Trigonometry Quizzes

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MATH

Topic: Naming angles

Question: Which kind of angle is shown below?

Answer choices:

A Zero angle

B Obtuse angle

C Right angle

D Straight angle

Solution: B

The angle is greater than a quarter circle but less than a half circle, so it’s an obtuse angle.

Topic: Naming angles

Question: What is the name of an angle with measure less than 90 degrees?

Answer choices:

A Acute angle

B Right angle

C Obtuse angle

D Straight angle

Solution: A

Any angle that's less than a quarter circle, or $0^\circ < \theta < 90^\circ$, is called an "acute" angle.

Topic: Naming angles

Question: Which angle represents an obtuse angle?

Answer choices:

A $\frac{\pi}{2}$

B 45°

C $\frac{2\pi}{3}$

D 180°

Solution: C

Any angle that's greater than a quarter circle but less than a half circle, $90^\circ < \theta < 180^\circ$ in degrees or $\pi/2 < \theta < \pi$ in radians, is an obtuse angle.

Because the angle $\theta = 2\pi/3$ is larger than $\pi/2$ but less than π,

$$\frac{\pi}{2} < \frac{2\pi}{3} < \pi$$

it's an obtuse angle, measured in radians.

Topic: Complementary and supplementary angles

Question: Find the angle θ that's supplementary to 126°.

Answer choices:

A $\theta = 154^\circ$

B $\theta = 36^\circ$

C $\theta = 54^\circ$

D $\theta = 180^\circ$

Solution: C

Since θ is supplementary to an angle of 126° we have

$$\theta + 126^\circ = 180^\circ$$

$$\theta = 180^\circ - 126^\circ$$

$$\theta = 54^\circ$$

Topic: Complementary and supplementary angles

Question: Find the complement θ of $\pi/12$.

Answer choices:

A $\theta = \frac{5\pi}{12}$

B $\theta = \frac{\pi}{2}$

C $\theta = \frac{5\pi}{6}$

D $\theta = \frac{\pi}{3}$

Solution: A

The angle θ and an angle of $\pi/12$ are complementary, so

$$\theta + \frac{\pi}{12} = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \frac{\pi}{12}$$

Find a common denominator.

$$\theta = \frac{\pi}{2}\left(\frac{6}{6}\right) - \frac{\pi}{12}$$

$$\theta = \frac{6\pi}{12} - \frac{\pi}{12}$$

$$\theta = \frac{5\pi}{12}$$

Topic: Complementary and supplementary angles

Question: Find the angle θ that's $1/3$ as large as the supplement of 87°.

Answer choices:

A $\theta = 1^\circ$

B $\theta = 31^\circ$

C $\theta = 37^\circ$

D $\theta = 13^\circ$

Solution: B

Let α be the angle that's supplementary to 87°.

$$\alpha + 87^\circ = 180^\circ$$

$$\alpha = 180^\circ - 87^\circ$$

$$\alpha = 93^\circ$$

To find the angle θ that's $1/3$ as large, we'll divide α by 3.

$$\theta = \frac{\alpha}{3}$$

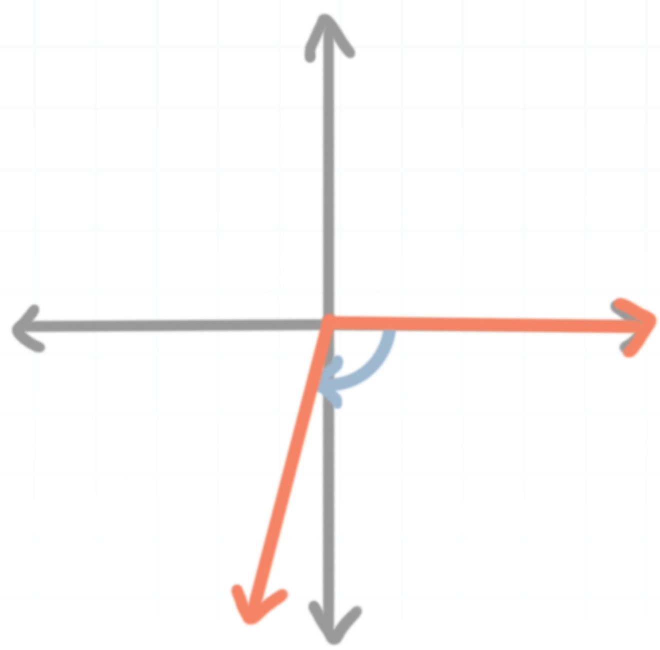
$$\theta = \frac{93^\circ}{3}$$

$$\theta = 31^\circ$$

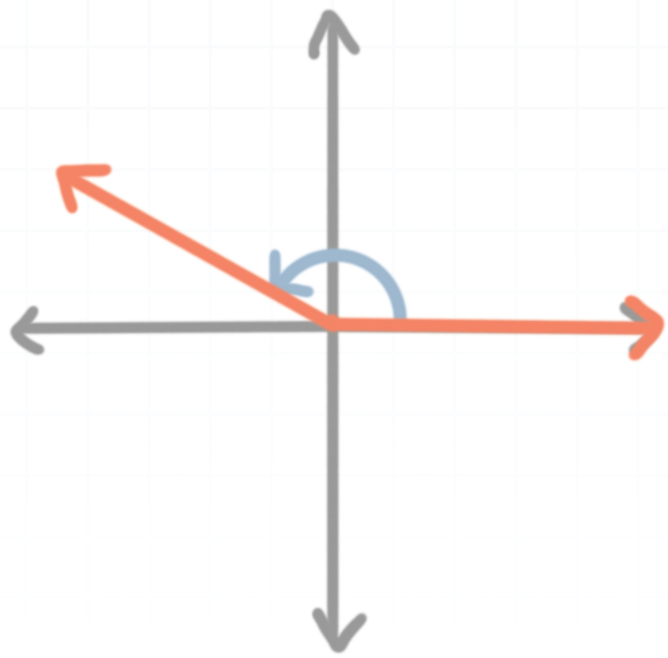
Topic: Positive and negative angles

Question: Which choice could be a sketch of -210° in standard position?

Answer choices:



A



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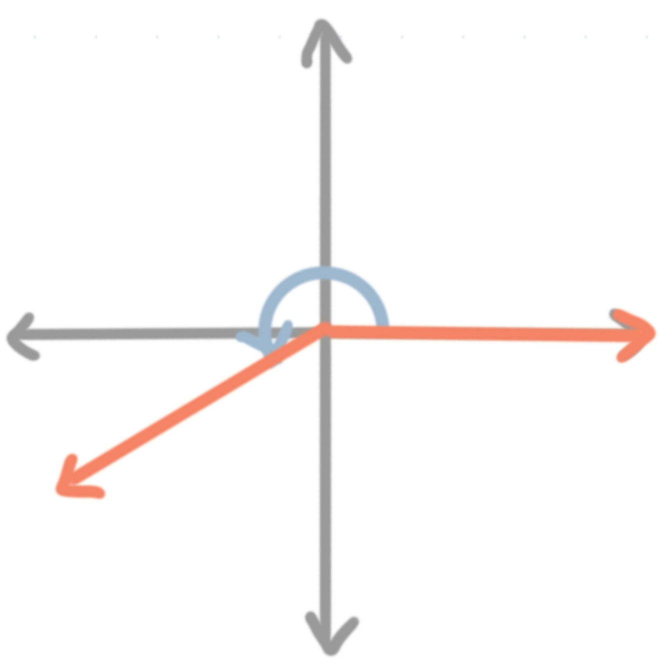
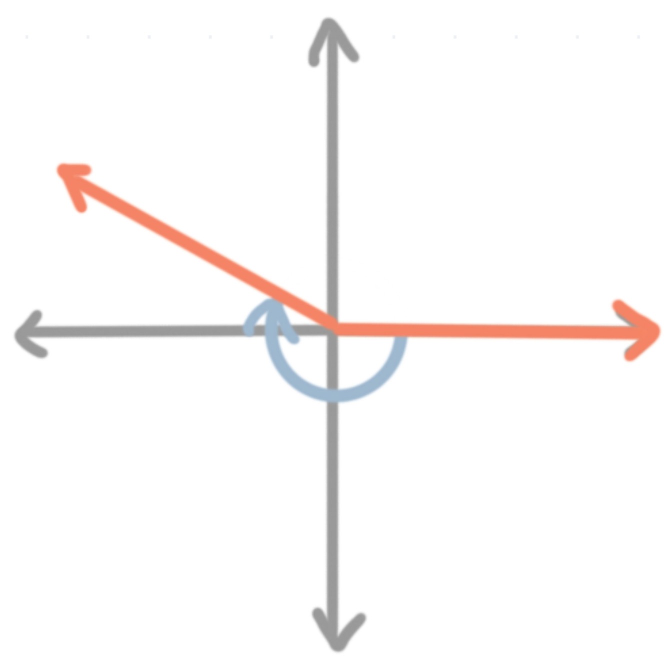


C



D

Solution: D

An angle of -210° is negative, so its terminal side is reached from its initial side by making a rotation of 210° in the negative (clockwise) direction about the origin. Based on only the direction of rotation, A and D are the only possible correct choices.

Rotating in the negative direction, we know that -90° gets us from the starting point on the positive x-axis to the negative y-axis, and -180° gets us to the negative x-axis. An angle of -210° has us rotating even further than -180°, which means D must be the correct answer choice. Answer choice A looks like an approximately -100° angle, since its rotation is just past the negative y-axis.

Topic: Positive and negative angles

Question: Where is the initial side of an angle located, if its sketched in standard position?

Answer choices:

A Along the positive side of the x-axis

B Along the negative side of the x-axis

C Along the positive side of the y-axis

D Along the negative side of the y-axis

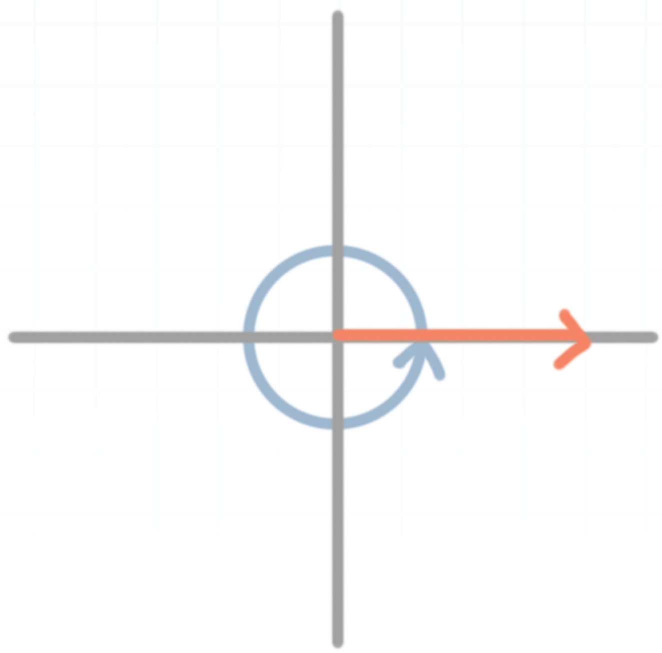
Solution: A

The “initial side” is the side where the angle begins. In standard position, this side is always sketched along the positive side of the x-axis.

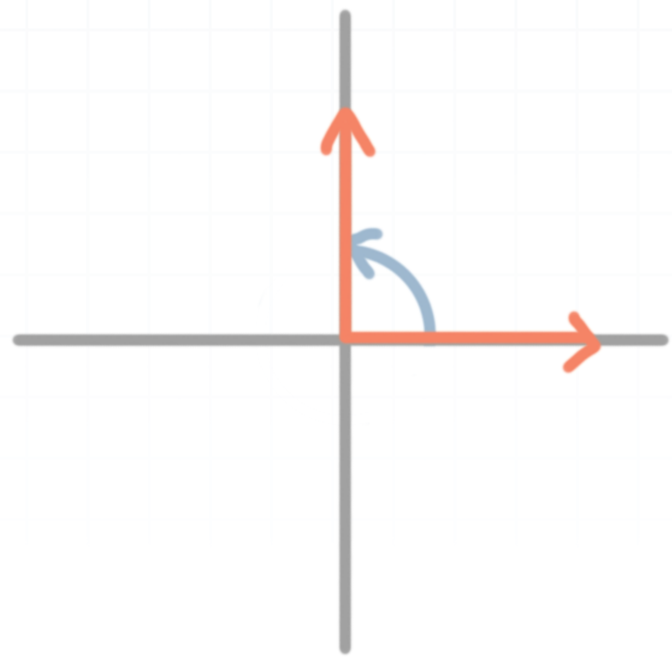
Topic: Positive and negative angles

Question: Which of the following could be a sketch of an 810° angle in standard position?

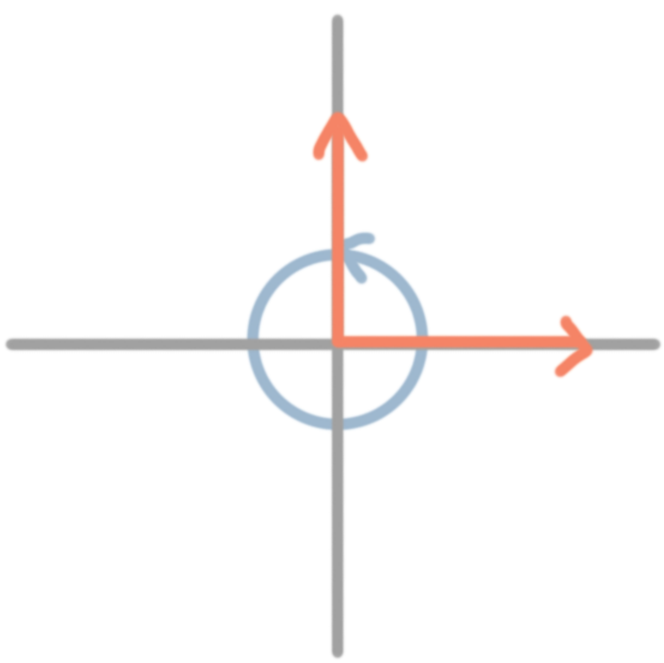
Answer choices:



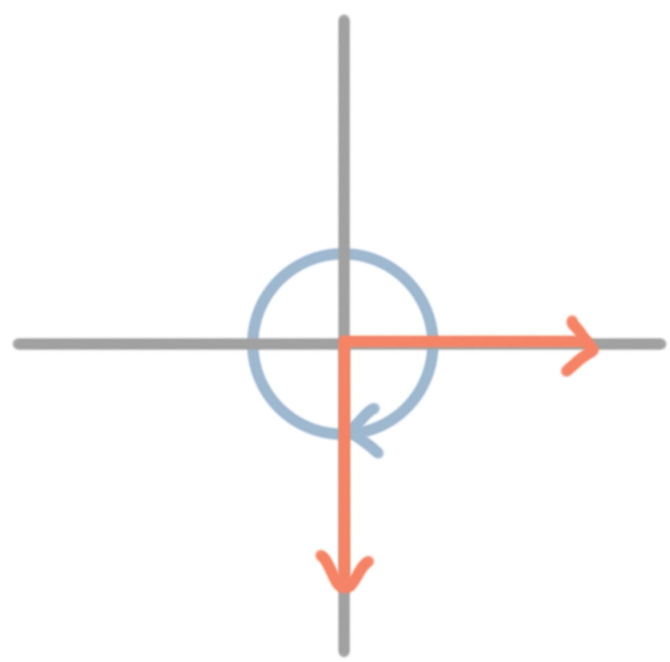
A



B



C



D

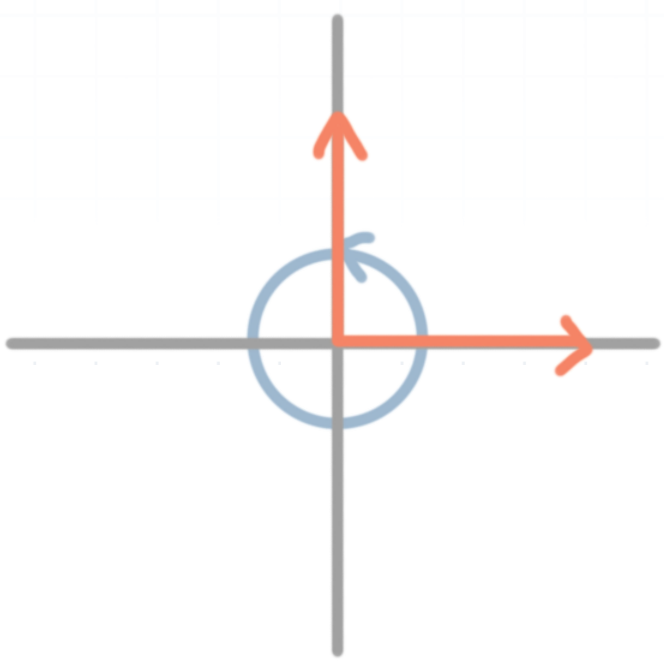
Solution: C

Since $720^\circ < 810^\circ$, the angle 810° is more than two full rotations. We'll find out how much more by finding the difference between the angles.

$$810^\circ - 720^\circ = 90^\circ$$

So to sketch the angle, we'll put the initial side of the angle along the positive side of the x-axis. Then we'll rotate counterclockwise, into the first quadrant, rotating two full rotations, all the way around the circle, and then an additional 90°.

Because 90° falls along the positive side of the y-axis, the terminal side of 810° should also fall along the positive side of the y-axis.



Topic: Quadrant of the angle

Question: Which two axes make up the boundary of the third quadrant?

Answer choices:

A The positive horizontal axis and the negative vertical axis

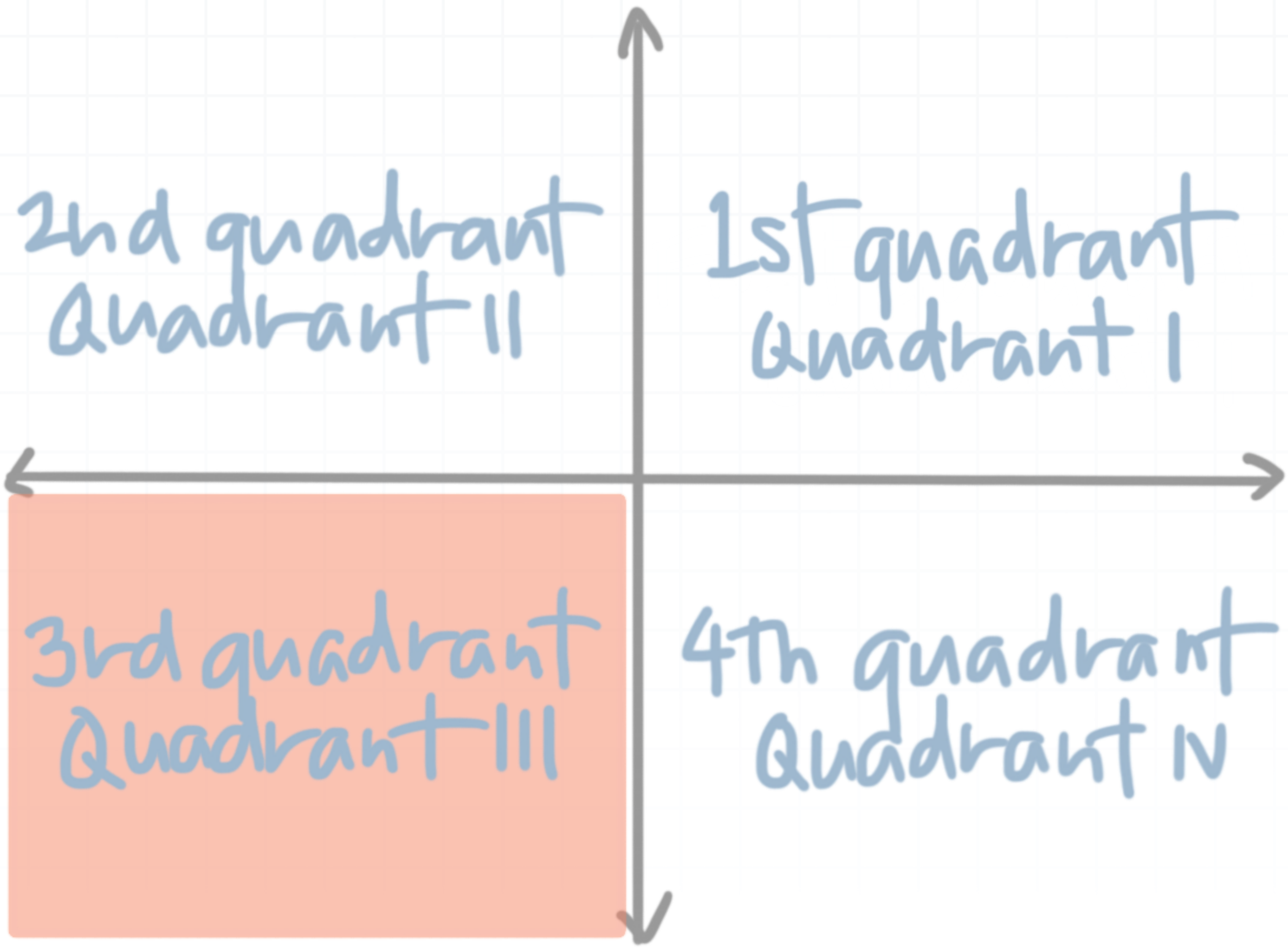
B The negative horizontal axis and the negative vertical axis

C The positive horizontal axis and the positive vertical axis

D The negative horizontal axis and the positive vertical axis

Solution: B

The third quadrant is bounded by the negative horizontal axis and the negative vertical axis.



Topic: Quadrant of the angle

Question: Where is an angle of 10.5π radians in standard position located?

Answer choices:

A On the positive vertical axis

B On the negative vertical axis

C In the second quadrant

D In the fourth quadrant

Solution: A

One full rotation is 2π radians, so we know 10.5π radians is more than one full rotation. To figure out how many rotations are made by 10.5π, divide 10.5π by 2π.

$$\frac{10.5\pi}{2\pi}$$

$$5.25$$

So 10.5π is five and a quarter rotations in the positive direction. If we start along the positive x-axis in standard position and make 5 full rotations, we'll end up right back in the same place, on the positive x-axis.

One more quarter rotation will then put us along the positive y-axis.

Topic: Quadrant of the angle

Question: In which quadrant is the angle located?

$$-1{,}600^\circ$$

Answer choices:

A First quadrant

B Second quadrant

C Third quadrant

D Fourth quadrant

Solution: C

One full rotation is -360°, so we know $-1{,}600^\circ$ is more than one full rotation. To figure out how many rotations are made by $-1{,}600^\circ$, divide $-1{,}600^\circ$ by -360°.

$$\frac{-1{,}600^\circ}{-360^\circ}$$

$$4.44$$

So $-1{,}600^\circ$ is almost four and a half rotations in the negative direction. If we start along the positive x-axis in standard position and make 4 full rotations in the negative direction, we'll end up right back in the same place, on the positive x-axis.

Four full rotations is

$$4(-360^\circ)$$

$$-1{,}440^\circ$$

and once we've made a $-1{,}440^\circ$ rotation, to get to $-1{,}600^\circ$, we'll need an additional -160° of rotation. From standard position, we know -90° puts us along the negative vertical axis, and then -180° puts us along the negative horizontal axis. So a rotation of -160° will put us in the third quadrant, just short of the negative horizontal axis.

Topic: Degrees, radians, and DMS

Question: Convert 220° to radians.

Answer choices:

A $\dfrac{9\pi}{11}$

B $\dfrac{5\pi}{4}$

C $\dfrac{11\pi}{9}$

D $\dfrac{5\pi}{6}$

Solution: C

Since there are π radians in 180°, we'll convert 220° to radians by multiplying by $\pi/180^\circ$.

$$220^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$\frac{220\pi}{180}$$

$$\frac{11\pi}{9}$$

Topic: Degrees, radians, and DMS

Question: Convert 38.27° to DMS.

Answer choices:

A $38^\circ 0' 0.27''$

B $38^\circ 16' 12''$

C $38^\circ 0.27' 0''$

D $38^\circ 12' 16''$

Solution: B

When we convert the angle 38.27° in degrees to DMS, the integer part, 38, will be the degree part of the DMS angle. So the only thing we really need to do is convert the 0.27° to minutes and seconds.

If we think about 0.27° as $(27/100)^\circ$, then we can convert it from degrees to minutes.

$$\left(\frac{27}{100}\right)^\circ \left(\frac{60'}{1^\circ}\right)$$

$$\left(\frac{27(60)}{100}\right)'$$

$$\left(\frac{1{,}620}{100}\right)'$$

$$16.2'$$

So the minutes portion of the DMS angle will be $16'$. Now we'll convert the $0.2'$ into seconds.

$$0.2'\left(\frac{60''}{1'}\right)$$

$$12''$$

So the angle 38.27° in degrees converts to the DMS angle $38^\circ 16' 12''$.

Topic: Degrees, radians, and DMS

Question: Convert $55^\circ 36' 18''$ to degrees.

Answer choices:

A $\quad 55.9^\circ$

B $\quad 55.54^\circ$

C $\quad 55.5^\circ$

D $\quad 55.605^\circ$

Solution: D

The 55° part of $55^\circ 36' 18''$ is already in degrees, so we only need to convert the minutes and seconds portions of the DMS angle.

First, we convert seconds to minutes, then minutes to degrees, so we'll convert the seconds part first. We need to convert $18''$ from seconds to minutes. We know that $1' = 60''$, so we'll multiply $18''$ by $1'/60''$ in order to cancel the seconds and be left with just minutes.

$$18''\left(\frac{1'}{60''}\right)$$

$$\left(\frac{18}{60}\right)'$$

$$0.3'$$

Then the total minutes in $55^\circ 36' 18''$ is

$$(36 + 0.3)'$$

$$36.3'$$

To convert this value for minutes into degrees, we'll multiply by $1^\circ/60'$ in order to cancel the minutes and be left with an approximate value for degrees.

$$36.3'\left(\frac{1^\circ}{60'}\right)$$

$$\left(\frac{36.3}{60}\right)^\circ$$

$$0.605^\circ$$

Putting this together with the 55° from the original angle, we get approximately

$$(55 + 0.605)^\circ$$

$$55.605^\circ$$

Topic: Coterminal angles

Question: Which angle is coterminal with -150°?

Answer choices:

A 120°

B -420°

C 570°

D 230°

Solution: C

We'll check each of the answer choices. We're looking for the angle that differs from -150° by an integer multiple of 360°.

There's only one answer choice that's a negative angle, so we'll check the negative angle first. If we subtract 360° from -150°, we get

$$-150^\circ - 360^\circ$$

$$-510^\circ$$

So -420° isn't coterminal with -150°.

Now let's check the three positive angle choices just by adding 360° a few times to -150°.

$$-150^\circ + 1(360^\circ) = 210^\circ$$

$$-150^\circ + 2(360^\circ) = 570^\circ$$

$$-150^\circ + 3(360^\circ) = 930^\circ$$

The only angle that matches up is 570°.

Topic: Coterminal angles

Question: Which angle is not coterminal with $-8\pi/3$?

Answer choices:

A $\frac{4\pi}{3}$

B $-\frac{4\pi}{3}$

C $-\frac{26\pi}{3}$

D $\frac{22\pi}{3}$

Solution: B

To find angles coterminal with $-8\pi/3$, we'll add or subtract multiples of 2π from the angle. Because the angle $-8\pi/3$ is given in thirds (the denominator is 3), it'll be easier to express 2π in thirds, so we'll rewrite 2π as

$$2\pi\left(\frac{3}{3}\right)$$

$$\frac{6\pi}{3}$$

So let's start by subtracting multiples of $6\pi/3$ from $-8\pi/3$ to find negative coterminal angles.

$$-\frac{8\pi}{3}-1\left(\frac{6\pi}{3}\right)=-\frac{14\pi}{3}$$

$$-\frac{8\pi}{3}-2\left(\frac{6\pi}{3}\right)=-\frac{20\pi}{3}$$

$$-\frac{8\pi}{3}-3\left(\frac{6\pi}{3}\right)=-\frac{26\pi}{3}$$

No we'll add multiples of $6\pi/3$ to $-8\pi/3$ to find more coterminal angles.

$$-\frac{8\pi}{3}+1\left(\frac{6\pi}{3}\right)=-\frac{2\pi}{3}$$

$$-\frac{8\pi}{3}+2\left(\frac{6\pi}{3}\right)=\frac{4\pi}{3}$$

$$-\frac{8\pi}{3}+3\left(\frac{6\pi}{3}\right)=\frac{10\pi}{3}$$

$$-\frac{8\pi}{3}+4\left(\frac{6\pi}{3}\right)=\frac{16\pi}{3}$$

$$-\frac{8\pi}{3}+5\left(\frac{6\pi}{3}\right)=\frac{22\pi}{3}$$

We've seen that each of the answer choices is coterminal with $-8\pi/3$, except $-4\pi/3$.

Topic: Coterminal angles

Question: Which angle α is coterminal with $71\pi/16$ if we find α by starting at $71\pi/16$ and making two full negative rotations?

Answer choices:

A $\alpha = \frac{7\pi}{16}$

B $\alpha = -\frac{7\pi}{16}$

C $\alpha = \frac{39\pi}{16}$

D $\alpha = -\frac{25\pi}{16}$

Solution: A

One full negative rotation is given by -2π, so two full negative rotations is given by -4π. Therefore, if we start with $71\pi/16$ and make two full negative rotations, we'll get an angle of

$$\alpha = \frac{71\pi}{16} - 4\pi$$

$$\alpha = \frac{71\pi}{16} - 4\pi\left(\frac{16}{16}\right)$$

$$\alpha = \frac{71\pi}{16} - \frac{64\pi}{16}$$

$$\alpha = \frac{71\pi - 64\pi}{16}$$

$$\alpha = \frac{7\pi}{16}$$

Topic: Sine, cosine, and tangent

Question: In the right triangle ABC, $\cos\angle C = 30/50$. What is the length of the hypotenuse of triangle ABC?

Answer choices:

A 30

B 50

C $\frac{1}{50}$

D 40

Solution: B

The cosine of an angle θ is equivalent to the length of the side adjacent to θ, divided by the length of the hypotenuse.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Therefore, the length of the hypotenuse is 50.

Topic: Sine, cosine, and tangent

Question: The ratio of the length of the side of a triangle opposite an angle, to the length of the longest side of the triangle (the hypotenuse) is equal to which value?

Answer choices:

A Tangent of the angle

B Sine of the angle

C Cosine of the angle

D Square of the other side

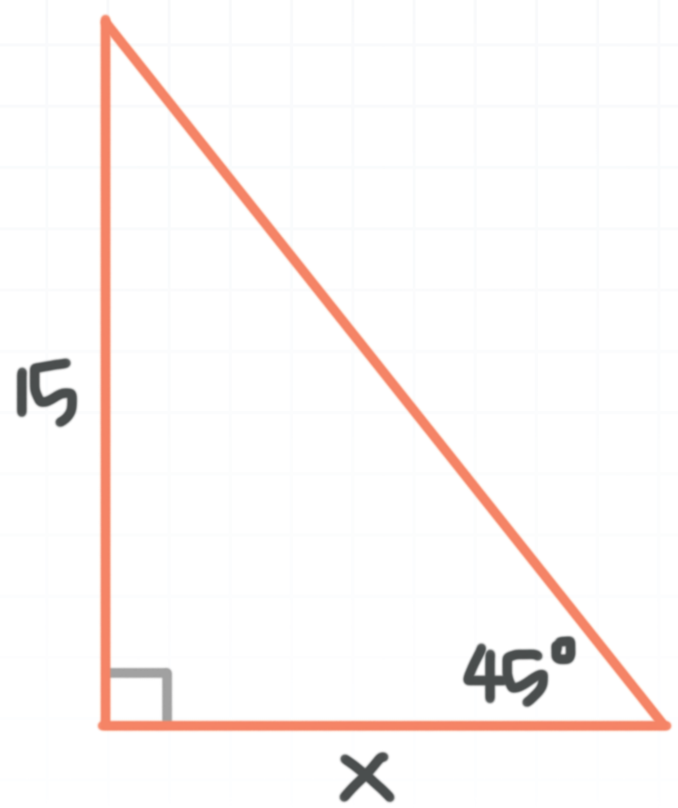
Solution: B

The sine of an angle θ is equivalent to the length of the side opposite the angle, divided by the length of the hypotenuse.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Topic: Sine, cosine, and tangent

Question: Which equation would be used to solve for x? Note: The figure may not be drawn to scale.



Answer choices:

A $\tan 45^\circ = \frac{15}{x}$

B $\sin 45^\circ = \frac{15}{x}$

C $\tan 45^\circ = \frac{x}{15}$

D $\cos 45^\circ = \frac{x}{15}$

Solution: A

Given the position of the angle $\theta = 45^\circ$ in the right triangle, the length of the opposite side is 15 and the length of the adjacent side is x.

Tangent of an angle gives the relationship between the side opposite to the angle and the side adjacent to the angle.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 45^\circ = \frac{15}{x}$$

Topic: Cosecant, secant, cotangent, and the reciprocal identities

Question: The cosecant of an angle θ is equal to which of the following?

Answer choices:

A $\quad \dfrac{1}{\sin\theta}$

B $\quad \dfrac{1}{\cos\theta}$

C $\quad \dfrac{1}{\tan\theta}$

D $\quad \dfrac{1}{\sec\theta}$

Solution: A

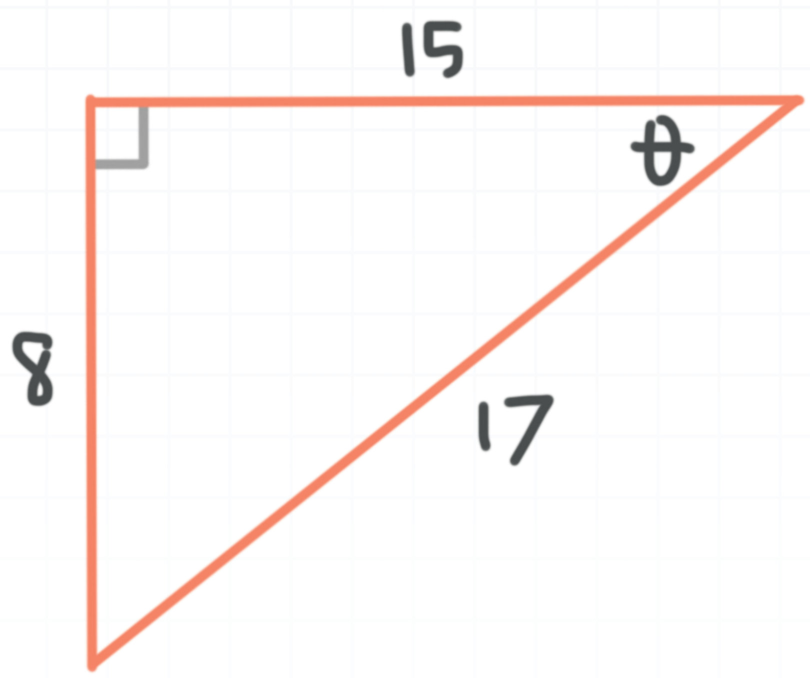
Sine is the reciprocal of cosecant, and vice versa.

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

Topic: Cosecant, secant, cotangent, and the reciprocal identities

Question: Find the cotangent of θ.



Answer choices:

A $\frac{15}{17}$

B $\frac{8}{15}$

C $\frac{1}{8}$

D $\frac{15}{8}$

Solution: D

Given the position of the angle θ in the right triangle, the length of the opposite side is 8, the length of the adjacent side is 15, and the length of the hypotenuse is 17.

Then the cotangent of the angle is

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

$$\cot\theta = \frac{15}{8}$$

Topic: Cosecant, secant, cotangent, and the reciprocal identities

Question: Which ratio defines the secant function?

