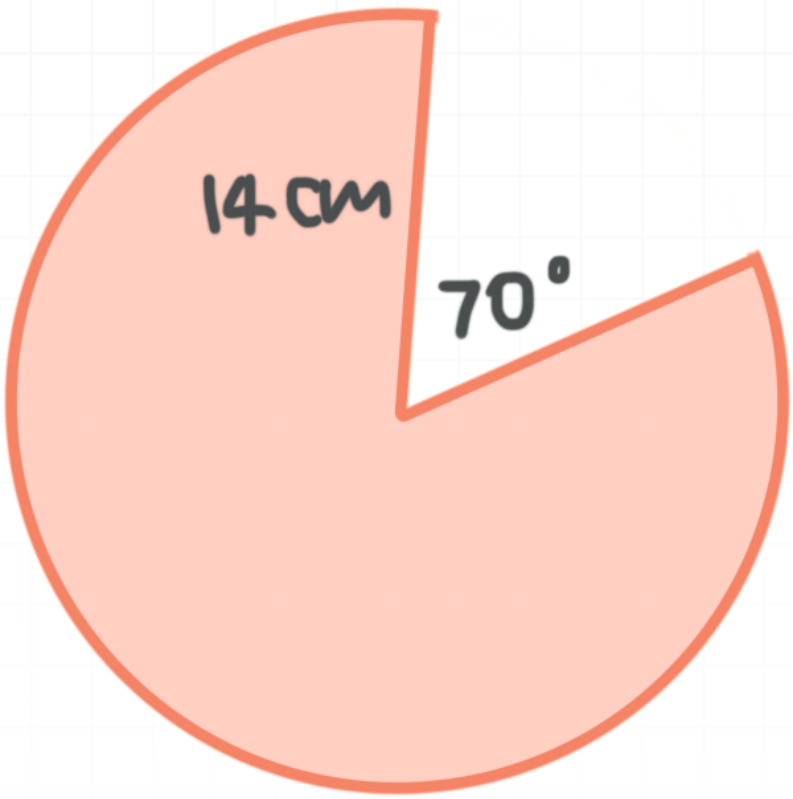


■ 6. Find the length of the arc AB given that the radius of the circle is 12 cm. Round the answer to one decimal place.

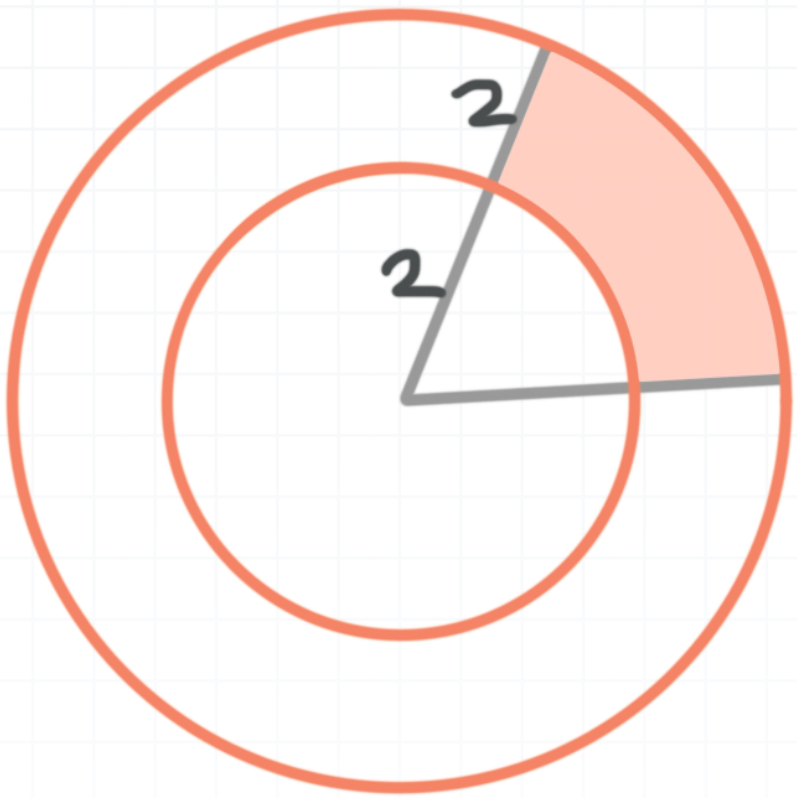


AREA OF A CIRCULAR SECTOR

1. Find the area of the shaded region.



2. Find the area of the shaded region between the concentric circles, if the angle that subtends the arc is $80°$.



■ 3. A circle has radius 13. Find the area A of a sector of the circle that has a central angle of $2\pi/5$.

■ 4. A pizza with 16 inch diameter is sliced into 8 equal slices. Find the area of one of the pizza slices.

■ 5. Find the area of a sector of a circle that has diameter $\overline{GH}$ with $G(-1,-1)$ and $H(5,7)$ if the arc which bounds that sector subtends a central angle of $4\pi/9$. Use the distance formula for d to find the length of the diameter.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

6. The area of a sector of a circle is formed with a central angle $3\pi/4$ and has area 54π. Find the diameter of the circle.

TRIG FUNCTIONS OF REAL NUMBERS

1. Find $\sec 1.56$ using a calculator to evaluate only cosine. Round the result to three decimal places.

2. Find $\cot 0.567$ using a calculator to evaluate only sine and cosine. Round the result to four decimal places.

$$\cot 0.567$$

3. Find the value of all six circular functions at $a = 1.273$ using a calculator to evaluate only sine and cosine.

4. Find the value of all six circular functions at $t = -0.2489$.

5. Find $\tan 3.49$ using a calculator to evaluate only sine and cosine. Round the result to two decimal places.

6. Find the value of all six circular functions at $s = -4.5$, using a calculator to evaluate only sine and cosine.

LINEAR AND ANGULAR VELOCITY

■ 1. What is the angular velocity, in radians per second, of a wheel that rotates at a constant rate and sweeps out an angle of $33\pi/4$ radians in 0.6 seconds?

■ 2. The wind turbine has a circular blade with diameter 154 meters that rotates at 18 rotations per minute. Find the angular velocity of the blade in degrees per second.

■ 3. What is the angular velocity, in radians per second, of a wheel that rotates at a constant rate and sweeps out an angle of $21\pi/5$ radians in 0.85 seconds?

■ 4. Suppose a frisbee rotates at a constant rate of 105 revolutions per minute. What is its angular velocity ω in radians per second?

■ 5. Find angular velocity, in radians per minute, of an object that rotates at a constant rate and sweeps out an angle of $985°$ in 8.4 seconds.

6. A cylinder with a 3.4 ft radius is rotating at 150 rpm. Give the angular velocity in rad/sec and in degrees per second.

RELATING LINEAR AND ANGULAR VELOCITY

1. A saw has a circular blade with diameter 10 inches that rotates at $5{,}000$ revolutions per minute. Find the approximate linear velocity of the saw teeth (in ft/sec) as they contact the wood being cut.

2. A car's tire has a radius of 12.5 inches and turns with an angular velocity of 84.5 radians per second. Find the approximate linear velocity of the car in miles per hour. (Use the fact that there are 12 inches in 1 foot, and approximately $5{,}280$ feet in 1 mile.)

3. A bicycle tire with a diameter of 26 inches turns with an angular velocity of 2 radians per seconds. Find the distance traveled in 5 minutes by a point on the tire.

4. A tire with a radius of 0.75 feet is rotating at 36 miles per hour. Find the angular velocity of a point on its rim, expressed in revolutions per minute.

5. The carousel at the county fair makes 3.5 revolutions per minute. The linear speed of a person riding inside the carousel is 2.9 ft/sec. How far is this person from the carousel's center?

6. A disk is spinning at 27 rpm. If a fly is sitting 9 cm from the center of the disk, what is the angular velocity of the fly in radians/sec? What is the speed of the fly in cm/sec? After 2 min, how far has the fly traveled?

SKETCHING SINE AND COSINE

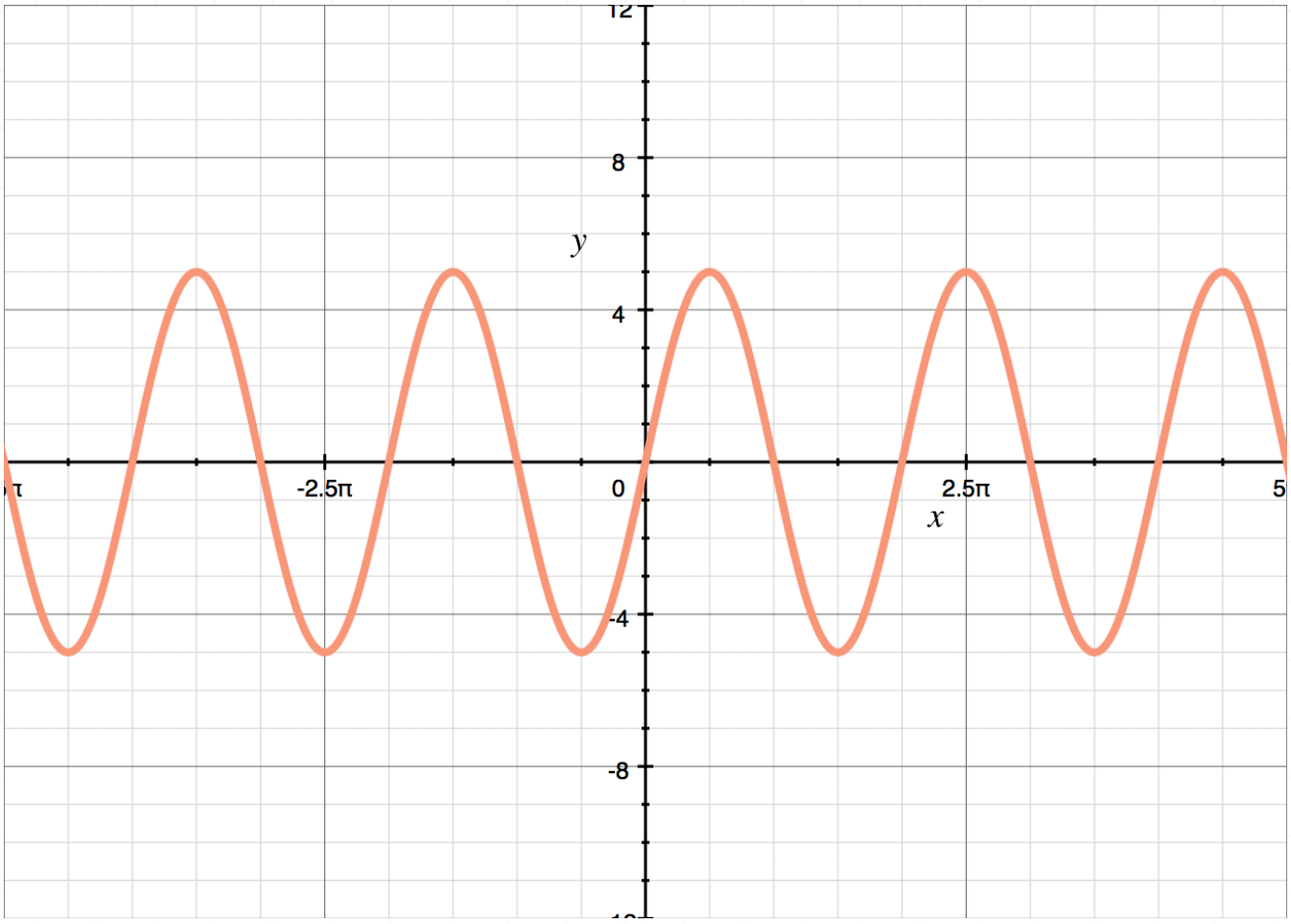
1. Sketch the graph of $y = 3\sin(\theta/2)$.

