# General Trigonometric Formulae 1

#### Addition Formulae 1.1

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \tag{1}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \tag{2}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \tag{3}$$

$$\cot(A \pm B) = \frac{\cot B \cot A \mp 1}{\cot B \pm \cot A} \tag{4}$$

### 1.2 Double Angle Formulae

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1 + \tan^2 A} \tag{5}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

$$1 - \tan^2 A$$

$$=\frac{1-\tan^2 A}{1+\tan^2 A}\tag{6}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A} \tag{7}$$

## Half Angle Formulae 1.3

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}} \qquad \frac{+ | + |}{- | - |} \tag{8}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \qquad \frac{-\ |\ +}{-\ |\ +} \tag{9}$$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} \qquad \frac{-\mid +\mid}{\mid -\mid}$$

$$= \frac{\sin A}{1 + \cos A}$$
$$1 - \cos A$$

$$= \frac{1 - \cos A}{\sin A}$$

$$= \csc A - \cot A$$

# 1.5 Multiple Angle Formulae

$$\sin 3A = 3\sin A - 4\sin^3 A \tag{14}$$

$$\cos 3A = 4\cos^3 A - 3\cos A \tag{15}$$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} \tag{16}$$

$$\tan\left(\sum_{i=1}^{n} a_i\right) = \frac{e_1 - e_3 + e_5 + \dots}{1 - e_2 + e_4 + \dots} \tag{17}$$

where

$$e_1 = \sum_{i=1}^n a_i$$

$$e_2 = \sum_{i < j} a_i a_j$$

$$e_3 = \sum_{i < j < k} a_i a_j a + k$$

and so on.

# Sum to Product Formulae

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \tag{18}$$

$$\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right) \tag{19}$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \tag{20}$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) \tag{21}$$

#### 1.7 Difference of Squares

$$\sin^2 A - \sin^2 B = \sin(A - B)\sin(A + B) \tag{22}$$

$$\cos^2 A - \sin^2 B = \cos(A - B)\cos(A + B) \tag{23}$$

# Product to Sum Formulae

$$2\sin A\sin B = \cos(A - B) - \cos(A + B) \tag{11}$$

$$2\cos A\cos B = \cos(A-B) + \cos(A+B) \tag{12}$$

$$2\sin A\cos B = \sin(A-B) + \sin(A+B) \tag{13}$$

# Special Results 1.8

$$\sin(A)\sin\left(\frac{\pi}{3} - A\right)\sin\left(\frac{\pi}{3} + A\right) = \frac{1}{4}\sin 3A \tag{24}$$

$$\cos(A)\cos\left(\frac{\pi}{3} - A\right)\cos\left(\frac{\pi}{3} + A\right) = \frac{1}{4}\cos 3A \tag{25}$$

$$\tan(A)\tan\left(\frac{\pi}{3} - A\right)\tan\left(\frac{\pi}{3} + A\right) = \tan 3A \tag{26}$$

# Sum and Product of Trigonometric Functions 2

# Product formulae 2.1

$$C = \cos \theta \cos 2\theta \cos 4\theta \cdots \cos 2^{n-1}\theta$$

$$= \frac{\sin \theta}{\sin \theta} \cos \theta \cos 2\theta \cos 4\theta \cdots \cos 2^{n-1}\theta$$

$$= \frac{\sin 2\theta}{2 \sin \theta} \cos 2\theta \cos 4\theta \cdots \cos 2^{n-1}\theta$$

$$C = \frac{\sin 2^{n}\theta}{2^{n} \sin \theta}$$
(27)

$$S = \cos\frac{\theta}{2}\cos\frac{\theta}{4}\cos\frac{\theta}{8}\cdots\cos\frac{\theta}{2^n}$$
$$= \frac{1}{\sin\frac{\theta}{2^n}}\cos\frac{\theta}{2}\cos\frac{\theta}{4}\cos\frac{\theta}{8}\cdots\cos\frac{\theta}{2^n}\sin\frac{\theta}{2^n}$$

(10)

$$S = \frac{1}{2^n \sin \frac{\theta}{2\pi}} \sin \theta \tag{28}$$

$$S \to \frac{\sin \theta}{\theta} \text{ as } n \to \infty$$
 (29)

### 2.2Sum formulae

$$\sum_{r=1}^{n} \sin(\alpha + (r-1)\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left(\alpha + (n-1)\frac{\beta}{2}\right)$$
 (30)

$$\sum_{r=1}^{n} \cos(\alpha + (r-1)\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha + (n-1)\frac{\beta}{2}\right)$$
(31)