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Subject: Statistical Methods

Tutorial: Tutorial 6

1. Calculate the coefficient of correlation from the following data

x	y	x^2	y^2	xy
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98
8	16	64	256	128
9	15	81	225	135

$$\text{Cov}(x,y) = E(xy) - E(x)E(y)$$

$$= \frac{\Sigma xy}{n} - \frac{\Sigma x}{n}\,\frac{\Sigma y}{n}$$

$$= \frac{597}{9} - \left(\frac{45}{9}\right)\left(\frac{108}{9}\right)$$

$$= 66.33 - (5)(12)$$

$$= 66.33 - 60$$

$$= 6.33$$

$$\sigma_x = \sqrt{E(x^2) - (E(x))^2}$$

$$= \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{285}{9} - \left(\frac{45}{9}\right)^2}$$

$$= \sqrt{\frac{240}{9}} \quad \sqrt{\frac{285}{9} - \frac{2025}{81}}$$

$$= \sqrt{\frac{1740}{9}} \quad \sqrt{31.66 - 25}$$

$$= \sqrt{193.33} \quad \sqrt{6.66}$$

$$= 13.90 \quad 2.58$$

$$r = \frac{\text{cov}(x, y)}{\sigma_x \cdot \sigma_y}$$

$$= \frac{6.33}{2.58 \times 2.58}$$

$$= 0.95$$

$$\sigma_y = \sqrt{E(Y^2) - (E(Y))^2}$$

$$= \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$$

$$= \sqrt{\frac{1356}{9} - \frac{11664}{81}}$$

$$= \sqrt{150.66 - 144}$$

$$= \sqrt{6.66}$$

$$= 2.58$$

Q1. b.

x	y	$u = x-5$	$v = y-12$	u^2	v^2	uv
1	9	-4	-3	16	9	12
2	8	-3	-4	9	16	12
3	10	-2	-2	4	4	4
4	12	-1	0	1	0	0
5	11	0	-1	0	1	0
6	13	1	1	1	1	1
7	14	2	2	4	4	4
8	16	3	4	9	16	12
9	15	4	3	16	9	12

$\Sigma x = 45$ $\Sigma y = 108$ $\Sigma u = 0$ $\Sigma v = 0$ $\Sigma u^2 = 60$ $\Sigma v^2 = 60$ $\Sigma uv = 57$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{45}{9} = 5$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{108}{9} = 12$$

$$r = r_{uv} = \frac{cov(u,v)}{\sigma_u \cdot \sigma_v}$$

$$cov(u,v) = \frac{\Sigma uv}{n} - \left(\frac{\Sigma u}{n}\right)\left(\frac{\Sigma v}{n}\right)$$

$$= \frac{57}{9} - (0)(0)$$

$$= 6.33$$

$$\sigma_u = \sqrt{\frac{\Sigma u^2}{n} - \left(\frac{\Sigma u}{n}\right)^2}$$

$$= \sqrt{\frac{60}{9} - 0}$$

$$= 2.562$$

$$r = r_{uv} = \frac{cov(u,v)}{\sigma_u \cdot \sigma_v}$$

$$= \frac{6.33}{(2.582)(2.582)}$$

$$= 0.95$$

X on Y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$\bar{u} = \bar{x} - 5$$
$$\bar{x} = \bar{u} + 5$$

$$\bar{v} = \bar{y} - 12$$
$$\bar{y} = \bar{v} + 12$$

$$\bar{x} - (\bar{u} + 5) = r_{uv} \frac{\sigma_u}{\sigma_v} (\bar{y} - (\bar{v} + 12))$$

$$x - 5 = 0.95 (y - 12)$$

$$x = 0.95y - 64$$

X on Y

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$y - (\bar{v} + 12) = r_{uv} \frac{\sigma_v}{\sigma_u} (\bar{x} - (\bar{u} + 5))$$

$$y - 12 = 0.95 (x - 5)$$

$$y = 0.95x + 7.25$$

Q2.