2. Sketch the graph of $y = 2.6\cos(3\theta)$.

3. Sketch the graph of $y = -4\cos(\theta/3)$.

4. On the same set of axes, graph $y = 2\cos\theta$ and $y = \sin 2\theta$.

5. Which function is represented by the curve?



6. Graph $y = -4\cos(\theta/2)$ and $y = 3\sin\theta$ on the same set of axes.

SKETCHING COSECANT AND SECANT

1. Sketch the graph of $y = \csc(\theta/2)$.

2. Sketch the graph of $y = -\sec(2\theta)$.

3. Sketch the graph of $y = 5\csc(2\theta)$.

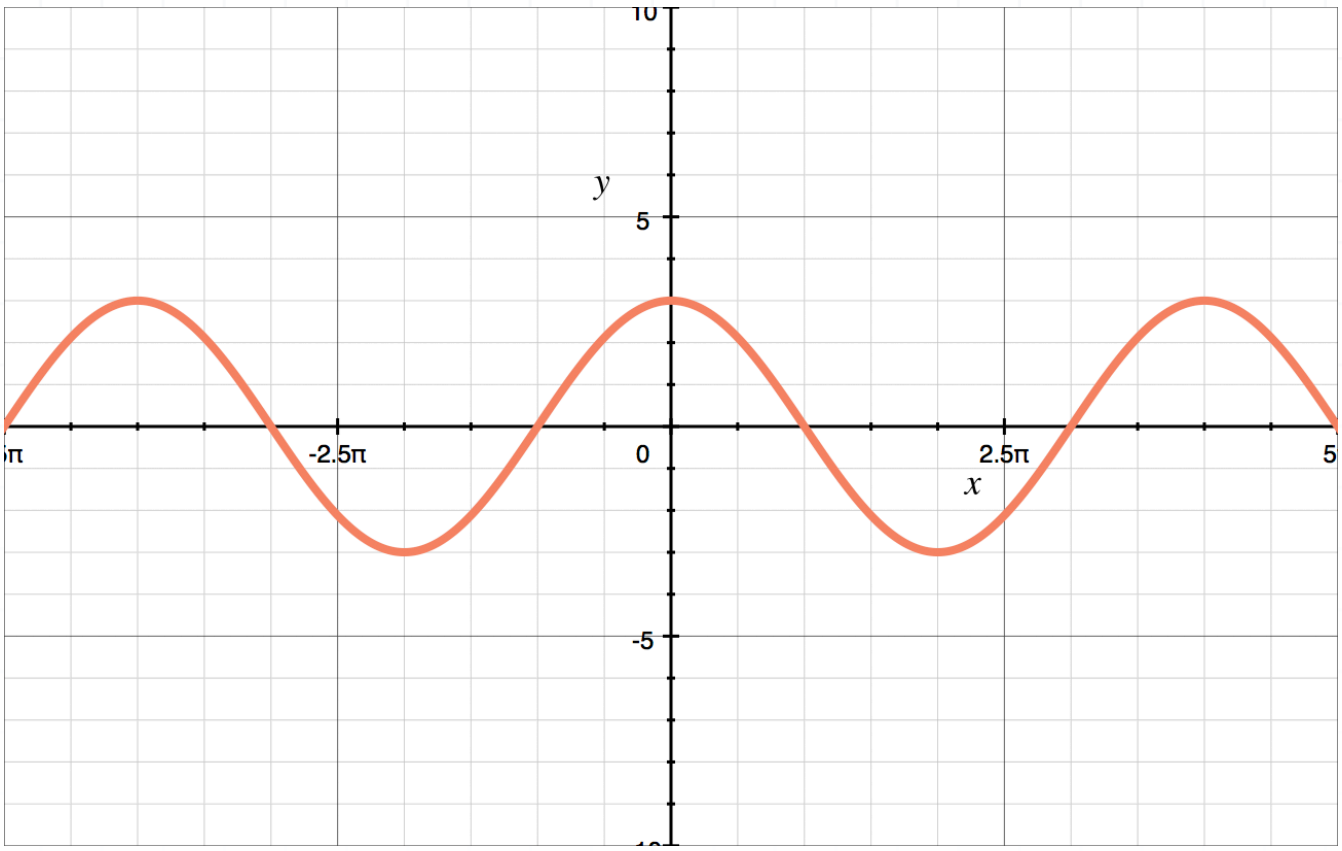
4. Sketch the graph of $y = (1/4)\sec(\theta/2)$.

5. Sketch the graph of $y = (1/2)\csc(-\theta)$.

6. Sketch the graph of $y = -2\sec(\theta/4)$.

PERIOD AND AMPLITUDE

■ 1. Find all possible cosine functions that could represent the graph.



■ 2. Modify the basic sine function so that it has a period of $60°$ and an amplitude of 3.

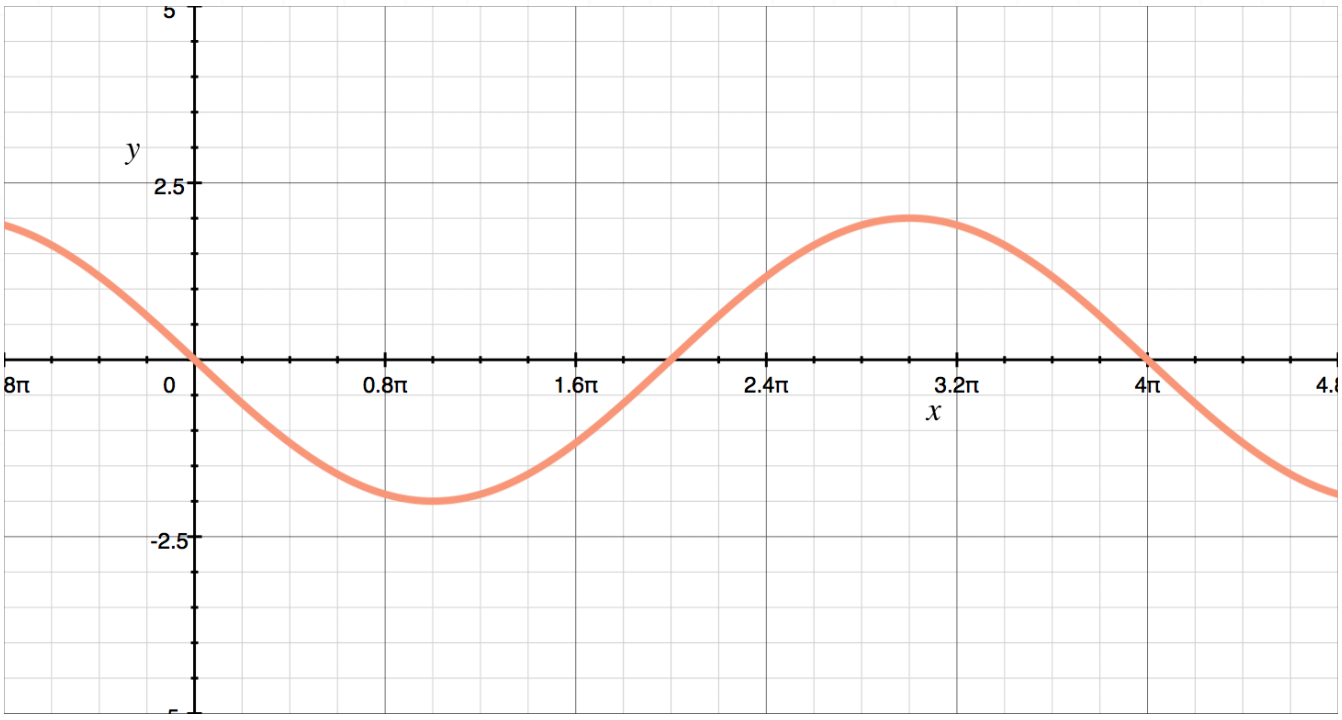
■ 3. Which one of these functions does not have a period of 3π?

$$y = -7\tan\left(\frac{x}{3}\right) \qquad y = -7\sec\left(\frac{2x}{3}\right)$$

