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Subject: Statistical Methods

Tutorial: Tutorial 7

1. In partially destroyed laboratory record of an analysis of correlation data, the following results only are legible

Variance of $X = 9$, regression equations are $8X - 10Y + 66 = 0$ & $40X - 18Y = 214$

What was

i) mean of X and Y

ii) the correlation between X and Y

iii) the SD of Y

Solution :

Variance of $X = 9$

$8X - 10Y + 66 = 0$

$40X - 18Y = 214$

$\sigma x = 9$

$8x - 10y = -66 \quad - (I)$

$40x - 18y = 214 \quad - (II)$

Multiplying eqn I by 5

$$\begin{array}{rcl} 40x - 50y & = & -330 \\ 40x - 18y & = & 214 \\ (-) \quad\quad (+) & & - \\ \hline -32y & = & -544 \end{array}$$

$y = 17$

$\therefore \bar{y} = 17$

Then $\bar{x} = \dfrac{10\bar{y} - 66}{8}$

$\bar{x} = \dfrac{170 - 66}{8}$

$\bar{x} = 13$

$(13, 17)$ is the point where two regression

Mean of $x = 13$
Mean of $y = 17$

ii) Correlation between X and Y

$8x - 10y = -66$

$10y = 66 + 8x$

$y = \dfrac{66}{10} + \dfrac{8}{10}x$

Slope $= b_{yx} = \dfrac{8}{10} = 0.8 \quad - (1)$

$40x - 18y = 214$

$40x = 214 + 18y$

$x = \frac{214}{40} + \frac{18}{40} y$

Slope $= b_{xy} = \frac{18}{40} = 0.45$ — (2)

Finding coefficient of correlation btw x & y.

$r = \sqrt{b_{xy} \times b_{yx}}$

$= \sqrt{0.45 \times 0.8}$

$= \frac{\sqrt{3600}}{10^4}$

$= 0.60$

Standard deviation of Y

$\sigma_y = \frac{b_{yx} \cdot \sigma_x}{r}$

$= 0.40$

Q2. In a study of a random sample of 120 students the following results are obtained

$\overline{X_1} = 68$
$\overline{X_2} = 70$
$\overline{X_3} = 74$
$S_1^2 = 100$
$S_2^2 = 25$
$S_3^2 = 81$
$r_{12} = 0.60$
$r_{13} = 0.70$
$r_{23} = 0.65$

$S_1^2 = Var(X_i)$

where x_1, x_2, x_3 denote percentage of marks obtained by student in in I test, II test and the final examination respectively

i) Obtain the least square regression equation of X_3 on X_1 and X_2

ii) Compute $r_{12.3}$ and $R_{3.12}$

iii) Estimate the percentage marks of a student in final examination if he gets 60 % and 67 % in I & II tests respectively

Solution

Regression equation of X_3 on X_1 & X_2 is :

$$(X_3 - \overline{X_3}) = b_{32\cdot1}(X - \overline{X_2}) + b_{31\cdot2}(X_1 - \overline{X_1})$$

$$= r_{32\cdot1}\left(\frac{\sigma_{3\cdot1}}{\sigma_{2\cdot1}}\right)(X_2 - \overline{X_2}) + r_{31\cdot2}\left(\frac{\sigma_{3\cdot2}}{\sigma_{1\cdot2}}\right)(X_1 - \overline{X_1})$$

$$= \frac{r_{32} - r_{31}r_{21}}{\sqrt{1-r_{31}^2}\sqrt{1-r_{21}^2}} \star \frac{\sigma_3\sqrt{1-r_{31}^2}}{\sigma_2\sqrt{1-r_{21}^2}}(X_2 - \overline{X_2})$$

$$+ \frac{r_{31} - r_{32}r_{12}}{\sqrt{1-r_{32}^2}\sqrt{1-r_{12}^2}} \star \frac{\sigma_3\sqrt{1-r_{32}^2}}{\sigma_1\sqrt{1-r_{12}^2}}(X_1 - \overline{X_1})$$

$$= \frac{r_{32} - r_{31}}{1-r_{21}^2} \star \frac{\sigma_3}{\sigma_2}(X_2 - \overline{X_2}) + \frac{r_{31} - r_{32}r_{12}}{1-r_{12}^2} \star \frac{\sigma_3}{\sigma_1}(X_1 - \overline{X_1})$$

$$= \frac{r_{32} - r_{31}r_{21}}{1-r_{21}^2} \times \frac{S_3}{S_2}(X_2 - \overline{X_2}) + \frac{r_{31} - r_{32}r_{12}}{1-r_{12}^2}\frac{S_3}{S_1}(X_1 - \overline{X_1})$$

