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Subject: Statistical Methods

Tutorial: Tutorial 10

Lab Exercise 10

1. Consider the following with respect to DV's

	DV 1	DV 2
Population 1	9, 6, 9	3, 2, 7
Population 2	0, 2	4, 0
Population 3	3, 1, 2	8, 9, 7

Find SStreatment, SS residual, Total SS corrected

Construct MANOVA table

Find Wilks Lambda

Conduct F Test

The sample sizes are $n_1 = 3$, $n_2 = 2$, $n_3 = 3$

Arranging the observation pairs x_{ij} in rows we get

$$\begin{bmatrix} 9 \\ 3 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 8 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 9 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

with $\bar{x}_1 = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

$$\bar{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\bar{x}_3 = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

For DV 1

Observation = mean + treatment + residual

$$x_{ij} = \bar{x} + (\bar{x}_i + \bar{x}) + (x_{ij} - \bar{x}_i)$$

$$\begin{pmatrix} 9 & 6 & 9 \\ 0 & 2 & \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & \\ 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 8-4 & 4 & 4 \\ 1-4 & -3 & \\ 2-4 & -2 & 2 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & \\ 1 & -1 & 0 \end{pmatrix}$$

$$SSobs = 9^2 + 6^2 + 9^2 + 0^2 + 2^2 + 3^2 + 1^2 + 2^2$$
$$= 81 + 36 + 81 + 0 + 4 + 9 + 1 + 4$$
$$= 216$$

$$SSmean = 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2$$
$$= 128$$

$$SStr = 4^2 + 4^2 + 4^2 + (-3)^2 + (-3)^2 + (-2)^2 + (-2)^2 + (-2)^2$$
$$= 78$$

$$SSres = 1^2 + (-2)^2 + 1^2 + (-1)^2 + 1^2 + 1^2 + (-1)^2 + 0^2$$
$$= 10$$

$$SS\ corrected = SSobs - SSmean$$
$$= 216 - 128$$
$$= 88$$
$$= SStr + SSres$$

For DV 2

observation = mean + treatment effect + residual

$$x_{ij} = \bar{x} + (\bar{x}_i - \bar{x}) + (x_{ij} - \bar{x}_e)$$

$$\begin{pmatrix} 3 & 2 & 7 \\ 4 & 0 & \\ 8 & 9 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & \\ 5 & 5 & 5 \end{pmatrix} + \begin{pmatrix} -1 & -1 & -1 \\ -3 & -3 & \\ 3 & 3 & 3 \end{pmatrix} + \begin{pmatrix} -1 & -2 & 3 \\ -2 & -2 & \\ 0 & 1 & -1 \end{pmatrix}$$

$SSobs = 3^2 + 2^2 + 7^2 + 4^2 + 0^2 + 8^2 + 9^2 + 7^2$
$= 9 + 4 + 49 + 16 + 0 + 64 + 81 + 49$
$= 272$

$SSmean = 8 \times 5^2 = 200$

$SSto = (-1)^2 + (-1)^2 + (-1)^2 + (-3)^2 + (-3)^2 + 3^2 + 3^2 + 3^2$
$= 48$

$SSres = (-1)^2 + (-2)^2 + 3^2 + (2)^2 + (-2)^2 + 0^2 + 1^2 + (-1)^2$
$= 24$

$SS connected = SS obs - SSmean$
$= SStr + SSres$
$= 24 + 48$
$= 72$

$SSobs = SSmean + SStr + SSres$

Cross product contributions

$Total = 9(3) + 6(2) + 9(7) + 0(4) + 2(0) + 3(8) + 1(9)$
$+ 2(7)$
$= 149$

$Mean = 4(5) + 4(5) + 4(5) + 4(5) + 4(5) + 4(5)$
$+ 4(5) + 4(5)$
$= 8 \times 4 \times 5$
$= 160$