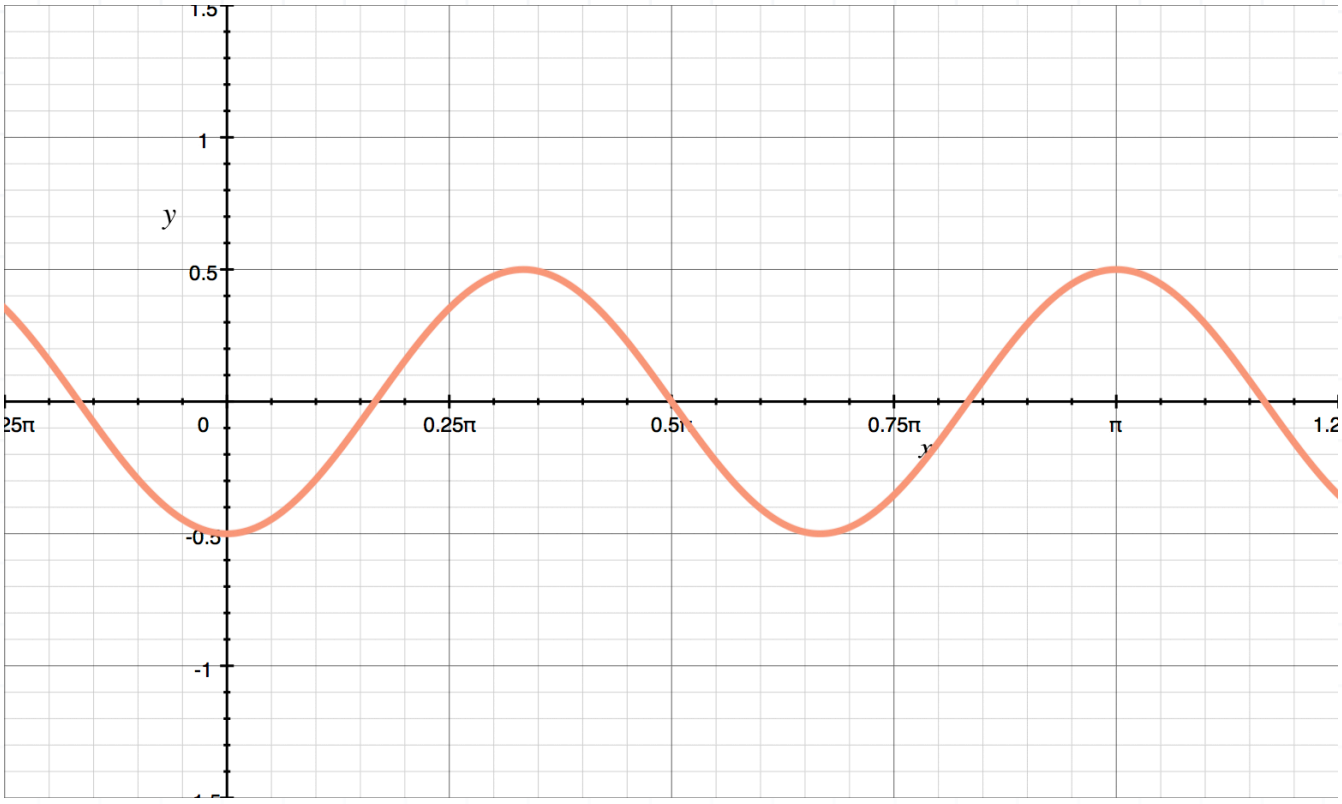
$$y = 7\tan\left(\frac{2x}{3}\right) \qquad y = 7\sec\left(\frac{2x}{3}\right)$$

■ 4. Find all possible sine and cosine functions $y = a\sin(bx)$ and $y = a\cos(bx)$ which have a period of $135°$ and an amplitude of 10.

■ 5. Give the amplitude and period of the function in the graph, then write an equation for the curve.



■ 6. Give the amplitude and period of function in the graph, then write the equation of the graph if we know that the function is not flipped across the y-axis.

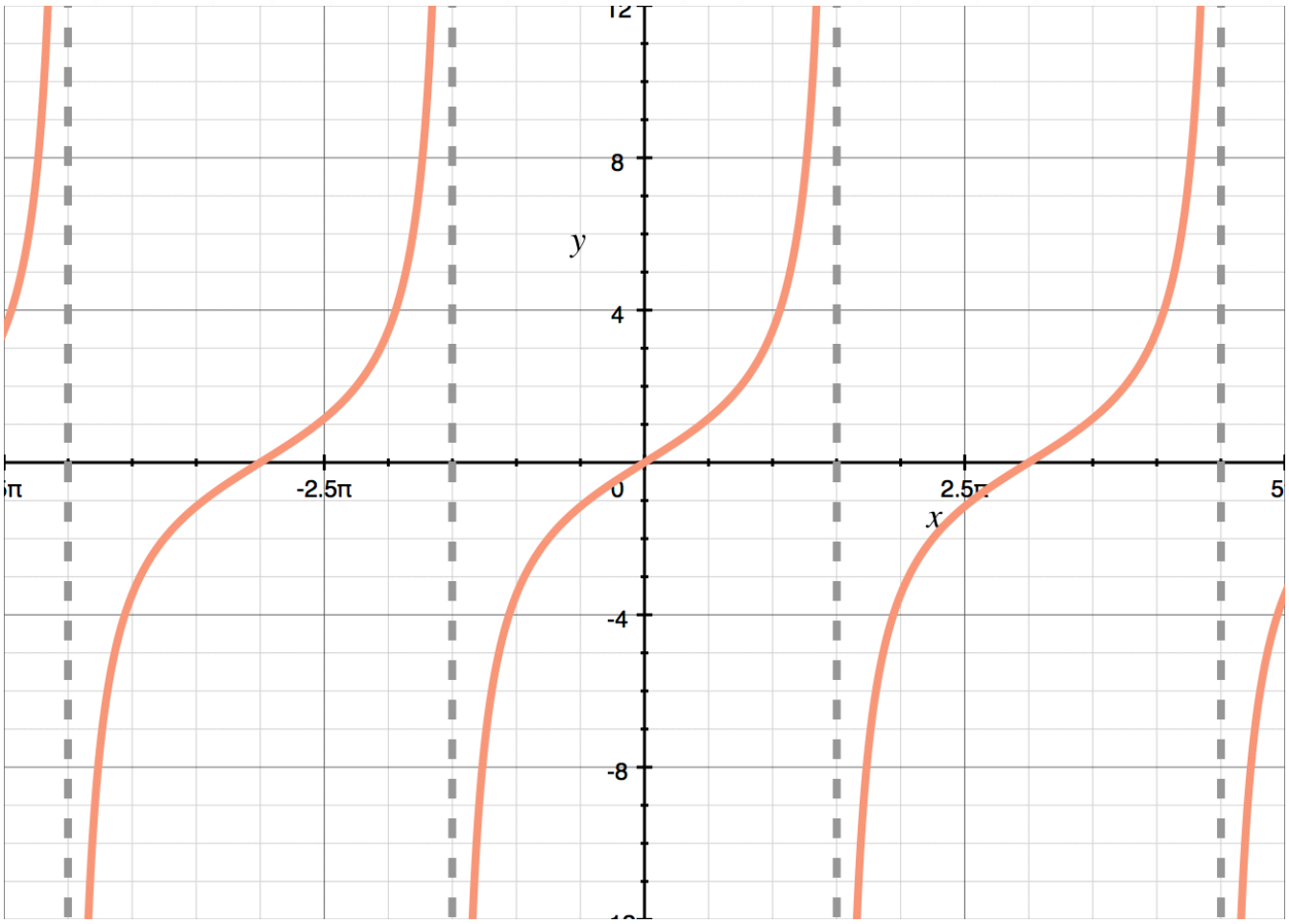


SKETCHING TANGENT AND COTANGENT

1. What are the vertical asymptotes of $y = -2\cot(3x)$?

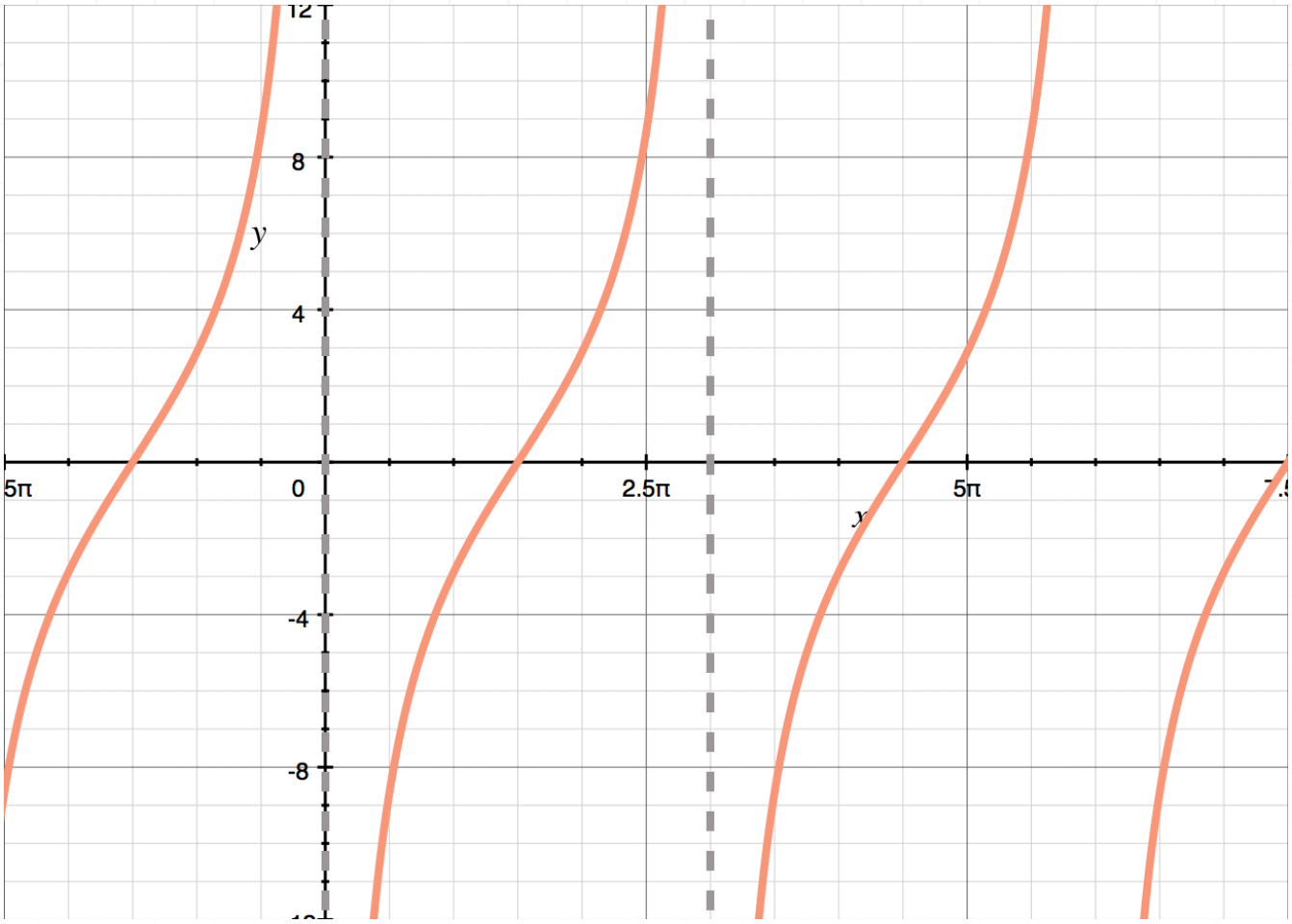
2. Sketch the graph of $y = -3\tan(2x)$.

