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Subject: Statistical Methods

Tutorial: Tutorial 12

SM Lab 12

Q1. $\begin{matrix} 12 & 6 & 9 & 15 & 13 & 5 \\ 19 & 22 & 6 & 3 & 2 & 20 \end{matrix}$

$Avg_1 = 10$

$Avg_2 = 12$

$$\text{Centered Matrix } (A) = \begin{bmatrix} 2 & -4 & -1 & 5 & 3 & -5 \\ 7 & 10 & -6 & -9 & -10 & 8 \end{bmatrix}$$

$$S = \frac{AA^T}{n-1} \qquad n = 6$$

$$AA^T = \begin{bmatrix} 2 & -4 & -1 & 5 & 3 & -5 \\ 7 & 10 & -6 & -9 & -10 & 8 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -4 & 10 \\ -1 & -6 \\ 5 & -9 \\ 3 & -10 \\ -5 & 8 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} -80 & -135 \\ -135 & 430 \end{bmatrix}$$

$$S = \begin{bmatrix} 16 & -27 \\ -27 & 86 \end{bmatrix}$$

Eigen values of S :

$$\begin{vmatrix} 6-\lambda & -27 \\ -27 & 86-\lambda \end{vmatrix} = 0$$

$$[16-\lambda][86-\lambda] - 729 = 0$$

$$\lambda^2 - 102\lambda + 647 = 0$$

$$\lambda = 95.2, 6.8$$

For $\lambda = 95.2$

$$\begin{bmatrix} 16 & -27 \\ -27 & 86-95.2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$e = (-0.32, -0.94)$$

ii) For $\lambda = 6.8$

$$\begin{bmatrix} 16-6.8 & -27 \\ -27 & 86-6.8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$9.2x - 27y = 0$$

~~$-27x +$~~

$$x = \frac{27}{79.2} y$$

Let $y=1$, $x = 0.34091$

Normalizing (dividing by 1.056512)

the vector is $(0.32, 0.95)$

first value is 93.33% of total sample variance, second is 6.67% of the same

So, the ~~fixed~~ ~~fix~~ first principal component axis

$$z_1 = -0.95x + 0.32y$$

$$z_1 = (e_1)^T X^*$$

$$= (-0.95, 0.32) \begin{pmatrix} 3 & 2 & 20 & 19 & 22 & 6 \\ 15 & 13 & 5 & 12 & 6 & 9 \end{pmatrix}$$

$$= (1.95 \quad 2.26 \quad -17.4 \quad -14.21 \quad -18.98 \quad -2.82$$