Answer choices:

A $\dfrac{\text{hypotenuse}}{\text{opposite}}$

B $\dfrac{\text{hypotenuse}}{\text{adjacent}}$

C $\dfrac{\text{adjacent}}{\text{opposite}}$

D $\dfrac{\text{opposite}}{\text{adjacent}}$

Solution: B

Cosine is the reciprocal of secant, and vice versa. Remember that the reciprocal of a fraction is what we get when we flip the fraction upside down. So the reciprocal of a/b is b/a. Therefore,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

Topic: The quotient identities

Question: Which of the following is equivalent to the ratio?

$$\frac{\cos\theta}{\sin\theta}$$

Answer choices:

A $\sec\theta$

B $\tan\theta$

C $\cot\theta$

D $\csc\theta$

Solution: C

The quotient identity for cotangent tells us that

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Topic: The quotient identities

Question: Which expression is equivalent to the given fraction?

$$\frac{1}{\tan\theta}$$

Answer choices:

A $\quad \dfrac{\sin\theta}{\cos\theta}$

B $\quad \dfrac{1}{\cos\theta}$

C $\quad \dfrac{1}{\sin\theta}$

D $\quad \dfrac{\cos\theta}{\sin\theta}$

Solution: D

Using the reciprocal identities, we remember that

$$\cot\theta = \frac{1}{\tan\theta}$$

And from the quotient identity for cotangent, we get

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Topic: The quotient identities

Question: If $\sin\theta = 8/17$ and $\cos\theta = 15/17$, what is the value of $\tan\theta$?

Answer choices:

A $\frac{8}{15}$

B $\frac{17}{15}$

C $\frac{15}{8}$

D $\frac{17}{8}$

Solution: A

We can find tangent of θ just by plugging these sine and cosine values into the quotient identity for tangent.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\frac{8}{17}}{\frac{15}{17}}$$

$$\tan\theta = \frac{8}{17} \cdot \frac{17}{15}$$

$$\tan\theta = \frac{8}{15}$$

Topic: The Pythagorean identities

Question: If $\cos(236^\circ) = -0.559$, find the value of $\sin(236^\circ)$.

Answer choices:

A $\sin(236^\circ) \approx 0.688$

B $\sin(236^\circ) \approx -0.688$

C $\sin(236^\circ) \approx 0.829$

D $\sin(236^\circ) \approx -0.829$

Solution: D

We'll use a rewritten form of the Pythagorean identity with sine and cosine.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2(236^\circ) = 1 - \cos^2(236^\circ)$$

$$\sin^2(236^\circ) = 1 - (-0.559)^2$$

$$\sin^2(236^\circ) \approx 1 - 0.312$$

$$\sin^2(236^\circ) \approx 0.688$$

$$\sin(236^\circ) \approx \pm\sqrt{0.688}$$

The angle 236° lies in the third quadrant, and all points in the third quadrant have a negative y-value, which means that the sine of any angle in the third quadrant will be negative. So we ignore the positive value and say

$$\sin(236^\circ) \approx -\sqrt{0.688}$$

$$\sin(236^\circ) \approx -0.829$$

Topic: The Pythagorean identities

Question: Which equation is not a Pythagorean identity?

Answer choices:

A $\sec^2\theta - \tan^2\theta = 1$

B $a^2 + b^2 = c^2$

C $\cot^2\theta + 1 = \csc^2\theta$

D $\sin^2\theta + \cos^2\theta = 1$

Solution: B

We know that the three Pythagorean identities are

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

We can rewrite the third identity as

$$\sec^2\theta - \tan^2\theta = 1$$

The equation $a^2 + b^2 = c^2$ is the Pythagorean Theorem, which tells us that, for any right triangle, the sum of the squares of the side lengths is equal to the square of the length of the hypotenuse. But the Pythagorean Theorem is not specifically one of the three Pythagorean identities.

Topic: The Pythagorean identities

Question: Find the value of the expression.

$$\frac{1}{\cos^2(25^\circ)} - \tan^2(25^\circ)$$

Answer choices:

A -1

B 0

C 1

D $\frac{\sqrt{2}}{2}$

Solution: C

Rewrite the expression using the reciprocal identity for secant.

$$\frac{1}{\cos^2(25^\circ)} - \tan^2(25^\circ)$$

$$\sec^2(25^\circ) - \tan^2(25^\circ)$$

Then the Pythagorean identity $1 + \tan^2\theta = \sec^2\theta$, which can be rewritten as $\sec^2\theta - \tan^2\theta = 1$, tells us that the value of the expression is 1.

Topic: Signs by quadrant

Question: If θ is an angle in the second quadrant and $\csc\theta = \sqrt{7}$, what is the value of $\sec\theta$?

Answer choices:

A $\sec\theta = -\frac{\sqrt{42}}{7}$

B $\sec\theta = -\frac{6}{7}$

C $\sec\theta = -\frac{7}{6}$

D $\sec\theta = -\frac{\sqrt{42}}{6}$

Solution: D

Use the reciprocal identity to find the value of $\sin\theta$.

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\sin\theta = \frac{1}{\sqrt{7}}$$

Then we can use the value of sine in the Pythagorean identity to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - \left(\frac{1}{\sqrt{7}}\right)^2$$

$$\cos^2\theta = 1 - \frac{1}{7}$$

$$\cos^2\theta = \frac{6}{7}$$

$$\cos\theta = \pm\sqrt{\frac{6}{7}}$$

The cosine of any angle in the second quadrant is negative, so

$$\cos\theta = -\sqrt{\frac{6}{7}}$$

Then we'll use the reciprocal identity to find $\sec\theta$.

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec\theta = \frac{1}{-\sqrt{\frac{6}{7}}}$$

$$\sec\theta = -\sqrt{\frac{7}{6}}$$

$$\sec\theta = -\frac{\sqrt{7}}{\sqrt{6}}$$

Rationalize the denominator by multiplying both the numerator and denominator by $\sqrt{6}$.

$$\sec\theta = -\frac{\sqrt{7}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$\sec\theta = -\frac{\sqrt{42}}{6}$$

Topic: Signs by quadrant

Question: If $\cos(11^\circ) \approx 0.981627$, what's the approximate value of $\cot(11^\circ)$?

Answer choices:

A $\cot(11^\circ) \approx 4.64$

B $\cot(11^\circ) \approx 4.98$

C $\cot(11^\circ) \approx 5.14$

D $\cot(11^\circ) \approx 5.26$

Solution: C

Start with a rewritten form of the Pythagorean identity with sine and cosine.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sqrt{\sin^2\theta} = \pm\sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$$

Plug $\cos(11^\circ) = 0.98$ into the equation we've created.

$$\sin\theta \approx \pm\sqrt{1 - 0.981627^2}$$

$$\sin\theta \approx \pm\sqrt{1 - 0.963592}$$

$$\sin\theta \approx \pm\sqrt{0.036408}$$

$$\sin\theta \approx \pm 0.190809$$

An angle of 11° is in the first quadrant. Since the sine of any angle in the first quadrant is positive, we get

$$\sin\theta \approx 0.190809$$

Plug this value for $\sin\theta$ into the quotient identity for $\cot\theta$.

$$\cot\theta \approx \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta \approx \frac{0.981627}{0.190809} \approx 5.14$$

Topic: Signs by quadrant

Question: What is the value of $\tan\theta$ if $\cos\theta = -0.218$ and θ is an angle in the third quadrant?

Answer choices:

A $\tan\theta \approx -5.46$

B $\tan\theta \approx \pm 4.48$

C $\tan\theta \approx 4.48$

D $\tan\theta \approx \pm 5.46$

Solution: C

We need to find any possible values of $\sin\theta$ using the Pythagorean identity with sine and cosine.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - (-0.218)^2$$

$$\sin^2\theta \approx 1 - 0.0475$$

$$\sin^2\theta = 0.953$$

$$\sin\theta = \pm\sqrt{0.953}$$

$$\sin\theta = \pm 0.976$$

Since θ is an angle in the third quadrant, $\sin\theta$ is negative and $\sin\theta \approx -0.976$.

If we plug this value of sine into the quotient identity for tangent, along with $\cos\theta = -0.218$, we get:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \approx \frac{-0.976}{-0.218} \approx 4.48$$

Topic: When the trig functions are undefined

Question: For what angle(s) is $\sec\theta$ undefined in the interval?

$$\left(-\frac{53\pi}{6},-\frac{25\pi}{3}\right)$$

Answer choices:

A $\theta=-9\pi$

B $\theta=-\dfrac{17\pi}{2}$

C $\theta=-\dfrac{15\pi}{2}$

D $\theta=-8\pi$

Solution: B

Let's start by multiplying $\theta = -25\pi/3$ by $2/2$, so that we have both the angles in the interval in terms of sixths. This will make the interval a little easier to envision.

$$\left(-\frac{53\pi}{6}, -\frac{25\pi}{3}\right)$$

$$\left(-\frac{53\pi}{6}, -\frac{25\pi}{3}\left(\frac{2}{2}\right)\right)$$

$$\left(-\frac{53\pi}{6}, -\frac{50\pi}{6}\right)$$

In sixths, we know that one full rotation in the negative direction is equivalent to $-2\pi = -12\pi/6$. Because $4(-12\pi/6) = -48\pi/6$ is four full rotations, and $-53\pi/6$ and $-50\pi/6$ are both a little more than four full negative rotations, let's add back four full rotations to this original interval.

$$\left(-\frac{53\pi}{6} + \frac{48\pi}{6}, -\frac{50\pi}{6} + \frac{48\pi}{6}\right)$$

$$\left(-\frac{5\pi}{6}, -\frac{2\pi}{6}\right)$$

$$\left(-\frac{5\pi}{6}, -\frac{\pi}{3}\right)$$

We know that $\theta = -5\pi/6$ is an angle slightly less than 180° in the negative direction (in the third quadrant). The angle $\theta = -\pi/3$ is an angle less than 90° in the negative direction (in the fourth quadrant). Therefore, we'll be starting in the third quadrant at $\theta = -5\pi/6$ and rotating in the positive direction until we arrive at $\theta = -\pi/3$ in the fourth quadrant.

We know by the reciprocal identity

$$\sec\theta = \frac{1}{\cos\theta}$$

that secant is undefined when $\cos\theta = 0$, and we know that $\cos\theta = 0$ along the vertical axis. As we rotate from $\theta = -5\pi/6$ to $\theta = -\pi/3$, we'll cross the vertical axis exactly once at $\theta = -\pi/2$.

But remember that we shifted the original interval by $\theta = 48\pi/6$, so we have to pull this back out to find the angle coterminal with $\theta = -\pi/2$ that falls within the original interval $(-53\pi/6, -25\pi/3)$.

When we do, we can say that $\sec\theta$ is undefined at

$$\theta = -\frac{\pi}{2} - \frac{48\pi}{6}$$

$$\theta = -\frac{\pi}{2} - \frac{16\pi}{2}$$

$$\theta = -\frac{17\pi}{2}$$

within the interval $(-53\pi/6, -25\pi/3)$.

Topic: When the trig functions are undefined

Question: Say whether $\tan(29\pi/6)$ is defined.

Answer choices:

A $\tan\left(\frac{29\pi}{6}\right)$ is defined

B $\tan\left(\frac{29\pi}{6}\right)$ is undefined

C We can't determine whether or not $\tan\left(\frac{29\pi}{6}\right)$ is defined

D $\tan\left(\frac{5\pi}{6}\right)$ is undefined

Solution: A

From the quotient identity for tangent,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

we know that $\tan\theta$ is undefined when $\cos\theta = 0$. We know $\cos\theta = 0$ anywhere along the vertical axis. So we need to check to see whether or not $29\pi/6$ lies along the vertical axis.

If we know one full rotation is $2\pi = 12\pi/6$, then we can say $29\pi/6$ is two full rotations ($24\pi/6$) and then an additional $5\pi/6$ rotations. The angle $5\pi/6$ doesn't fall on the vertical axis, which means $\tan(29\pi/6)$ will be defined.

Topic: When the trig functions are undefined

Question: At which angle is the cotangent function undefined?

Answer choices:

A $\theta = 3\pi$

B $\theta = \dfrac{5\pi}{3}$

C $\theta = \dfrac{7\pi}{2}$

D $\theta = \dfrac{9\pi}{4}$

Solution: A

We know from the quotient identity for cotangent,

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

cotangent of an angle θ is undefined when $\sin\theta = 0$. We know that the sine function, which represents the y-coordinate, is 0 at angles along the horizontal axis. Angles along the horizontal axis include the full set of angles coterminal with 0,

$$\theta = 0, \ \pm 2\pi, \ \pm 4\pi, \ \pm 6\pi, \ \ldots$$

and the full set of angles coterminal with π.

$$\theta = \pm\pi, \ \pm 3\pi, \ \pm 5\pi, \ \pm 7\pi, \ \ldots$$

Of the answer choices, the only angle contained in either of these sets is the angle $\theta = 3\pi$.

Topic: The unit circle

Question: What is the coordinate point associated with $\theta = 30^\circ$ along the unit circle?

Answer choices:

A $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

B $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

C $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

D $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Solution: A

Looking at the unit circle shows that the coordinate point associated with $\theta = 30^\circ$ in the first quadrant is

$$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Topic: The unit circle

Question: At which angles are sine and cosine equivalent?

Answer choices:

A $\frac{\pi}{4}$

B $\frac{\pi}{4}, \frac{3\pi}{4}$

C $\frac{\pi}{4}, \frac{5\pi}{4}$

D $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Solution: C

The sign of cosine and sine is the same in the first and third quadrants, so we need to find the angles in these quadrants. From the unit circle, we see that sine and cosine are the same at $\theta = \pi/4$ and $\theta = 5\pi/4$.

Topic: The unit circle

Question: Use the unit circle to find the value of tangent of the angle $\theta = 5\pi/3$.

Answer choices:

A -1

B $\frac{\sqrt{3}}{3}$

C $\sqrt{3}$

D $-\sqrt{3}$

Solution: D

Looking at the unit circle, we know that sine of $\theta = 5\pi/3$ is the y-value of the coordinate point at that angle, and that cosine of $\theta = 5\pi/3$ is the x-value of the coordinate point at that angle.

$$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$$

Use the quotient identity for tangent to find the value of tangent.

$$\tan\left(\frac{5\pi}{3}\right) = \frac{\sin\left(\frac{5\pi}{3}\right)}{\cos\left(\frac{5\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2}\left(\frac{2}{1}\right) = -\sqrt{3}$$

Topic: Negative angles and angles more than one rotation

Question: Which angle in $[0^\circ, 360^\circ)$ is coterminal with -116°?

Answer choices:

A 116°

B 326°

C 360°

D 244°

Solution: D

The angle $\theta = -116^\circ$ is close enough to the interval $[0^\circ, 360^\circ)$ that we'll just add 360° to $\theta = -116^\circ$ to find the coterminal angle.

$$-116^\circ + 360^\circ$$

$$244^\circ$$

Topic: Negative angles and angles more than one rotation

Question: Which angle in the interval $[0,2\pi)$ is coterminal with $-9\pi/4$?

Answer choices:

A $\frac{9\pi}{4}$

B $-\frac{7\pi}{2}$

C $\frac{7\pi}{4}$

D $\frac{5\pi}{4}$

Solution: C

To find the number of full rotations included in $\theta = -9\pi/4$, we'll divide the angle by 2π.

$$\frac{-\frac{9\pi}{4}}{2\pi}$$

$$-\frac{9\pi}{4} \cdot \frac{1}{2\pi}$$

$$-\frac{9\pi}{8\pi}$$

$$-1.125$$

So $\theta = -9\pi/4$ is 1 full rotation in the negative direction, and then an additional 0.125 of one more rotation in the negative direction. So to find a coterminal angle, we'll get rid of the 1 full rotation by adding $1(2\pi)$ to the angle.

$$-\frac{9\pi}{4} + 1(2\pi)$$

$$-\frac{9\pi}{4} + 2\pi$$

$$-\frac{9\pi}{4} + \frac{8\pi}{4}$$

$$-\frac{\pi}{4}$$

Now we have an angle that's less than one full rotation, but we'd still like to find a positive coterminal angle that's less than one full rotation. So we'll add 2π one more time.

$$-\frac{\pi}{4}+2\pi$$

$$-\frac{\pi}{4}+\frac{8\pi}{4}$$

$$\frac{7\pi}{4}$$

Therefore, we can say that $7\pi/4$ is coterminal with $\theta=-9\pi/4$ in the interval $[0,2\pi)$.

Topic: Negative angles and angles more than one rotation

Question: Find the value of sine of the angle $\theta = -71\pi/4$.

Answer choices:

A 1

B $\frac{\sqrt{2}}{2}$

C $-\frac{1}{2}$

D $-\frac{\sqrt{2}}{2}$

Solution: B

First we need to find the angle in the interval $[0,2\pi)$ that's coterminal with $\theta = -71\pi/4$.

To find the number of full rotations included in $\theta = -71\pi/4$, we'll divide the angle by 2π.

$$\frac{-\frac{71\pi}{4}}{2\pi}$$

$$-\frac{71\pi}{4} \cdot \frac{1}{2\pi}$$

$$-\frac{71\pi}{8\pi}$$

$$-8.875$$

So $\theta = -71\pi/4$ is 8 full rotations in the negative direction, and then an additional 0.875 of one more rotation in the negative direction. So to find a coterminal angle, we'll get rid of the 8 full rotations by adding $8(2\pi)$ to the angle.

$$-\frac{71\pi}{4} + 8(2\pi)$$

$$-\frac{71\pi}{4} + 16\pi$$

$$-\frac{71\pi}{4} + \frac{64\pi}{4}$$

$$-\frac{7\pi}{4}$$

Now we have an angle that's less than one full rotation, but we'd still like to find a positive coterminal angle that's less than one full rotation. So we'll add 2π one more time.

$$-\frac{7\pi}{4}+2\pi$$

$$-\frac{7\pi}{4}+\frac{8\pi}{4}$$

$$\frac{\pi}{4}$$

Therefore, we can say that $\pi/4$ is coterminal with $\theta=-71\pi/4$, and therefore that

$$\sin\left(-\frac{71\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

Topic: Coterminal angles in a particular interval

Question: Which angle in the interval $(-3\pi/5, 7\pi/5]$ is coterminal with $67\pi/5$?

Answer choices:

A $-\frac{2\pi}{5}$

B $-\frac{3\pi}{5}$

C $\frac{7\pi}{5}$

D $-\frac{\pi}{5}$

Solution: C

We'll let $\theta = 67\pi/5$ and α be the angle within $(-3\pi/5, 7\pi/5]$ that's coterminal with θ. We'll use $\alpha = \theta + n(2\pi)$ and solve for the value of n that makes α lie in that interval.

$$-3\pi/5 < \alpha \le 7\pi/5$$

$$-3\pi/5 < \theta + n(2\pi) \le 7\pi/5$$

$$-\frac{3\pi}{5} < \frac{67\pi}{5} + n(2\pi) \le \frac{7\pi}{5}$$

$$-\frac{70\pi}{5} < n(2\pi) \le -\frac{60\pi}{5}$$

$$-14\pi < n(2\pi) \le -12\pi$$

$$-7 < n \le -6$$

Because n has to be an integer, we know $n = -6$. To find α, we'll substitute $n = -6$ into $\theta + n(2\pi)$.

$$\alpha = \frac{67\pi}{5} + (-6)(2\pi)$$

$$\alpha = \frac{67\pi}{5} - \frac{60\pi}{5}$$

$$\alpha = \frac{7\pi}{5}$$

Topic: Coterminal angles in a particular interval

Question: Which angle in the interval $(380^\circ, 740^\circ]$ is coterminal with 145°?

Answer choices:

A 380°

B -215°

C 740°

D 505°

Solution: D

The interval $(380^\circ, 740^\circ]$ is a full 360° rotation. Notice how, because we have a parenthesis around the 380° and a bracket around the 740°, it means that the angle 380° exactly isn't included in the interval, but the angle 740° exactly *is* included.

We'll let $\theta = 145^\circ$, and then we'll say that α is the coterminal angle that lies within $(380^\circ, 740^\circ]$. Then we can say

$$380^\circ < \alpha \leq 740^\circ$$

But since α is coterminal with θ, we substitute $\alpha = \theta + n(360^\circ)$ into the inequality.

$$380^\circ < \theta + n(360^\circ) \leq 740^\circ$$

$$380^\circ < 145^\circ + n(360^\circ) \leq 740^\circ$$

$$235^\circ < n(360^\circ) \leq 595^\circ$$

$$0.65 < n \leq 1.65$$

Remember, n has to be an integer, which means $n = 1$. And therefore, to find α, we'll substitute $n = 1$ into $\alpha = \theta + n(360^\circ)$.

$$\alpha = 145^\circ + 1(360^\circ)$$

$$\alpha = 145^\circ + 360^\circ$$

$$\alpha = 505^\circ$$

Topic: Coterminal angles in a particular interval

Question: Which angle in the interval $[25\pi/4, 33\pi/4)$ is coterminal with $-33\pi/4$?

Answer choices:

A $\frac{31\pi}{4}$

B $\frac{25\pi}{4}$

C $\frac{23\pi}{4}$

D $\frac{33\pi}{4}$

Solution: A

We'll let $\theta = -33\pi/4$ and α be the angle within $[25\pi/4, 33\pi/4)$ that's coterminal with θ. We'll use $\alpha = \theta + n(2\pi)$ and solve for the value of n that makes α lie in that interval.

$$\frac{25\pi}{4} \le \alpha < \frac{33\pi}{4}$$

$$\frac{25\pi}{4} \le \theta + n(2\pi) < \frac{33\pi}{4}$$

$$\frac{25\pi}{4} \le -\frac{33\pi}{4} + n(2\pi) < \frac{33\pi}{4}$$

$$\frac{29\pi}{2} \le n(2\pi) < \frac{33\pi}{2}$$

$$7.25 \le n < 8.25$$

Because n has to be an integer, we know $n = 8$. To find α, we'll substitute $n = 8$ into $\theta + n(2\pi)$.

$$\alpha = \frac{-33\pi}{4} + 8(2\pi)$$

$$\alpha = -\frac{33\pi}{4} + \frac{64\pi}{4}$$

$$\alpha = \frac{31\pi}{4}$$

Topic: Reference angles

Question: What is the reference angle of $37\pi/5$?

Answer choices:

A $\frac{\pi}{5}$

B $\frac{2\pi}{5}$

C $\frac{3\pi}{5}$

D $\frac{4\pi}{5}$

Solution: B

The angle $\theta = 37\pi/5$ is three full rotations of 2π in the positive direction, and then an extra $7\pi/5$ in the positive direction, which means the angle is coterminal with $\theta = 7\pi/5$.

The angle $\theta = 7\pi/5$ is in the third quadrant, so the reference angle β is

$$\beta = \theta - \pi$$

$$\beta = \frac{7\pi}{5} - \pi$$

$$\beta = \frac{7\pi}{5} - \frac{5\pi}{5}$$

$$\beta = \frac{2\pi}{5}$$

Topic: Reference angles

Question: Which angle is a reference angle for $1{,}180^\circ$?

Answer choices:

A 10°

B 80°

C 100°

D 260°

Solution: B

The angle $\theta = 1{,}180^\circ$ is three full rotations of 360°, plus an extra 100° in the positive direction, which means the angle is coterminal with $\theta = 100^\circ$.

So the angle $\theta = 1{,}180^\circ$ is coterminal with $\theta = 100^\circ$. Now that we have a positive coterminal angle, we can find the reference angle.

Since $\theta = 100^\circ$ is in the second quadrant, the reference angle β is

$$\beta = 180^\circ - \theta$$

$$\beta = 180^\circ - 100^\circ$$

$$\beta = 80^\circ$$

Topic: Reference angles

Question: What is the reference angle of $-\pi/4$?

Answer choices:

A $\quad -\dfrac{\pi}{4}$

B $\quad \dfrac{5\pi}{4}$

C $\quad \dfrac{9\pi}{4}$

D $\quad \dfrac{\pi}{4}$

Solution: D

We want to convert this to a positive angle, which we can do by adding the negative angle to 2π.

$$\theta = -\frac{\pi}{4} + 2\pi$$

$$\theta = -\frac{\pi}{4} + \frac{8\pi}{4}$$

$$\theta = \frac{7\pi}{4}$$

So the angle $\theta = -\pi/4$ is coterminal with $\theta = 7\pi/4$. Now that we have a positive coterminal angle, we can find the reference angle.

Since $\theta = 7\pi/4$ is in the fourth quadrant, the reference angle β is

$$\beta = 2\pi - \theta$$

$$\beta = 2\pi - \frac{7\pi}{4}$$

$$\beta = \frac{8\pi}{4} - \frac{7\pi}{4}$$

$$\beta = \frac{\pi}{4}$$

Topic: Symmetry across axes

Question: Use the unit circle to find the angle that has the same sine as the angle $\theta = 123^\circ$.

Answer choices:

A -123°

B -57°

C 57°

D 33°

Solution: C

For any angle θ, the value of $\sin\theta$ is equal to the y-coordinate of the point at which the terminal side of θ intersects the unit circle.

Since $\theta = 123^\circ$ lies in the second quadrant, the sine of the angle will be positive. The only other quadrant in which the sine is positive is the first quadrant. Because $\theta = 123^\circ$ is $123^\circ - 90^\circ = 33^\circ$ past the positive y-axis, we need an angle that's 33° short of the positive y-axis, which is the angle

$$90^\circ - 33^\circ = 57^\circ$$

Topic: Symmetry across axes

Question: If θ is an angle such that $\sin\theta = 0.439$, what are two possible values of $\cos(\theta + 540^\circ)$?

Answer choices:

A $\cos(\theta + 540^\circ) = \pm 0.193$

B $\cos(\theta + 540^\circ) = \pm 0.807$

C $\cos(\theta + 540^\circ) = \pm 0.327$

D $\cos(\theta + 540^\circ) = \pm 0.898$

Solution: D

Use the Pythagorean identity with sine and cosine, and the given value $\sin\theta = 0.439$, to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$(0.439)^2 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - (0.439)^2$$

$$\cos^2\theta \approx 1 - 0.193$$

$$\cos^2\theta \approx 0.807$$

$$\cos\theta \approx \pm\sqrt{0.807}$$

$$\cos\theta \approx \pm 0.898$$

Since $\sin\theta$ is positive, θ is in either the first or second quadrant. In the first quadrant we'll have $\cos\theta \approx 0.898$, and in the second quadrant we'll have $\cos\theta \approx -0.898$.

We've been asked about the angle $\theta + 540^\circ$. If we realize that $540^\circ = 360^\circ + 180^\circ$, we realize that a 360° rotation puts us right back at the same angle θ, but that the additional 180° flips us across the y-axis and then across the x-axis, which means the signs on both x and y will flip.

Which means that the angle θ in the second quadrant associated with $(-0.898, 0.439)$ will become the angle $\theta + 540^\circ$ associated with $(0.898, -0.439)$. And the angle θ in the first quadrant associated with $(0.898, 0.439)$ will become the angle $\theta + 540^\circ$ associated with $(-0.898, -0.439)$.

Therefore, for an angle $\theta + 540^\circ$, the possible values of cosine of the angle are the x-values from $(0.898, -0.439)$ and $(-0.898, -0.439)$.

$$\cos(\theta + 540^\circ) = \pm 0.898$$

Topic: Symmetry across axes

Question: If θ is an angle in the second quadrant such that $\cos\theta = -0.713$, what is the value of $\sin(\theta + 5\pi)$?

Answer choices:

A $\sin(\theta + 5\pi) = 0.508$

B $\sin(\theta + 5\pi) = -0.713$

C $\sin(\theta + 5\pi) = -0.701$

D $\sin(\theta + 5\pi) = 0.693$

Solution: C

Use the Pythagorean identity with sine and cosine, and the given value $\cos\theta = -0.713$, to find $\sin\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + (-0.713)^2 = 1$$

$$\sin^2\theta = 1 - (-0.713)^2$$

$$\sin^2\theta \approx 1 - 0.508$$

$$\sin^2\theta \approx 0.492$$

$$\sin\theta \approx \pm\sqrt{0.492}$$

Since θ is in the second quadrant, sine of the angle must be positive.

$$\sin\theta \approx \sqrt{0.492}$$

$$\sin\theta \approx 0.701$$

We've been asked about the angle $\theta + 5\pi$. If we realize that $5\pi = 4\pi + \pi$, we realize that a 4π rotation puts us right back at the same angle θ, but that the additional π flips us across the y-axis and then across the x-axis, which means the signs on both x and y will flip.

Which means that the angle θ in the second quadrant associated with $(-0.714,0.701)$ will become the angle $\theta + 5\pi$ associated with $(0.714, -0.701)$.

Therefore, for an angle $\theta + 5\pi$, the value of sine of the angle is the y-value from $(0.714, -0.701)$.

$$\sin(\theta + 5\pi) = -0.701$$

Topic: Even-odd identities

Question: Use even-odd identities to find the values of $\sin(-855^\circ)$ and $\cos(-855^\circ)$.

Answer choices:

A $\quad \sin(-855^\circ) = \frac{\sqrt{3}}{2} \qquad \cos(-855^\circ) = -\frac{1}{2}$

B $\quad \sin(-855^\circ) = -\frac{\sqrt{2}}{2} \qquad \cos(-855^\circ) = \frac{\sqrt{2}}{2}$

C $\quad \sin(-855^\circ) = \frac{1}{2} \qquad \cos(-855^\circ) = -\frac{\sqrt{3}}{2}$

D $\quad \sin(-855^\circ) = -\frac{\sqrt{2}}{2} \qquad \cos(-855^\circ) = -\frac{\sqrt{2}}{2}$

Solution: D

Let's find an angle within $[0^\circ, 360^\circ)$ that's coterminal with 855°. We're using 855° instead of -855° because using the even-odd identities tell us that

$$\sin(-855^\circ) = -\sin(855^\circ)$$

$$\cos(-855^\circ) = \cos(855^\circ)$$

and we'll deal with the negative sign using even-odd identities later.

$$855^\circ - 360^\circ = 495^\circ$$

$$495^\circ - 360^\circ = 135^\circ$$

From the unit circle we know the sine and cosine of 135°.

$$\sin(135^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(135^\circ) = -\frac{\sqrt{2}}{2}$$

Because 855° is coterminal with 135°, these two angles will have equal sine and cosine values, so

$$\sin(855^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(855^\circ) = -\frac{\sqrt{2}}{2}$$

Substituting the values we already found for $\sin(855^\circ)$ and $\cos(855^\circ)$ into the right sides of the even-odd equations, we get

$$\sin(-855^\circ) = -\frac{\sqrt{2}}{2}$$

$$\cos(-855^\circ) = -\frac{\sqrt{2}}{2}$$

Topic: Even-odd identities

Question: Use reference angles and even-odd identities to find the values of tangent and secant at $\theta = -5\pi/6$.

Answer choices:

A $\tan\left(-\frac{5\pi}{6}\right) = \sqrt{3}$ $\sec\left(-\frac{5\pi}{6}\right) = \frac{2\sqrt{3}}{3}$

B $\tan\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3}$ $\sec\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

C $\tan\left(-\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{3}$ $\sec\left(-\frac{5\pi}{6}\right) = -\frac{2\sqrt{3}}{3}$

D $\tan\left(-\frac{5\pi}{6}\right) = -\sqrt{3}$ $\sec\left(-\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Solution: C

Since the even-odd identities for sine and cosine are

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

we can find the reference angle for $5\pi/6$. The angle $5\pi/6$ is in the second quadrant and has a reference angle of $\pi/6$. We could also consider the angle $\pi/6$ in the first quadrant that has the same reference angle of $\pi/6$.

Remember that sine and the cosine are positive in the first quadrant, and sine is positive while cosine is negative in the second quadrant. Therefore, the reference angle theorem tells us that

$$\cos\left(\frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{5\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

Then by the even-odd identities for sine and cosine,

$$\cos\left(-\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{5\pi}{6}\right) = -\frac{1}{2}$$

Then tangent and secant of $\theta = -5\pi/6$ must be

$$\tan\left(-\frac{5\pi}{6}\right)=\frac{\sin\left(-\frac{5\pi}{6}\right)}{\cos\left(-\frac{5\pi}{6}\right)}=\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\sec\left(-\frac{5\pi}{6}\right)=\frac{1}{\cos\left(-\frac{5\pi}{6}\right)}=\frac{1}{-\frac{\sqrt{3}}{2}}=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

Topic: Even-odd identities

Question: Find the values of cosecant and tangent at $\theta = -49\pi/3$.

Answer choices:

A $\quad \csc\left(-\frac{49\pi}{3}\right) = -\frac{2\sqrt{3}}{3} \qquad \tan\left(-\frac{49\pi}{3}\right) = -\sqrt{3}$

B $\quad \csc\left(-\frac{49\pi}{3}\right) = -\frac{\sqrt{3}}{2} \qquad \tan\left(-\frac{49\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

C $\quad \csc\left(-\frac{49\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \tan\left(-\frac{49\pi}{3}\right) = -\sqrt{3}$

D $\quad \csc\left(-\frac{49\pi}{3}\right) = \frac{2\sqrt{3}}{3} \qquad \tan\left(-\frac{49\pi}{3}\right) = \frac{\sqrt{3}}{3}$

Solution: A

Since the cosine function is even and the sine function is odd, we can say

$$\cos\left(-\frac{49\pi}{3}\right)=\cos\left(\frac{49\pi}{3}\right)$$

$$\sin\left(-\frac{49\pi}{3}\right)=-\sin\left(\frac{49\pi}{3}\right)$$

Then to find the cosine of this positive angle, we'll get the coterminal angle for $49\pi/3$ by dividing it by 2π.

$$\frac{\frac{49\pi}{3}}{2\pi}=\frac{49\pi}{3(2\pi)}=\frac{49}{6}\approx 8.167$$

So $\theta=49\pi/3$ is 8 full positive rotations, plus a little bit more. If we take away 8 full rotations, or 16π, from $\theta=49\pi/3$, we're left with

$$\frac{49\pi}{3}-16\pi=\frac{49\pi}{3}-\frac{48\pi}{3}=\frac{\pi}{3}$$

So $\theta=49\pi/3$ is coterminal with $\alpha=\pi/3$, which means the sine of these angles are equal, and the cosine of these angles are equal.

$$\sin\left(\frac{49\pi}{3}\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{49\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$$

Apply even-odd identities and reciprocal identities to find cosecant and tangent of $\theta=-49\pi/3$.

$$\csc\left(-\frac{49\pi}{3}\right) = -\csc\left(\frac{49\pi}{3}\right) = -\frac{1}{\sin\left(\frac{49\pi}{3}\right)} = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\tan\left(-\frac{49\pi}{3}\right) = -\tan\left(\frac{49\pi}{3}\right) = -\frac{\sin\left(\frac{49\pi}{3}\right)}{\cos\left(\frac{49\pi}{3}\right)} = -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

Topic: The set of all possible angles

Question: Which set of angles satisfies the equation $\sin\theta = \sin 60^\circ$?

Answer choices:

A $\quad \theta = \left\{ -\frac{2\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\}$

B $\quad \theta = \left\{ \frac{\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\} \cup \left\{ \frac{2\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\}$

C $\quad \theta = \left\{ -\frac{5\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\} \cup \left\{ -\frac{4\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\}$

D $\quad \theta = \left\{ \frac{4\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{3} + 2\pi k: \ k \in \mathbb{Z} \right\}$

Solution: B

The value of sine is positive, which means the y-coordinate is positive, and therefore that angles which satisfy the equation are found in the first and second quadrants.

We'll convert the degree angle to radians.

$$60^\circ = 60^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{60\pi}{180} = \frac{\pi}{3}$$

From the unit circle, we know that

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

So $\pi/3$ is one solution, but we need to include all other angles that are coterminal with $\pi/3$.

$$\left\{\frac{\pi}{3} + 2\pi k: \ k \in Z\right\}$$

But angles in the first and second quadrant have equivalent sine values, which means $2\pi/3$ is also a solution, as well as all angles that are coterminal with $2\pi/3$:

$$\left\{\frac{2\pi}{3} + 2\pi k: \ k \in Z\right\}$$

Topic: The set of all possible angles

Question: Which set of angles satisfies the equation?

$$\cos\theta = \cos\left(\frac{11\pi}{8}\right)$$

Answer choices:

A $\theta = \left\{-\frac{57\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

B $\theta = \left\{\frac{45\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

C $\theta = \left\{-\frac{75\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

D $\theta = \left\{\frac{29\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

Solution: C

The set that satisfies the equation could be the set of angles that are all coterminal with $11\pi/8$. So we'll check to see which of the angles given in the answer choices, if any, is coterminal with $11\pi/8$.

$$-\frac{57\pi}{8}-\frac{11\pi}{8}=\frac{(-57-11)\pi}{8}=-\frac{68\pi}{8}=-\frac{17\pi}{2}$$

$$\frac{45\pi}{8}-\frac{11\pi}{8}=\frac{(45-11)\pi}{8}=\frac{34\pi}{8}=\frac{17\pi}{4}$$

$$-\frac{75\pi}{8}-\frac{11\pi}{8}=\frac{(-75-11)\pi}{8}=-\frac{86\pi}{8}=-\frac{43\pi}{4}$$

$$\frac{29\pi}{8}-\frac{11\pi}{8}=\frac{(29-11)\pi}{8}=\frac{18\pi}{8}=\frac{9\pi}{4}$$

None of these results give an angle that's a multiple of 2π, which means none of the angles in the answer choices are coterminal with $11\pi/8$.