$$\cancel{(X_3 - 74) = \frac{0.65 - 0.6 \times 0.7}{1-(0.6)^2} \times \frac{9}{5}(X_2 - 70) + 0.7 - 0.65 \times}$$

$$= \frac{0.65 - 0.42}{0.64} \cdot \frac{9}{5}(X_2 - 70) + \frac{0.7 - 0.39}{0.64}\left(\frac{9}{10}\right)(X_1 - 68)$$

$$X_3 - 74 = 0.646(X_2 - 70) + 0.435(X_1 - 68)$$

$$x_3 = 74 + 0.646 X_2 - 45.22 + 0.435 X_1 - 29.58$$

$$X_3 = 0.646 X_2 + 0.435 X_1 - 0.8$$

$$r_{12\cdot3} = \frac{r_{12} - r_{13}\cdot r_{23}}{\sqrt{1-r_{13}^2}\sqrt{1-r_{23}^2}}$$

$$= \frac{0.6 - (0.7)(0.65)}{(0.71)(\sqrt{0.5775})}$$

$$= \frac{0.145}{0.539}$$

$$= 0.267$$

$$R_{3\cdot12} = \sqrt{\frac{r_{13}^2 + r_{23}^2 - 2 r_{12}\cdot r_{13}\cdot r_{23}}{1-r_{12}^2}}$$

$$= \sqrt{\frac{0.49 + 0.4225 - 2(0.6)(0.7)(0.65)}{1-0.36}}$$

$$= 0.7565$$

iii) $X_1 = 60$
$X_2 = 67$

$$X_3 = 0.646 X_2 + 0.435 X_1 - 0.8$$

$$X_3 = 43.282 + 26.1 - 0.8$$

$$X_3 = 68.582$$

∴ Predicted value of final exam is 68.582 %

Q3. Assuming X_1, X_2, X_3 measured as deviations from means we get $a = 0$

$$X_1 = a_{1.23} + b_{12.3} X_2 + b_{13.2} X_3$$

$$X_1 = b_{12.3} X_2 + b_{13.2} X_3 \quad \text{(Demand equation)}$$

$$b_{12.3} = \frac{\sigma_{1.23}}{\sigma_{2.13}} \; r_{12.3}$$

$$b_{12.3} = \frac{\sigma_1 \sqrt{1 - r_{13}^2}}{\sigma_2 \sqrt{1 - r_{23}^2}} \left(\frac{r_{12} - r_{13} r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}} \right)$$

$$b_{12.3} = \frac{7.22}{5.47} \left(\sqrt{\frac{0.1536}{0.6279}} \right) \left(\frac{-0.83 + 0.5612}{(0.391)(0.792)} \right)$$

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$$b_{12.3} = 1.3199 \times 0.494 \left(\frac{-0.2688}{0.3096} \right)$$

$$b_{12.3} = -0.566$$

$$b_{13.2} = \frac{\sigma_{1.32}}{\sigma_{3.12}} \quad r_{13.2}$$

$$= \frac{\sigma_1}{\sigma_2} \left(\frac{r_3 - r_{12} r_{23}}{1 - r_{23}^2} \right)$$

$$= \frac{7.22}{6.87} \left(\frac{0.92 - 0.5063}{0.6279} \right)$$

$$= 1.05 \left(\frac{0.4137}{0.6279} \right)$$

$$= 0.6918$$

$$X_1 = -0.566 X_2 + 0.6918 X_3$$

Q4. $\begin{vmatrix} x_1/\sigma_1 & x_2/\sigma_2 & x_3/\sigma_3 \\ r_{12} & 1 & r_{23} \\ r_{13} & r_{23} & 1 \end{vmatrix} = 0$

$$\frac{x_1}{\sigma_1}(1-r_{23}^2) - \frac{x_2}{\sigma_2}(r_{12}-r_{13}r_{23}) + \frac{x_3}{\sigma_3}(r_{12}r_{23}-r_{13}) = 0$$

$$\frac{x_1}{\sigma_1} \times \frac{\sigma_1}{\sigma_3}\left(\frac{r_{13}-r_{12}r_{23}}{b_{13\cdot2}}\right) - \frac{x_2}{\sigma_2} \times \frac{\sigma_2}{\sigma_1}(1-r_{23}^2)(b_{12\cdot3})$$

$$+ \frac{x_3}{\sigma_3} \times (-1)(r_{13}-r_{12}r_{23}) = 0$$

$$\Rightarrow \frac{x_1}{\sigma_3}\frac{(r_{13}-r_{12}r_{23})}{b_{13\cdot2}} - \frac{x_3}{\sigma_3}(r_{13}-r_{12}r_{23})$$