x	y	x^2	y^2	xy
56	147	3136	21609	8232
42	125	1764	15625	5250
72	160	5184	25600	11,520
36	118	1296	13924	4248
63	149	3969	22,201	~~6,016~~ 9,387
47	128	2209	16,384	6,016
55	150	3025	~~21,025~~ 22,500	8,250
49	145	2401	21,025	7,105
38	115	1444	13,225	4,370
42	140	1764	19,600	5,880
68	152	4624	23,104	10,336
60	155	3600	24,025	9,300
$\Sigma x = 628$	$\Sigma y = 1684$	$\Sigma x^2 = 34,416$	$\Sigma y^2 = 238,822$	$\Sigma xy = 89,894$

$$\text{Cov}(x,y) = E(xy) - E(x)E(y)$$

$$= \frac{\Sigma xy}{n} - \frac{\Sigma x}{n}\frac{\Sigma y}{n}$$

$$= \frac{89,894}{12} - \left(\frac{628}{12}\right)\left(\frac{1684}{12}\right)$$

$$= \frac{89894}{12} - (52.33)(140.33)$$

$$= 7491.166 - 7343.468$$

$$= 147.698$$

$$\sigma_x = \sqrt{E(x^2) - (E(x))^2}$$

$$= \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{34,416}{12} - \left(\frac{628}{12}\right)^2}$$

$$= \sqrt{\frac{34,416}{12} - \frac{3,94,384}{144}}$$

$$= \sqrt{2868 - 2738.77}$$

$$= \sqrt{129.23}$$

$$= 11.36$$

$$\sigma_y = \sqrt{E(Y^2) - (E(Y))^2}$$

$$= \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$$

$$= \sqrt{\frac{238,822}{12} - \left(\frac{1684}{12}\right)^2}$$

$$= \sqrt{\frac{238,822}{12} - \frac{28,35,856}{144}}$$

$$\sigma_y = \sqrt{19,901.83 - 19,693.44}$$

$$= \sqrt{208.39}$$

$$= 14.43$$

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$= \frac{147.698}{(11.36)(14.43)}$$

$$= \frac{147.698}{163.92}$$

$$= 0.90$$

$n = 12$

$$byx = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{12(89,894) - (628)(1684)}{12(34,416) - (628)^2}$$

$$= \frac{1,078,728 - 1,057,552}{412,992 - 394,384}$$

$$= \frac{21,176}{18,608}$$

$b_{yx} = 1.138$

Now $\bar{x} = \frac{\Sigma x}{n} = \frac{628}{12} = 52.33$

$\bar{y} = \frac{\Sigma y}{n} = \frac{1684}{12} = 140.33$

The line regression on y is

Y on X

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 140.33 = 1.138(x - 52.33)$

$y - 140.33 = 1.138x - 59.55$

$y = 1.138x + 80.78$

$$b_{xy} = \frac{(n\Sigma xy - \Sigma x \Sigma y)}{(n\Sigma y^2 - (\Sigma y)^2)}$$

$$= \frac{12(89,894) - (628)(1684)}{12(238,822) - (1684)^2}$$

$$= \frac{1,078,728 - 1,057,552}{2,865,864 - 2,835,856}$$

$$= \frac{21,176}{30,008}$$

$$= 0.705$$

The line regression on Y is

X on Y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

~~$x - 52.33 = 0.705(y -$~~

$$x - 52.33 = 0.705\,(y - 140.33)$$

$$x - 52.33 = 0.705y - 98.93$$

$$x = 0.705y - 46.6$$

Q3.