The given equation tells us that the cosines of the angles are equal, and we know that cosine always gives the x-value in our coordinate point in the unit circle.

Because the angle $11\pi/8$ is in the third quadrant, we can also find an equal cosine value in the second quadrant, where we're at the same point along the x-axis, but at the opposite point along the y-axis.

We know that $11\pi/8$ is $3\pi/8$ past π, which means by symmetry that the angle in the second quadrant with the same cosine is $3\pi/8$ short of π, or at

$$\pi-\frac{3\pi}{8}=\frac{8\pi}{8}-\frac{3\pi}{8}=\frac{5\pi}{8}$$

Which means that the correct angle set must be coterminal with $5\pi/8$. We'll check the answers again to see which angle is coterminal with $5\pi/8$.

$$-\frac{57\pi}{8}-\frac{5\pi}{8}=\frac{(-57-5)\pi}{8}=-\frac{62\pi}{8}=-\frac{31\pi}{4}$$

$$\frac{45\pi}{8}-\frac{5\pi}{8}=\frac{(45-5)\pi}{8}=\frac{40\pi}{8}=5\pi$$

$$-\frac{75\pi}{8}-\frac{5\pi}{8}=\frac{(-75-5)\pi}{8}=-\frac{80\pi}{8}=-10\pi$$

$$\frac{29\pi}{8}-\frac{5\pi}{8}=\frac{(29-5)\pi}{8}=\frac{24\pi}{8}=3\pi$$

Only $-75\pi/8$ gives an integer multiple of 2π (the angle is five full rotations in the negative direction, since $2\pi(-5)=-10\pi$).

Topic: The set of all possible angles

Question: Solve $\cos\theta = 1/2$ for all possible values of θ.

Answer choices:

A $\theta = \{60^\circ + (360^\circ)k: \ k \in Z\} \cup \{300^\circ + (360^\circ)k: \ k \in Z\}$

B $\theta = \{30^\circ + (360^\circ)k: \ k \in Z\} \cup \{330^\circ + (360^\circ)k: \ k \in Z\}$

C $\theta = \{60^\circ + (360^\circ)k: \ k \in Z\} \cup \{120^\circ + (360^\circ)k: \ k \in Z\}$

D $\theta = \{30^\circ + (360^\circ)k: \ k \in Z\} \cup \{150^\circ + (360^\circ)k: \ k \in Z\}$

Solution: A

The equation is telling us that the value of cosine is positive, which means the value of the x-coordinate is positive. Therefore, the solution will be limited to values of θ in the first and fourth quadrants.

From the unit circle, we know that

$$\cos 60^\circ = \frac{1}{2}$$

So 60° is one solution, but we need to include all other angles that are coterminal with 60°.

$$\{60^\circ \pm (360^\circ)k:\ k = 0,1,2,\ldots\} \text{ or } \theta = \{60^\circ + (360^\circ)k:\ k \in Z\}$$

We can find the other set of angles by symmetry. If 60° gives us one set of angles, and 60° is 60° "above" the x-axis, then the other set of angles is at 60° "below" the x-axis, so the other angle is $(360 - 60)^\circ = 300^\circ$. Which means the set

$$\{300^\circ \pm (360^\circ)k:\ k = 0,1,2,\ldots\} \text{ or } \theta = \{300^\circ + (360^\circ)k:\ k \in Z\}$$

also satisfies the equation. Combining these results, the complete solution set is

$$\theta = \{60^\circ + (360^\circ)k:\ k \in Z\} \cup \{300^\circ + (360^\circ)k:\ k \in Z\}$$

Topic: Points not on the unit circle

Question: What is the value of $\sin\theta$ for an angle θ whose terminal side contains the point $(7,-15)$?

Answer choices:

A $\sin\theta=\frac{15}{22}$

B $\sin\theta=-\frac{7}{15}$

C $\sin\theta=-\frac{15\sqrt{274}}{274}$

D $\sin\theta=-\frac{15}{7}$

Solution: C

Substitute $x = 7$ and $y = -15$ into formula for $\sin\theta$ of a point off the unit circle.

$$\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\sin\theta = \frac{-15}{\sqrt{7^2 + (-15)^2}}$$

$$\sin\theta = \frac{-15}{\sqrt{49 + 225}}$$

$$\sin\theta = -\frac{15}{\sqrt{274}}$$

$$\sin\theta = -\frac{15\sqrt{274}}{274}$$

Topic: Points not on the unit circle

Question: What is the value of $\cos\theta$ for an angle θ whose terminal side contains the point $(-16,-8)$?

Answer choices:

A $\quad \cos\theta = \frac{1}{2}$

B $\quad \cos\theta = -\frac{2\sqrt{5}}{5}$

C $\quad \cos\theta = -\frac{1}{3}$

D $\quad \cos\theta = \frac{4\sqrt{5}}{5}$

Solution: B

Substitute $x = -16$ and $y = -8$ into formula for $\cos\theta$ of a point off the unit circle.

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos\theta = \frac{-16}{\sqrt{(-16)^2 + (-8)^2}}$$

$$\cos\theta = \frac{-16}{\sqrt{256 + 64}}$$

$$\cos\theta = -\frac{16}{\sqrt{320}}$$

$$\cos\theta = -\frac{16}{8\sqrt{5}}$$

$$\cos\theta = -\frac{2}{\sqrt{5}}$$

$$\cos\theta = -\frac{2\sqrt{5}}{5}$$

Topic: Points not on the unit circle

Question: Let α be an angle whose terminal side contains the point $(12,5)$, and let $\theta = \alpha + \pi$. What are the values of $\sin\theta$ and $\cos\theta$?

Answer choices:

A $\sin\theta = -\frac{5}{13}$ $\cos\theta = -\frac{12}{13}$

B $\sin\theta = -\frac{12}{17}$ $\cos\theta = -\frac{5}{17}$

C $\sin\theta = \frac{5}{17}$ $\cos\theta = -\frac{12}{17}$

D $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$

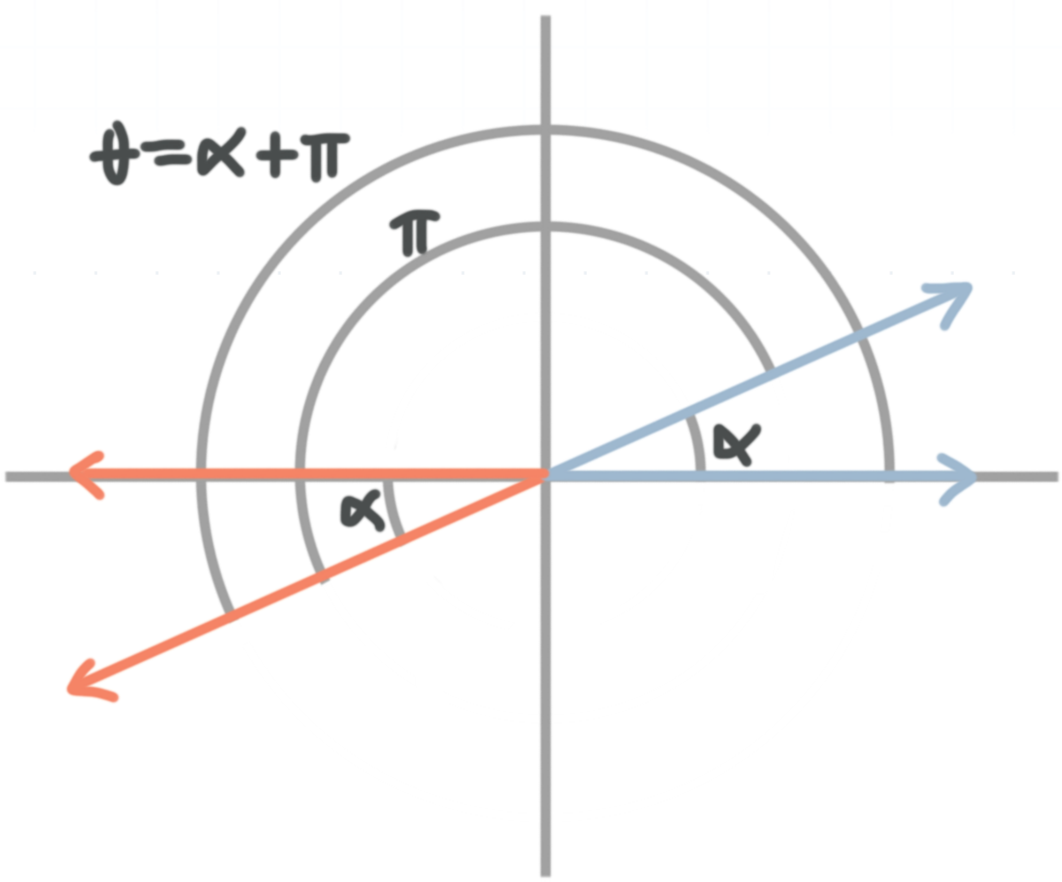
Solution: A

First we'll find $\sin\alpha$ and $\cos\alpha$ by substituting $x=12$ and $y=5$ into the following formulas for $\sin\alpha$ and $\cos\alpha$.

$$\sin\alpha=\frac{y}{\sqrt{x^2+y^2}}=\frac{5}{\sqrt{12^2+5^2}}=\frac{5}{\sqrt{144+25}}=\frac{5}{\sqrt{169}}=\frac{5}{13}$$

$$\cos\alpha=\frac{x}{\sqrt{x^2+y^2}}=\frac{12}{\sqrt{12^2+5^2}}=\frac{12}{\sqrt{144+25}}=\frac{12}{\sqrt{169}}=\frac{12}{13}$$

Because $(12,5)$ is on the terminal side of α, we know α is in the first quadrant, because x and y are positive, and therefore that $\alpha+\pi$ is in the third quadrant. Furthermore, the reference angle for both θ and α is α itself.



Then using the reference angle, we get

$$\sin\theta = -\sin\alpha = -\frac{5}{13}$$

$$\cos\theta = -\cos\alpha = -\frac{12}{13}$$

Topic: Solving right triangles

Question: If the measure of the acute angle of a right triangle that's opposite one of the legs is $2\pi/5$, what is the measure of the acute angle that's opposite the other leg?

Answer choices:

A $\frac{\pi}{10}$

B $\frac{6\pi}{5}$

C $\frac{3\pi}{5}$

D $\frac{11\pi}{10}$

Solution: A

Let $\theta = 2\pi/5$ and let α be the other acute angle. The measures of θ and α sum to 90°, so

$$\theta + \alpha = \frac{\pi}{2}$$

$$\frac{2\pi}{5} + \alpha = \frac{\pi}{2}$$

Solve for α.

$$\alpha = \frac{\pi}{2} - \frac{2\pi}{5}$$

$$\alpha = \frac{5\pi}{10} - \frac{4\pi}{10}$$

$$\alpha = \frac{\pi}{10}$$

Topic: Solving right triangles

Question: The length of one leg of a right triangle is 5 and the opposite angle has measure 58°. What is the length of the other leg?

Answer choices:

A $\quad 2.65$

B $\quad 8.00$

C $\quad 4.24$

D $\quad 3.12$

Solution: D

Use the definition of sine.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 58^\circ = \frac{5}{c}$$

$$c = \frac{5}{\sin 58^\circ} \approx \frac{5}{0.848408} \approx 5.8959$$

Now use the Pythagorean Theorem to find the length of the other leg.

$$a^2 + b^2 = c^2$$

$$b^2 = c^2 - a^2$$

$$b^2 \approx 5.8959^2 - 5^2$$

$$b^2 \approx 34.7615 - 25$$

$$b^2 \approx 9.7615$$

$$b \approx \sqrt{9.7615}$$

$$b \approx 3.12$$

Topic: Solving right triangles

Question: The length of one leg of a right triangle is 2.4, and the angle opposite the other leg has measure 28°. What is the length of the triangle's hypotenuse?

Answer choices:

A 2.72

B 4.51

C 1.28

D 5.11

Solution: A

Use the definition of sine.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 62^\circ = \frac{2.4}{c}$$

$$c = \frac{2.4}{\sin 62^\circ} \approx \frac{2.4}{0.883} \approx 2.72$$

Topic: Angles of elevation and depression

Question: Two cats, Fluffy and Muffy, each have their own cat tree and their favorite spot in their tree. The horizontal distance between their favorite spots is 4.8 feet and the angle of elevation of Fluffy's favorite spot to Muffy's is 35°. How far is Fluffy from Muffy?

Answer choices:

A 7.75 feet

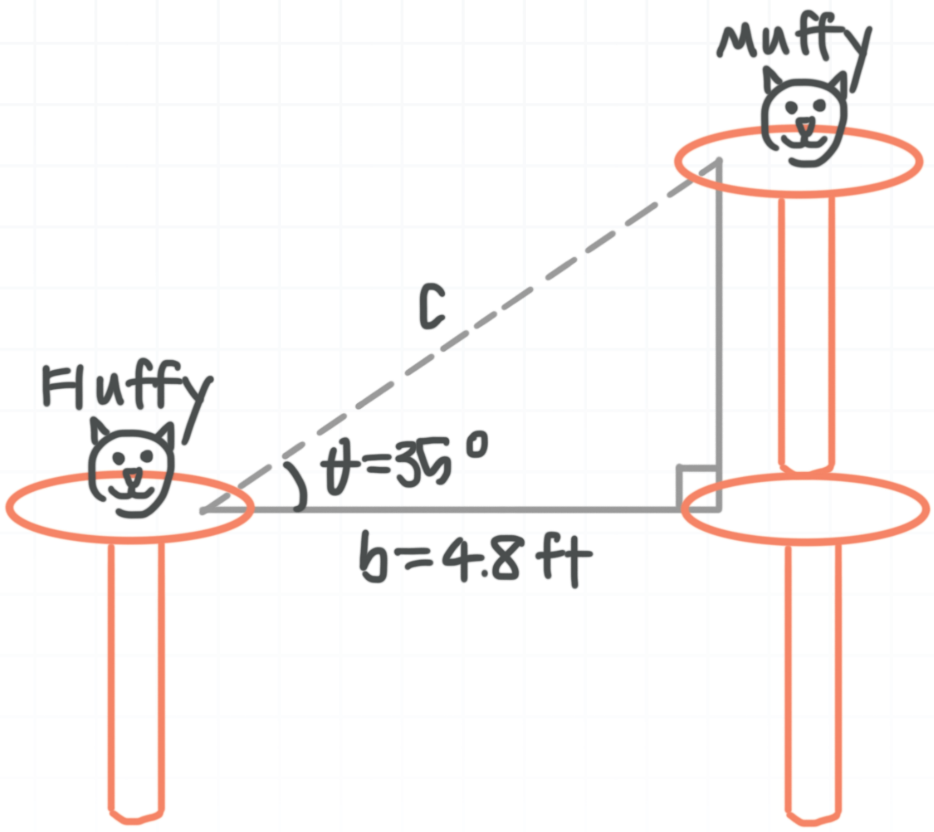
B 5.86 feet

C 8.37 feet

D 6.93 feet

Solution: B

Let b be the horizontal distance between the cat trees and c be the distance from one cat directly to the other.



We need to find c using $b=4.8$ ft and the angle of elevation $\theta=35^\circ$. Side b of the right triangle is the adjacent side, and side c is the hypotenuse, so

$$\cos 35^\circ = \frac{b}{c}$$

$$c\cos 35^\circ = b$$

$$c = \frac{b}{\cos 35^\circ}$$

Substitute $b=4.8$ ft.

$$c = \frac{4.8 \text{ ft}}{\cos 35^\circ} \approx \frac{4.8 \text{ ft}}{0.819} \approx 5.86 \text{ ft}$$

Topic: Angles of elevation and depression

Question: A drone is 750 feet above the ground. A piece of tracking apparatus on a stand 25 feet above ground has an angle of elevation to the drone of 49°. What is the horizontal distance between the drone and the tracking apparatus?

Answer choices:

A 863 feet

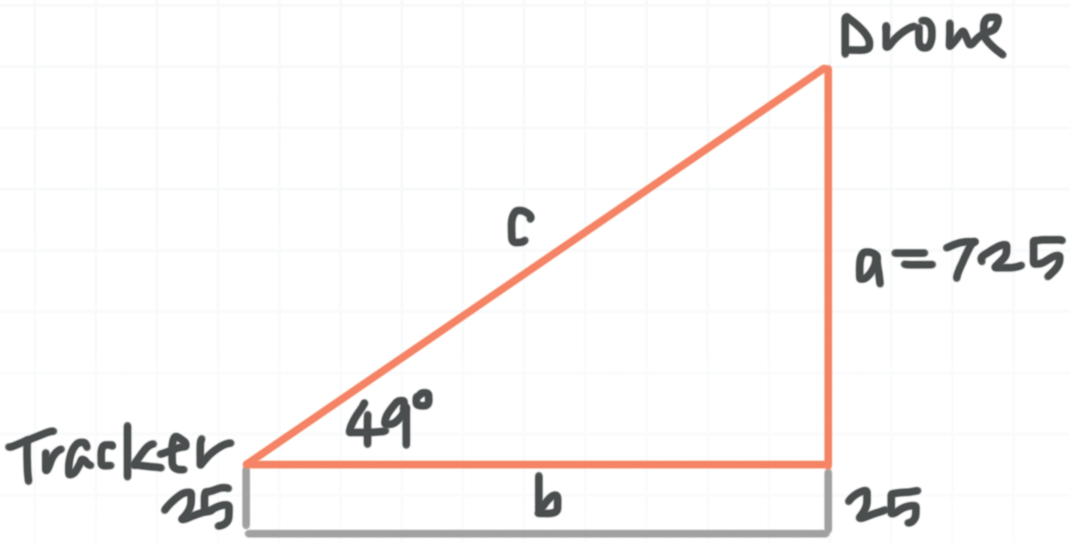
B 630 feet

C 652 feet

D 834 feet

Solution: B

Sketch a diagram of the situation.



Then we can say sine and cosine of the angle are given by

$$\sin 49^\circ = \frac{725}{c} \text{ and } \cos 49^\circ = \frac{b}{c}$$

Solve both equations for c.

$$\sin 49^\circ = \frac{725}{c}$$

$$c \sin 49^\circ = 725$$

$$c = \frac{725}{\sin 49^\circ}$$

and

$$\cos 49^\circ = \frac{b}{c}$$

$$c \cos 49^\circ = b$$

$$c = \frac{b}{\cos 49^\circ}$$

Because both equations are equal to c, they're equal to each other.

$$\frac{725}{\sin 49^\circ} = \frac{b}{\cos 49^\circ}$$

$$b = \left(\frac{725}{\sin 49^\circ}\right)(\cos 49^\circ)$$

$$b \approx \left(\frac{725}{0.755}\right)(0.656)$$

$b \approx 630$ feet

Topic: Angles of elevation and depression

Question: A man is trying to catch some fish and spots one with an angle of depression of 51° with respect to the end of his fishing rod. The fish is located 8.2 feet lower than the end of the fishing rod. How far away is the fish from the end of the rod?

Answer choices:

A 13.0 feet

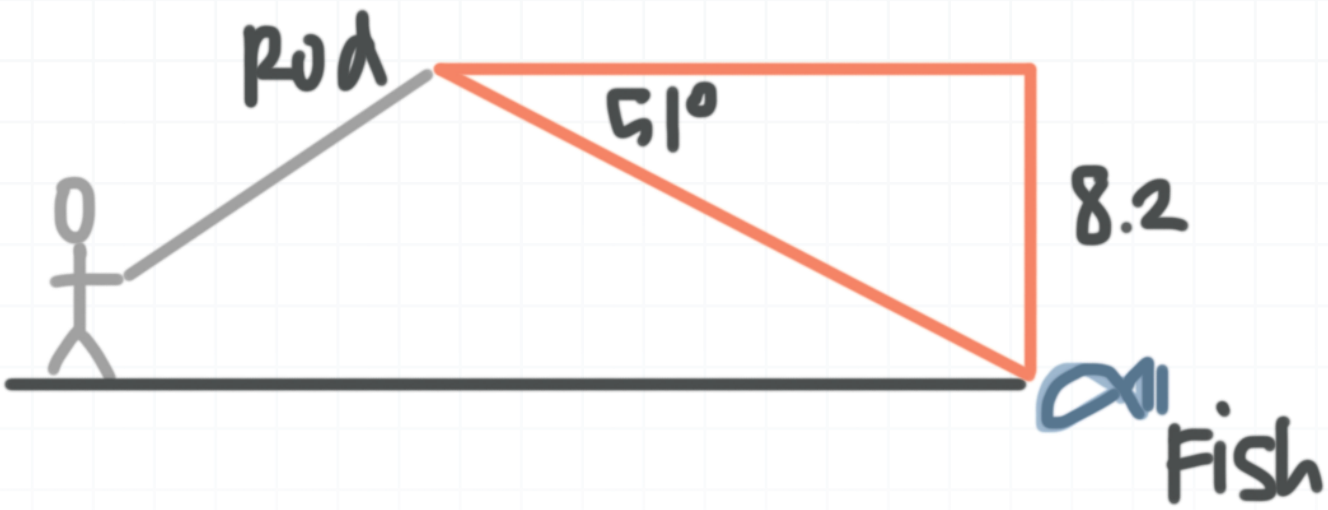
B 19.1 feet

C 9.16 feet

D 10.6 feet

Solution: D

Sketch a diagram of the situation.



Let a be the vertical distance between the fish and the end of the rod, and c be the direct distance between them.

$$\frac{a}{\sin 51^\circ} = \frac{c}{1}$$

$$c = \frac{a}{\sin 51^\circ}$$

Substituting $a = 8.2$ ft.

$$c = \frac{8.2}{\sin 51^\circ}$$

$$c \approx \frac{8.2}{0.777}$$

$c \approx 10.6$ feet

Topic: Radians and arc length

Question: What is the radian measure of an arc of a circle with a radius of 12 centimeters and an associated central angle of 30°?

Answer choices:

A π

B $\dfrac{\pi}{2}$

C $\dfrac{2\pi}{3}$

D 2π

Solution: D

We can only use an angle defined in radians in the arc length formula, so we'll need to convert 30° to radians.

$$30^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{6}$$

Now we'll plug what we know into the arc length formula.

$$s = r\theta$$

$$s = 12\left(\frac{\pi}{6}\right)$$

$$s = 2\pi$$

Topic: Radians and arc length

Question: In radius of circle O is 25, and the measure of arc AB is 150°. Find the approximate length of arc AB.

Answer choices:

A 65

B 94

C 21

D 131

Solution: A

We can only use an angle defined in radians in the arc length formula, so we'll need to convert 150° to radians.

$$150^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{6}$$

Now we'll plug what we know into the arc length formula.

$$s = r\theta$$

$$s = 25\left(\frac{5\pi}{6}\right)$$

$$s \approx 65$$

Topic: Radians and arc length

Question: Approximately how many radians make up 252°?

Answer choices:

A 1.4

B 4.4

C 0.4

D 2.2

Solution: B

If we want to convert an angle from degrees to radians, we multiply it by $\pi/180^\circ$.

$$252^\circ\left(\frac{\pi}{180^\circ}\right)$$

$$1.4\pi$$

$$1.4(3.14)$$

$$4.4$$

Topic: Area of a circular sector

Question: If a circle has radius 5, find the area A of a sector that subtends a central angle of 135°.

Answer choices:

A $\quad A = \frac{75}{8}$

B $\quad A = \frac{75\pi}{8}$

C $\quad A = \frac{5{,}625}{2}$

D $\quad A = \frac{5{,}625\pi}{2}$

Solution: B

Convert the central angle from degrees to radians.

$$135^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{3\pi}{4}$$

Then the area of the circular sector is

$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} (5)^2 \left(\frac{3\pi}{4} \right)$$

$$A = \frac{75\pi}{8}$$

Topic: Area of a circular sector

Question: Find the area A (in square centimeters) of the sector of a circle of radius 6 centimeters if that sector is bounded by an arc that subtends a central angle of $7\pi/4$ radians.

Answer choices:

A $A = \frac{63\pi}{2}$

B $A = \frac{63}{2}$

C $A = \frac{7\pi}{2}$

D $A = 7\pi$

Solution: A

The area of the circular sector is

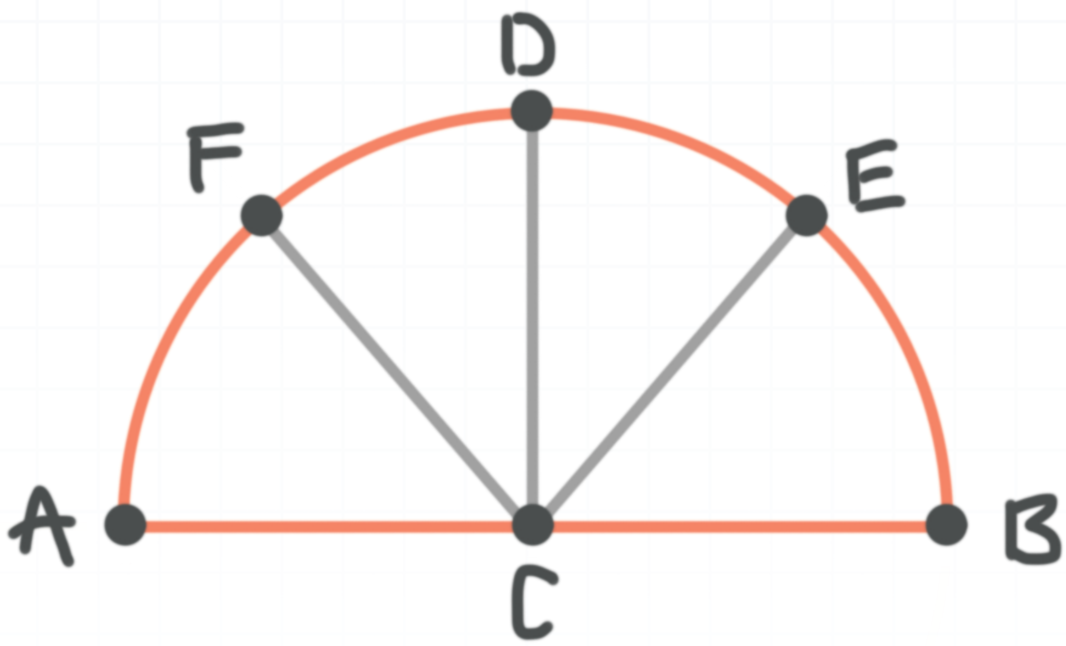
$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}(6)^2\left(\frac{7\pi}{4}\right)$$

$$A = \frac{63\pi}{2}$$

Topic: Area of a circular sector

Question: A semicircular window is shown below. If the window contains four stained glass sections of equal size and has a base length of 28 centimeters, what is the area of each stained glass section?



Answer choices:

A 49π

B 24.5π

C 73.5π

D 98π

Solution: B

The window is semicircular, so $\angle ACB = 180^\circ = \pi$. We can find the area of the whole window, and then divide it into four equal parts.

Since the base is 28 centimeters long, the diameter of the circle is 28 cm and the radius is $28/2 = 14$ cm.

The area of the circular sector is

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}(14)^2(\pi)$$

$$A = 98\pi$$

The area of one stained glass section will be

$$A_1 = \frac{98\pi}{4}$$

$$A_1 = 24.5\pi$$

Topic: Trig functions of real numbers

Question: Evaluate $\cos(1.25)$, rounded to four decimal places.

Answer choices:

A 0.9490

B 0.3

C 0.3153

D 0.9998

Solution: C

We'll use a calculator to evaluate the cosine function at 1.25, making sure the calculator is set to radian mode.

$$\cos(1.25) \approx 0.3153$$

Topic: Trig functions of real numbers

Question: Evaluate $\csc(0.87)$, rounded to four decimal places.

Answer choices:

A 1.3083

B 0.7643

C 0.0152

D 65.7895

Solution: A

We'll use a calculator to evaluate the circular function at 0.87, making sure the calculator is set to radian mode.

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\csc(0.87) = \frac{1}{\sin(0.87)}$$

$$\csc(0.87) \approx 1.3083$$

Topic: Trig functions of real numbers

Question: Evaluate $\tan(2.78)$, rounded to three decimal places.

Answer choices:

A 0.379

B 0.354

C -0.378

D -0.935

Solution: C

We'll use a calculator to evaluate the tangent function at 2.78, making sure the calculator is set to radian mode.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan(2.78) = \frac{\sin(2.78)}{\cos(2.78)}$$

$$\tan(2.78) \approx -0.378$$

Topic: Linear and angular velocity

Question: What is the angular velocity ω of a wheel that rotates at a constant rate and sweeps out an angle of 543π radians in 14.6 minutes?

Answer choices:

A $\omega = 37.2$ radians per second

B $\omega = 0.620$ radians per second

C $\omega = 0.620\pi$ radians per second

D $\omega = 1.83$ radians per second

Solution: C

Since the elapsed time t $(14.6$ minutes) is given in units of minutes and all the answer choices are given in units of radians per second (not radians per minute), we need to convert the elapsed time into seconds.

$$t = (14.6 \text{ min})\left(\frac{60 \text{ sec}}{1 \text{ min}}\right)$$

$$t = 14.6(60) \text{ sec}$$

$$t = 876 \text{ sec}$$

Now we compute the angular velocity in radians per second.

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{(543\pi) \text{ rad}}{876 \text{ sec}}$$

$$\omega = \frac{543\pi}{876} \text{ radians per second}$$

$$\omega \approx 0.620\pi \text{ radians per second}$$

Topic: Linear and angular velocity

Question: If a disc is rotating at a constant rate of 94.9 revolutions per minute, what is its angular velocity ω in units of radians per second?

Answer choices:

A $\omega = 8.62$ radians per second

B $\omega = 18.7\pi$ radians per second

C $\omega = 3.16\pi$ radians per second

D $\omega = 15.1$ radians per second

Solution: C

To convert from revolutions per minute to radians per second, we need to use the following facts: (a) There are 2π radians in 1 full revolution, and (b) there are 60 seconds in a minute.

$$\omega = \left(94.9 \ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \ \text{rad}}{1 \ \text{rev}}\right)\left(\frac{1 \ \text{min}}{60 \ \text{sec}}\right)$$

$$\omega = \frac{94.9(2)\pi}{60} \text{ radians per second}$$

$$\omega \approx 3.16\pi \text{ radians per second}$$

Topic: Linear and angular velocity

Question: Michael is running around a circular track. If he runs 5 laps in 20 minutes, what is his angular velocity?

Answer choices:

A 0.5π radians per minute

B 2π radians per minute

C 0.008π radians per minute

D 30π radians per minute

Solution: A

First we need to convert 5 revolutions to radians, remembering that there are 2π radians in 1 full revolution.

$$5 \text{ rev} = 5 \text{ rev} \left(2\pi \ \frac{\text{rad}}{\text{rev}}\right) = 10\pi \text{ rad}$$

Find angular velocity ω.

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{10\pi \text{ radians}}{20 \text{ minutes}}$$

$$\omega = \frac{\pi}{2} \text{ radians per minute}$$

$$\omega \approx 0.5\pi \text{ radians per minute}$$

Topic: Relating linear and angular velocity

Question: If a wheel of diameter 21 inches is rotating at a rate of 0.543π radians per second, what is the linear velocity v of a point on the outside edge of the wheel?

Answer choices:

A $v = 1.49$ ft/sec

B $v = 2.98$ ft/sec

C $v = 17.9$ ft/sec

D $v = 3.66$ ft/sec

Solution: A

The radius is half the diameter, so

$$r = \frac{21.0}{2} = 10.5 \text{ inches}$$

Use the formula relating linear velocity to angular velocity, and substitute the values we know.

$$v = r\omega$$

$$v = (10.5 \text{ in})\left(\frac{0.543\pi}{\text{sec}}\right)$$

Multiply by a conversion factor to change inches into feet.

$$v = (10.5 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{0.543\pi}{\text{sec}}\right)$$

$$v = \frac{10.5(0.543)}{12}\pi \text{ ft/sec}$$

$$v \approx 0.475\pi \text{ ft/sec}$$

$$v \approx 1.49 \text{ ft/sec}$$

Topic: Relating linear and angular velocity

Question: Determine the linear velocity in inches per minute of the tips of the $13''$ blades of a ceiling fan, if the blades are rotating at 52 revolutions per minute.

Answer choices:

A 431 in/min

B $1{,}352$ in/min

C 676 in/min

D $4{,}247$ in/min

Solution: D

First we need to convert the angular velocity from revolutions per minute to radians per minute.

$$\omega = \left(52\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$

$$\omega \approx 104\pi \text{ radians per minute}$$

Now we can find the linear speed.

$$v = r\omega$$

$$v \approx (13\ \text{in})\left(\frac{104\pi}{\text{min}}\right)$$

$$v \approx 13(104\pi) \text{ inches per minute}$$

$$v \approx 4{,}247 \text{ inches per minute}$$

Topic: Relating linear and angular velocity

Question: The wheels of a car have a diameter of 2.5 ft and are rotating at 4 revolutions per second. How fast is the car moving in miles per hour? Hint: There are $5,280$ feet in 1 mile.

Answer choices:

A 21.4 mi/hr

B 6.82 mi/hr

C 42.8 mi/hr

D 3.4 mi/hr

Solution: A

First we need to convert the angular velocity from revolutions per second to radians per second.

$$\omega = \left(4\ \frac{\text{rev}}{\text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$

$$\omega \approx 8\pi \text{ radians per second}$$

Because the diameter of the blade is 2.5 ft, its radius is 1.25 ft, so linear velocity is

$$v = r\omega$$

$$v = (1.25\ \text{ft})\left(\frac{8\pi}{\text{sec}}\right)$$

Multiply by a conversion factor to change feet into miles and seconds into hours.

$$v = (1.25\ \text{ft})\left(\frac{1\ \text{mi}}{5{,}280\ \text{ft}}\right)\left(\frac{8\pi}{\text{sec}}\right)\left(\frac{3{,}600\ \text{sec}}{\text{hr}}\right)$$

$$v = \frac{1.25(8)(3600)}{5280}\pi\ \text{mi/hr}$$

$$v \approx 6.82\pi\ \text{mi/hr}$$

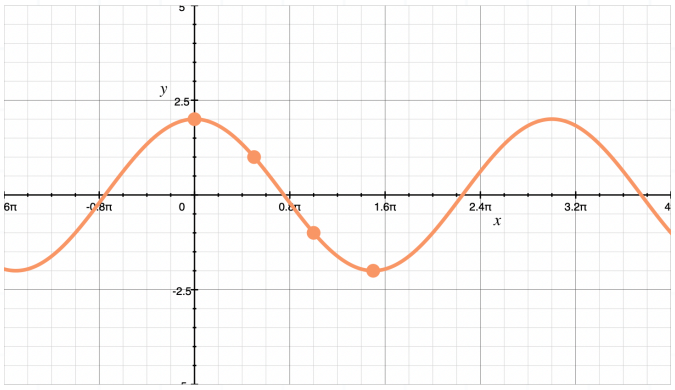
$$v \approx 21.4\ \text{mi/hr}$$

Topic: Sketching sine and cosine

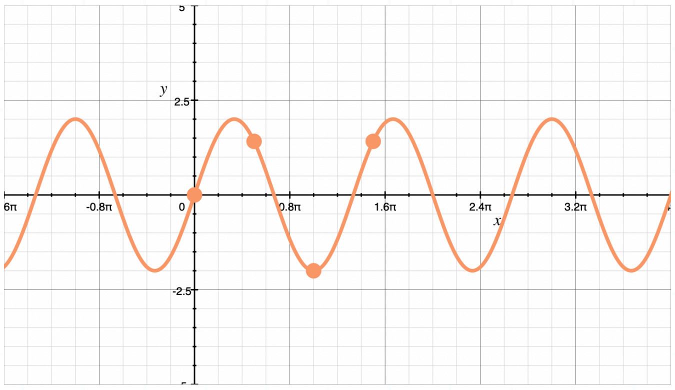
Question: Identify the graph of $y = -2\sin(2\theta/3)$.