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$$\text{Treatment} = 4(-1) + 4(-1) + 4(-1) + (-3)(-3) + (-3)(-3) + (-2)(3) + (-2)(3) - 2(3)$$

$$= 3(4)(-1) + 2(-3)(-3) + 3(-2)(3)$$

$$= -12$$

$$\text{Residual} = (-1) + (-2)(-2) + 1(3) + (-1)(2) + 1(-3) + 1(-1) + 0$$

$$= 1$$

Total corrected cross product = Total cross product − mean cross product

$$= 149 - 160$$

$$= -11$$

MANOVA TABLE

Source of Variation	Matrix of SS and cross product	Degrees of freedom
Treatment (W)	$\begin{bmatrix} 78 & -12 \\ -12 & 48 \end{bmatrix}$	$g - 1 = 3 - 1$ $g = 2$
Residual (B)	$\begin{bmatrix} 10 & 1 \\ 1 & 24 \end{bmatrix}$	$3+2+3-3$ $= 5$
Total connected	$\begin{bmatrix} 88 & -11 \\ -11 & 72 \end{bmatrix}$	$(3+2+3)-1$ $= 7$

Wilk's Lambda $= \dfrac{|W|}{|e+W|}$

$$= \frac{\begin{vmatrix} 10 & 1 \\ 1 & 24 \end{vmatrix}}{\begin{vmatrix} 88 & -11 \\ -11 & 72 \end{vmatrix}}$$

$$= \frac{10(24)-1}{88(72)-121}$$

$$= 0.0385$$

To carry out F-Test

$$\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right)\left(\frac{\Sigma n_0 - g - 1}{g-1}\right) = \left(\frac{1-\sqrt{0.0385}}{\sqrt{0.0385}}\right)\left(\frac{8-3-1}{3-1}\right)$$

$$= 8.19$$

$v_1 = 2(g-1) = 4$

$v_2 = 2(\Sigma n_e - g - 1) = 8$

Since $8.19 > F_{4,8}(0.01)$

$8.19 > 7.01$ we reject H_0 and conclude that treatment differences exist

2. Group | Observations | Sample mean vectors

$l=1$ (private) $n_1 = 271$ $\bar{x}_1 = \begin{bmatrix} 2.066 \\ 0.480 \\ -6.82 \\ 0.360 \end{bmatrix}$

$l=2$ (non profit) $n_2 = 138$

$l=3$ (govt) $n_3 = 107$

$\Sigma n_l = 516$

$\bar{x}_2 = \begin{bmatrix} 2.167 \\ 0.596 \\ 0.124 \\ 0.418 \end{bmatrix}$ $\bar{x}_3 = \begin{bmatrix} 2.273 \\ 0.521 \\ 0.125 \\ 0.383 \end{bmatrix}$

3 types of owners: private, non-profit & government

$g = 3$

4 dependent variables $\Rightarrow p = 4$

Since matrices S_l seem to be compatible, we can obtain W as

$$W = (n_1 - 1) S_1 + (n_2 - 1) S_2 + (n_3 - 1) S_3$$

$$= \begin{bmatrix} 182.962 & & & \\ 4.408 & 8.2 & & \\ 1.695 & 0.633 & 1.484 & \\ 9.581 & 2.488 & 0.394 & 6.538 \end{bmatrix}$$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3} = \begin{bmatrix} 2.136 \\ 0.579 \\ 0.102 \\ 0.380 \end{bmatrix}$$

$$B = \sum_{l=1}^{3} n_l(\bar{x}_l - \bar{x})(\bar{x}_l - \bar{x}) = \begin{bmatrix} 3.475 & & & \\ 1.111 & 1.225 & & \\ 0.821 & 0.453 & 0.235 & \\ 0.584 & 0.610 & 0.230 & 0.307 \end{bmatrix}$$

Ho there is no difference in average costs among the three types of owners : private, non profit, goverment

H1 : the average costs differ depending on the type of ownership

We have $\Lambda^* = \frac{|W|}{|W+B|} = 0.7714$

$p = 4$ & $g = 3$ gives

$$\left(\frac{\Sigma n_l - p - 2}{p}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right)$$

$$= \left(\frac{516 - 4 - 2}{4}\right)\left(\frac{1-\sqrt{0.7714}}{\sqrt{0.7714}}\right)$$

$$= 17.667824$$

Let $\alpha = 0.01$, so that $F_{2(4) \times 2(510)}(0.01)$

$\chi^2_8(0.01)/8 = 2.51$

Since $17.67 > F_{8,1020}(0.1) = 2.51$

we reject H_0 and conclude that average costs differ, depending upon type of ownership