Cellular Automata

An introduction to Conway's Game of Life

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April 16, 2021

Determinism and Non-Determinism

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A **Deterministic Algorithm** is an algorithm that repeatedly gives the same set of outputs for a given set of outputs. An example is a simple function $f(x) = x^2$

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A **Nondeterministic Algorithm** is an algorithm that does not give repeatable outputs for a given input. An example is a cup in which n dice are rolled. ($g : \mathbb{N} \to \mathbb{N}^n$)

$$1 \longrightarrow \boxed{\quad g \quad} \longrightarrow \{1, 2, 3, 4, 5, 6\}$$

State Machines

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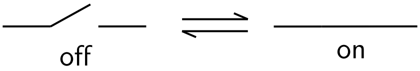

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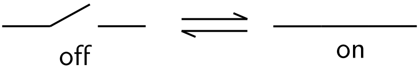

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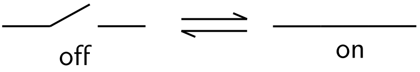

Can we make an infinite state machine by combining infinite finite state machines? Theoretically yes, but practically no. We'll cover this in more detail when we go to Cellular Automata.

Deterministic and Nondeterministic Finite State Machines

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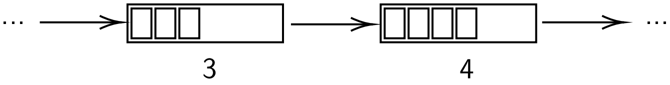
Combining both these concepts, a **Deterministic Finite State Automata (DFA)** is one whose next state is uniquely determined by the current state. As an example, a loading bar at n% can only progress to a state at (n+1)%



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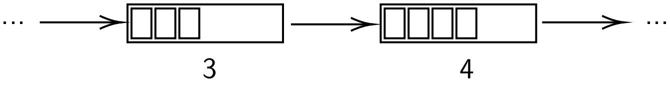
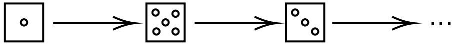


In contrast, a **Nondeterministic Finite State Automata (NFA)** is one whose next state is not uniquely determined by it's current state. An example would be the output of a dice roll: it's a NFA, because the number of states are finite, and the next state is independent of the current state



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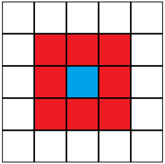
Each cell has two types of neighbourhoods: **Moore** and **Von Neumann**. Rules (either Deterministic or Nondeterministic) based on these neighbourhoods determine the next state of the cell.



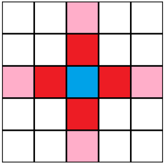


Figure: Moore(L) and Von Neumann(R) neighbourhoods

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Theoretically, nothing prevents cellular automata from being infinite-state automata. However, they would have a countably infinite number of states, which makes things (theoretically) simpler. For all practical purposes, however, cellular automata are finite and are bounded by the memory space of the computer simulating them.

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If we do have to bound an automata, we generally impose a different set of conditions on the boundary: something like reflection or stasis or vanishing conditions.

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Few other things: Life is Turing complete (proof left as an exercise), and Life is Undecidable: you cannot predict the initial state from the final state, or directly compute n states into the future (as a consequence of the Halting problem).

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Further reading/watching

- Numberphile video on Life (With Conway himself):
  `https://www.youtube.com/watch?v=R9Plq-D1gEk`
- LifeWiki, the reference on all Game of Life Patterns:
  `https://www.conwaylife.com/wiki/`
- Golly, an open-source program for simulating life:
  `http://golly.sourceforge.net`
- PyGameOfLife, a Game of Life program implemented in Python:
  `https://github.com/Aniruddha-Deb/PyGameOfLife`