3. Which function is represented by the curve if $a = 2$?

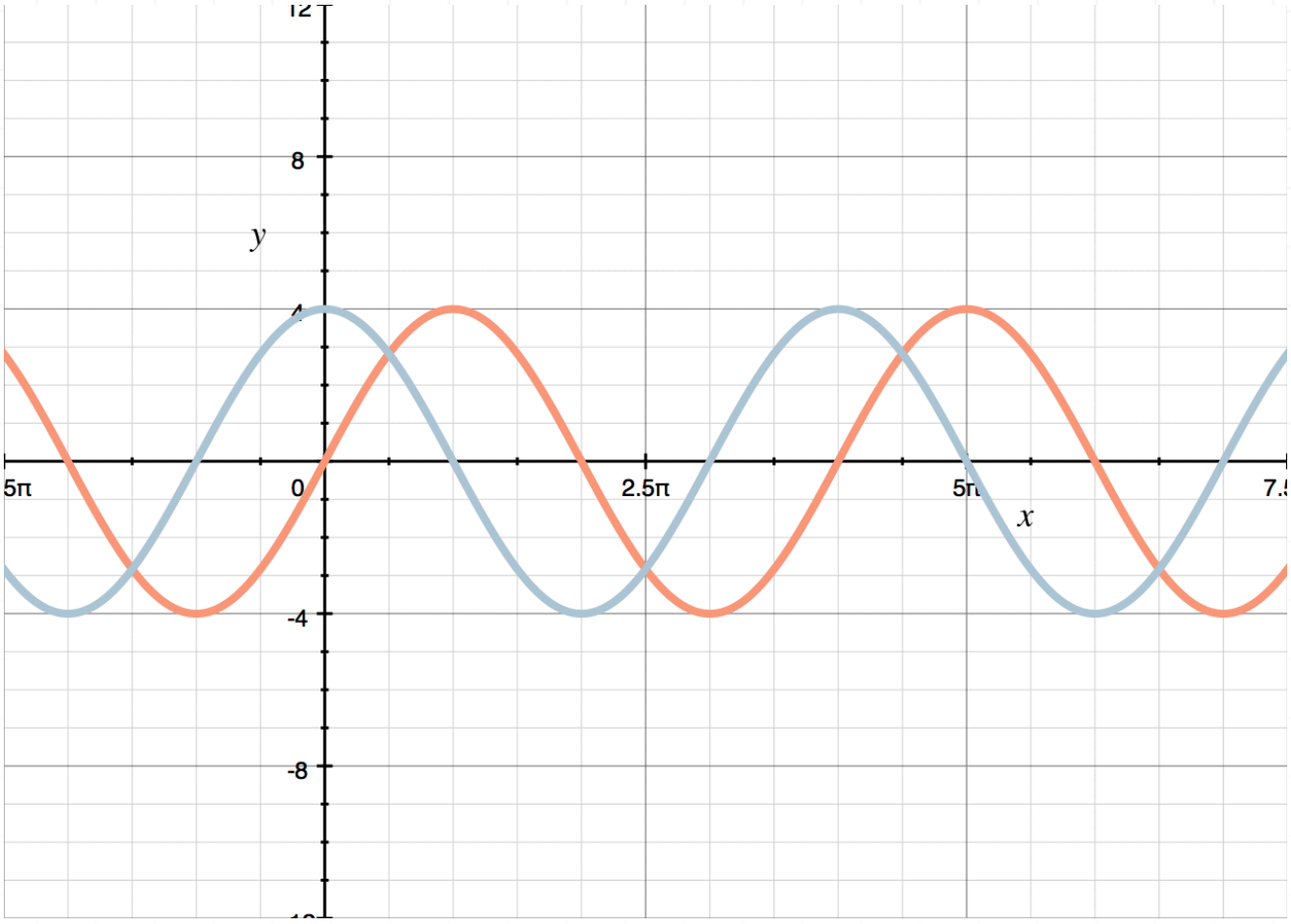


4. Sketch the graph of $y = 2\cot(-x/2)$.

5. Which function is represented by the curve if $a = -5$?

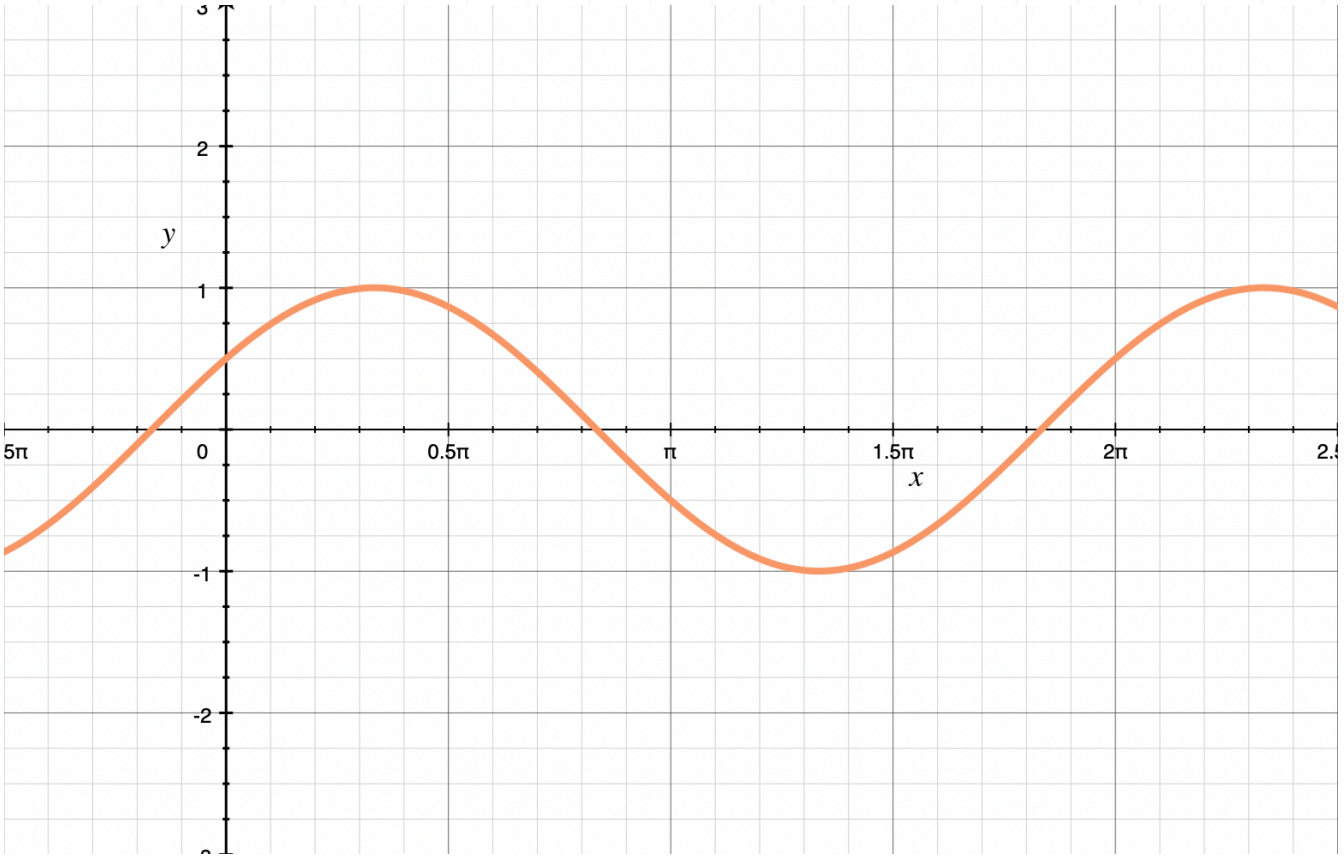


6. Sketch the graph of $y = 4\tan(x/2)$, using the graph of $y = 4\sin(x/2)$ in red and $y = 4\cos(x/2)$ in blue.



HORIZONTAL AND VERTICAL SHIFTS

1. Determine the equation of the cosine function shown in the graph.

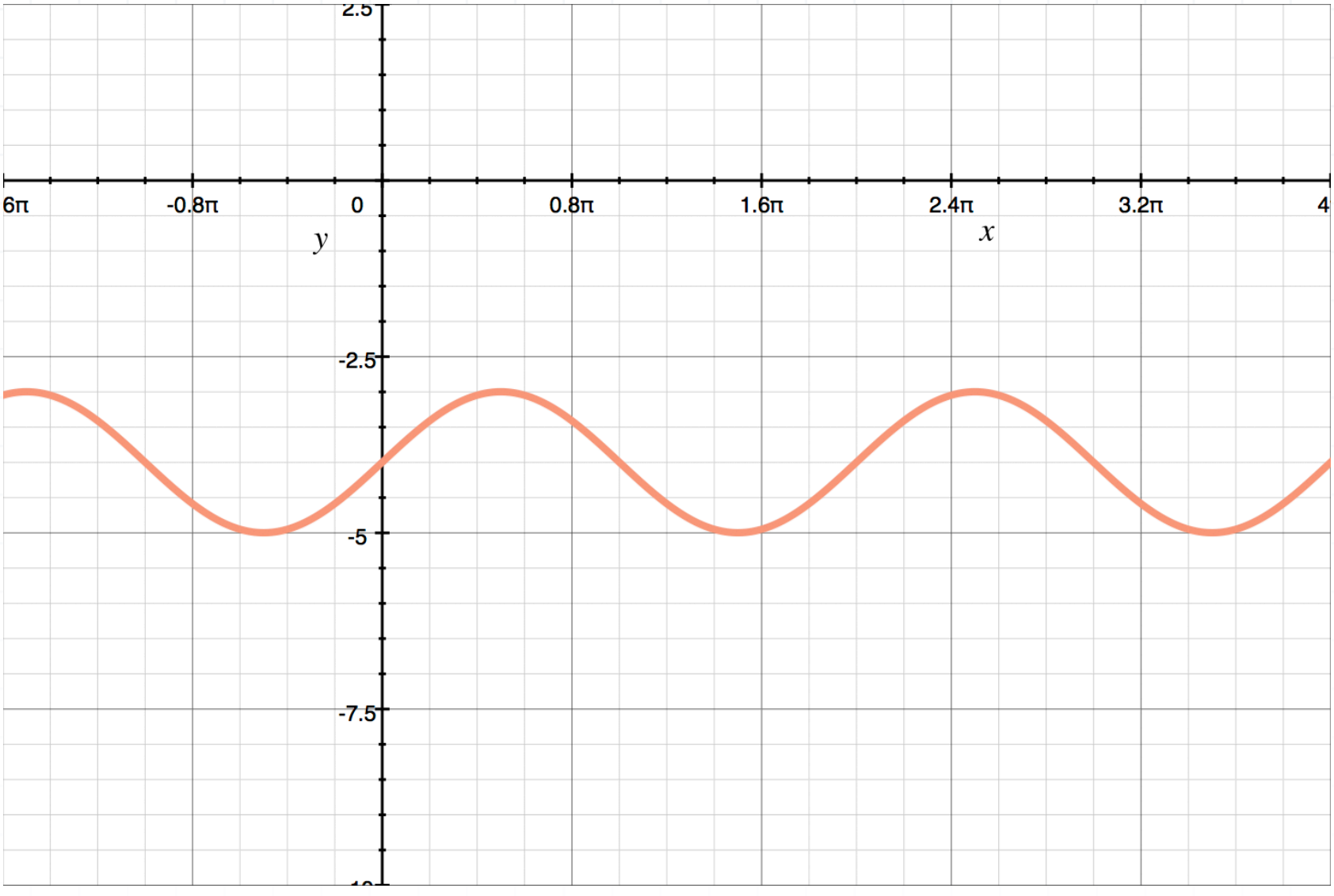


2. Determine the phase shift and the vertical shift of the sine function.

$$y = 2\sin\left(x + \frac{\pi}{6}\right) - 2$$

3. Sketch the graph of $y = \sin(\theta - \pi)$.

4. Determine the equation of the sine function shown in the graph.



■ 5. A trigonometric function has an amplitude of 3 units, a horizontal shift to the left by $\pi/4$, a vertical shift down by 7 units, and no reflections. Represent the curve with a cosine function.

■ 6. Sketch the graph of $y = \cos\theta - 5$.

GRAPHING TRANSFORMATIONS

■ 1. What are the period, amplitude, and range of the function?

$$y = -3\cos(2\pi x - 1) + 4$$

■ 2. Find the equation of the curve that's the result of applying the following sequence of transformations to $f(x) = \sin(x - \pi)$.

1. A horizontal compression by a factor of 2
2. A horizontal shift to the right by 3π
3. A vertical stretch by a factor of 5
4. A reflection over the x-axis
5. A vertical shift down by 2

■ 3. Which function has an amplitude of 2 and a range of $[-3,1]$?

$-2\sin(5x - 3\pi) - 2$

$2\sin(3x - 3\pi) - 1$

$-4\cos(2x + 3\pi) - 2$

$4\cos(2x + 3\pi) - 1$

■ 4. Find the equation of the curve that's the result of applying the following sequence of transformations to $f(x) = 2\sin(3x)$.

1. A horizontal shift to the left by $\pi/12$
2. A reflection over the y-axis
3. A reflection over the x-axis

■ 5. What will be the zeros of the function $f(x) = \cos x$ after the following sequence of transformations?

1. A horizontal compression by a factor of 2
2. A horizontal shift to the right by $\pi/6$
3. A vertical stretch by a factor of 5

■ 6. What transformations are applied to transform $y = \sin\theta$ into the given function?

$$y = 3\sin\left(3\theta + \frac{3\pi}{2}\right)$$

GRAPHING COMBINATIONS

1. Find the period of the function.

$$\tan(3\theta - \pi) - \sin(6\theta)$$

2. Find the period of the function.

$$\frac{\sin\left(5\theta - \frac{\pi}{2}\right)}{\cos(2\theta)}$$

3. Graph the combination function $2\cos(3\theta) + \sin(2\theta)$.

4. Graph the combination function.

$$\cos\left(\theta - \frac{\pi}{2}\right) - 5\sin\left(4\theta + \frac{3\pi}{2}\right)$$

5. Graph the combination function $(2\cos(3\theta - 2\pi))(\sin\theta + 2)$.

6. Graph the combination function.

$$\frac{\cos\left(3\theta+\frac{3\pi}{4}\right)}{3\sin\left(\theta-\frac{\pi}{2}\right)}$$

INVERSE TRIG RELATIONS

1. In degrees, use the unit circle to find the set of angles whose cosine is $-\sqrt{2}/2$.

2. In both radians and degrees, use the unit circle to find the set of angles whose sine is -1.

3. In both radians and degrees, use the unit circle to find the set of angles whose secant is 2.

4. In both radians and degrees, use the unit circle to find the set of angles whose cosecant is 1.

