1 General Hyperbolic Trigonometric Formulae

1.1 Hyperbolic Function definitions

$$\sinh x = \frac{e^x - e^{-x}}{2} \tag{1}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \tag{2}$$

$$\tanh x = \frac{\cosh x}{\sinh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{3}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} \tag{4}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} \tag{5}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \tag{6}$$

1.2 Hyperbolic Function identities

$$\cosh^2 x - \sinh^2 x = 1 \tag{7}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \tag{8}$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y \tag{9}$$

$$\sinh 2x = 2 \sinh x \cosh x \tag{10}$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x \tag{11}$$

$$= 2\cosh^2 x - 1 \tag{12}$$

$$= 2\sinh^2 x + 1 \tag{13}$$

$$\cosh \frac{x}{2} = \sqrt{\frac{\cosh u + 1}{2}} \tag{14}$$

$$\sinh \frac{x}{2} = \pm\sqrt{\frac{\cosh u - 1}{2}} \tag{15}$$

1.3 Differentiation of Hyperbolic functions

$$\frac{\mathrm{d} \sinh x}{\mathrm{d}x} = \cosh x \tag{16}$$

$$\frac{\mathrm{d} \cosh x}{\mathrm{d}x} = \sinh x \tag{17}$$

$$\frac{\mathrm{d} \tanh x}{\mathrm{d}x} = \operatorname{sech}^2 x \tag{18}$$

$$\frac{\mathrm{d} \operatorname{cosech} x}{\mathrm{d}x} = -\operatorname{cosech} x \coth x \tag{19}$$

$$\frac{\mathrm{d} \operatorname{sech} x}{\mathrm{d}x} = -\operatorname{sech} x \tanh x \tag{20}$$

$$\frac{\mathrm{d} \coth x}{\mathrm{d}x} = -\operatorname{cosech}^2 x \tag{21}$$

1.4 Integration of Hyperbolic Functions

$$\int \cosh x \; dx = \sinh x + c \tag{22}$$

$$\int \sinh x \; dx = \cosh x + c \tag{23}$$

$$\int \operatorname{sech}^2 x \; dx = \tanh x + c \tag{24}$$

$$\int \operatorname{cosech} x \coth x \; dx = -\operatorname{cosech} x + c \tag{25}$$

$$\int \operatorname{sech} x \tanh x \; dx = -\operatorname{sech} x + c \tag{26}$$

$$\int \operatorname{cosech}^2 x \; dx = -\coth x + c \tag{27}$$

1.5 Inverse Hyperbolic Function definitions

$$\operatorname{arsinh} x = \ln(x + \sqrt{x^2+1}) \tag{28}$$

$$\operatorname{arcosh} x = \ln(x + \sqrt{x^2-1}) \tag{29}$$

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), |x| < 1 \tag{30}$$

$$\operatorname{arcosech} x = \ln\left(\frac{1}{x} + \frac{\sqrt{x^2+1}}{|x|}\right) = \operatorname{arsinh}\frac{1}{x} \tag{31}$$

$$\operatorname{arsech} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) = \operatorname{arcosh}\frac{1}{x} \tag{32}$$

$$\operatorname{arcoth} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), |x| > 1 = \operatorname{artanh}\frac{1}{x} \tag{33}$$

* Note that arsinh is the formal notation for the inverse hyperbolic functions. This is not a typo; 'ar' stands for 'area' in this case and not for 'arc'. However, the notations arcsinh and $\sinh^{-1}$ are also used

1.6 Differentiation of Inverse Hyperbolic Functions

TODO