Answer choices:

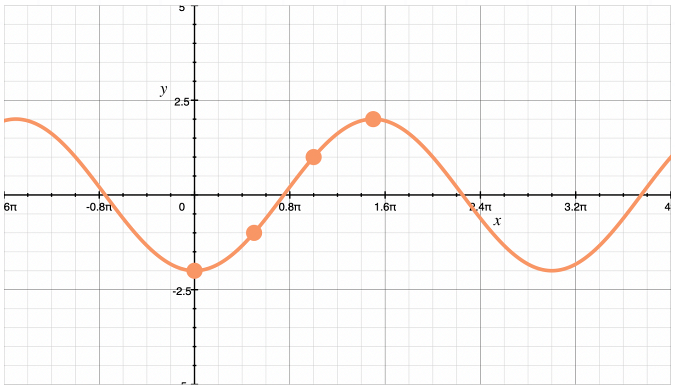
A



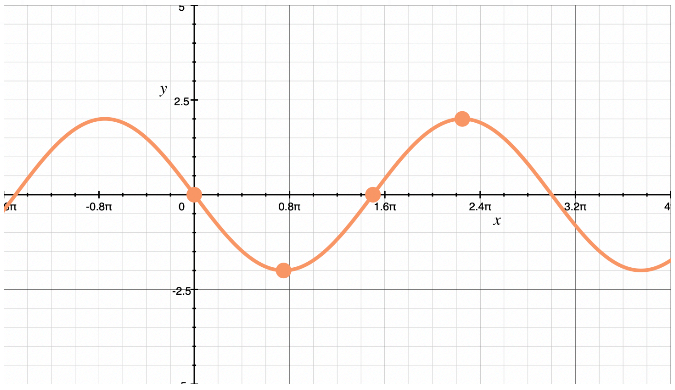
B



C



D



Solution: D

Setting $b = 2/3$ means we'll compress $y = \sin\theta$ horizontally by a factor of $2/3$. Pick a few points on $y = \sin\theta$,

$$(0,0) \qquad \left(\frac{\pi}{2},1\right) \qquad (\pi,0) \qquad \left(\frac{3\pi}{2},-1\right)$$

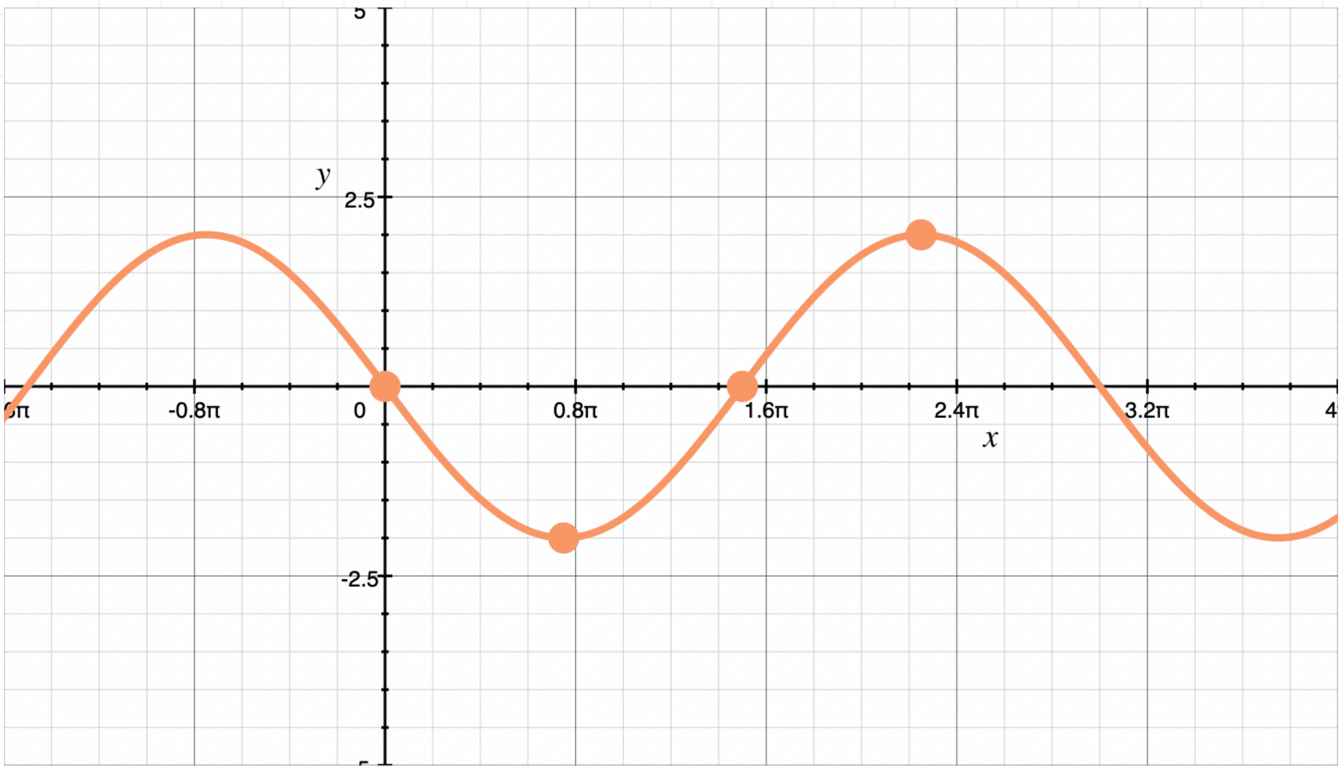
then horizontally compress the x-values by a factor of $2/3$, which means we'll multiply each x-value by $3/2$, while keeping the y-values the same.

$$(0,0) \qquad \left(\frac{3\pi}{4},1\right) \qquad \left(\frac{3\pi}{2},0\right) \qquad \left(\frac{9\pi}{4},-1\right)$$

Then setting $a = -2$ means we'll stretch $y = \sin\theta$ vertically by a factor of 2, and reflect it over the x-axis. So we'll take these points and multiply the y -values by -2.

$$(0,0) \qquad \left(\frac{3\pi}{4},-2\right) \qquad \left(\frac{3\pi}{2},0\right) \qquad \left(\frac{9\pi}{4},2\right)$$

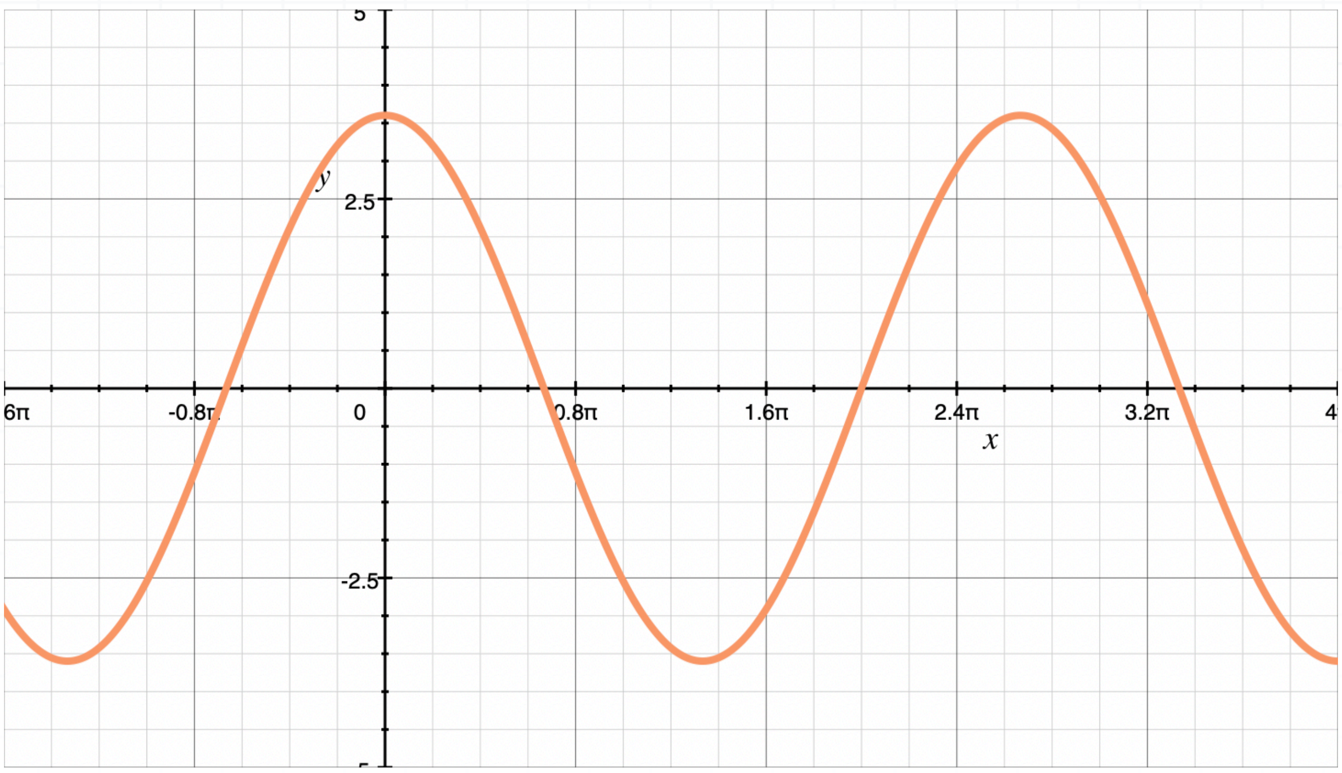
Then these four points are on the graph of $y = -2\sin(2\theta/3)$. If we plot the points and connect them, we get



y
x
5
2.5
-2.5
-5
-0.8π
0
0.8π
1.6π
2.4π
3.2π

Topic: Sketching sine and cosine

Question: Which function is represented by the curve?



Answer choices:

A $\quad 3.6\cos\left(\frac{3\theta}{4}\right)$

B $\quad 3.6\sin\left(\frac{6\theta}{5}\right)$

C $\quad -3.6\cos\left(\frac{4\theta}{3}\right)$

D $\quad -3.6\sin\left(\frac{3\theta}{2}\right)$

Solution: A

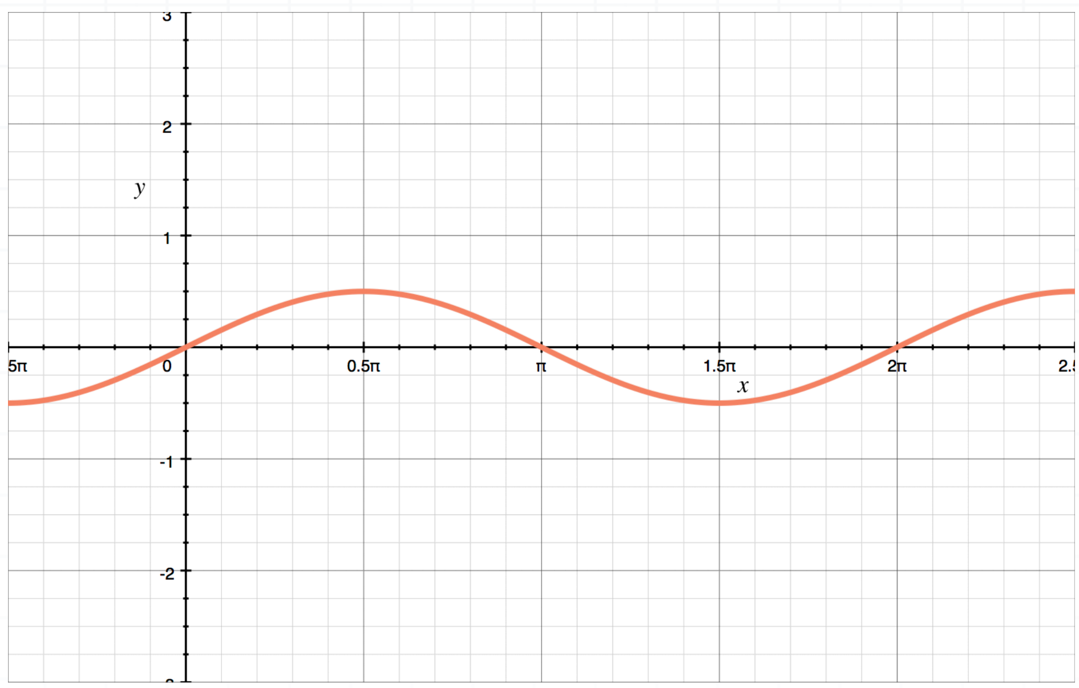
Because the value of the function at $\theta = 0$ isn't 0, the curve can't be the graph of either sine function. So we'll only need to determine which cosine function the graph represents.

When $\theta = 0$, the cosine function will always be 1, which means the graph of the curve in answer choice A will have a value of $3.6(1) = 3.6$, and the curve in answer choice C will have a value of $-3.6(1) = -3.6$.

So the function in answer choice A is the only one that could be represented by the graph.

Topic: Sketching sine and cosine

Question: Which function is represented by the curve?



Answer choices:

A $\frac{1}{2}\cos\left(\frac{\theta}{2}\right)$

B $2\sin\left(\frac{\theta}{2}\right)$

C $\frac{1}{2}\sin\theta$

D $2\cos(2\theta)$

Solution: C

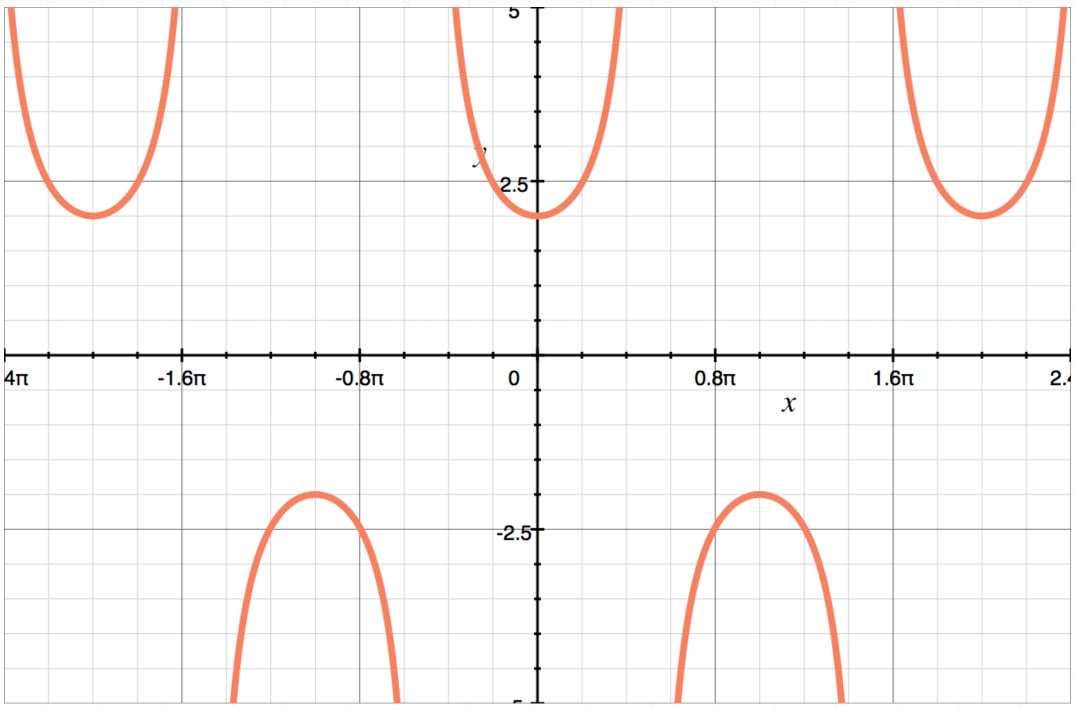
Because the value of the function at $\theta = 0$ is 0, the curve can't be the graph of either cosine function. So we'll only need to determine which sine function the graph represents.

When $\theta = \pi/2$, the sine function will always be 1, which means the graph of the curve in answer choice B will have a value of $2(1) = 2$, and the curve in answer choice C will have a value of $1/2(1) = 1/2$.

So the function in answer choice C is the only one that could be represented by the graph.

Topic: Sketching cosecant and secant

Question: Which trigonometric function is represented by the graph?



Answer choices:

A $2 \cos x$

B $2 \cot x$

C $2 \sec x$

D $2 \csc x$

Solution: C

Cosecant and secant are the only two of the six trig functions that have this general shape, so we only need to determine which of those is represented in this particular graph.

We see when $x = 0$, the function is 2. If this graph is the graph of cosecant, which is the reciprocal of sine, then at $x = 0$ we would expect the value of this curve to be

$$\csc x = 2\left(\frac{1}{\sin x}\right) = \frac{2}{\sin(0)} \neq \frac{2}{0}$$

In other words, the graph of cosecant is undefined at $x = 0$, so this must be the graph of secant. To check, we'll set up the same "expected value" equation for secant.

$$\sec x = 2\left(\frac{1}{\cos x}\right) = \frac{2}{\cos(0)} = \frac{2}{1} = 2$$

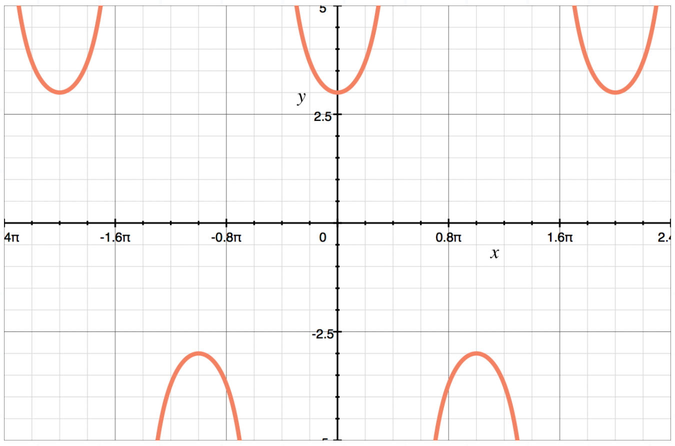
Therefore, this must be the graph of the secant function, $2\sec x$.

Topic: Sketching cosecant and secant

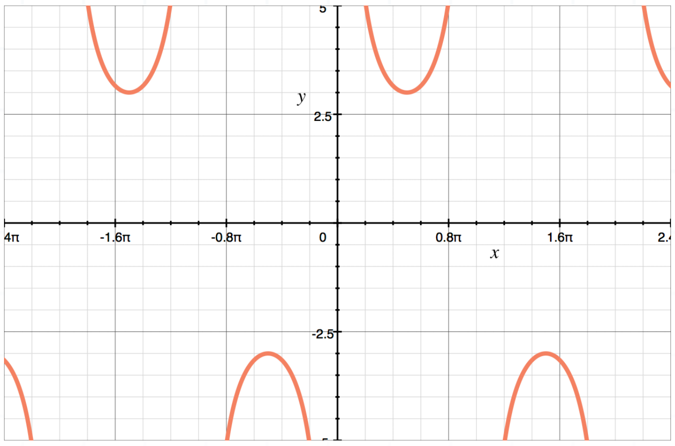
Question: Identify the graph of $y = 3\csc\theta$.

Answer choices:

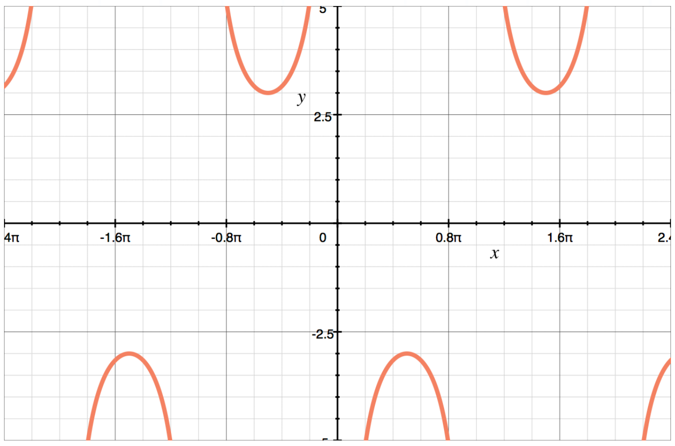
A



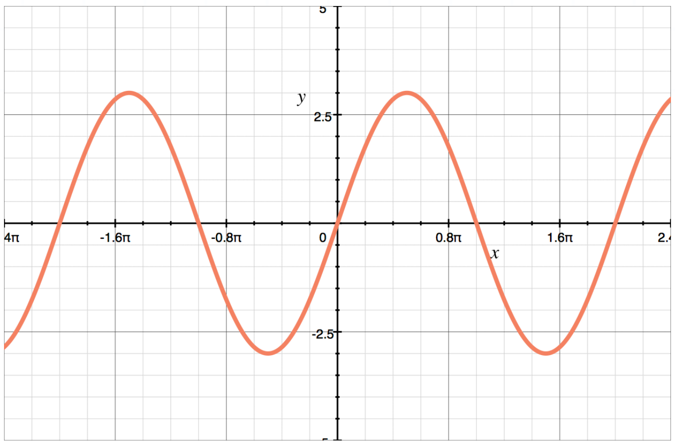
B



C



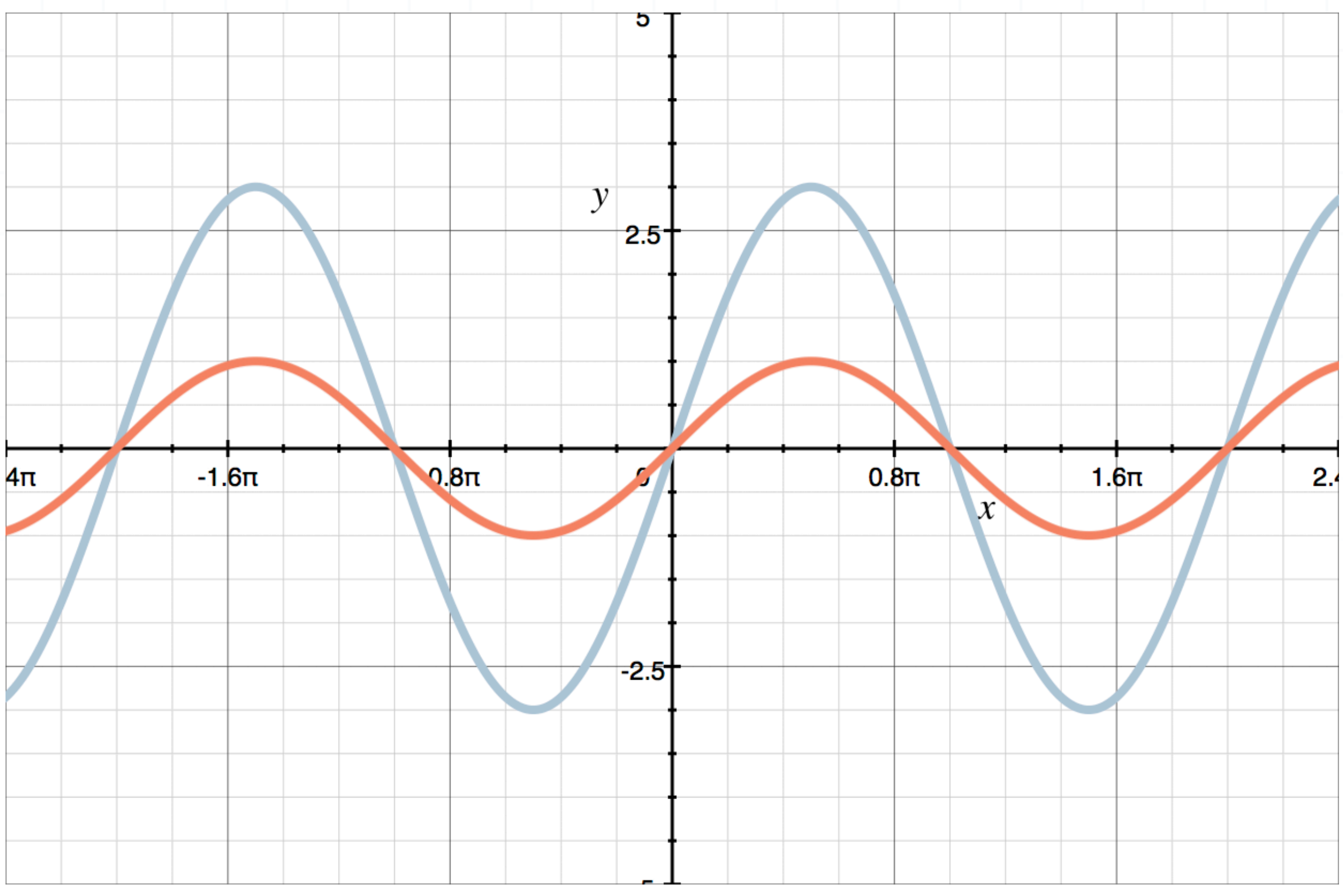
D



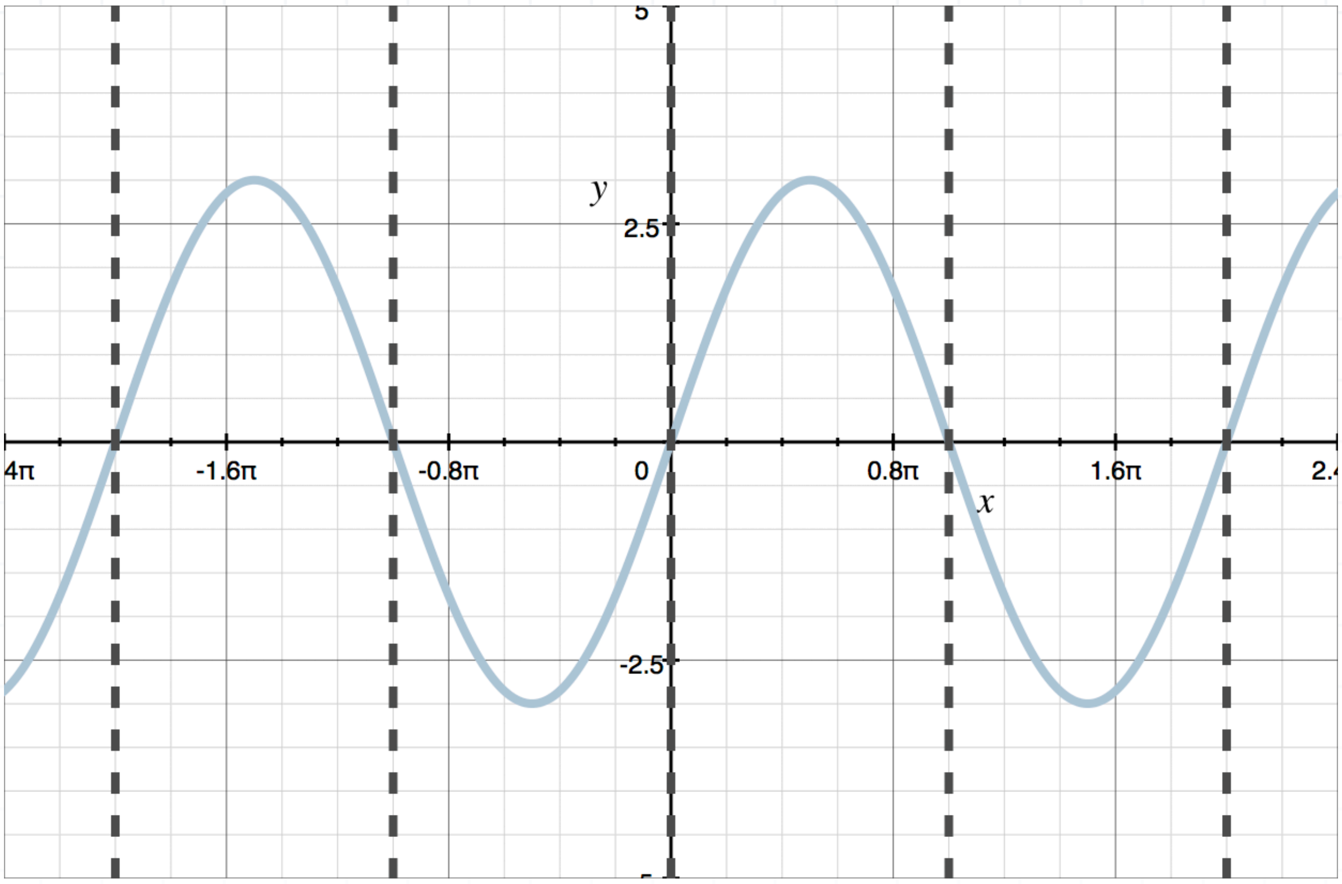
Solution: B

Because the reciprocal function of cosecant is sine, we want to start by replacing cosecant with sine in the function we've been given. In other words, the corresponding function is $y = 3 \sin x$.

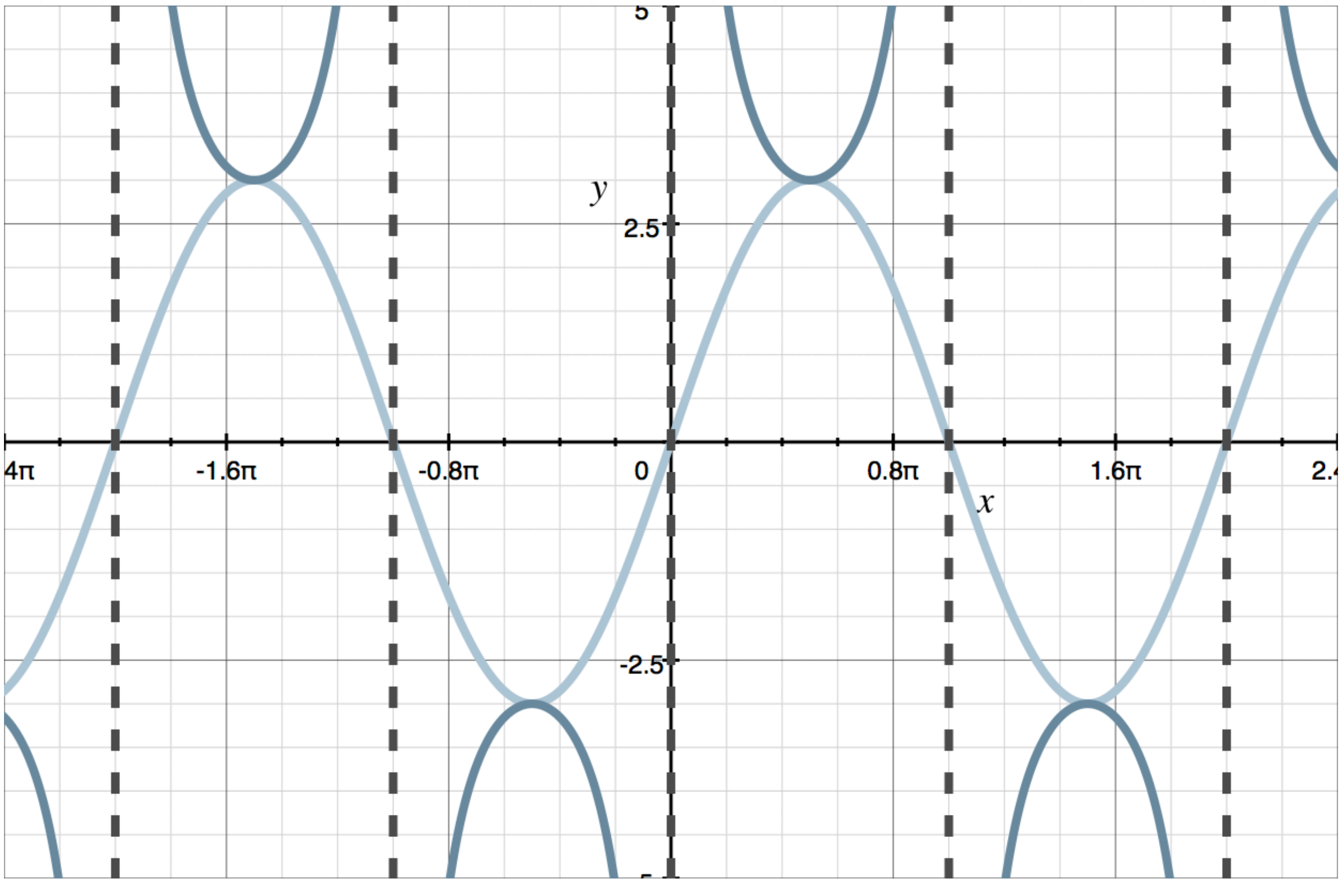
To graph $y = 3 \sin x$, we see that $a = 3$, so we'll vertically stretch the sine curve by tripling all the y-values. So if we sketch $y = \sin x$ in red and $y = 3 \sin x$ in blue, we get



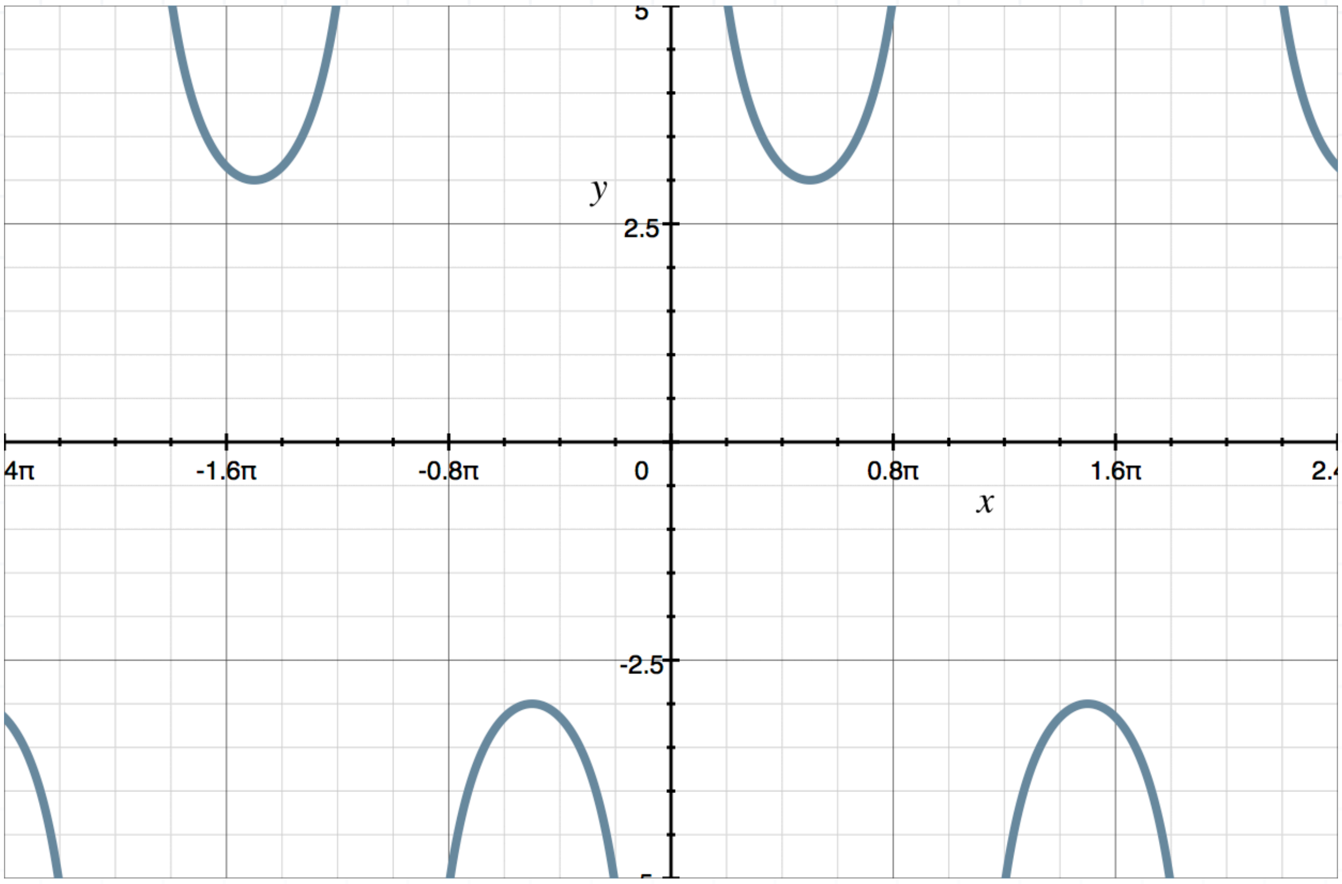
Then we'll sketch in vertical asymptotes at the midline of $y = 3 \sin x$.



