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Subject: Statistical Methods

Tutorial: Tutorial 4

Practical 4

Q1. The mean and variance of 8 observations are 9 and 9.25 respectively. If size of the observations are 6, 7, 10, 12, 12, 13. Find the remaining 2 observations

Let the remaining observations be x and y

$$\frac{6 + 7 + 10 + 12 + 12 + 13 + x + y}{8} = 9$$

$$72 = 60 + x + y$$

$$x + y = 12$$

$$9.25 = \left(\frac{36 + 49 + 100 + 144 + 144 + 169 + x^2 + y^2}{8} \right) - 81$$

$$(9.25) \times 8 = 642 + x^2 + y^2$$

$$722 - 642 = x^2 + y^2$$

$$80 = x^2 + y^2$$

From 1, $x^2 + (12 - x)^2 = 80$

$$2x^2 + 144 - 24x = 80$$

$$2x^2 - 24x + 64 = 0$$

$$x^2 - 12x + 32 = 0$$

$$(x - 8)(x - 4) = 0$$

$$x = 8, x = 4$$

$$y = 8, 4$$

∴ Remaining 2 observations are 8, 4

Q2) The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and standard deviation of the resulting observation

Let the observation be x_1, x_2, \dots, x_6

$$8 = \frac{x_1 + x_2 + \dots + x_6}{6}$$

$$\sum_{i=1}^6 x_i = 48$$

$$\text{new mean} = \frac{3(x_1 + x_2 + \dots + x_6)}{6}$$

$$= \frac{3(48)}{6}$$

$$\text{new mean} = 24$$

$$\text{variance} = 16$$

$$16 = \frac{x_1^2 + x_2^2 + \dots + x_6^2}{6} - (8)^2$$

$$(16 + 64) \times 6 = x_1^2 + x_2^2 + \dots + x_6^2$$
$$480 = x_1^2 + x_2^2 + \dots + x_6^2$$

$$\therefore \text{New variance} \Rightarrow (3)^2 [x_1^2 + x_2^2 + \dots + x_6^2] - (\text{new mean})^2$$

$$= \frac{9}{6} (480) - (24)^2 = 720 - 576$$

$$= 144$$

$$\text{new s.d} = \sqrt{144}$$

$$\text{new standard dev} = 12$$

Q3. The mean and standard deviation of 20 observations are found to be 10 and 2. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following

i) If wrong item is omitted

ii) If it is replaced by 12

$$i) 10 = \frac{x_1 + x_2 + \dots + x_{20}}{20}$$

$$200 = x_1 + x_2 + \dots + x_{20}$$

$$\text{var} = (2)^2 = 4$$

$$\text{correct mean} = \frac{(x_1 + x_2 + \dots + x_{20})}{20} - 8$$

$$= \frac{200 - 8}{19} = \frac{192}{19}$$

$$\text{correct mean} = 10.105$$

$$\text{Also, } 4 = \frac{x_1^2 + x_2^2 + \dots + x_{20}^2}{20} - 100$$

$$(104)(20) = x_1^2 + x_2^2 + \dots + x_{20}^2$$

$$2080 = x_1^2 + x_2^2 + \dots + x_{20}^2$$

$$\therefore \text{Correct value var} \Rightarrow \left(\frac{x_1^2 + x_2^2 + \dots + x_{20}^2}{19} - (\text{correct mean})^2 \right)$$

$$= \left(\frac{2080 - 64}{19} - (10.105)^2 \right)$$

$$= \left(\frac{106 \cdot 105}{19} - (10.105)^2 \right)$$

$$= 106 \cdot 105 - 102 \cdot 111$$

$$= 3.993$$

$$\therefore \text{Correct s.d} = \sqrt{3.993} = 1.998$$

$$\text{ii)} \quad x_1 + x_2 + x_3 \dots x_{20} = 200$$

$$\text{correct mean} = \frac{200 - 8 + 12}{20}$$

$$= \frac{204}{20}$$

$$\therefore \text{correct mean} = 10.2$$

$$x_1^2 + x_2^2 + \dots x_{20}^2 = 2080$$

$$\text{correct var} = \left[\frac{2080 - (8)^2 + (12)^2}{20} \right] - (10.2)^2$$

$$= \left(\frac{2160}{20} \right) - (10.2)^2$$

$$= 108 - 104.04$$

$$= 3.96$$

$$\text{correct sd} = \sqrt{3.96}$$

$$= 1.989$$

Q4. The mean and standard deviation of marks obtained by 50 students of a class in 3 subjects are given below