X	Y	x^2	y^2	XY
59	75	3481	5625	4425
65	70	4225	4900	4550
45	55	2025	3025	2475
52	65	2704	4225	3380
60	60	3600	3600	3600
62	69	3844	4761	4278
70	80	4900	6400	5600
55	65	3025	4225	3575
45	59	2025	3481	2655
49	61	2401	3721	2989

$\Sigma x = 562$ $\Sigma y = 659$ $\Sigma x^2 = 32,230$ $\Sigma y^2 = 43,963$ $\Sigma xy = 37,527$

$$\text{Cov}(x,y) = E(x,y) - E(x)E(y)$$

$$= \frac{\Sigma xy}{n} - \frac{\Sigma x}{n}\,\frac{\Sigma y}{n}$$

$$= \frac{37527}{10} - \left(\frac{562}{10}\right)\left(\frac{659}{10}\right)$$

$$= \frac{37527}{10} - \frac{370,358}{100}$$

$$= 3752.7 - 3703.58$$

$$= 49.12$$

$$\sigma_x = \sqrt{E(x^2) - (E(x))^2}$$

$$= \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{32,230}{10} - \left(\frac{562}{100}\right)^2}$$

$$= \sqrt{\frac{32,230}{10} - \frac{315,844}{100}}$$

$$= \sqrt{3223 - 3158.44}$$

$$= \sqrt{64.56}$$

$$= 8.03$$

$$\sigma_y = \sqrt{E(Y^2) - (E(Y))^2}$$

$$= \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$$

$$= \sqrt{\frac{43,963}{10} - \left(\frac{659}{10}\right)^2}$$

$$= \sqrt{\frac{43963}{10} - \frac{4,34,281}{100}}$$

$$= \sqrt{4696.3 - 4342.81}$$

$$= \sqrt{371.49}$$

$$= 19.27$$

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$= \frac{49.12}{(8.03)(19.27)}$$

$$= \frac{49.12}{154.7381}$$

$$= 0.317$$

$n = 10$

$$byx = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{10(37527) - (562)(659)}{10(32,230) - (562)^2}$$

$$= \frac{375270 - 370,358}{322,300 - 315,844}$$

$$= \frac{4912}{6456}$$

$byx = 0.760$

Now $\bar{x} = \frac{\Sigma x}{n} = \frac{562}{10} = 56.2$

$\bar{y} = \frac{\Sigma y}{n} = \frac{659}{10} = 65.9$

The line regression on x is

Y on X

$y - \bar{y} = byx\,(x - \bar{x})$

$y - 65.9 = 0.760(x - 56.2)$

$y - 65.9 = 0.760x - 42.712$

$y = 0.76x - 42.712 + 65.9$

$y = 0.76x + 23.188$

$bxy = \frac{(n\Sigma xy - \Sigma x \Sigma y)}{(n\Sigma y^2 - (\Sigma y)^2)}$

$= \frac{10(37527) - (562)(659)}{10(43,963) - (659)^2}$

$= \frac{375270 - 370,358}{439,630 - 434,281}$

$= \frac{4912}{5349}$

$b_{xy} = 0.918$

The line regression on Y is

X on Y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$
$$x - 56.2 = 0.918(y - 65.9)$$
$$x - 56.2 = 0.918y - 60.49$$
$$x = 0.918y - 4.29$$

If a student gets 61 masks in Economics, what would you estimate his marks in Maths to be?

Y on X

$$y = 0.76x + 23.188 \quad - (1)$$

Substituting $x = 61$ in eqn (1)

$$y = 0.76(61) + 23.188$$
$$y = 46.36 + 23.188$$
$$y = 69.548$$

If a student got 61 marks in Economics, I would estimate his marks in Maths to be 69.548

Q3.

Y/x →	20	30	40	50	60	fy	fv	fv^2
v/u →	2	3	4	5	6			
22.5 (2.25)	30	6	3	—	—	39	8775	197.4375
37.5 (3.75)	18	32	15	12	8	85	318.75	1195.312
52.5 (5.25)	2	28	40	16	9	95	498.75	2618.438
67.5 (6.75)	—	4	9	10	8	31	209.25	1412.438
						250	1114.5	5423.626

fx	50	70	67	38	25	250
fv	100	210	268	190	150	918
fv^2	200	630	1072	950	900	3752
fuv	291	922.5	133.5	982.5	787.5	4318.5