We'll sketch in the U-shapes for $y = 3\csc\theta$.



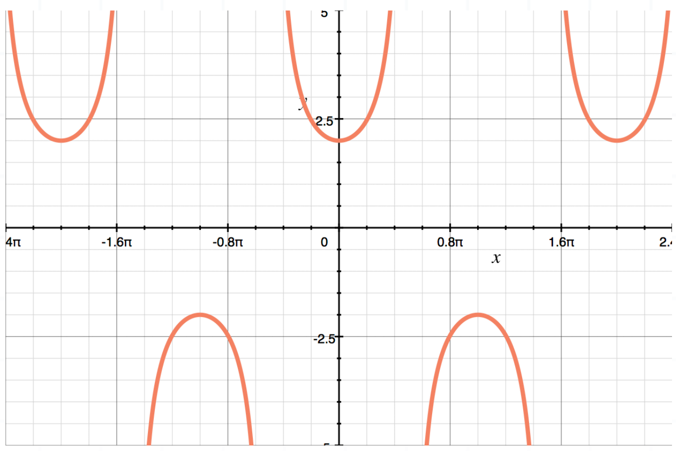
Finally, we'll take away the sine curve and the vertical asymptotes to get the final sketch of $y = 3\csc\theta$.

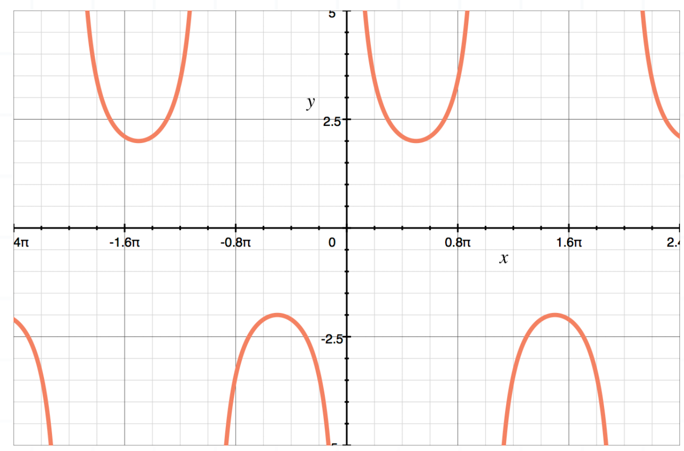

5
y
2.5
4π
-1.6π
-0.8π
0
0.8π
1.6π
2.
x
-2.5

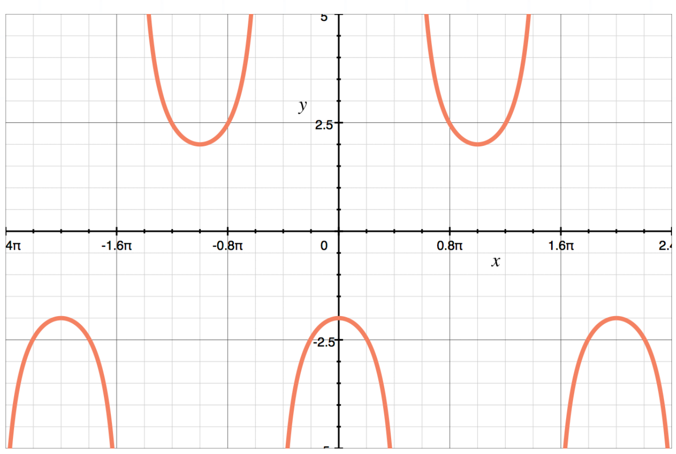
Topic: Sketching cosecant and secant

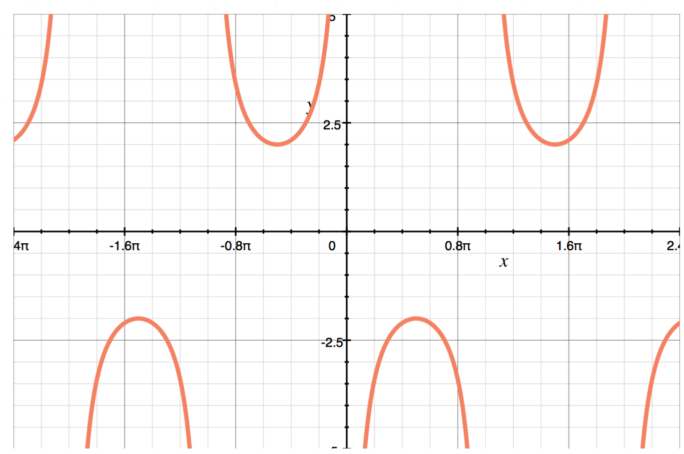
Question: Identify the graph of $y = -2\sec\theta$.

Answer choices:

A


B


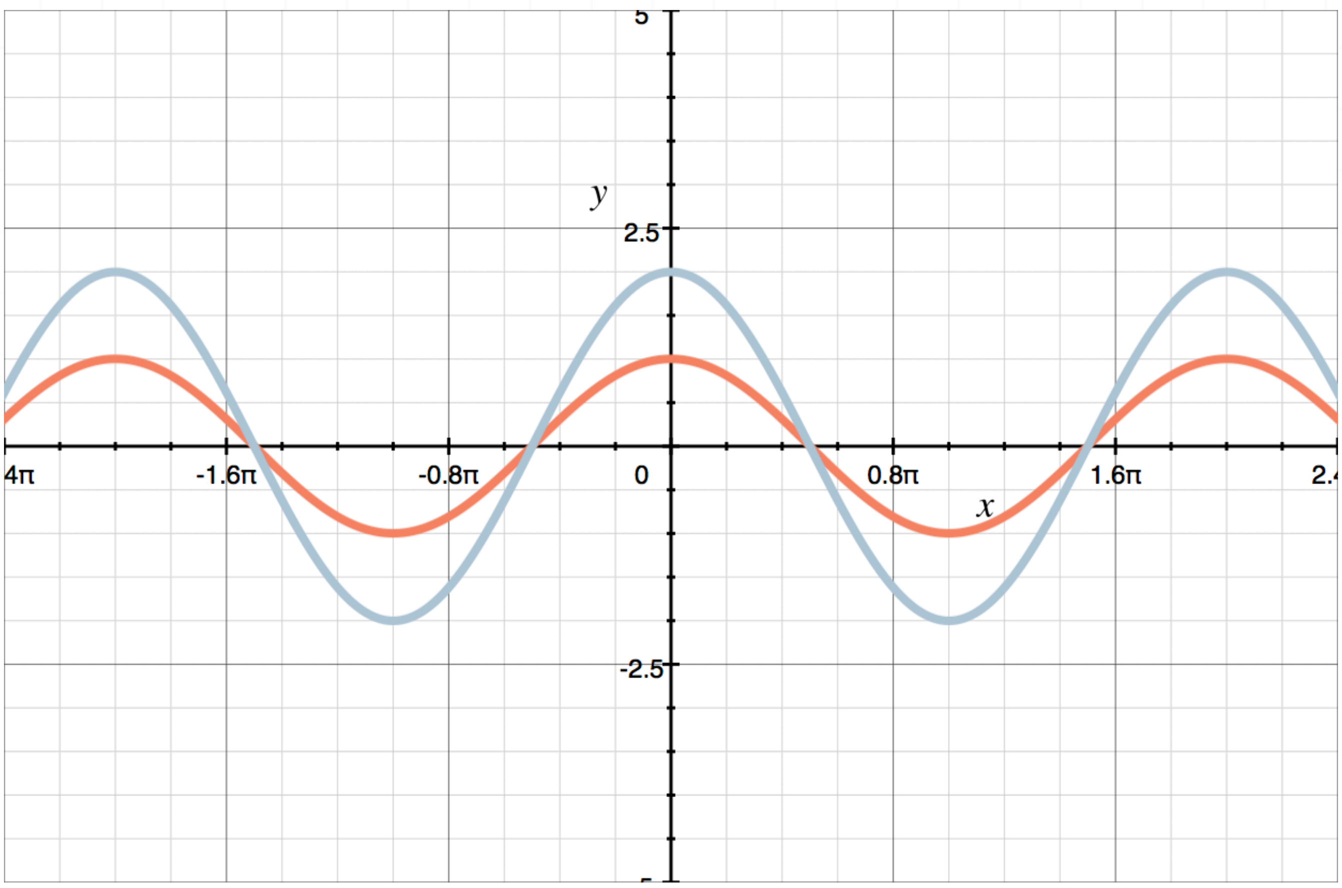
C


D


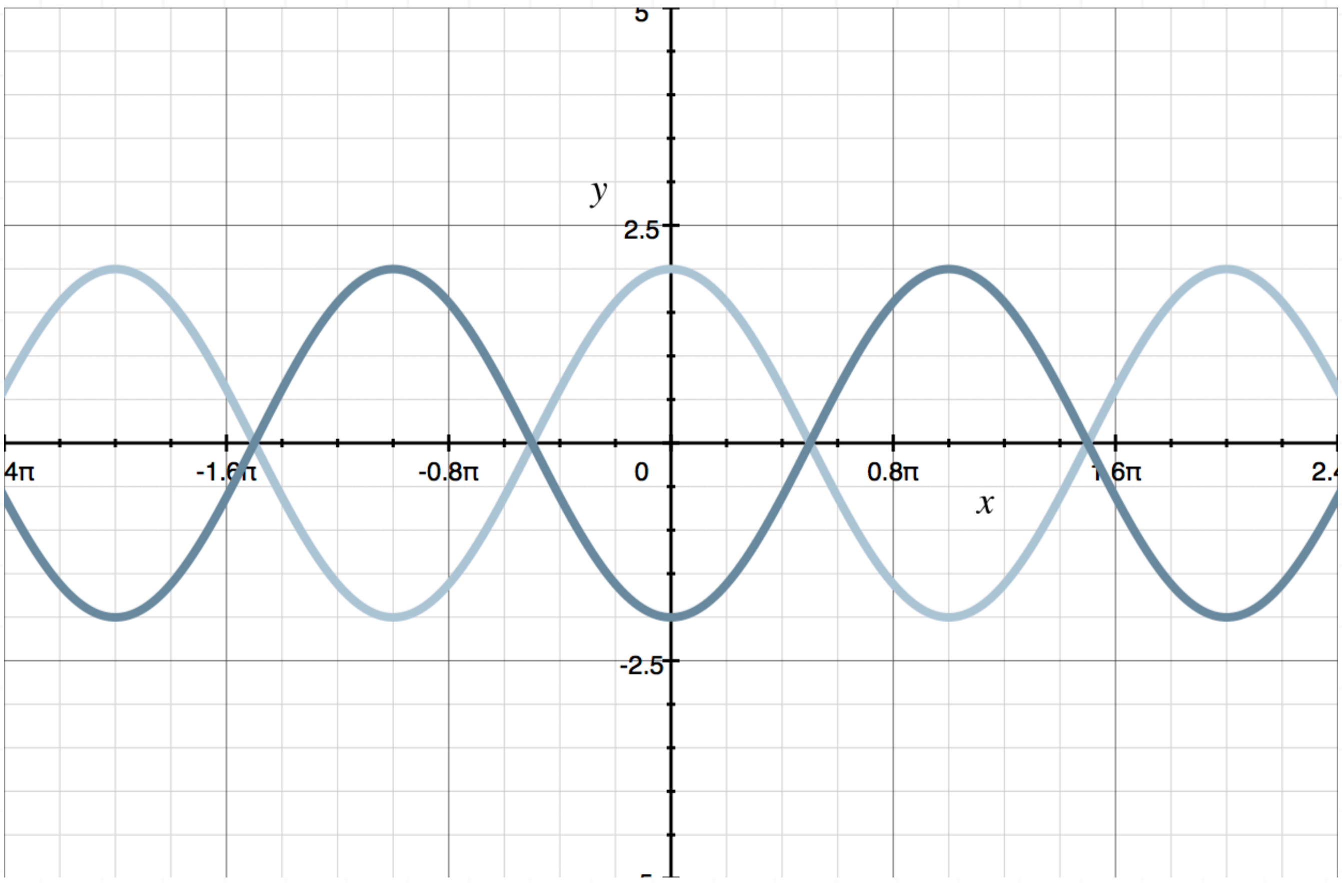
Solution: C

Because the reciprocal function of secant is cosine, we want to start by replacing secant with cosine in the function we've been given. In other words, the corresponding function is $y = -2\cos x$.

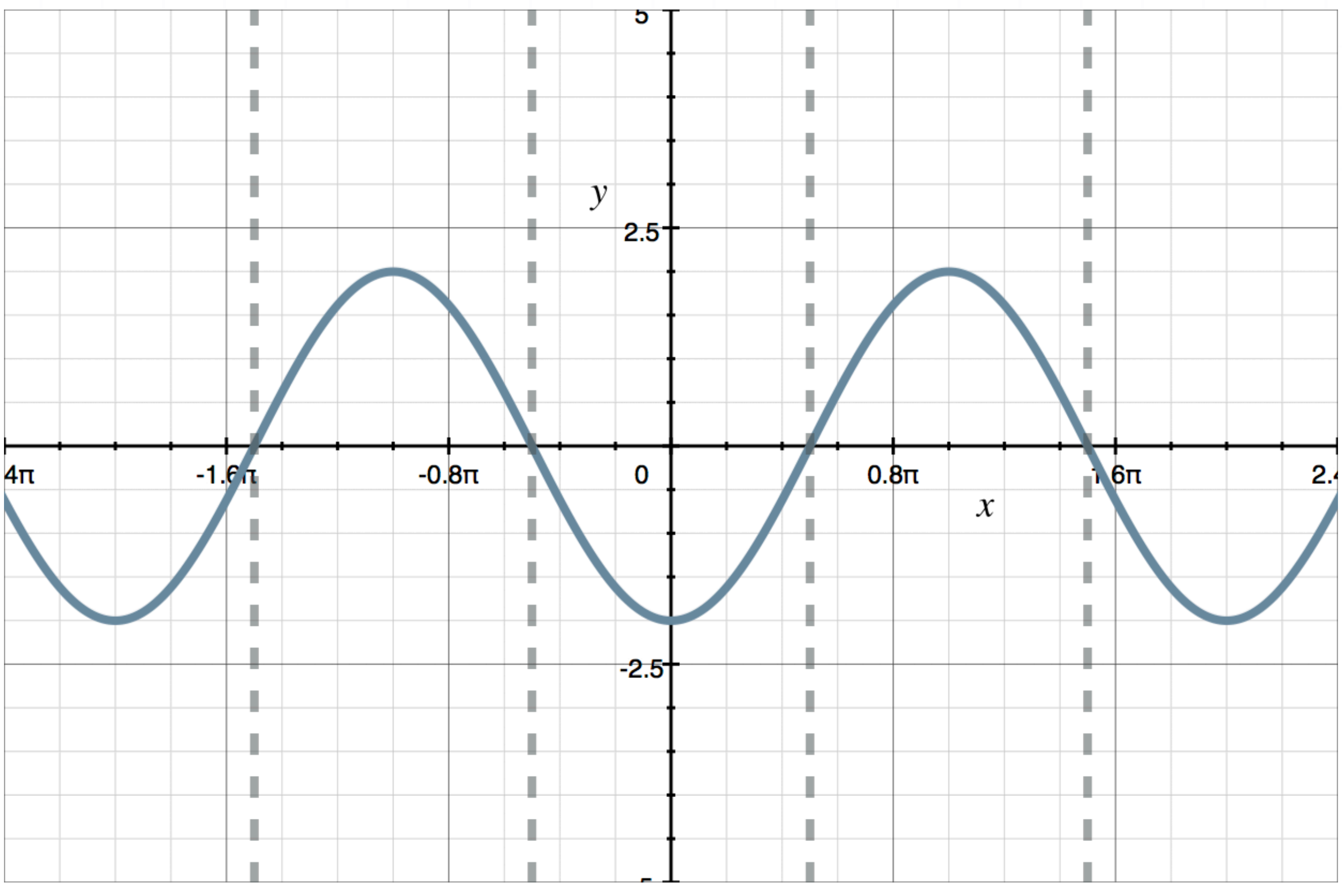
To graph $y = -2\cos x$, we see that $a = 2$, so we'll vertically stretch the cosine curve by doubling all the y-values. So if we sketch $y = \cos x$ in red and $y = 2\cos x$ in blue, we get



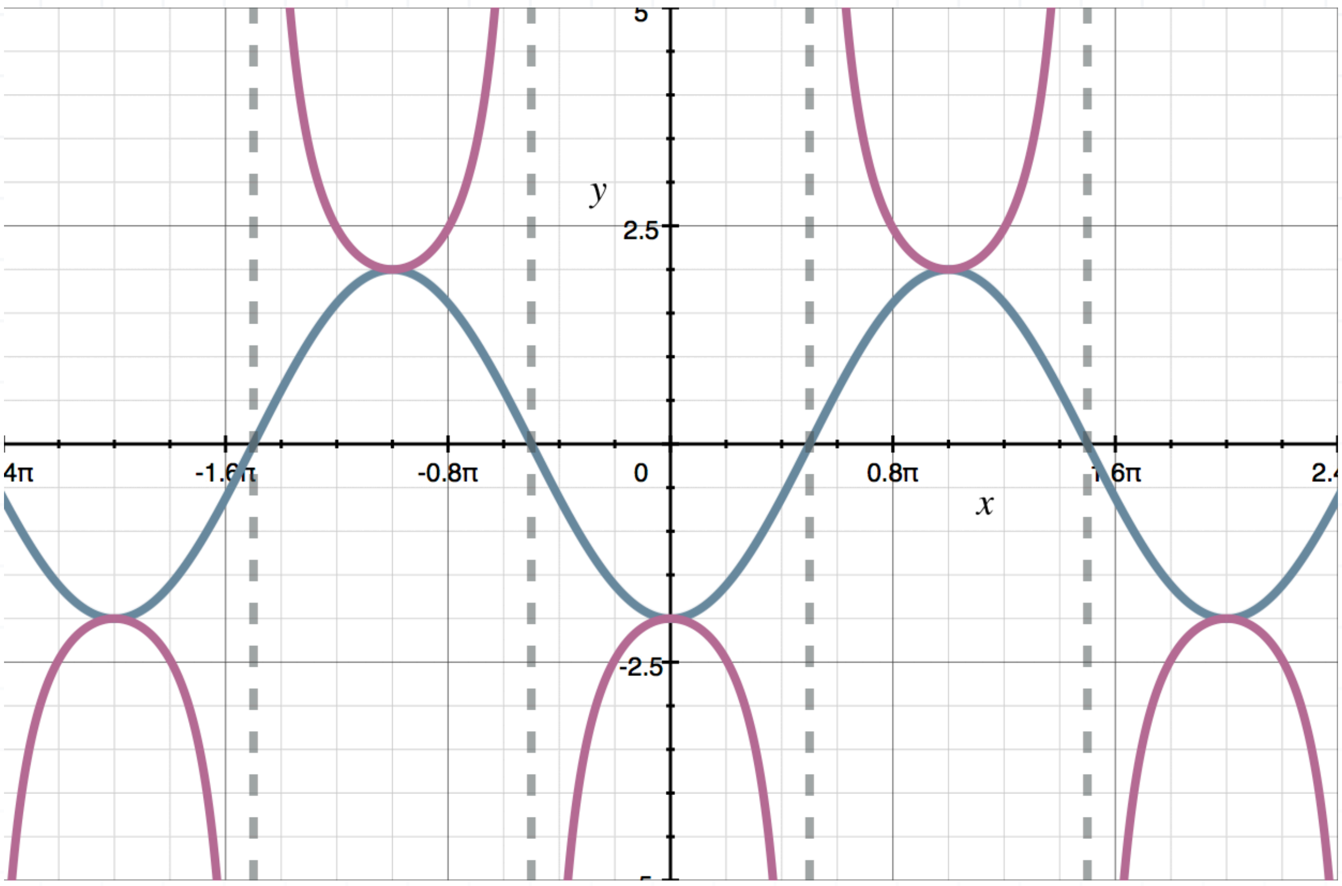
To get $y = -2\cos x$, we need to reflect the curve over the x-axis, which means we multiply the y-value in each point by -1. So if we sketch $y = 2\cos x$ in blue and $y = -2\cos x$ in dark blue, we get



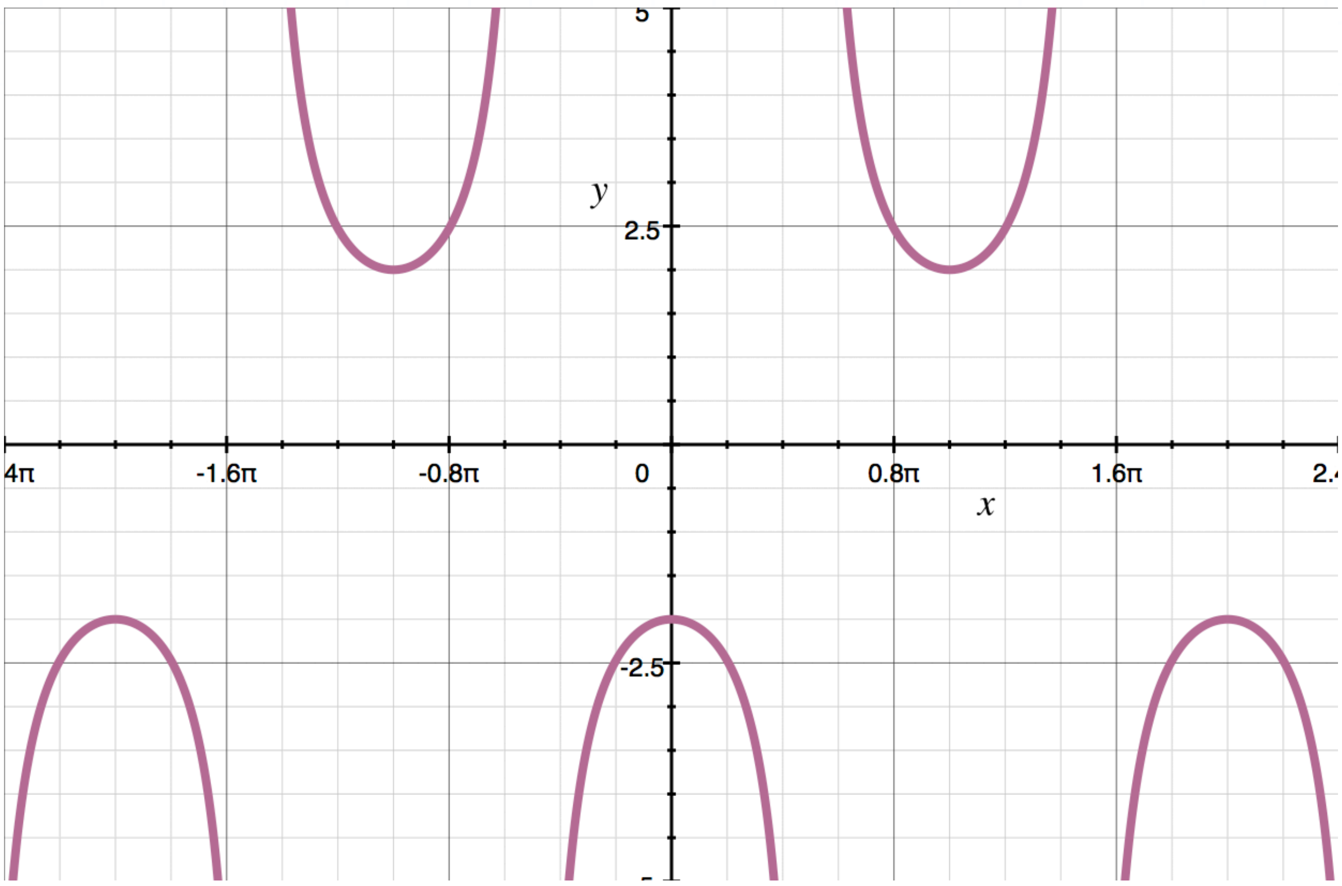
Then we'll sketch in vertical asymptotes at the midline of $y = -2\cos x$.



We'll sketch in the U-shapes for $y = -2\sec\theta$.



Finally, we'll take away the cosine curve and the vertical asymptotes to get the final sketch of $y = -2\sec\theta$.



Topic: Period and amplitude

Question: What are the amplitude and period of the function?

$$y = 6\csc(-2.9\theta)$$

Answer choices:

A The amplitude is -6, and its period is 2.9π.

B The amplitude is 6, and its period is 5.8π.

C The amplitude is undefined, and its period is $20\pi/29$.

D The amplitude is undefined, and its period is $\pi/2.9$.

Solution: C

Since we're dealing with a cosecant function, we can't consider the amplitude.

Because b is positive and the period of $y = \csc\theta$ is 2π, the period of $y = -6\csc(2.9\theta)$ is

$$\frac{2\pi}{|b|} = \frac{2\pi}{|2.9|} = \frac{20\pi}{29}$$

Topic: Period and amplitude

Question: What are the amplitude and period of the function?

$$y = -5\cos\frac{\theta}{7}$$

Answer choices:

A The amplitude is 5, and the period is 14π.

B The amplitude is -5, and the period is $2\pi/7$.

C The amplitude is 5, and the period is 2π.

D The amplitude is -5, and the period is 7π.

Solution: A

The function $y = -5\cos(\theta/7)$ is in the form $y = a\cos(b\theta)$ where $a = -5$ and $b = 1/7$. Since $a = -5$, the amplitude of the function is

$$|a| = |-5| = 5$$

Since b is positive and the period of $\cos\theta$ is 2π, the period of $y = -5\cos(\theta/7)$ is

$$\frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{1}{7}\right|} = 2\pi\left(\frac{7}{1}\right) = 14\pi$$

Topic: Period and amplitude

Question: Which statement is false?

Answer choices:

A The period of $-\pi \sin(\pi^2\theta)$ is $2/\pi$.

B The period of $\pi^3 \tan(4\theta/3)$ is $3\pi/4$.

C The period of $(-1.7/\pi)\sec(\theta/3)$ is 6π.

D The period of $2\cot(4\pi\theta/3)$ is $3/2$.

Solution: D

The function in answer choice D,

$$y = 2\cot\left(\frac{4\pi\theta}{3}\right)$$

is in the form $y = a\cot(b\theta)$ with $a = 2$ and $b = 4\pi/3$. Since b is positive and the period of $\cot\theta$ is π, the period of this function is

$$\frac{\pi}{|b|} = \frac{\pi}{\left|\frac{4\pi}{3}\right|} = \pi\left(\frac{3}{4\pi}\right) = \frac{3}{4} \neq \frac{3}{2}$$

Topic: Sketching tangent and cotangent

Question: What are the vertical asymptotes for of the function?

$$y = 3\cot\left(\frac{x}{2}\right)$$

Answer choices:

A $\dots, \ -\pi, \pi, 3\pi, \dots$

B $\dots, 0, 2\pi, 4\pi, \dots$

C $\dots, 0, \pi, 2\pi, \dots$

D There are no vertical asymptotes

Solution: B

To find adjacent vertical asymptotes for $y = a\cot(bx)$, solve $bx = 0$ and $bx = \pi$. The function $y = 3\cot(x/2)$ is in the form $y = a\cot(bx)$ where $a = 3$ and $b = 1/2$, so we find

$$\frac{x}{2} = 0$$

$$x = 0$$

and

$$\frac{x}{2} = \pi$$

$$x = 2\pi$$

Since b is positive and the period of $\cot\theta$ is π, the period is

$$\frac{\pi}{|b|} = \frac{\pi}{\left|\frac{1}{2}\right|} = \pi\left(\frac{2}{1}\right) = 2\pi$$

So based on everything we've found so far, we know the other asymptotes will be 2π more or less from the previous one, so we'll have asymptotes at

$$0 - 2\pi = -2\pi$$

$$2\pi + 2\pi = 4\pi$$

$$4\pi + 2\pi = 6\pi$$

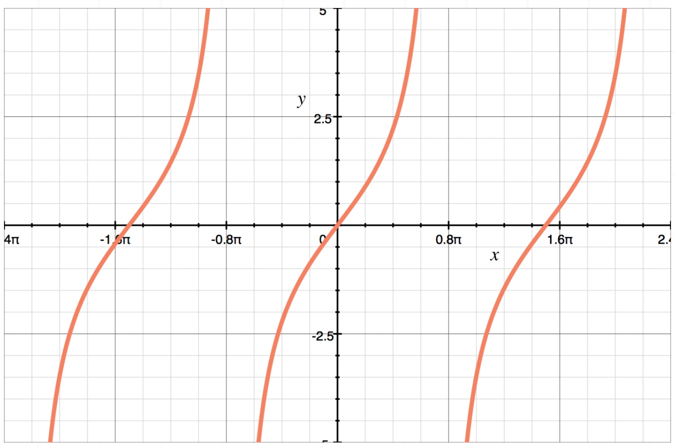
Therefore, the asymptotes for the function are

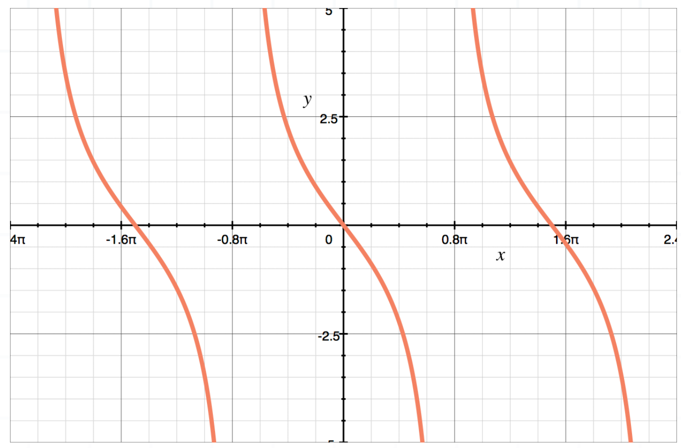
$$\ldots, \ -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$$

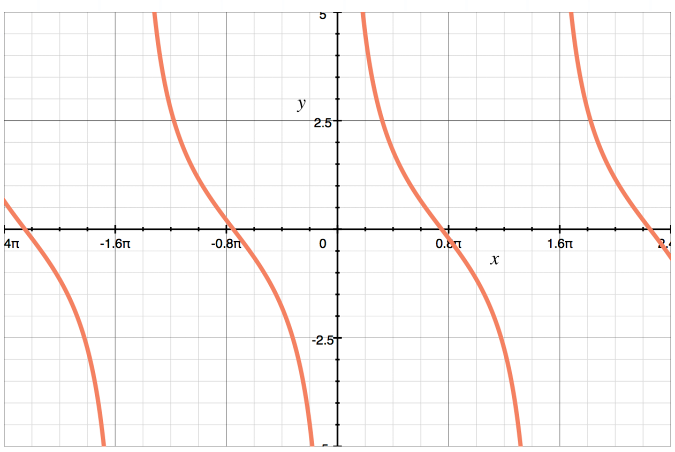
Topic: Sketching tangent and cotangent

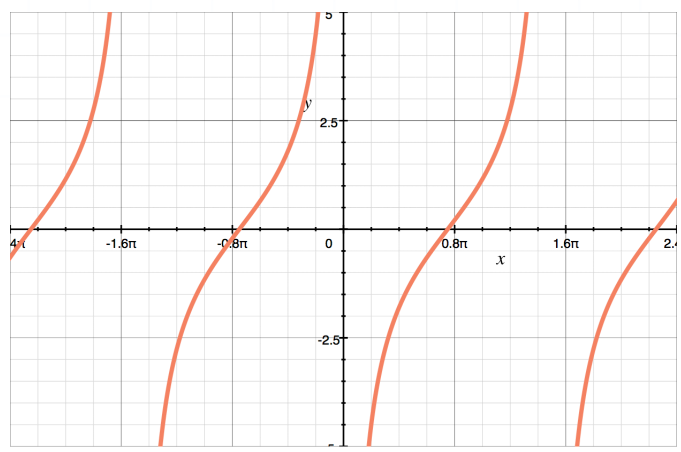
Question: Identify the graph of $y = 2\tan(2\theta/3)$.

Answer choices:

A


B


C


D


Solution: A

We'll find two adjacent vertical asymptotes by solving $bx = -\pi/2$ and $bx = \pi/2$ for x. With $b = 2/3$, we get

$$\frac{2x}{3} = -\frac{\pi}{2}$$

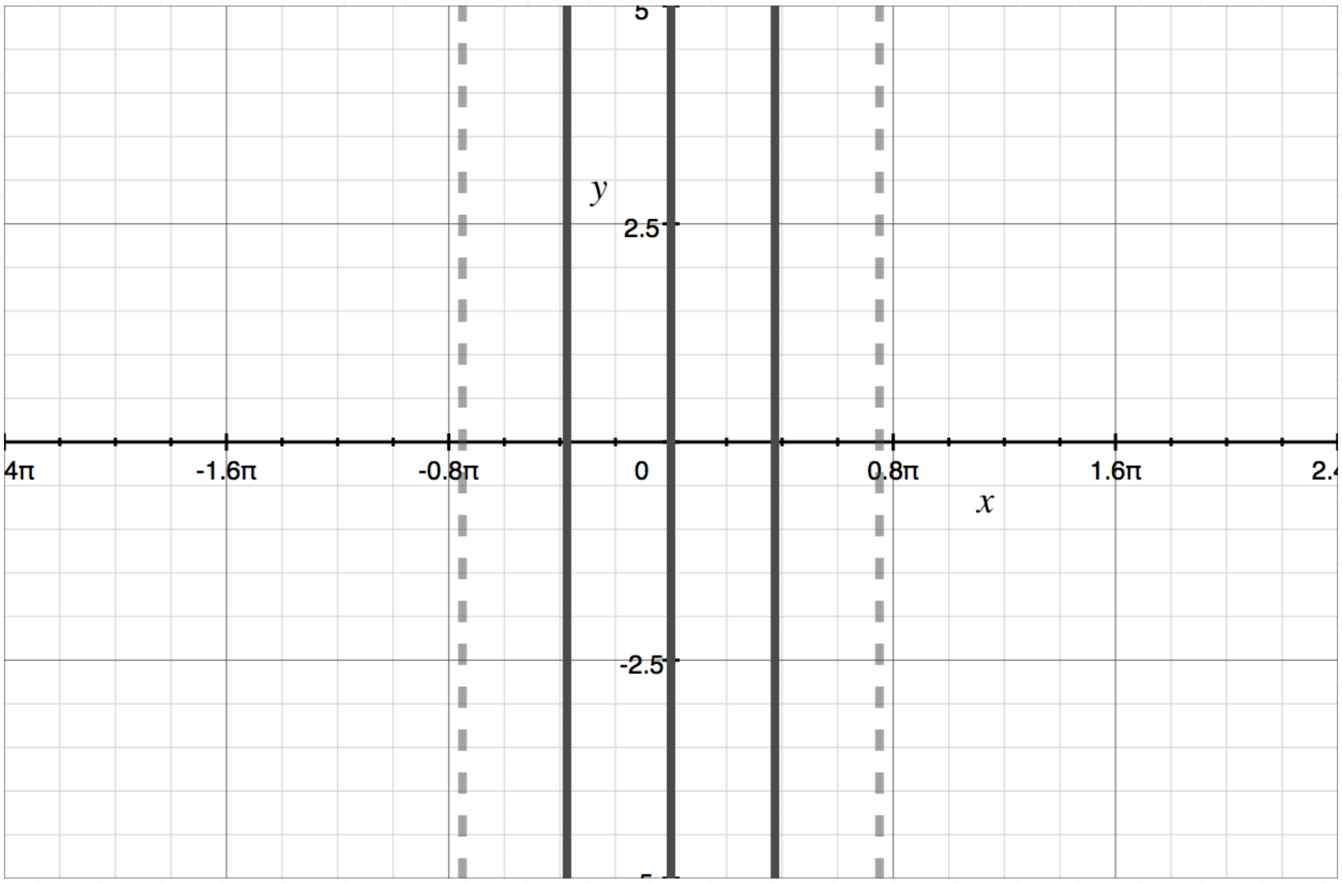
$$x = -\frac{3\pi}{4}$$

and

$$\frac{2x}{3} = \frac{\pi}{2}$$

$$x = \frac{3\pi}{4}$$

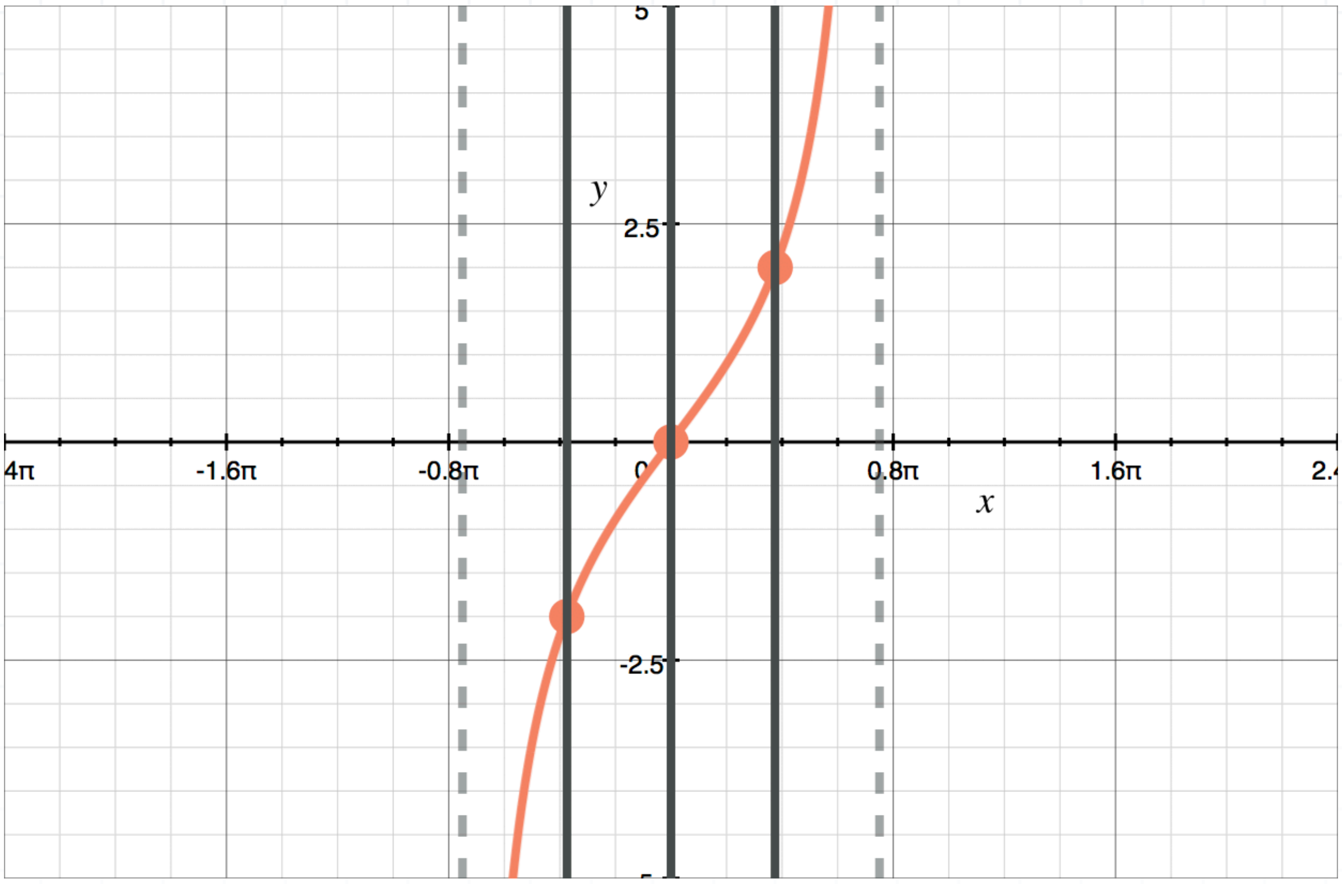
We'll sketch in the vertical asymptotes $x = -3\pi/4$ and $x = 3\pi/4$, and then divide the interval between $x = -3\pi/4$ and $x = 3\pi/4$ into four equal parts.