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

which of the following shows highest variability in marks and which shows the lowest? Give reasons

$$\text{Coefficient of variation} = \frac{s}{\bar{x}} \times 100$$

$$\text{Cov (Mathematics)} = \frac{12}{42} \times 100$$

$$= 28.57$$

$$\text{Cov (Physics)} = \frac{15}{32} \times 100$$

$$= 46.875$$

$$\text{Cov (Chemistry)} = \frac{20}{40.9} \times 100$$

Chemistry has the highest variability since its cov is the highest and Mathematics has the lowest variability since its cov is the lowest

Q5. Find mean and variance of the data

Class Interval	0-10	10-20	20-30	30-40	40-50
frequency	14	23	27	21	15

Draw the following

i) Histogram

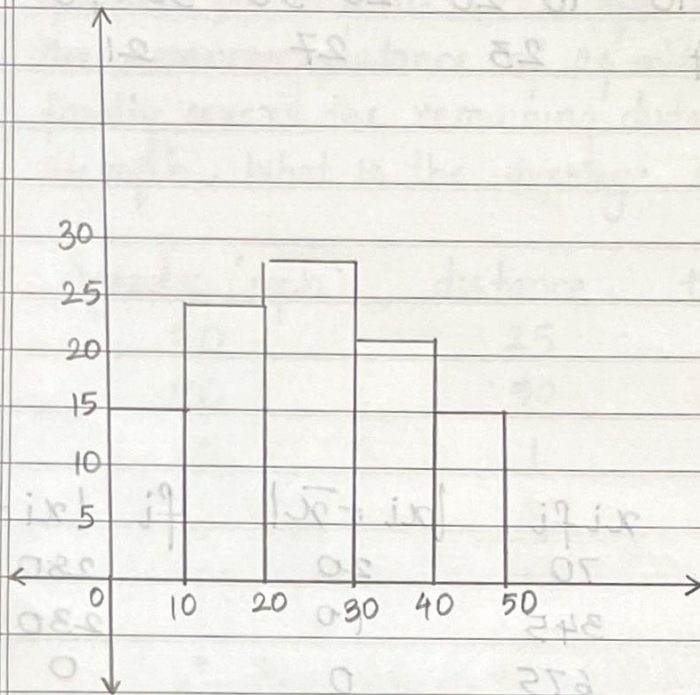
ii) Frequency Polygon

Classes	x_i	f_i	$x_i f_i$	$ x_i - \bar{x} $	f_i	$ x_i - \bar{x} $
0-10	5	14	70	20		280
10-20	15	23	345	10		230
20-30	25	27	675	0		0
30-40	35	21	735	10		210
40-50	45	15	675	20		300
		<u>100</u>	<u>2500</u>			<u>1020</u>

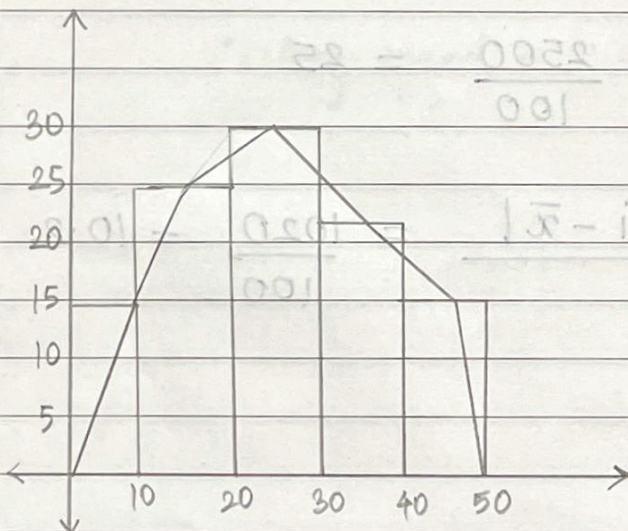
$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{2500}{100} = 25$$

$$\text{var} = \frac{\sum f_i x_i |x_i - \bar{x}|}{\sum f_i} = \frac{1020}{100} = 10.2$$

i) Histogram



ii) Frequency Polygon



Q6. A train runs 25 miles at a speed of 30mph, another 50 miles at a speed of 40 mph, then due to repairs of the track travels for 6 mins at a speed of 10 mph and finally covers the remaining distance of 24 miles at speed of 24 mph and finally covers the remaining distance of 24 miles at speed of 24 mph. What is the average speed in mph

Speeds (mph)	distance	time taken (hr)
30	25	0.853
40	50	1.25
10	1	6 mins \rightarrow 0.1 hr
	<u>24</u>	<u>1</u>
	100	3.183

$$\therefore \text{Avg speed} \Rightarrow \frac{\text{Total dist}}{\text{Total time}}$$

$$= \frac{100}{3.183}$$

$$\therefore \text{Avg speed} = 31.42 \text{ mph}$$