5. In both radians and degrees, use the unit circle to find the set of angles whose tangent is 1.

6. In both radians and degrees, use the unit circle to find the set of angles whose cotangent is 0.

INVERSE TRIG FUNCTIONS

1. Find the value of the inverse tangent function.

$$\tan^{-1}(0)$$

2. Find the value of the inverse cotangent function.

$$\cot^{-1}(-1)$$

3. Find the value of the inverse sine function.

$$\sin^{-1}\left(-\frac{1}{2}\right)$$

4. Find the value of the inverse secant function.

$$\sec^{-1}(-2)$$

5. Find the value of the inverse cosine function.

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

6. Find the value of the inverse cosecant function.

$$\csc^{-1}(-\sqrt{2})$$

TRIG FUNCTIONS OF INVERSE TRIG FUNCTIONS

1. Find the value of the expression.

$$\sin\left(\tan^{-1}\left(\frac{1}{3}\right)\right)$$

2. Find the value of $\tan^{-1}(\sin \pi)$.

3. Find the value of the expression.

$$\csc\left(\cot^{-1}\left(\frac{1}{x}\right)\right)$$

4. Find the value of the expression.

$$\cos\left(\sec^{-1}\left(-\frac{9}{2}\right)\right)$$

5. Find the value of the expression.

$$\cot\left(\cos^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)\right)$$

6. A 16-foot ladder leans against a brick wall. The base of the ladder is 8 feet from the wall. Find the angle the ladder makes with the wall.

SUM-DIFFERENCE IDENTITIES FOR SINE AND COSINE

1. Evaluate the expression.

$$\cos\left(\frac{13\pi}{12}\right)$$

2. Find $\sin 75°$.

3. Simplify the expressions.

$$\cos\left(\frac{\pi}{2}+\theta\right) \text{ and } \cos\left(\frac{\pi}{2}-\theta\right)$$

4. Find the value of $a - 2b$, if a and b are real numbers.

$$\sin(\theta-\alpha) = a\sin\theta\cos\alpha + b\cos\theta\sin\alpha$$

5. Find the exact value of the expression.

$$\cos\left(\sin^{-1}\frac{\sqrt{3}}{2} - \cos^{-1}\frac{4}{5}\right)$$

6. Find the solutions to the equation in the interval $[0,\pi)$.

$$\cos\left(\theta-\frac{\pi}{2}\right)+\sin\left(\theta-\frac{3\pi}{2}\right)=0$$

COFUNCTION IDENTITIES

1. Find an angle θ that satisfies the equation.

$$\tan\left(-\frac{3\pi}{4}\right) = \cot\theta$$

2. Find an acute angle that satisfies the equation.

$$\sin\left(2\alpha - \frac{5\pi}{6}\right) = \cos\left(4\alpha - \frac{\pi}{3}\right)$$

3. What is the value of θ?

$$\tan\left(\frac{\pi}{6} - \theta\right) = \cot\left(\frac{\pi}{6}\right)$$

4. Find the value of $\cos\theta$.

$$\sin\left(\frac{\pi}{2} - \theta\right) + \frac{1}{4}\csc\left(\frac{\pi}{2} - \theta\right) = 1$$

5. Rewrite the expression as the cosine of an angle in terms of α and β.

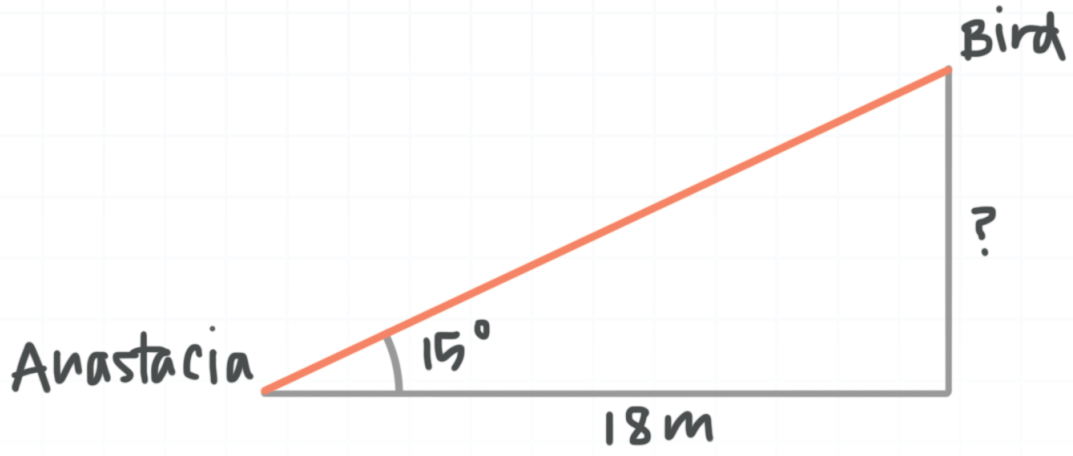
$$\sin\left(\frac{\pi}{2}-\alpha-\beta\right)$$

6. Find an angle θ that satisfies the equation.

$$\csc\left(\frac{\pi}{5}\right)=\sec\theta$$

SUM-DIFFERENCE IDENTITIES FOR TANGENT

1. Cara is watching a bird on a tree. She measured the angle of elevation of the bird as $15°$, and the distance to the tree as 18 meters. Find the exact altitude of the bird above the ground.



2. Find the exact value of $\tan 105°$.

3. Find the exact values of $\tan(\theta - \alpha)$ if θ is an angle in the first quadrant whose cosine is $3/5$ and α is an angle in the fourth quadrant whose sine is $-5/13$.