$$- \frac{x_2}{\sigma_1}(1-r_{23}^2)(b_{12\cdot3}) = 0$$

$$\frac{(r_{13}-r_{12}r_{23})}{\sigma_3}\left(\frac{x_1}{b_{13\cdot2}} - x_3\right) = \frac{x_2}{\sigma_1}(1-r_{23}^2)(b_{12\cdot3})$$

$$\frac{x_1}{b_{13\cdot2}} - x_3 = \frac{x_2}{b_{13\cdot2}} \times b_{12\cdot3}$$

$$\frac{x_1}{b_{13\cdot2}} = x_3 + \frac{x_2\, b_{12\cdot3}}{b_{13\cdot2}}$$

$X_1 = X_2\, b_{12\cdot 3} + X_3\, b_{13\cdot 2}$

$X_1 = b_{12\cdot 3}\, X_2 + b_{13\cdot 2}\, X_3$

Q5.

$X_1 = a_{1\cdot 23} + b_{12\cdot 3}\, X_2 + b_{13\cdot 2}\, X_3$

$\sum X_1 = 6a_{1\cdot 23} + b_{12\cdot 3} \sum X_2 + b_{13\cdot 2} \sum X_3$

$\sum X_1 X_2 = a_{1\cdot 23} \sum X_2 + b_{12\cdot 3} \sum X_2^2 + b_{13\cdot 2} \sum X_3 X_2$

$\sum X_1 X_3 = a_{1\cdot 23} \sum X_3 + b_{12\cdot 3} \sum X_2 X_3 + b_{13\cdot 2} \sum X_3 X_2$

$\sum X_1 = 54$

$\sum X_2 = 48$

$\sum X_3 = 102$

$\sum X_1 X_2 = 60 + 72 + 56 + 54 + 32 + 45$
$= 339$

$\sum X_2 X_3 = 1034$

$\sum X_1 X_3 = 720$

$\sum X_2^2 = 494$

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$\sum X_3^2 = 2188$

Substituting values

$54 = 6a_{1.23} + 48b_{12.3} + 102b_{13.2}$

$339 = 48a_{1.23} + 494b_{12.3} + 1034b_{13.2}$

$720 = 102a_{1.23} + 1034b_{12.3} + 2188b_{13.2}$

Solving 1,2,3 eqns

$a_{1.23} = 16.477$

$b_{12.3} = 0.3899$

$b_{13.2} = -0.62334$

MLR is

$X_1 = 16.447 + 0.3899X_2 - 0.62334X_3$

Q6. i) X_1 on X_2 and X_3

ii) X_2 on X_1 and X_3

$r_{12} = 0.8$

$r_{13} = 0.6$

$r_{23} = 0.5$

$\sigma_1 = 10$

$\sigma_2 = 8$

$\sigma_3 = 5$

$X_1 = b_{12.3} X_2 + b_{13.2} X_3$

$X_2 = b_{21.3} X_1 + b_{23.1} X_3$

$$b_{12.3} = \frac{\sigma_1}{\sigma_2}\left(\frac{r_{12} - r_{13} r_{23}}{1 - r_{23}^2}\right)$$

$$= \frac{10}{8}\left(\frac{0.8 - 0.3}{0.75}\right)$$

$$= 0.833$$

$$b_{13.2} = \frac{\sigma_1}{\sigma_3}\left(\frac{r_{13} - r_{12} \cdot r_{23}}{1 - r_{23}^2}\right)$$

$$= \frac{10}{5}\left(\frac{0.6 - 0.4}{0.75}\right)$$

$$= 0.533$$

$$b_{21\cdot 3} = \frac{\sigma_2}{\sigma_1}\left(\frac{r_{21} - r_{23}r_{13}}{1 - r_{13}^2}\right)$$

$$= \frac{8}{10}\left(\frac{0\cdot 8 - 0\cdot 3}{0\cdot 64}\right) = 0\cdot 625$$

$$b_{23\cdot 1} = \frac{\sigma_2}{\sigma_3}\left(\frac{r_{23} - r_{12}r_{13}}{1 - r_{13}^2}\right)$$

$$= \frac{8}{5}\left(\frac{0\cdot 5 - 0\cdot 48}{0\cdot 64}\right)$$

$$= 0\cdot 05$$

X_1 on X_2 and X_3

$$X_1 = 0\cdot 833 X_2 + 0\cdot 533 X_3$$

X_2 on X_1 and X_3 :

$$X_2 = 0\cdot 625 X_1 + 0\cdot 05 X_3$$

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