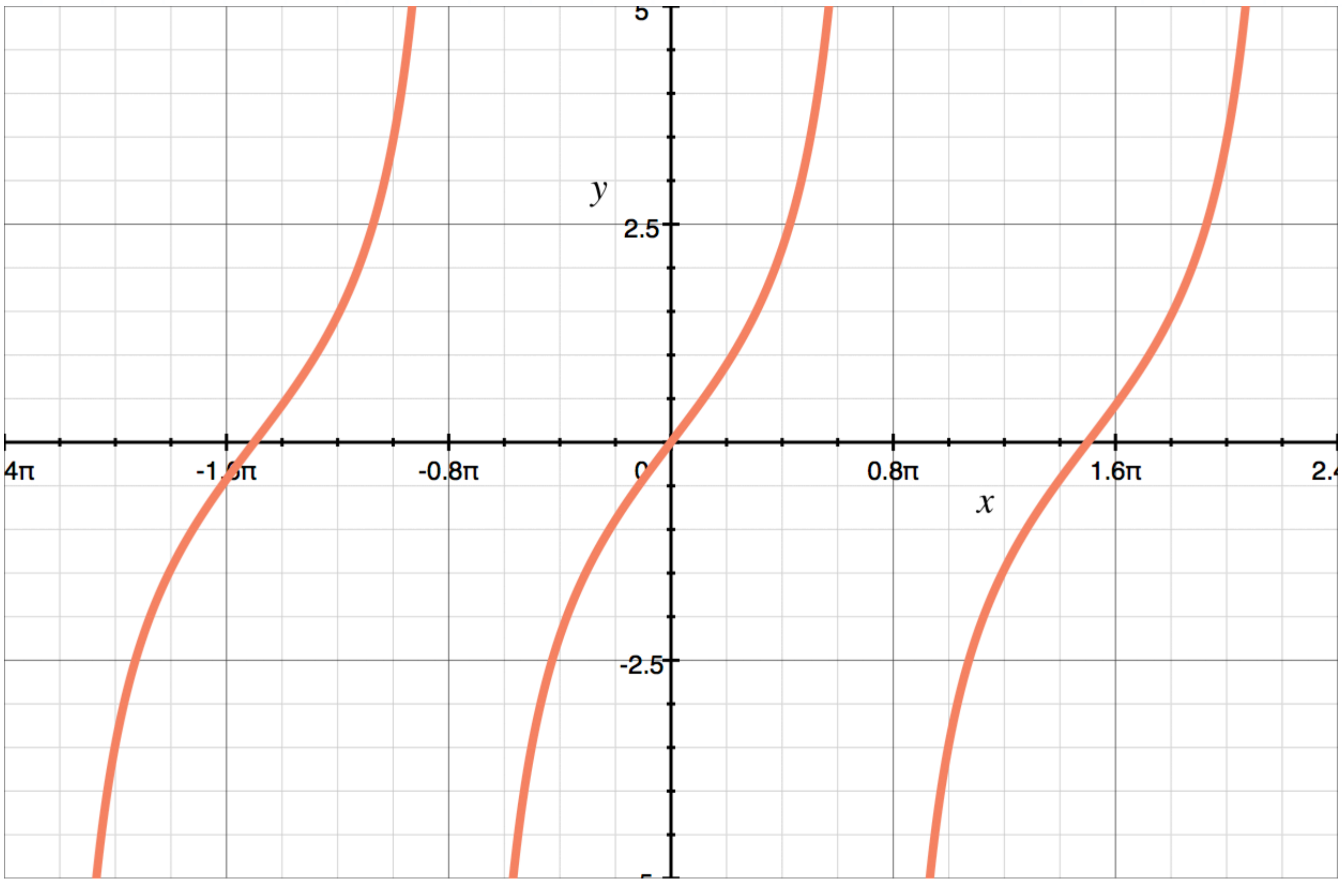
The dividing lines of each of these four sub-intervals are $x = -3\pi/8$, $x = 0$, and $x = 3\pi/8$, so we'll evaluate $y = 2\tan(2\theta/3)$ at those three values, and we'll get

$$\left(-\frac{3\pi}{8}, -2\right), (0{,}0) \text{ and } \left(\frac{3\pi}{8}, 2\right)$$

We'll plot those points, and then connect them with a smooth curve, respecting the asymptotes.



Finally, we'll repeat that pattern both to the left and right, and take away the asymptotes and other guiding lines that we sketched, and we'll get the final graph of $y = 2\tan(2\theta/3)$.



Topic: Sketching tangent and cotangent

Question: Where are the first pair of asymptotes of the function?

$$y = 5\tan\left(\frac{2\pi x}{15}\right)$$

Answer choices:

A $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

B $-\frac{15\pi}{4}$ and $\frac{15\pi}{4}$

C 0 and $\frac{15}{2}$

D $-\frac{15}{4}$ and $\frac{15}{4}$

Solution: D

To find adjacent vertical asymptotes for $y = a\tan(bx)$, solve the equations $bx = -\pi/2$ and $bx = \pi/2$. With $b = 2\pi/15$, we get

$$\frac{2\pi}{15}x = -\frac{\pi}{2}$$

$$x = -\frac{\pi}{2} \cdot \frac{15}{2\pi}$$

$$x = -\frac{15}{4}$$

and

$$\frac{2\pi}{15}x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} \cdot \frac{15}{2\pi}$$

$$x = \frac{15}{4}$$

Therefore, the first pair of asymptotes of the function are $-15/4$ and $15/4$.

Topic: Horizontal and vertical shifts

Question: Which table shows values that satisfy the function?

$$f(x) = -\sin\left(x - \frac{\pi}{6}\right)$$

Answer choices:

A

x	4π/3	3π/2	5π/3	11π/6	2π
f(x)	-1	-√3/2	-1/2	0	1/2

B

x	4π/3	3π/2	5π/3	11π/6	2π
f(x)	-√3/2	-1	-√3/2	-1/2	1/2

C

x	4π/3	3π/2	5π/3	11π/6	2π
f(x)	1/2	√3/2	1	√3/2	1/2

D

x	4π/3	3π/2	5π/3	11π/6	2π
f(x)	1/2	0	-1/2	-√3/2	-1

Solution: C

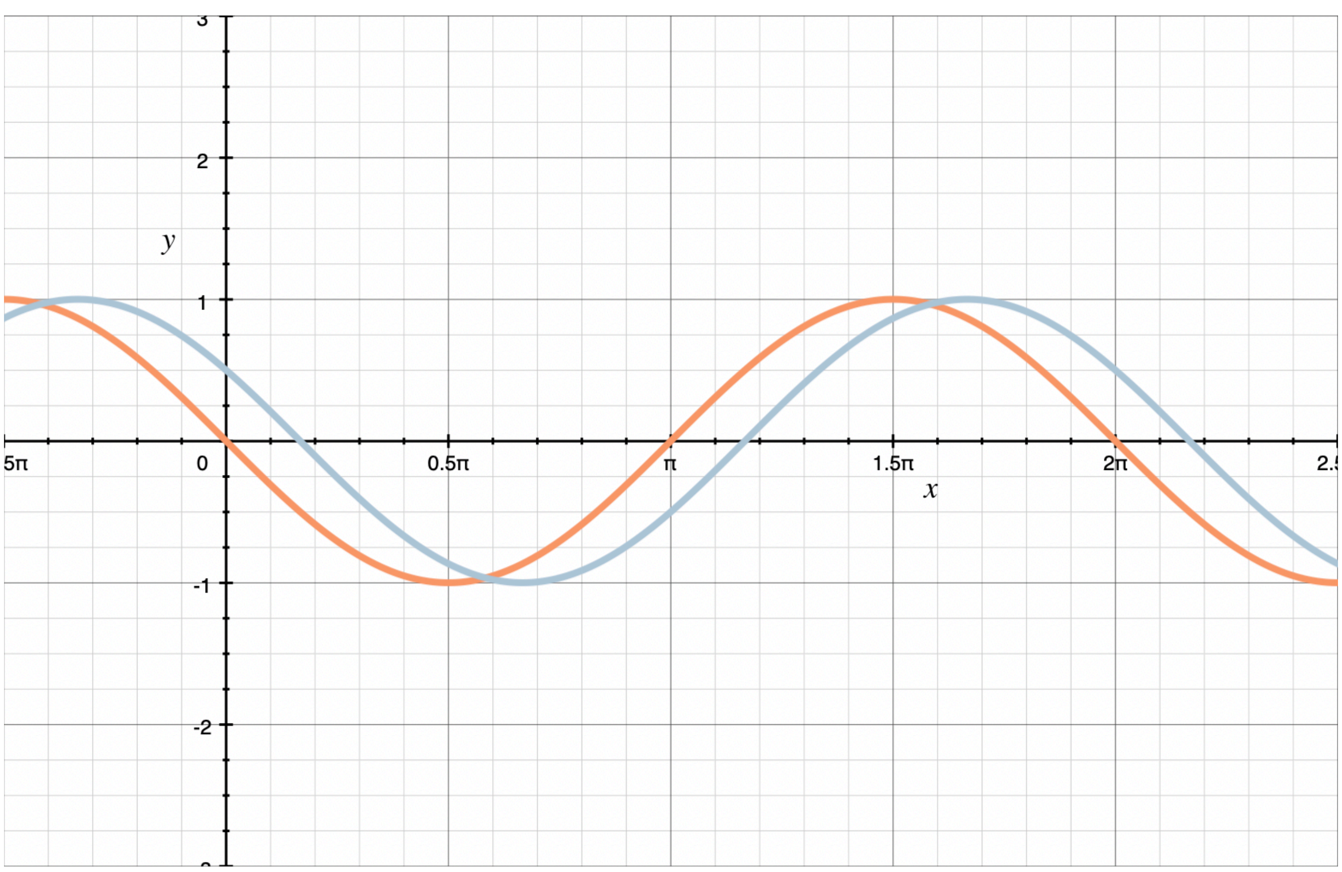
To start, ignore the negative sign in front of the sine function, and calculate values for $x - (\pi/6)$ and then $\sin(x - (\pi/6))$ at the indicated values of x.

x	4π/3	3π/2	5π/3	11π/6	2π
x-π/6	7π/6	4π/3	3π/2	5π/3	11π/6
sin(x-(π/6))	-1/2	-√3/2	-1	-√3/2	-1/2

Then to find the value of $f(x)$, we just need to multiply this last row by -1.

x	4π/3	3π/2	5π/3	11π/6	2π
f(x)	1/2	√3/2	1	√3/2	1/2

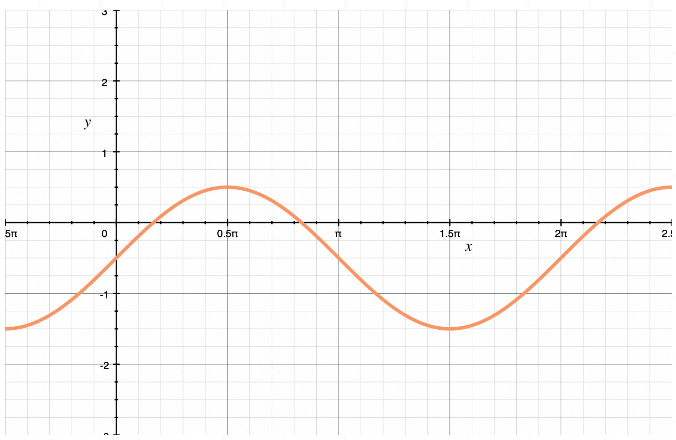
We can also visualize the $\pi/6$ shift to the right by graphing $y = -\sin x$ (in red) and the given function (in blue) on the same set of axes.

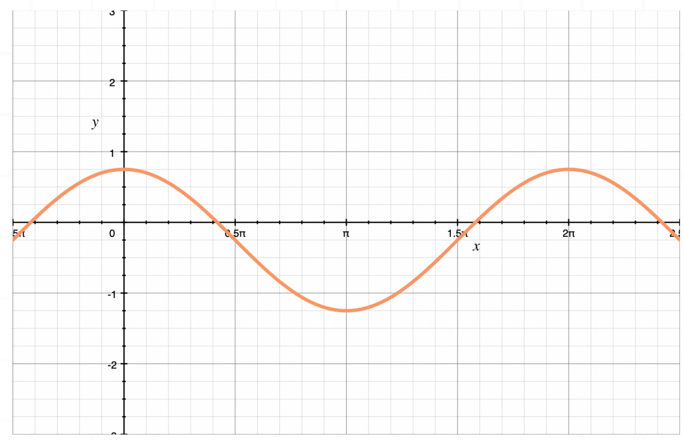


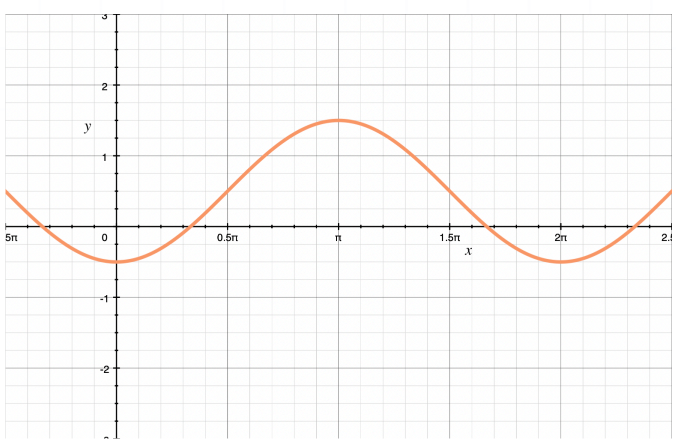
Topic: Horizontal and vertical shifts

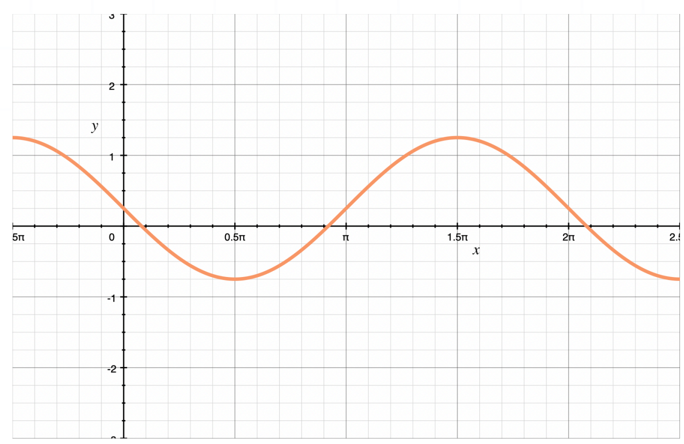
Question: Which curve is the graph of $y = \cos\theta - 0.25$?

Answer choices:

A 

B 

C 

D 

Solution: B

The function $y = \cos\theta - 0.25$ is the result of starting with the basic cosine function, $y = \cos\theta$ and shifting it down vertically by 0.25 units.

Therefore, we're looking for a cosine function that ranges from -1.25 to 0.75.

Topic: Horizontal and vertical shifts

Question: Consider the following statements.

I. The graph of $y = \sin(\theta - (\pi/2))$ on the interval $[0,2\pi)$ is identical to the graph of $y = \cos\theta$ on the same interval.

II. The graph of $y = \sin(\theta - (\pi/2))$ on the interval $[0,2\pi)$ is identical to the graph of $y = -\cos\theta$ on the same interval.

Answer choices:

A Statement I is true, and statement II is false.

B Statement I is false, and statement II is true.

C Statements I and II are both true.

D Statements I and II are both false.

Solution: B

Answer choice C can be eliminated, because it implies that the graphs of $y = \cos\theta$ and $y = -\cos\theta$ are identical, which they aren't.

Consider Statement II, and compare the values of $y = \sin(\theta - (\pi/2))$ and $y = -\cos\theta$ at key angles in the interval $[0,2\pi)$.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\left(\theta - \frac{\pi}{2}\right)$	-1	0	1	0
$-\cos\theta$	-1	0	1	0

It appears that Statement II is true, which means automatically that Statement I is false.

Topic: Graphing transformations

Question: Which transformations must be applied to $y = \sin\theta$ to get to the following function?

$$y = 6\left(\sin\left(\frac{2\theta}{5} - \frac{\pi}{10}\right) - 5\right)$$

Answer choices:

A A horizontal compression, a horizontal shift to the right, a vertical stretch, and a vertical shift down.

B A horizontal stretch, a horizontal shift to the left, a vertical compression, and a vertical shift up.

C A horizontal compression, a horizontal shift to the left, a vertical compression, and a vertical shift down.

D A horizontal stretch, a horizontal shift to the right, a vertical stretch, and a vertical shift down.

Solution: D

Rewrite the given function in the form $a\sin(b(\theta + c)) + d$.

$$y = 6\left(\sin\left(\frac{2\theta}{5} - \frac{\pi}{10}\right) - 5\right)$$

$$y = 6\sin\left(\frac{2\theta}{5} - \frac{\pi}{10}\right) - 30$$

$$y = 6\sin\left(\frac{2}{5}\left(\theta - \frac{\pi}{4}\right)\right) - 30$$

From this rewritten form, $a = 6$, $b = 2/5$, $c = -\pi/4$, and $d = -30$. Therefore, the series of transformations must be

1. Horizontally stretch $y = \sin\theta$ by a factor of $5/2$

2. Horizontally shift the result to the right by $\pi/4$

3. Vertically stretch the result by a factor of 6

4. Vertically shift the result down by 30

Topic: Graphing transformations

Question: Which function is the result of only a vertical shift and a horizontal compression applied to the basic cosine function?

Answer choices:

A $y = \cos\theta - 2.9$

B $y = 2\cos\left(\theta + \dfrac{\pi}{8}\right)$

C $y = \cos\dfrac{3\theta}{2} + 0.2$

D $y = \cos\left(\dfrac{\theta}{4} - \dfrac{\pi}{3}\right)$

Solution: C

Answer choices B and D don't have a vertical shift and answer choice A doesn't have a horizontal compression. We can rewrite the function in answer choice C as

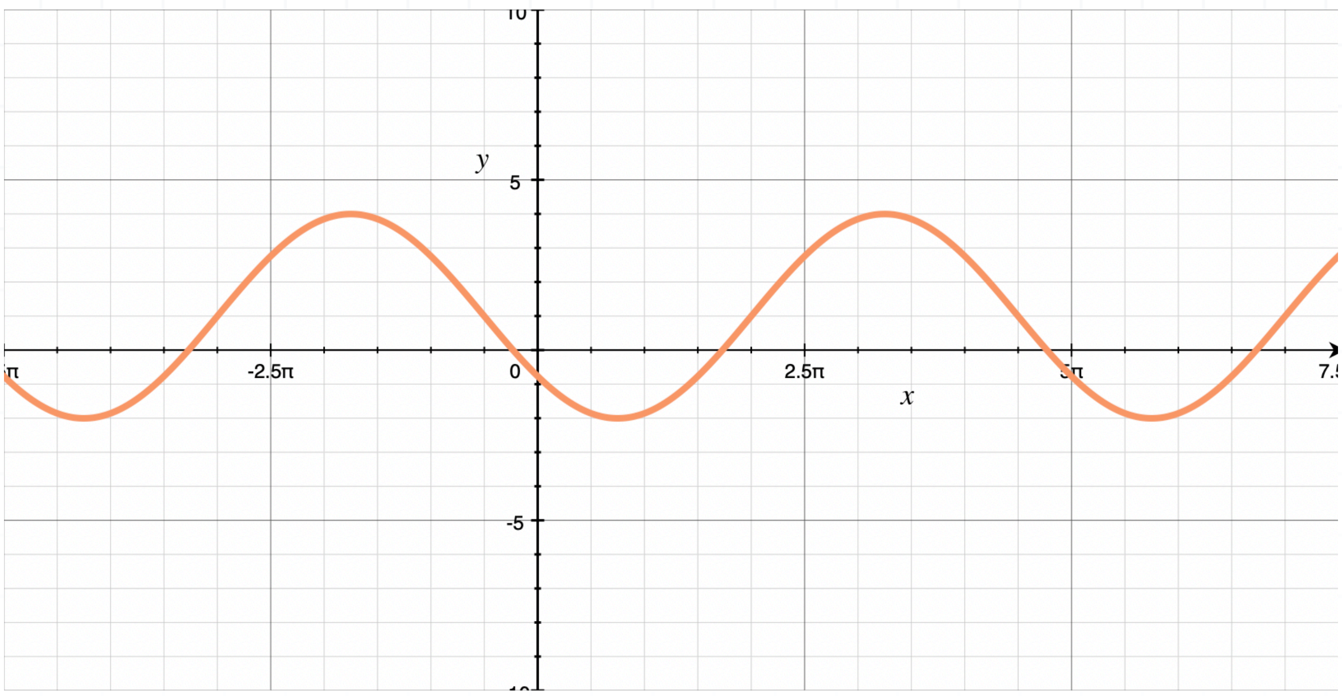
$$y = \cos\left(\frac{3}{2}(\theta + 0)\right) + 0.2$$

Now the function is in the form $y = a\cos(b(\theta + c)) + d$ with $a = 1$, $b = 3/2$, $c = 0$, and $d = 0.2$.

- Since $a = 1$, there's no vertical stretch or compression.
- Since $b = 3/2$, there's a horizontal compression.
- Since $c = 0$, there's no horizontal shift.
- Since $d = 0.2$, there is a vertical shift up.

Topic: Graphing transformations

Question: The graph shows a sine or cosine function with a period equal to 5π. What is the function?



Answer choices:

A $\quad 3\cos\left(\frac{\theta}{5}+\frac{\pi}{10}\right)-1$

B $\quad -3\sin\left(\frac{2\theta}{5}+\frac{\pi}{5}\right)+1$

C $\quad 2\sin\left(\frac{2\theta}{5}+\frac{\pi}{10}\right)+2$

D $\quad -2\cos\left(\frac{\theta}{10}+\frac{2\pi}{5}\right)-1$

Solution: B

The given curve ranges from -2 to 4, so the amplitude of the function must be

$$\frac{1}{2}(4-(-2))=\frac{1}{2}(6)=3$$

This rules out answer choices C and D, which both have an amplitude of 2.

If the amplitude is 3, and the curves ranges from -2 to 4, we can subtract the amplitude from the upper end of the range and add the amplitude to the lower end of the range.

$$4-3=1$$

$$-2+3=1$$

Because we get 1 in both cases, it tells us the function has a vertical shift of $+1$.

Since the period of sine and cosine functions is always given by $2\pi/|b|$, and the period of the given function is 5π, we set up an equation that lets us solve for the value of b.

$$\frac{2\pi}{|b|}=5\pi$$

$$|b|=\frac{2\pi}{5\pi}$$

$$|b|=\frac{2}{5}$$

$$b = \pm\frac{2}{5}$$

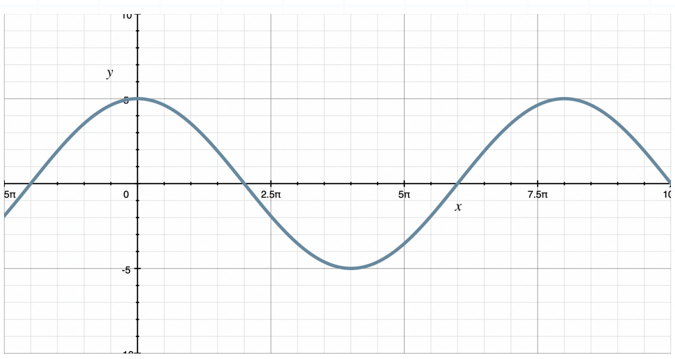
Answer choice B is the only function that meets both requirements.

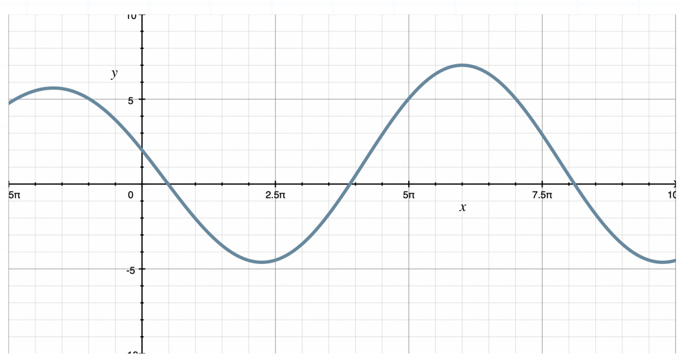
Topic: Graphing combinations

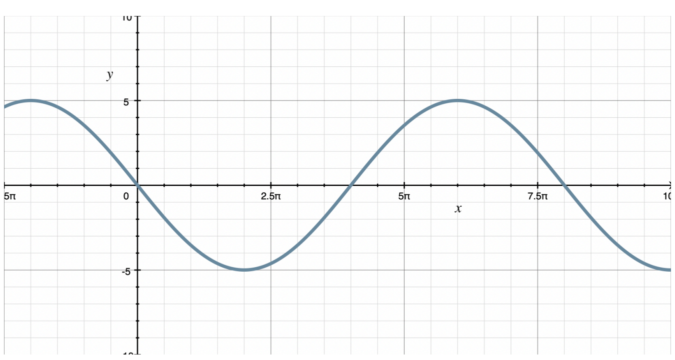
Question: Which is the graph of the function on the interval $[0,7\pi)$?

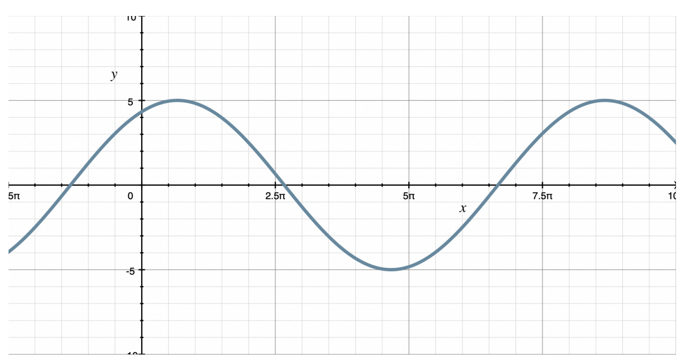
$$5\sin\left(\frac{\theta}{4}+\pi\right)-\left[\cos\left(\frac{\theta}{3}+\pi\right)-1\right]$$

Answer choices:

A 

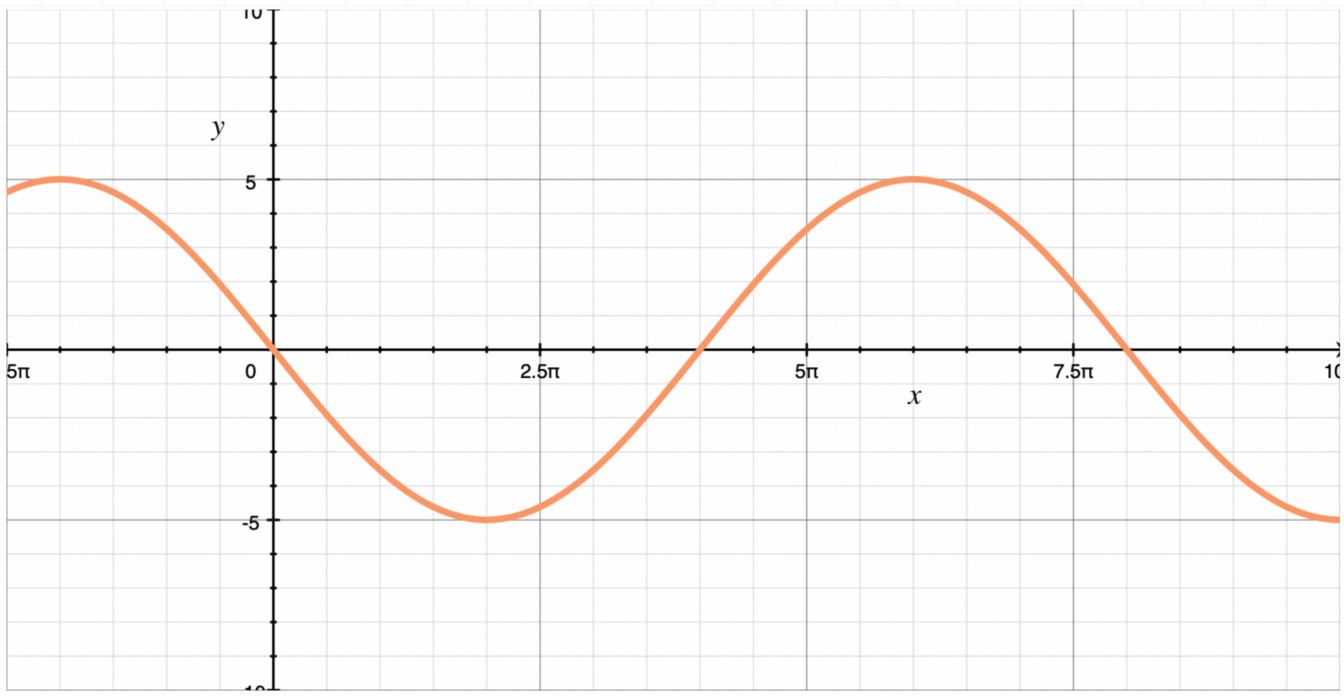
B 

C 

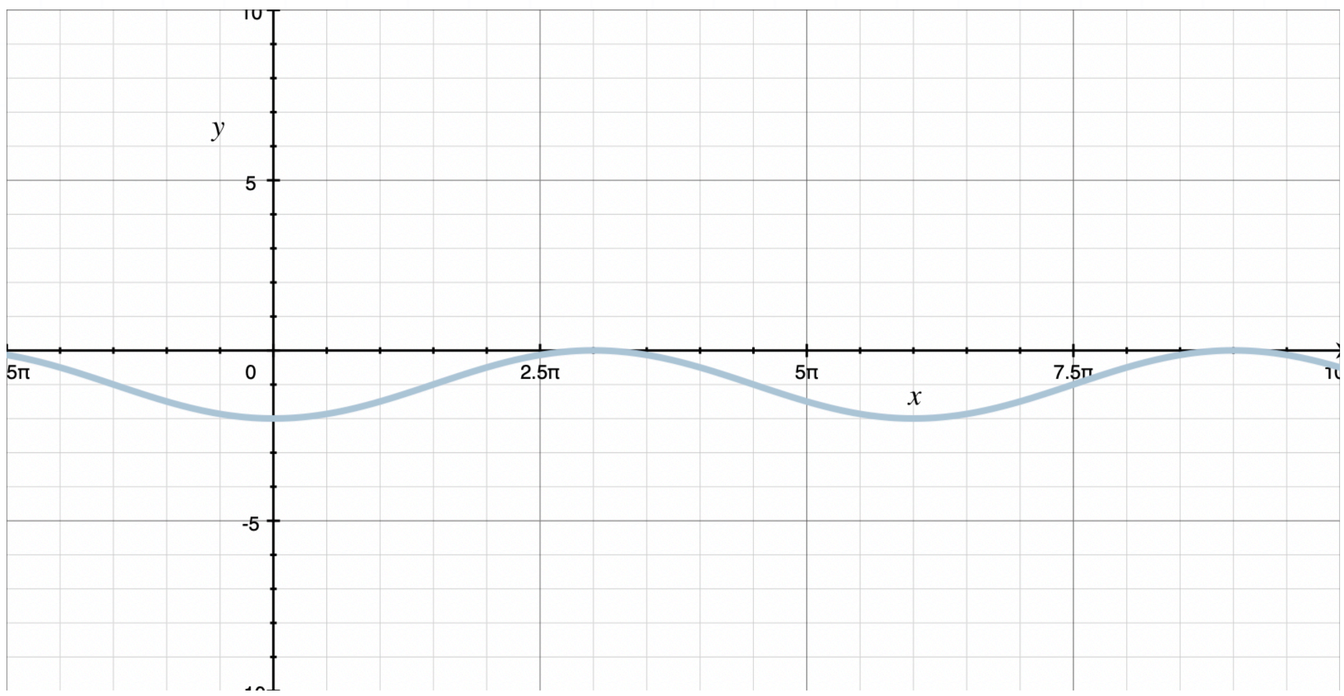
D 

Solution: B

If we graph both functions in the combination, the graph of $5\sin((\theta/4)+\pi)$ is



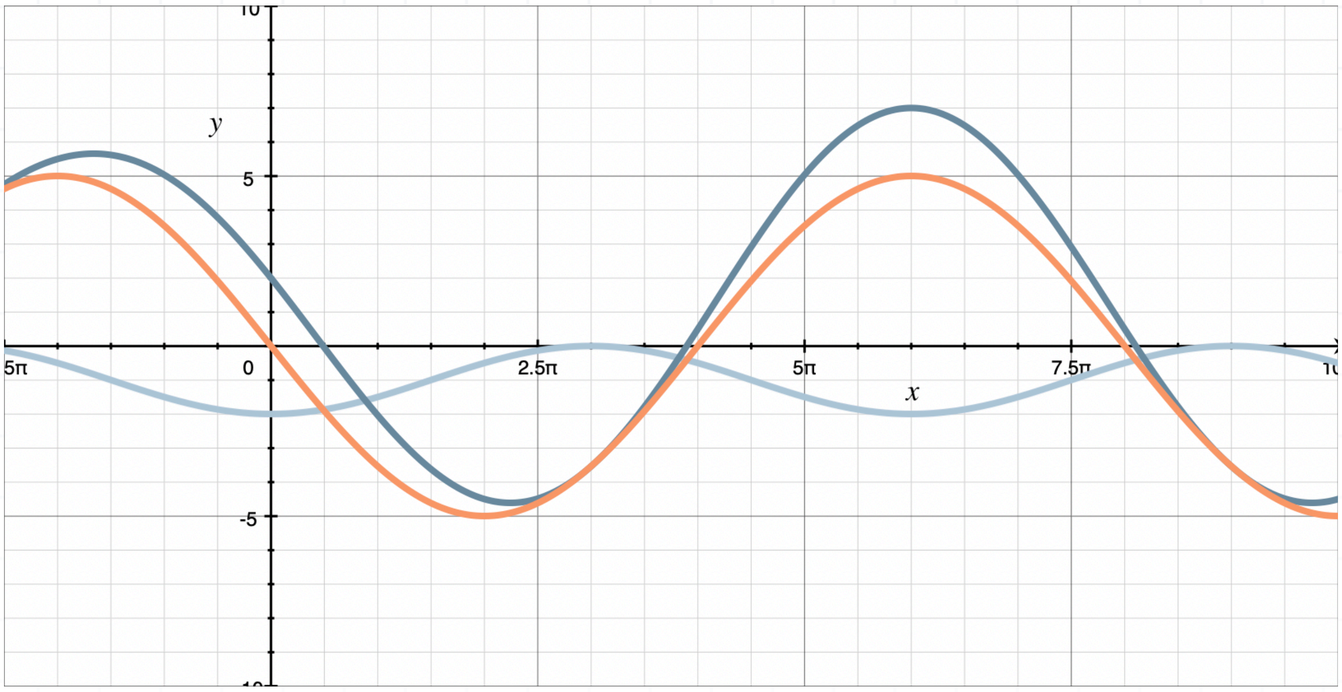
and the graph of $\cos((\theta/3)+\pi)-1$ is



From the graphs of these two functions, we can see that the value of the sine function is 0 at $\theta=0$, and that the value of the cosine function is -2 at $\theta=0$. Which means the combination at $\theta=0$ will have a value of

$$0-(-2)=0+2=2$$

The only curve that has that value at $\theta = 0$ is the curve in answer choice B.



