1 General Trigonometric Formulae

1.1 Addition Formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \tag{1}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \tag{2}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \tag{3}$$

$$\cot(A \pm B) = \frac{\cot B \cot A \mp 1}{\cot B \pm \cot A} \tag{4}$$

1.2 Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A} \tag{5}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \\ &= \frac{1 - \tan^2 A}{1 + \tan^2 A} \end{aligned} \tag{6}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \tag{7}$$

1.3 Half Angle Formulae

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}} \qquad \begin{array}{c|c} + & + \\ \hline - & - \end{array} \tag{8}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} \qquad \begin{array}{c|c} - & + \\ \hline - & + \end{array} \tag{9}$$

$$\begin{aligned} \tan \frac{A}{2} &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} \qquad \begin{array}{c|c} - & + \\ \hline + & - \end{array} \\ &= \frac{\sin A}{1 + \cos A} \\ &= \frac{1 - \cos A}{\sin A} \\ &= \operatorname{cosec} A - \cot A \end{aligned} \tag{10}$$

1.4 Product to Sum Formulae

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B) \tag{11}$$

$$2 \cos A \cos B = \cos(A - B) + \cos(A + B) \tag{12}$$

$$2 \sin A \cos B = \sin(A - B) + \sin(A + B) \tag{13}$$

1.5 Multiple Angle Formulae

$$\sin 3A = 3 \sin A - 4 \sin^3 A \tag{14}$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A \tag{15}$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \tag{16}$$

$$\tan\left(\sum_{i=1}^{n} a_i\right) = \frac{e_1 - e_3 + e_5 + ...}{1 - e_2 + e_4 + ...} \tag{17}$$

where

$$e_1 = \sum_{i=1}^{n} a_i$$

$$e_2 = \sum_{i<j} a_i a_j$$

$$e_3 = \sum_{i<j<k} a_i a_j a + k$$

and so on.

1.6 Sum to Product Formulae

$$\sin C + \sin D = 2 \sin\left(\frac{C + D}{2}\right) \cos\left(\frac{C - D}{2}\right) \tag{18}$$

$$\sin C - \sin D = 2 \sin\left(\frac{C - D}{2}\right) \cos\left(\frac{C + D}{2}\right) \tag{19}$$

$$\cos C + \cos D = 2 \cos\left(\frac{C + D}{2}\right) \cos\left(\frac{C - D}{2}\right) \tag{20}$$

$$\cos C - \cos D = -2 \sin\left(\frac{C + D}{2}\right) \sin\left(\frac{C - D}{2}\right) \tag{21}$$

1.7 Difference of Squares

$$\sin^2 A - \sin^2 B = \sin(A - B) \sin(A + B) \tag{22}$$

$$\cos^2 A - \sin^2 B = \cos(A - B) \cos(A + B) \tag{23}$$

1.8 Special Results

$$\sin(A) \sin\left(\frac{\pi}{3} - A\right) \sin\left(\frac{\pi}{3} + A\right) = \frac{1}{4} \sin 3A \tag{24}$$

$$\cos(A) \cos\left(\frac{\pi}{3} - A\right) \cos\left(\frac{\pi}{3} + A\right) = \frac{1}{4} \cos 3A \tag{25}$$

$$\tan(A) \tan\left(\frac{\pi}{3} - A\right) \tan\left(\frac{\pi}{3} + A\right) = \tan 3A \tag{26}$$

2 Sum and Product of Trigonometric Functions

2.1 Product formulae

$$\begin{aligned} C &= \cos\theta \cos 2\theta \cos 4\theta \cdots \cos 2^{n-1}\theta \\ &= \frac{\sin\theta}{\sin\theta} \cos\theta \cos 2\theta \cos 4\theta \cdots \cos 2^{n-1}\theta \\ &= \frac{\sin 2\theta}{2\sin\theta} \cos 2\theta \cos 4\theta \cdots \cos 2^{n-1}\theta \\ C &= \frac{\sin 2^n\theta}{2^n \sin\theta} \end{aligned} \tag{27}$$

$$\begin{aligned} S &= \cos\frac{\theta}{2} \cos\frac{\theta}{4} \cos\frac{\theta}{8} \cdots \cos\frac{\theta}{2^n} \\ &= \frac{1}{\sin\frac{\theta}{2^n}} \cos\frac{\theta}{2} \cos\frac{\theta}{4} \cos\frac{\theta}{8} \cdots \cos\frac{\theta}{2^n} \sin\frac{\theta}{2^n} \\ S &= \frac{1}{2^n \sin\frac{\theta}{2^n}} \sin\theta \end{aligned} \tag{28}$$

$$S \to \frac{\sin\theta}{\theta} \text{ as } n \to \infty \tag{29}$$

2.2 Sum formulae

$$\sum_{r=1}^{n} \sin(\alpha + (r-1)\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left(\alpha + (n-1)\frac{\beta}{2}\right) \tag{30}$$

$$\sum_{r=1}^{n} \cos(\alpha + (r-1)\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha + (n-1)\frac{\beta}{2}\right) \tag{31}$$