$$\bar{u} = \frac{\Sigma fu}{n} = \frac{918}{250} = 3.672$$

$$\bar{v} = \frac{\Sigma fv}{n} = \frac{1114.5}{250} = 4.458$$

$$\frac{\Sigma fv^2}{n} = \frac{3752}{250} = 15.003$$

$$\Sigma fv^2 = \frac{5423.626}{250} = 21.682$$

$$\frac{\Sigma fuv}{n} = \frac{4318.5}{250} = 17.274$$

$$r_{xy} = \frac{17.274 - 16.369}{(1.234)(1.3447)} = 0.545$$

$$b_{yx} = 0.545 \times \frac{1.3447}{1.234}$$

$$= 0.593$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 44.58 = 0.593\,(x - 36.72)$$

$$y = 0.593x + 22.8$$

$$b_{xy} = 0.545 \times \frac{1.234}{1.3447}$$

$$= 0.501$$

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 36.72 = 0.501\,(y - 44.58)$$

$$x = 0.501y + 14.385$$

When wife = 31 ($x = 31$)

Man Y = $0.593\,(31) + 22.8$

Husband y = 41.83 years

When Husband = $y = 40$

Wife x = $0.50\,(40) + 14.385$

= 34.425

Q4. $r(x,y) = ?$

$n = 69$ villagers

Linear reg Y on $x = ?$ Bigaud

Total Area (in Bighas)

Y \ x	0-500	500-1000	1000-1500	1500-2000	2000-2500
0-200	12	6	—	—	—
200-400	2	18	4	2	1
400-600	—	4	7	3	—
600-800	—	1	—	2	1
800-1000	—	—	1	2	3

Y \ X, V \ u	250	750	1250	1750	2250
	2·5	7·5	12·5	17·5	22·5
100	12	6	—	—	—
300	2	18	4	2	1
500	—	4	7	3	—
700	—	1	—	2	1
900	—	—	1	2	3

fy	fx	fv	fu	fv^2	fu^2	fuv
18	14	18	35	18	87.5	45
27	29	81	217.5	243	1631.25	652.5
14	12	70	150	350	1875	700
4	9	28	157.5	196	2756.25	927.5
6	5	54	112.5	486	2531.25	832.5
69	69	251	672.5	1293	8881.25	3157.5

$$\bar{u} = \frac{672.5}{69} = 9.746 = \bar{u}$$

$$\bar{v} = \frac{251}{69} = \bar{v} = 3.637$$

$$\frac{\sum f^2 u}{n} = \frac{8881.25}{69} = 128.713$$

$$\frac{\sum f^2 v}{n} = \frac{1293.0}{69} = 18.739$$

$$\frac{\sum fuv}{n} = \frac{3157.5}{69} = 45.7608$$

$$\sigma u = \sqrt{128.7 - (9.746)^2} = 5.8065$$

$$\sigma v = \sqrt{18.739 - (3.637)^2} = 2.3476$$

$$r_{xy} = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$r_{xy} = r_{uv} = \frac{cov(v,u)}{\sigma u \cdot \sigma v}$$

$$r_{uv} = \frac{45.7608 - 35.4462}{(5.8065)(2.3476)}$$

$$= \frac{10.3146}{13.631}$$

$$r_{uv} = 0.7567$$

$$b_{yx} = r_{xy} \cdot \frac{\sigma x}{\sigma y}$$

$$b_{yx} = 0.7567 \times \frac{2.3476}{5.8065}$$

$$b_{yx} = 0.3059$$

Y on X

$$(y - \bar{y}) = 0.3059 (x - \bar{x})$$

$$(y - 363.7) = 0.3059 (x - 974.6)$$

$$y = 0.3059x - 298.13014 + 363.7$$

$$y = 0.3059x + 65.57$$

X on Y

$$b_{xy} = 0.7567 \times \frac{5.8065}{2.3476}$$

$$b_{xy} = 1.8716$$

$$(x - 974.6) = 1.8716\,(y - 363.7)$$

$$x = 1.8716\,y + 293.89$$

For 100 Bighas $(x = 1000)$

$$y = 0.3059\,(1000) + 65.57$$

$$y = 306 + 65.57$$

$$y = 371.57$$