Topic: Graphing combinations

Question: What is the period of the function?

$$\sin(4x)\left(3\cos\left(\frac{x}{2}\right)\right)$$

Answer choices:

A $\frac{\pi}{2}$

B π

C 2π

D 4π

Solution: D

The period of $\sin(4x)$ is

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

and the period of $3\cos(x/2)$ is

$$\frac{2\pi}{\frac{1}{2}} = 4\pi$$

When the combination function is a product or quotient like it is in this problem, we take the least common multiple of the periods of the original functions as the period of the combination.

The least common multiple of $\pi/2$ and 4π is 4π, so the period of the combination is 4π.

Topic: Graphing combinations

Question: Which point does the function pass through at $\theta = 0$?

$$\cos(2\theta) + 2\sin\left(3\theta - \frac{\pi}{2}\right)$$

Answer choices:

A $(0, -1)$

B $(0, -2)$

C $(0,0)$

D $(0,1)$

Solution: A

We can find the value of the combination simply by adding together the values of the two functions at the given angle.

At $\theta = 0$, $\cos(2\theta)$ has a value of 1, and $2\sin(3\theta - (\pi/2))$ has a value of -2, which means the original expression will have a value of $1 + (-2) = -1$.

Therefore, when $\theta = 0$, the function passes through $(0, -1)$.

Topic: Inverse trig relations

Question: Which equation can't be used to express an inverse relation?

Answer choices:

A $\arcsin(x) = \arcsin(\sin y)$

B $y = \arccos x$

C $y = \csc^{-1} x$

D $y = \dfrac{1}{\tan x}$

Solution: D

We indicate inverse sine as either $\sin^{-1}$ or as $\arcsin$ and the same for the other functions.

In Algebra, we would make a negative exponent positive by moving the term to the denominator. For instance, x^{-2} could be rewritten as $1/(x^2)$. But the -1 in $\tan^{-1}$ isn't a negative exponent, it's simply notation to indicate "inverse tangent." So

$$\tan^{-1} x \neq \frac{1}{\tan x}$$

Topic: Inverse trig relations

Question: Use the unit circle to find the angle in degrees whose sine is $\sqrt{3}/2$.

Answer choices:

A 30°

B 120°

C 150°

D 330°

Solution: B

On the unit circle, we know that the y-value in the coordinate point is the value that gives us the sine of the angle. Therefore, because we're told that sine of the angle is $\sqrt{3}/2$, we need to find the angles in the unit circle where the corresponding coordinate point has a y-value equal to $\sqrt{3}/2$. Those angles are 60° and 120°.

Topic: Inverse trig relations

Question: Use the unit circle to say which angle has a cosine whose value isn't 0.

Answer choices:

A $\frac{\pi}{2}$

B π

C $\frac{3\pi}{2}$

D $-\frac{\pi}{2}$

Solution: B

On the unit circle, we know that the x-value in the coordinate point is the value that gives us the cosine of the angle. Therefore, because we're told that cosine of the angle is 0, we need to find the angles in the unit circle where the corresponding coordinate point has an x-value equal to 0.

Those angles are $\pi/2$ and $3\pi/2$. To give the full set of radian angles, we have to give all of the angles that are coterminal with these two.

$$\theta = \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{3\pi}{2} + 2n\pi\right\}$$

Because $\pi/2$ and $3\pi/2$ differ by an angle of just π, we can consolidate this set of angles into just

$$\theta = \left\{\frac{\pi}{2} + n\pi\right\}$$

Answer choice A is in this set when $n = 0$, answer choice C is in this set when $n = 1$, and answer choice D is in this set when $n = -1$. Only answer choice B is outside this angle set.

Topic: Inverse trig functions

Question: On what range is the inverse secant function defined?

Answer choices:

A $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

B $(-\infty,\infty)$

C $(-\infty,1]\cup[1,\infty)$

D $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$

Solution: D

The range of the secant function

$$y = \sec^{-1} x = \frac{1}{\cos^{-1} x}$$

is $0 \leq y \leq \pi$, excluding $y = \pi/2$.

Topic: Inverse trig functions

Question: Find the exact value of the inverse cosine function.

$$\arccos\left(-\frac{\sqrt{3}}{2}\right)$$

Answer choices:

A $\quad \frac{5\pi}{6}$

B $\quad \frac{4\pi}{3}$ and $\frac{5\pi}{3}$

C $\quad \frac{4\pi}{3}$

D $\quad \frac{5\pi}{6}$ and $\frac{7\pi}{6}$

Solution: A

If we look at the unit circle, we can see that the cosine function is $-\sqrt{3}/2$ when $\theta = 5\pi/6$ and when $\theta = 7\pi/6$. But because we're dealing with the inverse cosine function, we only want an angle in the interval $[0,\pi]$.

The angle $\theta = 5\pi/6$ is the only angle in $[0,\pi]$, so

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

Topic: Inverse trig functions

Question: Find the value of the inverse cosecant function.

$$\csc^{-1}(\sqrt{2})$$

Answer choices:

A $\dfrac{3\pi}{4}$

B $\dfrac{7\pi}{4}$

C $\dfrac{\pi}{4}$

D $\dfrac{\pi}{4}$ and $\dfrac{3\pi}{4}$

Solution: C

The reciprocal identity for cosecant tells us that

$$\csc\theta = \frac{1}{\sin\theta}$$

To get the inverses for the reciprocal functions, you do the same thing, but we'll take the reciprocal of what's in the parentheses and then use the "normal" trig functions.

To get $\csc^{-1}(\sqrt{2})$, we have to look for $\sin^{-1}(1/\sqrt{2})$, which gives $\pi/4$ and $3\pi/4$. But because we're dealing with the inverse sine function, we only want an angle in the interval $[-\pi/2,\pi/2]$. Therefore, $\csc^{-1}(\sqrt{2}) = \sin^{-1}(1/\sqrt{2}) = \pi/4$, or 45°.

Topic: Trig functions of inverse trig functions

Question: Find the value of the expression.

$$\cos^{-1}\left(\cos\left(\frac{3\pi}{8}\right)\right)$$

Answer choices:

A $\frac{3\pi}{8}$

B $\frac{8}{3\pi}$

C $-\frac{3\pi}{8}$

D $\frac{5\pi}{8}$

Solution: A

The inverse property $\cos^{-1}(\cos x) = x$ applies for every x in $[0,\pi]$. This value of $x = 3\pi/8$ lies in $[0,\pi]$, which is the domain of the cosine function. Therefore we can use the inverse property $\cos^{-1}(\cos x) = x$. Therefore,

$$\cos^{-1}\left(\cos\left(\frac{3\pi}{8}\right)\right) = \frac{3\pi}{8}$$

Topic: Trig functions of inverse trig functions

Question: Find the value of the expression.

$$\cos\left(\sin^{-1}\left(\frac{8}{17}\right)\right)$$

Answer choices:

A $\frac{8}{15}$

B $\frac{8}{17}$

C $\frac{15}{17}$

D $\frac{17}{15}$

Solution: C

Let θ represent the angle in the interval $[-\pi/2,\pi/2]$ whose sine is $8/17$. Then we can say

$$\theta = \sin^{-1}\left(\frac{8}{17}\right)$$

$$\sin\theta = \frac{8}{17}$$

Because $\sin\theta$ is positive, θ must be an angle in $(0,\pi/2]$, so θ is a positive angle that lies in quadrant I and x and y are both positive.

$$\theta = \sin^{-1}\left(\frac{8 = \text{opposite}}{17 = \text{hypotenuse}}\right)$$

Given a triangle with opposite leg 8 and hypotenuse 17, the adjacent leg must be

$$a^2 + b^2 = c^2$$

$$a^2 + 8^2 = 17^2$$

$$a^2 = 289 - 64$$

$$a^2 = 225$$

$$a = 15$$

Because cosine is equivalent to adjacent/hypotenuse, we get

$$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$$

$$\cos\left(\sin^{-1}\left(\frac{8}{17}\right)\right) = \cos\theta = \frac{15}{17}$$

Topic: Trig functions of inverse trig functions

Question: Find the value of $\csc(\sin^{-1} x)$.

Answer choices:

A x

B $\frac{1}{x}$

C $\frac{1}{\sqrt{1-x^2}}$

D $\sqrt{1-x^2}$

Solution: B

Set $\theta = \sin^{-1} x$. Then we can say

$$\theta = \sin^{-1}\left(\frac{x}{1}\right)$$

$$\theta = \sin^{-1}\left(\frac{x = \text{opposite}}{1 = \text{hypotenuse}}\right)$$

Because cosecant is equivalent to hypotenuse/opposite, we get

$$\frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{x}$$

Topic: Sum-difference identities for sine and cosine

Question: Simplify the sum.

$$(-\sin 70^\circ)(\cos 42^\circ) + (\cos 70^\circ)\sin(42^\circ)$$

Answer choices:

A $\cos 112^\circ$

B $\sin 28^\circ$

C $\sin(-28^\circ)$

D $\sin(-112^\circ)$

Solution: C

The expression is in the form

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

Substitute the angles from the expression.

$$\sin(-70^\circ + 42^\circ) = (\sin(-70^\circ))(\cos 42^\circ) + (\cos(-70^\circ))(\sin 42^\circ)$$

By the odd identity $\sin(-\theta) = -\sin\theta$ and the even identity $\cos(-\theta) = \cos\theta$, this equation becomes

$$\sin(-70^\circ + 42^\circ) = (-\sin 70^\circ)(\cos 42^\circ) + (\cos 70^\circ)(\sin 42^\circ)$$

$$\sin(-28^\circ) = (-\sin 70^\circ)(\cos 42^\circ) + (\cos 70^\circ)(\sin 42^\circ)$$

Topic: Sum-difference identities for sine and cosine

Question: Let θ be an angle in the second quadrant whose sine is $1/3$, and let α be an angle in the fourth quadrant whose cosine is $2/\sqrt{5}$. What are the exact values of $\sin(\theta+\alpha)$ and $\cos(\theta-\alpha)$?

Answer choices:

A $\quad \sin(\theta+\alpha)=\dfrac{2+2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=-\dfrac{4\sqrt{2}+1}{3\sqrt{5}}$

B $\quad \sin(\theta+\alpha)=\dfrac{2-2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=-\dfrac{4\sqrt{2}+1}{3\sqrt{5}}$

C $\quad \sin(\theta+\alpha)=\dfrac{2+2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=\dfrac{4\sqrt{2}-1}{3\sqrt{5}}$

D $\quad \sin(\theta+\alpha)=-\dfrac{2+2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=\dfrac{4\sqrt{2}-1}{3\sqrt{5}}$

Solution: A

Rewrite the Pythagorean identity with sine and cosine $\sin^2\theta + \cos^2\theta = 1$ as

$$\cos^2\theta = 1 - \sin^2\theta$$

Substitute $\sin\theta = 1/3$.

$$\cos^2\theta = 1 - \left(\frac{1}{3}\right)^2$$

$$\cos^2\theta = 1 - \frac{1}{9}$$

$$\cos^2\theta = \frac{8}{9}$$

$$\cos\theta = \pm\sqrt{\frac{8}{9}}$$

Since θ is in the second quadrant, $\cos\theta$ is negative.

$$\cos\theta = -\sqrt{\frac{8}{9}} = -\frac{\sqrt{8}}{\sqrt{9}} = -\frac{\sqrt{4(2)}}{3} = -\frac{2\sqrt{2}}{3}$$

Rewrite the Pythagorean identity with sine and cosine $\sin^2\alpha + \cos^2\alpha = 1$ as

$$\sin^2\alpha = 1 - \cos^2\alpha$$

Substitute $\cos\alpha = 2/\sqrt{5}$.

$$\sin^2\alpha = 1 - \left(\frac{2}{\sqrt{5}}\right)^2$$

$$\sin^2 \alpha = 1 - \frac{4}{5}$$

$$\sin^2 \alpha = \frac{1}{5}$$

$$\sin \alpha = \pm \sqrt{\frac{1}{5}}$$

Since α is in the fourth quadrant, $\sin \alpha$ is negative.

$$\sin \alpha = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}}$$

By the sum identity for the sine function,

$$\sin(\theta + \alpha) = \sin \theta \cos \alpha + \cos \theta \sin \alpha$$

$$\sin(\theta + \alpha) = \left(\frac{1}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(-\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{\sqrt{5}}\right)$$

$$\sin(\theta + \alpha) = \frac{2}{3\sqrt{5}} + \frac{2\sqrt{2}}{3\sqrt{5}}$$

$$\sin(\theta + \alpha) = \frac{2 + 2\sqrt{2}}{3\sqrt{5}}$$

By the difference identity for the cosine function,

$$\cos(\theta - \alpha) = \cos \theta \cos \alpha + \sin \theta \sin \alpha$$

$$\cos(\theta - \alpha) = \left(-\frac{2\sqrt{2}}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{1}{3}\right)\left(-\frac{1}{\sqrt{5}}\right)$$

$$\cos(\theta - \alpha) = -\frac{4\sqrt{2}}{3\sqrt{5}} - \frac{1}{3\sqrt{5}}$$

$$\cos(\theta - \alpha) = -\frac{1 + 4\sqrt{2}}{3\sqrt{5}}$$

Topic: Sum-difference identities for sine and cosine

Question: Simplify the expression.

$$\cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{11\pi}{6}\right)+\sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{11\pi}{6}\right)$$

Answer choices:

A $-\frac{\sqrt{3}}{2}$

B $-\frac{1}{2}$

C 1

D $\frac{1}{2}$

Solution: A

The expression is in the form of the difference identity for cosine,

$$\cos(\theta-\alpha)=\cos\theta\cos\alpha+\sin\theta\sin\alpha$$

so we'll substitute the angles from the expression.

$$\cos\left(\frac{11\pi}{6}-\frac{2\pi}{3}\right)=\cos\left(\frac{11\pi}{6}\right)\cos\left(\frac{2\pi}{3}\right)+\sin\left(\frac{11\pi}{6}\right)\sin\left(\frac{2\pi}{3}\right)$$

$$\cos\left(\frac{11\pi}{6}-\frac{4\pi}{6}\right)=\cos\left(\frac{11\pi}{6}\right)\cos\left(\frac{2\pi}{3}\right)+\sin\left(\frac{11\pi}{6}\right)\sin\left(\frac{2\pi}{3}\right)$$

$$\cos\left(\frac{7\pi}{6}\right)=\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\cos\left(\frac{7\pi}{6}\right)=-\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}$$

$$\cos\left(\frac{7\pi}{6}\right)=-\frac{\sqrt{3}}{2}$$

Topic: Cofunction identities

Question: Find an angle θ that satisfies the equation.

$$\cos\left(\frac{4\pi}{9}\right) = \sin\theta$$

Answer choices:

A $-\dfrac{\pi}{18}$

B $\dfrac{4\pi}{9}$

C $\dfrac{5\pi}{18}$

D $\dfrac{\pi}{18}$

Solution: D

The equation we're given tells us that the sine of some angle is equivalent to cosine of the angle $4\pi/9$. Sine and cosine are cofunctions, which means we can plug into the cofunction identity for cosine that relates them.

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{\pi}{2} - \frac{4\pi}{9}\right)$$

Find a common denominator.

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{\pi}{2}\left(\frac{9}{9}\right) - \frac{4\pi}{9}\left(\frac{2}{2}\right)\right)$$

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{9\pi}{18} - \frac{8\pi}{18}\right)$$

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{\pi}{18}\right)$$

So the angle θ that satisfies the equation is $\theta = \pi/18$.

Topic: Cofunction identities

Question: Find θ.

$$\sec\left(\frac{3\pi}{8}\right) = \csc\left(\frac{\pi}{2} - \theta\right)$$

Answer choices:

A $\frac{3\pi}{8}$

B $\frac{\pi}{8}$

C $\frac{\pi}{4}$

D $\frac{\pi}{2}$

Solution: A

Secant and cosecant are cofunctions, which means we can plug into the cofunction identity for secant that relates the two functions.

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\left(\frac{3\pi}{8}\right) = \csc\left(\frac{\pi}{2} - \frac{3\pi}{8}\right)$$

So the angle θ that satisfies the equation is $\theta = 3\pi/8$.

Topic: Cofunction identities

Question: Find an acute angle that satisfies the equation.

$$\tan(3\alpha + 27^\circ) = \cot(\alpha + 3^\circ)$$

Answer choices:

A 60°

B 18°

C 15°

D 72°

Solution: C

The equation matches the form of the cofunction identity for tangent, $\tan\theta = \cot(90^\circ - \theta)$, where $\theta = 3\alpha + 27^\circ$. Then

$$\alpha + 3^\circ = 90^\circ - (3\alpha + 27^\circ)$$

$$\alpha + 3^\circ = 90^\circ - 3\alpha - 27^\circ$$

$$\alpha + 3^\circ = 63^\circ - 3\alpha$$

$$4\alpha = 60^\circ$$

$$\alpha = 15^\circ$$

Topic: Sum-difference identities for tangent

Question: Simplify the expression.

$$\frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

Answer choices:

A $\tan 55^\circ$

B $\tan 45^\circ$

C $\tan 119^\circ$

D $\tan 109^\circ$

Solution: B

The expression matches the form of the right side of the sum identity for tangent.

$$\tan(\theta + \alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$$

Substitute the angles from the expression.

$$\tan(82^\circ + (-37^\circ)) = \frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

$$\tan(82^\circ - 37^\circ) = \frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

$$\tan 45^\circ = \frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

Topic: Sum-difference identities for tangent

Question: Find the exact value of $\tan(13\pi/12)$.

Answer choices:

A $2+\sqrt{3}$

B $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

C $2-\sqrt{3}$

D $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

Solution: C

From just the unit circle, we wouldn't know the value of tangent at $13\pi/12$, but we can rewrite $13\pi/12$ as

$$\frac{13\pi}{12}=\frac{(10+3)\pi}{12}=\frac{10\pi}{12}+\frac{3\pi}{12}=\frac{5\pi}{6}+\frac{\pi}{4}$$

So the original expression can be rewritten as

$$\tan\left(\frac{13\pi}{12}\right)=\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)$$

and we can plug this right side into the sum identity for the tangent function.

$$\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{\tan\left(\frac{5\pi}{6}\right)+\tan\left(\frac{\pi}{4}\right)}{1-\tan\left(\frac{5\pi}{6}\right)\tan\left(\frac{\pi}{4}\right)}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{-\frac{\sqrt{3}}{3}+1}{1-\left(-\frac{\sqrt{3}}{3}\right)(1)}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{-\frac{\sqrt{3}}{3}+1}{1+\frac{\sqrt{3}}{3}}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{3-\sqrt{3}}{3+\sqrt{3}}$$

Multiply both the numerator and denominator by the conjugate of the denominator.

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{3-\sqrt{3}}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{9-6\sqrt{3}+3}{9-3}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{12-6\sqrt{3}}{6}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=2-\sqrt{3}$$

Topic: Sum-difference identities for tangent

Question: Find the exact values of $\tan(\theta + \alpha)$ if $\tan\theta = -1/2$ and $\tan\alpha = 3/4$.

Answer choices:

A $\frac{2}{5}$

B $-\frac{2}{5}$

C 2

D $\frac{2}{11}$

Solution: D

Use the sum identity for the tangent function, substituting the values we've been given.

$$\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$$

$$\tan(\theta+\alpha)=\frac{-\frac{1}{2}+\frac{3}{4}}{1-\left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)}$$

Simplify the right side.

$$\tan(\theta+\alpha)=\frac{\frac{-2+3}{4}}{1+\frac{3}{8}}$$

$$\tan(\theta+\alpha)=\frac{\frac{1}{4}}{\frac{8+3}{8}}$$

$$\tan(\theta+\alpha)=\frac{\frac{1}{4}}{\frac{11}{8}}$$

$$\tan(\theta+\alpha)=\frac{1}{4}\cdot\frac{8}{11}$$

$$\tan(\theta+\alpha)=\frac{2}{11}$$

Topic: Double-angle identities

Question: If θ is an angle in the fourth quadrant whose sine is $-4/5$, what are the values of $\sin 2\theta$ and $\cos 2\theta$?

Answer choices:

A $\sin 2\theta = \frac{3}{5}$ $\cos 2\theta = -\frac{4}{5}$

B $\sin 2\theta = -\frac{24}{25}$ $\cos 2\theta = -\frac{7}{25}$

C $\sin 2\theta = \frac{2\sqrt{6}}{5}$ $\cos 2\theta = -\frac{1}{5}$

D $\sin 2\theta = \frac{7}{25}$ $\cos 2\theta = -\frac{24}{25}$

Solution: B

Substitute $\sin \theta = -4/5$ into the rewritten form of the Pythagorean identity with sine and cosine.

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\cos^2 \theta = 1 - \left(-\frac{4}{5}\right)^2$$

$$\cos^2 \theta = 1 - \frac{16}{25}$$

$$\cos^2 \theta = \frac{9}{25}$$

$$\cos \theta = \pm \sqrt{\frac{9}{25}}$$

Since θ is in the fourth quadrant, $\cos \theta$ is positive, so

$$\cos \theta = \sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

Substituting $\cos \theta = 3/5$ and $\sin \theta = -4/5$ into the double-angle identity for sine, $\sin 2\theta = 2 \sin \theta \cos \theta$, we get

$$\sin 2\theta = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$\sin 2\theta = -\frac{24}{25}$$

And using the double-angle identity for cosine, we get

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$$

$$\cos 2\theta = \frac{9}{25} - \frac{16}{25}$$

$$\cos 2\theta = -\frac{7}{25}$$

Topic: Double-angle identities

Question: Let θ be an angle in the third quadrant whose cosine is $-\sqrt{10}/11$. Which set of equations is true?

Answer choices:

A $\sin 2\theta = \frac{2\sqrt{1{,}110}}{121}$ $\cos 2\theta = -\frac{101}{121}$

B $\cos 2\theta = -\frac{101}{121}$ $\tan 2\theta = -\frac{\sqrt{1{,}110}}{101}$

C $\sin 2\theta = \frac{101}{121}$ $\tan 2\theta = -\frac{2\sqrt{1{,}110}}{101}$

D $\sin 2\theta = \frac{1}{121}$ $\cos 2\theta = -\frac{101}{121}$

Solution: A

Substitute $\cos\theta = -\sqrt{10}/11$ into the rewritten form of the Pythagorean identity with sine and cosine.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(-\frac{\sqrt{10}}{11}\right)^2$$

$$\sin^2\theta = 1 - \frac{10}{121}$$

$$\sin^2\theta = \frac{111}{121}$$

Substituting $\cos\theta = -\sqrt{10}/11$ and $\sin^2\theta = 111/121$ into the double-angle identity for cosine, $\cos 2\theta = \cos^2\theta - \sin^2\theta$, we get

$$\cos 2\theta = \left(-\frac{\sqrt{10}}{11}\right)^2 - \frac{111}{121}$$

$$\cos 2\theta = \frac{10}{121} - \frac{111}{121}$$

$$\cos 2\theta = -\frac{101}{121}$$

From the value of $\sin^2\theta$ we found earlier, we can say

$$\sin\theta = \pm\sqrt{\frac{111}{121}}$$

Since θ is in the third quadrant, $\sin\theta$ is negative. Therefore,

$$\sin\theta = -\sqrt{\frac{111}{121}} = -\frac{\sqrt{111}}{\sqrt{121}} = -\frac{\sqrt{111}}{11}$$

Apply the double-angle identity for sine, $\sin 2\theta = 2\sin\theta\cos\theta$.

$$\sin 2\theta = 2\left(-\frac{\sqrt{111}}{11}\right)\left(-\frac{\sqrt{10}}{11}\right)$$

$$\sin 2\theta = \frac{2\sqrt{111}\sqrt{10}}{121}$$

$$\sin 2\theta = \frac{2\sqrt{1,110}}{121}$$

Then tangent of the double angle will be

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{2\sqrt{1,110}}{121}}{-\frac{101}{121}} = -\frac{2\sqrt{1,110}}{101}$$

Topic: Double-angle identities

Question: Use double-angle identities to find the exact value of $\sin 240^\circ$.

Answer choices:

A $\frac{\sqrt{3}}{2}$

B $\sqrt{3}$

C $-\frac{1}{2}$

D $-\frac{\sqrt{3}}{2}$

Solution: D

We can rewrite $\sin 240^\circ$ as $\sin(2 \cdot 120^\circ)$ and use the double-angle identity for sine, $\sin 2\theta = 2 \sin \theta \cos \theta$. We'll substitute and get

$$\sin 240^\circ = \sin(2 \cdot 120^\circ) = 2 \sin 120^\circ \cos 120^\circ$$

Now we can use the double-angle identity for sine and cosine one more time, or if we remember $\sin 120^\circ = \sqrt{3}/2$ and $\cos(-1/2)$, we get

$$\sin 240^\circ = 2 \left(\frac{\sqrt{3}}{2} \right) \left(-\frac{1}{2} \right)$$

$$\sin 240^\circ = -\frac{\sqrt{3}}{2}$$

Topic: Half-angle identities

Question: Given the inequality, which pair of inequalities is true?

$$-\frac{27\pi}{11} < \theta < -\frac{23\pi}{11}$$

Answer choices:

A $\cos\frac{\theta}{2} > 0$ $\sin\frac{\theta}{2} > 0$

B $\cos\frac{\theta}{2} > 0$ $\sin\frac{\theta}{2} < 0$

C $\cos\frac{\theta}{2} < 0$ $\sin\frac{\theta}{2} > 0$

D $\cos\frac{\theta}{2} < 0$ $\sin\frac{\theta}{2} < 0$

Solution: C

Dividing through the inequality by 2.

$$-\frac{27\pi}{11} < \theta < -\frac{23\pi}{11}$$

$$-\frac{27\pi}{22} < \frac{\theta}{2} < -\frac{23\pi}{22}$$

$$-1.23\pi < \frac{\theta}{2} < -1.05\pi$$

Find coterminal angles for the bounds on this interval, in order to make the angles positive.

$$-1.23\pi + 2\pi = 0.77\pi$$

$$-1.05\pi + 2\pi = 0.95\pi$$

The value 0.77π falls in the second quadrant, and so does 0.95π. Which means the angle $\theta/2$ falls in the second quadrant, where sine must be positive and cosine must be negative.

Topic: Half-angle identities

Question: If θ is the angle in the interval $(3\pi/2, 2\pi)$ with $\sin\theta = -2/3$, what are the values of $\cos(\theta/2)$ and $\sin(\theta/2)$?

Answer choices:

A $\cos\frac{\theta}{2} = -\sqrt{\frac{3-\sqrt{7}}{6}}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3+\sqrt{7}}{6}}$

B $\cos\frac{\theta}{2} = \sqrt{\frac{3+\sqrt{7}}{6}}$ $\sin\frac{\theta}{2} = -\sqrt{\frac{3-\sqrt{7}}{6}}$

C $\cos\frac{\theta}{2} = -\sqrt{\frac{3+\sqrt{5}}{6}}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3-\sqrt{5}}{6}}$

D $\cos\frac{\theta}{2} = \sqrt{\frac{3-\sqrt{5}}{6}}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3+\sqrt{5}}{6}}$

Solution: C

Substitute $\sin\theta = -2/3$ into the rewritten form of the Pythagorean identity with sine and cosine.

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - \left(-\frac{2}{3}\right)^2$$

$$\cos^2\theta = 1 - \frac{4}{9}$$

$$\cos^2\theta = \frac{5}{9}$$

$$\cos\theta = \pm\sqrt{\frac{5}{9}}$$

We know θ is in the interval $(3\pi/2, 2\pi)$, which means θ is in the fourth quadrant, and therefore that $\cos\theta$ is positive.

$$\cos\theta = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$$

By the half-angle identities for cosine and sine,

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{5}}{3}}{2}} = \pm\sqrt{\frac{\frac{3}{3}+\frac{\sqrt{5}}{3}}{2}} = \pm\sqrt{\frac{3+\sqrt{5}}{6}}$$

$$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}=\pm\sqrt{\frac{1-\frac{\sqrt{5}}{3}}{2}}=\pm\sqrt{\frac{\frac{3}{3}-\frac{\sqrt{5}}{3}}{2}}=\pm\sqrt{\frac{3-\sqrt{5}}{6}}$$

Because θ is in the interval $(3\pi/2,2\pi)$, we know that $\theta/2$ must be in the interval

$$\left(\frac{\frac{3\pi}{2}}{2},\frac{2\pi}{2}\right)=\left(\frac{3\pi}{4},\pi\right)$$

The entire interval $(3\pi/4,\pi)$ is in the second quadrant, where sine is positive and cosine is negative, so

$$\cos\frac{\theta}{2}=-\sqrt{\frac{3+\sqrt{5}}{6}}$$

$$\sin\frac{\theta}{2}=\sqrt{\frac{3-\sqrt{5}}{6}}$$

Topic: Half-angle identities

Question: If θ is the angle in the interval $(17\pi/2, 9\pi)$ with $\cos\theta = -3/7$, what are the values of $\sin(\theta/2)$ and $\cos(\theta/2)$?

Answer choices:

A $\sin\frac{\theta}{2} = -\sqrt{\frac{1}{7}}$ $\cos\frac{\theta}{2} = \sqrt{\frac{6}{7}}$

B $\sin\frac{\theta}{2} = \sqrt{\frac{5}{7}}$ $\cos\frac{\theta}{2} = \sqrt{\frac{2}{7}}$

C $\sin\frac{\theta}{2} = \sqrt{\frac{3}{7}}$ $\cos\frac{\theta}{2} = \sqrt{\frac{4}{7}}$

D $\sin\frac{\theta}{2} = -\sqrt{\frac{5}{7}}$ $\cos\frac{\theta}{2} = -\sqrt{\frac{2}{7}}$

Solution: B

Substitute $\cos\theta = -3/7$ into the half-angle identities for cosine and sine,

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

we get

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\left(-\frac{3}{7}\right)}{2}} = \pm\sqrt{\frac{\frac{7(1)+1(-3)}{7}}{2}} = \pm\sqrt{\frac{7-3}{14}} = \pm\sqrt{\frac{4}{14}} = \pm\sqrt{\frac{2}{7}}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\left(-\frac{3}{7}\right)}{2}} = \pm\sqrt{\frac{\frac{7(1)+1(3)}{7}}{2}} = \pm\sqrt{\frac{7+3}{14}} = \pm\sqrt{\frac{10}{14}} = \pm\sqrt{\frac{5}{7}}$$

Because θ is in the interval $(17\pi/2, 9\pi)$, the half angle is in the interval

$$\frac{17\pi}{4} < \frac{\theta}{2} < \frac{9\pi}{2}$$

Find coterminal angles for the bounds on this interval.

$$\frac{17\pi}{4} = \frac{16\pi}{4} + \frac{\pi}{4} = 4\pi + \frac{\pi}{4}$$

$$\frac{9\pi}{2} = \frac{8\pi}{2} + \frac{\pi}{2} = 4\pi + \frac{\pi}{2}$$

Therefore, we can say

$$\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$$

So $\theta/2$ is in the first quadrant, which means both $\sin(\theta/2)$ and $\cos(\theta/2)$ are positive.

$$\sin\frac{\theta}{2} = \sqrt{\frac{5}{7}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{2}{7}}$$

Topic: Product-to-sum identities

Question: Rewrite $\cos(14\theta)\sin(-5\theta)$ as a sum.

Answer choices:

A $\frac{1}{2}\left[\sin(19\theta)+\sin(-5\theta)\right]$

B $\frac{1}{2}\left[\sin(9\theta)+\sin(-19\theta)\right]$

C $\frac{1}{2}\left[\sin(19\theta)+\sin(-14\theta)\right]$

D $\frac{1}{2}\left[\sin(14\theta)+\sin(-9\theta)\right]$

Solution: B

Using the product-to-sum identity,

$$\cos\theta\sin\alpha = \frac{1}{2}\left[\sin(\theta+\alpha) - \sin(\theta-\alpha)\right]$$

we can set $\theta = 14\theta$ and $\alpha = -5\theta$ and rewrite the product as

$$\frac{1}{2}\left[\sin(14\theta + (-5\theta)) - \sin(14\theta - (-5\theta))\right]$$

$$\frac{1}{2}\left[\sin(14\theta - 5\theta) - \sin(14\theta + 5\theta)\right]$$

$$\frac{1}{2}\left[\sin(9\theta) - \sin(19\theta)\right]$$

Use the even-odd identity $\sin(-\theta) = -\sin\theta$ to rewrite the difference as a sum.

$$\frac{1}{2}\left[\sin(9\theta) + \sin(-19\theta)\right]$$

Topic: Product-to-sum identities

Question: Find the value of the expression.

$$\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$$

Answer choices:

A $\frac{\sqrt{3}+2}{2}$

B $\frac{\sqrt{3}-2}{2}$

C $\frac{1}{2}$

D $\frac{1}{4}$

Solution: D

Using the product-to-sum identity,

$$\sin\theta\cos\alpha = \frac{1}{2}\left[\sin(\theta+\alpha)+\sin(\theta-\alpha)\right]$$

we can set $\theta = \pi/12$ and $\alpha = \pi/12$ and rewrite the product as

$$\frac{1}{2}\left[\sin\left(\frac{\pi}{12}+\frac{\pi}{12}\right)+\sin\left(\frac{\pi}{12}-\frac{\pi}{12}\right)\right]$$

$$\frac{1}{2}\left(\sin\frac{\pi}{6}+\sin 0\right)$$

$$\frac{1}{2}\left(\frac{1}{2}+0\right)$$

$$\frac{1}{4}$$

Topic: Product-to-sum identities

Question: Which angle pair is a solution to the equation?

$$\cos\theta\cos\alpha = -\frac{2+\sqrt{2}}{4}$$

Answer choices:

A $(\theta, \alpha) = \left(\frac{17\pi}{8}, \frac{25\pi}{8}\right)$

B $(\theta, \alpha) = \left(\frac{11\pi}{8}, \frac{7\pi}{8}\right)$

C $(\theta, \alpha) = \left(\frac{5\pi}{8}, \frac{7\pi}{8}\right)$

D $(\theta, \alpha) = \left(\frac{9\pi}{8}, \frac{11\pi}{8}\right)$

Solution: A

We need to test each answer choice using the product-to-sum identity for the product of two cosine functions.

$$\cos\theta\cos\alpha = \frac{1}{2}\left[\cos(\theta+\alpha)+\cos(\theta-\alpha)\right]$$

Test answer choice A with $\theta = 17\pi/8$ and $\alpha = 25\pi/8$.

$$\frac{1}{2}\left[\cos\left(\frac{17\pi}{8}+\frac{25\pi}{8}\right)+\cos\left(\frac{17\pi}{8}-\frac{25\pi}{8}\right)\right]$$

$$\frac{1}{2}\left[\cos\left(\frac{42\pi}{8}\right)+\cos\left(-\frac{8\pi}{8}\right)\right]$$

$$\frac{1}{2}\left[\cos\left(\frac{21\pi}{4}\right)+\cos(-\pi)\right]$$

The angle $21\pi/4$ is coterminal with $5\pi/4$, so both angles will have the same cosine, and we can rewrite the expression as

$$\frac{1}{2}\left[\cos\left(\frac{5\pi}{4}\right)+\cos(-\pi)\right]$$

Using the unit circle, we can find the value of each of the cosine functions.

$$\frac{1}{2}\left[-\frac{\sqrt{2}}{2}+(-1)\right]$$

$$-\frac{\sqrt{2}}{4}-\frac{1}{2}$$

Find a common denominator.

$$-\frac{\sqrt{2}}{4}-\frac{2}{4}$$

$$\frac{-2-\sqrt{2}}{4}$$

$$-\frac{2+\sqrt{2}}{4}$$

Topic: Sum-to-product identities

Question: Express $\sin(8\theta) - \sin(20\theta)$ as a product.

