SoCP Question 1

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As hinted in the problem, let's consider a recursive solution. For a sequence of size n, on placing the largest element at the nth position, we are left with an increasing sequence of $n-1$ elements. We would have $C_{n-1}C_0$ ways of arranging them. If we then place the largest element at the $n-1$th position, we obtain $C_{n-2}C_1$ ways of arranging the elements. If we continue to proceed like this, we get the following recurrence relation:

$$C_n = C_{n-1}C_0 + C_{n-2}C_1 + \cdots + C_1C_{n-2} + C_0C_{n-1} = \sum_{i=0}^{n-1} C_iC_{n-1-i}$$

This is the recurrence relation for the Catalan Numbers. The closed form for them is:

$$C_n = \frac{1}{2n+1}\binom{2n}{n}$$