4. Simplify the expressions $\tan(\pi + \theta)$ and $\tan(\pi - \theta)$.

■ 5. Find the exact values of $\tan(\theta + \alpha)$ if θ is an angle in the second quadrant whose cosine is $-4/7$ and α is an angle in the third quadrant whose cosine is $-9/10$.

■ 6. Find the exact value of the expression.

$$\tan\left(\sin^{-1}\frac{1}{2} - \cos^{-1}\frac{1}{2}\right)$$

DOUBLE-ANGLE IDENTITIES

■ 1. If θ is an angle in the fourth quadrant whose sine is $-3/5$, what are the values of $\tan 2\theta$?

■ 2. If θ is an angle in the third quadrant whose tangent is $3/4$, what are the values of $\cos 2\theta$?

■ 3. Use a double-angle identity to rewrite the expression.

$$(\sin x + \cos x)^2$$

■ 4. If θ is an angle in the third quadrant whose sine is $-1/\sqrt{5}$, what is the value of $\sin 2\theta$?

■ 5. If θ is an angle in the third quadrant whose tangent is $7/24$, what is the value of $\tan 2\theta$?

■ 6. Use a double-angle formula to rewrite the expression.

$$12\sin(4x)\cos(4x)$$

HALF-ANGLE IDENTITIES

1. Use a half-angle identity to find the exact value of the expression.

$$\sin 15^\circ$$

2. If θ is the angle in Quadrant II with $\sin\theta = 7/25$, what are the values of $\sin(\theta/2)$ and $\cos(\theta/2)$?

3. If θ is the angle in the interval $(0,\pi/2)$ with $\tan\theta = 2$, what are the values of $\sin(\theta/2)$ and $\cos(\theta/2)$?

4. If θ is the angle in the interval $(3\pi/2,2\pi)$ with $\sin\theta = -15/17$, what are the values of $\tan(\theta/2)$ and $\cot(\theta/2)$?

5. Use a half-angle identity to find the exact value of the expression.

$$\sec\left(\frac{7\pi}{8}\right)$$

6. Prove the identity.

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

PRODUCT-TO-SUM IDENTITIES

1. Rewrite $\cos(x-y)\cos(x+y)$ as a sum.

2. Rewrite $\cos(x-15°)\sin(x+15°)$ as a sum.

3. Find a sum equivalent to $\cos^3 x$.

4. Find the exact value of each expression.

$$\left(\sin\frac{3\pi}{8}\right)\left(\cos\frac{3\pi}{8}\right)$$

$$\sin^2\left(\frac{3\pi}{8}\right)$$

$$\cos^2\left(\frac{3\pi}{8}\right)$$

5. Simplify the expression.

$$\sin(x-y)\cos y+\cos(x-y)\sin y$$

6. Find the value of the expression.

$$\sin^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{3\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right)$$

SUM-TO-PRODUCT IDENTITIES

■ 1. Rewrite the function as a product.

$$f(x) = \sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right)$$

■ 2. Find a product equal to $\sin(x + y) + \sin(x - y)$.

■ 3. Find the exact value of the expression.

$$\frac{\cos 93^\circ + \cos 27^\circ}{\cos 33^\circ}$$

■ 4. Simplify the expression.

$$\frac{\sin(7\theta) + \sin(3\theta)}{\cos(7\theta) + \cos(3\theta)}$$

■ 5. Find a product equal to $\cos(3\theta) + \cos(5\theta) - 2\cos(\theta)\cos(8\theta)$.

■ 6. Find the exact value of the expression.

$$16 \sin 390^\circ + 22 \sin 240^\circ + 16 \sin 150^\circ - 22 \sin 120^\circ$$

PROVING THE TRIG EQUATION

1. Prove the trig equation.

$$\tan\left(\frac{x}{2}\right) = \frac{1 - \cos x}{\sin x}$$

2. Prove the trigonometric equation.

$$\frac{\sin(5x) - \sin x}{\cos(5x) + \cos x} = \tan(2x)$$

3. Prove the trigonometric equation.

$$\sin(x - \pi)\sin(x + \pi) = \sin^2 x$$

4. Prove the trigonometric equation.

$$\sin(-x)\cos(-x)\tan(-x)\csc(-x) = -\sin x$$

5. Prove the trigonometric equation.

$$(\sin t + \cos t)^2 - 1 = \sin(2t)$$

6. Prove the trigonometric equation.

$$\frac{\cos(270^\circ + x)}{\sin(180^\circ - x)} = 1$$

COMPLETE SOLUTION SET OF THE EQUATION

■ 1. Find the complete solution set of the equation $\cos^2 x - 3\cos x + 2 = 0$.

■ 2. Find all the solutions of the trig equation, then list only the solutions that lie in the interval $[0,2\pi)$.

$$3\csc^2\theta - 2\cot^2\theta - 4 = 0$$

■ 3. Find the complete solution set of the equation.

$$4\cos^3\theta - 2\cos^2\theta - 2\cos\theta + 1 = 0$$

■ 4. Find all the solutions of the trig equation, then list only the solutions that lie in the interval $[0,2\pi)$.

$$\cos\theta + 1 = \sin\theta$$

■ 5. Find all the solutions of the trig equation, then list only the solutions that lie in the interval $[0,2\pi)$.

$$2(\sin^2\theta - \cos^2\theta) = \sqrt{3}$$

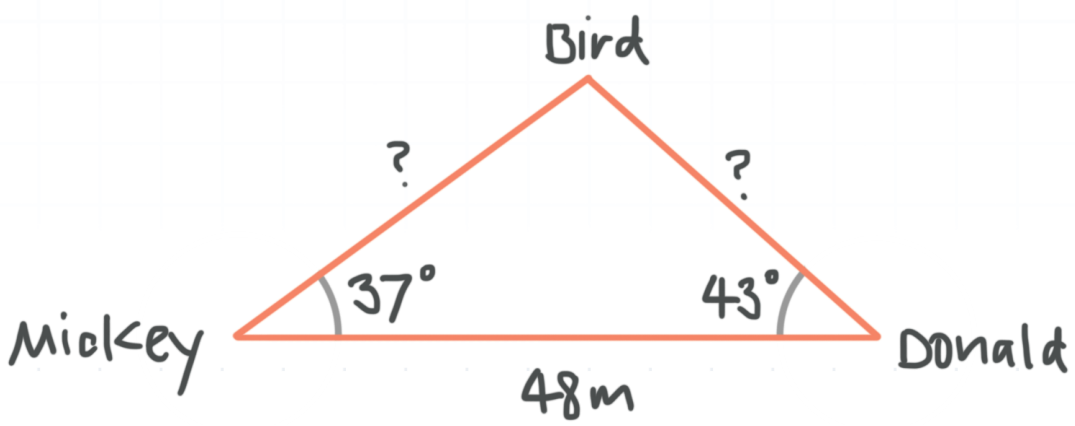
6. Find the complete solution set of the equation.

$$4\sin\left(\theta-\frac{\pi}{3}\right)\cos\left(\theta-\frac{\pi}{3}\right)=\sqrt{3}$$

LAW OF SINES

■ 1. The interior angle measures of a triangle are $97°$, $43°$, and $40°$. How many triangles can be made with these measurements?

■ 2. Mickey and Donald stand on different sides of a tree. Each of them sees the same bird in the tree. They measure the angles of elevation from themselves to the bird, and get $37°$ and $43°$ respectively. If Mickey and Donald are 48 m apart, find the distances from Mickey and Donald to the bird.



■ 3. If the measures of two interior angles of a triangle are $53°$ and $44°$, and the length of the side opposite the $44°$ angle is 7, find the length b of the side opposite the $53°$ angle and the length c of the third side.

■ 4. Solve the triangle with angle measures $A = 30°$ and $C = 90°$ and side length $c = 13$.

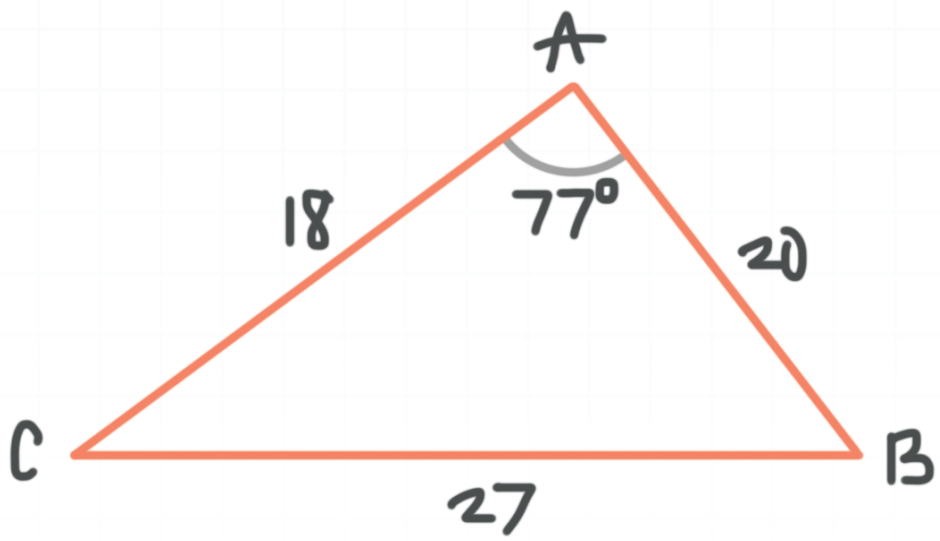
■ 5. Solve the triangle with angle measures $A = 45°$ and $B = 45°$ and side length $c = 10$.

■ 6. Find the lengths of the two unknown sides of a triangle with angle measures $A = 58°$ and $B = 42°$ and side length $a = 12$.