Answer choices:

A $-4\cos(7\theta)\sin(3\theta)$

B $-\cos(14\theta)\sin(-6\theta)$

C $-\cos(7\theta)\sin(3\theta)$

D $-2\cos(14\theta)\sin(6\theta)$

Solution: D

Using the sum-to-product identity,

$$\sin\theta - \sin\alpha = 2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right)$$

we can set $\theta = 8\theta$ and $\alpha = 20\theta$ and rewrite the product as

$$2\cos\left(\frac{8\theta+20\theta}{2}\right)\sin\left(\frac{8\theta-20\theta}{2}\right)$$

$$2\cos\left(\frac{28\theta}{2}\right)\sin\left(\frac{-12\theta}{2}\right)$$

$$2\cos(14\theta)\sin(-6\theta)$$

Using the even-odd identity $\sin(-\theta) = -\sin\theta$ to simplify the negative angle, we get

$$-2\cos(14\theta)\sin(6\theta)$$

Topic: Sum-to-product identities

Question: Find the exact value of the expression.

$$\cos\left(\frac{15\pi}{8}\right)+\cos\left(\frac{7\pi}{8}\right)$$

Answer choices:

A -2

B 1

C 0

D 2

Solution: C

Using the sum-to-product identity,

$$\cos\theta + \cos\alpha = 2\cos\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right)$$

we can set $\theta = 15\pi/8$ and $\alpha = 7\pi/8$ and rewrite the product as

$$2\cos\left(\frac{\frac{15\pi}{8}+\frac{7\pi}{8}}{2}\right)\cos\left(\frac{\frac{15\pi}{8}-\frac{7\pi}{8}}{2}\right)$$

$$2\cos\left(\frac{\frac{22\pi}{8}}{2}\right)\cos\left(\frac{\frac{8\pi}{8}}{2}\right)$$

$$2\cos\left(\frac{\frac{11\pi}{4}}{2}\right)\cos\left(\frac{\pi}{2}\right)$$

$$2\cos\left(\frac{11\pi}{4}\left(\frac{1}{2}\right)\right)\cos\left(\frac{\pi}{2}\right)$$

$$2\cos\left(\frac{11\pi}{8}\right)\cos\left(\frac{\pi}{2}\right)$$

Because $\cos(\pi/2) = 0$, we get

$$2\cos\left(\frac{11\pi}{8}\right)(0) = 0$$

Topic: Sum-to-product identities

Question: Find the exact value of the expression.

$$4\sin 45^\circ + 6\cos 165^\circ + 4\sin 45^\circ - 6\cos 105^\circ$$

Answer choices:

A $5\sqrt{2}$

B $\frac{\sqrt{2}}{2}$

C $\sqrt{2}$

D $2+\sqrt{3}$

Solution: C

First we need to rewrite our expression as

$$4\sin 45^\circ + 4\sin 45^\circ + 6\cos 165^\circ - 6\cos 105^\circ$$

$$4(\sin 45^\circ + \sin 45^\circ) + 6(\cos 165^\circ - \cos 105^\circ)$$

Using the sum-to-product identity,

$$\sin\theta + \sin\alpha = 2\sin\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right)$$

we can set $\theta = 45^\circ$ and $\alpha = 45^\circ$ and rewrite the product as

$$\sin 45^\circ + \sin 45^\circ = 2\sin\left(\frac{45^\circ + 45^\circ}{2}\right)\cos\left(\frac{45^\circ - 45^\circ}{2}\right)$$

$$\sin 45^\circ + \sin 45^\circ = 2\sin\left(\frac{90^\circ}{2}\right)\cos\left(\frac{0}{2}\right)$$

$$\sin 45^\circ + \sin 45^\circ = 2\sin 45^\circ\cos 0^\circ$$

$$\sin 45^\circ + \sin 45^\circ = 2\left(\frac{\sqrt{2}}{2}\right)(1)$$

$$\sin 45^\circ + \sin 45^\circ = \sqrt{2}$$

Now using the sum-to-product identity,

$$\cos\theta - \cos\alpha = -2\sin\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right)$$

we can set $\theta = 165^\circ$ and $\alpha = 105^\circ$ and rewrite the product as

$$\cos 165^\circ - \cos 105^\circ = -2\sin\left(\frac{165^\circ + 105^\circ}{2}\right)\sin\left(\frac{165^\circ - 105^\circ}{2}\right)$$

$$\cos 165^\circ - \cos 105^\circ = -2\sin\left(\frac{270^\circ}{2}\right)\sin\left(\frac{60^\circ}{2}\right)$$

$$\cos 165^\circ - \cos 105^\circ = -2\sin 135^\circ \sin 30^\circ$$

$$\cos 165^\circ - \cos 105^\circ = -2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\cos 165^\circ - \cos 105^\circ = -\frac{\sqrt{2}}{2}$$

Then the value of the original expression is

$$4(\sqrt{2}) + 6\left(-\frac{\sqrt{2}}{2}\right)$$

$$4\sqrt{2} - 3\sqrt{2}$$

$$\sqrt{2}$$

Topic: Proving the trig equation

Question: Choose the equivalent expression.

$$\frac{\sin\theta + \sin(3\theta)}{\cos\theta + \cos(3\theta)}$$

Answer choices:

A $\sin(2\theta)$

B $\tan(2\theta)$

C $\cos(2\theta)$

D $\cot(2\theta)$

Solution: B

Work on the numerator of the expression. By a sum-to-product identity,

$$\sin\theta+\sin(3\theta)=2\sin\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{\theta-3\theta}{2}\right)$$

$$\sin\theta+\sin(3\theta)=2\sin\left(\frac{4\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)$$

$$\sin\theta+\sin(3\theta)=2\sin(2\theta)\cos(-\theta)$$

Work on the denominator of the expression. By a sum-to-product identity,

$$\cos\theta+\cos(3\theta)=2\cos\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{\theta-3\theta}{2}\right)$$

$$\cos\theta+\cos(3\theta)=2\cos\left(\frac{4\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)$$

$$\cos\theta+\cos(3\theta)=2\cos(2\theta)\cos(-\theta)$$

Replacing both the numerator and denominator with these results gives

$$\frac{\sin\theta+\sin(3\theta)}{\cos\theta+\cos(3\theta)}=\frac{2\sin(2\theta)\cos(-\theta)}{2\cos(2\theta)\cos(-\theta)}$$

$$\frac{\sin\theta+\sin(3\theta)}{\cos\theta+\cos(3\theta)}=\frac{\sin(2\theta)}{\cos(2\theta)}$$

$$\frac{\sin\theta+\sin(3\theta)}{\cos\theta+\cos(3\theta)}=\tan(2\theta)$$

Topic: Proving the trig equation

Question: Choose the equivalent expression.

$$\sin\left(\frac{\theta}{4}\right)\cos\left(\frac{3\theta}{4}\right)$$

Answer choices:

A $\quad \frac{1}{2}\left[\sin\theta + \sin\left(\frac{\theta}{2}\right)\right]$

B $\quad \frac{1}{2}\left[\sin\theta - \sin\left(\frac{\theta}{2}\right)\right]$

C $\quad \frac{1}{2}\left[\cos\theta - \cos\left(\frac{\theta}{2}\right)\right]$

D $\quad \frac{1}{2}\left[\cos\theta + \cos\left(\frac{\theta}{2}\right)\right]$

Solution: B

Using a product-to-sum identity, we can rewrite the expression as

$$\frac{1}{2}\left[\sin\left(\frac{\theta}{4}+\frac{3\theta}{4}\right)+\sin\left(\frac{\theta}{4}-\frac{3\theta}{4}\right)\right]$$

$$\frac{1}{2}\left[\sin\left(\frac{\theta+3\theta}{4}\right)+\sin\left(\frac{\theta-3\theta}{4}\right)\right]$$

$$\frac{1}{2}\left[\sin\left(\frac{4\theta}{4}\right)+\sin\left(\frac{-2\theta}{4}\right)\right]$$

$$\frac{1}{2}\left[\sin\theta+\sin\left(-\frac{\theta}{2}\right)\right]$$

Using the odd identity for sine, we get

$$\frac{1}{2}\left[\sin\theta-\sin\left(\frac{\theta}{2}\right)\right]$$

Topic: Proving the trig equation

Question: Choose the equivalent expression.

$$\tan^2 x - \sin^2 x$$

Answer choices:

A $\sin^2 x \tan^2 x$

B $\cos^2 x \cot^2 x$

C $(1 - \sin^2 x)(\tan^2 x)$

D $\sin x \tan x$

Solution: A

Use the quotient identity to rewrite the tangent function.

$$\tan^2 x - \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

Factor out the sine function, then find a common denominator and combine the fractions.

$$\sin^2 x \left(\frac{1}{\cos^2 x} - 1 \right)$$

$$\sin^2 x \left(\frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} \right)$$

$$\sin^2 x \left(\frac{1 - \cos^2 x}{\cos^2 x} \right)$$

Use the Pythagorean identity with sine and cosine for the numerator of the fraction.

$$\sin^2 x \left(\frac{\sin^2 x}{\cos^2 x} \right)$$

$$\sin^2 x \tan^2 x$$

Topic: Complete solution set of the equation

Question: Find the complete solution set of the equation.

$$\cos\left(\theta + \frac{3\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

Answer choices:

A $\theta = \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}$ where n is any integer

B $\theta = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}$ where n is any integer

C $\theta = \left\{\frac{\pi}{4} + 2n\pi\right\} \cup \left\{\frac{3\pi}{4} + 2n\pi\right\}$ where n is any integer

D $\theta = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\}$ where n is any integer

Solution: D

By the sum identity for cosine,

$$\cos\left(\theta+\frac{3\pi}{2}\right)=\cos\theta\cos\left(\frac{3\pi}{2}\right)-\sin\theta\sin\left(\frac{3\pi}{2}\right)$$

$$\cos\left(\theta+\frac{3\pi}{2}\right)=(\cos\theta)(0)-(\sin\theta)(-1)$$

$$\cos\left(\theta+\frac{3\pi}{2}\right)=\sin\theta$$

Replacing the left side of the equation with $\sqrt{3}/2$, we realize that the equation we need to solve is

$$\sin\theta=\frac{\sqrt{3}}{2}$$

From the unit circle, we know this equation is true at $\pi/3$ and $2\pi/3$, so

$$\theta=\frac{\pi}{3}+2n\pi$$

$$\theta=\frac{2\pi}{3}+2n\pi$$

Therefore, the complete solution set is

$$\theta=\left\{\frac{\pi}{3}+2n\pi\right\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}\text{ where } n \text{ is any integer}$$

Topic: Complete solution set of the equation

Question: Find every angle in the interval $[0,2\pi)$ that satisfies $\csc^2\theta + \csc\theta = 2$.

Answer choices:

A $\quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

B $\quad \theta = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{3\pi}{2}$

C $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

D $\quad \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2}$

Solution: A

We'll start be rewriting $\csc^2\theta + \csc\theta = 2$ as

$$\csc^2\theta + \csc\theta - 2 = 0$$

The left side of this equation is a quadratic, which means it can be factored as

$$(\csc\theta + 2)(\csc\theta - 1) = 0$$

Now we can set each factor equal to 0 individually, and find the angles in the principal interval that satisfy each equation. We get

$$\csc\theta + 2 = 0$$

$$\csc\theta = -2$$

$$\frac{1}{\sin\theta} = -2$$

$$1 = -2\sin\theta$$

$$\sin\theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

and

$$\csc\theta - 1 = 0$$

$$\csc\theta = 1$$

$$\frac{1}{\sin\theta} = 1$$

$$\sin\theta = 1$$

$$\theta = \frac{\pi}{2}$$

Therefore, the full solution set of angles in $[0,2\pi)$ is $\pi/2$, $7\pi/6$, $11\pi/6$.

Topic: Complete solution set of the equation

Question: Find every angle in the interval $[0,2\pi)$ that satisfies $\tan(4\theta + \pi) = -1$.

Answer choices:

A $\theta = \frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16}$

B $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

C $\theta = \frac{3\pi}{16}, \frac{7\pi}{16}, \frac{11\pi}{16}, \frac{15\pi}{16}, \frac{19\pi}{16}, \frac{23\pi}{16}, \frac{27\pi}{16}, \frac{31\pi}{16}$

D $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

Solution: C

Using the sum identity for tangent, we can rewrite the left side of $\tan(4\theta + \pi) = -1$ as

$$\frac{\tan(4\theta) + \tan(\pi)}{1 - \tan(4\theta)\tan(\pi)}$$

$$\frac{\tan(4\theta) + 0}{1 - \tan(4\theta)(0)}$$

$$\frac{\tan(4\theta)}{1}$$

$$\tan(4\theta)$$

Therefore, the equation we need to solve is $\tan(4\theta) = -1$. The tangent of an angle will be -1 when the sine and cosine of the angle are equal, but with opposite signs, which will happen in the second quadrant at $3\pi/4$ and in the fourth quadrant at $7\pi/4$. So the angles that satisfy the equation will be these two, and any angles coterminal with these.

Therefore, we need to solve two equations:

$$4\theta = \frac{3\pi}{4} + 2n\pi$$

$$\theta_1 = \frac{3\pi}{16} + \frac{n\pi}{2}$$

and

$$4\theta = \frac{7\pi}{4} + 2n\pi$$

$$\theta_2 = \frac{7\pi}{16} + \frac{n\pi}{2}$$

Now we need to test different values of n to find angles that satisfy the equation in the interval $[0,2\pi)$. At $n = 0$, the angle equations give

$$\theta_1 = \frac{3\pi}{16} + \frac{0\pi}{2} = \frac{3\pi}{16}$$

$$\theta_2 = \frac{7\pi}{16} + \frac{0\pi}{2} = \frac{7\pi}{16}$$

At $n = 1$, the angle equations give

$$\theta_1 = \frac{3\pi}{16} + \frac{1\pi}{2} = \frac{3\pi}{16} + \frac{8\pi}{16} = \frac{11\pi}{16}$$

$$\theta_2 = \frac{7\pi}{16} + \frac{1\pi}{2} = \frac{7\pi}{16} + \frac{8\pi}{16} = \frac{15\pi}{16}$$

At $n = 2$, the angle equations give

$$\theta_1 = \frac{3\pi}{16} + \frac{2\pi}{2} = \frac{3\pi}{16} + \frac{16\pi}{16} = \frac{19\pi}{16}$$

$$\theta_2 = \frac{7\pi}{16} + \frac{2\pi}{2} = \frac{7\pi}{16} + \frac{16\pi}{16} = \frac{23\pi}{16}$$

At $n = 3$, the angle equations give

$$\theta_1 = \frac{3\pi}{16} + \frac{3\pi}{2} = \frac{3\pi}{16} + \frac{24\pi}{16} = \frac{27\pi}{16}$$

$$\theta_2 = \frac{7\pi}{16} + \frac{3\pi}{2} = \frac{7\pi}{16} + \frac{24\pi}{16} = \frac{31\pi}{16}$$

At $n = 4$, the angle equations give

$$\theta_1 = \frac{3\pi}{16} + \frac{4\pi}{2} = \frac{3\pi}{16} + \frac{32\pi}{16} = \frac{35\pi}{16}$$

$$\theta_2 = \frac{7\pi}{16} + \frac{4\pi}{2} = \frac{7\pi}{16} + \frac{32\pi}{16} = \frac{39\pi}{16}$$

These $35\pi/16$ and $39\pi/16$ angles are the first angles we found outside the interval $[0,2\pi)$, so we'll exclude these from the solution set. But all the angles we found previously are within the interval $[0,2\pi)$, so the full solution set is

$$\theta = \frac{3\pi}{16}, \frac{7\pi}{16}, \frac{11\pi}{16}, \frac{15\pi}{16}, \frac{19\pi}{16}, \frac{23\pi}{16}, \frac{27\pi}{16}, \frac{31\pi}{16}$$

Topic: Law of sines

Question: If the measures of two interior angles of a triangle are 70° and 43°, and the length of the side opposite the 70° angle is 12, find the length b of the side opposite the 43° angle and the length c of the third side.

Answer choices:

A $b \approx 11.8$ and $c \approx 16.5$

B $b \approx 16.5$ and $c \approx 12.2$

C $b \approx 12.2$ and $c \approx 8.71$

D $b \approx 8.71$ and $c \approx 11.8$

Solution: D

We know the third interior angle has measure

$$180^\circ - 70^\circ - 43^\circ$$

$$67^\circ$$

Then the law of sines gives

$$\frac{12}{\sin 70^\circ} = \frac{b}{\sin 43^\circ} = \frac{c}{\sin 67^\circ}$$

Find b using the first two parts of this three-part equation.

$$\frac{12}{\sin 70^\circ} = \frac{b}{\sin 43^\circ}$$

$$b = \frac{12 \sin 43^\circ}{\sin 70^\circ} \approx \frac{12(0.682)}{0.940} \approx 8.71$$

Find c using the first and third parts of the three-part equation.

$$\frac{12}{\sin 70^\circ} = \frac{c}{\sin 67^\circ}$$

$$c = \frac{12 \sin 67^\circ}{\sin 70^\circ} \approx \frac{12(0.921)}{0.940} \approx 11.8$$

Topic: Law of sines

Question: If the lengths of two sides of a triangle are 20 and 30, and the measure of the interior angle opposite the side of length 30 is $B = 95^\circ$, find the measures of angles A and C, where A is opposite the side of length 20.

Answer choices:

A $A \approx 48.9^\circ$, $C \approx 38.6^\circ$, and $c \approx 16.3$

B $A \approx 138.4^\circ$, $C \approx 22.9^\circ$, and $c \approx 45.2$

C $A \approx 37.6^\circ$, $C \approx 47.4^\circ$, and $c \approx 31.8$

D $A \approx 41.6^\circ$, $C \approx 43.4^\circ$, and $c \approx 20.7$

Solution: D

Plugging what we know into the law of sines gives

$$\frac{20}{\sin A}=\frac{30}{\sin 95^\circ}=\frac{c}{\sin C}$$

Find A using the first two parts of this three-part equation.

$$\frac{20}{\sin A}=\frac{30}{\sin 95^\circ}$$

$$\sin A=\frac{20\sin 95^\circ}{30}\approx\frac{2(0.996)}{3}\approx 0.664$$

If A is acute, then $A=41.6^\circ$, and if angle A is obtuse, then $A=138.4^\circ$. But it's impossible to have $A=138.4^\circ$ because the sum of interior angles A and B would be $138.4^\circ+95^\circ=233.4^\circ$, which exceeds 180°, so $A=41.6^\circ$.

Then the third interior angle has measure

$$180^\circ-95^\circ-41.6^\circ$$

$$43.4^\circ$$

Find c using the second and third parts of the three-part equation.

$$\frac{30}{\sin 95^\circ}\approx\frac{c}{\sin 43.4^\circ}$$

$$c\approx\frac{30\sin 43.4^\circ}{\sin 95^\circ}\approx\frac{30(0.687)}{0.996}\approx 20.7$$

Topic: Law of sines

Question: Solve the triangle with interior angles 68° and 79°, where the length of the side opposite the third angle is 18.

Answer choices:

A $C \approx 33^\circ$, $a \approx 30.6$, and $b \approx 32.4$

B $C \approx 112^\circ$, $a \approx 18$, and $b \approx 19.1$

C $C \approx 32^\circ$, $a \approx 31.5$, and $b \approx 9.7$

D $C \approx 101^\circ$, $a \approx 17$, and $b \approx 18$

Solution: A

We'll let angle $A = 68^\circ$ and angle $B = 79^\circ$, then we'll find the measure of the third angle.

$$A + B + C = 180^\circ$$

$$68^\circ + 79^\circ + C = 180^\circ$$

$$C = 180^\circ - 68^\circ - 79^\circ$$

$$C = 33^\circ$$

The known side is opposite this third angle C, so we'll say $c = 18$. Plugging this side length and all three angle measures into the law of sines gives

$$\frac{a}{\sin 68^\circ} = \frac{b}{\sin 79^\circ} = \frac{18}{\sin 33^\circ}$$

We'll use just the first and third parts of the three-part equation in order to solve for a.

$$\frac{a}{\sin 68^\circ} = \frac{18}{\sin 33^\circ}$$

$$a = \frac{18 \sin 68^\circ}{\sin 33^\circ}$$

$$a \approx 30.6$$

To solve for b, we'll use just the second and third parts of the three-part equation.

$$\frac{b}{\sin 79^\circ} = \frac{18}{\sin 33^\circ}$$

$$b = \frac{18 \sin 79^\circ}{\sin 33^\circ}$$

$$b \approx 32.4$$

Topic: The ambiguous case of the law of sines

Question: How many triangles are possible with side lengths 17 and 25, where the angle opposite the side with length 17 is 80°?

Answer choices:

A Two triangles are possible

B One triangle is possible

C No triangles are possible

D The number of triangles can't be determined

Solution: C

Let $a = 17$ and $b = 25$, and let angle $A = 80^\circ$. Then, plugging what we know into the law of sines gives

$$\frac{17}{\sin 80^\circ} = \frac{25}{\sin B} = \frac{c}{\sin C}$$

Find B using the first two parts of this three-part equation.

$$\frac{17}{\sin 80^\circ} = \frac{25}{\sin B}$$

$$\sin B = \frac{25 \sin 80^\circ}{17} \approx 1.45$$

Since the sine of an angle can't be greater than 1, it’s impossible to build a triangle with these properties.

Topic: The ambiguous case of the law of sines

Question: A triangle has side lengths $a = 20$ and $c = 16$ and interior angle $C = 35^\circ$. How many triangles can be made with these properties?

Answer choices:

A Two triangles can be made

B One triangle can be made

C No triangles can be made

D The number of triangles can't be determined

Solution: A

Plugging what we know into the law of sines gives

$$\frac{20}{\sin A} = \frac{b}{\sin B} = \frac{16}{\sin 35^\circ}$$

Find A using the first and third parts of this three-part equation.

$$\frac{20}{\sin A} = \frac{16}{\sin 35^\circ}$$

$$\sin A = \frac{20 \sin 35^\circ}{16} \approx \frac{5(0.574)}{4} \approx 0.718$$

If A is acute then $A = 45.9^\circ$, and if A is obtuse then $A = 134.1^\circ$. Both angle measures keep the sum of the first two interior angles at less than 180°, which means two triangles are possible.

We weren't asked in the question to solve the triangles, but if we do, we find that the two triangles are

1) a triangle with interior angles of 45.9°, 99.1°, and 35°, and corresponding side lengths 20, 27.5, and 16

2) a triangle with interior angles of 134.1°, 10.9°, and 35°, and corresponding side lengths 20, 5.27, and 16

Topic: The ambiguous case of the law of sines

Question: A triangle has side lengths $b = 90$ and $c = 45$ and interior angle $C = 30^\circ$. How many triangles can be made with these properties?

Answer choices:

A One triangle can be made

B Two triangles can be made

C No triangles can be made

D The number of triangles can't be determined

Solution: A

Plugging what we know into the law of sines gives

$$\frac{a}{\sin A} = \frac{90}{\sin B} = \frac{45}{\sin 30^\circ}$$

Find A using the first and third parts of this three-part equation.

$$\frac{90}{\sin B} = \frac{45}{\sin 30^\circ}$$

$$\sin A = \frac{90 \sin 30^\circ}{45} = \frac{90\left(\frac{1}{2}\right)}{45} = 1$$

$$\arcsin(1) = 90^\circ$$

Which means one triangle is possible.

Topic: Area from the law of sines

Question: What is the area of a triangle with side lengths 17 and 53, and an included angle of 81°?

Answer choices:

A $\quad 560$

B $\quad 468$

C $\quad 371$

D $\quad 445$

Solution: D

Let $a = 17$ and $b = 53$. Then the included angle is $C = 81^\circ$. Plugging what we know into the law of sines for the area of a triangle, we get

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$\text{Area} = \frac{1}{2}(17)(53)\sin 81^\circ$$

$$\text{Area} \approx \frac{901}{2}(0.988)$$

$$\text{Area} \approx 445$$

Topic: Area from the law of sines

Question: Find the area of a triangle with interior angles 77° and 56°, if the included side has length 39.

Answer choices:

A 841

B 492

C 571

D 708

Solution: A

The third angle in the triangle has measure

$$180^\circ - 77^\circ - 56^\circ$$

$$47^\circ$$

Then, plugging everything we know into the law of sines, we get

$$\frac{a}{\sin 77^\circ} = \frac{b}{\sin 56^\circ} = \frac{39}{\sin 47^\circ}$$

Find a using the first and third parts of this three-part equation.

$$\frac{a}{\sin 77^\circ} = \frac{39}{\sin 47^\circ}$$

$$a = \frac{39 \sin 77^\circ}{\sin 47^\circ} \approx \frac{39(0.974)}{0.731} \approx 52.0$$

Use the law of sines for the area of a triangle.

$$\text{Area} = \frac{1}{2}ac \sin B$$

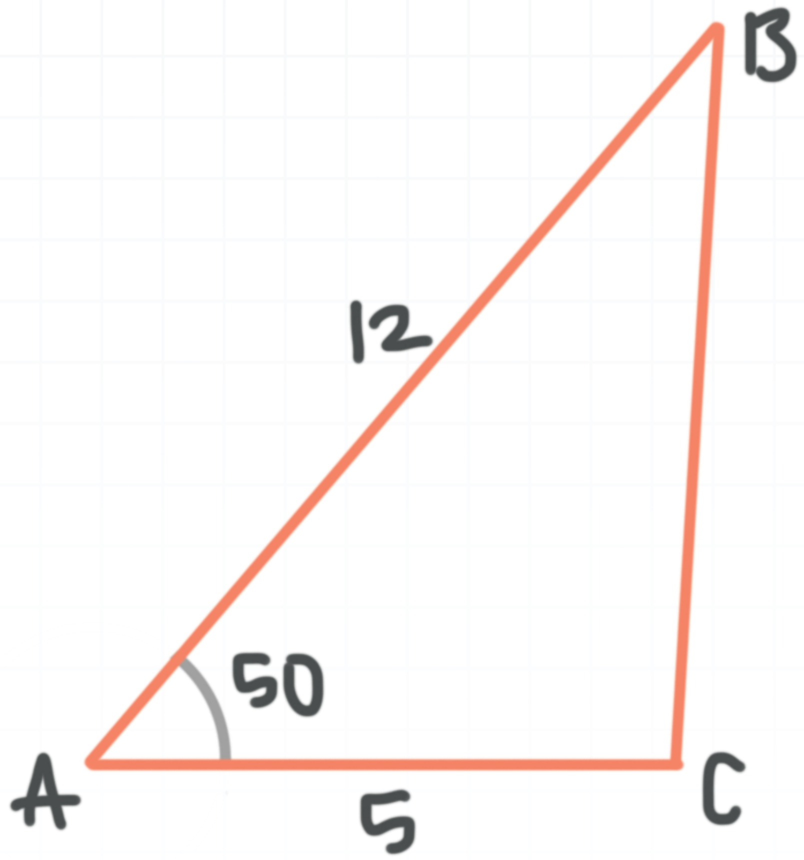
$$\text{Area} \approx \frac{1}{2}(52.0)(39)\sin 56^\circ$$

$$\text{Area} \approx \frac{2{,}028}{2}(0.829)$$

$$\text{Area} \approx 841$$

Topic: Area from the law of sines

Question: Find the area of the triangle to the nearest tenth.



Answer choices:

A 46

B 39

C 23

D 12

Solution: C

Plugging what we know into the law of sines for the area of a triangle, we get

$$\text{Area} = \frac{1}{2}bc\sin A$$

$$\text{Area} = \frac{1}{2}(5)(12)\sin 50^\circ$$

$$\text{Area} \approx 30(0.766)$$

$$\text{Area} \approx 23$$

Topic: Law of cosines

Question: Solve the triangle that has side lengths $a = 28$ and $b = 37$, with an included angle $C = 110^\circ$.

Answer choices:

A $c \approx 12.3$, and the other two angles are 11.9° and 58.1°

B $c \approx 39.8$ and the other two angles are 23.1° and 46.9°

C $c \approx 53.5$ and the other two angles are 29.4° and 40.6°

D $c \approx 66.2$ and the other two angles are 51.3° and 18.7°

Solution: C

Plugging what we know into the law of cosines with $\cos C$ gives

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 28^2 + 37^2 - 2(28)(37)\cos 110^\circ$$

$$c^2 = 784 + 1{,}369 - 56(37)\cos 110^\circ$$

$$c^2 = 2{,}153 - 2{,}072(-0.342)$$

$$c^2 \approx 2{,}862$$

$$c \approx 53.5$$

Rewrite the law of cosines with $\cos A$ by solving it for $\cos A$. (We could also use the law of sines.)

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Plug in what we know to find A.

$$\cos A \approx \frac{28^2 - (37^2 + 53.5^2)}{-2(37)(53.5)}$$

$$\cos A \approx \frac{784 - (1{,}369 + 2{,}862)}{-74(53.5)}$$

$$\cos A \approx \frac{784 - 4{,}231}{-3{,}959}$$

$$\cos A \approx \frac{-3{,}447}{-3{,}959}$$

$$\cos A \approx 0.871$$

$$A \approx \arccos 0.871$$

$$A \approx 29.4^\circ$$

The third angle is therefore

$$B \approx 180^\circ - 110^\circ - 29.4^\circ$$

$$B \approx 40.6^\circ$$

Topic: Law of cosines

Question: If the side lengths of a triangle are $a = 17$, $b = 24$, and $c = 31$, what are the measures of its three interior angles?

Answer choices:

A $A \approx 33.0^\circ$, $B \approx 50.3^\circ$, and $C \approx 96.7^\circ$

B $A \approx 46.3^\circ$, $B \approx 55.6^\circ$, and $C \approx 78.1^\circ$

C $A \approx 37.4^\circ$, $B \approx 63.5^\circ$, and $C \approx 79.1^\circ$

D $A \approx 33.9^\circ$, $B \approx 61.9^\circ$, and $C \approx 84.2^\circ$

Solution: A

Rewrite the law of cosines with $\cos A$ by solving it for $\cos A$.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Plugging in all three side lengths gives

$$\cos A \approx \frac{17^2 - (24^2 + 31^2)}{-2(24)(31)}$$

$$\cos A \approx \frac{289 - (576 + 961)}{-48(31)}$$

$$\cos A \approx \frac{289 - 1{,}537}{-1{,}488}$$

$$\cos A \approx \frac{-1{,}248}{-1{,}488}$$

$$\cos A \approx 0.839$$

$$A \approx \arccos 0.839$$

$$A \approx 33.0^\circ$$

Rewrite the law of cosines with $\cos B$ by solving it for $\cos B$. (We could also use the law of sines.)

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{b^2 - (a^2 + c^2)}{-2ac}$$

Plugging in all three side lengths gives

$$\cos B \approx \frac{24^2 - (17^2 + 31^2)}{-2(17)(31)}$$

$$\cos B \approx \frac{576 - (289 + 961)}{-34(31)}$$

$$\cos B \approx \frac{576 - 1{,}250}{-1{,}054}$$

$$\cos B \approx \frac{-674}{-1{,}054}$$

$$\cos B \approx 0.639$$

$$B \approx \arccos 0.639$$

$$B \approx 50.3^\circ$$

Then the third angle is

$$C = 180^\circ - 33.0^\circ - 50.3^\circ$$

$$C = 96.7^\circ$$

Topic: Law of cosines

Question: Find the length of a in the triangle that has side lengths $b = 14$ and $c = 31$, with an included angle $A = 135^\circ$.

Answer choices:

A $1{,}771$

B 23.3

C 14.3

D 42.1

Solution: D

Plugging what we know into the law of cosines with $\cos C$ gives

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 14^2 + 31^2 - 2(14)(31)\cos 135^\circ$$

$$a^2 = 196 + 961 - 28(31)\cos 135^\circ$$

$$a^2 = 1{,}157 - 868(-0.707)$$

$$a^2 \approx 1{,}771$$

$$a \approx 42.1$$

Topic: Heron's formula

Question: What is the area of a triangle with side lengths 44, 28, and 36?

Answer choices:

A 495

B 503

C 618

D 527

Solution: B

Find s, which is half the perimeter of the triangle.

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(44 + 28 + 36) = \frac{1}{2}(72 + 36) = \frac{1}{2}(108) = 54$$

Then by Heron's formula, the area of the triangle is

$$\text{Area} = \sqrt{54(54 - 44)(54 - 28)(54 - 36)}$$

$$\text{Area} = \sqrt{54(10)(26)(18)}$$

$$\text{Area} = \sqrt{252{,}720}$$

$$\text{Area} \approx 503$$

Topic: Heron's formula

Question: Find the area of the triangle with interior angles 59° and 67°, if the side opposite the 67° angle has length 22.

Answer choices:

A 111

B 182

C 216

D 364

Solution: B

Let $A = 59^\circ$ and $B = 67^\circ$. Then $b = 22$, and

$$C = 180^\circ - 59^\circ - 67^\circ$$

$$C = 54^\circ$$

Plug everything we know into the law of sines.

$$\frac{a}{\sin 59^\circ} = \frac{22}{\sin 67^\circ} = \frac{c}{\sin 54^\circ}$$

Find a using the first two parts of this three-part equation.

$$\frac{a}{\sin 59^\circ} = \frac{22}{\sin 67^\circ}$$

$$a = \frac{22 \sin 59^\circ}{\sin 67^\circ} \approx \frac{22(0.857)}{0.921} \approx 20.5$$

Plug what we know into the law of sines for the area of a triangle with $\sin C$.

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\text{Area} \approx \frac{1}{2}(20.5)(22)\sin 54^\circ \approx \frac{1}{2}(20.5)(22)(0.809) \approx 182$$

Topic: Heron's formula

Question: Find the area of a triangle with side lengths 25 cm and 12 cm and a perimeter of 54 cm.

Answer choices:

A 90

B $1{,}560$

C 515

D 371

Solution: A

The perimeter of the triangle is $p = a + b + c$, so we can find the length of the third side.

$$54 = 25 + 12 + c$$

$$54 = 37 + c$$

$$c = 17$$

Find s, which is half the perimeter of the triangle.

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(p) = \frac{1}{2}(54) = 27$$

Then by Heron's formula, the area of the triangle is

$$\text{Area} = \sqrt{27(27 - 25)(27 - 12)(27 - 17)}$$

$$\text{Area} = \sqrt{27(2)(15)(10)}$$

$$\text{Area} = \sqrt{8100}$$

$$\text{Area} = 90$$

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