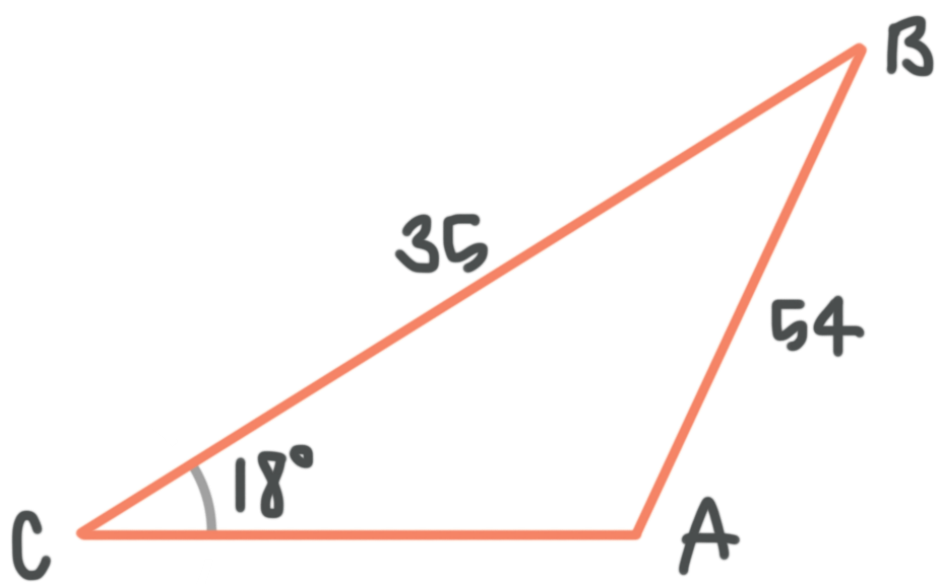
THE AMBIGUOUS CASE OF THE LAW OF SINES

■ 1. A triangle has one side with length 15 and another with length 28. The angle opposite the side with length 15 is $128°$. Complete the triangle.

■ 2. Find $\angle B$.



■ 3. Find $\angle A$.



4. If the lengths of two sides of a triangle are 18 and 34, and the measure of the interior angle opposite the side of length 34 is $B = 127°$, find the length of the third side and the measures of angles A and C, where A is opposite the side of length 18.

5. A triangle has side lengths $a = 27$ and $c = 15$ and interior angle $A = 55°$. Find all possible measures of the angle C to the nearest degree.

6. How many triangles are possible with side lengths 5 and 24, where the angle opposite the side with length 24 is $95°$?

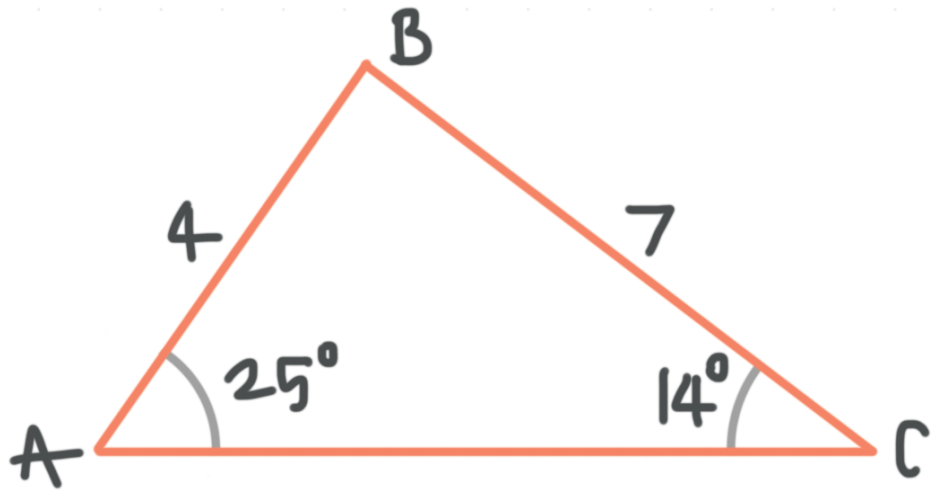
AREA FROM THE LAW OF SINES

■ 1. Find the area of the triangle in which two of the sides have lengths 15 and 24 and the measure of the included angle is $47°$.

■ 2. Find the area of the triangle with interior angles $101°$ and $25°$, if the included side has length 23.

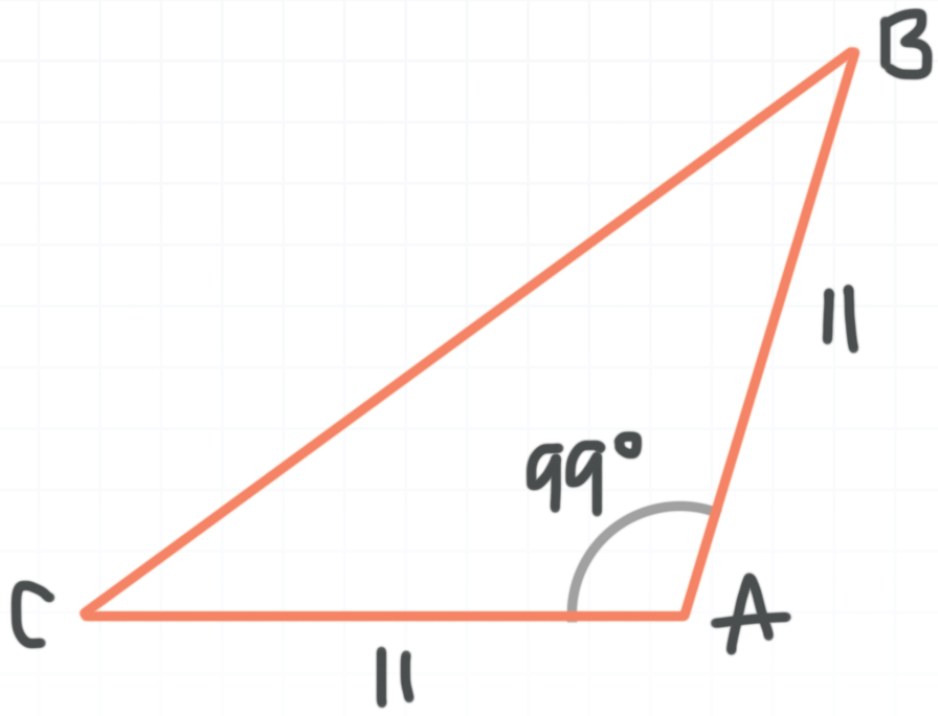
■ 3. Find the area of the triangle in which two of the sides have lengths 36 and 17 and the measure of the included angle is $90°$.

■ 4. Find the area of the triangle.



■ 5. Find the area of the triangle with interior angles $90°$ and $35°$, if the included side has length 7.

6. Find the area of a triangle.

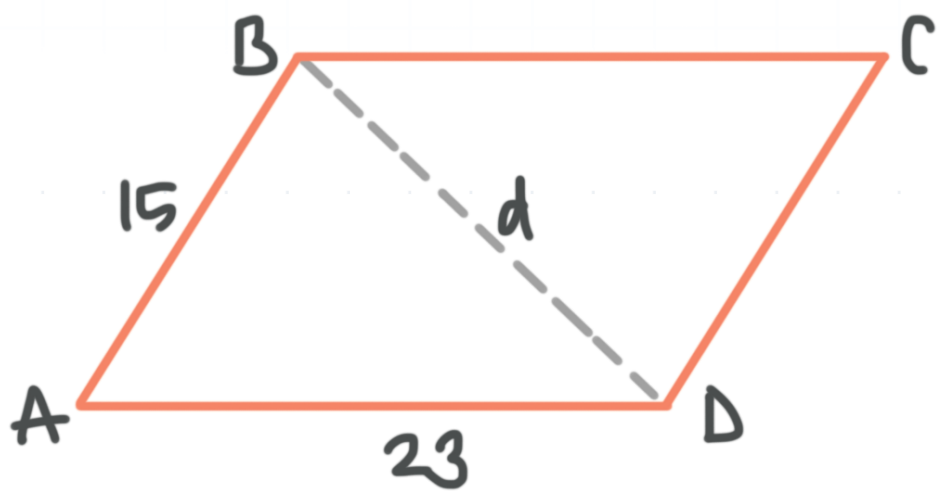


LAW OF COSINES

■ 1. Solve the triangle where two of the sides are 18 and 13 and the measure of their included angle is $121°$.

■ 2. If the side lengths of a triangle are $a = 15$, $b = 9$, and $c = 21$, what are the measures of its three interior angles?

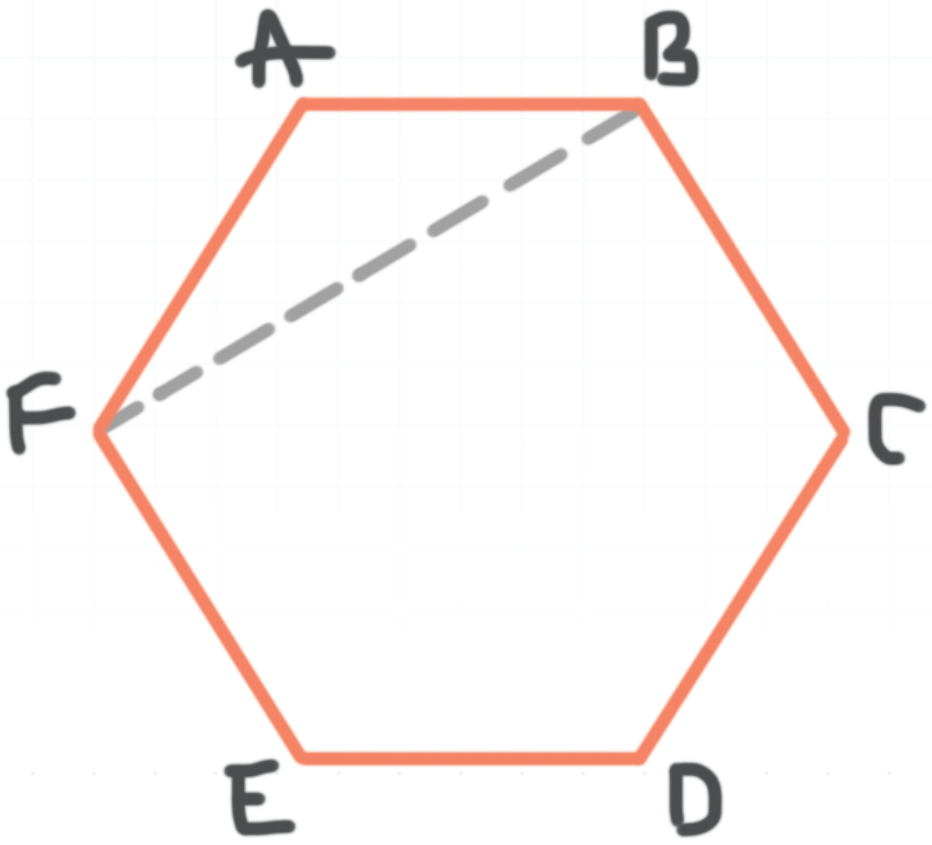
■ 3. If the measure of the angle B is $56°$, find the length of the parallelogram's diagonal, d, to the nearest centimeter. Hint: Consecutive angles of a parallelogram are supplementary, so $m\angle A + m\angle B = 180°$.



■ 4. Solve the triangle where two of the sides are 27 and 14 and the measure of their included angle is $33°$.

■ 5. If the side lengths of a triangle are $a = 17$, $b = 25$, and $c = 28$, what are the measures of its three interior angles?

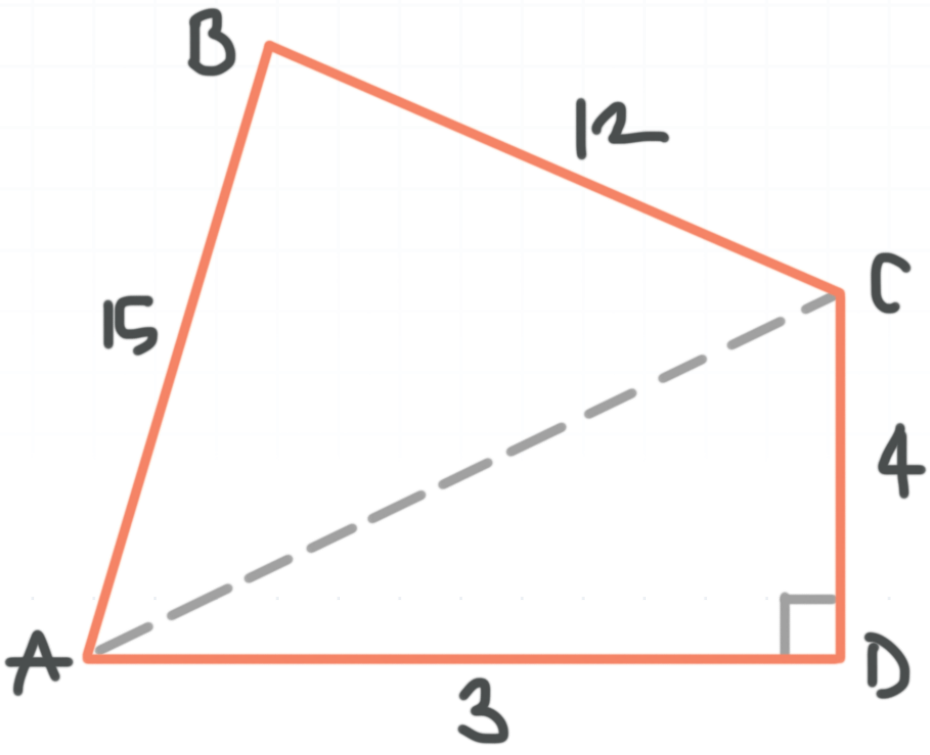
■ 6. A regular hexagon (all side lengths are equal, and all interior angles are equal) has side lengths of 20 inches. Find $\overline{FB}$ to the nearest tenth. Hint: The sum of the interior angles of a hexagon is $720°$.



HERON'S FORMULA

■ 1. The lengths of the sides of a triangle have a ratio of $5 : 8 : 12$, and the triangle's perimeter is 200 cm. Find the area of the triangle.

■ 2. Find the area of the quadrilateral, given that it's made of two separate triangles.

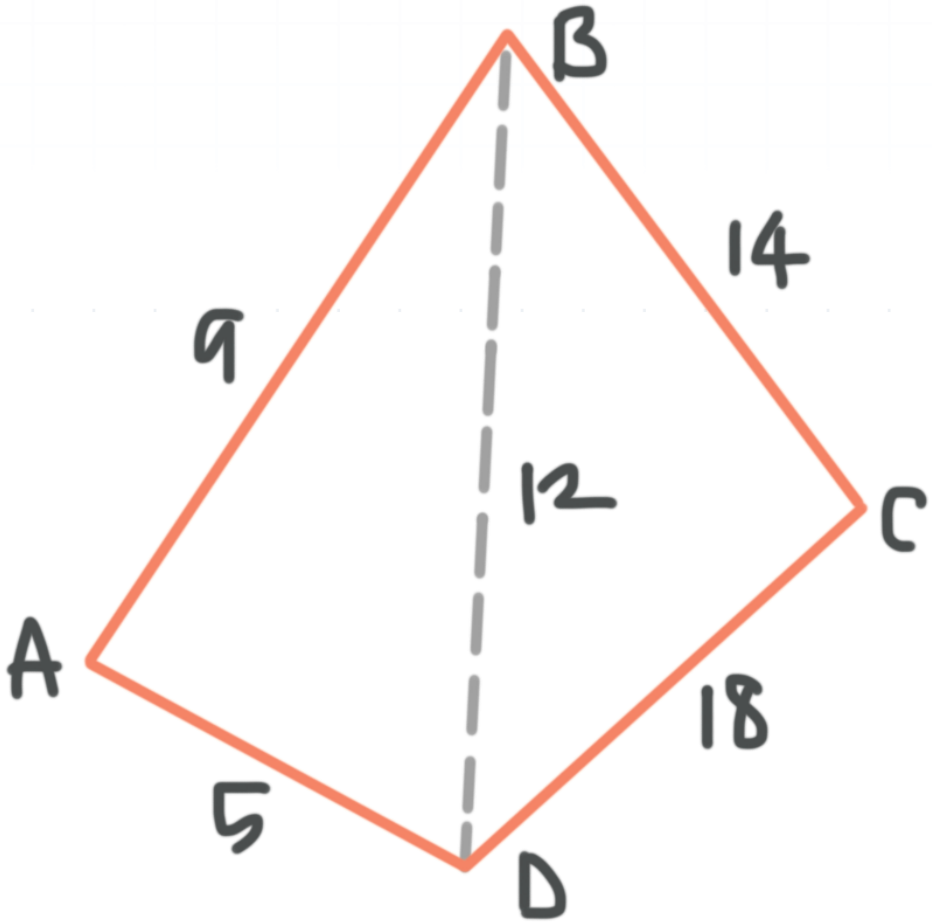


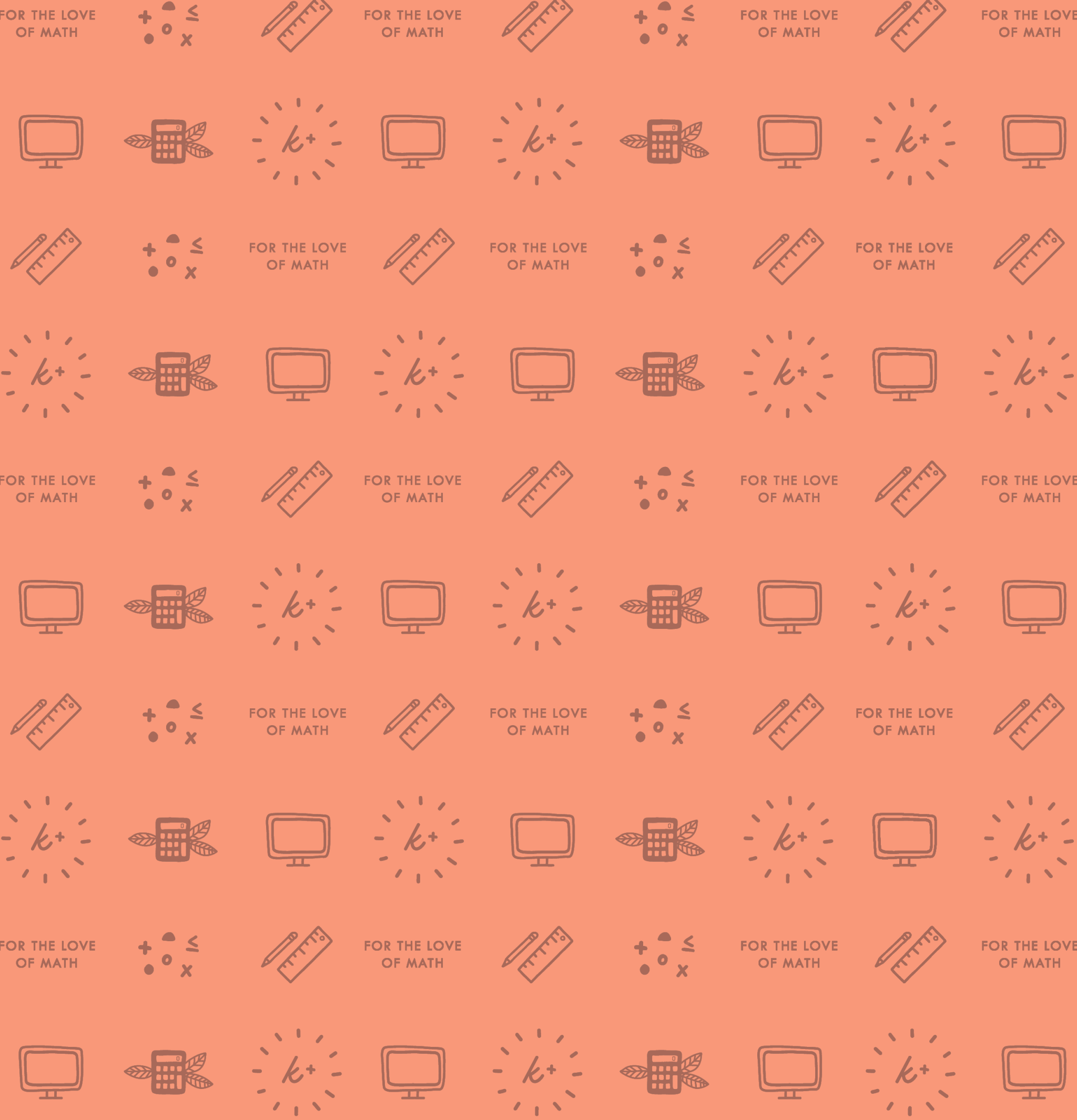
■ 3. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 12 cm, 14 cm, and 16 cm, and the parallelogram has a base of 14 cm, find the height of the parallelogram. Hint: The area of a parallelogram is $A = bh$, where b is its base and h is its height.

■ 4. Find the area of a triangle with side lengths 34 cm and 29 cm, if half its perimeter is 62 cm.

■ 5. An isosceles triangle (a triangle with two equal side lengths) has a half perimeter of 48 in. Its two equal sides measure 27 in each. Find the area of the triangle.

■ 6. Find the area of the quadrilateral by finding the sum of the areas of the triangles.





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